Computer algebra independent integration tests

4-Trig-functions/4.7-Miscellaneous/4.7.2-trig^m-a-trig+b-trig-^n

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Contents

Chapter 1

Introduction

This report gives the result of running the computer algebra independent integration problems. The listing of the problems are maintained by and can be downloaded from `https://rulebasedintegration.org`

The number of integrals in this report is [294]. This is test number [136].

1.1 Listing of CAS systems tested

The following systems were tested at this time.

1. Mathematica 12.3 (64 bit) on windows 10.
2. Rubi 4.16.1 in Mathematica 12.1 on windows 10.
3. Maple 2021.1 (64 bit) on windows 10.
4. Maxima 5.44 on Linux. (via sagemath 9.3)
5. Fricas 1.3.7 on Linux (via sagemath 9.3)
6. Giac/Xcas 1.7 on Linux. (via sagemath 9.3)
7. Sympy 1.8 under Python 3.8.8 using Anaconda distribution on Ubuntu.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 under windows 10 (64 bit)

Maxima, Fricas and Giac/Xcas were called from inside SageMath. This was done using SageMath integrate command by changing the name of the algorithm to use the different CAS systems.

Sympy was called directly using Python.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed.

If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	solved	Failed
Rubi	% 100.00 (294)	% 0.00 (0)
Mathematica	% 100.00 (294)	% 0.00 (0)
Maple	% 98.30 (289)	% 1.70 (5)
Maxima	% 92.18 (271)	% 7.82 (23)
Fricas	% 98.64 (290)	% 1.36 (4)
Sympy	% 21.77 (64)	% 78.23 (230)
Giac	% 94.90 (279)	% 5.10 (15)
Mupad	% 98.64 (290)	% 1.36 (4)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

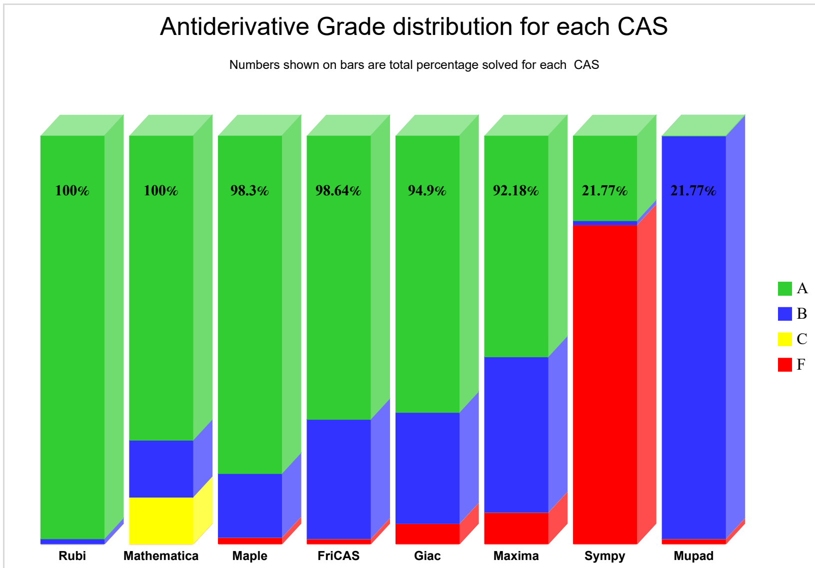
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

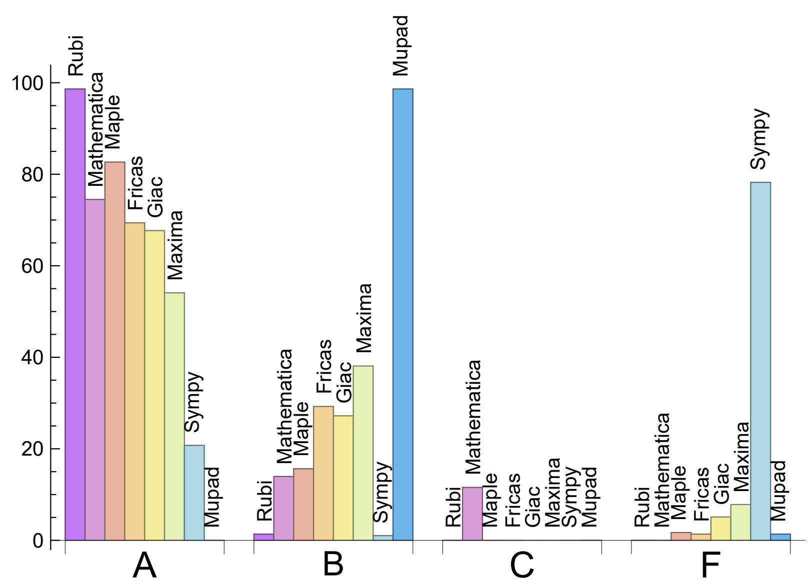
System	% A grade	% B grade	% C grade	% F grade
Rubi	98.64	1.36	0.00	0.00
Mathematica	74.49	13.95	11.56	0.00
Maple	82.65	15.65	0.00	1.70
Maxima	54.08	38.10	0.00	7.82
Fricas	69.39	29.25	0.00	1.36
Sympy	20.75	1.02	0.00	78.23
Giac	67.69	27.21	0.00	5.10
Mupad	0.00	98.64	0.00	1.36

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the CAS systems for each grade level.



The following table shows the distribution of the different types of failure for each CAS. There are 3 types of reasons why it can fail. The first is when CAS returns back the input within the time limit, which means it could not solve it. This the typical normal failure **F** .

The second is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned **F(-1)**.

The third is due to an exception generated. Assigned **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and Giac) or it could be an indication of an internal error in CAS. This type of error requires more investigations to determine the cause.

System	Number failed	Percentage normal failure	Percentage timeout failure	Percentage exception failure
Rubi	0	0.00 %	0.00 %	0.00 %
Mathematica	0	0.00 %	0.00 %	0.00 %
Maple	5	100.00 %	0.00 %	0.00 %
Maxima	23	17.39 %	0.00 %	82.61 %
Fricas	4	100.00 %	0.00 %	0.00 %
Sympy	230	60.87 %	36.96 %	2.17 %
Giac	15	33.33 %	26.67 %	40.00 %
Mupad	4	100.00 %	0.00 %	0.00 %

Table 1.4: Time and leaf size performance for each CAS

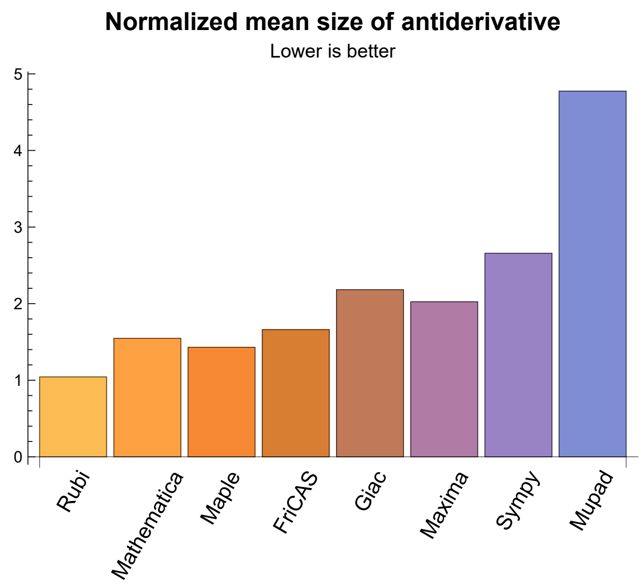
1.3 Performance

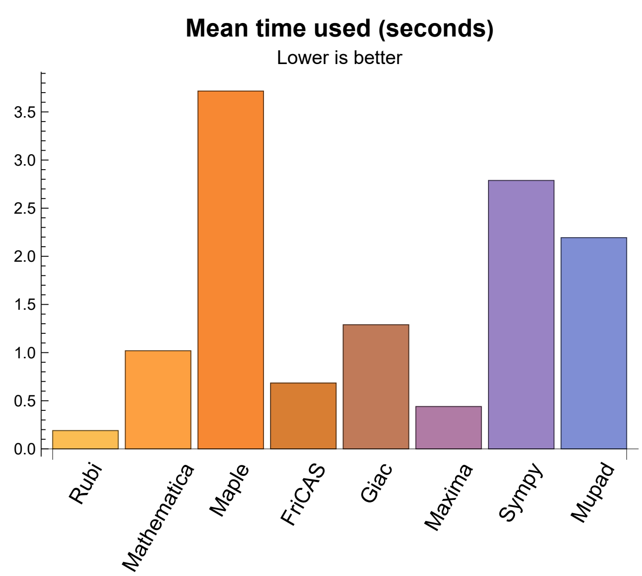
The table below summarizes the performance of each CAS system in terms of CPU time and leaf size of results.

System	Mean time (sec)	Mean size	Normalized mean	Median size	Normalized median
Rubi	0.19	112.64	1.04	86.50	1.00
Mathematica	1.02	175.96	1.55	95.00	1.02
Maple	3.72	158.37	1.43	109.00	1.22
Maxima	0.44	177.45	2.02	118.00	1.50
Fricas	0.68	171.98	1.66	115.50	1.20
Sympy	2.79	255.38	2.66	169.00	1.88
Giac	1.29	221.10	2.18	118.00	1.46
Mupad	2.19	610.58	4.77	149.50	1.79

Table 1.5: Time and leaf size performance for each CAS

The following are bar charts for the normalized leafsize and time used columns from the above table.





1.4 list of integrals that has no closed form antiderivative

{}

1.5 list of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

Mupad {}

1.6 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not mean necessarily that the anti-derivative is wrong, as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it easier to do further investigation to determine why it was not possible to verify the result produced.

Rubi {}

Mathematica {29, 100, 272}

Maple Verification phase not implemented yet.

Maxima Verification phase not implemented yet.

Fricas Verification phase not implemented yet.

Sympy Verification phase not implemented yet.

Giac Verification phase not implemented yet.

Mupad Verification phase not implemented yet.

1.7 Timing

The command `AboluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of _int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call has completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out is not counted in the final statistics.

1.8 Verification

A verification phase was applied on the result of integration for Rubi and Mathematica. Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative produced was correct.

Verification phase has 3 minutes time out. An integral whose result was not verified could still be correct. Further investigation is needed on those integrals which failed verifications. Such integrals are marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.9 Important notes about some of the results

1.9.1 Important note about Maxima results

Since these integrals are run in a batch mode, using an automated script, and by using sagemath (SageMath uses Maxima), then any integral where Maxima needs an interactive response from the user to answer a question during evaluation of the integral in order to complete the integration, will fail and is counted as failed.

The exception raised is `ValueError`. Therefore Maxima result below is lower than what could result if Maxima was run directly and each question Maxima asks was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the Timofeev test file, there were about 14 such integrals out of total 705, or about 2 percent. This pecrentage can be higher or lower depending on the specific input test file.

Such integrals can be indentified by looking at the output of the integration in each section for Maxima. The exception message will indicate of the error is due to the interactive question being asked or not.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath loading of Maxima `abs_integrate` was found to cause some problem. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See `https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/` for reference.

1.9.2 Important note about FriCAS and Giac/XCAS results

There are Few integrals which failed due to SageMath not able to translate the result back to SageMath syntax and not because these CAS system were not able to do the integrations.

These will fail With error `Exception raised: NotImplementedError`

The number of such cases seems to be very small. About 1 or 2 percent of all integrals.

Hopefully the next version of SageMath will have complete translation of FriCAS and XCAS syntax and I will re-run all the tests again when this happens.

1.9.3 Important note about finding leaf size of antiderivative

For Mathematica, Rubi and Maple, the buildin system function `LeafSize` is used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special buildin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy and Giac antiderivatives is determined using the following function, thanks to user `slelievre` at `https://`

ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which is called directly from Python, the following code is used to obtain the leafsize of its result

```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
         leafCount =1
```

1.9.4 Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative, Maple was used to determine the leaf size of Mupad output by post processing.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

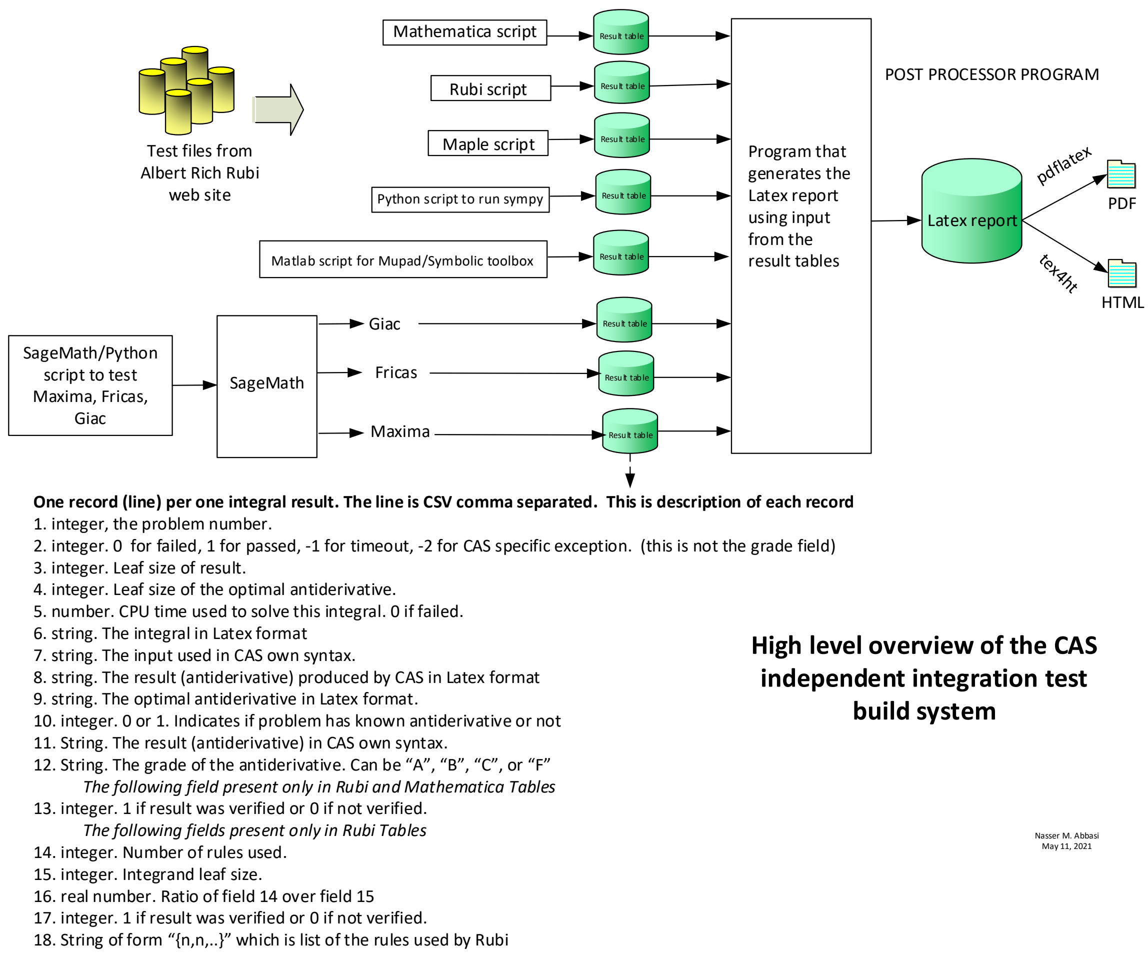
The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives `sin(x)^2/2`

1.10 Design of the test system

The following diagram gives a high level view of the current test build system.



Chapter 2

detailed summary tables of results

2.1 List of integrals sorted by grade for each CAS

2.1.1 Rubi

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 16, 17, 18, 19, 20, 21, 22, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294 }

B grade: { 15, 23, 131, 142 }

C grade: { }

F grade: { }

2.1.2 Mathematica

A grade: { 1, 2, 3, 4, 5, 7, 9, 11, 12, 13, 14, 15, 17, 18, 19, 20, 23, 26, 27, 28, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 64, 67, 69, 71, 73, 74, 75, 76, 78, 79, 80, 82, 87, 89, 90, 91, 92, 93, 95, 96, 97, 101, 106, 107, 108, 109, 110, 111, 113, 114, 115, 116, 117, 118, 120, 122, 123, 125, 126, 127, 128, 130, 133, 136, 137, 138, 140, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 170, 172, 174, 175, 176, 177, 178, 180, 181, 182, 183, 184, 186, 187, 189, 190, 194, 195, 196, 197, 198, 201, 202, 204, 206, 208, 210, 212, 213, 214, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 265, 266, 269, 270, 271, 273, 274, 275, 277, 279, 281, 283, 285, 287, 288, 289, 291, 293, 294 }

B grade: { 6, 21, 24, 63, 66, 68, 70, 72, 81, 84, 85, 86, 88, 98, 99, 100, 103, 104, 105, 119, 121, 134, 143, 169, 171, 173, 179, 185, 188, 191, 192, 193, 199, 200, 203, 205, 207, 209, 211, 241, 242 }

C grade: { 8, 10, 16, 22, 25, 29, 50, 65, 77, 83, 94, 102, 112, 124, 129, 131, 132, 135, 139, 141, 142, 215, 264, 267, 268, 272, 276, 278, 280, 282, 284, 286, 290, 292 }

F grade: { }

2.1.3 Maple

A grade: { 1, 2, 3, 4, 5, 6, 7, 9, 10, 11, 12, 14, 15, 16, 17, 18, 19, 20, 22, 26, 27, 28, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 105, 106, 107, 108, 109, 111, 113, 114, 115, 116, 118, 120, 123, 124, 125, 126, 127, 128, 130, 131, 132, 134, 135, 136, 138, 140, 141, 143, 145, 147, 149, 150, 152, 153, 154, 155, 156, 158, 160, 162, 164, 165, 166, 167, 168, 169, 170, 172, 174, 175, 176, 177, 178, 179, 181, 182, 183, 185, 188, 189, 191, 192, 193, 194, 195, 196, 198, 199, 200, 201, 202, 204, 206, 207, 208, 210, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 273, 275, 277, 278, 279, 281, 283, 284, 285, 286, 287, 289, 290, 291, 293, 294 }

B grade: { 8, 13, 21, 23, 24, 25, 67, 85, 103, 104, 110, 112, 117, 119, 121, 122, 129, 133, 137, 139, 142, 144, 146, 148, 151, 157, 159, 161, 163, 171, 173, 180, 184, 186, 190, 197, 203, 205, 209, 211, 249, 276, 280, 282, 288, 292 }

C grade: { }

F grade: { 29, 187, 271, 272, 274 }

2.1.4 Maxima

A grade: { 1, 2, 3, 4, 5, 6, 7, 9, 11, 16, 18, 20, 27, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 105, 106, 107, 108, 109, 113, 115, 122, 124, 126, 128, 130, 143, 145, 147, 149, 155, 167, 168, 170, 172, 174, 178, 179, 180, 181, 193, 198, 200, 201, 202, 206, 208, 212, 214, 217, 218, 219, 224, 225, 227, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 271, 273, 275, 284, 294 }

B grade: { 8, 10, 12, 13, 14, 15, 17, 19, 21, 22, 23, 24, 25, 26, 28, 67, 85, 104, 110, 111, 112, 114, 116, 117, 118, 119, 120, 121, 123, 125, 127, 129, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 144, 146, 148, 156, 157, 158, 159, 160, 161, 162, 169, 171, 173, 182, 183, 184, 185, 186, 188, 189, 190, 191, 192, 194, 195, 196, 197, 199, 203, 204, 205, 207, 209, 210, 211, 213, 215, 216, 220, 221, 222, 223, 226, 228, 264, 265, 266, 267, 268, 269, 270, 276, 277, 278, 279, 280, 281, 282, 283, 285, 286, 287, 288, 289, 290, 291, 292, 293 }

C grade: { }

F grade: { 29, 150, 151, 152, 153, 154, 163, 164, 165, 166, 175, 176, 177, 187, 257, 258, 259, 260, 261, 262, 263, 272, 274 }

2.1.5 FriCAS

A grade: { 1, 2, 3, 4, 5, 7, 8, 10, 12, 30, 31, 32, 33, 34, 35, 36, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 105, 106, 107, 108, 109, 110, 111, 112, 114, 116, 118, 119, 120, 121, 122, 126, 139, 150, 151, 152, 153, 154, 155, 156, 158, 160, 162, 163, 164, 165, 166, 167, 168, 169, 170, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 188, 189, 190, 193, 194, 195, 196, 197, 198, 200, 201, 202, 203, 204, 206, 207, 208, 209, 210, 212, 213, 214, 216, 217, 218, 219, 221, 222, 224, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 273, 276, 277, 278, 280, 281, 282, 283, 284, 286, 288, 290, 292 }

B grade: { 6, 9, 11, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 37, 67, 85, 104, 113, 115, 117, 123, 124, 125, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 157, 159, 161, 171, 172, 173, 184, 185, 186, 191, 192, 199, 205, 211, 215, 220, 223, 225, 226, 264, 265, 266, 267, 268, 269, 270, 271, 275, 279, 285, 287, 289, 291, 293, 294 }

C grade: { }

F grade: { 29, 187, 272, 274 }

2.1.6 Sympy

A grade: { 2, 3, 4, 5, 6, 7, 10, 16, 30, 31, 32, 33, 34, 35, 43, 44, 45, 46, 47, 48, 57, 58, 59, 60, 61, 62, 74, 75, 76, 77, 78, 79, 92, 93, 94, 95, 96, 97, 114, 124, 150, 151, 152, 153, 154, 155, 163, 164, 165, 166, 167, 168, 175, 176, 177, 178, 179, 180, 202, 232, 284 }

B grade: { 1, 188, 195 }

C grade: { }

F grade: { 8, 9, 11, 12, 13, 14, 15, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 36, 37, 38, 39, 40, 41, 42, 49, 50, 51, 52, 53, 54, 55, 56, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 115, 116, 117, 118, 119, 120, 121, 122, 123, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 156, 157, 158, 159, 160, 161, 162, 169, 170, 171, 172, 173, 174, 181, 182, 183, 184, 185, 186, 187, 189, 190, 191, 192, 193, 194, 196, 197, 198, 199, 200, 201, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294 }

2.1.7 Giac

A grade: { 1, 2, 3, 4, 7, 8, 9, 10, 11, 12, 14, 15, 17, 18, 19, 20, 21, 24, 26, 27, 28, 30, 31, 32, 33, 34, 35, 36, 38, 40, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 54, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 69, 71, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 87, 89, 91, 92, 93, 94, 95, 96, 98, 99, 100, 101, 102, 103, 106, 108, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 122, 124, 125, 126, 127, 128, 129, 130, 131, 134, 136, 137, 138, 139, 140, 143, 145, 147, 148, 149, 150, 151, 152, 153, 154, 155, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 174, 175, 176, 177, 178, 180, 181, 182, 183, 184, 185, 186, 189, 191, 192, 193, 194, 196, 198, 199, 200, 201, 202, 204, 206, 207, 208, 210, 212, 213, 214, 216, 217, 218, 219, 221, 223, 225, 228, 232, 241, 242, 253, 254, 256, 258, 259, 260, 263, 275, 276, 277, 278, 279, 280, 281, 282, 285, 286, 287, 288, 290, 291, 294 }

B grade: { 5, 6, 13, 16, 22, 23, 25, 37, 39, 41, 53, 55, 67, 68, 70, 72, 84, 85, 86, 88, 90, 97, 104, 105, 107, 109, 121, 123, 132, 133, 135, 141, 142, 144, 146, 156, 173, 179, 188, 190, 195, 197, 203, 205, 209, 211, 215, 220, 222, 224, 226, 227, 230, 231, 233, 234, 235, 237, 238, 239, 240, 250, 251, 252, 255, 257, 261, 262, 264, 265, 266, 267, 268, 269, 270, 283, 284, 289, 292, 293 }

C grade: { }

F grade: { 29, 187, 229, 236, 243, 244, 245, 246, 247, 248, 249, 271, 272, 273, 274 }

2.1.8 Mupad

A grade: { }

B grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 273, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294 }

C grade: { }

F grade: { 29, 187, 272, 274 }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by table below. The elapsed time is in seconds. For failed result it is given as F(-1) if the failure was due to timeout. It is given as F(-2) if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given just an F.

In this table,the column **normalized size** is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$

Problem 1	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	34	28	25	36	75	33	35
normalized size	1	1.00	0.94	0.78	0.69	1.00	2.08	0.92	0.97
time (sec)	N/A	0.049	0.006	0.839	0.312	0.416	0.531	2.726	0.526

Problem 2	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	26	20	20	27	27	25	32
normalized size	1	1.00	1.08	0.83	0.83	1.12	1.12	1.04	1.33
time (sec)	N/A	0.041	0.004	0.818	0.322	0.447	0.282	2.961	0.454

Problem 3	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	25	21	21	19	37	19	19
normalized size	1	1.00	1.00	0.84	0.84	0.76	1.48	0.76	0.76
time (sec)	N/A	0.027	0.004	0.157	0.320	0.403	0.163	0.168	0.412

Problem 4	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	10	10	10	11	10	10	8	10	10
normalized size	1	1.00	1.00	1.10	1.00	1.00	0.80	1.00	1.00
time (sec)	N/A	0.006	0.002	0.055	0.319	0.509	0.046	1.829	0.399

Problem 5	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	9	9	9	10	9	11	8	24	54
normalized size	1	1.00	1.00	1.11	1.00	1.22	0.89	2.67	6.00
time (sec)	N/A	0.019	0.006	0.451	0.312	0.504	1.140	0.206	0.540

Problem 6	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	25	19	24	33	24	33	24
normalized size	1	1.00	2.08	1.58	2.00	2.75	2.00	2.75	2.00
time (sec)	N/A	0.033	0.007	0.495	0.322	0.465	1.830	0.200	0.397

Problem 7	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	15	14	15	19	17	13	14
normalized size	1	1.00	1.00	0.93	1.00	1.27	1.13	0.87	0.93
time (sec)	N/A	0.043	0.008	0.824	0.323	0.434	3.878	0.228	0.409

Problem 8	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	B	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	91	91	94	173	209	93	0	148	3512
normalized size	1	1.00	1.03	1.90	2.30	1.02	0.00	1.63	38.59
time (sec)	N/A	0.107	0.198	0.473	0.428	0.479	0.000	0.214	7.501

Problem 9	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	68	62	84	106	144	0	94	94
normalized size	1	1.00	0.91	1.24	1.56	2.12	0.00	1.38	1.38
time (sec)	N/A	0.078	0.163	0.486	0.429	0.479	0.000	0.258	0.581

Problem 10	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	B	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	47	54	88	46	173	55	970
normalized size	1	1.00	1.34	1.54	2.51	1.31	4.94	1.57	27.71
time (sec)	N/A	0.057	0.059	0.458	0.425	0.475	0.772	1.456	2.141

Problem 11	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-2)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	38	35	61	96	0	61	31
normalized size	1	1.00	1.06	0.97	1.69	2.67	0.00	1.69	0.86
time (sec)	N/A	0.018	0.026	0.488	0.427	0.451	0.000	0.414	1.052

Problem 12	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	20	21	48	44	0	22	32
normalized size	1	1.00	0.87	0.91	2.09	1.91	0.00	0.96	1.39
time (sec)	N/A	0.070	0.048	0.713	0.324	0.454	0.000	1.718	0.575

Problem 13	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	67	107	107	133	0	108	170
normalized size	1	1.00	1.22	1.95	1.95	2.42	0.00	1.96	3.09
time (sec)	N/A	0.068	0.130	0.787	0.430	0.491	0.000	2.970	0.678

Problem 14	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	48	64	119	117	0	78	91
normalized size	1	1.00	0.87	1.16	2.16	2.13	0.00	1.42	1.65
time (sec)	N/A	0.121	0.161	11.609	0.324	0.433	0.000	0.187	0.571

Problem 15	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	B	A	A	B	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	107	283	107	141	253	240	0	186	224
normalized size	1	2.64	1.00	1.32	2.36	2.24	0.00	1.74	2.09
time (sec)	N/A	1.169	0.438	0.551	0.424	0.449	0.000	4.740	0.836

Problem 16	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	121	99	117	132	1017	139	626
normalized size	1	1.00	1.89	1.55	1.83	2.06	15.89	2.17	9.78
time (sec)	N/A	0.118	0.264	0.557	0.421	0.707	2.196	1.982	7.064

Problem 17	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	62	97	128	164	0	103	86
normalized size	1	1.00	1.03	1.62	2.13	2.73	0.00	1.72	1.43
time (sec)	N/A	0.046	0.165	0.547	0.420	0.455	0.000	0.257	0.612

Problem 18	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	17	14	14	39	0	13	29
normalized size	1	1.00	1.00	0.82	0.82	2.29	0.00	0.76	1.71
time (sec)	N/A	0.013	0.021	0.533	0.313	0.438	0.000	0.182	0.431

Problem 19	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	72	106	128	220	0	109	492
normalized size	1	1.00	1.14	1.68	2.03	3.49	0.00	1.73	7.81
time (sec)	N/A	0.060	0.331	0.808	0.436	0.562	0.000	4.461	0.812

Problem 20	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	76	60	62	134	0	63	114
normalized size	1	1.00	1.55	1.22	1.27	2.73	0.00	1.29	2.33
time (sec)	N/A	0.076	0.201	0.724	0.342	0.582	0.000	0.171	0.621

Problem 21	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	118	118	270	224	242	345	0	215	511
normalized size	1	1.00	2.29	1.90	2.05	2.92	0.00	1.82	4.33
time (sec)	N/A	0.180	1.909	0.721	0.435	0.596	0.000	8.783	0.744

Problem 22	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	B	B	F(-2)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	98	98	114	193	359	282	0	242	5324
normalized size	1	1.00	1.16	1.97	3.66	2.88	0.00	2.47	54.33
time (sec)	N/A	0.198	0.860	0.577	0.445	0.495	0.000	2.954	8.599

Problem 23	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	B	A	B	B	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	92	300	92	212	299	282	0	197	263
normalized size	1	3.26	1.00	2.30	3.25	3.07	0.00	2.14	2.86
time (sec)	N/A	0.695	0.429	0.587	0.435	1.405	0.000	0.281	0.797

Problem 24	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	19	47	29	84	116	0	20	48
normalized size	1	1.27	3.13	1.93	5.60	7.73	0.00	1.33	3.20
time (sec)	N/A	0.026	0.096	0.691	0.337	0.652	0.000	0.260	0.493

Problem 25	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	B	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	101	157	250	225	0	166	216
normalized size	1	1.00	1.38	2.15	3.42	3.08	0.00	2.27	2.96
time (sec)	N/A	0.035	0.171	0.538	0.443	0.594	0.000	2.382	0.755

Problem 26	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	96	73	172	220	0	77	131
normalized size	1	1.00	1.63	1.24	2.92	3.73	0.00	1.31	2.22
time (sec)	N/A	0.082	0.225	0.666	0.337	0.763	0.000	2.015	0.711

Problem 27	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	184	184	193	333	276	463	0	212	813
normalized size	1	1.00	1.05	1.81	1.50	2.52	0.00	1.15	4.42
time (sec)	N/A	0.224	0.795	0.749	0.435	1.918	0.000	3.963	1.005

Problem 28	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	117	117	208	151	308	385	0	146	253
normalized size	1	1.00	1.78	1.29	2.63	3.29	0.00	1.25	2.16
time (sec)	N/A	0.137	0.835	0.756	0.361	1.939	0.000	1.896	0.856

Problem 29	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	367	0	0	0	0	0	-1
normalized size	1	1.00	5.56	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.063	3.747	10.852	0.000	0.643	0.000	0.000	0.000

Problem 30	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	87	87	57	62	62	62	175	95	149
normalized size	1	1.00	0.66	0.71	0.71	0.71	2.01	1.09	1.71
time (sec)	N/A	0.091	0.117	1.087	0.326	0.605	3.105	0.414	4.205

Problem 31	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	60	46	49	51	87	85	67
normalized size	1	1.00	1.00	0.77	0.82	0.85	1.45	1.42	1.12
time (sec)	N/A	0.070	0.016	1.053	0.320	0.577	1.606	0.935	0.464

Problem 32	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	62	52	48	51	128	65	107
normalized size	1	1.00	0.95	0.80	0.74	0.78	1.97	1.00	1.65
time (sec)	N/A	0.078	0.090	10.813	0.330	0.665	0.876	0.981	4.106

Problem 33	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	44	36	35	38	63	55	47
normalized size	1	1.00	1.00	0.82	0.80	0.86	1.43	1.25	1.07
time (sec)	N/A	0.065	0.011	1.028	0.319	0.746	0.434	3.945	0.437

Problem 34	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	46	41	37	35	73	35	35
normalized size	1	1.00	1.07	0.95	0.86	0.81	1.70	0.81	0.81
time (sec)	N/A	0.043	0.049	0.708	0.317	0.590	0.210	3.987	0.428

Problem 35	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	46	25	24	23	31	24	38
normalized size	1	1.00	1.92	1.04	1.00	0.96	1.29	1.00	1.58
time (sec)	N/A	0.013	0.012	0.130	0.318	0.524	0.140	0.191	0.357

Problem 36	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	17	24	30	21	0	27	70
normalized size	1	1.00	1.00	1.41	1.76	1.24	0.00	1.59	4.12
time (sec)	N/A	0.026	0.017	1.338	0.334	0.751	0.000	0.161	0.572

Problem 37	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	24	34	40	54	0	54	38
normalized size	1	1.00	1.00	1.42	1.67	2.25	0.00	2.25	1.58
time (sec)	N/A	0.045	0.014	1.773	0.325	0.716	0.000	0.234	0.413

Problem 38	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	28	25	30	30	0	25	23
normalized size	1	1.00	1.00	0.89	1.07	1.07	0.00	0.89	0.82
time (sec)	N/A	0.058	0.014	1.695	0.322	1.258	0.000	0.259	0.399

Problem 39	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	52	52	52	54	61	74	0	99	105
normalized size	1	1.00	1.00	1.04	1.17	1.42	0.00	1.90	2.02
time (sec)	N/A	0.067	0.017	2.004	0.324	2.160	0.000	4.021	2.207

Problem 40	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	41	38	41	45	0	48	40
normalized size	1	1.00	0.93	0.86	0.93	1.02	0.00	1.09	0.91
time (sec)	N/A	0.063	0.089	1.746	0.326	0.628	0.000	0.246	0.523

Problem 41	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	68	74	86	88	0	141	175
normalized size	1	1.00	0.92	1.00	1.16	1.19	0.00	1.91	2.36
time (sec)	N/A	0.081	0.210	11.311	0.327	0.936	0.000	4.687	4.118

Problem 42	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	53	48	53	57	0	70	65
normalized size	1	1.00	0.88	0.80	0.88	0.95	0.00	1.17	1.08
time (sec)	N/A	0.071	0.160	1.952	0.329	0.721	0.000	0.241	0.674

Problem 43	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	137	137	154	108	98	94	187	155	176
normalized size	1	1.00	1.12	0.79	0.72	0.69	1.36	1.13	1.28
time (sec)	N/A	0.138	0.376	1.672	0.327	0.975	5.323	0.866	0.695

Problem 44	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	174	174	147	118	102	95	340	132	156
normalized size	1	1.00	0.84	0.68	0.59	0.55	1.95	0.76	0.90
time (sec)	N/A	0.170	0.248	11.253	0.339	0.498	3.480	0.242	0.610

Problem 45	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	103	103	116	88	77	74	138	114	115
normalized size	1	1.00	1.13	0.85	0.75	0.72	1.34	1.11	1.12
time (sec)	N/A	0.122	0.171	1.763	0.323	0.623	1.752	3.511	0.615

Problem 46	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	126	126	98	97	75	75	238	85	89
normalized size	1	1.00	0.78	0.77	0.60	0.60	1.89	0.67	0.71
time (sec)	N/A	0.135	0.227	1.425	0.318	0.537	1.046	4.968	0.584

Problem 47	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	67	64	52	52	53	85	73	77
normalized size	1	1.00	0.96	0.78	0.78	0.79	1.27	1.09	1.15
time (sec)	N/A	0.091	0.394	1.428	0.330	0.499	0.478	0.251	0.514

Problem 48	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	52	70	68	52	128	50	63
normalized size	1	1.00	0.95	1.27	1.24	0.95	2.33	0.91	1.15
time (sec)	N/A	0.020	0.098	1.189	0.319	0.607	0.274	3.728	0.482

Problem 49	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	84	63	60	62	0	89	66
normalized size	1	1.00	1.53	1.15	1.09	1.13	0.00	1.62	1.20
time (sec)	N/A	0.071	0.156	1.855	0.332	0.658	0.000	4.937	0.489

Problem 50	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	69	57	49	60	0	44	118
normalized size	1	1.00	1.77	1.46	1.26	1.54	0.00	1.13	3.03
time (sec)	N/A	0.056	0.135	3.741	0.431	0.525	0.000	0.215	0.679

Problem 51	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	67	67	98	89	96	0	122	106
normalized size	1	1.00	1.00	1.46	1.33	1.43	0.00	1.82	1.58
time (sec)	N/A	0.094	0.044	1.689	0.328	0.690	0.000	4.738	0.901

Problem 52	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	46	48	45	55	0	41	68
normalized size	1	1.00	1.53	1.60	1.50	1.83	0.00	1.37	2.27
time (sec)	N/A	0.046	0.042	10.894	0.328	0.464	0.000	0.289	0.491

Problem 53	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	120	120	120	143	129	120	0	249	216
normalized size	1	1.00	1.00	1.19	1.08	1.00	0.00	2.08	1.80
time (sec)	N/A	0.142	0.077	1.961	0.334	0.592	0.000	0.326	3.161

Problem 54	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	54	82	70	79	0	80	98
normalized size	1	1.00	0.64	0.96	0.82	0.93	0.00	0.94	1.15
time (sec)	N/A	0.077	0.191	1.971	0.340	0.673	0.000	0.583	0.621

Problem 55	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	168	168	104	189	180	142	0	343	328
normalized size	1	1.00	0.62	1.12	1.07	0.85	0.00	2.04	1.95
time (sec)	N/A	0.170	0.594	10.992	0.340	0.525	0.000	0.526	3.259

Problem 56	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	125	125	104	110	91	100	0	118	130
normalized size	1	1.00	0.83	0.88	0.73	0.80	0.00	0.94	1.04
time (sec)	N/A	0.104	0.689	2.100	0.342	0.667	0.000	0.312	0.827

Problem 57	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	265	265	235	175	163	150	532	218	523
normalized size	1	1.00	0.89	0.66	0.62	0.57	2.01	0.82	1.97
time (sec)	N/A	0.246	0.472	11.011	0.340	0.718	10.333	0.348	2.289

Problem 58	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	175	175	204	145	126	123	233	197	214
normalized size	1	1.00	1.17	0.83	0.72	0.70	1.33	1.13	1.22
time (sec)	N/A	0.184	0.416	10.379	0.321	0.704	5.528	0.317	0.784

Problem 59	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	216	216	171	155	131	128	400	157	407
normalized size	1	1.00	0.79	0.72	0.61	0.59	1.85	0.73	1.88
time (sec)	N/A	0.213	0.301	9.716	0.336	0.590	3.694	0.356	2.030

Problem 60	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	140	140	150	125	107	102	182	145	147
normalized size	1	1.00	1.07	0.89	0.76	0.73	1.30	1.04	1.05
time (sec)	N/A	0.163	0.290	10.500	0.320	0.696	1.928	0.483	0.698

Problem 61	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	78	78	94	114	91	100	272	104	281
normalized size	1	1.00	1.21	1.46	1.17	1.28	3.49	1.33	3.60
time (sec)	N/A	0.064	0.391	1.383	0.330	0.694	1.120	0.244	1.627

Problem 62	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	81	75	84	77	117	91	104
normalized size	1	1.00	1.40	1.29	1.45	1.33	2.02	1.57	1.79
time (sec)	N/A	0.023	0.340	9.743	0.322	0.873	0.520	0.204	0.572

Problem 63	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	91	91	401	123	91	79	0	93	156
normalized size	1	1.00	4.41	1.35	1.00	0.87	0.00	1.02	1.71
time (sec)	N/A	0.119	0.808	1.592	0.336	0.714	0.000	0.252	1.242

Problem 64	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	86	86	131	126	84	109	0	150	116
normalized size	1	1.00	1.52	1.47	0.98	1.27	0.00	1.74	1.35
time (sec)	N/A	0.112	1.071	10.959	0.330	0.648	0.000	0.293	1.051

Problem 65	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	79	93	85	88	0	71	183
normalized size	1	1.00	1.10	1.29	1.18	1.22	0.00	0.99	2.54
time (sec)	N/A	0.093	0.267	1.796	0.421	0.721	0.000	0.323	1.864

Problem 66	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	103	103	293	187	118	123	0	171	160
normalized size	1	1.00	2.84	1.82	1.15	1.19	0.00	1.66	1.55
time (sec)	N/A	0.123	1.631	10.246	0.329	0.719	0.000	0.329	2.378

Problem 67	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	57	72	87	78	0	57	88
normalized size	1	1.00	1.90	2.40	2.90	2.60	0.00	1.90	2.93
time (sec)	N/A	0.047	0.178	11.415	0.342	0.756	0.000	0.598	0.636

Problem 68	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	158	158	464	256	157	147	0	333	293
normalized size	1	1.00	2.94	1.62	0.99	0.93	0.00	2.11	1.85
time (sec)	N/A	0.181	1.325	2.360	0.330	0.735	0.000	0.519	4.278

Problem 69	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	120	120	54	127	122	105	0	112	123
normalized size	1	1.00	0.45	1.06	1.02	0.88	0.00	0.93	1.02
time (sec)	N/A	0.098	0.373	20.798	0.329	0.515	0.000	0.342	0.841

Problem 70	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	210	210	637	328	208	170	0	465	423
normalized size	1	1.00	3.03	1.56	0.99	0.81	0.00	2.21	2.01
time (sec)	N/A	0.220	2.131	11.101	0.333	0.733	0.000	0.385	4.282

Problem 71	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	174	174	115	173	154	128	0	166	156
normalized size	1	1.00	0.66	0.99	0.89	0.74	0.00	0.95	0.90
time (sec)	N/A	0.140	0.635	9.661	0.334	0.808	0.000	0.429	1.154

Problem 72	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	259	259	810	399	248	192	0	597	547
normalized size	1	1.00	3.13	1.54	0.96	0.74	0.00	2.31	2.11
time (sec)	N/A	0.268	4.025	21.070	0.339	0.763	0.000	0.906	4.557

Problem 73	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	213	213	177	219	184	150	0	220	189
normalized size	1	1.00	0.83	1.03	0.86	0.70	0.00	1.03	0.89
time (sec)	N/A	0.179	2.073	20.739	0.333	0.582	0.000	0.448	1.580

Problem 74	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	279	279	237	236	186	177	367	269	334
normalized size	1	1.00	0.85	0.85	0.67	0.63	1.32	0.96	1.20
time (sec)	N/A	0.258	0.709	11.184	0.330	0.662	15.615	0.702	2.001

Problem 75	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	381	381	222	250	199	184	760	245	343
normalized size	1	1.00	0.58	0.66	0.52	0.48	1.99	0.64	0.90
time (sec)	N/A	0.389	0.610	21.237	0.332	0.762	11.257	0.671	1.688

Problem 76	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	220	220	204	206	154	149	286	229	291
normalized size	1	1.00	0.93	0.94	0.70	0.68	1.30	1.04	1.32
time (sec)	N/A	0.234	0.537	39.350	0.330	0.621	5.708	0.491	1.277

Problem 77	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	301	301	178	219	170	151	563	187	471
normalized size	1	1.00	0.59	0.73	0.56	0.50	1.87	0.62	1.56
time (sec)	N/A	0.303	0.426	28.973	0.342	0.685	3.988	2.017	2.306

Problem 78	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	165	165	146	142	123	123	206	165	204
normalized size	1	1.00	0.88	0.86	0.75	0.75	1.25	1.00	1.24
time (sec)	N/A	0.181	0.428	39.384	0.332	0.776	1.994	2.941	0.828

Problem 79	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	108	108	107	153	136	121	381	122	320
normalized size	1	1.00	0.99	1.42	1.26	1.12	3.53	1.13	2.96
time (sec)	N/A	0.044	0.432	30.891	0.337	0.927	1.217	2.922	1.959

Problem 80	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	150	150	181	163	126	121	0	217	190
normalized size	1	1.00	1.21	1.09	0.84	0.81	0.00	1.45	1.27
time (sec)	N/A	0.153	0.953	11.205	0.342	0.827	0.000	0.396	2.750

Problem 81	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	119	119	477	210	135	136	0	128	255
normalized size	1	1.00	4.01	1.76	1.13	1.14	0.00	1.08	2.14
time (sec)	N/A	0.183	6.261	52.301	0.440	0.827	0.000	3.522	1.231

Problem 82	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	151	151	268	211	142	153	0	206	221
normalized size	1	1.00	1.77	1.40	0.94	1.01	0.00	1.36	1.46
time (sec)	N/A	0.164	2.556	29.434	0.349	1.052	0.000	0.399	2.962

Problem 83	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	103	103	105	145	116	119	0	104	546
normalized size	1	1.00	1.02	1.41	1.13	1.16	0.00	1.01	5.30
time (sec)	N/A	0.156	0.404	1.921	0.420	0.579	0.000	0.376	1.794

Problem 84	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	168	168	936	297	192	163	0	325	278
normalized size	1	1.00	5.57	1.77	1.14	0.97	0.00	1.93	1.65
time (sec)	N/A	0.195	6.235	37.628	0.335	0.759	0.000	0.478	4.215

Problem 85	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	73	96	103	109	0	73	139
normalized size	1	1.00	2.43	3.20	3.43	3.63	0.00	2.43	4.63
time (sec)	N/A	0.048	0.316	27.471	0.328	0.529	0.000	0.656	0.800

Problem 86	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	258	258	1342	394	251	187	0	536	419
normalized size	1	1.00	5.20	1.53	0.97	0.72	0.00	2.08	1.62
time (sec)	N/A	0.294	6.251	65.240	0.334	0.811	0.000	0.486	4.266

Problem 87	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	143	143	54	171	151	142	0	144	186
normalized size	1	1.00	0.38	1.20	1.06	0.99	0.00	1.01	1.30
time (sec)	N/A	0.123	0.552	52.372	0.328	0.493	0.000	0.435	1.098

Problem 88	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	330	330	1732	491	322	214	0	706	566
normalized size	1	1.00	5.25	1.49	0.98	0.65	0.00	2.14	1.72
time (sec)	N/A	0.343	6.391	68.458	0.335	0.555	0.000	0.507	4.400

Problem 89	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	201	201	115	236	193	167	0	214	447
normalized size	1	1.00	0.57	1.17	0.96	0.83	0.00	1.06	2.22
time (sec)	N/A	0.171	0.905	75.918	0.331	0.761	0.000	0.435	4.284

Problem 90	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	408	408	242	590	382	251	0	880	703
normalized size	1	1.00	0.59	1.45	0.94	0.62	0.00	2.16	1.72
time (sec)	N/A	0.398	1.312	74.032	0.344	0.899	0.000	0.524	5.147

Problem 91	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	254	254	175	300	233	194	0	284	560
normalized size	1	1.00	0.69	1.18	0.92	0.76	0.00	1.12	2.20
time (sec)	N/A	0.219	1.804	78.721	0.333	0.483	0.000	0.535	4.818

Problem 92	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	515	515	307	335	290	250	1037	342	801
normalized size	1	1.00	0.60	0.65	0.56	0.49	2.01	0.66	1.56
time (sec)	N/A	0.485	1.227	0.329	0.340	0.808	27.116	0.863	2.513

Problem 93	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	337	337	278	291	224	217	440	313	495
normalized size	1	1.00	0.82	0.86	0.66	0.64	1.31	0.93	1.47
time (sec)	N/A	0.300	1.017	0.237	0.331	0.677	16.229	0.700	4.342

Problem 94	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	426	426	259	305	228	220	826	278	650
normalized size	1	1.00	0.61	0.72	0.54	0.52	1.94	0.65	1.53
time (sec)	N/A	0.413	0.866	0.178	0.328	0.704	11.543	0.644	2.480

Problem 95	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	275	275	236	261	194	186	357	259	372
normalized size	1	1.00	0.86	0.95	0.71	0.68	1.30	0.94	1.35
time (sec)	N/A	0.281	0.746	0.159	0.328	0.630	6.049	0.597	4.421

Problem 96	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	126	126	188	236	187	182	609	211	472
normalized size	1	1.00	1.49	1.87	1.48	1.44	4.83	1.67	3.75
time (sec)	N/A	0.090	0.614	0.115	0.328	0.619	4.173	0.528	2.319

Problem 97	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	94	94	156	175	172	155	267	187	248
normalized size	1	1.00	1.66	1.86	1.83	1.65	2.84	1.99	2.64
time (sec)	N/A	0.047	0.464	0.150	0.328	0.679	2.141	0.328	0.938

Problem 98	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	170	170	711	272	170	160	0	199	297
normalized size	1	1.00	4.18	1.60	1.00	0.94	0.00	1.17	1.75
time (sec)	N/A	0.222	6.445	0.237	0.327	0.534	0.000	4.848	2.648

Problem 99	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	205	205	632	251	162	177	0	283	277
normalized size	1	1.00	3.08	1.22	0.79	0.86	0.00	1.38	1.35
time (sec)	N/A	0.216	6.296	0.239	0.331	0.609	0.000	0.559	3.978

Problem 100	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	F(-1)	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	169	169	571	291	179	177	0	173	354
normalized size	1	1.00	3.38	1.72	1.06	1.05	0.00	1.02	2.09
time (sec)	N/A	0.231	6.364	0.241	0.426	0.456	0.000	0.578	2.533

Problem 101	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	204	204	397	327	181	190	0	281	302
normalized size	1	1.00	1.95	1.60	0.89	0.93	0.00	1.38	1.48
time (sec)	N/A	0.208	5.899	0.233	0.335	0.497	0.000	0.599	4.047

Problem 102	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	147	147	126	202	174	155	0	144	971
normalized size	1	1.00	0.86	1.37	1.18	1.05	0.00	0.98	6.61
time (sec)	N/A	0.230	0.737	0.234	0.422	0.640	0.000	0.623	3.447

Problem 103	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	224	224	1219	440	230	196	0	410	345
normalized size	1	1.00	5.44	1.96	1.03	0.88	0.00	1.83	1.54
time (sec)	N/A	0.233	6.294	0.227	0.337	0.643	0.000	0.642	4.256

Problem 104	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	89	120	166	144	0	89	169
normalized size	1	1.00	2.97	4.00	5.53	4.80	0.00	2.97	5.63
time (sec)	N/A	0.048	0.495	0.282	0.347	0.522	0.000	2.263	0.976

Problem 105	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	318	318	1677	564	289	227	0	680	514
normalized size	1	1.00	5.27	1.77	0.91	0.71	0.00	2.14	1.62
time (sec)	N/A	0.337	6.335	0.285	0.341	0.594	0.000	6.462	4.205

Problem 106	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	177	177	54	217	223	176	0	176	419
normalized size	1	1.00	0.31	1.23	1.26	0.99	0.00	0.99	2.37
time (sec)	N/A	0.152	0.448	0.279	0.957	0.447	0.000	2.942	4.272

Problem 107	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	391	391	331	688	360	257	0	888	675
normalized size	1	1.00	0.85	1.76	0.92	0.66	0.00	2.27	1.73
time (sec)	N/A	0.389	2.138	0.280	1.009	0.624	0.000	0.739	4.709

Problem 108	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	242	242	115	299	275	207	0	262	548
normalized size	1	1.00	0.48	1.24	1.14	0.86	0.00	1.08	2.26
time (sec)	N/A	0.222	1.222	0.271	0.785	0.528	0.000	1.179	4.442

Problem 109	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	472	472	374	814	420	287	0	1096	831
normalized size	1	1.00	0.79	1.72	0.89	0.61	0.00	2.32	1.76
time (sec)	N/A	0.467	1.795	0.282	0.542	0.674	0.000	12.983	5.924

Problem 110	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	227	227	218	524	564	208	0	322	6099
normalized size	1	1.00	0.96	2.31	2.48	0.92	0.00	1.42	26.87
time (sec)	N/A	0.214	0.418	0.210	1.667	0.598	0.000	2.928	11.158

Problem 111	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	166	166	137	221	379	262	0	286	342
normalized size	1	1.00	0.83	1.33	2.28	1.58	0.00	1.72	2.06
time (sec)	N/A	0.175	0.986	0.179	0.534	0.573	0.000	0.303	3.316

Problem 112	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	B	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	119	119	143	236	284	119	0	182	3572
normalized size	1	1.00	1.20	1.98	2.39	1.00	0.00	1.53	30.02
time (sec)	N/A	0.129	0.231	0.162	2.008	0.587	0.000	0.282	6.163

Problem 113	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	91	91	79	90	142	187	0	118	110
normalized size	1	1.00	0.87	0.99	1.56	2.05	0.00	1.30	1.21
time (sec)	N/A	0.082	0.180	0.161	0.570	0.644	0.000	1.325	0.631

Problem 114	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	41	74	124	61	296	74	1069
normalized size	1	1.00	0.91	1.64	2.76	1.36	6.58	1.64	23.76
time (sec)	N/A	0.066	0.062	0.151	0.605	0.695	1.749	0.195	1.156

Problem 115	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-2)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	45	43	80	131	0	74	39
normalized size	1	1.00	0.96	0.91	1.70	2.79	0.00	1.57	0.83
time (sec)	N/A	0.023	0.032	0.144	0.731	0.764	0.000	2.751	0.467

Problem 116	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	18	19	103	59	0	19	62
normalized size	1	1.00	0.44	0.46	2.51	1.44	0.00	0.46	1.51
time (sec)	N/A	0.082	0.026	0.213	2.216	0.755	0.000	1.969	0.723

Problem 117	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	109	174	163	191	0	136	310
normalized size	1	1.00	1.36	2.18	2.04	2.39	0.00	1.70	3.88
time (sec)	N/A	0.084	0.139	0.227	1.693	0.718	0.000	0.322	0.713

Problem 118	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	52	72	238	117	0	54	300
normalized size	1	1.00	0.59	0.82	2.70	1.33	0.00	0.61	3.41
time (sec)	N/A	0.141	0.146	0.234	0.600	0.659	0.000	4.587	1.496

Problem 119	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	153	153	321	488	361	259	0	278	724
normalized size	1	1.00	2.10	3.19	2.36	1.69	0.00	1.82	4.73
time (sec)	N/A	0.157	2.000	0.244	0.419	0.715	0.000	3.037	2.213

Problem 120	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	158	158	99	162	462	183	0	120	575
normalized size	1	1.00	0.63	1.03	2.92	1.16	0.00	0.76	3.64
time (sec)	N/A	0.222	1.173	0.231	0.345	0.452	0.000	4.893	3.682

Problem 121	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	262	262	661	994	625	346	0	554	2979
normalized size	1	1.00	2.52	3.79	2.39	1.32	0.00	2.11	11.37
time (sec)	N/A	0.256	5.029	0.257	0.439	0.895	0.000	0.362	2.841

Problem 122	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	145	145	149	292	282	279	0	250	6604
normalized size	1	1.00	1.03	2.01	1.94	1.92	0.00	1.72	45.54
time (sec)	N/A	0.293	0.975	0.230	0.553	0.619	0.000	0.198	11.457

Problem 123	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	138	231	130	172	348	302	0	286	286
normalized size	1	1.67	0.94	1.25	2.52	2.19	0.00	2.07	2.07
time (sec)	N/A	1.048	0.756	0.235	2.551	0.566	0.000	0.322	2.813

Problem 124	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	82	192	130	131	173	1552	159	3114
normalized size	1	1.00	2.34	1.59	1.60	2.11	18.93	1.94	37.98
time (sec)	N/A	0.137	0.403	0.211	0.432	0.538	6.382	1.962	4.867

Problem 125	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	83	79	118	182	215	0	138	136
normalized size	1	1.00	0.95	1.42	2.19	2.59	0.00	1.66	1.64
time (sec)	N/A	0.067	0.217	0.207	0.441	0.553	0.000	0.303	0.840

Problem 126	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	32	21	21	57	0	20	47
normalized size	1	1.00	1.00	0.66	0.66	1.78	0.00	0.62	1.47
time (sec)	N/A	0.017	0.032	0.196	0.340	0.553	0.000	1.614	0.486

Problem 127	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	92	92	120	174	212	293	0	166	383
normalized size	1	1.00	1.30	1.89	2.30	3.18	0.00	1.80	4.16
time (sec)	N/A	0.079	0.787	0.276	0.426	0.589	0.000	0.355	1.171

Problem 128	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	75	75	51	78	60	178	0	71	382
normalized size	1	1.00	0.68	1.04	0.80	2.37	0.00	0.95	5.09
time (sec)	N/A	0.096	0.265	0.306	0.330	0.482	0.000	0.247	2.360

Problem 129	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	B	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	179	179	709	440	471	355	0	280	585
normalized size	1	1.00	3.96	2.46	2.63	1.98	0.00	1.56	3.27
time (sec)	N/A	0.240	6.115	0.323	0.438	0.615	0.000	0.343	1.900

Problem 130	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	141	141	122	174	115	281	0	149	1132
normalized size	1	1.00	0.87	1.23	0.82	1.99	0.00	1.06	8.03
time (sec)	N/A	0.150	2.779	0.342	0.330	0.458	0.000	0.248	4.203

Problem 131	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	B	C	A	B	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	216	492	211	283	658	480	0	399	610
normalized size	1	2.28	0.98	1.31	3.05	2.22	0.00	1.85	2.82
time (sec)	N/A	1.742	1.118	0.269	0.443	0.565	0.000	0.992	4.253

Problem 132	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	B	B	F(-2)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	122	122	154	219	481	341	0	265	6190
normalized size	1	1.00	1.26	1.80	3.94	2.80	0.00	2.17	50.74
time (sec)	N/A	0.212	1.260	0.253	0.464	0.565	0.000	0.387	8.539

Problem 133	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	119	225	119	280	412	352	0	293	443
normalized size	1	1.89	1.00	2.35	3.46	2.96	0.00	2.46	3.72
time (sec)	N/A	0.588	0.705	0.248	0.437	0.673	0.000	2.195	1.718

Problem 134	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	30	57	21	171	142	0	20	85
normalized size	1	1.36	2.59	0.95	7.77	6.45	0.00	0.91	3.86
time (sec)	N/A	0.031	0.122	0.230	0.356	0.528	0.000	0.439	0.615

Problem 135	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	B	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	103	103	132	191	326	294	0	221	260
normalized size	1	1.00	1.28	1.85	3.17	2.85	0.00	2.15	2.52
time (sec)	N/A	0.049	0.269	0.256	0.455	0.612	0.000	1.590	2.733

Problem 136	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	86	86	57	84	315	284	0	62	396
normalized size	1	1.00	0.66	0.98	3.66	3.30	0.00	0.72	4.60
time (sec)	N/A	0.103	0.521	0.394	0.338	0.596	0.000	1.833	2.595

Problem 137	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	260	260	396	611	518	513	0	314	1311
normalized size	1	1.00	1.52	2.35	1.99	1.97	0.00	1.21	5.04
time (sec)	N/A	0.286	2.440	0.409	0.457	0.616	0.000	1.323	2.620

Problem 138	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	161	161	140	184	652	354	0	140	1204
normalized size	1	1.00	0.87	1.14	4.05	2.20	0.00	0.87	7.48
time (sec)	N/A	0.168	2.998	0.404	0.373	0.692	0.000	0.598	4.744

Problem 139	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	B	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	383	383	688	1125	902	564	0	510	1203
normalized size	1	1.00	1.80	2.94	2.36	1.47	0.00	1.33	3.14
time (sec)	N/A	0.785	2.464	0.397	0.523	0.889	0.000	2.074	3.822

Problem 140	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	232	232	272	321	1053	476	0	243	1712
normalized size	1	1.00	1.17	1.38	4.54	2.05	0.00	1.05	7.38
time (sec)	N/A	0.244	1.336	0.431	0.389	0.777	0.000	1.004	7.638

Problem 141	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	B	B	F(-2)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	165	165	419	304	385	575	0	370	8586
normalized size	1	1.00	2.54	1.84	2.33	3.48	0.00	2.24	52.04
time (sec)	N/A	0.303	6.238	0.303	0.430	0.582	0.000	1.822	12.555

Problem 142	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	B	C	B	B	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	157	362	165	494	724	524	0	524	764
normalized size	1	2.31	1.05	3.15	4.61	3.34	0.00	3.34	4.87
time (sec)	N/A	1.173	1.118	0.309	0.445	0.758	0.000	0.435	2.691

Problem 143	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	124	21	53	255	0	20	224
normalized size	1	1.00	4.13	0.70	1.77	8.50	0.00	0.67	7.47
time (sec)	N/A	0.053	0.655	0.302	0.339	0.569	0.000	1.455	1.292

Problem 144	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	141	141	128	383	606	420	0	426	505
normalized size	1	1.00	0.91	2.72	4.30	2.98	0.00	3.02	3.58
time (sec)	N/A	0.111	0.728	0.303	0.446	0.741	0.000	4.080	3.821

Problem 145	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	98	98	85	64	85	217	0	50	222
normalized size	1	1.00	0.87	0.65	0.87	2.21	0.00	0.51	2.27
time (sec)	N/A	0.040	0.292	0.279	0.345	0.558	0.000	1.899	1.249

Problem 146	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	231	231	290	1367	661	745	0	527	2848
normalized size	1	1.00	1.26	5.92	2.86	3.23	0.00	2.28	12.33
time (sec)	N/A	0.170	3.276	0.420	0.469	0.781	0.000	4.072	4.830

Problem 147	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	138	138	133	188	144	537	0	138	666
normalized size	1	1.00	0.96	1.36	1.04	3.89	0.00	1.00	4.83
time (sec)	N/A	0.160	2.167	0.411	0.408	0.719	0.000	4.817	4.438

Problem 148	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	400	400	538	1255	936	820	0	548	1961
normalized size	1	1.00	1.34	3.14	2.34	2.05	0.00	1.37	4.90
time (sec)	N/A	0.796	3.465	0.473	0.484	0.813	0.000	3.103	4.593

Problem 149	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	232	232	295	330	217	553	0	249	1599
normalized size	1	1.00	1.27	1.42	0.94	2.38	0.00	1.07	6.89
time (sec)	N/A	0.249	2.014	0.428	0.334	0.776	0.000	0.273	7.970

Problem 150	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	99	99	82	137	0	76	223	116	164
normalized size	1	1.00	0.83	1.38	0.00	0.77	2.25	1.17	1.66
time (sec)	N/A	0.151	0.137	0.231	0.000	0.591	0.412	0.239	5.137

Problem 151	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	70	111	141	0	63	199	119	134
normalized size	1	1.00	1.59	2.01	0.00	0.90	2.84	1.70	1.91
time (sec)	N/A	0.128	0.067	0.165	0.000	0.538	0.468	1.226	2.087

Problem 152	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	75	75	60	98	0	54	155	99	111
normalized size	1	1.00	0.80	1.31	0.00	0.72	2.07	1.32	1.48
time (sec)	N/A	0.128	0.099	0.161	0.000	0.556	0.300	0.189	3.433

Problem 153	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	52	52	73	75	0	41	129	67	78
normalized size	1	1.00	1.40	1.44	0.00	0.79	2.48	1.29	1.50
time (sec)	N/A	0.120	0.069	0.161	0.000	0.493	0.483	1.796	0.772

Problem 154	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	38	59	0	32	61	60	39
normalized size	1	1.00	0.83	1.28	0.00	0.70	1.33	1.30	0.85
time (sec)	N/A	0.029	0.066	0.148	0.000	0.600	0.206	2.931	0.712

Problem 155	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	29	23	29	17	31	21	25
normalized size	1	1.00	1.00	0.79	1.00	0.59	1.07	0.72	0.86
time (sec)	N/A	0.016	0.033	0.134	0.316	0.565	0.141	0.163	0.613

Problem 156	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	23	22	101	26	0	57	41
normalized size	1	1.00	1.00	0.96	4.39	1.13	0.00	2.48	1.78
time (sec)	N/A	0.070	0.066	0.181	0.336	0.550	0.000	0.195	0.739

Problem 157	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	35	85	83	80	0	58	43
normalized size	1	1.00	1.13	2.74	2.68	2.58	0.00	1.87	1.39
time (sec)	N/A	0.094	0.219	0.211	0.417	0.608	0.000	2.581	0.670

Problem 158	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	35	26	108	33	0	27	25
normalized size	1	1.00	1.03	0.76	3.18	0.97	0.00	0.79	0.74
time (sec)	N/A	0.107	0.192	0.220	0.346	0.548	0.000	0.234	0.684

Problem 159	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	54	258	186	174	0	99	116
normalized size	1	1.00	0.90	4.30	3.10	2.90	0.00	1.65	1.93
time (sec)	N/A	0.117	0.246	0.231	0.331	0.518	0.000	2.725	2.552

Problem 160	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	52	52	53	47	211	72	0	47	99
normalized size	1	1.00	1.02	0.90	4.06	1.38	0.00	0.90	1.90
time (sec)	N/A	0.116	0.279	0.223	0.346	0.475	0.000	0.229	1.291

Problem 161	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	84	84	66	430	289	266	0	138	193
normalized size	1	1.00	0.79	5.12	3.44	3.17	0.00	1.64	2.30
time (sec)	N/A	0.133	0.465	0.237	0.353	0.599	0.000	0.232	4.246

Problem 162	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	70	67	68	313	109	0	67	139
normalized size	1	1.00	0.96	0.97	4.47	1.56	0.00	0.96	1.99
time (sec)	N/A	0.122	0.372	0.231	0.358	0.469	0.000	0.224	1.889

Problem 163	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	149	174	0	74	233	145	161
normalized size	1	1.00	1.75	2.05	0.00	0.87	2.74	1.71	1.89
time (sec)	N/A	0.186	0.105	0.194	0.000	0.581	0.590	1.205	4.114

Problem 164	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	101	82	117	0	65	190	103	138
normalized size	1	1.00	0.81	1.16	0.00	0.64	1.88	1.02	1.37
time (sec)	N/A	0.101	0.123	0.201	0.000	0.577	0.370	0.250	4.736

Problem 165	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	68	111	108	0	52	165	93	90
normalized size	1	1.00	1.63	1.59	0.00	0.76	2.43	1.37	1.32
time (sec)	N/A	0.176	0.082	0.184	0.000	0.422	0.411	0.235	1.015

Problem 166	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	60	79	0	43	119	72	69
normalized size	1	1.00	0.67	0.89	0.00	0.48	1.34	0.81	0.78
time (sec)	N/A	0.082	0.100	0.190	0.000	0.474	0.228	0.419	1.756

Problem 167	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	52	52	73	57	45	30	94	47	79
normalized size	1	1.00	1.40	1.10	0.87	0.58	1.81	0.90	1.52
time (sec)	N/A	0.114	0.057	0.168	0.328	0.585	0.234	1.068	0.647

Problem 168	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	31	23	22	17	46	30	31
normalized size	1	1.00	1.00	0.74	0.71	0.55	1.48	0.97	1.00
time (sec)	N/A	0.016	0.041	0.154	0.316	0.429	0.143	0.189	0.611

Problem 169	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	184	63	117	64	0	57	44
normalized size	1	1.00	4.00	1.37	2.54	1.39	0.00	1.24	0.96
time (sec)	N/A	0.115	0.242	0.227	0.425	0.487	0.000	1.682	0.672

Problem 170	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	71	35	30	68	0	100	83
normalized size	1	1.00	1.29	0.64	0.55	1.24	0.00	1.82	1.51
time (sec)	N/A	0.071	0.423	0.241	0.333	0.593	0.000	2.926	0.782

Problem 171	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	56	56	146	170	167	134	0	95	104
normalized size	1	1.00	2.61	3.04	2.98	2.39	0.00	1.70	1.86
time (sec)	N/A	0.146	0.432	0.250	0.333	0.889	0.000	0.335	1.181

Problem 172	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	68	36	35	54	0	35	49
normalized size	1	1.00	2.00	1.06	1.03	1.59	0.00	1.03	1.44
time (sec)	N/A	0.064	0.260	0.257	0.514	0.635	0.000	1.630	0.725

Problem 173	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	84	84	215	342	295	230	0	151	136
normalized size	1	1.00	2.56	4.07	3.51	2.74	0.00	1.80	1.62
time (sec)	N/A	0.192	0.997	0.275	0.344	0.508	0.000	0.260	3.208

Problem 174	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	70	77	47	47	97	0	47	76
normalized size	1	1.00	1.10	0.67	0.67	1.39	0.00	0.67	1.09
time (sec)	N/A	0.078	0.412	0.254	0.336	0.807	0.000	0.235	0.915

Problem 175	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	125	125	106	137	0	76	228	119	164
normalized size	1	1.00	0.85	1.10	0.00	0.61	1.82	0.95	1.31
time (sec)	N/A	0.111	0.186	0.198	0.000	1.259	0.444	0.283	4.785

Problem 176	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	106	106	149	141	0	63	201	119	134
normalized size	1	1.00	1.41	1.33	0.00	0.59	1.90	1.12	1.26
time (sec)	N/A	0.234	0.079	0.194	0.000	2.121	0.477	0.265	3.169

Problem 177	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	131	131	84	98	0	54	160	80	96
normalized size	1	1.00	0.64	0.75	0.00	0.41	1.22	0.61	0.73
time (sec)	N/A	0.141	0.113	0.194	0.000	0.620	0.313	0.255	3.487

Problem 178	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	90	90	111	90	69	41	136	73	133
normalized size	1	1.00	1.23	1.00	0.77	0.46	1.51	0.81	1.48
time (sec)	N/A	0.221	0.073	0.191	0.526	0.909	0.345	1.504	0.867

Problem 179	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	77	23	51	30	97	57	100
normalized size	1	1.00	2.41	0.72	1.59	0.94	3.03	1.78	3.12
time (sec)	N/A	0.032	0.058	0.167	0.367	0.670	0.221	0.974	0.754

Problem 180	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	31	57	29	17	46	36	68
normalized size	1	1.00	1.00	1.84	0.94	0.55	1.48	1.16	2.19
time (sec)	N/A	0.016	0.038	0.172	0.557	0.998	0.147	1.898	0.632

Problem 181	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	91	40	99	55	0	100	101
normalized size	1	1.00	1.49	0.66	1.62	0.90	0.00	1.64	1.66
time (sec)	N/A	0.064	0.282	0.257	0.791	0.561	0.000	0.311	0.773

Problem 182	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	62	62	109	108	329	112	0	110	105
normalized size	1	1.00	1.76	1.74	5.31	1.81	0.00	1.77	1.69
time (sec)	N/A	0.162	0.329	0.279	0.740	0.611	0.000	0.622	0.992

Problem 183	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	75	75	110	52	299	110	0	128	104
normalized size	1	1.00	1.47	0.69	3.99	1.47	0.00	1.71	1.39
time (sec)	N/A	0.081	0.616	0.294	0.504	1.080	0.000	0.296	0.924

Problem 184	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	76	76	64	258	215	182	0	112	135
normalized size	1	1.00	0.84	3.39	2.83	2.39	0.00	1.47	1.78
time (sec)	N/A	0.182	0.468	0.294	0.369	0.459	0.000	0.282	2.720

Problem 185	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	90	47	240	69	0	47	55
normalized size	1	1.00	2.65	1.38	7.06	2.03	0.00	1.38	1.62
time (sec)	N/A	0.063	0.466	0.301	0.379	1.424	0.000	0.407	0.883

Problem 186	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	104	104	115	430	341	278	0	164	150
normalized size	1	1.00	1.11	4.13	3.28	2.67	0.00	1.58	1.44
time (sec)	N/A	0.230	0.432	0.305	0.350	1.082	0.000	1.955	3.293

Problem 187	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	90	0	0	0	0	0	-1
normalized size	1	1.00	1.36	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.061	2.166	0.678	0.000	0.678	0.000	0.000	0.000

Problem 188	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	5	5	16	6	31	5	17	22	21
normalized size	1	1.00	3.20	1.20	6.20	1.00	3.40	4.40	4.20
time (sec)	N/A	0.023	0.019	0.080	0.321	1.046	0.134	5.739	1.108

Problem 189	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	10	10	19	11	54	10	0	10	21
normalized size	1	1.00	1.90	1.10	5.40	1.00	0.00	1.00	2.10
time (sec)	N/A	0.068	0.021	0.106	0.413	0.763	0.000	0.157	0.596

Problem 190	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	4	4	4	15	30	4	0	14	4
normalized size	1	1.00	1.00	3.75	7.50	1.00	0.00	3.50	1.00
time (sec)	N/A	0.060	0.018	0.109	0.424	1.371	0.000	0.828	0.554

Problem 191	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	11	11	25	13	28	24	0	12	12
normalized size	1	1.00	2.27	1.18	2.55	2.18	0.00	1.09	1.09
time (sec)	N/A	0.053	0.033	0.077	0.429	0.701	0.000	0.959	0.583

Problem 192	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	9	9	20	10	23	22	0	10	23
normalized size	1	1.00	2.22	1.11	2.56	2.44	0.00	1.11	2.56
time (sec)	N/A	0.078	0.022	0.130	0.491	0.680	0.000	3.984	0.593

Problem 193	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	10	10	23	11	15	18	0	10	10
normalized size	1	1.00	2.30	1.10	1.50	1.80	0.00	1.00	1.00
time (sec)	N/A	0.024	0.017	0.072	0.426	1.952	0.000	0.200	0.555

Problem 194	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	11	11	20	8	25	13	0	12	15
normalized size	1	1.00	1.82	0.73	2.27	1.18	0.00	1.09	1.36
time (sec)	N/A	0.055	0.022	0.119	0.570	0.499	0.000	0.161	0.574

Problem 195	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	9	9	18	8	29	9	17	20	19
normalized size	1	1.00	2.00	0.89	3.22	1.00	1.89	2.22	2.11
time (sec)	N/A	0.027	0.018	0.085	0.341	0.496	0.142	0.167	0.957

Problem 196	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	23	13	54	14	0	14	23
normalized size	1	1.00	1.64	0.93	3.86	1.00	0.00	1.00	1.64
time (sec)	N/A	0.082	0.021	0.105	1.027	1.463	0.000	2.392	0.608

Problem 197	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	6	6	6	15	30	6	0	14	6
normalized size	1	1.00	1.00	2.50	5.00	1.00	0.00	2.33	1.00
time (sec)	N/A	0.060	0.020	0.114	0.491	0.583	0.000	1.026	0.577

Problem 198	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	29	15	28	28	0	14	14
normalized size	1	1.00	1.93	1.00	1.87	1.87	0.00	0.93	0.93
time (sec)	N/A	0.060	0.035	0.070	1.340	0.534	0.000	1.954	0.577

Problem 199	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	7	7	18	8	23	20	0	8	23
normalized size	1	1.00	2.57	1.14	3.29	2.86	0.00	1.14	3.29
time (sec)	N/A	0.089	0.022	0.129	0.443	0.463	0.000	0.201	0.610

Problem 200	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	11	11	25	11	15	17	0	10	10
normalized size	1	1.00	2.27	1.00	1.36	1.55	0.00	0.91	0.91
time (sec)	N/A	0.029	0.016	0.070	0.328	0.485	0.000	1.668	0.563

Problem 201	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	22	8	25	15	0	14	15
normalized size	1	1.00	1.69	0.62	1.92	1.15	0.00	1.08	1.15
time (sec)	N/A	0.062	0.022	0.107	0.335	0.688	0.000	0.185	0.564

Problem 202	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	15	24	22	21	27	16	14
normalized size	1	1.00	0.65	1.04	0.96	0.91	1.17	0.70	0.61
time (sec)	N/A	0.094	0.015	0.103	0.338	0.502	2.106	0.261	0.589

Problem 203	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	6	6	14	19	38	6	0	18	6
normalized size	1	1.00	2.33	3.17	6.33	1.00	0.00	3.00	1.00
time (sec)	N/A	0.070	0.007	0.104	0.656	1.475	0.000	0.212	1.047

Problem 204	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	10	10	20	11	34	12	0	10	24
normalized size	1	1.00	2.00	1.10	3.40	1.20	0.00	1.00	2.40
time (sec)	N/A	0.067	0.007	0.097	0.431	2.001	0.000	0.427	0.591

Problem 205	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	7	7	36	21	39	20	0	22	11
normalized size	1	1.00	5.14	3.00	5.57	2.86	0.00	3.14	1.57
time (sec)	N/A	0.085	0.026	0.112	0.433	0.712	0.000	0.216	0.575

Problem 206	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	10	9	23	17	0	8	8
normalized size	1	1.00	0.83	0.75	1.92	1.42	0.00	0.67	0.67
time (sec)	N/A	0.054	0.017	0.065	0.430	0.593	0.000	0.216	0.573

Problem 207	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	11	11	25	6	29	15	0	12	11
normalized size	1	1.00	2.27	0.55	2.64	1.36	0.00	1.09	1.00
time (sec)	N/A	0.060	0.008	0.089	0.330	1.046	0.000	0.153	0.663

Problem 208	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	9	9	6	5	9	9	0	4	4
normalized size	1	1.00	0.67	0.56	1.00	1.00	0.00	0.44	0.44
time (sec)	N/A	0.026	0.019	0.059	0.330	0.577	0.000	0.215	0.542

Problem 209	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	4	4	14	19	38	4	0	18	4
normalized size	1	1.00	3.50	4.75	9.50	1.00	0.00	4.50	1.00
time (sec)	N/A	0.070	0.007	0.096	0.447	0.624	0.000	0.199	1.026

Problem 210	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	10	10	20	9	46	10	0	10	31
normalized size	1	1.00	2.00	0.90	4.60	1.00	0.00	1.00	3.10
time (sec)	N/A	0.068	0.010	0.096	0.652	0.803	0.000	0.196	0.618

Problem 211	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	5	5	46	19	39	18	0	20	9
normalized size	1	1.00	9.20	3.80	7.80	3.60	0.00	4.00	1.80
time (sec)	N/A	0.094	0.017	0.113	0.429	0.781	0.000	0.475	0.570

Problem 212	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	16	13	23	14	0	12	10
normalized size	1	1.00	1.00	0.81	1.44	0.88	0.00	0.75	0.62
time (sec)	N/A	0.060	0.015	0.069	0.449	0.689	0.000	0.158	0.578

Problem 213	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	25	6	41	15	0	14	19
normalized size	1	1.00	1.92	0.46	3.15	1.15	0.00	1.08	1.46
time (sec)	N/A	0.061	0.010	0.094	0.341	0.536	0.000	0.257	0.610

Problem 214	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	8	9	10	10	0	8	6
normalized size	1	1.00	0.67	0.75	0.83	0.83	0.00	0.67	0.50
time (sec)	N/A	0.030	0.007	0.059	0.339	0.907	0.000	0.202	0.534

Problem 215	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	61	21	176	44	0	68	42
normalized size	1	1.00	2.65	0.91	7.65	1.91	0.00	2.96	1.83
time (sec)	N/A	0.039	0.186	0.115	0.574	0.645	0.000	1.205	0.656

Problem 216	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	30	30	252	52	0	82	62
normalized size	1	1.00	0.59	0.59	4.94	1.02	0.00	1.61	1.22
time (sec)	N/A	0.172	0.070	0.131	0.576	0.612	0.000	0.223	0.619

Problem 217	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	20	17	16	18	0	16	16
normalized size	1	1.00	1.11	0.94	0.89	1.00	0.00	0.89	0.89
time (sec)	N/A	0.031	0.107	0.064	0.469	0.501	0.000	0.261	0.059

Problem 218	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	24	42	35	37	0	37	61
normalized size	1	1.00	0.83	1.45	1.21	1.28	0.00	1.28	2.10
time (sec)	N/A	0.038	0.035	0.158	0.551	0.436	0.000	0.391	0.695

Problem 219	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	11	11	11	13	11	11	0	11	45
normalized size	1	1.00	1.00	1.18	1.00	1.00	0.00	1.00	4.09
time (sec)	N/A	0.025	0.038	0.074	0.645	0.490	0.000	0.252	0.671

Problem 220	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	30	19	39	30	0	79	14
normalized size	1	1.00	1.88	1.19	2.44	1.88	0.00	4.94	0.88
time (sec)	N/A	0.032	0.045	0.088	0.472	0.500	0.000	0.278	0.607

Problem 221	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	22	20	245	46	0	72	56
normalized size	1	1.00	0.46	0.42	5.10	0.96	0.00	1.50	1.17
time (sec)	N/A	0.108	0.026	0.109	0.477	0.510	0.000	0.253	0.709

Problem 222	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	10	10	10	13	28	12	0	28	20
normalized size	1	1.00	1.00	1.30	2.80	1.20	0.00	2.80	2.00
time (sec)	N/A	0.025	0.010	0.121	0.419	0.459	0.000	0.191	0.663

Problem 223	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	23	24	64	31	0	18	33
normalized size	1	1.00	1.64	1.71	4.57	2.21	0.00	1.29	2.36
time (sec)	N/A	0.147	0.008	0.124	0.437	0.417	0.000	0.345	0.620

Problem 224	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	12	13	24	14	0	26	14
normalized size	1	1.00	1.00	1.08	2.00	1.17	0.00	2.17	1.17
time (sec)	N/A	0.031	0.008	0.063	0.337	0.456	0.000	0.199	0.059

Problem 225	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	34	60	46	61	0	48	69
normalized size	1	1.00	1.00	1.76	1.35	1.79	0.00	1.41	2.03
time (sec)	N/A	0.039	0.015	0.158	0.343	0.427	0.000	0.316	1.000

Problem 226	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	11	11	11	12	26	28	0	28	15
normalized size	1	1.00	1.00	1.09	2.36	2.55	0.00	2.55	1.36
time (sec)	N/A	0.026	0.002	0.118	0.328	0.448	0.000	0.988	0.639

Problem 227	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	15	14	17	13	0	46	13
normalized size	1	1.00	1.00	0.93	1.13	0.87	0.00	3.07	0.87
time (sec)	N/A	0.032	0.012	0.076	0.349	0.390	0.000	0.231	0.043

Problem 228	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	10	10	10	11	44	18	0	10	29
normalized size	1	1.00	1.00	1.10	4.40	1.80	0.00	1.00	2.90
time (sec)	N/A	0.086	0.004	0.107	0.335	0.415	0.000	0.204	0.580

Problem 229	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	33	29	28	28	0	0	29
normalized size	1	1.00	1.00	0.88	0.85	0.85	0.00	0.00	0.88
time (sec)	N/A	0.062	0.014	0.051	0.344	0.409	0.000	0.000	0.681

Problem 230	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	38	29	28	28	0	99	49
normalized size	1	1.00	1.15	0.88	0.85	0.85	0.00	3.00	1.48
time (sec)	N/A	0.054	0.115	0.044	0.339	0.491	0.000	3.260	0.644

Problem 231	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	29	40	26	25	25	0	102	28
normalized size	1	1.32	1.82	1.18	1.14	1.14	0.00	4.64	1.27
time (sec)	N/A	0.030	0.010	0.034	0.412	0.406	0.000	0.271	0.626

Problem 232	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	37	31	25	25	37	27	40
normalized size	1	1.00	1.42	1.19	0.96	0.96	1.42	1.04	1.54
time (sec)	N/A	0.013	0.018	0.008	0.354	0.424	0.163	0.157	0.655

Problem 233	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	25	25	32	34	0	107	40
normalized size	1	1.00	1.00	1.00	1.28	1.36	0.00	4.28	1.60
time (sec)	N/A	0.028	0.017	0.035	0.325	0.509	0.000	0.287	0.664

Problem 234	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	28	25	27	24	0	71	24
normalized size	1	1.00	1.00	0.89	0.96	0.86	0.00	2.54	0.86
time (sec)	N/A	0.050	0.019	0.039	0.332	0.620	0.000	0.293	0.635

Problem 235	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	33	28	32	26	0	97	29
normalized size	1	1.00	1.00	0.85	0.97	0.79	0.00	2.94	0.88
time (sec)	N/A	0.057	0.022	0.043	0.332	0.577	0.000	0.355	0.693

Problem 236	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	106	106	77	100	68	85	0	0	101
normalized size	1	1.00	0.73	0.94	0.64	0.80	0.00	0.00	0.95
time (sec)	N/A	0.349	0.385	0.092	0.370	0.592	0.000	0.000	0.793

Problem 237	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	86	86	82	86	66	74	0	5161	76
normalized size	1	1.00	0.95	1.00	0.77	0.86	0.00	60.01	0.88
time (sec)	N/A	0.190	0.181	0.082	0.334	0.681	0.000	7.762	0.715

Problem 238	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	87	87	117	83	76	83	0	5713	121
normalized size	1	1.00	1.34	0.95	0.87	0.95	0.00	65.67	1.39
time (sec)	N/A	0.321	0.146	0.074	0.339	0.547	0.000	4.221	0.840

Problem 239	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	77	116	99	84	108	0	2752	143
normalized size	1	1.00	1.51	1.29	1.09	1.40	0.00	35.74	1.86
time (sec)	N/A	0.116	0.581	0.059	0.687	0.650	0.000	1.276	0.797

Problem 240	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	90	90	75	123	102	126	0	171	147
normalized size	1	1.00	0.83	1.37	1.13	1.40	0.00	1.90	1.63
time (sec)	N/A	0.426	0.192	0.092	0.461	0.706	0.000	1.585	0.788

Problem 241	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	99	99	201	109	82	115	0	158	227
normalized size	1	1.00	2.03	1.10	0.83	1.16	0.00	1.60	2.29
time (sec)	N/A	0.467	1.118	0.108	0.436	0.654	0.000	2.093	1.026

Problem 242	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	125	125	336	169	129	129	0	226	177
normalized size	1	1.00	2.69	1.35	1.03	1.03	0.00	1.81	1.42
time (sec)	N/A	0.440	0.608	0.112	0.387	0.577	0.000	1.889	3.269

Problem 243	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	77	114	109	95	100	0	0	149
normalized size	1	1.00	1.48	1.42	1.23	1.30	0.00	0.00	1.94
time (sec)	N/A	0.189	0.233	0.106	0.377	1.268	0.000	0.000	0.800

Problem 244	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	120	120	106	128	94	101	0	0	237
normalized size	1	1.00	0.88	1.07	0.78	0.84	0.00	0.00	1.98
time (sec)	N/A	0.194	0.187	0.106	0.457	0.694	0.000	0.000	0.792

Problem 245	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	112	112	98	147	87	128	0	0	225
normalized size	1	1.00	0.88	1.31	0.78	1.14	0.00	0.00	2.01
time (sec)	N/A	0.177	0.268	0.078	0.352	0.611	0.000	0.000	4.182

Problem 246	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	116	116	102	164	113	123	0	0	219
normalized size	1	1.00	0.88	1.41	0.97	1.06	0.00	0.00	1.89
time (sec)	N/A	0.104	0.293	0.076	0.421	0.615	0.000	0.000	4.524

Problem 247	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	115	115	100	213	109	122	0	0	219
normalized size	1	1.00	0.87	1.85	0.95	1.06	0.00	0.00	1.90
time (sec)	N/A	0.242	0.512	0.114	0.353	0.492	0.000	0.000	4.447

Problem 248	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	111	111	97	204	96	107	0	0	223
normalized size	1	1.00	0.87	1.84	0.86	0.96	0.00	0.00	2.01
time (sec)	N/A	0.254	1.897	0.137	0.357	0.608	0.000	0.000	4.217

Problem 249	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F(-1)	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	119	119	99	252	128	109	0	0	220
normalized size	1	1.00	0.83	2.12	1.08	0.92	0.00	0.00	1.85
time (sec)	N/A	0.220	0.317	0.138	0.364	0.649	0.000	0.000	4.153

Problem 250	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	113	113	100	111	188	123	0	303	161
normalized size	1	1.00	0.88	0.98	1.66	1.09	0.00	2.68	1.42
time (sec)	N/A	0.341	0.368	0.184	0.428	0.636	0.000	0.397	1.160

Problem 251	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	92	92	80	93	129	98	0	190	117
normalized size	1	1.00	0.87	1.01	1.40	1.07	0.00	2.07	1.27
time (sec)	N/A	0.279	0.228	0.180	0.445	0.779	0.000	0.355	0.823

Problem 252	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	70	80	102	75	0	257	93
normalized size	1	1.00	0.88	1.00	1.28	0.94	0.00	3.21	1.16
time (sec)	N/A	0.237	0.098	0.154	0.424	0.556	0.000	0.387	0.868

Problem 253	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	63	75	71	64	0	100	67
normalized size	1	1.00	0.85	1.01	0.96	0.86	0.00	1.35	0.91
time (sec)	N/A	0.080	0.093	0.146	0.327	0.541	0.000	0.206	0.900

Problem 254	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	75	75	64	75	73	65	0	101	68
normalized size	1	1.00	0.85	1.00	0.97	0.87	0.00	1.35	0.91
time (sec)	N/A	0.186	0.063	0.190	0.360	0.775	0.000	0.321	0.708

Problem 255	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	94	94	103	95	125	96	0	253	93
normalized size	1	1.00	1.10	1.01	1.33	1.02	0.00	2.69	0.99
time (sec)	N/A	0.270	0.151	0.195	0.374	0.896	0.000	0.387	0.843

Problem 256	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	108	108	92	110	158	147	0	190	118
normalized size	1	1.00	0.85	1.02	1.46	1.36	0.00	1.76	1.09
time (sec)	N/A	0.284	0.290	0.197	0.438	0.672	0.000	0.388	0.827

Problem 257	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	243	243	164	291	0	857	0	1362	7329
normalized size	1	1.00	0.67	1.20	0.00	3.53	0.00	5.60	30.16
time (sec)	N/A	0.611	2.952	0.235	0.000	0.591	0.000	1.128	5.445

Problem 258	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	227	227	151	255	0	705	0	331	6093
normalized size	1	1.00	0.67	1.12	0.00	3.11	0.00	1.46	26.84
time (sec)	N/A	0.491	2.082	0.210	0.000	0.740	0.000	0.738	5.245

Problem 259	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	219	219	131	155	0	518	0	282	213
normalized size	1	1.00	0.60	0.71	0.00	2.37	0.00	1.29	0.97
time (sec)	N/A	0.385	1.288	0.175	0.000	0.541	0.000	0.666	1.126

Problem 260	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	203	203	128	162	0	526	0	289	245
normalized size	1	1.00	0.63	0.80	0.00	2.59	0.00	1.42	1.21
time (sec)	N/A	0.399	1.264	0.180	0.000	0.697	0.000	0.239	1.193

Problem 261	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	136	136	127	162	0	532	0	288	245
normalized size	1	1.00	0.93	1.19	0.00	3.91	0.00	2.12	1.80
time (sec)	N/A	0.318	1.133	0.220	0.000	0.880	0.000	0.498	1.207

Problem 262	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	131	131	121	155	0	516	0	284	215
normalized size	1	1.00	0.92	1.18	0.00	3.94	0.00	2.17	1.64
time (sec)	N/A	0.334	0.779	0.219	0.000	0.707	0.000	0.559	1.084

Problem 263	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	231	231	196	276	0	864	0	354	6056
normalized size	1	1.00	0.85	1.19	0.00	3.74	0.00	1.53	26.22
time (sec)	N/A	0.437	1.969	0.244	0.000	1.659	0.000	0.678	5.207

Problem 264	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	B	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	248	248	713	333	684	1180	0	848	527
normalized size	1	1.00	2.88	1.34	2.76	4.76	0.00	3.42	2.12
time (sec)	N/A	0.902	6.358	0.227	0.477	1.080	0.000	1.636	2.049

Problem 265	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	232	232	204	295	589	1045	0	676	491
normalized size	1	1.00	0.88	1.27	2.54	4.50	0.00	2.91	2.12
time (sec)	N/A	0.762	6.112	0.229	0.883	1.867	0.000	1.394	1.212

Problem 266	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	211	211	184	220	593	994	0	690	490
normalized size	1	1.00	0.87	1.04	2.81	4.71	0.00	3.27	2.32
time (sec)	N/A	0.668	5.327	0.191	0.621	0.736	0.000	1.437	1.109

Problem 267	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	229	229	696	322	601	1071	0	800	494
normalized size	1	1.00	3.04	1.41	2.62	4.68	0.00	3.49	2.16
time (sec)	N/A	0.480	6.321	0.205	0.415	0.972	0.000	0.415	1.130

Problem 268	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	231	231	703	324	602	1076	0	801	496
normalized size	1	1.00	3.04	1.40	2.61	4.66	0.00	3.47	2.15
time (sec)	N/A	0.621	6.380	0.234	0.376	0.840	0.000	0.942	1.122

Problem 269	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	212	212	217	251	596	939	0	689	492
normalized size	1	1.00	1.02	1.18	2.81	4.43	0.00	3.25	2.32
time (sec)	N/A	0.364	6.267	0.249	0.386	1.008	0.000	1.109	1.129

Problem 270	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	228	228	217	256	591	971	0	675	490
normalized size	1	1.00	0.95	1.12	2.59	4.26	0.00	2.96	2.15
time (sec)	N/A	0.408	6.233	0.248	0.375	0.782	0.000	1.252	1.133

Problem 271	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	B	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	155	155	246	0	180	412	0	0	861
normalized size	1	1.00	1.59	0.00	1.16	2.66	0.00	0.00	5.55
time (sec)	N/A	0.385	1.447	4.639	0.362	0.532	0.000	0.000	7.512

Problem 272	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	264	264	6669	0	0	0	0	0	-1
normalized size	1	1.00	25.26	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.764	30.597	2.247	0.000	1.471	0.000	0.000	0.000

Problem 273	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	35	40	36	35	0	0	35
normalized size	1	1.00	0.90	1.03	0.92	0.90	0.00	0.00	0.90
time (sec)	N/A	0.073	0.085	0.088	0.344	0.521	0.000	0.000	0.979

Problem 274	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	144	144	106	0	0	0	0	0	-1
normalized size	1	1.00	0.74	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.432	0.371	0.646	0.000	0.470	0.000	0.000	0.000

Problem 275	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	61	81	105	142	0	94	93
normalized size	1	1.00	0.94	1.25	1.62	2.18	0.00	1.45	1.43
time (sec)	N/A	0.065	0.136	0.075	0.448	0.649	0.000	5.985	1.381

Problem 276	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	B	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	92	92	153	174	211	94	0	152	3401
normalized size	1	1.00	1.66	1.89	2.29	1.02	0.00	1.65	36.97
time (sec)	N/A	0.139	0.337	0.085	0.463	0.467	0.000	2.008	6.233

Problem 277	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	122	122	113	166	278	210	0	190	286
normalized size	1	1.00	0.93	1.36	2.28	1.72	0.00	1.56	2.34
time (sec)	N/A	0.166	0.983	0.088	0.447	0.440	0.000	4.003	1.195

Problem 278	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	B	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	93	93	82	175	212	94	0	156	3419
normalized size	1	1.00	0.88	1.88	2.28	1.01	0.00	1.68	36.76
time (sec)	N/A	0.133	0.310	0.082	0.451	0.477	0.000	1.922	6.187

Problem 279	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	112	112	115	168	281	215	0	192	277
normalized size	1	1.00	1.03	1.50	2.51	1.92	0.00	1.71	2.47
time (sec)	N/A	0.196	0.629	0.087	0.465	0.771	0.000	8.002	1.001

Problem 280	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	B	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	176	176	178	363	431	174	0	275	5902
normalized size	1	1.00	1.01	2.06	2.45	0.99	0.00	1.56	33.53
time (sec)	N/A	0.278	0.522	0.086	0.451	0.726	0.000	1.916	11.949

Problem 281	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	123	123	112	170	281	213	0	201	291
normalized size	1	1.00	0.91	1.38	2.28	1.73	0.00	1.63	2.37
time (sec)	N/A	0.158	1.020	0.092	0.433	0.521	0.000	8.014	1.265

Problem 282	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	B	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	175	175	287	363	424	175	0	273	5870
normalized size	1	1.00	1.64	2.07	2.42	1.00	0.00	1.56	33.54
time (sec)	N/A	0.278	0.786	0.090	0.453	0.644	0.000	0.206	11.144

Problem 283	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	193	193	223	305	521	307	0	361	600
normalized size	1	1.00	1.16	1.58	2.70	1.59	0.00	1.87	3.11
time (sec)	N/A	0.359	1.476	0.110	0.450	0.601	0.000	1.688	1.643

Problem 284	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	87	144	120	118	138	991	144	1017
normalized size	1	1.24	2.06	1.71	1.69	1.97	14.16	2.06	14.53
time (sec)	N/A	0.166	0.257	0.103	0.452	0.510	2.102	0.189	5.161

Problem 285	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	110	152	111	142	265	252	0	209	249
normalized size	1	1.38	1.01	1.29	2.41	2.29	0.00	1.90	2.26
time (sec)	N/A	0.239	0.593	0.121	0.460	0.513	0.000	0.203	1.232

Problem 286	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	B	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	129	198	226	243	259	236	0	223	5431
normalized size	1	1.53	1.75	1.88	2.01	1.83	0.00	1.73	42.10
time (sec)	N/A	0.506	1.479	0.114	0.458	0.741	0.000	0.445	7.682

Problem 287	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	109	151	110	152	264	252	0	204	253
normalized size	1	1.39	1.01	1.39	2.42	2.31	0.00	1.87	2.32
time (sec)	N/A	0.259	0.682	0.112	0.443	0.662	0.000	0.989	1.158

Problem 288	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	131	186	145	260	257	244	0	219	6012
normalized size	1	1.42	1.11	1.98	1.96	1.86	0.00	1.67	45.89
time (sec)	N/A	0.545	1.659	0.124	0.459	0.650	0.000	0.147	12.131

Problem 289	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	172	238	200	269	611	360	0	342	594
normalized size	1	1.38	1.16	1.56	3.55	2.09	0.00	1.99	3.45
time (sec)	N/A	0.677	1.208	0.139	0.481	0.604	0.000	0.248	2.924

Problem 290	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	B	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	128	196	221	241	256	252	0	214	5428
normalized size	1	1.53	1.73	1.88	2.00	1.97	0.00	1.67	42.41
time (sec)	N/A	0.408	1.333	0.115	0.457	0.650	0.000	0.189	8.158

Problem 291	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	176	238	198	261	606	369	0	335	586
normalized size	1	1.35	1.12	1.48	3.44	2.10	0.00	1.90	3.33
time (sec)	N/A	0.697	1.268	0.142	0.485	0.560	0.000	0.255	2.875

Problem 292	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	B	A	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	210	289	409	515	456	371	0	435	8198
normalized size	1	1.38	1.95	2.45	2.17	1.77	0.00	2.07	39.04
time (sec)	N/A	1.251	2.757	0.130	0.470	0.499	0.000	0.156	13.920

Problem 293	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	76	63	98	140	0	90	408
normalized size	1	1.00	1.62	1.34	2.09	2.98	0.00	1.91	8.68
time (sec)	N/A	0.079	0.114	0.111	0.452	0.698	0.000	1.781	1.476

Problem 294	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	60	49	79	142	0	75	123
normalized size	1	1.00	1.25	1.02	1.65	2.96	0.00	1.56	2.56
time (sec)	N/A	0.078	0.079	0.101	0.458	0.468	0.000	0.211	0.984

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is given. The larger this ratio is, the harder the integral was to solve. In this test, problem number [286] had the largest ratio of [.7222]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	7	5	1.00	14	0.357
2	A	6	4	1.00	14	0.286
3	A	6	5	1.00	12	0.417
4	A	3	2	1.00	9	0.222
5	A	3	2	1.00	12	0.167
6	A	5	4	1.00	14	0.286
7	A	6	5	1.00	14	0.357
8	A	5	5	1.00	16	0.312
9	A	4	4	1.00	16	0.250
10	A	2	2	1.00	14	0.143

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
11	A	2	2	1.00	11	0.182
12	A	3	3	1.00	14	0.214
13	A	4	4	1.00	16	0.250
14	A	6	6	1.00	16	0.375
15	B	19	11	2.64	16	0.688
16	A	4	4	1.00	16	0.250
17	A	3	3	1.00	14	0.214
18	A	1	1	1.00	11	0.091
19	A	4	4	1.00	14	0.286
20	A	3	2	1.00	16	0.125
21	A	11	7	1.00	16	0.438
22	A	5	5	1.00	16	0.312
23	B	13	7	3.26	16	0.438
24	A	2	2	1.27	14	0.143
25	A	3	3	1.00	11	0.273
26	A	3	2	1.00	14	0.143
27	A	12	7	1.00	16	0.438
28	A	3	2	1.00	16	0.125
29	A	1	1	1.00	33	0.030
30	A	8	5	1.00	26	0.192
31	A	6	4	1.00	26	0.154
32	A	7	5	1.00	26	0.192
33	A	6	4	1.00	26	0.154
34	A	6	5	1.00	24	0.208
35	A	3	2	1.00	17	0.118
36	A	3	2	1.00	24	0.083

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
37	A	5	4	1.00	26	0.154
38	A	6	5	1.00	26	0.192
39	A	6	5	1.00	26	0.192
40	A	6	4	1.00	26	0.154
41	A	7	5	1.00	26	0.192
42	A	6	4	1.00	26	0.154
43	A	9	6	1.00	28	0.214
44	A	12	6	1.00	28	0.214
45	A	9	6	1.00	28	0.214
46	A	10	6	1.00	28	0.214
47	A	8	5	1.00	26	0.192
48	A	2	2	1.00	19	0.105
49	A	7	6	1.00	26	0.231
50	A	3	3	1.00	28	0.107
51	A	7	5	1.00	28	0.179
52	A	2	2	1.00	28	0.071
53	A	9	6	1.00	28	0.214
54	A	3	2	1.00	28	0.071
55	A	11	6	1.00	28	0.214
56	A	3	2	1.00	28	0.071
57	A	17	7	1.00	28	0.250
58	A	12	7	1.00	28	0.250
59	A	15	8	1.00	28	0.286
60	A	12	6	1.00	28	0.214
61	A	4	4	1.00	26	0.154
62	A	2	1	1.00	19	0.053

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
63	A	7	6	1.00	26	0.231
64	A	10	8	1.00	28	0.286
65	A	4	4	1.00	28	0.143
66	A	9	5	1.00	28	0.179
67	A	2	2	1.00	28	0.071
68	A	12	7	1.00	28	0.250
69	A	3	2	1.00	28	0.071
70	A	14	7	1.00	28	0.250
71	A	3	2	1.00	28	0.071
72	A	16	7	1.00	28	0.250
73	A	3	2	1.00	28	0.071
74	A	15	7	1.00	28	0.250
75	A	22	7	1.00	28	0.250
76	A	15	7	1.00	28	0.250
77	A	19	8	1.00	28	0.286
78	A	14	6	1.00	26	0.231
79	A	3	2	1.00	19	0.105
80	A	14	8	1.00	26	0.308
81	A	7	6	1.00	28	0.214
82	A	14	9	1.00	28	0.321
83	A	5	5	1.00	28	0.179
84	A	12	5	1.00	28	0.179
85	A	2	2	1.00	28	0.071
86	A	16	7	1.00	28	0.250
87	A	3	2	1.00	28	0.071
88	A	19	7	1.00	28	0.250

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
89	A	3	2	1.00	28	0.071
90	A	22	7	1.00	28	0.250
91	A	3	2	1.00	28	0.071
92	A	29	10	1.00	28	0.357
93	A	18	7	1.00	28	0.250
94	A	25	8	1.00	28	0.286
95	A	18	7	1.00	28	0.250
96	A	5	4	1.00	26	0.154
97	A	3	2	1.00	19	0.105
98	A	8	6	1.00	26	0.231
99	A	17	10	1.00	28	0.357
100	A	7	6	1.00	28	0.214
101	A	17	10	1.00	28	0.357
102	A	6	5	1.00	28	0.179
103	A	15	6	1.00	28	0.214
104	A	2	2	1.00	28	0.071
105	A	19	8	1.00	28	0.286
106	A	3	2	1.00	28	0.071
107	A	22	8	1.00	28	0.286
108	A	3	2	1.00	28	0.071
109	A	25	8	1.00	28	0.286
110	A	9	5	1.00	28	0.179
111	A	7	5	1.00	28	0.179
112	A	5	5	1.00	28	0.179
113	A	4	4	1.00	28	0.143
114	A	2	2	1.00	26	0.077

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
115	A	2	2	1.00	19	0.105
116	A	3	3	1.00	26	0.115
117	A	4	4	1.00	28	0.143
118	A	6	6	1.00	28	0.214
119	A	7	5	1.00	28	0.179
120	A	9	6	1.00	28	0.214
121	A	11	5	1.00	28	0.179
122	A	7	6	1.00	28	0.214
123	A	11	6	1.67	28	0.214
124	A	4	4	1.00	28	0.143
125	A	3	3	1.00	26	0.115
126	A	1	1	1.00	19	0.053
127	A	4	4	1.00	26	0.154
128	A	3	2	1.00	28	0.071
129	A	11	7	1.00	28	0.250
130	A	3	2	1.00	28	0.071
131	B	15	7	2.28	28	0.250
132	A	5	5	1.00	28	0.179
133	A	6	4	1.89	28	0.143
134	A	2	2	1.36	26	0.077
135	A	3	3	1.00	19	0.158
136	A	3	2	1.00	26	0.077
137	A	12	7	1.00	28	0.250
138	A	3	2	1.00	28	0.071
139	A	31	8	1.00	28	0.286
140	A	3	2	1.00	28	0.071

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
141	A	6	5	1.00	28	0.179
142	B	7	4	2.31	28	0.143
143	A	2	2	1.00	28	0.071
144	A	5	5	1.00	26	0.192
145	A	2	2	1.00	19	0.105
146	A	8	5	1.00	26	0.192
147	A	3	2	1.00	28	0.071
148	A	32	8	1.00	28	0.286
149	A	3	2	1.00	28	0.071
150	A	9	6	1.00	31	0.194
151	A	7	5	1.00	31	0.161
152	A	8	6	1.00	31	0.194
153	A	7	5	1.00	31	0.161
154	A	2	2	1.00	29	0.069
155	A	1	1	1.00	22	0.045
156	A	4	3	1.00	29	0.103
157	A	6	5	1.00	31	0.161
158	A	7	6	1.00	31	0.194
159	A	7	6	1.00	31	0.194
160	A	7	5	1.00	31	0.161
161	A	8	6	1.00	31	0.194
162	A	7	5	1.00	31	0.161
163	A	10	7	1.00	31	0.226
164	A	5	4	1.00	31	0.129
165	A	10	7	1.00	31	0.226
166	A	3	2	1.00	31	0.065

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
167	A	9	6	1.00	29	0.207
168	A	1	1	1.00	22	0.045
169	A	8	7	1.00	29	0.241
170	A	4	3	1.00	31	0.097
171	A	8	6	1.00	31	0.194
172	A	3	3	1.00	31	0.097
173	A	10	7	1.00	31	0.226
174	A	4	3	1.00	31	0.097
175	A	5	4	1.00	31	0.129
176	A	13	8	1.00	31	0.258
177	A	4	2	1.00	31	0.065
178	A	13	7	1.00	31	0.226
179	A	2	2	1.00	29	0.069
180	A	1	1	1.00	22	0.045
181	A	4	3	1.00	29	0.103
182	A	11	9	1.00	31	0.290
183	A	4	3	1.00	31	0.097
184	A	10	6	1.00	31	0.194
185	A	3	3	1.00	31	0.097
186	A	13	8	1.00	31	0.258
187	A	1	1	1.00	33	0.030
188	A	3	3	1.00	7	0.429
189	A	4	3	1.00	10	0.300
190	A	3	3	1.00	10	0.300
191	A	3	3	1.00	10	0.300
192	A	4	4	1.00	10	0.400

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
193	A	2	2	1.00	10	0.200
194	A	5	5	1.00	10	0.500
195	A	3	3	1.00	9	0.333
196	A	4	3	1.00	12	0.250
197	A	3	3	1.00	12	0.250
198	A	3	3	1.00	12	0.250
199	A	4	4	1.00	12	0.333
200	A	2	2	1.00	12	0.167
201	A	5	5	1.00	12	0.417
202	A	4	4	1.00	20	0.200
203	A	3	3	1.00	10	0.300
204	A	4	3	1.00	10	0.300
205	A	4	4	1.00	10	0.400
206	A	3	3	1.00	10	0.300
207	A	5	5	1.00	10	0.500
208	A	2	2	1.00	10	0.200
209	A	3	3	1.00	12	0.250
210	A	4	3	1.00	12	0.250
211	A	4	4	1.00	12	0.333
212	A	3	3	1.00	12	0.250
213	A	5	5	1.00	12	0.417
214	A	2	2	1.00	12	0.167
215	A	3	3	1.00	15	0.200
216	A	4	2	1.00	22	0.091
217	A	2	2	1.00	22	0.091
218	A	4	3	1.00	22	0.136

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
219	A	2	2	1.00	22	0.091
220	A	3	2	1.00	22	0.091
221	A	2	1	1.00	22	0.045
222	A	3	3	1.00	17	0.176
223	A	3	2	1.00	24	0.083
224	A	2	2	1.00	24	0.083
225	A	3	2	1.00	24	0.083
226	A	2	2	1.00	24	0.083
227	A	2	1	1.00	24	0.042
228	A	2	1	1.00	24	0.042
229	A	6	4	1.00	26	0.154
230	A	6	5	1.00	26	0.192
231	A	5	5	1.32	24	0.208
232	A	3	2	1.00	17	0.118
233	A	5	5	1.00	24	0.208
234	A	6	5	1.00	26	0.192
235	A	6	4	1.00	26	0.154
236	A	6	5	1.00	28	0.179
237	A	5	5	1.00	28	0.179
238	A	7	7	1.00	26	0.269
239	A	10	8	1.00	19	0.421
240	A	7	7	1.00	26	0.269
241	A	7	7	1.00	28	0.250
242	A	9	9	1.00	28	0.321
243	A	4	3	1.00	28	0.107
244	A	5	4	1.00	28	0.143

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
245	A	5	4	1.00	26	0.154
246	A	4	3	1.00	19	0.158
247	A	5	4	1.00	26	0.154
248	A	5	4	1.00	28	0.143
249	A	5	4	1.00	28	0.143
250	A	5	4	1.00	28	0.143
251	A	5	4	1.00	28	0.143
252	A	5	4	1.00	26	0.154
253	A	4	3	1.00	19	0.158
254	A	7	5	1.00	26	0.192
255	A	5	4	1.00	28	0.143
256	A	5	4	1.00	28	0.143
257	A	12	8	1.00	28	0.286
258	A	11	7	1.00	28	0.250
259	A	11	7	1.00	26	0.269
260	A	11	7	1.00	19	0.368
261	A	6	6	1.00	26	0.231
262	A	6	6	1.00	28	0.214
263	A	12	8	1.00	28	0.286
264	A	6	5	1.00	28	0.179
265	A	6	5	1.00	28	0.179
266	A	6	5	1.00	26	0.192
267	A	5	4	1.00	19	0.210
268	A	6	5	1.00	26	0.192
269	A	6	5	1.00	28	0.179
270	A	5	4	1.00	28	0.143

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
271	A	4	3	1.00	28	0.107
272	A	8	7	1.00	28	0.250
273	A	6	4	1.00	26	0.154
274	A	7	4	1.00	28	0.143
275	A	5	5	1.00	16	0.312
276	A	7	7	1.00	18	0.389
277	A	9	8	1.00	18	0.444
278	A	7	7	1.00	18	0.389
279	A	10	8	1.00	20	0.400
280	A	13	8	1.00	20	0.400
281	A	9	8	1.00	18	0.444
282	A	13	9	1.00	20	0.450
283	A	17	9	1.00	20	0.450
284	A	6	5	1.24	16	0.312
285	A	13	8	1.38	18	0.444
286	A	17	13	1.53	18	0.722
287	A	13	8	1.39	18	0.444
288	A	21	10	1.42	20	0.500
289	A	33	12	1.38	20	0.600
290	A	17	13	1.53	18	0.722
291	A	33	12	1.35	20	0.600
292	A	48	12	1.38	20	0.600
293	A	5	4	1.00	14	0.286
294	A	5	4	1.00	14	0.286

Chapter 3

Listing of integrals

3.1 $\int \sin^3(x)(a\cos(x)+b\sin(x))\,dx$

Optimal. Leaf size=36

$$\frac{1}{4}a\sin^4(x)+\frac{3bx}{8}-\frac{1}{4}b\sin^3(x)\cos(x)-\frac{3}{8}b\sin(x)\cos(x)$$

```
[Out] 3/8*b*x-3/8*b*cos(x)*sin(x)-1/4*b*cos(x)*sin(x)^3+1/4*a*sin(x)^4
```

Rubi [A] time = 0.05, antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {3089, 2564, 30, 2635, 8}

$$\frac{1}{4}a\sin^4(x)+\frac{3bx}{8}-\frac{1}{4}b\sin^3(x)\cos(x)-\frac{3}{8}b\sin(x)\cos(x)$$

Antiderivative was successfully verified.

```
[In] Int[Sin[x]^3*(a*Cos[x] + b*Sin[x]),x]
```

```
[Out] (3*b*x)/8 - (3*b*Cos[x]*Sin[x])/8 - (b*Cos[x]*Sin[x]^3)/4 + (a*Sin[x]^4)/4
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2564

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 3089

```
Int[sin[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_.), x_Symbol] :> Int[ExpandTrig[sin[c + d*x]^m*(a
*cos[c + d*x] + b*sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d}, x] && Inte
gerQ[m] && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\int \sin^3(x)(a\cos(x)+b\sin(x))\,dx &= \int \left(a\cos(x)\sin^3(x)+b\sin^4(x)\right)\,dx \\
&= a\int \cos(x)\sin^3(x)\,dx + b\int \sin^4(x)\,dx \\
&= -\frac{1}{4}b\cos(x)\sin^3(x) + a\,\text{Subst}\left(\int x^3\,dx, x, \sin(x)\right) + \frac{1}{4}(3b)\int \sin^2(x)\,dx \\
&= -\frac{3}{8}b\cos(x)\sin(x) - \frac{1}{4}b\cos(x)\sin^3(x) + \frac{1}{4}a\sin^4(x) + \frac{1}{8}(3b)\int 1\,dx \\
&= \frac{3bx}{8} - \frac{3}{8}b\cos(x)\sin(x) - \frac{1}{4}b\cos(x)\sin^3(x) + \frac{1}{4}a\sin^4(x)
\end{aligned}$$

Mathematica [A] time = 0.01, size = 34, normalized size = 0.94

$$\frac{1}{4}a\sin^4(x) + \frac{3bx}{8} - \frac{1}{4}b\sin(2x) + \frac{1}{32}b\sin(4x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[x]^3*(a*Cos[x] + b*Sin[x]),x]
```

```
[Out] (3*b*x)/8 + (a*Sin[x]^4)/4 - (b*Sin[2*x])/4 + (b*Sin[4*x])/32
```

fricas [A] time = 0.42, size = 36, normalized size = 1.00

$$\frac{1}{4}\,a\cos(x)^4-\frac{1}{2}\,a\cos(x)^2+\frac{3}{8}\,bx+\frac{1}{8}\left(2\,b\cos(x)^3-5\,b\cos(x)\right)\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^3*(a*cos(x)+b*sin(x)),x, algorithm="fricas")
```

```
[Out] 1/4*a*cos(x)^4 - 1/2*a*cos(x)^2 + 3/8*b*x + 1/8*(2*b*cos(x)^3 - 5*b*cos(x))
*sin(x)
```

giac [A] time = 2.73, size = 33, normalized size = 0.92

$$\frac{3}{8}\,bx+\frac{1}{32}\,a\cos\left(4\,x\right)-\frac{1}{8}\,a\cos\left(2\,x\right)+\frac{1}{32}\,b\sin\left(4\,x\right)-\frac{1}{4}\,b\sin\left(2\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^3*(a*cos(x)+b*sin(x)),x, algorithm="giac")
```

```
[Out] 3/8*b*x + 1/32*a*cos(4*x) - 1/8*a*cos(2*x) + 1/32*b*sin(4*x) - 1/4*b*sin(2*
x)
```

maple [A] time = 0.84, size = 28, normalized size = 0.78

$$b\left(-\frac{\left(\sin^3(x)+\frac{3\sin(x)}{2}\right)\cos(x)}{4}+\frac{3x}{8}\right)+\frac{a\left(\sin^4(x)\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)^3*(a*cos(x)+b*sin(x)),x)
```

```
[Out] b*(-1/4*(sin(x)^3+3/2*sin(x))*cos(x)+3/8*x)+1/4*a*sin(x)^4
```

maxima [A] time = 0.31, size = 25, normalized size = 0.69

$$\frac{1}{4}\,a\sin(x)^4+\frac{1}{32}\,b(12\,x+\sin\left(4\,x\right)-8\,\sin\left(2\,x\right))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^3*(a*cos(x)+b*sin(x)),x, algorithm="maxima")
```

```
[Out] 1/4*a*sin(x)^4 + 1/32*b*(12*x + sin(4*x) - 8*sin(2*x))
```

mupad [B] time = 0.53, size = 35, normalized size = 0.97

$$\frac{a\cos(x)^4}{4}+\frac{b\,\sin(x)\cos(x)^3}{4}-\frac{a\cos(x)^2}{2}-\frac{5\,b\,\sin(x)\,\cos(x)}{8}+\frac{3\,b\,x}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)^3*(a*cos(x) + b*sin(x)),x)
```

```
[Out] (3*b*x)/8 - (a*cos(x)^2)/2 + (a*cos(x)^4)/4 - (5*b*cos(x)*sin(x))/8 + (b*co
s(x)^3*sin(x))/4
```

sympy [B] time = 0.53, size = 75, normalized size = 2.08

$$\frac{a\sin^4(x)}{4}+\frac{3bx\sin^4(x)}{8}+\frac{3bx\sin^2(x)\cos^2(x)}{4}+\frac{3bx\cos^4(x)}{8}-\frac{5b\sin^3(x)\cos(x)}{8}-\frac{3b\sin(x)\cos^3(x)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)**3*(a*cos(x)+b*sin(x)),x)
```

```
[Out] a*sin(x)**4/4 + 3*b*x*sin(x)**4/8 + 3*b*x*sin(x)**2*cos(x)**2/4 + 3*b*x*cos
(x)**4/8 - 5*b*sin(x)**3*cos(x)/8 - 3*b*sin(x)*cos(x)**3/8
```

3.2 $\int \sin^2(x)(a\cos(x) + b\sin(x))\,dx$

Optimal. Leaf size=24

$$\frac{1}{3}a\sin^3(x) + \frac{1}{3}b\cos^3(x) - b\cos(x)$$

```
[Out] -b*cos(x)+1/3*b*cos(x)^3+1/3*a*sin(x)^3
```

Rubi [A] time = 0.04, antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3089, 2564, 30, 2633}

$$\frac{1}{3}a\sin^3(x) + \frac{1}{3}b\cos^3(x) - b\cos(x)$$

Antiderivative was successfully verified.

```
[In] Int[Sin[x]^2*(a*Cos[x] + b*Sin[x]),x]
```

```
[Out] -(b*Cos[x]) + (b*Cos[x]^3)/3 + (a*Sin[x]^3)/3
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2564

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 3089

```
Int[sin[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_.), x_Symbol] :> Int[ExpandTrig[sin[c + d*x]^m*(a
*cos[c + d*x] + b*sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d}, x] && Inte
gerQ[m] && IGtQ[n, 0]
```

Rubi steps

$$
\begin{aligned}
\int \sin^2(x)(a\cos(x)+b\sin(x))\,dx &= \int \left(a\cos(x)\sin^2(x)+b\sin^3(x)\right)\,dx \\
&= a\int \cos(x)\sin^2(x)\,dx + b\int \sin^3(x)\,dx \\
&= a\,\text{Subst}\left(\int x^2\,dx, x, \sin(x)\right) - b\,\text{Subst}\left(\int \left(1-x^2\right)\,dx, x, \cos(x)\right) \\
&= -b\cos(x)+\frac{1}{3}b\cos^3(x)+\frac{1}{3}a\sin^3(x)
\end{aligned}
$$

Mathematica [A] time = 0.00, size = 26, normalized size = 1.08

$$\frac{1}{3}a\sin^3(x)-\frac{3}{4}b\cos(x)+\frac{1}{12}b\cos(3x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[x]^2*(a*Cos[x] + b*Sin[x]),x]
[Out] (-3*b*Cos[x])/4 + (b*Cos[3*x])/12 + (a*Sin[x]^3)/3
```

fricas [A] time = 0.45, size = 27, normalized size = 1.12

$$\frac{1}{3}\,b\cos(x)^3-b\cos(x)-\frac{1}{3}\left(a\cos(x)^2-a\right)\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^2*(a*cos(x)+b*sin(x)),x, algorithm="fricas")
[Out] 1/3*b*cos(x)^3 - b*cos(x) - 1/3*(a*cos(x)^2 - a)*sin(x)
```

giac [A] time = 2.96, size = 25, normalized size = 1.04

$$\frac{1}{12}\,b\cos\left(3\,x\right)-\frac{3}{4}\,b\cos(x)-\frac{1}{12}\,a\sin\left(3\,x\right)+\frac{1}{4}\,a\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^2*(a*cos(x)+b*sin(x)),x, algorithm="giac")
[Out] 1/12*b*cos(3*x) - 3/4*b*cos(x) - 1/12*a*sin(3*x) + 1/4*a*sin(x)
```

maple [A] time = 0.82, size = 20, normalized size = 0.83

$$-\frac{b\left(2+\sin^2(x)\right)\cos(x)}{3}+\frac{a\left(\sin^3(x)\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)^2*(a*cos(x)+b*sin(x)),x)
```

```
[Out] -1/3*b*(2+sin(x)^2)*cos(x)+1/3*a*sin(x)^3
```

maxima [A] time = 0.32, size = 20, normalized size = 0.83

$$\frac{1}{3}\,a\sin(x)^3+\frac{1}{3}\left(\cos(x)^3-3\,\cos(x)\right)b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^2*(a*cos(x)+b*sin(x)),x, algorithm="maxima")
```

```
[Out] 1/3*a*sin(x)^3 + 1/3*(cos(x)^3 - 3*cos(x))*b
```

mupad [B] time = 0.45, size = 32, normalized size = 1.33

$$-\frac{4\left(-2\,a\tan\left(\frac{x}{2}\right)^3+3\,b\tan\left(\frac{x}{2}\right)^2+b\right)}{3\left(\tan\left(\frac{x}{2}\right)^2+1\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)^2*(a*cos(x) + b*sin(x)),x)
```

```
[Out] -(4*(b - 2*a*tan(x/2)^3 + 3*b*tan(x/2)^2))/(3*(tan(x/2)^2 + 1)^3)
```

sympy [A] time = 0.28, size = 27, normalized size = 1.12

$$\frac{a\sin^3(x)}{3}-b\sin^2(x)\cos(x)-\frac{2b\cos^3(x)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)**2*(a*cos(x)+b*sin(x)),x)
```

```
[Out] a*sin(x)**3/3 - b*sin(x)**2*cos(x) - 2*b*cos(x)**3/3
```

3.3 $\int \sin(x)(a\cos(x)+b\sin(x))\,dx$

Optimal. Leaf size=25

$$\frac{1}{2}a\sin^2(x)+\frac{bx}{2}-\frac{1}{2}b\sin(x)\cos(x)$$

```
[Out] 1/2*b*x-1/2*b*cos(x)*sin(x)+1/2*a*sin(x)^2
```

Rubi [A] time = 0.03, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {3089, 2564, 30, 2635, 8}

$$\frac{1}{2}a\sin^2(x)+\frac{bx}{2}-\frac{1}{2}b\sin(x)\cos(x)$$

Antiderivative was successfully verified.

```
[In] Int[Sin[x]*(a*Cos[x] + b*Sin[x]),x]
```

```
[Out] (b*x)/2 - (b*Cos[x]*Sin[x])/2 + (a*Sin[x]^2)/2
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2564

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 3089

```
Int[sin[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_.), x_Symbol] :> Int[ExpandTrig[sin[c + d*x]^m*(a
*cos[c + d*x] + b*sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d}, x] && Inte
gerQ[m] && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}\int \sin(x)(a\cos(x)+b\sin(x))\,dx &= \int \left(a\cos(x)\sin(x)+b\sin^2(x)\right)\,dx\\ &= a\int \cos(x)\sin(x)\,dx+b\int \sin^2(x)\,dx\\ &= -\frac{1}{2}b\cos(x)\sin(x)+a\,\text{Subst}\left(\int x\,dx,x,\sin(x)\right)+\frac{1}{2}b\int 1\,dx\\ &= \frac{bx}{2}-\frac{1}{2}b\cos(x)\sin(x)+\frac{1}{2}a\sin^2(x)\end{aligned}$$

Mathematica [A] time = 0.00, size = 25, normalized size = 1.00

$$-\frac{1}{2}a\cos^2(x)+\frac{bx}{2}-\frac{1}{4}b\sin(2x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[x]*(a*Cos[x] + b*Sin[x]),x]
```

```
[Out] (b*x)/2 - (a*Cos[x]^2)/2 - (b*Sin[2*x])/4
```

fricas [A] time = 0.40, size = 19, normalized size = 0.76

$$-\frac{1}{2}\,a\cos(x)^2-\frac{1}{2}\,b\cos(x)\sin(x)+\frac{1}{2}\,bx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)*(a*cos(x)+b*sin(x)),x, algorithm="fricas")
```

```
[Out] -1/2*a*cos(x)^2 - 1/2*b*cos(x)*sin(x) + 1/2*b*x
```

giac [A] time = 0.17, size = 19, normalized size = 0.76

$$\frac{1}{2}\,bx-\frac{1}{4}\,a\cos\left(2\,x\right)-\frac{1}{4}\,b\sin\left(2\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)*(a*cos(x)+b*sin(x)),x, algorithm="giac")
[Out] 1/2*b*x - 1/4*a*cos(2*x) - 1/4*b*sin(2*x)
```

maple [A] time = 0.16, size = 21, normalized size = 0.84

$$b\left(-\frac{\cos(x)\sin(x)}{2}+\frac{x}{2}\right)-\frac{a\left(\cos^2(x)\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)*(a*cos(x)+b*sin(x)),x)
[Out] b*(-1/2*cos(x)*sin(x)+1/2*x)-1/2*a*cos(x)^2
```

maxima [A] time = 0.32, size = 21, normalized size = 0.84

$$-\frac{1}{2}\,a\cos(x)^2+\frac{1}{4}\,b(2\,x-\sin\left(2\,x\right))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)*(a*cos(x)+b*sin(x)),x, algorithm="maxima")
[Out] -1/2*a*cos(x)^2 + 1/4*b*(2*x - sin(2*x))
```

mupad [B] time = 0.41, size = 19, normalized size = 0.76

$$\frac{a\sin(x)^2}{2}-\frac{b\,\cos(x)\,\sin(x)}{2}+\frac{b\,x}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)*(a*cos(x) + b*sin(x)),x)
[Out] (a*sin(x)^2)/2 + (b*x)/2 - (b*cos(x)*sin(x))/2
```

sympy [A] time = 0.16, size = 37, normalized size = 1.48

$$-\frac{a\cos^2\left(x\right)}{2}+\frac{bx\sin^2\left(x\right)}{2}+\frac{bx\cos^2\left(x\right)}{2}-\frac{b\sin\left(x\right)\cos\left(x\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)*(a*cos(x)+b*sin(x)),x)
[Out] -a*cos(x)**2/2 + b*x*sin(x)**2/2 + b*x*cos(x)**2/2 - b*sin(x)*cos(x)/2
```

3.4 $\int (a\cos(x)+b\sin(x))\,dx$

Optimal. Leaf size=10

$$a\sin(x)-b\cos(x)$$

```
[Out] -b*cos(x)+a*sin(x)
```

Rubi [A] time = 0.01, antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {2637, 2638}

$$a\sin(x)-b\cos(x)$$

Antiderivative was successfully verified.

```
[In] Int[a*Cos[x] + b*Sin[x],x]
[Out] -(b*Cos[x]) + a*Sin[x]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int (a\cos(x)+b\sin(x))\,dx &= a\int \cos(x)\,dx+b\int \sin(x)\,dx\\ &= -b\cos(x)+a\sin(x)\end{aligned}$$

Mathematica [A] time = 0.00, size = 10, normalized size = 1.00

$$a\sin(x)-b\cos(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[a*Cos[x] + b*Sin[x],x]
[Out] -(b*Cos[x]) + a*Sin[x]
```

fricas [A] time = 0.51, size = 10, normalized size = 1.00

$$-b\cos(x) + a\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a*cos(x)+b*sin(x),x, algorithm="fricas")
[Out] -b*cos(x) + a*sin(x)
```

giac [A] time = 1.83, size = 10, normalized size = 1.00

$$-b\cos(x) + a\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a*cos(x)+b*sin(x),x, algorithm="giac")
[Out] -b*cos(x) + a*sin(x)
```

maple [A] time = 0.06, size = 11, normalized size = 1.10

$$-b\cos(x) + a\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(a*cos(x)+b*sin(x),x)
[Out] -b*cos(x)+a*sin(x)
```

maxima [A] time = 0.32, size = 10, normalized size = 1.00

$$-b\cos(x) + a\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a*cos(x)+b*sin(x),x, algorithm="maxima")
[Out] -b*cos(x) + a*sin(x)
```

mupad [B] time = 0.40, size = 10, normalized size = 1.00

$$a\ \sin(x) - b\ \cos(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(a*cos(x) + b*sin(x),x)
[Out] a*sin(x) - b*cos(x)
```

sympy [A] time = 0.05, size = 8, normalized size = 0.80

$$a \sin(x) - b \cos(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a*cos(x)+b*sin(x),x)
```

```
[Out] a*sin(x) - b*cos(x)
```

3.5 $\int \csc(x)(a\cos(x)+b\sin(x))\,dx$

Optimal. Leaf size=9

$$a\log(\sin(x))+bx$$

```
[Out] b*x+a*ln(sin(x))
```

Rubi [A] time = 0.02, antiderivative size = 9, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3085, 3475}

$$a\log(\sin(x))+bx$$

Antiderivative was successfully verified.

```
[In] Int[Csc[x]*(a*Cos[x] + b*Sin[x]),x]
```

```
[Out] b*x + a*Log[Sin[x]]
```

Rule 3085

```
Int[sin[(c_.) + (d_.)*(x_)]^(m_)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin
[(c_.) + (d_.)*(x_)])^(n_.), x_Symbol] :> Int[(b + a*Cot[c + d*x])^n, x] /;
 FreeQ[{a, b, c, d}, x] && EqQ[m + n, 0] && IntegerQ[n] && NeQ[a^2 + b^2, 0
]
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \csc(x)(a\cos(x)+b\sin(x))\,dx &= \int (b+a\cot(x))\,dx\\ &= bx+a\int \cot(x)\,dx\\ &= bx+a\log(\sin(x))\end{aligned}$$

Mathematica [A] time = 0.01, size = 9, normalized size = 1.00

$$a\log(\sin(x))+bx$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[x]*(a*Cos[x] + b*Sin[x]),x]
[Out] b*x + a*Log[Sin[x]]
```

fricas [A] time = 0.50, size = 11, normalized size = 1.22

$$bx + a\log\left(\frac{1}{2}\sin(x)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)*(a*cos(x)+b*sin(x)),x, algorithm="fricas")
[Out] b*x + a*log(1/2*sin(x))
```

giac [B] time = 0.21, size = 24, normalized size = 2.67

$$bx - a\log\left(\tan\left(\frac{1}{2}x\right)^2 + 1\right) + a\log\left(\left|\tan\left(\frac{1}{2}x\right)\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)*(a*cos(x)+b*sin(x)),x, algorithm="giac")
[Out] b*x - a*log(tan(1/2*x)^2 + 1) + a*log(abs(tan(1/2*x)))
```

maple [A] time = 0.45, size = 10, normalized size = 1.11

$$bx + a\ln(\sin(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(x)*(a*cos(x)+b*sin(x)),x)
[Out] b*x+a*ln(sin(x))
```

maxima [A] time = 0.31, size = 9, normalized size = 1.00

$$bx + a\log(\sin(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)*(a*cos(x)+b*sin(x)),x, algorithm="maxima")
[Out] b*x + a*log(sin(x))
```

mupad [B] time = 0.54, size = 54, normalized size = 6.00

$$a\ln\left(\tan\left(\frac{x}{2}\right)\right) - a\ln\left(\tan\left(\frac{x}{2}\right) - \mathrm{i}\right) - a\ln\left(\tan\left(\frac{x}{2}\right) + 1\mathrm{i}\right) - b\ln\left(\tan\left(\frac{x}{2}\right) - \mathrm{i}\right)1\mathrm{i} + b\ln\left(\tan\left(\frac{x}{2}\right) + 1\mathrm{i}\right)1\mathrm{i}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*cos(x) + b*sin(x))/sin(x),x)
```

```
[Out] a*log(tan(x/2)) - a*log(tan(x/2) - 1i) - a*log(tan(x/2) + 1i) - b*log(tan(x
/2) - 1i)*1i + b*log(tan(x/2) + 1i)*1i
```

sympy [A] time = 1.14, size = 8, normalized size = 0.89

$$a\log\left(\sin\left(x\right)\right) + bx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)*(a*cos(x)+b*sin(x)),x)
```

```
[Out] a*log(sin(x)) + b*x
```

3.6 $\int \csc^2(x)(a\cos(x)+b\sin(x))\,dx$

Optimal. Leaf size=12

$$-a\csc(x)-b\tanh^{-1}(\cos(x))$$

```
[Out] -b*arctanh(cos(x))-a*csc(x)
```

Rubi [A] time = 0.03, antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3089, 3770, 2606, 8}

$$-a\csc(x)-b\tanh^{-1}(\cos(x))$$

Antiderivative was successfully verified.

```
[In] Int[Csc[x]^2*(a*Cos[x] + b*Sin[x]),x]
```

```
[Out] -(b*ArcTanh[Cos[x]]) - a*Csc[x]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 3089

```
Int[sin[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_.), x_Symbol] :> Int[ExpandTrig[sin[c + d*x]^m*(a
*cos[c + d*x] + b*sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d}, x] && Inte
gerQ[m] && IGtQ[n, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \csc^2(x)(a\cos(x)+b\sin(x))\,dx &= \int (b\csc(x)+a\cot(x)\csc(x))\,dx\\ &= a\int \cot(x)\csc(x)\,dx + b\int \csc(x)\,dx\\ &= -b\tanh^{-1}(\cos(x)) - a\,\text{Subst}(\int 1\,dx, x, \csc(x))\\ &= -b\tanh^{-1}(\cos(x)) - a\csc(x)\end{aligned}$$

Mathematica [B] time = 0.01, size = 25, normalized size = 2.08

$$-a\csc(x)+b\log\left(\sin\left(\frac{x}{2}\right)\right)-b\log\left(\cos\left(\frac{x}{2}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[x]^2*(a*Cos[x] + b*Sin[x]),x]
[Out] -(a*Csc[x]) - b*Log[Cos[x/2]] + b*Log[Sin[x/2]]
```

fricas [B] time = 0.46, size = 33, normalized size = 2.75

$$-\frac{b\log\left(\frac{1}{2}\cos(x)+\frac{1}{2}\right)\sin(x)-b\log\left(-\frac{1}{2}\cos(x)+\frac{1}{2}\right)\sin(x)+2\,a}{2\sin(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)^2*(a*cos(x)+b*sin(x)),x, algorithm="fricas")
[Out] -1/2*(b*log(1/2*cos(x) + 1/2)*sin(x) - b*log(-1/2*cos(x) + 1/2)*sin(x) + 2*
a)/sin(x)
```

giac [B] time = 0.20, size = 33, normalized size = 2.75

$$b\log\left(\left|\tan\left(\frac{1}{2}\,x\right)\right|\right)-\frac{1}{2}\,a\tan\left(\frac{1}{2}\,x\right)-\frac{2\,b\tan\left(\frac{1}{2}\,x\right)+a}{2\tan\left(\frac{1}{2}\,x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)^2*(a*cos(x)+b*sin(x)),x, algorithm="giac")
[Out] b*log(abs(tan(1/2*x))) - 1/2*a*tan(1/2*x) - 1/2*(2*b*tan(1/2*x) + a)/tan(1/
2*x)
```

maple [A] time = 0.50, size = 19, normalized size = 1.58

$$-\frac{a}{\sin(x)}+b\ln\left(-\cot(x)+\csc(x)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(x)^2*(a*cos(x)+b*sin(x)),x)
```

```
[Out] -a/sin(x)+b*ln(-cot(x)+csc(x))
```

maxima [A] time = 0.32, size = 24, normalized size = 2.00

$$-\frac{1}{2}\,b\Big(\log\left(\cos(x)+1\right)-\log\left(\cos(x)-1\right)\Big)-\frac{a}{\sin(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)^2*(a*cos(x)+b*sin(x)),x, algorithm="maxima")
```

```
[Out] -1/2*b*(log(cos(x) + 1) - log(cos(x) - 1)) - a/sin(x)
```

mupad [B] time = 0.40, size = 24, normalized size = 2.00

$$b\,\ln\left(\tan\left(\frac{x}{2}\right)\right)-\frac{a}{2\tan\left(\frac{x}{2}\right)}-\frac{a\tan\left(\frac{x}{2}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*cos(x) + b*sin(x))/sin(x)^2,x)
```

```
[Out] b*log(tan(x/2)) - a/(2*tan(x/2)) - (a*tan(x/2))/2
```

sympy [A] time = 1.83, size = 24, normalized size = 2.00

$$-\frac{a}{\sin\left(x\right)}+\frac{b\log\left(\cos\left(x\right)-1\right)}{2}-\frac{b\log\left(\cos\left(x\right)+1\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)**2*(a*cos(x)+b*sin(x)),x)
```

```
[Out] -a/sin(x) + b*log(cos(x) - 1)/2 - b*log(cos(x) + 1)/2
```

3.7 $\int \csc^3(x)(a\cos(x)+b\sin(x))\,dx$

Optimal. Leaf size=15

$$-\frac{1}{2}a\csc^2(x)-b\cot(x)$$

```
[Out] -b*cot(x)-1/2*a*csc(x)^2
```

Rubi [A] time = 0.04, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {3089, 3767, 8, 2606, 30}

$$-\frac{1}{2}a\csc^2(x)-b\cot(x)$$

Antiderivative was successfully verified.

```
[In] Int[Csc[x]^3*(a*Cos[x] + b*Sin[x]),x]
```

```
[Out] -(b*Cot[x]) - (a*Csc[x]^2)/2
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 3089

```
Int[sin[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_.), x_Symbol] :> Int[ExpandTrig[sin[c + d*x]^m*(a
*cos[c + d*x] + b*sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d}, x] && Inte
gerQ[m] && IGtQ[n, 0]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$
\begin{aligned}
\int \csc^3(x)(a\cos(x)+b\sin(x))\,dx &= \int \left(b\csc^2(x)+a\cot(x)\csc^2(x)\right)\,dx\\
&= a\int \cot(x)\csc^2(x)\,dx + b\int \csc^2(x)\,dx\\
&= -(a\,\text{Subst}(\int x\,dx, x, \csc(x))) - b\,\text{Subst}(\int 1\,dx, x, \cot(x))\\
&= -b\cot(x) - \frac{1}{2}a\csc^2(x)
\end{aligned}
$$

Mathematica [A] time = 0.01, size = 15, normalized size = 1.00

$$-\frac{1}{2}a\csc^2(x) - b\cot(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[x]^3*(a*Cos[x] + b*Sin[x]),x]
```

```
[Out] -(b*Cot[x]) - (a*Csc[x]^2)/2
```

fricas [A] time = 0.43, size = 19, normalized size = 1.27

$$\frac{2\,b\cos(x)\sin(x)+a}{2\left(\cos(x)^2-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)^3*(a*cos(x)+b*sin(x)),x, algorithm="fricas")
```

```
[Out] 1/2*(2*b*cos(x)*sin(x) + a)/(cos(x)^2 - 1)
```

giac [A] time = 0.23, size = 13, normalized size = 0.87

$$-\frac{2\,b\tan(x)+a}{2\,\tan(x)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)^3*(a*cos(x)+b*sin(x)),x, algorithm="giac")
[Out] -1/2*(2*b*tan(x) + a)/tan(x)^2
```

maple [A] time = 0.82, size = 14, normalized size = 0.93

$$-\frac{a}{2\sin(x)^2} - b\cot(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(x)^3*(a*cos(x)+b*sin(x)),x)
[Out] -1/2*a/sin(x)^2-b*cot(x)
```

maxima [A] time = 0.32, size = 15, normalized size = 1.00

$$-\frac{b}{\tan(x)} - \frac{a}{2\,\sin(x)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)^3*(a*cos(x)+b*sin(x)),x, algorithm="maxima")
[Out] -b/tan(x) - 1/2*a/sin(x)^2
```

mupad [B] time = 0.41, size = 14, normalized size = 0.93

$$-\frac{a + b\,\sin(2\,x)}{2\sin(x)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*cos(x) + b*sin(x))/sin(x)^3,x)
[Out] -(a + b*sin(2*x))/(2*sin(x)^2)
```

sympy [A] time = 3.88, size = 17, normalized size = 1.13

$$-\frac{a}{2\sin^2(x)} - \frac{b\cos(x)}{\sin(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)**3*(a*cos(x)+b*sin(x)),x)
[Out] -a/(2*sin(x)**2) - b*cos(x)/sin(x)
```

3.8 $\int \frac{\sin^3(x)}{a\cos(x)+b\sin(x)}\,dx$

Optimal. Leaf size=91

$$\frac{a^2bx}{\left(a^2+b^2\right)^2}+\frac{bx}{2\left(a^2+b^2\right)}-\frac{a\sin^2(x)}{2\left(a^2+b^2\right)}-\frac{b\sin(x)\cos(x)}{2\left(a^2+b^2\right)}-\frac{a^3\log(a\cos(x)+b\sin(x))}{\left(a^2+b^2\right)^2}$$

```
[Out] a^2*b*x/(a^2+b^2)^2+1/2*b*x/(a^2+b^2)-a^3*ln(a*cos(x)+b*sin(x))/(a^2+b^2)^2
-1/2*b*cos(x)*sin(x)/(a^2+b^2)-1/2*a*sin(x)^2/(a^2+b^2)
```

Rubi [A] time = 0.11, antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312, Rules used = {3099, 3097, 3133, 2635, 8}

$$\frac{a^2bx}{\left(a^2+b^2\right)^2}+\frac{bx}{2\left(a^2+b^2\right)}-\frac{a\sin^2(x)}{2\left(a^2+b^2\right)}-\frac{b\sin(x)\cos(x)}{2\left(a^2+b^2\right)}-\frac{a^3\log(a\cos(x)+b\sin(x))}{\left(a^2+b^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[x]^3/(a*Cos[x] + b*Sin[x]),x]
```

```
[Out] (a^2*b*x)/(a^2 + b^2)^2 + (b*x)/(2*(a^2 + b^2)) - (a^3*Log[a*Cos[x] + b*Sin
[x]])/(a^2 + b^2)^2 - (b*Cos[x]*Sin[x])/(2*(a^2 + b^2)) - (a*Sin[x]^2)/(2*(
a^2 + b^2))
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 3097

```
Int[sin[(c_.) + (d_.)*(x_)]/(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.
) + (d_.)*(x_)]), x_Symbol] :> Simp[(b*x)/(a^2 + b^2), x] - Dist[a/(a^2 + b
^2), Int[(b*Cos[c + d*x] - a*Sin[c + d*x])/(a*Cos[c + d*x] + b*Sin[c + d*x]
), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

Rule 3099

```
Int[sin[(c_.) + (d_.)*(x_)]^(m_)/(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin
[(c_.) + (d_.)*(x_)]), x_Symbol] :> -Simp[(a*Sin[c + d*x]^(m - 1))/(d*(a^2
+ b^2)*(m - 1)), x] + (Dist[a^2/(a^2 + b^2), Int[Sin[c + d*x]^(m - 2)/(a*Co
s[c + d*x] + b*Sin[c + d*x]), x], x] + Dist[b/(a^2 + b^2), Int[Sin[c + d*x]
^(m - 1), x], x]) /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0] && GtQ[m,
1]
```

Rule 3133

```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])
/((a_.) + cos[(d_.) + (e_.)*(x_)]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)]), x
_Symbol] :> Simp[((b*B + c*C)*x)/(b^2 + c^2), x] + Simp[((c*B - b*C)*Log[a
+ b*Cos[d + e*x] + c*Sin[d + e*x]])/(e*(b^2 + c^2)), x] /; FreeQ[{a, b, c,
d, e, A, B, C}, x] && NeQ[b^2 + c^2, 0] && EqQ[A*(b^2 + c^2) - a*(b*B + c*C
), 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sin^3(x)}{a\cos(x)+b\sin(x)}\,dx &= -\frac{a\sin^2(x)}{2\left(a^2+b^2\right)}+\frac{a^2\int \frac{\sin(x)}{a\cos(x)+b\sin(x)}\,dx}{a^2+b^2}+\frac{b\int \sin^2(x)\,dx}{a^2+b^2}\\
&= \frac{a^2bx}{\left(a^2+b^2\right)^2}-\frac{b\cos(x)\sin(x)}{2\left(a^2+b^2\right)}-\frac{a\sin^2(x)}{2\left(a^2+b^2\right)}-\frac{a^3\int \frac{b\cos(x)-a\sin(x)}{a\cos(x)+b\sin(x)}\,dx}{\left(a^2+b^2\right)^2}+\frac{b\int 1\,dx}{2\left(a^2+b^2\right)}\\
&= \frac{a^2bx}{\left(a^2+b^2\right)^2}+\frac{bx}{2\left(a^2+b^2\right)}-\frac{a^3\log(a\cos(x)+b\sin(x))}{\left(a^2+b^2\right)^2}-\frac{b\cos(x)\sin(x)}{2\left(a^2+b^2\right)}-\frac{a\sin^2(x)}{2\left(a^2+b^2\right)}
\end{aligned}
$$

Mathematica [C] time = 0.20, size = 94, normalized size = 1.03

$$
\frac{-2a^3\log\left((a\cos(x)+b\sin(x))^2\right)-4ia^3x+4ia^3\tan^{-1}(\tan(x))+a\left(a^2+b^2\right)\cos(2x)+6a^2bx-a^2b\sin(2x)+2b^3x}{4\left(a^2+b^2\right)^2}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[x]^3/(a*Cos[x] + b*Sin[x]),x]
```

```
[Out] ((-4*I)*a^3*x + 6*a^2*b*x + 2*b^3*x + (4*I)*a^3*ArcTan[Tan[x]] + a*(a^2 + b
^2)*Cos[2*x] - 2*a^3*Log[(a*Cos[x] + b*Sin[x])^2] - a^2*b*Sin[2*x] - b^3*Si
n[2*x])/(4*(a^2 + b^2)^2)
```

fricas [A] time = 0.48, size = 93, normalized size = 1.02

$$-\frac{a^3\log\left(2\,ab\cos(x)\sin(x)+\left(a^2-b^2\right)\cos(x)^2+b^2\right)-\left(a^3+ab^2\right)\cos(x)^2+\left(a^2b+b^3\right)\cos(x)\sin(x)-\left(3\,a^2b+b\right.}{2\left(a^4+2\,a^2b^2+b^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^3/(a*cos(x)+b*sin(x)),x, algorithm="fricas")
```

```
[Out] -1/2*(a^3*log(2*a*b*cos(x)*sin(x) + (a^2 - b^2)*cos(x)^2 + b^2) - (a^3 + a*
b^2)*cos(x)^2 + (a^2*b + b^3)*cos(x)*sin(x) - (3*a^2*b + b^3)*x)/(a^4 + 2*a
^2*b^2 + b^4)
```

giac [A] time = 0.21, size = 148, normalized size = 1.63

$$-\frac{a^3b\log\left(|b\tan(x)+a|\right)}{a^4b+2\,a^2b^3+b^5}+\frac{a^3\log\left(\tan(x)^2+1\right)}{2\left(a^4+2\,a^2b^2+b^4\right)}+\frac{\left(3\,a^2b+b^3\right)x}{2\left(a^4+2\,a^2b^2+b^4\right)}-\frac{a^3\tan(x)^2+a^2b\tan(x)+b^3\tan(x)-ab^2}{2\left(a^4+2\,a^2b^2+b^4\right)\left(\tan(x)^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^3/(a*cos(x)+b*sin(x)),x, algorithm="giac")
```

```
[Out] -a^3*b*log(abs(b*tan(x) + a))/(a^4*b + 2*a^2*b^3 + b^5) + 1/2*a^3*log(tan(x
)^2 + 1)/(a^4 + 2*a^2*b^2 + b^4) + 1/2*(3*a^2*b + b^3)*x/(a^4 + 2*a^2*b^2 +
 b^4) - 1/2*(a^3*tan(x)^2 + a^2*b*tan(x) + b^3*tan(x) - a*b^2)/((a^4 + 2*a^
2*b^2 + b^4)*(tan(x)^2 + 1))
```

maple [B] time = 0.47, size = 173, normalized size = 1.90

$$-\frac{a^3\ln\left(a+b\tan(x)\right)}{\left(a^2+b^2\right)^2}-\frac{\tan(x)a^2b}{2\left(a^2+b^2\right)^2\left(1+\tan^2(x)\right)}-\frac{\tan(x)b^3}{2\left(a^2+b^2\right)^2\left(1+\tan^2(x)\right)}+\frac{a^3}{2\left(a^2+b^2\right)^2\left(1+\tan^2(x)\right)}+\frac{}{2\left(a^2\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)^3/(a*cos(x)+b*sin(x)),x)
```

```
[Out] -a^3/(a^2+b^2)^2*ln(a+b*tan(x))-1/2/(a^2+b^2)^2/(1+tan(x)^2)*tan(x)*a^2*b-1
/2/(a^2+b^2)^2/(1+tan(x)^2)*tan(x)*b^3+1/2/(a^2+b^2)^2/(1+tan(x)^2)*a^3+1/2
/(a^2+b^2)^2/(1+tan(x)^2)*b^2*a+1/2/(a^2+b^2)^2*a^3*ln(1+tan(x)^2)+3/2/(a^2
+b^2)^2*arctan(tan(x))*a^2*b+1/2/(a^2+b^2)^2*arctan(tan(x))*b^3
```

maxima [B] time = 0.43, size = 209, normalized size = 2.30

$$-\frac{a^3\log\left(-a-\frac{2\,b\sin(x)}{\cos(x)+1}+\frac{a\sin(x)^2}{(\cos(x)+1)^2}\right)}{a^4+2\,a^2b^2+b^4}+\frac{a^3\log\left(\frac{\sin(x)^2}{(\cos(x)+1)^2}+1\right)}{a^4+2\,a^2b^2+b^4}+\frac{\left(3\,a^2b+b^3\right)\arctan\left(\frac{\sin(x)}{\cos(x)+1}\right)}{a^4+2\,a^2b^2+b^4}-\frac{\frac{b\sin(x)}{\cos(x)+1}+\frac{2\,a\sin}{(\cos(x)}}{a^2+b^2+\frac{2\left(a^2+b^2\right)\sin}{(\cos(x)+}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^3/(a*cos(x)+b*sin(x)),x, algorithm="maxima")
```

```
[Out] -a^3*log(-a - 2*b*sin(x)/(cos(x) + 1) + a*sin(x)^2/(cos(x) + 1)^2)/(a^4 + 2
*a^2*b^2 + b^4) + a^3*log(sin(x)^2/(cos(x) + 1)^2 + 1)/(a^4 + 2*a^2*b^2 + b
^4) + (3*a^2*b + b^3)*arctan(sin(x)/(cos(x) + 1))/(a^4 + 2*a^2*b^2 + b^4) -
 (b*sin(x)/(cos(x) + 1) + 2*a*sin(x)^2/(cos(x) + 1)^2 - b*sin(x)^3/(cos(x)
+ 1)^3)/(a^2 + b^2 + 2*(a^2 + b^2)*sin(x)^2/(cos(x) + 1)^2 + (a^2 + b^2)*si
n(x)^4/(cos(x) + 1)^4)
```

mupad [B] time = 7.50, size = 3512, normalized size = 38.59

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)^3/(a*cos(x) + b*sin(x)),x)
```

```
[Out] (4*a^3*log(1/(cos(x) + 1)))/(4*a^4 + 4*b^4 + 8*a^2*b^2) - ((b*tan(x/2))/(a^
2 + b^2) + (2*a*tan(x/2)^2)/(a^2 + b^2) - (b*tan(x/2)^3)/(a^2 + b^2))/(2*ta
n(x/2)^2 + tan(x/2)^4 + 1) - (a^3*log(a + 2*b*tan(x/2) - a*tan(x/2)^2))/(a^
4 + b^4 + 2*a^2*b^2) - (b*atan((tan(x/2)*((((4*a^3*((b*((8*(4*a^2*b^8 - 8*a
^10 + 16*a^4*b^6 + 12*a^6*b^4 - 8*a^8*b^2))/(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*
b^2) + (32*a^3*(12*a*b^10 + 48*a^3*b^8 + 72*a^5*b^6 + 48*a^7*b^4 + 12*a^9*b
^2))/((4*a^4 + 4*b^4 + 8*a^2*b^2)*(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2)))*(3*
a^2 + b^2))/(2*(a^4 + b^4 + 2*a^2*b^2)) + (16*a^3*b*(3*a^2 + b^2)*(12*a*b^1
0 + 48*a^3*b^8 + 72*a^5*b^6 + 48*a^7*b^4 + 12*a^9*b^2))/((4*a^4 + 4*b^4 + 8
*a^2*b^2)*(a^4 + b^4 + 2*a^2*b^2)*(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2))))/(4
*a^4 + 4*b^4 + 8*a^2*b^2) + (b*(3*a^2 + b^2)*((8*(2*a*b^8 + 13*a^3*b^6 + 32
*a^5*b^4 + 21*a^7*b^2))/(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2) + (4*a^3*((8*(4
*a^2*b^8 - 8*a^10 + 16*a^4*b^6 + 12*a^6*b^4 - 8*a^8*b^2))/(a^6 + b^6 + 3*a^
2*b^4 + 3*a^4*b^2) + (32*a^3*(12*a*b^10 + 48*a^3*b^8 + 72*a^5*b^6 + 48*a^7*
b^4 + 12*a^9*b^2))/((4*a^4 + 4*b^4 + 8*a^2*b^2)*(a^6 + b^6 + 3*a^2*b^4 + 3*
a^4*b^2))))/(4*a^4 + 4*b^4 + 8*a^2*b^2)))/(2*(a^4 + b^4 + 2*a^2*b^2)) - (b^
3*(3*a^2 + b^2)^3*(12*a*b^10 + 48*a^3*b^8 + 72*a^5*b^6 + 48*a^7*b^4 + 12*a^
9*b^2))/((a^4 + b^4 + 2*a^2*b^2)^3*(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2)))*(1
6*a^8 + b^8 + 5*a^2*b^6 - 13*a^4*b^4 - 73*a^6*b^2))/(16*a^8 + b^8 + 7*a^2*b
^6 + 15*a^4*b^4 + 25*a^6*b^2)^2 + (2*a*b*(b^6 - 28*a^6 + 10*a^2*b^4 + 17*a^
4*b^2)*((8*(8*a^8 + 2*a^4*b^4 + 9*a^6*b^2))/(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*
b^2) + (4*a^3*((8*(2*a*b^8 + 13*a^3*b^6 + 32*a^5*b^4 + 21*a^7*b^2))/(a^6 +
b^6 + 3*a^2*b^4 + 3*a^4*b^2) + (4*a^3*((8*(4*a^2*b^8 - 8*a^10 + 16*a^4*b^6
+ 12*a^6*b^4 - 8*a^8*b^2))/(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2) + (32*a^3*(1
2*a*b^10 + 48*a^3*b^8 + 72*a^5*b^6 + 48*a^7*b^4 + 12*a^9*b^2))/((4*a^4 + 4*
b^4 + 8*a^2*b^2)*(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2))))/(4*a^4 + 4*b^4 + 8*
a^2*b^2)))/(4*a^4 + 4*b^4 + 8*a^2*b^2) - (b*(3*a^2 + b^2)*((b*((8*(4*a^2*b^
```

```
8 - 8*a^10 + 16*a^4*b^6 + 12*a^6*b^4 - 8*a^8*b^2))/(a^6 + b^6 + 3*a^2*b^4 +
 3*a^4*b^2) + (32*a^3*(12*a*b^10 + 48*a^3*b^8 + 72*a^5*b^6 + 48*a^7*b^4 + 1
2*a^9*b^2))/((4*a^4 + 4*b^4 + 8*a^2*b^2)*(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2
)))*(3*a^2 + b^2))/(2*(a^4 + b^4 + 2*a^2*b^2)) + (16*a^3*b*(3*a^2 + b^2)*(1
2*a*b^10 + 48*a^3*b^8 + 72*a^5*b^6 + 48*a^7*b^4 + 12*a^9*b^2))/((4*a^4 + 4*
b^4 + 8*a^2*b^2)*(a^4 + b^4 + 2*a^2*b^2)*(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2
))))/(2*(a^4 + b^4 + 2*a^2*b^2)) - (8*a^3*b^2*(3*a^2 + b^2)^2*(12*a*b^10 +
48*a^3*b^8 + 72*a^5*b^6 + 48*a^7*b^4 + 12*a^9*b^2))/((4*a^4 + 4*b^4 + 8*a^2
*b^2)*(a^4 + b^4 + 2*a^2*b^2)^2*(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2))))/(16*
a^8 + b^8 + 7*a^2*b^6 + 15*a^4*b^4 + 25*a^6*b^2)^2)*(a^10 + b^10 + 5*a^2*b^
8 + 10*a^4*b^6 + 10*a^6*b^4 + 5*a^8*b^2))/(4*a*b^3 + 12*a^3*b) - (((4*a^3*(
(b*(3*a^2 + b^2)*((8*(2*a*b^9 - 10*a^9*b + 8*a^3*b^7 - 16*a^7*b^3))/(a^6 +
b^6 + 3*a^2*b^4 + 3*a^4*b^2) - (32*a^3*(12*a^10*b + 12*a^2*b^9 + 48*a^4*b^7
 + 72*a^6*b^5 + 48*a^8*b^3))/((4*a^4 + 4*b^4 + 8*a^2*b^2)*(a^6 + b^6 + 3*a^
2*b^4 + 3*a^4*b^2))))/(2*(a^4 + b^4 + 2*a^2*b^2)) - (16*a^3*b*(3*a^2 + b^2)
*(12*a^10*b + 12*a^2*b^9 + 48*a^4*b^7 + 72*a^6*b^5 + 48*a^8*b^3))/((4*a^4 +
 4*b^4 + 8*a^2*b^2)*(a^4 + b^4 + 2*a^2*b^2)*(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*
b^2))))/(4*a^4 + 4*b^4 + 8*a^2*b^2) - (b*((8*(a^2*b^7 + 2*a^4*b^5 + a^6*b^3
))/(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2) - (4*a^3*((8*(2*a*b^9 - 10*a^9*b + 8
*a^3*b^7 - 16*a^7*b^3))/(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2) - (32*a^3*(12*a
^10*b + 12*a^2*b^9 + 48*a^4*b^7 + 72*a^6*b^5 + 48*a^8*b^3))/((4*a^4 + 4*b^4
 + 8*a^2*b^2)*(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2))))/(4*a^4 + 4*b^4 + 8*a^2
*b^2))*(3*a^2 + b^2))/(2*(a^4 + b^4 + 2*a^2*b^2)) + (b^3*(3*a^2 + b^2)^3*(1
2*a^10*b + 12*a^2*b^9 + 48*a^4*b^7 + 72*a^6*b^5 + 48*a^8*b^3))/((a^4 + b^4
+ 2*a^2*b^2)^3*(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2)))*(16*a^8 + b^8 + 5*a^2*
b^6 - 13*a^4*b^4 - 73*a^6*b^2)*(a^10 + b^10 + 5*a^2*b^8 + 10*a^4*b^6 + 10*a
^6*b^4 + 5*a^8*b^2))/((4*a*b^3 + 12*a^3*b)*(16*a^8 + b^8 + 7*a^2*b^6 + 15*a
^4*b^4 + 25*a^6*b^2)^2) + (2*a*b*(b^6 - 28*a^6 + 10*a^2*b^4 + 17*a^4*b^2)*(
(8*(2*a^7*b + a^5*b^3))/(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2) + (4*a^3*((8*(a
^2*b^7 + 2*a^4*b^5 + a^6*b^3))/(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2) - (4*a^3
*((8*(2*a*b^9 - 10*a^9*b + 8*a^3*b^7 - 16*a^7*b^3))/(a^6 + b^6 + 3*a^2*b^4
+ 3*a^4*b^2) - (32*a^3*(12*a^10*b + 12*a^2*b^9 + 48*a^4*b^7 + 72*a^6*b^5 +
48*a^8*b^3))/((4*a^4 + 4*b^4 + 8*a^2*b^2)*(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^
2))))/(4*a^4 + 4*b^4 + 8*a^2*b^2)))/(4*a^4 + 4*b^4 + 8*a^2*b^2) + (b*((b*(3
*a^2 + b^2)*((8*(2*a*b^9 - 10*a^9*b + 8*a^3*b^7 - 16*a^7*b^3))/(a^6 + b^6 +
 3*a^2*b^4 + 3*a^4*b^2) - (32*a^3*(12*a^10*b + 12*a^2*b^9 + 48*a^4*b^7 + 72
*a^6*b^5 + 48*a^8*b^3))/((4*a^4 + 4*b^4 + 8*a^2*b^2)*(a^6 + b^6 + 3*a^2*b^4
 + 3*a^4*b^2))))/(2*(a^4 + b^4 + 2*a^2*b^2)) - (16*a^3*b*(3*a^2 + b^2)*(12*
a^10*b + 12*a^2*b^9 + 48*a^4*b^7 + 72*a^6*b^5 + 48*a^8*b^3))/((4*a^4 + 4*b^
4 + 8*a^2*b^2)*(a^4 + b^4 + 2*a^2*b^2)*(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2))
)*(3*a^2 + b^2))/(2*(a^4 + b^4 + 2*a^2*b^2)) - (8*a^3*b^2*(3*a^2 + b^2)^2*(
12*a^10*b + 12*a^2*b^9 + 48*a^4*b^7 + 72*a^6*b^5 + 48*a^8*b^3))/((4*a^4 + 4
*b^4 + 8*a^2*b^2)*(a^4 + b^4 + 2*a^2*b^2)^2*(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*
b^2)))*(a^10 + b^10 + 5*a^2*b^8 + 10*a^4*b^6 + 10*a^6*b^4 + 5*a^8*b^2))/((4
*a*b^3 + 12*a^3*b)*(16*a^8 + b^8 + 7*a^2*b^6 + 15*a^4*b^4 + 25*a^6*b^2)^2))
```

```
*(3*a^2 + b^2))/(a^4 + b^4 + 2*a^2*b^2)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)**3/(a*cos(x)+b*sin(x)),x)
[Out] Timed out
```

3.9 $\int \frac{\sin^2(x)}{a\cos(x)+b\sin(x)}\,dx$

Optimal. Leaf size=68

$$-\frac{a\sin(x)}{a^2+b^2}-\frac{b\cos(x)}{a^2+b^2}-\frac{a^2\tanh^{-1}\left(\frac{b\cos(x)-a\sin(x)}{\sqrt{a^2+b^2}}\right)}{\left(a^2+b^2\right)^{3/2}}$$

```
[Out] -a^2*arctanh((b*cos(x)-a*sin(x))/(a^2+b^2)^(1/2))/(a^2+b^2)^(3/2)-b*cos(x)/
(a^2+b^2)-a*sin(x)/(a^2+b^2)
```

Rubi [A] time = 0.08, antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {3099, 3074, 206, 2638}

$$-\frac{a\sin(x)}{a^2+b^2}-\frac{b\cos(x)}{a^2+b^2}-\frac{a^2\tanh^{-1}\left(\frac{b\cos(x)-a\sin(x)}{\sqrt{a^2+b^2}}\right)}{\left(a^2+b^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[x]^2/(a*Cos[x] + b*Sin[x]),x]
```

```
[Out] -((a^2*ArcTanh[(b*Cos[x] - a*Sin[x])/Sqrt[a^2 + b^2]])/(a^2 + b^2)^(3/2)) -
 (b*Cos[x])/(a^2 + b^2) - (a*Sin[x])/(a^2 + b^2)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3074

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x
_Symbol] :> -Dist[d^(-1), Subst[Int[1/(a^2 + b^2 - x^2), x], x, b*Cos[c + d
*x] - a*Sin[c + d*x]], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

Rule 3099

```
Int[sin[(c_.) + (d_.)*(x_)]^(m_)/(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin
[(c_.) + (d_.)*(x_)]), x_Symbol] :> -Simp[(a*Sin[c + d*x]^(m - 1))/(d*(a^2
+ b^2)*(m - 1)), x] + (Dist[a^2/(a^2 + b^2), Int[Sin[c + d*x]^(m - 2)/(a*Co
s[c + d*x] + b*Sin[c + d*x]), x], x] + Dist[b/(a^2 + b^2), Int[Sin[c + d*x]
^(m - 1), x], x]) /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0] && GtQ[m,
1]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sin^2(x)}{a\cos(x)+b\sin(x)}\,dx &= -\frac{a\sin(x)}{a^2+b^2}+\frac{a^2\int \frac{1}{a\cos(x)+b\sin(x)}\,dx}{a^2+b^2}+\frac{b\int \sin(x)\,dx}{a^2+b^2}\\
&= -\frac{b\cos(x)}{a^2+b^2}-\frac{a\sin(x)}{a^2+b^2}-\frac{a^2\,\mathrm{Subst}\left(\int \frac{1}{a^2+b^2-x^2}\,dx,x,b\cos(x)-a\sin(x)\right)}{a^2+b^2}\\
&= -\frac{a^2\tanh^{-1}\left(\frac{b\cos(x)-a\sin(x)}{\sqrt{a^2+b^2}}\right)}{\left(a^2+b^2\right)^{3/2}}-\frac{b\cos(x)}{a^2+b^2}-\frac{a\sin(x)}{a^2+b^2}
\end{aligned}$$

Mathematica [A] time = 0.16, size = 62, normalized size = 0.91

$$\frac{2a^2\tanh^{-1}\left(\frac{a\tan\left(\frac{x}{2}\right)-b}{\sqrt{a^2+b^2}}\right)}{\left(a^2+b^2\right)^{3/2}}-\frac{a\sin(x)+b\cos(x)}{a^2+b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[x]^2/(a*Cos[x] + b*Sin[x]),x]
```

```
[Out] (2*a^2*ArcTanh[(-b + a*Tan[x/2])/Sqrt[a^2 + b^2]])/(a^2 + b^2)^(3/2) - (b*C
os[x] + a*Sin[x])/(a^2 + b^2)
```

fricas [B] time = 0.48, size = 144, normalized size = 2.12

$$\frac{\sqrt{a^2+b^2}\,a^2\log\left(-\frac{2\,ab\cos(x)\sin(x)+\left(a^2-b^2\right)\cos(x)^2-2\,a^2-b^2+2\,\sqrt{a^2+b^2}\,(b\cos(x)-a\sin(x))}{2\,ab\cos(x)\sin(x)+\left(a^2-b^2\right)\cos(x)^2+b^2}\right)-2\left(a^2b+b^3\right)\cos(x)-2\left(a^3+ab^2\right)\mathrm{s}}{2\left(a^4+2\,a^2b^2+b^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^2/(a*cos(x)+b*sin(x)),x, algorithm="fricas")
```

```
[Out] 1/2*(sqrt(a^2 + b^2)*a^2*log(-(2*a*b*cos(x)*sin(x) + (a^2 - b^2)*cos(x)^2 -
 2*a^2 - b^2 + 2*sqrt(a^2 + b^2)*(b*cos(x) - a*sin(x)))/(2*a*b*cos(x)*sin(x
) + (a^2 - b^2)*cos(x)^2 + b^2)) - 2*(a^2*b + b^3)*cos(x) - 2*(a^3 + a*b^2)
*sin(x))/(a^4 + 2*a^2*b^2 + b^4)
```

giac [A] time = 0.26, size = 94, normalized size = 1.38

$$-\frac{a^2\log\left(\frac{\left|2\,a\tan\left(\frac{1}{2}\,x\right)-2\,b-2\,\sqrt{a^2+b^2}\right|}{\left|2\,a\tan\left(\frac{1}{2}\,x\right)-2\,b+2\,\sqrt{a^2+b^2}\right|}\right)}{\left(a^2+b^2\right)^{\frac{3}{2}}}-\frac{2\left(a\tan\left(\frac{1}{2}\,x\right)+b\right)}{\left(a^2+b^2\right)\left(\tan\left(\frac{1}{2}\,x\right)^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^2/(a*cos(x)+b*sin(x)),x, algorithm="giac")
```

```
[Out] -a^2*log(abs(2*a*tan(1/2*x) - 2*b - 2*sqrt(a^2 + b^2))/abs(2*a*tan(1/2*x) -
 2*b + 2*sqrt(a^2 + b^2)))/(a^2 + b^2)^(3/2) - 2*(a*tan(1/2*x) + b)/((a^2 +
 b^2)*(tan(1/2*x)^2 + 1))
```

maple [A] time = 0.49, size = 84, normalized size = 1.24

$$\frac{8a^2\,\mathrm{arctanh}\left(\frac{2a\tan\left(\frac{x}{2}\right)-2b}{2\sqrt{a^2+b^2}}\right)}{\left(4a^2+4b^2\right)\sqrt{a^2+b^2}}+\frac{-2a\tan\left(\frac{x}{2}\right)-2b}{\left(a^2+b^2\right)\left(\tan^2\left(\frac{x}{2}\right)+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)^2/(a*cos(x)+b*sin(x)),x)
```

```
[Out] 8*a^2/(4*a^2+4*b^2)/(a^2+b^2)^(1/2)*arctanh(1/2*(2*a*tan(1/2*x)-2*b)/(a^2+b
^2)^(1/2))+2/(a^2+b^2)*(-a*tan(1/2*x)-b)/(tan(1/2*x)^2+1)
```

maxima [A] time = 0.43, size = 106, normalized size = 1.56

$$-\frac{a^2\log\left(\frac{b-\frac{a\sin(x)}{\cos(x)+1}+\sqrt{a^2+b^2}}{b-\frac{a\sin(x)}{\cos(x)+1}-\sqrt{a^2+b^2}}\right)}{\left(a^2+b^2\right)^{\frac{3}{2}}}-\frac{2\left(b+\frac{a\sin(x)}{\cos(x)+1}\right)}{a^2+b^2+\frac{\left(a^2+b^2\right)\sin(x)^2}{\left(\cos(x)+1\right)^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^2/(a*cos(x)+b*sin(x)),x, algorithm="maxima")
```

```
[Out] -a^2*log((b - a*sin(x)/(cos(x) + 1) + sqrt(a^2 + b^2))/(b - a*sin(x)/(cos(x
) + 1) - sqrt(a^2 + b^2)))/(a^2 + b^2)^(3/2) - 2*(b + a*sin(x)/(cos(x) + 1)
)/(a^2 + b^2 + (a^2 + b^2)*sin(x)^2/(cos(x) + 1)^2)
```

mupad [B] time = 0.58, size = 94, normalized size = 1.38

$$-\frac{\frac{2\,b}{a^2+b^2}+\frac{2\,a\,\tan\left(\frac{x}{2}\right)}{a^2+b^2}}{\tan\left(\frac{x}{2}\right)^2+1}-\frac{2\,a^2\,\mathrm{atanh}\left(\frac{2\,a^2\,b+2\,b^3-2\,a\,\tan\left(\frac{x}{2}\right)\,\left(a^2+b^2\right)}{2\,{\left(a^2+b^2\right)}^{3/2}}\right)}{{\left(a^2+b^2\right)}^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)^2/(a*cos(x) + b*sin(x)),x)
```

```
[Out] - ((2*b)/(a^2 + b^2) + (2*a*tan(x/2))/(a^2 + b^2))/(tan(x/2)^2 + 1) - (2*a^
2*atanh((2*a^2*b + 2*b^3 - 2*a*tan(x/2)*(a^2 + b^2))/(2*(a^2 + b^2)^(3/2)))
)/(a^2 + b^2)^(3/2)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)**2/(a*cos(x)+b*sin(x)),x)
```

```
[Out] Timed out
```

3.10 $\int \frac{\sin(x)}{a\cos(x)+b\sin(x)}\,dx$

Optimal. Leaf size=35

$$\frac{bx}{a^2+b^2}-\frac{a\log(a\cos(x)+b\sin(x))}{a^2+b^2}$$

```
[Out] b*x/(a^2+b^2)-a*ln(a*cos(x)+b*sin(x))/(a^2+b^2)
```

Rubi [A] time = 0.06, antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3097, 3133}

$$\frac{bx}{a^2+b^2}-\frac{a\log(a\cos(x)+b\sin(x))}{a^2+b^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[x]/(a*Cos[x] + b*Sin[x]),x]
```

```
[Out] (b*x)/(a^2 + b^2) - (a*Log[a*Cos[x] + b*Sin[x]])/(a^2 + b^2)
```

Rule 3097

```
Int[sin[(c_.) + (d_.)*(x_)]/(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.
) + (d_.)*(x_)]), x_Symbol] :> Simp[(b*x)/(a^2 + b^2), x] - Dist[a/(a^2 + b
^2), Int[(b*Cos[c + d*x] - a*Sin[c + d*x])/(a*Cos[c + d*x] + b*Sin[c + d*x]
), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

Rule 3133

```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])
/((a_.) + cos[(d_.) + (e_.)*(x_)]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)]), x
_Symbol] :> Simp[((b*B + c*C)*x)/(b^2 + c^2), x] + Simp[((c*B - b*C)*Log[a
+ b*Cos[d + e*x] + c*Sin[d + e*x]])/(e*(b^2 + c^2)), x] /; FreeQ[{a, b, c,
d, e, A, B, C}, x] && NeQ[b^2 + c^2, 0] && EqQ[A*(b^2 + c^2) - a*(b*B + c*C
), 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\sin(x)}{a\cos(x)+b\sin(x)}\,dx &= \frac{bx}{a^2+b^2}-\frac{a\int \frac{b\cos(x)-a\sin(x)}{a\cos(x)+b\sin(x)}\,dx}{a^2+b^2}\\ &= \frac{bx}{a^2+b^2}-\frac{a\log(a\cos(x)+b\sin(x))}{a^2+b^2}\end{aligned}$$

Mathematica [C] time = 0.06, size = 47, normalized size = 1.34

$$\frac{2x(b-ia)-a\log\left((a\cos(x)+b\sin(x))^2\right)+2ia\tan^{-1}(\tan(x))}{2\left(a^2+b^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[x]/(a*Cos[x] + b*Sin[x]),x]
```

```
[Out] (2*((-I)*a + b)*x + (2*I)*a*ArcTan[Tan[x]] - a*Log[(a*Cos[x] + b*Sin[x])^2]
)/(2*(a^2 + b^2))
```

fricas [A] time = 0.48, size = 46, normalized size = 1.31

$$\frac{2\,bx-a\log\left(2\,ab\cos(x)\sin(x)+\left(a^2-b^2\right)\cos(x)^2+b^2\right)}{2\left(a^2+b^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)/(a*cos(x)+b*sin(x)),x, algorithm="fricas")
```

```
[Out] 1/2*(2*b*x - a*log(2*a*b*cos(x)*sin(x) + (a^2 - b^2)*cos(x)^2 + b^2))/(a^2
+ b^2)
```

giac [A] time = 1.46, size = 55, normalized size = 1.57

$$-\frac{ab\log\left(|b\tan(x)+a|\right)}{a^2b+b^3}+\frac{bx}{a^2+b^2}+\frac{a\log\left(\tan(x)^2+1\right)}{2\left(a^2+b^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)/(a*cos(x)+b*sin(x)),x, algorithm="giac")
```

```
[Out] -a*b*log(abs(b*tan(x) + a))/(a^2*b + b^3) + b*x/(a^2 + b^2) + 1/2*a*log(tan
(x)^2 + 1)/(a^2 + b^2)
```

maple [A] time = 0.46, size = 54, normalized size = 1.54

$$-\frac{a\ln\left(a+b\tan(x)\right)}{a^2+b^2}+\frac{a\ln\left(1+\tan^2(x)\right)}{2a^2+2b^2}+\frac{b\arctan\left(\tan(x)\right)}{a^2+b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)/(a*cos(x)+b*sin(x)),x)
```

```
[Out] -a/(a^2+b^2)*ln(a+b*tan(x))+1/2/(a^2+b^2)*a*ln(1+tan(x)^2)+1/(a^2+b^2)*b*ar
ctan(tan(x))
```

maxima [B] time = 0.43, size = 88, normalized size = 2.51

$$\frac{2\,b\arctan\left(\frac{\sin(x)}{\cos(x)+1}\right)}{a^2+b^2} - \frac{a\log\left(-a-\frac{2\,b\sin(x)}{\cos(x)+1}+\frac{a\sin(x)^2}{(\cos(x)+1)^2}\right)}{a^2+b^2} + \frac{a\log\left(\frac{\sin(x)^2}{(\cos(x)+1)^2}+1\right)}{a^2+b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)/(a*cos(x)+b*sin(x)),x, algorithm="maxima")
```

```
[Out] 2*b*arctan(sin(x)/(cos(x) + 1))/(a^2 + b^2) - a*log(-a - 2*b*sin(x)/(cos(x)
 + 1) + a*sin(x)^2/(cos(x) + 1)^2)/(a^2 + b^2) + a*log(sin(x)^2/(cos(x) + 1
)^2 + 1)/(a^2 + b^2)
```

mupad [B] time = 2.14, size = 970, normalized size = 27.71

$$-\frac{a\ln(a\cos(x)+b\sin(x))}{a^2+b^2} - \frac{2\,b\,\mathrm{atan}\left(\frac{\left(a^4+2\,a^2\,b^2+b^4\right)\tan\left(\frac{x}{2}\right)\left(\frac{\left(4\,a^4-13\,a^2\,b^2+b^4\right)\left(\frac{b\left(64\,a\,b^2+\frac{a\left(32\,a^2\,b^2-64\,a^4+\frac{a\left(96\,a^3\,b^2+96\,a\,b^4\right)}{a^2+b^2}\right)}{a^2+b^2}\right)}{a^2+b^2}-\frac{b^3\left(96\,\ldots\right.}{\left(\ldots\right.}\ldots\right)}{\left(4\,a^4+5\,\ldots\right.}\ldots\right)}{\ldots}\right)}{\ldots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)/(a*cos(x) + b*sin(x)),x)
```

```
[Out] - (a*log(a*cos(x) + b*sin(x)))/(a^2 + b^2) - (2*b*atan(((a^4 + b^4 + 2*a^2*
b^2)*(tan(x/2)*(((4*a^4 + b^4 - 13*a^2*b^2)*((b*(64*a*b^2 + (a*(32*a^2*b^2
- 64*a^4 + (a*(96*a*b^4 + 96*a^3*b^2))/(a^2 + b^2)))/(a^2 + b^2)))/(a^2 + b
^2) - (b^3*(96*a*b^4 + 96*a^3*b^2))/(a^2 + b^2)^3 + (a*((b*(32*a^2*b^2 - 64
*a^4 + (a*(96*a*b^4 + 96*a^3*b^2))/(a^2 + b^2)))/(a^2 + b^2) + (a*b*(96*a*b
^4 + 96*a^3*b^2))/(a^2 + b^2)^2))/(a^2 + b^2)))/(4*a^4 + b^4 + 5*a^2*b^2)^2
 - (6*a*b*(2*a^2 - b^2)*(64*a^2 + (a*(64*a*b^2 + (a*(32*a^2*b^2 - 64*a^4 +
(a*(96*a*b^4 + 96*a^3*b^2))/(a^2 + b^2)))/(a^2 + b^2)))/(a^2 + b^2) - (b*((
b*(32*a^2*b^2 - 64*a^4 + (a*(96*a*b^4 + 96*a^3*b^2))/(a^2 + b^2)))/(a^2 + b
^2) + (a*b*(96*a*b^4 + 96*a^3*b^2))/(a^2 + b^2)^2))/(a^2 + b^2) - (a*b^2*(9
6*a*b^4 + 96*a^3*b^2))/(a^2 + b^2)^3))/(4*a^4 + b^4 + 5*a^2*b^2)^2) - ((4*a
^4 + b^4 - 13*a^2*b^2)*((b*(32*a^2*b - (a*(64*a^3*b - 32*a*b^3 + (a*(96*a^4
*b + 96*a^2*b^3))/(a^2 + b^2)))/(a^2 + b^2)))/(a^2 + b^2) + (b^3*(96*a^4*b
+ 96*a^2*b^3))/(a^2 + b^2)^3 - (a*((b*(64*a^3*b - 32*a*b^3 + (a*(96*a^4*b +
 96*a^2*b^3))/(a^2 + b^2)))/(a^2 + b^2) + (a*b*(96*a^4*b + 96*a^2*b^3))/(a^
2 + b^2)^2))/(a^2 + b^2)))/(4*a^4 + b^4 + 5*a^2*b^2)^2 + (6*a*b*(2*a^2 - b^
2)*((a*(32*a^2*b - (a*(64*a^3*b - 32*a*b^3 + (a*(96*a^4*b + 96*a^2*b^3))/(a
^2 + b^2)))/(a^2 + b^2)))/(a^2 + b^2) + (b*((b*(64*a^3*b - 32*a*b^3 + (a*(9
6*a^4*b + 96*a^2*b^3))/(a^2 + b^2)))/(a^2 + b^2) + (a*b*(96*a^4*b + 96*a^2*
b^3))/(a^2 + b^2)^2))/(a^2 + b^2) + (a*b^2*(96*a^4*b + 96*a^2*b^3))/(a^2 +
b^2)^3))/(4*a^4 + b^4 + 5*a^2*b^2)^2))/(32*a*b)))/(a^2 + b^2)
```

sympy [A] time = 0.77, size = 173, normalized size = 4.94

$$
\begin{cases}
\tilde{\infty}x & \text{for } a = 0 \wedge b = 0 \\
-\frac{ix\sin(x)}{-2ib\sin(x)-2b\cos(x)} - \frac{x\cos(x)}{-2ib\sin(x)-2b\cos(x)} + \frac{i\cos(x)}{-2ib\sin(x)-2b\cos(x)} & \text{for } a = -ib \\
\frac{ix\sin(x)}{2ib\sin(x)-2b\cos(x)} - \frac{x\cos(x)}{2ib\sin(x)-2b\cos(x)} - \frac{i\cos(x)}{2ib\sin(x)-2b\cos(x)} & \text{for } a = ib \\
-\frac{\log(\cos(x))}{a} & \text{for } b = 0 \\
-\frac{a\log\left(\frac{a\cos(x)}{b}+\sin(x)\right)}{a^2+b^2} + \frac{bx}{a^2+b^2} & \text{otherwise}
\end{cases}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)/(a*cos(x)+b*sin(x)),x)
```

```
[Out] Piecewise((zoo*x, Eq(a, 0) & Eq(b, 0)), (-I*x*sin(x)/(-2*I*b*sin(x) - 2*b*c
os(x)) - x*cos(x)/(-2*I*b*sin(x) - 2*b*cos(x)) + I*cos(x)/(-2*I*b*sin(x) -
2*b*cos(x)), Eq(a, -I*b)), (I*x*sin(x)/(2*I*b*sin(x) - 2*b*cos(x)) - x*cos(
```

```
x)/(2*I*b*sin(x) - 2*b*cos(x)) - I*cos(x)/(2*I*b*sin(x) - 2*b*cos(x)), Eq(a
, I*b)), (-log(cos(x))/a, Eq(b, 0)), (-a*log(a*cos(x)/b + sin(x))/(a**2 + b
**2) + b*x/(a**2 + b**2), True))
```

3.11 $\int \frac{1}{a\cos(x)+b\sin(x)}\,dx$

Optimal. Leaf size=36

$$-\frac{\tanh^{-1}\left(\frac{b\cos(x)-a\sin(x)}{\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}}$$

```
[Out] -arctanh((b*cos(x)-a*sin(x))/(a^2+b^2)^(1/2))/(a^2+b^2)^(1/2)
```

Rubi [A] time = 0.02, antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {3074, 206}

$$-\frac{\tanh^{-1}\left(\frac{b\cos(x)-a\sin(x)}{\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Cos[x] + b*Sin[x])^(-1),x]
```

```
[Out] -(ArcTanh[(b*Cos[x] - a*Sin[x])/Sqrt[a^2 + b^2]]/Sqrt[a^2 + b^2])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3074

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x
_Symbol] :> -Dist[d^(-1), Subst[Int[1/(a^2 + b^2 - x^2), x], x, b*Cos[c + d
*x] - a*Sin[c + d*x]], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{a\cos(x)+b\sin(x)}\,dx &= -\text{Subst}\left(\int \frac{1}{a^2+b^2-x^2}\,dx, x, b\cos(x)-a\sin(x)\right)\\ &= -\frac{\tanh^{-1}\left(\frac{b\cos(x)-a\sin(x)}{\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}}\end{aligned}$$

Mathematica [A] time = 0.03, size = 38, normalized size = 1.06

$$\frac{2\tanh^{-1}\left(\frac{a\tan\left(\frac{x}{2}\right)-b}{\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Cos[x] + b*Sin[x])^(-1),x]
```

```
[Out] (2*ArcTanh[(-b + a*Tan[x/2])/Sqrt[a^2 + b^2]])/Sqrt[a^2 + b^2]
```

fricas [B] time = 0.45, size = 96, normalized size = 2.67

$$\frac{\log\left(-\frac{2\,ab\cos(x)\sin(x)+\left(a^2-b^2\right)\cos(x)^2-2\,a^2-b^2+2\,\sqrt{a^2+b^2}\,(b\cos(x)-a\sin(x))}{2\,ab\cos(x)\sin(x)+\left(a^2-b^2\right)\cos(x)^2+b^2}\right)}{2\,\sqrt{a^2+b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cos(x)+b*sin(x)),x, algorithm="fricas")
```

```
[Out] 1/2*log(-(2*a*b*cos(x)*sin(x) + (a^2 - b^2)*cos(x)^2 - 2*a^2 - b^2 + 2*sqrt
(a^2 + b^2)*(b*cos(x) - a*sin(x)))/(2*a*b*cos(x)*sin(x) + (a^2 - b^2)*cos(x
)^2 + b^2))/sqrt(a^2 + b^2)
```

giac [A] time = 0.41, size = 61, normalized size = 1.69

$$-\frac{\log\left(\frac{\left|2\,a\tan\left(\frac{1}{2}\,x\right)-2\,b-2\,\sqrt{a^2+b^2}\right|}{\left|2\,a\tan\left(\frac{1}{2}\,x\right)-2\,b+2\,\sqrt{a^2+b^2}\right|}\right)}{\sqrt{a^2+b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cos(x)+b*sin(x)),x, algorithm="giac")
```

```
[Out] -log(abs(2*a*tan(1/2*x) - 2*b - 2*sqrt(a^2 + b^2))/abs(2*a*tan(1/2*x) - 2*b
 + 2*sqrt(a^2 + b^2)))/sqrt(a^2 + b^2)
```

maple [A] time = 0.49, size = 35, normalized size = 0.97

$$\frac{2\,\mathrm{arctanh}\left(\frac{2a\tan\left(\frac{x}{2}\right)-2b}{2\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*cos(x)+b*sin(x)),x)
```

```
[Out] 2/(a^2+b^2)^(1/2)*arctanh(1/2*(2*a*tan(1/2*x)-2*b)/(a^2+b^2)^(1/2))
```

maxima [A] time = 0.43, size = 61, normalized size = 1.69

$$-\frac{\log\left(\frac{b-\frac{a\sin(x)}{\cos(x)+1}+\sqrt{a^2+b^2}}{b-\frac{a\sin(x)}{\cos(x)+1}-\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cos(x)+b*sin(x)),x, algorithm="maxima")
```

```
[Out] -log((b - a*sin(x)/(cos(x) + 1) + sqrt(a^2 + b^2))/(b - a*sin(x)/(cos(x) +
1) - sqrt(a^2 + b^2)))/sqrt(a^2 + b^2)
```

mupad [B] time = 1.05, size = 31, normalized size = 0.86

$$-\frac{2\,\mathrm{atanh}\left(\frac{b-a\tan\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*cos(x) + b*sin(x)),x)
```

```
[Out] -(2*atanh((b - a*tan(x/2))/(a^2 + b^2)^(1/2)))/(a^2 + b^2)^(1/2)
```

sympy [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: AttributeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cos(x)+b*sin(x)),x)
```

```
[Out] Exception raised: AttributeError
```

3.12 $\int \frac{\csc(x)}{a\cos(x)+b\sin(x)}\,dx$

Optimal. Leaf size=23

$$\frac{\log(\sin(x))}{a}-\frac{\log(a\cos(x)+b\sin(x))}{a}$$

```
[Out] ln(sin(x))/a-ln(a*cos(x)+b*sin(x))/a
```

Rubi [A] time = 0.07, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {3101, 3475, 3133}

$$\frac{\log(\sin(x))}{a}-\frac{\log(a\cos(x)+b\sin(x))}{a}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[x]/(a*Cos[x] + b*Sin[x]),x]
```

```
[Out] Log[Sin[x]]/a - Log[a*Cos[x] + b*Sin[x]]/a
```

Rule 3101

```
Int[1/(sin[(c_.) + (d_.)*(x_)]*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(
c_.) + (d_.)*(x_)])), x_Symbol] :> Dist[1/a, Int[Cot[c + d*x], x], x] - Dis
t[1/a, Int[(b*Cos[c + d*x] - a*Sin[c + d*x])/(a*Cos[c + d*x] + b*Sin[c + d*
x]), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

Rule 3133

```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])
/((a_.) + cos[(d_.) + (e_.)*(x_)]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)]), x
_Symbol] :> Simp[((b*B + c*C)*x)/(b^2 + c^2), x] + Simp[((c*B - b*C)*Log[a
+ b*Cos[d + e*x] + c*Sin[d + e*x]])/(e*(b^2 + c^2)), x] /; FreeQ[{a, b, c,
d, e, A, B, C}, x] && NeQ[b^2 + c^2, 0] && EqQ[A*(b^2 + c^2) - a*(b*B + c*C
), 0]
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\csc(x)}{a\cos(x)+b\sin(x)}\,dx = \frac{\int \cot(x)\,dx}{a} - \frac{\int \frac{b\cos(x)-a\sin(x)}{a\cos(x)+b\sin(x)}\,dx}{a}$$
$$= \frac{\log(\sin(x))}{a} - \frac{\log(a\cos(x)+b\sin(x))}{a}$$

Mathematica [A] time = 0.05, size = 20, normalized size = 0.87

$$\frac{\log(\sin(x)) - \log(a\cos(x)+b\sin(x))}{a}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[x]/(a*Cos[x] + b*Sin[x]),x]
[Out] (Log[Sin[x]] - Log[a*Cos[x] + b*Sin[x]])/a
```

fricas [A] time = 0.45, size = 44, normalized size = 1.91

$$-\frac{\log\left(2\,ab\cos(x)\sin(x)+\left(a^2-b^2\right)\cos(x)^2+b^2\right)-\log\left(-\frac{1}{4}\,\cos(x)^2+\frac{1}{4}\right)}{2\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)/(a*cos(x)+b*sin(x)),x, algorithm="fricas")
[Out] -1/2*(log(2*a*b*cos(x)*sin(x) + (a^2 - b^2)*cos(x)^2 + b^2) - log(-1/4*cos(
x)^2 + 1/4))/a
```

giac [A] time = 1.72, size = 22, normalized size = 0.96

$$-\frac{\log\left(|b\tan(x)+a|\right)}{a} + \frac{\log\left(|\tan(x)|\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)/(a*cos(x)+b*sin(x)),x, algorithm="giac")
[Out] -log(abs(b*tan(x) + a))/a + log(abs(tan(x)))/a
```

maple [A] time = 0.71, size = 21, normalized size = 0.91

$$-\frac{\ln\left(a+b\tan(x)\right)}{a} + \frac{\ln\left(\tan(x)\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(x)/(a*cos(x)+b*sin(x)),x)
```

```
[Out] -1/a*ln(a+b*tan(x))+1/a*ln(tan(x))
```

maxima [B] time = 0.32, size = 48, normalized size = 2.09

$$-\frac{\log\left(-a-\frac{2\,b\sin(x)}{\cos(x)+1}+\frac{a\sin(x)^2}{(\cos(x)+1)^2}\right)}{a}+\frac{\log\left(\frac{\sin(x)}{\cos(x)+1}\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)/(a*cos(x)+b*sin(x)),x, algorithm="maxima")
```

```
[Out] -log(-a - 2*b*sin(x)/(cos(x) + 1) + a*sin(x)^2/(cos(x) + 1)^2)/a + log(sin(
x)/(cos(x) + 1))/a
```

mupad [B] time = 0.58, size = 32, normalized size = 1.39

$$-\frac{\ln\left(-a\tan\left(\frac{x}{2}\right)^2+2\,b\tan\left(\frac{x}{2}\right)+a\right)-\ln\left(\tan\left(\frac{x}{2}\right)\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(x)*(a*cos(x) + b*sin(x))),x)
```

```
[Out] -(log(a + 2*b*tan(x/2) - a*tan(x/2)^2) - log(tan(x/2)))/a
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\csc(x)}{a\cos(x)+b\sin(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)/(a*cos(x)+b*sin(x)),x)
```

```
[Out] Integral(csc(x)/(a*cos(x) + b*sin(x)), x)
```

3.13 $\int \frac{\csc^2(x)}{a\cos(x)+b\sin(x)}\,dx$

Optimal. Leaf size=55

$$-\frac{\sqrt{a^2+b^2}\,\tanh^{-1}\left(\frac{b\cos(x)-a\sin(x)}{\sqrt{a^2+b^2}}\right)}{a^2}+\frac{b\tanh^{-1}(\cos(x))}{a^2}-\frac{\csc(x)}{a}$$

```
[Out] b*arctanh(cos(x))/a^2-csc(x)/a-arctanh((b*cos(x)-a*sin(x))/(a^2+b^2)^(1/2))
*(a^2+b^2)^(1/2)/a^2
```

Rubi [A] time = 0.07, antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {3103, 3770, 3074, 206}

$$-\frac{\sqrt{a^2+b^2}\,\tanh^{-1}\left(\frac{b\cos(x)-a\sin(x)}{\sqrt{a^2+b^2}}\right)}{a^2}+\frac{b\tanh^{-1}(\cos(x))}{a^2}-\frac{\csc(x)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[x]^2/(a*Cos[x] + b*Sin[x]),x]
```

```
[Out] (b*ArcTanh[Cos[x]])/a^2 - (Sqrt[a^2 + b^2]*ArcTanh[(b*Cos[x] - a*Sin[x])/Sq
rt[a^2 + b^2]])/a^2 - Csc[x]/a
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3074

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x
_Symbol] :> -Dist[d^(-1), Subst[Int[1/(a^2 + b^2 - x^2), x], x, b*Cos[c + d
*x] - a*Sin[c + d*x]], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

Rule 3103

```
Int[sin[(c_.) + (d_.)*(x_)]^(m_)/(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin
[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[Sin[c + d*x]^(m + 1)/(a*d*(m + 1))
, x] + (-Dist[b/a^2, Int[Sin[c + d*x]^(m + 1), x], x] + Dist[(a^2 + b^2)/a^
2, Int[Sin[c + d*x]^(m + 2)/(a*Cos[c + d*x] + b*Sin[c + d*x]), x], x]) /; F
reeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0] && LtQ[m, -1]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\csc^2(x)}{a\cos(x)+b\sin(x)}\,dx &= -\frac{\csc(x)}{a}-\frac{b\int \csc(x)\,dx}{a^2}+\frac{\left(a^2+b^2\right)\int \frac{1}{a\cos(x)+b\sin(x)}\,dx}{a^2}\\
&= \frac{b\tanh^{-1}(\cos(x))}{a^2}-\frac{\csc(x)}{a}-\frac{\left(a^2+b^2\right)\operatorname{Subst}\left(\int \frac{1}{a^2+b^2-x^2}\,dx,x,b\cos(x)-a\sin(x)\right)}{a^2}\\
&= \frac{b\tanh^{-1}(\cos(x))}{a^2}-\frac{\sqrt{a^2+b^2}\,\tanh^{-1}\left(\frac{b\cos(x)-a\sin(x)}{\sqrt{a^2+b^2}}\right)}{a^2}-\frac{\csc(x)}{a}
\end{aligned}
$$

Mathematica [A] time = 0.13, size = 67, normalized size = 1.22

$$
\frac{2\sqrt{a^2+b^2}\,\tanh^{-1}\left(\frac{a\tan\left(\frac{x}{2}\right)-b}{\sqrt{a^2+b^2}}\right)-a\csc(x)+b\left(\log\left(\cos\left(\frac{x}{2}\right)\right)-\log\left(\sin\left(\frac{x}{2}\right)\right)\right)}{a^2}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[x]^2/(a*Cos[x] + b*Sin[x]),x]
```

```
[Out] (2*Sqrt[a^2 + b^2]*ArcTanh[(-b + a*Tan[x/2])/Sqrt[a^2 + b^2]] - a*Csc[x] +
b*(Log[Cos[x/2]] - Log[Sin[x/2]]))/a^2
```

fricas [B] time = 0.49, size = 133, normalized size = 2.42

$$
\frac{b\log\left(\frac{1}{2}\cos(x)+\frac{1}{2}\right)\sin(x)-b\log\left(-\frac{1}{2}\cos(x)+\frac{1}{2}\right)\sin(x)+\sqrt{a^2+b^2}\,\log\left(-\frac{2\,ab\cos(x)\sin(x)+\left(a^2-b^2\right)\cos(x)^2-2\,a^2-b^2+\cdots}{2\,ab\cos(x)\sin(x)+\left(a^2-b^2\right)\cdots}\right.\cdots}{2\,a^2\sin(x)}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)^2/(a*cos(x)+b*sin(x)),x, algorithm="fricas")
```

```
[Out] 1/2*(b*log(1/2*cos(x) + 1/2)*sin(x) - b*log(-1/2*cos(x) + 1/2)*sin(x) + sqr
t(a^2 + b^2)*log(-(2*a*b*cos(x)*sin(x) + (a^2 - b^2)*cos(x)^2 - 2*a^2 - b^2
 + 2*sqrt(a^2 + b^2)*(b*cos(x) - a*sin(x)))/(2*a*b*cos(x)*sin(x) + (a^2 - b
^2)*cos(x)^2 + b^2))*sin(x) - 2*a)/(a^2*sin(x))
```

giac [B] time = 2.97, size = 108, normalized size = 1.96

$$-\frac{b\log\left(\left|\tan\left(\frac{1}{2}x\right)\right|\right)}{a^2}-\frac{\tan\left(\frac{1}{2}x\right)}{2a}-\frac{\sqrt{a^2+b^2}\log\left(\frac{\left|2a\tan\left(\frac{1}{2}x\right)-2b-2\sqrt{a^2+b^2}\right|}{\left|2a\tan\left(\frac{1}{2}x\right)-2b+2\sqrt{a^2+b^2}\right|}\right)}{a^2}+\frac{2b\tan\left(\frac{1}{2}x\right)-a}{2a^2\tan\left(\frac{1}{2}x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)^2/(a*cos(x)+b*sin(x)),x, algorithm="giac")
```

```
[Out] -b*log(abs(tan(1/2*x)))/a^2 - 1/2*tan(1/2*x)/a - sqrt(a^2 + b^2)*log(abs(2*
a*tan(1/2*x) - 2*b - 2*sqrt(a^2 + b^2))/abs(2*a*tan(1/2*x) - 2*b + 2*sqrt(a
^2 + b^2)))/a^2 + 1/2*(2*b*tan(1/2*x) - a)/(a^2*tan(1/2*x))
```

maple [B] time = 0.79, size = 107, normalized size = 1.95

$$-\frac{\tan\left(\frac{x}{2}\right)}{2a}-\frac{1}{2a\tan\left(\frac{x}{2}\right)}-\frac{b\ln\left(\tan\left(\frac{x}{2}\right)\right)}{a^2}+\frac{2\,\mathrm{arctanh}\left(\frac{2a\tan\left(\frac{x}{2}\right)-2b}{2\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}}+\frac{2\,\mathrm{arctanh}\left(\frac{2a\tan\left(\frac{x}{2}\right)-2b}{2\sqrt{a^2+b^2}}\right)b^2}{a^2\sqrt{a^2+b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(x)^2/(a*cos(x)+b*sin(x)),x)
```

```
[Out] -1/2/a*tan(1/2*x)-1/2/a/tan(1/2*x)-b/a^2*ln(tan(1/2*x))+2/(a^2+b^2)^(1/2)*a
rctanh(1/2*(2*a*tan(1/2*x)-2*b)/(a^2+b^2)^(1/2))+2/a^2/(a^2+b^2)^(1/2)*arct
anh(1/2*(2*a*tan(1/2*x)-2*b)/(a^2+b^2)^(1/2))*b^2
```

maxima [B] time = 0.43, size = 107, normalized size = 1.95

$$-\frac{b\log\left(\frac{\sin(x)}{\cos(x)+1}\right)}{a^2}-\frac{\sqrt{a^2+b^2}\log\left(\frac{b-\frac{a\sin(x)}{\cos(x)+1}+\sqrt{a^2+b^2}}{b-\frac{a\sin(x)}{\cos(x)+1}-\sqrt{a^2+b^2}}\right)}{a^2}-\frac{\cos(x)+1}{2a\sin(x)}-\frac{\sin(x)}{2a(\cos(x)+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)^2/(a*cos(x)+b*sin(x)),x, algorithm="maxima")
```

```
[Out] -b*log(sin(x)/(cos(x) + 1))/a^2 - sqrt(a^2 + b^2)*log((b - a*sin(x)/(cos(x)
 + 1) + sqrt(a^2 + b^2))/(b - a*sin(x)/(cos(x) + 1) - sqrt(a^2 + b^2)))/a^2
 - 1/2*(cos(x) + 1)/(a*sin(x)) - 1/2*sin(x)/(a*(cos(x) + 1))
```

mupad [B] time = 0.68, size = 170, normalized size = 3.09

$$\frac{2\,\mathrm{atanh}\left(\frac{a^3\cos\left(\frac{x}{2}\right)\sqrt{a^2+b^2}+4b^3\sin\left(\frac{x}{2}\right)\sqrt{a^2+b^2}+3a^2b\sin\left(\frac{x}{2}\right)\sqrt{a^2+b^2}+2ab^2\cos\left(\frac{x}{2}\right)\sqrt{a^2+b^2}}{\sin\left(\frac{x}{2}\right)a^4+2\cos\left(\frac{x}{2}\right)a^3b+5\sin\left(\frac{x}{2}\right)a^2b^2+2\cos\left(\frac{x}{2}\right)ab^3+4\sin\left(\frac{x}{2}\right)b^4}\right)\sqrt{a^2+b^2}}{a^2}-\frac{1}{a\sin(x)}-\frac{b\ln\left(\frac{\sin\left(\frac{x}{2}\right)}{\cos\left(\frac{x}{2}\right)}\right)}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(x)^2*(a*cos(x) + b*sin(x))),x)
```

```
[Out] (2*atanh((a^3*cos(x/2)*(a^2 + b^2)^(1/2) + 4*b^3*sin(x/2)*(a^2 + b^2)^(1/2)
 + 3*a^2*b*sin(x/2)*(a^2 + b^2)^(1/2) + 2*a*b^2*cos(x/2)*(a^2 + b^2)^(1/2))
/(a^4*sin(x/2) + 4*b^4*sin(x/2) + 5*a^2*b^2*sin(x/2) + 2*a*b^3*cos(x/2) + 2
*a^3*b*cos(x/2)))*(a^2 + b^2)^(1/2))/a^2 - 1/(a*sin(x)) - (b*log(sin(x/2)/c
os(x/2)))/a^2
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc^2(x)}{a\cos(x) + b\sin(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)**2/(a*cos(x)+b*sin(x)),x)
```

```
[Out] Integral(csc(x)**2/(a*cos(x) + b*sin(x)), x)
```

3.14 $\int \frac{\csc^3(x)}{a\cos(x)+b\sin(x)}\,dx$

Optimal. Leaf size=55

$$\frac{b\cot(x)}{a^2}+\frac{(a^2+b^2)\log(\sin(x))}{a^3}-\frac{(a^2+b^2)\log(a\cos(x)+b\sin(x))}{a^3}-\frac{\csc^2(x)}{2a}$$

```
[Out] b*cot(x)/a^2-1/2*csc(x)^2/a+(a^2+b^2)*ln(sin(x))/a^3-(a^2+b^2)*ln(a*cos(x)+
b*sin(x))/a^3
```

Rubi [A] time = 0.12, antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {3103, 3767, 8, 3101, 3475, 3133}

$$\frac{(a^2+b^2)\log(\sin(x))}{a^3}-\frac{(a^2+b^2)\log(a\cos(x)+b\sin(x))}{a^3}+\frac{b\cot(x)}{a^2}-\frac{\csc^2(x)}{2a}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[x]^3/(a*Cos[x] + b*Sin[x]),x]
```

```
[Out] (b*Cot[x])/a^2 - Csc[x]^2/(2*a) + ((a^2 + b^2)*Log[Sin[x]])/a^3 - ((a^2 + b
^2)*Log[a*Cos[x] + b*Sin[x]])/a^3
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3101

```
Int[1/(sin[(c_.) + (d_.)*(x_)]*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(
c_.) + (d_.)*(x_)])), x_Symbol] :> Dist[1/a, Int[Cot[c + d*x], x], x] - Dis
t[1/a, Int[(b*Cos[c + d*x] - a*Sin[c + d*x])/(a*Cos[c + d*x] + b*Sin[c + d*
x]), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

Rule 3103

```
Int[sin[(c_.) + (d_.)*(x_)]^(m_)/(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin
[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[Sin[c + d*x]^(m + 1)/(a*d*(m + 1))
, x] + (-Dist[b/a^2, Int[Sin[c + d*x]^(m + 1), x], x] + Dist[(a^2 + b^2)/a^
2, Int[Sin[c + d*x]^(m + 2)/(a*Cos[c + d*x] + b*Sin[c + d*x]), x], x]) /; F
reeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0] && LtQ[m, -1]
```

Rule 3133

```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])
/((a_.) + cos[(d_.) + (e_.)*(x_)]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)]), x
_Symbol] :> Simp[((b*B + c*C)*x)/(b^2 + c^2), x] + Simp[((c*B - b*C)*Log[a
+ b*Cos[d + e*x] + c*Sin[d + e*x]])/(e*(b^2 + c^2)), x] /; FreeQ[{a, b, c,
d, e, A, B, C}, x] && NeQ[b^2 + c^2, 0] && EqQ[A*(b^2 + c^2) - a*(b*B + c*C
), 0]
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\csc^3(x)}{a\cos(x)+b\sin(x)}\,dx &= -\frac{\csc^2(x)}{2a}-\frac{b\int \csc^2(x)\,dx}{a^2}+\frac{\left(a^2+b^2\right)\int \frac{\csc(x)}{a\cos(x)+b\sin(x)}\,dx}{a^2}\\ &= -\frac{\csc^2(x)}{2a}+\frac{b\,\text{Subst}\left(\int 1\,dx,x,\cot(x)\right)}{a^2}+\frac{\left(a^2+b^2\right)\int \cot(x)\,dx}{a^3}-\frac{\left(a^2+b^2\right)\int \frac{b\cos(x)-\ldots}{a\cos(x)+\ldots}}{a^3}\\ &= \frac{b\cot(x)}{a^2}-\frac{\csc^2(x)}{2a}+\frac{\left(a^2+b^2\right)\log(\sin(x))}{a^3}-\frac{\left(a^2+b^2\right)\log(a\cos(x)+b\sin(x))}{a^3}\end{aligned}$$

Mathematica [A] time = 0.16, size = 48, normalized size = 0.87

$$\frac{2\left(a^2+b^2\right)(\log(\sin(x))-\log(a\cos(x)+b\sin(x)))-a^2\csc^2(x)+2ab\cot(x)}{2a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[x]^3/(a*Cos[x] + b*Sin[x]),x]
```

```
[Out] (2*a*b*Cot[x] - a^2*Csc[x]^2 + 2*(a^2 + b^2)*(Log[Sin[x]] - Log[a*Cos[x] +
b*Sin[x]]))/(2*a^3)
```

fricas [B] time = 0.43, size = 117, normalized size = 2.13

$$-\frac{2\,ab\cos(x)\sin(x) - a^2 + \left(\left(a^2+b^2\right)\cos(x)^2 - a^2 - b^2\right)\log\left(2\,ab\cos(x)\sin(x) + \left(a^2-b^2\right)\cos(x)^2 + b^2\right) - \left(\left(a^2 + \cdots\right.\right.}{2\left(a^3\cos(x)^2 - a^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)^3/(a*cos(x)+b*sin(x)),x, algorithm="fricas")
```

```
[Out] -1/2*(2*a*b*cos(x)*sin(x) - a^2 + ((a^2 + b^2)*cos(x)^2 - a^2 - b^2)*log(2*
a*b*cos(x)*sin(x) + (a^2 - b^2)*cos(x)^2 + b^2) - ((a^2 + b^2)*cos(x)^2 - a
^2 - b^2)*log(-1/4*cos(x)^2 + 1/4))/(a^3*cos(x)^2 - a^3)
```

giac [A] time = 0.19, size = 78, normalized size = 1.42

$$\frac{\left(a^2+b^2\right)\log\left(|\tan(x)|\right)}{a^3} - \frac{\left(a^2b+b^3\right)\log\left(|b\tan(x)+a|\right)}{a^3b} - \frac{3\,a^2\tan(x)^2 + 3\,b^2\tan(x)^2 - 2\,ab\tan(x) + a^2}{2\,a^3\tan(x)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)^3/(a*cos(x)+b*sin(x)),x, algorithm="giac")
```

```
[Out] (a^2 + b^2)*log(abs(tan(x)))/a^3 - (a^2*b + b^3)*log(abs(b*tan(x) + a))/(a^
3*b) - 1/2*(3*a^2*tan(x)^2 + 3*b^2*tan(x)^2 - 2*a*b*tan(x) + a^2)/(a^3*tan(
x)^2)
```

maple [A] time = 11.61, size = 64, normalized size = 1.16

$$-\frac{\ln\left(a+b\tan(x)\right)}{a} - \frac{\ln\left(a+b\tan(x)\right)b^2}{a^3} - \frac{1}{2a\tan(x)^2} + \frac{\ln\left(\tan(x)\right)}{a} + \frac{\ln\left(\tan(x)\right)b^2}{a^3} + \frac{b}{a^2\tan(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(x)^3/(a*cos(x)+b*sin(x)),x)
```

```
[Out] -1/a*ln(a+b*tan(x))-1/a^3*ln(a+b*tan(x))*b^2-1/2/a/tan(x)^2+1/a*ln(tan(x))+
1/a^3*ln(tan(x))*b^2+b/a^2/tan(x)
```

maxima [B] time = 0.32, size = 119, normalized size = 2.16

$$-\frac{\frac{4\,b\sin(x)}{\cos(x)+1} + \frac{a\sin(x)^2}{(\cos(x)+1)^2}}{8\,a^2} - \frac{\left(a^2+b^2\right)\log\left(-a - \frac{2\,b\sin(x)}{\cos(x)+1} + \frac{a\sin(x)^2}{(\cos(x)+1)^2}\right)}{a^3} + \frac{\left(a^2+b^2\right)\log\left(\frac{\sin(x)}{\cos(x)+1}\right)}{a^3} - \frac{\left(a - \frac{4\,b\sin(x)}{\cos(x)+1}\right)(\cos(x) + \cdots}{8\,a^2\sin(x)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)^3/(a*cos(x)+b*sin(x)),x, algorithm="maxima")
```

```
[Out] -1/8*(4*b*sin(x)/(cos(x) + 1) + a*sin(x)^2/(cos(x) + 1)^2)/a^2 - (a^2 + b^2
)*log(-a - 2*b*sin(x)/(cos(x) + 1) + a*sin(x)^2/(cos(x) + 1)^2)/a^3 + (a^2
+ b^2)*log(sin(x)/(cos(x) + 1))/a^3 - 1/8*(a - 4*b*sin(x)/(cos(x) + 1))*(co
s(x) + 1)^2/(a^2*sin(x)^2)
```

mupad [B] time = 0.57, size = 91, normalized size = 1.65

$$\frac{\ln\left(\tan\left(\frac{x}{2}\right)\right)\left(a^2+b^2\right)}{a^3}-\frac{\ln\left(-a\tan\left(\frac{x}{2}\right)^2+2\,b\tan\left(\frac{x}{2}\right)+a\right)\left(a^2+b^2\right)}{a^3}-\frac{\tan\left(\frac{x}{2}\right)^2}{8\,a}-\frac{b\tan\left(\frac{x}{2}\right)}{2\,a^2}-\frac{\frac{a}{2}-2\,b\tan\left(\frac{x}{2}\right)}{4\,a^2\tan\left(\frac{x}{2}\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(x)^3*(a*cos(x) + b*sin(x))),x)
```

```
[Out] (log(tan(x/2))*(a^2 + b^2))/a^3 - (log(a + 2*b*tan(x/2) - a*tan(x/2)^2)*(a^
2 + b^2))/a^3 - tan(x/2)^2/(8*a) - (b*tan(x/2))/(2*a^2) - (a/2 - 2*b*tan(x/
2))/(4*a^2*tan(x/2)^2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc^3(x)}{a\cos(x)+b\sin(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)**3/(a*cos(x)+b*sin(x)),x)
```

```
[Out] Integral(csc(x)**3/(a*cos(x) + b*sin(x)), x)
```

3.15 $\int \frac{\sin^3(x)}{(a\cos(x)+b\sin(x))^2}\,dx$

Optimal. Leaf size=107

$$\frac{-b\left(a^2+b^2\right)\sin(2x)+a\left(a^2+b^2\right)\cos(2x)+3a\left(a^2-b^2\right)}{2\left(a^2+b^2\right)^2\left(a\cos(x)+b\sin(x)\right)}+\frac{6a^2b\tanh^{-1}\left(\frac{a\tan\left(\frac{x}{2}\right)-b}{\sqrt{a^2+b^2}}\right)}{\left(a^2+b^2\right)^{5/2}}$$

```
[Out] 6*a^2*b*arctanh((-b+a*tan(1/2*x))/(a^2+b^2)^(1/2))/(a^2+b^2)^(5/2)+1/2*(3*a
*(a^2-b^2)+a*(a^2+b^2)*cos(2*x)-b*(a^2+b^2)*sin(2*x))/(a^2+b^2)^2/(a*cos(x)
+b*sin(x))
```

Rubi [B] time = 1.17, antiderivative size = 283, normalized size of antiderivative = 2.64, number of steps used = 19, number of rules used = 11, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.688, Rules used = {4401, 2637, 2638, 6742, 639, 203, 638, 618, 206, 3100, 3074}

$$\frac{3a^3\sin(x)}{b^3\left(a^2+b^2\right)}+\frac{3a^2\cos(x)}{b^2\left(a^2+b^2\right)}+\frac{2a^2\left(a+b\tan\left(\frac{x}{2}\right)\right)}{\left(a^2+b^2\right)^2\left(-a\tan^2\left(\frac{x}{2}\right)+a+2b\tan\left(\frac{x}{2}\right)\right)}-\frac{2a^3\cos^2\left(\frac{x}{2}\right)\left(\left(a^2-b^2\right)\tan\left(\frac{x}{2}\right)+2ab\right)}{b^3\left(a^2+b^2\right)^2}+\frac{2a}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[x]^3/(a*Cos[x] + b*Sin[x])^2,x]
```

```
[Out] (-3*a^2*ArcTanh[(b*Cos[x] - a*Sin[x])/Sqrt[a^2 + b^2]])/(b*(a^2 + b^2)^(3/2
)) - (2*a^2*b*ArcTanh[(b - a*Tan[x/2])/Sqrt[a^2 + b^2]])/(a^2 + b^2)^(5/2)
+ (2*a^2*(3*a^2 + b^2)*ArcTanh[(b - a*Tan[x/2])/Sqrt[a^2 + b^2]])/(b*(a^2 +
 b^2)^(5/2)) - Cos[x]/b^2 + (3*a^2*Cos[x])/(b^2*(a^2 + b^2)) - (2*a*Sin[x])
/b^3 + (3*a^3*Sin[x])/(b^3*(a^2 + b^2)) - (2*a^3*Cos[x/2]^2*(2*a*b + (a^2 -
 b^2)*Tan[x/2]))/(b^3*(a^2 + b^2)^2) + (2*a^2*(a + b*Tan[x/2]))/((a^2 + b^2
)^2*(a + 2*b*Tan[x/2] - a*Tan[x/2]^2))
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 638

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[((b*d - 2*a*e + (2*c*d - b*e)*x)*(a + b*x + c*x^2)^(p + 1))/((p +
 1)*(b^2 - 4*a*c)), x] - Dist[((2*p + 3)*(2*c*d - b*e))/((p + 1)*(b^2 - 4*a
*c)), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] &&
 NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2]
```

Rule 639

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((a*e
 - c*d*x)*(a + c*x^2)^(p + 1))/(2*a*c*(p + 1)), x] + Dist[(d*(2*p + 3))/(2*
a*(p + 1)), Int[(a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e}, x] && Lt
Q[p, -1] && NeQ[p, -3/2]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3074

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x
_Symbol] :> -Dist[d^(-1), Subst[Int[1/(a^2 + b^2 - x^2), x], x, b*Cos[c + d
*x] - a*Sin[c + d*x]], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

Rule 3100

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_)/(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin
[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[(b*Cos[c + d*x]^(m - 1))/(d*(a^2 +
 b^2)*(m - 1)), x] + (Dist[a/(a^2 + b^2), Int[Cos[c + d*x]^(m - 1), x], x]
+ Dist[b^2/(a^2 + b^2), Int[Cos[c + d*x]^(m - 2)/(a*Cos[c + d*x] + b*Sin[c
+ d*x]), x], x]) /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0] && GtQ[m, 1
```

```
]
```

Rule 4401

```
Int[u_, x_Symbol] :> With[{v = ExpandTrig[u, x]}, Int[v, x] /; SumQ[v]] /;
 !InertTrigFreeQ[u]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int \frac{\sin^3(x)}{(a\cos(x)+b\sin(x))^2}\,dx = \int \left(-\frac{2a\cos(x)}{b^3}+\frac{\sin(x)}{b^2}-\frac{a^3\cos^3(x)}{b^3(a\cos(x)+b\sin(x))^2}+\frac{3a^2\cos^2(x)}{b^3(a\cos(x)+b\sin(x))}\right)dx$$

$$= -\frac{(2a)\int\cos(x)\,dx}{b^3}+\frac{\left(3a^2\right)\int\frac{\cos^2(x)}{a\cos(x)+b\sin(x)}\,dx}{b^3}-\frac{a^3\int\frac{\cos^3(x)}{(a\cos(x)+b\sin(x))^2}\,dx}{b^3}+\frac{\int\sin(}{b^2}$$

$$= -\frac{\cos(x)}{b^2}+\frac{3a^2\cos(x)}{b^2\left(a^2+b^2\right)}-\frac{2a\sin(x)}{b^3}-\frac{\left(2a^3\right)\mathrm{Subst}\left(\int\frac{\left(1-x^2\right)^3}{\left(1+x^2\right)^2\left(a+2bx-ax^2\right)^2}\,dx,x,\tan\right.}{b^3}$$

$$= -\frac{\cos(x)}{b^2}+\frac{3a^2\cos(x)}{b^2\left(a^2+b^2\right)}-\frac{2a\sin(x)}{b^3}+\frac{3a^3\sin(x)}{b^3\left(a^2+b^2\right)}-\frac{\left(2a^3\right)\mathrm{Subst}\left(\int\left(\frac{2\left(a^2-b^2-2ab\right.}{\left(a^2+b^2\right)^2\left(1+x\right.}\right.\right.}{}$$

$$= -\frac{3a^2\tanh^{-1}\left(\frac{b\cos(x)-a\sin(x)}{\sqrt{a^2+b^2}}\right)}{b\left(a^2+b^2\right)^{3/2}}-\frac{\cos(x)}{b^2}+\frac{3a^2\cos(x)}{b^2\left(a^2+b^2\right)}-\frac{2a\sin(x)}{b^3}+\frac{3a^3\sin(x)}{b^3\left(a^2+b^2\right)}$$

$$= \frac{a^3\left(a^2-b^2\right)x}{b^3\left(a^2+b^2\right)^2}-\frac{3a^2\tanh^{-1}\left(\frac{b\cos(x)-a\sin(x)}{\sqrt{a^2+b^2}}\right)}{b\left(a^2+b^2\right)^{3/2}}-\frac{\cos(x)}{b^2}+\frac{3a^2\cos(x)}{b^2\left(a^2+b^2\right)}-\frac{2a\sin(x)}{b^3}$$

$$= -\frac{3a^2\tanh^{-1}\left(\frac{b\cos(x)-a\sin(x)}{\sqrt{a^2+b^2}}\right)}{b\left(a^2+b^2\right)^{3/2}}+\frac{2a^2\left(3a^2+b^2\right)\tanh^{-1}\left(\frac{b-a\tan\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{b\left(a^2+b^2\right)^{5/2}}-\frac{\cos(x)}{b^2}+\frac{3a}{b^2(}$$

$$= -\frac{3a^2\tanh^{-1}\left(\frac{b\cos(x)-a\sin(x)}{\sqrt{a^2+b^2}}\right)}{b\left(a^2+b^2\right)^{3/2}}-\frac{2a^2b\tanh^{-1}\left(\frac{b-a\tan\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{\left(a^2+b^2\right)^{5/2}}+\frac{2a^2\left(3a^2+b^2\right)\tanh^{-1}}{b\left(a^2+b^2\right)^{5/}}$$

Mathematica [A] time = 0.44, size = 107, normalized size = 1.00

$$\frac{-b\left(a^2+b^2\right)\sin(2x)+a\left(a^2+b^2\right)\cos(2x)+3a\left(a^2-b^2\right)}{2\left(a^2+b^2\right)^2(a\cos(x)+b\sin(x))}+\frac{6a^2b\tanh^{-1}\left(\frac{a\tan\left(\frac{x}{2}\right)-b}{\sqrt{a^2+b^2}}\right)}{\left(a^2+b^2\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[x]^3/(a*Cos[x] + b*Sin[x])^2,x]
```

```
[Out] (6*a^2*b*ArcTanh[(-b + a*Tan[x/2])/Sqrt[a^2 + b^2]])/(a^2 + b^2)^(5/2) + (3
*a*(a^2 - b^2) + a*(a^2 + b^2)*Cos[2*x] - b*(a^2 + b^2)*Sin[2*x])/(2*(a^2 +
 b^2)^2*(a*Cos[x] + b*Sin[x]))
```

fricas [B] time = 0.45, size = 240, normalized size = 2.24

$$\frac{2\,a^{5}-2\,a^{3}b^{2}-4\,ab^{4}+2\left(a^{5}+2\,a^{3}b^{2}+ab^{4}\right)\cos(x)^{2}-2\left(a^{4}b+2\,a^{2}b^{3}+b^{5}\right)\cos(x)\sin(x)+3\left(a^{3}b\cos(x)+a^{2}b^{2}\,\mathrm{s}\right.}{2\left(\left(a^{7}+3\,a^{5}b^{2}+3\,a^{3}b^{4}+ab^{6}\right)\cos(x)+\left(a^{6}b+3\,a^{4}b^{3}\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^3/(a*cos(x)+b*sin(x))^2,x, algorithm="fricas")
```

```
[Out] 1/2*(2*a^5 - 2*a^3*b^2 - 4*a*b^4 + 2*(a^5 + 2*a^3*b^2 + a*b^4)*cos(x)^2 - 2
*(a^4*b + 2*a^2*b^3 + b^5)*cos(x)*sin(x) + 3*(a^3*b*cos(x) + a^2*b^2*sin(x)
)*sqrt(a^2 + b^2)*log(-(2*a*b*cos(x)*sin(x) + (a^2 - b^2)*cos(x)^2 - 2*a^2
- b^2 + 2*sqrt(a^2 + b^2)*(b*cos(x) - a*sin(x)))/(2*a*b*cos(x)*sin(x) + (a^
2 - b^2)*cos(x)^2 + b^2)))/((a^7 + 3*a^5*b^2 + 3*a^3*b^4 + a*b^6)*cos(x) +
(a^6*b + 3*a^4*b^3 + 3*a^2*b^5 + b^7)*sin(x))
```

giac [A] time = 4.74, size = 186, normalized size = 1.74

$$-\frac{3\,a^{2}b\log\left(\frac{\left|2\,a\tan\left(\frac{1}{2}\,x\right)-2\,b-2\,\sqrt{a^{2}+b^{2}}\right|}{\left|2\,a\tan\left(\frac{1}{2}\,x\right)-2\,b+2\,\sqrt{a^{2}+b^{2}}\right|}\right)}{\left(a^{4}+2\,a^{2}b^{2}+b^{4}\right)\sqrt{a^{2}+b^{2}}}-\frac{2\left(3\,a^{2}b\tan\left(\frac{1}{2}\,x\right)^{3}-3\,ab^{2}\tan\left(\frac{1}{2}\,x\right)^{2}+a^{2}b\tan\left(\frac{1}{2}\,x\right)-2\,b^{3}\tan\left(\frac{1}{2}\,x\right)+2\,a^{3}\right.}{\left(a\tan\left(\frac{1}{2}\,x\right)^{4}-2\,b\tan\left(\frac{1}{2}\,x\right)^{3}-2\,b\tan\left(\frac{1}{2}\,x\right)-a\right)\left(a^{4}+2\,a^{2}b^{2}+b^{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^3/(a*cos(x)+b*sin(x))^2,x, algorithm="giac")
```

```
[Out] -3*a^2*b*log(abs(2*a*tan(1/2*x) - 2*b - 2*sqrt(a^2 + b^2))/abs(2*a*tan(1/2*
x) - 2*b + 2*sqrt(a^2 + b^2)))/((a^4 + 2*a^2*b^2 + b^4)*sqrt(a^2 + b^2)) -
2*(3*a^2*b*tan(1/2*x)^3 - 3*a*b^2*tan(1/2*x)^2 + a^2*b*tan(1/2*x) - 2*b^3*t
an(1/2*x) + 2*a^3 - a*b^2)/((a*tan(1/2*x)^4 - 2*b*tan(1/2*x)^3 - 2*b*tan(1/
2*x) - a)*(a^4 + 2*a^2*b^2 + b^4))
```

maple [A] time = 0.55, size = 141, normalized size = 1.32

$$-\frac{4a^{2}\left(\frac{\frac{\tan\left(\frac{x}{2}\right)b}{2}+\frac{a}{2}}{a\left(\tan^{2}\left(\frac{x}{2}\right)\right)-2\tan\left(\frac{x}{2}\right)b-a}-\frac{3b\,\mathrm{arctanh}\left(\frac{2a\tan\left(\frac{x}{2}\right)-2b}{2\sqrt{a^{2}+b^{2}}}\right)}{2\sqrt{a^{2}+b^{2}}}\right)}{a^{4}+2a^{2}b^{2}+b^{4}}+\frac{-4ab\tan\left(\frac{x}{2}\right)+2a^{2}-2b^{2}}{\left(a^{4}+2a^{2}b^{2}+b^{4}\right)\left(\tan^{2}\left(\frac{x}{2}\right)+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)^3/(a*cos(x)+b*sin(x))^2,x)
```

```
[Out] -4*a^2/(a^4+2*a^2*b^2+b^4)*((1/2*tan(1/2*x)*b+1/2*a)/(a*tan(1/2*x)^2-2*tan(
1/2*x)*b-a)-3/2*b/(a^2+b^2)^(1/2)*arctanh(1/2*(2*a*tan(1/2*x)-2*b)/(a^2+b^2
)^(1/2)))+4/(a^4+2*a^2*b^2+b^4)*(-a*b*tan(1/2*x)+1/2*a^2-1/2*b^2)/(tan(1/2*
x)^2+1)
```

maxima [B] time = 0.42, size = 253, normalized size = 2.36

$$-\frac{3\,a^2b\log\left(\frac{b-\frac{a\sin(x)}{\cos(x)+1}+\sqrt{a^2+b^2}}{b-\frac{a\sin(x)}{\cos(x)+1}-\sqrt{a^2+b^2}}\right)}{\left(a^4+2\,a^2b^2+b^4\right)\sqrt{a^2+b^2}}+\frac{2\left(2\,a^3-ab^2-\frac{3\,ab^2\sin(x)^2}{(\cos(x)+1)^2}+\frac{3\,a^2b\sin(x)^3}{(\cos(x)+1)^3}+\frac{\left(a^2b-2\,b^3\right)\sin(x)}{\cos(x)+1}\right)}{a^5+2\,a^3b^2+ab^4+\frac{2\left(a^4b+2\,a^2b^3+b^5\right)\sin(x)}{\cos(x)+1}+\frac{2\left(a^4b+2\,a^2b^3+b^5\right)\sin(x)^3}{(\cos(x)+1)^3}-\frac{\left(a^5+2\,a^3b^2+ab^4\right)\sin(x)^4}{(\cos(x)+1)^4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^3/(a*cos(x)+b*sin(x))^2,x, algorithm="maxima")
```

```
[Out] -3*a^2*b*log((b - a*sin(x)/(cos(x) + 1) + sqrt(a^2 + b^2))/(b - a*sin(x)/(c
os(x) + 1) - sqrt(a^2 + b^2)))/((a^4 + 2*a^2*b^2 + b^4)*sqrt(a^2 + b^2)) +
2*(2*a^3 - a*b^2 - 3*a*b^2*sin(x)^2/(cos(x) + 1)^2 + 3*a^2*b*sin(x)^3/(cos(
x) + 1)^3 + (a^2*b - 2*b^3)*sin(x)/(cos(x) + 1))/(a^5 + 2*a^3*b^2 + a*b^4 +
 2*(a^4*b + 2*a^2*b^3 + b^5)*sin(x)/(cos(x) + 1) + 2*(a^4*b + 2*a^2*b^3 + b
^5)*sin(x)^3/(cos(x) + 1)^3 - (a^5 + 2*a^3*b^2 + a*b^4)*sin(x)^4/(cos(x) +
1)^4)
```

mupad [B] time = 0.84, size = 224, normalized size = 2.09

$$-\frac{\frac{2\left(a\,b^2-2\,a^3\right)}{a^4+2\,a^2\,b^2+b^4}-\frac{2\tan\left(\frac{x}{2}\right)\left(a^2\,b-2\,b^3\right)}{a^4+2\,a^2\,b^2+b^4}+\frac{6\,a\,b^2\tan\left(\frac{x}{2}\right)^2}{a^4+2\,a^2\,b^2+b^4}-\frac{6\,a^2\,b\tan\left(\frac{x}{2}\right)^3}{a^4+2\,a^2\,b^2+b^4}}{-a\tan\left(\frac{x}{2}\right)^4+2\,b\tan\left(\frac{x}{2}\right)^3+2\,b\tan\left(\frac{x}{2}\right)+a}-\frac{6\,a^2\,b\,\mathrm{atanh}\left(\frac{2\,a^4\,b+2\,b^5+4\,a^2\,b^3-2\,a\tan\left(\frac{x}{2}\right)\left(a^4+2\,a^2\,b^2+b^4\right)}{2\left(a^2+b^2\right)^{5/2}}\right)}{\left(a^2+b^2\right)^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)^3/(a*cos(x) + b*sin(x))^2,x)
```

```
[Out] - ((2*(a*b^2 - 2*a^3))/(a^4 + b^4 + 2*a^2*b^2) - (2*tan(x/2)*(a^2*b - 2*b^3
))/(a^4 + b^4 + 2*a^2*b^2) + (6*a*b^2*tan(x/2)^2)/(a^4 + b^4 + 2*a^2*b^2) -
 (6*a^2*b*tan(x/2)^3)/(a^4 + b^4 + 2*a^2*b^2))/(a + 2*b*tan(x/2) - a*tan(x/
2)^4 + 2*b*tan(x/2)^3) - (6*a^2*b*atanh((2*a^4*b + 2*b^5 + 4*a^2*b^3 - 2*a*
tan(x/2)*(a^4 + b^4 + 2*a^2*b^2))/(2*(a^2 + b^2)^(5/2))))/(a^2 + b^2)^(5/2)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)**3/(a*cos(x)+b*sin(x))**2,x)
```

```
[Out] Timed out
```

3.16 $\int \frac{\sin^2(x)}{(a\cos(x)+b\sin(x))^2}\,dx$

Optimal. Leaf size=64

$$-\frac{x\left(a^2-b^2\right)}{\left(a^2+b^2\right)^2}+\frac{a}{\left(a^2+b^2\right)(a\cot(x)+b)}-\frac{2ab\log(a\cos(x)+b\sin(x))}{\left(a^2+b^2\right)^2}$$

```
[Out] -(a^2-b^2)*x/(a^2+b^2)^2+a/(a^2+b^2)/(b+a*cot(x))-2*a*b*ln(a*cos(x)+b*sin(x
))/(a^2+b^2)^2
```

Rubi [A] time = 0.12, antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {3085, 3483, 3531, 3530}

$$-\frac{x\left(a^2-b^2\right)}{\left(a^2+b^2\right)^2}+\frac{a}{\left(a^2+b^2\right)(a\cot(x)+b)}-\frac{2ab\log(a\cos(x)+b\sin(x))}{\left(a^2+b^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[x]^2/(a*Cos[x] + b*Sin[x])^2,x]
```

```
[Out] -(((a^2 - b^2)*x)/(a^2 + b^2)^2) + a/((a^2 + b^2)*(b + a*Cot[x])) - (2*a*b*
Log[a*Cos[x] + b*Sin[x]])/(a^2 + b^2)^2
```

Rule 3085

```
Int[sin[(c_.) + (d_.)*(x_)]^(m_)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin
[(c_.) + (d_.)*(x_)])^(n_.), x_Symbol] :> Int[(b + a*Cot[c + d*x])^n, x] /;
 FreeQ[{a, b, c, d}, x] && EqQ[m + n, 0] && IntegerQ[n] && NeQ[a^2 + b^2, 0
]
```

Rule 3483

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(a +
b*Tan[c + d*x])^(n + 1))/(d*(n + 1)*(a^2 + b^2)), x] + Dist[1/(a^2 + b^2),
Int[(a - b*Tan[c + d*x])*(a + b*Tan[c + d*x])^(n + 1), x], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 + b^2, 0] && LtQ[n, -1]
```

Rule 3530

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_) + (b_.)*tan[(e_.) + (f_.)*
(x_)]), x_Symbol] :> Simp[(c*Log[RemoveContent[a*Cos[e + f*x] + b*Sin[e + f
*x], x]])/(b*f), x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
```

```
NeQ[a^2 + b^2, 0] && EqQ[a*c + b*d, 0]
```

Rule 3531

```
Int[((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_.) + (b_.)*tan[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[((a*c + b*d)*x)/(a^2 + b^2), x] + Dist[(b*c - a
*d)/(a^2 + b^2), Int[(b - a*Tan[e + f*x])/(a + b*Tan[e + f*x]), x], x] /; F
reeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && Ne
Q[a*c + b*d, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sin^2(x)}{(a\cos(x)+b\sin(x))^2}\,dx &= \int \frac{1}{(b+a\cot(x))^2}\,dx \\
&= \frac{a}{\left(a^2+b^2\right)(b+a\cot(x))} + \frac{\int \frac{b-a\cot(x)}{b+a\cot(x)}\,dx}{a^2+b^2} \\
&= -\frac{\left(a^2-b^2\right)x}{\left(a^2+b^2\right)^2} + \frac{a}{\left(a^2+b^2\right)(b+a\cot(x))} - \frac{(2ab)\int \frac{-a+b\cot(x)}{b+a\cot(x)}\,dx}{\left(a^2+b^2\right)^2} \\
&= -\frac{\left(a^2-b^2\right)x}{\left(a^2+b^2\right)^2} + \frac{a}{\left(a^2+b^2\right)(b+a\cot(x))} - \frac{2ab\log(a\cos(x)+b\sin(x))}{\left(a^2+b^2\right)^2}
\end{aligned}
$$

Mathematica [C] time = 0.26, size = 121, normalized size = 1.89

$$
\frac{\sin(x)\left(a^3-a^2bx+ab^2(1-2ix)-ab^2\log\left((a\cos(x)+b\sin(x))^2\right)+b^3x\right)-a\cos(x)\left(ab\log\left((a\cos(x)+b\sin(x))^{\cdots}\right.\right.}{\left(a^2+b^2\right)^2(a\cos(x)+b\sin(x))}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[x]^2/(a*Cos[x] + b*Sin[x])^2,x]
```

```
[Out] (-(a*Cos[x]*((a + I*b)^2*x + a*b*Log[(a*Cos[x] + b*Sin[x])^2])) + (a^3 + a*
b^2*(1 - (2*I)*x) - a^2*b*x + b^3*x - a*b^2*Log[(a*Cos[x] + b*Sin[x])^2])*S
in[x] + (2*I)*a*b*ArcTan[Tan[x]]*(a*Cos[x] + b*Sin[x]))/((a^2 + b^2)^2*(a*C
os[x] + b*Sin[x]))
```

fricas [B] time = 0.71, size = 132, normalized size = 2.06

$$
-\frac{\left(a^2b+\left(a^3-ab^2\right)x\right)\cos(x)+\left(a^2b\cos(x)+ab^2\sin(x)\right)\log\left(2\,ab\cos(x)\sin(x)+\left(a^2-b^2\right)\cos(x)^2+b^2\right)-\left(a^3-\cdots\right.}{\left(a^5+2\,a^3b^2+ab^4\right)\cos(x)+\left(a^4b+2\,a^2b^3+b^5\right)\sin(x)}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^2/(a*cos(x)+b*sin(x))^2,x, algorithm="fricas")
```

```
[Out] -((a^2*b + (a^3 - a*b^2)*x)*cos(x) + (a^2*b*cos(x) + a*b^2*sin(x))*log(2*a*
b*cos(x)*sin(x) + (a^2 - b^2)*cos(x)^2 + b^2) - (a^3 - (a^2*b - b^3)*x)*sin
(x))/((a^5 + 2*a^3*b^2 + a*b^4)*cos(x) + (a^4*b + 2*a^2*b^3 + b^5)*sin(x))
```

giac [B] time = 1.98, size = 139, normalized size = 2.17

$$-\frac{2\,ab^2\log\left(|b\tan(x)+a|\right)}{a^4b+2\,a^2b^3+b^5}+\frac{ab\log\left(\tan(x)^2+1\right)}{a^4+2\,a^2b^2+b^4}-\frac{\left(a^2-b^2\right)x}{a^4+2\,a^2b^2+b^4}+\frac{2\,ab^3\tan(x)-a^4+a^2b^2}{\left(a^4b+2\,a^2b^3+b^5\right)\left(b\tan(x)+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^2/(a*cos(x)+b*sin(x))^2,x, algorithm="giac")
```

```
[Out] -2*a*b^2*log(abs(b*tan(x) + a))/(a^4*b + 2*a^2*b^3 + b^5) + a*b*log(tan(x)^
2 + 1)/(a^4 + 2*a^2*b^2 + b^4) - (a^2 - b^2)*x/(a^4 + 2*a^2*b^2 + b^4) + (2
*a*b^3*tan(x) - a^4 + a^2*b^2)/((a^4*b + 2*a^2*b^3 + b^5)*(b*tan(x) + a))
```

maple [A] time = 0.56, size = 99, normalized size = 1.55

$$-\frac{a^2}{\left(a^2+b^2\right)b\left(a+b\tan(x)\right)}-\frac{2ab\ln\left(a+b\tan(x)\right)}{\left(a^2+b^2\right)^2}+\frac{ab\ln\left(1+\tan^2(x)\right)}{\left(a^2+b^2\right)^2}-\frac{\arctan\left(\tan(x)\right)a^2}{\left(a^2+b^2\right)^2}+\frac{\arctan\left(\tan(x)\right)b^2}{\left(a^2+b^2\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)^2/(a*cos(x)+b*sin(x))^2,x)
```

```
[Out] -a^2/(a^2+b^2)/b/(a+b*tan(x))-2*a*b/(a^2+b^2)^2*ln(a+b*tan(x))+1/(a^2+b^2)^
2*a*b*ln(1+tan(x)^2)-1/(a^2+b^2)^2*arctan(tan(x))*a^2+1/(a^2+b^2)^2*arctan(
tan(x))*b^2
```

maxima [A] time = 0.42, size = 117, normalized size = 1.83

$$-\frac{2\,ab\log\left(b\tan(x)+a\right)}{a^4+2\,a^2b^2+b^4}+\frac{ab\log\left(\tan(x)^2+1\right)}{a^4+2\,a^2b^2+b^4}-\frac{a^2}{a^3b+ab^3+\left(a^2b^2+b^4\right)\tan(x)}-\frac{\left(a^2-b^2\right)x}{a^4+2\,a^2b^2+b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^2/(a*cos(x)+b*sin(x))^2,x, algorithm="maxima")
```

```
[Out] -2*a*b*log(b*tan(x) + a)/(a^4 + 2*a^2*b^2 + b^4) + a*b*log(tan(x)^2 + 1)/(a
^4 + 2*a^2*b^2 + b^4) - a^2/(a^3*b + a*b^3 + (a^2*b^2 + b^4)*tan(x)) - (a^2
 - b^2)*x/(a^4 + 2*a^2*b^2 + b^4)
```

mupad [B] time = 7.06, size = 626, normalized size = 9.78

$$a^3\ \sin(x) + a\,b^2\ \sin(x) - 2\,a^3\,\mathrm{atan}\left(\frac{\sin\left(\frac{x}{2}\right)}{\cos\left(\frac{x}{2}\right)}\right)\,\cos(x) + 2\,b^3\,\mathrm{atan}\left(\frac{\sin\left(\frac{x}{2}\right)}{\cos\left(\frac{x}{2}\right)}\right)\,\sin(x) + 2\,a\,b^2\,\mathrm{atan}\left(\frac{\sin\left(\frac{x}{2}\right)}{\cos\left(\frac{x}{2}\right)}\right)\,\cos(x) - 2\,a^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)^2/(a*cos(x) + b*sin(x))^2,x)
```

```
[Out] (a^3*sin(x) + a*b^2*sin(x) - 2*a^3*atan(sin(x/2)/cos(x/2))*cos(x) + 2*b^3*a
tan(sin(x/2)/cos(x/2))*sin(x) + 2*a*b^2*atan(sin(x/2)/cos(x/2))*cos(x) - 2*
a^2*b*atan(sin(x/2)/cos(x/2))*sin(x) + 2*a^2*b*cos(x)*log((1024*a^14 + 1024
*a^2*b^12 + 26624*a^4*b^10 + 146432*a^6*b^8 - 348160*a^8*b^6 + 146432*a^10*
b^4 + 26624*a^12*b^2)/(a^16/2 + b^16/2 + 4*a^2*b^14 + 14*a^4*b^12 + 28*a^6*
b^10 + 35*a^8*b^8 + 28*a^10*b^6 + 14*a^12*b^4 + 4*a^14*b^2 + (a^16*cos(x))/
2 + (b^16*cos(x))/2 + 4*a^2*b^14*cos(x) + 14*a^4*b^12*cos(x) + 28*a^6*b^10*
cos(x) + 35*a^8*b^8*cos(x) + 28*a^10*b^6*cos(x) + 14*a^12*b^4*cos(x) + 4*a^
14*b^2*cos(x))) + 2*a*b^2*log((1024*a^14 + 1024*a^2*b^12 + 26624*a^4*b^10 +
 146432*a^6*b^8 - 348160*a^8*b^6 + 146432*a^10*b^4 + 26624*a^12*b^2)/(a^16/
2 + b^16/2 + 4*a^2*b^14 + 14*a^4*b^12 + 28*a^6*b^10 + 35*a^8*b^8 + 28*a^10*
b^6 + 14*a^12*b^4 + 4*a^14*b^2 + (a^16*cos(x))/2 + (b^16*cos(x))/2 + 4*a^2*
b^14*cos(x) + 14*a^4*b^12*cos(x) + 28*a^6*b^10*cos(x) + 35*a^8*b^8*cos(x) +
 28*a^10*b^6*cos(x) + 14*a^12*b^4*cos(x) + 4*a^14*b^2*cos(x)))*sin(x) - 2*a
^2*b*log((a*cos(x) + b*sin(x))/cos(x/2)^2)*cos(x) - 2*a*b^2*log((a*cos(x) +
 b*sin(x))/cos(x/2)^2)*sin(x))/(b^5*sin(x) + a^5*cos(x) + a*b^4*cos(x) + a^
4*b*sin(x) + 2*a^3*b^2*cos(x) + 2*a^2*b^3*sin(x))
```

sympy [A] time = 2.20, size = 1017, normalized size = 15.89

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)**2/(a*cos(x)+b*sin(x))**2,x)
```

```
[Out] Piecewise((zoo*x, Eq(a, 0) & Eq(b, 0)), (2*x*sin(x)**2/(8*b**2*sin(x)**2 -
16*I*b**2*sin(x)*cos(x) - 8*b**2*cos(x)**2) - 4*I*x*sin(x)*cos(x)/(8*b**2*s
in(x)**2 - 16*I*b**2*sin(x)*cos(x) - 8*b**2*cos(x)**2) - 2*x*cos(x)**2/(8*b
**2*sin(x)**2 - 16*I*b**2*sin(x)*cos(x) - 8*b**2*cos(x)**2) + 3*I*sin(x)**2
/(8*b**2*sin(x)**2 - 16*I*b**2*sin(x)*cos(x) - 8*b**2*cos(x)**2) + I*cos(x)
**2/(8*b**2*sin(x)**2 - 16*I*b**2*sin(x)*cos(x) - 8*b**2*cos(x)**2), Eq(a,
-I*b)), (-2*I*x*sin(x)**2/(-8*I*b**2*sin(x)**2 + 16*b**2*sin(x)*cos(x) + 8*
I*b**2*cos(x)**2) + 4*x*sin(x)*cos(x)/(-8*I*b**2*sin(x)**2 + 16*b**2*sin(x)
*cos(x) + 8*I*b**2*cos(x)**2) + 2*I*x*cos(x)**2/(-8*I*b**2*sin(x)**2 + 16*b
```

```
**2*sin(x)*cos(x) + 8*I*b**2*cos(x)**2) - 3*sin(x)**2/(-8*I*b**2*sin(x)**2
+ 16*b**2*sin(x)*cos(x) + 8*I*b**2*cos(x)**2) - cos(x)**2/(-8*I*b**2*sin(x)
**2 + 16*b**2*sin(x)*cos(x) + 8*I*b**2*cos(x)**2), Eq(a, I*b)), ((-x + sin(
x)/cos(x))/a**2, Eq(b, 0)), (-a**4*cos(x)/(a**5*b*cos(x) + a**4*b**2*sin(x)
 + 2*a**3*b**3*cos(x) + 2*a**2*b**4*sin(x) + a*b**5*cos(x) + b**6*sin(x)) -
 a**3*b*x*cos(x)/(a**5*b*cos(x) + a**4*b**2*sin(x) + 2*a**3*b**3*cos(x) + 2
*a**2*b**4*sin(x) + a*b**5*cos(x) + b**6*sin(x)) - a**2*b**2*x*sin(x)/(a**5
*b*cos(x) + a**4*b**2*sin(x) + 2*a**3*b**3*cos(x) + 2*a**2*b**4*sin(x) + a*
b**5*cos(x) + b**6*sin(x)) - 2*a**2*b**2*log(a*cos(x)/b + sin(x))*cos(x)/(a
**5*b*cos(x) + a**4*b**2*sin(x) + 2*a**3*b**3*cos(x) + 2*a**2*b**4*sin(x) +
 a*b**5*cos(x) + b**6*sin(x)) - a**2*b**2*cos(x)/(a**5*b*cos(x) + a**4*b**2
*sin(x) + 2*a**3*b**3*cos(x) + 2*a**2*b**4*sin(x) + a*b**5*cos(x) + b**6*si
n(x)) + a*b**3*x*cos(x)/(a**5*b*cos(x) + a**4*b**2*sin(x) + 2*a**3*b**3*cos
(x) + 2*a**2*b**4*sin(x) + a*b**5*cos(x) + b**6*sin(x)) - 2*a*b**3*log(a*co
s(x)/b + sin(x))*sin(x)/(a**5*b*cos(x) + a**4*b**2*sin(x) + 2*a**3*b**3*cos
(x) + 2*a**2*b**4*sin(x) + a*b**5*cos(x) + b**6*sin(x)) + b**4*x*sin(x)/(a*
*5*b*cos(x) + a**4*b**2*sin(x) + 2*a**3*b**3*cos(x) + 2*a**2*b**4*sin(x) +
a*b**5*cos(x) + b**6*sin(x)), True))
```

3.17 $\int \frac{\sin(x)}{(a\cos(x)+b\sin(x))^2}\,dx$

Optimal. Leaf size=60

$$\frac{a}{(a^2+b^2)(a\cos(x)+b\sin(x))}-\frac{b\tanh^{-1}\left(\frac{b\cos(x)-a\sin(x)}{\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{3/2}}$$

```
[Out] -b*arctanh((b*cos(x)-a*sin(x))/(a^2+b^2)^(1/2))/(a^2+b^2)^(3/2)+a/(a^2+b^2)
/(a*cos(x)+b*sin(x))
```

Rubi [A] time = 0.05, antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {3154, 3074, 206}

$$\frac{a}{(a^2+b^2)(a\cos(x)+b\sin(x))}-\frac{b\tanh^{-1}\left(\frac{b\cos(x)-a\sin(x)}{\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[x]/(a*Cos[x] + b*Sin[x])^2,x]
```

```
[Out] -((b*ArcTanh[(b*Cos[x] - a*Sin[x])/Sqrt[a^2 + b^2]])/(a^2 + b^2)^(3/2)) + a
/((a^2 + b^2)*(a*Cos[x] + b*Sin[x]))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3074

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x
_Symbol] :> -Dist[d^(-1), Subst[Int[1/(a^2 + b^2 - x^2), x], x, b*Cos[c + d
*x] - a*Sin[c + d*x]], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

Rule 3154

```
Int[((A_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])/((a_.) + cos[(d_.) + (e_.)*(x_)
]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)])^2, x_Symbol] :> -Simp[(b*C + (a*C
- c*A)*Cos[d + e*x] + b*A*Sin[d + e*x])/(e*(a^2 - b^2 - c^2)*(a + b*Cos[d +
 e*x] + c*Sin[d + e*x])), x] + Dist[(a*A - c*C)/(a^2 - b^2 - c^2), Int[1/(a
 + b*Cos[d + e*x] + c*Sin[d + e*x]), x], x] /; FreeQ[{a, b, c, d, e, A, C},
```

```
x] && NeQ[a^2 - b^2 - c^2, 0] && NeQ[a*A - c*C, 0]
```

Rubi steps

$$\int \frac{\sin(x)}{(a\cos(x)+b\sin(x))^2}\,dx = \frac{a}{(a^2+b^2)(a\cos(x)+b\sin(x))} + \frac{b\int \frac{1}{a\cos(x)+b\sin(x)}\,dx}{a^2+b^2}$$

$$= \frac{a}{(a^2+b^2)(a\cos(x)+b\sin(x))} - \frac{b\,\mathrm{Subst}\left(\int \frac{1}{a^2+b^2-x^2}\,dx, x, b\cos(x)-a\sin(x)\right)}{a^2+b^2}$$

$$= -\frac{b\tanh^{-1}\left(\frac{b\cos(x)-a\sin(x)}{\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{3/2}} + \frac{a}{(a^2+b^2)(a\cos(x)+b\sin(x))}$$

Mathematica [A] time = 0.16, size = 62, normalized size = 1.03

$$\frac{a}{(a^2+b^2)(a\cos(x)+b\sin(x))} + \frac{2b\tanh^{-1}\left(\frac{a\tan\left(\frac{x}{2}\right)-b}{\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[x]/(a*Cos[x] + b*Sin[x])^2,x]
```

```
[Out] (2*b*ArcTanh[(-b + a*Tan[x/2])/Sqrt[a^2 + b^2]])/(a^2 + b^2)^(3/2) + a/((a^
2 + b^2)*(a*Cos[x] + b*Sin[x]))
```

fricas [B] time = 0.45, size = 164, normalized size = 2.73

$$\frac{2\,a^3+2\,ab^2+\left(ab\cos(x)+b^2\sin(x)\right)\sqrt{a^2+b^2}\,\log\left(-\frac{2\,ab\cos(x)\sin(x)+\left(a^2-b^2\right)\cos(x)^2-2\,a^2-b^2+2\,\sqrt{a^2+b^2}\,(b\cos(x)-a\sin(x))}{2\,ab\cos(x)\sin(x)+\left(a^2-b^2\right)\cos(x)^2+b^2}\right)}{2\left(\left(a^5+2\,a^3b^2+ab^4\right)\cos(x)+\left(a^4b+2\,a^2b^3+b^5\right)\sin(x)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)/(a*cos(x)+b*sin(x))^2,x, algorithm="fricas")
```

```
[Out] 1/2*(2*a^3 + 2*a*b^2 + (a*b*cos(x) + b^2*sin(x))*sqrt(a^2 + b^2)*log(-(2*a*
b*cos(x)*sin(x) + (a^2 - b^2)*cos(x)^2 - 2*a^2 - b^2 + 2*sqrt(a^2 + b^2)*(b
*cos(x) - a*sin(x)))/(2*a*b*cos(x)*sin(x) + (a^2 - b^2)*cos(x)^2 + b^2)))/(
(a^5 + 2*a^3*b^2 + a*b^4)*cos(x) + (a^4*b + 2*a^2*b^3 + b^5)*sin(x))
```

giac [A] time = 0.26, size = 103, normalized size = 1.72

$$-\frac{b\log\left(\frac{\left|2\,a\tan\left(\frac{1}{2}\,x\right)-2\,b-2\,\sqrt{a^2+b^2}\right|}{\left|2\,a\tan\left(\frac{1}{2}\,x\right)-2\,b+2\,\sqrt{a^2+b^2}\right|}\right)}{\left(a^2+b^2\right)^{\frac{3}{2}}}-\frac{2\left(b\tan\left(\frac{1}{2}\,x\right)+a\right)}{\left(a\tan\left(\frac{1}{2}\,x\right)^2-2\,b\tan\left(\frac{1}{2}\,x\right)-a\right)\left(a^2+b^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)/(a*cos(x)+b*sin(x))^2,x, algorithm="giac")
```

```
[Out] -b*log(abs(2*a*tan(1/2*x) - 2*b - 2*sqrt(a^2 + b^2))/abs(2*a*tan(1/2*x) - 2
*b + 2*sqrt(a^2 + b^2)))/(a^2 + b^2)^(3/2) - 2*(b*tan(1/2*x) + a)/((a*tan(1
/2*x)^2 - 2*b*tan(1/2*x) - a)*(a^2 + b^2))
```

maple [A] time = 0.55, size = 97, normalized size = 1.62

$$\frac{8\tan\left(\frac{x}{2}\right)b+8a}{\left(-4a^2-4b^2\right)\left(a\left(\tan^2\left(\frac{x}{2}\right)\right)-2\tan\left(\frac{x}{2}\right)b-a\right)}-\frac{8b\,\mathrm{arctanh}\left(\frac{2a\tan\left(\frac{x}{2}\right)-2b}{2\sqrt{a^2+b^2}}\right)}{\left(-4a^2-4b^2\right)\sqrt{a^2+b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)/(a*cos(x)+b*sin(x))^2,x)
```

```
[Out] 4*(2*tan(1/2*x)*b+2*a)/(-4*a^2-4*b^2)/(a*tan(1/2*x)^2-2*tan(1/2*x)*b-a)-8*b
/(-4*a^2-4*b^2)/(a^2+b^2)^(1/2)*arctanh(1/2*(2*a*tan(1/2*x)-2*b)/(a^2+b^2)^
(1/2))
```

maxima [B] time = 0.42, size = 128, normalized size = 2.13

$$-\frac{b\log\left(\frac{b-\frac{a\sin(x)}{\cos(x)+1}+\sqrt{a^2+b^2}}{b-\frac{a\sin(x)}{\cos(x)+1}-\sqrt{a^2+b^2}}\right)}{\left(a^2+b^2\right)^{\frac{3}{2}}}+\frac{2\left(a+\frac{b\sin(x)}{\cos(x)+1}\right)}{a^3+ab^2+\frac{2\left(a^2b+b^3\right)\sin(x)}{\cos(x)+1}-\frac{\left(a^3+ab^2\right)\sin(x)^2}{\left(\cos(x)+1\right)^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)/(a*cos(x)+b*sin(x))^2,x, algorithm="maxima")
```

```
[Out] -b*log((b - a*sin(x)/(cos(x) + 1) + sqrt(a^2 + b^2))/(b - a*sin(x)/(cos(x)
+ 1) - sqrt(a^2 + b^2)))/(a^2 + b^2)^(3/2) + 2*(a + b*sin(x)/(cos(x) + 1))/
(a^3 + a*b^2 + 2*(a^2*b + b^3)*sin(x)/(cos(x) + 1) - (a^3 + a*b^2)*sin(x)^2
/(cos(x) + 1)^2)
```

mupad [B] time = 0.61, size = 86, normalized size = 1.43

$$\frac{\frac{2a}{a^2+b^2}+\frac{2b\tan\left(\frac{x}{2}\right)}{a^2+b^2}}{-a\tan\left(\frac{x}{2}\right)^2+2b\tan\left(\frac{x}{2}\right)+a}-\frac{2b\,\mathrm{atanh}\left(\frac{2b-2a\tan\left(\frac{x}{2}\right)}{2\sqrt{a^2+b^2}}\right)}{\left(a^2+b^2\right)^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)/(a*cos(x) + b*sin(x))^2,x)
```

```
[Out] ((2*a)/(a^2 + b^2) + (2*b*tan(x/2))/(a^2 + b^2))/(a + 2*b*tan(x/2) - a*tan(
x/2)^2) - (2*b*atanh((2*b - 2*a*tan(x/2))/(2*(a^2 + b^2)^(1/2))))/(a^2 + b^
2)^(3/2)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)/(a*cos(x)+b*sin(x))**2,x)
```

```
[Out] Timed out
```

3.18 $\int \frac{1}{(a\cos(x)+b\sin(x))^2}\,dx$

Optimal. Leaf size=17

$$\frac{\sin(x)}{a(a\cos(x)+b\sin(x))}$$

```
[Out] sin(x)/a/(a*cos(x)+b*sin(x))
```

Rubi [A] time = 0.01, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {3075}

$$\frac{\sin(x)}{a(a\cos(x)+b\sin(x))}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Cos[x] + b*Sin[x])^(-2),x]
[Out] Sin[x]/(a*(a*Cos[x] + b*Sin[x]))
```

Rule 3075

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-2), x
_Symbol] :> Simp[Sin[c + d*x]/(a*d*(a*Cos[c + d*x] + b*Sin[c + d*x])), x] /
; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

Rubi steps

$$\int \frac{1}{(a\cos(x)+b\sin(x))^2}\,dx=\frac{\sin(x)}{a(a\cos(x)+b\sin(x))}$$

Mathematica [A] time = 0.02, size = 17, normalized size = 1.00

$$\frac{\sin(x)}{a(a\cos(x)+b\sin(x))}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Cos[x] + b*Sin[x])^(-2),x]
[Out] Sin[x]/(a*(a*Cos[x] + b*Sin[x]))
```

fricas [B] time = 0.44, size = 39, normalized size = 2.29

$$-\frac{b\cos(x) - a\sin(x)}{\left(a^3 + ab^2\right)\cos(x) + \left(a^2b + b^3\right)\sin(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cos(x)+b*sin(x))^2,x, algorithm="fricas")
[Out] -(b*cos(x) - a*sin(x))/((a^3 + a*b^2)*cos(x) + (a^2*b + b^3)*sin(x))
```

giac [A] time = 0.18, size = 13, normalized size = 0.76

$$-\frac{1}{(b\tan(x) + a)b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cos(x)+b*sin(x))^2,x, algorithm="giac")
[Out] -1/((b*tan(x) + a)*b)
```

maple [A] time = 0.53, size = 14, normalized size = 0.82

$$-\frac{1}{b\,(a + b\tan(x))}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*cos(x)+b*sin(x))^2,x)
[Out] -1/b/(a+b*tan(x))
```

maxima [A] time = 0.31, size = 14, normalized size = 0.82

$$-\frac{1}{b^2\tan(x) + ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cos(x)+b*sin(x))^2,x, algorithm="maxima")
[Out] -1/(b^2*tan(x) + a*b)
```

mupad [B] time = 0.43, size = 29, normalized size = 1.71

$$\frac{2\tan\left(\frac{x}{2}\right)}{a\left(-a\tan\left(\frac{x}{2}\right)^2 + 2\,b\tan\left(\frac{x}{2}\right) + a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*cos(x) + b*sin(x))^2,x)
```

```
[Out] (2*tan(x/2))/(a*(a + 2*b*tan(x/2) - a*tan(x/2)^2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cos(x)+b*sin(x))**2,x)
```

```
[Out] Timed out
```

3.19 $\int \frac{\csc(x)}{(a\cos(x)+b\sin(x))^2}\,dx$

Optimal. Leaf size=63

$$\frac{b\tanh^{-1}\left(\frac{b\cos(x)-a\sin(x)}{\sqrt{a^2+b^2}}\right)}{a^2\sqrt{a^2+b^2}}-\frac{\tanh^{-1}(\cos(x))}{a^2}+\frac{1}{a(a\cos(x)+b\sin(x))}$$

```
[Out] -arctanh(cos(x))/a^2+1/a/(a*cos(x)+b*sin(x))+b*arctanh((b*cos(x)-a*sin(x))/
(a^2+b^2)^(1/2))/a^2/(a^2+b^2)^(1/2)
```

Rubi [A] time = 0.06, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3093, 3770, 3074, 206}

$$\frac{b\tanh^{-1}\left(\frac{b\cos(x)-a\sin(x)}{\sqrt{a^2+b^2}}\right)}{a^2\sqrt{a^2+b^2}}-\frac{\tanh^{-1}(\cos(x))}{a^2}+\frac{1}{a(a\cos(x)+b\sin(x))}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[x]/(a*Cos[x] + b*Sin[x])^2,x]
```

```
[Out] -(ArcTanh[Cos[x]]/a^2) + (b*ArcTanh[(b*Cos[x] - a*Sin[x])/Sqrt[a^2 + b^2]])
/(a^2*Sqrt[a^2 + b^2]) + 1/(a*(a*Cos[x] + b*Sin[x]))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3074

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x
_Symbol] :> -Dist[d^(-1), Subst[Int[1/(a^2 + b^2 - x^2), x], x, b*Cos[c + d
*x] - a*Sin[c + d*x]], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

Rule 3093

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_)/si
n[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[(a*Cos[c + d*x] + b*Sin[c + d*x])
^(n + 1)/(a*d*(n + 1)), x] + (Dist[1/a^2, Int[(a*Cos[c + d*x] + b*Sin[c + d
*x])^(n + 2)/Sin[c + d*x], x], x] - Dist[b/a^2, Int[(a*Cos[c + d*x] + b*Sin
[c + d*x])^(n + 1), x], x]) /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

```
&& LtQ[n, -1]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\csc(x)}{(a\cos(x)+b\sin(x))^2}\,dx = \frac{1}{a(a\cos(x)+b\sin(x))}+\frac{\int \csc(x)\,dx}{a^2}-\frac{b\int \frac{1}{a\cos(x)+b\sin(x)}\,dx}{a^2}$$
$$= -\frac{\tanh^{-1}(\cos(x))}{a^2}+\frac{1}{a(a\cos(x)+b\sin(x))}+\frac{b\,\text{Subst}\left(\int \frac{1}{a^2+b^2-x^2}\,dx, x, b\cos(x)-a\right.}{a^2}$$
$$= -\frac{\tanh^{-1}(\cos(x))}{a^2}+\frac{b\tanh^{-1}\left(\frac{b\cos(x)-a\sin(x)}{\sqrt{a^2+b^2}}\right)}{a^2\sqrt{a^2+b^2}}+\frac{1}{a(a\cos(x)+b\sin(x))}$$

Mathematica [A] time = 0.33, size = 72, normalized size = 1.14

$$\frac{-\frac{2b\tanh^{-1}\left(\frac{a\tan\left(\frac{x}{2}\right)-b}{\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}}+\frac{a\csc(x)}{a\cot(x)+b}+\log\left(\sin\left(\frac{x}{2}\right)\right)-\log\left(\cos\left(\frac{x}{2}\right)\right)}{a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[x]/(a*Cos[x] + b*Sin[x])^2,x]
```

```
[Out] ((-2*b*ArcTanh[(-b + a*Tan[x/2])/Sqrt[a^2 + b^2]])/Sqrt[a^2 + b^2] + (a*Csc
[x])/(b + a*Cot[x]) - Log[Cos[x/2]] + Log[Sin[x/2]])/a^2
```

fricas [B] time = 0.56, size = 220, normalized size = 3.49

$$\frac{2a^3+2ab^2+\left(ab\cos(x)+b^2\sin(x)\right)\sqrt{a^2+b^2}\,\log\left(\frac{2ab\cos(x)\sin(x)+\left(a^2-b^2\right)\cos(x)^2-2a^2-b^2-2\sqrt{a^2+b^2}\,(b\cos(x)-a\sin(x))}{2ab\cos(x)\sin(x)+\left(a^2-b^2\right)\cos(x)^2+b^2}\right)-\left(\left(\right.\right.}{2\left(\left(a^5+a^3b^2\right)\cos(\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)/(a*cos(x)+b*sin(x))^2,x, algorithm="fricas")
```

```
[Out] 1/2*(2*a^3 + 2*a*b^2 + (a*b*cos(x) + b^2*sin(x))*sqrt(a^2 + b^2)*log((2*a*b
*cos(x)*sin(x) + (a^2 - b^2)*cos(x)^2 - 2*a^2 - b^2 - 2*sqrt(a^2 + b^2)*(b*
cos(x) - a*sin(x)))/(2*a*b*cos(x)*sin(x) + (a^2 - b^2)*cos(x)^2 + b^2)) - (
(a^3 + a*b^2)*cos(x) + (a^2*b + b^3)*sin(x))*log(1/2*cos(x) + 1/2) + ((a^3
+ a*b^2)*cos(x) + (a^2*b + b^3)*sin(x))*log(-1/2*cos(x) + 1/2))/((a^5 + a^3
*b^2)*cos(x) + (a^4*b + a^2*b^3)*sin(x))
```

giac [A] time = 4.46, size = 109, normalized size = 1.73

$$\frac{b \log\left(\frac{\left|2\, a \tan\left(\frac{1}{2}\, x\right) - 2\, b - 2\, \sqrt{a^2+b^2}\right|}{\left|2\, a \tan\left(\frac{1}{2}\, x\right) - 2\, b + 2\, \sqrt{a^2+b^2}\right|}\right)}{\sqrt{a^2 + b^2}\, a^2} + \frac{\log\left(\left|\tan\left(\frac{1}{2}\, x\right)\right|\right)}{a^2} - \frac{2\left(b \tan\left(\frac{1}{2}\, x\right) + a\right)}{\left(a \tan\left(\frac{1}{2}\, x\right)^2 - 2\, b \tan\left(\frac{1}{2}\, x\right) - a\right) a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)/(a*cos(x)+b*sin(x))^2,x, algorithm="giac")
```

```
[Out] b*log(abs(2*a*tan(1/2*x) - 2*b - 2*sqrt(a^2 + b^2))/abs(2*a*tan(1/2*x) - 2*
b + 2*sqrt(a^2 + b^2)))/(sqrt(a^2 + b^2)*a^2) + log(abs(tan(1/2*x)))/a^2 -
2*(b*tan(1/2*x) + a)/((a*tan(1/2*x)^2 - 2*b*tan(1/2*x) - a)*a^2)
```

maple [A] time = 0.81, size = 106, normalized size = 1.68

$$\frac{\ln\left(\tan\left(\frac{x}{2}\right)\right)}{a^2} - \frac{2 \tan\left(\frac{x}{2}\right) b}{a^2 \left(a \left(\tan^2\left(\frac{x}{2}\right)\right) - 2 \tan\left(\frac{x}{2}\right) b - a\right)} - \frac{2}{a \left(a \left(\tan^2\left(\frac{x}{2}\right)\right) - 2 \tan\left(\frac{x}{2}\right) b - a\right)} - \frac{2b \operatorname{arctanh}\left(\frac{2a \tan\left(\frac{x}{2}\right) - 2b}{2\sqrt{a^2+b^2}}\right)}{a^2 \sqrt{a^2 + b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(x)/(a*cos(x)+b*sin(x))^2,x)
```

```
[Out] 1/a^2*ln(tan(1/2*x))-2/a^2/(a*tan(1/2*x)^2-2*tan(1/2*x)*b-a)*tan(1/2*x)*b-2
/a/(a*tan(1/2*x)^2-2*tan(1/2*x)*b-a)-2/a^2*b/(a^2+b^2)^(1/2)*arctanh(1/2*(2
*a*tan(1/2*x)-2*b)/(a^2+b^2)^(1/2))
```

maxima [B] time = 0.44, size = 128, normalized size = 2.03

$$\frac{2\left(a + \frac{b \sin(x)}{\cos(x)+1}\right)}{a^3 + \frac{2\, a^2 b \sin(x)}{\cos(x)+1} - \frac{a^3 \sin(x)^2}{(\cos(x)+1)^2}} + \frac{b \log\left(\frac{b - \frac{a \sin(x)}{\cos(x)+1} + \sqrt{a^2+b^2}}{b - \frac{a \sin(x)}{\cos(x)+1} - \sqrt{a^2+b^2}}\right)}{\sqrt{a^2 + b^2}\, a^2} + \frac{\log\left(\frac{\sin(x)}{\cos(x)+1}\right)}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)/(a*cos(x)+b*sin(x))^2,x, algorithm="maxima")
```

```
[Out] 2*(a + b*sin(x)/(cos(x) + 1))/(a^3 + 2*a^2*b*sin(x)/(cos(x) + 1) - a^3*sin(
x)^2/(cos(x) + 1)^2) + b*log((b - a*sin(x)/(cos(x) + 1) + sqrt(a^2 + b^2))/
(b - a*sin(x)/(cos(x) + 1) - sqrt(a^2 + b^2)))/(sqrt(a^2 + b^2)*a^2) + log(
sin(x)/(cos(x) + 1))/a^2
```

mupad [B] time = 0.81, size = 492, normalized size = 7.81

$$\frac{\frac{2}{a}+\frac{2\,b\tan\left(\frac{x}{2}\right)}{a^2}}{-a\tan\left(\frac{x}{2}\right)^2+2\,b\tan\left(\frac{x}{2}\right)+a}+\frac{\ln\left(\tan\left(\frac{x}{2}\right)\right)}{a^2}+\frac{b\,\mathrm{atan}\left(\frac{\frac{b\,\sqrt{a^2+b^2}\left(4\,b+\frac{2\tan\left(\frac{x}{2}\right)\left(a^2+4\,b^2\right)}{a}+\frac{b\left(2\,a^2\,b+\frac{2\tan\left(\frac{x}{2}\right)\left(3\,a^4+4\,a^2\,b^2\right)}{a}\right)\sqrt{a^2+b^2}}{a^4+a^2\,b^2}\right)1\mathrm{i}}{a^4+a^2\,b^2}+\frac{b\,\sqrt{a\cdots}}{\cdots}}{\frac{4\,b}{a^2}+\frac{b\,\sqrt{a^2+b^2}\left(4\,b+\frac{2\tan\left(\frac{x}{2}\right)\left(a^2+4\,b^2\right)}{a}+\frac{b\left(2\,a^2\,b+\frac{2\tan\left(\frac{x}{2}\right)\left(3\,a^4+4\,a^2\,b^2\right)}{a}\right)\sqrt{a^2+b^2}}{a^4+a^2\,b^2}\right)}{a^4+a^2\,b^2}-\frac{b\,\cdots}{\cdots}}\right)\cdots}{a^4+a\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(x)*(a*cos(x) + b*sin(x))^2),x)
```

```
[Out] (2/a + (2*b*tan(x/2))/a^2)/(a + 2*b*tan(x/2) - a*tan(x/2)^2) + log(tan(x/2)
)/a^2 + (b*atan(((b*(a^2 + b^2)^(1/2)*(4*b + (2*tan(x/2)*(a^2 + 4*b^2))/a +
 (b*(2*a^2*b + (2*tan(x/2)*(3*a^4 + 4*a^2*b^2))/a)*(a^2 + b^2)^(1/2))/(a^4
+ a^2*b^2))*1i)/(a^4 + a^2*b^2) + (b*(a^2 + b^2)^(1/2)*(4*b + (2*tan(x/2)*(
a^2 + 4*b^2))/a - (b*(2*a^2*b + (2*tan(x/2)*(3*a^4 + 4*a^2*b^2))/a)*(a^2 +
b^2)^(1/2))/(a^4 + a^2*b^2))*1i)/(a^4 + a^2*b^2))/((4*b)/a^2 + (b*(a^2 + b^
2)^(1/2)*(4*b + (2*tan(x/2)*(a^2 + 4*b^2))/a + (b*(2*a^2*b + (2*tan(x/2)*(3
*a^4 + 4*a^2*b^2))/a)*(a^2 + b^2)^(1/2))/(a^4 + a^2*b^2)))/(a^4 + a^2*b^2)
- (b*(a^2 + b^2)^(1/2)*(4*b + (2*tan(x/2)*(a^2 + 4*b^2))/a - (b*(2*a^2*b +
(2*tan(x/2)*(3*a^4 + 4*a^2*b^2))/a)*(a^2 + b^2)^(1/2))/(a^4 + a^2*b^2)))/(a
^4 + a^2*b^2)))*(a^2 + b^2)^(1/2)*2i)/(a^4 + a^2*b^2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\csc\left(x\right)}{\left(a\cos\left(x\right)+b\sin\left(x\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)/(a*cos(x)+b*sin(x))**2,x)
```

```
[Out] Integral(csc(x)/(a*cos(x) + b*sin(x))**2, x)
```

3.20 $\int \frac{\csc^2(x)}{(a\cos(x)+b\sin(x))^2}\,dx$

Optimal. Leaf size=49

$$-\frac{2b\log(\tan(x))}{a^3}+\frac{2b\log(a+b\tan(x))}{a^3}-\frac{\frac{b}{a^2}+\frac{1}{b}}{a+b\tan(x)}-\frac{\cot(x)}{a^2}$$

```
[Out] -cot(x)/a^2-2*b*ln(tan(x))/a^3+2*b*ln(a+b*tan(x))/a^3+(-1/b-b/a^2)/(a+b*tan
(x))
```

Rubi [A] time = 0.08, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {3087, 894}

$$-\frac{\frac{b}{a^2}+\frac{1}{b}}{a+b\tan(x)}-\frac{2b\log(\tan(x))}{a^3}+\frac{2b\log(a+b\tan(x))}{a^3}-\frac{\cot(x)}{a^2}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[x]^2/(a*Cos[x] + b*Sin[x])^2,x]
```

```
[Out] -(Cot[x]/a^2) - (2*b*Log[Tan[x]])/a^3 + (2*b*Log[a + b*Tan[x]])/a^3 - (b^(-
1) + b/a^2)/(a + b*Tan[x])
```

Rule 894

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ
[m, 0] && ILtQ[n, 0]))
```

Rule 3087

```
Int[sin[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[1/d, Subst[Int[(x^m*(a + b*x
)^n)/(1 + x^2)^((m + n + 2)/2), x], x, Tan[c + d*x]], x] /; FreeQ[{a, b, c,
 d}, x] && IntegerQ[n] && IntegerQ[(m + n)/2] && NeQ[n, -1] &&  !(GtQ[n, 0]
 && GtQ[m, 1])
```

Rubi steps

$$\int \frac{\csc^2(x)}{(a\cos(x)+b\sin(x))^2}\,dx = \text{Subst}\left(\int \frac{1+x^2}{x^2(a+bx)^2}\,dx, x, \tan(x)\right)$$
$$= \text{Subst}\left(\int \left(\frac{1}{a^2x^2} - \frac{2b}{a^3x} + \frac{a^2+b^2}{a^2(a+bx)^2} + \frac{2b^2}{a^3(a+bx)}\right) dx, x, \tan(x)\right)$$
$$= -\frac{\cot(x)}{a^2} - \frac{2b\log(\tan(x))}{a^3} + \frac{2b\log(a+b\tan(x))}{a^3} - \frac{\frac{1}{b}+\frac{b}{a^2}}{a+b\tan(x)}$$

Mathematica [**A**] time = 0.20, size = 76, normalized size = 1.55

$$\frac{a^2\left(-\cot^2(x)\right) + a^2 + 2b^2\log(a\cos(x)+b\sin(x)) - ab\cot(x)(-2\log(a\cos(x)+b\sin(x)) + 2\log(\sin(x)) + 1) - 2}{a^3(a\cot(x)+b)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[x]^2/(a*Cos[x] + b*Sin[x])^2,x]
```

```
[Out] (a^2 + b^2 - a^2*Cot[x]^2 - 2*b^2*Log[Sin[x]] - a*b*Cot[x]*(1 + 2*Log[Sin[x
]] - 2*Log[a*Cos[x] + b*Sin[x]]) + 2*b^2*Log[a*Cos[x] + b*Sin[x]])/(a^3*(b
+ a*Cot[x]))
```

fricas [**B**] time = 0.58, size = 134, normalized size = 2.73

$$\frac{2\,a^2\cos(x)^2 + 2\,ab\cos(x)\sin(x) - a^2 + \left(b^2\cos(x)^2 - ab\cos(x)\sin(x) - b^2\right)\log\left(2\,ab\cos(x)\sin(x) + \left(a^2-b^2\right)\cos\right.}{a^3b\cos(x)^2 - a^4\cos(x)\sin(x) - a^3b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)^2/(a*cos(x)+b*sin(x))^2,x, algorithm="fricas")
```

```
[Out] (2*a^2*cos(x)^2 + 2*a*b*cos(x)*sin(x) - a^2 + (b^2*cos(x)^2 - a*b*cos(x)*si
n(x) - b^2)*log(2*a*b*cos(x)*sin(x) + (a^2 - b^2)*cos(x)^2 + b^2) - (b^2*co
s(x)^2 - a*b*cos(x)*sin(x) - b^2)*log(-1/4*cos(x)^2 + 1/4))/(a^3*b*cos(x)^2
 - a^4*cos(x)*sin(x) - a^3*b)
```

giac [**A**] time = 0.17, size = 63, normalized size = 1.29

$$\frac{2\,b\log\left(|b\tan(x)+a|\right)}{a^3} - \frac{2\,b\log\left(|\tan(x)|\right)}{a^3} - \frac{a^2\tan(x) + 2\,b^2\tan(x) + ab}{\left(b\tan(x)^2 + a\tan(x)\right)a^2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)^2/(a*cos(x)+b*sin(x))^2,x, algorithm="giac")
```

```
[Out] 2*b*log(abs(b*tan(x) + a))/a^3 - 2*b*log(abs(tan(x)))/a^3 - (a^2*tan(x) + 2
*b^2*tan(x) + a*b)/((b*tan(x)^2 + a*tan(x))*a^2*b)
```

maple [A] time = 0.72, size = 60, normalized size = 1.22

$$-\frac{1}{b\left(a+b\tan(x)\right)}-\frac{b}{a^{2}\left(a+b\tan(x)\right)}+\frac{2b\ln\left(a+b\tan(x)\right)}{a^{3}}-\frac{1}{a^{2}\tan(x)}-\frac{2b\ln\left(\tan(x)\right)}{a^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(x)^2/(a*cos(x)+b*sin(x))^2,x)
```

```
[Out] -1/b/(a+b*tan(x))-1/a^2*b/(a+b*tan(x))+2*b*ln(a+b*tan(x))/a^3-1/a^2/tan(x)-
2*b*ln(tan(x))/a^3
```

maxima [A] time = 0.34, size = 62, normalized size = 1.27

$$-\frac{ab+\left(a^{2}+2\,b^{2}\right)\tan(x)}{a^{2}b^{2}\tan(x)^{2}+a^{3}b\tan(x)}+\frac{2\,b\log\left(b\tan(x)+a\right)}{a^{3}}-\frac{2\,b\log\left(\tan(x)\right)}{a^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)^2/(a*cos(x)+b*sin(x))^2,x, algorithm="maxima")
```

```
[Out] -(a*b + (a^2 + 2*b^2)*tan(x))/(a^2*b^2*tan(x)^2 + a^3*b*tan(x)) + 2*b*log(b
*tan(x) + a)/a^3 - 2*b*log(tan(x))/a^3
```

mupad [B] time = 0.62, size = 114, normalized size = 2.33

$$\frac{\tan\left(\frac{x}{2}\right)}{2\,a^{2}}-\frac{a+2\,b\tan\left(\frac{x}{2}\right)-\frac{\tan\left(\frac{x}{2}\right)^{2}\left(5\,a^{2}+4\,b^{2}\right)}{a}}{-2\,a^{3}\tan\left(\frac{x}{2}\right)^{3}+2\,a^{3}\tan\left(\frac{x}{2}\right)+4\,b\,a^{2}\tan\left(\frac{x}{2}\right)^{2}}+\frac{2\,b\,\ln\left(-a\tan\left(\frac{x}{2}\right)^{2}+2\,b\tan\left(\frac{x}{2}\right)+a\right)}{a^{3}}-\frac{2\,b\,\ln\left(\tan\left(\frac{x}{2}\right)\right)}{a^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(x)^2*(a*cos(x) + b*sin(x))^2),x)
```

```
[Out] tan(x/2)/(2*a^2) - (a + 2*b*tan(x/2) - (tan(x/2)^2*(5*a^2 + 4*b^2))/a)/(2*a
^3*tan(x/2) - 2*a^3*tan(x/2)^3 + 4*a^2*b*tan(x/2)^2) + (2*b*log(a + 2*b*tan
(x/2) - a*tan(x/2)^2))/a^3 - (2*b*log(tan(x/2)))/a^3
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\csc^{2}\left(x\right)}{\left(a\cos\left(x\right)+b\sin\left(x\right)\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)**2/(a*cos(x)+b*sin(x))**2,x)
```

```
[Out] Integral(csc(x)**2/(a*cos(x) + b*sin(x))**2, x)
```

3.21 $\int \frac{\csc^3(x)}{(a\cos(x)+b\sin(x))^2}\,dx$

Optimal. Leaf size=118

$$-\frac{2b^2\tanh^{-1}(\cos(x))}{a^4}+\frac{2b\csc(x)}{a^3}-\frac{\tanh^{-1}(\cos(x))}{2a^2}-\frac{\cot(x)\csc(x)}{2a^2}-\frac{(a^2+b^2)\tanh^{-1}(\cos(x))}{a^4}+\frac{3b\sqrt{a^2+b^2}\tanh^{-1}}{a^4}$$

```
[Out] -1/2*arctanh(cos(x))/a^2-2*b^2*arctanh(cos(x))/a^4-(a^2+b^2)*arctanh(cos(x)
)/a^4+2*b*csc(x)/a^3-1/2*cot(x)*csc(x)/a^2+(a^2+b^2)/a^3/(a*cos(x)+b*sin(x)
)+3*b*arctanh((b*cos(x)-a*sin(x))/(a^2+b^2)^(1/2))*(a^2+b^2)^(1/2)/a^4
```

Rubi [A] time = 0.18, antiderivative size = 118, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 7, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.438, Rules used = {3105, 3093, 3770, 3074, 206, 3768, 3103}

$$\frac{a^2+b^2}{a^3(a\cos(x)+b\sin(x))}-\frac{2b^2\tanh^{-1}(\cos(x))}{a^4}-\frac{(a^2+b^2)\tanh^{-1}(\cos(x))}{a^4}+\frac{3b\sqrt{a^2+b^2}\tanh^{-1}\left(\frac{b\cos(x)-a\sin(x)}{\sqrt{a^2+b^2}}\right)}{a^4}+\frac{2b}{}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[x]^3/(a*Cos[x] + b*Sin[x])^2,x]
```

```
[Out] -ArcTanh[Cos[x]]/(2*a^2) - (2*b^2*ArcTanh[Cos[x]])/a^4 - ((a^2 + b^2)*ArcTa
nh[Cos[x]])/a^4 + (3*b*Sqrt[a^2 + b^2]*ArcTanh[(b*Cos[x] - a*Sin[x])/Sqrt[a
^2 + b^2]])/a^4 + (2*b*Csc[x])/a^3 - (Cot[x]*Csc[x])/(2*a^2) + (a^2 + b^2)/
(a^3*(a*Cos[x] + b*Sin[x]))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3074

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x
_Symbol] :> -Dist[d^(-1), Subst[Int[1/(a^2 + b^2 - x^2), x], x, b*Cos[c + d
*x] - a*Sin[c + d*x]], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

Rule 3093

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_)/si
n[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[(a*Cos[c + d*x] + b*Sin[c + d*x])
```

```
^(n + 1)/(a*d*(n + 1)), x] + (Dist[1/a^2, Int[(a*Cos[c + d*x] + b*Sin[c + d
*x])^(n + 2)/Sin[c + d*x], x], x] - Dist[b/a^2, Int[(a*Cos[c + d*x] + b*Sin
[c + d*x])^(n + 1), x], x]) /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
&& LtQ[n, -1]
```

Rule 3103

```
Int[sin[(c_.) + (d_.)*(x_)]^(m_)/(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin
[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[Sin[c + d*x]^(m + 1)/(a*d*(m + 1))
, x] + (-Dist[b/a^2, Int[Sin[c + d*x]^(m + 1), x], x] + Dist[(a^2 + b^2)/a^
2, Int[Sin[c + d*x]^(m + 2)/(a*Cos[c + d*x] + b*Sin[c + d*x]), x], x]) /; F
reeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0] && LtQ[m, -1]
```

Rule 3105

```
Int[sin[(c_.) + (d_.)*(x_)]^(m_)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin
[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(a^2 + b^2)/a^2, Int[Sin[c +
d*x]^(m + 2)*(a*Cos[c + d*x] + b*Sin[c + d*x])^n, x], x] + (Dist[1/a^2, Int
[Sin[c + d*x]^m*(a*Cos[c + d*x] + b*Sin[c + d*x])^(n + 2), x], x] - Dist[(2
*b)/a^2, Int[Sin[c + d*x]^(m + 1)*(a*Cos[c + d*x] + b*Sin[c + d*x])^(n + 1)
, x], x]) /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0] && LtQ[n, -1] && L
tQ[m, -1]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\csc^3(x)}{(a\cos(x)+b\sin(x))^2}\,dx = \frac{\int \csc^3(x)\,dx}{a^2} - \frac{(2b)\int \frac{\csc^2(x)}{a\cos(x)+b\sin(x)}\,dx}{a^2} + \frac{\left(a^2+b^2\right)\int \frac{\csc(x)}{(a\cos(x)+b\sin(x))^2}\,dx}{a^2}$$

$$= \frac{2b\csc(x)}{a^3} - \frac{\cot(x)\csc(x)}{2a^2} + \frac{a^2+b^2}{a^3(a\cos(x)+b\sin(x))} + \frac{\int \csc(x)\,dx}{2a^2} + \frac{\left(2b^2\right)\int \csc(}{a^4}$$

$$= -\frac{\tanh^{-1}(\cos(x))}{2a^2} - \frac{2b^2\tanh^{-1}(\cos(x))}{a^4} - \frac{\left(a^2+b^2\right)\tanh^{-1}(\cos(x))}{a^4} + \frac{2b\csc(x)}{a^3}$$

$$= -\frac{\tanh^{-1}(\cos(x))}{2a^2} - \frac{2b^2\tanh^{-1}(\cos(x))}{a^4} - \frac{\left(a^2+b^2\right)\tanh^{-1}(\cos(x))}{a^4} + \frac{3b\sqrt{a^2+b}}{}$$

Mathematica [B] time = 1.91, size = 270, normalized size = 2.29

$$\frac{8a^3\csc(x) + a^3\cot(x)\sec^2\left(\frac{x}{2}\right) - 12a^3\cot(x)\log\left(\cos\left(\frac{x}{2}\right)\right) + 12a^3\cot(x)\log\left(\sin\left(\frac{x}{2}\right)\right) - 48b\sqrt{a^2+b^2}\,(a\cot(x)+}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[x]^3/(a*Cos[x] + b*Sin[x])^2,x]
```

```
[Out] (-48*b*Sqrt[a^2 + b^2]*ArcTanh[(-b + a*Tan[x/2])/Sqrt[a^2 + b^2]]*(b + a*Co
t[x]) + 8*a^3*Csc[x] + 8*a*b^2*Csc[x] - 12*a^2*b*Log[Cos[x/2]] - 24*b^3*Log
[Cos[x/2]] - 12*a^3*Cot[x]*Log[Cos[x/2]] - 24*a*b^2*Cot[x]*Log[Cos[x/2]] +
12*a^2*b*Log[Sin[x/2]] + 24*b^3*Log[Sin[x/2]] + 12*a^3*Cot[x]*Log[Sin[x/2]]
 + 24*a*b^2*Cot[x]*Log[Sin[x/2]] + a^2*b*Sec[x/2]^2 + a^3*Cot[x]*Sec[x/2]^2
 - a*Csc[x/2]^2*(-4*a*b*Cos[x] + a^2*Cot[x] + b*(a - 4*b*Sin[x])) + 8*a*b^2
*Tan[x/2] + 8*a^2*b*Cot[x]*Tan[x/2])/(8*a^4*(b + a*Cot[x]))
```

fricas [B] time = 0.60, size = 345, normalized size = 2.92

$$-\frac{6\,a^2b\cos(x)\sin(x) + 4\,a^3 + 12\,ab^2 - 6\left(a^3+2\,ab^2\right)\cos(x)^2 - 6\left(ab\cos(x)^3 - ab\cos(x) + \left(b^2\cos(x)^2 - b^2\right)\sin(\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)^3/(a*cos(x)+b*sin(x))^2,x, algorithm="fricas")
```

```
[Out] -1/4*(6*a^2*b*cos(x)*sin(x) + 4*a^3 + 12*a*b^2 - 6*(a^3 + 2*a*b^2)*cos(x)^2
 - 6*(a*b*cos(x)^3 - a*b*cos(x) + (b^2*cos(x)^2 - b^2)*sin(x))*sqrt(a^2 + b
```

```
^2)*log((2*a*b*cos(x)*sin(x) + (a^2 - b^2)*cos(x)^2 - 2*a^2 - b^2 - 2*sqrt(
a^2 + b^2)*(b*cos(x) - a*sin(x)))/(2*a*b*cos(x)*sin(x) + (a^2 - b^2)*cos(x)
^2 + b^2)) + 3*((a^3 + 2*a*b^2)*cos(x)^3 - (a^3 + 2*a*b^2)*cos(x) - (a^2*b
+ 2*b^3 - (a^2*b + 2*b^3)*cos(x)^2)*sin(x))*log(1/2*cos(x) + 1/2) - 3*((a^3
 + 2*a*b^2)*cos(x)^3 - (a^3 + 2*a*b^2)*cos(x) - (a^2*b + 2*b^3 - (a^2*b + 2
*b^3)*cos(x)^2)*sin(x))*log(-1/2*cos(x) + 1/2))/(a^5*cos(x)^3 - a^5*cos(x)
+ (a^4*b*cos(x)^2 - a^4*b)*sin(x))
```

giac [A] time = 8.78, size = 215, normalized size = 1.82

$$\frac{3\left(a^2+2\,b^2\right)\log\left(\left|\tan\left(\frac{1}{2}\,x\right)\right|\right)}{2\,a^4}+\frac{3\left(a^2b+b^3\right)\log\left(\frac{\left|2\,a\tan\left(\frac{1}{2}\,x\right)-2\,b-2\,\sqrt{a^2+b^2}\right|}{\left|2\,a\tan\left(\frac{1}{2}\,x\right)-2\,b+2\,\sqrt{a^2+b^2}\right|}\right)}{\sqrt{a^2+b^2}\,a^4}+\frac{a^2\tan\left(\frac{1}{2}\,x\right)^2+8\,ab\tan\left(\frac{1}{2}\,x\right)}{8\,a^4}-\frac{2\left(a^2b\,t\right.}{\left(a\,t\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)^3/(a*cos(x)+b*sin(x))^2,x, algorithm="giac")
```

```
[Out] 3/2*(a^2 + 2*b^2)*log(abs(tan(1/2*x)))/a^4 + 3*(a^2*b + b^3)*log(abs(2*a*ta
n(1/2*x) - 2*b - 2*sqrt(a^2 + b^2))/abs(2*a*tan(1/2*x) - 2*b + 2*sqrt(a^2 +
 b^2)))/(sqrt(a^2 + b^2)*a^4) + 1/8*(a^2*tan(1/2*x)^2 + 8*a*b*tan(1/2*x))/a
^4 - 2*(a^2*b*tan(1/2*x) + b^3*tan(1/2*x) + a^3 + a*b^2)/((a*tan(1/2*x)^2 -
 2*b*tan(1/2*x) - a)*a^4) - 1/8*(18*a^2*tan(1/2*x)^2 + 36*b^2*tan(1/2*x)^2
- 8*a*b*tan(1/2*x) + a^2)/(a^4*tan(1/2*x)^2)
```

maple [B] time = 0.72, size = 224, normalized size = 1.90

$$\frac{\tan^2\left(\frac{x}{2}\right)}{8a^2}+\frac{\tan\left(\frac{x}{2}\right)b}{a^3}-\frac{1}{8a^2\tan\left(\frac{x}{2}\right)^2}+\frac{3\ln\left(\tan\left(\frac{x}{2}\right)\right)}{2a^2}+\frac{3\ln\left(\tan\left(\frac{x}{2}\right)\right)b^2}{a^4}+\frac{b}{a^3\tan\left(\frac{x}{2}\right)}-\frac{2\tan\left(\frac{x}{2}\right)b}{a^2\left(a\left(\tan^2\left(\frac{x}{2}\right)\right)-2\tan\left(\frac{x}{2}\right)b-\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(x)^3/(a*cos(x)+b*sin(x))^2,x)
```

```
[Out] 1/8/a^2*tan(1/2*x)^2+1/a^3*tan(1/2*x)*b-1/8/a^2/tan(1/2*x)^2+3/2/a^2*ln(tan
(1/2*x))+3/a^4*ln(tan(1/2*x))*b^2+1/a^3*b/tan(1/2*x)-2/a^2/(a*tan(1/2*x)^2-
2*tan(1/2*x)*b-a)*tan(1/2*x)*b-2/a^4/(a*tan(1/2*x)^2-2*tan(1/2*x)*b-a)*tan(
1/2*x)*b^3-2/a/(a*tan(1/2*x)^2-2*tan(1/2*x)*b-a)-2/a^3/(a*tan(1/2*x)^2-2*ta
n(1/2*x)*b-a)*b^2-6/a^4*b*(a^2+b^2)^(1/2)*arctanh(1/2*(2*a*tan(1/2*x)-2*b)/
(a^2+b^2)^(1/2))
```

maxima [B] time = 0.43, size = 242, normalized size = 2.05

$$-\frac{a^3 - \frac{6\,a^2 b \sin(x)}{\cos(x)+1} - \frac{\left(17\,a^3+32\,ab^2\right)\sin(x)^2}{(\cos(x)+1)^2} - \frac{8\left(a^2b+2\,b^3\right)\sin(x)^3}{(\cos(x)+1)^3}}{8\left(\frac{a^5\sin(x)^2}{(\cos(x)+1)^2} + \frac{2\,a^4 b\sin(x)^3}{(\cos(x)+1)^3} - \frac{a^5\sin(x)^4}{(\cos(x)+1)^4}\right)} + \frac{\frac{8\,b\sin(x)}{\cos(x)+1} + \frac{a\sin(x)^2}{(\cos(x)+1)^2}}{8\,a^3} + \frac{3\left(a^2+2\,b^2\right)\log\left(\frac{\sin(x)}{\cos(x)+1}\right)}{2\,a^4} + \frac{3\left(a^2 b\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)^3/(a*cos(x)+b*sin(x))^2,x, algorithm="maxima")
```

```
[Out] -1/8*(a^3 - 6*a^2*b*sin(x)/(cos(x) + 1) - (17*a^3 + 32*a*b^2)*sin(x)^2/(cos
(x) + 1)^2 - 8*(a^2*b + 2*b^3)*sin(x)^3/(cos(x) + 1)^3)/(a^5*sin(x)^2/(cos(
x) + 1)^2 + 2*a^4*b*sin(x)^3/(cos(x) + 1)^3 - a^5*sin(x)^4/(cos(x) + 1)^4)
+ 1/8*(8*b*sin(x)/(cos(x) + 1) + a*sin(x)^2/(cos(x) + 1)^2)/a^3 + 3/2*(a^2
+ 2*b^2)*log(sin(x)/(cos(x) + 1))/a^4 + 3*(a^2*b + b^3)*log((b - a*sin(x)/(
cos(x) + 1) + sqrt(a^2 + b^2))/(b - a*sin(x)/(cos(x) + 1) - sqrt(a^2 + b^2)
))/(sqrt(a^2 + b^2)*a^4)
```

mupad [B] time = 0.74, size = 511, normalized size = 4.33

$$\frac{\tan\left(\frac{x}{2}\right)^2\left(\frac{17\,a^2}{2}+16\,b^2\right) - \frac{a^2}{2} + 3\,a\,b\tan\left(\frac{x}{2}\right) + \frac{4\tan\left(\frac{x}{2}\right)^3\left(a^2\,b+2\,b^3\right)}{a}}{-4\,a^4\tan\left(\frac{x}{2}\right)^4 + 4\,a^4\tan\left(\frac{x}{2}\right)^2 + 8\,b\,a^3\tan\left(\frac{x}{2}\right)^3} + \frac{\tan\left(\frac{x}{2}\right)^2}{8\,a^2} + \frac{\ln\left(\tan\left(\frac{x}{2}\right)\right)\left(3\,a^2+6\,b^2\right)}{2\,a^4} + \frac{b\tan\left(\frac{x}{2}\right)}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(x)^3*(a*cos(x) + b*sin(x))^2),x)
```

```
[Out] (tan(x/2)^2*((17*a^2)/2 + 16*b^2) - a^2/2 + 3*a*b*tan(x/2) + (4*tan(x/2)^3*
(a^2*b + 2*b^3))/a)/(4*a^4*tan(x/2)^2 - 4*a^4*tan(x/2)^4 + 8*a^3*b*tan(x/2)
^3) + tan(x/2)^2/(8*a^2) + (log(tan(x/2))*(3*a^2 + 6*b^2))/(2*a^4) + (b*tan
(x/2))/a^3 - (6*b*atanh((54*b^2*(a^2 + b^2)^(1/2))/(18*a^2*b + 90*b^3 + (72
*b^5)/a^2 + (216*b^4*tan(x/2))/a + (144*b^6*tan(x/2))/a^3 + 72*a*b^2*tan(x/
2)) + (72*b^4*(a^2 + b^2)^(1/2))/(18*a^4*b + 72*b^5 + 90*a^2*b^3 + 72*a^3*b
^2*tan(x/2) + (144*b^6*tan(x/2))/a + 216*a*b^4*tan(x/2)) + (144*b^3*tan(x/2
)*(a^2 + b^2)^(1/2))/(216*b^4*tan(x/2) + 90*a*b^3 + 18*a^3*b + (72*b^5)/a +
 72*a^2*b^2*tan(x/2) + (144*b^6*tan(x/2))/a^2) + (144*b^5*tan(x/2)*(a^2 + b
^2)^(1/2))/(144*b^6*tan(x/2) + 72*a*b^5 + 18*a^5*b + 90*a^3*b^3 + 216*a^2*b
^4*tan(x/2) + 72*a^4*b^2*tan(x/2)) + (18*b*tan(x/2)*(a^2 + b^2)^(1/2))/(18*
a*b + 72*b^2*tan(x/2) + (90*b^3)/a + (72*b^5)/a^3 + (216*b^4*tan(x/2))/a^2
+ (144*b^6*tan(x/2))/a^4))*(a^2 + b^2)^(1/2))/a^4
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc^3(x)}{(a\cos(x) + b\sin(x))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)**3/(a*cos(x)+b*sin(x))**2,x)
```

```
[Out] Integral(csc(x)**3/(a*cos(x) + b*sin(x))**2, x)
```

3.22 $\int \frac{\sin^3(x)}{(a\cos(x)+b\sin(x))^3}\,dx$

Optimal. Leaf size=98

$$-\frac{bx\left(3a^2-b^2\right)}{\left(a^2+b^2\right)^3}+\frac{2ab}{\left(a^2+b^2\right)^2(a\cot(x)+b)}+\frac{a}{2\left(a^2+b^2\right)(a\cot(x)+b)^2}+\frac{a\left(a^2-3b^2\right)\log(a\cos(x)+b\sin(x))}{\left(a^2+b^2\right)^3}$$

```
[Out] -b*(3*a^2-b^2)*x/(a^2+b^2)^3+1/2*a/(a^2+b^2)/(b+a*cot(x))^2+2*a*b/(a^2+b^2)
^2/(b+a*cot(x))+a*(a^2-3*b^2)*ln(a*cos(x)+b*sin(x))/(a^2+b^2)^3
```

Rubi [A] time = 0.20, antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312, Rules used = {3085, 3483, 3529, 3531, 3530}

$$-\frac{bx\left(3a^2-b^2\right)}{\left(a^2+b^2\right)^3}+\frac{2ab}{\left(a^2+b^2\right)^2(a\cot(x)+b)}+\frac{a}{2\left(a^2+b^2\right)(a\cot(x)+b)^2}+\frac{a\left(a^2-3b^2\right)\log(a\cos(x)+b\sin(x))}{\left(a^2+b^2\right)^3}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[x]^3/(a*Cos[x] + b*Sin[x])^3,x]
```

```
[Out] -((b*(3*a^2 - b^2)*x)/(a^2 + b^2)^3) + a/(2*(a^2 + b^2)*(b + a*Cot[x])^2) +
 (2*a*b)/((a^2 + b^2)^2*(b + a*Cot[x])) + (a*(a^2 - 3*b^2)*Log[a*Cos[x] + b
*Sin[x]])/(a^2 + b^2)^3
```

Rule 3085

```
Int[sin[(c_.) + (d_.)*(x_)]^(m_)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin
[(c_.) + (d_.)*(x_)])^(n_.), x_Symbol] :> Int[(b + a*Cot[c + d*x])^n, x] /;
 FreeQ[{a, b, c, d}, x] && EqQ[m + n, 0] && IntegerQ[n] && NeQ[a^2 + b^2, 0
]
```

Rule 3483

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(a +
b*Tan[c + d*x])^(n + 1))/(d*(n + 1)*(a^2 + b^2)), x] + Dist[1/(a^2 + b^2),
Int[(a - b*Tan[c + d*x])*(a + b*Tan[c + d*x])^(n + 1), x], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 + b^2, 0] && LtQ[n, -1]
```

Rule 3529

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[((b*c - a*d)*(a + b*Tan[e + f*x])^(m + 1))
```

```
/(f*(m + 1)*(a^2 + b^2)), x] + Dist[1/(a^2 + b^2), Int[(a + b*Tan[e + f*x])
^(m + 1)*Simp[a*c + b*d - (b*c - a*d)*Tan[e + f*x], x], x], x] /; FreeQ[{a,
 b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && LtQ[m, -1]
```

Rule 3530

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_) + (b_.)*tan[(e_.) + (f_.)*
(x_)]), x_Symbol] :> Simp[(c*Log[RemoveContent[a*Cos[e + f*x] + b*Sin[e + f
*x], x]])/(b*f), x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
 NeQ[a^2 + b^2, 0] && EqQ[a*c + b*d, 0]
```

Rule 3531

```
Int[((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_.) + (b_.)*tan[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[((a*c + b*d)*x)/(a^2 + b^2), x] + Dist[(b*c - a
*d)/(a^2 + b^2), Int[(b - a*Tan[e + f*x])/(a + b*Tan[e + f*x]), x], x] /; F
reeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && Ne
Q[a*c + b*d, 0]
```

Rubi steps

$$\int \frac{\sin^3(x)}{(a\cos(x)+b\sin(x))^3}\,dx = \int \frac{1}{(b+a\cot(x))^3}\,dx$$

$$= \frac{a}{2\left(a^2+b^2\right)(b+a\cot(x))^2} + \frac{\int \frac{b-a\cot(x)}{(b+a\cot(x))^2}\,dx}{a^2+b^2}$$

$$= \frac{a}{2\left(a^2+b^2\right)(b+a\cot(x))^2} + \frac{2ab}{\left(a^2+b^2\right)^2(b+a\cot(x))} + \frac{\int \frac{-a^2+b^2-2ab\cot(x)}{b+a\cot(x)}\,dx}{\left(a^2+b^2\right)^2}$$

$$= -\frac{b\left(3a^2-b^2\right)x}{\left(a^2+b^2\right)^3} + \frac{a}{2\left(a^2+b^2\right)(b+a\cot(x))^2} + \frac{2ab}{\left(a^2+b^2\right)^2(b+a\cot(x))} + \frac{(a\left(a^2-\right.}{}$$

$$= -\frac{b\left(3a^2-b^2\right)x}{\left(a^2+b^2\right)^3} + \frac{a}{2\left(a^2+b^2\right)(b+a\cot(x))^2} + \frac{2ab}{\left(a^2+b^2\right)^2(b+a\cot(x))} + \frac{a\left(a^2-\right.}{}$$

Mathematica [C] time = 0.86, size = 114, normalized size = 1.16

$$\frac{a^3}{2(a-ib)^2(a+ib)^2(a\cos(x)+b\sin(x))^2} + \frac{bx\left(b^2-3a^2\right)}{\left(a^2+b^2\right)^3} + \frac{3ab\sin(x)}{\left(a^2+b^2\right)^2(a\cos(x)+b\sin(x))} + \frac{a\left(a^2-3b^2\right)\log(a\cos(}{\left(a^2+b^2\right)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[x]^3/(a*Cos[x] + b*Sin[x])^3,x]
```

```
[Out] (b*(-3*a^2 + b^2)*x)/(a^2 + b^2)^3 + (a*(a^2 - 3*b^2)*Log[a*Cos[x] + b*Sin[
x]])/(a^2 + b^2)^3 + a^3/(2*(a - I*b)^2*(a + I*b)^2*(a*Cos[x] + b*Sin[x])^2
) + (3*a*b*Sin[x])/((a^2 + b^2)^2*(a*Cos[x] + b*Sin[x]))
```

fricas [B] time = 0.50, size = 282, normalized size = 2.88

$$\frac{a^5 + 7\,a^3b^2 - 2\left(6\,a^3b^2 + \left(3\,a^4b - 4\,a^2b^3 + b^5\right)x\right)\cos(x)^2 + 2\left(3\,a^4b - 3\,a^2b^3 - 2\left(3\,a^3b^2 - ab^4\right)x\right)\cos(x)\sin(x) -}{2\left(a^6b^2 + 3\,a^4b^4 + 3\,a^2b^6 + b^8 + \left(a^8 + 2\,a\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^3/(a*cos(x)+b*sin(x))^3,x, algorithm="fricas")
```

```
[Out] 1/2*(a^5 + 7*a^3*b^2 - 2*(6*a^3*b^2 + (3*a^4*b - 4*a^2*b^3 + b^5)*x)*cos(x)
^2 + 2*(3*a^4*b - 3*a^2*b^3 - 2*(3*a^3*b^2 - a*b^4)*x)*cos(x)*sin(x) - 2*(3
*a^2*b^3 - b^5)*x + (a^3*b^2 - 3*a*b^4 + (a^5 - 4*a^3*b^2 + 3*a*b^4)*cos(x)
^2 + 2*(a^4*b - 3*a^2*b^3)*cos(x)*sin(x))*log(2*a*b*cos(x)*sin(x) + (a^2 -
b^2)*cos(x)^2 + b^2))/(a^6*b^2 + 3*a^4*b^4 + 3*a^2*b^6 + b^8 + (a^8 + 2*a^6
*b^2 - 2*a^2*b^6 - b^8)*cos(x)^2 + 2*(a^7*b + 3*a^5*b^3 + 3*a^3*b^5 + a*b^7
)*cos(x)*sin(x))
```

giac [B] time = 2.95, size = 242, normalized size = 2.47

$$-\frac{\left(3\,a^2b - b^3\right)x}{a^6 + 3\,a^4b^2 + 3\,a^2b^4 + b^6} - \frac{\left(a^3 - 3\,ab^2\right)\log\left(\tan(x)^2 + 1\right)}{2\left(a^6 + 3\,a^4b^2 + 3\,a^2b^4 + b^6\right)} + \frac{\left(a^3b - 3\,ab^3\right)\log\left(|b\tan(x) + a|\right)}{a^6b + 3\,a^4b^3 + 3\,a^2b^5 + b^7} - \frac{3\,a^3b^4\tan(x)^2 - 9}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^3/(a*cos(x)+b*sin(x))^3,x, algorithm="giac")
```

```
[Out] -(3*a^2*b - b^3)*x/(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6) - 1/2*(a^3 - 3*a*b^2
)*log(tan(x)^2 + 1)/(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6) + (a^3*b - 3*a*b^3)
*log(abs(b*tan(x) + a))/(a^6*b + 3*a^4*b^3 + 3*a^2*b^5 + b^7) - 1/2*(3*a^3*
b^4*tan(x)^2 - 9*a*b^6*tan(x)^2 + 2*a^6*b*tan(x) + 14*a^4*b^3*tan(x) - 12*a
^2*b^5*tan(x) + a^7 + 9*a^5*b^2 - 4*a^3*b^4)/((a^6*b^2 + 3*a^4*b^4 + 3*a^2*
b^6 + b^8)*(b*tan(x) + a)^2)
```

maple [A] time = 0.58, size = 193, normalized size = 1.97

$$\frac{a^3\ln\left(a + b\tan(x)\right)}{\left(a^2 + b^2\right)^3} - \frac{3a\ln\left(a + b\tan(x)\right)b^2}{\left(a^2 + b^2\right)^3} - \frac{a^4}{\left(a^2 + b^2\right)^2 b^2\left(a + b\tan(x)\right)} - \frac{3a^2}{\left(a^2 + b^2\right)^2\left(a + b\tan(x)\right)} + \frac{}{2b^2\left(a^2 + b^2\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)^3/(a*cos(x)+b*sin(x))^3,x)
```

```
[Out] a^3/(a^2+b^2)^3*ln(a+b*tan(x))-3*a/(a^2+b^2)^3*ln(a+b*tan(x))*b^2-a^4/(a^2+
b^2)^2/b^2/(a+b*tan(x))-3*a^2/(a^2+b^2)^2/(a+b*tan(x))+1/2/b^2*a^3/(a^2+b^2
)/(a+b*tan(x))^2-1/2/(a^2+b^2)^3*ln(1+tan(x)^2)*a^3+3/2/(a^2+b^2)^3*ln(1+ta
n(x)^2)*b^2*a-3/(a^2+b^2)^3*arctan(tan(x))*a^2*b+1/(a^2+b^2)^3*arctan(tan(x
))*b^3
```

maxima [B] time = 0.45, size = 359, normalized size = 3.66

$$-\frac{2\left(3\,a^2b-b^3\right)\arctan\left(\frac{\sin(x)}{\cos(x)+1}\right)}{a^6+3\,a^4b^2+3\,a^2b^4+b^6}+\frac{\left(a^3-3\,ab^2\right)\log\left(-a-\frac{2\,b\sin(x)}{\cos(x)+1}+\frac{a\sin(x)^2}{(\cos(x)+1)^2}\right)}{a^6+3\,a^4b^2+3\,a^2b^4+b^6}-\frac{\left(a^3-3\,ab^2\right)\log\left(\frac{\sin(x)^2}{(\cos(x)+1)^2}+1\right)}{a^6+3\,a^4b^2+3\,a^2b^4+b^6}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^3/(a*cos(x)+b*sin(x))^3,x, algorithm="maxima")
```

```
[Out] -2*(3*a^2*b - b^3)*arctan(sin(x)/(cos(x) + 1))/(a^6 + 3*a^4*b^2 + 3*a^2*b^4
 + b^6) + (a^3 - 3*a*b^2)*log(-a - 2*b*sin(x)/(cos(x) + 1) + a*sin(x)^2/(co
s(x) + 1)^2)/(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6) - (a^3 - 3*a*b^2)*log(sin(
x)^2/(cos(x) + 1)^2 + 1)/(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6) + 2*(2*a^2*b*s
in(x)/(cos(x) + 1) - 2*a^2*b*sin(x)^3/(cos(x) + 1)^3 + (a^3 + 5*a*b^2)*sin(
x)^2/(cos(x) + 1)^2)/(a^6 + 2*a^4*b^2 + a^2*b^4 + 4*(a^5*b + 2*a^3*b^3 + a*
b^5)*sin(x)/(cos(x) + 1) - 2*(a^6 - 3*a^2*b^4 - 2*b^6)*sin(x)^2/(cos(x) + 1
)^2 - 4*(a^5*b + 2*a^3*b^3 + a*b^5)*sin(x)^3/(cos(x) + 1)^3 + (a^6 + 2*a^4*
b^2 + a^2*b^4)*sin(x)^4/(cos(x) + 1)^4)
```

mupad [B] time = 8.60, size = 5324, normalized size = 54.33

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)^3/(a*cos(x) + b*sin(x))^3,x)
```

```
[Out] ((2*tan(x/2)^2*(5*a*b^2 + a^3))/(a^4 + b^4 + 2*a^2*b^2) - (4*a^2*b*tan(x/2)
^3)/(a^4 + b^4 + 2*a^2*b^2) + (4*a^2*b*tan(x/2))/(a^4 + b^4 + 2*a^2*b^2))/(
a^2 - tan(x/2)^2*(2*a^2 - 4*b^2) + a^2*tan(x/2)^4 + 4*a*b*tan(x/2) - 4*a*b*
tan(x/2)^3) - (log(a + 2*b*tan(x/2) - a*tan(x/2)^2)*(3*a*b^2 - a^3))/(a^6 +
 b^6 + 3*a^2*b^4 + 3*a^4*b^2) + (log(1/(cos(x) + 1))*(6*a*b^2 - 2*a^3))/(2*
(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2)) + (2*b*atan((tan(x/2)*(((((6*a*b^2 - 2
*a^3)*((b*((32*(a^2*b^12 - 2*a^14 + 15*a^4*b^10 + 48*a^6*b^8 + 62*a^8*b^6 +
 33*a^10*b^4 + 3*a^12*b^2))/(a^12 + b^12 + 6*a^2*b^10 + 15*a^4*b^8 + 20*a^6
```

```
*b^6 + 15*a^8*b^4 + 6*a^10*b^2) - (16*(6*a*b^2 - 2*a^3)*(3*a*b^16 + 21*a^3*
b^14 + 63*a^5*b^12 + 105*a^7*b^10 + 105*a^9*b^8 + 63*a^11*b^6 + 21*a^13*b^4
 + 3*a^15*b^2))/((a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2)*(a^12 + b^12 + 6*a^2*b
^10 + 15*a^4*b^8 + 20*a^6*b^6 + 15*a^8*b^4 + 6*a^10*b^2)))*(3*a^2 - b^2))/(
a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2) - (16*b*(6*a*b^2 - 2*a^3)*(3*a^2 - b^2)*
(3*a*b^16 + 21*a^3*b^14 + 63*a^5*b^12 + 105*a^7*b^10 + 105*a^9*b^8 + 63*a^1
1*b^6 + 21*a^13*b^4 + 3*a^15*b^2))/((a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2)^2*(
a^12 + b^12 + 6*a^2*b^10 + 15*a^4*b^8 + 20*a^6*b^6 + 15*a^8*b^4 + 6*a^10*b^
2))))/(2*(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2)) - (b*((32*(2*a*b^10 - 24*a^3*
b^8 - 36*a^5*b^6 + 8*a^7*b^4 + 18*a^9*b^2))/(a^12 + b^12 + 6*a^2*b^10 + 15*
a^4*b^8 + 20*a^6*b^6 + 15*a^8*b^4 + 6*a^10*b^2) - ((6*a*b^2 - 2*a^3)*((32*(
a^2*b^12 - 2*a^14 + 15*a^4*b^10 + 48*a^6*b^8 + 62*a^8*b^6 + 33*a^10*b^4 + 3
*a^12*b^2))/(a^12 + b^12 + 6*a^2*b^10 + 15*a^4*b^8 + 20*a^6*b^6 + 15*a^8*b^
4 + 6*a^10*b^2) - (16*(6*a*b^2 - 2*a^3)*(3*a*b^16 + 21*a^3*b^14 + 63*a^5*b^
12 + 105*a^7*b^10 + 105*a^9*b^8 + 63*a^11*b^6 + 21*a^13*b^4 + 3*a^15*b^2))/
((a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2)*(a^12 + b^12 + 6*a^2*b^10 + 15*a^4*b^8
 + 20*a^6*b^6 + 15*a^8*b^4 + 6*a^10*b^2))))/(2*(a^6 + b^6 + 3*a^2*b^4 + 3*a
^4*b^2)))*(3*a^2 - b^2))/(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2) + (32*b^3*(3*a
^2 - b^2)^3*(3*a*b^16 + 21*a^3*b^14 + 63*a^5*b^12 + 105*a^7*b^10 + 105*a^9*
b^8 + 63*a^11*b^6 + 21*a^13*b^4 + 3*a^15*b^2))/((a^6 + b^6 + 3*a^2*b^4 + 3*
a^4*b^2)^3*(a^12 + b^12 + 6*a^2*b^10 + 15*a^4*b^8 + 20*a^6*b^6 + 15*a^8*b^4
 + 6*a^10*b^2)))*(4*a^8 + b^8 - 67*a^2*b^6 + 155*a^4*b^4 - 61*a^6*b^2))/(4*
a^8 + b^8 + 31*a^2*b^6 + 15*a^4*b^4 - 11*a^6*b^2)^2 + (2*a*b*(10*a^6 - 7*b^
6 + 68*a^2*b^4 - 59*a^4*b^2)*((32*(2*a^8 - 6*a^2*b^6 + 2*a^4*b^4 - 6*a^6*b^
2))/(a^12 + b^12 + 6*a^2*b^10 + 15*a^4*b^8 + 20*a^6*b^6 + 15*a^8*b^4 + 6*a^
10*b^2) - ((6*a*b^2 - 2*a^3)*((32*(2*a*b^10 - 24*a^3*b^8 - 36*a^5*b^6 + 8*a
^7*b^4 + 18*a^9*b^2))/(a^12 + b^12 + 6*a^2*b^10 + 15*a^4*b^8 + 20*a^6*b^6 +
 15*a^8*b^4 + 6*a^10*b^2) - ((6*a*b^2 - 2*a^3)*((32*(a^2*b^12 - 2*a^14 + 15
*a^4*b^10 + 48*a^6*b^8 + 62*a^8*b^6 + 33*a^10*b^4 + 3*a^12*b^2))/(a^12 + b^
12 + 6*a^2*b^10 + 15*a^4*b^8 + 20*a^6*b^6 + 15*a^8*b^4 + 6*a^10*b^2) - (16*
(6*a*b^2 - 2*a^3)*(3*a*b^16 + 21*a^3*b^14 + 63*a^5*b^12 + 105*a^7*b^10 + 10
5*a^9*b^8 + 63*a^11*b^6 + 21*a^13*b^4 + 3*a^15*b^2))/((a^6 + b^6 + 3*a^2*b^
4 + 3*a^4*b^2)*(a^12 + b^12 + 6*a^2*b^10 + 15*a^4*b^8 + 20*a^6*b^6 + 15*a^8
*b^4 + 6*a^10*b^2))))/(2*(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2))))/(2*(a^6 + b
^6 + 3*a^2*b^4 + 3*a^4*b^2)) - (b*((b*((32*(a^2*b^12 - 2*a^14 + 15*a^4*b^10
 + 48*a^6*b^8 + 62*a^8*b^6 + 33*a^10*b^4 + 3*a^12*b^2))/(a^12 + b^12 + 6*a^
2*b^10 + 15*a^4*b^8 + 20*a^6*b^6 + 15*a^8*b^4 + 6*a^10*b^2) - (16*(6*a*b^2
- 2*a^3)*(3*a*b^16 + 21*a^3*b^14 + 63*a^5*b^12 + 105*a^7*b^10 + 105*a^9*b^8
 + 63*a^11*b^6 + 21*a^13*b^4 + 3*a^15*b^2))/((a^6 + b^6 + 3*a^2*b^4 + 3*a^4
*b^2)*(a^12 + b^12 + 6*a^2*b^10 + 15*a^4*b^8 + 20*a^6*b^6 + 15*a^8*b^4 + 6*
a^10*b^2)))*(3*a^2 - b^2))/(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2) - (16*b*(6*a
*b^2 - 2*a^3)*(3*a^2 - b^2)*(3*a*b^16 + 21*a^3*b^14 + 63*a^5*b^12 + 105*a^7
*b^10 + 105*a^9*b^8 + 63*a^11*b^6 + 21*a^13*b^4 + 3*a^15*b^2))/((a^6 + b^6
+ 3*a^2*b^4 + 3*a^4*b^2)^2*(a^12 + b^12 + 6*a^2*b^10 + 15*a^4*b^8 + 20*a^6*
b^6 + 15*a^8*b^4 + 6*a^10*b^2)))*(3*a^2 - b^2))/(a^6 + b^6 + 3*a^2*b^4 + 3*
```

```
a^4*b^2) + (16*b^2*(6*a*b^2 - 2*a^3)*(3*a^2 - b^2)^2*(3*a*b^16 + 21*a^3*b^1
4 + 63*a^5*b^12 + 105*a^7*b^10 + 105*a^9*b^8 + 63*a^11*b^6 + 21*a^13*b^4 +
3*a^15*b^2))/((a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2)^3*(a^12 + b^12 + 6*a^2*b^
10 + 15*a^4*b^8 + 20*a^6*b^6 + 15*a^8*b^4 + 6*a^10*b^2))))/(4*a^8 + b^8 + 3
1*a^2*b^6 + 15*a^4*b^4 - 11*a^6*b^2)^2)*(a^16 + b^16 + 8*a^2*b^14 + 28*a^4*
b^12 + 56*a^6*b^10 + 70*a^8*b^8 + 56*a^10*b^6 + 28*a^12*b^4 + 8*a^14*b^2))/
(32*a*b^3 - 96*a^3*b) + (((b*((32*(5*a^2*b^9 - 3*a^10*b + 12*a^4*b^7 + 6*a^
6*b^5 - 4*a^8*b^3))/(a^12 + b^12 + 6*a^2*b^10 + 15*a^4*b^8 + 20*a^6*b^6 + 1
5*a^8*b^4 + 6*a^10*b^2) - ((6*a*b^2 - 2*a^3)*((32*(3*a^3*b^11 - 4*a^13*b -
a*b^13 + 18*a^5*b^9 + 22*a^7*b^7 + 3*a^9*b^5 - 9*a^11*b^3))/(a^12 + b^12 +
6*a^2*b^10 + 15*a^4*b^8 + 20*a^6*b^6 + 15*a^8*b^4 + 6*a^10*b^2) + (16*(6*a*
b^2 - 2*a^3)*(3*a^16*b + 3*a^2*b^15 + 21*a^4*b^13 + 63*a^6*b^11 + 105*a^8*b
^9 + 105*a^10*b^7 + 63*a^12*b^5 + 21*a^14*b^3))/((a^6 + b^6 + 3*a^2*b^4 + 3
*a^4*b^2)*(a^12 + b^12 + 6*a^2*b^10 + 15*a^4*b^8 + 20*a^6*b^6 + 15*a^8*b^4
 + 6*a^10*b^2))))/(2*(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2)))*(3*a^2 - b^2))/(a
^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2) - ((6*a*b^2 - 2*a^3)*((b*((32*(3*a^3*b^11
 - 4*a^13*b - a*b^13 + 18*a^5*b^9 + 22*a^7*b^7 + 3*a^9*b^5 - 9*a^11*b^3))/(
a^12 + b^12 + 6*a^2*b^10 + 15*a^4*b^8 + 20*a^6*b^6 + 15*a^8*b^4 + 6*a^10*b^
2) + (16*(6*a*b^2 - 2*a^3)*(3*a^16*b + 3*a^2*b^15 + 21*a^4*b^13 + 63*a^6*b^
11 + 105*a^8*b^9 + 105*a^10*b^7 + 63*a^12*b^5 + 21*a^14*b^3))/((a^6 + b^6 +
 3*a^2*b^4 + 3*a^4*b^2)*(a^12 + b^12 + 6*a^2*b^10 + 15*a^4*b^8 + 20*a^6*b^6
 + 15*a^8*b^4 + 6*a^10*b^2)))*(3*a^2 - b^2))/(a^6 + b^6 + 3*a^2*b^4 + 3*a^4
*b^2) + (16*b*(6*a*b^2 - 2*a^3)*(3*a^2 - b^2)*(3*a^16*b + 3*a^2*b^15 + 21*a
^4*b^13 + 63*a^6*b^11 + 105*a^8*b^9 + 105*a^10*b^7 + 63*a^12*b^5 + 21*a^14*
b^3))/((a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2)^2*(a^12 + b^12 + 6*a^2*b^10 + 15
*a^4*b^8 + 20*a^6*b^6 + 15*a^8*b^4 + 6*a^10*b^2))))/(2*(a^6 + b^6 + 3*a^2*b
^4 + 3*a^4*b^2)) + (32*b^3*(3*a^2 - b^2)^3*(3*a^16*b + 3*a^2*b^15 + 21*a^4*
b^13 + 63*a^6*b^11 + 105*a^8*b^9 + 105*a^10*b^7 + 63*a^12*b^5 + 21*a^14*b^3
))/((a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2)^3*(a^12 + b^12 + 6*a^2*b^10 + 15*a^
4*b^8 + 20*a^6*b^6 + 15*a^8*b^4 + 6*a^10*b^2)))*(4*a^8 + b^8 - 67*a^2*b^6 +
 155*a^4*b^4 - 61*a^6*b^2)*(a^16 + b^16 + 8*a^2*b^14 + 28*a^4*b^12 + 56*a^6
*b^10 + 70*a^8*b^8 + 56*a^10*b^6 + 28*a^12*b^4 + 8*a^14*b^2))/((32*a*b^3 -
96*a^3*b)*(4*a^8 + b^8 + 31*a^2*b^6 + 15*a^4*b^4 - 11*a^6*b^2)^2) + (2*a*b*
((32*(2*a^7*b + 6*a^3*b^5 - 8*a^5*b^3))/(a^12 + b^12 + 6*a^2*b^10 + 15*a^4*
b^8 + 20*a^6*b^6 + 15*a^8*b^4 + 6*a^10*b^2) + ((6*a*b^2 - 2*a^3)*((32*(5*a^
2*b^9 - 3*a^10*b + 12*a^4*b^7 + 6*a^6*b^5 - 4*a^8*b^3))/(a^12 + b^12 + 6*a^
2*b^10 + 15*a^4*b^8 + 20*a^6*b^6 + 15*a^8*b^4 + 6*a^10*b^2) - ((6*a*b^2 - 2
*a^3)*((32*(3*a^3*b^11 - 4*a^13*b - a*b^13 + 18*a^5*b^9 + 22*a^7*b^7 + 3*a^
9*b^5 - 9*a^11*b^3))/(a^12 + b^12 + 6*a^2*b^10 + 15*a^4*b^8 + 20*a^6*b^6 +
15*a^8*b^4 + 6*a^10*b^2) + (16*(6*a*b^2 - 2*a^3)*(3*a^16*b + 3*a^2*b^15 + 2
1*a^4*b^13 + 63*a^6*b^11 + 105*a^8*b^9 + 105*a^10*b^7 + 63*a^12*b^5 + 21*a^
14*b^3))/((a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2)*(a^12 + b^12 + 6*a^2*b^10 + 1
5*a^4*b^8 + 20*a^6*b^6 + 15*a^8*b^4 + 6*a^10*b^2))))/(2*(a^6 + b^6 + 3*a^2*
b^4 + 3*a^4*b^2))))/(2*(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2)) + (b*((b*((32*(
3*a^3*b^11 - 4*a^13*b - a*b^13 + 18*a^5*b^9 + 22*a^7*b^7 + 3*a^9*b^5 - 9*a^
```

```
11*b^3))/(a^12 + b^12 + 6*a^2*b^10 + 15*a^4*b^8 + 20*a^6*b^6 + 15*a^8*b^4 +
 6*a^10*b^2) + (16*(6*a*b^2 - 2*a^3)*(3*a^16*b + 3*a^2*b^15 + 21*a^4*b^13 +
 63*a^6*b^11 + 105*a^8*b^9 + 105*a^10*b^7 + 63*a^12*b^5 + 21*a^14*b^3))/((a
^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2)*(a^12 + b^12 + 6*a^2*b^10 + 15*a^4*b^8 +
20*a^6*b^6 + 15*a^8*b^4 + 6*a^10*b^2)))*(3*a^2 - b^2))/(a^6 + b^6 + 3*a^2*b
^4 + 3*a^4*b^2) + (16*b*(6*a*b^2 - 2*a^3)*(3*a^2 - b^2)*(3*a^16*b + 3*a^2*b
^15 + 21*a^4*b^13 + 63*a^6*b^11 + 105*a^8*b^9 + 105*a^10*b^7 + 63*a^12*b^5
+ 21*a^14*b^3))/((a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2)^2*(a^12 + b^12 + 6*a^2
*b^10 + 15*a^4*b^8 + 20*a^6*b^6 + 15*a^8*b^4 + 6*a^10*b^2)))*(3*a^2 - b^2))
/(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2) + (16*b^2*(6*a*b^2 - 2*a^3)*(3*a^2 - b
^2)^2*(3*a^16*b + 3*a^2*b^15 + 21*a^4*b^13 + 63*a^6*b^11 + 105*a^8*b^9 + 10
5*a^10*b^7 + 63*a^12*b^5 + 21*a^14*b^3))/((a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^
2)^3*(a^12 + b^12 + 6*a^2*b^10 + 15*a^4*b^8 + 20*a^6*b^6 + 15*a^8*b^4 + 6*a
^10*b^2)))*(10*a^6 - 7*b^6 + 68*a^2*b^4 - 59*a^4*b^2)*(a^16 + b^16 + 8*a^2*
b^14 + 28*a^4*b^12 + 56*a^6*b^10 + 70*a^8*b^8 + 56*a^10*b^6 + 28*a^12*b^4 +
 8*a^14*b^2))/((32*a*b^3 - 96*a^3*b)*(4*a^8 + b^8 + 31*a^2*b^6 + 15*a^4*b^4
 - 11*a^6*b^2)^2))*(3*a^2 - b^2))/(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2)
```

sympy [**F(-2)**] time = 0.00, size = 0, normalized size = 0.00

Exception raised: AttributeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)**3/(a*cos(x)+b*sin(x))**3,x)
[Out] Exception raised: AttributeError
```

3.23 $\int \frac{\sin^2(x)}{(a\cos(x)+b\sin(x))^3}\,dx$

Optimal. Leaf size=92

$$\frac{a\left(\left(a^2+4b^2\right)\sin(x)+3ab\cos(x)\right)}{2\left(a^2+b^2\right)^2(a\cos(x)+b\sin(x))^2}-\frac{\left(a^2-2b^2\right)\tanh^{-1}\left(\frac{a\tan\left(\frac{x}{2}\right)-b}{\sqrt{a^2+b^2}}\right)}{\left(a^2+b^2\right)^{5/2}}$$

```
[Out] -(a^2-2*b^2)*arctanh((-b+a*tan(1/2*x))/(a^2+b^2)^(1/2))/(a^2+b^2)^(5/2)+1/2
*a*(3*a*b*cos(x)+(a^2+4*b^2)*sin(x))/(a^2+b^2)^2/(a*cos(x)+b*sin(x))^2
```

Rubi [B] time = 0.70, antiderivative size = 300, normalized size of antiderivative = 3.26, number of steps used = 13, number of rules used = 7, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.438, Rules used = {4401, 1660, 12, 618, 206, 3155, 3074}

$$-\frac{ab\left(5a^2+2b^2\right)\tan\left(\frac{x}{2}\right)+3a^2b^2+4a^4+2b^4}{ab\left(a^2+b^2\right)^2\left(-a\tan^2\left(\frac{x}{2}\right)+a+2b\tan\left(\frac{x}{2}\right)\right)}+\frac{2\left(\left(a^2+2b^2\right)\tan\left(\frac{x}{2}\right)+ab\right)}{a\left(a^2+b^2\right)\left(-a\tan^2\left(\frac{x}{2}\right)+a+2b\tan\left(\frac{x}{2}\right)\right)^2}+\frac{2a}{b\left(a^2+b^2\right)(a\cos(x)}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[x]^2/(a*Cos[x] + b*Sin[x])^3,x]
```

```
[Out] (2*a^2*ArcTanh[(b*Cos[x] - a*Sin[x])/Sqrt[a^2 + b^2]])/(b^2*(a^2 + b^2)^(3/
2)) - ArcTanh[(b*Cos[x] - a*Sin[x])/Sqrt[a^2 + b^2]]/(b^2*Sqrt[a^2 + b^2])
- (a^2*(2*a^2 - b^2)*ArcTanh[(b - a*Tan[x/2])/Sqrt[a^2 + b^2]])/(b^2*(a^2 +
 b^2)^(5/2)) + (2*a)/(b*(a^2 + b^2)*(a*Cos[x] + b*Sin[x])) + (2*(a*b + (a^2
 + 2*b^2)*Tan[x/2]))/(a*(a^2 + b^2)*(a + 2*b*Tan[x/2] - a*Tan[x/2]^2)^2) -
(4*a^4 + 3*a^2*b^2 + 2*b^4 + a*b*(5*a^2 + 2*b^2)*Tan[x/2])/(a*b*(a^2 + b^2)
^2*(a + 2*b*Tan[x/2] - a*Tan[x/2]^2))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1660

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[((b*f - 2*a*g + (2*c*f - b*g)*x)*(a + b*x + c*x^2)^
(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*(
2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2
 - 4*a*c, 0] && LtQ[p, -1]
```

Rule 3074

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x
_Symbol] :> -Dist[d^(-1), Subst[Int[1/(a^2 + b^2 - x^2), x], x, b*Cos[c + d
*x] - a*Sin[c + d*x]], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

Rule 3155

```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.))/((a_.) + cos[(d_.) + (e_.)*(x_)
]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)])^2, x_Symbol] :> Simp[(c*B + c*A*Co
s[d + e*x] + (a*B - b*A)*Sin[d + e*x])/(e*(a^2 - b^2 - c^2)*(a + b*Cos[d +
e*x] + c*Sin[d + e*x])), x] + Dist[(a*A - b*B)/(a^2 - b^2 - c^2), Int[1/(a
+ b*Cos[d + e*x] + c*Sin[d + e*x]), x], x] /; FreeQ[{a, b, c, d, e, A, B},
x] && NeQ[a^2 - b^2 - c^2, 0] && NeQ[a*A - b*B, 0]
```

Rule 4401

```
Int[u_, x_Symbol] :> With[{v = ExpandTrig[u, x]}, Int[v, x] /; SumQ[v]] /;
 !InertTrigFreeQ[u]
```

Rubi steps

$$\int \frac{\sin^2(x)}{(a\cos(x)+b\sin(x))^3}\,dx = \int \left(\frac{a^2\cos^2(x)}{b^2(a\cos(x)+b\sin(x))^3} - \frac{2a\cos(x)}{b^2(a\cos(x)+b\sin(x))^2} + \frac{1}{b^2(a\cos(x)+b\sin(x))}\right)$$

$$= \frac{\int \frac{1}{a\cos(x)+b\sin(x)}\,dx}{b^2} - \frac{(2a)\int \frac{\cos(x)}{(a\cos(x)+b\sin(x))^2}\,dx}{b^2} + \frac{a^2\int \frac{\cos^2(x)}{(a\cos(x)+b\sin(x))^3}\,dx}{b^2}$$

$$= \frac{2a}{b\left(a^2+b^2\right)(a\cos(x)+b\sin(x))} - \frac{\text{Subst}\left(\int \frac{1}{a^2+b^2-x^2}\,dx, x, b\cos(x)-a\sin(x)\right)}{b^2} + -$$

$$= -\frac{\tanh^{-1}\left(\frac{b\cos(x)-a\sin(x)}{\sqrt{a^2+b^2}}\right)}{b^2\sqrt{a^2+b^2}} + \frac{2a}{b\left(a^2+b^2\right)(a\cos(x)+b\sin(x))} + \frac{2\left(ab+\left(a^2+\right.\right.}{a\left(a^2+b^2\right)\left(a+2b\,\mathrm{t}\right.}$$

$$= \frac{2a^2\tanh^{-1}\left(\frac{b\cos(x)-a\sin(x)}{\sqrt{a^2+b^2}}\right)}{b^2\left(a^2+b^2\right)^{3/2}} - \frac{\tanh^{-1}\left(\frac{b\cos(x)-a\sin(x)}{\sqrt{a^2+b^2}}\right)}{b^2\sqrt{a^2+b^2}} + \frac{2a}{b\left(a^2+b^2\right)(a\cos(x)+b\,\mathrm{si}}$$

$$= \frac{2a^2\tanh^{-1}\left(\frac{b\cos(x)-a\sin(x)}{\sqrt{a^2+b^2}}\right)}{b^2\left(a^2+b^2\right)^{3/2}} - \frac{\tanh^{-1}\left(\frac{b\cos(x)-a\sin(x)}{\sqrt{a^2+b^2}}\right)}{b^2\sqrt{a^2+b^2}} + \frac{2a}{b\left(a^2+b^2\right)(a\cos(x)+b\,\mathrm{si}}$$

$$= \frac{2a^2\tanh^{-1}\left(\frac{b\cos(x)-a\sin(x)}{\sqrt{a^2+b^2}}\right)}{b^2\left(a^2+b^2\right)^{3/2}} - \frac{\tanh^{-1}\left(\frac{b\cos(x)-a\sin(x)}{\sqrt{a^2+b^2}}\right)}{b^2\sqrt{a^2+b^2}} + \frac{2a}{b\left(a^2+b^2\right)(a\cos(x)+b\,\mathrm{si}}$$

$$= \frac{2a^2\tanh^{-1}\left(\frac{b\cos(x)-a\sin(x)}{\sqrt{a^2+b^2}}\right)}{b^2\left(a^2+b^2\right)^{3/2}} - \frac{\tanh^{-1}\left(\frac{b\cos(x)-a\sin(x)}{\sqrt{a^2+b^2}}\right)}{b^2\sqrt{a^2+b^2}} - \frac{a^2\left(2a^2-b^2\right)\tanh^{-1}\left(\frac{b-a\,\mathrm{ta}}{\sqrt{a^2}}\right.}{b^2\left(a^2+b^2\right)^{5/2}}$$

<u>**Mathematica**</u> [**A**]   time = 0.43, size = 92, normalized size = 1.00

$$\frac{a\left(\left(a^2+4b^2\right)\sin(x)+3ab\cos(x)\right)}{2\left(a^2+b^2\right)^2(a\cos(x)+b\sin(x))^2} - \frac{\left(a^2-2b^2\right)\tanh^{-1}\left(\frac{a\tan\left(\frac{x}{2}\right)-b}{\sqrt{a^2+b^2}}\right)}{\left(a^2+b^2\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[x]^2/(a*Cos[x] + b*Sin[x])^3,x]
```

```
[Out] -(((a^2 - 2*b^2)*ArcTanh[(-b + a*Tan[x/2])/Sqrt[a^2 + b^2]])/(a^2 + b^2)^(5
/2)) + (a*(3*a*b*Cos[x] + (a^2 + 4*b^2)*Sin[x]))/(2*(a^2 + b^2)^2*(a*Cos[x]
 + b*Sin[x])^2)
```

fricas [B] time = 1.41, size = 282, normalized size = 3.07

$$-\frac{\left(a^2b^2 - 2\,b^4 + \left(a^4 - 3\,a^2b^2 + 2\,b^4\right)\cos(x)^2 + 2\left(a^3b - 2\,ab^3\right)\cos(x)\sin(x)\right)\sqrt{a^2+b^2}\,\log\left(-\frac{2\,ab\cos(x)\sin(x)+\left(a^2-b^2\ldots\right.}{2\,ab\cos\ldots}\right.}{4\left(a^6b^2 + 3\,a^4b^4 + 3\,a^2b^6 + b^8 + \left(a^8 + 2\,a^6b^2 - 2\,a^2b^6 - b^8\right)\cos(x)\ldots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^2/(a*cos(x)+b*sin(x))^3,x, algorithm="fricas")
```

```
[Out] -1/4*((a^2*b^2 - 2*b^4 + (a^4 - 3*a^2*b^2 + 2*b^4)*cos(x)^2 + 2*(a^3*b - 2*
a*b^3)*cos(x)*sin(x))*sqrt(a^2 + b^2)*log(-(2*a*b*cos(x)*sin(x) + (a^2 - b^
2)*cos(x)^2 - 2*a^2 - b^2 + 2*sqrt(a^2 + b^2)*(b*cos(x) - a*sin(x)))/(2*a*b
*cos(x)*sin(x) + (a^2 - b^2)*cos(x)^2 + b^2)) - 6*(a^4*b + a^2*b^3)*cos(x)
- 2*(a^5 + 5*a^3*b^2 + 4*a*b^4)*sin(x))/(a^6*b^2 + 3*a^4*b^4 + 3*a^2*b^6 +
b^8 + (a^8 + 2*a^6*b^2 - 2*a^2*b^6 - b^8)*cos(x)^2 + 2*(a^7*b + 3*a^5*b^3 +
 3*a^3*b^5 + a*b^7)*cos(x)*sin(x))
```

giac [B] time = 0.28, size = 197, normalized size = 2.14

$$\frac{\left(a^2 - 2\,b^2\right)\log\left(\frac{\left|2\,a\tan\left(\frac{1}{2}\,x\right) - 2\,b - 2\,\sqrt{a^2+b^2}\right|}{\left|2\,a\tan\left(\frac{1}{2}\,x\right) - 2\,b + 2\,\sqrt{a^2+b^2}\right|}\right)}{2\left(a^4 + 2\,a^2b^2 + b^4\right)\sqrt{a^2+b^2}} + \frac{a^3\tan\left(\frac{1}{2}\,x\right)^3 - 2\,ab^2\tan\left(\frac{1}{2}\,x\right)^3 - 3\,a^2b\tan\left(\frac{1}{2}\,x\right)^2 + 6\,b^3\tan\left(\frac{1}{2}\,x\right)^2 + \ldots}{\left(a^4 + 2\,a^2b^2 + b^4\right)\left(a\tan\left(\frac{1}{2}\,x\right)^2 - 2\,b\tan\left(\ldots\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^2/(a*cos(x)+b*sin(x))^3,x, algorithm="giac")
```

```
[Out] 1/2*(a^2 - 2*b^2)*log(abs(2*a*tan(1/2*x) - 2*b - 2*sqrt(a^2 + b^2))/abs(2*a
*tan(1/2*x) - 2*b + 2*sqrt(a^2 + b^2)))/((a^4 + 2*a^2*b^2 + b^4)*sqrt(a^2 +
 b^2)) + (a^3*tan(1/2*x)^3 - 2*a*b^2*tan(1/2*x)^3 - 3*a^2*b*tan(1/2*x)^2 +
6*b^3*tan(1/2*x)^2 + a^3*tan(1/2*x) + 10*a*b^2*tan(1/2*x) + 3*a^2*b)/((a^4
+ 2*a^2*b^2 + b^4)*(a*tan(1/2*x)^2 - 2*b*tan(1/2*x) - a)^2)
```

maple [B] time = 0.59, size = 212, normalized size = 2.30

$$-\frac{8\left(-\frac{a\left(a^2-2b^2\right)\left(\tan^3\left(\frac{x}{2}\right)\right)}{8\left(a^4+2a^2b^2+b^4\right)} + \frac{3b\left(a^2-2b^2\right)\left(\tan^2\left(\frac{x}{2}\right)\right)}{8\left(a^4+2a^2b^2+b^4\right)} - \frac{\left(a^2+10b^2\right)a\tan\left(\frac{x}{2}\right)}{8\left(a^4+2a^2b^2+b^4\right)} - \frac{3a^2b}{8\left(a^4+2a^2b^2+b^4\right)}\right)}{\left(a\left(\tan^2\left(\frac{x}{2}\right)\right) - 2\tan\left(\frac{x}{2}\right)b - a\right)^2} - \frac{\left(a^2 - 2b^2\right)\operatorname{arctanh}\left(\frac{2a\tan\left(\frac{x}{2}\right) - 2b}{2\sqrt{a^2+b^2}}\right)}{\left(a^4 + 2a^2b^2 + b^4\right)\sqrt{a^2+b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)^2/(a*cos(x)+b*sin(x))^3,x)
```

```
[Out] -8*(-1/8*a*(a^2-2*b^2)/(a^4+2*a^2*b^2+b^4)*tan(1/2*x)^3+3/8*b*(a^2-2*b^2)/(
a^4+2*a^2*b^2+b^4)*tan(1/2*x)^2-1/8*(a^2+10*b^2)*a/(a^4+2*a^2*b^2+b^4)*tan(
1/2*x)-3/8*a^2*b/(a^4+2*a^2*b^2+b^4))/(a*tan(1/2*x)^2-2*tan(1/2*x)*b-a)^2-(
a^2-2*b^2)/(a^4+2*a^2*b^2+b^4)/(a^2+b^2)^(1/2)*arctanh(1/2*(2*a*tan(1/2*x)-
2*b)/(a^2+b^2)^(1/2))
```

maxima [B] time = 0.43, size = 299, normalized size = 3.25

$$\frac{\left(a^{2}-2\,b^{2}\right)\log\left(\frac{b-\frac{a\sin(x)}{\cos(x)+1}+\sqrt{a^{2}+b^{2}}}{b-\frac{a\sin(x)}{\cos(x)+1}-\sqrt{a^{2}+b^{2}}}\right)}{2\left(a^{4}+2\,a^{2}b^{2}+b^{4}\right)\sqrt{a^{2}+b^{2}}}+\frac{3\,a^{2}b+\frac{\left(a^{3}+10\,ab^{2}\right)\sin(x)}{\cos(x)+1}-\frac{3\left(a^{2}b-2\,b^{3}\right)\sin(x)^{2}}{(\cos(x)+1)^{2}}+\frac{\left(a^{3}-2\,ab^{2}\right)\ldots}{(\cos(x)\ldots}}{a^{6}+2\,a^{4}b^{2}+a^{2}b^{4}+\frac{4\left(a^{5}b+2\,a^{3}b^{3}+ab^{5}\right)\sin(x)}{\cos(x)+1}-\frac{2\left(a^{6}-3\,a^{2}b^{4}-2\,b^{6}\right)\sin(x)^{2}}{(\cos(x)+1)^{2}}-\frac{4\left(a^{5}b+2\,a^{3}b^{3}\ldots\right.}{(\cos(x)\ldots}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^2/(a*cos(x)+b*sin(x))^3,x, algorithm="maxima")
```

```
[Out] 1/2*(a^2 - 2*b^2)*log((b - a*sin(x)/(cos(x) + 1) + sqrt(a^2 + b^2))/(b - a*
sin(x)/(cos(x) + 1) - sqrt(a^2 + b^2)))/((a^4 + 2*a^2*b^2 + b^4)*sqrt(a^2 +
 b^2)) + (3*a^2*b + (a^3 + 10*a*b^2)*sin(x)/(cos(x) + 1) - 3*(a^2*b - 2*b^3
)*sin(x)^2/(cos(x) + 1)^2 + (a^3 - 2*a*b^2)*sin(x)^3/(cos(x) + 1)^3)/(a^6 +
 2*a^4*b^2 + a^2*b^4 + 4*(a^5*b + 2*a^3*b^3 + a*b^5)*sin(x)/(cos(x) + 1) -
2*(a^6 - 3*a^2*b^4 - 2*b^6)*sin(x)^2/(cos(x) + 1)^2 - 4*(a^5*b + 2*a^3*b^3
+ a*b^5)*sin(x)^3/(cos(x) + 1)^3 + (a^6 + 2*a^4*b^2 + a^2*b^4)*sin(x)^4/(co
s(x) + 1)^4)
```

mupad [B] time = 0.80, size = 263, normalized size = 2.86

$$\frac{\frac{3\,a^{2}\,b}{a^{4}+2\,a^{2}\,b^{2}+b^{4}}+\frac{a\tan\left(\frac{x}{2}\right)\left(a^{2}+10\,b^{2}\right)}{a^{4}+2\,a^{2}\,b^{2}+b^{4}}+\frac{a\tan\left(\frac{x}{2}\right)^{3}\left(a^{2}-2\,b^{2}\right)}{a^{4}+2\,a^{2}\,b^{2}+b^{4}}-\frac{3\,b\tan\left(\frac{x}{2}\right)^{2}\left(a^{2}-2\,b^{2}\right)}{a^{4}+2\,a^{2}\,b^{2}+b^{4}}}{a^{2}-\tan\left(\frac{x}{2}\right)^{2}\left(2\,a^{2}-4\,b^{2}\right)+a^{2}\tan\left(\frac{x}{2}\right)^{4}+4\,a\,b\tan\left(\frac{x}{2}\right)-4\,a\,b\tan\left(\frac{x}{2}\right)^{3}}+\frac{\mathrm{atanh}\left(\frac{2\,a^{4}\,b+4\,a^{2}\,b^{3}+2\,b^{5}}{2\left(a^{2}+b^{2}\right)^{5/2}}-\frac{a\tan\left(\frac{x}{2}\right)\left(a^{4}+2\,a^{2}\,b\ldots\right.}{\left(a^{2}+b^{2}\right)^{5/2}}\ldots}{\left(a^{2}+b^{2}\right)^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)^2/(a*cos(x) + b*sin(x))^3,x)
```

```
[Out] ((3*a^2*b)/(a^4 + b^4 + 2*a^2*b^2) + (a*tan(x/2)*(a^2 + 10*b^2))/(a^4 + b^4
 + 2*a^2*b^2) + (a*tan(x/2)^3*(a^2 - 2*b^2))/(a^4 + b^4 + 2*a^2*b^2) - (3*b
*tan(x/2)^2*(a^2 - 2*b^2))/(a^4 + b^4 + 2*a^2*b^2))/(a^2 - tan(x/2)^2*(2*a^
2 - 4*b^2) + a^2*tan(x/2)^4 + 4*a*b*tan(x/2) - 4*a*b*tan(x/2)^3) + (atanh((
2*a^4*b + 2*b^5 + 4*a^2*b^3)/(2*(a^2 + b^2)^(5/2)) - (a*tan(x/2)*(a^4 + b^4
 + 2*a^2*b^2))/(a^2 + b^2)^(5/2))*(a^2 - 2*b^2))/(a^2 + b^2)^(5/2)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)**2/(a*cos(x)+b*sin(x))**3,x)
```

```
[Out] Timed out
```

3.24 $\int \frac{\sin(x)}{(a\cos(x)+b\sin(x))^3}\,dx$

Optimal. Leaf size=15

$$\frac{1}{2a(a\cot(x)+b)^2}$$

```
[Out] 1/2/a/(b+a*cot(x))^2
```

Rubi [A] time = 0.03, antiderivative size = 19, normalized size of antiderivative = 1.27, number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3087, 37}

$$\frac{\tan^2(x)}{2a(a+b\tan(x))^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[x]/(a*Cos[x] + b*Sin[x])^3,x]
```

```
[Out] Tan[x]^2/(2*a*(a + b*Tan[x])^2)
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 3087

```
Int[sin[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[1/d, Subst[Int[(x^m*(a + b*x
)^n)/(1 + x^2)^((m + n + 2)/2), x], x, Tan[c + d*x]], x] /; FreeQ[{a, b, c,
 d}, x] && IntegerQ[n] && IntegerQ[(m + n)/2] && NeQ[n, -1] &&  !(GtQ[n, 0]
 && GtQ[m, 1])
```

Rubi steps

$$\begin{aligned}\int \frac{\sin(x)}{(a\cos(x)+b\sin(x))^3}\,dx &= \text{Subst}\left(\int \frac{x}{(a+bx)^3}\,dx, x, \tan(x)\right)\\ &= \frac{\tan^2(x)}{2a(a+b\tan(x))^2}\end{aligned}$$

Mathematica [B] time = 0.10, size = 47, normalized size = 3.13

$$\frac{a(a+b\sin(2x))+2b^2\sin^2(x)}{2a\left(a^2+b^2\right)(a\cos(x)+b\sin(x))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[x]/(a*Cos[x] + b*Sin[x])^3,x]
```

```
[Out] (2*b^2*Sin[x]^2 + a*(a + b*Sin[2*x]))/(2*a*(a^2 + b^2)*(a*Cos[x] + b*Sin[x]
)^2)
```

fricas [B] time = 0.65, size = 116, normalized size = 7.73

$$-\frac{4\,ab^2\cos(x)^2-a^3-3\,ab^2-2\left(a^2b-b^3\right)\cos(x)\sin(x)}{2\left(a^4b^2+2\,a^2b^4+b^6+\left(a^6+a^4b^2-a^2b^4-b^6\right)\cos(x)^2+2\left(a^5b+2\,a^3b^3+ab^5\right)\cos(x)\sin(x)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)/(a*cos(x)+b*sin(x))^3,x, algorithm="fricas")
```

```
[Out] -1/2*(4*a*b^2*cos(x)^2 - a^3 - 3*a*b^2 - 2*(a^2*b - b^3)*cos(x)*sin(x))/(a^
4*b^2 + 2*a^2*b^4 + b^6 + (a^6 + a^4*b^2 - a^2*b^4 - b^6)*cos(x)^2 + 2*(a^5
*b + 2*a^3*b^3 + a*b^5)*cos(x)*sin(x))
```

giac [A] time = 0.26, size = 20, normalized size = 1.33

$$-\frac{2\,b\tan(x)+a}{2\,(b\tan(x)+a)^2b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)/(a*cos(x)+b*sin(x))^3,x, algorithm="giac")
```

```
[Out] -1/2*(2*b*tan(x) + a)/((b*tan(x) + a)^2*b^2)
```

maple [B] time = 0.69, size = 29, normalized size = 1.93

$$\frac{a}{2b^2\,(a+b\tan(x))^2}-\frac{1}{b^2\,(a+b\tan(x))}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)/(a*cos(x)+b*sin(x))^3,x)
```

```
[Out] 1/2*a/b^2/(a+b*tan(x))^2-1/b^2/(a+b*tan(x))
```

maxima [B] time = 0.34, size = 84, normalized size = 5.60

$$\frac{2 \sin(x)^2}{\left(a^3 + \frac{4 a^2 b \sin(x)}{\cos(x)+1} - \frac{4 a^2 b \sin(x)^3}{(\cos(x)+1)^3} + \frac{a^3 \sin(x)^4}{(\cos(x)+1)^4} - \frac{2 \left(a^3-2 a b^2\right) \sin(x)^2}{(\cos(x)+1)^2}\right)(\cos(x) + 1)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)/(a*cos(x)+b*sin(x))^3,x, algorithm="maxima")
```

```
[Out] 2*sin(x)^2/((a^3 + 4*a^2*b*sin(x)/(cos(x) + 1) - 4*a^2*b*sin(x)^3/(cos(x) +
 1)^3 + a^3*sin(x)^4/(cos(x) + 1)^4 - 2*(a^3 - 2*a*b^2)*sin(x)^2/(cos(x) +
1)^2)*(cos(x) + 1)^2)
```

mupad [B] time = 0.49, size = 48, normalized size = 3.20

$$\frac{\tan\left(\frac{x}{2}\right)^2 \left(a - \frac{2 a^2-4 b^2}{2 a}\right)}{b^2 \left(-a \tan\left(\frac{x}{2}\right)^2 + 2 b \tan\left(\frac{x}{2}\right) + a\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)/(a*cos(x) + b*sin(x))^3,x)
```

```
[Out] (tan(x/2)^2*(a - (2*a^2 - 4*b^2)/(2*a)))/(b^2*(a + 2*b*tan(x/2) - a*tan(x/2
)^2)^2)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)/(a*cos(x)+b*sin(x))**3,x)
```

```
[Out] Timed out
```

3.25 $\int \frac{1}{(a\cos(x)+b\sin(x))^3}\,dx$

Optimal. Leaf size=73

$$-\frac{b\cos(x)-a\sin(x)}{2\left(a^2+b^2\right)(a\cos(x)+b\sin(x))^2}-\frac{\tanh^{-1}\left(\frac{b\cos(x)-a\sin(x)}{\sqrt{a^2+b^2}}\right)}{2\left(a^2+b^2\right)^{3/2}}$$

```
[Out] -1/2*arctanh((b*cos(x)-a*sin(x))/(a^2+b^2)^(1/2))/(a^2+b^2)^(3/2)+1/2*(-b*c
os(x)+a*sin(x))/(a^2+b^2)/(a*cos(x)+b*sin(x))^2
```

Rubi [A] time = 0.04, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {3076, 3074, 206}

$$-\frac{b\cos(x)-a\sin(x)}{2\left(a^2+b^2\right)(a\cos(x)+b\sin(x))^2}-\frac{\tanh^{-1}\left(\frac{b\cos(x)-a\sin(x)}{\sqrt{a^2+b^2}}\right)}{2\left(a^2+b^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Cos[x] + b*Sin[x])^(-3),x]
```

```
[Out] -ArcTanh[(b*Cos[x] - a*Sin[x])/Sqrt[a^2 + b^2]]/(2*(a^2 + b^2)^(3/2)) - (b*
Cos[x] - a*Sin[x])/(2*(a^2 + b^2)*(a*Cos[x] + b*Sin[x])^2)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3074

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x
_Symbol] :> -Dist[d^(-1), Subst[Int[1/(a^2 + b^2 - x^2), x], x, b*Cos[c + d
*x] - a*Sin[c + d*x]], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

Rule 3076

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x
_Symbol] :> Simp[((b*Cos[c + d*x] - a*Sin[c + d*x])*(a*Cos[c + d*x] + b*Sin
[c + d*x])^(n + 1))/(d*(n + 1)*(a^2 + b^2)), x] + Dist[(n + 2)/((n + 1)*(a^
2 + b^2)), Int[(a*Cos[c + d*x] + b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{
```

```
a, b, c, d}, x] && NeQ[a^2 + b^2, 0] && LtQ[n, -1] && NeQ[n, -2]
```

Rubi steps

$$\int \frac{1}{(a\cos(x)+b\sin(x))^3}\,dx = -\frac{b\cos(x)-a\sin(x)}{2\left(a^2+b^2\right)(a\cos(x)+b\sin(x))^2}+\frac{\int \frac{1}{a\cos(x)+b\sin(x)}\,dx}{2\left(a^2+b^2\right)}$$

$$= -\frac{b\cos(x)-a\sin(x)}{2\left(a^2+b^2\right)(a\cos(x)+b\sin(x))^2}-\frac{\text{Subst}\left(\int \frac{1}{a^2+b^2-x^2}\,dx, x, b\cos(x)-a\sin(x)\right)}{2\left(a^2+b^2\right)}$$

$$= -\frac{\tanh^{-1}\left(\frac{b\cos(x)-a\sin(x)}{\sqrt{a^2+b^2}}\right)}{2\left(a^2+b^2\right)^{3/2}}-\frac{b\cos(x)-a\sin(x)}{2\left(a^2+b^2\right)(a\cos(x)+b\sin(x))^2}$$

Mathematica [C] time = 0.17, size = 101, normalized size = 1.38

$$\frac{\left(a^2+b^2\right)(a\sin(x)-b\cos(x))+2\sqrt{a^2+b^2}\,(a\cos(x)+b\sin(x))^2\tanh^{-1}\left(\frac{a\tan\left(\frac{x}{2}\right)-b}{\sqrt{a^2+b^2}}\right)}{2(a-ib)^2(a+ib)^2(a\cos(x)+b\sin(x))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Cos[x] + b*Sin[x])^(-3),x]
```

```
[Out] ((a^2 + b^2)*(-(b*Cos[x]) + a*Sin[x]) + 2*Sqrt[a^2 + b^2]*ArcTanh[(-b + a*T
an[x/2])/Sqrt[a^2 + b^2]]*(a*Cos[x] + b*Sin[x])^2)/(2*(a - I*b)^2*(a + I*b)
^2*(a*Cos[x] + b*Sin[x])^2)
```

fricas [B] time = 0.59, size = 225, normalized size = 3.08

$$\frac{\left(2\,ab\cos(x)\sin(x)+\left(a^2-b^2\right)\cos(x)^2+b^2\right)\sqrt{a^2+b^2}\,\log\left(-\frac{2\,ab\cos(x)\sin(x)+\left(a^2-b^2\right)\cos(x)^2-2\,a^2-b^2+2\,\sqrt{a^2+b^2}\,(b\cos(x)-a\sin}{2\,ab\cos(x)\sin(x)+\left(a^2-b^2\right)\cos(x)^2+b^2}\right.}{4\left(a^4b^2+2\,a^2b^4+b^6+\left(a^6+a^4b^2-a^2b^4-b^6\right)\cos(x)^2+2\left(a^5b+2\,a^3b^3+ab^5\right)\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cos(x)+b*sin(x))^3,x, algorithm="fricas")
```

```
[Out] 1/4*((2*a*b*cos(x)*sin(x) + (a^2 - b^2)*cos(x)^2 + b^2)*sqrt(a^2 + b^2)*log
(-(2*a*b*cos(x)*sin(x) + (a^2 - b^2)*cos(x)^2 - 2*a^2 - b^2 + 2*sqrt(a^2 +
b^2)*(b*cos(x) - a*sin(x)))/(2*a*b*cos(x)*sin(x) + (a^2 - b^2)*cos(x)^2 + b
```

```
^2)) - 2*(a^2*b + b^3)*cos(x) + 2*(a^3 + a*b^2)*sin(x))/(a^4*b^2 + 2*a^2*b^
4 + b^6 + (a^6 + a^4*b^2 - a^2*b^4 - b^6)*cos(x)^2 + 2*(a^5*b + 2*a^3*b^3 +
 a*b^5)*cos(x)*sin(x))
```

giac [B] time = 2.38, size = 166, normalized size = 2.27

$$-\frac{\log\left(\frac{\left|2\,a\tan\left(\frac{1}{2}\,x\right)-2\,b-2\,\sqrt{a^2+b^2}\right|}{\left|2\,a\tan\left(\frac{1}{2}\,x\right)-2\,b+2\,\sqrt{a^2+b^2}\right|}\right)}{2\,\left(a^2+b^2\right)^{\frac{3}{2}}}+\frac{a^3\tan\left(\frac{1}{2}\,x\right)^3+2\,ab^2\tan\left(\frac{1}{2}\,x\right)^3+a^2b\tan\left(\frac{1}{2}\,x\right)^2-2\,b^3\tan\left(\frac{1}{2}\,x\right)^2+a^3\tan\left(\frac{1}{2}\,x\right.}{\left(a^4+a^2b^2\right)\left(a\tan\left(\frac{1}{2}\,x\right)^2-2\,b\tan\left(\frac{1}{2}\,x\right)-a\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cos(x)+b*sin(x))^3,x, algorithm="giac")
```

```
[Out] -1/2*log(abs(2*a*tan(1/2*x) - 2*b - 2*sqrt(a^2 + b^2))/abs(2*a*tan(1/2*x) -
 2*b + 2*sqrt(a^2 + b^2)))/(a^2 + b^2)^(3/2) + (a^3*tan(1/2*x)^3 + 2*a*b^2*
tan(1/2*x)^3 + a^2*b*tan(1/2*x)^2 - 2*b^3*tan(1/2*x)^2 + a^3*tan(1/2*x) - 2
*a*b^2*tan(1/2*x) - a^2*b)/((a^4 + a^2*b^2)*(a*tan(1/2*x)^2 - 2*b*tan(1/2*x
) - a)^2)
```

maple [B] time = 0.54, size = 157, normalized size = 2.15

$$-\frac{2\left(-\frac{\left(a^2+2b^2\right)\left(\tan^3\left(\frac{x}{2}\right)\right)}{2\left(a^2+b^2\right)a}-\frac{b\left(a^2-2b^2\right)\left(\tan^2\left(\frac{x}{2}\right)\right)}{2a^2\left(a^2+b^2\right)}-\frac{\left(a^2-2b^2\right)\tan\left(\frac{x}{2}\right)}{2\left(a^2+b^2\right)a}+\frac{b}{2a^2+2b^2}\right)}{\left(a\left(\tan^2\left(\frac{x}{2}\right)\right)-2\tan\left(\frac{x}{2}\right)b-a\right)^2}+\frac{\operatorname{arctanh}\left(\frac{2a\tan\left(\frac{x}{2}\right)-2b}{2\sqrt{a^2+b^2}}\right)}{\left(a^2+b^2\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*cos(x)+b*sin(x))^3,x)
```

```
[Out] -2*(-1/2*(a^2+2*b^2)/(a^2+b^2)/a*tan(1/2*x)^3-1/2*b*(a^2-2*b^2)/a^2/(a^2+b^
2)*tan(1/2*x)^2-1/2*(a^2-2*b^2)/(a^2+b^2)/a*tan(1/2*x)+1/2*b/(a^2+b^2))/(a*
tan(1/2*x)^2-2*tan(1/2*x)*b-a)^2+1/(a^2+b^2)^(3/2)*arctanh(1/2*(2*a*tan(1/2
*x)-2*b)/(a^2+b^2)^(1/2))
```

maxima [B] time = 0.44, size = 250, normalized size = 3.42

$$-\frac{a^2b-\frac{\left(a^3-2\,ab^2\right)\sin(x)}{\cos(x)+1}-\frac{\left(a^2b-2\,b^3\right)\sin(x)^2}{\left(\cos(x)+1\right)^2}-\frac{\left(a^3+2\,ab^2\right)\sin(x)^3}{\left(\cos(x)+1\right)^3}}{a^6+a^4b^2+\frac{4\left(a^5b+a^3b^3\right)\sin(x)}{\cos(x)+1}-\frac{2\left(a^6-a^4b^2-2\,a^2b^4\right)\sin(x)^2}{\left(\cos(x)+1\right)^2}-\frac{4\left(a^5b+a^3b^3\right)\sin(x)^3}{\left(\cos(x)+1\right)^3}+\frac{\left(a^6+a^4b^2\right)\sin(x)^4}{\left(\cos(x)+1\right)^4}}-\frac{\log\left(\frac{b-\frac{a\sin(x)}{\cos(x)+1}+\sqrt{a^2+b^2}}{b-\frac{a\sin(x)}{\cos(x)+1}-\sqrt{a^2+b^2}}\right)}{2\left(a^2+b^2\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cos(x)+b*sin(x))^3,x, algorithm="maxima")
```

```
[Out] -(a^2*b - (a^3 - 2*a*b^2)*sin(x)/(cos(x) + 1) - (a^2*b - 2*b^3)*sin(x)^2/(c
os(x) + 1)^2 - (a^3 + 2*a*b^2)*sin(x)^3/(cos(x) + 1)^3)/(a^6 + a^4*b^2 + 4*
(a^5*b + a^3*b^3)*sin(x)/(cos(x) + 1) - 2*(a^6 - a^4*b^2 - 2*a^2*b^4)*sin(x
)^2/(cos(x) + 1)^2 - 4*(a^5*b + a^3*b^3)*sin(x)^3/(cos(x) + 1)^3 + (a^6 + a
^4*b^2)*sin(x)^4/(cos(x) + 1)^4) - 1/2*log((b - a*sin(x)/(cos(x) + 1) + sqr
t(a^2 + b^2))/(b - a*sin(x)/(cos(x) + 1) - sqrt(a^2 + b^2)))/(a^2 + b^2)^(3
/2)
```

mupad [B] time = 0.76, size = 216, normalized size = 2.96

$$\frac{\frac{\tan\left(\frac{x}{2}\right)^3\,\left(a^2+2\,b^2\right)}{a\,\left(a^2+b^2\right)}-\frac{b}{a^2+b^2}+\frac{\tan\left(\frac{x}{2}\right)\,\left(a^2-2\,b^2\right)}{a\,\left(a^2+b^2\right)}+\frac{b\,\tan\left(\frac{x}{2}\right)^2\,\left(a^2-2\,b^2\right)}{a^2\,\left(a^2+b^2\right)}}{a^2-\tan\left(\frac{x}{2}\right)^2\,\left(2\,a^2-4\,b^2\right)+a^2\,\tan\left(\frac{x}{2}\right)^4+4\,a\,b\,\tan\left(\frac{x}{2}\right)-4\,a\,b\,\tan\left(\frac{x}{2}\right)^3}-\frac{\mathrm{atanh}\left(-\frac{\left(2\,a\,\tan\left(\frac{x}{2}\right)-\frac{2\,a^2\,b+2\,b^3}{a^2+b^2}\right)\,\left(\frac{a^2}{2}+\frac{b^2}{2}\right)}{{\left(a^2+b^2\right)}^{3/2}}\right)}{{\left(a^2+b^2\right)}^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*cos(x) + b*sin(x))^3,x)
```

```
[Out] ((tan(x/2)^3*(a^2 + 2*b^2))/(a*(a^2 + b^2)) - b/(a^2 + b^2) + (tan(x/2)*(a^
2 - 2*b^2))/(a*(a^2 + b^2)) + (b*tan(x/2)^2*(a^2 - 2*b^2))/(a^2*(a^2 + b^2)
))/(a^2 - tan(x/2)^2*(2*a^2 - 4*b^2) + a^2*tan(x/2)^4 + 4*a*b*tan(x/2) - 4*
a*b*tan(x/2)^3) - atanh(-((2*a*tan(x/2) - (2*a^2*b + 2*b^3)/(a^2 + b^2))*(a
^2/2 + b^2/2))/(a^2 + b^2)^(3/2))/(a^2 + b^2)^(3/2)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cos(x)+b*sin(x))**3,x)
```

```
[Out] Timed out
```

3.26 $\int \frac{\csc(x)}{(a\cos(x)+b\sin(x))^3}\,dx$

Optimal. Leaf size=59

$$-\frac{\log(a+b\tan(x))}{a^3}+\frac{\log(\tan(x))}{a^3}+\frac{\frac{1}{a^2}-\frac{1}{b^2}}{a+b\tan(x)}+\frac{\frac{a}{b^2}+\frac{1}{a}}{2(a+b\tan(x))^2}$$

```
[Out] ln(tan(x))/a^3-ln(a+b*tan(x))/a^3+1/2*(1/a+a/b^2)/(a+b*tan(x))^2+(1/a^2-1/b
^2)/(a+b*tan(x))
```

Rubi [A] time = 0.08, antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3087, 894}

$$\frac{\frac{1}{a^2}-\frac{1}{b^2}}{a+b\tan(x)}-\frac{\log(a+b\tan(x))}{a^3}+\frac{\log(\tan(x))}{a^3}+\frac{\frac{a}{b^2}+\frac{1}{a}}{2(a+b\tan(x))^2}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[x]/(a*Cos[x] + b*Sin[x])^3,x]
```

```
[Out] Log[Tan[x]]/a^3 - Log[a + b*Tan[x]]/a^3 + (a^(-1) + a/b^2)/(2*(a + b*Tan[x]
)^2) + (a^(-2) - b^(-2))/(a + b*Tan[x])
```

Rule 894

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ
[m, 0] && ILtQ[n, 0]))
```

Rule 3087

```
Int[sin[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[1/d, Subst[Int[(x^m*(a + b*x
)^n)/(1 + x^2)^((m + n + 2)/2), x], x, Tan[c + d*x]], x] /; FreeQ[{a, b, c,
 d}, x] && IntegerQ[n] && IntegerQ[(m + n)/2] && NeQ[n, -1] &&  !(GtQ[n, 0]
 && GtQ[m, 1])
```

Rubi steps

$$\int \frac{\csc(x)}{(a\cos(x)+b\sin(x))^3}\,dx = \text{Subst}\left(\int \frac{1+x^2}{x(a+bx)^3}\,dx, x, \tan(x)\right)$$
$$= \text{Subst}\left(\int \left(\frac{1}{a^3x}+\frac{-a^2-b^2}{ab(a+bx)^3}+\frac{a^2-b^2}{a^2b(a+bx)^2}-\frac{b}{a^3(a+bx)}\right)dx, x, \tan(x)\right)$$
$$= \frac{\log(\tan(x))}{a^3}-\frac{\log(a+b\tan(x))}{a^3}+\frac{\frac{1}{a}+\frac{a}{b^2}}{2(a+b\tan(x))^2}+\frac{\frac{1}{a^2}-\frac{1}{b^2}}{a+b\tan(x)}$$

Mathematica [A] time = 0.22, size = 96, normalized size = 1.63

$$\frac{2a^2\cot^2(x)(\log(\sin(x))-\log(a\cos(x)+b\sin(x)))+a^2\csc^2(x)+2b^2(-\log(a\cos(x)+b\sin(x))+\log(\sin(x))-1}{2a^3(a\cot(x)+b)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[x]/(a*Cos[x] + b*Sin[x])^3,x]
```

```
[Out] (a^2*Csc[x]^2 + 2*a*b*Cot[x]*(-1 + 2*Log[Sin[x]] - 2*Log[a*Cos[x] + b*Sin[x
]]) + 2*b^2*(-1 + Log[Sin[x]] - Log[a*Cos[x] + b*Sin[x]]) + 2*a^2*Cot[x]^2*
(Log[Sin[x]] - Log[a*Cos[x] + b*Sin[x]]))/(2*a^3*(b + a*Cot[x])^2)
```

fricas [B] time = 0.76, size = 220, normalized size = 3.73

$$\frac{4\,a^2b^2\cos(x)^2+a^4-a^2b^2-2\left(a^3b-ab^3\right)\cos(x)\sin(x)-\left(a^2b^2+b^4+\left(a^4-b^4\right)\cos(x)^2+2\left(a^3b+ab^3\right)\cos(x)\sin\right.}{2\left(a^5b^2+a^3b^4+\left(a^7\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)/(a*cos(x)+b*sin(x))^3,x, algorithm="fricas")
```

```
[Out] 1/2*(4*a^2*b^2*cos(x)^2 + a^4 - a^2*b^2 - 2*(a^3*b - a*b^3)*cos(x)*sin(x) -
 (a^2*b^2 + b^4 + (a^4 - b^4)*cos(x)^2 + 2*(a^3*b + a*b^3)*cos(x)*sin(x))*l
og(2*a*b*cos(x)*sin(x) + (a^2 - b^2)*cos(x)^2 + b^2) + (a^2*b^2 + b^4 + (a^
4 - b^4)*cos(x)^2 + 2*(a^3*b + a*b^3)*cos(x)*sin(x))*log(-1/4*cos(x)^2 + 1/
4))/(a^5*b^2 + a^3*b^4 + (a^7 - a^3*b^4)*cos(x)^2 + 2*(a^6*b + a^4*b^3)*cos
(x)*sin(x))
```

giac [A] time = 2.02, size = 77, normalized size = 1.31

$$-\frac{\log\left(|b\tan(x)+a|\right)}{a^3}+\frac{\log\left(|\tan(x)|\right)}{a^3}+\frac{3\,b^4\tan(x)^2-2\,a^3b\tan(x)+8\,ab^3\tan(x)-a^4+6\,a^2b^2}{2\left(b\tan(x)+a\right)^2a^3b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)/(a*cos(x)+b*sin(x))^3,x, algorithm="giac")
```

```
[Out] -log(abs(b*tan(x) + a))/a^3 + log(abs(tan(x)))/a^3 + 1/2*(3*b^4*tan(x)^2 -
2*a^3*b*tan(x) + 8*a*b^3*tan(x) - a^4 + 6*a^2*b^2)/((b*tan(x) + a)^2*a^3*b^
2)
```

maple [A] time = 0.67, size = 73, normalized size = 1.24

$$\frac{a}{2b^2\left(a+b\tan(x)\right)^2}+\frac{1}{2a\left(a+b\tan(x)\right)^2}-\frac{1}{b^2\left(a+b\tan(x)\right)}+\frac{1}{a^2\left(a+b\tan(x)\right)}-\frac{\ln\left(a+b\tan(x)\right)}{a^3}+\frac{\ln\left(\tan(x)\right)}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(x)/(a*cos(x)+b*sin(x))^3,x)
```

```
[Out] 1/2*a/b^2/(a+b*tan(x))^2+1/2/a/(a+b*tan(x))^2-1/b^2/(a+b*tan(x))+1/a^2/(a+b
*tan(x))-ln(a+b*tan(x))/a^3+ln(tan(x))/a^3
```

maxima [B] time = 0.34, size = 172, normalized size = 2.92

$$-\frac{2\left(\frac{2\,ab\sin(x)}{\cos(x)+1}-\frac{2\,ab\sin(x)^3}{(\cos(x)+1)^3}-\frac{\left(a^2-3\,b^2\right)\sin(x)^2}{(\cos(x)+1)^2}\right)}{a^5+\frac{4\,a^4b\sin(x)}{\cos(x)+1}-\frac{4\,a^4b\sin(x)^3}{(\cos(x)+1)^3}+\frac{a^5\sin(x)^4}{(\cos(x)+1)^4}-\frac{2\left(a^5-2\,a^3b^2\right)\sin(x)^2}{(\cos(x)+1)^2}}-\frac{\log\left(-a-\frac{2\,b\sin(x)}{\cos(x)+1}+\frac{a\sin(x)^2}{(\cos(x)+1)^2}\right)}{a^3}+\frac{\log\left(\frac{\sin(x)}{\cos(x)+1}\right)}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)/(a*cos(x)+b*sin(x))^3,x, algorithm="maxima")
```

```
[Out] -2*(2*a*b*sin(x)/(cos(x) + 1) - 2*a*b*sin(x)^3/(cos(x) + 1)^3 - (a^2 - 3*b^
2)*sin(x)^2/(cos(x) + 1)^2)/(a^5 + 4*a^4*b*sin(x)/(cos(x) + 1) - 4*a^4*b*si
n(x)^3/(cos(x) + 1)^3 + a^5*sin(x)^4/(cos(x) + 1)^4 - 2*(a^5 - 2*a^3*b^2)*s
in(x)^2/(cos(x) + 1)^2) - log(-a - 2*b*sin(x)/(cos(x) + 1) + a*sin(x)^2/(co
s(x) + 1)^2)/a^3 + log(sin(x)/(cos(x) + 1))/a^3
```

mupad [B] time = 0.71, size = 131, normalized size = 2.22

$$\frac{\ln\left(\tan\left(\frac{x}{2}\right)\right)}{a^3}-\frac{\ln\left(-a\tan\left(\frac{x}{2}\right)^2+2\,b\tan\left(\frac{x}{2}\right)+a\right)}{a^3}+\frac{\frac{2\tan\left(\frac{x}{2}\right)^2\left(a^2-3\,b^2\right)}{a^3}+\frac{4\,b\tan\left(\frac{x}{2}\right)^3}{a^2}-\frac{4\,b\tan\left(\frac{x}{2}\right)}{a^2}}{a^2-\tan\left(\frac{x}{2}\right)^2\,\left(2\,a^2-4\,b^2\right)+a^2\tan\left(\frac{x}{2}\right)^4+4\,a\,b\tan\left(\frac{x}{2}\right)-4\,a\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(x)*(a*cos(x) + b*sin(x))^3),x)
```

```
[Out] log(tan(x/2))/a^3 - log(a + 2*b*tan(x/2) - a*tan(x/2)^2)/a^3 + ((2*tan(x/2)
^2*(a^2 - 3*b^2))/a^3 + (4*b*tan(x/2)^3)/a^2 - (4*b*tan(x/2))/a^2)/(a^2 - t
an(x/2)^2*(2*a^2 - 4*b^2) + a^2*tan(x/2)^4 + 4*a*b*tan(x/2) - 4*a*b*tan(x/2
)^3)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc(x)}{(a\cos(x) + b\sin(x))^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)/(a*cos(x)+b*sin(x))**3,x)
```

```
[Out] Integral(csc(x)/(a*cos(x) + b*sin(x))**3, x)
```

3.27 $\int \frac{\csc^2(x)}{(a\cos(x)+b\sin(x))^3}\,dx$

Optimal. Leaf size=184

$$\frac{3b\tanh^{-1}(\cos(x))}{a^4}-\frac{2b}{a^3(a\cos(x)+b\sin(x))}-\frac{\csc(x)}{a^3}-\frac{\tanh^{-1}\left(\frac{b\cos(x)-a\sin(x)}{\sqrt{a^2+b^2}}\right)}{2a^2\sqrt{a^2+b^2}}-\frac{b\cos(x)-a\sin(x)}{2a^2(a\cos(x)+b\sin(x))^2}-\frac{2b^2\tan}{}$$

```
[Out] 3*b*arctanh(cos(x))/a^4-csc(x)/a^3+1/2*(-b*cos(x)+a*sin(x))/a^2/(a*cos(x)+b
*sin(x))^2-2*b/a^3/(a*cos(x)+b*sin(x))-1/2*arctanh((b*cos(x)-a*sin(x))/(a^2
+b^2)^(1/2))/a^2/(a^2+b^2)^(1/2)-2*b^2*arctanh((b*cos(x)-a*sin(x))/(a^2+b^2
)^(1/2))/a^4/(a^2+b^2)^(1/2)-arctanh((b*cos(x)-a*sin(x))/(a^2+b^2)^(1/2))*(
a^2+b^2)^(1/2)/a^4
```

Rubi [A] time = 0.22, antiderivative size = 184, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 7, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.438, Rules used = {3105, 3076, 3074, 206, 3103, 3770, 3093}

$$-\frac{2b^2\tanh^{-1}\left(\frac{b\cos(x)-a\sin(x)}{\sqrt{a^2+b^2}}\right)}{a^4\sqrt{a^2+b^2}}-\frac{\sqrt{a^2+b^2}\tanh^{-1}\left(\frac{b\cos(x)-a\sin(x)}{\sqrt{a^2+b^2}}\right)}{a^4}-\frac{\tanh^{-1}\left(\frac{b\cos(x)-a\sin(x)}{\sqrt{a^2+b^2}}\right)}{2a^2\sqrt{a^2+b^2}}-\frac{2b}{a^3(a\cos(x)+b\sin(x)}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[x]^2/(a*Cos[x] + b*Sin[x])^3,x]
```

```
[Out] (3*b*ArcTanh[Cos[x]])/a^4 - ArcTanh[(b*Cos[x] - a*Sin[x])/Sqrt[a^2 + b^2]]/
(2*a^2*Sqrt[a^2 + b^2]) - (2*b^2*ArcTanh[(b*Cos[x] - a*Sin[x])/Sqrt[a^2 + b
^2]])/(a^4*Sqrt[a^2 + b^2]) - (Sqrt[a^2 + b^2]*ArcTanh[(b*Cos[x] - a*Sin[x]
)/Sqrt[a^2 + b^2]])/a^4 - Csc[x]/a^3 - (b*Cos[x] - a*Sin[x])/(2*a^2*(a*Cos[
x] + b*Sin[x])^2) - (2*b)/(a^3*(a*Cos[x] + b*Sin[x]))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3074

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x
_Symbol] :> -Dist[d^(-1), Subst[Int[1/(a^2 + b^2 - x^2), x], x, b*Cos[c + d
*x] - a*Sin[c + d*x]], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

Rule 3076

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x
_Symbol] :> Simp[((b*Cos[c + d*x] - a*Sin[c + d*x])*(a*Cos[c + d*x] + b*Sin
[c + d*x])^(n + 1))/(d*(n + 1)*(a^2 + b^2)), x] + Dist[(n + 2)/((n + 1)*(a^
2 + b^2)), Int[(a*Cos[c + d*x] + b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{
a, b, c, d}, x] && NeQ[a^2 + b^2, 0] && LtQ[n, -1] && NeQ[n, -2]
```

Rule 3093

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_)/si
n[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[(a*Cos[c + d*x] + b*Sin[c + d*x])
^(n + 1)/(a*d*(n + 1)), x] + (Dist[1/a^2, Int[(a*Cos[c + d*x] + b*Sin[c + d
*x])^(n + 2)/Sin[c + d*x], x], x] - Dist[b/a^2, Int[(a*Cos[c + d*x] + b*Sin
[c + d*x])^(n + 1), x], x]) /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
&& LtQ[n, -1]
```

Rule 3103

```
Int[sin[(c_.) + (d_.)*(x_)]^(m_)/(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin
[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[Sin[c + d*x]^(m + 1)/(a*d*(m + 1))
, x] + (-Dist[b/a^2, Int[Sin[c + d*x]^(m + 1), x], x] + Dist[(a^2 + b^2)/a^
2, Int[Sin[c + d*x]^(m + 2)/(a*Cos[c + d*x] + b*Sin[c + d*x]), x], x]) /; F
reeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0] && LtQ[m, -1]
```

Rule 3105

```
Int[sin[(c_.) + (d_.)*(x_)]^(m_)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin
[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(a^2 + b^2)/a^2, Int[Sin[c +
d*x]^(m + 2)*(a*Cos[c + d*x] + b*Sin[c + d*x])^n, x], x] + (Dist[1/a^2, Int
[Sin[c + d*x]^m*(a*Cos[c + d*x] + b*Sin[c + d*x])^(n + 2), x], x] - Dist[(2
*b)/a^2, Int[Sin[c + d*x]^(m + 1)*(a*Cos[c + d*x] + b*Sin[c + d*x])^(n + 1)
, x], x]) /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0] && LtQ[n, -1] && L
tQ[m, -1]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\csc^2(x)}{(a\cos(x)+b\sin(x))^3}\,dx = \frac{\int \frac{\csc^2(x)}{a\cos(x)+b\sin(x)}\,dx}{a^2} - \frac{(2b)\int \frac{\csc(x)}{(a\cos(x)+b\sin(x))^2}\,dx}{a^2} + \frac{\left(a^2+b^2\right)\int \frac{1}{(a\cos(x)+b\sin(x))^3}\,dx}{a^2}$$

$$= -\frac{\csc(x)}{a^3} - \frac{b\cos(x)-a\sin(x)}{2a^2(a\cos(x)+b\sin(x))^2} - \frac{2b}{a^3(a\cos(x)+b\sin(x))} + \frac{\int \frac{1}{a\cos(x)+b\sin(x)}}{2a^2}$$

$$= \frac{3b\tanh^{-1}(\cos(x))}{a^4} - \frac{\csc(x)}{a^3} - \frac{b\cos(x)-a\sin(x)}{2a^2(a\cos(x)+b\sin(x))^2} - \frac{2b}{a^3(a\cos(x)+b\sin(x))}$$

$$= \frac{3b\tanh^{-1}(\cos(x))}{a^4} - \frac{\tanh^{-1}\left(\frac{b\cos(x)-a\sin(x)}{\sqrt{a^2+b^2}}\right)}{2a^2\sqrt{a^2+b^2}} - \frac{2b^2\tanh^{-1}\left(\frac{b\cos(x)-a\sin(x)}{\sqrt{a^2+b^2}}\right)}{a^4\sqrt{a^2+b^2}} - \frac{\sqrt{a^2}}{}$$

Mathematica [A] time = 0.80, size = 193, normalized size = 1.05

$$\frac{\csc^3(x)(a\cos(x)+b\sin(x))\left(a\left(a^2+b^2\right)\sin(x) + \frac{6\left(a^2+2b^2\right)(a\cos(x)+b\sin(x))^2\tanh^{-1}\left(\frac{a\tan\left(\frac{x}{2}\right)-b}{\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}} - 5ab(a\cos(x)+b\sin(x\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[x]^2/(a*Cos[x] + b*Sin[x])^3,x]
```

```
[Out] (Csc[x]^3*(a*Cos[x] + b*Sin[x])*(a*(a^2 + b^2)*Sin[x] - 5*a*b*(a*Cos[x] + b
*Sin[x]) + (6*(a^2 + 2*b^2)*ArcTanh[(-b + a*Tan[x/2])/Sqrt[a^2 + b^2]]*(a*C
os[x] + b*Sin[x])^2)/Sqrt[a^2 + b^2] - a*Cot[x/2]*(a*Cos[x] + b*Sin[x])^2 +
 6*b*Log[Cos[x/2]]*(a*Cos[x] + b*Sin[x])^2 - 6*b*Log[Sin[x/2]]*(a*Cos[x] +
b*Sin[x])^2 - a*(a*Cos[x] + b*Sin[x])^2*Tan[x/2]))/(2*a^4*(b + a*Cot[x])^3)
```

fricas [B] time = 1.92, size = 463, normalized size = 2.52

$$-\frac{2\,a^5 - 10\,a^3b^2 - 12\,ab^4 - 6\left(a^5 - a^3b^2 - 2\,ab^4\right)\cos(x)^2 - 18\left(a^4b + a^2b^3\right)\cos(x)\sin(x) - 3\left(2\left(a^3b + 2\,ab^3\right)\cos(\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)^2/(a*cos(x)+b*sin(x))^3,x, algorithm="fricas")
```

```
[Out] -1/4*(2*a^5 - 10*a^3*b^2 - 12*a*b^4 - 6*(a^5 - a^3*b^2 - 2*a*b^4)*cos(x)^2
- 18*(a^4*b + a^2*b^3)*cos(x)*sin(x) - 3*(2*(a^3*b + 2*a*b^3)*cos(x)^3 - 2*
```

```
(a^3*b + 2*a*b^3)*cos(x) - (a^2*b^2 + 2*b^4 + (a^4 + a^2*b^2 - 2*b^4)*cos(x
)^2)*sin(x))*sqrt(a^2 + b^2)*log(-(2*a*b*cos(x)*sin(x) + (a^2 - b^2)*cos(x)
^2 - 2*a^2 - b^2 + 2*sqrt(a^2 + b^2)*(b*cos(x) - a*sin(x)))/(2*a*b*cos(x)*s
in(x) + (a^2 - b^2)*cos(x)^2 + b^2)) - 6*(2*(a^3*b^2 + a*b^4)*cos(x)^3 - 2*
(a^3*b^2 + a*b^4)*cos(x) - (a^2*b^3 + b^5 + (a^4*b - b^5)*cos(x)^2)*sin(x))
*log(1/2*cos(x) + 1/2) + 6*(2*(a^3*b^2 + a*b^4)*cos(x)^3 - 2*(a^3*b^2 + a*b
^4)*cos(x) - (a^2*b^3 + b^5 + (a^4*b - b^5)*cos(x)^2)*sin(x))*log(-1/2*cos(
x) + 1/2))/(2*(a^7*b + a^5*b^3)*cos(x)^3 - 2*(a^7*b + a^5*b^3)*cos(x) - (a^
6*b^2 + a^4*b^4 + (a^8 - a^4*b^4)*cos(x)^2)*sin(x))
```

giac [A] time = 3.96, size = 212, normalized size = 1.15

$$-\frac{3\,b\log\left(\left|\tan\left(\frac{1}{2}\,x\right)\right|\right)}{a^4}-\frac{\tan\left(\frac{1}{2}\,x\right)}{2\,a^3}-\frac{3\left(a^2+2\,b^2\right)\log\left(\frac{\left|2\,a\tan\left(\frac{1}{2}\,x\right)-2\,b-2\,\sqrt{a^2+b^2}\right|}{\left|2\,a\tan\left(\frac{1}{2}\,x\right)-2\,b+2\,\sqrt{a^2+b^2}\right|}\right)}{2\,\sqrt{a^2+b^2}\,a^4}+\frac{6\,b\tan\left(\frac{1}{2}\,x\right)-a}{2\,a^4\tan\left(\frac{1}{2}\,x\right)}+\frac{a^3\tan\left(\frac{1}{2}\,x\right)^3+6\cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)^2/(a*cos(x)+b*sin(x))^3,x, algorithm="giac")
```

```
[Out] -3*b*log(abs(tan(1/2*x)))/a^4 - 1/2*tan(1/2*x)/a^3 - 3/2*(a^2 + 2*b^2)*log(
abs(2*a*tan(1/2*x) - 2*b - 2*sqrt(a^2 + b^2))/abs(2*a*tan(1/2*x) - 2*b + 2*
sqrt(a^2 + b^2)))/(sqrt(a^2 + b^2)*a^4) + 1/2*(6*b*tan(1/2*x) - a)/(a^4*tan
(1/2*x)) + (a^3*tan(1/2*x)^3 + 6*a*b^2*tan(1/2*x)^3 + 5*a^2*b*tan(1/2*x)^2
- 10*b^3*tan(1/2*x)^2 + a^3*tan(1/2*x) - 14*a*b^2*tan(1/2*x) - 5*a^2*b)/((a
*tan(1/2*x)^2 - 2*b*tan(1/2*x) - a)^2*a^4)
```

maple [A] time = 0.75, size = 333, normalized size = 1.81

$$-\frac{\tan\left(\frac{x}{2}\right)}{2a^3}-\frac{1}{2a^3\tan\left(\frac{x}{2}\right)}-\frac{3b\ln\left(\tan\left(\frac{x}{2}\right)\right)}{a^4}+\frac{\tan^3\left(\frac{x}{2}\right)}{a\left(a\left(\tan^2\left(\frac{x}{2}\right)\right)-2\tan\left(\frac{x}{2}\right)b-a\right)^2}+\frac{6\left(\tan^3\left(\frac{x}{2}\right)\right)b^2}{a^3\left(a\left(\tan^2\left(\frac{x}{2}\right)\right)-2\tan\left(\frac{x}{2}\right)b-a\right)^2}+\cdots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(x)^2/(a*cos(x)+b*sin(x))^3,x)
```

```
[Out] -1/2/a^3*tan(1/2*x)-1/2/a^3/tan(1/2*x)-3/a^4*b*ln(tan(1/2*x))+1/a/(a*tan(1/
2*x)^2-2*tan(1/2*x)*b-a)^2*tan(1/2*x)^3+6/a^3/(a*tan(1/2*x)^2-2*tan(1/2*x)*
b-a)^2*tan(1/2*x)^3*b^2+5/a^2/(a*tan(1/2*x)^2-2*tan(1/2*x)*b-a)^2*tan(1/2*x
)^2*b-10/a^4/(a*tan(1/2*x)^2-2*tan(1/2*x)*b-a)^2*tan(1/2*x)^2*b^3+1/a/(a*ta
n(1/2*x)^2-2*tan(1/2*x)*b-a)^2*tan(1/2*x)-14/a^3/(a*tan(1/2*x)^2-2*tan(1/2*
x)*b-a)^2*tan(1/2*x)*b^2-5/a^2/(a*tan(1/2*x)^2-2*tan(1/2*x)*b-a)^2*b+3/a^2/
```

```
(a^2+b^2)^(1/2)*arctanh(1/2*(2*a*tan(1/2*x)-2*b)/(a^2+b^2)^(1/2))+6/a^4/(a^
2+b^2)^(1/2)*arctanh(1/2*(2*a*tan(1/2*x)-2*b)/(a^2+b^2)^(1/2))*b^2
```

maxima [A] time = 0.44, size = 276, normalized size = 1.50

$$-\frac{a^3+\frac{14a^2b\sin(x)}{\cos(x)+1}-\frac{4(a^3-8ab^2)\sin(x)^2}{(\cos(x)+1)^2}-\frac{2(7a^2b-10b^3)\sin(x)^3}{(\cos(x)+1)^3}-\frac{(a^3+12ab^2)\sin(x)^4}{(\cos(x)+1)^4}}{2\left(\frac{a^6\sin(x)}{\cos(x)+1}+\frac{4a^5b\sin(x)^2}{(\cos(x)+1)^2}-\frac{4a^5b\sin(x)^4}{(\cos(x)+1)^4}+\frac{a^6\sin(x)^5}{(\cos(x)+1)^5}-\frac{2(a^6-2a^4b^2)\sin(x)^3}{(\cos(x)+1)^3}\right)}-\frac{3b\log\left(\frac{\sin(x)}{\cos(x)+1}\right)}{a^4}-\frac{3(a^2+2b^2)\log\left(\right.}{2\sqrt{a^2+}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)^2/(a*cos(x)+b*sin(x))^3,x, algorithm="maxima")
```

```
[Out] -1/2*(a^3 + 14*a^2*b*sin(x)/(cos(x) + 1) - 4*(a^3 - 8*a*b^2)*sin(x)^2/(cos(
x) + 1)^2 - 2*(7*a^2*b - 10*b^3)*sin(x)^3/(cos(x) + 1)^3 - (a^3 + 12*a*b^2)
*sin(x)^4/(cos(x) + 1)^4)/(a^6*sin(x)/(cos(x) + 1) + 4*a^5*b*sin(x)^2/(cos(
x) + 1)^2 - 4*a^5*b*sin(x)^4/(cos(x) + 1)^4 + a^6*sin(x)^5/(cos(x) + 1)^5 -
 2*(a^6 - 2*a^4*b^2)*sin(x)^3/(cos(x) + 1)^3) - 3*b*log(sin(x)/(cos(x) + 1)
)/a^4 - 3/2*(a^2 + 2*b^2)*log((b - a*sin(x)/(cos(x) + 1) + sqrt(a^2 + b^2))
/(b - a*sin(x)/(cos(x) + 1) - sqrt(a^2 + b^2)))/(sqrt(a^2 + b^2)*a^4) - 1/2
*sin(x)/(a^3*(cos(x) + 1))
```

mupad [B] time = 1.01, size = 813, normalized size = 4.42

$$\frac{\tan\left(\frac{x}{2}\right)^4\left(a^2+12b^2\right)+\tan\left(\frac{x}{2}\right)^2\left(4a^2-32b^2\right)-a^2-14ab\tan\left(\frac{x}{2}\right)+\frac{2\tan\left(\frac{x}{2}\right)^3\left(7a^2b-10b^3\right)}{a}}{2a^5\tan\left(\frac{x}{2}\right)-\tan\left(\frac{x}{2}\right)^3\left(4a^5-8a^3b^2\right)+2a^5\tan\left(\frac{x}{2}\right)^5+8a^4b\tan\left(\frac{x}{2}\right)^2-8a^4b\tan\left(\frac{x}{2}\right)^4}-\frac{\tan\left(\frac{x}{2}\right)}{2a^3}-\frac{3b\ln\left(\tan\right.}{a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(x)^2*(a*cos(x) + b*sin(x))^3),x)
```

```
[Out] (tan(x/2)^4*(a^2 + 12*b^2) + tan(x/2)^2*(4*a^2 - 32*b^2) - a^2 - 14*a*b*tan
(x/2) + (2*tan(x/2)^3*(7*a^2*b - 10*b^3))/a)/(2*a^5*tan(x/2) - tan(x/2)^3*(
4*a^5 - 8*a^3*b^2) + 2*a^5*tan(x/2)^5 + 8*a^4*b*tan(x/2)^2 - 8*a^4*b*tan(x/
```

```
2)^4) - tan(x/2)/(2*a^3) - (3*b*log(tan(x/2)))/a^4 - (atan((((a^2 + 2*b^2)*
(a^2 + b^2)^(1/2)*((3*a^6 + 12*a^4*b^2)/a^6 + (tan(x/2)*(12*a^4*b + 24*a^2*
b^3))/a^5 - (3*(a^2 + 2*b^2)*(2*a^2*b + (tan(x/2)*(6*a^8 + 8*a^6*b^2))/a^5)
*(a^2 + b^2)^(1/2))/(2*(a^6 + a^4*b^2)))*3i)/(2*(a^6 + a^4*b^2)) + ((a^2 +
2*b^2)*(a^2 + b^2)^(1/2)*((3*a^6 + 12*a^4*b^2)/a^6 + (tan(x/2)*(12*a^4*b +
24*a^2*b^3))/a^5 + (3*(a^2 + 2*b^2)*(2*a^2*b + (tan(x/2)*(6*a^8 + 8*a^6*b^2
))/a^5)*(a^2 + b^2)^(1/2))/(2*(a^6 + a^4*b^2)))*3i)/(2*(a^6 + a^4*b^2)))/((
2*(9*a^2*b + 18*b^3))/a^6 - (2*tan(x/2)*(9*a^2 + 18*b^2))/a^5 - (3*(a^2 + 2
*b^2)*(a^2 + b^2)^(1/2)*((3*a^6 + 12*a^4*b^2)/a^6 + (tan(x/2)*(12*a^4*b + 2
4*a^2*b^3))/a^5 - (3*(a^2 + 2*b^2)*(2*a^2*b + (tan(x/2)*(6*a^8 + 8*a^6*b^2)
)/a^5)*(a^2 + b^2)^(1/2))/(2*(a^6 + a^4*b^2))))/(2*(a^6 + a^4*b^2)) + (3*(a
^2 + 2*b^2)*(a^2 + b^2)^(1/2)*((3*a^6 + 12*a^4*b^2)/a^6 + (tan(x/2)*(12*a^4
*b + 24*a^2*b^3))/a^5 + (3*(a^2 + 2*b^2)*(2*a^2*b + (tan(x/2)*(6*a^8 + 8*a^
6*b^2))/a^5)*(a^2 + b^2)^(1/2))/(2*(a^6 + a^4*b^2))))/(2*(a^6 + a^4*b^2))))
*(a^2 + 2*b^2)*(a^2 + b^2)^(1/2)*3i)/(a^6 + a^4*b^2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc^2(x)}{(a\cos(x)+b\sin(x))^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)**2/(a*cos(x)+b*sin(x))**3,x)
```

```
[Out] Integral(csc(x)**2/(a*cos(x) + b*sin(x))**3, x)
```

3.28 $\int \frac{\csc^3(x)}{(a\cos(x)+b\sin(x))^3}\,dx$

Optimal. Leaf size=117

$$\frac{3b\cot(x)}{a^4}-\frac{\cot^2(x)}{2a^3}+\frac{2\left(a^2+3b^2\right)\log(\tan(x))}{a^5}-\frac{2\left(a^2+3b^2\right)\log(a+b\tan(x))}{a^5}-\frac{\left(a^2-3b^2\right)\left(a^2+b^2\right)}{a^4b^2(a+b\tan(x))}+\frac{\left(a^2+\right.}{2a^3b^2(a+b}$$

```
[Out] 3*b*cot(x)/a^4-1/2*cot(x)^2/a^3+2*(a^2+3*b^2)*ln(tan(x))/a^5-2*(a^2+3*b^2)*
ln(a+b*tan(x))/a^5+1/2*(a^2+b^2)^2/a^3/b^2/(a+b*tan(x))^2-(a^2-3*b^2)*(a^2+
b^2)/a^4/b^2/(a+b*tan(x))
```

Rubi [A] time = 0.14, antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {3087, 894}

$$\frac{\left(a^2+b^2\right)^2}{2a^3b^2(a+b\tan(x))^2}-\frac{\left(a^2-3b^2\right)\left(a^2+b^2\right)}{a^4b^2(a+b\tan(x))}+\frac{2\left(a^2+3b^2\right)\log(\tan(x))}{a^5}-\frac{2\left(a^2+3b^2\right)\log(a+b\tan(x))}{a^5}+\frac{3b\cot(x)}{a^4}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[x]^3/(a*Cos[x] + b*Sin[x])^3,x]
```

```
[Out] (3*b*Cot[x])/a^4 - Cot[x]^2/(2*a^3) + (2*(a^2 + 3*b^2)*Log[Tan[x]])/a^5 - (
2*(a^2 + 3*b^2)*Log[a + b*Tan[x]])/a^5 + (a^2 + b^2)^2/(2*a^3*b^2*(a + b*Ta
n[x])^2) - ((a^2 - 3*b^2)*(a^2 + b^2))/(a^4*b^2*(a + b*Tan[x]))
```

Rule 894

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ
[m, 0] && ILtQ[n, 0]))
```

Rule 3087

```
Int[sin[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[1/d, Subst[Int[(x^m*(a + b*x
)^n)/(1 + x^2)^((m + n + 2)/2), x], x, Tan[c + d*x]], x] /; FreeQ[{a, b, c,
 d}, x] && IntegerQ[n] && IntegerQ[(m + n)/2] && NeQ[n, -1] && !(GtQ[n, 0]
 && GtQ[m, 1])
```

Rubi steps

$$\int \frac{\csc^3(x)}{(a\cos(x)+b\sin(x))^3}\,dx = \text{Subst}\left(\int \frac{\left(1+x^2\right)^2}{x^3(a+bx)^3}\,dx, x, \tan(x)\right)$$

$$= \text{Subst}\left(\int \left(\frac{1}{a^3x^3} - \frac{3b}{a^4x^2} + \frac{2\left(a^2+3b^2\right)}{a^5x} - \frac{\left(a^2+b^2\right)^2}{a^3b(a+bx)^3} + \frac{a^4-2a^2b^2-3b^4}{a^4b(a+bx)^2} - \frac{2b\left(a\right.}{a^5(\right.}$$

$$= \frac{3b\cot(x)}{a^4} - \frac{\cot^2(x)}{2a^3} + \frac{2\left(a^2+3b^2\right)\log(\tan(x))}{a^5} - \frac{2\left(a^2+3b^2\right)\log(a+b\tan(x))}{a^5} +$$

Mathematica [A] time = 0.83, size = 208, normalized size = 1.78

$$\frac{a^4\csc^2(x) + 6a^3b\cot^3(x) + 2b^2\left(2\left(a^2+3b^2\right)\log(\sin(x)) - 2\left(a^2+3b^2\right)\log(a\cos(x)+b\sin(x)) - 3\left(a^2+b^2\right)\right) - 2}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[x]^3/(a*Cos[x] + b*Sin[x])^3,x]
```

```
[Out] (6*a^3*b*Cot[x]^3 + a^4*Csc[x]^2 - 2*a*b*Cot[x]*(3*a^2 + a^2*Csc[x]^2 - 4*(
a^2 + 3*b^2)*Log[Sin[x]] + 4*a^2*Log[a*Cos[x] + b*Sin[x]] + 12*b^2*Log[a*Co
s[x] + b*Sin[x]]) + 2*b^2*(-3*(a^2 + b^2) + 2*(a^2 + 3*b^2)*Log[Sin[x]] - 2
*(a^2 + 3*b^2)*Log[a*Cos[x] + b*Sin[x]]) + Cot[x]^2*(-(a^4*Csc[x]^2) + 4*a^
2*(3*b^2 + (a^2 + 3*b^2)*Log[Sin[x]] - (a^2 + 3*b^2)*Log[a*Cos[x] + b*Sin[x
]])))/(2*a^5*(b + a*Cot[x])^2)
```

fricas [B] time = 1.94, size = 385, normalized size = 3.29

$$-\frac{24\,a^2b^2\cos(x)^4 - a^4 + 6\,a^2b^2 + 2\left(a^4 - 15\,a^2b^2\right)\cos(x)^2 - 2\left(\left(a^4 + 2\,a^2b^2 - 3\,b^4\right)\cos(x)^4 - a^2b^2 - 3\,b^4 - \left(a^4 + a\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)^3/(a*cos(x)+b*sin(x))^3,x, algorithm="fricas")
```

```
[Out] -1/2*(24*a^2*b^2*cos(x)^4 - a^4 + 6*a^2*b^2 + 2*(a^4 - 15*a^2*b^2)*cos(x)^2
 - 2*((a^4 + 2*a^2*b^2 - 3*b^4)*cos(x)^4 - a^2*b^2 - 3*b^4 - (a^4 + a^2*b^2
 - 6*b^4)*cos(x)^2 + 2*((a^3*b + 3*a*b^3)*cos(x)^3 - (a^3*b + 3*a*b^3)*cos(
x))*sin(x))*log(2*a*b*cos(x)*sin(x) + (a^2 - b^2)*cos(x)^2 + b^2) + 2*((a^4
 + 2*a^2*b^2 - 3*b^4)*cos(x)^4 - a^2*b^2 - 3*b^4 - (a^4 + a^2*b^2 - 6*b^4)*
cos(x)^2 + 2*((a^3*b + 3*a*b^3)*cos(x)^3 - (a^3*b + 3*a*b^3)*cos(x))*sin(x)
```

```
)*log(-1/4*cos(x)^2 + 1/4) - 4*(3*(a^3*b - a*b^3)*cos(x)^3 - (2*a^3*b - 3*a
*b^3)*cos(x))*sin(x))/(a^5*b^2 - (a^7 - a^5*b^2)*cos(x)^4 + (a^7 - 2*a^5*b^
2)*cos(x)^2 - 2*(a^6*b*cos(x)^3 - a^6*b*cos(x))*sin(x))
```

giac [A] time = 1.90, size = 146, normalized size = 1.25

$$\frac{2\left(a^2+3\,b^2\right)\log\left(|\tan(x)|\right)}{a^5}-\frac{2\left(a^2b+3\,b^3\right)\log\left(|b\tan(x)+a|\right)}{a^5b}-\frac{2\,a^4b\tan(x)^3-4\,a^2b^3\tan(x)^3-12\,b^5\tan(x)^3+a}{2\left(b\,\mathrm{ta}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)^3/(a*cos(x)+b*sin(x))^3,x, algorithm="giac")
```

```
[Out] 2*(a^2 + 3*b^2)*log(abs(tan(x)))/a^5 - 2*(a^2*b + 3*b^3)*log(abs(b*tan(x) +
 a))/(a^5*b) - 1/2*(2*a^4*b*tan(x)^3 - 4*a^2*b^3*tan(x)^3 - 12*b^5*tan(x)^3
 + a^5*tan(x)^2 - 6*a^3*b^2*tan(x)^2 - 18*a*b^4*tan(x)^2 - 4*a^2*b^3*tan(x)
 + a^3*b^2)/((b*tan(x)^2 + a*tan(x))^2*a^4*b^2)
```

maple [A] time = 0.76, size = 151, normalized size = 1.29

$$\frac{a}{2b^2\left(a+b\tan(x)\right)^2}+\frac{1}{a\left(a+b\tan(x)\right)^2}+\frac{b^2}{2a^3\left(a+b\tan(x)\right)^2}-\frac{1}{b^2\left(a+b\tan(x)\right)}+\frac{2}{a^2\left(a+b\tan(x)\right)}+\frac{3b^2}{a^4\left(a+b\tan(x\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(x)^3/(a*cos(x)+b*sin(x))^3,x)
```

```
[Out] 1/2*a/b^2/(a+b*tan(x))^2+1/a/(a+b*tan(x))^2+1/2/a^3*b^2/(a+b*tan(x))^2-1/b^
2/(a+b*tan(x))+2/a^2/(a+b*tan(x))+3/a^4*b^2/(a+b*tan(x))-2*ln(a+b*tan(x))/a
^3-6/a^5*ln(a+b*tan(x))*b^2-1/2/a^3/tan(x)^2+2*ln(tan(x))/a^3+6/a^5*ln(tan(
x))*b^2+3/a^4*b/tan(x)
```

maxima [B] time = 0.36, size = 308, normalized size = 2.63

$$-\frac{a^4-\frac{8\,a^3b\sin(x)}{\cos(x)+1}-\frac{2\left(a^4+22\,a^2b^2\right)\sin(x)^2}{(\cos(x)+1)^2}+\frac{4\left(21\,a^3b+4\,ab^3\right)\sin(x)^3}{(\cos(x)+1)^3}-\frac{\left(15\,a^4-144\,a^2b^2-112\,b^4\right)\sin(x)^4}{(\cos(x)+1)^4}-\frac{4\left(19\,a^3b+16\,ab^3\right)\sin(x)^5}{(\cos(x)+1)^5}}{8\left(\frac{a^7\sin(x)^2}{(\cos(x)+1)^2}+\frac{4\,a^6b\sin(x)^3}{(\cos(x)+1)^3}-\frac{4\,a^6b\sin(x)^5}{(\cos(x)+1)^5}+\frac{a^7\sin(x)^6}{(\cos(x)+1)^6}-\frac{2\left(a^7-2\,a^5b^2\right)\sin(x)^4}{(\cos(x)+1)^4}\right)}-\frac{12\,b}{\cos}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)^3/(a*cos(x)+b*sin(x))^3,x, algorithm="maxima")
```

```
[Out] -1/8*(a^4 - 8*a^3*b*sin(x)/(cos(x) + 1) - 2*(a^4 + 22*a^2*b^2)*sin(x)^2/(co
s(x) + 1)^2 + 4*(21*a^3*b + 4*a*b^3)*sin(x)^3/(cos(x) + 1)^3 - (15*a^4 - 14
4*a^2*b^2 - 112*b^4)*sin(x)^4/(cos(x) + 1)^4 - 4*(19*a^3*b + 16*a*b^3)*sin(
```

```
x)^5/(cos(x) + 1)^5)/(a^7*sin(x)^2/(cos(x) + 1)^2 + 4*a^6*b*sin(x)^3/(cos(x
) + 1)^3 - 4*a^6*b*sin(x)^5/(cos(x) + 1)^5 + a^7*sin(x)^6/(cos(x) + 1)^6 -
2*(a^7 - 2*a^5*b^2)*sin(x)^4/(cos(x) + 1)^4) - 1/8*(12*b*sin(x)/(cos(x) + 1
) + a*sin(x)^2/(cos(x) + 1)^2)/a^4 - 2*(a^2 + 3*b^2)*log(-a - 2*b*sin(x)/(c
os(x) + 1) + a*sin(x)^2/(cos(x) + 1)^2)/a^5 + 2*(a^2 + 3*b^2)*log(sin(x)/(c
os(x) + 1))/a^5
```

mupad [B] time = 0.86, size = 253, normalized size = 2.16

$$\frac{\ln\left(\tan\left(\frac{x}{2}\right)\right)\,\left(2\,a^2+6\,b^2\right)}{a^5}-\frac{\tan\left(\frac{x}{2}\right)^3\,\left(42\,a^2\,b+8\,b^3\right)-\tan\left(\frac{x}{2}\right)^5\,\left(38\,a^2\,b+32\,b^3\right)-\tan\left(\frac{x}{2}\right)^2\,\left(a^3+22\,a\,b^2\right)+\frac{a^3}{2}+}{4\,a^6\,\tan\left(\frac{x}{2}\right)^2-\tan\left(\frac{x}{2}\right)^4\,\left(8\,a^6-16\,a^4\,b^2\right)+4\,a^6\,\tan\left(\frac{x}{2}\right)^6+16\,a^5\,b\,\mathrm{ta}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(x)^3*(a*cos(x) + b*sin(x))^3),x)
```

```
[Out] (log(tan(x/2))*(2*a^2 + 6*b^2))/a^5 - (tan(x/2)^3*(42*a^2*b + 8*b^3) - tan(
x/2)^5*(38*a^2*b + 32*b^3) - tan(x/2)^2*(22*a*b^2 + a^3) + a^3/2 + (tan(x/2
)^4*(112*b^4 - 15*a^4 + 144*a^2*b^2))/(2*a) - 4*a^2*b*tan(x/2))/(4*a^6*tan(
x/2)^2 - tan(x/2)^4*(8*a^6 - 16*a^4*b^2) + 4*a^6*tan(x/2)^6 + 16*a^5*b*tan(
x/2)^3 - 16*a^5*b*tan(x/2)^5) - (log(a + 2*b*tan(x/2) - a*tan(x/2)^2)*(2*a^
2 + 6*b^2))/a^5 - tan(x/2)^2/(8*a^3) - (3*b*tan(x/2))/(2*a^4)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc^3(x)}{(a\cos(x)+b\sin(x))^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)**3/(a*cos(x)+b*sin(x))**3,x)
```

```
[Out] Integral(csc(x)**3/(a*cos(x) + b*sin(x))**3, x)
```

3.29 $\int \sin^{-n}(c+dx)(a\cos(c+dx)+ia\sin(c+dx))^n\,dx$

Optimal. Leaf size=66

$$-\frac{i\sin^{-n}(c+dx)\,{}_2F_1\left(1,n;n+1;-\frac{1}{2}i(\cot(c+dx)+i)\right)(a\cos(c+dx)+ia\sin(c+dx))^n}{2dn}$$

```
[Out] -1/2*I*hypergeom([1, n],[1+n],-1/2*I*(I+cot(d*x+c)))*(a*cos(d*x+c)+I*a*sin(
d*x+c))^n/d/n/(sin(d*x+c)^n)
```

Rubi [A] time = 0.06, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.030, Rules used = {3083}

$$-\frac{i\sin^{-n}(c+dx)\,{}_2F_1\left(1,n;n+1;-\frac{1}{2}i(\cot(c+dx)+i)\right)(a\cos(c+dx)+ia\sin(c+dx))^n}{2dn}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Cos[c + d*x] + I*a*Sin[c + d*x])^n/Sin[c + d*x]^n,x]
```

```
[Out] ((-I/2)*Hypergeometric2F1[1, n, 1 + n, (-I/2)*(I + Cot[c + d*x])]*(a*Cos[c
+ d*x] + I*a*Sin[c + d*x])^n)/(d*n*Sin[c + d*x]^n)
```

Rule 3083

```
Int[sin[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(a*(a*Cos[c + d*x] + b*Sin[c
 + d*x])^n*Hypergeometric2F1[1, n, n + 1, (b + a*Cot[c + d*x])/(2*b)])/(2*b
*d*n*Sin[c + d*x]^n), x] /; FreeQ[{a, b, c, d, n}, x] && EqQ[m + n, 0] && E
qQ[a^2 + b^2, 0] &&  !IntegerQ[n]
```

Rubi steps

$$\int \sin^{-n}(c+dx)(a\cos(c+dx)+ia\sin(c+dx))^n\,dx = -\frac{i\,{}_2F_1\left(1,n;1+n;-\frac{1}{2}i(i+\cot(c+dx))\right)\sin^{-n}(c+dx)(a\,\mathrm{c}}{2dn}$$

Mathematica [C] time = 3.75, size = 367, normalized size = 5.56

$$-\frac{4\sin\left(\frac{1}{2}(c+dx)\right)\cos\left(\frac{1}{2}(c+dx)\right)\sin^{-n}(c+dx)(a(\cos(c+dx)+i\sin(c+dx)))^n\left(\right.}{d(n-1)\left(2F_1\left(1-n;-2n,1;2-n;-i\tan\left(\frac{1}{2}(c+dx)\right),i\tan\left(\frac{1}{2}(c+dx)\right)\right)+\frac{\left(1-i\tan\left(\frac{1}{2}(c+dx)\right)\right)\left(-2n(i\sin(c+dx)+\cos(c+d\right.}{}\right.}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a*Cos[c + d*x] + I*a*Sin[c + d*x])^n/Sin[c + d*x]^n,x]
```

```
[Out] (-4*Cos[(c + d*x)/2]*(AppellF1[1 - n, -2*n, 1, 2 - n, (-I)*Tan[(c + d*x)/2]
, I*Tan[(c + d*x)/2]] + Hypergeometric2F1[1 - 2*n, 1 - n, 2 - n, (-I)*Tan[(
c + d*x)/2]])*Sin[(c + d*x)/2]*(a*(Cos[c + d*x] + I*Sin[c + d*x]))^n)/(d*(-
1 + n)*Sin[c + d*x]^n*(2*AppellF1[1 - n, -2*n, 1, 2 - n, (-I)*Tan[(c + d*x)
/2], I*Tan[(c + d*x)/2]] + ((-2*n*AppellF1[2 - n, 1 - 2*n, 1, 3 - n, (-I)*T
an[(c + d*x)/2], I*Tan[(c + d*x)/2]]*(-1 + Cos[c + d*x] + I*Sin[c + d*x]) -
 AppellF1[2 - n, -2*n, 2, 3 - n, (-I)*Tan[(c + d*x)/2], I*Tan[(c + d*x)/2]]
*(-1 + Cos[c + d*x] + I*Sin[c + d*x]) + (-2 + n)*(1 + Cos[c + d*x])*(1 + I*
Tan[(c + d*x)/2])^(2*n))*(1 - I*Tan[(c + d*x)/2]))/(-2 + n)))
```

fricas [F] time = 0.64, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{e^{\left(i\,dnx+i\,cn+n\log(a)\right)}}{\left(\frac{1}{2}\left(-i\,e^{(2i\,dx+2i\,c)}+i\right)e^{(-i\,dx-i\,c)}\right)^{n}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cos(d*x+c)+I*a*sin(d*x+c))^n/(sin(d*x+c)^n),x, algorithm="fric
as")
```

```
[Out] integral(e^(I*d*n*x + I*c*n + n*log(a))/(1/2*(-I*e^(2*I*d*x + 2*I*c) + I)*e
^(-I*d*x - I*c))^n, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a\cos(dx+c)+i\,a\sin(dx+c))^{n}}{\sin(dx+c)^{n}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cos(d*x+c)+I*a*sin(d*x+c))^n/(sin(d*x+c)^n),x, algorithm="giac
")
```

```
[Out] integrate((a*cos(d*x + c) + I*a*sin(d*x + c))^n/sin(d*x + c)^n, x)
```

maple [F] time = 10.85, size = 0, normalized size = 0.00

$$\int (a\cos(dx+c)+ia\sin(dx+c))^{n}\left(\sin^{-n}(dx+c)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*cos(d*x+c)+I*a*sin(d*x+c))^n/(sin(d*x+c)^n),x)
```

```
[Out] int((a*cos(d*x+c)+I*a*sin(d*x+c))^n/(sin(d*x+c)^n),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a\cos(dx+c)+i\,a\sin(dx+c))^{n}\sin(dx+c)^{-n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cos(d*x+c)+I*a*sin(d*x+c))^n/(sin(d*x+c)^n),x, algorithm="maxi
ma")
```

```
[Out] integrate((a*cos(d*x + c) + I*a*sin(d*x + c))^n*sin(d*x + c)^(-n), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{(a\ \cos(c+d\,x)+a\ \sin(c+d\,x)\ 1\mathrm{i})^{n}}{\sin(c+d\,x)^{n}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*cos(c + d*x) + a*sin(c + d*x)*1i)^n/sin(c + d*x)^n,x)
```

```
[Out] int((a*cos(c + d*x) + a*sin(c + d*x)*1i)^n/sin(c + d*x)^n, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a\,(i\sin(c+dx)+\cos(c+dx)))^{n}\sin^{-n}(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cos(d*x+c)+I*a*sin(d*x+c))**n/(sin(d*x+c)**n),x)
```

```
[Out] Integral((a*(I*sin(c + d*x) + cos(c + d*x)))**n*sin(c + d*x)**(-n), x)
```

3.30 $\int \cos^5(c+dx)(a\cos(c+dx)+b\sin(c+dx))\,dx$

Optimal. Leaf size=87

$$\frac{a\sin(c+dx)\cos^5(c+dx)}{6d}+\frac{5a\sin(c+dx)\cos^3(c+dx)}{24d}+\frac{5a\sin(c+dx)\cos(c+dx)}{16d}+\frac{5ax}{16}-\frac{b\cos^6(c+dx)}{6d}$$

```
[Out] 5/16*a*x-1/6*b*cos(d*x+c)^6/d+5/16*a*cos(d*x+c)*sin(d*x+c)/d+5/24*a*cos(d*x
+c)^3*sin(d*x+c)/d+1/6*a*cos(d*x+c)^5*sin(d*x+c)/d
```

Rubi [A] time = 0.09, antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {3090, 2635, 8, 2565, 30}

$$\frac{a\sin(c+dx)\cos^5(c+dx)}{6d}+\frac{5a\sin(c+dx)\cos^3(c+dx)}{24d}+\frac{5a\sin(c+dx)\cos(c+dx)}{16d}+\frac{5ax}{16}-\frac{b\cos^6(c+dx)}{6d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^5*(a*Cos[c + d*x] + b*Sin[c + d*x]),x]
```

```
[Out] (5*a*x)/16 - (b*Cos[c + d*x]^6)/(6*d) + (5*a*Cos[c + d*x]*Sin[c + d*x])/(16
*d) + (5*a*Cos[c + d*x]^3*Sin[c + d*x])/(24*d) + (a*Cos[c + d*x]^5*Sin[c +
d*x])/(6*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2565

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> -Dist[(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
```

```
]
```

Rule 3090

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_.), x_Symbol] :> Int[ExpandTrig[cos[c + d*x]^m*(a
*cos[c + d*x] + b*sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d}, x] && Inte
gerQ[m] && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}\int \cos^5(c+dx)(a\cos(c+dx)+b\sin(c+dx))\,dx &= \int \left(a\cos^6(c+dx)+b\cos^5(c+dx)\sin(c+dx)\right)\,dx\\ &= a\int \cos^6(c+dx)\,dx+b\int \cos^5(c+dx)\sin(c+dx)\,dx\\ &= \frac{a\cos^5(c+dx)\sin(c+dx)}{6d}+\frac{1}{6}(5a)\int \cos^4(c+dx)\,dx-\frac{b\,\mathrm{Su}}{}\\ &= -\frac{b\cos^6(c+dx)}{6d}+\frac{5a\cos^3(c+dx)\sin(c+dx)}{24d}+\frac{a\cos^5(c+}{}\\ &= -\frac{b\cos^6(c+dx)}{6d}+\frac{5a\cos(c+dx)\sin(c+dx)}{16d}+\frac{5a\cos^3(c+}{}\\ &= \frac{5ax}{16}-\frac{b\cos^6(c+dx)}{6d}+\frac{5a\cos(c+dx)\sin(c+dx)}{16d}+\frac{5a\cos}{}\end{aligned}$$

Mathematica [A] time = 0.12, size = 57, normalized size = 0.66

$$\frac{a(45\sin(2(c+dx))+9\sin(4(c+dx))+\sin(6(c+dx))+60c+60dx)-32b\cos^6(c+dx)}{192d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^5*(a*Cos[c + d*x] + b*Sin[c + d*x]),x]
```

```
[Out] (-32*b*Cos[c + d*x]^6 + a*(60*c + 60*d*x + 45*Sin[2*(c + d*x)] + 9*Sin[4*(c
 + d*x)] + Sin[6*(c + d*x)]))/(192*d)
```

fricas [A] time = 0.60, size = 62, normalized size = 0.71

$$-\frac{8\,b\cos\left(dx+c\right)^{6}-15\,adx-\left(8\,a\cos\left(dx+c\right)^{5}+10\,a\cos\left(dx+c\right)^{3}+15\,a\cos\left(dx+c\right)\right)\sin\left(dx+c\right)}{48\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(a*cos(d*x+c)+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/48*(8*b*cos(d*x + c)^6 - 15*a*d*x - (8*a*cos(d*x + c)^5 + 10*a*cos(d*x +
 c)^3 + 15*a*cos(d*x + c))*sin(d*x + c))/d
```

giac [A] time = 0.41, size = 95, normalized size = 1.09

$$\frac{5}{16}\,ax - \frac{b\cos\left(6\,dx + 6\,c\right)}{192\,d} - \frac{b\cos\left(4\,dx + 4\,c\right)}{32\,d} - \frac{5\,b\cos\left(2\,dx + 2\,c\right)}{64\,d} + \frac{a\sin\left(6\,dx + 6\,c\right)}{192\,d} + \frac{3\,a\sin\left(4\,dx + 4\,c\right)}{64\,d} + \frac{15\,a\sin}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(a*cos(d*x+c)+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 5/16*a*x - 1/192*b*cos(6*d*x + 6*c)/d - 1/32*b*cos(4*d*x + 4*c)/d - 5/64*b*
cos(2*d*x + 2*c)/d + 1/192*a*sin(6*d*x + 6*c)/d + 3/64*a*sin(4*d*x + 4*c)/d
 + 15/64*a*sin(2*d*x + 2*c)/d
```

maple [A] time = 1.09, size = 62, normalized size = 0.71

$$\frac{a\left(\frac{\left(\cos^{5}(dx+c)+\frac{5\left(\cos^{3}(dx+c)\right)}{4}+\frac{15\cos(dx+c)}{8}\right)\sin(dx+c)}{6}+\frac{5dx}{16}+\frac{5c}{16}\right)-\frac{b\left(\cos^{6}(dx+c)\right)}{6}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*(a*cos(d*x+c)+b*sin(d*x+c)),x)
```

```
[Out] 1/d*(a*(1/6*(cos(d*x+c)^5+5/4*cos(d*x+c)^3+15/8*cos(d*x+c))*sin(d*x+c)+5/16
*d*x+5/16*c)-1/6*b*cos(d*x+c)^6)
```

maxima [A] time = 0.33, size = 62, normalized size = 0.71

$$-\frac{32\,b\cos\left(dx + c\right)^{6} + \left(4\,\sin\left(2\,dx + 2\,c\right)^{3} - 60\,dx - 60\,c - 9\,\sin\left(4\,dx + 4\,c\right) - 48\,\sin\left(2\,dx + 2\,c\right)\right)a}{192\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(a*cos(d*x+c)+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/192*(32*b*cos(d*x + c)^6 + (4*sin(2*d*x + 2*c)^3 - 60*d*x - 60*c - 9*sin
(4*d*x + 4*c) - 48*sin(2*d*x + 2*c))*a)/d
```

mupad [B] time = 4.20, size = 149, normalized size = 1.71

$$\frac{5\,a\,x}{16} + \frac{-\frac{11\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{11}}{8} + 2\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{10} + \frac{5\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{9}}{24} - \frac{15\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{7}}{4} + \frac{20\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{6}}{3} + \frac{15\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{5}}{4} - \frac{5\,a}{}}{d\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{2}+1\right)^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^5*(a*cos(c + d*x) + b*sin(c + d*x)),x)
```

```
[Out] (5*a*x)/16 + ((11*a*tan(c/2 + (d*x)/2))/8 - (5*a*tan(c/2 + (d*x)/2)^3)/24 +
 (15*a*tan(c/2 + (d*x)/2)^5)/4 - (15*a*tan(c/2 + (d*x)/2)^7)/4 + (5*a*tan(c
/2 + (d*x)/2)^9)/24 - (11*a*tan(c/2 + (d*x)/2)^11)/8 + 2*b*tan(c/2 + (d*x)/
2)^2 + (20*b*tan(c/2 + (d*x)/2)^6)/3 + 2*b*tan(c/2 + (d*x)/2)^10)/(d*(tan(c
/2 + (d*x)/2)^2 + 1)^6)
```

sympy [A] time = 3.10, size = 175, normalized size = 2.01

$$\begin{cases} \frac{5ax\sin^6(c+dx)}{16} + \frac{15ax\sin^4(c+dx)\cos^2(c+dx)}{16} + \frac{15ax\sin^2(c+dx)\cos^4(c+dx)}{16} + \frac{5ax\cos^6(c+dx)}{16} + \frac{5a\sin^5(c+dx)\cos(c+dx)}{16d} + \frac{5a\sin^3(c+ \\ x(a\cos(c) + b\sin(c))\cos^5(c) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*(a*cos(d*x+c)+b*sin(d*x+c)),x)
```

```
[Out] Piecewise((5*a*x*sin(c + d*x)**6/16 + 15*a*x*sin(c + d*x)**4*cos(c + d*x)**
2/16 + 15*a*x*sin(c + d*x)**2*cos(c + d*x)**4/16 + 5*a*x*cos(c + d*x)**6/16
 + 5*a*sin(c + d*x)**5*cos(c + d*x)/(16*d) + 5*a*sin(c + d*x)**3*cos(c + d*
x)**3/(6*d) + 11*a*sin(c + d*x)*cos(c + d*x)**5/(16*d) - b*cos(c + d*x)**6/
(6*d), Ne(d, 0)), (x*(a*cos(c) + b*sin(c))*cos(c)**5, True))
```

3.31 $\int \cos^4(c+dx)(a\cos(c+dx)+b\sin(c+dx))\,dx$

Optimal. Leaf size=60

$$\frac{a\sin^5(c+dx)}{5d}-\frac{2a\sin^3(c+dx)}{3d}+\frac{a\sin(c+dx)}{d}-\frac{b\cos^5(c+dx)}{5d}$$

```
[Out] -1/5*b*cos(d*x+c)^5/d+a*sin(d*x+c)/d-2/3*a*sin(d*x+c)^3/d+1/5*a*sin(d*x+c)^
5/d
```

Rubi [A] time = 0.07, antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {3090, 2633, 2565, 30}

$$\frac{a\sin^5(c+dx)}{5d}-\frac{2a\sin^3(c+dx)}{3d}+\frac{a\sin(c+dx)}{d}-\frac{b\cos^5(c+dx)}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^4*(a*Cos[c + d*x] + b*Sin[c + d*x]),x]
```

```
[Out] -(b*Cos[c + d*x]^5)/(5*d) + (a*Sin[c + d*x])/d - (2*a*Sin[c + d*x]^3)/(3*d)
 + (a*Sin[c + d*x]^5)/(5*d)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2565

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> -Dist[(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 3090

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_.), x_Symbol] :> Int[ExpandTrig[cos[c + d*x]^m*(a
```

```
*cos[c + d*x] + b*sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d}, x] && Inte
gerQ[m] && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}\int \cos^4(c+dx)(a\cos(c+dx)+b\sin(c+dx))\,dx &= \int \left(a\cos^5(c+dx)+b\cos^4(c+dx)\sin(c+dx)\right)\,dx\\ &= a\int \cos^5(c+dx)\,dx+b\int \cos^4(c+dx)\sin(c+dx)\,dx\\ &= -\frac{a\,\mathrm{Subst}\left(\int \left(1-2x^2+x^4\right)\,dx,x,-\sin(c+dx)\right)}{d}-\frac{b\,\mathrm{Subst}(\\ &= -\frac{b\cos^5(c+dx)}{5d}+\frac{a\sin(c+dx)}{d}-\frac{2a\sin^3(c+dx)}{3d}+\frac{a\sin^5(}{5}\end{aligned}$$

Mathematica [A] time = 0.02, size = 60, normalized size = 1.00

$$\frac{a\sin^5(c+dx)}{5d}-\frac{2a\sin^3(c+dx)}{3d}+\frac{a\sin(c+dx)}{d}-\frac{b\cos^5(c+dx)}{5d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^4*(a*Cos[c + d*x] + b*Sin[c + d*x]),x]
```

```
[Out] -1/5*(b*Cos[c + d*x]^5)/d + (a*Sin[c + d*x])/d - (2*a*Sin[c + d*x]^3)/(3*d)
 + (a*Sin[c + d*x]^5)/(5*d)
```

fricas [A] time = 0.58, size = 51, normalized size = 0.85

$$-\frac{3\,b\cos\left(dx+c\right)^5-\left(3\,a\cos\left(dx+c\right)^4+4\,a\cos\left(dx+c\right)^2+8\,a\right)\sin\left(dx+c\right)}{15\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a*cos(d*x+c)+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/15*(3*b*cos(d*x + c)^5 - (3*a*cos(d*x + c)^4 + 4*a*cos(d*x + c)^2 + 8*a)
*sin(d*x + c))/d
```

giac [A] time = 0.93, size = 85, normalized size = 1.42

$$-\frac{b\cos\left(5\,dx+5\,c\right)}{80\,d}-\frac{b\cos\left(3\,dx+3\,c\right)}{16\,d}-\frac{b\cos\left(dx+c\right)}{8\,d}+\frac{a\sin\left(5\,dx+5\,c\right)}{80\,d}+\frac{5\,a\sin\left(3\,dx+3\,c\right)}{48\,d}+\frac{5\,a\sin\left(dx+c\right)}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a*cos(d*x+c)+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/80*b*cos(5*d*x + 5*c)/d - 1/16*b*cos(3*d*x + 3*c)/d - 1/8*b*cos(d*x + c)
/d + 1/80*a*sin(5*d*x + 5*c)/d + 5/48*a*sin(3*d*x + 3*c)/d + 5/8*a*sin(d*x
+ c)/d
```

maple [A] time = 1.05, size = 46, normalized size = 0.77

$$\frac{\frac{a\left(\frac{8}{3}+\cos^4(dx+c)+\frac{4\left(\cos^2(dx+c)\right)}{3}\right)\sin(dx+c)}{5}-\frac{b\left(\cos^5(dx+c)\right)}{5}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*(a*cos(d*x+c)+b*sin(d*x+c)),x)
```

```
[Out] 1/d*(1/5*a*(8/3+cos(d*x+c)^4+4/3*cos(d*x+c)^2)*sin(d*x+c)-1/5*b*cos(d*x+c)^
5)
```

maxima [A] time = 0.32, size = 49, normalized size = 0.82

$$-\frac{3\,b\cos\left(dx+c\right)^5-\left(3\,\sin\left(dx+c\right)^5-10\,\sin\left(dx+c\right)^3+15\,\sin\left(dx+c\right)\right)a}{15\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a*cos(d*x+c)+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/15*(3*b*cos(d*x + c)^5 - (3*sin(d*x + c)^5 - 10*sin(d*x + c)^3 + 15*sin(
d*x + c))*a)/d
```

mupad [B] time = 0.46, size = 67, normalized size = 1.12

$$\frac{8\,a\,\sin\left(c+d\,x\right)}{15\,d}-\frac{b\cos\left(c+d\,x\right)^5}{5\,d}+\frac{4\,a\cos\left(c+d\,x\right)^2\,\sin\left(c+d\,x\right)}{15\,d}+\frac{a\cos\left(c+d\,x\right)^4\,\sin\left(c+d\,x\right)}{5\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^4*(a*cos(c + d*x) + b*sin(c + d*x)),x)
```

```
[Out] (8*a*sin(c + d*x))/(15*d) - (b*cos(c + d*x)^5)/(5*d) + (4*a*cos(c + d*x)^2*
sin(c + d*x))/(15*d) + (a*cos(c + d*x)^4*sin(c + d*x))/(5*d)
```

sympy [A] time = 1.61, size = 87, normalized size = 1.45

$$\begin{cases}\frac{8a\sin^5(c+dx)}{15d}+\frac{4a\sin^3(c+dx)\cos^2(c+dx)}{3d}+\frac{a\sin(c+dx)\cos^4(c+dx)}{d}-\frac{b\cos^5(c+dx)}{5d} & \text{for } d\neq 0\\ x\left(a\cos(c)+b\sin(c)\right)\cos^4(c) & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*(a*cos(d*x+c)+b*sin(d*x+c)),x)
```

```
[Out] Piecewise((8*a*sin(c + d*x)**5/(15*d) + 4*a*sin(c + d*x)**3*cos(c + d*x)**2
/(3*d) + a*sin(c + d*x)*cos(c + d*x)**4/d - b*cos(c + d*x)**5/(5*d), Ne(d,
0)), (x*(a*cos(c) + b*sin(c))*cos(c)**4, True))
```

3.32 $\int \cos^3(c+dx)(a\cos(c+dx)+b\sin(c+dx))\,dx$

Optimal. Leaf size=65

$$\frac{a\sin(c+dx)\cos^3(c+dx)}{4d}+\frac{3a\sin(c+dx)\cos(c+dx)}{8d}+\frac{3ax}{8}-\frac{b\cos^4(c+dx)}{4d}$$

```
[Out] 3/8*a*x-1/4*b*cos(d*x+c)^4/d+3/8*a*cos(d*x+c)*sin(d*x+c)/d+1/4*a*cos(d*x+c)
^3*sin(d*x+c)/d
```

Rubi [A] time = 0.08, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {3090, 2635, 8, 2565, 30}

$$\frac{a\sin(c+dx)\cos^3(c+dx)}{4d}+\frac{3a\sin(c+dx)\cos(c+dx)}{8d}+\frac{3ax}{8}-\frac{b\cos^4(c+dx)}{4d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^3*(a*Cos[c + d*x] + b*Sin[c + d*x]),x]
```

```
[Out] (3*a*x)/8 - (b*Cos[c + d*x]^4)/(4*d) + (3*a*Cos[c + d*x]*Sin[c + d*x])/(8*d
) + (a*Cos[c + d*x]^3*Sin[c + d*x])/(4*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2565

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> -Dist[(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
```

```
]
```

Rule 3090

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_.), x_Symbol] :> Int[ExpandTrig[cos[c + d*x]^m*(a
*cos[c + d*x] + b*sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d}, x] && Inte
gerQ[m] && IGtQ[n, 0]
```

Rubi steps

$$
\begin{aligned}
\int \cos^3(c+dx)(a\cos(c+dx)+b\sin(c+dx))\,dx &= \int \left(a\cos^4(c+dx)+b\cos^3(c+dx)\sin(c+dx)\right)\,dx\\
&= a\int \cos^4(c+dx)\,dx+b\int \cos^3(c+dx)\sin(c+dx)\,dx\\
&= \frac{a\cos^3(c+dx)\sin(c+dx)}{4d}+\frac{1}{4}(3a)\int \cos^2(c+dx)\,dx-\frac{b\,\mathrm{Su}}{}\\
&= -\frac{b\cos^4(c+dx)}{4d}+\frac{3a\cos(c+dx)\sin(c+dx)}{8d}+\frac{a\cos^3(c+}{}\\
&= \frac{3ax}{8}-\frac{b\cos^4(c+dx)}{4d}+\frac{3a\cos(c+dx)\sin(c+dx)}{8d}+\frac{a\cos^3}{}
\end{aligned}
$$

Mathematica [A] time = 0.09, size = 62, normalized size = 0.95

$$\frac{3a(c+dx)}{8d}+\frac{a\sin(2(c+dx))}{4d}+\frac{a\sin(4(c+dx))}{32d}-\frac{b\cos^4(c+dx)}{4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^3*(a*Cos[c + d*x] + b*Sin[c + d*x]),x]
```

```
[Out] (3*a*(c + d*x))/(8*d) - (b*Cos[c + d*x]^4)/(4*d) + (a*Sin[2*(c + d*x)])/(4*
d) + (a*Sin[4*(c + d*x)])/(32*d)
```

fricas [A] time = 0.67, size = 51, normalized size = 0.78

$$-\frac{2\,b\cos\left(dx+c\right)^4-3\,adx-\left(2\,a\cos\left(dx+c\right)^3+3\,a\cos\left(dx+c\right)\right)\sin\left(dx+c\right)}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a*cos(d*x+c)+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/8*(2*b*cos(d*x + c)^4 - 3*a*d*x - (2*a*cos(d*x + c)^3 + 3*a*cos(d*x + c)
)*sin(d*x + c))/d
```

giac [A] time = 0.98, size = 65, normalized size = 1.00

$$\frac{3}{8}ax - \frac{b\cos(4dx+4c)}{32d} - \frac{b\cos(2dx+2c)}{8d} + \frac{a\sin(4dx+4c)}{32d} + \frac{a\sin(2dx+2c)}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a*cos(d*x+c)+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 3/8*a*x - 1/32*b*cos(4*d*x + 4*c)/d - 1/8*b*cos(2*d*x + 2*c)/d + 1/32*a*sin
(4*d*x + 4*c)/d + 1/4*a*sin(2*d*x + 2*c)/d
```

maple [A] time = 10.81, size = 52, normalized size = 0.80

$$\frac{a\left(\frac{\left(\cos^3(dx+c)+\frac{3\cos(dx+c)}{2}\right)\sin(dx+c)}{4} + \frac{3dx}{8} + \frac{3c}{8}\right) - \frac{b\left(\cos^4(dx+c)\right)}{4}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3*(a*cos(d*x+c)+b*sin(d*x+c)),x)
```

```
[Out] 1/d*(a*(1/4*(cos(d*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+3/8*d*x+3/8*c)-1/4*b*c
os(d*x+c)^4)
```

maxima [A] time = 0.33, size = 48, normalized size = 0.74

$$-\frac{8\,b\cos(dx+c)^4 - (12\,dx + 12\,c + \sin(4\,dx+4\,c) + 8\,\sin(2\,dx+2\,c))a}{32\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a*cos(d*x+c)+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/32*(8*b*cos(d*x + c)^4 - (12*d*x + 12*c + sin(4*d*x + 4*c) + 8*sin(2*d*x
 + 2*c))*a)/d
```

mupad [B] time = 4.11, size = 107, normalized size = 1.65

$$\frac{3\,a\,x}{8} + \frac{-\frac{5\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7}{4} + 2\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6 + \frac{3\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5}{4} - \frac{3\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{4} + 2\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2 + \frac{5\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{4}}{d\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+1\right)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^3*(a*cos(c + d*x) + b*sin(c + d*x)),x)
```

```
[Out] (3*a*x)/8 + ((5*a*tan(c/2 + (d*x)/2))/4 - (3*a*tan(c/2 + (d*x)/2)^3)/4 + (3
*a*tan(c/2 + (d*x)/2)^5)/4 - (5*a*tan(c/2 + (d*x)/2)^7)/4 + 2*b*tan(c/2 + (
d*x)/2)^2 + 2*b*tan(c/2 + (d*x)/2)^6)/(d*(tan(c/2 + (d*x)/2)^2 + 1)^4)
```

sympy [A] time = 0.88, size = 128, normalized size = 1.97

$$\begin{cases} \frac{3ax\sin^4(c+dx)}{8} + \frac{3ax\sin^2(c+dx)\cos^2(c+dx)}{4} + \frac{3ax\cos^4(c+dx)}{8} + \frac{3a\sin^3(c+dx)\cos(c+dx)}{8d} + \frac{5a\sin(c+dx)\cos^3(c+dx)}{8d} - \frac{b\cos^4(c+dx)}{4d} \\ x(a\cos(c) + b\sin(c))\cos^3(c) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3*(a*cos(d*x+c)+b*sin(d*x+c)),x)
```

```
[Out] Piecewise((3*a*x*sin(c + d*x)**4/8 + 3*a*x*sin(c + d*x)**2*cos(c + d*x)**2/
4 + 3*a*x*cos(c + d*x)**4/8 + 3*a*sin(c + d*x)**3*cos(c + d*x)/(8*d) + 5*a*
sin(c + d*x)*cos(c + d*x)**3/(8*d) - b*cos(c + d*x)**4/(4*d), Ne(d, 0)), (x
*(a*cos(c) + b*sin(c))*cos(c)**3, True))
```

3.33 $\int \cos^2(c+dx)(a\cos(c+dx)+b\sin(c+dx))\,dx$

Optimal. Leaf size=44

$$-\frac{a\sin^3(c+dx)}{3d}+\frac{a\sin(c+dx)}{d}-\frac{b\cos^3(c+dx)}{3d}$$

```
[Out] -1/3*b*cos(d*x+c)^3/d+a*sin(d*x+c)/d-1/3*a*sin(d*x+c)^3/d
```

Rubi [A] time = 0.06, antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {3090, 2633, 2565, 30}

$$-\frac{a\sin^3(c+dx)}{3d}+\frac{a\sin(c+dx)}{d}-\frac{b\cos^3(c+dx)}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2*(a*Cos[c + d*x] + b*Sin[c + d*x]),x]
```

```
[Out] -(b*Cos[c + d*x]^3)/(3*d) + (a*Sin[c + d*x])/d - (a*Sin[c + d*x]^3)/(3*d)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2565

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> -Dist[(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 3090

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_.), x_Symbol] :> Int[ExpandTrig[cos[c + d*x]^m*(a
*cos[c + d*x] + b*sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d}, x] && Inte
```

```
gerQ[m] && IGtQ[n, 0]
```

Rubi steps

$$\int \cos^2(c+dx)(a\cos(c+dx)+b\sin(c+dx))\,dx = \int \left(a\cos^3(c+dx)+b\cos^2(c+dx)\sin(c+dx)\right)\,dx$$
$$= a\int \cos^3(c+dx)\,dx + b\int \cos^2(c+dx)\sin(c+dx)\,dx$$
$$= -\frac{a\,\mathrm{Subst}\left(\int \left(1-x^2\right)\,dx, x, -\sin(c+dx)\right)}{d} - \frac{b\,\mathrm{Subst}\left(\int x^2\,d\right.}{}$$
$$= -\frac{b\cos^3(c+dx)}{3d} + \frac{a\sin(c+dx)}{d} - \frac{a\sin^3(c+dx)}{3d}$$

Mathematica [A] time = 0.01, size = 44, normalized size = 1.00

$$-\frac{a\sin^3(c+dx)}{3d} + \frac{a\sin(c+dx)}{d} - \frac{b\cos^3(c+dx)}{3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2*(a*Cos[c + d*x] + b*Sin[c + d*x]),x]
[Out] -1/3*(b*Cos[c + d*x]^3)/d + (a*Sin[c + d*x])/d - (a*Sin[c + d*x]^3)/(3*d)
```

fricas [A] time = 0.75, size = 38, normalized size = 0.86

$$-\frac{b\cos\left(dx+c\right)^3 - \left(a\cos\left(dx+c\right)^2 + 2\,a\right)\sin\left(dx+c\right)}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a*cos(d*x+c)+b*sin(d*x+c)),x, algorithm="fricas")
[Out] -1/3*(b*cos(d*x + c)^3 - (a*cos(d*x + c)^2 + 2*a)*sin(d*x + c))/d
```

giac [A] time = 3.94, size = 55, normalized size = 1.25

$$-\frac{b\cos\left(3\,dx+3\,c\right)}{12\,d} - \frac{b\cos\left(dx+c\right)}{4\,d} + \frac{a\sin\left(3\,dx+3\,c\right)}{12\,d} + \frac{3\,a\sin\left(dx+c\right)}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a*cos(d*x+c)+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/12*b*cos(3*d*x + 3*c)/d - 1/4*b*cos(d*x + c)/d + 1/12*a*sin(3*d*x + 3*c)
/d + 3/4*a*sin(d*x + c)/d
```

maple [A] time = 1.03, size = 36, normalized size = 0.82

$$\frac{\frac{a\left(2+\cos^{2}(dx+c)\right)\sin(dx+c)}{3}-\frac{b\left(\cos^{3}(dx+c)\right)}{3}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(a*cos(d*x+c)+b*sin(d*x+c)),x)
```

```
[Out] 1/d*(1/3*a*(2+cos(d*x+c)^2)*sin(d*x+c)-1/3*b*cos(d*x+c)^3)
```

maxima [A] time = 0.32, size = 35, normalized size = 0.80

$$-\frac{b\cos\left(dx+c\right)^{3}+\left(\sin\left(dx+c\right)^{3}-3\,\sin\left(dx+c\right)\right)a}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a*cos(d*x+c)+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/3*(b*cos(d*x + c)^3 + (sin(d*x + c)^3 - 3*sin(d*x + c))*a)/d
```

mupad [B] time = 0.44, size = 47, normalized size = 1.07

$$\frac{2\,a\,\sin\left(c+d\,x\right)}{3\,d}-\frac{b\cos\left(c+d\,x\right)^{3}}{3\,d}+\frac{a\cos\left(c+d\,x\right)^{2}\,\sin\left(c+d\,x\right)}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^2*(a*cos(c + d*x) + b*sin(c + d*x)),x)
```

```
[Out] (2*a*sin(c + d*x))/(3*d) - (b*cos(c + d*x)^3)/(3*d) + (a*cos(c + d*x)^2*sin
(c + d*x))/(3*d)
```

sympy [A] time = 0.43, size = 63, normalized size = 1.43

$$\begin{cases}\frac{2a\sin^{3}\left(c+dx\right)}{3d}+\frac{a\sin\left(c+dx\right)\cos^{2}\left(c+dx\right)}{d}-\frac{b\cos^{3}\left(c+dx\right)}{3d} & \text{for}\: d\neq 0\\x\left(a\cos\left(c\right)+b\sin\left(c\right)\right)\cos^{2}\left(c\right) & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(a*cos(d*x+c)+b*sin(d*x+c)),x)
```

```
[Out] Piecewise((2*a*sin(c + d*x)**3/(3*d) + a*sin(c + d*x)*cos(c + d*x)**2/d - b
*cos(c + d*x)**3/(3*d), Ne(d, 0)), (x*(a*cos(c) + b*sin(c))*cos(c)**2, True
))
```

3.34 $\int \cos(c+dx)(a\cos(c+dx)+b\sin(c+dx))\,dx$

Optimal. Leaf size=43

$$\frac{a\sin(c+dx)\cos(c+dx)}{2d}+\frac{ax}{2}+\frac{b\sin^2(c+dx)}{2d}$$

```
[Out] 1/2*a*x+1/2*a*cos(d*x+c)*sin(d*x+c)/d+1/2*b*sin(d*x+c)^2/d
```

Rubi [A] time = 0.04, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {3090, 2635, 8, 2564, 30}

$$\frac{a\sin(c+dx)\cos(c+dx)}{2d}+\frac{ax}{2}+\frac{b\sin^2(c+dx)}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]*(a*Cos[c + d*x] + b*Sin[c + d*x]),x]
```

```
[Out] (a*x)/2 + (a*Cos[c + d*x]*Sin[c + d*x])/(2*d) + (b*Sin[c + d*x]^2)/(2*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2564

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 3090

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_.), x_Symbol] :> Int[ExpandTrig[cos[c + d*x]^m*(a
*cos[c + d*x] + b*sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d}, x] && Inte
gerQ[m] && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}\int \cos(c+dx)(a\cos(c+dx)+b\sin(c+dx))\,dx &= \int \left(a\cos^2(c+dx)+b\cos(c+dx)\sin(c+dx)\right)\,dx\\ &= a\int \cos^2(c+dx)\,dx+b\int \cos(c+dx)\sin(c+dx)\,dx\\ &= \frac{a\cos(c+dx)\sin(c+dx)}{2d}+\frac{1}{2}a\int 1\,dx+\frac{b\,\mathrm{Subst}(\int x\,dx,x,\mathrm{si}}{d}\\ &= \frac{ax}{2}+\frac{a\cos(c+dx)\sin(c+dx)}{2d}+\frac{b\sin^2(c+dx)}{2d}\end{aligned}$$

Mathematica [A] time = 0.05, size = 46, normalized size = 1.07

$$\frac{a(c+dx)}{2d}+\frac{a\sin(2(c+dx))}{4d}-\frac{b\cos^2(c+dx)}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]*(a*Cos[c + d*x] + b*Sin[c + d*x]),x]
[Out] (a*(c + d*x))/(2*d) - (b*Cos[c + d*x]^2)/(2*d) + (a*Sin[2*(c + d*x)])/(4*d)
```

fricas [A] time = 0.59, size = 35, normalized size = 0.81

$$\frac{adx-b\cos\left(dx+c\right)^2+a\cos\left(dx+c\right)\sin\left(dx+c\right)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a*cos(d*x+c)+b*sin(d*x+c)),x, algorithm="fricas")
[Out] 1/2*(a*d*x - b*cos(d*x + c)^2 + a*cos(d*x + c)*sin(d*x + c))/d
```

giac [A] time = 3.99, size = 35, normalized size = 0.81

$$\frac{1}{2}\,ax-\frac{b\cos\left(2\,dx+2\,c\right)}{4\,d}+\frac{a\sin\left(2\,dx+2\,c\right)}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a*cos(d*x+c)+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/2*a*x - 1/4*b*cos(2*d*x + 2*c)/d + 1/4*a*sin(2*d*x + 2*c)/d
```

maple [A] time = 0.71, size = 41, normalized size = 0.95

$$\frac{a\left(\frac{\cos(dx+c)\sin(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)-\frac{\left(\cos^2(dx+c)\right)b}{2}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(a*cos(d*x+c)+b*sin(d*x+c)),x)
```

```
[Out] 1/d*(a*(1/2*cos(d*x+c)*sin(d*x+c)+1/2*d*x+1/2*c)-1/2*cos(d*x+c)^2*b)
```

maxima [A] time = 0.32, size = 37, normalized size = 0.86

$$-\frac{2\,b\cos\left(dx+c\right)^2-\left(2\,dx+2\,c+\sin\left(2\,dx+2\,c\right)\right)a}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a*cos(d*x+c)+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/4*(2*b*cos(d*x + c)^2 - (2*d*x + 2*c + sin(2*d*x + 2*c))*a)/d
```

mupad [B] time = 0.43, size = 35, normalized size = 0.81

$$\frac{a\,x}{2}-\frac{b\,\cos\left(2\,c+2\,d\,x\right)}{4\,d}+\frac{a\,\sin\left(2\,c+2\,d\,x\right)}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)*(a*cos(c + d*x) + b*sin(c + d*x)),x)
```

```
[Out] (a*x)/2 - (b*cos(2*c + 2*d*x))/(4*d) + (a*sin(2*c + 2*d*x))/(4*d)
```

sympy [A] time = 0.21, size = 73, normalized size = 1.70

$$\begin{cases} \frac{ax\sin^2\left(c+dx\right)}{2}+\frac{ax\cos^2\left(c+dx\right)}{2}+\frac{a\sin\left(c+dx\right)\cos\left(c+dx\right)}{2d}-\frac{b\cos^2\left(c+dx\right)}{2d} & \text{for } d\neq 0 \\ x\left(a\cos\left(c\right)+b\sin\left(c\right)\right)\cos\left(c\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a*cos(d*x+c)+b*sin(d*x+c)),x)
```

```
[Out] Piecewise((a*x*sin(c + d*x)**2/2 + a*x*cos(c + d*x)**2/2 + a*sin(c + d*x)*c
os(c + d*x)/(2*d) - b*cos(c + d*x)**2/(2*d), Ne(d, 0)), (x*(a*cos(c) + b*si
n(c))*cos(c), True))
```

3.35 $\int (a\cos(c+dx)+b\sin(c+dx))\,dx$

Optimal. Leaf size=24

$$\frac{a\sin(c+dx)}{d}-\frac{b\cos(c+dx)}{d}$$

```
[Out] -b*cos(d*x+c)/d+a*sin(d*x+c)/d
```

Rubi [A] time = 0.01, antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {2637, 2638}

$$\frac{a\sin(c+dx)}{d}-\frac{b\cos(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[a*Cos[c + d*x] + b*Sin[c + d*x],x]
```

```
[Out] -((b*Cos[c + d*x])/d) + (a*Sin[c + d*x])/d
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int (a\cos(c+dx)+b\sin(c+dx))\,dx &= a\int \cos(c+dx)\,dx+b\int \sin(c+dx)\,dx\\ &= -\frac{b\cos(c+dx)}{d}+\frac{a\sin(c+dx)}{d}\end{aligned}$$

Mathematica [A] time = 0.01, size = 46, normalized size = 1.92

$$\frac{a\sin(c)\cos(dx)}{d}+\frac{a\cos(c)\sin(dx)}{d}+\frac{b\sin(c)\sin(dx)}{d}-\frac{b\cos(c)\cos(dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[a*Cos[c + d*x] + b*Sin[c + d*x],x]
```

```
[Out] -((b*Cos[c]*Cos[d*x])/d) + (a*Cos[d*x]*Sin[c])/d + (a*Cos[c]*Sin[d*x])/d +
(b*Sin[c]*Sin[d*x])/d
```

fricas [A] time = 0.52, size = 23, normalized size = 0.96

$$-\frac{b\cos(dx+c)-a\sin(dx+c)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a*cos(d*x+c)+b*sin(d*x+c),x, algorithm="fricas")
```

```
[Out] -(b*cos(d*x + c) - a*sin(d*x + c))/d
```

giac [A] time = 0.19, size = 24, normalized size = 1.00

$$-\frac{b\cos(dx+c)}{d}+\frac{a\sin(dx+c)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a*cos(d*x+c)+b*sin(d*x+c),x, algorithm="giac")
```

```
[Out] -b*cos(d*x + c)/d + a*sin(d*x + c)/d
```

maple [A] time = 0.13, size = 25, normalized size = 1.04

$$-\frac{b\cos(dx+c)}{d}+\frac{a\sin(dx+c)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(a*cos(d*x+c)+b*sin(d*x+c),x)
```

```
[Out] -b*cos(d*x+c)/d+a*sin(d*x+c)/d
```

maxima [A] time = 0.32, size = 24, normalized size = 1.00

$$-\frac{b\cos(dx+c)}{d}+\frac{a\sin(dx+c)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a*cos(d*x+c)+b*sin(d*x+c),x, algorithm="maxima")
```

```
[Out] -b*cos(d*x + c)/d + a*sin(d*x + c)/d
```

mupad [B] time = 0.36, size = 38, normalized size = 1.58

$$-\frac{2\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(b\,\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)-a\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(a*cos(c + d*x) + b*sin(c + d*x),x)
```

```
[Out] -(2*cos(c/2 + (d*x)/2)*(b*cos(c/2 + (d*x)/2) - a*sin(c/2 + (d*x)/2)))/d
```

sympy [A] time = 0.14, size = 31, normalized size = 1.29

$$a\left(\begin{cases}\frac{\sin(c+dx)}{d} & \text{for}\ d\neq 0\\ x\cos(c) & \text{otherwise}\end{cases}\right)+b\left(\begin{cases}-\frac{\cos(c+dx)}{d} & \text{for}\ d\neq 0\\ x\sin(c) & \text{otherwise}\end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a*cos(d*x+c)+b*sin(d*x+c),x)
```

```
[Out] a*Piecewise((sin(c + d*x)/d, Ne(d, 0)), (x*cos(c), True)) + b*Piecewise((-c
os(c + d*x)/d, Ne(d, 0)), (x*sin(c), True))
```

3.36 $\int \sec(c+dx)(a\cos(c+dx)+b\sin(c+dx))\,dx$

Optimal. Leaf size=17

$$ax-\frac{b\log(\cos(c+dx))}{d}$$

```
[Out] a*x-b*ln(cos(d*x+c))/d
```

Rubi [A] time = 0.03, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {3086, 3475}

$$ax-\frac{b\log(\cos(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]*(a*Cos[c + d*x] + b*Sin[c + d*x]),x]
```

```
[Out] a*x - (b*Log[Cos[c + d*x]])/d
```

Rule 3086

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin
[(c_.) + (d_.)*(x_)])^(n_.), x_Symbol] :> Int[(a + b*Tan[c + d*x])^n, x] /;
 FreeQ[{a, b, c, d}, x] && EqQ[m + n, 0] && IntegerQ[n] && NeQ[a^2 + b^2, 0
]
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \sec(c+dx)(a\cos(c+dx)+b\sin(c+dx))\,dx &= \int (a+b\tan(c+dx))\,dx\\ &= ax+b\int \tan(c+dx)\,dx\\ &= ax-\frac{b\log(\cos(c+dx))}{d}\end{aligned}$$

Mathematica [A] time = 0.02, size = 17, normalized size = 1.00

$$ax-\frac{b\log(\cos(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]*(a*Cos[c + d*x] + b*Sin[c + d*x]),x]
[Out] a*x - (b*Log[Cos[c + d*x]])/d
```

fricas [A] time = 0.75, size = 21, normalized size = 1.24

$$\frac{adx - b\log\left(-\cos\left(dx + c\right)\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a*cos(d*x+c)+b*sin(d*x+c)),x, algorithm="fricas")
[Out] (a*d*x - b*log(-cos(d*x + c)))/d
```

giac [A] time = 0.16, size = 27, normalized size = 1.59

$$\frac{2\left(dx + c\right)a + b\log\left(\tan\left(dx + c\right)^2 + 1\right)}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a*cos(d*x+c)+b*sin(d*x+c)),x, algorithm="giac")
[Out] 1/2*(2*(d*x + c)*a + b*log(tan(d*x + c)^2 + 1))/d
```

maple [A] time = 1.34, size = 24, normalized size = 1.41

$$ax - \frac{b\ln\left(\cos\left(dx + c\right)\right)}{d} + \frac{ca}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)*(a*cos(d*x+c)+b*sin(d*x+c)),x)
[Out] a*x-b*ln(cos(d*x+c))/d+1/d*c*a
```

maxima [A] time = 0.33, size = 30, normalized size = 1.76

$$\frac{2\left(dx + c\right)a - b\log\left(-\sin\left(dx + c\right)^2 + 1\right)}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a*cos(d*x+c)+b*sin(d*x+c)),x, algorithm="maxima")
[Out] 1/2*(2*(d*x + c)*a - b*log(-sin(d*x + c)^2 + 1))/d
```

mupad [B] time = 0.57, size = 70, normalized size = 4.12

$$\frac{b\,\ln\left(\frac{1}{{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2}\right)}{d}+\frac{2\,a\,\mathrm{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d}-\frac{b\,\ln\left(\frac{\cos\left(c+d\,x\right)}{\cos\left(c+d\,x\right)+1}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*cos(c + d*x) + b*sin(c + d*x))/cos(c + d*x),x)
```

```
[Out] (b*log(1/cos(c/2 + (d*x)/2)^2))/d + (2*a*atan(sin(c/2 + (d*x)/2)/cos(c/2 +
(d*x)/2)))/d - (b*log(cos(c + d*x)/(cos(c + d*x) + 1)))/d
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a\cos(c+dx)+b\sin(c+dx))\sec(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a*cos(d*x+c)+b*sin(d*x+c)),x)
```

```
[Out] Integral((a*cos(c + d*x) + b*sin(c + d*x))*sec(c + d*x), x)
```

3.37 $\int \sec^2(c+dx)(a\cos(c+dx)+b\sin(c+dx))\,dx$

Optimal. Leaf size=24

$$\frac{a\tanh^{-1}(\sin(c+dx))}{d}+\frac{b\sec(c+dx)}{d}$$

```
[Out] a*arctanh(sin(d*x+c))/d+b*sec(d*x+c)/d
```

Rubi [A] time = 0.05, antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {3090, 3770, 2606, 8}

$$\frac{a\tanh^{-1}(\sin(c+dx))}{d}+\frac{b\sec(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^2*(a*Cos[c + d*x] + b*Sin[c + d*x]),x]
```

```
[Out] (a*ArcTanh[Sin[c + d*x]])/d + (b*Sec[c + d*x])/d
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 && !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 3090

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_.), x_Symbol] :> Int[ExpandTrig[cos[c + d*x]^m*(a
*cos[c + d*x] + b*sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d}, x] && Inte
gerQ[m] && IGtQ[n, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \sec^2(c+dx)(a\cos(c+dx)+b\sin(c+dx))\,dx = \int (a\sec(c+dx)+b\sec(c+dx)\tan(c+dx))\,dx$$
$$= a\int \sec(c+dx)\,dx + b\int \sec(c+dx)\tan(c+dx)\,dx$$
$$= \frac{a\tanh^{-1}(\sin(c+dx))}{d} + \frac{b\,\text{Subst}(\int 1\,dx, x, \sec(c+dx))}{d}$$
$$= \frac{a\tanh^{-1}(\sin(c+dx))}{d} + \frac{b\sec(c+dx)}{d}$$

Mathematica [**A**] time = 0.01, size = 24, normalized size = 1.00

$$\frac{a\tanh^{-1}(\sin(c+dx))}{d} + \frac{b\sec(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^2*(a*Cos[c + d*x] + b*Sin[c + d*x]),x]
```

```
[Out] (a*ArcTanh[Sin[c + d*x]])/d + (b*Sec[c + d*x])/d
```

fricas [**B**] time = 0.72, size = 54, normalized size = 2.25

$$\frac{a\cos(dx+c)\log(\sin(dx+c)+1) - a\cos(dx+c)\log(-\sin(dx+c)+1) + 2\,b}{2\,d\cos(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a*cos(d*x+c)+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/2*(a*cos(d*x + c)*log(sin(d*x + c) + 1) - a*cos(d*x + c)*log(-sin(d*x + c
) + 1) + 2*b)/(d*cos(d*x + c))
```

giac [**B**] time = 0.23, size = 54, normalized size = 2.25

$$\frac{a\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right) - a\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right) - \frac{2\,b}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a*cos(d*x+c)+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] (a*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - a*log(abs(tan(1/2*d*x + 1/2*c) - 1)
) - 2*b/(tan(1/2*d*x + 1/2*c)^2 - 1))/d
```

maple [A] time = 1.77, size = 34, normalized size = 1.42

$$\frac{a\ln\left(\sec\left(dx+c\right)+\tan\left(dx+c\right)\right)}{d}+\frac{b}{d\cos\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2*(a*cos(d*x+c)+b*sin(d*x+c)),x)
[Out] 1/d*a*ln(sec(d*x+c)+tan(d*x+c))+1/d*b/cos(d*x+c)
```

maxima [A] time = 0.32, size = 40, normalized size = 1.67

$$\frac{a\left(\log\left(\sin\left(dx+c\right)+1\right)-\log\left(\sin\left(dx+c\right)-1\right)\right)+\frac{2\,b}{\cos(dx+c)}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a*cos(d*x+c)+b*sin(d*x+c)),x, algorithm="maxima")
[Out] 1/2*(a*(log(sin(d*x + c) + 1) - log(sin(d*x + c) - 1)) + 2*b/cos(d*x + c))/
d
```

mupad [B] time = 0.41, size = 38, normalized size = 1.58

$$\frac{2\,a\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{d}-\frac{2\,b}{d\,\left({\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*cos(c + d*x) + b*sin(c + d*x))/cos(c + d*x)^2,x)
[Out] (2*a*atanh(tan(c/2 + (d*x)/2)))/d - (2*b)/(d*(tan(c/2 + (d*x)/2)^2 - 1))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a\cos\left(c+dx\right)+b\sin\left(c+dx\right)\right)\sec^{2}\left(c+dx\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2*(a*cos(d*x+c)+b*sin(d*x+c)),x)
[Out] Integral((a*cos(c + d*x) + b*sin(c + d*x))*sec(c + d*x)**2, x)
```

3.38 $\int \sec^3(c+dx)(a\cos(c+dx)+b\sin(c+dx))\,dx$

Optimal. Leaf size=28

$$\frac{a\tan(c+dx)}{d}+\frac{b\sec^2(c+dx)}{2d}$$

[Out] 1/2*b*sec(d*x+c)^2/d+a*tan(d*x+c)/d

Rubi [A] time = 0.06, antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {3090, 3767, 8, 2606, 30}

$$\frac{a\tan(c+dx)}{d}+\frac{b\sec^2(c+dx)}{2d}$$

Antiderivative was successfully verified.

[In] Int[Sec[c + d*x]^3*(a*Cos[c + d*x] + b*Sin[c + d*x]),x]

[Out] (b*Sec[c + d*x]^2)/(2*d) + (a*Tan[c + d*x])/d

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 && !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 3090

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_.), x_Symbol] :> Int[ExpandTrig[cos[c + d*x]^m*(a
*cos[c + d*x] + b*sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d}, x] && Inte
gerQ[m] && IGtQ[n, 0]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$
\begin{aligned}
\int \sec^3(c+dx)(a\cos(c+dx)+b\sin(c+dx))\,dx &= \int \left(a\sec^2(c+dx)+b\sec^2(c+dx)\tan(c+dx)\right)\,dx\\
&= a\int \sec^2(c+dx)\,dx+b\int \sec^2(c+dx)\tan(c+dx)\,dx\\
&= -\frac{a\,\mathrm{Subst}(\int 1\,dx,x,-\tan(c+dx))}{d}+\frac{b\,\mathrm{Subst}(\int x\,dx,x,\sec(c
}{d}\\
&= \frac{b\sec^2(c+dx)}{2d}+\frac{a\tan(c+dx)}{d}
\end{aligned}
$$

Mathematica [A] time = 0.01, size = 28, normalized size = 1.00

$$\frac{a\tan(c+dx)}{d}+\frac{b\sec^2(c+dx)}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^3*(a*Cos[c + d*x] + b*Sin[c + d*x]),x]
[Out] (b*Sec[c + d*x]^2)/(2*d) + (a*Tan[c + d*x])/d
```

fricas [A] time = 1.26, size = 30, normalized size = 1.07

$$\frac{2\,a\cos\left(dx+c\right)\sin\left(dx+c\right)+b}{2\,d\cos\left(dx+c\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a*cos(d*x+c)+b*sin(d*x+c)),x, algorithm="fricas")
[Out] 1/2*(2*a*cos(d*x + c)*sin(d*x + c) + b)/(d*cos(d*x + c)^2)
```

giac [A] time = 0.26, size = 25, normalized size = 0.89

$$\frac{b\tan\left(dx+c\right)^2+2\,a\tan\left(dx+c\right)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a*cos(d*x+c)+b*sin(d*x+c)),x, algorithm="giac")
[Out] 1/2*(b*tan(d*x + c)^2 + 2*a*tan(d*x + c))/d
```

maple [A] time = 1.70, size = 25, normalized size = 0.89

$$\frac{a\tan\left(dx+c\right)+\frac{b}{2\cos\left(dx+c\right)^{2}}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^3*(a*cos(d*x+c)+b*sin(d*x+c)),x)
[Out] 1/d*(a*tan(d*x+c)+1/2*b/cos(d*x+c)^2)
```

maxima [A] time = 0.32, size = 30, normalized size = 1.07

$$\frac{2\,a\tan\left(dx+c\right)-\frac{b}{\sin\left(dx+c\right)^{2}-1}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a*cos(d*x+c)+b*sin(d*x+c)),x, algorithm="maxima")
[Out] 1/2*(2*a*tan(d*x + c) - b/(sin(d*x + c)^2 - 1))/d
```

mupad [B] time = 0.40, size = 23, normalized size = 0.82

$$\frac{\tan\left(c+d\,x\right)\,\left(2\,a+b\tan\left(c+d\,x\right)\right)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*cos(c + d*x) + b*sin(c + d*x))/cos(c + d*x)^3,x)
[Out] (tan(c + d*x)*(2*a + b*tan(c + d*x)))/(2*d)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a\cos\left(c+dx\right)+b\sin\left(c+dx\right)\right)\sec^{3}\left(c+dx\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**3*(a*cos(d*x+c)+b*sin(d*x+c)),x)
[Out] Integral((a*cos(c + d*x) + b*sin(c + d*x))*sec(c + d*x)**3, x)
```

3.39 $\int \sec^4(c+dx)(a\cos(c+dx)+b\sin(c+dx))\,dx$

Optimal. Leaf size=52

$$\frac{a\tanh^{-1}(\sin(c+dx))}{2d}+\frac{a\tan(c+dx)\sec(c+dx)}{2d}+\frac{b\sec^3(c+dx)}{3d}$$

```
[Out] 1/2*a*arctanh(sin(d*x+c))/d+1/3*b*sec(d*x+c)^3/d+1/2*a*sec(d*x+c)*tan(d*x+c
)/d
```

Rubi [A] time = 0.07, antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {3090, 3768, 3770, 2606, 30}

$$\frac{a\tanh^{-1}(\sin(c+dx))}{2d}+\frac{a\tan(c+dx)\sec(c+dx)}{2d}+\frac{b\sec^3(c+dx)}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^4*(a*Cos[c + d*x] + b*Sin[c + d*x]),x]
```

```
[Out] (a*ArcTanh[Sin[c + d*x]])/(2*d) + (b*Sec[c + d*x]^3)/(3*d) + (a*Sec[c + d*x
]*Tan[c + d*x])/(2*d)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 3090

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_.), x_Symbol] :> Int[ExpandTrig[cos[c + d*x]^m*(a
*cos[c + d*x] + b*sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d}, x] && Inte
gerQ[m] && IGtQ[n, 0]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \sec^4(c+dx)(a\cos(c+dx)+b\sin(c+dx))\,dx &= \int \left(a\sec^3(c+dx)+b\sec^3(c+dx)\tan(c+dx)\right)\,dx\\ &= a\int \sec^3(c+dx)\,dx+b\int \sec^3(c+dx)\tan(c+dx)\,dx\\ &= \frac{a\sec(c+dx)\tan(c+dx)}{2d}+\frac{1}{2}a\int \sec(c+dx)\,dx+\frac{b\,\mathrm{Subst}\left(\right.}{}\\ &= \frac{a\tanh^{-1}(\sin(c+dx))}{2d}+\frac{b\sec^3(c+dx)}{3d}+\frac{a\sec(c+dx)\tan(c}{2d}\end{aligned}$$

Mathematica [A] time = 0.02, size = 52, normalized size = 1.00

$$\frac{a\tanh^{-1}(\sin(c+dx))}{2d}+\frac{a\tan(c+dx)\sec(c+dx)}{2d}+\frac{b\sec^3(c+dx)}{3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^4*(a*Cos[c + d*x] + b*Sin[c + d*x]),x]
```

```
[Out] (a*ArcTanh[Sin[c + d*x]])/(2*d) + (b*Sec[c + d*x]^3)/(3*d) + (a*Sec[c + d*x
]*Tan[c + d*x])/(2*d)
```

fricas [A] time = 2.16, size = 74, normalized size = 1.42

$$\frac{3\,a\cos(dx+c)^3\log(\sin(dx+c)+1)-3\,a\cos(dx+c)^3\log(-\sin(dx+c)+1)+6\,a\cos(dx+c)\sin(dx+c)+}{12\,d\cos(dx+c)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(a*cos(d*x+c)+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/12*(3*a*cos(d*x + c)^3*log(sin(d*x + c) + 1) - 3*a*cos(d*x + c)^3*log(-si
n(d*x + c) + 1) + 6*a*cos(d*x + c)*sin(d*x + c) + 4*b)/(d*cos(d*x + c)^3)
```

giac [B] time = 4.02, size = 99, normalized size = 1.90

$$\frac{3\,a\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-3\,a\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)+\frac{2\left(3\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}-6\,b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{4}-3\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\right.}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-1\right)^{3}}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(a*cos(d*x+c)+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/6*(3*a*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 3*a*log(abs(tan(1/2*d*x + 1/2
*c) - 1)) + 2*(3*a*tan(1/2*d*x + 1/2*c)^5 - 6*b*tan(1/2*d*x + 1/2*c)^4 - 3*
a*tan(1/2*d*x + 1/2*c) - 2*b)/(tan(1/2*d*x + 1/2*c)^2 - 1)^3)/d
```

maple [A] time = 2.00, size = 54, normalized size = 1.04

$$\frac{a\sec\left(dx+c\right)\tan\left(dx+c\right)}{2d}+\frac{a\ln\left(\sec\left(dx+c\right)+\tan\left(dx+c\right)\right)}{2d}+\frac{b}{3d\cos\left(dx+c\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^4*(a*cos(d*x+c)+b*sin(d*x+c)),x)
```

```
[Out] 1/2*a*sec(d*x+c)*tan(d*x+c)/d+1/2/d*a*ln(sec(d*x+c)+tan(d*x+c))+1/3/d*b/cos
(d*x+c)^3
```

maxima [A] time = 0.32, size = 61, normalized size = 1.17

$$-\frac{3\,a\left(\frac{2\,\sin\left(dx+c\right)}{\sin\left(dx+c\right)^{2}-1}-\log\left(\sin\left(dx+c\right)+1\right)+\log\left(\sin\left(dx+c\right)-1\right)\right)-\frac{4\,b}{\cos\left(dx+c\right)^{3}}}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(a*cos(d*x+c)+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/12*(3*a*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) - log(sin(d*x + c) + 1) + l
og(sin(d*x + c) - 1)) - 4*b/cos(d*x + c)^3)/d
```

mupad [B] time = 2.21, size = 105, normalized size = 2.02

$$\frac{a\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{d}-\frac{-a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{5}+2\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{4}+a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+\frac{2\,b}{3}}{d\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{6}-3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{4}+3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{2}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*cos(c + d*x) + b*sin(c + d*x))/cos(c + d*x)^4,x)
```

```
[Out] (a*atanh(tan(c/2 + (d*x)/2)))/d - ((2*b)/3 + a*tan(c/2 + (d*x)/2) - a*tan(c
/2 + (d*x)/2)^5 + 2*b*tan(c/2 + (d*x)/2)^4)/(d*(3*tan(c/2 + (d*x)/2)^2 - 3*
tan(c/2 + (d*x)/2)^4 + tan(c/2 + (d*x)/2)^6 - 1))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a\cos(c+dx)+b\sin(c+dx))\sec^4(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**4*(a*cos(d*x+c)+b*sin(d*x+c)),x)
```

```
[Out] Integral((a*cos(c + d*x) + b*sin(c + d*x))*sec(c + d*x)**4, x)
```

3.40 $\int \sec^5(c+dx)(a\cos(c+dx)+b\sin(c+dx))\,dx$

Optimal. Leaf size=44

$$\frac{a\tan^3(c+dx)}{3d}+\frac{a\tan(c+dx)}{d}+\frac{b\sec^4(c+dx)}{4d}$$

```
[Out] 1/4*b*sec(d*x+c)^4/d+a*tan(d*x+c)/d+1/3*a*tan(d*x+c)^3/d
```

Rubi [A] time = 0.06, antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {3090, 3767, 2606, 30}

$$\frac{a\tan^3(c+dx)}{3d}+\frac{a\tan(c+dx)}{d}+\frac{b\sec^4(c+dx)}{4d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^5*(a*Cos[c + d*x] + b*Sin[c + d*x]),x]
```

```
[Out] (b*Sec[c + d*x]^4)/(4*d) + (a*Tan[c + d*x])/d + (a*Tan[c + d*x]^3)/(3*d)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 3090

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_.), x_Symbol] :> Int[ExpandTrig[cos[c + d*x]^m*(a
*cos[c + d*x] + b*sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d}, x] && Inte
gerQ[m] && IGtQ[n, 0]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
```

```
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\begin{aligned}\int \sec^5(c+dx)(a\cos(c+dx)+b\sin(c+dx))\,dx &= \int \left(a\sec^4(c+dx)+b\sec^4(c+dx)\tan(c+dx)\right)\,dx\\ &= a\int \sec^4(c+dx)\,dx+b\int \sec^4(c+dx)\tan(c+dx)\,dx\\ &= -\frac{a\,\text{Subst}\left(\int \left(1+x^2\right)\,dx,x,-\tan(c+dx)\right)}{d}+\frac{b\,\text{Subst}\left(\int x^3\,dx\right.}{d}\\ &= \frac{b\sec^4(c+dx)}{4d}+\frac{a\tan(c+dx)}{d}+\frac{a\tan^3(c+dx)}{3d}\end{aligned}$$

Mathematica [A] time = 0.09, size = 41, normalized size = 0.93

$$\frac{a\left(\frac{1}{3}\tan^3(c+dx)+\tan(c+dx)\right)}{d}+\frac{b\sec^4(c+dx)}{4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^5*(a*Cos[c + d*x] + b*Sin[c + d*x]),x]
```

```
[Out] (b*Sec[c + d*x]^4)/(4*d) + (a*(Tan[c + d*x] + Tan[c + d*x]^3/3))/d
```

fricas [A] time = 0.63, size = 45, normalized size = 1.02

$$\frac{4\left(2\,a\cos\left(dx+c\right)^3+a\cos\left(dx+c\right)\right)\sin\left(dx+c\right)+3\,b}{12\,d\cos\left(dx+c\right)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*(a*cos(d*x+c)+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/12*(4*(2*a*cos(d*x + c)^3 + a*cos(d*x + c))*sin(d*x + c) + 3*b)/(d*cos(d*
x + c)^4)
```

giac [A] time = 0.25, size = 48, normalized size = 1.09

$$\frac{3\,b\tan\left(dx+c\right)^4+4\,a\tan\left(dx+c\right)^3+6\,b\tan\left(dx+c\right)^2+12\,a\tan\left(dx+c\right)}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*(a*cos(d*x+c)+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/12*(3*b*tan(d*x + c)^4 + 4*a*tan(d*x + c)^3 + 6*b*tan(d*x + c)^2 + 12*a*t
an(d*x + c))/d
```

maple [A] time = 1.75, size = 38, normalized size = 0.86

$$\frac{-a\left(-\frac{2}{3}-\frac{\left(\sec^2(dx+c)\right)}{3}\right)\tan(dx+c)+\frac{b}{4\cos(dx+c)^4}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^5*(a*cos(d*x+c)+b*sin(d*x+c)),x)
```

```
[Out] 1/d*(-a*(-2/3-1/3*sec(d*x+c)^2)*tan(d*x+c)+1/4*b/cos(d*x+c)^4)
```

maxima [A] time = 0.33, size = 41, normalized size = 0.93

$$\frac{4\left(\tan(dx+c)^3+3\tan(dx+c)\right)a+\frac{3b}{\left(\sin(dx+c)^2-1\right)^2}}{12d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*(a*cos(d*x+c)+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/12*(4*(tan(d*x + c)^3 + 3*tan(d*x + c))*a + 3*b/(sin(d*x + c)^2 - 1)^2)/d
```

mupad [B] time = 0.52, size = 40, normalized size = 0.91

$$\frac{\frac{b}{4}+\frac{a\sin(2c+2dx)}{3}+\frac{a\sin(4c+4dx)}{12}}{d\cos(c+dx)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*cos(c + d*x) + b*sin(c + d*x))/cos(c + d*x)^5,x)
```

```
[Out] (b/4 + (a*sin(2*c + 2*d*x))/3 + (a*sin(4*c + 4*d*x))/12)/(d*cos(c + d*x)^4)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int(a\cos(c+dx)+b\sin(c+dx))\sec^5(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**5*(a*cos(d*x+c)+b*sin(d*x+c)),x)
```

```
[Out] Integral((a*cos(c + d*x) + b*sin(c + d*x))*sec(c + d*x)**5, x)
```

3.41 $\int \sec^6(c+dx)(a\cos(c+dx)+b\sin(c+dx))\,dx$

Optimal. Leaf size=74

$$\frac{3a\tanh^{-1}(\sin(c+dx))}{8d}+\frac{a\tan(c+dx)\sec^3(c+dx)}{4d}+\frac{3a\tan(c+dx)\sec(c+dx)}{8d}+\frac{b\sec^5(c+dx)}{5d}$$

```
[Out] 3/8*a*arctanh(sin(d*x+c))/d+1/5*b*sec(d*x+c)^5/d+3/8*a*sec(d*x+c)*tan(d*x+c
)/d+1/4*a*sec(d*x+c)^3*tan(d*x+c)/d
```

Rubi [A] time = 0.08, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {3090, 3768, 3770, 2606, 30}

$$\frac{3a\tanh^{-1}(\sin(c+dx))}{8d}+\frac{a\tan(c+dx)\sec^3(c+dx)}{4d}+\frac{3a\tan(c+dx)\sec(c+dx)}{8d}+\frac{b\sec^5(c+dx)}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^6*(a*Cos[c + d*x] + b*Sin[c + d*x]),x]
```

```
[Out] (3*a*ArcTanh[Sin[c + d*x]])/(8*d) + (b*Sec[c + d*x]^5)/(5*d) + (3*a*Sec[c +
 d*x]*Tan[c + d*x])/(8*d) + (a*Sec[c + d*x]^3*Tan[c + d*x])/(4*d)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 3090

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_.), x_Symbol] :> Int[ExpandTrig[cos[c + d*x]^m*(a
*cos[c + d*x] + b*sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d}, x] && Inte
gerQ[m] && IGtQ[n, 0]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \sec^6(c+dx)(a\cos(c+dx)+b\sin(c+dx))\,dx = \int \left(a\sec^5(c+dx)+b\sec^5(c+dx)\tan(c+dx)\right)\,dx$$
$$= a\int \sec^5(c+dx)\,dx + b\int \sec^5(c+dx)\tan(c+dx)\,dx$$
$$= \frac{a\sec^3(c+dx)\tan(c+dx)}{4d} + \frac{1}{4}(3a)\int \sec^3(c+dx)\,dx + \frac{b\,\mathrm{Su}}{}$$
$$= \frac{b\sec^5(c+dx)}{5d} + \frac{3a\sec(c+dx)\tan(c+dx)}{8d} + \frac{a\sec^3(c+dx}{4}$$
$$= \frac{3a\tanh^{-1}(\sin(c+dx))}{8d} + \frac{b\sec^5(c+dx)}{5d} + \frac{3a\sec(c+dx)\,\mathrm{ta}}{8d}$$

Mathematica [A] time = 0.21, size = 68, normalized size = 0.92

$$\frac{a\tan(c+dx)\sec^3(c+dx)}{4d}+\frac{3a\left(\tanh^{-1}(\sin(c+dx))+\tan(c+dx)\sec(c+dx)\right)}{8d}+\frac{b\sec^5(c+dx)}{5d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^6*(a*Cos[c + d*x] + b*Sin[c + d*x]),x]
```

```
[Out] (b*Sec[c + d*x]^5)/(5*d) + (a*Sec[c + d*x]^3*Tan[c + d*x])/(4*d) + (3*a*(Ar
cTanh[Sin[c + d*x]] + Sec[c + d*x]*Tan[c + d*x]))/(8*d)
```

fricas [A] time = 0.94, size = 88, normalized size = 1.19

$$\frac{15\,a\cos(dx+c)^5\log(\sin(dx+c)+1)-15\,a\cos(dx+c)^5\log(-\sin(dx+c)+1)+10\left(3\,a\cos(dx+c)^3+2\,a\right.}{80\,d\cos(dx+c)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^6*(a*cos(d*x+c)+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/80*(15*a*cos(d*x + c)^5*log(sin(d*x + c) + 1) - 15*a*cos(d*x + c)^5*log(-
sin(d*x + c) + 1) + 10*(3*a*cos(d*x + c)^3 + 2*a*cos(d*x + c))*sin(d*x + c)
 + 16*b)/(d*cos(d*x + c)^5)
```

giac [B] time = 4.69, size = 141, normalized size = 1.91

$$\frac{15\,a\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-15\,a\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)+\frac{2\left(25\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{9}-40\,b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{8}-10\,a\tan\left(\frac{1}{2}\,d\right.}{\left(t\right.}}{40\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^6*(a*cos(d*x+c)+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/40*(15*a*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 15*a*log(abs(tan(1/2*d*x +
1/2*c) - 1)) + 2*(25*a*tan(1/2*d*x + 1/2*c)^9 - 40*b*tan(1/2*d*x + 1/2*c)^8
 - 10*a*tan(1/2*d*x + 1/2*c)^7 - 80*b*tan(1/2*d*x + 1/2*c)^4 + 10*a*tan(1/2
*d*x + 1/2*c)^3 - 25*a*tan(1/2*d*x + 1/2*c) - 8*b)/(tan(1/2*d*x + 1/2*c)^2
- 1)^5)/d
```

maple [A] time = 11.31, size = 74, normalized size = 1.00

$$\frac{a\left(\sec^{3}\left(dx+c\right)\right)\tan\left(dx+c\right)}{4d}+\frac{3a\sec\left(dx+c\right)\tan\left(dx+c\right)}{8d}+\frac{3a\ln\left(\sec\left(dx+c\right)+\tan\left(dx+c\right)\right)}{8d}+\frac{b}{5d\cos\left(dx+c\right)^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^6*(a*cos(d*x+c)+b*sin(d*x+c)),x)
```

```
[Out] 1/4*a*sec(d*x+c)^3*tan(d*x+c)/d+3/8*a*sec(d*x+c)*tan(d*x+c)/d+3/8/d*a*ln(se
c(d*x+c)+tan(d*x+c))+1/5/d*b/cos(d*x+c)^5
```

maxima [A] time = 0.33, size = 86, normalized size = 1.16

$$-\frac{5\,a\left(\frac{2\left(3\,\sin\left(dx+c\right)^{3}-5\,\sin\left(dx+c\right)\right)}{\sin\left(dx+c\right)^{4}-2\,\sin\left(dx+c\right)^{2}+1}-3\,\log\left(\sin\left(dx+c\right)+1\right)+3\,\log\left(\sin\left(dx+c\right)-1\right)\right)-\frac{16\,b}{\cos\left(dx+c\right)^{5}}}{80\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^6*(a*cos(d*x+c)+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/80*(5*a*(2*(3*sin(d*x + c)^3 - 5*sin(d*x + c))/(sin(d*x + c)^4 - 2*sin(d
*x + c)^2 + 1) - 3*log(sin(d*x + c) + 1) + 3*log(sin(d*x + c) - 1)) - 16*b/
cos(d*x + c)^5)/d
```

mupad [B] time = 4.12, size = 175, normalized size = 2.36

$$\frac{3\,a\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{4\,d}-\frac{-\frac{5\,a\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^9}{4}+2\,b\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8+\frac{a\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7}{2}+4\,b\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4-\frac{a\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{2}+\cdots}{d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{10}-5\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8+10\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6-10\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4+5\,\tan\cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*cos(c + d*x) + b*sin(c + d*x))/cos(c + d*x)^6,x)
```

```
[Out] (3*a*atanh(tan(c/2 + (d*x)/2)))/(4*d) - ((2*b)/5 + (5*a*tan(c/2 + (d*x)/2))
/4 - (a*tan(c/2 + (d*x)/2)^3)/2 + (a*tan(c/2 + (d*x)/2)^7)/2 - (5*a*tan(c/2
 + (d*x)/2)^9)/4 + 4*b*tan(c/2 + (d*x)/2)^4 + 2*b*tan(c/2 + (d*x)/2)^8)/(d*
(5*tan(c/2 + (d*x)/2)^2 - 10*tan(c/2 + (d*x)/2)^4 + 10*tan(c/2 + (d*x)/2)^6
 - 5*tan(c/2 + (d*x)/2)^8 + tan(c/2 + (d*x)/2)^10 - 1))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**6*(a*cos(d*x+c)+b*sin(d*x+c)),x)
```

```
[Out] Timed out
```

3.42 $\int \sec^7(c+dx)(a\cos(c+dx)+b\sin(c+dx))\,dx$

Optimal. Leaf size=60

$$\frac{a\tan^5(c+dx)}{5d}+\frac{2a\tan^3(c+dx)}{3d}+\frac{a\tan(c+dx)}{d}+\frac{b\sec^6(c+dx)}{6d}$$

```
[Out] 1/6*b*sec(d*x+c)^6/d+a*tan(d*x+c)/d+2/3*a*tan(d*x+c)^3/d+1/5*a*tan(d*x+c)^5
/d
```

Rubi [A] time = 0.07, antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {3090, 3767, 2606, 30}

$$\frac{a\tan^5(c+dx)}{5d}+\frac{2a\tan^3(c+dx)}{3d}+\frac{a\tan(c+dx)}{d}+\frac{b\sec^6(c+dx)}{6d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^7*(a*Cos[c + d*x] + b*Sin[c + d*x]),x]
```

```
[Out] (b*Sec[c + d*x]^6)/(6*d) + (a*Tan[c + d*x])/d + (2*a*Tan[c + d*x]^3)/(3*d)
+ (a*Tan[c + d*x]^5)/(5*d)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 && !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 3090

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_.), x_Symbol] :> Int[ExpandTrig[cos[c + d*x]^m*(a
*cos[c + d*x] + b*sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d}, x] && Inte
gerQ[m] && IGtQ[n, 0]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
```

```
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\int \sec^7(c+dx)(a\cos(c+dx)+b\sin(c+dx))\,dx = \int \left(a\sec^6(c+dx)+b\sec^6(c+dx)\tan(c+dx)\right)\,dx$$
$$= a\int \sec^6(c+dx)\,dx + b\int \sec^6(c+dx)\tan(c+dx)\,dx$$
$$= -\frac{a\,\text{Subst}\left(\int \left(1+2x^2+x^4\right)\,dx, x, -\tan(c+dx)\right)}{d} + \frac{b\,\text{Subst}}{}$$
$$= \frac{b\sec^6(c+dx)}{6d} + \frac{a\tan(c+dx)}{d} + \frac{2a\tan^3(c+dx)}{3d} + \frac{a\tan^5(c}{5d}$$

Mathematica [A] time = 0.16, size = 53, normalized size = 0.88

$$\frac{a\left(\frac{1}{5}\tan^5(c+dx)+\frac{2}{3}\tan^3(c+dx)+\tan(c+dx)\right)}{d}+\frac{b\sec^6(c+dx)}{6d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^7*(a*Cos[c + d*x] + b*Sin[c + d*x]),x]
```

```
[Out] (b*Sec[c + d*x]^6)/(6*d) + (a*(Tan[c + d*x] + (2*Tan[c + d*x]^3)/3 + Tan[c
+ d*x]^5/5))/d
```

fricas [A] time = 0.72, size = 57, normalized size = 0.95

$$\frac{2\left(8\,a\cos\left(dx+c\right)^5+4\,a\cos\left(dx+c\right)^3+3\,a\cos\left(dx+c\right)\right)\sin\left(dx+c\right)+5\,b}{30\,d\cos\left(dx+c\right)^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^7*(a*cos(d*x+c)+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/30*(2*(8*a*cos(d*x + c)^5 + 4*a*cos(d*x + c)^3 + 3*a*cos(d*x + c))*sin(d*
x + c) + 5*b)/(d*cos(d*x + c)^6)
```

giac [A] time = 0.24, size = 70, normalized size = 1.17

$$\frac{5\,b\tan\left(dx+c\right)^6+6\,a\tan\left(dx+c\right)^5+15\,b\tan\left(dx+c\right)^4+20\,a\tan\left(dx+c\right)^3+15\,b\tan\left(dx+c\right)^2+30\,a\tan\left(dx-\right.}{30\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^7*(a*cos(d*x+c)+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/30*(5*b*tan(d*x + c)^6 + 6*a*tan(d*x + c)^5 + 15*b*tan(d*x + c)^4 + 20*a*
tan(d*x + c)^3 + 15*b*tan(d*x + c)^2 + 30*a*tan(d*x + c))/d
```

maple [A] time = 1.95, size = 48, normalized size = 0.80

$$\frac{-a\left(-\frac{8}{15}-\frac{\left(\sec^{4}(dx+c)\right)}{5}-\frac{4\left(\sec^{2}(dx+c)\right)}{15}\right)\tan\left(dx+c\right)+\frac{b}{6\cos(dx+c)^{6}}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^7*(a*cos(d*x+c)+b*sin(d*x+c)),x)
```

```
[Out] 1/d*(-a*(-8/15-1/5*sec(d*x+c)^4-4/15*sec(d*x+c)^2)*tan(d*x+c)+1/6*b/cos(d*x
+c)^6)
```

maxima [A] time = 0.33, size = 53, normalized size = 0.88

$$\frac{2\left(3\tan\left(dx+c\right)^{5}+10\tan\left(dx+c\right)^{3}+15\tan\left(dx+c\right)\right)a-\frac{5b}{\left(\sin(dx+c)^{2}-1\right)^{3}}}{30d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^7*(a*cos(d*x+c)+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/30*(2*(3*tan(d*x + c)^5 + 10*tan(d*x + c)^3 + 15*tan(d*x + c))*a - 5*b/(s
in(d*x + c)^2 - 1)^3)/d
```

mupad [B] time = 0.67, size = 65, normalized size = 1.08

$$\frac{\frac{8a\sin(c+dx)\cos(c+dx)^{5}}{15}+\frac{4a\sin(c+dx)\cos(c+dx)^{3}}{15}+\frac{a\sin(c+dx)\cos(c+dx)}{5}+\frac{b}{6}}{d\cos\left(c+dx\right)^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*cos(c + d*x) + b*sin(c + d*x))/cos(c + d*x)^7,x)
```

```
[Out] (b/6 + (a*cos(c + d*x)*sin(c + d*x))/5 + (4*a*cos(c + d*x)^3*sin(c + d*x))/
15 + (8*a*cos(c + d*x)^5*sin(c + d*x))/15)/(d*cos(c + d*x)^6)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**7*(a*cos(d*x+c)+b*sin(d*x+c)),x)
```

```
[Out] Timed out
```

3.43 $\int \cos^5(c+dx)(a\cos(c+dx)+b\sin(c+dx))^2\,dx$

Optimal. Leaf size=137

$$-\frac{a^2\sin^7(c+dx)}{7d}+\frac{3a^2\sin^5(c+dx)}{5d}-\frac{a^2\sin^3(c+dx)}{d}+\frac{a^2\sin(c+dx)}{d}-\frac{2ab\cos^7(c+dx)}{7d}+\frac{b^2\sin^7(c+dx)}{7d}-\frac{2b^2\sin^5}{5}$$

```
[Out] -2/7*a*b*cos(d*x+c)^7/d+a^2*sin(d*x+c)/d-a^2*sin(d*x+c)^3/d+1/3*b^2*sin(d*x
+c)^3/d+3/5*a^2*sin(d*x+c)^5/d-2/5*b^2*sin(d*x+c)^5/d-1/7*a^2*sin(d*x+c)^7/
d+1/7*b^2*sin(d*x+c)^7/d
```

Rubi [A] time = 0.14, antiderivative size = 137, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {3090, 2633, 2565, 30, 2564, 270}

$$-\frac{a^2\sin^7(c+dx)}{7d}+\frac{3a^2\sin^5(c+dx)}{5d}-\frac{a^2\sin^3(c+dx)}{d}+\frac{a^2\sin(c+dx)}{d}-\frac{2ab\cos^7(c+dx)}{7d}+\frac{b^2\sin^7(c+dx)}{7d}-\frac{2b^2\sin^5}{5}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^5*(a*Cos[c + d*x] + b*Sin[c + d*x])^2,x]
```

```
[Out] (-2*a*b*Cos[c + d*x]^7)/(7*d) + (a^2*Sin[c + d*x])/d - (a^2*Sin[c + d*x]^3)
/d + (b^2*Sin[c + d*x]^3)/(3*d) + (3*a^2*Sin[c + d*x]^5)/(5*d) - (2*b^2*Sin
[c + d*x]^5)/(5*d) - (a^2*Sin[c + d*x]^7)/(7*d) + (b^2*Sin[c + d*x]^7)/(7*d
)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2564

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 2565

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> -Dist[(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 3090

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_.), x_Symbol] :> Int[ExpandTrig[cos[c + d*x]^m*(a
*cos[c + d*x] + b*sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d}, x] && Inte
gerQ[m] && IGtQ[n, 0]
```

Rubi steps

$$\int \cos^5(c+dx)(a\cos(c+dx)+b\sin(c+dx))^2\,dx = \int \left(a^2\cos^7(c+dx)+2ab\cos^6(c+dx)\sin(c+dx)+b^2\cos$$

$$= a^2\int \cos^7(c+dx)\,dx+(2ab)\int \cos^6(c+dx)\sin(c+dx)\,dx$$

$$= -\frac{a^2\,\mathrm{Subst}\left(\int \left(1-3x^2+3x^4-x^6\right)\,dx,x,-\sin(c+dx)\right)}{d}-$$

$$= -\frac{2ab\cos^7(c+dx)}{7d}+\frac{a^2\sin(c+dx)}{d}-\frac{a^2\sin^3(c+dx)}{d}+\frac{3a}{}$$

$$= -\frac{2ab\cos^7(c+dx)}{7d}+\frac{a^2\sin(c+dx)}{d}-\frac{a^2\sin^3(c+dx)}{d}+\frac{b^2}{}$$

Mathematica [A] time = 0.38, size = 154, normalized size = 1.12

$$-\frac{-3675a^2\sin(c+dx)-735a^2\sin(3(c+dx))-147a^2\sin(5(c+dx))-15a^2\sin(7(c+dx))+1050ab\cos(c+dx)}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^5*(a*Cos[c + d*x] + b*Sin[c + d*x])^2,x]
```

```
[Out] -1/6720*(1050*a*b*Cos[c + d*x] + 630*a*b*Cos[3*(c + d*x)] + 210*a*b*Cos[5*(
c + d*x)] + 30*a*b*Cos[7*(c + d*x)] - 3675*a^2*Sin[c + d*x] - 525*b^2*Sin[c
```

```
 + d*x] - 735*a^2*Sin[3*(c + d*x)] + 35*b^2*Sin[3*(c + d*x)] - 147*a^2*Sin[
5*(c + d*x)] + 63*b^2*Sin[5*(c + d*x)] - 15*a^2*Sin[7*(c + d*x)] + 15*b^2*S
in[7*(c + d*x)])/d
```

fricas [A] time = 0.98, size = 94, normalized size = 0.69

$$-\frac{30\,ab\cos\left(dx+c\right)^{7}-\left(15\left(a^{2}-b^{2}\right)\cos\left(dx+c\right)^{6}+3\left(6\,a^{2}+b^{2}\right)\cos\left(dx+c\right)^{4}+4\left(6\,a^{2}+b^{2}\right)\cos\left(dx+c\right)^{2}+48\,a}{105\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(a*cos(d*x+c)+b*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -1/105*(30*a*b*cos(d*x + c)^7 - (15*(a^2 - b^2)*cos(d*x + c)^6 + 3*(6*a^2 +
 b^2)*cos(d*x + c)^4 + 4*(6*a^2 + b^2)*cos(d*x + c)^2 + 48*a^2 + 8*b^2)*sin
(d*x + c))/d
```

giac [A] time = 0.87, size = 155, normalized size = 1.13

$$-\frac{ab\cos\left(7\,dx+7\,c\right)}{224\,d}-\frac{ab\cos\left(5\,dx+5\,c\right)}{32\,d}-\frac{3\,ab\cos\left(3\,dx+3\,c\right)}{32\,d}-\frac{5\,ab\cos\left(dx+c\right)}{32\,d}+\frac{\left(a^{2}-b^{2}\right)\sin\left(7\,dx+7\,c\right)}{448\,d}+\frac{\left(7\,a^{2}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(a*cos(d*x+c)+b*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/224*a*b*cos(7*d*x + 7*c)/d - 1/32*a*b*cos(5*d*x + 5*c)/d - 3/32*a*b*cos(
3*d*x + 3*c)/d - 5/32*a*b*cos(d*x + c)/d + 1/448*(a^2 - b^2)*sin(7*d*x + 7*
c)/d + 1/320*(7*a^2 - 3*b^2)*sin(5*d*x + 5*c)/d + 1/192*(21*a^2 - b^2)*sin(
3*d*x + 3*c)/d + 5/64*(7*a^2 + b^2)*sin(d*x + c)/d
```

maple [A] time = 1.67, size = 108, normalized size = 0.79

$$\frac{b^{2}\left(-\frac{\sin(dx+c)\left(\cos^{6}(dx+c)\right)}{7}+\frac{\left(\frac{8}{3}+\cos^{4}(dx+c)+\frac{4\left(\cos^{2}(dx+c)\right)}{3}\right)\sin(dx+c)}{35}\right)-\frac{2ab\left(\cos^{7}(dx+c)\right)}{7}+\frac{a^{2}\left(\frac{16}{5}+\cos^{6}(dx+c)+\frac{6\left(\cos^{4}(dx+c)\right)}{5}+\frac{8\left(\cos^{2}(dx+c)\right)}{5}\right)}{7}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*(a*cos(d*x+c)+b*sin(d*x+c))^2,x)
```

```
[Out] 1/d*(b^2*(-1/7*sin(d*x+c)*cos(d*x+c)^6+1/35*(8/3+cos(d*x+c)^4+4/3*cos(d*x+c
)^2)*sin(d*x+c))-2/7*a*b*cos(d*x+c)^7+1/7*a^2*(16/5+cos(d*x+c)^6+6/5*cos(d*
x+c)^4+8/5*cos(d*x+c)^2)*sin(d*x+c))
```

maxima [A] time = 0.33, size = 98, normalized size = 0.72

$$-\frac{30\,ab\cos\left(dx+c\right)^{7}+3\left(5\,\sin\left(dx+c\right)^{7}-21\,\sin\left(dx+c\right)^{5}+35\,\sin\left(dx+c\right)^{3}-35\,\sin\left(dx+c\right)\right)a^{2}-\left(15\,\sin\left(dx\right.\right.}{105\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(a*cos(d*x+c)+b*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -1/105*(30*a*b*cos(d*x + c)^7 + 3*(5*sin(d*x + c)^7 - 21*sin(d*x + c)^5 + 3
5*sin(d*x + c)^3 - 35*sin(d*x + c))*a^2 - (15*sin(d*x + c)^7 - 42*sin(d*x +
 c)^5 + 35*sin(d*x + c)^3)*b^2)/d
```

mupad [B] time = 0.69, size = 176, normalized size = 1.28

$$\frac{16\,a^2\,\sin\left(c+d\,x\right)}{35\,d}+\frac{8\,b^2\,\sin\left(c+d\,x\right)}{105\,d}+\frac{8\,a^2\cos\left(c+d\,x\right)^2\,\sin\left(c+d\,x\right)}{35\,d}+\frac{6\,a^2\cos\left(c+d\,x\right)^4\,\sin\left(c+d\,x\right)}{35\,d}+\frac{a^2\cos\left(\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^5*(a*cos(c + d*x) + b*sin(c + d*x))^2,x)
```

```
[Out] (16*a^2*sin(c + d*x))/(35*d) + (8*b^2*sin(c + d*x))/(105*d) + (8*a^2*cos(c
+ d*x)^2*sin(c + d*x))/(35*d) + (6*a^2*cos(c + d*x)^4*sin(c + d*x))/(35*d)
+ (a^2*cos(c + d*x)^6*sin(c + d*x))/(7*d) + (4*b^2*cos(c + d*x)^2*sin(c + d
*x))/(105*d) + (b^2*cos(c + d*x)^4*sin(c + d*x))/(35*d) - (b^2*cos(c + d*x)
^6*sin(c + d*x))/(7*d) - (2*a*b*cos(c + d*x)^7)/(7*d)
```

sympy [A] time = 5.32, size = 187, normalized size = 1.36

$$\begin{cases} \frac{16a^2\sin^7(c+dx)}{35d}+\frac{8a^2\sin^5(c+dx)\cos^2(c+dx)}{5d}+\frac{2a^2\sin^3(c+dx)\cos^4(c+dx)}{d}+\frac{a^2\sin(c+dx)\cos^6(c+dx)}{d}-\frac{2ab\cos^7(c+dx)}{7d}+\frac{8b^2\sin^7(c+dx)}{105d} \\ x\left(a\cos(c)+b\sin(c)\right)^2\cos^5(c) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*(a*cos(d*x+c)+b*sin(d*x+c))**2,x)
```

```
[Out] Piecewise((16*a**2*sin(c + d*x)**7/(35*d) + 8*a**2*sin(c + d*x)**5*cos(c +
d*x)**2/(5*d) + 2*a**2*sin(c + d*x)**3*cos(c + d*x)**4/d + a**2*sin(c + d*x
)*cos(c + d*x)**6/d - 2*a*b*cos(c + d*x)**7/(7*d) + 8*b**2*sin(c + d*x)**7/
(105*d) + 4*b**2*sin(c + d*x)**5*cos(c + d*x)**2/(15*d) + b**2*sin(c + d*x)
**3*cos(c + d*x)**4/(3*d), Ne(d, 0)), (x*(a*cos(c) + b*sin(c))**2*cos(c)**5
, True))
```

3.44 $\int \cos^4(c+dx)(a\cos(c+dx)+b\sin(c+dx))^2\,dx$

Optimal. Leaf size=174

$$\frac{a^2\sin(c+dx)\cos^5(c+dx)}{6d}+\frac{5a^2\sin(c+dx)\cos^3(c+dx)}{24d}+\frac{5a^2\sin(c+dx)\cos(c+dx)}{16d}+\frac{5a^2x}{16}-\frac{ab\cos^6(c+dx)}{3d}-\frac{b}{}$$

```
[Out] 5/16*a^2*x+1/16*b^2*x-1/3*a*b*cos(d*x+c)^6/d+5/16*a^2*cos(d*x+c)*sin(d*x+c)
/d+1/16*b^2*cos(d*x+c)*sin(d*x+c)/d+5/24*a^2*cos(d*x+c)^3*sin(d*x+c)/d+1/24
*b^2*cos(d*x+c)^3*sin(d*x+c)/d+1/6*a^2*cos(d*x+c)^5*sin(d*x+c)/d-1/6*b^2*co
s(d*x+c)^5*sin(d*x+c)/d
```

Rubi [A] time = 0.17, antiderivative size = 174, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 6, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {3090, 2635, 8, 2565, 30, 2568}

$$\frac{a^2\sin(c+dx)\cos^5(c+dx)}{6d}+\frac{5a^2\sin(c+dx)\cos^3(c+dx)}{24d}+\frac{5a^2\sin(c+dx)\cos(c+dx)}{16d}+\frac{5a^2x}{16}-\frac{ab\cos^6(c+dx)}{3d}-\frac{b}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^4*(a*Cos[c + d*x] + b*Sin[c + d*x])^2,x]
```

```
[Out] (5*a^2*x)/16 + (b^2*x)/16 - (a*b*Cos[c + d*x]^6)/(3*d) + (5*a^2*Cos[c + d*x
]*Sin[c + d*x])/(16*d) + (b^2*Cos[c + d*x]*Sin[c + d*x])/(16*d) + (5*a^2*Co
s[c + d*x]^3*Sin[c + d*x])/(24*d) + (b^2*Cos[c + d*x]^3*Sin[c + d*x])/(24*d
) + (a^2*Cos[c + d*x]^5*Sin[c + d*x])/(6*d) - (b^2*Cos[c + d*x]^5*Sin[c + d
*x])/(6*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2565

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> -Dist[(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 2568

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> -Simp[(a*(b*Cos[e + f*x])^(n + 1)*(a*Sin[e + f*x])^(m - 1)
)/(b*f*(m + n)), x] + Dist[(a^2*(m - 1))/(m + n), Int[(b*Cos[e + f*x])^n*(a
*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1] &&
 NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 3090

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_.), x_Symbol] :> Int[ExpandTrig[cos[c + d*x]^m*(a
*cos[c + d*x] + b*sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d}, x] && Inte
gerQ[m] && IGtQ[n, 0]
```

Rubi steps

$$\int \cos^4(c+dx)(a\cos(c+dx)+b\sin(c+dx))^2\,dx = \int \left(a^2\cos^6(c+dx)+2ab\cos^5(c+dx)\sin(c+dx)+b^2\cos\right.$$

$$= a^2\int \cos^6(c+dx)\,dx+(2ab)\int \cos^5(c+dx)\sin(c+dx)\,dx$$

$$= \frac{a^2\cos^5(c+dx)\sin(c+dx)}{6d}-\frac{b^2\cos^5(c+dx)\sin(c+dx)}{6d}+$$

$$= -\frac{ab\cos^6(c+dx)}{3d}+\frac{5a^2\cos^3(c+dx)\sin(c+dx)}{24d}+\frac{b^2\cos^3}{}$$

$$= -\frac{ab\cos^6(c+dx)}{3d}+\frac{5a^2\cos(c+dx)\sin(c+dx)}{16d}+\frac{b^2\cos(c}{}$$

$$= \frac{5a^2x}{16}+\frac{b^2x}{16}-\frac{ab\cos^6(c+dx)}{3d}+\frac{5a^2\cos(c+dx)\sin(c+dx}{16d}$$

Mathematica [A] time = 0.25, size = 147, normalized size = 0.84

$$\frac{\left(5a^2+b^2\right)(c+dx)}{16d}+\frac{\left(15a^2+b^2\right)\sin(2(c+dx))}{64d}+\frac{\left(3a^2-b^2\right)\sin(4(c+dx))}{64d}+\frac{\left(a^2-b^2\right)\sin(6(c+dx))}{192d}-\frac{5ab\cos(2}{32}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^4*(a*Cos[c + d*x] + b*Sin[c + d*x])^2,x]
```

```
[Out] ((5*a^2 + b^2)*(c + d*x))/(16*d) - (5*a*b*Cos[2*(c + d*x)])/(32*d) - (a*b*C
os[4*(c + d*x)])/(16*d) - (a*b*Cos[6*(c + d*x)])/(96*d) + ((15*a^2 + b^2)*S
in[2*(c + d*x)])/(64*d) + ((3*a^2 - b^2)*Sin[4*(c + d*x)])/(64*d) + ((a^2 -
 b^2)*Sin[6*(c + d*x)])/(192*d)
```

fricas [A] time = 0.50, size = 95, normalized size = 0.55

$$-\frac{16\,ab\cos\left(dx+c\right)^{6}-3\left(5\,a^{2}+b^{2}\right)dx-\left(8\left(a^{2}-b^{2}\right)\cos\left(dx+c\right)^{5}+2\left(5\,a^{2}+b^{2}\right)\cos\left(dx+c\right)^{3}+3\left(5\,a^{2}+b^{2}\right)\cos}{48\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a*cos(d*x+c)+b*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -1/48*(16*a*b*cos(d*x + c)^6 - 3*(5*a^2 + b^2)*d*x - (8*(a^2 - b^2)*cos(d*x
 + c)^5 + 2*(5*a^2 + b^2)*cos(d*x + c)^3 + 3*(5*a^2 + b^2)*cos(d*x + c))*si
n(d*x + c))/d
```

giac [A] time = 0.24, size = 132, normalized size = 0.76

$$\frac{1}{16}\left(5\,a^{2}+b^{2}\right)x-\frac{ab\cos\left(6\,dx+6\,c\right)}{96\,d}-\frac{ab\cos\left(4\,dx+4\,c\right)}{16\,d}-\frac{5\,ab\cos\left(2\,dx+2\,c\right)}{32\,d}+\frac{\left(a^{2}-b^{2}\right)\sin\left(6\,dx+6\,c\right)}{192\,d}+\frac{\left(3\,a^{2}-\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a*cos(d*x+c)+b*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/16*(5*a^2 + b^2)*x - 1/96*a*b*cos(6*d*x + 6*c)/d - 1/16*a*b*cos(4*d*x + 4
*c)/d - 5/32*a*b*cos(2*d*x + 2*c)/d + 1/192*(a^2 - b^2)*sin(6*d*x + 6*c)/d
+ 1/64*(3*a^2 - b^2)*sin(4*d*x + 4*c)/d + 1/64*(15*a^2 + b^2)*sin(2*d*x + 2
*c)/d
```

maple [A] time = 11.25, size = 118, normalized size = 0.68

$$\frac{b^{2}\left(-\frac{\left(\cos^{5}(dx+c)\right)\sin(dx+c)}{6}+\frac{\left(\cos^{3}(dx+c)+\frac{3\cos(dx+c)}{2}\right)\sin(dx+c)}{24}+\frac{dx}{16}+\frac{c}{16}\right)-\frac{ab\left(\cos^{6}(dx+c)\right)}{3}+a^{2}\left(\frac{\left(\cos^{5}(dx+c)+\frac{5\left(\cos^{3}(dx+c)\right)}{4}+\frac{15\cos(dx}{8}\right.}{6}\right.}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*(a*cos(d*x+c)+b*sin(d*x+c))^2,x)
```

```
[Out] 1/d*(b^2*(-1/6*cos(d*x+c)^5*sin(d*x+c)+1/24*(cos(d*x+c)^3+3/2*cos(d*x+c))*s
in(d*x+c)+1/16*d*x+1/16*c)-1/3*a*b*cos(d*x+c)^6+a^2*(1/6*(cos(d*x+c)^5+5/4*
cos(d*x+c)^3+15/8*cos(d*x+c))*sin(d*x+c)+5/16*d*x+5/16*c))
```

maxima [A] time = 0.34, size = 102, normalized size = 0.59

$$-\frac{64\,ab\cos\left(dx+c\right)^{6}+\left(4\,\sin\left(2\,dx+2\,c\right)^{3}-60\,dx-60\,c-9\,\sin\left(4\,dx+4\,c\right)-48\,\sin\left(2\,dx+2\,c\right)\right)a^{2}-\left(4\,\sin\left(2\right.\right.}{192\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a*cos(d*x+c)+b*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -1/192*(64*a*b*cos(d*x + c)^6 + (4*sin(2*d*x + 2*c)^3 - 60*d*x - 60*c - 9*s
in(4*d*x + 4*c) - 48*sin(2*d*x + 2*c))*a^2 - (4*sin(2*d*x + 2*c)^3 + 12*d*x
 + 12*c - 3*sin(4*d*x + 4*c))*b^2)/d
```

mupad [B] time = 0.61, size = 156, normalized size = 0.90

$$\frac{5\,a^2\,x}{16}+\frac{b^2\,x}{16}+\frac{5\,a^2\cos\left(c+d\,x\right)^3\,\sin\left(c+d\,x\right)}{24\,d}+\frac{a^2\cos\left(c+d\,x\right)^5\,\sin\left(c+d\,x\right)}{6\,d}+\frac{b^2\cos\left(c+d\,x\right)^3\,\sin\left(c+d\,x\right)}{24\,d}-\frac{b^2\,c}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^4*(a*cos(c + d*x) + b*sin(c + d*x))^2,x)
```

```
[Out] (5*a^2*x)/16 + (b^2*x)/16 + (5*a^2*cos(c + d*x)^3*sin(c + d*x))/(24*d) + (a
^2*cos(c + d*x)^5*sin(c + d*x))/(6*d) + (b^2*cos(c + d*x)^3*sin(c + d*x))/(
24*d) - (b^2*cos(c + d*x)^5*sin(c + d*x))/(6*d) - (a*b*cos(c + d*x)^6)/(3*d
) + (5*a^2*cos(c + d*x)*sin(c + d*x))/(16*d) + (b^2*cos(c + d*x)*sin(c + d*
x))/(16*d)
```

sympy [A] time = 3.48, size = 340, normalized size = 1.95

$$\begin{cases}\frac{5a^2x\sin^6(c+dx)}{16}+\frac{15a^2x\sin^4(c+dx)\cos^2(c+dx)}{16}+\frac{15a^2x\sin^2(c+dx)\cos^4(c+dx)}{16}+\frac{5a^2x\cos^6(c+dx)}{16}+\frac{5a^2\sin^5(c+dx)\cos(c+dx)}{16d}+\frac{5a^2\sin}{} \\ x\left(a\cos\left(c\right)+b\sin\left(c\right)\right)^2\cos^4\left(c\right)\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*(a*cos(d*x+c)+b*sin(d*x+c))**2,x)
```

```
[Out] Piecewise((5*a**2*x*sin(c + d*x)**6/16 + 15*a**2*x*sin(c + d*x)**4*cos(c +
d*x)**2/16 + 15*a**2*x*sin(c + d*x)**2*cos(c + d*x)**4/16 + 5*a**2*x*cos(c
+ d*x)**6/16 + 5*a**2*sin(c + d*x)**5*cos(c + d*x)/(16*d) + 5*a**2*sin(c +
d*x)**3*cos(c + d*x)**3/(6*d) + 11*a**2*sin(c + d*x)*cos(c + d*x)**5/(16*d)
 - a*b*cos(c + d*x)**6/(3*d) + b**2*x*sin(c + d*x)**6/16 + 3*b**2*x*sin(c +
 d*x)**4*cos(c + d*x)**2/16 + 3*b**2*x*sin(c + d*x)**2*cos(c + d*x)**4/16 +
 b**2*x*cos(c + d*x)**6/16 + b**2*sin(c + d*x)**5*cos(c + d*x)/(16*d) + b**
2*sin(c + d*x)**3*cos(c + d*x)**3/(6*d) - b**2*sin(c + d*x)*cos(c + d*x)**5
/(16*d), Ne(d, 0)), (x*(a*cos(c) + b*sin(c))**2*cos(c)**4, True))
```

3.45 $\int \cos^3(c+dx)(a\cos(c+dx)+b\sin(c+dx))^2\,dx$

Optimal. Leaf size=103

$$\frac{a^2\sin^5(c+dx)}{5d}-\frac{2a^2\sin^3(c+dx)}{3d}+\frac{a^2\sin(c+dx)}{d}-\frac{2ab\cos^5(c+dx)}{5d}-\frac{b^2\sin^5(c+dx)}{5d}+\frac{b^2\sin^3(c+dx)}{3d}$$

```
[Out] -2/5*a*b*cos(d*x+c)^5/d+a^2*sin(d*x+c)/d-2/3*a^2*sin(d*x+c)^3/d+1/3*b^2*sin
(d*x+c)^3/d+1/5*a^2*sin(d*x+c)^5/d-1/5*b^2*sin(d*x+c)^5/d
```

Rubi [A] time = 0.12, antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {3090, 2633, 2565, 30, 2564, 14}

$$\frac{a^2\sin^5(c+dx)}{5d}-\frac{2a^2\sin^3(c+dx)}{3d}+\frac{a^2\sin(c+dx)}{d}-\frac{2ab\cos^5(c+dx)}{5d}-\frac{b^2\sin^5(c+dx)}{5d}+\frac{b^2\sin^3(c+dx)}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^3*(a*Cos[c + d*x] + b*Sin[c + d*x])^2,x]
```

```
[Out] (-2*a*b*Cos[c + d*x]^5)/(5*d) + (a^2*Sin[c + d*x])/d - (2*a^2*Sin[c + d*x]^
3)/(3*d) + (b^2*Sin[c + d*x]^3)/(3*d) + (a^2*Sin[c + d*x]^5)/(5*d) - (b^2*S
in[c + d*x]^5)/(5*d)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2564

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] && !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 2565

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> -Dist[(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
```

```
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 3090

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_.), x_Symbol] :> Int[ExpandTrig[cos[c + d*x]^m*(a
*cos[c + d*x] + b*sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d}, x] && Inte
gerQ[m] && IGtQ[n, 0]
```

Rubi steps

$$\int \cos^3(c+dx)(a\cos(c+dx)+b\sin(c+dx))^2\,dx = \int \left(a^2\cos^5(c+dx)+2ab\cos^4(c+dx)\sin(c+dx)+b^2\cos\right.$$

$$= a^2\int \cos^5(c+dx)\,dx + (2ab)\int \cos^4(c+dx)\sin(c+dx)\,dx$$

$$= -\frac{a^2\,\mathrm{Subst}\left(\int \left(1-2x^2+x^4\right)dx, x, -\sin(c+dx)\right)}{d} - \frac{(2ab)\,\mathrm{S}}{}$$

$$= -\frac{2ab\cos^5(c+dx)}{5d}+\frac{a^2\sin(c+dx)}{d}-\frac{2a^2\sin^3(c+dx)}{3d}+\frac{a}{}$$

$$= -\frac{2ab\cos^5(c+dx)}{5d}+\frac{a^2\sin(c+dx)}{d}-\frac{2a^2\sin^3(c+dx)}{3d}+\frac{b}{}$$

Mathematica [A] time = 0.17, size = 116, normalized size = 1.13

$$\frac{150a^2\sin(c+dx)+25a^2\sin(3(c+dx))+3a^2\sin(5(c+dx))-60ab\cos(c+dx)-30ab\cos(3(c+dx))-6ab\cos}{240d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^3*(a*Cos[c + d*x] + b*Sin[c + d*x])^2,x]
```

```
[Out] (-60*a*b*Cos[c + d*x] - 30*a*b*Cos[3*(c + d*x)] - 6*a*b*Cos[5*(c + d*x)] +
150*a^2*Sin[c + d*x] + 30*b^2*Sin[c + d*x] + 25*a^2*Sin[3*(c + d*x)] - 5*b^
2*Sin[3*(c + d*x)] + 3*a^2*Sin[5*(c + d*x)] - 3*b^2*Sin[5*(c + d*x)])/(240*
d)
```

fricas [A] time = 0.62, size = 74, normalized size = 0.72

$$-\frac{6\,ab\cos\left(dx+c\right)^{5}-\left(3\left(a^{2}-b^{2}\right)\cos\left(dx+c\right)^{4}+\left(4\,a^{2}+b^{2}\right)\cos\left(dx+c\right)^{2}+8\,a^{2}+2\,b^{2}\right)\sin\left(dx+c\right)}{15\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a*cos(d*x+c)+b*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -1/15*(6*a*b*cos(d*x + c)^5 - (3*(a^2 - b^2)*cos(d*x + c)^4 + (4*a^2 + b^2)
*cos(d*x + c)^2 + 8*a^2 + 2*b^2)*sin(d*x + c))/d
```

giac [A] time = 3.51, size = 114, normalized size = 1.11

$$-\frac{ab\cos\left(5\,dx+5\,c\right)}{40\,d}-\frac{ab\cos\left(3\,dx+3\,c\right)}{8\,d}-\frac{ab\cos\left(dx+c\right)}{4\,d}+\frac{\left(a^{2}-b^{2}\right)\sin\left(5\,dx+5\,c\right)}{80\,d}+\frac{\left(5\,a^{2}-b^{2}\right)\sin\left(3\,dx+3\,c\right)}{48\,d}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a*cos(d*x+c)+b*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/40*a*b*cos(5*d*x + 5*c)/d - 1/8*a*b*cos(3*d*x + 3*c)/d - 1/4*a*b*cos(d*x
 + c)/d + 1/80*(a^2 - b^2)*sin(5*d*x + 5*c)/d + 1/48*(5*a^2 - b^2)*sin(3*d*
x + 3*c)/d + 1/8*(5*a^2 + b^2)*sin(d*x + c)/d
```

maple [A] time = 1.76, size = 88, normalized size = 0.85

$$\frac{b^{2}\left(-\frac{\sin(dx+c)\left(\cos^{4}(dx+c)\right)}{5}+\frac{\left(2+\cos^{2}(dx+c)\right)\sin(dx+c)}{15}\right)-\frac{2ab\left(\cos^{5}(dx+c)\right)}{5}+\frac{a^{2}\left(\frac{8}{3}+\cos^{4}(dx+c)+\frac{4\left(\cos^{2}(dx+c)\right)}{3}\right)\sin(dx+c)}{5}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3*(a*cos(d*x+c)+b*sin(d*x+c))^2,x)
```

```
[Out] 1/d*(b^2*(-1/5*sin(d*x+c)*cos(d*x+c)^4+1/15*(2+cos(d*x+c)^2)*sin(d*x+c))-2/
5*a*b*cos(d*x+c)^5+1/5*a^2*(8/3+cos(d*x+c)^4+4/3*cos(d*x+c)^2)*sin(d*x+c))
```

maxima [A] time = 0.32, size = 77, normalized size = 0.75

$$-\frac{6\,ab\cos\left(dx+c\right)^{5}-\left(3\,\sin\left(dx+c\right)^{5}-10\,\sin\left(dx+c\right)^{3}+15\,\sin\left(dx+c\right)\right)a^{2}+\left(3\,\sin\left(dx+c\right)^{5}-5\,\sin\left(dx+c\right)^{3}\right.}{15\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a*cos(d*x+c)+b*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -1/15*(6*a*b*cos(d*x + c)^5 - (3*sin(d*x + c)^5 - 10*sin(d*x + c)^3 + 15*si
n(d*x + c))*a^2 + (3*sin(d*x + c)^5 - 5*sin(d*x + c)^3)*b^2)/d
```

mupad [B] time = 0.61, size = 115, normalized size = 1.12

$$\frac{2\left(\frac{3\sin(c+d\,x)\,a^2\cos(c+d\,x)^4}{2}+2\,\sin\left(c+d\,x\right)\,a^2\cos\left(c+d\,x\right)^2+4\,\sin\left(c+d\,x\right)\,a^2-3\,a\,b\cos\left(c+d\,x\right)^5-\frac{3\sin(c+d\,x)\,b}{2}\right.}{15\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^3*(a*cos(c + d*x) + b*sin(c + d*x))^2,x)
```

```
[Out] (2*(4*a^2*sin(c + d*x) + b^2*sin(c + d*x) + 2*a^2*cos(c + d*x)^2*sin(c + d*
x) + (3*a^2*cos(c + d*x)^4*sin(c + d*x))/2 + (b^2*cos(c + d*x)^2*sin(c + d*
x))/2 - (3*b^2*cos(c + d*x)^4*sin(c + d*x))/2 - 3*a*b*cos(c + d*x)^5))/(15*
d)
```

sympy [A] time = 1.75, size = 138, normalized size = 1.34

$$\begin{cases} \frac{8a^2\sin^5(c+dx)}{15d}+\frac{4a^2\sin^3(c+dx)\cos^2(c+dx)}{3d}+\frac{a^2\sin(c+dx)\cos^4(c+dx)}{d}-\frac{2ab\cos^5(c+dx)}{5d}+\frac{2b^2\sin^5(c+dx)}{15d}+\frac{b^2\sin^3(c+dx)\cos^2(c+dx)}{3d} \\ x\left(a\cos(c)+b\sin(c)\right)^2\cos^3(c) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3*(a*cos(d*x+c)+b*sin(d*x+c))**2,x)
```

```
[Out] Piecewise((8*a**2*sin(c + d*x)**5/(15*d) + 4*a**2*sin(c + d*x)**3*cos(c + d
*x)**2/(3*d) + a**2*sin(c + d*x)*cos(c + d*x)**4/d - 2*a*b*cos(c + d*x)**5/
(5*d) + 2*b**2*sin(c + d*x)**5/(15*d) + b**2*sin(c + d*x)**3*cos(c + d*x)**
2/(3*d), Ne(d, 0)), (x*(a*cos(c) + b*sin(c))**2*cos(c)**3, True))
```

3.46 $\int \cos^2(c+dx)(a\cos(c+dx)+b\sin(c+dx))^2\,dx$

Optimal. Leaf size=126

$$\frac{a^2\sin(c+dx)\cos^3(c+dx)}{4d}+\frac{3a^2\sin(c+dx)\cos(c+dx)}{8d}+\frac{3a^2x}{8}-\frac{ab\cos^4(c+dx)}{2d}-\frac{b^2\sin(c+dx)\cos^3(c+dx)}{4d}+\frac{b^2}{}$$

```
[Out] 3/8*a^2*x+1/8*b^2*x-1/2*a*b*cos(d*x+c)^4/d+3/8*a^2*cos(d*x+c)*sin(d*x+c)/d+
1/8*b^2*cos(d*x+c)*sin(d*x+c)/d+1/4*a^2*cos(d*x+c)^3*sin(d*x+c)/d-1/4*b^2*c
os(d*x+c)^3*sin(d*x+c)/d
```

Rubi [A] time = 0.13, antiderivative size = 126, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 6, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {3090, 2635, 8, 2565, 30, 2568}

$$\frac{a^2\sin(c+dx)\cos^3(c+dx)}{4d}+\frac{3a^2\sin(c+dx)\cos(c+dx)}{8d}+\frac{3a^2x}{8}-\frac{ab\cos^4(c+dx)}{2d}-\frac{b^2\sin(c+dx)\cos^3(c+dx)}{4d}+\frac{b^2}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2*(a*Cos[c + d*x] + b*Sin[c + d*x])^2,x]
```

```
[Out] (3*a^2*x)/8 + (b^2*x)/8 - (a*b*Cos[c + d*x]^4)/(2*d) + (3*a^2*Cos[c + d*x]*
Sin[c + d*x])/(8*d) + (b^2*Cos[c + d*x]*Sin[c + d*x])/(8*d) + (a^2*Cos[c +
d*x]^3*Sin[c + d*x])/(4*d) - (b^2*Cos[c + d*x]^3*Sin[c + d*x])/(4*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2565

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> -Dist[(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 2568

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> -Simp[(a*(b*Cos[e + f*x])^(n + 1)*(a*Sin[e + f*x])^(m - 1)
```

```
)/(b*f*(m + n)), x] + Dist[(a^2*(m - 1))/(m + n), Int[(b*Cos[e + f*x])^n*(a
*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1] &&
 NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 3090

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_.), x_Symbol] :> Int[ExpandTrig[cos[c + d*x]^m*(a
*cos[c + d*x] + b*sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d}, x] && Inte
gerQ[m] && IGtQ[n, 0]
```

Rubi steps

$$\int \cos^2(c+dx)(a\cos(c+dx)+b\sin(c+dx))^2\,dx = \int \left(a^2\cos^4(c+dx)+2ab\cos^3(c+dx)\sin(c+dx)+b^2\cos$$

$$= a^2\int \cos^4(c+dx)\,dx+(2ab)\int \cos^3(c+dx)\sin(c+dx)\,dx$$

$$= \frac{a^2\cos^3(c+dx)\sin(c+dx)}{4d}-\frac{b^2\cos^3(c+dx)\sin(c+dx)}{4d}+$$

$$= -\frac{ab\cos^4(c+dx)}{2d}+\frac{3a^2\cos(c+dx)\sin(c+dx)}{8d}+\frac{b^2\cos(c$$

$$= \frac{3a^2x}{8}+\frac{b^2x}{8}-\frac{ab\cos^4(c+dx)}{2d}+\frac{3a^2\cos(c+dx)\sin(c+dx}{8d}$$

Mathematica [A] time = 0.23, size = 98, normalized size = 0.78

$$\frac{\left(3a^2+b^2\right)(c+dx)}{8d}+\frac{\left(a^2-b^2\right)\sin(4(c+dx))}{32d}+\frac{a^2\sin(2(c+dx))}{4d}-\frac{ab\cos(2(c+dx))}{4d}-\frac{ab\cos(4(c+dx))}{16d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2*(a*Cos[c + d*x] + b*Sin[c + d*x])^2,x]
```

```
[Out] ((3*a^2 + b^2)*(c + d*x))/(8*d) - (a*b*Cos[2*(c + d*x)])/(4*d) - (a*b*Cos[4
*(c + d*x)])/(16*d) + (a^2*Sin[2*(c + d*x)])/(4*d) + ((a^2 - b^2)*Sin[4*(c
+ d*x)])/(32*d)
```

fricas [A] time = 0.54, size = 75, normalized size = 0.60

$$-\frac{4\,ab\cos\left(dx+c\right)^{4}-\left(3\,a^{2}+b^{2}\right)dx-\left(2\,\left(a^{2}-b^{2}\right)\cos\left(dx+c\right)^{3}+\left(3\,a^{2}+b^{2}\right)\cos\left(dx+c\right)\right)\sin\left(dx+c\right)}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a*cos(d*x+c)+b*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -1/8*(4*a*b*cos(d*x + c)^4 - (3*a^2 + b^2)*d*x - (2*(a^2 - b^2)*cos(d*x + c
)^3 + (3*a^2 + b^2)*cos(d*x + c))*sin(d*x + c))/d
```

giac [A] time = 4.97, size = 85, normalized size = 0.67

$$\frac{1}{8}\left(3\,a^{2}+b^{2}\right)x-\frac{ab\cos\left(4\,dx+4\,c\right)}{16\,d}-\frac{ab\cos\left(2\,dx+2\,c\right)}{4\,d}+\frac{a^{2}\sin\left(2\,dx+2\,c\right)}{4\,d}+\frac{\left(a^{2}-b^{2}\right)\sin\left(4\,dx+4\,c\right)}{32\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a*cos(d*x+c)+b*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/8*(3*a^2 + b^2)*x - 1/16*a*b*cos(4*d*x + 4*c)/d - 1/4*a*b*cos(2*d*x + 2*c
)/d + 1/4*a^2*sin(2*d*x + 2*c)/d + 1/32*(a^2 - b^2)*sin(4*d*x + 4*c)/d
```

maple [A] time = 1.42, size = 97, normalized size = 0.77

$$\frac{b^{2}\left(-\frac{\left(\cos^{3}(dx+c)\right)\sin(dx+c)}{4}+\frac{\cos(dx+c)\sin(dx+c)}{8}+\frac{dx}{8}+\frac{c}{8}\right)-\frac{ab\left(\cos^{4}(dx+c)\right)}{2}+a^{2}\left(\frac{\left(\cos^{3}(dx+c)+\frac{3\cos(dx+c)}{2}\right)\sin(dx+c)}{4}+\frac{3dx}{8}+\frac{3c}{8}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(a*cos(d*x+c)+b*sin(d*x+c))^2,x)
```

```
[Out] 1/d*(b^2*(-1/4*cos(d*x+c)^3*sin(d*x+c)+1/8*cos(d*x+c)*sin(d*x+c)+1/8*d*x+1/
8*c)-1/2*a*b*cos(d*x+c)^4+a^2*(1/4*(cos(d*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)
+3/8*d*x+3/8*c))
```

maxima [A] time = 0.32, size = 75, normalized size = 0.60

$$-\frac{16\,ab\cos\left(dx+c\right)^{4}-\left(12\,dx+12\,c+\sin\left(4\,dx+4\,c\right)+8\,\sin\left(2\,dx+2\,c\right)\right)a^{2}-\left(4\,dx+4\,c-\sin\left(4\,dx+4\,c\right)\right)b^{2}}{32\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a*cos(d*x+c)+b*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -1/32*(16*a*b*cos(d*x + c)^4 - (12*d*x + 12*c + sin(4*d*x + 4*c) + 8*sin(2*
d*x + 2*c))*a^2 - (4*d*x + 4*c - sin(4*d*x + 4*c))*b^2)/d
```

mupad [B] time = 0.58, size = 89, normalized size = 0.71

$$\frac{4\,a^2\,\sin\left(2\,c+2\,d\,x\right)+\frac{a^2\,\sin(4\,c+4\,d\,x)}{2}-\frac{b^2\,\sin(4\,c+4\,d\,x)}{2}+2\,a\,b\,\sin\left(2\,c+2\,d\,x\right)^2+8\,a\,b\,\sin\left(c+d\,x\right)^2+6\,a^2\,d\,x+2\,b}{16\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^2*(a*cos(c + d*x) + b*sin(c + d*x))^2,x)
```

```
[Out] (4*a^2*sin(2*c + 2*d*x) + (a^2*sin(4*c + 4*d*x))/2 - (b^2*sin(4*c + 4*d*x))
/2 + 2*a*b*sin(2*c + 2*d*x)^2 + 8*a*b*sin(c + d*x)^2 + 6*a^2*d*x + 2*b^2*d*
x)/(16*d)
```

sympy [A] time = 1.05, size = 238, normalized size = 1.89

$$\begin{cases} \frac{3a^2x\sin^4\left(c+dx\right)}{8}+\frac{3a^2x\sin^2\left(c+dx\right)\cos^2\left(c+dx\right)}{4}+\frac{3a^2x\cos^4\left(c+dx\right)}{8}+\frac{3a^2\sin^3\left(c+dx\right)\cos\left(c+dx\right)}{8d}+\frac{5a^2\sin\left(c+dx\right)\cos^3\left(c+dx\right)}{8d}-\frac{ab\cos^4\left(c+\right.}{2d} \\ x\left(a\cos\left(c\right)+b\sin\left(c\right)\right)^2\cos^2\left(c\right) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(a*cos(d*x+c)+b*sin(d*x+c))**2,x)
```

```
[Out] Piecewise((3*a**2*x*sin(c + d*x)**4/8 + 3*a**2*x*sin(c + d*x)**2*cos(c + d*
x)**2/4 + 3*a**2*x*cos(c + d*x)**4/8 + 3*a**2*sin(c + d*x)**3*cos(c + d*x)/
(8*d) + 5*a**2*sin(c + d*x)*cos(c + d*x)**3/(8*d) - a*b*cos(c + d*x)**4/(2*
d) + b**2*x*sin(c + d*x)**4/8 + b**2*x*sin(c + d*x)**2*cos(c + d*x)**2/4 +
b**2*x*cos(c + d*x)**4/8 + b**2*sin(c + d*x)**3*cos(c + d*x)/(8*d) - b**2*s
in(c + d*x)*cos(c + d*x)**3/(8*d), Ne(d, 0)), (x*(a*cos(c) + b*sin(c))**2*c
os(c)**2, True))
```

3.47 $\int \cos(c+dx)(a\cos(c+dx)+b\sin(c+dx))^2\,dx$

Optimal. Leaf size=67

$$-\frac{a^2\sin^3(c+dx)}{3d}+\frac{a^2\sin(c+dx)}{d}-\frac{2ab\cos^3(c+dx)}{3d}+\frac{b^2\sin^3(c+dx)}{3d}$$

```
[Out] -2/3*a*b*cos(d*x+c)^3/d+a^2*sin(d*x+c)/d-1/3*a^2*sin(d*x+c)^3/d+1/3*b^2*sin
(d*x+c)^3/d
```

Rubi [A] time = 0.09, antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {3090, 2633, 2565, 30, 2564}

$$-\frac{a^2\sin^3(c+dx)}{3d}+\frac{a^2\sin(c+dx)}{d}-\frac{2ab\cos^3(c+dx)}{3d}+\frac{b^2\sin^3(c+dx)}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]*(a*Cos[c + d*x] + b*Sin[c + d*x])^2,x]
```

```
[Out] (-2*a*b*Cos[c + d*x]^3)/(3*d) + (a^2*Sin[c + d*x])/d - (a^2*Sin[c + d*x]^3)
/(3*d) + (b^2*Sin[c + d*x]^3)/(3*d)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2564

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 2565

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> -Dist[(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
```

```
&& IGtQ[(n - 1)/2, 0]
```

Rule 3090

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_.), x_Symbol] :> Int[ExpandTrig[cos[c + d*x]^m*(a
*cos[c + d*x] + b*sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d}, x] && Inte
gerQ[m] && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}\int \cos(c+dx)(a\cos(c+dx)+b\sin(c+dx))^2\,dx &= \int \left(a^2\cos^3(c+dx)+2ab\cos^2(c+dx)\sin(c+dx)+b^2\cos(\right.\\ &= a^2\int \cos^3(c+dx)\,dx+(2ab)\int \cos^2(c+dx)\sin(c+dx)\,dx\\ &= -\frac{a^2\,\mathrm{Subst}\left(\int \left(1-x^2\right)dx,x,-\sin(c+dx)\right)}{d}-\frac{(2ab)\,\mathrm{Subst}\left(\right.}{}\\ &= -\frac{2ab\cos^3(c+dx)}{3d}+\frac{a^2\sin(c+dx)}{d}-\frac{a^2\sin^3(c+dx)}{3d}+\frac{b^2\,\mathrm{s}}{}\end{aligned}$$

Mathematica [A] time = 0.39, size = 64, normalized size = 0.96

$$\frac{\sin(c+dx)\left(\left(a^2-b^2\right)\cos(2(c+dx))+5a^2+b^2\right)-3ab\cos(c+dx)-ab\cos(3(c+dx))}{6d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]*(a*Cos[c + d*x] + b*Sin[c + d*x])^2,x]
```

```
[Out] (-3*a*b*Cos[c + d*x] - a*b*Cos[3*(c + d*x)] + (5*a^2 + b^2 + (a^2 - b^2)*Co
s[2*(c + d*x)])*Sin[c + d*x])/(6*d)
```

fricas [A] time = 0.50, size = 53, normalized size = 0.79

$$-\frac{2\,ab\cos\left(dx+c\right)^3-\left(\left(a^2-b^2\right)\cos\left(dx+c\right)^2+2\,a^2+b^2\right)\sin\left(dx+c\right)}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a*cos(d*x+c)+b*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -1/3*(2*a*b*cos(d*x + c)^3 - ((a^2 - b^2)*cos(d*x + c)^2 + 2*a^2 + b^2)*sin
(d*x + c))/d
```

giac [A] time = 0.25, size = 73, normalized size = 1.09

$$-\frac{ab\cos\left(3\,dx+3\,c\right)}{6\,d}-\frac{ab\cos\left(dx+c\right)}{2\,d}+\frac{\left(a^2-b^2\right)\sin\left(3\,dx+3\,c\right)}{12\,d}+\frac{\left(3\,a^2+b^2\right)\sin\left(dx+c\right)}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a*cos(d*x+c)+b*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/6*a*b*cos(3*d*x + 3*c)/d - 1/2*a*b*cos(d*x + c)/d + 1/12*(a^2 - b^2)*sin
(3*d*x + 3*c)/d + 1/4*(3*a^2 + b^2)*sin(d*x + c)/d
```

maple [A] time = 1.43, size = 52, normalized size = 0.78

$$\frac{\frac{b^2\left(\sin^3(dx+c)\right)}{3}-\frac{2\left(\cos^3(dx+c)\right)ab}{3}+\frac{a^2\left(2+\cos^2(dx+c)\right)\sin(dx+c)}{3}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(a*cos(d*x+c)+b*sin(d*x+c))^2,x)
```

```
[Out] 1/d*(1/3*b^2*sin(d*x+c)^3-2/3*cos(d*x+c)^3*a*b+1/3*a^2*(2+cos(d*x+c)^2)*sin
(d*x+c))
```

maxima [A] time = 0.33, size = 52, normalized size = 0.78

$$-\frac{2\,ab\cos\left(dx+c\right)^3-b^2\sin\left(dx+c\right)^3+\left(\sin\left(dx+c\right)^3-3\,\sin\left(dx+c\right)\right)a^2}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a*cos(d*x+c)+b*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -1/3*(2*a*b*cos(d*x + c)^3 - b^2*sin(d*x + c)^3 + (sin(d*x + c)^3 - 3*sin(d
*x + c))*a^2)/d
```

mupad [B] time = 0.51, size = 77, normalized size = 1.15

$$\frac{2\left(\frac{\sin(c+d\,x)\,a^2\cos(c+d\,x)^2}{2}+\sin\left(c+d\,x\right)\,a^2-a\,b\cos\left(c+d\,x\right)^3-\frac{\sin(c+d\,x)\,b^2\cos(c+d\,x)^2}{2}+\frac{\sin(c+d\,x)\,b^2}{2}\right)}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)*(a*cos(c + d*x) + b*sin(c + d*x))^2,x)
```

```
[Out] (2*(a^2*sin(c + d*x) + (b^2*sin(c + d*x))/2 + (a^2*cos(c + d*x)^2*sin(c + d
*x))/2 - (b^2*cos(c + d*x)^2*sin(c + d*x))/2 - a*b*cos(c + d*x)^3))/(3*d)
```

sympy [A] time = 0.48, size = 85, normalized size = 1.27

$$\begin{cases} \frac{2a^2\sin^3(c+dx)}{3d} + \frac{a^2\sin(c+dx)\cos^2(c+dx)}{d} - \frac{2ab\cos^3(c+dx)}{3d} + \frac{b^2\sin^3(c+dx)}{3d} & \text{for } d \neq 0 \\ x(a\cos(c) + b\sin(c))^2\cos(c) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a*cos(d*x+c)+b*sin(d*x+c))**2,x)
```

```
[Out] Piecewise((2*a**2*sin(c + d*x)**3/(3*d) + a**2*sin(c + d*x)*cos(c + d*x)**2
/d - 2*a*b*cos(c + d*x)**3/(3*d) + b**2*sin(c + d*x)**3/(3*d), Ne(d, 0)), (
x*(a*cos(c) + b*sin(c))**2*cos(c), True))
```

3.48 $\int (a\cos(c+dx)+b\sin(c+dx))^2\,dx$

Optimal. Leaf size=55

$$\frac{1}{2}x\left(a^2+b^2\right)-\frac{(b\cos(c+dx)-a\sin(c+dx))(a\cos(c+dx)+b\sin(c+dx))}{2d}$$

```
[Out] 1/2*(a^2+b^2)*x-1/2*(b*cos(d*x+c)-a*sin(d*x+c))*(a*cos(d*x+c)+b*sin(d*x+c))
/d
```

Rubi [A] time = 0.02, antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {3073, 8}

$$\frac{1}{2}x\left(a^2+b^2\right)-\frac{(b\cos(c+dx)-a\sin(c+dx))(a\cos(c+dx)+b\sin(c+dx))}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Cos[c + d*x] + b*Sin[c + d*x])^2,x]
```

```
[Out] ((a^2 + b^2)*x)/2 - ((b*Cos[c + d*x] - a*Sin[c + d*x])*(a*Cos[c + d*x] + b*
Sin[c + d*x]))/(2*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3073

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x
_Symbol] :> -Simp[((b*Cos[c + d*x] - a*Sin[c + d*x])*(a*Cos[c + d*x] + b*Si
n[c + d*x])^(n - 1))/(d*n), x] + Dist[((n - 1)*(a^2 + b^2))/n, Int[(a*Cos[c
 + d*x] + b*Sin[c + d*x])^(n - 2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 + b^2, 0] &&  !IntegerQ[(n - 1)/2] && GtQ[n, 1]
```

Rubi steps

$$\int (a\cos(c+dx)+b\sin(c+dx))^2\,dx=-\frac{(b\cos(c+dx)-a\sin(c+dx))(a\cos(c+dx)+b\sin(c+dx))}{2d}+\frac{1}{2}\left(a^2+\right.$$

$$=\frac{1}{2}\left(a^2+b^2\right)x-\frac{(b\cos(c+dx)-a\sin(c+dx))(a\cos(c+dx)+b\sin(c+d}{2d}$$

Mathematica [A] time = 0.10, size = 52, normalized size = 0.95

$$\frac{2\left(a^2+b^2\right)(c+dx)+\left(a^2-b^2\right)\sin(2(c+dx))-2ab\cos(2(c+dx))}{4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Cos[c + d*x] + b*Sin[c + d*x])^2,x]
```

```
[Out] (2*(a^2 + b^2)*(c + d*x) - 2*a*b*Cos[2*(c + d*x)] + (a^2 - b^2)*Sin[2*(c +
d*x)])/(4*d)
```

fricas [A] time = 0.61, size = 52, normalized size = 0.95

$$-\frac{2\,ab\cos\left(dx+c\right)^2-\left(a^2+b^2\right)dx-\left(a^2-b^2\right)\cos\left(dx+c\right)\sin\left(dx+c\right)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cos(d*x+c)+b*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -1/2*(2*a*b*cos(d*x + c)^2 - (a^2 + b^2)*d*x - (a^2 - b^2)*cos(d*x + c)*sin
(d*x + c))/d
```

giac [A] time = 3.73, size = 50, normalized size = 0.91

$$\frac{1}{2}\left(a^2+b^2\right)x-\frac{ab\cos\left(2\,dx+2\,c\right)}{2\,d}+\frac{\left(a^2-b^2\right)\sin\left(2\,dx+2\,c\right)}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cos(d*x+c)+b*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/2*(a^2 + b^2)*x - 1/2*a*b*cos(2*d*x + 2*c)/d + 1/4*(a^2 - b^2)*sin(2*d*x
+ 2*c)/d
```

maple [A] time = 1.19, size = 70, normalized size = 1.27

$$\frac{b^2\left(-\frac{\cos(dx+c)\sin(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)-\left(\cos^2\left(dx+c\right)\right)ab+a^2\left(\frac{\cos(dx+c)\sin(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*cos(d*x+c)+b*sin(d*x+c))^2,x)
```

```
[Out] 1/d*(b^2*(-1/2*cos(d*x+c)*sin(d*x+c)+1/2*d*x+1/2*c)-cos(d*x+c)^2*a*b+a^2*(1
/2*cos(d*x+c)*sin(d*x+c)+1/2*d*x+1/2*c))
```

maxima [A] time = 0.32, size = 68, normalized size = 1.24

$$-\frac{ab\cos(dx+c)^2}{d}+\frac{(2\,dx+2\,c+\sin(2\,dx+2\,c))a^2}{4\,d}+\frac{(2\,dx+2\,c-\sin(2\,dx+2\,c))b^2}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cos(d*x+c)+b*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -a*b*cos(d*x + c)^2/d + 1/4*(2*d*x + 2*c + sin(2*d*x + 2*c))*a^2/d + 1/4*(2
*d*x + 2*c - sin(2*d*x + 2*c))*b^2/d
```

mupad [B] time = 0.48, size = 63, normalized size = 1.15

$$\frac{a^2\,x}{2}+\frac{b^2\,x}{2}+\frac{a^2\,\sin(2\,c+2\,d\,x)}{4\,d}-\frac{b^2\,\sin(2\,c+2\,d\,x)}{4\,d}-\frac{a\,b\,\cos(2\,c+2\,d\,x)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*cos(c + d*x) + b*sin(c + d*x))^2,x)
```

```
[Out] (a^2*x)/2 + (b^2*x)/2 + (a^2*sin(2*c + 2*d*x))/(4*d) - (b^2*sin(2*c + 2*d*x
))/(4*d) - (a*b*cos(2*c + 2*d*x))/(2*d)
```

sympy [A] time = 0.27, size = 128, normalized size = 2.33

$$\begin{cases}\frac{a^2x\sin^2(c+dx)}{2}+\frac{a^2x\cos^2(c+dx)}{2}+\frac{a^2\sin(c+dx)\cos(c+dx)}{2d}-\frac{ab\cos^2(c+dx)}{d}+\frac{b^2x\sin^2(c+dx)}{2}+\frac{b^2x\cos^2(c+dx)}{2}-\frac{b^2\sin(c+dx)\cos(c+dx)}{2d} \\ x(a\cos(c)+b\sin(c))^2\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cos(d*x+c)+b*sin(d*x+c))**2,x)
```

```
[Out] Piecewise((a**2*x*sin(c + d*x)**2/2 + a**2*x*cos(c + d*x)**2/2 + a**2*sin(c
 + d*x)*cos(c + d*x)/(2*d) - a*b*cos(c + d*x)**2/d + b**2*x*sin(c + d*x)**2
/2 + b**2*x*cos(c + d*x)**2/2 - b**2*sin(c + d*x)*cos(c + d*x)/(2*d), Ne(d,
 0)), (x*(a*cos(c) + b*sin(c))**2, True))
```

3.49 $\int \sec(c+dx)(a\cos(c+dx)+b\sin(c+dx))^2\,dx$

Optimal. Leaf size=55

$$\frac{a^2\sin(c+dx)}{d}-\frac{2ab\cos(c+dx)}{d}-\frac{b^2\sin(c+dx)}{d}+\frac{b^2\tanh^{-1}(\sin(c+dx))}{d}$$

```
[Out] b^2*arctanh(sin(d*x+c))/d-2*a*b*cos(d*x+c)/d+a^2*sin(d*x+c)/d-b^2*sin(d*x+c
)/d
```

Rubi [A] time = 0.07, antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {3090, 2637, 2638, 2592, 321, 206}

$$\frac{a^2\sin(c+dx)}{d}-\frac{2ab\cos(c+dx)}{d}-\frac{b^2\sin(c+dx)}{d}+\frac{b^2\tanh^{-1}(\sin(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]*(a*Cos[c + d*x] + b*Sin[c + d*x])^2,x]
```

```
[Out] (b^2*ArcTanh[Sin[c + d*x]])/d - (2*a*b*Cos[c + d*x])/d + (a^2*Sin[c + d*x])
/d - (b^2*Sin[c + d*x])/d
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2592

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(
ff*x)^(m + n)/(a^2 - ff^2*x^2)^((n + 1)/2), x], x, (a*Sin[e + f*x])/ff], x]
] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3090

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_.), x_Symbol] :> Int[ExpandTrig[cos[c + d*x]^m*(a
*cos[c + d*x] + b*sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d}, x] && Inte
gerQ[m] && IGtQ[n, 0]
```

Rubi steps

$$\int \sec(c+dx)(a\cos(c+dx)+b\sin(c+dx))^2\,dx = \int \left(a^2\cos(c+dx)+2ab\sin(c+dx)+b^2\sin(c+dx)\tan(c+\right.$$

$$= a^2\int \cos(c+dx)\,dx+(2ab)\int \sin(c+dx)\,dx+b^2\int \sin(c+$$

$$= -\frac{2ab\cos(c+dx)}{d}+\frac{a^2\sin(c+dx)}{d}+\frac{b^2\,\mathrm{Subst}\left(\int \frac{x^2}{1-x^2}\,dx,x,\mathrm{s}\right.}{d}$$

$$= -\frac{2ab\cos(c+dx)}{d}+\frac{a^2\sin(c+dx)}{d}-\frac{b^2\sin(c+dx)}{d}+\frac{b^2\,\mathrm{Sub}}{}$$

$$= \frac{b^2\tanh^{-1}(\sin(c+dx))}{d}-\frac{2ab\cos(c+dx)}{d}+\frac{a^2\sin(c+dx)}{d}-$$

Mathematica [A] time = 0.16, size = 84, normalized size = 1.53

$$\frac{\left(a^2-b^2\right)\sin(c+dx)-2ab\cos(c+dx)+b^2\left(\log\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)-\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\right.\right.}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]*(a*Cos[c + d*x] + b*Sin[c + d*x])^2,x]
```

```
[Out] (-2*a*b*Cos[c + d*x] + b^2*(-Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + Log
[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]) + (a^2 - b^2)*Sin[c + d*x])/d
```

fricas [A] time = 0.66, size = 62, normalized size = 1.13

$$-\frac{4\,ab\cos\left(dx+c\right)-b^2\log\left(\sin\left(dx+c\right)+1\right)+b^2\log\left(-\sin\left(dx+c\right)+1\right)-2\left(a^2-b^2\right)\sin\left(dx+c\right)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a*cos(d*x+c)+b*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -1/2*(4*a*b*cos(d*x + c) - b^2*log(sin(d*x + c) + 1) + b^2*log(-sin(d*x + c
) + 1) - 2*(a^2 - b^2)*sin(d*x + c))/d
```

giac [A] time = 4.94, size = 89, normalized size = 1.62

$$\frac{b^2\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-b^2\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)+\frac{2\left(a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-2\,ab\right)}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a*cos(d*x+c)+b*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] (b^2*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - b^2*log(abs(tan(1/2*d*x + 1/2*c)
- 1)) + 2*(a^2*tan(1/2*d*x + 1/2*c) - b^2*tan(1/2*d*x + 1/2*c) - 2*a*b)/(ta
n(1/2*d*x + 1/2*c)^2 + 1))/d
```

maple [A] time = 1.86, size = 63, normalized size = 1.15

$$-\frac{2ab\cos\left(dx+c\right)}{d}+\frac{a^2\sin\left(dx+c\right)}{d}-\frac{b^2\sin\left(dx+c\right)}{d}+\frac{b^2\ln\left(\sec\left(dx+c\right)+\tan\left(dx+c\right)\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)*(a*cos(d*x+c)+b*sin(d*x+c))^2,x)
```

```
[Out] -2*a*b*cos(d*x+c)/d+a^2*sin(d*x+c)/d-b^2*sin(d*x+c)/d+1/d*b^2*ln(sec(d*x+c)
+tan(d*x+c))
```

maxima [A] time = 0.33, size = 60, normalized size = 1.09

$$\frac{b^2\left(\log\left(\sin\left(dx+c\right)+1\right)-\log\left(\sin\left(dx+c\right)-1\right)-2\,\sin\left(dx+c\right)\right)-4\,ab\cos\left(dx+c\right)+2\,a^2\sin\left(dx+c\right)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a*cos(d*x+c)+b*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 1/2*(b^2*(log(sin(d*x + c) + 1) - log(sin(d*x + c) - 1) - 2*sin(d*x + c)) -
 4*a*b*cos(d*x + c) + 2*a^2*sin(d*x + c))/d
```

mupad [B] time = 0.49, size = 66, normalized size = 1.20

$$\frac{2\,b^2\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{d}-\frac{4\,a\,b-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(2\,a^2-2\,b^2\right)}{d\,\left({\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*cos(c + d*x) + b*sin(c + d*x))^2/cos(c + d*x),x)
```

```
[Out] (2*b^2*atanh(tan(c/2 + (d*x)/2)))/d - (4*a*b - tan(c/2 + (d*x)/2)*(2*a^2 -
2*b^2))/(d*(tan(c/2 + (d*x)/2)^2 + 1))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a\cos\left(c+dx\right)+b\sin\left(c+dx\right)\right)^2\sec\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a*cos(d*x+c)+b*sin(d*x+c))**2,x)
```

```
[Out] Integral((a*cos(c + d*x) + b*sin(c + d*x))**2*sec(c + d*x), x)
```

3.50 $\int \sec^2(c+dx)(a\cos(c+dx)+b\sin(c+dx))^2\,dx$

Optimal. Leaf size=39

$$x\left(a^2-b^2\right)-\frac{2ab\log(\cos(c+dx))}{d}+\frac{b^2\tan(c+dx)}{d}$$

```
[Out] (a^2-b^2)*x-2*a*b*ln(cos(d*x+c))/d+b^2*tan(d*x+c)/d
```

Rubi [A] time = 0.06, antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.107, Rules used = {3086, 3477, 3475}

$$x\left(a^2-b^2\right)-\frac{2ab\log(\cos(c+dx))}{d}+\frac{b^2\tan(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^2*(a*Cos[c + d*x] + b*Sin[c + d*x])^2,x]
```

```
[Out] (a^2 - b^2)*x - (2*a*b*Log[Cos[c + d*x]])/d + (b^2*Tan[c + d*x])/d
```

Rule 3086

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin
[(c_.) + (d_.)*(x_)])^(n_.), x_Symbol] :> Int[(a + b*Tan[c + d*x])^n, x] /;
 FreeQ[{a, b, c, d}, x] && EqQ[m + n, 0] && IntegerQ[n] && NeQ[a^2 + b^2, 0
]
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3477

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^2, x_Symbol] :> Simp[(a^2 - b^2)
*x, x] + (Dist[2*a*b, Int[Tan[c + d*x], x], x] + Simp[(b^2*Tan[c + d*x])/d,
 x]) /; FreeQ[{a, b, c, d}, x]
```

Rubi steps

$$\int \sec^2(c+dx)(a\cos(c+dx)+b\sin(c+dx))^2\,dx = \int (a+b\tan(c+dx))^2\,dx$$
$$= \left(a^2-b^2\right)x + \frac{b^2\tan(c+dx)}{d} + (2ab)\int \tan(c+dx)\,dx$$
$$= \left(a^2-b^2\right)x - \frac{2ab\log(\cos(c+dx))}{d} + \frac{b^2\tan(c+dx)}{d}$$

Mathematica [C] time = 0.14, size = 69, normalized size = 1.77

$$\frac{2b^2\tan(c+dx) - i\left((a+ib)^2\log(-\tan(c+dx)+i) - (a-ib)^2\log(\tan(c+dx)+i)\right)}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^2*(a*Cos[c + d*x] + b*Sin[c + d*x])^2,x]
```

```
[Out] ((-I)*((a + I*b)^2*Log[I - Tan[c + d*x]] - (a - I*b)^2*Log[I + Tan[c + d*x]
]) + 2*b^2*Tan[c + d*x])/(2*d)
```

fricas [A] time = 0.52, size = 60, normalized size = 1.54

$$\frac{\left(a^2-b^2\right)dx\cos\left(dx+c\right) - 2\,ab\cos\left(dx+c\right)\log\left(-\cos\left(dx+c\right)\right) + b^2\sin\left(dx+c\right)}{d\cos\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a*cos(d*x+c)+b*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] ((a^2 - b^2)*d*x*cos(d*x + c) - 2*a*b*cos(d*x + c)*log(-cos(d*x + c)) + b^2
*sin(d*x + c))/(d*cos(d*x + c))
```

giac [A] time = 0.21, size = 44, normalized size = 1.13

$$\frac{ab\log\left(\tan\left(dx+c\right)^2+1\right) + b^2\tan\left(dx+c\right) + \left(a^2-b^2\right)(dx+c)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a*cos(d*x+c)+b*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] (a*b*log(tan(d*x + c)^2 + 1) + b^2*tan(d*x + c) + (a^2 - b^2)*(d*x + c))/d
```

maple [A] time = 3.74, size = 57, normalized size = 1.46

$$a^2x - b^2x + \frac{b^2\tan\left(dx+c\right)}{d} - \frac{2ab\ln\left(\cos\left(dx+c\right)\right)}{d} + \frac{a^2c}{d} - \frac{c\,b^2}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2*(a*cos(d*x+c)+b*sin(d*x+c))^2,x)
```

```
[Out] a^2*x-b^2*x+b^2*tan(d*x+c)/d-2*a*b*ln(cos(d*x+c))/d+1/d*a^2*c-1/d*c*b^2
```

maxima [A] time = 0.43, size = 49, normalized size = 1.26

$$\frac{(dx+c)a^2 - (dx+c-\tan(dx+c))b^2 - ab\log\left(-\sin(dx+c)^2+1\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a*cos(d*x+c)+b*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] ((d*x + c)*a^2 - (d*x + c - tan(d*x + c))*b^2 - a*b*log(-sin(d*x + c)^2 + 1
))/d
```

mupad [B] time = 0.68, size = 118, normalized size = 3.03

$$\frac{b^2\tan(c+d\,x)}{d}+\frac{2\,a^2\,\mathrm{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d}-\frac{2\,b^2\,\mathrm{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d}-\frac{2\,a\,b\,\ln\left(\frac{\cos(c+d\,x)}{\cos(c+d\,x)+1}\right)}{d}+\frac{2\,a\,b\,\ln\left(\frac{1}{{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*cos(c + d*x) + b*sin(c + d*x))^2/cos(c + d*x)^2,x)
```

```
[Out] (b^2*tan(c + d*x))/d + (2*a^2*atan(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/
d - (2*b^2*atan(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/d - (2*a*b*log(cos(
c + d*x)/(cos(c + d*x) + 1)))/d + (2*a*b*log(1/cos(c/2 + (d*x)/2)^2))/d
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a\cos(c+dx)+b\sin(c+dx))^2\sec^2(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2*(a*cos(d*x+c)+b*sin(d*x+c))**2,x)
```

```
[Out] Integral((a*cos(c + d*x) + b*sin(c + d*x))**2*sec(c + d*x)**2, x)
```

3.51 $\int \sec^3(c+dx)(a\cos(c+dx)+b\sin(c+dx))^2\,dx$

Optimal. Leaf size=67

$$\frac{a^2\tanh^{-1}(\sin(c+dx))}{d}+\frac{2ab\sec(c+dx)}{d}-\frac{b^2\tanh^{-1}(\sin(c+dx))}{2d}+\frac{b^2\tan(c+dx)\sec(c+dx)}{2d}$$

```
[Out] a^2*arctanh(sin(d*x+c))/d-1/2*b^2*arctanh(sin(d*x+c))/d+2*a*b*sec(d*x+c)/d+
1/2*b^2*sec(d*x+c)*tan(d*x+c)/d
```

Rubi [A] time = 0.09, antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.179, Rules used = {3090, 3770, 2606, 8, 2611}

$$\frac{a^2\tanh^{-1}(\sin(c+dx))}{d}+\frac{2ab\sec(c+dx)}{d}-\frac{b^2\tanh^{-1}(\sin(c+dx))}{2d}+\frac{b^2\tan(c+dx)\sec(c+dx)}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^3*(a*Cos[c + d*x] + b*Sin[c + d*x])^2,x]
```

```
[Out] (a^2*ArcTanh[Sin[c + d*x]])/d - (b^2*ArcTanh[Sin[c + d*x]])/(2*d) + (2*a*b*
Sec[c + d*x])/d + (b^2*Sec[c + d*x]*Tan[c + d*x])/(2*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 && !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 2611

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(b*(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*(
m + n - 1)), x] - Dist[(b^2*(n - 1))/(m + n - 1), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 3090

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_.), x_Symbol] :> Int[ExpandTrig[cos[c + d*x]^m*(a
*cos[c + d*x] + b*sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d}, x] && Inte
gerQ[m] && IGtQ[n, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \sec^3(c+dx)(a\cos(c+dx)+b\sin(c+dx))^2\,dx &= \int \left(a^2\sec(c+dx)+2ab\sec(c+dx)\tan(c+dx)+b^2\sec(c\right.\\ &= a^2\int \sec(c+dx)\,dx+(2ab)\int \sec(c+dx)\tan(c+dx)\,dx+\\ &= \frac{a^2\tanh^{-1}(\sin(c+dx))}{d}+\frac{b^2\sec(c+dx)\tan(c+dx)}{2d}-\frac{1}{2}b^2\\ &= \frac{a^2\tanh^{-1}(\sin(c+dx))}{d}-\frac{b^2\tanh^{-1}(\sin(c+dx))}{2d}+\frac{2ab\sec}{}\end{aligned}$$

Mathematica [A] time = 0.04, size = 67, normalized size = 1.00

$$\frac{a^2\tanh^{-1}(\sin(c+dx))}{d}+\frac{2ab\sec(c+dx)}{d}-\frac{b^2\tanh^{-1}(\sin(c+dx))}{2d}+\frac{b^2\tan(c+dx)\sec(c+dx)}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^3*(a*Cos[c + d*x] + b*Sin[c + d*x])^2,x]
```

```
[Out] (a^2*ArcTanh[Sin[c + d*x]])/d - (b^2*ArcTanh[Sin[c + d*x]])/(2*d) + (2*a*b*
Sec[c + d*x])/d + (b^2*Sec[c + d*x]*Tan[c + d*x])/(2*d)
```

fricas [A] time = 0.69, size = 96, normalized size = 1.43

$$\frac{\left(2\,a^2-b^2\right)\cos\left(dx+c\right)^2\log\left(\sin\left(dx+c\right)+1\right)-\left(2\,a^2-b^2\right)\cos\left(dx+c\right)^2\log\left(-\sin\left(dx+c\right)+1\right)+8\,ab\cos\left(dx\right.}{4\,d\cos\left(dx+c\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a*cos(d*x+c)+b*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/4*((2*a^2 - b^2)*cos(d*x + c)^2*log(sin(d*x + c) + 1) - (2*a^2 - b^2)*cos
(d*x + c)^2*log(-sin(d*x + c) + 1) + 8*a*b*cos(d*x + c) + 2*b^2*sin(d*x + c
))/(d*cos(d*x + c)^2)
```

giac [A] time = 4.74, size = 122, normalized size = 1.82

$$\frac{\left(2\,a^2-b^2\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-\left(2\,a^2-b^2\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)+\frac{2\left(b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-4\,ab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right.\right.}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2\right.}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a*cos(d*x+c)+b*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/2*((2*a^2 - b^2)*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - (2*a^2 - b^2)*log(a
bs(tan(1/2*d*x + 1/2*c) - 1)) + 2*(b^2*tan(1/2*d*x + 1/2*c)^3 - 4*a*b*tan(1
/2*d*x + 1/2*c)^2 + b^2*tan(1/2*d*x + 1/2*c) + 4*a*b)/(tan(1/2*d*x + 1/2*c)
^2 - 1)^2)/d
```

maple [A] time = 1.69, size = 98, normalized size = 1.46

$$\frac{a^2\ln\left(\sec\left(dx+c\right)+\tan\left(dx+c\right)\right)}{d}+\frac{2ab}{d\cos\left(dx+c\right)}+\frac{b^2\left(\sin^3\left(dx+c\right)\right)}{2d\cos\left(dx+c\right)^2}+\frac{b^2\sin\left(dx+c\right)}{2d}-\frac{b^2\ln\left(\sec\left(dx+c\right)+\tan\left(\right.\right.}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^3*(a*cos(d*x+c)+b*sin(d*x+c))^2,x)
```

```
[Out] 1/d*a^2*ln(sec(d*x+c)+tan(d*x+c))+2/d*a*b/cos(d*x+c)+1/2/d*b^2*sin(d*x+c)^3
/cos(d*x+c)^2+1/2*b^2*sin(d*x+c)/d-1/2/d*b^2*ln(sec(d*x+c)+tan(d*x+c))
```

maxima [A] time = 0.33, size = 89, normalized size = 1.33

$$-\frac{b^2\left(\frac{2\,\sin(dx+c)}{\sin(dx+c)^2-1}+\log\left(\sin\left(dx+c\right)+1\right)-\log\left(\sin\left(dx+c\right)-1\right)\right)-2\,a^2\left(\log\left(\sin\left(dx+c\right)+1\right)-\log\left(\sin\left(dx+c\right)-\right.\right.}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a*cos(d*x+c)+b*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -1/4*(b^2*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) + log(sin(d*x + c) + 1) - lo
g(sin(d*x + c) - 1)) - 2*a^2*(log(sin(d*x + c) + 1) - log(sin(d*x + c) - 1)
) - 8*a*b/cos(d*x + c))/d
```

mupad [B] time = 0.90, size = 106, normalized size = 1.58

$$\frac{b^2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3+b^2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)-4ab\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2+4ab}{d\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4-2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2+1\right)}+\frac{\operatorname{atanh}\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)\left(2a^2-b^2\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*cos(c + d*x) + b*sin(c + d*x))^2/cos(c + d*x)^3,x)
```

```
[Out] (4*a*b + b^2*tan(c/2 + (d*x)/2)^3 + b^2*tan(c/2 + (d*x)/2) - 4*a*b*tan(c/2
+ (d*x)/2)^2)/(d*(tan(c/2 + (d*x)/2)^4 - 2*tan(c/2 + (d*x)/2)^2 + 1)) + (at
anh(tan(c/2 + (d*x)/2))*(2*a^2 - b^2))/d
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a\cos(c+dx)+b\sin(c+dx))^2\sec^3(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**3*(a*cos(d*x+c)+b*sin(d*x+c))**2,x)
```

```
[Out] Integral((a*cos(c + d*x) + b*sin(c + d*x))**2*sec(c + d*x)**3, x)
```

3.52 $\int \sec^4(c+dx)(a\cos(c+dx)+b\sin(c+dx))^2\,dx$

Optimal. Leaf size=30

$$\frac{\tan^3(c+dx)(a\cot(c+dx)+b)^3}{3bd}$$

```
[Out] 1/3*(b+a*cot(d*x+c))^3*tan(d*x+c)^3/b/d
```

Rubi [A] time = 0.05, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {3088, 37}

$$\frac{\tan^3(c+dx)(a\cot(c+dx)+b)^3}{3bd}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^4*(a*Cos[c + d*x] + b*Sin[c + d*x])^2,x]
```

```
[Out] ((b + a*Cot[c + d*x])^3*Tan[c + d*x]^3)/(3*b*d)
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 3088

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[(x^m*(b +
 a*x)^n)/(1 + x^2)^((m + n + 2)/2), x], x, Cot[c + d*x]], x] /; FreeQ[{a, b
, c, d}, x] && IntegerQ[n] && IntegerQ[(m + n)/2] && NeQ[n, -1] &&  !(GtQ[n
, 0] && GtQ[m, 1])
```

Rubi steps

$$\begin{aligned}\int \sec^4(c+dx)(a\cos(c+dx)+b\sin(c+dx))^2\,dx &= -\frac{\text{Subst}\left(\int \frac{(b+ax)^2}{x^4}\,dx, x, \cot(c+dx)\right)}{d}\\ &= \frac{(b+a\cot(c+dx))^3\tan^3(c+dx)}{3bd}\end{aligned}$$

Mathematica [A] time = 0.04, size = 46, normalized size = 1.53

$$\frac{a^2\tan(c+dx)}{d}+\frac{ab\tan^2(c+dx)}{d}+\frac{b^2\tan^3(c+dx)}{3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^4*(a*Cos[c + d*x] + b*Sin[c + d*x])^2,x]
```

```
[Out] (a^2*Tan[c + d*x])/d + (a*b*Tan[c + d*x]^2)/d + (b^2*Tan[c + d*x]^3)/(3*d)
```

fricas [A] time = 0.46, size = 55, normalized size = 1.83

$$\frac{3\,ab\cos\left(dx+c\right)+\left(\left(3\,a^2-b^2\right)\cos\left(dx+c\right)^2+b^2\right)\sin\left(dx+c\right)}{3\,d\cos\left(dx+c\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(a*cos(d*x+c)+b*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/3*(3*a*b*cos(d*x + c) + ((3*a^2 - b^2)*cos(d*x + c)^2 + b^2)*sin(d*x + c)
)/(d*cos(d*x + c)^3)
```

giac [A] time = 0.29, size = 41, normalized size = 1.37

$$\frac{b^2\tan\left(dx+c\right)^3+3\,ab\tan\left(dx+c\right)^2+3\,a^2\tan\left(dx+c\right)}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(a*cos(d*x+c)+b*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/3*(b^2*tan(d*x + c)^3 + 3*a*b*tan(d*x + c)^2 + 3*a^2*tan(d*x + c))/d
```

maple [A] time = 10.89, size = 48, normalized size = 1.60

$$\frac{a^2\tan\left(dx+c\right)+\frac{ab}{\cos(dx+c)^2}+\frac{b^2\left(\sin^3(dx+c)\right)}{3\cos(dx+c)^3}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^4*(a*cos(d*x+c)+b*sin(d*x+c))^2,x)
```

```
[Out] 1/d*(a^2*tan(d*x+c)+a*b/cos(d*x+c)^2+1/3*b^2*sin(d*x+c)^3/cos(d*x+c)^3)
```

maxima [A] time = 0.33, size = 45, normalized size = 1.50

$$\frac{b^2\tan(dx+c)^3+3a^2\tan(dx+c)-\frac{3ab}{\sin(dx+c)^2-1}}{3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(a*cos(d*x+c)+b*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 1/3*(b^2*tan(d*x + c)^3 + 3*a^2*tan(d*x + c) - 3*a*b/(sin(d*x + c)^2 - 1))/
d
```

mupad [B] time = 0.49, size = 68, normalized size = 2.27

$$\frac{\frac{b^2\sin(c+d\,x)}{3}+\frac{\cos(c+d\,x)^2\sin(c+d\,x)\left(3a^2-b^2\right)}{3}+a\,b\,\cos(c+d\,x)\,\sin(c+d\,x)^2}{d\cos(c+d\,x)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*cos(c + d*x) + b*sin(c + d*x))^2/cos(c + d*x)^4,x)
```

```
[Out] ((b^2*sin(c + d*x))/3 + (cos(c + d*x)^2*sin(c + d*x)*(3*a^2 - b^2))/3 + a*b
*cos(c + d*x)*sin(c + d*x)^2)/(d*cos(c + d*x)^3)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a\cos(c+dx)+b\sin(c+dx))^2\sec^4(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**4*(a*cos(d*x+c)+b*sin(d*x+c))**2,x)
```

```
[Out] Integral((a*cos(c + d*x) + b*sin(c + d*x))**2*sec(c + d*x)**4, x)
```

3.53 $\int \sec^5(c+dx)(a\cos(c+dx)+b\sin(c+dx))^2\,dx$

Optimal. Leaf size=120

$$\frac{a^2\tanh^{-1}(\sin(c+dx))}{2d}+\frac{a^2\tan(c+dx)\sec(c+dx)}{2d}+\frac{2ab\sec^3(c+dx)}{3d}-\frac{b^2\tanh^{-1}(\sin(c+dx))}{8d}+\frac{b^2\tan(c+dx)\,\mathrm{s}}{4d}$$

```
[Out] 1/2*a^2*arctanh(sin(d*x+c))/d-1/8*b^2*arctanh(sin(d*x+c))/d+2/3*a*b*sec(d*x
+c)^3/d+1/2*a^2*sec(d*x+c)*tan(d*x+c)/d-1/8*b^2*sec(d*x+c)*tan(d*x+c)/d+1/4
*b^2*sec(d*x+c)^3*tan(d*x+c)/d
```

Rubi [A] time = 0.14, antiderivative size = 120, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {3090, 3768, 3770, 2606, 30, 2611}

$$\frac{a^2\tanh^{-1}(\sin(c+dx))}{2d}+\frac{a^2\tan(c+dx)\sec(c+dx)}{2d}+\frac{2ab\sec^3(c+dx)}{3d}-\frac{b^2\tanh^{-1}(\sin(c+dx))}{8d}+\frac{b^2\tan(c+dx)\,\mathrm{s}}{4d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^5*(a*Cos[c + d*x] + b*Sin[c + d*x])^2,x]
```

```
[Out] (a^2*ArcTanh[Sin[c + d*x]])/(2*d) - (b^2*ArcTanh[Sin[c + d*x]])/(8*d) + (2*
a*b*Sec[c + d*x]^3)/(3*d) + (a^2*Sec[c + d*x]*Tan[c + d*x])/(2*d) - (b^2*Se
c[c + d*x]*Tan[c + d*x])/(8*d) + (b^2*Sec[c + d*x]^3*Tan[c + d*x])/(4*d)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 2611

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(b*(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*(
m + n - 1)), x] - Dist[(b^2*(n - 1))/(m + n - 1), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 3090

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_.), x_Symbol] :> Int[ExpandTrig[cos[c + d*x]^m*(a
*cos[c + d*x] + b*sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d}, x] && Inte
gerQ[m] && IGtQ[n, 0]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \sec^5(c+dx)(a\cos(c+dx)+b\sin(c+dx))^2\,dx = \int \left(a^2\sec^3(c+dx)+2ab\sec^3(c+dx)\tan(c+dx)+b^2\sec^3(\right.$$

$$= a^2\int \sec^3(c+dx)\,dx+(2ab)\int \sec^3(c+dx)\tan(c+dx)\,dx$$

$$= \frac{a^2\sec(c+dx)\tan(c+dx)}{2d}+\frac{b^2\sec^3(c+dx)\tan(c+dx)}{4d}+\frac{1}{2}$$

$$= \frac{a^2\tanh^{-1}(\sin(c+dx))}{2d}+\frac{2ab\sec^3(c+dx)}{3d}+\frac{a^2\sec(c+dx)}{2d}$$

$$= \frac{a^2\tanh^{-1}(\sin(c+dx))}{2d}-\frac{b^2\tanh^{-1}(\sin(c+dx))}{8d}+\frac{2ab\sec^3(}{3d}$$

Mathematica [A] time = 0.08, size = 120, normalized size = 1.00

$$\frac{a^2\tanh^{-1}(\sin(c+dx))}{2d}+\frac{a^2\tan(c+dx)\sec(c+dx)}{2d}+\frac{2ab\sec^3(c+dx)}{3d}-\frac{b^2\tanh^{-1}(\sin(c+dx))}{8d}+\frac{b^2\tan(c+dx)\sec}{4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^5*(a*Cos[c + d*x] + b*Sin[c + d*x])^2,x]
```

```
[Out] (a^2*ArcTanh[Sin[c + d*x]])/(2*d) - (b^2*ArcTanh[Sin[c + d*x]])/(8*d) + (2*
a*b*Sec[c + d*x]^3)/(3*d) + (a^2*Sec[c + d*x]*Tan[c + d*x])/(2*d) - (b^2*Se
c[c + d*x]*Tan[c + d*x])/(8*d) + (b^2*Sec[c + d*x]^3*Tan[c + d*x])/(4*d)
```

fricas [A] time = 0.59, size = 120, normalized size = 1.00

$$\frac{3\left(4\,a^2-b^2\right)\cos\left(dx+c\right)^4\log\left(\sin\left(dx+c\right)+1\right)-3\left(4\,a^2-b^2\right)\cos\left(dx+c\right)^4\log\left(-\sin\left(dx+c\right)+1\right)+32\,ab\cos}{48\,d\cos\left(dx+c\right)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*(a*cos(d*x+c)+b*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/48*(3*(4*a^2 - b^2)*cos(d*x + c)^4*log(sin(d*x + c) + 1) - 3*(4*a^2 - b^2
)*cos(d*x + c)^4*log(-sin(d*x + c) + 1) + 32*a*b*cos(d*x + c) + 6*((4*a^2 -
 b^2)*cos(d*x + c)^2 + 2*b^2)*sin(d*x + c))/(d*cos(d*x + c)^4)
```

giac [B] time = 0.33, size = 249, normalized size = 2.08

$$3\left(4\,a^2-b^2\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-3\left(4\,a^2-b^2\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)+\frac{2\left(12\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7+3\,b^2\tan\left(\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*(a*cos(d*x+c)+b*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/24*(3*(4*a^2 - b^2)*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 3*(4*a^2 - b^2)*
log(abs(tan(1/2*d*x + 1/2*c) - 1)) + 2*(12*a^2*tan(1/2*d*x + 1/2*c)^7 + 3*b
^2*tan(1/2*d*x + 1/2*c)^7 - 48*a*b*tan(1/2*d*x + 1/2*c)^6 - 12*a^2*tan(1/2*
d*x + 1/2*c)^5 + 21*b^2*tan(1/2*d*x + 1/2*c)^5 + 48*a*b*tan(1/2*d*x + 1/2*c
)^4 - 12*a^2*tan(1/2*d*x + 1/2*c)^3 + 21*b^2*tan(1/2*d*x + 1/2*c)^3 - 16*a*
b*tan(1/2*d*x + 1/2*c)^2 + 12*a^2*tan(1/2*d*x + 1/2*c) + 3*b^2*tan(1/2*d*x
+ 1/2*c) + 16*a*b)/(tan(1/2*d*x + 1/2*c)^2 - 1)^4)/d
```

maple [A] time = 1.96, size = 143, normalized size = 1.19

$$\frac{a^2\sec\left(dx+c\right)\tan\left(dx+c\right)}{2d}+\frac{a^2\ln\left(\sec\left(dx+c\right)+\tan\left(dx+c\right)\right)}{2d}+\frac{2ab}{3d\cos\left(dx+c\right)^3}+\frac{b^2\left(\sin^3\left(dx+c\right)\right)}{4d\cos\left(dx+c\right)^4}+\frac{b^2\left(\sin^3\left(\right.\right.}{8d\cos\left(d\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^5*(a*cos(d*x+c)+b*sin(d*x+c))^2,x)
```

[Out] 1/2*a^2*sec(d*x+c)*tan(d*x+c)/d+1/2/d*a^2*ln(sec(d*x+c)+tan(d*x+c))+2/3/d*a
*b/cos(d*x+c)^3+1/4/d*b^2*sin(d*x+c)^3/cos(d*x+c)^4+1/8/d*b^2*sin(d*x+c)^3/
cos(d*x+c)^2+1/8*b^2*sin(d*x+c)/d-1/8/d*b^2*ln(sec(d*x+c)+tan(d*x+c))

maxima [A] time = 0.33, size = 129, normalized size = 1.08

$$\frac{3\,b^2\left(\frac{2\left(\sin(dx+c)^3+\sin(dx+c)\right)}{\sin(dx+c)^4-2\,\sin(dx+c)^2+1}-\log\left(\sin\left(dx+c\right)+1\right)+\log\left(\sin\left(dx+c\right)-1\right)\right)-12\,a^2\left(\frac{2\,\sin(dx+c)}{\sin(dx+c)^2-1}-\log\left(\sin\left(dx+c\right.\right.\right.}{48\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*(a*cos(d*x+c)+b*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 1/48*(3*b^2*(2*(sin(d*x + c)^3 + sin(d*x + c))/(sin(d*x + c)^4 - 2*sin(d*x
+ c)^2 + 1) - log(sin(d*x + c) + 1) + log(sin(d*x + c) - 1)) - 12*a^2*(2*si
n(d*x + c)/(sin(d*x + c)^2 - 1) - log(sin(d*x + c) + 1) + log(sin(d*x + c)
- 1)) + 32*a*b/cos(d*x + c)^3)/d
```

mupad [B] time = 3.16, size = 216, normalized size = 1.80

$$\frac{\left(a^2+\frac{b^2}{4}\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7-4\,a\,b\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6+\left(\frac{7\,b^2}{4}-a^2\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5+4\,a\,b\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4+\left(\frac{7\,b^2}{4}-a^2\right)\,\tan}{d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8-4\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6+6\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4-4\,\tan\left(\frac{c}{2}\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*cos(c + d*x) + b*sin(c + d*x))^2/cos(c + d*x)^5,x)
```

```
[Out] ((4*a*b)/3 + tan(c/2 + (d*x)/2)*(a^2 + b^2/4) + tan(c/2 + (d*x)/2)^7*(a^2 +
 b^2/4) - tan(c/2 + (d*x)/2)^3*(a^2 - (7*b^2)/4) - tan(c/2 + (d*x)/2)^5*(a^
2 - (7*b^2)/4) - (4*a*b*tan(c/2 + (d*x)/2)^2)/3 + 4*a*b*tan(c/2 + (d*x)/2)^
4 - 4*a*b*tan(c/2 + (d*x)/2)^6)/(d*(6*tan(c/2 + (d*x)/2)^4 - 4*tan(c/2 + (d
*x)/2)^2 - 4*tan(c/2 + (d*x)/2)^6 + tan(c/2 + (d*x)/2)^8 + 1)) + (atanh(tan
(c/2 + (d*x)/2))*(a^2 - b^2/4))/d
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**5*(a*cos(d*x+c)+b*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

3.54 $\int \sec^6(c+dx)(a\cos(c+dx)+b\sin(c+dx))^2\,dx$

Optimal. Leaf size=85

$$\frac{(a^2+b^2)\tan^3(c+dx)}{3d}+\frac{a^2\tan(c+dx)}{d}+\frac{ab\tan^4(c+dx)}{2d}+\frac{ab\tan^2(c+dx)}{d}+\frac{b^2\tan^5(c+dx)}{5d}$$

```
[Out] a^2*tan(d*x+c)/d+a*b*tan(d*x+c)^2/d+1/3*(a^2+b^2)*tan(d*x+c)^3/d+1/2*a*b*ta
n(d*x+c)^4/d+1/5*b^2*tan(d*x+c)^5/d
```

Rubi [A] time = 0.08, antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {3088, 894}

$$\frac{(a^2+b^2)\tan^3(c+dx)}{3d}+\frac{a^2\tan(c+dx)}{d}+\frac{ab\tan^4(c+dx)}{2d}+\frac{ab\tan^2(c+dx)}{d}+\frac{b^2\tan^5(c+dx)}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^6*(a*Cos[c + d*x] + b*Sin[c + d*x])^2,x]
```

```
[Out] (a^2*Tan[c + d*x])/d + (a*b*Tan[c + d*x]^2)/d + ((a^2 + b^2)*Tan[c + d*x]^3
)/(3*d) + (a*b*Tan[c + d*x]^4)/(2*d) + (b^2*Tan[c + d*x]^5)/(5*d)
```

Rule 894

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ
[m, 0] && ILtQ[n, 0]))
```

Rule 3088

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[(x^m*(b +
 a*x)^n)/(1 + x^2)^((m + n + 2)/2), x], x, Cot[c + d*x]], x] /; FreeQ[{a, b
, c, d}, x] && IntegerQ[n] && IntegerQ[(m + n)/2] && NeQ[n, -1] &&  !(GtQ[n
, 0] && GtQ[m, 1])
```

Rubi steps

$$\int \sec^6(c+dx)(a\cos(c+dx)+b\sin(c+dx))^2\,dx = -\frac{\text{Subst}\left(\int \frac{(b+ax)^2\left(1+x^2\right)}{x^6}\,dx,x,\cot(c+dx)\right)}{d}$$
$$= -\frac{\text{Subst}\left(\int \left(\frac{b^2}{x^6}+\frac{2ab}{x^5}+\frac{a^2+b^2}{x^4}+\frac{2ab}{x^3}+\frac{a^2}{x^2}\right)dx,x,\cot(c+dx)\right)}{d}$$
$$= \frac{a^2\tan(c+dx)}{d}+\frac{ab\tan^2(c+dx)}{d}+\frac{\left(a^2+b^2\right)\tan^3(c+dx)}{3d}+$$

Mathematica [A] time = 0.19, size = 54, normalized size = 0.64

$$\frac{(a+b\tan(c+dx))^3\left(a^2-3ab\tan(c+dx)+6b^2\tan^2(c+dx)+10b^2\right)}{30b^3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^6*(a*Cos[c + d*x] + b*Sin[c + d*x])^2,x]
```

```
[Out] ((a + b*Tan[c + d*x])^3*(a^2 + 10*b^2 - 3*a*b*Tan[c + d*x] + 6*b^2*Tan[c +
d*x]^2))/(30*b^3*d)
```

fricas [A] time = 0.67, size = 79, normalized size = 0.93

$$\frac{15\,ab\cos(dx+c)+2\left(2\left(5\,a^2-b^2\right)\cos(dx+c)^4+\left(5\,a^2-b^2\right)\cos(dx+c)^2+3\,b^2\right)\sin(dx+c)}{30\,d\cos(dx+c)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^6*(a*cos(d*x+c)+b*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/30*(15*a*b*cos(d*x + c) + 2*(2*(5*a^2 - b^2)*cos(d*x + c)^4 + (5*a^2 - b^
2)*cos(d*x + c)^2 + 3*b^2)*sin(d*x + c))/(d*cos(d*x + c)^5)
```

giac [A] time = 0.58, size = 80, normalized size = 0.94

$$\frac{6\,b^2\tan(dx+c)^5+15\,ab\tan(dx+c)^4+10\,a^2\tan(dx+c)^3+10\,b^2\tan(dx+c)^3+30\,ab\tan(dx+c)^2+30\,a^2\tan}{30\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^6*(a*cos(d*x+c)+b*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/30*(6*b^2*tan(d*x + c)^5 + 15*a*b*tan(d*x + c)^4 + 10*a^2*tan(d*x + c)^3
+ 10*b^2*tan(d*x + c)^3 + 30*a*b*tan(d*x + c)^2 + 30*a^2*tan(d*x + c))/d
```

maple [A] time = 1.97, size = 82, normalized size = 0.96

$$\frac{-a^2\left(-\frac{2}{3}-\frac{(\sec^2(dx+c))}{3}\right)\tan(dx+c)+\frac{ab}{2\cos(dx+c)^4}+b^2\left(\frac{\sin^3(dx+c)}{5\cos(dx+c)^5}+\frac{2(\sin^3(dx+c))}{15\cos(dx+c)^3}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^6*(a*cos(d*x+c)+b*sin(d*x+c))^2,x)
```

```
[Out] 1/d*(-a^2*(-2/3-1/3*sec(d*x+c)^2)*tan(d*x+c)+1/2*a*b/cos(d*x+c)^4+b^2*(1/5*
sin(d*x+c)^3/cos(d*x+c)^5+2/15*sin(d*x+c)^3/cos(d*x+c)^3))
```

maxima [A] time = 0.34, size = 70, normalized size = 0.82

$$\frac{10\left(\tan(dx+c)^3+3\tan(dx+c)\right)a^2+2\left(3\tan(dx+c)^5+5\tan(dx+c)^3\right)b^2+\frac{15ab}{\left(\sin(dx+c)^2-1\right)^2}}{30d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^6*(a*cos(d*x+c)+b*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 1/30*(10*(tan(d*x + c)^3 + 3*tan(d*x + c))*a^2 + 2*(3*tan(d*x + c)^5 + 5*ta
n(d*x + c)^3)*b^2 + 15*a*b/(sin(d*x + c)^2 - 1)^2)/d
```

mupad [B] time = 0.62, size = 98, normalized size = 1.15

$$\frac{\frac{b^2\sin(c+dx)}{5}+\cos(c+dx)^2\left(\frac{a^2\sin(c+dx)}{3}-\frac{b^2\sin(c+dx)}{15}\right)+\cos(c+dx)^4\left(\frac{2a^2\sin(c+dx)}{3}-\frac{2b^2\sin(c+dx)}{15}\right)+\frac{ab\cos(c+dx)}{2}}{d\cos(c+dx)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*cos(c + d*x) + b*sin(c + d*x))^2/cos(c + d*x)^6,x)
```

```
[Out] ((b^2*sin(c + d*x))/5 + cos(c + d*x)^2*((a^2*sin(c + d*x))/3 - (b^2*sin(c +
 d*x))/15) + cos(c + d*x)^4*((2*a^2*sin(c + d*x))/3 - (2*b^2*sin(c + d*x))/
15) + (a*b*cos(c + d*x))/2)/(d*cos(c + d*x)^5)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**6*(a*cos(d*x+c)+b*sin(d*x+c))**2,x)

[Out] Timed out
```

3.55 $\int \sec^7(c+dx)(a\cos(c+dx)+b\sin(c+dx))^2\,dx$

Optimal. Leaf size=168

$$\frac{3a^2\tanh^{-1}(\sin(c+dx))}{8d}+\frac{a^2\tan(c+dx)\sec^3(c+dx)}{4d}+\frac{3a^2\tan(c+dx)\sec(c+dx)}{8d}+\frac{2ab\sec^5(c+dx)}{5d}-\frac{b^2\tanh^{-}}{}$$

```
[Out] 3/8*a^2*arctanh(sin(d*x+c))/d-1/16*b^2*arctanh(sin(d*x+c))/d+2/5*a*b*sec(d*
x+c)^5/d+3/8*a^2*sec(d*x+c)*tan(d*x+c)/d-1/16*b^2*sec(d*x+c)*tan(d*x+c)/d+1
/4*a^2*sec(d*x+c)^3*tan(d*x+c)/d-1/24*b^2*sec(d*x+c)^3*tan(d*x+c)/d+1/6*b^2
*sec(d*x+c)^5*tan(d*x+c)/d
```

Rubi [A] time = 0.17, antiderivative size = 168, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 6, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {3090, 3768, 3770, 2606, 30, 2611}

$$\frac{3a^2\tanh^{-1}(\sin(c+dx))}{8d}+\frac{a^2\tan(c+dx)\sec^3(c+dx)}{4d}+\frac{3a^2\tan(c+dx)\sec(c+dx)}{8d}+\frac{2ab\sec^5(c+dx)}{5d}-\frac{b^2\tanh^{-}}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^7*(a*Cos[c + d*x] + b*Sin[c + d*x])^2,x]
```

```
[Out] (3*a^2*ArcTanh[Sin[c + d*x]])/(8*d) - (b^2*ArcTanh[Sin[c + d*x]])/(16*d) +
(2*a*b*Sec[c + d*x]^5)/(5*d) + (3*a^2*Sec[c + d*x]*Tan[c + d*x])/(8*d) - (b
^2*Sec[c + d*x]*Tan[c + d*x])/(16*d) + (a^2*Sec[c + d*x]^3*Tan[c + d*x])/(4
*d) - (b^2*Sec[c + d*x]^3*Tan[c + d*x])/(24*d) + (b^2*Sec[c + d*x]^5*Tan[c
+ d*x])/(6*d)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 && !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 2611

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(b*(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*(
m + n - 1)), x] - Dist[(b^2*(n - 1))/(m + n - 1), Int[(a*Sec[e + f*x])^m*(b
```

```
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 3090

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_.), x_Symbol] :> Int[ExpandTrig[cos[c + d*x]^m*(a
*cos[c + d*x] + b*sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d}, x] && Inte
gerQ[m] && IGtQ[n, 0]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \sec^7(c+dx)(a\cos(c+dx)+b\sin(c+dx))^2\,dx &= \int \left(a^2\sec^5(c+dx)+2ab\sec^5(c+dx)\tan(c+dx)+b^2\sec^5(\\
&= a^2\int \sec^5(c+dx)\,dx+(2ab)\int \sec^5(c+dx)\tan(c+dx)\,dx\\
&= \frac{a^2\sec^3(c+dx)\tan(c+dx)}{4d}+\frac{b^2\sec^5(c+dx)\tan(c+dx)}{6d}+\\
&= \frac{2ab\sec^5(c+dx)}{5d}+\frac{3a^2\sec(c+dx)\tan(c+dx)}{8d}+\frac{a^2\sec^3(c\\
&= \frac{3a^2\tanh^{-1}(\sin(c+dx))}{8d}+\frac{2ab\sec^5(c+dx)}{5d}+\frac{3a^2\sec(c+d\\
&= \frac{3a^2\tanh^{-1}(\sin(c+dx))}{8d}-\frac{b^2\tanh^{-1}(\sin(c+dx))}{16d}+\frac{2ab\sec\end{aligned}$$

Mathematica [A] time = 0.59, size = 104, normalized size = 0.62

$$\frac{15\left(6a^2-b^2\right)\tanh^{-1}(\sin(c+dx))+10\left(6a^2-b^2\right)\tan(c+dx)\sec^3(c+dx)+15\left(6a^2-b^2\right)\tan(c+dx)\sec(c+dx}{240d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^7*(a*Cos[c + d*x] + b*Sin[c + d*x])^2,x]
```

```
[Out] (15*(6*a^2 - b^2)*ArcTanh[Sin[c + d*x]] + 15*(6*a^2 - b^2)*Sec[c + d*x]*Tan
[c + d*x] + 10*(6*a^2 - b^2)*Sec[c + d*x]^3*Tan[c + d*x] + 8*b*Sec[c + d*x]
^5*(12*a + 5*b*Tan[c + d*x]))/(240*d)
```

fricas [A] time = 0.53, size = 142, normalized size = 0.85

$$\frac{15\left(6\,a^2-b^2\right)\cos\left(dx+c\right)^6\log\left(\sin\left(dx+c\right)+1\right)-15\left(6\,a^2-b^2\right)\cos\left(dx+c\right)^6\log\left(-\sin\left(dx+c\right)+1\right)+192\,ab\,c}{480\,d\cos\left(dx+c\right)^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^7*(a*cos(d*x+c)+b*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/480*(15*(6*a^2 - b^2)*cos(d*x + c)^6*log(sin(d*x + c) + 1) - 15*(6*a^2 -
b^2)*cos(d*x + c)^6*log(-sin(d*x + c) + 1) + 192*a*b*cos(d*x + c) + 10*(3*(
6*a^2 - b^2)*cos(d*x + c)^4 + 2*(6*a^2 - b^2)*cos(d*x + c)^2 + 8*b^2)*sin(d
*x + c))/(d*cos(d*x + c)^6)
```

giac [B] time = 0.53, size = 343, normalized size = 2.04

$$15\left(6\,a^2-b^2\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-15\left(6\,a^2-b^2\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)+\frac{2\left(150\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{11}+15\,b\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^7*(a*cos(d*x+c)+b*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/240*(15*(6*a^2 - b^2)*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 15*(6*a^2 - b^
2)*log(abs(tan(1/2*d*x + 1/2*c) - 1)) + 2*(150*a^2*tan(1/2*d*x + 1/2*c)^11
+ 15*b^2*tan(1/2*d*x + 1/2*c)^11 - 480*a*b*tan(1/2*d*x + 1/2*c)^10 - 210*a^
2*tan(1/2*d*x + 1/2*c)^9 + 235*b^2*tan(1/2*d*x + 1/2*c)^9 + 480*a*b*tan(1/2
*d*x + 1/2*c)^8 + 60*a^2*tan(1/2*d*x + 1/2*c)^7 + 390*b^2*tan(1/2*d*x + 1/2
*c)^7 - 960*a*b*tan(1/2*d*x + 1/2*c)^6 + 60*a^2*tan(1/2*d*x + 1/2*c)^5 + 39
0*b^2*tan(1/2*d*x + 1/2*c)^5 + 960*a*b*tan(1/2*d*x + 1/2*c)^4 - 210*a^2*tan
(1/2*d*x + 1/2*c)^3 + 235*b^2*tan(1/2*d*x + 1/2*c)^3 - 96*a*b*tan(1/2*d*x +
 1/2*c)^2 + 150*a^2*tan(1/2*d*x + 1/2*c) + 15*b^2*tan(1/2*d*x + 1/2*c) + 96
*a*b)/(tan(1/2*d*x + 1/2*c)^2 - 1)^6)/d
```

maple [A] time = 10.99, size = 189, normalized size = 1.12

$$\frac{a^2\left(\sec^3\left(dx+c\right)\right)\tan\left(dx+c\right)}{4d}+\frac{3a^2\sec\left(dx+c\right)\tan\left(dx+c\right)}{8d}+\frac{3a^2\ln\left(\sec\left(dx+c\right)+\tan\left(dx+c\right)\right)}{8d}+\frac{2ab}{5d\cos\left(dx+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^7*(a*cos(d*x+c)+b*sin(d*x+c))^2,x)
```

```
[Out] 1/4*a^2*sec(d*x+c)^3*tan(d*x+c)/d+3/8*a^2*sec(d*x+c)*tan(d*x+c)/d+3/8/d*a^2
*ln(sec(d*x+c)+tan(d*x+c))+2/5/d*a*b/cos(d*x+c)^5+1/6/d*b^2*sin(d*x+c)^3/co
s(d*x+c)^6+1/8/d*b^2*sin(d*x+c)^3/cos(d*x+c)^4+1/16/d*b^2*sin(d*x+c)^3/cos(
d*x+c)^2+1/16*b^2*sin(d*x+c)/d-1/16/d*b^2*ln(sec(d*x+c)+tan(d*x+c))
```

maxima [A] time = 0.34, size = 180, normalized size = 1.07

$$\frac{5\,b^2\left(\frac{2\left(3\,\sin(dx+c)^5-8\,\sin(dx+c)^3-3\,\sin(dx+c)\right)}{\sin(dx+c)^6-3\,\sin(dx+c)^4+3\,\sin(dx+c)^2-1}-3\,\log\left(\sin\left(dx+c\right)+1\right)+3\,\log\left(\sin\left(dx+c\right)-1\right)\right)-30\,a^2\left(\frac{2\left(3\,\sin(dx+c)^3-\cdots\right)}{\sin(dx+c)^4-2\,\sin\cdots}\right.}{480\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^7*(a*cos(d*x+c)+b*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 1/480*(5*b^2*(2*(3*sin(d*x + c)^5 - 8*sin(d*x + c)^3 - 3*sin(d*x + c))/(sin
(d*x + c)^6 - 3*sin(d*x + c)^4 + 3*sin(d*x + c)^2 - 1) - 3*log(sin(d*x + c)
 + 1) + 3*log(sin(d*x + c) - 1)) - 30*a^2*(2*(3*sin(d*x + c)^3 - 5*sin(d*x
+ c))/(sin(d*x + c)^4 - 2*sin(d*x + c)^2 + 1) - 3*log(sin(d*x + c) + 1) + 3
*log(sin(d*x + c) - 1)) + 192*a*b/cos(d*x + c)^5)/d
```

mupad [B] time = 3.26, size = 328, normalized size = 1.95

$$\frac{\left(\frac{5\,a^2}{4}+\frac{b^2}{8}\right)\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{11}-4\,a\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{10}+\left(\frac{47\,b^2}{24}-\frac{7\,a^2}{4}\right)\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{9}+4\,a\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{8}+\left(\frac{a^2}{2}+\frac{13\,b^2}{4}\cdots\right.}{d\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{12}-6\tan\left(\frac{c}{2}+\cdots\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*cos(c + d*x) + b*sin(c + d*x))^2/cos(c + d*x)^7,x)
```

```
[Out] ((4*a*b)/5 + tan(c/2 + (d*x)/2)^5*(a^2/2 + (13*b^2)/4) + tan(c/2 + (d*x)/2)
^7*(a^2/2 + (13*b^2)/4) + tan(c/2 + (d*x)/2)^11*((5*a^2)/4 + b^2/8) - tan(c
/2 + (d*x)/2)^3*((7*a^2)/4 - (47*b^2)/24) - tan(c/2 + (d*x)/2)^9*((7*a^2)/4
 - (47*b^2)/24) + tan(c/2 + (d*x)/2)*((5*a^2)/4 + b^2/8) - (4*a*b*tan(c/2 +
 (d*x)/2)^2)/5 + 8*a*b*tan(c/2 + (d*x)/2)^4 - 8*a*b*tan(c/2 + (d*x)/2)^6 +
4*a*b*tan(c/2 + (d*x)/2)^8 - 4*a*b*tan(c/2 + (d*x)/2)^10)/(d*(15*tan(c/2 +
(d*x)/2)^4 - 6*tan(c/2 + (d*x)/2)^2 - 20*tan(c/2 + (d*x)/2)^6 + 15*tan(c/2
+ (d*x)/2)^8 - 6*tan(c/2 + (d*x)/2)^10 + tan(c/2 + (d*x)/2)^12 + 1)) + (ata
nh(tan(c/2 + (d*x)/2))*((3*a^2)/4 - b^2/8))/d
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**7*(a*cos(d*x+c)+b*sin(d*x+c))**2,x)
[Out] Timed out
```

3.56 $\int \sec^8(c+dx)(a\cos(c+dx)+b\sin(c+dx))^2\,dx$

Optimal. Leaf size=125

$$\frac{(a^2+2b^2)\tan^5(c+dx)}{5d}+\frac{(2a^2+b^2)\tan^3(c+dx)}{3d}+\frac{a^2\tan(c+dx)}{d}+\frac{ab\tan^6(c+dx)}{3d}+\frac{ab\tan^4(c+dx)}{d}+\frac{ab\tan^2(c+\cdots}{d}$$

```
[Out] a^2*tan(d*x+c)/d+a*b*tan(d*x+c)^2/d+1/3*(2*a^2+b^2)*tan(d*x+c)^3/d+a*b*tan(
d*x+c)^4/d+1/5*(a^2+2*b^2)*tan(d*x+c)^5/d+1/3*a*b*tan(d*x+c)^6/d+1/7*b^2*ta
n(d*x+c)^7/d
```

Rubi [A] time = 0.10, antiderivative size = 125, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {3088, 948}

$$\frac{(a^2+2b^2)\tan^5(c+dx)}{5d}+\frac{(2a^2+b^2)\tan^3(c+dx)}{3d}+\frac{a^2\tan(c+dx)}{d}+\frac{ab\tan^6(c+dx)}{3d}+\frac{ab\tan^4(c+dx)}{d}+\frac{ab\tan^2(c+\cdots}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^8*(a*Cos[c + d*x] + b*Sin[c + d*x])^2,x]
```

```
[Out] (a^2*Tan[c + d*x])/d + (a*b*Tan[c + d*x]^2)/d + ((2*a^2 + b^2)*Tan[c + d*x]
^3)/(3*d) + (a*b*Tan[c + d*x]^4)/d + ((a^2 + 2*b^2)*Tan[c + d*x]^5)/(5*d) +
 (a*b*Tan[c + d*x]^6)/(3*d) + (b^2*Tan[c + d*x]^7)/(7*d)
```

Rule 948

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IGtQ[p, 0] && (IGtQ[m, 0] || (EqQ[m, -2] && EqQ[p, 1] &
& EqQ[d, 0]))
```

Rule 3088

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[(x^m*(b +
 a*x)^n)/(1 + x^2)^((m + n + 2)/2), x], x, Cot[c + d*x]], x] /; FreeQ[{a, b
, c, d}, x] && IntegerQ[n] && IntegerQ[(m + n)/2] && NeQ[n, -1] &&  !(GtQ[n
, 0] && GtQ[m, 1])
```

Rubi steps

$$\int \sec^8(c+dx)(a\cos(c+dx)+b\sin(c+dx))^2\,dx = -\frac{\text{Subst}\left(\int \frac{(b+ax)^2\left(1+x^2\right)^2}{x^8}\,dx, x, \cot(c+dx)\right)}{d}$$
$$= -\frac{\text{Subst}\left(\int \left(\frac{b^2}{x^8}+\frac{2ab}{x^7}+\frac{a^2+2b^2}{x^6}+\frac{4ab}{x^5}+\frac{2a^2+b^2}{x^4}+\frac{2ab}{x^3}+\frac{a^2}{x^2}\right) dx}{d}$$
$$= \frac{a^2\tan(c+dx)}{d}+\frac{ab\tan^2(c+dx)}{d}+\frac{\left(2a^2+b^2\right)\tan^3(c+dx}{3d}$$

Mathematica [A] time = 0.69, size = 104, normalized size = 0.83

$$\frac{\tan(c+dx)\left(21\left(a^2+2b^2\right)\tan^4(c+dx)+35\left(2a^2+b^2\right)\tan^2(c+dx)+105a^2+35ab\tan^5(c+dx)+105ab\tan^3(}{105d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^8*(a*Cos[c + d*x] + b*Sin[c + d*x])^2,x]
```

```
[Out] (Tan[c + d*x]*(105*a^2 + 105*a*b*Tan[c + d*x] + 35*(2*a^2 + b^2)*Tan[c + d*
x]^2 + 105*a*b*Tan[c + d*x]^3 + 21*(a^2 + 2*b^2)*Tan[c + d*x]^4 + 35*a*b*Ta
n[c + d*x]^5 + 15*b^2*Tan[c + d*x]^6))/(105*d)
```

fricas [A] time = 0.67, size = 100, normalized size = 0.80

$$\frac{35\,ab\cos(dx+c)+\left(8\left(7\,a^2-b^2\right)\cos(dx+c)^6+4\left(7\,a^2-b^2\right)\cos(dx+c)^4+3\left(7\,a^2-b^2\right)\cos(dx+c)^2+15\,b^2}{105\,d\cos(dx+c)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^8*(a*cos(d*x+c)+b*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/105*(35*a*b*cos(d*x + c) + (8*(7*a^2 - b^2)*cos(d*x + c)^6 + 4*(7*a^2 - b
^2)*cos(d*x + c)^4 + 3*(7*a^2 - b^2)*cos(d*x + c)^2 + 15*b^2)*sin(d*x + c))
/(d*cos(d*x + c)^7)
```

giac [A] time = 0.31, size = 118, normalized size = 0.94

$$\frac{15\,b^2\tan(dx+c)^7+35\,ab\tan(dx+c)^6+21\,a^2\tan(dx+c)^5+42\,b^2\tan(dx+c)^5+105\,ab\tan(dx+c)^4+70\,a}{105\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^8*(a*cos(d*x+c)+b*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/105*(15*b^2*tan(d*x + c)^7 + 35*a*b*tan(d*x + c)^6 + 21*a^2*tan(d*x + c)^
5 + 42*b^2*tan(d*x + c)^5 + 105*a*b*tan(d*x + c)^4 + 70*a^2*tan(d*x + c)^3
+ 35*b^2*tan(d*x + c)^3 + 105*a*b*tan(d*x + c)^2 + 105*a^2*tan(d*x + c))/d
```

maple [A] time = 2.10, size = 110, normalized size = 0.88

$$\frac{-a^2\left(-\frac{8}{15}-\frac{\left(\sec^4(dx+c)\right)}{5}-\frac{4\left(\sec^2(dx+c)\right)}{15}\right)\tan(dx+c)+\frac{ab}{3\cos(dx+c)^6}+b^2\left(\frac{\sin^3(dx+c)}{7\cos(dx+c)^7}+\frac{4\left(\sin^3(dx+c)\right)}{35\cos(dx+c)^5}+\frac{8\left(\sin^3(dx+c)\right)}{105\cos(dx+c)^3}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^8*(a*cos(d*x+c)+b*sin(d*x+c))^2,x)
```

```
[Out] 1/d*(-a^2*(-8/15-1/5*sec(d*x+c)^4-4/15*sec(d*x+c)^2)*tan(d*x+c)+1/3*a*b/cos
(d*x+c)^6+b^2*(1/7*sin(d*x+c)^3/cos(d*x+c)^7+4/35*sin(d*x+c)^3/cos(d*x+c)^5
+8/105*sin(d*x+c)^3/cos(d*x+c)^3))
```

maxima [A] time = 0.34, size = 91, normalized size = 0.73

$$\frac{7\left(3\tan(dx+c)^5+10\tan(dx+c)^3+15\tan(dx+c)\right)a^2+\left(15\tan(dx+c)^7+42\tan(dx+c)^5+35\tan(dx+\right.}{105\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^8*(a*cos(d*x+c)+b*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 1/105*(7*(3*tan(d*x + c)^5 + 10*tan(d*x + c)^3 + 15*tan(d*x + c))*a^2 + (15
*tan(d*x + c)^7 + 42*tan(d*x + c)^5 + 35*tan(d*x + c)^3)*b^2 - 35*a*b/(sin(
d*x + c)^2 - 1)^3)/d
```

mupad [B] time = 0.83, size = 130, normalized size = 1.04

$$\frac{\frac{b^2\sin(c+d\,x)}{7}+\cos(c+d\,x)^2\left(\frac{a^2\sin(c+d\,x)}{5}-\frac{b^2\sin(c+d\,x)}{35}\right)+\cos(c+d\,x)^4\left(\frac{4\,a^2\sin(c+d\,x)}{15}-\frac{4\,b^2\sin(c+d\,x)}{105}\right)+\cos(c+d\,x}{d\cos(c+d\,x)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*cos(c + d*x) + b*sin(c + d*x))^2/cos(c + d*x)^8,x)
```

```
[Out] ((b^2*sin(c + d*x))/7 + cos(c + d*x)^2*((a^2*sin(c + d*x))/5 - (b^2*sin(c +
 d*x))/35) + cos(c + d*x)^4*((4*a^2*sin(c + d*x))/15 - (4*b^2*sin(c + d*x))
/105) + cos(c + d*x)^6*((8*a^2*sin(c + d*x))/15 - (8*b^2*sin(c + d*x))/105)
 + (a*b*cos(c + d*x))/3)/(d*cos(c + d*x)^7)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**8*(a*cos(d*x+c)+b*sin(d*x+c))**2,x)
[Out] Timed out
```

3.57 $\int \cos^5(c+dx)(a\cos(c+dx)+b\sin(c+dx))^3\,dx$

Optimal. Leaf size=265

$$\frac{a^3\sin(c+dx)\cos^7(c+dx)}{8d}+\frac{7a^3\sin(c+dx)\cos^5(c+dx)}{48d}+\frac{35a^3\sin(c+dx)\cos^3(c+dx)}{192d}+\frac{35a^3\sin(c+dx)\cos(c}{128d}$$

```
[Out] 35/128*a^3*x+15/128*a*b^2*x-1/6*b^3*cos(d*x+c)^6/d-3/8*a^2*b*cos(d*x+c)^8/d
+1/8*b^3*cos(d*x+c)^8/d+35/128*a^3*cos(d*x+c)*sin(d*x+c)/d+15/128*a*b^2*cos
(d*x+c)*sin(d*x+c)/d+35/192*a^3*cos(d*x+c)^3*sin(d*x+c)/d+5/64*a*b^2*cos(d*
x+c)^3*sin(d*x+c)/d+7/48*a^3*cos(d*x+c)^5*sin(d*x+c)/d+1/16*a*b^2*cos(d*x+c
)^5*sin(d*x+c)/d+1/8*a^3*cos(d*x+c)^7*sin(d*x+c)/d-3/8*a*b^2*cos(d*x+c)^7*s
in(d*x+c)/d
```

Rubi [A] time = 0.25, antiderivative size = 265, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 7, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {3090, 2635, 8, 2565, 30, 2568, 14}

$$-\frac{3a^2b\cos^8(c+dx)}{8d}+\frac{a^3\sin(c+dx)\cos^7(c+dx)}{8d}+\frac{7a^3\sin(c+dx)\cos^5(c+dx)}{48d}+\frac{35a^3\sin(c+dx)\cos^3(c+dx)}{192d}+3$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^5*(a*Cos[c + d*x] + b*Sin[c + d*x])^3,x]
```

```
[Out] (35*a^3*x)/128 + (15*a*b^2*x)/128 - (b^3*Cos[c + d*x]^6)/(6*d) - (3*a^2*b*C
os[c + d*x]^8)/(8*d) + (b^3*Cos[c + d*x]^8)/(8*d) + (35*a^3*Cos[c + d*x]*Si
n[c + d*x])/(128*d) + (15*a*b^2*Cos[c + d*x]*Sin[c + d*x])/(128*d) + (35*a^
3*Cos[c + d*x]^3*Sin[c + d*x])/(192*d) + (5*a*b^2*Cos[c + d*x]^3*Sin[c + d*
x])/(64*d) + (7*a^3*Cos[c + d*x]^5*Sin[c + d*x])/(48*d) + (a*b^2*Cos[c + d*
x]^5*Sin[c + d*x])/(16*d) + (a^3*Cos[c + d*x]^7*Sin[c + d*x])/(8*d) - (3*a*
b^2*Cos[c + d*x]^7*Sin[c + d*x])/(8*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2565

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> -Dist[(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 2568

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> -Simp[(a*(b*Cos[e + f*x])^(n + 1)*(a*Sin[e + f*x])^(m - 1)
)/(b*f*(m + n)), x] + Dist[(a^2*(m - 1))/(m + n), Int[(b*Cos[e + f*x])^n*(a
*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1] &&
 NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 3090

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_.), x_Symbol] :> Int[ExpandTrig[cos[c + d*x]^m*(a
*cos[c + d*x] + b*sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d}, x] && Inte
gerQ[m] && IGtQ[n, 0]
```

Rubi steps

$$\int \cos^5(c+dx)(a\cos(c+dx)+b\sin(c+dx))^3\,dx = \int \left(a^3\cos^8(c+dx)+3a^2b\cos^7(c+dx)\sin(c+dx)+3ab^2\,\text{c}\right.$$
$$= a^3\int \cos^8(c+dx)\,dx + \left(3a^2b\right)\int \cos^7(c+dx)\sin(c+dx)\,dx$$
$$= \frac{a^3\cos^7(c+dx)\sin(c+dx)}{8d} - \frac{3ab^2\cos^7(c+dx)\sin(c+dx)}{8d}$$
$$= -\frac{3a^2b\cos^8(c+dx)}{8d} + \frac{7a^3\cos^5(c+dx)\sin(c+dx)}{48d} + \frac{ab^2\,\text{co}}{}$$
$$= -\frac{b^3\cos^6(c+dx)}{6d} - \frac{3a^2b\cos^8(c+dx)}{8d} + \frac{b^3\cos^8(c+dx)}{8d} + 3$$
$$= -\frac{b^3\cos^6(c+dx)}{6d} - \frac{3a^2b\cos^8(c+dx)}{8d} + \frac{b^3\cos^8(c+dx)}{8d} + 3$$
$$= \frac{35a^3x}{128} + \frac{15}{128}ab^2x - \frac{b^3\cos^6(c+dx)}{6d} - \frac{3a^2b\cos^8(c+dx)}{8d} + $$

Mathematica [A] time = 0.47, size = 235, normalized size = 0.89

$$\frac{5a\left(7a^2+3b^2\right)(c+dx)}{128d} + \frac{a\left(14a^2+3b^2\right)\sin(2(c+dx))}{64d} + \frac{a\left(7a^2-3b^2\right)\sin(4(c+dx))}{128d} + \frac{a\left(2a^2-3b^2\right)\sin(6(c+dx)}{192d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^5*(a*Cos[c + d*x] + b*Sin[c + d*x])^3,x]
```

```
[Out] (5*a*(7*a^2 + 3*b^2)*(c + d*x))/(128*d) - (3*b*(7*a^2 + b^2)*Cos[2*(c + d*x
)])/(128*d) - (b*(21*a^2 + b^2)*Cos[4*(c + d*x)])/(256*d) - (b*(9*a^2 - b^2
)*Cos[6*(c + d*x)])/(384*d) - (b*(3*a^2 - b^2)*Cos[8*(c + d*x)])/(1024*d) +
 (a*(14*a^2 + 3*b^2)*Sin[2*(c + d*x)])/(64*d) + (a*(7*a^2 - 3*b^2)*Sin[4*(c
 + d*x)])/(128*d) + (a*(2*a^2 - 3*b^2)*Sin[6*(c + d*x)])/(192*d) + (a*(a^2
- 3*b^2)*Sin[8*(c + d*x)])/(1024*d)
```

fricas [A] time = 0.72, size = 150, normalized size = 0.57

$$-\frac{64\,b^3\cos(dx+c)^6 + 48\left(3\,a^2b-b^3\right)\cos(dx+c)^8 - 15\left(7\,a^3+3\,ab^2\right)dx - \left(48\left(a^3-3\,ab^2\right)\cos(dx+c)^7 + 8\left(7\,a^3\right.\right.}{384\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(a*cos(d*x+c)+b*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] -1/384*(64*b^3*cos(d*x + c)^6 + 48*(3*a^2*b - b^3)*cos(d*x + c)^8 - 15*(7*a
^3 + 3*a*b^2)*d*x - (48*(a^3 - 3*a*b^2)*cos(d*x + c)^7 + 8*(7*a^3 + 3*a*b^2
```

)*cos(d*x + c)^5 + 10*(7*a^3 + 3*a*b^2)*cos(d*x + c)^3 + 15*(7*a^3 + 3*a*b^
2)*cos(d*x + c))*sin(d*x + c))/d

giac [A] time = 0.35, size = 218, normalized size = 0.82

$$\frac{5}{128}\left(7\,a^3+3\,ab^2\right)x-\frac{\left(3\,a^2b-b^3\right)\cos\left(8\,dx+8\,c\right)}{1024\,d}-\frac{\left(9\,a^2b-b^3\right)\cos\left(6\,dx+6\,c\right)}{384\,d}-\frac{\left(21\,a^2b+b^3\right)\cos\left(4\,dx+4\,c\right)}{256\,d}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(a*cos(d*x+c)+b*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 5/128*(7*a^3 + 3*a*b^2)*x - 1/1024*(3*a^2*b - b^3)*cos(8*d*x + 8*c)/d - 1/3
84*(9*a^2*b - b^3)*cos(6*d*x + 6*c)/d - 1/256*(21*a^2*b + b^3)*cos(4*d*x +
4*c)/d - 3/128*(7*a^2*b + b^3)*cos(2*d*x + 2*c)/d + 1/1024*(a^3 - 3*a*b^2)*
sin(8*d*x + 8*c)/d + 1/192*(2*a^3 - 3*a*b^2)*sin(6*d*x + 6*c)/d + 1/128*(7*
a^3 - 3*a*b^2)*sin(4*d*x + 4*c)/d + 1/64*(14*a^3 + 3*a*b^2)*sin(2*d*x + 2*c
)/d
```

maple [A] time = 11.01, size = 175, normalized size = 0.66

$$\frac{b^3\left(-\frac{\left(\sin^2(dx+c)\right)\left(\cos^6(dx+c)\right)}{8}-\frac{\left(\cos^6(dx+c)\right)}{24}\right)+3b^2a\left(-\frac{\sin(dx+c)\left(\cos^7(dx+c)\right)}{8}+\frac{\left(\cos^5(dx+c)+\frac{5\left(\cos^3(dx+c)\right)}{4}+\frac{15\cos(dx+c)}{8}\right)\sin(dx+c)}{48}+\right.}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*(a*cos(d*x+c)+b*sin(d*x+c))^3,x)
```

```
[Out] 1/d*(b^3*(-1/8*sin(d*x+c)^2*cos(d*x+c)^6-1/24*cos(d*x+c)^6)+3*b^2*a*(-1/8*s
in(d*x+c)*cos(d*x+c)^7+1/48*(cos(d*x+c)^5+5/4*cos(d*x+c)^3+15/8*cos(d*x+c))
*sin(d*x+c)+5/128*d*x+5/128*c)-3/8*a^2*b*cos(d*x+c)^8+a^3*(1/8*(cos(d*x+c)^
7+7/6*cos(d*x+c)^5+35/24*cos(d*x+c)^3+35/16*cos(d*x+c))*sin(d*x+c)+35/128*d
*x+35/128*c))
```

maxima [A] time = 0.34, size = 163, normalized size = 0.62

$$-\frac{1152\,a^2b\cos\left(dx+c\right)^8+\left(128\,\sin\left(2\,dx+2\,c\right)^3-840\,dx-840\,c-3\,\sin\left(8\,dx+8\,c\right)-168\,\sin\left(4\,dx+4\,c\right)-76\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(a*cos(d*x+c)+b*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -1/3072*(1152*a^2*b*cos(d*x + c)^8 + (128*sin(2*d*x + 2*c)^3 - 840*d*x - 84
0*c - 3*sin(8*d*x + 8*c) - 168*sin(4*d*x + 4*c) - 768*sin(2*d*x + 2*c))*a^3
```

```
 - 3*(64*sin(2*d*x + 2*c)^3 + 120*d*x + 120*c - 3*sin(8*d*x + 8*c) - 24*sin
(4*d*x + 4*c))*a*b^2 - 128*(3*sin(d*x + c)^8 - 8*sin(d*x + c)^6 + 6*sin(d*x
 + c)^4)*b^3)/d
```

mupad [B] time = 2.29, size = 523, normalized size = 1.97

$$\frac{4\,b^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\left(\frac{15\,a\,b^2}{64}-\frac{93\,a^3}{64}\right)+\frac{40\,b^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8}{3}+4\,b^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{12}+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{15}\left(\frac{15\,a\,b^2}{64}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^5*(a*cos(c + d*x) + b*sin(c + d*x))^3,x)
```

```
[Out] (4*b^3*tan(c/2 + (d*x)/2)^4 - tan(c/2 + (d*x)/2)*((15*a*b^2)/64 - (93*a^3)/
64) + (40*b^3*tan(c/2 + (d*x)/2)^8)/3 + 4*b^3*tan(c/2 + (d*x)/2)^12 + tan(c
/2 + (d*x)/2)^15*((15*a*b^2)/64 - (93*a^3)/64) + tan(c/2 + (d*x)/2)^3*((397
*a*b^2)/64 + (91*a^3)/192) - tan(c/2 + (d*x)/2)^13*((397*a*b^2)/64 + (91*a^
3)/192) - tan(c/2 + (d*x)/2)^5*((895*a*b^2)/64 - (1799*a^3)/192) + tan(c/2
+ (d*x)/2)^11*((895*a*b^2)/64 - (1799*a^3)/192) + tan(c/2 + (d*x)/2)^7*((17
65*a*b^2)/64 - (1085*a^3)/192) - tan(c/2 + (d*x)/2)^9*((1765*a*b^2)/64 - (1
085*a^3)/192) + tan(c/2 + (d*x)/2)^6*(42*a^2*b - (16*b^3)/3) + tan(c/2 + (d
*x)/2)^10*(42*a^2*b - (16*b^3)/3) + 6*a^2*b*tan(c/2 + (d*x)/2)^2 + 6*a^2*b*
tan(c/2 + (d*x)/2)^14)/(d*(8*tan(c/2 + (d*x)/2)^2 + 28*tan(c/2 + (d*x)/2)^4
 + 56*tan(c/2 + (d*x)/2)^6 + 70*tan(c/2 + (d*x)/2)^8 + 56*tan(c/2 + (d*x)/2
)^10 + 28*tan(c/2 + (d*x)/2)^12 + 8*tan(c/2 + (d*x)/2)^14 + tan(c/2 + (d*x)
/2)^16 + 1)) + (5*a*atan((5*a*tan(c/2 + (d*x)/2)*(7*a^2 + 3*b^2))/(64*((15*
a*b^2)/64 + (35*a^3)/64)))*(7*a^2 + 3*b^2))/(64*d) - (5*a*(7*a^2 + 3*b^2)*(
atan(tan(c/2 + (d*x)/2)) - (d*x)/2))/(64*d)
```

sympy [A] time = 10.33, size = 532, normalized size = 2.01

$$\begin{cases} \frac{35a^3x\sin^8(c+dx)}{128}+\frac{35a^3x\sin^6(c+dx)\cos^2(c+dx)}{32}+\frac{105a^3x\sin^4(c+dx)\cos^4(c+dx)}{64}+\frac{35a^3x\sin^2(c+dx)\cos^6(c+dx)}{32}+\frac{35a^3x\cos^8(c+dx)}{128}+ \\ x\left(a\cos(c)+b\sin(c)\right)^3\cos^5(c) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*(a*cos(d*x+c)+b*sin(d*x+c))**3,x)
```

```
[Out] Piecewise((35*a**3*x*sin(c + d*x)**8/128 + 35*a**3*x*sin(c + d*x)**6*cos(c
+ d*x)**2/32 + 105*a**3*x*sin(c + d*x)**4*cos(c + d*x)**4/64 + 35*a**3*x*si
n(c + d*x)**2*cos(c + d*x)**6/32 + 35*a**3*x*cos(c + d*x)**8/128 + 35*a**3*
```

```
sin(c + d*x)**7*cos(c + d*x)/(128*d) + 385*a**3*sin(c + d*x)**5*cos(c + d*x
)**3/(384*d) + 511*a**3*sin(c + d*x)**3*cos(c + d*x)**5/(384*d) + 93*a**3*s
in(c + d*x)*cos(c + d*x)**7/(128*d) - 3*a**2*b*cos(c + d*x)**8/(8*d) + 15*a
*b**2*x*sin(c + d*x)**8/128 + 15*a*b**2*x*sin(c + d*x)**6*cos(c + d*x)**2/3
2 + 45*a*b**2*x*sin(c + d*x)**4*cos(c + d*x)**4/64 + 15*a*b**2*x*sin(c + d*
x)**2*cos(c + d*x)**6/32 + 15*a*b**2*x*cos(c + d*x)**8/128 + 15*a*b**2*sin(
c + d*x)**7*cos(c + d*x)/(128*d) + 55*a*b**2*sin(c + d*x)**5*cos(c + d*x)**
3/(128*d) + 73*a*b**2*sin(c + d*x)**3*cos(c + d*x)**5/(128*d) - 15*a*b**2*s
in(c + d*x)*cos(c + d*x)**7/(128*d) + b**3*sin(c + d*x)**8/(24*d) + b**3*si
n(c + d*x)**6*cos(c + d*x)**2/(6*d) + b**3*sin(c + d*x)**4*cos(c + d*x)**4/
(4*d), Ne(d, 0)), (x*(a*cos(c) + b*sin(c))**3*cos(c)**5, True))
```

3.58 $\int \cos^4(c+dx)(a\cos(c+dx)+b\sin(c+dx))^3\,dx$

Optimal. Leaf size=175

$$-\frac{a^3\sin^7(c+dx)}{7d}+\frac{3a^3\sin^5(c+dx)}{5d}-\frac{a^3\sin^3(c+dx)}{d}+\frac{a^3\sin(c+dx)}{d}-\frac{3a^2b\cos^7(c+dx)}{7d}+\frac{3ab^2\sin^7(c+dx)}{7d}-\frac{6ab^2}{}$$

```
[Out] -1/5*b^3*cos(d*x+c)^5/d-3/7*a^2*b*cos(d*x+c)^7/d+1/7*b^3*cos(d*x+c)^7/d+a^3
*sin(d*x+c)/d-a^3*sin(d*x+c)^3/d+a*b^2*sin(d*x+c)^3/d+3/5*a^3*sin(d*x+c)^5/
d-6/5*a*b^2*sin(d*x+c)^5/d-1/7*a^3*sin(d*x+c)^7/d+3/7*a*b^2*sin(d*x+c)^7/d
```

Rubi [A] time = 0.18, antiderivative size = 175, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 7, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {3090, 2633, 2565, 30, 2564, 270, 14}

$$-\frac{3a^2b\cos^7(c+dx)}{7d}-\frac{a^3\sin^7(c+dx)}{7d}+\frac{3a^3\sin^5(c+dx)}{5d}-\frac{a^3\sin^3(c+dx)}{d}+\frac{a^3\sin(c+dx)}{d}+\frac{3ab^2\sin^7(c+dx)}{7d}-\frac{6ab^2}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^4*(a*Cos[c + d*x] + b*Sin[c + d*x])^3,x]
```

```
[Out] -(b^3*Cos[c + d*x]^5)/(5*d) - (3*a^2*b*Cos[c + d*x]^7)/(7*d) + (b^3*Cos[c +
 d*x]^7)/(7*d) + (a^3*Sin[c + d*x])/d - (a^3*Sin[c + d*x]^3)/d + (a*b^2*Sin
[c + d*x]^3)/d + (3*a^3*Sin[c + d*x]^5)/(5*d) - (6*a*b^2*Sin[c + d*x]^5)/(5
*d) - (a^3*Sin[c + d*x]^7)/(7*d) + (3*a*b^2*Sin[c + d*x]^7)/(7*d)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2564

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 2565

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> -Dist[(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 3090

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_.), x_Symbol] :> Int[ExpandTrig[cos[c + d*x]^m*(a
*cos[c + d*x] + b*sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d}, x] && Inte
gerQ[m] && IGtQ[n, 0]
```

Rubi steps

$$\int \cos^4(c+dx)(a\cos(c+dx)+b\sin(c+dx))^3\,dx = \int \left(a^3\cos^7(c+dx)+3a^2b\cos^6(c+dx)\sin(c+dx)+3ab^2\right.$$

$$= a^3\int \cos^7(c+dx)\,dx+\left(3a^2b\right)\int \cos^6(c+dx)\sin(c+dx)\,d$$

$$= -\frac{a^3\,\mathrm{Subst}\left(\int\left(1-3x^2+3x^4-x^6\right)dx,x,-\sin(c+dx)\right)}{d}-$$

$$= -\frac{3a^2b\cos^7(c+dx)}{7d}+\frac{a^3\sin(c+dx)}{d}-\frac{a^3\sin^3(c+dx)}{d}+\frac{3}{}$$

$$= -\frac{b^3\cos^5(c+dx)}{5d}-\frac{3a^2b\cos^7(c+dx)}{7d}+\frac{b^3\cos^7(c+dx)}{7d}+$$

Mathematica [A] time = 0.42, size = 204, normalized size = 1.17

$$1225a^3\sin(c+dx)+245a^3\sin(3(c+dx))+49a^3\sin(5(c+dx))+5a^3\sin(7(c+dx))-35\left(9a^2b+b^3\right)\cos(3(c+$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^4*(a*Cos[c + d*x] + b*Sin[c + d*x])^3,x]
```

```
[Out] (-105*b*(5*a^2 + b^2)*Cos[c + d*x] - 35*(9*a^2*b + b^3)*Cos[3*(c + d*x)] -
105*a^2*b*Cos[5*(c + d*x)] + 7*b^3*Cos[5*(c + d*x)] - 15*a^2*b*Cos[7*(c + d
*x)] + 5*b^3*Cos[7*(c + d*x)] + 1225*a^3*Sin[c + d*x] + 525*a*b^2*Sin[c + d
*x] + 245*a^3*Sin[3*(c + d*x)] - 35*a*b^2*Sin[3*(c + d*x)] + 49*a^3*Sin[5*(
c + d*x)] - 63*a*b^2*Sin[5*(c + d*x)] + 5*a^3*Sin[7*(c + d*x)] - 15*a*b^2*S
in[7*(c + d*x)])/(2240*d)
```

fricas [A] time = 0.70, size = 123, normalized size = 0.70

$$-\frac{7\,b^3\cos\left(dx+c\right)^5+5\left(3\,a^2b-b^3\right)\cos\left(dx+c\right)^7-\left(5\left(a^3-3\,ab^2\right)\cos\left(dx+c\right)^6+3\left(2\,a^3+ab^2\right)\cos\left(dx+c\right)^4+1\right.}{35\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a*cos(d*x+c)+b*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] -1/35*(7*b^3*cos(d*x + c)^5 + 5*(3*a^2*b - b^3)*cos(d*x + c)^7 - (5*(a^3 -
3*a*b^2)*cos(d*x + c)^6 + 3*(2*a^3 + a*b^2)*cos(d*x + c)^4 + 16*a^3 + 8*a*b
^2 + 4*(2*a^3 + a*b^2)*cos(d*x + c)^2)*sin(d*x + c))/d
```

giac [A] time = 0.32, size = 197, normalized size = 1.13

$$-\frac{\left(3\,a^2b-b^3\right)\cos\left(7\,dx+7\,c\right)}{448\,d}-\frac{\left(15\,a^2b-b^3\right)\cos\left(5\,dx+5\,c\right)}{320\,d}-\frac{\left(9\,a^2b+b^3\right)\cos\left(3\,dx+3\,c\right)}{64\,d}-\frac{3\left(5\,a^2b+b^3\right)\cos\left(dx\right.}{64\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a*cos(d*x+c)+b*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -1/448*(3*a^2*b - b^3)*cos(7*d*x + 7*c)/d - 1/320*(15*a^2*b - b^3)*cos(5*d*
x + 5*c)/d - 1/64*(9*a^2*b + b^3)*cos(3*d*x + 3*c)/d - 3/64*(5*a^2*b + b^3)
*cos(d*x + c)/d + 1/448*(a^3 - 3*a*b^2)*sin(7*d*x + 7*c)/d + 1/320*(7*a^3 -
 9*a*b^2)*sin(5*d*x + 5*c)/d + 1/64*(7*a^3 - a*b^2)*sin(3*d*x + 3*c)/d + 5/
64*(7*a^3 + 3*a*b^2)*sin(d*x + c)/d
```

maple [A] time = 10.38, size = 145, normalized size = 0.83

$$\frac{b^3\left(-\frac{\left(\sin^2(dx+c)\right)\left(\cos^5(dx+c)\right)}{7}-\frac{2\left(\cos^5(dx+c)\right)}{35}\right)+3b^2a\left(-\frac{\sin(dx+c)\left(\cos^6(dx+c)\right)}{7}+\frac{\left(\frac{8}{3}+\cos^4(dx+c)+\frac{4\left(\cos^2(dx+c)\right)}{3}\right)\sin(dx+c)}{35}\right)-\frac{3a^2b\left(\cos\right.}{}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*(a*cos(d*x+c)+b*sin(d*x+c))^3,x)
```

```
[Out] 1/d*(b^3*(-1/7*sin(d*x+c)^2*cos(d*x+c)^5-2/35*cos(d*x+c)^5)+3*b^2*a*(-1/7*s
in(d*x+c)*cos(d*x+c)^6+1/35*(8/3+cos(d*x+c)^4+4/3*cos(d*x+c)^2)*sin(d*x+c))
-3/7*a^2*b*cos(d*x+c)^7+1/7*a^3*(16/5+cos(d*x+c)^6+6/5*cos(d*x+c)^4+8/5*cos
(d*x+c)^2)*sin(d*x+c))
```

maxima [A] time = 0.32, size = 126, normalized size = 0.72

$$-\frac{15\,a^2b\cos\left(dx+c\right)^7+\left(5\,\sin\left(dx+c\right)^7-21\,\sin\left(dx+c\right)^5+35\,\sin\left(dx+c\right)^3-35\,\sin\left(dx+c\right)\right)a^3-\left(15\,\sin\left(dx\right.\right.}{35\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a*cos(d*x+c)+b*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -1/35*(15*a^2*b*cos(d*x + c)^7 + (5*sin(d*x + c)^7 - 21*sin(d*x + c)^5 + 35
*sin(d*x + c)^3 - 35*sin(d*x + c))*a^3 - (15*sin(d*x + c)^7 - 42*sin(d*x +
c)^5 + 35*sin(d*x + c)^3)*a*b^2 - (5*cos(d*x + c)^7 - 7*cos(d*x + c)^5)*b^3
)/d
```

mupad [B] time = 0.78, size = 214, normalized size = 1.22

$$\frac{16\,a^3\,\sin\left(c+d\,x\right)}{35\,d}-\frac{b^3\cos\left(c+d\,x\right)^5}{5\,d}+\frac{b^3\cos\left(c+d\,x\right)^7}{7\,d}-\frac{3\,a^2\,b\cos\left(c+d\,x\right)^7}{7\,d}+\frac{8\,a^3\cos\left(c+d\,x\right)^2\,\sin\left(c+d\,x\right)}{35\,d}+\frac{6}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^4*(a*cos(c + d*x) + b*sin(c + d*x))^3,x)
```

```
[Out] (16*a^3*sin(c + d*x))/(35*d) - (b^3*cos(c + d*x)^5)/(5*d) + (b^3*cos(c + d*
x)^7)/(7*d) - (3*a^2*b*cos(c + d*x)^7)/(7*d) + (8*a^3*cos(c + d*x)^2*sin(c
+ d*x))/(35*d) + (6*a^3*cos(c + d*x)^4*sin(c + d*x))/(35*d) + (a^3*cos(c +
d*x)^6*sin(c + d*x))/(7*d) + (8*a*b^2*sin(c + d*x))/(35*d) + (4*a*b^2*cos(c
 + d*x)^2*sin(c + d*x))/(35*d) + (3*a*b^2*cos(c + d*x)^4*sin(c + d*x))/(35*
d) - (3*a*b^2*cos(c + d*x)^6*sin(c + d*x))/(7*d)
```

sympy [A] time = 5.53, size = 233, normalized size = 1.33

$$\begin{cases}\frac{16a^3\sin^7\left(c+dx\right)}{35d}+\frac{8a^3\sin^5\left(c+dx\right)\cos^2\left(c+dx\right)}{5d}+\frac{2a^3\sin^3\left(c+dx\right)\cos^4\left(c+dx\right)}{d}+\frac{a^3\sin\left(c+dx\right)\cos^6\left(c+dx\right)}{d}-\frac{3a^2b\cos^7\left(c+dx\right)}{7d}+\frac{8ab^2\sin^7\left(c+\right.}{35d} \\ x\left(a\cos\left(c\right)+b\sin\left(c\right)\right)^3\cos^4\left(c\right)\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*(a*cos(d*x+c)+b*sin(d*x+c))**3,x)
```

```
[Out] Piecewise((16*a**3*sin(c + d*x)**7/(35*d) + 8*a**3*sin(c + d*x)**5*cos(c +
d*x)**2/(5*d) + 2*a**3*sin(c + d*x)**3*cos(c + d*x)**4/d + a**3*sin(c + d*x
)*cos(c + d*x)**6/d - 3*a**2*b*cos(c + d*x)**7/(7*d) + 8*a*b**2*sin(c + d*x
)**7/(35*d) + 4*a*b**2*sin(c + d*x)**5*cos(c + d*x)**2/(5*d) + a*b**2*sin(c
 + d*x)**3*cos(c + d*x)**4/d - b**3*sin(c + d*x)**2*cos(c + d*x)**5/(5*d) -
 2*b**3*cos(c + d*x)**7/(35*d), Ne(d, 0)), (x*(a*cos(c) + b*sin(c))**3*cos(
c)**4, True))
```

3.59 $\int \cos^3(c+dx)(a\cos(c+dx)+b\sin(c+dx))^3\,dx$

Optimal. Leaf size=216

$$\frac{a^3\sin(c+dx)\cos^5(c+dx)}{6d}+\frac{5a^3\sin(c+dx)\cos^3(c+dx)}{24d}+\frac{5a^3\sin(c+dx)\cos(c+dx)}{16d}+\frac{5a^3x}{16}-\frac{a^2b\cos^6(c+dx)}{2d}$$

```
[Out] 5/16*a^3*x+3/16*a*b^2*x-1/2*a^2*b*cos(d*x+c)^6/d+5/16*a^3*cos(d*x+c)*sin(d*
x+c)/d+3/16*a*b^2*cos(d*x+c)*sin(d*x+c)/d+5/24*a^3*cos(d*x+c)^3*sin(d*x+c)/
d+1/8*a*b^2*cos(d*x+c)^3*sin(d*x+c)/d+1/6*a^3*cos(d*x+c)^5*sin(d*x+c)/d-1/2
*a*b^2*cos(d*x+c)^5*sin(d*x+c)/d+1/4*b^3*sin(d*x+c)^4/d-1/6*b^3*sin(d*x+c)^
6/d
```

Rubi [A] time = 0.21, antiderivative size = 216, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 8, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3090, 2635, 8, 2565, 30, 2568, 2564, 14}

$$-\frac{a^2b\cos^6(c+dx)}{2d}+\frac{a^3\sin(c+dx)\cos^5(c+dx)}{6d}+\frac{5a^3\sin(c+dx)\cos^3(c+dx)}{24d}+\frac{5a^3\sin(c+dx)\cos(c+dx)}{16d}+\frac{5a^3}{16}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^3*(a*Cos[c + d*x] + b*Sin[c + d*x])^3,x]
```

```
[Out] (5*a^3*x)/16 + (3*a*b^2*x)/16 - (a^2*b*Cos[c + d*x]^6)/(2*d) + (5*a^3*Cos[c
 + d*x]*Sin[c + d*x])/(16*d) + (3*a*b^2*Cos[c + d*x]*Sin[c + d*x])/(16*d) +
 (5*a^3*Cos[c + d*x]^3*Sin[c + d*x])/(24*d) + (a*b^2*Cos[c + d*x]^3*Sin[c +
 d*x])/(8*d) + (a^3*Cos[c + d*x]^5*Sin[c + d*x])/(6*d) - (a*b^2*Cos[c + d*x
]^5*Sin[c + d*x])/(2*d) + (b^3*Sin[c + d*x]^4)/(4*d) - (b^3*Sin[c + d*x]^6)
/(6*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2564

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 2565

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> -Dist[(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 2568

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> -Simp[(a*(b*Cos[e + f*x])^(n + 1)*(a*Sin[e + f*x])^(m - 1)
)/(b*f*(m + n)), x] + Dist[(a^2*(m - 1))/(m + n), Int[(b*Cos[e + f*x])^n*(a
*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1] &&
 NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 3090

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_.), x_Symbol] :> Int[ExpandTrig[cos[c + d*x]^m*(a
*cos[c + d*x] + b*sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d}, x] && Inte
gerQ[m] && IGtQ[n, 0]
```

Rubi steps

$$\int \cos^3(c+dx)(a\cos(c+dx)+b\sin(c+dx))^3\,dx = \int \left(a^3\cos^6(c+dx)+3a^2b\cos^5(c+dx)\sin(c+dx)+3ab^2\right.$$
$$= a^3\int \cos^6(c+dx)\,dx + \left(3a^2b\right)\int \cos^5(c+dx)\sin(c+dx)$$
$$= \frac{a^3\cos^5(c+dx)\sin(c+dx)}{6d} - \frac{ab^2\cos^5(c+dx)\sin(c+dx)}{2d}$$
$$= -\frac{a^2b\cos^6(c+dx)}{2d} + \frac{5a^3\cos^3(c+dx)\sin(c+dx)}{24d} + \frac{ab^2\cos}{}$$
$$= -\frac{a^2b\cos^6(c+dx)}{2d} + \frac{5a^3\cos(c+dx)\sin(c+dx)}{16d} + \frac{3ab^2\cos}{}$$
$$= \frac{5a^3x}{16} + \frac{3}{16}ab^2x - \frac{a^2b\cos^6(c+dx)}{2d} + \frac{5a^3\cos(c+dx)\sin(c}{16d}$$

Mathematica [A] time = 0.30, size = 171, normalized size = 0.79

$$\frac{a\left(5a^2+3b^2\right)(c+dx)}{16d}+\frac{3a\left(5a^2+b^2\right)\sin(2(c+dx))}{64d}+\frac{3a\left(a^2-b^2\right)\sin(4(c+dx))}{64d}+\frac{a\left(a^2-3b^2\right)\sin(6(c+dx))}{192d}-$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^3*(a*Cos[c + d*x] + b*Sin[c + d*x])^3,x]
```

```
[Out] (a*(5*a^2 + 3*b^2)*(c + d*x))/(16*d) - (3*b*(5*a^2 + b^2)*Cos[2*(c + d*x)])
/(64*d) - (3*a^2*b*Cos[4*(c + d*x)])/(32*d) - (b*(3*a^2 - b^2)*Cos[6*(c + d
*x)])/(192*d) + (3*a*(5*a^2 + b^2)*Sin[2*(c + d*x)])/(64*d) + (3*a*(a^2 - b
^2)*Sin[4*(c + d*x)])/(64*d) + (a*(a^2 - 3*b^2)*Sin[6*(c + d*x)])/(192*d)
```

fricas [A] time = 0.59, size = 128, normalized size = 0.59

$$-\frac{12\,b^3\cos(dx+c)^4+8\left(3\,a^2b-b^3\right)\cos(dx+c)^6-3\left(5\,a^3+3\,ab^2\right)dx-\left(8\left(a^3-3\,ab^2\right)\cos(dx+c)^5+2\left(5\,a^3+\right.\right.}{48\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a*cos(d*x+c)+b*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] -1/48*(12*b^3*cos(d*x + c)^4 + 8*(3*a^2*b - b^3)*cos(d*x + c)^6 - 3*(5*a^3
+ 3*a*b^2)*d*x - (8*(a^3 - 3*a*b^2)*cos(d*x + c)^5 + 2*(5*a^3 + 3*a*b^2)*co
s(d*x + c)^3 + 3*(5*a^3 + 3*a*b^2)*cos(d*x + c))*sin(d*x + c))/d
```

giac [A] time = 0.36, size = 157, normalized size = 0.73

$$-\frac{3\,a^2b\cos(4\,dx+4\,c)}{32\,d}+\frac{1}{16}\left(5\,a^3+3\,ab^2\right)x-\frac{\left(3\,a^2b-b^3\right)\cos(6\,dx+6\,c)}{192\,d}-\frac{3\left(5\,a^2b+b^3\right)\cos(2\,dx+2\,c)}{64\,d}+\frac{\left(a^3-\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a*cos(d*x+c)+b*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -3/32*a^2*b*cos(4*d*x + 4*c)/d + 1/16*(5*a^3 + 3*a*b^2)*x - 1/192*(3*a^2*b
- b^3)*cos(6*d*x + 6*c)/d - 3/64*(5*a^2*b + b^3)*cos(2*d*x + 2*c)/d + 1/192
*(a^3 - 3*a*b^2)*sin(6*d*x + 6*c)/d + 3/64*(a^3 - a*b^2)*sin(4*d*x + 4*c)/d
 + 3/64*(5*a^3 + a*b^2)*sin(2*d*x + 2*c)/d
```

maple [A] time = 9.72, size = 155, normalized size = 0.72

$$\frac{b^3\left(-\frac{(\sin^2(dx+c))(\cos^4(dx+c))}{6}-\frac{(\cos^4(dx+c))}{12}\right)+3b^2a\left(-\frac{(\cos^5(dx+c))\sin(dx+c)}{6}+\frac{\left(\cos^3(dx+c)+\frac{3\cos(dx+c)}{2}\right)\sin(dx+c)}{24}+\frac{dx}{16}+\frac{c}{16}\right)-\cdots}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3*(a*cos(d*x+c)+b*sin(d*x+c))^3,x)
```

```
[Out] 1/d*(b^3*(-1/6*sin(d*x+c)^2*cos(d*x+c)^4-1/12*cos(d*x+c)^4)+3*b^2*a*(-1/6*c
os(d*x+c)^5*sin(d*x+c)+1/24*(cos(d*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+1/16*d
*x+1/16*c)-1/2*a^2*b*cos(d*x+c)^6+a^3*(1/6*(cos(d*x+c)^5+5/4*cos(d*x+c)^3+1
5/8*cos(d*x+c))*sin(d*x+c)+5/16*d*x+5/16*c))
```

maxima [A] time = 0.34, size = 131, normalized size = 0.61

$$-\frac{96\,a^2b\cos(dx+c)^6+\left(4\,\sin(2\,dx+2\,c)^3-60\,dx-60\,c-9\,\sin(4\,dx+4\,c)-48\,\sin(2\,dx+2\,c)\right)a^3-3\left(4\,\sin\cdots\right.}{192\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a*cos(d*x+c)+b*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -1/192*(96*a^2*b*cos(d*x + c)^6 + (4*sin(2*d*x + 2*c)^3 - 60*d*x - 60*c - 9
*sin(4*d*x + 4*c) - 48*sin(2*d*x + 2*c))*a^3 - 3*(4*sin(2*d*x + 2*c)^3 + 12
*d*x + 12*c - 3*sin(4*d*x + 4*c))*a*b^2 + 16*(2*sin(d*x + c)^6 - 3*sin(d*x
+ c)^4)*b^3)/d
```

mupad [B] time = 2.03, size = 407, normalized size = 1.88

$$\frac{4\,b^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\left(\frac{3\,a\,b^2}{8}-\frac{11\,a^3}{8}\right)+4\,b^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{11}\left(\frac{3\,a\,b^2}{8}-\frac{11\,a^3}{8}\right)-\tan\left(\frac{c}{2}+\cdots\right.}{d\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{12}+\cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^3*(a*cos(c + d*x) + b*sin(c + d*x))^3,x)
```

```
[Out] (4*b^3*tan(c/2 + (d*x)/2)^4 - tan(c/2 + (d*x)/2)*((3*a*b^2)/8 - (11*a^3)/8)
 + 4*b^3*tan(c/2 + (d*x)/2)^8 + tan(c/2 + (d*x)/2)^11*((3*a*b^2)/8 - (11*a^
3)/8) - tan(c/2 + (d*x)/2)^5*((39*a*b^2)/4 - (15*a^3)/4) + tan(c/2 + (d*x)/
2)^7*((39*a*b^2)/4 - (15*a^3)/4) + tan(c/2 + (d*x)/2)^3*((47*a*b^2)/8 - (5*
a^3)/24) - tan(c/2 + (d*x)/2)^9*((47*a*b^2)/8 - (5*a^3)/24) + tan(c/2 + (d*
x)/2)^6*(20*a^2*b - (8*b^3)/3) + 6*a^2*b*tan(c/2 + (d*x)/2)^2 + 6*a^2*b*tan
(c/2 + (d*x)/2)^10)/(d*(6*tan(c/2 + (d*x)/2)^2 + 15*tan(c/2 + (d*x)/2)^4 +
20*tan(c/2 + (d*x)/2)^6 + 15*tan(c/2 + (d*x)/2)^8 + 6*tan(c/2 + (d*x)/2)^10
 + tan(c/2 + (d*x)/2)^12 + 1)) + (a*atan((a*tan(c/2 + (d*x)/2)*(5*a^2 + 3*b
^2))/(8*((3*a*b^2)/8 + (5*a^3)/8)))*(5*a^2 + 3*b^2))/(8*d) - (a*(5*a^2 + 3*
b^2)*(atan(tan(c/2 + (d*x)/2)) - (d*x)/2))/(8*d)
```

sympy [A] time = 3.69, size = 400, normalized size = 1.85

$$\begin{cases} \frac{5a^3x\sin^6(c+dx)}{16} + \frac{15a^3x\sin^4(c+dx)\cos^2(c+dx)}{16} + \frac{15a^3x\sin^2(c+dx)\cos^4(c+dx)}{16} + \frac{5a^3x\cos^6(c+dx)}{16} + \frac{5a^3\sin^5(c+dx)\cos(c+dx)}{16d} + \frac{5a^3 \sin}{} \\ x(a\cos(c)+b\sin(c))^3\cos^3(c) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3*(a*cos(d*x+c)+b*sin(d*x+c))**3,x)
```

```
[Out] Piecewise((5*a**3*x*sin(c + d*x)**6/16 + 15*a**3*x*sin(c + d*x)**4*cos(c +
d*x)**2/16 + 15*a**3*x*sin(c + d*x)**2*cos(c + d*x)**4/16 + 5*a**3*x*cos(c
+ d*x)**6/16 + 5*a**3*sin(c + d*x)**5*cos(c + d*x)/(16*d) + 5*a**3*sin(c +
d*x)**3*cos(c + d*x)**3/(6*d) + 11*a**3*sin(c + d*x)*cos(c + d*x)**5/(16*d)
 - a**2*b*cos(c + d*x)**6/(2*d) + 3*a*b**2*x*sin(c + d*x)**6/16 + 9*a*b**2*
x*sin(c + d*x)**4*cos(c + d*x)**2/16 + 9*a*b**2*x*sin(c + d*x)**2*cos(c + d
*x)**4/16 + 3*a*b**2*x*cos(c + d*x)**6/16 + 3*a*b**2*sin(c + d*x)**5*cos(c
+ d*x)/(16*d) + a*b**2*sin(c + d*x)**3*cos(c + d*x)**3/(2*d) - 3*a*b**2*sin
(c + d*x)*cos(c + d*x)**5/(16*d) + b**3*sin(c + d*x)**6/(12*d) + b**3*sin(c
 + d*x)**4*cos(c + d*x)**2/(4*d), Ne(d, 0)), (x*(a*cos(c) + b*sin(c))**3*co
s(c)**3, True))
```

3.60 $\int \cos^2(c+dx)(a\cos(c+dx)+b\sin(c+dx))^3\,dx$

Optimal. Leaf size=140

$$\frac{a^3\sin^5(c+dx)}{5d}-\frac{2a^3\sin^3(c+dx)}{3d}+\frac{a^3\sin(c+dx)}{d}-\frac{3a^2b\cos^5(c+dx)}{5d}-\frac{3ab^2\sin^5(c+dx)}{5d}+\frac{ab^2\sin^3(c+dx)}{d}+\frac{b^3\text{cc}}{}$$

```
[Out] -1/3*b^3*cos(d*x+c)^3/d-3/5*a^2*b*cos(d*x+c)^5/d+1/5*b^3*cos(d*x+c)^5/d+a^3
*sin(d*x+c)/d-2/3*a^3*sin(d*x+c)^3/d+a*b^2*sin(d*x+c)^3/d+1/5*a^3*sin(d*x+c
)^5/d-3/5*a*b^2*sin(d*x+c)^5/d
```

Rubi [A] time = 0.16, antiderivative size = 140, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 6, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {3090, 2633, 2565, 30, 2564, 14}

$$-\frac{3a^2b\cos^5(c+dx)}{5d}+\frac{a^3\sin^5(c+dx)}{5d}-\frac{2a^3\sin^3(c+dx)}{3d}+\frac{a^3\sin(c+dx)}{d}-\frac{3ab^2\sin^5(c+dx)}{5d}+\frac{ab^2\sin^3(c+dx)}{d}+\frac{b^3\text{c}}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2*(a*Cos[c + d*x] + b*Sin[c + d*x])^3,x]
```

```
[Out] -(b^3*Cos[c + d*x]^3)/(3*d) - (3*a^2*b*Cos[c + d*x]^5)/(5*d) + (b^3*Cos[c +
 d*x]^5)/(5*d) + (a^3*Sin[c + d*x])/d - (2*a^3*Sin[c + d*x]^3)/(3*d) + (a*b
^2*Sin[c + d*x]^3)/d + (a^3*Sin[c + d*x]^5)/(5*d) - (3*a*b^2*Sin[c + d*x]^5
)/(5*d)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2564

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 2565

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> -Dist[(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 3090

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_.), x_Symbol] :> Int[ExpandTrig[cos[c + d*x]^m*(a
*cos[c + d*x] + b*sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d}, x] && Inte
gerQ[m] && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}\int \cos^2(c+dx)(a\cos(c+dx)+b\sin(c+dx))^3\,dx &= \int \left(a^3\cos^5(c+dx)+3a^2b\cos^4(c+dx)\sin(c+dx)+3ab^2\right.\\ &= a^3\int \cos^5(c+dx)\,dx+\left(3a^2b\right)\int \cos^4(c+dx)\sin(c+dx)\\ &= -\frac{a^3\,\text{Subst}\left(\int\left(1-2x^2+x^4\right)dx,x,-\sin(c+dx)\right)}{d}-\frac{\left(3a^2b\right)}{}\\ &= -\frac{3a^2b\cos^5(c+dx)}{5d}+\frac{a^3\sin(c+dx)}{d}-\frac{2a^3\sin^3(c+dx)}{3d}+\\ &= -\frac{b^3\cos^3(c+dx)}{3d}-\frac{3a^2b\cos^5(c+dx)}{5d}+\frac{b^3\cos^5(c+dx)}{5d}+\end{aligned}$$

Mathematica [A] time = 0.29, size = 150, normalized size = 1.07

$$\frac{150a^3\sin(c+dx)+25a^3\sin(3(c+dx))+3a^3\sin(5(c+dx))-5\left(9a^2b+b^3\right)\cos(3(c+dx))-30b\left(3a^2+b^2\right)\cos}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2*(a*Cos[c + d*x] + b*Sin[c + d*x])^3,x]
```

```
[Out] (-30*b*(3*a^2 + b^2)*Cos[c + d*x] - 5*(9*a^2*b + b^3)*Cos[3*(c + d*x)] - 9*
a^2*b*Cos[5*(c + d*x)] + 3*b^3*Cos[5*(c + d*x)] + 150*a^3*Sin[c + d*x] + 90
```

```
*a*b^2*Sin[c + d*x] + 25*a^3*Sin[3*(c + d*x)] - 15*a*b^2*Sin[3*(c + d*x)] +
 3*a^3*Sin[5*(c + d*x)] - 9*a*b^2*Sin[5*(c + d*x)])/(240*d)
```

fricas [A] time = 0.70, size = 102, normalized size = 0.73

$$-\frac{5\,b^3\cos\left(dx+c\right)^3+3\left(3\,a^2b-b^3\right)\cos\left(dx+c\right)^5-\left(3\left(a^3-3\,ab^2\right)\cos\left(dx+c\right)^4+8\,a^3+6\,ab^2+\left(4\,a^3+3\,ab^2\right)\cos}{15\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a*cos(d*x+c)+b*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] -1/15*(5*b^3*cos(d*x + c)^3 + 3*(3*a^2*b - b^3)*cos(d*x + c)^5 - (3*(a^3 -
3*a*b^2)*cos(d*x + c)^4 + 8*a^3 + 6*a*b^2 + (4*a^3 + 3*a*b^2)*cos(d*x + c)^
2)*sin(d*x + c))/d
```

giac [A] time = 0.48, size = 145, normalized size = 1.04

$$-\frac{\left(3\,a^2b-b^3\right)\cos\left(5\,dx+5\,c\right)}{80\,d}-\frac{\left(9\,a^2b+b^3\right)\cos\left(3\,dx+3\,c\right)}{48\,d}-\frac{\left(3\,a^2b+b^3\right)\cos\left(dx+c\right)}{8\,d}+\frac{\left(a^3-3\,ab^2\right)\sin\left(5\,dx+5\right.}{80\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a*cos(d*x+c)+b*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -1/80*(3*a^2*b - b^3)*cos(5*d*x + 5*c)/d - 1/48*(9*a^2*b + b^3)*cos(3*d*x +
 3*c)/d - 1/8*(3*a^2*b + b^3)*cos(d*x + c)/d + 1/80*(a^3 - 3*a*b^2)*sin(5*d
*x + 5*c)/d + 1/48*(5*a^3 - 3*a*b^2)*sin(3*d*x + 3*c)/d + 1/8*(5*a^3 + 3*a*
b^2)*sin(d*x + c)/d
```

maple [A] time = 10.50, size = 125, normalized size = 0.89

$$\frac{b^3\left(-\frac{\left(\sin^2(dx+c)\right)\left(\cos^3(dx+c)\right)}{5}-\frac{2\left(\cos^3(dx+c)\right)}{15}\right)+3b^2a\left(-\frac{\sin(dx+c)\left(\cos^4(dx+c)\right)}{5}+\frac{\left(2+\cos^2(dx+c)\right)\sin(dx+c)}{15}\right)-\frac{3a^2b\left(\cos^5(dx+c)\right)}{5}+-\frac{a}{}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(a*cos(d*x+c)+b*sin(d*x+c))^3,x)
```

```
[Out] 1/d*(b^3*(-1/5*sin(d*x+c)^2*cos(d*x+c)^3-2/15*cos(d*x+c)^3)+3*b^2*a*(-1/5*s
in(d*x+c)*cos(d*x+c)^4+1/15*(2+cos(d*x+c)^2)*sin(d*x+c))-3/5*a^2*b*cos(d*x+
c)^5+1/5*a^3*(8/3+cos(d*x+c)^4+4/3*cos(d*x+c)^2)*sin(d*x+c))
```

maxima [A] time = 0.32, size = 107, normalized size = 0.76

$$-\frac{9\,a^2b\cos\left(dx+c\right)^5-\left(3\,\sin\left(dx+c\right)^5-10\,\sin\left(dx+c\right)^3+15\,\sin\left(dx+c\right)\right)a^3+3\left(3\,\sin\left(dx+c\right)^5-5\,\sin\left(dx+c\right.}{15\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a*cos(d*x+c)+b*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -1/15*(9*a^2*b*cos(d*x + c)^5 - (3*sin(d*x + c)^5 - 10*sin(d*x + c)^3 + 15*
sin(d*x + c))*a^3 + 3*(3*sin(d*x + c)^5 - 5*sin(d*x + c)^3)*a*b^2 - (3*cos(
d*x + c)^5 - 5*cos(d*x + c)^3)*b^3)/d
```

mupad [B] time = 0.70, size = 147, normalized size = 1.05

$$\frac{2\left(\frac{3\,\sin(c+d\,x)\,a^3\cos(c+d\,x)^4}{2}+2\,\sin\left(c+d\,x\right)\,a^3\cos\left(c+d\,x\right)^2+4\,\sin\left(c+d\,x\right)\,a^3-\frac{9\,a^2\,b\cos(c+d\,x)^5}{2}-\frac{9\,\sin(c+d\,x)\,a\,b^2\cos}{2}\right.}{15\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^2*(a*cos(c + d*x) + b*sin(c + d*x))^3,x)
```

```
[Out] (2*(4*a^3*sin(c + d*x) - (5*b^3*cos(c + d*x)^3)/2 + (3*b^3*cos(c + d*x)^5)/
2 - (9*a^2*b*cos(c + d*x)^5)/2 + 2*a^3*cos(c + d*x)^2*sin(c + d*x) + (3*a^3
*cos(c + d*x)^4*sin(c + d*x))/2 + 3*a*b^2*sin(c + d*x) + (3*a*b^2*cos(c + d
*x)^2*sin(c + d*x))/2 - (9*a*b^2*cos(c + d*x)^4*sin(c + d*x))/2))/(15*d)
```

sympy [A] time = 1.93, size = 182, normalized size = 1.30

$$\begin{cases}\frac{8a^3\sin^5(c+dx)}{15d}+\frac{4a^3\sin^3(c+dx)\cos^2(c+dx)}{3d}+\frac{a^3\sin(c+dx)\cos^4(c+dx)}{d}-\frac{3a^2b\cos^5(c+dx)}{5d}+\frac{2ab^2\sin^5(c+dx)}{5d}+\frac{ab^2\sin^3(c+dx)\cos^2(c+d}{d} \\ x\left(a\cos(c)+b\sin(c)\right)^3\cos^2(c)\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(a*cos(d*x+c)+b*sin(d*x+c))**3,x)
```

```
[Out] Piecewise((8*a**3*sin(c + d*x)**5/(15*d) + 4*a**3*sin(c + d*x)**3*cos(c + d
*x)**2/(3*d) + a**3*sin(c + d*x)*cos(c + d*x)**4/d - 3*a**2*b*cos(c + d*x)*
*5/(5*d) + 2*a*b**2*sin(c + d*x)**5/(5*d) + a*b**2*sin(c + d*x)**3*cos(c +
d*x)**2/d - b**3*sin(c + d*x)**2*cos(c + d*x)**3/(3*d) - 2*b**3*cos(c + d*x
)**5/(15*d), Ne(d, 0)), (x*(a*cos(c) + b*sin(c))**3*cos(c)**2, True))
```

3.61 $\int \cos(c+dx)(a\cos(c+dx)+b\sin(c+dx))^3\,dx$

Optimal. Leaf size=78

$$\frac{3}{8}ax\left(a^2+b^2\right)+\frac{\sin^4(c+dx)(a\cot(c+dx)+b)^3}{4d}+\frac{3a\sin^2(c+dx)(a\cot(c+dx)+b)(a-b\cot(c+dx))}{8d}$$

```
[Out] 3/8*a*(a^2+b^2)*x+3/8*a*(b+a*cot(d*x+c))*(a-b*cot(d*x+c))*sin(d*x+c)^2/d+1/
4*(b+a*cot(d*x+c))^3*sin(d*x+c)^4/d
```

Rubi [A] time = 0.06, antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {3088, 805, 723, 203}

$$\frac{3}{8}ax\left(a^2+b^2\right)+\frac{\sin^4(c+dx)(a\cot(c+dx)+b)^3}{4d}+\frac{3a\sin^2(c+dx)(a\cot(c+dx)+b)(a-b\cot(c+dx))}{8d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]*(a*Cos[c + d*x] + b*Sin[c + d*x])^3,x]
```

```
[Out] (3*a*(a^2 + b^2)*x)/8 + (3*a*(b + a*Cot[c + d*x])*(a - b*Cot[c + d*x])*Sin[
c + d*x]^2)/(8*d) + ((b + a*Cot[c + d*x])^3*Sin[c + d*x]^4)/(4*d)
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 723

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
((d + e*x)^(m - 1)*(a*e - c*d*x)*(a + c*x^2)^(p + 1))/(2*a*c*(p + 1)), x] +
 Dist[((2*p + 3)*(c*d^2 + a*e^2))/(2*a*c*(p + 1)), Int[(d + e*x)^(m - 2)*(a
 + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0
] && EqQ[m + 2*p + 2, 0] && LtQ[p, -1]
```

Rule 805

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> Simp[((d + e*x)^m*(a + c*x^2)^(p + 1)*(a*g - c*f*x))/(2*a*
c*(p + 1)), x] - Dist[(m*(c*d*f + a*e*g))/(2*a*c*(p + 1)), Int[(d + e*x)^(m
 - 1)*(a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*
d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0] && LtQ[p, -1]
```

Rule 3088

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[(x^m*(b +
 a*x)^n)/(1 + x^2)^((m + n + 2)/2), x], x, Cot[c + d*x]], x] /; FreeQ[{a, b
, c, d}, x] && IntegerQ[n] && IntegerQ[(m + n)/2] && NeQ[n, -1] &&  !(GtQ[n
, 0] && GtQ[m, 1])
```

Rubi steps

$$\int \cos(c+dx)(a\cos(c+dx)+b\sin(c+dx))^3\,dx = -\frac{\text{Subst}\left(\int \frac{x(b+ax)^3}{\left(1+x^2\right)^3}\,dx, x, \cot(c+dx)\right)}{d}$$

$$= \frac{(b+a\cot(c+dx))^3\sin^4(c+dx)}{4d} - \frac{(3a)\,\text{Subst}\left(\int \frac{(b+ax)^2}{\left(1+x^2\right)^2}\,dx,\right.}{4d}$$

$$= \frac{3a(b+a\cot(c+dx))(a-b\cot(c+dx))\sin^2(c+dx)}{8d} + \frac{(b+}{}$$

$$= \frac{3}{8}a\left(a^2+b^2\right)x + \frac{3a(b+a\cot(c+dx))(a-b\cot(c+dx))\sin}{8d}$$

Mathematica [A] time = 0.39, size = 94, normalized size = 1.21

$$\frac{8a^3\sin(2(c+dx)) - 4\left(3a^2b+b^3\right)\cos(2(c+dx)) + \left(b^3-3a^2b\right)\cos(4(c+dx)) + 12a\left(a^2+b^2\right)(c+dx) + a\left(a^2-\right.}{32d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]*(a*Cos[c + d*x] + b*Sin[c + d*x])^3,x]
```

```
[Out] (12*a*(a^2 + b^2)*(c + d*x) - 4*(3*a^2*b + b^3)*Cos[2*(c + d*x)] + (-3*a^2*
b + b^3)*Cos[4*(c + d*x)] + 8*a^3*Sin[2*(c + d*x)] + a*(a^2 - 3*b^2)*Sin[4*
(c + d*x)])/(32*d)
```

fricas [A] time = 0.69, size = 100, normalized size = 1.28

$$-\frac{4\,b^3\cos(dx+c)^2 + 2\left(3\,a^2b-b^3\right)\cos(dx+c)^4 - 3\left(a^3+ab^2\right)dx - \left(2\left(a^3-3\,ab^2\right)\cos(dx+c)^3 + 3\left(a^3+ab^2\right)\right.}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a*cos(d*x+c)+b*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] -1/8*(4*b^3*cos(d*x + c)^2 + 2*(3*a^2*b - b^3)*cos(d*x + c)^4 - 3*(a^3 + a*
b^2)*d*x - (2*(a^3 - 3*a*b^2)*cos(d*x + c)^3 + 3*(a^3 + a*b^2)*cos(d*x + c)
)*sin(d*x + c))/d
```

giac [**A**] time = 0.24, size = 104, normalized size = 1.33

$$\frac{a^3\sin(2\,dx+2\,c)}{4\,d}+\frac{3}{8}\left(a^3+ab^2\right)x-\frac{\left(3\,a^2b-b^3\right)\cos(4\,dx+4\,c)}{32\,d}-\frac{\left(3\,a^2b+b^3\right)\cos(2\,dx+2\,c)}{8\,d}+\frac{\left(a^3-3\,ab^2\right)\sin(4}{32\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a*cos(d*x+c)+b*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/4*a^3*sin(2*d*x + 2*c)/d + 3/8*(a^3 + a*b^2)*x - 1/32*(3*a^2*b - b^3)*cos
(4*d*x + 4*c)/d - 1/8*(3*a^2*b + b^3)*cos(2*d*x + 2*c)/d + 1/32*(a^3 - 3*a*
b^2)*sin(4*d*x + 4*c)/d
```

maple [**A**] time = 1.38, size = 114, normalized size = 1.46

$$\frac{\frac{b^3\left(\sin^4(dx+c)\right)}{4}+3b^2a\left(-\frac{\left(\cos^3(dx+c)\right)\sin(dx+c)}{4}+\frac{\cos(dx+c)\sin(dx+c)}{8}+\frac{dx}{8}+\frac{c}{8}\right)-\frac{3a^2b\left(\cos^4(dx+c)\right)}{4}+a^3\left(\frac{\left(\cos^3(dx+c)+\frac{3\cos(dx+c)}{2}\right)\mathrm{si}}{4}\right.}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(a*cos(d*x+c)+b*sin(d*x+c))^3,x)
```

```
[Out] 1/d*(1/4*b^3*sin(d*x+c)^4+3*b^2*a*(-1/4*cos(d*x+c)^3*sin(d*x+c)+1/8*cos(d*x
+c)*sin(d*x+c)+1/8*d*x+1/8*c)-3/4*a^2*b*cos(d*x+c)^4+a^3*(1/4*(cos(d*x+c)^3
+3/2*cos(d*x+c))*sin(d*x+c)+3/8*d*x+3/8*c))
```

maxima [**A**] time = 0.33, size = 91, normalized size = 1.17

$$-\frac{24\,a^2b\cos(dx+c)^4-8\,b^3\sin(dx+c)^4-(12\,dx+12\,c+\sin(4\,dx+4\,c)+8\,\sin(2\,dx+2\,c))a^3-3\,(4\,dx+4\,c-}{32\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a*cos(d*x+c)+b*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -1/32*(24*a^2*b*cos(d*x + c)^4 - 8*b^3*sin(d*x + c)^4 - (12*d*x + 12*c + si
n(4*d*x + 4*c) + 8*sin(2*d*x + 2*c))*a^3 - 3*(4*d*x + 4*c - sin(4*d*x + 4*c
))*a*b^2)/d
```

mupad [B] time = 1.63, size = 281, normalized size = 3.60

$$\frac{4\,b^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\left(\frac{3\,a\,b^2}{4}-\frac{5\,a^3}{4}\right)+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7\left(\frac{3\,a\,b^2}{4}-\frac{5\,a^3}{4}\right)+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3\left(\frac{21\,a\,b^2}{4}-\frac{3\,a^3}{4}\right)-}{d\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8+4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6+6\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4+4\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)*(a*cos(c + d*x) + b*sin(c + d*x))^3,x)
```

```
[Out] (4*b^3*tan(c/2 + (d*x)/2)^4 - tan(c/2 + (d*x)/2)*((3*a*b^2)/4 - (5*a^3)/4)
+ tan(c/2 + (d*x)/2)^7*((3*a*b^2)/4 - (5*a^3)/4) + tan(c/2 + (d*x)/2)^3*((2
1*a*b^2)/4 - (3*a^3)/4) - tan(c/2 + (d*x)/2)^5*((21*a*b^2)/4 - (3*a^3)/4) +
 6*a^2*b*tan(c/2 + (d*x)/2)^2 + 6*a^2*b*tan(c/2 + (d*x)/2)^6)/(d*(4*tan(c/2
 + (d*x)/2)^2 + 6*tan(c/2 + (d*x)/2)^4 + 4*tan(c/2 + (d*x)/2)^6 + tan(c/2 +
 (d*x)/2)^8 + 1)) - (3*a*(atan(tan(c/2 + (d*x)/2)) - (d*x)/2)*(a^2 + b^2))/
(4*d) + (3*a*atan((3*a*tan(c/2 + (d*x)/2)*(a^2 + b^2))/(4*((3*a*b^2)/4 + (3
*a^3)/4)))*(a^2 + b^2))/(4*d)
```

sympy [A] time = 1.12, size = 272, normalized size = 3.49

$$\begin{cases} \frac{3a^3x\sin^4(c+dx)}{8}+\frac{3a^3x\sin^2(c+dx)\cos^2(c+dx)}{4}+\frac{3a^3x\cos^4(c+dx)}{8}+\frac{3a^3\sin^3(c+dx)\cos(c+dx)}{8d}+\frac{5a^3\sin(c+dx)\cos^3(c+dx)}{8d}-\frac{3a^2b\cos^4(}{4d} \\ x\left(a\cos(c)+b\sin(c)\right)^3\cos(c) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a*cos(d*x+c)+b*sin(d*x+c))**3,x)
```

```
[Out] Piecewise((3*a**3*x*sin(c + d*x)**4/8 + 3*a**3*x*sin(c + d*x)**2*cos(c + d*
x)**2/4 + 3*a**3*x*cos(c + d*x)**4/8 + 3*a**3*sin(c + d*x)**3*cos(c + d*x)/
(8*d) + 5*a**3*sin(c + d*x)*cos(c + d*x)**3/(8*d) - 3*a**2*b*cos(c + d*x)**
4/(4*d) + 3*a*b**2*x*sin(c + d*x)**4/8 + 3*a*b**2*x*sin(c + d*x)**2*cos(c +
 d*x)**2/4 + 3*a*b**2*x*cos(c + d*x)**4/8 + 3*a*b**2*sin(c + d*x)**3*cos(c
+ d*x)/(8*d) - 3*a*b**2*sin(c + d*x)*cos(c + d*x)**3/(8*d) + b**3*sin(c + d
*x)**4/(4*d), Ne(d, 0)), (x*(a*cos(c) + b*sin(c))**3*cos(c), True))
```

3.62 $\int (a\cos(c+dx)+b\sin(c+dx))^3\,dx$

Optimal. Leaf size=58

$$\frac{(b\cos(c+dx)-a\sin(c+dx))^3}{3d}-\frac{\left(a^2+b^2\right)(b\cos(c+dx)-a\sin(c+dx))}{d}$$

```
[Out] -(a^2+b^2)*(b*cos(d*x+c)-a*sin(d*x+c))/d+1/3*(b*cos(d*x+c)-a*sin(d*x+c))^3/
d
```

Rubi [A] time = 0.02, antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {3072}

$$\frac{(b\cos(c+dx)-a\sin(c+dx))^3}{3d}-\frac{\left(a^2+b^2\right)(b\cos(c+dx)-a\sin(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Cos[c + d*x] + b*Sin[c + d*x])^3,x]
```

```
[Out] -(((a^2 + b^2)*(b*Cos[c + d*x] - a*Sin[c + d*x]))/d) + (b*Cos[c + d*x] - a*
Sin[c + d*x])^3/(3*d)
```

Rule 3072

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x
_Symbol] :> -Dist[d^(-1), Subst[Int[(a^2 + b^2 - x^2)^((n - 1)/2), x], x, b
*Cos[c + d*x] - a*Sin[c + d*x]], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 +
b^2, 0] && IGtQ[(n - 1)/2, 0]
```

Rubi steps

$$\begin{aligned}\int (a\cos(c+dx)+b\sin(c+dx))^3\,dx &= -\frac{\text{Subst}\left(\int \left(a^2+b^2-x^2\right)\,dx, x, b\cos(c+dx)-a\sin(c+dx)\right)}{d}\\ &= -\frac{\left(a^2+b^2\right)(b\cos(c+dx)-a\sin(c+dx))}{d}+\frac{(b\cos(c+dx)-a\sin(c+dx))^3}{3d}\end{aligned}$$

Mathematica [A] time = 0.34, size = 81, normalized size = 1.40

$$\frac{\left(b^3-3a^2b\right)\cos(3(c+dx))-9b\left(a^2+b^2\right)\cos(c+dx)+2a\sin(c+dx)\left(\left(a^2-3b^2\right)\cos(2(c+dx))+5a^2+3b^2\right)}{12d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Cos[c + d*x] + b*Sin[c + d*x])^3,x]
```

```
[Out] (-9*b*(a^2 + b^2)*Cos[c + d*x] + (-3*a^2*b + b^3)*Cos[3*(c + d*x)] + 2*a*(5
*a^2 + 3*b^2 + (a^2 - 3*b^2)*Cos[2*(c + d*x)])*Sin[c + d*x])/(12*d)
```

fricas [A] time = 0.87, size = 77, normalized size = 1.33

$$-\frac{3\,b^{3}\cos\left(dx+c\right)+\left(3\,a^{2}b-b^{3}\right)\cos\left(dx+c\right)^{3}-\left(2\,a^{3}+3\,ab^{2}+\left(a^{3}-3\,ab^{2}\right)\cos\left(dx+c\right)^{2}\right)\sin\left(dx+c\right)}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cos(d*x+c)+b*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] -1/3*(3*b^3*cos(d*x + c) + (3*a^2*b - b^3)*cos(d*x + c)^3 - (2*a^3 + 3*a*b^
2 + (a^3 - 3*a*b^2)*cos(d*x + c)^2)*sin(d*x + c))/d
```

giac [A] time = 0.20, size = 91, normalized size = 1.57

$$-\frac{\left(3\,a^{2}b-b^{3}\right)\cos\left(3\,dx+3\,c\right)}{12\,d}-\frac{3\,\left(a^{2}b+b^{3}\right)\cos\left(dx+c\right)}{4\,d}+\frac{\left(a^{3}-3\,ab^{2}\right)\sin\left(3\,dx+3\,c\right)}{12\,d}+\frac{3\,\left(a^{3}+ab^{2}\right)\sin\left(dx+c\right)}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cos(d*x+c)+b*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -1/12*(3*a^2*b - b^3)*cos(3*d*x + 3*c)/d - 3/4*(a^2*b + b^3)*cos(d*x + c)/d
 + 1/12*(a^3 - 3*a*b^2)*sin(3*d*x + 3*c)/d + 3/4*(a^3 + a*b^2)*sin(d*x + c)
/d
```

maple [A] time = 9.74, size = 75, normalized size = 1.29

$$\frac{-\frac{b^{3}\left(2+\sin^{2}\left(dx+c\right)\right)\cos\left(dx+c\right)}{3}+b^{2}a\left(\sin^{3}\left(dx+c\right)\right)-\left(\cos^{3}\left(dx+c\right)\right)a^{2}b+\frac{a^{3}\left(2+\cos^{2}\left(dx+c\right)\right)\sin\left(dx+c\right)}{3}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*cos(d*x+c)+b*sin(d*x+c))^3,x)
```

```
[Out] 1/d*(-1/3*b^3*(2+sin(d*x+c)^2)*cos(d*x+c)+b^2*a*sin(d*x+c)^3-cos(d*x+c)^3*a
^2*b+1/3*a^3*(2+cos(d*x+c)^2)*sin(d*x+c))
```

maxima [A] time = 0.32, size = 84, normalized size = 1.45

$$-\frac{a^{2}b\cos\left(dx+c\right)^{3}}{d}+\frac{ab^{2}\sin\left(dx+c\right)^{3}}{d}-\frac{\left(\sin\left(dx+c\right)^{3}-3\,\sin\left(dx+c\right)\right)a^{3}}{3\,d}+\frac{\left(\cos\left(dx+c\right)^{3}-3\,\cos\left(dx+c\right)\right)b^{3}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cos(d*x+c)+b*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -a^2*b*cos(d*x + c)^3/d + a*b^2*sin(d*x + c)^3/d - 1/3*(sin(d*x + c)^3 - 3*
sin(d*x + c))*a^3/d + 1/3*(cos(d*x + c)^3 - 3*cos(d*x + c))*b^3/d
```

mupad [B] time = 0.57, size = 104, normalized size = 1.79

$$\frac{\frac{\sin(c+d\,x)\,a^3\cos(c+d\,x)^2}{3}+\frac{2\,\sin(c+d\,x)\,a^3}{3}-a^2\,b\cos\left(c+d\,x\right)^3-\sin\left(c+d\,x\right)\,a\,b^2\cos\left(c+d\,x\right)^2+\sin\left(c+d\,x\right)\,a\,b^2+\frac{b^3\,\mathrm{co}}{}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*cos(c + d*x) + b*sin(c + d*x))^3,x)
```

```
[Out] ((2*a^3*sin(c + d*x))/3 - b^3*cos(c + d*x) + (b^3*cos(c + d*x)^3)/3 - a^2*b
*cos(c + d*x)^3 + (a^3*cos(c + d*x)^2*sin(c + d*x))/3 + a*b^2*sin(c + d*x)
- a*b^2*cos(c + d*x)^2*sin(c + d*x))/d
```

sympy [A] time = 0.52, size = 117, normalized size = 2.02

$$\begin{cases} \frac{2a^3\sin^3(c+dx)}{3d}+\frac{a^3\sin(c+dx)\cos^2(c+dx)}{d}-\frac{a^2b\cos^3(c+dx)}{d}+\frac{ab^2\sin^3(c+dx)}{d}-\frac{b^3\sin^2(c+dx)\cos(c+dx)}{d}-\frac{2b^3\cos^3(c+dx)}{3d} & \text{for } d\neq 0 \\ x\left(a\cos(c)+b\sin(c)\right)^3 & \text{otherwis} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cos(d*x+c)+b*sin(d*x+c))**3,x)
```

```
[Out] Piecewise((2*a**3*sin(c + d*x)**3/(3*d) + a**3*sin(c + d*x)*cos(c + d*x)**2
/d - a**2*b*cos(c + d*x)**3/d + a*b**2*sin(c + d*x)**3/d - b**3*sin(c + d*x
)**2*cos(c + d*x)/d - 2*b**3*cos(c + d*x)**3/(3*d), Ne(d, 0)), (x*(a*cos(c)
 + b*sin(c))**3, True))
```

3.63 $\int \sec(c+dx)(a\cos(c+dx)+b\sin(c+dx))^3\,dx$

Optimal. Leaf size=91

$$\frac{\sin^2(c+dx)\left(a\left(a^2-3b^2\right)\cot(c+dx)+b\left(3a^2-b^2\right)\right)}{2d}+\frac{1}{2}ax\left(a^2+3b^2\right)-\frac{b^3\log(\sin(c+dx))}{d}+\frac{b^3\log(\tan(c+dx))}{d}$$

```
[Out] 1/2*a*(a^2+3*b^2)*x-b^3*ln(sin(d*x+c))/d+b^3*ln(tan(d*x+c))/d+1/2*(b*(3*a^2
-b^2)+a*(a^2-3*b^2)*cot(d*x+c))*sin(d*x+c)^2/d
```

Rubi [A] time = 0.12, antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {3088, 1805, 801, 635, 203, 260}

$$\frac{\sin^2(c+dx)\left(a\left(a^2-3b^2\right)\cot(c+dx)+b\left(3a^2-b^2\right)\right)}{2d}+\frac{1}{2}ax\left(a^2+3b^2\right)-\frac{b^3\log(\sin(c+dx))}{d}+\frac{b^3\log(\tan(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]*(a*Cos[c + d*x] + b*Sin[c + d*x])^3,x]
```

```
[Out] (a*(a^2 + 3*b^2)*x)/2 - (b^3*Log[Sin[c + d*x]])/d + (b^3*Log[Tan[c + d*x]])
/d + ((b*(3*a^2 - b^2) + a*(a^2 - 3*b^2)*Cot[c + d*x])*Sin[c + d*x]^2)/(2*d
)
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 801

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a + c*x^2), x],
```

```
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rule 1805

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[((a*g - b*f*x)*(a + b*x^2)^(p + 1))/(2*a
*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int[(c*x)^m*(a + b*x^2)^(p + 1)*Exp
andToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; Fr
eeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 3088

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[(x^m*(b +
 a*x)^n)/(1 + x^2)^((m + n + 2)/2), x], x, Cot[c + d*x]], x] /; FreeQ[{a, b
, c, d}, x] && IntegerQ[n] && IntegerQ[(m + n)/2] && NeQ[n, -1] &&  !(GtQ[n
, 0] && GtQ[m, 1])
```

Rubi steps

$$\int \sec(c+dx)(a\cos(c+dx)+b\sin(c+dx))^3\,dx = -\frac{\text{Subst}\left(\int \frac{(b+ax)^3}{x\left(1+x^2\right)^2}\,dx, x, \cot(c+dx)\right)}{d}$$

$$= \frac{\left(b\left(3a^2-b^2\right)+a\left(a^2-3b^2\right)\cot(c+dx)\right)\sin^2(c+dx)}{2d} + \frac{\text{Subs}}{}$$

$$= \frac{\left(b\left(3a^2-b^2\right)+a\left(a^2-3b^2\right)\cot(c+dx)\right)\sin^2(c+dx)}{2d} + \frac{\text{Subs}}{}$$

$$= \frac{b^3\log(\tan(c+dx))}{d} + \frac{\left(b\left(3a^2-b^2\right)+a\left(a^2-3b^2\right)\cot(c+dx\right.}{2d}$$

$$= \frac{b^3\log(\tan(c+dx))}{d} + \frac{\left(b\left(3a^2-b^2\right)+a\left(a^2-3b^2\right)\cot(c+dx\right.}{2d}$$

$$= \frac{1}{2}a\left(a^2+3b^2\right)x - \frac{b^3\log(\sin(c+dx))}{d} + \frac{b^3\log(\tan(c+dx))}{d} +$$

Mathematica [B] time = 0.81, size = 401, normalized size = 4.41

$$\frac{-a^5\sqrt{-b^2}\log\left(\sqrt{-b^2}-b\tan(c+dx)\right)+a^5\sqrt{-b^2}\log\left(\sqrt{-b^2}+b\tan(c+dx)\right)+5a^4b^2+4a^3\left(-b^2\right)^{3/2}\log\left(\sqrt{-b^2}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]*(a*Cos[c + d*x] + b*Sin[c + d*x])^3,x]
```

```
[Out] (5*a^4*b^2 + 2*a^2*b^4 - b^6 + (-3*a^4*b^2 - 2*a^2*b^4 + b^6)*Cos[2*(c + d*
x)] + 2*a^2*b^4*Log[Sqrt[-b^2] - b*Tan[c + d*x]] + 2*b^6*Log[Sqrt[-b^2] - b
*Tan[c + d*x]] - a^5*Sqrt[-b^2]*Log[Sqrt[-b^2] - b*Tan[c + d*x]] + 4*a^3*(-
b^2)^(3/2)*Log[Sqrt[-b^2] - b*Tan[c + d*x]] - 3*a*(-b^2)^(5/2)*Log[Sqrt[-b^
2] - b*Tan[c + d*x]] + 2*a^2*b^4*Log[Sqrt[-b^2] + b*Tan[c + d*x]] + 2*b^6*L
og[Sqrt[-b^2] + b*Tan[c + d*x]] + a^5*Sqrt[-b^2]*Log[Sqrt[-b^2] + b*Tan[c +
 d*x]] + 3*a*b^4*Sqrt[-b^2]*Log[Sqrt[-b^2] + b*Tan[c + d*x]] - 4*a^3*(-b^2)
^(3/2)*Log[Sqrt[-b^2] + b*Tan[c + d*x]] + a*b*(a^4 - 2*a^2*b^2 - 3*b^4)*Sin
[2*(c + d*x)])/(4*b*(a^2 + b^2)*d)
```

fricas [A] time = 0.71, size = 79, normalized size = 0.87

$$-\frac{2\,b^3\log\left(-\cos\left(dx+c\right)\right)-\left(a^3+3\,ab^2\right)dx+\left(3\,a^2b-b^3\right)\cos\left(dx+c\right)^2-\left(a^3-3\,ab^2\right)\cos\left(dx+c\right)\sin\left(dx+c\right)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a*cos(d*x+c)+b*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] -1/2*(2*b^3*log(-cos(d*x + c)) - (a^3 + 3*a*b^2)*d*x + (3*a^2*b - b^3)*cos(
d*x + c)^2 - (a^3 - 3*a*b^2)*cos(d*x + c)*sin(d*x + c))/d
```

giac [A] time = 0.25, size = 93, normalized size = 1.02

$$\frac{b^3\log\left(\tan\left(dx+c\right)^2+1\right)+\left(a^3+3\,ab^2\right)(dx+c)-\frac{b^3\tan(dx+c)^2-a^3\tan(dx+c)+3\,ab^2\tan(dx+c)+3\,a^2b}{\tan(dx+c)^2+1}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a*cos(d*x+c)+b*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/2*(b^3*log(tan(d*x + c)^2 + 1) + (a^3 + 3*a*b^2)*(d*x + c) - (b^3*tan(d*x
 + c)^2 - a^3*tan(d*x + c) + 3*a*b^2*tan(d*x + c) + 3*a^2*b)/(tan(d*x + c)^
2 + 1))/d
```

maple [A] time = 1.59, size = 123, normalized size = 1.35

$$\frac{a^3\cos(dx+c)\sin(dx+c)}{2d}+\frac{a^3x}{2}+\frac{a^3c}{2d}-\frac{3a^2b\left(\cos^2(dx+c)\right)}{2d}-\frac{3a\,b^2\cos(dx+c)\sin(dx+c)}{2d}+\frac{3a\,b^2x}{2}+\frac{3a\,b^2c}{2d}-\frac{(\mathrm{si}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)*(a*cos(d*x+c)+b*sin(d*x+c))^3,x)
```

```
[Out] 1/2*a^3*cos(d*x+c)*sin(d*x+c)/d+1/2*a^3*x+1/2/d*a^3*c-3/2*a^2*b*cos(d*x+c)^
2/d-3/2*a*b^2*cos(d*x+c)*sin(d*x+c)/d+3/2*a*b^2*x+3/2/d*a*b^2*c-1/2/d*sin(d
*x+c)^2*b^3-b^3*ln(cos(d*x+c))/d
```

maxima [A] time = 0.34, size = 91, normalized size = 1.00

$$\frac{6\,a^2b\sin(dx+c)^2+(2\,dx+2\,c+\sin(2\,dx+2\,c))a^3+3\,(2\,dx+2\,c-\sin(2\,dx+2\,c))ab^2-2\left(\sin(dx+c)^2+\log\right.}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a*cos(d*x+c)+b*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/4*(6*a^2*b*sin(d*x + c)^2 + (2*d*x + 2*c + sin(2*d*x + 2*c))*a^3 + 3*(2*d
*x + 2*c - sin(2*d*x + 2*c))*a*b^2 - 2*(sin(d*x + c)^2 + log(sin(d*x + c)^2
 - 1))*b^3)/d
```

mupad [B] time = 1.24, size = 156, normalized size = 1.71

$$\frac{b^3\,\ln\left(\frac{1}{{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2}\right)-b^3\,\ln\left(\frac{\cos(c+d\,x)}{\cos(c+d\,x)+1}\right)+a^3\,\mathrm{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)+\frac{b^3\,\cos(2\,c+2\,d\,x)}{4}+\frac{a^3\,\sin(2\,c+2\,d\,x)}{4}+3\,a\,b^2\,\mathrm{atan}\left(\frac{\sin\left(\frac{c}{2}+\right.}{\cos\left(\frac{c}{2}+\right.}\right.}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*cos(c + d*x) + b*sin(c + d*x))^3/cos(c + d*x),x)
```

```
[Out] (b^3*log(1/cos(c/2 + (d*x)/2)^2) - b^3*log(cos(c + d*x)/(cos(c + d*x) + 1))
 + a^3*atan(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)) + (b^3*cos(2*c + 2*d*x))
/4 + (a^3*sin(2*c + 2*d*x))/4 + 3*a*b^2*atan(sin(c/2 + (d*x)/2)/cos(c/2 + (
d*x)/2)) - (3*a^2*b*cos(2*c + 2*d*x))/4 - (3*a*b^2*sin(2*c + 2*d*x))/4)/d
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a\cos(c+dx)+b\sin(c+dx))^3\sec(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a*cos(d*x+c)+b*sin(d*x+c))**3,x)
[Out] Integral((a*cos(c + d*x) + b*sin(c + d*x))**3*sec(c + d*x), x)
```

3.64 $\int \sec^2(c+dx)(a\cos(c+dx)+b\sin(c+dx))^3\,dx$

Optimal. Leaf size=86

$$\frac{a^3\sin(c+dx)}{d}-\frac{3a^2b\cos(c+dx)}{d}-\frac{3ab^2\sin(c+dx)}{d}+\frac{3ab^2\tanh^{-1}(\sin(c+dx))}{d}+\frac{b^3\cos(c+dx)}{d}+\frac{b^3\sec(c+dx)}{d}$$

```
[Out] 3*a*b^2*arctanh(sin(d*x+c))/d-3*a^2*b*cos(d*x+c)/d+b^3*cos(d*x+c)/d+b^3*sec
(d*x+c)/d+a^3*sin(d*x+c)/d-3*a*b^2*sin(d*x+c)/d
```

Rubi [A] time = 0.11, antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 8, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3090, 2637, 2638, 2592, 321, 206, 2590, 14}

$$-\frac{3a^2b\cos(c+dx)}{d}+\frac{a^3\sin(c+dx)}{d}-\frac{3ab^2\sin(c+dx)}{d}+\frac{3ab^2\tanh^{-1}(\sin(c+dx))}{d}+\frac{b^3\cos(c+dx)}{d}+\frac{b^3\sec(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^2*(a*Cos[c + d*x] + b*Sin[c + d*x])^3,x]
```

```
[Out] (3*a*b^2*ArcTanh[Sin[c + d*x]])/d - (3*a^2*b*Cos[c + d*x])/d + (b^3*Cos[c +
 d*x])/d + (b^3*Sec[c + d*x])/d + (a^3*Sin[c + d*x])/d - (3*a*b^2*Sin[c + d
*x])/d
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2590

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> -Dist[f^(-1), Subst[Int[(1 - x^2)^((m + n - 1)/2)/x^n, x], x, Cos[e + f*
x]], x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n - 1)/2]
```

Rule 2592

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(
ff*x)^(m + n)/(a^2 - ff^2*x^2)^((n + 1)/2), x], x, (a*Sin[e + f*x])/ff], x]
] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3090

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_.), x_Symbol] :> Int[ExpandTrig[cos[c + d*x]^m*(a
*cos[c + d*x] + b*sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d}, x] && Inte
gerQ[m] && IGtQ[n, 0]
```

Rubi steps

$$\int \sec^2(c+dx)(a\cos(c+dx)+b\sin(c+dx))^3\,dx = \int \left(a^3\cos(c+dx)+3a^2b\sin(c+dx)+3ab^2\sin(c+dx)\tan\right.$$

$$= a^3\int\cos(c+dx)\,dx+\left(3a^2b\right)\int\sin(c+dx)\,dx+\left(3ab^2\right)\int$$

$$= -\frac{3a^2b\cos(c+dx)}{d}+\frac{a^3\sin(c+dx)}{d}+\frac{\left(3ab^2\right)\text{Subst}\left(\int\frac{x^2}{1-x^2}\right.}{d}$$

$$= -\frac{3a^2b\cos(c+dx)}{d}+\frac{a^3\sin(c+dx)}{d}-\frac{3ab^2\sin(c+dx)}{d}+$$

$$= \frac{3ab^2\tanh^{-1}(\sin(c+dx))}{d}-\frac{3a^2b\cos(c+dx)}{d}+\frac{b^3\cos(c+}{d}$$

Mathematica [A] time = 1.07, size = 131, normalized size = 1.52

$$\frac{\sec(c+dx)\left(a^3\sin(2(c+dx))+\left(b^3-3a^2b\right)\cos(2(c+dx))-3a^2b-3ab^2\sin(2(c+dx))-6ab^2\cos(c+dx)\left(\log\left(\right.\right.\right.}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^2*(a*Cos[c + d*x] + b*Sin[c + d*x])^3,x]
```

```
[Out] (Sec[c + d*x]*(-3*a^2*b + 3*b^3 + (-3*a^2*b + b^3)*Cos[2*(c + d*x)] - 6*a*b
^2*Cos[c + d*x]*(Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] - Log[Cos[(c + d*
x)/2] + Sin[(c + d*x)/2]]) + a^3*Sin[2*(c + d*x)] - 3*a*b^2*Sin[2*(c + d*x)
]))/(2*d)
```

fricas [A] time = 0.65, size = 109, normalized size = 1.27

$$\frac{3\,ab^2\cos\left(dx+c\right)\log\left(\sin\left(dx+c\right)+1\right)-3\,ab^2\cos\left(dx+c\right)\log\left(-\sin\left(dx+c\right)+1\right)+2\,b^3-2\left(3\,a^2b-b^3\right)\cos\left(dx\right.}{2\,d\cos\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a*cos(d*x+c)+b*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/2*(3*a*b^2*cos(d*x + c)*log(sin(d*x + c) + 1) - 3*a*b^2*cos(d*x + c)*log(
-sin(d*x + c) + 1) + 2*b^3 - 2*(3*a^2*b - b^3)*cos(d*x + c)^2 + 2*(a^3 - 3*
a*b^2)*cos(d*x + c)*sin(d*x + c))/(d*cos(d*x + c))
```

giac [A] time = 0.29, size = 150, normalized size = 1.74

$$\frac{3\,ab^2\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-3\,ab^2\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)+\frac{2\left(a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-3\,ab^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-3\,a^2b\tan\left(\right.\right.}{\tan\left(\right.}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a*cos(d*x+c)+b*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] (3*a*b^2*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 3*a*b^2*log(abs(tan(1/2*d*x +
 1/2*c) - 1)) + 2*(a^3*tan(1/2*d*x + 1/2*c)^3 - 3*a*b^2*tan(1/2*d*x + 1/2*c
)^3 - 3*a^2*b*tan(1/2*d*x + 1/2*c)^2 - a^3*tan(1/2*d*x + 1/2*c) + 3*a*b^2*t
an(1/2*d*x + 1/2*c) + 3*a^2*b - 2*b^3)/(tan(1/2*d*x + 1/2*c)^4 - 1))/d
```

maple [A] time = 10.96, size = 126, normalized size = 1.47

$$\frac{a^3\sin\left(dx+c\right)}{d}-\frac{3a^2b\cos\left(dx+c\right)}{d}-\frac{3a\,b^2\sin\left(dx+c\right)}{d}+\frac{3b^2a\ln\left(\sec\left(dx+c\right)+\tan\left(dx+c\right)\right)}{d}+\frac{b^3\left(\sin^4\left(dx+c\right)\right)}{d\cos\left(dx+c\right)}+\frac{b}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2*(a*cos(d*x+c)+b*sin(d*x+c))^3,x)
```

```
[Out] a^3*sin(d*x+c)/d-3*a^2*b*cos(d*x+c)/d-3*a*b^2*sin(d*x+c)/d+3/d*b^2*a*ln(sec
(d*x+c)+tan(d*x+c))+1/d*b^3*sin(d*x+c)^4/cos(d*x+c)+1/d*b^3*cos(d*x+c)*sin(
d*x+c)^2+2*b^3*cos(d*x+c)/d
```

maxima [A] time = 0.33, size = 84, normalized size = 0.98

$$\frac{2\,b^3\left(\frac{1}{\cos(dx+c)}+\cos\left(dx+c\right)\right)+3\,ab^2\left(\log\left(\sin\left(dx+c\right)+1\right)-\log\left(\sin\left(dx+c\right)-1\right)-2\,\sin\left(dx+c\right)\right)-6\,a^2b\cos}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a*cos(d*x+c)+b*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/2*(2*b^3*(1/cos(d*x + c) + cos(d*x + c)) + 3*a*b^2*(log(sin(d*x + c) + 1)
 - log(sin(d*x + c) - 1) - 2*sin(d*x + c)) - 6*a^2*b*cos(d*x + c) + 2*a^3*s
in(d*x + c))/d
```

mupad [B] time = 1.05, size = 116, normalized size = 1.35

$$\frac{6\,a\,b^2\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{d}-\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3\,\left(6\,a\,b^2-2\,a^3\right)-6\,a^2\,b-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(6\,a\,b^2-2\,a^3\right)+4\,b^3+6\,a^2\,b}{d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*cos(c + d*x) + b*sin(c + d*x))^3/cos(c + d*x)^2,x)
```

```
[Out] (6*a*b^2*atanh(tan(c/2 + (d*x)/2)))/d - (tan(c/2 + (d*x)/2)^3*(6*a*b^2 - 2*
a^3) - 6*a^2*b - tan(c/2 + (d*x)/2)*(6*a*b^2 - 2*a^3) + 4*b^3 + 6*a^2*b*tan
(c/2 + (d*x)/2)^2)/(d*(tan(c/2 + (d*x)/2)^4 - 1))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a\cos\left(c+dx\right)+b\sin\left(c+dx\right)\right)^3\sec^2\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2*(a*cos(d*x+c)+b*sin(d*x+c))**3,x)
```

```
[Out] Integral((a*cos(c + d*x) + b*sin(c + d*x))**3*sec(c + d*x)**2, x)
```

3.65 $\int \sec^3(c+dx)(a\cos(c+dx)+b\sin(c+dx))^3\,dx$

Optimal. Leaf size=72

$$-\frac{b\left(3a^2-b^2\right)\log(\cos(c+dx))}{d}+ax\left(a^2-3b^2\right)+\frac{2ab^2\tan(c+dx)}{d}+\frac{b(a+b\tan(c+dx))^2}{2d}$$

```
[Out] a*(a^2-3*b^2)*x-b*(3*a^2-b^2)*ln(cos(d*x+c))/d+2*a*b^2*tan(d*x+c)/d+1/2*b*(
a+b*tan(d*x+c))^2/d
```

Rubi [A] time = 0.09, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3086, 3482, 3525, 3475}

$$-\frac{b\left(3a^2-b^2\right)\log(\cos(c+dx))}{d}+ax\left(a^2-3b^2\right)+\frac{2ab^2\tan(c+dx)}{d}+\frac{b(a+b\tan(c+dx))^2}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^3*(a*Cos[c + d*x] + b*Sin[c + d*x])^3,x]
```

```
[Out] a*(a^2 - 3*b^2)*x - (b*(3*a^2 - b^2)*Log[Cos[c + d*x]])/d + (2*a*b^2*Tan[c
 + d*x])/d + (b*(a + b*Tan[c + d*x])^2)/(2*d)
```

Rule 3086

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin
[(c_.) + (d_.)*(x_)])^(n_.), x_Symbol] :> Int[(a + b*Tan[c + d*x])^n, x] /;
 FreeQ[{a, b, c, d}, x] && EqQ[m + n, 0] && IntegerQ[n] && NeQ[a^2 + b^2, 0
]
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3482

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(a +
b*Tan[c + d*x])^(n - 1))/(d*(n - 1)), x] + Int[(a^2 - b^2 + 2*a*b*Tan[c + d
*x])*(a + b*Tan[c + d*x])^(n - 2), x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2
+ b^2, 0] && GtQ[n, 1]
```

Rule 3525

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[(a*c - b*d)*x, x] + (Dist[b*c + a*d, Int[Tan[e +
 f*x], x], x] + Simp[(b*d*Tan[e + f*x])/f, x]) /; FreeQ[{a, b, c, d, e, f},
 x] && NeQ[b*c - a*d, 0] && NeQ[b*c + a*d, 0]
```

Rubi steps

$$\begin{aligned}
\int \sec^3(c+dx)(a\cos(c+dx)+b\sin(c+dx))^3\,dx &= \int (a+b\tan(c+dx))^3\,dx \\
&= \frac{b(a+b\tan(c+dx))^2}{2d} + \int (a+b\tan(c+dx))\left(a^2-b^2+2a\right. \\
&= a\left(a^2-3b^2\right)x + \frac{2ab^2\tan(c+dx)}{d} + \frac{b(a+b\tan(c+dx))^2}{2d} + \\
&= a\left(a^2-3b^2\right)x - \frac{b\left(3a^2-b^2\right)\log(\cos(c+dx))}{d} + \frac{2ab^2\tan(c}{d}
\end{aligned}$$

Mathematica [C] time = 0.27, size = 79, normalized size = 1.10

$$\frac{6ab^2\tan(c+dx)+(-b+ia)^3\log(-\tan(c+dx)+i)-(b+ia)^3\log(\tan(c+dx)+i)+b^3\tan^2(c+dx)}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^3*(a*Cos[c + d*x] + b*Sin[c + d*x])^3,x]
```

```
[Out] ((I*a - b)^3*Log[I - Tan[c + d*x]] - (I*a + b)^3*Log[I + Tan[c + d*x]] + 6*
a*b^2*Tan[c + d*x] + b^3*Tan[c + d*x]^2)/(2*d)
```

fricas [A] time = 0.72, size = 88, normalized size = 1.22

$$\frac{2\left(a^3-3\,ab^2\right)dx\cos\left(dx+c\right)^2+6\,ab^2\cos\left(dx+c\right)\sin\left(dx+c\right)-2\left(3\,a^2b-b^3\right)\cos\left(dx+c\right)^2\log\left(-\cos\left(dx+c\right)\right)}{2\,d\cos\left(dx+c\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a*cos(d*x+c)+b*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/2*(2*(a^3 - 3*a*b^2)*d*x*cos(d*x + c)^2 + 6*a*b^2*cos(d*x + c)*sin(d*x +
c) - 2*(3*a^2*b - b^3)*cos(d*x + c)^2*log(-cos(d*x + c)) + b^3)/(d*cos(d*x
+ c)^2)
```

giac [A] time = 0.32, size = 71, normalized size = 0.99

$$\frac{b^3\tan\left(dx+c\right)^2+6\,ab^2\tan\left(dx+c\right)+2\left(a^3-3\,ab^2\right)(dx+c)+\left(3\,a^2b-b^3\right)\log\left(\tan\left(dx+c\right)^2+1\right)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a*cos(d*x+c)+b*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/2*(b^3*tan(d*x + c)^2 + 6*a*b^2*tan(d*x + c) + 2*(a^3 - 3*a*b^2)*(d*x + c
) + (3*a^2*b - b^3)*log(tan(d*x + c)^2 + 1))/d
```

maple [A] time = 1.80, size = 93, normalized size = 1.29

$$a^3x+\frac{a^3c}{d}-\frac{3a^2b\ln(\cos(dx+c))}{d}-3a\,b^2x+\frac{3a\,b^2\tan(dx+c)}{d}-\frac{3a\,b^2c}{d}+\frac{b^3\left(\tan^2(dx+c)\right)}{2d}+\frac{b^3\ln(\cos(dx+c))}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^3*(a*cos(d*x+c)+b*sin(d*x+c))^3,x)
```

```
[Out] a^3*x+1/d*a^3*c-3*a^2*b*ln(cos(d*x+c))/d-3*a*b^2*x+3*a*b^2*tan(d*x+c)/d-3/d
*a*b^2*c+1/2*b^3*tan(d*x+c)^2/d+b^3*ln(cos(d*x+c))/d
```

maxima [A] time = 0.42, size = 85, normalized size = 1.18

$$\frac{2(dx+c)a^3-6(dx+c-\tan(dx+c))ab^2-b^3\left(\frac{1}{\sin(dx+c)^2-1}-\log\left(\sin(dx+c)^2-1\right)\right)-3a^2b\log\left(-\sin(dx+c)^2\right.}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a*cos(d*x+c)+b*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/2*(2*(d*x + c)*a^3 - 6*(d*x + c - tan(d*x + c))*a*b^2 - b^3*(1/(sin(d*x +
 c)^2 - 1) - log(sin(d*x + c)^2 - 1)) - 3*a^2*b*log(-sin(d*x + c)^2 + 1))/d
```

mupad [B] time = 1.86, size = 183, normalized size = 2.54

$$\frac{2\left(\frac{b^3\ln\left(\frac{\cos(c+dx)}{\cos(c+dx)+1}\right)}{2}-\frac{b^3\ln\left(\frac{1}{\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^2}\right)}{2}+a^3\,\mathrm{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{dx}{2}\right)}{\cos\left(\frac{c}{2}+\frac{dx}{2}\right)}\right)+\frac{3a^2b\ln\left(\frac{1}{\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^2}\right)}{2}-3ab^2\,\mathrm{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{dx}{2}\right)}{\cos\left(\frac{c}{2}+\frac{dx}{2}\right)}\right)-\frac{3a^2b\ln\left(\right.}{}\right.}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*cos(c + d*x) + b*sin(c + d*x))^3/cos(c + d*x)^3,x)
```

```
[Out] (2*((b^3*log(cos(c + d*x)/(cos(c + d*x) + 1)))/2 - (b^3*log(1/cos(c/2 + (d*
x)/2)^2))/2 + a^3*atan(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)) + (3*a^2*b*lo
```

```
g(1/cos(c/2 + (d*x)/2)^2))/2 - 3*a*b^2*atan(sin(c/2 + (d*x)/2)/cos(c/2 + (d
*x)/2)) - (3*a^2*b*log(cos(c + d*x)/(cos(c + d*x) + 1)))/2))/d + (b^3/2 + (
3*a*b^2*sin(2*c + 2*d*x))/2)/(d*(cos(2*c + 2*d*x)/2 + 1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a\cos(c+dx)+b\sin(c+dx))^3\sec^3(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**3*(a*cos(d*x+c)+b*sin(d*x+c))**3,x)
```

```
[Out] Integral((a*cos(c + d*x) + b*sin(c + d*x))**3*sec(c + d*x)**3, x)
```

3.66 $\int \sec^4(c+dx)(a\cos(c+dx)+b\sin(c+dx))^3\,dx$

Optimal. Leaf size=103

$$\frac{a^3\tanh^{-1}(\sin(c+dx))}{d}+\frac{3a^2b\sec(c+dx)}{d}-\frac{3ab^2\tanh^{-1}(\sin(c+dx))}{2d}+\frac{3ab^2\tan(c+dx)\sec(c+dx)}{2d}+\frac{b^3\sec^3(c+}{3d}$$

```
[Out] a^3*arctanh(sin(d*x+c))/d-3/2*a*b^2*arctanh(sin(d*x+c))/d+3*a^2*b*sec(d*x+c
)/d-b^3*sec(d*x+c)/d+1/3*b^3*sec(d*x+c)^3/d+3/2*a*b^2*sec(d*x+c)*tan(d*x+c)
/d
```

Rubi [A] time = 0.12, antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 5, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.179, Rules used = {3090, 3770, 2606, 8, 2611}

$$\frac{3a^2b\sec(c+dx)}{d}+\frac{a^3\tanh^{-1}(\sin(c+dx))}{d}-\frac{3ab^2\tanh^{-1}(\sin(c+dx))}{2d}+\frac{3ab^2\tan(c+dx)\sec(c+dx)}{2d}+\frac{b^3\sec^3(c+}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^4*(a*Cos[c + d*x] + b*Sin[c + d*x])^3,x]
```

```
[Out] (a^3*ArcTanh[Sin[c + d*x]])/d - (3*a*b^2*ArcTanh[Sin[c + d*x]])/(2*d) + (3*
a^2*b*Sec[c + d*x])/d - (b^3*Sec[c + d*x])/d + (b^3*Sec[c + d*x]^3)/(3*d) +
 (3*a*b^2*Sec[c + d*x]*Tan[c + d*x])/(2*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 2611

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(b*(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*(
m + n - 1)), x] - Dist[(b^2*(n - 1))/(m + n - 1), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 3090

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_.), x_Symbol] :> Int[ExpandTrig[cos[c + d*x]^m*(a
*cos[c + d*x] + b*sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d}, x] && Inte
gerQ[m] && IGtQ[n, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \sec^4(c+dx)(a\cos(c+dx)+b\sin(c+dx))^3\,dx = \int \left(a^3\sec(c+dx)+3a^2b\sec(c+dx)\tan(c+dx)+3ab^2\,\mathrm{se}\right.$$

$$= a^3\int \sec(c+dx)\,dx+\left(3a^2b\right)\int \sec(c+dx)\tan(c+dx)\,dx$$

$$= \frac{a^3\tanh^{-1}(\sin(c+dx))}{d}+\frac{3ab^2\sec(c+dx)\tan(c+dx)}{2d}-\frac{1}{2}$$

$$= \frac{a^3\tanh^{-1}(\sin(c+dx))}{d}-\frac{3ab^2\tanh^{-1}(\sin(c+dx))}{2d}+\frac{3a^2b}{}$$

Mathematica [B] time = 1.63, size = 293, normalized size = 2.84

$$\frac{12a^3\log\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)-6a\left(2a^2-3b^2\right)\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)+2b\sin^2}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^4*(a*Cos[c + d*x] + b*Sin[c + d*x])^3,x]
```

```
[Out] (36*a^2*b - 10*b^3 - 6*a*(2*a^2 - 3*b^2)*Log[Cos[(c + d*x)/2] - Sin[(c + d*
x)/2]] + 12*a^3*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] - 18*a*b^2*Log[Cos
[(c + d*x)/2] + Sin[(c + d*x)/2]] + (9*a*b^2)/(Cos[(c + d*x)/2] - Sin[(c +
d*x)/2])^2 + b^3/(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^2 + 2*b*(18*a^2 - b^
2 + 2*b^2*Cos[c + d*x] + (18*a^2 - 5*b^2)*Cos[2*(c + d*x)])*Sec[c + d*x]^3*
Sin[(c + d*x)/2]^2 - (9*a*b^2)/(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^2 + b^
3/(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^2)/(12*d)
```

fricas [A] time = 0.72, size = 123, normalized size = 1.19

$$\frac{3\left(2\,a^3-3\,ab^2\right)\cos\left(dx+c\right)^3\log\left(\sin\left(dx+c\right)+1\right)-3\left(2\,a^3-3\,ab^2\right)\cos\left(dx+c\right)^3\log\left(-\sin\left(dx+c\right)+1\right)+18\,a}{12\,d\cos\left(dx+c\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(a*cos(d*x+c)+b*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/12*(3*(2*a^3 - 3*a*b^2)*cos(d*x + c)^3*log(sin(d*x + c) + 1) - 3*(2*a^3 -
 3*a*b^2)*cos(d*x + c)^3*log(-sin(d*x + c) + 1) + 18*a*b^2*cos(d*x + c)*sin
(d*x + c) + 4*b^3 + 12*(3*a^2*b - b^3)*cos(d*x + c)^2)/(d*cos(d*x + c)^3)
```

giac [A] time = 0.33, size = 171, normalized size = 1.66

$$\frac{3\left(2a^3-3ab^2\right)\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right|\right)-3\left(2a^3-3ab^2\right)\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1\right|\right)+\frac{2\left(9ab^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5-18a\right.}{}}{6d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(a*cos(d*x+c)+b*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/6*(3*(2*a^3 - 3*a*b^2)*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 3*(2*a^3 - 3*
a*b^2)*log(abs(tan(1/2*d*x + 1/2*c) - 1)) + 2*(9*a*b^2*tan(1/2*d*x + 1/2*c)
^5 - 18*a^2*b*tan(1/2*d*x + 1/2*c)^4 + 36*a^2*b*tan(1/2*d*x + 1/2*c)^2 - 12
*b^3*tan(1/2*d*x + 1/2*c)^2 - 9*a*b^2*tan(1/2*d*x + 1/2*c) - 18*a^2*b + 4*b
^3)/(tan(1/2*d*x + 1/2*c)^2 - 1)^3)/d
```

maple [A] time = 10.25, size = 187, normalized size = 1.82

$$\frac{a^3\ln\left(\sec\left(dx+c\right)+\tan\left(dx+c\right)\right)}{d}+\frac{3a^2b}{d\cos\left(dx+c\right)}+\frac{3b^2a\left(\sin^3\left(dx+c\right)\right)}{2d\cos\left(dx+c\right)^2}+\frac{3a\,b^2\sin\left(dx+c\right)}{2d}-\frac{3b^2a\ln\left(\sec\left(dx+c\right)\right.}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^4*(a*cos(d*x+c)+b*sin(d*x+c))^3,x)
```

```
[Out] 1/d*a^3*ln(sec(d*x+c)+tan(d*x+c))+3/d*a^2*b/cos(d*x+c)+3/2/d*b^2*a*sin(d*x+
c)^3/cos(d*x+c)^2+3/2*a*b^2*sin(d*x+c)/d-3/2/d*b^2*a*ln(sec(d*x+c)+tan(d*x+
c))+1/3/d*b^3*sin(d*x+c)^4/cos(d*x+c)^3-1/3/d*b^3*sin(d*x+c)^4/cos(d*x+c)-1
/3/d*b^3*cos(d*x+c)*sin(d*x+c)^2-2/3*b^3*cos(d*x+c)/d
```

maxima [A] time = 0.33, size = 118, normalized size = 1.15

$$-\frac{9ab^2\left(\frac{2\sin\left(dx+c\right)}{\sin\left(dx+c\right)^2-1}+\log\left(\sin\left(dx+c\right)+1\right)-\log\left(\sin\left(dx+c\right)-1\right)\right)-6a^3\left(\log\left(\sin\left(dx+c\right)+1\right)-\log\left(\sin\left(dx+\right.\right.\right.}{12d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(a*cos(d*x+c)+b*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -1/12*(9*a*b^2*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) + log(sin(d*x + c) + 1)
 - log(sin(d*x + c) - 1)) - 6*a^3*(log(sin(d*x + c) + 1) - log(sin(d*x + c)
 - 1)) - 36*a^2*b/cos(d*x + c) + 4*(3*cos(d*x + c)^2 - 1)*b^3/cos(d*x + c)^
3)/d
```

mupad [B] time = 2.38, size = 160, normalized size = 1.55

$$-\frac{\operatorname{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)\,\left(3\,a\,b^2-2\,a^3\right)}{d}-\frac{6\,a^2\,b-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\,\left(12\,a^2\,b-4\,b^3\right)-\frac{4\,b^3}{3}+3\,a\,b^2\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+6\,a^2 \cdots}{d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6-3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4+3\,\tan\left(\cdots\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*cos(c + d*x) + b*sin(c + d*x))^3/cos(c + d*x)^4,x)
```

```
[Out] - (atanh(tan(c/2 + (d*x)/2))*(3*a*b^2 - 2*a^3))/d - (6*a^2*b - tan(c/2 + (d
*x)/2)^2*(12*a^2*b - 4*b^3) - (4*b^3)/3 + 3*a*b^2*tan(c/2 + (d*x)/2) + 6*a^
2*b*tan(c/2 + (d*x)/2)^4 - 3*a*b^2*tan(c/2 + (d*x)/2)^5)/(d*(3*tan(c/2 + (d
*x)/2)^2 - 3*tan(c/2 + (d*x)/2)^4 + tan(c/2 + (d*x)/2)^6 - 1))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**4*(a*cos(d*x+c)+b*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

3.67 $\int \sec^5(c+dx)(a\cos(c+dx)+b\sin(c+dx))^3\,dx$

Optimal. Leaf size=30

$$\frac{\tan^4(c+dx)(a\cot(c+dx)+b)^4}{4bd}$$

[Out] 1/4*(b+a*cot(d*x+c))^4*tan(d*x+c)^4/b/d

Rubi [A] time = 0.05, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {3088, 37}

$$\frac{\tan^4(c+dx)(a\cot(c+dx)+b)^4}{4bd}$$

Antiderivative was successfully verified.

[In] Int[Sec[c + d*x]^5*(a*Cos[c + d*x] + b*Sin[c + d*x])^3,x]

[Out] ((b + a*Cot[c + d*x])^4*Tan[c + d*x]^4)/(4*b*d)

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 3088

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[(x^m*(b +
 a*x)^n)/(1 + x^2)^((m + n + 2)/2), x], x, Cot[c + d*x]], x] /; FreeQ[{a, b
, c, d}, x] && IntegerQ[n] && IntegerQ[(m + n)/2] && NeQ[n, -1] &&  !(GtQ[n
, 0] && GtQ[m, 1])
```

Rubi steps

$$\begin{aligned}\int \sec^5(c+dx)(a\cos(c+dx)+b\sin(c+dx))^3\,dx &= -\frac{\text{Subst}\left(\int \frac{(b+ax)^3}{x^5}\,dx, x, \cot(c+dx)\right)}{d}\\ &= \frac{(b+a\cot(c+dx))^4\tan^4(c+dx)}{4bd}\end{aligned}$$

Mathematica [A] time = 0.18, size = 57, normalized size = 1.90

$$\frac{\tan(c+dx)\left(4a^3+6a^2b\tan(c+dx)+4ab^2\tan^2(c+dx)+b^3\tan^3(c+dx)\right)}{4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^5*(a*Cos[c + d*x] + b*Sin[c + d*x])^3,x]
```

```
[Out] (Tan[c + d*x]*(4*a^3 + 6*a^2*b*Tan[c + d*x] + 4*a*b^2*Tan[c + d*x]^2 + b^3*
Tan[c + d*x]^3))/(4*d)
```

fricas [B] time = 0.76, size = 78, normalized size = 2.60

$$\frac{b^3+2\left(3a^2b-b^3\right)\cos\left(dx+c\right)^2+4\left(ab^2\cos\left(dx+c\right)+\left(a^3-ab^2\right)\cos\left(dx+c\right)^3\right)\sin\left(dx+c\right)}{4d\cos\left(dx+c\right)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*(a*cos(d*x+c)+b*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/4*(b^3 + 2*(3*a^2*b - b^3)*cos(d*x + c)^2 + 4*(a*b^2*cos(d*x + c) + (a^3
- a*b^2)*cos(d*x + c)^3)*sin(d*x + c))/(d*cos(d*x + c)^4)
```

giac [B] time = 0.60, size = 57, normalized size = 1.90

$$\frac{b^3\tan\left(dx+c\right)^4+4ab^2\tan\left(dx+c\right)^3+6a^2b\tan\left(dx+c\right)^2+4a^3\tan\left(dx+c\right)}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*(a*cos(d*x+c)+b*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/4*(b^3*tan(d*x + c)^4 + 4*a*b^2*tan(d*x + c)^3 + 6*a^2*b*tan(d*x + c)^2 +
 4*a^3*tan(d*x + c))/d
```

maple [B] time = 11.42, size = 72, normalized size = 2.40

$$\frac{a^3\tan\left(dx+c\right)+\frac{3a^2b}{2\cos(dx+c)^2}+\frac{b^2a\left(\sin^3(dx+c)\right)}{\cos(dx+c)^3}+\frac{b^3\left(\sin^4(dx+c)\right)}{4\cos(dx+c)^4}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^5*(a*cos(d*x+c)+b*sin(d*x+c))^3,x)
```

```
[Out] 1/d*(a^3*tan(d*x+c)+3/2*a^2*b/cos(d*x+c)^2+b^2*a*sin(d*x+c)^3/cos(d*x+c)^3+
1/4*b^3*sin(d*x+c)^4/cos(d*x+c)^4)
```

maxima [B] time = 0.34, size = 87, normalized size = 2.90

$$\frac{4\,ab^2\tan\left(dx+c\right)^3+4\,a^3\tan\left(dx+c\right)+\frac{\left(2\,\sin\left(dx+c\right)^2-1\right)b^3}{\sin\left(dx+c\right)^4-2\,\sin\left(dx+c\right)^2+1}-\frac{6\,a^2b}{\sin\left(dx+c\right)^2-1}}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*(a*cos(d*x+c)+b*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/4*(4*a*b^2*tan(d*x + c)^3 + 4*a^3*tan(d*x + c) + (2*sin(d*x + c)^2 - 1)*b
^3/(sin(d*x + c)^4 - 2*sin(d*x + c)^2 + 1) - 6*a^2*b/(sin(d*x + c)^2 - 1))/
d
```

mupad [B] time = 0.64, size = 88, normalized size = 2.93

$$\frac{\cos\left(c+d\,x\right)^3\,\left(a^3\,\sin\left(c+d\,x\right)-a\,b^2\,\sin\left(c+d\,x\right)\right)+\cos\left(c+d\,x\right)^2\,\left(\frac{3\,a^2\,b}{2}-\frac{b^3}{2}\right)+\frac{b^3}{4}+a\,b^2\,\cos\left(c+d\,x\right)\,\sin\left(c+\right.}{d\cos\left(c+d\,x\right)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*cos(c + d*x) + b*sin(c + d*x))^3/cos(c + d*x)^5,x)
```

```
[Out] (cos(c + d*x)^3*(a^3*sin(c + d*x) - a*b^2*sin(c + d*x)) + cos(c + d*x)^2*((
3*a^2*b)/2 - b^3/2) + b^3/4 + a*b^2*cos(c + d*x)*sin(c + d*x))/(d*cos(c + d
*x)^4)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**5*(a*cos(d*x+c)+b*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

3.68 $\int \sec^6(c+dx)(a\cos(c+dx)+b\sin(c+dx))^3\,dx$

Optimal. Leaf size=158

$$\frac{a^3\tanh^{-1}(\sin(c+dx))}{2d}+\frac{a^3\tan(c+dx)\sec(c+dx)}{2d}+\frac{a^2b\sec^3(c+dx)}{d}-\frac{3ab^2\tanh^{-1}(\sin(c+dx))}{8d}+\frac{3ab^2\tan(c+$$

```
[Out] 1/2*a^3*arctanh(sin(d*x+c))/d-3/8*a*b^2*arctanh(sin(d*x+c))/d+a^2*b*sec(d*x
+c)^3/d-1/3*b^3*sec(d*x+c)^3/d+1/5*b^3*sec(d*x+c)^5/d+1/2*a^3*sec(d*x+c)*ta
n(d*x+c)/d-3/8*a*b^2*sec(d*x+c)*tan(d*x+c)/d+3/4*a*b^2*sec(d*x+c)^3*tan(d*x
+c)/d
```

Rubi [A] time = 0.18, antiderivative size = 158, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 7, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {3090, 3768, 3770, 2606, 30, 2611, 14}

$$\frac{a^2b\sec^3(c+dx)}{d}+\frac{a^3\tanh^{-1}(\sin(c+dx))}{2d}+\frac{a^3\tan(c+dx)\sec(c+dx)}{2d}-\frac{3ab^2\tanh^{-1}(\sin(c+dx))}{8d}+\frac{3ab^2\tan(c+$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^6*(a*Cos[c + d*x] + b*Sin[c + d*x])^3,x]
```

```
[Out] (a^3*ArcTanh[Sin[c + d*x]])/(2*d) - (3*a*b^2*ArcTanh[Sin[c + d*x]])/(8*d) +
 (a^2*b*Sec[c + d*x]^3)/d - (b^3*Sec[c + d*x]^3)/(3*d) + (b^3*Sec[c + d*x]^
5)/(5*d) + (a^3*Sec[c + d*x]*Tan[c + d*x])/(2*d) - (3*a*b^2*Sec[c + d*x]*Ta
n[c + d*x])/(8*d) + (3*a*b^2*Sec[c + d*x]^3*Tan[c + d*x])/(4*d)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 2611

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(b*(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*(
m + n - 1)), x] - Dist[(b^2*(n - 1))/(m + n - 1), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 3090

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_.), x_Symbol] :> Int[ExpandTrig[cos[c + d*x]^m*(a
*cos[c + d*x] + b*sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d}, x] && Inte
gerQ[m] && IGtQ[n, 0]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \sec^6(c+dx)(a\cos(c+dx)+b\sin(c+dx))^3\,dx = \int \left(a^3\sec^3(c+dx)+3a^2b\sec^3(c+dx)\tan(c+dx)+3ab^2\,\text{se}\right.$$

$$= a^3\int \sec^3(c+dx)\,dx+\left(3a^2b\right)\int \sec^3(c+dx)\tan(c+dx)\,dx$$

$$= \frac{a^3\sec(c+dx)\tan(c+dx)}{2d}+\frac{3ab^2\sec^3(c+dx)\tan(c+dx)}{4d}+$$

$$= \frac{a^3\tanh^{-1}(\sin(c+dx))}{2d}+\frac{a^2b\sec^3(c+dx)}{d}+\frac{a^3\sec(c+dx)\,\text{t}}{2d}$$

$$= \frac{a^3\tanh^{-1}(\sin(c+dx))}{2d}-\frac{3ab^2\tanh^{-1}(\sin(c+dx))}{8d}+\frac{a^2b\,\text{se}}{}$$

Mathematica [B] time = 1.32, size = 464, normalized size = 2.94

$$\frac{\sec^5(c+dx)\left(240a^3\sin(2(c+dx))+120a^3\sin(4(c+dx))-300a^3\cos(3(c+dx))\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(\right.\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^6*(a*Cos[c + d*x] + b*Sin[c + d*x])^3,x]
```

```
[Out] (Sec[c + d*x]^5*(960*a^2*b + 64*b^3 + 320*(3*a^2*b - b^3)*Cos[2*(c + d*x)]
- 300*a^3*Cos[3*(c + d*x)]*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + 225*a
*b^2*Cos[3*(c + d*x)]*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] - 60*a^3*Cos
[5*(c + d*x)]*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + 45*a*b^2*Cos[5*(c
+ d*x)]*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] - 150*a*(4*a^2 - 3*b^2)*Co
s[c + d*x]*(Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] - Log[Cos[(c + d*x)/2]
 + Sin[(c + d*x)/2]]) + 300*a^3*Cos[3*(c + d*x)]*Log[Cos[(c + d*x)/2] + Sin
[(c + d*x)/2]] - 225*a*b^2*Cos[3*(c + d*x)]*Log[Cos[(c + d*x)/2] + Sin[(c +
 d*x)/2]] + 60*a^3*Cos[5*(c + d*x)]*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]
] - 45*a*b^2*Cos[5*(c + d*x)]*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] + 24
0*a^3*Sin[2*(c + d*x)] + 540*a*b^2*Sin[2*(c + d*x)] + 120*a^3*Sin[4*(c + d*
x)] - 90*a*b^2*Sin[4*(c + d*x)]))/(1920*d)
```

fricas [A] time = 0.74, size = 147, normalized size = 0.93

$$\frac{15\left(4a^3-3ab^2\right)\cos\left(dx+c\right)^5\log\left(\sin\left(dx+c\right)+1\right)-15\left(4a^3-3ab^2\right)\cos\left(dx+c\right)^5\log\left(-\sin\left(dx+c\right)+1\right)+48}{240\,d\cos\left(dx+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^6*(a*cos(d*x+c)+b*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/240*(15*(4*a^3 - 3*a*b^2)*cos(d*x + c)^5*log(sin(d*x + c) + 1) - 15*(4*a^
3 - 3*a*b^2)*cos(d*x + c)^5*log(-sin(d*x + c) + 1) + 48*b^3 + 80*(3*a^2*b -
 b^3)*cos(d*x + c)^2 + 30*(6*a*b^2*cos(d*x + c) + (4*a^3 - 3*a*b^2)*cos(d*x
 + c)^3)*sin(d*x + c))/(d*cos(d*x + c)^5)
```

giac [B] time = 0.52, size = 333, normalized size = 2.11

$$\frac{15\left(4a^3-3ab^2\right)\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right|\right)-15\left(4a^3-3ab^2\right)\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1\right|\right)+\frac{2\left(60a^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^9\right.}{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^6*(a*cos(d*x+c)+b*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/120*(15*(4*a^3 - 3*a*b^2)*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 15*(4*a^3
- 3*a*b^2)*log(abs(tan(1/2*d*x + 1/2*c) - 1)) + 2*(60*a^3*tan(1/2*d*x + 1/2
*c)^9 + 45*a*b^2*tan(1/2*d*x + 1/2*c)^9 - 360*a^2*b*tan(1/2*d*x + 1/2*c)^8
- 120*a^3*tan(1/2*d*x + 1/2*c)^7 + 270*a*b^2*tan(1/2*d*x + 1/2*c)^7 + 720*a
^2*b*tan(1/2*d*x + 1/2*c)^6 - 240*b^3*tan(1/2*d*x + 1/2*c)^6 - 480*a^2*b*ta
n(1/2*d*x + 1/2*c)^4 - 80*b^3*tan(1/2*d*x + 1/2*c)^4 + 120*a^3*tan(1/2*d*x
+ 1/2*c)^3 - 270*a*b^2*tan(1/2*d*x + 1/2*c)^3 + 240*a^2*b*tan(1/2*d*x + 1/2
*c)^2 - 80*b^3*tan(1/2*d*x + 1/2*c)^2 - 60*a^3*tan(1/2*d*x + 1/2*c) - 45*a*
b^2*tan(1/2*d*x + 1/2*c) - 120*a^2*b + 16*b^3)/(tan(1/2*d*x + 1/2*c)^2 - 1)
^5)/d
```

maple [A] time = 2.36, size = 256, normalized size = 1.62

$$\frac{a^3 \sec(dx+c)\tan(dx+c)}{2d} + \frac{a^3 \ln(\sec(dx+c)+\tan(dx+c))}{2d} + \frac{a^2 b}{d\cos(dx+c)^3} + \frac{3b^2 a\left(\sin^3(dx+c)\right)}{4d\cos(dx+c)^4} + \frac{3b^2 a\left(\sin^3\right.}{8d\cos\left(\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^6*(a*cos(d*x+c)+b*sin(d*x+c))^3,x)
```

```
[Out] 1/2*a^3*sec(d*x+c)*tan(d*x+c)/d+1/2/d*a^3*ln(sec(d*x+c)+tan(d*x+c))+1/d*a^2
*b/cos(d*x+c)^3+3/4/d*b^2*a*sin(d*x+c)^3/cos(d*x+c)^4+3/8/d*b^2*a*sin(d*x+c
)^3/cos(d*x+c)^2+3/8*a*b^2*sin(d*x+c)/d-3/8/d*b^2*a*ln(sec(d*x+c)+tan(d*x+c
))+1/5/d*b^3*sin(d*x+c)^4/cos(d*x+c)^5+1/15/d*b^3*sin(d*x+c)^4/cos(d*x+c)^3
-1/15/d*b^3*sin(d*x+c)^4/cos(d*x+c)-1/15/d*b^3*cos(d*x+c)*sin(d*x+c)^2-2/15
*b^3*cos(d*x+c)/d
```

maxima [A] time = 0.33, size = 157, normalized size = 0.99

$$\frac{45\,ab^2\left(\frac{2\left(\sin(dx+c)^3+\sin(dx+c)\right)}{\sin(dx+c)^4-2\,\sin(dx+c)^2+1} - \log(\sin(dx+c)+1) + \log(\sin(dx+c)-1)\right) - 60\,a^3\left(\frac{2\,\sin(dx+c)}{\sin(dx+c)^2-1} - \log(\sin(dx+\right.}{240\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^6*(a*cos(d*x+c)+b*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/240*(45*a*b^2*(2*(sin(d*x + c)^3 + sin(d*x + c))/(sin(d*x + c)^4 - 2*sin(
d*x + c)^2 + 1) - log(sin(d*x + c) + 1) + log(sin(d*x + c) - 1)) - 60*a^3*(
2*sin(d*x + c)/(sin(d*x + c)^2 - 1) - log(sin(d*x + c) + 1) + log(sin(d*x +
 c) - 1)) + 240*a^2*b/cos(d*x + c)^3 - 16*(5*cos(d*x + c)^2 - 3)*b^3/cos(d*
x + c)^5)/d
```

mupad [B] time = 4.28, size = 293, normalized size = 1.85

$$\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^9\,\left(a^3+\frac{3\,a\,b^2}{4}\right)-2\,a^2\,b-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3\,\left(\frac{9\,a\,b^2}{2}-2\,a^3\right)+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7\,\left(\frac{9\,a\,b^2}{2}-2\,a^3\right)+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{10}-5\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*cos(c + d*x) + b*sin(c + d*x))^3/cos(c + d*x)^6,x)
```

```
[Out] (tan(c/2 + (d*x)/2)^9*((3*a*b^2)/4 + a^3) - 2*a^2*b - tan(c/2 + (d*x)/2)^3*
((9*a*b^2)/2 - 2*a^3) + tan(c/2 + (d*x)/2)^7*((9*a*b^2)/2 - 2*a^3) + tan(c/
2 + (d*x)/2)^2*(4*a^2*b - (4*b^3)/3) - tan(c/2 + (d*x)/2)^4*(8*a^2*b + (4*b
^3)/3) + tan(c/2 + (d*x)/2)^6*(12*a^2*b - 4*b^3) + (4*b^3)/15 - tan(c/2 + (
d*x)/2)*((3*a*b^2)/4 + a^3) - 6*a^2*b*tan(c/2 + (d*x)/2)^8)/(d*(5*tan(c/2 +
 (d*x)/2)^2 - 10*tan(c/2 + (d*x)/2)^4 + 10*tan(c/2 + (d*x)/2)^6 - 5*tan(c/2
 + (d*x)/2)^8 + tan(c/2 + (d*x)/2)^10 - 1)) - (atanh(tan(c/2 + (d*x)/2))*((
3*a*b^2)/4 - a^3))/d
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**6*(a*cos(d*x+c)+b*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

3.69 $\int \sec^7(c+dx)(a\cos(c+dx)+b\sin(c+dx))^3\,dx$

Optimal. Leaf size=120

$$\frac{a^3\tan(c+dx)}{d}+\frac{b\left(3a^2+b^2\right)\tan^4(c+dx)}{4d}+\frac{a\left(a^2+3b^2\right)\tan^3(c+dx)}{3d}+\frac{3a^2b\tan^2(c+dx)}{2d}+\frac{3ab^2\tan^5(c+dx)}{5d}+\frac{b^3 t}{}$$

```
[Out] a^3*tan(d*x+c)/d+3/2*a^2*b*tan(d*x+c)^2/d+1/3*a*(a^2+3*b^2)*tan(d*x+c)^3/d+
1/4*b*(3*a^2+b^2)*tan(d*x+c)^4/d+3/5*a*b^2*tan(d*x+c)^5/d+1/6*b^3*tan(d*x+c
)^6/d
```

Rubi [A] time = 0.10, antiderivative size = 120, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {3088, 894}

$$\frac{b\left(3a^2+b^2\right)\tan^4(c+dx)}{4d}+\frac{a\left(a^2+3b^2\right)\tan^3(c+dx)}{3d}+\frac{3a^2b\tan^2(c+dx)}{2d}+\frac{a^3\tan(c+dx)}{d}+\frac{3ab^2\tan^5(c+dx)}{5d}+\frac{b^3 t}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^7*(a*Cos[c + d*x] + b*Sin[c + d*x])^3,x]
```

```
[Out] (a^3*Tan[c + d*x])/d + (3*a^2*b*Tan[c + d*x]^2)/(2*d) + (a*(a^2 + 3*b^2)*Ta
n[c + d*x]^3)/(3*d) + (b*(3*a^2 + b^2)*Tan[c + d*x]^4)/(4*d) + (3*a*b^2*Tan
[c + d*x]^5)/(5*d) + (b^3*Tan[c + d*x]^6)/(6*d)
```

Rule 894

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ
[m, 0] && ILtQ[n, 0]))
```

Rule 3088

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[(x^m*(b +
 a*x)^n)/(1 + x^2)^((m + n + 2)/2), x], x, Cot[c + d*x]], x] /; FreeQ[{a, b
, c, d}, x] && IntegerQ[n] && IntegerQ[(m + n)/2] && NeQ[n, -1] &&  !(GtQ[n
, 0] && GtQ[m, 1])
```

Rubi steps

$$\int \sec^7(c+dx)(a\cos(c+dx)+b\sin(c+dx))^3\,dx = -\frac{\text{Subst}\left(\int \frac{(b+ax)^3(1+x^2)}{x^7}\,dx, x, \cot(c+dx)\right)}{d}$$
$$= -\frac{\text{Subst}\left(\int \left(\frac{b^3}{x^7}+\frac{3ab^2}{x^6}+\frac{3a^2b+b^3}{x^5}+\frac{a^3+3ab^2}{x^4}+\frac{3a^2b}{x^3}+\frac{a^3}{x^2}\right)dx, x\right.}{d}$$
$$= \frac{a^3\tan(c+dx)}{d}+\frac{3a^2b\tan^2(c+dx)}{2d}+\frac{a\left(a^2+3b^2\right)\tan^3(c+}{3d}$$

Mathematica [A] time = 0.37, size = 54, normalized size = 0.45

$$\frac{(a+b\tan(c+dx))^4\left(a^2-4ab\tan(c+dx)+10b^2\tan^2(c+dx)+15b^2\right)}{60b^3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^7*(a*Cos[c + d*x] + b*Sin[c + d*x])^3,x]
```

```
[Out] ((a + b*Tan[c + d*x])^4*(a^2 + 15*b^2 - 4*a*b*Tan[c + d*x] + 10*b^2*Tan[c +
 d*x]^2))/(60*b^3*d)
```

fricas [A] time = 0.52, size = 105, normalized size = 0.88

$$\frac{10\,b^3+15\left(3\,a^2b-b^3\right)\cos\left(dx+c\right)^2+4\left(2\left(5\,a^3-3\,ab^2\right)\cos\left(dx+c\right)^5+9\,ab^2\cos\left(dx+c\right)+\left(5\,a^3-3\,ab^2\right)\cos\left(\right.\right.}{60\,d\cos\left(dx+c\right)^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^7*(a*cos(d*x+c)+b*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/60*(10*b^3 + 15*(3*a^2*b - b^3)*cos(d*x + c)^2 + 4*(2*(5*a^3 - 3*a*b^2)*c
os(d*x + c)^5 + 9*a*b^2*cos(d*x + c) + (5*a^3 - 3*a*b^2)*cos(d*x + c)^3)*si
n(d*x + c))/(d*cos(d*x + c)^6)
```

giac [A] time = 0.34, size = 112, normalized size = 0.93

$$\frac{10\,b^3\tan\left(dx+c\right)^6+36\,ab^2\tan\left(dx+c\right)^5+45\,a^2b\tan\left(dx+c\right)^4+15\,b^3\tan\left(dx+c\right)^4+20\,a^3\tan\left(dx+c\right)^3+60\,}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^7*(a*cos(d*x+c)+b*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/60*(10*b^3*tan(d*x + c)^6 + 36*a*b^2*tan(d*x + c)^5 + 45*a^2*b*tan(d*x +
c)^4 + 15*b^3*tan(d*x + c)^4 + 20*a^3*tan(d*x + c)^3 + 60*a*b^2*tan(d*x + c
)^3 + 90*a^2*b*tan(d*x + c)^2 + 60*a^3*tan(d*x + c))/d
```

maple [A] time = 20.80, size = 127, normalized size = 1.06

$$\frac{-a^3\left(-\frac{2}{3}-\frac{\left(\sec^2(dx+c)\right)}{3}\right)\tan(dx+c)+\frac{3a^2b}{4\cos(dx+c)^4}+3b^2a\left(\frac{\sin^3(dx+c)}{5\cos(dx+c)^5}+\frac{2\left(\sin^3(dx+c)\right)}{15\cos(dx+c)^3}\right)+b^3\left(\frac{\sin^4(dx+c)}{6\cos(dx+c)^6}+\frac{\sin^4(dx+c)}{12\cos(dx+c)^4}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^7*(a*cos(d*x+c)+b*sin(d*x+c))^3,x)
```

```
[Out] 1/d*(-a^3*(-2/3-1/3*sec(d*x+c)^2)*tan(d*x+c)+3/4*a^2*b/cos(d*x+c)^4+3*b^2*a
*(1/5*sin(d*x+c)^3/cos(d*x+c)^5+2/15*sin(d*x+c)^3/cos(d*x+c)^3)+b^3*(1/6*si
n(d*x+c)^4/cos(d*x+c)^6+1/12*sin(d*x+c)^4/cos(d*x+c)^4))
```

maxima [A] time = 0.33, size = 122, normalized size = 1.02

$$\frac{20\left(\tan(dx+c)^3+3\tan(dx+c)\right)a^3+12\left(3\tan(dx+c)^5+5\tan(dx+c)^3\right)ab^2-\frac{5\left(3\sin(dx+c)^2-1\right)b^3}{\sin(dx+c)^6-3\sin(dx+c)^4+3\sin(dx+c)^2}}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^7*(a*cos(d*x+c)+b*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/60*(20*(tan(d*x + c)^3 + 3*tan(d*x + c))*a^3 + 12*(3*tan(d*x + c)^5 + 5*t
an(d*x + c)^3)*a*b^2 - 5*(3*sin(d*x + c)^2 - 1)*b^3/(sin(d*x + c)^6 - 3*sin
(d*x + c)^4 + 3*sin(d*x + c)^2 - 1) + 45*a^2*b/(sin(d*x + c)^2 - 1)^2)/d
```

mupad [B] time = 0.84, size = 123, normalized size = 1.02

$$\frac{\cos(c+d\,x)^3\,\left(\frac{a^3\,\sin(c+d\,x)}{3}-\frac{a\,b^2\,\sin(c+d\,x)}{5}\right)+\cos(c+d\,x)^5\,\left(\frac{2\,a^3\,\sin(c+d\,x)}{3}-\frac{2\,a\,b^2\,\sin(c+d\,x)}{5}\right)+\cos(c+d\,x)^2\,\left(\frac{3\,a^2\,b}{4}-\right.}{d\cos(c+d\,x)^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*cos(c + d*x) + b*sin(c + d*x))^3/cos(c + d*x)^7,x)
```

```
[Out] (cos(c + d*x)^3*((a^3*sin(c + d*x))/3 - (a*b^2*sin(c + d*x))/5) + cos(c + d
*x)^5*((2*a^3*sin(c + d*x))/3 - (2*a*b^2*sin(c + d*x))/5) + cos(c + d*x)^2*
((3*a^2*b)/4 - b^3/4) + b^3/6 + (3*a*b^2*cos(c + d*x)*sin(c + d*x))/5)/(d*c
os(c + d*x)^6)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**7*(a*cos(d*x+c)+b*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

3.70 $\int \sec^8(c+dx)(a\cos(c+dx)+b\sin(c+dx))^3\,dx$

Optimal. Leaf size=210

$$\frac{3a^3\tanh^{-1}(\sin(c+dx))}{8d}+\frac{a^3\tan(c+dx)\sec^3(c+dx)}{4d}+\frac{3a^3\tan(c+dx)\sec(c+dx)}{8d}+\frac{3a^2b\sec^5(c+dx)}{5d}-\frac{3ab^2\tanh}{}$$

```
[Out] 3/8*a^3*arctanh(sin(d*x+c))/d-3/16*a*b^2*arctanh(sin(d*x+c))/d+3/5*a^2*b*se
c(d*x+c)^5/d-1/5*b^3*sec(d*x+c)^5/d+1/7*b^3*sec(d*x+c)^7/d+3/8*a^3*sec(d*x+
c)*tan(d*x+c)/d-3/16*a*b^2*sec(d*x+c)*tan(d*x+c)/d+1/4*a^3*sec(d*x+c)^3*tan
(d*x+c)/d-1/8*a*b^2*sec(d*x+c)^3*tan(d*x+c)/d+1/2*a*b^2*sec(d*x+c)^5*tan(d*
x+c)/d
```

Rubi [A] time = 0.22, antiderivative size = 210, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 7, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {3090, 3768, 3770, 2606, 30, 2611, 14}

$$\frac{3a^2b\sec^5(c+dx)}{5d}+\frac{3a^3\tanh^{-1}(\sin(c+dx))}{8d}+\frac{a^3\tan(c+dx)\sec^3(c+dx)}{4d}+\frac{3a^3\tan(c+dx)\sec(c+dx)}{8d}-\frac{3ab^2\tanh}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^8*(a*Cos[c + d*x] + b*Sin[c + d*x])^3,x]
```

```
[Out] (3*a^3*ArcTanh[Sin[c + d*x]])/(8*d) - (3*a*b^2*ArcTanh[Sin[c + d*x]])/(16*d
) + (3*a^2*b*Sec[c + d*x]^5)/(5*d) - (b^3*Sec[c + d*x]^5)/(5*d) + (b^3*Sec[
c + d*x]^7)/(7*d) + (3*a^3*Sec[c + d*x]*Tan[c + d*x])/(8*d) - (3*a*b^2*Sec[
c + d*x]*Tan[c + d*x])/(16*d) + (a^3*Sec[c + d*x]^3*Tan[c + d*x])/(4*d) - (
a*b^2*Sec[c + d*x]^3*Tan[c + d*x])/(8*d) + (a*b^2*Sec[c + d*x]^5*Tan[c + d*
x])/(2*d)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
```

```
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 && !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 2611

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(b*(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*(
m + n - 1)), x] - Dist[(b^2*(n - 1))/(m + n - 1), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 3090

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_.), x_Symbol] :> Int[ExpandTrig[cos[c + d*x]^m*(a
*cos[c + d*x] + b*sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d}, x] && Inte
gerQ[m] && IGtQ[n, 0]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \sec^8(c+dx)(a\cos(c+dx)+b\sin(c+dx))^3\,dx = \int \left(a^3\sec^5(c+dx)+3a^2b\sec^5(c+dx)\tan(c+dx)+3ab^2\,\mathrm{se}\right.$$
$$= a^3\int \sec^5(c+dx)\,dx+\left(3a^2b\right)\int \sec^5(c+dx)\tan(c+dx)\,dx$$
$$= \frac{a^3\sec^3(c+dx)\tan(c+dx)}{4d}+\frac{ab^2\sec^5(c+dx)\tan(c+dx)}{2d}+$$
$$= \frac{3a^2b\sec^5(c+dx)}{5d}+\frac{3a^3\sec(c+dx)\tan(c+dx)}{8d}+\frac{a^3\sec^3(c}{}$$
$$= \frac{3a^3\tanh^{-1}(\sin(c+dx))}{8d}+\frac{3a^2b\sec^5(c+dx)}{5d}-\frac{b^3\sec^5(c+d}{5d}$$
$$= \frac{3a^3\tanh^{-1}(\sin(c+dx))}{8d}-\frac{3ab^2\tanh^{-1}(\sin(c+dx))}{16d}+\frac{3a^2b}{}$$

Mathematica [B] time = 2.13, size = 637, normalized size = 3.03

$$\frac{\sec^7(c+dx)\left(4340a^3\sin(2(c+dx))+2800a^3\sin(4(c+dx))+420a^3\sin(6(c+dx))-4410a^3\cos(3(c+dx))\log\left(\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^8*(a*Cos[c + d*x] + b*Sin[c + d*x])^3,x]
```

```
[Out] (Sec[c + d*x]^7*(10752*a^2*b + 1536*b^3 + 3584*(3*a^2*b - b^3)*Cos[2*(c + d
*x)] - 4410*a^3*Cos[3*(c + d*x)]*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] +
 2205*a*b^2*Cos[3*(c + d*x)]*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] - 147
0*a^3*Cos[5*(c + d*x)]*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + 735*a*b^2
*Cos[5*(c + d*x)]*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] - 210*a^3*Cos[7*
(c + d*x)]*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + 105*a*b^2*Cos[7*(c +
d*x)]*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] - 3675*a*(2*a^2 - b^2)*Cos[c
 + d*x]*(Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] - Log[Cos[(c + d*x)/2] +
Sin[(c + d*x)/2]]) + 4410*a^3*Cos[3*(c + d*x)]*Log[Cos[(c + d*x)/2] + Sin[(
c + d*x)/2]] - 2205*a*b^2*Cos[3*(c + d*x)]*Log[Cos[(c + d*x)/2] + Sin[(c +
d*x)/2]] + 1470*a^3*Cos[5*(c + d*x)]*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2
]] - 735*a*b^2*Cos[5*(c + d*x)]*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] +
210*a^3*Cos[7*(c + d*x)]*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] - 105*a*b
^2*Cos[7*(c + d*x)]*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] + 4340*a^3*Sin
[2*(c + d*x)] + 6790*a*b^2*Sin[2*(c + d*x)] + 2800*a^3*Sin[4*(c + d*x)] - 1
400*a*b^2*Sin[4*(c + d*x)] + 420*a^3*Sin[6*(c + d*x)] - 210*a*b^2*Sin[6*(c
+ d*x)]))/(35840*d)
```

fricas [A] time = 0.73, size = 170, normalized size = 0.81

$$\frac{105\left(2\,a^3-ab^2\right)\cos\left(dx+c\right)^7\log\left(\sin\left(dx+c\right)+1\right)-105\left(2\,a^3-ab^2\right)\cos\left(dx+c\right)^7\log\left(-\sin\left(dx+c\right)+1\right)+16\cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^8*(a*cos(d*x+c)+b*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/1120*(105*(2*a^3 - a*b^2)*cos(d*x + c)^7*log(sin(d*x + c) + 1) - 105*(2*a
^3 - a*b^2)*cos(d*x + c)^7*log(-sin(d*x + c) + 1) + 160*b^3 + 224*(3*a^2*b
- b^3)*cos(d*x + c)^2 + 70*(3*(2*a^3 - a*b^2)*cos(d*x + c)^5 + 8*a*b^2*cos(
d*x + c) + 2*(2*a^3 - a*b^2)*cos(d*x + c)^3)*sin(d*x + c))/(d*cos(d*x + c)^
7)
```

giac [B] time = 0.38, size = 465, normalized size = 2.21

$$\frac{105\left(2\,a^3-ab^2\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-105\left(2\,a^3-ab^2\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)+\frac{2\left(350\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{1\cdots}\cdots\right)}{\cdots}}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^8*(a*cos(d*x+c)+b*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/560*(105*(2*a^3 - a*b^2)*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 105*(2*a^3
- a*b^2)*log(abs(tan(1/2*d*x + 1/2*c) - 1)) + 2*(350*a^3*tan(1/2*d*x + 1/2*
c)^13 + 105*a*b^2*tan(1/2*d*x + 1/2*c)^13 - 1680*a^2*b*tan(1/2*d*x + 1/2*c)
^12 - 840*a^3*tan(1/2*d*x + 1/2*c)^11 + 1540*a*b^2*tan(1/2*d*x + 1/2*c)^11
+ 3360*a^2*b*tan(1/2*d*x + 1/2*c)^10 - 1120*b^3*tan(1/2*d*x + 1/2*c)^10 + 6
30*a^3*tan(1/2*d*x + 1/2*c)^9 + 1085*a*b^2*tan(1/2*d*x + 1/2*c)^9 - 5040*a^
2*b*tan(1/2*d*x + 1/2*c)^8 - 1120*b^3*tan(1/2*d*x + 1/2*c)^8 + 6720*a^2*b*t
an(1/2*d*x + 1/2*c)^6 - 2240*b^3*tan(1/2*d*x + 1/2*c)^6 - 630*a^3*tan(1/2*d
*x + 1/2*c)^5 - 1085*a*b^2*tan(1/2*d*x + 1/2*c)^5 - 3696*a^2*b*tan(1/2*d*x
+ 1/2*c)^4 - 448*b^3*tan(1/2*d*x + 1/2*c)^4 + 840*a^3*tan(1/2*d*x + 1/2*c)^
3 - 1540*a*b^2*tan(1/2*d*x + 1/2*c)^3 + 672*a^2*b*tan(1/2*d*x + 1/2*c)^2 -
224*b^3*tan(1/2*d*x + 1/2*c)^2 - 350*a^3*tan(1/2*d*x + 1/2*c) - 105*a*b^2*t
an(1/2*d*x + 1/2*c) - 336*a^2*b + 32*b^3)/(tan(1/2*d*x + 1/2*c)^2 - 1)^7)/d
```

maple [A] time = 11.10, size = 328, normalized size = 1.56

$$\frac{a^3\left(\sec^3\left(dx+c\right)\right)\tan\left(dx+c\right)}{4d}+\frac{3a^3\sec\left(dx+c\right)\tan\left(dx+c\right)}{8d}+\frac{3a^3\ln\left(\sec\left(dx+c\right)+\tan\left(dx+c\right)\right)}{8d}+\frac{3a^2b}{5d\cos\left(dx+\cdots\right.}\cdots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^8*(a*cos(d*x+c)+b*sin(d*x+c))^3,x)
```

```
[Out] 1/4*a^3*sec(d*x+c)^3*tan(d*x+c)/d+3/8*a^3*sec(d*x+c)*tan(d*x+c)/d+3/8/d*a^3
*ln(sec(d*x+c)+tan(d*x+c))+3/5/d*a^2*b/cos(d*x+c)^5+1/2/d*b^2*a*sin(d*x+c)^
3/cos(d*x+c)^6+3/8/d*b^2*a*sin(d*x+c)^3/cos(d*x+c)^4+3/16/d*b^2*a*sin(d*x+c
)^3/cos(d*x+c)^2+3/16*a*b^2*sin(d*x+c)/d-3/16/d*b^2*a*ln(sec(d*x+c)+tan(d*x
+c))+1/7/d*b^3*sin(d*x+c)^4/cos(d*x+c)^7+3/35/d*b^3*sin(d*x+c)^4/cos(d*x+c)
^5+1/35/d*b^3*sin(d*x+c)^4/cos(d*x+c)^3-1/35/d*b^3*sin(d*x+c)^4/cos(d*x+c)-
1/35/d*b^3*cos(d*x+c)*sin(d*x+c)^2-2/35*b^3*cos(d*x+c)/d
```

maxima [A] time = 0.33, size = 208, normalized size = 0.99

$$\frac{35\,ab^2\left(\frac{2\left(3\,\sin(dx+c)^5-8\,\sin(dx+c)^3-3\,\sin(dx+c)\right)}{\sin(dx+c)^6-3\,\sin(dx+c)^4+3\,\sin(dx+c)^2-1}-3\,\log\left(\sin\left(dx+c\right)+1\right)+3\,\log\left(\sin\left(dx+c\right)-1\right)\right)-70\,a^3\left(\frac{2\left(3\,\sin(dx+c\cdots\right.}{\sin(dx+c)^4-\cdots}\right.\cdots}{1120\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^8*(a*cos(d*x+c)+b*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/1120*(35*a*b^2*(2*(3*sin(d*x + c)^5 - 8*sin(d*x + c)^3 - 3*sin(d*x + c))/
(sin(d*x + c)^6 - 3*sin(d*x + c)^4 + 3*sin(d*x + c)^2 - 1) - 3*log(sin(d*x
+ c) + 1) + 3*log(sin(d*x + c) - 1)) - 70*a^3*(2*(3*sin(d*x + c)^3 - 5*sin(
d*x + c))/(sin(d*x + c)^4 - 2*sin(d*x + c)^2 + 1) - 3*log(sin(d*x + c) + 1)
 + 3*log(sin(d*x + c) - 1)) + 672*a^2*b/cos(d*x + c)^5 - 32*(7*cos(d*x + c)
^2 - 5)*b^3/cos(d*x + c)^7)/d
```

mupad [B] time = 4.28, size = 423, normalized size = 2.01

$$\frac{3\,a\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)\,\left(2\,a^2-b^2\right)}{8\,d}-\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(\frac{5\,a^3}{4}+\frac{3\,a\,b^2}{8}\right)+\frac{6\,a^2\,b}{5}+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3\,\left(\frac{11\,a\,b^2}{2}-3\,a^3\right)-\tan\left(\frac{c}{2}\cdots\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*cos(c + d*x) + b*sin(c + d*x))^3/cos(c + d*x)^8,x)
```

```
[Out] (3*a*atanh(tan(c/2 + (d*x)/2))*(2*a^2 - b^2))/(8*d) - (tan(c/2 + (d*x)/2)*(
(3*a*b^2)/8 + (5*a^3)/4) + (6*a^2*b)/5 + tan(c/2 + (d*x)/2)^3*((11*a*b^2)/2
 - 3*a^3) - tan(c/2 + (d*x)/2)^11*((11*a*b^2)/2 - 3*a^3) - tan(c/2 + (d*x)/
2)^13*((3*a*b^2)/8 + (5*a^3)/4) + tan(c/2 + (d*x)/2)^5*((31*a*b^2)/8 + (9*a
^3)/4) - tan(c/2 + (d*x)/2)^9*((31*a*b^2)/8 + (9*a^3)/4) - tan(c/2 + (d*x)/
2)^10*(12*a^2*b - 4*b^3) - tan(c/2 + (d*x)/2)^2*((12*a^2*b)/5 - (4*b^3)/5)
+ tan(c/2 + (d*x)/2)^8*(18*a^2*b + 4*b^3) - tan(c/2 + (d*x)/2)^6*(24*a^2*b
```

```
- 8*b^3) + tan(c/2 + (d*x)/2)^4*((66*a^2*b)/5 + (8*b^3)/5) - (4*b^3)/35 + 6
*a^2*b*tan(c/2 + (d*x)/2)^12)/(d*(7*tan(c/2 + (d*x)/2)^2 - 21*tan(c/2 + (d*
x)/2)^4 + 35*tan(c/2 + (d*x)/2)^6 - 35*tan(c/2 + (d*x)/2)^8 + 21*tan(c/2 +
(d*x)/2)^10 - 7*tan(c/2 + (d*x)/2)^12 + tan(c/2 + (d*x)/2)^14 - 1))
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**8*(a*cos(d*x+c)+b*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

3.71 $\int \sec^9(c+dx)(a\cos(c+dx)+b\sin(c+dx))^3\,dx$

Optimal. Leaf size=174

$$\frac{a^3\tan(c+dx)}{d}+\frac{b\left(3a^2+2b^2\right)\tan^6(c+dx)}{6d}+\frac{a\left(a^2+6b^2\right)\tan^5(c+dx)}{5d}+\frac{b\left(6a^2+b^2\right)\tan^4(c+dx)}{4d}+\frac{a\left(2a^2+3b^2\right)\tan^3(c+dx)}{3d}$$

```
[Out] a^3*tan(d*x+c)/d+3/2*a^2*b*tan(d*x+c)^2/d+1/3*a*(2*a^2+3*b^2)*tan(d*x+c)^3/
d+1/4*b*(6*a^2+b^2)*tan(d*x+c)^4/d+1/5*a*(a^2+6*b^2)*tan(d*x+c)^5/d+1/6*b*(
3*a^2+2*b^2)*tan(d*x+c)^6/d+3/7*a*b^2*tan(d*x+c)^7/d+1/8*b^3*tan(d*x+c)^8/d
```

Rubi [A] time = 0.14, antiderivative size = 174, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {3088, 948}

$$\frac{b\left(3a^2+2b^2\right)\tan^6(c+dx)}{6d}+\frac{a\left(a^2+6b^2\right)\tan^5(c+dx)}{5d}+\frac{b\left(6a^2+b^2\right)\tan^4(c+dx)}{4d}+\frac{a\left(2a^2+3b^2\right)\tan^3(c+dx)}{3d}+\frac{3a}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^9*(a*Cos[c + d*x] + b*Sin[c + d*x])^3,x]
```

```
[Out] (a^3*Tan[c + d*x])/d + (3*a^2*b*Tan[c + d*x]^2)/(2*d) + (a*(2*a^2 + 3*b^2)*
Tan[c + d*x]^3)/(3*d) + (b*(6*a^2 + b^2)*Tan[c + d*x]^4)/(4*d) + (a*(a^2 +
6*b^2)*Tan[c + d*x]^5)/(5*d) + (b*(3*a^2 + 2*b^2)*Tan[c + d*x]^6)/(6*d) + (
3*a*b^2*Tan[c + d*x]^7)/(7*d) + (b^3*Tan[c + d*x]^8)/(8*d)
```

Rule 948

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IGtQ[p, 0] && (IGtQ[m, 0] || (EqQ[m, -2] && EqQ[p, 1] &
& EqQ[d, 0]))
```

Rule 3088

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[(x^m*(b +
 a*x)^n)/(1 + x^2)^((m + n + 2)/2), x], x, Cot[c + d*x]], x] /; FreeQ[{a, b
, c, d}, x] && IntegerQ[n] && IntegerQ[(m + n)/2] && NeQ[n, -1] &&  !(GtQ[n
, 0] && GtQ[m, 1])
```

Rubi steps

$$\int \sec^9(c+dx)(a\cos(c+dx)+b\sin(c+dx))^3\,dx = -\frac{\text{Subst}\left(\int \frac{(b+ax)^3(1+x^2)^2}{x^9}\,dx, x, \cot(c+dx)\right)}{d}$$
$$= -\frac{\text{Subst}\left(\int \left(\frac{b^3}{x^9}+\frac{3ab^2}{x^8}+\frac{3a^2b+2b^3}{x^7}+\frac{a^3+6ab^2}{x^6}+\frac{6a^2b+b^3}{x^5}+\frac{2a^3+3}{x^4}\right.\right.}{d}$$
$$= \frac{a^3\tan(c+dx)}{d}+\frac{3a^2b\tan^2(c+dx)}{2d}+\frac{a\left(2a^2+3b^2\right)\tan^3(c}{3d}$$

Mathematica [A] time = 0.63, size = 115, normalized size = 0.66

$$\frac{\frac{1}{3}\left(3a^2+b^2\right)(a+b\tan(c+dx))^6-\frac{4}{5}a\left(a^2+b^2\right)(a+b\tan(c+dx))^5+\frac{1}{4}\left(a^2+b^2\right)^2(a+b\tan(c+dx))^4+\frac{1}{8}(a+}{b^5d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^9*(a*Cos[c + d*x] + b*Sin[c + d*x])^3,x]
```

```
[Out] (((a^2 + b^2)^2*(a + b*Tan[c + d*x])^4)/4 - (4*a*(a^2 + b^2)*(a + b*Tan[c +
 d*x])^5)/5 + ((3*a^2 + b^2)*(a + b*Tan[c + d*x])^6)/3 - (4*a*(a + b*Tan[c
+ d*x])^7)/7 + (a + b*Tan[c + d*x])^8/8)/(b^5*d)
```

fricas [A] time = 0.81, size = 128, normalized size = 0.74

$$\frac{105\,b^3+140\left(3\,a^2b-b^3\right)\cos(dx+c)^2+8\left(8\left(7\,a^3-3\,ab^2\right)\cos(dx+c)^7+4\left(7\,a^3-3\,ab^2\right)\cos(dx+c)^5+45\,ab\right.}{840\,d\cos(dx+c)^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^9*(a*cos(d*x+c)+b*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/840*(105*b^3 + 140*(3*a^2*b - b^3)*cos(d*x + c)^2 + 8*(8*(7*a^3 - 3*a*b^2
)*cos(d*x + c)^7 + 4*(7*a^3 - 3*a*b^2)*cos(d*x + c)^5 + 45*a*b^2*cos(d*x +
c) + 3*(7*a^3 - 3*a*b^2)*cos(d*x + c)^3)*sin(d*x + c))/(d*cos(d*x + c)^8)
```

giac [A] time = 0.43, size = 166, normalized size = 0.95

$$\frac{105\,b^3\tan(dx+c)^8+360\,ab^2\tan(dx+c)^7+420\,a^2b\tan(dx+c)^6+280\,b^3\tan(dx+c)^6+168\,a^3\tan(dx+c)^5}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^9*(a*cos(d*x+c)+b*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/840*(105*b^3*tan(d*x + c)^8 + 360*a*b^2*tan(d*x + c)^7 + 420*a^2*b*tan(d*
x + c)^6 + 280*b^3*tan(d*x + c)^6 + 168*a^3*tan(d*x + c)^5 + 1008*a*b^2*tan
(d*x + c)^5 + 1260*a^2*b*tan(d*x + c)^4 + 210*b^3*tan(d*x + c)^4 + 560*a^3*
tan(d*x + c)^3 + 840*a*b^2*tan(d*x + c)^3 + 1260*a^2*b*tan(d*x + c)^2 + 840
*a^3*tan(d*x + c))/d
```

maple [A] time = 9.66, size = 173, normalized size = 0.99

$$\frac{-a^3\left(-\frac{8}{15}-\frac{\left(\sec^4(dx+c)\right)}{5}-\frac{4\left(\sec^2(dx+c)\right)}{15}\right)\tan\left(dx+c\right)+\frac{a^2b}{2\cos(dx+c)^6}+3b^2a\left(\frac{\sin^3(dx+c)}{7\cos(dx+c)^7}+\frac{4\left(\sin^3(dx+c)\right)}{35\cos(dx+c)^5}+\frac{8\left(\sin^3(dx+c)\right)}{105\cos(dx+c)^3}\right)+}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^9*(a*cos(d*x+c)+b*sin(d*x+c))^3,x)
```

```
[Out] 1/d*(-a^3*(-8/15-1/5*sec(d*x+c)^4-4/15*sec(d*x+c)^2)*tan(d*x+c)+1/2*a^2*b/c
os(d*x+c)^6+3*b^2*a*(1/7*sin(d*x+c)^3/cos(d*x+c)^7+4/35*sin(d*x+c)^3/cos(d*
x+c)^5+8/105*sin(d*x+c)^3/cos(d*x+c)^3)+b^3*(1/8*sin(d*x+c)^4/cos(d*x+c)^8+
1/12*sin(d*x+c)^4/cos(d*x+c)^6+1/24*sin(d*x+c)^4/cos(d*x+c)^4))
```

maxima [A] time = 0.33, size = 154, normalized size = 0.89

$$\frac{56\left(3\tan\left(dx+c\right)^5+10\tan\left(dx+c\right)^3+15\tan\left(dx+c\right)\right)a^3+24\left(15\tan\left(dx+c\right)^7+42\tan\left(dx+c\right)^5+35\tan\left(d\right.\right.}{840\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^9*(a*cos(d*x+c)+b*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/840*(56*(3*tan(d*x + c)^5 + 10*tan(d*x + c)^3 + 15*tan(d*x + c))*a^3 + 24
*(15*tan(d*x + c)^7 + 42*tan(d*x + c)^5 + 35*tan(d*x + c)^3)*a*b^2 + 35*(4*
sin(d*x + c)^2 - 1)*b^3/(sin(d*x + c)^8 - 4*sin(d*x + c)^6 + 6*sin(d*x + c)
^4 - 4*sin(d*x + c)^2 + 1) - 420*a^2*b/(sin(d*x + c)^2 - 1)^3)/d
```

mupad [B] time = 1.15, size = 156, normalized size = 0.90

$$\frac{\cos\left(c+d\,x\right)^3\,\left(\frac{a^3\,\sin(c+d\,x)}{5}-\frac{3\,a\,b^2\,\sin(c+d\,x)}{35}\right)+\cos\left(c+d\,x\right)^5\,\left(\frac{4\,a^3\,\sin(c+d\,x)}{15}-\frac{4\,a\,b^2\,\sin(c+d\,x)}{35}\right)+\cos\left(c+d\,x\right)^7\,\left(\frac{8\,a^3\,\sin}{15}\right.}{d\cos\left(c+d\,x\right)^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*cos(c + d*x) + b*sin(c + d*x))^3/cos(c + d*x)^9,x)
```

```
[Out] (cos(c + d*x)^3*((a^3*sin(c + d*x))/5 - (3*a*b^2*sin(c + d*x))/35) + cos(c
+ d*x)^5*((4*a^3*sin(c + d*x))/15 - (4*a*b^2*sin(c + d*x))/35) + cos(c + d*
x)^7*((8*a^3*sin(c + d*x))/15 - (8*a*b^2*sin(c + d*x))/35) + cos(c + d*x)^2
*((a^2*b)/2 - b^3/6) + b^3/8 + (3*a*b^2*cos(c + d*x)*sin(c + d*x))/7)/(d*co
s(c + d*x)^8)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**9*(a*cos(d*x+c)+b*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

3.72 $\int \sec^{10}(c+dx)(a\cos(c+dx)+b\sin(c+dx))^3\,dx$

Optimal. Leaf size=259

$$\frac{5a^3\tanh^{-1}(\sin(c+dx))}{16d}+\frac{a^3\tan(c+dx)\sec^5(c+dx)}{6d}+\frac{5a^3\tan(c+dx)\sec^3(c+dx)}{24d}+\frac{5a^3\tan(c+dx)\sec(c+dx)}{16d}$$

```
[Out] 5/16*a^3*arctanh(sin(d*x+c))/d-15/128*a*b^2*arctanh(sin(d*x+c))/d+3/7*a^2*b
*sec(d*x+c)^7/d-1/7*b^3*sec(d*x+c)^7/d+1/9*b^3*sec(d*x+c)^9/d+5/16*a^3*sec(
d*x+c)*tan(d*x+c)/d-15/128*a*b^2*sec(d*x+c)*tan(d*x+c)/d+5/24*a^3*sec(d*x+c
)^3*tan(d*x+c)/d-5/64*a*b^2*sec(d*x+c)^3*tan(d*x+c)/d+1/6*a^3*sec(d*x+c)^5*
tan(d*x+c)/d-1/16*a*b^2*sec(d*x+c)^5*tan(d*x+c)/d+3/8*a*b^2*sec(d*x+c)^7*ta
n(d*x+c)/d
```

Rubi [A] time = 0.27, antiderivative size = 259, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 7, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {3090, 3768, 3770, 2606, 30, 2611, 14}

$$\frac{3a^2b\sec^7(c+dx)}{7d}+\frac{5a^3\tanh^{-1}(\sin(c+dx))}{16d}+\frac{a^3\tan(c+dx)\sec^5(c+dx)}{6d}+\frac{5a^3\tan(c+dx)\sec^3(c+dx)}{24d}+\frac{5a^3\tan(}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^10*(a*Cos[c + d*x] + b*Sin[c + d*x])^3,x]
```

```
[Out] (5*a^3*ArcTanh[Sin[c + d*x]])/(16*d) - (15*a*b^2*ArcTanh[Sin[c + d*x]])/(12
8*d) + (3*a^2*b*Sec[c + d*x]^7)/(7*d) - (b^3*Sec[c + d*x]^7)/(7*d) + (b^3*S
ec[c + d*x]^9)/(9*d) + (5*a^3*Sec[c + d*x]*Tan[c + d*x])/(16*d) - (15*a*b^2
*Sec[c + d*x]*Tan[c + d*x])/(128*d) + (5*a^3*Sec[c + d*x]^3*Tan[c + d*x])/(
24*d) - (5*a*b^2*Sec[c + d*x]^3*Tan[c + d*x])/(64*d) + (a^3*Sec[c + d*x]^5*
Tan[c + d*x])/(6*d) - (a*b^2*Sec[c + d*x]^5*Tan[c + d*x])/(16*d) + (3*a*b^2
*Sec[c + d*x]^7*Tan[c + d*x])/(8*d)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 2611

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(b*(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*(
m + n - 1)), x] - Dist[(b^2*(n - 1))/(m + n - 1), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 3090

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_.), x_Symbol] :> Int[ExpandTrig[cos[c + d*x]^m*(a
*cos[c + d*x] + b*sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d}, x] && Inte
gerQ[m] && IGtQ[n, 0]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \sec^{10}(c+dx)(a\cos(c+dx)+b\sin(c+dx))^3\,dx = \int \left(a^3\sec^7(c+dx)+3a^2b\sec^7(c+dx)\tan(c+dx)+3ab^2\text{ s}\right.$$
$$= a^3\int \sec^7(c+dx)\,dx + \left(3a^2b\right)\int \sec^7(c+dx)\tan(c+dx)\,d$$
$$= \frac{a^3\sec^5(c+dx)\tan(c+dx)}{6d} + \frac{3ab^2\sec^7(c+dx)\tan(c+dx)}{8d}$$
$$= \frac{3a^2b\sec^7(c+dx)}{7d} + \frac{5a^3\sec^3(c+dx)\tan(c+dx)}{24d} + \frac{a^3\sec^5}{}$$
$$= \frac{3a^2b\sec^7(c+dx)}{7d} - \frac{b^3\sec^7(c+dx)}{7d} + \frac{b^3\sec^9(c+dx)}{9d} + \frac{5a}{}$$
$$= \frac{5a^3\tanh^{-1}(\sin(c+dx))}{16d} + \frac{3a^2b\sec^7(c+dx)}{7d} - \frac{b^3\sec^7(c+}{7d}$$
$$= \frac{5a^3\tanh^{-1}(\sin(c+dx))}{16d} - \frac{15ab^2\tanh^{-1}(\sin(c+dx))}{128d} + \frac{3a^2}{}$$

Mathematica [B] time = 4.02, size = 810, normalized size = 3.13

$$\frac{\sec^9(c+dx)\left(-211680\cos(3(c+dx))\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)a^3-90720\cos(5(c+dx))\log\left(\cos\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^10*(a*Cos[c + d*x] + b*Sin[c + d*x])^3,x]
```

```
[Out] (Sec[c + d*x]^9*(442368*a^2*b + 81920*b^3 + 147456*(3*a^2*b - b^3)*Cos[2*(c
 + d*x)] - 211680*a^3*Cos[3*(c + d*x)]*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)
/2]] + 79380*a*b^2*Cos[3*(c + d*x)]*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]
] - 90720*a^3*Cos[5*(c + d*x)]*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + 3
4020*a*b^2*Cos[5*(c + d*x)]*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] - 2268
0*a^3*Cos[7*(c + d*x)]*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + 8505*a*b^
2*Cos[7*(c + d*x)]*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] - 2520*a^3*Cos[
9*(c + d*x)]*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + 945*a*b^2*Cos[9*(c
+ d*x)]*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] - 39690*a*(8*a^2 - 3*b^2)*
Cos[c + d*x]*(Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] - Log[Cos[(c + d*x)/
2] + Sin[(c + d*x)/2]]) + 211680*a^3*Cos[3*(c + d*x)]*Log[Cos[(c + d*x)/2]
+ Sin[(c + d*x)/2]] - 79380*a*b^2*Cos[3*(c + d*x)]*Log[Cos[(c + d*x)/2] + S
in[(c + d*x)/2]] + 90720*a^3*Cos[5*(c + d*x)]*Log[Cos[(c + d*x)/2] + Sin[(c
 + d*x)/2]] - 34020*a*b^2*Cos[5*(c + d*x)]*Log[Cos[(c + d*x)/2] + Sin[(c +
d*x)/2]] + 22680*a^3*Cos[7*(c + d*x)]*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/
2]] - 8505*a*b^2*Cos[7*(c + d*x)]*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]
```

```
+ 2520*a^3*Cos[9*(c + d*x)]*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] - 945*
a*b^2*Cos[9*(c + d*x)]*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] + 223776*a^
3*Sin[2*(c + d*x)] + 303156*a*b^2*Sin[2*(c + d*x)] + 167328*a^3*Sin[4*(c +
d*x)] - 62748*a*b^2*Sin[4*(c + d*x)] + 43680*a^3*Sin[6*(c + d*x)] - 16380*a
*b^2*Sin[6*(c + d*x)] + 5040*a^3*Sin[8*(c + d*x)] - 1890*a*b^2*Sin[8*(c + d
*x)]))/(2064384*d)
```

fricas [A] time = 0.76, size = 192, normalized size = 0.74

$$\frac{315\left(8\,a^3-3\,ab^2\right)\cos\left(dx+c\right)^9\log\left(\sin\left(dx+c\right)+1\right)-315\left(8\,a^3-3\,ab^2\right)\cos\left(dx+c\right)^9\log\left(-\sin\left(dx+c\right)+1\right)+\cdots}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^10*(a*cos(d*x+c)+b*sin(d*x+c))^3,x, algorithm="fricas"
)
```

```
[Out] 1/16128*(315*(8*a^3 - 3*a*b^2)*cos(d*x + c)^9*log(sin(d*x + c) + 1) - 315*(
8*a^3 - 3*a*b^2)*cos(d*x + c)^9*log(-sin(d*x + c) + 1) + 1792*b^3 + 2304*(3
*a^2*b - b^3)*cos(d*x + c)^2 + 42*(15*(8*a^3 - 3*a*b^2)*cos(d*x + c)^7 + 10
*(8*a^3 - 3*a*b^2)*cos(d*x + c)^5 + 144*a*b^2*cos(d*x + c) + 8*(8*a^3 - 3*a
*b^2)*cos(d*x + c)^3)*sin(d*x + c))/(d*cos(d*x + c)^9)
```

giac [B] time = 0.91, size = 597, normalized size = 2.31

$$\frac{315\left(8\,a^3-3\,ab^2\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-315\left(8\,a^3-3\,ab^2\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)+\frac{2\left(5544\,a^3\tan\left(\frac{1}{2}\,dx\cdots\right.\right.}{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^10*(a*cos(d*x+c)+b*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/8064*(315*(8*a^3 - 3*a*b^2)*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 315*(8*a
^3 - 3*a*b^2)*log(abs(tan(1/2*d*x + 1/2*c) - 1)) + 2*(5544*a^3*tan(1/2*d*x
+ 1/2*c)^17 + 945*a*b^2*tan(1/2*d*x + 1/2*c)^17 - 24192*a^2*b*tan(1/2*d*x +
 1/2*c)^16 - 15792*a^3*tan(1/2*d*x + 1/2*c)^15 + 24066*a*b^2*tan(1/2*d*x +
1/2*c)^15 + 48384*a^2*b*tan(1/2*d*x + 1/2*c)^14 - 16128*b^3*tan(1/2*d*x + 1
/2*c)^14 + 29232*a^3*tan(1/2*d*x + 1/2*c)^13 + 31374*a*b^2*tan(1/2*d*x + 1/
2*c)^13 - 145152*a^2*b*tan(1/2*d*x + 1/2*c)^12 - 26880*b^3*tan(1/2*d*x + 1/
2*c)^12 - 33264*a^3*tan(1/2*d*x + 1/2*c)^11 + 54810*a*b^2*tan(1/2*d*x + 1/2
*c)^11 + 241920*a^2*b*tan(1/2*d*x + 1/2*c)^10 - 80640*b^3*tan(1/2*d*x + 1/2
*c)^10 - 193536*a^2*b*tan(1/2*d*x + 1/2*c)^8 - 48384*b^3*tan(1/2*d*x + 1/2*
c)^8 + 33264*a^3*tan(1/2*d*x + 1/2*c)^7 - 54810*a*b^2*tan(1/2*d*x + 1/2*c)^
```

```
7 + 145152*a^2*b*tan(1/2*d*x + 1/2*c)^6 - 48384*b^3*tan(1/2*d*x + 1/2*c)^6
- 29232*a^3*tan(1/2*d*x + 1/2*c)^5 - 31374*a*b^2*tan(1/2*d*x + 1/2*c)^5 - 7
6032*a^2*b*tan(1/2*d*x + 1/2*c)^4 - 6912*b^3*tan(1/2*d*x + 1/2*c)^4 + 15792
*a^3*tan(1/2*d*x + 1/2*c)^3 - 24066*a*b^2*tan(1/2*d*x + 1/2*c)^3 + 6912*a^2
*b*tan(1/2*d*x + 1/2*c)^2 - 2304*b^3*tan(1/2*d*x + 1/2*c)^2 - 5544*a^3*tan(
1/2*d*x + 1/2*c) - 945*a*b^2*tan(1/2*d*x + 1/2*c) - 3456*a^2*b + 256*b^3)/(
tan(1/2*d*x + 1/2*c)^2 - 1)^9)/d
```

maple [A] time = 21.07, size = 399, normalized size = 1.54

$$\frac{a^3 \left(\sec^5(dx+c)\right)\tan(dx+c)}{6d}+\frac{5a^3\left(\sec^3(dx+c)\right)\tan(dx+c)}{24d}+\frac{5a^3\sec(dx+c)\tan(dx+c)}{16d}+\frac{5a^3\ln(\sec(dx+c}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^10*(a*cos(d*x+c)+b*sin(d*x+c))^3,x)
```

```
[Out] 1/6*a^3*sec(d*x+c)^5*tan(d*x+c)/d+5/24*a^3*sec(d*x+c)^3*tan(d*x+c)/d+5/16*a
^3*sec(d*x+c)*tan(d*x+c)/d+5/16/d*a^3*ln(sec(d*x+c)+tan(d*x+c))+3/7/d*a^2*b
/cos(d*x+c)^7+3/8/d*b^2*a*sin(d*x+c)^3/cos(d*x+c)^8+5/16/d*b^2*a*sin(d*x+c)
^3/cos(d*x+c)^6+15/64/d*b^2*a*sin(d*x+c)^3/cos(d*x+c)^4+15/128/d*b^2*a*sin(
d*x+c)^3/cos(d*x+c)^2+15/128*a*b^2*sin(d*x+c)/d-15/128/d*b^2*a*ln(sec(d*x+c
)+tan(d*x+c))+1/9/d*b^3*sin(d*x+c)^4/cos(d*x+c)^9+5/63/d*b^3*sin(d*x+c)^4/c
os(d*x+c)^7+1/21/d*b^3*sin(d*x+c)^4/cos(d*x+c)^5+1/63/d*b^3*sin(d*x+c)^4/co
s(d*x+c)^3-1/63/d*b^3*sin(d*x+c)^4/cos(d*x+c)-1/63/d*b^3*cos(d*x+c)*sin(d*x
+c)^2-2/63*b^3*cos(d*x+c)/d
```

maxima [A] time = 0.34, size = 248, normalized size = 0.96

$$63\,ab^2\left(\frac{2\left(15\,\sin(dx+c)^7-55\,\sin(dx+c)^5+73\,\sin(dx+c)^3+15\,\sin(dx+c)\right)}{\sin(dx+c)^8-4\,\sin(dx+c)^6+6\,\sin(dx+c)^4-4\,\sin(dx+c)^2+1}-15\,\log(\sin(dx+c)+1)+15\,\log(\sin(dx+c)-1)\right)-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^10*(a*cos(d*x+c)+b*sin(d*x+c))^3,x, algorithm="maxima"
)
```

```
[Out] 1/16128*(63*a*b^2*(2*(15*sin(d*x + c)^7 - 55*sin(d*x + c)^5 + 73*sin(d*x +
c)^3 + 15*sin(d*x + c))/(sin(d*x + c)^8 - 4*sin(d*x + c)^6 + 6*sin(d*x + c)
^4 - 4*sin(d*x + c)^2 + 1) - 15*log(sin(d*x + c) + 1) + 15*log(sin(d*x + c)
 - 1)) - 168*a^3*(2*(15*sin(d*x + c)^5 - 40*sin(d*x + c)^3 + 33*sin(d*x + c
))/(sin(d*x + c)^6 - 3*sin(d*x + c)^4 + 3*sin(d*x + c)^2 - 1) - 15*log(sin(
d*x + c) + 1) + 15*log(sin(d*x + c) - 1)) + 6912*a^2*b/cos(d*x + c)^7 - 256
*(9*cos(d*x + c)^2 - 7)*b^3/cos(d*x + c)^9)/d
```

mupad [B] time = 4.56, size = 547, normalized size = 2.11

$$-\frac{\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)\,\left(\frac{15\,a\,b^2}{64}-\frac{5\,a^3}{8}\right)}{d}-\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(\frac{11\,a^3}{8}+\frac{15\,a\,b^2}{64}\right)+\frac{6\,a^2\,b}{7}-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{17}\,\left(\frac{11\,a^3}{8}+\frac{15\,a\,b^2}{64}\right)+}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*cos(c + d*x) + b*sin(c + d*x))^3/cos(c + d*x)^10,x)
```

```
[Out] - (atanh(tan(c/2 + (d*x)/2))*((15*a*b^2)/64 - (5*a^3)/8))/d - (tan(c/2 + (d
*x)/2)*((15*a*b^2)/64 + (11*a^3)/8) + (6*a^2*b)/7 - tan(c/2 + (d*x)/2)^17*(
(15*a*b^2)/64 + (11*a^3)/8) + tan(c/2 + (d*x)/2)^3*((191*a*b^2)/32 - (47*a^
3)/12) - tan(c/2 + (d*x)/2)^15*((191*a*b^2)/32 - (47*a^3)/12) + tan(c/2 + (
d*x)/2)^5*((249*a*b^2)/32 + (29*a^3)/4) - tan(c/2 + (d*x)/2)^13*((249*a*b^2
)/32 + (29*a^3)/4) + tan(c/2 + (d*x)/2)^7*((435*a*b^2)/32 - (33*a^3)/4) - t
an(c/2 + (d*x)/2)^11*((435*a*b^2)/32 - (33*a^3)/4) - tan(c/2 + (d*x)/2)^14*
(12*a^2*b - 4*b^3) - tan(c/2 + (d*x)/2)^2*((12*a^2*b)/7 - (4*b^3)/7) - tan(
c/2 + (d*x)/2)^6*(36*a^2*b - 12*b^3) + tan(c/2 + (d*x)/2)^8*(48*a^2*b + 12*
b^3) + tan(c/2 + (d*x)/2)^12*(36*a^2*b + (20*b^3)/3) - tan(c/2 + (d*x)/2)^1
0*(60*a^2*b - 20*b^3) + tan(c/2 + (d*x)/2)^4*((132*a^2*b)/7 + (12*b^3)/7) -
 (4*b^3)/63 + 6*a^2*b*tan(c/2 + (d*x)/2)^16)/(d*(9*tan(c/2 + (d*x)/2)^2 - 3
6*tan(c/2 + (d*x)/2)^4 + 84*tan(c/2 + (d*x)/2)^6 - 126*tan(c/2 + (d*x)/2)^8
 + 126*tan(c/2 + (d*x)/2)^10 - 84*tan(c/2 + (d*x)/2)^12 + 36*tan(c/2 + (d*x
)/2)^14 - 9*tan(c/2 + (d*x)/2)^16 + tan(c/2 + (d*x)/2)^18 - 1))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**10*(a*cos(d*x+c)+b*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

3.73 $\int \sec^{11}(c+dx)(a\cos(c+dx)+b\sin(c+dx))^3\,dx$

Optimal. Leaf size=213

$$\frac{a^3\tan(c+dx)}{d}+\frac{3b\left(a^2+b^2\right)\tan^8(c+dx)}{8d}+\frac{a\left(a^2+9b^2\right)\tan^7(c+dx)}{7d}+\frac{b\left(3a^2+b^2\right)\tan^6(c+dx)}{2d}+\frac{3a\left(a^2+3b^2\right)\text{ta}}{5d}$$

```
[Out] a^3*tan(d*x+c)/d+3/2*a^2*b*tan(d*x+c)^2/d+a*(a^2+b^2)*tan(d*x+c)^3/d+1/4*b*
(9*a^2+b^2)*tan(d*x+c)^4/d+3/5*a*(a^2+3*b^2)*tan(d*x+c)^5/d+1/2*b*(3*a^2+b^
2)*tan(d*x+c)^6/d+1/7*a*(a^2+9*b^2)*tan(d*x+c)^7/d+3/8*b*(a^2+b^2)*tan(d*x+
c)^8/d+1/3*a*b^2*tan(d*x+c)^9/d+1/10*b^3*tan(d*x+c)^10/d
```

Rubi [A] time = 0.18, antiderivative size = 213, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {3088, 948}

$$\frac{3b\left(a^2+b^2\right)\tan^8(c+dx)}{8d}+\frac{a\left(a^2+9b^2\right)\tan^7(c+dx)}{7d}+\frac{b\left(3a^2+b^2\right)\tan^6(c+dx)}{2d}+\frac{3a\left(a^2+3b^2\right)\tan^5(c+dx)}{5d}+\frac{b\left(9\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^11*(a*Cos[c + d*x] + b*Sin[c + d*x])^3,x]
```

```
[Out] (a^3*Tan[c + d*x])/d + (3*a^2*b*Tan[c + d*x]^2)/(2*d) + (a*(a^2 + b^2)*Tan[
c + d*x]^3)/d + (b*(9*a^2 + b^2)*Tan[c + d*x]^4)/(4*d) + (3*a*(a^2 + 3*b^2)
*Tan[c + d*x]^5)/(5*d) + (b*(3*a^2 + b^2)*Tan[c + d*x]^6)/(2*d) + (a*(a^2 +
 9*b^2)*Tan[c + d*x]^7)/(7*d) + (3*b*(a^2 + b^2)*Tan[c + d*x]^8)/(8*d) + (a
*b^2*Tan[c + d*x]^9)/(3*d) + (b^3*Tan[c + d*x]^10)/(10*d)
```

Rule 948

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IGtQ[p, 0] && (IGtQ[m, 0] || (EqQ[m, -2] && EqQ[p, 1] &
& EqQ[d, 0]))
```

Rule 3088

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[(x^m*(b +
 a*x)^n)/(1 + x^2)^((m + n + 2)/2), x], x, Cot[c + d*x]], x] /; FreeQ[{a, b
, c, d}, x] && IntegerQ[n] && IntegerQ[(m + n)/2] && NeQ[n, -1] &&  !(GtQ[n
, 0] && GtQ[m, 1])
```

Rubi steps

$$\int \sec^{11}(c+dx)(a\cos(c+dx)+b\sin(c+dx))^3\,dx = -\frac{\text{Subst}\left(\int \frac{(b+ax)^3\left(1+x^2\right)^3}{x^{11}}\,dx, x, \cot(c+dx)\right)}{d}$$

$$= -\frac{\text{Subst}\left(\int \left(\frac{b^3}{x^{11}}+\frac{3ab^2}{x^{10}}+\frac{3b\left(a^2+b^2\right)}{x^9}+\frac{a^3+9ab^2}{x^8}+\frac{3\left(3a^2b+b^3\right)}{x^7}+\right.\right.}{}$$

$$= \frac{a^3\tan(c+dx)}{d}+\frac{3a^2b\tan^2(c+dx)}{2d}+\frac{a\left(a^2+b^2\right)\tan^3(c+}{d}$$

Mathematica [A] time = 2.07, size = 177, normalized size = 0.83

$$\frac{\frac{3}{8}\left(5a^2+b^2\right)(a+b\tan(c+dx))^8-\frac{4}{7}a\left(5a^2+3b^2\right)(a+b\tan(c+dx))^7+\frac{1}{2}\left(a^2+b^2\right)\left(5a^2+b^2\right)(a+b\tan(c+dx}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^11*(a*Cos[c + d*x] + b*Sin[c + d*x])^3,x]
```

```
[Out] (((a^2 + b^2)^3*(a + b*Tan[c + d*x])^4)/4 - (6*a*(a^2 + b^2)^2*(a + b*Tan[c
 + d*x])^5)/5 + ((a^2 + b^2)*(5*a^2 + b^2)*(a + b*Tan[c + d*x])^6)/2 - (4*a
*(5*a^2 + 3*b^2)*(a + b*Tan[c + d*x])^7)/7 + (3*(5*a^2 + b^2)*(a + b*Tan[c
+ d*x])^8)/8 - (2*a*(a + b*Tan[c + d*x])^9)/3 + (a + b*Tan[c + d*x])^10/10)
/(b^7*d)
```

fricas [A] time = 0.58, size = 150, normalized size = 0.70

$$\frac{84\,b^3+105\left(3\,a^2b-b^3\right)\cos\left(dx+c\right)^2+8\left(16\left(3\,a^3-ab^2\right)\cos\left(dx+c\right)^9+8\left(3\,a^3-ab^2\right)\cos\left(dx+c\right)^7+6\left(3\,a^3-\right.\right.}{840\,d\cos\left(dx+c\right)^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^11*(a*cos(d*x+c)+b*sin(d*x+c))^3,x, algorithm="fricas"
)
```

```
[Out] 1/840*(84*b^3 + 105*(3*a^2*b - b^3)*cos(d*x + c)^2 + 8*(16*(3*a^3 - a*b^2)*
cos(d*x + c)^9 + 8*(3*a^3 - a*b^2)*cos(d*x + c)^7 + 6*(3*a^3 - a*b^2)*cos(d
*x + c)^5 + 35*a*b^2*cos(d*x + c) + 5*(3*a^3 - a*b^2)*cos(d*x + c)^3)*sin(d
*x + c))/(d*cos(d*x + c)^10)
```

giac [A] time = 0.45, size = 220, normalized size = 1.03

$$\frac{84\,b^3\tan\left(dx+c\right)^{10}+280\,ab^2\tan\left(dx+c\right)^9+315\,a^2b\tan\left(dx+c\right)^8+315\,b^3\tan\left(dx+c\right)^8+120\,a^3\tan\left(dx+c\right)^7+\cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^11*(a*cos(d*x+c)+b*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/840*(84*b^3*tan(d*x + c)^10 + 280*a*b^2*tan(d*x + c)^9 + 315*a^2*b*tan(d*
x + c)^8 + 315*b^3*tan(d*x + c)^8 + 120*a^3*tan(d*x + c)^7 + 1080*a*b^2*tan
(d*x + c)^7 + 1260*a^2*b*tan(d*x + c)^6 + 420*b^3*tan(d*x + c)^6 + 504*a^3*
tan(d*x + c)^5 + 1512*a*b^2*tan(d*x + c)^5 + 1890*a^2*b*tan(d*x + c)^4 + 21
0*b^3*tan(d*x + c)^4 + 840*a^3*tan(d*x + c)^3 + 840*a*b^2*tan(d*x + c)^3 +
1260*a^2*b*tan(d*x + c)^2 + 840*a^3*tan(d*x + c))/d
```

maple [A] time = 20.74, size = 219, normalized size = 1.03

$$\frac{-a^3\left(-\frac{16}{35}-\frac{\left(\sec^6(dx+c)\right)}{7}-\frac{6\left(\sec^4(dx+c)\right)}{35}-\frac{8\left(\sec^2(dx+c)\right)}{35}\right)\tan\left(dx+c\right)+\frac{3a^2b}{8\cos(dx+c)^8}+3b^2a\left(\frac{\sin^3(dx+c)}{9\cos(dx+c)^9}+\frac{2\left(\sin^3(dx+c)\right)}{21\cos(dx+c)^7}+\cdots\right.}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^11*(a*cos(d*x+c)+b*sin(d*x+c))^3,x)
```

```
[Out] 1/d*(-a^3*(-16/35-1/7*sec(d*x+c)^6-6/35*sec(d*x+c)^4-8/35*sec(d*x+c)^2)*tan
(d*x+c)+3/8*a^2*b/cos(d*x+c)^8+3*b^2*a*(1/9*sin(d*x+c)^3/cos(d*x+c)^9+2/21*
sin(d*x+c)^3/cos(d*x+c)^7+8/105*sin(d*x+c)^3/cos(d*x+c)^5+16/315*sin(d*x+c)
^3/cos(d*x+c)^3)+b^3*(1/10*sin(d*x+c)^4/cos(d*x+c)^10+3/40*sin(d*x+c)^4/cos
(d*x+c)^8+1/20*sin(d*x+c)^4/cos(d*x+c)^6+1/40*sin(d*x+c)^4/cos(d*x+c)^4))
```

maxima [A] time = 0.33, size = 184, normalized size = 0.86

$$\frac{24\left(5\,\tan\left(dx+c\right)^7+21\,\tan\left(dx+c\right)^5+35\,\tan\left(dx+c\right)^3+35\,\tan\left(dx+c\right)\right)a^3+8\left(35\,\tan\left(dx+c\right)^9+135\,\tan\left(d\cdots\right.\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^11*(a*cos(d*x+c)+b*sin(d*x+c))^3,x, algorithm="maxima"
)
```

```
[Out] 1/840*(24*(5*tan(d*x + c)^7 + 21*tan(d*x + c)^5 + 35*tan(d*x + c)^3 + 35*ta
n(d*x + c))*a^3 + 8*(35*tan(d*x + c)^9 + 135*tan(d*x + c)^7 + 189*tan(d*x +
 c)^5 + 105*tan(d*x + c)^3)*a*b^2 - 21*(5*sin(d*x + c)^2 - 1)*b^3/(sin(d*x
```

```
+ c)^10 - 5*sin(d*x + c)^8 + 10*sin(d*x + c)^6 - 10*sin(d*x + c)^4 + 5*sin(
d*x + c)^2 - 1) + 315*a^2*b/(sin(d*x + c)^2 - 1)^4)/d
```

mupad [B] time = 1.58, size = 189, normalized size = 0.89

$$\frac{\cos\left(c+d\,x\right)^3\,\left(\frac{a^3\,\sin\left(c+d\,x\right)}{7}-\frac{a\,b^2\,\sin\left(c+d\,x\right)}{21}\right)+\cos\left(c+d\,x\right)^5\,\left(\frac{6\,a^3\,\sin\left(c+d\,x\right)}{35}-\frac{2\,a\,b^2\,\sin\left(c+d\,x\right)}{35}\right)+\cos\left(c+d\,x\right)^7\,\left(\frac{8\,a^3\,\sin}{3}\right.}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*cos(c + d*x) + b*sin(c + d*x))^3/cos(c + d*x)^11,x)
```

```
[Out] (cos(c + d*x)^3*((a^3*sin(c + d*x))/7 - (a*b^2*sin(c + d*x))/21) + cos(c +
d*x)^5*((6*a^3*sin(c + d*x))/35 - (2*a*b^2*sin(c + d*x))/35) + cos(c + d*x)
^7*((8*a^3*sin(c + d*x))/35 - (8*a*b^2*sin(c + d*x))/105) + cos(c + d*x)^9*
((16*a^3*sin(c + d*x))/35 - (16*a*b^2*sin(c + d*x))/105) + cos(c + d*x)^2*(
(3*a^2*b)/8 - b^3/8) + b^3/10 + (a*b^2*cos(c + d*x)*sin(c + d*x))/3)/(d*cos
(c + d*x)^10)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**11*(a*cos(d*x+c)+b*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

3.74 $\int \cos^5(c+dx)(a\cos(c+dx)+b\sin(c+dx))^4\,dx$

Optimal. Leaf size=279

$$\frac{a^4\sin^9(c+dx)}{9d}-\frac{4a^4\sin^7(c+dx)}{7d}+\frac{6a^4\sin^5(c+dx)}{5d}-\frac{4a^4\sin^3(c+dx)}{3d}+\frac{a^4\sin(c+dx)}{d}-\frac{4a^3b\cos^9(c+dx)}{9d}-\frac{2a^2b^2}{}$$

```
[Out] -4/7*a*b^3*cos(d*x+c)^7/d-4/9*a^3*b*cos(d*x+c)^9/d+4/9*a*b^3*cos(d*x+c)^9/d
+a^4*sin(d*x+c)/d-4/3*a^4*sin(d*x+c)^3/d+2*a^2*b^2*sin(d*x+c)^3/d+6/5*a^4*s
in(d*x+c)^5/d-18/5*a^2*b^2*sin(d*x+c)^5/d+1/5*b^4*sin(d*x+c)^5/d-4/7*a^4*si
n(d*x+c)^7/d+18/7*a^2*b^2*sin(d*x+c)^7/d-2/7*b^4*sin(d*x+c)^7/d+1/9*a^4*sin
(d*x+c)^9/d-2/3*a^2*b^2*sin(d*x+c)^9/d+1/9*b^4*sin(d*x+c)^9/d
```

Rubi [A] time = 0.26, antiderivative size = 279, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 7, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {3090, 2633, 2565, 30, 2564, 270, 14}

$$-\frac{2a^2b^2\sin^9(c+dx)}{3d}+\frac{18a^2b^2\sin^7(c+dx)}{7d}-\frac{18a^2b^2\sin^5(c+dx)}{5d}+\frac{2a^2b^2\sin^3(c+dx)}{d}-\frac{4a^3b\cos^9(c+dx)}{9d}+\frac{a^4\sin^9(}{9d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^5*(a*Cos[c + d*x] + b*Sin[c + d*x])^4,x]
```

```
[Out] (-4*a*b^3*Cos[c + d*x]^7)/(7*d) - (4*a^3*b*Cos[c + d*x]^9)/(9*d) + (4*a*b^3
*Cos[c + d*x]^9)/(9*d) + (a^4*Sin[c + d*x])/d - (4*a^4*Sin[c + d*x]^3)/(3*d
) + (2*a^2*b^2*Sin[c + d*x]^3)/d + (6*a^4*Sin[c + d*x]^5)/(5*d) - (18*a^2*b
^2*Sin[c + d*x]^5)/(5*d) + (b^4*Sin[c + d*x]^5)/(5*d) - (4*a^4*Sin[c + d*x]
^7)/(7*d) + (18*a^2*b^2*Sin[c + d*x]^7)/(7*d) - (2*b^4*Sin[c + d*x]^7)/(7*d
) + (a^4*Sin[c + d*x]^9)/(9*d) - (2*a^2*b^2*Sin[c + d*x]^9)/(3*d) + (b^4*Si
n[c + d*x]^9)/(9*d)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
```

```
IGtQ[p, 0]
```

Rule 2564

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 2565

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> -Dist[(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 3090

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_.), x_Symbol] :> Int[ExpandTrig[cos[c + d*x]^m*(a
*cos[c + d*x] + b*sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d}, x] && Inte
gerQ[m] && IGtQ[n, 0]
```

Rubi steps

$$\int \cos^5(c+dx)(a\cos(c+dx)+b\sin(c+dx))^4\,dx = \int \left(a^4\cos^9(c+dx)+4a^3b\cos^8(c+dx)\sin(c+dx)+6a^2b^2\right.$$

$$= a^4\int \cos^9(c+dx)\,dx+\left(4a^3b\right)\int \cos^8(c+dx)\sin(c+dx)\,d$$

$$= -\frac{a^4\,\mathrm{Subst}\left(\int\left(1-4x^2+6x^4-4x^6+x^8\right)dx,x,-\sin(c+d}{d}$$

$$= -\frac{4a^3b\cos^9(c+dx)}{9d}+\frac{a^4\sin(c+dx)}{d}-\frac{4a^4\sin^3(c+dx)}{3d}+$$

$$= -\frac{4ab^3\cos^7(c+dx)}{7d}-\frac{4a^3b\cos^9(c+dx)}{9d}+\frac{4ab^3\cos^9(c+d}{9d}$$

Mathematica [A] time = 0.71, size = 237, normalized size = 0.85

$$\frac{420\left(21a^4-b^4\right)\sin(3(c+dx))-5040a^3b\cos(5(c+dx))-2520ab\left(7a^2+3b^2\right)\cos(c+dx)-1680ab\left(7a^2+2b^2\right)c}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^5*(a*Cos[c + d*x] + b*Sin[c + d*x])^4,x]
```

```
[Out] (-2520*a*b*(7*a^2 + 3*b^2)*Cos[c + d*x] - 1680*a*b*(7*a^2 + 2*b^2)*Cos[3*(c
 + d*x)] - 5040*a^3*b*Cos[5*(c + d*x)] - 180*a*b*(7*a^2 - 3*b^2)*Cos[7*(c +
 d*x)] - 140*a*b*(a^2 - b^2)*Cos[9*(c + d*x)] + 1890*(21*a^4 + 14*a^2*b^2 +
 b^4)*Sin[c + d*x] + 420*(21*a^4 - b^4)*Sin[3*(c + d*x)] + 252*(9*a^4 - 12*
a^2*b^2 - b^4)*Sin[5*(c + d*x)] + 45*(9*a^4 - 30*a^2*b^2 + b^4)*Sin[7*(c +
d*x)] + 35*(a^4 - 6*a^2*b^2 + b^4)*Sin[9*(c + d*x)])/(80640*d)
```

fricas [A] time = 0.66, size = 177, normalized size = 0.63

$$-\frac{180\,ab^3\cos(dx+c)^7+140\left(a^3b-ab^3\right)\cos(dx+c)^9-\left(35\left(a^4-6\,a^2b^2+b^4\right)\cos(dx+c)^8+10\left(4\,a^4+3\,a^2b^2-\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(a*cos(d*x+c)+b*sin(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] -1/315*(180*a*b^3*cos(d*x + c)^7 + 140*(a^3*b - a*b^3)*cos(d*x + c)^9 - (35
*(a^4 - 6*a^2*b^2 + b^4)*cos(d*x + c)^8 + 10*(4*a^4 + 3*a^2*b^2 - 5*b^4)*co
s(d*x + c)^6 + 3*(16*a^4 + 12*a^2*b^2 + b^4)*cos(d*x + c)^4 + 128*a^4 + 96*
a^2*b^2 + 8*b^4 + 4*(16*a^4 + 12*a^2*b^2 + b^4)*cos(d*x + c)^2)*sin(d*x + c
))/d
```

giac [A] time = 0.70, size = 269, normalized size = 0.96

$$-\frac{a^3b\cos(5\,dx+5\,c)}{16\,d}-\frac{\left(a^3b-ab^3\right)\cos(9\,dx+9\,c)}{576\,d}-\frac{\left(7\,a^3b-3\,ab^3\right)\cos(7\,dx+7\,c)}{448\,d}-\frac{\left(7\,a^3b+2\,ab^3\right)\cos(3\,dx+3\,}{48\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(a*cos(d*x+c)+b*sin(d*x+c))^4,x, algorithm="giac")
```

```
[Out] -1/16*a^3*b*cos(5*d*x + 5*c)/d - 1/576*(a^3*b - a*b^3)*cos(9*d*x + 9*c)/d -
 1/448*(7*a^3*b - 3*a*b^3)*cos(7*d*x + 7*c)/d - 1/48*(7*a^3*b + 2*a*b^3)*co
s(3*d*x + 3*c)/d - 1/32*(7*a^3*b + 3*a*b^3)*cos(d*x + c)/d + 1/2304*(a^4 -
6*a^2*b^2 + b^4)*sin(9*d*x + 9*c)/d + 1/1792*(9*a^4 - 30*a^2*b^2 + b^4)*sin
(7*d*x + 7*c)/d + 1/320*(9*a^4 - 12*a^2*b^2 - b^4)*sin(5*d*x + 5*c)/d + 1/1
92*(21*a^4 - b^4)*sin(3*d*x + 3*c)/d + 3/128*(21*a^4 + 14*a^2*b^2 + b^4)*si
n(d*x + c)/d
```

maple [A] time = 11.18, size = 236, normalized size = 0.85

$$\frac{b^4\left(-\frac{\left(\sin^3(dx+c)\right)\left(\cos^6(dx+c)\right)}{9}-\frac{\sin(dx+c)\left(\cos^6(dx+c)\right)}{21}+\frac{\left(\frac{8}{3}+\cos^4(dx+c)+\frac{4\left(\cos^2(dx+c)\right)}{3}\right)\sin(dx+c)}{105}\right)+4a\,b^3\left(-\frac{\left(\sin^2(dx+c)\right)\left(\cos^7(dx+c)\right)}{9}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*(a*cos(d*x+c)+b*sin(d*x+c))^4,x)
```

```
[Out] 1/d*(b^4*(-1/9*sin(d*x+c)^3*cos(d*x+c)^6-1/21*sin(d*x+c)*cos(d*x+c)^6+1/105
*(8/3+cos(d*x+c)^4+4/3*cos(d*x+c)^2)*sin(d*x+c))+4*a*b^3*(-1/9*sin(d*x+c)^2
*cos(d*x+c)^7-2/63*cos(d*x+c)^7)+6*a^2*b^2*(-1/9*sin(d*x+c)*cos(d*x+c)^8+1/
63*(16/5+cos(d*x+c)^6+6/5*cos(d*x+c)^4+8/5*cos(d*x+c)^2)*sin(d*x+c))-4/9*a^
3*b*cos(d*x+c)^9+1/9*a^4*(128/35+cos(d*x+c)^8+8/7*cos(d*x+c)^6+48/35*cos(d*
x+c)^4+64/35*cos(d*x+c)^2)*sin(d*x+c))
```

maxima [A] time = 0.33, size = 186, normalized size = 0.67

$$-\frac{140\,a^3 b\cos(dx+c)^9-\left(35\,\sin(dx+c)^9-180\,\sin(dx+c)^7+378\,\sin(dx+c)^5-420\,\sin(dx+c)^3+315\,\sin\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(a*cos(d*x+c)+b*sin(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] -1/315*(140*a^3*b*cos(d*x + c)^9 - (35*sin(d*x + c)^9 - 180*sin(d*x + c)^7
+ 378*sin(d*x + c)^5 - 420*sin(d*x + c)^3 + 315*sin(d*x + c))*a^4 + 6*(35*s
in(d*x + c)^9 - 135*sin(d*x + c)^7 + 189*sin(d*x + c)^5 - 105*sin(d*x + c)^
3)*a^2*b^2 - 20*(7*cos(d*x + c)^9 - 9*cos(d*x + c)^7)*a*b^3 - (35*sin(d*x +
 c)^9 - 90*sin(d*x + c)^7 + 63*sin(d*x + c)^5)*b^4)/d
```

mupad [B] time = 2.00, size = 334, normalized size = 1.20

$$-\frac{\frac{b^4\,\sin(3\,c+3\,d\,x)}{192}-\frac{3\,b^4\,\sin(c+d\,x)}{128}-\frac{7\,a^4\,\sin(3\,c+3\,d\,x)}{64}-\frac{9\,a^4\,\sin(5\,c+5\,d\,x)}{320}-\frac{9\,a^4\,\sin(7\,c+7\,d\,x)}{1792}-\frac{a^4\,\sin(9\,c+9\,d\,x)}{2304}-\frac{63\,a^4\,\sin(c+d\,x)}{128}+}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^5*(a*cos(c + d*x) + b*sin(c + d*x))^4,x)
```

```
[Out] -((b^4*sin(3*c + 3*d*x))/192 - (3*b^4*sin(c + d*x))/128 - (7*a^4*sin(3*c +
3*d*x))/64 - (9*a^4*sin(5*c + 5*d*x))/320 - (9*a^4*sin(7*c + 7*d*x))/1792 -
 (a^4*sin(9*c + 9*d*x))/2304 - (63*a^4*sin(c + d*x))/128 + (b^4*sin(5*c + 5
*d*x))/320 - (b^4*sin(7*c + 7*d*x))/1792 - (b^4*sin(9*c + 9*d*x))/2304 + (a
```

```
*b^3*cos(3*c + 3*d*x))/24 + (7*a^3*b*cos(3*c + 3*d*x))/48 + (a^3*b*cos(5*c
+ 5*d*x))/16 - (3*a*b^3*cos(7*c + 7*d*x))/448 + (a^3*b*cos(7*c + 7*d*x))/64
 - (a*b^3*cos(9*c + 9*d*x))/576 + (a^3*b*cos(9*c + 9*d*x))/576 - (21*a^2*b^
2*sin(c + d*x))/64 + (3*a^2*b^2*sin(5*c + 5*d*x))/80 + (15*a^2*b^2*sin(7*c
+ 7*d*x))/896 + (a^2*b^2*sin(9*c + 9*d*x))/384 + (3*a*b^3*cos(c + d*x))/32
+ (7*a^3*b*cos(c + d*x))/32)/d
```

sympy [A] time = 15.62, size = 367, normalized size = 1.32

$$\begin{cases} \frac{128a^4\sin^9(c+dx)}{315d} + \frac{64a^4\sin^7(c+dx)\cos^2(c+dx)}{35d} + \frac{16a^4\sin^5(c+dx)\cos^4(c+dx)}{5d} + \frac{8a^4\sin^3(c+dx)\cos^6(c+dx)}{3d} + \frac{a^4\sin(c+dx)\cos^8(c+dx)}{d} - \\ x\,(a\cos(c) + b\sin(c))^4\cos^5(c) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*(a*cos(d*x+c)+b*sin(d*x+c))**4,x)
```

```
[Out] Piecewise((128*a**4*sin(c + d*x)**9/(315*d) + 64*a**4*sin(c + d*x)**7*cos(c
 + d*x)**2/(35*d) + 16*a**4*sin(c + d*x)**5*cos(c + d*x)**4/(5*d) + 8*a**4*
sin(c + d*x)**3*cos(c + d*x)**6/(3*d) + a**4*sin(c + d*x)*cos(c + d*x)**8/d
 - 4*a**3*b*cos(c + d*x)**9/(9*d) + 32*a**2*b**2*sin(c + d*x)**9/(105*d) +
48*a**2*b**2*sin(c + d*x)**7*cos(c + d*x)**2/(35*d) + 12*a**2*b**2*sin(c +
d*x)**5*cos(c + d*x)**4/(5*d) + 2*a**2*b**2*sin(c + d*x)**3*cos(c + d*x)**6
/d - 4*a*b**3*sin(c + d*x)**2*cos(c + d*x)**7/(7*d) - 8*a*b**3*cos(c + d*x)
**9/(63*d) + 8*b**4*sin(c + d*x)**9/(315*d) + 4*b**4*sin(c + d*x)**7*cos(c
+ d*x)**2/(35*d) + b**4*sin(c + d*x)**5*cos(c + d*x)**4/(5*d), Ne(d, 0)), (
x*(a*cos(c) + b*sin(c))**4*cos(c)**5, True))
```

3.75 $\int \cos^4(c+dx)(a\cos(c+dx)+b\sin(c+dx))^4\,dx$

Optimal. Leaf size=381

$$\frac{a^4\sin(c+dx)\cos^7(c+dx)}{8d}+\frac{7a^4\sin(c+dx)\cos^5(c+dx)}{48d}+\frac{35a^4\sin(c+dx)\cos^3(c+dx)}{192d}+\frac{35a^4\sin(c+dx)\cos(}{128d}$$

```
[Out] 35/128*a^4*x+15/64*a^2*b^2*x+3/128*b^4*x-2/3*a*b^3*cos(d*x+c)^6/d-1/2*a^3*b
*cos(d*x+c)^8/d+1/2*a*b^3*cos(d*x+c)^8/d+35/128*a^4*cos(d*x+c)*sin(d*x+c)/d
+15/64*a^2*b^2*cos(d*x+c)*sin(d*x+c)/d+3/128*b^4*cos(d*x+c)*sin(d*x+c)/d+35
/192*a^4*cos(d*x+c)^3*sin(d*x+c)/d+5/32*a^2*b^2*cos(d*x+c)^3*sin(d*x+c)/d+1
/64*b^4*cos(d*x+c)^3*sin(d*x+c)/d+7/48*a^4*cos(d*x+c)^5*sin(d*x+c)/d+1/8*a^
2*b^2*cos(d*x+c)^5*sin(d*x+c)/d-1/16*b^4*cos(d*x+c)^5*sin(d*x+c)/d+1/8*a^4*
cos(d*x+c)^7*sin(d*x+c)/d-3/4*a^2*b^2*cos(d*x+c)^7*sin(d*x+c)/d-1/8*b^4*cos
(d*x+c)^5*sin(d*x+c)^3/d
```

Rubi [A] time = 0.39, antiderivative size = 381, normalized size of antiderivative = 1.00, number of steps used = 22, number of rules used = 7, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {3090, 2635, 8, 2565, 30, 2568, 14}

$$-\frac{3a^2b^2\sin(c+dx)\cos^7(c+dx)}{4d}+\frac{a^2b^2\sin(c+dx)\cos^5(c+dx)}{8d}+\frac{5a^2b^2\sin(c+dx)\cos^3(c+dx)}{32d}+\frac{15a^2b^2\sin(c+}{6}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^4*(a*Cos[c + d*x] + b*Sin[c + d*x])^4,x]
```

```
[Out] (35*a^4*x)/128 + (15*a^2*b^2*x)/64 + (3*b^4*x)/128 - (2*a*b^3*Cos[c + d*x]^
6)/(3*d) - (a^3*b*Cos[c + d*x]^8)/(2*d) + (a*b^3*Cos[c + d*x]^8)/(2*d) + (3
5*a^4*Cos[c + d*x]*Sin[c + d*x])/(128*d) + (15*a^2*b^2*Cos[c + d*x]*Sin[c +
 d*x])/(64*d) + (3*b^4*Cos[c + d*x]*Sin[c + d*x])/(128*d) + (35*a^4*Cos[c +
 d*x]^3*Sin[c + d*x])/(192*d) + (5*a^2*b^2*Cos[c + d*x]^3*Sin[c + d*x])/(32
*d) + (b^4*Cos[c + d*x]^3*Sin[c + d*x])/(64*d) + (7*a^4*Cos[c + d*x]^5*Sin[
c + d*x])/(48*d) + (a^2*b^2*Cos[c + d*x]^5*Sin[c + d*x])/(8*d) - (b^4*Cos[c
 + d*x]^5*Sin[c + d*x])/(16*d) + (a^4*Cos[c + d*x]^7*Sin[c + d*x])/(8*d) -
(3*a^2*b^2*Cos[c + d*x]^7*Sin[c + d*x])/(4*d) - (b^4*Cos[c + d*x]^5*Sin[c +
 d*x]^3)/(8*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_)
```

```
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2565

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> -Dist[(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 2568

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> -Simp[(a*(b*Cos[e + f*x])^(n + 1)*(a*Sin[e + f*x])^(m - 1)
)/(b*f*(m + n)), x] + Dist[(a^2*(m - 1))/(m + n), Int[(b*Cos[e + f*x])^n*(a
*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1] &&
 NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 3090

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_.), x_Symbol] :> Int[ExpandTrig[cos[c + d*x]^m*(a
*cos[c + d*x] + b*sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d}, x] && Inte
gerQ[m] && IGtQ[n, 0]
```

Rubi steps

$$\int \cos^4(c+dx)(a\cos(c+dx)+b\sin(c+dx))^4\,dx = \int \left(a^4\cos^8(c+dx)+4a^3b\cos^7(c+dx)\sin(c+dx)+6a^2b^2\right.$$
$$= a^4\int \cos^8(c+dx)\,dx + \left(4a^3b\right)\int \cos^7(c+dx)\sin(c+dx)$$
$$= \frac{a^4\cos^7(c+dx)\sin(c+dx)}{8d} - \frac{3a^2b^2\cos^7(c+dx)\sin(c+d}{4d}$$
$$= -\frac{a^3b\cos^8(c+dx)}{2d} + \frac{7a^4\cos^5(c+dx)\sin(c+dx)}{48d} + \frac{a^2b^2\,\mathrm{c}}{}$$
$$= -\frac{2ab^3\cos^6(c+dx)}{3d} - \frac{a^3b\cos^8(c+dx)}{2d} + \frac{ab^3\cos^8(c+dx)}{2d}$$
$$= -\frac{2ab^3\cos^6(c+dx)}{3d} - \frac{a^3b\cos^8(c+dx)}{2d} + \frac{ab^3\cos^8(c+dx)}{2d}$$
$$= \frac{35a^4x}{128} + \frac{15}{64}a^2b^2x + \frac{3b^4x}{128} - \frac{2ab^3\cos^6(c+dx)}{3d} - \frac{a^3b\cos^8(}{2d}$$

Mathematica [A] time = 0.61, size = 222, normalized size = 0.58

$$\frac{96a^2\left(7a^2+3b^2\right)\sin(2(c+dx))+32a^2\left(a^2-3b^2\right)\sin(6(c+dx))-96ab\left(7a^2+3b^2\right)\cos(2(c+dx))-48ab\left(7a^2\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^4*(a*Cos[c + d*x] + b*Sin[c + d*x])^4,x]
```

```
[Out] (24*(35*a^4 + 30*a^2*b^2 + 3*b^4)*(c + d*x) - 96*a*b*(7*a^2 + 3*b^2)*Cos[2*
(c + d*x)] - 48*a*b*(7*a^2 + b^2)*Cos[4*(c + d*x)] - 32*a*b*(3*a^2 - b^2)*C
os[6*(c + d*x)] - 12*a*b*(a^2 - b^2)*Cos[8*(c + d*x)] + 96*a^2*(7*a^2 + 3*b
^2)*Sin[2*(c + d*x)] + 24*(7*a^4 - 6*a^2*b^2 - b^4)*Sin[4*(c + d*x)] + 32*a
^2*(a^2 - 3*b^2)*Sin[6*(c + d*x)] + 3*(a^4 - 6*a^2*b^2 + b^4)*Sin[8*(c + d*
x)])/(3072*d)
```

fricas [A] time = 0.76, size = 184, normalized size = 0.48

$$-\frac{256\,ab^3\cos(dx+c)^6+192\left(a^3b-ab^3\right)\cos(dx+c)^8-3\left(35\,a^4+30\,a^2b^2+3\,b^4\right)dx-\left(48\left(a^4-6\,a^2b^2+b^4\right)\mathrm{cc}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a*cos(d*x+c)+b*sin(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] -1/384*(256*a*b^3*cos(d*x + c)^6 + 192*(a^3*b - a*b^3)*cos(d*x + c)^8 - 3*(
35*a^4 + 30*a^2*b^2 + 3*b^4)*d*x - (48*(a^4 - 6*a^2*b^2 + b^4)*cos(d*x + c)
```

```
^7 + 8*(7*a^4 + 6*a^2*b^2 - 9*b^4)*cos(d*x + c)^5 + 2*(35*a^4 + 30*a^2*b^2
+ 3*b^4)*cos(d*x + c)^3 + 3*(35*a^4 + 30*a^2*b^2 + 3*b^4)*cos(d*x + c))*sin
(d*x + c))/d
```

giac [A] time = 0.67, size = 245, normalized size = 0.64

$$\frac{1}{128}\left(35\,a^4+30\,a^2b^2+3\,b^4\right)x-\frac{\left(a^3b-ab^3\right)\cos\left(8\,dx+8\,c\right)}{256\,d}-\frac{\left(3\,a^3b-ab^3\right)\cos\left(6\,dx+6\,c\right)}{96\,d}-\frac{\left(7\,a^3b+ab^3\right)\cos\left(4\,dx\right.}{64\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a*cos(d*x+c)+b*sin(d*x+c))^4,x, algorithm="giac")
```

```
[Out] 1/128*(35*a^4 + 30*a^2*b^2 + 3*b^4)*x - 1/256*(a^3*b - a*b^3)*cos(8*d*x + 8
*c)/d - 1/96*(3*a^3*b - a*b^3)*cos(6*d*x + 6*c)/d - 1/64*(7*a^3*b + a*b^3)*
cos(4*d*x + 4*c)/d - 1/32*(7*a^3*b + 3*a*b^3)*cos(2*d*x + 2*c)/d + 1/1024*(
a^4 - 6*a^2*b^2 + b^4)*sin(8*d*x + 8*c)/d + 1/96*(a^4 - 3*a^2*b^2)*sin(6*d*
x + 6*c)/d + 1/128*(7*a^4 - 6*a^2*b^2 - b^4)*sin(4*d*x + 4*c)/d + 1/32*(7*a
^4 + 3*a^2*b^2)*sin(2*d*x + 2*c)/d
```

maple [A] time = 21.24, size = 250, normalized size = 0.66

$$\frac{b^4\left(-\frac{\left(\sin^3(dx+c)\right)\left(\cos^5(dx+c)\right)}{8}-\frac{\left(\cos^5(dx+c)\right)\sin(dx+c)}{16}+\frac{\left(\cos^3(dx+c)+\frac{3\cos(dx+c)}{2}\right)\sin(dx+c)}{64}+\frac{3dx}{128}+\frac{3c}{128}\right)+4a\,b^3\left(-\frac{\left(\sin^2(dx+c)\right)\left(\cos\right.}{8}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*(a*cos(d*x+c)+b*sin(d*x+c))^4,x)
```

```
[Out] 1/d*(b^4*(-1/8*sin(d*x+c)^3*cos(d*x+c)^5-1/16*cos(d*x+c)^5*sin(d*x+c)+1/64*
(cos(d*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+3/128*d*x+3/128*c)+4*a*b^3*(-1/8*s
in(d*x+c)^2*cos(d*x+c)^6-1/24*cos(d*x+c)^6)+6*a^2*b^2*(-1/8*sin(d*x+c)*cos(
d*x+c)^7+1/48*(cos(d*x+c)^5+5/4*cos(d*x+c)^3+15/8*cos(d*x+c))*sin(d*x+c)+5/
128*d*x+5/128*c)-1/2*a^3*b*cos(d*x+c)^8+a^4*(1/8*(cos(d*x+c)^7+7/6*cos(d*x+
c)^5+35/24*cos(d*x+c)^3+35/16*cos(d*x+c))*sin(d*x+c)+35/128*d*x+35/128*c))
```

maxima [A] time = 0.33, size = 199, normalized size = 0.52

$$-\frac{1536\,a^3b\cos\left(dx+c\right)^8+\left(128\,\sin\left(2\,dx+2\,c\right)^3-840\,dx-840\,c-3\,\sin\left(8\,dx+8\,c\right)-168\,\sin\left(4\,dx+4\,c\right)-768\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a*cos(d*x+c)+b*sin(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] -1/3072*(1536*a^3*b*cos(d*x + c)^8 + (128*sin(2*d*x + 2*c)^3 - 840*d*x - 84
0*c - 3*sin(8*d*x + 8*c) - 168*sin(4*d*x + 4*c) - 768*sin(2*d*x + 2*c))*a^4
 - 6*(64*sin(2*d*x + 2*c)^3 + 120*d*x + 120*c - 3*sin(8*d*x + 8*c) - 24*sin
(4*d*x + 4*c))*a^2*b^2 - 512*(3*sin(d*x + c)^8 - 8*sin(d*x + c)^6 + 6*sin(d
*x + c)^4)*a*b^3 - 3*(24*d*x + 24*c + sin(8*d*x + 8*c) - 8*sin(4*d*x + 4*c)
)*b^4)/d
```

mupad [B] time = 1.69, size = 343, normalized size = 0.90

$$\frac{35\,a^4\,x}{128}+\frac{3\,b^4\,x}{128}+\frac{15\,a^2\,b^2\,x}{64}-\frac{2\,a\,b^3\cos\left(c+d\,x\right)^6}{3\,d}+\frac{a\,b^3\cos\left(c+d\,x\right)^8}{2\,d}-\frac{a^3\,b\cos\left(c+d\,x\right)^8}{2\,d}+\frac{35\,a^4\cos\left(c+d\,x\right)^3\,\sin}{192\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^4*(a*cos(c + d*x) + b*sin(c + d*x))^4,x)
```

```
[Out] (35*a^4*x)/128 + (3*b^4*x)/128 + (15*a^2*b^2*x)/64 - (2*a*b^3*cos(c + d*x)^
6)/(3*d) + (a*b^3*cos(c + d*x)^8)/(2*d) - (a^3*b*cos(c + d*x)^8)/(2*d) + (3
5*a^4*cos(c + d*x)^3*sin(c + d*x))/(192*d) + (7*a^4*cos(c + d*x)^5*sin(c +
d*x))/(48*d) + (a^4*cos(c + d*x)^7*sin(c + d*x))/(8*d) + (b^4*cos(c + d*x)^
3*sin(c + d*x))/(64*d) - (3*b^4*cos(c + d*x)^5*sin(c + d*x))/(16*d) + (b^4*
cos(c + d*x)^7*sin(c + d*x))/(8*d) + (35*a^4*cos(c + d*x)*sin(c + d*x))/(12
8*d) + (3*b^4*cos(c + d*x)*sin(c + d*x))/(128*d) + (15*a^2*b^2*cos(c + d*x)
*sin(c + d*x))/(64*d) + (5*a^2*b^2*cos(c + d*x)^3*sin(c + d*x))/(32*d) + (a
^2*b^2*cos(c + d*x)^5*sin(c + d*x))/(8*d) - (3*a^2*b^2*cos(c + d*x)^7*sin(c
 + d*x))/(4*d)
```

sympy [A] time = 11.26, size = 760, normalized size = 1.99

$$\begin{cases} \frac{35a^4x\sin^8(c+dx)}{128}+\frac{35a^4x\sin^6(c+dx)\cos^2(c+dx)}{32}+\frac{105a^4x\sin^4(c+dx)\cos^4(c+dx)}{64}+\frac{35a^4x\sin^2(c+dx)\cos^6(c+dx)}{32}+\frac{35a^4x\cos^8(c+dx)}{128}+ \\ x\left(a\cos(c)+b\sin(c)\right)^4\cos^4(c) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*(a*cos(d*x+c)+b*sin(d*x+c))**4,x)
```

```
[Out] Piecewise((35*a**4*x*sin(c + d*x)**8/128 + 35*a**4*x*sin(c + d*x)**6*cos(c
+ d*x)**2/32 + 105*a**4*x*sin(c + d*x)**4*cos(c + d*x)**4/64 + 35*a**4*x*si
n(c + d*x)**2*cos(c + d*x)**6/32 + 35*a**4*x*cos(c + d*x)**8/128 + 35*a**4*
sin(c + d*x)**7*cos(c + d*x)/(128*d) + 385*a**4*sin(c + d*x)**5*cos(c + d*x
)**3/(384*d) + 511*a**4*sin(c + d*x)**3*cos(c + d*x)**5/(384*d) + 93*a**4*s
in(c + d*x)*cos(c + d*x)**7/(128*d) - a**3*b*cos(c + d*x)**8/(2*d) + 15*a**
2*b**2*x*sin(c + d*x)**8/64 + 15*a**2*b**2*x*sin(c + d*x)**6*cos(c + d*x)**
2/16 + 45*a**2*b**2*x*sin(c + d*x)**4*cos(c + d*x)**4/32 + 15*a**2*b**2*x*s
```

```
in(c + d*x)**2*cos(c + d*x)**6/16 + 15*a**2*b**2*x*cos(c + d*x)**8/64 + 15*
a**2*b**2*sin(c + d*x)**7*cos(c + d*x)/(64*d) + 55*a**2*b**2*sin(c + d*x)**
5*cos(c + d*x)**3/(64*d) + 73*a**2*b**2*sin(c + d*x)**3*cos(c + d*x)**5/(64
*d) - 15*a**2*b**2*sin(c + d*x)*cos(c + d*x)**7/(64*d) + a*b**3*sin(c + d*x
)**8/(6*d) + 2*a*b**3*sin(c + d*x)**6*cos(c + d*x)**2/(3*d) + a*b**3*sin(c
+ d*x)**4*cos(c + d*x)**4/d + 3*b**4*x*sin(c + d*x)**8/128 + 3*b**4*x*sin(c
 + d*x)**6*cos(c + d*x)**2/32 + 9*b**4*x*sin(c + d*x)**4*cos(c + d*x)**4/64
 + 3*b**4*x*sin(c + d*x)**2*cos(c + d*x)**6/32 + 3*b**4*x*cos(c + d*x)**8/1
28 + 3*b**4*sin(c + d*x)**7*cos(c + d*x)/(128*d) + 11*b**4*sin(c + d*x)**5*
cos(c + d*x)**3/(128*d) - 11*b**4*sin(c + d*x)**3*cos(c + d*x)**5/(128*d) -
 3*b**4*sin(c + d*x)*cos(c + d*x)**7/(128*d), Ne(d, 0)), (x*(a*cos(c) + b*s
in(c))**4*cos(c)**4, True))
```

3.76 $\int \cos^3(c+dx)(a\cos(c+dx)+b\sin(c+dx))^4\,dx$

Optimal. Leaf size=220

$$-\frac{a^4\sin^7(c+dx)}{7d}+\frac{3a^4\sin^5(c+dx)}{5d}-\frac{a^4\sin^3(c+dx)}{d}+\frac{a^4\sin(c+dx)}{d}-\frac{4a^3b\cos^7(c+dx)}{7d}+\frac{6a^2b^2\sin^7(c+dx)}{7d}-12$$

```
[Out] -4/5*a*b^3*cos(d*x+c)^5/d-4/7*a^3*b*cos(d*x+c)^7/d+4/7*a*b^3*cos(d*x+c)^7/d
+a^4*sin(d*x+c)/d-a^4*sin(d*x+c)^3/d+2*a^2*b^2*sin(d*x+c)^3/d+3/5*a^4*sin(d
*x+c)^5/d-12/5*a^2*b^2*sin(d*x+c)^5/d+1/5*b^4*sin(d*x+c)^5/d-1/7*a^4*sin(d*
x+c)^7/d+6/7*a^2*b^2*sin(d*x+c)^7/d-1/7*b^4*sin(d*x+c)^7/d
```

Rubi [A] time = 0.23, antiderivative size = 220, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 7, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {3090, 2633, 2565, 30, 2564, 270, 14}

$$\frac{6a^2b^2\sin^7(c+dx)}{7d}-\frac{12a^2b^2\sin^5(c+dx)}{5d}+\frac{2a^2b^2\sin^3(c+dx)}{d}-\frac{4a^3b\cos^7(c+dx)}{7d}-\frac{a^4\sin^7(c+dx)}{7d}+\frac{3a^4\sin^5(c+}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^3*(a*Cos[c + d*x] + b*Sin[c + d*x])^4,x]
```

```
[Out] (-4*a*b^3*Cos[c + d*x]^5)/(5*d) - (4*a^3*b*Cos[c + d*x]^7)/(7*d) + (4*a*b^3
*Cos[c + d*x]^7)/(7*d) + (a^4*Sin[c + d*x])/d - (a^4*Sin[c + d*x]^3)/d + (2
*a^2*b^2*Sin[c + d*x]^3)/d + (3*a^4*Sin[c + d*x]^5)/(5*d) - (12*a^2*b^2*Sin
[c + d*x]^5)/(5*d) + (b^4*Sin[c + d*x]^5)/(5*d) - (a^4*Sin[c + d*x]^7)/(7*d
) + (6*a^2*b^2*Sin[c + d*x]^7)/(7*d) - (b^4*Sin[c + d*x]^7)/(7*d)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2564

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 2565

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> -Dist[(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 3090

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_.), x_Symbol] :> Int[ExpandTrig[cos[c + d*x]^m*(a
*cos[c + d*x] + b*sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d}, x] && Inte
gerQ[m] && IGtQ[n, 0]
```

Rubi steps

$$\int \cos^3(c+dx)(a\cos(c+dx)+b\sin(c+dx))^4\,dx = \int \left(a^4\cos^7(c+dx)+4a^3b\cos^6(c+dx)\sin(c+dx)+6a^2b^2 \right.$$

$$= a^4\int \cos^7(c+dx)\,dx+\left(4a^3b\right)\int \cos^6(c+dx)\sin(c+dx)\,dx$$

$$= -\frac{a^4\operatorname{Subst}\left(\int\left(1-3x^2+3x^4-x^6\right)dx,x,-\sin(c+dx)\right)}{d}-\frac{(4}{}$$

$$= -\frac{4a^3b\cos^7(c+dx)}{7d}+\frac{a^4\sin(c+dx)}{d}-\frac{a^4\sin^3(c+dx)}{d}+\frac{3a}{}$$

$$= -\frac{4ab^3\cos^5(c+dx)}{5d}-\frac{4a^3b\cos^7(c+dx)}{7d}+\frac{4ab^3\cos^7(c+dx)}{7d}$$

Mathematica [A] time = 0.54, size = 204, normalized size = 0.93

$$\frac{-140ab\left(5a^2+3b^2\right)\cos(c+dx)-140ab\left(3a^2+b^2\right)\cos(3(c+dx))-28ab\left(5a^2-b^2\right)\cos(5(c+dx))-20ab\left(a^2-\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^3*(a*Cos[c + d*x] + b*Sin[c + d*x])^4,x]
```

```
[Out] (-140*a*b*(5*a^2 + 3*b^2)*Cos[c + d*x] - 140*a*b*(3*a^2 + b^2)*Cos[3*(c + d
*x)] - 28*a*b*(5*a^2 - b^2)*Cos[5*(c + d*x)] - 20*a*b*(a^2 - b^2)*Cos[7*(c
+ d*x)] + 35*(35*a^4 + 30*a^2*b^2 + 3*b^4)*Sin[c + d*x] + 35*(7*a^4 - 2*a^2
*b^2 - b^4)*Sin[3*(c + d*x)] + 7*(7*a^4 - 18*a^2*b^2 - b^4)*Sin[5*(c + d*x)
] + 5*(a^4 - 6*a^2*b^2 + b^4)*Sin[7*(c + d*x)])/(2240*d)
```

fricas [A] time = 0.62, size = 149, normalized size = 0.68

$$-\frac{28\,ab^3\cos\left(dx+c\right)^5+20\left(a^3b-ab^3\right)\cos\left(dx+c\right)^7-\left(5\left(a^4-6\,a^2b^2+b^4\right)\cos\left(dx+c\right)^6+2\left(3\,a^4+3\,a^2b^2-4\,b\right.\right.}{35\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a*cos(d*x+c)+b*sin(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] -1/35*(28*a*b^3*cos(d*x + c)^5 + 20*(a^3*b - a*b^3)*cos(d*x + c)^7 - (5*(a^
4 - 6*a^2*b^2 + b^4)*cos(d*x + c)^6 + 2*(3*a^4 + 3*a^2*b^2 - 4*b^4)*cos(d*x
 + c)^4 + 16*a^4 + 16*a^2*b^2 + 2*b^4 + (8*a^4 + 8*a^2*b^2 + b^4)*cos(d*x +
 c)^2)*sin(d*x + c))/d
```

giac [A] time = 0.49, size = 229, normalized size = 1.04

$$-\frac{\left(a^3b-ab^3\right)\cos\left(7\,dx+7\,c\right)}{112\,d}-\frac{\left(5\,a^3b-ab^3\right)\cos\left(5\,dx+5\,c\right)}{80\,d}-\frac{\left(3\,a^3b+ab^3\right)\cos\left(3\,dx+3\,c\right)}{16\,d}-\frac{\left(5\,a^3b+3\,ab^3\right)\cos}{16\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a*cos(d*x+c)+b*sin(d*x+c))^4,x, algorithm="giac")
```

```
[Out] -1/112*(a^3*b - a*b^3)*cos(7*d*x + 7*c)/d - 1/80*(5*a^3*b - a*b^3)*cos(5*d*
x + 5*c)/d - 1/16*(3*a^3*b + a*b^3)*cos(3*d*x + 3*c)/d - 1/16*(5*a^3*b + 3*
a*b^3)*cos(d*x + c)/d + 1/448*(a^4 - 6*a^2*b^2 + b^4)*sin(7*d*x + 7*c)/d +
1/320*(7*a^4 - 18*a^2*b^2 - b^4)*sin(5*d*x + 5*c)/d + 1/64*(7*a^4 - 2*a^2*b
^2 - b^4)*sin(3*d*x + 3*c)/d + 1/64*(35*a^4 + 30*a^2*b^2 + 3*b^4)*sin(d*x +
 c)/d
```

maple [A] time = 39.35, size = 206, normalized size = 0.94

$$\frac{b^4\left(-\frac{\left(\sin^3(dx+c)\right)\left(\cos^4(dx+c)\right)}{7}-\frac{3\sin(dx+c)\left(\cos^4(dx+c)\right)}{35}+\frac{\left(2+\cos^2(dx+c)\right)\sin(dx+c)}{35}\right)+4a\,b^3\left(-\frac{\left(\sin^2(dx+c)\right)\left(\cos^5(dx+c)\right)}{7}-\frac{2\left(\cos^5(dx+\right.}{35}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3*(a*cos(d*x+c)+b*sin(d*x+c))^4,x)
```

```
[Out] 1/d*(b^4*(-1/7*sin(d*x+c)^3*cos(d*x+c)^4-3/35*sin(d*x+c)*cos(d*x+c)^4+1/35*
(2+cos(d*x+c)^2)*sin(d*x+c))+4*a*b^3*(-1/7*sin(d*x+c)^2*cos(d*x+c)^5-2/35*c
os(d*x+c)^5)+6*a^2*b^2*(-1/7*sin(d*x+c)*cos(d*x+c)^6+1/35*(8/3+cos(d*x+c)^4
+4/3*cos(d*x+c)^2)*sin(d*x+c))-4/7*a^3*b*cos(d*x+c)^7+1/7*a^4*(16/5+cos(d*x
+c)^6+6/5*cos(d*x+c)^4+8/5*cos(d*x+c)^2)*sin(d*x+c))
```

maxima [A] time = 0.33, size = 154, normalized size = 0.70

$$-\frac{20\,a^3b\cos\left(dx+c\right)^7+\left(5\,\sin\left(dx+c\right)^7-21\,\sin\left(dx+c\right)^5+35\,\sin\left(dx+c\right)^3-35\,\sin\left(dx+c\right)\right)a^4-2\left(15\,\sin\left(dx\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a*cos(d*x+c)+b*sin(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] -1/35*(20*a^3*b*cos(d*x + c)^7 + (5*sin(d*x + c)^7 - 21*sin(d*x + c)^5 + 35
*sin(d*x + c)^3 - 35*sin(d*x + c))*a^4 - 2*(15*sin(d*x + c)^7 - 42*sin(d*x
+ c)^5 + 35*sin(d*x + c)^3)*a^2*b^2 - 4*(5*cos(d*x + c)^7 - 7*cos(d*x + c)^
5)*a*b^3 + (5*sin(d*x + c)^7 - 7*sin(d*x + c)^5)*b^4)/d
```

mupad [B] time = 1.28, size = 291, normalized size = 1.32

$$-\frac{\frac{b^4\sin(3c+3dx)}{64}-\frac{3b^4\sin(c+dx)}{64}-\frac{7a^4\sin(3c+3dx)}{64}-\frac{7a^4\sin(5c+5dx)}{320}-\frac{a^4\sin(7c+7dx)}{448}-\frac{35a^4\sin(c+dx)}{64}+\frac{b^4\sin(5c+5dx)}{320}-\frac{b^4}{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^3*(a*cos(c + d*x) + b*sin(c + d*x))^4,x)
```

```
[Out] -((b^4*sin(3*c + 3*d*x))/64 - (3*b^4*sin(c + d*x))/64 - (7*a^4*sin(3*c + 3*
d*x))/64 - (7*a^4*sin(5*c + 5*d*x))/320 - (a^4*sin(7*c + 7*d*x))/448 - (35*
a^4*sin(c + d*x))/64 + (b^4*sin(5*c + 5*d*x))/320 - (b^4*sin(7*c + 7*d*x))/
448 + (a*b^3*cos(3*c + 3*d*x))/16 + (3*a^3*b*cos(3*c + 3*d*x))/16 - (a*b^3*
cos(5*c + 5*d*x))/80 + (a^3*b*cos(5*c + 5*d*x))/16 - (a*b^3*cos(7*c + 7*d*x
))/112 + (a^3*b*cos(7*c + 7*d*x))/112 - (15*a^2*b^2*sin(c + d*x))/32 + (a^2
```

```
*b^2*sin(3*c + 3*d*x))/32 + (9*a^2*b^2*sin(5*c + 5*d*x))/160 + (3*a^2*b^2*s
in(7*c + 7*d*x))/224 + (3*a*b^3*cos(c + d*x))/16 + (5*a^3*b*cos(c + d*x))/1
6)/d
```

sympy [A] time = 5.71, size = 286, normalized size = 1.30

$$\begin{cases} \frac{16a^4\sin^7(c+dx)}{35d} + \frac{8a^4\sin^5(c+dx)\cos^2(c+dx)}{5d} + \frac{2a^4\sin^3(c+dx)\cos^4(c+dx)}{d} + \frac{a^4\sin(c+dx)\cos^6(c+dx)}{d} - \frac{4a^3b\cos^7(c+dx)}{7d} + \frac{16a^2b^2\sin^7(}{35d} \\ x\left(a\cos(c)+b\sin(c)\right)^4\cos^3(c) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3*(a*cos(d*x+c)+b*sin(d*x+c))**4,x)
```

```
[Out] Piecewise((16*a**4*sin(c + d*x)**7/(35*d) + 8*a**4*sin(c + d*x)**5*cos(c +
d*x)**2/(5*d) + 2*a**4*sin(c + d*x)**3*cos(c + d*x)**4/d + a**4*sin(c + d*x
)*cos(c + d*x)**6/d - 4*a**3*b*cos(c + d*x)**7/(7*d) + 16*a**2*b**2*sin(c +
 d*x)**7/(35*d) + 8*a**2*b**2*sin(c + d*x)**5*cos(c + d*x)**2/(5*d) + 2*a**
2*b**2*sin(c + d*x)**3*cos(c + d*x)**4/d - 4*a*b**3*sin(c + d*x)**2*cos(c +
 d*x)**5/(5*d) - 8*a*b**3*cos(c + d*x)**7/(35*d) + 2*b**4*sin(c + d*x)**7/(
35*d) + b**4*sin(c + d*x)**5*cos(c + d*x)**2/(5*d), Ne(d, 0)), (x*(a*cos(c)
 + b*sin(c))**4*cos(c)**3, True))
```

3.77 $\int \cos^2(c+dx)(a\cos(c+dx)+b\sin(c+dx))^4\,dx$

Optimal. Leaf size=301

$$\frac{a^4\sin(c+dx)\cos^5(c+dx)}{6d}+\frac{5a^4\sin(c+dx)\cos^3(c+dx)}{24d}+\frac{5a^4\sin(c+dx)\cos(c+dx)}{16d}+\frac{5a^4x}{16}-\frac{2a^3b\cos^6(c+dx)}{3d}$$

```
[Out] 5/16*a^4*x+3/8*a^2*b^2*x+1/16*b^4*x-2/3*a^3*b*cos(d*x+c)^6/d+5/16*a^4*cos(d
*x+c)*sin(d*x+c)/d+3/8*a^2*b^2*cos(d*x+c)*sin(d*x+c)/d+1/16*b^4*cos(d*x+c)*
sin(d*x+c)/d+5/24*a^4*cos(d*x+c)^3*sin(d*x+c)/d+1/4*a^2*b^2*cos(d*x+c)^3*si
n(d*x+c)/d-1/8*b^4*cos(d*x+c)^3*sin(d*x+c)/d+1/6*a^4*cos(d*x+c)^5*sin(d*x+c
)/d-a^2*b^2*cos(d*x+c)^5*sin(d*x+c)/d-1/6*b^4*cos(d*x+c)^3*sin(d*x+c)^3/d+a
*b^3*sin(d*x+c)^4/d-2/3*a*b^3*sin(d*x+c)^6/d
```

Rubi **[A]** time = 0.30, antiderivative size = 301, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 8, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3090, 2635, 8, 2565, 30, 2568, 2564, 14}

$$-\frac{a^2b^2\sin(c+dx)\cos^5(c+dx)}{d}+\frac{a^2b^2\sin(c+dx)\cos^3(c+dx)}{4d}+\frac{3a^2b^2\sin(c+dx)\cos(c+dx)}{8d}+\frac{3}{8}a^2b^2x-\frac{2a^3b\cos^6}{3}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2*(a*Cos[c + d*x] + b*Sin[c + d*x])^4,x]
```

```
[Out] (5*a^4*x)/16 + (3*a^2*b^2*x)/8 + (b^4*x)/16 - (2*a^3*b*Cos[c + d*x]^6)/(3*d
) + (5*a^4*Cos[c + d*x]*Sin[c + d*x])/(16*d) + (3*a^2*b^2*Cos[c + d*x]*Sin[
c + d*x])/(8*d) + (b^4*Cos[c + d*x]*Sin[c + d*x])/(16*d) + (5*a^4*Cos[c + d
*x]^3*Sin[c + d*x])/(24*d) + (a^2*b^2*Cos[c + d*x]^3*Sin[c + d*x])/(4*d) -
(b^4*Cos[c + d*x]^3*Sin[c + d*x])/(8*d) + (a^4*Cos[c + d*x]^5*Sin[c + d*x])
/(6*d) - (a^2*b^2*Cos[c + d*x]^5*Sin[c + d*x])/d - (b^4*Cos[c + d*x]^3*Sin[
c + d*x]^3)/(6*d) + (a*b^3*Sin[c + d*x]^4)/d - (2*a*b^3*Sin[c + d*x]^6)/(3*
d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2564

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 2565

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> -Dist[(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 2568

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> -Simp[(a*(b*Cos[e + f*x])^(n + 1)*(a*Sin[e + f*x])^(m - 1)
)/(b*f*(m + n)), x] + Dist[(a^2*(m - 1))/(m + n), Int[(b*Cos[e + f*x])^n*(a
*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1] &&
 NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 3090

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_.), x_Symbol] :> Int[ExpandTrig[cos[c + d*x]^m*(a
*cos[c + d*x] + b*sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d}, x] && Inte
gerQ[m] && IGtQ[n, 0]
```

Rubi steps

$$\int \cos^2(c+dx)(a\cos(c+dx)+b\sin(c+dx))^4\,dx = \int \left(a^4\cos^6(c+dx)+4a^3b\cos^5(c+dx)\sin(c+dx)+6a^2b^2\right.$$
$$= a^4\int \cos^6(c+dx)\,dx+\left(4a^3b\right)\int \cos^5(c+dx)\sin(c+dx)\,dx$$
$$= \frac{a^4\cos^5(c+dx)\sin(c+dx)}{6d}-\frac{a^2b^2\cos^5(c+dx)\sin(c+dx)}{d}$$
$$= -\frac{2a^3b\cos^6(c+dx)}{3d}+\frac{5a^4\cos^3(c+dx)\sin(c+dx)}{24d}+\frac{a^2b^2}{}$$
$$= -\frac{2a^3b\cos^6(c+dx)}{3d}+\frac{5a^4\cos(c+dx)\sin(c+dx)}{16d}+\frac{3a^2b^2}{}$$
$$= \frac{5a^4x}{16}+\frac{3}{8}a^2b^2x+\frac{b^4x}{16}-\frac{2a^3b\cos^6(c+dx)}{3d}+\frac{5a^4\cos(c+dx}{16}$$

Mathematica [C] time = 0.43, size = 178, normalized size = 0.59

$$\frac{-24a^3b\cos(4(c+dx))+12(a-ib)(a+ib)\left(5a^2+b^2\right)(c+dx)-12ab\left(5a^2+3b^2\right)\cos(2(c+dx))-4ab\left(a^2-b^2\right)}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2*(a*Cos[c + d*x] + b*Sin[c + d*x])^4,x]
```

```
[Out] (12*(a - I*b)*(a + I*b)*(5*a^2 + b^2)*(c + d*x) - 12*a*b*(5*a^2 + 3*b^2)*Co
s[2*(c + d*x)] - 24*a^3*b*Cos[4*(c + d*x)] - 4*a*b*(a^2 - b^2)*Cos[6*(c + d
*x)] + 3*(15*a^4 + 6*a^2*b^2 - b^4)*Sin[2*(c + d*x)] + 3*(3*a^4 - 6*a^2*b^2
 - b^4)*Sin[4*(c + d*x)] + (a^4 - 6*a^2*b^2 + b^4)*Sin[6*(c + d*x)])/(192*d
)
```

fricas [A] time = 0.69, size = 151, normalized size = 0.50

$$-\frac{48\,ab^3\cos(dx+c)^4+32\left(a^3b-ab^3\right)\cos(dx+c)^6-3\left(5\,a^4+6\,a^2b^2+b^4\right)dx-\left(8\left(a^4-6\,a^2b^2+b^4\right)\cos(dx+c\right.}{48\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a*cos(d*x+c)+b*sin(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] -1/48*(48*a*b^3*cos(d*x + c)^4 + 32*(a^3*b - a*b^3)*cos(d*x + c)^6 - 3*(5*a
^4 + 6*a^2*b^2 + b^4)*d*x - (8*(a^4 - 6*a^2*b^2 + b^4)*cos(d*x + c)^5 + 2*(
5*a^4 + 6*a^2*b^2 - 7*b^4)*cos(d*x + c)^3 + 3*(5*a^4 + 6*a^2*b^2 + b^4)*cos
(d*x + c))*sin(d*x + c))/d
```

giac [A] time = 2.02, size = 187, normalized size = 0.62

$$-\frac{a^3b\cos(4\,dx+4\,c)}{8\,d}+\frac{1}{16}\left(5\,a^4+6\,a^2b^2+b^4\right)x-\frac{\left(a^3b-ab^3\right)\cos(6\,dx+6\,c)}{48\,d}-\frac{\left(5\,a^3b+3\,ab^3\right)\cos(2\,dx+2\,c)}{16\,d}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a*cos(d*x+c)+b*sin(d*x+c))^4,x, algorithm="giac")
```

```
[Out] -1/8*a^3*b*cos(4*d*x + 4*c)/d + 1/16*(5*a^4 + 6*a^2*b^2 + b^4)*x - 1/48*(a^
3*b - a*b^3)*cos(6*d*x + 6*c)/d - 1/16*(5*a^3*b + 3*a*b^3)*cos(2*d*x + 2*c)
/d + 1/192*(a^4 - 6*a^2*b^2 + b^4)*sin(6*d*x + 6*c)/d + 1/64*(3*a^4 - 6*a^2
*b^2 - b^4)*sin(4*d*x + 4*c)/d + 1/64*(15*a^4 + 6*a^2*b^2 - b^4)*sin(2*d*x
+ 2*c)/d
```

maple [A] time = 28.97, size = 219, normalized size = 0.73

$$b^4\left(-\frac{\left(\sin^3(dx+c)\right)\left(\cos^3(dx+c)\right)}{6}-\frac{\left(\cos^3(dx+c)\right)\sin(dx+c)}{8}+\frac{\cos(dx+c)\sin(dx+c)}{16}+\frac{dx}{16}+\frac{c}{16}\right)+4a\,b^3\left(-\frac{\left(\sin^2(dx+c)\right)\left(\cos^4(dx+c)\right)}{6}-\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(a*cos(d*x+c)+b*sin(d*x+c))^4,x)
```

```
[Out] 1/d*(b^4*(-1/6*sin(d*x+c)^3*cos(d*x+c)^3-1/8*cos(d*x+c)^3*sin(d*x+c)+1/16*c
os(d*x+c)*sin(d*x+c)+1/16*d*x+1/16*c)+4*a*b^3*(-1/6*sin(d*x+c)^2*cos(d*x+c)
^4-1/12*cos(d*x+c)^4)+6*a^2*b^2*(-1/6*cos(d*x+c)^5*sin(d*x+c)+1/24*(cos(d*x
+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+1/16*d*x+1/16*c)-2/3*a^3*b*cos(d*x+c)^6+a^
4*(1/6*(cos(d*x+c)^5+5/4*cos(d*x+c)^3+15/8*cos(d*x+c))*sin(d*x+c)+5/16*d*x+
5/16*c))
```

maxima [A] time = 0.34, size = 170, normalized size = 0.56

$$-\frac{128\,a^3b\cos(dx+c)^6+\left(4\,\sin(2\,dx+2\,c)^3-60\,dx-60\,c-9\,\sin(4\,dx+4\,c)-48\,\sin(2\,dx+2\,c)\right)a^4-6\left(4\,\sin\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a*cos(d*x+c)+b*sin(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] -1/192*(128*a^3*b*cos(d*x + c)^6 + (4*sin(2*d*x + 2*c)^3 - 60*d*x - 60*c -
9*sin(4*d*x + 4*c) - 48*sin(2*d*x + 2*c))*a^4 - 6*(4*sin(2*d*x + 2*c)^3 + 1
2*d*x + 12*c - 3*sin(4*d*x + 4*c))*a^2*b^2 + 64*(2*sin(d*x + c)^6 - 3*sin(d
```

```
*x + c)^4)*a*b^3 + (4*sin(2*d*x + 2*c)^3 - 12*d*x - 12*c + 3*sin(4*d*x + 4*
c))*b^4)/d
```

mupad [B] time = 2.31, size = 471, normalized size = 1.56

$$\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{11}\,\left(-\frac{11\,a^4}{8}+\frac{3\,a^2\,b^2}{4}+\frac{b^4}{8}\right)+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5\,\left(\frac{15\,a^4}{4}-\frac{39\,a^2\,b^2}{2}+\frac{19\,b^4}{4}\right)-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7\,\left(\frac{15\,a^4}{4}-\frac{39\,a^2\,b^2}{2}+\cdots\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^2*(a*cos(c + d*x) + b*sin(c + d*x))^4,x)
```

```
[Out] (tan(c/2 + (d*x)/2)^11*(b^4/8 - (11*a^4)/8 + (3*a^2*b^2)/4) + tan(c/2 + (d*
x)/2)^5*((15*a^4)/4 + (19*b^4)/4 - (39*a^2*b^2)/2) - tan(c/2 + (d*x)/2)^7*(
(15*a^4)/4 + (19*b^4)/4 - (39*a^2*b^2)/2) - tan(c/2 + (d*x)/2)^3*((5*a^4)/2
4 + (17*b^4)/24 - (47*a^2*b^2)/4) + tan(c/2 + (d*x)/2)^9*((5*a^4)/24 + (17*
b^4)/24 - (47*a^2*b^2)/4) - tan(c/2 + (d*x)/2)*(b^4/8 - (11*a^4)/8 + (3*a^2
*b^2)/4) - tan(c/2 + (d*x)/2)^6*((32*a*b^3)/3 - (80*a^3*b)/3) + 8*a^3*b*tan
(c/2 + (d*x)/2)^2 + 16*a*b^3*tan(c/2 + (d*x)/2)^4 + 16*a*b^3*tan(c/2 + (d*x
)/2)^8 + 8*a^3*b*tan(c/2 + (d*x)/2)^10)/(d*(6*tan(c/2 + (d*x)/2)^2 + 15*tan
(c/2 + (d*x)/2)^4 + 20*tan(c/2 + (d*x)/2)^6 + 15*tan(c/2 + (d*x)/2)^8 + 6*t
an(c/2 + (d*x)/2)^10 + tan(c/2 + (d*x)/2)^12 + 1)) - ((atan(tan(c/2 + (d*x)
/2)) - (d*x)/2)*(5*a^4 + b^4 + 6*a^2*b^2))/(8*d) + (atan((tan(c/2 + (d*x)/2
)*(5*a^2 + b^2)*(a^2 + b^2))/(8*((5*a^4)/8 + b^4/8 + (3*a^2*b^2)/4)))*(5*a^
2 + b^2)*(a^2 + b^2))/(8*d)
```

sympy [A] time = 3.99, size = 563, normalized size = 1.87

$$\begin{cases} \frac{5a^4x\sin^6(c+dx)}{16}+\frac{15a^4x\sin^4(c+dx)\cos^2(c+dx)}{16}+\frac{15a^4x\sin^2(c+dx)\cos^4(c+dx)}{16}+\frac{5a^4x\cos^6(c+dx)}{16}+\frac{5a^4\sin^5(c+dx)\cos(c+dx)}{16d}+\frac{5a^4\sin^3\cdots}{} \\ x\,(a\cos(c)+b\sin(c))^4\cos^2(c) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(a*cos(d*x+c)+b*sin(d*x+c))**4,x)
```

```
[Out] Piecewise((5*a**4*x*sin(c + d*x)**6/16 + 15*a**4*x*sin(c + d*x)**4*cos(c +
d*x)**2/16 + 15*a**4*x*sin(c + d*x)**2*cos(c + d*x)**4/16 + 5*a**4*x*cos(c
+ d*x)**6/16 + 5*a**4*sin(c + d*x)**5*cos(c + d*x)/(16*d) + 5*a**4*sin(c +
d*x)**3*cos(c + d*x)**3/(6*d) + 11*a**4*sin(c + d*x)*cos(c + d*x)**5/(16*d)
 - 2*a**3*b*cos(c + d*x)**6/(3*d) + 3*a**2*b**2*x*sin(c + d*x)**6/8 + 9*a**
2*b**2*x*sin(c + d*x)**4*cos(c + d*x)**2/8 + 9*a**2*b**2*x*sin(c + d*x)**2*
```

```
cos(c + d*x)**4/8 + 3*a**2*b**2*x*cos(c + d*x)**6/8 + 3*a**2*b**2*sin(c + d
*x)**5*cos(c + d*x)/(8*d) + a**2*b**2*sin(c + d*x)**3*cos(c + d*x)**3/d - 3
*a**2*b**2*sin(c + d*x)*cos(c + d*x)**5/(8*d) + a*b**3*sin(c + d*x)**6/(3*d
) + a*b**3*sin(c + d*x)**4*cos(c + d*x)**2/d + b**4*x*sin(c + d*x)**6/16 +
3*b**4*x*sin(c + d*x)**4*cos(c + d*x)**2/16 + 3*b**4*x*sin(c + d*x)**2*cos(
c + d*x)**4/16 + b**4*x*cos(c + d*x)**6/16 + b**4*sin(c + d*x)**5*cos(c + d
*x)/(16*d) - b**4*sin(c + d*x)**3*cos(c + d*x)**3/(6*d) - b**4*sin(c + d*x)
*cos(c + d*x)**5/(16*d), Ne(d, 0)), (x*(a*cos(c) + b*sin(c))**4*cos(c)**2,
True))
```

3.78 $\int \cos(c+dx)(a\cos(c+dx)+b\sin(c+dx))^4\,dx$

Optimal. Leaf size=165

$$\frac{a^4\sin^5(c+dx)}{5d}-\frac{2a^4\sin^3(c+dx)}{3d}+\frac{a^4\sin(c+dx)}{d}-\frac{4a^3b\cos^5(c+dx)}{5d}-\frac{6a^2b^2\sin^5(c+dx)}{5d}+\frac{2a^2b^2\sin^3(c+dx)}{d}+\frac{4a}{}$$

```
[Out] -4/3*a*b^3*cos(d*x+c)^3/d-4/5*a^3*b*cos(d*x+c)^5/d+4/5*a*b^3*cos(d*x+c)^5/d
+a^4*sin(d*x+c)/d-2/3*a^4*sin(d*x+c)^3/d+2*a^2*b^2*sin(d*x+c)^3/d+1/5*a^4*s
in(d*x+c)^5/d-6/5*a^2*b^2*sin(d*x+c)^5/d+1/5*b^4*sin(d*x+c)^5/d
```

Rubi [A] time = 0.18, antiderivative size = 165, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {3090, 2633, 2565, 30, 2564, 14}

$$-\frac{6a^2b^2\sin^5(c+dx)}{5d}+\frac{2a^2b^2\sin^3(c+dx)}{d}-\frac{4a^3b\cos^5(c+dx)}{5d}+\frac{a^4\sin^5(c+dx)}{5d}-\frac{2a^4\sin^3(c+dx)}{3d}+\frac{a^4\sin(c+dx)}{d}+$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]*(a*Cos[c + d*x] + b*Sin[c + d*x])^4,x]
```

```
[Out] (-4*a*b^3*Cos[c + d*x]^3)/(3*d) - (4*a^3*b*Cos[c + d*x]^5)/(5*d) + (4*a*b^3
*Cos[c + d*x]^5)/(5*d) + (a^4*Sin[c + d*x])/d - (2*a^4*Sin[c + d*x]^3)/(3*d
) + (2*a^2*b^2*Sin[c + d*x]^3)/d + (a^4*Sin[c + d*x]^5)/(5*d) - (6*a^2*b^2*
Sin[c + d*x]^5)/(5*d) + (b^4*Sin[c + d*x]^5)/(5*d)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2564

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 2565

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> -Dist[(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 3090

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_.), x_Symbol] :> Int[ExpandTrig[cos[c + d*x]^m*(a
*cos[c + d*x] + b*sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d}, x] && Inte
gerQ[m] && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}\int \cos(c+dx)(a\cos(c+dx)+b\sin(c+dx))^4\,dx &= \int \left(a^4\cos^5(c+dx)+4a^3b\cos^4(c+dx)\sin(c+dx)+6a^2b^2\right.\\
&= a^4\int \cos^5(c+dx)\,dx+\left(4a^3b\right)\int \cos^4(c+dx)\sin(c+dx)\,d\\
&= -\frac{a^4\,\mathrm{Subst}\left(\int \left(1-2x^2+x^4\right)dx,x,-\sin(c+dx)\right)}{d}-\frac{\left(4a^3b\right)\mathrm{S}}{}\\
&= -\frac{4a^3b\cos^5(c+dx)}{5d}+\frac{a^4\sin(c+dx)}{d}-\frac{2a^4\sin^3(c+dx)}{3d}+\frac{a}{}\\
&= -\frac{4ab^3\cos^3(c+dx)}{3d}-\frac{4a^3b\cos^5(c+dx)}{5d}+\frac{4ab^3\cos^5(c+dx}{5d}\end{aligned}$$

Mathematica [A] time = 0.43, size = 146, normalized size = 0.88

$$\frac{-120ab\left(a^2+b^2\right)\cos(c+dx)-20ab\left(3a^2+b^2\right)\cos(3(c+dx))-12ab\left(a^2-b^2\right)\cos(5(c+dx))+30\left(5a^4+6a^2b\right.}{240d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]*(a*Cos[c + d*x] + b*Sin[c + d*x])^4,x]
```

```
[Out] (-120*a*b*(a^2 + b^2)*Cos[c + d*x] - 20*a*b*(3*a^2 + b^2)*Cos[3*(c + d*x)]
- 12*a*b*(a^2 - b^2)*Cos[5*(c + d*x)] + 30*(5*a^4 + 6*a^2*b^2 + b^4)*Sin[c
```

```
+ d*x] + 5*(5*a^4 - 6*a^2*b^2 - 3*b^4)*Sin[3*(c + d*x)] + 3*(a^4 - 6*a^2*b^
2 + b^4)*Sin[5*(c + d*x)])/(240*d)
```

fricas [A] time = 0.78, size = 123, normalized size = 0.75

$$-\frac{20\,ab^3\cos(dx+c)^3+12\left(a^3b-ab^3\right)\cos(dx+c)^5-\left(3\left(a^4-6\,a^2b^2+b^4\right)\cos(dx+c)^4+8\,a^4+12\,a^2b^2+3\,b^4+\right.}{15\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a*cos(d*x+c)+b*sin(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] -1/15*(20*a*b^3*cos(d*x + c)^3 + 12*(a^3*b - a*b^3)*cos(d*x + c)^5 - (3*(a^
4 - 6*a^2*b^2 + b^4)*cos(d*x + c)^4 + 8*a^4 + 12*a^2*b^2 + 3*b^4 + 2*(2*a^4
 + 3*a^2*b^2 - 3*b^4)*cos(d*x + c)^2)*sin(d*x + c))/d
```

giac [A] time = 2.94, size = 165, normalized size = 1.00

$$-\frac{\left(a^3b-ab^3\right)\cos(5\,dx+5\,c)}{20\,d}-\frac{\left(3\,a^3b+ab^3\right)\cos(3\,dx+3\,c)}{12\,d}-\frac{\left(a^3b+ab^3\right)\cos(dx+c)}{2\,d}+\frac{\left(a^4-6\,a^2b^2+b^4\right)\sin(5\,d}{80\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a*cos(d*x+c)+b*sin(d*x+c))^4,x, algorithm="giac")
```

```
[Out] -1/20*(a^3*b - a*b^3)*cos(5*d*x + 5*c)/d - 1/12*(3*a^3*b + a*b^3)*cos(3*d*x
 + 3*c)/d - 1/2*(a^3*b + a*b^3)*cos(d*x + c)/d + 1/80*(a^4 - 6*a^2*b^2 + b^
4)*sin(5*d*x + 5*c)/d + 1/48*(5*a^4 - 6*a^2*b^2 - 3*b^4)*sin(3*d*x + 3*c)/d
 + 1/8*(5*a^4 + 6*a^2*b^2 + b^4)*sin(d*x + c)/d
```

maple [A] time = 39.38, size = 142, normalized size = 0.86

$$\frac{\frac{b^4\left(\sin^5(dx+c)\right)}{5}+4a\,b^3\left(-\frac{\left(\sin^2(dx+c)\right)\left(\cos^3(dx+c)\right)}{5}-\frac{2\left(\cos^3(dx+c)\right)}{15}\right)+6a^2b^2\left(-\frac{\sin(dx+c)\left(\cos^4(dx+c)\right)}{5}+\frac{\left(2+\cos^2(dx+c)\right)\sin(dx+c)}{15}\right)-}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(a*cos(d*x+c)+b*sin(d*x+c))^4,x)
```

```
[Out] 1/d*(1/5*b^4*sin(d*x+c)^5+4*a*b^3*(-1/5*sin(d*x+c)^2*cos(d*x+c)^3-2/15*cos(
d*x+c)^3)+6*a^2*b^2*(-1/5*sin(d*x+c)*cos(d*x+c)^4+1/15*(2+cos(d*x+c)^2)*sin
(d*x+c))-4/5*a^3*b*cos(d*x+c)^5+1/5*a^4*(8/3+cos(d*x+c)^4+4/3*cos(d*x+c)^2)
*sin(d*x+c))
```

maxima [A] time = 0.33, size = 123, normalized size = 0.75

$$-\frac{12\,a^3b\cos\left(dx+c\right)^5-3\,b^4\sin\left(dx+c\right)^5-\left(3\,\sin\left(dx+c\right)^5-10\,\sin\left(dx+c\right)^3+15\,\sin\left(dx+c\right)\right)a^4+6\left(3\,\sin\left(d\right.\right.}{15\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a*cos(d*x+c)+b*sin(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] -1/15*(12*a^3*b*cos(d*x + c)^5 - 3*b^4*sin(d*x + c)^5 - (3*sin(d*x + c)^5 -
 10*sin(d*x + c)^3 + 15*sin(d*x + c))*a^4 + 6*(3*sin(d*x + c)^5 - 5*sin(d*x
 + c)^3)*a^2*b^2 - 4*(3*cos(d*x + c)^5 - 5*cos(d*x + c)^3)*a*b^3)/d
```

mupad [B] time = 0.83, size = 204, normalized size = 1.24

$$\frac{2\left(\frac{3\,\sin(c+d\,x)\,a^4\cos(c+d\,x)^4}{2}+2\,\sin\left(c+d\,x\right)\,a^4\cos\left(c+d\,x\right)^2+4\,\sin\left(c+d\,x\right)\,a^4-6\,a^3\,b\cos\left(c+d\,x\right)^5-9\,\sin\left(c+\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)*(a*cos(c + d*x) + b*sin(c + d*x))^4,x)
```

```
[Out] (2*(4*a^4*sin(c + d*x) + (3*b^4*sin(c + d*x))/2 - 10*a*b^3*cos(c + d*x)^3 +
 6*a*b^3*cos(c + d*x)^5 - 6*a^3*b*cos(c + d*x)^5 + 2*a^4*cos(c + d*x)^2*sin
(c + d*x) + (3*a^4*cos(c + d*x)^4*sin(c + d*x))/2 + 6*a^2*b^2*sin(c + d*x)
- 3*b^4*cos(c + d*x)^2*sin(c + d*x) + (3*b^4*cos(c + d*x)^4*sin(c + d*x))/2
 + 3*a^2*b^2*cos(c + d*x)^2*sin(c + d*x) - 9*a^2*b^2*cos(c + d*x)^4*sin(c +
 d*x)))/(15*d)
```

sympy [A] time = 1.99, size = 206, normalized size = 1.25

$$\begin{cases} \frac{8a^4\sin^5\left(c+dx\right)}{15d}+\frac{4a^4\sin^3\left(c+dx\right)\cos^2\left(c+dx\right)}{3d}+\frac{a^4\sin\left(c+dx\right)\cos^4\left(c+dx\right)}{d}-\frac{4a^3b\cos^5\left(c+dx\right)}{5d}+\frac{4a^2b^2\sin^5\left(c+dx\right)}{5d}+\frac{2a^2b^2\sin^3\left(c+dx\right)\cos^2\left(\right.}{d} \\ x\left(a\cos\left(c\right)+b\sin\left(c\right)\right)^4\cos\left(c\right) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a*cos(d*x+c)+b*sin(d*x+c))**4,x)
```

```
[Out] Piecewise((8*a**4*sin(c + d*x)**5/(15*d) + 4*a**4*sin(c + d*x)**3*cos(c + d
*x)**2/(3*d) + a**4*sin(c + d*x)*cos(c + d*x)**4/d - 4*a**3*b*cos(c + d*x)*
*5/(5*d) + 4*a**2*b**2*sin(c + d*x)**5/(5*d) + 2*a**2*b**2*sin(c + d*x)**3*
cos(c + d*x)**2/d - 4*a*b**3*sin(c + d*x)**2*cos(c + d*x)**3/(3*d) - 8*a*b*
*3*cos(c + d*x)**5/(15*d) + b**4*sin(c + d*x)**5/(5*d), Ne(d, 0)), (x*(a*co
s(c) + b*sin(c))**4*cos(c), True))
```

3.79 $\int (a\cos(c+dx)+b\sin(c+dx))^4\,dx$

Optimal. Leaf size=108

$$-\frac{3\left(a^2+b^2\right)(b\cos(c+dx)-a\sin(c+dx))(a\cos(c+dx)+b\sin(c+dx))}{8d}+\frac{3}{8}x\left(a^2+b^2\right)^2-\frac{(b\cos(c+dx)-a\sin(}{}$$

```
[Out] 3/8*(a^2+b^2)^2*x-3/8*(a^2+b^2)*(b*cos(d*x+c)-a*sin(d*x+c))*(a*cos(d*x+c)+b
*sin(d*x+c))/d-1/4*(b*cos(d*x+c)-a*sin(d*x+c))*(a*cos(d*x+c)+b*sin(d*x+c))^
3/d
```

Rubi [A] time = 0.04, antiderivative size = 108, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {3073, 8}

$$-\frac{3\left(a^2+b^2\right)(b\cos(c+dx)-a\sin(c+dx))(a\cos(c+dx)+b\sin(c+dx))}{8d}+\frac{3}{8}x\left(a^2+b^2\right)^2-\frac{(b\cos(c+dx)-a\sin(}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Cos[c + d*x] + b*Sin[c + d*x])^4,x]
```

```
[Out] (3*(a^2 + b^2)^2*x)/8 - (3*(a^2 + b^2)*(b*Cos[c + d*x] - a*Sin[c + d*x])*(a
*Cos[c + d*x] + b*Sin[c + d*x]))/(8*d) - ((b*Cos[c + d*x] - a*Sin[c + d*x])
*(a*Cos[c + d*x] + b*Sin[c + d*x])^3)/(4*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3073

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x
_Symbol] :> -Simp[((b*Cos[c + d*x] - a*Sin[c + d*x])*(a*Cos[c + d*x] + b*Si
n[c + d*x])^(n - 1))/(d*n), x] + Dist[((n - 1)*(a^2 + b^2))/n, Int[(a*Cos[c
 + d*x] + b*Sin[c + d*x])^(n - 2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 + b^2, 0] &&  !IntegerQ[(n - 1)/2] && GtQ[n, 1]
```

Rubi steps

$$\int (a\cos(c+dx)+b\sin(c+dx))^4\,dx = -\frac{(b\cos(c+dx)-a\sin(c+dx))(a\cos(c+dx)+b\sin(c+dx))^3}{4d}+\frac{1}{4}\Big(3\Big($$
$$= -\frac{3\left(a^2+b^2\right)(b\cos(c+dx)-a\sin(c+dx))(a\cos(c+dx)+b\sin(c+dx$$
$$= \frac{3}{8}\left(a^2+b^2\right)^2 x-\frac{3\left(a^2+b^2\right)(b\cos(c+dx)-a\sin(c+dx))(a\cos(c+dx$$

Mathematica [A] time = 0.43, size = 107, normalized size = 0.99

$$\frac{8\left(a^4-b^4\right)\sin(2(c+dx))+12\left(a^2+b^2\right)^2(c+dx)-16ab\left(a^2+b^2\right)\cos(2(c+dx))-4ab\left(a^2-b^2\right)\cos(4(c+dx))}{32d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Cos[c + d*x] + b*Sin[c + d*x])^4,x]
```

```
[Out] (12*(a^2 + b^2)^2*(c + d*x) - 16*a*b*(a^2 + b^2)*Cos[2*(c + d*x)] - 4*a*b*(
a^2 - b^2)*Cos[4*(c + d*x)] + 8*(a^4 - b^4)*Sin[2*(c + d*x)] + (a^4 - 6*a^2
*b^2 + b^4)*Sin[4*(c + d*x)])/(32*d)
```

fricas [A] time = 0.93, size = 121, normalized size = 1.12

$$-\frac{16\,ab^3\cos(dx+c)^2+8\left(a^3b-ab^3\right)\cos(dx+c)^4-3\left(a^4+2\,a^2b^2+b^4\right)dx-\left(2\left(a^4-6\,a^2b^2+b^4\right)\cos(dx+c)^3\right.}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cos(d*x+c)+b*sin(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] -1/8*(16*a*b^3*cos(d*x + c)^2 + 8*(a^3*b - a*b^3)*cos(d*x + c)^4 - 3*(a^4 +
 2*a^2*b^2 + b^4)*d*x - (2*(a^4 - 6*a^2*b^2 + b^4)*cos(d*x + c)^3 + (3*a^4
+ 6*a^2*b^2 - 5*b^4)*cos(d*x + c))*sin(d*x + c))/d
```

giac [A] time = 2.92, size = 122, normalized size = 1.13

$$\frac{3}{8}\left(a^4+2\,a^2b^2+b^4\right)x-\frac{\left(a^3b-ab^3\right)\cos(4\,dx+4\,c)}{8\,d}-\frac{\left(a^3b+ab^3\right)\cos(2\,dx+2\,c)}{2\,d}+\frac{\left(a^4-6\,a^2b^2+b^4\right)\sin(4\,dx+}{32\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cos(d*x+c)+b*sin(d*x+c))^4,x, algorithm="giac")
```

```
[Out] 3/8*(a^4 + 2*a^2*b^2 + b^4)*x - 1/8*(a^3*b - a*b^3)*cos(4*d*x + 4*c)/d - 1/
2*(a^3*b + a*b^3)*cos(2*d*x + 2*c)/d + 1/32*(a^4 - 6*a^2*b^2 + b^4)*sin(4*d
*x + 4*c)/d + 1/4*(a^4 - b^4)*sin(2*d*x + 2*c)/d
```

maple [A] time = 30.89, size = 153, normalized size = 1.42

$$\frac{b^4\left(-\frac{\left(\sin^3(dx+c)+\frac{3\sin(dx+c)}{2}\right)\cos(dx+c)}{4}+\frac{3dx}{8}+\frac{3c}{8}\right)+a\,b^3\left(\sin^4(dx+c)\right)+6a^2b^2\left(-\frac{\left(\cos^3(dx+c)\right)\sin(dx+c)}{4}+\frac{\cos(dx+c)\sin(dx+c}{8}\right.}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*cos(d*x+c)+b*sin(d*x+c))^4,x)
```

```
[Out] 1/d*(b^4*(-1/4*(sin(d*x+c)^3+3/2*sin(d*x+c))*cos(d*x+c)+3/8*d*x+3/8*c)+a*b^
3*sin(d*x+c)^4+6*a^2*b^2*(-1/4*cos(d*x+c)^3*sin(d*x+c)+1/8*cos(d*x+c)*sin(d
*x+c)+1/8*d*x+1/8*c)-cos(d*x+c)^4*a^3*b+a^4*(1/4*(cos(d*x+c)^3+3/2*cos(d*x+
c))*sin(d*x+c)+3/8*d*x+3/8*c))
```

maxima [A] time = 0.34, size = 136, normalized size = 1.26

$$-\frac{a^3b\cos(dx+c)^4}{d}+\frac{ab^3\sin(dx+c)^4}{d}+\frac{(12\,dx+12\,c+\sin(4\,dx+4\,c)+8\,\sin(2\,dx+2\,c))a^4}{32\,d}+\frac{3\,(4\,dx+4\,c-\sin}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cos(d*x+c)+b*sin(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] -a^3*b*cos(d*x + c)^4/d + a*b^3*sin(d*x + c)^4/d + 1/32*(12*d*x + 12*c + si
n(4*d*x + 4*c) + 8*sin(2*d*x + 2*c))*a^4/d + 3/16*(4*d*x + 4*c - sin(4*d*x
+ 4*c))*a^2*b^2/d + 1/32*(12*d*x + 12*c + sin(4*d*x + 4*c) - 8*sin(2*d*x +
2*c))*b^4/d
```

mupad [B] time = 1.96, size = 320, normalized size = 2.96

$$\frac{3\,\mathrm{atan}\left(\frac{3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\left(a^2+b^2\right)^2}{4\left(\frac{3\,a^4}{4}+\frac{3\,a^2\,b^2}{2}+\frac{3\,b^4}{4}\right)}\right)\left(a^2+b^2\right)^2}{4\,d}+\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7\left(-\frac{5\,a^4}{4}+\frac{3\,a^2\,b^2}{2}+\frac{3\,b^4}{4}\right)-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3\left(\frac{3\,a^4}{4}-\frac{21\,a^2\,b^2}{2}+\frac{11\,b}{4}\right.}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*cos(c + d*x) + b*sin(c + d*x))^4,x)
```

```
[Out] (3*atan((3*tan(c/2 + (d*x)/2)*(a^2 + b^2)^2)/(4*((3*a^4)/4 + (3*b^4)/4 + (3
*a^2*b^2)/2)))*(a^2 + b^2)^2)/(4*d) + (tan(c/2 + (d*x)/2)^7*((3*b^4)/4 - (5
```

```
*a^4)/4 + (3*a^2*b^2)/2) - tan(c/2 + (d*x)/2)^3*((3*a^4)/4 + (11*b^4)/4 - (
21*a^2*b^2)/2) + tan(c/2 + (d*x)/2)^5*((3*a^4)/4 + (11*b^4)/4 - (21*a^2*b^2
)/2) - tan(c/2 + (d*x)/2)*((3*b^4)/4 - (5*a^4)/4 + (3*a^2*b^2)/2) + 8*a^3*b
*tan(c/2 + (d*x)/2)^2 + 16*a*b^3*tan(c/2 + (d*x)/2)^4 + 8*a^3*b*tan(c/2 + (
d*x)/2)^6)/(d*(4*tan(c/2 + (d*x)/2)^2 + 6*tan(c/2 + (d*x)/2)^4 + 4*tan(c/2
+ (d*x)/2)^6 + tan(c/2 + (d*x)/2)^8 + 1)) - (3*(atan(tan(c/2 + (d*x)/2)) -
(d*x)/2)*(a^2 + b^2)^2)/(4*d)
```

sympy [A] time = 1.22, size = 381, normalized size = 3.53

$$\begin{cases} \frac{3a^4x\sin^4(c+dx)}{8} + \frac{3a^4x\sin^2(c+dx)\cos^2(c+dx)}{4} + \frac{3a^4x\cos^4(c+dx)}{8} + \frac{3a^4\sin^3(c+dx)\cos(c+dx)}{8d} + \frac{5a^4\sin(c+dx)\cos^3(c+dx)}{8d} - \frac{a^3b\cos^4(c}{d} \\ x\left(a\cos(c) + b\sin(c)\right)^4 \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cos(d*x+c)+b*sin(d*x+c))**4,x)
```

```
[Out] Piecewise((3*a**4*x*sin(c + d*x)**4/8 + 3*a**4*x*sin(c + d*x)**2*cos(c + d*
x)**2/4 + 3*a**4*x*cos(c + d*x)**4/8 + 3*a**4*sin(c + d*x)**3*cos(c + d*x)/
(8*d) + 5*a**4*sin(c + d*x)*cos(c + d*x)**3/(8*d) - a**3*b*cos(c + d*x)**4/
d + 3*a**2*b**2*x*sin(c + d*x)**4/4 + 3*a**2*b**2*x*sin(c + d*x)**2*cos(c +
 d*x)**2/2 + 3*a**2*b**2*x*cos(c + d*x)**4/4 + 3*a**2*b**2*sin(c + d*x)**3*
cos(c + d*x)/(4*d) - 3*a**2*b**2*sin(c + d*x)*cos(c + d*x)**3/(4*d) + a*b**
3*sin(c + d*x)**4/d + 3*b**4*x*sin(c + d*x)**4/8 + 3*b**4*x*sin(c + d*x)**2
*cos(c + d*x)**2/4 + 3*b**4*x*cos(c + d*x)**4/8 - 5*b**4*sin(c + d*x)**3*co
s(c + d*x)/(8*d) - 3*b**4*sin(c + d*x)*cos(c + d*x)**3/(8*d), Ne(d, 0)), (x
*(a*cos(c) + b*sin(c))**4, True))
```

3.80 $\int \sec(c+dx)(a\cos(c+dx)+b\sin(c+dx))^4\,dx$

Optimal. Leaf size=150

$$-\frac{a^4\sin^3(c+dx)}{3d}+\frac{a^4\sin(c+dx)}{d}-\frac{4a^3b\cos^3(c+dx)}{3d}+\frac{2a^2b^2\sin^3(c+dx)}{d}+\frac{4ab^3\cos^3(c+dx)}{3d}-\frac{4ab^3\cos(c+dx)}{d}-$$

```
[Out] b^4*arctanh(sin(d*x+c))/d-4*a*b^3*cos(d*x+c)/d-4/3*a^3*b*cos(d*x+c)^3/d+4/3
*a*b^3*cos(d*x+c)^3/d+a^4*sin(d*x+c)/d-b^4*sin(d*x+c)/d-1/3*a^4*sin(d*x+c)^
3/d+2*a^2*b^2*sin(d*x+c)^3/d-1/3*b^4*sin(d*x+c)^3/d
```

Rubi [A] time = 0.15, antiderivative size = 150, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 8, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {3090, 2633, 2565, 30, 2564, 2592, 302, 206}

$$\frac{2a^2b^2\sin^3(c+dx)}{d}-\frac{4a^3b\cos^3(c+dx)}{3d}-\frac{a^4\sin^3(c+dx)}{3d}+\frac{a^4\sin(c+dx)}{d}+\frac{4ab^3\cos^3(c+dx)}{3d}-\frac{4ab^3\cos(c+dx)}{d}-$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]*(a*Cos[c + d*x] + b*Sin[c + d*x])^4,x]
```

```
[Out] (b^4*ArcTanh[Sin[c + d*x]])/d - (4*a*b^3*Cos[c + d*x])/d - (4*a^3*b*Cos[c +
 d*x]^3)/(3*d) + (4*a*b^3*Cos[c + d*x]^3)/(3*d) + (a^4*Sin[c + d*x])/d - (b
^4*Sin[c + d*x])/d - (a^4*Sin[c + d*x]^3)/(3*d) + (2*a^2*b^2*Sin[c + d*x]^3
)/d - (b^4*Sin[c + d*x]^3)/(3*d)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 2564

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 2565

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> -Dist[(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 2592

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(
ff*x)^(m + n)/(a^2 - ff^2*x^2)^((n + 1)/2), x], x, (a*Sin[e + f*x])/ff], x]
] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 3090

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_.), x_Symbol] :> Int[ExpandTrig[cos[c + d*x]^m*(a
*cos[c + d*x] + b*sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d}, x] && Inte
gerQ[m] && IGtQ[n, 0]
```

Rubi steps

$$\int \sec(c+dx)(a\cos(c+dx)+b\sin(c+dx))^4\,dx = \int \left(a^4\cos^3(c+dx)+4a^3b\cos^2(c+dx)\sin(c+dx)+6a^2b^2\,\mathrm{co}\right.$$
$$= a^4\int \cos^3(c+dx)\,dx + \left(4a^3b\right)\int \cos^2(c+dx)\sin(c+dx)\,dx$$
$$= -\frac{a^4\,\mathrm{Subst}\left(\int\left(1-x^2\right)\,dx,x,-\sin(c+dx)\right)}{d} - \frac{\left(4a^3b\right)\mathrm{Subst}\left(\int\right.}{}$$
$$= -\frac{4ab^3\cos(c+dx)}{d} - \frac{4a^3b\cos^3(c+dx)}{3d} + \frac{4ab^3\cos^3(c+dx)}{3d} +$$
$$= -\frac{4ab^3\cos(c+dx)}{d} - \frac{4a^3b\cos^3(c+dx)}{3d} + \frac{4ab^3\cos^3(c+dx)}{3d} +$$
$$= \frac{b^4\tanh^{-1}(\sin(c+dx))}{d} - \frac{4ab^3\cos(c+dx)}{d} - \frac{4a^3b\cos^3(c+d}{3d}$$

Mathematica [A] time = 0.95, size = 181, normalized size = 1.21

$$\frac{9a^4\sin(c+dx)+a^4\sin(3(c+dx))+\left(4ab^3-4a^3b\right)\cos(3(c+dx))+18a^2b^2\sin(c+dx)-6a^2b^2\sin(3(c+dx))-}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]*(a*Cos[c + d*x] + b*Sin[c + d*x])^4,x]
```

```
[Out] (-12*a*b*(a^2 + 3*b^2)*Cos[c + d*x] + (-4*a^3*b + 4*a*b^3)*Cos[3*(c + d*x)]
 - 12*b^4*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + 12*b^4*Log[Cos[(c + d*
x)/2] + Sin[(c + d*x)/2]] + 9*a^4*Sin[c + d*x] + 18*a^2*b^2*Sin[c + d*x] -
15*b^4*Sin[c + d*x] + a^4*Sin[3*(c + d*x)] - 6*a^2*b^2*Sin[3*(c + d*x)] + b
^4*Sin[3*(c + d*x)])/(12*d)
```

fricas [A] time = 0.83, size = 121, normalized size = 0.81

$$-\frac{24\,ab^3\cos(dx+c)-3\,b^4\log(\sin(dx+c)+1)+3\,b^4\log(-\sin(dx+c)+1)+8\left(a^3b-ab^3\right)\cos(dx+c)^3-2\,(}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a*cos(d*x+c)+b*sin(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] -1/6*(24*a*b^3*cos(d*x + c) - 3*b^4*log(sin(d*x + c) + 1) + 3*b^4*log(-sin(
d*x + c) + 1) + 8*(a^3*b - a*b^3)*cos(d*x + c)^3 - 2*(2*a^4 + 6*a^2*b^2 - 4
*b^4 + (a^4 - 6*a^2*b^2 + b^4)*cos(d*x + c)^2)*sin(d*x + c))/d
```

giac [A] time = 0.40, size = 217, normalized size = 1.45

$$3\,b^4\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-3\,b^4\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)+\frac{2\left(3\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-3\,b^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-12\,a^3b\tan\left(\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a*cos(d*x+c)+b*sin(d*x+c))^4,x, algorithm="giac")
```

```
[Out] 1/3*(3*b^4*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 3*b^4*log(abs(tan(1/2*d*x +
 1/2*c) - 1)) + 2*(3*a^4*tan(1/2*d*x + 1/2*c)^5 - 3*b^4*tan(1/2*d*x + 1/2*c
)^5 - 12*a^3*b*tan(1/2*d*x + 1/2*c)^4 + 2*a^4*tan(1/2*d*x + 1/2*c)^3 + 24*a
^2*b^2*tan(1/2*d*x + 1/2*c)^3 - 10*b^4*tan(1/2*d*x + 1/2*c)^3 - 24*a*b^3*ta
n(1/2*d*x + 1/2*c)^2 + 3*a^4*tan(1/2*d*x + 1/2*c) - 3*b^4*tan(1/2*d*x + 1/2
*c) - 4*a^3*b - 8*a*b^3)/(tan(1/2*d*x + 1/2*c)^2 + 1)^3)/d
```

maple [A] time = 11.20, size = 163, normalized size = 1.09

$$\frac{\sin\left(dx+c\right)\left(\cos^2\left(dx+c\right)\right)a^4}{3d}+\frac{2a^4\sin\left(dx+c\right)}{3d}-\frac{4a^3b\left(\cos^3\left(dx+c\right)\right)}{3d}+\frac{2a^2b^2\left(\sin^3\left(dx+c\right)\right)}{d}-\frac{4\left(\sin^2\left(dx+c\right)\right)c}{3a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)*(a*cos(d*x+c)+b*sin(d*x+c))^4,x)
```

```
[Out] 1/3/d*sin(d*x+c)*cos(d*x+c)^2*a^4+2/3*a^4*sin(d*x+c)/d-4/3*a^3*b*cos(d*x+c)
^3/d+2*a^2*b^2*sin(d*x+c)^3/d-4/3/d*sin(d*x+c)^2*cos(d*x+c)*a*b^3-8/3*a*b^3
*cos(d*x+c)/d-1/3*b^4*sin(d*x+c)^3/d-b^4*sin(d*x+c)/d+1/d*b^4*ln(sec(d*x+c)
+tan(d*x+c))
```

maxima [A] time = 0.34, size = 126, normalized size = 0.84

$$-\frac{8\,a^3b\cos\left(dx+c\right)^3-12\,a^2b^2\sin\left(dx+c\right)^3+2\left(\sin\left(dx+c\right)^3-3\,\sin\left(dx+c\right)\right)a^4-8\left(\cos\left(dx+c\right)^3-3\,\cos\left(dx\right.\right.}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a*cos(d*x+c)+b*sin(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] -1/6*(8*a^3*b*cos(d*x + c)^3 - 12*a^2*b^2*sin(d*x + c)^3 + 2*(sin(d*x + c)^
3 - 3*sin(d*x + c))*a^4 - 8*(cos(d*x + c)^3 - 3*cos(d*x + c))*a*b^3 + (2*si
n(d*x + c)^3 - 3*log(sin(d*x + c) + 1) + 3*log(sin(d*x + c) - 1) + 6*sin(d*
x + c))*b^4)/d
```

mupad [B] time = 2.75, size = 190, normalized size = 1.27

$$\frac{2\,b^4\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{d}-\frac{\frac{16\,a\,b^3}{3}-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5\,\left(2\,a^4-2\,b^4\right)-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3\,\left(\frac{4\,a^4}{3}+16\,a^2\,b^2-\frac{20\,b^4}{3}\right)+\frac{8\,a^3\,b}{3}-}{d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6+3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4 \cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*cos(c + d*x) + b*sin(c + d*x))^4/cos(c + d*x),x)
```

```
[Out] (2*b^4*atanh(tan(c/2 + (d*x)/2)))/d - ((16*a*b^3)/3 - tan(c/2 + (d*x)/2)^5*
(2*a^4 - 2*b^4) - tan(c/2 + (d*x)/2)^3*((4*a^4)/3 - (20*b^4)/3 + 16*a^2*b^2
) + (8*a^3*b)/3 - tan(c/2 + (d*x)/2)*(2*a^4 - 2*b^4) + 16*a*b^3*tan(c/2 + (
d*x)/2)^2 + 8*a^3*b*tan(c/2 + (d*x)/2)^4)/(d*(3*tan(c/2 + (d*x)/2)^2 + 3*ta
n(c/2 + (d*x)/2)^4 + tan(c/2 + (d*x)/2)^6 + 1))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a\cos(c+dx)+b\sin(c+dx))^4\sec(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a*cos(d*x+c)+b*sin(d*x+c))**4,x)
```

```
[Out] Integral((a*cos(c + d*x) + b*sin(c + d*x))**4*sec(c + d*x), x)
```

3.81 $\int \sec^2(c+dx)(a\cos(c+dx)+b\sin(c+dx))^4\,dx$

Optimal. Leaf size=119

$$\frac{\sin^2(c+dx)\left(4ab\left(a^2-b^2\right)+\left(a^4-6a^2b^2+b^4\right)\cot(c+dx)\right)}{2d}+\frac{1}{2}x\left(a^4+6a^2b^2-3b^4\right)-\frac{4ab^3\log(\sin(c+dx))}{d}+\frac{4a$$

```
[Out] 1/2*(a^4+6*a^2*b^2-3*b^4)*x-4*a*b^3*ln(sin(d*x+c))/d+4*a*b^3*ln(tan(d*x+c))
/d+1/2*(4*a*b*(a^2-b^2)+(a^4-6*a^2*b^2+b^4)*cot(d*x+c))*sin(d*x+c)^2/d+b^4*
tan(d*x+c)/d
```

Rubi [A] time = 0.18, antiderivative size = 119, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {3088, 1805, 1802, 635, 203, 260}

$$\frac{\sin^2(c+dx)\left(\left(-6a^2b^2+a^4+b^4\right)\cot(c+dx)+4ab\left(a^2-b^2\right)\right)}{2d}+\frac{1}{2}x\left(6a^2b^2+a^4-3b^4\right)-\frac{4ab^3\log(\sin(c+dx))}{d}+\frac{4$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^2*(a*Cos[c + d*x] + b*Sin[c + d*x])^4,x]
```

```
[Out] ((a^4 + 6*a^2*b^2 - 3*b^4)*x)/2 - (4*a*b^3*Log[Sin[c + d*x]])/d + (4*a*b^3*
Log[Tan[c + d*x]])/d + ((4*a*b*(a^2 - b^2) + (a^4 - 6*a^2*b^2 + b^4)*Cot[c
+ d*x])*Sin[c + d*x]^2)/(2*d) + (b^4*Tan[c + d*x])/d
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] && !NiceSqrtQ[-(a*c)]
```

Rule 1802

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 1805

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[((a*g - b*f*x)*(a + b*x^2)^(p + 1))/(2*a
*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int[(c*x)^m*(a + b*x^2)^(p + 1)*Exp
andToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; Fr
eeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 3088

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[(x^m*(b +
 a*x)^n)/(1 + x^2)^((m + n + 2)/2), x], x, Cot[c + d*x]], x] /; FreeQ[{a, b
, c, d}, x] && IntegerQ[n] && IntegerQ[(m + n)/2] && NeQ[n, -1] &&  !(GtQ[n
, 0] && GtQ[m, 1])
```

Rubi steps

$$\int \sec^2(c+dx)(a\cos(c+dx)+b\sin(c+dx))^4\,dx = -\frac{\text{Subst}\left(\int \frac{(b+ax)^4}{x^2\left(1+x^2\right)^2}\,dx, x, \cot(c+dx)\right)}{d}$$

$$= \frac{\left(4ab\left(a^2-b^2\right)+\left(a^4-6a^2b^2+b^4\right)\cot(c+dx)\right)\sin^2(c+dx)}{2d}$$

$$= \frac{\left(4ab\left(a^2-b^2\right)+\left(a^4-6a^2b^2+b^4\right)\cot(c+dx)\right)\sin^2(c+dx)}{2d}$$

$$= \frac{4ab^3\log(\tan(c+dx))}{d} + \frac{\left(4ab\left(a^2-b^2\right)+\left(a^4-6a^2b^2+b^4\right)\text{c}\right.}{2d}$$

$$= \frac{4ab^3\log(\tan(c+dx))}{d} + \frac{\left(4ab\left(a^2-b^2\right)+\left(a^4-6a^2b^2+b^4\right)\text{c}\right.}{2d}$$

$$= \frac{1}{2}\left(a^4+6a^2b^2-3b^4\right)x - \frac{4ab^3\log(\sin(c+dx))}{d} + \frac{4ab^3\log(\text{t}}{}$$

Mathematica [B] time = 6.26, size = 477, normalized size = 4.01

$$\frac{b^3\left(\frac{\cos^2(c+dx)(a+b\tan(c+dx))^5\left(ab\tan(c+dx)+b^2\right)}{2b^4\left(a^2+b^2\right)}-\frac{\left(3b^2-5a^2\right)\left(b\left(6a^2-b^2\right)\tan(c+dx)+\frac{1}{2}\left(\frac{a^4-6a^2b^2+b^4}{\sqrt{-b^2}}+4a(a-b)(a+b)\right)\log\left(\sqrt{-b^2}-b\tan(c+dx)\right)+\frac{}{}\right.}{}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^2*(a*Cos[c + d*x] + b*Sin[c + d*x])^4,x]
```

```
[Out] (b^3*((Cos[c + d*x]^2*(a + b*Tan[c + d*x])^5*(b^2 + a*b*Tan[c + d*x]))/(2*b
^4*(a^2 + b^2)) - ((-5*a^2 + 3*b^2)*(((4*a*(a - b)*(a + b) + (a^4 - 6*a^2*b
^2 + b^4)/Sqrt[-b^2])*Log[Sqrt[-b^2] - b*Tan[c + d*x]])/2 + ((4*a*(a - b)*(
a + b) - (a^4 - 6*a^2*b^2 + b^4)/Sqrt[-b^2])*Log[Sqrt[-b^2] + b*Tan[c + d*x
]])/2 + b*(6*a^2 - b^2)*Tan[c + d*x] + 2*a*b^2*Tan[c + d*x]^2 + (b^3*Tan[c
+ d*x]^3)/3) + 4*a*(((5*a^4 - 10*a^2*b^2 + b^4 + (a^5 - 10*a^3*b^2 + 5*a*b^
4)/Sqrt[-b^2])*Log[Sqrt[-b^2] - b*Tan[c + d*x]])/2 + ((5*a^4 - 10*a^2*b^2 +
 b^4 - (a^5 - 10*a^3*b^2 + 5*a*b^4)/Sqrt[-b^2])*Log[Sqrt[-b^2] + b*Tan[c +
d*x]])/2 + 5*a*b*(2*a^2 - b^2)*Tan[c + d*x] + (b^2*(10*a^2 - b^2)*Tan[c + d
*x]^2)/2 + (5*a*b^3*Tan[c + d*x]^3)/3 + (b^4*Tan[c + d*x]^4)/4))/(2*b^2*(a^
2 + b^2))))/d
```

fricas [A] time = 0.83, size = 136, normalized size = 1.14

$$-\frac{8\,ab^3\cos\left(dx+c\right)\log\left(-\cos\left(dx+c\right)\right)+4\left(a^3b-ab^3\right)\cos\left(dx+c\right)^3-\left(2\,a^3b-2\,ab^3+\left(a^4+6\,a^2b^2-3\,b^4\right)dx\right)c}{2\,d\cos\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a*cos(d*x+c)+b*sin(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] -1/2*(8*a*b^3*cos(d*x + c)*log(-cos(d*x + c)) + 4*(a^3*b - a*b^3)*cos(d*x +
 c)^3 - (2*a^3*b - 2*a*b^3 + (a^4 + 6*a^2*b^2 - 3*b^4)*d*x)*cos(d*x + c) -
(2*b^4 + (a^4 - 6*a^2*b^2 + b^4)*cos(d*x + c)^2)*sin(d*x + c))/(d*cos(d*x +
 c))
```

giac [A] time = 3.52, size = 128, normalized size = 1.08

$$\frac{4\,ab^3\log\left(\tan\left(dx+c\right)^2+1\right)+2\,b^4\tan\left(dx+c\right)+\left(a^4+6\,a^2b^2-3\,b^4\right)\left(dx+c\right)-\frac{4\,ab^3\tan(dx+c)^2-a^4\tan(dx+c)+6\,a^2b^2\tan}{\tan(dx+c)^2+1}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a*cos(d*x+c)+b*sin(d*x+c))^4,x, algorithm="giac")
```

```
[Out] 1/2*(4*a*b^3*log(tan(d*x + c)^2 + 1) + 2*b^4*tan(d*x + c) + (a^4 + 6*a^2*b^
2 - 3*b^4)*(d*x + c) - (4*a*b^3*tan(d*x + c)^2 - a^4*tan(d*x + c) + 6*a^2*b
^2*tan(d*x + c) - b^4*tan(d*x + c) + 4*a^3*b)/(tan(d*x + c)^2 + 1))/d
```

maple [A] time = 52.30, size = 210, normalized size = 1.76

$$\frac{a^4\cos(dx+c)\sin(dx+c)}{2d}+\frac{a^4x}{2}+\frac{a^4c}{2d}-\frac{2\left(\cos^2(dx+c)\right)a^3b}{d}-\frac{3a^2b^2\cos(dx+c)\sin(dx+c)}{d}+3a^2b^2x+\frac{3a^2b^2c}{d}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2*(a*cos(d*x+c)+b*sin(d*x+c))^4,x)
```

```
[Out] 1/2*a^4*cos(d*x+c)*sin(d*x+c)/d+1/2*a^4*x+1/2/d*a^4*c-2/d*cos(d*x+c)^2*a^3*
b-3*a^2*b^2*cos(d*x+c)*sin(d*x+c)/d+3*a^2*b^2*x+3/d*a^2*b^2*c-2/d*a*b^3*sin
(d*x+c)^2-4*a*b^3*ln(cos(d*x+c))/d+1/d*b^4*sin(d*x+c)^5/cos(d*x+c)+1/d*b^4*
cos(d*x+c)*sin(d*x+c)^3+3/2*b^4*cos(d*x+c)*sin(d*x+c)/d-3/2*b^4*x-3/2/d*b^4
*c
```

maxima [A] time = 0.44, size = 135, normalized size = 1.13

$$\frac{8\,a^3b\sin(dx+c)^2+(2\,dx+2\,c+\sin(2\,dx+2\,c))a^4+6\,(2\,dx+2\,c-\sin(2\,dx+2\,c))a^2b^2-8\left(\sin(dx+c)^2+\log\right.}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a*cos(d*x+c)+b*sin(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] 1/4*(8*a^3*b*sin(d*x + c)^2 + (2*d*x + 2*c + sin(2*d*x + 2*c))*a^4 + 6*(2*d
*x + 2*c - sin(2*d*x + 2*c))*a^2*b^2 - 8*(sin(d*x + c)^2 + log(sin(d*x + c)
^2 - 1))*a*b^3 - 2*(3*d*x + 3*c - tan(d*x + c)/(tan(d*x + c)^2 + 1) - 2*tan
(d*x + c))*b^4)/d
```

mupad [B] time = 1.23, size = 255, normalized size = 2.14

$$\frac{a^4\,\mathrm{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)-3\,b^4\,\mathrm{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)+4\,a\,b^3\,\ln\left(\frac{1}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}\right)+6\,a^2\,b^2\,\mathrm{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)-4\,a\,b^3\,\ln\left(\frac{\cos(c+d}{\cos(c+d\,x}\right.}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*cos(c + d*x) + b*sin(c + d*x))^4/cos(c + d*x)^2,x)
```

```
[Out] (a^4*atan(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)) - 3*b^4*atan(sin(c/2 + (d*
x)/2)/cos(c/2 + (d*x)/2)) + 4*a*b^3*log(1/cos(c/2 + (d*x)/2)^2) + 6*a^2*b^2
```

```
*atan(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)) - 4*a*b^3*log(cos(c + d*x)/(co
s(c + d*x) + 1)))/d + ((a^4*sin(c + d*x))/8 + (9*b^4*sin(c + d*x))/8 + (a^4
*sin(3*c + 3*d*x))/8 + (b^4*sin(3*c + 3*d*x))/8 + (a*b^3*cos(3*c + 3*d*x))/
2 - (a^3*b*cos(3*c + 3*d*x))/2 - (3*a^2*b^2*sin(c + d*x))/4 - (3*a^2*b^2*si
n(3*c + 3*d*x))/4)/(d*cos(c + d*x))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2*(a*cos(d*x+c)+b*sin(d*x+c))**4,x)
```

```
[Out] Timed out
```

3.82 $\int \sec^3(c+dx)(a\cos(c+dx)+b\sin(c+dx))^4\,dx$

Optimal. Leaf size=151

$$\frac{a^4\sin(c+dx)}{d}-\frac{4a^3b\cos(c+dx)}{d}-\frac{6a^2b^2\sin(c+dx)}{d}+\frac{6a^2b^2\tanh^{-1}(\sin(c+dx))}{d}+\frac{4ab^3\cos(c+dx)}{d}+\frac{4ab^3\sec(c+}{d}$$

```
[Out] 6*a^2*b^2*arctanh(sin(d*x+c))/d-3/2*b^4*arctanh(sin(d*x+c))/d-4*a^3*b*cos(d
*x+c)/d+4*a*b^3*cos(d*x+c)/d+4*a*b^3*sec(d*x+c)/d+a^4*sin(d*x+c)/d-6*a^2*b^
2*sin(d*x+c)/d+3/2*b^4*sin(d*x+c)/d+1/2*b^4*sin(d*x+c)*tan(d*x+c)^2/d
```

Rubi [A] time = 0.16, antiderivative size = 151, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 9, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.321, Rules used = {3090, 2637, 2638, 2592, 321, 206, 2590, 14, 288}

$$-\frac{6a^2b^2\sin(c+dx)}{d}+\frac{6a^2b^2\tanh^{-1}(\sin(c+dx))}{d}-\frac{4a^3b\cos(c+dx)}{d}+\frac{a^4\sin(c+dx)}{d}+\frac{4ab^3\cos(c+dx)}{d}+\frac{4ab^3\sec(c}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^3*(a*Cos[c + d*x] + b*Sin[c + d*x])^4,x]
```

```
[Out] (6*a^2*b^2*ArcTanh[Sin[c + d*x]])/d - (3*b^4*ArcTanh[Sin[c + d*x]])/(2*d) -
 (4*a^3*b*Cos[c + d*x])/d + (4*a*b^3*Cos[c + d*x])/d + (4*a*b^3*Sec[c + d*x
])/d + (a^4*Sin[c + d*x])/d - (6*a^2*b^2*Sin[c + d*x])/d + (3*b^4*Sin[c + d
*x])/(2*d) + (b^4*Sin[c + d*x]*Tan[c + d*x]^2)/(2*d)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
```

```
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2590

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> -Dist[f^(-1), Subst[Int[(1 - x^2)^((m + n - 1)/2)/x^n, x], x, Cos[e + f*
x]], x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n - 1)/2]
```

Rule 2592

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(
ff*x)^(m + n)/(a^2 - ff^2*x^2)^((n + 1)/2), x], x, (a*Sin[e + f*x])/ff], x]
] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3090

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_.), x_Symbol] :> Int[ExpandTrig[cos[c + d*x]^m*(a
*cos[c + d*x] + b*sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d}, x] && Inte
gerQ[m] && IGtQ[n, 0]
```

Rubi steps

$$\int \sec^3(c+dx)(a\cos(c+dx)+b\sin(c+dx))^4\,dx = \int \left(a^4\cos(c+dx)+4a^3b\sin(c+dx)+6a^2b^2\sin(c+dx)\tan\right.$$
$$= a^4\int\cos(c+dx)\,dx+\left(4a^3b\right)\int\sin(c+dx)\,dx+\left(6a^2b^2\right)\int$$
$$= -\frac{4a^3b\cos(c+dx)}{d}+\frac{a^4\sin(c+dx)}{d}+\frac{\left(6a^2b^2\right)\text{Subst}\left(\int\frac{x^2}{1-x^2}\right.}{d}$$
$$= -\frac{4a^3b\cos(c+dx)}{d}+\frac{a^4\sin(c+dx)}{d}-\frac{6a^2b^2\sin(c+dx)}{d}+\frac{b}{}$$
$$= \frac{6a^2b^2\tanh^{-1}(\sin(c+dx))}{d}-\frac{4a^3b\cos(c+dx)}{d}+\frac{4ab^3\cos(c}{d}$$
$$= \frac{6a^2b^2\tanh^{-1}(\sin(c+dx))}{d}-\frac{3b^4\tanh^{-1}(\sin(c+dx))}{2d}-\frac{4a^3b}{}$$

Mathematica [A] time = 2.56, size = 268, normalized size = 1.77

$$\frac{4a^4\sin(c+dx)-24a^2b^2\sin(c+dx)-16ab\left(a^2-b^2\right)\cos(c+dx)-24a^2b^2\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^3*(a*Cos[c + d*x] + b*Sin[c + d*x])^4,x]
```

```
[Out] (16*a*b^3 - 16*a*b*(a^2 - b^2)*Cos[c + d*x] - 24*a^2*b^2*Log[Cos[(c + d*x)/
2] - Sin[(c + d*x)/2]] + 6*b^4*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + 2
4*a^2*b^2*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] - 6*b^4*Log[Cos[(c + d*x
)/2] + Sin[(c + d*x)/2]] + b^4/(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^2 + 32
*a*b^3*Sec[c + d*x]*Sin[(c + d*x)/2]^2 - b^4/(Cos[(c + d*x)/2] + Sin[(c + d
*x)/2])^2 + 4*a^4*Sin[c + d*x] - 24*a^2*b^2*Sin[c + d*x] + 4*b^4*Sin[c + d*
x])/(4*d)
```

fricas [A] time = 1.05, size = 153, normalized size = 1.01

$$\frac{16\,ab^3\cos(dx+c)-16\left(a^3b-ab^3\right)\cos(dx+c)^3+3\left(4\,a^2b^2-b^4\right)\cos(dx+c)^2\log(\sin(dx+c)+1)-3\left(4\,a^2b^2\right.}{4\,d\cos(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a*cos(d*x+c)+b*sin(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] 1/4*(16*a*b^3*cos(d*x + c) - 16*(a^3*b - a*b^3)*cos(d*x + c)^3 + 3*(4*a^2*b
^2 - b^4)*cos(d*x + c)^2*log(sin(d*x + c) + 1) - 3*(4*a^2*b^2 - b^4)*cos(d*
x + c)^2*log(-sin(d*x + c) + 1) + 2*(b^4 + 2*(a^4 - 6*a^2*b^2 + b^4)*cos(d*
x + c)^2)*sin(d*x + c))/(d*cos(d*x + c)^2)
```

giac [A] time = 0.40, size = 206, normalized size = 1.36

$$\frac{3\left(4\,a^2b^2-b^4\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-3\left(4\,a^2b^2-b^4\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)+\frac{4\left(a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-6\,a^2b^2\,\mathrm{t}\cdots\right.}{\cdots}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a*cos(d*x+c)+b*sin(d*x+c))^4,x, algorithm="giac")
```

```
[Out] 1/2*(3*(4*a^2*b^2 - b^4)*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 3*(4*a^2*b^2
- b^4)*log(abs(tan(1/2*d*x + 1/2*c) - 1)) + 4*(a^4*tan(1/2*d*x + 1/2*c) - 6
*a^2*b^2*tan(1/2*d*x + 1/2*c) + b^4*tan(1/2*d*x + 1/2*c) - 4*a^3*b + 4*a*b^
3)/(tan(1/2*d*x + 1/2*c)^2 + 1) + 2*(b^4*tan(1/2*d*x + 1/2*c)^3 - 8*a*b^3*t
an(1/2*d*x + 1/2*c)^2 + b^4*tan(1/2*d*x + 1/2*c) + 8*a*b^3)/(tan(1/2*d*x +
1/2*c)^2 - 1)^2)/d
```

maple [A] time = 29.43, size = 211, normalized size = 1.40

$$\frac{a^4\sin\left(dx+c\right)}{d}-\frac{4a^3b\cos\left(dx+c\right)}{d}+\frac{6a^2b^2\ln\left(\sec\left(dx+c\right)+\tan\left(dx+c\right)\right)}{d}-\frac{6a^2b^2\sin\left(dx+c\right)}{d}+\frac{4a\,b^3\left(\sin^4\left(dx+\cdots\right.\right.}{d\cos\left(dx+c\right)}\cdots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^3*(a*cos(d*x+c)+b*sin(d*x+c))^4,x)
```

```
[Out] a^4*sin(d*x+c)/d-4*a^3*b*cos(d*x+c)/d+6/d*a^2*b^2*ln(sec(d*x+c)+tan(d*x+c))
-6*a^2*b^2*sin(d*x+c)/d+4/d*a*b^3*sin(d*x+c)^4/cos(d*x+c)+4/d*sin(d*x+c)^2*
cos(d*x+c)*a*b^3+8*a*b^3*cos(d*x+c)/d+1/2/d*b^4*sin(d*x+c)^5/cos(d*x+c)^2+1
/2*b^4*sin(d*x+c)^3/d+3/2*b^4*sin(d*x+c)/d-3/2/d*b^4*ln(sec(d*x+c)+tan(d*x+
c))
```

maxima [A] time = 0.35, size = 142, normalized size = 0.94

$$-\frac{b^4\left(\frac{2\,\sin\left(dx+c\right)}{\sin\left(dx+c\right)^2-1}+3\,\log\left(\sin\left(dx+c\right)+1\right)-3\,\log\left(\sin\left(dx+c\right)-1\right)-4\,\sin\left(dx+c\right)\right)-16\,ab^3\left(\frac{1}{\cos\left(dx+c\right)}+\cos\left(d\cdots\right.\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a*cos(d*x+c)+b*sin(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] -1/4*(b^4*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) + 3*log(sin(d*x + c) + 1) -
3*log(sin(d*x + c) - 1) - 4*sin(d*x + c)) - 16*a*b^3*(1/cos(d*x + c) + cos(
d*x + c)) - 12*a^2*b^2*(log(sin(d*x + c) + 1) - log(sin(d*x + c) - 1) - 2*s
in(d*x + c)) + 16*a^3*b*cos(d*x + c) - 4*a^4*sin(d*x + c))/d
```

mupad [B] time = 2.96, size = 221, normalized size = 1.46

$$\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3\,\left(4\,a^4-24\,a^2\,b^2+2\,b^4\right)-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5\,\left(2\,a^4-12\,a^2\,b^2+3\,b^4\right)-16\,a\,b^3+8\,a^3\,b-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(2\right.}{d\,\left(-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4+\tan\left(\frac{c}{2}\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*cos(c + d*x) + b*sin(c + d*x))^4/cos(c + d*x)^3,x)
```

```
[Out] (tan(c/2 + (d*x)/2)^3*(4*a^4 + 2*b^4 - 24*a^2*b^2) - tan(c/2 + (d*x)/2)^5*(
2*a^4 + 3*b^4 - 12*a^2*b^2) - 16*a*b^3 + 8*a^3*b - tan(c/2 + (d*x)/2)*(2*a^
4 + 3*b^4 - 12*a^2*b^2) + tan(c/2 + (d*x)/2)^2*(16*a*b^3 - 16*a^3*b) + 8*a^
3*b*tan(c/2 + (d*x)/2)^4)/(d*(tan(c/2 + (d*x)/2)^2 + tan(c/2 + (d*x)/2)^4 -
 tan(c/2 + (d*x)/2)^6 - 1)) - (atanh(tan(c/2 + (d*x)/2))*(3*b^4 - 12*a^2*b^
2))/d
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**3*(a*cos(d*x+c)+b*sin(d*x+c))**4,x)
```

```
[Out] Timed out
```

3.83 $\int \sec^4(c+dx)(a\cos(c+dx)+b\sin(c+dx))^4\,dx$

Optimal. Leaf size=103

$$\frac{b^2\left(3a^2-b^2\right)\tan(c+dx)}{d}-\frac{4ab\left(a^2-b^2\right)\log(\cos(c+dx))}{d}+x\left(a^4-6a^2b^2+b^4\right)+\frac{b(a+b\tan(c+dx))^3}{3d}+\frac{ab(a+b$$

```
[Out] (a^4-6*a^2*b^2+b^4)*x-4*a*b*(a^2-b^2)*ln(cos(d*x+c))/d+b^2*(3*a^2-b^2)*tan(
d*x+c)/d+a*b*(a+b*tan(d*x+c))^2/d+1/3*b*(a+b*tan(d*x+c))^3/d
```

Rubi [A] time = 0.16, antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.179, Rules used = {3086, 3482, 3528, 3525, 3475}

$$\frac{b^2\left(3a^2-b^2\right)\tan(c+dx)}{d}-\frac{4ab\left(a^2-b^2\right)\log(\cos(c+dx))}{d}+x\left(-6a^2b^2+a^4+b^4\right)+\frac{b(a+b\tan(c+dx))^3}{3d}+\frac{ab(a+$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^4*(a*Cos[c + d*x] + b*Sin[c + d*x])^4,x]
```

```
[Out] (a^4 - 6*a^2*b^2 + b^4)*x - (4*a*b*(a^2 - b^2)*Log[Cos[c + d*x]])/d + (b^2*
(3*a^2 - b^2)*Tan[c + d*x])/d + (a*b*(a + b*Tan[c + d*x])^2)/d + (b*(a + b*
Tan[c + d*x])^3)/(3*d)
```

Rule 3086

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin
[(c_.) + (d_.)*(x_)])^(n_.), x_Symbol] :> Int[(a + b*Tan[c + d*x])^n, x] /;
 FreeQ[{a, b, c, d}, x] && EqQ[m + n, 0] && IntegerQ[n] && NeQ[a^2 + b^2, 0
]
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3482

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(a +
b*Tan[c + d*x])^(n - 1))/(d*(n - 1)), x] + Int[(a^2 - b^2 + 2*a*b*Tan[c + d
*x])*(a + b*Tan[c + d*x])^(n - 2), x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2
+ b^2, 0] && GtQ[n, 1]
```

Rule 3525

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[(a*c - b*d)*x, x] + (Dist[b*c + a*d, Int[Tan[e +
 f*x], x], x] + Simp[(b*d*Tan[e + f*x])/f, x]) /; FreeQ[{a, b, c, d, e, f},
 x] && NeQ[b*c - a*d, 0] && NeQ[b*c + a*d, 0]
```

Rule 3528

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[(d*(a + b*Tan[e + f*x])^m)/(f*m), x] + Int
[(a + b*Tan[e + f*x])^(m - 1)*Simp[a*c - b*d + (b*c + a*d)*Tan[e + f*x], x]
, x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2,
 0] && GtQ[m, 0]
```

Rubi steps

$$\int \sec^4(c+dx)(a\cos(c+dx)+b\sin(c+dx))^4\,dx = \int (a+b\tan(c+dx))^4\,dx$$
$$= \frac{b(a+b\tan(c+dx))^3}{3d} + \int (a+b\tan(c+dx))^2\left(a^2-b^2+2ab\right.$$
$$= \frac{ab(a+b\tan(c+dx))^2}{d} + \frac{b(a+b\tan(c+dx))^3}{3d} + \int (a+b\,\mathrm{ta}$$
$$= \left(a^4-6a^2b^2+b^4\right)x + \frac{b^2\left(3a^2-b^2\right)\tan(c+dx)}{d} + \frac{ab(a+b\,\mathrm{ta}}{}$$
$$= \left(a^4-6a^2b^2+b^4\right)x - \frac{4ab\left(a^2-b^2\right)\log(\cos(c+dx))}{d} + \frac{b^2\left(3a\right.}{}$$

Mathematica [C] time = 0.40, size = 105, normalized size = 1.02

$$\frac{-6b^2\left(b^2-6a^2\right)\tan(c+dx)+12ab^3\tan^2(c+dx)+3i(a-ib)^4\log(\tan(c+dx)+i)-3i(a+ib)^4\log(-\tan(c+dx}{6d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^4*(a*Cos[c + d*x] + b*Sin[c + d*x])^4,x]
```

```
[Out] ((-3*I)*(a + I*b)^4*Log[I - Tan[c + d*x]] + (3*I)*(a - I*b)^4*Log[I + Tan[c
 + d*x]] - 6*b^2*(-6*a^2 + b^2)*Tan[c + d*x] + 12*a*b^3*Tan[c + d*x]^2 + 2*
b^4*Tan[c + d*x]^3)/(6*d)
```

fricas [A] time = 0.58, size = 119, normalized size = 1.16

$$\frac{3\left(a^4-6\,a^2b^2+b^4\right)dx\cos\left(dx+c\right)^3+6\,ab^3\cos\left(dx+c\right)-12\left(a^3b-ab^3\right)\cos\left(dx+c\right)^3\log\left(-\cos\left(dx+c\right)\right)+\left(b^4+\right.}{3\,d\cos\left(dx+c\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(a*cos(d*x+c)+b*sin(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] 1/3*(3*(a^4 - 6*a^2*b^2 + b^4)*d*x*cos(d*x + c)^3 + 6*a*b^3*cos(d*x + c) -
12*(a^3*b - a*b^3)*cos(d*x + c)^3*log(-cos(d*x + c)) + (b^4 + 2*(9*a^2*b^2
- 2*b^4)*cos(d*x + c)^2)*sin(d*x + c))/(d*cos(d*x + c)^3)
```

giac [A] time = 0.38, size = 104, normalized size = 1.01

$$\frac{b^4 \tan(dx+c)^3 + 6\,ab^3 \tan(dx+c)^2 + 18\,a^2b^2 \tan(dx+c) - 3\,b^4 \tan(dx+c) + 3\left(a^4 - 6\,a^2b^2 + b^4\right)(dx+c) + 6}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(a*cos(d*x+c)+b*sin(d*x+c))^4,x, algorithm="giac")
```

```
[Out] 1/3*(b^4*tan(d*x + c)^3 + 6*a*b^3*tan(d*x + c)^2 + 18*a^2*b^2*tan(d*x + c)
- 3*b^4*tan(d*x + c) + 3*(a^4 - 6*a^2*b^2 + b^4)*(d*x + c) + 6*(a^3*b - a*b
^3)*log(tan(d*x + c)^2 + 1))/d
```

maple [A] time = 1.92, size = 145, normalized size = 1.41

$$a^4x+\frac{a^4c}{d}-\frac{4a^3b\ln(\cos(dx+c))}{d}-6a^2b^2x+\frac{6a^2b^2\tan(dx+c)}{d}-\frac{6a^2b^2c}{d}+\frac{2a\,b^3\left(\tan^2(dx+c)\right)}{d}+\frac{4a\,b^3\ln(\cos(dx+}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^4*(a*cos(d*x+c)+b*sin(d*x+c))^4,x)
```

```
[Out] a^4*x+1/d*a^4*c-4/d*a^3*b*ln(cos(d*x+c))-6*a^2*b^2*x+6*a^2*b^2*tan(d*x+c)/d
-6/d*a^2*b^2*c+2*a*b^3*tan(d*x+c)^2/d+4*a*b^3*ln(cos(d*x+c))/d+1/3*b^4*tan(
d*x+c)^3/d-b^4*tan(d*x+c)/d+b^4*x+1/d*b^4*c
```

maxima [A] time = 0.42, size = 116, normalized size = 1.13

$$\frac{3\,(dx+c)a^4 - 18\,(dx+c-\tan(dx+c))a^2b^2 + \left(\tan(dx+c)^3 + 3\,dx + 3\,c - 3\,\tan(dx+c)\right)b^4 - 6\,ab^3\left(\frac{1}{\sin(dx+c)^2-}\right.}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(a*cos(d*x+c)+b*sin(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] 1/3*(3*(d*x + c)*a^4 - 18*(d*x + c - tan(d*x + c))*a^2*b^2 + (tan(d*x + c)^
3 + 3*d*x + 3*c - 3*tan(d*x + c))*b^4 - 6*a*b^3*(1/(sin(d*x + c)^2 - 1) - l
og(sin(d*x + c)^2 - 1)) - 6*a^3*b*log(-sin(d*x + c)^2 + 1))/d
```

mupad [B] time = 1.79, size = 546, normalized size = 5.30

$$\frac{\frac{3\,a^4\,\cos(c+d\,x)\,\mathrm{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{2}-\frac{b^4\,\sin(3\,c+3\,d\,x)}{3}+\frac{3\,b^4\,\cos(c+d\,x)\,\mathrm{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{2}-\frac{a\,b^3\,\cos(3\,c+3\,d\,x)}{2}+\frac{3\,a^2\,b^2\,\sin(c+d\,x)}{2}+\frac{a^4\,\mathrm{atan}\left(\cdots\right.}{\cdots}}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*cos(c + d*x) + b*sin(c + d*x))^4/cos(c + d*x)^4,x)
```

```
[Out] ((3*a^4*cos(c + d*x)*atan(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/2 - (b^4*
sin(3*c + 3*d*x))/3 + (3*b^4*cos(c + d*x)*atan(sin(c/2 + (d*x)/2)/cos(c/2 +
 (d*x)/2)))/2 - (a*b^3*cos(3*c + 3*d*x))/2 + (3*a^2*b^2*sin(c + d*x))/2 + (
a^4*atan(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2))*cos(3*c + 3*d*x))/2 + (b^4*
atan(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2))*cos(3*c + 3*d*x))/2 + (3*a^2*b^
2*sin(3*c + 3*d*x))/2 + (a*b^3*cos(c + d*x))/2 + 3*a*b^3*log(-cos(c + d*x)/
cos(c/2 + (d*x)/2)^2)*cos(c + d*x) - 3*a^3*b*log(-cos(c + d*x)/cos(c/2 + (d
*x)/2)^2)*cos(c + d*x) - 3*a^2*b^2*atan(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/
2))*cos(3*c + 3*d*x) - 3*a*b^3*cos(c + d*x)*log(1/cos(c/2 + (d*x)/2)^2) + 3
*a^3*b*cos(c + d*x)*log(1/cos(c/2 + (d*x)/2)^2) + a*b^3*log(-cos(c + d*x)/c
os(c/2 + (d*x)/2)^2)*cos(3*c + 3*d*x) - a^3*b*log(-cos(c + d*x)/cos(c/2 + (
d*x)/2)^2)*cos(3*c + 3*d*x) - a*b^3*log(1/cos(c/2 + (d*x)/2)^2)*cos(3*c + 3
*d*x) + a^3*b*log(1/cos(c/2 + (d*x)/2)^2)*cos(3*c + 3*d*x) - 9*a^2*b^2*cos(
c + d*x)*atan(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/(d*((3*cos(c + d*x))/
4 + cos(3*c + 3*d*x)/4))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**4*(a*cos(d*x+c)+b*sin(d*x+c))**4,x)
```

```
[Out] Timed out
```

3.84 $\int \sec^5(c+dx)(a\cos(c+dx)+b\sin(c+dx))^4\,dx$

Optimal. Leaf size=168

$$\frac{a^4\tanh^{-1}(\sin(c+dx))}{d}+\frac{4a^3b\sec(c+dx)}{d}-\frac{3a^2b^2\tanh^{-1}(\sin(c+dx))}{d}+\frac{3a^2b^2\tan(c+dx)\sec(c+dx)}{d}+\frac{4ab^3\sec}{3}$$

```
[Out] a^4*arctanh(sin(d*x+c))/d-3*a^2*b^2*arctanh(sin(d*x+c))/d+3/8*b^4*arctanh(s
in(d*x+c))/d+4*a^3*b*sec(d*x+c)/d-4*a*b^3*sec(d*x+c)/d+4/3*a*b^3*sec(d*x+c)
^3/d+3*a^2*b^2*sec(d*x+c)*tan(d*x+c)/d-3/8*b^4*sec(d*x+c)*tan(d*x+c)/d+1/4*
b^4*sec(d*x+c)*tan(d*x+c)^3/d
```

Rubi [A] time = 0.19, antiderivative size = 168, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 5, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.179, Rules used = {3090, 3770, 2606, 8, 2611}

$$-\frac{3a^2b^2\tanh^{-1}(\sin(c+dx))}{d}+\frac{3a^2b^2\tan(c+dx)\sec(c+dx)}{d}+\frac{4a^3b\sec(c+dx)}{d}+\frac{a^4\tanh^{-1}(\sin(c+dx))}{d}+\frac{4ab^3\text{se}}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^5*(a*Cos[c + d*x] + b*Sin[c + d*x])^4,x]
```

```
[Out] (a^4*ArcTanh[Sin[c + d*x]])/d - (3*a^2*b^2*ArcTanh[Sin[c + d*x]])/d + (3*b^
4*ArcTanh[Sin[c + d*x]])/(8*d) + (4*a^3*b*Sec[c + d*x])/d - (4*a*b^3*Sec[c
+ d*x])/d + (4*a*b^3*Sec[c + d*x]^3)/(3*d) + (3*a^2*b^2*Sec[c + d*x]*Tan[c
+ d*x])/d - (3*b^4*Sec[c + d*x]*Tan[c + d*x])/(8*d) + (b^4*Sec[c + d*x]*Tan
[c + d*x]^3)/(4*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 2611

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(b*(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*(
m + n - 1)), x] - Dist[(b^2*(n - 1))/(m + n - 1), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
```

```
NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 3090

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_.), x_Symbol] :> Int[ExpandTrig[cos[c + d*x]^m*(a
*cos[c + d*x] + b*sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d}, x] && Inte
gerQ[m] && IGtQ[n, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \sec^5(c+dx)(a\cos(c+dx)+b\sin(c+dx))^4\,dx = \int \left(a^4\sec(c+dx)+4a^3b\sec(c+dx)\tan(c+dx)+6a^2b^2\sec\right.$$

$$= a^4\int \sec(c+dx)\,dx+\left(4a^3b\right)\int \sec(c+dx)\tan(c+dx)\,dx+$$

$$= \frac{a^4\tanh^{-1}(\sin(c+dx))}{d}+\frac{3a^2b^2\sec(c+dx)\tan(c+dx)}{d}+\frac{b^4}{}$$

$$= \frac{a^4\tanh^{-1}(\sin(c+dx))}{d}-\frac{3a^2b^2\tanh^{-1}(\sin(c+dx))}{d}+\frac{4a^3b}{}$$

$$= \frac{a^4\tanh^{-1}(\sin(c+dx))}{d}-\frac{3a^2b^2\tanh^{-1}(\sin(c+dx))}{d}+\frac{3b^4\,\mathrm{ta}}{}$$

Mathematica [B] time = 6.23, size = 936, normalized size = 5.57

$$\frac{2ab\left(6a^2-5b^2\right)\cos^4(c+dx)(a+b\tan(c+dx))^4}{3d(a\cos(c+dx)+b\sin(c+dx))^4}+\frac{\left(-8a^4+24b^2a^2-3b^4\right)\cos^4(c+dx)\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\right.}{8d(a\cos(c+dx)+b\sin(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^5*(a*Cos[c + d*x] + b*Sin[c + d*x])^4,x]
```

```
[Out] (2*a*b*(6*a^2 - 5*b^2)*Cos[c + d*x]^4*(a + b*Tan[c + d*x])^4)/(3*d*(a*Cos[c
 + d*x] + b*Sin[c + d*x])^4) + ((-8*a^4 + 24*a^2*b^2 - 3*b^4)*Cos[c + d*x]^
4*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]]*(a + b*Tan[c + d*x])^4)/(8*d*(a*
Cos[c + d*x] + b*Sin[c + d*x])^4) + ((8*a^4 - 24*a^2*b^2 + 3*b^4)*Cos[c + d
```

```
*x]^4*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]*(a + b*Tan[c + d*x])^4)/(8*d
*(a*Cos[c + d*x] + b*Sin[c + d*x])^4) + (b^4*Cos[c + d*x]^4*(a + b*Tan[c +
d*x])^4)/(16*d*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^4*(a*Cos[c + d*x] + b*
Sin[c + d*x])^4) + ((72*a^2*b^2 + 16*a*b^3 - 15*b^4)*Cos[c + d*x]^4*(a + b*
Tan[c + d*x])^4)/(48*d*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^2*(a*Cos[c + d
*x] + b*Sin[c + d*x])^4) + (2*a*b^3*Cos[c + d*x]^4*Sin[(c + d*x)/2]*(a + b*
Tan[c + d*x])^4)/(3*d*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^3*(a*Cos[c + d*
x] + b*Sin[c + d*x])^4) - (b^4*Cos[c + d*x]^4*(a + b*Tan[c + d*x])^4)/(16*d
*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^4*(a*Cos[c + d*x] + b*Sin[c + d*x])^
4) - (2*a*b^3*Cos[c + d*x]^4*Sin[(c + d*x)/2]*(a + b*Tan[c + d*x])^4)/(3*d*
(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^3*(a*Cos[c + d*x] + b*Sin[c + d*x])^4
) + ((-72*a^2*b^2 + 16*a*b^3 + 15*b^4)*Cos[c + d*x]^4*(a + b*Tan[c + d*x])^
4)/(48*d*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^2*(a*Cos[c + d*x] + b*Sin[c
+ d*x])^4) + (2*Cos[c + d*x]^4*(6*a^3*b*Sin[(c + d*x)/2] - 5*a*b^3*Sin[(c +
 d*x)/2])*(a + b*Tan[c + d*x])^4)/(3*d*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2]
)*(a*Cos[c + d*x] + b*Sin[c + d*x])^4) - (2*Cos[c + d*x]^4*(6*a^3*b*Sin[(c
+ d*x)/2] - 5*a*b^3*Sin[(c + d*x)/2])*(a + b*Tan[c + d*x])^4)/(3*d*(Cos[(c
+ d*x)/2] + Sin[(c + d*x)/2])*(a*Cos[c + d*x] + b*Sin[c + d*x])^4)
```

fricas [A] time = 0.76, size = 163, normalized size = 0.97

$$\frac{3\left(8\,a^4-24\,a^2b^2+3\,b^4\right)\cos\left(dx+c\right)^4\log\left(\sin\left(dx+c\right)+1\right)-3\left(8\,a^4-24\,a^2b^2+3\,b^4\right)\cos\left(dx+c\right)^4\log\left(-\sin\left(\right.\right.}{48\,d\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*(a*cos(d*x+c)+b*sin(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] 1/48*(3*(8*a^4 - 24*a^2*b^2 + 3*b^4)*cos(d*x + c)^4*log(sin(d*x + c) + 1) -
 3*(8*a^4 - 24*a^2*b^2 + 3*b^4)*cos(d*x + c)^4*log(-sin(d*x + c) + 1) + 64*
a*b^3*cos(d*x + c) + 192*(a^3*b - a*b^3)*cos(d*x + c)^3 + 6*(2*b^4 + (24*a^
2*b^2 - 5*b^4)*cos(d*x + c)^2)*sin(d*x + c))/(d*cos(d*x + c)^4)
```

giac [B] time = 0.48, size = 325, normalized size = 1.93

$$\frac{3\left(8\,a^4-24\,a^2b^2+3\,b^4\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-3\left(8\,a^4-24\,a^2b^2+3\,b^4\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)+\frac{2(}{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*(a*cos(d*x+c)+b*sin(d*x+c))^4,x, algorithm="giac")
```

```
[Out] 1/24*(3*(8*a^4 - 24*a^2*b^2 + 3*b^4)*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 3
*(8*a^4 - 24*a^2*b^2 + 3*b^4)*log(abs(tan(1/2*d*x + 1/2*c) - 1)) + 2*(72*a^
```

```
2*b^2*tan(1/2*d*x + 1/2*c)^7 - 9*b^4*tan(1/2*d*x + 1/2*c)^7 - 96*a^3*b*tan(
1/2*d*x + 1/2*c)^6 - 72*a^2*b^2*tan(1/2*d*x + 1/2*c)^5 + 33*b^4*tan(1/2*d*x
 + 1/2*c)^5 + 288*a^3*b*tan(1/2*d*x + 1/2*c)^4 - 192*a*b^3*tan(1/2*d*x + 1/
2*c)^4 - 72*a^2*b^2*tan(1/2*d*x + 1/2*c)^3 + 33*b^4*tan(1/2*d*x + 1/2*c)^3
- 288*a^3*b*tan(1/2*d*x + 1/2*c)^2 + 256*a*b^3*tan(1/2*d*x + 1/2*c)^2 + 72*
a^2*b^2*tan(1/2*d*x + 1/2*c) - 9*b^4*tan(1/2*d*x + 1/2*c) + 96*a^3*b - 64*a
*b^3)/(tan(1/2*d*x + 1/2*c)^2 - 1)^4)/d
```

maple [A] time = 37.63, size = 297, normalized size = 1.77

$$\frac{a^4 \ln\left(\sec\left(dx+c\right)+\tan\left(dx+c\right)\right)}{d}+\frac{4a^3b}{d\cos\left(dx+c\right)}+\frac{3a^2b^2\left(\sin^3\left(dx+c\right)\right)}{d\cos\left(dx+c\right)^2}+\frac{3a^2b^2\sin\left(dx+c\right)}{d}-\frac{3a^2b^2\ln\left(\sec\left(dx+\right.\right.}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^5*(a*cos(d*x+c)+b*sin(d*x+c))^4,x)
```

```
[Out] 1/d*a^4*ln(sec(d*x+c)+tan(d*x+c))+4/d*a^3*b/cos(d*x+c)+3/d*a^2*b^2*sin(d*x+
c)^3/cos(d*x+c)^2+3*a^2*b^2*sin(d*x+c)/d-3/d*a^2*b^2*ln(sec(d*x+c)+tan(d*x+
c))+4/3/d*a*b^3*sin(d*x+c)^4/cos(d*x+c)^3-4/3/d*a*b^3*sin(d*x+c)^4/cos(d*x+
c)-4/3/d*sin(d*x+c)^2*cos(d*x+c)*a*b^3-8/3*a*b^3*cos(d*x+c)/d+1/4/d*b^4*sin
(d*x+c)^5/cos(d*x+c)^4-1/8/d*b^4*sin(d*x+c)^5/cos(d*x+c)^2-1/8*b^4*sin(d*x+
c)^3/d-3/8*b^4*sin(d*x+c)/d+3/8/d*b^4*ln(sec(d*x+c)+tan(d*x+c))
```

maxima [A] time = 0.33, size = 192, normalized size = 1.14

$$\frac{3\,b^4\left(\frac{2\left(5\,\sin(dx+c)^3-3\,\sin(dx+c)\right)}{\sin(dx+c)^4-2\,\sin(dx+c)^2+1}+3\,\log\left(\sin\left(dx+c\right)+1\right)-3\,\log\left(\sin\left(dx+c\right)-1\right)\right)-72\,a^2b^2\left(\frac{2\,\sin(dx+c)}{\sin(dx+c)^2-1}+\log\left(\sin\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*(a*cos(d*x+c)+b*sin(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] 1/48*(3*b^4*(2*(5*sin(d*x + c)^3 - 3*sin(d*x + c))/(sin(d*x + c)^4 - 2*sin(
d*x + c)^2 + 1) + 3*log(sin(d*x + c) + 1) - 3*log(sin(d*x + c) - 1)) - 72*a
^2*b^2*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) + log(sin(d*x + c) + 1) - log(s
in(d*x + c) - 1)) + 24*a^4*(log(sin(d*x + c) + 1) - log(sin(d*x + c) - 1))
+ 192*a^3*b/cos(d*x + c) - 64*(3*cos(d*x + c)^2 - 1)*a*b^3/cos(d*x + c)^3)/
d
```

mupad [B] time = 4.21, size = 278, normalized size = 1.65

$$\frac{\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)\,\left(2\,a^4-6\,a^2\,b^2+\frac{3\,b^4}{4}\right)}{d}-\frac{\frac{16\,a\,b^3}{3}-8\,a^3\,b+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(\frac{3\,b^4}{4}-6\,a^2\,b^2\right)+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7\,\left(\frac{3\,b^4}{4}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*cos(c + d*x) + b*sin(c + d*x))^4/cos(c + d*x)^5,x)
```

```
[Out] (atanh(tan(c/2 + (d*x)/2))*(2*a^4 + (3*b^4)/4 - 6*a^2*b^2))/d - ((16*a*b^3)
/3 - 8*a^3*b + tan(c/2 + (d*x)/2)*((3*b^4)/4 - 6*a^2*b^2) + tan(c/2 + (d*x)
/2)^7*((3*b^4)/4 - 6*a^2*b^2) - tan(c/2 + (d*x)/2)^3*((11*b^4)/4 - 6*a^2*b^
2) - tan(c/2 + (d*x)/2)^5*((11*b^4)/4 - 6*a^2*b^2) + tan(c/2 + (d*x)/2)^4*(
16*a*b^3 - 24*a^3*b) - tan(c/2 + (d*x)/2)^2*((64*a*b^3)/3 - 24*a^3*b) + 8*a
^3*b*tan(c/2 + (d*x)/2)^6)/(d*(6*tan(c/2 + (d*x)/2)^4 - 4*tan(c/2 + (d*x)/2
)^2 - 4*tan(c/2 + (d*x)/2)^6 + tan(c/2 + (d*x)/2)^8 + 1))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**5*(a*cos(d*x+c)+b*sin(d*x+c))**4,x)
```

```
[Out] Timed out
```

3.85 $\int \sec^6(c+dx)(a\cos(c+dx)+b\sin(c+dx))^4\,dx$

Optimal. Leaf size=30

$$\frac{\tan^5(c+dx)(a\cot(c+dx)+b)^5}{5bd}$$

```
[Out] 1/5*(b+a*cot(d*x+c))^5*tan(d*x+c)^5/b/d
```

Rubi [A] time = 0.05, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {3088, 37}

$$\frac{\tan^5(c+dx)(a\cot(c+dx)+b)^5}{5bd}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^6*(a*Cos[c + d*x] + b*Sin[c + d*x])^4,x]
```

```
[Out] ((b + a*Cot[c + d*x])^5*Tan[c + d*x]^5)/(5*b*d)
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 3088

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[(x^m*(b +
 a*x)^n)/(1 + x^2)^((m + n + 2)/2), x], x, Cot[c + d*x]], x] /; FreeQ[{a, b
, c, d}, x] && IntegerQ[n] && IntegerQ[(m + n)/2] && NeQ[n, -1] &&  !(GtQ[n
, 0] && GtQ[m, 1])
```

Rubi steps

$$\begin{aligned}\int \sec^6(c+dx)(a\cos(c+dx)+b\sin(c+dx))^4\,dx &= -\frac{\text{Subst}\left(\int \frac{(b+ax)^4}{x^6}\,dx, x, \cot(c+dx)\right)}{d}\\ &= \frac{(b+a\cot(c+dx))^5\tan^5(c+dx)}{5bd}\end{aligned}$$

Mathematica [B] time = 0.32, size = 73, normalized size = 2.43

$$\frac{\tan(c+dx)\left(5a^4+10a^3b\tan(c+dx)+10a^2b^2\tan^2(c+dx)+5ab^3\tan^3(c+dx)+b^4\tan^4(c+dx)\right)}{5d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^6*(a*Cos[c + d*x] + b*Sin[c + d*x])^4,x]
```

```
[Out] (Tan[c + d*x]*(5*a^4 + 10*a^3*b*Tan[c + d*x] + 10*a^2*b^2*Tan[c + d*x]^2 +
5*a*b^3*Tan[c + d*x]^3 + b^4*Tan[c + d*x]^4))/(5*d)
```

fricas [B] time = 0.53, size = 109, normalized size = 3.63

$$\frac{5\,ab^3\cos\left(dx+c\right)+10\left(a^3b-ab^3\right)\cos\left(dx+c\right)^3+\left(\left(5\,a^4-10\,a^2b^2+b^4\right)\cos\left(dx+c\right)^4+b^4+2\left(5\,a^2b^2-b^4\right)\cos\right.}{5\,d\cos\left(dx+c\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^6*(a*cos(d*x+c)+b*sin(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] 1/5*(5*a*b^3*cos(d*x + c) + 10*(a^3*b - a*b^3)*cos(d*x + c)^3 + ((5*a^4 - 1
0*a^2*b^2 + b^4)*cos(d*x + c)^4 + b^4 + 2*(5*a^2*b^2 - b^4)*cos(d*x + c)^2)
*sin(d*x + c))/(d*cos(d*x + c)^5)
```

giac [B] time = 0.66, size = 73, normalized size = 2.43

$$\frac{b^4\tan\left(dx+c\right)^5+5\,ab^3\tan\left(dx+c\right)^4+10\,a^2b^2\tan\left(dx+c\right)^3+10\,a^3b\tan\left(dx+c\right)^2+5\,a^4\tan\left(dx+c\right)}{5\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^6*(a*cos(d*x+c)+b*sin(d*x+c))^4,x, algorithm="giac")
```

```
[Out] 1/5*(b^4*tan(d*x + c)^5 + 5*a*b^3*tan(d*x + c)^4 + 10*a^2*b^2*tan(d*x + c)^
3 + 10*a^3*b*tan(d*x + c)^2 + 5*a^4*tan(d*x + c))/d
```

maple [B] time = 27.47, size = 96, normalized size = 3.20

$$\frac{a^4\tan\left(dx+c\right)+\frac{2a^3b}{\cos(dx+c)^2}+\frac{2a^2b^2\left(\sin^3(dx+c)\right)}{\cos(dx+c)^3}+\frac{a\,b^3\left(\sin^4(dx+c)\right)}{\cos(dx+c)^4}+\frac{b^4\left(\sin^5(dx+c)\right)}{5\cos(dx+c)^5}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^6*(a*cos(d*x+c)+b*sin(d*x+c))^4,x)
```

```
[Out] 1/d*(a^4*tan(d*x+c)+2*a^3*b/cos(d*x+c)^2+2*a^2*b^2*sin(d*x+c)^3/cos(d*x+c)^
3+a*b^3*sin(d*x+c)^4/cos(d*x+c)^4+1/5*b^4*sin(d*x+c)^5/cos(d*x+c)^5)
```

maxima [B] time = 0.33, size = 103, normalized size = 3.43

$$\frac{b^4 \tan(dx+c)^5 + 10\,a^2 b^2 \tan(dx+c)^3 + 5\,a^4 \tan(dx+c) + \frac{5\left(2\,\sin(dx+c)^2-1\right)ab^3}{\sin(dx+c)^4-2\,\sin(dx+c)^2+1} - \frac{10\,a^3 b}{\sin(dx+c)^2-1}}{5\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^6*(a*cos(d*x+c)+b*sin(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] 1/5*(b^4*tan(d*x + c)^5 + 10*a^2*b^2*tan(d*x + c)^3 + 5*a^4*tan(d*x + c) +
5*(2*sin(d*x + c)^2 - 1)*a*b^3/(sin(d*x + c)^4 - 2*sin(d*x + c)^2 + 1) - 10
*a^3*b/(sin(d*x + c)^2 - 1))/d
```

mupad [B] time = 0.80, size = 139, normalized size = 4.63

$$\frac{\frac{b^4\,\sin(c+d\,x)}{5} - \cos(c+d\,x)^3\,\left(2\,a\,b^3-2\,a^3\,b\right) - \cos(c+d\,x)^2\,\left(\frac{2\,b^4\,\sin(c+d\,x)}{5}-2\,a^2\,b^2\,\sin(c+d\,x)\right) + \cos(c+d\,x)^4\,\left(\right.}{d\,\cos(c+d\,x)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*cos(c + d*x) + b*sin(c + d*x))^4/cos(c + d*x)^6,x)
```

```
[Out] ((b^4*sin(c + d*x))/5 - cos(c + d*x)^3*(2*a*b^3 - 2*a^3*b) - cos(c + d*x)^2
*((2*b^4*sin(c + d*x))/5 - 2*a^2*b^2*sin(c + d*x)) + cos(c + d*x)^4*(a^4*si
n(c + d*x) + (b^4*sin(c + d*x))/5 - 2*a^2*b^2*sin(c + d*x)) + a*b^3*cos(c +
 d*x))/(d*cos(c + d*x)^5)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**6*(a*cos(d*x+c)+b*sin(d*x+c))**4,x)
```

```
[Out] Timed out
```

3.86 $\int \sec^7(c+dx)(a\cos(c+dx)+b\sin(c+dx))^4\,dx$

Optimal. Leaf size=258

$$\frac{a^4\tanh^{-1}(\sin(c+dx))}{2d}+\frac{a^4\tan(c+dx)\sec(c+dx)}{2d}+\frac{4a^3b\sec^3(c+dx)}{3d}-\frac{3a^2b^2\tanh^{-1}(\sin(c+dx))}{4d}+\frac{3a^2b^2\tan(c}{}$$

```
[Out] 1/2*a^4*arctanh(sin(d*x+c))/d-3/4*a^2*b^2*arctanh(sin(d*x+c))/d+1/16*b^4*ar
ctanh(sin(d*x+c))/d+4/3*a^3*b*sec(d*x+c)^3/d-4/3*a*b^3*sec(d*x+c)^3/d+4/5*a
*b^3*sec(d*x+c)^5/d+1/2*a^4*sec(d*x+c)*tan(d*x+c)/d-3/4*a^2*b^2*sec(d*x+c)*
tan(d*x+c)/d+1/16*b^4*sec(d*x+c)*tan(d*x+c)/d+3/2*a^2*b^2*sec(d*x+c)^3*tan(
d*x+c)/d-1/8*b^4*sec(d*x+c)^3*tan(d*x+c)/d+1/6*b^4*sec(d*x+c)^3*tan(d*x+c)^
3/d
```

Rubi [A] time = 0.29, antiderivative size = 258, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 7, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {3090, 3768, 3770, 2606, 30, 2611, 14}

$$-\frac{3a^2b^2\tanh^{-1}(\sin(c+dx))}{4d}+\frac{3a^2b^2\tan(c+dx)\sec^3(c+dx)}{2d}-\frac{3a^2b^2\tan(c+dx)\sec(c+dx)}{4d}+\frac{4a^3b\sec^3(c+dx)}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^7*(a*Cos[c + d*x] + b*Sin[c + d*x])^4,x]
```

```
[Out] (a^4*ArcTanh[Sin[c + d*x]])/(2*d) - (3*a^2*b^2*ArcTanh[Sin[c + d*x]])/(4*d)
 + (b^4*ArcTanh[Sin[c + d*x]])/(16*d) + (4*a^3*b*Sec[c + d*x]^3)/(3*d) - (4
*a*b^3*Sec[c + d*x]^3)/(3*d) + (4*a*b^3*Sec[c + d*x]^5)/(5*d) + (a^4*Sec[c
+ d*x]*Tan[c + d*x])/(2*d) - (3*a^2*b^2*Sec[c + d*x]*Tan[c + d*x])/(4*d) +
(b^4*Sec[c + d*x]*Tan[c + d*x])/(16*d) + (3*a^2*b^2*Sec[c + d*x]^3*Tan[c +
d*x])/(2*d) - (b^4*Sec[c + d*x]^3*Tan[c + d*x])/(8*d) + (b^4*Sec[c + d*x]^3
*Tan[c + d*x]^3)/(6*d)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 && !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 2611

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(b*(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*(
m + n - 1)), x] - Dist[(b^2*(n - 1))/(m + n - 1), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 3090

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_.), x_Symbol] :> Int[ExpandTrig[cos[c + d*x]^m*(a
*cos[c + d*x] + b*sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d}, x] && Inte
gerQ[m] && IGtQ[n, 0]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \sec^7(c+dx)(a\cos(c+dx)+b\sin(c+dx))^4\,dx = \int \left(a^4\sec^3(c+dx)+4a^3b\sec^3(c+dx)\tan(c+dx)+6a^2b^2\right.$$

$$= a^4\int \sec^3(c+dx)\,dx + \left(4a^3b\right)\int \sec^3(c+dx)\tan(c+dx)\,d$$

$$= \frac{a^4\sec(c+dx)\tan(c+dx)}{2d} + \frac{3a^2b^2\sec^3(c+dx)\tan(c+dx}{2d}$$

$$= \frac{a^4\tanh^{-1}(\sin(c+dx))}{2d} + \frac{4a^3b\sec^3(c+dx)}{3d} + \frac{a^4\sec(c+d}{2}$$

$$= \frac{a^4\tanh^{-1}(\sin(c+dx))}{2d} - \frac{3a^2b^2\tanh^{-1}(\sin(c+dx))}{4d} + \frac{4a^3}{}$$

$$= \frac{a^4\tanh^{-1}(\sin(c+dx))}{2d} - \frac{3a^2b^2\tanh^{-1}(\sin(c+dx))}{4d} + \frac{b^4\,t}{}$$

Mathematica [B] time = 6.25, size = 1342, normalized size = 5.20

result too large to display

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^7*(a*Cos[c + d*x] + b*Sin[c + d*x])^4,x]
```

```
[Out] (a*b*(20*a^2 - 11*b^2)*Cos[c + d*x]^4*(a + b*Tan[c + d*x])^4)/(30*d*(a*Cos[
c + d*x] + b*Sin[c + d*x])^4) + ((-8*a^4 + 12*a^2*b^2 - b^4)*Cos[c + d*x]^4
*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]]*(a + b*Tan[c + d*x])^4)/(16*d*(a*
Cos[c + d*x] + b*Sin[c + d*x])^4) + ((8*a^4 - 12*a^2*b^2 + b^4)*Cos[c + d*x
]^4*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]*(a + b*Tan[c + d*x])^4)/(16*d*
(a*Cos[c + d*x] + b*Sin[c + d*x])^4) + (b^4*Cos[c + d*x]^4*(a + b*Tan[c + d
*x])^4)/(48*d*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^6*(a*Cos[c + d*x] + b*S
in[c + d*x])^4) + ((30*a^2*b^2 + 8*a*b^3 - 5*b^4)*Cos[c + d*x]^4*(a + b*Tan
[c + d*x])^4)/(80*d*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^4*(a*Cos[c + d*x]
 + b*Sin[c + d*x])^4) + ((120*a^4 + 160*a^3*b - 180*a^2*b^2 - 88*a*b^3 + 15
*b^4)*Cos[c + d*x]^4*(a + b*Tan[c + d*x])^4)/(480*d*(Cos[(c + d*x)/2] - Sin
[(c + d*x)/2])^2*(a*Cos[c + d*x] + b*Sin[c + d*x])^4) + (a*b^3*Cos[c + d*x]
^4*Sin[(c + d*x)/2]*(a + b*Tan[c + d*x])^4)/(5*d*(Cos[(c + d*x)/2] - Sin[(c
 + d*x)/2])^5*(a*Cos[c + d*x] + b*Sin[c + d*x])^4) - (b^4*Cos[c + d*x]^4*(a
 + b*Tan[c + d*x])^4)/(48*d*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^6*(a*Cos[
c + d*x] + b*Sin[c + d*x])^4) - (a*b^3*Cos[c + d*x]^4*Sin[(c + d*x)/2]*(a +
 b*Tan[c + d*x])^4)/(5*d*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^5*(a*Cos[c +
 d*x] + b*Sin[c + d*x])^4) + ((-30*a^2*b^2 + 8*a*b^3 + 5*b^4)*Cos[c + d*x]^
4*(a + b*Tan[c + d*x])^4)/(80*d*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^4*(a*
Cos[c + d*x] + b*Sin[c + d*x])^4) + ((-120*a^4 + 160*a^3*b + 180*a^2*b^2 -
```

```
88*a*b^3 - 15*b^4)*Cos[c + d*x]^4*(a + b*Tan[c + d*x])^4)/(480*d*(Cos[(c +
d*x)/2] + Sin[(c + d*x)/2])^2*(a*Cos[c + d*x] + b*Sin[c + d*x])^4) + (Cos[c
 + d*x]^4*(20*a^3*b*Sin[(c + d*x)/2] - 11*a*b^3*Sin[(c + d*x)/2])*(a + b*Ta
n[c + d*x])^4)/(30*d*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^3*(a*Cos[c + d*x
] + b*Sin[c + d*x])^4) + (Cos[c + d*x]^4*(20*a^3*b*Sin[(c + d*x)/2] - 11*a*
b^3*Sin[(c + d*x)/2])*(a + b*Tan[c + d*x])^4)/(30*d*(Cos[(c + d*x)/2] - Sin
[(c + d*x)/2])*(a*Cos[c + d*x] + b*Sin[c + d*x])^4) + (Cos[c + d*x]^4*(-20*
a^3*b*Sin[(c + d*x)/2] + 11*a*b^3*Sin[(c + d*x)/2])*(a + b*Tan[c + d*x])^4)
/(30*d*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^3*(a*Cos[c + d*x] + b*Sin[c +
d*x])^4) + (Cos[c + d*x]^4*(-20*a^3*b*Sin[(c + d*x)/2] + 11*a*b^3*Sin[(c +
d*x)/2])*(a + b*Tan[c + d*x])^4)/(30*d*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2]
)*(a*Cos[c + d*x] + b*Sin[c + d*x])^4)
```

fricas [A] time = 0.81, size = 187, normalized size = 0.72

$$\frac{15\left(8\,a^4-12\,a^2b^2+b^4\right)\cos\left(dx+c\right)^6\log\left(\sin\left(dx+c\right)+1\right)-15\left(8\,a^4-12\,a^2b^2+b^4\right)\cos\left(dx+c\right)^6\log\left(-\sin\left(dx\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^7*(a*cos(d*x+c)+b*sin(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] 1/480*(15*(8*a^4 - 12*a^2*b^2 + b^4)*cos(d*x + c)^6*log(sin(d*x + c) + 1) -
 15*(8*a^4 - 12*a^2*b^2 + b^4)*cos(d*x + c)^6*log(-sin(d*x + c) + 1) + 384*
a*b^3*cos(d*x + c) + 640*(a^3*b - a*b^3)*cos(d*x + c)^3 + 10*(3*(8*a^4 - 12
*a^2*b^2 + b^4)*cos(d*x + c)^4 + 8*b^4 + 2*(36*a^2*b^2 - 7*b^4)*cos(d*x + c
)^2)*sin(d*x + c))/(d*cos(d*x + c)^6)
```

giac [B] time = 0.49, size = 536, normalized size = 2.08

$$\frac{15\left(8\,a^4-12\,a^2b^2+b^4\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-15\left(8\,a^4-12\,a^2b^2+b^4\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)+\frac{2\left(120\right.}{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^7*(a*cos(d*x+c)+b*sin(d*x+c))^4,x, algorithm="giac")
```

```
[Out] 1/240*(15*(8*a^4 - 12*a^2*b^2 + b^4)*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 1
5*(8*a^4 - 12*a^2*b^2 + b^4)*log(abs(tan(1/2*d*x + 1/2*c) - 1)) + 2*(120*a^
4*tan(1/2*d*x + 1/2*c)^11 + 180*a^2*b^2*tan(1/2*d*x + 1/2*c)^11 - 15*b^4*ta
n(1/2*d*x + 1/2*c)^11 - 960*a^3*b*tan(1/2*d*x + 1/2*c)^10 - 360*a^4*tan(1/2
*d*x + 1/2*c)^9 + 900*a^2*b^2*tan(1/2*d*x + 1/2*c)^9 + 85*b^4*tan(1/2*d*x +
 1/2*c)^9 + 2880*a^3*b*tan(1/2*d*x + 1/2*c)^8 - 1920*a*b^3*tan(1/2*d*x + 1/
```

```
2*c)^8 + 240*a^4*tan(1/2*d*x + 1/2*c)^7 - 1080*a^2*b^2*tan(1/2*d*x + 1/2*c)
^7 + 570*b^4*tan(1/2*d*x + 1/2*c)^7 - 3200*a^3*b*tan(1/2*d*x + 1/2*c)^6 + 1
280*a*b^3*tan(1/2*d*x + 1/2*c)^6 + 240*a^4*tan(1/2*d*x + 1/2*c)^5 - 1080*a^
2*b^2*tan(1/2*d*x + 1/2*c)^5 + 570*b^4*tan(1/2*d*x + 1/2*c)^5 + 1920*a^3*b*
tan(1/2*d*x + 1/2*c)^4 - 360*a^4*tan(1/2*d*x + 1/2*c)^3 + 900*a^2*b^2*tan(1
/2*d*x + 1/2*c)^3 + 85*b^4*tan(1/2*d*x + 1/2*c)^3 - 960*a^3*b*tan(1/2*d*x +
 1/2*c)^2 + 768*a*b^3*tan(1/2*d*x + 1/2*c)^2 + 120*a^4*tan(1/2*d*x + 1/2*c)
 + 180*a^2*b^2*tan(1/2*d*x + 1/2*c) - 15*b^4*tan(1/2*d*x + 1/2*c) + 320*a^3
*b - 128*a*b^3)/(tan(1/2*d*x + 1/2*c)^2 - 1)^6)/d
```

maple [A] time = 65.24, size = 394, normalized size = 1.53

$$\frac{a^4 \sec(dx+c)\tan(dx+c)}{2d}+\frac{a^4 \ln(\sec(dx+c)+\tan(dx+c))}{2d}+\frac{4a^3b}{3d\cos(dx+c)^3}+\frac{3a^2b^2\left(\sin^3(dx+c)\right)}{2d\cos(dx+c)^4}+\frac{3a^2b^2}{4d\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^7*(a*cos(d*x+c)+b*sin(d*x+c))^4,x)
```

```
[Out] 1/2*a^4*sec(d*x+c)*tan(d*x+c)/d+1/2/d*a^4*ln(sec(d*x+c)+tan(d*x+c))+4/3/d*a
^3*b/cos(d*x+c)^3+3/2/d*a^2*b^2*sin(d*x+c)^3/cos(d*x+c)^4+3/4/d*a^2*b^2*sin
(d*x+c)^3/cos(d*x+c)^2+3/4*a^2*b^2*sin(d*x+c)/d-3/4/d*a^2*b^2*ln(sec(d*x+c)
+tan(d*x+c))+4/5/d*a*b^3*sin(d*x+c)^4/cos(d*x+c)^5+4/15/d*a*b^3*sin(d*x+c)^
4/cos(d*x+c)^3-4/15/d*a*b^3*sin(d*x+c)^4/cos(d*x+c)-4/15/d*sin(d*x+c)^2*cos
(d*x+c)*a*b^3-8/15*a*b^3*cos(d*x+c)/d+1/6/d*b^4*sin(d*x+c)^5/cos(d*x+c)^6+1
/24/d*b^4*sin(d*x+c)^5/cos(d*x+c)^4-1/48/d*b^4*sin(d*x+c)^5/cos(d*x+c)^2-1/
48*b^4*sin(d*x+c)^3/d-1/16*b^4*sin(d*x+c)/d+1/16/d*b^4*ln(sec(d*x+c)+tan(d*
x+c))
```

maxima [A] time = 0.33, size = 251, normalized size = 0.97

$$-\frac{5\,b^4\left(\frac{2\left(3\,\sin(dx+c)^5+8\,\sin(dx+c)^3-3\,\sin(dx+c)\right)}{\sin(dx+c)^6-3\,\sin(dx+c)^4+3\,\sin(dx+c)^2-1}-3\,\log(\sin(dx+c)+1)+3\,\log(\sin(dx+c)-1)\right)-180\,a^2b^2\left(\frac{2\left(\sin(\right.}{\sin(dx+}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^7*(a*cos(d*x+c)+b*sin(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] -1/480*(5*b^4*(2*(3*sin(d*x + c)^5 + 8*sin(d*x + c)^3 - 3*sin(d*x + c))/(si
n(d*x + c)^6 - 3*sin(d*x + c)^4 + 3*sin(d*x + c)^2 - 1) - 3*log(sin(d*x + c
) + 1) + 3*log(sin(d*x + c) - 1)) - 180*a^2*b^2*(2*(sin(d*x + c)^3 + sin(d*
x + c))/(sin(d*x + c)^4 - 2*sin(d*x + c)^2 + 1) - log(sin(d*x + c) + 1) + l
og(sin(d*x + c) - 1)) + 120*a^4*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) - log(
sin(d*x + c) + 1) + log(sin(d*x + c) - 1)) - 640*a^3*b/cos(d*x + c)^3 + 128
*(5*cos(d*x + c)^2 - 3)*a*b^3/cos(d*x + c)^5)/d
```

mupad [B] time = 4.27, size = 419, normalized size = 1.62

$$\frac{\operatorname{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)\left(a^4-\frac{3\,a^2\,b^2}{2}+\frac{b^4}{8}\right)}{d}+\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5\left(2\,a^4-9\,a^2\,b^2+\frac{19\,b^4}{4}\right)+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7\left(2\,a^4-9\,a^2\,b^2+\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*cos(c + d*x) + b*sin(c + d*x))^4/cos(c + d*x)^7,x)
```

```
[Out] (atanh(tan(c/2 + (d*x)/2))*(a^4 + b^4/8 - (3*a^2*b^2)/2))/d + (tan(c/2 + (d
*x)/2)^5*(2*a^4 + (19*b^4)/4 - 9*a^2*b^2) + tan(c/2 + (d*x)/2)^7*(2*a^4 + (
19*b^4)/4 - 9*a^2*b^2) + tan(c/2 + (d*x)/2)^3*((17*b^4)/24 - 3*a^4 + (15*a^
2*b^2)/2) + tan(c/2 + (d*x)/2)^9*((17*b^4)/24 - 3*a^4 + (15*a^2*b^2)/2) + t
an(c/2 + (d*x)/2)*(a^4 - b^4/8 + (3*a^2*b^2)/2) - (16*a*b^3)/15 + (8*a^3*b)
/3 + tan(c/2 + (d*x)/2)^11*(a^4 - b^4/8 + (3*a^2*b^2)/2) + tan(c/2 + (d*x)/
2)^2*((32*a*b^3)/5 - 8*a^3*b) - tan(c/2 + (d*x)/2)^8*(16*a*b^3 - 24*a^3*b)
+ tan(c/2 + (d*x)/2)^6*((32*a*b^3)/3 - (80*a^3*b)/3) + 16*a^3*b*tan(c/2 + (
d*x)/2)^4 - 8*a^3*b*tan(c/2 + (d*x)/2)^10)/(d*(15*tan(c/2 + (d*x)/2)^4 - 6*
tan(c/2 + (d*x)/2)^2 - 20*tan(c/2 + (d*x)/2)^6 + 15*tan(c/2 + (d*x)/2)^8 -
6*tan(c/2 + (d*x)/2)^10 + tan(c/2 + (d*x)/2)^12 + 1))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**7*(a*cos(d*x+c)+b*sin(d*x+c))**4,x)
```

```
[Out] Timed out
```

3.87 $\int \sec^8(c+dx)(a\cos(c+dx)+b\sin(c+dx))^4\,dx$

Optimal. Leaf size=143

$$\frac{a^4\tan(c+dx)}{d}+\frac{2a^3b\tan^2(c+dx)}{d}+\frac{b^2\left(6a^2+b^2\right)\tan^5(c+dx)}{5d}+\frac{ab\left(a^2+b^2\right)\tan^4(c+dx)}{d}+\frac{a^2\left(a^2+6b^2\right)\tan^3(c$$

```
[Out] a^4*tan(d*x+c)/d+2*a^3*b*tan(d*x+c)^2/d+1/3*a^2*(a^2+6*b^2)*tan(d*x+c)^3/d+
a*b*(a^2+b^2)*tan(d*x+c)^4/d+1/5*b^2*(6*a^2+b^2)*tan(d*x+c)^5/d+2/3*a*b^3*t
an(d*x+c)^6/d+1/7*b^4*tan(d*x+c)^7/d
```

Rubi [A] time = 0.12, antiderivative size = 143, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {3088, 894}

$$\frac{b^2\left(6a^2+b^2\right)\tan^5(c+dx)}{5d}+\frac{ab\left(a^2+b^2\right)\tan^4(c+dx)}{d}+\frac{a^2\left(a^2+6b^2\right)\tan^3(c+dx)}{3d}+\frac{2a^3b\tan^2(c+dx)}{d}+\frac{a^4\tan(c}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^8*(a*Cos[c + d*x] + b*Sin[c + d*x])^4,x]
```

```
[Out] (a^4*Tan[c + d*x])/d + (2*a^3*b*Tan[c + d*x]^2)/d + (a^2*(a^2 + 6*b^2)*Tan[
c + d*x]^3)/(3*d) + (a*b*(a^2 + b^2)*Tan[c + d*x]^4)/d + (b^2*(6*a^2 + b^2)
*Tan[c + d*x]^5)/(5*d) + (2*a*b^3*Tan[c + d*x]^6)/(3*d) + (b^4*Tan[c + d*x]
^7)/(7*d)
```

Rule 894

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ
[m, 0] && ILtQ[n, 0]))
```

Rule 3088

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[(x^m*(b +
 a*x)^n)/(1 + x^2)^((m + n + 2)/2), x], x, Cot[c + d*x]], x] /; FreeQ[{a, b
, c, d}, x] && IntegerQ[n] && IntegerQ[(m + n)/2] && NeQ[n, -1] && !(GtQ[n
, 0] && GtQ[m, 1])
```

Rubi steps

$$\int \sec^8(c+dx)(a\cos(c+dx)+b\sin(c+dx))^4\,dx = -\frac{\text{Subst}\left(\int \frac{(b+ax)^4(1+x^2)}{x^8}\,dx, x, \cot(c+dx)\right)}{d}$$

$$= -\frac{\text{Subst}\left(\int \left(\frac{b^4}{x^8}+\frac{4ab^3}{x^7}+\frac{6a^2b^2+b^4}{x^6}+\frac{4ab(a^2+b^2)}{x^5}+\frac{a^4+6a^2b^2}{x^4}+\frac{4a^3b}{x^3}\right.\right.}{d}$$

$$= \frac{a^4\tan(c+dx)}{d}+\frac{2a^3b\tan^2(c+dx)}{d}+\frac{a^2\left(a^2+6b^2\right)\tan^3(c+}{3d}$$

Mathematica [A] time = 0.55, size = 54, normalized size = 0.38

$$\frac{(a+b\tan(c+dx))^5\left(a^2-5ab\tan(c+dx)+15b^2\tan^2(c+dx)+21b^2\right)}{105b^3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^8*(a*Cos[c + d*x] + b*Sin[c + d*x])^4,x]
```

```
[Out] ((a + b*Tan[c + d*x])^5*(a^2 + 21*b^2 - 5*a*b*Tan[c + d*x] + 15*b^2*Tan[c +
 d*x]^2))/(105*b^3*d)
```

fricas [A] time = 0.49, size = 142, normalized size = 0.99

$$\frac{70\,ab^3\cos(dx+c)+105\left(a^3b-ab^3\right)\cos(dx+c)^3+\left(2\left(35\,a^4-42\,a^2b^2+3\,b^4\right)\cos(dx+c)^6+\left(35\,a^4-42\,a^2b^2+\right.\right.}{105\,d\cos(dx+c)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^8*(a*cos(d*x+c)+b*sin(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] 1/105*(70*a*b^3*cos(d*x + c) + 105*(a^3*b - a*b^3)*cos(d*x + c)^3 + (2*(35*
a^4 - 42*a^2*b^2 + 3*b^4)*cos(d*x + c)^6 + (35*a^4 - 42*a^2*b^2 + 3*b^4)*co
s(d*x + c)^4 + 15*b^4 + 6*(21*a^2*b^2 - 4*b^4)*cos(d*x + c)^2)*sin(d*x + c)
)/(d*cos(d*x + c)^7)
```

giac [A] time = 0.43, size = 144, normalized size = 1.01

$$\frac{15\,b^4\tan(dx+c)^7+70\,ab^3\tan(dx+c)^6+126\,a^2b^2\tan(dx+c)^5+21\,b^4\tan(dx+c)^5+105\,a^3b\tan(dx+c)^4+}{105}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^8*(a*cos(d*x+c)+b*sin(d*x+c))^4,x, algorithm="giac")
```

```
[Out] 1/105*(15*b^4*tan(d*x + c)^7 + 70*a*b^3*tan(d*x + c)^6 + 126*a^2*b^2*tan(d*
x + c)^5 + 21*b^4*tan(d*x + c)^5 + 105*a^3*b*tan(d*x + c)^4 + 105*a*b^3*tan
(d*x + c)^4 + 35*a^4*tan(d*x + c)^3 + 210*a^2*b^2*tan(d*x + c)^3 + 210*a^3*
b*tan(d*x + c)^2 + 105*a^4*tan(d*x + c))/d
```

maple [A] time = 52.37, size = 171, normalized size = 1.20

$$\frac{-a^4\left(-\frac{2}{3}-\frac{\left(\sec^2(dx+c)\right)}{3}\right)\tan(dx+c)+\frac{a^3b}{\cos(dx+c)^4}+6a^2b^2\left(\frac{\sin^3(dx+c)}{5\cos(dx+c)^5}+\frac{2\left(\sin^3(dx+c)\right)}{15\cos(dx+c)^3}\right)+4a\,b^3\left(\frac{\sin^4(dx+c)}{6\cos(dx+c)^6}+\frac{\sin^4(dx+\ldots}{12\cos(dx\ldots}\right.}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^8*(a*cos(d*x+c)+b*sin(d*x+c))^4,x)
```

```
[Out] 1/d*(-a^4*(-2/3-1/3*sec(d*x+c)^2)*tan(d*x+c)+a^3*b/cos(d*x+c)^4+6*a^2*b^2*(
1/5*sin(d*x+c)^3/cos(d*x+c)^5+2/15*sin(d*x+c)^3/cos(d*x+c)^3)+4*a*b^3*(1/6*
sin(d*x+c)^4/cos(d*x+c)^6+1/12*sin(d*x+c)^4/cos(d*x+c)^4)+b^4*(1/7*sin(d*x+
c)^5/cos(d*x+c)^7+2/35*sin(d*x+c)^5/cos(d*x+c)^5))
```

maxima [A] time = 0.33, size = 151, normalized size = 1.06

$$\frac{35\left(\tan(dx+c)^3+3\tan(dx+c)\right)a^4+42\left(3\tan(dx+c)^5+5\tan(dx+c)^3\right)a^2b^2+3\left(5\tan(dx+c)^7+7\tan\ldots\right.}{105\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^8*(a*cos(d*x+c)+b*sin(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] 1/105*(35*(tan(d*x + c)^3 + 3*tan(d*x + c))*a^4 + 42*(3*tan(d*x + c)^5 + 5*
tan(d*x + c)^3)*a^2*b^2 + 3*(5*tan(d*x + c)^7 + 7*tan(d*x + c)^5)*b^4 - 35*
(3*sin(d*x + c)^2 - 1)*a*b^3/(sin(d*x + c)^6 - 3*sin(d*x + c)^4 + 3*sin(d*x
 + c)^2 - 1) + 105*a^3*b/(sin(d*x + c)^2 - 1)^2)/d
```

mupad [B] time = 1.10, size = 186, normalized size = 1.30

$$\frac{\frac{b^4\sin(c+d\,x)}{7}-\cos(c+d\,x)^3\,\left(a\,b^3-a^3\,b\right)-\cos(c+d\,x)^2\,\left(\frac{8\,b^4\sin(c+d\,x)}{35}-\frac{6\,a^2\,b^2\sin(c+d\,x)}{5}\right)+\cos(c+d\,x)^4\,\left(\frac{\sin(c+d\,x\ldots}{3}\right.}{d\cos(c\ldots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*cos(c + d*x) + b*sin(c + d*x))^4/cos(c + d*x)^8,x)
```

```
[Out] ((b^4*sin(c + d*x))/7 - cos(c + d*x)^3*(a*b^3 - a^3*b) - cos(c + d*x)^2*((8
*b^4*sin(c + d*x))/35 - (6*a^2*b^2*sin(c + d*x))/5) + cos(c + d*x)^4*((a^4*
sin(c + d*x))/3 + (b^4*sin(c + d*x))/35 - (2*a^2*b^2*sin(c + d*x))/5) + cos
(c + d*x)^6*((2*a^4*sin(c + d*x))/3 + (2*b^4*sin(c + d*x))/35 - (4*a^2*b^2*
sin(c + d*x))/5) + (2*a*b^3*cos(c + d*x))/3)/(d*cos(c + d*x)^7)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**8*(a*cos(d*x+c)+b*sin(d*x+c))**4,x)
```

```
[Out] Timed out
```

3.88 $\int \sec^9(c+dx)(a\cos(c+dx)+b\sin(c+dx))^4\,dx$

Optimal. Leaf size=330

$$\frac{3a^4\tanh^{-1}(\sin(c+dx))}{8d}+\frac{a^4\tan(c+dx)\sec^3(c+dx)}{4d}+\frac{3a^4\tan(c+dx)\sec(c+dx)}{8d}+\frac{4a^3b\sec^5(c+dx)}{5d}-\frac{3a^2b^2\,\mathrm{ta}}{}$$

```
[Out] 3/8*a^4*arctanh(sin(d*x+c))/d-3/8*a^2*b^2*arctanh(sin(d*x+c))/d+3/128*b^4*a
rctanh(sin(d*x+c))/d+4/5*a^3*b*sec(d*x+c)^5/d-4/5*a*b^3*sec(d*x+c)^5/d+4/7*
a*b^3*sec(d*x+c)^7/d+3/8*a^4*sec(d*x+c)*tan(d*x+c)/d-3/8*a^2*b^2*sec(d*x+c)
*tan(d*x+c)/d+3/128*b^4*sec(d*x+c)*tan(d*x+c)/d+1/4*a^4*sec(d*x+c)^3*tan(d*
x+c)/d-1/4*a^2*b^2*sec(d*x+c)^3*tan(d*x+c)/d+1/64*b^4*sec(d*x+c)^3*tan(d*x+
c)/d+a^2*b^2*sec(d*x+c)^5*tan(d*x+c)/d-1/16*b^4*sec(d*x+c)^5*tan(d*x+c)/d+1
/8*b^4*sec(d*x+c)^5*tan(d*x+c)^3/d
```

Rubi [A] time = 0.34, antiderivative size = 330, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 7, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {3090, 3768, 3770, 2606, 30, 2611, 14}

$$-\frac{3a^2b^2\tanh^{-1}(\sin(c+dx))}{8d}+\frac{a^2b^2\tan(c+dx)\sec^5(c+dx)}{d}-\frac{a^2b^2\tan(c+dx)\sec^3(c+dx)}{4d}-\frac{3a^2b^2\tan(c+dx)\,\mathrm{se}}{8d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^9*(a*Cos[c + d*x] + b*Sin[c + d*x])^4,x]
```

```
[Out] (3*a^4*ArcTanh[Sin[c + d*x]])/(8*d) - (3*a^2*b^2*ArcTanh[Sin[c + d*x]])/(8*
d) + (3*b^4*ArcTanh[Sin[c + d*x]])/(128*d) + (4*a^3*b*Sec[c + d*x]^5)/(5*d)
 - (4*a*b^3*Sec[c + d*x]^5)/(5*d) + (4*a*b^3*Sec[c + d*x]^7)/(7*d) + (3*a^4
*Sec[c + d*x]*Tan[c + d*x])/(8*d) - (3*a^2*b^2*Sec[c + d*x]*Tan[c + d*x])/(
8*d) + (3*b^4*Sec[c + d*x]*Tan[c + d*x])/(128*d) + (a^4*Sec[c + d*x]^3*Tan[
c + d*x])/(4*d) - (a^2*b^2*Sec[c + d*x]^3*Tan[c + d*x])/(4*d) + (b^4*Sec[c
+ d*x]^3*Tan[c + d*x])/(64*d) + (a^2*b^2*Sec[c + d*x]^5*Tan[c + d*x])/d - (
b^4*Sec[c + d*x]^5*Tan[c + d*x])/(16*d) + (b^4*Sec[c + d*x]^5*Tan[c + d*x]^
3)/(8*d)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 2611

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(b*(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*(
m + n - 1)), x] - Dist[(b^2*(n - 1))/(m + n - 1), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 3090

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_.), x_Symbol] :> Int[ExpandTrig[cos[c + d*x]^m*(a
*cos[c + d*x] + b*sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d}, x] && Inte
gerQ[m] && IGtQ[n, 0]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \sec^9(c+dx)(a\cos(c+dx)+b\sin(c+dx))^4\,dx = \int \left(a^4\sec^5(c+dx)+4a^3b\sec^5(c+dx)\tan(c+dx)+6a^2b^2\right.$$
$$= a^4\int \sec^5(c+dx)\,dx+\left(4a^3b\right)\int \sec^5(c+dx)\tan(c+dx)\,d$$
$$= \frac{a^4\sec^3(c+dx)\tan(c+dx)}{4d}+\frac{a^2b^2\sec^5(c+dx)\tan(c+dx)}{d}$$
$$= \frac{4a^3b\sec^5(c+dx)}{5d}+\frac{3a^4\sec(c+dx)\tan(c+dx)}{8d}+\frac{a^4\sec^3(}{}$$
$$= \frac{3a^4\tanh^{-1}(\sin(c+dx))}{8d}+\frac{4a^3b\sec^5(c+dx)}{5d}-\frac{4ab^3\sec^5(c}{5d}$$
$$= \frac{3a^4\tanh^{-1}(\sin(c+dx))}{8d}-\frac{3a^2b^2\tanh^{-1}(\sin(c+dx))}{8d}+\frac{4a}{}$$
$$= \frac{3a^4\tanh^{-1}(\sin(c+dx))}{8d}-\frac{3a^2b^2\tanh^{-1}(\sin(c+dx))}{8d}+\frac{3b}{}$$

Mathematica [B] time = 6.39, size = 1732, normalized size = 5.25

result too large to display

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^9*(a*Cos[c + d*x] + b*Sin[c + d*x])^4,x]
```

```
[Out] (a*b*(42*a^2 - 17*b^2)*Cos[c + d*x]^4*(a + b*Tan[c + d*x])^4)/(140*d*(a*Cos
[c + d*x] + b*Sin[c + d*x])^4) - (3*(16*a^4 - 16*a^2*b^2 + b^4)*Cos[c + d*x
]^4*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]]*(a + b*Tan[c + d*x])^4)/(128*d
*(a*Cos[c + d*x] + b*Sin[c + d*x])^4) + (3*(16*a^4 - 16*a^2*b^2 + b^4)*Cos[
c + d*x]^4*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]*(a + b*Tan[c + d*x])^4)
/(128*d*(a*Cos[c + d*x] + b*Sin[c + d*x])^4) + (b^4*Cos[c + d*x]^4*(a + b*T
an[c + d*x])^4)/(128*d*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^8*(a*Cos[c + d
*x] + b*Sin[c + d*x])^4) + ((56*a^2*b^2 + 16*a*b^3 - 7*b^4)*Cos[c + d*x]^4*
(a + b*Tan[c + d*x])^4)/(448*d*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^6*(a*C
os[c + d*x] + b*Sin[c + d*x])^4) + ((560*a^4 + 896*a^3*b - 256*a*b^3 - 35*b
^4)*Cos[c + d*x]^4*(a + b*Tan[c + d*x])^4)/(8960*d*(Cos[(c + d*x)/2] - Sin[
(c + d*x)/2])^4*(a*Cos[c + d*x] + b*Sin[c + d*x])^4) + ((1680*a^4 + 1344*a^
3*b - 1680*a^2*b^2 - 544*a*b^3 + 105*b^4)*Cos[c + d*x]^4*(a + b*Tan[c + d*x
])^4)/(8960*d*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^2*(a*Cos[c + d*x] + b*S
in[c + d*x])^4) + (a*b^3*Cos[c + d*x]^4*Sin[(c + d*x)/2]*(a + b*Tan[c + d*x
])^4)/(14*d*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^7*(a*Cos[c + d*x] + b*Sin
[c + d*x])^4) - (b^4*Cos[c + d*x]^4*(a + b*Tan[c + d*x])^4)/(128*d*(Cos[(c
+ d*x)/2] + Sin[(c + d*x)/2])^8*(a*Cos[c + d*x] + b*Sin[c + d*x])^4) - (a*b
```

```
^3*Cos[c + d*x]^4*Sin[(c + d*x)/2]*(a + b*Tan[c + d*x])^4)/(14*d*(Cos[(c +
d*x)/2] + Sin[(c + d*x)/2])^7*(a*Cos[c + d*x] + b*Sin[c + d*x])^4) + ((-56*
a^2*b^2 + 16*a*b^3 + 7*b^4)*Cos[c + d*x]^4*(a + b*Tan[c + d*x])^4)/(448*d*(
Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^6*(a*Cos[c + d*x] + b*Sin[c + d*x])^4)
 + ((-560*a^4 + 896*a^3*b - 256*a*b^3 + 35*b^4)*Cos[c + d*x]^4*(a + b*Tan[c
 + d*x])^4)/(8960*d*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^4*(a*Cos[c + d*x]
 + b*Sin[c + d*x])^4) + ((-1680*a^4 + 1344*a^3*b + 1680*a^2*b^2 - 544*a*b^3
 - 105*b^4)*Cos[c + d*x]^4*(a + b*Tan[c + d*x])^4)/(8960*d*(Cos[(c + d*x)/2
] + Sin[(c + d*x)/2])^2*(a*Cos[c + d*x] + b*Sin[c + d*x])^4) + (Cos[c + d*x
]^4*(42*a^3*b*Sin[(c + d*x)/2] - 17*a*b^3*Sin[(c + d*x)/2])*(a + b*Tan[c +
d*x])^4)/(140*d*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^3*(a*Cos[c + d*x] + b
*Sin[c + d*x])^4) + (Cos[c + d*x]^4*(42*a^3*b*Sin[(c + d*x)/2] - 17*a*b^3*S
in[(c + d*x)/2])*(a + b*Tan[c + d*x])^4)/(140*d*(Cos[(c + d*x)/2] - Sin[(c
+ d*x)/2])*(a*Cos[c + d*x] + b*Sin[c + d*x])^4) + (Cos[c + d*x]^4*(7*a^3*b*
Sin[(c + d*x)/2] - 2*a*b^3*Sin[(c + d*x)/2])*(a + b*Tan[c + d*x])^4)/(35*d*
(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^5*(a*Cos[c + d*x] + b*Sin[c + d*x])^4
) + (Cos[c + d*x]^4*(-7*a^3*b*Sin[(c + d*x)/2] + 2*a*b^3*Sin[(c + d*x)/2])*
(a + b*Tan[c + d*x])^4)/(35*d*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^5*(a*Co
s[c + d*x] + b*Sin[c + d*x])^4) + (Cos[c + d*x]^4*(-42*a^3*b*Sin[(c + d*x)/
2] + 17*a*b^3*Sin[(c + d*x)/2])*(a + b*Tan[c + d*x])^4)/(140*d*(Cos[(c + d*
x)/2] + Sin[(c + d*x)/2])^3*(a*Cos[c + d*x] + b*Sin[c + d*x])^4) + (Cos[c +
 d*x]^4*(-42*a^3*b*Sin[(c + d*x)/2] + 17*a*b^3*Sin[(c + d*x)/2])*(a + b*Tan
[c + d*x])^4)/(140*d*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])*(a*Cos[c + d*x]
+ b*Sin[c + d*x])^4)
```

fricas [A] time = 0.56, size = 214, normalized size = 0.65

$$\frac{105\left(16a^4-16a^2b^2+b^4\right)\cos\left(dx+c\right)^8\log\left(\sin\left(dx+c\right)+1\right)-105\left(16a^4-16a^2b^2+b^4\right)\cos\left(dx+c\right)^8\log\left(-\sin\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^9*(a*cos(d*x+c)+b*sin(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] 1/8960*(105*(16*a^4 - 16*a^2*b^2 + b^4)*cos(d*x + c)^8*log(sin(d*x + c) + 1
) - 105*(16*a^4 - 16*a^2*b^2 + b^4)*cos(d*x + c)^8*log(-sin(d*x + c) + 1) +
 5120*a*b^3*cos(d*x + c) + 7168*(a^3*b - a*b^3)*cos(d*x + c)^3 + 70*(3*(16*
a^4 - 16*a^2*b^2 + b^4)*cos(d*x + c)^6 + 2*(16*a^4 - 16*a^2*b^2 + b^4)*cos(
d*x + c)^4 + 16*b^4 + 8*(16*a^2*b^2 - 3*b^4)*cos(d*x + c)^2)*sin(d*x + c))/
(d*cos(d*x + c)^8)
```

giac [B] time = 0.51, size = 706, normalized size = 2.14

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^9*(a*cos(d*x+c)+b*sin(d*x+c))^4,x, algorithm="giac")
```

```
[Out] 1/4480*(105*(16*a^4 - 16*a^2*b^2 + b^4)*log(abs(tan(1/2*d*x + 1/2*c) + 1))
- 105*(16*a^4 - 16*a^2*b^2 + b^4)*log(abs(tan(1/2*d*x + 1/2*c) - 1)) + 2*(2
800*a^4*tan(1/2*d*x + 1/2*c)^15 + 1680*a^2*b^2*tan(1/2*d*x + 1/2*c)^15 - 10
5*b^4*tan(1/2*d*x + 1/2*c)^15 - 17920*a^3*b*tan(1/2*d*x + 1/2*c)^14 - 9520*
a^4*tan(1/2*d*x + 1/2*c)^13 + 22960*a^2*b^2*tan(1/2*d*x + 1/2*c)^13 + 805*b
^4*tan(1/2*d*x + 1/2*c)^13 + 53760*a^3*b*tan(1/2*d*x + 1/2*c)^12 - 35840*a*
b^3*tan(1/2*d*x + 1/2*c)^12 + 11760*a^4*tan(1/2*d*x + 1/2*c)^11 - 7280*a^2*
b^2*tan(1/2*d*x + 1/2*c)^11 + 11655*b^4*tan(1/2*d*x + 1/2*c)^11 - 89600*a^3
*b*tan(1/2*d*x + 1/2*c)^10 - 5040*a^4*tan(1/2*d*x + 1/2*c)^9 - 17360*a^2*b^
2*tan(1/2*d*x + 1/2*c)^9 + 23485*b^4*tan(1/2*d*x + 1/2*c)^9 + 125440*a^3*b*
tan(1/2*d*x + 1/2*c)^8 - 35840*a*b^3*tan(1/2*d*x + 1/2*c)^8 - 5040*a^4*tan(
1/2*d*x + 1/2*c)^7 - 17360*a^2*b^2*tan(1/2*d*x + 1/2*c)^7 + 23485*b^4*tan(1
/2*d*x + 1/2*c)^7 - 111104*a^3*b*tan(1/2*d*x + 1/2*c)^6 + 57344*a*b^3*tan(1
/2*d*x + 1/2*c)^6 + 11760*a^4*tan(1/2*d*x + 1/2*c)^5 - 7280*a^2*b^2*tan(1/2
*d*x + 1/2*c)^5 + 11655*b^4*tan(1/2*d*x + 1/2*c)^5 + 46592*a^3*b*tan(1/2*d*
x + 1/2*c)^4 + 7168*a*b^3*tan(1/2*d*x + 1/2*c)^4 - 9520*a^4*tan(1/2*d*x + 1
/2*c)^3 + 22960*a^2*b^2*tan(1/2*d*x + 1/2*c)^3 + 805*b^4*tan(1/2*d*x + 1/2*
c)^3 - 10752*a^3*b*tan(1/2*d*x + 1/2*c)^2 + 8192*a*b^3*tan(1/2*d*x + 1/2*c)
^2 + 2800*a^4*tan(1/2*d*x + 1/2*c) + 1680*a^2*b^2*tan(1/2*d*x + 1/2*c) - 10
5*b^4*tan(1/2*d*x + 1/2*c) + 3584*a^3*b - 1024*a*b^3)/(tan(1/2*d*x + 1/2*c)
^2 - 1)^8)/d
```

maple [A] time = 68.46, size = 491, normalized size = 1.49

$$\frac{a^4\left(\sec^3(dx+c)\right)\tan(dx+c)}{4d}+\frac{3a^4\sec(dx+c)\tan(dx+c)}{8d}+\frac{3a^4\ln(\sec(dx+c)+\tan(dx+c))}{8d}+\frac{4a^3b}{5d\cos(dx+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^9*(a*cos(d*x+c)+b*sin(d*x+c))^4,x)
```

```
[Out] 1/4*a^4*sec(d*x+c)^3*tan(d*x+c)/d+3/8*a^4*sec(d*x+c)*tan(d*x+c)/d+3/8/d*a^4
*ln(sec(d*x+c)+tan(d*x+c))+4/5/d*a^3*b/cos(d*x+c)^5+1/d*a^2*b^2*sin(d*x+c)^
3/cos(d*x+c)^6+3/4/d*a^2*b^2*sin(d*x+c)^3/cos(d*x+c)^4+3/8/d*a^2*b^2*sin(d*
x+c)^3/cos(d*x+c)^2+3/8*a^2*b^2*sin(d*x+c)/d-3/8/d*a^2*b^2*ln(sec(d*x+c)+ta
n(d*x+c))+4/7/d*a*b^3*sin(d*x+c)^4/cos(d*x+c)^7+12/35/d*a*b^3*sin(d*x+c)^4/
cos(d*x+c)^5+4/35/d*a*b^3*sin(d*x+c)^4/cos(d*x+c)^3-4/35/d*a*b^3*sin(d*x+c)
^4/cos(d*x+c)-4/35/d*sin(d*x+c)^2*cos(d*x+c)*a*b^3-8/35*a*b^3*cos(d*x+c)/d+
1/8/d*b^4*sin(d*x+c)^5/cos(d*x+c)^8+1/16/d*b^4*sin(d*x+c)^5/cos(d*x+c)^6+1/
64/d*b^4*sin(d*x+c)^5/cos(d*x+c)^4-1/128/d*b^4*sin(d*x+c)^5/cos(d*x+c)^2-1/
128*b^4*sin(d*x+c)^3/d-3/128*b^4*sin(d*x+c)/d+3/128/d*b^4*ln(sec(d*x+c)+tan
(d*x+c))
```

maxima [A] time = 0.34, size = 322, normalized size = 0.98

$$-\frac{35\,b^4\left(\frac{2\left(3\,\sin(dx+c)^7-11\,\sin(dx+c)^5-11\,\sin(dx+c)^3+3\,\sin(dx+c)\right)}{\sin(dx+c)^8-4\,\sin(dx+c)^6+6\,\sin(dx+c)^4-4\,\sin(dx+c)^2+1}-3\,\log\left(\sin\left(dx+c\right)+1\right)+3\,\log\left(\sin\left(dx+c\right)-1\right)\right)-560}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^9*(a*cos(d*x+c)+b*sin(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] -1/8960*(35*b^4*(2*(3*sin(d*x + c)^7 - 11*sin(d*x + c)^5 - 11*sin(d*x + c)^
3 + 3*sin(d*x + c))/(sin(d*x + c)^8 - 4*sin(d*x + c)^6 + 6*sin(d*x + c)^4 -
 4*sin(d*x + c)^2 + 1) - 3*log(sin(d*x + c) + 1) + 3*log(sin(d*x + c) - 1))
 - 560*a^2*b^2*(2*(3*sin(d*x + c)^5 - 8*sin(d*x + c)^3 - 3*sin(d*x + c))/(s
in(d*x + c)^6 - 3*sin(d*x + c)^4 + 3*sin(d*x + c)^2 - 1) - 3*log(sin(d*x +
c) + 1) + 3*log(sin(d*x + c) - 1)) + 560*a^4*(2*(3*sin(d*x + c)^3 - 5*sin(d
*x + c))/(sin(d*x + c)^4 - 2*sin(d*x + c)^2 + 1) - 3*log(sin(d*x + c) + 1)
+ 3*log(sin(d*x + c) - 1)) - 7168*a^3*b/cos(d*x + c)^5 + 1024*(7*cos(d*x +
c)^2 - 5)*a*b^3/cos(d*x + c)^7)/d
```

mupad [B] time = 4.40, size = 566, normalized size = 1.72

$$\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{15}\,\left(\frac{5\,a^4}{4}+\frac{3\,a^2\,b^2}{4}-\frac{3\,b^4}{64}\right)+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3\,\left(-\frac{17\,a^4}{4}+\frac{41\,a^2\,b^2}{4}+\frac{23\,b^4}{64}\right)+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{13}\,\left(-\frac{17\,a^4}{4}+\frac{41\,a^2\,b^2}{4}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*cos(c + d*x) + b*sin(c + d*x))^4/cos(c + d*x)^9,x)
```

```
[Out] (tan(c/2 + (d*x)/2)^15*((5*a^4)/4 - (3*b^4)/64 + (3*a^2*b^2)/4) + tan(c/2 +
 (d*x)/2)^3*((23*b^4)/64 - (17*a^4)/4 + (41*a^2*b^2)/4) + tan(c/2 + (d*x)/2
)^13*((23*b^4)/64 - (17*a^4)/4 + (41*a^2*b^2)/4) + tan(c/2 + (d*x)/2)^5*((2
1*a^4)/4 + (333*b^4)/64 - (13*a^2*b^2)/4) + tan(c/2 + (d*x)/2)^11*((21*a^4)
/4 + (333*b^4)/64 - (13*a^2*b^2)/4) - tan(c/2 + (d*x)/2)^7*((9*a^4)/4 - (67
1*b^4)/64 + (31*a^2*b^2)/4) - tan(c/2 + (d*x)/2)^9*((9*a^4)/4 - (671*b^4)/6
4 + (31*a^2*b^2)/4) - (16*a*b^3)/35 + (8*a^3*b)/5 + tan(c/2 + (d*x)/2)*((5*
a^4)/4 - (3*b^4)/64 + (3*a^2*b^2)/4) - tan(c/2 + (d*x)/2)^12*(16*a*b^3 - 24
*a^3*b) - tan(c/2 + (d*x)/2)^8*(16*a*b^3 - 56*a^3*b) + tan(c/2 + (d*x)/2)^4
*((16*a*b^3)/5 + (104*a^3*b)/5) + tan(c/2 + (d*x)/2)^2*((128*a*b^3)/35 - (2
4*a^3*b)/5) + tan(c/2 + (d*x)/2)^6*((128*a*b^3)/5 - (248*a^3*b)/5) - 40*a^3
*b*tan(c/2 + (d*x)/2)^10 - 8*a^3*b*tan(c/2 + (d*x)/2)^14)/(d*(28*tan(c/2 +
(d*x)/2)^4 - 8*tan(c/2 + (d*x)/2)^2 - 56*tan(c/2 + (d*x)/2)^6 + 70*tan(c/2
+ (d*x)/2)^8 - 56*tan(c/2 + (d*x)/2)^10 + 28*tan(c/2 + (d*x)/2)^12 - 8*tan(
```

```
c/2 + (d*x)/2)^14 + tan(c/2 + (d*x)/2)^16 + 1)) + (atanh(tan(c/2 + (d*x)/2)
)*((3*a^4)/4 + (3*b^4)/64 - (3*a^2*b^2)/4))/d
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**9*(a*cos(d*x+c)+b*sin(d*x+c))**4,x)
[Out] Timed out
```

3.89 $\int \sec^{10}(c+dx)(a\cos(c+dx)+b\sin(c+dx))^4\,dx$

Optimal. Leaf size=201

$$\frac{a^4\tan(c+dx)}{d}+\frac{2a^3b\tan^2(c+dx)}{d}+\frac{2b^2\left(3a^2+b^2\right)\tan^7(c+dx)}{7d}+\frac{2ab\left(a^2+2b^2\right)\tan^6(c+dx)}{3d}+\frac{ab\left(2a^2+b^2\right)\tan^4}{d}$$

```
[Out] a^4*tan(d*x+c)/d+2*a^3*b*tan(d*x+c)^2/d+2/3*a^2*(a^2+3*b^2)*tan(d*x+c)^3/d+
a*b*(2*a^2+b^2)*tan(d*x+c)^4/d+1/5*(a^4+12*a^2*b^2+b^4)*tan(d*x+c)^5/d+2/3*
a*b*(a^2+2*b^2)*tan(d*x+c)^6/d+2/7*b^2*(3*a^2+b^2)*tan(d*x+c)^7/d+1/2*a*b^3
*tan(d*x+c)^8/d+1/9*b^4*tan(d*x+c)^9/d
```

Rubi [A] time = 0.17, antiderivative size = 201, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {3088, 948}

$$\frac{2b^2\left(3a^2+b^2\right)\tan^7(c+dx)}{7d}+\frac{2ab\left(a^2+2b^2\right)\tan^6(c+dx)}{3d}+\frac{\left(12a^2b^2+a^4+b^4\right)\tan^5(c+dx)}{5d}+\frac{ab\left(2a^2+b^2\right)\tan^4(c}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^10*(a*Cos[c + d*x] + b*Sin[c + d*x])^4,x]
```

```
[Out] (a^4*Tan[c + d*x])/d + (2*a^3*b*Tan[c + d*x]^2)/d + (2*a^2*(a^2 + 3*b^2)*Ta
n[c + d*x]^3)/(3*d) + (a*b*(2*a^2 + b^2)*Tan[c + d*x]^4)/d + ((a^4 + 12*a^2
*b^2 + b^4)*Tan[c + d*x]^5)/(5*d) + (2*a*b*(a^2 + 2*b^2)*Tan[c + d*x]^6)/(3
*d) + (2*b^2*(3*a^2 + b^2)*Tan[c + d*x]^7)/(7*d) + (a*b^3*Tan[c + d*x]^8)/(
2*d) + (b^4*Tan[c + d*x]^9)/(9*d)
```

Rule 948

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IGtQ[p, 0] && (IGtQ[m, 0] || (EqQ[m, -2] && EqQ[p, 1] &
& EqQ[d, 0]))
```

Rule 3088

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[(x^m*(b +
 a*x)^n)/(1 + x^2)^((m + n + 2)/2), x], x, Cot[c + d*x]], x] /; FreeQ[{a, b
, c, d}, x] && IntegerQ[n] && IntegerQ[(m + n)/2] && NeQ[n, -1] &&  !(GtQ[n
, 0] && GtQ[m, 1])
```

Rubi steps

$$\int \sec^{10}(c+dx)(a\cos(c+dx)+b\sin(c+dx))^4\,dx = -\frac{\text{Subst}\left(\int \frac{(b+ax)^4(1+x^2)^2}{x^{10}}\,dx, x, \cot(c+dx)\right)}{d}$$

$$= -\frac{\text{Subst}\left(\int \left(\frac{b^4}{x^{10}} + \frac{4ab^3}{x^9} + \frac{2(3a^2b^2+b^4)}{x^8} + \frac{4ab(a^2+2b^2)}{x^7} + \frac{a^4+12a^2b}{x^6}\right.\right.}{}$$

$$= \frac{a^4\tan(c+dx)}{d} + \frac{2a^3b\tan^2(c+dx)}{d} + \frac{2a^2\left(a^2+3b^2\right)\tan^3(}{3d}$$

Mathematica [A] time = 0.90, size = 115, normalized size = 0.57

$$\frac{\frac{2}{7}\left(3a^2+b^2\right)(a+b\tan(c+dx))^7 - \frac{2}{3}a\left(a^2+b^2\right)(a+b\tan(c+dx))^6 + \frac{1}{5}\left(a^2+b^2\right)^2(a+b\tan(c+dx))^5 + \frac{1}{9}(a+}{b^5d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^10*(a*Cos[c + d*x] + b*Sin[c + d*x])^4,x]
```

```
[Out] (((a^2 + b^2)^2*(a + b*Tan[c + d*x])^5)/5 - (2*a*(a^2 + b^2)*(a + b*Tan[c +
 d*x])^6)/3 + (2*(3*a^2 + b^2)*(a + b*Tan[c + d*x])^7)/7 - (a*(a + b*Tan[c
+ d*x])^8)/2 + (a + b*Tan[c + d*x])^9/9)/(b^5*d)
```

fricas [A] time = 0.76, size = 167, normalized size = 0.83

$$\frac{315\,ab^3\cos(dx+c) + 420\left(a^3b - ab^3\right)\cos(dx+c)^3 + 2\left(8\left(21\,a^4 - 18\,a^2b^2 + b^4\right)\cos(dx+c)^8 + 4\left(21\,a^4 - 18\,a^2\right.\right.}{630\,d\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^10*(a*cos(d*x+c)+b*sin(d*x+c))^4,x, algorithm="fricas"
)
```

```
[Out] 1/630*(315*a*b^3*cos(d*x + c) + 420*(a^3*b - a*b^3)*cos(d*x + c)^3 + 2*(8*(
21*a^4 - 18*a^2*b^2 + b^4)*cos(d*x + c)^8 + 4*(21*a^4 - 18*a^2*b^2 + b^4)*c
os(d*x + c)^6 + 3*(21*a^4 - 18*a^2*b^2 + b^4)*cos(d*x + c)^4 + 35*b^4 + 10*
(27*a^2*b^2 - 5*b^4)*cos(d*x + c)^2)*sin(d*x + c))/(d*cos(d*x + c)^9)
```

giac [A] time = 0.43, size = 214, normalized size = 1.06

$$\frac{70\,b^4\tan(dx+c)^9 + 315\,ab^3\tan(dx+c)^8 + 540\,a^2b^2\tan(dx+c)^7 + 180\,b^4\tan(dx+c)^7 + 420\,a^3b\tan(dx+c)}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^10*(a*cos(d*x+c)+b*sin(d*x+c))^4,x, algorithm="giac")
```

```
[Out] 1/630*(70*b^4*tan(d*x + c)^9 + 315*a*b^3*tan(d*x + c)^8 + 540*a^2*b^2*tan(d
*x + c)^7 + 180*b^4*tan(d*x + c)^7 + 420*a^3*b*tan(d*x + c)^6 + 840*a*b^3*t
an(d*x + c)^6 + 126*a^4*tan(d*x + c)^5 + 1512*a^2*b^2*tan(d*x + c)^5 + 126*
b^4*tan(d*x + c)^5 + 1260*a^3*b*tan(d*x + c)^4 + 630*a*b^3*tan(d*x + c)^4 +
 420*a^4*tan(d*x + c)^3 + 1260*a^2*b^2*tan(d*x + c)^3 + 1260*a^3*b*tan(d*x
+ c)^2 + 630*a^4*tan(d*x + c))/d
```

maple [A] time = 75.92, size = 236, normalized size = 1.17

$$\frac{-a^4\left(-\frac{8}{15}-\frac{\left(\sec^4(dx+c)\right)}{5}-\frac{4\left(\sec^2(dx+c)\right)}{15}\right)\tan(dx+c)+\frac{2a^3b}{3\cos(dx+c)^6}+6a^2b^2\left(\frac{\sin^3(dx+c)}{7\cos(dx+c)^7}+\frac{4\left(\sin^3(dx+c)\right)}{35\cos(dx+c)^5}+\frac{8\left(\sin^3(dx+c)\right)}{105\cos(dx+c)^3}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^10*(a*cos(d*x+c)+b*sin(d*x+c))^4,x)
```

```
[Out] 1/d*(-a^4*(-8/15-1/5*sec(d*x+c)^4-4/15*sec(d*x+c)^2)*tan(d*x+c)+2/3*a^3*b/c
os(d*x+c)^6+6*a^2*b^2*(1/7*sin(d*x+c)^3/cos(d*x+c)^7+4/35*sin(d*x+c)^3/cos(
d*x+c)^5+8/105*sin(d*x+c)^3/cos(d*x+c)^3)+4*a*b^3*(1/8*sin(d*x+c)^4/cos(d*x
+c)^8+1/12*sin(d*x+c)^4/cos(d*x+c)^6+1/24*sin(d*x+c)^4/cos(d*x+c)^4)+b^4*(1
/9*sin(d*x+c)^5/cos(d*x+c)^9+4/63*sin(d*x+c)^5/cos(d*x+c)^7+8/315*sin(d*x+c
)^5/cos(d*x+c)^5))
```

maxima [A] time = 0.33, size = 193, normalized size = 0.96

$$42\left(3\tan(dx+c)^5+10\tan(dx+c)^3+15\tan(dx+c)\right)a^4+36\left(15\tan(dx+c)^7+42\tan(dx+c)^5+35\tan(d\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^10*(a*cos(d*x+c)+b*sin(d*x+c))^4,x, algorithm="maxima"
)
```

```
[Out] 1/630*(42*(3*tan(d*x + c)^5 + 10*tan(d*x + c)^3 + 15*tan(d*x + c))*a^4 + 36
*(15*tan(d*x + c)^7 + 42*tan(d*x + c)^5 + 35*tan(d*x + c)^3)*a^2*b^2 + 2*(3
5*tan(d*x + c)^9 + 90*tan(d*x + c)^7 + 63*tan(d*x + c)^5)*b^4 + 105*(4*sin(
d*x + c)^2 - 1)*a*b^3/(sin(d*x + c)^8 - 4*sin(d*x + c)^6 + 6*sin(d*x + c)^4
 - 4*sin(d*x + c)^2 + 1) - 420*a^3*b/(sin(d*x + c)^2 - 1)^3)/d
```

mupad [B] time = 4.28, size = 447, normalized size = 2.22

$$-\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5\,\left(\frac{152\,a^4}{5}-\frac{96\,a^2\,b^2}{5}+\frac{32\,b^4}{5}\right)+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{13}\,\left(\frac{152\,a^4}{5}-\frac{96\,a^2\,b^2}{5}+\frac{32\,b^4}{5}\right)+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7\,\left(-\frac{288\,a^4}{5}+\frac{14}{}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*cos(c + d*x) + b*sin(c + d*x))^4/cos(c + d*x)^10,x)
```

```
[Out] -(tan(c/2 + (d*x)/2)^5*((152*a^4)/5 + (32*b^4)/5 - (96*a^2*b^2)/5) + tan(c/
2 + (d*x)/2)^13*((152*a^4)/5 + (32*b^4)/5 - (96*a^2*b^2)/5) + tan(c/2 + (d*
x)/2)^7*((384*b^4)/35 - (288*a^4)/5 + (1488*a^2*b^2)/35) + tan(c/2 + (d*x)/
2)^11*((384*b^4)/35 - (288*a^4)/5 + (1488*a^2*b^2)/35) + tan(c/2 + (d*x)/2)
^9*((1076*a^4)/15 + (6976*b^4)/315 - (2752*a^2*b^2)/35) + 2*a^4*tan(c/2 + (
d*x)/2)^17 - tan(c/2 + (d*x)/2)^3*((32*a^4)/3 - 16*a^2*b^2) - tan(c/2 + (d*
x)/2)^15*((32*a^4)/3 - 16*a^2*b^2) + 2*a^4*tan(c/2 + (d*x)/2) + tan(c/2 + (
d*x)/2)^4*(16*a*b^3 - 24*a^3*b) - tan(c/2 + (d*x)/2)^14*(16*a*b^3 - 24*a^3*
b) + tan(c/2 + (d*x)/2)^8*(32*a*b^3 - 88*a^3*b) - tan(c/2 + (d*x)/2)^10*(32
*a*b^3 - 88*a^3*b) + tan(c/2 + (d*x)/2)^6*((16*a*b^3)/3 + (152*a^3*b)/3) -
tan(c/2 + (d*x)/2)^12*((16*a*b^3)/3 + (152*a^3*b)/3) + 8*a^3*b*tan(c/2 + (d
*x)/2)^2 - 8*a^3*b*tan(c/2 + (d*x)/2)^16)/(d*(tan(c/2 + (d*x)/2)^2 - 1)^9)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**10*(a*cos(d*x+c)+b*sin(d*x+c))**4,x)
```

```
[Out] Timed out
```

3.90 $\int \sec^{11}(c+dx)(a\cos(c+dx)+b\sin(c+dx))^4\,dx$

Optimal. Leaf size=408

$$\frac{5a^4\tanh^{-1}(\sin(c+dx))}{16d}+\frac{a^4\tan(c+dx)\sec^5(c+dx)}{6d}+\frac{5a^4\tan(c+dx)\sec^3(c+dx)}{24d}+\frac{5a^4\tan(c+dx)\sec(c+dx)}{16d}$$

```
[Out] 5/16*a^4*arctanh(sin(d*x+c))/d-15/64*a^2*b^2*arctanh(sin(d*x+c))/d+3/256*b^
4*arctanh(sin(d*x+c))/d+4/7*a^3*b*sec(d*x+c)^7/d-4/7*a*b^3*sec(d*x+c)^7/d+4
/9*a*b^3*sec(d*x+c)^9/d+5/16*a^4*sec(d*x+c)*tan(d*x+c)/d-15/64*a^2*b^2*sec(
d*x+c)*tan(d*x+c)/d+3/256*b^4*sec(d*x+c)*tan(d*x+c)/d+5/24*a^4*sec(d*x+c)^3
*tan(d*x+c)/d-5/32*a^2*b^2*sec(d*x+c)^3*tan(d*x+c)/d+1/128*b^4*sec(d*x+c)^3
*tan(d*x+c)/d+1/6*a^4*sec(d*x+c)^5*tan(d*x+c)/d-1/8*a^2*b^2*sec(d*x+c)^5*ta
n(d*x+c)/d+1/160*b^4*sec(d*x+c)^5*tan(d*x+c)/d+3/4*a^2*b^2*sec(d*x+c)^7*tan
(d*x+c)/d-3/80*b^4*sec(d*x+c)^7*tan(d*x+c)/d+1/10*b^4*sec(d*x+c)^7*tan(d*x+
c)^3/d
```

Rubi [A] time = 0.40, antiderivative size = 408, normalized size of antiderivative = 1.00, number of steps used = 22, number of rules used = 7, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {3090, 3768, 3770, 2606, 30, 2611, 14}

$$-\frac{15a^2b^2\tanh^{-1}(\sin(c+dx))}{64d}+\frac{3a^2b^2\tan(c+dx)\sec^7(c+dx)}{4d}-\frac{a^2b^2\tan(c+dx)\sec^5(c+dx)}{8d}-\frac{5a^2b^2\tan(c+dx)\text{s}}{32d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^11*(a*Cos[c + d*x] + b*Sin[c + d*x])^4,x]
```

```
[Out] (5*a^4*ArcTanh[Sin[c + d*x]])/(16*d) - (15*a^2*b^2*ArcTanh[Sin[c + d*x]])/(
64*d) + (3*b^4*ArcTanh[Sin[c + d*x]])/(256*d) + (4*a^3*b*Sec[c + d*x]^7)/(7
*d) - (4*a*b^3*Sec[c + d*x]^7)/(7*d) + (4*a*b^3*Sec[c + d*x]^9)/(9*d) + (5*
a^4*Sec[c + d*x]*Tan[c + d*x])/(16*d) - (15*a^2*b^2*Sec[c + d*x]*Tan[c + d*
x])/(64*d) + (3*b^4*Sec[c + d*x]*Tan[c + d*x])/(256*d) + (5*a^4*Sec[c + d*x
]^3*Tan[c + d*x])/(24*d) - (5*a^2*b^2*Sec[c + d*x]^3*Tan[c + d*x])/(32*d) +
 (b^4*Sec[c + d*x]^3*Tan[c + d*x])/(128*d) + (a^4*Sec[c + d*x]^5*Tan[c + d*
x])/(6*d) - (a^2*b^2*Sec[c + d*x]^5*Tan[c + d*x])/(8*d) + (b^4*Sec[c + d*x]
^5*Tan[c + d*x])/(160*d) + (3*a^2*b^2*Sec[c + d*x]^7*Tan[c + d*x])/(4*d) -
(3*b^4*Sec[c + d*x]^7*Tan[c + d*x])/(80*d) + (b^4*Sec[c + d*x]^7*Tan[c + d*
x]^3)/(10*d)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 2611

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(b*(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*(
m + n - 1)), x] - Dist[(b^2*(n - 1))/(m + n - 1), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 3090

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_.), x_Symbol] :> Int[ExpandTrig[cos[c + d*x]^m*(a
*cos[c + d*x] + b*sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d}, x] && Inte
gerQ[m] && IGtQ[n, 0]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \sec^{11}(c+dx)(a\cos(c+dx)+b\sin(c+dx))^4\,dx = \int \left(a^4\sec^7(c+dx)+4a^3b\sec^7(c+dx)\tan(c+dx)+6a^2b^2\right.$$
$$= a^4\int \sec^7(c+dx)\,dx+\left(4a^3b\right)\int \sec^7(c+dx)\tan(c+dx)\,d$$
$$= \frac{a^4\sec^5(c+dx)\tan(c+dx)}{6d}+\frac{3a^2b^2\sec^7(c+dx)\tan(c+dx}{4d}$$
$$= \frac{4a^3b\sec^7(c+dx)}{7d}+\frac{5a^4\sec^3(c+dx)\tan(c+dx)}{24d}+\frac{a^4\sec^5}{}$$
$$= \frac{4a^3b\sec^7(c+dx)}{7d}-\frac{4ab^3\sec^7(c+dx)}{7d}+\frac{4ab^3\sec^9(c+dx)}{9d}$$
$$= \frac{5a^4\tanh^{-1}(\sin(c+dx))}{16d}+\frac{4a^3b\sec^7(c+dx)}{7d}-\frac{4ab^3\sec^7(c}{7d}$$
$$= \frac{5a^4\tanh^{-1}(\sin(c+dx))}{16d}-\frac{15a^2b^2\tanh^{-1}(\sin(c+dx))}{64d}+\frac{4a}{}$$
$$= \frac{5a^4\tanh^{-1}(\sin(c+dx))}{16d}-\frac{15a^2b^2\tanh^{-1}(\sin(c+dx))}{64d}+\frac{3b}{}$$

Mathematica [A] time = 1.31, size = 242, normalized size = 0.59

$$\frac{10\sec^9(c+dx)\left(32768ab\left(27a^2+b^2\right)+189\left(592a^4+1604a^2b^2+739b^4\right)\tan(c+dx)\right)-80640\left(80a^4-60a^2b^2+\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^11*(a*Cos[c + d*x] + b*Sin[c + d*x])^4,x]
```

```
[Out] (-80640*(80*a^4 - 60*a^2*b^2 + 3*b^4)*(Log[Cos[(c + d*x)/2] - Sin[(c + d*x)
/2]] - Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]) + 3*Sec[c + d*x]^10*(98304
0*a*b*(a^2 - b^2)*Cos[3*(c + d*x)] + 420*(1552*a^4 + 1908*a^2*b^2 - 505*b^4
)*Sin[3*(c + d*x)] + 7*(80*a^4 - 60*a^2*b^2 + 3*b^4)*(628*Sin[5*(c + d*x)]
+ 145*Sin[7*(c + d*x)] + 15*Sin[9*(c + d*x)])) + 10*Sec[c + d*x]^9*(32768*a
*b*(27*a^2 + b^2) + 189*(592*a^4 + 1604*a^2*b^2 + 739*b^4)*Tan[c + d*x]))/(
20643840*d)
```

fricas [A] time = 0.90, size = 251, normalized size = 0.62

$$\frac{315\left(80\,a^4-60\,a^2b^2+3\,b^4\right)\cos\left(dx+c\right)^{10}\log\left(\sin\left(dx+c\right)+1\right)-315\left(80\,a^4-60\,a^2b^2+3\,b^4\right)\cos\left(dx+c\right)^{10}\log\left(\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^11*(a*cos(d*x+c)+b*sin(d*x+c))^4,x, algorithm="fricas"
)
```

```
[Out] 1/161280*(315*(80*a^4 - 60*a^2*b^2 + 3*b^4)*cos(d*x + c)^10*log(sin(d*x + c
) + 1) - 315*(80*a^4 - 60*a^2*b^2 + 3*b^4)*cos(d*x + c)^10*log(-sin(d*x + c
) + 1) + 71680*a*b^3*cos(d*x + c) + 92160*(a^3*b - a*b^3)*cos(d*x + c)^3 +
42*(15*(80*a^4 - 60*a^2*b^2 + 3*b^4)*cos(d*x + c)^8 + 10*(80*a^4 - 60*a^2*b
^2 + 3*b^4)*cos(d*x + c)^6 + 8*(80*a^4 - 60*a^2*b^2 + 3*b^4)*cos(d*x + c)^4
 + 384*b^4 + 48*(60*a^2*b^2 - 11*b^4)*cos(d*x + c)^2)*sin(d*x + c))/(d*cos(
d*x + c)^10)
```

giac [B] time = 0.52, size = 880, normalized size = 2.16

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^11*(a*cos(d*x+c)+b*sin(d*x+c))^4,x, algorithm="giac")
```

```
[Out] 1/80640*(315*(80*a^4 - 60*a^2*b^2 + 3*b^4)*log(abs(tan(1/2*d*x + 1/2*c) + 1
)) - 315*(80*a^4 - 60*a^2*b^2 + 3*b^4)*log(abs(tan(1/2*d*x + 1/2*c) - 1)) +
 2*(55440*a^4*tan(1/2*d*x + 1/2*c)^19 + 18900*a^2*b^2*tan(1/2*d*x + 1/2*c)^
19 - 945*b^4*tan(1/2*d*x + 1/2*c)^19 - 322560*a^3*b*tan(1/2*d*x + 1/2*c)^18
 - 213360*a^4*tan(1/2*d*x + 1/2*c)^17 + 462420*a^2*b^2*tan(1/2*d*x + 1/2*c)
^17 + 9135*b^4*tan(1/2*d*x + 1/2*c)^17 + 967680*a^3*b*tan(1/2*d*x + 1/2*c)^
16 - 645120*a*b^3*tan(1/2*d*x + 1/2*c)^16 + 450240*a^4*tan(1/2*d*x + 1/2*c)
^15 + 146160*a^2*b^2*tan(1/2*d*x + 1/2*c)^15 + 218484*b^4*tan(1/2*d*x + 1/2
*c)^15 - 2580480*a^3*b*tan(1/2*d*x + 1/2*c)^14 - 430080*a*b^3*tan(1/2*d*x +
 1/2*c)^14 - 624960*a^4*tan(1/2*d*x + 1/2*c)^13 + 468720*a^2*b^2*tan(1/2*d*
x + 1/2*c)^13 + 653940*b^4*tan(1/2*d*x + 1/2*c)^13 + 5160960*a^3*b*tan(1/2*
d*x + 1/2*c)^12 - 2150400*a*b^3*tan(1/2*d*x + 1/2*c)^12 + 332640*a^4*tan(1/
2*d*x + 1/2*c)^11 - 1096200*a^2*b^2*tan(1/2*d*x + 1/2*c)^11 + 1183770*b^4*t
an(1/2*d*x + 1/2*c)^11 - 5806080*a^3*b*tan(1/2*d*x + 1/2*c)^10 + 1290240*a*
b^3*tan(1/2*d*x + 1/2*c)^10 + 332640*a^4*tan(1/2*d*x + 1/2*c)^9 - 1096200*a
^2*b^2*tan(1/2*d*x + 1/2*c)^9 + 1183770*b^4*tan(1/2*d*x + 1/2*c)^9 + 451584
0*a^3*b*tan(1/2*d*x + 1/2*c)^8 - 624960*a^4*tan(1/2*d*x + 1/2*c)^7 + 468720
*a^2*b^2*tan(1/2*d*x + 1/2*c)^7 + 653940*b^4*tan(1/2*d*x + 1/2*c)^7 - 29491
20*a^3*b*tan(1/2*d*x + 1/2*c)^6 + 1658880*a*b^3*tan(1/2*d*x + 1/2*c)^6 + 45
0240*a^4*tan(1/2*d*x + 1/2*c)^5 + 146160*a^2*b^2*tan(1/2*d*x + 1/2*c)^5 + 2
18484*b^4*tan(1/2*d*x + 1/2*c)^5 + 1105920*a^3*b*tan(1/2*d*x + 1/2*c)^4 + 1
84320*a*b^3*tan(1/2*d*x + 1/2*c)^4 - 213360*a^4*tan(1/2*d*x + 1/2*c)^3 + 46
2420*a^2*b^2*tan(1/2*d*x + 1/2*c)^3 + 9135*b^4*tan(1/2*d*x + 1/2*c)^3 - 138
240*a^3*b*tan(1/2*d*x + 1/2*c)^2 + 102400*a*b^3*tan(1/2*d*x + 1/2*c)^2 + 55
440*a^4*tan(1/2*d*x + 1/2*c) + 18900*a^2*b^2*tan(1/2*d*x + 1/2*c) - 945*b^4
*tan(1/2*d*x + 1/2*c) + 46080*a^3*b - 10240*a*b^3)/(tan(1/2*d*x + 1/2*c)^2
- 1)^10)/d
```

maple [A] time = 74.03, size = 590, normalized size = 1.45

$$\frac{b^4\left(\sin^5(dx+c)\right)}{10d\cos(dx+c)^{10}}+\frac{a^4\left(\sec^5(dx+c)\right)\tan(dx+c)}{6d}+\frac{b^4\left(\sin^5(dx+c)\right)}{32d\cos(dx+c)^6}+\frac{4a^3b}{7d\cos(dx+c)^7}-\frac{15a^2b^2\ln(\sec(dx+c)-}{64d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^11*(a*cos(d*x+c)+b*sin(d*x+c))^4,x)
```

```
[Out] 4/7/d*a^3*b/cos(d*x+c)^7+1/10/d*b^4*sin(d*x+c)^5/cos(d*x+c)^10-15/64/d*a^2*
b^2*ln(sec(d*x+c)+tan(d*x+c))-1/256/d*b^4*sin(d*x+c)^5/cos(d*x+c)^2+3/256/d
*b^4*ln(sec(d*x+c)+tan(d*x+c))+5/16/d*a^4*ln(sec(d*x+c)+tan(d*x+c))+5/16*a^
4*sec(d*x+c)*tan(d*x+c)/d+5/24*a^4*sec(d*x+c)^3*tan(d*x+c)/d+1/6*a^4*sec(d*
x+c)^5*tan(d*x+c)/d-8/63*a*b^3*cos(d*x+c)/d+15/64*a^2*b^2*sin(d*x+c)/d+20/6
3/d*a*b^3*sin(d*x+c)^4/cos(d*x+c)^7+15/64/d*a^2*b^2*sin(d*x+c)^3/cos(d*x+c)
^2-4/63/d*sin(d*x+c)^2*cos(d*x+c)*a*b^3-1/256*b^4*sin(d*x+c)^3/d-3/256*b^4*
sin(d*x+c)/d+1/32/d*b^4*sin(d*x+c)^5/cos(d*x+c)^6+4/21/d*a*b^3*sin(d*x+c)^4
/cos(d*x+c)^5+4/63/d*a*b^3*sin(d*x+c)^4/cos(d*x+c)^3-4/63/d*a*b^3*sin(d*x+c
)^4/cos(d*x+c)+15/32/d*a^2*b^2*sin(d*x+c)^3/cos(d*x+c)^4+5/8/d*a^2*b^2*sin(
d*x+c)^3/cos(d*x+c)^6+1/16/d*b^4*sin(d*x+c)^5/cos(d*x+c)^8+3/4/d*a^2*b^2*si
n(d*x+c)^3/cos(d*x+c)^8+4/9/d*a*b^3*sin(d*x+c)^4/cos(d*x+c)^9+1/128/d*b^4*s
in(d*x+c)^5/cos(d*x+c)^4
```

maxima [A] time = 0.34, size = 382, normalized size = 0.94

$$-\frac{63\,b^4\left(\frac{2\left(15\,\sin(dx+c)^9-70\,\sin(dx+c)^7+128\,\sin(dx+c)^5+70\,\sin(dx+c)^3-15\,\sin(dx+c)\right)}{\sin(dx+c)^{10}-5\,\sin(dx+c)^8+10\,\sin(dx+c)^6-10\,\sin(dx+c)^4+5\,\sin(dx+c)^2-1}-15\,\log(\sin(dx+c)+1)+15\,\log(\sin(dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^11*(a*cos(d*x+c)+b*sin(d*x+c))^4,x, algorithm="maxima"
)
```

```
[Out] -1/161280*(63*b^4*(2*(15*sin(d*x + c)^9 - 70*sin(d*x + c)^7 + 128*sin(d*x +
 c)^5 + 70*sin(d*x + c)^3 - 15*sin(d*x + c))/(sin(d*x + c)^10 - 5*sin(d*x +
 c)^8 + 10*sin(d*x + c)^6 - 10*sin(d*x + c)^4 + 5*sin(d*x + c)^2 - 1) - 15*
log(sin(d*x + c) + 1) + 15*log(sin(d*x + c) - 1)) - 1260*a^2*b^2*(2*(15*sin
(d*x + c)^7 - 55*sin(d*x + c)^5 + 73*sin(d*x + c)^3 + 15*sin(d*x + c))/(sin
(d*x + c)^8 - 4*sin(d*x + c)^6 + 6*sin(d*x + c)^4 - 4*sin(d*x + c)^2 + 1) -
 15*log(sin(d*x + c) + 1) + 15*log(sin(d*x + c) - 1)) + 1680*a^4*(2*(15*sin
(d*x + c)^5 - 40*sin(d*x + c)^3 + 33*sin(d*x + c))/(sin(d*x + c)^6 - 3*sin(
d*x + c)^4 + 3*sin(d*x + c)^2 - 1) - 15*log(sin(d*x + c) + 1) + 15*log(sin(
d*x + c) - 1)) - 92160*a^3*b/cos(d*x + c)^7 + 10240*(9*cos(d*x + c)^2 - 7)*
a*b^3/cos(d*x + c)^9)/d
```

mupad [B] time = 5.15, size = 703, normalized size = 1.72

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*cos(c + d*x) + b*sin(c + d*x))^4/cos(c + d*x)^11,x)
```

```
[Out] (atanh(tan(c/2 + (d*x)/2))*((5*a^4)/8 + (3*b^4)/128 - (15*a^2*b^2)/32))/d +
 (tan(c/2 + (d*x)/2)^19*((11*a^4)/8 - (3*b^4)/128 + (15*a^2*b^2)/32) + tan(
c/2 + (d*x)/2)^7*((519*b^4)/32 - (31*a^4)/2 + (93*a^2*b^2)/8) + tan(c/2 + (
d*x)/2)^13*((519*b^4)/32 - (31*a^4)/2 + (93*a^2*b^2)/8) + tan(c/2 + (d*x)/2
)^3*((29*b^4)/128 - (127*a^4)/24 + (367*a^2*b^2)/32) + tan(c/2 + (d*x)/2)^1
7*((29*b^4)/128 - (127*a^4)/24 + (367*a^2*b^2)/32) + tan(c/2 + (d*x)/2)^5*(
(67*a^4)/6 + (867*b^4)/160 + (29*a^2*b^2)/8) + tan(c/2 + (d*x)/2)^15*((67*a
^4)/6 + (867*b^4)/160 + (29*a^2*b^2)/8) + tan(c/2 + (d*x)/2)^9*((33*a^4)/4
+ (1879*b^4)/64 - (435*a^2*b^2)/16) + tan(c/2 + (d*x)/2)^11*((33*a^4)/4 + (
1879*b^4)/64 - (435*a^2*b^2)/16) - (16*a*b^3)/63 + (8*a^3*b)/7 + tan(c/2 +
(d*x)/2)*((11*a^4)/8 - (3*b^4)/128 + (15*a^2*b^2)/32) - tan(c/2 + (d*x)/2)^
16*(16*a*b^3 - 24*a^3*b) - tan(c/2 + (d*x)/2)^14*((32*a*b^3)/3 + 64*a^3*b)
+ tan(c/2 + (d*x)/2)^10*(32*a*b^3 - 144*a^3*b) + tan(c/2 + (d*x)/2)^4*((32*
a*b^3)/7 + (192*a^3*b)/7) + tan(c/2 + (d*x)/2)^2*((160*a*b^3)/63 - (24*a^3*
b)/7) - tan(c/2 + (d*x)/2)^12*((160*a*b^3)/3 - 128*a^3*b) + tan(c/2 + (d*x)
/2)^6*((288*a*b^3)/7 - (512*a^3*b)/7) + 112*a^3*b*tan(c/2 + (d*x)/2)^8 - 8*
a^3*b*tan(c/2 + (d*x)/2)^18)/(d*(45*tan(c/2 + (d*x)/2)^4 - 10*tan(c/2 + (d*
x)/2)^2 - 120*tan(c/2 + (d*x)/2)^6 + 210*tan(c/2 + (d*x)/2)^8 - 252*tan(c/2
 + (d*x)/2)^10 + 210*tan(c/2 + (d*x)/2)^12 - 120*tan(c/2 + (d*x)/2)^14 + 45
*tan(c/2 + (d*x)/2)^16 - 10*tan(c/2 + (d*x)/2)^18 + tan(c/2 + (d*x)/2)^20 +
 1))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**11*(a*cos(d*x+c)+b*sin(d*x+c))**4,x)
```

```
[Out] Timed out
```

3.91 $\int \sec^{12}(c+dx)(a\cos(c+dx)+b\sin(c+dx))^4\,dx$

Optimal. Leaf size=254

$$\frac{a^4\tan(c+dx)}{d}+\frac{2a^3b\tan^2(c+dx)}{d}+\frac{b^2\left(2a^2+b^2\right)\tan^9(c+dx)}{3d}+\frac{ab\left(a^2+3b^2\right)\tan^8(c+dx)}{2d}+\frac{2ab\left(a^2+b^2\right)\tan^6(c$$

```
[Out] a^4*tan(d*x+c)/d+2*a^3*b*tan(d*x+c)^2/d+a^2*(a^2+2*b^2)*tan(d*x+c)^3/d+a*b*
(3*a^2+b^2)*tan(d*x+c)^4/d+1/5*(3*a^4+18*a^2*b^2+b^4)*tan(d*x+c)^5/d+2*a*b*
(a^2+b^2)*tan(d*x+c)^6/d+1/7*(a^4+18*a^2*b^2+3*b^4)*tan(d*x+c)^7/d+1/2*a*b*
(a^2+3*b^2)*tan(d*x+c)^8/d+1/3*b^2*(2*a^2+b^2)*tan(d*x+c)^9/d+2/5*a*b^3*tan
(d*x+c)^10/d+1/11*b^4*tan(d*x+c)^11/d
```

Rubi [A] time = 0.22, antiderivative size = 254, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {3088, 948}

$$\frac{b^2\left(2a^2+b^2\right)\tan^9(c+dx)}{3d}+\frac{ab\left(a^2+3b^2\right)\tan^8(c+dx)}{2d}+\frac{\left(18a^2b^2+a^4+3b^4\right)\tan^7(c+dx)}{7d}+\frac{2ab\left(a^2+b^2\right)\tan^6(c+$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^12*(a*Cos[c + d*x] + b*Sin[c + d*x])^4,x]
```

```
[Out] (a^4*Tan[c + d*x])/d + (2*a^3*b*Tan[c + d*x]^2)/d + (a^2*(a^2 + 2*b^2)*Tan[
c + d*x]^3)/d + (a*b*(3*a^2 + b^2)*Tan[c + d*x]^4)/d + ((3*a^4 + 18*a^2*b^2
 + b^4)*Tan[c + d*x]^5)/(5*d) + (2*a*b*(a^2 + b^2)*Tan[c + d*x]^6)/d + ((a^
4 + 18*a^2*b^2 + 3*b^4)*Tan[c + d*x]^7)/(7*d) + (a*b*(a^2 + 3*b^2)*Tan[c +
d*x]^8)/(2*d) + (b^2*(2*a^2 + b^2)*Tan[c + d*x]^9)/(3*d) + (2*a*b^3*Tan[c +
 d*x]^10)/(5*d) + (b^4*Tan[c + d*x]^11)/(11*d)
```

Rule 948

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IGtQ[p, 0] && (IGtQ[m, 0] || (EqQ[m, -2] && EqQ[p, 1] &
& EqQ[d, 0]))
```

Rule 3088

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[(x^m*(b +
 a*x)^n)/(1 + x^2)^((m + n + 2)/2), x], x, Cot[c + d*x]], x] /; FreeQ[{a, b
, c, d}, x] && IntegerQ[n] && IntegerQ[(m + n)/2] && NeQ[n, -1] &&  !(GtQ[n
```

```
, 0] && GtQ[m, 1])
```

Rubi steps

$$\int \sec^{12}(c+dx)(a\cos(c+dx)+b\sin(c+dx))^4\,dx = -\frac{\text{Subst}\left(\int \frac{(b+ax)^4\left(1+x^2\right)^3}{x^{12}}\,dx, x, \cot(c+dx)\right)}{d}$$

$$= -\frac{\text{Subst}\left(\int \left(\frac{b^4}{x^{12}}+\frac{4ab^3}{x^{11}}+\frac{3\left(2a^2b^2+b^4\right)}{x^{10}}+\frac{4ab\left(a^2+3b^2\right)}{x^9}+\frac{a^4+18a^2b}{x^8}\right.\right.}{}$$

$$= \frac{a^4\tan(c+dx)}{d}+\frac{2a^3b\tan^2(c+dx)}{d}+\frac{a^2\left(a^2+2b^2\right)\tan^3(c}{d}$$

Mathematica [A] time = 1.80, size = 175, normalized size = 0.69

$$\frac{\frac{1}{3}\left(5a^2+b^2\right)(a+b\tan(c+dx))^9-\frac{1}{2}a\left(5a^2+3b^2\right)(a+b\tan(c+dx))^8+\frac{3}{7}\left(a^2+b^2\right)\left(5a^2+b^2\right)(a+b\tan(c+dx}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^12*(a*Cos[c + d*x] + b*Sin[c + d*x])^4,x]
```

```
[Out] (((a^2 + b^2)^3*(a + b*Tan[c + d*x])^5)/5 - a*(a^2 + b^2)^2*(a + b*Tan[c +
d*x])^6 + (3*(a^2 + b^2)*(5*a^2 + b^2)*(a + b*Tan[c + d*x])^7)/7 - (a*(5*a^
2 + 3*b^2)*(a + b*Tan[c + d*x])^8)/2 + ((5*a^2 + b^2)*(a + b*Tan[c + d*x])^
9)/3 - (3*a*(a + b*Tan[c + d*x])^10)/5 + (a + b*Tan[c + d*x])^11/11)/(b^7*d
)
```

fricas [A] time = 0.48, size = 194, normalized size = 0.76

$$\frac{924\,ab^3\cos(dx+c)+1155\left(a^3b-ab^3\right)\cos(dx+c)^3+2\left(16\left(33\,a^4-22\,a^2b^2+b^4\right)\cos(dx+c)^{10}+8\left(33\,a^4-2\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^12*(a*cos(d*x+c)+b*sin(d*x+c))^4,x, algorithm="fricas"
)
```

```
[Out] 1/2310*(924*a*b^3*cos(d*x + c) + 1155*(a^3*b - a*b^3)*cos(d*x + c)^3 + 2*(1
6*(33*a^4 - 22*a^2*b^2 + b^4)*cos(d*x + c)^10 + 8*(33*a^4 - 22*a^2*b^2 + b^
4)*cos(d*x + c)^8 + 6*(33*a^4 - 22*a^2*b^2 + b^4)*cos(d*x + c)^6 + 5*(33*a^
```

```
4 - 22*a^2*b^2 + b^4)*cos(d*x + c)^4 + 105*b^4 + 70*(11*a^2*b^2 - 2*b^4)*co
s(d*x + c)^2)*sin(d*x + c))/(d*cos(d*x + c)^11)
```

giac [A] time = 0.54, size = 284, normalized size = 1.12

$$\frac{210\,b^4\tan\left(dx+c\right)^{11}+924\,ab^3\tan\left(dx+c\right)^{10}+1540\,a^2b^2\tan\left(dx+c\right)^9+770\,b^4\tan\left(dx+c\right)^9+1155\,a^3b\tan\left(dx\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^12*(a*cos(d*x+c)+b*sin(d*x+c))^4,x, algorithm="giac")
```

```
[Out] 1/2310*(210*b^4*tan(d*x + c)^11 + 924*a*b^3*tan(d*x + c)^10 + 1540*a^2*b^2*
tan(d*x + c)^9 + 770*b^4*tan(d*x + c)^9 + 1155*a^3*b*tan(d*x + c)^8 + 3465*
a*b^3*tan(d*x + c)^8 + 330*a^4*tan(d*x + c)^7 + 5940*a^2*b^2*tan(d*x + c)^7
 + 990*b^4*tan(d*x + c)^7 + 4620*a^3*b*tan(d*x + c)^6 + 4620*a*b^3*tan(d*x
+ c)^6 + 1386*a^4*tan(d*x + c)^5 + 8316*a^2*b^2*tan(d*x + c)^5 + 462*b^4*ta
n(d*x + c)^5 + 6930*a^3*b*tan(d*x + c)^4 + 2310*a*b^3*tan(d*x + c)^4 + 2310
*a^4*tan(d*x + c)^3 + 4620*a^2*b^2*tan(d*x + c)^3 + 4620*a^3*b*tan(d*x + c)
^2 + 2310*a^4*tan(d*x + c))/d
```

maple [A] time = 78.72, size = 300, normalized size = 1.18

$$\frac{-a^4\left(-\frac{16}{35}-\frac{\left(\sec^6(dx+c)\right)}{7}-\frac{6\left(\sec^4(dx+c)\right)}{35}-\frac{8\left(\sec^2(dx+c)\right)}{35}\right)\tan\left(dx+c\right)+\frac{a^3b}{2\cos(dx+c)^8}+6a^2b^2\left(\frac{\sin^3(dx+c)}{9\cos(dx+c)^9}+\frac{2\left(\sin^3(dx+c)\right)}{21\cos(dx+c)^7}+\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^12*(a*cos(d*x+c)+b*sin(d*x+c))^4,x)
```

```
[Out] 1/d*(-a^4*(-16/35-1/7*sec(d*x+c)^6-6/35*sec(d*x+c)^4-8/35*sec(d*x+c)^2)*tan
(d*x+c)+1/2*a^3*b/cos(d*x+c)^8+6*a^2*b^2*(1/9*sin(d*x+c)^3/cos(d*x+c)^9+2/2
1*sin(d*x+c)^3/cos(d*x+c)^7+8/105*sin(d*x+c)^3/cos(d*x+c)^5+16/315*sin(d*x+
c)^3/cos(d*x+c)^3)+4*a*b^3*(1/10*sin(d*x+c)^4/cos(d*x+c)^10+3/40*sin(d*x+c)
^4/cos(d*x+c)^8+1/20*sin(d*x+c)^4/cos(d*x+c)^6+1/40*sin(d*x+c)^4/cos(d*x+c)
^4)+b^4*(1/11*sin(d*x+c)^5/cos(d*x+c)^11+2/33*sin(d*x+c)^5/cos(d*x+c)^9+8/2
31*sin(d*x+c)^5/cos(d*x+c)^7+16/1155*sin(d*x+c)^5/cos(d*x+c)^5))
```

maxima [A] time = 0.33, size = 233, normalized size = 0.92

$$\frac{66\left(5\,\tan\left(dx+c\right)^7+21\,\tan\left(dx+c\right)^5+35\,\tan\left(dx+c\right)^3+35\,\tan\left(dx+c\right)\right)a^4+44\left(35\,\tan\left(dx+c\right)^9+135\,\tan\left(\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^12*(a*cos(d*x+c)+b*sin(d*x+c))^4,x, algorithm="maxima"
)
```

```
[Out] 1/2310*(66*(5*tan(d*x + c)^7 + 21*tan(d*x + c)^5 + 35*tan(d*x + c)^3 + 35*t
an(d*x + c))*a^4 + 44*(35*tan(d*x + c)^9 + 135*tan(d*x + c)^7 + 189*tan(d*x
 + c)^5 + 105*tan(d*x + c)^3)*a^2*b^2 + 2*(105*tan(d*x + c)^11 + 385*tan(d*
x + c)^9 + 495*tan(d*x + c)^7 + 231*tan(d*x + c)^5)*b^4 - 231*(5*sin(d*x +
c)^2 - 1)*a*b^3/(sin(d*x + c)^10 - 5*sin(d*x + c)^8 + 10*sin(d*x + c)^6 - 1
0*sin(d*x + c)^4 + 5*sin(d*x + c)^2 - 1) + 1155*a^3*b/(sin(d*x + c)^2 - 1)^
4)/d
```

mupad [B] time = 4.82, size = 560, normalized size = 2.20

$$-\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5\,\left(\frac{226\,a^4}{5}-\frac{64\,a^2\,b^2}{5}+\frac{32\,b^4}{5}\right)+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{17}\,\left(\frac{226\,a^4}{5}-\frac{64\,a^2\,b^2}{5}+\frac{32\,b^4}{5}\right)+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^9\,\left(\frac{1308\,a^4}{7}-\frac{300}{}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*cos(c + d*x) + b*sin(c + d*x))^4/cos(c + d*x)^12,x)
```

```
[Out] -(tan(c/2 + (d*x)/2)^5*((226*a^4)/5 + (32*b^4)/5 - (64*a^2*b^2)/5) + tan(c/
2 + (d*x)/2)^17*((226*a^4)/5 + (32*b^4)/5 - (64*a^2*b^2)/5) + tan(c/2 + (d*
x)/2)^9*((1308*a^4)/7 + (992*b^4)/21 - (3008*a^2*b^2)/21) + tan(c/2 + (d*x)
/2)^13*((1308*a^4)/7 + (992*b^4)/21 - (3008*a^2*b^2)/21) + tan(c/2 + (d*x)/
2)^7*((576*b^4)/35 - (3952*a^4)/35 + (3008*a^2*b^2)/35) + tan(c/2 + (d*x)/2
)^15*((576*b^4)/35 - (3952*a^4)/35 + (3008*a^2*b^2)/35) + tan(c/2 + (d*x)/2
)^11*((10624*b^4)/231 - (1528*a^4)/7 + (2272*a^2*b^2)/21) + 2*a^4*tan(c/2 +
 (d*x)/2)^21 - tan(c/2 + (d*x)/2)^3*(12*a^4 - 16*a^2*b^2) - tan(c/2 + (d*x)
/2)^19*(12*a^4 - 16*a^2*b^2) + 2*a^4*tan(c/2 + (d*x)/2) + tan(c/2 + (d*x)/2
)^4*(16*a*b^3 - 24*a^3*b) - tan(c/2 + (d*x)/2)^18*(16*a*b^3 - 24*a^3*b) + t
an(c/2 + (d*x)/2)^6*(16*a*b^3 + 80*a^3*b) - tan(c/2 + (d*x)/2)^16*(16*a*b^3
 + 80*a^3*b) + tan(c/2 + (d*x)/2)^8*(80*a*b^3 - 176*a^3*b) - tan(c/2 + (d*x
)/2)^14*(80*a*b^3 - 176*a^3*b) - tan(c/2 + (d*x)/2)^10*((112*a*b^3)/5 - 224
*a^3*b) + tan(c/2 + (d*x)/2)^12*((112*a*b^3)/5 - 224*a^3*b) + 8*a^3*b*tan(c
/2 + (d*x)/2)^2 - 8*a^3*b*tan(c/2 + (d*x)/2)^20)/(d*(tan(c/2 + (d*x)/2)^2 -
 1)^11)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**12*(a*cos(d*x+c)+b*sin(d*x+c))**4,x)
```

```
[Out] Timed out
```

3.92 $\int \cos^5(c+dx)(a\cos(c+dx)+b\sin(c+dx))^5\,dx$

Optimal. Leaf size=515

$$\frac{a^5\sin(c+dx)\cos^9(c+dx)}{10d}+\frac{9a^5\sin(c+dx)\cos^7(c+dx)}{80d}+\frac{21a^5\sin(c+dx)\cos^5(c+dx)}{160d}+\frac{21a^5\sin(c+dx)\cos^3(c$$

```
[Out] 63/256*a^5*x+35/128*a^3*b^2*x+15/256*a*b^4*x-5/4*a^2*b^3*cos(d*x+c)^8/d-1/2
*a^4*b*cos(d*x+c)^10/d+a^2*b^3*cos(d*x+c)^10/d+63/256*a^5*cos(d*x+c)*sin(d*
x+c)/d+35/128*a^3*b^2*cos(d*x+c)*sin(d*x+c)/d+15/256*a*b^4*cos(d*x+c)*sin(d
*x+c)/d+21/128*a^5*cos(d*x+c)^3*sin(d*x+c)/d+35/192*a^3*b^2*cos(d*x+c)^3*si
n(d*x+c)/d+5/128*a*b^4*cos(d*x+c)^3*sin(d*x+c)/d+21/160*a^5*cos(d*x+c)^5*si
n(d*x+c)/d+7/48*a^3*b^2*cos(d*x+c)^5*sin(d*x+c)/d+1/32*a*b^4*cos(d*x+c)^5*s
in(d*x+c)/d+9/80*a^5*cos(d*x+c)^7*sin(d*x+c)/d+1/8*a^3*b^2*cos(d*x+c)^7*sin
(d*x+c)/d-3/16*a*b^4*cos(d*x+c)^7*sin(d*x+c)/d+1/10*a^5*cos(d*x+c)^9*sin(d*
x+c)/d-a^3*b^2*cos(d*x+c)^9*sin(d*x+c)/d-1/2*a*b^4*cos(d*x+c)^7*sin(d*x+c)^
3/d+1/6*b^5*sin(d*x+c)^6/d-1/4*b^5*sin(d*x+c)^8/d+1/10*b^5*sin(d*x+c)^10/d
```

Rubi [A] time = 0.48, antiderivative size = 515, normalized size of antiderivative = 1.00, number of steps used = 29, number of rules used = 10, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {3090, 2635, 8, 2565, 30, 2568, 14, 2564, 266, 43}

$$\frac{a^2b^3\cos^{10}(c+dx)}{d}-\frac{5a^2b^3\cos^8(c+dx)}{4d}-\frac{a^3b^2\sin(c+dx)\cos^9(c+dx)}{d}+\frac{a^3b^2\sin(c+dx)\cos^7(c+dx)}{8d}+\frac{7a^3b^2\sin($$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^5*(a*Cos[c + d*x] + b*Sin[c + d*x])^5,x]
```

```
[Out] (63*a^5*x)/256 + (35*a^3*b^2*x)/128 + (15*a*b^4*x)/256 - (5*a^2*b^3*Cos[c +
 d*x]^8)/(4*d) - (a^4*b*Cos[c + d*x]^10)/(2*d) + (a^2*b^3*Cos[c + d*x]^10)/
d + (63*a^5*Cos[c + d*x]*Sin[c + d*x])/(256*d) + (35*a^3*b^2*Cos[c + d*x]*S
in[c + d*x])/(128*d) + (15*a*b^4*Cos[c + d*x]*Sin[c + d*x])/(256*d) + (21*a
^5*Cos[c + d*x]^3*Sin[c + d*x])/(128*d) + (35*a^3*b^2*Cos[c + d*x]^3*Sin[c
+ d*x])/(192*d) + (5*a*b^4*Cos[c + d*x]^3*Sin[c + d*x])/(128*d) + (21*a^5*C
os[c + d*x]^5*Sin[c + d*x])/(160*d) + (7*a^3*b^2*Cos[c + d*x]^5*Sin[c + d*x
])/(48*d) + (a*b^4*Cos[c + d*x]^5*Sin[c + d*x])/(32*d) + (9*a^5*Cos[c + d*x
]^7*Sin[c + d*x])/(80*d) + (a^3*b^2*Cos[c + d*x]^7*Sin[c + d*x])/(8*d) - (3
*a*b^4*Cos[c + d*x]^7*Sin[c + d*x])/(16*d) + (a^5*Cos[c + d*x]^9*Sin[c + d*
x])/(10*d) - (a^3*b^2*Cos[c + d*x]^9*Sin[c + d*x])/d - (a*b^4*Cos[c + d*x]^
7*Sin[c + d*x]^3)/(2*d) + (b^5*Sin[c + d*x]^6)/(6*d) - (b^5*Sin[c + d*x]^8)
/(4*d) + (b^5*Sin[c + d*x]^10)/(10*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 2564

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 2565

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> -Dist[(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 2568

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> -Simp[(a*(b*Cos[e + f*x])^(n + 1)*(a*Sin[e + f*x])^(m - 1)
)/(b*f*(m + n)), x] + Dist[(a^2*(m - 1))/(m + n), Int[(b*Cos[e + f*x])^n*(a
*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1] &&
```

```
NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 3090

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_.), x_Symbol] :> Int[ExpandTrig[cos[c + d*x]^m*(a
*cos[c + d*x] + b*sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d}, x] && Inte
gerQ[m] && IGtQ[n, 0]
```

Rubi steps

$$\int \cos^5(c+dx)(a\cos(c+dx)+b\sin(c+dx))^5\,dx = \int \left(a^5\cos^{10}(c+dx)+5a^4b\cos^9(c+dx)\sin(c+dx)+10a^3b\right.$$

$$= a^5\int \cos^{10}(c+dx)\,dx+\left(5a^4b\right)\int \cos^9(c+dx)\sin(c+dx)\,d$$

$$= \frac{a^5\cos^9(c+dx)\sin(c+dx)}{10d}-\frac{a^3b^2\cos^9(c+dx)\sin(c+dx)}{d}$$

$$= -\frac{a^4b\cos^{10}(c+dx)}{2d}+\frac{9a^5\cos^7(c+dx)\sin(c+dx)}{80d}+\frac{a^3b^2\,\mathrm{co}}{}$$

$$= -\frac{5a^2b^3\cos^8(c+dx)}{4d}-\frac{a^4b\cos^{10}(c+dx)}{2d}+\frac{a^2b^3\cos^{10}(c+d}{d}$$

$$= -\frac{5a^2b^3\cos^8(c+dx)}{4d}-\frac{a^4b\cos^{10}(c+dx)}{2d}+\frac{a^2b^3\cos^{10}(c+d}{d}$$

$$= -\frac{5a^2b^3\cos^8(c+dx)}{4d}-\frac{a^4b\cos^{10}(c+dx)}{2d}+\frac{a^2b^3\cos^{10}(c+d}{d}$$

$$= \frac{63a^5x}{256}+\frac{35}{128}a^3b^2x+\frac{15}{256}ab^4x-\frac{5a^2b^3\cos^8(c+dx)}{4d}-\frac{a^4b\,\mathrm{c}}{}$$

Mathematica [A] time = 1.23, size = 307, normalized size = 0.60

$$\frac{-1200a^2b\left(3a^2+b^2\right)\cos(4(c+dx))-300a^2b\left(a^2-b^2\right)\cos(8(c+dx))+120a\left(63a^4+70a^2b^2+15b^4\right)(c+dx)+3}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^5*(a*Cos[c + d*x] + b*Sin[c + d*x])^5,x]
```

```
[Out] (120*a*(63*a^4 + 70*a^2*b^2 + 15*b^4)*(c + d*x) - 300*b*(21*a^4 + 14*a^2*b^
2 + b^4)*Cos[2*(c + d*x)] - 1200*a^2*b*(3*a^2 + b^2)*Cos[4*(c + d*x)] + 50*
b*(-27*a^4 + 6*a^2*b^2 + b^4)*Cos[6*(c + d*x)] - 300*a^2*b*(a^2 - b^2)*Cos[
8*(c + d*x)] - 6*b*(5*a^4 - 10*a^2*b^2 + b^4)*Cos[10*(c + d*x)] + 300*a*(21
*a^4 + 14*a^2*b^2 + b^4)*Sin[2*(c + d*x)] + 600*a*(3*a^4 - 2*a^2*b^2 - b^4)
*Sin[4*(c + d*x)] + 50*a*(9*a^4 - 26*a^2*b^2 - 3*b^4)*Sin[6*(c + d*x)] + 75
*a*(a^4 - 6*a^2*b^2 + b^4)*Sin[8*(c + d*x)] + 6*a*(a^4 - 10*a^2*b^2 + 5*b^4
)*Sin[10*(c + d*x)])/(30720*d)
```

fricas [A] time = 0.81, size = 250, normalized size = 0.49

$$-\frac{640\,b^5\cos\left(dx+c\right)^6+384\left(5\,a^4b-10\,a^2b^3+b^5\right)\cos\left(dx+c\right)^{10}+960\left(5\,a^2b^3-b^5\right)\cos\left(dx+c\right)^8-15\left(63\,a^5+\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(a*cos(d*x+c)+b*sin(d*x+c))^5,x, algorithm="fricas")
```

```
[Out] -1/3840*(640*b^5*cos(d*x + c)^6 + 384*(5*a^4*b - 10*a^2*b^3 + b^5)*cos(d*x
+ c)^10 + 960*(5*a^2*b^3 - b^5)*cos(d*x + c)^8 - 15*(63*a^5 + 70*a^3*b^2 +
15*a*b^4)*d*x - (384*(a^5 - 10*a^3*b^2 + 5*a*b^4)*cos(d*x + c)^9 + 48*(9*a^
5 + 10*a^3*b^2 - 55*a*b^4)*cos(d*x + c)^7 + 8*(63*a^5 + 70*a^3*b^2 + 15*a*b
^4)*cos(d*x + c)^5 + 10*(63*a^5 + 70*a^3*b^2 + 15*a*b^4)*cos(d*x + c)^3 + 1
5*(63*a^5 + 70*a^3*b^2 + 15*a*b^4)*cos(d*x + c))*sin(d*x + c))/d
```

giac [A] time = 0.86, size = 342, normalized size = 0.66

$$\frac{1}{256}\left(63\,a^5+70\,a^3b^2+15\,ab^4\right)x-\frac{\left(5\,a^4b-10\,a^2b^3+b^5\right)\cos\left(10\,dx+10\,c\right)}{5120\,d}-\frac{5\left(a^4b-a^2b^3\right)\cos\left(8\,dx+8\,c\right)}{512\,d}-\frac{5\left(27\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(a*cos(d*x+c)+b*sin(d*x+c))^5,x, algorithm="giac")
```

```
[Out] 1/256*(63*a^5 + 70*a^3*b^2 + 15*a*b^4)*x - 1/5120*(5*a^4*b - 10*a^2*b^3 + b
^5)*cos(10*d*x + 10*c)/d - 5/512*(a^4*b - a^2*b^3)*cos(8*d*x + 8*c)/d - 5/3
072*(27*a^4*b - 6*a^2*b^3 - b^5)*cos(6*d*x + 6*c)/d - 5/128*(3*a^4*b + a^2*
b^3)*cos(4*d*x + 4*c)/d - 5/512*(21*a^4*b + 14*a^2*b^3 + b^5)*cos(2*d*x + 2
*c)/d + 1/5120*(a^5 - 10*a^3*b^2 + 5*a*b^4)*sin(10*d*x + 10*c)/d + 5/2048*(
a^5 - 6*a^3*b^2 + a*b^4)*sin(8*d*x + 8*c)/d + 5/3072*(9*a^5 - 26*a^3*b^2 -
3*a*b^4)*sin(6*d*x + 6*c)/d + 5/256*(3*a^5 - 2*a^3*b^2 - a*b^4)*sin(4*d*x +
 4*c)/d + 5/512*(21*a^5 + 14*a^3*b^2 + a*b^4)*sin(2*d*x + 2*c)/d
```

maple [A] time = 0.33, size = 335, normalized size = 0.65

$$\frac{b^5\left(-\frac{\left(\sin^4(dx+c)\right)\left(\cos^6(dx+c)\right)}{10}-\frac{\left(\sin^2(dx+c)\right)\left(\cos^6(dx+c)\right)}{20}-\frac{\left(\cos^6(dx+c)\right)}{60}\right)+5a\,b^4\left(-\frac{\left(\sin^3(dx+c)\right)\left(\cos^7(dx+c)\right)}{10}-\frac{3\sin(dx+c)\left(\cos^7(dx+c\right.}{80}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*(a*cos(d*x+c)+b*sin(d*x+c))^5,x)
```

```
[Out] 1/d*(b^5*(-1/10*sin(d*x+c)^4*cos(d*x+c)^6-1/20*sin(d*x+c)^2*cos(d*x+c)^6-1/
60*cos(d*x+c)^6)+5*a*b^4*(-1/10*sin(d*x+c)^3*cos(d*x+c)^7-3/80*sin(d*x+c)*c
os(d*x+c)^7+1/160*(cos(d*x+c)^5+5/4*cos(d*x+c)^3+15/8*cos(d*x+c))*sin(d*x+c
)+3/256*d*x+3/256*c)+10*a^2*b^3*(-1/10*sin(d*x+c)^2*cos(d*x+c)^8-1/40*cos(d
*x+c)^8)+10*a^3*b^2*(-1/10*sin(d*x+c)*cos(d*x+c)^9+1/80*(cos(d*x+c)^7+7/6*c
os(d*x+c)^5+35/24*cos(d*x+c)^3+35/16*cos(d*x+c))*sin(d*x+c)+7/256*d*x+7/256
*c)-1/2*a^4*b*cos(d*x+c)^10+a^5*(1/10*(cos(d*x+c)^9+9/8*cos(d*x+c)^7+21/16*
cos(d*x+c)^5+105/64*cos(d*x+c)^3+315/128*cos(d*x+c))*sin(d*x+c)+63/256*d*x+
63/256*c))
```

maxima [A] time = 0.34, size = 290, normalized size = 0.56

$$-\frac{15360\,a^4b\cos\left(dx+c\right)^{10}-3\left(32\,\sin\left(2\,dx+2\,c\right)^5-640\,\sin\left(2\,dx+2\,c\right)^3+2520\,dx+2520\,c+25\,\sin\left(8\,dx+8\,c\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(a*cos(d*x+c)+b*sin(d*x+c))^5,x, algorithm="maxima")
```

```
[Out] -1/30720*(15360*a^4*b*cos(d*x + c)^10 - 3*(32*sin(2*d*x + 2*c)^5 - 640*sin(
2*d*x + 2*c)^3 + 2520*d*x + 2520*c + 25*sin(8*d*x + 8*c) + 600*sin(4*d*x +
4*c) + 2560*sin(2*d*x + 2*c))*a^5 + 10*(96*sin(2*d*x + 2*c)^5 - 640*sin(2*d
*x + 2*c)^3 - 840*d*x - 840*c + 45*sin(8*d*x + 8*c) + 120*sin(4*d*x + 4*c))
*a^3*b^2 + 7680*(4*sin(d*x + c)^10 - 15*sin(d*x + c)^8 + 20*sin(d*x + c)^6
- 10*sin(d*x + c)^4)*a^2*b^3 - 15*(32*sin(2*d*x + 2*c)^5 + 120*d*x + 120*c
+ 5*sin(8*d*x + 8*c) - 40*sin(4*d*x + 4*c))*a*b^4 - 512*(6*sin(d*x + c)^10
- 15*sin(d*x + c)^8 + 10*sin(d*x + c)^6)*b^5)/d
```

mupad [B] time = 2.51, size = 801, normalized size = 1.56

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^5*(a*cos(c + d*x) + b*sin(c + d*x))^5,x)
```

```
[Out] (tan(c/2 + (d*x)/2)^19*((15*a*b^4)/128 - (193*a^5)/128 + (35*a^3*b^2)/64) -
 tan(c/2 + (d*x)/2)*((15*a*b^4)/128 - (193*a^5)/128 + (35*a^3*b^2)/64) + ta
n(c/2 + (d*x)/2)^3*((159*a^5)/128 - (145*a*b^4)/128 + (4105*a^3*b^2)/192) -
 tan(c/2 + (d*x)/2)^17*((159*a^5)/128 - (145*a*b^4)/128 + (4105*a^3*b^2)/19
2) - tan(c/2 + (d*x)/2)^7*((2595*a*b^4)/32 + (147*a^5)/32 - (2905*a^3*b^2)/
16) + tan(c/2 + (d*x)/2)^13*((2595*a*b^4)/32 + (147*a^5)/32 - (2905*a^3*b^2
)/16) + tan(c/2 + (d*x)/2)^5*((867*a*b^4)/32 + (2847*a^5)/160 - (2891*a^3*b
^2)/48) - tan(c/2 + (d*x)/2)^15*((867*a*b^4)/32 + (2847*a^5)/160 - (2891*a^
3*b^2)/48) + tan(c/2 + (d*x)/2)^9*((9395*a*b^4)/64 + (1827*a^5)/64 - (7945*
a^3*b^2)/32) - tan(c/2 + (d*x)/2)^11*((9395*a*b^4)/64 + (1827*a^5)/64 - (79
45*a^3*b^2)/32) + tan(c/2 + (d*x)/2)^6*(120*a^4*b + (32*b^5)/3 - 80*a^2*b^3
) + tan(c/2 + (d*x)/2)^14*(120*a^4*b + (32*b^5)/3 - 80*a^2*b^3) + tan(c/2 +
 (d*x)/2)^10*(252*a^4*b + (192*b^5)/5 - 224*a^2*b^3) - tan(c/2 + (d*x)/2)^8
*((64*b^5)/3 - 280*a^2*b^3) - tan(c/2 + (d*x)/2)^12*((64*b^5)/3 - 280*a^2*b
^3) + 40*a^2*b^3*tan(c/2 + (d*x)/2)^4 + 40*a^2*b^3*tan(c/2 + (d*x)/2)^16 +
10*a^4*b*tan(c/2 + (d*x)/2)^2 + 10*a^4*b*tan(c/2 + (d*x)/2)^18)/(d*(10*tan(
c/2 + (d*x)/2)^2 + 45*tan(c/2 + (d*x)/2)^4 + 120*tan(c/2 + (d*x)/2)^6 + 210
*tan(c/2 + (d*x)/2)^8 + 252*tan(c/2 + (d*x)/2)^10 + 210*tan(c/2 + (d*x)/2)^
12 + 120*tan(c/2 + (d*x)/2)^14 + 45*tan(c/2 + (d*x)/2)^16 + 10*tan(c/2 + (d
*x)/2)^18 + tan(c/2 + (d*x)/2)^20 + 1)) + (a*atan((a*tan(c/2 + (d*x)/2)*(63
*a^4 + 15*b^4 + 70*a^2*b^2))/(128*((15*a*b^4)/128 + (63*a^5)/128 + (35*a^3*
b^2)/64)))*(63*a^4 + 15*b^4 + 70*a^2*b^2))/(128*d) - (a*(atan(tan(c/2 + (d*
x)/2)) - (d*x)/2)*(63*a^4 + 15*b^4 + 70*a^2*b^2))/(128*d)
```

sympy [A] time = 27.12, size = 1037, normalized size = 2.01

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*(a*cos(d*x+c)+b*sin(d*x+c))**5,x)
```

```
[Out] Piecewise((63*a**5*x*sin(c + d*x)**10/256 + 315*a**5*x*sin(c + d*x)**8*cos(
c + d*x)**2/256 + 315*a**5*x*sin(c + d*x)**6*cos(c + d*x)**4/128 + 315*a**5
*x*sin(c + d*x)**4*cos(c + d*x)**6/128 + 315*a**5*x*sin(c + d*x)**2*cos(c +
 d*x)**8/256 + 63*a**5*x*cos(c + d*x)**10/256 + 63*a**5*sin(c + d*x)**9*cos
(c + d*x)/(256*d) + 147*a**5*sin(c + d*x)**7*cos(c + d*x)**3/(128*d) + 21*a
**5*sin(c + d*x)**5*cos(c + d*x)**5/(10*d) + 237*a**5*sin(c + d*x)**3*cos(c
 + d*x)**7/(128*d) + 193*a**5*sin(c + d*x)*cos(c + d*x)**9/(256*d) - a**4*b
*cos(c + d*x)**10/(2*d) + 35*a**3*b**2*x*sin(c + d*x)**10/128 + 175*a**3*b*
*2*x*sin(c + d*x)**8*cos(c + d*x)**2/128 + 175*a**3*b**2*x*sin(c + d*x)**6*
cos(c + d*x)**4/64 + 175*a**3*b**2*x*sin(c + d*x)**4*cos(c + d*x)**6/64 + 1
75*a**3*b**2*x*sin(c + d*x)**2*cos(c + d*x)**8/128 + 35*a**3*b**2*x*cos(c +
 d*x)**10/128 + 35*a**3*b**2*sin(c + d*x)**9*cos(c + d*x)/(128*d) + 245*a**
3*b**2*sin(c + d*x)**7*cos(c + d*x)**3/(192*d) + 7*a**3*b**2*sin(c + d*x)**
5*cos(c + d*x)**5/(3*d) + 395*a**3*b**2*sin(c + d*x)**3*cos(c + d*x)**7/(19
```

```
2*d) - 35*a**3*b**2*sin(c + d*x)*cos(c + d*x)**9/(128*d) + a**2*b**3*sin(c
+ d*x)**10/(4*d) + 5*a**2*b**3*sin(c + d*x)**8*cos(c + d*x)**2/(4*d) + 5*a*
*2*b**3*sin(c + d*x)**6*cos(c + d*x)**4/(2*d) + 5*a**2*b**3*sin(c + d*x)**4
*cos(c + d*x)**6/(2*d) + 15*a*b**4*x*sin(c + d*x)**10/256 + 75*a*b**4*x*sin
(c + d*x)**8*cos(c + d*x)**2/256 + 75*a*b**4*x*sin(c + d*x)**6*cos(c + d*x)
**4/128 + 75*a*b**4*x*sin(c + d*x)**4*cos(c + d*x)**6/128 + 75*a*b**4*x*sin
(c + d*x)**2*cos(c + d*x)**8/256 + 15*a*b**4*x*cos(c + d*x)**10/256 + 15*a*
b**4*sin(c + d*x)**9*cos(c + d*x)/(256*d) + 35*a*b**4*sin(c + d*x)**7*cos(c
 + d*x)**3/(128*d) + a*b**4*sin(c + d*x)**5*cos(c + d*x)**5/(2*d) - 35*a*b*
*4*sin(c + d*x)**3*cos(c + d*x)**7/(128*d) - 15*a*b**4*sin(c + d*x)*cos(c +
 d*x)**9/(256*d) + b**5*sin(c + d*x)**10/(60*d) + b**5*sin(c + d*x)**8*cos(
c + d*x)**2/(12*d) + b**5*sin(c + d*x)**6*cos(c + d*x)**4/(6*d), Ne(d, 0)),
 (x*(a*cos(c) + b*sin(c))**5*cos(c)**5, True))
```

3.93 $\int \cos^4(c+dx)(a\cos(c+dx)+b\sin(c+dx))^5\,dx$

Optimal. Leaf size=337

$$\frac{a^5\sin^9(c+dx)}{9d}-\frac{4a^5\sin^7(c+dx)}{7d}+\frac{6a^5\sin^5(c+dx)}{5d}-\frac{4a^5\sin^3(c+dx)}{3d}+\frac{a^5\sin(c+dx)}{d}-\frac{5a^4b\cos^9(c+dx)}{9d}-\frac{10a^3}{}$$

```
[Out] -1/5*b^5*cos(d*x+c)^5/d-10/7*a^2*b^3*cos(d*x+c)^7/d+2/7*b^5*cos(d*x+c)^7/d-
5/9*a^4*b*cos(d*x+c)^9/d+10/9*a^2*b^3*cos(d*x+c)^9/d-1/9*b^5*cos(d*x+c)^9/d
+a^5*sin(d*x+c)/d-4/3*a^5*sin(d*x+c)^3/d+10/3*a^3*b^2*sin(d*x+c)^3/d+6/5*a^
5*sin(d*x+c)^5/d-6*a^3*b^2*sin(d*x+c)^5/d+a*b^4*sin(d*x+c)^5/d-4/7*a^5*sin(
d*x+c)^7/d+30/7*a^3*b^2*sin(d*x+c)^7/d-10/7*a*b^4*sin(d*x+c)^7/d+1/9*a^5*si
n(d*x+c)^9/d-10/9*a^3*b^2*sin(d*x+c)^9/d+5/9*a*b^4*sin(d*x+c)^9/d
```

Rubi [A] time = 0.30, antiderivative size = 337, normalized size of antiderivative = 1.00, number of steps used = 18, number of rules used = 7, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {3090, 2633, 2565, 30, 2564, 270, 14}

$$-\frac{10a^3b^2\sin^9(c+dx)}{9d}+\frac{30a^3b^2\sin^7(c+dx)}{7d}-\frac{6a^3b^2\sin^5(c+dx)}{d}+\frac{10a^3b^2\sin^3(c+dx)}{3d}+\frac{10a^2b^3\cos^9(c+dx)}{9d}-\frac{10a}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^4*(a*Cos[c + d*x] + b*Sin[c + d*x])^5,x]
```

```
[Out] -(b^5*Cos[c + d*x]^5)/(5*d) - (10*a^2*b^3*Cos[c + d*x]^7)/(7*d) + (2*b^5*Co
s[c + d*x]^7)/(7*d) - (5*a^4*b*Cos[c + d*x]^9)/(9*d) + (10*a^2*b^3*Cos[c +
d*x]^9)/(9*d) - (b^5*Cos[c + d*x]^9)/(9*d) + (a^5*Sin[c + d*x])/d - (4*a^5*
Sin[c + d*x]^3)/(3*d) + (10*a^3*b^2*Sin[c + d*x]^3)/(3*d) + (6*a^5*Sin[c +
d*x]^5)/(5*d) - (6*a^3*b^2*Sin[c + d*x]^5)/d + (a*b^4*Sin[c + d*x]^5)/d - (
4*a^5*Sin[c + d*x]^7)/(7*d) + (30*a^3*b^2*Sin[c + d*x]^7)/(7*d) - (10*a*b^4
*Sin[c + d*x]^7)/(7*d) + (a^5*Sin[c + d*x]^9)/(9*d) - (10*a^3*b^2*Sin[c + d
*x]^9)/(9*d) + (5*a*b^4*Sin[c + d*x]^9)/(9*d)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2564

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 2565

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> -Dist[(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 3090

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_.), x_Symbol] :> Int[ExpandTrig[cos[c + d*x]^m*(a
*cos[c + d*x] + b*sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d}, x] && Inte
gerQ[m] && IGtQ[n, 0]
```

Rubi steps

$$\int \cos^4(c+dx)(a\cos(c+dx)+b\sin(c+dx))^5\,dx = \int \left(a^5\cos^9(c+dx)+5a^4b\cos^8(c+dx)\sin(c+dx)+10a^3b\right.$$

$$= a^5\int \cos^9(c+dx)\,dx + \left(5a^4b\right)\int \cos^8(c+dx)\sin(c+dx)\,d$$

$$= -\frac{a^5\,\mathrm{Subst}\left(\int \left(1-4x^2+6x^4-4x^6+x^8\right)\,dx, x, -\sin(c+dx\right.}{d}$$

$$= -\frac{5a^4b\cos^9(c+dx)}{9d} + \frac{a^5\sin(c+dx)}{d} - \frac{4a^5\sin^3(c+dx)}{3d} +$$

$$= -\frac{b^5\cos^5(c+dx)}{5d} - \frac{10a^2b^3\cos^7(c+dx)}{7d} + \frac{2b^5\cos^7(c+dx)}{7d}$$

Mathematica [A] time = 1.02, size = 278, normalized size = 0.82

$$\frac{420a\left(21a^4-5b^4\right)\sin(3(c+dx))+252b\left(b^4-25a^4\right)\cos(5(c+dx))+630a\left(63a^4+70a^2b^2+15b^4\right)\sin(c+dx)}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^4*(a*Cos[c + d*x] + b*Sin[c + d*x])^5,x]
```

```
[Out] (-630*b*(35*a^4 + 30*a^2*b^2 + 3*b^4)*Cos[c + d*x] - 420*b*(35*a^4 + 20*a^2
*b^2 + b^4)*Cos[3*(c + d*x)] + 252*b*(-25*a^4 + b^4)*Cos[5*(c + d*x)] + 45*
b*(-35*a^4 + 30*a^2*b^2 + b^4)*Cos[7*(c + d*x)] - 35*b*(5*a^4 - 10*a^2*b^2
+ b^4)*Cos[9*(c + d*x)] + 630*a*(63*a^4 + 70*a^2*b^2 + 15*b^4)*Sin[c + d*x]
 + 420*a*(21*a^4 - 5*b^4)*Sin[3*(c + d*x)] + 252*a*(9*a^4 - 20*a^2*b^2 - 5*
b^4)*Sin[5*(c + d*x)] + 45*a*(9*a^4 - 50*a^2*b^2 + 5*b^4)*Sin[7*(c + d*x)]
+ 35*a*(a^4 - 10*a^2*b^2 + 5*b^4)*Sin[9*(c + d*x)])/(80640*d)
```

fricas [A] time = 0.68, size = 217, normalized size = 0.64

$$-\frac{63\,b^5\cos\left(dx+c\right)^5+35\left(5\,a^4b-10\,a^2b^3+b^5\right)\cos\left(dx+c\right)^9+90\left(5\,a^2b^3-b^5\right)\cos\left(dx+c\right)^7-\left(35\left(a^5-10\,a^3b\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a*cos(d*x+c)+b*sin(d*x+c))^5,x, algorithm="fricas")
```

```
[Out] -1/315*(63*b^5*cos(d*x + c)^5 + 35*(5*a^4*b - 10*a^2*b^3 + b^5)*cos(d*x + c
)^9 + 90*(5*a^2*b^3 - b^5)*cos(d*x + c)^7 - (35*(a^5 - 10*a^3*b^2 + 5*a*b^4
)*cos(d*x + c)^8 + 10*(4*a^5 + 5*a^3*b^2 - 25*a*b^4)*cos(d*x + c)^6 + 128*a
^5 + 160*a^3*b^2 + 40*a*b^4 + 3*(16*a^5 + 20*a^3*b^2 + 5*a*b^4)*cos(d*x + c
)^4 + 4*(16*a^5 + 20*a^3*b^2 + 5*a*b^4)*cos(d*x + c)^2)*sin(d*x + c))/d
```

giac [A] time = 0.70, size = 313, normalized size = 0.93

$$-\frac{\left(5\,a^{4}b-10\,a^{2}b^{3}+b^{5}\right)\cos\left(9\,dx+9\,c\right)}{2304\,d}-\frac{\left(35\,a^{4}b-30\,a^{2}b^{3}-b^{5}\right)\cos\left(7\,dx+7\,c\right)}{1792\,d}-\frac{\left(25\,a^{4}b-b^{5}\right)\cos\left(5\,dx+5\,c\right)}{320\,d}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a*cos(d*x+c)+b*sin(d*x+c))^5,x, algorithm="giac")
```

```
[Out] -1/2304*(5*a^4*b - 10*a^2*b^3 + b^5)*cos(9*d*x + 9*c)/d - 1/1792*(35*a^4*b
- 30*a^2*b^3 - b^5)*cos(7*d*x + 7*c)/d - 1/320*(25*a^4*b - b^5)*cos(5*d*x +
 5*c)/d - 1/192*(35*a^4*b + 20*a^2*b^3 + b^5)*cos(3*d*x + 3*c)/d - 1/128*(3
5*a^4*b + 30*a^2*b^3 + 3*b^5)*cos(d*x + c)/d + 1/2304*(a^5 - 10*a^3*b^2 + 5
*a*b^4)*sin(9*d*x + 9*c)/d + 1/1792*(9*a^5 - 50*a^3*b^2 + 5*a*b^4)*sin(7*d*
x + 7*c)/d + 1/320*(9*a^5 - 20*a^3*b^2 - 5*a*b^4)*sin(5*d*x + 5*c)/d + 1/19
2*(21*a^5 - 5*a*b^4)*sin(3*d*x + 3*c)/d + 1/128*(63*a^5 + 70*a^3*b^2 + 15*a
*b^4)*sin(d*x + c)/d
```

maple [A] time = 0.24, size = 291, normalized size = 0.86

$$\frac{b^{5}\left(-\frac{\left(\sin^{4}(dx+c)\right)\left(\cos^{5}(dx+c)\right)}{9}-\frac{4\left(\sin^{2}(dx+c)\right)\left(\cos^{5}(dx+c)\right)}{63}-\frac{8\left(\cos^{5}(dx+c)\right)}{315}\right)+5a\,b^{4}\left(-\frac{\left(\sin^{3}(dx+c)\right)\left(\cos^{6}(dx+c)\right)}{9}-\frac{\sin(dx+c)\left(\cos^{6}(dx+c\right.}{21}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*(a*cos(d*x+c)+b*sin(d*x+c))^5,x)
```

```
[Out] 1/d*(b^5*(-1/9*sin(d*x+c)^4*cos(d*x+c)^5-4/63*sin(d*x+c)^2*cos(d*x+c)^5-8/3
15*cos(d*x+c)^5)+5*a*b^4*(-1/9*sin(d*x+c)^3*cos(d*x+c)^6-1/21*sin(d*x+c)*co
s(d*x+c)^6+1/105*(8/3+cos(d*x+c)^4+4/3*cos(d*x+c)^2)*sin(d*x+c))+10*a^2*b^3
*(-1/9*sin(d*x+c)^2*cos(d*x+c)^7-2/63*cos(d*x+c)^7)+10*a^3*b^2*(-1/9*sin(d*
x+c)*cos(d*x+c)^8+1/63*(16/5+cos(d*x+c)^6+6/5*cos(d*x+c)^4+8/5*cos(d*x+c)^2
)*sin(d*x+c))-5/9*a^4*b*cos(d*x+c)^9+1/9*a^5*(128/35+cos(d*x+c)^8+8/7*cos(d
*x+c)^6+48/35*cos(d*x+c)^4+64/35*cos(d*x+c)^2)*sin(d*x+c))
```

maxima [A] time = 0.33, size = 224, normalized size = 0.66

$$-\frac{175\,a^{4}b\cos\left(dx+c\right)^{9}-\left(35\,\sin\left(dx+c\right)^{9}-180\,\sin\left(dx+c\right)^{7}+378\,\sin\left(dx+c\right)^{5}-420\,\sin\left(dx+c\right)^{3}+315\,\sin\left(\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a*cos(d*x+c)+b*sin(d*x+c))^5,x, algorithm="maxima")
```

```
[Out] -1/315*(175*a^4*b*cos(d*x + c)^9 - (35*sin(d*x + c)^9 - 180*sin(d*x + c)^7
+ 378*sin(d*x + c)^5 - 420*sin(d*x + c)^3 + 315*sin(d*x + c))*a^5 + 10*(35*
sin(d*x + c)^9 - 135*sin(d*x + c)^7 + 189*sin(d*x + c)^5 - 105*sin(d*x + c)
^3)*a^3*b^2 - 50*(7*cos(d*x + c)^9 - 9*cos(d*x + c)^7)*a^2*b^3 - 5*(35*sin(
d*x + c)^9 - 90*sin(d*x + c)^7 + 63*sin(d*x + c)^5)*a*b^4 + (35*cos(d*x + c
)^9 - 90*cos(d*x + c)^7 + 63*cos(d*x + c)^5)*b^5)/d
```

mupad [B] time = 4.34, size = 495, normalized size = 1.47

$$\frac{2\,a^5\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{17}+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5\,\left(\frac{152\,a^5}{5}-32\,a^3\,b^2+32\,a\,b^4\right)+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{13}\,\left(\frac{152\,a^5}{5}-32\,a^3\,b^2+32\,a\,b^4\right)+}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^4*(a*cos(c + d*x) + b*sin(c + d*x))^5,x)
```

```
[Out] (2*a^5*tan(c/2 + (d*x)/2)^17 + tan(c/2 + (d*x)/2)^5*(32*a*b^4 + (152*a^5)/5
 - 32*a^3*b^2) + tan(c/2 + (d*x)/2)^13*(32*a*b^4 + (152*a^5)/5 - 32*a^3*b^2
) + tan(c/2 + (d*x)/2)^7*((1136*a^5)/35 - (384*a*b^4)/7 + (1264*a^3*b^2)/7)
 + tan(c/2 + (d*x)/2)^11*((1136*a^5)/35 - (384*a*b^4)/7 + (1264*a^3*b^2)/7)
 + tan(c/2 + (d*x)/2)^9*((6976*a*b^4)/63 + (21316*a^5)/315 - (5696*a^3*b^2)
/63) - tan(c/2 + (d*x)/2)^4*(40*a^4*b + (64*b^5)/35 - (120*a^2*b^3)/7) - ta
n(c/2 + (d*x)/2)^8*(140*a^4*b + (112*b^5)/5 - 120*a^2*b^3) - tan(c/2 + (d*x
)/2)^12*((280*a^4*b)/3 + (32*b^5)/3 - (200*a^2*b^3)/3) - (10*a^4*b)/9 - (16
*b^5)/315 - (40*a^2*b^3)/63 + tan(c/2 + (d*x)/2)^3*((16*a^5)/3 + (80*a^3*b^
2)/3) + tan(c/2 + (d*x)/2)^15*((16*a^5)/3 + (80*a^3*b^2)/3) + 2*a^5*tan(c/2
 + (d*x)/2) - tan(c/2 + (d*x)/2)^2*((16*b^5)/35 + (40*a^2*b^3)/7) + tan(c/2
 + (d*x)/2)^6*((32*b^5)/5 - 120*a^2*b^3) + tan(c/2 + (d*x)/2)^10*(16*b^5 -
200*a^2*b^3) - 40*a^2*b^3*tan(c/2 + (d*x)/2)^14 - 10*a^4*b*tan(c/2 + (d*x)/
2)^16)/(d*(tan(c/2 + (d*x)/2)^2 + 1)^9)
```

sympy [A] time = 16.23, size = 440, normalized size = 1.31

$$\begin{cases} \frac{128a^5\sin^9(c+dx)}{315d}+\frac{64a^5\sin^7(c+dx)\cos^2(c+dx)}{35d}+\frac{16a^5\sin^5(c+dx)\cos^4(c+dx)}{5d}+\frac{8a^5\sin^3(c+dx)\cos^6(c+dx)}{3d}+\frac{a^5\sin(c+dx)\cos^8(c+dx)}{d} - \\ x\left(a\cos(c)+b\sin(c)\right)^5\cos^4(c) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*(a*cos(d*x+c)+b*sin(d*x+c))**5,x)
```

```
[Out] Piecewise((128*a**5*sin(c + d*x)**9/(315*d) + 64*a**5*sin(c + d*x)**7*cos(c
 + d*x)**2/(35*d) + 16*a**5*sin(c + d*x)**5*cos(c + d*x)**4/(5*d) + 8*a**5*
```

```
sin(c + d*x)**3*cos(c + d*x)**6/(3*d) + a**5*sin(c + d*x)*cos(c + d*x)**8/d
 - 5*a**4*b*cos(c + d*x)**9/(9*d) + 32*a**3*b**2*sin(c + d*x)**9/(63*d) + 1
6*a**3*b**2*sin(c + d*x)**7*cos(c + d*x)**2/(7*d) + 4*a**3*b**2*sin(c + d*x
)**5*cos(c + d*x)**4/d + 10*a**3*b**2*sin(c + d*x)**3*cos(c + d*x)**6/(3*d)
 - 10*a**2*b**3*sin(c + d*x)**2*cos(c + d*x)**7/(7*d) - 20*a**2*b**3*cos(c
+ d*x)**9/(63*d) + 8*a*b**4*sin(c + d*x)**9/(63*d) + 4*a*b**4*sin(c + d*x)*
*7*cos(c + d*x)**2/(7*d) + a*b**4*sin(c + d*x)**5*cos(c + d*x)**4/d - b**5*
sin(c + d*x)**4*cos(c + d*x)**5/(5*d) - 4*b**5*sin(c + d*x)**2*cos(c + d*x)
**7/(35*d) - 8*b**5*cos(c + d*x)**9/(315*d), Ne(d, 0)), (x*(a*cos(c) + b*si
n(c))**5*cos(c)**4, True))
```

3.94 $\int \cos^3(c+dx)(a\cos(c+dx)+b\sin(c+dx))^5\,dx$

Optimal. Leaf size=426

$$\frac{a^5\sin(c+dx)\cos^7(c+dx)}{8d}+\frac{7a^5\sin(c+dx)\cos^5(c+dx)}{48d}+\frac{35a^5\sin(c+dx)\cos^3(c+dx)}{192d}+\frac{35a^5\sin(c+dx)\cos(\ldots}{128d}$$

```
[Out] 35/128*a^5*x+25/64*a^3*b^2*x+15/128*a*b^4*x-5/3*a^2*b^3*cos(d*x+c)^6/d-5/8*
a^4*b*cos(d*x+c)^8/d+5/4*a^2*b^3*cos(d*x+c)^8/d+35/128*a^5*cos(d*x+c)*sin(d
*x+c)/d+25/64*a^3*b^2*cos(d*x+c)*sin(d*x+c)/d+15/128*a*b^4*cos(d*x+c)*sin(d
*x+c)/d+35/192*a^5*cos(d*x+c)^3*sin(d*x+c)/d+25/96*a^3*b^2*cos(d*x+c)^3*sin
(d*x+c)/d+5/64*a*b^4*cos(d*x+c)^3*sin(d*x+c)/d+7/48*a^5*cos(d*x+c)^5*sin(d*
x+c)/d+5/24*a^3*b^2*cos(d*x+c)^5*sin(d*x+c)/d-5/16*a*b^4*cos(d*x+c)^5*sin(d
*x+c)/d+1/8*a^5*cos(d*x+c)^7*sin(d*x+c)/d-5/4*a^3*b^2*cos(d*x+c)^7*sin(d*x+
c)/d-5/8*a*b^4*cos(d*x+c)^5*sin(d*x+c)^3/d+1/6*b^5*sin(d*x+c)^6/d-1/8*b^5*s
in(d*x+c)^8/d
```

Rubi [A] time = 0.41, antiderivative size = 426, normalized size of antiderivative = 1.00, number of steps used = 25, number of rules used = 8, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3090, 2635, 8, 2565, 30, 2568, 14, 2564}

$$\frac{5a^2b^3\cos^8(c+dx)}{4d}-\frac{5a^2b^3\cos^6(c+dx)}{3d}-\frac{5a^3b^2\sin(c+dx)\cos^7(c+dx)}{4d}+\frac{5a^3b^2\sin(c+dx)\cos^5(c+dx)}{24d}+\frac{25a^3b\ldots}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^3*(a*Cos[c + d*x] + b*Sin[c + d*x])^5,x]
```

```
[Out] (35*a^5*x)/128 + (25*a^3*b^2*x)/64 + (15*a*b^4*x)/128 - (5*a^2*b^3*Cos[c +
d*x]^6)/(3*d) - (5*a^4*b*Cos[c + d*x]^8)/(8*d) + (5*a^2*b^3*Cos[c + d*x]^8)
/(4*d) + (35*a^5*Cos[c + d*x]*Sin[c + d*x])/(128*d) + (25*a^3*b^2*Cos[c + d
*x]*Sin[c + d*x])/(64*d) + (15*a*b^4*Cos[c + d*x]*Sin[c + d*x])/(128*d) + (
35*a^5*Cos[c + d*x]^3*Sin[c + d*x])/(192*d) + (25*a^3*b^2*Cos[c + d*x]^3*Si
n[c + d*x])/(96*d) + (5*a*b^4*Cos[c + d*x]^3*Sin[c + d*x])/(64*d) + (7*a^5*
Cos[c + d*x]^5*Sin[c + d*x])/(48*d) + (5*a^3*b^2*Cos[c + d*x]^5*Sin[c + d*x
])/(24*d) - (5*a*b^4*Cos[c + d*x]^5*Sin[c + d*x])/(16*d) + (a^5*Cos[c + d*x
]^7*Sin[c + d*x])/(8*d) - (5*a^3*b^2*Cos[c + d*x]^7*Sin[c + d*x])/(4*d) - (
5*a*b^4*Cos[c + d*x]^5*Sin[c + d*x]^3)/(8*d) + (b^5*Sin[c + d*x]^6)/(6*d) -
 (b^5*Sin[c + d*x]^8)/(8*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2564

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 2565

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> -Dist[(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 2568

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> -Simp[(a*(b*Cos[e + f*x])^(n + 1)*(a*Sin[e + f*x])^(m - 1)
)/(b*f*(m + n)), x] + Dist[(a^2*(m - 1))/(m + n), Int[(b*Cos[e + f*x])^n*(a
*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1] &&
 NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 3090

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_.), x_Symbol] :> Int[ExpandTrig[cos[c + d*x]^m*(a
*cos[c + d*x] + b*sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d}, x] && Inte
gerQ[m] && IGtQ[n, 0]
```

Rubi steps

$$\int \cos^3(c+dx)(a\cos(c+dx)+b\sin(c+dx))^5\,dx = \int \left(a^5\cos^8(c+dx)+5a^4b\cos^7(c+dx)\sin(c+dx)+10a^3b\right.$$

$$= a^5\int \cos^8(c+dx)\,dx + \left(5a^4b\right)\int \cos^7(c+dx)\sin(c+dx)\,d$$

$$= \frac{a^5\cos^7(c+dx)\sin(c+dx)}{8d} - \frac{5a^3b^2\cos^7(c+dx)\sin(c+dx}{4d}$$

$$= -\frac{5a^4b\cos^8(c+dx)}{8d} + \frac{7a^5\cos^5(c+dx)\sin(c+dx)}{48d} + \frac{5a^3b^2}{}$$

$$= -\frac{5a^2b^3\cos^6(c+dx)}{3d} - \frac{5a^4b\cos^8(c+dx)}{8d} + \frac{5a^2b^3\cos^8(c+}{4d}$$

$$= -\frac{5a^2b^3\cos^6(c+dx)}{3d} - \frac{5a^4b\cos^8(c+dx)}{8d} + \frac{5a^2b^3\cos^8(c+}{4d}$$

$$= \frac{35a^5x}{128} + \frac{25}{64}a^3b^2x + \frac{15}{128}ab^4x - \frac{5a^2b^3\cos^6(c+dx)}{3d} - \frac{5a^4b}{}$$

Mathematica [C] time = 0.87, size = 259, normalized size = 0.61

$$\frac{120a(a-ib)(a+ib)\left(7a^2+3b^2\right)(c+dx)+24a\left(7a^4-10a^2b^2-5b^4\right)\sin(4(c+dx))+3a\left(a^4-10a^2b^2+5b^4\right)\sin}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^3*(a*Cos[c + d*x] + b*Sin[c + d*x])^5,x]
```

```
[Out] (120*a*(a - I*b)*(a + I*b)*(7*a^2 + 3*b^2)*(c + d*x) - 24*b*(35*a^4 + 30*a^
2*b^2 + 3*b^4)*Cos[2*(c + d*x)] + 12*b*(-35*a^4 - 10*a^2*b^2 + b^4)*Cos[4*(
c + d*x)] + 8*b*(-15*a^4 + 10*a^2*b^2 + b^4)*Cos[6*(c + d*x)] - 3*b*(5*a^4
- 10*a^2*b^2 + b^4)*Cos[8*(c + d*x)] + 96*a^3*(7*a^2 + 5*b^2)*Sin[2*(c + d*
x)] + 24*a*(7*a^4 - 10*a^2*b^2 - 5*b^4)*Sin[4*(c + d*x)] + 32*a^3*(a^2 - 5*
b^2)*Sin[6*(c + d*x)] + 3*a*(a^4 - 10*a^2*b^2 + 5*b^4)*Sin[8*(c + d*x)])/(3
072*d)
```

fricas [A] time = 0.70, size = 220, normalized size = 0.52

$$-\frac{96\,b^5\cos(dx+c)^4+48\left(5\,a^4b-10\,a^2b^3+b^5\right)\cos(dx+c)^8+128\left(5\,a^2b^3-b^5\right)\cos(dx+c)^6-15\left(7\,a^5+10\,a^3\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a*cos(d*x+c)+b*sin(d*x+c))^5,x, algorithm="fricas")
```

```
[Out] -1/384*(96*b^5*cos(d*x + c)^4 + 48*(5*a^4*b - 10*a^2*b^3 + b^5)*cos(d*x + c
)^8 + 128*(5*a^2*b^3 - b^5)*cos(d*x + c)^6 - 15*(7*a^5 + 10*a^3*b^2 + 3*a*b
^4)*d*x - (48*(a^5 - 10*a^3*b^2 + 5*a*b^4)*cos(d*x + c)^7 + 8*(7*a^5 + 10*a
^3*b^2 - 45*a*b^4)*cos(d*x + c)^5 + 10*(7*a^5 + 10*a^3*b^2 + 3*a*b^4)*cos(d
*x + c)^3 + 15*(7*a^5 + 10*a^3*b^2 + 3*a*b^4)*cos(d*x + c))*sin(d*x + c))/d
```

giac [A] time = 0.64, size = 278, normalized size = 0.65

$$\frac{5}{128}\left(7\,a^5+10\,a^3b^2+3\,ab^4\right)x-\frac{\left(5\,a^4b-10\,a^2b^3+b^5\right)\cos\left(8\,dx+8\,c\right)}{1024\,d}-\frac{\left(15\,a^4b-10\,a^2b^3-b^5\right)\cos\left(6\,dx+6\,c\right)}{384\,d}-\frac{(3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a*cos(d*x+c)+b*sin(d*x+c))^5,x, algorithm="giac")
```

```
[Out] 5/128*(7*a^5 + 10*a^3*b^2 + 3*a*b^4)*x - 1/1024*(5*a^4*b - 10*a^2*b^3 + b^5
)*cos(8*d*x + 8*c)/d - 1/384*(15*a^4*b - 10*a^2*b^3 - b^5)*cos(6*d*x + 6*c)
/d - 1/256*(35*a^4*b + 10*a^2*b^3 - b^5)*cos(4*d*x + 4*c)/d - 1/128*(35*a^4
*b + 30*a^2*b^3 + 3*b^5)*cos(2*d*x + 2*c)/d + 1/1024*(a^5 - 10*a^3*b^2 + 5*
a*b^4)*sin(8*d*x + 8*c)/d + 1/96*(a^5 - 5*a^3*b^2)*sin(6*d*x + 6*c)/d + 1/1
28*(7*a^5 - 10*a^3*b^2 - 5*a*b^4)*sin(4*d*x + 4*c)/d + 1/32*(7*a^5 + 5*a^3*
b^2)*sin(2*d*x + 2*c)/d
```

maple [A] time = 0.18, size = 305, normalized size = 0.72

$$\frac{b^5\left(-\frac{\left(\sin^4(dx+c)\right)\left(\cos^4(dx+c)\right)}{8}-\frac{\left(\sin^2(dx+c)\right)\left(\cos^4(dx+c)\right)}{12}-\frac{\left(\cos^4(dx+c)\right)}{24}\right)+5a\,b^4\left(-\frac{\left(\sin^3(dx+c)\right)\left(\cos^5(dx+c)\right)}{8}-\frac{\sin(dx+c)\left(\cos^5(dx+c)\right)}{16}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3*(a*cos(d*x+c)+b*sin(d*x+c))^5,x)
```

```
[Out] 1/d*(b^5*(-1/8*sin(d*x+c)^4*cos(d*x+c)^4-1/12*sin(d*x+c)^2*cos(d*x+c)^4-1/2
4*cos(d*x+c)^4)+5*a*b^4*(-1/8*sin(d*x+c)^3*cos(d*x+c)^5-1/16*sin(d*x+c)*cos
(d*x+c)^5+1/64*(cos(d*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+3/128*d*x+3/128*c)+
10*a^2*b^3*(-1/8*sin(d*x+c)^2*cos(d*x+c)^6-1/24*cos(d*x+c)^6)+10*a^3*b^2*(-
1/8*sin(d*x+c)*cos(d*x+c)^7+1/48*(cos(d*x+c)^5+5/4*cos(d*x+c)^3+15/8*cos(d*
x+c))*sin(d*x+c)+5/128*d*x+5/128*c)-5/8*a^4*b*cos(d*x+c)^8+a^5*(1/8*(cos(d*
x+c)^7+7/6*cos(d*x+c)^5+35/24*cos(d*x+c)^3+35/16*cos(d*x+c))*sin(d*x+c)+35/
128*d*x+35/128*c))
```

maxima [A] time = 0.33, size = 228, normalized size = 0.54

$$-\frac{1920\,a^4b\cos\left(dx+c\right)^8+\left(128\,\sin\left(2\,dx+2\,c\right)^3-840\,dx-840\,c-3\,\sin\left(8\,dx+8\,c\right)-168\,\sin\left(4\,dx+4\,c\right)-768}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a*cos(d*x+c)+b*sin(d*x+c))^5,x, algorithm="maxima")
```

```
[Out] -1/3072*(1920*a^4*b*cos(d*x + c)^8 + (128*sin(2*d*x + 2*c)^3 - 840*d*x - 84
0*c - 3*sin(8*d*x + 8*c) - 168*sin(4*d*x + 4*c) - 768*sin(2*d*x + 2*c))*a^5
 - 10*(64*sin(2*d*x + 2*c)^3 + 120*d*x + 120*c - 3*sin(8*d*x + 8*c) - 24*si
n(4*d*x + 4*c))*a^3*b^2 - 1280*(3*sin(d*x + c)^8 - 8*sin(d*x + c)^6 + 6*sin
(d*x + c)^4)*a^2*b^3 - 15*(24*d*x + 24*c + sin(8*d*x + 8*c) - 8*sin(4*d*x +
 4*c))*a*b^4 + 128*(3*sin(d*x + c)^8 - 4*sin(d*x + c)^6)*b^5)/d
```

mupad [B] time = 2.48, size = 650, normalized size = 1.53

$$\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{15}\,\left(-\frac{93\,a^5}{64}+\frac{25\,a^3\,b^2}{32}+\frac{15\,a\,b^4}{64}\right)-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(-\frac{93\,a^5}{64}+\frac{25\,a^3\,b^2}{32}+\frac{15\,a\,b^4}{64}\right)+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3\,\left(\frac{91\,a^5}{192}+\frac{198}{}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^3*(a*cos(c + d*x) + b*sin(c + d*x))^5,x)
```

```
[Out] (tan(c/2 + (d*x)/2)^15*((15*a*b^4)/64 - (93*a^5)/64 + (25*a^3*b^2)/32) - ta
n(c/2 + (d*x)/2)*((15*a*b^4)/64 - (93*a^5)/64 + (25*a^3*b^2)/32) + tan(c/2
+ (d*x)/2)^3*((91*a^5)/192 - (115*a*b^4)/64 + (1985*a^3*b^2)/96) - tan(c/2
+ (d*x)/2)^13*((91*a^5)/192 - (115*a*b^4)/64 + (1985*a^3*b^2)/96) + tan(c/2
 + (d*x)/2)^5*((1665*a*b^4)/64 + (1799*a^5)/192 - (4475*a^3*b^2)/96) - tan(
c/2 + (d*x)/2)^11*((1665*a*b^4)/64 + (1799*a^5)/192 - (4475*a^3*b^2)/96) -
tan(c/2 + (d*x)/2)^7*((3355*a*b^4)/64 + (1085*a^5)/192 - (8825*a^3*b^2)/96)
 + tan(c/2 + (d*x)/2)^9*((3355*a*b^4)/64 + (1085*a^5)/192 - (8825*a^3*b^2)/
96) + tan(c/2 + (d*x)/2)^6*(70*a^4*b + (32*b^5)/3 - (160*a^2*b^3)/3) + tan(
c/2 + (d*x)/2)^10*(70*a^4*b + (32*b^5)/3 - (160*a^2*b^3)/3) - tan(c/2 + (d*
x)/2)^8*((32*b^5)/3 - (400*a^2*b^3)/3) + 40*a^2*b^3*tan(c/2 + (d*x)/2)^4 +
40*a^2*b^3*tan(c/2 + (d*x)/2)^12 + 10*a^4*b*tan(c/2 + (d*x)/2)^2 + 10*a^4*b
*tan(c/2 + (d*x)/2)^14)/(d*(8*tan(c/2 + (d*x)/2)^2 + 28*tan(c/2 + (d*x)/2)^
4 + 56*tan(c/2 + (d*x)/2)^6 + 70*tan(c/2 + (d*x)/2)^8 + 56*tan(c/2 + (d*x)/
2)^10 + 28*tan(c/2 + (d*x)/2)^12 + 8*tan(c/2 + (d*x)/2)^14 + tan(c/2 + (d*x
)/2)^16 + 1)) - (5*a*(atan(tan(c/2 + (d*x)/2)) - (d*x)/2)*(7*a^4 + 3*b^4 +
10*a^2*b^2))/(64*d) + (5*a*atan((5*a*tan(c/2 + (d*x)/2)*(7*a^2 + 3*b^2)*(a^
2 + b^2))/(64*((15*a*b^4)/64 + (35*a^5)/64 + (25*a^3*b^2)/32)))*(7*a^2 + 3*
b^2)*(a^2 + b^2))/(64*d)
```

sympy [A] time = 11.54, size = 826, normalized size = 1.94

$$\begin{cases} \frac{35a^5x\sin^8(c+dx)}{128} + \frac{35a^5x\sin^6(c+dx)\cos^2(c+dx)}{32} + \frac{105a^5x\sin^4(c+dx)\cos^4(c+dx)}{64} + \frac{35a^5x\sin^2(c+dx)\cos^6(c+dx)}{32} + \frac{35a^5x\cos^8(c+dx)}{128} + \cdots \\ x(a\cos(c) + b\sin(c))^5\cos^3(c) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3*(a*cos(d*x+c)+b*sin(d*x+c))**5,x)
```

```
[Out] Piecewise((35*a**5*x*sin(c + d*x)**8/128 + 35*a**5*x*sin(c + d*x)**6*cos(c
+ d*x)**2/32 + 105*a**5*x*sin(c + d*x)**4*cos(c + d*x)**4/64 + 35*a**5*x*si
n(c + d*x)**2*cos(c + d*x)**6/32 + 35*a**5*x*cos(c + d*x)**8/128 + 35*a**5*
sin(c + d*x)**7*cos(c + d*x)/(128*d) + 385*a**5*sin(c + d*x)**5*cos(c + d*x
)**3/(384*d) + 511*a**5*sin(c + d*x)**3*cos(c + d*x)**5/(384*d) + 93*a**5*s
in(c + d*x)*cos(c + d*x)**7/(128*d) - 5*a**4*b*cos(c + d*x)**8/(8*d) + 25*a
**3*b**2*x*sin(c + d*x)**8/64 + 25*a**3*b**2*x*sin(c + d*x)**6*cos(c + d*x)
**2/16 + 75*a**3*b**2*x*sin(c + d*x)**4*cos(c + d*x)**4/32 + 25*a**3*b**2*x
*sin(c + d*x)**2*cos(c + d*x)**6/16 + 25*a**3*b**2*x*cos(c + d*x)**8/64 + 2
5*a**3*b**2*sin(c + d*x)**7*cos(c + d*x)/(64*d) + 275*a**3*b**2*sin(c + d*x
)**5*cos(c + d*x)**3/(192*d) + 365*a**3*b**2*sin(c + d*x)**3*cos(c + d*x)**
5/(192*d) - 25*a**3*b**2*sin(c + d*x)*cos(c + d*x)**7/(64*d) + 5*a**2*b**3*
sin(c + d*x)**8/(12*d) + 5*a**2*b**3*sin(c + d*x)**6*cos(c + d*x)**2/(3*d)
+ 5*a**2*b**3*sin(c + d*x)**4*cos(c + d*x)**4/(2*d) + 15*a*b**4*x*sin(c + d
*x)**8/128 + 15*a*b**4*x*sin(c + d*x)**6*cos(c + d*x)**2/32 + 45*a*b**4*x*s
in(c + d*x)**4*cos(c + d*x)**4/64 + 15*a*b**4*x*sin(c + d*x)**2*cos(c + d*x
)**6/32 + 15*a*b**4*x*cos(c + d*x)**8/128 + 15*a*b**4*sin(c + d*x)**7*cos(c
 + d*x)/(128*d) + 55*a*b**4*sin(c + d*x)**5*cos(c + d*x)**3/(128*d) - 55*a*
b**4*sin(c + d*x)**3*cos(c + d*x)**5/(128*d) - 15*a*b**4*sin(c + d*x)*cos(c
 + d*x)**7/(128*d) + b**5*sin(c + d*x)**8/(24*d) + b**5*sin(c + d*x)**6*cos
(c + d*x)**2/(6*d), Ne(d, 0)), (x*(a*cos(c) + b*sin(c))**5*cos(c)**3, True)
)
```

3.95 $\int \cos^2(c+dx)(a\cos(c+dx)+b\sin(c+dx))^5\,dx$

Optimal. Leaf size=275

$$-\frac{a^5\sin^7(c+dx)}{7d}+\frac{3a^5\sin^5(c+dx)}{5d}-\frac{a^5\sin^3(c+dx)}{d}+\frac{a^5\sin(c+dx)}{d}-\frac{5a^4b\cos^7(c+dx)}{7d}+\frac{10a^3b^2\sin^7(c+dx)}{7d}-$$

```
[Out] -1/3*b^5*cos(d*x+c)^3/d-2*a^2*b^3*cos(d*x+c)^5/d+2/5*b^5*cos(d*x+c)^5/d-5/7
*a^4*b*cos(d*x+c)^7/d+10/7*a^2*b^3*cos(d*x+c)^7/d-1/7*b^5*cos(d*x+c)^7/d+a^
5*sin(d*x+c)/d-a^5*sin(d*x+c)^3/d+10/3*a^3*b^2*sin(d*x+c)^3/d+3/5*a^5*sin(d
*x+c)^5/d-4*a^3*b^2*sin(d*x+c)^5/d+a*b^4*sin(d*x+c)^5/d-1/7*a^5*sin(d*x+c)^
7/d+10/7*a^3*b^2*sin(d*x+c)^7/d-5/7*a*b^4*sin(d*x+c)^7/d
```

Rubi [A] time = 0.28, antiderivative size = 275, normalized size of antiderivative = 1.00, number of steps used = 18, number of rules used = 7, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {3090, 2633, 2565, 30, 2564, 270, 14}

$$\frac{10a^3b^2\sin^7(c+dx)}{7d}-\frac{4a^3b^2\sin^5(c+dx)}{d}+\frac{10a^3b^2\sin^3(c+dx)}{3d}+\frac{10a^2b^3\cos^7(c+dx)}{7d}-\frac{2a^2b^3\cos^5(c+dx)}{d}-\frac{5a^4b\,c}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2*(a*Cos[c + d*x] + b*Sin[c + d*x])^5,x]
```

```
[Out] -(b^5*Cos[c + d*x]^3)/(3*d) - (2*a^2*b^3*Cos[c + d*x]^5)/d + (2*b^5*Cos[c +
 d*x]^5)/(5*d) - (5*a^4*b*Cos[c + d*x]^7)/(7*d) + (10*a^2*b^3*Cos[c + d*x]^
7)/(7*d) - (b^5*Cos[c + d*x]^7)/(7*d) + (a^5*Sin[c + d*x])/d - (a^5*Sin[c +
 d*x]^3)/d + (10*a^3*b^2*Sin[c + d*x]^3)/(3*d) + (3*a^5*Sin[c + d*x]^5)/(5*
d) - (4*a^3*b^2*Sin[c + d*x]^5)/d + (a*b^4*Sin[c + d*x]^5)/d - (a^5*Sin[c +
 d*x]^7)/(7*d) + (10*a^3*b^2*Sin[c + d*x]^7)/(7*d) - (5*a*b^4*Sin[c + d*x]^
7)/(7*d)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
```

```
IGtQ[p, 0]
```

Rule 2564

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 2565

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> -Dist[(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 3090

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_.), x_Symbol] :> Int[ExpandTrig[cos[c + d*x]^m*(a
*cos[c + d*x] + b*sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d}, x] && Inte
gerQ[m] && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\int \cos^2(c+dx)(a\cos(c+dx)+b\sin(c+dx))^5\,dx &= \int \left(a^5\cos^7(c+dx)+5a^4b\cos^6(c+dx)\sin(c+dx)+10a^3b^2\right.\\
&= a^5\int \cos^7(c+dx)\,dx+\left(5a^4b\right)\int \cos^6(c+dx)\sin(c+dx)\,dx\\
&= -\frac{a^5\,\mathrm{Subst}\left(\int\left(1-3x^2+3x^4-x^6\right)dx,x,-\sin(c+dx)\right)}{d}-\frac{(5}{}\\
&= -\frac{5a^4b\cos^7(c+dx)}{7d}+\frac{a^5\sin(c+dx)}{d}-\frac{a^5\sin^3(c+dx)}{d}+\frac{3a}{}\\
&= -\frac{b^5\cos^3(c+dx)}{3d}-\frac{2a^2b^3\cos^5(c+dx)}{d}+\frac{2b^5\cos^5(c+dx)}{5d}-
\end{aligned}$$

Mathematica [A] time = 0.75, size = 236, normalized size = 0.86

$$\frac{525a\left(7a^4+10a^2b^2+3b^4\right)\sin(c+dx)+35a\left(21a^4-10a^2b^2-15b^4\right)\sin(3(c+dx))+21a\left(7a^4-30a^2b^2-5b^4\right)s}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2*(a*Cos[c + d*x] + b*Sin[c + d*x])^5,x]
```

```
[Out] (-525*b*(5*a^4 + 6*a^2*b^2 + b^4)*Cos[c + d*x] - 35*b*(45*a^4 + 30*a^2*b^2
+ b^4)*Cos[3*(c + d*x)] + 21*b*(-25*a^4 + 10*a^2*b^2 + 3*b^4)*Cos[5*(c + d*
x)] - 15*b*(5*a^4 - 10*a^2*b^2 + b^4)*Cos[7*(c + d*x)] + 525*a*(7*a^4 + 10*
a^2*b^2 + 3*b^4)*Sin[c + d*x] + 35*a*(21*a^4 - 10*a^2*b^2 - 15*b^4)*Sin[3*(
c + d*x)] + 21*a*(7*a^4 - 30*a^2*b^2 - 5*b^4)*Sin[5*(c + d*x)] + 15*a*(a^4
- 10*a^2*b^2 + 5*b^4)*Sin[7*(c + d*x)])/(6720*d)
```

fricas [A] time = 0.63, size = 186, normalized size = 0.68

$$-\frac{35\,b^5\cos\left(dx+c\right)^3+15\left(5\,a^4b-10\,a^2b^3+b^5\right)\cos\left(dx+c\right)^7+42\left(5\,a^2b^3-b^5\right)\cos\left(dx+c\right)^5-\left(15\left(a^5-10\,a^3b^2\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a*cos(d*x+c)+b*sin(d*x+c))^5,x, algorithm="fricas")
```

```
[Out] -1/105*(35*b^5*cos(d*x + c)^3 + 15*(5*a^4*b - 10*a^2*b^3 + b^5)*cos(d*x + c
)^7 + 42*(5*a^2*b^3 - b^5)*cos(d*x + c)^5 - (15*(a^5 - 10*a^3*b^2 + 5*a*b^4
)*cos(d*x + c)^6 + 48*a^5 + 80*a^3*b^2 + 30*a*b^4 + 6*(3*a^5 + 5*a^3*b^2 -
20*a*b^4)*cos(d*x + c)^4 + (24*a^5 + 40*a^3*b^2 + 15*a*b^4)*cos(d*x + c)^2)
*sin(d*x + c))/d
```

giac [A] time = 0.60, size = 259, normalized size = 0.94

$$-\frac{\left(5\,a^4b-10\,a^2b^3+b^5\right)\cos\left(7\,dx+7\,c\right)}{448\,d}-\frac{\left(25\,a^4b-10\,a^2b^3-3\,b^5\right)\cos\left(5\,dx+5\,c\right)}{320\,d}-\frac{\left(45\,a^4b+30\,a^2b^3+b^5\right)\cos\left(\right.}{192\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a*cos(d*x+c)+b*sin(d*x+c))^5,x, algorithm="giac")
```

```
[Out] -1/448*(5*a^4*b - 10*a^2*b^3 + b^5)*cos(7*d*x + 7*c)/d - 1/320*(25*a^4*b -
10*a^2*b^3 - 3*b^5)*cos(5*d*x + 5*c)/d - 1/192*(45*a^4*b + 30*a^2*b^3 + b^5
)*cos(3*d*x + 3*c)/d - 5/64*(5*a^4*b + 6*a^2*b^3 + b^5)*cos(d*x + c)/d + 1/
448*(a^5 - 10*a^3*b^2 + 5*a*b^4)*sin(7*d*x + 7*c)/d + 1/320*(7*a^5 - 30*a^3
*b^2 - 5*a*b^4)*sin(5*d*x + 5*c)/d + 1/192*(21*a^5 - 10*a^3*b^2 - 15*a*b^4)
*sin(3*d*x + 3*c)/d + 5/64*(7*a^5 + 10*a^3*b^2 + 3*a*b^4)*sin(d*x + c)/d
```

maple [A] time = 0.16, size = 261, normalized size = 0.95

$$\frac{b^5\left(-\frac{\left(\sin^4(dx+c)\right)\left(\cos^3(dx+c)\right)}{7}-\frac{4\left(\sin^2(dx+c)\right)\left(\cos^3(dx+c)\right)}{35}-\frac{8\left(\cos^3(dx+c)\right)}{105}\right)+5a\,b^4\left(-\frac{\left(\sin^3(dx+c)\right)\left(\cos^4(dx+c)\right)}{7}-\frac{3\sin(dx+c)\left(\cos^4(dx\right.}{35}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(a*cos(d*x+c)+b*sin(d*x+c))^5,x)
```

```
[Out] 1/d*(b^5*(-1/7*sin(d*x+c)^4*cos(d*x+c)^3-4/35*sin(d*x+c)^2*cos(d*x+c)^3-8/1
05*cos(d*x+c)^3)+5*a*b^4*(-1/7*sin(d*x+c)^3*cos(d*x+c)^4-3/35*sin(d*x+c)*co
s(d*x+c)^4+1/35*(2+cos(d*x+c)^2)*sin(d*x+c))+10*a^2*b^3*(-1/7*sin(d*x+c)^2*
cos(d*x+c)^5-2/35*cos(d*x+c)^5)+10*a^3*b^2*(-1/7*sin(d*x+c)*cos(d*x+c)^6+1/
35*(8/3+cos(d*x+c)^4+4/3*cos(d*x+c)^2)*sin(d*x+c))-5/7*a^4*b*cos(d*x+c)^7+1
/7*a^5*(16/5+cos(d*x+c)^6+6/5*cos(d*x+c)^4+8/5*cos(d*x+c)^2)*sin(d*x+c))
```

maxima [A] time = 0.33, size = 194, normalized size = 0.71

$$-\frac{75\,a^4 b\cos\left(dx+c\right)^7+3\left(5\,\sin\left(dx+c\right)^7-21\,\sin\left(dx+c\right)^5+35\,\sin\left(dx+c\right)^3-35\,\sin\left(dx+c\right)\right)a^5-10\left(15\,\sin\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a*cos(d*x+c)+b*sin(d*x+c))^5,x, algorithm="maxima")
```

```
[Out] -1/105*(75*a^4*b*cos(d*x + c)^7 + 3*(5*sin(d*x + c)^7 - 21*sin(d*x + c)^5 +
 35*sin(d*x + c)^3 - 35*sin(d*x + c))*a^5 - 10*(15*sin(d*x + c)^7 - 42*sin(
d*x + c)^5 + 35*sin(d*x + c)^3)*a^3*b^2 - 30*(5*cos(d*x + c)^7 - 7*cos(d*x
+ c)^5)*a^2*b^3 + 15*(5*sin(d*x + c)^7 - 7*sin(d*x + c)^5)*a*b^4 + (15*cos(
d*x + c)^7 - 42*cos(d*x + c)^5 + 35*cos(d*x + c)^3)*b^5)/d
```

mupad [B] time = 4.42, size = 372, normalized size = 1.35

$$\frac{2\,a^5\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{13}+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5\left(\frac{86\,a^5}{5}-\frac{64\,a^3\,b^2}{3}+32\,a\,b^4\right)+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^9\left(\frac{86\,a^5}{5}-\frac{64\,a^3\,b^2}{3}+32\,a\,b^4\right)+\tan\left(\frac{c}{2}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^2*(a*cos(c + d*x) + b*sin(c + d*x))^5,x)
```

```
[Out] (2*a^5*tan(c/2 + (d*x)/2)^13 + tan(c/2 + (d*x)/2)^5*(32*a*b^4 + (86*a^5)/5
- (64*a^3*b^2)/3) + tan(c/2 + (d*x)/2)^9*(32*a*b^4 + (86*a^5)/5 - (64*a^3*b
```

```
^2)/3) + tan(c/2 + (d*x)/2)^7*((424*a^5)/35 - (192*a*b^4)/7 + (608*a^3*b^2)
/7) - tan(c/2 + (d*x)/2)^4*(30*a^4*b + (16*b^5)/5 - 16*a^2*b^3) - tan(c/2 +
 (d*x)/2)^8*(50*a^4*b + (32*b^5)/3 - 40*a^2*b^3) - (10*a^4*b)/7 - (16*b^5)/
105 - (8*a^2*b^3)/7 + tan(c/2 + (d*x)/2)^3*(4*a^5 + (80*a^3*b^2)/3) + tan(c
/2 + (d*x)/2)^11*(4*a^5 + (80*a^3*b^2)/3) + 2*a^5*tan(c/2 + (d*x)/2) - tan(
c/2 + (d*x)/2)^2*((16*b^5)/15 + 8*a^2*b^3) + tan(c/2 + (d*x)/2)^6*((16*b^5)
/3 - 80*a^2*b^3) - 40*a^2*b^3*tan(c/2 + (d*x)/2)^10 - 10*a^4*b*tan(c/2 + (d
*x)/2)^12)/(d*(tan(c/2 + (d*x)/2)^2 + 1)^7)
```

sympy [A] time = 6.05, size = 357, normalized size = 1.30

$$\begin{cases} \frac{16a^5 \sin^7(c+dx)}{35d} + \frac{8a^5 \sin^5(c+dx)\cos^2(c+dx)}{5d} + \frac{2a^5 \sin^3(c+dx)\cos^4(c+dx)}{d} + \frac{a^5 \sin(c+dx)\cos^6(c+dx)}{d} - \frac{5a^4 b \cos^7(c+dx)}{7d} + \frac{16a^3b^2 \sin^7(}{21d} \\ x\left(a\cos(c) + b\sin(c)\right)^5 \cos^2(c) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(a*cos(d*x+c)+b*sin(d*x+c))**5,x)
```

```
[Out] Piecewise((16*a**5*sin(c + d*x)**7/(35*d) + 8*a**5*sin(c + d*x)**5*cos(c +
d*x)**2/(5*d) + 2*a**5*sin(c + d*x)**3*cos(c + d*x)**4/d + a**5*sin(c + d*x
)*cos(c + d*x)**6/d - 5*a**4*b*cos(c + d*x)**7/(7*d) + 16*a**3*b**2*sin(c +
 d*x)**7/(21*d) + 8*a**3*b**2*sin(c + d*x)**5*cos(c + d*x)**2/(3*d) + 10*a*
*3*b**2*sin(c + d*x)**3*cos(c + d*x)**4/(3*d) - 2*a**2*b**3*sin(c + d*x)**2
*cos(c + d*x)**5/d - 4*a**2*b**3*cos(c + d*x)**7/(7*d) + 2*a*b**4*sin(c + d
*x)**7/(7*d) + a*b**4*sin(c + d*x)**5*cos(c + d*x)**2/d - b**5*sin(c + d*x)
**4*cos(c + d*x)**3/(3*d) - 4*b**5*sin(c + d*x)**2*cos(c + d*x)**5/(15*d) -
 8*b**5*cos(c + d*x)**7/(105*d), Ne(d, 0)), (x*(a*cos(c) + b*sin(c))**5*cos
(c)**2, True))
```

3.96 $\int \cos(c+dx)(a\cos(c+dx)+b\sin(c+dx))^5\,dx$

Optimal. Leaf size=126

$$\frac{5a\left(a^2+b^2\right)\sin^2(c+dx)(a\cot(c+dx)+b)(a-b\cot(c+dx))}{16d}+\frac{5}{16}ax\left(a^2+b^2\right)^2+\frac{\sin^6(c+dx)(a\cot(c+dx)+b)^5}{6d}$$

```
[Out] 5/16*a*(a^2+b^2)^2*x+5/16*a*(a^2+b^2)*(b+a*cot(d*x+c))*(a-b*cot(d*x+c))*sin
(d*x+c)^2/d+5/24*a*(b+a*cot(d*x+c))^3*(a-b*cot(d*x+c))*sin(d*x+c)^4/d+1/6*(
b+a*cot(d*x+c))^5*sin(d*x+c)^6/d
```

Rubi [A] time = 0.09, antiderivative size = 126, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {3088, 805, 723, 203}

$$\frac{5a\left(a^2+b^2\right)\sin^2(c+dx)(a\cot(c+dx)+b)(a-b\cot(c+dx))}{16d}+\frac{5}{16}ax\left(a^2+b^2\right)^2+\frac{\sin^6(c+dx)(a\cot(c+dx)+b)^5}{6d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]*(a*Cos[c + d*x] + b*Sin[c + d*x])^5,x]
```

```
[Out] (5*a*(a^2 + b^2)^2*x)/16 + (5*a*(a^2 + b^2)*(b + a*Cot[c + d*x])*(a - b*Cot
[c + d*x])*Sin[c + d*x]^2)/(16*d) + (5*a*(b + a*Cot[c + d*x])^3*(a - b*Cot[
c + d*x])*Sin[c + d*x]^4)/(24*d) + ((b + a*Cot[c + d*x])^5*Sin[c + d*x]^6)/
(6*d)
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 723

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
((d + e*x)^(m - 1)*(a*e - c*d*x)*(a + c*x^2)^(p + 1))/(2*a*c*(p + 1)), x] +
 Dist[((2*p + 3)*(c*d^2 + a*e^2))/(2*a*c*(p + 1)), Int[(d + e*x)^(m - 2)*(a
 + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0
] && EqQ[m + 2*p + 2, 0] && LtQ[p, -1]
```

Rule 805

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> Simp[((d + e*x)^m*(a + c*x^2)^(p + 1)*(a*g - c*f*x))/(2*a*
c*(p + 1)), x] - Dist[(m*(c*d*f + a*e*g))/(2*a*c*(p + 1)), Int[(d + e*x)^(m
```

```
 - 1)*(a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*
d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0] && LtQ[p, -1]
```

Rule 3088

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[(x^m*(b +
 a*x)^n)/(1 + x^2)^((m + n + 2)/2), x], x, Cot[c + d*x]], x] /; FreeQ[{a, b
, c, d}, x] && IntegerQ[n] && IntegerQ[(m + n)/2] && NeQ[n, -1] &&  !(GtQ[n
, 0] && GtQ[m, 1])
```

Rubi steps

$$\int \cos(c+dx)(a\cos(c+dx)+b\sin(c+dx))^5\,dx = -\frac{\text{Subst}\left(\int \frac{x(b+ax)^5}{\left(1+x^2\right)^4}\,dx, x, \cot(c+dx)\right)}{d}$$

$$= \frac{(b+a\cot(c+dx))^5\sin^6(c+dx)}{6d} - \frac{(5a)\,\text{Subst}\left(\int \frac{(b+ax)^4}{\left(1+x^2\right)^3}\,dx,\right.}{6d}$$

$$= \frac{5a(b+a\cot(c+dx))^3(a-b\cot(c+dx))\sin^4(c+dx)}{24d} + \frac{(b+}{}$$

$$= \frac{5a\left(a^2+b^2\right)(b+a\cot(c+dx))(a-b\cot(c+dx))\sin^2(c+d}{16d}$$

$$= \frac{5}{16}a\left(a^2+b^2\right)^2x + \frac{5a\left(a^2+b^2\right)(b+a\cot(c+dx))(a-b\cot}{16d}$$

Mathematica [A] time = 0.61, size = 188, normalized size = 1.49

$$\frac{6b\left(b^4-5a^4\right)\cos(4(c+dx)) + 60a\left(a^2+b^2\right)^2(c+dx) + 15a\left(3a^4+2a^2b^2-b^4\right)\sin(2(c+dx)) + 3a\left(3a^4-10a^2b\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]*(a*Cos[c + d*x] + b*Sin[c + d*x])^5,x]
```

```
[Out] (60*a*(a^2 + b^2)^2*(c + d*x) - 15*b*(5*a^4 + 6*a^2*b^2 + b^4)*Cos[2*(c + d
*x)] + 6*b*(-5*a^4 + b^4)*Cos[4*(c + d*x)] - b*(5*a^4 - 10*a^2*b^2 + b^4)*C
os[6*(c + d*x)] + 15*a*(3*a^4 + 2*a^2*b^2 - b^4)*Sin[2*(c + d*x)] + 3*a*(3*
```

```
a^4 - 10*a^2*b^2 - 5*b^4)*Sin[4*(c + d*x)] + a*(a^4 - 10*a^2*b^2 + 5*b^4)*S
in[6*(c + d*x)])/(192*d)
```

fricas [A] time = 0.62, size = 182, normalized size = 1.44

$$-\frac{24\,b^5\cos\left(dx+c\right)^2+8\left(5\,a^4b-10\,a^2b^3+b^5\right)\cos\left(dx+c\right)^6+24\left(5\,a^2b^3-b^5\right)\cos\left(dx+c\right)^4-15\left(a^5+2\,a^3b^2+a\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a*cos(d*x+c)+b*sin(d*x+c))^5,x, algorithm="fricas")
```

```
[Out] -1/48*(24*b^5*cos(d*x + c)^2 + 8*(5*a^4*b - 10*a^2*b^3 + b^5)*cos(d*x + c)^
6 + 24*(5*a^2*b^3 - b^5)*cos(d*x + c)^4 - 15*(a^5 + 2*a^3*b^2 + a*b^4)*d*x
- (8*(a^5 - 10*a^3*b^2 + 5*a*b^4)*cos(d*x + c)^5 + 10*(a^5 + 2*a^3*b^2 - 7*
a*b^4)*cos(d*x + c)^3 + 15*(a^5 + 2*a^3*b^2 + a*b^4)*cos(d*x + c))*sin(d*x
+ c))/d
```

giac [A] time = 0.53, size = 211, normalized size = 1.67

$$\frac{5}{16}\left(a^5+2\,a^3b^2+ab^4\right)x-\frac{\left(5\,a^4b-10\,a^2b^3+b^5\right)\cos\left(6\,dx+6\,c\right)}{192\,d}-\frac{\left(5\,a^4b-b^5\right)\cos\left(4\,dx+4\,c\right)}{32\,d}-\frac{5\left(5\,a^4b+6\,a^2b^3+\right.}{64}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a*cos(d*x+c)+b*sin(d*x+c))^5,x, algorithm="giac")
```

```
[Out] 5/16*(a^5 + 2*a^3*b^2 + a*b^4)*x - 1/192*(5*a^4*b - 10*a^2*b^3 + b^5)*cos(6
*d*x + 6*c)/d - 1/32*(5*a^4*b - b^5)*cos(4*d*x + 4*c)/d - 5/64*(5*a^4*b + 6
*a^2*b^3 + b^5)*cos(2*d*x + 2*c)/d + 1/192*(a^5 - 10*a^3*b^2 + 5*a*b^4)*sin
(6*d*x + 6*c)/d + 1/64*(3*a^5 - 10*a^3*b^2 - 5*a*b^4)*sin(4*d*x + 4*c)/d +
5/64*(3*a^5 + 2*a^3*b^2 - a*b^4)*sin(2*d*x + 2*c)/d
```

maple [A] time = 0.12, size = 236, normalized size = 1.87

$$\frac{\frac{b^5\left(\sin^6\left(dx+c\right)\right)}{6}+5a\,b^4\left(-\frac{\left(\sin^3\left(dx+c\right)\right)\left(\cos^3\left(dx+c\right)\right)}{6}-\frac{\sin\left(dx+c\right)\left(\cos^3\left(dx+c\right)\right)}{8}+\frac{\cos\left(dx+c\right)\sin\left(dx+c\right)}{16}+\frac{dx}{16}+\frac{c}{16}\right)+10a^2b^3\left(-\frac{\left(\sin^2\left(dx+\right.\right.}{}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(a*cos(d*x+c)+b*sin(d*x+c))^5,x)
```

```
[Out] 1/d*(1/6*b^5*sin(d*x+c)^6+5*a*b^4*(-1/6*sin(d*x+c)^3*cos(d*x+c)^3-1/8*sin(d
*x+c)*cos(d*x+c)^3+1/16*cos(d*x+c)*sin(d*x+c)+1/16*d*x+1/16*c)+10*a^2*b^3*(
```

```
-1/6*sin(d*x+c)^2*cos(d*x+c)^4-1/12*cos(d*x+c)^4)+10*a^3*b^2*(-1/6*sin(d*x+
c)*cos(d*x+c)^5+1/24*(cos(d*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+1/16*d*x+1/16
*c)-5/6*a^4*b*cos(d*x+c)^6+a^5*(1/6*(cos(d*x+c)^5+5/4*cos(d*x+c)^3+15/8*cos
(d*x+c))*sin(d*x+c)+5/16*d*x+5/16*c))
```

maxima [A] time = 0.33, size = 187, normalized size = 1.48

$$-\frac{160\,a^4 b\cos(dx+c)^6 - 32\,b^5\sin(dx+c)^6 + \left(4\,\sin(2\,dx+2\,c)^3 - 60\,dx - 60\,c - 9\,\sin(4\,dx+4\,c) - 48\,\sin(2}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a*cos(d*x+c)+b*sin(d*x+c))^5,x, algorithm="maxima")
```

```
[Out] -1/192*(160*a^4*b*cos(d*x + c)^6 - 32*b^5*sin(d*x + c)^6 + (4*sin(2*d*x + 2
*c)^3 - 60*d*x - 60*c - 9*sin(4*d*x + 4*c) - 48*sin(2*d*x + 2*c))*a^5 - 10*
(4*sin(2*d*x + 2*c)^3 + 12*d*x + 12*c - 3*sin(4*d*x + 4*c))*a^3*b^2 + 160*(
2*sin(d*x + c)^6 - 3*sin(d*x + c)^4)*a^2*b^3 + 5*(4*sin(2*d*x + 2*c)^3 - 12
*d*x - 12*c + 3*sin(4*d*x + 4*c))*a*b^4)/d
```

mupad [B] time = 2.32, size = 472, normalized size = 3.75

$$\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{11}\,\left(-\frac{11\,a^5}{8}+\frac{5\,a^3\,b^2}{4}+\frac{5\,a\,b^4}{8}\right)-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(-\frac{11\,a^5}{8}+\frac{5\,a^3\,b^2}{4}+\frac{5\,a\,b^4}{8}\right)+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5\,\left(\frac{15\,a^5}{4}-\frac{65\,a^3\,b^2}{2}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)*(a*cos(c + d*x) + b*sin(c + d*x))^5,x)
```

```
[Out] (tan(c/2 + (d*x)/2)^11*((5*a*b^4)/8 - (11*a^5)/8 + (5*a^3*b^2)/4) - tan(c/2
 + (d*x)/2)*((5*a*b^4)/8 - (11*a^5)/8 + (5*a^3*b^2)/4) + tan(c/2 + (d*x)/2)
^5*((95*a*b^4)/4 + (15*a^5)/4 - (65*a^3*b^2)/2) - tan(c/2 + (d*x)/2)^7*((95
*a*b^4)/4 + (15*a^5)/4 - (65*a^3*b^2)/2) - tan(c/2 + (d*x)/2)^3*((85*a*b^4)
/24 + (5*a^5)/24 - (235*a^3*b^2)/12) + tan(c/2 + (d*x)/2)^9*((85*a*b^4)/24
+ (5*a^5)/24 - (235*a^3*b^2)/12) + tan(c/2 + (d*x)/2)^6*((100*a^4*b)/3 + (3
2*b^5)/3 - (80*a^2*b^3)/3) + 40*a^2*b^3*tan(c/2 + (d*x)/2)^4 + 40*a^2*b^3*t
an(c/2 + (d*x)/2)^8 + 10*a^4*b*tan(c/2 + (d*x)/2)^2 + 10*a^4*b*tan(c/2 + (d
*x)/2)^10)/(d*(6*tan(c/2 + (d*x)/2)^2 + 15*tan(c/2 + (d*x)/2)^4 + 20*tan(c/
2 + (d*x)/2)^6 + 15*tan(c/2 + (d*x)/2)^8 + 6*tan(c/2 + (d*x)/2)^10 + tan(c/
2 + (d*x)/2)^12 + 1)) + (5*a*atan((5*a*tan(c/2 + (d*x)/2)*(a^2 + b^2)^2)/(8
*((5*a*b^4)/8 + (5*a^5)/8 + (5*a^3*b^2)/4)))*(a^2 + b^2)^2)/(8*d) - (5*a*(a
tan(tan(c/2 + (d*x)/2)) - (d*x)/2)*(a^2 + b^2)^2)/(8*d)
```

sympy [A] time = 4.17, size = 609, normalized size = 4.83

$$\begin{cases} \frac{5a^5x\sin^6(c+dx)}{16} + \frac{15a^5x\sin^4(c+dx)\cos^2(c+dx)}{16} + \frac{15a^5x\sin^2(c+dx)\cos^4(c+dx)}{16} + \frac{5a^5x\cos^6(c+dx)}{16} + \frac{5a^5\sin^5(c+dx)\cos(c+dx)}{16d} + \frac{5a^5\sin^3}{} \\ x\left(a\cos(c) + b\sin(c)\right)^5\cos(c) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a*cos(d*x+c)+b*sin(d*x+c))**5,x)
```

```
[Out] Piecewise((5*a**5*x*sin(c + d*x)**6/16 + 15*a**5*x*sin(c + d*x)**4*cos(c +
d*x)**2/16 + 15*a**5*x*sin(c + d*x)**2*cos(c + d*x)**4/16 + 5*a**5*x*cos(c
+ d*x)**6/16 + 5*a**5*sin(c + d*x)**5*cos(c + d*x)/(16*d) + 5*a**5*sin(c +
d*x)**3*cos(c + d*x)**3/(6*d) + 11*a**5*sin(c + d*x)*cos(c + d*x)**5/(16*d)
 - 5*a**4*b*cos(c + d*x)**6/(6*d) + 5*a**3*b**2*x*sin(c + d*x)**6/8 + 15*a*
*3*b**2*x*sin(c + d*x)**4*cos(c + d*x)**2/8 + 15*a**3*b**2*x*sin(c + d*x)**
2*cos(c + d*x)**4/8 + 5*a**3*b**2*x*cos(c + d*x)**6/8 + 5*a**3*b**2*sin(c +
 d*x)**5*cos(c + d*x)/(8*d) + 5*a**3*b**2*sin(c + d*x)**3*cos(c + d*x)**3/(
3*d) - 5*a**3*b**2*sin(c + d*x)*cos(c + d*x)**5/(8*d) + 5*a**2*b**3*sin(c +
 d*x)**6/(6*d) + 5*a**2*b**3*sin(c + d*x)**4*cos(c + d*x)**2/(2*d) + 5*a*b*
*4*x*sin(c + d*x)**6/16 + 15*a*b**4*x*sin(c + d*x)**4*cos(c + d*x)**2/16 +
15*a*b**4*x*sin(c + d*x)**2*cos(c + d*x)**4/16 + 5*a*b**4*x*cos(c + d*x)**6
/16 + 5*a*b**4*sin(c + d*x)**5*cos(c + d*x)/(16*d) - 5*a*b**4*sin(c + d*x)*
*3*cos(c + d*x)**3/(6*d) - 5*a*b**4*sin(c + d*x)*cos(c + d*x)**5/(16*d) + b
**5*sin(c + d*x)**6/(6*d), Ne(d, 0)), (x*(a*cos(c) + b*sin(c))**5*cos(c), T
rue))
```

3.97 $\int (a\cos(c+dx)+b\sin(c+dx))^5\,dx$

Optimal. Leaf size=94

$$\frac{2\left(a^2+b^2\right)(b\cos(c+dx)-a\sin(c+dx))^3}{3d}-\frac{\left(a^2+b^2\right)^2(b\cos(c+dx)-a\sin(c+dx))}{d}-\frac{(b\cos(c+dx)-a\sin(c+dx))^5}{5d}$$

```
[Out] -(a^2+b^2)^2*(b*cos(d*x+c)-a*sin(d*x+c))/d+2/3*(a^2+b^2)*(b*cos(d*x+c)-a*si
n(d*x+c))^3/d-1/5*(b*cos(d*x+c)-a*sin(d*x+c))^5/d
```

Rubi [A] time = 0.05, antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {3072, 194}

$$\frac{2\left(a^2+b^2\right)(b\cos(c+dx)-a\sin(c+dx))^3}{3d}-\frac{\left(a^2+b^2\right)^2(b\cos(c+dx)-a\sin(c+dx))}{d}-\frac{(b\cos(c+dx)-a\sin(c+dx))^5}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Cos[c + d*x] + b*Sin[c + d*x])^5,x]
```

```
[Out] -(((a^2 + b^2)^2*(b*Cos[c + d*x] - a*Sin[c + d*x]))/d) + (2*(a^2 + b^2)*(b*
Cos[c + d*x] - a*Sin[c + d*x])^3)/(3*d) - (b*Cos[c + d*x] - a*Sin[c + d*x])
^5/(5*d)
```

Rule 194

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 3072

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x
_Symbol] :> -Dist[d^(-1), Subst[Int[(a^2 + b^2 - x^2)^((n - 1)/2), x], x, b
*Cos[c + d*x] - a*Sin[c + d*x]], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 +
b^2, 0] && IGtQ[(n - 1)/2, 0]
```

Rubi steps

$$\int (a\cos(c+dx)+b\sin(c+dx))^5\,dx = -\frac{\text{Subst}\left(\int \left(a^2+b^2-x^2\right)^2\,dx, x, b\cos(c+dx)-a\sin(c+dx)\right)}{d}$$

$$= -\frac{\text{Subst}\left(\int \left(a^4\left(1+\frac{2a^2b^2+b^4}{a^4}\right)-2a^2\left(1+\frac{b^2}{a^2}\right)x^2+x^4\right)dx, x, b\cos(c+dx) -}{d}$$

$$= -\frac{\left(a^2+b^2\right)^2(b\cos(c+dx)-a\sin(c+dx))}{d} + \frac{2\left(a^2+b^2\right)(b\cos(c+dx) -}{3d}$$

Mathematica [A] time = 0.46, size = 156, normalized size = 1.66

$$\frac{150a\left(a^2+b^2\right)^2\sin(c+dx)-150b\left(a^2+b^2\right)^2\cos(c+dx)+25a\left(a^4-2a^2b^2-3b^4\right)\sin(3(c+dx))+3a\left(a^4-10a^2\right.}{240d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Cos[c + d*x] + b*Sin[c + d*x])^5,x]
```

```
[Out] (-150*b*(a^2 + b^2)^2*Cos[c + d*x] + 25*b*(-3*a^4 - 2*a^2*b^2 + b^4)*Cos[3*
(c + d*x)] - 3*b*(5*a^4 - 10*a^2*b^2 + b^4)*Cos[5*(c + d*x)] + 150*a*(a^2 +
 b^2)^2*Sin[c + d*x] + 25*a*(a^4 - 2*a^2*b^2 - 3*b^4)*Sin[3*(c + d*x)] + 3*
a*(a^4 - 10*a^2*b^2 + 5*b^4)*Sin[5*(c + d*x)])/(240*d)
```

fricas [A] time = 0.68, size = 155, normalized size = 1.65

$$-\frac{15\,b^5\cos(dx+c)+3\left(5\,a^4b-10\,a^2b^3+b^5\right)\cos(dx+c)^5+10\left(5\,a^2b^3-b^5\right)\cos(dx+c)^3-\left(8\,a^5+20\,a^3b^2+15\right.}{15\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cos(d*x+c)+b*sin(d*x+c))^5,x, algorithm="fricas")
```

```
[Out] -1/15*(15*b^5*cos(d*x + c) + 3*(5*a^4*b - 10*a^2*b^3 + b^5)*cos(d*x + c)^5
+ 10*(5*a^2*b^3 - b^5)*cos(d*x + c)^3 - (8*a^5 + 20*a^3*b^2 + 15*a*b^4 + 3*
(a^5 - 10*a^3*b^2 + 5*a*b^4)*cos(d*x + c)^4 + 2*(2*a^5 + 5*a^3*b^2 - 15*a*b
^4)*cos(d*x + c)^2)*sin(d*x + c))/d
```

giac [B] time = 0.33, size = 187, normalized size = 1.99

$$-\frac{\left(5\,a^4b-10\,a^2b^3+b^5\right)\cos(5\,dx+5\,c)}{80\,d}-\frac{5\left(3\,a^4b+2\,a^2b^3-b^5\right)\cos(3\,dx+3\,c)}{48\,d}-\frac{5\left(a^4b+2\,a^2b^3+b^5\right)\cos(dx+}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cos(d*x+c)+b*sin(d*x+c))^5,x, algorithm="giac")
```

```
[Out] -1/80*(5*a^4*b - 10*a^2*b^3 + b^5)*cos(5*d*x + 5*c)/d - 5/48*(3*a^4*b + 2*a
^2*b^3 - b^5)*cos(3*d*x + 3*c)/d - 5/8*(a^4*b + 2*a^2*b^3 + b^5)*cos(d*x +
c)/d + 1/80*(a^5 - 10*a^3*b^2 + 5*a*b^4)*sin(5*d*x + 5*c)/d + 5/48*(a^5 - 2
*a^3*b^2 - 3*a*b^4)*sin(3*d*x + 3*c)/d + 5/8*(a^5 + 2*a^3*b^2 + a*b^4)*sin(
d*x + c)/d
```

maple [A] time = 0.15, size = 175, normalized size = 1.86

$$\frac{-\frac{b^5\left(\frac{8}{3}+\sin^4(dx+c)+\frac{4\left(\sin^2(dx+c)\right)}{3}\right)\cos(dx+c)}{5}+a\,b^4\left(\sin^5(dx+c)\right)+10a^2b^3\left(-\frac{\left(\sin^2(dx+c)\right)\left(\cos^3(dx+c)\right)}{5}-\frac{2\left(\cos^3(dx+c)\right)}{15}\right)+10a^3 \cdots}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*cos(d*x+c)+b*sin(d*x+c))^5,x)
```

```
[Out] 1/d*(-1/5*b^5*(8/3+sin(d*x+c)^4+4/3*sin(d*x+c)^2)*cos(d*x+c)+a*b^4*sin(d*x+
c)^5+10*a^2*b^3*(-1/5*sin(d*x+c)^2*cos(d*x+c)^3-2/15*cos(d*x+c)^3)+10*a^3*b
^2*(-1/5*sin(d*x+c)*cos(d*x+c)^4+1/15*(2+cos(d*x+c)^2)*sin(d*x+c))-a^4*b*co
s(d*x+c)^5+1/5*a^5*(8/3+cos(d*x+c)^4+4/3*cos(d*x+c)^2)*sin(d*x+c))
```

maxima [A] time = 0.33, size = 172, normalized size = 1.83

$$-\frac{a^4b\cos(dx+c)^5}{d}+\frac{ab^4\sin(dx+c)^5}{d}+\frac{\left(3\sin(dx+c)^5-10\sin(dx+c)^3+15\sin(dx+c)\right)a^5}{15\,d}-\frac{2\left(3\sin(dx+c \cdots\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cos(d*x+c)+b*sin(d*x+c))^5,x, algorithm="maxima")
```

```
[Out] -a^4*b*cos(d*x + c)^5/d + a*b^4*sin(d*x + c)^5/d + 1/15*(3*sin(d*x + c)^5 -
 10*sin(d*x + c)^3 + 15*sin(d*x + c))*a^5/d - 2/3*(3*sin(d*x + c)^5 - 5*sin
(d*x + c)^3)*a^3*b^2/d + 2/3*(3*cos(d*x + c)^5 - 5*cos(d*x + c)^3)*a^2*b^3/
d - 1/15*(3*cos(d*x + c)^5 - 10*cos(d*x + c)^3 + 15*cos(d*x + c))*b^5/d
```

mupad [B] time = 0.94, size = 248, normalized size = 2.64

$$\frac{2\left(\frac{3\sin(c+d\,x)\,a^5\cos(c+d\,x)^4}{2}+2\sin(c+d\,x)\,a^5\cos(c+d\,x)^2+4\sin(c+d\,x)\,a^5-\frac{15\,a^4\,b\cos(c+d\,x)^5}{2}-15\sin(c+d\,x \cdots\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*cos(c + d*x) + b*sin(c + d*x))^5,x)
```

```
[Out] (2*(4*a^5*sin(c + d*x) - (15*b^5*cos(c + d*x))/2 + 5*b^5*cos(c + d*x)^3 - (
3*b^5*cos(c + d*x)^5)/2 - (15*a^4*b*cos(c + d*x)^5)/2 + 2*a^5*cos(c + d*x)^
2*sin(c + d*x) + (3*a^5*cos(c + d*x)^4*sin(c + d*x))/2 + 10*a^3*b^2*sin(c +
 d*x) - 25*a^2*b^3*cos(c + d*x)^3 + 15*a^2*b^3*cos(c + d*x)^5 + (15*a*b^4*s
in(c + d*x))/2 + 5*a^3*b^2*cos(c + d*x)^2*sin(c + d*x) - 15*a^3*b^2*cos(c +
 d*x)^4*sin(c + d*x) - 15*a*b^4*cos(c + d*x)^2*sin(c + d*x) + (15*a*b^4*cos
(c + d*x)^4*sin(c + d*x))/2))/(15*d)
```

sympy [A] time = 2.14, size = 267, normalized size = 2.84

$$\begin{cases} \frac{8a^5\sin^5(c+dx)}{15d} + \frac{4a^5\sin^3(c+dx)\cos^2(c+dx)}{3d} + \frac{a^5\sin(c+dx)\cos^4(c+dx)}{d} - \frac{a^4b\cos^5(c+dx)}{d} + \frac{4a^3b^2\sin^5(c+dx)}{3d} + \frac{10a^3b^2\sin^3(c+dx)\cos^2(c+ \\ x\,(a\cos(c) + b\sin(c))^5 \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cos(d*x+c)+b*sin(d*x+c))**5,x)
```

```
[Out] Piecewise((8*a**5*sin(c + d*x)**5/(15*d) + 4*a**5*sin(c + d*x)**3*cos(c + d
*x)**2/(3*d) + a**5*sin(c + d*x)*cos(c + d*x)**4/d - a**4*b*cos(c + d*x)**5
/d + 4*a**3*b**2*sin(c + d*x)**5/(3*d) + 10*a**3*b**2*sin(c + d*x)**3*cos(c
 + d*x)**2/(3*d) - 10*a**2*b**3*sin(c + d*x)**2*cos(c + d*x)**3/(3*d) - 4*a
**2*b**3*cos(c + d*x)**5/(3*d) + a*b**4*sin(c + d*x)**5/d - b**5*sin(c + d*
x)**4*cos(c + d*x)/d - 4*b**5*sin(c + d*x)**2*cos(c + d*x)**3/(3*d) - 8*b**
5*cos(c + d*x)**5/(15*d), Ne(d, 0)), (x*(a*cos(c) + b*sin(c))**5, True))
```

3.98 $\int \sec(c+dx)(a\cos(c+dx)+b\sin(c+dx))^5\,dx$

Optimal. Leaf size=170

$$-\frac{\sin^4(c+dx)\left(a\left(a^4-10a^2b^2+5b^4\right)\cot(c+dx)+b\left(5a^4-10a^2b^2+b^4\right)\right)}{4d}+\frac{\sin^2(c+dx)\left(4b\left(5a^4-b^4\right)+5a\left(a^2\right.\right.}{8d}$$

```
[Out] 1/8*a*(3*a^4+10*a^2*b^2+15*b^4)*x-b^5*ln(sin(d*x+c))/d+b^5*ln(tan(d*x+c))/d
+1/8*(4*b*(5*a^4-b^4)+5*a*(a^2-3*b^2)*(a^2+b^2)*cot(d*x+c))*sin(d*x+c)^2/d-
1/4*(b*(5*a^4-10*a^2*b^2+b^4)+a*(a^4-10*a^2*b^2+5*b^4)*cot(d*x+c))*sin(d*x+
c)^4/d
```

Rubi [A] time = 0.22, antiderivative size = 170, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {3088, 1805, 801, 635, 203, 260}

$$-\frac{\sin^4(c+dx)\left(a\left(-10a^2b^2+a^4+5b^4\right)\cot(c+dx)+b\left(-10a^2b^2+5a^4+b^4\right)\right)}{4d}+\frac{\sin^2(c+dx)\left(5a\left(a^2-3b^2\right)\left(a^2+\right.\right.}{8}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]*(a*Cos[c + d*x] + b*Sin[c + d*x])^5,x]
```

```
[Out] (a*(3*a^4 + 10*a^2*b^2 + 15*b^4)*x)/8 - (b^5*Log[Sin[c + d*x]])/d + (b^5*Lo
g[Tan[c + d*x]])/d + ((4*b*(5*a^4 - b^4) + 5*a*(a^2 - 3*b^2)*(a^2 + b^2)*Co
t[c + d*x])*Sin[c + d*x]^2)/(8*d) - ((b*(5*a^4 - 10*a^2*b^2 + b^4) + a*(a^4
 - 10*a^2*b^2 + 5*b^4)*Cot[c + d*x])*Sin[c + d*x]^4)/(4*d)
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] && !NiceSqrtQ[-(a*c)]
```

Rule 801

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a + c*x^2), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rule 1805

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[((a*g - b*f*x)*(a + b*x^2)^(p + 1))/(2*a
*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int[(c*x)^m*(a + b*x^2)^(p + 1)*Exp
andToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; Fr
eeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 3088

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[(x^m*(b +
 a*x)^n)/(1 + x^2)^((m + n + 2)/2), x], x, Cot[c + d*x]], x] /; FreeQ[{a, b
, c, d}, x] && IntegerQ[n] && IntegerQ[(m + n)/2] && NeQ[n, -1] &&  !(GtQ[n
, 0] && GtQ[m, 1])
```

Rubi steps

$$\int \sec(c+dx)(a\cos(c+dx)+b\sin(c+dx))^5\,dx = -\frac{\mathrm{Subst}\left(\int \frac{(b+ax)^5}{x(1+x^2)^3}\,dx, x, \cot(c+dx)\right)}{d}$$

$$= -\frac{\left(b\left(5a^4-10a^2b^2+b^4\right)+a\left(a^4-10a^2b^2+5b^4\right)\cot(c+dx)\right.}{4d}$$

$$= \frac{\left(4b\left(5a^4-b^4\right)+5a\left(a^2-3b^2\right)\left(a^2+b^2\right)\cot(c+dx)\right)\sin^2(c}{8d}$$

$$= \frac{\left(4b\left(5a^4-b^4\right)+5a\left(a^2-3b^2\right)\left(a^2+b^2\right)\cot(c+dx)\right)\sin^2(c}{8d}$$

$$= \frac{b^5\log(\tan(c+dx))}{d} + \frac{\left(4b\left(5a^4-b^4\right)+5a\left(a^2-3b^2\right)\left(a^2+b\right.\right.}{8d}$$

$$= \frac{b^5\log(\tan(c+dx))}{d} + \frac{\left(4b\left(5a^4-b^4\right)+5a\left(a^2-3b^2\right)\left(a^2+b\right.\right.}{8d}$$

$$= \frac{1}{8}a\left(3a^4+10a^2b^2+15b^4\right)x - \frac{b^5\log(\sin(c+dx))}{d} + \frac{b^5\log(t}{}$$

Mathematica [B] time = 6.44, size = 711, normalized size = 4.18

$$\frac{b^5\left(\frac{\cos^4(c+dx)(a+b\tan(c+dx))^6\left(ab\tan(c+dx)+b^2\right)}{4b^6\left(a^2+b^2\right)} - \frac{\frac{\cos^2(c+dx)(a+b\tan(c+dx))^6\left(b\left(a\left(2b^2-3a^2\right)+3ab^2\right)\tan(c+dx)-3a^2b^2+b^2\left(2b^2-3a^2\right)\right)}{2b^4\left(a^2+b^2\right)} - \frac{\left(3a^4-29a^2b^2+5a^2\left(3a^2\right.\right.}{}}{}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]*(a*Cos[c + d*x] + b*Sin[c + d*x])^5,x]
```

```
[Out] (b^5*((Cos[c + d*x]^4*(a + b*Tan[c + d*x])^6*(b^2 + a*b*Tan[c + d*x]))/(4*b
^6*(a^2 + b^2)) - ((Cos[c + d*x]^2*(a + b*Tan[c + d*x])^6*(-3*a^2*b^2 + b^2
*(-3*a^2 + 2*b^2) + b*(3*a*b^2 + a*(-3*a^2 + 2*b^2))*Tan[c + d*x]))/(2*b^4*
(a^2 + b^2)) - ((3*a^4 - 29*a^2*b^2 + 8*b^4 + 5*a^2*(3*a^2 - 5*b^2))*(((5*a
^4 - 10*a^2*b^2 + b^4 + (a^5 - 10*a^3*b^2 + 5*a*b^4)/Sqrt[-b^2])*Log[Sqrt[-
b^2] - b*Tan[c + d*x]])/2 + ((5*a^4 - 10*a^2*b^2 + b^4 - (a^5 - 10*a^3*b^2
+ 5*a*b^4)/Sqrt[-b^2])*Log[Sqrt[-b^2] + b*Tan[c + d*x]])/2 + 5*a*b*(2*a^2 -
```

```
 b^2)*Tan[c + d*x] + (b^2*(10*a^2 - b^2)*Tan[c + d*x]^2)/2 + (5*a*b^3*Tan[c
 + d*x]^3)/3 + (b^4*Tan[c + d*x]^4)/4) - 5*a*(3*a^2 - 5*b^2)*(((6*a^5 - 20*
a^3*b^2 + 6*a*b^4 + (a^6 - 15*a^4*b^2 + 15*a^2*b^4 - b^6)/Sqrt[-b^2])*Log[S
qrt[-b^2] - b*Tan[c + d*x]])/2 + ((6*a^5 - 20*a^3*b^2 + 6*a*b^4 - (a^6 - 15
*a^4*b^2 + 15*a^2*b^4 - b^6)/Sqrt[-b^2])*Log[Sqrt[-b^2] + b*Tan[c + d*x]])/
2 + b*(15*a^4 - 15*a^2*b^2 + b^4)*Tan[c + d*x] + a*b^2*(10*a^2 - 3*b^2)*Tan
[c + d*x]^2 + (b^3*(15*a^2 - b^2)*Tan[c + d*x]^3)/3 + (3*a*b^4*Tan[c + d*x]
^4)/2 + (b^5*Tan[c + d*x]^5)/5))/(2*b^2*(a^2 + b^2)))/(4*b^2*(a^2 + b^2))))
/d
```

fricas [A] time = 0.53, size = 160, normalized size = 0.94

$$-\frac{8\,b^5\log\left(-\cos\left(dx+c\right)\right)+2\left(5\,a^4b-10\,a^2b^3+b^5\right)\cos\left(dx+c\right)^4-\left(3\,a^5+10\,a^3b^2+15\,ab^4\right)dx+8\left(5\,a^2b^3-b^5\right)c}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a*cos(d*x+c)+b*sin(d*x+c))^5,x, algorithm="fricas")
```

```
[Out] -1/8*(8*b^5*log(-cos(d*x + c)) + 2*(5*a^4*b - 10*a^2*b^3 + b^5)*cos(d*x + c
)^4 - (3*a^5 + 10*a^3*b^2 + 15*a*b^4)*d*x + 8*(5*a^2*b^3 - b^5)*cos(d*x + c
)^2 - (2*(a^5 - 10*a^3*b^2 + 5*a*b^4)*cos(d*x + c)^3 + (3*a^5 + 10*a^3*b^2
- 25*a*b^4)*cos(d*x + c))*sin(d*x + c))/d
```

giac [A] time = 4.85, size = 199, normalized size = 1.17

$$\frac{4\,b^5\log\left(\tan\left(dx+c\right)^2+1\right)+\left(3\,a^5+10\,a^3b^2+15\,ab^4\right)\left(dx+c\right)-\frac{6\,b^5\tan(dx+c)^4-3\,a^5\tan(dx+c)^3-10\,a^3b^2\tan(dx+c)^3+25\,ab^4\tan}{}}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a*cos(d*x+c)+b*sin(d*x+c))^5,x, algorithm="giac")
```

```
[Out] 1/8*(4*b^5*log(tan(d*x + c)^2 + 1) + (3*a^5 + 10*a^3*b^2 + 15*a*b^4)*(d*x +
 c) - (6*b^5*tan(d*x + c)^4 - 3*a^5*tan(d*x + c)^3 - 10*a^3*b^2*tan(d*x + c
)^3 + 25*a*b^4*tan(d*x + c)^3 + 40*a^2*b^3*tan(d*x + c)^2 + 4*b^5*tan(d*x +
 c)^2 - 5*a^5*tan(d*x + c) + 10*a^3*b^2*tan(d*x + c) + 15*a*b^4*tan(d*x + c
) + 10*a^4*b + 20*a^2*b^3)/(tan(d*x + c)^2 + 1)^2)/d
```

maple [A] time = 0.24, size = 272, normalized size = 1.60

$$\frac{a^5\left(\cos^3\left(dx+c\right)\right)\sin\left(dx+c\right)}{4d}+\frac{3a^5\cos\left(dx+c\right)\sin\left(dx+c\right)}{8d}+\frac{3a^5x}{8}+\frac{3a^5c}{8d}-\frac{5a^4\left(\cos^4\left(dx+c\right)\right)b}{4d}-\frac{5a^3b^2\left(\cos^3\left(dx\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)*(a*cos(d*x+c)+b*sin(d*x+c))^5,x)
```

```
[Out] 1/4*a^5*cos(d*x+c)^3*sin(d*x+c)/d+3/8*a^5*cos(d*x+c)*sin(d*x+c)/d+3/8*a^5*x
+3/8/d*a^5*c-5/4/d*a^4*cos(d*x+c)^4*b-5/2*a^3*b^2*cos(d*x+c)^3*sin(d*x+c)/d
+5/4*a^3*b^2*cos(d*x+c)*sin(d*x+c)/d+5/4*a^3*b^2*x+5/4/d*a^3*b^2*c+5/2/d*a^
2*b^3*sin(d*x+c)^4-5/4/d*a*b^4*cos(d*x+c)*sin(d*x+c)^3-15/8*a*b^4*cos(d*x+c
)*sin(d*x+c)/d+15/8*a*b^4*x+15/8/d*a*b^4*c-1/4/d*b^5*sin(d*x+c)^4-1/2/d*sin
(d*x+c)^2*b^5-1/d*b^5*ln(cos(d*x+c))
```

maxima [A] time = 0.33, size = 170, normalized size = 1.00

$$\frac{80\,a^2b^3\sin(dx+c)^4 - 40\left(\sin(dx+c)^2-1\right)^2a^4b + (12\,dx+12\,c+\sin(4\,dx+4\,c)+8\,\sin(2\,dx+2\,c))a^5 + 10\,(4}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a*cos(d*x+c)+b*sin(d*x+c))^5,x, algorithm="maxima")
```

```
[Out] 1/32*(80*a^2*b^3*sin(d*x + c)^4 - 40*(sin(d*x + c)^2 - 1)^2*a^4*b + (12*d*x
 + 12*c + sin(4*d*x + 4*c) + 8*sin(2*d*x + 2*c))*a^5 + 10*(4*d*x + 4*c - si
n(4*d*x + 4*c))*a^3*b^2 + 5*(12*d*x + 12*c + sin(4*d*x + 4*c) - 8*sin(2*d*x
 + 2*c))*a*b^4 - 8*(sin(d*x + c)^4 + 2*sin(d*x + c)^2 + 2*log(sin(d*x + c)^
2 - 1))*b^5)/d
```

mupad [B] time = 2.65, size = 297, normalized size = 1.75

$$\frac{4\,b^5\,\ln\left(\frac{1}{{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2}\right) - 4\,b^5\,\ln\left(\frac{\cos(c+d\,x)}{\cos(c+d\,x)+1}\right) + 3\,a^5\,\mathrm{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right) + \frac{3\,b^5\,\cos(2\,c+2\,d\,x)}{2} - \frac{b^5\,\cos(4\,c+4\,d\,x)}{8} + a^5\,\sin(2\,c}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*cos(c + d*x) + b*sin(c + d*x))^5/cos(c + d*x),x)
```

```
[Out] (4*b^5*log(1/cos(c/2 + (d*x)/2)^2) - 4*b^5*log(cos(c + d*x)/(cos(c + d*x) +
 1)) + 3*a^5*atan(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)) + (3*b^5*cos(2*c +
 2*d*x))/2 - (b^5*cos(4*c + 4*d*x))/8 + a^5*sin(2*c + 2*d*x) + (a^5*sin(4*c
 + 4*d*x))/8 + 15*a*b^4*atan(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)) - (5*a^
4*b*cos(2*c + 2*d*x))/2 - (5*a^4*b*cos(4*c + 4*d*x))/8 - 5*a*b^4*sin(2*c +
2*d*x) + (5*a*b^4*sin(4*c + 4*d*x))/8 + 10*a^3*b^2*atan(sin(c/2 + (d*x)/2)/
cos(c/2 + (d*x)/2)) - 5*a^2*b^3*cos(2*c + 2*d*x) + (5*a^2*b^3*cos(4*c + 4*d
*x))/4 - (5*a^3*b^2*sin(4*c + 4*d*x))/4)/(4*d)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a*cos(d*x+c)+b*sin(d*x+c))**5,x)
```

```
[Out] Timed out
```

3.99 $\int \sec^2(c+dx)(a\cos(c+dx)+b\sin(c+dx))^5\,dx$

Optimal. Leaf size=205

$$-\frac{a^5\sin^3(c+dx)}{3d}+\frac{a^5\sin(c+dx)}{d}-\frac{5a^4b\cos^3(c+dx)}{3d}+\frac{10a^3b^2\sin^3(c+dx)}{3d}+\frac{10a^2b^3\cos^3(c+dx)}{3d}-\frac{10a^2b^3\cos(c+}{d}$$

```
[Out] 5*a*b^4*arctanh(sin(d*x+c))/d-10*a^2*b^3*cos(d*x+c)/d+2*b^5*cos(d*x+c)/d-5/
3*a^4*b*cos(d*x+c)^3/d+10/3*a^2*b^3*cos(d*x+c)^3/d-1/3*b^5*cos(d*x+c)^3/d+b
^5*sec(d*x+c)/d+a^5*sin(d*x+c)/d-5*a*b^4*sin(d*x+c)/d-1/3*a^5*sin(d*x+c)^3/
d+10/3*a^3*b^2*sin(d*x+c)^3/d-5/3*a*b^4*sin(d*x+c)^3/d
```

Rubi [A] time = 0.22, antiderivative size = 205, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 10, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.357$, Rules used = {3090, 2633, 2565, 30, 2564, 2592, 302, 206, 2590, 270}

$$\frac{10a^3b^2\sin^3(c+dx)}{3d}+\frac{10a^2b^3\cos^3(c+dx)}{3d}-\frac{10a^2b^3\cos(c+dx)}{d}-\frac{5a^4b\cos^3(c+dx)}{3d}-\frac{a^5\sin^3(c+dx)}{3d}+\frac{a^5\sin(c+}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^2*(a*Cos[c + d*x] + b*Sin[c + d*x])^5,x]
```

```
[Out] (5*a*b^4*ArcTanh[Sin[c + d*x]])/d - (10*a^2*b^3*Cos[c + d*x])/d + (2*b^5*Co
s[c + d*x])/d - (5*a^4*b*Cos[c + d*x]^3)/(3*d) + (10*a^2*b^3*Cos[c + d*x]^3
)/(3*d) - (b^5*Cos[c + d*x]^3)/(3*d) + (b^5*Sec[c + d*x])/d + (a^5*Sin[c +
d*x])/d - (5*a*b^4*Sin[c + d*x])/d - (a^5*Sin[c + d*x]^3)/(3*d) + (10*a^3*b
^2*Sin[c + d*x]^3)/(3*d) - (5*a*b^4*Sin[c + d*x]^3)/(3*d)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 2564

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 2565

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> -Dist[(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 2590

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> -Dist[f^(-1), Subst[Int[(1 - x^2)^((m + n - 1)/2)/x^n, x], x, Cos[e + f*
x]], x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n - 1)/2]
```

Rule 2592

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(
ff*x)^(m + n)/(a^2 - ff^2*x^2)^((n + 1)/2), x], x, (a*Sin[e + f*x])/ff], x]
] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 3090

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_.), x_Symbol] :> Int[ExpandTrig[cos[c + d*x]^m*(a
*cos[c + d*x] + b*sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d}, x] && Inte
```

```
gerQ[m] && IGtQ[n, 0]
```

Rubi steps

$$\int \sec^2(c+dx)(a\cos(c+dx)+b\sin(c+dx))^5\,dx = \int \left(a^5\cos^3(c+dx)+5a^4b\cos^2(c+dx)\sin(c+dx)+10a^3b$$

$$= a^5\int \cos^3(c+dx)\,dx+\left(5a^4b\right)\int \cos^2(c+dx)\sin(c+dx)\,d$$

$$= -\frac{a^5\,\mathrm{Subst}\left(\int \left(1-x^2\right)\,dx,x,-\sin(c+dx)\right)}{d}-\frac{\left(5a^4b\right)\mathrm{Subst}}{}$$

$$= -\frac{10a^2b^3\cos(c+dx)}{d}-\frac{5a^4b\cos^3(c+dx)}{3d}+\frac{10a^2b^3\cos^3(c+}{3d}$$

$$= -\frac{10a^2b^3\cos(c+dx)}{d}+\frac{2b^5\cos(c+dx)}{d}-\frac{5a^4b\cos^3(c+dx}{3d}$$

$$= \frac{5ab^4\tanh^{-1}(\sin(c+dx))}{d}-\frac{10a^2b^3\cos(c+dx)}{d}+\frac{2b^5\cos(c}{d}$$

Mathematica [B] time = 6.30, size = 632, normalized size = 3.08

$$-\frac{b\left(5a^4+30a^2b^2-7b^4\right)\cos^6(c+dx)(a+b\tan(c+dx))^5}{4d(a\cos(c+dx)+b\sin(c+dx))^5}+\frac{a\left(a^4-10a^2b^2+5b^4\right)\sin(3(c+dx))\cos^5(c+dx)(a+}{12d(a\cos(c+dx)+b\sin(c+dx))^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^2*(a*Cos[c + d*x] + b*Sin[c + d*x])^5,x]
```

```
[Out] (b^5*Cos[c + d*x]^5*(a + b*Tan[c + d*x])^5)/(d*(a*Cos[c + d*x] + b*Sin[c +
d*x])^5) - (b*(5*a^4 + 30*a^2*b^2 - 7*b^4)*Cos[c + d*x]^6*(a + b*Tan[c + d*
x])^5)/(4*d*(a*Cos[c + d*x] + b*Sin[c + d*x])^5) - (b*(5*a^4 - 10*a^2*b^2 +
 b^4)*Cos[c + d*x]^5*Cos[3*(c + d*x)]*(a + b*Tan[c + d*x])^5)/(12*d*(a*Cos[
c + d*x] + b*Sin[c + d*x])^5) - (5*a*b^4*Cos[c + d*x]^5*Log[Cos[(c + d*x)/2
] - Sin[(c + d*x)/2]]*(a + b*Tan[c + d*x])^5)/(d*(a*Cos[c + d*x] + b*Sin[c
+ d*x])^5) + (5*a*b^4*Cos[c + d*x]^5*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2
]]*(a + b*Tan[c + d*x])^5)/(d*(a*Cos[c + d*x] + b*Sin[c + d*x])^5) + (b^5*C
os[c + d*x]^5*Sin[(c + d*x)/2]*(a + b*Tan[c + d*x])^5)/(d*(Cos[(c + d*x)/2]
 - Sin[(c + d*x)/2])*(a*Cos[c + d*x] + b*Sin[c + d*x])^5) - (b^5*Cos[c + d*
x]^5*Sin[(c + d*x)/2]*(a + b*Tan[c + d*x])^5)/(d*(Cos[(c + d*x)/2] + Sin[(c
```

```
 + d*x)/2])*(a*Cos[c + d*x] + b*Sin[c + d*x])^5) + (a*(3*a^4 + 10*a^2*b^2 -
 25*b^4)*Cos[c + d*x]^5*Sin[c + d*x]*(a + b*Tan[c + d*x])^5)/(4*d*(a*Cos[c
+ d*x] + b*Sin[c + d*x])^5) + (a*(a^4 - 10*a^2*b^2 + 5*b^4)*Cos[c + d*x]^5*
Sin[3*(c + d*x)]*(a + b*Tan[c + d*x])^5)/(12*d*(a*Cos[c + d*x] + b*Sin[c +
d*x])^5)
```

fricas [A] time = 0.61, size = 177, normalized size = 0.86

$$\frac{15\,ab^4\cos\left(dx+c\right)\log\left(\sin\left(dx+c\right)+1\right)-15\,ab^4\cos\left(dx+c\right)\log\left(-\sin\left(dx+c\right)+1\right)+6\,b^5-2\left(5\,a^4b-10\,a^2b^3+\cdots\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a*cos(d*x+c)+b*sin(d*x+c))^5,x, algorithm="fricas")
```

```
[Out] 1/6*(15*a*b^4*cos(d*x + c)*log(sin(d*x + c) + 1) - 15*a*b^4*cos(d*x + c)*lo
g(-sin(d*x + c) + 1) + 6*b^5 - 2*(5*a^4*b - 10*a^2*b^3 + b^5)*cos(d*x + c)^
4 - 12*(5*a^2*b^3 - b^5)*cos(d*x + c)^2 + 2*((a^5 - 10*a^3*b^2 + 5*a*b^4)*c
os(d*x + c)^3 + 2*(a^5 + 5*a^3*b^2 - 10*a*b^4)*cos(d*x + c))*sin(d*x + c))/
(d*cos(d*x + c))
```

giac [A] time = 0.56, size = 283, normalized size = 1.38

$$\frac{15\,ab^4\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-15\,ab^4\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)-\frac{6\,b^5}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1}+\frac{2\left(3\,a^5\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-15\,ab^4\cdots\right.}{\cdots}}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a*cos(d*x+c)+b*sin(d*x+c))^5,x, algorithm="giac")
```

```
[Out] 1/3*(15*a*b^4*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 15*a*b^4*log(abs(tan(1/2
*d*x + 1/2*c) - 1)) - 6*b^5/(tan(1/2*d*x + 1/2*c)^2 - 1) + 2*(3*a^5*tan(1/2
*d*x + 1/2*c)^5 - 15*a*b^4*tan(1/2*d*x + 1/2*c)^5 - 15*a^4*b*tan(1/2*d*x +
1/2*c)^4 + 3*b^5*tan(1/2*d*x + 1/2*c)^4 + 2*a^5*tan(1/2*d*x + 1/2*c)^3 + 40
*a^3*b^2*tan(1/2*d*x + 1/2*c)^3 - 50*a*b^4*tan(1/2*d*x + 1/2*c)^3 - 60*a^2*
b^3*tan(1/2*d*x + 1/2*c)^2 + 12*b^5*tan(1/2*d*x + 1/2*c)^2 + 3*a^5*tan(1/2*
d*x + 1/2*c) - 15*a*b^4*tan(1/2*d*x + 1/2*c) - 5*a^4*b - 20*a^2*b^3 + 5*b^5
)/(tan(1/2*d*x + 1/2*c)^2 + 1)^3)/d
```

maple [A] time = 0.24, size = 251, normalized size = 1.22

$$\frac{\left(\cos^2\left(dx+c\right)\right)\sin\left(dx+c\right)a^5}{3d}+\frac{2a^5\sin\left(dx+c\right)}{3d}-\frac{5a^4b\left(\cos^3\left(dx+c\right)\right)}{3d}+\frac{10a^3b^2\left(\sin^3\left(dx+c\right)\right)}{3d}-\frac{10\cos\left(dx+c\right)\left(\sin\cdots\right.}{3d}\cdots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2*(a*cos(d*x+c)+b*sin(d*x+c))^5,x)
```

```
[Out] 1/3/d*cos(d*x+c)^2*sin(d*x+c)*a^5+2/3*a^5*sin(d*x+c)/d-5/3*a^4*b*cos(d*x+c)
^3/d+10/3*a^3*b^2*sin(d*x+c)^3/d-10/3/d*cos(d*x+c)*sin(d*x+c)^2*a^2*b^3-20/
3*a^2*b^3*cos(d*x+c)/d-5/3*a*b^4*sin(d*x+c)^3/d-5*a*b^4*sin(d*x+c)/d+5/d*a*
b^4*ln(sec(d*x+c)+tan(d*x+c))+1/d*b^5*sin(d*x+c)^6/cos(d*x+c)+8/3*b^5*cos(d
*x+c)/d+1/d*b^5*cos(d*x+c)*sin(d*x+c)^4+4/3/d*cos(d*x+c)*sin(d*x+c)^2*b^5
```

maxima [A] time = 0.33, size = 162, normalized size = 0.79

$$-\frac{10\,a^4b\cos(dx+c)^3-20\,a^3b^2\sin(dx+c)^3+2\left(\sin(dx+c)^3-3\,\sin(dx+c)\right)a^5-20\left(\cos(dx+c)^3-3\,\cos(d\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a*cos(d*x+c)+b*sin(d*x+c))^5,x, algorithm="maxima")
```

```
[Out] -1/6*(10*a^4*b*cos(d*x + c)^3 - 20*a^3*b^2*sin(d*x + c)^3 + 2*(sin(d*x + c)
^3 - 3*sin(d*x + c))*a^5 - 20*(cos(d*x + c)^3 - 3*cos(d*x + c))*a^2*b^3 + 5
*(2*sin(d*x + c)^3 - 3*log(sin(d*x + c) + 1) + 3*log(sin(d*x + c) - 1) + 6*
sin(d*x + c))*a*b^4 + 2*(cos(d*x + c)^3 - 3/cos(d*x + c) - 6*cos(d*x + c))*
b^5)/d
```

mupad [B] time = 3.98, size = 277, normalized size = 1.35

$$\frac{10\,a\,b^4\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{d}-\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(10\,a\,b^4-2\,a^5\right)+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4\,\left(10\,a^4\,b-40\,a^2\,b^3\right)+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*cos(c + d*x) + b*sin(c + d*x))^5/cos(c + d*x)^2,x)
```

```
[Out] (10*a*b^4*atanh(tan(c/2 + (d*x)/2)))/d - (tan(c/2 + (d*x)/2)*(10*a*b^4 - 2*
a^5) + tan(c/2 + (d*x)/2)^4*(10*a^4*b - 40*a^2*b^3) + tan(c/2 + (d*x)/2)^3*
((70*a*b^4)/3 + (2*a^5)/3 - (80*a^3*b^2)/3) - tan(c/2 + (d*x)/2)^5*((70*a*b
^4)/3 + (2*a^5)/3 - (80*a^3*b^2)/3) - tan(c/2 + (d*x)/2)^2*((10*a^4*b)/3 +
(32*b^5)/3 - (80*a^2*b^3)/3) + (10*a^4*b)/3 - tan(c/2 + (d*x)/2)^7*(10*a*b^
4 - 2*a^5) - (16*b^5)/3 + (40*a^2*b^3)/3 - 10*a^4*b*tan(c/2 + (d*x)/2)^6)/(
d*(2*tan(c/2 + (d*x)/2)^2 - 2*tan(c/2 + (d*x)/2)^6 - tan(c/2 + (d*x)/2)^8 +
 1))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2*(a*cos(d*x+c)+b*sin(d*x+c))**5,x)
```

```
[Out] Timed out
```

3.100 $\int \sec^3(c+dx)(a\cos(c+dx)+b\sin(c+dx))^5\,dx$

Optimal. Leaf size=169

$$-\frac{2b^3\left(5a^2-b^2\right)\log(\sin(c+dx))}{d}+\frac{2b^3\left(5a^2-b^2\right)\log(\tan(c+dx))}{d}+\frac{\sin^2(c+dx)\left(a\left(a^4-10a^2b^2+5b^4\right)\cot(c+\right.}{2d}$$

```
[Out] 1/2*a*(a^4+10*a^2*b^2-15*b^4)*x-2*b^3*(5*a^2-b^2)*ln(sin(d*x+c))/d+2*b^3*(5
*a^2-b^2)*ln(tan(d*x+c))/d+1/2*(b*(5*a^4-10*a^2*b^2+b^4)+a*(a^4-10*a^2*b^2+
5*b^4)*cot(d*x+c))*sin(d*x+c)^2/d+5*a*b^4*tan(d*x+c)/d+1/2*b^5*tan(d*x+c)^2
/d
```

Rubi [A] time = 0.23, antiderivative size = 169, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {3088, 1805, 1802, 635, 203, 260}

$$-\frac{2b^3\left(5a^2-b^2\right)\log(\sin(c+dx))}{d}+\frac{2b^3\left(5a^2-b^2\right)\log(\tan(c+dx))}{d}+\frac{\sin^2(c+dx)\left(a\left(-10a^2b^2+a^4+5b^4\right)\cot(c\right.}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^3*(a*Cos[c + d*x] + b*Sin[c + d*x])^5,x]
```

```
[Out] (a*(a^4 + 10*a^2*b^2 - 15*b^4)*x)/2 - (2*b^3*(5*a^2 - b^2)*Log[Sin[c + d*x]
])/d + (2*b^3*(5*a^2 - b^2)*Log[Tan[c + d*x]])/d + ((b*(5*a^4 - 10*a^2*b^2
+ b^4) + a*(a^4 - 10*a^2*b^2 + 5*b^4)*Cot[c + d*x])*Sin[c + d*x]^2)/(2*d) +
 (5*a*b^4*Tan[c + d*x])/d + (b^5*Tan[c + d*x]^2)/(2*d)
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] && !NiceSqrtQ[-(a*c)]
```

Rule 1802

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 1805

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[((a*g - b*f*x)*(a + b*x^2)^(p + 1))/(2*a
*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int[(c*x)^m*(a + b*x^2)^(p + 1)*Exp
andToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; Fr
eeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 3088

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[(x^m*(b +
 a*x)^n)/(1 + x^2)^((m + n + 2)/2), x], x, Cot[c + d*x]], x] /; FreeQ[{a, b
, c, d}, x] && IntegerQ[n] && IntegerQ[(m + n)/2] && NeQ[n, -1] &&  !(GtQ[n
, 0] && GtQ[m, 1])
```

Rubi steps

$$\int \sec^3(c+dx)(a\cos(c+dx)+b\sin(c+dx))^5\,dx = -\frac{\text{Subst}\left(\int \frac{(b+ax)^5}{x^3(1+x^2)^2}\,dx, x, \cot(c+dx)\right)}{d}$$

$$= \frac{\left(b\left(5a^4-10a^2b^2+b^4\right)+a\left(a^4-10a^2b^2+5b^4\right)\cot(c+dx)\right.}{2d}$$

$$= \frac{\left(b\left(5a^4-10a^2b^2+b^4\right)+a\left(a^4-10a^2b^2+5b^4\right)\cot(c+dx)\right.}{2d}$$

$$= \frac{2b^3\left(5a^2-b^2\right)\log(\tan(c+dx))}{d} + \frac{\left(b\left(5a^4-10a^2b^2+b^4\right)+\right.}{}$$

$$= \frac{2b^3\left(5a^2-b^2\right)\log(\tan(c+dx))}{d} + \frac{\left(b\left(5a^4-10a^2b^2+b^4\right)+\right.}{}$$

$$= \frac{1}{2}a\left(a^4+10a^2b^2-15b^4\right)x - \frac{2b^3\left(5a^2-b^2\right)\log(\sin(c+dx)}{d}$$

Mathematica [B] time = 6.36, size = 571, normalized size = 3.38

$$b^3\left(\frac{\cos^2(c+dx)(a+b\tan(c+dx))^6\left(ab\tan(c+dx)+b^2\right)}{2b^4\left(a^2+b^2\right)} - \frac{\left(4b^2-6a^2\right)\left(\frac{1}{2}b^2\left(10a^2-b^2\right)\tan^2(c+dx)+5ab\left(2a^2-b^2\right)\tan(c+dx)+\frac{1}{2}\left(5a^4-10a^2b^2+\frac{a^5-10a^3b^2+5a}{\sqrt{-b^2}}\right.\right.}{}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sec[c + d*x]^3*(a*Cos[c + d*x] + b*Sin[c + d*x])^5,x]
```

```
[Out] (b^3*((Cos[c + d*x]^2*(a + b*Tan[c + d*x])^6*(b^2 + a*b*Tan[c + d*x]))/(2*b
^4*(a^2 + b^2)) - ((-6*a^2 + 4*b^2)*(((5*a^4 - 10*a^2*b^2 + b^4 + (a^5 - 10
*a^3*b^2 + 5*a*b^4)/Sqrt[-b^2])*Log[Sqrt[-b^2] - b*Tan[c + d*x]])/2 + ((5*a
^4 - 10*a^2*b^2 + b^4 - (a^5 - 10*a^3*b^2 + 5*a*b^4)/Sqrt[-b^2])*Log[Sqrt[-
b^2] + b*Tan[c + d*x]])/2 + 5*a*b*(2*a^2 - b^2)*Tan[c + d*x] + (b^2*(10*a^2
 - b^2)*Tan[c + d*x]^2)/2 + (5*a*b^3*Tan[c + d*x]^3)/3 + (b^4*Tan[c + d*x]^
4)/4) + 5*a*(((6*a^5 - 20*a^3*b^2 + 6*a*b^4 + (a^6 - 15*a^4*b^2 + 15*a^2*b^
4 - b^6)/Sqrt[-b^2])*Log[Sqrt[-b^2] - b*Tan[c + d*x]])/2 + ((6*a^5 - 20*a^3
*b^2 + 6*a*b^4 - (a^6 - 15*a^4*b^2 + 15*a^2*b^4 - b^6)/Sqrt[-b^2])*Log[Sqrt
[-b^2] + b*Tan[c + d*x]])/2 + b*(15*a^4 - 15*a^2*b^2 + b^4)*Tan[c + d*x] +
a*b^2*(10*a^2 - 3*b^2)*Tan[c + d*x]^2 + (b^3*(15*a^2 - b^2)*Tan[c + d*x]^3)
```

```
/3 + (3*a*b^4*Tan[c + d*x]^4)/2 + (b^5*Tan[c + d*x]^5)/5))/(2*b^2*(a^2 + b^
2))))/d
```

fricas [A] time = 0.46, size = 177, normalized size = 1.05

$$\frac{2\,b^5 - 2\left(5\,a^4b - 10\,a^2b^3 + b^5\right)\cos\left(dx+c\right)^4 - 8\left(5\,a^2b^3 - b^5\right)\cos\left(dx+c\right)^2\log\left(-\cos\left(dx+c\right)\right) + \left(5\,a^4b - 10\,a^2b^3\right.}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a*cos(d*x+c)+b*sin(d*x+c))^5,x, algorithm="fricas")
```

```
[Out] 1/4*(2*b^5 - 2*(5*a^4*b - 10*a^2*b^3 + b^5)*cos(d*x + c)^4 - 8*(5*a^2*b^3 -
 b^5)*cos(d*x + c)^2*log(-cos(d*x + c)) + (5*a^4*b - 10*a^2*b^3 + b^5 + 2*(
a^5 + 10*a^3*b^2 - 15*a*b^4)*d*x)*cos(d*x + c)^2 + 2*(10*a*b^4*cos(d*x + c)
 + (a^5 - 10*a^3*b^2 + 5*a*b^4)*cos(d*x + c)^3)*sin(d*x + c))/(d*cos(d*x +
c)^2)
```

giac [A] time = 0.58, size = 173, normalized size = 1.02

$$\frac{b^5\tan\left(dx+c\right)^2 + 10\,ab^4\tan\left(dx+c\right) + \left(a^5 + 10\,a^3b^2 - 15\,ab^4\right)\left(dx+c\right) + 2\left(5\,a^2b^3 - b^5\right)\log\left(\tan\left(dx+c\right)^2 + 1\right) -}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a*cos(d*x+c)+b*sin(d*x+c))^5,x, algorithm="giac")
```

```
[Out] 1/2*(b^5*tan(d*x + c)^2 + 10*a*b^4*tan(d*x + c) + (a^5 + 10*a^3*b^2 - 15*a*
b^4)*(d*x + c) + 2*(5*a^2*b^3 - b^5)*log(tan(d*x + c)^2 + 1) - (10*a^2*b^3*
tan(d*x + c)^2 - 2*b^5*tan(d*x + c)^2 - a^5*tan(d*x + c) + 10*a^3*b^2*tan(d
*x + c) - 5*a*b^4*tan(d*x + c) + 5*a^4*b - b^5)/(tan(d*x + c)^2 + 1))/d
```

maple [A] time = 0.24, size = 291, normalized size = 1.72

$$\frac{a^5\cos\left(dx+c\right)\sin\left(dx+c\right)}{2d}+\frac{a^5x}{2}+\frac{a^5c}{2d}-\frac{5a^4b\left(\cos^2\left(dx+c\right)\right)}{2d}-\frac{5a^3b^2\cos\left(dx+c\right)\sin\left(dx+c\right)}{d}+5a^3b^2x+\frac{5a^3b^2c}{d}-\frac{5a}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^3*(a*cos(d*x+c)+b*sin(d*x+c))^5,x)
```

```
[Out] 1/2*a^5*cos(d*x+c)*sin(d*x+c)/d+1/2*a^5*x+1/2/d*a^5*c-5/2/d*a^4*b*cos(d*x+c
)^2-5*a^3*b^2*cos(d*x+c)*sin(d*x+c)/d+5*a^3*b^2*x+5/d*a^3*b^2*c-5/d*a^2*b^3
*sin(d*x+c)^2-10/d*a^2*b^3*ln(cos(d*x+c))+5/d*a*b^4*sin(d*x+c)^5/cos(d*x+c)
+5/d*a*b^4*cos(d*x+c)*sin(d*x+c)^3+15/2*a*b^4*cos(d*x+c)*sin(d*x+c)/d-15/2*
```

```
a*b^4*x-15/2/d*a*b^4*c+1/2/d*b^5*sin(d*x+c)^6/cos(d*x+c)^2+1/2/d*b^5*sin(d*
x+c)^4+1/d*sin(d*x+c)^2*b^5+2/d*b^5*ln(cos(d*x+c))
```

maxima [A] time = 0.43, size = 179, normalized size = 1.06

$$\frac{10\,a^4b\sin\left(dx+c\right)^2+\left(2\,dx+2\,c+\sin\left(2\,dx+2\,c\right)\right)a^5+10\left(2\,dx+2\,c-\sin\left(2\,dx+2\,c\right)\right)a^3b^2-20\left(\sin\left(dx+c\right)^2\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a*cos(d*x+c)+b*sin(d*x+c))^5,x, algorithm="maxima")
```

```
[Out] 1/4*(10*a^4*b*sin(d*x + c)^2 + (2*d*x + 2*c + sin(2*d*x + 2*c))*a^5 + 10*(2
*d*x + 2*c - sin(2*d*x + 2*c))*a^3*b^2 - 20*(sin(d*x + c)^2 + log(sin(d*x +
 c)^2 - 1))*a^2*b^3 - 10*(3*d*x + 3*c - tan(d*x + c)/(tan(d*x + c)^2 + 1) -
 2*tan(d*x + c))*a*b^4 + 2*(sin(d*x + c)^2 - 1/(sin(d*x + c)^2 - 1) + 2*log
(sin(d*x + c)^2 - 1))*b^5)/d
```

mupad [B] time = 2.53, size = 354, normalized size = 2.09

$$\frac{2\left(b^5\ln\left(\frac{\cos(c+d\,x)}{\cos(c+d\,x)+1}\right)-b^5\ln\left(\frac{1}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}\right)+\frac{a^5\operatorname{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{2}-5\,a^2\,b^3\ln\left(\frac{\cos(c+d\,x)}{\cos(c+d\,x)+1}\right)-\frac{15\,a\,b^4\operatorname{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{2}+5\right.}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*cos(c + d*x) + b*sin(c + d*x))^5/cos(c + d*x)^3,x)
```

```
[Out] (2*(b^5*log(cos(c + d*x)/(cos(c + d*x) + 1)) - b^5*log(1/cos(c/2 + (d*x)/2)
^2) + (a^5*atan(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/2 - 5*a^2*b^3*log(c
os(c + d*x)/(cos(c + d*x) + 1)) - (15*a*b^4*atan(sin(c/2 + (d*x)/2)/cos(c/2
 + (d*x)/2)))/2 + 5*a^2*b^3*log(1/cos(c/2 + (d*x)/2)^2) + 5*a^3*b^2*atan(si
n(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2))))/d + ((5*a^4*b)/16 + (9*b^5)/16 - (5*
a^2*b^3)/8 - (b^5*cos(4*c + 4*d*x))/16 + (a^5*sin(2*c + 2*d*x))/8 + (a^5*si
n(4*c + 4*d*x))/16 - (5*a^4*b*cos(4*c + 4*d*x))/16 + (25*a*b^4*sin(2*c + 2*
d*x))/8 + (5*a*b^4*sin(4*c + 4*d*x))/16 + (5*a^2*b^3*cos(4*c + 4*d*x))/8 -
(5*a^3*b^2*sin(2*c + 2*d*x))/4 - (5*a^3*b^2*sin(4*c + 4*d*x))/8)/(d*(cos(2*
c + 2*d*x)/2 + 1/2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**3*(a*cos(d*x+c)+b*sin(d*x+c))**5,x)
```

```
[Out] Timed out
```

3.101 $\int \sec^4(c+dx)(a\cos(c+dx)+b\sin(c+dx))^5\,dx$

Optimal. Leaf size=204

$$\frac{a^5\sin(c+dx)}{d}-\frac{5a^4b\cos(c+dx)}{d}-\frac{10a^3b^2\sin(c+dx)}{d}+\frac{10a^3b^2\tanh^{-1}(\sin(c+dx))}{d}+\frac{10a^2b^3\cos(c+dx)}{d}+\frac{10a^2b^3}{}$$

```
[Out] 10*a^3*b^2*arctanh(sin(d*x+c))/d-15/2*a*b^4*arctanh(sin(d*x+c))/d-5*a^4*b*c
os(d*x+c)/d+10*a^2*b^3*cos(d*x+c)/d-b^5*cos(d*x+c)/d+10*a^2*b^3*sec(d*x+c)/
d-2*b^5*sec(d*x+c)/d+1/3*b^5*sec(d*x+c)^3/d+a^5*sin(d*x+c)/d-10*a^3*b^2*sin
(d*x+c)/d+15/2*a*b^4*sin(d*x+c)/d+5/2*a*b^4*sin(d*x+c)*tan(d*x+c)^2/d
```

Rubi [A] time = 0.21, antiderivative size = 204, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 10, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.357$, Rules used = {3090, 2637, 2638, 2592, 321, 206, 2590, 14, 288, 270}

$$-\frac{10a^3b^2\sin(c+dx)}{d}+\frac{10a^2b^3\cos(c+dx)}{d}+\frac{10a^2b^3\sec(c+dx)}{d}+\frac{10a^3b^2\tanh^{-1}(\sin(c+dx))}{d}-\frac{5a^4b\cos(c+dx)}{d}+$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^4*(a*Cos[c + d*x] + b*Sin[c + d*x])^5,x]
```

```
[Out] (10*a^3*b^2*ArcTanh[Sin[c + d*x]])/d - (15*a*b^4*ArcTanh[Sin[c + d*x]])/(2*
d) - (5*a^4*b*Cos[c + d*x])/d + (10*a^2*b^3*Cos[c + d*x])/d - (b^5*Cos[c +
d*x])/d + (10*a^2*b^3*Sec[c + d*x])/d - (2*b^5*Sec[c + d*x])/d + (b^5*Sec[c
 + d*x]^3)/(3*d) + (a^5*Sin[c + d*x])/d - (10*a^3*b^2*Sin[c + d*x])/d + (15
*a*b^4*Sin[c + d*x])/(2*d) + (5*a*b^4*Sin[c + d*x]*Tan[c + d*x]^2)/(2*d)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
```

```
IGtQ[p, 0]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2590

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> -Dist[f^(-1), Subst[Int[(1 - x^2)^((m + n - 1)/2)/x^n, x], x, Cos[e + f*
x]], x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n - 1)/2]
```

Rule 2592

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(
ff*x)^(m + n)/(a^2 - ff^2*x^2)^((n + 1)/2), x], x, (a*Sin[e + f*x])/ff], x]
] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3090

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_.), x_Symbol] :> Int[ExpandTrig[cos[c + d*x]^m*(a
```

```
*cos[c + d*x] + b*sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d}, x] && Inte
gerQ[m] && IGtQ[n, 0]
```

Rubi steps

$$\int \sec^4(c+dx)(a\cos(c+dx)+b\sin(c+dx))^5\,dx = \int \left(a^5\cos(c+dx)+5a^4b\sin(c+dx)+10a^3b^2\sin(c+dx)\right.$$

$$= a^5\int\cos(c+dx)\,dx+\left(5a^4b\right)\int\sin(c+dx)\,dx+\left(10a^3b^2\right)$$

$$= -\frac{5a^4b\cos(c+dx)}{d}+\frac{a^5\sin(c+dx)}{d}+\frac{\left(10a^3b^2\right)\text{Subst}\left(\int\right.}{}$$

$$= -\frac{5a^4b\cos(c+dx)}{d}+\frac{a^5\sin(c+dx)}{d}-\frac{10a^3b^2\sin(c+dx)}{d}+$$

$$= \frac{10a^3b^2\tanh^{-1}(\sin(c+dx))}{d}-\frac{5a^4b\cos(c+dx)}{d}+\frac{10a^2b^3}{}$$

$$= \frac{10a^3b^2\tanh^{-1}(\sin(c+dx))}{d}-\frac{15ab^4\tanh^{-1}(\sin(c+dx))}{2d}-$$

Mathematica [A] time = 5.90, size = 397, normalized size = 1.95

$$120a^2b^3-30ab^2\left(4a^2-3b^2\right)\log\left(\cos\left(\tfrac{1}{2}(c+dx)\right)-\sin\left(\tfrac{1}{2}(c+dx)\right)\right)+30ab^2\left(4a^2-3b^2\right)\log\left(\sin\left(\tfrac{1}{2}(c+dx)\right)+\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^4*(a*Cos[c + d*x] + b*Sin[c + d*x])^5,x]
```

```
[Out] (120*a^2*b^3 - 22*b^5 - 12*b*(5*a^4 - 10*a^2*b^2 + b^4)*Cos[c + d*x] - 30*a
*b^2*(4*a^2 - 3*b^2)*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + 30*a*b^2*(4
*a^2 - 3*b^2)*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] + (b^4*(15*a + b))/(
Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^2 + (2*b^5*Sin[(c + d*x)/2])/(Cos[(c +
 d*x)/2] - Sin[(c + d*x)/2])^3 + (2*b^3*(60*a^2 - 11*b^2)*Sin[(c + d*x)/2])
/(Cos[(c + d*x)/2] - Sin[(c + d*x)/2]) - (2*b^5*Sin[(c + d*x)/2])/(Cos[(c +
 d*x)/2] + Sin[(c + d*x)/2])^3 + (b^4*(-15*a + b))/(Cos[(c + d*x)/2] + Sin[
(c + d*x)/2])^2 + (2*b^3*(-60*a^2 + 11*b^2)*Sin[(c + d*x)/2])/(Cos[(c + d*x
)/2] + Sin[(c + d*x)/2]) + 12*a*(a^4 - 10*a^2*b^2 + 5*b^4)*Sin[c + d*x])/(1
2*d)
```

fricas [A] time = 0.50, size = 190, normalized size = 0.93

$$\frac{4\,b^5 - 12\left(5\,a^4b - 10\,a^2b^3 + b^5\right)\cos\left(dx+c\right)^4 + 15\left(4\,a^3b^2 - 3\,ab^4\right)\cos\left(dx+c\right)^3\log\left(\sin\left(dx+c\right)+1\right) - 15\left(4\,a^3b^2 \right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(a*cos(d*x+c)+b*sin(d*x+c))^5,x, algorithm="fricas")
```

```
[Out] 1/12*(4*b^5 - 12*(5*a^4*b - 10*a^2*b^3 + b^5)*cos(d*x + c)^4 + 15*(4*a^3*b^
2 - 3*a*b^4)*cos(d*x + c)^3*log(sin(d*x + c) + 1) - 15*(4*a^3*b^2 - 3*a*b^4
)*cos(d*x + c)^3*log(-sin(d*x + c) + 1) + 24*(5*a^2*b^3 - b^5)*cos(d*x + c)
^2 + 6*(5*a*b^4*cos(d*x + c) + 2*(a^5 - 10*a^3*b^2 + 5*a*b^4)*cos(d*x + c)^
3)*sin(d*x + c))/(d*cos(d*x + c)^3)
```

giac [A] time = 0.60, size = 281, normalized size = 1.38

$$\frac{15\left(4\,a^3b^2 - 3\,ab^4\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right)+1\right|\right) - 15\left(4\,a^3b^2 - 3\,ab^4\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right)-1\right|\right) + \frac{12\left(a^5\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right.\right.}{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(a*cos(d*x+c)+b*sin(d*x+c))^5,x, algorithm="giac")
```

```
[Out] 1/6*(15*(4*a^3*b^2 - 3*a*b^4)*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 15*(4*a^
3*b^2 - 3*a*b^4)*log(abs(tan(1/2*d*x + 1/2*c) - 1)) + 12*(a^5*tan(1/2*d*x +
 1/2*c) - 10*a^3*b^2*tan(1/2*d*x + 1/2*c) + 5*a*b^4*tan(1/2*d*x + 1/2*c) -
5*a^4*b + 10*a^2*b^3 - b^5)/(tan(1/2*d*x + 1/2*c)^2 + 1) + 2*(15*a*b^4*tan(
1/2*d*x + 1/2*c)^5 - 60*a^2*b^3*tan(1/2*d*x + 1/2*c)^4 + 6*b^5*tan(1/2*d*x
+ 1/2*c)^4 + 120*a^2*b^3*tan(1/2*d*x + 1/2*c)^2 - 24*b^5*tan(1/2*d*x + 1/2*
c)^2 - 15*a*b^4*tan(1/2*d*x + 1/2*c) - 60*a^2*b^3 + 10*b^5)/(tan(1/2*d*x +
1/2*c)^2 - 1)^3)/d
```

maple [A] time = 0.23, size = 327, normalized size = 1.60

$$\frac{a^5\sin\left(dx+c\right)}{d} - \frac{5a^4b\cos\left(dx+c\right)}{d} - \frac{10a^3b^2\sin\left(dx+c\right)}{d} + \frac{10a^3b^2\ln\left(\sec\left(dx+c\right)+\tan\left(dx+c\right)\right)}{d} + \frac{10a^2b^3\left(\sin^4\left(dx\right.\right.}{d\cos\left(dx+c\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^4*(a*cos(d*x+c)+b*sin(d*x+c))^5,x)
```

```
[Out] a^5*sin(d*x+c)/d-5*a^4*b*cos(d*x+c)/d-10*a^3*b^2*sin(d*x+c)/d+10/d*a^3*b^2*
ln(sec(d*x+c)+tan(d*x+c))+10/d*a^2*b^3*sin(d*x+c)^4/cos(d*x+c)+10/d*cos(d*x
```

```
+c)*sin(d*x+c)^2*a^2*b^3+20*a^2*b^3*cos(d*x+c)/d+5/2/d*a*b^4*sin(d*x+c)^5/c
os(d*x+c)^2+5/2*a*b^4*sin(d*x+c)^3/d+15/2*a*b^4*sin(d*x+c)/d-15/2/d*a*b^4*l
n(sec(d*x+c)+tan(d*x+c))+1/3/d*b^5*sin(d*x+c)^6/cos(d*x+c)^3-1/d*b^5*sin(d*
x+c)^6/cos(d*x+c)-8/3*b^5*cos(d*x+c)/d-1/d*b^5*cos(d*x+c)*sin(d*x+c)^4-4/3/
d*cos(d*x+c)*sin(d*x+c)^2*b^5
```

maxima [A] time = 0.33, size = 181, normalized size = 0.89

$$-\frac{15\,ab^4\left(\frac{2\,\sin(dx+c)}{\sin(dx+c)^2-1}+3\,\log\left(\sin\left(dx+c\right)+1\right)-3\,\log\left(\sin\left(dx+c\right)-1\right)-4\,\sin\left(dx+c\right)\right)-120\,a^2b^3\left(\frac{1}{\cos(dx+c)}+\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(a*cos(d*x+c)+b*sin(d*x+c))^5,x, algorithm="maxima")
```

```
[Out] -1/12*(15*a*b^4*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) + 3*log(sin(d*x + c) +
 1) - 3*log(sin(d*x + c) - 1) - 4*sin(d*x + c)) - 120*a^2*b^3*(1/cos(d*x +
c) + cos(d*x + c)) + 4*b^5*((6*cos(d*x + c)^2 - 1)/cos(d*x + c)^3 + 3*cos(d
*x + c)) - 60*a^3*b^2*(log(sin(d*x + c) + 1) - log(sin(d*x + c) - 1) - 2*si
n(d*x + c)) + 60*a^4*b*cos(d*x + c) - 12*a^5*sin(d*x + c))/d
```

mupad [B] time = 4.05, size = 302, normalized size = 1.48

$$-\frac{\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)\,\left(15\,a\,b^4-20\,a^3\,b^2\right)}{d}-\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(2\,a^5-20\,a^3\,b^2+15\,a\,b^4\right)-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4\,\left(30\,a^4\,b-\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*cos(c + d*x) + b*sin(c + d*x))^5/cos(c + d*x)^4,x)
```

```
[Out] - (atanh(tan(c/2 + (d*x)/2))*(15*a*b^4 - 20*a^3*b^2))/d - (tan(c/2 + (d*x)/
2)*(15*a*b^4 + 2*a^5 - 20*a^3*b^2) - tan(c/2 + (d*x)/2)^4*(30*a^4*b - 40*a^
2*b^3) - tan(c/2 + (d*x)/2)^7*(15*a*b^4 + 2*a^5 - 20*a^3*b^2) - tan(c/2 + (
d*x)/2)^3*(25*a*b^4 + 6*a^5 - 60*a^3*b^2) + tan(c/2 + (d*x)/2)^5*(25*a*b^4
+ 6*a^5 - 60*a^3*b^2) + tan(c/2 + (d*x)/2)^2*(30*a^4*b + (32*b^5)/3 - 80*a^
2*b^3) - 10*a^4*b - (16*b^5)/3 + 40*a^2*b^3 + 10*a^4*b*tan(c/2 + (d*x)/2)^6
)/(d*(2*tan(c/2 + (d*x)/2)^2 - 2*tan(c/2 + (d*x)/2)^6 + tan(c/2 + (d*x)/2)^
8 - 1))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**4*(a*cos(d*x+c)+b*sin(d*x+c))**5,x)
```

```
[Out] Timed out
```

3.102 $\int \sec^5(c+dx)(a\cos(c+dx)+b\sin(c+dx))^5\,dx$

Optimal. Leaf size=147

$$\frac{b\left(3a^2-b^2\right)(a+b\tan(c+dx))^2}{2d}+\frac{4ab^2\left(a^2-b^2\right)\tan(c+dx)}{d}-\frac{b\left(5a^4-10a^2b^2+b^4\right)\log(\cos(c+dx))}{d}+ax\left(a^4-1\right.$$

```
[Out] a*(a^4-10*a^2*b^2+5*b^4)*x-b*(5*a^4-10*a^2*b^2+b^4)*ln(cos(d*x+c))/d+4*a*b^
2*(a^2-b^2)*tan(d*x+c)/d+1/2*b*(3*a^2-b^2)*(a+b*tan(d*x+c))^2/d+2/3*a*b*(a+
b*tan(d*x+c))^3/d+1/4*b*(a+b*tan(d*x+c))^4/d
```

Rubi [A] time = 0.23, antiderivative size = 147, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.179, Rules used = {3086, 3482, 3528, 3525, 3475}

$$\frac{b\left(3a^2-b^2\right)(a+b\tan(c+dx))^2}{2d}+\frac{4ab^2\left(a^2-b^2\right)\tan(c+dx)}{d}-\frac{b\left(-10a^2b^2+5a^4+b^4\right)\log(\cos(c+dx))}{d}+ax\left(-10\right.$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^5*(a*Cos[c + d*x] + b*Sin[c + d*x])^5,x]
```

```
[Out] a*(a^4 - 10*a^2*b^2 + 5*b^4)*x - (b*(5*a^4 - 10*a^2*b^2 + b^4)*Log[Cos[c +
d*x]])/d + (4*a*b^2*(a^2 - b^2)*Tan[c + d*x])/d + (b*(3*a^2 - b^2)*(a + b*T
an[c + d*x])^2)/(2*d) + (2*a*b*(a + b*Tan[c + d*x])^3)/(3*d) + (b*(a + b*Ta
n[c + d*x])^4)/(4*d)
```

Rule 3086

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin
[(c_.) + (d_.)*(x_)])^(n_.), x_Symbol] :> Int[(a + b*Tan[c + d*x])^n, x] /;
 FreeQ[{a, b, c, d}, x] && EqQ[m + n, 0] && IntegerQ[n] && NeQ[a^2 + b^2, 0
]
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3482

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(a +
b*Tan[c + d*x])^(n - 1))/(d*(n - 1)), x] + Int[(a^2 - b^2 + 2*a*b*Tan[c + d
*x])*(a + b*Tan[c + d*x])^(n - 2), x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2
+ b^2, 0] && GtQ[n, 1]
```

Rule 3525

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[(a*c - b*d)*x, x] + (Dist[b*c + a*d, Int[Tan[e +
 f*x], x], x] + Simp[(b*d*Tan[e + f*x])/f, x]) /; FreeQ[{a, b, c, d, e, f},
 x] && NeQ[b*c - a*d, 0] && NeQ[b*c + a*d, 0]
```

Rule 3528

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[(d*(a + b*Tan[e + f*x])^m)/(f*m), x] + Int
[(a + b*Tan[e + f*x])^(m - 1)*Simp[a*c - b*d + (b*c + a*d)*Tan[e + f*x], x]
, x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2,
 0] && GtQ[m, 0]
```

Rubi steps

$$\int \sec^5(c+dx)(a\cos(c+dx)+b\sin(c+dx))^5\,dx = \int (a+b\tan(c+dx))^5\,dx$$
$$= \frac{b(a+b\tan(c+dx))^4}{4d} + \int (a+b\tan(c+dx))^3\left(a^2-b^2+2ab\right.$$
$$= \frac{2ab(a+b\tan(c+dx))^3}{3d} + \frac{b(a+b\tan(c+dx))^4}{4d} + \int (a+b\,\mathrm{t}$$
$$= \frac{b\left(3a^2-b^2\right)(a+b\tan(c+dx))^2}{2d} + \frac{2ab(a+b\tan(c+dx))^3}{3d} +$$
$$= a\left(a^4-10a^2b^2+5b^4\right)x + \frac{4ab^2\left(a^2-b^2\right)\tan(c+dx)}{d} + \frac{b\left(3a\right.}{}$$
$$= a\left(a^4-10a^2b^2+5b^4\right)x - \frac{b\left(5a^4-10a^2b^2+b^4\right)\log(\cos(c+}{d}$$

Mathematica [C] time = 0.74, size = 126, normalized size = 0.86

$$\frac{60ab^2\left(2a^2-b^2\right)\tan(c+dx)-6b^3\left(b^2-10a^2\right)\tan^2(c+dx)+20ab^4\tan^3(c+dx)+6(b-ia)^5\log(-\tan(c+dx)+}{12d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^5*(a*Cos[c + d*x] + b*Sin[c + d*x])^5,x]
```

```
[Out] (6*((-I)*a + b)^5*Log[I - Tan[c + d*x]] + 6*(I*a + b)^5*Log[I + Tan[c + d*x
]] + 60*a*b^2*(2*a^2 - b^2)*Tan[c + d*x] - 6*b^3*(-10*a^2 + b^2)*Tan[c + d*
x]^2 + 20*a*b^4*Tan[c + d*x]^3 + 3*b^5*Tan[c + d*x]^4)/(12*d)
```

fricas [A] time = 0.64, size = 155, normalized size = 1.05

$$\frac{12\left(a^5-10\,a^3b^2+5\,ab^4\right)dx\cos\left(dx+c\right)^4-12\left(5\,a^4b-10\,a^2b^3+b^5\right)\cos\left(dx+c\right)^4\log\left(-\cos\left(dx+c\right)\right)+3\,b^5+1}{12\,d\cos\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*(a*cos(d*x+c)+b*sin(d*x+c))^5,x, algorithm="fricas")
```

```
[Out] 1/12*(12*(a^5 - 10*a^3*b^2 + 5*a*b^4)*d*x*cos(d*x + c)^4 - 12*(5*a^4*b - 10
*a^2*b^3 + b^5)*cos(d*x + c)^4*log(-cos(d*x + c)) + 3*b^5 + 12*(5*a^2*b^3 -
 b^5)*cos(d*x + c)^2 + 20*(a*b^4*cos(d*x + c) + 2*(3*a^3*b^2 - 2*a*b^4)*cos
(d*x + c)^3)*sin(d*x + c))/(d*cos(d*x + c)^4)
```

giac [A] time = 0.62, size = 144, normalized size = 0.98

$$\frac{3\,b^5\tan\left(dx+c\right)^4+20\,ab^4\tan\left(dx+c\right)^3+60\,a^2b^3\tan\left(dx+c\right)^2-6\,b^5\tan\left(dx+c\right)^2+120\,a^3b^2\tan\left(dx+c\right)-60}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*(a*cos(d*x+c)+b*sin(d*x+c))^5,x, algorithm="giac")
```

```
[Out] 1/12*(3*b^5*tan(d*x + c)^4 + 20*a*b^4*tan(d*x + c)^3 + 60*a^2*b^3*tan(d*x +
 c)^2 - 6*b^5*tan(d*x + c)^2 + 120*a^3*b^2*tan(d*x + c) - 60*a*b^4*tan(d*x
+ c) + 12*(a^5 - 10*a^3*b^2 + 5*a*b^4)*(d*x + c) + 6*(5*a^4*b - 10*a^2*b^3
+ b^5)*log(tan(d*x + c)^2 + 1))/d
```

maple [A] time = 0.23, size = 202, normalized size = 1.37

$$a^5x+\frac{a^5c}{d}-\frac{5a^4b\ln\left(\cos\left(dx+c\right)\right)}{d}-10a^3b^2x+\frac{10\tan\left(dx+c\right)a^3b^2}{d}-\frac{10a^3b^2c}{d}+\frac{5a^2b^3\left(\tan^2\left(dx+c\right)\right)}{d}+\frac{10a^2b^3\ln\left(\cos\right.}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^5*(a*cos(d*x+c)+b*sin(d*x+c))^5,x)
```

```
[Out] a^5*x+1/d*a^5*c-5/d*a^4*b*ln(cos(d*x+c))-10*a^3*b^2*x+10/d*tan(d*x+c)*a^3*b
^2-10/d*a^3*b^2*c+5/d*a^2*b^3*tan(d*x+c)^2+10/d*a^2*b^3*ln(cos(d*x+c))+5/3/
d*a*b^4*tan(d*x+c)^3-5*a*b^4*tan(d*x+c)/d+5*a*b^4*x+5/d*a*b^4*c+1/4/d*b^5*t
an(d*x+c)^4-1/2*b^5*tan(d*x+c)^2/d-1/d*b^5*ln(cos(d*x+c))
```

maxima [A] time = 0.42, size = 174, normalized size = 1.18

$$\frac{12\left(dx+c\right)a^5-120\left(dx+c-\tan\left(dx+c\right)\right)a^3b^2+20\left(\tan\left(dx+c\right)^3+3\,dx+3\,c-3\,\tan\left(dx+c\right)\right)ab^4+3\,b^5\left(\frac{}{\sin(d}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*(a*cos(d*x+c)+b*sin(d*x+c))^5,x, algorithm="maxima")
```

```
[Out] 1/12*(12*(d*x + c)*a^5 - 120*(d*x + c - tan(d*x + c))*a^3*b^2 + 20*(tan(d*x
 + c)^3 + 3*d*x + 3*c - 3*tan(d*x + c))*a*b^4 + 3*b^5*((4*sin(d*x + c)^2 -
3)/(sin(d*x + c)^4 - 2*sin(d*x + c)^2 + 1) - 2*log(sin(d*x + c)^2 - 1)) - 6
0*a^2*b^3*(1/(sin(d*x + c)^2 - 1) - log(sin(d*x + c)^2 - 1)) - 30*a^4*b*log
(-sin(d*x + c)^2 + 1))/d
```

mupad [B] time = 3.45, size = 971, normalized size = 6.61

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*cos(c + d*x) + b*sin(c + d*x))^5/cos(c + d*x)^5,x)
```

```
[Out] ((3*b^5*log(1/cos(c/2 + (d*x)/2)^2))/8 - (3*b^5*log(-cos(c + d*x)/cos(c/2 +
 (d*x)/2)^2))/8 + b^5/32 + (3*a^5*atan(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2
)))/4 + (5*a^2*b^3)/8 - (b^5*cos(2*c + 2*d*x))/8 + (3*b^5*cos(4*c + 4*d*x))
/32 + (15*a^4*b*log(1/cos(c/2 + (d*x)/2)^2))/8 + (15*a^2*b^3*log(-cos(c + d
*x)/cos(c/2 + (d*x)/2)^2))/4 - (b^5*log(-cos(c + d*x)/cos(c/2 + (d*x)/2)^2)
*cos(2*c + 2*d*x))/2 - (b^5*log(-cos(c + d*x)/cos(c/2 + (d*x)/2)^2)*cos(4*c
 + 4*d*x))/8 + (15*a*b^4*atan(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/4 - (
5*a*b^4*sin(2*c + 2*d*x))/6 - (5*a*b^4*sin(4*c + 4*d*x))/6 - (15*a^2*b^3*lo
g(1/cos(c/2 + (d*x)/2)^2))/4 + (b^5*log(1/cos(c/2 + (d*x)/2)^2)*cos(2*c + 2
*d*x))/2 + (b^5*log(1/cos(c/2 + (d*x)/2)^2)*cos(4*c + 4*d*x))/8 + a^5*atan(
sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2))*cos(2*c + 2*d*x) + (a^5*atan(sin(c/2
 + (d*x)/2)/cos(c/2 + (d*x)/2))*cos(4*c + 4*d*x))/4 - (15*a^3*b^2*atan(sin(
c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/2 - (5*a^2*b^3*cos(4*c + 4*d*x))/8 + (5
*a^3*b^2*sin(2*c + 2*d*x))/2 + (5*a^3*b^2*sin(4*c + 4*d*x))/4 - (15*a^4*b*l
og(-cos(c + d*x)/cos(c/2 + (d*x)/2)^2))/8 - 5*a^2*b^3*log(1/cos(c/2 + (d*x)
/2)^2)*cos(2*c + 2*d*x) - (5*a^2*b^3*log(1/cos(c/2 + (d*x)/2)^2)*cos(4*c +
4*d*x))/4 - 10*a^3*b^2*atan(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2))*cos(2*c
+ 2*d*x) - (5*a^3*b^2*atan(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2))*cos(4*c +
 4*d*x))/2 - (5*a^4*b*log(-cos(c + d*x)/cos(c/2 + (d*x)/2)^2)*cos(2*c + 2*d
*x))/2 - (5*a^4*b*log(-cos(c + d*x)/cos(c/2 + (d*x)/2)^2)*cos(4*c + 4*d*x))
/8 + (5*a^4*b*log(1/cos(c/2 + (d*x)/2)^2)*cos(2*c + 2*d*x))/2 + (5*a^4*b*lo
g(1/cos(c/2 + (d*x)/2)^2)*cos(4*c + 4*d*x))/8 + 5*a^2*b^3*log(-cos(c + d*x)
/cos(c/2 + (d*x)/2)^2)*cos(2*c + 2*d*x) + (5*a^2*b^3*log(-cos(c + d*x)/cos(
c/2 + (d*x)/2)^2)*cos(4*c + 4*d*x))/4 + 5*a*b^4*atan(sin(c/2 + (d*x)/2)/cos
(c/2 + (d*x)/2))*cos(2*c + 2*d*x) + (5*a*b^4*atan(sin(c/2 + (d*x)/2)/cos(c/
2 + (d*x)/2))*cos(4*c + 4*d*x))/4)/(d*(cos(2*c + 2*d*x)/2 + cos(4*c + 4*d*x
)/8 + 3/8))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**5*(a*cos(d*x+c)+b*sin(d*x+c))**5,x)
```

```
[Out] Timed out
```

3.103 $\int \sec^6(c+dx)(a\cos(c+dx)+b\sin(c+dx))^5\,dx$

Optimal. Leaf size=224

$$\frac{a^5\tanh^{-1}(\sin(c+dx))}{d}+\frac{5a^4b\sec(c+dx)}{d}-\frac{5a^3b^2\tanh^{-1}(\sin(c+dx))}{d}+\frac{5a^3b^2\tan(c+dx)\sec(c+dx)}{d}+\frac{10a^2b^3\sec}{3}$$

```
[Out] a^5*arctanh(sin(d*x+c))/d-5*a^3*b^2*arctanh(sin(d*x+c))/d+15/8*a*b^4*arctan
h(sin(d*x+c))/d+5*a^4*b*sec(d*x+c)/d-10*a^2*b^3*sec(d*x+c)/d+b^5*sec(d*x+c)
/d+10/3*a^2*b^3*sec(d*x+c)^3/d-2/3*b^5*sec(d*x+c)^3/d+1/5*b^5*sec(d*x+c)^5/
d+5*a^3*b^2*sec(d*x+c)*tan(d*x+c)/d-15/8*a*b^4*sec(d*x+c)*tan(d*x+c)/d+5/4*
a*b^4*sec(d*x+c)*tan(d*x+c)^3/d
```

Rubi [A] time = 0.23, antiderivative size = 224, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 6, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {3090, 3770, 2606, 8, 2611, 194}

$$\frac{10a^2b^3\sec^3(c+dx)}{3d}-\frac{10a^2b^3\sec(c+dx)}{d}-\frac{5a^3b^2\tanh^{-1}(\sin(c+dx))}{d}+\frac{5a^3b^2\tan(c+dx)\sec(c+dx)}{d}+\frac{5a^4b\sec(c}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^6*(a*Cos[c + d*x] + b*Sin[c + d*x])^5,x]
```

```
[Out] (a^5*ArcTanh[Sin[c + d*x]])/d - (5*a^3*b^2*ArcTanh[Sin[c + d*x]])/d + (15*a
*b^4*ArcTanh[Sin[c + d*x]])/(8*d) + (5*a^4*b*Sec[c + d*x])/d - (10*a^2*b^3*
Sec[c + d*x])/d + (b^5*Sec[c + d*x])/d + (10*a^2*b^3*Sec[c + d*x]^3)/(3*d)
- (2*b^5*Sec[c + d*x]^3)/(3*d) + (b^5*Sec[c + d*x]^5)/(5*d) + (5*a^3*b^2*Se
c[c + d*x]*Tan[c + d*x])/d - (15*a*b^4*Sec[c + d*x]*Tan[c + d*x])/(8*d) + (
5*a*b^4*Sec[c + d*x]*Tan[c + d*x]^3)/(4*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 194

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
```

```
&& !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 2611

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(b*(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*(
m + n - 1)), x] - Dist[(b^2*(n - 1))/(m + n - 1), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 3090

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_.), x_Symbol] :> Int[ExpandTrig[cos[c + d*x]^m*(a
*cos[c + d*x] + b*sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d}, x] && Inte
gerQ[m] && IGtQ[n, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \sec^6(c+dx)(a\cos(c+dx)+b\sin(c+dx))^5\,dx = \int \left(a^5\sec(c+dx)+5a^4b\sec(c+dx)\tan(c+dx)+10a^3b^2\right.$$

$$= a^5\int \sec(c+dx)\,dx+\left(5a^4b\right)\int \sec(c+dx)\tan(c+dx)\,dx$$

$$= \frac{a^5\tanh^{-1}(\sin(c+dx))}{d}+\frac{5a^3b^2\sec(c+dx)\tan(c+dx)}{d}+$$

$$= \frac{a^5\tanh^{-1}(\sin(c+dx))}{d}-\frac{5a^3b^2\tanh^{-1}(\sin(c+dx))}{d}+\frac{5a^4}{}$$

$$= \frac{a^5\tanh^{-1}(\sin(c+dx))}{d}-\frac{5a^3b^2\tanh^{-1}(\sin(c+dx))}{d}+\frac{15a}{}$$

Mathematica [B] time = 6.29, size = 1219, normalized size = 5.44

result too large to display

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^6*(a*Cos[c + d*x] + b*Sin[c + d*x])^5,x]
```

```
[Out] (b*(600*a^4 - 1000*a^2*b^2 + 89*b^4)*Cos[c + d*x]^5*(a + b*Tan[c + d*x])^5)
/(120*d*(a*Cos[c + d*x] + b*Sin[c + d*x])^5) + ((-8*a^5 + 40*a^3*b^2 - 15*a
*b^4)*Cos[c + d*x]^5*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]]*(a + b*Tan[c
+ d*x])^5)/(8*d*(a*Cos[c + d*x] + b*Sin[c + d*x])^5) + ((8*a^5 - 40*a^3*b^2
 + 15*a*b^4)*Cos[c + d*x]^5*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]*(a + b
*Tan[c + d*x])^5)/(8*d*(a*Cos[c + d*x] + b*Sin[c + d*x])^5) + ((25*a*b^4 +
2*b^5)*Cos[c + d*x]^5*(a + b*Tan[c + d*x])^5)/(80*d*(Cos[(c + d*x)/2] - Sin
[(c + d*x)/2])^4*(a*Cos[c + d*x] + b*Sin[c + d*x])^5) + ((600*a^3*b^2 + 200
*a^2*b^3 - 375*a*b^4 - 31*b^5)*Cos[c + d*x]^5*(a + b*Tan[c + d*x])^5)/(240*
d*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^2*(a*Cos[c + d*x] + b*Sin[c + d*x])
^5) + (b^5*Cos[c + d*x]^5*Sin[(c + d*x)/2]*(a + b*Tan[c + d*x])^5)/(20*d*(C
os[(c + d*x)/2] - Sin[(c + d*x)/2])^5*(a*Cos[c + d*x] + b*Sin[c + d*x])^5)
- (b^5*Cos[c + d*x]^5*Sin[(c + d*x)/2]*(a + b*Tan[c + d*x])^5)/(20*d*(Cos[(
c + d*x)/2] + Sin[(c + d*x)/2])^5*(a*Cos[c + d*x] + b*Sin[c + d*x])^5) + ((
-25*a*b^4 + 2*b^5)*Cos[c + d*x]^5*(a + b*Tan[c + d*x])^5)/(80*d*(Cos[(c + d
*x)/2] + Sin[(c + d*x)/2])^4*(a*Cos[c + d*x] + b*Sin[c + d*x])^5) + ((-600*
a^3*b^2 + 200*a^2*b^3 + 375*a*b^4 - 31*b^5)*Cos[c + d*x]^5*(a + b*Tan[c + d
*x])^5)/(240*d*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^2*(a*Cos[c + d*x] + b*
Sin[c + d*x])^5) + (Cos[c + d*x]^5*(-600*a^4*b*Sin[(c + d*x)/2] + 1000*a^2*
b^3*Sin[(c + d*x)/2] - 89*b^5*Sin[(c + d*x)/2])*(a + b*Tan[c + d*x])^5)/(12
0*d*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])*(a*Cos[c + d*x] + b*Sin[c + d*x])
^5) + (Cos[c + d*x]^5*(200*a^2*b^3*Sin[(c + d*x)/2] - 31*b^5*Sin[(c + d*x)/
2])*(a + b*Tan[c + d*x])^5)/(120*d*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^3*
(a*Cos[c + d*x] + b*Sin[c + d*x])^5) + (Cos[c + d*x]^5*(-200*a^2*b^3*Sin[(c
 + d*x)/2] + 31*b^5*Sin[(c + d*x)/2])*(a + b*Tan[c + d*x])^5)/(120*d*(Cos[(
c + d*x)/2] + Sin[(c + d*x)/2])^3*(a*Cos[c + d*x] + b*Sin[c + d*x])^5) + (C
os[c + d*x]^5*(600*a^4*b*Sin[(c + d*x)/2] - 1000*a^2*b^3*Sin[(c + d*x)/2] +
 89*b^5*Sin[(c + d*x)/2])*(a + b*Tan[c + d*x])^5)/(120*d*(Cos[(c + d*x)/2]
- Sin[(c + d*x)/2])*(a*Cos[c + d*x] + b*Sin[c + d*x])^5)
```

fricas [A] time = 0.64, size = 196, normalized size = 0.88

$$\frac{15\left(8\,a^5-40\,a^3b^2+15\,ab^4\right)\cos\left(dx+c\right)^5\log\left(\sin\left(dx+c\right)+1\right)-15\left(8\,a^5-40\,a^3b^2+15\,ab^4\right)\cos\left(dx+c\right)^5\log\left(-\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^6*(a*cos(d*x+c)+b*sin(d*x+c))^5,x, algorithm="fricas")
```

```
[Out] 1/240*(15*(8*a^5 - 40*a^3*b^2 + 15*a*b^4)*cos(d*x + c)^5*log(sin(d*x + c) +
 1) - 15*(8*a^5 - 40*a^3*b^2 + 15*a*b^4)*cos(d*x + c)^5*log(-sin(d*x + c) +
 1) + 48*b^5 + 240*(5*a^4*b - 10*a^2*b^3 + b^5)*cos(d*x + c)^4 + 160*(5*a^2
*b^3 - b^5)*cos(d*x + c)^2 + 150*(2*a*b^4*cos(d*x + c) + (8*a^3*b^2 - 5*a*b
^4)*cos(d*x + c)^3)*sin(d*x + c))/(d*cos(d*x + c)^5)
```

giac [A] time = 0.64, size = 410, normalized size = 1.83

$$15\left(8\,a^5-40\,a^3b^2+15\,ab^4\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-15\left(8\,a^5-40\,a^3b^2+15\,ab^4\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^6*(a*cos(d*x+c)+b*sin(d*x+c))^5,x, algorithm="giac")
```

```
[Out] 1/120*(15*(8*a^5 - 40*a^3*b^2 + 15*a*b^4)*log(abs(tan(1/2*d*x + 1/2*c) + 1)
) - 15*(8*a^5 - 40*a^3*b^2 + 15*a*b^4)*log(abs(tan(1/2*d*x + 1/2*c) - 1)) +
 2*(600*a^3*b^2*tan(1/2*d*x + 1/2*c)^9 - 225*a*b^4*tan(1/2*d*x + 1/2*c)^9 -
 600*a^4*b*tan(1/2*d*x + 1/2*c)^8 - 1200*a^3*b^2*tan(1/2*d*x + 1/2*c)^7 + 1
050*a*b^4*tan(1/2*d*x + 1/2*c)^7 + 2400*a^4*b*tan(1/2*d*x + 1/2*c)^6 - 2400
*a^2*b^3*tan(1/2*d*x + 1/2*c)^6 - 3600*a^4*b*tan(1/2*d*x + 1/2*c)^4 + 5600*
a^2*b^3*tan(1/2*d*x + 1/2*c)^4 - 640*b^5*tan(1/2*d*x + 1/2*c)^4 + 1200*a^3*
b^2*tan(1/2*d*x + 1/2*c)^3 - 1050*a*b^4*tan(1/2*d*x + 1/2*c)^3 + 2400*a^4*b
*tan(1/2*d*x + 1/2*c)^2 - 4000*a^2*b^3*tan(1/2*d*x + 1/2*c)^2 + 320*b^5*tan
(1/2*d*x + 1/2*c)^2 - 600*a^3*b^2*tan(1/2*d*x + 1/2*c) + 225*a*b^4*tan(1/2*
d*x + 1/2*c) - 600*a^4*b + 800*a^2*b^3 - 64*b^5)/(tan(1/2*d*x + 1/2*c)^2 -
1)^5)/d
```

maple [B] time = 0.23, size = 440, normalized size = 1.96

$$\frac{a^5\ln\left(\sec\left(dx+c\right)+\tan\left(dx+c\right)\right)}{d}+\frac{5a^4b}{d\cos\left(dx+c\right)}+\frac{5a^3b^2\left(\sin^3\left(dx+c\right)\right)}{d\cos\left(dx+c\right)^2}+\frac{5a^3b^2\sin\left(dx+c\right)}{d}-\frac{5a^3b^2\ln\left(\sec\left(dx+\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^6*(a*cos(d*x+c)+b*sin(d*x+c))^5,x)
```

```
[Out] 1/d*a^5*ln(sec(d*x+c)+tan(d*x+c))+5/d*a^4*b/cos(d*x+c)+5/d*a^3*b^2*sin(d*x+
c)^3/cos(d*x+c)^2+5*a^3*b^2*sin(d*x+c)/d-5/d*a^3*b^2*ln(sec(d*x+c)+tan(d*x+
c))+10/3/d*a^2*b^3*sin(d*x+c)^4/cos(d*x+c)^3-10/3/d*a^2*b^3*sin(d*x+c)^4/co
s(d*x+c)-10/3/d*cos(d*x+c)*sin(d*x+c)^2*a^2*b^3-20/3*a^2*b^3*cos(d*x+c)/d+5
/4/d*a*b^4*sin(d*x+c)^5/cos(d*x+c)^4-5/8/d*a*b^4*sin(d*x+c)^5/cos(d*x+c)^2-
5/8*a*b^4*sin(d*x+c)^3/d-15/8*a*b^4*sin(d*x+c)/d+15/8/d*a*b^4*ln(sec(d*x+c)
+tan(d*x+c))+1/5/d*b^5*sin(d*x+c)^6/cos(d*x+c)^5-1/15/d*b^5*sin(d*x+c)^6/co
s(d*x+c)^3+1/5/d*b^5*sin(d*x+c)^6/cos(d*x+c)+8/15*b^5*cos(d*x+c)/d+1/5/d*b^
5*cos(d*x+c)*sin(d*x+c)^4+4/15/d*cos(d*x+c)*sin(d*x+c)^2*b^5
```

maxima [A] time = 0.34, size = 230, normalized size = 1.03

$$\frac{75\,ab^4\left(\frac{2\left(5\,\sin(dx+c)^3-3\,\sin(dx+c)\right)}{\sin(dx+c)^4-2\,\sin(dx+c)^2+1}+3\,\log\left(\sin\left(dx+c\right)+1\right)-3\,\log\left(\sin\left(dx+c\right)-1\right)\right)-600\,a^3b^2\left(\frac{2\,\sin(dx+c)}{\sin(dx+c)^2-1}+\log\left(\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^6*(a*cos(d*x+c)+b*sin(d*x+c))^5,x, algorithm="maxima")
```

```
[Out] 1/240*(75*a*b^4*(2*(5*sin(d*x + c)^3 - 3*sin(d*x + c))/(sin(d*x + c)^4 - 2*
sin(d*x + c)^2 + 1) + 3*log(sin(d*x + c) + 1) - 3*log(sin(d*x + c) - 1)) -
600*a^3*b^2*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) + log(sin(d*x + c) + 1) -
log(sin(d*x + c) - 1)) + 120*a^5*(log(sin(d*x + c) + 1) - log(sin(d*x + c)
- 1)) + 1200*a^4*b/cos(d*x + c) - 800*(3*cos(d*x + c)^2 - 1)*a^2*b^3/cos(d*
x + c)^3 + 16*(15*cos(d*x + c)^4 - 10*cos(d*x + c)^2 + 3)*b^5/cos(d*x + c)^
5)/d
```

mupad [B] time = 4.26, size = 345, normalized size = 1.54

$$\frac{\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)\,\left(2\,a^5-10\,a^3\,b^2+\frac{15\,a\,b^4}{4}\right)}{d}-\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^9\,\left(\frac{15\,a\,b^4}{4}-10\,a^3\,b^2\right)+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3\,\left(\frac{35\,a\,b^4}{2}-20\,a^3\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*cos(c + d*x) + b*sin(c + d*x))^5/cos(c + d*x)^6,x)
```

```
[Out] (atanh(tan(c/2 + (d*x)/2))*((15*a*b^4)/4 + 2*a^5 - 10*a^3*b^2))/d - (tan(c/
2 + (d*x)/2)^9*((15*a*b^4)/4 - 10*a^3*b^2) + tan(c/2 + (d*x)/2)^3*((35*a*b^
4)/2 - 20*a^3*b^2) - tan(c/2 + (d*x)/2)^7*((35*a*b^4)/2 - 20*a^3*b^2) - tan
(c/2 + (d*x)/2)^6*(40*a^4*b - 40*a^2*b^3) - tan(c/2 + (d*x)/2)^2*(40*a^4*b
+ (16*b^5)/3 - (200*a^2*b^3)/3) + tan(c/2 + (d*x)/2)^4*(60*a^4*b + (32*b^5)
/3 - (280*a^2*b^3)/3) + 10*a^4*b + (16*b^5)/15 - (40*a^2*b^3)/3 - tan(c/2 +
 (d*x)/2)*((15*a*b^4)/4 - 10*a^3*b^2) + 10*a^4*b*tan(c/2 + (d*x)/2)^8)/(d*(
5*tan(c/2 + (d*x)/2)^2 - 10*tan(c/2 + (d*x)/2)^4 + 10*tan(c/2 + (d*x)/2)^6
- 5*tan(c/2 + (d*x)/2)^8 + tan(c/2 + (d*x)/2)^10 - 1))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**6*(a*cos(d*x+c)+b*sin(d*x+c))**5,x)
```

```
[Out] Timed out
```

3.104 $\int \sec^7(c+dx)(a\cos(c+dx)+b\sin(c+dx))^5\,dx$

Optimal. Leaf size=30

$$\frac{\tan^6(c+dx)(a\cot(c+dx)+b)^6}{6bd}$$

```
[Out] 1/6*(b+a*cot(d*x+c))^6*tan(d*x+c)^6/b/d
```

Rubi [A] time = 0.05, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} =$ 0.071, Rules used = {3088, 37}

$$\frac{\tan^6(c+dx)(a\cot(c+dx)+b)^6}{6bd}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^7*(a*Cos[c + d*x] + b*Sin[c + d*x])^5,x]
```

```
[Out] ((b + a*Cot[c + d*x])^6*Tan[c + d*x]^6)/(6*b*d)
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 3088

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[(x^m*(b +
 a*x)^n)/(1 + x^2)^((m + n + 2)/2), x], x, Cot[c + d*x]], x] /; FreeQ[{a, b
, c, d}, x] && IntegerQ[n] && IntegerQ[(m + n)/2] && NeQ[n, -1] &&  !(GtQ[n
, 0] && GtQ[m, 1])
```

Rubi steps

$$\begin{aligned}\int \sec^7(c+dx)(a\cos(c+dx)+b\sin(c+dx))^5\,dx &= -\frac{\text{Subst}\left(\int \frac{(b+ax)^5}{x^7}\,dx, x, \cot(c+dx)\right)}{d}\\ &= \frac{(b+a\cot(c+dx))^6\tan^6(c+dx)}{6bd}\end{aligned}$$

Mathematica [B] time = 0.49, size = 89, normalized size = 2.97

$$\frac{\tan(c+dx)\left(6a^5+15a^4b\tan(c+dx)+20a^3b^2\tan^2(c+dx)+15a^2b^3\tan^3(c+dx)+6ab^4\tan^4(c+dx)+b^5\tan^5(\right.}{6d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^7*(a*Cos[c + d*x] + b*Sin[c + d*x])^5,x]
```

```
[Out] (Tan[c + d*x]*(6*a^5 + 15*a^4*b*Tan[c + d*x] + 20*a^3*b^2*Tan[c + d*x]^2 +
15*a^2*b^3*Tan[c + d*x]^3 + 6*a*b^4*Tan[c + d*x]^4 + b^5*Tan[c + d*x]^5))/(
6*d)
```

fricas [B] time = 0.52, size = 144, normalized size = 4.80

$$\frac{b^5+3\left(5a^4b-10a^2b^3+b^5\right)\cos(dx+c)^4+3\left(5a^2b^3-b^5\right)\cos(dx+c)^2+2\left(3ab^4\cos(dx+c)+\left(3a^5-10a^3b^2\right.\right.}{6d\cos(dx+c)^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^7*(a*cos(d*x+c)+b*sin(d*x+c))^5,x, algorithm="fricas")
```

```
[Out] 1/6*(b^5 + 3*(5*a^4*b - 10*a^2*b^3 + b^5)*cos(d*x + c)^4 + 3*(5*a^2*b^3 - b
^5)*cos(d*x + c)^2 + 2*(3*a*b^4*cos(d*x + c) + (3*a^5 - 10*a^3*b^2 + 3*a*b^
4)*cos(d*x + c)^5 + 2*(5*a^3*b^2 - 3*a*b^4)*cos(d*x + c)^3)*sin(d*x + c))/(
d*cos(d*x + c)^6)
```

giac [B] time = 2.26, size = 89, normalized size = 2.97

$$\frac{b^5\tan(dx+c)^6+6ab^4\tan(dx+c)^5+15a^2b^3\tan(dx+c)^4+20a^3b^2\tan(dx+c)^3+15a^4b\tan(dx+c)^2+6a^5}{6d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^7*(a*cos(d*x+c)+b*sin(d*x+c))^5,x, algorithm="giac")
```

```
[Out] 1/6*(b^5*tan(d*x + c)^6 + 6*a*b^4*tan(d*x + c)^5 + 15*a^2*b^3*tan(d*x + c)^
4 + 20*a^3*b^2*tan(d*x + c)^3 + 15*a^4*b*tan(d*x + c)^2 + 6*a^5*tan(d*x + c
))/d
```

maple [B] time = 0.28, size = 120, normalized size = 4.00

$$\frac{a^5\tan(dx+c)+\frac{5a^4b}{2\cos(dx+c)^2}+\frac{10a^3b^2\left(\sin^3(dx+c)\right)}{3\cos(dx+c)^3}+\frac{5a^2b^3\left(\sin^4(dx+c)\right)}{2\cos(dx+c)^4}+\frac{ab^4\left(\sin^5(dx+c)\right)}{\cos(dx+c)^5}+\frac{b^5\left(\sin^6(dx+c)\right)}{6\cos(dx+c)^6}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^7*(a*cos(d*x+c)+b*sin(d*x+c))^5,x)
```

```
[Out] 1/d*(a^5*tan(d*x+c)+5/2*a^4*b/cos(d*x+c)^2+10/3*a^3*b^2*sin(d*x+c)^3/cos(d*
x+c)^3+5/2*a^2*b^3*sin(d*x+c)^4/cos(d*x+c)^4+a*b^4*sin(d*x+c)^5/cos(d*x+c)^
5+1/6*b^5*sin(d*x+c)^6/cos(d*x+c)^6)
```

maxima [B] time = 0.35, size = 166, normalized size = 5.53

$$\frac{6\,ab^4\tan\left(dx+c\right)^5 + 20\,a^3b^2\tan\left(dx+c\right)^3 + 6\,a^5\tan\left(dx+c\right) + \frac{15\left(2\,\sin(dx+c)^2-1\right)a^2b^3}{\sin(dx+c)^4-2\,\sin(dx+c)^2+1} - \frac{\left(3\,\sin(dx+c)^4-3\,\sin(dx+c)\right.}{\sin(dx+c)^6-3\,\sin(dx+c)^4+3\,\sin}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^7*(a*cos(d*x+c)+b*sin(d*x+c))^5,x, algorithm="maxima")
```

```
[Out] 1/6*(6*a*b^4*tan(d*x + c)^5 + 20*a^3*b^2*tan(d*x + c)^3 + 6*a^5*tan(d*x + c
) + 15*(2*sin(d*x + c)^2 - 1)*a^2*b^3/(sin(d*x + c)^4 - 2*sin(d*x + c)^2 +
1) - (3*sin(d*x + c)^4 - 3*sin(d*x + c)^2 + 1)*b^5/(sin(d*x + c)^6 - 3*sin(
d*x + c)^4 + 3*sin(d*x + c)^2 - 1) - 15*a^4*b/(sin(d*x + c)^2 - 1))/d
```

mupad [B] time = 0.98, size = 169, normalized size = 5.63

$$\frac{\cos\left(c+d\,x\right)^4\,\left(\frac{5\,a^4\,b}{2}-5\,a^2\,b^3+\frac{b^5}{2}\right)+\cos\left(c+d\,x\right)^5\,\left(\sin\left(c+d\,x\right)\,a^5-\frac{10\,\sin(c+d\,x)\,a^3\,b^2}{3}+\sin\left(c+d\,x\right)\,a\,b^4\right)-\cos}{d\cos}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*cos(c + d*x) + b*sin(c + d*x))^5/cos(c + d*x)^7,x)
```

```
[Out] (cos(c + d*x)^4*((5*a^4*b)/2 + b^5/2 - 5*a^2*b^3) + cos(c + d*x)^5*(a^5*sin
(c + d*x) - (10*a^3*b^2*sin(c + d*x))/3 + a*b^4*sin(c + d*x)) - cos(c + d*x
)^2*(b^5/2 - (5*a^2*b^3)/2) + b^5/6 + cos(c + d*x)^3*((10*a^3*b^2*sin(c + d
*x))/3 - 2*a*b^4*sin(c + d*x)) + a*b^4*cos(c + d*x)*sin(c + d*x))/(d*cos(c
+ d*x)^6)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**7*(a*cos(d*x+c)+b*sin(d*x+c))**5,x)
```

```
[Out] Timed out
```

3.105 $\int \sec^8(c+dx)(a\cos(c+dx)+b\sin(c+dx))^5\,dx$

Optimal. Leaf size=318

$$\frac{a^5\tanh^{-1}(\sin(c+dx))}{2d}+\frac{a^5\tan(c+dx)\sec(c+dx)}{2d}+\frac{5a^4b\sec^3(c+dx)}{3d}-\frac{5a^3b^2\tanh^{-1}(\sin(c+dx))}{4d}+\frac{5a^3b^2\tan(c+$$

```
[Out] 1/2*a^5*arctanh(sin(d*x+c))/d-5/4*a^3*b^2*arctanh(sin(d*x+c))/d+5/16*a*b^4*
arctanh(sin(d*x+c))/d+5/3*a^4*b*sec(d*x+c)^3/d-10/3*a^2*b^3*sec(d*x+c)^3/d+
1/3*b^5*sec(d*x+c)^3/d+2*a^2*b^3*sec(d*x+c)^5/d-2/5*b^5*sec(d*x+c)^5/d+1/7*
b^5*sec(d*x+c)^7/d+1/2*a^5*sec(d*x+c)*tan(d*x+c)/d-5/4*a^3*b^2*sec(d*x+c)*t
an(d*x+c)/d+5/16*a*b^4*sec(d*x+c)*tan(d*x+c)/d+5/2*a^3*b^2*sec(d*x+c)^3*tan
(d*x+c)/d-5/8*a*b^4*sec(d*x+c)^3*tan(d*x+c)/d+5/6*a*b^4*sec(d*x+c)^3*tan(d*
x+c)^3/d
```

Rubi [A] time = 0.34, antiderivative size = 318, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 8, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3090, 3768, 3770, 2606, 30, 2611, 14, 270}

$$\frac{2a^2b^3\sec^5(c+dx)}{d}-\frac{10a^2b^3\sec^3(c+dx)}{3d}-\frac{5a^3b^2\tanh^{-1}(\sin(c+dx))}{4d}+\frac{5a^3b^2\tan(c+dx)\sec^3(c+dx)}{2d}-\frac{5a^3b^2\tan($$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^8*(a*Cos[c + d*x] + b*Sin[c + d*x])^5,x]
```

```
[Out] (a^5*ArcTanh[Sin[c + d*x]])/(2*d) - (5*a^3*b^2*ArcTanh[Sin[c + d*x]])/(4*d)
 + (5*a*b^4*ArcTanh[Sin[c + d*x]])/(16*d) + (5*a^4*b*Sec[c + d*x]^3)/(3*d)
- (10*a^2*b^3*Sec[c + d*x]^3)/(3*d) + (b^5*Sec[c + d*x]^3)/(3*d) + (2*a^2*b
^3*Sec[c + d*x]^5)/d - (2*b^5*Sec[c + d*x]^5)/(5*d) + (b^5*Sec[c + d*x]^7)/
(7*d) + (a^5*Sec[c + d*x]*Tan[c + d*x])/(2*d) - (5*a^3*b^2*Sec[c + d*x]*Tan
[c + d*x])/(4*d) + (5*a*b^4*Sec[c + d*x]*Tan[c + d*x])/(16*d) + (5*a^3*b^2*
Sec[c + d*x]^3*Tan[c + d*x])/(2*d) - (5*a*b^4*Sec[c + d*x]^3*Tan[c + d*x])/
(8*d) + (5*a*b^4*Sec[c + d*x]^3*Tan[c + d*x]^3)/(6*d)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 2611

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(b*(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*(
m + n - 1)), x] - Dist[(b^2*(n - 1))/(m + n - 1), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 3090

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_.), x_Symbol] :> Int[ExpandTrig[cos[c + d*x]^m*(a
*cos[c + d*x] + b*sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d}, x] && Inte
gerQ[m] && IGtQ[n, 0]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \sec^8(c+dx)(a\cos(c+dx)+b\sin(c+dx))^5\,dx = \int \left(a^5\sec^3(c+dx)+5a^4b\sec^3(c+dx)\tan(c+dx)+10a^3b^2\right.$$
$$= a^5\int \sec^3(c+dx)\,dx + \left(5a^4b\right)\int \sec^3(c+dx)\tan(c+dx)\,dx$$
$$= \frac{a^5\sec(c+dx)\tan(c+dx)}{2d} + \frac{5a^3b^2\sec^3(c+dx)\tan(c+dx)}{2d}$$
$$= \frac{a^5\tanh^{-1}(\sin(c+dx))}{2d} + \frac{5a^4b\sec^3(c+dx)}{3d} + \frac{a^5\sec(c+dx)}{2d}$$
$$= \frac{a^5\tanh^{-1}(\sin(c+dx))}{2d} - \frac{5a^3b^2\tanh^{-1}(\sin(c+dx))}{4d} + \frac{5a^4b}{}$$
$$= \frac{a^5\tanh^{-1}(\sin(c+dx))}{2d} - \frac{5a^3b^2\tanh^{-1}(\sin(c+dx))}{4d} + \frac{5ab^4}{}$$

Mathematica [B] time = 6.34, size = 1677, normalized size = 5.27

result too large to display

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^8*(a*Cos[c + d*x] + b*Sin[c + d*x])^5,x]
```

```
[Out] (b*(1400*a^4 - 1540*a^2*b^2 + 103*b^4)*Cos[c + d*x]^5*(a + b*Tan[c + d*x])^
5)/(1680*d*(a*Cos[c + d*x] + b*Sin[c + d*x])^5) + ((-8*a^5 + 20*a^3*b^2 - 5
*a*b^4)*Cos[c + d*x]^5*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]]*(a + b*Tan[
c + d*x])^5)/(16*d*(a*Cos[c + d*x] + b*Sin[c + d*x])^5) + ((8*a^5 - 20*a^3*
b^2 + 5*a*b^4)*Cos[c + d*x]^5*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]*(a +
 b*Tan[c + d*x])^5)/(16*d*(a*Cos[c + d*x] + b*Sin[c + d*x])^5) + ((35*a*b^4
 + 3*b^5)*Cos[c + d*x]^5*(a + b*Tan[c + d*x])^5)/(336*d*(Cos[(c + d*x)/2] -
 Sin[(c + d*x)/2])^6*(a*Cos[c + d*x] + b*Sin[c + d*x])^5) + ((350*a^3*b^2 +
 140*a^2*b^3 - 175*a*b^4 - 18*b^5)*Cos[c + d*x]^5*(a + b*Tan[c + d*x])^5)/(
560*d*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^4*(a*Cos[c + d*x] + b*Sin[c + d
*x])^5) + ((840*a^5 + 1400*a^4*b - 2100*a^3*b^2 - 1540*a^2*b^3 + 525*a*b^4
+ 103*b^5)*Cos[c + d*x]^5*(a + b*Tan[c + d*x])^5)/(3360*d*(Cos[(c + d*x)/2]
 - Sin[(c + d*x)/2])^2*(a*Cos[c + d*x] + b*Sin[c + d*x])^5) + (b^5*Cos[c +
d*x]^5*Sin[(c + d*x)/2]*(a + b*Tan[c + d*x])^5)/(56*d*(Cos[(c + d*x)/2] - S
in[(c + d*x)/2])^7*(a*Cos[c + d*x] + b*Sin[c + d*x])^5) - (b^5*Cos[c + d*x]
^5*Sin[(c + d*x)/2]*(a + b*Tan[c + d*x])^5)/(56*d*(Cos[(c + d*x)/2] + Sin[(
c + d*x)/2])^7*(a*Cos[c + d*x] + b*Sin[c + d*x])^5) + ((-35*a*b^4 + 3*b^5)*
Cos[c + d*x]^5*(a + b*Tan[c + d*x])^5)/(336*d*(Cos[(c + d*x)/2] + Sin[(c +
d*x)/2])^6*(a*Cos[c + d*x] + b*Sin[c + d*x])^5) + ((-350*a^3*b^2 + 140*a^2*
b^3 + 175*a*b^4 - 18*b^5)*Cos[c + d*x]^5*(a + b*Tan[c + d*x])^5)/(560*d*(Co
```

```
s[(c + d*x)/2] + Sin[(c + d*x)/2])^4*(a*Cos[c + d*x] + b*Sin[c + d*x])^5) +
 ((-840*a^5 + 1400*a^4*b + 2100*a^3*b^2 - 1540*a^2*b^3 - 525*a*b^4 + 103*b^
5)*Cos[c + d*x]^5*(a + b*Tan[c + d*x])^5)/(3360*d*(Cos[(c + d*x)/2] + Sin[(
c + d*x)/2])^2*(a*Cos[c + d*x] + b*Sin[c + d*x])^5) + (Cos[c + d*x]^5*(-140
0*a^4*b*Sin[(c + d*x)/2] + 1540*a^2*b^3*Sin[(c + d*x)/2] - 103*b^5*Sin[(c +
 d*x)/2])*(a + b*Tan[c + d*x])^5)/(1680*d*(Cos[(c + d*x)/2] + Sin[(c + d*x)
/2])^3*(a*Cos[c + d*x] + b*Sin[c + d*x])^5) + (Cos[c + d*x]^5*(-1400*a^4*b*
Sin[(c + d*x)/2] + 1540*a^2*b^3*Sin[(c + d*x)/2] - 103*b^5*Sin[(c + d*x)/2]
)*(a + b*Tan[c + d*x])^5)/(1680*d*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])*(a*
Cos[c + d*x] + b*Sin[c + d*x])^5) + (Cos[c + d*x]^5*(70*a^2*b^3*Sin[(c + d*
x)/2] - 9*b^5*Sin[(c + d*x)/2])*(a + b*Tan[c + d*x])^5)/(140*d*(Cos[(c + d*
x)/2] - Sin[(c + d*x)/2])^5*(a*Cos[c + d*x] + b*Sin[c + d*x])^5) + (Cos[c +
 d*x]^5*(-70*a^2*b^3*Sin[(c + d*x)/2] + 9*b^5*Sin[(c + d*x)/2])*(a + b*Tan[
c + d*x])^5)/(140*d*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^5*(a*Cos[c + d*x]
 + b*Sin[c + d*x])^5) + (Cos[c + d*x]^5*(1400*a^4*b*Sin[(c + d*x)/2] - 1540
*a^2*b^3*Sin[(c + d*x)/2] + 103*b^5*Sin[(c + d*x)/2])*(a + b*Tan[c + d*x])^
5)/(1680*d*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^3*(a*Cos[c + d*x] + b*Sin[
c + d*x])^5) + (Cos[c + d*x]^5*(1400*a^4*b*Sin[(c + d*x)/2] - 1540*a^2*b^3*
Sin[(c + d*x)/2] + 103*b^5*Sin[(c + d*x)/2])*(a + b*Tan[c + d*x])^5)/(1680*
d*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])*(a*Cos[c + d*x] + b*Sin[c + d*x])^5
)
```

fricas [A] time = 0.59, size = 227, normalized size = 0.71

$$\frac{105 \left(8 a^5 - 20 a^3 b^2 + 5 ab^4\right) \cos(dx+c)^7 \log(\sin(dx+c)+1) - 105 \left(8 a^5 - 20 a^3 b^2 + 5 ab^4\right) \cos(dx+c)^7 \log(}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^8*(a*cos(d*x+c)+b*sin(d*x+c))^5,x, algorithm="fricas")
```

```
[Out] 1/3360*(105*(8*a^5 - 20*a^3*b^2 + 5*a*b^4)*cos(d*x + c)^7*log(sin(d*x + c)
+ 1) - 105*(8*a^5 - 20*a^3*b^2 + 5*a*b^4)*cos(d*x + c)^7*log(-sin(d*x + c)
+ 1) + 480*b^5 + 1120*(5*a^4*b - 10*a^2*b^3 + b^5)*cos(d*x + c)^4 + 1344*(5
*a^2*b^3 - b^5)*cos(d*x + c)^2 + 70*(40*a*b^4*cos(d*x + c) + 3*(8*a^5 - 20*
a^3*b^2 + 5*a*b^4)*cos(d*x + c)^5 + 10*(12*a^3*b^2 - 7*a*b^4)*cos(d*x + c)^
3)*sin(d*x + c))/(d*cos(d*x + c)^7)
```

giac [B] time = 6.46, size = 680, normalized size = 2.14

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^8*(a*cos(d*x+c)+b*sin(d*x+c))^5,x, algorithm="giac")
```

```
[Out] 1/1680*(105*(8*a^5 - 20*a^3*b^2 + 5*a*b^4)*log(abs(tan(1/2*d*x + 1/2*c) + 1
)) - 105*(8*a^5 - 20*a^3*b^2 + 5*a*b^4)*log(abs(tan(1/2*d*x + 1/2*c) - 1))
+ 2*(840*a^5*tan(1/2*d*x + 1/2*c)^13 + 2100*a^3*b^2*tan(1/2*d*x + 1/2*c)^13
 - 525*a*b^4*tan(1/2*d*x + 1/2*c)^13 - 8400*a^4*b*tan(1/2*d*x + 1/2*c)^12 -
 3360*a^5*tan(1/2*d*x + 1/2*c)^11 + 8400*a^3*b^2*tan(1/2*d*x + 1/2*c)^11 +
3500*a*b^4*tan(1/2*d*x + 1/2*c)^11 + 33600*a^4*b*tan(1/2*d*x + 1/2*c)^10 -
33600*a^2*b^3*tan(1/2*d*x + 1/2*c)^10 + 4200*a^5*tan(1/2*d*x + 1/2*c)^9 - 2
3100*a^3*b^2*tan(1/2*d*x + 1/2*c)^9 + 16975*a*b^4*tan(1/2*d*x + 1/2*c)^9 -
53200*a^4*b*tan(1/2*d*x + 1/2*c)^8 + 56000*a^2*b^3*tan(1/2*d*x + 1/2*c)^8 -
 8960*b^5*tan(1/2*d*x + 1/2*c)^8 + 44800*a^4*b*tan(1/2*d*x + 1/2*c)^6 - 224
00*a^2*b^3*tan(1/2*d*x + 1/2*c)^6 - 4480*b^5*tan(1/2*d*x + 1/2*c)^6 - 4200*
a^5*tan(1/2*d*x + 1/2*c)^5 + 23100*a^3*b^2*tan(1/2*d*x + 1/2*c)^5 - 16975*a
*b^4*tan(1/2*d*x + 1/2*c)^5 - 25200*a^4*b*tan(1/2*d*x + 1/2*c)^4 + 13440*a^
2*b^3*tan(1/2*d*x + 1/2*c)^4 - 2688*b^5*tan(1/2*d*x + 1/2*c)^4 + 3360*a^5*t
an(1/2*d*x + 1/2*c)^3 - 8400*a^3*b^2*tan(1/2*d*x + 1/2*c)^3 - 3500*a*b^4*ta
n(1/2*d*x + 1/2*c)^3 + 11200*a^4*b*tan(1/2*d*x + 1/2*c)^2 - 15680*a^2*b^3*t
an(1/2*d*x + 1/2*c)^2 + 896*b^5*tan(1/2*d*x + 1/2*c)^2 - 840*a^5*tan(1/2*d*
x + 1/2*c) - 2100*a^3*b^2*tan(1/2*d*x + 1/2*c) + 525*a*b^4*tan(1/2*d*x + 1/
2*c) - 2800*a^4*b + 2240*a^2*b^3 - 128*b^5)/(tan(1/2*d*x + 1/2*c)^2 - 1)^7)
/d
```

maple [A] time = 0.28, size = 564, normalized size = 1.77

$$\frac{8b^5\cos(dx+c)}{105d}-\frac{5a\,b^4\sin(dx+c)}{16d}+\frac{5a^3b^2\sin(dx+c)}{4d}+\frac{a^5\sec(dx+c)\tan(dx+c)}{2d}-\frac{4a^2b^3\cos(dx+c)}{3d}-\frac{2\cos(d}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^8*(a*cos(d*x+c)+b*sin(d*x+c))^5,x)
```

```
[Out] 8/105*b^5*cos(d*x+c)/d-5/16*a*b^4*sin(d*x+c)/d-5/48*a*b^4*sin(d*x+c)^3/d+5/
4*a^3*b^2*sin(d*x+c)/d+1/2*a^5*sec(d*x+c)*tan(d*x+c)/d-4/3*a^2*b^3*cos(d*x+
c)/d+1/35/d*b^5*sin(d*x+c)^6/cos(d*x+c)^5-5/4/d*a^3*b^2*ln(sec(d*x+c)+tan(d
*x+c))-1/105/d*b^5*sin(d*x+c)^6/cos(d*x+c)^3+5/4/d*a^3*b^2*sin(d*x+c)^3/cos
(d*x+c)^2+2/3/d*a^2*b^3*sin(d*x+c)^4/cos(d*x+c)^3+5/24/d*a*b^4*sin(d*x+c)^5
/cos(d*x+c)^4-2/3/d*cos(d*x+c)*sin(d*x+c)^2*a^2*b^3+5/2/d*a^3*b^2*sin(d*x+c
)^3/cos(d*x+c)^4+2/d*a^2*b^3*sin(d*x+c)^4/cos(d*x+c)^5+5/6/d*a*b^4*sin(d*x+
c)^5/cos(d*x+c)^6+1/2/d*a^5*ln(sec(d*x+c)+tan(d*x+c))+5/16/d*a*b^4*ln(sec(d
*x+c)+tan(d*x+c))+1/35/d*b^5*sin(d*x+c)^6/cos(d*x+c)+1/35/d*b^5*cos(d*x+c)*
sin(d*x+c)^4+4/105/d*cos(d*x+c)*sin(d*x+c)^2*b^5-2/3/d*a^2*b^3*sin(d*x+c)^4
/cos(d*x+c)-5/48/d*a*b^4*sin(d*x+c)^5/cos(d*x+c)^2+5/3/d*a^4*b/cos(d*x+c)^3
+1/7/d*b^5*sin(d*x+c)^6/cos(d*x+c)^7
```

maxima [A] time = 0.34, size = 289, normalized size = 0.91

$$-\frac{175\,ab^4\left(\frac{2\left(3\,\sin(dx+c)^5+8\,\sin(dx+c)^3-3\,\sin(dx+c)\right)}{\sin(dx+c)^6-3\,\sin(dx+c)^4+3\,\sin(dx+c)^2-1}-3\,\log\left(\sin\left(dx+c\right)+1\right)+3\,\log\left(\sin\left(dx+c\right)-1\right)\right)-2100\,a^3b^2\left(\cdots\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^8*(a*cos(d*x+c)+b*sin(d*x+c))^5,x, algorithm="maxima")
```

```
[Out] -1/3360*(175*a*b^4*(2*(3*sin(d*x + c)^5 + 8*sin(d*x + c)^3 - 3*sin(d*x + c)
)/(sin(d*x + c)^6 - 3*sin(d*x + c)^4 + 3*sin(d*x + c)^2 - 1) - 3*log(sin(d*
x + c) + 1) + 3*log(sin(d*x + c) - 1)) - 2100*a^3*b^2*(2*(sin(d*x + c)^3 +
sin(d*x + c))/(sin(d*x + c)^4 - 2*sin(d*x + c)^2 + 1) - log(sin(d*x + c) +
1) + log(sin(d*x + c) - 1)) + 840*a^5*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1)
- log(sin(d*x + c) + 1) + log(sin(d*x + c) - 1)) - 5600*a^4*b/cos(d*x + c)^
3 + 2240*(5*cos(d*x + c)^2 - 3)*a^2*b^3/cos(d*x + c)^5 - 32*(35*cos(d*x + c
)^4 - 42*cos(d*x + c)^2 + 15)*b^5/cos(d*x + c)^7)/d
```

mupad [B] time = 4.21, size = 514, normalized size = 1.62

$$\frac{\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)\,\left(a^5-\frac{5\,a^3\,b^2}{2}+\frac{5\,a\,b^4}{8}\right)}{d}-\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3\,\left(-4\,a^5+10\,a^3\,b^2+\frac{25\,a\,b^4}{6}\right)-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{10}\,\left(40\,a^4\,b\cdots\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*cos(c + d*x) + b*sin(c + d*x))^5/cos(c + d*x)^8,x)
```

```
[Out] (atanh(tan(c/2 + (d*x)/2))*((5*a*b^4)/8 + a^5 - (5*a^3*b^2)/2))/d - (tan(c/
2 + (d*x)/2)^3*((25*a*b^4)/6 - 4*a^5 + 10*a^3*b^2) - tan(c/2 + (d*x)/2)^10*
(40*a^4*b - 40*a^2*b^3) - tan(c/2 + (d*x)/2)^13*(a^5 - (5*a*b^4)/8 + (5*a^3
*b^2)/2) - tan(c/2 + (d*x)/2)^11*((25*a*b^4)/6 - 4*a^5 + 10*a^3*b^2) + tan(
c/2 + (d*x)/2)^5*((485*a*b^4)/24 + 5*a^5 - (55*a^3*b^2)/2) - tan(c/2 + (d*x
)/2)^9*((485*a*b^4)/24 + 5*a^5 - (55*a^3*b^2)/2) + tan(c/2 + (d*x)/2)^4*(30
*a^4*b + (16*b^5)/5 - 16*a^2*b^3) - tan(c/2 + (d*x)/2)^2*((40*a^4*b)/3 + (1
6*b^5)/15 - (56*a^2*b^3)/3) + tan(c/2 + (d*x)/2)^6*((16*b^5)/3 - (160*a^4*b
)/3 + (80*a^2*b^3)/3) + tan(c/2 + (d*x)/2)^8*((190*a^4*b)/3 + (32*b^5)/3 -
(200*a^2*b^3)/3) + (10*a^4*b)/3 + tan(c/2 + (d*x)/2)*(a^5 - (5*a*b^4)/8 + (
5*a^3*b^2)/2) + (16*b^5)/105 - (8*a^2*b^3)/3 + 10*a^4*b*tan(c/2 + (d*x)/2)^
12)/(d*(7*tan(c/2 + (d*x)/2)^2 - 21*tan(c/2 + (d*x)/2)^4 + 35*tan(c/2 + (d*
x)/2)^6 - 35*tan(c/2 + (d*x)/2)^8 + 21*tan(c/2 + (d*x)/2)^10 - 7*tan(c/2 +
(d*x)/2)^12 + tan(c/2 + (d*x)/2)^14 - 1))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**8*(a*cos(d*x+c)+b*sin(d*x+c))**5,x)
```

```
[Out] Timed out
```

3.106 $\int \sec^9(c+dx)(a\cos(c+dx)+b\sin(c+dx))^5\,dx$

Optimal. Leaf size=177

$$\frac{a^5\tan(c+dx)}{d}+\frac{5a^4b\tan^2(c+dx)}{2d}+\frac{ab^2\left(2a^2+b^2\right)\tan^5(c+dx)}{d}+\frac{5a^2b\left(a^2+2b^2\right)\tan^4(c+dx)}{4d}+\frac{b^3\left(10a^2+b^2\right)\text{t}}{6d}$$

```
[Out] a^5*tan(d*x+c)/d+5/2*a^4*b*tan(d*x+c)^2/d+1/3*a^3*(a^2+10*b^2)*tan(d*x+c)^3
/d+5/4*a^2*b*(a^2+2*b^2)*tan(d*x+c)^4/d+a*b^2*(2*a^2+b^2)*tan(d*x+c)^5/d+1/
6*b^3*(10*a^2+b^2)*tan(d*x+c)^6/d+5/7*a*b^4*tan(d*x+c)^7/d+1/8*b^5*tan(d*x+
c)^8/d
```

Rubi [A] time = 0.15, antiderivative size = 177, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {3088, 894}

$$\frac{b^3\left(10a^2+b^2\right)\tan^6(c+dx)}{6d}+\frac{ab^2\left(2a^2+b^2\right)\tan^5(c+dx)}{d}+\frac{5a^2b\left(a^2+2b^2\right)\tan^4(c+dx)}{4d}+\frac{a^3\left(a^2+10b^2\right)\tan^3(c+}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^9*(a*Cos[c + d*x] + b*Sin[c + d*x])^5,x]
```

```
[Out] (a^5*Tan[c + d*x])/d + (5*a^4*b*Tan[c + d*x]^2)/(2*d) + (a^3*(a^2 + 10*b^2)
*Tan[c + d*x]^3)/(3*d) + (5*a^2*b*(a^2 + 2*b^2)*Tan[c + d*x]^4)/(4*d) + (a*
b^2*(2*a^2 + b^2)*Tan[c + d*x]^5)/d + (b^3*(10*a^2 + b^2)*Tan[c + d*x]^6)/(
6*d) + (5*a*b^4*Tan[c + d*x]^7)/(7*d) + (b^5*Tan[c + d*x]^8)/(8*d)
```

Rule 894

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ
[m, 0] && ILtQ[n, 0]))
```

Rule 3088

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[(x^m*(b +
 a*x)^n)/(1 + x^2)^((m + n + 2)/2), x], x, Cot[c + d*x]], x] /; FreeQ[{a, b
, c, d}, x] && IntegerQ[n] && IntegerQ[(m + n)/2] && NeQ[n, -1] &&  !(GtQ[n
, 0] && GtQ[m, 1])
```

Rubi steps

$$\int \sec^9(c+dx)(a\cos(c+dx)+b\sin(c+dx))^5\,dx = -\frac{\text{Subst}\left(\int \frac{(b+ax)^5(1+x^2)}{x^9}\,dx, x, \cot(c+dx)\right)}{d}$$
$$= -\frac{\text{Subst}\left(\int \left(\frac{b^5}{x^9}+\frac{5ab^4}{x^8}+\frac{10a^2b^3+b^5}{x^7}+\frac{5ab^2(2a^2+b^2)}{x^6}+\frac{5a^2b(a^2+2b^2)}{x^5}\right.\right.}{d}$$
$$= \frac{a^5\tan(c+dx)}{d}+\frac{5a^4b\tan^2(c+dx)}{2d}+\frac{a^3\left(a^2+10b^2\right)\tan^3(c}{3d}$$

Mathematica [A] time = 0.45, size = 54, normalized size = 0.31

$$\frac{(a+b\tan(c+dx))^6\left(a^2-6ab\tan(c+dx)+21b^2\tan^2(c+dx)+28b^2\right)}{168b^3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^9*(a*Cos[c + d*x] + b*Sin[c + d*x])^5,x]
```

```
[Out] ((a + b*Tan[c + d*x])^6*(a^2 + 28*b^2 - 6*a*b*Tan[c + d*x] + 21*b^2*Tan[c +
 d*x]^2))/(168*b^3*d)
```

fricas [A] time = 0.45, size = 176, normalized size = 0.99

$$\frac{21\,b^5+42\left(5\,a^4b-10\,a^2b^3+b^5\right)\cos(dx+c)^4+56\left(5\,a^2b^3-b^5\right)\cos(dx+c)^2+8\left(2\left(7\,a^5-14\,a^3b^2+3\,ab^4\right)\cos\right.}{168\,d\,\mathrm{co}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^9*(a*cos(d*x+c)+b*sin(d*x+c))^5,x, algorithm="fricas")
```

```
[Out] 1/168*(21*b^5 + 42*(5*a^4*b - 10*a^2*b^3 + b^5)*cos(d*x + c)^4 + 56*(5*a^2*
b^3 - b^5)*cos(d*x + c)^2 + 8*(2*(7*a^5 - 14*a^3*b^2 + 3*a*b^4)*cos(d*x + c
)^7 + 15*a*b^4*cos(d*x + c) + (7*a^5 - 14*a^3*b^2 + 3*a*b^4)*cos(d*x + c)^5
 + 6*(7*a^3*b^2 - 4*a*b^4)*cos(d*x + c)^3)*sin(d*x + c))/(d*cos(d*x + c)^8)
```

giac [A] time = 2.94, size = 176, normalized size = 0.99

$$\frac{21\,b^5\tan(dx+c)^8+120\,ab^4\tan(dx+c)^7+280\,a^2b^3\tan(dx+c)^6+28\,b^5\tan(dx+c)^6+336\,a^3b^2\tan(dx+c)^5}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^9*(a*cos(d*x+c)+b*sin(d*x+c))^5,x, algorithm="giac")
```

```
[Out] 1/168*(21*b^5*tan(d*x + c)^8 + 120*a*b^4*tan(d*x + c)^7 + 280*a^2*b^3*tan(d
*x + c)^6 + 28*b^5*tan(d*x + c)^6 + 336*a^3*b^2*tan(d*x + c)^5 + 168*a*b^4*
tan(d*x + c)^5 + 210*a^4*b*tan(d*x + c)^4 + 420*a^2*b^3*tan(d*x + c)^4 + 56
*a^5*tan(d*x + c)^3 + 560*a^3*b^2*tan(d*x + c)^3 + 420*a^4*b*tan(d*x + c)^2
 + 168*a^5*tan(d*x + c))/d
```

maple [A] time = 0.28, size = 217, normalized size = 1.23

$$\frac{-a^5\left(-\frac{2}{3}-\frac{\left(\sec^2(dx+c)\right)}{3}\right)\tan\left(dx+c\right)+\frac{5a^4b}{4\cos(dx+c)^4}+10a^3b^2\left(\frac{\sin^3(dx+c)}{5\cos(dx+c)^5}+\frac{2\left(\sin^3(dx+c)\right)}{15\cos(dx+c)^3}\right)+10a^2b^3\left(\frac{\sin^4(dx+c)}{6\cos(dx+c)^6}+\frac{\sin^4(dx+c)}{12\cos(dx+c)^4}\right)+\cdots}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^9*(a*cos(d*x+c)+b*sin(d*x+c))^5,x)
```

```
[Out] 1/d*(-a^5*(-2/3-1/3*sec(d*x+c)^2)*tan(d*x+c)+5/4*a^4*b/cos(d*x+c)^4+10*a^3*
b^2*(1/5*sin(d*x+c)^3/cos(d*x+c)^5+2/15*sin(d*x+c)^3/cos(d*x+c)^3)+10*a^2*b
^3*(1/6*sin(d*x+c)^4/cos(d*x+c)^6+1/12*sin(d*x+c)^4/cos(d*x+c)^4)+5*a*b^4*(
1/7*sin(d*x+c)^5/cos(d*x+c)^7+2/35*sin(d*x+c)^5/cos(d*x+c)^5)+b^5*(1/8*sin(
d*x+c)^6/cos(d*x+c)^8+1/24*sin(d*x+c)^6/cos(d*x+c)^6))
```

maxima [A] time = 0.96, size = 223, normalized size = 1.26

$$\frac{56\left(\tan\left(dx+c\right)^3+3\,\tan\left(dx+c\right)\right)a^5+112\left(3\,\tan\left(dx+c\right)^5+5\,\tan\left(dx+c\right)^3\right)a^3b^2+24\left(5\,\tan\left(dx+c\right)^7+7\,\tan\cdots\right.}{16\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^9*(a*cos(d*x+c)+b*sin(d*x+c))^5,x, algorithm="maxima")
```

```
[Out] 1/168*(56*(tan(d*x + c)^3 + 3*tan(d*x + c))*a^5 + 112*(3*tan(d*x + c)^5 + 5
*tan(d*x + c)^3)*a^3*b^2 + 24*(5*tan(d*x + c)^7 + 7*tan(d*x + c)^5)*a*b^4 -
 140*(3*sin(d*x + c)^2 - 1)*a^2*b^3/(sin(d*x + c)^6 - 3*sin(d*x + c)^4 + 3*
sin(d*x + c)^2 - 1) + 7*(6*sin(d*x + c)^4 - 4*sin(d*x + c)^2 + 1)*b^5/(sin(
d*x + c)^8 - 4*sin(d*x + c)^6 + 6*sin(d*x + c)^4 - 4*sin(d*x + c)^2 + 1) +
210*a^4*b/(sin(d*x + c)^2 - 1)^2)/d
```

mupad [B] time = 4.27, size = 419, normalized size = 2.37

$$\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5\,\left(\frac{86\,a^5}{3}-\frac{208\,a^3\,b^2}{3}+32\,a\,b^4\right)-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4\,\left(40\,a^4\,b-40\,a^2\,b^3\right)-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{12}\,\left(40\,a^4\,b-40\,a^2\cdots\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*cos(c + d*x) + b*sin(c + d*x))^5/cos(c + d*x)^9,x)
```

```
[Out] (tan(c/2 + (d*x)/2)^5*(32*a*b^4 + (86*a^5)/3 - (208*a^3*b^2)/3) - tan(c/2 +
 (d*x)/2)^4*(40*a^4*b - 40*a^2*b^3) - tan(c/2 + (d*x)/2)^12*(40*a^4*b - 40*
a^2*b^3) - 2*a^5*tan(c/2 + (d*x)/2)^15 - tan(c/2 + (d*x)/2)^11*(32*a*b^4 +
(86*a^5)/3 - (208*a^3*b^2)/3) - tan(c/2 + (d*x)/2)^7*((32*a*b^4)/7 + (130*a
^5)/3 - (224*a^3*b^2)/3) + tan(c/2 + (d*x)/2)^9*((32*a*b^4)/7 + (130*a^5)/3
 - (224*a^3*b^2)/3) + tan(c/2 + (d*x)/2)^8*((32*b^5)/3 - 80*a^4*b + (80*a^2
*b^3)/3) + tan(c/2 + (d*x)/2)^6*(70*a^4*b + (32*b^5)/3 - (160*a^2*b^3)/3) +
 tan(c/2 + (d*x)/2)^10*(70*a^4*b + (32*b^5)/3 - (160*a^2*b^3)/3) - tan(c/2
+ (d*x)/2)^3*((34*a^5)/3 - (80*a^3*b^2)/3) + tan(c/2 + (d*x)/2)^13*((34*a^5
)/3 - (80*a^3*b^2)/3) + 2*a^5*tan(c/2 + (d*x)/2) + 10*a^4*b*tan(c/2 + (d*x)
/2)^2 + 10*a^4*b*tan(c/2 + (d*x)/2)^14)/(d*(tan(c/2 + (d*x)/2)^2 - 1)^8)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**9*(a*cos(d*x+c)+b*sin(d*x+c))**5,x)
```

```
[Out] Timed out
```

3.107 $\int \sec^{10}(c+dx)(a\cos(c+dx)+b\sin(c+dx))^5\,dx$

Optimal. Leaf size=391

$$\frac{3a^5\tanh^{-1}(\sin(c+dx))}{8d}+\frac{a^5\tan(c+dx)\sec^3(c+dx)}{4d}+\frac{3a^5\tan(c+dx)\sec(c+dx)}{8d}+\frac{a^4b\sec^5(c+dx)}{d}-\frac{5a^3b^2\tan}{}$$

```
[Out] 3/8*a^5*arctanh(sin(d*x+c))/d-5/8*a^3*b^2*arctanh(sin(d*x+c))/d+15/128*a*b^
4*arctanh(sin(d*x+c))/d+a^4*b*sec(d*x+c)^5/d-2*a^2*b^3*sec(d*x+c)^5/d+1/5*b
^5*sec(d*x+c)^5/d+10/7*a^2*b^3*sec(d*x+c)^7/d-2/7*b^5*sec(d*x+c)^7/d+1/9*b^
5*sec(d*x+c)^9/d+3/8*a^5*sec(d*x+c)*tan(d*x+c)/d-5/8*a^3*b^2*sec(d*x+c)*tan
(d*x+c)/d+15/128*a*b^4*sec(d*x+c)*tan(d*x+c)/d+1/4*a^5*sec(d*x+c)^3*tan(d*x
+c)/d-5/12*a^3*b^2*sec(d*x+c)^3*tan(d*x+c)/d+5/64*a*b^4*sec(d*x+c)^3*tan(d*
x+c)/d+5/3*a^3*b^2*sec(d*x+c)^5*tan(d*x+c)/d-5/16*a*b^4*sec(d*x+c)^5*tan(d*
x+c)/d+5/8*a*b^4*sec(d*x+c)^5*tan(d*x+c)^3/d
```

Rubi [A] time = 0.39, antiderivative size = 391, normalized size of antiderivative = 1.00, number of steps used = 22, number of rules used = 8, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3090, 3768, 3770, 2606, 30, 2611, 14, 270}

$$\frac{10a^2b^3\sec^7(c+dx)}{7d}-\frac{2a^2b^3\sec^5(c+dx)}{d}-\frac{5a^3b^2\tanh^{-1}(\sin(c+dx))}{8d}+\frac{5a^3b^2\tan(c+dx)\sec^5(c+dx)}{3d}-\frac{5a^3b^2\tan}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^10*(a*Cos[c + d*x] + b*Sin[c + d*x])^5,x]
```

```
[Out] (3*a^5*ArcTanh[Sin[c + d*x]])/(8*d) - (5*a^3*b^2*ArcTanh[Sin[c + d*x]])/(8*
d) + (15*a*b^4*ArcTanh[Sin[c + d*x]])/(128*d) + (a^4*b*Sec[c + d*x]^5)/d -
(2*a^2*b^3*Sec[c + d*x]^5)/d + (b^5*Sec[c + d*x]^5)/(5*d) + (10*a^2*b^3*Sec
[c + d*x]^7)/(7*d) - (2*b^5*Sec[c + d*x]^7)/(7*d) + (b^5*Sec[c + d*x]^9)/(9
*d) + (3*a^5*Sec[c + d*x]*Tan[c + d*x])/(8*d) - (5*a^3*b^2*Sec[c + d*x]*Tan
[c + d*x])/(8*d) + (15*a*b^4*Sec[c + d*x]*Tan[c + d*x])/(128*d) + (a^5*Sec[
c + d*x]^3*Tan[c + d*x])/(4*d) - (5*a^3*b^2*Sec[c + d*x]^3*Tan[c + d*x])/(1
2*d) + (5*a*b^4*Sec[c + d*x]^3*Tan[c + d*x])/(64*d) + (5*a^3*b^2*Sec[c + d*
x]^5*Tan[c + d*x])/(3*d) - (5*a*b^4*Sec[c + d*x]^5*Tan[c + d*x])/(16*d) + (
5*a*b^4*Sec[c + d*x]^5*Tan[c + d*x]^3)/(8*d)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 2611

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(b*(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*(
m + n - 1)), x] - Dist[(b^2*(n - 1))/(m + n - 1), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 3090

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_.), x_Symbol] :> Int[ExpandTrig[cos[c + d*x]^m*(a
*cos[c + d*x] + b*sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d}, x] && Inte
gerQ[m] && IGtQ[n, 0]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \sec^{10}(c+dx)(a\cos(c+dx)+b\sin(c+dx))^5\,dx = \int \left(a^5\sec^5(c+dx)+5a^4b\sec^5(c+dx)\tan(c+dx)+10a^3\right.$$
$$= a^5\int \sec^5(c+dx)\,dx + \left(5a^4b\right)\int \sec^5(c+dx)\tan(c+dx)$$
$$= \frac{a^5\sec^3(c+dx)\tan(c+dx)}{4d} + \frac{5a^3b^2\sec^5(c+dx)\tan(c+}{3d}$$
$$= \frac{a^4b\sec^5(c+dx)}{d} + \frac{3a^5\sec(c+dx)\tan(c+dx)}{8d} + \frac{a^5\sec^3(}{}$$
$$= \frac{3a^5\tanh^{-1}(\sin(c+dx))}{8d} + \frac{a^4b\sec^5(c+dx)}{d} - \frac{2a^2b^3\sec^5(}{d}$$
$$= \frac{3a^5\tanh^{-1}(\sin(c+dx))}{8d} - \frac{5a^3b^2\tanh^{-1}(\sin(c+dx))}{8d} + \frac{a^4}{}$$
$$= \frac{3a^5\tanh^{-1}(\sin(c+dx))}{8d} - \frac{5a^3b^2\tanh^{-1}(\sin(c+dx))}{8d} + \frac{15}{}$$

Mathematica [A] time = 2.14, size = 331, normalized size = 0.85

$$\frac{1260a\left(656a^4+2320a^2b^2+845b^4\right)\tan(c+dx)\sec^7(c+dx)-40320a\left(48a^4-80a^2b^2+15b^4\right)\left(\log\left(\cos\left(\frac{1}{2}(c+\right.\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^10*(a*Cos[c + d*x] + b*Sin[c + d*x])^5,x]
```

```
[Out] (-40320*a*(48*a^4 - 80*a^2*b^2 + 15*b^4)*(Log[Cos[(c + d*x)/2] - Sin[(c + d
*x)/2]] - Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]) + Sec[c + d*x]^9*(19353
60*a^4*b - 184320*a^2*b^3 + 223232*b^5 + 73728*(35*a^4*b - 20*a^2*b^3 - 3*b
^5)*Cos[2*(c + d*x)] + 129024*(5*a^4*b - 10*a^2*b^3 + b^5)*Cos[4*(c + d*x)]
 + 372960*a^5*Sin[4*(c + d*x)] + 453600*a^3*b^2*Sin[4*(c + d*x)] - 488250*a
*b^4*Sin[4*(c + d*x)] + 131040*a^5*Sin[6*(c + d*x)] - 218400*a^3*b^2*Sin[6*
(c + d*x)] + 40950*a*b^4*Sin[6*(c + d*x)] + 15120*a^5*Sin[8*(c + d*x)] - 25
200*a^3*b^2*Sin[8*(c + d*x)] + 4725*a*b^4*Sin[8*(c + d*x)]) + 1260*a*(656*a
^4 + 2320*a^2*b^2 + 845*b^4)*Sec[c + d*x]^7*Tan[c + d*x])/(5160960*d)
```

fricas [A] time = 0.62, size = 257, normalized size = 0.66

$$\frac{315\left(48\,a^5-80\,a^3b^2+15\,ab^4\right)\cos(dx+c)^9\log(\sin(dx+c)+1)-315\left(48\,a^5-80\,a^3b^2+15\,ab^4\right)\cos(dx+c)^9}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^10*(a*cos(d*x+c)+b*sin(d*x+c))^5,x, algorithm="fricas"
)
```

```
[Out] 1/80640*(315*(48*a^5 - 80*a^3*b^2 + 15*a*b^4)*cos(d*x + c)^9*log(sin(d*x +
c) + 1) - 315*(48*a^5 - 80*a^3*b^2 + 15*a*b^4)*cos(d*x + c)^9*log(-sin(d*x
+ c) + 1) + 8960*b^5 + 16128*(5*a^4*b - 10*a^2*b^3 + b^5)*cos(d*x + c)^4 +
23040*(5*a^2*b^3 - b^5)*cos(d*x + c)^2 + 210*(3*(48*a^5 - 80*a^3*b^2 + 15*a
*b^4)*cos(d*x + c)^7 + 240*a*b^4*cos(d*x + c) + 2*(48*a^5 - 80*a^3*b^2 + 15
*a*b^4)*cos(d*x + c)^5 + 40*(16*a^3*b^2 - 9*a*b^4)*cos(d*x + c)^3)*sin(d*x
+ c))/(d*cos(d*x + c)^9)
```

giac [B] time = 0.74, size = 888, normalized size = 2.27

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^10*(a*cos(d*x+c)+b*sin(d*x+c))^5,x, algorithm="giac")
```

```
[Out] 1/40320*(315*(48*a^5 - 80*a^3*b^2 + 15*a*b^4)*log(abs(tan(1/2*d*x + 1/2*c)
+ 1)) - 315*(48*a^5 - 80*a^3*b^2 + 15*a*b^4)*log(abs(tan(1/2*d*x + 1/2*c) -
 1)) + 2*(25200*a^5*tan(1/2*d*x + 1/2*c)^17 + 25200*a^3*b^2*tan(1/2*d*x + 1
/2*c)^17 - 4725*a*b^4*tan(1/2*d*x + 1/2*c)^17 - 201600*a^4*b*tan(1/2*d*x +
1/2*c)^16 - 110880*a^5*tan(1/2*d*x + 1/2*c)^15 + 319200*a^3*b^2*tan(1/2*d*x
 + 1/2*c)^15 + 40950*a*b^4*tan(1/2*d*x + 1/2*c)^15 + 806400*a^4*b*tan(1/2*d
*x + 1/2*c)^14 - 806400*a^2*b^3*tan(1/2*d*x + 1/2*c)^14 + 191520*a^5*tan(1/
2*d*x + 1/2*c)^13 - 453600*a^3*b^2*tan(1/2*d*x + 1/2*c)^13 + 488250*a*b^4*t
an(1/2*d*x + 1/2*c)^13 - 1612800*a^4*b*tan(1/2*d*x + 1/2*c)^12 + 806400*a^2
*b^3*tan(1/2*d*x + 1/2*c)^12 - 215040*b^5*tan(1/2*d*x + 1/2*c)^12 - 151200*
a^5*tan(1/2*d*x + 1/2*c)^11 - 151200*a^3*b^2*tan(1/2*d*x + 1/2*c)^11 + 5323
50*a*b^4*tan(1/2*d*x + 1/2*c)^11 + 2419200*a^4*b*tan(1/2*d*x + 1/2*c)^10 -
806400*a^2*b^3*tan(1/2*d*x + 1/2*c)^10 - 322560*b^5*tan(1/2*d*x + 1/2*c)^10
 - 2661120*a^4*b*tan(1/2*d*x + 1/2*c)^8 + 2096640*a^2*b^3*tan(1/2*d*x + 1/2
*c)^8 - 451584*b^5*tan(1/2*d*x + 1/2*c)^8 + 151200*a^5*tan(1/2*d*x + 1/2*c)
^7 + 151200*a^3*b^2*tan(1/2*d*x + 1/2*c)^7 - 532350*a*b^4*tan(1/2*d*x + 1/2
*c)^7 + 1774080*a^4*b*tan(1/2*d*x + 1/2*c)^6 - 1128960*a^2*b^3*tan(1/2*d*x
+ 1/2*c)^6 - 129024*b^5*tan(1/2*d*x + 1/2*c)^6 - 191520*a^5*tan(1/2*d*x + 1
/2*c)^5 + 453600*a^3*b^2*tan(1/2*d*x + 1/2*c)^5 - 488250*a*b^4*tan(1/2*d*x
+ 1/2*c)^5 - 645120*a^4*b*tan(1/2*d*x + 1/2*c)^4 + 23040*a^2*b^3*tan(1/2*d*
x + 1/2*c)^4 - 36864*b^5*tan(1/2*d*x + 1/2*c)^4 + 110880*a^5*tan(1/2*d*x +
1/2*c)^3 - 319200*a^3*b^2*tan(1/2*d*x + 1/2*c)^3 - 40950*a*b^4*tan(1/2*d*x
+ 1/2*c)^3 + 161280*a^4*b*tan(1/2*d*x + 1/2*c)^2 - 207360*a^2*b^3*tan(1/2*d
*x + 1/2*c)^2 + 9216*b^5*tan(1/2*d*x + 1/2*c)^2 - 25200*a^5*tan(1/2*d*x + 1
/2*c) - 25200*a^3*b^2*tan(1/2*d*x + 1/2*c) + 4725*a*b^4*tan(1/2*d*x + 1/2*c
) - 40320*a^4*b + 23040*a^2*b^3 - 1024*b^5)/(tan(1/2*d*x + 1/2*c)^2 - 1)^9)
/d
```

maple [A] time = 0.28, size = 688, normalized size = 1.76

$$\frac{8b^5\cos(dx+c)}{315d}-\frac{15a\,b^4\sin(dx+c)}{128d}+\frac{5a^3b^2\sin(dx+c)}{8d}+\frac{3a^5\sec(dx+c)\tan(dx+c)}{8d}-\frac{4a^2b^3\cos(dx+c)}{7d}-\frac{2\,\mathrm{co}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^10*(a*cos(d*x+c)+b*sin(d*x+c))^5,x)
```

```
[Out] 8/315*b^5*cos(d*x+c)/d-15/128*a*b^4*sin(d*x+c)/d-5/128*a*b^4*sin(d*x+c)^3/d
+5/8*a^3*b^2*sin(d*x+c)/d+3/8*a^5*sec(d*x+c)*tan(d*x+c)/d+1/4*a^5*sec(d*x+c
)^3*tan(d*x+c)/d-4/7*a^2*b^3*cos(d*x+c)/d+1/d*a^4*b/cos(d*x+c)^5+1/9/d*b^5*
sin(d*x+c)^6/cos(d*x+c)^9+1/105/d*b^5*sin(d*x+c)^6/cos(d*x+c)^5-5/8/d*a^3*b
^2*ln(sec(d*x+c)+tan(d*x+c))-1/315/d*b^5*sin(d*x+c)^6/cos(d*x+c)^3+5/8/d*a^
3*b^2*sin(d*x+c)^3/cos(d*x+c)^2+2/7/d*a^2*b^3*sin(d*x+c)^4/cos(d*x+c)^3+5/6
4/d*a*b^4*sin(d*x+c)^5/cos(d*x+c)^4-2/7/d*cos(d*x+c)*sin(d*x+c)^2*a^2*b^3+5
/4/d*a^3*b^2*sin(d*x+c)^3/cos(d*x+c)^4+6/7/d*a^2*b^3*sin(d*x+c)^4/cos(d*x+c
)^5+5/16/d*a*b^4*sin(d*x+c)^5/cos(d*x+c)^6+3/8/d*a^5*ln(sec(d*x+c)+tan(d*x+
c))+15/128/d*a*b^4*ln(sec(d*x+c)+tan(d*x+c))+1/105/d*b^5*sin(d*x+c)^6/cos(d
*x+c)+1/105/d*b^5*cos(d*x+c)*sin(d*x+c)^4+4/315/d*cos(d*x+c)*sin(d*x+c)^2*b
^5-2/7/d*a^2*b^3*sin(d*x+c)^4/cos(d*x+c)-5/128/d*a*b^4*sin(d*x+c)^5/cos(d*x
+c)^2+1/21/d*b^5*sin(d*x+c)^6/cos(d*x+c)^7+5/3/d*a^3*b^2*sin(d*x+c)^3/cos(d
*x+c)^6+10/7/d*a^2*b^3*sin(d*x+c)^4/cos(d*x+c)^7+5/8/d*a*b^4*sin(d*x+c)^5/c
os(d*x+c)^8
```

maxima [A] time = 1.01, size = 360, normalized size = 0.92

$$-\frac{1575\,ab^4\left(\frac{2\left(3\,\sin(dx+c)^7-11\,\sin(dx+c)^5-11\,\sin(dx+c)^3+3\,\sin(dx+c)\right)}{\sin(dx+c)^8-4\,\sin(dx+c)^6+6\,\sin(dx+c)^4-4\,\sin(dx+c)^2+1}-3\,\log(\sin(dx+c)+1)+3\,\log(\sin(dx+c)-1)\right)}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^10*(a*cos(d*x+c)+b*sin(d*x+c))^5,x, algorithm="maxima"
)
```

```
[Out] -1/80640*(1575*a*b^4*(2*(3*sin(d*x + c)^7 - 11*sin(d*x + c)^5 - 11*sin(d*x
+ c)^3 + 3*sin(d*x + c))/(sin(d*x + c)^8 - 4*sin(d*x + c)^6 + 6*sin(d*x + c
)^4 - 4*sin(d*x + c)^2 + 1) - 3*log(sin(d*x + c) + 1) + 3*log(sin(d*x + c)
- 1)) - 8400*a^3*b^2*(2*(3*sin(d*x + c)^5 - 8*sin(d*x + c)^3 - 3*sin(d*x +
c))/(sin(d*x + c)^6 - 3*sin(d*x + c)^4 + 3*sin(d*x + c)^2 - 1) - 3*log(sin(
d*x + c) + 1) + 3*log(sin(d*x + c) - 1)) + 5040*a^5*(2*(3*sin(d*x + c)^3 -
5*sin(d*x + c))/(sin(d*x + c)^4 - 2*sin(d*x + c)^2 + 1) - 3*log(sin(d*x + c
) + 1) + 3*log(sin(d*x + c) - 1)) - 80640*a^4*b/cos(d*x + c)^5 + 23040*(7*c
os(d*x + c)^2 - 5)*a^2*b^3/cos(d*x + c)^7 - 256*(63*cos(d*x + c)^4 - 90*cos
(d*x + c)^2 + 35)*b^5/cos(d*x + c)^9)/d
```

mupad [B] time = 4.71, size = 675, normalized size = 1.73

$$\frac{\operatorname{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)\,\left(\frac{3\,a^5}{4}-\frac{5\,a^3\,b^2}{4}+\frac{15\,a\,b^4}{64}\right)}{d}-\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(\frac{5\,a^5}{4}+\frac{5\,a^3\,b^2}{4}-\frac{15\,a\,b^4}{64}\right)-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{14}\,\left(40\,a^4\,b-40\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*cos(c + d*x) + b*sin(c + d*x))^5/cos(c + d*x)^10,x)
```

```
[Out] (atanh(tan(c/2 + (d*x)/2))*((15*a*b^4)/64 + (3*a^5)/4 - (5*a^3*b^2)/4))/d -
 (tan(c/2 + (d*x)/2)*((5*a^5)/4 - (15*a*b^4)/64 + (5*a^3*b^2)/4) - tan(c/2
+ (d*x)/2)^14*(40*a^4*b - 40*a^2*b^3) - tan(c/2 + (d*x)/2)^17*((5*a^5)/4 -
(15*a*b^4)/64 + (5*a^3*b^2)/4) + tan(c/2 + (d*x)/2)^3*((65*a*b^4)/32 - (11*
a^5)/2 + (95*a^3*b^2)/6) - tan(c/2 + (d*x)/2)^15*((65*a*b^4)/32 - (11*a^5)/
2 + (95*a^3*b^2)/6) + tan(c/2 + (d*x)/2)^5*((775*a*b^4)/32 + (19*a^5)/2 - (
45*a^3*b^2)/2) - tan(c/2 + (d*x)/2)^13*((775*a*b^4)/32 + (19*a^5)/2 - (45*a
^3*b^2)/2) - tan(c/2 + (d*x)/2)^7*((15*a^5)/2 - (845*a*b^4)/32 + (15*a^3*b^
2)/2) + tan(c/2 + (d*x)/2)^11*((15*a^5)/2 - (845*a*b^4)/32 + (15*a^3*b^2)/2
) - tan(c/2 + (d*x)/2)^2*(8*a^4*b + (16*b^5)/35 - (72*a^2*b^3)/7) + tan(c/2
 + (d*x)/2)^4*(32*a^4*b + (64*b^5)/35 - (8*a^2*b^3)/7) + tan(c/2 + (d*x)/2)
^12*(80*a^4*b + (32*b^5)/3 - 40*a^2*b^3) + tan(c/2 + (d*x)/2)^10*(16*b^5 -
120*a^4*b + 40*a^2*b^3) + tan(c/2 + (d*x)/2)^6*((32*b^5)/5 - 88*a^4*b + 56*
a^2*b^3) + tan(c/2 + (d*x)/2)^8*(132*a^4*b + (112*b^5)/5 - 104*a^2*b^3) + 2
*a^4*b + (16*b^5)/315 - (8*a^2*b^3)/7 + 10*a^4*b*tan(c/2 + (d*x)/2)^16)/(d*
(9*tan(c/2 + (d*x)/2)^2 - 36*tan(c/2 + (d*x)/2)^4 + 84*tan(c/2 + (d*x)/2)^6
 - 126*tan(c/2 + (d*x)/2)^8 + 126*tan(c/2 + (d*x)/2)^10 - 84*tan(c/2 + (d*x
)/2)^12 + 36*tan(c/2 + (d*x)/2)^14 - 9*tan(c/2 + (d*x)/2)^16 + tan(c/2 + (d
*x)/2)^18 - 1))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**10*(a*cos(d*x+c)+b*sin(d*x+c))**5,x)
```

```
[Out] Timed out
```

3.108 $\int \sec^{11}(c+dx)(a\cos(c+dx)+b\sin(c+dx))^5\,dx$

Optimal. Leaf size=242

$$\frac{a^5\tan(c+dx)}{d}+\frac{5a^4b\tan^2(c+dx)}{2d}+\frac{10ab^2\left(a^2+b^2\right)\tan^7(c+dx)}{7d}+\frac{5a^2b\left(a^2+b^2\right)\tan^4(c+dx)}{2d}+\frac{b^3\left(5a^2+b^2\right)\tan}{4d}$$

```
[Out] a^5*tan(d*x+c)/d+5/2*a^4*b*tan(d*x+c)^2/d+2/3*a^3*(a^2+5*b^2)*tan(d*x+c)^3/
d+5/2*a^2*b*(a^2+b^2)*tan(d*x+c)^4/d+1/5*a*(a^4+20*a^2*b^2+5*b^4)*tan(d*x+c
)^5/d+1/6*b*(5*a^4+20*a^2*b^2+b^4)*tan(d*x+c)^6/d+10/7*a*b^2*(a^2+b^2)*tan(
d*x+c)^7/d+1/4*b^3*(5*a^2+b^2)*tan(d*x+c)^8/d+5/9*a*b^4*tan(d*x+c)^9/d+1/10
*b^5*tan(d*x+c)^10/d
```

Rubi [A] time = 0.22, antiderivative size = 242, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {3088, 948}

$$\frac{b^3\left(5a^2+b^2\right)\tan^8(c+dx)}{4d}+\frac{10ab^2\left(a^2+b^2\right)\tan^7(c+dx)}{7d}+\frac{b\left(20a^2b^2+5a^4+b^4\right)\tan^6(c+dx)}{6d}+\frac{a\left(20a^2b^2+a^4+\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^11*(a*Cos[c + d*x] + b*Sin[c + d*x])^5,x]
```

```
[Out] (a^5*Tan[c + d*x])/d + (5*a^4*b*Tan[c + d*x]^2)/(2*d) + (2*a^3*(a^2 + 5*b^2
)*Tan[c + d*x]^3)/(3*d) + (5*a^2*b*(a^2 + b^2)*Tan[c + d*x]^4)/(2*d) + (a*(
a^4 + 20*a^2*b^2 + 5*b^4)*Tan[c + d*x]^5)/(5*d) + (b*(5*a^4 + 20*a^2*b^2 +
b^4)*Tan[c + d*x]^6)/(6*d) + (10*a*b^2*(a^2 + b^2)*Tan[c + d*x]^7)/(7*d) +
(b^3*(5*a^2 + b^2)*Tan[c + d*x]^8)/(4*d) + (5*a*b^4*Tan[c + d*x]^9)/(9*d) +
 (b^5*Tan[c + d*x]^10)/(10*d)
```

Rule 948

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IGtQ[p, 0] && (IGtQ[m, 0] || (EqQ[m, -2] && EqQ[p, 1] &
& EqQ[d, 0]))
```

Rule 3088

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[(x^m*(b +
 a*x)^n)/(1 + x^2)^((m + n + 2)/2), x], x, Cot[c + d*x]], x] /; FreeQ[{a, b
, c, d}, x] && IntegerQ[n] && IntegerQ[(m + n)/2] && NeQ[n, -1] &&  !(GtQ[n
```

```
, 0] && GtQ[m, 1])
```

Rubi steps

$$\int \sec^{11}(c+dx)(a\cos(c+dx)+b\sin(c+dx))^5\,dx = -\frac{\text{Subst}\left(\int \frac{(b+ax)^5\left(1+x^2\right)^2}{x^{11}}\,dx, x, \cot(c+dx)\right)}{d}$$
$$= -\frac{\text{Subst}\left(\int \left(\frac{b^5}{x^{11}}+\frac{5ab^4}{x^{10}}+\frac{2\left(5a^2b^3+b^5\right)}{x^9}+\frac{10ab^2\left(a^2+b^2\right)}{x^8}+\frac{5a^4b+20a^2}{x^7}\right.\right.}{}$$
$$= \frac{a^5\tan(c+dx)}{d}+\frac{5a^4b\tan^2(c+dx)}{2d}+\frac{2a^3\left(a^2+5b^2\right)\tan^3(c}{3d}$$

Mathematica [A] time = 1.22, size = 115, normalized size = 0.48

$$\frac{\frac{1}{4}\left(3a^2+b^2\right)(a+b\tan(c+dx))^8-\frac{4}{7}a\left(a^2+b^2\right)(a+b\tan(c+dx))^7+\frac{1}{6}\left(a^2+b^2\right)^2(a+b\tan(c+dx))^6+\frac{1}{10}(a+b}{b^5d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^11*(a*Cos[c + d*x] + b*Sin[c + d*x])^5,x]
```

```
[Out] (((a^2 + b^2)^2*(a + b*Tan[c + d*x])^6)/6 - (4*a*(a^2 + b^2)*(a + b*Tan[c +
 d*x])^7)/7 + ((3*a^2 + b^2)*(a + b*Tan[c + d*x])^8)/4 - (4*a*(a + b*Tan[c
+ d*x])^9)/9 + (a + b*Tan[c + d*x])^10/10)/(b^5*d)
```

fricas [A] time = 0.53, size = 207, normalized size = 0.86

$$\frac{126\,b^5+210\left(5\,a^4b-10\,a^2b^3+b^5\right)\cos\left(dx+c\right)^4+315\left(5\,a^2b^3-b^5\right)\cos\left(dx+c\right)^2+4\left(8\left(21\,a^5-30\,a^3b^2+5\,ab^4\right)\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^11*(a*cos(d*x+c)+b*sin(d*x+c))^5,x, algorithm="fricas"
)
```

```
[Out] 1/1260*(126*b^5 + 210*(5*a^4*b - 10*a^2*b^3 + b^5)*cos(d*x + c)^4 + 315*(5*
a^2*b^3 - b^5)*cos(d*x + c)^2 + 4*(8*(21*a^5 - 30*a^3*b^2 + 5*a*b^4)*cos(d*
x + c)^9 + 4*(21*a^5 - 30*a^3*b^2 + 5*a*b^4)*cos(d*x + c)^7 + 175*a*b^4*cos
(d*x + c) + 3*(21*a^5 - 30*a^3*b^2 + 5*a*b^4)*cos(d*x + c)^5 + 50*(9*a^3*b^
2 - 5*a*b^4)*cos(d*x + c)^3)*sin(d*x + c))/(d*cos(d*x + c)^10)
```

giac [A] time = 1.18, size = 262, normalized size = 1.08

$$126\,b^5\tan(dx+c)^{10}+700\,ab^4\tan(dx+c)^9+1575\,a^2b^3\tan(dx+c)^8+315\,b^5\tan(dx+c)^8+1800\,a^3b^2\tan(d$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^11*(a*cos(d*x+c)+b*sin(d*x+c))^5,x, algorithm="giac")
```

```
[Out] 1/1260*(126*b^5*tan(d*x + c)^10 + 700*a*b^4*tan(d*x + c)^9 + 1575*a^2*b^3*t
an(d*x + c)^8 + 315*b^5*tan(d*x + c)^8 + 1800*a^3*b^2*tan(d*x + c)^7 + 1800
*a*b^4*tan(d*x + c)^7 + 1050*a^4*b*tan(d*x + c)^6 + 4200*a^2*b^3*tan(d*x +
c)^6 + 210*b^5*tan(d*x + c)^6 + 252*a^5*tan(d*x + c)^5 + 5040*a^3*b^2*tan(d
*x + c)^5 + 1260*a*b^4*tan(d*x + c)^5 + 3150*a^4*b*tan(d*x + c)^4 + 3150*a^
2*b^3*tan(d*x + c)^4 + 840*a^5*tan(d*x + c)^3 + 4200*a^3*b^2*tan(d*x + c)^3
 + 3150*a^4*b*tan(d*x + c)^2 + 1260*a^5*tan(d*x + c))/d
```

maple [A] time = 0.27, size = 299, normalized size = 1.24

$$-a^5\left(-\frac{8}{15}-\frac{\left(\sec^4(dx+c)\right)}{5}-\frac{4\left(\sec^2(dx+c)\right)}{15}\right)\tan(dx+c)+\frac{5a^4b}{6\cos(dx+c)^6}+10a^3b^2\left(\frac{\sin^3(dx+c)}{7\cos(dx+c)^7}+\frac{4\left(\sin^3(dx+c)\right)}{35\cos(dx+c)^5}+\frac{8\left(\sin^3(dx+c)\right)}{105\cos(dx+c)^3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^11*(a*cos(d*x+c)+b*sin(d*x+c))^5,x)
```

```
[Out] 1/d*(-a^5*(-8/15-1/5*sec(d*x+c)^4-4/15*sec(d*x+c)^2)*tan(d*x+c)+5/6*a^4*b/c
os(d*x+c)^6+10*a^3*b^2*(1/7*sin(d*x+c)^3/cos(d*x+c)^7+4/35*sin(d*x+c)^3/cos
(d*x+c)^5+8/105*sin(d*x+c)^3/cos(d*x+c)^3)+10*a^2*b^3*(1/8*sin(d*x+c)^4/cos
(d*x+c)^8+1/12*sin(d*x+c)^4/cos(d*x+c)^6+1/24*sin(d*x+c)^4/cos(d*x+c)^4)+5*
a*b^4*(1/9*sin(d*x+c)^5/cos(d*x+c)^9+4/63*sin(d*x+c)^5/cos(d*x+c)^7+8/315*s
in(d*x+c)^5/cos(d*x+c)^5)+b^5*(1/10*sin(d*x+c)^6/cos(d*x+c)^10+1/20*sin(d*x
+c)^6/cos(d*x+c)^8+1/60*sin(d*x+c)^6/cos(d*x+c)^6))
```

maxima [A] time = 0.78, size = 275, normalized size = 1.14

$$84\left(3\tan(dx+c)^5+10\tan(dx+c)^3+15\tan(dx+c)\right)a^5+120\left(15\tan(dx+c)^7+42\tan(dx+c)^5+35\tan\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^11*(a*cos(d*x+c)+b*sin(d*x+c))^5,x, algorithm="maxima"
)
```

```
[Out] 1/1260*(84*(3*tan(d*x + c)^5 + 10*tan(d*x + c)^3 + 15*tan(d*x + c))*a^5 + 1
20*(15*tan(d*x + c)^7 + 42*tan(d*x + c)^5 + 35*tan(d*x + c)^3)*a^3*b^2 + 20
*(35*tan(d*x + c)^9 + 90*tan(d*x + c)^7 + 63*tan(d*x + c)^5)*a*b^4 + 525*(4
*sin(d*x + c)^2 - 1)*a^2*b^3/(sin(d*x + c)^8 - 4*sin(d*x + c)^6 + 6*sin(d*x
 + c)^4 - 4*sin(d*x + c)^2 + 1) - 21*(10*sin(d*x + c)^4 - 5*sin(d*x + c)^2
+ 1)*b^5/(sin(d*x + c)^10 - 5*sin(d*x + c)^8 + 10*sin(d*x + c)^6 - 10*sin(d
*x + c)^4 + 5*sin(d*x + c)^2 - 1) - 1050*a^4*b/(sin(d*x + c)^2 - 1)^3)/d
```

mupad [B] time = 4.44, size = 548, normalized size = 2.26

$$\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5\,\left(\frac{616\,a^5}{15}-\frac{176\,a^3\,b^2}{3}+32\,a\,b^4\right)-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4\,\left(40\,a^4\,b-40\,a^2\,b^3\right)-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{16}\,\left(40\,a^4\,b-40\,a^2\,b\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*cos(c + d*x) + b*sin(c + d*x))^5/cos(c + d*x)^11,x)
```

```
[Out] (tan(c/2 + (d*x)/2)^5*(32*a*b^4 + (616*a^5)/15 - (176*a^3*b^2)/3) - tan(c/2
 + (d*x)/2)^4*(40*a^4*b - 40*a^2*b^3) - tan(c/2 + (d*x)/2)^16*(40*a^4*b - 4
0*a^2*b^3) - 2*a^5*tan(c/2 + (d*x)/2)^19 - tan(c/2 + (d*x)/2)^15*(32*a*b^4
+ (616*a^5)/15 - (176*a^3*b^2)/3) + tan(c/2 + (d*x)/2)^7*((160*a*b^4)/7 - 8
8*a^5 + (720*a^3*b^2)/7) - tan(c/2 + (d*x)/2)^13*((160*a*b^4)/7 - 88*a^5 +
(720*a^3*b^2)/7) + tan(c/2 + (d*x)/2)^9*((3520*a*b^4)/63 + (388*a^5)/3 - (4
240*a^3*b^2)/21) - tan(c/2 + (d*x)/2)^11*((3520*a*b^4)/63 + (388*a^5)/3 - (
4240*a^3*b^2)/21) + tan(c/2 + (d*x)/2)^6*((280*a^4*b)/3 + (32*b^5)/3 - (80*
a^2*b^3)/3) + tan(c/2 + (d*x)/2)^14*((280*a^4*b)/3 + (32*b^5)/3 - (80*a^2*b
^3)/3) + tan(c/2 + (d*x)/2)^10*(220*a^4*b + (192*b^5)/5 - 160*a^2*b^3) + ta
n(c/2 + (d*x)/2)^8*((64*b^5)/3 - (520*a^4*b)/3 + (200*a^2*b^3)/3) + tan(c/2
 + (d*x)/2)^12*((64*b^5)/3 - (520*a^4*b)/3 + (200*a^2*b^3)/3) - tan(c/2 + (
d*x)/2)^3*((38*a^5)/3 - (80*a^3*b^2)/3) + tan(c/2 + (d*x)/2)^17*((38*a^5)/3
 - (80*a^3*b^2)/3) + 2*a^5*tan(c/2 + (d*x)/2) + 10*a^4*b*tan(c/2 + (d*x)/2)
^2 + 10*a^4*b*tan(c/2 + (d*x)/2)^18)/(d*(tan(c/2 + (d*x)/2)^2 - 1)^10)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**11*(a*cos(d*x+c)+b*sin(d*x+c))**5,x)
```

```
[Out] Timed out
```

3.109 $\int \sec^{12}(c+dx)(a\cos(c+dx)+b\sin(c+dx))^5\,dx$

Optimal. Leaf size=472

$$\frac{5a^5\tanh^{-1}(\sin(c+dx))}{16d}+\frac{a^5\tan(c+dx)\sec^5(c+dx)}{6d}+\frac{5a^5\tan(c+dx)\sec^3(c+dx)}{24d}+\frac{5a^5\tan(c+dx)\sec(c+dx)}{16d}$$

```
[Out] 5/16*a^5*arctanh(sin(d*x+c))/d-25/64*a^3*b^2*arctanh(sin(d*x+c))/d+15/256*a
*b^4*arctanh(sin(d*x+c))/d+5/7*a^4*b*sec(d*x+c)^7/d-10/7*a^2*b^3*sec(d*x+c)
^7/d+1/7*b^5*sec(d*x+c)^7/d+10/9*a^2*b^3*sec(d*x+c)^9/d-2/9*b^5*sec(d*x+c)^
9/d+1/11*b^5*sec(d*x+c)^11/d+5/16*a^5*sec(d*x+c)*tan(d*x+c)/d-25/64*a^3*b^2
*sec(d*x+c)*tan(d*x+c)/d+15/256*a*b^4*sec(d*x+c)*tan(d*x+c)/d+5/24*a^5*sec(
d*x+c)^3*tan(d*x+c)/d-25/96*a^3*b^2*sec(d*x+c)^3*tan(d*x+c)/d+5/128*a*b^4*s
ec(d*x+c)^3*tan(d*x+c)/d+1/6*a^5*sec(d*x+c)^5*tan(d*x+c)/d-5/24*a^3*b^2*sec
(d*x+c)^5*tan(d*x+c)/d+1/32*a*b^4*sec(d*x+c)^5*tan(d*x+c)/d+5/4*a^3*b^2*sec
(d*x+c)^7*tan(d*x+c)/d-3/16*a*b^4*sec(d*x+c)^7*tan(d*x+c)/d+1/2*a*b^4*sec(d
*x+c)^7*tan(d*x+c)^3/d
```

Rubi [A] time = 0.47, antiderivative size = 472, normalized size of antiderivative = 1.00, number of steps used = 25, number of rules used = 8, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3090, 3768, 3770, 2606, 30, 2611, 14, 270}

$$\frac{10a^2b^3\sec^9(c+dx)}{9d}-\frac{10a^2b^3\sec^7(c+dx)}{7d}-\frac{25a^3b^2\tanh^{-1}(\sin(c+dx))}{64d}+\frac{5a^3b^2\tan(c+dx)\sec^7(c+dx)}{4d}-\frac{5a^3b^2}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^12*(a*Cos[c + d*x] + b*Sin[c + d*x])^5,x]
```

```
[Out] (5*a^5*ArcTanh[Sin[c + d*x]])/(16*d) - (25*a^3*b^2*ArcTanh[Sin[c + d*x]])/(
64*d) + (15*a*b^4*ArcTanh[Sin[c + d*x]])/(256*d) + (5*a^4*b*Sec[c + d*x]^7)
/(7*d) - (10*a^2*b^3*Sec[c + d*x]^7)/(7*d) + (b^5*Sec[c + d*x]^7)/(7*d) + (
10*a^2*b^3*Sec[c + d*x]^9)/(9*d) - (2*b^5*Sec[c + d*x]^9)/(9*d) + (b^5*Sec[
c + d*x]^11)/(11*d) + (5*a^5*Sec[c + d*x]*Tan[c + d*x])/(16*d) - (25*a^3*b^
2*Sec[c + d*x]*Tan[c + d*x])/(64*d) + (15*a*b^4*Sec[c + d*x]*Tan[c + d*x])/
(256*d) + (5*a^5*Sec[c + d*x]^3*Tan[c + d*x])/(24*d) - (25*a^3*b^2*Sec[c +
d*x]^3*Tan[c + d*x])/(96*d) + (5*a*b^4*Sec[c + d*x]^3*Tan[c + d*x])/(128*d)
 + (a^5*Sec[c + d*x]^5*Tan[c + d*x])/(6*d) - (5*a^3*b^2*Sec[c + d*x]^5*Tan[
c + d*x])/(24*d) + (a*b^4*Sec[c + d*x]^5*Tan[c + d*x])/(32*d) + (5*a^3*b^2*
Sec[c + d*x]^7*Tan[c + d*x])/(4*d) - (3*a*b^4*Sec[c + d*x]^7*Tan[c + d*x])/
(16*d) + (a*b^4*Sec[c + d*x]^7*Tan[c + d*x]^3)/(2*d)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
```

```
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 2611

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(b*(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*(
m + n - 1)), x] - Dist[(b^2*(n - 1))/(m + n - 1), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 3090

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_.), x_Symbol] :> Int[ExpandTrig[cos[c + d*x]^m*(a
*cos[c + d*x] + b*sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d}, x] && Inte
gerQ[m] && IGtQ[n, 0]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \sec^{12}(c+dx)(a\cos(c+dx)+b\sin(c+dx))^5\,dx = \int \left(a^5\sec^7(c+dx)+5a^4b\sec^7(c+dx)\tan(c+dx)+10a^3\right.$$

$$= a^5\int \sec^7(c+dx)\,dx+\left(5a^4b\right)\int \sec^7(c+dx)\tan(c+dx)$$

$$= \frac{a^5\sec^5(c+dx)\tan(c+dx)}{6d}+\frac{5a^3b^2\sec^7(c+dx)\tan(c+}{4d}$$

$$= \frac{5a^4b\sec^7(c+dx)}{7d}+\frac{5a^5\sec^3(c+dx)\tan(c+dx)}{24d}+\frac{a^5\sec}{}$$

$$= \frac{5a^4b\sec^7(c+dx)}{7d}-\frac{10a^2b^3\sec^7(c+dx)}{7d}+\frac{b^5\sec^7(c+dx)}{7d}$$

$$= \frac{5a^5\tanh^{-1}(\sin(c+dx))}{16d}+\frac{5a^4b\sec^7(c+dx)}{7d}-\frac{10a^2b^3\sec}{7}$$

$$= \frac{5a^5\tanh^{-1}(\sin(c+dx))}{16d}-\frac{25a^3b^2\tanh^{-1}(\sin(c+dx))}{64d}+$$

$$= \frac{5a^5\tanh^{-1}(\sin(c+dx))}{16d}-\frac{25a^3b^2\tanh^{-1}(\sin(c+dx))}{64d}+$$

Mathematica [A] time = 1.79, size = 374, normalized size = 0.79

$$\frac{13860a\left(976a^4+2876a^2b^2+1207b^4\right)\tan(c+dx)\sec^9(c+dx)-1774080a\left(16a^4-20a^2b^2+3b^4\right)\left(\log\left(\cos\left(\frac{1}{2}(\right.\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^12*(a*Cos[c + d*x] + b*Sin[c + d*x])^5,x]
```

```
[Out] (-1774080*a*(16*a^4 - 20*a^2*b^2 + 3*b^4)*(Log[Cos[(c + d*x)/2] - Sin[(c +
d*x)/2]] - Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]) + Sec[c + d*x]^11*(243
30240*a^4*b + 1802240*a^2*b^3 + 3031040*b^5 + 3604480*(9*a^4*b - 4*a^2*b^3
- b^5)*Cos[2*(c + d*x)] + 1622016*(5*a^4*b - 10*a^2*b^3 + b^5)*Cos[4*(c + d
*x)] + 6623232*a^5*Sin[4*(c + d*x)] + 5913600*a^3*b^2*Sin[4*(c + d*x)] - 65
64096*a*b^4*Sin[4*(c + d*x)] + 2857008*a^5*Sin[6*(c + d*x)] - 3571260*a^3*b
^2*Sin[6*(c + d*x)] + 535689*a*b^4*Sin[6*(c + d*x)] + 591360*a^5*Sin[8*(c +
 d*x)] - 739200*a^3*b^2*Sin[8*(c + d*x)] + 110880*a*b^4*Sin[8*(c + d*x)] +
55440*a^5*Sin[10*(c + d*x)] - 69300*a^3*b^2*Sin[10*(c + d*x)] + 10395*a*b^4
```

```
*Sin[10*(c + d*x)]) + 13860*a*(976*a^4 + 2876*a^2*b^2 + 1207*b^4)*Sec[c + d
*x]^9*Tan[c + d*x])/(90832896*d)
```

fricas [A] time = 0.67, size = 287, normalized size = 0.61

$$\frac{3465\left(16\,a^5-20\,a^3b^2+3\,ab^4\right)\cos\left(dx+c\right)^{11}\log\left(\sin\left(dx+c\right)+1\right)-3465\left(16\,a^5-20\,a^3b^2+3\,ab^4\right)\cos\left(dx+c\right)^{11}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^12*(a*cos(d*x+c)+b*sin(d*x+c))^5,x, algorithm="fricas"
)
```

```
[Out] 1/354816*(3465*(16*a^5 - 20*a^3*b^2 + 3*a*b^4)*cos(d*x + c)^11*log(sin(d*x
+ c) + 1) - 3465*(16*a^5 - 20*a^3*b^2 + 3*a*b^4)*cos(d*x + c)^11*log(-sin(d
*x + c) + 1) + 32256*b^5 + 50688*(5*a^4*b - 10*a^2*b^3 + b^5)*cos(d*x + c)^
4 + 78848*(5*a^2*b^3 - b^5)*cos(d*x + c)^2 + 462*(15*(16*a^5 - 20*a^3*b^2 +
 3*a*b^4)*cos(d*x + c)^9 + 10*(16*a^5 - 20*a^3*b^2 + 3*a*b^4)*cos(d*x + c)^
7 + 384*a*b^4*cos(d*x + c) + 8*(16*a^5 - 20*a^3*b^2 + 3*a*b^4)*cos(d*x + c)
^5 + 48*(20*a^3*b^2 - 11*a*b^4)*cos(d*x + c)^3)*sin(d*x + c))/(d*cos(d*x +
c)^11)
```

giac [B] time = 12.98, size = 1096, normalized size = 2.32

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^12*(a*cos(d*x+c)+b*sin(d*x+c))^5,x, algorithm="giac")
```

```
[Out] 1/177408*(3465*(16*a^5 - 20*a^3*b^2 + 3*a*b^4)*log(abs(tan(1/2*d*x + 1/2*c)
 + 1)) - 3465*(16*a^5 - 20*a^3*b^2 + 3*a*b^4)*log(abs(tan(1/2*d*x + 1/2*c)
- 1)) + 2*(121968*a^5*tan(1/2*d*x + 1/2*c)^21 + 69300*a^3*b^2*tan(1/2*d*x +
 1/2*c)^21 - 10395*a*b^4*tan(1/2*d*x + 1/2*c)^21 - 887040*a^4*b*tan(1/2*d*x
 + 1/2*c)^20 - 591360*a^5*tan(1/2*d*x + 1/2*c)^19 + 1626240*a^3*b^2*tan(1/2
*d*x + 1/2*c)^19 + 110880*a*b^4*tan(1/2*d*x + 1/2*c)^19 + 3548160*a^4*b*tan
(1/2*d*x + 1/2*c)^18 - 3548160*a^2*b^3*tan(1/2*d*x + 1/2*c)^18 + 1459920*a^
5*tan(1/2*d*x + 1/2*c)^17 - 1159620*a^3*b^2*tan(1/2*d*x + 1/2*c)^17 + 23028
39*a*b^4*tan(1/2*d*x + 1/2*c)^17 - 9757440*a^4*b*tan(1/2*d*x + 1/2*c)^16 +
1182720*a^2*b^3*tan(1/2*d*x + 1/2*c)^16 - 946176*b^5*tan(1/2*d*x + 1/2*c)^1
6 - 2365440*a^5*tan(1/2*d*x + 1/2*c)^15 + 1182720*a^3*b^2*tan(1/2*d*x + 1/2
*c)^15 + 4790016*a*b^4*tan(1/2*d*x + 1/2*c)^15 + 21288960*a^4*b*tan(1/2*d*x
 + 1/2*c)^14 - 9461760*a^2*b^3*tan(1/2*d*x + 1/2*c)^14 - 2365440*b^5*tan(1/
2*d*x + 1/2*c)^14 + 2106720*a^5*tan(1/2*d*x + 1/2*c)^13 - 5738040*a^3*b^2*t
an(1/2*d*x + 1/2*c)^13 + 5828130*a*b^4*tan(1/2*d*x + 1/2*c)^13 - 30159360*a
^4*b*tan(1/2*d*x + 1/2*c)^12 + 18923520*a^2*b^3*tan(1/2*d*x + 1/2*c)^12 - 5
```

```
203968*b^5*tan(1/2*d*x + 1/2*c)^12 + 28385280*a^4*b*tan(1/2*d*x + 1/2*c)^10
 - 7096320*a^2*b^3*tan(1/2*d*x + 1/2*c)^10 - 4257792*b^5*tan(1/2*d*x + 1/2*
c)^10 - 2106720*a^5*tan(1/2*d*x + 1/2*c)^9 + 5738040*a^3*b^2*tan(1/2*d*x +
1/2*c)^9 - 5828130*a*b^4*tan(1/2*d*x + 1/2*c)^9 - 20528640*a^4*b*tan(1/2*d*
x + 1/2*c)^8 + 9123840*a^2*b^3*tan(1/2*d*x + 1/2*c)^8 - 3041280*b^5*tan(1/2
*d*x + 1/2*c)^8 + 2365440*a^5*tan(1/2*d*x + 1/2*c)^7 - 1182720*a^3*b^2*tan(
1/2*d*x + 1/2*c)^7 - 4790016*a*b^4*tan(1/2*d*x + 1/2*c)^7 + 11151360*a^4*b*
tan(1/2*d*x + 1/2*c)^6 - 8110080*a^2*b^3*tan(1/2*d*x + 1/2*c)^6 - 608256*b^
5*tan(1/2*d*x + 1/2*c)^6 - 1459920*a^5*tan(1/2*d*x + 1/2*c)^5 + 1159620*a^3
*b^2*tan(1/2*d*x + 1/2*c)^5 - 2302839*a*b^4*tan(1/2*d*x + 1/2*c)^5 - 342144
0*a^4*b*tan(1/2*d*x + 1/2*c)^4 - 450560*a^2*b^3*tan(1/2*d*x + 1/2*c)^4 - 11
2640*b^5*tan(1/2*d*x + 1/2*c)^4 + 591360*a^5*tan(1/2*d*x + 1/2*c)^3 - 16262
40*a^3*b^2*tan(1/2*d*x + 1/2*c)^3 - 110880*a*b^4*tan(1/2*d*x + 1/2*c)^3 + 5
06880*a^4*b*tan(1/2*d*x + 1/2*c)^2 - 619520*a^2*b^3*tan(1/2*d*x + 1/2*c)^2
+ 22528*b^5*tan(1/2*d*x + 1/2*c)^2 - 121968*a^5*tan(1/2*d*x + 1/2*c) - 6930
0*a^3*b^2*tan(1/2*d*x + 1/2*c) + 10395*a*b^4*tan(1/2*d*x + 1/2*c) - 126720*
a^4*b + 56320*a^2*b^3 - 2048*b^5)/(tan(1/2*d*x + 1/2*c)^2 - 1)^11)/d
```

maple [A] time = 0.28, size = 814, normalized size = 1.72

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^12*(a*cos(d*x+c)+b*sin(d*x+c))^5,x)
```

```
[Out] 8/693*b^5*cos(d*x+c)/d-15/256*a*b^4*sin(d*x+c)/d-5/256*a*b^4*sin(d*x+c)^3/d
+25/64*a^3*b^2*sin(d*x+c)/d+5/16*a^5*sec(d*x+c)*tan(d*x+c)/d+5/24*a^5*sec(d
*x+c)^3*tan(d*x+c)/d+1/6*a^5*sec(d*x+c)^5*tan(d*x+c)/d-20/63*a^2*b^3*cos(d*
x+c)/d+5/99/d*b^5*sin(d*x+c)^6/cos(d*x+c)^9+5/7/d*a^4*b/cos(d*x+c)^7+1/11/d
*b^5*sin(d*x+c)^6/cos(d*x+c)^11+5/4/d*a^3*b^2*sin(d*x+c)^3/cos(d*x+c)^8+10/
9/d*a^2*b^3*sin(d*x+c)^4/cos(d*x+c)^9+1/2/d*a*b^4*sin(d*x+c)^5/cos(d*x+c)^1
0+1/231/d*b^5*sin(d*x+c)^6/cos(d*x+c)^5-25/64/d*a^3*b^2*ln(sec(d*x+c)+tan(d
*x+c))-1/693/d*b^5*sin(d*x+c)^6/cos(d*x+c)^3+25/64/d*a^3*b^2*sin(d*x+c)^3/c
os(d*x+c)^2+10/63/d*a^2*b^3*sin(d*x+c)^4/cos(d*x+c)^3+5/128/d*a*b^4*sin(d*x
+c)^5/cos(d*x+c)^4-10/63/d*cos(d*x+c)*sin(d*x+c)^2*a^2*b^3+25/32/d*a^3*b^2*
sin(d*x+c)^3/cos(d*x+c)^4+10/21/d*a^2*b^3*sin(d*x+c)^4/cos(d*x+c)^5+5/32/d*
a*b^4*sin(d*x+c)^5/cos(d*x+c)^6+5/16/d*a^5*ln(sec(d*x+c)+tan(d*x+c))+15/256
/d*a*b^4*ln(sec(d*x+c)+tan(d*x+c))+1/231/d*b^5*sin(d*x+c)^6/cos(d*x+c)+1/23
1/d*b^5*cos(d*x+c)*sin(d*x+c)^4+4/693/d*cos(d*x+c)*sin(d*x+c)^2*b^5-10/63/d
*a^2*b^3*sin(d*x+c)^4/cos(d*x+c)-5/256/d*a*b^4*sin(d*x+c)^5/cos(d*x+c)^2+5/
231/d*b^5*sin(d*x+c)^6/cos(d*x+c)^7+25/24/d*a^3*b^2*sin(d*x+c)^3/cos(d*x+c)
^6+50/63/d*a^2*b^3*sin(d*x+c)^4/cos(d*x+c)^7+5/16/d*a*b^4*sin(d*x+c)^5/cos(
d*x+c)^8
```

maxima [A] time = 0.54, size = 420, normalized size = 0.89

$$-\frac{693\,ab^4\left(\frac{2\left(15\,\sin(dx+c)^9-70\,\sin(dx+c)^7+128\,\sin(dx+c)^5+70\,\sin(dx+c)^3-15\,\sin(dx+c)\right)}{\sin(dx+c)^{10}-5\,\sin(dx+c)^8+10\,\sin(dx+c)^6-10\,\sin(dx+c)^4+5\,\sin(dx+c)^2-1}-15\,\log\left(\sin\left(dx+c\right)+1\right)+15\,\log\left(\sin\left(\right.\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^12*(a*cos(d*x+c)+b*sin(d*x+c))^5,x, algorithm="maxima"
)
```

```
[Out] -1/354816*(693*a*b^4*(2*(15*sin(d*x + c)^9 - 70*sin(d*x + c)^7 + 128*sin(d*
x + c)^5 + 70*sin(d*x + c)^3 - 15*sin(d*x + c))/(sin(d*x + c)^10 - 5*sin(d*
x + c)^8 + 10*sin(d*x + c)^6 - 10*sin(d*x + c)^4 + 5*sin(d*x + c)^2 - 1) -
15*log(sin(d*x + c) + 1) + 15*log(sin(d*x + c) - 1)) - 4620*a^3*b^2*(2*(15*
sin(d*x + c)^7 - 55*sin(d*x + c)^5 + 73*sin(d*x + c)^3 + 15*sin(d*x + c))/(
sin(d*x + c)^8 - 4*sin(d*x + c)^6 + 6*sin(d*x + c)^4 - 4*sin(d*x + c)^2 + 1
) - 15*log(sin(d*x + c) + 1) + 15*log(sin(d*x + c) - 1)) + 3696*a^5*(2*(15*
sin(d*x + c)^5 - 40*sin(d*x + c)^3 + 33*sin(d*x + c))/(sin(d*x + c)^6 - 3*s
in(d*x + c)^4 + 3*sin(d*x + c)^2 - 1) - 15*log(sin(d*x + c) + 1) + 15*log(s
in(d*x + c) - 1)) - 253440*a^4*b/cos(d*x + c)^7 + 56320*(9*cos(d*x + c)^2 -
 7)*a^2*b^3/cos(d*x + c)^9 - 512*(99*cos(d*x + c)^4 - 154*cos(d*x + c)^2 +
63)*b^5/cos(d*x + c)^11)/d
```

mupad [B] time = 5.92, size = 831, normalized size = 1.76

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*cos(c + d*x) + b*sin(c + d*x))^5/cos(c + d*x)^12,x)
```

```
[Out] (5*a*atanh(tan(c/2 + (d*x)/2))*(16*a^4 + 3*b^4 - 20*a^2*b^2))/(128*d) - (ta
n(c/2 + (d*x)/2)*((11*a^5)/8 - (15*a*b^4)/128 + (25*a^3*b^2)/32) - tan(c/2
+ (d*x)/2)^18*(40*a^4*b - 40*a^2*b^3) + tan(c/2 + (d*x)/2)^3*((5*a*b^4)/4 -
 (20*a^5)/3 + (55*a^3*b^2)/3) - tan(c/2 + (d*x)/2)^19*((5*a*b^4)/4 - (20*a^
5)/3 + (55*a^3*b^2)/3) + tan(c/2 + (d*x)/2)^7*(54*a*b^4 - (80*a^5)/3 + (40*
a^3*b^2)/3) - tan(c/2 + (d*x)/2)^15*(54*a*b^4 - (80*a^5)/3 + (40*a^3*b^2)/3
) - tan(c/2 + (d*x)/2)^21*((11*a^5)/8 - (15*a*b^4)/128 + (25*a^3*b^2)/32) +
 tan(c/2 + (d*x)/2)^5*((3323*a*b^4)/128 + (395*a^5)/24 - (1255*a^3*b^2)/96)
 - tan(c/2 + (d*x)/2)^17*((3323*a*b^4)/128 + (395*a^5)/24 - (1255*a^3*b^2)/
96) + tan(c/2 + (d*x)/2)^9*((4205*a*b^4)/64 + (95*a^5)/4 - (1035*a^3*b^2)/1
6) - tan(c/2 + (d*x)/2)^13*((4205*a*b^4)/64 + (95*a^5)/4 - (1035*a^3*b^2)/1
6) + tan(c/2 + (d*x)/2)^16*(110*a^4*b + (32*b^5)/3 - (40*a^2*b^3)/3) + tan(
c/2 + (d*x)/2)^10*(48*b^5 - 320*a^4*b + 80*a^2*b^3) - tan(c/2 + (d*x)/2)^2*
((40*a^4*b)/7 + (16*b^5)/63 - (440*a^2*b^3)/63) + tan(c/2 + (d*x)/2)^14*((8
```

```
0*b^5)/3 - 240*a^4*b + (320*a^2*b^3)/3) + tan(c/2 + (d*x)/2)^4*((270*a^4*b)
/7 + (80*b^5)/63 + (320*a^2*b^3)/63) + tan(c/2 + (d*x)/2)^12*(340*a^4*b + (
176*b^5)/3 - (640*a^2*b^3)/3) + tan(c/2 + (d*x)/2)^6*((48*b^5)/7 - (880*a^4
*b)/7 + (640*a^2*b^3)/7) + tan(c/2 + (d*x)/2)^8*((1620*a^4*b)/7 + (240*b^5)
/7 - (720*a^2*b^3)/7) + (10*a^4*b)/7 + (16*b^5)/693 - (40*a^2*b^3)/63 + 10*
a^4*b*tan(c/2 + (d*x)/2)^20)/(d*(11*tan(c/2 + (d*x)/2)^2 - 55*tan(c/2 + (d*
x)/2)^4 + 165*tan(c/2 + (d*x)/2)^6 - 330*tan(c/2 + (d*x)/2)^8 + 462*tan(c/2
 + (d*x)/2)^10 - 462*tan(c/2 + (d*x)/2)^12 + 330*tan(c/2 + (d*x)/2)^14 - 16
5*tan(c/2 + (d*x)/2)^16 + 55*tan(c/2 + (d*x)/2)^18 - 11*tan(c/2 + (d*x)/2)^
20 + tan(c/2 + (d*x)/2)^22 - 1))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**12*(a*cos(d*x+c)+b*sin(d*x+c))**5,x)
```

```
[Out] Timed out
```

3.110 $\int \frac{\cos^5(c+dx)}{a\cos(c+dx)+b\sin(c+dx)}\,dx$

Optimal. Leaf size=227

$$\frac{b\cos^4(c+dx)}{4d\left(a^2+b^2\right)}+\frac{a\sin(c+dx)\cos^3(c+dx)}{4d\left(a^2+b^2\right)}+\frac{ab^2\sin(c+dx)\cos(c+dx)}{2d\left(a^2+b^2\right)^2}+\frac{3a\sin(c+dx)\cos(c+dx)}{8d\left(a^2+b^2\right)}+\frac{ab^2x}{2\left(a^2+b^2\right)^2}$$

```
[Out] a*b^4*x/(a^2+b^2)^3+1/2*a*b^2*x/(a^2+b^2)^2+3/8*a*x/(a^2+b^2)+1/2*b^3*cos(d
*x+c)^2/(a^2+b^2)^2/d+1/4*b*cos(d*x+c)^4/(a^2+b^2)/d+b^5*ln(a*cos(d*x+c)+b*
sin(d*x+c))/(a^2+b^2)^3/d+1/2*a*b^2*cos(d*x+c)*sin(d*x+c)/(a^2+b^2)^2/d+3/8
*a*cos(d*x+c)*sin(d*x+c)/(a^2+b^2)/d+1/4*a*cos(d*x+c)^3*sin(d*x+c)/(a^2+b^2
)/d
```

Rubi [A] time = 0.21, antiderivative size = 227, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 5, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.179, Rules used = {3100, 2635, 8, 3098, 3133}

$$\frac{b^3\cos^2(c+dx)}{2d\left(a^2+b^2\right)^2}+\frac{b\cos^4(c+dx)}{4d\left(a^2+b^2\right)}+\frac{a\sin(c+dx)\cos^3(c+dx)}{4d\left(a^2+b^2\right)}+\frac{ab^2\sin(c+dx)\cos(c+dx)}{2d\left(a^2+b^2\right)^2}+\frac{3a\sin(c+dx)\cos(c+dx)}{8d\left(a^2+b^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^5/(a*Cos[c + d*x] + b*Sin[c + d*x]),x]
```

```
[Out] (a*b^4*x)/(a^2 + b^2)^3 + (a*b^2*x)/(2*(a^2 + b^2)^2) + (3*a*x)/(8*(a^2 + b
^2)) + (b^3*Cos[c + d*x]^2)/(2*(a^2 + b^2)^2*d) + (b*Cos[c + d*x]^4)/(4*(a^
2 + b^2)*d) + (b^5*Log[a*Cos[c + d*x] + b*Sin[c + d*x]])/((a^2 + b^2)^3*d)
+ (a*b^2*Cos[c + d*x]*Sin[c + d*x])/(2*(a^2 + b^2)^2*d) + (3*a*Cos[c + d*x]
*Sin[c + d*x])/(8*(a^2 + b^2)*d) + (a*Cos[c + d*x]^3*Sin[c + d*x])/(4*(a^2
+ b^2)*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 3098

```
Int[cos[(c_.) + (d_.)*(x_)]/(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.
) + (d_.)*(x_)]), x_Symbol] :> Simp[(a*x)/(a^2 + b^2), x] + Dist[b/(a^2 + b
^2), Int[(b*Cos[c + d*x] - a*Sin[c + d*x])/(a*Cos[c + d*x] + b*Sin[c + d*x]
), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

Rule 3100

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_)/(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin
[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[(b*Cos[c + d*x]^(m - 1))/(d*(a^2 +
 b^2)*(m - 1)), x] + (Dist[a/(a^2 + b^2), Int[Cos[c + d*x]^(m - 1), x], x]
+ Dist[b^2/(a^2 + b^2), Int[Cos[c + d*x]^(m - 2)/(a*Cos[c + d*x] + b*Sin[c
+ d*x]), x], x]) /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0] && GtQ[m, 1
]
```

Rule 3133

```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])
/((a_.) + cos[(d_.) + (e_.)*(x_)]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)]), x
_Symbol] :> Simp[((b*B + c*C)*x)/(b^2 + c^2), x] + Simp[((c*B - b*C)*Log[a
+ b*Cos[d + e*x] + c*Sin[d + e*x]])/(e*(b^2 + c^2)), x] /; FreeQ[{a, b, c,
d, e, A, B, C}, x] && NeQ[b^2 + c^2, 0] && EqQ[A*(b^2 + c^2) - a*(b*B + c*C
), 0]
```

Rubi steps

$$\int \frac{\cos^5(c+dx)}{a\cos(c+dx)+b\sin(c+dx)}\,dx = \frac{b\cos^4(c+dx)}{4\left(a^2+b^2\right)d}+\frac{a\int \cos^4(c+dx)\,dx}{a^2+b^2}+\frac{b^2\int \frac{\cos^3(c+dx)}{a\cos(c+dx)+b\sin(c+dx)}\,dx}{a^2+b^2}$$

$$= \frac{b^3\cos^2(c+dx)}{2\left(a^2+b^2\right)^2 d}+\frac{b\cos^4(c+dx)}{4\left(a^2+b^2\right)d}+\frac{a\cos^3(c+dx)\sin(c+dx)}{4\left(a^2+b^2\right)d}+\frac{\left(ab^2\right)\int \cos}{(a}$$

$$= \frac{ab^4x}{\left(a^2+b^2\right)^3}+\frac{b^3\cos^2(c+dx)}{2\left(a^2+b^2\right)^2 d}+\frac{b\cos^4(c+dx)}{4\left(a^2+b^2\right)d}+\frac{ab^2\cos(c+dx)\sin(c+}{2\left(a^2+b^2\right)^2 d}$$

$$= \frac{ab^4x}{\left(a^2+b^2\right)^3}+\frac{ab^2x}{2\left(a^2+b^2\right)^2}+\frac{3ax}{8\left(a^2+b^2\right)}+\frac{b^3\cos^2(c+dx)}{2\left(a^2+b^2\right)^2 d}+\frac{b\cos^4(c+d}{4\left(a^2+b^2\right)}$$

Mathematica [A] time = 0.42, size = 218, normalized size = 0.96

$$\frac{8a^5\sin(2(c+dx))+a^5\sin(4(c+dx))+12a^5c+12a^5dx+24a^3b^2\sin(2(c+dx))+2a^3b^2\sin(4(c+dx))+40a^3b^2 \cdots}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^5/(a*Cos[c + d*x] + b*Sin[c + d*x]),x]
```

```
[Out] (12*a^5*c + 40*a^3*b^2*c + 60*a*b^4*c + 12*a^5*d*x + 40*a^3*b^2*d*x + 60*a*
b^4*d*x + 4*b*(a^4 + 4*a^2*b^2 + 3*b^4)*Cos[2*(c + d*x)] + b*(a^2 + b^2)^2*
Cos[4*(c + d*x)] + 32*b^5*Log[a*Cos[c + d*x] + b*Sin[c + d*x]] + 8*a^5*Sin[
2*(c + d*x)] + 24*a^3*b^2*Sin[2*(c + d*x)] + 16*a*b^4*Sin[2*(c + d*x)] + a^
5*Sin[4*(c + d*x)] + 2*a^3*b^2*Sin[4*(c + d*x)] + a*b^4*Sin[4*(c + d*x)])/(
32*(a^2 + b^2)^3*d)
```

fricas [A] time = 0.60, size = 208, normalized size = 0.92

$$\frac{4\,b^5\log\left(2\,ab\cos\left(dx+c\right)\sin\left(dx+c\right)+\left(a^2-b^2\right)\cos\left(dx+c\right)^2+b^2\right)+2\left(a^4b+2\,a^2b^3+b^5\right)\cos\left(dx+c\right)^4+\left(3\,a^5 \cdots\right.}{8 \cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5/(a*cos(d*x+c)+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/8*(4*b^5*log(2*a*b*cos(d*x + c)*sin(d*x + c) + (a^2 - b^2)*cos(d*x + c)^2
 + b^2) + 2*(a^4*b + 2*a^2*b^3 + b^5)*cos(d*x + c)^4 + (3*a^5 + 10*a^3*b^2
+ 15*a*b^4)*d*x + 4*(a^2*b^3 + b^5)*cos(d*x + c)^2 + (2*(a^5 + 2*a^3*b^2 +
a*b^4)*cos(d*x + c)^3 + (3*a^5 + 10*a^3*b^2 + 7*a*b^4)*cos(d*x + c))*sin(d*
x + c))/((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*d)
```

giac [A] time = 2.93, size = 322, normalized size = 1.42

$$\frac{\frac{8\,b^6\log\left(\left|b\tan\left(dx+c\right)+a\right|\right)}{a^6b+3\,a^4b^3+3\,a^2b^5+b^7}-\frac{4\,b^5\log\left(\tan\left(dx+c\right)^2+1\right)}{a^6+3\,a^4b^2+3\,a^2b^4+b^6}+\frac{\left(3\,a^5+10\,a^3b^2+15\,ab^4\right)\left(dx+c\right)}{a^6+3\,a^4b^2+3\,a^2b^4+b^6}+\frac{6\,b^5\tan\left(dx+c\right)^4+3\,a^5\tan\left(dx+c\right)^3+10\,a^3b^2\tan\left(dx+c\right)^3+7\,ab^4 \cdots}{\cdots}}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5/(a*cos(d*x+c)+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/8*(8*b^6*log(abs(b*tan(d*x + c) + a))/(a^6*b + 3*a^4*b^3 + 3*a^2*b^5 + b^
7) - 4*b^5*log(tan(d*x + c)^2 + 1)/(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6) + (3
*a^5 + 10*a^3*b^2 + 15*a*b^4)*(d*x + c)/(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)
 + (6*b^5*tan(d*x + c)^4 + 3*a^5*tan(d*x + c)^3 + 10*a^3*b^2*tan(d*x + c)^3
```

```
 + 7*a*b^4*tan(d*x + c)^3 + 4*a^2*b^3*tan(d*x + c)^2 + 16*b^5*tan(d*x + c)^
2 + 5*a^5*tan(d*x + c) + 14*a^3*b^2*tan(d*x + c) + 9*a*b^4*tan(d*x + c) + 2
*a^4*b + 8*a^2*b^3 + 12*b^5)/((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*(tan(d*x
+ c)^2 + 1)^2))/d
```

maple [B] time = 0.21, size = 524, normalized size = 2.31

$$\frac{b^5 \ln(a + b\tan(dx+c))}{d\left(a^2+b^2\right)^3} + \frac{3\left(\tan^3(dx+c)\right)a^5}{8d\left(a^2+b^2\right)^3\left(\tan^2(dx+c)+1\right)^2} + \frac{5\left(\tan^3(dx+c)\right)a^3b^2}{4d\left(a^2+b^2\right)^3\left(\tan^2(dx+c)+1\right)^2} + \frac{7\left(\tan^3\right.}{8d\left(a^2+b^2\right)^3\left(\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5/(a*cos(d*x+c)+b*sin(d*x+c)),x)
```

```
[Out] 1/d*b^5/(a^2+b^2)^3*ln(a+b*tan(d*x+c))+3/8/d/(a^2+b^2)^3/(tan(d*x+c)^2+1)^2
*tan(d*x+c)^3*a^5+5/4/d/(a^2+b^2)^3/(tan(d*x+c)^2+1)^2*tan(d*x+c)^3*a^3*b^2
+7/8/d/(a^2+b^2)^3/(tan(d*x+c)^2+1)^2*tan(d*x+c)^3*a*b^4+1/2/d/(a^2+b^2)^3/
(tan(d*x+c)^2+1)^2*tan(d*x+c)^2*a^2*b^3+1/2/d/(a^2+b^2)^3/(tan(d*x+c)^2+1)^
2*tan(d*x+c)^2*b^5+7/4/d/(a^2+b^2)^3/(tan(d*x+c)^2+1)^2*tan(d*x+c)*a^3*b^2+
9/8/d/(a^2+b^2)^3/(tan(d*x+c)^2+1)^2*tan(d*x+c)*a*b^4+5/8/d/(a^2+b^2)^3/(ta
n(d*x+c)^2+1)^2*tan(d*x+c)*a^5+1/4/d/(a^2+b^2)^3/(tan(d*x+c)^2+1)^2*a^4*b+1
/d/(a^2+b^2)^3/(tan(d*x+c)^2+1)^2*a^2*b^3+3/4/d/(a^2+b^2)^3/(tan(d*x+c)^2+1
)^2*b^5-1/2/d/(a^2+b^2)^3*b^5*ln(tan(d*x+c)^2+1)+15/8/d/(a^2+b^2)^3*arctan(
tan(d*x+c))*a*b^4+3/8/d/(a^2+b^2)^3*arctan(tan(d*x+c))*a^5+5/4/d/(a^2+b^2)^
3*arctan(tan(d*x+c))*a^3*b^2
```

maxima [B] time = 1.67, size = 564, normalized size = 2.48

$$\frac{\frac{4\,b^5 \log\left(-a-\frac{2\,b\sin(dx+c)}{\cos(dx+c)+1}+\frac{a\sin(dx+c)^2}{(\cos(dx+c)+1)^2}\right)}{a^6+3\,a^4b^2+3\,a^2b^4+b^6} - \frac{4\,b^5\log\left(\frac{\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+1\right)}{a^6+3\,a^4b^2+3\,a^2b^4+b^6} + \frac{\left(3\,a^5+10\,a^3b^2+15\,ab^4\right)\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^6+3\,a^4b^2+3\,a^2b^4+b^6} - \frac{\frac{16\,b^3\sin(dx+c)^4}{(\cos(dx+c)+1)^4} - \frac{\left(5\,a^3+9\,ab^2\right.}{\cos(d}}{a^4+2\,a^2b^2+b}}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5/(a*cos(d*x+c)+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/4*(4*b^5*log(-a - 2*b*sin(d*x + c)/(cos(d*x + c) + 1) + a*sin(d*x + c)^2/
(cos(d*x + c) + 1)^2)/(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6) - 4*b^5*log(sin(d
*x + c)^2/(cos(d*x + c) + 1)^2 + 1)/(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6) + (
3*a^5 + 10*a^3*b^2 + 15*a*b^4)*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/(a^6
 + 3*a^4*b^2 + 3*a^2*b^4 + b^6) - (16*b^3*sin(d*x + c)^4/(cos(d*x + c) + 1)
^4 - (5*a^3 + 9*a*b^2)*sin(d*x + c)/(cos(d*x + c) + 1) + 8*(a^2*b + 2*b^3)*
sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + (3*a^3 - a*b^2)*sin(d*x + c)^3/(cos(d
*x + c) + 1)^3 - (3*a^3 - a*b^2)*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + 8*(a
```

```
^2*b + 2*b^3)*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 + (5*a^3 + 9*a*b^2)*sin(d
*x + c)^7/(cos(d*x + c) + 1)^7)/(a^4 + 2*a^2*b^2 + b^4 + 4*(a^4 + 2*a^2*b^2
 + b^4)*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 6*(a^4 + 2*a^2*b^2 + b^4)*sin
(d*x + c)^4/(cos(d*x + c) + 1)^4 + 4*(a^4 + 2*a^2*b^2 + b^4)*sin(d*x + c)^6
/(cos(d*x + c) + 1)^6 + (a^4 + 2*a^2*b^2 + b^4)*sin(d*x + c)^8/(cos(d*x + c
) + 1)^8))/d
```

mupad [B] time = 11.16, size = 6099, normalized size = 26.87

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^5/(a*cos(c + d*x) + b*sin(c + d*x)),x)
```

```
[Out] (b^5*log(a + 2*b*tan(c/2 + (d*x)/2) - a*tan(c/2 + (d*x)/2)^2))/(d*(a^6 + b^
6 + 3*a^2*b^4 + 3*a^4*b^2)) - (64*b^5*log(1/(cos(c + d*x) + 1)))/(d*(64*a^6
 + 64*b^6 + 192*a^2*b^4 + 192*a^4*b^2)) - ((4*b^3*tan(c/2 + (d*x)/2)^4)/(a^
4 + b^4 + 2*a^2*b^2) - (tan(c/2 + (d*x)/2)*(9*a*b^2 + 5*a^3))/(4*(a^4 + b^4
 + 2*a^2*b^2)) - (tan(c/2 + (d*x)/2)^3*(a*b^2 - 3*a^3))/(4*(a^4 + b^4 + 2*a
^2*b^2)) + (tan(c/2 + (d*x)/2)^5*(a*b^2 - 3*a^3))/(4*(a^4 + b^4 + 2*a^2*b^2
)) + (tan(c/2 + (d*x)/2)^7*(9*a*b^2 + 5*a^3))/(4*(a^4 + b^4 + 2*a^2*b^2)) +
 (2*b*tan(c/2 + (d*x)/2)^2*(a^2 + 2*b^2))/(a^4 + b^4 + 2*a^2*b^2) + (2*b*ta
n(c/2 + (d*x)/2)^6*(a^2 + 2*b^2))/(a^4 + b^4 + 2*a^2*b^2))/(d*(4*tan(c/2 +
(d*x)/2)^2 + 6*tan(c/2 + (d*x)/2)^4 + 4*tan(c/2 + (d*x)/2)^6 + tan(c/2 + (d
*x)/2)^8 + 1)) - (a*atan((tan(c/2 + (d*x)/2)*((((64*b^5*((a*((64*a*b^15 + 4
8*a^15*b + 624*a^3*b^13 + 2016*a^5*b^11 + 3152*a^7*b^9 + 2688*a^9*b^7 + 129
6*a^11*b^5 + 352*a^13*b^3)/(2*(a^12 + b^12 + 6*a^2*b^10 + 15*a^4*b^8 + 20*a
^6*b^6 + 15*a^8*b^4 + 6*a^10*b^2)) - (32*b^5*(192*a*b^16 + 1344*a^3*b^14 +
4032*a^5*b^12 + 6720*a^7*b^10 + 6720*a^9*b^8 + 4032*a^11*b^6 + 1344*a^13*b^
4 + 192*a^15*b^2))/((64*a^6 + 64*b^6 + 192*a^2*b^4 + 192*a^4*b^2)*(a^12 + b
^12 + 6*a^2*b^10 + 15*a^4*b^8 + 20*a^6*b^6 + 15*a^8*b^4 + 6*a^10*b^2)))*(3*
a^4 + 15*b^4 + 10*a^2*b^2))/(8*(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2)) - (4*a*
b^5*(3*a^4 + 15*b^4 + 10*a^2*b^2)*(192*a*b^16 + 1344*a^3*b^14 + 4032*a^5*b^
12 + 6720*a^7*b^10 + 6720*a^9*b^8 + 4032*a^11*b^6 + 1344*a^13*b^4 + 192*a^1
5*b^2))/((64*a^6 + 64*b^6 + 192*a^2*b^4 + 192*a^4*b^2)*(a^6 + b^6 + 3*a^2*b
^4 + 3*a^4*b^2)*(a^12 + b^12 + 6*a^2*b^10 + 15*a^4*b^8 + 20*a^6*b^6 + 15*a^
8*b^4 + 6*a^10*b^2))))/(64*a^6 + 64*b^6 + 192*a^2*b^4 + 192*a^4*b^2) - (a*(
(9*a^15 - 192*a*b^14 - 222*a^3*b^12 + 475*a^5*b^10 + 1089*a^7*b^8 + 894*a^9
*b^6 + 388*a^11*b^4 + 87*a^13*b^2)/(2*(a^12 + b^12 + 6*a^2*b^10 + 15*a^4*b^
8 + 20*a^6*b^6 + 15*a^8*b^4 + 6*a^10*b^2)) - (64*b^5*((64*a*b^15 + 48*a^15*
b + 624*a^3*b^13 + 2016*a^5*b^11 + 3152*a^7*b^9 + 2688*a^9*b^7 + 1296*a^11*
b^5 + 352*a^13*b^3)/(2*(a^12 + b^12 + 6*a^2*b^10 + 15*a^4*b^8 + 20*a^6*b^6
+ 15*a^8*b^4 + 6*a^10*b^2)) - (32*b^5*(192*a*b^16 + 1344*a^3*b^14 + 4032*a^
5*b^12 + 6720*a^7*b^10 + 6720*a^9*b^8 + 4032*a^11*b^6 + 1344*a^13*b^4 + 192
*a^15*b^2))/((64*a^6 + 64*b^6 + 192*a^2*b^4 + 192*a^4*b^2)*(a^12 + b^12 + 6
```

*a^2*b^10 + 15*a^4*b^8 + 20*a^6*b^6 + 15*a^8*b^4 + 6*a^10*b^2))))/(64*a^6 + 64*b^6 + 192*a^2*b^4 + 192*a^4*b^2))*(3*a^4 + 15*b^4 + 10*a^2*b^2))/(8*(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2)) + (a^3*(3*a^4 + 15*b^4 + 10*a^2*b^2)^3*(192*a*b^16 + 1344*a^3*b^14 + 4032*a^5*b^12 + 6720*a^7*b^10 + 6720*a^9*b^8 + 4032*a^11*b^6 + 1344*a^13*b^4 + 192*a^15*b^2))/(1024*(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2)^3*(a^12 + b^12 + 6*a^2*b^10 + 15*a^4*b^8 + 20*a^6*b^6 + 15*a^8*b^4 + 6*a^10*b^2)))*(9*a^12 + 256*b^12 - 1441*a^2*b^10 - 715*a^4*b^8 - 82*a^6*b^6 + 130*a^8*b^4 + 51*a^10*b^2))/(9*a^12 + 256*b^12 + 481*a^2*b^10 + 525*a^4*b^8 + 490*a^6*b^6 + 250*a^8*b^4 + 69*a^10*b^2)^2 - (2*a*b*((64*a*b^13 + 210*a^3*b^11 + 181*a^5*b^9 + 60*a^7*b^7 + 9*a^9*b^5)/(2*(a^12 + b^12 + 6*a^2*b^10 + 15*a^4*b^8 + 20*a^6*b^6 + 15*a^8*b^4 + 6*a^10*b^2)) + (64*b^5*((9*a^15 - 192*a*b^14 - 222*a^3*b^12 + 475*a^5*b^10 + 1089*a^7*b^8 + 894*a^9*b^6 + 388*a^11*b^4 + 87*a^13*b^2)/(2*(a^12 + b^12 + 6*a^2*b^10 + 15*a^4*b^8 + 20*a^6*b^6 + 15*a^8*b^4 + 6*a^10*b^2)) - (64*b^5*((64*a*b^15 + 48*a^15*b + 624*a^3*b^13 + 2016*a^5*b^11 + 3152*a^7*b^9 + 2688*a^9*b^7 + 1296*a^11*b^5 + 352*a^13*b^3)/(2*(a^12 + b^12 + 6*a^2*b^10 + 15*a^4*b^8 + 20*a^6*b^6 + 15*a^8*b^4 + 6*a^10*b^2)) - (32*b^5*(192*a*b^16 + 1344*a^3*b^14 + 4032*a^5*b^12 + 6720*a^7*b^10 + 6720*a^9*b^8 + 4032*a^11*b^6 + 1344*a^13*b^4 + 192*a^15*b^2))/((64*a^6 + 64*b^6 + 192*a^2*b^4 + 192*a^4*b^2)*(a^12 + b^12 + 6*a^2*b^10 + 15*a^4*b^8 + 20*a^6*b^6 + 15*a^8*b^4 + 6*a^10*b^2))))/(64*a^6 + 64*b^6 + 192*a^2*b^4 + 192*a^4*b^2)))/(64*a^6 + 64*b^6 + 192*a^2*b^4 + 192*a^4*b^2) + (a*((a*((64*a*b^15 + 48*a^15*b + 624*a^3*b^13 + 2016*a^5*b^11 + 3152*a^7*b^9 + 2688*a^9*b^7 + 1296*a^11*b^5 + 352*a^13*b^3)/(2*(a^12 + b^12 + 6*a^2*b^10 + 15*a^4*b^8 + 20*a^6*b^6 + 15*a^8*b^4 + 6*a^10*b^2)) - (32*b^5*(192*a*b^16 + 1344*a^3*b^14 + 4032*a^5*b^12 + 6720*a^7*b^10 + 6720*a^9*b^8 + 4032*a^11*b^6 + 1344*a^13*b^4 + 192*a^15*b^2))/((64*a^6 + 64*b^6 + 192*a^2*b^4 + 192*a^4*b^2)*(a^12 + b^12 + 6*a^2*b^10 + 15*a^4*b^8 + 20*a^6*b^6 + 15*a^8*b^4 + 6*a^10*b^2)))*(3*a^4 + 15*b^4 + 10*a^2*b^2))/(8*(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2)) - (4*a*b^5*(3*a^4 + 15*b^4 + 10*a^2*b^2)*(192*a*b^16 + 1344*a^3*b^14 + 4032*a^5*b^12 + 6720*a^7*b^10 + 6720*a^9*b^8 + 4032*a^11*b^6 + 1344*a^13*b^4 + 192*a^15*b^2))/((64*a^6 + 64*b^6 + 192*a^2*b^4 + 192*a^4*b^2)*(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2)*(a^12 + b^12 + 6*a^2*b^10 + 15*a^4*b^8 + 20*a^6*b^6 + 15*a^8*b^4 + 6*a^10*b^2)))*(3*a^4 + 15*b^4 + 10*a^2*b^2))/(8*(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2)) - (a^2*b^5*(3*a^4 + 15*b^4 + 10*a^2*b^2)^2*(192*a*b^16 + 1344*a^3*b^14 + 4032*a^5*b^12 + 6720*a^7*b^10 + 6720*a^9*b^8 + 4032*a^11*b^6 + 1344*a^13*b^4 + 192*a^15*b^2))/(2*(64*a^6 + 64*b^6 + 192*a^2*b^4 + 192*a^4*b^2)*(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2)^2*(a^12 + b^12 + 6*a^2*b^10 + 15*a^4*b^8 + 20*a^6*b^6 + 15*a^8*b^4 + 6*a^10*b^2)))*(9*a^10 - 496*b^10 + 305*a^2*b^8 + 412*a^4*b^6 + 238*a^6*b^4 + 60*a^8*b^2))/(9*a^12 + 256*b^12 + 481*a^2*b^10 + 525*a^4*b^8 + 490*a^6*b^6 + 250*a^8*b^4 + 69*a^10*b^2)^2)*(16*a^16 + 16*b^16 + 128*a^2*b^14 + 448*a^4*b^12 + 896*a^6*b^10 + 1120*a^8*b^8 + 896*a^10*b^6 + 448*a^12*b^4 + 128*a^14*b^2))/(3*a^6 + 15*a^2*b^4 + 10*a^4*b^2) + (((64*b^5*((a*((24*a^16 - 184*a^2*b^14 - 696*a^4*b^12 - 888*a^6*b^10 - 248*a^8*b^8 + 408*a^10*b^6 + 408*a^12*b^4 + 152*a^14*b^2)/(2*(a^12 + b^12 + 6*a^2*b^10 + 15*a^4*b^8 + 20*a^6

*b^6 + 15*a^8*b^4 + 6*a^10*b^2)) - (32*b^5*(192*a^16*b + 192*a^2*b^15 + 134
4*a^4*b^13 + 4032*a^6*b^11 + 6720*a^8*b^9 + 6720*a^10*b^7 + 4032*a^12*b^5 +
 1344*a^14*b^3))/((64*a^6 + 64*b^6 + 192*a^2*b^4 + 192*a^4*b^2)*(a^12 + b^1
2 + 6*a^2*b^10 + 15*a^4*b^8 + 20*a^6*b^6 + 15*a^8*b^4 + 6*a^10*b^2)))*(3*a^
4 + 15*b^4 + 10*a^2*b^2))/(8*(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2)) - (4*a*b^
5*(3*a^4 + 15*b^4 + 10*a^2*b^2)*(192*a^16*b + 192*a^2*b^15 + 1344*a^4*b^13
+ 4032*a^6*b^11 + 6720*a^8*b^9 + 6720*a^10*b^7 + 4032*a^12*b^5 + 1344*a^14*
b^3))/((64*a^6 + 64*b^6 + 192*a^2*b^4 + 192*a^4*b^2)*(a^6 + b^6 + 3*a^2*b^4
 + 3*a^4*b^2)*(a^12 + b^12 + 6*a^2*b^10 + 15*a^4*b^8 + 20*a^6*b^6 + 15*a^8*
b^4 + 6*a^10*b^2))))/(64*a^6 + 64*b^6 + 192*a^2*b^4 + 192*a^4*b^2) - (a*((9
*a^14*b + 48*a^2*b^13 + 193*a^4*b^11 + 333*a^6*b^9 + 330*a^8*b^7 + 202*a^10
*b^5 + 69*a^12*b^3)/(2*(a^12 + b^12 + 6*a^2*b^10 + 15*a^4*b^8 + 20*a^6*b^6
+ 15*a^8*b^4 + 6*a^10*b^2)) - (64*b^5*((24*a^16 - 184*a^2*b^14 - 696*a^4*b^
12 - 888*a^6*b^10 - 248*a^8*b^8 + 408*a^10*b^6 + 408*a^12*b^4 + 152*a^14*b^
2)/(2*(a^12 + b^12 + 6*a^2*b^10 + 15*a^4*b^8 + 20*a^6*b^6 + 15*a^8*b^4 + 6*
a^10*b^2)) - (32*b^5*(192*a^16*b + 192*a^2*b^15 + 1344*a^4*b^13 + 4032*a^6*
b^11 + 6720*a^8*b^9 + 6720*a^10*b^7 + 4032*a^12*b^5 + 1344*a^14*b^3))/((64*
a^6 + 64*b^6 + 192*a^2*b^4 + 192*a^4*b^2)*(a^12 + b^12 + 6*a^2*b^10 + 15*a^
4*b^8 + 20*a^6*b^6 + 15*a^8*b^4 + 6*a^10*b^2))))/(64*a^6 + 64*b^6 + 192*a^2
*b^4 + 192*a^4*b^2))*(3*a^4 + 15*b^4 + 10*a^2*b^2))/(8*(a^6 + b^6 + 3*a^2*b
^4 + 3*a^4*b^2)) + (a^3*(3*a^4 + 15*b^4 + 10*a^2*b^2)^3*(192*a^16*b + 192*a
^2*b^15 + 1344*a^4*b^13 + 4032*a^6*b^11 + 6720*a^8*b^9 + 6720*a^10*b^7 + 40
32*a^12*b^5 + 1344*a^14*b^3))/(1024*(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2)^3*(
a^12 + b^12 + 6*a^2*b^10 + 15*a^4*b^8 + 20*a^6*b^6 + 15*a^8*b^4 + 6*a^10*b^
2)))*(9*a^12 + 256*b^12 - 1441*a^2*b^10 - 715*a^4*b^8 - 82*a^6*b^6 + 130*a^
8*b^4 + 51*a^10*b^2)*(16*a^16 + 16*b^16 + 128*a^2*b^14 + 448*a^4*b^12 + 896
*a^6*b^10 + 1120*a^8*b^8 + 896*a^10*b^6 + 448*a^12*b^4 + 128*a^14*b^2))/((3
*a^6 + 15*a^2*b^4 + 10*a^4*b^2)*(9*a^12 + 256*b^12 + 481*a^2*b^10 + 525*a^4
*b^8 + 490*a^6*b^6 + 250*a^8*b^4 + 69*a^10*b^2)^2) - (2*a*b*((56*a^2*b^12 +
 73*a^4*b^10 + 42*a^6*b^8 + 9*a^8*b^6)/(2*(a^12 + b^12 + 6*a^2*b^10 + 15*a^
4*b^8 + 20*a^6*b^6 + 15*a^8*b^4 + 6*a^10*b^2)) + (64*b^5*((9*a^14*b + 48*a^
2*b^13 + 193*a^4*b^11 + 333*a^6*b^9 + 330*a^8*b^7 + 202*a^10*b^5 + 69*a^12*
b^3)/(2*(a^12 + b^12 + 6*a^2*b^10 + 15*a^4*b^8 + 20*a^6*b^6 + 15*a^8*b^4 +
6*a^10*b^2)) - (64*b^5*((24*a^16 - 184*a^2*b^14 - 696*a^4*b^12 - 888*a^6*b^
10 - 248*a^8*b^8 + 408*a^10*b^6 + 408*a^12*b^4 + 152*a^14*b^2)/(2*(a^12 + b
^12 + 6*a^2*b^10 + 15*a^4*b^8 + 20*a^6*b^6 + 15*a^8*b^4 + 6*a^10*b^2)) - (3
2*b^5*(192*a^16*b + 192*a^2*b^15 + 1344*a^4*b^13 + 4032*a^6*b^11 + 6720*a^8
*b^9 + 6720*a^10*b^7 + 4032*a^12*b^5 + 1344*a^14*b^3))/((64*a^6 + 64*b^6 +
192*a^2*b^4 + 192*a^4*b^2)*(a^12 + b^12 + 6*a^2*b^10 + 15*a^4*b^8 + 20*a^6*
b^6 + 15*a^8*b^4 + 6*a^10*b^2))))/(64*a^6 + 64*b^6 + 192*a^2*b^4 + 192*a^4*
b^2)))/(64*a^6 + 64*b^6 + 192*a^2*b^4 + 192*a^4*b^2) + (a*((a*((24*a^16 - 1
84*a^2*b^14 - 696*a^4*b^12 - 888*a^6*b^10 - 248*a^8*b^8 + 408*a^10*b^6 + 40
8*a^12*b^4 + 152*a^14*b^2)/(2*(a^12 + b^12 + 6*a^2*b^10 + 15*a^4*b^8 + 20*a
^6*b^6 + 15*a^8*b^4 + 6*a^10*b^2)) - (32*b^5*(192*a^16*b + 192*a^2*b^15 + 1
344*a^4*b^13 + 4032*a^6*b^11 + 6720*a^8*b^9 + 6720*a^10*b^7 + 4032*a^12*b^5

```
 + 1344*a^14*b^3))/((64*a^6 + 64*b^6 + 192*a^2*b^4 + 192*a^4*b^2)*(a^12 + b
^12 + 6*a^2*b^10 + 15*a^4*b^8 + 20*a^6*b^6 + 15*a^8*b^4 + 6*a^10*b^2)))*(3*
a^4 + 15*b^4 + 10*a^2*b^2))/(8*(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2)) - (4*a*
b^5*(3*a^4 + 15*b^4 + 10*a^2*b^2)*(192*a^16*b + 192*a^2*b^15 + 1344*a^4*b^1
3 + 4032*a^6*b^11 + 6720*a^8*b^9 + 6720*a^10*b^7 + 4032*a^12*b^5 + 1344*a^1
4*b^3))/((64*a^6 + 64*b^6 + 192*a^2*b^4 + 192*a^4*b^2)*(a^6 + b^6 + 3*a^2*b
^4 + 3*a^4*b^2)*(a^12 + b^12 + 6*a^2*b^10 + 15*a^4*b^8 + 20*a^6*b^6 + 15*a^
8*b^4 + 6*a^10*b^2)))*(3*a^4 + 15*b^4 + 10*a^2*b^2))/(8*(a^6 + b^6 + 3*a^2*
b^4 + 3*a^4*b^2)) - (a^2*b^5*(3*a^4 + 15*b^4 + 10*a^2*b^2)^2*(192*a^16*b +
192*a^2*b^15 + 1344*a^4*b^13 + 4032*a^6*b^11 + 6720*a^8*b^9 + 6720*a^10*b^7
 + 4032*a^12*b^5 + 1344*a^14*b^3))/(2*(64*a^6 + 64*b^6 + 192*a^2*b^4 + 192*
a^4*b^2)*(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2)^2*(a^12 + b^12 + 6*a^2*b^10 +
15*a^4*b^8 + 20*a^6*b^6 + 15*a^8*b^4 + 6*a^10*b^2)))*(9*a^10 - 496*b^10 + 3
05*a^2*b^8 + 412*a^4*b^6 + 238*a^6*b^4 + 60*a^8*b^2)*(16*a^16 + 16*b^16 + 1
28*a^2*b^14 + 448*a^4*b^12 + 896*a^6*b^10 + 1120*a^8*b^8 + 896*a^10*b^6 + 4
48*a^12*b^4 + 128*a^14*b^2))/((3*a^6 + 15*a^2*b^4 + 10*a^4*b^2)*(9*a^12 + 2
56*b^12 + 481*a^2*b^10 + 525*a^4*b^8 + 490*a^6*b^6 + 250*a^8*b^4 + 69*a^10*
b^2)^2))*(3*a^4 + 15*b^4 + 10*a^2*b^2))/(4*d*(a^6 + b^6 + 3*a^2*b^4 + 3*a^4
*b^2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5/(a*cos(d*x+c)+b*sin(d*x+c)),x)
```

```
[Out] Timed out
```

3.111 $\int \frac{\cos^4(c+dx)}{a\cos(c+dx)+b\sin(c+dx)}\,dx$

Optimal. Leaf size=166

$$-\frac{a\sin^3(c+dx)}{3d\left(a^2+b^2\right)}+\frac{ab^2\sin(c+dx)}{d\left(a^2+b^2\right)^2}+\frac{a\sin(c+dx)}{d\left(a^2+b^2\right)}+\frac{b\cos^3(c+dx)}{3d\left(a^2+b^2\right)}-\frac{b^4\tanh^{-1}\left(\frac{b\cos(c+dx)-a\sin(c+dx)}{\sqrt{a^2+b^2}}\right)}{d\left(a^2+b^2\right)^{5/2}}+\frac{b^3\cos(c+dx)}{d\left(a^2+b^2\right)^2}$$

```
[Out] -b^4*arctanh((b*cos(d*x+c)-a*sin(d*x+c))/(a^2+b^2)^(1/2))/(a^2+b^2)^(5/2)/d
+b^3*cos(d*x+c)/(a^2+b^2)^2/d+1/3*b*cos(d*x+c)^3/(a^2+b^2)/d+a*b^2*sin(d*x+
c)/(a^2+b^2)^2/d+a*sin(d*x+c)/(a^2+b^2)/d-1/3*a*sin(d*x+c)^3/(a^2+b^2)/d
```

Rubi [A] time = 0.17, antiderivative size = 166, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.179, Rules used = {3100, 2633, 2637, 3074, 206}

$$-\frac{a\sin^3(c+dx)}{3d\left(a^2+b^2\right)}+\frac{ab^2\sin(c+dx)}{d\left(a^2+b^2\right)^2}+\frac{a\sin(c+dx)}{d\left(a^2+b^2\right)}+\frac{b\cos^3(c+dx)}{3d\left(a^2+b^2\right)}+\frac{b^3\cos(c+dx)}{d\left(a^2+b^2\right)^2}-\frac{b^4\tanh^{-1}\left(\frac{b\cos(c+dx)-a\sin(c+dx)}{\sqrt{a^2+b^2}}\right)}{d\left(a^2+b^2\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^4/(a*Cos[c + d*x] + b*Sin[c + d*x]),x]
```

```
[Out] -((b^4*ArcTanh[(b*Cos[c + d*x] - a*Sin[c + d*x])/Sqrt[a^2 + b^2]])/((a^2 +
b^2)^(5/2)*d)) + (b^3*Cos[c + d*x])/((a^2 + b^2)^2*d) + (b*Cos[c + d*x]^3)/
(3*(a^2 + b^2)*d) + (a*b^2*Sin[c + d*x])/((a^2 + b^2)^2*d) + (a*Sin[c + d*x
])/((a^2 + b^2)*d) - (a*Sin[c + d*x]^3)/(3*(a^2 + b^2)*d)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3074

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x
_Symbol] :> -Dist[d^(-1), Subst[Int[1/(a^2 + b^2 - x^2), x], x, b*Cos[c + d
*x] - a*Sin[c + d*x]], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

Rule 3100

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_)/(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin
[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[(b*Cos[c + d*x]^(m - 1))/(d*(a^2 +
 b^2)*(m - 1)), x] + (Dist[a/(a^2 + b^2), Int[Cos[c + d*x]^(m - 1), x], x]
+ Dist[b^2/(a^2 + b^2), Int[Cos[c + d*x]^(m - 2)/(a*Cos[c + d*x] + b*Sin[c
+ d*x]), x], x]) /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0] && GtQ[m, 1
]
```

Rubi steps

$$\int \frac{\cos^4(c+dx)}{a\cos(c+dx)+b\sin(c+dx)}\,dx = \frac{b\cos^3(c+dx)}{3\left(a^2+b^2\right)d}+\frac{a\int\cos^3(c+dx)\,dx}{a^2+b^2}+\frac{b^2\int\frac{\cos^2(c+dx)}{a\cos(c+dx)+b\sin(c+dx)}\,dx}{a^2+b^2}$$

$$= \frac{b^3\cos(c+dx)}{\left(a^2+b^2\right)^2 d}+\frac{b\cos^3(c+dx)}{3\left(a^2+b^2\right)d}+\frac{\left(ab^2\right)\int\cos(c+dx)\,dx}{\left(a^2+b^2\right)^2}+\frac{b^4\int\frac{1}{a\cos(c+dx}$$

$$= \frac{b^3\cos(c+dx)}{\left(a^2+b^2\right)^2 d}+\frac{b\cos^3(c+dx)}{3\left(a^2+b^2\right)d}+\frac{ab^2\sin(c+dx)}{\left(a^2+b^2\right)^2 d}+\frac{a\sin(c+dx)}{\left(a^2+b^2\right)d}-\frac{a\sin}{3(a}$$

$$= -\frac{b^4\tanh^{-1}\left(\frac{b\cos(c+dx)-a\sin(c+dx)}{\sqrt{a^2+b^2}}\right)}{\left(a^2+b^2\right)^{5/2}d}+\frac{b^3\cos(c+dx)}{\left(a^2+b^2\right)^2 d}+\frac{b\cos^3(c+dx)}{3\left(a^2+b^2\right)d}+\frac{ab^2}{(}$$

Mathematica [A] time = 0.99, size = 137, normalized size = 0.83

$$\frac{\sqrt{a^2+b^2}\left(3b\left(a^2+5b^2\right)\cos(c+dx)+b\left(a^2+b^2\right)\cos(3(c+dx))+2a\sin(c+dx)\left(\left(a^2+b^2\right)\cos(2(c+dx))+5a\right.\right.}{12d\left(a^2+b^2\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^4/(a*Cos[c + d*x] + b*Sin[c + d*x]),x]
```

```
[Out] (24*b^4*ArcTanh[(-b + a*Tan[(c + d*x)/2])/Sqrt[a^2 + b^2]] + Sqrt[a^2 + b^2
]*(3*b*(a^2 + 5*b^2)*Cos[c + d*x] + b*(a^2 + b^2)*Cos[3*(c + d*x)] + 2*a*(5
*a^2 + 11*b^2 + (a^2 + b^2)*Cos[2*(c + d*x)])*Sin[c + d*x]))/(12*(a^2 + b^2
)^(5/2)*d)
```

fricas [A] time = 0.57, size = 262, normalized size = 1.58

$$\frac{3\sqrt{a^2+b^2}\,b^4\log\left(-\frac{2\,ab\cos(dx+c)\sin(dx+c)+\left(a^2-b^2\right)\cos(dx+c)^2-2\,a^2-b^2+2\,\sqrt{a^2+b^2}\,(b\cos(dx+c)-a\sin(dx+c))}{2\,ab\cos(dx+c)\sin(dx+c)+\left(a^2-b^2\right)\cos(dx+c)^2+b^2}\right)+2\left(a^4b+2\,a^2b^3+b\right.}{6\left(a^6+3\,a^4b\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4/(a*cos(d*x+c)+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/6*(3*sqrt(a^2 + b^2)*b^4*log(-(2*a*b*cos(d*x + c)*sin(d*x + c) + (a^2 - b
^2)*cos(d*x + c)^2 - 2*a^2 - b^2 + 2*sqrt(a^2 + b^2)*(b*cos(d*x + c) - a*si
n(d*x + c)))/(2*a*b*cos(d*x + c)*sin(d*x + c) + (a^2 - b^2)*cos(d*x + c)^2
+ b^2)) + 2*(a^4*b + 2*a^2*b^3 + b^5)*cos(d*x + c)^3 + 6*(a^2*b^3 + b^5)*co
s(d*x + c) + 2*(2*a^5 + 7*a^3*b^2 + 5*a*b^4 + (a^5 + 2*a^3*b^2 + a*b^4)*cos
(d*x + c)^2)*sin(d*x + c))/((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*d)
```

giac [A] time = 0.30, size = 286, normalized size = 1.72

$$-\frac{\frac{3\,b^4\log\left(\frac{\left|2\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-2\,b-2\,\sqrt{a^2+b^2}\right|}{\left|2\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-2\,b+2\,\sqrt{a^2+b^2}\right|}\right)}{\left(a^4+2\,a^2b^2+b^4\right)\sqrt{a^2+b^2}}-\frac{2\left(3\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+6\,ab^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+3\,a^2b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4+6\,b^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4+2\,a^3\tan\left(\frac{1}{2}\,\right.\right.}{\left(a^4+2\,a^2b^2+b^4\right)\left(\right.}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4/(a*cos(d*x+c)+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/3*(3*b^4*log(abs(2*a*tan(1/2*d*x + 1/2*c) - 2*b - 2*sqrt(a^2 + b^2))/abs
(2*a*tan(1/2*d*x + 1/2*c) - 2*b + 2*sqrt(a^2 + b^2)))/((a^4 + 2*a^2*b^2 + b
^4)*sqrt(a^2 + b^2)) - 2*(3*a^3*tan(1/2*d*x + 1/2*c)^5 + 6*a*b^2*tan(1/2*d*
x + 1/2*c)^5 + 3*a^2*b*tan(1/2*d*x + 1/2*c)^4 + 6*b^3*tan(1/2*d*x + 1/2*c)^
4 + 2*a^3*tan(1/2*d*x + 1/2*c)^3 + 8*a*b^2*tan(1/2*d*x + 1/2*c)^3 + 6*b^3*t
an(1/2*d*x + 1/2*c)^2 + 3*a^3*tan(1/2*d*x + 1/2*c) + 6*a*b^2*tan(1/2*d*x +
1/2*c) + a^2*b + 4*b^3)/((a^4 + 2*a^2*b^2 + b^4)*(tan(1/2*d*x + 1/2*c)^2 +
1)^3))/d
```

maple [A] time = 0.18, size = 221, normalized size = 1.33

$$-\frac{-\frac{2\left(\left(-a^{3}-2a\,b^{2}\right)\left(\tan^{5}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\left(-a^{2}b-2b^{3}\right)\left(\tan^{4}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\left(-\frac{2}{3}a^{3}-\frac{8}{3}a\,b^{2}\right)\left(\tan^{3}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-2b^{3}\left(\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\left(-a^{3}-2a\,b^{2}\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{a^{2}b}{3}-\frac{4b^{3}}{3}\right)}{\left(a^{4}+2a^{2}b^{2}+b^{4}\right)\left(\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{3}} - \cdots}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4/(a*cos(d*x+c)+b*sin(d*x+c)),x)
```

```
[Out] 1/d*(-2/(a^4+2*a^2*b^2+b^4)*((-a^3-2*a*b^2)*tan(1/2*d*x+1/2*c)^5+(-a^2*b-2*
b^3)*tan(1/2*d*x+1/2*c)^4+(-2/3*a^3-8/3*a*b^2)*tan(1/2*d*x+1/2*c)^3-2*b^3*t
an(1/2*d*x+1/2*c)^2+(-a^3-2*a*b^2)*tan(1/2*d*x+1/2*c)-1/3*a^2*b-4/3*b^3)/(t
an(1/2*d*x+1/2*c)^2+1)^3+2*b^4/(a^4+2*a^2*b^2+b^4)/(a^2+b^2)^(1/2)*arctanh(
1/2*(2*a*tan(1/2*d*x+1/2*c)-2*b)/(a^2+b^2)^(1/2)))
```

maxima [B] time = 0.53, size = 379, normalized size = 2.28

$$-\frac{\frac{3\,b^{4}\log\left(\frac{b-\frac{a\sin(dx+c)}{\cos(dx+c)+1}+\sqrt{a^{2}+b^{2}}}{b-\frac{a\sin(dx+c)}{\cos(dx+c)+1}-\sqrt{a^{2}+b^{2}}}\right)}{\left(a^{4}+2\,a^{2}b^{2}+b^{4}\right)\sqrt{a^{2}+b^{2}}}-\frac{2\left(a^{2}b+4\,b^{3}+\frac{6\,b^{3}\sin(dx+c)^{2}}{(\cos(dx+c)+1)^{2}}+\frac{3\left(a^{3}+2\,ab^{2}\right)\sin(dx+c)}{\cos(dx+c)+1}+\frac{2\left(a^{3}+4\,ab^{2}\right)\sin(dx+c)^{3}}{(\cos(dx+c)+1)^{3}}+\frac{3\left(a^{2}b+2\,b^{3}\right)\sin(dx+c)^{4}}{(\cos(dx+c)+1)^{4}}+\frac{3\left(a^{3}+2\,ab^{2}\right)\sin(\cdots}{(\cos(dx+c)+1\cdots}\cdots\right)}{a^{4}+2\,a^{2}b^{2}+b^{4}+\frac{3\left(a^{4}+2\,a^{2}b^{2}+b^{4}\right)\sin(dx+c)^{2}}{(\cos(dx+c)+1)^{2}}+\frac{3\left(a^{4}+2\,a^{2}b^{2}+b^{4}\right)\sin(dx+c)^{4}}{(\cos(dx+c)+1)^{4}}+\frac{\left(a^{4}+2\,a^{2}b^{2}+b^{4}\right)\sin(dx+c)^{6}}{(\cos(dx+c)+1)^{6}}}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4/(a*cos(d*x+c)+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/3*(3*b^4*log((b - a*sin(d*x + c)/(cos(d*x + c) + 1) + sqrt(a^2 + b^2))/(
b - a*sin(d*x + c)/(cos(d*x + c) + 1) - sqrt(a^2 + b^2)))/((a^4 + 2*a^2*b^2
 + b^4)*sqrt(a^2 + b^2)) - 2*(a^2*b + 4*b^3 + 6*b^3*sin(d*x + c)^2/(cos(d*x
 + c) + 1)^2 + 3*(a^3 + 2*a*b^2)*sin(d*x + c)/(cos(d*x + c) + 1) + 2*(a^3 +
 4*a*b^2)*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 3*(a^2*b + 2*b^3)*sin(d*x +
 c)^4/(cos(d*x + c) + 1)^4 + 3*(a^3 + 2*a*b^2)*sin(d*x + c)^5/(cos(d*x + c)
 + 1)^5)/(a^4 + 2*a^2*b^2 + b^4 + 3*(a^4 + 2*a^2*b^2 + b^4)*sin(d*x + c)^2/
(cos(d*x + c) + 1)^2 + 3*(a^4 + 2*a^2*b^2 + b^4)*sin(d*x + c)^4/(cos(d*x +
c) + 1)^4 + (a^4 + 2*a^2*b^2 + b^4)*sin(d*x + c)^6/(cos(d*x + c) + 1)^6))/d
```

mupad [B] time = 3.32, size = 342, normalized size = 2.06

$$\frac{\frac{\frac{2\,a^{2}\,b}{3}+\frac{8\,b^{3}}{3}}{a^{4}+2\,a^{2}\,b^{2}+b^{4}}+\frac{4\,b^{3}\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{2}}{a^{4}+2\,a^{2}\,b^{2}+b^{4}}+\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{5}\left(2\,a^{3}+4\,a\,b^{2}\right)}{a^{4}+2\,a^{2}\,b^{2}+b^{4}}+\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{3}\left(\frac{4\,a^{3}}{3}+\frac{16\,a\,b^{2}}{3}\right)}{a^{4}+2\,a^{2}\,b^{2}+b^{4}}+\frac{2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\left(a^{3}+2\,a\,b^{2}\right)}{a^{4}+2\,a^{2}\,b^{2}+b^{4}}+\frac{2\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{4}\cdots}{a^{4}+2\,a^{2}\,b^{2}\cdots}\cdots}{d\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{6}+3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{4}+3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{2}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^4/(a*cos(c + d*x) + b*sin(c + d*x)),x)
```

```
[Out] (((2*a^2*b)/3 + (8*b^3)/3)/(a^4 + b^4 + 2*a^2*b^2) + (4*b^3*tan(c/2 + (d*x)
/2)^2)/(a^4 + b^4 + 2*a^2*b^2) + (tan(c/2 + (d*x)/2)^5*(4*a*b^2 + 2*a^3))/(
a^4 + b^4 + 2*a^2*b^2) + (tan(c/2 + (d*x)/2)^3*((16*a*b^2)/3 + (4*a^3)/3))/
(a^4 + b^4 + 2*a^2*b^2) + (2*tan(c/2 + (d*x)/2)*(2*a*b^2 + a^3))/(a^4 + b^4
 + 2*a^2*b^2) + (2*b*tan(c/2 + (d*x)/2)^4*(a^2 + 2*b^2))/(a^4 + b^4 + 2*a^2
*b^2))/(d*(3*tan(c/2 + (d*x)/2)^2 + 3*tan(c/2 + (d*x)/2)^4 + tan(c/2 + (d*x
)/2)^6 + 1)) - (2*b^4*atanh((a^4*b + b^5 + 2*a^2*b^3 - a*tan(c/2 + (d*x)/2)
*(a^4 + b^4 + 2*a^2*b^2))/(a^2 + b^2)^(5/2)))/(d*(a^2 + b^2)^(5/2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4/(a*cos(d*x+c)+b*sin(d*x+c)),x)
```

```
[Out] Timed out
```

3.112 $\int \frac{\cos^3(c+dx)}{a\cos(c+dx)+b\sin(c+dx)}\,dx$

Optimal. Leaf size=119

$$\frac{b\cos^2(c+dx)}{2d(a^2+b^2)}+\frac{a\sin(c+dx)\cos(c+dx)}{2d(a^2+b^2)}+\frac{ab^2x}{(a^2+b^2)^2}+\frac{ax}{2(a^2+b^2)}+\frac{b^3\log(a\cos(c+dx)+b\sin(c+dx))}{d(a^2+b^2)^2}$$

```
[Out] a*b^2*x/(a^2+b^2)^2+1/2*a*x/(a^2+b^2)+1/2*b*cos(d*x+c)^2/(a^2+b^2)/d+b^3*ln
(a*cos(d*x+c)+b*sin(d*x+c))/(a^2+b^2)^2/d+1/2*a*cos(d*x+c)*sin(d*x+c)/(a^2+
b^2)/d
```

Rubi [A] time = 0.13, antiderivative size = 119, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.179, Rules used = {3100, 2635, 8, 3098, 3133}

$$\frac{b\cos^2(c+dx)}{2d(a^2+b^2)}+\frac{a\sin(c+dx)\cos(c+dx)}{2d(a^2+b^2)}+\frac{b^3\log(a\cos(c+dx)+b\sin(c+dx))}{d(a^2+b^2)^2}+\frac{ab^2x}{(a^2+b^2)^2}+\frac{ax}{2(a^2+b^2)}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^3/(a*Cos[c + d*x] + b*Sin[c + d*x]),x]
```

```
[Out] (a*b^2*x)/(a^2 + b^2)^2 + (a*x)/(2*(a^2 + b^2)) + (b*Cos[c + d*x]^2)/(2*(a^
2 + b^2)*d) + (b^3*Log[a*Cos[c + d*x] + b*Sin[c + d*x]])/((a^2 + b^2)^2*d)
 + (a*Cos[c + d*x]*Sin[c + d*x])/(2*(a^2 + b^2)*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 3098

```
Int[cos[(c_.) + (d_.)*(x_)]/(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.
) + (d_.)*(x_)]), x_Symbol] :> Simp[(a*x)/(a^2 + b^2), x] + Dist[b/(a^2 + b
^2), Int[(b*Cos[c + d*x] - a*Sin[c + d*x])/(a*Cos[c + d*x] + b*Sin[c + d*x]
), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

Rule 3100

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_)/(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin
[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[(b*Cos[c + d*x]^(m - 1))/(d*(a^2 +
 b^2)*(m - 1)), x] + (Dist[a/(a^2 + b^2), Int[Cos[c + d*x]^(m - 1), x], x]
+ Dist[b^2/(a^2 + b^2), Int[Cos[c + d*x]^(m - 2)/(a*Cos[c + d*x] + b*Sin[c
+ d*x]), x], x]) /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0] && GtQ[m, 1
]
```

Rule 3133

```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])
/((a_.) + cos[(d_.) + (e_.)*(x_)]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)]), x
_Symbol] :> Simp[((b*B + c*C)*x)/(b^2 + c^2), x] + Simp[((c*B - b*C)*Log[a
+ b*Cos[d + e*x] + c*Sin[d + e*x]])/(e*(b^2 + c^2)), x] /; FreeQ[{a, b, c,
d, e, A, B, C}, x] && NeQ[b^2 + c^2, 0] && EqQ[A*(b^2 + c^2) - a*(b*B + c*C
), 0]
```

Rubi steps

$$\int \frac{\cos^3(c+dx)}{a\cos(c+dx)+b\sin(c+dx)}\,dx = \frac{b\cos^2(c+dx)}{2\left(a^2+b^2\right)d}+\frac{a\int \cos^2(c+dx)\,dx}{a^2+b^2}+\frac{b^2\int \frac{\cos(c+dx)}{a\cos(c+dx)+b\sin(c+dx)}\,dx}{a^2+b^2}$$

$$= \frac{ab^2x}{\left(a^2+b^2\right)^2}+\frac{b\cos^2(c+dx)}{2\left(a^2+b^2\right)d}+\frac{a\cos(c+dx)\sin(c+dx)}{2\left(a^2+b^2\right)d}+\frac{b^3\int \frac{b\cos(c+dx)-a}{a\cos(c+dx)+b}}{\left(a^2+b^2\right.}$$

$$= \frac{ab^2x}{\left(a^2+b^2\right)^2}+\frac{ax}{2\left(a^2+b^2\right)}+\frac{b\cos^2(c+dx)}{2\left(a^2+b^2\right)d}+\frac{b^3\log(a\cos(c+dx)+b\sin(c}{\left(a^2+b^2\right)^2 d}$$

Mathematica [C] time = 0.23, size = 143, normalized size = 1.20

$$\frac{a^3\sin(2(c+dx))+2a^3c+2a^3dx+b\left(a^2+b^2\right)\cos(2(c+dx))+2b^3\log\left((a\cos(c+dx)+b\sin(c+dx))^2\right)+ab^2\,\mathrm{s}}{4d\left(a^2+b^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^3/(a*Cos[c + d*x] + b*Sin[c + d*x]),x]
```

```
[Out] (2*a^3*c + 6*a*b^2*c + (4*I)*b^3*c + 2*a^3*d*x + 6*a*b^2*d*x + (4*I)*b^3*d*
x - (4*I)*b^3*ArcTan[Tan[c + d*x]] + b*(a^2 + b^2)*Cos[2*(c + d*x)] + 2*b^3
```

```
*Log[(a*Cos[c + d*x] + b*Sin[c + d*x])^2] + a^3*Sin[2*(c + d*x)] + a*b^2*Si
n[2*(c + d*x)])/(4*(a^2 + b^2)^2*d)
```

fricas [A] time = 0.59, size = 119, normalized size = 1.00

$$\frac{b^3 \log\left(2\,ab\cos(dx+c)\sin(dx+c) + \left(a^2-b^2\right)\cos(dx+c)^2 + b^2\right) + \left(a^3+3\,ab^2\right)dx + \left(a^2b+b^3\right)\cos(dx+c)^2}{2\left(a^4+2\,a^2b^2+b^4\right)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3/(a*cos(d*x+c)+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/2*(b^3*log(2*a*b*cos(d*x + c)*sin(d*x + c) + (a^2 - b^2)*cos(d*x + c)^2 +
 b^2) + (a^3 + 3*a*b^2)*d*x + (a^2*b + b^3)*cos(d*x + c)^2 + (a^3 + a*b^2)*
cos(d*x + c)*sin(d*x + c))/((a^4 + 2*a^2*b^2 + b^4)*d)
```

giac [A] time = 0.28, size = 182, normalized size = 1.53

$$\frac{\frac{2\,b^4\log(|b\tan(dx+c)+a|)}{a^4b+2\,a^2b^3+b^5} - \frac{b^3\log\left(\tan(dx+c)^2+1\right)}{a^4+2\,a^2b^2+b^4} + \frac{\left(a^3+3\,ab^2\right)(dx+c)}{a^4+2\,a^2b^2+b^4} + \frac{b^3\tan(dx+c)^2+a^3\tan(dx+c)+ab^2\tan(dx+c)+a^2b+2\,b^3}{\left(a^4+2\,a^2b^2+b^4\right)\left(\tan(dx+c)^2+1\right)}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3/(a*cos(d*x+c)+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/2*(2*b^4*log(abs(b*tan(d*x + c) + a))/(a^4*b + 2*a^2*b^3 + b^5) - b^3*log
(tan(d*x + c)^2 + 1)/(a^4 + 2*a^2*b^2 + b^4) + (a^3 + 3*a*b^2)*(d*x + c)/(a
^4 + 2*a^2*b^2 + b^4) + (b^3*tan(d*x + c)^2 + a^3*tan(d*x + c) + a*b^2*tan(
d*x + c) + a^2*b + 2*b^3)/((a^4 + 2*a^2*b^2 + b^4)*(tan(d*x + c)^2 + 1)))/d
```

maple [B] time = 0.16, size = 236, normalized size = 1.98

$$\frac{b^3\ln(a+b\tan(dx+c))}{d\left(a^2+b^2\right)^2} + \frac{\tan(dx+c)\,a^3}{2d\left(a^2+b^2\right)^2\left(\tan^2(dx+c)+1\right)} + \frac{\tan(dx+c)\,a\,b^2}{2d\left(a^2+b^2\right)^2\left(\tan^2(dx+c)+1\right)} + \frac{a^2}{2d\left(a^2+b^2\right)^2\left(\tan^2\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3/(a*cos(d*x+c)+b*sin(d*x+c)),x)
```

```
[Out] 1/d*b^3/(a^2+b^2)^2*ln(a+b*tan(d*x+c))+1/2/d/(a^2+b^2)^2/(tan(d*x+c)^2+1)*t
an(d*x+c)*a^3+1/2/d/(a^2+b^2)^2/(tan(d*x+c)^2+1)*tan(d*x+c)*a*b^2+1/2/d/(a^
2+b^2)^2/(tan(d*x+c)^2+1)*a^2*b+1/2/d/(a^2+b^2)^2/(tan(d*x+c)^2+1)*b^3-1/2/
d/(a^2+b^2)^2*b^3*ln(tan(d*x+c)^2+1)+3/2/d/(a^2+b^2)^2*arctan(tan(d*x+c))*a
*b^2+1/2/d/(a^2+b^2)^2*arctan(tan(d*x+c))*a^3
```

maxima [B] time = 2.01, size = 284, normalized size = 2.39

$$\frac{\frac{b^3\log\left(-a-\frac{2\,b\sin(dx+c)}{\cos(dx+c)+1}+\frac{a\sin(dx+c)^2}{(\cos(dx+c)+1)^2}\right)}{a^4+2\,a^2b^2+b^4}-\frac{b^3\log\left(\frac{\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+1\right)}{a^4+2\,a^2b^2+b^4}+\frac{\left(a^3+3\,ab^2\right)\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^4+2\,a^2b^2+b^4}+\frac{\frac{a\sin(dx+c)}{\cos(dx+c)+1}-\frac{2\,b\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-\frac{a\sin(dx+c)^3}{(\cos(dx+c)+1)^3}}{a^2+b^2+\frac{2\left(a^2+b^2\right)\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{\left(a^2+b^2\right)\sin(dx+c)^4}{(\cos(dx+c)+1)^4}}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3/(a*cos(d*x+c)+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] (b^3*log(-a - 2*b*sin(d*x + c)/(cos(d*x + c) + 1) + a*sin(d*x + c)^2/(cos(d
*x + c) + 1)^2)/(a^4 + 2*a^2*b^2 + b^4) - b^3*log(sin(d*x + c)^2/(cos(d*x +
 c) + 1)^2 + 1)/(a^4 + 2*a^2*b^2 + b^4) + (a^3 + 3*a*b^2)*arctan(sin(d*x +
c)/(cos(d*x + c) + 1))/(a^4 + 2*a^2*b^2 + b^4) + (a*sin(d*x + c)/(cos(d*x +
 c) + 1) - 2*b*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 - a*sin(d*x + c)^3/(cos(
d*x + c) + 1)^3)/(a^2 + b^2 + 2*(a^2 + b^2)*sin(d*x + c)^2/(cos(d*x + c) +
1)^2 + (a^2 + b^2)*sin(d*x + c)^4/(cos(d*x + c) + 1)^4))/d
```

mupad [B] time = 6.16, size = 3572, normalized size = 30.02

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^3/(a*cos(c + d*x) + b*sin(c + d*x)),x)
```

```
[Out] (b^3*log(a + 2*b*tan(c/2 + (d*x)/2) - a*tan(c/2 + (d*x)/2)^2))/(d*(a^4 + b^
4 + 2*a^2*b^2)) - (4*b^3*log(1/(cos(c + d*x) + 1)))/(d*(4*a^4 + 4*b^4 + 8*a
^2*b^2)) - ((a*tan(c/2 + (d*x)/2)^3)/(a^2 + b^2) + (2*b*tan(c/2 + (d*x)/2)^
2)/(a^2 + b^2) - (a*tan(c/2 + (d*x)/2))/(a^2 + b^2))/(d*(2*tan(c/2 + (d*x)/
2)^2 + tan(c/2 + (d*x)/2)^4 + 1)) - (a*atan((tan(c/2 + (d*x)/2)*((((4*b^3*(
(a*((8*(4*a*b^9 + 4*a^9*b + 28*a^3*b^7 + 48*a^5*b^5 + 28*a^7*b^3))/(a^6 + b
^6 + 3*a^2*b^4 + 3*a^4*b^2) - (32*b^3*(12*a*b^10 + 48*a^3*b^8 + 72*a^5*b^6
+ 48*a^7*b^4 + 12*a^9*b^2))/((4*a^4 + 4*b^4 + 8*a^2*b^2)*(a^6 + b^6 + 3*a^2
*b^4 + 3*a^4*b^2)))*(a^2 + 3*b^2))/(2*(a^4 + b^4 + 2*a^2*b^2)) - (16*a*b^3*
(a^2 + 3*b^2)*(12*a*b^10 + 48*a^3*b^8 + 72*a^5*b^6 + 48*a^7*b^4 + 12*a^9*b^
2))/((4*a^4 + 4*b^4 + 8*a^2*b^2)*(a^4 + b^4 + 2*a^2*b^2)*(a^6 + b^6 + 3*a^2
*b^4 + 3*a^4*b^2))))/(4*a^4 + 4*b^4 + 8*a^2*b^2) - (a*((8*(a^9 - 12*a*b^8 -
 6*a^3*b^6 + 13*a^5*b^4 + 8*a^7*b^2))/(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2) -
 (4*b^3*((8*(4*a*b^9 + 4*a^9*b + 28*a^3*b^7 + 48*a^5*b^5 + 28*a^7*b^3))/(a^
6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2) - (32*b^3*(12*a*b^10 + 48*a^3*b^8 + 72*a^5
*b^6 + 48*a^7*b^4 + 12*a^9*b^2))/((4*a^4 + 4*b^4 + 8*a^2*b^2)*(a^6 + b^6 +
3*a^2*b^4 + 3*a^4*b^2))))/(4*a^4 + 4*b^4 + 8*a^2*b^2))*(a^2 + 3*b^2))/(2*(a
^4 + b^4 + 2*a^2*b^2)) + (a^3*(a^2 + 3*b^2)^3*(12*a*b^10 + 48*a^3*b^8 + 72*
a^5*b^6 + 48*a^7*b^4 + 12*a^9*b^2))/((a^4 + b^4 + 2*a^2*b^2)^3*(a^6 + b^6 +
```

```
 3*a^2*b^4 + 3*a^4*b^2)))*(a^8 + 16*b^8 - 73*a^2*b^6 - 13*a^4*b^4 + 5*a^6*b
^2))/(a^8 + 16*b^8 + 25*a^2*b^6 + 15*a^4*b^4 + 7*a^6*b^2)^2 - (2*a*b*(a^6 -
 28*b^6 + 17*a^2*b^4 + 10*a^4*b^2)*((8*(4*a*b^7 + 6*a^3*b^5 + a^5*b^3))/(a^
6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2) + (4*b^3*((8*(a^9 - 12*a*b^8 - 6*a^3*b^6 +
 13*a^5*b^4 + 8*a^7*b^2))/(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2) - (4*b^3*((8*
(4*a*b^9 + 4*a^9*b + 28*a^3*b^7 + 48*a^5*b^5 + 28*a^7*b^3))/(a^6 + b^6 + 3*
a^2*b^4 + 3*a^4*b^2) - (32*b^3*(12*a*b^10 + 48*a^3*b^8 + 72*a^5*b^6 + 48*a^
7*b^4 + 12*a^9*b^2))/((4*a^4 + 4*b^4 + 8*a^2*b^2)*(a^6 + b^6 + 3*a^2*b^4 +
3*a^4*b^2))))/(4*a^4 + 4*b^4 + 8*a^2*b^2)))/(4*a^4 + 4*b^4 + 8*a^2*b^2) + (
a*(a^2 + 3*b^2)*((a*((8*(4*a*b^9 + 4*a^9*b + 28*a^3*b^7 + 48*a^5*b^5 + 28*a
^7*b^3))/(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2) - (32*b^3*(12*a*b^10 + 48*a^3*
b^8 + 72*a^5*b^6 + 48*a^7*b^4 + 12*a^9*b^2))/((4*a^4 + 4*b^4 + 8*a^2*b^2)*(
a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2)))*(a^2 + 3*b^2))/(2*(a^4 + b^4 + 2*a^2*b
^2)) - (16*a*b^3*(a^2 + 3*b^2)*(12*a*b^10 + 48*a^3*b^8 + 72*a^5*b^6 + 48*a^
7*b^4 + 12*a^9*b^2))/((4*a^4 + 4*b^4 + 8*a^2*b^2)*(a^4 + b^4 + 2*a^2*b^2)*(
a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2))))/(2*(a^4 + b^4 + 2*a^2*b^2)) - (8*a^2*
b^3*(a^2 + 3*b^2)^2*(12*a*b^10 + 48*a^3*b^8 + 72*a^5*b^6 + 48*a^7*b^4 + 12*
a^9*b^2))/((4*a^4 + 4*b^4 + 8*a^2*b^2)*(a^4 + b^4 + 2*a^2*b^2)^2*(a^6 + b^6
 + 3*a^2*b^4 + 3*a^4*b^2))))/(a^8 + 16*b^8 + 25*a^2*b^6 + 15*a^4*b^4 + 7*a^
6*b^2)^2)*(a^10 + b^10 + 5*a^2*b^8 + 10*a^4*b^6 + 10*a^6*b^4 + 5*a^8*b^2))/
(4*a^4 + 12*a^2*b^2) + (((4*b^3*((a*(a^2 + 3*b^2)*((8*(2*a^10 - 10*a^2*b^8
- 16*a^4*b^6 + 8*a^8*b^2))/(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2) - (32*b^3*(1
2*a^10*b + 12*a^2*b^9 + 48*a^4*b^7 + 72*a^6*b^5 + 48*a^8*b^3))/((4*a^4 + 4*
b^4 + 8*a^2*b^2)*(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2))))/(2*(a^4 + b^4 + 2*a
^2*b^2)) - (16*a*b^3*(a^2 + 3*b^2)*(12*a^10*b + 12*a^2*b^9 + 48*a^4*b^7 + 7
2*a^6*b^5 + 48*a^8*b^3))/((4*a^4 + 4*b^4 + 8*a^2*b^2)*(a^4 + b^4 + 2*a^2*b^
2)*(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2))))/(4*a^4 + 4*b^4 + 8*a^2*b^2) - (a*
(a^2 + 3*b^2)*((8*(a^8*b + a^4*b^5 + 2*a^6*b^3))/(a^6 + b^6 + 3*a^2*b^4 + 3
*a^4*b^2) - (4*b^3*((8*(2*a^10 - 10*a^2*b^8 - 16*a^4*b^6 + 8*a^8*b^2))/(a^6
 + b^6 + 3*a^2*b^4 + 3*a^4*b^2) - (32*b^3*(12*a^10*b + 12*a^2*b^9 + 48*a^4*
b^7 + 72*a^6*b^5 + 48*a^8*b^3))/((4*a^4 + 4*b^4 + 8*a^2*b^2)*(a^6 + b^6 + 3
*a^2*b^4 + 3*a^4*b^2))))/(4*a^4 + 4*b^4 + 8*a^2*b^2)))/(2*(a^4 + b^4 + 2*a^
2*b^2)) + (a^3*(a^2 + 3*b^2)^3*(12*a^10*b + 12*a^2*b^9 + 48*a^4*b^7 + 72*a^
6*b^5 + 48*a^8*b^3))/((a^4 + b^4 + 2*a^2*b^2)^3*(a^6 + b^6 + 3*a^2*b^4 + 3*
a^4*b^2)))*(a^8 + 16*b^8 - 73*a^2*b^6 - 13*a^4*b^4 + 5*a^6*b^2)*(a^10 + b^1
0 + 5*a^2*b^8 + 10*a^4*b^6 + 10*a^6*b^4 + 5*a^8*b^2))/((4*a^4 + 12*a^2*b^2)
*(a^8 + 16*b^8 + 25*a^2*b^6 + 15*a^4*b^4 + 7*a^6*b^2)^2) - (2*a*b*(a^6 - 28
*b^6 + 17*a^2*b^4 + 10*a^4*b^2)*((8*(2*a^2*b^6 + a^4*b^4))/(a^6 + b^6 + 3*a
^2*b^4 + 3*a^4*b^2) + (4*b^3*((8*(a^8*b + a^4*b^5 + 2*a^6*b^3))/(a^6 + b^6
+ 3*a^2*b^4 + 3*a^4*b^2) - (4*b^3*((8*(2*a^10 - 10*a^2*b^8 - 16*a^4*b^6 + 8
*a^8*b^2))/(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2) - (32*b^3*(12*a^10*b + 12*a^
2*b^9 + 48*a^4*b^7 + 72*a^6*b^5 + 48*a^8*b^3))/((4*a^4 + 4*b^4 + 8*a^2*b^2)
*(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2))))/(4*a^4 + 4*b^4 + 8*a^2*b^2)))/(4*a^
4 + 4*b^4 + 8*a^2*b^2) + (a*((a*(a^2 + 3*b^2)*((8*(2*a^10 - 10*a^2*b^8 - 16
*a^4*b^6 + 8*a^8*b^2))/(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2) - (32*b^3*(12*a^
```

```
10*b + 12*a^2*b^9 + 48*a^4*b^7 + 72*a^6*b^5 + 48*a^8*b^3))/((4*a^4 + 4*b^4
+ 8*a^2*b^2)*(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2))))/(2*(a^4 + b^4 + 2*a^2*b
^2)) - (16*a*b^3*(a^2 + 3*b^2)*(12*a^10*b + 12*a^2*b^9 + 48*a^4*b^7 + 72*a^
6*b^5 + 48*a^8*b^3))/((4*a^4 + 4*b^4 + 8*a^2*b^2)*(a^4 + b^4 + 2*a^2*b^2)*(
a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2)))*(a^2 + 3*b^2))/(2*(a^4 + b^4 + 2*a^2*b
^2)) - (8*a^2*b^3*(a^2 + 3*b^2)^2*(12*a^10*b + 12*a^2*b^9 + 48*a^4*b^7 + 72
*a^6*b^5 + 48*a^8*b^3))/((4*a^4 + 4*b^4 + 8*a^2*b^2)*(a^4 + b^4 + 2*a^2*b^2
)^2*(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2)))*(a^10 + b^10 + 5*a^2*b^8 + 10*a^4
*b^6 + 10*a^6*b^4 + 5*a^8*b^2))/((4*a^4 + 12*a^2*b^2)*(a^8 + 16*b^8 + 25*a^
2*b^6 + 15*a^4*b^4 + 7*a^6*b^2)^2))*(a^2 + 3*b^2))/(d*(a^4 + b^4 + 2*a^2*b^
2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3/(a*cos(d*x+c)+b*sin(d*x+c)),x)
```

```
[Out] Timed out
```

3.113 $\int \frac{\cos^2(c+dx)}{a\cos(c+dx)+b\sin(c+dx)}\,dx$

Optimal. Leaf size=91

$$\frac{a\sin(c+dx)}{d\left(a^2+b^2\right)}+\frac{b\cos(c+dx)}{d\left(a^2+b^2\right)}-\frac{b^2\tanh^{-1}\left(\frac{b\cos(c+dx)-a\sin(c+dx)}{\sqrt{a^2+b^2}}\right)}{d\left(a^2+b^2\right)^{3/2}}$$

```
[Out] -b^2*arctanh((b*cos(d*x+c)-a*sin(d*x+c))/(a^2+b^2)^(1/2))/(a^2+b^2)^(3/2)/d
+b*cos(d*x+c)/(a^2+b^2)/d+a*sin(d*x+c)/(a^2+b^2)/d
```

Rubi [A] time = 0.08, antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3100, 2637, 3074, 206}

$$\frac{a\sin(c+dx)}{d\left(a^2+b^2\right)}+\frac{b\cos(c+dx)}{d\left(a^2+b^2\right)}-\frac{b^2\tanh^{-1}\left(\frac{b\cos(c+dx)-a\sin(c+dx)}{\sqrt{a^2+b^2}}\right)}{d\left(a^2+b^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2/(a*Cos[c + d*x] + b*Sin[c + d*x]),x]
```

```
[Out] -((b^2*ArcTanh[(b*Cos[c + d*x] - a*Sin[c + d*x])/Sqrt[a^2 + b^2]])/((a^2 +
b^2)^(3/2)*d)) + (b*Cos[c + d*x])/((a^2 + b^2)*d) + (a*Sin[c + d*x])/((a^2
+ b^2)*d)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3074

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x
_Symbol] :> -Dist[d^(-1), Subst[Int[1/(a^2 + b^2 - x^2), x], x, b*Cos[c + d
*x] - a*Sin[c + d*x]], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

Rule 3100

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_)/(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin
[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[(b*Cos[c + d*x]^(m - 1))/(d*(a^2 +
 b^2)*(m - 1)), x] + (Dist[a/(a^2 + b^2), Int[Cos[c + d*x]^(m - 1), x], x]
+ Dist[b^2/(a^2 + b^2), Int[Cos[c + d*x]^(m - 2)/(a*Cos[c + d*x] + b*Sin[c
+ d*x]), x], x]) /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0] && GtQ[m, 1
]
```

Rubi steps

$$\int \frac{\cos^2(c+dx)}{a\cos(c+dx)+b\sin(c+dx)}\,dx = \frac{b\cos(c+dx)}{\left(a^2+b^2\right)d}+\frac{a\int\cos(c+dx)\,dx}{a^2+b^2}+\frac{b^2\int\frac{1}{a\cos(c+dx)+b\sin(c+dx)}\,dx}{a^2+b^2}$$

$$= \frac{b\cos(c+dx)}{\left(a^2+b^2\right)d}+\frac{a\sin(c+dx)}{\left(a^2+b^2\right)d}-\frac{b^2\,\mathrm{Subst}\left(\int\frac{1}{a^2+b^2-x^2}\,dx,x,b\cos(c+dx)-a\right.}{\left(a^2+b^2\right)d}$$

$$= -\frac{b^2\tanh^{-1}\left(\frac{b\cos(c+dx)-a\sin(c+dx)}{\sqrt{a^2+b^2}}\right)}{\left(a^2+b^2\right)^{3/2}d}+\frac{b\cos(c+dx)}{\left(a^2+b^2\right)d}+\frac{a\sin(c+dx)}{\left(a^2+b^2\right)d}$$

Mathematica [A] time = 0.18, size = 79, normalized size = 0.87

$$\frac{\sqrt{a^2+b^2}\,(a\sin(c+dx)+b\cos(c+dx))+2b^2\tanh^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)-b}{\sqrt{a^2+b^2}}\right)}{d\left(a^2+b^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2/(a*Cos[c + d*x] + b*Sin[c + d*x]),x]
```

```
[Out] (2*b^2*ArcTanh[(-b + a*Tan[(c + d*x)/2])/Sqrt[a^2 + b^2]] + Sqrt[a^2 + b^2]
*(b*Cos[c + d*x] + a*Sin[c + d*x]))/((a^2 + b^2)^(3/2)*d)
```

fricas [B] time = 0.64, size = 187, normalized size = 2.05

$$\frac{\sqrt{a^2+b^2}\,b^2\log\left(-\frac{2\,ab\cos(dx+c)\sin(dx+c)+\left(a^2-b^2\right)\cos(dx+c)^2-2\,a^2-b^2+2\,\sqrt{a^2+b^2}\,(b\cos(dx+c)-a\sin(dx+c))}{2\,ab\cos(dx+c)\sin(dx+c)+\left(a^2-b^2\right)\cos(dx+c)^2+b^2}\right)+2\left(a^2b+b^3\right)\cos(dx}{2\left(a^4+2\,a^2b^2+b^4\right)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(a*cos(d*x+c)+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/2*(sqrt(a^2 + b^2)*b^2*log(-(2*a*b*cos(d*x + c)*sin(d*x + c) + (a^2 - b^2
)*cos(d*x + c)^2 - 2*a^2 - b^2 + 2*sqrt(a^2 + b^2)*(b*cos(d*x + c) - a*sin(
d*x + c)))/(2*a*b*cos(d*x + c)*sin(d*x + c) + (a^2 - b^2)*cos(d*x + c)^2 +
b^2)) + 2*(a^2*b + b^3)*cos(d*x + c) + 2*(a^3 + a*b^2)*sin(d*x + c))/((a^4
+ 2*a^2*b^2 + b^4)*d)
```

giac [A] time = 1.33, size = 118, normalized size = 1.30

$$-\frac{\frac{b^2\log\left(\frac{\left|2\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-2\,b-2\,\sqrt{a^2+b^2}\right|}{\left|2\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-2\,b+2\,\sqrt{a^2+b^2}\right|}\right)}{\left(a^2+b^2\right)^{\frac{3}{2}}}-\frac{2\left(a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+b\right)}{\left(a^2+b^2\right)\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(a*cos(d*x+c)+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -(b^2*log(abs(2*a*tan(1/2*d*x + 1/2*c) - 2*b - 2*sqrt(a^2 + b^2))/abs(2*a*t
an(1/2*d*x + 1/2*c) - 2*b + 2*sqrt(a^2 + b^2)))/(a^2 + b^2)^(3/2) - 2*(a*ta
n(1/2*d*x + 1/2*c) + b)/((a^2 + b^2)*(tan(1/2*d*x + 1/2*c)^2 + 1)))/d
```

maple [A] time = 0.16, size = 90, normalized size = 0.99

$$\frac{-\frac{2\left(-a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-b\right)}{\left(a^2+b^2\right)\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}+\frac{2b^2\,\mathrm{arctanh}\left(\frac{2a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-2b}{2\sqrt{a^2+b^2}}\right)}{\left(a^2+b^2\right)^{\frac{3}{2}}}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2/(a*cos(d*x+c)+b*sin(d*x+c)),x)
```

```
[Out] 1/d*(-2/(a^2+b^2)*(-a*tan(1/2*d*x+1/2*c)-b)/(tan(1/2*d*x+1/2*c)^2+1)+2*b^2/
(a^2+b^2)^(3/2)*arctanh(1/2*(2*a*tan(1/2*d*x+1/2*c)-2*b)/(a^2+b^2)^(1/2)))
```

maxima [A] time = 0.57, size = 142, normalized size = 1.56

$$-\frac{\frac{b^2\log\left(\frac{b-\frac{a\sin(dx+c)}{\cos(dx+c)+1}+\sqrt{a^2+b^2}}{b-\frac{a\sin(dx+c)}{\cos(dx+c)+1}-\sqrt{a^2+b^2}}\right)}{\left(a^2+b^2\right)^{\frac{3}{2}}}-\frac{2\left(b+\frac{a\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^2+b^2+\frac{\left(a^2+b^2\right)\sin(dx+c)^2}{(\cos(dx+c)+1)^2}}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(a*cos(d*x+c)+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -(b^2*log((b - a*sin(d*x + c)/(cos(d*x + c) + 1) + sqrt(a^2 + b^2))/(b - a*
sin(d*x + c)/(cos(d*x + c) + 1) - sqrt(a^2 + b^2)))/(a^2 + b^2)^(3/2) - 2*(
b + a*sin(d*x + c)/(cos(d*x + c) + 1))/(a^2 + b^2 + (a^2 + b^2)*sin(d*x + c
)^2/(cos(d*x + c) + 1)^2))/d
```

mupad [B] time = 0.63, size = 110, normalized size = 1.21

$$\frac{\frac{2\,b}{a^2+b^2}+\frac{2\,a\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{a^2+b^2}}{d\,\left({\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2+1\right)}-\frac{2\,b^2\,\mathrm{atanh}\left(\frac{a^2\,b+b^3-a\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(a^2+b^2\right)}{{\left(a^2+b^2\right)}^{3/2}}\right)}{d\,{\left(a^2+b^2\right)}^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^2/(a*cos(c + d*x) + b*sin(c + d*x)),x)
```

```
[Out] ((2*b)/(a^2 + b^2) + (2*a*tan(c/2 + (d*x)/2))/(a^2 + b^2))/(d*(tan(c/2 + (d
*x)/2)^2 + 1)) - (2*b^2*atanh((a^2*b + b^3 - a*tan(c/2 + (d*x)/2)*(a^2 + b^
2))/(a^2 + b^2)^(3/2)))/(d*(a^2 + b^2)^(3/2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2/(a*cos(d*x+c)+b*sin(d*x+c)),x)
```

```
[Out] Timed out
```

3.114 $\int \frac{\cos(c+dx)}{a\cos(c+dx)+b\sin(c+dx)}\,dx$

Optimal. Leaf size=45

$$\frac{b\log(a\cos(c+dx)+b\sin(c+dx))}{d\left(a^2+b^2\right)}+\frac{ax}{a^2+b^2}$$

```
[Out] a*x/(a^2+b^2)+b*ln(a*cos(d*x+c)+b*sin(d*x+c))/(a^2+b^2)/d
```

Rubi [A] time = 0.07, antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {3098, 3133}

$$\frac{b\log(a\cos(c+dx)+b\sin(c+dx))}{d\left(a^2+b^2\right)}+\frac{ax}{a^2+b^2}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]/(a*Cos[c + d*x] + b*Sin[c + d*x]),x]
```

```
[Out] (a*x)/(a^2 + b^2) + (b*Log[a*Cos[c + d*x] + b*Sin[c + d*x]])/((a^2 + b^2)*d
)
```

Rule 3098

```
Int[cos[(c_.) + (d_.)*(x_)]/(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.
) + (d_.)*(x_)]), x_Symbol] :> Simp[(a*x)/(a^2 + b^2), x] + Dist[b/(a^2 + b
^2), Int[(b*Cos[c + d*x] - a*Sin[c + d*x])/(a*Cos[c + d*x] + b*Sin[c + d*x]
), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

Rule 3133

```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])
/((a_.) + cos[(d_.) + (e_.)*(x_)]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)]), x
_Symbol] :> Simp[((b*B + c*C)*x)/(b^2 + c^2), x] + Simp[((c*B - b*C)*Log[a
+ b*Cos[d + e*x] + c*Sin[d + e*x]])/(e*(b^2 + c^2)), x] /; FreeQ[{a, b, c,
d, e, A, B, C}, x] && NeQ[b^2 + c^2, 0] && EqQ[A*(b^2 + c^2) - a*(b*B + c*C
), 0]
```

Rubi steps

$$\int \frac{\cos(c+dx)}{a\cos(c+dx)+b\sin(c+dx)}\,dx = \frac{ax}{a^2+b^2}+\frac{b\int \frac{b\cos(c+dx)-a\sin(c+dx)}{a\cos(c+dx)+b\sin(c+dx)}\,dx}{a^2+b^2}$$
$$= \frac{ax}{a^2+b^2}+\frac{b\log(a\cos(c+dx)+b\sin(c+dx))}{(a^2+b^2)\,d}$$

Mathematica [A] time = 0.06, size = 41, normalized size = 0.91

$$\frac{b\log(a\cos(c+dx)+b\sin(c+dx))+a(c+dx)}{d\,(a^2+b^2)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]/(a*Cos[c + d*x] + b*Sin[c + d*x]),x]
```

```
[Out] (a*(c + d*x) + b*Log[a*Cos[c + d*x] + b*Sin[c + d*x]])/((a^2 + b^2)*d)
```

fricas [A] time = 0.69, size = 61, normalized size = 1.36

$$\frac{2\,adx+b\log\left(2\,ab\cos\left(dx+c\right)\sin\left(dx+c\right)+\left(a^2-b^2\right)\cos\left(dx+c\right)^2+b^2\right)}{2\left(a^2+b^2\right)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a*cos(d*x+c)+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/2*(2*a*d*x + b*log(2*a*b*cos(d*x + c)*sin(d*x + c) + (a^2 - b^2)*cos(d*x
+ c)^2 + b^2))/((a^2 + b^2)*d)
```

giac [A] time = 0.20, size = 74, normalized size = 1.64

$$\frac{\frac{2\,b^2\log(|b\tan(dx+c)+a|)}{a^2b+b^3}+\frac{2\,(dx+c)a}{a^2+b^2}-\frac{b\log\left(\tan(dx+c)^2+1\right)}{a^2+b^2}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a*cos(d*x+c)+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/2*(2*b^2*log(abs(b*tan(d*x + c) + a))/(a^2*b + b^3) + 2*(d*x + c)*a/(a^2
+ b^2) - b*log(tan(d*x + c)^2 + 1)/(a^2 + b^2))/d
```

maple [A] time = 0.15, size = 74, normalized size = 1.64

$$\frac{b\ln\left(a+b\tan\left(dx+c\right)\right)}{d\left(a^{2}+b^{2}\right)}-\frac{b\ln\left(\tan^{2}\left(dx+c\right)+1\right)}{2d\left(a^{2}+b^{2}\right)}+\frac{a\arctan\left(\tan\left(dx+c\right)\right)}{d\left(a^{2}+b^{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)/(a*cos(d*x+c)+b*sin(d*x+c)),x)
```

```
[Out] 1/d*b/(a^2+b^2)*ln(a+b*tan(d*x+c))-1/2/d/(a^2+b^2)*b*ln(tan(d*x+c)^2+1)+1/d
/(a^2+b^2)*a*arctan(tan(d*x+c))
```

maxima [B] time = 0.60, size = 124, normalized size = 2.76

$$\frac{\frac{2\,a\arctan\left(\frac{\sin\left(dx+c\right)}{\cos\left(dx+c\right)+1}\right)}{a^{2}+b^{2}}+\frac{b\log\left(-a-\frac{2\,b\sin\left(dx+c\right)}{\cos\left(dx+c\right)+1}+\frac{a\sin\left(dx+c\right)^{2}}{\left(\cos\left(dx+c\right)+1\right)^{2}}\right)}{a^{2}+b^{2}}-\frac{b\log\left(\frac{\sin\left(dx+c\right)^{2}}{\left(\cos\left(dx+c\right)+1\right)^{2}}+1\right)}{a^{2}+b^{2}}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a*cos(d*x+c)+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] (2*a*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/(a^2 + b^2) + b*log(-a - 2*b*s
in(d*x + c)/(cos(d*x + c) + 1) + a*sin(d*x + c)^2/(cos(d*x + c) + 1)^2)/(a^
2 + b^2) - b*log(sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 1)/(a^2 + b^2))/d
```

mupad [B] time = 1.16, size = 1069, normalized size = 23.76

$$\frac{b\,\ln\left(-a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+2\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+a\right)}{d\,\left(a^2+b^2\right)}-\frac{2\,a\,\mathrm{atan}\left(\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\left(\left(a^4-13\,a^2\,b^2+4\,b^4\right)\left(\frac{a^3\,\left(96\,a^3\,b^2+96\,a\,b^4\right)}{\left(a^2+b^2\right)^3}+\frac{a\left(96\,a\,b^2-32\,a^3+\frac{b\left(32\,a\,b\cdots\right.}{\cdots}\right.}{\cdots}\right.\right.}{\cdots}\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)/(a*cos(c + d*x) + b*sin(c + d*x)),x)
```

```
[Out] (b*log(a + 2*b*tan(c/2 + (d*x)/2) - a*tan(c/2 + (d*x)/2)^2))/(d*(a^2 + b^2)
) - (2*a*atan((tan(c/2 + (d*x)/2)*(((a^4 + 4*b^4 - 13*a^2*b^2)*((a^3*(96*a*
b^4 + 96*a^3*b^2))/(a^2 + b^2)^3 + (a*(96*a*b^2 - 32*a^3 + (b*(32*a*b^3 + 1
28*a^3*b - (b*(96*a*b^4 + 96*a^3*b^2))/(a^2 + b^2)))/(a^2 + b^2)))/(a^2 + b
^2) + (b*((a*(32*a*b^3 + 128*a^3*b - (b*(96*a*b^4 + 96*a^3*b^2))/(a^2 + b^2
)))/(a^2 + b^2) - (a*b*(96*a*b^4 + 96*a^3*b^2))/(a^2 + b^2)^2))/(a^2 + b^2)
))/(a^4 + 4*b^4 + 5*a^2*b^2)^2 - (6*a*b*(a^2 - 2*b^2)*(32*a*b - (b*(96*a*b^
2 - 32*a^3 + (b*(32*a*b^3 + 128*a^3*b - (b*(96*a*b^4 + 96*a^3*b^2))/(a^2 +
b^2)))/(a^2 + b^2)))/(a^2 + b^2) + (a*((a*(32*a*b^3 + 128*a^3*b - (b*(96*a*
b^4 + 96*a^3*b^2))/(a^2 + b^2)))/(a^2 + b^2) - (a*b*(96*a*b^4 + 96*a^3*b^2)
```

```
)/(a^2 + b^2)^2))/(a^2 + b^2) - (a^2*b*(96*a*b^4 + 96*a^3*b^2))/(a^2 + b^2)
^3))/(a^4 + 4*b^4 + 5*a^2*b^2)^2)*(a^4 + b^4 + 2*a^2*b^2))/(32*a^2) + ((a^4
 + 4*b^4 - 13*a^2*b^2)*((a*(32*a^2*b - (b*(64*a^2*b^2 - 32*a^4 + (b*(96*a^4
*b + 96*a^2*b^3))/(a^2 + b^2)))/(a^2 + b^2)))/(a^2 + b^2) + (a^3*(96*a^4*b
+ 96*a^2*b^3))/(a^2 + b^2)^3 - (b*((a*(64*a^2*b^2 - 32*a^4 + (b*(96*a^4*b +
 96*a^2*b^3))/(a^2 + b^2)))/(a^2 + b^2) + (a*b*(96*a^4*b + 96*a^2*b^3))/(a^
2 + b^2)^2))/(a^2 + b^2))*(a^4 + b^4 + 2*a^2*b^2))/(32*a^2*(a^4 + 4*b^4 + 5
*a^2*b^2)^2) + (3*b*(a^2 - 2*b^2)*((b*(32*a^2*b - (b*(64*a^2*b^2 - 32*a^4 +
 (b*(96*a^4*b + 96*a^2*b^3))/(a^2 + b^2)))/(a^2 + b^2)))/(a^2 + b^2) + (a*(
(a*(64*a^2*b^2 - 32*a^4 + (b*(96*a^4*b + 96*a^2*b^3))/(a^2 + b^2)))/(a^2 +
b^2) + (a*b*(96*a^4*b + 96*a^2*b^3))/(a^2 + b^2)^2))/(a^2 + b^2) + (a^2*b*(
96*a^4*b + 96*a^2*b^3))/(a^2 + b^2)^3)*(a^4 + b^4 + 2*a^2*b^2))/(16*a*(a^4
+ 4*b^4 + 5*a^2*b^2)^2)))/(d*(a^2 + b^2)) - (b*log(tan(c/2 + (d*x)/2)^2 + 1
))/(d*(a^2 + b^2))
```

sympy [A] time = 1.75, size = 296, normalized size = 6.58

$$\begin{cases} \frac{\tilde{\infty} x \cos(c)}{\sin(c)} & \text{for } a = 0 \wedge b = 0 \wedge d = 0 \\ -\frac{dx \sin(c+dx)}{2ibd \sin(c+dx) + 2bd \cos(c+dx)} + \frac{idx \cos(c+dx)}{2ibd \sin(c+dx) + 2bd \cos(c+dx)} - \frac{\cos(c+dx)}{2ibd \sin(c+dx) + 2bd \cos(c+dx)} & \text{for } a = -ib \\ -\frac{dx \sin(c+dx)}{-2ibd \sin(c+dx) + 2bd \cos(c+dx)} - \frac{idx \cos(c+dx)}{-2ibd \sin(c+dx) + 2bd \cos(c+dx)} - \frac{\cos(c+dx)}{-2ibd \sin(c+dx) + 2bd \cos(c+dx)} & \text{for } a = ib \\ \frac{x \cos(c)}{a \cos(c) + b \sin(c)} & \text{for } d = 0 \\ \frac{\log(\sin(c+dx))}{bd} & \text{for } a = 0 \\ \frac{adx}{a^2 d + b^2 d} + \frac{b \log\left(\cos(c+dx) + \frac{b \sin(c+dx)}{a}\right)}{a^2 d + b^2 d} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a*cos(d*x+c)+b*sin(d*x+c)),x)
```

```
[Out] Piecewise((zoo*x*cos(c)/sin(c), Eq(a, 0) & Eq(b, 0) & Eq(d, 0)), (-d*x*sin(
c + d*x)/(2*I*b*d*sin(c + d*x) + 2*b*d*cos(c + d*x)) + I*d*x*cos(c + d*x)/(
2*I*b*d*sin(c + d*x) + 2*b*d*cos(c + d*x)) - cos(c + d*x)/(2*I*b*d*sin(c +
d*x) + 2*b*d*cos(c + d*x)), Eq(a, -I*b)), (-d*x*sin(c + d*x)/(-2*I*b*d*sin(
c + d*x) + 2*b*d*cos(c + d*x)) - I*d*x*cos(c + d*x)/(-2*I*b*d*sin(c + d*x)
+ 2*b*d*cos(c + d*x)) - cos(c + d*x)/(-2*I*b*d*sin(c + d*x) + 2*b*d*cos(c +
 d*x)), Eq(a, I*b)), (x*cos(c)/(a*cos(c) + b*sin(c)), Eq(d, 0)), (log(sin(c
 + d*x))/(b*d), Eq(a, 0)), (a*d*x/(a**2*d + b**2*d) + b*log(cos(c + d*x) +
b*sin(c + d*x)/a)/(a**2*d + b**2*d), True))
```

3.115 $\int \frac{1}{a\cos(c+dx)+b\sin(c+dx)}\,dx$

Optimal. Leaf size=47

$$-\frac{\tanh^{-1}\left(\frac{b\cos(c+dx)-a\sin(c+dx)}{\sqrt{a^2+b^2}}\right)}{d\sqrt{a^2+b^2}}$$

```
[Out] -arctanh((b*cos(d*x+c)-a*sin(d*x+c))/(a^2+b^2)^(1/2))/d/(a^2+b^2)^(1/2)
```

Rubi [A] time = 0.02, antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {3074, 206}

$$-\frac{\tanh^{-1}\left(\frac{b\cos(c+dx)-a\sin(c+dx)}{\sqrt{a^2+b^2}}\right)}{d\sqrt{a^2+b^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Cos[c + d*x] + b*Sin[c + d*x])^(-1),x]
```

```
[Out] -(ArcTanh[(b*Cos[c + d*x] - a*Sin[c + d*x])/Sqrt[a^2 + b^2]]/(Sqrt[a^2 + b^
2]*d))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3074

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x
_Symbol] :> -Dist[d^(-1), Subst[Int[1/(a^2 + b^2 - x^2), x], x, b*Cos[c + d
*x] - a*Sin[c + d*x]], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{a\cos(c+dx)+b\sin(c+dx)}\,dx &= -\frac{\text{Subst}\left(\int \frac{1}{a^2+b^2-x^2}\,dx, x, b\cos(c+dx)-a\sin(c+dx)\right)}{d}\\
&= -\frac{\tanh^{-1}\left(\frac{b\cos(c+dx)-a\sin(c+dx)}{\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}\,d}
\end{aligned}$$

Mathematica [A] time = 0.03, size = 45, normalized size = 0.96

$$\frac{2\tanh^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)-b}{\sqrt{a^2+b^2}}\right)}{d\sqrt{a^2+b^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Cos[c + d*x] + b*Sin[c + d*x])^(-1),x]
```

```
[Out] (2*ArcTanh[(-b + a*Tan[(c + d*x)/2])/Sqrt[a^2 + b^2]])/(Sqrt[a^2 + b^2]*d)
```

fricas [B] time = 0.76, size = 131, normalized size = 2.79

$$\frac{\log\left(-\frac{2\,ab\cos(dx+c)\sin(dx+c)+\left(a^2-b^2\right)\cos(dx+c)^2-2\,a^2-b^2+2\,\sqrt{a^2+b^2}\,(b\cos(dx+c)-a\sin(dx+c))}{2\,ab\cos(dx+c)\sin(dx+c)+\left(a^2-b^2\right)\cos(dx+c)^2+b^2}\right)}{2\,\sqrt{a^2+b^2}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cos(d*x+c)+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/2*log(-(2*a*b*cos(d*x + c)*sin(d*x + c) + (a^2 - b^2)*cos(d*x + c)^2 - 2*
a^2 - b^2 + 2*sqrt(a^2 + b^2)*(b*cos(d*x + c) - a*sin(d*x + c)))/(2*a*b*cos
(d*x + c)*sin(d*x + c) + (a^2 - b^2)*cos(d*x + c)^2 + b^2))/(sqrt(a^2 + b^2
)*d)
```

giac [A] time = 2.75, size = 74, normalized size = 1.57

$$-\frac{\log\left(\frac{\left|2\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-2\,b-2\,\sqrt{a^2+b^2}\right|}{\left|2\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-2\,b+2\,\sqrt{a^2+b^2}\right|}\right)}{\sqrt{a^2+b^2}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cos(d*x+c)+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -log(abs(2*a*tan(1/2*d*x + 1/2*c) - 2*b - 2*sqrt(a^2 + b^2))/abs(2*a*tan(1/
2*d*x + 1/2*c) - 2*b + 2*sqrt(a^2 + b^2)))/(sqrt(a^2 + b^2)*d)
```

maple [A] time = 0.14, size = 43, normalized size = 0.91

$$\frac{2\,\mathrm{arctanh}\left(\frac{2a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-2b}{2\sqrt{a^2+b^2}}\right)}{d\sqrt{a^2+b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*cos(d*x+c)+b*sin(d*x+c)),x)
```

```
[Out] 2/d/(a^2+b^2)^(1/2)*arctanh(1/2*(2*a*tan(1/2*d*x+1/2*c)-2*b)/(a^2+b^2)^(1/2
))
```

maxima [A] time = 0.73, size = 80, normalized size = 1.70

$$-\frac{\log\left(\frac{b-\frac{a\sin(dx+c)}{\cos(dx+c)+1}+\sqrt{a^2+b^2}}{b-\frac{a\sin(dx+c)}{\cos(dx+c)+1}-\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cos(d*x+c)+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -log((b - a*sin(d*x + c)/(cos(d*x + c) + 1) + sqrt(a^2 + b^2))/(b - a*sin(d
*x + c)/(cos(d*x + c) + 1) - sqrt(a^2 + b^2)))/(sqrt(a^2 + b^2)*d)
```

mupad [B] time = 0.47, size = 39, normalized size = 0.83

$$-\frac{2\,\mathrm{atanh}\left(\frac{b-a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{d\,\sqrt{a^2+b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*cos(c + d*x) + b*sin(c + d*x)),x)
```

```
[Out] -(2*atanh((b - a*tan(c/2 + (d*x)/2))/(a^2 + b^2)^(1/2)))/(d*(a^2 + b^2)^(1/
2))
```

sympy [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: AttributeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cos(d*x+c)+b*sin(d*x+c)),x)
```

```
[Out] Exception raised: AttributeError
```

3.116 $\int \frac{\sec(c+dx)}{a\cos(c+dx)+b\sin(c+dx)}\,dx$

Optimal. Leaf size=41

$$\frac{\log(a\cos(c+dx)+b\sin(c+dx))}{bd}-\frac{\log(\cos(c+dx))}{bd}$$

```
[Out] -ln(cos(d*x+c))/b/d+ln(a*cos(d*x+c)+b*sin(d*x+c))/b/d
```

Rubi [A] time = 0.08, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {3102, 3475, 3133}

$$\frac{\log(a\cos(c+dx)+b\sin(c+dx))}{bd}-\frac{\log(\cos(c+dx))}{bd}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]/(a*Cos[c + d*x] + b*Sin[c + d*x]),x]
```

```
[Out] -(Log[Cos[c + d*x]]/(b*d)) + Log[a*Cos[c + d*x] + b*Sin[c + d*x]]/(b*d)
```

Rule 3102

```
Int[1/(cos[(c_.) + (d_.)*(x_)]*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(
c_.) + (d_.)*(x_)])), x_Symbol] :> Dist[1/b, Int[Tan[c + d*x], x], x] + Dis
t[1/b, Int[(b*Cos[c + d*x] - a*Sin[c + d*x])/(a*Cos[c + d*x] + b*Sin[c + d*
x]), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

Rule 3133

```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])
/((a_.) + cos[(d_.) + (e_.)*(x_)]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)]), x
_Symbol] :> Simp[((b*B + c*C)*x)/(b^2 + c^2), x] + Simp[((c*B - b*C)*Log[a
+ b*Cos[d + e*x] + c*Sin[d + e*x]])/(e*(b^2 + c^2)), x] /; FreeQ[{a, b, c,
d, e, A, B, C}, x] && NeQ[b^2 + c^2, 0] && EqQ[A*(b^2 + c^2) - a*(b*B + c*C
), 0]
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\sec(c+dx)}{a\cos(c+dx)+b\sin(c+dx)}\,dx = \frac{\int \frac{b\cos(c+dx)-a\sin(c+dx)}{a\cos(c+dx)+b\sin(c+dx)}\,dx}{b} + \frac{\int \tan(c+dx)\,dx}{b}$$
$$= -\frac{\log(\cos(c+dx))}{bd} + \frac{\log(a\cos(c+dx)+b\sin(c+dx))}{bd}$$

Mathematica [A] time = 0.03, size = 18, normalized size = 0.44

$$\frac{\log(a+b\tan(c+dx))}{bd}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]/(a*Cos[c + d*x] + b*Sin[c + d*x]),x]
```

```
[Out] Log[a + b*Tan[c + d*x]]/(b*d)
```

fricas [A] time = 0.75, size = 59, normalized size = 1.44

$$\frac{\log\left(2\,ab\cos\left(dx+c\right)\sin\left(dx+c\right)+\left(a^2-b^2\right)\cos\left(dx+c\right)^2+b^2\right)-\log\left(\cos\left(dx+c\right)^2\right)}{2\,bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a*cos(d*x+c)+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/2*(log(2*a*b*cos(d*x + c)*sin(d*x + c) + (a^2 - b^2)*cos(d*x + c)^2 + b^2
) - log(cos(d*x + c)^2))/(b*d)
```

giac [A] time = 1.97, size = 19, normalized size = 0.46

$$\frac{\log\left(|b\tan\left(dx+c\right)+a|\right)}{bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a*cos(d*x+c)+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] log(abs(b*tan(d*x + c) + a))/(b*d)
```

maple [A] time = 0.21, size = 19, normalized size = 0.46

$$\frac{\ln\left(a+b\tan\left(dx+c\right)\right)}{db}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)/(a*cos(d*x+c)+b*sin(d*x+c)),x)
```

```
[Out] 1/d/b*ln(a+b*tan(d*x+c))
```

maxima [B] time = 2.22, size = 103, normalized size = 2.51

$$\frac{\frac{\log\left(-a-\frac{2\,b\sin(dx+c)}{\cos(dx+c)+1}+\frac{a\sin(dx+c)^2}{(\cos(dx+c)+1)^2}\right)}{b}-\frac{\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)}{b}-\frac{\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}-1\right)}{b}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a*cos(d*x+c)+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] (log(-a - 2*b*sin(d*x + c)/(cos(d*x + c) + 1) + a*sin(d*x + c)^2/(cos(d*x +
 c) + 1)^2)/b - log(sin(d*x + c)/(cos(d*x + c) + 1) + 1)/b - log(sin(d*x +
c)/(cos(d*x + c) + 1) - 1)/b)/d
```

mupad [B] time = 0.72, size = 62, normalized size = 1.51

$$-\frac{2\,\mathrm{atanh}\left(\frac{b\,(b\,\cos(c+d\,x)-a\,\sin(c+d\,x))}{2\,\cos(c+d\,x)\,a^2+\sin(c+d\,x)\,a\,b+\cos(c+d\,x)\,b^2}\right)}{b\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)*(a*cos(c + d*x) + b*sin(c + d*x))),x)
```

```
[Out] -(2*atanh((b*(b*cos(c + d*x) - a*sin(c + d*x)))/(2*a^2*cos(c + d*x) + b^2*c
os(c + d*x) + a*b*sin(c + d*x))))/(b*d)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec(c+dx)}{a\cos(c+dx)+b\sin(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a*cos(d*x+c)+b*sin(d*x+c)),x)
```

```
[Out] Integral(sec(c + d*x)/(a*cos(c + d*x) + b*sin(c + d*x)), x)
```

3.117 $\int \frac{\sec^2(c+dx)}{a\cos(c+dx)+b\sin(c+dx)}\,dx$

Optimal. Leaf size=80

$$-\frac{\sqrt{a^2+b^2}\,\tanh^{-1}\left(\frac{b\cos(c+dx)-a\sin(c+dx)}{\sqrt{a^2+b^2}}\right)}{b^2d}-\frac{a\tanh^{-1}(\sin(c+dx))}{b^2d}+\frac{\sec(c+dx)}{bd}$$

```
[Out] -a*arctanh(sin(d*x+c))/b^2/d+sec(d*x+c)/b/d-arctanh((b*cos(d*x+c)-a*sin(d*x
+c))/(a^2+b^2)^(1/2))*(a^2+b^2)^(1/2)/b^2/d
```

Rubi [A] time = 0.08, antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3104, 3770, 3074, 206}

$$-\frac{\sqrt{a^2+b^2}\,\tanh^{-1}\left(\frac{b\cos(c+dx)-a\sin(c+dx)}{\sqrt{a^2+b^2}}\right)}{b^2d}-\frac{a\tanh^{-1}(\sin(c+dx))}{b^2d}+\frac{\sec(c+dx)}{bd}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^2/(a*Cos[c + d*x] + b*Sin[c + d*x]),x]
```

```
[Out] -((a*ArcTanh[Sin[c + d*x]])/(b^2*d)) - (Sqrt[a^2 + b^2]*ArcTanh[(b*Cos[c +
d*x] - a*Sin[c + d*x])/Sqrt[a^2 + b^2]])/(b^2*d) + Sec[c + d*x]/(b*d)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3074

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x
_Symbol] :> -Dist[d^(-1), Subst[Int[1/(a^2 + b^2 - x^2), x], x, b*Cos[c + d
*x] - a*Sin[c + d*x]], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

Rule 3104

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_)/(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin
[(c_.) + (d_.)*(x_)]), x_Symbol] :> -Simp[Cos[c + d*x]^(m + 1)/(b*d*(m + 1)
), x] + (-Dist[a/b^2, Int[Cos[c + d*x]^(m + 1), x], x] + Dist[(a^2 + b^2)/b
^2, Int[Cos[c + d*x]^(m + 2)/(a*Cos[c + d*x] + b*Sin[c + d*x]), x], x]) /;
FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0] && LtQ[m, -1]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\sec^2(c+dx)}{a\cos(c+dx)+b\sin(c+dx)}\,dx = \frac{\sec(c+dx)}{bd} - \frac{a\int \sec(c+dx)\,dx}{b^2} + \frac{\left(a^2+b^2\right)\int \frac{1}{a\cos(c+dx)+b\sin(c+dx)}\,dx}{b^2}$$

$$= -\frac{a\tanh^{-1}(\sin(c+dx))}{b^2d} + \frac{\sec(c+dx)}{bd} - \frac{\left(a^2+b^2\right)\text{Subst}\left(\int \frac{1}{a^2+b^2-x^2}\,dx, x$$

$$= -\frac{a\tanh^{-1}(\sin(c+dx))}{b^2d} - \frac{\sqrt{a^2+b^2}\tanh^{-1}\left(\frac{b\cos(c+dx)-a\sin(c+dx)}{\sqrt{a^2+b^2}}\right)}{b^2d} + \frac{\sec(c$$

Mathematica [A] time = 0.14, size = 109, normalized size = 1.36

$$\frac{2\sqrt{a^2+b^2}\tanh^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)-b}{\sqrt{a^2+b^2}}\right) + a\left(\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)-\log\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}\right.\right.\right.}{b^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^2/(a*Cos[c + d*x] + b*Sin[c + d*x]),x]
```

```
[Out] (2*Sqrt[a^2 + b^2]*ArcTanh[(-b + a*Tan[(c + d*x)/2])/Sqrt[a^2 + b^2]] + a*(
Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] - Log[Cos[(c + d*x)/2] + Sin[(c +
d*x)/2]]) + b*Sec[c + d*x])/(b^2*d)
```

fricas [B] time = 0.72, size = 191, normalized size = 2.39

$$-\frac{a\cos(dx+c)\log(\sin(dx+c)+1) - a\cos(dx+c)\log(-\sin(dx+c)+1) - \sqrt{a^2+b^2}\cos(dx+c)\log\left(-\frac{2ab\,c}{}\right.}{2\,b^2d\cos(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a*cos(d*x+c)+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/2*(a*cos(d*x + c)*log(sin(d*x + c) + 1) - a*cos(d*x + c)*log(-sin(d*x +
c) + 1) - sqrt(a^2 + b^2)*cos(d*x + c)*log(-(2*a*b*cos(d*x + c)*sin(d*x + c
```

```
) + (a^2 - b^2)*cos(d*x + c)^2 - 2*a^2 - b^2 + 2*sqrt(a^2 + b^2)*(b*cos(d*x
 + c) - a*sin(d*x + c)))/(2*a*b*cos(d*x + c)*sin(d*x + c) + (a^2 - b^2)*cos
(d*x + c)^2 + b^2)) - 2*b)/(b^2*d*cos(d*x + c))
```

giac [A] time = 0.32, size = 136, normalized size = 1.70

$$-\frac{\frac{a\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)}{b^2}-\frac{a\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)}{b^2}+\frac{\sqrt{a^2+b^2}\,\log\left(\frac{\left|2\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-2\,b-2\,\sqrt{a^2+b^2}\right|}{\left|2\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-2\,b+2\,\sqrt{a^2+b^2}\right|}\right)}{b^2}+\frac{2}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1\right)b}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a*cos(d*x+c)+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -(a*log(abs(tan(1/2*d*x + 1/2*c) + 1))/b^2 - a*log(abs(tan(1/2*d*x + 1/2*c)
 - 1))/b^2 + sqrt(a^2 + b^2)*log(abs(2*a*tan(1/2*d*x + 1/2*c) - 2*b - 2*sqr
t(a^2 + b^2))/abs(2*a*tan(1/2*d*x + 1/2*c) - 2*b + 2*sqrt(a^2 + b^2)))/b^2
+ 2/((tan(1/2*d*x + 1/2*c)^2 - 1)*b))/d
```

maple [B] time = 0.23, size = 174, normalized size = 2.18

$$-\frac{1}{db\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}+\frac{a\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{d\,b^2}+\frac{1}{db\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}-\frac{a\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{d\,b^2}+\frac{2\,\mathrm{arctanh}\left(\frac{2a\,\mathrm{t}}{}\right.}{d\,b^2\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2/(a*cos(d*x+c)+b*sin(d*x+c)),x)
```

```
[Out] -1/d/b/(tan(1/2*d*x+1/2*c)-1)+1/d*a/b^2*ln(tan(1/2*d*x+1/2*c)-1)+1/d/b/(tan
(1/2*d*x+1/2*c)+1)-1/d*a/b^2*ln(tan(1/2*d*x+1/2*c)+1)+2/d/b^2/(a^2+b^2)^(1/
2)*arctanh(1/2*(2*a*tan(1/2*d*x+1/2*c)-2*b)/(a^2+b^2)^(1/2))*a^2+2/d/(a^2+b
^2)^(1/2)*arctanh(1/2*(2*a*tan(1/2*d*x+1/2*c)-2*b)/(a^2+b^2)^(1/2))
```

maxima [B] time = 1.69, size = 163, normalized size = 2.04

$$-\frac{\frac{a\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)}{b^2}-\frac{a\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}-1\right)}{b^2}+\frac{\sqrt{a^2+b^2}\,\log\left(\frac{b-\frac{a\sin(dx+c)}{\cos(dx+c)+1}+\sqrt{a^2+b^2}}{b-\frac{a\sin(dx+c)}{\cos(dx+c)+1}-\sqrt{a^2+b^2}}\right)}{b^2}-\frac{2}{b-\frac{b\sin(dx+c)^2}{(\cos(dx+c)+1)^2}}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a*cos(d*x+c)+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -(a*log(sin(d*x + c)/(cos(d*x + c) + 1) + 1)/b^2 - a*log(sin(d*x + c)/(cos(
d*x + c) + 1) - 1)/b^2 + sqrt(a^2 + b^2)*log((b - a*sin(d*x + c)/(cos(d*x +
 c) + 1) + sqrt(a^2 + b^2))/(b - a*sin(d*x + c)/(cos(d*x + c) + 1) - sqrt(a
^2 + b^2)))/b^2 - 2/(b - b*sin(d*x + c)^2/(cos(d*x + c) + 1)^2))/d
```

mupad [B] time = 0.71, size = 310, normalized size = 3.88

$$\frac{2\,\mathrm{atanh}\left(\frac{64\,a^2\,\sqrt{a^2+b^2}}{64\,a^2\,b+\frac{64\,a^4}{b}+128\,a^3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+128\,a\,b^2\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}+\frac{128\,a\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\sqrt{a^2+b^2}}{64\,a^2+\frac{64\,a^4}{b^2}+\frac{128\,a^3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{b}+128\,a\,b\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}+\frac{64\,a^3}{64\,a^4+128\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right.}{b^2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^2*(a*cos(c + d*x) + b*sin(c + d*x))),x)
```

```
[Out] (2*atanh((64*a^2*(a^2 + b^2)^(1/2))/(64*a^2*b + (64*a^4)/b + 128*a^3*tan(c/
2 + (d*x)/2) + 128*a*b^2*tan(c/2 + (d*x)/2)) + (128*a*tan(c/2 + (d*x)/2)*(a
^2 + b^2)^(1/2))/(64*a^2 + (64*a^4)/b^2 + (128*a^3*tan(c/2 + (d*x)/2))/b +
128*a*b*tan(c/2 + (d*x)/2)) + (64*a^3*tan(c/2 + (d*x)/2)*(a^2 + b^2)^(1/2))
/(64*a^4 + 64*a^2*b^2 + 128*a*b^3*tan(c/2 + (d*x)/2) + 128*a^3*b*tan(c/2 +
(d*x)/2)))*(a^2 + b^2)^(1/2))/(b^2*d) - (2*a*atanh((64*a^2*tan(c/2 + (d*x)/
2))/(64*a^2 + (64*a^4)/b^2) + (64*a^4*tan(c/2 + (d*x)/2))/(64*a^4 + 64*a^2*
b^2)))/(b^2*d) - 2/(b*d*(tan(c/2 + (d*x)/2)^2 - 1))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec^2(c+dx)}{a\cos(c+dx)+b\sin(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2/(a*cos(d*x+c)+b*sin(d*x+c)),x)
```

```
[Out] Integral(sec(c + d*x)**2/(a*cos(c + d*x) + b*sin(c + d*x)), x)
```

3.118 $\int \frac{\sec^3(c+dx)}{a\cos(c+dx)+b\sin(c+dx)}\,dx$

Optimal. Leaf size=88

$$-\frac{(a^2+b^2)\log(\cos(c+dx))}{b^3d}+\frac{(a^2+b^2)\log(a\cos(c+dx)+b\sin(c+dx))}{b^3d}-\frac{a\tan(c+dx)}{b^2d}+\frac{\sec^2(c+dx)}{2bd}$$

```
[Out] -(a^2+b^2)*ln(cos(d*x+c))/b^3/d+(a^2+b^2)*ln(a*cos(d*x+c)+b*sin(d*x+c))/b^3
/d+1/2*sec(d*x+c)^2/b/d-a*tan(d*x+c)/b^2/d
```

Rubi [A] time = 0.14, antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {3104, 3767, 8, 3102, 3475, 3133}

$$-\frac{(a^2+b^2)\log(\cos(c+dx))}{b^3d}+\frac{(a^2+b^2)\log(a\cos(c+dx)+b\sin(c+dx))}{b^3d}-\frac{a\tan(c+dx)}{b^2d}+\frac{\sec^2(c+dx)}{2bd}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^3/(a*Cos[c + d*x] + b*Sin[c + d*x]),x]
```

```
[Out] -(((a^2 + b^2)*Log[Cos[c + d*x]])/(b^3*d)) + ((a^2 + b^2)*Log[a*Cos[c + d*x
] + b*Sin[c + d*x]])/(b^3*d) + Sec[c + d*x]^2/(2*b*d) - (a*Tan[c + d*x])/(b
^2*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3102

```
Int[1/(cos[(c_.) + (d_.)*(x_)]*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(
c_.) + (d_.)*(x_)])), x_Symbol] :> Dist[1/b, Int[Tan[c + d*x], x], x] + Dis
t[1/b, Int[(b*Cos[c + d*x] - a*Sin[c + d*x])/(a*Cos[c + d*x] + b*Sin[c + d*
x]), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

Rule 3104

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_)/(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin
[(c_.) + (d_.)*(x_)]), x_Symbol] :> -Simp[Cos[c + d*x]^(m + 1)/(b*d*(m + 1)
), x] + (-Dist[a/b^2, Int[Cos[c + d*x]^(m + 1), x], x] + Dist[(a^2 + b^2)/b
^2, Int[Cos[c + d*x]^(m + 2)/(a*Cos[c + d*x] + b*Sin[c + d*x]), x], x]) /;
FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0] && LtQ[m, -1]
```

Rule 3133

```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])
/((a_.) + cos[(d_.) + (e_.)*(x_)]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)]), x
_Symbol] :> Simp[((b*B + c*C)*x)/(b^2 + c^2), x] + Simp[((c*B - b*C)*Log[a
+ b*Cos[d + e*x] + c*Sin[d + e*x]])/(e*(b^2 + c^2)), x] /; FreeQ[{a, b, c,
d, e, A, B, C}, x] && NeQ[b^2 + c^2, 0] && EqQ[A*(b^2 + c^2) - a*(b*B + c*C
), 0]
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\int \frac{\sec^3(c+dx)}{a\cos(c+dx)+b\sin(c+dx)}\,dx = \frac{\sec^2(c+dx)}{2bd} - \frac{a\int \sec^2(c+dx)\,dx}{b^2} + \frac{\left(a^2+b^2\right)\int \frac{\sec(c+dx)}{a\cos(c+dx)+b\sin(c+dx)}\,dx}{b^2}$$
$$= \frac{\sec^2(c+dx)}{2bd} + \frac{\left(a^2+b^2\right)\int \frac{b\cos(c+dx)-a\sin(c+dx)}{a\cos(c+dx)+b\sin(c+dx)}\,dx}{b^3} + \frac{\left(a^2+b^2\right)\int \tan(c+a$$
$$= -\frac{\left(a^2+b^2\right)\log(\cos(c+dx))}{b^3d} + \frac{\left(a^2+b^2\right)\log(a\cos(c+dx)+b\sin(c+dx)}{b^3d}$$

Mathematica [A] time = 0.15, size = 52, normalized size = 0.59

$$\frac{\left(a^2+b^2\right)\log(a+b\tan(c+dx)) - ab\tan(c+dx) + \frac{1}{2}b^2\tan^2(c+dx)}{b^3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^3/(a*Cos[c + d*x] + b*Sin[c + d*x]),x]
```

```
[Out] ((a^2 + b^2)*Log[a + b*Tan[c + d*x]] - a*b*Tan[c + d*x] + (b^2*Tan[c + d*x]
^2)/2)/(b^3*d)
```

fricas [A] time = 0.66, size = 117, normalized size = 1.33

$$\frac{\left(a^{2}+b^{2}\right)\cos\left(dx+c\right)^{2}\log\left(2\,ab\cos\left(dx+c\right)\sin\left(dx+c\right)+\left(a^{2}-b^{2}\right)\cos\left(dx+c\right)^{2}+b^{2}\right)-\left(a^{2}+b^{2}\right)\cos\left(dx+c\right)^{2}}{2\,b^{3}d\cos\left(dx+c\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a*cos(d*x+c)+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/2*((a^2 + b^2)*cos(d*x + c)^2*log(2*a*b*cos(d*x + c)*sin(d*x + c) + (a^2
- b^2)*cos(d*x + c)^2 + b^2) - (a^2 + b^2)*cos(d*x + c)^2*log(cos(d*x + c)^
2) - 2*a*b*cos(d*x + c)*sin(d*x + c) + b^2)/(b^3*d*cos(d*x + c)^2)
```

giac [A] time = 4.59, size = 54, normalized size = 0.61

$$\frac{\frac{b\tan(dx+c)^{2}-2\,a\tan(dx+c)}{b^{2}}+\frac{2\left(a^{2}+b^{2}\right)\log(|b\tan(dx+c)+a|)}{b^{3}}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a*cos(d*x+c)+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/2*((b*tan(d*x + c)^2 - 2*a*tan(d*x + c))/b^2 + 2*(a^2 + b^2)*log(abs(b*ta
n(d*x + c) + a))/b^3)/d
```

maple [A] time = 0.23, size = 72, normalized size = 0.82

$$\frac{\tan^{2}\left(dx+c\right)}{2db}-\frac{a\tan\left(dx+c\right)}{b^{2}d}+\frac{\ln\left(a+b\tan\left(dx+c\right)\right)a^{2}}{d\,b^{3}}+\frac{\ln\left(a+b\tan\left(dx+c\right)\right)}{db}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^3/(a*cos(d*x+c)+b*sin(d*x+c)),x)
```

```
[Out] 1/2/d/b*tan(d*x+c)^2-a*tan(d*x+c)/b^2/d+1/d/b^3*ln(a+b*tan(d*x+c))*a^2+1/d/
b*ln(a+b*tan(d*x+c))
```

maxima [B] time = 0.60, size = 238, normalized size = 2.70

$$-\frac{\frac{2\left(\frac{a\sin(dx+c)}{\cos(dx+c)+1}-\frac{b\sin(dx+c)^{2}}{(\cos(dx+c)+1)^{2}}-\frac{a\sin(dx+c)^{3}}{(\cos(dx+c)+1)^{3}}\right)}{b^{2}-\frac{2\,b^{2}\sin(dx+c)^{2}}{(\cos(dx+c)+1)^{2}}+\frac{b^{2}\sin(dx+c)^{4}}{(\cos(dx+c)+1)^{4}}}-\frac{\left(a^{2}+b^{2}\right)\log\left(-a-\frac{2\,b\sin(dx+c)}{\cos(dx+c)+1}+\frac{a\sin(dx+c)^{2}}{(\cos(dx+c)+1)^{2}}\right)}{b^{3}}+\frac{\left(a^{2}+b^{2}\right)\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)}{b^{3}}+\frac{\left(a^{2}+b^{2}\right)\log\left(\frac{\sin}{\cos(}\right.}{b^{3}}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a*cos(d*x+c)+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -(2*(a*sin(d*x + c)/(cos(d*x + c) + 1) - b*sin(d*x + c)^2/(cos(d*x + c) + 1
)^2 - a*sin(d*x + c)^3/(cos(d*x + c) + 1)^3)/(b^2 - 2*b^2*sin(d*x + c)^2/(c
os(d*x + c) + 1)^2 + b^2*sin(d*x + c)^4/(cos(d*x + c) + 1)^4) - (a^2 + b^2)
*log(-a - 2*b*sin(d*x + c)/(cos(d*x + c) + 1) + a*sin(d*x + c)^2/(cos(d*x +
 c) + 1)^2)/b^3 + (a^2 + b^2)*log(sin(d*x + c)/(cos(d*x + c) + 1) + 1)/b^3
+ (a^2 + b^2)*log(sin(d*x + c)/(cos(d*x + c) + 1) - 1)/b^3)/d
```

mupad [**B**] time = 1.50, size = 300, normalized size = 3.41

$$\frac{2\,b^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+2\,a\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3-2\,a\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{d\,\left(b^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4-2\,b^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+b^3\right)}-\frac{a^2\,\mathrm{atan}\left(\frac{b^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\,1\mathrm{i}-b^2\,1\mathrm{i}+2\mathrm{i}\,a\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{-2\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+2\,a^2+2\,a\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)-b^2\tan\left(\frac{c}{2}+\right.}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^3*(a*cos(c + d*x) + b*sin(c + d*x))),x)
```

```
[Out] (2*b^2*tan(c/2 + (d*x)/2)^2 + 2*a*b*tan(c/2 + (d*x)/2)^3 - 2*a*b*tan(c/2 +
(d*x)/2))/(d*(b^3*tan(c/2 + (d*x)/2)^4 - 2*b^3*tan(c/2 + (d*x)/2)^2 + b^3))
 - (a^2*atan((b^2*tan(c/2 + (d*x)/2)^2*1i - b^2*1i + a*b*tan(c/2 + (d*x)/2)
*2i)/(2*a^2 - b^2*tan(c/2 + (d*x)/2)^2 - 2*a^2*tan(c/2 + (d*x)/2)^2 + b^2 +
 2*a*b*tan(c/2 + (d*x)/2)))*2i + b^2*atan((b^2*tan(c/2 + (d*x)/2)^2*1i - b^
2*1i + a*b*tan(c/2 + (d*x)/2)*2i)/(2*a^2 - b^2*tan(c/2 + (d*x)/2)^2 - 2*a^2
*tan(c/2 + (d*x)/2)^2 + b^2 + 2*a*b*tan(c/2 + (d*x)/2)))*2i)/(b^3*d)
```

sympy [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec^3(c+dx)}{a\cos(c+dx)+b\sin(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**3/(a*cos(d*x+c)+b*sin(d*x+c)),x)
```

```
[Out] Integral(sec(c + d*x)**3/(a*cos(c + d*x) + b*sin(c + d*x)), x)
```

3.119 $\int \frac{\sec^4(c+dx)}{a\cos(c+dx)+b\sin(c+dx)}\,dx$

Optimal. Leaf size=153

$$-\frac{a\left(a^2+b^2\right)\tanh^{-1}(\sin(c+dx))}{b^4d}-\frac{\left(a^2+b^2\right)^{3/2}\tanh^{-1}\left(\frac{b\cos(c+dx)-a\sin(c+dx)}{\sqrt{a^2+b^2}}\right)}{b^4d}+\frac{\left(a^2+b^2\right)\sec(c+dx)}{b^3d}-\frac{a\tanh^{-1}(\sin}{2b^2 d}$$

```
[Out] -1/2*a*arctanh(sin(d*x+c))/b^2/d-a*(a^2+b^2)*arctanh(sin(d*x+c))/b^4/d-(a^2
+b^2)^(3/2)*arctanh((b*cos(d*x+c)-a*sin(d*x+c))/(a^2+b^2)^(1/2))/b^4/d+(a^2
+b^2)*sec(d*x+c)/b^3/d+1/3*sec(d*x+c)^3/b/d-1/2*a*sec(d*x+c)*tan(d*x+c)/b^2
/d
```

Rubi [A] time = 0.16, antiderivative size = 153, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.179, Rules used = {3104, 3768, 3770, 3074, 206}

$$\frac{\left(a^2+b^2\right)\sec(c+dx)}{b^3d}-\frac{a\left(a^2+b^2\right)\tanh^{-1}(\sin(c+dx))}{b^4d}-\frac{\left(a^2+b^2\right)^{3/2}\tanh^{-1}\left(\frac{b\cos(c+dx)-a\sin(c+dx)}{\sqrt{a^2+b^2}}\right)}{b^4d}-\frac{a\tanh^{-1}(\sin(}{2b^2d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^4/(a*Cos[c + d*x] + b*Sin[c + d*x]),x]
```

```
[Out] -(a*ArcTanh[Sin[c + d*x]])/(2*b^2*d) - (a*(a^2 + b^2)*ArcTanh[Sin[c + d*x]]
)/(b^4*d) - ((a^2 + b^2)^(3/2)*ArcTanh[(b*Cos[c + d*x] - a*Sin[c + d*x])/Sq
rt[a^2 + b^2]])/(b^4*d) + ((a^2 + b^2)*Sec[c + d*x])/(b^3*d) + Sec[c + d*x]
^3/(3*b*d) - (a*Sec[c + d*x]*Tan[c + d*x])/(2*b^2*d)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3074

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x
_Symbol] :> -Dist[d^(-1), Subst[Int[1/(a^2 + b^2 - x^2), x], x, b*Cos[c + d
*x] - a*Sin[c + d*x]], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

Rule 3104

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_)/(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin
[(c_.) + (d_.)*(x_)]), x_Symbol] :> -Simp[Cos[c + d*x]^(m + 1)/(b*d*(m + 1)
```

```
), x] + (-Dist[a/b^2, Int[Cos[c + d*x]^(m + 1), x], x] + Dist[(a^2 + b^2)/b
^2, Int[Cos[c + d*x]^(m + 2)/(a*Cos[c + d*x] + b*Sin[c + d*x]), x], x]) /;
FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0] && LtQ[m, -1]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\sec^4(c+dx)}{a\cos(c+dx)+b\sin(c+dx)}\,dx = \frac{\sec^3(c+dx)}{3bd} - \frac{a\int \sec^3(c+dx)\,dx}{b^2} + \frac{\left(a^2+b^2\right)\int \frac{\sec^2(c+dx)}{a\cos(c+dx)+b\sin(c+dx)}\,dx}{b^2}$$

$$= \frac{\left(a^2+b^2\right)\sec(c+dx)}{b^3d} + \frac{\sec^3(c+dx)}{3bd} - \frac{a\sec(c+dx)\tan(c+dx)}{2b^2d} - \frac{a\int \text{se}$$

$$= -\frac{a\tanh^{-1}(\sin(c+dx))}{2b^2d} - \frac{a\left(a^2+b^2\right)\tanh^{-1}(\sin(c+dx))}{b^4d} + \frac{\left(a^2+b^2\right)\text{sec}}{b^3d}$$

$$= -\frac{a\tanh^{-1}(\sin(c+dx))}{2b^2d} - \frac{a\left(a^2+b^2\right)\tanh^{-1}(\sin(c+dx))}{b^4d} - \frac{\left(a^2+b^2\right)^{3/2}\text{t}}{}$$

Mathematica [B] time = 2.00, size = 321, normalized size = 2.10

$$\frac{48\left(a^2+b^2\right)^{3/2}\tanh^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)-b}{\sqrt{a^2+b^2}}\right) + \sec^3(c+dx)\left(6a^3\cos(3(c+dx))\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^4/(a*Cos[c + d*x] + b*Sin[c + d*x]),x]
```

```
[Out] (48*(a^2 + b^2)^(3/2)*ArcTanh[(-b + a*Tan[(c + d*x)/2])/Sqrt[a^2 + b^2]] +
Sec[c + d*x]^3*(12*a^2*b + 20*b^3 + 12*b*(a^2 + b^2)*Cos[2*(c + d*x)] + 6*a
```

```
^3*Cos[3*(c + d*x)]*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + 9*a*b^2*Cos[
3*(c + d*x)]*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + 9*a*(2*a^2 + 3*b^2)
*Cos[c + d*x]*(Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] - Log[Cos[(c + d*x)
/2] + Sin[(c + d*x)/2]]) - 6*a^3*Cos[3*(c + d*x)]*Log[Cos[(c + d*x)/2] + Si
n[(c + d*x)/2]] - 9*a*b^2*Cos[3*(c + d*x)]*Log[Cos[(c + d*x)/2] + Sin[(c +
d*x)/2]] - 6*a*b^2*Sin[2*(c + d*x)]))/(24*b^4*d)
```

fricas [A] time = 0.72, size = 259, normalized size = 1.69

$$\frac{6\left(a^2+b^2\right)^{\frac{3}{2}}\cos\left(dx+c\right)^3\log\left(-\frac{2\,ab\cos(dx+c)\sin(dx+c)+\left(a^2-b^2\right)\cos(dx+c)^2-2\,a^2-b^2+2\,\sqrt{a^2+b^2}\,\left(b\cos(dx+c)-a\sin(dx+c)\right)}{2\,ab\cos(dx+c)\sin(dx+c)+\left(a^2-b^2\right)\cos(dx+c)^2+b^2}\right)-3\left(2\,a^3\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4/(a*cos(d*x+c)+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/12*(6*(a^2 + b^2)^(3/2)*cos(d*x + c)^3*log(-(2*a*b*cos(d*x + c)*sin(d*x +
 c) + (a^2 - b^2)*cos(d*x + c)^2 - 2*a^2 - b^2 + 2*sqrt(a^2 + b^2)*(b*cos(d
*x + c) - a*sin(d*x + c)))/(2*a*b*cos(d*x + c)*sin(d*x + c) + (a^2 - b^2)*c
os(d*x + c)^2 + b^2)) - 3*(2*a^3 + 3*a*b^2)*cos(d*x + c)^3*log(sin(d*x + c)
 + 1) + 3*(2*a^3 + 3*a*b^2)*cos(d*x + c)^3*log(-sin(d*x + c) + 1) - 6*a*b^2
*cos(d*x + c)*sin(d*x + c) + 4*b^3 + 12*(a^2*b + b^3)*cos(d*x + c)^2)/(b^4*
d*cos(d*x + c)^3)
```

giac [A] time = 3.04, size = 278, normalized size = 1.82

$$-\frac{\frac{3\left(2\,a^3+3\,ab^2\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)}{b^4}-\frac{3\left(2\,a^3+3\,ab^2\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)}{b^4}+\frac{6\left(a^4+2\,a^2b^2+b^4\right)\log\left(\frac{\left|2\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-2\,b-2\,\sqrt{a^2+b^2}\right|}{\left|2\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-2\,b+2\,\sqrt{a^2+b^2}\right|}\right)}{\sqrt{a^2+b^2}\,b^4}+\frac{2\left(3\right.}{}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4/(a*cos(d*x+c)+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/6*(3*(2*a^3 + 3*a*b^2)*log(abs(tan(1/2*d*x + 1/2*c) + 1))/b^4 - 3*(2*a^3
 + 3*a*b^2)*log(abs(tan(1/2*d*x + 1/2*c) - 1))/b^4 + 6*(a^4 + 2*a^2*b^2 + b
^4)*log(abs(2*a*tan(1/2*d*x + 1/2*c) - 2*b - 2*sqrt(a^2 + b^2))/abs(2*a*tan
(1/2*d*x + 1/2*c) - 2*b + 2*sqrt(a^2 + b^2)))/(sqrt(a^2 + b^2)*b^4) + 2*(3*
a*b*tan(1/2*d*x + 1/2*c)^5 + 6*a^2*tan(1/2*d*x + 1/2*c)^4 + 12*b^2*tan(1/2*
d*x + 1/2*c)^4 - 12*a^2*tan(1/2*d*x + 1/2*c)^2 - 12*b^2*tan(1/2*d*x + 1/2*c
)^2 - 3*a*b*tan(1/2*d*x + 1/2*c) + 6*a^2 + 8*b^2)/((tan(1/2*d*x + 1/2*c)^2
- 1)^3*b^3))/d
```

maple [B] time = 0.24, size = 488, normalized size = 3.19

$$-\frac{1}{3db\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{3}}-\frac{a}{2d\,b^{2}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{2}}-\frac{1}{2db\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{2}}-\frac{a^{2}}{d\,b^{3}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}-\frac{}{2d\,b^{2}\left(\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^4/(a*cos(d*x+c)+b*sin(d*x+c)),x)
```

```
[Out] -1/3/d/b/(tan(1/2*d*x+1/2*c)-1)^3-1/2/d/b^2/(tan(1/2*d*x+1/2*c)-1)^2*a-1/2/
d/b/(tan(1/2*d*x+1/2*c)-1)^2-1/d/b^3/(tan(1/2*d*x+1/2*c)-1)*a^2-1/2/d/b^2/(
tan(1/2*d*x+1/2*c)-1)*a-3/2/d/b/(tan(1/2*d*x+1/2*c)-1)+1/d*a^3/b^4*ln(tan(1
/2*d*x+1/2*c)-1)+3/2/d*a/b^2*ln(tan(1/2*d*x+1/2*c)-1)+1/3/d/b/(tan(1/2*d*x+
1/2*c)+1)^3-1/2/d/b/(tan(1/2*d*x+1/2*c)+1)^2+1/2/d/b^2/(tan(1/2*d*x+1/2*c)+
1)^2*a+1/d/b^3/(tan(1/2*d*x+1/2*c)+1)*a^2-1/2/d/b^2/(tan(1/2*d*x+1/2*c)+1)*
a+3/2/d/b/(tan(1/2*d*x+1/2*c)+1)-1/d*a^3/b^4*ln(tan(1/2*d*x+1/2*c)+1)-3/2/d
*a/b^2*ln(tan(1/2*d*x+1/2*c)+1)+2/d/b^4/(a^2+b^2)^(1/2)*arctanh(1/2*(2*a*ta
n(1/2*d*x+1/2*c)-2*b)/(a^2+b^2)^(1/2))*a^4+4/d/b^2/(a^2+b^2)^(1/2)*arctanh(
1/2*(2*a*tan(1/2*d*x+1/2*c)-2*b)/(a^2+b^2)^(1/2))*a^2+2/d/(a^2+b^2)^(1/2)*a
rctanh(1/2*(2*a*tan(1/2*d*x+1/2*c)-2*b)/(a^2+b^2)^(1/2))
```

maxima [B] time = 0.42, size = 361, normalized size = 2.36

$$\frac{\frac{2\left(6\,a^{2}+8\,b^{2}-\frac{3\,ab\sin(dx+c)}{\cos(dx+c)+1}+\frac{3\,ab\sin(dx+c)^{5}}{(\cos(dx+c)+1)^{5}}-\frac{12\left(a^{2}+b^{2}\right)\sin(dx+c)^{2}}{(\cos(dx+c)+1)^{2}}+\frac{6\left(a^{2}+2\,b^{2}\right)\sin(dx+c)^{4}}{(\cos(dx+c)+1)^{4}}\right)}{b^{3}-\frac{3\,b^{3}\sin(dx+c)^{2}}{(\cos(dx+c)+1)^{2}}+\frac{3\,b^{3}\sin(dx+c)^{4}}{(\cos(dx+c)+1)^{4}}-\frac{b^{3}\sin(dx+c)^{6}}{(\cos(dx+c)+1)^{6}}}-\frac{3\left(2\,a^{3}+3\,ab^{2}\right)\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)}{b^{4}}+\frac{3\left(2\,a^{3}+3\,ab^{2}\right)\log\left(\right.}{b^{4}}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4/(a*cos(d*x+c)+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/6*(2*(6*a^2 + 8*b^2 - 3*a*b*sin(d*x + c)/(cos(d*x + c) + 1) + 3*a*b*sin(d
*x + c)^5/(cos(d*x + c) + 1)^5 - 12*(a^2 + b^2)*sin(d*x + c)^2/(cos(d*x + c
) + 1)^2 + 6*(a^2 + 2*b^2)*sin(d*x + c)^4/(cos(d*x + c) + 1)^4)/(b^3 - 3*b^
3*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 3*b^3*sin(d*x + c)^4/(cos(d*x + c)
+ 1)^4 - b^3*sin(d*x + c)^6/(cos(d*x + c) + 1)^6) - 3*(2*a^3 + 3*a*b^2)*log
(sin(d*x + c)/(cos(d*x + c) + 1) + 1)/b^4 + 3*(2*a^3 + 3*a*b^2)*log(sin(d*x
 + c)/(cos(d*x + c) + 1) - 1)/b^4 - 6*(a^4 + 2*a^2*b^2 + b^4)*log((b - a*si
n(d*x + c)/(cos(d*x + c) + 1) + sqrt(a^2 + b^2))/(b - a*sin(d*x + c)/(cos(d
*x + c) + 1) - sqrt(a^2 + b^2)))/(sqrt(a^2 + b^2)*b^4))/d
```

mupad [B] time = 2.21, size = 724, normalized size = 4.73

$$b^3\left(\cos(c+d\,x)+\frac{\cos(2\,c+2\,d\,x)}{2}+\frac{\cos(3\,c+3\,d\,x)}{3}+\frac{5}{6}\right)-b^2\left(\frac{a\,\sin(2\,c+2\,d\,x)}{4}+\frac{3\,a\,\mathrm{atanh}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)\cos(3\,c+3\,d\,x)}{4}+\frac{9\,a\,\cos(c+d\,x)}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^4*(a*cos(c + d*x) + b*sin(c + d*x))),x)
```

```
[Out] (b^3*(cos(c + d*x) + cos(2*c + 2*d*x)/2 + cos(3*c + 3*d*x)/3 + 5/6) - b^2*(
(a*sin(2*c + 2*d*x))/4 + (3*a*atanh(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2))*
cos(3*c + 3*d*x))/4 + (9*a*cos(c + d*x)*atanh(sin(c/2 + (d*x)/2)/cos(c/2 +
(d*x)/2)))/4) + b*((3*a^2*cos(c + d*x))/4 + a^2/2 + (a^2*cos(2*c + 2*d*x))/
2 + (a^2*cos(3*c + 3*d*x))/4) + (atanh((a^2*sin(c/2 + (d*x)/2)*(a^6 + b^6 +
 3*a^2*b^4 + 3*a^4*b^2)^(1/2) + 2*b^2*sin(c/2 + (d*x)/2)*(a^6 + b^6 + 3*a^2
*b^4 + 3*a^4*b^2)^(1/2) + a*b*cos(c/2 + (d*x)/2)*(a^6 + b^6 + 3*a^2*b^4 + 3
*a^4*b^2)^(1/2))/(a^5*cos(c/2 + (d*x)/2) + 2*b^5*sin(c/2 + (d*x)/2) + a*b^4
*cos(c/2 + (d*x)/2) + 2*a^4*b*sin(c/2 + (d*x)/2) + 2*a^3*b^2*cos(c/2 + (d*x
)/2) + 4*a^2*b^3*sin(c/2 + (d*x)/2)))*cos(3*c + 3*d*x)*((a^2 + b^2)^3)^(1/2
))/2 - (3*a^3*cos(c + d*x)*atanh(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/2
- (a^3*atanh(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2))*cos(3*c + 3*d*x))/2 + (
3*cos(c + d*x)*atanh((a^2*sin(c/2 + (d*x)/2)*(a^6 + b^6 + 3*a^2*b^4 + 3*a^4
*b^2)^(1/2) + 2*b^2*sin(c/2 + (d*x)/2)*(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2)^
(1/2) + a*b*cos(c/2 + (d*x)/2)*(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2)^(1/2))/(
a^5*cos(c/2 + (d*x)/2) + 2*b^5*sin(c/2 + (d*x)/2) + a*b^4*cos(c/2 + (d*x)/2
) + 2*a^4*b*sin(c/2 + (d*x)/2) + 2*a^3*b^2*cos(c/2 + (d*x)/2) + 4*a^2*b^3*s
in(c/2 + (d*x)/2)))*((a^2 + b^2)^3)^(1/2))/2)/(b^4*d*((3*cos(c + d*x))/4 +
cos(3*c + 3*d*x)/4))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec^4(c+dx)}{a\cos(c+dx)+b\sin(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**4/(a*cos(d*x+c)+b*sin(d*x+c)),x)
```

```
[Out] Integral(sec(c + d*x)**4/(a*cos(c + d*x) + b*sin(c + d*x)), x)
```

3.120 $\int \frac{\sec^5(c+dx)}{a\cos(c+dx)+b\sin(c+dx)}\,dx$

Optimal. Leaf size=158

$$-\frac{\left(a^2+b^2\right)^2\log(\cos(c+dx))}{b^5d}+\frac{\left(a^2+b^2\right)^2\log(a\cos(c+dx)+b\sin(c+dx))}{b^5d}-\frac{a\left(a^2+b^2\right)\tan(c+dx)}{b^4d}+\frac{\left(a^2+b^2\right)}{2}$$

```
[Out] -(a^2+b^2)^2*ln(cos(d*x+c))/b^5/d+(a^2+b^2)^2*ln(a*cos(d*x+c)+b*sin(d*x+c))
/b^5/d+1/2*(a^2+b^2)*sec(d*x+c)^2/b^3/d+1/4*sec(d*x+c)^4/b/d-a*tan(d*x+c)/b
^2/d-a*(a^2+b^2)*tan(d*x+c)/b^4/d-1/3*a*tan(d*x+c)^3/b^2/d
```

Rubi [A] time = 0.22, antiderivative size = 158, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {3104, 3767, 8, 3102, 3475, 3133}

$$-\frac{a\left(a^2+b^2\right)\tan(c+dx)}{b^4d}+\frac{\left(a^2+b^2\right)\sec^2(c+dx)}{2b^3d}-\frac{\left(a^2+b^2\right)^2\log(\cos(c+dx))}{b^5d}+\frac{\left(a^2+b^2\right)^2\log(a\cos(c+dx)+b}{b^5d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^5/(a*Cos[c + d*x] + b*Sin[c + d*x]),x]
```

```
[Out] -(((a^2 + b^2)^2*Log[Cos[c + d*x]])/(b^5*d)) + ((a^2 + b^2)^2*Log[a*Cos[c +
 d*x] + b*Sin[c + d*x]])/(b^5*d) + ((a^2 + b^2)*Sec[c + d*x]^2)/(2*b^3*d) +
 Sec[c + d*x]^4/(4*b*d) - (a*Tan[c + d*x])/(b^2*d) - (a*(a^2 + b^2)*Tan[c +
 d*x])/(b^4*d) - (a*Tan[c + d*x]^3)/(3*b^2*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3102

```
Int[1/(cos[(c_.) + (d_.)*(x_)]*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(
c_.) + (d_.)*(x_)])), x_Symbol] :> Dist[1/b, Int[Tan[c + d*x], x], x] + Dis
t[1/b, Int[(b*Cos[c + d*x] - a*Sin[c + d*x])/(a*Cos[c + d*x] + b*Sin[c + d*
x]), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

Rule 3104

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_)/(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin
[(c_.) + (d_.)*(x_)]), x_Symbol] :> -Simp[Cos[c + d*x]^(m + 1)/(b*d*(m + 1)
), x] + (-Dist[a/b^2, Int[Cos[c + d*x]^(m + 1), x], x] + Dist[(a^2 + b^2)/b
^2, Int[Cos[c + d*x]^(m + 2)/(a*Cos[c + d*x] + b*Sin[c + d*x]), x], x]) /;
```

```
FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0] && LtQ[m, -1]
```

Rule 3133

```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])
/((a_.) + cos[(d_.) + (e_.)*(x_)]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)]), x
_Symbol] :> Simp[((b*B + c*C)*x)/(b^2 + c^2), x] + Simp[((c*B - b*C)*Log[a
+ b*Cos[d + e*x] + c*Sin[d + e*x]])/(e*(b^2 + c^2)), x] /; FreeQ[{a, b, c,
d, e, A, B, C}, x] && NeQ[b^2 + c^2, 0] && EqQ[A*(b^2 + c^2) - a*(b*B + c*C
), 0]
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\int \frac{\sec^5(c+dx)}{a\cos(c+dx)+b\sin(c+dx)}\,dx = \frac{\sec^4(c+dx)}{4bd} - \frac{a\int \sec^4(c+dx)\,dx}{b^2} + \frac{\left(a^2+b^2\right)\int \frac{\sec^3(c+dx)}{a\cos(c+dx)+b\sin(c+dx)}\,dx}{b^2}$$

$$= \frac{\left(a^2+b^2\right)\sec^2(c+dx)}{2b^3d} + \frac{\sec^4(c+dx)}{4bd} - \frac{\left(a\left(a^2+b^2\right)\right)\int \sec^2(c+dx)\,dx}{b^4} + \cdots$$

$$= \frac{\left(a^2+b^2\right)\sec^2(c+dx)}{2b^3d} + \frac{\sec^4(c+dx)}{4bd} - \frac{a\tan(c+dx)}{b^2d} - \frac{a\tan^3(c+dx)}{3b^2d} + \cdots$$

$$= -\frac{\left(a^2+b^2\right)^2\log(\cos(c+dx))}{b^5d} + \frac{\left(a^2+b^2\right)^2\log(a\cos(c+dx)+b\sin(c+dx\cdots}{b^5d}$$

Mathematica [A] time = 1.17, size = 99, normalized size = 0.63

$$\frac{6b^2\left(a^2+b^2\right)\tan^2(c+dx) - 12ab\left(a^2+2b^2\right)\tan(c+dx) + 12\left(a^2+b^2\right)^2\log(a+b\tan(c+dx)) - 4ab^3\tan^3(c+dx\cdots}{12b^5d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^5/(a*Cos[c + d*x] + b*Sin[c + d*x]),x]
```

```
[Out] (12*(a^2 + b^2)^2*Log[a + b*Tan[c + d*x]] + 3*b^4*Sec[c + d*x]^4 - 12*a*b*(
a^2 + 2*b^2)*Tan[c + d*x] + 6*b^2*(a^2 + b^2)*Tan[c + d*x]^2 - 4*a*b^3*Tan[
c + d*x]^3)/(12*b^5*d)
```

fricas [A] time = 0.45, size = 183, normalized size = 1.16

$$\frac{6\left(a^4+2\,a^2b^2+b^4\right)\cos\left(dx+c\right)^4\log\left(2\,ab\cos\left(dx+c\right)\sin\left(dx+c\right)+\left(a^2-b^2\right)\cos\left(dx+c\right)^2+b^2\right)-6\left(a^4+2\,a^2\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5/(a*cos(d*x+c)+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/12*(6*(a^4 + 2*a^2*b^2 + b^4)*cos(d*x + c)^4*log(2*a*b*cos(d*x + c)*sin(d
*x + c) + (a^2 - b^2)*cos(d*x + c)^2 + b^2) - 6*(a^4 + 2*a^2*b^2 + b^4)*cos
(d*x + c)^4*log(cos(d*x + c)^2) + 3*b^4 + 6*(a^2*b^2 + b^4)*cos(d*x + c)^2
- 4*(a*b^3*cos(d*x + c) + (3*a^3*b + 5*a*b^3)*cos(d*x + c)^3)*sin(d*x + c))
/(b^5*d*cos(d*x + c)^4)
```

giac [A] time = 4.89, size = 120, normalized size = 0.76

$$\frac{\frac{3\,b^3\tan(dx+c)^4-4\,ab^2\tan(dx+c)^3+6\,a^2b\tan(dx+c)^2+12\,b^3\tan(dx+c)^2-12\,a^3\tan(dx+c)-24\,ab^2\tan(dx+c)}{b^4}+\frac{12\left(a^4+2\,a^2b^2+b^4\right)\log(|b\tan(dx+c)+a}{b^5}}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5/(a*cos(d*x+c)+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/12*((3*b^3*tan(d*x + c)^4 - 4*a*b^2*tan(d*x + c)^3 + 6*a^2*b*tan(d*x + c)
^2 + 12*b^3*tan(d*x + c)^2 - 12*a^3*tan(d*x + c) - 24*a*b^2*tan(d*x + c))/b
^4 + 12*(a^4 + 2*a^2*b^2 + b^4)*log(abs(b*tan(d*x + c) + a))/b^5)/d
```

maple [A] time = 0.23, size = 162, normalized size = 1.03

$$\frac{\tan^4(dx+c)}{4db}-\frac{a\left(\tan^3(dx+c)\right)}{3b^2d}+\frac{\left(\tan^2(dx+c)\right)a^2}{2d\,b^3}+\frac{\tan^2(dx+c)}{db}-\frac{\tan(dx+c)\,a^3}{d\,b^4}-\frac{2a\tan(dx+c)}{b^2d}+\frac{\ln(a+b\,\mathrm{t}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^5/(a*cos(d*x+c)+b*sin(d*x+c)),x)
```

```
[Out] 1/4/d/b*tan(d*x+c)^4-1/3*a*tan(d*x+c)^3/b^2/d+1/2/d/b^3*tan(d*x+c)^2*a^2+1/
d/b*tan(d*x+c)^2-1/d/b^4*tan(d*x+c)*a^3-2*a*tan(d*x+c)/b^2/d+1/d/b^5*ln(a+b
*tan(d*x+c))*a^4+2/d/b^3*ln(a+b*tan(d*x+c))*a^2+1/d/b*ln(a+b*tan(d*x+c))
```

maxima [B] time = 0.34, size = 462, normalized size = 2.92

$$-\frac{2\left(\frac{3\left(a^3+2ab^2\right)\sin(dx+c)}{\cos(dx+c)+1}-\frac{3\left(a^2b+2b^3\right)\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-\frac{\left(9a^3+14ab^2\right)\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{6\left(a^2b+b^3\right)\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{\left(9a^3+14ab^2\right)\sin(dx+c)^5}{(\cos(dx+c)+1)^5}-\frac{3\left(a^2b+2b^3\right)\sin(dx+c)^6}{(\cos(dx+c)+1)^6}-\frac{3\left(a^3+2ab^2\right)\sin}{(\cos(dx+c}\right.}{b^4-\frac{4b^4\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{6b^4\sin(dx+c)^4}{(\cos(dx+c)+1)^4}-\frac{4b^4\sin(dx+c)^6}{(\cos(dx+c)+1)^6}+\frac{b^4\sin(dx+c)^8}{(\cos(dx+c)+1)^8}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5/(a*cos(d*x+c)+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/3*(2*(3*(a^3 + 2*a*b^2)*sin(d*x + c)/(cos(d*x + c) + 1) - 3*(a^2*b + 2*b
^3)*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 - (9*a^3 + 14*a*b^2)*sin(d*x + c)^3
/(cos(d*x + c) + 1)^3 + 6*(a^2*b + b^3)*sin(d*x + c)^4/(cos(d*x + c) + 1)^4
 + (9*a^3 + 14*a*b^2)*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 3*(a^2*b + 2*b^
3)*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 - 3*(a^3 + 2*a*b^2)*sin(d*x + c)^7/(
cos(d*x + c) + 1)^7)/(b^4 - 4*b^4*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 6*b
^4*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 - 4*b^4*sin(d*x + c)^6/(cos(d*x + c)
 + 1)^6 + b^4*sin(d*x + c)^8/(cos(d*x + c) + 1)^8) - 3*(a^4 + 2*a^2*b^2 + b
^4)*log(-a - 2*b*sin(d*x + c)/(cos(d*x + c) + 1) + a*sin(d*x + c)^2/(cos(d*
x + c) + 1)^2)/b^5 + 3*(a^4 + 2*a^2*b^2 + b^4)*log(sin(d*x + c)/(cos(d*x +
c) + 1) + 1)/b^5 + 3*(a^4 + 2*a^2*b^2 + b^4)*log(sin(d*x + c)/(cos(d*x + c)
 + 1) - 1)/b^5)/d
```

mupad [B] time = 3.68, size = 575, normalized size = 3.64

$$\frac{\left(6a^3b+12ab^3\right)\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^7+\left(6a^2b^2+12b^4\right)\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^6+\left(-18a^3b-28ab^3\right)\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^5+\left(-12a^2\right.}{d\left(3b^5\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^8-12b^5\tan\left(\frac{c}{2}+\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^5*(a*cos(c + d*x) + b*sin(c + d*x))),x)
```

```
[Out] (tan(c/2 + (d*x)/2)^2*(12*b^4 + 6*a^2*b^2) - tan(c/2 + (d*x)/2)*(12*a*b^3 +
 6*a^3*b) + tan(c/2 + (d*x)/2)^6*(12*b^4 + 6*a^2*b^2) - tan(c/2 + (d*x)/2)^
4*(12*b^4 + 12*a^2*b^2) + tan(c/2 + (d*x)/2)^7*(12*a*b^3 + 6*a^3*b) + tan(c
/2 + (d*x)/2)^3*(28*a*b^3 + 18*a^3*b) - tan(c/2 + (d*x)/2)^5*(28*a*b^3 + 18
*a^3*b))/(d*(18*b^5*tan(c/2 + (d*x)/2)^4 - 12*b^5*tan(c/2 + (d*x)/2)^2 - 12
*b^5*tan(c/2 + (d*x)/2)^6 + 3*b^5*tan(c/2 + (d*x)/2)^8 + 3*b^5)) - (a^4*ata
n((b^2*tan(c/2 + (d*x)/2)^2*1i - b^2*1i + a*b*tan(c/2 + (d*x)/2)*2i)/(2*a^2
 - b^2*tan(c/2 + (d*x)/2)^2 - 2*a^2*tan(c/2 + (d*x)/2)^2 + b^2 + 2*a*b*tan(
c/2 + (d*x)/2))*2i + b^4*atan((b^2*tan(c/2 + (d*x)/2)^2*1i - b^2*1i + a*b*
```

```
tan(c/2 + (d*x)/2)*2i)/(2*a^2 - b^2*tan(c/2 + (d*x)/2)^2 - 2*a^2*tan(c/2 +
(d*x)/2)^2 + b^2 + 2*a*b*tan(c/2 + (d*x)/2)))*2i + a^2*b^2*atan((b^2*tan(c/
2 + (d*x)/2)^2*1i - b^2*1i + a*b*tan(c/2 + (d*x)/2)*2i)/(2*a^2 - b^2*tan(c/
2 + (d*x)/2)^2 - 2*a^2*tan(c/2 + (d*x)/2)^2 + b^2 + 2*a*b*tan(c/2 + (d*x)/2
)))*4i)/(b^5*d)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec^5(c+dx)}{a\cos(c+dx)+b\sin(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**5/(a*cos(d*x+c)+b*sin(d*x+c)),x)
```

```
[Out] Integral(sec(c + d*x)**5/(a*cos(c + d*x) + b*sin(c + d*x)), x)
```

3.121 $\int \frac{\sec^6(c+dx)}{a\cos(c+dx)+b\sin(c+dx)}\,dx$

Optimal. Leaf size=262

$$-\frac{a\left(a^2+b^2\right)^2\tanh^{-1}(\sin(c+dx))}{b^6 d}-\frac{\left(a^2+b^2\right)^{5/2}\tanh^{-1}\left(\frac{b\cos(c+dx)-a\sin(c+dx)}{\sqrt{a^2+b^2}}\right)}{b^6 d}+\frac{\left(a^2+b^2\right)^2\sec(c+dx)}{b^5 d}-\frac{a\left(a^2+b^2\right)\ldots}{\ldots}$$

```
[Out] -3/8*a*arctanh(sin(d*x+c))/b^2/d-1/2*a*(a^2+b^2)*arctanh(sin(d*x+c))/b^4/d-
a*(a^2+b^2)^2*arctanh(sin(d*x+c))/b^6/d-(a^2+b^2)^(5/2)*arctanh((b*cos(d*x+
c)-a*sin(d*x+c))/(a^2+b^2)^(1/2))/b^6/d+(a^2+b^2)^2*sec(d*x+c)/b^5/d+1/3*(a
^2+b^2)*sec(d*x+c)^3/b^3/d+1/5*sec(d*x+c)^5/b/d-3/8*a*sec(d*x+c)*tan(d*x+c)
/b^2/d-1/2*a*(a^2+b^2)*sec(d*x+c)*tan(d*x+c)/b^4/d-1/4*a*sec(d*x+c)^3*tan(d
*x+c)/b^2/d
```

Rubi [A] time = 0.26, antiderivative size = 262, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 5, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.179$, Rules used = {3104, 3768, 3770, 3074, 206}

$$\frac{\left(a^2+b^2\right)\sec^3(c+dx)}{3b^3 d}+\frac{\left(a^2+b^2\right)^2\sec(c+dx)}{b^5 d}-\frac{a\left(a^2+b^2\right)^2\tanh^{-1}(\sin(c+dx))}{b^6 d}-\frac{a\left(a^2+b^2\right)\tanh^{-1}(\sin(c+dx))}{2b^4 d}\ldots$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^6/(a*Cos[c + d*x] + b*Sin[c + d*x]),x]
```

```
[Out] (-3*a*ArcTanh[Sin[c + d*x]])/(8*b^2*d) - (a*(a^2 + b^2)*ArcTanh[Sin[c + d*x
]])/(2*b^4*d) - (a*(a^2 + b^2)^2*ArcTanh[Sin[c + d*x]])/(b^6*d) - ((a^2 + b
^2)^(5/2)*ArcTanh[(b*Cos[c + d*x] - a*Sin[c + d*x])/Sqrt[a^2 + b^2]])/(b^6*
d) + ((a^2 + b^2)^2*Sec[c + d*x])/(b^5*d) + ((a^2 + b^2)*Sec[c + d*x]^3)/(3
*b^3*d) + Sec[c + d*x]^5/(5*b*d) - (3*a*Sec[c + d*x]*Tan[c + d*x])/(8*b^2*d
) - (a*(a^2 + b^2)*Sec[c + d*x]*Tan[c + d*x])/(2*b^4*d) - (a*Sec[c + d*x]^3
*Tan[c + d*x])/(4*b^2*d)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3074

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x
_Symbol] :> -Dist[d^(-1), Subst[Int[1/(a^2 + b^2 - x^2), x], x, b*Cos[c + d
```

```
*x] - a*Sin[c + d*x]], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

Rule 3104

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_)/(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin
[(c_.) + (d_.)*(x_)]), x_Symbol] :> -Simp[Cos[c + d*x]^(m + 1)/(b*d*(m + 1)
), x] + (-Dist[a/b^2, Int[Cos[c + d*x]^(m + 1), x], x] + Dist[(a^2 + b^2)/b
^2, Int[Cos[c + d*x]^(m + 2)/(a*Cos[c + d*x] + b*Sin[c + d*x]), x], x]) /;
FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0] && LtQ[m, -1]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\sec^6(c+dx)}{a\cos(c+dx)+b\sin(c+dx)}\,dx = \frac{\sec^5(c+dx)}{5bd} - \frac{a\int \sec^5(c+dx)\,dx}{b^2} + \frac{\left(a^2+b^2\right)\int \frac{\sec^4(c+dx)}{a\cos(c+dx)+b\sin(c+dx)}\,dx}{b^2}$$

$$= \frac{\left(a^2+b^2\right)\sec^3(c+dx)}{3b^3d} + \frac{\sec^5(c+dx)}{5bd} - \frac{a\sec^3(c+dx)\tan(c+dx)}{4b^2d} - \frac{(3a}{}$$

$$= \frac{\left(a^2+b^2\right)^2\sec(c+dx)}{b^5d} + \frac{\left(a^2+b^2\right)\sec^3(c+dx)}{3b^3d} + \frac{\sec^5(c+dx)}{5bd} - \frac{3a\sec(}{}$$

$$= -\frac{3a\tanh^{-1}(\sin(c+dx))}{8b^2d} - \frac{a\left(a^2+b^2\right)\tanh^{-1}(\sin(c+dx))}{2b^4d} - \frac{a\left(a^2+b^2\right)^2}{}$$

$$= -\frac{3a\tanh^{-1}(\sin(c+dx))}{8b^2d} - \frac{a\left(a^2+b^2\right)\tanh^{-1}(\sin(c+dx))}{2b^4d} - \frac{a\left(a^2+b^2\right)^2}{}$$

Mathematica [B] time = 5.03, size = 661, normalized size = 2.52

$$\sec(c+dx)(a\cos(c+dx)+b\sin(c+dx))\left(240a^4b+520a^2b^3+480\left(a^2+b^2\right)^{5/2}\tanh^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)-b}{\sqrt{a^2+b^2}}\right)+\frac{2b^3(20a}{\left(\cos\left(\frac{1}{2}(\right.\right.}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^6/(a*Cos[c + d*x] + b*Sin[c + d*x]),x]
```

```
[Out] (Sec[c + d*x]*(240*a^4*b + 520*a^2*b^3 + 298*b^5 + 480*(a^2 + b^2)^(5/2)*Ar
cTanh[(-b + a*Tan[(c + d*x)/2])/Sqrt[a^2 + b^2]] + 30*a*(8*a^4 + 20*a^2*b^2
 + 15*b^4)*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] - 30*a*(8*a^4 + 20*a^2*
b^2 + 15*b^4)*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] + (3*b^4*(-5*a + 2*b
))/(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^4 + (b^2*(-60*a^3 + 20*a^2*b - 105
*a*b^2 + 29*b^3))/(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^2 + (12*b^5*Sin[(c
+ d*x)/2])/(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^5 + (2*b^3*(20*a^2 + 29*b^
2)*Sin[(c + d*x)/2])/(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^3 + (2*b*(120*a^
4 + 260*a^2*b^2 + 149*b^4)*Sin[(c + d*x)/2])/(Cos[(c + d*x)/2] - Sin[(c + d
*x)/2]) - (12*b^5*Sin[(c + d*x)/2])/(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^5
 + (3*b^4*(5*a + 2*b))/(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^4 - (2*b^3*(20
*a^2 + 29*b^2)*Sin[(c + d*x)/2])/(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^3 +
(b^2*(60*a^3 + 20*a^2*b + 105*a*b^2 + 29*b^3))/(Cos[(c + d*x)/2] + Sin[(c +
 d*x)/2])^2 - (2*b*(120*a^4 + 260*a^2*b^2 + 149*b^4)*Sin[(c + d*x)/2])/(Cos
[(c + d*x)/2] + Sin[(c + d*x)/2]))*(a*Cos[c + d*x] + b*Sin[c + d*x]))/(240*
b^6*d*(a + b*Tan[c + d*x]))
```

fricas [A] time = 0.90, size = 346, normalized size = 1.32

$$120\left(a^4+2a^2b^2+b^4\right)\sqrt{a^2+b^2}\cos\left(dx+c\right)^5\log\left(-\frac{2ab\cos(dx+c)\sin(dx+c)+\left(a^2-b^2\right)\cos(dx+c)^2-2a^2-b^2+2\sqrt{a^2+b^2}\,(b\cos(dx+c)-a}{2ab\cos(dx+c)\sin(dx+c)+\left(a^2-b^2\right)\cos(dx+c)^2+b^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^6/(a*cos(d*x+c)+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/240*(120*(a^4 + 2*a^2*b^2 + b^4)*sqrt(a^2 + b^2)*cos(d*x + c)^5*log(-(2*a
*b*cos(d*x + c)*sin(d*x + c) + (a^2 - b^2)*cos(d*x + c)^2 - 2*a^2 - b^2 + 2
*sqrt(a^2 + b^2)*(b*cos(d*x + c) - a*sin(d*x + c)))/(2*a*b*cos(d*x + c)*sin
(d*x + c) + (a^2 - b^2)*cos(d*x + c)^2 + b^2)) - 15*(8*a^5 + 20*a^3*b^2 + 1
5*a*b^4)*cos(d*x + c)^5*log(sin(d*x + c) + 1) + 15*(8*a^5 + 20*a^3*b^2 + 15
*a*b^4)*cos(d*x + c)^5*log(-sin(d*x + c) + 1) + 48*b^5 + 240*(a^4*b + 2*a^2
*b^3 + b^5)*cos(d*x + c)^4 + 80*(a^2*b^3 + b^5)*cos(d*x + c)^2 - 30*(2*a*b^
```

```
4*cos(d*x + c) + (4*a^3*b^2 + 7*a*b^4)*cos(d*x + c)^3)*sin(d*x + c))/(b^6*d
*cos(d*x + c)^5)
```

giac [B] time = 0.36, size = 554, normalized size = 2.11

$$-\frac{\frac{15\left(8a^5+20a^3b^2+15ab^4\right)\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right|\right)}{b^6}-\frac{15\left(8a^5+20a^3b^2+15ab^4\right)\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1\right|\right)}{b^6}+\frac{120\left(a^6+3a^4b^2+3a^2b^4+b^6\right)\log\left(\frac{|2a\tan}{|2a\tan}\right.}{\sqrt{a^2+b^2}\,b^6}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^6/(a*cos(d*x+c)+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/120*(15*(8*a^5 + 20*a^3*b^2 + 15*a*b^4)*log(abs(tan(1/2*d*x + 1/2*c) + 1
))/b^6 - 15*(8*a^5 + 20*a^3*b^2 + 15*a*b^4)*log(abs(tan(1/2*d*x + 1/2*c) -
1))/b^6 + 120*(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*log(abs(2*a*tan(1/2*d*x +
 1/2*c) - 2*b - 2*sqrt(a^2 + b^2))/abs(2*a*tan(1/2*d*x + 1/2*c) - 2*b + 2*s
qrt(a^2 + b^2)))/(sqrt(a^2 + b^2)*b^6) + 2*(60*a^3*b*tan(1/2*d*x + 1/2*c)^9
 + 135*a*b^3*tan(1/2*d*x + 1/2*c)^9 + 120*a^4*tan(1/2*d*x + 1/2*c)^8 + 360*
a^2*b^2*tan(1/2*d*x + 1/2*c)^8 + 360*b^4*tan(1/2*d*x + 1/2*c)^8 - 120*a^3*b
*tan(1/2*d*x + 1/2*c)^7 - 150*a*b^3*tan(1/2*d*x + 1/2*c)^7 - 480*a^4*tan(1/
2*d*x + 1/2*c)^6 - 1200*a^2*b^2*tan(1/2*d*x + 1/2*c)^6 - 720*b^4*tan(1/2*d*
x + 1/2*c)^6 + 720*a^4*tan(1/2*d*x + 1/2*c)^4 + 1600*a^2*b^2*tan(1/2*d*x +
1/2*c)^4 + 1120*b^4*tan(1/2*d*x + 1/2*c)^4 + 120*a^3*b*tan(1/2*d*x + 1/2*c)
^3 + 150*a*b^3*tan(1/2*d*x + 1/2*c)^3 - 480*a^4*tan(1/2*d*x + 1/2*c)^2 - 10
40*a^2*b^2*tan(1/2*d*x + 1/2*c)^2 - 560*b^4*tan(1/2*d*x + 1/2*c)^2 - 60*a^3
*b*tan(1/2*d*x + 1/2*c) - 135*a*b^3*tan(1/2*d*x + 1/2*c) + 120*a^4 + 280*a^
2*b^2 + 184*b^4)/((tan(1/2*d*x + 1/2*c)^2 - 1)^5*b^5))/d
```

maple [B] time = 0.26, size = 994, normalized size = 3.79

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^6/(a*cos(d*x+c)+b*sin(d*x+c)),x)
```

```
[Out] 2/d/(a^2+b^2)^(1/2)*arctanh(1/2*(2*a*tan(1/2*d*x+1/2*c)-2*b)/(a^2+b^2)^(1/2
))-15/8/d/b/(tan(1/2*d*x+1/2*c)-1)+15/8/d/b/(tan(1/2*d*x+1/2*c)+1)+6/d/b^4/
(a^2+b^2)^(1/2)*arctanh(1/2*(2*a*tan(1/2*d*x+1/2*c)-2*b)/(a^2+b^2)^(1/2))*a
^4+6/d/b^2/(a^2+b^2)^(1/2)*arctanh(1/2*(2*a*tan(1/2*d*x+1/2*c)-2*b)/(a^2+b^
2)^(1/2))*a^2+2/d/b^6/(a^2+b^2)^(1/2)*arctanh(1/2*(2*a*tan(1/2*d*x+1/2*c)-2
*b)/(a^2+b^2)^(1/2))*a^6-13/12/d/b/(tan(1/2*d*x+1/2*c)-1)^3-9/8/d/b/(tan(1/
2*d*x+1/2*c)-1)^2+13/12/d/b/(tan(1/2*d*x+1/2*c)+1)^3-9/8/d/b/(tan(1/2*d*x+1
```

```
/2*c)+1)^2-1/4/d/b^2/(tan(1/2*d*x+1/2*c)-1)^4*a-1/3/d/b^3/(tan(1/2*d*x+1/2*
c)-1)^3*a^2-1/2/d/b^2/(tan(1/2*d*x+1/2*c)-1)^3*a-1/2/d/b^4/(tan(1/2*d*x+1/2
*c)-1)^2*a^3-1/2/d/b^3/(tan(1/2*d*x+1/2*c)-1)^2*a^2-1/d/b^5/(tan(1/2*d*x+1/
2*c)-1)*a^4-1/2/d/b^4/(tan(1/2*d*x+1/2*c)-1)*a^3+1/d*a^5/b^6*ln(tan(1/2*d*x
+1/2*c)-1)+1/4/d/b^2/(tan(1/2*d*x+1/2*c)+1)^4*a+1/3/d/b^3/(tan(1/2*d*x+1/2*
c)+1)^3*a^2-1/2/d/b^2/(tan(1/2*d*x+1/2*c)+1)^3*a+1/2/d/b^4/(tan(1/2*d*x+1/2
*c)+1)^2*a^3-1/2/d/b^3/(tan(1/2*d*x+1/2*c)+1)^2*a^2+1/d/b^5/(tan(1/2*d*x+1/
2*c)+1)*a^4-1/2/d/b^4/(tan(1/2*d*x+1/2*c)+1)*a^3-5/2/d/b^3/(tan(1/2*d*x+1/2
*c)-1)*a^2-9/8/d/b^2/(tan(1/2*d*x+1/2*c)-1)*a+5/2/d*a^3/b^4*ln(tan(1/2*d*x+
1/2*c)-1)+11/8/d/b^2/(tan(1/2*d*x+1/2*c)+1)^2*a+5/2/d/b^3/(tan(1/2*d*x+1/2*
c)+1)*a^2-9/8/d/b^2/(tan(1/2*d*x+1/2*c)+1)*a-5/2/d*a^3/b^4*ln(tan(1/2*d*x+1
/2*c)+1)-11/8/d/b^2/(tan(1/2*d*x+1/2*c)-1)^2*a-1/d*a^5/b^6*ln(tan(1/2*d*x+1
/2*c)+1)-1/5/d/b/(tan(1/2*d*x+1/2*c)-1)^5-1/2/d/b/(tan(1/2*d*x+1/2*c)-1)^4+
1/5/d/b/(tan(1/2*d*x+1/2*c)+1)^5-1/2/d/b/(tan(1/2*d*x+1/2*c)+1)^4+15/8/d*a/
b^2*ln(tan(1/2*d*x+1/2*c)-1)-15/8/d*a/b^2*ln(tan(1/2*d*x+1/2*c)+1)
```

maxima [B] time = 0.44, size = 625, normalized size = 2.39

$$\frac{2\left(120\,a^4+280\,a^2b^2+184\,b^4-\frac{15\left(4\,a^3b+9\,ab^3\right)\sin(dx+c)}{\cos(dx+c)+1}-\frac{80\left(6\,a^4+13\,a^2b^2+7\,b^4\right)\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{30\left(4\,a^3b+5\,ab^3\right)\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{80\left(9\,a^4+20\,a^2b^2+14\,b^4\right)\sin(dx+c)^4}{(\cos(dx+c)+1)^4}-\frac{240\left(2\,a^4\right.}{}}{b^5-\frac{5\,b^5\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{10\,b^5\sin(dx+c)^4}{(\cos(dx+c)+1)^4}-\frac{10\,b^5\sin(dx+c)^6}{(\cos(dx+c)+1)^6}+\frac{5\,b^5\sin(dx+c)}{(\cos(dx+c)+1)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^6/(a*cos(d*x+c)+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/120*(2*(120*a^4 + 280*a^2*b^2 + 184*b^4 - 15*(4*a^3*b + 9*a*b^3)*sin(d*x
+ c)/(cos(d*x + c) + 1) - 80*(6*a^4 + 13*a^2*b^2 + 7*b^4)*sin(d*x + c)^2/(c
os(d*x + c) + 1)^2 + 30*(4*a^3*b + 5*a*b^3)*sin(d*x + c)^3/(cos(d*x + c) +
1)^3 + 80*(9*a^4 + 20*a^2*b^2 + 14*b^4)*sin(d*x + c)^4/(cos(d*x + c) + 1)^4
 - 240*(2*a^4 + 5*a^2*b^2 + 3*b^4)*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 - 30
*(4*a^3*b + 5*a*b^3)*sin(d*x + c)^7/(cos(d*x + c) + 1)^7 + 120*(a^4 + 3*a^2
*b^2 + 3*b^4)*sin(d*x + c)^8/(cos(d*x + c) + 1)^8 + 15*(4*a^3*b + 9*a*b^3)*
sin(d*x + c)^9/(cos(d*x + c) + 1)^9)/(b^5 - 5*b^5*sin(d*x + c)^2/(cos(d*x +
 c) + 1)^2 + 10*b^5*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 - 10*b^5*sin(d*x +
c)^6/(cos(d*x + c) + 1)^6 + 5*b^5*sin(d*x + c)^8/(cos(d*x + c) + 1)^8 - b^5
*sin(d*x + c)^10/(cos(d*x + c) + 1)^10) - 15*(8*a^5 + 20*a^3*b^2 + 15*a*b^4
)*log(sin(d*x + c)/(cos(d*x + c) + 1) + 1)/b^6 + 15*(8*a^5 + 20*a^3*b^2 + 1
5*a*b^4)*log(sin(d*x + c)/(cos(d*x + c) + 1) - 1)/b^6 - 120*(a^6 + 3*a^4*b^
2 + 3*a^2*b^4 + b^6)*log((b - a*sin(d*x + c)/(cos(d*x + c) + 1) + sqrt(a^2
+ b^2))/(b - a*sin(d*x + c)/(cos(d*x + c) + 1) - sqrt(a^2 + b^2)))/(sqrt(a^
2 + b^2)*b^6))/d
```

mupad [B] time = 2.84, size = 2979, normalized size = 11.37

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^6*(a*cos(c + d*x) + b*sin(c + d*x))),x)
```

```
[Out] (atan(((((a^2 + b^2)^5)^(1/2)*(((225*a^4*b^13)/2 + 300*a^6*b^11 + 320*a^8*b
^9 + 160*a^10*b^7 + 32*a^12*b^5)/b^14 + (tan(c/2 + (d*x)/2)*(64*a*b^17 + 83
4*a^3*b^15 + 2385*a^5*b^13 + 3160*a^7*b^11 + 2240*a^9*b^9 + 832*a^11*b^7 +
128*a^13*b^5))/(2*b^15) - (((a^2 + b^2)^5)^(1/2)*((28*a^2*b^16 + 44*a^4*b^1
4 + 16*a^6*b^12)/b^14 - (tan(c/2 + (d*x)/2)*(128*a*b^18 + 384*a^3*b^16 + 38
4*a^5*b^14 + 128*a^7*b^12))/(2*b^15) + (((a^2 + b^2)^5)^(1/2)*(32*a^2*b^3 +
 (tan(c/2 + (d*x)/2)*(192*a*b^19 + 128*a^3*b^17))/(2*b^15)))/b^6))/b^6)*1i)
/b^6 + (((a^2 + b^2)^5)^(1/2)*(((225*a^4*b^13)/2 + 300*a^6*b^11 + 320*a^8*b
^9 + 160*a^10*b^7 + 32*a^12*b^5)/b^14 + (tan(c/2 + (d*x)/2)*(64*a*b^17 + 83
4*a^3*b^15 + 2385*a^5*b^13 + 3160*a^7*b^11 + 2240*a^9*b^9 + 832*a^11*b^7 +
128*a^13*b^5))/(2*b^15) - (((a^2 + b^2)^5)^(1/2)*((tan(c/2 + (d*x)/2)*(128*
a*b^18 + 384*a^3*b^16 + 384*a^5*b^14 + 128*a^7*b^12))/(2*b^15) - (28*a^2*b^
16 + 44*a^4*b^14 + 16*a^6*b^12)/b^14 + (((a^2 + b^2)^5)^(1/2)*(32*a^2*b^3 +
 (tan(c/2 + (d*x)/2)*(192*a*b^19 + 128*a^3*b^17))/(2*b^15)))/b^6))/b^6)*1i)
/b^6)/((32*a^16 + 120*a^2*b^14 + 655*a^4*b^12 + 1549*a^6*b^10 + 2069*a^8*b^
8 + 1695*a^10*b^6 + 856*a^12*b^4 + 248*a^14*b^2)/b^14 + (((a^2 + b^2)^5)^(1
/2)*(((225*a^4*b^13)/2 + 300*a^6*b^11 + 320*a^8*b^9 + 160*a^10*b^7 + 32*a^1
2*b^5)/b^14 + (tan(c/2 + (d*x)/2)*(64*a*b^17 + 834*a^3*b^15 + 2385*a^5*b^13
 + 3160*a^7*b^11 + 2240*a^9*b^9 + 832*a^11*b^7 + 128*a^13*b^5))/(2*b^15) -
(((a^2 + b^2)^5)^(1/2)*((28*a^2*b^16 + 44*a^4*b^14 + 16*a^6*b^12)/b^14 - (t
an(c/2 + (d*x)/2)*(128*a*b^18 + 384*a^3*b^16 + 384*a^5*b^14 + 128*a^7*b^12)
)/(2*b^15) + (((a^2 + b^2)^5)^(1/2)*(32*a^2*b^3 + (tan(c/2 + (d*x)/2)*(192*
a*b^19 + 128*a^3*b^17))/(2*b^15)))/b^6))/b^6))/b^6 - (((a^2 + b^2)^5)^(1/2)
*(((225*a^4*b^13)/2 + 300*a^6*b^11 + 320*a^8*b^9 + 160*a^10*b^7 + 32*a^12*b
^5)/b^14 + (tan(c/2 + (d*x)/2)*(64*a*b^17 + 834*a^3*b^15 + 2385*a^5*b^13 +
3160*a^7*b^11 + 2240*a^9*b^9 + 832*a^11*b^7 + 128*a^13*b^5))/(2*b^15) - (((
a^2 + b^2)^5)^(1/2)*((tan(c/2 + (d*x)/2)*(128*a*b^18 + 384*a^3*b^16 + 384*a
^5*b^14 + 128*a^7*b^12))/(2*b^15) - (28*a^2*b^16 + 44*a^4*b^14 + 16*a^6*b^1
2)/b^14 + (((a^2 + b^2)^5)^(1/2)*(32*a^2*b^3 + (tan(c/2 + (d*x)/2)*(192*a*b
^19 + 128*a^3*b^17))/(2*b^15)))/b^6))/b^6))/b^6 - (tan(c/2 + (d*x)/2)*(128*
a^17 + 450*a^3*b^14 + 2550*a^5*b^12 + 6230*a^7*b^10 + 8530*a^9*b^8 + 7088*a
^11*b^6 + 3584*a^13*b^4 + 1024*a^15*b^2))/b^15))*((a^2 + b^2)^5)^(1/2)*2i)/
(b^6*d) - ((2*(15*a^4 + 23*b^4 + 35*a^2*b^2))/(15*b^5) + (tan(c/2 + (d*x)/2
)^3*(5*a*b^2 + 4*a^3))/(2*b^4) - (tan(c/2 + (d*x)/2)^7*(5*a*b^2 + 4*a^3))/(
2*b^4) + (tan(c/2 + (d*x)/2)^9*(9*a*b^2 + 4*a^3))/(4*b^4) + (2*tan(c/2 + (d
*x)/2)^8*(a^4 + 3*b^4 + 3*a^2*b^2))/b^5 - (4*tan(c/2 + (d*x)/2)^6*(2*a^4 +
3*b^4 + 5*a^2*b^2))/b^5 - (4*tan(c/2 + (d*x)/2)^2*(6*a^4 + 7*b^4 + 13*a^2*b
```

```
^2))/(3*b^5) + (4*tan(c/2 + (d*x)/2)^4*(9*a^4 + 14*b^4 + 20*a^2*b^2))/(3*b^
5) - (tan(c/2 + (d*x)/2)*(9*a*b^2 + 4*a^3))/(4*b^4))/(d*(5*tan(c/2 + (d*x)/
2)^2 - 10*tan(c/2 + (d*x)/2)^4 + 10*tan(c/2 + (d*x)/2)^6 - 5*tan(c/2 + (d*x
)/2)^8 + tan(c/2 + (d*x)/2)^10 - 1)) + (atan(((((15*a*b^4)/8 + a^5 + (5*a^3
*b^2)/2)*(((225*a^4*b^13)/2 + 300*a^6*b^11 + 320*a^8*b^9 + 160*a^10*b^7 + 3
2*a^12*b^5)/b^14 + (tan(c/2 + (d*x)/2)*(64*a*b^17 + 834*a^3*b^15 + 2385*a^5
*b^13 + 3160*a^7*b^11 + 2240*a^9*b^9 + 832*a^11*b^7 + 128*a^13*b^5))/(2*b^1
5) - (((15*a*b^4)/8 + a^5 + (5*a^3*b^2)/2)*((28*a^2*b^16 + 44*a^4*b^14 + 16
*a^6*b^12)/b^14 - (tan(c/2 + (d*x)/2)*(128*a*b^18 + 384*a^3*b^16 + 384*a^5*
b^14 + 128*a^7*b^12))/(2*b^15) + ((32*a^2*b^3 + (tan(c/2 + (d*x)/2)*(192*a*
b^19 + 128*a^3*b^17))/(2*b^15))*((15*a*b^4)/8 + a^5 + (5*a^3*b^2)/2))/b^6))
/b^6)*1i)/b^6 + (((15*a*b^4)/8 + a^5 + (5*a^3*b^2)/2)*(((225*a^4*b^13)/2 +
300*a^6*b^11 + 320*a^8*b^9 + 160*a^10*b^7 + 32*a^12*b^5)/b^14 + (tan(c/2 +
(d*x)/2)*(64*a*b^17 + 834*a^3*b^15 + 2385*a^5*b^13 + 3160*a^7*b^11 + 2240*a
^9*b^9 + 832*a^11*b^7 + 128*a^13*b^5))/(2*b^15) - (((15*a*b^4)/8 + a^5 + (5
*a^3*b^2)/2)*((tan(c/2 + (d*x)/2)*(128*a*b^18 + 384*a^3*b^16 + 384*a^5*b^14
 + 128*a^7*b^12))/(2*b^15) - (28*a^2*b^16 + 44*a^4*b^14 + 16*a^6*b^12)/b^14
 + ((32*a^2*b^3 + (tan(c/2 + (d*x)/2)*(192*a*b^19 + 128*a^3*b^17))/(2*b^15)
)*((15*a*b^4)/8 + a^5 + (5*a^3*b^2)/2))/b^6))/b^6)*1i)/b^6)/((32*a^16 + 120
*a^2*b^14 + 655*a^4*b^12 + 1549*a^6*b^10 + 2069*a^8*b^8 + 1695*a^10*b^6 + 8
56*a^12*b^4 + 248*a^14*b^2)/b^14 + (((15*a*b^4)/8 + a^5 + (5*a^3*b^2)/2)*((
(225*a^4*b^13)/2 + 300*a^6*b^11 + 320*a^8*b^9 + 160*a^10*b^7 + 32*a^12*b^5)
/b^14 + (tan(c/2 + (d*x)/2)*(64*a*b^17 + 834*a^3*b^15 + 2385*a^5*b^13 + 316
0*a^7*b^11 + 2240*a^9*b^9 + 832*a^11*b^7 + 128*a^13*b^5))/(2*b^15) - (((15*
a*b^4)/8 + a^5 + (5*a^3*b^2)/2)*((28*a^2*b^16 + 44*a^4*b^14 + 16*a^6*b^12)/
b^14 - (tan(c/2 + (d*x)/2)*(128*a*b^18 + 384*a^3*b^16 + 384*a^5*b^14 + 128*
a^7*b^12))/(2*b^15) + ((32*a^2*b^3 + (tan(c/2 + (d*x)/2)*(192*a*b^19 + 128*
a^3*b^17))/(2*b^15))*((15*a*b^4)/8 + a^5 + (5*a^3*b^2)/2))/b^6))/b^6))/b^6
- (((15*a*b^4)/8 + a^5 + (5*a^3*b^2)/2)*(((225*a^4*b^13)/2 + 300*a^6*b^11 +
 320*a^8*b^9 + 160*a^10*b^7 + 32*a^12*b^5)/b^14 + (tan(c/2 + (d*x)/2)*(64*a
*b^17 + 834*a^3*b^15 + 2385*a^5*b^13 + 3160*a^7*b^11 + 2240*a^9*b^9 + 832*a
^11*b^7 + 128*a^13*b^5))/(2*b^15) - (((15*a*b^4)/8 + a^5 + (5*a^3*b^2)/2)*(
(tan(c/2 + (d*x)/2)*(128*a*b^18 + 384*a^3*b^16 + 384*a^5*b^14 + 128*a^7*b^1
2))/(2*b^15) - (28*a^2*b^16 + 44*a^4*b^14 + 16*a^6*b^12)/b^14 + ((32*a^2*b^
3 + (tan(c/2 + (d*x)/2)*(192*a*b^19 + 128*a^3*b^17))/(2*b^15))*((15*a*b^4)/
8 + a^5 + (5*a^3*b^2)/2))/b^6))/b^6))/b^6 - (tan(c/2 + (d*x)/2)*(128*a^17 +
 450*a^3*b^14 + 2550*a^5*b^12 + 6230*a^7*b^10 + 8530*a^9*b^8 + 7088*a^11*b^
6 + 3584*a^13*b^4 + 1024*a^15*b^2))/b^15))*((15*a*b^4)/8 + a^5 + (5*a^3*b^2
)/2)*2i)/(b^6*d)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec^6(c+dx)}{a\cos(c+dx)+b\sin(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**6/(a*cos(d*x+c)+b*sin(d*x+c)),x)
```

```
[Out] Integral(sec(c + d*x)**6/(a*cos(c + d*x) + b*sin(c + d*x)), x)
```

3.122 $\int \frac{\cos^4(c+dx)}{(a\cos(c+dx)+b\sin(c+dx))^2}\,dx$

Optimal. Leaf size=145

$$-\frac{\sin^2(c+dx)\left(2ab-\left(a^2-b^2\right)\cot(c+dx)\right)}{2d\left(a^2+b^2\right)^2}+\frac{b^4}{ad\left(a^2+b^2\right)^2(a\cot(c+dx)+b)}+\frac{4ab^3\log(a\cos(c+dx)+b\sin(c+$$

```
[Out] 1/2*(a^4+6*a^2*b^2-3*b^4)*x/(a^2+b^2)^3+b^4/a/(a^2+b^2)^2/d/(b+a*cot(d*x+c)
)+4*a*b^3*ln(a*cos(d*x+c)+b*sin(d*x+c))/(a^2+b^2)^3/d-1/2*(2*a*b-(a^2-b^2)*
cot(d*x+c))*sin(d*x+c)^2/(a^2+b^2)^2/d
```

Rubi [A] time = 0.29, antiderivative size = 145, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {3088, 1647, 1629, 635, 203, 260}

$$\frac{b^4}{ad\left(a^2+b^2\right)^2(a\cot(c+dx)+b)}-\frac{\sin^2(c+dx)\left(2ab-\left(a^2-b^2\right)\cot(c+dx)\right)}{2d\left(a^2+b^2\right)^2}+\frac{4ab^3\log(a\cos(c+dx)+b\sin(c+a$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^4/(a*Cos[c + d*x] + b*Sin[c + d*x])^2,x]
```

```
[Out] ((a^4 + 6*a^2*b^2 - 3*b^4)*x)/(2*(a^2 + b^2)^3) + b^4/(a*(a^2 + b^2)^2*d*(b
 + a*Cot[c + d*x])) + (4*a*b^3*Log[a*Cos[c + d*x] + b*Sin[c + d*x]])/((a^2
+ b^2)^3*d) - ((2*a*b - (a^2 - b^2)*Cot[c + d*x])*Sin[c + d*x]^2)/(2*(a^2 +
 b^2)^2*d)
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] && !NiceSqrtQ[-(a*c)]
```

Rule 1629

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol]
:> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
 d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 1647

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[(d + e*x)^m*Pq, a + c*x^2, x], f = Coeff[Pol
ynomialRemainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 0], g = Coeff[Polynomial
Remainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 1]}, Simp[((a*g - c*f*x)*(a + c
*x^2)^(p + 1))/(2*a*c*(p + 1)), x] + Dist[1/(2*a*c*(p + 1)), Int[(d + e*x)^
m*(a + c*x^2)^(p + 1)*ExpandToSum[(2*a*c*(p + 1)*Q)/(d + e*x)^m + (c*f*(2*p
 + 3))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, c, d, e}, x] && PolyQ[Pq, x] &
& NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 3088

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[(x^m*(b +
 a*x)^n)/(1 + x^2)^((m + n + 2)/2), x], x, Cot[c + d*x]], x] /; FreeQ[{a, b
, c, d}, x] && IntegerQ[n] && IntegerQ[(m + n)/2] && NeQ[n, -1] &&  !(GtQ[n
, 0] && GtQ[m, 1])
```

Rubi steps

$$\int \frac{\cos^4(c+dx)}{(a\cos(c+dx)+b\sin(c+dx))^2}\,dx = -\frac{\text{Subst}\left(\int \frac{x^4}{(b+ax)^2(1+x^2)^2}\,dx, x, \cot(c+dx)\right)}{d}$$

$$= -\frac{\left(2ab-\left(a^2-b^2\right)\cot(c+dx)\right)\sin^2(c+dx)}{2\left(a^2+b^2\right)^2 d} + \frac{\text{Subst}\left(\int \frac{-\frac{b^2\left(a^2-b^2\right)}{\left(a^2+b^2\right)^2}-\frac{2abx}{a^2+b^2}-\frac{(}{}}{(b+ax)^2(1}\right.}{}$$

$$= -\frac{\left(2ab-\left(a^2-b^2\right)\cot(c+dx)\right)\sin^2(c+dx)}{2\left(a^2+b^2\right)^2 d} + \frac{\text{Subst}\left(\int\left(-\frac{2b^4}{\left(a^2+b^2\right)^2(b+ax)^2}\right.\right.}{}$$

$$= \frac{b^4}{a\left(a^2+b^2\right)^2 d(b+a\cot(c+dx))} + \frac{4ab^3\log(b+a\cot(c+dx))}{\left(a^2+b^2\right)^3 d} - \frac{(2ab-}{}$$

$$= \frac{b^4}{a\left(a^2+b^2\right)^2 d(b+a\cot(c+dx))} + \frac{4ab^3\log(b+a\cot(c+dx))}{\left(a^2+b^2\right)^3 d} - \frac{(2ab-}{}$$

$$= \frac{\left(a^4+6a^2b^2-3b^4\right)x}{2\left(a^2+b^2\right)^3} + \frac{b^4}{a\left(a^2+b^2\right)^2 d(b+a\cot(c+dx))} + \frac{4ab^3\log(b+}{\left(a^2+\right.}$$

Mathematica [A] time = 0.98, size = 149, normalized size = 1.03

$$\frac{\left(a^2-b^2\right)\left(a^2+b^2\right)\sin(2(c+dx))+2ab\left(a^2+b^2\right)\cos(2(c+dx))+\frac{4b^4\left(a^2+b^2\right)\sin(c+dx)}{a(a\cos(c+dx)+b\sin(c+dx))}+2\left(a^4+6a^2b^2-3b^4\right)(c}{4d\left(a^2+b^2\right)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^4/(a*Cos[c + d*x] + b*Sin[c + d*x])^2,x]
```

```
[Out] (2*(a^4 + 6*a^2*b^2 - 3*b^4)*(c + d*x) + 2*a*b*(a^2 + b^2)*Cos[2*(c + d*x)]
 + 16*a*b^3*Log[a*Cos[c + d*x] + b*Sin[c + d*x]] + (4*b^4*(a^2 + b^2)*Sin[c
 + d*x])/(a*(a*Cos[c + d*x] + b*Sin[c + d*x])) + (a^2 - b^2)*(a^2 + b^2)*Si
n[2*(c + d*x)])/(4*(a^2 + b^2)^3*d)
```

fricas [A] time = 0.62, size = 279, normalized size = 1.92

$$\frac{\left(a^4b+2a^2b^3+b^5\right)\cos(dx+c)^3-\left(a^2b^3+3b^5-\left(a^5+6a^3b^2-3ab^4\right)dx\right)\cos(dx+c)+4\left(a^2b^3\cos(dx+c)+ab\right.}{2\left(\left(a^7+3a^5b^2+\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4/(a*cos(d*x+c)+b*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/2*((a^4*b + 2*a^2*b^3 + b^5)*cos(d*x + c)^3 - (a^2*b^3 + 3*b^5 - (a^5 + 6
*a^3*b^2 - 3*a*b^4)*d*x)*cos(d*x + c) + 4*(a^2*b^3*cos(d*x + c) + a*b^4*sin
(d*x + c))*log(2*a*b*cos(d*x + c)*sin(d*x + c) + (a^2 - b^2)*cos(d*x + c)^2
 + b^2) - (a^3*b^2 - a*b^4 - (a^4*b + 6*a^2*b^3 - 3*b^5)*d*x - (a^5 + 2*a^3
*b^2 + a*b^4)*cos(d*x + c)^2)*sin(d*x + c))/((a^7 + 3*a^5*b^2 + 3*a^3*b^4 +
 a*b^6)*d*cos(d*x + c) + (a^6*b + 3*a^4*b^3 + 3*a^2*b^5 + b^7)*d*sin(d*x +
c))
```

giac [A] time = 0.20, size = 250, normalized size = 1.72

$$\frac{\frac{8\,ab^4\log(|b\tan(dx+c)+a|)}{a^6b+3\,a^4b^3+3\,a^2b^5+b^7}-\frac{4\,ab^3\log\left(\tan(dx+c)^2+1\right)}{a^6+3\,a^4b^2+3\,a^2b^4+b^6}+\frac{\left(a^4+6\,a^2b^2-3\,b^4\right)(dx+c)}{a^6+3\,a^4b^2+3\,a^2b^4+b^6}+\frac{a^2b\tan(dx+c)^2-3\,b^3\tan(dx+c)^2+a^3\tan(dx+c)+ab^2\tan(dx+c}{\left(a^4+2\,a^2b^2+b^4\right)\left(b\tan(dx+c)^3+a\tan(dx+c)^2+b\tan(dx}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4/(a*cos(d*x+c)+b*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/2*(8*a*b^4*log(abs(b*tan(d*x + c) + a))/(a^6*b + 3*a^4*b^3 + 3*a^2*b^5 +
b^7) - 4*a*b^3*log(tan(d*x + c)^2 + 1)/(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)
+ (a^4 + 6*a^2*b^2 - 3*b^4)*(d*x + c)/(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6) +
 (a^2*b*tan(d*x + c)^2 - 3*b^3*tan(d*x + c)^2 + a^3*tan(d*x + c) + a*b^2*ta
n(d*x + c) + 2*a^2*b - 2*b^3)/((a^4 + 2*a^2*b^2 + b^4)*(b*tan(d*x + c)^3 +
a*tan(d*x + c)^2 + b*tan(d*x + c) + a)))/d
```

maple [B] time = 0.23, size = 292, normalized size = 2.01

$$-\frac{b^3}{d\left(a^2+b^2\right)^2\left(a+b\tan(dx+c)\right)}+\frac{4b^3a\ln\left(a+b\tan(dx+c)\right)}{d\left(a^2+b^2\right)^3}+\frac{\tan(dx+c)\,a^4}{2d\left(a^2+b^2\right)^3\left(\tan^2(dx+c)+1\right)}-\frac{\tan}{2d\left(a^2+b^2\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4/(a*cos(d*x+c)+b*sin(d*x+c))^2,x)
```

```
[Out] -1/d*b^3/(a^2+b^2)^2/(a+b*tan(d*x+c))+4/d*b^3/(a^2+b^2)^3*a*ln(a+b*tan(d*x+
c))+1/2/d/(a^2+b^2)^3/(tan(d*x+c)^2+1)*tan(d*x+c)*a^4-1/2/d/(a^2+b^2)^3/(ta
n(d*x+c)^2+1)*tan(d*x+c)*b^4+1/d/(a^2+b^2)^3/(tan(d*x+c)^2+1)*a^3*b+1/d/(a^
2+b^2)^3/(tan(d*x+c)^2+1)*b^3*a-2/d/(a^2+b^2)^3*b^3*a*ln(tan(d*x+c)^2+1)+3/
d/(a^2+b^2)^3*arctan(tan(d*x+c))*a^2*b^2-3/2/d/(a^2+b^2)^3*arctan(tan(d*x+c
))*b^4+1/2/d/(a^2+b^2)^3*arctan(tan(d*x+c))*a^4
```

maxima [A] time = 0.55, size = 282, normalized size = 1.94

$$\frac{\frac{8\,ab^3\log(b\tan(dx+c)+a)}{a^6+3\,a^4b^2+3\,a^2b^4+b^6}-\frac{4\,ab^3\log\left(\tan(dx+c)^2+1\right)}{a^6+3\,a^4b^2+3\,a^2b^4+b^6}+\frac{\left(a^4+6\,a^2b^2-3\,b^4\right)(dx+c)}{a^6+3\,a^4b^2+3\,a^2b^4+b^6}+\frac{2\,a^2b-2\,b^3+\left(a^2b-3\,b^3\right)\tan(dx+c)^2+\left(a^3\right.}{a^5+2\,a^3b^2+ab^4+\left(a^4b+2\,a^2b^3+b^5\right)\tan(dx+c)^3+\left(a^5+2\,a^3b^2+ab^4\right)}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4/(a*cos(d*x+c)+b*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 1/2*(8*a*b^3*log(b*tan(d*x + c) + a)/(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6) -
4*a*b^3*log(tan(d*x + c)^2 + 1)/(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6) + (a^4
+ 6*a^2*b^2 - 3*b^4)*(d*x + c)/(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6) + (2*a^2
*b - 2*b^3 + (a^2*b - 3*b^3)*tan(d*x + c)^2 + (a^3 + a*b^2)*tan(d*x + c))/(
a^5 + 2*a^3*b^2 + a*b^4 + (a^4*b + 2*a^2*b^3 + b^5)*tan(d*x + c)^3 + (a^5 +
 2*a^3*b^2 + a*b^4)*tan(d*x + c)^2 + (a^4*b + 2*a^2*b^3 + b^5)*tan(d*x + c)
))/d
```

mupad [B] time = 11.46, size = 6604, normalized size = 45.54

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^4/(a*cos(c + d*x) + b*sin(c + d*x))^2,x)
```

```
[Out] ((2*b*tan(c/2 + (d*x)/2)^4)/(a^2 + b^2) - (2*b*tan(c/2 + (d*x)/2)^2)/(a^2 +
 b^2) + (tan(c/2 + (d*x)/2)*(a^4 + 2*b^4 - a^2*b^2))/(a*(a^2 + b^2)^2) + (t
an(c/2 + (d*x)/2)^5*(a^4 + 2*b^4 - a^2*b^2))/(a*(a^4 + b^4 + 2*a^2*b^2)) -
(2*tan(c/2 + (d*x)/2)^3*(a^4 - 2*b^4 + 3*a^2*b^2))/(a*(a^2 + b^2)^2))/(d*(a
 + 2*b*tan(c/2 + (d*x)/2) + a*tan(c/2 + (d*x)/2)^2 - a*tan(c/2 + (d*x)/2)^4
 - a*tan(c/2 + (d*x)/2)^6 + 4*b*tan(c/2 + (d*x)/2)^3 + 2*b*tan(c/2 + (d*x)/
2)^5)) - (atan((tan(c/2 + (d*x)/2)*(((((a^4 - 3*b^4 + 6*a^2*b^2)^3*(12*a*b^
16 + 84*a^3*b^14 + 252*a^5*b^12 + 420*a^7*b^10 + 420*a^9*b^8 + 252*a^11*b^6
 + 84*a^13*b^4 + 12*a^15*b^2))/((a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2)^3*(a^12
 + b^12 + 6*a^2*b^10 + 15*a^4*b^8 + 20*a^6*b^6 + 15*a^8*b^4 + 6*a^10*b^2))
- (((8*(18*a*b^12 + a^13 - 141*a^3*b^10 - 327*a^5*b^8 - 146*a^7*b^6 + 36*a^
9*b^4 + 15*a^11*b^2))/(a^12 + b^12 + 6*a^2*b^10 + 15*a^4*b^8 + 20*a^6*b^6 +
 15*a^8*b^4 + 6*a^10*b^2) - (16*a*b^3*((8*(4*a^14*b + 4*a^2*b^13 + 72*a^4*b
^11 + 252*a^6*b^9 + 368*a^8*b^7 + 252*a^10*b^5 + 72*a^12*b^3))/(a^12 + b^12
 + 6*a^2*b^10 + 15*a^4*b^8 + 20*a^6*b^6 + 15*a^8*b^4 + 6*a^10*b^2) - (128*a
*b^3*(12*a*b^16 + 84*a^3*b^14 + 252*a^5*b^12 + 420*a^7*b^10 + 420*a^9*b^8 +
 252*a^11*b^6 + 84*a^13*b^4 + 12*a^15*b^2))/((4*a^6 + 4*b^6 + 12*a^2*b^4 +
12*a^4*b^2)*(a^12 + b^12 + 6*a^2*b^10 + 15*a^4*b^8 + 20*a^6*b^6 + 15*a^8*b^
4 + 6*a^10*b^2))))/(4*a^6 + 4*b^6 + 12*a^2*b^4 + 12*a^4*b^2))*(a^4 - 3*b^4
+ 6*a^2*b^2))/(2*(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2)) + (16*a*b^3*((((8*(4*
```

a^14*b + 4*a^2*b^13 + 72*a^4*b^11 + 252*a^6*b^9 + 368*a^8*b^7 + 252*a^10*b^
5 + 72*a^12*b^3))/(a^12 + b^12 + 6*a^2*b^10 + 15*a^4*b^8 + 20*a^6*b^6 + 15*
a^8*b^4 + 6*a^10*b^2) - (128*a*b^3*(12*a*b^16 + 84*a^3*b^14 + 252*a^5*b^12
+ 420*a^7*b^10 + 420*a^9*b^8 + 252*a^11*b^6 + 84*a^13*b^4 + 12*a^15*b^2))/(
(4*a^6 + 4*b^6 + 12*a^2*b^4 + 12*a^4*b^2)*(a^12 + b^12 + 6*a^2*b^10 + 15*a^
4*b^8 + 20*a^6*b^6 + 15*a^8*b^4 + 6*a^10*b^2)))*(a^4 - 3*b^4 + 6*a^2*b^2))/
(2*(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2)) - (64*a*b^3*(a^4 - 3*b^4 + 6*a^2*b^
2)*(12*a*b^16 + 84*a^3*b^14 + 252*a^5*b^12 + 420*a^7*b^10 + 420*a^9*b^8 + 2
52*a^11*b^6 + 84*a^13*b^4 + 12*a^15*b^2))/((4*a^6 + 4*b^6 + 12*a^2*b^4 + 12
*a^4*b^2)*(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2)*(a^12 + b^12 + 6*a^2*b^10 + 1
5*a^4*b^8 + 20*a^6*b^6 + 15*a^8*b^4 + 6*a^10*b^2))))/(4*a^6 + 4*b^6 + 12*a^
2*b^4 + 12*a^4*b^2))*(a^10 - 9*b^10 + 493*a^2*b^8 - 706*a^4*b^6 - 46*a^6*b^
4 + 11*a^8*b^2))/(a^10 + 9*b^10 + 229*a^2*b^8 + 250*a^4*b^6 + 42*a^6*b^4 +
13*a^8*b^2)^2 - (2*a*b*((8*(72*a^2*b^9 + 52*a^4*b^7 + 48*a^6*b^5 + 4*a^8*b^
3))/(a^12 + b^12 + 6*a^2*b^10 + 15*a^4*b^8 + 20*a^6*b^6 + 15*a^8*b^4 + 6*a^
10*b^2) + (((((8*(4*a^14*b + 4*a^2*b^13 + 72*a^4*b^11 + 252*a^6*b^9 + 368*a
^8*b^7 + 252*a^10*b^5 + 72*a^12*b^3))/(a^12 + b^12 + 6*a^2*b^10 + 15*a^4*b^
8 + 20*a^6*b^6 + 15*a^8*b^4 + 6*a^10*b^2) - (128*a*b^3*(12*a*b^16 + 84*a^3*
b^14 + 252*a^5*b^12 + 420*a^7*b^10 + 420*a^9*b^8 + 252*a^11*b^6 + 84*a^13*b
^4 + 12*a^15*b^2))/((4*a^6 + 4*b^6 + 12*a^2*b^4 + 12*a^4*b^2)*(a^12 + b^12
+ 6*a^2*b^10 + 15*a^4*b^8 + 20*a^6*b^6 + 15*a^8*b^4 + 6*a^10*b^2)))*(a^4 -
3*b^4 + 6*a^2*b^2))/(2*(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2)) - (64*a*b^3*(a^
4 - 3*b^4 + 6*a^2*b^2)*(12*a*b^16 + 84*a^3*b^14 + 252*a^5*b^12 + 420*a^7*b^
10 + 420*a^9*b^8 + 252*a^11*b^6 + 84*a^13*b^4 + 12*a^15*b^2))/((4*a^6 + 4*b
^6 + 12*a^2*b^4 + 12*a^4*b^2)*(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2)*(a^12 + b
^12 + 6*a^2*b^10 + 15*a^4*b^8 + 20*a^6*b^6 + 15*a^8*b^4 + 6*a^10*b^2)))*(a^
4 - 3*b^4 + 6*a^2*b^2))/(2*(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2)) + (16*a*b^3
((8(18*a*b^12 + a^13 - 141*a^3*b^10 - 327*a^5*b^8 - 146*a^7*b^6 + 36*a^9*
b^4 + 15*a^11*b^2))/(a^12 + b^12 + 6*a^2*b^10 + 15*a^4*b^8 + 20*a^6*b^6 + 1
5*a^8*b^4 + 6*a^10*b^2) - (16*a*b^3*((8*(4*a^14*b + 4*a^2*b^13 + 72*a^4*b^1
1 + 252*a^6*b^9 + 368*a^8*b^7 + 252*a^10*b^5 + 72*a^12*b^3))/(a^12 + b^12 +
6*a^2*b^10 + 15*a^4*b^8 + 20*a^6*b^6 + 15*a^8*b^4 + 6*a^10*b^2) - (128*a*b
^3*(12*a*b^16 + 84*a^3*b^14 + 252*a^5*b^12 + 420*a^7*b^10 + 420*a^9*b^8 + 2
52*a^11*b^6 + 84*a^13*b^4 + 12*a^15*b^2))/((4*a^6 + 4*b^6 + 12*a^2*b^4 + 12
*a^4*b^2)*(a^12 + b^12 + 6*a^2*b^10 + 15*a^4*b^8 + 20*a^6*b^6 + 15*a^8*b^4
+ 6*a^10*b^2))))/(4*a^6 + 4*b^6 + 12*a^2*b^4 + 12*a^4*b^2)))/(4*a^6 + 4*b^6
+ 12*a^2*b^4 + 12*a^4*b^2) - (32*a*b^3*(a^4 - 3*b^4 + 6*a^2*b^2)^2*(12*a*b
^16 + 84*a^3*b^14 + 252*a^5*b^12 + 420*a^7*b^10 + 420*a^9*b^8 + 252*a^11*b^
6 + 84*a^13*b^4 + 12*a^15*b^2))/((4*a^6 + 4*b^6 + 12*a^2*b^4 + 12*a^4*b^2)*
(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2)^2*(a^12 + b^12 + 6*a^2*b^10 + 15*a^4*b^
8 + 20*a^6*b^6 + 15*a^8*b^4 + 6*a^10*b^2)))*(a^8 + 57*b^8 - 436*a^2*b^6 + 1
10*a^4*b^4 + 28*a^6*b^2))/(a^10 + 9*b^10 + 229*a^2*b^8 + 250*a^4*b^6 + 42*a
^6*b^4 + 13*a^8*b^2)^2)*(a^16 + b^16 + 8*a^2*b^14 + 28*a^4*b^12 + 56*a^6*b^
10 + 70*a^8*b^8 + 56*a^10*b^6 + 28*a^12*b^4 + 8*a^14*b^2))/(4*a^5 - 12*a*b^
4 + 24*a^3*b^2) + (((((8*(39*a^2*b^11 - a^12*b + 123*a^4*b^9 + 134*a^6*b^7

```
+ 54*a^8*b^5 + 3*a^10*b^3))/(a^12 + b^12 + 6*a^2*b^10 + 15*a^4*b^8 + 20*a^6
*b^6 + 15*a^8*b^4 + 6*a^10*b^2) + (16*a*b^3*((8*(6*a*b^14 + 2*a^15 - 10*a^3
*b^12 - 90*a^5*b^10 - 138*a^7*b^8 - 62*a^9*b^6 + 18*a^11*b^4 + 18*a^13*b^2)
)/(a^12 + b^12 + 6*a^2*b^10 + 15*a^4*b^8 + 20*a^6*b^6 + 15*a^8*b^4 + 6*a^10
*b^2) - (128*a*b^3*(12*a^16*b + 12*a^2*b^15 + 84*a^4*b^13 + 252*a^6*b^11 +
420*a^8*b^9 + 420*a^10*b^7 + 252*a^12*b^5 + 84*a^14*b^3))/((4*a^6 + 4*b^6 +
 12*a^2*b^4 + 12*a^4*b^2)*(a^12 + b^12 + 6*a^2*b^10 + 15*a^4*b^8 + 20*a^6*b
^6 + 15*a^8*b^4 + 6*a^10*b^2))))/(4*a^6 + 4*b^6 + 12*a^2*b^4 + 12*a^4*b^2))
*(a^4 - 3*b^4 + 6*a^2*b^2))/(2*(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2)) + ((a^4
 - 3*b^4 + 6*a^2*b^2)^3*(12*a^16*b + 12*a^2*b^15 + 84*a^4*b^13 + 252*a^6*b^
11 + 420*a^8*b^9 + 420*a^10*b^7 + 252*a^12*b^5 + 84*a^14*b^3))/((a^6 + b^6
+ 3*a^2*b^4 + 3*a^4*b^2)^3*(a^12 + b^12 + 6*a^2*b^10 + 15*a^4*b^8 + 20*a^6*
b^6 + 15*a^8*b^4 + 6*a^10*b^2)) + (16*a*b^3*((((8*(6*a*b^14 + 2*a^15 - 10*a
^3*b^12 - 90*a^5*b^10 - 138*a^7*b^8 - 62*a^9*b^6 + 18*a^11*b^4 + 18*a^13*b^
2))/(a^12 + b^12 + 6*a^2*b^10 + 15*a^4*b^8 + 20*a^6*b^6 + 15*a^8*b^4 + 6*a^
10*b^2) - (128*a*b^3*(12*a^16*b + 12*a^2*b^15 + 84*a^4*b^13 + 252*a^6*b^11
+ 420*a^8*b^9 + 420*a^10*b^7 + 252*a^12*b^5 + 84*a^14*b^3))/((4*a^6 + 4*b^6
 + 12*a^2*b^4 + 12*a^4*b^2)*(a^12 + b^12 + 6*a^2*b^10 + 15*a^4*b^8 + 20*a^6
*b^6 + 15*a^8*b^4 + 6*a^10*b^2)))*(a^4 - 3*b^4 + 6*a^2*b^2))/(2*(a^6 + b^6
+ 3*a^2*b^4 + 3*a^4*b^2)) - (64*a*b^3*(a^4 - 3*b^4 + 6*a^2*b^2)*(12*a^16*b
+ 12*a^2*b^15 + 84*a^4*b^13 + 252*a^6*b^11 + 420*a^8*b^9 + 420*a^10*b^7 + 2
52*a^12*b^5 + 84*a^14*b^3))/((4*a^6 + 4*b^6 + 12*a^2*b^4 + 12*a^4*b^2)*(a^6
 + b^6 + 3*a^2*b^4 + 3*a^4*b^2)*(a^12 + b^12 + 6*a^2*b^10 + 15*a^4*b^8 + 20
*a^6*b^6 + 15*a^8*b^4 + 6*a^10*b^2))))/(4*a^6 + 4*b^6 + 12*a^2*b^4 + 12*a^4
*b^2))*(a^10 - 9*b^10 + 493*a^2*b^8 - 706*a^4*b^6 - 46*a^6*b^4 + 11*a^8*b^2
)*(a^16 + b^16 + 8*a^2*b^14 + 28*a^4*b^12 + 56*a^6*b^10 + 70*a^8*b^8 + 56*a
^10*b^6 + 28*a^12*b^4 + 8*a^14*b^2))/((4*a^5 - 12*a*b^4 + 24*a^3*b^2)*(a^10
 + 9*b^10 + 229*a^2*b^8 + 250*a^4*b^6 + 42*a^6*b^4 + 13*a^8*b^2)^2) - (2*a*
b*((8*(8*a^5*b^6 - 60*a^3*b^8 + 4*a^7*b^4))/(a^12 + b^12 + 6*a^2*b^10 + 15*
a^4*b^8 + 20*a^6*b^6 + 15*a^8*b^4 + 6*a^10*b^2) + (((((8*(6*a*b^14 + 2*a^15
 - 10*a^3*b^12 - 90*a^5*b^10 - 138*a^7*b^8 - 62*a^9*b^6 + 18*a^11*b^4 + 18*
a^13*b^2))/(a^12 + b^12 + 6*a^2*b^10 + 15*a^4*b^8 + 20*a^6*b^6 + 15*a^8*b^4
 + 6*a^10*b^2) - (128*a*b^3*(12*a^16*b + 12*a^2*b^15 + 84*a^4*b^13 + 252*a^
6*b^11 + 420*a^8*b^9 + 420*a^10*b^7 + 252*a^12*b^5 + 84*a^14*b^3))/((4*a^6
+ 4*b^6 + 12*a^2*b^4 + 12*a^4*b^2)*(a^12 + b^12 + 6*a^2*b^10 + 15*a^4*b^8 +
 20*a^6*b^6 + 15*a^8*b^4 + 6*a^10*b^2)))*(a^4 - 3*b^4 + 6*a^2*b^2))/(2*(a^6
 + b^6 + 3*a^2*b^4 + 3*a^4*b^2)) - (64*a*b^3*(a^4 - 3*b^4 + 6*a^2*b^2)*(12*
a^16*b + 12*a^2*b^15 + 84*a^4*b^13 + 252*a^6*b^11 + 420*a^8*b^9 + 420*a^10*
b^7 + 252*a^12*b^5 + 84*a^14*b^3))/((4*a^6 + 4*b^6 + 12*a^2*b^4 + 12*a^4*b^
2)*(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2)*(a^12 + b^12 + 6*a^2*b^10 + 15*a^4*b
^8 + 20*a^6*b^6 + 15*a^8*b^4 + 6*a^10*b^2)))*(a^4 - 3*b^4 + 6*a^2*b^2))/(2*
(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2)) - (16*a*b^3*((8*(39*a^2*b^11 - a^12*b
+ 123*a^4*b^9 + 134*a^6*b^7 + 54*a^8*b^5 + 3*a^10*b^3))/(a^12 + b^12 + 6*a^
2*b^10 + 15*a^4*b^8 + 20*a^6*b^6 + 15*a^8*b^4 + 6*a^10*b^2) + (16*a*b^3*((8
*(6*a*b^14 + 2*a^15 - 10*a^3*b^12 - 90*a^5*b^10 - 138*a^7*b^8 - 62*a^9*b^6
```

```
+ 18*a^11*b^4 + 18*a^13*b^2))/(a^12 + b^12 + 6*a^2*b^10 + 15*a^4*b^8 + 20*a
^6*b^6 + 15*a^8*b^4 + 6*a^10*b^2) - (128*a*b^3*(12*a^16*b + 12*a^2*b^15 + 8
4*a^4*b^13 + 252*a^6*b^11 + 420*a^8*b^9 + 420*a^10*b^7 + 252*a^12*b^5 + 84*
a^14*b^3))/((4*a^6 + 4*b^6 + 12*a^2*b^4 + 12*a^4*b^2)*(a^12 + b^12 + 6*a^2*
b^10 + 15*a^4*b^8 + 20*a^6*b^6 + 15*a^8*b^4 + 6*a^10*b^2))))/(4*a^6 + 4*b^6
 + 12*a^2*b^4 + 12*a^4*b^2)))/(4*a^6 + 4*b^6 + 12*a^2*b^4 + 12*a^4*b^2) - (
32*a*b^3*(a^4 - 3*b^4 + 6*a^2*b^2)^2*(12*a^16*b + 12*a^2*b^15 + 84*a^4*b^13
 + 252*a^6*b^11 + 420*a^8*b^9 + 420*a^10*b^7 + 252*a^12*b^5 + 84*a^14*b^3))
/((4*a^6 + 4*b^6 + 12*a^2*b^4 + 12*a^4*b^2)*(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*
b^2)^2*(a^12 + b^12 + 6*a^2*b^10 + 15*a^4*b^8 + 20*a^6*b^6 + 15*a^8*b^4 + 6
*a^10*b^2)))*(a^8 + 57*b^8 - 436*a^2*b^6 + 110*a^4*b^4 + 28*a^6*b^2)*(a^16
+ b^16 + 8*a^2*b^14 + 28*a^4*b^12 + 56*a^6*b^10 + 70*a^8*b^8 + 56*a^10*b^6
+ 28*a^12*b^4 + 8*a^14*b^2))/((4*a^5 - 12*a*b^4 + 24*a^3*b^2)*(a^10 + 9*b^1
0 + 229*a^2*b^8 + 250*a^4*b^6 + 42*a^6*b^4 + 13*a^8*b^2)^2))*(a^4 - 3*b^4 +
 6*a^2*b^2))/(d*(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2)) - (16*a*b^3*log(((32*a
^3*b^4*(a^4 - 15*b^4 + 2*a^2*b^2))/(a^2 + b^2)^6 - ((-(a^4 - 3*b^4 + 6*a^2*
b^2)^2/(a^2 + b^2)^6)^(1/2)/2 - (4*a*b^3)/(a^2 + b^2)^3)*(((-(a^4 - 3*b^4 +
 6*a^2*b^2)^2/(a^2 + b^2)^6)^(1/2)/2 - (4*a*b^3)/(a^2 + b^2)^3)*((16*a*(a^6
 + 3*b^6 - 17*a^2*b^4 + 5*a^4*b^2))/(a^2 + b^2)^2 + (32*a^2*b*tan(c/2 + (d*
x)/2)*(a^4 + b^4 + 14*a^2*b^2))/(a^2 + b^2)^2 + 96*a*b*((-(a^4 - 3*b^4 + 6*
a^2*b^2)^2/(a^2 + b^2)^6)^(1/2)/2 - (4*a*b^3)/(a^2 + b^2)^3)*(a + b*tan(c/2
 + (d*x)/2))*(a^2 + b^2)) - (8*a^2*b*(39*b^4 - a^4 + 6*a^2*b^2))/(a^2 + b^2
)^3 + (8*a*tan(c/2 + (d*x)/2)*(a^8 + 18*b^8 - 177*a^2*b^6 + 9*a^4*b^4 + 13*
a^6*b^2))/(a^2 + b^2)^4) + (32*a^2*b^3*tan(c/2 + (d*x)/2)*(a^6 + 18*b^6 + 1
3*a^2*b^4 + 12*a^4*b^2))/(a^2 + b^2)^6)*((32*a^3*b^4*(a^4 - 15*b^4 + 2*a^2*
b^2))/(a^2 + b^2)^6 - ((-(a^4 - 3*b^4 + 6*a^2*b^2)^2/(a^2 + b^2)^6)^(1/2)/2
 + (4*a*b^3)/(a^2 + b^2)^3)*(((-(a^4 - 3*b^4 + 6*a^2*b^2)^2/(a^2 + b^2)^6)^
(1/2)/2 + (4*a*b^3)/(a^2 + b^2)^3)*((16*a*(a^6 + 3*b^6 - 17*a^2*b^4 + 5*a^4
*b^2))/(a^2 + b^2)^2 + (32*a^2*b*tan(c/2 + (d*x)/2)*(a^4 + b^4 + 14*a^2*b^2
))/(a^2 + b^2)^2 - 96*a*b*((-(a^4 - 3*b^4 + 6*a^2*b^2)^2/(a^2 + b^2)^6)^(1/
2)/2 + (4*a*b^3)/(a^2 + b^2)^3)*(a + b*tan(c/2 + (d*x)/2))*(a^2 + b^2)) + (
8*a^2*b*(39*b^4 - a^4 + 6*a^2*b^2))/(a^2 + b^2)^3 - (8*a*tan(c/2 + (d*x)/2)
*(a^8 + 18*b^8 - 177*a^2*b^6 + 9*a^4*b^4 + 13*a^6*b^2))/(a^2 + b^2)^4) + (3
2*a^2*b^3*tan(c/2 + (d*x)/2)*(a^6 + 18*b^6 + 13*a^2*b^4 + 12*a^4*b^2))/(a^2
 + b^2)^6)))/(d*(4*a^6 + 4*b^6 + 12*a^2*b^4 + 12*a^4*b^2)) + (4*a*b^3*log(a
 + 2*b*tan(c/2 + (d*x)/2) - a*tan(c/2 + (d*x)/2)^2))/(d*(a^6 + b^6 + 3*a^2*
b^4 + 3*a^4*b^2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4/(a*cos(d*x+c)+b*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

3.123 $\int \frac{\cos^3(c+dx)}{(a\cos(c+dx)+b\sin(c+dx))^2}\,dx$

Optimal. Leaf size=138

$$\frac{(a^2-b^2)\sin(c+dx)}{d(a^2+b^2)^2}+\frac{2ab\cos(c+dx)}{d(a^2+b^2)^2}-\frac{3ab^2\tanh^{-1}\left(\frac{b\cos(c+dx)-a\sin(c+dx)}{\sqrt{a^2+b^2}}\right)}{d(a^2+b^2)^{5/2}}-\frac{b^3}{d(a^2+b^2)^2(a\cos(c+dx)+b\sin(c$$

```
[Out] -3*a*b^2*arctanh((b*cos(d*x+c)-a*sin(d*x+c))/(a^2+b^2)^(1/2))/(a^2+b^2)^(5/
2)/d+2*a*b*cos(d*x+c)/(a^2+b^2)^2/d+(a^2-b^2)*sin(d*x+c)/(a^2+b^2)^2/d-b^3/
(a^2+b^2)^2/d/(a*cos(d*x+c)+b*sin(d*x+c))
```

Rubi [A] time = 1.05, antiderivative size = 231, normalized size of antiderivative = 1.67, number of steps used = 11, number of rules used = 6, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {6742, 639, 203, 638, 618, 206}

$$-\frac{2b^3\left(a+b\tan\left(\frac{1}{2}(c+dx)\right)\right)}{ad(a^2+b^2)^2\left(-a\tan^2\left(\frac{1}{2}(c+dx)\right)+a+2b\tan\left(\frac{1}{2}(c+dx)\right)\right)}+\frac{2\left((a^2-b^2)\tan\left(\frac{1}{2}(c+dx)\right)+2ab\right)}{d(a^2+b^2)^2\left(\tan^2\left(\frac{1}{2}(c+dx)\right)+1\right)}+\frac{2b^4\tanh^{-}}{ad(}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^3/(a*Cos[c + d*x] + b*Sin[c + d*x])^2,x]
```

```
[Out] (2*b^4*ArcTanh[(b - a*Tan[(c + d*x)/2])/Sqrt[a^2 + b^2]])/(a*(a^2 + b^2)^(5
/2)*d) - (2*b^2*(3*a^2 + b^2)*ArcTanh[(b - a*Tan[(c + d*x)/2])/Sqrt[a^2 + b
^2]])/(a*(a^2 + b^2)^(5/2)*d) + (2*(2*a*b + (a^2 - b^2)*Tan[(c + d*x)/2]))/
((a^2 + b^2)^2*d*(1 + Tan[(c + d*x)/2]^2)) - (2*b^3*(a + b*Tan[(c + d*x)/2]
))/(a*(a^2 + b^2)^2*d*(a + 2*b*Tan[(c + d*x)/2] - a*Tan[(c + d*x)/2]^2))
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 638

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[((b*d - 2*a*e + (2*c*d - b*e)*x)*(a + b*x + c*x^2)^(p + 1))/((p +
 1)*(b^2 - 4*a*c)), x] - Dist[((2*p + 3)*(2*c*d - b*e))/((p + 1)*(b^2 - 4*a
*c)), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] &&
 NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2]
```

Rule 639

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((a*e
 - c*d*x)*(a + c*x^2)^(p + 1))/(2*a*c*(p + 1)), x] + Dist[(d*(2*p + 3))/(2*
a*(p + 1)), Int[(a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e}, x] && Lt
Q[p, -1] && NeQ[p, -3/2]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int \frac{\cos^3(c+dx)}{(a\cos(c+dx)+b\sin(c+dx))^2}\,dx = \frac{2\,\text{Subst}\left(\int \frac{\left(1-x^2\right)^3}{\left(1+x^2\right)^2\left(a+2bx-ax^2\right)^2}\,dx, x, \tan\left(\frac{1}{2}(c+dx)\right)\right)}{d}$$

$$= \frac{2\,\text{Subst}\left(\int \left(\frac{2\left(a^2-b^2-2abx\right)}{\left(a^2+b^2\right)^2\left(1+x^2\right)^2} + \frac{-a^2+b^2}{\left(a^2+b^2\right)^2\left(1+x^2\right)} - \frac{2b^3x}{a\left(a^2+b^2\right)\left(-a-2bx+ax^2\right)^2} - \frac{}{a\left(a^2+\right.}\right.\right.}{d}$$

$$= \frac{4\,\text{Subst}\left(\int \frac{a^2-b^2-2abx}{\left(1+x^2\right)^2}\,dx, x, \tan\left(\frac{1}{2}(c+dx)\right)\right)}{\left(a^2+b^2\right)^2 d} - \frac{\left(2\left(a^2-b^2\right)\right)\text{Subst}\left(\int \frac{}{1}\right.}{\left(a^2+\right.}$$

$$= -\frac{\left(a^2-b^2\right)x}{\left(a^2+b^2\right)^2} + \frac{2\left(2ab+\left(a^2-b^2\right)\tan\left(\frac{1}{2}(c+dx)\right)\right)}{\left(a^2+b^2\right)^2 d\left(1+\tan^2\left(\frac{1}{2}(c+dx)\right)\right)} - \frac{2}{a\left(a^2+b^2\right)^2 d\left(a\right.}$$

$$= -\frac{2b^2\left(3a^2+b^2\right)\tanh^{-1}\left(\frac{b-a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2+b^2}}\right)}{a\left(a^2+b^2\right)^{5/2} d} + \frac{2\left(2ab+\left(a^2-b^2\right)\tan\left(\frac{1}{2}(c\right.\right.}{\left(a^2+b^2\right)^2 d\left(1+\tan^2\left(\frac{1}{2}(c\right.\right.}$$

$$= \frac{2b^4\tanh^{-1}\left(\frac{b-a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2+b^2}}\right)}{a\left(a^2+b^2\right)^{5/2} d} - \frac{2b^2\left(3a^2+b^2\right)\tanh^{-1}\left(\frac{b-a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2+b^2}}\right)}{a\left(a^2+b^2\right)^{5/2} d} +$$

Mathematica [A] time = 0.76, size = 130, normalized size = 0.94

$$\frac{\frac{a\left(a^2+b^2\right)\sin(2(c+dx))+b\left(a^2+b^2\right)\cos(2(c+dx))+3b\left(a^2-b^2\right)}{\left(a^2+b^2\right)^2(a\cos(c+dx)+b\sin(c+dx))} + \frac{12ab^2\tanh^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)-b}{\sqrt{a^2+b^2}}\right)}{\left(a^2+b^2\right)^{5/2}}}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^3/(a*Cos[c + d*x] + b*Sin[c + d*x])^2,x]
```

```
[Out] ((12*a*b^2*ArcTanh[(-b + a*Tan[(c + d*x)/2])/Sqrt[a^2 + b^2]])/(a^2 + b^2)^
(5/2) + (3*b*(a^2 - b^2) + b*(a^2 + b^2)*Cos[2*(c + d*x)] + a*(a^2 + b^2)*S
in[2*(c + d*x)])/((a^2 + b^2)^2*(a*Cos[c + d*x] + b*Sin[c + d*x])))/(2*d)
```

fricas [B] time = 0.57, size = 302, normalized size = 2.19

$$\frac{2\,a^{4}b-2\,a^{2}b^{3}-4\,b^{5}+2\left(a^{4}b+2\,a^{2}b^{3}+b^{5}\right)\cos\left(dx+c\right)^{2}+2\left(a^{5}+2\,a^{3}b^{2}+ab^{4}\right)\cos\left(dx+c\right)\sin\left(dx+c\right)+3\left(a^{2}\cdots\right.}{2\left(\left(a^{7}+3\,a^{5}b^{2}+3\,a^{3}b^{4}+ab^{6}\right)d\cos\left(dx\cdots\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3/(a*cos(d*x+c)+b*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/2*(2*a^4*b - 2*a^2*b^3 - 4*b^5 + 2*(a^4*b + 2*a^2*b^3 + b^5)*cos(d*x + c)
^2 + 2*(a^5 + 2*a^3*b^2 + a*b^4)*cos(d*x + c)*sin(d*x + c) + 3*(a^2*b^2*cos
(d*x + c) + a*b^3*sin(d*x + c))*sqrt(a^2 + b^2)*log(-(2*a*b*cos(d*x + c)*si
n(d*x + c) + (a^2 - b^2)*cos(d*x + c)^2 - 2*a^2 - b^2 + 2*sqrt(a^2 + b^2)*(
b*cos(d*x + c) - a*sin(d*x + c)))/(2*a*b*cos(d*x + c)*sin(d*x + c) + (a^2 -
 b^2)*cos(d*x + c)^2 + b^2)))/((a^7 + 3*a^5*b^2 + 3*a^3*b^4 + a*b^6)*d*cos(
d*x + c) + (a^6*b + 3*a^4*b^3 + 3*a^2*b^5 + b^7)*d*sin(d*x + c))
```

giac [B] time = 0.32, size = 286, normalized size = 2.07

$$-\frac{\frac{3\,ab^{2}\log\left(\frac{\left|2\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-2\,b-2\,\sqrt{a^{2}+b^{2}}\right|}{\left|2\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-2\,b+2\,\sqrt{a^{2}+b^{2}}\right|}\right)}{\left(a^{4}+2\,a^{2}b^{2}+b^{4}\right)\sqrt{a^{2}+b^{2}}}-\frac{2\left(a^{4}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}-a^{2}b^{2}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}+b^{4}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}+3\,ab^{3}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-a^{4}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\cdots\right.\right.}{\left(a^{5}+2\,a^{3}b^{2}+ab^{4}\right)\left(a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{4}-2\,b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}-2\,b\tan\cdots\right.}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3/(a*cos(d*x+c)+b*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -(3*a*b^2*log(abs(2*a*tan(1/2*d*x + 1/2*c) - 2*b - 2*sqrt(a^2 + b^2))/abs(2
*a*tan(1/2*d*x + 1/2*c) - 2*b + 2*sqrt(a^2 + b^2)))/((a^4 + 2*a^2*b^2 + b^4
)*sqrt(a^2 + b^2)) - 2*(a^4*tan(1/2*d*x + 1/2*c)^3 - a^2*b^2*tan(1/2*d*x +
1/2*c)^3 + b^4*tan(1/2*d*x + 1/2*c)^3 + 3*a*b^3*tan(1/2*d*x + 1/2*c)^2 - a^
4*tan(1/2*d*x + 1/2*c) - 3*a^2*b^2*tan(1/2*d*x + 1/2*c) + b^4*tan(1/2*d*x +
 1/2*c) - 2*a^3*b + a*b^3)/((a^5 + 2*a^3*b^2 + a*b^4)*(a*tan(1/2*d*x + 1/2*
c)^4 - 2*b*tan(1/2*d*x + 1/2*c)^3 - 2*b*tan(1/2*d*x + 1/2*c) - a)))/d
```

maple [A] time = 0.24, size = 172, normalized size = 1.25

$$\frac{-\frac{2\left(\left(-a^{2}+b^{2}\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-2ab\right)}{\left(a^{4}+2a^{2}b^{2}+b^{4}\right)\left(\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}-\frac{2b^{2}\left(\frac{-\frac{b^{2}\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{a}-b}{\left(\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a-2b\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-a}-\frac{3a\,\mathrm{arctanh}\left(\frac{2a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-2b}{2\sqrt{a^{2}+b^{2}}}\right)}{\sqrt{a^{2}+b^{2}}}\right)}{\left(a^{2}+b^{2}\right)^{2}}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3/(a*cos(d*x+c)+b*sin(d*x+c))^2,x)
```

```
[Out] 1/d*(-2/(a^4+2*a^2*b^2+b^4)*((-a^2+b^2)*tan(1/2*d*x+1/2*c)-2*a*b)/(tan(1/2*
d*x+1/2*c)^2+1)-2*b^2/(a^2+b^2)^2*((-b^2/a*tan(1/2*d*x+1/2*c)-b)/(tan(1/2*d
*x+1/2*c)^2*a-2*b*tan(1/2*d*x+1/2*c)-a)-3*a/(a^2+b^2)^(1/2)*arctanh(1/2*(2*
a*tan(1/2*d*x+1/2*c)-2*b)/(a^2+b^2)^(1/2))))
```

maxima [B] time = 2.55, size = 348, normalized size = 2.52

$$-\frac{\frac{3\,ab^2\log\left(\frac{b-\frac{a\sin(dx+c)}{\cos(dx+c)+1}+\sqrt{a^2+b^2}}{b-\frac{a\sin(dx+c)}{\cos(dx+c)+1}-\sqrt{a^2+b^2}}\right)}{\left(a^4+2\,a^2b^2+b^4\right)\sqrt{a^2+b^2}}-\frac{2\left(2\,a^3b-ab^3-\frac{3\,ab^3\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{\left(a^4+3\,a^2b^2-b^4\right)\sin(dx+c)}{\cos(dx+c)+1}-\frac{\left(a^4-a^2b^2+b^4\right)\sin(dx+c)^3}{(\cos(dx+c)+1)^3}\right)}{a^6+2\,a^4b^2+a^2b^4+\frac{2\left(a^5b+2\,a^3b^3+ab^5\right)\sin(dx+c)}{\cos(dx+c)+1}+\frac{2\left(a^5b+2\,a^3b^3+ab^5\right)\sin(dx+c)^3}{(\cos(dx+c)+1)^3}-\frac{\left(a^6+2\,a^4b^2+a^2b^4\right)\sin(dx+c)^4}{(\cos(dx+c)+1)^4}}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3/(a*cos(d*x+c)+b*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -(3*a*b^2*log((b - a*sin(d*x + c)/(cos(d*x + c) + 1) + sqrt(a^2 + b^2))/(b
- a*sin(d*x + c)/(cos(d*x + c) + 1) - sqrt(a^2 + b^2)))/((a^4 + 2*a^2*b^2 +
 b^4)*sqrt(a^2 + b^2)) - 2*(2*a^3*b - a*b^3 - 3*a*b^3*sin(d*x + c)^2/(cos(d
*x + c) + 1)^2 + (a^4 + 3*a^2*b^2 - b^4)*sin(d*x + c)/(cos(d*x + c) + 1) -
(a^4 - a^2*b^2 + b^4)*sin(d*x + c)^3/(cos(d*x + c) + 1)^3)/(a^6 + 2*a^4*b^2
 + a^2*b^4 + 2*(a^5*b + 2*a^3*b^3 + a*b^5)*sin(d*x + c)/(cos(d*x + c) + 1)
+ 2*(a^5*b + 2*a^3*b^3 + a*b^5)*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 - (a^6
+ 2*a^4*b^2 + a^2*b^4)*sin(d*x + c)^4/(cos(d*x + c) + 1)^4))/d
```

mupad [B] time = 2.81, size = 286, normalized size = 2.07

$$\frac{\frac{4\,a^2\,b-2\,b^3}{a^4+2\,a^2\,b^2+b^4}-\frac{6\,b^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{a^4+2\,a^2\,b^2+b^4}+\frac{2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\left(a^4+3\,a^2\,b^2-b^4\right)}{a\left(a^4+2\,a^2\,b^2+b^4\right)}-\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3\left(2\,a^4-2\,a^2\,b^2+2\,b^4\right)}{a\left(a^4+2\,a^2\,b^2+b^4\right)}}{d\left(-a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4+2\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3+2\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+a\right)}-\frac{6\,a\,b^2\,\mathrm{atanh}\left(\frac{a^4\,b+b^5+2\,a^2\,b^3-a\,\mathrm{ta}\cdots}{(a\cdots}\right)}{d\left(a^2+b^2\cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^3/(a*cos(c + d*x) + b*sin(c + d*x))^2,x)
```

```
[Out] ((4*a^2*b - 2*b^3)/(a^4 + b^4 + 2*a^2*b^2) - (6*b^3*tan(c/2 + (d*x)/2)^2)/(
a^4 + b^4 + 2*a^2*b^2) + (2*tan(c/2 + (d*x)/2)*(a^4 - b^4 + 3*a^2*b^2))/(a*
(a^4 + b^4 + 2*a^2*b^2)) - (tan(c/2 + (d*x)/2)^3*(2*a^4 + 2*b^4 - 2*a^2*b^2
))/(a*(a^4 + b^4 + 2*a^2*b^2)))/(d*(a + 2*b*tan(c/2 + (d*x)/2) - a*tan(c/2
+ (d*x)/2)^4 + 2*b*tan(c/2 + (d*x)/2)^3)) - (6*a*b^2*atanh((a^4*b + b^5 + 2
```

```
*a^2*b^3 - a*tan(c/2 + (d*x)/2)*(a^4 + b^4 + 2*a^2*b^2))/(a^2 + b^2)^(5/2))
)/(d*(a^2 + b^2)^(5/2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3/(a*cos(d*x+c)+b*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

3.124 $\int \frac{\cos^2(c+dx)}{(a\cos(c+dx)+b\sin(c+dx))^2}\,dx$

Optimal. Leaf size=82

$$-\frac{b}{d\left(a^2+b^2\right)(a+b\tan(c+dx))}+\frac{2ab\log(a\cos(c+dx)+b\sin(c+dx))}{d\left(a^2+b^2\right)^2}+\frac{x\left(a^2-b^2\right)}{\left(a^2+b^2\right)^2}$$

```
[Out] (a^2-b^2)*x/(a^2+b^2)^2+2*a*b*ln(a*cos(d*x+c)+b*sin(d*x+c))/(a^2+b^2)^2/d-b
/(a^2+b^2)/d/(a+b*tan(d*x+c))
```

Rubi [A] time = 0.14, antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3086, 3483, 3531, 3530}

$$-\frac{b}{d\left(a^2+b^2\right)(a+b\tan(c+dx))}+\frac{2ab\log(a\cos(c+dx)+b\sin(c+dx))}{d\left(a^2+b^2\right)^2}+\frac{x\left(a^2-b^2\right)}{\left(a^2+b^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2/(a*Cos[c + d*x] + b*Sin[c + d*x])^2,x]
```

```
[Out] ((a^2 - b^2)*x)/(a^2 + b^2)^2 + (2*a*b*Log[a*Cos[c + d*x] + b*Sin[c + d*x]]
)/((a^2 + b^2)^2*d) - b/((a^2 + b^2)*d*(a + b*Tan[c + d*x]))
```

Rule 3086

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin
[(c_.) + (d_.)*(x_)])^(n_.), x_Symbol] :> Int[(a + b*Tan[c + d*x])^n, x] /;
 FreeQ[{a, b, c, d}, x] && EqQ[m + n, 0] && IntegerQ[n] && NeQ[a^2 + b^2, 0
]
```

Rule 3483

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(a +
b*Tan[c + d*x])^(n + 1))/(d*(n + 1)*(a^2 + b^2)), x] + Dist[1/(a^2 + b^2),
Int[(a - b*Tan[c + d*x])*(a + b*Tan[c + d*x])^(n + 1), x], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 + b^2, 0] && LtQ[n, -1]
```

Rule 3530

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_) + (b_.)*tan[(e_.) + (f_.)*
(x_)]), x_Symbol] :> Simp[(c*Log[RemoveContent[a*Cos[e + f*x] + b*Sin[e + f
*x], x]])/(b*f), x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
```

```
NeQ[a^2 + b^2, 0] && EqQ[a*c + b*d, 0]
```

Rule 3531

```
Int[((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_.) + (b_.)*tan[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[((a*c + b*d)*x)/(a^2 + b^2), x] + Dist[(b*c - a
*d)/(a^2 + b^2), Int[(b - a*Tan[e + f*x])/(a + b*Tan[e + f*x]), x], x] /; F
reeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && Ne
Q[a*c + b*d, 0]
```

Rubi steps

$$\int \frac{\cos^2(c+dx)}{(a\cos(c+dx)+b\sin(c+dx))^2}\,dx = \int \frac{1}{(a+b\tan(c+dx))^2}\,dx$$
$$= -\frac{b}{\left(a^2+b^2\right)d(a+b\tan(c+dx))} + \frac{\int \frac{a-b\tan(c+dx)}{a+b\tan(c+dx)}\,dx}{a^2+b^2}$$
$$= \frac{\left(a^2-b^2\right)x}{\left(a^2+b^2\right)^2} - \frac{b}{\left(a^2+b^2\right)d(a+b\tan(c+dx))} + \frac{(2ab)\int \frac{b-a\tan(c+dx)}{a+b\tan(c+dx)}\,dx}{\left(a^2+b^2\right)^2}$$
$$= \frac{\left(a^2-b^2\right)x}{\left(a^2+b^2\right)^2} + \frac{2ab\log(a\cos(c+dx)+b\sin(c+dx))}{\left(a^2+b^2\right)^2 d} - \frac{b}{\left(a^2+b^2\right)d(a+b$$

Mathematica [C] time = 0.40, size = 192, normalized size = 2.34

$$\frac{b\sin(c+dx)\left(a^2b\log\left((a\cos(c+dx)+b\sin(c+dx))^2\right)+(a+ib)\left(a^2(c+dx)+ab(ic+idx+1)-ib^2\right)\right)+a^2\cos(}{ad\left(a^2+b^2\right)^2(a}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2/(a*Cos[c + d*x] + b*Sin[c + d*x])^2,x]
```

```
[Out] (a^2*Cos[c + d*x]*((a + I*b)^2*(c + d*x) + a*b*Log[(a*Cos[c + d*x] + b*Sin[
c + d*x])^2]) + b*((a + I*b)*((-I)*b^2 + a*b*(1 + I*c + I*d*x) + a^2*(c + d
*x)) + a^2*b*Log[(a*Cos[c + d*x] + b*Sin[c + d*x])^2])*Sin[c + d*x] - (2*I)
*a^2*b*ArcTan[Tan[c + d*x]]*(a*Cos[c + d*x] + b*Sin[c + d*x]))/(a*(a^2 + b^
2)^2*d*(a*Cos[c + d*x] + b*Sin[c + d*x]))
```

fricas [B] time = 0.54, size = 173, normalized size = 2.11

$$-\frac{\left(b^3-\left(a^3-ab^2\right)dx\right)\cos\left(dx+c\right)-\left(a^2b\cos\left(dx+c\right)+ab^2\sin\left(dx+c\right)\right)\log\left(2\,ab\cos\left(dx+c\right)\sin\left(dx+c\right)+\left(a^2\right.\right.}{\left(a^5+2\,a^3b^2+ab^4\right)d\cos\left(dx+c\right)+\left(a^4b+2\,a^2b^3+b^5\right)d\sin}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(a*cos(d*x+c)+b*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -((b^3 - (a^3 - a*b^2)*d*x)*cos(d*x + c) - (a^2*b*cos(d*x + c) + a*b^2*sin(
d*x + c))*log(2*a*b*cos(d*x + c)*sin(d*x + c) + (a^2 - b^2)*cos(d*x + c)^2
+ b^2) - (a*b^2 + (a^2*b - b^3)*d*x)*sin(d*x + c))/((a^5 + 2*a^3*b^2 + a*b^
4)*d*cos(d*x + c) + (a^4*b + 2*a^2*b^3 + b^5)*d*sin(d*x + c))
```

giac [A] time = 1.96, size = 159, normalized size = 1.94

$$\frac{\frac{2\,ab^2\log\left(|b\tan\left(dx+c\right)+a|\right)}{a^4b+2\,a^2b^3+b^5}-\frac{ab\log\left(\tan\left(dx+c\right)^2+1\right)}{a^4+2\,a^2b^2+b^4}+\frac{\left(a^2-b^2\right)\left(dx+c\right)}{a^4+2\,a^2b^2+b^4}-\frac{2\,ab^2\tan\left(dx+c\right)+3\,a^2b+b^3}{\left(a^4+2\,a^2b^2+b^4\right)\left(b\tan\left(dx+c\right)+a\right)}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(a*cos(d*x+c)+b*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] (2*a*b^2*log(abs(b*tan(d*x + c) + a))/(a^4*b + 2*a^2*b^3 + b^5) - a*b*log(t
an(d*x + c)^2 + 1)/(a^4 + 2*a^2*b^2 + b^4) + (a^2 - b^2)*(d*x + c)/(a^4 + 2
*a^2*b^2 + b^4) - (2*a*b^2*tan(d*x + c) + 3*a^2*b + b^3)/((a^4 + 2*a^2*b^2
+ b^4)*(b*tan(d*x + c) + a)))/d
```

maple [A] time = 0.21, size = 130, normalized size = 1.59

$$-\frac{b}{\left(a^2+b^2\right)d\left(a+b\tan\left(dx+c\right)\right)}+\frac{2ba\ln\left(a+b\tan\left(dx+c\right)\right)}{d\left(a^2+b^2\right)^2}-\frac{ab\ln\left(\tan^2\left(dx+c\right)+1\right)}{d\left(a^2+b^2\right)^2}+\frac{\arctan\left(\tan\left(dx+c\right)\right)a^2}{d\left(a^2+b^2\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2/(a*cos(d*x+c)+b*sin(d*x+c))^2,x)
```

```
[Out] -b/(a^2+b^2)/d/(a+b*tan(d*x+c))+2/d*b*a/(a^2+b^2)^2*ln(a+b*tan(d*x+c))-1/d/
(a^2+b^2)^2*a*b*ln(tan(d*x+c)^2+1)+1/d/(a^2+b^2)^2*arctan(tan(d*x+c))*a^2-1
/d/(a^2+b^2)^2*arctan(tan(d*x+c))*b^2
```

maxima [A] time = 0.43, size = 131, normalized size = 1.60

$$\frac{\frac{2\,ab\log\left(b\tan\left(dx+c\right)+a\right)}{a^4+2\,a^2b^2+b^4}-\frac{ab\log\left(\tan\left(dx+c\right)^2+1\right)}{a^4+2\,a^2b^2+b^4}+\frac{\left(a^2-b^2\right)\left(dx+c\right)}{a^4+2\,a^2b^2+b^4}-\frac{b}{a^3+ab^2+\left(a^2b+b^3\right)\tan\left(dx+c\right)}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(a*cos(d*x+c)+b*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] (2*a*b*log(b*tan(d*x + c) + a)/(a^4 + 2*a^2*b^2 + b^4) - a*b*log(tan(d*x +
c)^2 + 1)/(a^4 + 2*a^2*b^2 + b^4) + (a^2 - b^2)*(d*x + c)/(a^4 + 2*a^2*b^2
+ b^4) - b/(a^3 + a*b^2 + (a^2*b + b^3)*tan(d*x + c)))/d
```

mupad [B] time = 4.87, size = 3114, normalized size = 37.98

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^2/(a*cos(c + d*x) + b*sin(c + d*x))^2,x)
```

```
[Out] (2*a*b*log(a + 2*b*tan(c/2 + (d*x)/2) - a*tan(c/2 + (d*x)/2)^2))/(d*(a^4 +
b^4 + 2*a^2*b^2)) - (2*a*b*log(1/(cos(c + d*x) + 1)))/(d*(a^4 + b^4 + 2*a^2
*b^2)) + (2*atan((tan(c/2 + (d*x)/2)*((((2*a*b*((((32*(6*a^8*b + 6*a^4*b^5
+ 12*a^6*b^3))/(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2) - (64*a*b*(3*a*b^10 + 12
*a^3*b^8 + 18*a^5*b^6 + 12*a^7*b^4 + 3*a^9*b^2))/((a^4 + b^4 + 2*a^2*b^2)*(
a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2)))*(a + b)*(a - b))/(a^4 + b^4 + 2*a^2*b^
2) - (64*a*b*(a + b)*(a - b)*(3*a*b^10 + 12*a^3*b^8 + 18*a^5*b^6 + 12*a^7*b
^4 + 3*a^9*b^2))/((a^4 + b^4 + 2*a^2*b^2)^2*(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*
b^2))))/(a^4 + b^4 + 2*a^2*b^2) - ((a + b)*((32*(2*a*b^6 + a^7 - 7*a^3*b^4
- 8*a^5*b^2))/(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2) - (2*a*b*((32*(6*a^8*b +
6*a^4*b^5 + 12*a^6*b^3))/(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2) - (64*a*b*(3*a
*b^10 + 12*a^3*b^8 + 18*a^5*b^6 + 12*a^7*b^4 + 3*a^9*b^2))/((a^4 + b^4 + 2*
a^2*b^2)*(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2))))/(a^4 + b^4 + 2*a^2*b^2))*(a
 - b))/(a^4 + b^4 + 2*a^2*b^2) + (32*(a + b)^3*(a - b)^3*(3*a*b^10 + 12*a^3
*b^8 + 18*a^5*b^6 + 12*a^7*b^4 + 3*a^9*b^2))/((a^4 + b^4 + 2*a^2*b^2)^3*(a^
6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2)))*(a^6 - b^6 + 35*a^2*b^4 - 35*a^4*b^2))/(
a^6 + b^6 + 15*a^2*b^4 + 15*a^4*b^2)^2 - (2*a*b*(5*a^4 + 5*b^4 - 26*a^2*b^2
)*((32*(2*a^4*b + 4*a^2*b^3))/(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2) + ((a + b
)*(a - b)*((((32*(6*a^8*b + 6*a^4*b^5 + 12*a^6*b^3))/(a^6 + b^6 + 3*a^2*b^4
 + 3*a^4*b^2) - (64*a*b*(3*a*b^10 + 12*a^3*b^8 + 18*a^5*b^6 + 12*a^7*b^4 +
3*a^9*b^2))/((a^4 + b^4 + 2*a^2*b^2)*(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2)))*
(a + b)*(a - b))/(a^4 + b^4 + 2*a^2*b^2) - (64*a*b*(a + b)*(a - b)*(3*a*b^1
0 + 12*a^3*b^8 + 18*a^5*b^6 + 12*a^7*b^4 + 3*a^9*b^2))/((a^4 + b^4 + 2*a^2*
b^2)^2*(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2))))/(a^4 + b^4 + 2*a^2*b^2) + (2*
a*b*((32*(2*a*b^6 + a^7 - 7*a^3*b^4 - 8*a^5*b^2))/(a^6 + b^6 + 3*a^2*b^4 +
3*a^4*b^2) - (2*a*b*((32*(6*a^8*b + 6*a^4*b^5 + 12*a^6*b^3))/(a^6 + b^6 + 3
*a^2*b^4 + 3*a^4*b^2) - (64*a*b*(3*a*b^10 + 12*a^3*b^8 + 18*a^5*b^6 + 12*a^
7*b^4 + 3*a^9*b^2))/((a^4 + b^4 + 2*a^2*b^2)*(a^6 + b^6 + 3*a^2*b^4 + 3*a^4
*b^2))))/(a^4 + b^4 + 2*a^2*b^2)))/(a^4 + b^4 + 2*a^2*b^2) - (64*a*b*(a + b
)^2*(a - b)^2*(3*a*b^10 + 12*a^3*b^8 + 18*a^5*b^6 + 12*a^7*b^4 + 3*a^9*b^2)
```

```
)/((a^4 + b^4 + 2*a^2*b^2)^3*(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2))))/(a^6 +
b^6 + 15*a^2*b^4 + 15*a^4*b^2)^2)*(a^10 + b^10 + 5*a^2*b^8 + 10*a^4*b^6 + 1
0*a^6*b^4 + 5*a^8*b^2))/(32*a*b^2 - 32*a^3) + ((((a + b)*(a - b)*((32*(3*a^
6*b + 3*a^2*b^5 + 6*a^4*b^3))/(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2) - (2*a*b*
((32*(2*a^3*b^6 - a^9 - a*b^8 + 6*a^5*b^4 + 2*a^7*b^2))/(a^6 + b^6 + 3*a^2*
b^4 + 3*a^4*b^2) + (64*a*b*(3*a^10*b + 3*a^2*b^9 + 12*a^4*b^7 + 18*a^6*b^5
+ 12*a^8*b^3))/((a^4 + b^4 + 2*a^2*b^2)*(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2)
)))/(a^4 + b^4 + 2*a^2*b^2)))/(a^4 + b^4 + 2*a^2*b^2) - (2*a*b*((((32*(2*a^
3*b^6 - a^9 - a*b^8 + 6*a^5*b^4 + 2*a^7*b^2))/(a^6 + b^6 + 3*a^2*b^4 + 3*a^
4*b^2) + (64*a*b*(3*a^10*b + 3*a^2*b^9 + 12*a^4*b^7 + 18*a^6*b^5 + 12*a^8*b
^3))/((a^4 + b^4 + 2*a^2*b^2)*(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2)))*(a + b)
*(a - b))/(a^4 + b^4 + 2*a^2*b^2) + (64*a*b*(a + b)*(a - b)*(3*a^10*b + 3*a
^2*b^9 + 12*a^4*b^7 + 18*a^6*b^5 + 12*a^8*b^3))/((a^4 + b^4 + 2*a^2*b^2)^2*
(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2))))/(a^4 + b^4 + 2*a^2*b^2) + (32*(a + b
)^3*(a - b)^3*(3*a^10*b + 3*a^2*b^9 + 12*a^4*b^7 + 18*a^6*b^5 + 12*a^8*b^3)
)/((a^4 + b^4 + 2*a^2*b^2)^3*(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2)))*(a^6 - b
^6 + 35*a^2*b^4 - 35*a^4*b^2)*(a^10 + b^10 + 5*a^2*b^8 + 10*a^4*b^6 + 10*a^
6*b^4 + 5*a^8*b^2))/((32*a*b^2 - 32*a^3)*(a^6 + b^6 + 15*a^2*b^4 + 15*a^4*b
^2)^2) + (2*a*b*(5*a^4 + 5*b^4 - 26*a^2*b^2)*((64*a^3*b^2)/(a^6 + b^6 + 3*a
^2*b^4 + 3*a^4*b^2) + (2*a*b*((32*(3*a^6*b + 3*a^2*b^5 + 6*a^4*b^3))/(a^6 +
 b^6 + 3*a^2*b^4 + 3*a^4*b^2) - (2*a*b*((32*(2*a^3*b^6 - a^9 - a*b^8 + 6*a^
5*b^4 + 2*a^7*b^2))/(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2) + (64*a*b*(3*a^10*b
 + 3*a^2*b^9 + 12*a^4*b^7 + 18*a^6*b^5 + 12*a^8*b^3))/((a^4 + b^4 + 2*a^2*b
^2)*(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2))))/(a^4 + b^4 + 2*a^2*b^2)))/(a^4 +
 b^4 + 2*a^2*b^2) + ((a + b)*(a - b)*((((32*(2*a^3*b^6 - a^9 - a*b^8 + 6*a^
5*b^4 + 2*a^7*b^2))/(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2) + (64*a*b*(3*a^10*b
 + 3*a^2*b^9 + 12*a^4*b^7 + 18*a^6*b^5 + 12*a^8*b^3))/((a^4 + b^4 + 2*a^2*b
^2)*(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2)))*(a + b)*(a - b))/(a^4 + b^4 + 2*a
^2*b^2) + (64*a*b*(a + b)*(a - b)*(3*a^10*b + 3*a^2*b^9 + 12*a^4*b^7 + 18*a
^6*b^5 + 12*a^8*b^3))/((a^4 + b^4 + 2*a^2*b^2)^2*(a^6 + b^6 + 3*a^2*b^4 + 3
*a^4*b^2))))/(a^4 + b^4 + 2*a^2*b^2) + (64*a*b*(a + b)^2*(a - b)^2*(3*a^10*
b + 3*a^2*b^9 + 12*a^4*b^7 + 18*a^6*b^5 + 12*a^8*b^3))/((a^4 + b^4 + 2*a^2*
b^2)^3*(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2)))*(a^10 + b^10 + 5*a^2*b^8 + 10*
a^4*b^6 + 10*a^6*b^4 + 5*a^8*b^2))/((32*a*b^2 - 32*a^3)*(a^6 + b^6 + 15*a^2
*b^4 + 15*a^4*b^2)^2))*(a + b)*(a - b))/(d*(a^4 + b^4 + 2*a^2*b^2)) + (2*b^
2*tan(c/2 + (d*x)/2))/(a*d*(a^2 + b^2)*(a + 2*b*tan(c/2 + (d*x)/2) - a*tan(
c/2 + (d*x)/2)^2))
```

sympy [A] time = 6.38, size = 1552, normalized size = 18.93

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2/(a*cos(d*x+c)+b*sin(d*x+c))**2,x)
```

```
[Out] Piecewise((zoo*x*cos(c)**2/sin(c)**2, Eq(a, 0) & Eq(b, 0) & Eq(d, 0)), ((-x
 - cos(c + d*x)/(d*sin(c + d*x)))/b**2, Eq(a, 0)), (2*d*x*sin(c + d*x)**2/(
-8*b**2*d*sin(c + d*x)**2 + 16*I*b**2*d*sin(c + d*x)*cos(c + d*x) + 8*b**2*
d*cos(c + d*x)**2) - 4*I*d*x*sin(c + d*x)*cos(c + d*x)/(-8*b**2*d*sin(c + d
*x)**2 + 16*I*b**2*d*sin(c + d*x)*cos(c + d*x) + 8*b**2*d*cos(c + d*x)**2)
- 2*d*x*cos(c + d*x)**2/(-8*b**2*d*sin(c + d*x)**2 + 16*I*b**2*d*sin(c + d*
x)*cos(c + d*x) + 8*b**2*d*cos(c + d*x)**2) - I*sin(c + d*x)**2/(-8*b**2*d*
sin(c + d*x)**2 + 16*I*b**2*d*sin(c + d*x)*cos(c + d*x) + 8*b**2*d*cos(c +
d*x)**2) - 3*I*cos(c + d*x)**2/(-8*b**2*d*sin(c + d*x)**2 + 16*I*b**2*d*sin
(c + d*x)*cos(c + d*x) + 8*b**2*d*cos(c + d*x)**2), Eq(a, -I*b)), (2*I*d*x*
sin(c + d*x)**2/(-8*I*b**2*d*sin(c + d*x)**2 + 16*b**2*d*sin(c + d*x)*cos(c
 + d*x) + 8*I*b**2*d*cos(c + d*x)**2) - 4*d*x*sin(c + d*x)*cos(c + d*x)/(-8
*I*b**2*d*sin(c + d*x)**2 + 16*b**2*d*sin(c + d*x)*cos(c + d*x) + 8*I*b**2*
d*cos(c + d*x)**2) - 2*I*d*x*cos(c + d*x)**2/(-8*I*b**2*d*sin(c + d*x)**2 +
 16*b**2*d*sin(c + d*x)*cos(c + d*x) + 8*I*b**2*d*cos(c + d*x)**2) - sin(c
+ d*x)**2/(-8*I*b**2*d*sin(c + d*x)**2 + 16*b**2*d*sin(c + d*x)*cos(c + d*x
) + 8*I*b**2*d*cos(c + d*x)**2) - 3*cos(c + d*x)**2/(-8*I*b**2*d*sin(c + d*
x)**2 + 16*b**2*d*sin(c + d*x)*cos(c + d*x) + 8*I*b**2*d*cos(c + d*x)**2),
Eq(a, I*b)), (x*cos(c)**2/(a*cos(c) + b*sin(c))**2, Eq(d, 0)), (a**3*d*x*co
s(c + d*x)/(a**5*d*cos(c + d*x) + a**4*b*d*sin(c + d*x) + 2*a**3*b**2*d*cos
(c + d*x) + 2*a**2*b**3*d*sin(c + d*x) + a*b**4*d*cos(c + d*x) + b**5*d*sin
(c + d*x)) + a**2*b*d*x*sin(c + d*x)/(a**5*d*cos(c + d*x) + a**4*b*d*sin(c
+ d*x) + 2*a**3*b**2*d*cos(c + d*x) + 2*a**2*b**3*d*sin(c + d*x) + a*b**4*d
*cos(c + d*x) + b**5*d*sin(c + d*x)) + 2*a**2*b*log(cos(c + d*x) + b*sin(c
+ d*x)/a)*cos(c + d*x)/(a**5*d*cos(c + d*x) + a**4*b*d*sin(c + d*x) + 2*a**
3*b**2*d*cos(c + d*x) + 2*a**2*b**3*d*sin(c + d*x) + a*b**4*d*cos(c + d*x)
+ b**5*d*sin(c + d*x)) - a**2*b*cos(c + d*x)/(a**5*d*cos(c + d*x) + a**4*b*
d*sin(c + d*x) + 2*a**3*b**2*d*cos(c + d*x) + 2*a**2*b**3*d*sin(c + d*x) +
a*b**4*d*cos(c + d*x) + b**5*d*sin(c + d*x)) - a*b**2*d*x*cos(c + d*x)/(a**
5*d*cos(c + d*x) + a**4*b*d*sin(c + d*x) + 2*a**3*b**2*d*cos(c + d*x) + 2*a
**2*b**3*d*sin(c + d*x) + a*b**4*d*cos(c + d*x) + b**5*d*sin(c + d*x)) + 2*
a*b**2*log(cos(c + d*x) + b*sin(c + d*x)/a)*sin(c + d*x)/(a**5*d*cos(c + d*
x) + a**4*b*d*sin(c + d*x) + 2*a**3*b**2*d*cos(c + d*x) + 2*a**2*b**3*d*sin
(c + d*x) + a*b**4*d*cos(c + d*x) + b**5*d*sin(c + d*x)) - b**3*d*x*sin(c +
 d*x)/(a**5*d*cos(c + d*x) + a**4*b*d*sin(c + d*x) + 2*a**3*b**2*d*cos(c +
d*x) + 2*a**2*b**3*d*sin(c + d*x) + a*b**4*d*cos(c + d*x) + b**5*d*sin(c +
d*x)) - b**3*cos(c + d*x)/(a**5*d*cos(c + d*x) + a**4*b*d*sin(c + d*x) + 2*
a**3*b**2*d*cos(c + d*x) + 2*a**2*b**3*d*sin(c + d*x) + a*b**4*d*cos(c + d*
x) + b**5*d*sin(c + d*x)), True))
```

3.125 $\int \frac{\cos(c+dx)}{(a\cos(c+dx)+b\sin(c+dx))^2}\,dx$

Optimal. Leaf size=83

$$-\frac{b}{d\left(a^2+b^2\right)\left(a\cos(c+dx)+b\sin(c+dx)\right)}-\frac{a\tanh^{-1}\left(\frac{b\cos(c+dx)-a\sin(c+dx)}{\sqrt{a^2+b^2}}\right)}{d\left(a^2+b^2\right)^{3/2}}$$

```
[Out] -a*arctanh((b*cos(d*x+c)-a*sin(d*x+c))/(a^2+b^2)^(1/2))/(a^2+b^2)^(3/2)/d-b
/(a^2+b^2)/d/(a*cos(d*x+c)+b*sin(d*x+c))
```

Rubi [A] time = 0.07, antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {3155, 3074, 206}

$$-\frac{b}{d\left(a^2+b^2\right)\left(a\cos(c+dx)+b\sin(c+dx)\right)}-\frac{a\tanh^{-1}\left(\frac{b\cos(c+dx)-a\sin(c+dx)}{\sqrt{a^2+b^2}}\right)}{d\left(a^2+b^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]/(a*Cos[c + d*x] + b*Sin[c + d*x])^2,x]
```

```
[Out] -((a*ArcTanh[(b*Cos[c + d*x] - a*Sin[c + d*x])/Sqrt[a^2 + b^2]])/((a^2 + b^
2)^(3/2)*d)) - b/((a^2 + b^2)*d*(a*Cos[c + d*x] + b*Sin[c + d*x]))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3074

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x
_Symbol] :> -Dist[d^(-1), Subst[Int[1/(a^2 + b^2 - x^2), x], x, b*Cos[c + d
*x] - a*Sin[c + d*x]], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

Rule 3155

```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.))/((a_.) + cos[(d_.) + (e_.)*(x_)
]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)])^2, x_Symbol] :> Simp[(c*B + c*A*Co
s[d + e*x] + (a*B - b*A)*Sin[d + e*x])/(e*(a^2 - b^2 - c^2)*(a + b*Cos[d +
e*x] + c*Sin[d + e*x])), x] + Dist[(a*A - b*B)/(a^2 - b^2 - c^2), Int[1/(a
+ b*Cos[d + e*x] + c*Sin[d + e*x]), x], x] /; FreeQ[{a, b, c, d, e, A, B},
```

```
x] && NeQ[a^2 - b^2 - c^2, 0] && NeQ[a*A - b*B, 0]
```

Rubi steps

$$\int \frac{\cos(c+dx)}{(a\cos(c+dx)+b\sin(c+dx))^2}\,dx = -\frac{b}{\left(a^2+b^2\right)d(a\cos(c+dx)+b\sin(c+dx))} + \frac{a\int \frac{1}{a\cos(c+dx)+b\sin(c+dx)}\,dx}{a^2+b^2}$$

$$= -\frac{b}{\left(a^2+b^2\right)d(a\cos(c+dx)+b\sin(c+dx))} - \frac{a\,\mathrm{Subst}\left(\int \frac{1}{a^2+b^2-x^2}\,dx, x, b$$

$$= -\frac{a\tanh^{-1}\left(\frac{b\cos(c+dx)-a\sin(c+dx)}{\sqrt{a^2+b^2}}\right)}{\left(a^2+b^2\right)^{3/2}d} - \frac{b}{\left(a^2+b^2\right)d(a\cos(c+dx)+b\sin(c+$$

Mathematica [A] time = 0.22, size = 79, normalized size = 0.95

$$\frac{\frac{2a\tanh^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)-b}{\sqrt{a^2+b^2}}\right)}{\left(a^2+b^2\right)^{3/2}} - \frac{b}{\left(a^2+b^2\right)(a\cos(c+dx)+b\sin(c+dx))}}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]/(a*Cos[c + d*x] + b*Sin[c + d*x])^2,x]
```

```
[Out] ((2*a*ArcTanh[(-b + a*Tan[(c + d*x)/2])/Sqrt[a^2 + b^2]])/(a^2 + b^2)^(3/2)
 - b/((a^2 + b^2)*(a*Cos[c + d*x] + b*Sin[c + d*x])))/d
```

fricas [B] time = 0.55, size = 215, normalized size = 2.59

$$-\frac{2\,a^2b+2\,b^3-\left(a^2\cos\left(dx+c\right)+ab\sin\left(dx+c\right)\right)\sqrt{a^2+b^2}\,\log\left(-\frac{2\,ab\cos(dx+c)\sin(dx+c)+\left(a^2-b^2\right)\cos(dx+c)^2-2\,a^2-b^2+2\,\sqrt{a^2}}{2\,ab\cos(dx+c)\sin(dx+c)+\left(a^2-b^2\right)\cos(dx}\right.}{2\left(\left(a^5+2\,a^3b^2+ab^4\right)d\cos\left(dx+c\right)+\left(a^4b+2\,a^2b^3+b^5\right)d\sin\left(dx+c\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a*cos(d*x+c)+b*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -1/2*(2*a^2*b + 2*b^3 - (a^2*cos(d*x + c) + a*b*sin(d*x + c))*sqrt(a^2 + b^
2)*log(-(2*a*b*cos(d*x + c)*sin(d*x + c) + (a^2 - b^2)*cos(d*x + c)^2 - 2*a
^2 - b^2 + 2*sqrt(a^2 + b^2)*(b*cos(d*x + c) - a*sin(d*x + c)))/(2*a*b*cos(
```

```
d*x + c)*sin(d*x + c) + (a^2 - b^2)*cos(d*x + c)^2 + b^2)))/((a^5 + 2*a^3*b
^2 + a*b^4)*d*cos(d*x + c) + (a^4*b + 2*a^2*b^3 + b^5)*d*sin(d*x + c))
```

giac [A] time = 0.30, size = 138, normalized size = 1.66

$$-\frac{\frac{a\log\left(\frac{\left|2\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-2\,b-2\,\sqrt{a^2+b^2}\right|}{\left|2\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-2\,b+2\,\sqrt{a^2+b^2}\right|}\right)}{\left(a^2+b^2\right)^{\frac{3}{2}}}-\frac{2\left(b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+ab\right)}{\left(a^3+ab^2\right)\left(a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-2\,b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-a\right)}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a*cos(d*x+c)+b*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -(a*log(abs(2*a*tan(1/2*d*x + 1/2*c) - 2*b - 2*sqrt(a^2 + b^2))/abs(2*a*tan
(1/2*d*x + 1/2*c) - 2*b + 2*sqrt(a^2 + b^2)))/(a^2 + b^2)^(3/2) - 2*(b^2*ta
n(1/2*d*x + 1/2*c) + a*b)/((a^3 + a*b^2)*(a*tan(1/2*d*x + 1/2*c)^2 - 2*b*ta
n(1/2*d*x + 1/2*c) - a)))/d
```

maple [A] time = 0.21, size = 118, normalized size = 1.42

$$\frac{-\frac{2\left(-\frac{b^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{a\left(a^2+b^2\right)}-\frac{b}{a^2+b^2}\right)}{\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a-2b\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-a}+\frac{2a\,\mathrm{arctanh}\left(\frac{2a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-2b}{2\sqrt{a^2+b^2}}\right)}{\left(a^2+b^2\right)^{\frac{3}{2}}}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)/(a*cos(d*x+c)+b*sin(d*x+c))^2,x)
```

```
[Out] 1/d*(-2*(-b^2/a/(a^2+b^2)*tan(1/2*d*x+1/2*c)-b/(a^2+b^2))/(tan(1/2*d*x+1/2*
c)^2*a-2*b*tan(1/2*d*x+1/2*c)-a)+2*a/(a^2+b^2)^(3/2)*arctanh(1/2*(2*a*tan(1
/2*d*x+1/2*c)-2*b)/(a^2+b^2)^(1/2)))
```

maxima [B] time = 0.44, size = 182, normalized size = 2.19

$$-\frac{\frac{a\log\left(\frac{b-\frac{a\sin(dx+c)}{\cos(dx+c)+1}+\sqrt{a^2+b^2}}{b-\frac{a\sin(dx+c)}{\cos(dx+c)+1}-\sqrt{a^2+b^2}}\right)}{\left(a^2+b^2\right)^{\frac{3}{2}}}+\frac{2\left(ab+\frac{b^2\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^4+a^2b^2+\frac{2\left(a^3b+ab^3\right)\sin(dx+c)}{\cos(dx+c)+1}-\frac{\left(a^4+a^2b^2\right)\sin(dx+c)^2}{(\cos(dx+c)+1)^2}}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a*cos(d*x+c)+b*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -(a*log((b - a*sin(d*x + c)/(cos(d*x + c) + 1) + sqrt(a^2 + b^2))/(b - a*si
n(d*x + c)/(cos(d*x + c) + 1) - sqrt(a^2 + b^2)))/(a^2 + b^2)^(3/2) + 2*(a*
b + b^2*sin(d*x + c)/(cos(d*x + c) + 1))/(a^4 + a^2*b^2 + 2*(a^3*b + a*b^3)
*sin(d*x + c)/(cos(d*x + c) + 1) - (a^4 + a^2*b^2)*sin(d*x + c)^2/(cos(d*x
+ c) + 1)^2))/d
```

mupad [B] time = 0.84, size = 136, normalized size = 1.64

$$-\frac{\frac{2\,b}{a^2+b^2}+\frac{2\,b^2\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{a\,\left(a^2+b^2\right)}}{d\,\left(-a\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+2\,b\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+a\right)}+\frac{a\,\mathrm{atan}\left(\frac{a^2\,b\,1\mathrm{i}+b^3\,1\mathrm{i}-a\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(a^2+b^2\right)\,1\mathrm{i}}{\left(a^2+b^2\right)^{3/2}}\right)\,2\mathrm{i}}{d\,\left(a^2+b^2\right)^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)/(a*cos(c + d*x) + b*sin(c + d*x))^2,x)
```

```
[Out] (a*atan((a^2*b*1i + b^3*1i - a*tan(c/2 + (d*x)/2)*(a^2 + b^2)*1i)/(a^2 + b^
2)^(3/2))*2i)/(d*(a^2 + b^2)^(3/2)) - ((2*b)/(a^2 + b^2) + (2*b^2*tan(c/2 +
 (d*x)/2))/(a*(a^2 + b^2)))/(d*(a + 2*b*tan(c/2 + (d*x)/2) - a*tan(c/2 + (d
*x)/2)^2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a*cos(d*x+c)+b*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

3.126 $\int \frac{1}{(a\cos(c+dx)+b\sin(c+dx))^2}\,dx$

Optimal. Leaf size=32

$$\frac{\sin(c+dx)}{ad(a\cos(c+dx)+b\sin(c+dx))}$$

```
[Out] sin(d*x+c)/a/d/(a*cos(d*x+c)+b*sin(d*x+c))
```

Rubi [A] time = 0.02, antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {3075}

$$\frac{\sin(c+dx)}{ad(a\cos(c+dx)+b\sin(c+dx))}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Cos[c + d*x] + b*Sin[c + d*x])^(-2),x]
```

```
[Out] Sin[c + d*x]/(a*d*(a*Cos[c + d*x] + b*Sin[c + d*x]))
```

Rule 3075

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-2), x
_Symbol] :> Simp[Sin[c + d*x]/(a*d*(a*Cos[c + d*x] + b*Sin[c + d*x])), x] /
; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

Rubi steps

$$\int \frac{1}{(a\cos(c+dx)+b\sin(c+dx))^2}\,dx = \frac{\sin(c+dx)}{ad(a\cos(c+dx)+b\sin(c+dx))}$$

Mathematica [A] time = 0.03, size = 32, normalized size = 1.00

$$\frac{\sin(c+dx)}{ad(a\cos(c+dx)+b\sin(c+dx))}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Cos[c + d*x] + b*Sin[c + d*x])^(-2),x]
```

```
[Out] Sin[c + d*x]/(a*d*(a*Cos[c + d*x] + b*Sin[c + d*x]))
```

fricas [A] time = 0.55, size = 57, normalized size = 1.78

$$-\frac{b\cos(dx+c)-a\sin(dx+c)}{\left(a^3+ab^2\right)d\cos(dx+c)+\left(a^2b+b^3\right)d\sin(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cos(d*x+c)+b*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -(b*cos(d*x + c) - a*sin(d*x + c))/((a^3 + a*b^2)*d*cos(d*x + c) + (a^2*b +
 b^3)*d*sin(d*x + c))
```

giac [A] time = 1.61, size = 20, normalized size = 0.62

$$-\frac{1}{(b\tan(dx+c)+a)bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cos(d*x+c)+b*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/((b*tan(d*x + c) + a)*b*d)
```

maple [A] time = 0.20, size = 21, normalized size = 0.66

$$-\frac{1}{db\,(a+b\tan(dx+c))}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*cos(d*x+c)+b*sin(d*x+c))^2,x)
```

```
[Out] -1/d/b/(a+b*tan(d*x+c))
```

maxima [A] time = 0.34, size = 21, normalized size = 0.66

$$-\frac{1}{\left(b^2\tan(dx+c)+ab\right)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cos(d*x+c)+b*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -1/((b^2*tan(d*x + c) + a*b)*d)
```

mupad [B] time = 0.49, size = 47, normalized size = 1.47

$$\frac{2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{a\,d\,\left(-a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+2\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*cos(c + d*x) + b*sin(c + d*x))^2,x)
```

```
[Out] (2*tan(c/2 + (d*x)/2))/(a*d*(a + 2*b*tan(c/2 + (d*x)/2) - a*tan(c/2 + (d*x)
/2)^2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cos(d*x+c)+b*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

3.127 $\int \frac{\sec(c+dx)}{(a\cos(c+dx)+b\sin(c+dx))^2}\,dx$

Optimal. Leaf size=92

$$\frac{a\tanh^{-1}\left(\frac{b\cos(c+dx)-a\sin(c+dx)}{\sqrt{a^2+b^2}}\right)}{b^2d\sqrt{a^2+b^2}}-\frac{1}{bd(a\cos(c+dx)+b\sin(c+dx))}+\frac{\tanh^{-1}(\sin(c+dx))}{b^2d}$$

```
[Out] arctanh(sin(d*x+c))/b^2/d-1/b/d/(a*cos(d*x+c)+b*sin(d*x+c))+a*arctanh((b*co
s(d*x+c)-a*sin(d*x+c))/(a^2+b^2)^(1/2))/b^2/d/(a^2+b^2)^(1/2)
```

Rubi [A] time = 0.08, antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {3094, 3770, 3074, 206}

$$\frac{a\tanh^{-1}\left(\frac{b\cos(c+dx)-a\sin(c+dx)}{\sqrt{a^2+b^2}}\right)}{b^2d\sqrt{a^2+b^2}}-\frac{1}{bd(a\cos(c+dx)+b\sin(c+dx))}+\frac{\tanh^{-1}(\sin(c+dx))}{b^2d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]/(a*Cos[c + d*x] + b*Sin[c + d*x])^2,x]
```

```
[Out] ArcTanh[Sin[c + d*x]]/(b^2*d) + (a*ArcTanh[(b*Cos[c + d*x] - a*Sin[c + d*x]
)/Sqrt[a^2 + b^2]])/(b^2*Sqrt[a^2 + b^2]*d) - 1/(b*d*(a*Cos[c + d*x] + b*Si
n[c + d*x]))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3074

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x
_Symbol] :> -Dist[d^(-1), Subst[Int[1/(a^2 + b^2 - x^2), x], x, b*Cos[c + d
*x] - a*Sin[c + d*x]], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

Rule 3094

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_)/co
s[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[(a*Cos[c + d*x] + b*Sin[c + d*x])^
(n + 1)/(b*d*(n + 1)), x] + (Dist[1/b^2, Int[(a*Cos[c + d*x] + b*Sin[c + d*
x])^(n + 2)/Cos[c + d*x], x], x] - Dist[a/b^2, Int[(a*Cos[c + d*x] + b*Sin[
```

```
c + d*x])^(n + 1), x], x]) /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0] &
& LtQ[n, -1]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\sec(c+dx)}{(a\cos(c+dx)+b\sin(c+dx))^2}\,dx = -\frac{1}{bd(a\cos(c+dx)+b\sin(c+dx))} + \frac{\int \sec(c+dx)\,dx}{b^2} - \frac{a\int \frac{1}{a\cos(c+dx)+\cdots}}{b^2\cdots}$$

$$= \frac{\tanh^{-1}(\sin(c+dx))}{b^2d} - \frac{1}{bd(a\cos(c+dx)+b\sin(c+dx))} + \frac{a\,\mathrm{Subst}\left(\int \cdots\right.}{\cdots}$$

$$= \frac{\tanh^{-1}(\sin(c+dx))}{b^2d} + \frac{a\tanh^{-1}\left(\frac{b\cos(c+dx)-a\sin(c+dx)}{\sqrt{a^2+b^2}}\right)}{b^2\sqrt{a^2+b^2}\,d} - \frac{\cdots}{bd(a\cos(c+d\cdots}$$

Mathematica [A] time = 0.79, size = 120, normalized size = 1.30

$$-\frac{\frac{2a\tanh^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)-b}{\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}} + \frac{b\sec(c+dx)}{a+b\tan(c+dx)} + \log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right) - \log\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}\cdots\right.\right.}{b^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]/(a*Cos[c + d*x] + b*Sin[c + d*x])^2,x]
```

```
[Out] -((((2*a*ArcTanh[(-b + a*Tan[(c + d*x)/2])/Sqrt[a^2 + b^2]])/Sqrt[a^2 + b^2]
 + Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] - Log[Cos[(c + d*x)/2] + Sin[(c
 + d*x)/2]] + (b*Sec[c + d*x])/(a + b*Tan[c + d*x]))/(b^2*d))
```

fricas [B] time = 0.59, size = 293, normalized size = 3.18

$$-\frac{2a^2b+2b^3-\left(a^2\cos(dx+c)+ab\sin(dx+c)\right)\sqrt{a^2+b^2}\,\log\left(\frac{2ab\cos(dx+c)\sin(dx+c)+\left(a^2-b^2\right)\cos(dx+c)^2-2a^2-b^2-2\sqrt{a^2\cdots}}{2ab\cos(dx+c)\sin(dx+c)+\left(a^2-b^2\right)\cos(d\cdots}\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a*cos(d*x+c)+b*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -1/2*(2*a^2*b + 2*b^3 - (a^2*cos(d*x + c) + a*b*sin(d*x + c))*sqrt(a^2 + b^
2)*log((2*a*b*cos(d*x + c)*sin(d*x + c) + (a^2 - b^2)*cos(d*x + c)^2 - 2*a^
2 - b^2 - 2*sqrt(a^2 + b^2)*(b*cos(d*x + c) - a*sin(d*x + c)))/(2*a*b*cos(d
*x + c)*sin(d*x + c) + (a^2 - b^2)*cos(d*x + c)^2 + b^2)) - ((a^3 + a*b^2)*
cos(d*x + c) + (a^2*b + b^3)*sin(d*x + c))*log(sin(d*x + c) + 1) + ((a^3 +
a*b^2)*cos(d*x + c) + (a^2*b + b^3)*sin(d*x + c))*log(-sin(d*x + c) + 1))/(
(a^3*b^2 + a*b^4)*d*cos(d*x + c) + (a^2*b^3 + b^5)*d*sin(d*x + c))
```

giac [A] time = 0.35, size = 166, normalized size = 1.80

$$\frac{\frac{a\log\left(\frac{\left|2\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-2\,b-2\,\sqrt{a^2+b^2}\right|}{\left|2\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-2\,b+2\,\sqrt{a^2+b^2}\right|}\right)}{\sqrt{a^2+b^2}\,b^2}+\frac{\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)}{b^2}-\frac{\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)}{b^2}+\frac{2\left(b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+a\right)}{\left(a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-2\,b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-a\right)ab}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a*cos(d*x+c)+b*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] (a*log(abs(2*a*tan(1/2*d*x + 1/2*c) - 2*b - 2*sqrt(a^2 + b^2))/abs(2*a*tan(
1/2*d*x + 1/2*c) - 2*b + 2*sqrt(a^2 + b^2)))/(sqrt(a^2 + b^2)*b^2) + log(ab
s(tan(1/2*d*x + 1/2*c) + 1))/b^2 - log(abs(tan(1/2*d*x + 1/2*c) - 1))/b^2 +
 2*(b*tan(1/2*d*x + 1/2*c) + a)/((a*tan(1/2*d*x + 1/2*c)^2 - 2*b*tan(1/2*d*
x + 1/2*c) - a)*a*b))/d
```

maple [A] time = 0.28, size = 174, normalized size = 1.89

$$-\frac{\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{d\,b^2}+\frac{\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{d\,b^2}+\frac{2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d\left(\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a-2b\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-a\right)a}+\frac{}{db\left(\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right.\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)/(a*cos(d*x+c)+b*sin(d*x+c))^2,x)
```

```
[Out] -1/d/b^2*ln(tan(1/2*d*x+1/2*c)-1)+1/d/b^2*ln(tan(1/2*d*x+1/2*c)+1)+2/d/(tan
(1/2*d*x+1/2*c)^2*a-2*b*tan(1/2*d*x+1/2*c)-a)/a*tan(1/2*d*x+1/2*c)+2/d/b/(t
an(1/2*d*x+1/2*c)^2*a-2*b*tan(1/2*d*x+1/2*c)-a)-2/d/b^2*a/(a^2+b^2)^(1/2)*a
rctanh(1/2*(2*a*tan(1/2*d*x+1/2*c)-2*b)/(a^2+b^2)^(1/2))
```

maxima [B] time = 0.43, size = 212, normalized size = 2.30

$$-\frac{\frac{2\left(a+\frac{b\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^2b+\frac{2ab^2\sin(dx+c)}{\cos(dx+c)+1}-\frac{a^2b\sin(dx+c)^2}{(\cos(dx+c)+1)^2}} - \frac{a\log\left(\frac{b-\frac{a\sin(dx+c)}{\cos(dx+c)+1}+\sqrt{a^2+b^2}}{b-\frac{a\sin(dx+c)}{\cos(dx+c)+1}-\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}\,b^2} - \frac{\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)}{b^2}+\frac{\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}-1\right)}{b^2}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a*cos(d*x+c)+b*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -(2*(a + b*sin(d*x + c)/(cos(d*x + c) + 1))/(a^2*b + 2*a*b^2*sin(d*x + c)/(
cos(d*x + c) + 1) - a^2*b*sin(d*x + c)^2/(cos(d*x + c) + 1)^2) - a*log((b -
 a*sin(d*x + c)/(cos(d*x + c) + 1) + sqrt(a^2 + b^2))/(b - a*sin(d*x + c)/(
cos(d*x + c) + 1) - sqrt(a^2 + b^2)))/(sqrt(a^2 + b^2)*b^2) - log(sin(d*x +
 c)/(cos(d*x + c) + 1) + 1)/b^2 + log(sin(d*x + c)/(cos(d*x + c) + 1) - 1)/
b^2)/d
```

mupad [B] time = 1.17, size = 383, normalized size = 4.16

$$-\frac{b^2\sin(c+d\,x) - \frac{2\left(a^2\cos(c+d\,x)\,\mathrm{atanh}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)\sqrt{a^2+b^2}+a^3\,\mathrm{atan}\left(\frac{1\mathrm{i}\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)a^2+1\mathrm{i}\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)a\,b+2\mathrm{i}\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)b^2}{a\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)\sqrt{a^2+b^2}+2\,b\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)\sqrt{a^2+b^2}}\right)\cos(c+d\,x)\,1\mathrm{i}\right)}{\sqrt{a^2+b^2}}+}{a\,b^2\,d\,(a\cos(c+}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)*(a*cos(c + d*x) + b*sin(c + d*x))^2),x)
```

```
[Out] -(b^2*sin(c + d*x) - (2*(a^3*atan((a^2*sin(c/2 + (d*x)/2)*1i + b^2*sin(c/2
+ (d*x)/2)*2i + a*b*cos(c/2 + (d*x)/2)*1i)/(a*cos(c/2 + (d*x)/2)*(a^2 + b^2
)^(1/2) + 2*b*sin(c/2 + (d*x)/2)*(a^2 + b^2)^(1/2)))*cos(c + d*x)*1i + a^2*
cos(c + d*x)*atanh(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2))*(a^2 + b^2)^(1/2)
))/(a^2 + b^2)^(1/2) + (2*b*((a*(a^2 + b^2)^(1/2))/2 + (a*cos(c + d*x)*(a^2
 + b^2)^(1/2))/2 - a^2*atan((a^2*sin(c/2 + (d*x)/2)*1i + b^2*sin(c/2 + (d*x
)/2)*2i + a*b*cos(c/2 + (d*x)/2)*1i)/(a*cos(c/2 + (d*x)/2)*(a^2 + b^2)^(1/2
) + 2*b*sin(c/2 + (d*x)/2)*(a^2 + b^2)^(1/2)))*sin(c + d*x)*1i - a*sin(c +
d*x)*atanh(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2))*(a^2 + b^2)^(1/2)))/(a^2
+ b^2)^(1/2))/(a*b^2*d*(a*cos(c + d*x) + b*sin(c + d*x)))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec(c+dx)}{(a\cos(c+dx)+b\sin(c+dx))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a*cos(d*x+c)+b*sin(d*x+c))**2,x)
```

```
[Out] Integral(sec(c + d*x)/(a*cos(c + d*x) + b*sin(c + d*x))**2, x)
```

3.128 $\int \frac{\sec^2(c+dx)}{(a\cos(c+dx)+b\sin(c+dx))^2}\,dx$

Optimal. Leaf size=75

$$-\frac{2a\log(\tan(c+dx))}{b^3d}-\frac{2a\log(a\cot(c+dx)+b)}{b^3d}+\frac{\frac{a}{b^2}+\frac{1}{a}}{d(a\cot(c+dx)+b)}+\frac{\tan(c+dx)}{b^2d}$$

```
[Out] (1/a+a/b^2)/d/(b+a*cot(d*x+c))-2*a*ln(b+a*cot(d*x+c))/b^3/d-2*a*ln(tan(d*x+
c))/b^3/d+tan(d*x+c)/b^2/d
```

Rubi [A] time = 0.10, antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {3088, 894}

$$\frac{\frac{a}{b^2}+\frac{1}{a}}{d(a\cot(c+dx)+b)}-\frac{2a\log(\tan(c+dx))}{b^3d}-\frac{2a\log(a\cot(c+dx)+b)}{b^3d}+\frac{\tan(c+dx)}{b^2d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^2/(a*Cos[c + d*x] + b*Sin[c + d*x])^2,x]
```

```
[Out] (a^(-1) + a/b^2)/(d*(b + a*Cot[c + d*x])) - (2*a*Log[b + a*Cot[c + d*x]])/(
b^3*d) - (2*a*Log[Tan[c + d*x]])/(b^3*d) + Tan[c + d*x]/(b^2*d)
```

Rule 894

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ
[m, 0] && ILtQ[n, 0]))
```

Rule 3088

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[(x^m*(b +
 a*x)^n)/(1 + x^2)^((m + n + 2)/2), x], x, Cot[c + d*x]], x] /; FreeQ[{a, b
, c, d}, x] && IntegerQ[n] && IntegerQ[(m + n)/2] && NeQ[n, -1] && !(GtQ[n
, 0] && GtQ[m, 1])
```

Rubi steps

$$\int \frac{\sec^2(c+dx)}{(a\cos(c+dx)+b\sin(c+dx))^2}\,dx = -\frac{\text{Subst}\left(\int \frac{1+x^2}{x^2(b+ax)^2}\,dx, x, \cot(c+dx)\right)}{d}$$
$$= -\frac{\text{Subst}\left(\int \left(\frac{1}{b^2x^2}-\frac{2a}{b^3x}+\frac{a^2+b^2}{b^2(b+ax)^2}+\frac{2a^2}{b^3(b+ax)}\right)dx, x, \cot(c+dx)\right)}{d}$$
$$= \frac{\frac{1}{a}+\frac{a}{b^2}}{d(b+a\cot(c+dx))} - \frac{2a\log(b+a\cot(c+dx))}{b^3d} - \frac{2a\log(\tan(c+dx))}{b^3d} +$$

Mathematica [A] time = 0.27, size = 51, normalized size = 0.68

$$-\frac{\frac{a^2+b^2}{a+b\tan(c+dx)} - 2a\log(a+b\tan(c+dx)) + b\tan(c+dx)}{b^3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^2/(a*Cos[c + d*x] + b*Sin[c + d*x])^2,x]
```

```
[Out] (-2*a*Log[a + b*Tan[c + d*x]] + b*Tan[c + d*x] - (a^2 + b^2)/(a + b*Tan[c +
 d*x]))/(b^3*d)
```

fricas [B] time = 0.48, size = 178, normalized size = 2.37

$$-\frac{2\,b^2\cos(dx+c)^2 - 2\,ab\cos(dx+c)\sin(dx+c) - b^2 + \left(a^2\cos(dx+c)^2 + ab\cos(dx+c)\sin(dx+c)\right)\log\left(2\,a\right.}{ab^3d\cos(dx+c)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a*cos(d*x+c)+b*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -(2*b^2*cos(d*x + c)^2 - 2*a*b*cos(d*x + c)*sin(d*x + c) - b^2 + (a^2*cos(d
*x + c)^2 + a*b*cos(d*x + c)*sin(d*x + c))*log(2*a*b*cos(d*x + c)*sin(d*x +
 c) + (a^2 - b^2)*cos(d*x + c)^2 + b^2) - (a^2*cos(d*x + c)^2 + a*b*cos(d*x
 + c)*sin(d*x + c))*log(cos(d*x + c)^2))/(a*b^3*d*cos(d*x + c)^2 + b^4*d*co
s(d*x + c)*sin(d*x + c))
```

giac [A] time = 0.25, size = 71, normalized size = 0.95

$$-\frac{\frac{2\,a\log(|b\tan(dx+c)+a|)}{b^3} - \frac{\tan(dx+c)}{b^2} - \frac{2\,ab\tan(dx+c)+a^2-b^2}{(b\tan(dx+c)+a)b^3}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a*cos(d*x+c)+b*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -(2*a*log(abs(b*tan(d*x + c) + a))/b^3 - tan(d*x + c)/b^2 - (2*a*b*tan(d*x
+ c) + a^2 - b^2)/((b*tan(d*x + c) + a)*b^3))/d
```

maple [A] time = 0.31, size = 78, normalized size = 1.04

$$\frac{\tan\left(dx+c\right)}{b^{2}d}-\frac{2a\ln\left(a+b\tan\left(dx+c\right)\right)}{d\,b^{3}}-\frac{a^{2}}{d\,b^{3}\left(a+b\tan\left(dx+c\right)\right)}-\frac{1}{db\left(a+b\tan\left(dx+c\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2/(a*cos(d*x+c)+b*sin(d*x+c))^2,x)
```

```
[Out] tan(d*x+c)/b^2/d-2/d*a/b^3*ln(a+b*tan(d*x+c))-1/d/b^3/(a+b*tan(d*x+c))*a^2-
1/d/b/(a+b*tan(d*x+c))
```

maxima [A] time = 0.33, size = 60, normalized size = 0.80

$$-\frac{\frac{a^{2}+b^{2}}{b^{4}\tan\left(dx+c\right)+ab^{3}}+\frac{2\,a\log\left(b\tan\left(dx+c\right)+a\right)}{b^{3}}-\frac{\tan\left(dx+c\right)}{b^{2}}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a*cos(d*x+c)+b*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -((a^2 + b^2)/(b^4*tan(d*x + c) + a*b^3) + 2*a*log(b*tan(d*x + c) + a)/b^3
- tan(d*x + c)/b^2)/d
```

mupad [B] time = 2.36, size = 382, normalized size = 5.09

$$\frac{\frac{4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{2}}{b}-\frac{2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{3}\left(2\,a^{2}+b^{2}\right)}{a\,b^{2}}+\frac{2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\left(2\,a^{2}+b^{2}\right)}{a\,b^{2}}}{d\,\left(a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{4}-2\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{3}-2\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{2}+2\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+a\right)}-\frac{4\,a\,\mathrm{atanh}\left(\frac{\quad}{64\,a^{3}-64\,a^{3}\tan\left(\frac{c}{2}+\right.}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^2*(a*cos(c + d*x) + b*sin(c + d*x))^2),x)
```

```
[Out] ((4*tan(c/2 + (d*x)/2)^2)/b - (2*tan(c/2 + (d*x)/2)^3*(2*a^2 + b^2))/(a*b^2
) + (2*tan(c/2 + (d*x)/2)*(2*a^2 + b^2))/(a*b^2))/(d*(a + 2*b*tan(c/2 + (d*
x)/2) - 2*a*tan(c/2 + (d*x)/2)^2 + a*tan(c/2 + (d*x)/2)^4 - 2*b*tan(c/2 + (
d*x)/2)^3)) - (4*a*atanh((64*a^3*tan(c/2 + (d*x)/2)^2)/(64*a^3 - 64*a^3*tan
(c/2 + (d*x)/2)^2 + (128*a^5)/b^2 - (128*a^5*tan(c/2 + (d*x)/2)^2)/b^2 + (1
```

```
28*a^4*tan(c/2 + (d*x)/2))/b) - (64*a^3)/(64*a^3 - 64*a^3*tan(c/2 + (d*x)/2
)^2 + (128*a^5)/b^2 - (128*a^5*tan(c/2 + (d*x)/2)^2)/b^2 + (128*a^4*tan(c/2
 + (d*x)/2))/b) + (128*a^4*tan(c/2 + (d*x)/2))/(64*a^3*b + (128*a^5)/b + 12
8*a^4*tan(c/2 + (d*x)/2) - (128*a^5*tan(c/2 + (d*x)/2)^2)/b - 64*a^3*b*tan(
c/2 + (d*x)/2)^2)))/(b^3*d)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec^2(c+dx)}{(a\cos(c+dx)+b\sin(c+dx))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2/(a*cos(d*x+c)+b*sin(d*x+c))**2,x)
```

```
[Out] Integral(sec(c + d*x)**2/(a*cos(c + d*x) + b*sin(c + d*x))**2, x)
```

3.129 $\int \frac{\sec^3(c+dx)}{(a\cos(c+dx)+b\sin(c+dx))^2}\,dx$

Optimal. Leaf size=179

$$\frac{2a^2\tanh^{-1}(\sin(c+dx))}{b^4d}+\frac{(a^2+b^2)\tanh^{-1}(\sin(c+dx))}{b^4d}+\frac{3a\sqrt{a^2+b^2}\tanh^{-1}\left(\frac{b\cos(c+dx)-a\sin(c+dx)}{\sqrt{a^2+b^2}}\right)}{b^4d}-\frac{}{b^3d(a\cos(c}$$

```
[Out] 2*a^2*arctanh(sin(d*x+c))/b^4/d+1/2*arctanh(sin(d*x+c))/b^2/d+(a^2+b^2)*arc
tanh(sin(d*x+c))/b^4/d-2*a*sec(d*x+c)/b^3/d+(-a^2-b^2)/b^3/d/(a*cos(d*x+c)+
b*sin(d*x+c))+3*a*arctanh((b*cos(d*x+c)-a*sin(d*x+c))/(a^2+b^2)^(1/2))*(a^2
+b^2)^(1/2)/b^4/d+1/2*sec(d*x+c)*tan(d*x+c)/b^2/d
```

Rubi [A] time = 0.24, antiderivative size = 179, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 7, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {3106, 3094, 3770, 3074, 206, 3768, 3104}

$$\frac{2a^2\tanh^{-1}(\sin(c+dx))}{b^4d}+\frac{(a^2+b^2)\tanh^{-1}(\sin(c+dx))}{b^4d}-\frac{a^2+b^2}{b^3d(a\cos(c+dx)+b\sin(c+dx))}+\frac{3a\sqrt{a^2+b^2}\tanh}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^3/(a*Cos[c + d*x] + b*Sin[c + d*x])^2,x]
```

```
[Out] (2*a^2*ArcTanh[Sin[c + d*x]])/(b^4*d) + ArcTanh[Sin[c + d*x]]/(2*b^2*d) + (
(a^2 + b^2)*ArcTanh[Sin[c + d*x]])/(b^4*d) + (3*a*Sqrt[a^2 + b^2]*ArcTanh[(
b*Cos[c + d*x] - a*Sin[c + d*x])/Sqrt[a^2 + b^2]])/(b^4*d) - (2*a*Sec[c + d
*x])/(b^3*d) - (a^2 + b^2)/(b^3*d*(a*Cos[c + d*x] + b*Sin[c + d*x])) + (Sec
[c + d*x]*Tan[c + d*x])/(2*b^2*d)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3074

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x
_Symbol] :> -Dist[d^(-1), Subst[Int[1/(a^2 + b^2 - x^2), x], x, b*Cos[c + d
*x] - a*Sin[c + d*x]], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

Rule 3094

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_)/co
s[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[(a*Cos[c + d*x] + b*Sin[c + d*x])^
(n + 1)/(b*d*(n + 1)), x] + (Dist[1/b^2, Int[(a*Cos[c + d*x] + b*Sin[c + d*
x])^(n + 2)/Cos[c + d*x], x], x] - Dist[a/b^2, Int[(a*Cos[c + d*x] + b*Sin[
c + d*x])^(n + 1), x], x]) /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0] &
& LtQ[n, -1]
```

Rule 3104

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_)/(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin
[(c_.) + (d_.)*(x_)]), x_Symbol] :> -Simp[Cos[c + d*x]^(m + 1)/(b*d*(m + 1)
), x] + (-Dist[a/b^2, Int[Cos[c + d*x]^(m + 1), x], x] + Dist[(a^2 + b^2)/b
^2, Int[Cos[c + d*x]^(m + 2)/(a*Cos[c + d*x] + b*Sin[c + d*x]), x], x]) /;
FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0] && LtQ[m, -1]
```

Rule 3106

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin
[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(a^2 + b^2)/b^2, Int[Cos[c +
d*x]^(m + 2)*(a*Cos[c + d*x] + b*Sin[c + d*x])^n, x], x] + (Dist[1/b^2, Int
[Cos[c + d*x]^m*(a*Cos[c + d*x] + b*Sin[c + d*x])^(n + 2), x], x] - Dist[(2
*a)/b^2, Int[Cos[c + d*x]^(m + 1)*(a*Cos[c + d*x] + b*Sin[c + d*x])^(n + 1)
, x], x]) /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0] && LtQ[n, -1] && L
tQ[m, -1]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\sec^3(c+dx)}{(a\cos(c+dx)+b\sin(c+dx))^2}\,dx = \frac{\int \sec^3(c+dx)\,dx}{b^2} - \frac{(2a)\int \frac{\sec^2(c+dx)}{a\cos(c+dx)+b\sin(c+dx)}\,dx}{b^2} + \frac{\left(a^2+b^2\right)\int \frac{\cdot}{(a\cos(c\cdot}}{b^{\cdot}}$$

$$= -\frac{2a\sec(c+dx)}{b^3d} - \frac{a^2+b^2}{b^3d(a\cos(c+dx)+b\sin(c+dx))} + \frac{\sec(c+dx)\tan\cdots}{2b^2d}$$

$$= \frac{2a^2\tanh^{-1}(\sin(c+dx))}{b^4d} + \frac{\tanh^{-1}(\sin(c+dx))}{2b^2d} + \frac{\left(a^2+b^2\right)\tanh^{-1}(\sin\cdots}{b^4d}$$

$$= \frac{2a^2\tanh^{-1}(\sin(c+dx))}{b^4d} + \frac{\tanh^{-1}(\sin(c+dx))}{2b^2d} + \frac{\left(a^2+b^2\right)\tanh^{-1}(\sin\cdots}{b^4d}$$

Mathematica [C] time = 6.12, size = 709, normalized size = 3.96

$$-\frac{3\left(2a^2+b^2\right)\sec^2(c+dx)\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)(a\cos(c+dx)+b\sin(c+dx))^2}{2b^4d(a+b\tan(c+dx))^2} + \frac{3\left(2a^2+b^2\right)\cdots}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^3/(a*Cos[c + d*x] + b*Sin[c + d*x])^2,x]
```

```
[Out] -(((a - I*b)*(a + I*b)*Sec[c + d*x]^2*(a*Cos[c + d*x] + b*Sin[c + d*x]))/(b
^3*d*(a + b*Tan[c + d*x])^2)) - (2*a*Sec[c + d*x]^2*(a*Cos[c + d*x] + b*Sin
[c + d*x])^2)/(b^3*d*(a + b*Tan[c + d*x])^2) - (6*a*Sqrt[a^2 + b^2]*ArcTanh
[(Sqrt[a^2 + b^2]*(-(b*Cos[(c + d*x)/2]) + a*Sin[(c + d*x)/2]))/(a^2*Cos[(c
 + d*x)/2] + b^2*Cos[(c + d*x)/2])]*Sec[c + d*x]^2*(a*Cos[c + d*x] + b*Sin[
c + d*x])^2)/(b^4*d*(a + b*Tan[c + d*x])^2) - (3*(2*a^2 + b^2)*Log[Cos[(c +
 d*x)/2] - Sin[(c + d*x)/2]]*Sec[c + d*x]^2*(a*Cos[c + d*x] + b*Sin[c + d*x
])^2)/(2*b^4*d*(a + b*Tan[c + d*x])^2) + (3*(2*a^2 + b^2)*Log[Cos[(c + d*x)
/2] + Sin[(c + d*x)/2]]*Sec[c + d*x]^2*(a*Cos[c + d*x] + b*Sin[c + d*x])^2)
/(2*b^4*d*(a + b*Tan[c + d*x])^2) + (Sec[c + d*x]^2*(a*Cos[c + d*x] + b*Sin
[c + d*x])^2)/(4*b^2*d*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^2*(a + b*Tan[c
 + d*x])^2) - (2*a*Sec[c + d*x]^2*Sin[(c + d*x)/2]*(a*Cos[c + d*x] + b*Sin[
c + d*x])^2)/(b^3*d*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])*(a + b*Tan[c + d*
x])^2) - (Sec[c + d*x]^2*(a*Cos[c + d*x] + b*Sin[c + d*x])^2)/(4*b^2*d*(Cos
[(c + d*x)/2] + Sin[(c + d*x)/2])^2*(a + b*Tan[c + d*x])^2) + (2*a*Sec[c +
d*x]^2*Sin[(c + d*x)/2]*(a*Cos[c + d*x] + b*Sin[c + d*x])^2)/(b^3*d*(Cos[(c
 + d*x)/2] + Sin[(c + d*x)/2])*(a + b*Tan[c + d*x])^2)
```

fricas [B] time = 0.61, size = 355, normalized size = 1.98

$$-\frac{6\,ab^2\cos(dx+c)\sin(dx+c) - 2\,b^3 + 6\left(2\,a^2b + b^3\right)\cos(dx+c)^2 - 6\left(a^2\cos(dx+c)^3 + ab\cos(dx+c)^2\sin(d\ldots\right.}{\ldots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a*cos(d*x+c)+b*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -1/4*(6*a*b^2*cos(d*x + c)*sin(d*x + c) - 2*b^3 + 6*(2*a^2*b + b^3)*cos(d*x
 + c)^2 - 6*(a^2*cos(d*x + c)^3 + a*b*cos(d*x + c)^2*sin(d*x + c))*sqrt(a^2
 + b^2)*log((2*a*b*cos(d*x + c)*sin(d*x + c) + (a^2 - b^2)*cos(d*x + c)^2 -
 2*a^2 - b^2 - 2*sqrt(a^2 + b^2)*(b*cos(d*x + c) - a*sin(d*x + c)))/(2*a*b*
cos(d*x + c)*sin(d*x + c) + (a^2 - b^2)*cos(d*x + c)^2 + b^2)) - 3*((2*a^3
+ a*b^2)*cos(d*x + c)^3 + (2*a^2*b + b^3)*cos(d*x + c)^2*sin(d*x + c))*log(
sin(d*x + c) + 1) + 3*((2*a^3 + a*b^2)*cos(d*x + c)^3 + (2*a^2*b + b^3)*cos
(d*x + c)^2*sin(d*x + c))*log(-sin(d*x + c) + 1))/(a*b^4*d*cos(d*x + c)^3 +
 b^5*d*cos(d*x + c)^2*sin(d*x + c))
```

giac [A] time = 0.34, size = 280, normalized size = 1.56

$$\frac{\frac{3\left(2\,a^2+b^2\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)}{b^4} - \frac{3\left(2\,a^2+b^2\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)}{b^4} + \frac{6\left(a^3+ab^2\right)\log\left(\frac{\left|2\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-2\,b-2\,\sqrt{a^2+b^2}\right|}{\left|2\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-2\,b+2\,\sqrt{a^2+b^2}\right|}\right)}{\sqrt{a^2+b^2}\,b^4} + \frac{2\left(b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\ldots\right.\right.}{\ldots}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a*cos(d*x+c)+b*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/2*(3*(2*a^2 + b^2)*log(abs(tan(1/2*d*x + 1/2*c) + 1))/b^4 - 3*(2*a^2 + b^
2)*log(abs(tan(1/2*d*x + 1/2*c) - 1))/b^4 + 6*(a^3 + a*b^2)*log(abs(2*a*tan
(1/2*d*x + 1/2*c) - 2*b - 2*sqrt(a^2 + b^2))/abs(2*a*tan(1/2*d*x + 1/2*c) -
 2*b + 2*sqrt(a^2 + b^2)))/(sqrt(a^2 + b^2)*b^4) + 2*(b*tan(1/2*d*x + 1/2*c
)^3 + 4*a*tan(1/2*d*x + 1/2*c)^2 + b*tan(1/2*d*x + 1/2*c) - 4*a)/((tan(1/2*
d*x + 1/2*c)^2 - 1)^2*b^3) + 4*(a^2*b*tan(1/2*d*x + 1/2*c) + b^3*tan(1/2*d*
x + 1/2*c) + a^3 + a*b^2)/((a*tan(1/2*d*x + 1/2*c)^2 - 2*b*tan(1/2*d*x + 1/
2*c) - a)*a*b^3))/d
```

maple [B] time = 0.32, size = 440, normalized size = 2.46

$$\frac{1}{2d\,b^2\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^2} + \frac{2a}{d\,b^3\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)} + \frac{1}{2d\,b^2\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)} - \frac{3\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)a^2}{d\,b^4} - \frac{3\ln\left(\mathrm{ta}\ldots\right.}{\ldots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^3/(a*cos(d*x+c)+b*sin(d*x+c))^2,x)
```

```
[Out] 1/2/d/b^2/(tan(1/2*d*x+1/2*c)-1)^2+2/d/b^3/(tan(1/2*d*x+1/2*c)-1)*a+1/2/d/b
^2/(tan(1/2*d*x+1/2*c)-1)-3/d/b^4*ln(tan(1/2*d*x+1/2*c)-1)*a^2-3/2/d/b^2*ln
(tan(1/2*d*x+1/2*c)-1)-1/2/d/b^2/(tan(1/2*d*x+1/2*c)+1)^2-2/d/b^3/(tan(1/2*
d*x+1/2*c)+1)*a+1/2/d/b^2/(tan(1/2*d*x+1/2*c)+1)+3/d/b^4*ln(tan(1/2*d*x+1/2
*c)+1)*a^2+3/2/d/b^2*ln(tan(1/2*d*x+1/2*c)+1)+2/d/b^2/(tan(1/2*d*x+1/2*c)^2
*a-2*b*tan(1/2*d*x+1/2*c)-a)*a*tan(1/2*d*x+1/2*c)+2/d/(tan(1/2*d*x+1/2*c)^2
*a-2*b*tan(1/2*d*x+1/2*c)-a)/a*tan(1/2*d*x+1/2*c)+2/d/b^3/(tan(1/2*d*x+1/2*
c)^2*a-2*b*tan(1/2*d*x+1/2*c)-a)*a^2+2/d/b/(tan(1/2*d*x+1/2*c)^2*a-2*b*tan(
1/2*d*x+1/2*c)-a)-6/d/b^4*a*(a^2+b^2)^(1/2)*arctanh(1/2*(2*a*tan(1/2*d*x+1/
2*c)-2*b)/(a^2+b^2)^(1/2))
```

maxima [B] time = 0.44, size = 471, normalized size = 2.63

$$-\frac{\frac{2\left(6a^3+2ab^2+\frac{6a^3\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{(9a^2b+2b^3)\sin(dx+c)}{\cos(dx+c)+1}-\frac{6(2a^3+ab^2)\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-\frac{4(3a^2b+b^3)\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{(3a^2b+2b^3)\sin(dx+c)^5}{(\cos(dx+c)+1)^5}\right)}{a^2b^3+\frac{2ab^4\sin(dx+c)}{\cos(dx+c)+1}-\frac{3a^2b^3\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-\frac{4ab^4\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{3a^2b^3\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{2ab^4\sin(dx+c)^5}{(\cos(dx+c)+1)^5}-\frac{a^2b^3\sin(dx+c)^6}{(\cos(dx+c)+1)^6}}-\frac{6\sqrt{a^2+b^2}\,a\log\left(\frac{b-\frac{a\sin(\cdots}{\cos(dx\cdots}}{b-\frac{a\sin(\cdots}{\cos(dx\cdots}}\right.}{b^4}\cdots}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a*cos(d*x+c)+b*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -1/2*(2*(6*a^3 + 2*a*b^2 + 6*a^3*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + (9*a
^2*b + 2*b^3)*sin(d*x + c)/(cos(d*x + c) + 1) - 6*(2*a^3 + a*b^2)*sin(d*x +
 c)^2/(cos(d*x + c) + 1)^2 - 4*(3*a^2*b + b^3)*sin(d*x + c)^3/(cos(d*x + c)
 + 1)^3 + (3*a^2*b + 2*b^3)*sin(d*x + c)^5/(cos(d*x + c) + 1)^5)/(a^2*b^3 +
 2*a*b^4*sin(d*x + c)/(cos(d*x + c) + 1) - 3*a^2*b^3*sin(d*x + c)^2/(cos(d*
x + c) + 1)^2 - 4*a*b^4*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 3*a^2*b^3*sin
(d*x + c)^4/(cos(d*x + c) + 1)^4 + 2*a*b^4*sin(d*x + c)^5/(cos(d*x + c) + 1
)^5 - a^2*b^3*sin(d*x + c)^6/(cos(d*x + c) + 1)^6) - 6*sqrt(a^2 + b^2)*a*lo
g((b - a*sin(d*x + c)/(cos(d*x + c) + 1) + sqrt(a^2 + b^2))/(b - a*sin(d*x
+ c)/(cos(d*x + c) + 1) - sqrt(a^2 + b^2)))/b^4 - 3*(2*a^2 + b^2)*log(sin(d
*x + c)/(cos(d*x + c) + 1) + 1)/b^4 + 3*(2*a^2 + b^2)*log(sin(d*x + c)/(cos
(d*x + c) + 1) - 1)/b^4)/d
```

mupad [B] time = 1.90, size = 585, normalized size = 3.27

$$\frac{\operatorname{atanh}\left(\frac{648a^3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{216ab^2+648a^3+\frac{432a^5}{b^2}}+\frac{432a^5\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{432a^5+648a^3b^2+216ab^4}+\frac{216a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{216a+\frac{648a^3}{b^2}+\frac{432a^5}{b^4}}\right)\left(6a^2+3b^2\right)}{b^4d}-\frac{\frac{2(3a^2+b^2)}{b^3}+\frac{6a^2\tan\left(\frac{c}{2}+\cdots\right.}{b^3}\cdots}{d\left(-a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^6+2b\,\mathrm{ta}\cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^3*(a*cos(c + d*x) + b*sin(c + d*x))^2),x)
```

```
[Out] (atanh((648*a^3*tan(c/2 + (d*x)/2))/(216*a*b^2 + 648*a^3 + (432*a^5)/b^2) +
 (432*a^5*tan(c/2 + (d*x)/2))/(216*a*b^4 + 432*a^5 + 648*a^3*b^2) + (216*a*
tan(c/2 + (d*x)/2))/(216*a + (648*a^3)/b^2 + (432*a^5)/b^4))*(6*a^2 + 3*b^2
))/(b^4*d) - ((2*(3*a^2 + b^2))/b^3 + (6*a^2*tan(c/2 + (d*x)/2)^4)/b^3 - (6
*tan(c/2 + (d*x)/2)^2*(2*a^2 + b^2))/b^3 + (tan(c/2 + (d*x)/2)*(9*a^2 + 2*b
^2))/(a*b^2) - (4*tan(c/2 + (d*x)/2)^3*(3*a^2 + b^2))/(a*b^2) + (tan(c/2 +
(d*x)/2)^5*(3*a^2 + 2*b^2))/(a*b^2))/(d*(a + 2*b*tan(c/2 + (d*x)/2) - 3*a*t
an(c/2 + (d*x)/2)^2 + 3*a*tan(c/2 + (d*x)/2)^4 - a*tan(c/2 + (d*x)/2)^6 - 4
*b*tan(c/2 + (d*x)/2)^3 + 2*b*tan(c/2 + (d*x)/2)^5)) - (6*a*atanh((432*a^3*
(a^2 + b^2)^(1/2))/(432*a^3*b + (432*a^5)/b + 864*a^4*tan(c/2 + (d*x)/2) +
864*a^2*b^2*tan(c/2 + (d*x)/2)) + (864*a^2*tan(c/2 + (d*x)/2)*(a^2 + b^2)^(
1/2))/(432*a^3 + (432*a^5)/b^2 + 864*a^2*b*tan(c/2 + (d*x)/2) + (864*a^4*ta
n(c/2 + (d*x)/2))/b) + (432*a^4*tan(c/2 + (d*x)/2)*(a^2 + b^2)^(1/2))/(432*
a^5 + 432*a^3*b^2 + 864*a^4*b*tan(c/2 + (d*x)/2) + 864*a^2*b^3*tan(c/2 + (d
*x)/2)))*(a^2 + b^2)^(1/2))/(b^4*d)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec^3(c+dx)}{(a\cos(c+dx)+b\sin(c+dx))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**3/(a*cos(d*x+c)+b*sin(d*x+c))**2,x)
```

```
[Out] Integral(sec(c + d*x)**3/(a*cos(c + d*x) + b*sin(c + d*x))**2, x)
```

3.130 $\int \frac{\sec^4(c+dx)}{(a\cos(c+dx)+b\sin(c+dx))^2}\,dx$

Optimal. Leaf size=141

$$-\frac{4a\left(a^2+b^2\right)\log(\tan(c+dx))}{b^5 d}-\frac{4a\left(a^2+b^2\right)\log(a\cot(c+dx)+b)}{b^5 d}+\frac{\left(3a^2+2b^2\right)\tan(c+dx)}{b^4 d}+\frac{\left(a^2+b^2\right)^2}{ab^4 d(a\cot(c+d$$

```
[Out] (a^2+b^2)^2/a/b^4/d/(b+a*cot(d*x+c))-4*a*(a^2+b^2)*ln(b+a*cot(d*x+c))/b^5/d
-4*a*(a^2+b^2)*ln(tan(d*x+c))/b^5/d+(3*a^2+2*b^2)*tan(d*x+c)/b^4/d-a*tan(d*
x+c)^2/b^3/d+1/3*tan(d*x+c)^3/b^2/d
```

Rubi [A] time = 0.15, antiderivative size = 141, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {3088, 894}

$$\frac{\left(3a^2+2b^2\right)\tan(c+dx)}{b^4 d}+\frac{\left(a^2+b^2\right)^2}{ab^4 d(a\cot(c+dx)+b)}-\frac{4a\left(a^2+b^2\right)\log(\tan(c+dx))}{b^5 d}-\frac{4a\left(a^2+b^2\right)\log(a\cot(c+dx)}{b^5 d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^4/(a*Cos[c + d*x] + b*Sin[c + d*x])^2,x]
```

```
[Out] (a^2 + b^2)^2/(a*b^4*d*(b + a*Cot[c + d*x])) - (4*a*(a^2 + b^2)*Log[b + a*C
ot[c + d*x]])/(b^5*d) - (4*a*(a^2 + b^2)*Log[Tan[c + d*x]])/(b^5*d) + ((3*a
^2 + 2*b^2)*Tan[c + d*x])/(b^4*d) - (a*Tan[c + d*x]^2)/(b^3*d) + Tan[c + d*
x]^3/(3*b^2*d)
```

Rule 894

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ
[m, 0] && ILtQ[n, 0]))
```

Rule 3088

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[(x^m*(b +
 a*x)^n)/(1 + x^2)^((m + n + 2)/2), x], x, Cot[c + d*x]], x] /; FreeQ[{a, b
, c, d}, x] && IntegerQ[n] && IntegerQ[(m + n)/2] && NeQ[n, -1] &&  !(GtQ[n
, 0] && GtQ[m, 1])
```

Rubi steps

$$\int \frac{\sec^4(c+dx)}{(a\cos(c+dx)+b\sin(c+dx))^2}\,dx = -\frac{\text{Subst}\left(\int \frac{\left(1+x^2\right)^2}{x^4(b+ax)^2}\,dx, x, \cot(c+dx)\right)}{d}$$

$$= -\frac{\text{Subst}\left(\int \left(\frac{1}{b^2x^4} - \frac{2a}{b^3x^3} + \frac{3a^2+2b^2}{b^4x^2} - \frac{4a\left(a^2+b^2\right)}{b^5x} + \frac{\left(a^2+b^2\right)^2}{b^4(b+ax)^2} + \frac{4a^2\left(a^2+b^2\right)}{b^5(b+ax)}\right) dx, x}{d}$$

$$= \frac{\left(a^2+b^2\right)^2}{ab^4d(b+a\cot(c+dx))} - \frac{4a\left(a^2+b^2\right)\log(b+a\cot(c+dx))}{b^5d} - \frac{4a\left(a^2+\right.}{}$$

Mathematica [**A**] time = 2.78, size = 122, normalized size = 0.87

$$\frac{4b\left(2a^2+b^2\right)\tan(c+dx) + \frac{b^4\sec^4(c+dx) - 4\left(a^2+b^2\right)\left(3a^2\log(a+b\tan(c+dx)) + a^2 + 3ab\tan(c+dx)\log(a+b\tan(c+dx)) + b^2\right)}{a+b\tan(c+dx)} - 2ab^2\tan^2(c}{3b^5d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^4/(a*Cos[c + d*x] + b*Sin[c + d*x])^2,x]
```

```
[Out] (4*b*(2*a^2 + b^2)*Tan[c + d*x] - 2*a*b^2*Tan[c + d*x]^2 + (b^4*Sec[c + d*x
]^4 - 4*(a^2 + b^2)*(a^2 + b^2 + 3*a^2*Log[a + b*Tan[c + d*x]] + 3*a*b*Log[
a + b*Tan[c + d*x]]*Tan[c + d*x]))/(a + b*Tan[c + d*x]))/(3*b^5*d)
```

fricas [**B**] time = 0.46, size = 281, normalized size = 1.99

$$-\frac{4\left(3\,a^2b^2+2\,b^4\right)\cos\left(dx+c\right)^4 - b^4 - 2\left(3\,a^2b^2+2\,b^4\right)\cos\left(dx+c\right)^2 + 6\left(\left(a^4+a^2b^2\right)\cos\left(dx+c\right)^4 + \left(a^3b+ab^3\right)\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4/(a*cos(d*x+c)+b*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -1/3*(4*(3*a^2*b^2 + 2*b^4)*cos(d*x + c)^4 - b^4 - 2*(3*a^2*b^2 + 2*b^4)*co
s(d*x + c)^2 + 6*((a^4 + a^2*b^2)*cos(d*x + c)^4 + (a^3*b + a*b^3)*cos(d*x
+ c)^3*sin(d*x + c))*log(2*a*b*cos(d*x + c)*sin(d*x + c) + (a^2 - b^2)*cos(
d*x + c)^2 + b^2) - 6*((a^4 + a^2*b^2)*cos(d*x + c)^4 + (a^3*b + a*b^3)*cos
(d*x + c)^3*sin(d*x + c))*log(cos(d*x + c)^2) + 2*(a*b^3*cos(d*x + c) - 2*(
3*a^3*b + 2*a*b^3)*cos(d*x + c)^3)*sin(d*x + c))/(a*b^5*d*cos(d*x + c)^4 +
b^6*d*cos(d*x + c)^3*sin(d*x + c))
```

giac [A] time = 0.25, size = 149, normalized size = 1.06

$$-\frac{\frac{12\left(a^3+ab^2\right)\log(|b\tan(dx+c)+a|)}{b^5}-\frac{b^4\tan(dx+c)^3-3\,ab^3\tan(dx+c)^2+9\,a^2b^2\tan(dx+c)+6\,b^4\tan(dx+c)}{b^6}-\frac{3\left(4\,a^3b\tan(dx+c)+4\,ab^3\tan(dx+c)+3\right.}{(b\tan(dx+c)+a)b^5}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4/(a*cos(d*x+c)+b*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/3*(12*(a^3 + a*b^2)*log(abs(b*tan(d*x + c) + a))/b^5 - (b^4*tan(d*x + c)
^3 - 3*a*b^3*tan(d*x + c)^2 + 9*a^2*b^2*tan(d*x + c) + 6*b^4*tan(d*x + c))/
b^6 - 3*(4*a^3*b*tan(d*x + c) + 4*a*b^3*tan(d*x + c) + 3*a^4 + 2*a^2*b^2 -
b^4)/((b*tan(d*x + c) + a)*b^5))/d
```

maple [A] time = 0.34, size = 174, normalized size = 1.23

$$\frac{\tan^3(dx+c)}{3b^2d}-\frac{a\left(\tan^2(dx+c)\right)}{b^3d}+\frac{3a^2\tan(dx+c)}{d\,b^4}+\frac{2\tan(dx+c)}{b^2d}-\frac{4a^3\ln(a+b\tan(dx+c))}{d\,b^5}-\frac{4a\ln(a+b\tan(}{d\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^4/(a*cos(d*x+c)+b*sin(d*x+c))^2,x)
```

```
[Out] 1/3*tan(d*x+c)^3/b^2/d-a*tan(d*x+c)^2/b^3/d+3/d/b^4*a^2*tan(d*x+c)+2*tan(d*
x+c)/b^2/d-4/d*a^3/b^5*ln(a+b*tan(d*x+c))-4/d*a/b^3*ln(a+b*tan(d*x+c))-1/d/
b^5/(a+b*tan(d*x+c))*a^4-2/d/b^3/(a+b*tan(d*x+c))*a^2-1/d/b/(a+b*tan(d*x+c)
)
```

maxima [A] time = 0.33, size = 115, normalized size = 0.82

$$-\frac{\frac{3\left(a^4+2\,a^2b^2+b^4\right)}{b^6\tan(dx+c)+ab^5}-\frac{b^2\tan(dx+c)^3-3\,ab\tan(dx+c)^2+3\left(3\,a^2+2\,b^2\right)\tan(dx+c)}{b^4}+\frac{12\left(a^3+ab^2\right)\log(b\tan(dx+c)+a)}{b^5}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4/(a*cos(d*x+c)+b*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -1/3*(3*(a^4 + 2*a^2*b^2 + b^4)/(b^6*tan(d*x + c) + a*b^5) - (b^2*tan(d*x +
 c)^3 - 3*a*b*tan(d*x + c)^2 + 3*(3*a^2 + 2*b^2)*tan(d*x + c))/b^4 + 12*(a^
3 + a*b^2)*log(b*tan(d*x + c) + a)/b^5)/d
```

mupad [B] time = 4.20, size = 1132, normalized size = 8.03

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^4*(a*cos(c + d*x) + b*sin(c + d*x))^2),x)
```

```
[Out] ((8*tan(c/2 + (d*x)/2)^2*(a^2 + b^2))/b^3 + (8*tan(c/2 + (d*x)/2)^6*(a^2 +
b^2))/b^3 - (16*tan(c/2 + (d*x)/2)^4*(3*a^2 + 2*b^2))/(3*b^3) - (2*tan(c/2
+ (d*x)/2)^7*(4*a^4 + b^4 + 4*a^2*b^2))/(a*b^4) - (2*tan(c/2 + (d*x)/2)^3*(
36*a^4 + 9*b^4 + 44*a^2*b^2))/(3*a*b^4) + (2*tan(c/2 + (d*x)/2)^5*(36*a^4 +
 9*b^4 + 44*a^2*b^2))/(3*a*b^4) + (2*tan(c/2 + (d*x)/2)*(4*a^4 + b^4 + 4*a^
2*b^2))/(a*b^4))/(d*(a + 2*b*tan(c/2 + (d*x)/2) - 4*a*tan(c/2 + (d*x)/2)^2
+ 6*a*tan(c/2 + (d*x)/2)^4 - 4*a*tan(c/2 + (d*x)/2)^6 + a*tan(c/2 + (d*x)/2
)^8 - 6*b*tan(c/2 + (d*x)/2)^3 + 6*b*tan(c/2 + (d*x)/2)^5 - 2*b*tan(c/2 + (
d*x)/2)^7)) + (a*atan(((a*(a^2 + b^2)*((16*tan(c/2 + (d*x)/2)*(4*a^5 + 4*a^
3*b^2))/b^4 - (4*(8*a^2*b^7 + 8*a^4*b^5))/b^8 + (4*tan(c/2 + (d*x)/2)^2*(8*
a^2*b^7 + 8*a^4*b^5))/b^8 + (4*a*(a^2 + b^2)*((4*(a*b^10 + 4*a^3*b^8))/b^8
- (4*tan(c/2 + (d*x)/2)^2*(3*a*b^10 + 4*a^3*b^8))/b^8 + 16*a^2*b*tan(c/2 +
(d*x)/2)))/b^5)*4i)/b^5 - (a*(a^2 + b^2)*((4*(8*a^2*b^7 + 8*a^4*b^5))/b^8 -
 (16*tan(c/2 + (d*x)/2)*(4*a^5 + 4*a^3*b^2))/b^4 - (4*tan(c/2 + (d*x)/2)^2*
(8*a^2*b^7 + 8*a^4*b^5))/b^8 + (4*a*(a^2 + b^2)*((4*(a*b^10 + 4*a^3*b^8))/b
^8 - (4*tan(c/2 + (d*x)/2)^2*(3*a*b^10 + 4*a^3*b^8))/b^8 + 16*a^2*b*tan(c/2
 + (d*x)/2)))/b^5)*4i)/b^5)/((8*(16*a^7 + 16*a^3*b^4 + 32*a^5*b^2))/b^8 + (
8*tan(c/2 + (d*x)/2)^2*(16*a^7 + 16*a^3*b^4 + 32*a^5*b^2))/b^8 + (4*a*(a^2
+ b^2)*((16*tan(c/2 + (d*x)/2)*(4*a^5 + 4*a^3*b^2))/b^4 - (4*(8*a^2*b^7 + 8
*a^4*b^5))/b^8 + (4*tan(c/2 + (d*x)/2)^2*(8*a^2*b^7 + 8*a^4*b^5))/b^8 + (4*
a*(a^2 + b^2)*((4*(a*b^10 + 4*a^3*b^8))/b^8 - (4*tan(c/2 + (d*x)/2)^2*(3*a*
b^10 + 4*a^3*b^8))/b^8 + 16*a^2*b*tan(c/2 + (d*x)/2)))/b^5))/b^5 + (4*a*(a^
2 + b^2)*((4*(8*a^2*b^7 + 8*a^4*b^5))/b^8 - (16*tan(c/2 + (d*x)/2)*(4*a^5 +
 4*a^3*b^2))/b^4 - (4*tan(c/2 + (d*x)/2)^2*(8*a^2*b^7 + 8*a^4*b^5))/b^8 + (
4*a*(a^2 + b^2)*((4*(a*b^10 + 4*a^3*b^8))/b^8 - (4*tan(c/2 + (d*x)/2)^2*(3*
a*b^10 + 4*a^3*b^8))/b^8 + 16*a^2*b*tan(c/2 + (d*x)/2)))/b^5))/b^5))*(a^2 +
 b^2)*8i)/(b^5*d)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec^4(c+dx)}{(a\cos(c+dx)+b\sin(c+dx))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**4/(a*cos(d*x+c)+b*sin(d*x+c))**2,x)
```

```
[Out] Integral(sec(c + d*x)**4/(a*cos(c + d*x) + b*sin(c + d*x))**2, x)
```

3.131 $\int \frac{\cos^4(c+dx)}{(a\cos(c+dx)+b\sin(c+dx))^3}\,dx$

Optimal. Leaf size=216

$$\frac{a\left(a^2-3b^2\right)\sin(c+dx)}{d\left(a^2+b^2\right)^3}+\frac{b\left(3a^2-b^2\right)\cos(c+dx)}{d\left(a^2+b^2\right)^3}-\frac{3b^2\left(4a^2-b^2\right)\tanh^{-1}\left(\frac{b-a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2+b^2}}\right)}{d\left(a^2+b^2\right)^{7/2}}+\frac{b^4\,\mathrm{s}}{2ad\left(a^2+b^2\right)^2(a\,\mathrm{co}}$$

```
[Out] -3*b^2*(4*a^2-b^2)*arctanh((b-a*tan(1/2*d*x+1/2*c))/(a^2+b^2)^(1/2))/(a^2+b
^2)^(7/2)/d+b*(3*a^2-b^2)*cos(d*x+c)/(a^2+b^2)^3/d+a*(a^2-3*b^2)*sin(d*x+c)
/(a^2+b^2)^3/d+1/2*b^4*sin(d*x+c)/a/(a^2+b^2)^2/d/(a*cos(d*x+c)+b*sin(d*x+c
))^2-1/2*b^3*(8*a^2+b^2)/a/(a^2+b^2)^3/d/(a*cos(d*x+c)+b*sin(d*x+c))
```

Rubi [B] time = 1.74, antiderivative size = 492, normalized size of antiderivative = 2.28, number of steps used = 15, number of rules used = 7, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {6742, 639, 203, 638, 618, 206, 614}

$$-\frac{3b^4\left(a^2+2b^2\right)\left(b-a\tan\left(\frac{1}{2}(c+dx)\right)\right)}{a^3d\left(a^2+b^2\right)^3\left(-a\tan^2\left(\frac{1}{2}(c+dx)\right)+a+2b\tan\left(\frac{1}{2}(c+dx)\right)\right)}+\frac{2b^4\left(\left(a^2+2b^2\right)\tan\left(\frac{1}{2}(c+dx)\right)+\right.}{a^3d\left(a^2+b^2\right)^2\left(-a\tan^2\left(\frac{1}{2}(c+dx)\right)+a+2b\,\mathrm{ta}\right.}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^4/(a*Cos[c + d*x] + b*Sin[c + d*x])^3,x]
```

```
[Out] (-3*b^4*(a^2 + 2*b^2)*ArcTanh[(b - a*Tan[(c + d*x)/2])/Sqrt[a^2 + b^2]])/(a
^2*(a^2 + b^2)^(7/2)*d) + (4*b^4*(3*a^2 + 2*b^2)*ArcTanh[(b - a*Tan[(c + d*
x)/2])/Sqrt[a^2 + b^2]])/(a^2*(a^2 + b^2)^(7/2)*d) - (2*b^2*(6*a^4 + 3*a^2*
b^2 + b^4)*ArcTanh[(b - a*Tan[(c + d*x)/2])/Sqrt[a^2 + b^2]])/(a^2*(a^2 + b
^2)^(7/2)*d) + (2*(b*(3*a^2 - b^2) + a*(a^2 - 3*b^2)*Tan[(c + d*x)/2]))/((a
^2 + b^2)^3*d*(1 + Tan[(c + d*x)/2]^2)) + (2*b^4*(a*b + (a^2 + 2*b^2)*Tan[(
c + d*x)/2]))/(a^3*(a^2 + b^2)^2*d*(a + 2*b*Tan[(c + d*x)/2] - a*Tan[(c + d
*x)/2]^2)^2) - (3*b^4*(a^2 + 2*b^2)*(b - a*Tan[(c + d*x)/2]))/(a^3*(a^2 + b
^2)^3*d*(a + 2*b*Tan[(c + d*x)/2] - a*Tan[(c + d*x)/2]^2)) - (4*b^3*(2*a^4
- b^4 + a*b*(3*a^2 + 2*b^2)*Tan[(c + d*x)/2]))/(a^3*(a^2 + b^2)^3*d*(a + 2*
b*Tan[(c + d*x)/2] - a*Tan[(c + d*x)/2]^2))
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 614

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] - Dist[(2*c*(2*p +
 3))/((p + 1)*(b^2 - 4*a*c)), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2] && Int
egerQ[4*p]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 638

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[((b*d - 2*a*e + (2*c*d - b*e)*x)*(a + b*x + c*x^2)^(p + 1))/((p +
 1)*(b^2 - 4*a*c)), x] - Dist[((2*p + 3)*(2*c*d - b*e))/((p + 1)*(b^2 - 4*a
*c)), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] &&
 NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2]
```

Rule 639

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((a*e
 - c*d*x)*(a + c*x^2)^(p + 1))/(2*a*c*(p + 1)), x] + Dist[(d*(2*p + 3))/(2*
a*(p + 1)), Int[(a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e}, x] && Lt
Q[p, -1] && NeQ[p, -3/2]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int \frac{\cos^4(c+dx)}{(a\cos(c+dx)+b\sin(c+dx))^3}\,dx = \frac{2\,\text{Subst}\left(\int \frac{(1-x^2)^4}{(1+x^2)^2(a+2bx-ax^2)^3}\,dx, x, \tan\left(\frac{1}{2}(c+dx)\right)\right)}{d}$$

$$= \frac{2\,\text{Subst}\left(\int \left(\frac{2\left(a\left(a^2-3b^2\right)-b\left(3a^2-b^2\right)x\right)}{\left(a^2+b^2\right)^3(1+x^2)^2} - \frac{a\left(a^2-3b^2\right)}{\left(a^2+b^2\right)^3(1+x^2)} + \frac{4b^3\left(-b\left(a^2+b^2\right)-a\left(2a^2+b^2\right.\right.}{a^3\left(a^2+b^2\right)^2\left(a+2bx-ax^2\right)}\right.\right.}{}$$

$$= \frac{4\,\text{Subst}\left(\int \frac{a\left(a^2-3b^2\right)-b\left(3a^2-b^2\right)x}{(1+x^2)^2}\,dx, x, \tan\left(\frac{1}{2}(c+dx)\right)\right)}{\left(a^2+b^2\right)^3 d} - \frac{\left(2a\left(a^2-3b^2\right)\right)}{}$$

$$= -\frac{a\left(a^2-3b^2\right)x}{\left(a^2+b^2\right)^3} + \frac{2\left(b\left(3a^2-b^2\right)+a\left(a^2-3b^2\right)\tan\left(\frac{1}{2}(c+dx)\right)\right)}{\left(a^2+b^2\right)^3 d\left(1+\tan^2\left(\frac{1}{2}(c+dx)\right)\right)} + \frac{}{a^3}$$

$$= -\frac{2b^2\left(6a^4+3a^2b^2+b^4\right)\tanh^{-1}\left(\frac{b-a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2+b^2}}\right)}{a^2\left(a^2+b^2\right)^{7/2}d} + \frac{2\left(b\left(3a^2-b^2\right)+a\right.}{\left(a^2+b^2\right)^3 d\left(\right.}$$

$$= \frac{4b^4\left(3a^2+2b^2\right)\tanh^{-1}\left(\frac{b-a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2+b^2}}\right)}{a^2\left(a^2+b^2\right)^{7/2}d} - \frac{2b^2\left(6a^4+3a^2b^2+b^4\right)\tanh}{a^2\left(a^2+b^2\right)^{7}}$$

$$= -\frac{3b^4\left(a^2+2b^2\right)\tanh^{-1}\left(\frac{b-a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2+b^2}}\right)}{a^2\left(a^2+b^2\right)^{7/2}d} + \frac{4b^4\left(3a^2+2b^2\right)\tanh^{-1}\left(\frac{b-a}{}\right.}{a^2\left(a^2+b^2\right)^{7/2}d}$$

Mathematica [C] time = 1.12, size = 211, normalized size = 0.98

$$\frac{\frac{2a\left(a^2-3b^2\right)\sin(c+dx)}{\left(a^2+b^2\right)^3} - \frac{2b\left(b^2-3a^2\right)\cos(c+dx)}{\left(a^2+b^2\right)^3} - \frac{6b^2\left(b^2-4a^2\right)\tanh^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)-b}{\sqrt{a^2+b^2}}\right)}{\left(a^2+b^2\right)^{7/2}} - \frac{b^3\left(8a^2+b^2\right)}{a\left(a^2+b^2\right)^3(a\cos(c+dx)+b\sin(c+dx))} + \frac{}{a(a-ib)^2(a+ib}}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^4/(a*Cos[c + d*x] + b*Sin[c + d*x])^3,x]
```

```
[Out] ((-6*b^2*(-4*a^2 + b^2)*ArcTanh[(-b + a*Tan[(c + d*x)/2])/Sqrt[a^2 + b^2]])
/(a^2 + b^2)^(7/2) - (2*b*(-3*a^2 + b^2)*Cos[c + d*x])/(a^2 + b^2)^3 + (2*a
*(a^2 - 3*b^2)*Sin[c + d*x])/(a^2 + b^2)^3 + (b^4*Sin[c + d*x])/(a*(a - I*b
)^2*(a + I*b)^2*(a*Cos[c + d*x] + b*Sin[c + d*x])^2) - (b^3*(8*a^2 + b^2))/
(a*(a^2 + b^2)^3*(a*Cos[c + d*x] + b*Sin[c + d*x])))/(2*d)
```

fricas [B] time = 0.57, size = 480, normalized size = 2.22

$$\frac{4\left(a^{6}b+3\,a^{4}b^{3}+3\,a^{2}b^{5}+b^{7}\right)\cos\left(dx+c\right)^{3}-3\left(4\,a^{2}b^{4}-b^{6}+\left(4\,a^{4}b^{2}-5\,a^{2}b^{4}+b^{6}\right)\cos\left(dx+c\right)^{2}+2\left(4\,a^{3}b^{3}-ab\right.\right.}{4\left(\left(a^{10}+3\,a^{8}b^{2}+\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4/(a*cos(d*x+c)+b*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/4*(4*(a^6*b + 3*a^4*b^3 + 3*a^2*b^5 + b^7)*cos(d*x + c)^3 - 3*(4*a^2*b^4
- b^6 + (4*a^4*b^2 - 5*a^2*b^4 + b^6)*cos(d*x + c)^2 + 2*(4*a^3*b^3 - a*b^5
)*cos(d*x + c)*sin(d*x + c))*sqrt(a^2 + b^2)*log((2*a*b*cos(d*x + c)*sin(d*
x + c) + (a^2 - b^2)*cos(d*x + c)^2 - 2*a^2 - b^2 - 2*sqrt(a^2 + b^2)*(b*co
s(d*x + c) - a*sin(d*x + c)))/(2*a*b*cos(d*x + c)*sin(d*x + c) + (a^2 - b^2
)*cos(d*x + c)^2 + b^2)) + 2*(4*a^6*b - 10*a^4*b^3 - 17*a^2*b^5 - 3*b^7)*co
s(d*x + c) + 2*(2*a^5*b^2 - 11*a^3*b^4 - 13*a*b^6 + 2*(a^7 + 3*a^5*b^2 + 3*
a^3*b^4 + a*b^6)*cos(d*x + c)^2)*sin(d*x + c))/((a^10 + 3*a^8*b^2 + 2*a^6*b
^4 - 2*a^4*b^6 - 3*a^2*b^8 - b^10)*d*cos(d*x + c)^2 + 2*(a^9*b + 4*a^7*b^3
+ 6*a^5*b^5 + 4*a^3*b^7 + a*b^9)*d*cos(d*x + c)*sin(d*x + c) + (a^8*b^2 + 4
*a^6*b^4 + 6*a^4*b^6 + 4*a^2*b^8 + b^10)*d)
```

giac [A] time = 0.99, size = 399, normalized size = 1.85

$$-\frac{\frac{3\left(4\,a^{2}b^{2}-b^{4}\right)\log\left(\frac{\left|2\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-2\,b-2\,\sqrt{a^{2}+b^{2}}\right|}{\left|2\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-2\,b+2\,\sqrt{a^{2}+b^{2}}\right|}\right)}{\left(a^{6}+3\,a^{4}b^{2}+3\,a^{2}b^{4}+b^{6}\right)\sqrt{a^{2}+b^{2}}}-\frac{4\left(a^{3}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-3\,ab^{2}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+3\,a^{2}b-b^{3}\right)}{\left(a^{6}+3\,a^{4}b^{2}+3\,a^{2}b^{4}+b^{6}\right)\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+1\right)}-\frac{2\left(9\,a^{3}b^{4}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}+2\,ab^{6}\tan\left(\right.\right.}{}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4/(a*cos(d*x+c)+b*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -1/2*(3*(4*a^2*b^2 - b^4)*log(abs(2*a*tan(1/2*d*x + 1/2*c) - 2*b - 2*sqrt(a
^2 + b^2))/abs(2*a*tan(1/2*d*x + 1/2*c) - 2*b + 2*sqrt(a^2 + b^2)))/((a^6 +
 3*a^4*b^2 + 3*a^2*b^4 + b^6)*sqrt(a^2 + b^2)) - 4*(a^3*tan(1/2*d*x + 1/2*c
) - 3*a*b^2*tan(1/2*d*x + 1/2*c) + 3*a^2*b - b^3)/((a^6 + 3*a^4*b^2 + 3*a^2
*b^4 + b^6)*(tan(1/2*d*x + 1/2*c)^2 + 1)) - 2*(9*a^3*b^4*tan(1/2*d*x + 1/2*
c)^3 + 2*a*b^6*tan(1/2*d*x + 1/2*c)^3 + 8*a^4*b^3*tan(1/2*d*x + 1/2*c)^2 -
```

```
15*a^2*b^5*tan(1/2*d*x + 1/2*c)^2 - 2*b^7*tan(1/2*d*x + 1/2*c)^2 - 23*a^3*b
^4*tan(1/2*d*x + 1/2*c) - 2*a*b^6*tan(1/2*d*x + 1/2*c) - 8*a^4*b^3 - a^2*b^
5)/((a^8 + 3*a^6*b^2 + 3*a^4*b^4 + a^2*b^6)*(a*tan(1/2*d*x + 1/2*c)^2 - 2*b
*tan(1/2*d*x + 1/2*c) - a)^2))/d
```

maple [A] time = 0.27, size = 283, normalized size = 1.31

$$\frac{-\frac{2\left(\left(-a^{3}+3ab^{2}\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-3a^{2}b+b^{3}\right)}{\left(a^{6}+3a^{4}b^{2}+3a^{2}b^{4}+b^{6}\right)\left(\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}-\frac{2b^{2}\left(\frac{-\frac{b^{2}\left(9a^{2}+2b^{2}\right)\left(\tan^{3}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2a}-\frac{b\left(8a^{4}-15a^{2}b^{2}-2b^{4}\right)\left(\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2a^{2}}+\frac{b^{2}\left(23a^{2}+2b^{2}\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2a}+4a^{2}b+\frac{b^{3}}{2}}{\left(\left(\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a-2b\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-a\right)^{2}}-\frac{3(4a}{}\right.}{\left(a^{2}+b^{2}\right)^{3}}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4/(a*cos(d*x+c)+b*sin(d*x+c))^3,x)
```

```
[Out] 1/d*(-2/(a^6+3*a^4*b^2+3*a^2*b^4+b^6)*((-a^3+3*a*b^2)*tan(1/2*d*x+1/2*c)-3*
a^2*b+b^3)/(tan(1/2*d*x+1/2*c)^2+1)-2*b^2/(a^2+b^2)^3*((-1/2*b^2*(9*a^2+2*b
^2)/a*tan(1/2*d*x+1/2*c)^3-1/2*b*(8*a^4-15*a^2*b^2-2*b^4)/a^2*tan(1/2*d*x+1
/2*c)^2+1/2*b^2*(23*a^2+2*b^2)/a*tan(1/2*d*x+1/2*c)+4*a^2*b+1/2*b^3)/(tan(1
/2*d*x+1/2*c)^2*a-2*b*tan(1/2*d*x+1/2*c)-a)^2-3/2*(4*a^2-b^2)/(a^2+b^2)^(1/
2)*arctanh(1/2*(2*a*tan(1/2*d*x+1/2*c)-2*b)/(a^2+b^2)^(1/2))))
```

maxima [B] time = 0.44, size = 658, normalized size = 3.05

$$-\frac{\frac{3\left(4a^{2}b^{2}-b^{4}\right)\log\left(\frac{b-\frac{a\sin(dx+c)}{\cos(dx+c)+1}+\sqrt{a^{2}+b^{2}}}{b-\frac{a\sin(dx+c)}{\cos(dx+c)+1}-\sqrt{a^{2}+b^{2}}}\right)}{\left(a^{6}+3a^{4}b^{2}+3a^{2}b^{4}+b^{6}\right)\sqrt{a^{2}+b^{2}}}-\frac{2\left(6a^{6}b-10a^{4}b^{3}-a^{2}b^{5}+\frac{\left(2a^{7}+18a^{5}b^{2}-31a^{3}b^{4}-2ab^{6}\right)\sin(dx+c)}{\cos(dx+c)+1}-\frac{2\left(2a^{6}b-2a^{4}b^{3}+12a^{2}b^{5}+b^{7}\right)\sin(dx+}{(\cos(dx+c)+1)^{2}}\right.}{a^{10}+3a^{8}b^{2}+3a^{6}b^{4}+a^{4}b^{6}+\frac{4\left(a^{9}b+3a^{7}b^{3}+3a^{5}b^{5}+a^{3}b^{7}\right)\sin(dx+c)}{\cos(dx+c)+1}-\frac{\left(a^{10}-a^{8}b^{2}-9a^{6}b^{4}-11a^{4}b^{6}-4a^{2}b^{8}\right)\sin(dx+}{(\cos(dx+c)+1)^{2}}}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4/(a*cos(d*x+c)+b*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -1/2*(3*(4*a^2*b^2 - b^4)*log((b - a*sin(d*x + c)/(cos(d*x + c) + 1) + sqrt
(a^2 + b^2))/(b - a*sin(d*x + c)/(cos(d*x + c) + 1) - sqrt(a^2 + b^2)))/((a
^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*sqrt(a^2 + b^2)) - 2*(6*a^6*b - 10*a^4*b^
3 - a^2*b^5 + (2*a^7 + 18*a^5*b^2 - 31*a^3*b^4 - 2*a*b^6)*sin(d*x + c)/(cos
(d*x + c) + 1) - 2*(2*a^6*b - 2*a^4*b^3 + 12*a^2*b^5 + b^7)*sin(d*x + c)^2/
(cos(d*x + c) + 1)^2 - 2*(2*a^7 + 2*a^5*b^2 + 15*a^3*b^4)*sin(d*x + c)^3/(c
os(d*x + c) + 1)^3 - (2*a^6*b - 30*a^4*b^3 + 15*a^2*b^5 + 2*b^7)*sin(d*x +
c)^4/(cos(d*x + c) + 1)^4 + (2*a^7 - 6*a^5*b^2 + 9*a^3*b^4 + 2*a*b^6)*sin(d
```

```
*x + c)^5/(cos(d*x + c) + 1)^5)/(a^10 + 3*a^8*b^2 + 3*a^6*b^4 + a^4*b^6 + 4
*(a^9*b + 3*a^7*b^3 + 3*a^5*b^5 + a^3*b^7)*sin(d*x + c)/(cos(d*x + c) + 1)
- (a^10 - a^8*b^2 - 9*a^6*b^4 - 11*a^4*b^6 - 4*a^2*b^8)*sin(d*x + c)^2/(cos
(d*x + c) + 1)^2 - (a^10 - a^8*b^2 - 9*a^6*b^4 - 11*a^4*b^6 - 4*a^2*b^8)*si
n(d*x + c)^4/(cos(d*x + c) + 1)^4 - 4*(a^9*b + 3*a^7*b^3 + 3*a^5*b^5 + a^3*
b^7)*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + (a^10 + 3*a^8*b^2 + 3*a^6*b^4 +
a^4*b^6)*sin(d*x + c)^6/(cos(d*x + c) + 1)^6))/d
```

mupad [B] time = 4.25, size = 610, normalized size = 2.82

$$-\frac{\frac{-6a^4b+10a^2b^3+b^5}{a^6+3a^4b^2+3a^2b^4+b^6}+\frac{2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3\left(2a^5+2a^3b^2+15ab^4\right)}{a^6+3a^4b^2+3a^2b^4+b^6}+\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4\left(2a^6b-30a^4b^3+15a^2b^5+2b^7\right)}{a^2\left(a^6+3a^4b^2+3a^2b^4+b^6\right)}-\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\left(2a^6+18a^4b^2-31\cdots\right)}{a\left(a^6+3a^4b^2+3a^2b^4+\cdots\right)}\cdots}{d\left(a^2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^6+a^2-\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2\left(a^2-4b^2\right)-\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4\left(a^2-4b\cdots\right)\cdots\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^4/(a*cos(c + d*x) + b*sin(c + d*x))^3,x)
```

```
[Out] - ((b^5 - 6*a^4*b + 10*a^2*b^3)/(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2) + (2*ta
n(c/2 + (d*x)/2)^3*(15*a*b^4 + 2*a^5 + 2*a^3*b^2))/(a^6 + b^6 + 3*a^2*b^4 +
 3*a^4*b^2) + (tan(c/2 + (d*x)/2)^4*(2*a^6*b + 2*b^7 + 15*a^2*b^5 - 30*a^4*
b^3))/(a^2*(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2)) - (tan(c/2 + (d*x)/2)*(2*a^
6 - 2*b^6 - 31*a^2*b^4 + 18*a^4*b^2))/(a*(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2
)) - (tan(c/2 + (d*x)/2)^5*(2*a^6 + 2*b^6 + 9*a^2*b^4 - 6*a^4*b^2))/(a*(a^6
 + b^6 + 3*a^2*b^4 + 3*a^4*b^2)) + (2*tan(c/2 + (d*x)/2)^2*(2*a^6*b + b^7 +
 12*a^2*b^5 - 2*a^4*b^3))/(a^2*(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2)))/(d*(a^
2*tan(c/2 + (d*x)/2)^6 + a^2 - tan(c/2 + (d*x)/2)^2*(a^2 - 4*b^2) - tan(c/2
 + (d*x)/2)^4*(a^2 - 4*b^2) - 4*a*b*tan(c/2 + (d*x)/2)^5 + 4*a*b*tan(c/2 +
(d*x)/2))) - (atan((a^6*b*1i + b^7*1i + a^2*b^5*3i + a^4*b^3*3i - a^7*tan(c
/2 + (d*x)/2)*1i - a*b^6*tan(c/2 + (d*x)/2)*1i - a^3*b^4*tan(c/2 + (d*x)/2)
*3i - a^5*b^2*tan(c/2 + (d*x)/2)*3i)/(a^2 + b^2)^(7/2))*(3*b^4 - 12*a^2*b^2
)*1i)/(d*(a^2 + b^2)^(7/2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4/(a*cos(d*x+c)+b*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

3.132 $\int \frac{\cos^3(c+dx)}{(a\cos(c+dx)+b\sin(c+dx))^3}\,dx$

Optimal. Leaf size=122

$$-\frac{2ab}{d\left(a^2+b^2\right)^2\left(a+b\tan(c+dx)\right)}-\frac{b}{2d\left(a^2+b^2\right)\left(a+b\tan(c+dx)\right)^2}+\frac{b\left(3a^2-b^2\right)\log(a\cos(c+dx)+b\sin(c+}{d\left(a^2+b^2\right)^3}$$

```
[Out] a*(a^2-3*b^2)*x/(a^2+b^2)^3+b*(3*a^2-b^2)*ln(a*cos(d*x+c)+b*sin(d*x+c))/(a^
2+b^2)^3/d-1/2*b/(a^2+b^2)/d/(a+b*tan(d*x+c))^2-2*a*b/(a^2+b^2)^2/d/(a+b*ta
n(d*x+c))
```

Rubi [**A**] time = 0.21, antiderivative size = 122, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.179, Rules used = {3086, 3483, 3529, 3531, 3530}

$$-\frac{2ab}{d\left(a^2+b^2\right)^2\left(a+b\tan(c+dx)\right)}-\frac{b}{2d\left(a^2+b^2\right)\left(a+b\tan(c+dx)\right)^2}+\frac{b\left(3a^2-b^2\right)\log(a\cos(c+dx)+b\sin(c+}{d\left(a^2+b^2\right)^3}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^3/(a*Cos[c + d*x] + b*Sin[c + d*x])^3,x]
```

```
[Out] (a*(a^2 - 3*b^2)*x)/(a^2 + b^2)^3 + (b*(3*a^2 - b^2)*Log[a*Cos[c + d*x] + b
*Sin[c + d*x]])/((a^2 + b^2)^3*d) - b/(2*(a^2 + b^2)*d*(a + b*Tan[c + d*x])
^2) - (2*a*b)/((a^2 + b^2)^2*d*(a + b*Tan[c + d*x]))
```

Rule 3086

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin
[(c_.) + (d_.)*(x_)])^(n_.), x_Symbol] :> Int[(a + b*Tan[c + d*x])^n, x] /;
 FreeQ[{a, b, c, d}, x] && EqQ[m + n, 0] && IntegerQ[n] && NeQ[a^2 + b^2, 0
]
```

Rule 3483

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(a +
b*Tan[c + d*x])^(n + 1))/(d*(n + 1)*(a^2 + b^2)), x] + Dist[1/(a^2 + b^2),
Int[(a - b*Tan[c + d*x])*(a + b*Tan[c + d*x])^(n + 1), x], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 + b^2, 0] && LtQ[n, -1]
```

Rule 3529

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[((b*c - a*d)*(a + b*Tan[e + f*x])^(m + 1))
```

```
/(f*(m + 1)*(a^2 + b^2)), x] + Dist[1/(a^2 + b^2), Int[(a + b*Tan[e + f*x])
^(m + 1)*Simp[a*c + b*d - (b*c - a*d)*Tan[e + f*x], x], x], x] /; FreeQ[{a,
 b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && LtQ[m, -1]
```

Rule 3530

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_) + (b_.)*tan[(e_.) + (f_.)*
(x_)]), x_Symbol] :> Simp[(c*Log[RemoveContent[a*Cos[e + f*x] + b*Sin[e + f
*x], x]])/(b*f), x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
 NeQ[a^2 + b^2, 0] && EqQ[a*c + b*d, 0]
```

Rule 3531

```
Int[((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_.) + (b_.)*tan[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[((a*c + b*d)*x)/(a^2 + b^2), x] + Dist[(b*c - a
*d)/(a^2 + b^2), Int[(b - a*Tan[e + f*x])/(a + b*Tan[e + f*x]), x], x] /; F
reeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && Ne
Q[a*c + b*d, 0]
```

Rubi steps

$$\int \frac{\cos^3(c+dx)}{(a\cos(c+dx)+b\sin(c+dx))^3}\,dx = \int \frac{1}{(a+b\tan(c+dx))^3}\,dx$$

$$= -\frac{b}{2\left(a^2+b^2\right)d(a+b\tan(c+dx))^2} + \frac{\int \frac{a-b\tan(c+dx)}{(a+b\tan(c+dx))^2}\,dx}{a^2+b^2}$$

$$= -\frac{b}{2\left(a^2+b^2\right)d(a+b\tan(c+dx))^2} - \frac{2ab}{\left(a^2+b^2\right)^2 d(a+b\tan(c+dx))} + \frac{\int}{}$$

$$= \frac{a\left(a^2-3b^2\right)x}{\left(a^2+b^2\right)^3} - \frac{b}{2\left(a^2+b^2\right)d(a+b\tan(c+dx))^2} - \frac{2ab}{\left(a^2+b^2\right)^2 d(a+b\,\mathrm{ta}}$$

$$= \frac{a\left(a^2-3b^2\right)x}{\left(a^2+b^2\right)^3} + \frac{b\left(3a^2-b^2\right)\log(a\cos(c+dx)+b\sin(c+dx))}{\left(a^2+b^2\right)^3 d} - \frac{}{2\left(a^2\right.}$$

Mathematica [C] time = 1.26, size = 154, normalized size = 1.26

$$\frac{\frac{2a\left(a^2-3b^2\right)(c+dx)}{\left(a^2+b^2\right)^3} + \frac{6b^2\sin(c+dx)}{\left(a^2+b^2\right)^2(a\cos(c+dx)+b\sin(c+dx))} - \frac{2b\left(b^2-3a^2\right)\log(a\cos(c+dx)+b\sin(c+dx))}{\left(a^2+b^2\right)^3} - \frac{b^3}{(a-ib)^2(a+ib)^2(a\cos(c+dx)+b\sin(c+dx))^2}}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^3/(a*Cos[c + d*x] + b*Sin[c + d*x])^3,x]
```

```
[Out] ((2*a*(a^2 - 3*b^2)*(c + d*x))/(a^2 + b^2)^3 - (2*b*(-3*a^2 + b^2)*Log[a*Co
s[c + d*x] + b*Sin[c + d*x]])/(a^2 + b^2)^3 - b^3/((a - I*b)^2*(a + I*b)^2*
(a*Cos[c + d*x] + b*Sin[c + d*x])^2) + (6*b^2*Sin[c + d*x])/((a^2 + b^2)^2*
(a*Cos[c + d*x] + b*Sin[c + d*x])))/(2*d)
```

fricas [B] time = 0.57, size = 341, normalized size = 2.80

$$\frac{5\,a^2b^3 - b^5 + 2\left(a^3b^2 - 3\,ab^4\right)dx - 2\left(6\,a^2b^3 - \left(a^5 - 4\,a^3b^2 + 3\,ab^4\right)dx\right)\cos\left(dx+c\right)^2 + 2\left(3\,a^3b^2 - 3\,ab^4 + 2\left(a^4b \cdots\right.\right.}{2\left(\left(a^8 + 2\,a^6b^2 - 2\,a^2b^6 - \cdots\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3/(a*cos(d*x+c)+b*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/2*(5*a^2*b^3 - b^5 + 2*(a^3*b^2 - 3*a*b^4)*d*x - 2*(6*a^2*b^3 - (a^5 - 4*
a^3*b^2 + 3*a*b^4)*d*x)*cos(d*x + c)^2 + 2*(3*a^3*b^2 - 3*a*b^4 + 2*(a^4*b
- 3*a^2*b^3)*d*x)*cos(d*x + c)*sin(d*x + c) + (3*a^2*b^3 - b^5 + (3*a^4*b -
 4*a^2*b^3 + b^5)*cos(d*x + c)^2 + 2*(3*a^3*b^2 - a*b^4)*cos(d*x + c)*sin(d
*x + c))*log(2*a*b*cos(d*x + c)*sin(d*x + c) + (a^2 - b^2)*cos(d*x + c)^2 +
 b^2))/((a^8 + 2*a^6*b^2 - 2*a^2*b^6 - b^8)*d*cos(d*x + c)^2 + 2*(a^7*b + 3
*a^5*b^3 + 3*a^3*b^5 + a*b^7)*d*cos(d*x + c)*sin(d*x + c) + (a^6*b^2 + 3*a^
4*b^4 + 3*a^2*b^6 + b^8)*d)
```

giac [B] time = 0.39, size = 265, normalized size = 2.17

$$\frac{\frac{2\left(a^3-3\,ab^2\right)(dx+c)}{a^6+3\,a^4b^2+3\,a^2b^4+b^6} - \frac{\left(3\,a^2b-b^3\right)\log\left(\tan(dx+c)^2+1\right)}{a^6+3\,a^4b^2+3\,a^2b^4+b^6} + \frac{2\left(3\,a^2b^2-b^4\right)\log\left(\left|b\tan(dx+c)+a\right|\right)}{a^6b+3\,a^4b^3+3\,a^2b^5+b^7} - \frac{9\,a^2b^3\tan(dx+c)^2-3\,b^5\tan(dx+c)^2+22\,a^3b^2\tan(\cdots}{\left(a^6+3\,a^4b^2+3\,a^2b^4+b^6\right)(\cdots}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3/(a*cos(d*x+c)+b*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/2*(2*(a^3 - 3*a*b^2)*(d*x + c)/(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6) - (3*a
^2*b - b^3)*log(tan(d*x + c)^2 + 1)/(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6) + 2
*(3*a^2*b^2 - b^4)*log(abs(b*tan(d*x + c) + a))/(a^6*b + 3*a^4*b^3 + 3*a^2*
b^5 + b^7) - (9*a^2*b^3*tan(d*x + c)^2 - 3*b^5*tan(d*x + c)^2 + 22*a^3*b^2*
tan(d*x + c) - 2*a*b^4*tan(d*x + c) + 14*a^4*b + 3*a^2*b^3 + b^5)/((a^6 + 3
*a^4*b^2 + 3*a^2*b^4 + b^6)*(b*tan(d*x + c) + a)^2))/d
```

maple [A] time = 0.25, size = 219, normalized size = 1.80

$$-\frac{b}{2\left(a^2+b^2\right)d\left(a+b\tan\left(dx+c\right)\right)^2}+\frac{3b\ln\left(a+b\tan\left(dx+c\right)\right)a^2}{d\left(a^2+b^2\right)^3}-\frac{b^3\ln\left(a+b\tan\left(dx+c\right)\right)}{d\left(a^2+b^2\right)^3}-\frac{2ab}{\left(a^2+b^2\right)^2 d\left(a+b\tan\cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3/(a*cos(d*x+c)+b*sin(d*x+c))^3,x)
```

```
[Out] -1/2*b/(a^2+b^2)/d/(a+b*tan(d*x+c))^2+3/d*b/(a^2+b^2)^3*ln(a+b*tan(d*x+c))*
a^2-1/d*b^3/(a^2+b^2)^3*ln(a+b*tan(d*x+c))-2*a*b/(a^2+b^2)^2/d/(a+b*tan(d*x
+c))-3/2/d/(a^2+b^2)^3*ln(tan(d*x+c)^2+1)*a^2*b+1/2/d/(a^2+b^2)^3*ln(tan(d*
x+c)^2+1)*b^3+1/d/(a^2+b^2)^3*arctan(tan(d*x+c))*a^3-3/d/(a^2+b^2)^3*arctan
(tan(d*x+c))*a*b^2
```

maxima [B] time = 0.46, size = 481, normalized size = 3.94

$$\frac{\frac{2\left(a^3-3\,ab^2\right)\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^6+3\,a^4b^2+3\,a^2b^4+b^6}+\frac{\left(3\,a^2b-b^3\right)\log\left(-a-\frac{2\,b\sin(dx+c)}{\cos(dx+c)+1}+\frac{a\sin(dx+c)^2}{(\cos(dx+c)+1)^2}\right)}{a^6+3\,a^4b^2+3\,a^2b^4+b^6}-\frac{\left(3\,a^2b-b^3\right)\log\left(\frac{\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+1\right)}{a^6+3\,a^4b^2+3\,a^2b^4+b^6}+\frac{}{a^8+2\,a^6b^2+a^4b^4+\frac{4\left(a^7b\cdots\right.}{}}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3/(a*cos(d*x+c)+b*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] (2*(a^3 - 3*a*b^2)*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/(a^6 + 3*a^4*b^2
 + 3*a^2*b^4 + b^6) + (3*a^2*b - b^3)*log(-a - 2*b*sin(d*x + c)/(cos(d*x +
c) + 1) + a*sin(d*x + c)^2/(cos(d*x + c) + 1)^2)/(a^6 + 3*a^4*b^2 + 3*a^2*b
^4 + b^6) - (3*a^2*b - b^3)*log(sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 1)/(a
^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6) + 2*((3*a^3*b^2 + a*b^4)*sin(d*x + c)/(co
s(d*x + c) + 1) + (5*a^2*b^3 + b^5)*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 - (
3*a^3*b^2 + a*b^4)*sin(d*x + c)^3/(cos(d*x + c) + 1)^3)/(a^8 + 2*a^6*b^2 +
a^4*b^4 + 4*(a^7*b + 2*a^5*b^3 + a^3*b^5)*sin(d*x + c)/(cos(d*x + c) + 1) -
 2*(a^8 - 3*a^4*b^4 - 2*a^2*b^6)*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 - 4*(a
^7*b + 2*a^5*b^3 + a^3*b^5)*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + (a^8 + 2*
a^6*b^2 + a^4*b^4)*sin(d*x + c)^4/(cos(d*x + c) + 1)^4))/d
```

mupad [B] time = 8.54, size = 6190, normalized size = 50.74

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^3/(a*cos(c + d*x) + b*sin(c + d*x))^3,x)
```

```
[Out] ((2*tan(c/2 + (d*x)/2)^2*(b^5 + 5*a^2*b^3))/(a^2*(a^4 + b^4 + 2*a^2*b^2)) +
 (2*b*tan(c/2 + (d*x)/2)*(3*a^2*b + b^3))/(a*(a^4 + b^4 + 2*a^2*b^2)) - (2*
b*tan(c/2 + (d*x)/2)^3*(3*a^2*b + b^3))/(a*(a^4 + b^4 + 2*a^2*b^2)))/(d*(a^
2*tan(c/2 + (d*x)/2)^4 - tan(c/2 + (d*x)/2)^2*(2*a^2 - 4*b^2) + a^2 - 4*a*b
*tan(c/2 + (d*x)/2)^3 + 4*a*b*tan(c/2 + (d*x)/2))) - (log((((-(a^2*(a^2 - 3
```

*b^2)^2)/(a^2 + b^2)^6)^(1/2) + (3*a^2*b - b^3)/(a^2 + b^2)^3)*(((-(a^2*(a^
2 - 3*b^2)^2)/(a^2 + b^2)^6)^(1/2) + (3*a^2*b - b^3)/(a^2 + b^2)^3)*((32*a*
b*tan(c/2 + (d*x)/2)*(b^4 - 8*a^4 + 5*a^2*b^2))/(a^2 + b^2)^2 - (32*a^2*(a^
4 + 4*b^4 - 7*a^2*b^2))/(a^2 + b^2)^2 + 96*a*b*(a + b*tan(c/2 + (d*x)/2))*(
(-(a^2*(a^2 - 3*b^2)^2)/(a^2 + b^2)^6)^(1/2) + (3*a^2*b - b^3)/(a^2 + b^2)^
3)*(a^2 + b^2)) - (32*a^2*b*(5*a^2 - 3*b^2))/(a^2 + b^2)^3 + (32*a*tan(c/2
+ (d*x)/2)*(a^6 - 3*b^6 + 27*a^2*b^4 - 17*a^4*b^2))/(a^2 + b^2)^4) - (64*a^
2*b^2*(3*a^4 + b^4 - 4*a^2*b^2))/(a^2 + b^2)^6 + (32*a*b*tan(c/2 + (d*x)/2)
*(3*a^6 - b^6 - 3*a^2*b^4 + 17*a^4*b^2))/(a^2 + b^2)^6)*(((-(a^2*(a^2 - 3*b
^2)^2)/(a^2 + b^2)^6)^(1/2) - (3*a^2*b - b^3)/(a^2 + b^2)^3)*(((-(a^2*(a^2
- 3*b^2)^2)/(a^2 + b^2)^6)^(1/2) - (3*a^2*b - b^3)/(a^2 + b^2)^3)*((32*a^2*
(a^4 + 4*b^4 - 7*a^2*b^2))/(a^2 + b^2)^2 - (32*a*b*tan(c/2 + (d*x)/2)*(b^4
- 8*a^4 + 5*a^2*b^2))/(a^2 + b^2)^2 + 96*a*b*(a + b*tan(c/2 + (d*x)/2))*((-
(a^2*(a^2 - 3*b^2)^2)/(a^2 + b^2)^6)^(1/2) - (3*a^2*b - b^3)/(a^2 + b^2)^3)
*(a^2 + b^2)) - (32*a^2*b*(5*a^2 - 3*b^2))/(a^2 + b^2)^3 + (32*a*tan(c/2 +
(d*x)/2)*(a^6 - 3*b^6 + 27*a^2*b^4 - 17*a^4*b^2))/(a^2 + b^2)^4) + (64*a^2*
b^2*(3*a^4 + b^4 - 4*a^2*b^2))/(a^2 + b^2)^6 - (32*a*b*tan(c/2 + (d*x)/2)*(
3*a^6 - b^6 - 3*a^2*b^4 + 17*a^4*b^2))/(a^2 + b^2)^6))*(6*a^2*b - 2*b^3))/(
2*d*(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2)) + (log(a + 2*b*tan(c/2 + (d*x)/2)
- a*tan(c/2 + (d*x)/2)^2)*(3*a^2*b - b^3))/(d*(a^6 + b^6 + 3*a^2*b^4 + 3*a^
4*b^2)) - (2*a*atan((tan(c/2 + (d*x)/2)*((((a*(a^2 - 3*b^2)*((32*(3*a*b^10
- a^11 - 21*a^3*b^8 - 34*a^5*b^6 + 6*a^7*b^4 + 15*a^9*b^2))/(a^12 + b^12 +
6*a^2*b^10 + 15*a^4*b^8 + 20*a^6*b^6 + 15*a^8*b^4 + 6*a^10*b^2) - ((6*a^2*b
 - 2*b^3)*((32*(a*b^13 - 8*a^13*b + 9*a^3*b^11 + 18*a^5*b^9 + 2*a^7*b^7 - 2
7*a^9*b^5 - 27*a^11*b^3))/(a^12 + b^12 + 6*a^2*b^10 + 15*a^4*b^8 + 20*a^6*b
^6 + 15*a^8*b^4 + 6*a^10*b^2) + (16*(6*a^2*b - 2*b^3)*(3*a*b^16 + 21*a^3*b^
14 + 63*a^5*b^12 + 105*a^7*b^10 + 105*a^9*b^8 + 63*a^11*b^6 + 21*a^13*b^4 +
 3*a^15*b^2))/((a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2)*(a^12 + b^12 + 6*a^2*b^1
0 + 15*a^4*b^8 + 20*a^6*b^6 + 15*a^8*b^4 + 6*a^10*b^2))))/(2*(a^6 + b^6 + 3
*a^2*b^4 + 3*a^4*b^2))))/(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2) - ((6*a^2*b -
2*b^3)*((a*(a^2 - 3*b^2)*((32*(a*b^13 - 8*a^13*b + 9*a^3*b^11 + 18*a^5*b^9
+ 2*a^7*b^7 - 27*a^9*b^5 - 27*a^11*b^3))/(a^12 + b^12 + 6*a^2*b^10 + 15*a^4
*b^8 + 20*a^6*b^6 + 15*a^8*b^4 + 6*a^10*b^2) + (16*(6*a^2*b - 2*b^3)*(3*a*b
^16 + 21*a^3*b^14 + 63*a^5*b^12 + 105*a^7*b^10 + 105*a^9*b^8 + 63*a^11*b^6
+ 21*a^13*b^4 + 3*a^15*b^2))/((a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2)*(a^12 + b
^12 + 6*a^2*b^10 + 15*a^4*b^8 + 20*a^6*b^6 + 15*a^8*b^4 + 6*a^10*b^2))))/(a
^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2) + (16*a*(6*a^2*b - 2*b^3)*(a^2 - 3*b^2)*(
3*a*b^16 + 21*a^3*b^14 + 63*a^5*b^12 + 105*a^7*b^10 + 105*a^9*b^8 + 63*a^11
*b^6 + 21*a^13*b^4 + 3*a^15*b^2))/((a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2)^2*(a
^12 + b^12 + 6*a^2*b^10 + 15*a^4*b^8 + 20*a^6*b^6 + 15*a^8*b^4 + 6*a^10*b^2
))))/(2*(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2)) + (32*a^3*(a^2 - 3*b^2)^3*(3*a
*b^16 + 21*a^3*b^14 + 63*a^5*b^12 + 105*a^7*b^10 + 105*a^9*b^8 + 63*a^11*b^
6 + 21*a^13*b^4 + 3*a^15*b^2))/((a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2)^3*(a^12
 + b^12 + 6*a^2*b^10 + 15*a^4*b^8 + 20*a^6*b^6 + 15*a^8*b^4 + 6*a^10*b^2)))
*(a^8 + 4*b^8 - 61*a^2*b^6 + 155*a^4*b^4 - 67*a^6*b^2))/(a^8 + 4*b^8 - 11*a

```
^2*b^6 + 15*a^4*b^4 + 31*a^6*b^2)^2 + (2*a*b*(7*a^6 - 10*b^6 + 59*a^2*b^4 -
 68*a^4*b^2)*((32*(a*b^7 - 3*a^7*b + 3*a^3*b^5 - 17*a^5*b^3))/(a^12 + b^12
+ 6*a^2*b^10 + 15*a^4*b^8 + 20*a^6*b^6 + 15*a^8*b^4 + 6*a^10*b^2) + ((6*a^2
*b - 2*b^3)*((32*(3*a*b^10 - a^11 - 21*a^3*b^8 - 34*a^5*b^6 + 6*a^7*b^4 + 1
5*a^9*b^2))/(a^12 + b^12 + 6*a^2*b^10 + 15*a^4*b^8 + 20*a^6*b^6 + 15*a^8*b^
4 + 6*a^10*b^2) - ((6*a^2*b - 2*b^3)*((32*(a*b^13 - 8*a^13*b + 9*a^3*b^11 +
 18*a^5*b^9 + 2*a^7*b^7 - 27*a^9*b^5 - 27*a^11*b^3))/(a^12 + b^12 + 6*a^2*b
^10 + 15*a^4*b^8 + 20*a^6*b^6 + 15*a^8*b^4 + 6*a^10*b^2) + (16*(6*a^2*b - 2
*b^3)*(3*a*b^16 + 21*a^3*b^14 + 63*a^5*b^12 + 105*a^7*b^10 + 105*a^9*b^8 +
63*a^11*b^6 + 21*a^13*b^4 + 3*a^15*b^2))/((a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^
2)*(a^12 + b^12 + 6*a^2*b^10 + 15*a^4*b^8 + 20*a^6*b^6 + 15*a^8*b^4 + 6*a^1
0*b^2))))/(2*(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2))))/(2*(a^6 + b^6 + 3*a^2*b
^4 + 3*a^4*b^2)) + (a*(a^2 - 3*b^2)*((a*(a^2 - 3*b^2)*((32*(a*b^13 - 8*a^13
*b + 9*a^3*b^11 + 18*a^5*b^9 + 2*a^7*b^7 - 27*a^9*b^5 - 27*a^11*b^3))/(a^12
 + b^12 + 6*a^2*b^10 + 15*a^4*b^8 + 20*a^6*b^6 + 15*a^8*b^4 + 6*a^10*b^2) +
 (16*(6*a^2*b - 2*b^3)*(3*a*b^16 + 21*a^3*b^14 + 63*a^5*b^12 + 105*a^7*b^10
 + 105*a^9*b^8 + 63*a^11*b^6 + 21*a^13*b^4 + 3*a^15*b^2))/((a^6 + b^6 + 3*a
^2*b^4 + 3*a^4*b^2)*(a^12 + b^12 + 6*a^2*b^10 + 15*a^4*b^8 + 20*a^6*b^6 + 1
5*a^8*b^4 + 6*a^10*b^2))))/(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2) + (16*a*(6*a
^2*b - 2*b^3)*(a^2 - 3*b^2)*(3*a*b^16 + 21*a^3*b^14 + 63*a^5*b^12 + 105*a^7
*b^10 + 105*a^9*b^8 + 63*a^11*b^6 + 21*a^13*b^4 + 3*a^15*b^2))/((a^6 + b^6
+ 3*a^2*b^4 + 3*a^4*b^2)^2*(a^12 + b^12 + 6*a^2*b^10 + 15*a^4*b^8 + 20*a^6*
b^6 + 15*a^8*b^4 + 6*a^10*b^2))))/(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2) + (16
*a^2*(6*a^2*b - 2*b^3)*(a^2 - 3*b^2)^2*(3*a*b^16 + 21*a^3*b^14 + 63*a^5*b^1
2 + 105*a^7*b^10 + 105*a^9*b^8 + 63*a^11*b^6 + 21*a^13*b^4 + 3*a^15*b^2))/(
(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2)^3*(a^12 + b^12 + 6*a^2*b^10 + 15*a^4*b^
8 + 20*a^6*b^6 + 15*a^8*b^4 + 6*a^10*b^2))))/(a^8 + 4*b^8 - 11*a^2*b^6 + 15
*a^4*b^4 + 31*a^6*b^2)^2)*(a^16 + b^16 + 8*a^2*b^14 + 28*a^4*b^12 + 56*a^6*
b^10 + 70*a^8*b^8 + 56*a^10*b^6 + 28*a^12*b^4 + 8*a^14*b^2))/(32*a^4 - 96*a
^2*b^2) + (((a*(a^2 - 3*b^2)*((32*(5*a^10*b - 3*a^2*b^9 - 4*a^4*b^7 + 6*a^6
*b^5 + 12*a^8*b^3))/(a^12 + b^12 + 6*a^2*b^10 + 15*a^4*b^8 + 20*a^6*b^6 + 1
5*a^8*b^4 + 6*a^10*b^2) - ((6*a^2*b - 2*b^3)*((32*(3*a^6*b^8 - 4*a^2*b^12 -
 9*a^4*b^10 - a^14 + 22*a^8*b^6 + 18*a^10*b^4 + 3*a^12*b^2))/(a^12 + b^12 +
 6*a^2*b^10 + 15*a^4*b^8 + 20*a^6*b^6 + 15*a^8*b^4 + 6*a^10*b^2) + (16*(6*a
^2*b - 2*b^3)*(3*a^16*b + 3*a^2*b^15 + 21*a^4*b^13 + 63*a^6*b^11 + 105*a^8*
b^9 + 105*a^10*b^7 + 63*a^12*b^5 + 21*a^14*b^3))/((a^6 + b^6 + 3*a^2*b^4 +
3*a^4*b^2)*(a^12 + b^12 + 6*a^2*b^10 + 15*a^4*b^8 + 20*a^6*b^6 + 15*a^8*b^4
 + 6*a^10*b^2))))/(2*(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2))))/(a^6 + b^6 + 3*
a^2*b^4 + 3*a^4*b^2) - ((6*a^2*b - 2*b^3)*((a*(a^2 - 3*b^2)*((32*(3*a^6*b^8
 - 4*a^2*b^12 - 9*a^4*b^10 - a^14 + 22*a^8*b^6 + 18*a^10*b^4 + 3*a^12*b^2))
/(a^12 + b^12 + 6*a^2*b^10 + 15*a^4*b^8 + 20*a^6*b^6 + 15*a^8*b^4 + 6*a^10*
b^2) + (16*(6*a^2*b - 2*b^3)*(3*a^16*b + 3*a^2*b^15 + 21*a^4*b^13 + 63*a^6*
b^11 + 105*a^8*b^9 + 105*a^10*b^7 + 63*a^12*b^5 + 21*a^14*b^3))/((a^6 + b^6
 + 3*a^2*b^4 + 3*a^4*b^2)*(a^12 + b^12 + 6*a^2*b^10 + 15*a^4*b^8 + 20*a^6*b
^6 + 15*a^8*b^4 + 6*a^10*b^2))))/(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2) + (16*
```

```
a*(6*a^2*b - 2*b^3)*(a^2 - 3*b^2)*(3*a^16*b + 3*a^2*b^15 + 21*a^4*b^13 + 63
*a^6*b^11 + 105*a^8*b^9 + 105*a^10*b^7 + 63*a^12*b^5 + 21*a^14*b^3))/((a^6
+ b^6 + 3*a^2*b^4 + 3*a^4*b^2)^2*(a^12 + b^12 + 6*a^2*b^10 + 15*a^4*b^8 + 2
0*a^6*b^6 + 15*a^8*b^4 + 6*a^10*b^2))))/(2*(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b
^2)) + (32*a^3*(a^2 - 3*b^2)^3*(3*a^16*b + 3*a^2*b^15 + 21*a^4*b^13 + 63*a^
6*b^11 + 105*a^8*b^9 + 105*a^10*b^7 + 63*a^12*b^5 + 21*a^14*b^3))/((a^6 + b
^6 + 3*a^2*b^4 + 3*a^4*b^2)^3*(a^12 + b^12 + 6*a^2*b^10 + 15*a^4*b^8 + 20*a
^6*b^6 + 15*a^8*b^4 + 6*a^10*b^2)))*(a^8 + 4*b^8 - 61*a^2*b^6 + 155*a^4*b^4
 - 67*a^6*b^2)*(a^16 + b^16 + 8*a^2*b^14 + 28*a^4*b^12 + 56*a^6*b^10 + 70*a
^8*b^8 + 56*a^10*b^6 + 28*a^12*b^4 + 8*a^14*b^2))/((32*a^4 - 96*a^2*b^2)*(a
^8 + 4*b^8 - 11*a^2*b^6 + 15*a^4*b^4 + 31*a^6*b^2)^2) + (2*a*b*((32*(2*a^2*
b^6 - 8*a^4*b^4 + 6*a^6*b^2))/(a^12 + b^12 + 6*a^2*b^10 + 15*a^4*b^8 + 20*a
^6*b^6 + 15*a^8*b^4 + 6*a^10*b^2) + ((6*a^2*b - 2*b^3)*((32*(5*a^10*b - 3*a
^2*b^9 - 4*a^4*b^7 + 6*a^6*b^5 + 12*a^8*b^3))/(a^12 + b^12 + 6*a^2*b^10 + 1
5*a^4*b^8 + 20*a^6*b^6 + 15*a^8*b^4 + 6*a^10*b^2) - ((6*a^2*b - 2*b^3)*((32
*(3*a^6*b^8 - 4*a^2*b^12 - 9*a^4*b^10 - a^14 + 22*a^8*b^6 + 18*a^10*b^4 + 3
*a^12*b^2))/(a^12 + b^12 + 6*a^2*b^10 + 15*a^4*b^8 + 20*a^6*b^6 + 15*a^8*b^
4 + 6*a^10*b^2) + (16*(6*a^2*b - 2*b^3)*(3*a^16*b + 3*a^2*b^15 + 21*a^4*b^1
3 + 63*a^6*b^11 + 105*a^8*b^9 + 105*a^10*b^7 + 63*a^12*b^5 + 21*a^14*b^3))/
((a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2)*(a^12 + b^12 + 6*a^2*b^10 + 15*a^4*b^8
 + 20*a^6*b^6 + 15*a^8*b^4 + 6*a^10*b^2))))/(2*(a^6 + b^6 + 3*a^2*b^4 + 3*a
^4*b^2))))/(2*(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2)) + (a*(a^2 - 3*b^2)*((a*(
a^2 - 3*b^2)*((32*(3*a^6*b^8 - 4*a^2*b^12 - 9*a^4*b^10 - a^14 + 22*a^8*b^6
+ 18*a^10*b^4 + 3*a^12*b^2))/(a^12 + b^12 + 6*a^2*b^10 + 15*a^4*b^8 + 20*a^
6*b^6 + 15*a^8*b^4 + 6*a^10*b^2) + (16*(6*a^2*b - 2*b^3)*(3*a^16*b + 3*a^2*
b^15 + 21*a^4*b^13 + 63*a^6*b^11 + 105*a^8*b^9 + 105*a^10*b^7 + 63*a^12*b^5
 + 21*a^14*b^3))/((a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2)*(a^12 + b^12 + 6*a^2*
b^10 + 15*a^4*b^8 + 20*a^6*b^6 + 15*a^8*b^4 + 6*a^10*b^2))))/(a^6 + b^6 + 3
*a^2*b^4 + 3*a^4*b^2) + (16*a*(6*a^2*b - 2*b^3)*(a^2 - 3*b^2)*(3*a^16*b + 3
*a^2*b^15 + 21*a^4*b^13 + 63*a^6*b^11 + 105*a^8*b^9 + 105*a^10*b^7 + 63*a^1
2*b^5 + 21*a^14*b^3))/((a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2)^2*(a^12 + b^12 +
 6*a^2*b^10 + 15*a^4*b^8 + 20*a^6*b^6 + 15*a^8*b^4 + 6*a^10*b^2))))/(a^6 +
b^6 + 3*a^2*b^4 + 3*a^4*b^2) + (16*a^2*(6*a^2*b - 2*b^3)*(a^2 - 3*b^2)^2*(3
*a^16*b + 3*a^2*b^15 + 21*a^4*b^13 + 63*a^6*b^11 + 105*a^8*b^9 + 105*a^10*b
^7 + 63*a^12*b^5 + 21*a^14*b^3))/((a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2)^3*(a^
12 + b^12 + 6*a^2*b^10 + 15*a^4*b^8 + 20*a^6*b^6 + 15*a^8*b^4 + 6*a^10*b^2)
))*(7*a^6 - 10*b^6 + 59*a^2*b^4 - 68*a^4*b^2)*(a^16 + b^16 + 8*a^2*b^14 + 2
8*a^4*b^12 + 56*a^6*b^10 + 70*a^8*b^8 + 56*a^10*b^6 + 28*a^12*b^4 + 8*a^14*
b^2))/((32*a^4 - 96*a^2*b^2)*(a^8 + 4*b^8 - 11*a^2*b^6 + 15*a^4*b^4 + 31*a^
6*b^2)^2))*(a^2 - 3*b^2))/(d*(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2))
```

sympy [**F(-2)**] time = 0.00, size = 0, normalized size = 0.00

Exception raised: AttributeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3/(a*cos(d*x+c)+b*sin(d*x+c))**3,x)
```

```
[Out] Exception raised: AttributeError
```

3.133 $\int \frac{\cos^2(c+dx)}{(a\cos(c+dx)+b\sin(c+dx))^3}\,dx$

Optimal. Leaf size=119

$$\frac{(2a^2-b^2)\tanh^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)-b}{\sqrt{a^2+b^2}}\right)}{d\left(a^2+b^2\right)^{5/2}}-\frac{b\left(\left(4a^2+b^2\right)\cos(c+dx)+3ab\sin(c+dx)\right)}{2d\left(a^2+b^2\right)^2(a\cos(c+dx)+b\sin(c+dx))^2}$$

```
[Out] (2*a^2-b^2)*arctanh((-b+a*tan(1/2*d*x+1/2*c))/(a^2+b^2)^(1/2))/(a^2+b^2)^(5
/2)/d-1/2*b*((4*a^2+b^2)*cos(d*x+c)+3*a*b*sin(d*x+c))/(a^2+b^2)^2/d/(a*cos(
d*x+c)+b*sin(d*x+c))^2
```

Rubi [A] time = 0.59, antiderivative size = 225, normalized size of antiderivative = 1.89, number of steps used = 6, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {1660, 12, 618, 206}

$$\frac{2b^2\left(\left(a^2+2b^2\right)\tan\left(\frac{1}{2}(c+dx)\right)+ab\right)}{a^3d\left(a^2+b^2\right)\left(-a\tan^2\left(\frac{1}{2}(c+dx)\right)+a+2b\tan\left(\frac{1}{2}(c+dx)\right)\right)^2}-\frac{b\left(ab\left(5a^2+2b^2\right)\tan\left(\frac{1}{2}(c+dx)\right)+3a^2b^2+4\right.}{a^3d\left(a^2+b^2\right)^2\left(-a\tan^2\left(\frac{1}{2}(c+dx)\right)+a+2b\tan\right.}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2/(a*Cos[c + d*x] + b*Sin[c + d*x])^3,x]
```

```
[Out] -(((2*a^2 - b^2)*ArcTanh[(b - a*Tan[(c + d*x)/2])/Sqrt[a^2 + b^2]])/((a^2 +
 b^2)^(5/2)*d)) + (2*b^2*(a*b + (a^2 + 2*b^2)*Tan[(c + d*x)/2]))/(a^3*(a^2
+ b^2)*d*(a + 2*b*Tan[(c + d*x)/2] - a*Tan[(c + d*x)/2]^2)^2) - (b*(4*a^4 +
 3*a^2*b^2 + 2*b^4 + a*b*(5*a^2 + 2*b^2)*Tan[(c + d*x)/2]))/(a^3*(a^2 + b^2
)^2*d*(a + 2*b*Tan[(c + d*x)/2] - a*Tan[(c + d*x)/2]^2))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1660

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[((b*f - 2*a*g + (2*c*f - b*g)*x)*(a + b*x + c*x^2)^
(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*(
2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2
 - 4*a*c, 0] && LtQ[p, -1]
```

Rubi steps

$$\int \frac{\cos^2(c+dx)}{(a\cos(c+dx)+b\sin(c+dx))^3}\,dx = \frac{2\,\mathrm{Subst}\left(\int \frac{\left(1-x^2\right)^2}{\left(a+2bx-ax^2\right)^3}\,dx, x, \tan\left(\frac{1}{2}(c+dx)\right)\right)}{d}$$

$$= \frac{2b^2\left(ab+\left(a^2+2b^2\right)\tan\left(\frac{1}{2}(c+dx)\right)\right)}{a^3\left(a^2+b^2\right)d\left(a+2b\tan\left(\frac{1}{2}(c+dx)\right)-a\tan^2\left(\frac{1}{2}(c+dx)\right)\right)^2} - \frac{\mathrm{Subst}\left(\right.}{}$$

$$= \frac{2b^2\left(ab+\left(a^2+2b^2\right)\tan\left(\frac{1}{2}(c+dx)\right)\right)}{a^3\left(a^2+b^2\right)d\left(a+2b\tan\left(\frac{1}{2}(c+dx)\right)-a\tan^2\left(\frac{1}{2}(c+dx)\right)\right)^2} - \frac{b\left(4a\right.}{a^3\left(a^2+\right.}$$

$$= \frac{2b^2\left(ab+\left(a^2+2b^2\right)\tan\left(\frac{1}{2}(c+dx)\right)\right)}{a^3\left(a^2+b^2\right)d\left(a+2b\tan\left(\frac{1}{2}(c+dx)\right)-a\tan^2\left(\frac{1}{2}(c+dx)\right)\right)^2} - \frac{b\left(4a\right.}{a^3\left(a^2+\right.}$$

$$= \frac{2b^2\left(ab+\left(a^2+2b^2\right)\tan\left(\frac{1}{2}(c+dx)\right)\right)}{a^3\left(a^2+b^2\right)d\left(a+2b\tan\left(\frac{1}{2}(c+dx)\right)-a\tan^2\left(\frac{1}{2}(c+dx)\right)\right)^2} - \frac{b\left(4a\right.}{a^3\left(a^2+\right.}$$

$$= -\frac{\left(2a^2-b^2\right)\tanh^{-1}\left(\frac{b-a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2+b^2}}\right)}{\left(a^2+b^2\right)^{5/2}d} + \frac{2b^2\left(ab+\left(a^2+2b^2\right)\right.}{a^3\left(a^2+b^2\right)d\left(a+2b\tan\left(\frac{1}{2}(c+\right.\right.}$$

Mathematica [A] time = 0.71, size = 119, normalized size = 1.00

$$\frac{\frac{2\left(2a^2-b^2\right)\tanh^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)-b}{\sqrt{a^2+b^2}}\right)}{\left(a^2+b^2\right)^{5/2}} - \frac{b\left(\left(4a^2+b^2\right)\cos(c+dx)+3ab\sin(c+dx)\right)}{\left(a^2+b^2\right)^2(a\cos(c+dx)+b\sin(c+dx))^2}}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2/(a*Cos[c + d*x] + b*Sin[c + d*x])^3,x]
```

```
[Out] ((2*(2*a^2 - b^2)*ArcTanh[(-b + a*Tan[(c + d*x)/2])/Sqrt[a^2 + b^2]])/(a^2
+ b^2)^(5/2) - (b*((4*a^2 + b^2)*Cos[c + d*x] + 3*a*b*Sin[c + d*x]))/((a^2
+ b^2)^2*(a*Cos[c + d*x] + b*Sin[c + d*x])^2))/(2*d)
```

fricas [B] time = 0.67, size = 352, normalized size = 2.96

$$-\frac{\left(2\,a^{2}b^{2}-b^{4}+\left(2\,a^{4}-3\,a^{2}b^{2}+b^{4}\right)\cos\left(dx+c\right)^{2}+2\left(2\,a^{3}b-ab^{3}\right)\cos\left(dx+c\right)\sin\left(dx+c\right)\right)\sqrt{a^{2}+b^{2}}\,\log\left(\frac{2\,ab\cos\cdots}{\cdots}\right.}{4\left(\left(a^{8}+2\,a^{6}b^{2}-2\,a^{2}b^{6}-b^{8}\right)d\cos\left(dx+c\right)^{2}+2\left(a^{7}b+3\,a\cdots\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(a*cos(d*x+c)+b*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] -1/4*((2*a^2*b^2 - b^4 + (2*a^4 - 3*a^2*b^2 + b^4)*cos(d*x + c)^2 + 2*(2*a^
3*b - a*b^3)*cos(d*x + c)*sin(d*x + c))*sqrt(a^2 + b^2)*log((2*a*b*cos(d*x
+ c)*sin(d*x + c) + (a^2 - b^2)*cos(d*x + c)^2 - 2*a^2 - b^2 - 2*sqrt(a^2 +
 b^2)*(b*cos(d*x + c) - a*sin(d*x + c)))/(2*a*b*cos(d*x + c)*sin(d*x + c) +
 (a^2 - b^2)*cos(d*x + c)^2 + b^2)) + 2*(4*a^4*b + 5*a^2*b^3 + b^5)*cos(d*x
 + c) + 6*(a^3*b^2 + a*b^4)*sin(d*x + c))/((a^8 + 2*a^6*b^2 - 2*a^2*b^6 - b
^8)*d*cos(d*x + c)^2 + 2*(a^7*b + 3*a^5*b^3 + 3*a^3*b^5 + a*b^7)*d*cos(d*x
+ c)*sin(d*x + c) + (a^6*b^2 + 3*a^4*b^4 + 3*a^2*b^6 + b^8)*d)
```

giac [B] time = 2.19, size = 293, normalized size = 2.46

$$-\frac{\frac{\left(2\,a^{2}-b^{2}\right)\log\left(\frac{\left|2\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-2\,b-2\,\sqrt{a^{2}+b^{2}}\right|}{\left|2\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-2\,b+2\,\sqrt{a^{2}+b^{2}}\right|}\right)}{\left(a^{4}+2\,a^{2}b^{2}+b^{4}\right)\sqrt{a^{2}+b^{2}}}-\frac{2\left(5\,a^{3}b^{2}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}+2\,ab^{4}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}+4\,a^{4}b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-7\,a^{2}b^{3}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-2\cdots\right.}{\left(a^{6}+2\,a^{4}b^{2}+a^{2}b^{4}\right)\left(a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-2\,b\cdots\right.}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(a*cos(d*x+c)+b*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -1/2*((2*a^2 - b^2)*log(abs(2*a*tan(1/2*d*x + 1/2*c) - 2*b - 2*sqrt(a^2 + b
^2))/abs(2*a*tan(1/2*d*x + 1/2*c) - 2*b + 2*sqrt(a^2 + b^2)))/((a^4 + 2*a^2
*b^2 + b^4)*sqrt(a^2 + b^2)) - 2*(5*a^3*b^2*tan(1/2*d*x + 1/2*c)^3 + 2*a*b^
4*tan(1/2*d*x + 1/2*c)^3 + 4*a^4*b*tan(1/2*d*x + 1/2*c)^2 - 7*a^2*b^3*tan(1
/2*d*x + 1/2*c)^2 - 2*b^5*tan(1/2*d*x + 1/2*c)^2 - 11*a^3*b^2*tan(1/2*d*x +
 1/2*c) - 2*a*b^4*tan(1/2*d*x + 1/2*c) - 4*a^4*b - a^2*b^3)/((a^6 + 2*a^4*b
^2 + a^2*b^4)*(a*tan(1/2*d*x + 1/2*c)^2 - 2*b*tan(1/2*d*x + 1/2*c) - a)^2))
/d
```

maple [B] time = 0.25, size = 280, normalized size = 2.35

$$-\frac{\frac{2\left(-\frac{b^{2}\left(5a^{2}+2b^{2}\right)\left(\tan^{3}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2a\left(a^{4}+2a^{2}b^{2}+b^{4}\right)}-\frac{b\left(4a^{4}-7a^{2}b^{2}-2b^{4}\right)\left(\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2\left(a^{4}+2a^{2}b^{2}+b^{4}\right)a^{2}}+\frac{b^{2}\left(11a^{2}+2b^{2}\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2\left(a^{4}+2a^{2}b^{2}+b^{4}\right)a}+\frac{b\left(4a^{2}+b^{2}\right)}{2a^{4}+4a^{2}b^{2}+2b^{4}}\right)}{\left(\left(\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a-2b\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-a\right)^{2}}+\frac{\left(2a^{2}-b^{2}\right)\operatorname{arctanh}\left(\frac{2a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-2b}{2\sqrt{a^{2}+b^{2}}}\right)}{\left(a^{4}+2a^{2}b^{2}+b^{4}\right)\sqrt{a^{2}+b^{2}}}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2/(a*cos(d*x+c)+b*sin(d*x+c))^3,x)
```

```
[Out] 1/d*(-2*(-1/2*b^2*(5*a^2+2*b^2)/a/(a^4+2*a^2*b^2+b^4)*tan(1/2*d*x+1/2*c)^3-
1/2*b*(4*a^4-7*a^2*b^2-2*b^4)/(a^4+2*a^2*b^2+b^4)/a^2*tan(1/2*d*x+1/2*c)^2+
1/2*b^2*(11*a^2+2*b^2)/(a^4+2*a^2*b^2+b^4)/a*tan(1/2*d*x+1/2*c)+1/2*b*(4*a^
2+b^2)/(a^4+2*a^2*b^2+b^4))/(tan(1/2*d*x+1/2*c)^2*a-2*b*tan(1/2*d*x+1/2*c)-
a)^2+(2*a^2-b^2)/(a^4+2*a^2*b^2+b^4)/(a^2+b^2)^(1/2)*arctanh(1/2*(2*a*tan(1
/2*d*x+1/2*c)-2*b)/(a^2+b^2)^(1/2)))
```

maxima [B] time = 0.44, size = 412, normalized size = 3.46

$$-\frac{\frac{\left(2\,a^2-b^2\right)\log\left(\frac{b-\frac{a\sin(dx+c)}{\cos(dx+c)+1}+\sqrt{a^2+b^2}}{b-\frac{a\sin(dx+c)}{\cos(dx+c)+1}-\sqrt{a^2+b^2}}\right)}{\left(a^4+2\,a^2b^2+b^4\right)\sqrt{a^2+b^2}}+\frac{2\left(4\,a^4b+a^2b^3+\frac{\left(11\,a^3b^2+2\,ab^4\right)\sin(dx+c)}{\cos(dx+c)+1}-\frac{\left(4\,a^4b-7\,a^2b^3-2\,b^5\right)\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-\frac{\left(5\,a^3b^2+2\,ab^4\right)\sin \cdots}{(\cos(dx+c)+\cdots}\right.}{a^8+2\,a^6b^2+a^4b^4+\frac{4\left(a^7b+2\,a^5b^3+a^3b^5\right)\sin(dx+c)}{\cos(dx+c)+1}-\frac{2\left(a^8-3\,a^4b^4-2\,a^2b^6\right)\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-\frac{4\left(a^7b+2\,a^5b^3+a^3b^5\right)\sin(dx+c)^3}{(\cos(dx+c)+1)^3}\cdots}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(a*cos(d*x+c)+b*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -1/2*((2*a^2 - b^2)*log((b - a*sin(d*x + c)/(cos(d*x + c) + 1) + sqrt(a^2 +
 b^2))/(b - a*sin(d*x + c)/(cos(d*x + c) + 1) - sqrt(a^2 + b^2)))/((a^4 + 2
*a^2*b^2 + b^4)*sqrt(a^2 + b^2)) + 2*(4*a^4*b + a^2*b^3 + (11*a^3*b^2 + 2*a
*b^4)*sin(d*x + c)/(cos(d*x + c) + 1) - (4*a^4*b - 7*a^2*b^3 - 2*b^5)*sin(d
*x + c)^2/(cos(d*x + c) + 1)^2 - (5*a^3*b^2 + 2*a*b^4)*sin(d*x + c)^3/(cos(
d*x + c) + 1)^3)/(a^8 + 2*a^6*b^2 + a^4*b^4 + 4*(a^7*b + 2*a^5*b^3 + a^3*b^
5)*sin(d*x + c)/(cos(d*x + c) + 1) - 2*(a^8 - 3*a^4*b^4 - 2*a^2*b^6)*sin(d*
x + c)^2/(cos(d*x + c) + 1)^2 - 4*(a^7*b + 2*a^5*b^3 + a^3*b^5)*sin(d*x + c
)^3/(cos(d*x + c) + 1)^3 + (a^8 + 2*a^6*b^2 + a^4*b^4)*sin(d*x + c)^4/(cos(
d*x + c) + 1)^4))/d
```

mupad [B] time = 1.72, size = 443, normalized size = 3.72

$$\frac{\ln\left(\left(a^2+b^2\right)^{5/2}-a^4\,b-b^5-2\,a^2\,b^3+a^5\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+a\,b^4\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+2\,a^3\,b^2\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)\left(a^2-\frac{b^2}{2}\right)}{d\left(a^2+b^2\right)^{5/2}}-\frac{\ln \cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^2/(a*cos(c + d*x) + b*sin(c + d*x))^3,x)
```

```
[Out] (log((a^2 + b^2)^(5/2) - a^4*b - b^5 - 2*a^2*b^3 + a^5*tan(c/2 + (d*x)/2) +
 a*b^4*tan(c/2 + (d*x)/2) + 2*a^3*b^2*tan(c/2 + (d*x)/2))*(a^2 - b^2/2))/(d
```

```
*(a^2 + b^2)^(5/2)) - (log((a^2 + b^2)^(5/2) + a^4*b + b^5 + 2*a^2*b^3 - a^
5*tan(c/2 + (d*x)/2) - a*b^4*tan(c/2 + (d*x)/2) - 2*a^3*b^2*tan(c/2 + (d*x)
/2))*(2*a^2 - b^2))/(2*d*(a^2 + b^2)^(5/2)) - ((4*a^2*b + b^3)/(a^4 + b^4 +
 2*a^2*b^2) - (tan(c/2 + (d*x)/2)^2*(a^2 - 2*b^2)*(4*a^2*b + b^3))/(a^2*(a^
4 + b^4 + 2*a^2*b^2)) + (b*tan(c/2 + (d*x)/2)*(11*a^2*b + 2*b^3))/(a*(a^4 +
 b^4 + 2*a^2*b^2)) - (b*tan(c/2 + (d*x)/2)^3*(5*a^2*b + 2*b^3))/(a*(a^4 + b
^4 + 2*a^2*b^2)))/(d*(a^2*tan(c/2 + (d*x)/2)^4 - tan(c/2 + (d*x)/2)^2*(2*a^
2 - 4*b^2) + a^2 - 4*a*b*tan(c/2 + (d*x)/2)^3 + 4*a*b*tan(c/2 + (d*x)/2)))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2/(a*cos(d*x+c)+b*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

3.134 $\int \frac{\cos(c+dx)}{(a\cos(c+dx)+b\sin(c+dx))^3}\,dx$

Optimal. Leaf size=22

$$-\frac{1}{2bd(a+b\tan(c+dx))^2}$$

```
[Out] -1/2/b/d/(a+b*tan(d*x+c))^2
```

Rubi [A] time = 0.03, antiderivative size = 30, normalized size of antiderivative = 1.36, number of steps used = 2, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} =$ 0.077, Rules used = {3088, 37}

$$-\frac{\cot^2(c+dx)}{2bd(a\cot(c+dx)+b)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]/(a*Cos[c + d*x] + b*Sin[c + d*x])^3,x]
```

```
[Out] -Cot[c + d*x]^2/(2*b*d*(b + a*Cot[c + d*x])^2)
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 3088

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[(x^m*(b +
 a*x)^n)/(1 + x^2)^((m + n + 2)/2), x], x, Cot[c + d*x]], x] /; FreeQ[{a, b
, c, d}, x] && IntegerQ[n] && IntegerQ[(m + n)/2] && NeQ[n, -1] &&  !(GtQ[n
, 0] && GtQ[m, 1])
```

Rubi steps

$$\begin{aligned}
\int \frac{\cos(c+dx)}{(a\cos(c+dx)+b\sin(c+dx))^3}\,dx &= -\frac{\operatorname{Subst}\left(\int \frac{x}{(b+ax)^3}\,dx, x, \cot(c+dx)\right)}{d} \\
&= -\frac{\cot^2(c+dx)}{2bd(b+a\cot(c+dx))^2}
\end{aligned}$$

Mathematica [**B**] time = 0.12, size = 57, normalized size = 2.59

$$\frac{a\sin(2(c+dx))-b\cos(2(c+dx))}{2d\left(a^2+b^2\right)(a\cos(c+dx)+b\sin(c+dx))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]/(a*Cos[c + d*x] + b*Sin[c + d*x])^3,x]
```

```
[Out] (-(b*Cos[2*(c + d*x)]) + a*Sin[2*(c + d*x)])/(2*(a^2 + b^2)*d*(a*Cos[c + d*
x] + b*Sin[c + d*x])^2)
```

fricas [**B**] time = 0.53, size = 142, normalized size = 6.45

$$-\frac{4\,a^2b\cos\left(dx+c\right)^2-a^2b+b^3-2\left(a^3-ab^2\right)\cos\left(dx+c\right)\sin\left(dx+c\right)}{2\left(\left(a^6+a^4b^2-a^2b^4-b^6\right)d\cos\left(dx+c\right)^2+2\left(a^5b+2\,a^3b^3+ab^5\right)d\cos\left(dx+c\right)\sin\left(dx+c\right)+\left(a^4b^2+2\,a^2b^4+b\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a*cos(d*x+c)+b*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] -1/2*(4*a^2*b*cos(d*x + c)^2 - a^2*b + b^3 - 2*(a^3 - a*b^2)*cos(d*x + c)*s
in(d*x + c))/((a^6 + a^4*b^2 - a^2*b^4 - b^6)*d*cos(d*x + c)^2 + 2*(a^5*b +
 2*a^3*b^3 + a*b^5)*d*cos(d*x + c)*sin(d*x + c) + (a^4*b^2 + 2*a^2*b^4 + b^
6)*d)
```

giac [**A**] time = 0.44, size = 20, normalized size = 0.91

$$-\frac{1}{2\left(b\tan\left(dx+c\right)+a\right)^2bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a*cos(d*x+c)+b*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -1/2/((b*tan(d*x + c) + a)^2*b*d)
```

maple [**A**] time = 0.23, size = 21, normalized size = 0.95

$$-\frac{1}{2bd\left(a+b\tan\left(dx+c\right)\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)/(a*cos(d*x+c)+b*sin(d*x+c))^3,x)
```

```
[Out] -1/2/b/d/(a+b*tan(d*x+c))^2
```

maxima [B] time = 0.36, size = 171, normalized size = 7.77

$$\frac{2\left(\frac{a\sin(dx+c)}{\cos(dx+c)+1}+\frac{b\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-\frac{a\sin(dx+c)^3}{(\cos(dx+c)+1)^3}\right)}{\left(a^4+\frac{4a^3b\sin(dx+c)}{\cos(dx+c)+1}-\frac{4a^3b\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{a^4\sin(dx+c)^4}{(\cos(dx+c)+1)^4}-\frac{2(a^4-2a^2b^2)\sin(dx+c)^2}{(\cos(dx+c)+1)^2}\right)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a*cos(d*x+c)+b*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 2*(a*sin(d*x + c)/(cos(d*x + c) + 1) + b*sin(d*x + c)^2/(cos(d*x + c) + 1)^
2 - a*sin(d*x + c)^3/(cos(d*x + c) + 1)^3)/((a^4 + 4*a^3*b*sin(d*x + c)/(co
s(d*x + c) + 1) - 4*a^3*b*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + a^4*sin(d*x
 + c)^4/(cos(d*x + c) + 1)^4 - 2*(a^4 - 2*a^2*b^2)*sin(d*x + c)^2/(cos(d*x
+ c) + 1)^2)*d)
```

mupad [B] time = 0.61, size = 85, normalized size = 3.86

$$-\frac{b\left(\frac{\cos(2c+2dx)}{2}-\frac{1}{2}\right)-a\sin(2c+2dx)}{a^2d\left(a^2+b^2+a^2\cos(2c+2dx)-b^2\cos(2c+2dx)+2ab\sin(2c+2dx)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)/(a*cos(c + d*x) + b*sin(c + d*x))^3,x)
```

```
[Out] -(b*(cos(2*c + 2*d*x)/2 - 1/2) - a*sin(2*c + 2*d*x))/(a^2*d*(a^2 + b^2 + a^
2*cos(2*c + 2*d*x) - b^2*cos(2*c + 2*d*x) + 2*a*b*sin(2*c + 2*d*x)))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a*cos(d*x+c)+b*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

3.135 $\int \frac{1}{(a\cos(c+dx)+b\sin(c+dx))^3}\,dx$

Optimal. Leaf size=103

$$-\frac{b\cos(c+dx)-a\sin(c+dx)}{2d\left(a^2+b^2\right)(a\cos(c+dx)+b\sin(c+dx))^2}-\frac{\tanh^{-1}\left(\frac{b\cos(c+dx)-a\sin(c+dx)}{\sqrt{a^2+b^2}}\right)}{2d\left(a^2+b^2\right)^{3/2}}$$

```
[Out] -1/2*arctanh((b*cos(d*x+c)-a*sin(d*x+c))/(a^2+b^2)^(1/2))/(a^2+b^2)^(3/2)/d
+1/2*(-b*cos(d*x+c)+a*sin(d*x+c))/(a^2+b^2)/d/(a*cos(d*x+c)+b*sin(d*x+c))^2
```

Rubi [A] time = 0.05, antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {3076, 3074, 206}

$$-\frac{b\cos(c+dx)-a\sin(c+dx)}{2d\left(a^2+b^2\right)(a\cos(c+dx)+b\sin(c+dx))^2}-\frac{\tanh^{-1}\left(\frac{b\cos(c+dx)-a\sin(c+dx)}{\sqrt{a^2+b^2}}\right)}{2d\left(a^2+b^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Cos[c + d*x] + b*Sin[c + d*x])^(-3),x]
```

```
[Out] -ArcTanh[(b*Cos[c + d*x] - a*Sin[c + d*x])/Sqrt[a^2 + b^2]]/(2*(a^2 + b^2)^
(3/2)*d) - (b*Cos[c + d*x] - a*Sin[c + d*x])/(2*(a^2 + b^2)*d*(a*Cos[c + d*
x] + b*Sin[c + d*x])^2)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3074

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x
_Symbol] :> -Dist[d^(-1), Subst[Int[1/(a^2 + b^2 - x^2), x], x, b*Cos[c + d
*x] - a*Sin[c + d*x]], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

Rule 3076

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x
_Symbol] :> Simp[((b*Cos[c + d*x] - a*Sin[c + d*x])*(a*Cos[c + d*x] + b*Sin
[c + d*x])^(n + 1))/(d*(n + 1)*(a^2 + b^2)), x] + Dist[(n + 2)/((n + 1)*(a^
2 + b^2)), Int[(a*Cos[c + d*x] + b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{
```

```
a, b, c, d}, x] && NeQ[a^2 + b^2, 0] && LtQ[n, -1] && NeQ[n, -2]
```

Rubi steps

$$\int \frac{1}{(a\cos(c+dx)+b\sin(c+dx))^3}\,dx = -\frac{b\cos(c+dx)-a\sin(c+dx)}{2\left(a^2+b^2\right)d(a\cos(c+dx)+b\sin(c+dx))^2}+\frac{\int \frac{1}{a\cos(c+dx)+b\sin(c+dx)}\,\ldots}{2\left(a^2+b^2\right)}$$

$$= -\frac{b\cos(c+dx)-a\sin(c+dx)}{2\left(a^2+b^2\right)d(a\cos(c+dx)+b\sin(c+dx))^2}-\frac{\text{Subst}\left(\int \frac{1}{a^2+b^2-x^2}\,dx,\ldots\right.}{2\ldots}$$

$$= -\frac{\tanh^{-1}\left(\frac{b\cos(c+dx)-a\sin(c+dx)}{\sqrt{a^2+b^2}}\right)}{2\left(a^2+b^2\right)^{3/2}d}-\frac{b\cos(c+dx)-a\sin(c+dx)}{2\left(a^2+b^2\right)d(a\cos(c+dx)+b\sin(c+\ldots}$$

Mathematica [C] time = 0.27, size = 132, normalized size = 1.28

$$\frac{\left(a^2+b^2\right)(a\sin(c+dx)-b\cos(c+dx))+2\sqrt{a^2+b^2}\,(a\cos(c+dx)+b\sin(c+dx))^2\tanh^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)-b}{\sqrt{a^2+b^2}}\right)}{2d(a-ib)^2(a+ib)^2(a\cos(c+dx)+b\sin(c+dx))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Cos[c + d*x] + b*Sin[c + d*x])^(-3),x]
```

```
[Out] ((a^2 + b^2)*(-(b*Cos[c + d*x]) + a*Sin[c + d*x]) + 2*Sqrt[a^2 + b^2]*ArcTa
nh[(-b + a*Tan[(c + d*x)/2])/Sqrt[a^2 + b^2]]*(a*Cos[c + d*x] + b*Sin[c + d
*x])^2)/(2*(a - I*b)^2*(a + I*b)^2*d*(a*Cos[c + d*x] + b*Sin[c + d*x])^2)
```

fricas [B] time = 0.61, size = 294, normalized size = 2.85

$$\frac{\left(2\,ab\cos(dx+c)\sin(dx+c)+\left(a^2-b^2\right)\cos(dx+c)^2+b^2\right)\sqrt{a^2+b^2}\,\log\left(-\frac{2\,ab\cos(dx+c)\sin(dx+c)+\left(a^2-b^2\right)\cos(dx+c)^2\ldots}{2\,ab\cos(dx+c)\sin(dx+c)\ldots}\right.}{4\left(\left(a^6+a^4b^2-a^2b^4-b^6\right)d\cos(dx+c)^2+2\left(a^5b+2\,a^3b^3+ab^5\right)d\,\mathrm{c}\ldots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cos(d*x+c)+b*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/4*((2*a*b*cos(d*x + c)*sin(d*x + c) + (a^2 - b^2)*cos(d*x + c)^2 + b^2)*s
qrt(a^2 + b^2)*log(-(2*a*b*cos(d*x + c)*sin(d*x + c) + (a^2 - b^2)*cos(d*x
+ c)^2 - 2*a^2 - b^2 + 2*sqrt(a^2 + b^2)*(b*cos(d*x + c) - a*sin(d*x + c)))
```

```
/(2*a*b*cos(d*x + c)*sin(d*x + c) + (a^2 - b^2)*cos(d*x + c)^2 + b^2)) - 2*
(a^2*b + b^3)*cos(d*x + c) + 2*(a^3 + a*b^2)*sin(d*x + c))/((a^6 + a^4*b^2
- a^2*b^4 - b^6)*d*cos(d*x + c)^2 + 2*(a^5*b + 2*a^3*b^3 + a*b^5)*d*cos(d*x
 + c)*sin(d*x + c) + (a^4*b^2 + 2*a^2*b^4 + b^6)*d)
```

giac [B] time = 1.59, size = 221, normalized size = 2.15

$$-\frac{\frac{\log\left(\frac{\left|2\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-2\,b-2\,\sqrt{a^2+b^2}\right|}{\left|2\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-2\,b+2\,\sqrt{a^2+b^2}\right|}\right)}{\left(a^2+b^2\right)^{\frac{3}{2}}}-\frac{2\left(a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+2\,ab^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+a^2b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-2\,b^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-\right.}{\left(a^4+a^2b^2\right)\left(a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-2\,b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-a\right)^2}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cos(d*x+c)+b*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -1/2*(log(abs(2*a*tan(1/2*d*x + 1/2*c) - 2*b - 2*sqrt(a^2 + b^2))/abs(2*a*t
an(1/2*d*x + 1/2*c) - 2*b + 2*sqrt(a^2 + b^2)))/(a^2 + b^2)^(3/2) - 2*(a^3*
tan(1/2*d*x + 1/2*c)^3 + 2*a*b^2*tan(1/2*d*x + 1/2*c)^3 + a^2*b*tan(1/2*d*x
 + 1/2*c)^2 - 2*b^3*tan(1/2*d*x + 1/2*c)^2 + a^3*tan(1/2*d*x + 1/2*c) - 2*a
*b^2*tan(1/2*d*x + 1/2*c) - a^2*b)/((a^4 + a^2*b^2)*(a*tan(1/2*d*x + 1/2*c)
^2 - 2*b*tan(1/2*d*x + 1/2*c) - a)^2))/d
```

maple [A] time = 0.26, size = 191, normalized size = 1.85

$$-\frac{\frac{2\left(-\frac{\left(a^2+2b^2\right)\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2a\left(a^2+b^2\right)}-\frac{b\left(a^2-2b^2\right)\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2\left(a^2+b^2\right)a^2}-\frac{\left(a^2-2b^2\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2a\left(a^2+b^2\right)}+\frac{b}{2a^2+2b^2}\right)}{\left(\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a-2b\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-a\right)^2}+\frac{\operatorname{arctanh}\left(\frac{2a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-2b}{2\sqrt{a^2+b^2}}\right)}{\left(a^2+b^2\right)^{\frac{3}{2}}}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*cos(d*x+c)+b*sin(d*x+c))^3,x)
```

```
[Out] 1/d*(-2*(-1/2*(a^2+2*b^2)/a/(a^2+b^2)*tan(1/2*d*x+1/2*c)^3-1/2*b*(a^2-2*b^2
)/(a^2+b^2)/a^2*tan(1/2*d*x+1/2*c)^2-1/2*(a^2-2*b^2)/a/(a^2+b^2)*tan(1/2*d*
x+1/2*c)+1/2*b/(a^2+b^2))/(tan(1/2*d*x+1/2*c)^2*a-2*b*tan(1/2*d*x+1/2*c)-a)
^2+1/(a^2+b^2)^(3/2)*arctanh(1/2*(2*a*tan(1/2*d*x+1/2*c)-2*b)/(a^2+b^2)^(1/
2)))
```

maxima [B] time = 0.46, size = 326, normalized size = 3.17

$$-\frac{\frac{2\left(a^2b-\frac{\left(a^3-2\,ab^2\right)\sin(dx+c)}{\cos(dx+c)+1}-\frac{\left(a^2b-2\,b^3\right)\sin(dx+c)^2}{\left(\cos(dx+c)+1\right)^2}-\frac{\left(a^3+2\,ab^2\right)\sin(dx+c)^3}{\left(\cos(dx+c)+1\right)^3}\right)}{a^6+a^4b^2+\frac{4\left(a^5b+a^3b^3\right)\sin(dx+c)}{\cos(dx+c)+1}-\frac{2\left(a^6-a^4b^2-2\,a^2b^4\right)\sin(dx+c)^2}{\left(\cos(dx+c)+1\right)^2}-\frac{4\left(a^5b+a^3b^3\right)\sin(dx+c)^3}{\left(\cos(dx+c)+1\right)^3}+\frac{\left(a^6+a^4b^2\right)\sin(dx+c)^4}{\left(\cos(dx+c)+1\right)^4}}+\frac{\log\left(\frac{b-\frac{a\sin(dx+c)}{\cos(dx+c)+1}+\sqrt{a^2+b^2}}{b-\frac{a\sin(dx+c)}{\cos(dx+c)+1}-\sqrt{a^2+b^2}}\right)}{\left(a^2+b^2\right)^{\frac{3}{2}}}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cos(d*x+c)+b*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -1/2*(2*(a^2*b - (a^3 - 2*a*b^2)*sin(d*x + c)/(cos(d*x + c) + 1) - (a^2*b -
 2*b^3)*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 - (a^3 + 2*a*b^2)*sin(d*x + c)^
3/(cos(d*x + c) + 1)^3)/(a^6 + a^4*b^2 + 4*(a^5*b + a^3*b^3)*sin(d*x + c)/(
cos(d*x + c) + 1) - 2*(a^6 - a^4*b^2 - 2*a^2*b^4)*sin(d*x + c)^2/(cos(d*x +
 c) + 1)^2 - 4*(a^5*b + a^3*b^3)*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + (a^6
 + a^4*b^2)*sin(d*x + c)^4/(cos(d*x + c) + 1)^4) + log((b - a*sin(d*x + c)/
(cos(d*x + c) + 1) + sqrt(a^2 + b^2))/(b - a*sin(d*x + c)/(cos(d*x + c) + 1
) - sqrt(a^2 + b^2)))/(a^2 + b^2)^(3/2))/d
```

mupad [B] time = 2.73, size = 260, normalized size = 2.52

$$\frac{\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\left(a^2-2\,b^2\right)}{a\,\left(a^2+b^2\right)}-\frac{b}{a^2+b^2}+\frac{{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^3\,\left(a^2+2\,b^2\right)}{a\,\left(a^2+b^2\right)}+\frac{b\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2\,\left(a^2-2\,b^2\right)}{a^2\,\left(a^2+b^2\right)}}{d\,\left(a^2\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^4-{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2\,\left(2\,a^2-4\,b^2\right)+a^2-4\,a\,b\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^3+4\,a\,b\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}+\frac{\mathrm{atanh}\left(\frac{\left(2\,a\,t\right.}{}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*cos(c + d*x) + b*sin(c + d*x))^3,x)
```

```
[Out] ((tan(c/2 + (d*x)/2)*(a^2 - 2*b^2))/(a*(a^2 + b^2)) - b/(a^2 + b^2) + (tan(
c/2 + (d*x)/2)^3*(a^2 + 2*b^2))/(a*(a^2 + b^2)) + (b*tan(c/2 + (d*x)/2)^2*(
a^2 - 2*b^2))/(a^2*(a^2 + b^2)))/(d*(a^2*tan(c/2 + (d*x)/2)^4 - tan(c/2 + (
d*x)/2)^2*(2*a^2 - 4*b^2) + a^2 - 4*a*b*tan(c/2 + (d*x)/2)^3 + 4*a*b*tan(c/
2 + (d*x)/2))) + atanh(((2*a*tan(c/2 + (d*x)/2) - (2*a^2*b + 2*b^3)/(a^2 +
b^2))*(a^2/2 + b^2/2))/(a^2 + b^2)^(3/2))/(d*(a^2 + b^2)^(3/2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cos(d*x+c)+b*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

3.136 $\int \frac{\sec(c+dx)}{(a\cos(c+dx)+b\sin(c+dx))^3}\,dx$

Optimal. Leaf size=86

$$\frac{\frac{1}{a^2}-\frac{1}{b^2}}{d(a\cot(c+dx)+b)}-\frac{\frac{b}{a^2}+\frac{1}{b}}{2d(a\cot(c+dx)+b)^2}+\frac{\log(a\cot(c+dx)+b)}{b^3d}+\frac{\log(\tan(c+dx))}{b^3d}$$

```
[Out] 1/2*(-1/b-b/a^2)/d/(b+a*cot(d*x+c))^2+(1/a^2-1/b^2)/d/(b+a*cot(d*x+c))+ln(b
+a*cot(d*x+c))/b^3/d+ln(tan(d*x+c))/b^3/d
```

Rubi [A] time = 0.10, antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {3088, 894}

$$\frac{\frac{1}{a^2}-\frac{1}{b^2}}{d(a\cot(c+dx)+b)}-\frac{\frac{b}{a^2}+\frac{1}{b}}{2d(a\cot(c+dx)+b)^2}+\frac{\log(a\cot(c+dx)+b)}{b^3d}+\frac{\log(\tan(c+dx))}{b^3d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]/(a*Cos[c + d*x] + b*Sin[c + d*x])^3,x]
```

```
[Out] -(b^(-1) + b/a^2)/(2*d*(b + a*Cot[c + d*x])^2) + (a^(-2) - b^(-2))/(d*(b +
a*Cot[c + d*x])) + Log[b + a*Cot[c + d*x]]/(b^3*d) + Log[Tan[c + d*x]]/(b^3
*d)
```

Rule 894

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ
[m, 0] && ILtQ[n, 0]))
```

Rule 3088

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[(x^m*(b +
 a*x)^n)/(1 + x^2)^((m + n + 2)/2), x], x, Cot[c + d*x]], x] /; FreeQ[{a, b
, c, d}, x] && IntegerQ[n] && IntegerQ[(m + n)/2] && NeQ[n, -1] &&  !(GtQ[n
, 0] && GtQ[m, 1])
```

Rubi steps

$$\int \frac{\sec(c+dx)}{(a\cos(c+dx)+b\sin(c+dx))^3}\,dx = -\frac{\text{Subst}\left(\int \frac{1+x^2}{x(b+ax)^3}\,dx, x, \cot(c+dx)\right)}{d}$$
$$= -\frac{\text{Subst}\left(\int \left(\frac{1}{b^3x}+\frac{-a^2-b^2}{ab(b+ax)^3}+\frac{-a^2+b^2}{ab^2(b+ax)^2}-\frac{a}{b^3(b+ax)}\right)dx, x, \cot(c+dx)\right)}{d}$$
$$= -\frac{\frac{1}{b}+\frac{b}{a^2}}{2d(b+a\cot(c+dx))^2}+\frac{\frac{1}{a^2}-\frac{1}{b^2}}{d(b+a\cot(c+dx))}+\frac{\log(b+a\cot(c+dx)}{b^3d}$$

Mathematica [A] time = 0.52, size = 57, normalized size = 0.66

$$\frac{-\frac{a^2+b^2}{2(a+b\tan(c+dx))^2}+\frac{2a}{a+b\tan(c+dx)}+\log(a+b\tan(c+dx))}{b^3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]/(a*Cos[c + d*x] + b*Sin[c + d*x])^3,x]
```

```
[Out] (Log[a + b*Tan[c + d*x]] - (a^2 + b^2)/(2*(a + b*Tan[c + d*x])^2) + (2*a)/(
a + b*Tan[c + d*x]))/(b^3*d)
```

fricas [B] time = 0.60, size = 284, normalized size = 3.30

$$\frac{4\,a^2b^2\cos(dx+c)^2-3\,a^2b^2-b^4-2\left(a^3b-ab^3\right)\cos(dx+c)\sin(dx+c)+\left(a^2b^2+b^4+\left(a^4-b^4\right)\cos(dx+c)^2\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a*cos(d*x+c)+b*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/2*(4*a^2*b^2*cos(d*x + c)^2 - 3*a^2*b^2 - b^4 - 2*(a^3*b - a*b^3)*cos(d*x
 + c)*sin(d*x + c) + (a^2*b^2 + b^4 + (a^4 - b^4)*cos(d*x + c)^2 + 2*(a^3*b
 + a*b^3)*cos(d*x + c)*sin(d*x + c))*log(2*a*b*cos(d*x + c)*sin(d*x + c) +
(a^2 - b^2)*cos(d*x + c)^2 + b^2) - (a^2*b^2 + b^4 + (a^4 - b^4)*cos(d*x +
c)^2 + 2*(a^3*b + a*b^3)*cos(d*x + c)*sin(d*x + c))*log(cos(d*x + c)^2))/((
a^4*b^3 - b^7)*d*cos(d*x + c)^2 + 2*(a^3*b^4 + a*b^6)*d*cos(d*x + c)*sin(d*
x + c) + (a^2*b^5 + b^7)*d)
```

giac [A] time = 1.83, size = 62, normalized size = 0.72

$$\frac{\frac{2\log(|b\tan(dx+c)+a|)}{b^3}-\frac{3\,b\tan(dx+c)^2+2\,a\tan(dx+c)+b}{(b\tan(dx+c)+a)^2b^2}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a*cos(d*x+c)+b*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/2*(2*log(abs(b*tan(d*x + c) + a))/b^3 - (3*b*tan(d*x + c)^2 + 2*a*tan(d*x
 + c) + b)/((b*tan(d*x + c) + a)^2*b^2))/d
```

maple [A] time = 0.39, size = 84, normalized size = 0.98

$$\frac{\ln\left(a+b\tan\left(dx+c\right)\right)}{d\,b^{3}}+\frac{2a}{d\,b^{3}\left(a+b\tan\left(dx+c\right)\right)}-\frac{a^{2}}{2d\,b^{3}\left(a+b\tan\left(dx+c\right)\right)^{2}}-\frac{1}{2bd\left(a+b\tan\left(dx+c\right)\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)/(a*cos(d*x+c)+b*sin(d*x+c))^3,x)
```

```
[Out] 1/d/b^3*ln(a+b*tan(d*x+c))+2/d*a/b^3/(a+b*tan(d*x+c))-1/2/d/b^3/(a+b*tan(d*
x+c))^2*a^2-1/2/b/d/(a+b*tan(d*x+c))^2
```

maxima [B] time = 0.34, size = 315, normalized size = 3.66

$$-\frac{\frac{2\left(\frac{\left(a^{3}-ab^{2}\right)\sin(dx+c)}{\cos(dx+c)+1}+\frac{\left(3\,a^{2}b-b^{3}\right)\sin(dx+c)^{2}}{(\cos(dx+c)+1)^{2}}-\frac{\left(a^{3}-ab^{2}\right)\sin(dx+c)^{3}}{(\cos(dx+c)+1)^{3}}\right)}{a^{4}b^{2}+\frac{4\,a^{3}b^{3}\sin(dx+c)}{\cos(dx+c)+1}-\frac{4\,a^{3}b^{3}\sin(dx+c)^{3}}{(\cos(dx+c)+1)^{3}}+\frac{a^{4}b^{2}\sin(dx+c)^{4}}{(\cos(dx+c)+1)^{4}}-\frac{2\left(a^{4}b^{2}-2\,a^{2}b^{4}\right)\sin(dx+c)^{2}}{(\cos(dx+c)+1)^{2}}}-\frac{\log\left(-a-\frac{2\,b\sin(dx+c)}{\cos(dx+c)+1}+\frac{a\sin(dx+c)^{2}}{(\cos(dx+c)+1)^{2}}\right)}{b^{3}}+\frac{\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)}{b^{3}}+\cdots}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a*cos(d*x+c)+b*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -(2*((a^3 - a*b^2)*sin(d*x + c)/(cos(d*x + c) + 1) + (3*a^2*b - b^3)*sin(d*
x + c)^2/(cos(d*x + c) + 1)^2 - (a^3 - a*b^2)*sin(d*x + c)^3/(cos(d*x + c)
+ 1)^3)/(a^4*b^2 + 4*a^3*b^3*sin(d*x + c)/(cos(d*x + c) + 1) - 4*a^3*b^3*si
n(d*x + c)^3/(cos(d*x + c) + 1)^3 + a^4*b^2*sin(d*x + c)^4/(cos(d*x + c) +
1)^4 - 2*(a^4*b^2 - 2*a^2*b^4)*sin(d*x + c)^2/(cos(d*x + c) + 1)^2) - log(-
a - 2*b*sin(d*x + c)/(cos(d*x + c) + 1) + a*sin(d*x + c)^2/(cos(d*x + c) +
1)^2)/b^3 + log(sin(d*x + c)/(cos(d*x + c) + 1) + 1)/b^3 + log(sin(d*x + c)
/(cos(d*x + c) + 1) - 1)/b^3)/d
```

mupad [B] time = 2.59, size = 396, normalized size = 4.60

$$\frac{2\,\mathrm{atanh}\left(\frac{16\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{2}}{16\,a+\frac{32\,a^{3}}{b^{2}}-16\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{2}-\frac{32\,a^{3}\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{2}}{b^{2}}+\frac{32\,a^{2}\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{b}}-\frac{16\,a}{16\,a+\frac{32\,a^{3}}{b^{2}}-16\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{2}-\frac{32\,a^{3}\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{2}}{b^{2}}+\frac{32\,a^{2}\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{b}}+\cdots\right)}{b^{3}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)*(a*cos(c + d*x) + b*sin(c + d*x))^3),x)
```

```
[Out] (2*atanh((16*a*tan(c/2 + (d*x)/2)^2)/(16*a + (32*a^3)/b^2 - 16*a*tan(c/2 +
(d*x)/2)^2 - (32*a^3*tan(c/2 + (d*x)/2)^2)/b^2 + (32*a^2*tan(c/2 + (d*x)/2)
)/b) - (16*a)/(16*a + (32*a^3)/b^2 - 16*a*tan(c/2 + (d*x)/2)^2 - (32*a^3*ta
n(c/2 + (d*x)/2)^2)/b^2 + (32*a^2*tan(c/2 + (d*x)/2))/b) + (32*a^2*tan(c/2
+ (d*x)/2))/(16*a*b + (32*a^3)/b + 32*a^2*tan(c/2 + (d*x)/2) - (32*a^3*tan(
c/2 + (d*x)/2)^2)/b - 16*a*b*tan(c/2 + (d*x)/2)^2)))/(b^3*d) - ((2*tan(c/2
+ (d*x)/2)^2*(3*a^2 - b^2))/(a^2*b) - (2*tan(c/2 + (d*x)/2)^3*(a^2 - b^2))/
(a*b^2) + (2*tan(c/2 + (d*x)/2)*(a^2 - b^2))/(a*b^2))/(d*(a^2*tan(c/2 + (d*
x)/2)^4 - tan(c/2 + (d*x)/2)^2*(2*a^2 - 4*b^2) + a^2 - 4*a*b*tan(c/2 + (d*x
)/2)^3 + 4*a*b*tan(c/2 + (d*x)/2)))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec(c+dx)}{(a\cos(c+dx)+b\sin(c+dx))^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a*cos(d*x+c)+b*sin(d*x+c))**3,x)
```

```
[Out] Integral(sec(c + d*x)/(a*cos(c + d*x) + b*sin(c + d*x))**3, x)
```

3.137 $\int \frac{\sec^2(c+dx)}{(a\cos(c+dx)+b\sin(c+dx))^3}\,dx$

Optimal. Leaf size=260

$$-\frac{\tanh^{-1}\left(\frac{b\cos(c+dx)-a\sin(c+dx)}{\sqrt{a^2+b^2}}\right)}{2b^2d\sqrt{a^2+b^2}}-\frac{2a^2\tanh^{-1}\left(\frac{b\cos(c+dx)-a\sin(c+dx)}{\sqrt{a^2+b^2}}\right)}{b^4d\sqrt{a^2+b^2}}-\frac{\sqrt{a^2+b^2}\,\tanh^{-1}\left(\frac{b\cos(c+dx)-a\sin(c+dx)}{\sqrt{a^2+b^2}}\right)}{b^4d}-\frac{3a\tan}{}$$

```
[Out] -3*a*arctanh(sin(d*x+c))/b^4/d+sec(d*x+c)/b^3/d+1/2*(-b*cos(d*x+c)+a*sin(d*
x+c))/b^2/d/(a*cos(d*x+c)+b*sin(d*x+c))^2+2*a/b^3/d/(a*cos(d*x+c)+b*sin(d*x
+c))-2*a^2*arctanh((b*cos(d*x+c)-a*sin(d*x+c))/(a^2+b^2)^(1/2))/b^4/d/(a^2+
b^2)^(1/2)-1/2*arctanh((b*cos(d*x+c)-a*sin(d*x+c))/(a^2+b^2)^(1/2))/b^2/d/(
a^2+b^2)^(1/2)-arctanh((b*cos(d*x+c)-a*sin(d*x+c))/(a^2+b^2)^(1/2))*(a^2+b^
2)^(1/2)/b^4/d
```

Rubi [A] time = 0.29, antiderivative size = 260, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 7, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {3106, 3076, 3074, 206, 3104, 3770, 3094}

$$-\frac{2a^2\tanh^{-1}\left(\frac{b\cos(c+dx)-a\sin(c+dx)}{\sqrt{a^2+b^2}}\right)}{b^4d\sqrt{a^2+b^2}}-\frac{\sqrt{a^2+b^2}\,\tanh^{-1}\left(\frac{b\cos(c+dx)-a\sin(c+dx)}{\sqrt{a^2+b^2}}\right)}{b^4d}-\frac{\tanh^{-1}\left(\frac{b\cos(c+dx)-a\sin(c+dx)}{\sqrt{a^2+b^2}}\right)}{2b^2d\sqrt{a^2+b^2}}-\frac{3a\tan}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^2/(a*Cos[c + d*x] + b*Sin[c + d*x])^3,x]
```

```
[Out] (-3*a*ArcTanh[Sin[c + d*x]])/(b^4*d) - (2*a^2*ArcTanh[(b*Cos[c + d*x] - a*S
in[c + d*x])/Sqrt[a^2 + b^2]])/(b^4*Sqrt[a^2 + b^2]*d) - ArcTanh[(b*Cos[c +
 d*x] - a*Sin[c + d*x])/Sqrt[a^2 + b^2]]/(2*b^2*Sqrt[a^2 + b^2]*d) - (Sqrt[
a^2 + b^2]*ArcTanh[(b*Cos[c + d*x] - a*Sin[c + d*x])/Sqrt[a^2 + b^2]])/(b^4
*d) + Sec[c + d*x]/(b^3*d) - (b*Cos[c + d*x] - a*Sin[c + d*x])/(2*b^2*d*(a*
Cos[c + d*x] + b*Sin[c + d*x])^2) + (2*a)/(b^3*d*(a*Cos[c + d*x] + b*Sin[c
+ d*x]))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3074

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x
_Symbol] :> -Dist[d^(-1), Subst[Int[1/(a^2 + b^2 - x^2), x], x, b*Cos[c + d
```

```
*x] - a*Sin[c + d*x]], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

Rule 3076

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x
_Symbol] :> Simp[((b*Cos[c + d*x] - a*Sin[c + d*x])*(a*Cos[c + d*x] + b*Sin
[c + d*x])^(n + 1))/(d*(n + 1)*(a^2 + b^2)), x] + Dist[(n + 2)/((n + 1)*(a^
2 + b^2)), Int[(a*Cos[c + d*x] + b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{
a, b, c, d}, x] && NeQ[a^2 + b^2, 0] && LtQ[n, -1] && NeQ[n, -2]
```

Rule 3094

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_)/co
s[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[(a*Cos[c + d*x] + b*Sin[c + d*x])^
(n + 1)/(b*d*(n + 1)), x] + (Dist[1/b^2, Int[(a*Cos[c + d*x] + b*Sin[c + d*
x])^(n + 2)/Cos[c + d*x], x], x] - Dist[a/b^2, Int[(a*Cos[c + d*x] + b*Sin[
c + d*x])^(n + 1), x], x]) /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0] &
& LtQ[n, -1]
```

Rule 3104

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_)/(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin
[(c_.) + (d_.)*(x_)]), x_Symbol] :> -Simp[Cos[c + d*x]^(m + 1)/(b*d*(m + 1)
), x] + (-Dist[a/b^2, Int[Cos[c + d*x]^(m + 1), x], x] + Dist[(a^2 + b^2)/b
^2, Int[Cos[c + d*x]^(m + 2)/(a*Cos[c + d*x] + b*Sin[c + d*x]), x], x]) /;
FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0] && LtQ[m, -1]
```

Rule 3106

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin
[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(a^2 + b^2)/b^2, Int[Cos[c +
d*x]^(m + 2)*(a*Cos[c + d*x] + b*Sin[c + d*x])^n, x], x] + (Dist[1/b^2, Int
[Cos[c + d*x]^m*(a*Cos[c + d*x] + b*Sin[c + d*x])^(n + 2), x], x] - Dist[(2
*a)/b^2, Int[Cos[c + d*x]^(m + 1)*(a*Cos[c + d*x] + b*Sin[c + d*x])^(n + 1)
, x], x]) /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0] && LtQ[n, -1] && L
tQ[m, -1]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\sec^2(c+dx)}{(a\cos(c+dx)+b\sin(c+dx))^3}\,dx = \frac{\int \frac{\sec^2(c+dx)}{a\cos(c+dx)+b\sin(c+dx)}\,dx}{b^2} - \frac{(2a)\int \frac{\sec(c+dx)}{(a\cos(c+dx)+b\sin(c+dx))^2}\,dx}{b^2} + \frac{\left(a^2+b^2\right)\int}{}$$

$$= \frac{\sec(c+dx)}{b^3d} - \frac{b\cos(c+dx)-a\sin(c+dx)}{2b^2d(a\cos(c+dx)+b\sin(c+dx))^2} + \frac{2a}{b^3d(a\cos(c+dx)}$$

$$= -\frac{3a\tanh^{-1}(\sin(c+dx))}{b^4d} + \frac{\sec(c+dx)}{b^3d} - \frac{b\cos(c+dx)-a\sin(c+d}{2b^2d(a\cos(c+dx)+b\sin(c+}$$

$$= -\frac{3a\tanh^{-1}(\sin(c+dx))}{b^4d} - \frac{2a^2\tanh^{-1}\left(\frac{b\cos(c+dx)-a\sin(c+dx)}{\sqrt{a^2+b^2}}\right)}{b^4\sqrt{a^2+b^2}\,d} - \frac{\tanh^{-1}\left(\right.}{2b}$$

Mathematica [A] time = 2.44, size = 396, normalized size = 1.52

$$\frac{\sec^3(c+dx)(a\cos(c+dx)+b\sin(c+dx))\left(\frac{b^2\left(a^2+b^2\right)\sin(c+dx)}{a} + \frac{6\left(2a^2+b^2\right)(a\cos(c+dx)+b\sin(c+dx))^2\tanh^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)-b}{\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}} + \right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^2/(a*Cos[c + d*x] + b*Sin[c + d*x])^3,x]
```

```
[Out] (Sec[c + d*x]^3*(a*Cos[c + d*x] + b*Sin[c + d*x])*((b^2*(a^2 + b^2)*Sin[c +
 d*x])/a + ((2*a - b)*b*(2*a + b)*(a*Cos[c + d*x] + b*Sin[c + d*x]))/a + 2*
b*(a*Cos[c + d*x] + b*Sin[c + d*x])^2 + (6*(2*a^2 + b^2)*ArcTanh[(-b + a*Ta
n[(c + d*x)/2])/Sqrt[a^2 + b^2]]*(a*Cos[c + d*x] + b*Sin[c + d*x])^2)/Sqrt[
a^2 + b^2] + 6*a*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]]*(a*Cos[c + d*x] +
 b*Sin[c + d*x])^2 - 6*a*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]*(a*Cos[c
+ d*x] + b*Sin[c + d*x])^2 + (2*b*Sin[(c + d*x)/2]*(a*Cos[c + d*x] + b*Sin[
c + d*x])^2)/(Cos[(c + d*x)/2] - Sin[(c + d*x)/2]) - (2*b*Sin[(c + d*x)/2]*
(a*Cos[c + d*x] + b*Sin[c + d*x])^2)/(Cos[(c + d*x)/2] + Sin[(c + d*x)/2]))
)/(2*b^4*d*(a + b*Tan[c + d*x])^3)
```

fricas [B] time = 0.62, size = 513, normalized size = 1.97

$$\frac{4\,a^2b^3 + 4\,b^5 + 6\left(2\,a^4b + a^2b^3 - b^5\right)\cos(dx+c)^2 + 18\left(a^3b^2 + ab^4\right)\cos(dx+c)\sin(dx+c) + 3\left(\left(2\,a^4 - a^2b^2 - b\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a*cos(d*x+c)+b*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/4*(4*a^2*b^3 + 4*b^5 + 6*(2*a^4*b + a^2*b^3 - b^5)*cos(d*x + c)^2 + 18*(a
^3*b^2 + a*b^4)*cos(d*x + c)*sin(d*x + c) + 3*((2*a^4 - a^2*b^2 - b^4)*cos(
d*x + c)^3 + 2*(2*a^3*b + a*b^3)*cos(d*x + c)^2*sin(d*x + c) + (2*a^2*b^2 +
 b^4)*cos(d*x + c))*sqrt(a^2 + b^2)*log(-(2*a*b*cos(d*x + c)*sin(d*x + c) +
 (a^2 - b^2)*cos(d*x + c)^2 - 2*a^2 - b^2 + 2*sqrt(a^2 + b^2)*(b*cos(d*x +
c) - a*sin(d*x + c)))/(2*a*b*cos(d*x + c)*sin(d*x + c) + (a^2 - b^2)*cos(d*
x + c)^2 + b^2)) - 6*((a^5 - a*b^4)*cos(d*x + c)^3 + 2*(a^4*b + a^2*b^3)*co
s(d*x + c)^2*sin(d*x + c) + (a^3*b^2 + a*b^4)*cos(d*x + c))*log(sin(d*x + c
) + 1) + 6*((a^5 - a*b^4)*cos(d*x + c)^3 + 2*(a^4*b + a^2*b^3)*cos(d*x + c)
^2*sin(d*x + c) + (a^3*b^2 + a*b^4)*cos(d*x + c))*log(-sin(d*x + c) + 1))/(
(a^4*b^4 - b^8)*d*cos(d*x + c)^3 + 2*(a^3*b^5 + a*b^7)*d*cos(d*x + c)^2*sin
(d*x + c) + (a^2*b^6 + b^8)*d*cos(d*x + c))
```

giac [A] time = 1.32, size = 314, normalized size = 1.21

$$-\frac{\frac{6\,a\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)}{b^4}-\frac{6\,a\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)}{b^4}+\frac{3\left(2\,a^2+b^2\right)\log\left(\frac{\left|2\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-2\,b-2\,\sqrt{a^2+b^2}\right|}{\left|2\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-2\,b+2\,\sqrt{a^2+b^2}\right|}\right)}{\sqrt{a^2+b^2}\,b^4}+\frac{4}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1\right)b^3}+\frac{2\left(\cdots\right.}{\cdots}}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a*cos(d*x+c)+b*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -1/2*(6*a*log(abs(tan(1/2*d*x + 1/2*c) + 1))/b^4 - 6*a*log(abs(tan(1/2*d*x
+ 1/2*c) - 1))/b^4 + 3*(2*a^2 + b^2)*log(abs(2*a*tan(1/2*d*x + 1/2*c) - 2*b
 - 2*sqrt(a^2 + b^2))/abs(2*a*tan(1/2*d*x + 1/2*c) - 2*b + 2*sqrt(a^2 + b^2
)))/(sqrt(a^2 + b^2)*b^4) + 4/((tan(1/2*d*x + 1/2*c)^2 - 1)*b^3) + 2*(3*a^3
*b*tan(1/2*d*x + 1/2*c)^3 - 2*a*b^3*tan(1/2*d*x + 1/2*c)^3 + 4*a^4*tan(1/2*
d*x + 1/2*c)^2 - 9*a^2*b^2*tan(1/2*d*x + 1/2*c)^2 + 2*b^4*tan(1/2*d*x + 1/2
*c)^2 - 13*a^3*b*tan(1/2*d*x + 1/2*c) + 2*a*b^3*tan(1/2*d*x + 1/2*c) - 4*a^
4 + a^2*b^2)/((a*tan(1/2*d*x + 1/2*c)^2 - 2*b*tan(1/2*d*x + 1/2*c) - a)^2*a
^2*b^3))/d
```

maple [B] time = 0.41, size = 611, normalized size = 2.35

$$-\frac{1}{d\,b^3\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}+\frac{3a\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{d\,b^4}+\frac{1}{d\,b^3\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}-\frac{3a\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{d\,b^4}-\frac{\cdots}{d\,b^2\left(\left(\cdots\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2/(a*cos(d*x+c)+b*sin(d*x+c))^3,x)
```

```
[Out] -1/d/b^3/(tan(1/2*d*x+1/2*c)-1)+3/d*a/b^4*ln(tan(1/2*d*x+1/2*c)-1)+1/d/b^3/
(tan(1/2*d*x+1/2*c)+1)-3/d*a/b^4*ln(tan(1/2*d*x+1/2*c)+1)-3/d/b^2/(tan(1/2*
d*x+1/2*c)^2*a-2*b*tan(1/2*d*x+1/2*c)-a)^2*a*tan(1/2*d*x+1/2*c)^3+2/d/(tan(
1/2*d*x+1/2*c)^2*a-2*b*tan(1/2*d*x+1/2*c)-a)^2/a*tan(1/2*d*x+1/2*c)^3-4/d/b
^3/(tan(1/2*d*x+1/2*c)^2*a-2*b*tan(1/2*d*x+1/2*c)-a)^2*a^2*tan(1/2*d*x+1/2*
c)^2+9/d/b/(tan(1/2*d*x+1/2*c)^2*a-2*b*tan(1/2*d*x+1/2*c)-a)^2*tan(1/2*d*x+
1/2*c)^2-2/d*b/(tan(1/2*d*x+1/2*c)^2*a-2*b*tan(1/2*d*x+1/2*c)-a)^2/a^2*tan(
1/2*d*x+1/2*c)^2+13/d/b^2/(tan(1/2*d*x+1/2*c)^2*a-2*b*tan(1/2*d*x+1/2*c)-a)
^2*a*tan(1/2*d*x+1/2*c)-2/d/(tan(1/2*d*x+1/2*c)^2*a-2*b*tan(1/2*d*x+1/2*c)-
a)^2/a*tan(1/2*d*x+1/2*c)+4/d/b^3/(tan(1/2*d*x+1/2*c)^2*a-2*b*tan(1/2*d*x+1
/2*c)-a)^2*a^2-1/d/b/(tan(1/2*d*x+1/2*c)^2*a-2*b*tan(1/2*d*x+1/2*c)-a)^2+6/
d/b^4/(a^2+b^2)^(1/2)*arctanh(1/2*(2*a*tan(1/2*d*x+1/2*c)-2*b)/(a^2+b^2)^(1
/2))*a^2+3/d/b^2/(a^2+b^2)^(1/2)*arctanh(1/2*(2*a*tan(1/2*d*x+1/2*c)-2*b)/(
a^2+b^2)^(1/2))
```

maxima [B] time = 0.46, size = 518, normalized size = 1.99

$$\frac{\frac{2\left(6\,a^{4}-a^{2}b^{2}+\frac{\left(21\,a^{3}b-2\,ab^{3}\right)\sin(dx+c)}{\cos(dx+c)+1}-\frac{2\left(6\,a^{4}-9\,a^{2}b^{2}+b^{4}\right)\sin(dx+c)^{2}}{(\cos(dx+c)+1)^{2}}-\frac{4\left(6\,a^{3}b-ab^{3}\right)\sin(dx+c)^{3}}{(\cos(dx+c)+1)^{3}}+\frac{\left(6\,a^{4}-9\,a^{2}b^{2}+2\,b^{4}\right)\sin(dx+c)^{4}}{(\cos(dx+c)+1)^{4}}+\frac{\left(3\,a^{3}b-2\,ab^{3}\right)\sin(dx+c)^{5}}{(\cos(dx+c)+1)^{5}}\right)}{a^{4}b^{3}+\frac{4\,a^{3}b^{4}\sin(dx+c)}{\cos(dx+c)+1}-\frac{8\,a^{3}b^{4}\sin(dx+c)^{3}}{(\cos(dx+c)+1)^{3}}+\frac{4\,a^{3}b^{4}\sin(dx+c)^{5}}{(\cos(dx+c)+1)^{5}}-\frac{a^{4}b^{3}\sin(dx+c)^{6}}{(\cos(dx+c)+1)^{6}}-\frac{\left(3\,a^{4}b^{3}-4\,a^{2}b^{5}\right)\sin(dx+c)^{2}}{(\cos(dx+c)+1)^{2}}+\frac{\left(3\,a^{4}b^{3}-4\,a^{2}b^{5}\right)\sin(dx+c)^{4}}{(\cos(dx+c)+1)^{4}}}-\frac{6\,a\log\Big(}{}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a*cos(d*x+c)+b*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/2*(2*(6*a^4 - a^2*b^2 + (21*a^3*b - 2*a*b^3)*sin(d*x + c)/(cos(d*x + c) +
 1) - 2*(6*a^4 - 9*a^2*b^2 + b^4)*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 - 4*(
6*a^3*b - a*b^3)*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + (6*a^4 - 9*a^2*b^2 +
 2*b^4)*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + (3*a^3*b - 2*a*b^3)*sin(d*x +
 c)^5/(cos(d*x + c) + 1)^5)/(a^4*b^3 + 4*a^3*b^4*sin(d*x + c)/(cos(d*x + c)
 + 1) - 8*a^3*b^4*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 4*a^3*b^4*sin(d*x +
 c)^5/(cos(d*x + c) + 1)^5 - a^4*b^3*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 -
(3*a^4*b^3 - 4*a^2*b^5)*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + (3*a^4*b^3 -
4*a^2*b^5)*sin(d*x + c)^4/(cos(d*x + c) + 1)^4) - 6*a*log(sin(d*x + c)/(cos
(d*x + c) + 1) + 1)/b^4 + 6*a*log(sin(d*x + c)/(cos(d*x + c) + 1) - 1)/b^4
- 3*(2*a^2 + b^2)*log((b - a*sin(d*x + c)/(cos(d*x + c) + 1) + sqrt(a^2 + b
^2))/(b - a*sin(d*x + c)/(cos(d*x + c) + 1) - sqrt(a^2 + b^2)))/(sqrt(a^2 +
 b^2)*b^4))/d
```

mupad [B] time = 2.62, size = 1311, normalized size = 5.04

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^2*(a*cos(c + d*x) + b*sin(c + d*x))^3),x)
```

```
[Out] ((6*a^2 - b^2)/b^3 - (2*tan(c/2 + (d*x)/2)^2*(6*a^4 + b^4 - 9*a^2*b^2))/(a^
2*b^3) + (tan(c/2 + (d*x)/2)*(21*a^2 - 2*b^2))/(a*b^2) + (tan(c/2 + (d*x)/2
)^4*(6*a^4 + 2*b^4 - 9*a^2*b^2))/(a^2*b^3) - (4*tan(c/2 + (d*x)/2)^3*(6*a^2
 - b^2))/(a*b^2) + (tan(c/2 + (d*x)/2)^5*(3*a^2 - 2*b^2))/(a*b^2))/(d*(tan(
c/2 + (d*x)/2)^4*(3*a^2 - 4*b^2) - tan(c/2 + (d*x)/2)^2*(3*a^2 - 4*b^2) - a
^2*tan(c/2 + (d*x)/2)^6 + a^2 - 8*a*b*tan(c/2 + (d*x)/2)^3 + 4*a*b*tan(c/2
+ (d*x)/2)^5 + 4*a*b*tan(c/2 + (d*x)/2))) - (6*a*atanh(tan(c/2 + (d*x)/2)))
/(b^4*d) + (atan((((2*a^2 + b^2)*(a^2 + b^2)^(1/2)*((288*a^4)/b^5 + (8*tan(
c/2 + (d*x)/2)*(9*a*b^7 + 108*a^3*b^5 + 72*a^5*b^3))/b^9 - (3*(2*a^2 + b^2)
*(a^2 + b^2)^(1/2)*((8*tan(c/2 + (d*x)/2)*(12*a*b^10 + 24*a^3*b^8))/b^9 - 4
8*a^2 + (3*(2*a^2 + b^2)*(a^2 + b^2)^(1/2)*(32*a^2*b^3 + (8*tan(c/2 + (d*x)
/2)*(12*a*b^13 + 8*a^3*b^11))/b^9))/(2*(b^6 + a^2*b^4))))/(2*(b^6 + a^2*b^4
)))*3i)/(2*(b^6 + a^2*b^4)) + ((2*a^2 + b^2)*(a^2 + b^2)^(1/2)*((288*a^4)/b
^5 + (8*tan(c/2 + (d*x)/2)*(9*a*b^7 + 108*a^3*b^5 + 72*a^5*b^3))/b^9 - (3*(
2*a^2 + b^2)*(a^2 + b^2)^(1/2)*(48*a^2 - (8*tan(c/2 + (d*x)/2)*(12*a*b^10 +
 24*a^3*b^8))/b^9 + (3*(2*a^2 + b^2)*(a^2 + b^2)^(1/2)*(32*a^2*b^3 + (8*tan
(c/2 + (d*x)/2)*(12*a*b^13 + 8*a^3*b^11))/b^9))/(2*(b^6 + a^2*b^4))))/(2*(b
^6 + a^2*b^4)))*3i)/(2*(b^6 + a^2*b^4)))/((16*(54*a^4 + 27*a^2*b^2))/b^8 -
(16*tan(c/2 + (d*x)/2)*(216*a^5 + 108*a^3*b^2))/b^9 - (3*(2*a^2 + b^2)*(a^2
 + b^2)^(1/2)*((288*a^4)/b^5 + (8*tan(c/2 + (d*x)/2)*(9*a*b^7 + 108*a^3*b^5
 + 72*a^5*b^3))/b^9 - (3*(2*a^2 + b^2)*(a^2 + b^2)^(1/2)*((8*tan(c/2 + (d*x
)/2)*(12*a*b^10 + 24*a^3*b^8))/b^9 - 48*a^2 + (3*(2*a^2 + b^2)*(a^2 + b^2)^
(1/2)*(32*a^2*b^3 + (8*tan(c/2 + (d*x)/2)*(12*a*b^13 + 8*a^3*b^11))/b^9))/(
2*(b^6 + a^2*b^4))))/(2*(b^6 + a^2*b^4))))/(2*(b^6 + a^2*b^4)) + (3*(2*a^2
+ b^2)*(a^2 + b^2)^(1/2)*((288*a^4)/b^5 + (8*tan(c/2 + (d*x)/2)*(9*a*b^7 +
108*a^3*b^5 + 72*a^5*b^3))/b^9 - (3*(2*a^2 + b^2)*(a^2 + b^2)^(1/2)*(48*a^2
 - (8*tan(c/2 + (d*x)/2)*(12*a*b^10 + 24*a^3*b^8))/b^9 + (3*(2*a^2 + b^2)*(
a^2 + b^2)^(1/2)*(32*a^2*b^3 + (8*tan(c/2 + (d*x)/2)*(12*a*b^13 + 8*a^3*b^1
1))/b^9))/(2*(b^6 + a^2*b^4))))/(2*(b^6 + a^2*b^4))))/(2*(b^6 + a^2*b^4))))
*(2*a^2 + b^2)*(a^2 + b^2)^(1/2)*3i)/(d*(b^6 + a^2*b^4))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec^2(c+dx)}{(a\cos(c+dx)+b\sin(c+dx))^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2/(a*cos(d*x+c)+b*sin(d*x+c))**3,x)
```

```
[Out] Integral(sec(c + d*x)**2/(a*cos(c + d*x) + b*sin(c + d*x))**3, x)
```

3.138 $\int \frac{\sec^3(c+dx)}{(a\cos(c+dx)+b\sin(c+dx))^3}\,dx$

Optimal. Leaf size=161

$$\frac{2\left(3a^2+b^2\right)\log(\tan(c+dx))}{b^5d}+\frac{2\left(3a^2+b^2\right)\log(a\cot(c+dx)+b)}{b^5d}-\frac{\left(3a^2-b^2\right)\left(a^2+b^2\right)}{a^2b^4d(a\cot(c+dx)+b)}-\frac{\left(a^2+b^2\right)^2}{2a^2b^3d(a\cot(c+dx)+b)^2}$$

```
[Out] -1/2*(a^2+b^2)^2/a^2/b^3/d/(b+a*cot(d*x+c))^2-(3*a^2-b^2)*(a^2+b^2)/a^2/b^4
/d/(b+a*cot(d*x+c))+2*(3*a^2+b^2)*ln(b+a*cot(d*x+c))/b^5/d+2*(3*a^2+b^2)*ln
(tan(d*x+c))/b^5/d-3*a*tan(d*x+c)/b^4/d+1/2*tan(d*x+c)^2/b^3/d
```

Rubi [A] time = 0.17, antiderivative size = 161, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {3088, 894}

$$-\frac{\left(a^2+b^2\right)^2}{2a^2b^3d(a\cot(c+dx)+b)^2}-\frac{\left(3a^2-b^2\right)\left(a^2+b^2\right)}{a^2b^4d(a\cot(c+dx)+b)}+\frac{2\left(3a^2+b^2\right)\log(\tan(c+dx))}{b^5d}+\frac{2\left(3a^2+b^2\right)\log(a\cot(c+dx)+b)}{b^5d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^3/(a*Cos[c + d*x] + b*Sin[c + d*x])^3,x]
```

```
[Out] -(a^2 + b^2)^2/(2*a^2*b^3*d*(b + a*Cot[c + d*x])^2) - ((3*a^2 - b^2)*(a^2 +
 b^2))/(a^2*b^4*d*(b + a*Cot[c + d*x])) + (2*(3*a^2 + b^2)*Log[b + a*Cot[c
+ d*x]])/(b^5*d) + (2*(3*a^2 + b^2)*Log[Tan[c + d*x]])/(b^5*d) - (3*a*Tan[c
 + d*x])/(b^4*d) + Tan[c + d*x]^2/(2*b^3*d)
```

Rule 894

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ
[m, 0] && ILtQ[n, 0]))
```

Rule 3088

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[(x^m*(b +
 a*x)^n)/(1 + x^2)^((m + n + 2)/2), x], x, Cot[c + d*x]], x] /; FreeQ[{a, b
, c, d}, x] && IntegerQ[n] && IntegerQ[(m + n)/2] && NeQ[n, -1] &&  !(GtQ[n
, 0] && GtQ[m, 1])
```

Rubi steps

$$\int \frac{\sec^3(c+dx)}{(a\cos(c+dx)+b\sin(c+dx))^3}\,dx = -\frac{\text{Subst}\left(\int \frac{(1+x^2)^2}{x^3(b+ax)^3}\,dx, x, \cot(c+dx)\right)}{d}$$
$$= -\frac{\text{Subst}\left(\int \left(\frac{1}{b^3x^3} - \frac{3a}{b^4x^2} + \frac{2(3a^2+b^2)}{b^5x} - \frac{(a^2+b^2)^2}{ab^3(b+ax)^3} + \frac{-3a^4-2a^2b^2+b^4}{ab^4(b+ax)^2} - \frac{2a(3a^2+b}{b^5(b+ax}\right.\right.}{d}$$
$$= -\frac{(a^2+b^2)^2}{2a^2b^3d(b+a\cot(c+dx))^2} - \frac{(3a^2-b^2)(a^2+b^2)}{a^2b^4d(b+a\cot(c+dx))} + \frac{2(3a^2+b^2)}{}$$

Mathematica [A] time = 3.00, size = 140, normalized size = 0.87

$$\frac{-2a\left(-\frac{a^2+b^2}{a+b\tan(c+dx)} - 2a\log(a+b\tan(c+dx)) + b\tan(c+dx)\right) + 2(a^2+b^2)\left(\frac{3a^2+4ab\tan(c+dx)-b^2}{2(a+b\tan(c+dx))^2} + \log(a+b\tan\right.}{b^5d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^3/(a*Cos[c + d*x] + b*Sin[c + d*x])^3,x]
```

```
[Out] ((b^4*Sec[c + d*x]^4)/(2*(a + b*Tan[c + d*x])^2) - 2*a*(-2*a*Log[a + b*Tan[
c + d*x]] + b*Tan[c + d*x] - (a^2 + b^2)/(a + b*Tan[c + d*x])) + 2*(a^2 + b
^2)*(Log[a + b*Tan[c + d*x]] + (3*a^2 - b^2 + 4*a*b*Tan[c + d*x])/(2*(a + b
*Tan[c + d*x])^2)))/(b^5*d)
```

fricas [B] time = 0.69, size = 354, normalized size = 2.20

$$\frac{24a^2b^2\cos(dx+c)^4 + b^4 - 2(9a^2b^2+b^4)\cos(dx+c)^2 + 2\left((3a^4-2a^2b^2-b^4)\cos(dx+c)^4 + 2(3a^3b+ab^3\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a*cos(d*x+c)+b*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/2*(24*a^2*b^2*cos(d*x + c)^4 + b^4 - 2*(9*a^2*b^2 + b^4)*cos(d*x + c)^2 +
 2*((3*a^4 - 2*a^2*b^2 - b^4)*cos(d*x + c)^4 + 2*(3*a^3*b + a*b^3)*cos(d*x
+ c)^3*sin(d*x + c) + (3*a^2*b^2 + b^4)*cos(d*x + c)^2)*log(2*a*b*cos(d*x +
 c)*sin(d*x + c) + (a^2 - b^2)*cos(d*x + c)^2 + b^2) - 2*((3*a^4 - 2*a^2*b^
2 - b^4)*cos(d*x + c)^4 + 2*(3*a^3*b + a*b^3)*cos(d*x + c)^3*sin(d*x + c) +
 (3*a^2*b^2 + b^4)*cos(d*x + c)^2)*log(cos(d*x + c)^2) - 4*(a*b^3*cos(d*x +
 c) + 3*(a^3*b - a*b^3)*cos(d*x + c)^3)*sin(d*x + c))/(2*a*b^6*d*cos(d*x +
```

```
c)^3*sin(d*x + c) + b^7*d*cos(d*x + c)^2 + (a^2*b^5 - b^7)*d*cos(d*x + c)^4
)
```

giac [A] time = 0.60, size = 140, normalized size = 0.87

$$\frac{\frac{4\left(3a^2+b^2\right)\log(|b\tan(dx+c)+a|)}{b^5}+\frac{b^3\tan(dx+c)^2-6ab^2\tan(dx+c)}{b^6}-\frac{18a^2b^2\tan(dx+c)^2+6b^4\tan(dx+c)^2+28a^3b\tan(dx+c)+4ab^3\tan(dx+c)+11a^4\cdots}{(b\tan(dx+c)+a)^2b^5}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a*cos(d*x+c)+b*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/2*(4*(3*a^2 + b^2)*log(abs(b*tan(d*x + c) + a))/b^5 + (b^3*tan(d*x + c)^2
 - 6*a*b^2*tan(d*x + c))/b^6 - (18*a^2*b^2*tan(d*x + c)^2 + 6*b^4*tan(d*x +
 c)^2 + 28*a^3*b*tan(d*x + c) + 4*a*b^3*tan(d*x + c) + 11*a^4 + b^4)/((b*ta
n(d*x + c) + a)^2*b^5))/d
```

maple [A] time = 0.40, size = 184, normalized size = 1.14

$$\frac{\tan^2(dx+c)}{2b^3d}-\frac{3a\tan(dx+c)}{b^4d}+\frac{4a^3}{d\,b^5\,(a+b\tan(dx+c))}+\frac{4a}{d\,b^3\,(a+b\tan(dx+c))}+\frac{6\ln(a+b\tan(dx+c))\,a^2}{d\,b^5}+\frac{2}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^3/(a*cos(d*x+c)+b*sin(d*x+c))^3,x)
```

```
[Out] 1/2*tan(d*x+c)^2/b^3/d-3*a*tan(d*x+c)/b^4/d+4/d*a^3/b^5/(a+b*tan(d*x+c))+4/
d*a/b^3/(a+b*tan(d*x+c))+6/d/b^5*ln(a+b*tan(d*x+c))*a^2+2/d/b^3*ln(a+b*tan(
d*x+c))-1/2/d/b^5/(a+b*tan(d*x+c))^2*a^4-1/d/b^3/(a+b*tan(d*x+c))^2*a^2-1/2
/b/d/(a+b*tan(d*x+c))^2
```

maxima [B] time = 0.37, size = 652, normalized size = 4.05

$$-\frac{2\left(\frac{\frac{\left(6a^5+2a^3b^2-ab^4\right)\sin(dx+c)}{\cos(dx+c)+1}+\frac{\left(18a^4b+6a^2b^3-b^5\right)\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-\frac{\left(18a^5-2a^3b^2-3ab^4\right)\sin(dx+c)^3}{(\cos(dx+c)+1)^3}-\frac{2\left(18a^4b+8a^2b^3-b^5\right)\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{\left(18a^5-2a^3b^2-3ab^4\right)\sin(dx+c)^5}{(\cos(dx+c)+1)^5}+\cdots}{a^4b^4+\frac{4a^3b^5\sin(dx+c)}{\cos(dx+c)+1}-\frac{12a^3b^5\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{12a^3b^5\sin(dx+c)^5}{(\cos(dx+c)+1)^5}-\frac{4a^3b^5\sin(dx+c)^7}{(\cos(dx+c)+1)^7}+\frac{a^4b^4\sin(dx+c)^8}{(\cos(dx+c)+1)^8}-\frac{4\left(a^4b^4-a^2b^6\right)\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{2\left(3a^4b^4-4a^2\cdots\right)}{(\cos(dx\cdots}}\cdots\right)}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a*cos(d*x+c)+b*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -2*(((6*a^5 + 2*a^3*b^2 - a*b^4)*sin(d*x + c)/(cos(d*x + c) + 1) + (18*a^4*
b + 6*a^2*b^3 - b^5)*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 - (18*a^5 - 2*a^3*
b^2 - 3*a*b^4)*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 - 2*(18*a^4*b + 8*a^2*b^
```

```
3 - b^5)*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + (18*a^5 - 2*a^3*b^2 - 3*a*b^
4)*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + (18*a^4*b + 6*a^2*b^3 - b^5)*sin(d
*x + c)^6/(cos(d*x + c) + 1)^6 - (6*a^5 + 2*a^3*b^2 - a*b^4)*sin(d*x + c)^7
/(cos(d*x + c) + 1)^7)/(a^4*b^4 + 4*a^3*b^5*sin(d*x + c)/(cos(d*x + c) + 1)
 - 12*a^3*b^5*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 12*a^3*b^5*sin(d*x + c)
^5/(cos(d*x + c) + 1)^5 - 4*a^3*b^5*sin(d*x + c)^7/(cos(d*x + c) + 1)^7 + a
^4*b^4*sin(d*x + c)^8/(cos(d*x + c) + 1)^8 - 4*(a^4*b^4 - a^2*b^6)*sin(d*x
+ c)^2/(cos(d*x + c) + 1)^2 + 2*(3*a^4*b^4 - 4*a^2*b^6)*sin(d*x + c)^4/(cos
(d*x + c) + 1)^4 - 4*(a^4*b^4 - a^2*b^6)*sin(d*x + c)^6/(cos(d*x + c) + 1)^
6) - (3*a^2 + b^2)*log(-a - 2*b*sin(d*x + c)/(cos(d*x + c) + 1) + a*sin(d*x
 + c)^2/(cos(d*x + c) + 1)^2)/b^5 + (3*a^2 + b^2)*log(sin(d*x + c)/(cos(d*x
 + c) + 1) + 1)/b^5 + (3*a^2 + b^2)*log(sin(d*x + c)/(cos(d*x + c) + 1) - 1
)/b^5)/d
```

mupad [B] time = 4.74, size = 1204, normalized size = 7.48

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^3*(a*cos(c + d*x) + b*sin(c + d*x))^3),x)
```

```
[Out] - ((2*tan(c/2 + (d*x)/2)*(6*a^4 - b^4 + 2*a^2*b^2))/(a*b^4) - (2*tan(c/2 +
(d*x)/2)^7*(6*a^4 - b^4 + 2*a^2*b^2))/(a*b^4) + (2*tan(c/2 + (d*x)/2)^3*(3*
b^4 - 18*a^4 + 2*a^2*b^2))/(a*b^4) + (2*tan(c/2 + (d*x)/2)^2*(18*a^4 - b^4
+ 6*a^2*b^2))/(a^2*b^3) - (2*tan(c/2 + (d*x)/2)^5*(3*b^4 - 18*a^4 + 2*a^2*b
^2))/(a*b^4) - (4*tan(c/2 + (d*x)/2)^4*(18*a^4 - b^4 + 8*a^2*b^2))/(a^2*b^3
) + (2*tan(c/2 + (d*x)/2)^6*(18*a^4 - b^4 + 6*a^2*b^2))/(a^2*b^3))/(d*(tan(
c/2 + (d*x)/2)^4*(6*a^2 - 8*b^2) - tan(c/2 + (d*x)/2)^6*(4*a^2 - 4*b^2) - t
an(c/2 + (d*x)/2)^2*(4*a^2 - 4*b^2) + a^2*tan(c/2 + (d*x)/2)^8 + a^2 - 12*a
*b*tan(c/2 + (d*x)/2)^3 + 12*a*b*tan(c/2 + (d*x)/2)^5 - 4*a*b*tan(c/2 + (d*
x)/2)^7 + 4*a*b*tan(c/2 + (d*x)/2))) - (atan((((3*a^2 + b^2)*((2*(3*a^2 + b
^2)*((4*(a*b^10 + 4*a^3*b^8))/b^8 - (4*tan(c/2 + (d*x)/2)^2*(3*a*b^10 + 4*a
^3*b^8))/b^8 + 16*a^2*b*tan(c/2 + (d*x)/2)))/b^5 - (4*(4*a*b^7 + 12*a^3*b^5
))/b^8 + (4*tan(c/2 + (d*x)/2)^2*(4*a*b^7 + 12*a^3*b^5))/b^8 + (16*tan(c/2
+ (d*x)/2)*(6*a^4 + 2*a^2*b^2))/b^4)*2i)/b^5 - ((3*a^2 + b^2)*((4*(4*a*b^7
+ 12*a^3*b^5))/b^8 + (2*(3*a^2 + b^2)*((4*(a*b^10 + 4*a^3*b^8))/b^8 - (4*ta
n(c/2 + (d*x)/2)^2*(3*a*b^10 + 4*a^3*b^8))/b^8 + 16*a^2*b*tan(c/2 + (d*x)/2
)))/b^5 - (4*tan(c/2 + (d*x)/2)^2*(4*a*b^7 + 12*a^3*b^5))/b^8 - (16*tan(c/2
 + (d*x)/2)*(6*a^4 + 2*a^2*b^2))/b^4)*2i)/b^5)/((8*(4*a*b^4 + 36*a^5 + 24*a
^3*b^2))/b^8 + (2*(3*a^2 + b^2)*((2*(3*a^2 + b^2)*((4*(a*b^10 + 4*a^3*b^8))
/b^8 - (4*tan(c/2 + (d*x)/2)^2*(3*a*b^10 + 4*a^3*b^8))/b^8 + 16*a^2*b*tan(c
/2 + (d*x)/2)))/b^5 - (4*(4*a*b^7 + 12*a^3*b^5))/b^8 + (4*tan(c/2 + (d*x)/2
)^2*(4*a*b^7 + 12*a^3*b^5))/b^8 + (16*tan(c/2 + (d*x)/2)*(6*a^4 + 2*a^2*b^2
))/b^4))/b^5 + (2*(3*a^2 + b^2)*((4*(4*a*b^7 + 12*a^3*b^5))/b^8 + (2*(3*a^2
 + b^2)*((4*(a*b^10 + 4*a^3*b^8))/b^8 - (4*tan(c/2 + (d*x)/2)^2*(3*a*b^10 +
```

```
 4*a^3*b^8))/b^8 + 16*a^2*b*tan(c/2 + (d*x)/2)))/b^5 - (4*tan(c/2 + (d*x)/2
)^2*(4*a*b^7 + 12*a^3*b^5))/b^8 - (16*tan(c/2 + (d*x)/2)*(6*a^4 + 2*a^2*b^2
))/b^4))/b^5 + (8*tan(c/2 + (d*x)/2)^2*(4*a*b^4 + 36*a^5 + 24*a^3*b^2))/b^8
))*(3*a^2 + b^2)*4i)/(b^5*d)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec^3(c+dx)}{(a\cos(c+dx)+b\sin(c+dx))^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**3/(a*cos(d*x+c)+b*sin(d*x+c))**3,x)
```

```
[Out] Integral(sec(c + d*x)**3/(a*cos(c + d*x) + b*sin(c + d*x))**3, x)
```

3.139 $\int \frac{\sec^4(c+dx)}{(a\cos(c+dx)+b\sin(c+dx))^3}\,dx$

Optimal. Leaf size=383

$$-\frac{4a^3\tanh^{-1}(\sin(c+dx))}{b^6d}+\frac{4a^2\sec(c+dx)}{b^5d}-\frac{6a\left(a^2+b^2\right)\tanh^{-1}(\sin(c+dx))}{b^6d}-\frac{8a^2\sqrt{a^2+b^2}\,\tanh^{-1}\left(\frac{b\cos(c+dx)-}{\sqrt{a^2+}}\right.}{b^6d}$$

```
[Out] -4*a^3*arctanh(sin(d*x+c))/b^6/d-3/2*a*arctanh(sin(d*x+c))/b^4/d-6*a*(a^2+b
^2)*arctanh(sin(d*x+c))/b^6/d-2*(a^2+b^2)^(3/2)*arctanh((b*cos(d*x+c)-a*sin
(d*x+c))/(a^2+b^2)^(1/2))/b^6/d+4*a^2*sec(d*x+c)/b^5/d+2*(a^2+b^2)*sec(d*x+
c)/b^5/d+1/3*sec(d*x+c)^3/b^3/d-1/2*(a^2+b^2)*(b*cos(d*x+c)-a*sin(d*x+c))/b
^4/d/(a*cos(d*x+c)+b*sin(d*x+c))^2+4*a*(a^2+b^2)/b^5/d/(a*cos(d*x+c)+b*sin(
d*x+c))-8*a^2*arctanh((b*cos(d*x+c)-a*sin(d*x+c))/(a^2+b^2)^(1/2))*(a^2+b^2
)^(1/2)/b^6/d-1/2*arctanh((b*cos(d*x+c)-a*sin(d*x+c))/(a^2+b^2)^(1/2))*(a^2
+b^2)^(1/2)/b^4/d-3/2*a*sec(d*x+c)*tan(d*x+c)/b^4/d
```

Rubi [A] time = 0.79, antiderivative size = 383, normalized size of antiderivative = 1.00, number of steps used = 31, number of rules used = 8, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3106, 3076, 3074, 206, 3104, 3770, 3094, 3768}

$$\frac{4a^2\sec(c+dx)}{b^5d}+\frac{2\left(a^2+b^2\right)\sec(c+dx)}{b^5d}-\frac{4a^3\tanh^{-1}(\sin(c+dx))}{b^6d}-\frac{6a\left(a^2+b^2\right)\tanh^{-1}(\sin(c+dx))}{b^6d}+\frac{}{b^5d(a\cos}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^4/(a*Cos[c + d*x] + b*Sin[c + d*x])^3,x]
```

```
[Out] (-4*a^3*ArcTanh[Sin[c + d*x]])/(b^6*d) - (3*a*ArcTanh[Sin[c + d*x]])/(2*b^4
*d) - (6*a*(a^2 + b^2)*ArcTanh[Sin[c + d*x]])/(b^6*d) - (8*a^2*Sqrt[a^2 + b
^2]*ArcTanh[(b*Cos[c + d*x] - a*Sin[c + d*x])/Sqrt[a^2 + b^2]])/(b^6*d) - (
Sqrt[a^2 + b^2]*ArcTanh[(b*Cos[c + d*x] - a*Sin[c + d*x])/Sqrt[a^2 + b^2]])
/(2*b^4*d) - (2*(a^2 + b^2)^(3/2)*ArcTanh[(b*Cos[c + d*x] - a*Sin[c + d*x])
/Sqrt[a^2 + b^2]])/(b^6*d) + (4*a^2*Sec[c + d*x])/(b^5*d) + (2*(a^2 + b^2)*
Sec[c + d*x])/(b^5*d) + Sec[c + d*x]^3/(3*b^3*d) - ((a^2 + b^2)*(b*Cos[c +
d*x] - a*Sin[c + d*x]))/(2*b^4*d*(a*Cos[c + d*x] + b*Sin[c + d*x])^2) + (4*
a*(a^2 + b^2))/(b^5*d*(a*Cos[c + d*x] + b*Sin[c + d*x])) - (3*a*Sec[c + d*x
]*Tan[c + d*x])/(2*b^4*d)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3074

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x
_Symbol] :> -Dist[d^(-1), Subst[Int[1/(a^2 + b^2 - x^2), x], x, b*Cos[c + d
*x] - a*Sin[c + d*x]], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

Rule 3076

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x
_Symbol] :> Simp[((b*Cos[c + d*x] - a*Sin[c + d*x])*(a*Cos[c + d*x] + b*Sin
[c + d*x])^(n + 1))/(d*(n + 1)*(a^2 + b^2)), x] + Dist[(n + 2)/((n + 1)*(a^
2 + b^2)), Int[(a*Cos[c + d*x] + b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{
a, b, c, d}, x] && NeQ[a^2 + b^2, 0] && LtQ[n, -1] && NeQ[n, -2]
```

Rule 3094

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_)/co
s[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[(a*Cos[c + d*x] + b*Sin[c + d*x])^
(n + 1)/(b*d*(n + 1)), x] + (Dist[1/b^2, Int[(a*Cos[c + d*x] + b*Sin[c + d*
x])^(n + 2)/Cos[c + d*x], x], x] - Dist[a/b^2, Int[(a*Cos[c + d*x] + b*Sin[
c + d*x])^(n + 1), x], x]) /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0] &
& LtQ[n, -1]
```

Rule 3104

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_)/(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin
[(c_.) + (d_.)*(x_)]), x_Symbol] :> -Simp[Cos[c + d*x]^(m + 1)/(b*d*(m + 1)
), x] + (-Dist[a/b^2, Int[Cos[c + d*x]^(m + 1), x], x] + Dist[(a^2 + b^2)/b
^2, Int[Cos[c + d*x]^(m + 2)/(a*Cos[c + d*x] + b*Sin[c + d*x]), x], x]) /;
FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0] && LtQ[m, -1]
```

Rule 3106

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin
[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(a^2 + b^2)/b^2, Int[Cos[c +
d*x]^(m + 2)*(a*Cos[c + d*x] + b*Sin[c + d*x])^n, x], x] + (Dist[1/b^2, Int
[Cos[c + d*x]^m*(a*Cos[c + d*x] + b*Sin[c + d*x])^(n + 2), x], x] - Dist[(2
*a)/b^2, Int[Cos[c + d*x]^(m + 1)*(a*Cos[c + d*x] + b*Sin[c + d*x])^(n + 1)
, x], x]) /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0] && LtQ[n, -1] && L
tQ[m, -1]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
```

```
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\sec^4(c+dx)}{(a\cos(c+dx)+b\sin(c+dx))^3}\,dx = \frac{\int \frac{\sec^4(c+dx)}{a\cos(c+dx)+b\sin(c+dx)}\,dx}{b^2} - \frac{(2a)\int \frac{\sec^3(c+dx)}{(a\cos(c+dx)+b\sin(c+dx))^2}\,dx}{b^2} + \frac{\left(a^2+b^2\right)}{}$$

$$= \frac{\sec^3(c+dx)}{3b^3d} - \frac{a\int \sec^3(c+dx)\,dx}{b^4} - \frac{(2a)\int \sec^3(c+dx)\,dx}{b^4} + \frac{\left(4a^2\right)\int}{}$$

$$= \frac{4a^2\sec(c+dx)}{b^5d} + \frac{\sec^3(c+dx)}{3b^3d} - \frac{\left(a^2+b^2\right)(b\cos(c+dx)-a\sin(c+d}{2b^4d(a\cos(c+dx)+b\sin(c+dx))}$$

$$= -\frac{4a^3\tanh^{-1}(\sin(c+dx))}{b^6d} - \frac{3a\tanh^{-1}(\sin(c+dx))}{2b^4d} + \frac{4a^2\sec(c+dx)}{b^5d}$$

$$= -\frac{4a^3\tanh^{-1}(\sin(c+dx))}{b^6d} - \frac{3a\tanh^{-1}(\sin(c+dx))}{2b^4d} - \frac{4a^2\sqrt{a^2+b^2}\ \tan}{}$$

Mathematica [C] time = 2.46, size = 688, normalized size = 1.80

$$\frac{\sec^3(c+dx)(a\cos(c+dx)+b\sin(c+dx))\left(\frac{6b^2\left(a^2+b^2\right)^2\sin(c+dx)}{a} + 2b\left(36a^2+13b^2\right)(a\cos(c+dx)+b\sin(c+dx)\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^4/(a*Cos[c + d*x] + b*Sin[c + d*x])^3,x]
```

```
[Out] (Sec[c + d*x]^3*(a*Cos[c + d*x] + b*Sin[c + d*x])*((6*b^2*(a^2 + b^2)^2*Sin
[c + d*x])/a + (6*(a - I*b)*(a + I*b)*b*(8*a^2 - b^2)*(a*Cos[c + d*x] + b*S
in[c + d*x]))/a + 2*b*(36*a^2 + 13*b^2)*(a*Cos[c + d*x] + b*Sin[c + d*x])^2
```

```
 + 60*Sqrt[a^2 + b^2]*(4*a^2 + b^2)*ArcTanh[(-b + a*Tan[(c + d*x)/2])/Sqrt[
a^2 + b^2]]*(a*Cos[c + d*x] + b*Sin[c + d*x])^2 + 30*a*(4*a^2 + 3*b^2)*Log[
Cos[(c + d*x)/2] - Sin[(c + d*x)/2]]*(a*Cos[c + d*x] + b*Sin[c + d*x])^2 -
30*a*(4*a^2 + 3*b^2)*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]*(a*Cos[c + d*
x] + b*Sin[c + d*x])^2 + (b^2*(-9*a + b)*(a*Cos[c + d*x] + b*Sin[c + d*x])^
2)/(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^2 + (2*b^3*Sin[(c + d*x)/2]*(a*Cos
[c + d*x] + b*Sin[c + d*x])^2)/(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^3 + (2
*b*(36*a^2 + 13*b^2)*Sin[(c + d*x)/2]*(a*Cos[c + d*x] + b*Sin[c + d*x])^2)/
(Cos[(c + d*x)/2] - Sin[(c + d*x)/2]) - (2*b^3*Sin[(c + d*x)/2]*(a*Cos[c +
d*x] + b*Sin[c + d*x])^2)/(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^3 + (b^2*(9
*a + b)*(a*Cos[c + d*x] + b*Sin[c + d*x])^2)/(Cos[(c + d*x)/2] + Sin[(c + d
*x)/2])^2 - (2*b*(36*a^2 + 13*b^2)*Sin[(c + d*x)/2]*(a*Cos[c + d*x] + b*Sin
[c + d*x])^2)/(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])))/(12*b^6*d*(a + b*Tan[
c + d*x])^3)
```

fricas [A] time = 0.89, size = 564, normalized size = 1.47

$$\frac{4\,b^5 + 30\left(4\,a^4 b + a^2 b^3 - b^5\right)\cos\left(dx+c\right)^4 + 20\left(2\,a^2 b^3 + b^5\right)\cos\left(dx+c\right)^2 + 15\left(\left(4\,a^4 - 3\,a^2 b^2 - b^4\right)\cos\left(dx+c\right)^5\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4/(a*cos(d*x+c)+b*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/12*(4*b^5 + 30*(4*a^4*b + a^2*b^3 - b^5)*cos(d*x + c)^4 + 20*(2*a^2*b^3 +
 b^5)*cos(d*x + c)^2 + 15*((4*a^4 - 3*a^2*b^2 - b^4)*cos(d*x + c)^5 + 2*(4*
a^3*b + a*b^3)*cos(d*x + c)^4*sin(d*x + c) + (4*a^2*b^2 + b^4)*cos(d*x + c)
^3)*sqrt(a^2 + b^2)*log(-(2*a*b*cos(d*x + c)*sin(d*x + c) + (a^2 - b^2)*cos
(d*x + c)^2 - 2*a^2 - b^2 + 2*sqrt(a^2 + b^2)*(b*cos(d*x + c) - a*sin(d*x +
 c)))/(2*a*b*cos(d*x + c)*sin(d*x + c) + (a^2 - b^2)*cos(d*x + c)^2 + b^2))
 - 15*((4*a^5 - a^3*b^2 - 3*a*b^4)*cos(d*x + c)^5 + 2*(4*a^4*b + 3*a^2*b^3)
*cos(d*x + c)^4*sin(d*x + c) + (4*a^3*b^2 + 3*a*b^4)*cos(d*x + c)^3)*log(si
n(d*x + c) + 1) + 15*((4*a^5 - a^3*b^2 - 3*a*b^4)*cos(d*x + c)^5 + 2*(4*a^4
*b + 3*a^2*b^3)*cos(d*x + c)^4*sin(d*x + c) + (4*a^3*b^2 + 3*a*b^4)*cos(d*x
 + c)^3)*log(-sin(d*x + c) + 1) - 10*(a*b^4*cos(d*x + c) - 6*(3*a^3*b^2 + 2
*a*b^4)*cos(d*x + c)^3)*sin(d*x + c))/(2*a*b^7*d*cos(d*x + c)^4*sin(d*x + c
) + b^8*d*cos(d*x + c)^3 + (a^2*b^6 - b^8)*d*cos(d*x + c)^5)
```

giac [A] time = 2.07, size = 510, normalized size = 1.33

$$-\frac{\frac{15\left(4\,a^3+3\,ab^2\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)}{b^6} - \frac{15\left(4\,a^3+3\,ab^2\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)}{b^6} + \frac{15\left(4\,a^4+5\,a^2b^2+b^4\right)\log\left(\frac{\left|2\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-2\,b-2\,\sqrt{a^2+b^2}\right|}{\left|2\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-2\,b+2\,\sqrt{a^2+b^2}\right|}\right)}{\sqrt{a^2+b^2}\,b^6} + }{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4/(a*cos(d*x+c)+b*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -1/6*(15*(4*a^3 + 3*a*b^2)*log(abs(tan(1/2*d*x + 1/2*c) + 1))/b^6 - 15*(4*a
^3 + 3*a*b^2)*log(abs(tan(1/2*d*x + 1/2*c) - 1))/b^6 + 15*(4*a^4 + 5*a^2*b^
2 + b^4)*log(abs(2*a*tan(1/2*d*x + 1/2*c) - 2*b - 2*sqrt(a^2 + b^2))/abs(2*
a*tan(1/2*d*x + 1/2*c) - 2*b + 2*sqrt(a^2 + b^2)))/(sqrt(a^2 + b^2)*b^6) +
2*(9*a*b*tan(1/2*d*x + 1/2*c)^5 + 36*a^2*tan(1/2*d*x + 1/2*c)^4 + 18*b^2*ta
n(1/2*d*x + 1/2*c)^4 - 72*a^2*tan(1/2*d*x + 1/2*c)^2 - 24*b^2*tan(1/2*d*x +
 1/2*c)^2 - 9*a*b*tan(1/2*d*x + 1/2*c) + 36*a^2 + 14*b^2)/((tan(1/2*d*x + 1
/2*c)^2 - 1)^3*b^5) + 6*(7*a^5*b*tan(1/2*d*x + 1/2*c)^3 + 5*a^3*b^3*tan(1/2
*d*x + 1/2*c)^3 - 2*a*b^5*tan(1/2*d*x + 1/2*c)^3 + 8*a^6*tan(1/2*d*x + 1/2*
c)^2 - 9*a^4*b^2*tan(1/2*d*x + 1/2*c)^2 - 15*a^2*b^4*tan(1/2*d*x + 1/2*c)^2
 + 2*b^6*tan(1/2*d*x + 1/2*c)^2 - 25*a^5*b*tan(1/2*d*x + 1/2*c) - 23*a^3*b^
3*tan(1/2*d*x + 1/2*c) + 2*a*b^5*tan(1/2*d*x + 1/2*c) - 8*a^6 - 7*a^4*b^2 +
 a^2*b^4)/((a*tan(1/2*d*x + 1/2*c)^2 - 2*b*tan(1/2*d*x + 1/2*c) - a)^2*a^2*
b^5))/d
```

maple [B] time = 0.40, size = 1125, normalized size = 2.94

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^4/(a*cos(d*x+c)+b*sin(d*x+c))^3,x)
```

```
[Out] 15/2/d*a/b^4*ln(tan(1/2*d*x+1/2*c)-1)-15/2/d*a/b^4*ln(tan(1/2*d*x+1/2*c)+1)
+2/d/(tan(1/2*d*x+1/2*c)^2*a-2*b*tan(1/2*d*x+1/2*c)-a)^2/a*tan(1/2*d*x+1/2*
c)^3+15/d/b/(tan(1/2*d*x+1/2*c)^2*a-2*b*tan(1/2*d*x+1/2*c)-a)^2*tan(1/2*d*x
+1/2*c)^2-2/d/(tan(1/2*d*x+1/2*c)^2*a-2*b*tan(1/2*d*x+1/2*c)-a)^2/a*tan(1/2
*d*x+1/2*c)+7/d/b^3/(tan(1/2*d*x+1/2*c)^2*a-2*b*tan(1/2*d*x+1/2*c)-a)^2*a^2
+5/d/b^2/(a^2+b^2)^(1/2)*arctanh(1/2*(2*a*tan(1/2*d*x+1/2*c)-2*b)/(a^2+b^2)
^(1/2))-5/2/d/b^3/(tan(1/2*d*x+1/2*c)-1)+5/2/d/b^3/(tan(1/2*d*x+1/2*c)+1)-1
/d/b/(tan(1/2*d*x+1/2*c)^2*a-2*b*tan(1/2*d*x+1/2*c)-a)^2+8/d/b^5/(tan(1/2*d
*x+1/2*c)^2*a-2*b*tan(1/2*d*x+1/2*c)-a)^2*a^4-3/2/d/b^4/(tan(1/2*d*x+1/2*c)
-1)^2*a-6/d/b^5/(tan(1/2*d*x+1/2*c)-1)*a^2-3/2/d/b^4/(tan(1/2*d*x+1/2*c)-1)
*a+10/d*a^3/b^6*ln(tan(1/2*d*x+1/2*c)-1)+3/2/d/b^4/(tan(1/2*d*x+1/2*c)+1)^2
*a+6/d/b^5/(tan(1/2*d*x+1/2*c)+1)*a^2-3/2/d/b^4/(tan(1/2*d*x+1/2*c)+1)*a-10
/d*a^3/b^6*ln(tan(1/2*d*x+1/2*c)+1)-1/3/d/b^3/(tan(1/2*d*x+1/2*c)-1)^3-1/2/
d/b^3/(tan(1/2*d*x+1/2*c)-1)^2+1/3/d/b^3/(tan(1/2*d*x+1/2*c)+1)^3-1/2/d/b^3
/(tan(1/2*d*x+1/2*c)+1)^2-5/d/b^2/(tan(1/2*d*x+1/2*c)^2*a-2*b*tan(1/2*d*x+1
/2*c)-a)^2*a*tan(1/2*d*x+1/2*c)^3+9/d/b^3/(tan(1/2*d*x+1/2*c)^2*a-2*b*tan(1
/2*d*x+1/2*c)-a)^2*a^2*tan(1/2*d*x+1/2*c)^2-2/d*b/(tan(1/2*d*x+1/2*c)^2*a-2
*b*tan(1/2*d*x+1/2*c)-a)^2/a^2*tan(1/2*d*x+1/2*c)^2+23/d/b^2/(tan(1/2*d*x+1
/2*c)^2*a-2*b*tan(1/2*d*x+1/2*c)-a)^2*a*tan(1/2*d*x+1/2*c)+25/d/b^4/(a^2+b^
```

```
2)^(1/2)*arctanh(1/2*(2*a*tan(1/2*d*x+1/2*c)-2*b)/(a^2+b^2)^(1/2))*a^2+25/d
/b^4/(tan(1/2*d*x+1/2*c)^2*a-2*b*tan(1/2*d*x+1/2*c)-a)^2*a^3*tan(1/2*d*x+1/
2*c)+20/d/b^6/(a^2+b^2)^(1/2)*arctanh(1/2*(2*a*tan(1/2*d*x+1/2*c)-2*b)/(a^2
+b^2)^(1/2))*a^4-7/d/b^4/(tan(1/2*d*x+1/2*c)^2*a-2*b*tan(1/2*d*x+1/2*c)-a)^
2*a^3*tan(1/2*d*x+1/2*c)^3-8/d/b^5/(tan(1/2*d*x+1/2*c)^2*a-2*b*tan(1/2*d*x+
1/2*c)-a)^2*a^4*tan(1/2*d*x+1/2*c)^2
```

maxima [B] time = 0.52, size = 902, normalized size = 2.36

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4/(a*cos(d*x+c)+b*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/6*(2*(60*a^6 + 35*a^4*b^2 - 3*a^2*b^4 + (210*a^5*b + 125*a^3*b^3 - 6*a*b^
5)*sin(d*x + c)/(cos(d*x + c) + 1) - 2*(120*a^6 - 10*a^4*b^2 - 55*a^2*b^4 +
 3*b^6)*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 - 2*(330*a^5*b + 205*a^3*b^3 -
12*a*b^5)*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 2*(180*a^6 - 95*a^4*b^2 - 1
20*a^2*b^4 + 9*b^6)*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + 12*(60*a^5*b + 35
*a^3*b^3 - 3*a*b^5)*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 6*(40*a^6 - 30*a^
4*b^2 - 35*a^2*b^4 + 3*b^6)*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 - 6*(50*a^5
*b + 25*a^3*b^3 - 4*a*b^5)*sin(d*x + c)^7/(cos(d*x + c) + 1)^7 + 3*(20*a^6
- 15*a^4*b^2 - 15*a^2*b^4 + 2*b^6)*sin(d*x + c)^8/(cos(d*x + c) + 1)^8 + 3*
(10*a^5*b + 5*a^3*b^3 - 2*a*b^5)*sin(d*x + c)^9/(cos(d*x + c) + 1)^9)/(a^4*
b^5 + 4*a^3*b^6*sin(d*x + c)/(cos(d*x + c) + 1) - 16*a^3*b^6*sin(d*x + c)^3
/(cos(d*x + c) + 1)^3 + 24*a^3*b^6*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 16
*a^3*b^6*sin(d*x + c)^7/(cos(d*x + c) + 1)^7 + 4*a^3*b^6*sin(d*x + c)^9/(co
s(d*x + c) + 1)^9 - a^4*b^5*sin(d*x + c)^10/(cos(d*x + c) + 1)^10 - (5*a^4*
b^5 - 4*a^2*b^7)*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 2*(5*a^4*b^5 - 6*a^2
*b^7)*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 - 2*(5*a^4*b^5 - 6*a^2*b^7)*sin(d
*x + c)^6/(cos(d*x + c) + 1)^6 + (5*a^4*b^5 - 4*a^2*b^7)*sin(d*x + c)^8/(co
s(d*x + c) + 1)^8) - 15*(4*a^3 + 3*a*b^2)*log(sin(d*x + c)/(cos(d*x + c) +
1) + 1)/b^6 + 15*(4*a^3 + 3*a*b^2)*log(sin(d*x + c)/(cos(d*x + c) + 1) - 1)
/b^6 - 15*(4*a^4 + 5*a^2*b^2 + b^4)*log((b - a*sin(d*x + c)/(cos(d*x + c) +
 1) + sqrt(a^2 + b^2))/(b - a*sin(d*x + c)/(cos(d*x + c) + 1) - sqrt(a^2 +
b^2)))/(sqrt(a^2 + b^2)*b^6))/d
```

mupad [B] time = 3.82, size = 1203, normalized size = 3.14

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^4*(a*cos(c + d*x) + b*sin(c + d*x))^3),x)
```

```
[Out] ((60*a^4 - 3*b^4 + 35*a^2*b^2)/(3*b^5) + (tan(c/2 + (d*x)/2)*(210*a^4 - 6*b
^4 + 125*a^2*b^2))/(3*a*b^4) + (tan(c/2 + (d*x)/2)^8*(20*a^6 + 2*b^6 - 15*a
^2*b^4 - 15*a^4*b^2))/(a^2*b^5) - (2*tan(c/2 + (d*x)/2)^6*(40*a^6 + 3*b^6 -
 35*a^2*b^4 - 30*a^4*b^2))/(a^2*b^5) - (2*tan(c/2 + (d*x)/2)^2*(120*a^6 + 3
*b^6 - 55*a^2*b^4 - 10*a^4*b^2))/(3*a^2*b^5) + (2*tan(c/2 + (d*x)/2)^4*(180
*a^6 + 9*b^6 - 120*a^2*b^4 - 95*a^4*b^2))/(3*a^2*b^5) + (tan(c/2 + (d*x)/2)
^9*(10*a^4 - 2*b^4 + 5*a^2*b^2))/(a*b^4) - (2*tan(c/2 + (d*x)/2)^7*(50*a^4
- 4*b^4 + 25*a^2*b^2))/(a*b^4) + (4*tan(c/2 + (d*x)/2)^5*(60*a^4 - 3*b^4 +
35*a^2*b^2))/(a*b^4) - (2*tan(c/2 + (d*x)/2)^3*(330*a^4 - 12*b^4 + 205*a^2*
b^2))/(3*a*b^4))/(d*(tan(c/2 + (d*x)/2)^8*(5*a^2 - 4*b^2) - tan(c/2 + (d*x)
/2)^2*(5*a^2 - 4*b^2) + tan(c/2 + (d*x)/2)^4*(10*a^2 - 12*b^2) - tan(c/2 +
(d*x)/2)^6*(10*a^2 - 12*b^2) - a^2*tan(c/2 + (d*x)/2)^10 + a^2 - 16*a*b*tan
(c/2 + (d*x)/2)^3 + 24*a*b*tan(c/2 + (d*x)/2)^5 - 16*a*b*tan(c/2 + (d*x)/2)
^7 + 4*a*b*tan(c/2 + (d*x)/2)^9 + 4*a*b*tan(c/2 + (d*x)/2))) - (atanh((3000
*a^2*tan(c/2 + (d*x)/2))/(3000*a^2 + (7000*a^4)/b^2 + (4000*a^6)/b^4) + (70
00*a^4*tan(c/2 + (d*x)/2))/(7000*a^4 + 3000*a^2*b^2 + (4000*a^6)/b^2) + (40
00*a^6*tan(c/2 + (d*x)/2))/(4000*a^6 + 3000*a^2*b^4 + 7000*a^4*b^2))*(15*a*
b^2 + 20*a^3))/(b^6*d) + (5*atanh((1000*a^2*(a^2 + b^2)^(1/2))/(1000*a^2*b
+ (5000*a^4)/b + (4000*a^6)/b^3 + 10000*a^3*tan(c/2 + (d*x)/2) + 2000*a*b^2
*tan(c/2 + (d*x)/2) + (8000*a^5*tan(c/2 + (d*x)/2))/b^2) + (4000*a^4*(a^2 +
 b^2)^(1/2))/(5000*a^4*b + 1000*a^2*b^3 + (4000*a^6)/b + 8000*a^5*tan(c/2 +
 (d*x)/2) + 2000*a*b^4*tan(c/2 + (d*x)/2) + 10000*a^3*b^2*tan(c/2 + (d*x)/2
)) + (9000*a^3*tan(c/2 + (d*x)/2)*(a^2 + b^2)^(1/2))/(5000*a^4 + 1000*a^2*b
^2 + (4000*a^6)/b^2 + 2000*a*b^3*tan(c/2 + (d*x)/2) + 10000*a^3*b*tan(c/2 +
 (d*x)/2) + (8000*a^5*tan(c/2 + (d*x)/2))/b) + (4000*a^5*tan(c/2 + (d*x)/2)
*(a^2 + b^2)^(1/2))/(4000*a^6 + 1000*a^2*b^4 + 5000*a^4*b^2 + 2000*a*b^5*ta
n(c/2 + (d*x)/2) + 8000*a^5*b*tan(c/2 + (d*x)/2) + 10000*a^3*b^3*tan(c/2 +
(d*x)/2)) + (2000*a*tan(c/2 + (d*x)/2)*(a^2 + b^2)^(1/2))/(1000*a^2 + (5000
*a^4)/b^2 + (4000*a^6)/b^4 + (10000*a^3*tan(c/2 + (d*x)/2))/b + (8000*a^5*t
an(c/2 + (d*x)/2))/b^3 + 2000*a*b*tan(c/2 + (d*x)/2)))*(4*a^2 + b^2)*(a^2 +
 b^2)^(1/2))/(b^6*d)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec^4(c+dx)}{(a\cos(c+dx)+b\sin(c+dx))^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**4/(a*cos(d*x+c)+b*sin(d*x+c))**3,x)
```

```
[Out] Integral(sec(c + d*x)**4/(a*cos(c + d*x) + b*sin(c + d*x))**3, x)
```

3.140 $\int \frac{\sec^5(c+dx)}{(a\cos(c+dx)+b\sin(c+dx))^3}\,dx$

Optimal. Leaf size=232

$$\frac{3\left(a^2+b^2\right)\left(5a^2+b^2\right)\log(\tan(c+dx))}{b^7d}+\frac{3\left(a^2+b^2\right)\left(5a^2+b^2\right)\log(a\cot(c+dx)+b)}{b^7d}-\frac{a\left(10a^2+9b^2\right)\tan(c+dx)}{b^6d}$$

```
[Out] -1/2*(a^2+b^2)^3/a^2/b^5/d/(b+a*cot(d*x+c))^2-(5*a^2-b^2)*(a^2+b^2)^2/a^2/b
^6/d/(b+a*cot(d*x+c))+3*(a^2+b^2)*(5*a^2+b^2)*ln(b+a*cot(d*x+c))/b^7/d+3*(a
^2+b^2)*(5*a^2+b^2)*ln(tan(d*x+c))/b^7/d-a*(10*a^2+9*b^2)*tan(d*x+c)/b^6/d+
3/2*(2*a^2+b^2)*tan(d*x+c)^2/b^5/d-a*tan(d*x+c)^3/b^4/d+1/4*tan(d*x+c)^4/b^
3/d
```

Rubi [A] time = 0.24, antiderivative size = 232, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {3088, 894}

$$\frac{3\left(2a^2+b^2\right)\tan^2(c+dx)}{2b^5d}-\frac{a\left(10a^2+9b^2\right)\tan(c+dx)}{b^6d}-\frac{\left(5a^2-b^2\right)\left(a^2+b^2\right)^2}{a^2b^6d(a\cot(c+dx)+b)}-\frac{\left(a^2+b^2\right)^3}{2a^2b^5d(a\cot(c+dx)+b)^2}+\frac{3\left(\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^5/(a*Cos[c + d*x] + b*Sin[c + d*x])^3,x]
```

```
[Out] -(a^2 + b^2)^3/(2*a^2*b^5*d*(b + a*Cot[c + d*x])^2) - ((5*a^2 - b^2)*(a^2 +
 b^2)^2)/(a^2*b^6*d*(b + a*Cot[c + d*x])) + (3*(a^2 + b^2)*(5*a^2 + b^2)*Lo
g[b + a*Cot[c + d*x]])/(b^7*d) + (3*(a^2 + b^2)*(5*a^2 + b^2)*Log[Tan[c + d
*x]])/(b^7*d) - (a*(10*a^2 + 9*b^2)*Tan[c + d*x])/(b^6*d) + (3*(2*a^2 + b^2
)*Tan[c + d*x]^2)/(2*b^5*d) - (a*Tan[c + d*x]^3)/(b^4*d) + Tan[c + d*x]^4/(
4*b^3*d)
```

Rule 894

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ
[m, 0] && ILtQ[n, 0]))
```

Rule 3088

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[(x^m*(b +
 a*x)^n)/(1 + x^2)^((m + n + 2)/2), x], x, Cot[c + d*x]], x] /; FreeQ[{a, b
```

```
, c, d}, x] && IntegerQ[n] && IntegerQ[(m + n)/2] && NeQ[n, -1] &&  !(GtQ[n
, 0] && GtQ[m, 1])
```

Rubi steps

$$\int \frac{\sec^5(c+dx)}{(a\cos(c+dx)+b\sin(c+dx))^3}\,dx = -\frac{\text{Subst}\left(\int \frac{\left(1+x^2\right)^3}{x^5(b+ax)^3}\,dx, x, \cot(c+dx)\right)}{d}$$

$$= -\frac{\text{Subst}\left(\int \left(\frac{1}{b^3x^5} - \frac{3a}{b^4x^4} + \frac{3\left(2a^2+b^2\right)}{b^5x^3} + \frac{-10a^3-9ab^2}{b^6x^2} + \frac{3\left(5a^4+6a^2b^2+b^4\right)}{b^7x} - \frac{\left(a^2+b\right.}{ab^5(b+}\right.\right.}{d}$$

$$= -\frac{\left(a^2+b^2\right)^3}{2a^2b^5d(b+a\cot(c+dx))^2} - \frac{\left(5a^2-b^2\right)\left(a^2+b^2\right)^2}{a^2b^6d(b+a\cot(c+dx))} + \frac{3\left(a^2+b^2\right)\left(\right.}{}$$

Mathematica [A] time = 1.34, size = 272, normalized size = 1.17

$$\frac{4a^2b^4\tan^4(c+dx) + b^4\sec^4(c+dx)\left(a^2-2ab\tan(c+dx)+3b^2\right) - 20ab^3\left(a^2+b^2\right)\tan^3(c+dx) + 4b^2\tan^2(c+}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^5/(a*Cos[c + d*x] + b*Sin[c + d*x])^3,x]
```

```
[Out] (2*(a^2 + b^2)*(19*a^4 + 16*a^2*b^2 - 3*b^4 + 6*a^2*(5*a^2 + b^2)*Log[a + b
*Tan[c + d*x]]) + b^6*Sec[c + d*x]^6 + 4*a*b*(4*a^4 + 17*a^2*b^2 + 11*b^4 +
 6*(5*a^4 + 6*a^2*b^2 + b^4)*Log[a + b*Tan[c + d*x]])*Tan[c + d*x] + 4*b^2*
(-13*a^4 - 10*a^2*b^2 + 3*(5*a^4 + 6*a^2*b^2 + b^4)*Log[a + b*Tan[c + d*x]]
)*Tan[c + d*x]^2 - 20*a*b^3*(a^2 + b^2)*Tan[c + d*x]^3 + 4*a^2*b^4*Tan[c +
d*x]^4 + b^4*Sec[c + d*x]^4*(a^2 + 3*b^2 - 2*a*b*Tan[c + d*x]))/(4*b^7*d*(a
 + b*Tan[c + d*x])^2)
```

fricas [B] time = 0.78, size = 476, normalized size = 2.05

$$\frac{8\left(15\,a^4b^2+13\,a^2b^4\right)\cos\left(dx+c\right)^6 + b^6 - 2\left(45\,a^4b^2+44\,a^2b^4+3\,b^6\right)\cos\left(dx+c\right)^4 + \left(5\,a^2b^4+3\,b^6\right)\cos\left(dx+c\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5/(a*cos(d*x+c)+b*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/4*(8*(15*a^4*b^2 + 13*a^2*b^4)*cos(d*x + c)^6 + b^6 - 2*(45*a^4*b^2 + 44*
a^2*b^4 + 3*b^6)*cos(d*x + c)^4 + (5*a^2*b^4 + 3*b^6)*cos(d*x + c)^2 + 6*((
5*a^6 + a^4*b^2 - 5*a^2*b^4 - b^6)*cos(d*x + c)^6 + 2*(5*a^5*b + 6*a^3*b^3
+ a*b^5)*cos(d*x + c)^5*sin(d*x + c) + (5*a^4*b^2 + 6*a^2*b^4 + b^6)*cos(d*
x + c)^4)*log(2*a*b*cos(d*x + c)*sin(d*x + c) + (a^2 - b^2)*cos(d*x + c)^2
+ b^2) - 6*((5*a^6 + a^4*b^2 - 5*a^2*b^4 - b^6)*cos(d*x + c)^6 + 2*(5*a^5*b
 + 6*a^3*b^3 + a*b^5)*cos(d*x + c)^5*sin(d*x + c) + (5*a^4*b^2 + 6*a^2*b^4
+ b^6)*cos(d*x + c)^4)*log(cos(d*x + c)^2) - 2*(a*b^5*cos(d*x + c) + 2*(15*
a^5*b - 2*a^3*b^3 - 13*a*b^5)*cos(d*x + c)^5 + 10*(a^3*b^3 + a*b^5)*cos(d*x
 + c)^3)*sin(d*x + c))/(2*a*b^8*d*cos(d*x + c)^5*sin(d*x + c) + b^9*d*cos(d
*x + c)^4 + (a^2*b^7 - b^9)*d*cos(d*x + c)^6)
```

giac [A] time = 1.00, size = 243, normalized size = 1.05

$$\frac{\frac{12\left(5a^4+6a^2b^2+b^4\right)\log\left(|b\tan(dx+c)+a|\right)}{b^7}-\frac{2\left(45a^4b^2\tan(dx+c)^2+54a^2b^4\tan(dx+c)^2+9b^6\tan(dx+c)^2+78a^5b\tan(dx+c)+84a^3b^3\tan(dx+c)+6ab^5\,t\right.}{(b\tan(dx+c)+a)^2b^7}}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5/(a*cos(d*x+c)+b*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/4*(12*(5*a^4 + 6*a^2*b^2 + b^4)*log(abs(b*tan(d*x + c) + a))/b^7 - 2*(45*
a^4*b^2*tan(d*x + c)^2 + 54*a^2*b^4*tan(d*x + c)^2 + 9*b^6*tan(d*x + c)^2 +
 78*a^5*b*tan(d*x + c) + 84*a^3*b^3*tan(d*x + c) + 6*a*b^5*tan(d*x + c) + 3
4*a^6 + 33*a^4*b^2 + b^6)/((b*tan(d*x + c) + a)^2*b^7) + (b^9*tan(d*x + c)^
4 - 4*a*b^8*tan(d*x + c)^3 + 12*a^2*b^7*tan(d*x + c)^2 + 6*b^9*tan(d*x + c)
^2 - 40*a^3*b^6*tan(d*x + c) - 36*a*b^8*tan(d*x + c))/b^12)/d
```

maple [A] time = 0.43, size = 321, normalized size = 1.38

$$\frac{\tan^4(dx+c)}{4b^3d}-\frac{a\left(\tan^3(dx+c)\right)}{b^4d}+\frac{3\left(\tan^2(dx+c)\right)a^2}{d\,b^5}+\frac{3\left(\tan^2(dx+c)\right)}{2b^3d}-\frac{10\tan(dx+c)\,a^3}{d\,b^6}-\frac{9a\tan(dx+c)}{b^4d}+\frac{15}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^5/(a*cos(d*x+c)+b*sin(d*x+c))^3,x)
```

```
[Out] 1/4*tan(d*x+c)^4/b^3/d-a*tan(d*x+c)^3/b^4/d+3/d/b^5*tan(d*x+c)^2*a^2+3/2*ta
n(d*x+c)^2/b^3/d-10/d/b^6*tan(d*x+c)*a^3-9*a*tan(d*x+c)/b^4/d+15/d/b^7*ln(a
+b*tan(d*x+c))*a^4+18/d/b^5*ln(a+b*tan(d*x+c))*a^2+3/d/b^3*ln(a+b*tan(d*x+c
))-1/2/d/b^7/(a+b*tan(d*x+c))^2*a^6-3/2/d/b^5/(a+b*tan(d*x+c))^2*a^4-3/2/d/
b^3/(a+b*tan(d*x+c))^2*a^2-1/2/b/d/(a+b*tan(d*x+c))^2+6/d*a^5/b^7/(a+b*tan(
d*x+c))+12/d*a^3/b^5/(a+b*tan(d*x+c))+6/d*a/b^3/(a+b*tan(d*x+c))
```

maxima [B] time = 0.39, size = 1053, normalized size = 4.54

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5/(a*cos(d*x+c)+b*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -(2*((15*a^7 + 18*a^5*b^2 + 3*a^3*b^4 - a*b^6)*sin(d*x + c)/(cos(d*x + c) +
 1) + (45*a^6*b + 54*a^4*b^3 + 9*a^2*b^5 - b^7)*sin(d*x + c)^2/(cos(d*x + c
) + 1)^2 - (75*a^7 + 70*a^5*b^2 - 9*a^3*b^4 - 5*a*b^6)*sin(d*x + c)^3/(cos(
d*x + c) + 1)^3 - 2*(90*a^6*b + 113*a^4*b^3 + 24*a^2*b^5 - 2*b^7)*sin(d*x +
 c)^4/(cos(d*x + c) + 1)^4 + 2*(75*a^7 + 60*a^5*b^2 - 17*a^3*b^4 - 5*a*b^6)
*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + 2*(135*a^6*b + 172*a^4*b^3 + 35*a^2*
b^5 - 3*b^7)*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 - 2*(75*a^7 + 60*a^5*b^2 -
 17*a^3*b^4 - 5*a*b^6)*sin(d*x + c)^7/(cos(d*x + c) + 1)^7 - 2*(90*a^6*b +
113*a^4*b^3 + 24*a^2*b^5 - 2*b^7)*sin(d*x + c)^8/(cos(d*x + c) + 1)^8 + (75
*a^7 + 70*a^5*b^2 - 9*a^3*b^4 - 5*a*b^6)*sin(d*x + c)^9/(cos(d*x + c) + 1)^
9 + (45*a^6*b + 54*a^4*b^3 + 9*a^2*b^5 - b^7)*sin(d*x + c)^10/(cos(d*x + c)
 + 1)^10 - (15*a^7 + 18*a^5*b^2 + 3*a^3*b^4 - a*b^6)*sin(d*x + c)^11/(cos(d
*x + c) + 1)^11)/(a^4*b^6 + 4*a^3*b^7*sin(d*x + c)/(cos(d*x + c) + 1) - 20*
a^3*b^7*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 40*a^3*b^7*sin(d*x + c)^5/(co
s(d*x + c) + 1)^5 - 40*a^3*b^7*sin(d*x + c)^7/(cos(d*x + c) + 1)^7 + 20*a^3
*b^7*sin(d*x + c)^9/(cos(d*x + c) + 1)^9 - 4*a^3*b^7*sin(d*x + c)^11/(cos(d
*x + c) + 1)^11 + a^4*b^6*sin(d*x + c)^12/(cos(d*x + c) + 1)^12 - 2*(3*a^4*
b^6 - 2*a^2*b^8)*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + (15*a^4*b^6 - 16*a^2
*b^8)*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 - 4*(5*a^4*b^6 - 6*a^2*b^8)*sin(d
*x + c)^6/(cos(d*x + c) + 1)^6 + (15*a^4*b^6 - 16*a^2*b^8)*sin(d*x + c)^8/(
cos(d*x + c) + 1)^8 - 2*(3*a^4*b^6 - 2*a^2*b^8)*sin(d*x + c)^10/(cos(d*x +
c) + 1)^10) - 3*(5*a^4 + 6*a^2*b^2 + b^4)*log(-a - 2*b*sin(d*x + c)/(cos(d*
x + c) + 1) + a*sin(d*x + c)^2/(cos(d*x + c) + 1)^2)/b^7 + 3*(5*a^4 + 6*a^2
*b^2 + b^4)*log(sin(d*x + c)/(cos(d*x + c) + 1) + 1)/b^7 + 3*(5*a^4 + 6*a^2
*b^2 + b^4)*log(sin(d*x + c)/(cos(d*x + c) + 1) - 1)/b^7)/d
```

mupad [B] time = 7.64, size = 1712, normalized size = 7.38

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^5*(a*cos(c + d*x) + b*sin(c + d*x))^3),x)
```

```
[Out] - ((2*tan(c/2 + (d*x)/2)*(15*a^6 - b^6 + 3*a^2*b^4 + 18*a^4*b^2))/(a*b^6) -
 (2*tan(c/2 + (d*x)/2)^11*(15*a^6 - b^6 + 3*a^2*b^4 + 18*a^4*b^2))/(a*b^6)
+ (2*tan(c/2 + (d*x)/2)^2*(45*a^6 - b^6 + 9*a^2*b^4 + 54*a^4*b^2))/(a^2*b^5
) + (2*tan(c/2 + (d*x)/2)^10*(45*a^6 - b^6 + 9*a^2*b^4 + 54*a^4*b^2))/(a^2*
b^5) - (2*tan(c/2 + (d*x)/2)^3*(75*a^6 - 5*b^6 - 9*a^2*b^4 + 70*a^4*b^2))/(
a*b^6) + (4*tan(c/2 + (d*x)/2)^5*(75*a^6 - 5*b^6 - 17*a^2*b^4 + 60*a^4*b^2)
)/(a*b^6) - (4*tan(c/2 + (d*x)/2)^7*(75*a^6 - 5*b^6 - 17*a^2*b^4 + 60*a^4*b
^2))/(a*b^6) + (2*tan(c/2 + (d*x)/2)^9*(75*a^6 - 5*b^6 - 9*a^2*b^4 + 70*a^4
```

```
*b^2))/(a*b^6) - (4*tan(c/2 + (d*x)/2)^4*(90*a^6 - 2*b^6 + 24*a^2*b^4 + 113
*a^4*b^2))/(a^2*b^5) - (4*tan(c/2 + (d*x)/2)^8*(90*a^6 - 2*b^6 + 24*a^2*b^4
 + 113*a^4*b^2))/(a^2*b^5) + (4*tan(c/2 + (d*x)/2)^6*(135*a^6 - 3*b^6 + 35*
a^2*b^4 + 172*a^4*b^2))/(a^2*b^5))/(d*(tan(c/2 + (d*x)/2)^4*(15*a^2 - 16*b^
2) - tan(c/2 + (d*x)/2)^10*(6*a^2 - 4*b^2) - tan(c/2 + (d*x)/2)^2*(6*a^2 -
4*b^2) + tan(c/2 + (d*x)/2)^8*(15*a^2 - 16*b^2) - tan(c/2 + (d*x)/2)^6*(20*
a^2 - 24*b^2) + a^2*tan(c/2 + (d*x)/2)^12 + a^2 - 20*a*b*tan(c/2 + (d*x)/2)
^3 + 40*a*b*tan(c/2 + (d*x)/2)^5 - 40*a*b*tan(c/2 + (d*x)/2)^7 + 20*a*b*tan
(c/2 + (d*x)/2)^9 - 4*a*b*tan(c/2 + (d*x)/2)^11 + 4*a*b*tan(c/2 + (d*x)/2))
) - (atan((((5*a^2 + b^2)*(a^2 + b^2)*((16*tan(c/2 + (d*x)/2)*(15*a^6 + 3*a
^2*b^4 + 18*a^4*b^2))/b^6 - (4*(6*a*b^11 + 36*a^3*b^9 + 30*a^5*b^7))/b^12 +
 (4*tan(c/2 + (d*x)/2)^2*(6*a*b^11 + 36*a^3*b^9 + 30*a^5*b^7))/b^12 + (3*(5
*a^2 + b^2)*(a^2 + b^2)*((4*(a*b^14 + 4*a^3*b^12))/b^12 - (4*tan(c/2 + (d*x
)/2)^2*(3*a*b^14 + 4*a^3*b^12))/b^12 + 16*a^2*b*tan(c/2 + (d*x)/2)))/b^7)*3
i)/b^7 - ((5*a^2 + b^2)*(a^2 + b^2)*((4*(6*a*b^11 + 36*a^3*b^9 + 30*a^5*b^7
))/b^12 - (16*tan(c/2 + (d*x)/2)*(15*a^6 + 3*a^2*b^4 + 18*a^4*b^2))/b^6 - (
4*tan(c/2 + (d*x)/2)^2*(6*a*b^11 + 36*a^3*b^9 + 30*a^5*b^7))/b^12 + (3*(5*a
^2 + b^2)*(a^2 + b^2)*((4*(a*b^14 + 4*a^3*b^12))/b^12 - (4*tan(c/2 + (d*x)/
2)^2*(3*a*b^14 + 4*a^3*b^12))/b^12 + 16*a^2*b*tan(c/2 + (d*x)/2)))/b^7)*3i)
/b^7)/((8*(9*a*b^8 + 225*a^9 + 108*a^3*b^6 + 414*a^5*b^4 + 540*a^7*b^2))/b^
12 + (8*tan(c/2 + (d*x)/2)^2*(9*a*b^8 + 225*a^9 + 108*a^3*b^6 + 414*a^5*b^4
 + 540*a^7*b^2))/b^12 + (3*(5*a^2 + b^2)*(a^2 + b^2)*((16*tan(c/2 + (d*x)/2
)*(15*a^6 + 3*a^2*b^4 + 18*a^4*b^2))/b^6 - (4*(6*a*b^11 + 36*a^3*b^9 + 30*a
^5*b^7))/b^12 + (4*tan(c/2 + (d*x)/2)^2*(6*a*b^11 + 36*a^3*b^9 + 30*a^5*b^7
))/b^12 + (3*(5*a^2 + b^2)*(a^2 + b^2)*((4*(a*b^14 + 4*a^3*b^12))/b^12 - (4
*tan(c/2 + (d*x)/2)^2*(3*a*b^14 + 4*a^3*b^12))/b^12 + 16*a^2*b*tan(c/2 + (d
*x)/2)))/b^7))/b^7 + (3*(5*a^2 + b^2)*(a^2 + b^2)*((4*(6*a*b^11 + 36*a^3*b^
9 + 30*a^5*b^7))/b^12 - (16*tan(c/2 + (d*x)/2)*(15*a^6 + 3*a^2*b^4 + 18*a^4
*b^2))/b^6 - (4*tan(c/2 + (d*x)/2)^2*(6*a*b^11 + 36*a^3*b^9 + 30*a^5*b^7))/
b^12 + (3*(5*a^2 + b^2)*(a^2 + b^2)*((4*(a*b^14 + 4*a^3*b^12))/b^12 - (4*ta
n(c/2 + (d*x)/2)^2*(3*a*b^14 + 4*a^3*b^12))/b^12 + 16*a^2*b*tan(c/2 + (d*x)
/2)))/b^7))/b^7))*(5*a^2 + b^2)*(a^2 + b^2)*6i)/(b^7*d)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec^5(c+dx)}{(a\cos(c+dx)+b\sin(c+dx))^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**5/(a*cos(d*x+c)+b*sin(d*x+c))**3,x)
```

```
[Out] Integral(sec(c + d*x)**5/(a*cos(c + d*x) + b*sin(c + d*x))**3, x)
```

3.141 $\int \frac{\cos^4(c+dx)}{(a\cos(c+dx)+b\sin(c+dx))^4}\,dx$

Optimal. Leaf size=165

$$-\frac{b\left(3a^2-b^2\right)}{d\left(a^2+b^2\right)^3(a+b\tan(c+dx))}-\frac{ab}{d\left(a^2+b^2\right)^2(a+b\tan(c+dx))^2}-\frac{b}{3d\left(a^2+b^2\right)(a+b\tan(c+dx))^3}+\frac{4ab\left(a^2-\right.}{}$$

```
[Out] (a^4-6*a^2*b^2+b^4)*x/(a^2+b^2)^4+4*a*b*(a^2-b^2)*ln(a*cos(d*x+c)+b*sin(d*x
+c))/(a^2+b^2)^4/d-1/3*b/(a^2+b^2)/d/(a+b*tan(d*x+c))^3-a*b/(a^2+b^2)^2/d/(
a+b*tan(d*x+c))^2-b*(3*a^2-b^2)/(a^2+b^2)^3/d/(a+b*tan(d*x+c))
```

Rubi [A] time = 0.30, antiderivative size = 165, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.179, Rules used = {3086, 3483, 3529, 3531, 3530}

$$-\frac{b\left(3a^2-b^2\right)}{d\left(a^2+b^2\right)^3(a+b\tan(c+dx))}-\frac{ab}{d\left(a^2+b^2\right)^2(a+b\tan(c+dx))^2}-\frac{b}{3d\left(a^2+b^2\right)(a+b\tan(c+dx))^3}+\frac{4ab\left(a^2-\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^4/(a*Cos[c + d*x] + b*Sin[c + d*x])^4,x]
```

```
[Out] ((a^4 - 6*a^2*b^2 + b^4)*x)/(a^2 + b^2)^4 + (4*a*b*(a^2 - b^2)*Log[a*Cos[c
+ d*x] + b*Sin[c + d*x]])/((a^2 + b^2)^4*d) - b/(3*(a^2 + b^2)*d*(a + b*Tan
[c + d*x])^3) - (a*b)/((a^2 + b^2)^2*d*(a + b*Tan[c + d*x])^2) - (b*(3*a^2
- b^2))/((a^2 + b^2)^3*d*(a + b*Tan[c + d*x]))
```

Rule 3086

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin
[(c_.) + (d_.)*(x_)])^(n_.), x_Symbol] :> Int[(a + b*Tan[c + d*x])^n, x] /;
 FreeQ[{a, b, c, d}, x] && EqQ[m + n, 0] && IntegerQ[n] && NeQ[a^2 + b^2, 0
]
```

Rule 3483

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(a +
b*Tan[c + d*x])^(n + 1))/(d*(n + 1)*(a^2 + b^2)), x] + Dist[1/(a^2 + b^2),
Int[(a - b*Tan[c + d*x])*(a + b*Tan[c + d*x])^(n + 1), x], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 + b^2, 0] && LtQ[n, -1]
```

Rule 3529

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[((b*c - a*d)*(a + b*Tan[e + f*x])^(m + 1))
/(f*(m + 1)*(a^2 + b^2)), x] + Dist[1/(a^2 + b^2), Int[(a + b*Tan[e + f*x])
^(m + 1)*Simp[a*c + b*d - (b*c - a*d)*Tan[e + f*x], x], x], x] /; FreeQ[{a,
 b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && LtQ[m, -1]
```

Rule 3530

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_) + (b_.)*tan[(e_.) + (f_.)*
(x_)]), x_Symbol] :> Simp[(c*Log[RemoveContent[a*Cos[e + f*x] + b*Sin[e + f
*x], x]])/(b*f), x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
 NeQ[a^2 + b^2, 0] && EqQ[a*c + b*d, 0]
```

Rule 3531

```
Int[((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_.) + (b_.)*tan[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[((a*c + b*d)*x)/(a^2 + b^2), x] + Dist[(b*c - a
*d)/(a^2 + b^2), Int[(b - a*Tan[e + f*x])/(a + b*Tan[e + f*x]), x], x] /; F
reeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && Ne
Q[a*c + b*d, 0]
```

Rubi steps

$$\int \frac{\cos^4(c+dx)}{(a\cos(c+dx)+b\sin(c+dx))^4}\,dx = \int \frac{1}{(a+b\tan(c+dx))^4}\,dx$$

$$= -\frac{b}{3\left(a^2+b^2\right)d(a+b\tan(c+dx))^3} + \frac{\int \frac{a-b\tan(c+dx)}{(a+b\tan(c+dx))^3}\,dx}{a^2+b^2}$$

$$= -\frac{b}{3\left(a^2+b^2\right)d(a+b\tan(c+dx))^3} - \frac{ab}{\left(a^2+b^2\right)^2 d(a+b\tan(c+dx))^2} + \cdots$$

$$= -\frac{b}{3\left(a^2+b^2\right)d(a+b\tan(c+dx))^3} - \frac{ab}{\left(a^2+b^2\right)^2 d(a+b\tan(c+dx))^2} - \cdots$$

$$= \frac{\left(a^4-6a^2b^2+b^4\right)x}{\left(a^2+b^2\right)^4} - \frac{b}{3\left(a^2+b^2\right)d(a+b\tan(c+dx))^3} - \frac{\cdots}{\left(a^2+b^2\right)^2 d(a \cdots}$$

$$= \frac{\left(a^4-6a^2b^2+b^4\right)x}{\left(a^2+b^2\right)^4} + \frac{4ab\left(a^2-b^2\right)\log(a\cos(c+dx)+b\sin(c+dx))}{\left(a^2+b^2\right)^4 d} - \cdots$$

Mathematica [C] time = 6.24, size = 419, normalized size = 2.54

$$\frac{(a^2-2ab-b^2)(a^2+2ab-b^2)(c+dx)}{d(a-ib)^4(a+ib)^4}+\frac{2(9a^2b^2\sin(c+dx)-2b^4\sin(c+dx))}{3ad(a-ib)^3(a+ib)^3(a\cos(c+dx)+b\sin(c+dx))}-\frac{}{3ad(a-ib)^3(a+}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^4/(a*Cos[c + d*x] + b*Sin[c + d*x])^4,x]
```

```
[Out] ((a^2 - 2*a*b - b^2)*(a^2 + 2*a*b - b^2)*(c + d*x))/((a - I*b)^4*(a + I*b)^
4*d) + (4*(I*a^10*b + a^9*b^2 + (2*I)*a^8*b^3 + 2*a^7*b^4 - (2*I)*a^4*b^7 -
 2*a^3*b^8 - I*a^2*b^9 - a*b^10)*(c + d*x))/((a - I*b)^8*(a + I*b)^7*d) - (
(4*I)*(a^3*b - a*b^3)*ArcTan[Tan[c + d*x]])/((a^2 + b^2)^4*d) + (2*(a^3*b -
 a*b^3)*Log[(a*Cos[c + d*x] + b*Sin[c + d*x])^2])/((a^2 + b^2)^4*d) + (b^4*
Sin[c + d*x])/(3*a*(a - I*b)^2*(a + I*b)^2*d*(a*Cos[c + d*x] + b*Sin[c + d*
x])^3) - (b^3*(6*a^2 + b^2))/(3*a*(a - I*b)^3*(a + I*b)^3*d*(a*Cos[c + d*x]
 + b*Sin[c + d*x])^2) + (2*(9*a^2*b^2*Sin[c + d*x] - 2*b^4*Sin[c + d*x]))/(
3*a*(a - I*b)^3*(a + I*b)^3*d*(a*Cos[c + d*x] + b*Sin[c + d*x]))
```

fricas [B] time = 0.58, size = 575, normalized size = 3.48

$$-\frac{\left(54\,a^4b^3-30\,a^2b^5+4\,b^7-3\left(a^7-9\,a^5b^2+19\,a^3b^4-3\,ab^6\right)dx\right)\cos(dx+c)^3-3\left(10\,a^4b^3-11\,a^2b^5+b^7+3\left(a\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4/(a*cos(d*x+c)+b*sin(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] -1/3*((54*a^4*b^3 - 30*a^2*b^5 + 4*b^7 - 3*(a^7 - 9*a^5*b^2 + 19*a^3*b^4 -
3*a*b^6)*d*x)*cos(d*x + c)^3 - 3*(10*a^4*b^3 - 11*a^2*b^5 + b^7 + 3*(a^5*b^
2 - 6*a^3*b^4 + a*b^6)*d*x)*cos(d*x + c) - 6*((a^6*b - 4*a^4*b^3 + 3*a^2*b^
5)*cos(d*x + c)^3 + 3*(a^4*b^3 - a^2*b^5)*cos(d*x + c) + (a^3*b^4 - a*b^6 +
 (3*a^5*b^2 - 4*a^3*b^4 + a*b^6)*cos(d*x + c)^2)*sin(d*x + c))*log(2*a*b*co
s(d*x + c)*sin(d*x + c) + (a^2 - b^2)*cos(d*x + c)^2 + b^2) - (13*a^3*b^4 -
 9*a*b^6 + 3*(a^4*b^3 - 6*a^2*b^5 + b^7)*d*x + (18*a^5*b^2 - 58*a^3*b^4 + 1
2*a*b^6 + 3*(3*a^6*b - 19*a^4*b^3 + 9*a^2*b^5 - b^7)*d*x)*cos(d*x + c)^2)*s
in(d*x + c))/((a^11 + a^9*b^2 - 6*a^7*b^4 - 14*a^5*b^6 - 11*a^3*b^8 - 3*a*b
^10)*d*cos(d*x + c)^3 + 3*(a^9*b^2 + 4*a^7*b^4 + 6*a^5*b^6 + 4*a^3*b^8 + a*
b^10)*d*cos(d*x + c) + ((3*a^10*b + 11*a^8*b^3 + 14*a^6*b^5 + 6*a^4*b^7 - a
^2*b^9 - b^11)*d*cos(d*x + c)^2 + (a^8*b^3 + 4*a^6*b^5 + 6*a^4*b^7 + 4*a^2*
b^9 + b^11)*d)*sin(d*x + c))
```

giac [B] time = 1.82, size = 370, normalized size = 2.24

$$\frac{\frac{3\left(a^4-6\,a^2b^2+b^4\right)(dx+c)}{a^8+4\,a^6b^2+6\,a^4b^4+4\,a^2b^6+b^8}-\frac{6\left(a^3b-ab^3\right)\log\left(\tan(dx+c)^2+1\right)}{a^8+4\,a^6b^2+6\,a^4b^4+4\,a^2b^6+b^8}+\frac{12\left(a^3b^2-ab^4\right)\log(|b\tan(dx+c)+a|)}{a^8b+4\,a^6b^3+6\,a^4b^5+4\,a^2b^7+b^9}-\frac{22\,a^3b^4\tan(dx+c)^3-22\,ab^6\tan(dx+c)^3+}{}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4/(a*cos(d*x+c)+b*sin(d*x+c))^4,x, algorithm="giac")
```

```
[Out] 1/3*(3*(a^4 - 6*a^2*b^2 + b^4)*(d*x + c)/(a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a
^2*b^6 + b^8) - 6*(a^3*b - a*b^3)*log(tan(d*x + c)^2 + 1)/(a^8 + 4*a^6*b^2
+ 6*a^4*b^4 + 4*a^2*b^6 + b^8) + 12*(a^3*b^2 - a*b^4)*log(abs(b*tan(d*x + c
) + a))/(a^8*b + 4*a^6*b^3 + 6*a^4*b^5 + 4*a^2*b^7 + b^9) - (22*a^3*b^4*tan
(d*x + c)^3 - 22*a*b^6*tan(d*x + c)^3 + 75*a^4*b^3*tan(d*x + c)^2 - 60*a^2*
b^5*tan(d*x + c)^2 - 3*b^7*tan(d*x + c)^2 + 87*a^5*b^2*tan(d*x + c) - 48*a^
3*b^4*tan(d*x + c) - 3*a*b^6*tan(d*x + c) + 35*a^6*b - 7*a^4*b^3 + 3*a^2*b^
5 + b^7)/((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)*(b*tan(d*x + c) +
 a)^3))/d
```

maple [A] time = 0.30, size = 304, normalized size = 1.84

$$-\frac{b}{3\left(a^{2}+b^{2}\right)d\left(a+b\tan\left(dx+c\right)\right)^{3}}-\frac{3b\,a^{2}}{d\left(a^{2}+b^{2}\right)^{3}\left(a+b\tan\left(dx+c\right)\right)}+\frac{b^{3}}{d\left(a^{2}+b^{2}\right)^{3}\left(a+b\tan\left(dx+c\right)\right)}-\frac{\cdots}{\left(a^{2}+b^{2}\right)^{\cdots}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4/(a*cos(d*x+c)+b*sin(d*x+c))^4,x)
```

```
[Out] -1/3*b/(a^2+b^2)/d/(a+b*tan(d*x+c))^3-3/d*b/(a^2+b^2)^3/(a+b*tan(d*x+c))*a^
2+1/d*b^3/(a^2+b^2)^3/(a+b*tan(d*x+c))-a*b/(a^2+b^2)^2/d/(a+b*tan(d*x+c))^2
+4/d*b*a^3/(a^2+b^2)^4*ln(a+b*tan(d*x+c))-4/d*b^3*a/(a^2+b^2)^4*ln(a+b*tan(
d*x+c))-2/d/(a^2+b^2)^4*ln(tan(d*x+c)^2+1)*a^3*b+2/d/(a^2+b^2)^4*ln(tan(d*x
+c)^2+1)*a*b^3+1/d/(a^2+b^2)^4*arctan(tan(d*x+c))*a^4-6/d/(a^2+b^2)^4*arcta
n(tan(d*x+c))*a^2*b^2+1/d/(a^2+b^2)^4*arctan(tan(d*x+c))*b^4
```

maxima [B] time = 0.43, size = 385, normalized size = 2.33

$$\frac{\frac{3\left(a^{4}-6\,a^{2}b^{2}+b^{4}\right)(dx+c)}{a^{8}+4\,a^{6}b^{2}+6\,a^{4}b^{4}+4\,a^{2}b^{6}+b^{8}}+\frac{12\left(a^{3}b-ab^{3}\right)\log(b\tan(dx+c)+a)}{a^{8}+4\,a^{6}b^{2}+6\,a^{4}b^{4}+4\,a^{2}b^{6}+b^{8}}-\frac{6\left(a^{3}b-ab^{3}\right)\log\left(\tan(dx+c)^{2}+1\right)}{a^{8}+4\,a^{6}b^{2}+6\,a^{4}b^{4}+4\,a^{2}b^{6}+b^{8}}-\frac{13\,a^{4}b\cdots}{a^{9}+3\,a^{7}b^{2}+3\,a^{5}b^{4}+a^{3}b^{6}+\left(a^{6}b^{3}+3\,a^{4}b^{5}+3\,a^{2}\cdots\right.}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4/(a*cos(d*x+c)+b*sin(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] 1/3*(3*(a^4 - 6*a^2*b^2 + b^4)*(d*x + c)/(a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a
^2*b^6 + b^8) + 12*(a^3*b - a*b^3)*log(b*tan(d*x + c) + a)/(a^8 + 4*a^6*b^2
 + 6*a^4*b^4 + 4*a^2*b^6 + b^8) - 6*(a^3*b - a*b^3)*log(tan(d*x + c)^2 + 1)
/(a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8) - (13*a^4*b + 2*a^2*b^3 +
b^5 + 3*(3*a^2*b^3 - b^5)*tan(d*x + c)^2 + 3*(7*a^3*b^2 - a*b^4)*tan(d*x +
c))/(a^9 + 3*a^7*b^2 + 3*a^5*b^4 + a^3*b^6 + (a^6*b^3 + 3*a^4*b^5 + 3*a^2*b
```

```
^7 + b^9)*tan(d*x + c)^3 + 3*(a^7*b^2 + 3*a^5*b^4 + 3*a^3*b^6 + a*b^8)*tan(
d*x + c)^2 + 3*(a^8*b + 3*a^6*b^3 + 3*a^4*b^5 + a^2*b^7)*tan(d*x + c)))/d
```

mupad [B] time = 12.55, size = 8586, normalized size = 52.04

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^4/(a*cos(c + d*x) + b*sin(c + d*x))^4,x)
```

```
[Out] ((4*tan(c/2 + (d*x)/2)^2*(b^7 + 3*a^2*b^5 + 10*a^4*b^3))/(a^2*(a^6 + b^6 +
3*a^2*b^4 + 3*a^4*b^2)) - (4*tan(c/2 + (d*x)/2)^4*(b^7 + 3*a^2*b^5 + 10*a^4
*b^3))/(a^2*(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2)) + (2*b*tan(c/2 + (d*x)/2)^
5*(6*a^4*b + b^5 + 3*a^2*b^3))/(a*(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2)) + (4
*b*tan(c/2 + (d*x)/2)^3*(2*b^7 - 18*a^6*b + a^2*b^5 + 17*a^4*b^3))/(3*a^3*(
a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2)) + (2*b*tan(c/2 + (d*x)/2)*(6*a^4*b + b^
5 + 3*a^2*b^3))/(a*(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2)))/(d*(tan(c/2 + (d*x
)/2)^2*(12*a*b^2 - 3*a^3) - a^3*tan(c/2 + (d*x)/2)^6 - tan(c/2 + (d*x)/2)^4
*(12*a*b^2 - 3*a^3) - tan(c/2 + (d*x)/2)^3*(12*a^2*b - 8*b^3) + a^3 + 6*a^2
*b*tan(c/2 + (d*x)/2) + 6*a^2*b*tan(c/2 + (d*x)/2)^5)) - (log(a + 2*b*tan(c
/2 + (d*x)/2) - a*tan(c/2 + (d*x)/2)^2)*(4*a*b^3 - 4*a^3*b))/(d*(a^8 + b^8
+ 4*a^2*b^6 + 6*a^4*b^4 + 4*a^6*b^2)) + (log((((-(a^4 + b^4 - 6*a^2*b^2)^2/
(a^2 + b^2)^8)^(1/2) - (4*a*b*(a^2 - b^2))/(a^2 + b^2)^4)*(((-(a^4 + b^4 -
6*a^2*b^2)^2/(a^2 + b^2)^8)^(1/2) - (4*a*b*(a^2 - b^2))/(a^2 + b^2)^4)*((32
*a*(a^6 - b^6 + 11*a^2*b^4 - 11*a^4*b^2))/(a^2 + b^2)^3 + 96*a*b*((-(a^4 +
b^4 - 6*a^2*b^2)^2/(a^2 + b^2)^8)^(1/2) - (4*a*b*(a^2 - b^2))/(a^2 + b^2)^4
)*(a + b*tan(c/2 + (d*x)/2))*(a^2 + b^2) - (64*a^2*b*tan(c/2 + (d*x)/2)*(b^
4 - 5*a^4 + 8*a^2*b^2))/(a^2 + b^2)^3) - (32*a^2*b*(7*a^4 + 7*b^4 - 18*a^2*
b^2))/(a^2 + b^2)^5 + (32*a*tan(c/2 + (d*x)/2)*(a^8 + 2*b^8 - 57*a^2*b^6 +
105*a^4*b^4 - 27*a^6*b^2))/(a^2 + b^2)^6) + (128*a^3*b^2*(3*a^6 - 3*b^6 + 1
3*a^2*b^4 - 13*a^4*b^2))/(a^2 + b^2)^9 - (128*a^2*b*tan(c/2 + (d*x)/2)*(a^8
 - 2*b^8 + 5*a^2*b^6 - 15*a^4*b^4 + 11*a^6*b^2))/(a^2 + b^2)^9)*(((-(a^4 +
b^4 - 6*a^2*b^2)^2/(a^2 + b^2)^8)^(1/2) + (4*a*b*(a^2 - b^2))/(a^2 + b^2)^4
)*(((-(a^4 + b^4 - 6*a^2*b^2)^2/(a^2 + b^2)^8)^(1/2) + (4*a*b*(a^2 - b^2))/
(a^2 + b^2)^4)*(96*a*b*((-(a^4 + b^4 - 6*a^2*b^2)^2/(a^2 + b^2)^8)^(1/2) +
(4*a*b*(a^2 - b^2))/(a^2 + b^2)^4)*(a + b*tan(c/2 + (d*x)/2))*(a^2 + b^2) -
 (32*a*(a^6 - b^6 + 11*a^2*b^4 - 11*a^4*b^2))/(a^2 + b^2)^3 + (64*a^2*b*tan
(c/2 + (d*x)/2)*(b^4 - 5*a^4 + 8*a^2*b^2))/(a^2 + b^2)^3) - (32*a^2*b*(7*a^
4 + 7*b^4 - 18*a^2*b^2))/(a^2 + b^2)^5 + (32*a*tan(c/2 + (d*x)/2)*(a^8 + 2*
b^8 - 57*a^2*b^6 + 105*a^4*b^4 - 27*a^6*b^2))/(a^2 + b^2)^6) - (128*a^3*b^2
*(3*a^6 - 3*b^6 + 13*a^2*b^4 - 13*a^4*b^2))/(a^2 + b^2)^9 + (128*a^2*b*tan(
c/2 + (d*x)/2)*(a^8 - 2*b^8 + 5*a^2*b^6 - 15*a^4*b^4 + 11*a^6*b^2))/(a^2 +
b^2)^9))*(8*a*b^3 - 8*a^3*b))/(2*d*(a^8 + b^8 + 4*a^2*b^6 + 6*a^4*b^4 + 4*a
^6*b^2)) - (2*atan((tan(c/2 + (d*x)/2)*((((32*(4*a^10*b - 8*a^2*b^9 + 20*a^
4*b^7 - 60*a^6*b^5 + 44*a^8*b^3))/(a^18 + b^18 + 9*a^2*b^16 + 36*a^4*b^14 +
```

```
 84*a^6*b^12 + 126*a^8*b^10 + 126*a^10*b^8 + 84*a^12*b^6 + 36*a^14*b^4 + 9*
a^16*b^2) - (((32*(2*a*b^14 + a^15 - 51*a^3*b^12 - 60*a^5*b^10 + 119*a^7*b^
8 + 178*a^9*b^6 + 27*a^11*b^4 - 24*a^13*b^2))/(a^18 + b^18 + 9*a^2*b^16 + 3
6*a^4*b^14 + 84*a^6*b^12 + 126*a^8*b^10 + 126*a^10*b^8 + 84*a^12*b^6 + 36*a
^14*b^4 + 9*a^16*b^2) - ((8*a*b^3 - 8*a^3*b)*((32*(2*a^2*b^17 - 10*a^18*b +
 28*a^4*b^15 + 116*a^6*b^13 + 220*a^8*b^11 + 200*a^10*b^9 + 52*a^12*b^7 - 5
2*a^14*b^5 - 44*a^16*b^3))/(a^18 + b^18 + 9*a^2*b^16 + 36*a^4*b^14 + 84*a^6
*b^12 + 126*a^8*b^10 + 126*a^10*b^8 + 84*a^12*b^6 + 36*a^14*b^4 + 9*a^16*b^
2) - (16*(8*a*b^3 - 8*a^3*b)*(3*a*b^22 + 30*a^3*b^20 + 135*a^5*b^18 + 360*a
^7*b^16 + 630*a^9*b^14 + 756*a^11*b^12 + 630*a^13*b^10 + 360*a^15*b^8 + 135
*a^17*b^6 + 30*a^19*b^4 + 3*a^21*b^2))/((a^8 + b^8 + 4*a^2*b^6 + 6*a^4*b^4
+ 4*a^6*b^2)*(a^18 + b^18 + 9*a^2*b^16 + 36*a^4*b^14 + 84*a^6*b^12 + 126*a^
8*b^10 + 126*a^10*b^8 + 84*a^12*b^6 + 36*a^14*b^4 + 9*a^16*b^2))))/(2*(a^8
+ b^8 + 4*a^2*b^6 + 6*a^4*b^4 + 4*a^6*b^2)))*(8*a*b^3 - 8*a^3*b))/(2*(a^8 +
 b^8 + 4*a^2*b^6 + 6*a^4*b^4 + 4*a^6*b^2)) - (((((32*(2*a^2*b^17 - 10*a^18*
b + 28*a^4*b^15 + 116*a^6*b^13 + 220*a^8*b^11 + 200*a^10*b^9 + 52*a^12*b^7
- 52*a^14*b^5 - 44*a^16*b^3))/(a^18 + b^18 + 9*a^2*b^16 + 36*a^4*b^14 + 84*
a^6*b^12 + 126*a^8*b^10 + 126*a^10*b^8 + 84*a^12*b^6 + 36*a^14*b^4 + 9*a^16
*b^2) - (16*(8*a*b^3 - 8*a^3*b)*(3*a*b^22 + 30*a^3*b^20 + 135*a^5*b^18 + 36
0*a^7*b^16 + 630*a^9*b^14 + 756*a^11*b^12 + 630*a^13*b^10 + 360*a^15*b^8 +
135*a^17*b^6 + 30*a^19*b^4 + 3*a^21*b^2))/((a^8 + b^8 + 4*a^2*b^6 + 6*a^4*b
^4 + 4*a^6*b^2)*(a^18 + b^18 + 9*a^2*b^16 + 36*a^4*b^14 + 84*a^6*b^12 + 126
*a^8*b^10 + 126*a^10*b^8 + 84*a^12*b^6 + 36*a^14*b^4 + 9*a^16*b^2)))*(2*a*b
 - a^2 + b^2)*(2*a*b + a^2 - b^2))/(a^8 + b^8 + 4*a^2*b^6 + 6*a^4*b^4 + 4*a
^6*b^2) - (16*(8*a*b^3 - 8*a^3*b)*(2*a*b - a^2 + b^2)*(2*a*b + a^2 - b^2)*(
3*a*b^22 + 30*a^3*b^20 + 135*a^5*b^18 + 360*a^7*b^16 + 630*a^9*b^14 + 756*a
^11*b^12 + 630*a^13*b^10 + 360*a^15*b^8 + 135*a^17*b^6 + 30*a^19*b^4 + 3*a^
21*b^2))/((a^8 + b^8 + 4*a^2*b^6 + 6*a^4*b^4 + 4*a^6*b^2)^2*(a^18 + b^18 +
9*a^2*b^16 + 36*a^4*b^14 + 84*a^6*b^12 + 126*a^8*b^10 + 126*a^10*b^8 + 84*a
^12*b^6 + 36*a^14*b^4 + 9*a^16*b^2)))*(2*a*b - a^2 + b^2)*(2*a*b + a^2 - b^
2))/(a^8 + b^8 + 4*a^2*b^6 + 6*a^4*b^4 + 4*a^6*b^2) + (16*(8*a*b^3 - 8*a^3*
b)*(2*a*b - a^2 + b^2)^2*(2*a*b + a^2 - b^2)^2*(3*a*b^22 + 30*a^3*b^20 + 13
5*a^5*b^18 + 360*a^7*b^16 + 630*a^9*b^14 + 756*a^11*b^12 + 630*a^13*b^10 +
360*a^15*b^8 + 135*a^17*b^6 + 30*a^19*b^4 + 3*a^21*b^2))/((a^8 + b^8 + 4*a^
2*b^6 + 6*a^4*b^4 + 4*a^6*b^2)^3*(a^18 + b^18 + 9*a^2*b^16 + 36*a^4*b^14 +
84*a^6*b^12 + 126*a^8*b^10 + 126*a^10*b^8 + 84*a^12*b^6 + 36*a^14*b^4 + 9*a
^16*b^2)))*(18*a*b^9 + 18*a^9*b - 280*a^3*b^7 + 556*a^5*b^5 - 280*a^7*b^3))
/(a^10 + b^10 + 53*a^2*b^8 - 38*a^4*b^6 - 38*a^6*b^4 + 53*a^8*b^2)^2 + ((((
8*a*b^3 - 8*a^3*b)*((((32*(2*a^2*b^17 - 10*a^18*b + 28*a^4*b^15 + 116*a^6*b
^13 + 220*a^8*b^11 + 200*a^10*b^9 + 52*a^12*b^7 - 52*a^14*b^5 - 44*a^16*b^3
))/(a^18 + b^18 + 9*a^2*b^16 + 36*a^4*b^14 + 84*a^6*b^12 + 126*a^8*b^10 + 1
26*a^10*b^8 + 84*a^12*b^6 + 36*a^14*b^4 + 9*a^16*b^2) - (16*(8*a*b^3 - 8*a^
3*b)*(3*a*b^22 + 30*a^3*b^20 + 135*a^5*b^18 + 360*a^7*b^16 + 630*a^9*b^14 +
 756*a^11*b^12 + 630*a^13*b^10 + 360*a^15*b^8 + 135*a^17*b^6 + 30*a^19*b^4
+ 3*a^21*b^2))/((a^8 + b^8 + 4*a^2*b^6 + 6*a^4*b^4 + 4*a^6*b^2)*(a^18 + b^1
```

8 + 9*a^2*b^16 + 36*a^4*b^14 + 84*a^6*b^12 + 126*a^8*b^10 + 126*a^10*b^8 +
84*a^12*b^6 + 36*a^14*b^4 + 9*a^16*b^2)))*(2*a*b - a^2 + b^2)*(2*a*b + a^2
- b^2))/(a^8 + b^8 + 4*a^2*b^6 + 6*a^4*b^4 + 4*a^6*b^2) - (16*(8*a*b^3 - 8*
a^3*b)*(2*a*b - a^2 + b^2)*(2*a*b + a^2 - b^2)*(3*a*b^22 + 30*a^3*b^20 + 13
5*a^5*b^18 + 360*a^7*b^16 + 630*a^9*b^14 + 756*a^11*b^12 + 630*a^13*b^10 +
360*a^15*b^8 + 135*a^17*b^6 + 30*a^19*b^4 + 3*a^21*b^2))/((a^8 + b^8 + 4*a^
2*b^6 + 6*a^4*b^4 + 4*a^6*b^2)^2*(a^18 + b^18 + 9*a^2*b^16 + 36*a^4*b^14 +
84*a^6*b^12 + 126*a^8*b^10 + 126*a^10*b^8 + 84*a^12*b^6 + 36*a^14*b^4 + 9*a
^16*b^2))))/(2*(a^8 + b^8 + 4*a^2*b^6 + 6*a^4*b^4 + 4*a^6*b^2)) - (((32*(2*
a*b^14 + a^15 - 51*a^3*b^12 - 60*a^5*b^10 + 119*a^7*b^8 + 178*a^9*b^6 + 27*
a^11*b^4 - 24*a^13*b^2))/(a^18 + b^18 + 9*a^2*b^16 + 36*a^4*b^14 + 84*a^6*b
^12 + 126*a^8*b^10 + 126*a^10*b^8 + 84*a^12*b^6 + 36*a^14*b^4 + 9*a^16*b^2)
 - ((8*a*b^3 - 8*a^3*b)*((32*(2*a^2*b^17 - 10*a^18*b + 28*a^4*b^15 + 116*a^
6*b^13 + 220*a^8*b^11 + 200*a^10*b^9 + 52*a^12*b^7 - 52*a^14*b^5 - 44*a^16*
b^3))/(a^18 + b^18 + 9*a^2*b^16 + 36*a^4*b^14 + 84*a^6*b^12 + 126*a^8*b^10
+ 126*a^10*b^8 + 84*a^12*b^6 + 36*a^14*b^4 + 9*a^16*b^2) - (16*(8*a*b^3 - 8
*a^3*b)*(3*a*b^22 + 30*a^3*b^20 + 135*a^5*b^18 + 360*a^7*b^16 + 630*a^9*b^1
4 + 756*a^11*b^12 + 630*a^13*b^10 + 360*a^15*b^8 + 135*a^17*b^6 + 30*a^19*b
^4 + 3*a^21*b^2))/((a^8 + b^8 + 4*a^2*b^6 + 6*a^4*b^4 + 4*a^6*b^2)*(a^18 +
b^18 + 9*a^2*b^16 + 36*a^4*b^14 + 84*a^6*b^12 + 126*a^8*b^10 + 126*a^10*b^8
 + 84*a^12*b^6 + 36*a^14*b^4 + 9*a^16*b^2))))/(2*(a^8 + b^8 + 4*a^2*b^6 + 6
*a^4*b^4 + 4*a^6*b^2)))*(2*a*b - a^2 + b^2)*(2*a*b + a^2 - b^2))/(a^8 + b^8
 + 4*a^2*b^6 + 6*a^4*b^4 + 4*a^6*b^2) + (32*(2*a*b - a^2 + b^2)^3*(2*a*b +
a^2 - b^2)^3*(3*a*b^22 + 30*a^3*b^20 + 135*a^5*b^18 + 360*a^7*b^16 + 630*a^
9*b^14 + 756*a^11*b^12 + 630*a^13*b^10 + 360*a^15*b^8 + 135*a^17*b^6 + 30*a
^19*b^4 + 3*a^21*b^2))/((a^8 + b^8 + 4*a^2*b^6 + 6*a^4*b^4 + 4*a^6*b^2)^3*(
a^18 + b^18 + 9*a^2*b^16 + 36*a^4*b^14 + 84*a^6*b^12 + 126*a^8*b^10 + 126*a
^10*b^8 + 84*a^12*b^6 + 36*a^14*b^4 + 9*a^16*b^2)))*(a^10 - b^10 + 109*a^2*
b^8 - 466*a^4*b^6 + 466*a^6*b^4 - 109*a^8*b^2))/(a^10 + b^10 + 53*a^2*b^8 -
 38*a^4*b^6 - 38*a^6*b^4 + 53*a^8*b^2)^2)*(a^22 + b^22 + 11*a^2*b^20 + 55*a
^4*b^18 + 165*a^6*b^16 + 330*a^8*b^14 + 462*a^10*b^12 + 462*a^12*b^10 + 330
*a^14*b^8 + 165*a^16*b^6 + 55*a^18*b^4 + 11*a^20*b^2))/(32*a*b^4 + 32*a^5 -
 192*a^3*b^2) + ((((8*a*b^3 - 8*a^3*b)*((((32*(a*b^18 - a^19 - 5*a^3*b^16 -
 40*a^5*b^14 - 80*a^7*b^12 - 46*a^9*b^10 + 46*a^11*b^8 + 80*a^13*b^6 + 40*a
^15*b^4 + 5*a^17*b^2))/(a^18 + b^18 + 9*a^2*b^16 + 36*a^4*b^14 + 84*a^6*b^1
2 + 126*a^8*b^10 + 126*a^10*b^8 + 84*a^12*b^6 + 36*a^14*b^4 + 9*a^16*b^2) -
 (16*(8*a*b^3 - 8*a^3*b)*(3*a^22*b + 3*a^2*b^21 + 30*a^4*b^19 + 135*a^6*b^1
7 + 360*a^8*b^15 + 630*a^10*b^13 + 756*a^12*b^11 + 630*a^14*b^9 + 360*a^16*
b^7 + 135*a^18*b^5 + 30*a^20*b^3))/((a^8 + b^8 + 4*a^2*b^6 + 6*a^4*b^4 + 4*
a^6*b^2)*(a^18 + b^18 + 9*a^2*b^16 + 36*a^4*b^14 + 84*a^6*b^12 + 126*a^8*b^
10 + 126*a^10*b^8 + 84*a^12*b^6 + 36*a^14*b^4 + 9*a^16*b^2)))*(2*a*b - a^2
+ b^2)*(2*a*b + a^2 - b^2))/(a^8 + b^8 + 4*a^2*b^6 + 6*a^4*b^4 + 4*a^6*b^2)
 - (16*(8*a*b^3 - 8*a^3*b)*(2*a*b - a^2 + b^2)*(2*a*b + a^2 - b^2)*(3*a^22*
b + 3*a^2*b^21 + 30*a^4*b^19 + 135*a^6*b^17 + 360*a^8*b^15 + 630*a^10*b^13
+ 756*a^12*b^11 + 630*a^14*b^9 + 360*a^16*b^7 + 135*a^18*b^5 + 30*a^20*b^3)

)/((a^8 + b^8 + 4*a^2*b^6 + 6*a^4*b^4 + 4*a^6*b^2)^2*(a^18 + b^18 + 9*a^2*b
^16 + 36*a^4*b^14 + 84*a^6*b^12 + 126*a^8*b^10 + 126*a^10*b^8 + 84*a^12*b^6
 + 36*a^14*b^4 + 9*a^16*b^2))))/(2*(a^8 + b^8 + 4*a^2*b^6 + 6*a^4*b^4 + 4*a
^6*b^2)) + (((32*(7*a^14*b + 7*a^2*b^13 + 10*a^4*b^11 - 23*a^6*b^9 - 52*a^8
*b^7 - 23*a^10*b^5 + 10*a^12*b^3))/(a^18 + b^18 + 9*a^2*b^16 + 36*a^4*b^14
+ 84*a^6*b^12 + 126*a^8*b^10 + 126*a^10*b^8 + 84*a^12*b^6 + 36*a^14*b^4 + 9
*a^16*b^2) + ((8*a*b^3 - 8*a^3*b)*((32*(a*b^18 - a^19 - 5*a^3*b^16 - 40*a^5
*b^14 - 80*a^7*b^12 - 46*a^9*b^10 + 46*a^11*b^8 + 80*a^13*b^6 + 40*a^15*b^4
 + 5*a^17*b^2))/(a^18 + b^18 + 9*a^2*b^16 + 36*a^4*b^14 + 84*a^6*b^12 + 126
*a^8*b^10 + 126*a^10*b^8 + 84*a^12*b^6 + 36*a^14*b^4 + 9*a^16*b^2) - (16*(8
*a*b^3 - 8*a^3*b)*(3*a^22*b + 3*a^2*b^21 + 30*a^4*b^19 + 135*a^6*b^17 + 360
*a^8*b^15 + 630*a^10*b^13 + 756*a^12*b^11 + 630*a^14*b^9 + 360*a^16*b^7 + 1
35*a^18*b^5 + 30*a^20*b^3))/((a^8 + b^8 + 4*a^2*b^6 + 6*a^4*b^4 + 4*a^6*b^2
)*(a^18 + b^18 + 9*a^2*b^16 + 36*a^4*b^14 + 84*a^6*b^12 + 126*a^8*b^10 + 12
6*a^10*b^8 + 84*a^12*b^6 + 36*a^14*b^4 + 9*a^16*b^2))))/(2*(a^8 + b^8 + 4*a
^2*b^6 + 6*a^4*b^4 + 4*a^6*b^2)))*(2*a*b - a^2 + b^2)*(2*a*b + a^2 - b^2))/
(a^8 + b^8 + 4*a^2*b^6 + 6*a^4*b^4 + 4*a^6*b^2) + (32*(2*a*b - a^2 + b^2)^3
*(2*a*b + a^2 - b^2)^3*(3*a^22*b + 3*a^2*b^21 + 30*a^4*b^19 + 135*a^6*b^17
+ 360*a^8*b^15 + 630*a^10*b^13 + 756*a^12*b^11 + 630*a^14*b^9 + 360*a^16*b^
7 + 135*a^18*b^5 + 30*a^20*b^3))/((a^8 + b^8 + 4*a^2*b^6 + 6*a^4*b^4 + 4*a^
6*b^2)^3*(a^18 + b^18 + 9*a^2*b^16 + 36*a^4*b^14 + 84*a^6*b^12 + 126*a^8*b^
10 + 126*a^10*b^8 + 84*a^12*b^6 + 36*a^14*b^4 + 9*a^16*b^2)))*(a^10 - b^10
+ 109*a^2*b^8 - 466*a^4*b^6 + 466*a^6*b^4 - 109*a^8*b^2)*(a^22 + b^22 + 11*
a^2*b^20 + 55*a^4*b^18 + 165*a^6*b^16 + 330*a^8*b^14 + 462*a^10*b^12 + 462*
a^12*b^10 + 330*a^14*b^8 + 165*a^16*b^6 + 55*a^18*b^4 + 11*a^20*b^2))/((32*
a*b^4 + 32*a^5 - 192*a^3*b^2)*(a^10 + b^10 + 53*a^2*b^8 - 38*a^4*b^6 - 38*a
^6*b^4 + 53*a^8*b^2)^2) + (((32*(12*a^3*b^8 - 52*a^5*b^6 + 52*a^7*b^4 - 12*
a^9*b^2))/(a^18 + b^18 + 9*a^2*b^16 + 36*a^4*b^14 + 84*a^6*b^12 + 126*a^8*b
^10 + 126*a^10*b^8 + 84*a^12*b^6 + 36*a^14*b^4 + 9*a^16*b^2) + ((8*a*b^3 -
8*a^3*b)*((32*(7*a^14*b + 7*a^2*b^13 + 10*a^4*b^11 - 23*a^6*b^9 - 52*a^8*b^
7 - 23*a^10*b^5 + 10*a^12*b^3))/(a^18 + b^18 + 9*a^2*b^16 + 36*a^4*b^14 + 8
4*a^6*b^12 + 126*a^8*b^10 + 126*a^10*b^8 + 84*a^12*b^6 + 36*a^14*b^4 + 9*a^
16*b^2) + ((8*a*b^3 - 8*a^3*b)*((32*(a*b^18 - a^19 - 5*a^3*b^16 - 40*a^5*b^
14 - 80*a^7*b^12 - 46*a^9*b^10 + 46*a^11*b^8 + 80*a^13*b^6 + 40*a^15*b^4 +
5*a^17*b^2))/(a^18 + b^18 + 9*a^2*b^16 + 36*a^4*b^14 + 84*a^6*b^12 + 126*a^
8*b^10 + 126*a^10*b^8 + 84*a^12*b^6 + 36*a^14*b^4 + 9*a^16*b^2) - (16*(8*a*
b^3 - 8*a^3*b)*(3*a^22*b + 3*a^2*b^21 + 30*a^4*b^19 + 135*a^6*b^17 + 360*a^
8*b^15 + 630*a^10*b^13 + 756*a^12*b^11 + 630*a^14*b^9 + 360*a^16*b^7 + 135*
a^18*b^5 + 30*a^20*b^3))/((a^8 + b^8 + 4*a^2*b^6 + 6*a^4*b^4 + 4*a^6*b^2)*(
a^18 + b^18 + 9*a^2*b^16 + 36*a^4*b^14 + 84*a^6*b^12 + 126*a^8*b^10 + 126*a
^10*b^8 + 84*a^12*b^6 + 36*a^14*b^4 + 9*a^16*b^2))))/(2*(a^8 + b^8 + 4*a^2*
b^6 + 6*a^4*b^4 + 4*a^6*b^2))))/(2*(a^8 + b^8 + 4*a^2*b^6 + 6*a^4*b^4 + 4*a
^6*b^2)) - (((((32*(a*b^18 - a^19 - 5*a^3*b^16 - 40*a^5*b^14 - 80*a^7*b^12
- 46*a^9*b^10 + 46*a^11*b^8 + 80*a^13*b^6 + 40*a^15*b^4 + 5*a^17*b^2))/(a^1
8 + b^18 + 9*a^2*b^16 + 36*a^4*b^14 + 84*a^6*b^12 + 126*a^8*b^10 + 126*a^10

```
*b^8 + 84*a^12*b^6 + 36*a^14*b^4 + 9*a^16*b^2) - (16*(8*a*b^3 - 8*a^3*b)*(3
*a^22*b + 3*a^2*b^21 + 30*a^4*b^19 + 135*a^6*b^17 + 360*a^8*b^15 + 630*a^10
*b^13 + 756*a^12*b^11 + 630*a^14*b^9 + 360*a^16*b^7 + 135*a^18*b^5 + 30*a^2
0*b^3))/((a^8 + b^8 + 4*a^2*b^6 + 6*a^4*b^4 + 4*a^6*b^2)*(a^18 + b^18 + 9*a
^2*b^16 + 36*a^4*b^14 + 84*a^6*b^12 + 126*a^8*b^10 + 126*a^10*b^8 + 84*a^12
*b^6 + 36*a^14*b^4 + 9*a^16*b^2)))*(2*a*b - a^2 + b^2)*(2*a*b + a^2 - b^2))
/(a^8 + b^8 + 4*a^2*b^6 + 6*a^4*b^4 + 4*a^6*b^2) - (16*(8*a*b^3 - 8*a^3*b)*
(2*a*b - a^2 + b^2)*(2*a*b + a^2 - b^2)*(3*a^22*b + 3*a^2*b^21 + 30*a^4*b^1
9 + 135*a^6*b^17 + 360*a^8*b^15 + 630*a^10*b^13 + 756*a^12*b^11 + 630*a^14*
b^9 + 360*a^16*b^7 + 135*a^18*b^5 + 30*a^20*b^3))/((a^8 + b^8 + 4*a^2*b^6 +
 6*a^4*b^4 + 4*a^6*b^2)^2*(a^18 + b^18 + 9*a^2*b^16 + 36*a^4*b^14 + 84*a^6*
b^12 + 126*a^8*b^10 + 126*a^10*b^8 + 84*a^12*b^6 + 36*a^14*b^4 + 9*a^16*b^2
)))*(2*a*b - a^2 + b^2)*(2*a*b + a^2 - b^2))/(a^8 + b^8 + 4*a^2*b^6 + 6*a^4
*b^4 + 4*a^6*b^2) + (16*(8*a*b^3 - 8*a^3*b)*(2*a*b - a^2 + b^2)^2*(2*a*b +
a^2 - b^2)^2*(3*a^22*b + 3*a^2*b^21 + 30*a^4*b^19 + 135*a^6*b^17 + 360*a^8*
b^15 + 630*a^10*b^13 + 756*a^12*b^11 + 630*a^14*b^9 + 360*a^16*b^7 + 135*a^
18*b^5 + 30*a^20*b^3))/((a^8 + b^8 + 4*a^2*b^6 + 6*a^4*b^4 + 4*a^6*b^2)^3*(
a^18 + b^18 + 9*a^2*b^16 + 36*a^4*b^14 + 84*a^6*b^12 + 126*a^8*b^10 + 126*a
^10*b^8 + 84*a^12*b^6 + 36*a^14*b^4 + 9*a^16*b^2)))*(18*a*b^9 + 18*a^9*b -
280*a^3*b^7 + 556*a^5*b^5 - 280*a^7*b^3)*(a^22 + b^22 + 11*a^2*b^20 + 55*a^
4*b^18 + 165*a^6*b^16 + 330*a^8*b^14 + 462*a^10*b^12 + 462*a^12*b^10 + 330*
a^14*b^8 + 165*a^16*b^6 + 55*a^18*b^4 + 11*a^20*b^2))/((32*a*b^4 + 32*a^5 -
 192*a^3*b^2)*(a^10 + b^10 + 53*a^2*b^8 - 38*a^4*b^6 - 38*a^6*b^4 + 53*a^8*
b^2)^2))*(2*a*b - a^2 + b^2)*(2*a*b + a^2 - b^2))/(d*(a^8 + b^8 + 4*a^2*b^6
 + 6*a^4*b^4 + 4*a^6*b^2))
```

sympy [**F(-2)**] time = 0.00, size = 0, normalized size = 0.00

Exception raised: AttributeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4/(a*cos(d*x+c)+b*sin(d*x+c))**4,x)
```

```
[Out] Exception raised: AttributeError
```

3.142 $\int \frac{\cos^3(c+dx)}{(a\cos(c+dx)+b\sin(c+dx))^4}\,dx$

Optimal. Leaf size=157

$$\frac{a\left(2a^2-3b^2\right)\tanh^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)-b}{\sqrt{a^2+b^2}}\right)}{d\left(a^2+b^2\right)^{7/2}}+\frac{\frac{1}{2}b\left(b^2-9a^2\right)\left(2\left(a^2+b^2\right)+3ab\sin(2(c+dx))\right)-3\left(3a^4b-a^2b^3+b^5\right)\cos}{6d\left(a^2+b^2\right)^3(a\cos(c+dx)+b\sin(c+dx))^3}$$

```
[Out] a*(2*a^2-3*b^2)*arctanh((-b+a*tan(1/2*d*x+1/2*c))/(a^2+b^2)^(1/2))/(a^2+b^2
)^(7/2)/d+1/6*(-3*(3*a^4*b-a^2*b^3+b^5)*cos(2*d*x+2*c)+1/2*b*(-9*a^2+b^2)*(
2*a^2+2*b^2+3*a*b*sin(2*d*x+2*c)))/(a^2+b^2)^3/d/(a*cos(d*x+c)+b*sin(d*x+c)
)^3
```

Rubi [B] time = 1.17, antiderivative size = 362, normalized size of antiderivative = 2.31, number of steps used = 7, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {1660, 12, 618, 206}

$$-\frac{8b^3\left(b\left(3a^2+4b^2\right)\tan\left(\frac{1}{2}(c+dx)\right)+a\left(a^2+2b^2\right)\right)}{3a^5d\left(a^2+b^2\right)\left(-a\tan^2\left(\frac{1}{2}(c+dx)\right)+a+2b\tan\left(\frac{1}{2}(c+dx)\right)\right)^3}+\frac{2b^2\left(a\left(30a^2b^2+9a^4+16b^4\right)\tan\left(\frac{1}{2}(c+dx)\right)+\right.}{3a^5d\left(a^2+b^2\right)^2\left(-a\tan^2\left(\frac{1}{2}(c+dx)\right)+a+\right.}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^3/(a*Cos[c + d*x] + b*Sin[c + d*x])^4,x]
```

```
[Out] -((a*(2*a^2 - 3*b^2)*ArcTanh[(b - a*Tan[(c + d*x)/2])/Sqrt[a^2 + b^2]])/((a
^2 + b^2)^(7/2)*d)) - (8*b^3*(a*(a^2 + 2*b^2) + b*(3*a^2 + 4*b^2)*Tan[(c +
d*x)/2]))/(3*a^5*(a^2 + b^2)*d*(a + 2*b*Tan[(c + d*x)/2] - a*Tan[(c + d*x)/
2]^2)^3) + (2*b^2*(b*(15*a^4 + 18*a^2*b^2 + 8*b^4) + a*(9*a^4 + 30*a^2*b^2
+ 16*b^4)*Tan[(c + d*x)/2]))/(3*a^5*(a^2 + b^2)^2*d*(a + 2*b*Tan[(c + d*x)/
2] - a*Tan[(c + d*x)/2]^2)^2) - (b*(6*a^6 + 9*a^4*b^2 + 12*a^2*b^4 + 4*b^6
+ a*b*(9*a^4 + 6*a^2*b^2 + 2*b^4)*Tan[(c + d*x)/2]))/(a^4*(a^2 + b^2)^3*d*(
a + 2*b*Tan[(c + d*x)/2] - a*Tan[(c + d*x)/2]^2))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1660

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[((b*f - 2*a*g + (2*c*f - b*g)*x)*(a + b*x + c*x^2)^
(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*(
2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2
 - 4*a*c, 0] && LtQ[p, -1]
```

Rubi steps

$$\int \frac{\cos^3(c+dx)}{(a\cos(c+dx)+b\sin(c+dx))^4}\,dx = \frac{2\,\text{Subst}\left(\int \frac{\left(1-x^2\right)^3}{\left(a+2bx-ax^2\right)^4}\,dx, x, \tan\left(\frac{1}{2}(c+dx)\right)\right)}{d}$$

$$= -\frac{8b^3\left(a\left(a^2+2b^2\right)+b\left(3a^2+4b^2\right)\tan\left(\frac{1}{2}(c+dx)\right)\right)}{3a^5\left(a^2+b^2\right)d\left(a+2b\tan\left(\frac{1}{2}(c+dx)\right)-a\tan^2\left(\frac{1}{2}(c+dx)\right)\right)^3} - \frac{\text{Subst}}{}$$

$$= -\frac{8b^3\left(a\left(a^2+2b^2\right)+b\left(3a^2+4b^2\right)\tan\left(\frac{1}{2}(c+dx)\right)\right)}{3a^5\left(a^2+b^2\right)d\left(a+2b\tan\left(\frac{1}{2}(c+dx)\right)-a\tan^2\left(\frac{1}{2}(c+dx)\right)\right)^3} + \frac{2b^2\left(b\right.}{3}$$

$$= -\frac{8b^3\left(a\left(a^2+2b^2\right)+b\left(3a^2+4b^2\right)\tan\left(\frac{1}{2}(c+dx)\right)\right)}{3a^5\left(a^2+b^2\right)d\left(a+2b\tan\left(\frac{1}{2}(c+dx)\right)-a\tan^2\left(\frac{1}{2}(c+dx)\right)\right)^3} + \frac{2b^2\left(b\right.}{3}$$

$$= -\frac{8b^3\left(a\left(a^2+2b^2\right)+b\left(3a^2+4b^2\right)\tan\left(\frac{1}{2}(c+dx)\right)\right)}{3a^5\left(a^2+b^2\right)d\left(a+2b\tan\left(\frac{1}{2}(c+dx)\right)-a\tan^2\left(\frac{1}{2}(c+dx)\right)\right)^3} + \frac{2b^2\left(b\right.}{3}$$

$$= -\frac{8b^3\left(a\left(a^2+2b^2\right)+b\left(3a^2+4b^2\right)\tan\left(\frac{1}{2}(c+dx)\right)\right)}{3a^5\left(a^2+b^2\right)d\left(a+2b\tan\left(\frac{1}{2}(c+dx)\right)-a\tan^2\left(\frac{1}{2}(c+dx)\right)\right)^3} + \frac{2b^2\left(b\right.}{3}$$

$$= -\frac{a\left(2a^2-3b^2\right)\tanh^{-1}\left(\frac{b-a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2+b^2}}\right)}{\left(a^2+b^2\right)^{7/2}d} - \frac{8b^3\left(a\left(a^2+2b^2\right)+b\left(3a\right.\right.}{3a^5\left(a^2+b^2\right)d\left(a+2b\tan\left(\frac{1}{2}(\right.\right.}$$

Mathematica [C] time = 1.12, size = 165, normalized size = 1.05

$$\frac{\frac{6a\left(2a^2-3b^2\right)\tanh^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)-b}{\sqrt{a^2+b^2}}\right)}{\left(a^2+b^2\right)^{7/2}} + \frac{\frac{1}{2}b\left(b^2-9a^2\right)\left(2\left(a^2+b^2\right)+3ab\sin(2(c+dx))\right)-3\left(3a^4b-a^2b^3+b^5\right)\cos(2(c+dx))}{(a-ib)^3(a+ib)^3(a\cos(c+dx)+b\sin(c+dx))^3}}{6d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^3/(a*Cos[c + d*x] + b*Sin[c + d*x])^4,x]
```

```
[Out] ((6*a*(2*a^2 - 3*b^2)*ArcTanh[(-b + a*Tan[(c + d*x)/2])/Sqrt[a^2 + b^2]])/(
a^2 + b^2)^(7/2) + (-3*(3*a^4*b - a^2*b^3 + b^5)*Cos[2*(c + d*x)] + (b*(-9*
a^2 + b^2)*(2*(a^2 + b^2) + 3*a*b*Sin[2*(c + d*x)]))/2)/((a - I*b)^3*(a + I
*b)^3*(a*Cos[c + d*x] + b*Sin[c + d*x])^3))/(6*d)
```

fricas [B] time = 0.76, size = 524, normalized size = 3.34

$$-\frac{22\,a^4b^3+14\,a^2b^5-8\,b^7+12\left(3\,a^6b+2\,a^4b^3+b^7\right)\cos\left(dx+c\right)^2+6\left(9\,a^5b^2+8\,a^3b^4-ab^6\right)\cos\left(dx+c\right)\sin\left(dx\right.}{12\left(\left(a^{11}+a^9b^2-6\,a^7b^4-14\,a^5b^6-11\,a^3b^8-3\,ab^{10}\right)d\cos\left(d\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3/(a*cos(d*x+c)+b*sin(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] -1/12*(22*a^4*b^3 + 14*a^2*b^5 - 8*b^7 + 12*(3*a^6*b + 2*a^4*b^3 + b^7)*cos
(d*x + c)^2 + 6*(9*a^5*b^2 + 8*a^3*b^4 - a*b^6)*cos(d*x + c)*sin(d*x + c) +
 3*((2*a^6 - 9*a^4*b^2 + 9*a^2*b^4)*cos(d*x + c)^3 + 3*(2*a^4*b^2 - 3*a^2*b
^4)*cos(d*x + c) + (2*a^3*b^3 - 3*a*b^5 + (6*a^5*b - 11*a^3*b^3 + 3*a*b^5)*
cos(d*x + c)^2)*sin(d*x + c))*sqrt(a^2 + b^2)*log((2*a*b*cos(d*x + c)*sin(d
*x + c) + (a^2 - b^2)*cos(d*x + c)^2 - 2*a^2 - b^2 - 2*sqrt(a^2 + b^2)*(b*c
os(d*x + c) - a*sin(d*x + c)))/(2*a*b*cos(d*x + c)*sin(d*x + c) + (a^2 - b^
2)*cos(d*x + c)^2 + b^2)))/((a^11 + a^9*b^2 - 6*a^7*b^4 - 14*a^5*b^6 - 11*a
^3*b^8 - 3*a*b^10)*d*cos(d*x + c)^3 + 3*(a^9*b^2 + 4*a^7*b^4 + 6*a^5*b^6 +
4*a^3*b^8 + a*b^10)*d*cos(d*x + c) + ((3*a^10*b + 11*a^8*b^3 + 14*a^6*b^5 +
 6*a^4*b^7 - a^2*b^9 - b^11)*d*cos(d*x + c)^2 + (a^8*b^3 + 4*a^6*b^5 + 6*a^
4*b^7 + 4*a^2*b^9 + b^11)*d)*sin(d*x + c))
```

giac [B] time = 0.44, size = 524, normalized size = 3.34

$$-\frac{\frac{3\left(2\,a^3-3\,ab^2\right)\log\left(\frac{\left|2\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-2\,b-2\,\sqrt{a^2+b^2}\right|}{\left|2\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-2\,b+2\,\sqrt{a^2+b^2}\right|}\right)}{\left(a^6+3\,a^4b^2+3\,a^2b^4+b^6\right)\sqrt{a^2+b^2}}-\frac{2\left(27\,a^6b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+18\,a^4b^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+6\,a^2b^6\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+18\,a^7b\tan\left(\frac{1}{2}\,d\right.\right.}{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3/(a*cos(d*x+c)+b*sin(d*x+c))^4,x, algorithm="giac")
```

```
[Out] -1/6*(3*(2*a^3 - 3*a*b^2)*log(abs(2*a*tan(1/2*d*x + 1/2*c) - 2*b - 2*sqrt(a
^2 + b^2))/abs(2*a*tan(1/2*d*x + 1/2*c) - 2*b + 2*sqrt(a^2 + b^2)))/((a^6 +
 3*a^4*b^2 + 3*a^2*b^4 + b^6)*sqrt(a^2 + b^2)) - 2*(27*a^6*b^2*tan(1/2*d*x
+ 1/2*c)^5 + 18*a^4*b^4*tan(1/2*d*x + 1/2*c)^5 + 6*a^2*b^6*tan(1/2*d*x + 1/
```

```
2*c)^5 + 18*a^7*b*tan(1/2*d*x + 1/2*c)^4 - 81*a^5*b^3*tan(1/2*d*x + 1/2*c)^
4 - 36*a^3*b^5*tan(1/2*d*x + 1/2*c)^4 - 12*a*b^7*tan(1/2*d*x + 1/2*c)^4 - 1
08*a^6*b^2*tan(1/2*d*x + 1/2*c)^3 + 42*a^4*b^4*tan(1/2*d*x + 1/2*c)^3 + 8*a
^2*b^6*tan(1/2*d*x + 1/2*c)^3 + 8*b^8*tan(1/2*d*x + 1/2*c)^3 - 36*a^7*b*tan
(1/2*d*x + 1/2*c)^2 + 120*a^5*b^3*tan(1/2*d*x + 1/2*c)^2 + 18*a^3*b^5*tan(1
/2*d*x + 1/2*c)^2 + 12*a*b^7*tan(1/2*d*x + 1/2*c)^2 + 81*a^6*b^2*tan(1/2*d*
x + 1/2*c) + 12*a^4*b^4*tan(1/2*d*x + 1/2*c) + 6*a^2*b^6*tan(1/2*d*x + 1/2*
c) + 18*a^7*b + 5*a^5*b^3 + 2*a^3*b^5)/((a^9 + 3*a^7*b^2 + 3*a^5*b^4 + a^3*
b^6)*(a*tan(1/2*d*x + 1/2*c)^2 - 2*b*tan(1/2*d*x + 1/2*c) - a)^3))/d
```

maple [B] time = 0.31, size = 494, normalized size = 3.15

$$-\frac{\frac{2\left(-\frac{b^2(9a^4+6a^2b^2+2b^4)\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2a(a^6+3a^4b^2+3a^2b^4+b^6)}-\frac{b(6a^6-27a^4b^2-12a^2b^4-4b^6)\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2a^2(a^6+3a^4b^2+3a^2b^4+b^6)}+\frac{b^2(54a^6-21a^4b^2-4a^2b^4-4b^6)\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3a^3(a^6+3a^4b^2+3a^2b^4+b^6)}+\frac{b(6a^6-20a^4b^2-3a^2b^4-2b^6)\left(\tan^2\left(\frac{dx}{2}\right.\right.}{a^2(a^6+3a^4b^2+3a^2b^4+b^6)}}{\left(\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a-2b\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-a\right)^3}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3/(a*cos(d*x+c)+b*sin(d*x+c))^4,x)
```

```
[Out] 1/d*(-2*(-1/2*b^2*(9*a^4+6*a^2*b^2+2*b^4)/a/(a^6+3*a^4*b^2+3*a^2*b^4+b^6)*t
an(1/2*d*x+1/2*c)^5-1/2*b*(6*a^6-27*a^4*b^2-12*a^2*b^4-4*b^6)/a^2/(a^6+3*a^
4*b^2+3*a^2*b^4+b^6)*tan(1/2*d*x+1/2*c)^4+1/3/a^3*b^2*(54*a^6-21*a^4*b^2-4*
a^2*b^4-4*b^6)/(a^6+3*a^4*b^2+3*a^2*b^4+b^6)*tan(1/2*d*x+1/2*c)^3+1/a^2*b*(
6*a^6-20*a^4*b^2-3*a^2*b^4-2*b^6)/(a^6+3*a^4*b^2+3*a^2*b^4+b^6)*tan(1/2*d*x
+1/2*c)^2-1/2/a*b^2*(27*a^4+4*a^2*b^2+2*b^4)/(a^6+3*a^4*b^2+3*a^2*b^4+b^6)*
tan(1/2*d*x+1/2*c)-1/6*b*(18*a^4+5*a^2*b^2+2*b^4)/(a^6+3*a^4*b^2+3*a^2*b^4+
b^6))/(tan(1/2*d*x+1/2*c)^2*a-2*b*tan(1/2*d*x+1/2*c)-a)^3+a*(2*a^2-3*b^2)/(
a^6+3*a^4*b^2+3*a^2*b^4+b^6)/(a^2+b^2)^(1/2)*arctanh(1/2*(2*a*tan(1/2*d*x+1
/2*c)-2*b)/(a^2+b^2)^(1/2)))
```

maxima [B] time = 0.45, size = 724, normalized size = 4.61

$$-\frac{\frac{3\left(2a^2-3b^2\right)a\log\left(\frac{b-\frac{a\sin(dx+c)}{\cos(dx+c)+1}+\sqrt{a^2+b^2}}{b-\frac{a\sin(dx+c)}{\cos(dx+c)+1}-\sqrt{a^2+b^2}}\right)}{\left(a^6+3a^4b^2+3a^2b^4+b^6\right)\sqrt{a^2+b^2}}+\frac{2\left(18a^7b+5a^5b^3+2a^3b^5+\frac{3\left(27a^6b^2+4a^4b^4+2a^2b^6\right)\sin(dx+c)}{\cos(dx+c)+1}-\frac{6\left(6a^7b-20a\right.}{}\right.}{a^{12}+3a^{10}b^2+3a^8b^4+a^6b^6+\frac{6\left(a^{11}b+3a^9b^3+3a^7b^5+a^5b^7\right)\sin(dx+c)}{\cos(dx+c)+1}-\frac{3\left(a^{12}-a^{10}b^2-9a^8b^4-11a^6b^6-4a^4b^8\right)\sin(}{(\cos(dx+c)+1)^2}}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3/(a*cos(d*x+c)+b*sin(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] -1/6*(3*(2*a^2 - 3*b^2)*a*log((b - a*sin(d*x + c)/(cos(d*x + c) + 1) + sqrt
(a^2 + b^2))/(b - a*sin(d*x + c)/(cos(d*x + c) + 1) - sqrt(a^2 + b^2)))/((a
```

```
^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*sqrt(a^2 + b^2)) + 2*(18*a^7*b + 5*a^5*b^
3 + 2*a^3*b^5 + 3*(27*a^6*b^2 + 4*a^4*b^4 + 2*a^2*b^6)*sin(d*x + c)/(cos(d*
x + c) + 1) - 6*(6*a^7*b - 20*a^5*b^3 - 3*a^3*b^5 - 2*a*b^7)*sin(d*x + c)^2
/(cos(d*x + c) + 1)^2 - 2*(54*a^6*b^2 - 21*a^4*b^4 - 4*a^2*b^6 - 4*b^8)*sin
(d*x + c)^3/(cos(d*x + c) + 1)^3 + 3*(6*a^7*b - 27*a^5*b^3 - 12*a^3*b^5 - 4
*a*b^7)*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + 3*(9*a^6*b^2 + 6*a^4*b^4 + 2*
a^2*b^6)*sin(d*x + c)^5/(cos(d*x + c) + 1)^5)/(a^12 + 3*a^10*b^2 + 3*a^8*b^
4 + a^6*b^6 + 6*(a^11*b + 3*a^9*b^3 + 3*a^7*b^5 + a^5*b^7)*sin(d*x + c)/(co
s(d*x + c) + 1) - 3*(a^12 - a^10*b^2 - 9*a^8*b^4 - 11*a^6*b^6 - 4*a^4*b^8)*
sin(d*x + c)^2/(cos(d*x + c) + 1)^2 - 4*(3*a^11*b + 7*a^9*b^3 + 3*a^7*b^5 -
 3*a^5*b^7 - 2*a^3*b^9)*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 3*(a^12 - a^1
0*b^2 - 9*a^8*b^4 - 11*a^6*b^6 - 4*a^4*b^8)*sin(d*x + c)^4/(cos(d*x + c) +
1)^4 + 6*(a^11*b + 3*a^9*b^3 + 3*a^7*b^5 + a^5*b^7)*sin(d*x + c)^5/(cos(d*x
 + c) + 1)^5 - (a^12 + 3*a^10*b^2 + 3*a^8*b^4 + a^6*b^6)*sin(d*x + c)^6/(co
s(d*x + c) + 1)^6))/d
```

mupad [B] time = 2.69, size = 764, normalized size = 4.87

$$\frac{\ln\left(\left(a^2+b^2\right)^{7/2}+a^6\,b+b^7+3\,a^2\,b^5+3\,a^4\,b^3-a^7\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)-a\,b^6\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)-3\,a^3\,b^4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)-3\,a\right.}{d\left(a^2+b^2\right)^{7/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^3/(a*cos(c + d*x) + b*sin(c + d*x))^4,x)
```

```
[Out] (log((a^2 + b^2)^(7/2) + a^6*b + b^7 + 3*a^2*b^5 + 3*a^4*b^3 - a^7*tan(c/2
+ (d*x)/2) - a*b^6*tan(c/2 + (d*x)/2) - 3*a^3*b^4*tan(c/2 + (d*x)/2) - 3*a^
5*b^2*tan(c/2 + (d*x)/2))*((3*a*b^2)/2 - a^3))/(d*(a^2 + b^2)^(7/2)) - ((18
*a^4*b + 2*b^5 + 5*a^2*b^3)/(3*(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2)) + (2*ta
n(c/2 + (d*x)/2)^2*(2*b^7 - 6*a^6*b + 3*a^2*b^5 + 20*a^4*b^3))/(a^2*(a^6 +
b^6 + 3*a^2*b^4 + 3*a^4*b^2)) - (tan(c/2 + (d*x)/2)^4*(4*b^7 - 6*a^6*b + 12
*a^2*b^5 + 27*a^4*b^3))/(a^2*(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2)) + (b*tan(
c/2 + (d*x)/2)*(27*a^4*b + 2*b^5 + 4*a^2*b^3))/(a*(a^6 + b^6 + 3*a^2*b^4 +
3*a^4*b^2)) + (b*tan(c/2 + (d*x)/2)^5*(9*a^4*b + 2*b^5 + 6*a^2*b^3))/(a*(a^
6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2)) - (2*b*tan(c/2 + (d*x)/2)^3*(3*a^2 - 2*b^
2)*(18*a^4*b + 2*b^5 + 5*a^2*b^3))/(3*a^3*(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^
2)))/(d*(tan(c/2 + (d*x)/2)^2*(12*a*b^2 - 3*a^3) - a^3*tan(c/2 + (d*x)/2)^6
 - tan(c/2 + (d*x)/2)^4*(12*a*b^2 - 3*a^3) - tan(c/2 + (d*x)/2)^3*(12*a^2*b
 - 8*b^3) + a^3 + 6*a^2*b*tan(c/2 + (d*x)/2) + 6*a^2*b*tan(c/2 + (d*x)/2)^5
)) + (a*log((a^2 + b^2)^(7/2) - a^6*b - b^7 - 3*a^2*b^5 - 3*a^4*b^3 + a^7*t
an(c/2 + (d*x)/2) + a*b^6*tan(c/2 + (d*x)/2) + 3*a^3*b^4*tan(c/2 + (d*x)/2)
 + 3*a^5*b^2*tan(c/2 + (d*x)/2))*(2*a^2 - 3*b^2))/(2*d*(a^2 + b^2)^(7/2))
```

<u>**sympy**</u> **[F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3/(a*cos(d*x+c)+b*sin(d*x+c))**4,x)
```

```
[Out] Timed out
```

3.143 $\int \frac{\cos^2(c+dx)}{(a\cos(c+dx)+b\sin(c+dx))^4}\,dx$

Optimal. Leaf size=30

$$-\frac{\cot^3(c+dx)}{3bd(a\cot(c+dx)+b)^3}$$

```
[Out] -1/3*cot(d*x+c)^3/b/d/(b+a*cot(d*x+c))^3
```

Rubi [A] time = 0.05, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {3088, 37}

$$-\frac{\cot^3(c+dx)}{3bd(a\cot(c+dx)+b)^3}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2/(a*Cos[c + d*x] + b*Sin[c + d*x])^4,x]
```

```
[Out] -Cot[c + d*x]^3/(3*b*d*(b + a*Cot[c + d*x])^3)
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 3088

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[(x^m*(b +
 a*x)^n)/(1 + x^2)^((m + n + 2)/2), x], x, Cot[c + d*x]], x] /; FreeQ[{a, b
, c, d}, x] && IntegerQ[n] && IntegerQ[(m + n)/2] && NeQ[n, -1] &&  !(GtQ[n
, 0] && GtQ[m, 1])
```

Rubi steps

$$\begin{aligned}\int \frac{\cos^2(c+dx)}{(a\cos(c+dx)+b\sin(c+dx))^4}\,dx &= -\frac{\text{Subst}\left(\int \frac{x^2}{(b+ax)^4}\,dx, x, \cot(c+dx)\right)}{d}\\ &= -\frac{\cot^3(c+dx)}{3bd(b+a\cot(c+dx))^3}\end{aligned}$$

Mathematica [B] time = 0.66, size = 124, normalized size = 4.13

$$\frac{\left(2ab^3-6a^3b\right)\cos(3(c+dx))-6ab\left(a^2+b^2\right)\cos(c+dx)+2\left(a^2-b^2\right)\sin(c+dx)\left(\left(3a^2-b^2\right)\cos(2(c+dx))+3a\right.}{12ad\left(a^2+b^2\right)^2(a\cos(c+dx)+b\sin(c+dx))^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2/(a*Cos[c + d*x] + b*Sin[c + d*x])^4,x]
```

```
[Out] (-6*a*b*(a^2 + b^2)*Cos[c + d*x] + (-6*a^3*b + 2*a*b^3)*Cos[3*(c + d*x)] +
2*(a^2 - b^2)*(3*a^2 + b^2 + (3*a^2 - b^2)*Cos[2*(c + d*x)])*Sin[c + d*x])/
(12*a*(a^2 + b^2)^2*d*(a*Cos[c + d*x] + b*Sin[c + d*x])^3)
```

fricas [B] time = 0.57, size = 255, normalized size = 8.50

$$-\frac{\left(9\,a^4b-6\,a^2b^3+b^5\right)\cos\left(dx+c\right)^3-3\left(a^4b-3\,a^2b^3\right)\cos\left(dx+c\right)-\left(a^3b^2-3\,ab^4+\right.}{3\left(\left(a^9-6\,a^5b^4-8\,a^3b^6-3\,ab^8\right)d\cos\left(dx+c\right)^3+3\left(a^7b^2+3\,a^5b^4+3\,a^3b^6+ab^8\right)d\cos\left(dx+c\right)+\left(\left(3\,a^8b+8\,a^6\right.\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(a*cos(d*x+c)+b*sin(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] -1/3*((9*a^4*b - 6*a^2*b^3 + b^5)*cos(d*x + c)^3 - 3*(a^4*b - 3*a^2*b^3)*co
s(d*x + c) - (a^3*b^2 - 3*a*b^4 + (3*a^5 - 10*a^3*b^2 + 3*a*b^4)*cos(d*x +
c)^2)*sin(d*x + c))/((a^9 - 6*a^5*b^4 - 8*a^3*b^6 - 3*a*b^8)*d*cos(d*x + c)
^3 + 3*(a^7*b^2 + 3*a^5*b^4 + 3*a^3*b^6 + a*b^8)*d*cos(d*x + c) + ((3*a^8*b
 + 8*a^6*b^3 + 6*a^4*b^5 - b^9)*d*cos(d*x + c)^2 + (a^6*b^3 + 3*a^4*b^5 + 3
*a^2*b^7 + b^9)*d)*sin(d*x + c))
```

giac [A] time = 1.45, size = 20, normalized size = 0.67

$$-\frac{1}{3\left(b\tan\left(dx+c\right)+a\right)^3bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(a*cos(d*x+c)+b*sin(d*x+c))^4,x, algorithm="giac")
```

```
[Out] -1/3/((b*tan(d*x + c) + a)^3*b*d)
```

maple [A] time = 0.30, size = 21, normalized size = 0.70

$$-\frac{1}{3db\left(a+b\tan\left(dx+c\right)\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2/(a*cos(d*x+c)+b*sin(d*x+c))^4,x)
```

```
[Out] -1/3/d/b/(a+b*tan(d*x+c))^3
```

maxima [A] time = 0.34, size = 53, normalized size = 1.77

$$-\frac{1}{3\left(b^{4}\tan\left(dx+c\right)^{3}+3\,ab^{3}\tan\left(dx+c\right)^{2}+3\,a^{2}b^{2}\tan\left(dx+c\right)+a^{3}b\right)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(a*cos(d*x+c)+b*sin(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] -1/3/((b^4*tan(d*x + c)^3 + 3*a*b^3*tan(d*x + c)^2 + 3*a^2*b^2*tan(d*x + c)
 + a^3*b)*d)
```

mupad [B] time = 1.29, size = 224, normalized size = 7.47

$$\frac{\frac{2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{5}}{a}+\frac{2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{a}+\frac{4\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{2}}{a^{2}}-\frac{4\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{4}}{a^{2}}-\frac{4\tan\left(\frac{c}{2}+\frac{d}{2}\right.}{3}}{d\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{2}\left(12\,a\,b^{2}-3\,a^{3}\right)-a^{3}\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{6}-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{4}\left(12\,a\,b^{2}-3\,a^{3}\right)-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{3}\left(12\,a^{2}\,b-\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^2/(a*cos(c + d*x) + b*sin(c + d*x))^4,x)
```

```
[Out] ((2*tan(c/2 + (d*x)/2)^5)/a + (2*tan(c/2 + (d*x)/2))/a + (4*b*tan(c/2 + (d*
x)/2)^2)/a^2 - (4*b*tan(c/2 + (d*x)/2)^4)/a^2 - (4*tan(c/2 + (d*x)/2)^3*(3*
a^2 - 2*b^2))/(3*a^3))/(d*(tan(c/2 + (d*x)/2)^2*(12*a*b^2 - 3*a^3) - a^3*ta
n(c/2 + (d*x)/2)^6 - tan(c/2 + (d*x)/2)^4*(12*a*b^2 - 3*a^3) - tan(c/2 + (d
*x)/2)^3*(12*a^2*b - 8*b^3) + a^3 + 6*a^2*b*tan(c/2 + (d*x)/2) + 6*a^2*b*ta
n(c/2 + (d*x)/2)^5))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2/(a*cos(d*x+c)+b*sin(d*x+c))**4,x)
```

```
[Out] Timed out
```

3.144 $\int \frac{\cos(c+dx)}{(a\cos(c+dx)+b\sin(c+dx))^4}\,dx$

Optimal. Leaf size=141

$$-\frac{b}{3d\left(a^2+b^2\right)(a\cos(c+dx)+b\sin(c+dx))^3}-\frac{a(b\cos(c+dx)-a\sin(c+dx))}{2d\left(a^2+b^2\right)^2(a\cos(c+dx)+b\sin(c+dx))^2}-\frac{a\tanh^{-1}\left(\frac{b\cos(c+dx}{\sqrt{a}}\right.}{2d\left(a^2+b^2\right.}$$

```
[Out] -1/2*a*arctanh((b*cos(d*x+c)-a*sin(d*x+c))/(a^2+b^2)^(1/2))/(a^2+b^2)^(5/2)
/d-1/3*b/(a^2+b^2)/d/(a*cos(d*x+c)+b*sin(d*x+c))^3-1/2*a*(b*cos(d*x+c)-a*si
n(d*x+c))/(a^2+b^2)^2/d/(a*cos(d*x+c)+b*sin(d*x+c))^2
```

Rubi [A] time = 0.11, antiderivative size = 141, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {3158, 12, 3076, 3074, 206}

$$-\frac{b}{3d\left(a^2+b^2\right)(a\cos(c+dx)+b\sin(c+dx))^3}-\frac{a(b\cos(c+dx)-a\sin(c+dx))}{2d\left(a^2+b^2\right)^2(a\cos(c+dx)+b\sin(c+dx))^2}-\frac{a\tanh^{-1}\left(\frac{b\cos(c+dx}{\sqrt{a}}\right.}{2d\left(a^2+b^2\right.}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]/(a*Cos[c + d*x] + b*Sin[c + d*x])^4,x]
```

```
[Out] -(a*ArcTanh[(b*Cos[c + d*x] - a*Sin[c + d*x])/Sqrt[a^2 + b^2]])/(2*(a^2 + b
^2)^(5/2)*d) - b/(3*(a^2 + b^2)*d*(a*Cos[c + d*x] + b*Sin[c + d*x])^3) - (a
*(b*Cos[c + d*x] - a*Sin[c + d*x]))/(2*(a^2 + b^2)^2*d*(a*Cos[c + d*x] + b*
Sin[c + d*x])^2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3074

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x
_Symbol] :> -Dist[d^(-1), Subst[Int[1/(a^2 + b^2 - x^2), x], x, b*Cos[c + d
```

```
*x] - a*Sin[c + d*x]], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

Rule 3076

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x
_Symbol] :> Simp[((b*Cos[c + d*x] - a*Sin[c + d*x])*(a*Cos[c + d*x] + b*Sin
[c + d*x])^(n + 1))/(d*(n + 1)*(a^2 + b^2)), x] + Dist[(n + 2)/((n + 1)*(a^
2 + b^2)), Int[(a*Cos[c + d*x] + b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{
a, b, c, d}, x] && NeQ[a^2 + b^2, 0] && LtQ[n, -1] && NeQ[n, -2]
```

Rule 3158

```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.))*((a_.) + cos[(d_.) + (e_.)*(x_)
]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)])^(n_), x_Symbol] :> -Simp[((c*B + c
*A*Cos[d + e*x] + (a*B - b*A)*Sin[d + e*x])*(a + b*Cos[d + e*x] + c*Sin[d +
 e*x])^(n + 1))/(e*(n + 1)*(a^2 - b^2 - c^2)), x] + Dist[1/((n + 1)*(a^2 -
b^2 - c^2)), Int[(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n + 1)*Simp[(n + 1)
*(a*A - b*B) + (n + 2)*(a*B - b*A)*Cos[d + e*x] - (n + 2)*c*A*Sin[d + e*x],
 x], x], x] /; FreeQ[{a, b, c, d, e, A, B}, x] && LtQ[n, -1] && NeQ[a^2 - b
^2 - c^2, 0] && NeQ[n, -2]
```

Rubi steps

$$\int \frac{\cos(c+dx)}{(a\cos(c+dx)+b\sin(c+dx))^4}\,dx = -\frac{b}{3\left(a^2+b^2\right)d(a\cos(c+dx)+b\sin(c+dx))^3}+\frac{\int \frac{3a}{(a\cos(c+dx)+b\sin(c+dx))}}{3\left(a^2+b^2\right)}$$

$$= -\frac{b}{3\left(a^2+b^2\right)d(a\cos(c+dx)+b\sin(c+dx))^3}+\frac{a\int \frac{1}{(a\cos(c+dx)+b\sin(c+dx}}{a^2+b^2}$$

$$= -\frac{b}{3\left(a^2+b^2\right)d(a\cos(c+dx)+b\sin(c+dx))^3}-\frac{a(b\cos(c+dx)}{2\left(a^2+b^2\right)^2 d(a\cos(c}$$

$$= -\frac{b}{3\left(a^2+b^2\right)d(a\cos(c+dx)+b\sin(c+dx))^3}-\frac{a(b\cos(c+dx)}{2\left(a^2+b^2\right)^2 d(a\cos(c}$$

$$= -\frac{a\tanh^{-1}\left(\frac{b\cos(c+dx)-a\sin(c+dx)}{\sqrt{a^2+b^2}}\right)}{2\left(a^2+b^2\right)^{5/2}d}-\frac{b}{3\left(a^2+b^2\right)d(a\cos(c+dx)+b\sin(c}$$

Mathematica [A] time = 0.73, size = 128, normalized size = 0.91

$$\frac{\frac{6a\tanh^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)-b}{\sqrt{a^2+b^2}}\right)}{\left(a^2+b^2\right)^{5/2}}+\frac{3\left(a^3-ab^2\right)\sin(2(c+dx))-4b\left(a^2+b^2\right)-6a^2b\cos(2(c+dx))}{2\left(a^2+b^2\right)^2(a\cos(c+dx)+b\sin(c+dx))^3}}{6d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]/(a*Cos[c + d*x] + b*Sin[c + d*x])^4,x]
```

```
[Out] ((6*a*ArcTanh[(-b + a*Tan[(c + d*x)/2])/Sqrt[a^2 + b^2]])/(a^2 + b^2)^(5/2)
 + (-4*b*(a^2 + b^2) - 6*a^2*b*Cos[2*(c + d*x)] + 3*(a^3 - a*b^2)*Sin[2*(c
+ d*x)])/(2*(a^2 + b^2)^2*(a*Cos[c + d*x] + b*Sin[c + d*x])^3))/(6*d)
```

fricas [B] time = 0.74, size = 420, normalized size = 2.98

$$\frac{2\,a^4b-2\,a^2b^3-4\,b^5-12\left(a^4b+a^2b^3\right)\cos\left(dx+c\right)^2+6\left(a^5-ab^4\right)\cos\left(dx+c\right)\sin\left(dx+c\right)+3\left(3\,a^2b^2\cos\left(dx+c\right.\right.}{12\left(\left(a^9-6\,a^5b^4-8\,a^3b^6-3\,ab^8\right)d\cos\left(dx+c\right)^3+3\left(a^7b^2+3\,a^5b^4\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a*cos(d*x+c)+b*sin(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] 1/12*(2*a^4*b - 2*a^2*b^3 - 4*b^5 - 12*(a^4*b + a^2*b^3)*cos(d*x + c)^2 + 6
*(a^5 - a*b^4)*cos(d*x + c)*sin(d*x + c) + 3*(3*a^2*b^2*cos(d*x + c) + (a^4
 - 3*a^2*b^2)*cos(d*x + c)^3 + (a*b^3 + (3*a^3*b - a*b^3)*cos(d*x + c)^2)*s
in(d*x + c))*sqrt(a^2 + b^2)*log(-(2*a*b*cos(d*x + c)*sin(d*x + c) + (a^2 -
 b^2)*cos(d*x + c)^2 - 2*a^2 - b^2 + 2*sqrt(a^2 + b^2)*(b*cos(d*x + c) - a*
sin(d*x + c)))/(2*a*b*cos(d*x + c)*sin(d*x + c) + (a^2 - b^2)*cos(d*x + c)^
2 + b^2)))/((a^9 - 6*a^5*b^4 - 8*a^3*b^6 - 3*a*b^8)*d*cos(d*x + c)^3 + 3*(a
^7*b^2 + 3*a^5*b^4 + 3*a^3*b^6 + a*b^8)*d*cos(d*x + c) + ((3*a^8*b + 8*a^6*
b^3 + 6*a^4*b^5 - b^9)*d*cos(d*x + c)^2 + (a^6*b^3 + 3*a^4*b^5 + 3*a^2*b^7
+ b^9)*d)*sin(d*x + c))
```

giac [B] time = 4.08, size = 426, normalized size = 3.02

$$-\frac{\frac{3\,a\log\left(\frac{\left|2\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-2\,b-2\,\sqrt{a^2+b^2}\right|}{\left|2\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-2\,b+2\,\sqrt{a^2+b^2}\right|}\right)}{\left(a^4+2\,a^2b^2+b^4\right)\sqrt{a^2+b^2}}-\frac{2\left(3\,a^6\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+12\,a^4b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+6\,a^2b^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+3\,a^5b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4-24\,a^3b^3\,\mathrm{t}\right.}{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a*cos(d*x+c)+b*sin(d*x+c))^4,x, algorithm="giac")
```

```
[Out] -1/6*(3*a*log(abs(2*a*tan(1/2*d*x + 1/2*c) - 2*b - 2*sqrt(a^2 + b^2))/abs(2
*a*tan(1/2*d*x + 1/2*c) - 2*b + 2*sqrt(a^2 + b^2)))/((a^4 + 2*a^2*b^2 + b^4
)*sqrt(a^2 + b^2)) - 2*(3*a^6*tan(1/2*d*x + 1/2*c)^5 + 12*a^4*b^2*tan(1/2*d
*x + 1/2*c)^5 + 6*a^2*b^4*tan(1/2*d*x + 1/2*c)^5 + 3*a^5*b*tan(1/2*d*x + 1/
2*c)^4 - 24*a^3*b^3*tan(1/2*d*x + 1/2*c)^4 - 12*a*b^5*tan(1/2*d*x + 1/2*c)^
4 - 30*a^4*b^2*tan(1/2*d*x + 1/2*c)^3 + 8*a^2*b^4*tan(1/2*d*x + 1/2*c)^3 +
8*b^6*tan(1/2*d*x + 1/2*c)^3 - 12*a^5*b*tan(1/2*d*x + 1/2*c)^2 + 30*a^3*b^3
*tan(1/2*d*x + 1/2*c)^2 + 12*a*b^5*tan(1/2*d*x + 1/2*c)^2 - 3*a^6*tan(1/2*d
*x + 1/2*c) + 18*a^4*b^2*tan(1/2*d*x + 1/2*c) + 6*a^2*b^4*tan(1/2*d*x + 1/2
*c) + 5*a^5*b + 2*a^3*b^3)/((a^7 + 2*a^5*b^2 + a^3*b^4)*(a*tan(1/2*d*x + 1/
2*c)^2 - 2*b*tan(1/2*d*x + 1/2*c) - a)^3))/d
```

maple [B] time = 0.30, size = 383, normalized size = 2.72

$$-\frac{\frac{2\left(-\frac{\left(a^{4}+4a^{2}b^{2}+2b^{4}\right)\left(\tan^{5}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2a\left(a^{4}+2a^{2}b^{2}+b^{4}\right)}-\frac{b\left(a^{4}-8a^{2}b^{2}-4b^{4}\right)\left(\tan^{4}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2a^{2}\left(a^{4}+2a^{2}b^{2}+b^{4}\right)}+\frac{b^{2}\left(15a^{4}-4a^{2}b^{2}-4b^{4}\right)\left(\tan^{3}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3a^{3}\left(a^{4}+2a^{2}b^{2}+b^{4}\right)}+\frac{b\left(2a^{4}-5a^{2}b^{2}-2b^{4}\right)\left(\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a^{2}\left(a^{4}+2a^{2}b^{2}+b^{4}\right)}+\frac{\left(a^{4}-6a^{2}b^{2}-2b^{4}\right)\tan\cdots}{2a\left(a^{4}+2a^{2}b^{2}+\cdots\right.}\cdots}{\left(\left(\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a-2b\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-a\right)^{3}}\cdots}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)/(a*cos(d*x+c)+b*sin(d*x+c))^4,x)
```

```
[Out] 1/d*(-2*(-1/2*(a^4+4*a^2*b^2+2*b^4)/a/(a^4+2*a^2*b^2+b^4)*tan(1/2*d*x+1/2*c
)^5-1/2*b*(a^4-8*a^2*b^2-4*b^4)/a^2/(a^4+2*a^2*b^2+b^4)*tan(1/2*d*x+1/2*c)^
4+1/3/a^3*b^2*(15*a^4-4*a^2*b^2-4*b^4)/(a^4+2*a^2*b^2+b^4)*tan(1/2*d*x+1/2*
c)^3+1/a^2*b*(2*a^4-5*a^2*b^2-2*b^4)/(a^4+2*a^2*b^2+b^4)*tan(1/2*d*x+1/2*c)
^2+1/2/a*(a^4-6*a^2*b^2-2*b^4)/(a^4+2*a^2*b^2+b^4)*tan(1/2*d*x+1/2*c)-1/6*b
*(5*a^2+2*b^2)/(a^4+2*a^2*b^2+b^4))/(tan(1/2*d*x+1/2*c)^2*a-2*b*tan(1/2*d*x
+1/2*c)-a)^3+a/(a^4+2*a^2*b^2+b^4)/(a^2+b^2)^(1/2)*arctanh(1/2*(2*a*tan(1/2
*d*x+1/2*c)-2*b)/(a^2+b^2)^(1/2)))
```

maxima [B] time = 0.45, size = 606, normalized size = 4.30

$$-\frac{\frac{3\,a\log\left(\frac{b-\frac{a\sin\left(dx+c\right)}{\cos\left(dx+c\right)+1}+\sqrt{a^{2}+b^{2}}}{b-\frac{a\sin\left(dx+c\right)}{\cos\left(dx+c\right)+1}-\sqrt{a^{2}+b^{2}}}\right)}{\left(a^{4}+2\,a^{2}b^{2}+b^{4}\right)\sqrt{a^{2}+b^{2}}}+\frac{2\left(5\,a^{5}b+2\,a^{3}b^{3}-\frac{3\left(a^{6}-6\,a^{4}b^{2}-2\,a^{2}b^{4}\right)\sin\left(dx+c\right)}{\cos\left(dx+c\right)+1}-\frac{6\left(2\,a^{5}b-5\,a^{3}b^{3}-2\,ab^{5}\right)\sin\left(dx+c\right)^{2}}{\left(\cos\left(dx+c\right)+1\right)^{2}}-\frac{2\left(15\,a^{4}b^{2}-\cdots\right.}{\cdots}\cdots}{a^{10}+2\,a^{8}b^{2}+a^{6}b^{4}+\frac{6\left(a^{9}b+2\,a^{7}b^{3}+a^{5}b^{5}\right)\sin\left(dx+c\right)}{\cos\left(dx+c\right)+1}-\frac{3\left(a^{10}-2\,a^{8}b^{2}-7\,a^{6}b^{4}-4\,a^{4}b^{6}\right)\sin\left(dx+c\right)^{2}}{\left(\cos\left(dx+c\right)+1\right)^{2}}-\frac{4\left(3\,a^{9}b+4\,a^{7}b^{3}-a^{5}b^{5}-2\,a^{3}b^{7}\cdots\right.}{\left(\cos\left(dx+c\right)+1\right)^{3}}\cdots}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a*cos(d*x+c)+b*sin(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] -1/6*(3*a*log((b - a*sin(d*x + c)/(cos(d*x + c) + 1) + sqrt(a^2 + b^2))/(b
- a*sin(d*x + c)/(cos(d*x + c) + 1) - sqrt(a^2 + b^2)))/((a^4 + 2*a^2*b^2 +
 b^4)*sqrt(a^2 + b^2)) + 2*(5*a^5*b + 2*a^3*b^3 - 3*(a^6 - 6*a^4*b^2 - 2*a^
2*b^4)*sin(d*x + c)/(cos(d*x + c) + 1) - 6*(2*a^5*b - 5*a^3*b^3 - 2*a*b^5)*
sin(d*x + c)^2/(cos(d*x + c) + 1)^2 - 2*(15*a^4*b^2 - 4*a^2*b^4 - 4*b^6)*si
n(d*x + c)^3/(cos(d*x + c) + 1)^3 + 3*(a^5*b - 8*a^3*b^3 - 4*a*b^5)*sin(d*x
 + c)^4/(cos(d*x + c) + 1)^4 + 3*(a^6 + 4*a^4*b^2 + 2*a^2*b^4)*sin(d*x + c)
^5/(cos(d*x + c) + 1)^5)/(a^10 + 2*a^8*b^2 + a^6*b^4 + 6*(a^9*b + 2*a^7*b^3
 + a^5*b^5)*sin(d*x + c)/(cos(d*x + c) + 1) - 3*(a^10 - 2*a^8*b^2 - 7*a^6*b
^4 - 4*a^4*b^6)*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 - 4*(3*a^9*b + 4*a^7*b^
3 - a^5*b^5 - 2*a^3*b^7)*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 3*(a^10 - 2*
a^8*b^2 - 7*a^6*b^4 - 4*a^4*b^6)*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + 6*(a
^9*b + 2*a^7*b^3 + a^5*b^5)*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - (a^10 + 2
*a^8*b^2 + a^6*b^4)*sin(d*x + c)^6/(cos(d*x + c) + 1)^6))/d
```

mupad [B] time = 3.82, size = 505, normalized size = 3.58

$$-\frac{\frac{\frac{5\,a^2\,b}{3}+\frac{2\,b^3}{3}}{a^4+2\,a^2\,b^2+b^4}-\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4\,\left(-a^4\,b+8\,a^2\,b^3+4\,b^5\right)}{a^2\,\left(a^4+2\,a^2\,b^2+b^4\right)}+\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\,\left(-4\,a^4\,b+10\,a^2\,b^3+4\,b^5\right)}{a^2\,\left(a^4+2\,a^2\,b^2+b^4\right)}+\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(-a^4+6\,a^2\,b^2+2\,b^4\right)}{a\,\left(a^4+2\,a^2\,b^2+b^4\right)}+\frac{\tan\left(\frac{c}{2}+\right.}{a}}{d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\,\left(12\,a\,b^2-3\,a^3\right)-a^3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4\,\left(12\,a\,b^2-3\,a^3\right)-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3\,\left(12\,a^2\,b-\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)/(a*cos(c + d*x) + b*sin(c + d*x))^4,x)
```

```
[Out] - (((5*a^2*b)/3 + (2*b^3)/3)/(a^4 + b^4 + 2*a^2*b^2) - (tan(c/2 + (d*x)/2)^
4*(4*b^5 - a^4*b + 8*a^2*b^3))/(a^2*(a^4 + b^4 + 2*a^2*b^2)) + (tan(c/2 + (
d*x)/2)^2*(4*b^5 - 4*a^4*b + 10*a^2*b^3))/(a^2*(a^4 + b^4 + 2*a^2*b^2)) + (
tan(c/2 + (d*x)/2)*(2*b^4 - a^4 + 6*a^2*b^2))/(a*(a^4 + b^4 + 2*a^2*b^2)) +
 (tan(c/2 + (d*x)/2)^5*(a^4 + 2*b^4 + 4*a^2*b^2))/(a*(a^4 + b^4 + 2*a^2*b^2
)) - (2*b*tan(c/2 + (d*x)/2)^3*(5*a^2*b + 2*b^3)*(3*a^2 - 2*b^2))/(3*a^3*(a
^4 + b^4 + 2*a^2*b^2)))/(d*(tan(c/2 + (d*x)/2)^2*(12*a*b^2 - 3*a^3) - a^3*t
an(c/2 + (d*x)/2)^6 - tan(c/2 + (d*x)/2)^4*(12*a*b^2 - 3*a^3) - tan(c/2 + (
d*x)/2)^3*(12*a^2*b - 8*b^3) + a^3 + 6*a^2*b*tan(c/2 + (d*x)/2) + 6*a^2*b*t
an(c/2 + (d*x)/2)^5)) - (a*atanh((a^4*b + b^5 + 2*a^2*b^3)/(a^2 + b^2)^(5/2
) - (a*tan(c/2 + (d*x)/2)*(a^4 + b^4 + 2*a^2*b^2))/(a^2 + b^2)^(5/2)))/(d*(
a^2 + b^2)^(5/2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a*cos(d*x+c)+b*sin(d*x+c))**4,x)
```

```
[Out] Timed out
```

3.145 $\int \frac{1}{(a\cos(c+dx)+b\sin(c+dx))^4}\,dx$

Optimal. Leaf size=98

$$\frac{2\sin(c+dx)}{3ad\left(a^2+b^2\right)(a\cos(c+dx)+b\sin(c+dx))}-\frac{b\cos(c+dx)-a\sin(c+dx)}{3d\left(a^2+b^2\right)(a\cos(c+dx)+b\sin(c+dx))^3}$$

```
[Out] 1/3*(-b*cos(d*x+c)+a*sin(d*x+c))/(a^2+b^2)/d/(a*cos(d*x+c)+b*sin(d*x+c))^3+
2/3*sin(d*x+c)/a/(a^2+b^2)/d/(a*cos(d*x+c)+b*sin(d*x+c))
```

Rubi [A] time = 0.04, antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {3076, 3075}

$$\frac{2\sin(c+dx)}{3ad\left(a^2+b^2\right)(a\cos(c+dx)+b\sin(c+dx))}-\frac{b\cos(c+dx)-a\sin(c+dx)}{3d\left(a^2+b^2\right)(a\cos(c+dx)+b\sin(c+dx))^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Cos[c + d*x] + b*Sin[c + d*x])^(-4),x]
```

```
[Out] -(b*Cos[c + d*x] - a*Sin[c + d*x])/(3*(a^2 + b^2)*d*(a*Cos[c + d*x] + b*Sin
[c + d*x])^3) + (2*Sin[c + d*x])/(3*a*(a^2 + b^2)*d*(a*Cos[c + d*x] + b*Sin
[c + d*x]))
```

Rule 3075

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-2), x
_Symbol] :> Simp[Sin[c + d*x]/(a*d*(a*Cos[c + d*x] + b*Sin[c + d*x])), x] /
; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

Rule 3076

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x
_Symbol] :> Simp[((b*Cos[c + d*x] - a*Sin[c + d*x])*(a*Cos[c + d*x] + b*Sin
[c + d*x])^(n + 1))/(d*(n + 1)*(a^2 + b^2)), x] + Dist[(n + 2)/((n + 1)*(a^
2 + b^2)), Int[(a*Cos[c + d*x] + b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{
a, b, c, d}, x] && NeQ[a^2 + b^2, 0] && LtQ[n, -1] && NeQ[n, -2]
```

Rubi steps

$$\int \frac{1}{(a\cos(c+dx)+b\sin(c+dx))^4}\,dx = -\frac{b\cos(c+dx)-a\sin(c+dx)}{3\left(a^2+b^2\right)d(a\cos(c+dx)+b\sin(c+dx))^3} + \frac{2\int \frac{1}{(a\cos(c+dx)+b\sin(c+dx}}{3\left(a^2+b^2\right)}$$
$$= -\frac{b\cos(c+dx)-a\sin(c+dx)}{3\left(a^2+b^2\right)d(a\cos(c+dx)+b\sin(c+dx))^3} + \frac{2\sin(c}{3a\left(a^2+b^2\right)d(a\cos(c}$$

Mathematica [A] time = 0.29, size = 85, normalized size = 0.87

$$\frac{\sin(c+dx)\left(\left(a^2-b^2\right)\cos(2(c+dx))+2a^2+b^2\right)-ab\cos(3(c+dx))}{3ad\left(a^2+b^2\right)(a\cos(c+dx)+b\sin(c+dx))^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Cos[c + d*x] + b*Sin[c + d*x])^(-4),x]
```

```
[Out] (-(a*b*Cos[3*(c + d*x)]) + (2*a^2 + b^2 + (a^2 - b^2)*Cos[2*(c + d*x)])*Sin
[c + d*x])/(3*a*(a^2 + b^2)*d*(a*Cos[c + d*x] + b*Sin[c + d*x])^3)
```

fricas [B] time = 0.56, size = 217, normalized size = 2.21

$$-\frac{2\left(3a^2b-b^3\right)\cos(dx+c)^3-3\left(a^2b-b^3\right)\cos(dx+c)-\left(a^3+3ab^2+2\left(a^3-3a\right.\right.}{3\left(\left(a^7-a^5b^2-5a^3b^4-3ab^6\right)d\cos(dx+c)^3+3\left(a^5b^2+2a^3b^4+ab^6\right)d\cos(dx+c)+\left(\left(3a^6b+5a^4b^3+a^2b^5\right.\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cos(d*x+c)+b*sin(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] -1/3*(2*(3*a^2*b - b^3)*cos(d*x + c)^3 - 3*(a^2*b - b^3)*cos(d*x + c) - (a^
3 + 3*a*b^2 + 2*(a^3 - 3*a*b^2)*cos(d*x + c)^2)*sin(d*x + c))/((a^7 - a^5*b
^2 - 5*a^3*b^4 - 3*a*b^6)*d*cos(d*x + c)^3 + 3*(a^5*b^2 + 2*a^3*b^4 + a*b^6
)*d*cos(d*x + c) + ((3*a^6*b + 5*a^4*b^3 + a^2*b^5 - b^7)*d*cos(d*x + c)^2
+ (a^4*b^3 + 2*a^2*b^5 + b^7)*d)*sin(d*x + c))
```

giac [A] time = 1.90, size = 50, normalized size = 0.51

$$-\frac{3b^2\tan(dx+c)^2+3ab\tan(dx+c)+a^2+b^2}{3(b\tan(dx+c)+a)^3b^3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cos(d*x+c)+b*sin(d*x+c))^4,x, algorithm="giac")
```

```
[Out] -1/3*(3*b^2*tan(d*x + c)^2 + 3*a*b*tan(d*x + c) + a^2 + b^2)/((b*tan(d*x +
c) + a)^3*b^3*d)
```

maple [A] time = 0.28, size = 64, normalized size = 0.65

$$\frac{-\frac{1}{b^3(a+b\tan(dx+c))}-\frac{a^2+b^2}{3b^3(a+b\tan(dx+c))^3}+\frac{a}{b^3(a+b\tan(dx+c))^2}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*cos(d*x+c)+b*sin(d*x+c))^4,x)
```

```
[Out] 1/d*(-1/b^3/(a+b*tan(d*x+c))-1/3*(a^2+b^2)/b^3/(a+b*tan(d*x+c))^3+a/b^3/(a+
b*tan(d*x+c))^2)
```

maxima [A] time = 0.35, size = 85, normalized size = 0.87

$$-\frac{3\,b^2\tan\left(dx+c\right)^2+3\,ab\tan\left(dx+c\right)+a^2+b^2}{3\left(b^6\tan\left(dx+c\right)^3+3\,ab^5\tan\left(dx+c\right)^2+3\,a^2b^4\tan\left(dx+c\right)+a^3b^3\right)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cos(d*x+c)+b*sin(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] -1/3*(3*b^2*tan(d*x + c)^2 + 3*a*b*tan(d*x + c) + a^2 + b^2)/((b^6*tan(d*x
+ c)^3 + 3*a*b^5*tan(d*x + c)^2 + 3*a^2*b^4*tan(d*x + c) + a^3*b^3)*d)
```

mupad [B] time = 1.25, size = 222, normalized size = 2.27

$$\frac{\frac{2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5}{a}+\frac{2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{a}-\frac{4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3\left(a^2-2\,b^2\right)}{3\,a^3}+\frac{4\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{a^2}-\frac{4\,b\,t}{}}{d\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\left(12\,a\,b^2-3\,a^3\right)-a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4\left(12\,a\,b^2-3\,a^3\right)-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3\left(12\,a^2\,b-8\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*cos(c + d*x) + b*sin(c + d*x))^4,x)
```

```
[Out] ((2*tan(c/2 + (d*x)/2)^5)/a + (2*tan(c/2 + (d*x)/2))/a - (4*tan(c/2 + (d*x)
/2)^3*(a^2 - 2*b^2))/(3*a^3) + (4*b*tan(c/2 + (d*x)/2)^2)/a^2 - (4*b*tan(c/
2 + (d*x)/2)^4)/a^2)/(d*(tan(c/2 + (d*x)/2)^2*(12*a*b^2 - 3*a^3) - a^3*tan(
c/2 + (d*x)/2)^6 - tan(c/2 + (d*x)/2)^4*(12*a*b^2 - 3*a^3) - tan(c/2 + (d*x
)/2)^3*(12*a^2*b - 8*b^3) + a^3 + 6*a^2*b*tan(c/2 + (d*x)/2) + 6*a^2*b*tan(
c/2 + (d*x)/2)^5))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cos(d*x+c)+b*sin(d*x+c))**4,x)
```

```
[Out] Timed out
```

3.146 $\int \frac{\sec(c+dx)}{(a\cos(c+dx)+b\sin(c+dx))^4}\,dx$

Optimal. Leaf size=231

$$\frac{a(b\cos(c+dx)-a\sin(c+dx))}{2b^2d\left(a^2+b^2\right)(a\cos(c+dx)+b\sin(c+dx))^2}+\frac{a\tanh^{-1}\left(\frac{b\cos(c+dx)-a\sin(c+dx)}{\sqrt{a^2+b^2}}\right)}{2b^2d\left(a^2+b^2\right)^{3/2}}+\frac{a\tanh^{-1}\left(\frac{b\cos(c+dx)-a\sin(c+dx)}{\sqrt{a^2+b^2}}\right)}{b^4d\sqrt{a^2+b^2}}-$$

```
[Out] arctanh(sin(d*x+c))/b^4/d+1/2*a*arctanh((b*cos(d*x+c)-a*sin(d*x+c))/(a^2+b^
2)^(1/2))/b^2/(a^2+b^2)^(3/2)/d-1/3/b/d/(a*cos(d*x+c)+b*sin(d*x+c))^3+1/2*a
*(b*cos(d*x+c)-a*sin(d*x+c))/b^2/(a^2+b^2)/d/(a*cos(d*x+c)+b*sin(d*x+c))^2-
1/b^3/d/(a*cos(d*x+c)+b*sin(d*x+c))+a*arctanh((b*cos(d*x+c)-a*sin(d*x+c))/(
a^2+b^2)^(1/2))/b^4/d/(a^2+b^2)^(1/2)
```

Rubi [A] time = 0.17, antiderivative size = 231, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {3094, 3770, 3074, 206, 3076}

$$\frac{a(b\cos(c+dx)-a\sin(c+dx))}{2b^2d\left(a^2+b^2\right)(a\cos(c+dx)+b\sin(c+dx))^2}+\frac{a\tanh^{-1}\left(\frac{b\cos(c+dx)-a\sin(c+dx)}{\sqrt{a^2+b^2}}\right)}{b^4d\sqrt{a^2+b^2}}+\frac{a\tanh^{-1}\left(\frac{b\cos(c+dx)-a\sin(c+dx)}{\sqrt{a^2+b^2}}\right)}{2b^2d\left(a^2+b^2\right)^{3/2}}-$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]/(a*Cos[c + d*x] + b*Sin[c + d*x])^4,x]
```

```
[Out] ArcTanh[Sin[c + d*x]]/(b^4*d) + (a*ArcTanh[(b*Cos[c + d*x] - a*Sin[c + d*x]
)/Sqrt[a^2 + b^2]])/(2*b^2*(a^2 + b^2)^(3/2)*d) + (a*ArcTanh[(b*Cos[c + d*x
] - a*Sin[c + d*x])/Sqrt[a^2 + b^2]])/(b^4*Sqrt[a^2 + b^2]*d) - 1/(3*b*d*(a
*Cos[c + d*x] + b*Sin[c + d*x])^3) + (a*(b*Cos[c + d*x] - a*Sin[c + d*x]))/
(2*b^2*(a^2 + b^2)*d*(a*Cos[c + d*x] + b*Sin[c + d*x])^2) - 1/(b^3*d*(a*Cos
[c + d*x] + b*Sin[c + d*x]))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3074

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x
_Symbol] :> -Dist[d^(-1), Subst[Int[1/(a^2 + b^2 - x^2), x], x, b*Cos[c + d
*x] - a*Sin[c + d*x]], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

Rule 3076

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x
_Symbol] :> Simp[((b*Cos[c + d*x] - a*Sin[c + d*x])*(a*Cos[c + d*x] + b*Sin
[c + d*x])^(n + 1))/(d*(n + 1)*(a^2 + b^2)), x] + Dist[(n + 2)/((n + 1)*(a^
2 + b^2)), Int[(a*Cos[c + d*x] + b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{
a, b, c, d}, x] && NeQ[a^2 + b^2, 0] && LtQ[n, -1] && NeQ[n, -2]
```

Rule 3094

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_)/co
s[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[(a*Cos[c + d*x] + b*Sin[c + d*x])^
(n + 1)/(b*d*(n + 1)), x] + (Dist[1/b^2, Int[(a*Cos[c + d*x] + b*Sin[c + d*
x])^(n + 2)/Cos[c + d*x], x], x] - Dist[a/b^2, Int[(a*Cos[c + d*x] + b*Sin[
c + d*x])^(n + 1), x], x]) /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0] &
& LtQ[n, -1]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\sec(c+dx)}{(a\cos(c+dx)+b\sin(c+dx))^4}\,dx = -\frac{1}{3bd(a\cos(c+dx)+b\sin(c+dx))^3} + \frac{\int \frac{\sec(c+dx)}{(a\cos(c+dx)+b\sin(c+dx))^2}\,dx}{b^2} - \frac{a}{\cdots}$$

$$= -\frac{1}{3bd(a\cos(c+dx)+b\sin(c+dx))^3} + \frac{a(b\cos(c+dx)-a\sin(\cdots}{2b^2\left(a^2+b^2\right)d(a\cos(c+dx)+\cdots}$$

$$= \frac{\tanh^{-1}(\sin(c+dx))}{b^4 d} - \frac{1}{3bd(a\cos(c+dx)+b\sin(c+dx))^3} + \frac{a(\cdots}{2b^2\left(a^2+\cdots\right.}$$

$$= \frac{\tanh^{-1}(\sin(c+dx))}{b^4 d} + \frac{a\tanh^{-1}\left(\frac{b\cos(c+dx)-a\sin(c+dx)}{\sqrt{a^2+b^2}}\right)}{2b^2\left(a^2+b^2\right)^{3/2}d} + \frac{a\tanh^{-1}\left(\frac{b\cos\cdots}{\cdots}\right.}{b^4\sqrt{a\cdots}}$$

Mathematica [A] time = 3.28, size = 290, normalized size = 1.26

$$-\frac{\sec^3(c+dx)(a\cos(c+dx)+b\sin(c+dx))\left(\frac{3b(2a^2+b^2)\cos(c+dx)(a+b\tan(c+dx))^2}{a^2+b^2}+\frac{6a(2a^2+3b^2)\cos^2(c+dx)(a+b\tan(c+dx))^3\tan}{(a^2+b^2)^{3/2}}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]/(a*Cos[c + d*x] + b*Sin[c + d*x])^4,x]
```

```
[Out] -1/6*(Sec[c + d*x]^3*(a*Cos[c + d*x] + b*Sin[c + d*x])*(2*b^3*Sec[c + d*x]
+ 3*b^2*(a*Cos[c + d*x] + b*Sin[c + d*x])*Tan[c + d*x] + (3*b*(2*a^2 + b^2)
*Cos[c + d*x]*(a + b*Tan[c + d*x])^2)/(a^2 + b^2) + (6*a*(2*a^2 + 3*b^2)*Ar
cTanh[(-b + a*Tan[(c + d*x)/2])/Sqrt[a^2 + b^2]]*Cos[c + d*x]^2*(a + b*Tan[
c + d*x])^3)/(a^2 + b^2)^(3/2) + 6*Cos[c + d*x]^2*Log[Cos[(c + d*x)/2] - Si
n[(c + d*x)/2]]*(a + b*Tan[c + d*x])^3 - 6*Cos[c + d*x]^2*Log[Cos[(c + d*x)
/2] + Sin[(c + d*x)/2]]*(a + b*Tan[c + d*x])^3))/(b^4*d*(a + b*Tan[c + d*x]
)^4)
```

fricas [B] time = 0.78, size = 745, normalized size = 3.23

$$-\frac{22\,a^4b^3+38\,a^2b^5+16\,b^7+12\left(a^6b-2\,a^2b^5-b^7\right)\cos\left(dx+c\right)^2+6\left(5\,a^5b^2+8\,a^3b^4+3\,ab^6\right)\cos\left(dx+c\right)\sin\left(dx\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a*cos(d*x+c)+b*sin(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] -1/12*(22*a^4*b^3 + 38*a^2*b^5 + 16*b^7 + 12*(a^6*b - 2*a^2*b^5 - b^7)*cos(
d*x + c)^2 + 6*(5*a^5*b^2 + 8*a^3*b^4 + 3*a*b^6)*cos(d*x + c)*sin(d*x + c)
- 3*((2*a^6 - 3*a^4*b^2 - 9*a^2*b^4)*cos(d*x + c)^3 + 3*(2*a^4*b^2 + 3*a^2*
b^4)*cos(d*x + c) + (2*a^3*b^3 + 3*a*b^5 + (6*a^5*b + 7*a^3*b^3 - 3*a*b^5)*
cos(d*x + c)^2)*sin(d*x + c))*sqrt(a^2 + b^2)*log((2*a*b*cos(d*x + c)*sin(d
*x + c) + (a^2 - b^2)*cos(d*x + c)^2 - 2*a^2 - b^2 - 2*sqrt(a^2 + b^2)*(b*c
os(d*x + c) - a*sin(d*x + c)))/(2*a*b*cos(d*x + c)*sin(d*x + c) + (a^2 - b^
2)*cos(d*x + c)^2 + b^2)) - 6*((a^7 - a^5*b^2 - 5*a^3*b^4 - 3*a*b^6)*cos(d*
x + c)^3 + 3*(a^5*b^2 + 2*a^3*b^4 + a*b^6)*cos(d*x + c) + (a^4*b^3 + 2*a^2*
b^5 + b^7 + (3*a^6*b + 5*a^4*b^3 + a^2*b^5 - b^7)*cos(d*x + c)^2)*sin(d*x +
 c))*log(sin(d*x + c) + 1) + 6*((a^7 - a^5*b^2 - 5*a^3*b^4 - 3*a*b^6)*cos(d
*x + c)^3 + 3*(a^5*b^2 + 2*a^3*b^4 + a*b^6)*cos(d*x + c) + (a^4*b^3 + 2*a^2
*b^5 + b^7 + (3*a^6*b + 5*a^4*b^3 + a^2*b^5 - b^7)*cos(d*x + c)^2)*sin(d*x
+ c))*log(-sin(d*x + c) + 1))/((a^7*b^4 - a^5*b^6 - 5*a^3*b^8 - 3*a*b^10)*d
```

```
*cos(d*x + c)^3 + 3*(a^5*b^6 + 2*a^3*b^8 + a*b^10)*d*cos(d*x + c) + ((3*a^6
*b^5 + 5*a^4*b^7 + a^2*b^9 - b^11)*d*cos(d*x + c)^2 + (a^4*b^7 + 2*a^2*b^9
+ b^11)*d)*sin(d*x + c))
```

giac [B] time = 4.07, size = 527, normalized size = 2.28

$$\frac{\frac{3\left(2\,a^{3}+3\,ab^{2}\right)\log\left(\frac{\left|2\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-2\,b-2\,\sqrt{a^{2}+b^{2}}\right|}{\left|2\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-2\,b+2\,\sqrt{a^{2}+b^{2}}\right|}\right)}{\left(a^{2}b^{4}+b^{6}\right)\sqrt{a^{2}+b^{2}}}+\frac{2\left(3\,a^{6}b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}+6\,a^{4}b^{3}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}+6\,a^{2}b^{5}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}+6\,a^{7}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{4}\cdots}{\cdots}}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a*cos(d*x+c)+b*sin(d*x+c))^4,x, algorithm="giac")
```

```
[Out] 1/6*(3*(2*a^3 + 3*a*b^2)*log(abs(2*a*tan(1/2*d*x + 1/2*c) - 2*b - 2*sqrt(a^
2 + b^2))/abs(2*a*tan(1/2*d*x + 1/2*c) - 2*b + 2*sqrt(a^2 + b^2)))/((a^2*b^
4 + b^6)*sqrt(a^2 + b^2)) + 2*(3*a^6*b*tan(1/2*d*x + 1/2*c)^5 + 6*a^4*b^3*t
an(1/2*d*x + 1/2*c)^5 + 6*a^2*b^5*tan(1/2*d*x + 1/2*c)^5 + 6*a^7*tan(1/2*d*
x + 1/2*c)^4 - 9*a^5*b^2*tan(1/2*d*x + 1/2*c)^4 - 12*a^3*b^4*tan(1/2*d*x +
1/2*c)^4 - 12*a*b^6*tan(1/2*d*x + 1/2*c)^4 - 36*a^6*b*tan(1/2*d*x + 1/2*c)^
3 - 6*a^4*b^3*tan(1/2*d*x + 1/2*c)^3 + 8*a^2*b^5*tan(1/2*d*x + 1/2*c)^3 + 8
*b^7*tan(1/2*d*x + 1/2*c)^3 - 12*a^7*tan(1/2*d*x + 1/2*c)^2 + 48*a^5*b^2*ta
n(1/2*d*x + 1/2*c)^2 + 42*a^3*b^4*tan(1/2*d*x + 1/2*c)^2 + 12*a*b^6*tan(1/2
*d*x + 1/2*c)^2 + 33*a^6*b*tan(1/2*d*x + 1/2*c) + 24*a^4*b^3*tan(1/2*d*x +
1/2*c) + 6*a^2*b^5*tan(1/2*d*x + 1/2*c) + 6*a^7 + 5*a^5*b^2 + 2*a^3*b^4)/((
a^5*b^3 + a^3*b^5)*(a*tan(1/2*d*x + 1/2*c)^2 - 2*b*tan(1/2*d*x + 1/2*c) - a
)^3) + 6*log(abs(tan(1/2*d*x + 1/2*c) + 1))/b^4 - 6*log(abs(tan(1/2*d*x + 1
/2*c) - 1))/b^4)/d
```

maple [B] time = 0.42, size = 1367, normalized size = 5.92

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)/(a*cos(d*x+c)+b*sin(d*x+c))^4,x)
```

```
[Out] 2/d/(tan(1/2*d*x+1/2*c)^2*a-2*b*tan(1/2*d*x+1/2*c)-a)^3*a/(a^2+b^2)*tan(1/2
*d*x+1/2*c)^5+16/d/b/(tan(1/2*d*x+1/2*c)^2*a-2*b*tan(1/2*d*x+1/2*c)-a)^3*a^
2/(a^2+b^2)*tan(1/2*d*x+1/2*c)^2+4/d*b^3/(tan(1/2*d*x+1/2*c)^2*a-2*b*tan(1/
2*d*x+1/2*c)-a)^3/a^2/(a^2+b^2)*tan(1/2*d*x+1/2*c)^2+11/d/b^2/(tan(1/2*d*x+
1/2*c)^2*a-2*b*tan(1/2*d*x+1/2*c)-a)^3*a^3/(a^2+b^2)*tan(1/2*d*x+1/2*c)+2/d
*b^2/(tan(1/2*d*x+1/2*c)^2*a-2*b*tan(1/2*d*x+1/2*c)-a)^3/a/(a^2+b^2)*tan(1/
2*d*x+1/2*c)+1/d/b^2/(tan(1/2*d*x+1/2*c)^2*a-2*b*tan(1/2*d*x+1/2*c)-a)^3*a^
```

3/(a^2+b^2)*tan(1/2*d*x+1/2*c)^5+2/d*b^2/(tan(1/2*d*x+1/2*c)^2*a-2*b*tan(1/2*d*x+1/2*c)-a)^3/a/(a^2+b^2)*tan(1/2*d*x+1/2*c)^5+2/d/b^3/(tan(1/2*d*x+1/2*c)^2*a-2*b*tan(1/2*d*x+1/2*c)-a)^3/(a^2+b^2)*a^4*tan(1/2*d*x+1/2*c)^4-3/d/b/(tan(1/2*d*x+1/2*c)^2*a-2*b*tan(1/2*d*x+1/2*c)-a)^3/(a^2+b^2)*a^2*tan(1/2*d*x+1/2*c)^4-4/d*b^3/(tan(1/2*d*x+1/2*c)^2*a-2*b*tan(1/2*d*x+1/2*c)-a)^3/(a^2+b^2)/a^2*tan(1/2*d*x+1/2*c)^4-12/d/b^2/(tan(1/2*d*x+1/2*c)^2*a-2*b*tan(1/2*d*x+1/2*c)-a)^3*a^3/(a^2+b^2)*tan(1/2*d*x+1/2*c)^3+8/3/d*b^2/(tan(1/2*d*x+1/2*c)^2*a-2*b*tan(1/2*d*x+1/2*c)-a)^3/a/(a^2+b^2)*tan(1/2*d*x+1/2*c)^3+8/3/d*b^4/(tan(1/2*d*x+1/2*c)^2*a-2*b*tan(1/2*d*x+1/2*c)-a)^3/a^3/(a^2+b^2)*tan(1/2*d*x+1/2*c)^3-4/d/b^3/(tan(1/2*d*x+1/2*c)^2*a-2*b*tan(1/2*d*x+1/2*c)-a)^3*a^4/(a^2+b^2)*tan(1/2*d*x+1/2*c)^2+2/3/d*b/(tan(1/2*d*x+1/2*c)^2*a-2*b*tan(1/2*d*x+1/2*c)-a)^3/(a^2+b^2)-2/d/(tan(1/2*d*x+1/2*c)^2*a-2*b*tan(1/2*d*x+1/2*c)-a)^3*a/(a^2+b^2)*tan(1/2*d*x+1/2*c)^3+8/d/(tan(1/2*d*x+1/2*c)^2*a-2*b*tan(1/2*d*x+1/2*c)-a)^3*a/(a^2+b^2)*tan(1/2*d*x+1/2*c)-4/d*b/(tan(1/2*d*x+1/2*c)^2*a-2*b*tan(1/2*d*x+1/2*c)-a)^3/(a^2+b^2)*tan(1/2*d*x+1/2*c)^4+14/d*b/(tan(1/2*d*x+1/2*c)^2*a-2*b*tan(1/2*d*x+1/2*c)-a)^3/(a^2+b^2)*tan(1/2*d*x+1/2*c)^2+2/d/b^3/(tan(1/2*d*x+1/2*c)^2*a-2*b*tan(1/2*d*x+1/2*c)-a)^3/(a^2+b^2)*a^4+5/3/d/b/(tan(1/2*d*x+1/2*c)^2*a-2*b*tan(1/2*d*x+1/2*c)-a)^3/(a^2+b^2)*a^2-2/d/b^4*a^3/(a^2+b^2)^(3/2)*arctanh(1/2*(2*a*tan(1/2*d*x+1/2*c)-2*b)/(a^2+b^2)^(1/2))-3/d/b^2*a/(a^2+b^2)^(3/2)*arctanh(1/2*(2*a*tan(1/2*d*x+1/2*c)-2*b)/(a^2+b^2)^(1/2))-1/d/b^4*ln(tan(1/2*d*x+1/2*c)-1)+1/d/b^4*ln(tan(1/2*d*x+1/2*c)+1)

maxima [B] time = 0.47, size = 661, normalized size = 2.86

$$\frac{\frac{3\left(2a^2+3b^2\right)a\log\left(\frac{b-\frac{a\sin(dx+c)}{\cos(dx+c)+1}+\sqrt{a^2+b^2}}{b-\frac{a\sin(dx+c)}{\cos(dx+c)+1}-\sqrt{a^2+b^2}}\right)}{\left(a^2b^4+b^6\right)\sqrt{a^2+b^2}}-\frac{2\left(6a^7+5a^5b^2+2a^3b^4+\frac{3\left(11a^6b+8a^4b^3+2a^2b^5\right)\sin(dx+c)}{\cos(dx+c)+1}-\frac{6\left(2a^7-8a^5b^2-7a^3b^4-2ab^6\right)\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-\frac{2\left(18a^6b+3\cdots\right.}{\cdots}\right.}{a^8b^3+a^6b^5+\frac{6\left(a^7b^4+a^5b^6\right)\sin(dx+c)}{\cos(dx+c)+1}-\frac{3\left(a^8b^3-3a^6b^5-4a^4b^7\right)\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-\frac{4\left(3a^7b^4+a^5b^6-2a^3b^8\right)\sin(d\cdots}{(\cos(dx+c)+1)^3}}}{6d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)/(a*cos(d*x+c)+b*sin(d*x+c))^4,x, algorithm="maxima")

[Out] 1/6*(3*(2*a^2 + 3*b^2)*a*log((b - a*sin(d*x + c)/(cos(d*x + c) + 1) + sqrt(a^2 + b^2))/(b - a*sin(d*x + c)/(cos(d*x + c) + 1) - sqrt(a^2 + b^2)))/((a^2*b^4 + b^6)*sqrt(a^2 + b^2)) - 2*(6*a^7 + 5*a^5*b^2 + 2*a^3*b^4 + 3*(11*a^6*b + 8*a^4*b^3 + 2*a^2*b^5)*sin(d*x + c)/(cos(d*x + c) + 1) - 6*(2*a^7 - 8*a^5*b^2 - 7*a^3*b^4 - 2*a*b^6)*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 - 2*(18*a^6*b + 3*a^4*b^3 - 4*a^2*b^5 - 4*b^7)*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 3*(2*a^7 - 3*a^5*b^2 - 4*a^3*b^4 - 4*a*b^6)*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + 3*(a^6*b + 2*a^4*b^3 + 2*a^2*b^5)*sin(d*x + c)^5/(cos(d*x + c) + 1)^5)/(a^8*b^3 + a^6*b^5 + 6*(a^7*b^4 + a^5*b^6)*sin(d*x + c)/(cos(d*x + c) + 1) - 3*(a^8*b^3 - 3*a^6*b^5 - 4*a^4*b^7)*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 - 4*(3*a^7*b^4 + a^5*b^6 - 2*a^3*b^8)*sin(d*x + c)^3/(cos(d*x + c) + 1

```
)^3 + 3*(a^8*b^3 - 3*a^6*b^5 - 4*a^4*b^7)*sin(d*x + c)^4/(cos(d*x + c) + 1)
^4 + 6*(a^7*b^4 + a^5*b^6)*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - (a^8*b^3 +
 a^6*b^5)*sin(d*x + c)^6/(cos(d*x + c) + 1)^6) + 6*log(sin(d*x + c)/(cos(d*
x + c) + 1) + 1)/b^4 - 6*log(sin(d*x + c)/(cos(d*x + c) + 1) - 1)/b^4)/d
```

mupad [B] time = 4.83, size = 2848, normalized size = 12.33

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)*(a*cos(c + d*x) + b*sin(c + d*x))^4),x)
```

```
[Out] (2*atanh((64*a*b^5*tan(c/2 + (d*x)/2))/((176*a^3*b^15)/(b^12 + 2*a^2*b^10 +
 a^4*b^8) + (160*a^5*b^13)/(b^12 + 2*a^2*b^10 + a^4*b^8) + (48*a^7*b^11)/(b
^12 + 2*a^2*b^10 + a^4*b^8) + (64*a*b^17)/(b^12 + 2*a^2*b^10 + a^4*b^8)) +
(48*a^3*b^3*tan(c/2 + (d*x)/2))/((176*a^3*b^15)/(b^12 + 2*a^2*b^10 + a^4*b^
8) + (160*a^5*b^13)/(b^12 + 2*a^2*b^10 + a^4*b^8) + (48*a^7*b^11)/(b^12 + 2
*a^2*b^10 + a^4*b^8) + (64*a*b^17)/(b^12 + 2*a^2*b^10 + a^4*b^8))))/(b^4*d)
 - ((6*a^4 + 2*b^4 + 5*a^2*b^2)/(3*b^3*(a^2 + b^2)) + (tan(c/2 + (d*x)/2)*(
11*a^4 + 2*b^4 + 8*a^2*b^2))/(a*b^2*(a^2 + b^2)) + (tan(c/2 + (d*x)/2)^5*(a
^4 + 2*b^4 + 2*a^2*b^2))/(a*b^2*(a^2 + b^2)) - (tan(c/2 + (d*x)/2)^4*(4*b^6
 - 2*a^6 + 4*a^2*b^4 + 3*a^4*b^2))/(a^2*b^3*(a^2 + b^2)) + (2*tan(c/2 + (d*
x)/2)^2*(2*b^6 - 2*a^6 + 7*a^2*b^4 + 8*a^4*b^2))/(a^2*b^3*(a^2 + b^2)) - (2
*tan(c/2 + (d*x)/2)^3*(3*a^2 - 2*b^2)*(6*a^4 + 2*b^4 + 5*a^2*b^2))/(3*a^3*b
^2*(a^2 + b^2)))/(d*(tan(c/2 + (d*x)/2)^2*(12*a*b^2 - 3*a^3) - a^3*tan(c/2
+ (d*x)/2)^6 - tan(c/2 + (d*x)/2)^4*(12*a*b^2 - 3*a^3) - tan(c/2 + (d*x)/2)
^3*(12*a^2*b - 8*b^3) + a^3 + 6*a^2*b*tan(c/2 + (d*x)/2) + 6*a^2*b*tan(c/2
+ (d*x)/2)^5)) - (a*atan(((a*((a^2 + b^2)^3)^(1/2)*(2*a^2 + 3*b^2)*((8*(4*a
^2*b^7 + 8*a^4*b^5 + 4*a^6*b^3))/(b^12 + 2*a^2*b^10 + a^4*b^8) + (8*tan(c/2
 + (d*x)/2)*(8*a*b^9 + 29*a^3*b^7 + 28*a^5*b^5 + 8*a^7*b^3))/(b^13 + 2*a^2*
b^11 + a^4*b^9) - (a*((a^2 + b^2)^3)^(1/2)*(2*a^2 + 3*b^2)*((8*tan(c/2 + (d
*x)/2)*(12*a^2*b^12 + 20*a^4*b^10 + 8*a^6*b^8))/(b^13 + 2*a^2*b^11 + a^4*b^
9) - (8*(4*a*b^12 + 6*a^3*b^10 + 2*a^5*b^8))/(b^12 + 2*a^2*b^10 + a^4*b^8)
+ (a*((a^2 + b^2)^3)^(1/2)*(2*a^2 + 3*b^2)*((8*(4*a^2*b^15 + 8*a^4*b^13 + 4
*a^6*b^11))/(b^12 + 2*a^2*b^10 + a^4*b^8) + (8*tan(c/2 + (d*x)/2)*(12*a*b^1
7 + 32*a^3*b^15 + 28*a^5*b^13 + 8*a^7*b^11))/(b^13 + 2*a^2*b^11 + a^4*b^9))
)/(2*(b^10 + 3*a^2*b^8 + 3*a^4*b^6 + a^6*b^4))))/(2*(b^10 + 3*a^2*b^8 + 3*a
^4*b^6 + a^6*b^4)))*1i)/(2*(b^10 + 3*a^2*b^8 + 3*a^4*b^6 + a^6*b^4)) + (a*(
(a^2 + b^2)^3)^(1/2)*(2*a^2 + 3*b^2)*((8*(4*a^2*b^7 + 8*a^4*b^5 + 4*a^6*b^3
))/(b^12 + 2*a^2*b^10 + a^4*b^8) + (8*tan(c/2 + (d*x)/2)*(8*a*b^9 + 29*a^3*
b^7 + 28*a^5*b^5 + 8*a^7*b^3))/(b^13 + 2*a^2*b^11 + a^4*b^9) - (a*((a^2 + b
^2)^3)^(1/2)*(2*a^2 + 3*b^2)*((8*(4*a*b^12 + 6*a^3*b^10 + 2*a^5*b^8))/(b^12
 + 2*a^2*b^10 + a^4*b^8) - (8*tan(c/2 + (d*x)/2)*(12*a^2*b^12 + 20*a^4*b^10
 + 8*a^6*b^8))/(b^13 + 2*a^2*b^11 + a^4*b^9) + (a*((a^2 + b^2)^3)^(1/2)*(2*
a^2 + 3*b^2)*((8*(4*a^2*b^15 + 8*a^4*b^13 + 4*a^6*b^11))/(b^12 + 2*a^2*b^10
```

```
 + a^4*b^8) + (8*tan(c/2 + (d*x)/2)*(12*a*b^17 + 32*a^3*b^15 + 28*a^5*b^13
+ 8*a^7*b^11))/(b^13 + 2*a^2*b^11 + a^4*b^9)))/(2*(b^10 + 3*a^2*b^8 + 3*a^4
*b^6 + a^6*b^4))))/(2*(b^10 + 3*a^2*b^8 + 3*a^4*b^6 + a^6*b^4)))*1i)/(2*(b^
10 + 3*a^2*b^8 + 3*a^4*b^6 + a^6*b^4)))/((16*(2*a^5 + 3*a^3*b^2))/(b^12 + 2
*a^2*b^10 + a^4*b^8) - (16*tan(c/2 + (d*x)/2)*(8*a^6 + 12*a^2*b^4 + 20*a^4*
b^2))/(b^13 + 2*a^2*b^11 + a^4*b^9) - (a*((a^2 + b^2)^3)^(1/2)*(2*a^2 + 3*b
^2)*((8*(4*a^2*b^7 + 8*a^4*b^5 + 4*a^6*b^3))/(b^12 + 2*a^2*b^10 + a^4*b^8)
+ (8*tan(c/2 + (d*x)/2)*(8*a*b^9 + 29*a^3*b^7 + 28*a^5*b^5 + 8*a^7*b^3))/(b
^13 + 2*a^2*b^11 + a^4*b^9) - (a*((a^2 + b^2)^3)^(1/2)*(2*a^2 + 3*b^2)*((8*
tan(c/2 + (d*x)/2)*(12*a^2*b^12 + 20*a^4*b^10 + 8*a^6*b^8))/(b^13 + 2*a^2*b
^11 + a^4*b^9) - (8*(4*a*b^12 + 6*a^3*b^10 + 2*a^5*b^8))/(b^12 + 2*a^2*b^10
 + a^4*b^8) + (a*((a^2 + b^2)^3)^(1/2)*(2*a^2 + 3*b^2)*((8*(4*a^2*b^15 + 8*
a^4*b^13 + 4*a^6*b^11))/(b^12 + 2*a^2*b^10 + a^4*b^8) + (8*tan(c/2 + (d*x)/
2)*(12*a*b^17 + 32*a^3*b^15 + 28*a^5*b^13 + 8*a^7*b^11))/(b^13 + 2*a^2*b^11
 + a^4*b^9)))/(2*(b^10 + 3*a^2*b^8 + 3*a^4*b^6 + a^6*b^4))))/(2*(b^10 + 3*a
^2*b^8 + 3*a^4*b^6 + a^6*b^4))))/(2*(b^10 + 3*a^2*b^8 + 3*a^4*b^6 + a^6*b^4
)) + (a*((a^2 + b^2)^3)^(1/2)*(2*a^2 + 3*b^2)*((8*(4*a^2*b^7 + 8*a^4*b^5 +
4*a^6*b^3))/(b^12 + 2*a^2*b^10 + a^4*b^8) + (8*tan(c/2 + (d*x)/2)*(8*a*b^9
+ 29*a^3*b^7 + 28*a^5*b^5 + 8*a^7*b^3))/(b^13 + 2*a^2*b^11 + a^4*b^9) - (a*
((a^2 + b^2)^3)^(1/2)*(2*a^2 + 3*b^2)*((8*(4*a*b^12 + 6*a^3*b^10 + 2*a^5*b^
8))/(b^12 + 2*a^2*b^10 + a^4*b^8) - (8*tan(c/2 + (d*x)/2)*(12*a^2*b^12 + 20
*a^4*b^10 + 8*a^6*b^8))/(b^13 + 2*a^2*b^11 + a^4*b^9) + (a*((a^2 + b^2)^3)^
(1/2)*(2*a^2 + 3*b^2)*((8*(4*a^2*b^15 + 8*a^4*b^13 + 4*a^6*b^11))/(b^12 + 2
*a^2*b^10 + a^4*b^8) + (8*tan(c/2 + (d*x)/2)*(12*a*b^17 + 32*a^3*b^15 + 28*
a^5*b^13 + 8*a^7*b^11))/(b^13 + 2*a^2*b^11 + a^4*b^9)))/(2*(b^10 + 3*a^2*b^
8 + 3*a^4*b^6 + a^6*b^4))))/(2*(b^10 + 3*a^2*b^8 + 3*a^4*b^6 + a^6*b^4))))/
(2*(b^10 + 3*a^2*b^8 + 3*a^4*b^6 + a^6*b^4))))*((a^2 + b^2)^3)^(1/2)*(2*a^2
 + 3*b^2)*1i)/(d*(b^10 + 3*a^2*b^8 + 3*a^4*b^6 + a^6*b^4))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec(c+dx)}{(a\cos(c+dx)+b\sin(c+dx))^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a*cos(d*x+c)+b*sin(d*x+c))**4,x)
```

```
[Out] Integral(sec(c + d*x)/(a*cos(c + d*x) + b*sin(c + d*x))**4, x)
```

3.147 $\int \frac{\sec^2(c+dx)}{(a\cos(c+dx)+b\sin(c+dx))^4}\,dx$

Optimal. Leaf size=138

$$\frac{\frac{1}{a^3}+\frac{3a}{b^4}}{d(a\cot(c+dx)+b)}+\frac{\frac{a}{b^3}-\frac{b}{a^3}}{d(a\cot(c+dx)+b)^2}+\frac{\left(a^2+b^2\right)^2}{3a^3b^2d(a\cot(c+dx)+b)^3}-\frac{4a\log(\tan(c+dx))}{b^5d}-\frac{4a\log(a\cot(c+\ldots}{b^5d}$$

```
[Out] 1/3*(a^2+b^2)^2/a^3/b^2/d/(b+a*cot(d*x+c))^3+(a/b^3-b/a^3)/d/(b+a*cot(d*x+c
))^2+(1/a^3+3*a/b^4)/d/(b+a*cot(d*x+c))-4*a*ln(b+a*cot(d*x+c))/b^5/d-4*a*ln
(tan(d*x+c))/b^5/d+tan(d*x+c)/b^4/d
```

Rubi [A] time = 0.16, antiderivative size = 138, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {3088, 894}

$$\frac{\left(a^2+b^2\right)^2}{3a^3b^2d(a\cot(c+dx)+b)^3}+\frac{\frac{1}{a^3}+\frac{3a}{b^4}}{d(a\cot(c+dx)+b)}+\frac{\frac{a}{b^3}-\frac{b}{a^3}}{d(a\cot(c+dx)+b)^2}-\frac{4a\log(\tan(c+dx))}{b^5d}-\frac{4a\log(a\cot(c+\ldots}{b^5d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^2/(a*Cos[c + d*x] + b*Sin[c + d*x])^4,x]
```

```
[Out] (a^2 + b^2)^2/(3*a^3*b^2*d*(b + a*Cot[c + d*x])^3) + (a/b^3 - b/a^3)/(d*(b
+ a*Cot[c + d*x])^2) + (a^(-3) + (3*a)/b^4)/(d*(b + a*Cot[c + d*x])) - (4*a
*Log[b + a*Cot[c + d*x]])/(b^5*d) - (4*a*Log[Tan[c + d*x]])/(b^5*d) + Tan[c
 + d*x]/(b^4*d)
```

Rule 894

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ
[m, 0] && ILtQ[n, 0]))
```

Rule 3088

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[(x^m*(b +
 a*x)^n)/(1 + x^2)^((m + n + 2)/2), x], x, Cot[c + d*x]], x] /; FreeQ[{a, b
, c, d}, x] && IntegerQ[n] && IntegerQ[(m + n)/2] && NeQ[n, -1] && !(GtQ[n
, 0] && GtQ[m, 1])
```

Rubi steps

$$\int \frac{\sec^2(c+dx)}{(a\cos(c+dx)+b\sin(c+dx))^4}\,dx = -\frac{\text{Subst}\left(\int \frac{(1+x^2)^2}{x^2(b+ax)^4}\,dx, x, \cot(c+dx)\right)}{d}$$
$$= -\frac{\text{Subst}\left(\int \left(\frac{1}{b^4x^2}-\frac{4a}{b^5x}+\frac{(a^2+b^2)^2}{a^2b^2(b+ax)^4}+\frac{2(a^4-b^4)}{a^2b^3(b+ax)^3}+\frac{3a^4+b^4}{a^2b^4(b+ax)^2}+\frac{4a^2}{b^5(b+ax)}\right) c}{d}$$
$$= \frac{(a^2+b^2)^2}{3a^3b^2d(b+a\cot(c+dx))^3}+\frac{\frac{a}{b^3}-\frac{b}{a^3}}{d(b+a\cot(c+dx))^2}+\frac{\frac{1}{a^3}+\frac{3a}{b^4}}{d(b+a\cot(c+dx)}$$

Mathematica [A] time = 2.17, size = 133, normalized size = 0.96

$$\frac{-4(a^2+b^2)(a^2+3ab\tan(c+dx)+3b^2\tan^2(c+dx)+b^2)+6a(a+b\tan(c+dx))(a^2-4a(a+b\tan(c+dx))-}{3b^5d(a+b\tan(c+dx))^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^2/(a*Cos[c + d*x] + b*Sin[c + d*x])^4,x]
```

```
[Out] (3*b^4*Sec[c + d*x]^4 - 4*(a^2 + b^2)*(a^2 + b^2 + 3*a*b*Tan[c + d*x] + 3*b
^2*Tan[c + d*x]^2) + 6*a*(a + b*Tan[c + d*x])*(a^2 + b^2 - 4*a*(a + b*Tan[c
 + d*x]) - 2*Log[a + b*Tan[c + d*x]]*(a + b*Tan[c + d*x])^2))/(3*b^5*d*(a +
 b*Tan[c + d*x])^3)
```

fricas [B] time = 0.72, size = 537, normalized size = 3.89

$$\frac{3a^2b^4+3b^6-4(9a^4b^2+3a^2b^4-2b^6)\cos(dx+c)^4+6(5a^4b^2+a^2b^4-2b^6)\cos(dx+c)^2-6((a^6-2a^4b^2-}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a*cos(d*x+c)+b*sin(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] 1/3*(3*a^2*b^4 + 3*b^6 - 4*(9*a^4*b^2 + 3*a^2*b^4 - 2*b^6)*cos(d*x + c)^4 +
 6*(5*a^4*b^2 + a^2*b^4 - 2*b^6)*cos(d*x + c)^2 - 6*((a^6 - 2*a^4*b^2 - 3*a
^2*b^4)*cos(d*x + c)^4 + 3*(a^4*b^2 + a^2*b^4)*cos(d*x + c)^2 + ((3*a^5*b +
 2*a^3*b^3 - a*b^5)*cos(d*x + c)^3 + (a^3*b^3 + a*b^5)*cos(d*x + c))*sin(d*
x + c))*log(2*a*b*cos(d*x + c)*sin(d*x + c) + (a^2 - b^2)*cos(d*x + c)^2 +
b^2) + 6*((a^6 - 2*a^4*b^2 - 3*a^2*b^4)*cos(d*x + c)^4 + 3*(a^4*b^2 + a^2*b
^4)*cos(d*x + c)^2 + ((3*a^5*b + 2*a^3*b^3 - a*b^5)*cos(d*x + c)^3 + (a^3*b
^3 + a*b^5)*cos(d*x + c))*sin(d*x + c))*log(cos(d*x + c)^2) + 2*(2*(3*a^5*b
```

```
 - 7*a^3*b^3 - 6*a*b^5)*cos(d*x + c)^3 + (11*a^3*b^3 + 9*a*b^5)*cos(d*x + c
))*sin(d*x + c))/((a^5*b^5 - 2*a^3*b^7 - 3*a*b^9)*d*cos(d*x + c)^4 + 3*(a^3
*b^7 + a*b^9)*d*cos(d*x + c)^2 + ((3*a^4*b^6 + 2*a^2*b^8 - b^10)*d*cos(d*x
+ c)^3 + (a^2*b^8 + b^10)*d*cos(d*x + c))*sin(d*x + c))
```

giac [A] time = 4.82, size = 138, normalized size = 1.00

$$-\frac{\frac{12\,a\log(|b\tan(dx+c)+a|)}{b^5}-\frac{3\,\tan(dx+c)}{b^4}-\frac{22\,ab^3\tan(dx+c)^3+48\,a^2b^2\tan(dx+c)^2-6\,b^4\tan(dx+c)^2+36\,a^3b\tan(dx+c)-6\,ab^3\tan(dx+c)+9\,a^4-2\,a^2b^2-b^4}{(b\tan(dx+c)+a)^3b^5}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a*cos(d*x+c)+b*sin(d*x+c))^4,x, algorithm="giac")
```

```
[Out] -1/3*(12*a*log(abs(b*tan(d*x + c) + a))/b^5 - 3*tan(d*x + c)/b^4 - (22*a*b^
3*tan(d*x + c)^3 + 48*a^2*b^2*tan(d*x + c)^2 - 6*b^4*tan(d*x + c)^2 + 36*a^
3*b*tan(d*x + c) - 6*a*b^3*tan(d*x + c) + 9*a^4 - 2*a^2*b^2 - b^4)/((b*tan(
d*x + c) + a)^3*b^5))/d
```

maple [A] time = 0.41, size = 188, normalized size = 1.36

$$\frac{\tan(dx+c)}{b^4d}-\frac{4a\ln(a+b\tan(dx+c))}{d\,b^5}-\frac{6a^2}{d\,b^5(a+b\tan(dx+c))}-\frac{2}{d\,b^3(a+b\tan(dx+c))}-\frac{a^4}{3d\,b^5(a+b\tan(dx+c))^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2/(a*cos(d*x+c)+b*sin(d*x+c))^4,x)
```

```
[Out] tan(d*x+c)/b^4/d-4/d*a/b^5*ln(a+b*tan(d*x+c))-6/d/b^5/(a+b*tan(d*x+c))*a^2-
2/d/b^3/(a+b*tan(d*x+c))-1/3/d/b^5/(a+b*tan(d*x+c))^3*a^4-2/3/d/b^3/(a+b*ta
n(d*x+c))^3*a^2-1/3/d/b/(a+b*tan(d*x+c))^3+2/d*a^3/b^5/(a+b*tan(d*x+c))^2+2
/d*a/b^3/(a+b*tan(d*x+c))^2
```

maxima [A] time = 0.41, size = 144, normalized size = 1.04

$$-\frac{\frac{13\,a^4+2\,a^2b^2+b^4+6\left(3\,a^2b^2+b^4\right)\tan(dx+c)^2+6\left(5\,a^3b+ab^3\right)\tan(dx+c)}{b^8\tan(dx+c)^3+3\,ab^7\tan(dx+c)^2+3\,a^2b^6\tan(dx+c)+a^3b^5}+\frac{12\,a\log(b\tan(dx+c)+a)}{b^5}-\frac{3\,\tan(dx+c)}{b^4}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a*cos(d*x+c)+b*sin(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] -1/3*((13*a^4 + 2*a^2*b^2 + b^4 + 6*(3*a^2*b^2 + b^4)*tan(d*x + c)^2 + 6*(5
*a^3*b + a*b^3)*tan(d*x + c))/(b^8*tan(d*x + c)^3 + 3*a*b^7*tan(d*x + c)^2
+ 3*a^2*b^6*tan(d*x + c) + a^3*b^5) + 12*a*log(b*tan(d*x + c) + a)/b^5 - 3*
tan(d*x + c)/b^4)/d
```

mupad [B] time = 4.44, size = 666, normalized size = 4.83

$$\frac{\frac{4\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2\left(10a^4+b^4\right)}{a^2b^3}-\frac{2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^7\left(4a^4+b^4\right)}{ab^4}-\frac{8\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4\left(10a^4-2a^2b^2+b^4\right)}{a^2b^3}+\frac{4\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^6\left(10a^4\right.}{a^2b^3}}{d\left(a^3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^8+\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2\left(12ab^2-4a^3\right)+\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^6\left(12ab^2-4a^3\right)-\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4\left(24ab^2-\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^2*(a*cos(c + d*x) + b*sin(c + d*x))^4),x)
```

```
[Out] ((4*tan(c/2 + (d*x)/2)^2*(10*a^4 + b^4))/(a^2*b^3) - (2*tan(c/2 + (d*x)/2)^
7*(4*a^4 + b^4))/(a*b^4) - (8*tan(c/2 + (d*x)/2)^4*(10*a^4 + b^4 - 2*a^2*b^
2))/(a^2*b^3) + (4*tan(c/2 + (d*x)/2)^6*(10*a^4 + b^4))/(a^2*b^3) - (2*tan(
c/2 + (d*x)/2)^3*(36*a^6 - 4*b^6 + a^2*b^4 - 88*a^4*b^2))/(3*a^3*b^4) + (2*
tan(c/2 + (d*x)/2)^5*(36*a^6 - 4*b^6 + a^2*b^4 - 88*a^4*b^2))/(3*a^3*b^4) +
 (2*tan(c/2 + (d*x)/2)*(4*a^4 + b^4))/(a*b^4))/(d*(a^3*tan(c/2 + (d*x)/2)^8
 + tan(c/2 + (d*x)/2)^2*(12*a*b^2 - 4*a^3) + tan(c/2 + (d*x)/2)^6*(12*a*b^2
 - 4*a^3) - tan(c/2 + (d*x)/2)^4*(24*a*b^2 - 6*a^3) - tan(c/2 + (d*x)/2)^3*
(18*a^2*b - 8*b^3) + tan(c/2 + (d*x)/2)^5*(18*a^2*b - 8*b^3) + a^3 + 6*a^2*
b*tan(c/2 + (d*x)/2) - 6*a^2*b*tan(c/2 + (d*x)/2)^7)) - (8*a*atanh((256*a^3
*tan(c/2 + (d*x)/2)^2)/(256*a^3 - 256*a^3*tan(c/2 + (d*x)/2)^2 + (512*a^5)/
b^2 - (512*a^5*tan(c/2 + (d*x)/2)^2)/b^2 + (512*a^4*tan(c/2 + (d*x)/2))/b)
- (256*a^3)/(256*a^3 - 256*a^3*tan(c/2 + (d*x)/2)^2 + (512*a^5)/b^2 - (512*
a^5*tan(c/2 + (d*x)/2)^2)/b^2 + (512*a^4*tan(c/2 + (d*x)/2))/b) + (512*a^4*
tan(c/2 + (d*x)/2))/(256*a^3*b + (512*a^5)/b + 512*a^4*tan(c/2 + (d*x)/2) -
 (512*a^5*tan(c/2 + (d*x)/2)^2)/b - 256*a^3*b*tan(c/2 + (d*x)/2)^2)))/(b^5*
d)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec^2(c+dx)}{(a\cos(c+dx)+b\sin(c+dx))^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2/(a*cos(d*x+c)+b*sin(d*x+c))**4,x)
```

```
[Out] Integral(sec(c + d*x)**2/(a*cos(c + d*x) + b*sin(c + d*x))**4, x)
```

3.148 $\int \frac{\sec^3(c+dx)}{(a\cos(c+dx)+b\sin(c+dx))^4}\,dx$

Optimal. Leaf size=400

$$\frac{8a^2\tanh^{-1}(\sin(c+dx))}{b^6d}-\frac{4a^2}{b^5d(a\cos(c+dx)+b\sin(c+dx))}+\frac{2\left(a^2+b^2\right)\tanh^{-1}(\sin(c+dx))}{b^6d}+\frac{6a\sqrt{a^2+b^2}\tan}{}$$

```
[Out] 8*a^2*arctanh(sin(d*x+c))/b^6/d+1/2*arctanh(sin(d*x+c))/b^4/d+2*(a^2+b^2)*a
rctanh(sin(d*x+c))/b^6/d-4*a*sec(d*x+c)/b^5/d+1/3*(-a^2-b^2)/b^3/d/(a*cos(d
*x+c)+b*sin(d*x+c))^3+3/2*a*(b*cos(d*x+c)-a*sin(d*x+c))/b^4/d/(a*cos(d*x+c)
+b*sin(d*x+c))^2-4*a^2/b^5/d/(a*cos(d*x+c)+b*sin(d*x+c))-2*(a^2+b^2)/b^5/d/
(a*cos(d*x+c)+b*sin(d*x+c))+4*a^3*arctanh((b*cos(d*x+c)-a*sin(d*x+c))/(a^2+
b^2)^(1/2))/b^6/d/(a^2+b^2)^(1/2)+3/2*a*arctanh((b*cos(d*x+c)-a*sin(d*x+c))
/(a^2+b^2)^(1/2))/b^4/d/(a^2+b^2)^(1/2)+6*a*arctanh((b*cos(d*x+c)-a*sin(d*x
+c))/(a^2+b^2)^(1/2))*(a^2+b^2)^(1/2)/b^6/d+1/2*sec(d*x+c)*tan(d*x+c)/b^4/d
```

Rubi [A] time = 0.80, antiderivative size = 400, normalized size of antiderivative = 1.00, number of steps used = 32, number of rules used = 8, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3106, 3094, 3770, 3074, 206, 3076, 3768, 3104}

$$\frac{8a^2\tanh^{-1}(\sin(c+dx))}{b^6d}+\frac{2\left(a^2+b^2\right)\tanh^{-1}(\sin(c+dx))}{b^6d}-\frac{4a^2}{b^5d(a\cos(c+dx)+b\sin(c+dx))}-\frac{2\left(a^2\right.}{b^5d(a\cos(c+dx}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^3/(a*Cos[c + d*x] + b*Sin[c + d*x])^4,x]
```

```
[Out] (8*a^2*ArcTanh[Sin[c + d*x]])/(b^6*d) + ArcTanh[Sin[c + d*x]]/(2*b^4*d) + (
2*(a^2 + b^2)*ArcTanh[Sin[c + d*x]])/(b^6*d) + (4*a^3*ArcTanh[(b*Cos[c + d*
x] - a*Sin[c + d*x])/Sqrt[a^2 + b^2]])/(b^6*Sqrt[a^2 + b^2]*d) + (3*a*ArcTa
nh[(b*Cos[c + d*x] - a*Sin[c + d*x])/Sqrt[a^2 + b^2]])/(2*b^4*Sqrt[a^2 + b^
2]*d) + (6*a*Sqrt[a^2 + b^2]*ArcTanh[(b*Cos[c + d*x] - a*Sin[c + d*x])/Sqrt
[a^2 + b^2]])/(b^6*d) - (4*a*Sec[c + d*x])/(b^5*d) - (a^2 + b^2)/(3*b^3*d*(
a*Cos[c + d*x] + b*Sin[c + d*x])^3) + (3*a*(b*Cos[c + d*x] - a*Sin[c + d*x]
))/(2*b^4*d*(a*Cos[c + d*x] + b*Sin[c + d*x])^2) - (4*a^2)/(b^5*d*(a*Cos[c
+ d*x] + b*Sin[c + d*x])) - (2*(a^2 + b^2))/(b^5*d*(a*Cos[c + d*x] + b*Sin[
c + d*x])) + (Sec[c + d*x]*Tan[c + d*x])/(2*b^4*d)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3074

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x
_Symbol] :> -Dist[d^(-1), Subst[Int[1/(a^2 + b^2 - x^2), x], x, b*Cos[c + d
*x] - a*Sin[c + d*x]], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

Rule 3076

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x
_Symbol] :> Simp[((b*Cos[c + d*x] - a*Sin[c + d*x])*(a*Cos[c + d*x] + b*Sin
[c + d*x])^(n + 1))/(d*(n + 1)*(a^2 + b^2)), x] + Dist[(n + 2)/((n + 1)*(a^
2 + b^2)), Int[(a*Cos[c + d*x] + b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{
a, b, c, d}, x] && NeQ[a^2 + b^2, 0] && LtQ[n, -1] && NeQ[n, -2]
```

Rule 3094

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_)/co
s[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[(a*Cos[c + d*x] + b*Sin[c + d*x])^
(n + 1)/(b*d*(n + 1)), x] + (Dist[1/b^2, Int[(a*Cos[c + d*x] + b*Sin[c + d*
x])^(n + 2)/Cos[c + d*x], x], x] - Dist[a/b^2, Int[(a*Cos[c + d*x] + b*Sin[
c + d*x])^(n + 1), x], x]) /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0] &
& LtQ[n, -1]
```

Rule 3104

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_)/(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin
[(c_.) + (d_.)*(x_)]), x_Symbol] :> -Simp[Cos[c + d*x]^(m + 1)/(b*d*(m + 1)
), x] + (-Dist[a/b^2, Int[Cos[c + d*x]^(m + 1), x], x] + Dist[(a^2 + b^2)/b
^2, Int[Cos[c + d*x]^(m + 2)/(a*Cos[c + d*x] + b*Sin[c + d*x]), x], x]) /;
FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0] && LtQ[m, -1]
```

Rule 3106

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin
[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(a^2 + b^2)/b^2, Int[Cos[c +
d*x]^(m + 2)*(a*Cos[c + d*x] + b*Sin[c + d*x])^n, x], x] + (Dist[1/b^2, Int
[Cos[c + d*x]^m*(a*Cos[c + d*x] + b*Sin[c + d*x])^(n + 2), x], x] - Dist[(2
*a)/b^2, Int[Cos[c + d*x]^(m + 1)*(a*Cos[c + d*x] + b*Sin[c + d*x])^(n + 1)
, x], x]) /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0] && LtQ[n, -1] && L
tQ[m, -1]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
```

```
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\sec^3(c+dx)}{(a\cos(c+dx)+b\sin(c+dx))^4}\,dx = \frac{\int \frac{\sec^3(c+dx)}{(a\cos(c+dx)+b\sin(c+dx))^2}\,dx}{b^2} - \frac{(2a)\int \frac{\sec^2(c+dx)}{(a\cos(c+dx)+b\sin(c+dx))^3}\,dx}{b^2} + \frac{(a^2+b^2 \ldots}{\ldots}$$

$$= -\frac{a^2+b^2}{3b^3d(a\cos(c+dx)+b\sin(c+dx))^3} + \frac{\int \sec^3(c+dx)\,dx}{b^4} - 2\frac{(2a)\int \ldots}{\ldots}$$

$$= -\frac{a^2+b^2}{3b^3d(a\cos(c+dx)+b\sin(c+dx))^3} + \frac{3a(b\cos(c+dx)-a\sin(c+ \ldots}{2b^4d(a\cos(c+dx)+b\sin(c+ \ldots}$$

$$= \frac{4a^2\tanh^{-1}(\sin(c+dx))}{b^6d} + \frac{\tanh^{-1}(\sin(c+dx))}{2b^4d} - \frac{a^2+b \ldots}{3b^3d(a\cos(c+dx)+ \ldots}$$

$$= \frac{4a^2\tanh^{-1}(\sin(c+dx))}{b^6d} + \frac{\tanh^{-1}(\sin(c+dx))}{2b^4d} + \frac{4a^3\tanh^{-1}\left(\frac{b\cos(c+d \ldots}{\sqrt{\ldots}}\right)}{b^6\sqrt{a^2+b^2 \ldots}}$$

Mathematica [A] time = 3.47, size = 538, normalized size = 1.34

$$-\frac{\sec^4(c+dx)(a\cos(c+dx)+b\sin(c+dx))\Big(18b^2\left(a^2+b^2\right)\sin(c+dx)(a\cos(c+dx)+b\sin(c+dx))+6b\left(12a \ldots\right.}{\ldots}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^3/(a*Cos[c + d*x] + b*Sin[c + d*x])^4,x]
```

```
[Out] -1/12*(Sec[c + d*x]^4*(a*Cos[c + d*x] + b*Sin[c + d*x])*(4*b^3*(a^2 + b^2)
+ 18*b^2*(a^2 + b^2)*Sin[c + d*x]*(a*Cos[c + d*x] + b*Sin[c + d*x]) + 6*b*(
```

```
12*a^2 + b^2)*(a*Cos[c + d*x] + b*Sin[c + d*x])^2 + 48*a*b*(a*Cos[c + d*x]
+ b*Sin[c + d*x])^3 + (60*a*(4*a^2 + 3*b^2)*ArcTanh[(-b + a*Tan[(c + d*x)/2
])/Sqrt[a^2 + b^2]]*(a*Cos[c + d*x] + b*Sin[c + d*x])^3)/Sqrt[a^2 + b^2] +
30*(4*a^2 + b^2)*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]]*(a*Cos[c + d*x] +
 b*Sin[c + d*x])^3 - 30*(4*a^2 + b^2)*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/
2]]*(a*Cos[c + d*x] + b*Sin[c + d*x])^3 - (3*b^2*(a*Cos[c + d*x] + b*Sin[c
+ d*x])^3)/(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^2 + (48*a*b*Sin[(c + d*x)/
2]*(a*Cos[c + d*x] + b*Sin[c + d*x])^3)/(Cos[(c + d*x)/2] - Sin[(c + d*x)/2
]) + (3*b^2*(a*Cos[c + d*x] + b*Sin[c + d*x])^3)/(Cos[(c + d*x)/2] + Sin[(c
 + d*x)/2])^2 - (48*a*b*Sin[(c + d*x)/2]*(a*Cos[c + d*x] + b*Sin[c + d*x])^
3)/(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])))/(b^6*d*(a + b*Tan[c + d*x])^4)
```

fricas [B] time = 0.81, size = 820, normalized size = 2.05

$$\frac{6\,a^2b^5 + 6\,b^7 - 30\left(4\,a^6b - 3\,a^4b^3 - 8\,a^2b^5 - b^7\right)\cos\left(dx + c\right)^4 - 20\left(11\,a^4b^3 + 13\,a^2b^5 + 2\,b^7\right)\cos\left(dx + c\right)^2 + 15\left(\left(\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a*cos(d*x+c)+b*sin(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] 1/12*(6*a^2*b^5 + 6*b^7 - 30*(4*a^6*b - 3*a^4*b^3 - 8*a^2*b^5 - b^7)*cos(d*
x + c)^4 - 20*(11*a^4*b^3 + 13*a^2*b^5 + 2*b^7)*cos(d*x + c)^2 + 15*((4*a^6
 - 9*a^4*b^2 - 9*a^2*b^4)*cos(d*x + c)^5 + 3*(4*a^4*b^2 + 3*a^2*b^4)*cos(d*
x + c)^3 + ((12*a^5*b + 5*a^3*b^3 - 3*a*b^5)*cos(d*x + c)^4 + (4*a^3*b^3 +
3*a*b^5)*cos(d*x + c)^2)*sin(d*x + c))*sqrt(a^2 + b^2)*log((2*a*b*cos(d*x +
 c)*sin(d*x + c) + (a^2 - b^2)*cos(d*x + c)^2 - 2*a^2 - b^2 - 2*sqrt(a^2 +
b^2)*(b*cos(d*x + c) - a*sin(d*x + c)))/(2*a*b*cos(d*x + c)*sin(d*x + c) +
(a^2 - b^2)*cos(d*x + c)^2 + b^2)) + 15*((4*a^7 - 7*a^5*b^2 - 14*a^3*b^4 -
3*a*b^6)*cos(d*x + c)^5 + 3*(4*a^5*b^2 + 5*a^3*b^4 + a*b^6)*cos(d*x + c)^3
+ ((12*a^6*b + 11*a^4*b^3 - 2*a^2*b^5 - b^7)*cos(d*x + c)^4 + (4*a^4*b^3 +
5*a^2*b^5 + b^7)*cos(d*x + c)^2)*sin(d*x + c))*log(sin(d*x + c) + 1) - 15*(
(4*a^7 - 7*a^5*b^2 - 14*a^3*b^4 - 3*a*b^6)*cos(d*x + c)^5 + 3*(4*a^5*b^2 +
5*a^3*b^4 + a*b^6)*cos(d*x + c)^3 + ((12*a^6*b + 11*a^4*b^3 - 2*a^2*b^5 - b
^7)*cos(d*x + c)^4 + (4*a^4*b^3 + 5*a^2*b^5 + b^7)*cos(d*x + c)^2)*sin(d*x
+ c))*log(-sin(d*x + c) + 1) - 30*(10*(a^5*b^2 + a^3*b^4)*cos(d*x + c)^3 +
(a^3*b^4 + a*b^6)*cos(d*x + c))*sin(d*x + c))/((a^5*b^6 - 2*a^3*b^8 - 3*a*b
^10)*d*cos(d*x + c)^5 + 3*(a^3*b^8 + a*b^10)*d*cos(d*x + c)^3 + ((3*a^4*b^7
 + 2*a^2*b^9 - b^11)*d*cos(d*x + c)^4 + (a^2*b^9 + b^11)*d*cos(d*x + c)^2)*
sin(d*x + c))
```

giac [A] time = 3.10, size = 548, normalized size = 1.37

$$\frac{15\left(4\,a^{2}+b^{2}\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)}{b^{6}}-\frac{15\left(4\,a^{2}+b^{2}\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)}{b^{6}}+\frac{15\left(4\,a^{3}+3\,ab^{2}\right)\log\left(\frac{\left|2\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-2\,b-2\,\sqrt{a^{2}+b^{2}}\right|}{\left|2\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-2\,b+2\,\sqrt{a^{2}+b^{2}}\right|}\right)}{\sqrt{a^{2}+b^{2}}\,b^{6}}+\frac{6\left(b\tan\left(\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a*cos(d*x+c)+b*sin(d*x+c))^4,x, algorithm="giac")
```

```
[Out] 1/6*(15*(4*a^2 + b^2)*log(abs(tan(1/2*d*x + 1/2*c) + 1))/b^6 - 15*(4*a^2 +
b^2)*log(abs(tan(1/2*d*x + 1/2*c) - 1))/b^6 + 15*(4*a^3 + 3*a*b^2)*log(abs(
2*a*tan(1/2*d*x + 1/2*c) - 2*b - 2*sqrt(a^2 + b^2))/abs(2*a*tan(1/2*d*x + 1
/2*c) - 2*b + 2*sqrt(a^2 + b^2)))/(sqrt(a^2 + b^2)*b^6) + 6*(b*tan(1/2*d*x
+ 1/2*c)^3 + 8*a*tan(1/2*d*x + 1/2*c)^2 + b*tan(1/2*d*x + 1/2*c) - 8*a)/((t
an(1/2*d*x + 1/2*c)^2 - 1)^2*b^5) + 2*(27*a^6*b*tan(1/2*d*x + 1/2*c)^5 + 6*
a^2*b^5*tan(1/2*d*x + 1/2*c)^5 + 36*a^7*tan(1/2*d*x + 1/2*c)^4 - 117*a^5*b^
2*tan(1/2*d*x + 1/2*c)^4 - 12*a*b^6*tan(1/2*d*x + 1/2*c)^4 - 216*a^6*b*tan(
1/2*d*x + 1/2*c)^3 + 114*a^4*b^3*tan(1/2*d*x + 1/2*c)^3 + 8*a^2*b^5*tan(1/2
*d*x + 1/2*c)^3 + 8*b^7*tan(1/2*d*x + 1/2*c)^3 - 72*a^7*tan(1/2*d*x + 1/2*c
)^2 + 300*a^5*b^2*tan(1/2*d*x + 1/2*c)^2 + 54*a^3*b^4*tan(1/2*d*x + 1/2*c)^
2 + 12*a*b^6*tan(1/2*d*x + 1/2*c)^2 + 189*a^6*b*tan(1/2*d*x + 1/2*c) + 30*a
^4*b^3*tan(1/2*d*x + 1/2*c) + 6*a^2*b^5*tan(1/2*d*x + 1/2*c) + 36*a^7 + 5*a
^5*b^2 + 2*a^3*b^4)/((a*tan(1/2*d*x + 1/2*c)^2 - 2*b*tan(1/2*d*x + 1/2*c) -
 a)^3*a^3*b^5))/d
```

maple [B] time = 0.47, size = 1255, normalized size = 3.14

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^3/(a*cos(d*x+c)+b*sin(d*x+c))^4,x)
```

```
[Out] 2/3/d/b/(tan(1/2*d*x+1/2*c)^2*a-2*b*tan(1/2*d*x+1/2*c)-a)^3+1/2/d/b^4/(tan(
1/2*d*x+1/2*c)-1)^2+1/2/d/b^4/(tan(1/2*d*x+1/2*c)-1)-1/2/d/b^4/(tan(1/2*d*x
+1/2*c)+1)^2+1/2/d/b^4/(tan(1/2*d*x+1/2*c)+1)+18/d/b/(tan(1/2*d*x+1/2*c)^2*
a-2*b*tan(1/2*d*x+1/2*c)-a)^3*tan(1/2*d*x+1/2*c)^2+12/d/b^5/(tan(1/2*d*x+1/
2*c)^2*a-2*b*tan(1/2*d*x+1/2*c)-a)^3*a^4+5/3/d/b^3/(tan(1/2*d*x+1/2*c)^2*a-
2*b*tan(1/2*d*x+1/2*c)-a)^3*a^2+2/d/(tan(1/2*d*x+1/2*c)^2*a-2*b*tan(1/2*d*x
+1/2*c)-a)^3/a*tan(1/2*d*x+1/2*c)+2/d/(tan(1/2*d*x+1/2*c)^2*a-2*b*tan(1/2*d
*x+1/2*c)-a)^3/a*tan(1/2*d*x+1/2*c)^5+8/3/d/(tan(1/2*d*x+1/2*c)^2*a-2*b*tan
(1/2*d*x+1/2*c)-a)^3/a*tan(1/2*d*x+1/2*c)^3+4/d/b^5/(tan(1/2*d*x+1/2*c)-1)*
a-10/d/b^6*ln(tan(1/2*d*x+1/2*c)-1)*a^2-4/d/b^5/(tan(1/2*d*x+1/2*c)+1)*a+10
```

```
/d/b^6*ln(tan(1/2*d*x+1/2*c)+1)*a^2+8/3/d*b^2/(tan(1/2*d*x+1/2*c)^2*a-2*b*t
an(1/2*d*x+1/2*c)-a)^3/a^3*tan(1/2*d*x+1/2*c)^3-24/d/b^5/(tan(1/2*d*x+1/2*c
)^2*a-2*b*tan(1/2*d*x+1/2*c)-a)^3*a^4*tan(1/2*d*x+1/2*c)^2+100/d/b^3/(tan(1
/2*d*x+1/2*c)^2*a-2*b*tan(1/2*d*x+1/2*c)-a)^3*a^2*tan(1/2*d*x+1/2*c)^2+4/d*
b/(tan(1/2*d*x+1/2*c)^2*a-2*b*tan(1/2*d*x+1/2*c)-a)^3/a^2*tan(1/2*d*x+1/2*c
)^2+63/d/b^4/(tan(1/2*d*x+1/2*c)^2*a-2*b*tan(1/2*d*x+1/2*c)-a)^3*a^3*tan(1/
2*d*x+1/2*c)+10/d/b^2/(tan(1/2*d*x+1/2*c)^2*a-2*b*tan(1/2*d*x+1/2*c)-a)^3*a
*tan(1/2*d*x+1/2*c)-20/d/b^6*a^3/(a^2+b^2)^(1/2)*arctanh(1/2*(2*a*tan(1/2*d
*x+1/2*c)-2*b)/(a^2+b^2)^(1/2))-15/d/b^4*a/(a^2+b^2)^(1/2)*arctanh(1/2*(2*a
*tan(1/2*d*x+1/2*c)-2*b)/(a^2+b^2)^(1/2))+9/d/b^4/(tan(1/2*d*x+1/2*c)^2*a-2
*b*tan(1/2*d*x+1/2*c)-a)^3*a^3*tan(1/2*d*x+1/2*c)^5+12/d/b^5/(tan(1/2*d*x+1
/2*c)^2*a-2*b*tan(1/2*d*x+1/2*c)-a)^3*a^4*tan(1/2*d*x+1/2*c)^4-39/d/b^3/(ta
n(1/2*d*x+1/2*c)^2*a-2*b*tan(1/2*d*x+1/2*c)-a)^3*a^2*tan(1/2*d*x+1/2*c)^4-4
/d*b/(tan(1/2*d*x+1/2*c)^2*a-2*b*tan(1/2*d*x+1/2*c)-a)^3/a^2*tan(1/2*d*x+1/
2*c)^4-72/d/b^4/(tan(1/2*d*x+1/2*c)^2*a-2*b*tan(1/2*d*x+1/2*c)-a)^3*a^3*tan
(1/2*d*x+1/2*c)^3+38/d/b^2/(tan(1/2*d*x+1/2*c)^2*a-2*b*tan(1/2*d*x+1/2*c)-a
)^3*a*tan(1/2*d*x+1/2*c)^3-5/2/d/b^4*ln(tan(1/2*d*x+1/2*c)-1)+5/2/d/b^4*ln(
tan(1/2*d*x+1/2*c)+1)
```

maxima [B] time = 0.48, size = 936, normalized size = 2.34

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a*cos(d*x+c)+b*sin(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] -1/6*(2*(60*a^7 + 5*a^5*b^2 + 2*a^3*b^4 + 6*(55*a^6*b + 5*a^4*b^3 + a^2*b^5
)*sin(d*x + c)/(cos(d*x + c) + 1) - 2*(120*a^7 - 280*a^5*b^2 - 25*a^3*b^4 -
 6*a*b^6)*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 - 2*(510*a^6*b - 105*a^4*b^3
+ 2*a^2*b^5 - 4*b^7)*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 2*(180*a^7 - 635
*a^5*b^2 - 65*a^3*b^4 - 18*a*b^6)*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + 2*(
540*a^6*b - 195*a^4*b^3 - 2*a^2*b^5 - 8*b^7)*sin(d*x + c)^5/(cos(d*x + c) +
 1)^5 - 6*(40*a^7 - 140*a^5*b^2 - 5*a^3*b^4 - 6*a*b^6)*sin(d*x + c)^6/(cos(
d*x + c) + 1)^6 - 2*(210*a^6*b - 75*a^4*b^3 + 2*a^2*b^5 - 4*b^7)*sin(d*x +
c)^7/(cos(d*x + c) + 1)^7 + 3*(20*a^7 - 45*a^5*b^2 - 4*a*b^6)*sin(d*x + c)^
8/(cos(d*x + c) + 1)^8 + 6*(5*a^6*b + a^2*b^5)*sin(d*x + c)^9/(cos(d*x + c)
 + 1)^9)/(a^6*b^5 + 6*a^5*b^6*sin(d*x + c)/(cos(d*x + c) + 1) + 6*a^5*b^6*s
in(d*x + c)^9/(cos(d*x + c) + 1)^9 - a^6*b^5*sin(d*x + c)^10/(cos(d*x + c)
+ 1)^10 - (5*a^6*b^5 - 12*a^4*b^7)*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 - 8*
(3*a^5*b^6 - a^3*b^8)*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 2*(5*a^6*b^5 -
18*a^4*b^7)*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + 4*(9*a^5*b^6 - 4*a^3*b^8)
*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 2*(5*a^6*b^5 - 18*a^4*b^7)*sin(d*x +
 c)^6/(cos(d*x + c) + 1)^6 - 8*(3*a^5*b^6 - a^3*b^8)*sin(d*x + c)^7/(cos(d*
x + c) + 1)^7 + (5*a^6*b^5 - 12*a^4*b^7)*sin(d*x + c)^8/(cos(d*x + c) + 1)^
8) - 15*(4*a^2 + 3*b^2)*a*log((b - a*sin(d*x + c)/(cos(d*x + c) + 1) + sqrt
```

```
(a^2 + b^2))/(b - a*sin(d*x + c)/(cos(d*x + c) + 1) - sqrt(a^2 + b^2)))/(sq
rt(a^2 + b^2)*b^6) - 15*(4*a^2 + b^2)*log(sin(d*x + c)/(cos(d*x + c) + 1) +
 1)/b^6 + 15*(4*a^2 + b^2)*log(sin(d*x + c)/(cos(d*x + c) + 1) - 1)/b^6)/d
```

mupad [B] time = 4.59, size = 1961, normalized size = 4.90

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^3*(a*cos(c + d*x) + b*sin(c + d*x))^4),x)
```

```
[Out] (atanh((4000*a^3*tan(c/2 + (d*x)/2))/(1000*a*b^2 + 4000*a^3) + (1000*a*tan(
c/2 + (d*x)/2))/(1000*a + (4000*a^3)/b^2))*(20*a^2 + 5*b^2))/(b^6*d) - ((60
*a^4 + 2*b^4 + 5*a^2*b^2)/(3*b^5) + (2*tan(c/2 + (d*x)/2)^9*(5*a^4 + b^4))/
(a*b^4) + (2*tan(c/2 + (d*x)/2)^6*(6*b^6 - 40*a^6 + 5*a^2*b^4 + 140*a^4*b^2
))/(a^2*b^5) - (2*tan(c/2 + (d*x)/2)^7*(210*a^6 - 4*b^6 + 2*a^2*b^4 - 75*a^
4*b^2))/(3*a^3*b^4) + (2*tan(c/2 + (d*x)/2)^2*(6*b^6 - 120*a^6 + 25*a^2*b^4
 + 280*a^4*b^2))/(3*a^2*b^5) - (2*tan(c/2 + (d*x)/2)^3*(510*a^6 - 4*b^6 + 2
*a^2*b^4 - 105*a^4*b^2))/(3*a^3*b^4) - (2*tan(c/2 + (d*x)/2)^4*(18*b^6 - 18
0*a^6 + 65*a^2*b^4 + 635*a^4*b^2))/(3*a^2*b^5) - (tan(c/2 + (d*x)/2)^8*(4*b
^6 - 20*a^6 + 45*a^4*b^2))/(a^2*b^5) + (2*tan(c/2 + (d*x)/2)*(55*a^4 + b^4
+ 5*a^2*b^2))/(a*b^4) + (2*tan(c/2 + (d*x)/2)^5*(9*a^2 - 4*b^2)*(60*a^4 + 2
*b^4 + 5*a^2*b^2))/(3*a^3*b^4))/(d*(tan(c/2 + (d*x)/2)^2*(12*a*b^2 - 5*a^3)
 - a^3*tan(c/2 + (d*x)/2)^10 - tan(c/2 + (d*x)/2)^8*(12*a*b^2 - 5*a^3) - ta
n(c/2 + (d*x)/2)^4*(36*a*b^2 - 10*a^3) + tan(c/2 + (d*x)/2)^6*(36*a*b^2 - 1
0*a^3) - tan(c/2 + (d*x)/2)^3*(24*a^2*b - 8*b^3) - tan(c/2 + (d*x)/2)^7*(24
*a^2*b - 8*b^3) + tan(c/2 + (d*x)/2)^5*(36*a^2*b - 16*b^3) + a^3 + 6*a^2*b*
tan(c/2 + (d*x)/2) + 6*a^2*b*tan(c/2 + (d*x)/2)^9)) - (a*atan(((a*(4*a^2 +
3*b^2)*(a^2 + b^2)^(1/2)*((8*(25*a^2*b^9 + 200*a^4*b^7 + 400*a^6*b^5))/b^14
 + (8*tan(c/2 + (d*x)/2)*(50*a*b^11 + 650*a^3*b^9 + 1600*a^5*b^7 + 800*a^7*
b^5))/b^15 - (5*a*(4*a^2 + 3*b^2)*(a^2 + b^2)^(1/2)*((8*tan(c/2 + (d*x)/2)*
(60*a^2*b^14 + 80*a^4*b^12))/b^15 - (8*(10*a*b^14 + 20*a^3*b^12))/b^14 + (5
*a*(4*a^2 + 3*b^2)*(a^2 + b^2)^(1/2)*(32*a^2*b^3 + (8*tan(c/2 + (d*x)/2)*(1
2*a*b^19 + 8*a^3*b^17))/b^15))/(2*(b^8 + a^2*b^6))))/(2*(b^8 + a^2*b^6)))*5
i)/(2*(b^8 + a^2*b^6)) + (a*(4*a^2 + 3*b^2)*(a^2 + b^2)^(1/2)*((8*(25*a^2*b
^9 + 200*a^4*b^7 + 400*a^6*b^5))/b^14 + (8*tan(c/2 + (d*x)/2)*(50*a*b^11 +
650*a^3*b^9 + 1600*a^5*b^7 + 800*a^7*b^5))/b^15 - (5*a*(4*a^2 + 3*b^2)*(a^2
 + b^2)^(1/2)*((8*(10*a*b^14 + 20*a^3*b^12))/b^14 - (8*tan(c/2 + (d*x)/2)*(
60*a^2*b^14 + 80*a^4*b^12))/b^15 + (5*a*(4*a^2 + 3*b^2)*(a^2 + b^2)^(1/2)*(
32*a^2*b^3 + (8*tan(c/2 + (d*x)/2)*(12*a*b^19 + 8*a^3*b^17))/b^15))/(2*(b^8
 + a^2*b^6))))/(2*(b^8 + a^2*b^6)))*5i)/(2*(b^8 + a^2*b^6)))/((16*(2000*a^7
 + 375*a^3*b^4 + 2000*a^5*b^2))/b^14 - (16*tan(c/2 + (d*x)/2)*(8000*a^8 + 3
75*a^2*b^6 + 3500*a^4*b^4 + 10000*a^6*b^2))/b^15 - (5*a*(4*a^2 + 3*b^2)*(a^
2 + b^2)^(1/2)*((8*(25*a^2*b^9 + 200*a^4*b^7 + 400*a^6*b^5))/b^14 + (8*tan(
c/2 + (d*x)/2)*(50*a*b^11 + 650*a^3*b^9 + 1600*a^5*b^7 + 800*a^7*b^5))/b^15
```

```
 - (5*a*(4*a^2 + 3*b^2)*(a^2 + b^2)^(1/2)*((8*tan(c/2 + (d*x)/2)*(60*a^2*b^
14 + 80*a^4*b^12))/b^15 - (8*(10*a*b^14 + 20*a^3*b^12))/b^14 + (5*a*(4*a^2
+ 3*b^2)*(a^2 + b^2)^(1/2)*(32*a^2*b^3 + (8*tan(c/2 + (d*x)/2)*(12*a*b^19 +
 8*a^3*b^17))/b^15))/(2*(b^8 + a^2*b^6))))/(2*(b^8 + a^2*b^6))))/(2*(b^8 +
a^2*b^6)) + (5*a*(4*a^2 + 3*b^2)*(a^2 + b^2)^(1/2)*((8*(25*a^2*b^9 + 200*a^
4*b^7 + 400*a^6*b^5))/b^14 + (8*tan(c/2 + (d*x)/2)*(50*a*b^11 + 650*a^3*b^9
 + 1600*a^5*b^7 + 800*a^7*b^5))/b^15 - (5*a*(4*a^2 + 3*b^2)*(a^2 + b^2)^(1/
2)*((8*(10*a*b^14 + 20*a^3*b^12))/b^14 - (8*tan(c/2 + (d*x)/2)*(60*a^2*b^14
 + 80*a^4*b^12))/b^15 + (5*a*(4*a^2 + 3*b^2)*(a^2 + b^2)^(1/2)*(32*a^2*b^3
+ (8*tan(c/2 + (d*x)/2)*(12*a*b^19 + 8*a^3*b^17))/b^15))/(2*(b^8 + a^2*b^6)
)))/(2*(b^8 + a^2*b^6))))/(2*(b^8 + a^2*b^6))))*(4*a^2 + 3*b^2)*(a^2 + b^2)
^(1/2)*5i)/(d*(b^8 + a^2*b^6))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec^3(c+dx)}{(a\cos(c+dx)+b\sin(c+dx))^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**3/(a*cos(d*x+c)+b*sin(d*x+c))**4,x)
```

```
[Out] Integral(sec(c + d*x)**3/(a*cos(c + d*x) + b*sin(c + d*x))**4, x)
```

3.149 $\int \frac{\sec^4(c+dx)}{(a\cos(c+dx)+b\sin(c+dx))^4}\,dx$

Optimal. Leaf size=232

$$-\frac{4a\left(5a^2+3b^2\right)\log(\tan(c+dx))}{b^7d}-\frac{4a\left(5a^2+3b^2\right)\log(a\cot(c+dx)+b)}{b^7d}+\frac{\left(10a^2+3b^2\right)\tan(c+dx)}{b^6d}+\frac{(a}{3a^3b^4d(a\,c}$$

```
[Out] 1/3*(a^2+b^2)^3/a^3/b^4/d/(b+a*cot(d*x+c))^3+(2*a^6+3*a^4*b^2-b^6)/a^3/b^5/
d/(b+a*cot(d*x+c))^2+(10*a^6+9*a^4*b^2+b^6)/a^3/b^6/d/(b+a*cot(d*x+c))-4*a*
(5*a^2+3*b^2)*ln(b+a*cot(d*x+c))/b^7/d-4*a*(5*a^2+3*b^2)*ln(tan(d*x+c))/b^7
/d+(10*a^2+3*b^2)*tan(d*x+c)/b^6/d-2*a*tan(d*x+c)^2/b^5/d+1/3*tan(d*x+c)^3/
b^4/d
```

Rubi [A] time = 0.25, antiderivative size = 232, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {3088, 894}

$$\frac{\left(10a^2+3b^2\right)\tan(c+dx)}{b^6d}+\frac{\left(a^2+b^2\right)^3}{3a^3b^4d(a\cot(c+dx)+b)^3}+\frac{9a^4b^2+10a^6+b^6}{a^3b^6d(a\cot(c+dx)+b)}+\frac{3a^4b^2+2a^6-b^6}{a^3b^5d(a\cot(c+dx)+b)^2}-\frac{4a}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^4/(a*Cos[c + d*x] + b*Sin[c + d*x])^4,x]
```

```
[Out] (a^2 + b^2)^3/(3*a^3*b^4*d*(b + a*Cot[c + d*x])^3) + (2*a^6 + 3*a^4*b^2 - b
^6)/(a^3*b^5*d*(b + a*Cot[c + d*x])^2) + (10*a^6 + 9*a^4*b^2 + b^6)/(a^3*b^
6*d*(b + a*Cot[c + d*x])) - (4*a*(5*a^2 + 3*b^2)*Log[b + a*Cot[c + d*x]])/(
b^7*d) - (4*a*(5*a^2 + 3*b^2)*Log[Tan[c + d*x]])/(b^7*d) + ((10*a^2 + 3*b^2
)*Tan[c + d*x])/(b^6*d) - (2*a*Tan[c + d*x]^2)/(b^5*d) + Tan[c + d*x]^3/(3*
b^4*d)
```

Rule 894

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ
[m, 0] && ILtQ[n, 0]))
```

Rule 3088

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[(x^m*(b +
 a*x)^n)/(1 + x^2)^((m + n + 2)/2), x], x, Cot[c + d*x]], x] /; FreeQ[{a, b
```

```
, c, d}, x] && IntegerQ[n] && IntegerQ[(m + n)/2] && NeQ[n, -1] &&  !(GtQ[n
, 0] && GtQ[m, 1])
```

Rubi steps

$$\int \frac{\sec^4(c+dx)}{(a\cos(c+dx)+b\sin(c+dx))^4}\,dx = -\frac{\text{Subst}\left(\int \frac{\left(1+x^2\right)^3}{x^4(b+ax)^4}\,dx, x, \cot(c+dx)\right)}{d}$$

$$= -\frac{\text{Subst}\left(\int \left(\frac{1}{b^4x^4} - \frac{4a}{b^5x^3} + \frac{10a^2+3b^2}{b^6x^2} - \frac{4\left(5a^3+3ab^2\right)}{b^7x} + \frac{\left(a^2+b^2\right)^3}{a^2b^4(b+ax)^4} + \frac{2\left(2a^6+3a^4b^2-\right.}{a^2b^5(b+ax)}\right.\right.}{d}$$

$$= \frac{\left(a^2+b^2\right)^3}{3a^3b^4d(b+a\cot(c+dx))^3} + \frac{2a^6+3a^4b^2-b^6}{a^3b^5d(b+a\cot(c+dx))^2} + \frac{10a^6+9a^4}{a^3b^6d(b+a\,\text{co}}$$

Mathematica [A] time = 2.01, size = 295, normalized size = 1.27

$$\frac{3b^4\sec^4(c+dx)\left(a^2-ab\tan(c+dx)+2b^2\right)-2\left(37a^6+36a^4b^2-6a^2b^4\tan^4(c+dx)+3a^2b^4+6ab^3\tan^3(c+dx\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^4/(a*Cos[c + d*x] + b*Sin[c + d*x])^4,x]
```

```
[Out] (b^6*Sec[c + d*x]^6 + 3*b^4*Sec[c + d*x]^4*(a^2 + 2*b^2 - a*b*Tan[c + d*x])
 - 2*(37*a^6 + 36*a^4*b^2 + 3*a^2*b^4 + 4*b^6 + 6*a^4*(5*a^2 + 3*b^2)*Log[a
 + b*Tan[c + d*x]] + 3*a*b*(27*a^4 + 30*a^2*b^2 + b^4 + 6*a^2*(5*a^2 + 3*b^
2)*Log[a + b*Tan[c + d*x]])*Tan[c + d*x] + 6*b^2*(6*a^4 + 11*a^2*b^2 + 2*b^
4 + 3*a^2*(5*a^2 + 3*b^2)*Log[a + b*Tan[c + d*x]])*Tan[c + d*x]^2 + 6*a*b^3
*(-3*a^2 + (5*a^2 + 3*b^2)*Log[a + b*Tan[c + d*x]])*Tan[c + d*x]^3 - 6*a^2*
b^4*Tan[c + d*x]^4))/(3*b^7*d*(a + b*Tan[c + d*x])^3)
```

fricas [B] time = 0.78, size = 553, normalized size = 2.38

$$-\frac{4\left(45\,a^4b^2-3\,a^2b^4-4\,b^6\right)\cos\left(dx+c\right)^6-b^6-6\left(25\,a^4b^2-5\,a^2b^4-4\,b^6\right)\cos\left(dx+c\right)^4-3\left(5\,a^2b^4+2\,b^6\right)\cos\left(d\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4/(a*cos(d*x+c)+b*sin(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] -1/3*(4*(45*a^4*b^2 - 3*a^2*b^4 - 4*b^6)*cos(d*x + c)^6 - b^6 - 6*(25*a^4*b
^2 - 5*a^2*b^4 - 4*b^6)*cos(d*x + c)^4 - 3*(5*a^2*b^4 + 2*b^6)*cos(d*x + c)
^2 + 6*((5*a^6 - 12*a^4*b^2 - 9*a^2*b^4)*cos(d*x + c)^6 + 3*(5*a^4*b^2 + 3*
a^2*b^4)*cos(d*x + c)^4 + ((15*a^5*b + 4*a^3*b^3 - 3*a*b^5)*cos(d*x + c)^5
+ (5*a^3*b^3 + 3*a*b^5)*cos(d*x + c)^3)*sin(d*x + c))*log(2*a*b*cos(d*x + c
)*sin(d*x + c) + (a^2 - b^2)*cos(d*x + c)^2 + b^2) - 6*((5*a^6 - 12*a^4*b^2
 - 9*a^2*b^4)*cos(d*x + c)^6 + 3*(5*a^4*b^2 + 3*a^2*b^4)*cos(d*x + c)^4 + (
(15*a^5*b + 4*a^3*b^3 - 3*a*b^5)*cos(d*x + c)^5 + (5*a^3*b^3 + 3*a*b^5)*cos
(d*x + c)^3)*sin(d*x + c))*log(cos(d*x + c)^2) + (3*a*b^5*cos(d*x + c) - 4*
(15*a^5*b - 41*a^3*b^3 - 12*a*b^5)*cos(d*x + c)^5 - 2*(55*a^3*b^3 + 21*a*b^
5)*cos(d*x + c)^3)*sin(d*x + c))/(3*a*b^9*d*cos(d*x + c)^4 + (a^3*b^7 - 3*a
*b^9)*d*cos(d*x + c)^6 + (b^10*d*cos(d*x + c)^3 + (3*a^2*b^8 - b^10)*d*cos(
d*x + c)^5)*sin(d*x + c))
```

giac [A] time = 0.27, size = 249, normalized size = 1.07

$$-\frac{\frac{12\left(5a^3+3ab^2\right)\log(|b\tan(dx+c)+a|)}{b^7}-\frac{110a^3b^3\tan(dx+c)^3+66ab^5\tan(dx+c)^3+285a^4b^2\tan(dx+c)^2+144a^2b^4\tan(dx+c)^2-9b^6\tan(dx+c)^2+249}{(b\tan(dx+c)+a)^3b^7}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4/(a*cos(d*x+c)+b*sin(d*x+c))^4,x, algorithm="giac")
```

```
[Out] -1/3*(12*(5*a^3 + 3*a*b^2)*log(abs(b*tan(d*x + c) + a))/b^7 - (110*a^3*b^3*
tan(d*x + c)^3 + 66*a*b^5*tan(d*x + c)^3 + 285*a^4*b^2*tan(d*x + c)^2 + 144
*a^2*b^4*tan(d*x + c)^2 - 9*b^6*tan(d*x + c)^2 + 249*a^5*b*tan(d*x + c) + 1
08*a^3*b^3*tan(d*x + c) - 9*a*b^5*tan(d*x + c) + 73*a^6 + 27*a^4*b^2 - 3*a^
2*b^4 - b^6)/((b*tan(d*x + c) + a)^3*b^7) - (b^8*tan(d*x + c)^3 - 6*a*b^7*t
an(d*x + c)^2 + 30*a^2*b^6*tan(d*x + c) + 9*b^8*tan(d*x + c))/b^12)/d
```

maple [A] time = 0.43, size = 330, normalized size = 1.42

$$\frac{\tan^3(dx+c)}{3b^4d}-\frac{2a\left(\tan^2(dx+c)\right)}{b^5d}+\frac{10a^2\tan(dx+c)}{d\,b^6}+\frac{3\tan(dx+c)}{b^4d}-\frac{20a^3\ln(a+b\tan(dx+c))}{d\,b^7}-\frac{12a\ln(a+b}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^4/(a*cos(d*x+c)+b*sin(d*x+c))^4,x)
```

```
[Out] 1/3*tan(d*x+c)^3/b^4/d-2*a*tan(d*x+c)^2/b^5/d+10/d/b^6*a^2*tan(d*x+c)+3*tan
(d*x+c)/b^4/d-20/d/b^7*a^3*ln(a+b*tan(d*x+c))-12/d*a/b^5*ln(a+b*tan(d*x+c))
+3/d*a^5/b^7/(a+b*tan(d*x+c))^2+6/d*a^3/b^5/(a+b*tan(d*x+c))^2+3/d*a/b^3/(a
+b*tan(d*x+c))^2-1/3/d/b^7/(a+b*tan(d*x+c))^3*a^6-1/d/b^5/(a+b*tan(d*x+c))^
3*a^4-1/d/b^3/(a+b*tan(d*x+c))^3*a^2-1/3/d/b/(a+b*tan(d*x+c))^3-15/d/b^7/(a
+b*tan(d*x+c))*a^4-18/d/b^5/(a+b*tan(d*x+c))*a^2-3/d/b^3/(a+b*tan(d*x+c))
```

maxima [A] time = 0.33, size = 217, normalized size = 0.94

$$-\frac{\frac{37\,a^6+39\,a^4b^2+3\,a^2b^4+b^6+9\left(5\,a^4b^2+6\,a^2b^4+b^6\right)\tan(dx+c)^2+9\left(9\,a^5b+10\,a^3b^3+ab^5\right)\tan(dx+c)}{b^{10}\tan(dx+c)^3+3\,ab^9\tan(dx+c)^2+3\,a^2b^8\tan(dx+c)+a^3b^7}-\frac{b^2\tan(dx+c)^3-6\,ab\tan(dx+c)^2+3\left(10\,a^2+3\,b^2\right)\tan}{b^6}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4/(a*cos(d*x+c)+b*sin(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] -1/3*((37*a^6 + 39*a^4*b^2 + 3*a^2*b^4 + b^6 + 9*(5*a^4*b^2 + 6*a^2*b^4 + b
^6)*tan(d*x + c)^2 + 9*(9*a^5*b + 10*a^3*b^3 + a*b^5)*tan(d*x + c))/(b^10*t
an(d*x + c)^3 + 3*a*b^9*tan(d*x + c)^2 + 3*a^2*b^8*tan(d*x + c) + a^3*b^7)
- (b^2*tan(d*x + c)^3 - 6*a*b*tan(d*x + c)^2 + 3*(10*a^2 + 3*b^2)*tan(d*x +
 c))/b^6 + 12*(5*a^3 + 3*a*b^2)*log(b*tan(d*x + c) + a)/b^7)/d
```

mupad [B] time = 7.97, size = 1599, normalized size = 6.89

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^4*(a*cos(c + d*x) + b*sin(c + d*x))^4),x)
```

```
[Out] ((4*tan(c/2 + (d*x)/2)^2*(50*a^6 + b^6 + 30*a^4*b^2))/(a^2*b^5) - (16*tan(c
/2 + (d*x)/2)^8*(50*a^6 + b^6 - 3*a^2*b^4 + 25*a^4*b^2))/(a^2*b^5) - (2*tan
(c/2 + (d*x)/2)^11*(20*a^6 + b^6 + 12*a^4*b^2))/(a*b^6) - (16*tan(c/2 + (d*
x)/2)^4*(50*a^6 + b^6 - 3*a^2*b^4 + 25*a^4*b^2))/(a^2*b^5) + (4*tan(c/2 + (
d*x)/2)^10*(50*a^6 + b^6 + 30*a^4*b^2))/(a^2*b^5) - (4*tan(c/2 + (d*x)/2)^5
*(2*b^8 - 100*a^8 + a^2*b^6 + 140*a^4*b^4 + 160*a^6*b^2))/(a^3*b^6) + (4*ta
n(c/2 + (d*x)/2)^7*(2*b^8 - 100*a^8 + a^2*b^6 + 140*a^4*b^4 + 160*a^6*b^2))
/(a^3*b^6) + (2*tan(c/2 + (d*x)/2)^3*(4*b^8 - 300*a^8 - 3*a^2*b^6 + 264*a^4
*b^4 + 260*a^6*b^2))/(3*a^3*b^6) - (2*tan(c/2 + (d*x)/2)^9*(4*b^8 - 300*a^8
 - 3*a^2*b^6 + 264*a^4*b^4 + 260*a^6*b^2))/(3*a^3*b^6) + (8*tan(c/2 + (d*x)
/2)^6*(450*a^6 + 9*b^6 - 28*a^2*b^4 + 210*a^4*b^2))/(3*a^2*b^5) + (2*tan(c/
2 + (d*x)/2)*(20*a^6 + b^6 + 12*a^4*b^2))/(a*b^6))/(d*(a^3*tan(c/2 + (d*x)/
2)^12 + tan(c/2 + (d*x)/2)^2*(12*a*b^2 - 6*a^3) + tan(c/2 + (d*x)/2)^10*(12
*a*b^2 - 6*a^3) - tan(c/2 + (d*x)/2)^4*(48*a*b^2 - 15*a^3) - tan(c/2 + (d*x
)/2)^8*(48*a*b^2 - 15*a^3) + tan(c/2 + (d*x)/2)^6*(72*a*b^2 - 20*a^3) - tan
(c/2 + (d*x)/2)^3*(30*a^2*b - 8*b^3) + tan(c/2 + (d*x)/2)^9*(30*a^2*b - 8*b
^3) + tan(c/2 + (d*x)/2)^5*(60*a^2*b - 24*b^3) - tan(c/2 + (d*x)/2)^7*(60*a
^2*b - 24*b^3) + a^3 + 6*a^2*b*tan(c/2 + (d*x)/2) - 6*a^2*b*tan(c/2 + (d*x)
/2)^11)) + (a*atan(((a*(5*a^2 + 3*b^2)*((16*tan(c/2 + (d*x)/2)*(20*a^5 + 12
*a^3*b^2))/b^6 - (4*(24*a^2*b^9 + 40*a^4*b^7))/b^12 + (4*tan(c/2 + (d*x)/2)
^2*(24*a^2*b^9 + 40*a^4*b^7))/b^12 + (4*a*(5*a^2 + 3*b^2)*((4*(a*b^14 + 4*a
^3*b^12))/b^12 - (4*tan(c/2 + (d*x)/2)^2*(3*a*b^14 + 4*a^3*b^12))/b^12 + 16
```

```
*a^2*b*tan(c/2 + (d*x)/2)))/b^7)*4i)/b^7 - (a*(5*a^2 + 3*b^2)*((4*(24*a^2*b
^9 + 40*a^4*b^7))/b^12 - (16*tan(c/2 + (d*x)/2)*(20*a^5 + 12*a^3*b^2))/b^6
- (4*tan(c/2 + (d*x)/2)^2*(24*a^2*b^9 + 40*a^4*b^7))/b^12 + (4*a*(5*a^2 + 3
*b^2)*((4*(a*b^14 + 4*a^3*b^12))/b^12 - (4*tan(c/2 + (d*x)/2)^2*(3*a*b^14 +
 4*a^3*b^12))/b^12 + 16*a^2*b*tan(c/2 + (d*x)/2)))/b^7)*4i)/b^7)/((8*(400*a
^7 + 144*a^3*b^4 + 480*a^5*b^2))/b^12 + (8*tan(c/2 + (d*x)/2)^2*(400*a^7 +
144*a^3*b^4 + 480*a^5*b^2))/b^12 + (4*a*(5*a^2 + 3*b^2)*((16*tan(c/2 + (d*x
)/2)*(20*a^5 + 12*a^3*b^2))/b^6 - (4*(24*a^2*b^9 + 40*a^4*b^7))/b^12 + (4*t
an(c/2 + (d*x)/2)^2*(24*a^2*b^9 + 40*a^4*b^7))/b^12 + (4*a*(5*a^2 + 3*b^2)*
((4*(a*b^14 + 4*a^3*b^12))/b^12 - (4*tan(c/2 + (d*x)/2)^2*(3*a*b^14 + 4*a^3
*b^12))/b^12 + 16*a^2*b*tan(c/2 + (d*x)/2)))/b^7))/b^7 + (4*a*(5*a^2 + 3*b^
2)*((4*(24*a^2*b^9 + 40*a^4*b^7))/b^12 - (16*tan(c/2 + (d*x)/2)*(20*a^5 + 1
2*a^3*b^2))/b^6 - (4*tan(c/2 + (d*x)/2)^2*(24*a^2*b^9 + 40*a^4*b^7))/b^12 +
 (4*a*(5*a^2 + 3*b^2)*((4*(a*b^14 + 4*a^3*b^12))/b^12 - (4*tan(c/2 + (d*x)/
2)^2*(3*a*b^14 + 4*a^3*b^12))/b^12 + 16*a^2*b*tan(c/2 + (d*x)/2)))/b^7))/b^
7))*(5*a^2 + 3*b^2)*8i)/(b^7*d)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec^4(c+dx)}{(a\cos(c+dx)+b\sin(c+dx))^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**4/(a*cos(d*x+c)+b*sin(d*x+c))**4,x)
```

```
[Out] Integral(sec(c + d*x)**4/(a*cos(c + d*x) + b*sin(c + d*x))**4, x)
```

3.150 $\int \frac{\cos^5(c+dx)}{a\cos(c+dx)+ia\sin(c+dx)}\,dx$

Optimal. Leaf size=99

$$\frac{i\cos^6(c+dx)}{6ad}+\frac{\sin(c+dx)\cos^5(c+dx)}{6ad}+\frac{5\sin(c+dx)\cos^3(c+dx)}{24ad}+\frac{5\sin(c+dx)\cos(c+dx)}{16ad}+\frac{5x}{16a}$$

```
[Out] 5/16*x/a+1/6*I*cos(d*x+c)^6/a/d+5/16*cos(d*x+c)*sin(d*x+c)/a/d+5/24*cos(d*x
+c)^3*sin(d*x+c)/a/d+1/6*cos(d*x+c)^5*sin(d*x+c)/a/d
```

Rubi [A] time = 0.15, antiderivative size = 99, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.194, Rules used = {3092, 3090, 2635, 8, 2565, 30}

$$\frac{i\cos^6(c+dx)}{6ad}+\frac{\sin(c+dx)\cos^5(c+dx)}{6ad}+\frac{5\sin(c+dx)\cos^3(c+dx)}{24ad}+\frac{5\sin(c+dx)\cos(c+dx)}{16ad}+\frac{5x}{16a}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^5/(a*Cos[c + d*x] + I*a*Sin[c + d*x]),x]
```

```
[Out] (5*x)/(16*a) + ((I/6)*Cos[c + d*x]^6)/(a*d) + (5*Cos[c + d*x]*Sin[c + d*x])
/(16*a*d) + (5*Cos[c + d*x]^3*Sin[c + d*x])/(24*a*d) + (Cos[c + d*x]^5*Sin[
c + d*x])/(6*a*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2565

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> -Dist[(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
```

```
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 3090

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_.), x_Symbol] :> Int[ExpandTrig[cos[c + d*x]^m*(a
*cos[c + d*x] + b*sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d}, x] && Inte
gerQ[m] && IGtQ[n, 0]
```

Rule 3092

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[a^n*b^n, Int[Cos[c + d*x]^m/
(b*Cos[c + d*x] + a*Sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, m}, x] &&
 EqQ[a^2 + b^2, 0] && ILtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\cos^5(c+dx)}{a\cos(c+dx)+ia\sin(c+dx)}\,dx &= -\frac{i\int \cos^5(c+dx)(ia\cos(c+dx)+a\sin(c+dx))\,dx}{a^2} \\
&= -\frac{i\int \left(ia\cos^6(c+dx)+a\cos^5(c+dx)\sin(c+dx)\right)\,dx}{a^2} \\
&= -\frac{i\int \cos^5(c+dx)\sin(c+dx)\,dx}{a}+\frac{\int \cos^6(c+dx)\,dx}{a} \\
&= \frac{\cos^5(c+dx)\sin(c+dx)}{6ad}+\frac{5\int \cos^4(c+dx)\,dx}{6a}+\frac{i\,\mathrm{Subst}\left(\int x^5\,dx,x,\cos\right.}{ad} \\
&= \frac{i\cos^6(c+dx)}{6ad}+\frac{5\cos^3(c+dx)\sin(c+dx)}{24ad}+\frac{\cos^5(c+dx)\sin(c+dx)}{6ad}+ \\
&= \frac{i\cos^6(c+dx)}{6ad}+\frac{5\cos(c+dx)\sin(c+dx)}{16ad}+\frac{5\cos^3(c+dx)\sin(c+dx)}{24ad}+ \\
&= \frac{5x}{16a}+\frac{i\cos^6(c+dx)}{6ad}+\frac{5\cos(c+dx)\sin(c+dx)}{16ad}+\frac{5\cos^3(c+dx)\sin(c}{24ad}
\end{aligned}$$

Mathematica [A] time = 0.14, size = 82, normalized size = 0.83

$$\frac{45\sin(2(c+dx))+9\sin(4(c+dx))+\sin(6(c+dx))+15i\cos(2(c+dx))+6i\cos(4(c+dx))+i\cos(6(c+dx))}{192ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^5/(a*Cos[c + d*x] + I*a*Sin[c + d*x]),x]
```

```
[Out] (60*c + 60*d*x + (15*I)*Cos[2*(c + d*x)] + (6*I)*Cos[4*(c + d*x)] + I*Cos[6
*(c + d*x)] + 45*Sin[2*(c + d*x)] + 9*Sin[4*(c + d*x)] + Sin[6*(c + d*x)])/
(192*a*d)
```

fricas [A] time = 0.59, size = 76, normalized size = 0.77

$$\frac{\left(120\,dxe^{(6i\,dx+6i\,c)} - 3i\,e^{(10i\,dx+10i\,c)} - 30i\,e^{(8i\,dx+8i\,c)} + 60i\,e^{(4i\,dx+4i\,c)} + 15i\,e^{(2i\,dx+2i\,c)} + 2i\right)e^{(-6i\,dx-6i\,c)}}{384\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5/(a*cos(d*x+c)+I*a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/384*(120*d*x*e^(6*I*d*x + 6*I*c) - 3*I*e^(10*I*d*x + 10*I*c) - 30*I*e^(8*
I*d*x + 8*I*c) + 60*I*e^(4*I*d*x + 4*I*c) + 15*I*e^(2*I*d*x + 2*I*c) + 2*I)
*e^(-6*I*d*x - 6*I*c)/(a*d)
```

giac [A] time = 0.24, size = 116, normalized size = 1.17

$$-\frac{-\frac{30i\,\log(\tan(dx+c)+i)}{a} + \frac{30i\,\log(\tan(dx+c)-i)}{a} + \frac{3\left(-15i\,\tan(dx+c)^2+38\,\tan(dx+c)+25i\right)}{a(-i\,\tan(dx+c)+1)^2} - \frac{55i\,\tan(dx+c)^3+201\,\tan(dx+c)^2-255i\,\tan(dx+c)-}{a(\tan(dx+c)-i)^3}}{192\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5/(a*cos(d*x+c)+I*a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/192*(-30*I*log(tan(d*x + c) + I)/a + 30*I*log(tan(d*x + c) - I)/a + 3*(-
15*I*tan(d*x + c)^2 + 38*tan(d*x + c) + 25*I)/(a*(-I*tan(d*x + c) + 1)^2) -
 (55*I*tan(d*x + c)^3 + 201*tan(d*x + c)^2 - 255*I*tan(d*x + c) - 117)/(a*(
tan(d*x + c) - I)^3))/d
```

maple [A] time = 0.23, size = 137, normalized size = 1.38

$$\frac{i}{32ad\left(\tan\left(dx+c\right)+i\right)^2} + \frac{5i\ln\left(\tan\left(dx+c\right)+i\right)}{32ad} + \frac{1}{8ad\left(\tan\left(dx+c\right)+i\right)} - \frac{5i\ln\left(\tan\left(dx+c\right)-i\right)}{32ad} - \frac{3i}{32ad\left(\tan\left(dx+\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5/(a*cos(d*x+c)+I*a*sin(d*x+c)),x)
```

```
[Out] 1/32*I/a/d/(tan(d*x+c)+I)^2+5/32*I/a/d*ln(tan(d*x+c)+I)+1/8/a/d/(tan(d*x+c)
+I)-5/32*I/a/d*ln(tan(d*x+c)-I)-3/32*I/a/d/(tan(d*x+c)-I)^2-1/24/a/d/(tan(d
*x+c)-I)^3+3/16/a/d/(tan(d*x+c)-I)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5/(a*cos(d*x+c)+I*a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

mupad [B] time = 5.14, size = 164, normalized size = 1.66

$$\frac{5x}{16a}+\frac{\frac{11\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^9}{8}-\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^8 3i}{4}-\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^7}{3}-\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^6 1i}{12}+\frac{13\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^5}{4}+\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4 1i}{12}-\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3}{3}+\frac{\tan\left(\frac{c}{2}+\cdots\right.}{\cdots}}{a\,d\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)+1i\right)^4\left(1+\tan\left(\frac{c}{2}+\frac{dx}{2}\right)1i\right)^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^5/(a*cos(c + d*x) + a*sin(c + d*x)*1i),x)
```

```
[Out] (5*x)/(16*a) + ((11*tan(c/2 + (d*x)/2))/8 + (tan(c/2 + (d*x)/2)^2*3i)/4 - t
an(c/2 + (d*x)/2)^3/3 + (tan(c/2 + (d*x)/2)^4*1i)/12 + (13*tan(c/2 + (d*x)/
2)^5)/4 - (tan(c/2 + (d*x)/2)^6*1i)/12 - tan(c/2 + (d*x)/2)^7/3 - (tan(c/2
+ (d*x)/2)^8*3i)/4 + (11*tan(c/2 + (d*x)/2)^9)/8)/(a*d*(tan(c/2 + (d*x)/2)
+ 1i)^4*(tan(c/2 + (d*x)/2)*1i + 1)^6)
```

sympy [A] time = 0.41, size = 223, normalized size = 2.25

$$\begin{cases} -\frac{\left(50331648ia^4d^4e^{16ic}e^{4idx}+503316480ia^4d^4e^{14ic}e^{2idx}-1006632960ia^4d^4e^{10ic}e^{-2idx}-251658240ia^4d^4e^{8ic}e^{-4idx}-33554432ia^4d^4e^{6ic}e^{-6idx}\right)e^{-12ic}}{6442450944a^5d^5} & \text{f}\cdots \\ x\left(\frac{\left(e^{10ic}+5e^{8ic}+10e^{6ic}+10e^{4ic}+5e^{2ic}+1\right)e^{-6ic}}{32a}-\frac{5}{16a}\right) & \text{o}\cdots \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5/(a*cos(d*x+c)+I*a*sin(d*x+c)),x)
```

```
[Out] Piecewise((-(50331648*I*a**4*d**4*exp(16*I*c)*exp(4*I*d*x) + 503316480*I*a*
*4*d**4*exp(14*I*c)*exp(2*I*d*x) - 1006632960*I*a**4*d**4*exp(10*I*c)*exp(-
2*I*d*x) - 251658240*I*a**4*d**4*exp(8*I*c)*exp(-4*I*d*x) - 33554432*I*a**4
*d**4*exp(6*I*c)*exp(-6*I*d*x))*exp(-12*I*c)/(6442450944*a**5*d**5), Ne(644
2450944*a**5*d**5*exp(12*I*c), 0)), (x*((exp(10*I*c) + 5*exp(8*I*c) + 10*ex
p(6*I*c) + 10*exp(4*I*c) + 5*exp(2*I*c) + 1)*exp(-6*I*c)/(32*a) - 5/(16*a))
, True)) + 5*x/(16*a)
```

3.151 $\int \frac{\cos^4(c+dx)}{a\cos(c+dx)+ia\sin(c+dx)}\,dx$

Optimal. Leaf size=70

$$\frac{\sin^5(c+dx)}{5ad}-\frac{2\sin^3(c+dx)}{3ad}+\frac{\sin(c+dx)}{ad}+\frac{i\cos^5(c+dx)}{5ad}$$

```
[Out] 1/5*I*cos(d*x+c)^5/a/d+sin(d*x+c)/a/d-2/3*sin(d*x+c)^3/a/d+1/5*sin(d*x+c)^5
/a/d
```

Rubi [A] time = 0.13, antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.161, Rules used = {3092, 3090, 2633, 2565, 30}

$$\frac{\sin^5(c+dx)}{5ad}-\frac{2\sin^3(c+dx)}{3ad}+\frac{\sin(c+dx)}{ad}+\frac{i\cos^5(c+dx)}{5ad}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^4/(a*Cos[c + d*x] + I*a*Sin[c + d*x]),x]
```

```
[Out] ((I/5)*Cos[c + d*x]^5)/(a*d) + Sin[c + d*x]/(a*d) - (2*Sin[c + d*x]^3)/(3*a
*d) + Sin[c + d*x]^5/(5*a*d)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2565

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> -Dist[(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 3090

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_.), x_Symbol] :> Int[ExpandTrig[cos[c + d*x]^m*(a
```

```
*cos[c + d*x] + b*sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d}, x] && Inte
gerQ[m] && IGtQ[n, 0]
```

Rule 3092

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[a^n*b^n, Int[Cos[c + d*x]^m/
(b*Cos[c + d*x] + a*Sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, m}, x] &&
 EqQ[a^2 + b^2, 0] && ILtQ[n, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\cos^4(c+dx)}{a\cos(c+dx)+ia\sin(c+dx)}\,dx &= -\frac{i\int \cos^4(c+dx)(ia\cos(c+dx)+a\sin(c+dx))\,dx}{a^2}\\ &= -\frac{i\int \left(ia\cos^5(c+dx)+a\cos^4(c+dx)\sin(c+dx)\right)\,dx}{a^2}\\ &= -\frac{i\int \cos^4(c+dx)\sin(c+dx)\,dx}{a}+\frac{\int \cos^5(c+dx)\,dx}{a}\\ &= \frac{i\,\mathrm{Subst}\left(\int x^4\,dx,x,\cos(c+dx)\right)}{ad}-\frac{\mathrm{Subst}\left(\int \left(1-2x^2+x^4\right)\,dx,x,-\sin(c\right.}{ad}\\ &= \frac{i\cos^5(c+dx)}{5ad}+\frac{\sin(c+dx)}{ad}-\frac{2\sin^3(c+dx)}{3ad}+\frac{\sin^5(c+dx)}{5ad}\end{aligned}$$

Mathematica [A] time = 0.07, size = 111, normalized size = 1.59

$$\frac{5\sin(c+dx)}{8ad}+\frac{5\sin(3(c+dx))}{48ad}+\frac{\sin(5(c+dx))}{80ad}+\frac{i\cos(c+dx)}{8ad}+\frac{i\cos(3(c+dx))}{16ad}+\frac{i\cos(5(c+dx))}{80ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^4/(a*Cos[c + d*x] + I*a*Sin[c + d*x]),x]
```

```
[Out] ((I/8)*Cos[c + d*x])/(a*d) + ((I/16)*Cos[3*(c + d*x)])/(a*d) + ((I/80)*Cos[
5*(c + d*x)])/(a*d) + (5*Sin[c + d*x])/(8*a*d) + (5*Sin[3*(c + d*x)])/(48*a
*d) + Sin[5*(c + d*x)]/(80*a*d)
```

fricas [A] time = 0.54, size = 63, normalized size = 0.90

$$\frac{\left(-5i\,e^{(8i\,dx+8i\,c)}-60i\,e^{(6i\,dx+6i\,c)}+90i\,e^{(4i\,dx+4i\,c)}+20i\,e^{(2i\,dx+2i\,c)}+3i\right)e^{(-5i\,dx-5i\,c)}}{240\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4/(a*cos(d*x+c)+I*a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/240*(-5*I*e^(8*I*d*x + 8*I*c) - 60*I*e^(6*I*d*x + 6*I*c) + 90*I*e^(4*I*d*
x + 4*I*c) + 20*I*e^(2*I*d*x + 2*I*c) + 3*I)*e^(-5*I*d*x - 5*I*c)/(a*d)
```

giac [A] time = 1.23, size = 119, normalized size = 1.70

$$\frac{\frac{5\left(15 \tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{2}+24i\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-13\right)}{a\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+i\right)^{3}}+\frac{165\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{4}-480i\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{3}-650\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{2}+400i\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+113}{a\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-i\right)^{5}}}{120\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4/(a*cos(d*x+c)+I*a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/120*(5*(15*tan(1/2*d*x + 1/2*c)^2 + 24*I*tan(1/2*d*x + 1/2*c) - 13)/(a*(t
an(1/2*d*x + 1/2*c) + I)^3) + (165*tan(1/2*d*x + 1/2*c)^4 - 480*I*tan(1/2*d
*x + 1/2*c)^3 - 650*tan(1/2*d*x + 1/2*c)^2 + 400*I*tan(1/2*d*x + 1/2*c) + 1
13)/(a*(tan(1/2*d*x + 1/2*c) - I)^5))/d
```

maple [B] time = 0.16, size = 141, normalized size = 2.01

$$\frac{-\frac{i}{4\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+i\right)^{2}}-\frac{1}{6\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+i\right)^{3}}+\frac{5}{8\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+i\right)}-\frac{i}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-i\right)^{4}}+\frac{3i}{2\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-i\right)^{2}}+\frac{2}{5\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-i\right)^{5}}-\frac{5}{3\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-i\right.}}{ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4/(a*cos(d*x+c)+I*a*sin(d*x+c)),x)
```

```
[Out] 2/d/a*(-1/8*I/(tan(1/2*d*x+1/2*c)+I)^2-1/12/(tan(1/2*d*x+1/2*c)+I)^3+5/16/(
tan(1/2*d*x+1/2*c)+I)-1/2*I/(tan(1/2*d*x+1/2*c)-I)^4+3/4*I/(tan(1/2*d*x+1/2
*c)-I)^2+1/5/(tan(1/2*d*x+1/2*c)-I)^5-5/6/(tan(1/2*d*x+1/2*c)-I)^3+11/16/(t
an(1/2*d*x+1/2*c)-I))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4/(a*cos(d*x+c)+I*a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

mupad [B] time = 2.09, size = 134, normalized size = 1.91

$$-\frac{\left(-15\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6 15\mathrm{i}-5\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4 25\mathrm{i}-13\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3+\tan\left(\frac{c}{2}+\right.\right.}{15\,a\,d\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+1\mathrm{i}\right)^3\left(1+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)1\mathrm{i}\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^4/(a*cos(c + d*x) + a*sin(c + d*x)*1i),x)
```

```
[Out] -((9*tan(c/2 + (d*x)/2) + tan(c/2 + (d*x)/2)^2*21i - 13*tan(c/2 + (d*x)/2)^
3 + tan(c/2 + (d*x)/2)^4*25i - 5*tan(c/2 + (d*x)/2)^5 + tan(c/2 + (d*x)/2)^
6*15i - 15*tan(c/2 + (d*x)/2)^7 + 3i)*2i)/(15*a*d*(tan(c/2 + (d*x)/2) + 1i)
^3*(tan(c/2 + (d*x)/2)*1i + 1)^5)
```

sympy [A] time = 0.47, size = 199, normalized size = 2.84

$$\begin{cases} -\frac{\left(30720ia^4d^4e^{12ic}e^{3idx}+368640ia^4d^4e^{10ic}e^{idx}-552960ia^4d^4e^{8ic}e^{-idx}-122880ia^4d^4e^{6ic}e^{-3idx}-18432ia^4d^4e^{4ic}e^{-5idx}\right)e^{-9ic}}{1474560a^5d^5} & \text{for } 1474560a^5d^5e^{9 \\ \frac{x\left(e^{8ic}+4e^{6ic}+6e^{4ic}+4e^{2ic}+1\right)e^{-5ic}}{16a} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4/(a*cos(d*x+c)+I*a*sin(d*x+c)),x)
```

```
[Out] Piecewise((-(30720*I*a**4*d**4*exp(12*I*c)*exp(3*I*d*x) + 368640*I*a**4*d**
4*exp(10*I*c)*exp(I*d*x) - 552960*I*a**4*d**4*exp(8*I*c)*exp(-I*d*x) - 1228
80*I*a**4*d**4*exp(6*I*c)*exp(-3*I*d*x) - 18432*I*a**4*d**4*exp(4*I*c)*exp(
-5*I*d*x))*exp(-9*I*c)/(1474560*a**5*d**5), Ne(1474560*a**5*d**5*exp(9*I*c)
, 0)), (x*(exp(8*I*c) + 4*exp(6*I*c) + 6*exp(4*I*c) + 4*exp(2*I*c) + 1)*exp
(-5*I*c)/(16*a), True))
```

3.152 $\int \frac{\cos^3(c+dx)}{a\cos(c+dx)+ia\sin(c+dx)}\,dx$

Optimal. Leaf size=75

$$\frac{i\cos^4(c+dx)}{4ad}+\frac{\sin(c+dx)\cos^3(c+dx)}{4ad}+\frac{3\sin(c+dx)\cos(c+dx)}{8ad}+\frac{3x}{8a}$$

```
[Out] 3/8*x/a+1/4*I*cos(d*x+c)^4/a/d+3/8*cos(d*x+c)*sin(d*x+c)/a/d+1/4*cos(d*x+c)
^3*sin(d*x+c)/a/d
```

Rubi [A] time = 0.13, antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.194, Rules used = {3092, 3090, 2635, 8, 2565, 30}

$$\frac{i\cos^4(c+dx)}{4ad}+\frac{\sin(c+dx)\cos^3(c+dx)}{4ad}+\frac{3\sin(c+dx)\cos(c+dx)}{8ad}+\frac{3x}{8a}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^3/(a*Cos[c + d*x] + I*a*Sin[c + d*x]),x]
```

```
[Out] (3*x)/(8*a) + ((I/4)*Cos[c + d*x]^4)/(a*d) + (3*Cos[c + d*x]*Sin[c + d*x])/
(8*a*d) + (Cos[c + d*x]^3*Sin[c + d*x])/(4*a*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2565

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> -Dist[(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
```

```
]
```

Rule 3090

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_.), x_Symbol] :> Int[ExpandTrig[cos[c + d*x]^m*(a
*cos[c + d*x] + b*sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d}, x] && Inte
gerQ[m] && IGtQ[n, 0]
```

Rule 3092

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[a^n*b^n, Int[Cos[c + d*x]^m/
(b*Cos[c + d*x] + a*Sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, m}, x] &&
 EqQ[a^2 + b^2, 0] && ILtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\cos^3(c+dx)}{a\cos(c+dx)+ia\sin(c+dx)}\,dx &= -\frac{i\int \cos^3(c+dx)(ia\cos(c+dx)+a\sin(c+dx))\,dx}{a^2}\\
&= -\frac{i\int \left(ia\cos^4(c+dx)+a\cos^3(c+dx)\sin(c+dx)\right)\,dx}{a^2}\\
&= -\frac{i\int \cos^3(c+dx)\sin(c+dx)\,dx}{a}+\frac{\int \cos^4(c+dx)\,dx}{a}\\
&= \frac{\cos^3(c+dx)\sin(c+dx)}{4ad}+\frac{3\int \cos^2(c+dx)\,dx}{4a}+\frac{i\,\mathrm{Subst}\left(\int x^3\,dx,x,\cos\right.}{ad}\\
&= \frac{i\cos^4(c+dx)}{4ad}+\frac{3\cos(c+dx)\sin(c+dx)}{8ad}+\frac{\cos^3(c+dx)\sin(c+dx)}{4ad}+\\
&= \frac{3x}{8a}+\frac{i\cos^4(c+dx)}{4ad}+\frac{3\cos(c+dx)\sin(c+dx)}{8ad}+\frac{\cos^3(c+dx)\sin(c+d}{4ad}
\end{aligned}$$

Mathematica [A] time = 0.10, size = 60, normalized size = 0.80

$$\frac{8\sin(2(c+dx))+\sin(4(c+dx))+4i\cos(2(c+dx))+i\cos(4(c+dx))+12c+12dx}{32ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^3/(a*Cos[c + d*x] + I*a*Sin[c + d*x]),x]
```

```
[Out] (12*c + 12*d*x + (4*I)*Cos[2*(c + d*x)] + I*Cos[4*(c + d*x)] + 8*Sin[2*(c +
 d*x)] + Sin[4*(c + d*x)])/(32*a*d)
```

fricas [A] time = 0.56, size = 54, normalized size = 0.72

$$\frac{\left(12\,dxe^{(4i\,dx+4i\,c)} - 2i\,e^{(6i\,dx+6i\,c)} + 6i\,e^{(2i\,dx+2i\,c)} + i\right)e^{(-4i\,dx-4i\,c)}}{32\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3/(a*cos(d*x+c)+I*a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/32*(12*d*x*e^(4*I*d*x + 4*I*c) - 2*I*e^(6*I*d*x + 6*I*c) + 6*I*e^(2*I*d*x
 + 2*I*c) + I)*e^(-4*I*d*x - 4*I*c)/(a*d)
```

giac [A] time = 0.19, size = 99, normalized size = 1.32

$$-\frac{\frac{6i\log(i\tan(dx+c)+1)}{a} - \frac{6i\log(i\tan(dx+c)-1)}{a} + \frac{2(3\tan(dx+c)+5i)}{a(-i\tan(dx+c)+1)} + \frac{-9i\tan(dx+c)^2-26\tan(dx+c)+21i}{a(\tan(dx+c)-i)^2}}{32\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3/(a*cos(d*x+c)+I*a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/32*(6*I*log(I*tan(d*x + c) + 1)/a - 6*I*log(I*tan(d*x + c) - 1)/a + 2*(3
*tan(d*x + c) + 5*I)/(a*(-I*tan(d*x + c) + 1)) + (-9*I*tan(d*x + c)^2 - 26*
tan(d*x + c) + 21*I)/(a*(tan(d*x + c) - I)^2))/d
```

maple [A] time = 0.16, size = 98, normalized size = 1.31

$$\frac{3i\ln(\tan(dx+c)+i)}{16ad} + \frac{1}{8ad(\tan(dx+c)+i)} - \frac{3i\ln(\tan(dx+c)-i)}{16ad} - \frac{i}{8ad(\tan(dx+c)-i)^2} + \frac{1}{4ad(\tan(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3/(a*cos(d*x+c)+I*a*sin(d*x+c)),x)
```

```
[Out] 3/16*I/a/d*ln(tan(d*x+c)+I)+1/8/a/d/(tan(d*x+c)+I)-3/16*I/a/d*ln(tan(d*x+c)
-I)-1/8*I/a/d/(tan(d*x+c)-I)^2+1/4/a/d/(tan(d*x+c)-I)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3/(a*cos(d*x+c)+I*a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

mupad [B] time = 3.43, size = 111, normalized size = 1.48

$$\frac{3x}{8a}-\frac{\frac{5\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^5}{4}-\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4\,1\mathrm{i}}{2}-\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3}{2}+\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2\,1\mathrm{i}}{2}+\frac{5\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{4}}{a\,d\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)+1\mathrm{i}\right)^2\left(1+\tan\left(\frac{c}{2}+\frac{dx}{2}\right)1\mathrm{i}\right)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^3/(a*cos(c + d*x) + a*sin(c + d*x)*1i),x)
```

```
[Out] (3*x)/(8*a) - ((5*tan(c/2 + (d*x)/2))/4 + (tan(c/2 + (d*x)/2)^2*1i)/2 - tan
(c/2 + (d*x)/2)^3/2 - (tan(c/2 + (d*x)/2)^4*1i)/2 + (5*tan(c/2 + (d*x)/2)^5
)/4)/(a*d*(tan(c/2 + (d*x)/2) + 1i)^2*(tan(c/2 + (d*x)/2)*1i + 1)^4)
```

sympy [A] time = 0.30, size = 155, normalized size = 2.07

$$\begin{cases} -\frac{\left(512ia^2d^2e^{8ic}e^{2idx}-1536ia^2d^2e^{4ic}e^{-2idx}-256ia^2d^2e^{2ic}e^{-4idx}\right)e^{-6ic}}{8192a^3d^3} & \text{for } 8192a^3d^3e^{6ic}\neq 0 \\ x\left(\frac{\left(e^{6ic}+3e^{4ic}+3e^{2ic}+1\right)e^{-4ic}}{8a}-\frac{3}{8a}\right) & \text{otherwise} \end{cases}+\frac{3x}{8a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3/(a*cos(d*x+c)+I*a*sin(d*x+c)),x)
```

```
[Out] Piecewise((-(512*I*a**2*d**2*exp(8*I*c)*exp(2*I*d*x) - 1536*I*a**2*d**2*exp
(4*I*c)*exp(-2*I*d*x) - 256*I*a**2*d**2*exp(2*I*c)*exp(-4*I*d*x))*exp(-6*I*
c)/(8192*a**3*d**3), Ne(8192*a**3*d**3*exp(6*I*c), 0)), (x*((exp(6*I*c) + 3
*exp(4*I*c) + 3*exp(2*I*c) + 1)*exp(-4*I*c)/(8*a) - 3/(8*a)), True)) + 3*x/
(8*a)
```

3.153 $\int \frac{\cos^2(c+dx)}{a\cos(c+dx)+ia\sin(c+dx)}\,dx$

Optimal. Leaf size=52

$$-\frac{\sin^3(c+dx)}{3ad}+\frac{\sin(c+dx)}{ad}+\frac{i\cos^3(c+dx)}{3ad}$$

```
[Out] 1/3*I*cos(d*x+c)^3/a/d+sin(d*x+c)/a/d-1/3*sin(d*x+c)^3/a/d
```

Rubi [A] time = 0.12, antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.161, Rules used = {3092, 3090, 2633, 2565, 30}

$$-\frac{\sin^3(c+dx)}{3ad}+\frac{\sin(c+dx)}{ad}+\frac{i\cos^3(c+dx)}{3ad}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2/(a*Cos[c + d*x] + I*a*Sin[c + d*x]),x]
```

```
[Out] ((I/3)*Cos[c + d*x]^3)/(a*d) + Sin[c + d*x]/(a*d) - Sin[c + d*x]^3/(3*a*d)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2565

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> -Dist[(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 3090

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_.), x_Symbol] :> Int[ExpandTrig[cos[c + d*x]^m*(a
*cos[c + d*x] + b*sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d}, x] && Inte
```

```
gerQ[m] && IGtQ[n, 0]
```

Rule 3092

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[a^n*b^n, Int[Cos[c + d*x]^m/
(b*Cos[c + d*x] + a*Sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, m}, x] &&
 EqQ[a^2 + b^2, 0] && ILtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\cos^2(c+dx)}{a\cos(c+dx)+ia\sin(c+dx)}\,dx &= -\frac{i\int \cos^2(c+dx)(ia\cos(c+dx)+a\sin(c+dx))\,dx}{a^2}\\
&= -\frac{i\int \left(ia\cos^3(c+dx)+a\cos^2(c+dx)\sin(c+dx)\right)\,dx}{a^2}\\
&= -\frac{i\int \cos^2(c+dx)\sin(c+dx)\,dx}{a}+\frac{\int \cos^3(c+dx)\,dx}{a}\\
&= \frac{i\,\mathrm{Subst}\left(\int x^2\,dx,x,\cos(c+dx)\right)}{ad}-\frac{\mathrm{Subst}\left(\int \left(1-x^2\right)\,dx,x,-\sin(c+dx)\right)}{ad}\\
&= \frac{i\cos^3(c+dx)}{3ad}+\frac{\sin(c+dx)}{ad}-\frac{\sin^3(c+dx)}{3ad}
\end{aligned}$$

Mathematica [A] time = 0.07, size = 73, normalized size = 1.40

$$\frac{3\sin(c+dx)}{4ad}+\frac{\sin(3(c+dx))}{12ad}+\frac{i\cos(c+dx)}{4ad}+\frac{i\cos(3(c+dx))}{12ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2/(a*Cos[c + d*x] + I*a*Sin[c + d*x]),x]
```

```
[Out] ((I/4)*Cos[c + d*x])/(a*d) + ((I/12)*Cos[3*(c + d*x)])/(a*d) + (3*Sin[c + d
*x])/(4*a*d) + Sin[3*(c + d*x)]/(12*a*d)
```

fricas [A] time = 0.49, size = 41, normalized size = 0.79

$$\frac{\left(-3i\,e^{(4i\,dx+4i\,c)}+6i\,e^{(2i\,dx+2i\,c)}+i\right)e^{(-3i\,dx-3i\,c)}}{12\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(a*cos(d*x+c)+I*a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/12*(-3*I*e^(4*I*d*x + 4*I*c) + 6*I*e^(2*I*d*x + 2*I*c) + I)*e^(-3*I*d*x -
 3*I*c)/(a*d)
```

giac [A] time = 1.80, size = 67, normalized size = 1.29

$$\frac{\frac{3}{a\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+i\right)}+\frac{9\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-12i\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-7}{a\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-i\right)^{3}}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(a*cos(d*x+c)+I*a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/6*(3/(a*(tan(1/2*d*x + 1/2*c) + I)) + (9*tan(1/2*d*x + 1/2*c)^2 - 12*I*ta
n(1/2*d*x + 1/2*c) - 7)/(a*(tan(1/2*d*x + 1/2*c) - I)^3))/d
```

maple [A] time = 0.16, size = 75, normalized size = 1.44

$$\frac{\frac{2}{4\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+4i}-\frac{2}{3\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-i\right)^{3}}+\frac{i}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-i\right)^{2}}+\frac{3}{2\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-i\right)}}{ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2/(a*cos(d*x+c)+I*a*sin(d*x+c)),x)
```

```
[Out] 2/d/a*(1/4/(tan(1/2*d*x+1/2*c)+I)-1/3/(tan(1/2*d*x+1/2*c)-I)^3+1/2*I/(tan(1
/2*d*x+1/2*c)-I)^2+3/4/(tan(1/2*d*x+1/2*c)-I))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(a*cos(d*x+c)+I*a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

mupad [B] time = 0.77, size = 78, normalized size = 1.50

$$\frac{\left(-3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{3}+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{2}\,3\mathrm{i}+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+1\mathrm{i}\right)\,2\mathrm{i}}{3\,a\,d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+1\mathrm{i}\right)\,{\left(1+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,1\mathrm{i}\right)}^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^2/(a*cos(c + d*x) + a*sin(c + d*x)*1i),x)
```

```
[Out] ((tan(c/2 + (d*x)/2) + tan(c/2 + (d*x)/2)^2*3i - 3*tan(c/2 + (d*x)/2)^3 + 1
i)*2i)/(3*a*d*(tan(c/2 + (d*x)/2) + 1i)*(tan(c/2 + (d*x)/2)*1i + 1)^3)
```

sympy [A] time = 0.48, size = 129, normalized size = 2.48

$$\begin{cases} -\frac{\left(24ia^2d^2e^{5ic}e^{idx}-48ia^2d^2e^{3ic}e^{-idx}-8ia^2d^2e^{ic}e^{-3idx}\right)e^{-4ic}}{96a^3d^3} & \text{for } 96a^3d^3e^{4ic} \neq 0 \\ \frac{x\left(e^{4ic}+2e^{2ic}+1\right)e^{-3ic}}{4a} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2/(a*cos(d*x+c)+I*a*sin(d*x+c)),x)
```

```
[Out] Piecewise((-(24*I*a**2*d**2*exp(5*I*c)*exp(I*d*x) - 48*I*a**2*d**2*exp(3*I*
c)*exp(-I*d*x) - 8*I*a**2*d**2*exp(I*c)*exp(-3*I*d*x))*exp(-4*I*c)/(96*a**3
*d**3), Ne(96*a**3*d**3*exp(4*I*c), 0)), (x*(exp(4*I*c) + 2*exp(2*I*c) + 1)
*exp(-3*I*c)/(4*a), True))
```

3.154 $\int \frac{\cos(c+dx)}{a\cos(c+dx)+ia\sin(c+dx)}\,dx$

Optimal. Leaf size=46

$$\frac{x}{2a}+\frac{i\cos(c+dx)}{2d(a\cos(c+dx)+ia\sin(c+dx))}$$

```
[Out] 1/2*x/a+1/2*I*cos(d*x+c)/d/(a*cos(d*x+c)+I*a*sin(d*x+c))
```

Rubi [A] time = 0.03, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.069, Rules used = {3082, 8}

$$\frac{x}{2a}+\frac{i\cos(c+dx)}{2d(a\cos(c+dx)+ia\sin(c+dx))}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]/(a*Cos[c + d*x] + I*a*Sin[c + d*x]),x]
```

```
[Out] x/(2*a) + ((I/2)*Cos[c + d*x])/(d*(a*Cos[c + d*x] + I*a*Sin[c + d*x]))
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3082

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*(a*Cos[c + d*x] + b*Sin[
c + d*x])^n)/(2*a*d*n*Cos[c + d*x]^n), x] + Dist[1/(2*a), Int[(a*Cos[c + d*
x] + b*Sin[c + d*x])^(n + 1)/Cos[c + d*x]^(n + 1), x], x] /; FreeQ[{a, b, c
, d}, x] && EqQ[m + n, 0] && EqQ[a^2 + b^2, 0] && LtQ[n, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\cos(c+dx)}{a\cos(c+dx)+ia\sin(c+dx)}\,dx &= \frac{i\cos(c+dx)}{2d(a\cos(c+dx)+ia\sin(c+dx))}+\frac{\int 1\,dx}{2a}\\ &= \frac{x}{2a}+\frac{i\cos(c+dx)}{2d(a\cos(c+dx)+ia\sin(c+dx))}\end{aligned}$$

Mathematica [A] time = 0.07, size = 38, normalized size = 0.83

$$\frac{2(c+dx)+\sin(2(c+dx))+i\cos(2(c+dx))}{4ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]/(a*Cos[c + d*x] + I*a*Sin[c + d*x]),x]
[Out] (2*(c + d*x) + I*Cos[2*(c + d*x)] + Sin[2*(c + d*x)])/(4*a*d)
```

fricas [A] time = 0.60, size = 32, normalized size = 0.70

$$\frac{\left(2\,dxe^{(2i\,dx+2i\,c)}+i\right)e^{(-2i\,dx-2i\,c)}}{4\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a*cos(d*x+c)+I*a*sin(d*x+c)),x, algorithm="fricas")
[Out] 1/4*(2*d*x*e^(2*I*d*x + 2*I*c) + I)*e^(-2*I*d*x - 2*I*c)/(a*d)
```

giac [A] time = 2.93, size = 60, normalized size = 1.30

$$-\frac{\frac{i\,\log(\tan(dx+c)-i)}{a}-\frac{i\,\log(-i\,\tan(dx+c)+1)}{a}+\frac{-i\,\tan(dx+c)-3}{a(\tan(dx+c)-i)}}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a*cos(d*x+c)+I*a*sin(d*x+c)),x, algorithm="giac")
[Out] -1/4*(I*log(tan(d*x + c) - I)/a - I*log(-I*tan(d*x + c) + 1)/a + (-I*tan(d*
x + c) - 3)/(a*(tan(d*x + c) - I)))/d
```

maple [A] time = 0.15, size = 59, normalized size = 1.28

$$\frac{i\ln(\tan(dx+c)+i)}{4ad}-\frac{i\ln(\tan(dx+c)-i)}{4ad}+\frac{1}{2ad(\tan(dx+c)-i)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)/(a*cos(d*x+c)+I*a*sin(d*x+c)),x)
[Out] 1/4*I/a/d*ln(tan(d*x+c)+I)-1/4*I/a/d*ln(tan(d*x+c)-I)+1/2/a/d/(tan(d*x+c)-I
)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a*cos(d*x+c)+I*a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

mupad [B] time = 0.71, size = 39, normalized size = 0.85

$$\frac{x}{2\,a}+\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{a\,d\,{\left(1+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,1{}\mathrm{i}\right)}^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)/(a*cos(c + d*x) + a*sin(c + d*x)*1i),x)
```

```
[Out] x/(2*a) + tan(c/2 + (d*x)/2)/(a*d*(tan(c/2 + (d*x)/2)*1i + 1)^2)
```

sympy [A] time = 0.21, size = 61, normalized size = 1.33

$$\begin{cases} \frac{ie^{-2ic}e^{-2idx}}{4ad} & \text{for}\: 4ade^{2ic} \neq 0 \\ x \left(\frac{\left(e^{2ic}+1\right)e^{-2ic}}{2a} - \frac{1}{2a}\right) & \text{otherwise} \end{cases} + \frac{x}{2a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a*cos(d*x+c)+I*a*sin(d*x+c)),x)
```

```
[Out] Piecewise((I*exp(-2*I*c)*exp(-2*I*d*x)/(4*a*d), Ne(4*a*d*exp(2*I*c), 0)), (
x*((exp(2*I*c) + 1)*exp(-2*I*c)/(2*a) - 1/(2*a)), True)) + x/(2*a)
```

3.155 $\int \frac{1}{a\cos(c+dx)+ia\sin(c+dx)}\,dx$

Optimal. Leaf size=29

$$\frac{i}{d(a\cos(c+dx)+ia\sin(c+dx))}$$

```
[Out] I/d/(a*cos(d*x+c)+I*a*sin(d*x+c))
```

Rubi [A] time = 0.02, antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {3071}

$$\frac{i}{d(a\cos(c+dx)+ia\sin(c+dx))}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Cos[c + d*x] + I*a*Sin[c + d*x])^(-1),x]
```

```
[Out] I/(d*(a*Cos[c + d*x] + I*a*Sin[c + d*x]))
```

Rule 3071

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x
_Symbol] :> Simp[(a*(a*Cos[c + d*x] + b*Sin[c + d*x])^n)/(b*d*n), x] /; Fre
eQ[{a, b, c, d, n}, x] && EqQ[a^2 + b^2, 0]
```

Rubi steps

$$\int \frac{1}{a\cos(c+dx)+ia\sin(c+dx)}\,dx = \frac{i}{d(a\cos(c+dx)+ia\sin(c+dx))}$$

Mathematica [A] time = 0.03, size = 29, normalized size = 1.00

$$\frac{i}{d(a\cos(c+dx)+ia\sin(c+dx))}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Cos[c + d*x] + I*a*Sin[c + d*x])^(-1),x]
```

```
[Out] I/(d*(a*Cos[c + d*x] + I*a*Sin[c + d*x]))
```

fricas [A] time = 0.57, size = 17, normalized size = 0.59

$$\frac{i\,e^{(-i\,dx-i\,c)}}{ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cos(d*x+c)+I*a*sin(d*x+c)),x, algorithm="fricas")
[Out] I*e^(-I*d*x - I*c)/(a*d)
```

giac [A] time = 0.16, size = 21, normalized size = 0.72

$$\frac{2}{ad\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-i\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cos(d*x+c)+I*a*sin(d*x+c)),x, algorithm="giac")
[Out] 2/(a*d*(tan(1/2*d*x + 1/2*c) - I))
```

maple [A] time = 0.13, size = 23, normalized size = 0.79

$$\frac{2}{da\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-i\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*cos(d*x+c)+I*a*sin(d*x+c)),x)
[Out] 2/d/a/(tan(1/2*d*x+1/2*c)-I)
```

maxima [A] time = 0.32, size = 29, normalized size = 1.00

$$\frac{2}{\left(-i\,a+\frac{a\sin(dx+c)}{\cos(dx+c)+1}\right)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cos(d*x+c)+I*a*sin(d*x+c)),x, algorithm="maxima")
[Out] 2/((-I*a + a*sin(d*x + c)/(cos(d*x + c) + 1))*d)
```

mupad [B] time = 0.61, size = 25, normalized size = 0.86

$$\frac{2\mathrm{i}}{a\,d\,\left(1+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,1\mathrm{i}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*cos(c + d*x) + a*sin(c + d*x)*1i),x)
```

```
[Out] 2i/(a*d*(tan(c/2 + (d*x)/2)*1i + 1))
```

sympy [A] time = 0.14, size = 31, normalized size = 1.07

$$\begin{cases} \frac{ie^{-ic}e^{-idx}}{ad} & \text{for } ade^{ic} \neq 0 \\ \frac{xe^{-ic}}{a} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cos(d*x+c)+I*a*sin(d*x+c)),x)
```

```
[Out] Piecewise((I*exp(-I*c)*exp(-I*d*x)/(a*d), Ne(a*d*exp(I*c), 0)), (x*exp(-I*c
)/a, True))
```

3.156 $\int \frac{\sec(c+dx)}{a\cos(c+dx)+ia\sin(c+dx)}\,dx$

Optimal. Leaf size=23

$$\frac{x}{a}+\frac{i\log(\cos(c+dx))}{ad}$$

```
[Out] x/a+I*ln(cos(d*x+c))/a/d
```

Rubi [A] time = 0.07, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {3092, 3090, 3475}

$$\frac{x}{a}+\frac{i\log(\cos(c+dx))}{ad}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]/(a*Cos[c + d*x] + I*a*Sin[c + d*x]),x]
```

```
[Out] x/a + (I*Log[Cos[c + d*x]])/(a*d)
```

Rule 3090

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_.), x_Symbol] :> Int[ExpandTrig[cos[c + d*x]^m*(a
*cos[c + d*x] + b*sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d}, x] && Inte
gerQ[m] && IGtQ[n, 0]
```

Rule 3092

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[a^n*b^n, Int[Cos[c + d*x]^m/
(b*Cos[c + d*x] + a*Sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, m}, x] &&
 EqQ[a^2 + b^2, 0] && ILtQ[n, 0]
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\sec(c+dx)}{a\cos(c+dx)+ia\sin(c+dx)}\,dx = -\frac{i\int \sec(c+dx)(ia\cos(c+dx)+a\sin(c+dx))\,dx}{a^2}$$
$$= -\frac{i\int (ia+a\tan(c+dx))\,dx}{a^2}$$
$$= \frac{x}{a} - \frac{i\int \tan(c+dx)\,dx}{a}$$
$$= \frac{x}{a} + \frac{i\log(\cos(c+dx))}{ad}$$

Mathematica [A] time = 0.07, size = 23, normalized size = 1.00

$$\frac{i\log(\cos(c+dx))+c+dx}{ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]/(a*Cos[c + d*x] + I*a*Sin[c + d*x]),x]
```

```
[Out] (c + d*x + I*Log[Cos[c + d*x]])/(a*d)
```

fricas [A] time = 0.55, size = 26, normalized size = 1.13

$$\frac{2\,dx + i\,\log\left(e^{(2i\,dx+2i\,c)}+1\right)}{ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a*cos(d*x+c)+I*a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] (2*d*x + I*log(e^(2*I*d*x + 2*I*c) + 1))/(a*d)
```

giac [B] time = 0.20, size = 57, normalized size = 2.48

$$-\frac{-\frac{i\,\log\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)}{a} + \frac{2i\,\log\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-i\right)}{a} - \frac{i\,\log\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right)}{a}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a*cos(d*x+c)+I*a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -(-I*log(tan(1/2*d*x + 1/2*c) + 1)/a + 2*I*log(tan(1/2*d*x + 1/2*c) - I)/a
- I*log(tan(1/2*d*x + 1/2*c) - 1)/a)/d
```

maple [A] time = 0.18, size = 22, normalized size = 0.96

$$-\frac{i \ln \left(i \tan \left(d x+c\right)+1\right)}{d a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)/(a*cos(d*x+c)+I*a*sin(d*x+c)),x)
```

```
[Out] -I/d/a*ln(I*tan(d*x+c)+1)
```

maxima [B] time = 0.34, size = 101, normalized size = 4.39

$$-\frac{-\frac{i \log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)}{a}-\frac{i \log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}-1\right)}{a}+\frac{i \log\left(-\frac{2i \sin(dx+c)}{\cos(dx+c)+1}+\frac{\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-1\right)}{a}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a*cos(d*x+c)+I*a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -(-I*log(sin(d*x + c)/(cos(d*x + c) + 1) + 1)/a - I*log(sin(d*x + c)/(cos(d
*x + c) + 1) - 1)/a + I*log(-2*I*sin(d*x + c)/(cos(d*x + c) + 1) + sin(d*x
+ c)^2/(cos(d*x + c) + 1)^2 - 1)/a)/d
```

mupad [B] time = 0.74, size = 41, normalized size = 1.78

$$-\frac{\left(2\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)-\mathrm{i}\right)-\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2-1\right)\right)\,1{}\mathrm{i}}{a\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)*(a*cos(c + d*x) + a*sin(c + d*x)*1i)),x)
```

```
[Out] -((2*log(tan(c/2 + (d*x)/2) - 1i) - log(tan(c/2 + (d*x)/2)^2 - 1))*1i)/(a*d
)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\sec\left(c+dx\right)}{i \sin\left(c+dx\right)+\cos\left(c+dx\right)}\, dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a*cos(d*x+c)+I*a*sin(d*x+c)),x)
```

```
[Out] Integral(sec(c + d*x)/(I*sin(c + d*x) + cos(c + d*x)), x)/a
```

3.157 $\int \frac{\sec^2(c+dx)}{a\cos(c+dx)+ia\sin(c+dx)}\,dx$

Optimal. Leaf size=31

$$\frac{\tanh^{-1}(\sin(c+dx))}{ad}-\frac{i\sec(c+dx)}{ad}$$

```
[Out] arctanh(sin(d*x+c))/a/d-I*sec(d*x+c)/a/d
```

Rubi [A] time = 0.09, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.161, Rules used = {3092, 3090, 3770, 2606, 8}

$$\frac{\tanh^{-1}(\sin(c+dx))}{ad}-\frac{i\sec(c+dx)}{ad}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^2/(a*Cos[c + d*x] + I*a*Sin[c + d*x]),x]
```

```
[Out] ArcTanh[Sin[c + d*x]]/(a*d) - (I*Sec[c + d*x])/(a*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 3090

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_.), x_Symbol] :> Int[ExpandTrig[cos[c + d*x]^m*(a
*cos[c + d*x] + b*sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d}, x] && Inte
gerQ[m] && IGtQ[n, 0]
```

Rule 3092

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[a^n*b^n, Int[Cos[c + d*x]^m/
(b*Cos[c + d*x] + a*Sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, m}, x] &&
```

```
EqQ[a^2 + b^2, 0] && ILtQ[n, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sec^2(c+dx)}{a\cos(c+dx)+ia\sin(c+dx)}\,dx &= -\frac{i\int \sec^2(c+dx)(ia\cos(c+dx)+a\sin(c+dx))\,dx}{a^2}\\
&= -\frac{i\int (ia\sec(c+dx)+a\sec(c+dx)\tan(c+dx))\,dx}{a^2}\\
&= -\frac{i\int \sec(c+dx)\tan(c+dx)\,dx}{a}+\frac{\int \sec(c+dx)\,dx}{a}\\
&= \frac{\tanh^{-1}(\sin(c+dx))}{ad}-\frac{i\,\mathrm{Subst}(\int 1\,dx,x,\sec(c+dx))}{ad}\\
&= \frac{\tanh^{-1}(\sin(c+dx))}{ad}-\frac{i\sec(c+dx)}{ad}
\end{aligned}
$$

Mathematica [A] time = 0.22, size = 35, normalized size = 1.13

$$
-\frac{i\left(\sec(c+dx)+2i\tanh^{-1}\left(\cos(c)\tan\left(\frac{dx}{2}\right)+\sin(c)\right)\right)}{ad}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^2/(a*Cos[c + d*x] + I*a*Sin[c + d*x]),x]
[Out] ((-I)*((2*I)*ArcTanh[Sin[c] + Cos[c]*Tan[(d*x)/2]] + Sec[c + d*x]))/(a*d)
```

fricas [B] time = 0.61, size = 80, normalized size = 2.58

$$
\frac{\left(e^{(2i\,dx+2i\,c)}+1\right)\log\left(e^{(i\,dx+i\,c)}+i\right)-\left(e^{(2i\,dx+2i\,c)}+1\right)\log\left(e^{(i\,dx+i\,c)}-i\right)-2i\,e^{(i\,dx+i\,c)}}{ade^{(2i\,dx+2i\,c)}+ad}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a*cos(d*x+c)+I*a*sin(d*x+c)),x, algorithm="fricas")
```

[Out] ((e^(2*I*d*x + 2*I*c) + 1)*log(e^(I*d*x + I*c) + I) - (e^(2*I*d*x + 2*I*c) + 1)*log(e^(I*d*x + I*c) - I) - 2*I*e^(I*d*x + I*c))/(a*d*e^(2*I*d*x + 2*I*c) + a*d)

giac [A] time = 2.58, size = 58, normalized size = 1.87

$$\frac{\frac{\log\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)}{a}-\frac{\log\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right)}{a}+\frac{2i}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-1\right)a}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)^2/(a*cos(d*x+c)+I*a*sin(d*x+c)),x, algorithm="giac")

[Out] (log(tan(1/2*d*x + 1/2*c) + 1)/a - log(tan(1/2*d*x + 1/2*c) - 1)/a + 2*I/((tan(1/2*d*x + 1/2*c)^2 - 1)*a))/d

maple [B] time = 0.21, size = 85, normalized size = 2.74

$$\frac{i}{ad\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}-\frac{\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{ad}-\frac{i}{ad\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}+\frac{\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{ad}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sec(d*x+c)^2/(a*cos(d*x+c)+I*a*sin(d*x+c)),x)

[Out] I/a/d/(tan(1/2*d*x+1/2*c)-1)-1/a/d*ln(tan(1/2*d*x+1/2*c)-1)-I/a/d/(tan(1/2*d*x+1/2*c)+1)+1/a/d*ln(tan(1/2*d*x+1/2*c)+1)

maxima [B] time = 0.42, size = 83, normalized size = 2.68

$$\frac{\frac{\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)}{a}-\frac{\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}-1\right)}{a}-\frac{2}{-i\,a+\frac{i\,a\sin(dx+c)^{2}}{(\cos(dx+c)+1)^{2}}}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)^2/(a*cos(d*x+c)+I*a*sin(d*x+c)),x, algorithm="maxima")

[Out] (log(sin(d*x + c)/(cos(d*x + c) + 1) + 1)/a - log(sin(d*x + c)/(cos(d*x + c) + 1) - 1)/a - 2/(-I*a + I*a*sin(d*x + c)^2/(cos(d*x + c) + 1)^2))/d

mupad [B] time = 0.67, size = 43, normalized size = 1.39

$$\frac{2\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{a\,d}+\frac{2\mathrm{i}}{a\,d\,\left({\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^2*(a*cos(c + d*x) + a*sin(c + d*x)*1i)),x)
```

```
[Out] (2*atanh(tan(c/2 + (d*x)/2)))/(a*d) + 2i/(a*d*(tan(c/2 + (d*x)/2)^2 - 1))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\sec^2(c+dx)}{i\sin(c+dx)+\cos(c+dx)}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2/(a*cos(d*x+c)+I*a*sin(d*x+c)),x)
```

```
[Out] Integral(sec(c + d*x)**2/(I*sin(c + d*x) + cos(c + d*x)), x)/a
```

3.158 $\int \frac{\sec^3(c+dx)}{a\cos(c+dx)+ia\sin(c+dx)}\,dx$

Optimal. Leaf size=34

$$\frac{\tan(c+dx)}{ad}-\frac{i\sec^2(c+dx)}{2ad}$$

```
[Out] -1/2*I*sec(d*x+c)^2/a/d+tan(d*x+c)/a/d
```

Rubi [A] time = 0.11, antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.194, Rules used = {3092, 3090, 3767, 8, 2606, 30}

$$\frac{\tan(c+dx)}{ad}-\frac{i\sec^2(c+dx)}{2ad}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^3/(a*Cos[c + d*x] + I*a*Sin[c + d*x]),x]
```

```
[Out] ((-I/2)*Sec[c + d*x]^2)/(a*d) + Tan[c + d*x]/(a*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 3090

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_.), x_Symbol] :> Int[ExpandTrig[cos[c + d*x]^m*(a
*cos[c + d*x] + b*sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d}, x] && Inte
gerQ[m] && IGtQ[n, 0]
```

Rule 3092

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[a^n*b^n, Int[Cos[c + d*x]^m/
(b*Cos[c + d*x] + a*Sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, m}, x] &&
 EqQ[a^2 + b^2, 0] && ILtQ[n, 0]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sec^3(c+dx)}{a\cos(c+dx)+ia\sin(c+dx)}\,dx &= -\frac{i\int \sec^3(c+dx)(ia\cos(c+dx)+a\sin(c+dx))\,dx}{a^2}\\
&= -\frac{i\int \left(ia\sec^2(c+dx)+a\sec^2(c+dx)\tan(c+dx)\right)\,dx}{a^2}\\
&= -\frac{i\int \sec^2(c+dx)\tan(c+dx)\,dx}{a}+\frac{\int \sec^2(c+dx)\,dx}{a}\\
&= -\frac{i\,\mathrm{Subst}(\int x\,dx, x, \sec(c+dx))}{ad}-\frac{\mathrm{Subst}(\int 1\,dx, x, -\tan(c+dx))}{ad}\\
&= -\frac{i\sec^2(c+dx)}{2ad}+\frac{\tan(c+dx)}{ad}
\end{aligned}$$

Mathematica [A] time = 0.19, size = 35, normalized size = 1.03

$$-\frac{i\sec(c+dx)(\sec(c+dx)+2i\sec(c)\sin(dx))}{2ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^3/(a*Cos[c + d*x] + I*a*Sin[c + d*x]),x]
[Out] ((-1/2*I)*Sec[c + d*x]*(Sec[c + d*x] + (2*I)*Sec[c]*Sin[d*x]))/(a*d)
```

fricas [A] time = 0.55, size = 33, normalized size = 0.97

$$\frac{2i}{ade^{(4i\,dx+4i\,c)}+2\,ade^{(2i\,dx+2i\,c)}+ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a*cos(d*x+c)+I*a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 2*I/(a*d*e^(4*I*d*x + 4*I*c) + 2*a*d*e^(2*I*d*x + 2*I*c) + a*d)
```

giac [A] time = 0.23, size = 27, normalized size = 0.79

$$-\frac{i \tan (dx+c)^2 - 2 \tan (dx+c)}{2\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a*cos(d*x+c)+I*a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/2*(I*tan(d*x + c)^2 - 2*tan(d*x + c))/(a*d)
```

maple [A] time = 0.22, size = 26, normalized size = 0.76

$$\frac{\tan (dx+c) - \frac{i\left(\tan^2(dx+c)\right)}{2}}{da}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^3/(a*cos(d*x+c)+I*a*sin(d*x+c)),x)
```

```
[Out] 1/d/a*(tan(d*x+c)-1/2*I*tan(d*x+c)^2)
```

maxima [B] time = 0.35, size = 108, normalized size = 3.18

$$\frac{2\left(\frac{\sin(dx+c)}{\cos(dx+c)+1} - \frac{i\,\sin(dx+c)^2}{(\cos(dx+c)+1)^2} - \frac{\sin(dx+c)^3}{(\cos(dx+c)+1)^3}\right)}{\left(a - \frac{2\,a\sin(dx+c)^2}{(\cos(dx+c)+1)^2} + \frac{a\sin(dx+c)^4}{(\cos(dx+c)+1)^4}\right)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a*cos(d*x+c)+I*a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 2*(sin(d*x + c)/(cos(d*x + c) + 1) - I*sin(d*x + c)^2/(cos(d*x + c) + 1)^2
- sin(d*x + c)^3/(cos(d*x + c) + 1)^3)/((a - 2*a*sin(d*x + c)^2/(cos(d*x +
c) + 1)^2 + a*sin(d*x + c)^4/(cos(d*x + c) + 1)^4)*d)
```

mupad [B] time = 0.68, size = 25, normalized size = 0.74

$$-\frac{\tan (c+d\,x)\,\left(-2+\tan (c+d\,x)\,1\mathrm{i}\right)}{2\,a\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^3*(a*cos(c + d*x) + a*sin(c + d*x)*1i)),x)
```

```
[Out] -(tan(c + d*x)*(tan(c + d*x)*1i - 2))/(2*a*d)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\sec^3(c+dx)}{i\sin(c+dx)+\cos(c+dx)}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**3/(a*cos(d*x+c)+I*a*sin(d*x+c)),x)
```

```
[Out] Integral(sec(c + d*x)**3/(I*sin(c + d*x) + cos(c + d*x)), x)/a
```

3.159 $\int \frac{\sec^4(c+dx)}{a\cos(c+dx)+ia\sin(c+dx)}\,dx$

Optimal. Leaf size=60

$$-\frac{i\sec^3(c+dx)}{3ad}+\frac{\tanh^{-1}(\sin(c+dx))}{2ad}+\frac{\tan(c+dx)\sec(c+dx)}{2ad}$$

```
[Out] 1/2*arctanh(sin(d*x+c))/a/d-1/3*I*sec(d*x+c)^3/a/d+1/2*sec(d*x+c)*tan(d*x+c
)/a/d
```

Rubi [A] time = 0.12, antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.194, Rules used = {3092, 3090, 3768, 3770, 2606, 30}

$$-\frac{i\sec^3(c+dx)}{3ad}+\frac{\tanh^{-1}(\sin(c+dx))}{2ad}+\frac{\tan(c+dx)\sec(c+dx)}{2ad}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^4/(a*Cos[c + d*x] + I*a*Sin[c + d*x]),x]
```

```
[Out] ArcTanh[Sin[c + d*x]]/(2*a*d) - ((I/3)*Sec[c + d*x]^3)/(a*d) + (Sec[c + d*x
]*Tan[c + d*x])/(2*a*d)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 3090

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_.), x_Symbol] :> Int[ExpandTrig[cos[c + d*x]^m*(a
*cos[c + d*x] + b*sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d}, x] && Inte
gerQ[m] && IGtQ[n, 0]
```

Rule 3092

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[a^n*b^n, Int[Cos[c + d*x]^m/
(b*Cos[c + d*x] + a*Sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, m}, x] &&
 EqQ[a^2 + b^2, 0] && ILtQ[n, 0]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sec^4(c+dx)}{a\cos(c+dx)+ia\sin(c+dx)}\,dx &= -\frac{i\int \sec^4(c+dx)(ia\cos(c+dx)+a\sin(c+dx))\,dx}{a^2}\\
&= -\frac{i\int \left(ia\sec^3(c+dx)+a\sec^3(c+dx)\tan(c+dx)\right)\,dx}{a^2}\\
&= -\frac{i\int \sec^3(c+dx)\tan(c+dx)\,dx}{a}+\frac{\int \sec^3(c+dx)\,dx}{a}\\
&= \frac{\sec(c+dx)\tan(c+dx)}{2ad}+\frac{\int \sec(c+dx)\,dx}{2a}-\frac{i\,\mathrm{Subst}\left(\int x^2\,dx,x,\sec(c+d\right.}{ad}\\
&= \frac{\tanh^{-1}(\sin(c+dx))}{2ad}-\frac{i\sec^3(c+dx)}{3ad}+\frac{\sec(c+dx)\tan(c+dx)}{2ad}
\end{aligned}$$

Mathematica [**A**] time = 0.25, size = 54, normalized size = 0.90

$$-\frac{i\left((4+3i\sin(2(c+dx)))\sec^3(c+dx)+12i\tanh^{-1}\left(\cos(c)\tan\left(\frac{dx}{2}\right)+\sin(c)\right)\right)}{12ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^4/(a*Cos[c + d*x] + I*a*Sin[c + d*x]),x]
```

```
[Out] ((-1/12*I)*((12*I)*ArcTanh[Sin[c] + Cos[c]*Tan[(d*x)/2]] + Sec[c + d*x]^3*(
4 + (3*I)*Sin[2*(c + d*x)])))/(a*d)
```

fricas [B] time = 0.52, size = 174, normalized size = 2.90

$$\frac{3\left(e^{(6i\,dx+6i\,c)}+3\,e^{(4i\,dx+4i\,c)}+3\,e^{(2i\,dx+2i\,c)}+1\right)\log\left(e^{(i\,dx+i\,c)}+i\right)-3\left(e^{(6i\,dx+6i\,c)}+3\,e^{(4i\,dx+4i\,c)}+3\,e^{(2i\,dx+2i\,c)}+1\right)1}{6\left(ade^{(6i\,dx+6i\,c)}+3\,ade^{(4i\,dx+4i\,c)}+3\,ade^{(2i\,dx+2i\,c)}+ad\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4/(a*cos(d*x+c)+I*a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/6*(3*(e^(6*I*d*x + 6*I*c) + 3*e^(4*I*d*x + 4*I*c) + 3*e^(2*I*d*x + 2*I*c)
 + 1)*log(e^(I*d*x + I*c) + I) - 3*(e^(6*I*d*x + 6*I*c) + 3*e^(4*I*d*x + 4*
I*c) + 3*e^(2*I*d*x + 2*I*c) + 1)*log(e^(I*d*x + I*c) - I) - 6*I*e^(5*I*d*x
 + 5*I*c) - 16*I*e^(3*I*d*x + 3*I*c) + 6*I*e^(I*d*x + I*c))/(a*d*e^(6*I*d*x
 + 6*I*c) + 3*a*d*e^(4*I*d*x + 4*I*c) + 3*a*d*e^(2*I*d*x + 2*I*c) + a*d)
```

giac [A] time = 2.72, size = 99, normalized size = 1.65

$$\frac{\frac{3\,\log\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)}{a}-\frac{3\,\log\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right)}{a}+\frac{2\left(3\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}+6i\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{4}-3\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+2i\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-1\right)^{3}a}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4/(a*cos(d*x+c)+I*a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/6*(3*log(tan(1/2*d*x + 1/2*c) + 1)/a - 3*log(tan(1/2*d*x + 1/2*c) - 1)/a
+ 2*(3*tan(1/2*d*x + 1/2*c)^5 + 6*I*tan(1/2*d*x + 1/2*c)^4 - 3*tan(1/2*d*x
+ 1/2*c) + 2*I)/((tan(1/2*d*x + 1/2*c)^2 - 1)^3*a))/d
```

maple [B] time = 0.23, size = 258, normalized size = 4.30

$$\frac{i}{3ad\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{3}}+\frac{1}{2ad\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{2}}+\frac{i}{2ad\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{2}}+\frac{1}{2ad\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}+\frac{}{2ad\left(\tan\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^4/(a*cos(d*x+c)+I*a*sin(d*x+c)),x)
```

```
[Out] 1/3*I/a/d/(tan(1/2*d*x+1/2*c)-1)^3+1/2/a/d/(tan(1/2*d*x+1/2*c)-1)^2+1/2*I/a
/d/(tan(1/2*d*x+1/2*c)-1)^2+1/2/a/d/(tan(1/2*d*x+1/2*c)-1)+1/2*I/a/d/(tan(1
```

```
/2*d*x+1/2*c)-1)-1/2/a/d*ln(tan(1/2*d*x+1/2*c)-1)-1/3*I/a/d/(tan(1/2*d*x+1/
2*c)+1)^3+1/2/a/d/(tan(1/2*d*x+1/2*c)+1)-1/2*I/a/d/(tan(1/2*d*x+1/2*c)+1)-1
/2/a/d/(tan(1/2*d*x+1/2*c)+1)^2+1/2*I/a/d/(tan(1/2*d*x+1/2*c)+1)^2+1/2/a/d*
ln(tan(1/2*d*x+1/2*c)+1)
```

maxima [B] time = 0.33, size = 186, normalized size = 3.10

$$\frac{\frac{4\left(\frac{3i\sin(dx+c)}{\cos(dx+c)+1}+\frac{6\sin(dx+c)^4}{(\cos(dx+c)+1)^4}-\frac{3i\sin(dx+c)^5}{(\cos(dx+c)+1)^5}+2\right)}{6ia-\frac{18ia\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{18ia\sin(dx+c)^4}{(\cos(dx+c)+1)^4}-\frac{6ia\sin(dx+c)^6}{(\cos(dx+c)+1)^6}}+\frac{\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)}{a}-\frac{\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}-1\right)}{a}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4/(a*cos(d*x+c)+I*a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/2*(4*(3*I*sin(d*x + c)/(cos(d*x + c) + 1) + 6*sin(d*x + c)^4/(cos(d*x + c
) + 1)^4 - 3*I*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + 2)/(6*I*a - 18*I*a*sin
(d*x + c)^2/(cos(d*x + c) + 1)^2 + 18*I*a*sin(d*x + c)^4/(cos(d*x + c) + 1)
^4 - 6*I*a*sin(d*x + c)^6/(cos(d*x + c) + 1)^6) + log(sin(d*x + c)/(cos(d*x
 + c) + 1) + 1)/a - log(sin(d*x + c)/(cos(d*x + c) + 1) - 1)/a)/d
```

mupad [B] time = 2.55, size = 116, normalized size = 1.93

$$\frac{\operatorname{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{a\,d}+\frac{\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5}{a}-\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{a}+\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4\,2\mathrm{i}}{a}+\frac{2\mathrm{i}}{3\,a}}{d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6-3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4+3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^4*(a*cos(c + d*x) + a*sin(c + d*x)*1i)),x)
```

```
[Out] atanh(tan(c/2 + (d*x)/2))/(a*d) + ((tan(c/2 + (d*x)/2)^4*2i)/a + tan(c/2 +
(d*x)/2)^5/a + 2i/(3*a) - tan(c/2 + (d*x)/2)/a)/(d*(3*tan(c/2 + (d*x)/2)^2
- 3*tan(c/2 + (d*x)/2)^4 + tan(c/2 + (d*x)/2)^6 - 1))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\sec^4(c+dx)}{i\sin(c+dx)+\cos(c+dx)}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**4/(a*cos(d*x+c)+I*a*sin(d*x+c)),x)
```

```
[Out] Integral(sec(c + d*x)**4/(I*sin(c + d*x) + cos(c + d*x)), x)/a
```

3.160 $\int \frac{\sec^5(c+dx)}{a\cos(c+dx)+ia\sin(c+dx)}\,dx$

Optimal. Leaf size=52

$$\frac{\tan^3(c+dx)}{3ad}+\frac{\tan(c+dx)}{ad}-\frac{i\sec^4(c+dx)}{4ad}$$

```
[Out] -1/4*I*sec(d*x+c)^4/a/d+tan(d*x+c)/a/d+1/3*tan(d*x+c)^3/a/d
```

Rubi [A] time = 0.12, antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.161, Rules used = {3092, 3090, 3767, 2606, 30}

$$\frac{\tan^3(c+dx)}{3ad}+\frac{\tan(c+dx)}{ad}-\frac{i\sec^4(c+dx)}{4ad}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^5/(a*Cos[c + d*x] + I*a*Sin[c + d*x]),x]
```

```
[Out] ((-I/4)*Sec[c + d*x]^4)/(a*d) + Tan[c + d*x]/(a*d) + Tan[c + d*x]^3/(3*a*d)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 && !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 3090

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_.), x_Symbol] :> Int[ExpandTrig[cos[c + d*x]^m*(a
*cos[c + d*x] + b*sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d}, x] && Inte
gerQ[m] && IGtQ[n, 0]
```

Rule 3092

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[a^n*b^n, Int[Cos[c + d*x]^m/
```

```
(b*Cos[c + d*x] + a*Sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, m}, x] &&
 EqQ[a^2 + b^2, 0] && ILtQ[n, 0]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sec^5(c+dx)}{a\cos(c+dx)+ia\sin(c+dx)}\,dx &= -\frac{i\int \sec^5(c+dx)(ia\cos(c+dx)+a\sin(c+dx))\,dx}{a^2} \\
&= -\frac{i\int \left(ia\sec^4(c+dx)+a\sec^4(c+dx)\tan(c+dx)\right)\,dx}{a^2} \\
&= -\frac{i\int \sec^4(c+dx)\tan(c+dx)\,dx}{a}+\frac{\int \sec^4(c+dx)\,dx}{a} \\
&= -\frac{i\,\mathrm{Subst}\left(\int x^3\,dx,x,\sec(c+dx)\right)}{ad}-\frac{\mathrm{Subst}\left(\int \left(1+x^2\right)\,dx,x,-\tan(c+dx)\right)}{ad} \\
&= -\frac{i\sec^4(c+dx)}{4ad}+\frac{\tan(c+dx)}{ad}+\frac{\tan^3(c+dx)}{3ad}
\end{aligned}$$

Mathematica [A] time = 0.28, size = 53, normalized size = 1.02

$$-\frac{i\sec^4(c+dx)(i\sec(c)(4\sin(c+2dx)+\sin(3c+4dx))-3i\tan(c)+3)}{12ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^5/(a*Cos[c + d*x] + I*a*Sin[c + d*x]),x]
```

```
[Out] ((-1/12*I)*Sec[c + d*x]^4*(3 + I*Sec[c]*(4*Sin[c + 2*d*x] + Sin[3*c + 4*d*x
]) - (3*I)*Tan[c]))/(a*d)
```

fricas [A] time = 0.47, size = 72, normalized size = 1.38

$$\frac{16i\,e^{(2i\,dx+2i\,c)}+4i}{3\left(ade^{(8i\,dx+8i\,c)}+4\,ade^{(6i\,dx+6i\,c)}+6\,ade^{(4i\,dx+4i\,c)}+4\,ade^{(2i\,dx+2i\,c)}+ad\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5/(a*cos(d*x+c)+I*a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/3*(16*I*e^(2*I*d*x + 2*I*c) + 4*I)/(a*d*e^(8*I*d*x + 8*I*c) + 4*a*d*e^(6*
I*d*x + 6*I*c) + 6*a*d*e^(4*I*d*x + 4*I*c) + 4*a*d*e^(2*I*d*x + 2*I*c) + a*
d)
```

giac [A] time = 0.23, size = 47, normalized size = 0.90

$$-\frac{3i\,\tan\left(dx+c\right)^{4}-4\,\tan\left(dx+c\right)^{3}+6i\,\tan\left(dx+c\right)^{2}-12\,\tan\left(dx+c\right)}{12\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5/(a*cos(d*x+c)+I*a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/12*(3*I*tan(d*x + c)^4 - 4*tan(d*x + c)^3 + 6*I*tan(d*x + c)^2 - 12*tan(
d*x + c))/(a*d)
```

maple [A] time = 0.22, size = 47, normalized size = 0.90

$$\frac{\tan\left(dx+c\right)-\frac{i\left(\tan^{4}\left(dx+c\right)\right)}{4}+\frac{\left(\tan^{3}\left(dx+c\right)\right)}{3}-\frac{i\left(\tan^{2}\left(dx+c\right)\right)}{2}}{da}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^5/(a*cos(d*x+c)+I*a*sin(d*x+c)),x)
```

```
[Out] 1/d/a*(tan(d*x+c)-1/4*I*tan(d*x+c)^4+1/3*tan(d*x+c)^3-1/2*I*tan(d*x+c)^2)
```

maxima [B] time = 0.35, size = 211, normalized size = 4.06

$$\frac{2\left(\frac{3\,\sin\left(dx+c\right)}{\cos\left(dx+c\right)+1}-\frac{3i\,\sin\left(dx+c\right)^{2}}{\left(\cos\left(dx+c\right)+1\right)^{2}}-\frac{5\,\sin\left(dx+c\right)^{3}}{\left(\cos\left(dx+c\right)+1\right)^{3}}+\frac{5\,\sin\left(dx+c\right)^{5}}{\left(\cos\left(dx+c\right)+1\right)^{5}}-\frac{3i\,\sin\left(dx+c\right)^{6}}{\left(\cos\left(dx+c\right)+1\right)^{6}}-\frac{3\,\sin\left(dx+c\right)^{7}}{\left(\cos\left(dx+c\right)+1\right)^{7}}\right)}{3\left(a-\frac{4\,a\sin\left(dx+c\right)^{2}}{\left(\cos\left(dx+c\right)+1\right)^{2}}+\frac{6\,a\sin\left(dx+c\right)^{4}}{\left(\cos\left(dx+c\right)+1\right)^{4}}-\frac{4\,a\sin\left(dx+c\right)^{6}}{\left(\cos\left(dx+c\right)+1\right)^{6}}+\frac{a\sin\left(dx+c\right)^{8}}{\left(\cos\left(dx+c\right)+1\right)^{8}}\right)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5/(a*cos(d*x+c)+I*a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 2/3*(3*sin(d*x + c)/(cos(d*x + c) + 1) - 3*I*sin(d*x + c)^2/(cos(d*x + c) +
 1)^2 - 5*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 5*sin(d*x + c)^5/(cos(d*x +
 c) + 1)^5 - 3*I*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 - 3*sin(d*x + c)^7/(co
s(d*x + c) + 1)^7)/((a - 4*a*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 6*a*sin(
d*x + c)^4/(cos(d*x + c) + 1)^4 - 4*a*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 +
 a*sin(d*x + c)^8/(cos(d*x + c) + 1)^8)*d)
```

mupad [B] time = 1.29, size = 99, normalized size = 1.90

$$-\frac{2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5\,3\mathrm{i}-5\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4+5\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,3\mathrm{i}-3\right)}{3\,a\,d\,{\left({\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2-1\right)}^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^5*(a*cos(c + d*x) + a*sin(c + d*x)*1i)),x)
```

```
[Out] -(2*tan(c/2 + (d*x)/2)*(tan(c/2 + (d*x)/2)*3i + 5*tan(c/2 + (d*x)/2)^2 - 5*
tan(c/2 + (d*x)/2)^4 + tan(c/2 + (d*x)/2)^5*3i + 3*tan(c/2 + (d*x)/2)^6 - 3
))/(3*a*d*(tan(c/2 + (d*x)/2)^2 - 1)^4)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\sec^5(c+dx)}{i\sin(c+dx)+\cos(c+dx)}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**5/(a*cos(d*x+c)+I*a*sin(d*x+c)),x)
```

```
[Out] Integral(sec(c + d*x)**5/(I*sin(c + d*x) + cos(c + d*x)), x)/a
```

3.161 $\int \frac{\sec^6(c+dx)}{a\cos(c+dx)+ia\sin(c+dx)}\,dx$

Optimal. Leaf size=84

$$-\frac{i\sec^5(c+dx)}{5ad}+\frac{3\tanh^{-1}(\sin(c+dx))}{8ad}+\frac{\tan(c+dx)\sec^3(c+dx)}{4ad}+\frac{3\tan(c+dx)\sec(c+dx)}{8ad}$$

```
[Out] 3/8*arctanh(sin(d*x+c))/a/d-1/5*I*sec(d*x+c)^5/a/d+3/8*sec(d*x+c)*tan(d*x+c
)/a/d+1/4*sec(d*x+c)^3*tan(d*x+c)/a/d
```

Rubi [A] time = 0.13, antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.194, Rules used = {3092, 3090, 3768, 3770, 2606, 30}

$$-\frac{i\sec^5(c+dx)}{5ad}+\frac{3\tanh^{-1}(\sin(c+dx))}{8ad}+\frac{\tan(c+dx)\sec^3(c+dx)}{4ad}+\frac{3\tan(c+dx)\sec(c+dx)}{8ad}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^6/(a*Cos[c + d*x] + I*a*Sin[c + d*x]),x]
```

```
[Out] (3*ArcTanh[Sin[c + d*x]])/(8*a*d) - ((I/5)*Sec[c + d*x]^5)/(a*d) + (3*Sec[c
 + d*x]*Tan[c + d*x])/(8*a*d) + (Sec[c + d*x]^3*Tan[c + d*x])/(4*a*d)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 && !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 3090

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_.), x_Symbol] :> Int[ExpandTrig[cos[c + d*x]^m*(a
*cos[c + d*x] + b*sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d}, x] && Inte
gerQ[m] && IGtQ[n, 0]
```

Rule 3092

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[a^n*b^n, Int[Cos[c + d*x]^m/
(b*Cos[c + d*x] + a*Sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, m}, x] &&
 EqQ[a^2 + b^2, 0] && ILtQ[n, 0]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\sec^6(c+dx)}{a\cos(c+dx)+ia\sin(c+dx)}\,dx &= -\frac{i\int \sec^6(c+dx)(ia\cos(c+dx)+a\sin(c+dx))\,dx}{a^2}\\ &= -\frac{i\int \left(ia\sec^5(c+dx)+a\sec^5(c+dx)\tan(c+dx)\right)\,dx}{a^2}\\ &= -\frac{i\int \sec^5(c+dx)\tan(c+dx)\,dx}{a}+\frac{\int \sec^5(c+dx)\,dx}{a}\\ &= \frac{\sec^3(c+dx)\tan(c+dx)}{4ad}+\frac{3\int \sec^3(c+dx)\,dx}{4a}-\frac{i\,\mathrm{Subst}\left(\int x^4\,dx,x,\sec(c\right.}{ad}\\ &= -\frac{i\sec^5(c+dx)}{5ad}+\frac{3\sec(c+dx)\tan(c+dx)}{8ad}+\frac{\sec^3(c+dx)\tan(c+dx)}{4ad}+\\ &= \frac{3\tanh^{-1}(\sin(c+dx))}{8ad}-\frac{i\sec^5(c+dx)}{5ad}+\frac{3\sec(c+dx)\tan(c+dx)}{8ad}+\sec^3\end{aligned}$$

Mathematica [A] time = 0.46, size = 66, normalized size = 0.79

$$-\frac{i\left((70i\sin(2(c+dx))+15i\sin(4(c+dx))+64)\sec^5(c+dx)+240i\tanh^{-1}\left(\cos(c)\tan\left(\frac{dx}{2}\right)+\sin(c)\right)\right)}{320ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^6/(a*Cos[c + d*x] + I*a*Sin[c + d*x]),x]
```

```
[Out] ((-1/320*I)*((240*I)*ArcTanh[Sin[c] + Cos[c]*Tan[(d*x)/2]] + Sec[c + d*x]^5
*(64 + (70*I)*Sin[2*(c + d*x)] + (15*I)*Sin[4*(c + d*x)])))/(a*d)
```

fricas [B] time = 0.60, size = 266, normalized size = 3.17

$$\frac{15\left(e^{(10i\,dx+10i\,c)}+5\,e^{(8i\,dx+8i\,c)}+10\,e^{(6i\,dx+6i\,c)}+10\,e^{(4i\,dx+4i\,c)}+5\,e^{(2i\,dx+2i\,c)}+1\right)\log\left(e^{(i\,dx+i\,c)}+i\right)-15\left(e^{(10i\,dx+10i\,c)}\right.}{40\left(ade^{(10i\,dx+10i\,c)}+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^6/(a*cos(d*x+c)+I*a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/40*(15*(e^(10*I*d*x + 10*I*c) + 5*e^(8*I*d*x + 8*I*c) + 10*e^(6*I*d*x + 6
*I*c) + 10*e^(4*I*d*x + 4*I*c) + 5*e^(2*I*d*x + 2*I*c) + 1)*log(e^(I*d*x +
I*c) + I) - 15*(e^(10*I*d*x + 10*I*c) + 5*e^(8*I*d*x + 8*I*c) + 10*e^(6*I*d
*x + 6*I*c) + 10*e^(4*I*d*x + 4*I*c) + 5*e^(2*I*d*x + 2*I*c) + 1)*log(e^(I*
d*x + I*c) - I) - 30*I*e^(9*I*d*x + 9*I*c) - 140*I*e^(7*I*d*x + 7*I*c) - 25
6*I*e^(5*I*d*x + 5*I*c) + 140*I*e^(3*I*d*x + 3*I*c) + 30*I*e^(I*d*x + I*c))
/(a*d*e^(10*I*d*x + 10*I*c) + 5*a*d*e^(8*I*d*x + 8*I*c) + 10*a*d*e^(6*I*d*x
 + 6*I*c) + 10*a*d*e^(4*I*d*x + 4*I*c) + 5*a*d*e^(2*I*d*x + 2*I*c) + a*d)
```

giac [A] time = 0.23, size = 138, normalized size = 1.64

$$\frac{\frac{15\,\log\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)}{a}-\frac{15\,\log\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right)}{a}+\frac{2\left(25\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{9}+40i\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{8}-10\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{7}+80i\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{4}+\right.}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-1\right)^{5}a}}{40\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^6/(a*cos(d*x+c)+I*a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/40*(15*log(tan(1/2*d*x + 1/2*c) + 1)/a - 15*log(tan(1/2*d*x + 1/2*c) - 1)
/a + 2*(25*tan(1/2*d*x + 1/2*c)^9 + 40*I*tan(1/2*d*x + 1/2*c)^8 - 10*tan(1/
2*d*x + 1/2*c)^7 + 80*I*tan(1/2*d*x + 1/2*c)^4 + 10*tan(1/2*d*x + 1/2*c)^3
- 25*tan(1/2*d*x + 1/2*c) + 8*I)/((tan(1/2*d*x + 1/2*c)^2 - 1)^5*a))/d
```

maple [B] time = 0.24, size = 430, normalized size = 5.12

$$\frac{5i}{8ad\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{2}}+\frac{7}{8ad\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{2}}-\frac{3i}{4ad\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{3}}+\frac{5}{8ad\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}+\frac{}{5ad\left(\tan\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^6/(a*cos(d*x+c)+I*a*sin(d*x+c)),x)
```

```
[Out] 5/8*I/a/d/(tan(1/2*d*x+1/2*c)-1)^2+7/8/a/d/(tan(1/2*d*x+1/2*c)-1)^2-3/4*I/a
/d/(tan(1/2*d*x+1/2*c)+1)^3+5/8/a/d/(tan(1/2*d*x+1/2*c)-1)+1/5*I/a/d/(tan(1
/2*d*x+1/2*c)-1)^5+1/2/a/d/(tan(1/2*d*x+1/2*c)-1)^3-1/5*I/a/d/(tan(1/2*d*x+
1/2*c)+1)^5+1/4/a/d/(tan(1/2*d*x+1/2*c)-1)^4+3/8*I/a/d/(tan(1/2*d*x+1/2*c)-
1)-3/8/a/d*ln(tan(1/2*d*x+1/2*c)-1)+1/2*I/a/d/(tan(1/2*d*x+1/2*c)+1)^4+5/8/
a/d/(tan(1/2*d*x+1/2*c)+1)+3/4*I/a/d/(tan(1/2*d*x+1/2*c)-1)^3+1/2/a/d/(tan(
1/2*d*x+1/2*c)+1)^3+1/2*I/a/d/(tan(1/2*d*x+1/2*c)-1)^4-1/4/a/d/(tan(1/2*d*x
+1/2*c)+1)^4-3/8*I/a/d/(tan(1/2*d*x+1/2*c)+1)-7/8/a/d/(tan(1/2*d*x+1/2*c)+1
)^2+5/8*I/a/d/(tan(1/2*d*x+1/2*c)+1)^2+3/8/a/d*ln(tan(1/2*d*x+1/2*c)+1)
```

maxima [B] time = 0.35, size = 289, normalized size = 3.44

$$\frac{\frac{16\left(-\frac{75i\sin(dx+c)}{\cos(dx+c)+1}+\frac{30i\sin(dx+c)^3}{(\cos(dx+c)+1)^3}-\frac{240\sin(dx+c)^4}{(\cos(dx+c)+1)^4}-\frac{30i\sin(dx+c)^7}{(\cos(dx+c)+1)^7}-\frac{120\sin(dx+c)^8}{(\cos(dx+c)+1)^8}+\frac{75i\sin(dx+c)^9}{(\cos(dx+c)+1)^9}-24\right)}{-120ia+\frac{600ia\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-\frac{1200ia\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{1200ia\sin(dx+c)^6}{(\cos(dx+c)+1)^6}-\frac{600ia\sin(dx+c)^8}{(\cos(dx+c)+1)^8}+\frac{120ia\sin(dx+c)^{10}}{(\cos(dx+c)+1)^{10}}}+\frac{3\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)}{a}-\frac{3\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}-1\right)}{a}}{8d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^6/(a*cos(d*x+c)+I*a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/8*(16*(-75*I*sin(d*x + c)/(cos(d*x + c) + 1) + 30*I*sin(d*x + c)^3/(cos(d
*x + c) + 1)^3 - 240*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 - 30*I*sin(d*x + c
)^7/(cos(d*x + c) + 1)^7 - 120*sin(d*x + c)^8/(cos(d*x + c) + 1)^8 + 75*I*s
in(d*x + c)^9/(cos(d*x + c) + 1)^9 - 24)/(-120*I*a + 600*I*a*sin(d*x + c)^2
/(cos(d*x + c) + 1)^2 - 1200*I*a*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + 1200
*I*a*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 - 600*I*a*sin(d*x + c)^8/(cos(d*x
+ c) + 1)^8 + 120*I*a*sin(d*x + c)^10/(cos(d*x + c) + 1)^10) + 3*log(sin(d*
x + c)/(cos(d*x + c) + 1) + 1)/a - 3*log(sin(d*x + c)/(cos(d*x + c) + 1) -
1)/a)/d
```

mupad [B] time = 4.25, size = 193, normalized size = 2.30

$$\frac{3\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{4\,a\,d}+\frac{\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{2\,a}-\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7}{2\,a}+\frac{5\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^9}{4\,a}-\frac{5\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{4\,a}+\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4\,4\mathrm{i}}{a}+\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8\,2\mathrm{i}}{a}}{d\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{10}-5\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8+10\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6-10\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4+5\tan\left(\frac{c}{2}+\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^6*(a*cos(c + d*x) + a*sin(c + d*x)*1i)),x)
```

```
[Out] (3*atanh(tan(c/2 + (d*x)/2)))/(4*a*d) + (tan(c/2 + (d*x)/2)^3/(2*a) + (tan(
c/2 + (d*x)/2)^4*4i)/a - tan(c/2 + (d*x)/2)^7/(2*a) + (tan(c/2 + (d*x)/2)^8
```

```
*2i)/a + (5*tan(c/2 + (d*x)/2)^9)/(4*a) + 2i/(5*a) - (5*tan(c/2 + (d*x)/2))
/(4*a))/(d*(5*tan(c/2 + (d*x)/2)^2 - 10*tan(c/2 + (d*x)/2)^4 + 10*tan(c/2 +
 (d*x)/2)^6 - 5*tan(c/2 + (d*x)/2)^8 + tan(c/2 + (d*x)/2)^10 - 1))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\sec^6(c+dx)}{i\sin(c+dx)+\cos(c+dx)}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**6/(a*cos(d*x+c)+I*a*sin(d*x+c)),x)
```

```
[Out] Integral(sec(c + d*x)**6/(I*sin(c + d*x) + cos(c + d*x)), x)/a
```

3.162 $\int \frac{\sec^7(c+dx)}{a\cos(c+dx)+ia\sin(c+dx)}\,dx$

Optimal. Leaf size=70

$$\frac{\tan^5(c+dx)}{5ad}+\frac{2\tan^3(c+dx)}{3ad}+\frac{\tan(c+dx)}{ad}-\frac{i\sec^6(c+dx)}{6ad}$$

```
[Out] -1/6*I*sec(d*x+c)^6/a/d+tan(d*x+c)/a/d+2/3*tan(d*x+c)^3/a/d+1/5*tan(d*x+c)^
5/a/d
```

Rubi [A] time = 0.12, antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.161, Rules used = {3092, 3090, 3767, 2606, 30}

$$\frac{\tan^5(c+dx)}{5ad}+\frac{2\tan^3(c+dx)}{3ad}+\frac{\tan(c+dx)}{ad}-\frac{i\sec^6(c+dx)}{6ad}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^7/(a*Cos[c + d*x] + I*a*Sin[c + d*x]),x]
```

```
[Out] ((-I/6)*Sec[c + d*x]^6)/(a*d) + Tan[c + d*x]/(a*d) + (2*Tan[c + d*x]^3)/(3*
a*d) + Tan[c + d*x]^5/(5*a*d)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 3090

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_.), x_Symbol] :> Int[ExpandTrig[cos[c + d*x]^m*(a
*cos[c + d*x] + b*sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d}, x] && Inte
gerQ[m] && IGtQ[n, 0]
```

Rule 3092

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[a^n*b^n, Int[Cos[c + d*x]^m/
(b*Cos[c + d*x] + a*Sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, m}, x] &&
 EqQ[a^2 + b^2, 0] && ILtQ[n, 0]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\sec^7(c+dx)}{a\cos(c+dx)+ia\sin(c+dx)}\,dx &= -\frac{i\int \sec^7(c+dx)(ia\cos(c+dx)+a\sin(c+dx))\,dx}{a^2}\\ &= -\frac{i\int \left(ia\sec^6(c+dx)+a\sec^6(c+dx)\tan(c+dx)\right)\,dx}{a^2}\\ &= -\frac{i\int \sec^6(c+dx)\tan(c+dx)\,dx}{a}+\frac{\int \sec^6(c+dx)\,dx}{a}\\ &= -\frac{i\,\text{Subst}\left(\int x^5\,dx,x,\sec(c+dx)\right)}{ad}-\frac{\text{Subst}\left(\int \left(1+2x^2+x^4\right)\,dx,x,-\tan\right.}{ad}\\ &= -\frac{i\sec^6(c+dx)}{6ad}+\frac{\tan(c+dx)}{ad}+\frac{2\tan^3(c+dx)}{3ad}+\frac{\tan^5(c+dx)}{5ad}\end{aligned}$$

Mathematica [A] time = 0.37, size = 67, normalized size = 0.96

$$-\frac{i\sec(c)\sec^6(c+dx)(10\cos(c)-i(-15\sin(c+2dx)-6\sin(3c+4dx)-\sin(5c+6dx)+10\sin(c)))}{60ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^7/(a*Cos[c + d*x] + I*a*Sin[c + d*x]),x]
```

```
[Out] ((-1/60*I)*Sec[c]*Sec[c + d*x]^6*(10*Cos[c] - I*(10*Sin[c] - 15*Sin[c + 2*d
*x] - 6*Sin[3*c + 4*d*x] - Sin[5*c + 6*d*x])))/(a*d)
```

fricas [A] time = 0.47, size = 109, normalized size = 1.56

$$\frac{240i\,e^{(4i\,dx+4i\,c)}+96i\,e^{(2i\,dx+2i\,c)}+16i}{15\left(ade^{(12i\,dx+12i\,c)}+6\,ade^{(10i\,dx+10i\,c)}+15\,ade^{(8i\,dx+8i\,c)}+20\,ade^{(6i\,dx+6i\,c)}+15\,ade^{(4i\,dx+4i\,c)}+6\,ade^{(2i\,dx+2i\,c)}+ad\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^7/(a*cos(d*x+c)+I*a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/15*(240*I*e^(4*I*d*x + 4*I*c) + 96*I*e^(2*I*d*x + 2*I*c) + 16*I)/(a*d*e^(
12*I*d*x + 12*I*c) + 6*a*d*e^(10*I*d*x + 10*I*c) + 15*a*d*e^(8*I*d*x + 8*I*
c) + 20*a*d*e^(6*I*d*x + 6*I*c) + 15*a*d*e^(4*I*d*x + 4*I*c) + 6*a*d*e^(2*I
*d*x + 2*I*c) + a*d)
```

giac [A] time = 0.22, size = 67, normalized size = 0.96

$$-\frac{5i\,\tan\left(dx+c\right)^{6}-6\,\tan\left(dx+c\right)^{5}+15i\,\tan\left(dx+c\right)^{4}-20\,\tan\left(dx+c\right)^{3}+15i\,\tan\left(dx+c\right)^{2}-30\,\tan\left(dx+c\right)}{30\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^7/(a*cos(d*x+c)+I*a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/30*(5*I*tan(d*x + c)^6 - 6*tan(d*x + c)^5 + 15*I*tan(d*x + c)^4 - 20*tan
(d*x + c)^3 + 15*I*tan(d*x + c)^2 - 30*tan(d*x + c))/(a*d)
```

maple [A] time = 0.23, size = 68, normalized size = 0.97

$$\frac{\tan\left(dx+c\right)-\frac{i\left(\tan^{6}\left(dx+c\right)\right)}{6}+\frac{\left(\tan^{5}\left(dx+c\right)\right)}{5}-\frac{i\left(\tan^{4}\left(dx+c\right)\right)}{2}+\frac{2\left(\tan^{3}\left(dx+c\right)\right)}{3}-\frac{i\left(\tan^{2}\left(dx+c\right)\right)}{2}}{da}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^7/(a*cos(d*x+c)+I*a*sin(d*x+c)),x)
```

```
[Out] 1/d/a*(tan(d*x+c)-1/6*I*tan(d*x+c)^6+1/5*tan(d*x+c)^5-1/2*I*tan(d*x+c)^4+2/
3*tan(d*x+c)^3-1/2*I*tan(d*x+c)^2)
```

maxima [B] time = 0.36, size = 313, normalized size = 4.47

$$\frac{2\left(\frac{15\,\sin(dx+c)}{\cos(dx+c)+1}-\frac{15i\,\sin(dx+c)^{2}}{(\cos(dx+c)+1)^{2}}-\frac{35\,\sin(dx+c)^{3}}{(\cos(dx+c)+1)^{3}}+\frac{78\,\sin(dx+c)^{5}}{(\cos(dx+c)+1)^{5}}-\frac{50i\,\sin(dx+c)^{6}}{(\cos(dx+c)+1)^{6}}-\frac{78\,\sin(dx+c)^{7}}{(\cos(dx+c)+1)^{7}}+\frac{35\,\sin(dx+c)^{9}}{(\cos(dx+c)+1)^{9}}-\frac{15i\,\sin(d}{(\cos(dx+c}\right.}{15\left(a-\frac{6\,a\sin(dx+c)^{2}}{(\cos(dx+c)+1)^{2}}+\frac{15\,a\sin(dx+c)^{4}}{(\cos(dx+c)+1)^{4}}-\frac{20\,a\sin(dx+c)^{6}}{(\cos(dx+c)+1)^{6}}+\frac{15\,a\sin(dx+c)^{8}}{(\cos(dx+c)+1)^{8}}-\frac{6\,a\sin(dx+c)^{10}}{(\cos(dx+c)+1)^{10}}+\frac{a\sin(dx+c)^{12}}{(\cos(dx+c)+1)}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^7/(a*cos(d*x+c)+I*a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 2/15*(15*sin(d*x + c)/(cos(d*x + c) + 1) - 15*I*sin(d*x + c)^2/(cos(d*x + c
) + 1)^2 - 35*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 78*sin(d*x + c)^5/(cos(
d*x + c) + 1)^5 - 50*I*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 - 78*sin(d*x + c
```

```
)^7/(cos(d*x + c) + 1)^7 + 35*sin(d*x + c)^9/(cos(d*x + c) + 1)^9 - 15*I*si
n(d*x + c)^10/(cos(d*x + c) + 1)^10 - 15*sin(d*x + c)^11/(cos(d*x + c) + 1)
^11)/((a - 6*a*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 15*a*sin(d*x + c)^4/(c
os(d*x + c) + 1)^4 - 20*a*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 + 15*a*sin(d*
x + c)^8/(cos(d*x + c) + 1)^8 - 6*a*sin(d*x + c)^10/(cos(d*x + c) + 1)^10 +
 a*sin(d*x + c)^12/(cos(d*x + c) + 1)^12)*d)
```

mupad [B] time = 1.89, size = 139, normalized size = 1.99

$$-\frac{2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\left(15\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{10}+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{9}15\mathrm{i}-35\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{8}+78\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{6}+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{5}\right.}{15\,a\,d\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{2}-1\right)^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^7*(a*cos(c + d*x) + a*sin(c + d*x)*1i)),x)
```

```
[Out] -(2*tan(c/2 + (d*x)/2)*(tan(c/2 + (d*x)/2)*15i + 35*tan(c/2 + (d*x)/2)^2 -
78*tan(c/2 + (d*x)/2)^4 + tan(c/2 + (d*x)/2)^5*50i + 78*tan(c/2 + (d*x)/2)^
6 - 35*tan(c/2 + (d*x)/2)^8 + tan(c/2 + (d*x)/2)^9*15i + 15*tan(c/2 + (d*x)
/2)^10 - 15))/(15*a*d*(tan(c/2 + (d*x)/2)^2 - 1)^6)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**7/(a*cos(d*x+c)+I*a*sin(d*x+c)),x)
```

```
[Out] Timed out
```

3.163 $\int \frac{\cos^5(c+dx)}{(a\cos(c+dx)+ia\sin(c+dx))^2}\,dx$

Optimal. Leaf size=85

$$-\frac{2\sin^7(c+dx)}{7a^2d}+\frac{\sin^5(c+dx)}{a^2d}-\frac{4\sin^3(c+dx)}{3a^2d}+\frac{\sin(c+dx)}{a^2d}+\frac{2i\cos^7(c+dx)}{7a^2d}$$

```
[Out] 2/7*I*cos(d*x+c)^7/a^2/d+sin(d*x+c)/a^2/d-4/3*sin(d*x+c)^3/a^2/d+sin(d*x+c)
^5/a^2/d-2/7*sin(d*x+c)^7/a^2/d
```

Rubi [A] time = 0.19, antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.226, Rules used = {3092, 3090, 2633, 2565, 30, 2564, 270}

$$-\frac{2\sin^7(c+dx)}{7a^2d}+\frac{\sin^5(c+dx)}{a^2d}-\frac{4\sin^3(c+dx)}{3a^2d}+\frac{\sin(c+dx)}{a^2d}+\frac{2i\cos^7(c+dx)}{7a^2d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^5/(a*Cos[c + d*x] + I*a*Sin[c + d*x])^2,x]
```

```
[Out] (((2*I)/7)*Cos[c + d*x]^7)/(a^2*d) + Sin[c + d*x]/(a^2*d) - (4*Sin[c + d*x]
^3)/(3*a^2*d) + Sin[c + d*x]^5/(a^2*d) - (2*Sin[c + d*x]^7)/(7*a^2*d)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2564

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 2565

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> -Dist[(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
```

```
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 3090

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_.), x_Symbol] :> Int[ExpandTrig[cos[c + d*x]^m*(a
*cos[c + d*x] + b*sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d}, x] && Inte
gerQ[m] && IGtQ[n, 0]
```

Rule 3092

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[a^n*b^n, Int[Cos[c + d*x]^m/
(b*Cos[c + d*x] + a*Sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, m}, x] &&
 EqQ[a^2 + b^2, 0] && ILtQ[n, 0]
```

Rubi steps

$$\int \frac{\cos^5(c+dx)}{(a\cos(c+dx)+ia\sin(c+dx))^2}\,dx = -\frac{\int \cos^5(c+dx)(ia\cos(c+dx)+a\sin(c+dx))^2\,dx}{a^4}$$

$$= -\frac{\int \left(-a^2\cos^7(c+dx)+2ia^2\cos^6(c+dx)\sin(c+dx)+a^2\cos^5(c+dx\right.}{a^4}$$

$$= -\frac{(2i)\int \cos^6(c+dx)\sin(c+dx)\,dx}{a^2}+\frac{\int \cos^7(c+dx)\,dx}{a^2}-\frac{\int \cos^5(c+}{}$$

$$= \frac{(2i)\,\text{Subst}\left(\int x^6\,dx,x,\cos(c+dx)\right)}{a^2d}-\frac{\text{Subst}\left(\int x^2\left(1-x^2\right)^2\,dx,x,\sin(\right.}{a^2d}$$

$$= \frac{2i\cos^7(c+dx)}{7a^2d}+\frac{\sin(c+dx)}{a^2d}-\frac{\sin^3(c+dx)}{a^2d}+\frac{3\sin^5(c+dx)}{5a^2d}-\frac{\sin^7(}{7}$$

$$= \frac{2i\cos^7(c+dx)}{7a^2d}+\frac{\sin(c+dx)}{a^2d}-\frac{4\sin^3(c+dx)}{3a^2d}+\frac{\sin^5(c+dx)}{a^2d}-\frac{2\sin}{}$$

Mathematica [A] time = 0.11, size = 149, normalized size = 1.75

$$\frac{15\sin(c+dx)}{32a^2d}+\frac{11\sin(3(c+dx))}{96a^2d}+\frac{\sin(5(c+dx))}{32a^2d}+\frac{\sin(7(c+dx))}{224a^2d}+\frac{5i\cos(c+dx)}{32a^2d}+\frac{3i\cos(3(c+dx))}{32a^2d}+\frac{i\cos(5(c+}{32a^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^5/(a*Cos[c + d*x] + I*a*Sin[c + d*x])^2,x]
```

```
[Out] (((5*I)/32)*Cos[c + d*x])/(a^2*d) + (((3*I)/32)*Cos[3*(c + d*x)])/(a^2*d) +
 ((I/32)*Cos[5*(c + d*x)])/(a^2*d) + ((I/224)*Cos[7*(c + d*x)])/(a^2*d) + (
15*Sin[c + d*x])/(32*a^2*d) + (11*Sin[3*(c + d*x)])/(96*a^2*d) + Sin[5*(c +
 d*x)]/(32*a^2*d) + Sin[7*(c + d*x)]/(224*a^2*d)
```

fricas [A] time = 0.58, size = 74, normalized size = 0.87

$$\frac{\left(-7i\,e^{(10i\,dx+10i\,c)}-105i\,e^{(8i\,dx+8i\,c)}+210i\,e^{(6i\,dx+6i\,c)}+70i\,e^{(4i\,dx+4i\,c)}+21i\,e^{(2i\,dx+2i\,c)}+3i\right)e^{(-7i\,dx-7i\,c)}}{672\,a^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5/(a*cos(d*x+c)+I*a*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] 1/672*(-7*I*e^(10*I*d*x + 10*I*c) - 105*I*e^(8*I*d*x + 8*I*c) + 210*I*e^(6*
I*d*x + 6*I*c) + 70*I*e^(4*I*d*x + 4*I*c) + 21*I*e^(2*I*d*x + 2*I*c) + 3*I)
*e^(-7*I*d*x - 7*I*c)/(a^2*d)
```

giac [A] time = 1.20, size = 145, normalized size = 1.71

$$\frac{\frac{7\left(9\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+15i\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-8\right)}{a^2\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+i\right)^3}+\frac{273\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^6-1155i\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5-2450\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^4+2870i\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3+2037\tan}{a^2\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-i\right)^7}}{168\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5/(a*cos(d*x+c)+I*a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/168*(7*(9*tan(1/2*d*x + 1/2*c)^2 + 15*I*tan(1/2*d*x + 1/2*c) - 8)/(a^2*(t
an(1/2*d*x + 1/2*c) + I)^3) + (273*tan(1/2*d*x + 1/2*c)^6 - 1155*I*tan(1/2*
d*x + 1/2*c)^5 - 2450*tan(1/2*d*x + 1/2*c)^4 + 2870*I*tan(1/2*d*x + 1/2*c)^
3 + 2037*tan(1/2*d*x + 1/2*c)^2 - 791*I*tan(1/2*d*x + 1/2*c) - 152)/(a^2*(t
an(1/2*d*x + 1/2*c) - I)^7))/d
```

maple [B] time = 0.19, size = 174, normalized size = 2.05

$$\frac{-\frac{i}{8\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+i\right)^2}-\frac{1}{12\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+i\right)^3}+\frac{3}{8\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+i\right)}+\frac{2i}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-i\right)^6}-\frac{5i}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-i\right)^4}+\frac{23i}{8\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-i\right)^2}-\frac{4}{7\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-i\right.}}{d\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5/(a*cos(d*x+c)+I*a*sin(d*x+c))^2,x)
```

```
[Out] 2/d/a^2*(-1/16*I/(tan(1/2*d*x+1/2*c)+I)^2-1/24/(tan(1/2*d*x+1/2*c)+I)^3+3/1
6/(tan(1/2*d*x+1/2*c)+I)+I/(tan(1/2*d*x+1/2*c)-I)^6-5/2*I/(tan(1/2*d*x+1/2*
c)-I)^4+23/16*I/(tan(1/2*d*x+1/2*c)-I)^2-2/7/(tan(1/2*d*x+1/2*c)-I)^7+2/(ta
n(1/2*d*x+1/2*c)-I)^5-55/24/(tan(1/2*d*x+1/2*c)-I)^3+13/16/(tan(1/2*d*x+1/2
*c)-I))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5/(a*cos(d*x+c)+I*a*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

mupad [B] time = 4.11, size = 161, normalized size = 1.89

$$\frac{\left(-21\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^9+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8 42\mathrm{i}+28\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6 56\mathrm{i}+42\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5+\tan\left(\frac{c}{2}+\right.\right.}{21\,a^2\,d\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+1\mathrm{i}\right)^3\left(1+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^5/(a*cos(c + d*x) + a*sin(c + d*x)*1i)^2,x)
```

```
[Out] ((3*tan(c/2 + (d*x)/2) - tan(c/2 + (d*x)/2)^2*24i + 76*tan(c/2 + (d*x)/2)^3
 + tan(c/2 + (d*x)/2)^4*28i + 42*tan(c/2 + (d*x)/2)^5 + tan(c/2 + (d*x)/2)^
6*56i + 28*tan(c/2 + (d*x)/2)^7 + tan(c/2 + (d*x)/2)^8*42i - 21*tan(c/2 + (
d*x)/2)^9 - 6i)*2i)/(21*a^2*d*(tan(c/2 + (d*x)/2) + 1i)^3*(tan(c/2 + (d*x)/
2)*1i + 1)^7)
```

sympy [A] time = 0.59, size = 233, normalized size = 2.74

$$\begin{cases} \frac{\left(-176160768ia^{10}d^5e^{19ic}e^{3idx}-2642411520ia^{10}d^5e^{17ic}e^{idx}+5284823040ia^{10}d^5e^{15ic}e^{-idx}+1761607680ia^{10}d^5e^{13ic}e^{-3idx}+528482304ia^{10}d^5e^{11ic}e^{-5idx}+\right.}{16911433728a^{12}d^6} \\ \frac{x\left(e^{10ic}+5e^{8ic}+10e^{6ic}+10e^{4ic}+5e^{2ic}+1\right)e^{-7ic}}{32a^2} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5/(a*cos(d*x+c)+I*a*sin(d*x+c))**2,x)
```

```
[Out] Piecewise(((-176160768*I*a**10*d**5*exp(19*I*c)*exp(3*I*d*x) - 2642411520*I
*a**10*d**5*exp(17*I*c)*exp(I*d*x) + 5284823040*I*a**10*d**5*exp(15*I*c)*ex
p(-I*d*x) + 1761607680*I*a**10*d**5*exp(13*I*c)*exp(-3*I*d*x) + 528482304*I
*a**10*d**5*exp(11*I*c)*exp(-5*I*d*x) + 75497472*I*a**10*d**5*exp(9*I*c)*ex
p(-7*I*d*x))*exp(-16*I*c)/(16911433728*a**12*d**6), Ne(16911433728*a**12*d*
*6*exp(16*I*c), 0)), (x*(exp(10*I*c) + 5*exp(8*I*c) + 10*exp(6*I*c) + 10*ex
p(4*I*c) + 5*exp(2*I*c) + 1)*exp(-7*I*c)/(32*a**2), True))
```

3.164 $\int \frac{\cos^4(c+dx)}{(a\cos(c+dx)+ia\sin(c+dx))^2}\,dx$

Optimal. Leaf size=101

$$-\frac{1}{16a^2d(-\cot(c+dx)+i)}+\frac{11}{16a^2d(\cot(c+dx)+i)}-\frac{3i}{8a^2d(\cot(c+dx)+i)^2}-\frac{1}{12a^2d(\cot(c+dx)+i)^3}+\frac{x}{4a^2}$$

```
[Out] 1/4*x/a^2-1/16/a^2/d/(I-cot(d*x+c))-1/12/a^2/d/(I+cot(d*x+c))^3-3/8*I/a^2/d
/(I+cot(d*x+c))^2+11/16/a^2/d/(I+cot(d*x+c))
```

Rubi [A] time = 0.10, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.129, Rules used = {3088, 848, 88, 203}

$$-\frac{1}{16a^2d(-\cot(c+dx)+i)}+\frac{11}{16a^2d(\cot(c+dx)+i)}-\frac{3i}{8a^2d(\cot(c+dx)+i)^2}-\frac{1}{12a^2d(\cot(c+dx)+i)^3}+\frac{x}{4a^2}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^4/(a*Cos[c + d*x] + I*a*Sin[c + d*x])^2,x]
```

```
[Out] x/(4*a^2) - 1/(16*a^2*d*(I - Cot[c + d*x])) - 1/(12*a^2*d*(I + Cot[c + d*x]
)^3) - ((3*I)/8)/(a^2*d*(I + Cot[c + d*x])^2) + 11/(16*a^2*d*(I + Cot[c + d
*x]))
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 848

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_.), x_Symbol] :> Int[(d + e*x)^(m + p)*(f + g*x)^n*(a/d + (c*x)/e)^p,
x] /; FreeQ[{a, c, d, e, f, g, m, n}, x] && NeQ[e*f - d*g, 0] && EqQ[c*d^2
+ a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && EqQ[m + p, 0]))
```

Rule 3088

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[(x^m*(b +
 a*x)^n)/(1 + x^2)^((m + n + 2)/2), x], x, Cot[c + d*x]], x] /; FreeQ[{a, b
, c, d}, x] && IntegerQ[n] && IntegerQ[(m + n)/2] && NeQ[n, -1] &&  !(GtQ[n
, 0] && GtQ[m, 1])
```

Rubi steps

$$\int \frac{\cos^4(c+dx)}{(a\cos(c+dx)+ia\sin(c+dx))^2}\,dx = -\frac{\text{Subst}\left(\int \frac{x^4}{(ia+ax)^2\left(1+x^2\right)^2}\,dx, x, \cot(c+dx)\right)}{d}$$

$$= -\frac{\text{Subst}\left(\int \frac{x^4}{\left(-\frac{i}{a}+\frac{x}{a}\right)^2(ia+ax)^4}\,dx, x, \cot(c+dx)\right)}{d}$$

$$= -\frac{\text{Subst}\left(\int \left(\frac{1}{16a^2(-i+x)^2}-\frac{1}{4a^2(i+x)^4}-\frac{3i}{4a^2(i+x)^3}+\frac{11}{16a^2(i+x)^2}+\frac{1}{4a^2\left(1+x^2\right)}\right) dx, \ldots}{d}$$

$$= -\frac{1}{16a^2d(i-\cot(c+dx))}-\frac{1}{12a^2d(i+\cot(c+dx))^3}-\frac{3i}{8a^2d(i+\cot(c+d\ldots}$$

$$= \frac{x}{4a^2}-\frac{1}{16a^2d(i-\cot(c+dx))}-\frac{1}{12a^2d(i+\cot(c+dx))^3}-\frac{3i}{8a^2d(i+\cot(\ldots}$$

Mathematica [A] time = 0.12, size = 82, normalized size = 0.81

$$\frac{21\sin(2(c+dx))+6\sin(4(c+dx))+\sin(6(c+dx))+15i\cos(2(c+dx))+6i\cos(4(c+dx))+i\cos(6(c+dx))+\ldots}{96a^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^4/(a*Cos[c + d*x] + I*a*Sin[c + d*x])^2,x]
```

```
[Out] (24*c + 24*d*x + (15*I)*Cos[2*(c + d*x)] + (6*I)*Cos[4*(c + d*x)] + I*Cos[6
*(c + d*x)] + 21*Sin[2*(c + d*x)] + 6*Sin[4*(c + d*x)] + Sin[6*(c + d*x)])/
(96*a^2*d)
```

fricas [A] time = 0.58, size = 65, normalized size = 0.64

$$\frac{\left(24\,dxe^{(6i\,dx+6i\,c)}-3i\,e^{(8i\,dx+8i\,c)}+18i\,e^{(4i\,dx+4i\,c)}+6i\,e^{(2i\,dx+2i\,c)}+i\right)e^{(-6i\,dx-6i\,c)}}{96\,a^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4/(a*cos(d*x+c)+I*a*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] 1/96*(24*d*x*e^(6*I*d*x + 6*I*c) - 3*I*e^(8*I*d*x + 8*I*c) + 18*I*e^(4*I*d*
x + 4*I*c) + 6*I*e^(2*I*d*x + 2*I*c) + I)*e^(-6*I*d*x - 6*I*c)/(a^2*d)
```

giac [A] time = 0.25, size = 103, normalized size = 1.02

$$-\frac{-\frac{6i\log(\tan(dx+c)+i)}{a^2}+\frac{6i\log(\tan(dx+c)-i)}{a^2}+\frac{3(2i\tan(dx+c)-3)}{a^2(\tan(dx+c)+i)}+\frac{-11i\tan(dx+c)^3-42\tan(dx+c)^2+57i\tan(dx+c)+30}{a^2(\tan(dx+c)-i)^3}}{48d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4/(a*cos(d*x+c)+I*a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/48*(-6*I*log(tan(d*x + c) + I)/a^2 + 6*I*log(tan(d*x + c) - I)/a^2 + 3*(
2*I*tan(d*x + c) - 3)/(a^2*(tan(d*x + c) + I)) + (-11*I*tan(d*x + c)^3 - 42
*tan(d*x + c)^2 + 57*I*tan(d*x + c) + 30)/(a^2*(tan(d*x + c) - I)^3))/d
```

maple [A] time = 0.20, size = 117, normalized size = 1.16

$$\frac{i\ln(\tan(dx+c)+i)}{8a^2d}+\frac{1}{16a^2d(\tan(dx+c)+i)}-\frac{i\ln(\tan(dx+c)-i)}{8a^2d}-\frac{i}{8a^2d(\tan(dx+c)-i)^2}-\frac{1}{12a^2d(\tan(dx+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4/(a*cos(d*x+c)+I*a*sin(d*x+c))^2,x)
```

```
[Out] 1/8*I/a^2/d*ln(tan(d*x+c)+I)+1/16/a^2/d/(tan(d*x+c)+I)-1/8*I/a^2/d*ln(tan(d
*x+c)-I)-1/8*I/a^2/d/(tan(d*x+c)-I)^2-1/12/a^2/d/(tan(d*x+c)-I)^3+3/16/a^2/
d/(tan(d*x+c)-I)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4/(a*cos(d*x+c)+I*a*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

mupad [B] time = 4.74, size = 138, normalized size = 1.37

$$\frac{x}{4\,a^2}-\frac{-\frac{3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7}{2}+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6 2\mathrm{i}+\frac{7\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5}{6}+\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4 4\mathrm{i}}{3}-\frac{7\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{6}+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2 2\mathrm{i}+\frac{3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{2}}{a^2\,d\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+1\mathrm{i}\right)^2\left(1+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)1\mathrm{i}\right)^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^4/(a*cos(c + d*x) + a*sin(c + d*x)*1i)^2,x)
```

```
[Out] x/(4*a^2) - ((3*tan(c/2 + (d*x)/2))/2 + tan(c/2 + (d*x)/2)^2*2i - (7*tan(c/
2 + (d*x)/2)^3)/6 + (tan(c/2 + (d*x)/2)^4*4i)/3 + (7*tan(c/2 + (d*x)/2)^5)/
6 + tan(c/2 + (d*x)/2)^6*2i - (3*tan(c/2 + (d*x)/2)^7)/2)/(a^2*d*(tan(c/2 +
 (d*x)/2) + 1i)^2*(tan(c/2 + (d*x)/2)*1i + 1)^6)
```

sympy [A] time = 0.37, size = 190, normalized size = 1.88

$$\begin{cases}\frac{\left(-24576ia^6d^3e^{14ic}e^{2idx}+147456ia^6d^3e^{10ic}e^{-2idx}+49152ia^6d^3e^{8ic}e^{-4idx}+8192ia^6d^3e^{6ic}e^{-6idx}\right)e^{-12ic}}{786432a^8d^4} & \text{for } 786432a^8d^4e^{12ic}\neq 0\\ x\left(\frac{\left(e^{8ic}+4e^{6ic}+6e^{4ic}+4e^{2ic}+1\right)e^{-6ic}}{16a^2}-\frac{1}{4a^2}\right) & \text{otherwise}\end{cases}+\frac{x}{4a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4/(a*cos(d*x+c)+I*a*sin(d*x+c))**2,x)
```

```
[Out] Piecewise(((-24576*I*a**6*d**3*exp(14*I*c)*exp(2*I*d*x) + 147456*I*a**6*d**
3*exp(10*I*c)*exp(-2*I*d*x) + 49152*I*a**6*d**3*exp(8*I*c)*exp(-4*I*d*x) +
8192*I*a**6*d**3*exp(6*I*c)*exp(-6*I*d*x))*exp(-12*I*c)/(786432*a**8*d**4),
 Ne(786432*a**8*d**4*exp(12*I*c), 0)), (x*((exp(8*I*c) + 4*exp(6*I*c) + 6*e
xp(4*I*c) + 4*exp(2*I*c) + 1)*exp(-6*I*c)/(16*a**2) - 1/(4*a**2)), True)) +
 x/(4*a**2)
```

3.165 $\int \frac{\cos^3(c+dx)}{(a\cos(c+dx)+ia\sin(c+dx))^2}\,dx$

Optimal. Leaf size=68

$$\frac{2\sin^5(c+dx)}{5a^2d}-\frac{\sin^3(c+dx)}{a^2d}+\frac{\sin(c+dx)}{a^2d}+\frac{2i\cos^5(c+dx)}{5a^2d}$$

```
[Out] 2/5*I*cos(d*x+c)^5/a^2/d+sin(d*x+c)/a^2/d-sin(d*x+c)^3/a^2/d+2/5*sin(d*x+c)
^5/a^2/d
```

Rubi [A] time = 0.18, antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.226, Rules used = {3092, 3090, 2633, 2565, 30, 2564, 14}

$$\frac{2\sin^5(c+dx)}{5a^2d}-\frac{\sin^3(c+dx)}{a^2d}+\frac{\sin(c+dx)}{a^2d}+\frac{2i\cos^5(c+dx)}{5a^2d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^3/(a*Cos[c + d*x] + I*a*Sin[c + d*x])^2,x]
```

```
[Out] (((2*I)/5)*Cos[c + d*x]^5)/(a^2*d) + Sin[c + d*x]/(a^2*d) - Sin[c + d*x]^3/
(a^2*d) + (2*Sin[c + d*x]^5)/(5*a^2*d)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2564

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 2565

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> -Dist[(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
```

```
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 3090

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_.), x_Symbol] :> Int[ExpandTrig[cos[c + d*x]^m*(a
*cos[c + d*x] + b*sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d}, x] && Inte
gerQ[m] && IGtQ[n, 0]
```

Rule 3092

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[a^n*b^n, Int[Cos[c + d*x]^m/
(b*Cos[c + d*x] + a*Sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, m}, x] &&
 EqQ[a^2 + b^2, 0] && ILtQ[n, 0]
```

Rubi steps

$$\int \frac{\cos^3(c+dx)}{(a\cos(c+dx)+ia\sin(c+dx))^2}\,dx = -\frac{\int \cos^3(c+dx)(ia\cos(c+dx)+a\sin(c+dx))^2\,dx}{a^4}$$

$$= -\frac{\int \left(-a^2\cos^5(c+dx)+2ia^2\cos^4(c+dx)\sin(c+dx)+a^2\cos^3(c+dx)\right.}{a^4}$$

$$= -\frac{(2i)\int \cos^4(c+dx)\sin(c+dx)\,dx}{a^2}+\frac{\int \cos^5(c+dx)\,dx}{a^2}-\frac{\int \cos^3(c+d}{}$$

$$= \frac{(2i)\,\text{Subst}\left(\int x^4\,dx, x, \cos(c+dx)\right)}{a^2d}-\frac{\text{Subst}\left(\int x^2\left(1-x^2\right)\,dx, x, \sin(c+\right.}{a^2d}$$

$$= \frac{2i\cos^5(c+dx)}{5a^2d}+\frac{\sin(c+dx)}{a^2d}-\frac{2\sin^3(c+dx)}{3a^2d}+\frac{\sin^5(c+dx)}{5a^2d}-\text{Subst}$$

$$= \frac{2i\cos^5(c+dx)}{5a^2d}+\frac{\sin(c+dx)}{a^2d}-\frac{\sin^3(c+dx)}{a^2d}+\frac{2\sin^5(c+dx)}{5a^2d}$$

Mathematica [A] time = 0.08, size = 111, normalized size = 1.63

$$\frac{\sin(c+dx)}{2a^2d}+\frac{\sin(3(c+dx))}{8a^2d}+\frac{\sin(5(c+dx))}{40a^2d}+\frac{i\cos(c+dx)}{4a^2d}+\frac{i\cos(3(c+dx))}{8a^2d}+\frac{i\cos(5(c+dx))}{40a^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^3/(a*Cos[c + d*x] + I*a*Sin[c + d*x])^2,x]
```

```
[Out] ((I/4)*Cos[c + d*x])/(a^2*d) + ((I/8)*Cos[3*(c + d*x)])/(a^2*d) + ((I/40)*C
os[5*(c + d*x)])/(a^2*d) + Sin[c + d*x]/(2*a^2*d) + Sin[3*(c + d*x)]/(8*a^2
*d) + Sin[5*(c + d*x)]/(40*a^2*d)
```

fricas [A] time = 0.42, size = 52, normalized size = 0.76

$$\frac{\left(-5i\,e^{(6i\,dx+6i\,c)}+15i\,e^{(4i\,dx+4i\,c)}+5i\,e^{(2i\,dx+2i\,c)}+i\right)e^{(-5i\,dx-5i\,c)}}{40\,a^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3/(a*cos(d*x+c)+I*a*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] 1/40*(-5*I*e^(6*I*d*x + 6*I*c) + 15*I*e^(4*I*d*x + 4*I*c) + 5*I*e^(2*I*d*x
+ 2*I*c) + I)*e^(-5*I*d*x - 5*I*c)/(a^2*d)
```

giac [A] time = 0.23, size = 93, normalized size = 1.37

$$\frac{\frac{5}{a^2\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+i\right)}+\frac{35\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4-90i\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-120\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+70i\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+21}{a^2\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-i\right)^5}}{20\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3/(a*cos(d*x+c)+I*a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/20*(5/(a^2*(tan(1/2*d*x + 1/2*c) + I)) + (35*tan(1/2*d*x + 1/2*c)^4 - 90*
I*tan(1/2*d*x + 1/2*c)^3 - 120*tan(1/2*d*x + 1/2*c)^2 + 70*I*tan(1/2*d*x +
1/2*c) + 21)/(a^2*(tan(1/2*d*x + 1/2*c) - I)^5))/d
```

maple [A] time = 0.18, size = 108, normalized size = 1.59

$$\frac{\frac{2}{8\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+8i}-\frac{2i}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-i\right)^4}+\frac{5i}{2\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-i\right)^2}+\frac{4}{5\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-i\right)^5}-\frac{3}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-i\right)^3}+\frac{7}{4\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-i\right)}}{d\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3/(a*cos(d*x+c)+I*a*sin(d*x+c))^2,x)
```

```
[Out] 2/d/a^2*(1/8/(tan(1/2*d*x+1/2*c)+I)-I/(tan(1/2*d*x+1/2*c)-I)^4+5/4*I/(tan(1
/2*d*x+1/2*c)-I)^2+2/5/(tan(1/2*d*x+1/2*c)-I)^5-3/2/(tan(1/2*d*x+1/2*c)-I)^
3+7/8/(tan(1/2*d*x+1/2*c)-I))
```

maxima [**F(-2)**] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3/(a*cos(d*x+c)+I*a*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

mupad [**B**] time = 1.02, size = 90, normalized size = 1.32

$$-\frac{2\left(-5\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4 10\mathrm{i}+10\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3+3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)-2\mathrm{i}\right)}{5\,a^2\,d\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)-\mathrm{i}\right)^5\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+1\mathrm{i}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^3/(a*cos(c + d*x) + a*sin(c + d*x)*1i)^2,x)
```

```
[Out] -(2*(3*tan(c/2 + (d*x)/2) + 10*tan(c/2 + (d*x)/2)^3 + tan(c/2 + (d*x)/2)^4*
10i - 5*tan(c/2 + (d*x)/2)^5 - 2i))/(5*a^2*d*(tan(c/2 + (d*x)/2) - 1i)^5*(t
an(c/2 + (d*x)/2) + 1i))
```

sympy [**A**] time = 0.41, size = 165, normalized size = 2.43

$$\begin{cases} \frac{\left(-2560ia^6d^3e^{10ic}e^{idx}+7680ia^6d^3e^{8ic}e^{-idx}+2560ia^6d^3e^{6ic}e^{-3idx}+512ia^6d^3e^{4ic}e^{-5idx}\right)e^{-9ic}}{20480a^8d^4} & \text{for}\ 20480a^8d^4e^{9ic}\neq 0 \\ \frac{x\left(e^{6ic}+3e^{4ic}+3e^{2ic}+1\right)e^{-5ic}}{8a^2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3/(a*cos(d*x+c)+I*a*sin(d*x+c))**2,x)
```

```
[Out] Piecewise(((-2560*I*a**6*d**3*exp(10*I*c)*exp(I*d*x) + 7680*I*a**6*d**3*exp
(8*I*c)*exp(-I*d*x) + 2560*I*a**6*d**3*exp(6*I*c)*exp(-3*I*d*x) + 512*I*a**
6*d**3*exp(4*I*c)*exp(-5*I*d*x))*exp(-9*I*c)/(20480*a**8*d**4), Ne(20480*a*
*8*d**4*exp(9*I*c), 0)), (x*(exp(6*I*c) + 3*exp(4*I*c) + 3*exp(2*I*c) + 1)*
exp(-5*I*c)/(8*a**2), True))
```

3.166 $\int \frac{\cos^2(c+dx)}{(a\cos(c+dx)+ia\sin(c+dx))^2}\,dx$

Optimal. Leaf size=89

$$\frac{i\cos(c+dx)}{4d\left(a^2\cos(c+dx)+ia^2\sin(c+dx)\right)}+\frac{x}{4a^2}+\frac{i\cos^2(c+dx)}{4d(a\cos(c+dx)+ia\sin(c+dx))^2}$$

```
[Out] 1/4*x/a^2+1/4*I*cos(d*x+c)^2/d/(a*cos(d*x+c)+I*a*sin(d*x+c))^2+1/4*I*cos(d*
x+c)/d/(a^2*cos(d*x+c)+I*a^2*sin(d*x+c))
```

Rubi [A] time = 0.08, antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.065, Rules used = {3082, 8}

$$\frac{i\cos(c+dx)}{4d\left(a^2\cos(c+dx)+ia^2\sin(c+dx)\right)}+\frac{x}{4a^2}+\frac{i\cos^2(c+dx)}{4d(a\cos(c+dx)+ia\sin(c+dx))^2}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2/(a*Cos[c + d*x] + I*a*Sin[c + d*x])^2,x]
```

```
[Out] x/(4*a^2) + ((I/4)*Cos[c + d*x]^2)/(d*(a*Cos[c + d*x] + I*a*Sin[c + d*x])^2
) + ((I/4)*Cos[c + d*x])/(d*(a^2*Cos[c + d*x] + I*a^2*Sin[c + d*x]))
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3082

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*(a*Cos[c + d*x] + b*Sin[
c + d*x])^n)/(2*a*d*n*Cos[c + d*x]^n), x] + Dist[1/(2*a), Int[(a*Cos[c + d*
x] + b*Sin[c + d*x])^(n + 1)/Cos[c + d*x]^(n + 1), x], x] /; FreeQ[{a, b, c
, d}, x] && EqQ[m + n, 0] && EqQ[a^2 + b^2, 0] && LtQ[n, 0]
```

Rubi steps

$$\int \frac{\cos^2(c+dx)}{(a\cos(c+dx)+ia\sin(c+dx))^2}\,dx = \frac{i\cos^2(c+dx)}{4d(a\cos(c+dx)+ia\sin(c+dx))^2} + \frac{\int \frac{\cos(c+dx)}{a\cos(c+dx)+ia\sin(c+dx)}\,dx}{2a}$$
$$= \frac{i\cos^2(c+dx)}{4d(a\cos(c+dx)+ia\sin(c+dx))^2} + \frac{i\cos(c+dx)}{4d\left(a^2\cos(c+dx)+ia^2\sin(c+d$$
$$= \frac{x}{4a^2} + \frac{i\cos^2(c+dx)}{4d(a\cos(c+dx)+ia\sin(c+dx))^2} + \frac{i\cos(c+dx)}{4d\left(a^2\cos(c+dx)+ia^2\sin$$

Mathematica [A] time = 0.10, size = 60, normalized size = 0.67

$$\frac{4\sin(2(c+dx))+\sin(4(c+dx))+4i\cos(2(c+dx))+i\cos(4(c+dx))+4c+4dx}{16a^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2/(a*Cos[c + d*x] + I*a*Sin[c + d*x])^2,x]
```

```
[Out] (4*c + 4*d*x + (4*I)*Cos[2*(c + d*x)] + I*Cos[4*(c + d*x)] + 4*Sin[2*(c + d
*x)] + Sin[4*(c + d*x)])/(16*a^2*d)
```

fricas [A] time = 0.47, size = 43, normalized size = 0.48

$$\frac{\left(4\,dxe^{(4i\,dx+4i\,c)}+4i\,e^{(2i\,dx+2i\,c)}+i\right)e^{(-4i\,dx-4i\,c)}}{16\,a^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(a*cos(d*x+c)+I*a*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] 1/16*(4*d*x*e^(4*I*d*x + 4*I*c) + 4*I*e^(2*I*d*x + 2*I*c) + I)*e^(-4*I*d*x
- 4*I*c)/(a^2*d)
```

giac [A] time = 0.42, size = 72, normalized size = 0.81

$$-\frac{\frac{2i\log(i\tan(dx+c)+1)}{a^2}-\frac{2i\log(i\tan(dx+c)-1)}{a^2}+\frac{-3i\tan(dx+c)^2-10\tan(dx+c)+11i}{a^2(\tan(dx+c)-i)^2}}{16\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(a*cos(d*x+c)+I*a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/16*(2*I*log(I*tan(d*x + c) + 1)/a^2 - 2*I*log(I*tan(d*x + c) - 1)/a^2 +
(-3*I*tan(d*x + c)^2 - 10*tan(d*x + c) + 11*I)/(a^2*(tan(d*x + c) - I)^2))/
d
```

maple [A] time = 0.19, size = 79, normalized size = 0.89

$$\frac{i\ln\left(\tan\left(dx+c\right)+i\right)}{8a^2d}-\frac{i\ln\left(\tan\left(dx+c\right)-i\right)}{8a^2d}-\frac{i}{4a^2d\left(\tan\left(dx+c\right)-i\right)^2}+\frac{1}{4a^2d\left(\tan\left(dx+c\right)-i\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2/(a*cos(d*x+c)+I*a*sin(d*x+c))^2,x)
```

```
[Out] 1/8*I/a^2/d*ln(tan(d*x+c)+I)-1/8*I/a^2/d*ln(tan(d*x+c)-I)-1/4*I/a^2/d/(tan(
d*x+c)-I)^2+1/4/a^2/d/(tan(d*x+c)-I)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(a*cos(d*x+c)+I*a*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

mupad [B] time = 1.76, size = 69, normalized size = 0.78

$$\frac{x}{4\,a^2}+\frac{-\frac{3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{2}+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\,2\mathrm{i}+\frac{3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{2}}{a^2\,d\,{\left(1+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,1\mathrm{i}\right)}^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^2/(a*cos(c + d*x) + a*sin(c + d*x)*1i)^2,x)
```

```
[Out] x/(4*a^2) + ((3*tan(c/2 + (d*x)/2))/2 + tan(c/2 + (d*x)/2)^2*2i - (3*tan(c/
2 + (d*x)/2)^3)/2)/(a^2*d*(tan(c/2 + (d*x)/2)*1i + 1)^4)
```

sympy [A] time = 0.23, size = 119, normalized size = 1.34

$$\begin{cases}\frac{\left(16ia^2de^{4ic}e^{-2idx}+4ia^2de^{2ic}e^{-4idx}\right)e^{-6ic}}{64a^4d^2} & \text{for}\ 64a^4d^2e^{6ic}\neq 0\\ x\left(\frac{\left(e^{4ic}+2e^{2ic}+1\right)e^{-4ic}}{4a^2}-\frac{1}{4a^2}\right) & \text{otherwise}\end{cases}+\frac{x}{4a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2/(a*cos(d*x+c)+I*a*sin(d*x+c))**2,x)
```

```
[Out] Piecewise((((16*I*a**2*d*exp(4*I*c)*exp(-2*I*d*x) + 4*I*a**2*d*exp(2*I*c)*ex
p(-4*I*d*x))*exp(-6*I*c)/(64*a**4*d**2), Ne(64*a**4*d**2*exp(6*I*c), 0)), (
x*((exp(4*I*c) + 2*exp(2*I*c) + 1)*exp(-4*I*c)/(4*a**2) - 1/(4*a**2)), True
)) + x/(4*a**2)
```

3.167 $\int \frac{\cos(c+dx)}{(a\cos(c+dx)+ia\sin(c+dx))^2} dx$

Optimal. Leaf size=52

$$-\frac{2\sin^3(c+dx)}{3a^2d}+\frac{\sin(c+dx)}{a^2d}+\frac{2i\cos^3(c+dx)}{3a^2d}$$

```
[Out] 2/3*I*cos(d*x+c)^3/a^2/d+sin(d*x+c)/a^2/d-2/3*sin(d*x+c)^3/a^2/d
```

Rubi [A] time = 0.11, antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.207, Rules used = {3092, 3090, 2633, 2565, 30, 2564}

$$-\frac{2\sin^3(c+dx)}{3a^2d}+\frac{\sin(c+dx)}{a^2d}+\frac{2i\cos^3(c+dx)}{3a^2d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]/(a*Cos[c + d*x] + I*a*Sin[c + d*x])^2,x]
```

```
[Out] (((2*I)/3)*Cos[c + d*x]^3)/(a^2*d) + Sin[c + d*x]/(a^2*d) - (2*Sin[c + d*x]
^3)/(3*a^2*d)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2564

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 2565

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> -Dist[(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
```

```
&& IGtQ[(n - 1)/2, 0]
```

Rule 3090

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_.), x_Symbol] :> Int[ExpandTrig[cos[c + d*x]^m*(a
*cos[c + d*x] + b*sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d}, x] && Inte
gerQ[m] && IGtQ[n, 0]
```

Rule 3092

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[a^n*b^n, Int[Cos[c + d*x]^m/
(b*Cos[c + d*x] + a*Sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, m}, x] &&
 EqQ[a^2 + b^2, 0] && ILtQ[n, 0]
```

Rubi steps

$$\int \frac{\cos(c+dx)}{(a\cos(c+dx)+ia\sin(c+dx))^2}\,dx = -\frac{\int \cos(c+dx)(ia\cos(c+dx)+a\sin(c+dx))^2\,dx}{a^4}$$

$$= -\frac{\int \left(-a^2\cos^3(c+dx)+2ia^2\cos^2(c+dx)\sin(c+dx)+a^2\cos(c+dx)\,\mathrm{s}\right.}{a^4}$$

$$= -\frac{(2i)\int \cos^2(c+dx)\sin(c+dx)\,dx}{a^2}+\frac{\int \cos^3(c+dx)\,dx}{a^2}-\frac{\int \cos(c+dx}{}$$

$$= \frac{(2i)\,\mathrm{Subst}\left(\int x^2\,dx,x,\cos(c+dx)\right)}{a^2d}-\frac{\mathrm{Subst}\left(\int x^2\,dx,x,\sin(c+dx)\right)}{a^2d}-$$

$$= \frac{2i\cos^3(c+dx)}{3a^2d}+\frac{\sin(c+dx)}{a^2d}-\frac{2\sin^3(c+dx)}{3a^2d}$$

Mathematica [A] time = 0.06, size = 73, normalized size = 1.40

$$\frac{\sin(c+dx)}{2a^2d}+\frac{\sin(3(c+dx))}{6a^2d}+\frac{i\cos(c+dx)}{2a^2d}+\frac{i\cos(3(c+dx))}{6a^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]/(a*Cos[c + d*x] + I*a*Sin[c + d*x])^2,x]
```

```
[Out] ((I/2)*Cos[c + d*x])/(a^2*d) + ((I/6)*Cos[3*(c + d*x)])/(a^2*d) + Sin[c + d
*x]/(2*a^2*d) + Sin[3*(c + d*x)]/(6*a^2*d)
```

fricas [A] time = 0.59, size = 30, normalized size = 0.58

$$\frac{\left(3i\,e^{(2i\,dx+2i\,c)}+i\right)e^{(-3i\,dx-3i\,c)}}{6\,a^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a*cos(d*x+c)+I*a*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/6*(3*I*e^(2*I*d*x + 2*I*c) + I)*e^(-3*I*d*x - 3*I*c)/(a^2*d)
```

giac [A] time = 1.07, size = 47, normalized size = 0.90

$$\frac{2\left(3\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-3i\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-2\right)}{3\,a^2d\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-i\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a*cos(d*x+c)+I*a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 2/3*(3*tan(1/2*d*x + 1/2*c)^2 - 3*I*tan(1/2*d*x + 1/2*c) - 2)/(a^2*d*(tan(1
/2*d*x + 1/2*c) - I)^3)
```

maple [A] time = 0.17, size = 57, normalized size = 1.10

$$\frac{\frac{2}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-i}+\frac{2i}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-i\right)^2}-\frac{4}{3\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-i\right)^3}}{d\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)/(a*cos(d*x+c)+I*a*sin(d*x+c))^2,x)
```

```
[Out] 2/d/a^2*(1/(tan(1/2*d*x+1/2*c)-I)+I/(tan(1/2*d*x+1/2*c)-I)^2-2/3/(tan(1/2*d
*x+1/2*c)-I)^3)
```

maxima [A] time = 0.33, size = 45, normalized size = 0.87

$$\frac{i\,\cos\left(3\,dx+3\,c\right)+3i\,\cos\left(dx+c\right)+\sin\left(3\,dx+3\,c\right)+3\,\sin\left(dx+c\right)}{6\,a^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a*cos(d*x+c)+I*a*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 1/6*(I*cos(3*d*x + 3*c) + 3*I*cos(d*x + c) + sin(3*d*x + 3*c) + 3*sin(d*x +
 c))/(a^2*d)
```

mupad [B] time = 0.65, size = 79, normalized size = 1.52

$$-\frac{2\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\,3\mathrm{i}+3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)-2\mathrm{i}\right)}{3\,a^2\,d\,\left(-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3\,1\mathrm{i}-3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,3\mathrm{i}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)/(a*cos(c + d*x) + a*sin(c + d*x)*1i)^2,x)
```

```
[Out] -(2*(3*tan(c/2 + (d*x)/2) + tan(c/2 + (d*x)/2)^2*3i - 2i))/(3*a^2*d*(tan(c/
2 + (d*x)/2)*3i - 3*tan(c/2 + (d*x)/2)^2 - tan(c/2 + (d*x)/2)^3*1i + 1))
```

sympy [A] time = 0.23, size = 94, normalized size = 1.81

$$\begin{cases} \frac{\left(6ia^2de^{3ic}e^{-idx}+2ia^2de^{ic}e^{-3idx}\right)e^{-4ic}}{12a^4d^2} & \text{for } 12a^4d^2e^{4ic}\neq 0 \\ \frac{x\left(e^{2ic}+1\right)e^{-3ic}}{2a^2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a*cos(d*x+c)+I*a*sin(d*x+c))**2,x)
```

```
[Out] Piecewise(((6*I*a**2*d*exp(3*I*c)*exp(-I*d*x) + 2*I*a**2*d*exp(I*c)*exp(-3*
I*d*x))*exp(-4*I*c)/(12*a**4*d**2), Ne(12*a**4*d**2*exp(4*I*c), 0)), (x*(ex
p(2*I*c) + 1)*exp(-3*I*c)/(2*a**2), True))
```

3.168 $\int \frac{1}{(a\cos(c+dx)+ia\sin(c+dx))^2}\,dx$

Optimal. Leaf size=31

$$\frac{i}{2d(a\cos(c+dx)+ia\sin(c+dx))^2}$$

```
[Out] 1/2*I/d/(a*cos(d*x+c)+I*a*sin(d*x+c))^2
```

Rubi [A] time = 0.02, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {3071}

$$\frac{i}{2d(a\cos(c+dx)+ia\sin(c+dx))^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Cos[c + d*x] + I*a*Sin[c + d*x])^(-2),x]
```

```
[Out] (I/2)/(d*(a*Cos[c + d*x] + I*a*Sin[c + d*x])^2)
```

Rule 3071

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x
_Symbol] :> Simp[(a*(a*Cos[c + d*x] + b*Sin[c + d*x])^n)/(b*d*n), x] /; Fre
eQ[{a, b, c, d, n}, x] && EqQ[a^2 + b^2, 0]
```

Rubi steps

$$\int \frac{1}{(a\cos(c+dx)+ia\sin(c+dx))^2}\,dx=\frac{i}{2d(a\cos(c+dx)+ia\sin(c+dx))^2}$$

Mathematica [A] time = 0.04, size = 31, normalized size = 1.00

$$\frac{i}{2d(a\cos(c+dx)+ia\sin(c+dx))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Cos[c + d*x] + I*a*Sin[c + d*x])^(-2),x]
```

```
[Out] (I/2)/(d*(a*Cos[c + d*x] + I*a*Sin[c + d*x])^2)
```

fricas [A] time = 0.43, size = 17, normalized size = 0.55

$$\frac{i\,e^{(-2i\,dx-2i\,c)}}{2\,a^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cos(d*x+c)+I*a*sin(d*x+c))^2,x, algorithm="fricas")
[Out] 1/2*I*e^(-2*I*d*x - 2*I*c)/(a^2*d)
```

giac [A] time = 0.19, size = 30, normalized size = 0.97

$$-\frac{2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{a^2d\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-i\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cos(d*x+c)+I*a*sin(d*x+c))^2,x, algorithm="giac")
[Out] -2*tan(1/2*d*x + 1/2*c)/(a^2*d*(tan(1/2*d*x + 1/2*c) - I)^2)
```

maple [A] time = 0.15, size = 23, normalized size = 0.74

$$\frac{i}{d\,a^2\,(i\tan(dx+c)+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*cos(d*x+c)+I*a*sin(d*x+c))^2,x)
[Out] I/d/a^2/(I*tan(d*x+c)+1)
```

maxima [A] time = 0.32, size = 22, normalized size = 0.71

$$\frac{1}{\left(a^2\tan(dx+c)-i\,a^2\right)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cos(d*x+c)+I*a*sin(d*x+c))^2,x, algorithm="maxima")
[Out] 1/((a^2*tan(d*x + c) - I*a^2)*d)
```

mupad [B] time = 0.61, size = 31, normalized size = 1.00

$$-\frac{2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{a^2\,d\,{\left(\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)-\mathrm{i}\right)}^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*cos(c + d*x) + a*sin(c + d*x)*1i)^2,x)
```

```
[Out] -(2*tan(c/2 + (d*x)/2))/(a^2*d*(tan(c/2 + (d*x)/2) - 1i)^2)
```

sympy [A] time = 0.14, size = 46, normalized size = 1.48

$$\begin{cases} \frac{ie^{-2ic}e^{-2idx}}{2a^2d} & \text{for}\ 2a^2de^{2ic} \neq 0 \\ \frac{xe^{-2ic}}{a^2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cos(d*x+c)+I*a*sin(d*x+c))**2,x)
```

```
[Out] Piecewise((I*exp(-2*I*c)*exp(-2*I*d*x)/(2*a**2*d), Ne(2*a**2*d*exp(2*I*c),
0)), (x*exp(-2*I*c)/a**2, True))
```

3.169 $\int \frac{\sec(c+dx)}{(a\cos(c+dx)+ia\sin(c+dx))^2}\,dx$

Optimal. Leaf size=46

$$\frac{2\sin(c+dx)}{a^2d}+\frac{2i\cos(c+dx)}{a^2d}-\frac{\tanh^{-1}(\sin(c+dx))}{a^2d}$$

```
[Out] -arctanh(sin(d*x+c))/a^2/d+2*I*cos(d*x+c)/a^2/d+2*sin(d*x+c)/a^2/d
```

Rubi [A] time = 0.11, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.241, Rules used = {3092, 3090, 2637, 2638, 2592, 321, 206}

$$\frac{2\sin(c+dx)}{a^2d}+\frac{2i\cos(c+dx)}{a^2d}-\frac{\tanh^{-1}(\sin(c+dx))}{a^2d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]/(a*Cos[c + d*x] + I*a*Sin[c + d*x])^2,x]
```

```
[Out] -(ArcTanh[Sin[c + d*x]]/(a^2*d)) + ((2*I)*Cos[c + d*x])/(a^2*d) + (2*Sin[c
+ d*x])/(a^2*d)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2592

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(
ff*x)^(m + n)/(a^2 - ff^2*x^2)^((n + 1)/2), x], x, (a*Sin[e + f*x])/ff], x]
] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3090

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_.), x_Symbol] :> Int[ExpandTrig[cos[c + d*x]^m*(a
*cos[c + d*x] + b*sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d}, x] && Inte
gerQ[m] && IGtQ[n, 0]
```

Rule 3092

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[a^n*b^n, Int[Cos[c + d*x]^m/
(b*Cos[c + d*x] + a*Sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, m}, x] &&
 EqQ[a^2 + b^2, 0] && ILtQ[n, 0]
```

Rubi steps

$$\int \frac{\sec(c+dx)}{(a\cos(c+dx)+ia\sin(c+dx))^2}\,dx = -\frac{\int \sec(c+dx)(ia\cos(c+dx)+a\sin(c+dx))^2\,dx}{a^4}$$

$$= -\frac{\int \left(-a^2\cos(c+dx)+2ia^2\sin(c+dx)+a^2\sin(c+dx)\tan(c+dx)\right)\,d}{a^4}$$

$$= -\frac{(2i)\int \sin(c+dx)\,dx}{a^2}+\frac{\int \cos(c+dx)\,dx}{a^2}-\frac{\int \sin(c+dx)\tan(c+dx)}{a^2}$$

$$= \frac{2i\cos(c+dx)}{a^2d}+\frac{\sin(c+dx)}{a^2d}-\frac{\text{Subst}\left(\int \frac{x^2}{1-x^2}\,dx,x,\sin(c+dx)\right)}{a^2d}$$

$$= \frac{2i\cos(c+dx)}{a^2d}+\frac{2\sin(c+dx)}{a^2d}-\frac{\text{Subst}\left(\int \frac{1}{1-x^2}\,dx,x,\sin(c+dx)\right)}{a^2d}$$

$$= -\frac{\tanh^{-1}(\sin(c+dx))}{a^2d}+\frac{2i\cos(c+dx)}{a^2d}+\frac{2\sin(c+dx)}{a^2d}$$

Mathematica [B] time = 0.24, size = 184, normalized size = 4.00

$$-\frac{\sec^2(c+dx)\left(\cos\left(\frac{3}{2}(c+dx)\right)+i\sin\left(\frac{3}{2}(c+dx)\right)\right)\left(\cos\left(\frac{1}{2}(c+dx)\right)\left(\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)-\log\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]/(a*Cos[c + d*x] + I*a*Sin[c + d*x])^2,x]
```

```
[Out] -((Sec[c + d*x]^2*(Cos[(c + d*x)/2]*(2*I + Log[Cos[(c + d*x)/2] - Sin[(c +
d*x)/2]] - Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]) + (2 + I*Log[Cos[(c +
d*x)/2] - Sin[(c + d*x)/2]] - I*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]])*S
in[(c + d*x)/2])*(Cos[(3*(c + d*x))/2] + I*Sin[(3*(c + d*x))/2]))/(a^2*d*(-
I + Tan[c + d*x])^2))
```

fricas [A] time = 0.49, size = 64, normalized size = 1.39

$$-\frac{\left(e^{(i\,dx+i\,c)}\log\left(e^{(i\,dx+i\,c)}+i\right)-e^{(i\,dx+i\,c)}\log\left(e^{(i\,dx+i\,c)}-i\right)-2i\right)e^{(-i\,dx-i\,c)}}{a^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a*cos(d*x+c)+I*a*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -(e^(I*d*x + I*c)*log(e^(I*d*x + I*c) + I) - e^(I*d*x + I*c)*log(e^(I*d*x +
 I*c) - I) - 2*I)*e^(-I*d*x - I*c)/(a^2*d)
```

giac [A] time = 1.68, size = 57, normalized size = 1.24

$$-\frac{\frac{\log\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)}{a^2}-\frac{\log\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right)}{a^2}-\frac{4}{a^2\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-i\right)}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a*cos(d*x+c)+I*a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -(log(tan(1/2*d*x + 1/2*c) + 1)/a^2 - log(tan(1/2*d*x + 1/2*c) - 1)/a^2 - 4
/(a^2*(tan(1/2*d*x + 1/2*c) - I)))/d
```

maple [A] time = 0.23, size = 63, normalized size = 1.37

$$\frac{\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{a^2d}-\frac{\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{a^2d}+\frac{4}{a^2d\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-i\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)/(a*cos(d*x+c)+I*a*sin(d*x+c))^2,x)
```

```
[Out] 1/a^2/d*ln(tan(1/2*d*x+1/2*c)-1)-1/a^2/d*ln(tan(1/2*d*x+1/2*c)+1)+4/a^2/d/(
tan(1/2*d*x+1/2*c)-I)
```

maxima [B] time = 0.43, size = 117, normalized size = 2.54

$$-\frac{-2i\arctan\left(\cos\left(dx+c\right),\sin\left(dx+c\right)+1\right)-2i\arctan\left(\cos\left(dx+c\right),-\sin\left(dx+c\right)+1\right)-4i\cos\left(dx+c\right)+\log}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a*cos(d*x+c)+I*a*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -1/2*(-2*I*arctan2(cos(d*x + c), sin(d*x + c) + 1) - 2*I*arctan2(cos(d*x +
c), -sin(d*x + c) + 1) - 4*I*cos(d*x + c) + log(cos(d*x + c)^2 + sin(d*x +
c)^2 + 2*sin(d*x + c) + 1) - log(cos(d*x + c)^2 + sin(d*x + c)^2 - 2*sin(d*
x + c) + 1) - 4*sin(d*x + c))/(a^2*d)
```

mupad [B] time = 0.67, size = 44, normalized size = 0.96

$$-\frac{2\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{a^2\,d}+\frac{4\mathrm{i}}{a^2\,d\,\left(1+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,1\mathrm{i}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)*(a*cos(c + d*x) + a*sin(c + d*x)*1i)^2),x)
```

```
[Out] 4i/(a^2*d*(tan(c/2 + (d*x)/2)*1i + 1)) - (2*atanh(tan(c/2 + (d*x)/2)))/(a^2
*d)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\sec\left(c+dx\right)}{-\sin^2\left(c+dx\right)+2i\sin\left(c+dx\right)\cos\left(c+dx\right)+\cos^2\left(c+dx\right)}\,dx}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a*cos(d*x+c)+I*a*sin(d*x+c))**2,x)
```

```
[Out] Integral(sec(c + d*x)/(-sin(c + d*x)**2 + 2*I*sin(c + d*x)*cos(c + d*x) + c
os(c + d*x)**2), x)/a**2
```

3.170 $\int \frac{\sec^2(c+dx)}{(a\cos(c+dx)+ia\sin(c+dx))^2}\,dx$

Optimal. Leaf size=55

$$-\frac{\tan(c+dx)}{a^2d}+\frac{2i\log(\sin(c+dx))}{a^2d}-\frac{2i\log(\tan(c+dx))}{a^2d}+\frac{2x}{a^2}$$

```
[Out] 2*x/a^2+2*I*ln(sin(d*x+c))/a^2/d-2*I*ln(tan(d*x+c))/a^2/d-tan(d*x+c)/a^2/d
```

Rubi [A] time = 0.07, antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.097, Rules used = {3088, 848, 77}

$$-\frac{\tan(c+dx)}{a^2d}+\frac{2i\log(\sin(c+dx))}{a^2d}-\frac{2i\log(\tan(c+dx))}{a^2d}+\frac{2x}{a^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^2/(a*Cos[c + d*x] + I*a*Sin[c + d*x])^2,x]
```

```
[Out] (2*x)/a^2 + ((2*I)*Log[Sin[c + d*x]])/(a^2*d) - ((2*I)*Log[Tan[c + d*x]])/(
a^2*d) - Tan[c + d*x]/(a^2*d)
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 848

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_.), x_Symbol] :> Int[(d + e*x)^(m + p)*(f + g*x)^n*(a/d + (c*x)/e)^p,
x] /; FreeQ[{a, c, d, e, f, g, m, n}, x] && NeQ[e*f - d*g, 0] && EqQ[c*d^2
+ a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && EqQ[m + p, 0]))
```

Rule 3088

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[(x^m*(b +
 a*x)^n)/(1 + x^2)^((m + n + 2)/2), x], x, Cot[c + d*x]], x] /; FreeQ[{a, b
, c, d}, x] && IntegerQ[n] && IntegerQ[(m + n)/2] && NeQ[n, -1] &&  !(GtQ[n
```

```
, 0] && GtQ[m, 1])
```

Rubi steps

$$\int \frac{\sec^2(c+dx)}{(a\cos(c+dx)+ia\sin(c+dx))^2}\,dx = -\frac{\text{Subst}\left(\int \frac{1+x^2}{x^2(ia+ax)^2}\,dx, x, \cot(c+dx)\right)}{d}$$
$$= -\frac{\text{Subst}\left(\int \frac{-\frac{i}{a}+\frac{x}{a}}{x^2(ia+ax)}\,dx, x, \cot(c+dx)\right)}{d}$$
$$= -\frac{\text{Subst}\left(\int \left(-\frac{1}{a^2x^2}-\frac{2i}{a^2x}+\frac{2i}{a^2(i+x)}\right)dx, x, \cot(c+dx)\right)}{d}$$
$$= \frac{2x}{a^2}+\frac{2i\log(\sin(c+dx))}{a^2d}-\frac{2i\log(\tan(c+dx))}{a^2d}-\frac{\tan(c+dx)}{a^2d}$$

Mathematica [A] time = 0.42, size = 71, normalized size = 1.29

$$\frac{4\tan^{-1}(\tan(dx))+i\sec(c)\sec(c+dx)\left(\cos(dx)\log\left(\cos^2(c+dx)\right)+\cos(2c+dx)\log\left(\cos^2(c+dx)\right)+2i\sin(d\right.}{2a^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^2/(a*Cos[c + d*x] + I*a*Sin[c + d*x])^2,x]
```

```
[Out] (4*ArcTan[Tan[d*x]] + I*Sec[c]*Sec[c + d*x]*(Cos[d*x]*Log[Cos[c + d*x]^2] +
 Cos[2*c + d*x]*Log[Cos[c + d*x]^2] + (2*I)*Sin[d*x]))/(2*a^2*d)
```

fricas [A] time = 0.59, size = 68, normalized size = 1.24

$$\frac{4\,dxe^{(2i\,dx+2i\,c)}+4\,dx+\left(2i\,e^{(2i\,dx+2i\,c)}+2i\right)\log\left(e^{(2i\,dx+2i\,c)}+1\right)-2i}{a^2de^{(2i\,dx+2i\,c)}+a^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a*cos(d*x+c)+I*a*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] (4*d*x*e^(2*I*d*x + 2*I*c) + 4*d*x + (2*I*e^(2*I*d*x + 2*I*c) + 2*I)*log(e^
(2*I*d*x + 2*I*c) + 1) - 2*I)/(a^2*d*e^(2*I*d*x + 2*I*c) + a^2*d)
```

giac [A] time = 2.93, size = 100, normalized size = 1.82

$$\frac{2\left(\frac{i\log\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right)}{a^2}-\frac{2i\log\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-i\right)}{a^2}+\frac{i\log\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1\right)}{a^2}+\frac{-i\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+i}{\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)a^2}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a*cos(d*x+c)+I*a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 2*(I*log(tan(1/2*d*x + 1/2*c) + 1)/a^2 - 2*I*log(tan(1/2*d*x + 1/2*c) - I)/
a^2 + I*log(tan(1/2*d*x + 1/2*c) - 1)/a^2 + (-I*tan(1/2*d*x + 1/2*c)^2 + ta
n(1/2*d*x + 1/2*c) + I)/((tan(1/2*d*x + 1/2*c)^2 - 1)*a^2))/d
```

maple [A] time = 0.24, size = 35, normalized size = 0.64

$$-\frac{2i\ln\left(\tan\left(dx+c\right)-i\right)}{a^2d}-\frac{\tan\left(dx+c\right)}{a^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2/(a*cos(d*x+c)+I*a*sin(d*x+c))^2,x)
```

```
[Out] -2*I/a^2/d*ln(tan(d*x+c)-I)-tan(d*x+c)/a^2/d
```

maxima [A] time = 0.33, size = 30, normalized size = 0.55

$$\frac{-\frac{2i\log\left(\tan\left(dx+c\right)-i\right)}{a^2}-\frac{\tan\left(dx+c\right)}{a^2}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a*cos(d*x+c)+I*a*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] (-2*I*log(tan(d*x + c) - I)/a^2 - tan(d*x + c)/a^2)/d
```

mupad [B] time = 0.78, size = 83, normalized size = 1.51

$$\frac{2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{d\,\left(a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2-a^2\right)}-\frac{\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)-\mathrm{i}\right)\,4\mathrm{i}}{a^2\,d}+\frac{\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2-1\right)\,2\mathrm{i}}{a^2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^2*(a*cos(c + d*x) + a*sin(c + d*x)*1i)^2),x)
```

```
[Out] (2*tan(c/2 + (d*x)/2))/(d*(a^2*tan(c/2 + (d*x)/2)^2 - a^2)) - (log(tan(c/2
+ (d*x)/2) - 1i)*4i)/(a^2*d) + (log(tan(c/2 + (d*x)/2)^2 - 1)*2i)/(a^2*d)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\sec^2(c+dx)}{-\sin^2(c+dx)+2i\sin(c+dx)\cos(c+dx)+\cos^2(c+dx)}\,dx}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2/(a*cos(d*x+c)+I*a*sin(d*x+c))**2,x)
```

```
[Out] Integral(sec(c + d*x)**2/(-sin(c + d*x)**2 + 2*I*sin(c + d*x)*cos(c + d*x)
+ cos(c + d*x)**2), x)/a**2
```

3.171 $\int \frac{\sec^3(c+dx)}{(a\cos(c+dx)+ia\sin(c+dx))^2}\,dx$

Optimal. Leaf size=56

$$-\frac{2i\sec(c+dx)}{a^2d}+\frac{3\tanh^{-1}(\sin(c+dx))}{2a^2d}-\frac{\tan(c+dx)\sec(c+dx)}{2a^2d}$$

```
[Out] 3/2*arctanh(sin(d*x+c))/a^2/d-2*I*sec(d*x+c)/a^2/d-1/2*sec(d*x+c)*tan(d*x+c
)/a^2/d
```

Rubi [A] time = 0.15, antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.194, Rules used = {3092, 3090, 3770, 2606, 8, 2611}

$$-\frac{2i\sec(c+dx)}{a^2d}+\frac{3\tanh^{-1}(\sin(c+dx))}{2a^2d}-\frac{\tan(c+dx)\sec(c+dx)}{2a^2d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^3/(a*Cos[c + d*x] + I*a*Sin[c + d*x])^2,x]
```

```
[Out] (3*ArcTanh[Sin[c + d*x]])/(2*a^2*d) - ((2*I)*Sec[c + d*x])/(a^2*d) - (Sec[c
 + d*x]*Tan[c + d*x])/(2*a^2*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 2611

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(b*(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*(
m + n - 1)), x] - Dist[(b^2*(n - 1))/(m + n - 1), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 3090

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_.), x_Symbol] :> Int[ExpandTrig[cos[c + d*x]^m*(a
*cos[c + d*x] + b*sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d}, x] && Inte
gerQ[m] && IGtQ[n, 0]
```

Rule 3092

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[a^n*b^n, Int[Cos[c + d*x]^m/
(b*Cos[c + d*x] + a*Sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, m}, x] &&
 EqQ[a^2 + b^2, 0] && ILtQ[n, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\sec^3(c+dx)}{(a\cos(c+dx)+ia\sin(c+dx))^2}\,dx &= -\frac{\int \sec^3(c+dx)(ia\cos(c+dx)+a\sin(c+dx))^2\,dx}{a^4}\\ &= -\frac{\int \left(-a^2\sec(c+dx)+2ia^2\sec(c+dx)\tan(c+dx)+a^2\sec(c+dx)\,\mathrm{ta}\right.}{a^4}\\ &= -\frac{(2i)\int \sec(c+dx)\tan(c+dx)\,dx}{a^2}+\frac{\int \sec(c+dx)\,dx}{a^2}-\frac{\int \sec(c+dx)}{}\\ &= \frac{\tanh^{-1}(\sin(c+dx))}{a^2d}-\frac{\sec(c+dx)\tan(c+dx)}{2a^2d}+\frac{\int \sec(c+dx)\,dx}{2a^2}-\\ &= \frac{3\tanh^{-1}(\sin(c+dx))}{2a^2d}-\frac{2i\sec(c+dx)}{a^2d}-\frac{\sec(c+dx)\tan(c+dx)}{2a^2d}\end{aligned}$$

Mathematica [B] time = 0.43, size = 146, normalized size = 2.61

$$-\frac{\sec^2(c+dx)\left(2\sin(c+dx)+8i\cos(c+dx)+3\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)+3\cos(2(c+dx))\left(\mathrm{lo}\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^3/(a*Cos[c + d*x] + I*a*Sin[c + d*x])^2,x]
```

```
[Out] -1/4*(Sec[c + d*x]^2*((8*I)*Cos[c + d*x] + 3*Log[Cos[(c + d*x)/2] - Sin[(c
+ d*x)/2]] + 3*Cos[2*(c + d*x)]*(Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] -
```

```
 Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]) - 3*Log[Cos[(c + d*x)/2] + Sin[(
c + d*x)/2]] + 2*Sin[c + d*x]))/(a^2*d)
```

fricas [B] time = 0.89, size = 134, normalized size = 2.39

$$\frac{3\left(e^{(4i\,dx+4i\,c)}+2\,e^{(2i\,dx+2i\,c)}+1\right)\log\left(e^{(i\,dx+i\,c)}+i\right)-3\left(e^{(4i\,dx+4i\,c)}+2\,e^{(2i\,dx+2i\,c)}+1\right)\log\left(e^{(i\,dx+i\,c)}-i\right)-6i\,e^{(3i\,dx+3i\,c)}}{2\left(a^2de^{(4i\,dx+4i\,c)}+2\,a^2de^{(2i\,dx+2i\,c)}+a^2d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a*cos(d*x+c)+I*a*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] 1/2*(3*(e^(4*I*d*x + 4*I*c) + 2*e^(2*I*d*x + 2*I*c) + 1)*log(e^(I*d*x + I*c
) + I) - 3*(e^(4*I*d*x + 4*I*c) + 2*e^(2*I*d*x + 2*I*c) + 1)*log(e^(I*d*x +
 I*c) - I) - 6*I*e^(3*I*d*x + 3*I*c) - 10*I*e^(I*d*x + I*c))/(a^2*d*e^(4*I*
d*x + 4*I*c) + 2*a^2*d*e^(2*I*d*x + 2*I*c) + a^2*d)
```

giac [A] time = 0.34, size = 95, normalized size = 1.70

$$\frac{\frac{3\,\log\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)}{a^2}-\frac{3\,\log\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right)}{a^2}-\frac{2\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-4i\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+4i\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1\right)^2a^2}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a*cos(d*x+c)+I*a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/2*(3*log(tan(1/2*d*x + 1/2*c) + 1)/a^2 - 3*log(tan(1/2*d*x + 1/2*c) - 1)/
a^2 - 2*(tan(1/2*d*x + 1/2*c)^3 - 4*I*tan(1/2*d*x + 1/2*c)^2 + tan(1/2*d*x
+ 1/2*c) + 4*I)/((tan(1/2*d*x + 1/2*c)^2 - 1)^2*a^2))/d
```

maple [B] time = 0.25, size = 170, normalized size = 3.04

$$-\frac{1}{2a^2d\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}+\frac{2i}{a^2d\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}-\frac{1}{2a^2d\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^2}-\frac{3\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{2a^2d}-\frac{}{2a^2d\left(\tan\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^3/(a*cos(d*x+c)+I*a*sin(d*x+c))^2,x)
```

```
[Out] -1/2/a^2/d/(tan(1/2*d*x+1/2*c)-1)+2*I/a^2/d/(tan(1/2*d*x+1/2*c)-1)-1/2/a^2/
d/(tan(1/2*d*x+1/2*c)-1)^2-3/2/a^2/d*ln(tan(1/2*d*x+1/2*c)-1)-1/2/a^2/d/(ta
```

```
n(1/2*d*x+1/2*c)+1)-2*I/a^2/d/(tan(1/2*d*x+1/2*c)+1)+1/2/a^2/d/(tan(1/2*d*x
+1/2*c)+1)^2+3/2/a^2/d*ln(tan(1/2*d*x+1/2*c)+1)
```

maxima [B] time = 0.33, size = 167, normalized size = 2.98

$$-\frac{\frac{2\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}-\frac{4i\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+4i\right)}{a^2-\frac{2a^2\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{a^2\sin(dx+c)^4}{(\cos(dx+c)+1)^4}}-\frac{3\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)}{a^2}+\frac{3\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}-1\right)}{a^2}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a*cos(d*x+c)+I*a*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] -1/2*(2*(sin(d*x + c)/(cos(d*x + c) + 1) - 4*I*sin(d*x + c)^2/(cos(d*x + c)
 + 1)^2 + sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 4*I)/(a^2 - 2*a^2*sin(d*x +
 c)^2/(cos(d*x + c) + 1)^2 + a^2*sin(d*x + c)^4/(cos(d*x + c) + 1)^4) - 3*l
og(sin(d*x + c)/(cos(d*x + c) + 1) + 1)/a^2 + 3*log(sin(d*x + c)/(cos(d*x +
 c) + 1) - 1)/a^2)/d
```

mupad [B] time = 1.18, size = 104, normalized size = 1.86

$$\frac{3\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{a^2\,d}-\frac{\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{a^2}+\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{a^2}-\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\,4\mathrm{i}}{a^2}+\frac{4\mathrm{i}}{a^2}}{d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4-2\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^3*(a*cos(c + d*x) + a*sin(c + d*x)*1i)^2),x)
```

```
[Out] (3*atanh(tan(c/2 + (d*x)/2)))/(a^2*d) - (tan(c/2 + (d*x)/2)^3/a^2 - (tan(c/
2 + (d*x)/2)^2*4i)/a^2 + 4i/a^2 + tan(c/2 + (d*x)/2)/a^2)/(d*(tan(c/2 + (d*
x)/2)^4 - 2*tan(c/2 + (d*x)/2)^2 + 1))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\sec^3(c+dx)}{-\sin^2(c+dx)+2i\sin(c+dx)\cos(c+dx)+\cos^2(c+dx)}\,dx}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**3/(a*cos(d*x+c)+I*a*sin(d*x+c))**2,x)
```

```
[Out] Integral(sec(c + d*x)**3/(-sin(c + d*x)**2 + 2*I*sin(c + d*x)*cos(c + d*x)
+ cos(c + d*x)**2), x)/a**2
```

3.172 $\int \frac{\sec^4(c+dx)}{(a\cos(c+dx)+ia\sin(c+dx))^2}\,dx$

Optimal. Leaf size=34

$$-\frac{i\tan^3(c+dx)(-\cot(c+dx)+i)^3}{3a^2d}$$

```
[Out] -1/3*I*(I-cot(d*x+c))^3*tan(d*x+c)^3/a^2/d
```

Rubi [A] time = 0.06, antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.097, Rules used = {3088, 848, 37}

$$-\frac{i\tan^3(c+dx)(-\cot(c+dx)+i)^3}{3a^2d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^4/(a*Cos[c + d*x] + I*a*Sin[c + d*x])^2,x]
```

```
[Out] ((-I/3)*(I - Cot[c + d*x])^3*Tan[c + d*x]^3)/(a^2*d)
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 848

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_.), x_Symbol] :> Int[(d + e*x)^(m + p)*(f + g*x)^n*(a/d + (c*x)/e)^p,
x] /; FreeQ[{a, c, d, e, f, g, m, n}, x] && NeQ[e*f - d*g, 0] && EqQ[c*d^2
+ a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && EqQ[m + p, 0]))
```

Rule 3088

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[(x^m*(b +
 a*x)^n)/(1 + x^2)^((m + n + 2)/2), x], x, Cot[c + d*x]], x] /; FreeQ[{a, b
, c, d}, x] && IntegerQ[n] && IntegerQ[(m + n)/2] && NeQ[n, -1] &&  !(GtQ[n
, 0] && GtQ[m, 1])
```

Rubi steps

$$\int \frac{\sec^4(c+dx)}{(a\cos(c+dx)+ia\sin(c+dx))^2}\,dx = -\frac{\text{Subst}\left(\int \frac{(1+x^2)^2}{x^4(ia+ax)^2}\,dx, x, \cot(c+dx)\right)}{d}$$
$$= -\frac{\text{Subst}\left(\int \frac{\left(-\frac{i}{a}+\frac{x}{a}\right)^2}{x^4}\,dx, x, \cot(c+dx)\right)}{d}$$
$$= -\frac{i(i-\cot(c+dx))^3\tan^3(c+dx)}{3a^2d}$$

Mathematica [A] time = 0.26, size = 68, normalized size = 2.00

$$-\frac{\sec(c)\sec^3(c+dx)(3\sin(2c+dx)-2\sin(2c+3dx)+3i\cos(2c+dx)-3\sin(dx)+3i\cos(dx))}{6a^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^4/(a*Cos[c + d*x] + I*a*Sin[c + d*x])^2,x]
```

```
[Out] -1/6*(Sec[c]*Sec[c + d*x]^3*((3*I)*Cos[d*x] + (3*I)*Cos[2*c + d*x] - 3*Sin[
d*x] + 3*Sin[2*c + d*x] - 2*Sin[2*c + 3*d*x]))/(a^2*d)
```

fricas [B] time = 0.64, size = 54, normalized size = 1.59

$$\frac{8i}{3\left(a^2de^{(6i\,dx+6i\,c)}+3\,a^2de^{(4i\,dx+4i\,c)}+3\,a^2de^{(2i\,dx+2i\,c)}+a^2d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4/(a*cos(d*x+c)+I*a*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] 8/3*I/(a^2*d*e^(6*I*d*x + 6*I*c) + 3*a^2*d*e^(4*I*d*x + 4*I*c) + 3*a^2*d*e^
(2*I*d*x + 2*I*c) + a^2*d)
```

giac [A] time = 1.63, size = 35, normalized size = 1.03

$$-\frac{\tan(dx+c)^3+3i\,\tan(dx+c)^2-3\,\tan(dx+c)}{3\,a^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4/(a*cos(d*x+c)+I*a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/3*(tan(d*x + c)^3 + 3*I*tan(d*x + c)^2 - 3*tan(d*x + c))/(a^2*d)
```

maple [A] time = 0.26, size = 36, normalized size = 1.06

$$\frac{\tan\left(dx+c\right)-\frac{\left(\tan^{3}\left(dx+c\right)\right)}{3}-i\left(\tan^{2}\left(dx+c\right)\right)}{d\,a^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^4/(a*cos(d*x+c)+I*a*sin(d*x+c))^2,x)
```

```
[Out] 1/d/a^2*(tan(d*x+c)-1/3*tan(d*x+c)^3-I*tan(d*x+c)^2)
```

maxima [A] time = 0.51, size = 35, normalized size = 1.03

$$-\frac{\tan\left(dx+c\right)^{3}+3i\,\tan\left(dx+c\right)^{2}-3\,\tan\left(dx+c\right)}{3\,a^{2}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4/(a*cos(d*x+c)+I*a*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] -1/3*(tan(d*x + c)^3 + 3*I*tan(d*x + c)^2 - 3*tan(d*x + c))/(a^2*d)
```

mupad [B] time = 0.73, size = 49, normalized size = 1.44

$$-\frac{\sin\left(c+d\,x\right)\,\left(-4\cos\left(c+d\,x\right)^{2}+3\mathrm{i}\,\sin\left(c+d\,x\right)\,\cos\left(c+d\,x\right)+1\right)}{3\,a^{2}\,d\cos\left(c+d\,x\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^4*(a*cos(c + d*x) + a*sin(c + d*x)*1i)^2),x)
```

```
[Out] -(sin(c + d*x)*(cos(c + d*x)*sin(c + d*x)*3i - 4*cos(c + d*x)^2 + 1))/(3*a^
2*d*cos(c + d*x)^3)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int\frac{\sec^{4}\left(c+dx\right)}{-\sin^{2}\left(c+dx\right)+2i\sin\left(c+dx\right)\cos\left(c+dx\right)+\cos^{2}\left(c+dx\right)}\,dx}{a^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**4/(a*cos(d*x+c)+I*a*sin(d*x+c))**2,x)
```

```
[Out] Integral(sec(c + d*x)**4/(-sin(c + d*x)**2 + 2*I*sin(c + d*x)*cos(c + d*x)
+ cos(c + d*x)**2), x)/a**2
```

3.173 $\int \frac{\sec^5(c+dx)}{(a\cos(c+dx)+ia\sin(c+dx))^2}\,dx$

Optimal. Leaf size=84

$$-\frac{2i\sec^3(c+dx)}{3a^2d}+\frac{5\tanh^{-1}(\sin(c+dx))}{8a^2d}-\frac{\tan(c+dx)\sec^3(c+dx)}{4a^2d}+\frac{5\tan(c+dx)\sec(c+dx)}{8a^2d}$$

```
[Out] 5/8*arctanh(sin(d*x+c))/a^2/d-2/3*I*sec(d*x+c)^3/a^2/d+5/8*sec(d*x+c)*tan(d
*x+c)/a^2/d-1/4*sec(d*x+c)^3*tan(d*x+c)/a^2/d
```

Rubi [A] time = 0.19, antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.226, Rules used = {3092, 3090, 3768, 3770, 2606, 30, 2611}

$$-\frac{2i\sec^3(c+dx)}{3a^2d}+\frac{5\tanh^{-1}(\sin(c+dx))}{8a^2d}-\frac{\tan(c+dx)\sec^3(c+dx)}{4a^2d}+\frac{5\tan(c+dx)\sec(c+dx)}{8a^2d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^5/(a*Cos[c + d*x] + I*a*Sin[c + d*x])^2,x]
```

```
[Out] (5*ArcTanh[Sin[c + d*x]])/(8*a^2*d) - (((2*I)/3)*Sec[c + d*x]^3)/(a^2*d) +
(5*Sec[c + d*x]*Tan[c + d*x])/(8*a^2*d) - (Sec[c + d*x]^3*Tan[c + d*x])/(4*
a^2*d)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 2611

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(b*(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*(
m + n - 1)), x] - Dist[(b^2*(n - 1))/(m + n - 1), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 3090

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_.), x_Symbol] :> Int[ExpandTrig[cos[c + d*x]^m*(a
*cos[c + d*x] + b*sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d}, x] && Inte
gerQ[m] && IGtQ[n, 0]
```

Rule 3092

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[a^n*b^n, Int[Cos[c + d*x]^m/
(b*Cos[c + d*x] + a*Sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, m}, x] &&
 EqQ[a^2 + b^2, 0] && ILtQ[n, 0]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\sec^5(c+dx)}{(a\cos(c+dx)+ia\sin(c+dx))^2}\,dx = -\frac{\int \sec^5(c+dx)(ia\cos(c+dx)+a\sin(c+dx))^2\,dx}{a^4}$$

$$= -\frac{\int \left(-a^2\sec^3(c+dx)+2ia^2\sec^3(c+dx)\tan(c+dx)+a^2\sec^3(c+dx)\right.}{a^4}$$

$$= -\frac{(2i)\int \sec^3(c+dx)\tan(c+dx)\,dx}{a^2}+\frac{\int \sec^3(c+dx)\,dx}{a^2}-\frac{\int \sec^3(c+d}{}$$

$$= \frac{\sec(c+dx)\tan(c+dx)}{2a^2d}-\frac{\sec^3(c+dx)\tan(c+dx)}{4a^2d}+\frac{\int \sec^3(c+dx)\,dx}{4a^2}$$

$$= \frac{\tanh^{-1}(\sin(c+dx))}{2a^2d}-\frac{2i\sec^3(c+dx)}{3a^2d}+\frac{5\sec(c+dx)\tan(c+dx)}{8a^2d}-\text{se}$$

$$= \frac{5\tanh^{-1}(\sin(c+dx))}{8a^2d}-\frac{2i\sec^3(c+dx)}{3a^2d}+\frac{5\sec(c+dx)\tan(c+dx)}{8a^2d}-$$

Mathematica [B] time = 1.00, size = 215, normalized size = 2.56

$$-\frac{\sec^4(c+dx)\left(18\sin(c+dx)-30\sin(3(c+dx))+128i\cos(c+dx)+45\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^5/(a*Cos[c + d*x] + I*a*Sin[c + d*x])^2,x]
```

```
[Out] -1/192*(Sec[c + d*x]^4*((128*I)*Cos[c + d*x] + 45*Log[Cos[(c + d*x)/2] - Si
n[(c + d*x)/2]] + 60*Cos[2*(c + d*x)]*(Log[Cos[(c + d*x)/2] - Sin[(c + d*x)
/2]] - Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]) + 15*Cos[4*(c + d*x)]*(Log
[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] - Log[Cos[(c + d*x)/2] + Sin[(c + d*x
)/2]]) - 45*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] + 18*Sin[c + d*x] - 30
*Sin[3*(c + d*x)]))/(a^2*d)
```

fricas [B] time = 0.51, size = 230, normalized size = 2.74

$$\frac{15\left(e^{(8i\,dx+8i\,c)}+4\,e^{(6i\,dx+6i\,c)}+6\,e^{(4i\,dx+4i\,c)}+4\,e^{(2i\,dx+2i\,c)}+1\right)\log\left(e^{(i\,dx+i\,c)}+i\right)-15\left(e^{(8i\,dx+8i\,c)}+4\,e^{(6i\,dx+6i\,c)}+6\right.}{24\left(a^2de^{(8i\,dx+8i\,c)}+4\,a^2de^{(6i\,dx+6i\,c)}+6\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5/(a*cos(d*x+c)+I*a*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] 1/24*(15*(e^(8*I*d*x + 8*I*c) + 4*e^(6*I*d*x + 6*I*c) + 6*e^(4*I*d*x + 4*I*
c) + 4*e^(2*I*d*x + 2*I*c) + 1)*log(e^(I*d*x + I*c) + I) - 15*(e^(8*I*d*x +
 8*I*c) + 4*e^(6*I*d*x + 6*I*c) + 6*e^(4*I*d*x + 4*I*c) + 4*e^(2*I*d*x + 2*
I*c) + 1)*log(e^(I*d*x + I*c) - I) - 30*I*e^(7*I*d*x + 7*I*c) - 110*I*e^(5*
I*d*x + 5*I*c) - 146*I*e^(3*I*d*x + 3*I*c) + 30*I*e^(I*d*x + I*c))/(a^2*d*e
^(8*I*d*x + 8*I*c) + 4*a^2*d*e^(6*I*d*x + 6*I*c) + 6*a^2*d*e^(4*I*d*x + 4*I
*c) + 4*a^2*d*e^(2*I*d*x + 2*I*c) + a^2*d)
```

giac [B] time = 0.26, size = 151, normalized size = 1.80

$$\frac{\frac{15\log\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)}{a^2}-\frac{15\log\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right)}{a^2}+\frac{2\left(9\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7+48i\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^6-33\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-48i\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4-33\right.}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1\right)^4a^2}}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5/(a*cos(d*x+c)+I*a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/24*(15*log(tan(1/2*d*x + 1/2*c) + 1)/a^2 - 15*log(tan(1/2*d*x + 1/2*c) -
1)/a^2 + 2*(9*tan(1/2*d*x + 1/2*c)^7 + 48*I*tan(1/2*d*x + 1/2*c)^6 - 33*tan
(1/2*d*x + 1/2*c)^5 - 48*I*tan(1/2*d*x + 1/2*c)^4 - 33*tan(1/2*d*x + 1/2*c)
^3 + 16*I*tan(1/2*d*x + 1/2*c)^2 + 9*tan(1/2*d*x + 1/2*c) - 16*I)/((tan(1/2
*d*x + 1/2*c)^2 - 1)^4*a^2))/d
```

maple [B] time = 0.28, size = 342, normalized size = 4.07

$$\frac{3}{8a^2d\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}+\frac{i}{a^2d\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^2}+\frac{1}{8a^2d\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^2}-\frac{2i}{3a^2d\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^3}-\frac{}{2a^2d\left(\tan\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^5/(a*cos(d*x+c)+I*a*sin(d*x+c))^2,x)
```

```
[Out] 3/8/a^2/d/(tan(1/2*d*x+1/2*c)-1)+I/a^2/d/(tan(1/2*d*x+1/2*c)+1)^2+1/8/a^2/d
/(tan(1/2*d*x+1/2*c)-1)^2-2/3*I/a^2/d/(tan(1/2*d*x+1/2*c)+1)^3-1/2/a^2/d/(t
an(1/2*d*x+1/2*c)-1)^3-I/a^2/d/(tan(1/2*d*x+1/2*c)+1)-1/4/a^2/d/(tan(1/2*d*
x+1/2*c)-1)^4-5/8/a^2/d*ln(tan(1/2*d*x+1/2*c)-1)+3/8/a^2/d/(tan(1/2*d*x+1/2
*c)+1)+I/a^2/d/(tan(1/2*d*x+1/2*c)-1)^2-1/8/a^2/d/(tan(1/2*d*x+1/2*c)+1)^2+
I/a^2/d/(tan(1/2*d*x+1/2*c)-1)-1/2/a^2/d/(tan(1/2*d*x+1/2*c)+1)^3+2/3*I/a^2
/d/(tan(1/2*d*x+1/2*c)-1)^3+1/4/a^2/d/(tan(1/2*d*x+1/2*c)+1)^4+5/8/a^2/d*ln
(tan(1/2*d*x+1/2*c)+1)
```

maxima [B] time = 0.34, size = 295, normalized size = 3.51

$$\frac{\frac{2\left(\frac{9\sin(dx+c)}{\cos(dx+c)+1}+\frac{16i\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-\frac{33\sin(dx+c)^3}{(\cos(dx+c)+1)^3}-\frac{48i\sin(dx+c)^4}{(\cos(dx+c)+1)^4}-\frac{33\sin(dx+c)^5}{(\cos(dx+c)+1)^5}+\frac{48i\sin(dx+c)^6}{(\cos(dx+c)+1)^6}+\frac{9\sin(dx+c)^7}{(\cos(dx+c)+1)^7}-16i\right)}{a^2-\frac{4a^2\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{6a^2\sin(dx+c)^4}{(\cos(dx+c)+1)^4}-\frac{4a^2\sin(dx+c)^6}{(\cos(dx+c)+1)^6}+\frac{a^2\sin(dx+c)^8}{(\cos(dx+c)+1)^8}}+\frac{15\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)}{a^2}-\frac{15\log\left(\right.}{}}{24d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5/(a*cos(d*x+c)+I*a*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] 1/24*(2*(9*sin(d*x + c)/(cos(d*x + c) + 1) + 16*I*sin(d*x + c)^2/(cos(d*x +
 c) + 1)^2 - 33*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 - 48*I*sin(d*x + c)^4/(
cos(d*x + c) + 1)^4 - 33*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + 48*I*sin(d*x
 + c)^6/(cos(d*x + c) + 1)^6 + 9*sin(d*x + c)^7/(cos(d*x + c) + 1)^7 - 16*I
)/(a^2 - 4*a^2*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 6*a^2*sin(d*x + c)^4/(
cos(d*x + c) + 1)^4 - 4*a^2*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 + a^2*sin(d
*x + c)^8/(cos(d*x + c) + 1)^8) + 15*log(sin(d*x + c)/(cos(d*x + c) + 1) +
1)/a^2 - 15*log(sin(d*x + c)/(cos(d*x + c) + 1) - 1)/a^2)/d
```

mupad [B] time = 3.21, size = 136, normalized size = 1.62

$$\frac{5\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{4\,a^2\,d}+\frac{\frac{3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7}{4}+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6\,4\mathrm{i}-\frac{11\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5}{4}-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4\,4\mathrm{i}-\frac{11\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{4}+\frac{\tan(}{}}{a^2\,d\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2-1\right)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^5*(a*cos(c + d*x) + a*sin(c + d*x)*1i)^2),x)
```

```
[Out] (5*atanh(tan(c/2 + (d*x)/2)))/(4*a^2*d) + ((3*tan(c/2 + (d*x)/2))/4 + (tan(
c/2 + (d*x)/2)^2*4i)/3 - (11*tan(c/2 + (d*x)/2)^3)/4 - tan(c/2 + (d*x)/2)^4
*4i - (11*tan(c/2 + (d*x)/2)^5)/4 + tan(c/2 + (d*x)/2)^6*4i + (3*tan(c/2 +
(d*x)/2)^7)/4 - 4i/3)/(a^2*d*(tan(c/2 + (d*x)/2)^2 - 1)^4)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\sec^5(c+dx)}{-\sin^2(c+dx)+2i\sin(c+dx)\cos(c+dx)+\cos^2(c+dx)}\,dx}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**5/(a*cos(d*x+c)+I*a*sin(d*x+c))**2,x)
```

```
[Out] Integral(sec(c + d*x)**5/(-sin(c + d*x)**2 + 2*I*sin(c + d*x)*cos(c + d*x)
+ cos(c + d*x)**2), x)/a**2
```

3.174 $\int \frac{\sec^6(c+dx)}{(a\cos(c+dx)+ia\sin(c+dx))^2}\,dx$

Optimal. Leaf size=70

$$-\frac{\tan^5(c+dx)}{5a^2d}-\frac{i\tan^4(c+dx)}{2a^2d}-\frac{i\tan^2(c+dx)}{a^2d}+\frac{\tan(c+dx)}{a^2d}$$

```
[Out] tan(d*x+c)/a^2/d-I*tan(d*x+c)^2/a^2/d-1/2*I*tan(d*x+c)^4/a^2/d-1/5*tan(d*x+
c)^5/a^2/d
```

Rubi [A] time = 0.08, antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.097, Rules used = {3088, 848, 75}

$$-\frac{\tan^5(c+dx)}{5a^2d}-\frac{i\tan^4(c+dx)}{2a^2d}-\frac{i\tan^2(c+dx)}{a^2d}+\frac{\tan(c+dx)}{a^2d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^6/(a*Cos[c + d*x] + I*a*Sin[c + d*x])^2,x]
```

```
[Out] Tan[c + d*x]/(a^2*d) - (I*Tan[c + d*x]^2)/(a^2*d) - ((I/2)*Tan[c + d*x]^4)/
(a^2*d) - Tan[c + d*x]^5/(5*a^2*d)
```

Rule 75

```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_Sym
bol] :> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[
{a, b, d, e, f, n}, x] && IGtQ[p, 0] && EqQ[b*e + a*f, 0] &&  !(ILtQ[n + p
+ 2, 0] && GtQ[n + 2*p, 0])
```

Rule 848

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_.), x_Symbol] :> Int[(d + e*x)^(m + p)*(f + g*x)^n*(a/d + (c*x)/e)^p,
x] /; FreeQ[{a, c, d, e, f, g, m, n}, x] && NeQ[e*f - d*g, 0] && EqQ[c*d^2
+ a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && EqQ[m + p, 0]))
```

Rule 3088

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[(x^m*(b +
 a*x)^n)/(1 + x^2)^((m + n + 2)/2), x], x, Cot[c + d*x]], x] /; FreeQ[{a, b
, c, d}, x] && IntegerQ[n] && IntegerQ[(m + n)/2] && NeQ[n, -1] &&  !(GtQ[n
, 0] && GtQ[m, 1])
```

Rubi steps

$$\int \frac{\sec^6(c+dx)}{(a\cos(c+dx)+ia\sin(c+dx))^2}\,dx = -\frac{\text{Subst}\left(\int \frac{(1+x^2)^3}{x^6(ia+ax)^2}\,dx, x, \cot(c+dx)\right)}{d}$$
$$= -\frac{\text{Subst}\left(\int \frac{\left(-\frac{i}{a}+\frac{x}{a}\right)^3(ia+ax)}{x^6}\,dx, x, \cot(c+dx)\right)}{d}$$
$$= -\frac{\text{Subst}\left(\int \left(-\frac{1}{a^2x^6}-\frac{2i}{a^2x^5}-\frac{2i}{a^2x^3}+\frac{1}{a^2x^2}\right)dx, x, \cot(c+dx)\right)}{d}$$
$$= \frac{\tan(c+dx)}{a^2d}-\frac{i\tan^2(c+dx)}{a^2d}-\frac{i\tan^4(c+dx)}{2a^2d}-\frac{\tan^5(c+dx)}{5a^2d}$$

Mathematica [A] time = 0.41, size = 77, normalized size = 1.10

$$\frac{\sec(c)\sec^5(c+dx)(-5\sin(2c+dx)+5\sin(2c+3dx)+\sin(4c+5dx)-5i\cos(2c+dx)+5\sin(dx)-5i\cos(dx}{20a^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^6/(a*Cos[c + d*x] + I*a*Sin[c + d*x])^2,x]

[Out] (Sec[c]*Sec[c + d*x]^5*((-5*I)*Cos[d*x] - (5*I)*Cos[2*c + d*x] + 5*Sin[d*x]
 - 5*Sin[2*c + d*x] + 5*Sin[2*c + 3*d*x] + Sin[4*c + 5*d*x]))/(20*a^2*d)
```

fricas [A] time = 0.81, size = 97, normalized size = 1.39

$$\frac{40i\,e^{(2i\,dx+2i\,c)}+8i}{5\left(a^2de^{(10i\,dx+10i\,c)}+5\,a^2de^{(8i\,dx+8i\,c)}+10\,a^2de^{(6i\,dx+6i\,c)}+10\,a^2de^{(4i\,dx+4i\,c)}+5\,a^2de^{(2i\,dx+2i\,c)}+a^2d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^6/(a*cos(d*x+c)+I*a*sin(d*x+c))^2,x, algorithm="fricas
")

[Out] 1/5*(40*I*e^(2*I*d*x + 2*I*c) + 8*I)/(a^2*d*e^(10*I*d*x + 10*I*c) + 5*a^2*d
*e^(8*I*d*x + 8*I*c) + 10*a^2*d*e^(6*I*d*x + 6*I*c) + 10*a^2*d*e^(4*I*d*x +
 4*I*c) + 5*a^2*d*e^(2*I*d*x + 2*I*c) + a^2*d)
```

giac [A] time = 0.23, size = 47, normalized size = 0.67

$$-\frac{2\tan(dx+c)^5+5i\tan(dx+c)^4+10i\tan(dx+c)^2-10\tan(dx+c)}{10\,a^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^6/(a*cos(d*x+c)+I*a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/10*(2*tan(d*x + c)^5 + 5*I*tan(d*x + c)^4 + 10*I*tan(d*x + c)^2 - 10*tan
(d*x + c))/(a^2*d)
```

maple [A] time = 0.25, size = 47, normalized size = 0.67

$$\frac{\tan(dx+c) - \frac{\left(\tan^5(dx+c)\right)}{5} - \frac{i\left(\tan^4(dx+c)\right)}{2} - i\left(\tan^2(dx+c)\right)}{d\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^6/(a*cos(d*x+c)+I*a*sin(d*x+c))^2,x)
```

```
[Out] 1/d/a^2*(tan(d*x+c)-1/5*tan(d*x+c)^5-1/2*I*tan(d*x+c)^4-I*tan(d*x+c)^2)
```

maxima [A] time = 0.34, size = 47, normalized size = 0.67

$$-\frac{6\tan(dx+c)^5 + 15i\tan(dx+c)^4 + 30i\tan(dx+c)^2 - 30\tan(dx+c)}{30\,a^2 d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^6/(a*cos(d*x+c)+I*a*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] -1/30*(6*tan(d*x + c)^5 + 15*I*tan(d*x + c)^4 + 30*I*tan(d*x + c)^2 - 30*ta
n(d*x + c))/(a^2*d)
```

mupad [B] time = 0.92, size = 76, normalized size = 1.09

$$-\frac{\sin(c+d\,x)\left(-4\cos(c+d\,x)^4 + \frac{5\mathrm{i}\,\sin(c+d\,x)\cos(c+d\,x)^3}{2} - 2\cos(c+d\,x)^2 + \frac{5\mathrm{i}\,\sin(c+d\,x)\,\cos(c+d\,x)}{2} + 1\right)}{5\,a^2\,d\cos(c+d\,x)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^6*(a*cos(c + d*x) + a*sin(c + d*x)*1i)^2),x)
```

```
[Out] -(sin(c + d*x)*((cos(c + d*x)*sin(c + d*x)*5i)/2 + (cos(c + d*x)^3*sin(c +
d*x)*5i)/2 - 2*cos(c + d*x)^2 - 4*cos(c + d*x)^4 + 1))/(5*a^2*d*cos(c + d*x
)^5)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**6/(a*cos(d*x+c)+I*a*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

3.175 $\int \frac{\cos^5(c+dx)}{(a\cos(c+dx)+ia\sin(c+dx))^3}\,dx$

Optimal. Leaf size=125

$$-\frac{1}{32a^3d(-\cot(c+dx)+i)}+\frac{13}{16a^3d(\cot(c+dx)+i)}-\frac{23i}{32a^3d(\cot(c+dx)+i)^2}-\frac{1}{3a^3d(\cot(c+dx)+i)^3}+\frac{}{16a^3d(\cot(}$$

```
[Out] 5/32*x/a^3-1/32/a^3/d/(I-cot(d*x+c))+1/16*I/a^3/d/(I+cot(d*x+c))^4-1/3/a^3/
d/(I+cot(d*x+c))^3-23/32*I/a^3/d/(I+cot(d*x+c))^2+13/16/a^3/d/(I+cot(d*x+c)
)
```

Rubi [A] time = 0.11, antiderivative size = 125, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.129, Rules used = {3088, 848, 88, 203}

$$-\frac{1}{32a^3d(-\cot(c+dx)+i)}+\frac{13}{16a^3d(\cot(c+dx)+i)}-\frac{23i}{32a^3d(\cot(c+dx)+i)^2}-\frac{1}{3a^3d(\cot(c+dx)+i)^3}+\frac{}{16a^3d(\cot(}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^5/(a*Cos[c + d*x] + I*a*Sin[c + d*x])^3,x]
```

```
[Out] (5*x)/(32*a^3) - 1/(32*a^3*d*(I - Cot[c + d*x])) + (I/16)/(a^3*d*(I + Cot[c
 + d*x])^4) - 1/(3*a^3*d*(I + Cot[c + d*x])^3) - ((23*I)/32)/(a^3*d*(I + Co
t[c + d*x])^2) + 13/(16*a^3*d*(I + Cot[c + d*x]))
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 848

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_.), x_Symbol] :> Int[(d + e*x)^(m + p)*(f + g*x)^n*(a/d + (c*x)/e)^p,
x] /; FreeQ[{a, c, d, e, f, g, m, n}, x] && NeQ[e*f - d*g, 0] && EqQ[c*d^2
+ a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && EqQ[m + p, 0]))
```

Rule 3088

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[(x^m*(b +
 a*x)^n)/(1 + x^2)^((m + n + 2)/2), x], x, Cot[c + d*x]], x] /; FreeQ[{a, b
, c, d}, x] && IntegerQ[n] && IntegerQ[(m + n)/2] && NeQ[n, -1] &&  !(GtQ[n
, 0] && GtQ[m, 1])
```

Rubi steps

$$\int \frac{\cos^5(c+dx)}{(a\cos(c+dx)+ia\sin(c+dx))^3}\,dx = -\frac{\text{Subst}\left(\int \frac{x^5}{(ia+ax)^3\left(1+x^2\right)^2}\,dx, x, \cot(c+dx)\right)}{d}$$

$$= -\frac{\text{Subst}\left(\int \frac{x^5}{\left(-\frac{i}{a}+\frac{x}{a}\right)^2(ia+ax)^5}\,dx, x, \cot(c+dx)\right)}{d}$$

$$= -\frac{\text{Subst}\left(\int \left(\frac{1}{32a^3(-i+x)^2}+\frac{i}{4a^3(i+x)^5}-\frac{1}{a^3(i+x)^4}-\frac{23i}{16a^3(i+x)^3}+\frac{13}{16a^3(i+x)^2}+\frac{}{32}\right.\right.}{d}$$

$$= -\frac{1}{32a^3d(i-\cot(c+dx))}+\frac{i}{16a^3d(i+\cot(c+dx))^4}-\frac{1}{3a^3d(i+\cot(c+}$$

$$= \frac{5x}{32a^3}-\frac{1}{32a^3d(i-\cot(c+dx))}+\frac{i}{16a^3d(i+\cot(c+dx))^4}-\frac{}{3a^3d(i+c}$$

Mathematica [A] time = 0.19, size = 106, normalized size = 0.85

$$\frac{132\sin(2(c+dx))+60\sin(4(c+dx))+20\sin(6(c+dx))+3\sin(8(c+dx))+108i\cos(2(c+dx))+60i\cos(4(}{768a^3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^5/(a*Cos[c + d*x] + I*a*Sin[c + d*x])^3,x]
```

```
[Out] (120*c + 120*d*x + (108*I)*Cos[2*(c + d*x)] + (60*I)*Cos[4*(c + d*x)] + (20
*I)*Cos[6*(c + d*x)] + (3*I)*Cos[8*(c + d*x)] + 132*Sin[2*(c + d*x)] + 60*S
in[4*(c + d*x)] + 20*Sin[6*(c + d*x)] + 3*Sin[8*(c + d*x)])/(768*a^3*d)
```

fricas [A] time = 1.26, size = 76, normalized size = 0.61

$$\frac{\left(120\,dxe^{(8i\,dx+8i\,c)}-12i\,e^{(10i\,dx+10i\,c)}+120i\,e^{(6i\,dx+6i\,c)}+60i\,e^{(4i\,dx+4i\,c)}+20i\,e^{(2i\,dx+2i\,c)}+3i\right)e^{(-8i\,dx-8i\,c)}}{768\,a^3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5/(a*cos(d*x+c)+I*a*sin(d*x+c))^3,x, algorithm="fricas
")
```

```
[Out] 1/768*(120*d*x*e^(8*I*d*x + 8*I*c) - 12*I*e^(10*I*d*x + 10*I*c) + 120*I*e^(
6*I*d*x + 6*I*c) + 60*I*e^(4*I*d*x + 4*I*c) + 20*I*e^(2*I*d*x + 2*I*c) + 3*
I)*e^(-8*I*d*x - 8*I*c)/(a^3*d)
```

giac [A] time = 0.28, size = 119, normalized size = 0.95

$$-\frac{-\frac{60i\log(-i\tan(dx+c)+1)}{a^3}+\frac{60i\log(-i\tan(dx+c)-1)}{a^3}-\frac{12(5\tan(dx+c)+7i)}{a^3(i\tan(dx+c)-1)}+\frac{-125i\tan(dx+c)^4-596\tan(dx+c)^3+1110i\tan(dx+c)^2+996\tan}{a^3(\tan(dx+c)-i)^4}}{768d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5/(a*cos(d*x+c)+I*a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -1/768*(-60*I*log(-I*tan(d*x + c) + 1)/a^3 + 60*I*log(-I*tan(d*x + c) - 1)/
a^3 - 12*(5*tan(d*x + c) + 7*I)/(a^3*(I*tan(d*x + c) - 1)) + (-125*I*tan(d*
x + c)^4 - 596*tan(d*x + c)^3 + 1110*I*tan(d*x + c)^2 + 996*tan(d*x + c) -
405*I)/(a^3*(tan(d*x + c) - I)^4))/d
```

maple [A] time = 0.20, size = 137, normalized size = 1.10

$$\frac{5i\ln(\tan(dx+c)+i)}{64a^3d}+\frac{1}{32a^3d(\tan(dx+c)+i)}-\frac{5i\ln(\tan(dx+c)-i)}{64a^3d}+\frac{i}{16a^3d(\tan(dx+c)-i)^4}-\frac{3i}{32a^3d(\tan(d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5/(a*cos(d*x+c)+I*a*sin(d*x+c))^3,x)
```

```
[Out] 5/64*I/a^3/d*ln(tan(d*x+c)+I)+1/32/a^3/d/(tan(d*x+c)+I)-5/64*I/a^3/d*ln(tan
(d*x+c)-I)+1/16*I/a^3/d/(tan(d*x+c)-I)^4-3/32*I/a^3/d/(tan(d*x+c)-I)^2-1/12
/a^3/d/(tan(d*x+c)-I)^3+1/8/a^3/d/(tan(d*x+c)-I)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5/(a*cos(d*x+c)+I*a*sin(d*x+c))^3,x, algorithm="maxima
")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

mupad [B] time = 4.79, size = 164, normalized size = 1.31

$$\frac{5\,x}{32\,a^3}+\frac{-\frac{27\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^9}{16}+\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8\,33\mathrm{i}}{8}+\frac{31\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7}{6}+\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6\,9\mathrm{i}}{8}+\frac{89\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5}{24}-\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4\,9\mathrm{i}}{8}+\frac{31\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{6}}{a^3\,d\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+1\mathrm{i}\right)^2\left(1+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,1\mathrm{i}\right)^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^5/(a*cos(c + d*x) + a*sin(c + d*x)*1i)^3,x)
```

```
[Out] (5*x)/(32*a^3) + ((31*tan(c/2 + (d*x)/2)^3)/6 - (tan(c/2 + (d*x)/2)^2*33i)/
8 - (27*tan(c/2 + (d*x)/2))/16 - (tan(c/2 + (d*x)/2)^4*9i)/8 + (89*tan(c/2
+ (d*x)/2)^5)/24 + (tan(c/2 + (d*x)/2)^6*9i)/8 + (31*tan(c/2 + (d*x)/2)^7)/
6 + (tan(c/2 + (d*x)/2)^8*33i)/8 - (27*tan(c/2 + (d*x)/2)^9)/16)/(a^3*d*(ta
n(c/2 + (d*x)/2) + 1i)^2*(tan(c/2 + (d*x)/2)*1i + 1)^8)
```

sympy [A] time = 0.44, size = 228, normalized size = 1.82

$$\begin{cases} -\frac{\left(100663296ia^{12}d^4e^{22ic}e^{2idx}-1006632960ia^{12}d^4e^{18ic}e^{-2idx}-503316480ia^{12}d^4e^{16ic}e^{-4idx}-167772160ia^{12}d^4e^{14ic}e^{-6idx}-25165824ia^{12}d^4e^{12ic}e^{-8idx}\right)e}{6442450944a^{15}d^5} \\ x\left(\frac{\left(e^{10ic}+5e^{8ic}+10e^{6ic}+10e^{4ic}+5e^{2ic}+1\right)e^{-8ic}}{32a^3}-\frac{5}{32a^3}\right) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5/(a*cos(d*x+c)+I*a*sin(d*x+c))**3,x)
```

```
[Out] Piecewise((-(100663296*I*a**12*d**4*exp(22*I*c)*exp(2*I*d*x) - 1006632960*I
*a**12*d**4*exp(18*I*c)*exp(-2*I*d*x) - 503316480*I*a**12*d**4*exp(16*I*c)*
exp(-4*I*d*x) - 167772160*I*a**12*d**4*exp(14*I*c)*exp(-6*I*d*x) - 25165824
*I*a**12*d**4*exp(12*I*c)*exp(-8*I*d*x))*exp(-20*I*c)/(6442450944*a**15*d**
5), Ne(6442450944*a**15*d**5*exp(20*I*c), 0)), (x*((exp(10*I*c) + 5*exp(8*I
*c) + 10*exp(6*I*c) + 10*exp(4*I*c) + 5*exp(2*I*c) + 1)*exp(-8*I*c)/(32*a**
3) - 5/(32*a**3)), True)) + 5*x/(32*a**3)
```

3.176 $\int \frac{\cos^4(c+dx)}{(a\cos(c+dx)+ia\sin(c+dx))^3}\,dx$

Optimal. Leaf size=106

$$-\frac{4\sin^7(c+dx)}{7a^3d}+\frac{9\sin^5(c+dx)}{5a^3d}-\frac{2\sin^3(c+dx)}{a^3d}+\frac{\sin(c+dx)}{a^3d}+\frac{4i\cos^7(c+dx)}{7a^3d}-\frac{i\cos^5(c+dx)}{5a^3d}$$

```
[Out] -1/5*I*cos(d*x+c)^5/a^3/d+4/7*I*cos(d*x+c)^7/a^3/d+sin(d*x+c)/a^3/d-2*sin(d
*x+c)^3/a^3/d+9/5*sin(d*x+c)^5/a^3/d-4/7*sin(d*x+c)^7/a^3/d
```

Rubi [A] time = 0.23, antiderivative size = 106, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 8, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.258, Rules used = {3092, 3090, 2633, 2565, 30, 2564, 270, 14}

$$-\frac{4\sin^7(c+dx)}{7a^3d}+\frac{9\sin^5(c+dx)}{5a^3d}-\frac{2\sin^3(c+dx)}{a^3d}+\frac{\sin(c+dx)}{a^3d}+\frac{4i\cos^7(c+dx)}{7a^3d}-\frac{i\cos^5(c+dx)}{5a^3d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^4/(a*Cos[c + d*x] + I*a*Sin[c + d*x])^3,x]
```

```
[Out] ((-I/5)*Cos[c + d*x]^5)/(a^3*d) + (((4*I)/7)*Cos[c + d*x]^7)/(a^3*d) + Sin[
c + d*x]/(a^3*d) - (2*Sin[c + d*x]^3)/(a^3*d) + (9*Sin[c + d*x]^5)/(5*a^3*d
) - (4*Sin[c + d*x]^7)/(7*a^3*d)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2564

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
```

```
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 2565

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> -Dist[(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 3090

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_.), x_Symbol] :> Int[ExpandTrig[cos[c + d*x]^m*(a
*cos[c + d*x] + b*sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d}, x] && Inte
gerQ[m] && IGtQ[n, 0]
```

Rule 3092

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[a^n*b^n, Int[Cos[c + d*x]^m/
(b*Cos[c + d*x] + a*Sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, m}, x] &&
 EqQ[a^2 + b^2, 0] && ILtQ[n, 0]
```

Rubi steps

$$\int \frac{\cos^4(c+dx)}{(a\cos(c+dx)+ia\sin(c+dx))^3}\,dx = \frac{i\int \cos^4(c+dx)(ia\cos(c+dx)+a\sin(c+dx))^3\,dx}{a^6}$$

$$= \frac{i\int \left(-ia^3\cos^7(c+dx)-3a^3\cos^6(c+dx)\sin(c+dx)+3ia^3\cos^5(c+dx\right.}{a^6}$$

$$= \frac{i\int \cos^4(c+dx)\sin^3(c+dx)\,dx}{a^3} - \frac{(3i)\int \cos^6(c+dx)\sin(c+dx)\,dx}{a^3} + $$

$$= -\frac{i\,\text{Subst}\left(\int x^4\left(1-x^2\right)\,dx, x, \cos(c+dx)\right)}{a^3d} + \frac{(3i)\,\text{Subst}\left(\int x^6\,dx, x, \cos\right.}{a^3d}$$

$$= \frac{3i\cos^7(c+dx)}{7a^3d} + \frac{\sin(c+dx)}{a^3d} - \frac{\sin^3(c+dx)}{a^3d} + \frac{3\sin^5(c+dx)}{5a^3d} - \frac{\sin^7(c}{7a}$$

$$= -\frac{i\cos^5(c+dx)}{5a^3d} + \frac{4i\cos^7(c+dx)}{7a^3d} + \frac{\sin(c+dx)}{a^3d} - \frac{2\sin^3(c+dx)}{a^3d} + \frac{9\,\text{si}}{}$$

Mathematica [A] time = 0.08, size = 149, normalized size = 1.41

$$\frac{5\sin(c+dx)}{16a^3d}+\frac{\sin(3(c+dx))}{8a^3d}+\frac{\sin(5(c+dx))}{20a^3d}+\frac{\sin(7(c+dx))}{112a^3d}+\frac{3i\cos(c+dx)}{16a^3d}+\frac{i\cos(3(c+dx))}{8a^3d}+\frac{i\cos(5(c+dx))}{20a^3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^4/(a*Cos[c + d*x] + I*a*Sin[c + d*x])^3,x]
```

```
[Out] (((3*I)/16)*Cos[c + d*x])/(a^3*d) + ((I/8)*Cos[3*(c + d*x)])/(a^3*d) + ((I/
20)*Cos[5*(c + d*x)])/(a^3*d) + ((I/112)*Cos[7*(c + d*x)])/(a^3*d) + (5*Sin
[c + d*x])/(16*a^3*d) + Sin[3*(c + d*x)]/(8*a^3*d) + Sin[5*(c + d*x)]/(20*a
^3*d) + Sin[7*(c + d*x)]/(112*a^3*d)
```

fricas [A] time = 2.12, size = 63, normalized size = 0.59

$$\frac{\left(-35i\,e^{(8i\,dx+8i\,c)}+140i\,e^{(6i\,dx+6i\,c)}+70i\,e^{(4i\,dx+4i\,c)}+28i\,e^{(2i\,dx+2i\,c)}+5i\right)e^{(-7i\,dx-7i\,c)}}{560\,a^3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4/(a*cos(d*x+c)+I*a*sin(d*x+c))^3,x, algorithm="fricas
")
```

```
[Out] 1/560*(-35*I*e^(8*I*d*x + 8*I*c) + 140*I*e^(6*I*d*x + 6*I*c) + 70*I*e^(4*I*
d*x + 4*I*c) + 28*I*e^(2*I*d*x + 2*I*c) + 5*I)*e^(-7*I*d*x - 7*I*c)/(a^3*d)
```

giac [A] time = 0.26, size = 119, normalized size = 1.12

$$\frac{\frac{35}{a^3\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+i\right)}+\frac{525\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^6-1960i\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-4025\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4+4480i\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+3143\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1176i\,\mathrm{t}}{a^3\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-i\right)^7}}{280\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4/(a*cos(d*x+c)+I*a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/280*(35/(a^3*(tan(1/2*d*x + 1/2*c) + I)) + (525*tan(1/2*d*x + 1/2*c)^6 -
1960*I*tan(1/2*d*x + 1/2*c)^5 - 4025*tan(1/2*d*x + 1/2*c)^4 + 4480*I*tan(1/
2*d*x + 1/2*c)^3 + 3143*tan(1/2*d*x + 1/2*c)^2 - 1176*I*tan(1/2*d*x + 1/2*c
) - 243)/(a^3*(tan(1/2*d*x + 1/2*c) - I)^7))/d
```

maple [A] time = 0.19, size = 141, normalized size = 1.33

$$\frac{\frac{2}{16\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+16i}+\frac{4i}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-i\right)^6}-\frac{9i}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-i\right)^4}+\frac{17i}{4\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-i\right)^2}-\frac{8}{7\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-i\right)^7}+\frac{38}{5\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-i\right)^5}-\frac{15}{2\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-i\right.}}{d\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4/(a*cos(d*x+c)+I*a*sin(d*x+c))^3,x)
```

```
[Out] 2/d/a^3*(1/16/(tan(1/2*d*x+1/2*c)+I)+2*I/(tan(1/2*d*x+1/2*c)-I)^6-9/2*I/(ta
n(1/2*d*x+1/2*c)-I)^4+17/8*I/(tan(1/2*d*x+1/2*c)-I)^2-4/7/(tan(1/2*d*x+1/2*
c)-I)^7+19/5/(tan(1/2*d*x+1/2*c)-I)^5-15/4/(tan(1/2*d*x+1/2*c)-I)^3+15/16/(
tan(1/2*d*x+1/2*c)-I))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4/(a*cos(d*x+c)+I*a*sin(d*x+c))^3,x, algorithm="maxima
")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

mupad [B] time = 3.17, size = 134, normalized size = 1.26

$$-\frac{\left(35\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6 105\mathrm{i}-175\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4 105\mathrm{i}-7\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3+\tan\left(\frac{c}{2}\right.\right.}{35\,a^3\,d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+1\mathrm{i}\right)\,\left(1+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,1\mathrm{i}\right)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^4/(a*cos(c + d*x) + a*sin(c + d*x)*1i)^3,x)
```

```
[Out] -((43*tan(c/2 + (d*x)/2) + tan(c/2 + (d*x)/2)^2*77i - 7*tan(c/2 + (d*x)/2)^
3 + tan(c/2 + (d*x)/2)^4*105i - 175*tan(c/2 + (d*x)/2)^5 - tan(c/2 + (d*x)/
2)^6*105i + 35*tan(c/2 + (d*x)/2)^7 - 13i)*2i)/(35*a^3*d*(tan(c/2 + (d*x)/2
) + 1i)*(tan(c/2 + (d*x)/2)*1i + 1)^7)
```

sympy [A] time = 0.48, size = 201, normalized size = 1.90

$$\begin{cases} -\frac{\left(71680ia^{12}d^4e^{17ic}e^{idx}-286720ia^{12}d^4e^{15ic}e^{-idx}-143360ia^{12}d^4e^{13ic}e^{-3idx}-57344ia^{12}d^4e^{11ic}e^{-5idx}-10240ia^{12}d^4e^{9ic}e^{-7idx}\right)e^{-16ic}}{1146880a^{15}d^5} & \text{for } 1146880a^{1 \\ \frac{x\left(e^{8ic}+4e^{6ic}+6e^{4ic}+4e^{2ic}+1\right)e^{-7ic}}{16a^3} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4/(a*cos(d*x+c)+I*a*sin(d*x+c))**3,x)
```

```
[Out] Piecewise((-(71680*I*a**12*d**4*exp(17*I*c)*exp(I*d*x) - 286720*I*a**12*d**
4*exp(15*I*c)*exp(-I*d*x) - 143360*I*a**12*d**4*exp(13*I*c)*exp(-3*I*d*x) -
 57344*I*a**12*d**4*exp(11*I*c)*exp(-5*I*d*x) - 10240*I*a**12*d**4*exp(9*I*
c)*exp(-7*I*d*x))*exp(-16*I*c)/(1146880*a**15*d**5), Ne(1146880*a**15*d**5*
exp(16*I*c), 0)), (x*(exp(8*I*c) + 4*exp(6*I*c) + 6*exp(4*I*c) + 4*exp(2*I*
c) + 1)*exp(-7*I*c)/(16*a**3), True))
```

3.177 $\int \frac{\cos^3(c+dx)}{(a\cos(c+dx)+ia\sin(c+dx))^3}\,dx$

Optimal. Leaf size=131

$$\frac{i\cos(c+dx)}{8d\left(a^3\cos(c+dx)+ia^3\sin(c+dx)\right)}+\frac{x}{8a^3}+\frac{i\cos^3(c+dx)}{6d(a\cos(c+dx)+ia\sin(c+dx))^3}+\frac{i\cos^2(c+dx)}{8ad(a\cos(c+dx)+ia\sin(c+}$$

```
[Out] 1/8*x/a^3+1/6*I*cos(d*x+c)^3/d/(a*cos(d*x+c)+I*a*sin(d*x+c))^3+1/8*I*cos(d*
x+c)^2/a/d/(a*cos(d*x+c)+I*a*sin(d*x+c))^2+1/8*I*cos(d*x+c)/d/(a^3*cos(d*x+
c)+I*a^3*sin(d*x+c))
```

Rubi [A] time = 0.14, antiderivative size = 131, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.065, Rules used = {3082, 8}

$$\frac{i\cos(c+dx)}{8d\left(a^3\cos(c+dx)+ia^3\sin(c+dx)\right)}+\frac{x}{8a^3}+\frac{i\cos^3(c+dx)}{6d(a\cos(c+dx)+ia\sin(c+dx))^3}+\frac{i\cos^2(c+dx)}{8ad(a\cos(c+dx)+ia\sin(c+}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^3/(a*Cos[c + d*x] + I*a*Sin[c + d*x])^3,x]
```

```
[Out] x/(8*a^3) + ((I/6)*Cos[c + d*x]^3)/(d*(a*Cos[c + d*x] + I*a*Sin[c + d*x])^3
) + ((I/8)*Cos[c + d*x]^2)/(a*d*(a*Cos[c + d*x] + I*a*Sin[c + d*x])^2) + ((
I/8)*Cos[c + d*x])/(d*(a^3*Cos[c + d*x] + I*a^3*Sin[c + d*x]))
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3082

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*(a*Cos[c + d*x] + b*Sin[
c + d*x])^n)/(2*a*d*n*Cos[c + d*x]^n), x] + Dist[1/(2*a), Int[(a*Cos[c + d*
x] + b*Sin[c + d*x])^(n + 1)/Cos[c + d*x]^(n + 1), x], x] /; FreeQ[{a, b, c
, d}, x] && EqQ[m + n, 0] && EqQ[a^2 + b^2, 0] && LtQ[n, 0]
```

Rubi steps

$$\int \frac{\cos^3(c+dx)}{(a\cos(c+dx)+ia\sin(c+dx))^3}\,dx = \frac{i\cos^3(c+dx)}{6d(a\cos(c+dx)+ia\sin(c+dx))^3} + \frac{\int \frac{\cos^2(c+dx)}{(a\cos(c+dx)+ia\sin(c+dx))^2}\,dx}{2a}$$

$$= \frac{i\cos^3(c+dx)}{6d(a\cos(c+dx)+ia\sin(c+dx))^3} + \frac{i\cos^2(c+dx)}{8ad(a\cos(c+dx)+ia\sin(c+dx)}$$

$$= \frac{i\cos^3(c+dx)}{6d(a\cos(c+dx)+ia\sin(c+dx))^3} + \frac{i\cos^2(c+dx)}{8ad(a\cos(c+dx)+ia\sin(c+dx)}$$

$$= \frac{x}{8a^3} + \frac{i\cos^3(c+dx)}{6d(a\cos(c+dx)+ia\sin(c+dx))^3} + \frac{i\cos^2(c+dx)}{8ad(a\cos(c+dx)+ia\sin(}$$

Mathematica [A] time = 0.11, size = 84, normalized size = 0.64

$$\frac{18\sin(2(c+dx))+9\sin(4(c+dx))+2\sin(6(c+dx))+18i\cos(2(c+dx))+9i\cos(4(c+dx))+2i\cos(6(c+dx)}{96a^3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^3/(a*Cos[c + d*x] + I*a*Sin[c + d*x])^3,x]
```

```
[Out] (12*c + 12*d*x + (18*I)*Cos[2*(c + d*x)] + (9*I)*Cos[4*(c + d*x)] + (2*I)*C
os[6*(c + d*x)] + 18*Sin[2*(c + d*x)] + 9*Sin[4*(c + d*x)] + 2*Sin[6*(c + d
*x)])/(96*a^3*d)
```

fricas [A] time = 0.62, size = 54, normalized size = 0.41

$$\frac{\left(12\,dxe^{(6i\,dx+6i\,c)} + 18i\,e^{(4i\,dx+4i\,c)} + 9i\,e^{(2i\,dx+2i\,c)} + 2i\right)e^{(-6i\,dx-6i\,c)}}{96\,a^3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3/(a*cos(d*x+c)+I*a*sin(d*x+c))^3,x, algorithm="fricas
")
```

```
[Out] 1/96*(12*d*x*e^(6*I*d*x + 6*I*c) + 18*I*e^(4*I*d*x + 4*I*c) + 9*I*e^(2*I*d*
x + 2*I*c) + 2*I)*e^(-6*I*d*x - 6*I*c)/(a^3*d)
```

giac [A] time = 0.25, size = 80, normalized size = 0.61

$$-\frac{\frac{6i\,\log(\tan(dx+c)-i)}{a^3} - \frac{6i\,\log(i\,\tan(dx+c)-1)}{a^3} + \frac{-11i\,\tan(dx+c)^3-45\,\tan(dx+c)^2+69i\,\tan(dx+c)+51}{a^3(\tan(dx+c)-i)^3}}{96\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3/(a*cos(d*x+c)+I*a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -1/96*(6*I*log(tan(d*x + c) - I)/a^3 - 6*I*log(I*tan(d*x + c) - 1)/a^3 + (-
11*I*tan(d*x + c)^3 - 45*tan(d*x + c)^2 + 69*I*tan(d*x + c) + 51)/(a^3*(tan
(d*x + c) - I)^3))/d
```

maple [A] time = 0.19, size = 98, normalized size = 0.75

$$\frac{i\ln\left(\tan\left(dx+c\right)+i\right)}{16a^{3}d}-\frac{i\ln\left(\tan\left(dx+c\right)-i\right)}{16a^{3}d}-\frac{i}{8a^{3}d\left(\tan\left(dx+c\right)-i\right)^{2}}-\frac{1}{6a^{3}d\left(\tan\left(dx+c\right)-i\right)^{3}}+\frac{1}{8a^{3}d\left(\tan\left(dx+\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3/(a*cos(d*x+c)+I*a*sin(d*x+c))^3,x)
```

```
[Out] 1/16*I/a^3/d*ln(tan(d*x+c)+I)-1/16*I/a^3/d*ln(tan(d*x+c)-I)-1/8*I/a^3/d/(ta
n(d*x+c)-I)^2-1/6/a^3/d/(tan(d*x+c)-I)^3+1/8/a^3/d/(tan(d*x+c)-I)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3/(a*cos(d*x+c)+I*a*sin(d*x+c))^3,x, algorithm="maxima
")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

mupad [B] time = 3.49, size = 96, normalized size = 0.73

$$\frac{x}{8\,a^3}+\frac{\frac{7\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5}{4}-\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4\,9{}\mathrm{i}}{2}-\frac{41\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{6}+\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\,9{}\mathrm{i}}{2}+\frac{7\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{4}}{a^3\,d\,{\left(1+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,1{}\mathrm{i}\right)}^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^3/(a*cos(c + d*x) + a*sin(c + d*x)*1i)^3,x)
```

```
[Out] x/(8*a^3) + ((7*tan(c/2 + (d*x)/2))/4 + (tan(c/2 + (d*x)/2)^2*9i)/2 - (41*t
an(c/2 + (d*x)/2)^3)/6 - (tan(c/2 + (d*x)/2)^4*9i)/2 + (7*tan(c/2 + (d*x)/2
)^5)/4)/(a^3*d*(tan(c/2 + (d*x)/2)*1i + 1)^6)
```

<u>**sympy**</u> **[A]** time = 0.31, size = 160, normalized size = 1.22

$$\begin{cases} -\frac{\left(-4608ia^6d^2e^{10ic}e^{-2idx}-2304ia^6d^2e^{8ic}e^{-4idx}-512ia^6d^2e^{6ic}e^{-6idx}\right)e^{-12ic}}{24576a^9d^3} & \text{for } 24576a^9d^3e^{12ic}\neq 0 \\ x\left(\frac{\left(e^{6ic}+3e^{4ic}+3e^{2ic}+1\right)e^{-6ic}}{8a^3}-\frac{1}{8a^3}\right) & \text{otherwise} \end{cases} + \frac{x}{8a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3/(a*cos(d*x+c)+I*a*sin(d*x+c))**3,x)
```

```
[Out] Piecewise((-(-4608*I*a**6*d**2*exp(10*I*c)*exp(-2*I*d*x) - 2304*I*a**6*d**2
*exp(8*I*c)*exp(-4*I*d*x) - 512*I*a**6*d**2*exp(6*I*c)*exp(-6*I*d*x))*exp(-
12*I*c)/(24576*a**9*d**3), Ne(24576*a**9*d**3*exp(12*I*c), 0)), (x*((exp(6*
I*c) + 3*exp(4*I*c) + 3*exp(2*I*c) + 1)*exp(-6*I*c)/(8*a**3) - 1/(8*a**3)),
 True)) + x/(8*a**3)
```

3.178 $\int \frac{\cos^2(c+dx)}{(a\cos(c+dx)+ia\sin(c+dx))^3}\,dx$

Optimal. Leaf size=90

$$\frac{4\sin^5(c+dx)}{5a^3d}-\frac{5\sin^3(c+dx)}{3a^3d}+\frac{\sin(c+dx)}{a^3d}+\frac{4i\cos^5(c+dx)}{5a^3d}-\frac{i\cos^3(c+dx)}{3a^3d}$$

```
[Out] -1/3*I*cos(d*x+c)^3/a^3/d+4/5*I*cos(d*x+c)^5/a^3/d+sin(d*x+c)/a^3/d-5/3*sin
(d*x+c)^3/a^3/d+4/5*sin(d*x+c)^5/a^3/d
```

Rubi [A] time = 0.22, antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 7, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.226, Rules used = {3092, 3090, 2633, 2565, 30, 2564, 14}

$$\frac{4\sin^5(c+dx)}{5a^3d}-\frac{5\sin^3(c+dx)}{3a^3d}+\frac{\sin(c+dx)}{a^3d}+\frac{4i\cos^5(c+dx)}{5a^3d}-\frac{i\cos^3(c+dx)}{3a^3d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2/(a*Cos[c + d*x] + I*a*Sin[c + d*x])^3,x]
```

```
[Out] ((-I/3)*Cos[c + d*x]^3)/(a^3*d) + (((4*I)/5)*Cos[c + d*x]^5)/(a^3*d) + Sin[
c + d*x]/(a^3*d) - (5*Sin[c + d*x]^3)/(3*a^3*d) + (4*Sin[c + d*x]^5)/(5*a^3
*d)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2564

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 2565

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> -Dist[(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 3090

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_.), x_Symbol] :> Int[ExpandTrig[cos[c + d*x]^m*(a
*cos[c + d*x] + b*sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d}, x] && Inte
gerQ[m] && IGtQ[n, 0]
```

Rule 3092

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[a^n*b^n, Int[Cos[c + d*x]^m/
(b*Cos[c + d*x] + a*Sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, m}, x] &&
 EqQ[a^2 + b^2, 0] && ILtQ[n, 0]
```

Rubi steps

$$\int \frac{\cos^2(c+dx)}{(a\cos(c+dx)+ia\sin(c+dx))^3}\,dx = \frac{i\int \cos^2(c+dx)(ia\cos(c+dx)+a\sin(c+dx))^3\,dx}{a^6}$$

$$= \frac{i\int \left(-ia^3\cos^5(c+dx)-3a^3\cos^4(c+dx)\sin(c+dx)+3ia^3\cos^3(c+dx\right.}{a^6}$$

$$= \frac{i\int \cos^2(c+dx)\sin^3(c+dx)\,dx}{a^3}-\frac{(3i)\int \cos^4(c+dx)\sin(c+dx)\,dx}{a^3}+$$

$$= -\frac{i\,\text{Subst}\left(\int x^2\left(1-x^2\right)dx,x,\cos(c+dx)\right)}{a^3d}+\frac{(3i)\,\text{Subst}\left(\int x^4\,dx,x,\cos(\right.}{a^3d}$$

$$= \frac{3i\cos^5(c+dx)}{5a^3d}+\frac{\sin(c+dx)}{a^3d}-\frac{2\sin^3(c+dx)}{3a^3d}+\frac{\sin^5(c+dx)}{5a^3d}-\frac{i\,\text{Subs}}{}$$

$$= -\frac{i\cos^3(c+dx)}{3a^3d}+\frac{4i\cos^5(c+dx)}{5a^3d}+\frac{\sin(c+dx)}{a^3d}-\frac{5\sin^3(c+dx)}{3a^3d}+\frac{4\,\text{si}}{}$$

Mathematica [A] time = 0.07, size = 111, normalized size = 1.23

$$\frac{\sin(c+dx)}{4a^3d}+\frac{\sin(3(c+dx))}{6a^3d}+\frac{\sin(5(c+dx))}{20a^3d}+\frac{i\cos(c+dx)}{4a^3d}+\frac{i\cos(3(c+dx))}{6a^3d}+\frac{i\cos(5(c+dx))}{20a^3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2/(a*Cos[c + d*x] + I*a*Sin[c + d*x])^3,x]
```

```
[Out] ((I/4)*Cos[c + d*x])/(a^3*d) + ((I/6)*Cos[3*(c + d*x)])/(a^3*d) + ((I/20)*C
os[5*(c + d*x)])/(a^3*d) + Sin[c + d*x]/(4*a^3*d) + Sin[3*(c + d*x)]/(6*a^3
*d) + Sin[5*(c + d*x)]/(20*a^3*d)
```

fricas [A] time = 0.91, size = 41, normalized size = 0.46

$$\frac{\left(15i\,e^{(4i\,dx+4i\,c)}+10i\,e^{(2i\,dx+2i\,c)}+3i\right)e^{(-5i\,dx-5i\,c)}}{60\,a^3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(a*cos(d*x+c)+I*a*sin(d*x+c))^3,x, algorithm="fricas
")
```

```
[Out] 1/60*(15*I*e^(4*I*d*x + 4*I*c) + 10*I*e^(2*I*d*x + 2*I*c) + 3*I)*e^(-5*I*d*
x - 5*I*c)/(a^3*d)
```

giac [A] time = 1.50, size = 73, normalized size = 0.81

$$\frac{2\left(15\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4-30i\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-40\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+20i\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+7\right)}{15\,a^3d\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-i\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(a*cos(d*x+c)+I*a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 2/15*(15*tan(1/2*d*x + 1/2*c)^4 - 30*I*tan(1/2*d*x + 1/2*c)^3 - 40*tan(1/2*
d*x + 1/2*c)^2 + 20*I*tan(1/2*d*x + 1/2*c) + 7)/(a^3*d*(tan(1/2*d*x + 1/2*c
) - I)^5)
```

maple [A] time = 0.19, size = 90, normalized size = 1.00

$$\frac{-\frac{16}{3\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-i\right)^3}-\frac{4i}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-i\right)^4}+\frac{4i}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-i\right)^2}+\frac{2}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-i}+\frac{8}{5\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-i\right)^5}}{d\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2/(a*cos(d*x+c)+I*a*sin(d*x+c))^3,x)
```

```
[Out] 2/d/a^3*(-8/3/(tan(1/2*d*x+1/2*c)-I)^3-2*I/(tan(1/2*d*x+1/2*c)-I)^4+2*I/(ta
n(1/2*d*x+1/2*c)-I)^2+1/(tan(1/2*d*x+1/2*c)-I)+4/5/(tan(1/2*d*x+1/2*c)-I)^5
)
```

maxima [A] time = 0.53, size = 69, normalized size = 0.77

$$\frac{3i\cos(5dx+5c)+10i\cos(3dx+3c)+15i\cos(dx+c)+3\sin(5dx+5c)+10\sin(3dx+3c)+15\sin(dx}{60a^3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(a*cos(d*x+c)+I*a*sin(d*x+c))^3,x, algorithm="maxima
")
```

```
[Out] 1/60*(3*I*cos(5*d*x + 5*c) + 10*I*cos(3*d*x + 3*c) + 15*I*cos(d*x + c) + 3*
sin(5*d*x + 5*c) + 10*sin(3*d*x + 3*c) + 15*sin(d*x + c))/(a^3*d)
```

mupad [B] time = 0.87, size = 133, normalized size = 1.48

$$\frac{2\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4 15\mathrm{i}+30\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3-\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2 40\mathrm{i}-20\tan\left(\frac{c}{2}+\frac{dx}{2}\right)+7\mathrm{i}\right)}{15a^3d\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^5 1\mathrm{i}+5\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4-\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3 10\mathrm{i}-10\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2+\tan\left(\frac{c}{2}+\frac{dx}{2}\right)5\mathrm{i}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^2/(a*cos(c + d*x) + a*sin(c + d*x)*1i)^3,x)
```

```
[Out] (2*(30*tan(c/2 + (d*x)/2)^3 - tan(c/2 + (d*x)/2)^2*40i - 20*tan(c/2 + (d*x)
/2) + tan(c/2 + (d*x)/2)^4*15i + 7i))/(15*a^3*d*(tan(c/2 + (d*x)/2)*5i - 10
*tan(c/2 + (d*x)/2)^2 - tan(c/2 + (d*x)/2)^3*10i + 5*tan(c/2 + (d*x)/2)^4 +
 tan(c/2 + (d*x)/2)^5*1i + 1))
```

sympy [A] time = 0.35, size = 136, normalized size = 1.51

$$\begin{cases} -\frac{\left(-120ia^6d^2e^{8ic}e^{-idx}-80ia^6d^2e^{6ic}e^{-3idx}-24ia^6d^2e^{4ic}e^{-5idx}\right)e^{-9ic}}{480a^9d^3} & \text{for } 480a^9d^3e^{9ic}\neq 0 \\ \frac{x\left(e^{4ic}+2e^{2ic}+1\right)e^{-5ic}}{4a^3} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2/(a*cos(d*x+c)+I*a*sin(d*x+c))**3,x)
```

```
[Out] Piecewise((-(-120*I*a**6*d**2*exp(8*I*c)*exp(-I*d*x) - 80*I*a**6*d**2*exp(6
*I*c)*exp(-3*I*d*x) - 24*I*a**6*d**2*exp(4*I*c)*exp(-5*I*d*x))*exp(-9*I*c)/
(480*a**9*d**3), Ne(480*a**9*d**3*exp(9*I*c), 0)), (x*(exp(4*I*c) + 2*exp(2
*I*c) + 1)*exp(-5*I*c)/(4*a**3), True))
```

3.179 $\int \frac{\cos(c+dx)}{(a\cos(c+dx)+ia\sin(c+dx))^3}\,dx$

Optimal. Leaf size=32

$$\frac{i\cot^2(c+dx)}{2a^3d(\cot(c+dx)+i)^2}$$

```
[Out] 1/2*I*cot(d*x+c)^2/a^3/d/(I+cot(d*x+c))^2
```

Rubi [A] time = 0.03, antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.069, Rules used = {3088, 37}

$$\frac{i\cot^2(c+dx)}{2a^3d(\cot(c+dx)+i)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]/(a*Cos[c + d*x] + I*a*Sin[c + d*x])^3,x]
```

```
[Out] ((I/2)*Cot[c + d*x]^2)/(a^3*d*(I + Cot[c + d*x])^2)
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 3088

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[(x^m*(b +
 a*x)^n)/(1 + x^2)^((m + n + 2)/2), x], x, Cot[c + d*x]], x] /; FreeQ[{a, b
, c, d}, x] && IntegerQ[n] && IntegerQ[(m + n)/2] && NeQ[n, -1] &&  !(GtQ[n
, 0] && GtQ[m, 1])
```

Rubi steps

$$\begin{aligned}
\int \frac{\cos(c+dx)}{(a\cos(c+dx)+ia\sin(c+dx))^3}\,dx &= -\frac{\text{Subst}\left(\int \frac{x}{(ia+ax)^3}\,dx, x, \cot(c+dx)\right)}{d}\\
&= \frac{i\cot^2(c+dx)}{2a^3d(i+\cot(c+dx))^2}
\end{aligned}$$

Mathematica [B] time = 0.06, size = 77, normalized size = 2.41

$$\frac{\sin(2(c+dx))}{4a^3d}+\frac{\sin(4(c+dx))}{8a^3d}+\frac{i\cos(2(c+dx))}{4a^3d}+\frac{i\cos(4(c+dx))}{8a^3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]/(a*Cos[c + d*x] + I*a*Sin[c + d*x])^3,x]
```

```
[Out] ((I/4)*Cos[2*(c + d*x)])/(a^3*d) + ((I/8)*Cos[4*(c + d*x)])/(a^3*d) + Sin[2
*(c + d*x)]/(4*a^3*d) + Sin[4*(c + d*x)]/(8*a^3*d)
```

fricas [A] time = 0.67, size = 30, normalized size = 0.94

$$\frac{\left(2i\,e^{(2i\,dx+2i\,c)}+i\right)e^{(-4i\,dx-4i\,c)}}{8\,a^3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a*cos(d*x+c)+I*a*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/8*(2*I*e^(2*I*d*x + 2*I*c) + I)*e^(-4*I*d*x - 4*I*c)/(a^3*d)
```

giac [B] time = 0.97, size = 57, normalized size = 1.78

$$-\frac{2\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-i\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{a^3d\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-i\right)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a*cos(d*x+c)+I*a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -2*(tan(1/2*d*x + 1/2*c)^3 - I*tan(1/2*d*x + 1/2*c)^2 - tan(1/2*d*x + 1/2*c
))/(a^3*d*(tan(1/2*d*x + 1/2*c) - I)^4)
```

maple [A] time = 0.17, size = 23, normalized size = 0.72

$$\frac{i}{2d\,a^3\,(i\tan\,(dx+c)+1)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)/(a*cos(d*x+c)+I*a*sin(d*x+c))^3,x)
```

```
[Out] 1/2*I/d/a^3/(I*tan(d*x+c)+1)^2
```

maxima [A] time = 0.37, size = 51, normalized size = 1.59

$$\frac{i \cos(4\,dx+4\,c)+2i \cos(2\,dx+2\,c)+\sin(4\,dx+4\,c)+2 \sin(2\,dx+2\,c)}{8\,a^3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a*cos(d*x+c)+I*a*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/8*(I*cos(4*d*x + 4*c) + 2*I*cos(2*d*x + 2*c) + sin(4*d*x + 4*c) + 2*sin(2
*d*x + 2*c))/(a^3*d)
```

mupad [B] time = 0.75, size = 100, normalized size = 3.12

$$-\frac{2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2 1\mathrm{i}+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)-\mathrm{i}\right)}{a^3\,d\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4 1\mathrm{i}+4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2 6\mathrm{i}-4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+1\mathrm{i}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)/(a*cos(c + d*x) + a*sin(c + d*x)*1i)^3,x)
```

```
[Out] -(2*tan(c/2 + (d*x)/2)*(tan(c/2 + (d*x)/2) + tan(c/2 + (d*x)/2)^2*1i - 1i))
/(a^3*d*(4*tan(c/2 + (d*x)/2)^3 - tan(c/2 + (d*x)/2)^2*6i - 4*tan(c/2 + (d*
x)/2) + tan(c/2 + (d*x)/2)^4*1i + 1i))
```

sympy [A] time = 0.22, size = 97, normalized size = 3.03

$$\begin{cases} \frac{\left(8ia^3de^{4ic}e^{-2idx}+4ia^3de^{2ic}e^{-4idx}\right)e^{-6ic}}{32a^6d^2} & \text{for } 32a^6d^2e^{6ic}\neq 0 \\ \frac{x\left(e^{2ic}+1\right)e^{-4ic}}{2a^3} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a*cos(d*x+c)+I*a*sin(d*x+c))**3,x)
```

```
[Out] Piecewise((((8*I*a**3*d*exp(4*I*c)*exp(-2*I*d*x) + 4*I*a**3*d*exp(2*I*c)*exp
(-4*I*d*x))*exp(-6*I*c)/(32*a**6*d**2), Ne(32*a**6*d**2*exp(6*I*c), 0)), (x
*(exp(2*I*c) + 1)*exp(-4*I*c)/(2*a**3), True))
```

3.180 $\int \frac{1}{(a\cos(c+dx)+ia\sin(c+dx))^3}\,dx$

Optimal. Leaf size=31

$$\frac{i}{3d(a\cos(c+dx)+ia\sin(c+dx))^3}$$

```
[Out] 1/3*I/d/(a*cos(d*x+c)+I*a*sin(d*x+c))^3
```

Rubi [A] time = 0.02, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {3071}

$$\frac{i}{3d(a\cos(c+dx)+ia\sin(c+dx))^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Cos[c + d*x] + I*a*Sin[c + d*x])^(-3),x]
[Out] (I/3)/(d*(a*Cos[c + d*x] + I*a*Sin[c + d*x])^3)
```

Rule 3071

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x
_Symbol] :> Simp[(a*(a*Cos[c + d*x] + b*Sin[c + d*x])^n)/(b*d*n), x] /; Fre
eQ[{a, b, c, d, n}, x] && EqQ[a^2 + b^2, 0]
```

Rubi steps

$$\int \frac{1}{(a\cos(c+dx)+ia\sin(c+dx))^3}\,dx = \frac{i}{3d(a\cos(c+dx)+ia\sin(c+dx))^3}$$

Mathematica [A] time = 0.04, size = 31, normalized size = 1.00

$$\frac{i}{3d(a\cos(c+dx)+ia\sin(c+dx))^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Cos[c + d*x] + I*a*Sin[c + d*x])^(-3),x]
[Out] (I/3)/(d*(a*Cos[c + d*x] + I*a*Sin[c + d*x])^3)
```

fricas [A] time = 1.00, size = 17, normalized size = 0.55

$$\frac{i\,e^{(-3i\,dx-3i\,c)}}{3\,a^3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cos(d*x+c)+I*a*sin(d*x+c))^3,x, algorithm="fricas")
[Out] 1/3*I*e^(-3*I*d*x - 3*I*c)/(a^3*d)
```

giac [A] time = 1.90, size = 36, normalized size = 1.16

$$\frac{2\left(3\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1\right)}{3\,a^3d\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-i\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cos(d*x+c)+I*a*sin(d*x+c))^3,x, algorithm="giac")
[Out] 2/3*(3*tan(1/2*d*x + 1/2*c)^2 - 1)/(a^3*d*(tan(1/2*d*x + 1/2*c) - I)^3)
```

maple [B] time = 0.17, size = 57, normalized size = 1.84

$$\frac{\frac{4i}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-i\right)^2}+\frac{2}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-i}-\frac{8}{3\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-i\right)^3}}{d\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*cos(d*x+c)+I*a*sin(d*x+c))^3,x)
[Out] 2/d/a^3*(2*I/(tan(1/2*d*x+1/2*c)-I)^2+1/(tan(1/2*d*x+1/2*c)-I)-4/3/(tan(1/2
*d*x+1/2*c)-I)^3)
```

maxima [A] time = 0.56, size = 29, normalized size = 0.94

$$\frac{i\,\cos\left(3\,dx+3\,c\right)+\sin\left(3\,dx+3\,c\right)}{3\,a^3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cos(d*x+c)+I*a*sin(d*x+c))^3,x, algorithm="maxima")
[Out] 1/3*(I*cos(3*d*x + 3*c) + sin(3*d*x + 3*c))/(a^3*d)
```

mupad [B] time = 0.63, size = 68, normalized size = 2.19

$$-\frac{2\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2 3\mathrm{i}-\mathrm{i}\right)}{3\,a^3\,d\,\left(-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3 1\mathrm{i}-3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)3\mathrm{i}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*cos(c + d*x) + a*sin(c + d*x)*1i)^3,x)
```

```
[Out] -(2*(tan(c/2 + (d*x)/2)^2*3i - 1i))/(3*a^3*d*(tan(c/2 + (d*x)/2)*3i - 3*tan
(c/2 + (d*x)/2)^2 - tan(c/2 + (d*x)/2)^3*1i + 1))
```

sympy [A] time = 0.15, size = 46, normalized size = 1.48

$$\begin{cases} \frac{ie^{-3ic}e^{-3idx}}{3a^3d} & \text{for } 3a^3de^{3ic} \neq 0 \\ \frac{xe^{-3ic}}{a^3} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cos(d*x+c)+I*a*sin(d*x+c))**3,x)
```

```
[Out] Piecewise((I*exp(-3*I*c)*exp(-3*I*d*x)/(3*a**3*d), Ne(3*a**3*d*exp(3*I*c),
0)), (x*exp(-3*I*c)/a**3, True))
```

3.181 $\int \frac{\sec(c+dx)}{(a\cos(c+dx)+ia\sin(c+dx))^3}\,dx$

Optimal. Leaf size=61

$$\frac{2}{a^3d(\cot(c+dx)+i)}-\frac{i\log(\sin(c+dx))}{a^3d}+\frac{i\log(\tan(c+dx))}{a^3d}-\frac{x}{a^3}$$

```
[Out] -x/a^3+2/a^3/d/(I+cot(d*x+c))-I*ln(sin(d*x+c))/a^3/d+I*ln(tan(d*x+c))/a^3/d
```

Rubi **[A]** time = 0.06, antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {3088, 848, 77}

$$\frac{2}{a^3d(\cot(c+dx)+i)}-\frac{i\log(\sin(c+dx))}{a^3d}+\frac{i\log(\tan(c+dx))}{a^3d}-\frac{x}{a^3}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]/(a*Cos[c + d*x] + I*a*Sin[c + d*x])^3,x]
```

```
[Out] -(x/a^3) + 2/(a^3*d*(I + Cot[c + d*x])) - (I*Log[Sin[c + d*x]])/(a^3*d) + (
I*Log[Tan[c + d*x]])/(a^3*d)
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 848

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_.), x_Symbol] :> Int[(d + e*x)^(m + p)*(f + g*x)^n*(a/d + (c*x)/e)^p,
x] /; FreeQ[{a, c, d, e, f, g, m, n}, x] && NeQ[e*f - d*g, 0] && EqQ[c*d^2
+ a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && EqQ[m + p, 0]))
```

Rule 3088

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[(x^m*(b +
 a*x)^n)/(1 + x^2)^((m + n + 2)/2), x], x, Cot[c + d*x]], x] /; FreeQ[{a, b
, c, d}, x] && IntegerQ[n] && IntegerQ[(m + n)/2] && NeQ[n, -1] &&  !(GtQ[n
```

```
, 0] && GtQ[m, 1])
```

Rubi steps

$$\begin{aligned}\int \frac{\sec(c+dx)}{(a\cos(c+dx)+ia\sin(c+dx))^3}\,dx &= -\frac{\text{Subst}\left(\int \frac{1+x^2}{x(ia+ax)^3}\,dx, x, \cot(c+dx)\right)}{d}\\ &= -\frac{\text{Subst}\left(\int \frac{-\frac{i}{a}+\frac{x}{a}}{x(ia+ax)^2}\,dx, x, \cot(c+dx)\right)}{d}\\ &= -\frac{\text{Subst}\left(\int \left(\frac{i}{a^3x}+\frac{2}{a^3(i+x)^2}-\frac{i}{a^3(i+x)}\right)dx, x, \cot(c+dx)\right)}{d}\\ &= -\frac{x}{a^3}+\frac{2}{a^3d(i+\cot(c+dx))}-\frac{i\log(\sin(c+dx))}{a^3d}+\frac{i\log(\tan(c+dx))}{a^3d}\end{aligned}$$

Mathematica [A] time = 0.28, size = 91, normalized size = 1.49

$$\frac{i\sec^2(c+dx)(\sin(2(c+dx))-i\cos(2(c+dx)))(\log(\cos(c+dx))+\tan(c+dx)(i\log(\cos(c+dx))+dx+i)-ic}{a^3d(\tan(c+dx)-i)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]/(a*Cos[c + d*x] + I*a*Sin[c + d*x])^3,x]
```

```
[Out] (I*Sec[c + d*x]^2*((-I)*Cos[2*(c + d*x)] + Sin[2*(c + d*x)])*(-1 - I*d*x +
Log[Cos[c + d*x]] + (I + d*x + I*Log[Cos[c + d*x]])*Tan[c + d*x]))/(a^3*d*(
-I + Tan[c + d*x])^3)
```

fricas [A] time = 0.56, size = 55, normalized size = 0.90

$$-\frac{\left(2\,dxe^{(2i\,dx+2i\,c)}+i\,e^{(2i\,dx+2i\,c)}\log\left(e^{(2i\,dx+2i\,c)}+1\right)-i\right)e^{(-2i\,dx-2i\,c)}}{a^3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a*cos(d*x+c)+I*a*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] -(2*d*x*e^(2*I*d*x + 2*I*c) + I*e^(2*I*d*x + 2*I*c)*log(e^(2*I*d*x + 2*I*c)
 + 1) - I)*e^(-2*I*d*x - 2*I*c)/(a^3*d)
```

giac [A] time = 0.31, size = 100, normalized size = 1.64

$$-\frac{\frac{i \log\left(\tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)+1\right)}{a^3} - \frac{2i \log\left(\tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)-i\right)}{a^3} + \frac{i \log\left(\tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)-1\right)}{a^3} + \frac{3i \tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)^2 + 10 \tan\left(\frac{1}{2} dx+\frac{1}{2} c\right) - 3i}{a^3\left(\tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)-i\right)^2}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a*cos(d*x+c)+I*a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -(I*log(tan(1/2*d*x + 1/2*c) + 1)/a^3 - 2*I*log(tan(1/2*d*x + 1/2*c) - I)/a
^3 + I*log(tan(1/2*d*x + 1/2*c) - 1)/a^3 + (3*I*tan(1/2*d*x + 1/2*c)^2 + 10
*tan(1/2*d*x + 1/2*c) - 3*I)/(a^3*(tan(1/2*d*x + 1/2*c) - I)^2))/d
```

maple [A] time = 0.26, size = 40, normalized size = 0.66

$$\frac{2}{a^3 d\left(\tan(dx+c)-i\right)} + \frac{i \ln\left(\tan(dx+c)-i\right)}{a^3 d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)/(a*cos(d*x+c)+I*a*sin(d*x+c))^3,x)
```

```
[Out] 2/a^3/d/(tan(d*x+c)-I)+I/a^3/d*ln(tan(d*x+c)-I)
```

maxima [A] time = 0.79, size = 99, normalized size = 1.62

$$-\frac{4\,dx + 4\,c - 2\,\arctan\left(\sin\left(2\,dx+2\,c\right), \cos\left(2\,dx+2\,c\right)+1\right) - 2i\,\cos\left(2\,dx+2\,c\right) + i\,\log\left(\cos\left(2\,dx+2\,c\right)^2 + \sin\right.}{2\,a^3 d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a*cos(d*x+c)+I*a*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -1/2*(4*d*x + 4*c - 2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1) - 2*I
*cos(2*d*x + 2*c) + I*log(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2
*d*x + 2*c) + 1) - 2*sin(2*d*x + 2*c))/(a^3*d)
```

mupad [B] time = 0.77, size = 101, normalized size = 1.66

$$-\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,4\mathrm{i}}{d\,\left(a^3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\,1\mathrm{i}+2\,a^3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)-a^3\,1\mathrm{i}\right)}+\frac{\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)-\mathrm{i}\right)\,2\mathrm{i}}{a^3\,d}-\frac{\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2-1\right)\,1\mathrm{i}}{a^3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)*(a*cos(c + d*x) + a*sin(c + d*x)*1i)^3),x)
```

```
[Out] (log(tan(c/2 + (d*x)/2) - 1i)*2i)/(a^3*d) - (tan(c/2 + (d*x)/2)*4i)/(d*(a^3
*tan(c/2 + (d*x)/2)^2*1i - a^3*1i + 2*a^3*tan(c/2 + (d*x)/2))) - (log(tan(c
/2 + (d*x)/2)^2 - 1)*1i)/(a^3*d)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\sec (c+dx)}{-i\sin^3(c+dx)-3\sin^2(c+dx)\cos(c+dx)+3i\sin(c+dx)\cos^2(c+dx)+\cos^3(c+dx)}\,dx}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a*cos(d*x+c)+I*a*sin(d*x+c))**3,x)
```

```
[Out] Integral(sec(c + d*x)/(-I*sin(c + d*x)**3 - 3*sin(c + d*x)**2*cos(c + d*x)
+ 3*I*sin(c + d*x)*cos(c + d*x)**2 + cos(c + d*x)**3), x)/a**3
```

3.182 $\int \frac{\sec^2(c+dx)}{(a\cos(c+dx)+ia\sin(c+dx))^3}\,dx$

Optimal. Leaf size=62

$$\frac{4\sin(c+dx)}{a^3d}+\frac{4i\cos(c+dx)}{a^3d}+\frac{i\sec(c+dx)}{a^3d}-\frac{3\tanh^{-1}(\sin(c+dx))}{a^3d}$$

```
[Out] -3*arctanh(sin(d*x+c))/a^3/d+4*I*cos(d*x+c)/a^3/d+I*sec(d*x+c)/a^3/d+4*sin(
d*x+c)/a^3/d
```

Rubi [A] time = 0.16, antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 9, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.290, Rules used = {3092, 3090, 2637, 2638, 2592, 321, 206, 2590, 14}

$$\frac{4\sin(c+dx)}{a^3d}+\frac{4i\cos(c+dx)}{a^3d}+\frac{i\sec(c+dx)}{a^3d}-\frac{3\tanh^{-1}(\sin(c+dx))}{a^3d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^2/(a*Cos[c + d*x] + I*a*Sin[c + d*x])^3,x]
```

```
[Out] (-3*ArcTanh[Sin[c + d*x]])/(a^3*d) + ((4*I)*Cos[c + d*x])/(a^3*d) + (I*Sec[
c + d*x])/(a^3*d) + (4*Sin[c + d*x])/(a^3*d)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2590

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> -Dist[f^(-1), Subst[Int[(1 - x^2)^((m + n - 1)/2)/x^n, x], x, Cos[e + f*
x]], x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n - 1)/2]
```

Rule 2592

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(
ff*x)^(m + n)/(a^2 - ff^2*x^2)^((n + 1)/2), x], x, (a*Sin[e + f*x])/ff], x]
] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3090

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_.), x_Symbol] :> Int[ExpandTrig[cos[c + d*x]^m*(a
*cos[c + d*x] + b*sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d}, x] && Inte
gerQ[m] && IGtQ[n, 0]
```

Rule 3092

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[a^n*b^n, Int[Cos[c + d*x]^m/
(b*Cos[c + d*x] + a*Sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, m}, x] &&
 EqQ[a^2 + b^2, 0] && ILtQ[n, 0]
```

Rubi steps

$$\int \frac{\sec^2(c+dx)}{(a\cos(c+dx)+ia\sin(c+dx))^3}\,dx = \frac{i\int \sec^2(c+dx)(ia\cos(c+dx)+a\sin(c+dx))^3\,dx}{a^6}$$
$$= \frac{i\int \left(-ia^3\cos(c+dx)-3a^3\sin(c+dx)+3ia^3\sin(c+dx)\tan(c+dx)+\right.}{a^6}$$
$$= \frac{i\int \sin(c+dx)\tan^2(c+dx)\,dx}{a^3} - \frac{(3i)\int \sin(c+dx)\,dx}{a^3} + \frac{\int \cos(c+dx)}{a^3}$$
$$= \frac{3i\cos(c+dx)}{a^3d} + \frac{\sin(c+dx)}{a^3d} - \frac{i\,\mathrm{Subst}\left(\int \frac{1-x^2}{x^2}\,dx, x, \cos(c+dx)\right)}{a^3d} - \frac{3\,\mathrm{S}}{}$$
$$= \frac{3i\cos(c+dx)}{a^3d} + \frac{4\sin(c+dx)}{a^3d} - \frac{i\,\mathrm{Subst}\left(\int \left(-1+\frac{1}{x^2}\right)dx, x, \cos(c+dx\right.}{a^3d}$$
$$= -\frac{3\tanh^{-1}(\sin(c+dx))}{a^3d} + \frac{4i\cos(c+dx)}{a^3d} + \frac{i\sec(c+dx)}{a^3d} + \frac{4\sin(c+dx}{a^3d}$$

Mathematica [A] time = 0.33, size = 109, normalized size = 1.76

$$-\frac{i\sec^3(c+dx)(\cos(dx)+i\sin(dx))^3\left((\tan(c+dx)-5i)(\cos(2c-dx)+i\sin(2c-dx))+6(\cos(3c)+i\sin(3c))\,\mathrm{ta}\right.}{a^3d(\tan(c+dx)-i)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^2/(a*Cos[c + d*x] + I*a*Sin[c + d*x])^3,x]
```

```
[Out] ((-I)*Sec[c + d*x]^3*(Cos[d*x] + I*Sin[d*x])^3*(6*ArcTanh[Sin[c] + Cos[c]*T
an[(d*x)/2]]*(Cos[3*c] + I*Sin[3*c]) + (Cos[2*c - d*x] + I*Sin[2*c - d*x])*
(-5*I + Tan[c + d*x])))/(a^3*d*(-I + Tan[c + d*x])^3)
```

fricas [A] time = 0.61, size = 112, normalized size = 1.81

$$-\frac{3\left(e^{(3i\,dx+3i\,c)}+e^{(i\,dx+i\,c)}\right)\log\left(e^{(i\,dx+i\,c)}+i\right)-3\left(e^{(3i\,dx+3i\,c)}+e^{(i\,dx+i\,c)}\right)\log\left(e^{(i\,dx+i\,c)}-i\right)-6i\,e^{(2i\,dx+2i\,c)}-4i}{a^3de^{(3i\,dx+3i\,c)}+a^3de^{(i\,dx+i\,c)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a*cos(d*x+c)+I*a*sin(d*x+c))^3,x, algorithm="fricas
")
```

```
[Out] -(3*(e^(3*I*d*x + 3*I*c) + e^(I*d*x + I*c))*log(e^(I*d*x + I*c) + I) - 3*(e
^(3*I*d*x + 3*I*c) + e^(I*d*x + I*c))*log(e^(I*d*x + I*c) - I) - 6*I*e^(2*I
*d*x + 2*I*c) - 4*I)/(a^3*d*e^(3*I*d*x + 3*I*c) + a^3*d*e^(I*d*x + I*c))
```

giac [A] time = 0.62, size = 110, normalized size = 1.77

$$-\frac{\frac{3\log\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right)}{a^3}-\frac{3\log\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1\right)}{a^3}-\frac{2\left(4\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-i\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-5\right)}{\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3-i\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+i\right)a^3}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a*cos(d*x+c)+I*a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -(3*log(tan(1/2*d*x + 1/2*c) + 1)/a^3 - 3*log(tan(1/2*d*x + 1/2*c) - 1)/a^3
 - 2*(4*tan(1/2*d*x + 1/2*c)^2 - I*tan(1/2*d*x + 1/2*c) - 5)/((tan(1/2*d*x
+ 1/2*c)^3 - I*tan(1/2*d*x + 1/2*c)^2 - tan(1/2*d*x + 1/2*c) + I)*a^3))/d
```

maple [A] time = 0.28, size = 108, normalized size = 1.74

$$-\frac{i}{a^3d\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}+\frac{3\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{a^3d}+\frac{i}{a^3d\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}-\frac{3\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{a^3d}+\frac{8}{a^3d\left(\tan\left(\frac{d}{}\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2/(a*cos(d*x+c)+I*a*sin(d*x+c))^3,x)
```

```
[Out] -I/a^3/d/(tan(1/2*d*x+1/2*c)-1)+3/a^3/d*ln(tan(1/2*d*x+1/2*c)-1)+I/a^3/d/(t
an(1/2*d*x+1/2*c)+1)-3/a^3/d*ln(tan(1/2*d*x+1/2*c)+1)+8/a^3/d/(tan(1/2*d*x+
1/2*c)-I)
```

maxima [B] time = 0.74, size = 329, normalized size = 5.31

$$\frac{(6\cos(3dx+3c)+6\cos(dx+c)+6i\sin(3dx+3c)+6i\sin(dx+c))\arctan(\cos(dx+c),\sin(dx+c)+1)}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a*cos(d*x+c)+I*a*sin(d*x+c))^3,x, algorithm="maxima
")
```

```
[Out] ((6*cos(3*d*x + 3*c) + 6*cos(d*x + c) + 6*I*sin(3*d*x + 3*c) + 6*I*sin(d*x
+ c))*arctan2(cos(d*x + c), sin(d*x + c) + 1) + (6*cos(3*d*x + 3*c) + 6*cos
(d*x + c) + 6*I*sin(3*d*x + 3*c) + 6*I*sin(d*x + c))*arctan2(cos(d*x + c),
-sin(d*x + c) + 1) - (-3*I*cos(3*d*x + 3*c) - 3*I*cos(d*x + c) + 3*sin(3*d*
x + 3*c) + 3*sin(d*x + c))*log(cos(d*x + c)^2 + sin(d*x + c)^2 + 2*sin(d*x
+ c) + 1) - (3*I*cos(3*d*x + 3*c) + 3*I*cos(d*x + c) - 3*sin(3*d*x + 3*c) -
```

```
 3*sin(d*x + c))*log(cos(d*x + c)^2 + sin(d*x + c)^2 - 2*sin(d*x + c) + 1)
+ 12*cos(2*d*x + 2*c) + 12*I*sin(2*d*x + 2*c) + 8)/((-2*I*a^3*cos(3*d*x + 3
*c) - 2*I*a^3*cos(d*x + c) + 2*a^3*sin(3*d*x + 3*c) + 2*a^3*sin(d*x + c))*d
)
```

mupad [B] time = 0.99, size = 105, normalized size = 1.69

$$-\frac{6\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{a^3\,d}-\frac{\frac{2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{a^3}+\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\,8\mathrm{i}}{a^3}-\frac{10\mathrm{i}}{a^3}}{d\,\left(-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3\,1\mathrm{i}-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,1\mathrm{i}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^2*(a*cos(c + d*x) + a*sin(c + d*x)*1i)^3),x)
```

```
[Out] - (6*atanh(tan(c/2 + (d*x)/2)))/(a^3*d) - ((tan(c/2 + (d*x)/2)^2*8i)/a^3 -
10i/a^3 + (2*tan(c/2 + (d*x)/2))/a^3)/(d*(tan(c/2 + (d*x)/2)*1i - tan(c/2 +
 (d*x)/2)^2 - tan(c/2 + (d*x)/2)^3*1i + 1))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\sec^2(c+dx)}{-i\sin^3(c+dx)-3\sin^2(c+dx)\cos(c+dx)+3i\sin(c+dx)\cos^2(c+dx)+\cos^3(c+dx)}\,dx}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2/(a*cos(d*x+c)+I*a*sin(d*x+c))**3,x)
```

```
[Out] Integral(sec(c + d*x)**2/(-I*sin(c + d*x)**3 - 3*sin(c + d*x)**2*cos(c + d*
x) + 3*I*sin(c + d*x)*cos(c + d*x)**2 + cos(c + d*x)**3), x)/a**3
```

3.183 $\int \frac{\sec^3(c+dx)}{(a\cos(c+dx)+ia\sin(c+dx))^3}\,dx$

Optimal. Leaf size=75

$$\frac{i\tan^2(c+dx)}{2a^3d}-\frac{3\tan(c+dx)}{a^3d}+\frac{4i\log(\sin(c+dx))}{a^3d}-\frac{4i\log(\tan(c+dx))}{a^3d}+\frac{4x}{a^3}$$

```
[Out] 4*x/a^3+4*I*ln(sin(d*x+c))/a^3/d-4*I*ln(tan(d*x+c))/a^3/d-3*tan(d*x+c)/a^3/
d+1/2*I*tan(d*x+c)^2/a^3/d
```

Rubi [A] time = 0.08, antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.097, Rules used = {3088, 848, 88}

$$\frac{i\tan^2(c+dx)}{2a^3d}-\frac{3\tan(c+dx)}{a^3d}+\frac{4i\log(\sin(c+dx))}{a^3d}-\frac{4i\log(\tan(c+dx))}{a^3d}+\frac{4x}{a^3}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^3/(a*Cos[c + d*x] + I*a*Sin[c + d*x])^3,x]
```

```
[Out] (4*x)/a^3 + ((4*I)*Log[Sin[c + d*x]])/(a^3*d) - ((4*I)*Log[Tan[c + d*x]])/(
a^3*d) - (3*Tan[c + d*x])/(a^3*d) + ((I/2)*Tan[c + d*x]^2)/(a^3*d)
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 848

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_.), x_Symbol] :> Int[(d + e*x)^(m + p)*(f + g*x)^n*(a/d + (c*x)/e)^p,
x] /; FreeQ[{a, c, d, e, f, g, m, n}, x] && NeQ[e*f - d*g, 0] && EqQ[c*d^2
+ a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && EqQ[m + p, 0]))
```

Rule 3088

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[(x^m*(b +
 a*x)^n)/(1 + x^2)^((m + n + 2)/2), x], x, Cot[c + d*x]], x] /; FreeQ[{a, b
, c, d}, x] && IntegerQ[n] && IntegerQ[(m + n)/2] && NeQ[n, -1] &&  !(GtQ[n
, 0] && GtQ[m, 1])
```

Rubi steps

$$\int \frac{\sec^3(c+dx)}{(a\cos(c+dx)+ia\sin(c+dx))^3}\,dx = -\frac{\text{Subst}\left(\int \frac{(1+x^2)^2}{x^3(ia+ax)^3}\,dx, x, \cot(c+dx)\right)}{d}$$
$$= -\frac{\text{Subst}\left(\int \frac{\left(-\frac{i}{a}+\frac{x}{a}\right)^2}{x^3(ia+ax)}\,dx, x, \cot(c+dx)\right)}{d}$$
$$= -\frac{\text{Subst}\left(\int \left(\frac{i}{a^3x^3}-\frac{3}{a^3x^2}-\frac{4i}{a^3x}+\frac{4i}{a^3(i+x)}\right)dx, x, \cot(c+dx)\right)}{d}$$
$$= \frac{4x}{a^3}+\frac{4i\log(\sin(c+dx))}{a^3d}-\frac{4i\log(\tan(c+dx))}{a^3d}-\frac{3\tan(c+dx)}{a^3d}+\frac{i\tan^2}{2}$$

Mathematica [A] time = 0.62, size = 110, normalized size = 1.47

$$\frac{i\sec(c)\sec^2(c+dx)(\cos(c)(4\log(\cos(c+dx))-4idx+1)-i(2\cos(c+2dx)(dx+i\log(\cos(c+dx)))+2\cos(3c}{2a^3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^3/(a*Cos[c + d*x] + I*a*Sin[c + d*x])^3,x]
```

```
[Out] ((I/2)*Sec[c]*Sec[c + d*x]^2*(Cos[c]*(1 - (4*I)*d*x + 4*Log[Cos[c + d*x]])
- I*(2*Cos[c + 2*d*x]*(d*x + I*Log[Cos[c + d*x]]) + 2*Cos[3*c + 2*d*x]*(d*x
 + I*Log[Cos[c + d*x]]) - 6*Cos[c + d*x]*Sin[d*x])))/(a^3*d)
```

fricas [A] time = 1.08, size = 110, normalized size = 1.47

$$\frac{8\,dxe^{(4i\,dx+4i\,c)}+8\,dx+(16\,dx-4i)e^{(2i\,dx+2i\,c)}+\left(4i\,e^{(4i\,dx+4i\,c)}+8i\,e^{(2i\,dx+2i\,c)}+4i\right)\log\left(e^{(2i\,dx+2i\,c)}+1\right)-6i}{a^3de^{(4i\,dx+4i\,c)}+2\,a^3de^{(2i\,dx+2i\,c)}+a^3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a*cos(d*x+c)+I*a*sin(d*x+c))^3,x, algorithm="fricas
")
```

```
[Out] (8*d*x*e^(4*I*d*x + 4*I*c) + 8*d*x + (16*d*x - 4*I)*e^(2*I*d*x + 2*I*c) + (
4*I*e^(4*I*d*x + 4*I*c) + 8*I*e^(2*I*d*x + 2*I*c) + 4*I)*log(e^(2*I*d*x + 2
*I*c) + 1) - 6*I)/(a^3*d*e^(4*I*d*x + 4*I*c) + 2*a^3*d*e^(2*I*d*x + 2*I*c)
+ a^3*d)
```

giac [A] time = 0.30, size = 128, normalized size = 1.71

$$\frac{2\left(\frac{2i\log\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right)}{a^3}-\frac{4i\log\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-i\right)}{a^3}+\frac{2i\log\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1\right)}{a^3}+\frac{-3i\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^4+3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3+7i\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2 \cdots}{\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)^2a^3}\right.}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a*cos(d*x+c)+I*a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 2*(2*I*log(tan(1/2*d*x + 1/2*c) + 1)/a^3 - 4*I*log(tan(1/2*d*x + 1/2*c) - I
)/a^3 + 2*I*log(tan(1/2*d*x + 1/2*c) - 1)/a^3 + (-3*I*tan(1/2*d*x + 1/2*c)^
4 + 3*tan(1/2*d*x + 1/2*c)^3 + 7*I*tan(1/2*d*x + 1/2*c)^2 - 3*tan(1/2*d*x +
 1/2*c) - 3*I)/((tan(1/2*d*x + 1/2*c)^2 - 1)^2*a^3))/d
```

maple [A] time = 0.29, size = 52, normalized size = 0.69

$$-\frac{3\tan\left(dx+c\right)}{a^3d}+\frac{i\left(\tan^2\left(dx+c\right)\right)}{2a^3d}-\frac{4i\ln\left(\tan\left(dx+c\right)-i\right)}{a^3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^3/(a*cos(d*x+c)+I*a*sin(d*x+c))^3,x)
```

```
[Out] -3*tan(d*x+c)/a^3/d+1/2*I*tan(d*x+c)^2/a^3/d-4*I/a^3/d*ln(tan(d*x+c)-I)
```

maxima [B] time = 0.50, size = 299, normalized size = 3.99

$$\frac{-8i\,dx+\left(4i\cos\left(4\,dx+4\,c\right)+8i\cos\left(2\,dx+2\,c\right)-4\sin\left(4\,dx+4\,c\right)-8\sin\left(2\,dx+2\,c\right)+4i\right)\arctan\left(\sin\left(2\,dx \cdots\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a*cos(d*x+c)+I*a*sin(d*x+c))^3,x, algorithm="maxima
")
```

```
[Out] (-8*I*d*x + (4*I*cos(4*d*x + 4*c) + 8*I*cos(2*d*x + 2*c) - 4*sin(4*d*x + 4*
c) - 8*sin(2*d*x + 2*c) + 4*I)*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) +
 1) + (-8*I*d*x - 8*I*c)*cos(4*d*x + 4*c) + (-16*I*d*x - 16*I*c - 4)*cos(2*
d*x + 2*c) + (2*cos(4*d*x + 4*c) + 4*cos(2*d*x + 2*c) + 2*I*sin(4*d*x + 4*c
) + 4*I*sin(2*d*x + 2*c) + 2)*log(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 +
 2*cos(2*d*x + 2*c) + 1) + 8*(d*x + c)*sin(4*d*x + 4*c) + (16*d*x + 16*c -
4*I)*sin(2*d*x + 2*c) - 8*I*c - 6)/((-I*a^3*cos(4*d*x + 4*c) - 2*I*a^3*cos(
2*d*x + 2*c) + a^3*sin(4*d*x + 4*c) + 2*a^3*sin(2*d*x + 2*c) - I*a^3)*d)
```

mupad [B] time = 0.92, size = 104, normalized size = 1.39

$$-\frac{\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)-\mathrm{i}\right)\,8\mathrm{i}-\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2-1\right)\,4\mathrm{i}}{a^3\,d}+\frac{6\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\,2\mathrm{i}-6\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{a^3\,d\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2-1\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^3*(a*cos(c + d*x) + a*sin(c + d*x)*1i)^3),x)
```

```
[Out] (tan(c/2 + (d*x)/2)^2*2i - 6*tan(c/2 + (d*x)/2) + 6*tan(c/2 + (d*x)/2)^3)/(
a^3*d*(tan(c/2 + (d*x)/2)^2 - 1)^2) - (log(tan(c/2 + (d*x)/2) - 1i)*8i - lo
g(tan(c/2 + (d*x)/2)^2 - 1)*4i)/(a^3*d)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\sec^3(c+dx)}{-i\sin^3(c+dx)-3\sin^2(c+dx)\cos(c+dx)+3i\sin(c+dx)\cos^2(c+dx)+\cos^3(c+dx)}\,dx}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**3/(a*cos(d*x+c)+I*a*sin(d*x+c))**3,x)
```

```
[Out] Integral(sec(c + d*x)**3/(-I*sin(c + d*x)**3 - 3*sin(c + d*x)**2*cos(c + d*
x) + 3*I*sin(c + d*x)*cos(c + d*x)**2 + cos(c + d*x)**3), x)/a**3
```

3.184 $\int \frac{\sec^4(c+dx)}{(a\cos(c+dx)+ia\sin(c+dx))^3}\,dx$

Optimal. Leaf size=76

$$\frac{i\sec^3(c+dx)}{3a^3d}-\frac{4i\sec(c+dx)}{a^3d}+\frac{5\tanh^{-1}(\sin(c+dx))}{2a^3d}-\frac{3\tan(c+dx)\sec(c+dx)}{2a^3d}$$

```
[Out] 5/2*arctanh(sin(d*x+c))/a^3/d-4*I*sec(d*x+c)/a^3/d+1/3*I*sec(d*x+c)^3/a^3/d
-3/2*sec(d*x+c)*tan(d*x+c)/a^3/d
```

Rubi [A] time = 0.18, antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 6, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.194, Rules used = {3092, 3090, 3770, 2606, 8, 2611}

$$\frac{i\sec^3(c+dx)}{3a^3d}-\frac{4i\sec(c+dx)}{a^3d}+\frac{5\tanh^{-1}(\sin(c+dx))}{2a^3d}-\frac{3\tan(c+dx)\sec(c+dx)}{2a^3d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^4/(a*Cos[c + d*x] + I*a*Sin[c + d*x])^3,x]
```

```
[Out] (5*ArcTanh[Sin[c + d*x]])/(2*a^3*d) - ((4*I)*Sec[c + d*x])/(a^3*d) + ((I/3)
*Sec[c + d*x]^3)/(a^3*d) - (3*Sec[c + d*x]*Tan[c + d*x])/(2*a^3*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 && !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 2611

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(b*(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*(
m + n - 1)), x] - Dist[(b^2*(n - 1))/(m + n - 1), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 3090

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_.), x_Symbol] :> Int[ExpandTrig[cos[c + d*x]^m*(a
*cos[c + d*x] + b*sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d}, x] && Inte
gerQ[m] && IGtQ[n, 0]
```

Rule 3092

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[a^n*b^n, Int[Cos[c + d*x]^m/
(b*Cos[c + d*x] + a*Sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, m}, x] &&
 EqQ[a^2 + b^2, 0] && ILtQ[n, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\sec^4(c+dx)}{(a\cos(c+dx)+ia\sin(c+dx))^3}\,dx = \frac{i\int \sec^4(c+dx)(ia\cos(c+dx)+a\sin(c+dx))^3\,dx}{a^6}$$
$$= \frac{i\int \left(-ia^3\sec(c+dx)-3a^3\sec(c+dx)\tan(c+dx)+3ia^3\sec(c+dx)\tan\right.}{a^6}$$
$$= \frac{i\int \sec(c+dx)\tan^3(c+dx)\,dx}{a^3} - \frac{(3i)\int \sec(c+dx)\tan(c+dx)\,dx}{a^3} + \frac{\int}{}$$
$$= \frac{\tanh^{-1}(\sin(c+dx))}{a^3d} - \frac{3\sec(c+dx)\tan(c+dx)}{2a^3d} + \frac{3\int \sec(c+dx)\,dx}{2a^3} +$$
$$= \frac{5\tanh^{-1}(\sin(c+dx))}{2a^3d} - \frac{4i\sec(c+dx)}{a^3d} + \frac{i\sec^3(c+dx)}{3a^3d} - \frac{3\sec(c+dx)}{2a^3}$$

Mathematica [A] time = 0.47, size = 64, normalized size = 0.84

$$\frac{i\left(\sec^3(c+dx)(9i\sin(2(c+dx))-24\cos(2(c+dx))-20)-60i\tanh^{-1}\left(\cos(c)\tan\left(\frac{dx}{2}\right)+\sin(c)\right)\right)}{12a^3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^4/(a*Cos[c + d*x] + I*a*Sin[c + d*x])^3,x]
```

```
[Out] ((I/12)*((-60*I)*ArcTanh[Sin[c] + Cos[c]*Tan[(d*x)/2]] + Sec[c + d*x]^3*(-2
0 - 24*Cos[2*(c + d*x)] + (9*I)*Sin[2*(c + d*x)])))/(a^3*d)
```

fricas [B] time = 0.46, size = 182, normalized size = 2.39

$$\frac{15\left(e^{(6i\,dx+6i\,c)}+3\,e^{(4i\,dx+4i\,c)}+3\,e^{(2i\,dx+2i\,c)}+1\right)\log\left(e^{(i\,dx+i\,c)}+i\right)-15\left(e^{(6i\,dx+6i\,c)}+3\,e^{(4i\,dx+4i\,c)}+3\,e^{(2i\,dx+2i\,c)}+1\right.}{6\left(a^{3}de^{(6i\,dx+6i\,c)}+3\,a^{3}de^{(4i\,dx+4i\,c)}+3\,a^{3}de^{(2i\,dx+2i\,c)}+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4/(a*cos(d*x+c)+I*a*sin(d*x+c))^3,x, algorithm="fricas
")
```

```
[Out] 1/6*(15*(e^(6*I*d*x + 6*I*c) + 3*e^(4*I*d*x + 4*I*c) + 3*e^(2*I*d*x + 2*I*c
) + 1)*log(e^(I*d*x + I*c) + I) - 15*(e^(6*I*d*x + 6*I*c) + 3*e^(4*I*d*x +
4*I*c) + 3*e^(2*I*d*x + 2*I*c) + 1)*log(e^(I*d*x + I*c) - I) - 30*I*e^(5*I*
d*x + 5*I*c) - 80*I*e^(3*I*d*x + 3*I*c) - 66*I*e^(I*d*x + I*c))/(a^3*d*e^(6
*I*d*x + 6*I*c) + 3*a^3*d*e^(4*I*d*x + 4*I*c) + 3*a^3*d*e^(2*I*d*x + 2*I*c)
 + a^3*d)
```

giac [A] time = 0.28, size = 112, normalized size = 1.47

$$\frac{\frac{15\,\log\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)}{a^{3}}-\frac{15\,\log\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right)}{a^{3}}-\frac{2\left(9\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}-18i\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{4}+48i\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-9\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-22i\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-1\right)^{3}a^{3}}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4/(a*cos(d*x+c)+I*a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/6*(15*log(tan(1/2*d*x + 1/2*c) + 1)/a^3 - 15*log(tan(1/2*d*x + 1/2*c) - 1
)/a^3 - 2*(9*tan(1/2*d*x + 1/2*c)^5 - 18*I*tan(1/2*d*x + 1/2*c)^4 + 48*I*ta
n(1/2*d*x + 1/2*c)^2 - 9*tan(1/2*d*x + 1/2*c) - 22*I)/((tan(1/2*d*x + 1/2*c
)^2 - 1)^3*a^3))/d
```

maple [B] time = 0.29, size = 258, normalized size = 3.39

$$-\frac{i}{3a^{3}d\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{3}}-\frac{3}{2a^{3}d\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{2}}-\frac{i}{2a^{3}d\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{2}}-\frac{3}{2a^{3}d\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}+\frac{}{2a^{3}c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^4/(a*cos(d*x+c)+I*a*sin(d*x+c))^3,x)
```

```
[Out] -1/3*I/a^3/d/(tan(1/2*d*x+1/2*c)-1)^3-3/2/a^3/d/(tan(1/2*d*x+1/2*c)-1)^2-1/
2*I/a^3/d/(tan(1/2*d*x+1/2*c)-1)^2-3/2/a^3/d/(tan(1/2*d*x+1/2*c)-1)+7/2*I/a
^3/d/(tan(1/2*d*x+1/2*c)-1)-5/2/a^3/d*ln(tan(1/2*d*x+1/2*c)-1)+1/3*I/a^3/d/
(tan(1/2*d*x+1/2*c)+1)^3+3/2/a^3/d/(tan(1/2*d*x+1/2*c)+1)^2-1/2*I/a^3/d/(ta
n(1/2*d*x+1/2*c)+1)^2-3/2/a^3/d/(tan(1/2*d*x+1/2*c)+1)-7/2*I/a^3/d/(tan(1/2
*d*x+1/2*c)+1)+5/2/a^3/d*ln(tan(1/2*d*x+1/2*c)+1)
```

maxima [B] time = 0.37, size = 215, normalized size = 2.83

$$\frac{\frac{4\left(-\frac{9i\sin(dx+c)}{\cos(dx+c)+1}-\frac{48\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{18\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{9i\sin(dx+c)^5}{(\cos(dx+c)+1)^5}+22\right)}{6ia^3-\frac{18ia^3\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{18ia^3\sin(dx+c)^4}{(\cos(dx+c)+1)^4}-\frac{6ia^3\sin(dx+c)^6}{(\cos(dx+c)+1)^6}}+\frac{5\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)}{a^3}-\frac{5\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}-1\right)}{a^3}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4/(a*cos(d*x+c)+I*a*sin(d*x+c))^3,x, algorithm="maxima
")
```

```
[Out] 1/2*(4*(-9*I*sin(d*x + c)/(cos(d*x + c) + 1) - 48*sin(d*x + c)^2/(cos(d*x +
 c) + 1)^2 + 18*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + 9*I*sin(d*x + c)^5/(c
os(d*x + c) + 1)^5 + 22)/(6*I*a^3 - 18*I*a^3*sin(d*x + c)^2/(cos(d*x + c) +
 1)^2 + 18*I*a^3*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 - 6*I*a^3*sin(d*x + c)
^6/(cos(d*x + c) + 1)^6) + 5*log(sin(d*x + c)/(cos(d*x + c) + 1) + 1)/a^3 -
 5*log(sin(d*x + c)/(cos(d*x + c) + 1) - 1)/a^3)/d
```

mupad [B] time = 2.72, size = 135, normalized size = 1.78

$$\frac{5\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{a^3\,d}+\frac{\frac{3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{a^3}-\frac{3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5}{a^3}-\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\,16\mathrm{i}}{a^3}+\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4\,6\mathrm{i}}{a^3}+\frac{22\mathrm{i}}{3\,a^3}}{d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6-3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4+3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^4*(a*cos(c + d*x) + a*sin(c + d*x)*1i)^3),x)
```

```
[Out] (5*atanh(tan(c/2 + (d*x)/2)))/(a^3*d) + ((tan(c/2 + (d*x)/2)^4*6i)/a^3 - (t
an(c/2 + (d*x)/2)^2*16i)/a^3 - (3*tan(c/2 + (d*x)/2)^5)/a^3 + 22i/(3*a^3) +
 (3*tan(c/2 + (d*x)/2))/a^3)/(d*(3*tan(c/2 + (d*x)/2)^2 - 3*tan(c/2 + (d*x)
/2)^4 + tan(c/2 + (d*x)/2)^6 - 1))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\sec^4(c+dx)}{-i\sin^3(c+dx)-3\sin^2(c+dx)\cos(c+dx)+3i\sin(c+dx)\cos^2(c+dx)+\cos^3(c+dx)}\,dx}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**4/(a*cos(d*x+c)+I*a*sin(d*x+c))**3,x)
```

```
[Out] Integral(sec(c + d*x)**4/(-I*sin(c + d*x)**3 - 3*sin(c + d*x)**2*cos(c + d*
x) + 3*I*sin(c + d*x)*cos(c + d*x)**2 + cos(c + d*x)**3), x)/a**3
```

3.185 $\int \frac{\sec^5(c+dx)}{(a\cos(c+dx)+ia\sin(c+dx))^3}\,dx$

Optimal. Leaf size=34

$$\frac{i\tan^4(c+dx)(-\cot(c+dx)+i)^4}{4a^3d}$$

```
[Out] 1/4*I*(I-cot(d*x+c))^4*tan(d*x+c)^4/a^3/d
```

Rubi [A] time = 0.06, antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.097, Rules used = {3088, 848, 37}

$$\frac{i\tan^4(c+dx)(-\cot(c+dx)+i)^4}{4a^3d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^5/(a*Cos[c + d*x] + I*a*Sin[c + d*x])^3,x]
```

```
[Out] ((I/4)*(I - Cot[c + d*x])^4*Tan[c + d*x]^4)/(a^3*d)
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 848

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_.), x_Symbol] :> Int[(d + e*x)^(m + p)*(f + g*x)^n*(a/d + (c*x)/e)^p,
x] /; FreeQ[{a, c, d, e, f, g, m, n}, x] && NeQ[e*f - d*g, 0] && EqQ[c*d^2
+ a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && EqQ[m + p, 0]))
```

Rule 3088

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[(x^m*(b +
 a*x)^n)/(1 + x^2)^((m + n + 2)/2), x], x, Cot[c + d*x]], x] /; FreeQ[{a, b
, c, d}, x] && IntegerQ[n] && IntegerQ[(m + n)/2] && NeQ[n, -1] &&  !(GtQ[n
, 0] && GtQ[m, 1])
```

Rubi steps

$$\int \frac{\sec^5(c+dx)}{(a\cos(c+dx)+ia\sin(c+dx))^3}\,dx = -\frac{\text{Subst}\left(\int \frac{(1+x^2)^3}{x^5(ia+ax)^3}\,dx,x,\cot(c+dx)\right)}{d}$$
$$= -\frac{\text{Subst}\left(\int \frac{\left(-\frac{i}{a}+\frac{x}{a}\right)^3}{x^5}\,dx,x,\cot(c+dx)\right)}{d}$$
$$= \frac{i(i-\cot(c+dx))^4\tan^4(c+dx)}{4a^3d}$$

Mathematica [B] time = 0.47, size = 90, normalized size = 2.65

$$-\frac{i\sec(c)\sec^4(c+dx)(2i\sin(c+2dx)-2i\sin(3c+2dx)+i\sin(3c+4dx)+2\cos(c+2dx)+2\cos(3c+2dx)-}{4a^3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^5/(a*Cos[c + d*x] + I*a*Sin[c + d*x])^3,x]
```

```
[Out] ((-1/4*I)*Sec[c]*Sec[c + d*x]^4*(3*Cos[c] + 2*Cos[c + 2*d*x] + 2*Cos[3*c +
2*d*x] - (3*I)*Sin[c] + (2*I)*Sin[c + 2*d*x] - (2*I)*Sin[3*c + 2*d*x] + I*S
in[3*c + 4*d*x]))/(a^3*d)
```

fricas [B] time = 1.42, size = 69, normalized size = 2.03

$$\frac{4i}{a^3de^{(8i\,dx+8i\,c)}+4\,a^3de^{(6i\,dx+6i\,c)}+6\,a^3de^{(4i\,dx+4i\,c)}+4\,a^3de^{(2i\,dx+2i\,c)}+a^3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5/(a*cos(d*x+c)+I*a*sin(d*x+c))^3,x, algorithm="fricas
")
```

```
[Out] 4*I/(a^3*d*e^(8*I*d*x + 8*I*c) + 4*a^3*d*e^(6*I*d*x + 6*I*c) + 6*a^3*d*e^(4
*I*d*x + 4*I*c) + 4*a^3*d*e^(2*I*d*x + 2*I*c) + a^3*d)
```

giac [A] time = 0.41, size = 47, normalized size = 1.38

$$-\frac{-i\,\tan(dx+c)^4+4\,\tan(dx+c)^3+6i\,\tan(dx+c)^2-4\,\tan(dx+c)}{4\,a^3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5/(a*cos(d*x+c)+I*a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -1/4*(-I*tan(d*x + c)^4 + 4*tan(d*x + c)^3 + 6*I*tan(d*x + c)^2 - 4*tan(d*x
 + c))/(a^3*d)
```

maple [A] time = 0.30, size = 47, normalized size = 1.38

$$\frac{\tan\left(dx+c\right)+\frac{i\left(\tan^{4}\left(dx+c\right)\right)}{4}-\left(\tan^{3}\left(dx+c\right)\right)-\frac{3i\left(\tan^{2}\left(dx+c\right)\right)}{2}}{d\,a^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^5/(a*cos(d*x+c)+I*a*sin(d*x+c))^3,x)
```

```
[Out] 1/d/a^3*(tan(d*x+c)+1/4*I*tan(d*x+c)^4-tan(d*x+c)^3-3/2*I*tan(d*x+c)^2)
```

maxima [B] time = 0.38, size = 240, normalized size = 7.06

$$\frac{2\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}-\frac{3i\,\sin(dx+c)^{2}}{(\cos(dx+c)+1)^{2}}-\frac{7\,\sin(dx+c)^{3}}{(\cos(dx+c)+1)^{3}}+\frac{8i\,\sin(dx+c)^{4}}{(\cos(dx+c)+1)^{4}}+\frac{7\,\sin(dx+c)^{5}}{(\cos(dx+c)+1)^{5}}-\frac{3i\,\sin(dx+c)^{6}}{(\cos(dx+c)+1)^{6}}-\frac{\sin(dx+c)^{7}}{(\cos(dx+c)+1)^{7}}\right)}{\left(a^{3}-\frac{4\,a^{3}\sin(dx+c)^{2}}{(\cos(dx+c)+1)^{2}}+\frac{6\,a^{3}\sin(dx+c)^{4}}{(\cos(dx+c)+1)^{4}}-\frac{4\,a^{3}\sin(dx+c)^{6}}{(\cos(dx+c)+1)^{6}}+\frac{a^{3}\sin(dx+c)^{8}}{(\cos(dx+c)+1)^{8}}\right)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5/(a*cos(d*x+c)+I*a*sin(d*x+c))^3,x, algorithm="maxima
")
```

```
[Out] 2*(sin(d*x + c)/(cos(d*x + c) + 1) - 3*I*sin(d*x + c)^2/(cos(d*x + c) + 1)^
2 - 7*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 8*I*sin(d*x + c)^4/(cos(d*x + c
) + 1)^4 + 7*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 3*I*sin(d*x + c)^6/(cos(
d*x + c) + 1)^6 - sin(d*x + c)^7/(cos(d*x + c) + 1)^7)/((a^3 - 4*a^3*sin(d*
x + c)^2/(cos(d*x + c) + 1)^2 + 6*a^3*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 -
 4*a^3*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 + a^3*sin(d*x + c)^8/(cos(d*x +
c) + 1)^8)*d)
```

mupad [B] time = 0.88, size = 55, normalized size = 1.62

$$\frac{\sin\left(c+d\,x\right)^{2}\,1\mathrm{i}-\frac{\sin\left(2\,c+2\,d\,x\right)^{2}\,7\mathrm{i}}{4}+\sin\left(4\,c+4\,d\,x\right)}{4\,a^{3}\,d\,{\left(\sin\left(c+d\,x\right)^{2}-1\right)}^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^5*(a*cos(c + d*x) + a*sin(c + d*x)*1i)^3),x)
```

```
[Out] (sin(4*c + 4*d*x) - (sin(2*c + 2*d*x)^2*7i)/4 + sin(c + d*x)^2*1i)/(4*a^3*d
*(sin(c + d*x)^2 - 1)^2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\sec^5(c+dx)}{-i\sin^3(c+dx)-3\sin^2(c+dx)\cos(c+dx)+3i\sin(c+dx)\cos^2(c+dx)+\cos^3(c+dx)}\,dx}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**5/(a*cos(d*x+c)+I*a*sin(d*x+c))**3,x)
```

```
[Out] Integral(sec(c + d*x)**5/(-I*sin(c + d*x)**3 - 3*sin(c + d*x)**2*cos(c + d*
x) + 3*I*sin(c + d*x)*cos(c + d*x)**2 + cos(c + d*x)**3), x)/a**3
```

3.186 $\int \frac{\sec^6(c+dx)}{(a\cos(c+dx)+ia\sin(c+dx))^3}\,dx$

Optimal. Leaf size=104

$$\frac{i\sec^5(c+dx)}{5a^3d}-\frac{4i\sec^3(c+dx)}{3a^3d}+\frac{7\tanh^{-1}(\sin(c+dx))}{8a^3d}-\frac{3\tan(c+dx)\sec^3(c+dx)}{4a^3d}+\frac{7\tan(c+dx)\sec(c+dx)}{8a^3d}$$

```
[Out] 7/8*arctanh(sin(d*x+c))/a^3/d-4/3*I*sec(d*x+c)^3/a^3/d+1/5*I*sec(d*x+c)^5/a
^3/d+7/8*sec(d*x+c)*tan(d*x+c)/a^3/d-3/4*sec(d*x+c)^3*tan(d*x+c)/a^3/d
```

Rubi [A] time = 0.23, antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 8, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.258, Rules used = {3092, 3090, 3768, 3770, 2606, 30, 2611, 14}

$$\frac{i\sec^5(c+dx)}{5a^3d}-\frac{4i\sec^3(c+dx)}{3a^3d}+\frac{7\tanh^{-1}(\sin(c+dx))}{8a^3d}-\frac{3\tan(c+dx)\sec^3(c+dx)}{4a^3d}+\frac{7\tan(c+dx)\sec(c+dx)}{8a^3d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^6/(a*Cos[c + d*x] + I*a*Sin[c + d*x])^3,x]
```

```
[Out] (7*ArcTanh[Sin[c + d*x]])/(8*a^3*d) - (((4*I)/3)*Sec[c + d*x]^3)/(a^3*d) +
((I/5)*Sec[c + d*x]^5)/(a^3*d) + (7*Sec[c + d*x]*Tan[c + d*x])/(8*a^3*d) -
(3*Sec[c + d*x]^3*Tan[c + d*x])/(4*a^3*d)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 2611

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(b*(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*(
m + n - 1)), x] - Dist[(b^2*(n - 1))/(m + n - 1), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 3090

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_.), x_Symbol] :> Int[ExpandTrig[cos[c + d*x]^m*(a
*cos[c + d*x] + b*sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d}, x] && Inte
gerQ[m] && IGtQ[n, 0]
```

Rule 3092

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[a^n*b^n, Int[Cos[c + d*x]^m/
(b*Cos[c + d*x] + a*Sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, m}, x] &&
 EqQ[a^2 + b^2, 0] && ILtQ[n, 0]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\sec^6(c+dx)}{(a\cos(c+dx)+ia\sin(c+dx))^3}\,dx = \frac{i\int \sec^6(c+dx)(ia\cos(c+dx)+a\sin(c+dx))^3\,dx}{a^6}$$
$$= \frac{i\int \left(-ia^3\sec^3(c+dx)-3a^3\sec^3(c+dx)\tan(c+dx)+3ia^3\sec^3(c+dx\right.}{a^6}$$
$$= \frac{i\int \sec^3(c+dx)\tan^3(c+dx)\,dx}{a^3} - \frac{(3i)\int \sec^3(c+dx)\tan(c+dx)\,dx}{a^3} + \cdot$$
$$= \frac{\sec(c+dx)\tan(c+dx)}{2a^3d} - \frac{3\sec^3(c+dx)\tan(c+dx)}{4a^3d} + \frac{\int \sec(c+dx)\,dx}{2a^3}$$
$$= \frac{\tanh^{-1}(\sin(c+dx))}{2a^3d} - \frac{i\sec^3(c+dx)}{a^3d} + \frac{7\sec(c+dx)\tan(c+dx)}{8a^3d} - \frac{3\,\mathrm{se}}{}$$
$$= \frac{7\tanh^{-1}(\sin(c+dx))}{8a^3d} - \frac{4i\sec^3(c+dx)}{3a^3d} + \frac{i\sec^5(c+dx)}{5a^3d} + \frac{7\sec(c+dx}{8a}$$

Mathematica [A] time = 0.43, size = 115, normalized size = 1.11

$$\frac{i\sec^8(c+dx)(\sin(3(c+dx))-i\cos(3(c+dx)))\left(-150i\sin(2(c+dx))+105i\sin(4(c+dx))+640\cos(2(c+dx))\right.}{960a^3d(\tan(c+dx)-i)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^6/(a*Cos[c + d*x] + I*a*Sin[c + d*x])^3,x]
```

```
[Out] ((I/960)*Sec[c + d*x]^8*((-I)*Cos[3*(c + d*x)] + Sin[3*(c + d*x)])*(448 + (
1680*I)*ArcTanh[Sin[c] + Cos[c]*Tan[(d*x)/2]]*Cos[c + d*x]^5 + 640*Cos[2*(c
 + d*x)] - (150*I)*Sin[2*(c + d*x)] + (105*I)*Sin[4*(c + d*x)]))/(a^3*d*(-I
 + Tan[c + d*x])^3)
```

fricas [B] time = 1.08, size = 278, normalized size = 2.67

$$\frac{105\left(e^{(10i\,dx+10i\,c)}+5\,e^{(8i\,dx+8i\,c)}+10\,e^{(6i\,dx+6i\,c)}+10\,e^{(4i\,dx+4i\,c)}+5\,e^{(2i\,dx+2i\,c)}+1\right)\log\left(e^{(i\,dx+i\,c)}+i\right)-105\left(e^{(10i\,dx+10}\right.}{120\left(a^3de^{(10i\,dx+10i\,c)}+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^6/(a*cos(d*x+c)+I*a*sin(d*x+c))^3,x, algorithm="fricas
")
```

```
[Out] 1/120*(105*(e^(10*I*d*x + 10*I*c) + 5*e^(8*I*d*x + 8*I*c) + 10*e^(6*I*d*x +
 6*I*c) + 10*e^(4*I*d*x + 4*I*c) + 5*e^(2*I*d*x + 2*I*c) + 1)*log(e^(I*d*x
```

```
+ I*c) + I) - 105*(e^(10*I*d*x + 10*I*c) + 5*e^(8*I*d*x + 8*I*c) + 10*e^(6*
I*d*x + 6*I*c) + 10*e^(4*I*d*x + 4*I*c) + 5*e^(2*I*d*x + 2*I*c) + 1)*log(e^
(I*d*x + I*c) - I) - 210*I*e^(9*I*d*x + 9*I*c) - 980*I*e^(7*I*d*x + 7*I*c)
- 1792*I*e^(5*I*d*x + 5*I*c) - 1580*I*e^(3*I*d*x + 3*I*c) + 210*I*e^(I*d*x
+ I*c))/(a^3*d*e^(10*I*d*x + 10*I*c) + 5*a^3*d*e^(8*I*d*x + 8*I*c) + 10*a^3
*d*e^(6*I*d*x + 6*I*c) + 10*a^3*d*e^(4*I*d*x + 4*I*c) + 5*a^3*d*e^(2*I*d*x
+ 2*I*c) + a^3*d)
```

giac [A] time = 1.95, size = 164, normalized size = 1.58

$$\frac{\frac{105\,\log\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)}{a^3} - \frac{105\,\log\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right)}{a^3} + \frac{2\left(15\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^9+360i\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^8-390\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7-960i\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\right.\right.}{\left(\tan\left(\right.\right.}}{120\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^6/(a*cos(d*x+c)+I*a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/120*(105*log(tan(1/2*d*x + 1/2*c) + 1)/a^3 - 105*log(tan(1/2*d*x + 1/2*c)
 - 1)/a^3 + 2*(15*tan(1/2*d*x + 1/2*c)^9 + 360*I*tan(1/2*d*x + 1/2*c)^8 - 3
90*tan(1/2*d*x + 1/2*c)^7 - 960*I*tan(1/2*d*x + 1/2*c)^6 + 400*I*tan(1/2*d*
x + 1/2*c)^4 + 390*tan(1/2*d*x + 1/2*c)^3 - 320*I*tan(1/2*d*x + 1/2*c)^2 -
15*tan(1/2*d*x + 1/2*c) + 136*I)/((tan(1/2*d*x + 1/2*c)^2 - 1)^5*a^3))/d
```

maple [B] time = 0.30, size = 430, normalized size = 4.13

$$\frac{i}{5a^3d\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^5}+\frac{1}{8a^3d\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}-\frac{13i}{8a^3d\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}-\frac{3}{4a^3d\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^4}-\frac{}{5a^3d\left(\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^6/(a*cos(d*x+c)+I*a*sin(d*x+c))^3,x)
```

```
[Out] 1/5*I/a^3/d/(tan(1/2*d*x+1/2*c)+1)^5+1/8/a^3/d/(tan(1/2*d*x+1/2*c)-1)-13/8*
I/a^3/d/(tan(1/2*d*x+1/2*c)+1)-3/4/a^3/d/(tan(1/2*d*x+1/2*c)-1)^4-1/5*I/a^3
/d/(tan(1/2*d*x+1/2*c)-1)^5-5/8/a^3/d/(tan(1/2*d*x+1/2*c)-1)^2+7/12*I/a^3/d
/(tan(1/2*d*x+1/2*c)-1)^3-3/2/a^3/d/(tan(1/2*d*x+1/2*c)-1)^3+11/8*I/a^3/d/(
tan(1/2*d*x+1/2*c)-1)^2-7/8/a^3/d*ln(tan(1/2*d*x+1/2*c)-1)-1/2*I/a^3/d/(tan
(1/2*d*x+1/2*c)-1)^4+5/8/a^3/d/(tan(1/2*d*x+1/2*c)+1)^2+13/8*I/a^3/d/(tan(1
/2*d*x+1/2*c)-1)+3/4/a^3/d/(tan(1/2*d*x+1/2*c)+1)^4-7/12*I/a^3/d/(tan(1/2*d
*x+1/2*c)+1)^3+1/8/a^3/d/(tan(1/2*d*x+1/2*c)+1)+11/8*I/a^3/d/(tan(1/2*d*x+1
/2*c)+1)^2-3/2/a^3/d/(tan(1/2*d*x+1/2*c)+1)^3-1/2*I/a^3/d/(tan(1/2*d*x+1/2*
c)+1)^4+7/8/a^3/d*ln(tan(1/2*d*x+1/2*c)+1)
```

maxima [B] time = 0.35, size = 341, normalized size = 3.28

$$\frac{\frac{16\left(-\frac{15i\sin(dx+c)}{\cos(dx+c)+1}+\frac{320\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{390i\sin(dx+c)^3}{(\cos(dx+c)+1)^3}-\frac{400\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{960\sin(dx+c)^6}{(\cos(dx+c)+1)^6}-\frac{390i\sin(dx+c)^7}{(\cos(dx+c)+1)^7}-\frac{360\sin(dx+c)^8}{(\cos(dx+c)+1)^8}+\frac{15i\sin(dx+c)^9}{(\cos(dx+c)+1)^9}-136\right)}{-120ia^3+\frac{600ia^3\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-\frac{1200ia^3\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{1200ia^3\sin(dx+c)^6}{(\cos(dx+c)+1)^6}-\frac{600ia^3\sin(dx+c)^8}{(\cos(dx+c)+1)^8}+\frac{120ia^3\sin(dx+c)^{10}}{(\cos(dx+c)+1)^{10}}}+\frac{7\log\left(\frac{\sin(dx+}{\cos(dx+c}\right.}{a^3}}{8d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^6/(a*cos(d*x+c)+I*a*sin(d*x+c))^3,x, algorithm="maxima
")
```

```
[Out] 1/8*(16*(-15*I*sin(d*x + c)/(cos(d*x + c) + 1) + 320*sin(d*x + c)^2/(cos(d*
x + c) + 1)^2 + 390*I*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 - 400*sin(d*x + c
)^4/(cos(d*x + c) + 1)^4 + 960*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 - 390*I*
sin(d*x + c)^7/(cos(d*x + c) + 1)^7 - 360*sin(d*x + c)^8/(cos(d*x + c) + 1)
^8 + 15*I*sin(d*x + c)^9/(cos(d*x + c) + 1)^9 - 136)/(-120*I*a^3 + 600*I*a^
3*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 - 1200*I*a^3*sin(d*x + c)^4/(cos(d*x
+ c) + 1)^4 + 1200*I*a^3*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 - 600*I*a^3*si
n(d*x + c)^8/(cos(d*x + c) + 1)^8 + 120*I*a^3*sin(d*x + c)^10/(cos(d*x + c)
 + 1)^10) + 7*log(sin(d*x + c)/(cos(d*x + c) + 1) + 1)/a^3 - 7*log(sin(d*x
+ c)/(cos(d*x + c) + 1) - 1)/a^3)/d
```

mupad [B] time = 3.29, size = 150, normalized size = 1.44

$$\frac{7\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{4\,a^3\,d}+\frac{\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^9}{4}+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8\,6\mathrm{i}-\frac{13\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7}{2}-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6\,16\mathrm{i}+\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4\,20\mathrm{i}}{3}+\frac{13\tan}{}}{a^3\,d\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2-1\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^6*(a*cos(c + d*x) + a*sin(c + d*x)*1i)^3),x)
```

```
[Out] (7*atanh(tan(c/2 + (d*x)/2)))/(4*a^3*d) + ((13*tan(c/2 + (d*x)/2)^3)/2 - (t
an(c/2 + (d*x)/2)^2*16i)/3 - tan(c/2 + (d*x)/2)/4 + (tan(c/2 + (d*x)/2)^4*2
0i)/3 - tan(c/2 + (d*x)/2)^6*16i - (13*tan(c/2 + (d*x)/2)^7)/2 + tan(c/2 +
(d*x)/2)^8*6i + tan(c/2 + (d*x)/2)^9/4 + 34i/15)/(a^3*d*(tan(c/2 + (d*x)/2)
^2 - 1)^5)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**6/(a*cos(d*x+c)+I*a*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

3.187 $\int \cos^{-n}(c+dx)(a\cos(c+dx)+ia\sin(c+dx))^n\,dx$

Optimal. Leaf size=66

$$-\frac{i\cos^{-n}(c+dx)\,{}_2F_1\left(1,n;n+1;\frac{1}{2}(i\tan(c+dx)+1)\right)(a\cos(c+dx)+ia\sin(c+dx))^n}{2dn}$$

```
[Out] -1/2*I*hypergeom([1, n],[1+n],1/2+1/2*I*tan(d*x+c))*(a*cos(d*x+c)+I*a*sin(d
*x+c))^n/d/n/(cos(d*x+c)^n)
```

Rubi [A] time = 0.06, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.030, Rules used = {3084}

$$-\frac{i\cos^{-n}(c+dx)\,{}_2F_1\left(1,n;n+1;\frac{1}{2}(i\tan(c+dx)+1)\right)(a\cos(c+dx)+ia\sin(c+dx))^n}{2dn}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Cos[c + d*x] + I*a*Sin[c + d*x])^n/Cos[c + d*x]^n,x]
```

```
[Out] ((-I/2)*Hypergeometric2F1[1, n, 1 + n, (1 + I*Tan[c + d*x])/2]*(a*Cos[c + d
*x] + I*a*Sin[c + d*x])^n)/(d*n*Cos[c + d*x]^n)
```

Rule 3084

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*(a*Cos[c + d*x] + b*Sin[
c + d*x])^n*Hypergeometric2F1[1, n, n + 1, (a + b*Tan[c + d*x])/(2*a)])/(2*
a*d*n*Cos[c + d*x]^n), x] /; FreeQ[{a, b, c, d, n}, x] && EqQ[m + n, 0] &&
EqQ[a^2 + b^2, 0] &&  !IntegerQ[n]
```

Rubi steps

$$\int \cos^{-n}(c+dx)(a\cos(c+dx)+ia\sin(c+dx))^n\,dx = -\frac{i\cos^{-n}(c+dx)\,{}_2F_1\left(1,n;1+n;\frac{1}{2}(1+i\tan(c+dx))\right)(a\,\mathrm{co}}{2dn}$$

Mathematica [A] time = 2.17, size = 90, normalized size = 1.36

$$\frac{\cos^{-n}(c+dx)\left(n(\tan(c+dx)-i)\,{}_2F_1\left(1,n+1;n+2;\frac{1}{2}(i\tan(c+dx)+1)\right)-2i(n+1)\right)(a(\cos(c+dx)+i\sin(c+}{4dn(n+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Cos[c + d*x] + I*a*Sin[c + d*x])^n/Cos[c + d*x]^n,x]
```

```
[Out] ((a*(Cos[c + d*x] + I*Sin[c + d*x]))^n*((-2*I)*(1 + n) + n*Hypergeometric2F
1[1, 1 + n, 2 + n, (1 + I*Tan[c + d*x])/2]*(-I + Tan[c + d*x])))/(4*d*n*(1
+ n)*Cos[c + d*x]^n)
```

fricas [F] time = 0.68, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{e^{\left(i\,dnx+i\,cn+n\log(a)\right)}}{\left(\frac{1}{2}\left(e^{(2i\,dx+2i\,c)}+1\right)e^{(-i\,dx-i\,c)}\right)^{n}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cos(d*x+c)+I*a*sin(d*x+c))^n/(cos(d*x+c)^n),x, algorithm="fric
as")
```

```
[Out] integral(e^(I*d*n*x + I*c*n + n*log(a))/(1/2*(e^(2*I*d*x + 2*I*c) + 1)*e^(-
I*d*x - I*c))^n, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a\cos(dx+c)+i\,a\sin(dx+c))^{n}}{\cos(dx+c)^{n}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cos(d*x+c)+I*a*sin(d*x+c))^n/(cos(d*x+c)^n),x, algorithm="giac
")
```

```
[Out] integrate((a*cos(d*x + c) + I*a*sin(d*x + c))^n/cos(d*x + c)^n, x)
```

maple [F] time = 0.68, size = 0, normalized size = 0.00

$$\int (a\cos(dx+c)+ia\sin(dx+c))^{n}\left(\cos^{-n}(dx+c)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*cos(d*x+c)+I*a*sin(d*x+c))^n/(cos(d*x+c)^n),x)
```

```
[Out] int((a*cos(d*x+c)+I*a*sin(d*x+c))^n/(cos(d*x+c)^n),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a\cos(dx+c)+i\,a\sin(dx+c))^{n}\cos(dx+c)^{-n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cos(d*x+c)+I*a*sin(d*x+c))^n/(cos(d*x+c)^n),x, algorithm="maxi
ma")
```

```
[Out] integrate((a*cos(d*x + c) + I*a*sin(d*x + c))^n*cos(d*x + c)^(-n), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{{\left(a\,\cos\left(c+d\,x\right)+a\,\sin\left(c+d\,x\right)\,1{}\mathrm{i}\right)}^n}{{\cos\left(c+d\,x\right)}^n} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*cos(c + d*x) + a*sin(c + d*x)*1i)^n/cos(c + d*x)^n,x)
```

```
[Out] int((a*cos(c + d*x) + a*sin(c + d*x)*1i)^n/cos(c + d*x)^n, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a \left(i \sin{\left(c + d x \right)} + \cos{\left(c + d x \right)}\right)\right)^{n} \cos^{- n}{\left(c + d x \right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cos(d*x+c)+I*a*sin(d*x+c))**n/(cos(d*x+c)**n),x)
```

```
[Out] Integral((a*(I*sin(c + d*x) + cos(c + d*x)))**n*cos(c + d*x)**(-n), x)
```

3.188 $\int \frac{1}{\sec(x)+\tan(x)}\,dx$

Optimal. Leaf size=5

$$\log(\sin(x)+1)$$

[Out] ln(1+sin(x))

Rubi [A] time = 0.02, antiderivative size = 5, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {3159, 2667, 31}

$$\log(\sin(x)+1)$$

Antiderivative was successfully verified.

[In] Int[(Sec[x] + Tan[x])^(-1),x]

[Out] Log[1 + Sin[x]]

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2667

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rule 3159

```
Int[((a_.) + (b_.)*sec[(d_.) + (e_.)*(x_)] + (c_.)*tan[(d_.) + (e_.)*(x_)])
^(-1), x_Symbol] :> Int[Cos[d + e*x]/(b + a*Cos[d + e*x] + c*Sin[d + e*x]),
 x] /; FreeQ[{a, b, c, d, e}, x]
```

Rubi steps

$$\int \frac{1}{\sec(x)+\tan(x)}\,dx = \int \frac{\cos(x)}{1+\sin(x)}\,dx$$
$$= \text{Subst}\left(\int \frac{1}{1+x}\,dx, x, \sin(x)\right)$$
$$= \log(1+\sin(x))$$

<u>**Mathematica**</u> [**B**] time = 0.02, size = 16, normalized size = 3.20

$$2\log\left(\sin\left(\frac{x}{2}\right)+\cos\left(\frac{x}{2}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[x] + Tan[x])^(-1),x]
[Out] 2*Log[Cos[x/2] + Sin[x/2]]
```

<u>**fricas**</u> [**A**] time = 1.05, size = 5, normalized size = 1.00

$$\log\left(\sin(x)+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sec(x)+tan(x)),x, algorithm="fricas")
[Out] log(sin(x) + 1)
```

<u>**giac**</u> [**B**] time = 5.74, size = 22, normalized size = 4.40

$$-\log\left(\tan\left(\frac{1}{2}\,x\right)^2+1\right)+2\,\log\left(\left|\tan\left(\frac{1}{2}\,x\right)+1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sec(x)+tan(x)),x, algorithm="giac")
[Out] -log(tan(1/2*x)^2 + 1) + 2*log(abs(tan(1/2*x) + 1))
```

<u>**maple**</u> [**A**] time = 0.08, size = 6, normalized size = 1.20

$$\ln\left(1+\sin(x)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sec(x)+tan(x)),x)
```

```
[Out] ln(1+sin(x))
```

maxima [B] time = 0.32, size = 31, normalized size = 6.20

$$2\log\left(\frac{\sin(x)}{\cos(x)+1}+1\right)-\log\left(\frac{\sin(x)^2}{(\cos(x)+1)^2}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sec(x)+tan(x)),x, algorithm="maxima")
```

```
[Out] 2*log(sin(x)/(cos(x) + 1) + 1) - log(sin(x)^2/(cos(x) + 1)^2 + 1)
```

mupad [B] time = 1.11, size = 21, normalized size = 4.20

$$2\ln\left(\tan\left(\frac{x}{2}\right)+1\right)-\ln\left(\tan\left(\frac{x}{2}\right)^2+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(tan(x) + 1/cos(x)),x)
```

```
[Out] 2*log(tan(x/2) + 1) - log(tan(x/2)^2 + 1)
```

sympy [B] time = 0.13, size = 17, normalized size = 3.40

$$\log\left(\tan\left(x\right)+\sec\left(x\right)\right)-\frac{\log\left(\tan^{2}\left(x\right)+1\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sec(x)+tan(x)),x)
```

```
[Out] log(tan(x) + sec(x)) - log(tan(x)**2 + 1)/2
```

3.189 $\int \frac{\sin(x)}{\sec(x)+\tan(x)}\,dx$

Optimal. Leaf size=10

$$\sin(x) - \log(\sin(x) + 1)$$

```
[Out] -ln(1+sin(x))+sin(x)
```

Rubi [A] time = 0.07, antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {4391, 2833, 43}

$$\sin(x) - \log(\sin(x) + 1)$$

Antiderivative was successfully verified.

```
[In] Int[Sin[x]/(Sec[x] + Tan[x]),x]
```

```
[Out] -Log[1 + Sin[x]] + Sin[x]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2833

```
Int[cos[(e_.) + (f_.)*(x_)]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((
c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b*f), Sub
st[Int[(a + x)^m*(c + (d*x)/b)^n, x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b
, c, d, e, f, m, n}, x]
```

Rule 4391

```
Int[(u_.)*((b_.)*sec[(c_.) + (d_.)*(x_)]^(n_.) + (a_.)*tan[(c_.) + (d_.)*(x
_)]^(n_.))^(p_), x_Symbol] :> Int[ActivateTrig[u]*Sec[c + d*x]^(n*p)*(b + a
*Sin[c + d*x]^n)^p, x] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p]
```

Rubi steps

$$\begin{aligned}\int \frac{\sin(x)}{\sec(x)+\tan(x)}\,dx &= \int \frac{\cos(x)\sin(x)}{1+\sin(x)}\,dx\\ &= \text{Subst}\left(\int \frac{x}{1+x}\,dx, x, \sin(x)\right)\\ &= \text{Subst}\left(\int \left(1+\frac{1}{-1-x}\right)dx, x, \sin(x)\right)\\ &= -\log(1+\sin(x))+\sin(x)\end{aligned}$$

Mathematica [A] time = 0.02, size = 19, normalized size = 1.90

$$\sin(x) - 2\log\left(\sin\left(\frac{x}{2}\right)+\cos\left(\frac{x}{2}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[x]/(Sec[x] + Tan[x]),x]
[Out] -2*Log[Cos[x/2] + Sin[x/2]] + Sin[x]
```

fricas [A] time = 0.76, size = 10, normalized size = 1.00

$$-\log\left(\sin(x)+1\right)+\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)/(sec(x)+tan(x)),x, algorithm="fricas")
[Out] -log(sin(x) + 1) + sin(x)
```

giac [A] time = 0.16, size = 10, normalized size = 1.00

$$-\log\left(\sin(x)+1\right)+\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)/(sec(x)+tan(x)),x, algorithm="giac")
[Out] -log(sin(x) + 1) + sin(x)
```

maple [A] time = 0.11, size = 11, normalized size = 1.10

$$-\ln\left(1+\sin(x)\right)+\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)/(sec(x)+tan(x)),x)
[Out] -ln(1+sin(x))+sin(x)
```

maxima [B] time = 0.41, size = 54, normalized size = 5.40

$$\frac{2\sin(x)}{\left(\frac{\sin(x)^2}{(\cos(x)+1)^2}+1\right)(\cos(x)+1)} - 2\log\left(\frac{\sin(x)}{\cos(x)+1}+1\right) + \log\left(\frac{\sin(x)^2}{(\cos(x)+1)^2}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)/(sec(x)+tan(x)),x, algorithm="maxima")
[Out] 2*sin(x)/((sin(x)^2/(cos(x) + 1)^2 + 1)*(cos(x) + 1)) - 2*log(sin(x)/(cos(x
) + 1) + 1) + log(sin(x)^2/(cos(x) + 1)^2 + 1)
```

mupad [B] time = 0.60, size = 21, normalized size = 2.10

$$\ln\left(\tan\left(\frac{x}{2}\right)^2+1\right) - 2\ln\left(\tan\left(\frac{x}{2}\right)+1\right) + \sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)/(tan(x) + 1/cos(x)),x)
[Out] log(tan(x/2)^2 + 1) - 2*log(tan(x/2) + 1) + sin(x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin(x)}{\tan(x)+\sec(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)/(sec(x)+tan(x)),x)
[Out] Integral(sin(x)/(tan(x) + sec(x)), x)
```

3.190 $\int \frac{\cos(x)}{\sec(x)+\tan(x)}\,dx$

Optimal. Leaf size=4

$$x+\cos(x)$$

```
[Out] x+cos(x)
```

Rubi [A] time = 0.06, antiderivative size = 4, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {4391, 2682, 8}

$$x+\cos(x)$$

Antiderivative was successfully verified.

```
[In] Int[Cos[x]/(Sec[x] + Tan[x]),x]
```

```
[Out] x + Cos[x]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2682

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[(g*(g*Cos[e + f*x])^(p - 1))/(b*f*(p - 1)), x] + Di
st[g^2/a, Int[(g*Cos[e + f*x])^(p - 2), x], x] /; FreeQ[{a, b, e, f, g}, x]
 && EqQ[a^2 - b^2, 0] && GtQ[p, 1] && IntegerQ[2*p]
```

Rule 4391

```
Int[(u_.)*((b_.)*sec[(c_.) + (d_.)*(x_)]^(n_.) + (a_.)*tan[(c_.) + (d_.)*(x
_)]^(n_.))^(p_), x_Symbol] :> Int[ActivateTrig[u]*Sec[c + d*x]^(n*p)*(b + a
*Sin[c + d*x]^n)^p, x] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p]
```

Rubi steps

$$\begin{aligned}
\int \frac{\cos(x)}{\sec(x)+\tan(x)}\,dx &= \int \frac{\cos^2(x)}{1+\sin(x)}\,dx\\
&= \cos(x)+\int 1\,dx\\
&= x+\cos(x)
\end{aligned}$$

Mathematica [A] time = 0.02, size = 4, normalized size = 1.00

$$x + \cos(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[x]/(Sec[x] + Tan[x]),x]
[Out] x + Cos[x]
```

fricas [A] time = 1.37, size = 4, normalized size = 1.00

$$x + \cos(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)/(sec(x)+tan(x)),x, algorithm="fricas")
[Out] x + cos(x)
```

giac [B] time = 0.83, size = 14, normalized size = 3.50

$$x + \frac{2}{\tan\left(\frac{1}{2}x\right)^2 + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)/(sec(x)+tan(x)),x, algorithm="giac")
[Out] x + 2/(tan(1/2*x)^2 + 1)
```

maple [B] time = 0.11, size = 15, normalized size = 3.75

$$\frac{2}{\tan^2\left(\frac{x}{2}\right) + 1} + x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)/(sec(x)+tan(x)),x)
[Out] 2/(tan(1/2*x)^2+1)+x
```

maxima [B] time = 0.42, size = 30, normalized size = 7.50

$$\frac{2}{\frac{\sin(x)^2}{(\cos(x)+1)^2} + 1} + 2 \arctan\left(\frac{\sin(x)}{\cos(x) + 1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)/(sec(x)+tan(x)),x, algorithm="maxima")
[Out] 2/(sin(x)^2/(cos(x) + 1)^2 + 1) + 2*arctan(sin(x)/(cos(x) + 1))
```

mupad [B] time = 0.55, size = 4, normalized size = 1.00

$$x + \cos(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)/(tan(x) + 1/cos(x)),x)
[Out] x + cos(x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos(x)}{\tan(x) + \sec(x)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)/(sec(x)+tan(x)),x)
[Out] Integral(cos(x)/(tan(x) + sec(x)), x)
```

3.191 $\int \frac{\tan(x)}{\sec(x)+\tan(x)}\,dx$

Optimal. Leaf size=11

$$x+\frac{\cos(x)}{\sin(x)+1}$$

```
[Out] x+cos(x)/(1+sin(x))
```

Rubi [A] time = 0.05, antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {4391, 2735, 2648}

$$x+\frac{\cos(x)}{\sin(x)+1}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[x]/(Sec[x] + Tan[x]),x]
```

```
[Out] x + Cos[x]/(1 + Sin[x])
```

Rule 2648

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> -Simp[Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 4391

```
Int[(u_.)*((b_.)*sec[(c_.) + (d_.)*(x_)]^(n_.) + (a_.)*tan[(c_.) + (d_.)*(x
_)]^(n_.))^(p_), x_Symbol] :> Int[ActivateTrig[u]*Sec[c + d*x]^(n*p)*(b + a
*Sin[c + d*x]^n)^p, x] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p]
```

Rubi steps

$$\int \frac{\tan(x)}{\sec(x)+\tan(x)}\,dx = \int \frac{\sin(x)}{1+\sin(x)}\,dx$$
$$= x - \int \frac{1}{1+\sin(x)}\,dx$$
$$= x + \frac{\cos(x)}{1+\sin(x)}$$

Mathematica [B] time = 0.03, size = 25, normalized size = 2.27

$$x - \frac{2\sin\left(\frac{x}{2}\right)}{\sin\left(\frac{x}{2}\right)+\cos\left(\frac{x}{2}\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[x]/(Sec[x] + Tan[x]),x]
[Out] x - (2*Sin[x/2])/(Cos[x/2] + Sin[x/2])
```

fricas [B] time = 0.70, size = 24, normalized size = 2.18

$$\frac{(x+1)\cos(x)+(x-1)\sin(x)+x+1}{\cos(x)+\sin(x)+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)/(sec(x)+tan(x)),x, algorithm="fricas")
[Out] ((x + 1)*cos(x) + (x - 1)*sin(x) + x + 1)/(cos(x) + sin(x) + 1)
```

giac [A] time = 0.96, size = 12, normalized size = 1.09

$$x + \frac{2}{\tan\left(\frac{1}{2}x\right)+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)/(sec(x)+tan(x)),x, algorithm="giac")
[Out] x + 2/(tan(1/2*x) + 1)
```

maple [A] time = 0.08, size = 13, normalized size = 1.18

$$\frac{2}{\tan\left(\frac{x}{2}\right)+1} + x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(x)/(sec(x)+tan(x)),x)
```

```
[Out] 2/(tan(1/2*x)+1)+x
```

maxima [B] time = 0.43, size = 28, normalized size = 2.55

$$\frac{2}{\frac{\sin(x)}{\cos(x)+1}+1}+2\arctan\left(\frac{\sin(x)}{\cos(x)+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)/(sec(x)+tan(x)),x, algorithm="maxima")
```

```
[Out] 2/(sin(x)/(cos(x) + 1) + 1) + 2*arctan(sin(x)/(cos(x) + 1))
```

mupad [B] time = 0.58, size = 12, normalized size = 1.09

$$x+\frac{2}{\tan\left(\frac{x}{2}\right)+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(x)/(tan(x) + 1/cos(x)),x)
```

```
[Out] x + 2/(tan(x/2) + 1)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\tan(x)}{\tan(x)+\sec(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)/(sec(x)+tan(x)),x)
```

```
[Out] Integral(tan(x)/(tan(x) + sec(x)), x)
```

3.192 $\int \frac{\cot(x)}{\sec(x)+\tan(x)}\,dx$

Optimal. Leaf size=9

$$-x-\tanh^{-1}(\cos(x))$$

```
[Out] -x-arctanh(cos(x))
```

Rubi [A] time = 0.08, antiderivative size = 9, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {4391, 2839, 3770, 8}

$$-x-\tanh^{-1}(\cos(x))$$

Antiderivative was successfully verified.

```
[In] Int[Cot[x]/(Sec[x] + Tan[x]),x]
```

```
[Out] -x - ArcTanh[Cos[x]]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2839

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(
n_.))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[g^2/a, Int[
(g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^n, x], x] - Dist[g^2/(b*d), Int[(
g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d,
 e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 4391

```
Int[(u_.)*((b_.)*sec[(c_.) + (d_.)*(x_)]^(n_.) + (a_.)*tan[(c_.) + (d_.)*(x
_)]^(n_.))^(p_), x_Symbol] :> Int[ActivateTrig[u]*Sec[c + d*x]^(n*p)*(b + a
*Sin[c + d*x]^n)^p, x] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p]
```

Rubi steps

$$\int \frac{\cot(x)}{\sec(x)+\tan(x)}\,dx = \int \frac{\cos(x)\cot(x)}{1+\sin(x)}\,dx$$
$$= -\int 1\,dx + \int \csc(x)\,dx$$
$$= -x - \tanh^{-1}(\cos(x))$$

Mathematica [B] time = 0.02, size = 20, normalized size = 2.22

$$-x + \log\left(\sin\left(\frac{x}{2}\right)\right) - \log\left(\cos\left(\frac{x}{2}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[x]/(Sec[x] + Tan[x]),x]
[Out] -x - Log[Cos[x/2]] + Log[Sin[x/2]]
```

fricas [B] time = 0.68, size = 22, normalized size = 2.44

$$-x - \frac{1}{2}\log\left(\frac{1}{2}\cos(x) + \frac{1}{2}\right) + \frac{1}{2}\log\left(-\frac{1}{2}\cos(x) + \frac{1}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)/(sec(x)+tan(x)),x, algorithm="fricas")
[Out] -x - 1/2*log(1/2*cos(x) + 1/2) + 1/2*log(-1/2*cos(x) + 1/2)
```

giac [A] time = 3.98, size = 10, normalized size = 1.11

$$-x + \log\left(\left|\tan\left(\frac{1}{2}x\right)\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)/(sec(x)+tan(x)),x, algorithm="giac")
[Out] -x + log(abs(tan(1/2*x)))
```

maple [A] time = 0.13, size = 10, normalized size = 1.11

$$\ln\left(\tan\left(\frac{x}{2}\right)\right) - x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(x)/(sec(x)+tan(x)),x)
```

```
[Out] ln(tan(1/2*x))-x
```

maxima [B] time = 0.49, size = 23, normalized size = 2.56

$$-2 \arctan\left(\frac{\sin(x)}{\cos(x)+1}\right) + \log\left(\frac{\sin(x)}{\cos(x)+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)/(sec(x)+tan(x)),x, algorithm="maxima")
```

```
[Out] -2*arctan(sin(x)/(cos(x) + 1)) + log(sin(x)/(cos(x) + 1))
```

mupad [B] time = 0.59, size = 23, normalized size = 2.56

$$2\,\mathrm{atan}\left(\frac{8}{4\tan\left(\frac{x}{2}\right)+4}-1\right) + \ln\left(\tan\left(\frac{x}{2}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(x)/(tan(x) + 1/cos(x)),x)
```

```
[Out] 2*atan(8/(4*tan(x/2) + 4) - 1) + log(tan(x/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cot(x)}{\tan(x)+\sec(x)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)/(sec(x)+tan(x)),x)
```

```
[Out] Integral(cot(x)/(tan(x) + sec(x)), x)
```

3.193 $\int \frac{\sec(x)}{\sec(x)+\tan(x)}\,dx$

Optimal. Leaf size=10

$$-\frac{\cos(x)}{\sin(x)+1}$$

```
[Out] -cos(x)/(1+sin(x))
```

Rubi [A] time = 0.02, antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {3165, 2648}

$$-\frac{\cos(x)}{\sin(x)+1}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[x]/(Sec[x] + Tan[x]),x]
```

```
[Out] -(Cos[x]/(1 + Sin[x]))
```

Rule 2648

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> -Simp[Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rule 3165

```
Int[sec[(d_.) + (e_.)*(x_)]^(n_.)*((a_.) + (b_.)*sec[(d_.) + (e_.)*(x_)] +
(c_.)*tan[(d_.) + (e_.)*(x_)])^(m_), x_Symbol] :> Int[1/(b + a*Cos[d + e*x]
 + c*Sin[d + e*x])^n, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[m + n, 0] && I
ntegerQ[n]
```

Rubi steps

$$\begin{aligned}\int \frac{\sec(x)}{\sec(x)+\tan(x)}\,dx &= \int \frac{1}{1+\sin(x)}\,dx\\ &= -\frac{\cos(x)}{1+\sin(x)}\end{aligned}$$

Mathematica [B] time = 0.02, size = 23, normalized size = 2.30

$$\frac{2\sin\left(\frac{x}{2}\right)}{\sin\left(\frac{x}{2}\right)+\cos\left(\frac{x}{2}\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[x]/(Sec[x] + Tan[x]),x]
[Out] (2*Sin[x/2])/(Cos[x/2] + Sin[x/2])
```

fricas [A] time = 1.95, size = 18, normalized size = 1.80

$$-\frac{\cos(x)-\sin(x)+1}{\cos(x)+\sin(x)+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)/(sec(x)+tan(x)),x, algorithm="fricas")
[Out] -(cos(x) - sin(x) + 1)/(cos(x) + sin(x) + 1)
```

giac [A] time = 0.20, size = 10, normalized size = 1.00

$$-\frac{2}{\tan\left(\frac{1}{2}x\right)+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)/(sec(x)+tan(x)),x, algorithm="giac")
[Out] -2/(tan(1/2*x) + 1)
```

maple [A] time = 0.07, size = 11, normalized size = 1.10

$$-\frac{2}{\tan\left(\frac{x}{2}\right)+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(x)/(sec(x)+tan(x)),x)
[Out] -2/(tan(1/2*x)+1)
```

maxima [A] time = 0.43, size = 15, normalized size = 1.50

$$-\frac{2}{\frac{\sin(x)}{\cos(x)+1}+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)/(sec(x)+tan(x)),x, algorithm="maxima")
```

```
[Out] -2/(sin(x)/(cos(x) + 1) + 1)
```

mupad [B] time = 0.55, size = 10, normalized size = 1.00

$$-\frac{2}{\tan\left(\frac{x}{2}\right)+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(x)*(tan(x) + 1/cos(x))),x)
```

```
[Out] -2/(tan(x/2) + 1)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec(x)}{\tan(x)+\sec(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)/(sec(x)+tan(x)),x)
```

```
[Out] Integral(sec(x)/(tan(x) + sec(x)), x)
```

3.194 $\int \frac{\csc(x)}{\sec(x)+\tan(x)}\,dx$

Optimal. Leaf size=11

$$\log(\sin(x)) - \log(\sin(x) + 1)$$

```
[Out] ln(sin(x))-ln(1+sin(x))
```

Rubi [A] time = 0.05, antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {4391, 2707, 36, 29, 31}

$$\log(\sin(x)) - \log(\sin(x) + 1)$$

Antiderivative was successfully verified.

```
[In] Int[Csc[x]/(Sec[x] + Tan[x]),x]
```

```
[Out] Log[Sin[x]] - Log[1 + Sin[x]]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 2707

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Dist[1/f, Subst[Int[(x^p*(a + x)^(m - (p + 1)/2))/(a - x)
^((p + 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && Eq
Q[a^2 - b^2, 0] && IntegerQ[(p + 1)/2]
```

Rule 4391

```
Int[(u_.)*((b_.)*sec[(c_.) + (d_.)*(x_)]^(n_.) + (a_.)*tan[(c_.) + (d_.)*(x
_)]^(n_.))^(p_), x_Symbol] :> Int[ActivateTrig[u]*Sec[c + d*x]^(n*p)*(b + a
```

```
*Sin[c + d*x]^n)^p, x] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p]
```

Rubi steps

$$\begin{aligned}\int \frac{\csc(x)}{\sec(x)+\tan(x)}\,dx &= \int \frac{\cot(x)}{1+\sin(x)}\,dx\\ &= \text{Subst}\left(\int \frac{1}{x(1+x)}\,dx, x, \sin(x)\right)\\ &= \text{Subst}\left(\int \frac{1}{x}\,dx, x, \sin(x)\right) - \text{Subst}\left(\int \frac{1}{1+x}\,dx, x, \sin(x)\right)\\ &= \log(\sin(x)) - \log(1+\sin(x))\end{aligned}$$

Mathematica [A] time = 0.02, size = 20, normalized size = 1.82

$$\log(\sin(x)) - 2\log\left(\sin\left(\frac{x}{2}\right) + \cos\left(\frac{x}{2}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[x]/(Sec[x] + Tan[x]),x]
[Out] -2*Log[Cos[x/2] + Sin[x/2]] + Log[Sin[x]]
```

fricas [A] time = 0.50, size = 13, normalized size = 1.18

$$\log\left(\frac{1}{2}\,\sin(x)\right) - \log\left(\sin(x) + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)/(sec(x)+tan(x)),x, algorithm="fricas")
[Out] log(1/2*sin(x)) - log(sin(x) + 1)
```

giac [A] time = 0.16, size = 12, normalized size = 1.09

$$-\log\left(\sin(x) + 1\right) + \log\left(|\sin(x)|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)/(sec(x)+tan(x)),x, algorithm="giac")
[Out] -log(sin(x) + 1) + log(abs(sin(x)))
```

maple [A] time = 0.12, size = 8, normalized size = 0.73

$$-\ln(1+\csc(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(x)/(sec(x)+tan(x)),x)
[Out] -ln(1+csc(x))
```

maxima [B] time = 0.57, size = 25, normalized size = 2.27

$$-2\,\log\left(\frac{\sin(x)}{\cos(x)+1}+1\right)+\log\left(\frac{\sin(x)}{\cos(x)+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)/(sec(x)+tan(x)),x, algorithm="maxima")
[Out] -2*log(sin(x)/(cos(x) + 1) + 1) + log(sin(x)/(cos(x) + 1))
```

mupad [B] time = 0.57, size = 15, normalized size = 1.36

$$\ln\left(\tan\left(\frac{x}{2}\right)\right)-2\,\ln\left(\tan\left(\frac{x}{2}\right)+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(x)*(tan(x) + 1/cos(x))),x)
[Out] log(tan(x/2)) - 2*log(tan(x/2) + 1)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\csc(x)}{\tan(x)+\sec(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)/(sec(x)+tan(x)),x)
[Out] Integral(csc(x)/(tan(x) + sec(x)), x)
```

3.195 $\int \frac{1}{\sec(x)-\tan(x)}\,dx$

Optimal. Leaf size=9

$$-\log(1-\sin(x))$$

```
[Out] -ln(1-sin(x))
```

Rubi [A] time = 0.03, antiderivative size = 9, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3159, 2667, 31}

$$-\log(1-\sin(x))$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[x] - Tan[x])^(-1),x]
```

```
[Out] -Log[1 - Sin[x]]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2667

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2])
```

Rule 3159

```
Int[((a_.) + (b_.)*sec[(d_.) + (e_.)*(x_)] + (c_.)*tan[(d_.) + (e_.)*(x_)])^(-1), x_Symbol] :> Int[Cos[d + e*x]/(b + a*Cos[d + e*x] + c*Sin[d + e*x]), x] /; FreeQ[{a, b, c, d, e}, x]
```

Rubi steps

$$\int \frac{1}{\sec(x)-\tan(x)}\,dx = \int \frac{\cos(x)}{1-\sin(x)}\,dx$$
$$= -\operatorname{Subst}\left(\int \frac{1}{1+x}\,dx, x, -\sin(x)\right)$$
$$= -\log(1-\sin(x))$$

Mathematica [A] time = 0.02, size = 18, normalized size = 2.00

$$-2\log\left(\cos\left(\frac{x}{2}\right)-\sin\left(\frac{x}{2}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[x] - Tan[x])^(-1),x]
[Out] -2*Log[Cos[x/2] - Sin[x/2]]
```

fricas [A] time = 0.50, size = 9, normalized size = 1.00

$$-\log\left(-\sin(x)+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sec(x)-tan(x)),x, algorithm="fricas")
[Out] -log(-sin(x) + 1)
```

giac [B] time = 0.17, size = 20, normalized size = 2.22

$$\log\left(\tan\left(\frac{1}{2}\,x\right)^2+1\right)-2\,\log\left(\left|\tan\left(\frac{1}{2}\,x\right)-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sec(x)-tan(x)),x, algorithm="giac")
[Out] log(tan(1/2*x)^2 + 1) - 2*log(abs(tan(1/2*x) - 1))
```

maple [A] time = 0.08, size = 8, normalized size = 0.89

$$-\ln\left(\sin(x)-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sec(x)-tan(x)),x)
```

```
[Out] -ln(sin(x)-1)
```

maxima [B] time = 0.34, size = 29, normalized size = 3.22

$$-2\log\left(\frac{\sin(x)}{\cos(x)+1}-1\right)+\log\left(\frac{\sin(x)^2}{(\cos(x)+1)^2}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sec(x)-tan(x)),x, algorithm="maxima")
```

```
[Out] -2*log(sin(x)/(cos(x) + 1) - 1) + log(sin(x)^2/(cos(x) + 1)^2 + 1)
```

mupad [B] time = 0.96, size = 19, normalized size = 2.11

$$\ln\left(\tan\left(\frac{x}{2}\right)^2+1\right)-2\ln\left(\tan\left(\frac{x}{2}\right)-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-1/(tan(x) - 1/cos(x)),x)
```

```
[Out] log(tan(x/2)^2 + 1) - 2*log(tan(x/2) - 1)
```

sympy [B] time = 0.14, size = 17, normalized size = 1.89

$$-\log\left(-\tan\left(x\right)+\sec\left(x\right)\right)+\frac{\log\left(\tan^2\left(x\right)+1\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sec(x)-tan(x)),x)
```

```
[Out] -log(-tan(x) + sec(x)) + log(tan(x)**2 + 1)/2
```

3.196 $\int \frac{\sin(x)}{\sec(x)-\tan(x)}\,dx$

Optimal. Leaf size=14

$$-\sin(x)-\log(1-\sin(x))$$

```
[Out] -ln(1-sin(x))-sin(x)
```

Rubi [A] time = 0.08, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {4391, 2833, 43}

$$-\sin(x)-\log(1-\sin(x))$$

Antiderivative was successfully verified.

```
[In] Int[Sin[x]/(Sec[x] - Tan[x]),x]
```

```
[Out] -Log[1 - Sin[x]] - Sin[x]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2833

```
Int[cos[(e_.) + (f_.)*(x_)]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((
c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b*f), Sub
st[Int[(a + x)^m*(c + (d*x)/b)^n, x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b
, c, d, e, f, m, n}, x]
```

Rule 4391

```
Int[(u_.)*((b_.)*sec[(c_.) + (d_.)*(x_)]^(n_.) + (a_.)*tan[(c_.) + (d_.)*(x
_)]^(n_.))^(p_), x_Symbol] :> Int[ActivateTrig[u]*Sec[c + d*x]^(n*p)*(b + a
*Sin[c + d*x]^n)^p, x] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p]
```

Rubi steps

$$\int \frac{\sin(x)}{\sec(x)-\tan(x)}\,dx = \int \frac{\cos(x)\sin(x)}{1-\sin(x)}\,dx$$
$$= \text{Subst}\left(\int \frac{x}{1+x}\,dx, x, -\sin(x)\right)$$
$$= \text{Subst}\left(\int \left(1+\frac{1}{-1-x}\right)dx, x, -\sin(x)\right)$$
$$= -\log(1-\sin(x))-\sin(x)$$

Mathematica [A] time = 0.02, size = 23, normalized size = 1.64

$$-\sin(x)-2\log\left(\cos\left(\frac{x}{2}\right)-\sin\left(\frac{x}{2}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[x]/(Sec[x] - Tan[x]),x]
[Out] -2*Log[Cos[x/2] - Sin[x/2]] - Sin[x]
```

fricas [A] time = 1.46, size = 14, normalized size = 1.00

$$-\log(-\sin(x)+1)-\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)/(sec(x)-tan(x)),x, algorithm="fricas")
[Out] -log(-sin(x) + 1) - sin(x)
```

giac [A] time = 2.39, size = 14, normalized size = 1.00

$$-\log(-\sin(x)+1)-\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)/(sec(x)-tan(x)),x, algorithm="giac")
[Out] -log(-sin(x) + 1) - sin(x)
```

maple [A] time = 0.10, size = 13, normalized size = 0.93

$$-\sin(x)-\ln(\sin(x)-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)/(sec(x)-tan(x)),x)
```

```
[Out] -sin(x)-ln(sin(x)-1)
```

maxima [B] time = 1.03, size = 54, normalized size = 3.86

$$-\frac{2\,\sin(x)}{\left(\frac{\sin(x)^2}{(\cos(x)+1)^2}+1\right)(\cos(x)+1)}-2\,\log\left(\frac{\sin(x)}{\cos(x)+1}-1\right)+\log\left(\frac{\sin(x)^2}{(\cos(x)+1)^2}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)/(sec(x)-tan(x)),x, algorithm="maxima")
```

```
[Out] -2*sin(x)/((sin(x)^2/(cos(x) + 1)^2 + 1)*(cos(x) + 1)) - 2*log(sin(x)/(cos(
x) + 1) - 1) + log(sin(x)^2/(cos(x) + 1)^2 + 1)
```

mupad [B] time = 0.61, size = 23, normalized size = 1.64

$$\ln\left(\tan\left(\frac{x}{2}\right)^2+1\right)-2\,\ln\left(\tan\left(\frac{x}{2}\right)-1\right)-\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-sin(x)/(tan(x) - 1/cos(x)),x)
```

```
[Out] log(tan(x/2)^2 + 1) - 2*log(tan(x/2) - 1) - sin(x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sin(x)}{-\tan(x)+\sec(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)/(sec(x)-tan(x)),x)
```

```
[Out] Integral(sin(x)/(-tan(x) + sec(x)), x)
```

3.197 $\int \frac{\cos(x)}{\sec(x)-\tan(x)}\,dx$

Optimal. Leaf size=6

$$x - \cos(x)$$

```
[Out] x-cos(x)
```

Rubi [A] time = 0.06, antiderivative size = 6, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {4391, 2682, 8}

$$x - \cos(x)$$

Antiderivative was successfully verified.

```
[In] Int[Cos[x]/(Sec[x] - Tan[x]),x]
```

```
[Out] x - Cos[x]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2682

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[(g*(g*Cos[e + f*x])^(p - 1))/(b*f*(p - 1)), x] + Di
st[g^2/a, Int[(g*Cos[e + f*x])^(p - 2), x], x] /; FreeQ[{a, b, e, f, g}, x]
 && EqQ[a^2 - b^2, 0] && GtQ[p, 1] && IntegerQ[2*p]
```

Rule 4391

```
Int[(u_.)*((b_.)*sec[(c_.) + (d_.)*(x_)]^(n_.) + (a_.)*tan[(c_.) + (d_.)*(x
_)]^(n_.))^(p_), x_Symbol] :> Int[ActivateTrig[u]*Sec[c + d*x]^(n*p)*(b + a
*Sin[c + d*x]^n)^p, x] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p]
```

Rubi steps

$$\begin{aligned} \int \frac{\cos(x)}{\sec(x)-\tan(x)}\,dx &= \int \frac{\cos^2(x)}{1-\sin(x)}\,dx \\ &= -\cos(x) + \int 1\,dx \\ &= x - \cos(x) \end{aligned}$$

Mathematica [A] time = 0.02, size = 6, normalized size = 1.00

$$x - \cos(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[x]/(Sec[x] - Tan[x]),x]
[Out] x - Cos[x]
```

fricas [A] time = 0.58, size = 6, normalized size = 1.00

$$x - \cos(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)/(sec(x)-tan(x)),x, algorithm="fricas")
[Out] x - cos(x)
```

giac [B] time = 1.03, size = 14, normalized size = 2.33

$$x - \frac{2}{\tan\left(\frac{1}{2}\,x\right)^2 + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)/(sec(x)-tan(x)),x, algorithm="giac")
[Out] x - 2/(tan(1/2*x)^2 + 1)
```

maple [B] time = 0.11, size = 15, normalized size = 2.50

$$-\frac{2}{\tan^2\left(\frac{x}{2}\right) + 1} + x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)/(sec(x)-tan(x)),x)
[Out] -2/(tan(1/2*x)^2+1)+x
```

maxima [B] time = 0.49, size = 30, normalized size = 5.00

$$-\frac{2}{\frac{\sin(x)^2}{(\cos(x)+1)^2} + 1} + 2\arctan\left(\frac{\sin(x)}{\cos(x) + 1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)/(sec(x)-tan(x)),x, algorithm="maxima")
[Out] -2/(sin(x)^2/(cos(x) + 1)^2 + 1) + 2*arctan(sin(x)/(cos(x) + 1))
```

mupad [B] time = 0.58, size = 6, normalized size = 1.00

$$x - \cos(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-cos(x)/(tan(x) - 1/cos(x)),x)
[Out] x - cos(x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos(x)}{-\tan(x) + \sec(x)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)/(sec(x)-tan(x)),x)
[Out] Integral(cos(x)/(-tan(x) + sec(x)), x)
```

3.198 $\int \frac{\tan(x)}{\sec(x)-\tan(x)}\,dx$

Optimal. Leaf size=15

$$\frac{\cos(x)}{1-\sin(x)}-x$$

```
[Out] -x+cos(x)/(1-sin(x))
```

Rubi [A] time = 0.06, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {4391, 2735, 2648}

$$\frac{\cos(x)}{1-\sin(x)}-x$$

Antiderivative was successfully verified.

```
[In] Int[Tan[x]/(Sec[x] - Tan[x]),x]
```

```
[Out] -x + Cos[x]/(1 - Sin[x])
```

Rule 2648

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> -Simp[Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 4391

```
Int[(u_.)*((b_.)*sec[(c_.) + (d_.)*(x_)]^(n_.) + (a_.)*tan[(c_.) + (d_.)*(x
_)]^(n_.))^(p_), x_Symbol] :> Int[ActivateTrig[u]*Sec[c + d*x]^(n*p)*(b + a
*Sin[c + d*x]^n)^p, x] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p]
```

Rubi steps

$$\int \frac{\tan(x)}{\sec(x)-\tan(x)}\,dx = \int \frac{\sin(x)}{1-\sin(x)}\,dx$$
$$= -x + \int \frac{1}{1-\sin(x)}\,dx$$
$$= -x + \frac{\cos(x)}{1-\sin(x)}$$

Mathematica [A] time = 0.03, size = 29, normalized size = 1.93

$$\frac{2\sin\left(\frac{x}{2}\right)}{\cos\left(\frac{x}{2}\right)-\sin\left(\frac{x}{2}\right)}-x$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[x]/(Sec[x] - Tan[x]),x]
[Out] -x + (2*Sin[x/2])/(Cos[x/2] - Sin[x/2])
```

fricas [A] time = 0.53, size = 28, normalized size = 1.87

$$-\frac{(x-1)\cos(x)-(x+1)\sin(x)+x-1}{\cos(x)-\sin(x)+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)/(sec(x)-tan(x)),x, algorithm="fricas")
[Out] -((x - 1)*cos(x) - (x + 1)*sin(x) + x - 1)/(cos(x) - sin(x) + 1)
```

giac [A] time = 1.95, size = 14, normalized size = 0.93

$$-x-\frac{2}{\tan\left(\frac{1}{2}x\right)-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)/(sec(x)-tan(x)),x, algorithm="giac")
[Out] -x - 2/(tan(1/2*x) - 1)
```

maple [A] time = 0.07, size = 15, normalized size = 1.00

$$-\frac{2}{\tan\left(\frac{x}{2}\right)-1}-x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(x)/(sec(x)-tan(x)),x)
```

```
[Out] -2/(tan(1/2*x)-1)-x
```

maxima [A] time = 1.34, size = 28, normalized size = 1.87

$$-\frac{2}{\frac{\sin(x)}{\cos(x)+1}-1}-2\arctan\left(\frac{\sin(x)}{\cos(x)+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)/(sec(x)-tan(x)),x, algorithm="maxima")
```

```
[Out] -2/(sin(x)/(cos(x) + 1) - 1) - 2*arctan(sin(x)/(cos(x) + 1))
```

mupad [B] time = 0.58, size = 14, normalized size = 0.93

$$-x-\frac{2}{\tan\left(\frac{x}{2}\right)-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-tan(x)/(tan(x) - 1/cos(x)),x)
```

```
[Out] - x - 2/(tan(x/2) - 1)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan(x)}{-\tan(x)+\sec(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)/(sec(x)-tan(x)),x)
```

```
[Out] Integral(tan(x)/(-tan(x) + sec(x)), x)
```

3.199 $\int \frac{\cot(x)}{\sec(x)-\tan(x)}\,dx$

Optimal. Leaf size=7

$$x - \tanh^{-1}(\cos(x))$$

```
[Out] x-arctanh(cos(x))
```

Rubi [A] time = 0.09, antiderivative size = 7, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {4391, 2839, 3770, 8}

$$x - \tanh^{-1}(\cos(x))$$

Antiderivative was successfully verified.

```
[In] Int[Cot[x]/(Sec[x] - Tan[x]),x]
```

```
[Out] x - ArcTanh[Cos[x]]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2839

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(
n_.))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[g^2/a, Int[
(g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^n, x], x] - Dist[g^2/(b*d), Int[(
g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d,
 e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 4391

```
Int[(u_.)*((b_.)*sec[(c_.) + (d_.)*(x_)]^(n_.) + (a_.)*tan[(c_.) + (d_.)*(x
_)]^(n_.))^(p_), x_Symbol] :> Int[ActivateTrig[u]*Sec[c + d*x]^(n*p)*(b + a
*Sin[c + d*x]^n)^p, x] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p]
```

Rubi steps

$$\int \frac{\cot(x)}{\sec(x)-\tan(x)}\,dx = \int \frac{\cos(x)\cot(x)}{1-\sin(x)}\,dx$$
$$= \int 1\,dx + \int \csc(x)\,dx$$
$$= x - \tanh^{-1}(\cos(x))$$

Mathematica [B] time = 0.02, size = 18, normalized size = 2.57

$$x + \log\left(\sin\left(\frac{x}{2}\right)\right) - \log\left(\cos\left(\frac{x}{2}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[x]/(Sec[x] - Tan[x]),x]
[Out] x - Log[Cos[x/2]] + Log[Sin[x/2]]
```

fricas [B] time = 0.46, size = 20, normalized size = 2.86

$$x - \frac{1}{2}\log\left(\frac{1}{2}\cos(x) + \frac{1}{2}\right) + \frac{1}{2}\log\left(-\frac{1}{2}\cos(x) + \frac{1}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)/(sec(x)-tan(x)),x, algorithm="fricas")
[Out] x - 1/2*log(1/2*cos(x) + 1/2) + 1/2*log(-1/2*cos(x) + 1/2)
```

giac [A] time = 0.20, size = 8, normalized size = 1.14

$$x + \log\left(\left|\tan\left(\frac{1}{2}x\right)\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)/(sec(x)-tan(x)),x, algorithm="giac")
[Out] x + log(abs(tan(1/2*x)))
```

maple [A] time = 0.13, size = 8, normalized size = 1.14

$$\ln\left(\tan\left(\frac{x}{2}\right)\right) + x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(x)/(sec(x)-tan(x)),x)
```

```
[Out] ln(tan(1/2*x))+x
```

maxima [B] time = 0.44, size = 23, normalized size = 3.29

$$2 \arctan\left(\frac{\sin(x)}{\cos(x)+1}\right) + \log\left(\frac{\sin(x)}{\cos(x)+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)/(sec(x)-tan(x)),x, algorithm="maxima")
```

```
[Out] 2*arctan(sin(x)/(cos(x) + 1)) + log(sin(x)/(cos(x) + 1))
```

mupad [B] time = 0.61, size = 23, normalized size = 3.29

$$\ln\left(\tan\left(\frac{x}{2}\right)\right) - 2\,\mathrm{atan}\left(\frac{8}{4\tan\left(\frac{x}{2}\right)-4}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-cot(x)/(tan(x) - 1/cos(x)),x)
```

```
[Out] log(tan(x/2)) - 2*atan(8/(4*tan(x/2) - 4) + 1)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cot(x)}{-\tan(x)+\sec(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)/(sec(x)-tan(x)),x)
```

```
[Out] Integral(cot(x)/(-tan(x) + sec(x)), x)
```

3.200 $\int \frac{\sec(x)}{\sec(x)-\tan(x)}\,dx$

Optimal. Leaf size=11

$$\frac{\cos(x)}{1-\sin(x)}$$

```
[Out] cos(x)/(1-sin(x))
```

Rubi [A] time = 0.03, antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3165, 2648}

$$\frac{\cos(x)}{1-\sin(x)}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[x]/(Sec[x] - Tan[x]),x]
```

```
[Out] Cos[x]/(1 - Sin[x])
```

Rule 2648

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> -Simp[Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rule 3165

```
Int[sec[(d_.) + (e_.)*(x_)]^(n_.)*((a_.) + (b_.)*sec[(d_.) + (e_.)*(x_)] +
(c_.)*tan[(d_.) + (e_.)*(x_)])^(m_), x_Symbol] :> Int[1/(b + a*Cos[d + e*x]
 + c*Sin[d + e*x])^n, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[m + n, 0] && I
ntegerQ[n]
```

Rubi steps

$$\begin{aligned}\int \frac{\sec(x)}{\sec(x)-\tan(x)}\,dx &= \int \frac{1}{1-\sin(x)}\,dx\\ &= \frac{\cos(x)}{1-\sin(x)}\end{aligned}$$

Mathematica [B] time = 0.02, size = 25, normalized size = 2.27

$$\frac{2\sin\left(\frac{x}{2}\right)}{\cos\left(\frac{x}{2}\right)-\sin\left(\frac{x}{2}\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[x]/(Sec[x] - Tan[x]),x]
[Out] (2*Sin[x/2])/(Cos[x/2] - Sin[x/2])
```

fricas [A] time = 0.49, size = 17, normalized size = 1.55

$$\frac{\cos(x)+\sin(x)+1}{\cos(x)-\sin(x)+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)/(sec(x)-tan(x)),x, algorithm="fricas")
[Out] (cos(x) + sin(x) + 1)/(cos(x) - sin(x) + 1)
```

giac [A] time = 1.67, size = 10, normalized size = 0.91

$$-\frac{2}{\tan\left(\frac{1}{2}x\right)-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)/(sec(x)-tan(x)),x, algorithm="giac")
[Out] -2/(tan(1/2*x) - 1)
```

maple [A] time = 0.07, size = 11, normalized size = 1.00

$$-\frac{2}{\tan\left(\frac{x}{2}\right)-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(x)/(sec(x)-tan(x)),x)
[Out] -2/(tan(1/2*x)-1)
```

maxima [A] time = 0.33, size = 15, normalized size = 1.36

$$-\frac{2}{\frac{\sin(x)}{\cos(x)+1}-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)/(sec(x)-tan(x)),x, algorithm="maxima")
```

```
[Out] -2/(sin(x)/(cos(x) + 1) - 1)
```

mupad [B] time = 0.56, size = 10, normalized size = 0.91

$$-\frac{2}{\tan\left(\frac{x}{2}\right)-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-1/(cos(x)*(tan(x) - 1/cos(x))),x)
```

```
[Out] -2/(tan(x/2) - 1)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sec(x)}{-\tan(x)+\sec(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)/(sec(x)-tan(x)),x)
```

```
[Out] Integral(sec(x)/(-tan(x) + sec(x)), x)
```

3.201 $\int \frac{\csc(x)}{\sec(x)-\tan(x)} \, dx$

Optimal. Leaf size=13

$$\log(\sin(x)) - \log(1 - \sin(x))$$

```
[Out] -ln(1-sin(x))+ln(sin(x))
```

Rubi [A] time = 0.06, antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {4391, 2707, 36, 29, 31}

$$\log(\sin(x)) - \log(1 - \sin(x))$$

Antiderivative was successfully verified.

```
[In] Int[Csc[x]/(Sec[x] - Tan[x]),x]
```

```
[Out] -Log[1 - Sin[x]] + Log[Sin[x]]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 2707

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Dist[1/f, Subst[Int[(x^p*(a + x)^(m - (p + 1)/2))/(a - x)
^((p + 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && Eq
Q[a^2 - b^2, 0] && IntegerQ[(p + 1)/2]
```

Rule 4391

```
Int[(u_.)*((b_.)*sec[(c_.) + (d_.)*(x_)]^(n_.) + (a_.)*tan[(c_.) + (d_.)*(x
_)]^(n_.))^(p_), x_Symbol] :> Int[ActivateTrig[u]*Sec[c + d*x]^(n*p)*(b + a
```

```
*Sin[c + d*x]^n)^p, x] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p]
```

Rubi steps

$$\begin{aligned}
\int \frac{\csc(x)}{\sec(x)-\tan(x)}\,dx &= \int \frac{\cot(x)}{1-\sin(x)}\,dx\\
&= \text{Subst}\left(\int \frac{1}{x(1+x)}\,dx, x, -\sin(x)\right)\\
&= \text{Subst}\left(\int \frac{1}{x}\,dx, x, -\sin(x)\right) - \text{Subst}\left(\int \frac{1}{1+x}\,dx, x, -\sin(x)\right)\\
&= -\log(1-\sin(x)) + \log(\sin(x))
\end{aligned}$$

Mathematica [A] time = 0.02, size = 22, normalized size = 1.69

$$\log(\sin(x)) - 2\log\left(\cos\left(\frac{x}{2}\right) - \sin\left(\frac{x}{2}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[x]/(Sec[x] - Tan[x]),x]
[Out] -2*Log[Cos[x/2] - Sin[x/2]] + Log[Sin[x]]
```

fricas [A] time = 0.69, size = 15, normalized size = 1.15

$$\log\left(\frac{1}{2}\sin(x)\right) - \log\left(-\sin(x) + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)/(sec(x)-tan(x)),x, algorithm="fricas")
[Out] log(1/2*sin(x)) - log(-sin(x) + 1)
```

giac [A] time = 0.18, size = 14, normalized size = 1.08

$$-\log\left(-\sin(x) + 1\right) + \log\left(|\sin(x)|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)/(sec(x)-tan(x)),x, algorithm="giac")
[Out] -log(-sin(x) + 1) + log(abs(sin(x)))
```

maple [A] time = 0.11, size = 8, normalized size = 0.62

$$-\ln\left(-1+\csc(x)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(x)/(sec(x)-tan(x)),x)
[Out] -ln(-1+csc(x))
```

maxima [A] time = 0.34, size = 25, normalized size = 1.92

$$-2\log\left(\frac{\sin(x)}{\cos(x)+1}-1\right)+\log\left(\frac{\sin(x)}{\cos(x)+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)/(sec(x)-tan(x)),x, algorithm="maxima")
[Out] -2*log(sin(x)/(cos(x) + 1) - 1) + log(sin(x)/(cos(x) + 1))
```

mupad [B] time = 0.56, size = 15, normalized size = 1.15

$$\ln\left(\tan\left(\frac{x}{2}\right)\right)-2\ln\left(\tan\left(\frac{x}{2}\right)-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-1/(sin(x)*(tan(x) - 1/cos(x))),x)
[Out] log(tan(x/2)) - 2*log(tan(x/2) - 1)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\csc(x)}{-\tan(x)+\sec(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)/(sec(x)-tan(x)),x)
[Out] Integral(csc(x)/(-tan(x) + sec(x)), x)
```

3.202 $\int \csc(c+dx)(\cot(c+dx)+\csc(c+dx))\,dx$

Optimal. Leaf size=23

$$-\frac{\cot(c+dx)}{d}-\frac{\csc(c+dx)}{d}$$

```
[Out] -cot(d*x+c)/d-csc(d*x+c)/d
```

Rubi [A] time = 0.09, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {4397, 2669, 3767, 8}

$$-\frac{\cot(c+dx)}{d}-\frac{\csc(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]*(Cot[c + d*x] + Csc[c + d*x]),x]
```

```
[Out] -(Cot[c + d*x]/d) - Csc[c + d*x]/d
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2669

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1))/(f*g*(p + 1)), x] + D
ist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] && (I
ntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 4397

```
Int[u_, x_Symbol] :> Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]
```

Rubi steps

$$
\begin{aligned}
\int \csc(c+dx)(\cot(c+dx)+\csc(c+dx))\,dx &= \int (1+\cos(c+dx))\csc^2(c+dx)\,dx\\
&= -\frac{\csc(c+dx)}{d}+\int \csc^2(c+dx)\,dx\\
&= -\frac{\csc(c+dx)}{d}-\frac{\text{Subst}(\int 1\,dx,x,\cot(c+dx))}{d}\\
&= -\frac{\cot(c+dx)}{d}-\frac{\csc(c+dx)}{d}
\end{aligned}
$$

Mathematica [A] time = 0.01, size = 15, normalized size = 0.65

$$
-\frac{\cot\left(\frac{1}{2}(c+dx)\right)}{d}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]*(Cot[c + d*x] + Csc[c + d*x]),x]
[Out] -(Cot[(c + d*x)/2]/d)
```

fricas [A] time = 0.50, size = 21, normalized size = 0.91

$$
-\frac{\cos(dx+c)+1}{d\sin(dx+c)}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*(cot(d*x+c)+csc(d*x+c)),x, algorithm="fricas")
[Out] -(cos(d*x + c) + 1)/(d*sin(d*x + c))
```

giac [A] time = 0.26, size = 16, normalized size = 0.70

$$
-\frac{1}{d\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*(cot(d*x+c)+csc(d*x+c)),x, algorithm="giac")
[Out] -1/(d*tan(1/2*d*x + 1/2*c))
```

maple [A] time = 0.10, size = 24, normalized size = 1.04

$$\frac{-\frac{1}{\sin(dx+c)} - \cot(dx+c)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)*(cot(d*x+c)+csc(d*x+c)),x)
[Out] 1/d*(-1/sin(d*x+c)-cot(d*x+c))
```

maxima [A] time = 0.34, size = 22, normalized size = 0.96

$$-\frac{\frac{1}{\sin(dx+c)} + \frac{1}{\tan(dx+c)}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*(cot(d*x+c)+csc(d*x+c)),x, algorithm="maxima")
[Out] -(1/sin(d*x + c) + 1/tan(d*x + c))/d
```

mupad [B] time = 0.59, size = 14, normalized size = 0.61

$$-\frac{\cot\left(\frac{c}{2} + \frac{d\,x}{2}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cot(c + d*x) + 1/sin(c + d*x))/sin(c + d*x),x)
[Out] -cot(c/2 + (d*x)/2)/d
```

sympy [A] time = 2.11, size = 27, normalized size = 1.17

$$\begin{cases} \frac{-\cot(c+dx) - \csc(c+dx)}{d} & \text{for}\ d \neq 0 \\ x(\cot(c) + \csc(c))\csc(c) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*(cot(d*x+c)+csc(d*x+c)),x)
[Out] Piecewise(((-cot(c + d*x) - csc(c + d*x))/d, Ne(d, 0)), (x*(cot(c) + csc(c)
)*csc(c), True))
```

3.203 $\int \frac{\sin(x)}{\cot(x)+\csc(x)}\,dx$

Optimal. Leaf size=6

$$x-\sin(x)$$

```
[Out] x-sin(x)
```

Rubi [A] time = 0.07, antiderivative size = 6, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {4392, 2682, 8}

$$x-\sin(x)$$

Antiderivative was successfully verified.

```
[In] Int[Sin[x]/(Cot[x] + Csc[x]),x]
```

```
[Out] x - Sin[x]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2682

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[(g*(g*Cos[e + f*x])^(p - 1))/(b*f*(p - 1)), x] + Di
st[g^2/a, Int[(g*Cos[e + f*x])^(p - 2), x], x] /; FreeQ[{a, b, e, f, g}, x]
 && EqQ[a^2 - b^2, 0] && GtQ[p, 1] && IntegerQ[2*p]
```

Rule 4392

```
Int[(cot[(c_.) + (d_.)*(x_)]^(n_.)*(a_.) + csc[(c_.) + (d_.)*(x_)]^(n_.)*(b
_.))^(p_)*(u_.), x_Symbol] :> Int[ActivateTrig[u]*Csc[c + d*x]^(n*p)*(b + a
*Cos[c + d*x]^n)^p, x] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p]
```

Rubi steps

$$\begin{aligned}\int \frac{\sin(x)}{\cot(x)+\csc(x)}\,dx &= \int \frac{\sin^2(x)}{1+\cos(x)}\,dx\\ &= -\sin(x)+\int 1\,dx\\ &= x-\sin(x)\end{aligned}$$

Mathematica [B] time = 0.01, size = 14, normalized size = 2.33

$$2\left(\frac{x}{2}-\frac{\sin(x)}{2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[x]/(Cot[x] + Csc[x]),x]
[Out] 2*(x/2 - Sin[x]/2)
```

fricas [A] time = 1.47, size = 6, normalized size = 1.00

$$x-\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)/(cot(x)+csc(x)),x, algorithm="fricas")
[Out] x - sin(x)
```

giac [B] time = 0.21, size = 18, normalized size = 3.00

$$x-\frac{2\tan\left(\frac{1}{2}x\right)}{\tan\left(\frac{1}{2}x\right)^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)/(cot(x)+csc(x)),x, algorithm="giac")
[Out] x - 2*tan(1/2*x)/(tan(1/2*x)^2 + 1)
```

maple [B] time = 0.10, size = 19, normalized size = 3.17

$$-\frac{2\tan\left(\frac{x}{2}\right)}{\tan^2\left(\frac{x}{2}\right)+1}+x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)/(cot(x)+csc(x)),x)
[Out] -2*tan(1/2*x)/(tan(1/2*x)^2+1)+x
```

maxima [B] time = 0.66, size = 38, normalized size = 6.33

$$-\frac{2\sin(x)}{\left(\frac{\sin(x)^2}{(\cos(x)+1)^2}+1\right)(\cos(x)+1)}+2\arctan\left(\frac{\sin(x)}{\cos(x)+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)/(cot(x)+csc(x)),x, algorithm="maxima")
```

```
[Out] -2*sin(x)/((sin(x)^2/(cos(x) + 1)^2 + 1)*(cos(x) + 1)) + 2*arctan(sin(x)/(c
os(x) + 1))
```

mupad [B] time = 1.05, size = 6, normalized size = 1.00

$$x - \sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)/(cot(x) + 1/sin(x)),x)
```

```
[Out] x - sin(x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin(x)}{\cot(x) + \csc(x)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)/(cot(x)+csc(x)),x)
```

```
[Out] Integral(sin(x)/(cot(x) + csc(x)), x)
```

3.204 $\int \frac{\cos(x)}{\cot(x)+\csc(x)}\,dx$

Optimal. Leaf size=10

$$\log(\cos(x)+1)-\cos(x)$$

```
[Out] -cos(x)+ln(1+cos(x))
```

Rubi [A] time = 0.07, antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {4392, 2833, 43}

$$\log(\cos(x)+1)-\cos(x)$$

Antiderivative was successfully verified.

```
[In] Int[Cos[x]/(Cot[x] + Csc[x]),x]
```

```
[Out] -Cos[x] + Log[1 + Cos[x]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2833

```
Int[cos[(e_.) + (f_.)*(x_)]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((
c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b*f), Sub
st[Int[(a + x)^m*(c + (d*x)/b)^n, x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b
, c, d, e, f, m, n}, x]
```

Rule 4392

```
Int[(cot[(c_.) + (d_.)*(x_)]^(n_.)*(a_.) + csc[(c_.) + (d_.)*(x_)]^(n_.)*(b
_.))^(p_)*(u_.), x_Symbol] :> Int[ActivateTrig[u]*Csc[c + d*x]^(n*p)*(b + a
*Cos[c + d*x]^n)^p, x] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p]
```

Rubi steps

$$\int \frac{\cos(x)}{\cot(x)+\csc(x)}\,dx = \int \frac{\cos(x)\sin(x)}{1+\cos(x)}\,dx$$
$$= -\operatorname{Subst}\left(\int \frac{x}{1+x}\,dx, x, \cos(x)\right)$$
$$= -\operatorname{Subst}\left(\int \left(1+\frac{1}{-1-x}\right)dx, x, \cos(x)\right)$$
$$= -\cos(x)+\log(1+\cos(x))$$

Mathematica [**A**] time = 0.01, size = 20, normalized size = 2.00

$$2\log\left(\cos\left(\frac{x}{2}\right)\right)-2\cos^2\left(\frac{x}{2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[x]/(Cot[x] + Csc[x]),x]
[Out] -2*Cos[x/2]^2 + 2*Log[Cos[x/2]]
```

fricas [**A**] time = 2.00, size = 12, normalized size = 1.20

$$-\cos(x)+\log\left(\frac{1}{2}\cos(x)+\frac{1}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)/(cot(x)+csc(x)),x, algorithm="fricas")
[Out] -cos(x) + log(1/2*cos(x) + 1/2)
```

giac [**A**] time = 0.43, size = 10, normalized size = 1.00

$$-\cos(x)+\log(\cos(x)+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)/(cot(x)+csc(x)),x, algorithm="giac")
[Out] -cos(x) + log(cos(x) + 1)
```

maple [**A**] time = 0.10, size = 11, normalized size = 1.10

$$-\cos(x)+\ln(1+\cos(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)/(cot(x)+csc(x)),x)
```

```
[Out] -cos(x)+ln(1+cos(x))
```

maxima [B] time = 0.43, size = 34, normalized size = 3.40

$$-\frac{2}{\frac{\sin(x)^2}{(\cos(x)+1)^2}+1}-\log\left(\frac{\sin(x)^2}{(\cos(x)+1)^2}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)/(cot(x)+csc(x)),x, algorithm="maxima")
```

```
[Out] -2/(sin(x)^2/(cos(x) + 1)^2 + 1) - log(sin(x)^2/(cos(x) + 1)^2 + 1)
```

mupad [B] time = 0.59, size = 24, normalized size = 2.40

$$-\ln\left(\tan\left(\frac{x}{2}\right)^2+1\right)-\frac{2}{\tan\left(\frac{x}{2}\right)^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)/(cot(x) + 1/sin(x)),x)
```

```
[Out] - log(tan(x/2)^2 + 1) - 2/(tan(x/2)^2 + 1)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\cos(x)}{\cot(x)+\csc(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)/(cot(x)+csc(x)),x)
```

```
[Out] Integral(cos(x)/(cot(x) + csc(x)), x)
```

3.205 $\int \frac{\tan(x)}{\cot(x)+\csc(x)}\,dx$

Optimal. Leaf size=7

$$\tanh^{-1}(\sin(x)) - x$$

```
[Out] -x+arctanh(sin(x))
```

Rubi [A] time = 0.08, antiderivative size = 7, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {4392, 2839, 3770, 8}

$$\tanh^{-1}(\sin(x)) - x$$

Antiderivative was successfully verified.

```
[In] Int[Tan[x]/(Cot[x] + Csc[x]),x]
```

```
[Out] -x + ArcTanh[Sin[x]]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2839

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(
n_.))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[g^2/a, Int[
(g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^n, x], x] - Dist[g^2/(b*d), Int[(
g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d,
 e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 4392

```
Int[(cot[(c_.) + (d_.)*(x_)]^(n_.)*(a_.) + csc[(c_.) + (d_.)*(x_)]^(n_.)*(b
_.))^(p_)*(u_.), x_Symbol] :> Int[ActivateTrig[u]*Csc[c + d*x]^(n*p)*(b + a
*Cos[c + d*x]^n)^p, x] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p]
```

Rubi steps

$$\int \frac{\tan(x)}{\cot(x)+\csc(x)}\,dx = \int \frac{\sin(x)\tan(x)}{1+\cos(x)}\,dx$$
$$= -\int 1\,dx + \int \sec(x)\,dx$$
$$= -x + \tanh^{-1}(\sin(x))$$

Mathematica [B] time = 0.03, size = 36, normalized size = 5.14

$$-x-\log\left(\cos\left(\frac{x}{2}\right)-\sin\left(\frac{x}{2}\right)\right)+\log\left(\sin\left(\frac{x}{2}\right)+\cos\left(\frac{x}{2}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[x]/(Cot[x] + Csc[x]),x]
[Out] -x - Log[Cos[x/2] - Sin[x/2]] + Log[Cos[x/2] + Sin[x/2]]
```

fricas [B] time = 0.71, size = 20, normalized size = 2.86

$$-x+\frac{1}{2}\log\left(\sin(x)+1\right)-\frac{1}{2}\log\left(-\sin(x)+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)/(cot(x)+csc(x)),x, algorithm="fricas")
[Out] -x + 1/2*log(sin(x) + 1) - 1/2*log(-sin(x) + 1)
```

giac [B] time = 0.22, size = 22, normalized size = 3.14

$$-x+\log\left(\left|\tan\left(\frac{1}{2}x\right)+1\right|\right)-\log\left(\left|\tan\left(\frac{1}{2}x\right)-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)/(cot(x)+csc(x)),x, algorithm="giac")
[Out] -x + log(abs(tan(1/2*x) + 1)) - log(abs(tan(1/2*x) - 1))
```

maple [B] time = 0.11, size = 21, normalized size = 3.00

$$-\ln\left(\tan\left(\frac{x}{2}\right)-1\right)+\ln\left(\tan\left(\frac{x}{2}\right)+1\right)-x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(x)/(cot(x)+csc(x)),x)
```

```
[Out] -ln(tan(1/2*x)-1)+ln(tan(1/2*x)+1)-x
```

maxima [B] time = 0.43, size = 39, normalized size = 5.57

$$-2\arctan\left(\frac{\sin(x)}{\cos(x)+1}\right)+\log\left(\frac{\sin(x)}{\cos(x)+1}+1\right)-\log\left(\frac{\sin(x)}{\cos(x)+1}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)/(cot(x)+csc(x)),x, algorithm="maxima")
```

```
[Out] -2*arctan(sin(x)/(cos(x) + 1)) + log(sin(x)/(cos(x) + 1) + 1) - log(sin(x)/
(cos(x) + 1) - 1)
```

mupad [B] time = 0.57, size = 11, normalized size = 1.57

$$2\operatorname{atanh}\left(\tan\left(\frac{x}{2}\right)\right)-x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(x)/(cot(x) + 1/sin(x)),x)
```

```
[Out] 2*atanh(tan(x/2)) - x
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\tan(x)}{\cot(x)+\csc(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)/(cot(x)+csc(x)),x)
```

```
[Out] Integral(tan(x)/(cot(x) + csc(x)), x)
```

3.206 $\int \frac{\cot(x)}{\cot(x)+\csc(x)}\,dx$

Optimal. Leaf size=12

$$x - \frac{\sin(x)}{\cos(x)+1}$$

```
[Out] x-sin(x)/(1+cos(x))
```

Rubi [A] time = 0.05, antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {4392, 2735, 2648}

$$x - \frac{\sin(x)}{\cos(x)+1}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[x]/(Cot[x] + Csc[x]),x]
```

```
[Out] x - Sin[x]/(1 + Cos[x])
```

Rule 2648

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> -Simp[Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 4392

```
Int[(cot[(c_.) + (d_.)*(x_)]^(n_.)*(a_.) + csc[(c_.) + (d_.)*(x_)]^(n_.)*(b
_.))^(p_)*(u_.), x_Symbol] :> Int[ActivateTrig[u]*Csc[c + d*x]^(n*p)*(b + a
*Cos[c + d*x]^n)^p, x] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p]
```

Rubi steps

$$\int \frac{\cot(x)}{\cot(x)+\csc(x)}\,dx = \int \frac{\cos(x)}{1+\cos(x)}\,dx$$
$$= x - \int \frac{1}{1+\cos(x)}\,dx$$
$$= x - \frac{\sin(x)}{1+\cos(x)}$$

Mathematica [A] time = 0.02, size = 10, normalized size = 0.83

$$x - \tan\left(\frac{x}{2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[x]/(Cot[x] + Csc[x]),x]
[Out] x - Tan[x/2]
```

fricas [A] time = 0.59, size = 17, normalized size = 1.42

$$\frac{x\cos(x) + x - \sin(x)}{\cos(x) + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)/(cot(x)+csc(x)),x, algorithm="fricas")
[Out] (x*cos(x) + x - sin(x))/(cos(x) + 1)
```

giac [A] time = 0.22, size = 8, normalized size = 0.67

$$x - \tan\left(\frac{1}{2}\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)/(cot(x)+csc(x)),x, algorithm="giac")
[Out] x - tan(1/2*x)
```

maple [A] time = 0.06, size = 9, normalized size = 0.75

$$-\tan\left(\frac{x}{2}\right) + x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(x)/(cot(x)+csc(x)),x)
[Out] -tan(1/2*x)+x
```

maxima [A] time = 0.43, size = 23, normalized size = 1.92

$$-\frac{\sin(x)}{\cos(x)+1} + 2\arctan\left(\frac{\sin(x)}{\cos(x)+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)/(cot(x)+csc(x)),x, algorithm="maxima")
[Out] -sin(x)/(cos(x) + 1) + 2*arctan(sin(x)/(cos(x) + 1))
```

mupad [B] time = 0.57, size = 8, normalized size = 0.67

$$x - \tan\left(\frac{x}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(x)/(cot(x) + 1/sin(x)),x)
[Out] x - tan(x/2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cot(x)}{\cot(x)+\csc(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)/(cot(x)+csc(x)),x)
[Out] Integral(cot(x)/(cot(x) + csc(x)), x)
```

3.207 $\int \frac{\sec(x)}{\cot(x)+\csc(x)} \, dx$

Optimal. Leaf size=11

$$\log(\cos(x)+1)-\log(\cos(x))$$

```
[Out] -ln(cos(x))+ln(1+cos(x))
```

Rubi [A] time = 0.06, antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {4392, 2707, 36, 29, 31}

$$\log(\cos(x)+1)-\log(\cos(x))$$

Antiderivative was successfully verified.

```
[In] Int[Sec[x]/(Cot[x] + Csc[x]),x]
```

```
[Out] -Log[Cos[x]] + Log[1 + Cos[x]]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 2707

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Dist[1/f, Subst[Int[(x^p*(a + x)^(m - (p + 1)/2))/(a - x)
^((p + 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && Eq
Q[a^2 - b^2, 0] && IntegerQ[(p + 1)/2]
```

Rule 4392

```
Int[(cot[(c_.) + (d_.)*(x_)]^(n_.)*(a_.) + csc[(c_.) + (d_.)*(x_)]^(n_.)*(b
_.))^(p_)*(u_.), x_Symbol] :> Int[ActivateTrig[u]*Csc[c + d*x]^(n*p)*(b + a
```

```
*Cos[c + d*x]^n)^p, x] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p]
```

Rubi steps

$$\begin{aligned}\int \frac{\sec(x)}{\cot(x)+\csc(x)}\,dx &= \int \frac{\tan(x)}{1+\cos(x)}\,dx\\ &= -\operatorname{Subst}\left(\int \frac{1}{x(1+x)}\,dx, x, \cos(x)\right)\\ &= -\operatorname{Subst}\left(\int \frac{1}{x}\,dx, x, \cos(x)\right)+\operatorname{Subst}\left(\int \frac{1}{1+x}\,dx, x, \cos(x)\right)\\ &= -\log(\cos(x))+\log(1+\cos(x))\end{aligned}$$

Mathematica [B] time = 0.01, size = 25, normalized size = 2.27

$$2\log\left(\cos\left(\frac{x}{2}\right)\right)-\log\left(1-2\cos^2\left(\frac{x}{2}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[x]/(Cot[x] + Csc[x]),x]
[Out] 2*Log[Cos[x/2]] - Log[1 - 2*Cos[x/2]^2]
```

fricas [A] time = 1.05, size = 15, normalized size = 1.36

$$-\log\left(-\cos(x)\right)+\log\left(\frac{1}{2}\,\cos(x)+\frac{1}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)/(cot(x)+csc(x)),x, algorithm="fricas")
[Out] -log(-cos(x)) + log(1/2*cos(x) + 1/2)
```

giac [A] time = 0.15, size = 12, normalized size = 1.09

$$\log\left(\cos(x)+1\right)-\log\left(|\cos(x)|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)/(cot(x)+csc(x)),x, algorithm="giac")
[Out] log(cos(x) + 1) - log(abs(cos(x)))
```

maple [A] time = 0.09, size = 6, normalized size = 0.55

$$\ln(1+\sec(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(x)/(cot(x)+csc(x)),x)
[Out] ln(1+sec(x))
```

maxima [B] time = 0.33, size = 29, normalized size = 2.64

$$-\log\left(\frac{\sin(x)}{\cos(x)+1}+1\right)-\log\left(\frac{\sin(x)}{\cos(x)+1}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)/(cot(x)+csc(x)),x, algorithm="maxima")
[Out] -log(sin(x)/(cos(x) + 1) + 1) - log(sin(x)/(cos(x) + 1) - 1)
```

mupad [B] time = 0.66, size = 11, normalized size = 1.00

$$-\ln\left(\tan\left(\frac{x}{2}\right)^2-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(x)*(cot(x) + 1/sin(x))),x)
[Out] -log(tan(x/2)^2 - 1)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sec(x)}{\cot(x)+\csc(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)/(cot(x)+csc(x)),x)
[Out] Integral(sec(x)/(cot(x) + csc(x)), x)
```

3.208 $\int \frac{\csc(x)}{\cot(x)+\csc(x)}\,dx$

Optimal. Leaf size=9

$$\frac{\sin(x)}{\cos(x)+1}$$

```
[Out] sin(x)/(1+cos(x))
```

Rubi [A] time = 0.03, antiderivative size = 9, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {3166, 2648}

$$\frac{\sin(x)}{\cos(x)+1}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[x]/(Cot[x] + Csc[x]),x]
```

```
[Out] Sin[x]/(1 + Cos[x])
```

Rule 2648

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> -Simp[Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rule 3166

```
Int[csc[(d_.) + (e_.)*(x_)]^(n_.)*((a_.) + csc[(d_.) + (e_.)*(x_)]*(b_.) +
cot[(d_.) + (e_.)*(x_)]*(c_.))^(m_), x_Symbol] :> Int[1/(b + a*Sin[d + e*x]
 + c*Cos[d + e*x])^n, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[m + n, 0] && I
ntegerQ[n]
```

Rubi steps

$$\begin{aligned}\int \frac{\csc(x)}{\cot(x)+\csc(x)}\,dx &= \int \frac{1}{1+\cos(x)}\,dx\\ &= \frac{\sin(x)}{1+\cos(x)}\end{aligned}$$

Mathematica [A] time = 0.02, size = 6, normalized size = 0.67

$$\tan\left(\frac{x}{2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[x]/(Cot[x] + Csc[x]),x]
```

```
[Out] Tan[x/2]
```

fricas [A] time = 0.58, size = 9, normalized size = 1.00

$$\frac{\sin(x)}{\cos(x)+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)/(cot(x)+csc(x)),x, algorithm="fricas")
```

```
[Out] sin(x)/(cos(x) + 1)
```

giac [A] time = 0.21, size = 4, normalized size = 0.44

$$\tan\left(\frac{1}{2}\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)/(cot(x)+csc(x)),x, algorithm="giac")
```

```
[Out] tan(1/2*x)
```

maple [A] time = 0.06, size = 5, normalized size = 0.56

$$\tan\left(\frac{x}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(x)/(cot(x)+csc(x)),x)
```

```
[Out] tan(1/2*x)
```

maxima [A] time = 0.33, size = 9, normalized size = 1.00

$$\frac{\sin(x)}{\cos(x)+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)/(cot(x)+csc(x)),x, algorithm="maxima")
```

```
[Out] sin(x)/(cos(x) + 1)
```

mupad [B]   time = 0.54, size = 4, normalized size = 0.44

$$\tan\left(\frac{x}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(x)*(cot(x) + 1/sin(x))),x)
```

```
[Out] tan(x/2)
```

sympy [F]   time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc(x)}{\cot(x) + \csc(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)/(cot(x)+csc(x)),x)
```

```
[Out] Integral(csc(x)/(cot(x) + csc(x)), x)
```

3.209 $\int \frac{\sin(x)}{-\cot(x)+\csc(x)}\,dx$

Optimal. Leaf size=4

$$x+\sin(x)$$

[Out] x+sin(x)

Rubi [A] time = 0.07, antiderivative size = 4, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {4392, 2682, 8}

$$x+\sin(x)$$

Antiderivative was successfully verified.

[In] Int[Sin[x]/(-Cot[x] + Csc[x]),x]

[Out] x + Sin[x]

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2682

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[(g*(g*Cos[e + f*x])^(p - 1))/(b*f*(p - 1)), x] + Di
st[g^2/a, Int[(g*Cos[e + f*x])^(p - 2), x], x] /; FreeQ[{a, b, e, f, g}, x]
 && EqQ[a^2 - b^2, 0] && GtQ[p, 1] && IntegerQ[2*p]
```

Rule 4392

```
Int[(cot[(c_.) + (d_.)*(x_)]^(n_.)*(a_.) + csc[(c_.) + (d_.)*(x_)]^(n_.)*(b
_.))^(p_)*(u_.), x_Symbol] :> Int[ActivateTrig[u]*Csc[c + d*x]^(n*p)*(b + a
*Cos[c + d*x]^n)^p, x] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sin(x)}{-\cot(x)+\csc(x)}\,dx &= \int \frac{\sin^2(x)}{1-\cos(x)}\,dx\\
&= \sin(x)+\int 1\,dx\\
&= x+\sin(x)
\end{aligned}$$

Mathematica [B] time = 0.01, size = 14, normalized size = 3.50

$$2\left(\frac{x}{2}+\frac{\sin(x)}{2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[x]/(-Cot[x] + Csc[x]),x]
[Out] 2*(x/2 + Sin[x]/2)
```

fricas [A] time = 0.62, size = 4, normalized size = 1.00

$$x+\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)/(-cot(x)+csc(x)),x, algorithm="fricas")
[Out] x + sin(x)
```

giac [B] time = 0.20, size = 18, normalized size = 4.50

$$x+\frac{2\tan\left(\frac{1}{2}x\right)}{\tan\left(\frac{1}{2}x\right)^{2}+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)/(-cot(x)+csc(x)),x, algorithm="giac")
[Out] x + 2*tan(1/2*x)/(tan(1/2*x)^2 + 1)
```

maple [B] time = 0.10, size = 19, normalized size = 4.75

$$\frac{2\tan\left(\frac{x}{2}\right)}{\tan^{2}\left(\frac{x}{2}\right)+1}+x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)/(-cot(x)+csc(x)),x)
[Out] 2*tan(1/2*x)/(tan(1/2*x)^2+1)+x
```

maxima [B] time = 0.45, size = 38, normalized size = 9.50

$$\frac{2\sin(x)}{\left(\frac{\sin(x)^{2}}{(\cos(x)+1)^{2}}+1\right)(\cos(x)+1)}+2\arctan\left(\frac{\sin(x)}{\cos(x)+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)/(-cot(x)+csc(x)),x, algorithm="maxima")
```

```
[Out] 2*sin(x)/((sin(x)^2/(cos(x) + 1)^2 + 1)*(cos(x) + 1)) + 2*arctan(sin(x)/(co
s(x) + 1))
```

mupad [B] time = 1.03, size = 4, normalized size = 1.00

$$x + \sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-sin(x)/(cot(x) - 1/sin(x)),x)
```

```
[Out] x + sin(x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{\sin(x)}{\cot(x) - \csc(x)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)/(-cot(x)+csc(x)),x)
```

```
[Out] -Integral(sin(x)/(cot(x) - csc(x)), x)
```

3.210 $\int \frac{\cos(x)}{-\cot(x)+\csc(x)}\,dx$

Optimal. Leaf size=10

$$\cos(x) + \log(1 - \cos(x))$$

```
[Out] cos(x)+ln(1-cos(x))
```

Rubi [A] time = 0.07, antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {4392, 2833, 43}

$$\cos(x) + \log(1 - \cos(x))$$

Antiderivative was successfully verified.

```
[In] Int[Cos[x]/(-Cot[x] + Csc[x]),x]
```

```
[Out] Cos[x] + Log[1 - Cos[x]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2833

```
Int[cos[(e_.) + (f_.)*(x_)]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((
c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b*f), Sub
st[Int[(a + x)^m*(c + (d*x)/b)^n, x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b
, c, d, e, f, m, n}, x]
```

Rule 4392

```
Int[(cot[(c_.) + (d_.)*(x_)]^(n_.)*(a_.) + csc[(c_.) + (d_.)*(x_)]^(n_.)*(b
_.))^(p_)*(u_.), x_Symbol] :> Int[ActivateTrig[u]*Csc[c + d*x]^(n*p)*(b + a
*Cos[c + d*x]^n)^p, x] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p]
```

Rubi steps

$$\begin{aligned}\int \frac{\cos(x)}{-\cot(x)+\csc(x)}\,dx &= \int \frac{\cos(x)\sin(x)}{1-\cos(x)}\,dx\\ &= -\operatorname{Subst}\left(\int \frac{x}{1+x}\,dx, x, -\cos(x)\right)\\ &= -\operatorname{Subst}\left(\int \left(1+\frac{1}{-1-x}\right)dx, x, -\cos(x)\right)\\ &= \cos(x)+\log(1-\cos(x))\end{aligned}$$

Mathematica [A] time = 0.01, size = 20, normalized size = 2.00

$$2\log\left(\sin\left(\frac{x}{2}\right)\right)-2\sin^2\left(\frac{x}{2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[x]/(-Cot[x] + Csc[x]),x]
[Out] 2*Log[Sin[x/2]] - 2*Sin[x/2]^2
```

fricas [A] time = 0.80, size = 10, normalized size = 1.00

$$\cos(x)+\log\left(-\frac{1}{2}\cos(x)+\frac{1}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)/(-cot(x)+csc(x)),x, algorithm="fricas")
[Out] cos(x) + log(-1/2*cos(x) + 1/2)
```

giac [A] time = 0.20, size = 10, normalized size = 1.00

$$\cos(x)+\log(-\cos(x)+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)/(-cot(x)+csc(x)),x, algorithm="giac")
[Out] cos(x) + log(-cos(x) + 1)
```

maple [A] time = 0.10, size = 9, normalized size = 0.90

$$\cos(x)+\ln(\cos(x)-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)/(-cot(x)+csc(x)),x)
```

```
[Out] cos(x)+ln(cos(x)-1)
```

maxima [B] time = 0.65, size = 46, normalized size = 4.60

$$\frac{2}{\frac{\sin(x)^2}{(\cos(x)+1)^2}+1} + 2\log\left(\frac{\sin(x)}{\cos(x)+1}\right) - \log\left(\frac{\sin(x)^2}{(\cos(x)+1)^2}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)/(-cot(x)+csc(x)),x, algorithm="maxima")
```

```
[Out] 2/(sin(x)^2/(cos(x) + 1)^2 + 1) + 2*log(sin(x)/(cos(x) + 1)) - log(sin(x)^2
/(cos(x) + 1)^2 + 1)
```

mupad [B] time = 0.62, size = 31, normalized size = 3.10

$$2\,\ln\left(\tan\left(\frac{x}{2}\right)\right) - \ln\left(\tan\left(\frac{x}{2}\right)^2+1\right) + \frac{2}{\tan\left(\frac{x}{2}\right)^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-cos(x)/(cot(x) - 1/sin(x)),x)
```

```
[Out] 2*log(tan(x/2)) - log(tan(x/2)^2 + 1) + 2/(tan(x/2)^2 + 1)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{\cos(x)}{\cot(x)-\csc(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)/(-cot(x)+csc(x)),x)
```

```
[Out] -Integral(cos(x)/(cot(x) - csc(x)), x)
```

3.211 $\int \frac{\tan(x)}{-\cot(x)+\csc(x)} \, dx$

Optimal. Leaf size=5

$$x + \tanh^{-1}(\sin(x))$$

```
[Out] x+arctanh(sin(x))
```

Rubi [A] time = 0.09, antiderivative size = 5, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {4392, 2839, 3770, 8}

$$x + \tanh^{-1}(\sin(x))$$

Antiderivative was successfully verified.

```
[In] Int[Tan[x]/(-Cot[x] + Csc[x]),x]
```

```
[Out] x + ArcTanh[Sin[x]]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2839

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(
n_.))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[g^2/a, Int[
(g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^n, x], x] - Dist[g^2/(b*d), Int[(
g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d,
 e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 4392

```
Int[(cot[(c_.) + (d_.)*(x_)]^(n_.)*(a_.) + csc[(c_.) + (d_.)*(x_)]^(n_.)*(b
_.))^(p_)*(u_.), x_Symbol] :> Int[ActivateTrig[u]*Csc[c + d*x]^(n*p)*(b + a
*Cos[c + d*x]^n)^p, x] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p]
```

Rubi steps

$$\int \frac{\tan(x)}{-\cot(x)+\csc(x)}\,dx = \int \frac{\sin(x)\tan(x)}{1-\cos(x)}\,dx$$
$$= \int 1\,dx + \int \sec(x)\,dx$$
$$= x + \tanh^{-1}(\sin(x))$$

Mathematica [B] time = 0.02, size = 46, normalized size = 9.20

$$2\left(\frac{x}{2} - \frac{1}{2}\log\left(\cos\left(\frac{x}{2}\right) - \sin\left(\frac{x}{2}\right)\right) + \frac{1}{2}\log\left(\sin\left(\frac{x}{2}\right) + \cos\left(\frac{x}{2}\right)\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[x]/(-Cot[x] + Csc[x]),x]
[Out] 2*(x/2 - Log[Cos[x/2] - Sin[x/2]]/2 + Log[Cos[x/2] + Sin[x/2]]/2)
```

fricas [B] time = 0.78, size = 18, normalized size = 3.60

$$x + \frac{1}{2}\,\log\left(\sin(x)+1\right) - \frac{1}{2}\,\log\left(-\sin(x)+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)/(-cot(x)+csc(x)),x, algorithm="fricas")
[Out] x + 1/2*log(sin(x) + 1) - 1/2*log(-sin(x) + 1)
```

giac [B] time = 0.48, size = 20, normalized size = 4.00

$$x + \log\left(\left|\tan\left(\frac{1}{2}\,x\right)+1\right|\right) - \log\left(\left|\tan\left(\frac{1}{2}\,x\right)-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)/(-cot(x)+csc(x)),x, algorithm="giac")
[Out] x + log(abs(tan(1/2*x) + 1)) - log(abs(tan(1/2*x) - 1))
```

maple [B] time = 0.11, size = 19, normalized size = 3.80

$$-\ln\left(\tan\left(\frac{x}{2}\right)-1\right) + \ln\left(\tan\left(\frac{x}{2}\right)+1\right) + x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(x)/(-cot(x)+csc(x)),x)
```

```
[Out] -ln(tan(1/2*x)-1)+ln(tan(1/2*x)+1)+x
```

maxima [B] time = 0.43, size = 39, normalized size = 7.80

$$2 \arctan\left(\frac{\sin(x)}{\cos(x)+1}\right) + \log\left(\frac{\sin(x)}{\cos(x)+1} + 1\right) - \log\left(\frac{\sin(x)}{\cos(x)+1} - 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)/(-cot(x)+csc(x)),x, algorithm="maxima")
```

```
[Out] 2*arctan(sin(x)/(cos(x) + 1)) + log(sin(x)/(cos(x) + 1) + 1) - log(sin(x)/(
cos(x) + 1) - 1)
```

mupad [B] time = 0.57, size = 9, normalized size = 1.80

$$x + 2\,\mathrm{atanh}\left(\tan\left(\frac{x}{2}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-tan(x)/(cot(x) - 1/sin(x)),x)
```

```
[Out] x + 2*atanh(tan(x/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{\tan(x)}{\cot(x) - \csc(x)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)/(-cot(x)+csc(x)),x)
```

```
[Out] -Integral(tan(x)/(cot(x) - csc(x)), x)
```

3.212 $\int \frac{\cot(x)}{-\cot(x)+\csc(x)}\,dx$

Optimal. Leaf size=16

$$-x-\frac{\sin(x)}{1-\cos(x)}$$

```
[Out] -x-sin(x)/(1-cos(x))
```

Rubi [A] time = 0.06, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {4392, 2735, 2648}

$$-x-\frac{\sin(x)}{1-\cos(x)}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[x]/(-Cot[x] + Csc[x]),x]
```

```
[Out] -x - Sin[x]/(1 - Cos[x])
```

Rule 2648

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> -Simp[Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 4392

```
Int[(cot[(c_.) + (d_.)*(x_)]^(n_.)*(a_.) + csc[(c_.) + (d_.)*(x_)]^(n_.)*(b
_.))^(p_)*(u_.), x_Symbol] :> Int[ActivateTrig[u]*Csc[c + d*x]^(n*p)*(b + a
*Cos[c + d*x]^n)^p, x] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p]
```

Rubi steps

$$\int \frac{\cot(x)}{-\cot(x)+\csc(x)}\,dx = \int \frac{\cos(x)}{1-\cos(x)}\,dx$$
$$= -x + \int \frac{1}{1-\cos(x)}\,dx$$
$$= -x - \frac{\sin(x)}{1-\cos(x)}$$

Mathematica [A] time = 0.02, size = 16, normalized size = 1.00

$$\frac{1}{2}\left(-2x - 2\cot\left(\frac{x}{2}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[x]/(-Cot[x] + Csc[x]),x]
[Out] (-2*x - 2*Cot[x/2])/2
```

fricas [A] time = 0.69, size = 14, normalized size = 0.88

$$-\frac{x\sin(x)+\cos(x)+1}{\sin(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)/(-cot(x)+csc(x)),x, algorithm="fricas")
[Out] -(x*sin(x) + cos(x) + 1)/sin(x)
```

giac [A] time = 0.16, size = 12, normalized size = 0.75

$$-x - \frac{1}{\tan\left(\frac{1}{2}\,x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)/(-cot(x)+csc(x)),x, algorithm="giac")
[Out] -x - 1/tan(1/2*x)
```

maple [A] time = 0.07, size = 13, normalized size = 0.81

$$-\frac{1}{\tan\left(\frac{x}{2}\right)} - x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(x)/(-cot(x)+csc(x)),x)
```

```
[Out] -1/tan(1/2*x)-x
```

maxima [A] time = 0.45, size = 23, normalized size = 1.44

$$-\frac{\cos(x)+1}{\sin(x)} - 2\arctan\left(\frac{\sin(x)}{\cos(x)+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)/(-cot(x)+csc(x)),x, algorithm="maxima")
```

```
[Out] -(cos(x) + 1)/sin(x) - 2*arctan(sin(x)/(cos(x) + 1))
```

mupad [B] time = 0.58, size = 10, normalized size = 0.62

$$-x - \cot\left(\frac{x}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-cot(x)/(cot(x) - 1/sin(x)),x)
```

```
[Out] - x - cot(x/2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{\cot(x)}{\cot(x) - \csc(x)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)/(-cot(x)+csc(x)),x)
```

```
[Out] -Integral(cot(x)/(cot(x) - csc(x)), x)
```

3.213 $\int \frac{\sec(x)}{-\cot(x)+\csc(x)}\,dx$

Optimal. Leaf size=13

$$\log(1-\cos(x)) - \log(\cos(x))$$

```
[Out] ln(1-cos(x))-ln(cos(x))
```

Rubi [A] time = 0.06, antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {4392, 2707, 36, 29, 31}

$$\log(1-\cos(x)) - \log(\cos(x))$$

Antiderivative was successfully verified.

```
[In] Int[Sec[x]/(-Cot[x] + Csc[x]),x]
```

```
[Out] Log[1 - Cos[x]] - Log[Cos[x]]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 2707

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Dist[1/f, Subst[Int[(x^p*(a + x)^(m - (p + 1)/2))/(a - x)
^((p + 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && Eq
Q[a^2 - b^2, 0] && IntegerQ[(p + 1)/2]
```

Rule 4392

```
Int[(cot[(c_.) + (d_.)*(x_)]^(n_.)*(a_.) + csc[(c_.) + (d_.)*(x_)]^(n_.)*(b
_.))^(p_)*(u_.), x_Symbol] :> Int[ActivateTrig[u]*Csc[c + d*x]^(n*p)*(b + a
```

```
*Cos[c + d*x]^n)^p, x] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p]
```

Rubi steps

$$\int \frac{\sec(x)}{-\cot(x)+\csc(x)}\,dx = \int \frac{\tan(x)}{1-\cos(x)}\,dx$$
$$= -\text{Subst}\left(\int \frac{1}{x(1+x)}\,dx, x, -\cos(x)\right)$$
$$= -\text{Subst}\left(\int \frac{1}{x}\,dx, x, -\cos(x)\right) + \text{Subst}\left(\int \frac{1}{1+x}\,dx, x, -\cos(x)\right)$$
$$= \log(1-\cos(x)) - \log(\cos(x))$$

Mathematica [A] time = 0.01, size = 25, normalized size = 1.92

$$2\log\left(\sin\left(\frac{x}{2}\right)\right) - \log\left(1-2\sin^2\left(\frac{x}{2}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[x]/(-Cot[x] + Csc[x]),x]
[Out] 2*Log[Sin[x/2]] - Log[1 - 2*Sin[x/2]^2]
```

fricas [A] time = 0.54, size = 15, normalized size = 1.15

$$-\log\left(-\cos(x)\right) + \log\left(-\frac{1}{2}\,\cos(x) + \frac{1}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)/(-cot(x)+csc(x)),x, algorithm="fricas")
[Out] -log(-cos(x)) + log(-1/2*cos(x) + 1/2)
```

giac [A] time = 0.26, size = 14, normalized size = 1.08

$$\log\left(-\cos(x)+1\right) - \log\left(|\cos(x)|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)/(-cot(x)+csc(x)),x, algorithm="giac")
[Out] log(-cos(x) + 1) - log(abs(cos(x)))
```

maple [A] time = 0.09, size = 6, normalized size = 0.46

$$\ln\left(-1+\sec(x)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(x)/(-cot(x)+csc(x)),x)
```

```
[Out] ln(-1+sec(x))
```

maxima [B] time = 0.34, size = 41, normalized size = 3.15

$$-\log\left(\frac{\sin(x)}{\cos(x)+1}+1\right)-\log\left(\frac{\sin(x)}{\cos(x)+1}-1\right)+2\,\log\left(\frac{\sin(x)}{\cos(x)+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)/(-cot(x)+csc(x)),x, algorithm="maxima")
```

```
[Out] -log(sin(x)/(cos(x) + 1) + 1) - log(sin(x)/(cos(x) + 1) - 1) + 2*log(sin(x)
/(cos(x) + 1))
```

mupad [B] time = 0.61, size = 19, normalized size = 1.46

$$2\,\ln\left(\tan\left(\frac{x}{2}\right)\right)-\ln\left(\tan\left(\frac{x}{2}\right)^2-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-1/(cos(x)*(cot(x) - 1/sin(x))),x)
```

```
[Out] 2*log(tan(x/2)) - log(tan(x/2)^2 - 1)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int\frac{\sec(x)}{\cot(x)-\csc(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)/(-cot(x)+csc(x)),x)
```

```
[Out] -Integral(sec(x)/(cot(x) - csc(x)), x)
```

3.214 $\int \frac{\csc(x)}{-\cot(x)+\csc(x)}\,dx$

Optimal. Leaf size=12

$$-\frac{\sin(x)}{1-\cos(x)}$$

```
[Out] -sin(x)/(1-cos(x))
```

Rubi [A] time = 0.03, antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3166, 2648}

$$-\frac{\sin(x)}{1-\cos(x)}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[x]/(-Cot[x] + Csc[x]),x]
```

```
[Out] -(Sin[x]/(1 - Cos[x]))
```

Rule 2648

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> -Simp[Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rule 3166

```
Int[csc[(d_.) + (e_.)*(x_)]^(n_.)*((a_.) + csc[(d_.) + (e_.)*(x_)]*(b_.) +
cot[(d_.) + (e_.)*(x_)]*(c_.))^(m_), x_Symbol] :> Int[1/(b + a*Sin[d + e*x]
 + c*Cos[d + e*x])^n, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[m + n, 0] && I
ntegerQ[n]
```

Rubi steps

$$\begin{aligned}
\int \frac{\csc(x)}{-\cot(x)+\csc(x)}\,dx &= \int \frac{1}{1-\cos(x)}\,dx \\
&= -\frac{\sin(x)}{1-\cos(x)}
\end{aligned}$$

Mathematica [A] time = 0.01, size = 8, normalized size = 0.67

$$-\cot\left(\frac{x}{2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[x]/(-Cot[x] + Csc[x]),x]
```

```
[Out] -Cot[x/2]
```

fricas [A] time = 0.91, size = 10, normalized size = 0.83

$$-\frac{\cos(x)+1}{\sin(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)/(-cot(x)+csc(x)),x, algorithm="fricas")
```

```
[Out] -(cos(x) + 1)/sin(x)
```

giac [A] time = 0.20, size = 8, normalized size = 0.67

$$-\frac{1}{\tan\left(\frac{1}{2}x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)/(-cot(x)+csc(x)),x, algorithm="giac")
```

```
[Out] -1/tan(1/2*x)
```

maple [A] time = 0.06, size = 9, normalized size = 0.75

$$-\frac{1}{\tan\left(\frac{x}{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(x)/(-cot(x)+csc(x)),x)
```

```
[Out] -1/tan(1/2*x)
```

maxima [A] time = 0.34, size = 10, normalized size = 0.83

$$-\frac{\cos(x)+1}{\sin(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)/(-cot(x)+csc(x)),x, algorithm="maxima")
```

```
[Out] -(cos(x) + 1)/sin(x)
```

mupad [B] time = 0.53, size = 6, normalized size = 0.50

$$-\cot\left(\frac{x}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-1/(sin(x)*(cot(x) - 1/sin(x))),x)
```

```
[Out] -cot(x/2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int\frac{\csc\left(x\right)}{\cot\left(x\right)-\csc\left(x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)/(-cot(x)+csc(x)),x)
```

```
[Out] -Integral(csc(x)/(cot(x) - csc(x)), x)
```

3.215 $\int \frac{1}{\csc(c+dx)+\sin(c+dx)}\,dx$

Optimal. Leaf size=23

$$-\frac{\tanh^{-1}\left(\frac{\cos(c+dx)}{\sqrt{2}}\right)}{\sqrt{2}\,d}$$

```
[Out] -1/2*arctanh(1/2*cos(d*x+c)*2^(1/2))/d*2^(1/2)
```

Rubi [A] time = 0.04, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {4397, 3186, 206}

$$-\frac{\tanh^{-1}\left(\frac{\cos(c+dx)}{\sqrt{2}}\right)}{\sqrt{2}\,d}$$

Antiderivative was successfully verified.

```
[In] Int[(Csc[c + d*x] + Sin[c + d*x])^(-1),x]
```

```
[Out] -(ArcTanh[Cos[c + d*x]/Sqrt[2]]/(Sqrt[2]*d))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3186

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[ff/f, S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e +
f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 4397

```
Int[u_, x_Symbol] :> Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]
```

Rubi steps

$$\int \frac{1}{\csc(c+dx)+\sin(c+dx)}\,dx = \int \frac{\sin(c+dx)}{1+\sin^2(c+dx)}\,dx$$
$$= -\frac{\text{Subst}\left(\int \frac{1}{2-x^2}\,dx, x, \cos(c+dx)\right)}{d}$$
$$= -\frac{\tanh^{-1}\left(\frac{\cos(c+dx)}{\sqrt{2}}\right)}{\sqrt{2}\,d}$$

Mathematica [C] time = 0.19, size = 61, normalized size = 2.65

$$-\frac{\tanh^{-1}\left(\frac{\cos(c)-(\sin(c)-i)\tan\left(\frac{dx}{2}\right)}{\sqrt{2}}\right)+\tanh^{-1}\left(\frac{\cos(c)-(\sin(c)+i)\tan\left(\frac{dx}{2}\right)}{\sqrt{2}}\right)}{\sqrt{2}\,d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Csc[c + d*x] + Sin[c + d*x])^(-1),x]
[Out] -((ArcTanh[(Cos[c] - (-I + Sin[c])*Tan[(d*x)/2])/Sqrt[2]] + ArcTanh[(Cos[c]
 - (I + Sin[c])*Tan[(d*x)/2])/Sqrt[2]])/(Sqrt[2]*d))
```

fricas [B] time = 0.65, size = 44, normalized size = 1.91

$$\frac{\sqrt{2}\,\log\left(-\frac{\cos(dx+c)^2-2\,\sqrt{2}\,\cos(dx+c)+2}{\cos(dx+c)^2-2}\right)}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(csc(d*x+c)+sin(d*x+c)),x, algorithm="fricas")
[Out] 1/4*sqrt(2)*log(-(cos(d*x + c)^2 - 2*sqrt(2)*cos(d*x + c) + 2)/(cos(d*x + c
)^2 - 2))/d
```

giac [B] time = 1.20, size = 68, normalized size = 2.96

$$\frac{\sqrt{2}\,\log\left(\frac{\left|-4\,\sqrt{2}-\frac{2\,(\cos(dx+c)-1)}{\cos(dx+c)+1}+6\right|}{\left|4\,\sqrt{2}-\frac{2\,(\cos(dx+c)-1)}{\cos(dx+c)+1}+6\right|}\right)}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(csc(d*x+c)+sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/4*sqrt(2)*log(abs(-4*sqrt(2) - 2*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) +
6)/abs(4*sqrt(2) - 2*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 6))/d
```

maple [A] time = 0.12, size = 21, normalized size = 0.91

$$-\frac{\operatorname{arctanh}\left(\frac{\cos(dx+c)\sqrt{2}}{2}\right)\sqrt{2}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(csc(d*x+c)+sin(d*x+c)),x)
```

```
[Out] -1/2*arctanh(1/2*cos(d*x+c)*2^(1/2))/d*2^(1/2)
```

maxima [B] time = 0.57, size = 176, normalized size = 7.65

$$\frac{\sqrt{2}\log\left(-\frac{2\left(\sqrt{2}+1\right)\cos(dx+c)-\cos(dx+c)^2-\sin(dx+c)^2-2\sqrt{2}-3}{2\left(\sqrt{2}-1\right)\cos(dx+c)+\cos(dx+c)^2+\sin(dx+c)^2-2\sqrt{2}+3}\right)+\sqrt{2}\log\left(-\frac{2\left(\sqrt{2}-1\right)\cos(dx+c)-\cos(dx+c)^2-\sin(dx+c)^2+2\sqrt{2}-3}{2\left(\sqrt{2}+1\right)\cos(dx+c)+\cos(dx+c)^2+\sin(dx+c)^2+2\sqrt{2}+3}\right)}{8d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(csc(d*x+c)+sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/8*(sqrt(2)*log(-(2*(sqrt(2) + 1)*cos(d*x + c) - cos(d*x + c)^2 - sin(d*x
+ c)^2 - 2*sqrt(2) - 3)/(2*(sqrt(2) - 1)*cos(d*x + c) + cos(d*x + c)^2 + si
n(d*x + c)^2 - 2*sqrt(2) + 3)) + sqrt(2)*log(-(2*(sqrt(2) - 1)*cos(d*x + c)
 - cos(d*x + c)^2 - sin(d*x + c)^2 + 2*sqrt(2) - 3)/(2*(sqrt(2) + 1)*cos(d*
x + c) + cos(d*x + c)^2 + sin(d*x + c)^2 + 2*sqrt(2) + 3)))/d
```

mupad [B] time = 0.66, size = 42, normalized size = 1.83

$$\frac{\sqrt{2}\operatorname{atanh}\left(\frac{2\sqrt{2}\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{2\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+1}\right)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(c + d*x) + 1/sin(c + d*x)),x)
```

```
[Out] (2^(1/2)*atanh((2*2^(1/2)*sin(c/2 + (d*x)/2)^2)/(2*sin(c/2 + (d*x)/2)^2 + 1
)))/(2*d)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sin(c+dx)+\csc(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(csc(d*x+c)+sin(d*x+c)),x)
```

```
[Out] Integral(1/(sin(c + d*x) + csc(c + d*x)), x)
```

3.216 $\int \frac{\sin(c+dx)}{\csc(c+dx)+\sin(c+dx)}\,dx$

Optimal. Leaf size=51

$$-\frac{\tan^{-1}\left(\frac{\sin(c+dx)\cos(c+dx)}{\sin^2(c+dx)+\sqrt{2}+1}\right)}{\sqrt{2}\,d}-\frac{x}{\sqrt{2}}+x$$

```
[Out] x-1/2*x*2^(1/2)-1/2*arctan(cos(d*x+c)*sin(d*x+c)/(1+sin(d*x+c)^2+2^(1/2)))/
d*2^(1/2)
```

Rubi [A] time = 0.17, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {1130, 203}

$$-\frac{\tan^{-1}\left(\frac{\sin(c+dx)\cos(c+dx)}{\sin^2(c+dx)+\sqrt{2}+1}\right)}{\sqrt{2}\,d}-\frac{x}{\sqrt{2}}+x$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]/(Csc[c + d*x] + Sin[c + d*x]),x]
```

```
[Out] x - x/Sqrt[2] - ArcTan[(Cos[c + d*x]*Sin[c + d*x])/(1 + Sqrt[2] + Sin[c + d
*x]^2)]/(Sqrt[2]*d)
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 1130

```
Int[((d_.)*(x_))^(m_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> Wi
th[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(d^2*(b/q + 1))/2, Int[(d*x)^(m - 2)/(b/2
 + q/2 + c*x^2), x], x] - Dist[(d^2*(b/q - 1))/2, Int[(d*x)^(m - 2)/(b/2 -
q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - 4*a*c, 0] && G
eQ[m, 2]
```

Rubi steps

$$\int \frac{\sin(c+dx)}{\csc(c+dx)+\sin(c+dx)}\,dx = \frac{\text{Subst}\left(\int \frac{x^2}{1+3x^2+2x^4}\,dx, x, \tan(c+dx)\right)}{d}$$
$$= -\frac{\text{Subst}\left(\int \frac{1}{1+2x^2}\,dx, x, \tan(c+dx)\right)}{d} + \frac{2\,\text{Subst}\left(\int \frac{1}{2+2x^2}\,dx, x, \tan(c+dx)\right)}{d}$$
$$= x - \frac{x}{\sqrt{2}} - \frac{\tan^{-1}\left(\frac{\cos(c+dx)\sin(c+dx)}{1+\sqrt{2}+\sin^2(c+dx)}\right)}{\sqrt{2}\,d}$$

Mathematica [A] time = 0.07, size = 30, normalized size = 0.59

$$-\frac{\tan^{-1}\left(\sqrt{2}\,\tan(c+dx)\right)}{\sqrt{2}\,d} + \frac{c}{d} + x$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]/(Csc[c + d*x] + Sin[c + d*x]),x]
```

```
[Out] c/d + x - ArcTan[Sqrt[2]*Tan[c + d*x]]/(Sqrt[2]*d)
```

fricas [A] time = 0.61, size = 52, normalized size = 1.02

$$\frac{4\,dx + \sqrt{2}\,\arctan\left(\frac{3\,\sqrt{2}\,\cos(dx+c)^2-2\,\sqrt{2}}{4\,\cos(dx+c)\sin(dx+c)}\right)}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)/(csc(d*x+c)+sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/4*(4*d*x + sqrt(2)*arctan(1/4*(3*sqrt(2)*cos(d*x + c)^2 - 2*sqrt(2))/(cos
(d*x + c)*sin(d*x + c))))/d
```

giac [A] time = 0.22, size = 82, normalized size = 1.61

$$\frac{2\,dx - \sqrt{2}\left(dx + c + \arctan\left(-\frac{\sqrt{2}\,\sin(2\,dx+2\,c)-2\,\sin(2\,dx+2\,c)}{\sqrt{2}\,\cos(2\,dx+2\,c)+\sqrt{2}-2\,\cos(2\,dx+2\,c)+2}\right)\right) + 2\,c}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)/(csc(d*x+c)+sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/2*(2*d*x - sqrt(2)*(d*x + c + arctan(-(sqrt(2)*sin(2*d*x + 2*c) - 2*sin(2
*d*x + 2*c))/(sqrt(2)*cos(2*d*x + 2*c) + sqrt(2) - 2*cos(2*d*x + 2*c) + 2))
) + 2*c)/d
```

maple [A] time = 0.13, size = 30, normalized size = 0.59

$$-\frac{\sqrt{2}\ \arctan\left(\tan\left(dx+c\right)\sqrt{2}\right)}{2d}+\frac{dx+c}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)/(csc(d*x+c)+sin(d*x+c)),x)
```

```
[Out] -1/2/d*2^(1/2)*arctan(tan(d*x+c)*2^(1/2))+1/d*(d*x+c)
```

maxima [B] time = 0.58, size = 252, normalized size = 4.94

$$\frac{4\,dx-\sqrt{2}\ \arctan\left(\frac{2\,\sqrt{2}\ \sin(dx+c)}{2\left(\sqrt{2}+1\right)\cos(dx+c)+\cos(dx+c)^2+\sin(dx+c)^2+2\,\sqrt{2}+3},\frac{\cos(dx+c)^2+\sin(dx+c)^2+2\,\cos(dx+c)-1}{2\left(\sqrt{2}+1\right)\cos(dx+c)+\cos(dx+c)^2+\sin(dx+c)^2+2\,\sqrt{2}+3}\right)+\sqrt{2}\ a}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)/(csc(d*x+c)+sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/4*(4*d*x - sqrt(2)*arctan2(2*sqrt(2)*sin(d*x + c)/(2*(sqrt(2) + 1)*cos(d*
x + c) + cos(d*x + c)^2 + sin(d*x + c)^2 + 2*sqrt(2) + 3), (cos(d*x + c)^2
+ sin(d*x + c)^2 + 2*cos(d*x + c) - 1)/(2*(sqrt(2) + 1)*cos(d*x + c) + cos(
d*x + c)^2 + sin(d*x + c)^2 + 2*sqrt(2) + 3)) + sqrt(2)*arctan2(2*sqrt(2)*s
in(d*x + c)/(2*(sqrt(2) - 1)*cos(d*x + c) + cos(d*x + c)^2 + sin(d*x + c)^2
 - 2*sqrt(2) + 3), (cos(d*x + c)^2 + sin(d*x + c)^2 - 2*cos(d*x + c) - 1)/(
2*(sqrt(2) - 1)*cos(d*x + c) + cos(d*x + c)^2 + sin(d*x + c)^2 - 2*sqrt(2)
+ 3)) + 4*c)/d
```

mupad [B] time = 0.62, size = 62, normalized size = 1.22

$$x-\frac{\sqrt{2}\ \left(2\,\mathrm{atan}\left(\frac{\sqrt{2}\ \tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{4}+\frac{7\,\sqrt{2}\ \tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{4}\right)+2\,\mathrm{atan}\left(\frac{\sqrt{2}\ \tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{4}\right)\right)}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)/(sin(c + d*x) + 1/sin(c + d*x)),x)
```

```
[Out] x - (2^(1/2)*(2*atan((7*2^(1/2)*tan(c/2 + (d*x)/2))/4 + (2^(1/2)*tan(c/2 +
(d*x)/2)^3)/4) + 2*atan((2^(1/2)*tan(c/2 + (d*x)/2))/4)))/(4*d)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin(c+dx)}{\sin(c+dx)+\csc(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)/(csc(d*x+c)+sin(d*x+c)),x)
```

```
[Out] Integral(sin(c + d*x)/(sin(c + d*x) + csc(c + d*x)), x)
```

3.217 $\int \frac{\cos(c+dx)}{\csc(c+dx)+\sin(c+dx)}\,dx$

Optimal. Leaf size=18

$$\frac{\log\left(\sin^2(c+dx)+1\right)}{2d}$$

```
[Out] 1/2*ln(1+sin(d*x+c)^2)/d
```

Rubi [A] time = 0.03, antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {4334, 260}

$$\frac{\log\left(\sin^2(c+dx)+1\right)}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]/(Csc[c + d*x] + Sin[c + d*x]),x]
```

```
[Out] Log[1 + Sin[c + d*x]^2]/(2*d)
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 4334

```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{d = FreeFacto
rs[Sin[c*(a + b*x)], x]}, Dist[d/(b*c), Subst[Int[SubstFor[1, Sin[c*(a + b*
x)]/d, u, x], x], x, Sin[c*(a + b*x)]/d], x] /; FunctionOfQ[Sin[c*(a + b*x)
]/d, u, x, True]] /; FreeQ[{a, b, c}, x] && (EqQ[F, Cos] || EqQ[F, cos])
```

Rubi steps

$$\begin{aligned}\int \frac{\cos(c+dx)}{\csc(c+dx)+\sin(c+dx)}\,dx &= \frac{\text{Subst}\left(\int \frac{x}{1+x^2}\,dx, x, \sin(c+dx)\right)}{d}\\ &= \frac{\log\left(1+\sin^2(c+dx)\right)}{2d}\end{aligned}$$

Mathematica [A] time = 0.11, size = 20, normalized size = 1.11

$$\frac{\log(3-\cos(2(c+dx)))}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]/(Csc[c + d*x] + Sin[c + d*x]),x]
[Out] Log[3 - Cos[2*(c + d*x)]]/(2*d)
```

fricas [A] time = 0.50, size = 18, normalized size = 1.00

$$\frac{\log\left(-\cos\left(dx+c\right)^2+2\right)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(csc(d*x+c)+sin(d*x+c)),x, algorithm="fricas")
[Out] 1/2*log(-cos(d*x + c)^2 + 2)/d
```

giac [A] time = 0.26, size = 16, normalized size = 0.89

$$\frac{\log\left(\sin\left(dx+c\right)^2+1\right)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(csc(d*x+c)+sin(d*x+c)),x, algorithm="giac")
[Out] 1/2*log(sin(d*x + c)^2 + 1)/d
```

maple [A] time = 0.06, size = 17, normalized size = 0.94

$$\frac{\ln\left(\cos^2\left(dx+c\right)-2\right)}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)/(csc(d*x+c)+sin(d*x+c)),x)
[Out] 1/2/d*ln(cos(d*x+c)^2-2)
```

maxima [A] time = 0.47, size = 16, normalized size = 0.89

$$\frac{\log\left(\sin\left(dx+c\right)^2+1\right)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(csc(d*x+c)+sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/2*log(sin(d*x + c)^2 + 1)/d
```

mupad [B] time = 0.06, size = 16, normalized size = 0.89

$$\frac{\ln\left(\sin\left(c+d\,x\right)^2+1\right)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)/(sin(c + d*x) + 1/sin(c + d*x)),x)
```

```
[Out] log(sin(c + d*x)^2 + 1)/(2*d)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos\left(c+dx\right)}{\sin\left(c+dx\right)+\csc\left(c+dx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(csc(d*x+c)+sin(d*x+c)),x)
```

```
[Out] Integral(cos(c + d*x)/(sin(c + d*x) + csc(c + d*x)), x)
```

3.218 $\int \frac{\tan(c+dx)}{\csc(c+dx)+\sin(c+dx)}\,dx$

Optimal. Leaf size=29

$$\frac{\tanh^{-1}(\sin(c+dx))}{2d}-\frac{\tan^{-1}(\sin(c+dx))}{2d}$$

```
[Out] -1/2*arctan(sin(d*x+c))/d+1/2*arctanh(sin(d*x+c))/d
```

Rubi [A] time = 0.04, antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {298, 203, 206}

$$\frac{\tanh^{-1}(\sin(c+dx))}{2d}-\frac{\tan^{-1}(\sin(c+dx))}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]/(Csc[c + d*x] + Sin[c + d*x]),x]
```

```
[Out] -ArcTan[Sin[c + d*x]]/(2*d) + ArcTanh[Sin[c + d*x]]/(2*d)
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rubi steps

$$\int \frac{\tan(c+dx)}{\csc(c+dx)+\sin(c+dx)}\,dx = \frac{\text{Subst}\left(\int \frac{x^2}{1-x^4}\,dx, x, \sin(c+dx)\right)}{d}$$
$$= \frac{\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \sin(c+dx)\right)}{2d} - \frac{\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \sin(c+dx)\right)}{2d}$$
$$= -\frac{\tan^{-1}(\sin(c+dx))}{2d} + \frac{\tanh^{-1}(\sin(c+dx))}{2d}$$

Mathematica [A] time = 0.03, size = 24, normalized size = 0.83

$$\frac{\tanh^{-1}(\sin(c+dx)) - \tan^{-1}(\sin(c+dx))}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]/(Csc[c + d*x] + Sin[c + d*x]),x]
```

```
[Out] (-ArcTan[Sin[c + d*x]] + ArcTanh[Sin[c + d*x]])/(2*d)
```

fricas [A] time = 0.44, size = 37, normalized size = 1.28

$$-\frac{2\arctan(\sin(dx+c)) - \log(\sin(dx+c)+1) + \log(-\sin(dx+c)+1)}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)/(csc(d*x+c)+sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/4*(2*arctan(sin(d*x + c)) - log(sin(d*x + c) + 1) + log(-sin(d*x + c) +
1))/d
```

giac [A] time = 0.39, size = 37, normalized size = 1.28

$$-\frac{2\arctan(\sin(dx+c)) - \log(|\sin(dx+c)+1|) + \log(|\sin(dx+c)-1|)}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)/(csc(d*x+c)+sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/4*(2*arctan(sin(d*x + c)) - log(abs(sin(d*x + c) + 1)) + log(abs(sin(d*x
 + c) - 1)))/d
```

maple [A] time = 0.16, size = 42, normalized size = 1.45

$$-\frac{\ln(\sin(dx+c)-1)}{4d}+\frac{\ln(\sin(dx+c)+1)}{4d}-\frac{\arctan(\sin(dx+c))}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)/(csc(d*x+c)+sin(d*x+c)),x)
```

```
[Out] -1/4/d*ln(sin(d*x+c)-1)+1/4/d*ln(sin(d*x+c)+1)-1/2*arctan(sin(d*x+c))/d
```

maxima [A] time = 0.55, size = 35, normalized size = 1.21

$$-\frac{2\arctan(\sin(dx+c))-\log(\sin(dx+c)+1)+\log(\sin(dx+c)-1)}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)/(csc(d*x+c)+sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/4*(2*arctan(sin(d*x + c)) - log(sin(d*x + c) + 1) + log(sin(d*x + c) - 1
))/d
```

mupad [B] time = 0.69, size = 61, normalized size = 2.10

$$\frac{\operatorname{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{d}-\frac{\operatorname{atan}\left(\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{2}+\frac{5\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{2}\right)-\operatorname{atan}\left(\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{2}\right)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)/(sin(c + d*x) + 1/sin(c + d*x)),x)
```

```
[Out] atanh(tan(c/2 + (d*x)/2))/d - (atan((5*tan(c/2 + (d*x)/2))/2 + tan(c/2 + (d
*x)/2)^3/2) - atan(tan(c/2 + (d*x)/2)/2))/(2*d)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\tan(c+dx)}{\sin(c+dx)+\csc(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)/(csc(d*x+c)+sin(d*x+c)),x)
```

```
[Out] Integral(tan(c + d*x)/(sin(c + d*x) + csc(c + d*x)), x)
```

3.219 $\int \frac{\cot(c+dx)}{\csc(c+dx)+\sin(c+dx)}\,dx$

Optimal. Leaf size=11

$$\frac{\tan^{-1}(\sin(c+dx))}{d}$$

```
[Out] arctan(sin(d*x+c))/d
```

Rubi [A] time = 0.03, antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {4338, 203}

$$\frac{\tan^{-1}(\sin(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]/(Csc[c + d*x] + Sin[c + d*x]),x]
```

```
[Out] ArcTan[Sin[c + d*x]]/d
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 4338

```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{d = FreeFacto
rs[Sin[c*(a + b*x)], x]}, Dist[1/(b*c), Subst[Int[SubstFor[1/x, Sin[c*(a +
b*x)]/d, u, x], x], x, Sin[c*(a + b*x)]/d], x] /; FunctionOfQ[Sin[c*(a + b*
x)]/d, u, x, True]] /; FreeQ[{a, b, c}, x] && (EqQ[F, Cot] || EqQ[F, cot])
```

Rubi steps

$$\begin{aligned} \int \frac{\cot(c+dx)}{\csc(c+dx)+\sin(c+dx)}\,dx &= \frac{\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \sin(c+dx)\right)}{d} \\ &= \frac{\tan^{-1}(\sin(c+dx))}{d} \end{aligned}$$

Mathematica [A] time = 0.04, size = 11, normalized size = 1.00

$$\frac{\tan^{-1}(\sin(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]/(Csc[c + d*x] + Sin[c + d*x]),x]
[Out] ArcTan[Sin[c + d*x]]/d
```

fricas [A] time = 0.49, size = 11, normalized size = 1.00

$$\frac{\arctan\left(\sin\left(dx+c\right)\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)/(csc(d*x+c)+sin(d*x+c)),x, algorithm="fricas")
[Out] arctan(sin(d*x + c))/d
```

giac [A] time = 0.25, size = 11, normalized size = 1.00

$$\frac{\arctan\left(\sin\left(dx+c\right)\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)/(csc(d*x+c)+sin(d*x+c)),x, algorithm="giac")
[Out] arctan(sin(d*x + c))/d
```

maple [A] time = 0.07, size = 13, normalized size = 1.18

$$-\frac{\arctan\left(\csc\left(dx+c\right)\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)/(csc(d*x+c)+sin(d*x+c)),x)
[Out] -1/d*arctan(csc(d*x+c))
```

maxima [A] time = 0.64, size = 11, normalized size = 1.00

$$\frac{\arctan\left(\sin\left(dx+c\right)\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)/(csc(d*x+c)+sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] arctan(sin(d*x + c))/d
```

mupad [B] time = 0.67, size = 45, normalized size = 4.09

$$\frac{\operatorname{atan}\left(\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{2}+\frac{5\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{2}\right)-\operatorname{atan}\left(\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{2}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)/(sin(c + d*x) + 1/sin(c + d*x)),x)
```

```
[Out] (atan((5*tan(c/2 + (d*x)/2))/2 + tan(c/2 + (d*x)/2)^3/2) - atan(tan(c/2 + (
d*x)/2)/2))/d
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cot(c+dx)}{\sin(c+dx)+\csc(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)/(csc(d*x+c)+sin(d*x+c)),x)
```

```
[Out] Integral(cot(c + d*x)/(sin(c + d*x) + csc(c + d*x)), x)
```

3.220 $\int \frac{\sec(c+dx)}{\csc(c+dx)+\sin(c+dx)}\,dx$

Optimal. Leaf size=16

$$\frac{\tanh^{-1}\left(\sin^2(c+dx)\right)}{2d}$$

```
[Out] 1/2*arctanh(sin(d*x+c)^2)/d
```

Rubi **[A]** time = 0.03, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {275, 206}

$$\frac{\tanh^{-1}\left(\sin^2(c+dx)\right)}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]/(Csc[c + d*x] + Sin[c + d*x]),x]
```

```
[Out] ArcTanh[Sin[c + d*x]^2]/(2*d)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sec(c+dx)}{\csc(c+dx)+\sin(c+dx)}\,dx &= \frac{\text{Subst}\left(\int \frac{x}{1-x^4}\,dx, x, \sin(c+dx)\right)}{d}\\
&= \frac{\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \sin^2(c+dx)\right)}{2d}\\
&= \frac{\tanh^{-1}\left(\sin^2(c+dx)\right)}{2d}
\end{aligned}$$

Mathematica [A] time = 0.05, size = 30, normalized size = 1.88

$$\frac{\log\left(2-\cos^2(c+dx)\right)-2\log(\cos(c+dx))}{4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]/(Csc[c + d*x] + Sin[c + d*x]),x]
[Out] (-2*Log[Cos[c + d*x]] + Log[2 - Cos[c + d*x]^2])/(4*d)
```

fricas [B] time = 0.50, size = 30, normalized size = 1.88

$$\frac{\log\left(-\cos\left(dx+c\right)^2+2\right)-2\,\log\left(-\cos\left(dx+c\right)\right)}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(csc(d*x+c)+sin(d*x+c)),x, algorithm="fricas")
[Out] 1/4*(log(-cos(d*x + c)^2 + 2) - 2*log(-cos(d*x + c)))/d
```

giac [B] time = 0.28, size = 79, normalized size = 4.94

$$-\frac{2\,\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}-1\right|\right)-\log\left(\left|-\frac{6\,(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}+1\right|\right)}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(csc(d*x+c)+sin(d*x+c)),x, algorithm="giac")
[Out] -1/4*(2*log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 1)) - log(abs(-6*(
cos(d*x + c) - 1)/(cos(d*x + c) + 1) + (cos(d*x + c) - 1)^2/(cos(d*x + c) +
 1)^2 + 1)))/d
```

maple [A] time = 0.09, size = 19, normalized size = 1.19

$$\frac{\ln\left(2\left(\sec^2\left(dx+c\right)\right)-1\right)}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)/(csc(d*x+c)+sin(d*x+c)),x)
[Out] 1/4/d*ln(2*sec(d*x+c)^2-1)
```

maxima [B] time = 0.47, size = 39, normalized size = 2.44

$$\frac{\log\left(\sin\left(dx+c\right)^2+1\right)-\log\left(\sin\left(dx+c\right)+1\right)-\log\left(\sin\left(dx+c\right)-1\right)}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(csc(d*x+c)+sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/4*(log(sin(d*x + c)^2 + 1) - log(sin(d*x + c) + 1) - log(sin(d*x + c) - 1
))/d
```

mupad [B] time = 0.61, size = 14, normalized size = 0.88

$$\frac{\operatorname{atanh}\left(\sin\left(c+d\,x\right)^2\right)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)*(sin(c + d*x) + 1/sin(c + d*x))),x)
```

```
[Out] atanh(sin(c + d*x)^2)/(2*d)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec\left(c+dx\right)}{\sin\left(c+dx\right)+\csc\left(c+dx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(csc(d*x+c)+sin(d*x+c)),x)
```

```
[Out] Integral(sec(c + d*x)/(sin(c + d*x) + csc(c + d*x)), x)
```

3.221 $\int \frac{\csc(c+dx)}{\csc(c+dx)+\sin(c+dx)}\,dx$

Optimal. Leaf size=48

$$\frac{\tan^{-1}\left(\frac{\sin(c+dx)\cos(c+dx)}{\sin^2(c+dx)+\sqrt{2}+1}\right)}{\sqrt{2}\,d}+\frac{x}{\sqrt{2}}$$

```
[Out] 1/2*x*2^(1/2)+1/2*arctan(cos(d*x+c)*sin(d*x+c)/(1+sin(d*x+c)^2+2^(1/2)))/d*
2^(1/2)
```

Rubi [A] time = 0.11, antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {203}

$$\frac{\tan^{-1}\left(\frac{\sin(c+dx)\cos(c+dx)}{\sin^2(c+dx)+\sqrt{2}+1}\right)}{\sqrt{2}\,d}+\frac{x}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]/(Csc[c + d*x] + Sin[c + d*x]),x]
```

```
[Out] x/Sqrt[2] + ArcTan[(Cos[c + d*x]*Sin[c + d*x])/(1 + Sqrt[2] + Sin[c + d*x]^
2)]/(Sqrt[2]*d)
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{\csc(c+dx)}{\csc(c+dx)+\sin(c+dx)}\,dx &= \frac{\text{Subst}\left(\int \frac{1}{1+2x^2}\,dx,x,\tan(c+dx)\right)}{d}\\ &= \frac{x}{\sqrt{2}}+\frac{\tan^{-1}\left(\frac{\cos(c+dx)\sin(c+dx)}{1+\sqrt{2}+\sin^2(c+dx)}\right)}{\sqrt{2}\,d}\end{aligned}$$

Mathematica [A] time = 0.03, size = 22, normalized size = 0.46

$$\frac{\tan^{-1}\left(\sqrt{2}\tan(c+dx)\right)}{\sqrt{2}\,d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]/(Csc[c + d*x] + Sin[c + d*x]),x]
```

```
[Out] ArcTan[Sqrt[2]*Tan[c + d*x]]/(Sqrt[2]*d)
```

fricas [A] time = 0.51, size = 46, normalized size = 0.96

$$-\frac{\sqrt{2}\arctan\left(\frac{3\sqrt{2}\cos(dx+c)^2-2\sqrt{2}}{4\cos(dx+c)\sin(dx+c)}\right)}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)/(csc(d*x+c)+sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/4*sqrt(2)*arctan(1/4*(3*sqrt(2)*cos(d*x + c)^2 - 2*sqrt(2))/(cos(d*x + c
)*sin(d*x + c)))/d
```

giac [A] time = 0.25, size = 72, normalized size = 1.50

$$\frac{\sqrt{2}\left(dx+c+\arctan\left(-\frac{\sqrt{2}\sin(2dx+2c)-2\sin(2dx+2c)}{\sqrt{2}\cos(2dx+2c)+\sqrt{2}-2\cos(2dx+2c)+2}\right)\right)}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)/(csc(d*x+c)+sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(2)*(d*x + c + arctan(-(sqrt(2)*sin(2*d*x + 2*c) - 2*sin(2*d*x + 2*
c))/(sqrt(2)*cos(2*d*x + 2*c) + sqrt(2) - 2*cos(2*d*x + 2*c) + 2)))/d
```

maple [A] time = 0.11, size = 20, normalized size = 0.42

$$\frac{\sqrt{2}\arctan\left(\tan(dx+c)\sqrt{2}\right)}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)/(csc(d*x+c)+sin(d*x+c)),x)
```

```
[Out] 1/2/d*2^(1/2)*arctan(tan(d*x+c)*2^(1/2))
```

maxima [B] time = 0.48, size = 245, normalized size = 5.10

$$\frac{\sqrt{2}\arctan\left(\frac{2\sqrt{2}\sin(dx+c)}{2\left(\sqrt{2}+1\right)\cos(dx+c)+\cos(dx+c)^2+\sin(dx+c)^2+2\sqrt{2}+3},\frac{\cos(dx+c)^2+\sin(dx+c)^2+2\cos(dx+c)-1}{2\left(\sqrt{2}+1\right)\cos(dx+c)+\cos(dx+c)^2+\sin(dx+c)^2+2\sqrt{2}+3}\right)-\sqrt{2}\,\mathrm{arctan}}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)/(csc(d*x+c)+sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/4*(sqrt(2)*arctan2(2*sqrt(2)*sin(d*x + c)/(2*(sqrt(2) + 1)*cos(d*x + c) +
 cos(d*x + c)^2 + sin(d*x + c)^2 + 2*sqrt(2) + 3), (cos(d*x + c)^2 + sin(d*
x + c)^2 + 2*cos(d*x + c) - 1)/(2*(sqrt(2) + 1)*cos(d*x + c) + cos(d*x + c)
^2 + sin(d*x + c)^2 + 2*sqrt(2) + 3)) - sqrt(2)*arctan2(2*sqrt(2)*sin(d*x +
 c)/(2*(sqrt(2) - 1)*cos(d*x + c) + cos(d*x + c)^2 + sin(d*x + c)^2 - 2*sqr
t(2) + 3), (cos(d*x + c)^2 + sin(d*x + c)^2 - 2*cos(d*x + c) - 1)/(2*(sqrt(
2) - 1)*cos(d*x + c) + cos(d*x + c)^2 + sin(d*x + c)^2 - 2*sqrt(2) + 3)))/d
```

mupad [B] time = 0.71, size = 56, normalized size = 1.17

$$\frac{\sqrt{2}\left(\operatorname{atan}\left(\frac{\sqrt{2}\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3}{4}+\frac{7\sqrt{2}\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{4}\right)+\operatorname{atan}\left(\frac{\sqrt{2}\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{4}\right)\right)}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(c + d*x)*(sin(c + d*x) + 1/sin(c + d*x))),x)
```

```
[Out] (2^(1/2)*(atan((7*2^(1/2)*tan(c/2 + (d*x)/2))/4 + (2^(1/2)*tan(c/2 + (d*x)/
2)^3)/4) + atan((2^(1/2)*tan(c/2 + (d*x)/2))/4)))/(2*d)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc(c+dx)}{\sin(c+dx)+\csc(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)/(csc(d*x+c)+sin(d*x+c)),x)
```

```
[Out] Integral(csc(c + d*x)/(sin(c + d*x) + csc(c + d*x)), x)
```

3.222 $\int \frac{1}{\csc(c+dx)-\sin(c+dx)}\,dx$

Optimal. Leaf size=10

$$\frac{\sec(c+dx)}{d}$$

```
[Out] sec(d*x+c)/d
```

Rubi [A] time = 0.03, antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {4397, 2606, 8}

$$\frac{\sec(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(Csc[c + d*x] - Sin[c + d*x])^(-1),x]
```

```
[Out] Sec[c + d*x]/d
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 4397

```
Int[u_, x_Symbol] :> Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\csc(c+dx)-\sin(c+dx)}\,dx &= \int \sec(c+dx)\tan(c+dx)\,dx\\ &= \frac{\text{Subst}\left(\int 1\,dx, x, \sec(c+dx)\right)}{d}\\ &= \frac{\sec(c+dx)}{d}\end{aligned}$$

Mathematica [A] time = 0.01, size = 10, normalized size = 1.00

$$\frac{\sec(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Csc[c + d*x] - Sin[c + d*x])^(-1),x]
[Out] Sec[c + d*x]/d
```

fricas [A] time = 0.46, size = 12, normalized size = 1.20

$$\frac{1}{d\cos(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(csc(d*x+c)-sin(d*x+c)),x, algorithm="fricas")
[Out] 1/(d*cos(d*x + c))
```

giac [B] time = 0.19, size = 28, normalized size = 2.80

$$\frac{2}{d\left(\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(csc(d*x+c)-sin(d*x+c)),x, algorithm="giac")
[Out] 2/(d*((cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1))
```

maple [A] time = 0.12, size = 13, normalized size = 1.30

$$\frac{1}{d\cos(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(csc(d*x+c)-sin(d*x+c)),x)
[Out] 1/d/cos(d*x+c)
```

maxima [B] time = 0.42, size = 28, normalized size = 2.80

$$-\frac{2}{d\left(\frac{\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(csc(d*x+c)-sin(d*x+c)),x, algorithm="maxima")
[Out] -2/(d*(sin(d*x + c)^2/(cos(d*x + c) + 1)^2 - 1))
```

mupad [B] time = 0.66, size = 20, normalized size = 2.00

$$-\frac{2}{d\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-1/(sin(c + d*x) - 1/sin(c + d*x)),x)
[Out] -2/(d*(tan(c/2 + (d*x)/2)^2 - 1))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{-\sin\left(c+dx\right)+\csc\left(c+dx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(csc(d*x+c)-sin(d*x+c)),x)
[Out] Integral(1/(-sin(c + d*x) + csc(c + d*x)), x)
```

3.223 $\int \frac{\sin(c+dx)}{\csc(c+dx)-\sin(c+dx)}\,dx$

Optimal. Leaf size=14

$$\frac{\tan(c+dx)}{d}-x$$

```
[Out] -x+tan(d*x+c)/d
```

Rubi [A] time = 0.15, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {321, 203}

$$\frac{\tan(c+dx)}{d}-x$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]/(Csc[c + d*x] - Sin[c + d*x]),x]
```

```
[Out] -x + Tan[c + d*x]/d
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sin(c+dx)}{\csc(c+dx)-\sin(c+dx)}\,dx &= \frac{\text{Subst}\left(\int \frac{x^2}{1+x^2}\,dx, x, \tan(c+dx)\right)}{d}\\
&= \frac{\tan(c+dx)}{d}-\frac{\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \tan(c+dx)\right)}{d}\\
&= -x+\frac{\tan(c+dx)}{d}
\end{aligned}$$

Mathematica [A] time = 0.01, size = 23, normalized size = 1.64

$$\frac{\tan(c+dx)}{d}-\frac{\tan^{-1}(\tan(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]/(Csc[c + d*x] - Sin[c + d*x]),x]
[Out] -(ArcTan[Tan[c + d*x]]/d) + Tan[c + d*x]/d
```

fricas [B] time = 0.42, size = 31, normalized size = 2.21

$$-\frac{dx\cos\left(dx+c\right)-\sin\left(dx+c\right)}{d\cos\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)/(csc(d*x+c)-sin(d*x+c)),x, algorithm="fricas")
[Out] -(d*x*cos(d*x + c) - sin(d*x + c))/(d*cos(d*x + c))
```

giac [A] time = 0.35, size = 18, normalized size = 1.29

$$-\frac{dx+c-\tan\left(dx+c\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)/(csc(d*x+c)-sin(d*x+c)),x, algorithm="giac")
[Out] -(d*x + c - tan(d*x + c))/d
```

maple [A] time = 0.12, size = 24, normalized size = 1.71

$$\frac{\tan\left(dx+c\right)}{d}-\frac{\arctan\left(\tan\left(dx+c\right)\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)/(csc(d*x+c)-sin(d*x+c)),x)
[Out] tan(d*x+c)/d-1/d*arctan(tan(d*x+c))
```

maxima [B] time = 0.44, size = 64, normalized size = 4.57

$$-\frac{2\left(\frac{\sin(dx+c)}{\left(\frac{\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-1\right)(\cos(dx+c)+1)}+\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)/(csc(d*x+c)-sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -2*(sin(d*x + c)/((sin(d*x + c)^2/(cos(d*x + c) + 1)^2 - 1)*(cos(d*x + c) +
 1)) + arctan(sin(d*x + c)/(cos(d*x + c) + 1)))/d
```

mupad [B] time = 0.62, size = 33, normalized size = 2.36

$$-x-\frac{2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{d\,\left({\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-sin(c + d*x)/(sin(c + d*x) - 1/sin(c + d*x)),x)
```

```
[Out] - x - (2*tan(c/2 + (d*x)/2))/(d*(tan(c/2 + (d*x)/2)^2 - 1))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin(c+dx)}{-\sin(c+dx)+\csc(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)/(csc(d*x+c)-sin(d*x+c)),x)
```

```
[Out] Integral(sin(c + d*x)/(-sin(c + d*x) + csc(c + d*x)), x)
```

3.224 $\int \frac{\cos(c+dx)}{\csc(c+dx)-\sin(c+dx)}\,dx$

Optimal. Leaf size=12

$$-\frac{\log(\cos(c+dx))}{d}$$

```
[Out] -ln(cos(d*x+c))/d
```

Rubi [A] time = 0.03, antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {4334, 260}

$$-\frac{\log(\cos(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]/(Csc[c + d*x] - Sin[c + d*x]),x]
```

```
[Out] -(Log[Cos[c + d*x]]/d)
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 4334

```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{d = FreeFacto
rs[Sin[c*(a + b*x)], x]}, Dist[d/(b*c), Subst[Int[SubstFor[1, Sin[c*(a + b*
x)]/d, u, x], x], x, Sin[c*(a + b*x)]/d], x] /; FunctionOfQ[Sin[c*(a + b*x)
]/d, u, x, True]] /; FreeQ[{a, b, c}, x] && (EqQ[F, Cos] || EqQ[F, cos])
```

Rubi steps

$$\begin{aligned}\int \frac{\cos(c+dx)}{\csc(c+dx)-\sin(c+dx)}\,dx &= \frac{\text{Subst}\left(\int \frac{x}{1-x^2}\,dx, x, \sin(c+dx)\right)}{d}\\ &= -\frac{\log(\cos(c+dx))}{d}\end{aligned}$$

Mathematica [A] time = 0.01, size = 12, normalized size = 1.00

$$-\frac{\log(\cos(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]/(Csc[c + d*x] - Sin[c + d*x]),x]
[Out] -(Log[Cos[c + d*x]]/d)
```

fricas [A] time = 0.46, size = 14, normalized size = 1.17

$$-\frac{\log\left(-\cos\left(dx+c\right)\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(csc(d*x+c)-sin(d*x+c)),x, algorithm="fricas")
[Out] -log(-cos(d*x + c))/d
```

giac [B] time = 0.20, size = 26, normalized size = 2.17

$$-\frac{\log\left(|\sin\left(dx+c\right)+1|\right)+\log\left(|\sin\left(dx+c\right)-1|\right)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(csc(d*x+c)-sin(d*x+c)),x, algorithm="giac")
[Out] -1/2*(log(abs(sin(d*x + c) + 1)) + log(abs(sin(d*x + c) - 1)))/d
```

maple [A] time = 0.06, size = 13, normalized size = 1.08

$$-\frac{\ln\left(\cos\left(dx+c\right)\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)/(csc(d*x+c)-sin(d*x+c)),x)
[Out] -ln(cos(d*x+c))/d
```

maxima [A] time = 0.34, size = 24, normalized size = 2.00

$$-\frac{\log\left(\sin\left(dx+c\right)+1\right)+\log\left(\sin\left(dx+c\right)-1\right)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(csc(d*x+c)-sin(d*x+c)),x, algorithm="maxima")
[Out] -1/2*(log(sin(d*x + c) + 1) + log(sin(d*x + c) - 1))/d
```

mupad [B] time = 0.06, size = 14, normalized size = 1.17

$$-\frac{\ln\left(\cos\left(c+d\,x\right)^2\right)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-cos(c + d*x)/(sin(c + d*x) - 1/sin(c + d*x)),x)
[Out] -log(cos(c + d*x)^2)/(2*d)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos\left(c+dx\right)}{-\sin\left(c+dx\right)+\csc\left(c+dx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(csc(d*x+c)-sin(d*x+c)),x)
[Out] Integral(cos(c + d*x)/(-sin(c + d*x) + csc(c + d*x)), x)
```

3.225 $\int \frac{\tan(c+dx)}{\csc(c+dx)-\sin(c+dx)}\,dx$

Optimal. Leaf size=34

$$\frac{\tan(c+dx)\sec(c+dx)}{2d}-\frac{\tanh^{-1}(\sin(c+dx))}{2d}$$

```
[Out] -1/2*arctanh(sin(d*x+c))/d+1/2*sec(d*x+c)*tan(d*x+c)/d
```

Rubi [A] time = 0.04, antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {288, 206}

$$\frac{\tan(c+dx)\sec(c+dx)}{2d}-\frac{\tanh^{-1}(\sin(c+dx))}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]/(Csc[c + d*x] - Sin[c + d*x]),x]
```

```
[Out] -ArcTanh[Sin[c + d*x]]/(2*d) + (Sec[c + d*x]*Tan[c + d*x])/(2*d)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\int \frac{\tan(c+dx)}{\csc(c+dx)-\sin(c+dx)}\,dx = \frac{\text{Subst}\left(\int \frac{x^2}{(1-x^2)^2}\,dx, x, \sin(c+dx)\right)}{d}$$
$$= \frac{\sec(c+dx)\tan(c+dx)}{2d} - \frac{\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \sin(c+dx)\right)}{2d}$$
$$= -\frac{\tanh^{-1}(\sin(c+dx))}{2d} + \frac{\sec(c+dx)\tan(c+dx)}{2d}$$

Mathematica [A] time = 0.01, size = 34, normalized size = 1.00

$$\frac{\tan(c+dx)\sec(c+dx)}{2d} - \frac{\tanh^{-1}(\sin(c+dx))}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]/(Csc[c + d*x] - Sin[c + d*x]),x]
[Out] -1/2*ArcTanh[Sin[c + d*x]]/d + (Sec[c + d*x]*Tan[c + d*x])/(2*d)
```

fricas [B] time = 0.43, size = 61, normalized size = 1.79

$$-\frac{\cos(dx+c)^2\log(\sin(dx+c)+1) - \cos(dx+c)^2\log(-\sin(dx+c)+1) - 2\sin(dx+c)}{4\,d\cos(dx+c)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)/(csc(d*x+c)-sin(d*x+c)),x, algorithm="fricas")
[Out] -1/4*(cos(d*x + c)^2*log(sin(d*x + c) + 1) - cos(d*x + c)^2*log(-sin(d*x +
c) + 1) - 2*sin(d*x + c))/(d*cos(d*x + c)^2)
```

giac [A] time = 0.32, size = 48, normalized size = 1.41

$$-\frac{\frac{2\sin(dx+c)}{\sin(dx+c)^2-1} + \log(|\sin(dx+c)+1|) - \log(|\sin(dx+c)-1|)}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)/(csc(d*x+c)-sin(d*x+c)),x, algorithm="giac")
[Out] -1/4*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) + log(abs(sin(d*x + c) + 1)) - lo
g(abs(sin(d*x + c) - 1)))/d
```

maple [A] time = 0.16, size = 60, normalized size = 1.76

$$-\frac{1}{4d\left(\sin\left(dx+c\right)-1\right)}+\frac{\ln\left(\sin\left(dx+c\right)-1\right)}{4d}-\frac{1}{4d\left(\sin\left(dx+c\right)+1\right)}-\frac{\ln\left(\sin\left(dx+c\right)+1\right)}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)/(csc(d*x+c)-sin(d*x+c)),x)
```

```
[Out] -1/4/d/(sin(d*x+c)-1)+1/4/d*ln(sin(d*x+c)-1)-1/4/d/(sin(d*x+c)+1)-1/4/d*ln(
sin(d*x+c)+1)
```

maxima [A] time = 0.34, size = 46, normalized size = 1.35

$$-\frac{\frac{2\,\sin(dx+c)}{\sin(dx+c)^2-1}+\log\left(\sin\left(dx+c\right)+1\right)-\log\left(\sin\left(dx+c\right)-1\right)}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)/(csc(d*x+c)-sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/4*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) + log(sin(d*x + c) + 1) - log(sin
(d*x + c) - 1))/d
```

mupad [B] time = 1.00, size = 69, normalized size = 2.03

$$\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4-2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+1\right)}-\frac{\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-tan(c + d*x)/(sin(c + d*x) - 1/sin(c + d*x)),x)
```

```
[Out] (tan(c/2 + (d*x)/2) + tan(c/2 + (d*x)/2)^3)/(d*(tan(c/2 + (d*x)/2)^4 - 2*ta
n(c/2 + (d*x)/2)^2 + 1)) - atanh(tan(c/2 + (d*x)/2))/d
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\tan\left(c+dx\right)}{-\sin\left(c+dx\right)+\csc\left(c+dx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)/(csc(d*x+c)-sin(d*x+c)),x)
```

```
[Out] Integral(tan(c + d*x)/(-sin(c + d*x) + csc(c + d*x)), x)
```

3.226 $\int \frac{\cot(c+dx)}{\csc(c+dx)-\sin(c+dx)}\,dx$

Optimal. Leaf size=11

$$\frac{\tanh^{-1}(\sin(c+dx))}{d}$$

```
[Out] arctanh(sin(d*x+c))/d
```

Rubi [A] time = 0.03, antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {4338, 206}

$$\frac{\tanh^{-1}(\sin(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]/(Csc[c + d*x] - Sin[c + d*x]),x]
```

```
[Out] ArcTanh[Sin[c + d*x]]/d
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 4338

```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{d = FreeFacto
rs[Sin[c*(a + b*x)], x]}, Dist[1/(b*c), Subst[Int[SubstFor[1/x, Sin[c*(a +
b*x)]/d, u, x], x], x, Sin[c*(a + b*x)]/d], x] /; FunctionOfQ[Sin[c*(a + b*
x)]/d, u, x, True]] /; FreeQ[{a, b, c}, x] && (EqQ[F, Cot] || EqQ[F, cot])
```

Rubi steps

$$\begin{aligned}\int \frac{\cot(c+dx)}{\csc(c+dx)-\sin(c+dx)}\,dx &= \frac{\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \sin(c+dx)\right)}{d}\\ &= \frac{\tanh^{-1}(\sin(c+dx))}{d}\end{aligned}$$

Mathematica [**A**] time = 0.00, size = 11, normalized size = 1.00

$$\frac{\tanh^{-1}(\sin(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]/(Csc[c + d*x] - Sin[c + d*x]),x]
```

```
[Out] ArcTanh[Sin[c + d*x]]/d
```

fricas [**B**] time = 0.45, size = 28, normalized size = 2.55

$$\frac{\log\left(\sin\left(dx+c\right)+1\right)-\log\left(-\sin\left(dx+c\right)+1\right)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)/(csc(d*x+c)-sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/2*(log(sin(d*x + c) + 1) - log(-sin(d*x + c) + 1))/d
```

giac [**B**] time = 0.99, size = 28, normalized size = 2.55

$$\frac{\log\left(|\sin\left(dx+c\right)+1|\right)-\log\left(|\sin\left(dx+c\right)-1|\right)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)/(csc(d*x+c)-sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/2*(log(abs(sin(d*x + c) + 1)) - log(abs(sin(d*x + c) - 1)))/d
```

maple [**A**] time = 0.12, size = 12, normalized size = 1.09

$$\frac{\operatorname{arctanh}\left(\sin\left(dx+c\right)\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)/(csc(d*x+c)-sin(d*x+c)),x)
```

```
[Out] arctanh(sin(d*x+c))/d
```

maxima [**B**] time = 0.33, size = 26, normalized size = 2.36

$$\frac{\log\left(\sin\left(dx+c\right)+1\right)-\log\left(\sin\left(dx+c\right)-1\right)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)/(csc(d*x+c)-sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/2*(log(sin(d*x + c) + 1) - log(sin(d*x + c) - 1))/d
```

mupad [B] time = 0.64, size = 15, normalized size = 1.36

$$\frac{2\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-cot(c + d*x)/(sin(c + d*x) - 1/sin(c + d*x)),x)
```

```
[Out] (2*atanh(tan(c/2 + (d*x)/2)))/d
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cot(c+dx)}{-\sin(c+dx)+\csc(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)/(csc(d*x+c)-sin(d*x+c)),x)
```

```
[Out] Integral(cot(c + d*x)/(-sin(c + d*x) + csc(c + d*x)), x)
```

3.227 $\int \frac{\sec(c+dx)}{\csc(c+dx)-\sin(c+dx)}\,dx$

Optimal. Leaf size=15

$$\frac{\sec^2(c+dx)}{2d}$$

```
[Out] 1/2*sec(d*x+c)^2/d
```

Rubi [A] time = 0.03, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.042, Rules used = {261}

$$\frac{\sec^2(c+dx)}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]/(Csc[c + d*x] - Sin[c + d*x]),x]
```

```
[Out] Sec[c + d*x]^2/(2*d)
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\int \frac{\sec(c+dx)}{\csc(c+dx)-\sin(c+dx)}\,dx = \frac{\text{Subst}\left(\int \frac{x}{(1-x^2)^2}\,dx, x, \sin(c+dx)\right)}{d}$$
$$= \frac{\sec^2(c+dx)}{2d}$$

Mathematica [A] time = 0.01, size = 15, normalized size = 1.00

$$\frac{\sec^2(c+dx)}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]/(Csc[c + d*x] - Sin[c + d*x]),x]
```

```
[Out] Sec[c + d*x]^2/(2*d)
```

fricas [A] time = 0.39, size = 13, normalized size = 0.87

$$\frac{1}{2\,d\cos\left(dx+c\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(csc(d*x+c)-sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/2/(d*cos(d*x + c)^2)
```

giac [B] time = 0.23, size = 46, normalized size = 3.07

$$-\frac{2\left(\cos\left(dx+c\right)-1\right)}{d\left(\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right)^{2}\left(\cos\left(dx+c\right)+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(csc(d*x+c)-sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -2*(cos(d*x + c) - 1)/(d*((cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1)^2*(cos
(d*x + c) + 1))
```

maple [A] time = 0.08, size = 14, normalized size = 0.93

$$\frac{\sec^{2}\left(dx+c\right)}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)/(csc(d*x+c)-sin(d*x+c)),x)
```

```
[Out] 1/2*sec(d*x+c)^2/d
```

maxima [A] time = 0.35, size = 17, normalized size = 1.13

$$-\frac{1}{2\left(\sin\left(dx+c\right)^{2}-1\right)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(csc(d*x+c)-sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/2/((sin(d*x + c)^2 - 1)*d)
```

mupad [B] time = 0.04, size = 13, normalized size = 0.87

$$\frac{1}{2\,d\cos\left(c+d\,x\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-1/(cos(c + d*x)*(sin(c + d*x) - 1/sin(c + d*x))),x)
[Out] 1/(2*d*cos(c + d*x)^2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec\left(c+dx\right)}{-\sin\left(c+dx\right)+\csc\left(c+dx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(csc(d*x+c)-sin(d*x+c)),x)
[Out] Integral(sec(c + d*x)/(-sin(c + d*x) + csc(c + d*x)), x)
```

3.228 $\int \frac{\csc(c+dx)}{\csc(c+dx)-\sin(c+dx)}\,dx$

Optimal. Leaf size=10

$$\frac{\tan(c+dx)}{d}$$

[Out] tan(d*x+c)/d

Rubi [A] time = 0.09, antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.042, Rules used = {8}

$$\frac{\tan(c+dx)}{d}$$

Antiderivative was successfully verified.

[In] Int[Csc[c + d*x]/(Csc[c + d*x] - Sin[c + d*x]),x]

[Out] Tan[c + d*x]/d

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\csc(c+dx)}{\csc(c+dx)-\sin(c+dx)}\,dx &= \frac{\operatorname{Subst}\left(\int 1\,dx, x, \tan(c+dx)\right)}{d}\\ &= \frac{\tan(c+dx)}{d}\end{aligned}$$

Mathematica [A] time = 0.00, size = 10, normalized size = 1.00

$$\frac{\tan(c+dx)}{d}$$

Antiderivative was successfully verified.

[In] Integrate[Csc[c + d*x]/(Csc[c + d*x] - Sin[c + d*x]),x]

[Out] Tan[c + d*x]/d

fricas [A] time = 0.42, size = 18, normalized size = 1.80

$$\frac{\sin(dx+c)}{d\cos(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)/(csc(d*x+c)-sin(d*x+c)),x, algorithm="fricas")
[Out] sin(d*x + c)/(d*cos(d*x + c))
```

giac [A] time = 0.20, size = 10, normalized size = 1.00

$$\frac{\tan(dx+c)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)/(csc(d*x+c)-sin(d*x+c)),x, algorithm="giac")
[Out] tan(d*x + c)/d
```

maple [A] time = 0.11, size = 11, normalized size = 1.10

$$\frac{\tan(dx+c)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)/(csc(d*x+c)-sin(d*x+c)),x)
[Out] tan(d*x+c)/d
```

maxima [B] time = 0.34, size = 44, normalized size = 4.40

$$-\frac{2\sin(dx+c)}{d\left(\frac{\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-1\right)(\cos(dx+c)+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)/(csc(d*x+c)-sin(d*x+c)),x, algorithm="maxima")
[Out] -2*sin(d*x + c)/(d*(sin(d*x + c)^2/(cos(d*x + c) + 1)^2 - 1)*(cos(d*x + c)
+ 1))
```

mupad [B] time = 0.58, size = 29, normalized size = 2.90

$$-\frac{2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{d\,\left({\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-1/(sin(c + d*x)*(sin(c + d*x) - 1/sin(c + d*x))),x)
```

```
[Out] -(2*tan(c/2 + (d*x)/2))/(d*(tan(c/2 + (d*x)/2)^2 - 1))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc(c+dx)}{-\sin(c+dx)+\csc(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)/(csc(d*x+c)-sin(d*x+c)),x)
```

```
[Out] Integral(csc(c + d*x)/(-sin(c + d*x) + csc(c + d*x)), x)
```

3.229 $\int \cos^3(c+dx)(a\sin(c+dx)+b\tan(c+dx))\,dx$

Optimal. Leaf size=33

$$-\frac{a\cos^4(c+dx)}{4d}-\frac{b\cos^3(c+dx)}{3d}$$

```
[Out] -1/3*b*cos(d*x+c)^3/d-1/4*a*cos(d*x+c)^4/d
```

Rubi [A] time = 0.06, antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {4377, 12, 2565, 30}

$$-\frac{a\cos^4(c+dx)}{4d}-\frac{b\cos^3(c+dx)}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^3*(a*Sin[c + d*x] + b*Tan[c + d*x]),x]
```

```
[Out] -(b*Cos[c + d*x]^3)/(3*d) - (a*Cos[c + d*x]^4)/(4*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2565

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> -Dist[(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 4377

```
Int[(u_)*((v_) + (d_.)*(F_)[(c_.)*((a_.) + (b_.)*(x_))]^(n_.)), x_Symbol] :
> With[{e = FreeFactors[Cos[c*(a + b*x)], x]}, Int[ActivateTrig[u*v], x] +
Dist[d, Int[ActivateTrig[u]*Sin[c*(a + b*x)]^n, x], x] /; FunctionOfQ[Cos[c
*(a + b*x)]/e, u, x]] /; FreeQ[{a, b, c, d}, x] &&  !FreeQ[v, x] && Integer
Q[(n - 1)/2] && NonsumQ[u] && (EqQ[F, Sin] || EqQ[F, sin])
```

Rubi steps

$$\int \cos^3(c+dx)(a\sin(c+dx)+b\tan(c+dx))\,dx = a\int \cos^3(c+dx)\sin(c+dx)\,dx + \int b\cos^2(c+dx)\sin(c+$$

$$= b\int \cos^2(c+dx)\sin(c+dx)\,dx - \frac{a\,\mathrm{Subst}\left(\int x^3\,dx, x, \cos(c\right.}{d}$$

$$= -\frac{a\cos^4(c+dx)}{4d} - \frac{b\,\mathrm{Subst}\left(\int x^2\,dx, x, \cos(c+dx)\right)}{d}$$

$$= -\frac{b\cos^3(c+dx)}{3d} - \frac{a\cos^4(c+dx)}{4d}$$

Mathematica [A] time = 0.01, size = 33, normalized size = 1.00

$$-\frac{a\cos^4(c+dx)}{4d} - \frac{b\cos^3(c+dx)}{3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^3*(a*Sin[c + d*x] + b*Tan[c + d*x]),x]
[Out] -1/3*(b*Cos[c + d*x]^3)/d - (a*Cos[c + d*x]^4)/(4*d)
```

fricas [A] time = 0.41, size = 28, normalized size = 0.85

$$-\frac{3\,a\cos\left(dx+c\right)^4 + 4\,b\cos\left(dx+c\right)^3}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a*sin(d*x+c)+b*tan(d*x+c)),x, algorithm="fricas")
[Out] -1/12*(3*a*cos(d*x + c)^4 + 4*b*cos(d*x + c)^3)/d
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a*sin(d*x+c)+b*tan(d*x+c)),x, algorithm="giac")
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
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4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to

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/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/
x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check
 sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Un
able to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>
(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign
: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable
to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*p
i/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*
pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to ch
eck sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2
)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/
2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check s
ign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unab
le to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-
4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign:
(4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to
 check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/
x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)-a*8*d/(8*d)^2*cos(2*c+2*d*
x)-a*32*d/(32*d)^2*cos(4*c+4*d*x)+(3*b*pi*sign(tan(c/2)^2*tan(d*x/2)^2+2*ta
n(c/2)^2*tan(d*x/2)+tan(c/2)^2-tan(d*x/2)^2+2*tan(d*x/2)-1)*sign(tan(c/2)^2
*tan(d*x/2)^2-tan(c/2)^2+2*tan(c/2)*tan(d*x/2)^2+2*tan(c/2)+tan(d*x/2)^2-1)
*tan(c/2)^6*tan(d*x/2)^6+9*b*pi*sign(tan(c/2)^2*tan(d*x/2)^2+2*tan(c/2)^2*t
an(d*x/2)+tan(c/2)^2-tan(d*x/2)^2+2*tan(d*x/2)-1)*sign(tan(c/2)^2*tan(d*x/2
)^2-tan(c/2)^2+2*tan(c/2)*tan(d*x/2)^2+2*tan(c/2)+tan(d*x/2)^2-1)*tan(c/2)^
6*tan(d*x/2)^4+9*b*pi*sign(tan(c/2)^2*tan(d*x/2)^2+2*tan(c/2)^2*tan(d*x/2)+
tan(c/2)^2-tan(d*x/2)^2+2*tan(d*x/2)-1)*sign(tan(c/2)^2*tan(d*x/2)^2-tan(c/
2)^2+2*tan(c/2)*tan(d*x/2)^2+2*tan(c/2)+tan(d*x/2)^2-1)*tan(c/2)^6*tan(d*x/
2)^2+3*b*pi*sign(tan(c/2)^2*tan(d*x/2)^2+2*tan(c/2)^2*tan(d*x/2)+tan(c/2)^2
-tan(d*x/2)^2+2*tan(d*x/2)-1)*sign(tan(c/2)^2*tan(d*x/2)^2-tan(c/2)^2+2*tan
(c/2)*tan(d*x/2)^2+2*tan(c/2)+tan(d*x/2)^2-1)*tan(c/2)^6+9*b*pi*sign(tan(c/
2)^2*tan(d*x/2)^2+2*tan(c/2)^2*tan(d*x/2)+tan(c/2)^2-tan(d*x/2)^2+2*tan(d*x
/2)-1)*sign(tan(c/2)^2*tan(d*x/2)^2-tan(c/2)^2+2*tan(c/2)*tan(d*x/2)^2+2*ta
n(c/2)+tan(d*x/2)^2-1)*tan(c/2)^4*tan(d*x/2)^6+27*b*pi*sign(tan(c/2)^2*tan(
d*x/2)^2+2*tan(c/2)^2*tan(d*x/2)+tan(c/2)^2-tan(d*x/2)^2+2*tan(d*x/2)-1)*si
gn(tan(c/2)^2*tan(d*x/2)^2-tan(c/2)^2+2*tan(c/2)*tan(d*x/2)^2+2*tan(c/2)+ta
n(d*x/2)^2-1)*tan(c/2)^4*tan(d*x/2)^4+27*b*pi*sign(tan(c/2)^2*tan(d*x/2)^2+
2*tan(c/2)^2*tan(d*x/2)+tan(c/2)^2-tan(d*x/2)^2+2*tan(d*x/2)-1)*sign(tan(c/
2)^2*tan(d*x/2)^2-tan(c/2)^2+2*tan(c/2)*tan(d*x/2)^2+2*tan(c/2)+tan(d*x/2)^
2-1)*tan(c/2)^4*tan(d*x/2)^2+9*b*pi*sign(tan(c/2)^2*tan(d*x/2)^2+2*tan(c/2)
^2*tan(d*x/2)+tan(c/2)^2-tan(d*x/2)^2+2*tan(d*x/2)-1)*sign(tan(c/2)^2*tan(d
*x/2)^2-tan(c/2)^2+2*tan(c/2)*tan(d*x/2)^2+2*tan(c/2)+tan(d*x/2)^2-1)*tan(c
/2)^4+9*b*pi*sign(tan(c/2)^2*tan(d*x/2)^2+2*tan(c/2)^2*tan(d*x/2)+tan(c/2)^
2-tan(d*x/2)^2+2*tan(d*x/2)-1)*sign(tan(c/2)^2*tan(d*x/2)^2-tan(c/2)^2+2*ta
n(c/2)*tan(d*x/2)^2+2*tan(c/2)+tan(d*x/2)^2-1)*tan(c/2)^2*tan(d*x/2)^6+27*b

```
*pi*sign(tan(c/2)^2*tan(d*x/2)^2+2*tan(c/2)^2*tan(d*x/2)+tan(c/2)^2-tan(d*x
/2)^2+2*tan(d*x/2)-1)*sign(tan(c/2)^2*tan(d*x/2)^2-tan(c/2)^2+2*tan(c/2)*ta
n(d*x/2)^2+2*tan(c/2)+tan(d*x/2)^2-1)*tan(c/2)^2*tan(d*x/2)^4+27*b*pi*sign(
tan(c/2)^2*tan(d*x/2)^2+2*tan(c/2)^2*tan(d*x/2)+tan(c/2)^2-tan(d*x/2)^2+2*t
an(d*x/2)-1)*sign(tan(c/2)^2*tan(d*x/2)^2-tan(c/2)^2+2*tan(c/2)*tan(d*x/2)^
2+2*tan(c/2)+tan(d*x/2)^2-1)*tan(c/2)^2*tan(d*x/2)^2+9*b*pi*sign(tan(c/2)^2
*tan(d*x/2)^2+2*tan(c/2)^2*tan(d*x/2)+tan(c/2)^2-tan(d*x/2)^2+2*tan(d*x/2)-
1)*sign(tan(c/2)^2*tan(d*x/2)^2-tan(c/2)^2+2*tan(c/2)*tan(d*x/2)^2+2*tan(c/
2)+tan(d*x/2)^2-1)*tan(c/2)^2+3*b*pi*sign(tan(c/2)^2*tan(d*x/2)^2+2*tan(c/2
)^2*tan(d*x/2)+tan(c/2)^2-tan(d*x/2)^2+2*tan(d*x/2)-1)*sign(tan(c/2)^2*tan(
d*x/2)^2-tan(c/2)^2+2*tan(c/2)*tan(d*x/2)^2+2*tan(c/2)+tan(d*x/2)^2-1)*tan(
d*x/2)^6+9*b*pi*sign(tan(c/2)^2*tan(d*x/2)^2+2*tan(c/2)^2*tan(d*x/2)+tan(c/
2)^2-tan(d*x/2)^2+2*tan(d*x/2)-1)*sign(tan(c/2)^2*tan(d*x/2)^2-tan(c/2)^2+2
*tan(c/2)*tan(d*x/2)^2+2*tan(c/2)+tan(d*x/2)^2-1)*tan(d*x/2)^4+9*b*pi*sign(
tan(c/2)^2*tan(d*x/2)^2+2*tan(c/2)^2*tan(d*x/2)+tan(c/2)^2-tan(d*x/2)^2+2*t
an(d*x/2)-1)*sign(tan(c/2)^2*tan(d*x/2)^2-tan(c/2)^2+2*tan(c/2)*tan(d*x/2)^
2+2*tan(c/2)+tan(d*x/2)^2-1)*tan(d*x/2)^2+3*b*pi*sign(tan(c/2)^2*tan(d*x/2)
^2+2*tan(c/2)^2*tan(d*x/2)+tan(c/2)^2-tan(d*x/2)^2+2*tan(d*x/2)-1)*sign(tan
(c/2)^2*tan(d*x/2)^2-tan(c/2)^2+2*tan(c/2)*tan(d*x/2)^2+2*tan(c/2)+tan(d*x/
2)^2-1)-3*b*pi*sign(tan(c/2)^2*tan(d*x/2)^2+2*tan(c/2)^2*tan(d*x/2)+tan(c/2
)^2-tan(d*x/2)^2+2*tan(d*x/2)-1)*tan(c/2)^6*tan(d*x/2)^6-9*b*pi*sign(tan(c/
2)^2*tan(d*x/2)^2+2*tan(c/2)^2*tan(d*x/2)+tan(c/2)^2-tan(d*x/2)^2+2*tan(d*x
/2)-1)*tan(c/2)^6*tan(d*x/2)^4-9*b*pi*sign(tan(c/2)^2*tan(d*x/2)^2+2*tan(c/
2)^2*tan(d*x/2)+tan(c/2)^2-tan(d*x/2)^2+2*tan(d*x/2)-1)*tan(c/2)^6*tan(d*x/
2)^2-3*b*pi*sign(tan(c/2)^2*tan(d*x/2)^2+2*tan(c/2)^2*tan(d*x/2)+tan(c/2)^2
-tan(d*x/2)^2+2*tan(d*x/2)-1)*tan(c/2)^6-9*b*pi*sign(tan(c/2)^2*tan(d*x/2)^
2+2*tan(c/2)^2*tan(d*x/2)+tan(c/2)^2-tan(d*x/2)^2+2*tan(d*x/2)-1)*tan(c/2)^
4*tan(d*x/2)^6-27*b*pi*sign(tan(c/2)^2*tan(d*x/2)^2+2*tan(c/2)^2*tan(d*x/2)
+tan(c/2)^2-tan(d*x/2)^2+2*tan(d*x/2)-1)*tan(c/2)^4*tan(d*x/2)^4-27*b*pi*si
gn(tan(c/2)^2*tan(d*x/2)^2+2*tan(c/2)^2*tan(d*x/2)+tan(c/2)^2-tan(d*x/2)^2+
2*tan(d*x/2)-1)*tan(c/2)^4*tan(d*x/2)^2-9*b*pi*sign(tan(c/2)^2*tan(d*x/2)^2
+2*tan(c/2)^2*tan(d*x/2)+tan(c/2)^2-tan(d*x/2)^2+2*tan(d*x/2)-1)*tan(c/2)^4
-9*b*pi*sign(tan(c/2)^2*tan(d*x/2)^2+2*tan(c/2)^2*tan(d*x/2)+tan(c/2)^2-tan
(d*x/2)^2+2*tan(d*x/2)-1)*tan(c/2)^2*tan(d*x/2)^6-27*b*pi*sign(tan(c/2)^2*t
an(d*x/2)^2+2*tan(c/2)^2*tan(d*x/2)+tan(c/2)^2-tan(d*x/2)^2+2*tan(d*x/2)-1)
*tan(c/2)^2*tan(d*x/2)^4-27*b*pi*sign(tan(c/2)^2*tan(d*x/2)^2+2*tan(c/2)^2*
tan(d*x/2)+tan(c/2)^2-tan(d*x/2)^2+2*tan(d*x/2)-1)*tan(c/2)^2*tan(d*x/2)^2-
9*b*pi*sign(tan(c/2)^2*tan(d*x/2)^2+2*tan(c/2)^2*tan(d*x/2)+tan(c/2)^2-tan(
d*x/2)^2+2*tan(d*x/2)-1)*tan(c/2)^2-3*b*pi*sign(tan(c/2)^2*tan(d*x/2)^2+2*t
an(c/2)^2*tan(d*x/2)+tan(c/2)^2-tan(d*x/2)^2+2*tan(d*x/2)-1)*tan(d*x/2)^6-9
*b*pi*sign(tan(c/2)^2*tan(d*x/2)^2+2*tan(c/2)^2*tan(d*x/2)+tan(c/2)^2-tan(d
*x/2)^2+2*tan(d*x/2)-1)*tan(d*x/2)^4-9*b*pi*sign(tan(c/2)^2*tan(d*x/2)^2+2*
tan(c/2)^2*tan(d*x/2)+tan(c/2)^2-tan(d*x/2)^2+2*tan(d*x/2)-1)*tan(d*x/2)^2-
3*b*pi*sign(tan(c/2)^2*tan(d*x/2)^2+2*tan(c/2)^2*tan(d*x/2)+tan(c/2)^2-tan(
d*x/2)^2+2*tan(d*x/2)-1)+3*b*pi*sign(tan(c/2)^2*tan(d*x/2)^2-2*tan(c/2)^2*t
```

an(d*x/2)+tan(c/2)^2-tan(d*x/2)^2-2*tan(d*x/2)-1)*sign(tan(c/2)^2*tan(d*x/2
)^2-tan(c/2)^2-2*tan(c/2)*tan(d*x/2)^2-2*tan(c/2)+tan(d*x/2)^2-1)*tan(c/2)^
6*tan(d*x/2)^6+9*b*pi*sign(tan(c/2)^2*tan(d*x/2)^2-2*tan(c/2)^2*tan(d*x/2)+
tan(c/2)^2-tan(d*x/2)^2-2*tan(d*x/2)-1)*sign(tan(c/2)^2*tan(d*x/2)^2-tan(c/
2)^2-2*tan(c/2)*tan(d*x/2)^2-2*tan(c/2)+tan(d*x/2)^2-1)*tan(c/2)^6*tan(d*x/
2)^4+9*b*pi*sign(tan(c/2)^2*tan(d*x/2)^2-2*tan(c/2)^2*tan(d*x/2)+tan(c/2)^2
-tan(d*x/2)^2-2*tan(d*x/2)-1)*sign(tan(c/2)^2*tan(d*x/2)^2-tan(c/2)^2-2*tan
(c/2)*tan(d*x/2)^2-2*tan(c/2)+tan(d*x/2)^2-1)*tan(c/2)^6*tan(d*x/2)^2+3*b*p
i*sign(tan(c/2)^2*tan(d*x/2)^2-2*tan(c/2)^2*tan(d*x/2)+tan(c/2)^2-tan(d*x/2
)^2-2*tan(d*x/2)-1)*sign(tan(c/2)^2*tan(d*x/2)^2-tan(c/2)^2-2*tan(c/2)*tan(
d*x/2)^2-2*tan(c/2)+tan(d*x/2)^2-1)*tan(c/2)^6+9*b*pi*sign(tan(c/2)^2*tan(d
*x/2)^2-2*tan(c/2)^2*tan(d*x/2)+tan(c/2)^2-tan(d*x/2)^2-2*tan(d*x/2)-1)*sig
n(tan(c/2)^2*tan(d*x/2)^2-tan(c/2)^2-2*tan(c/2)*tan(d*x/2)^2-2*tan(c/2)+tan
(d*x/2)^2-1)*tan(c/2)^4*tan(d*x/2)^6+27*b*pi*sign(tan(c/2)^2*tan(d*x/2)^2-2
*tan(c/2)^2*tan(d*x/2)+tan(c/2)^2-tan(d*x/2)^2-2*tan(d*x/2)-1)*sign(tan(c/2
)^2*tan(d*x/2)^2-tan(c/2)^2-2*tan(c/2)*tan(d*x/2)^2-2*tan(c/2)+tan(d*x/2)^2
-1)*tan(c/2)^4*tan(d*x/2)^4+27*b*pi*sign(tan(c/2)^2*tan(d*x/2)^2-2*tan(c/2)
^2*tan(d*x/2)+tan(c/2)^2-tan(d*x/2)^2-2*tan(d*x/2)-1)*sign(tan(c/2)^2*tan(d
*x/2)^2-tan(c/2)^2-2*tan(c/2)*tan(d*x/2)^2-2*tan(c/2)+tan(d*x/2)^2-1)*tan(c
/2)^4*tan(d*x/2)^2+9*b*pi*sign(tan(c/2)^2*tan(d*x/2)^2-2*tan(c/2)^2*tan(d*x
/2)+tan(c/2)^2-tan(d*x/2)^2-2*tan(d*x/2)-1)*sign(tan(c/2)^2*tan(d*x/2)^2-ta
n(c/2)^2-2*tan(c/2)*tan(d*x/2)^2-2*tan(c/2)+tan(d*x/2)^2-1)*tan(c/2)^4+9*b*
pi*sign(tan(c/2)^2*tan(d*x/2)^2-2*tan(c/2)^2*tan(d*x/2)+tan(c/2)^2-tan(d*x/
2)^2-2*tan(d*x/2)-1)*sign(tan(c/2)^2*tan(d*x/2)^2-tan(c/2)^2-2*tan(c/2)*tan
(d*x/2)^2-2*tan(c/2)+tan(d*x/2)^2-1)*tan(c/2)^2*tan(d*x/2)^6+27*b*pi*sign(t
an(c/2)^2*tan(d*x/2)^2-2*tan(c/2)^2*tan(d*x/2)+tan(c/2)^2-tan(d*x/2)^2-2*ta
n(d*x/2)-1)*sign(tan(c/2)^2*tan(d*x/2)^2-tan(c/2)^2-2*tan(c/2)*tan(d*x/2)^2
-2*tan(c/2)+tan(d*x/2)^2-1)*tan(c/2)^2*tan(d*x/2)^4+27*b*pi*sign(tan(c/2)^2
*tan(d*x/2)^2-2*tan(c/2)^2*tan(d*x/2)+tan(c/2)^2-tan(d*x/2)^2-2*tan(d*x/2)-
1)*sign(tan(c/2)^2*tan(d*x/2)^2-tan(c/2)^2-2*tan(c/2)*tan(d*x/2)^2-2*tan(c/
2)+tan(d*x/2)^2-1)*tan(c/2)^2*tan(d*x/2)^2+9*b*pi*sign(tan(c/2)^2*tan(d*x/2
)^2-2*tan(c/2)^2*tan(d*x/2)+tan(c/2)^2-tan(d*x/2)^2-2*tan(d*x/2)-1)*sign(ta
n(c/2)^2*tan(d*x/2)^2-tan(c/2)^2-2*tan(c/2)*tan(d*x/2)^2-2*tan(c/2)+tan(d*x
/2)^2-1)*tan(c/2)^2+3*b*pi*sign(tan(c/2)^2*tan(d*x/2)^2-2*tan(c/2)^2*tan(d*
x/2)+tan(c/2)^2-tan(d*x/2)^2-2*tan(d*x/2)-1)*sign(tan(c/2)^2*tan(d*x/2)^2-t
an(c/2)^2-2*tan(c/2)*tan(d*x/2)^2-2*tan(c/2)+tan(d*x/2)^2-1)*tan(d*x/2)^6+9
*b*pi*sign(tan(c/2)^2*tan(d*x/2)^2-2*tan(c/2)^2*tan(d*x/2)+tan(c/2)^2-tan(d
*x/2)^2-2*tan(d*x/2)-1)*sign(tan(c/2)^2*tan(d*x/2)^2-tan(c/2)^2-2*tan(c/2)*
tan(d*x/2)^2-2*tan(c/2)+tan(d*x/2)^2-1)*tan(d*x/2)^4+9*b*pi*sign(tan(c/2)^2
*tan(d*x/2)^2-2*tan(c/2)^2*tan(d*x/2)+tan(c/2)^2-tan(d*x/2)^2-2*tan(d*x/2)-
1)*sign(tan(c/2)^2*tan(d*x/2)^2-tan(c/2)^2-2*tan(c/2)*tan(d*x/2)^2-2*tan(c/
2)+tan(d*x/2)^2-1)*tan(d*x/2)^2+3*b*pi*sign(tan(c/2)^2*tan(d*x/2)^2-2*tan(c
/2)^2*tan(d*x/2)+tan(c/2)^2-tan(d*x/2)^2-2*tan(d*x/2)-1)*sign(tan(c/2)^2*ta
n(d*x/2)^2-tan(c/2)^2-2*tan(c/2)*tan(d*x/2)^2-2*tan(c/2)+tan(d*x/2)^2-1)+3*
b*pi*sign(tan(c/2)^2*tan(d*x/2)^2-2*tan(c/2)^2*tan(d*x/2)+tan(c/2)^2-tan(d*

x/2)^2-2*tan(d*x/2)-1)*tan(c/2)^6*tan(d*x/2)^6+9*b*pi*sign(tan(c/2)^2*tan(d
*x/2)^2-2*tan(c/2)^2*tan(d*x/2)+tan(c/2)^2-tan(d*x/2)^2-2*tan(d*x/2)-1)*tan
(c/2)^6*tan(d*x/2)^4+9*b*pi*sign(tan(c/2)^2*tan(d*x/2)^2-2*tan(c/2)^2*tan(d
*x/2)+tan(c/2)^2-tan(d*x/2)^2-2*tan(d*x/2)-1)*tan(c/2)^6*tan(d*x/2)^2+3*b*p
i*sign(tan(c/2)^2*tan(d*x/2)^2-2*tan(c/2)^2*tan(d*x/2)+tan(c/2)^2-tan(d*x/2
)^2-2*tan(d*x/2)-1)*tan(c/2)^6+9*b*pi*sign(tan(c/2)^2*tan(d*x/2)^2-2*tan(c/
2)^2*tan(d*x/2)+tan(c/2)^2-tan(d*x/2)^2-2*tan(d*x/2)-1)*tan(c/2)^4*tan(d*x/
2)^6+27*b*pi*sign(tan(c/2)^2*tan(d*x/2)^2-2*tan(c/2)^2*tan(d*x/2)+tan(c/2)^
2-tan(d*x/2)^2-2*tan(d*x/2)-1)*tan(c/2)^4*tan(d*x/2)^4+27*b*pi*sign(tan(c/2
)^2*tan(d*x/2)^2-2*tan(c/2)^2*tan(d*x/2)+tan(c/2)^2-tan(d*x/2)^2-2*tan(d*x/
2)-1)*tan(c/2)^4*tan(d*x/2)^2+9*b*pi*sign(tan(c/2)^2*tan(d*x/2)^2-2*tan(c/2
)^2*tan(d*x/2)+tan(c/2)^2-tan(d*x/2)^2-2*tan(d*x/2)-1)*tan(c/2)^4+9*b*pi*si
gn(tan(c/2)^2*tan(d*x/2)^2-2*tan(c/2)^2*tan(d*x/2)+tan(c/2)^2-tan(d*x/2)^2-
2*tan(d*x/2)-1)*tan(c/2)^2*tan(d*x/2)^6+27*b*pi*sign(tan(c/2)^2*tan(d*x/2)^
2-2*tan(c/2)^2*tan(d*x/2)+tan(c/2)^2-tan(d*x/2)^2-2*tan(d*x/2)-1)*tan(c/2)^
2*tan(d*x/2)^4+27*b*pi*sign(tan(c/2)^2*tan(d*x/2)^2-2*tan(c/2)^2*tan(d*x/2)
+tan(c/2)^2-tan(d*x/2)^2-2*tan(d*x/2)-1)*tan(c/2)^2*tan(d*x/2)^2+9*b*pi*sig
n(tan(c/2)^2*tan(d*x/2)^2-2*tan(c/2)^2*tan(d*x/2)+tan(c/2)^2-tan(d*x/2)^2-2
*tan(d*x/2)-1)*tan(c/2)^2+3*b*pi*sign(tan(c/2)^2*tan(d*x/2)^2-2*tan(c/2)^2*
tan(d*x/2)+tan(c/2)^2-tan(d*x/2)^2-2*tan(d*x/2)-1)*tan(d*x/2)^6+9*b*pi*sign
(tan(c/2)^2*tan(d*x/2)^2-2*tan(c/2)^2*tan(d*x/2)+tan(c/2)^2-tan(d*x/2)^2-2*
tan(d*x/2)-1)*tan(d*x/2)^4+9*b*pi*sign(tan(c/2)^2*tan(d*x/2)^2-2*tan(c/2)^2
*tan(d*x/2)+tan(c/2)^2-tan(d*x/2)^2-2*tan(d*x/2)-1)*tan(d*x/2)^2+3*b*pi*sig
n(tan(c/2)^2*tan(d*x/2)^2-2*tan(c/2)^2*tan(d*x/2)+tan(c/2)^2-tan(d*x/2)^2-2
*tan(d*x/2)-1)-6*b*pi*sign(tan(c/2)^2*tan(d*x/2)^2-tan(c/2)^2-4*tan(c/2)*ta
n(d*x/2)-tan(d*x/2)^2+1)*tan(c/2)^6*tan(d*x/2)^6-18*b*pi*sign(tan(c/2)^2*ta
n(d*x/2)^2-tan(c/2)^2-4*tan(c/2)*tan(d*x/2)-tan(d*x/2)^2+1)*tan(c/2)^6*tan(
d*x/2)^4-18*b*pi*sign(tan(c/2)^2*tan(d*x/2)^2-tan(c/2)^2-4*tan(c/2)*tan(d*x
/2)-tan(d*x/2)^2+1)*tan(c/2)^6*tan(d*x/2)^2-6*b*pi*sign(tan(c/2)^2*tan(d*x/
2)^2-tan(c/2)^2-4*tan(c/2)*tan(d*x/2)-tan(d*x/2)^2+1)*tan(c/2)^6-18*b*pi*si
gn(tan(c/2)^2*tan(d*x/2)^2-tan(c/2)^2-4*tan(c/2)*tan(d*x/2)-tan(d*x/2)^2+1)
*tan(c/2)^4*tan(d*x/2)^6-54*b*pi*sign(tan(c/2)^2*tan(d*x/2)^2-tan(c/2)^2-4*
tan(c/2)*tan(d*x/2)-tan(d*x/2)^2+1)*tan(c/2)^4*tan(d*x/2)^4-54*b*pi*sign(ta
n(c/2)^2*tan(d*x/2)^2-tan(c/2)^2-4*tan(c/2)*tan(d*x/2)-tan(d*x/2)^2+1)*tan(
c/2)^4*tan(d*x/2)^2-18*b*pi*sign(tan(c/2)^2*tan(d*x/2)^2-tan(c/2)^2-4*tan(c
/2)*tan(d*x/2)-tan(d*x/2)^2+1)*tan(c/2)^4-18*b*pi*sign(tan(c/2)^2*tan(d*x/2
)^2-tan(c/2)^2-4*tan(c/2)*tan(d*x/2)-tan(d*x/2)^2+1)*tan(c/2)^2*tan(d*x/2)^
6-54*b*pi*sign(tan(c/2)^2*tan(d*x/2)^2-tan(c/2)^2-4*tan(c/2)*tan(d*x/2)-tan
(d*x/2)^2+1)*tan(c/2)^2*tan(d*x/2)^4-54*b*pi*sign(tan(c/2)^2*tan(d*x/2)^2-t
an(c/2)^2-4*tan(c/2)*tan(d*x/2)-tan(d*x/2)^2+1)*tan(c/2)^2*tan(d*x/2)^2-18*
b*pi*sign(tan(c/2)^2*tan(d*x/2)^2-tan(c/2)^2-4*tan(c/2)*tan(d*x/2)-tan(d*x/
2)^2+1)*tan(c/2)^2-6*b*pi*sign(tan(c/2)^2*tan(d*x/2)^2-tan(c/2)^2-4*tan(c/2
)*tan(d*x/2)-tan(d*x/2)^2+1)*tan(d*x/2)^6-18*b*pi*sign(tan(c/2)^2*tan(d*x/2
)^2-tan(c/2)^2-4*tan(c/2)*tan(d*x/2)-tan(d*x/2)^2+1)*tan(d*x/2)^4-18*b*pi*s
ign(tan(c/2)^2*tan(d*x/2)^2-tan(c/2)^2-4*tan(c/2)*tan(d*x/2)-tan(d*x/2)^2+1

)*tan(d*x/2)^2-6*b*pi*sign(tan(c/2)^2*tan(d*x/2)^2-tan(c/2)^2-4*tan(c/2)*ta
n(d*x/2)-tan(d*x/2)^2+1)+6*b*pi*tan(c/2)^6*tan(d*x/2)^6+18*b*pi*tan(c/2)^6*
tan(d*x/2)^4+18*b*pi*tan(c/2)^6*tan(d*x/2)^2+6*b*pi*tan(c/2)^6+18*b*pi*tan(
c/2)^4*tan(d*x/2)^6+54*b*pi*tan(c/2)^4*tan(d*x/2)^4+54*b*pi*tan(c/2)^4*tan(
d*x/2)^2+18*b*pi*tan(c/2)^4+18*b*pi*tan(c/2)^2*tan(d*x/2)^6+54*b*pi*tan(c/2
)^2*tan(d*x/2)^4+54*b*pi*tan(c/2)^2*tan(d*x/2)^2+18*b*pi*tan(c/2)^2+6*b*pi*
tan(d*x/2)^6+18*b*pi*tan(d*x/2)^4+18*b*pi*tan(d*x/2)^2+6*b*pi+6*b*atan((tan
(c/2)*tan(d*x/2)+tan(c/2)+tan(d*x/2)-1)/(tan(c/2)*tan(d*x/2)-tan(c/2)-tan(d
*x/2)-1))*tan(c/2)^6*tan(d*x/2)^6+18*b*atan((tan(c/2)*tan(d*x/2)+tan(c/2)+t
an(d*x/2)-1)/(tan(c/2)*tan(d*x/2)-tan(c/2)-tan(d*x/2)-1))*tan(c/2)^6*tan(d*
x/2)^4+18*b*atan((tan(c/2)*tan(d*x/2)+tan(c/2)+tan(d*x/2)-1)/(tan(c/2)*tan(
d*x/2)-tan(c/2)-tan(d*x/2)-1))*tan(c/2)^6*tan(d*x/2)^2+6*b*atan((tan(c/2)*t
an(d*x/2)+tan(c/2)+tan(d*x/2)-1)/(tan(c/2)*tan(d*x/2)-tan(c/2)-tan(d*x/2)-1
))*tan(c/2)^6+18*b*atan((tan(c/2)*tan(d*x/2)+tan(c/2)+tan(d*x/2)-1)/(tan(c/
2)*tan(d*x/2)-tan(c/2)-tan(d*x/2)-1))*tan(c/2)^4*tan(d*x/2)^6+54*b*atan((ta
n(c/2)*tan(d*x/2)+tan(c/2)+tan(d*x/2)-1)/(tan(c/2)*tan(d*x/2)-tan(c/2)-tan(
d*x/2)-1))*tan(c/2)^4*tan(d*x/2)^4+54*b*atan((tan(c/2)*tan(d*x/2)+tan(c/2)+
tan(d*x/2)-1)/(tan(c/2)*tan(d*x/2)-tan(c/2)-tan(d*x/2)-1))*tan(c/2)^4*tan(d
*x/2)^2+18*b*atan((tan(c/2)*tan(d*x/2)+tan(c/2)+tan(d*x/2)-1)/(tan(c/2)*tan
(d*x/2)-tan(c/2)-tan(d*x/2)-1))*tan(c/2)^4+18*b*atan((tan(c/2)*tan(d*x/2)+t
an(c/2)+tan(d*x/2)-1)/(tan(c/2)*tan(d*x/2)-tan(c/2)-tan(d*x/2)-1))*tan(c/2)
^2*tan(d*x/2)^6+54*b*atan((tan(c/2)*tan(d*x/2)+tan(c/2)+tan(d*x/2)-1)/(tan(
c/2)*tan(d*x/2)-tan(c/2)-tan(d*x/2)-1))*tan(c/2)^2*tan(d*x/2)^4+54*b*atan((
tan(c/2)*tan(d*x/2)+tan(c/2)+tan(d*x/2)-1)/(tan(c/2)*tan(d*x/2)-tan(c/2)-ta
n(d*x/2)-1))*tan(c/2)^2*tan(d*x/2)^2+18*b*atan((tan(c/2)*tan(d*x/2)+tan(c/2
)+tan(d*x/2)-1)/(tan(c/2)*tan(d*x/2)-tan(c/2)-tan(d*x/2)-1))*tan(c/2)^2+6*b
*atan((tan(c/2)*tan(d*x/2)+tan(c/2)+tan(d*x/2)-1)/(tan(c/2)*tan(d*x/2)-tan(
c/2)-tan(d*x/2)-1))*tan(d*x/2)^6+18*b*atan((tan(c/2)*tan(d*x/2)+tan(c/2)+ta
n(d*x/2)-1)/(tan(c/2)*tan(d*x/2)-tan(c/2)-tan(d*x/2)-1))*tan(d*x/2)^4+18*b*
atan((tan(c/2)*tan(d*x/2)+tan(c/2)+tan(d*x/2)-1)/(tan(c/2)*tan(d*x/2)-tan(c
/2)-tan(d*x/2)-1))*tan(d*x/2)^2+6*b*atan((tan(c/2)*tan(d*x/2)+tan(c/2)+tan(
d*x/2)-1)/(tan(c/2)*tan(d*x/2)-tan(c/2)-tan(d*x/2)-1))-6*b*atan((tan(c/2)*t
an(d*x/2)+tan(c/2)-tan(d*x/2)+1)/(tan(c/2)*tan(d*x/2)-tan(c/2)+tan(d*x/2)+1
))*tan(c/2)^6*tan(d*x/2)^6-18*b*atan((tan(c/2)*tan(d*x/2)+tan(c/2)-tan(d*x/
2)+1)/(tan(c/2)*tan(d*x/2)-tan(c/2)+tan(d*x/2)+1))*tan(c/2)^6*tan(d*x/2)^4-
18*b*atan((tan(c/2)*tan(d*x/2)+tan(c/2)-tan(d*x/2)+1)/(tan(c/2)*tan(d*x/2)-
tan(c/2)+tan(d*x/2)+1))*tan(c/2)^6*tan(d*x/2)^2-6*b*atan((tan(c/2)*tan(d*x/
2)+tan(c/2)-tan(d*x/2)+1)/(tan(c/2)*tan(d*x/2)-tan(c/2)+tan(d*x/2)+1))*tan(
c/2)^6-18*b*atan((tan(c/2)*tan(d*x/2)+tan(c/2)-tan(d*x/2)+1)/(tan(c/2)*tan(
d*x/2)-tan(c/2)+tan(d*x/2)+1))*tan(c/2)^4*tan(d*x/2)^6-54*b*atan((tan(c/2)*
tan(d*x/2)+tan(c/2)-tan(d*x/2)+1)/(tan(c/2)*tan(d*x/2)-tan(c/2)+tan(d*x/2)+
1))*tan(c/2)^4*tan(d*x/2)^4-54*b*atan((tan(c/2)*tan(d*x/2)+tan(c/2)-tan(d*x
/2)+1)/(tan(c/2)*tan(d*x/2)-tan(c/2)+tan(d*x/2)+1))*tan(c/2)^4*tan(d*x/2)^2
-18*b*atan((tan(c/2)*tan(d*x/2)+tan(c/2)-tan(d*x/2)+1)/(tan(c/2)*tan(d*x/2)
-tan(c/2)+tan(d*x/2)+1))*tan(c/2)^4-18*b*atan((tan(c/2)*tan(d*x/2)+tan(c/2)

-tan(d*x/2)+1)/(tan(c/2)*tan(d*x/2)-tan(c/2)+tan(d*x/2)+1))*tan(c/2)^2*tan(
d*x/2)^6-54*b*atan((tan(c/2)*tan(d*x/2)+tan(c/2)-tan(d*x/2)+1)/(tan(c/2)*ta
n(d*x/2)-tan(c/2)+tan(d*x/2)+1))*tan(c/2)^2*tan(d*x/2)^4-54*b*atan((tan(c/2
)*tan(d*x/2)+tan(c/2)-tan(d*x/2)+1)/(tan(c/2)*tan(d*x/2)-tan(c/2)+tan(d*x/2
)+1))*tan(c/2)^2*tan(d*x/2)^2-18*b*atan((tan(c/2)*tan(d*x/2)+tan(c/2)-tan(d
*x/2)+1)/(tan(c/2)*tan(d*x/2)-tan(c/2)+tan(d*x/2)+1))*tan(c/2)^2-6*b*atan((
tan(c/2)*tan(d*x/2)+tan(c/2)-tan(d*x/2)+1)/(tan(c/2)*tan(d*x/2)-tan(c/2)+ta
n(d*x/2)+1))*tan(d*x/2)^6-18*b*atan((tan(c/2)*tan(d*x/2)+tan(c/2)-tan(d*x/2
)+1)/(tan(c/2)*tan(d*x/2)-tan(c/2)+tan(d*x/2)+1))*tan(d*x/2)^4-18*b*atan((t
an(c/2)*tan(d*x/2)+tan(c/2)-tan(d*x/2)+1)/(tan(c/2)*tan(d*x/2)-tan(c/2)+tan
(d*x/2)+1))*tan(d*x/2)^2-6*b*atan((tan(c/2)*tan(d*x/2)+tan(c/2)-tan(d*x/2)+
1)/(tan(c/2)*tan(d*x/2)-tan(c/2)+tan(d*x/2)+1))-6*b*atan((tan(c/2)*tan(d*x/
2)-tan(c/2)+tan(d*x/2)+1)/(tan(c/2)*tan(d*x/2)+tan(c/2)-tan(d*x/2)+1))*tan(
c/2)^6*tan(d*x/2)^6-18*b*atan((tan(c/2)*tan(d*x/2)-tan(c/2)+tan(d*x/2)+1)/(
tan(c/2)*tan(d*x/2)+tan(c/2)-tan(d*x/2)+1))*tan(c/2)^6*tan(d*x/2)^4-18*b*at
an((tan(c/2)*tan(d*x/2)-tan(c/2)+tan(d*x/2)+1)/(tan(c/2)*tan(d*x/2)+tan(c/2
)-tan(d*x/2)+1))*tan(c/2)^6*tan(d*x/2)^2-6*b*atan((tan(c/2)*tan(d*x/2)-tan(
c/2)+tan(d*x/2)+1)/(tan(c/2)*tan(d*x/2)+tan(c/2)-tan(d*x/2)+1))*tan(c/2)^6-
18*b*atan((tan(c/2)*tan(d*x/2)-tan(c/2)+tan(d*x/2)+1)/(tan(c/2)*tan(d*x/2)+
tan(c/2)-tan(d*x/2)+1))*tan(c/2)^4*tan(d*x/2)^6-54*b*atan((tan(c/2)*tan(d*x
/2)-tan(c/2)+tan(d*x/2)+1)/(tan(c/2)*tan(d*x/2)+tan(c/2)-tan(d*x/2)+1))*tan
(c/2)^4*tan(d*x/2)^4-54*b*atan((tan(c/2)*tan(d*x/2)-tan(c/2)+tan(d*x/2)+1)/
(tan(c/2)*tan(d*x/2)+tan(c/2)-tan(d*x/2)+1))*tan(c/2)^4*tan(d*x/2)^2-18*b*a
tan((tan(c/2)*tan(d*x/2)-tan(c/2)+tan(d*x/2)+1)/(tan(c/2)*tan(d*x/2)+tan(c/
2)-tan(d*x/2)+1))*tan(c/2)^4-18*b*atan((tan(c/2)*tan(d*x/2)-tan(c/2)+tan(d*
x/2)+1)/(tan(c/2)*tan(d*x/2)+tan(c/2)-tan(d*x/2)+1))*tan(c/2)^2*tan(d*x/2)^
6-54*b*atan((tan(c/2)*tan(d*x/2)-tan(c/2)+tan(d*x/2)+1)/(tan(c/2)*tan(d*x/2
)+tan(c/2)-tan(d*x/2)+1))*tan(c/2)^2*tan(d*x/2)^4-54*b*atan((tan(c/2)*tan(d
*x/2)-tan(c/2)+tan(d*x/2)+1)/(tan(c/2)*tan(d*x/2)+tan(c/2)-tan(d*x/2)+1))*t
an(c/2)^2*tan(d*x/2)^2-18*b*atan((tan(c/2)*tan(d*x/2)-tan(c/2)+tan(d*x/2)+1
)/(tan(c/2)*tan(d*x/2)+tan(c/2)-tan(d*x/2)+1))*tan(c/2)^2-6*b*atan((tan(c/2
)*tan(d*x/2)-tan(c/2)+tan(d*x/2)+1)/(tan(c/2)*tan(d*x/2)+tan(c/2)-tan(d*x/2
)+1))*tan(d*x/2)^6-18*b*atan((tan(c/2)*tan(d*x/2)-tan(c/2)+tan(d*x/2)+1)/(t
an(c/2)*tan(d*x/2)+tan(c/2)-tan(d*x/2)+1))*tan(d*x/2)^4-18*b*atan((tan(c/2)
*tan(d*x/2)-tan(c/2)+tan(d*x/2)+1)/(tan(c/2)*tan(d*x/2)+tan(c/2)-tan(d*x/2)
+1))*tan(d*x/2)^2-6*b*atan((tan(c/2)*tan(d*x/2)-tan(c/2)+tan(d*x/2)+1)/(tan
(c/2)*tan(d*x/2)+tan(c/2)-tan(d*x/2)+1))+6*b*atan((tan(c/2)*tan(d*x/2)-tan(
c/2)-tan(d*x/2)-1)/(tan(c/2)*tan(d*x/2)+tan(c/2)+tan(d*x/2)-1))*tan(c/2)^6*
tan(d*x/2)^6+18*b*atan((tan(c/2)*tan(d*x/2)-tan(c/2)-tan(d*x/2)-1)/(tan(c/2
)*tan(d*x/2)+tan(c/2)+tan(d*x/2)-1))*tan(c/2)^6*tan(d*x/2)^4+18*b*atan((tan
(c/2)*tan(d*x/2)-tan(c/2)-tan(d*x/2)-1)/(tan(c/2)*tan(d*x/2)+tan(c/2)+tan(d
*x/2)-1))*tan(c/2)^6*tan(d*x/2)^2+6*b*atan((tan(c/2)*tan(d*x/2)-tan(c/2)-ta
n(d*x/2)-1)/(tan(c/2)*tan(d*x/2)+tan(c/2)+tan(d*x/2)-1))*tan(c/2)^6+18*b*at
an((tan(c/2)*tan(d*x/2)-tan(c/2)-tan(d*x/2)-1)/(tan(c/2)*tan(d*x/2)+tan(c/2
)+tan(d*x/2)-1))*tan(c/2)^4*tan(d*x/2)^6+54*b*atan((tan(c/2)*tan(d*x/2)-tan

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(c/2)-tan(d*x/2)-1)/(tan(c/2)*tan(d*x/2)+tan(c/2)+tan(d*x/2)-1))*tan(c/2)^4
*tan(d*x/2)^4+54*b*atan((tan(c/2)*tan(d*x/2)-tan(c/2)-tan(d*x/2)-1)/(tan(c/
2)*tan(d*x/2)+tan(c/2)+tan(d*x/2)-1))*tan(c/2)^4*tan(d*x/2)^2+18*b*atan((ta
n(c/2)*tan(d*x/2)-tan(c/2)-tan(d*x/2)-1)/(tan(c/2)*tan(d*x/2)+tan(c/2)+tan(
d*x/2)-1))*tan(c/2)^4+18*b*atan((tan(c/2)*tan(d*x/2)-tan(c/2)-tan(d*x/2)-1)
/(tan(c/2)*tan(d*x/2)+tan(c/2)+tan(d*x/2)-1))*tan(c/2)^2*tan(d*x/2)^6+54*b*
atan((tan(c/2)*tan(d*x/2)-tan(c/2)-tan(d*x/2)-1)/(tan(c/2)*tan(d*x/2)+tan(c
/2)+tan(d*x/2)-1))*tan(c/2)^2*tan(d*x/2)^4+54*b*atan((tan(c/2)*tan(d*x/2)-t
an(c/2)-tan(d*x/2)-1)/(tan(c/2)*tan(d*x/2)+tan(c/2)+tan(d*x/2)-1))*tan(c/2)
^2*tan(d*x/2)^2+18*b*atan((tan(c/2)*tan(d*x/2)-tan(c/2)-tan(d*x/2)-1)/(tan(
c/2)*tan(d*x/2)+tan(c/2)+tan(d*x/2)-1))*tan(c/2)^2+6*b*atan((tan(c/2)*tan(d
*x/2)-tan(c/2)-tan(d*x/2)-1)/(tan(c/2)*tan(d*x/2)+tan(c/2)+tan(d*x/2)-1))*t
an(d*x/2)^6+18*b*atan((tan(c/2)*tan(d*x/2)-tan(c/2)-tan(d*x/2)-1)/(tan(c/2)
*tan(d*x/2)+tan(c/2)+tan(d*x/2)-1))*tan(d*x/2)^4+18*b*atan((tan(c/2)*tan(d*
x/2)-tan(c/2)-tan(d*x/2)-1)/(tan(c/2)*tan(d*x/2)+tan(c/2)+tan(d*x/2)-1))*ta
n(d*x/2)^2+6*b*atan((tan(c/2)*tan(d*x/2)-tan(c/2)-tan(d*x/2)-1)/(tan(c/2)*t
an(d*x/2)+tan(c/2)+tan(d*x/2)-1))-32*b*tan(c/2)^6*tan(d*x/2)^6+96*b*tan(c/2
)^6*tan(d*x/2)^4-96*b*tan(c/2)^6*tan(d*x/2)^2+32*b*tan(c/2)^6+384*b*tan(c/2
)^5*tan(d*x/2)^5-768*b*tan(c/2)^5*tan(d*x/2)^3+384*b*tan(c/2)^5*tan(d*x/2)+
96*b*tan(c/2)^4*tan(d*x/2)^6-1824*b*tan(c/2)^4*tan(d*x/2)^4+1824*b*tan(c/2)
^4*tan(d*x/2)^2-96*b*tan(c/2)^4-768*b*tan(c/2)^3*tan(d*x/2)^5+3584*b*tan(c/
2)^3*tan(d*x/2)^3-768*b*tan(c/2)^3*tan(d*x/2)-96*b*tan(c/2)^2*tan(d*x/2)^6+
1824*b*tan(c/2)^2*tan(d*x/2)^4-1824*b*tan(c/2)^2*tan(d*x/2)^2+96*b*tan(c/2)
^2+384*b*tan(c/2)*tan(d*x/2)^5-768*b*tan(c/2)*tan(d*x/2)^3+384*b*tan(c/2)*t
an(d*x/2)+32*b*tan(d*x/2)^6-96*b*tan(d*x/2)^4+96*b*tan(d*x/2)^2-32*b)/(96*d
*tan(c/2)^6*tan(d*x/2)^6+288*d*tan(c/2)^6*tan(d*x/2)^4+288*d*tan(c/2)^6*tan
(d*x/2)^2+96*d*tan(c/2)^6+288*d*tan(c/2)^4*tan(d*x/2)^6+864*d*tan(c/2)^4*ta
n(d*x/2)^4+864*d*tan(c/2)^4*tan(d*x/2)^2+288*d*tan(c/2)^4+288*d*tan(c/2)^2*
tan(d*x/2)^6+864*d*tan(c/2)^2*tan(d*x/2)^4+864*d*tan(c/2)^2*tan(d*x/2)^2+28
8*d*tan(c/2)^2+96*d*tan(d*x/2)^6+288*d*tan(d*x/2)^4+288*d*tan(d*x/2)^2+96*d
)
```

maple [A] time = 0.05, size = 29, normalized size = 0.88

$$-\frac{\frac{a\left(\cos^{4}(dx+c)\right)}{4}+\frac{b\left(\cos^{3}(dx+c)\right)}{3}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3*(a*sin(d*x+c)+b*tan(d*x+c)),x)
```

```
[Out] -1/d*(1/4*a*cos(d*x+c)^4+1/3*b*cos(d*x+c)^3)
```

maxima [A] time = 0.34, size = 28, normalized size = 0.85

$$-\frac{3\,a\cos\left(dx+c\right)^{4}+4\,b\cos\left(dx+c\right)^{3}}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a*sin(d*x+c)+b*tan(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/12*(3*a*cos(d*x + c)^4 + 4*b*cos(d*x + c)^3)/d
```

mupad [B] time = 0.68, size = 29, normalized size = 0.88

$$-\frac{a\cos\left(c+d\,x\right)^4}{4\,d}-\frac{b\cos\left(c+d\,x\right)^3}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^3*(a*sin(c + d*x) + b*tan(c + d*x)),x)
```

```
[Out] - (a*cos(c + d*x)^4)/(4*d) - (b*cos(c + d*x)^3)/(3*d)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a\sin\left(c+dx\right)+b\tan\left(c+dx\right)\right)\cos^3\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3*(a*sin(d*x+c)+b*tan(d*x+c)),x)
```

```
[Out] Integral((a*sin(c + d*x) + b*tan(c + d*x))*cos(c + d*x)**3, x)
```

3.230 $\int \cos^2(c+dx)(a\sin(c+dx)+b\tan(c+dx))\,dx$

Optimal. Leaf size=33

$$\frac{b\sin^2(c+dx)}{2d}-\frac{a\cos^3(c+dx)}{3d}$$

```
[Out] -1/3*a*cos(d*x+c)^3/d+1/2*b*sin(d*x+c)^2/d
```

Rubi [A] time = 0.05, antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {4377, 12, 2564, 30, 2565}

$$\frac{b\sin^2(c+dx)}{2d}-\frac{a\cos^3(c+dx)}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2*(a*Sin[c + d*x] + b*Tan[c + d*x]),x]
```

```
[Out] -(a*Cos[c + d*x]^3)/(3*d) + (b*Sin[c + d*x]^2)/(2*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2564

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 2565

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> -Dist[(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 4377

```
Int[(u_)*((v_) + (d_.)*(F_)[(c_.)*((a_.) + (b_.)*(x_))]^(n_.)), x_Symbol] :
> With[{e = FreeFactors[Cos[c*(a + b*x)], x]}, Int[ActivateTrig[u*v], x] +
Dist[d, Int[ActivateTrig[u]*Sin[c*(a + b*x)]^n, x], x] /; FunctionOfQ[Cos[c
*(a + b*x)]/e, u, x]] /; FreeQ[{a, b, c, d}, x] && !FreeQ[v, x] && Integer
Q[(n - 1)/2] && NonsumQ[u] && (EqQ[F, Sin] || EqQ[F, sin])
```

Rubi steps

$$\begin{aligned}\int \cos^2(c+dx)(a\sin(c+dx)+b\tan(c+dx))\,dx &= a\int \cos^2(c+dx)\sin(c+dx)\,dx+\int b\cos(c+dx)\sin(c+dx \\ &= b\int \cos(c+dx)\sin(c+dx)\,dx-\frac{a\,\mathrm{Subst}\left(\int x^2\,dx,x,\cos(c+\right.}{d} \\ &= -\frac{a\cos^3(c+dx)}{3d}+\frac{b\,\mathrm{Subst}(\int x\,dx,x,\sin(c+dx))}{d} \\ &= -\frac{a\cos^3(c+dx)}{3d}+\frac{b\sin^2(c+dx)}{2d}\end{aligned}$$

Mathematica [A] time = 0.12, size = 38, normalized size = 1.15

$$-\frac{3a\cos(c+dx)+a\cos(3(c+dx))+3b\cos(2(c+dx))}{12d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2*(a*Sin[c + d*x] + b*Tan[c + d*x]),x]
[Out] -1/12*(3*a*Cos[c + d*x] + 3*b*Cos[2*(c + d*x)] + a*Cos[3*(c + d*x)])/d
```

fricas [A] time = 0.49, size = 28, normalized size = 0.85

$$-\frac{2\,a\cos\left(dx+c\right)^3+3\,b\cos\left(dx+c\right)^2}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a*sin(d*x+c)+b*tan(d*x+c)),x, algorithm="fricas")
[Out] -1/6*(2*a*cos(d*x + c)^3 + 3*b*cos(d*x + c)^2)/d
```

giac [B] time = 3.26, size = 99, normalized size = 3.00

$$-\frac{a\cos\left(3\,dx+3\,c\right)}{12\,d}-\frac{a\cos\left(dx+c\right)}{4\,d}-\frac{b\tan\left(dx\right)^2\tan(c)^2-b\tan\left(dx\right)^2-4\,b\tan\left(dx\right)\tan(c)-b\tan(c)^2+b}{4\left(d\tan\left(dx\right)^2\tan(c)^2+d\tan\left(dx\right)^2+d\tan(c)^2+d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a*sin(d*x+c)+b*tan(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/12*a*cos(3*d*x + 3*c)/d - 1/4*a*cos(d*x + c)/d - 1/4*(b*tan(d*x)^2*tan(c
)^2 - b*tan(d*x)^2 - 4*b*tan(d*x)*tan(c) - b*tan(c)^2 + b)/(d*tan(d*x)^2*ta
n(c)^2 + d*tan(d*x)^2 + d*tan(c)^2 + d)
```

maple [A] time = 0.04, size = 29, normalized size = 0.88

$$-\frac{\frac{a\left(\cos^{3}(dx+c)\right)}{3}+\frac{b\left(\cos^{2}(dx+c)\right)}{2}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(a*sin(d*x+c)+b*tan(d*x+c)),x)
```

```
[Out] -1/d*(1/3*a*cos(d*x+c)^3+1/2*b*cos(d*x+c)^2)
```

maxima [A] time = 0.34, size = 28, normalized size = 0.85

$$-\frac{2\,a\cos\left(dx+c\right)^{3}-3\,b\sin\left(dx+c\right)^{2}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a*sin(d*x+c)+b*tan(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/6*(2*a*cos(d*x + c)^3 - 3*b*sin(d*x + c)^2)/d
```

mupad [B] time = 0.64, size = 49, normalized size = 1.48

$$-\frac{\left(\cos\left(c+d\,x\right)+1\right)\,\left(2\,a-3\,b-2\,a\,\cos\left(c+d\,x\right)+3\,b\,\cos\left(c+d\,x\right)+2\,a\,\cos\left(c+d\,x\right)^{2}\right)}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^2*(a*sin(c + d*x) + b*tan(c + d*x)),x)
```

```
[Out] -((cos(c + d*x) + 1)*(2*a - 3*b - 2*a*cos(c + d*x) + 3*b*cos(c + d*x) + 2*a
*cos(c + d*x)^2))/(6*d)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a\sin\left(c+dx\right)+b\tan\left(c+dx\right)\right)\cos^{2}\left(c+dx\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(a*sin(d*x+c)+b*tan(d*x+c)),x)
```

```
[Out] Integral((a*sin(c + d*x) + b*tan(c + d*x))*cos(c + d*x)**2, x)
```

3.231 $\int \cos(c+dx)(a\sin(c+dx)+b\tan(c+dx))\,dx$

Optimal. Leaf size=22

$$-\frac{(a\cos(c+dx)+b)^2}{2ad}$$

[Out] -1/2*(b+a*cos(d*x+c))^2/a/d

Rubi [A] time = 0.03, antiderivative size = 29, normalized size of antiderivative = 1.32, number of steps used = 5, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {4377, 12, 2638, 2564, 30}

$$\frac{a\sin^2(c+dx)}{2d}-\frac{b\cos(c+dx)}{d}$$

Antiderivative was successfully verified.

[In] Int[Cos[c + d*x]*(a*Sin[c + d*x] + b*Tan[c + d*x]),x]

[Out] -((b*Cos[c + d*x])/d) + (a*Sin[c + d*x]^2)/(2*d)

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2564

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 4377

```
Int[(u_)*((v_) + (d_.)*(F_)[(c_.)*((a_.) + (b_.)*(x_))]^(n_.)), x_Symbol] :
> With[{e = FreeFactors[Cos[c*(a + b*x)], x]}, Int[ActivateTrig[u*v], x] +
```

```
Dist[d, Int[ActivateTrig[u]*Sin[c*(a + b*x)]^n, x], x] /; FunctionOfQ[Cos[c
*(a + b*x)]/e, u, x]] /; FreeQ[{a, b, c, d}, x] && !FreeQ[v, x] && Integer
Q[(n - 1)/2] && NonsumQ[u] && (EqQ[F, Sin] || EqQ[F, sin])
```

Rubi steps

$$\begin{aligned}\int \cos(c+dx)(a\sin(c+dx)+b\tan(c+dx))\,dx &= a\int \cos(c+dx)\sin(c+dx)\,dx+\int b\sin(c+dx)\,dx\\ &= b\int \sin(c+dx)\,dx+\frac{a\,\text{Subst}\left(\int x\,dx,x,\sin(c+dx)\right)}{d}\\ &= -\frac{b\cos(c+dx)}{d}+\frac{a\sin^2(c+dx)}{2d}\end{aligned}$$

Mathematica [A] time = 0.01, size = 40, normalized size = 1.82

$$-\frac{a\cos^2(c+dx)}{2d}+\frac{b\sin(c)\sin(dx)}{d}-\frac{b\cos(c)\cos(dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]*(a*Sin[c + d*x] + b*Tan[c + d*x]),x]
[Out] -((b*Cos[c]*Cos[d*x])/d) - (a*Cos[c + d*x]^2)/(2*d) + (b*Sin[c]*Sin[d*x])/d
```

fricas [A] time = 0.41, size = 25, normalized size = 1.14

$$-\frac{a\cos\left(dx+c\right)^2+2\,b\cos\left(dx+c\right)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a*sin(d*x+c)+b*tan(d*x+c)),x, algorithm="fricas")
[Out] -1/2*(a*cos(d*x + c)^2 + 2*b*cos(d*x + c))/d
```

giac [B] time = 0.27, size = 102, normalized size = 4.64

$$-\frac{a\cos\left(2\,dx+2\,c\right)}{4\,d}-\frac{b\tan\left(\frac{1}{2}\,dx\right)^2\tan\left(\frac{1}{2}\,c\right)^2-b\tan\left(\frac{1}{2}\,dx\right)^2-4\,b\tan\left(\frac{1}{2}\,dx\right)\tan\left(\frac{1}{2}\,c\right)-b\tan\left(\frac{1}{2}\,c\right)^2+b}{d\tan\left(\frac{1}{2}\,dx\right)^2\tan\left(\frac{1}{2}\,c\right)^2+d\tan\left(\frac{1}{2}\,dx\right)^2+d\tan\left(\frac{1}{2}\,c\right)^2+d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a*sin(d*x+c)+b*tan(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/4*a*cos(2*d*x + 2*c)/d - (b*tan(1/2*d*x)^2*tan(1/2*c)^2 - b*tan(1/2*d*x)
^2 - 4*b*tan(1/2*d*x)*tan(1/2*c) - b*tan(1/2*c)^2 + b)/(d*tan(1/2*d*x)^2*ta
n(1/2*c)^2 + d*tan(1/2*d*x)^2 + d*tan(1/2*c)^2 + d)
```

maple [A] time = 0.03, size = 26, normalized size = 1.18

$$-\frac{\frac{\left(\cos^2(dx+c)\right)a}{2}+b\cos\left(dx+c\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(a*sin(d*x+c)+b*tan(d*x+c)),x)
```

```
[Out] -1/d*(1/2*cos(d*x+c)^2*a+b*cos(d*x+c))
```

maxima [A] time = 0.41, size = 25, normalized size = 1.14

$$-\frac{a\cos\left(dx+c\right)^2+2\,b\cos\left(dx+c\right)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a*sin(d*x+c)+b*tan(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/2*(a*cos(d*x + c)^2 + 2*b*cos(d*x + c))/d
```

mupad [B] time = 0.63, size = 28, normalized size = 1.27

$$-\frac{\left(\cos\left(c+d\,x\right)+1\right)\,\left(2\,b-a+a\,\cos\left(c+d\,x\right)\right)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)*(a*sin(c + d*x) + b*tan(c + d*x)),x)
```

```
[Out] -((cos(c + d*x) + 1)*(2*b - a + a*cos(c + d*x)))/(2*d)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a\sin\left(c+dx\right)+b\tan\left(c+dx\right)\right)\cos\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a*sin(d*x+c)+b*tan(d*x+c)),x)
```

```
[Out] Integral((a*sin(c + d*x) + b*tan(c + d*x))*cos(c + d*x), x)
```

3.232 $\int (a\sin(c+dx)+b\tan(c+dx))\,dx$

Optimal. Leaf size=26

$$-\frac{a\cos(c+dx)}{d}-\frac{b\log(\cos(c+dx))}{d}$$

```
[Out] -a*cos(d*x+c)/d-b*ln(cos(d*x+c))/d
```

Rubi [A] time = 0.01, antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {2638, 3475}

$$-\frac{a\cos(c+dx)}{d}-\frac{b\log(\cos(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[a*Sin[c + d*x] + b*Tan[c + d*x],x]
```

```
[Out] -((a*Cos[c + d*x])/d) - (b*Log[Cos[c + d*x]])/d
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int (a\sin(c+dx)+b\tan(c+dx))\,dx &= a\int \sin(c+dx)\,dx+b\int \tan(c+dx)\,dx\\ &= -\frac{a\cos(c+dx)}{d}-\frac{b\log(\cos(c+dx))}{d}\end{aligned}$$

Mathematica [A] time = 0.02, size = 37, normalized size = 1.42

$$\frac{a\sin(c)\sin(dx)}{d}-\frac{a\cos(c)\cos(dx)}{d}-\frac{b\log(\cos(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[a*Sin[c + d*x] + b*Tan[c + d*x],x]
[Out] -((a*Cos[c]*Cos[d*x])/d) - (b*Log[Cos[c + d*x]])/d + (a*Sin[c]*Sin[d*x])/d
```

fricas [A] time = 0.42, size = 25, normalized size = 0.96

$$-\frac{a\cos(dx+c)+b\log(-\cos(dx+c))}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a*sin(d*x+c)+b*tan(d*x+c),x, algorithm="fricas")
[Out] -(a*cos(d*x + c) + b*log(-cos(d*x + c)))/d
```

giac [A] time = 0.16, size = 27, normalized size = 1.04

$$-\frac{a\cos(dx+c)}{d}-\frac{b\log(|\cos(dx+c)|)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a*sin(d*x+c)+b*tan(d*x+c),x, algorithm="giac")
[Out] -a*cos(d*x + c)/d - b*log(abs(cos(d*x + c)))/d
```

maple [A] time = 0.01, size = 31, normalized size = 1.19

$$\frac{b\ln\left(\tan^2(dx+c)+1\right)}{2d}-\frac{a\cos(dx+c)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(a*sin(d*x+c)+b*tan(d*x+c),x)
[Out] 1/2*b/d*ln(tan(d*x+c)^2+1)-a*cos(d*x+c)/d
```

maxima [A] time = 0.35, size = 25, normalized size = 0.96

$$-\frac{a\cos(dx+c)}{d}+\frac{b\log(\sec(dx+c))}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a*sin(d*x+c)+b*tan(d*x+c),x, algorithm="maxima")
[Out] -a*cos(d*x + c)/d + b*log(sec(d*x + c))/d
```

mupad [B] time = 0.65, size = 40, normalized size = 1.54

$$\frac{2\,b\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\right)}{d}-\frac{2\,a}{d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(a*sin(c + d*x) + b*tan(c + d*x),x)
```

```
[Out] (2*b*atanh(tan(c/2 + (d*x)/2)^2))/d - (2*a)/(d*(tan(c/2 + (d*x)/2)^2 + 1))
```

sympy [A] time = 0.16, size = 37, normalized size = 1.42

$$a\left(\begin{cases}-\frac{\cos\left(c+dx\right)}{d} & \text{for}\ d\neq 0\\ x\sin\left(c\right) & \text{otherwise}\end{cases}\right)+b\left(\begin{cases}\frac{\log\left(\tan^2\left(c+dx\right)+1\right)}{2d} & \text{for}\ d\neq 0\\ x\tan\left(c\right) & \text{otherwise}\end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a*sin(d*x+c)+b*tan(d*x+c),x)
```

```
[Out] a*Piecewise((-cos(c + d*x)/d, Ne(d, 0)), (x*sin(c), True)) + b*Piecewise((l
og(tan(c + d*x)**2 + 1)/(2*d), Ne(d, 0)), (x*tan(c), True))
```

3.233 $\int \sec(c+dx)(a\sin(c+dx)+b\tan(c+dx))\,dx$

Optimal. Leaf size=25

$$\frac{b\sec(c+dx)}{d}-\frac{a\log(\cos(c+dx))}{d}$$

```
[Out] -a*ln(cos(d*x+c))/d+b*sec(d*x+c)/d
```

Rubi [A] time = 0.03, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {4377, 12, 2606, 8, 3475}

$$\frac{b\sec(c+dx)}{d}-\frac{a\log(\cos(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]*(a*Sin[c + d*x] + b*Tan[c + d*x]),x]
```

```
[Out] -((a*Log[Cos[c + d*x]])/d) + (b*Sec[c + d*x])/d
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 4377

```
Int[(u_)*((v_) + (d_.)*(F_)[(c_.)*((a_.) + (b_.)*(x_))]^(n_.)), x_Symbol] :
> With[{e = FreeFactors[Cos[c*(a + b*x)], x]}, Int[ActivateTrig[u*v], x] +
```

```
Dist[d, Int[ActivateTrig[u]*Sin[c*(a + b*x)]^n, x], x] /; FunctionOfQ[Cos[c
*(a + b*x)]/e, u, x]] /; FreeQ[{a, b, c, d}, x] &&  !FreeQ[v, x] && Integer
Q[(n - 1)/2] && NonsumQ[u] && (EqQ[F, Sin] || EqQ[F, sin])
```

Rubi steps

$$\begin{aligned}
\int \sec(c+dx)(a\sin(c+dx)+b\tan(c+dx))\,dx &= a\int \tan(c+dx)\,dx+\int b\sec(c+dx)\tan(c+dx)\,dx \\
&= -\frac{a\log(\cos(c+dx))}{d}+b\int \sec(c+dx)\tan(c+dx)\,dx \\
&= -\frac{a\log(\cos(c+dx))}{d}+\frac{b\,\mathrm{Subst}(\int 1\,dx,x,\sec(c+dx))}{d} \\
&= -\frac{a\log(\cos(c+dx))}{d}+\frac{b\sec(c+dx)}{d}
\end{aligned}$$

Mathematica [A] time = 0.02, size = 25, normalized size = 1.00

$$\frac{b\sec(c+dx)}{d}-\frac{a\log(\cos(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]*(a*Sin[c + d*x] + b*Tan[c + d*x]),x]
[Out] -((a*Log[Cos[c + d*x]])/d) + (b*Sec[c + d*x])/d
```

fricas [A] time = 0.51, size = 34, normalized size = 1.36

$$-\frac{a\cos\left(dx+c\right)\log\left(-\cos\left(dx+c\right)\right)-b}{d\cos\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a*sin(d*x+c)+b*tan(d*x+c)),x, algorithm="fricas")
[Out] -(a*cos(d*x + c)*log(-cos(d*x + c)) - b)/(d*cos(d*x + c))
```

giac [B] time = 0.29, size = 107, normalized size = 4.28

$$\frac{a\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right|\right)-a\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}-1\right|\right)+\frac{a+2\,b+\frac{a(\cos(dx+c)-1)}{\cos(dx+c)+1}}{\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a*sin(d*x+c)+b*tan(d*x+c)),x, algorithm="giac")
```

```
[Out] (a*log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1)) - a*log(abs(-(cos(d
*x + c) - 1)/(cos(d*x + c) + 1) - 1)) + (a + 2*b + a*(cos(d*x + c) - 1)/(co
s(d*x + c) + 1))/((cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1))/d
```

maple [A] time = 0.04, size = 25, normalized size = 1.00

$$\frac{b\sec(dx+c)}{d}+\frac{a\ln(\sec(dx+c))}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)*(a*sin(d*x+c)+b*tan(d*x+c)),x)
```

```
[Out] b*sec(d*x+c)/d+1/d*a*ln(sec(d*x+c))
```

maxima [A] time = 0.33, size = 32, normalized size = 1.28

$$-\frac{a\log\left(-\sin(dx+c)^2+1\right)-\frac{2b}{\cos(dx+c)}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a*sin(d*x+c)+b*tan(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/2*(a*log(-sin(d*x + c)^2 + 1) - 2*b/cos(d*x + c))/d
```

mupad [B] time = 0.66, size = 40, normalized size = 1.60

$$\frac{2a\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2\right)}{d}-\frac{2b}{d\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*sin(c + d*x) + b*tan(c + d*x))/cos(c + d*x),x)
```

```
[Out] (2*a*atanh(tan(c/2 + (d*x)/2)^2))/d - (2*b)/(d*(tan(c/2 + (d*x)/2)^2 - 1))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int(a\sin(c+dx)+b\tan(c+dx))\sec(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a*sin(d*x+c)+b*tan(d*x+c)),x)
```

```
[Out] Integral((a*sin(c + d*x) + b*tan(c + d*x))*sec(c + d*x), x)
```

3.234 $\int \sec^2(c+dx)(a\sin(c+dx)+b\tan(c+dx))\,dx$

Optimal. Leaf size=28

$$\frac{a\sec(c+dx)}{d}+\frac{b\sec^2(c+dx)}{2d}$$

```
[Out] a*sec(d*x+c)/d+1/2*b*sec(d*x+c)^2/d
```

Rubi [A] time = 0.05, antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {4377, 12, 2606, 30, 8}

$$\frac{a\sec(c+dx)}{d}+\frac{b\sec^2(c+dx)}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^2*(a*Sin[c + d*x] + b*Tan[c + d*x]),x]
```

```
[Out] (a*Sec[c + d*x])/d + (b*Sec[c + d*x]^2)/(2*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 4377

```
Int[(u_)*((v_) + (d_.)*(F_)[(c_.)*((a_.) + (b_.)*(x_))]^(n_.)), x_Symbol] :
> With[{e = FreeFactors[Cos[c*(a + b*x)], x]}, Int[ActivateTrig[u*v], x] +
```

```
Dist[d, Int[ActivateTrig[u]*Sin[c*(a + b*x)]^n, x], x] /; FunctionOfQ[Cos[c
*(a + b*x)]/e, u, x]] /; FreeQ[{a, b, c, d}, x] &&  !FreeQ[v, x] && Integer
Q[(n - 1)/2] && NonsumQ[u] && (EqQ[F, Sin] || EqQ[F, sin])
```

Rubi steps

$$\int \sec^2(c+dx)(a\sin(c+dx)+b\tan(c+dx))\,dx = a\int \sec(c+dx)\tan(c+dx)\,dx + \int b\sec^2(c+dx)\tan(c+d$$

$$= b\int \sec^2(c+dx)\tan(c+dx)\,dx + \frac{a\,\text{Subst}(\int 1\,dx, x, \sec(c+}{d}$$

$$= \frac{a\sec(c+dx)}{d} + \frac{b\,\text{Subst}(\int x\,dx, x, \sec(c+dx))}{d}$$

$$= \frac{a\sec(c+dx)}{d} + \frac{b\sec^2(c+dx)}{2d}$$

Mathematica [A] time = 0.02, size = 28, normalized size = 1.00

$$\frac{a\sec(c+dx)}{d} + \frac{b\sec^2(c+dx)}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^2*(a*Sin[c + d*x] + b*Tan[c + d*x]),x]
[Out] (a*Sec[c + d*x])/d + (b*Sec[c + d*x]^2)/(2*d)
```

fricas [A] time = 0.62, size = 24, normalized size = 0.86

$$\frac{2\,a\cos(dx+c)+b}{2\,d\cos(dx+c)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a*sin(d*x+c)+b*tan(d*x+c)),x, algorithm="fricas")
[Out] 1/2*(2*a*cos(d*x + c) + b)/(d*cos(d*x + c)^2)
```

giac [B] time = 0.29, size = 71, normalized size = 2.54

$$\frac{2\left(a + \frac{a(\cos(dx+c)-1)}{\cos(dx+c)+1} - \frac{b(\cos(dx+c)-1)}{\cos(dx+c)+1}\right)}{d\left(\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a*sin(d*x+c)+b*tan(d*x+c)),x, algorithm="giac")
```

```
[Out] 2*(a + a*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - b*(cos(d*x + c) - 1)/(cos(
d*x + c) + 1))/(d*((cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1)^2)
```

maple [A] time = 0.04, size = 25, normalized size = 0.89

$$\frac{\frac{b\left(\sec^2(dx+c)\right)}{2} + a\sec(dx+c)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2*(a*sin(d*x+c)+b*tan(d*x+c)),x)
```

```
[Out] 1/d*(1/2*b*sec(d*x+c)^2+a*sec(d*x+c))
```

maxima [A] time = 0.33, size = 27, normalized size = 0.96

$$\frac{b\tan(dx+c)^2 + \frac{2a}{\cos(dx+c)}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a*sin(d*x+c)+b*tan(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/2*(b*tan(d*x + c)^2 + 2*a/cos(d*x + c))/d
```

mupad [B] time = 0.63, size = 24, normalized size = 0.86

$$\frac{\frac{b}{2} + a\,\cos(c+d\,x)}{d\cos(c+d\,x)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*sin(c + d*x) + b*tan(c + d*x))/cos(c + d*x)^2,x)
```

```
[Out] (b/2 + a*cos(c + d*x))/(d*cos(c + d*x)^2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a\sin(c+dx) + b\tan(c+dx))\sec^2(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2*(a*sin(d*x+c)+b*tan(d*x+c)),x)
```

```
[Out] Integral((a*sin(c + d*x) + b*tan(c + d*x))*sec(c + d*x)**2, x)
```

3.235 $\int \sec^3(c+dx)(a\sin(c+dx)+b\tan(c+dx))\,dx$

Optimal. Leaf size=33

$$\frac{a\sec^2(c+dx)}{2d}+\frac{b\sec^3(c+dx)}{3d}$$

```
[Out] 1/2*a*sec(d*x+c)^2/d+1/3*b*sec(d*x+c)^3/d
```

Rubi [A] time = 0.06, antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {4377, 12, 2606, 30}

$$\frac{a\sec^2(c+dx)}{2d}+\frac{b\sec^3(c+dx)}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^3*(a*Sin[c + d*x] + b*Tan[c + d*x]),x]
```

```
[Out] (a*Sec[c + d*x]^2)/(2*d) + (b*Sec[c + d*x]^3)/(3*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 4377

```
Int[(u_)*((v_) + (d_.)*(F_)[(c_.)*((a_.) + (b_.)*(x_))]^(n_.)), x_Symbol] :
> With[{e = FreeFactors[Cos[c*(a + b*x)], x]}, Int[ActivateTrig[u*v], x] +
Dist[d, Int[ActivateTrig[u]*Sin[c*(a + b*x)]^n, x], x] /; FunctionOfQ[Cos[c
*(a + b*x)]/e, u, x]] /; FreeQ[{a, b, c, d}, x] &&  !FreeQ[v, x] && Integer
Q[(n - 1)/2] && NonsumQ[u] && (EqQ[F, Sin] || EqQ[F, sin])
```

Rubi steps

$$\begin{aligned}\int \sec^3(c+dx)(a\sin(c+dx)+b\tan(c+dx))\,dx &= a\int \sec^2(c+dx)\tan(c+dx)\,dx+\int b\sec^3(c+dx)\tan(c+dx)\,dx\\ &= b\int \sec^3(c+dx)\tan(c+dx)\,dx+\frac{a\,\text{Subst}(\int x\,dx,x,\sec(c+dx))}{d}\\ &= \frac{a\sec^2(c+dx)}{2d}+\frac{b\,\text{Subst}\left(\int x^2\,dx,x,\sec(c+dx)\right)}{d}\\ &= \frac{a\sec^2(c+dx)}{2d}+\frac{b\sec^3(c+dx)}{3d}\end{aligned}$$

Mathematica [A] time = 0.02, size = 33, normalized size = 1.00

$$\frac{a\sec^2(c+dx)}{2d}+\frac{b\sec^3(c+dx)}{3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^3*(a*Sin[c + d*x] + b*Tan[c + d*x]),x]
[Out] (a*Sec[c + d*x]^2)/(2*d) + (b*Sec[c + d*x]^3)/(3*d)
```

fricas [A] time = 0.58, size = 26, normalized size = 0.79

$$\frac{3\,a\cos(dx+c)+2\,b}{6\,d\cos(dx+c)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a*sin(d*x+c)+b*tan(d*x+c)),x, algorithm="fricas")
[Out] 1/6*(3*a*cos(d*x + c) + 2*b)/(d*cos(d*x + c)^3)
```

giac [B] time = 0.35, size = 97, normalized size = 2.94

$$\frac{2\left(b-\frac{3\,a(\cos(dx+c)-1)}{\cos(dx+c)+1}-\frac{3\,a(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}+\frac{3\,b(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}\right)}{3\,d\left(\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a*sin(d*x+c)+b*tan(d*x+c)),x, algorithm="giac")
```

```
[Out] 2/3*(b - 3*a*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 3*a*(cos(d*x + c) - 1)
^2/(cos(d*x + c) + 1)^2 + 3*b*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2)/(d
*((cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1)^3)
```

maple [A] time = 0.04, size = 28, normalized size = 0.85

$$\frac{\frac{\left(\sec^{3}(dx+c)\right)b}{3}+\frac{\left(\sec^{2}(dx+c)\right)a}{2}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^3*(a*sin(d*x+c)+b*tan(d*x+c)),x)
```

```
[Out] 1/d*(1/3*sec(d*x+c)^3*b+1/2*sec(d*x+c)^2*a)
```

maxima [A] time = 0.33, size = 32, normalized size = 0.97

$$-\frac{\frac{3\,a}{\sin(dx+c)^{2}-1}-\frac{2\,b}{\cos(dx+c)^{3}}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a*sin(d*x+c)+b*tan(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/6*(3*a/(sin(d*x + c)^2 - 1) - 2*b/cos(d*x + c)^3)/d
```

mupad [B] time = 0.69, size = 29, normalized size = 0.88

$$\frac{a}{2\,d\cos\left(c+d\,x\right)^{2}}+\frac{b}{3\,d\cos\left(c+d\,x\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*sin(c + d*x) + b*tan(c + d*x))/cos(c + d*x)^3,x)
```

```
[Out] a/(2*d*cos(c + d*x)^2) + b/(3*d*cos(c + d*x)^3)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a\sin\left(c+dx\right)+b\tan\left(c+dx\right)\right)\sec^{3}\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**3*(a*sin(d*x+c)+b*tan(d*x+c)),x)
```

```
[Out] Integral((a*sin(c + d*x) + b*tan(c + d*x))*sec(c + d*x)**3, x)
```

3.236 $\int \cos^3(c+dx)(a\sin(c+dx)+b\tan(c+dx))^2\,dx$

Optimal. Leaf size=106

$$\frac{(4a^2+b^2)\sin^3(c+dx)}{30d}+\frac{\sin^3(c+dx)(a\cos(c+dx)+b)^2}{5d}+\frac{b\sin^3(c+dx)(a\cos(c+dx)+b)}{10d}-\frac{ab\sin(c+dx)\cos(c+dx)}{4d}$$

```
[Out] 1/4*a*b*x-1/4*a*b*cos(d*x+c)*sin(d*x+c)/d+1/30*(4*a^2+b^2)*sin(d*x+c)^3/d+1
/10*b*(b+a*cos(d*x+c))*sin(d*x+c)^3/d+1/5*(b+a*cos(d*x+c))^2*sin(d*x+c)^3/d
```

Rubi [A] time = 0.35, antiderivative size = 106, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.179, Rules used = {4397, 2862, 2669, 2635, 8}

$$\frac{(4a^2+b^2)\sin^3(c+dx)}{30d}+\frac{\sin^3(c+dx)(a\cos(c+dx)+b)^2}{5d}+\frac{b\sin^3(c+dx)(a\cos(c+dx)+b)}{10d}-\frac{ab\sin(c+dx)\cos(c+dx)}{4d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^3*(a*Sin[c + d*x] + b*Tan[c + d*x])^2,x]
```

```
[Out] (a*b*x)/4 - (a*b*Cos[c + d*x]*Sin[c + d*x])/(4*d) + ((4*a^2 + b^2)*Sin[c +
d*x]^3)/(30*d) + (b*(b + a*Cos[c + d*x])*Sin[c + d*x]^3)/(10*d) + ((b + a*C
os[c + d*x])^2*Sin[c + d*x]^3)/(5*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2669

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1))/(f*g*(p + 1)), x] + D
ist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] && (I
ntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

Rule 2862

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> -Simp[(d*(
g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m)/(f*g*(m + p + 1)), x] + Dis
t[1/(m + p + 1), Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1)*Simp[a
*c*(m + p + 1) + b*d*m + (a*d*m + b*c*(m + p + 1))*Sin[e + f*x], x], x], x]
 /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[a^2 - b^2, 0] && GtQ[m, 0] &&
  !LtQ[p, -1] && IntegerQ[2*m] &&  !(EqQ[m, 1] && NeQ[c^2 - d^2, 0] && Simp
lerQ[c + d*x, a + b*x])
```

Rule 4397

```
Int[u_, x_Symbol] :> Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]
```

Rubi steps

$$\int \cos^3(c+dx)(a\sin(c+dx)+b\tan(c+dx))^2\,dx = \int \cos(c+dx)(b+a\cos(c+dx))^2\sin^2(c+dx)\,dx$$
$$= \frac{(b+a\cos(c+dx))^2\sin^3(c+dx)}{5d} + \frac{1}{5}\int (b+a\cos(c+dx))$$
$$= \frac{b(b+a\cos(c+dx))\sin^3(c+dx)}{10d} + \frac{(b+a\cos(c+dx))^2\sin}{5d}$$
$$= \frac{\left(4a^2+b^2\right)\sin^3(c+dx)}{30d} + \frac{b(b+a\cos(c+dx))\sin^3(c+dx)}{10d}$$
$$= -\frac{ab\cos(c+dx)\sin(c+dx)}{4d} + \frac{\left(4a^2+b^2\right)\sin^3(c+dx)}{30d} + \frac{b(}{}$$
$$= \frac{abx}{4} - \frac{ab\cos(c+dx)\sin(c+dx)}{4d} + \frac{\left(4a^2+b^2\right)\sin^3(c+dx)}{30d}$$

Mathematica [A] time = 0.38, size = 77, normalized size = 0.73

$$\frac{30\left(a^2+2b^2\right)\sin(c+dx)-5\left(a^2+4b^2\right)\sin(3(c+dx))-3a(a\sin(5(c+dx))-20b(c+dx)+5b\sin(4(c+dx)))}{240d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^3*(a*Sin[c + d*x] + b*Tan[c + d*x])^2,x]
```

```
[Out] (30*(a^2 + 2*b^2)*Sin[c + d*x] - 5*(a^2 + 4*b^2)*Sin[3*(c + d*x)] - 3*a*(-2
0*b*(c + d*x) + 5*b*Sin[4*(c + d*x)] + a*Sin[5*(c + d*x)]))/(240*d)
```

fricas [A] time = 0.59, size = 85, normalized size = 0.80

$$\frac{15\,abdx - \left(12\,a^2\cos\left(dx+c\right)^4 + 30\,ab\cos\left(dx+c\right)^3 - 15\,ab\cos\left(dx+c\right) - 4\left(a^2 - 5\,b^2\right)\cos\left(dx+c\right)^2 - 8\,a^2 - 20\,b^2}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a*sin(d*x+c)+b*tan(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/60*(15*a*b*d*x - (12*a^2*cos(d*x + c)^4 + 30*a*b*cos(d*x + c)^3 - 15*a*b*
cos(d*x + c) - 4*(a^2 - 5*b^2)*cos(d*x + c)^2 - 8*a^2 - 20*b^2)*sin(d*x + c
))/d
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a*sin(d*x+c)+b*tan(d*x+c))^2,x, algorithm="giac")
```

```
[Out] Timed out
```

maple [A] time = 0.09, size = 100, normalized size = 0.94

$$\frac{a^2\left(-\frac{\sin(dx+c)\left(\cos^4(dx+c)\right)}{5} + \frac{\left(2+\cos^2(dx+c)\right)\sin(dx+c)}{15}\right) + 2ab\left(-\frac{\sin(dx+c)\left(\cos^3(dx+c)\right)}{4} + \frac{\cos(dx+c)\sin(dx+c)}{8} + \frac{dx}{8} + \frac{c}{8}\right) + \frac{b^2\left(\sin^3(\right.}{3}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3*(a*sin(d*x+c)+b*tan(d*x+c))^2,x)
```

```
[Out] 1/d*(a^2*(-1/5*sin(d*x+c)*cos(d*x+c)^4+1/15*(2+cos(d*x+c)^2)*sin(d*x+c))+2*
a*b*(-1/4*sin(d*x+c)*cos(d*x+c)^3+1/8*cos(d*x+c)*sin(d*x+c)+1/8*d*x+1/8*c)+
1/3*b^2*sin(d*x+c)^3)
```

maxima [A] time = 0.37, size = 68, normalized size = 0.64

$$\frac{80\,b^2\sin\left(dx+c\right)^3 - 16\left(3\,\sin\left(dx+c\right)^5 - 5\,\sin\left(dx+c\right)^3\right)a^2 + 15\left(4\,dx + 4\,c - \sin\left(4\,dx + 4\,c\right)\right)ab}{240\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a*sin(d*x+c)+b*tan(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 1/240*(80*b^2*sin(d*x + c)^3 - 16*(3*sin(d*x + c)^5 - 5*sin(d*x + c)^3)*a^2
 + 15*(4*d*x + 4*c - sin(4*d*x + 4*c))*a*b)/d
```

mupad [B] time = 0.79, size = 101, normalized size = 0.95

$$\frac{a^2 \sin(c+d x)}{8 d}+\frac{b^2 \sin(c+d x)}{4 d}+\frac{a b x}{4}-\frac{a^2 \sin(3 c+3 d x)}{48 d}-\frac{a^2 \sin(5 c+5 d x)}{80 d}-\frac{b^2 \sin(3 c+3 d x)}{12 d}-\frac{a b \sin(4 c+4 d x)}{16 d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^3*(a*sin(c + d*x) + b*tan(c + d*x))^2,x)
```

```
[Out] (a^2*sin(c + d*x))/(8*d) + (b^2*sin(c + d*x))/(4*d) + (a*b*x)/4 - (a^2*sin(
3*c + 3*d*x))/(48*d) - (a^2*sin(5*c + 5*d*x))/(80*d) - (b^2*sin(3*c + 3*d*x
))/(12*d) - (a*b*sin(4*c + 4*d*x))/(16*d)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a\sin(c+dx)+b\tan(c+dx))^2\cos^3(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3*(a*sin(d*x+c)+b*tan(d*x+c))**2,x)
```

```
[Out] Integral((a*sin(c + d*x) + b*tan(c + d*x))**2*cos(c + d*x)**3, x)
```

3.237 $\int \cos^2(c+dx)(a\sin(c+dx)+b\tan(c+dx))^2\,dx$

Optimal. Leaf size=86

$$-\frac{\left(a^2+4b^2\right)\sin(c+dx)\cos(c+dx)}{8d}+\frac{1}{8}x\left(a^2+4b^2\right)+\frac{5ab\sin^3(c+dx)}{12d}+\frac{a\sin^3(c+dx)(a\cos(c+dx)+b)}{4d}$$

```
[Out] 1/8*(a^2+4*b^2)*x-1/8*(a^2+4*b^2)*cos(d*x+c)*sin(d*x+c)/d+5/12*a*b*sin(d*x+
c)^3/d+1/4*a*(b+a*cos(d*x+c))*sin(d*x+c)^3/d
```

Rubi [A] time = 0.19, antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.179, Rules used = {4397, 2692, 2669, 2635, 8}

$$-\frac{\left(a^2+4b^2\right)\sin(c+dx)\cos(c+dx)}{8d}+\frac{1}{8}x\left(a^2+4b^2\right)+\frac{5ab\sin^3(c+dx)}{12d}+\frac{a\sin^3(c+dx)(a\cos(c+dx)+b)}{4d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2*(a*Sin[c + d*x] + b*Tan[c + d*x])^2,x]
```

```
[Out] ((a^2 + 4*b^2)*x)/8 - ((a^2 + 4*b^2)*Cos[c + d*x]*Sin[c + d*x])/(8*d) + (5*
a*b*Sin[c + d*x]^3)/(12*d) + (a*(b + a*Cos[c + d*x])*Sin[c + d*x]^3)/(4*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2669

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1))/(f*g*(p + 1)), x] + D
ist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] && (I
ntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

Rule 2692

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*
```

```
x])^(m - 1))/(f*g*(m + p)), x] + Dist[1/(m + p), Int[(g*Cos[e + f*x])^p*(a
+ b*Sin[e + f*x])^(m - 2)*(b^2*(m - 1) + a^2*(m + p) + a*b*(2*m + p - 1)*Si
n[e + f*x]), x], x] /; FreeQ[{a, b, e, f, g, p}, x] && NeQ[a^2 - b^2, 0] &&
 GtQ[m, 1] && NeQ[m + p, 0] && (IntegersQ[2*m, 2*p] || IntegerQ[m])
```

Rule 4397

```
Int[u_, x_Symbol] :> Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]
```

Rubi steps

$$\int \cos^2(c+dx)(a\sin(c+dx)+b\tan(c+dx))^2\,dx = \int (b+a\cos(c+dx))^2\sin^2(c+dx)\,dx$$
$$= \frac{a(b+a\cos(c+dx))\sin^3(c+dx)}{4d} + \frac{1}{4}\int \left(a^2+4b^2+5ab\,\mathrm{co}\right.$$
$$= \frac{5ab\sin^3(c+dx)}{12d} + \frac{a(b+a\cos(c+dx))\sin^3(c+dx)}{4d} + \frac{1}{4}\left(\right.$$
$$= -\frac{\left(a^2+4b^2\right)\cos(c+dx)\sin(c+dx)}{8d} + \frac{5ab\sin^3(c+dx)}{12d} + $$
$$= \frac{1}{8}\left(a^2+4b^2\right)x - \frac{\left(a^2+4b^2\right)\cos(c+dx)\sin(c+dx)}{8d} + \frac{5ab\,\mathrm{s}}{}$$

Mathematica [A] time = 0.18, size = 82, normalized size = 0.95

$$\frac{-3a^2\sin(4(c+dx))+12a^2c+12a^2dx+48ab\sin(c+dx)-16ab\sin(3(c+dx))-24b^2\sin(2(c+dx))+48b^2c+}{96d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2*(a*Sin[c + d*x] + b*Tan[c + d*x])^2,x]
```

```
[Out] (12*a^2*c + 48*b^2*c + 12*a^2*d*x + 48*b^2*d*x + 48*a*b*Sin[c + d*x] - 24*b
^2*Sin[2*(c + d*x)] - 16*a*b*Sin[3*(c + d*x)] - 3*a^2*Sin[4*(c + d*x)])/(96
*d)
```

fricas [A] time = 0.68, size = 74, normalized size = 0.86

$$\frac{3\left(a^2+4\,b^2\right)dx-\left(6\,a^2\cos\left(dx+c\right)^3+16\,ab\cos\left(dx+c\right)^2-16\,ab-3\left(a^2-4\,b^2\right)\cos\left(dx+c\right)\right)\sin\left(dx+c\right)}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a*sin(d*x+c)+b*tan(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/24*(3*(a^2 + 4*b^2)*d*x - (6*a^2*cos(d*x + c)^3 + 16*a*b*cos(d*x + c)^2 -
 16*a*b - 3*(a^2 - 4*b^2)*cos(d*x + c))*sin(d*x + c))/d
```

giac [B] time = 7.76, size = 5161, normalized size = 60.01

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a*sin(d*x+c)+b*tan(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/8*a^2*x - 1/32*a^2*sin(4*d*x + 4*c)/d + 1/6*(3*b^2*d*x*tan(d*x)^2*tan(1/2
*d*x)^6*tan(1/2*c)^6*tan(c)^2 + 3*b^2*d*x*tan(d*x)^2*tan(1/2*d*x)^6*tan(1/2
*c)^6 + 9*b^2*d*x*tan(d*x)^2*tan(1/2*d*x)^6*tan(1/2*c)^4*tan(c)^2 + 9*b^2*d
*x*tan(d*x)^2*tan(1/2*d*x)^4*tan(1/2*c)^6*tan(c)^2 + 3*b^2*d*x*tan(1/2*d*x)
^6*tan(1/2*c)^6*tan(c)^2 + 3*b^2*tan(d*x)^2*tan(1/2*d*x)^6*tan(1/2*c)^6*tan
(c) + 3*b^2*tan(d*x)*tan(1/2*d*x)^6*tan(1/2*c)^6*tan(c)^2 + 9*b^2*d*x*tan(d
*x)^2*tan(1/2*d*x)^6*tan(1/2*c)^4 + 9*b^2*d*x*tan(d*x)^2*tan(1/2*d*x)^4*tan
(1/2*c)^6 + 3*b^2*d*x*tan(1/2*d*x)^6*tan(1/2*c)^6 + 9*b^2*d*x*tan(d*x)^2*ta
n(1/2*d*x)^6*tan(1/2*c)^2*tan(c)^2 + 27*b^2*d*x*tan(d*x)^2*tan(1/2*d*x)^4*t
an(1/2*c)^4*tan(c)^2 + 9*b^2*d*x*tan(1/2*d*x)^6*tan(1/2*c)^4*tan(c)^2 + 9*b
^2*d*x*tan(d*x)^2*tan(1/2*d*x)^2*tan(1/2*c)^6*tan(c)^2 + 9*b^2*d*x*tan(1/2*
d*x)^4*tan(1/2*c)^6*tan(c)^2 - 3*b^2*tan(d*x)*tan(1/2*d*x)^6*tan(1/2*c)^6 +
 9*b^2*tan(d*x)^2*tan(1/2*d*x)^6*tan(1/2*c)^4*tan(c) + 9*b^2*tan(d*x)^2*tan
(1/2*d*x)^4*tan(1/2*c)^6*tan(c) - 3*b^2*tan(1/2*d*x)^6*tan(1/2*c)^6*tan(c)
- 32*a*b*tan(d*x)^2*tan(1/2*d*x)^6*tan(1/2*c)^3*tan(c)^2 - 96*a*b*tan(d*x)^
2*tan(1/2*d*x)^5*tan(1/2*c)^4*tan(c)^2 + 9*b^2*tan(d*x)*tan(1/2*d*x)^6*tan(
1/2*c)^4*tan(c)^2 - 96*a*b*tan(d*x)^2*tan(1/2*d*x)^4*tan(1/2*c)^5*tan(c)^2
- 32*a*b*tan(d*x)^2*tan(1/2*d*x)^3*tan(1/2*c)^6*tan(c)^2 + 9*b^2*tan(d*x)*t
an(1/2*d*x)^4*tan(1/2*c)^6*tan(c)^2 + 9*b^2*d*x*tan(d*x)^2*tan(1/2*d*x)^6*t
an(1/2*c)^2 + 27*b^2*d*x*tan(d*x)^2*tan(1/2*d*x)^4*tan(1/2*c)^4 + 9*b^2*d*x
*tan(1/2*d*x)^6*tan(1/2*c)^4 + 9*b^2*d*x*tan(d*x)^2*tan(1/2*d*x)^2*tan(1/2*
c)^6 + 9*b^2*d*x*tan(1/2*d*x)^4*tan(1/2*c)^6 + 3*b^2*d*x*tan(d*x)^2*tan(1/2
*d*x)^6*tan(c)^2 + 27*b^2*d*x*tan(d*x)^2*tan(1/2*d*x)^4*tan(1/2*c)^2*tan(c)
^2 + 9*b^2*d*x*tan(1/2*d*x)^6*tan(1/2*c)^2*tan(c)^2 + 27*b^2*d*x*tan(d*x)^2
*tan(1/2*d*x)^2*tan(1/2*c)^4*tan(c)^2 + 27*b^2*d*x*tan(1/2*d*x)^4*tan(1/2*c
)^4*tan(c)^2 + 3*b^2*d*x*tan(d*x)^2*tan(1/2*c)^6*tan(c)^2 + 9*b^2*d*x*tan(1
/2*d*x)^2*tan(1/2*c)^6*tan(c)^2 - 32*a*b*tan(d*x)^2*tan(1/2*d*x)^6*tan(1/2*
c)^3 - 96*a*b*tan(d*x)^2*tan(1/2*d*x)^5*tan(1/2*c)^4 - 9*b^2*tan(d*x)*tan(1
/2*d*x)^6*tan(1/2*c)^4 - 96*a*b*tan(d*x)^2*tan(1/2*d*x)^4*tan(1/2*c)^5 - 32
*a*b*tan(d*x)^2*tan(1/2*d*x)^3*tan(1/2*c)^6 - 9*b^2*tan(d*x)*tan(1/2*d*x)^4
*tan(1/2*c)^6 + 9*b^2*tan(d*x)^2*tan(1/2*d*x)^6*tan(1/2*c)^2*tan(c) + 27*b^
2*tan(d*x)^2*tan(1/2*d*x)^4*tan(1/2*c)^4*tan(c) - 9*b^2*tan(1/2*d*x)^6*tan(
1/2*c)^4*tan(c) + 9*b^2*tan(d*x)^2*tan(1/2*d*x)^2*tan(1/2*c)^6*tan(c) - 9*b
```

^2*tan(1/2*d*x)^4*tan(1/2*c)^6*tan(c) + 96*a*b*tan(d*x)^2*tan(1/2*d*x)^5*ta
n(1/2*c)^2*tan(c)^2 + 9*b^2*tan(d*x)*tan(1/2*d*x)^6*tan(1/2*c)^2*tan(c)^2 +
288*a*b*tan(d*x)^2*tan(1/2*d*x)^4*tan(1/2*c)^3*tan(c)^2 - 32*a*b*tan(1/2*d
*x)^6*tan(1/2*c)^3*tan(c)^2 + 288*a*b*tan(d*x)^2*tan(1/2*d*x)^3*tan(1/2*c)^
4*tan(c)^2 + 27*b^2*tan(d*x)*tan(1/2*d*x)^4*tan(1/2*c)^4*tan(c)^2 - 96*a*b*
tan(1/2*d*x)^5*tan(1/2*c)^4*tan(c)^2 + 96*a*b*tan(d*x)^2*tan(1/2*d*x)^2*tan
(1/2*c)^5*tan(c)^2 - 96*a*b*tan(1/2*d*x)^4*tan(1/2*c)^5*tan(c)^2 + 9*b^2*ta
n(d*x)*tan(1/2*d*x)^2*tan(1/2*c)^6*tan(c)^2 - 32*a*b*tan(1/2*d*x)^3*tan(1/2
*c)^6*tan(c)^2 + 3*b^2*d*x*tan(d*x)^2*tan(1/2*d*x)^6 + 27*b^2*d*x*tan(d*x)^
2*tan(1/2*d*x)^4*tan(1/2*c)^2 + 9*b^2*d*x*tan(1/2*d*x)^6*tan(1/2*c)^2 + 27*
b^2*d*x*tan(d*x)^2*tan(1/2*d*x)^2*tan(1/2*c)^4 + 27*b^2*d*x*tan(1/2*d*x)^4*
tan(1/2*c)^4 + 3*b^2*d*x*tan(d*x)^2*tan(1/2*c)^6 + 9*b^2*d*x*tan(1/2*d*x)^2
*tan(1/2*c)^6 + 9*b^2*d*x*tan(d*x)^2*tan(1/2*d*x)^4*tan(c)^2 + 3*b^2*d*x*ta
n(1/2*d*x)^6*tan(c)^2 + 27*b^2*d*x*tan(d*x)^2*tan(1/2*d*x)^2*tan(1/2*c)^2*t
an(c)^2 + 27*b^2*d*x*tan(1/2*d*x)^4*tan(1/2*c)^2*tan(c)^2 + 9*b^2*d*x*tan(d
*x)^2*tan(1/2*c)^4*tan(c)^2 + 27*b^2*d*x*tan(1/2*d*x)^2*tan(1/2*c)^4*tan(c)
^2 + 3*b^2*d*x*tan(1/2*c)^6*tan(c)^2 + 96*a*b*tan(d*x)^2*tan(1/2*d*x)^5*tan
(1/2*c)^2 - 9*b^2*tan(d*x)*tan(1/2*d*x)^6*tan(1/2*c)^2 + 288*a*b*tan(d*x)^2
*tan(1/2*d*x)^4*tan(1/2*c)^3 - 32*a*b*tan(1/2*d*x)^6*tan(1/2*c)^3 + 288*a*b
*tan(d*x)^2*tan(1/2*d*x)^3*tan(1/2*c)^4 - 27*b^2*tan(d*x)*tan(1/2*d*x)^4*ta
n(1/2*c)^4 - 96*a*b*tan(1/2*d*x)^5*tan(1/2*c)^4 + 96*a*b*tan(d*x)^2*tan(1/2
*d*x)^2*tan(1/2*c)^5 - 96*a*b*tan(1/2*d*x)^4*tan(1/2*c)^5 - 9*b^2*tan(d*x)*
tan(1/2*d*x)^2*tan(1/2*c)^6 - 32*a*b*tan(1/2*d*x)^3*tan(1/2*c)^6 + 3*b^2*ta
n(d*x)^2*tan(1/2*d*x)^6*tan(c) + 27*b^2*tan(d*x)^2*tan(1/2*d*x)^4*tan(1/2*c
)^2*tan(c) - 9*b^2*tan(1/2*d*x)^6*tan(1/2*c)^2*tan(c) + 27*b^2*tan(d*x)^2*t
an(1/2*d*x)^2*tan(1/2*c)^4*tan(c) - 27*b^2*tan(1/2*d*x)^4*tan(1/2*c)^4*tan(
c) + 3*b^2*tan(d*x)^2*tan(1/2*c)^6*tan(c) - 9*b^2*tan(1/2*d*x)^2*tan(1/2*c)
^6*tan(c) + 3*b^2*tan(d*x)*tan(1/2*d*x)^6*tan(c)^2 - 96*a*b*tan(d*x)^2*tan(
1/2*d*x)^4*tan(1/2*c)*tan(c)^2 - 288*a*b*tan(d*x)^2*tan(1/2*d*x)^3*tan(1/2*
c)^2*tan(c)^2 + 27*b^2*tan(d*x)*tan(1/2*d*x)^4*tan(1/2*c)^2*tan(c)^2 + 96*a
*b*tan(1/2*d*x)^5*tan(1/2*c)^2*tan(c)^2 - 288*a*b*tan(d*x)^2*tan(1/2*d*x)^2
*tan(1/2*c)^3*tan(c)^2 + 288*a*b*tan(1/2*d*x)^4*tan(1/2*c)^3*tan(c)^2 - 96*
a*b*tan(d*x)^2*tan(1/2*d*x)*tan(1/2*c)^4*tan(c)^2 + 27*b^2*tan(d*x)*tan(1/2
*d*x)^2*tan(1/2*c)^4*tan(c)^2 + 288*a*b*tan(1/2*d*x)^3*tan(1/2*c)^4*tan(c)^
2 + 96*a*b*tan(1/2*d*x)^2*tan(1/2*c)^5*tan(c)^2 + 3*b^2*tan(d*x)*tan(1/2*c)
^6*tan(c)^2 + 9*b^2*d*x*tan(d*x)^2*tan(1/2*d*x)^4 + 3*b^2*d*x*tan(1/2*d*x)^
6 + 27*b^2*d*x*tan(d*x)^2*tan(1/2*d*x)^2*tan(1/2*c)^2 + 27*b^2*d*x*tan(1/2*
d*x)^4*tan(1/2*c)^2 + 9*b^2*d*x*tan(d*x)^2*tan(1/2*c)^4 + 27*b^2*d*x*tan(1/
2*d*x)^2*tan(1/2*c)^4 + 3*b^2*d*x*tan(1/2*c)^6 + 9*b^2*d*x*tan(d*x)^2*tan(1
/2*d*x)^2*tan(c)^2 + 9*b^2*d*x*tan(1/2*d*x)^4*tan(c)^2 + 9*b^2*d*x*tan(d*x)
^2*tan(1/2*c)^2*tan(c)^2 + 27*b^2*d*x*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(c)^2
+ 9*b^2*d*x*tan(1/2*c)^4*tan(c)^2 - 3*b^2*tan(d*x)*tan(1/2*d*x)^6 - 96*a*b*
tan(d*x)^2*tan(1/2*d*x)^4*tan(1/2*c) - 288*a*b*tan(d*x)^2*tan(1/2*d*x)^3*ta
n(1/2*c)^2 - 27*b^2*tan(d*x)*tan(1/2*d*x)^4*tan(1/2*c)^2 + 96*a*b*tan(1/2*d
*x)^5*tan(1/2*c)^2 - 288*a*b*tan(d*x)^2*tan(1/2*d*x)^2*tan(1/2*c)^3 + 288*a

*b*tan(1/2*d*x)^4*tan(1/2*c)^3 - 96*a*b*tan(d*x)^2*tan(1/2*d*x)*tan(1/2*c)^4 - 27*b^2*tan(d*x)*tan(1/2*d*x)^2*tan(1/2*c)^4 + 288*a*b*tan(1/2*d*x)^3*tan(1/2*c)^4 + 96*a*b*tan(1/2*d*x)^2*tan(1/2*c)^5 - 3*b^2*tan(d*x)*tan(1/2*c)^6 + 9*b^2*tan(d*x)^2*tan(1/2*d*x)^4*tan(c) - 3*b^2*tan(1/2*d*x)^6*tan(c) + 27*b^2*tan(d*x)^2*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(c) - 27*b^2*tan(1/2*d*x)^4*tan(1/2*c)^2*tan(c) + 9*b^2*tan(d*x)^2*tan(1/2*c)^4*tan(c) - 27*b^2*tan(1/2*d*x)^2*tan(1/2*c)^4*tan(c) - 3*b^2*tan(1/2*c)^6*tan(c) + 32*a*b*tan(d*x)^2*tan(1/2*d*x)^3*tan(c)^2 + 9*b^2*tan(d*x)*tan(1/2*d*x)^4*tan(c)^2 + 96*a*b*tan(d*x)^2*tan(1/2*d*x)^2*tan(1/2*c)*tan(c)^2 - 96*a*b*tan(1/2*d*x)^4*tan(1/2*c)*tan(c)^2 + 96*a*b*tan(d*x)^2*tan(1/2*d*x)*tan(1/2*c)^2*tan(c)^2 + 27*b^2*tan(d*x)*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(c)^2 - 288*a*b*tan(1/2*d*x)^3*tan(1/2*c)^2*tan(c)^2 + 32*a*b*tan(d*x)^2*tan(1/2*c)^3*tan(c)^2 - 288*a*b*tan(1/2*d*x)^2*tan(1/2*c)^3*tan(c)^2 + 9*b^2*tan(d*x)*tan(1/2*c)^4*tan(c)^2 - 96*a*b*tan(1/2*d*x)*tan(1/2*c)^4*tan(c)^2 + 9*b^2*d*x*tan(d*x)^2*tan(1/2*d*x)^2 + 9*b^2*d*x*tan(1/2*d*x)^4 + 9*b^2*d*x*tan(d*x)^2*tan(1/2*c)^2 + 27*b^2*d*x*tan(1/2*d*x)^2*tan(1/2*c)^2 + 9*b^2*d*x*tan(1/2*c)^4 + 3*b^2*d*x*tan(d*x)^2*tan(c)^2 + 9*b^2*d*x*tan(1/2*d*x)^2*tan(c)^2 + 9*b^2*d*x*tan(1/2*c)^2*tan(c)^2 + 32*a*b*tan(d*x)^2*tan(1/2*d*x)^3 - 9*b^2*tan(d*x)*tan(1/2*d*x)^4 + 96*a*b*tan(d*x)^2*tan(1/2*d*x)^2*tan(1/2*c) - 96*a*b*tan(1/2*d*x)^4*tan(1/2*c) + 96*a*b*tan(d*x)^2*tan(1/2*d*x)*tan(1/2*c)^2 - 27*b^2*tan(d*x)*tan(1/2*d*x)^2*tan(1/2*c)^2 - 288*a*b*tan(1/2*d*x)^3*tan(1/2*c)^2 + 32*a*b*tan(d*x)^2*tan(1/2*c)^3 - 288*a*b*tan(1/2*d*x)^2*tan(1/2*c)^3 - 9*b^2*tan(d*x)*tan(1/2*c)^4 - 96*a*b*tan(1/2*d*x)*tan(1/2*c)^4 + 9*b^2*tan(d*x)^2*tan(1/2*d*x)^2*tan(c) - 9*b^2*tan(1/2*d*x)^4*tan(c) + 9*b^2*tan(d*x)^2*tan(1/2*c)^2*tan(c) - 27*b^2*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(c) - 9*b^2*tan(1/2*c)^4*tan(c) + 9*b^2*tan(d*x)*tan(1/2*d*x)^2*tan(c)^2 + 32*a*b*tan(1/2*d*x)^3*tan(c)^2 + 96*a*b*tan(1/2*d*x)^2*tan(1/2*c)*tan(c)^2 + 9*b^2*tan(d*x)*tan(1/2*c)^2*tan(c)^2 + 96*a*b*tan(1/2*d*x)*tan(1/2*c)^2*tan(c)^2 + 32*a*b*tan(1/2*c)^3*tan(c)^2 + 3*b^2*d*x*tan(d*x)^2 + 9*b^2*d*x*tan(1/2*d*x)^2 + 9*b^2*d*x*tan(1/2*c)^2 + 3*b^2*d*x*tan(c)^2 - 9*b^2*tan(d*x)*tan(1/2*d*x)^2 + 32*a*b*tan(1/2*d*x)^3 + 96*a*b*tan(1/2*d*x)^2*tan(1/2*c) - 9*b^2*tan(d*x)*tan(1/2*c)^2 + 96*a*b*tan(1/2*d*x)*tan(1/2*c)^2 + 32*a*b*tan(1/2*c)^3 + 3*b^2*tan(d*x)^2*tan(c) - 9*b^2*tan(1/2*d*x)^2*tan(c) - 9*b^2*tan(1/2*c)^2*tan(c) + 3*b^2*tan(d*x)*tan(c)^2 + 3*b^2*d*x - 3*b^2*tan(d*x) - 3*b^2*tan(c))/(d*tan(d*x)^2*tan(1/2*d*x)^6*tan(1/2*c)^6*tan(c)^2 + d*tan(d*x)^2*tan(1/2*d*x)^6*tan(1/2*c)^6 + 3*d*tan(d*x)^2*tan(1/2*d*x)^6*tan(1/2*c)^4*tan(c)^2 + 3*d*tan(d*x)^2*tan(1/2*d*x)^4*tan(1/2*c)^6*tan(c)^2 + d*tan(1/2*d*x)^6*tan(1/2*c)^6*tan(c)^2 + 3*d*tan(d*x)^2*tan(1/2*d*x)^6*tan(1/2*c)^4 + 3*d*tan(d*x)^2*tan(1/2*d*x)^4*tan(1/2*c)^6 + d*tan(1/2*d*x)^6*tan(1/2*c)^6 + 3*d*tan(d*x)^2*tan(1/2*d*x)^6*tan(1/2*c)^2*tan(c)^2 + 9*d*tan(d*x)^2*tan(1/2*d*x)^4*tan(1/2*c)^4*tan(c)^2 + 3*d*tan(1/2*d*x)^6*tan(1/2*c)^4*tan(c)^2 + 3*d*tan(d*x)^2*tan(1/2*d*x)^2*tan(1/2*c)^6*tan(c)^2 + 3*d*tan(1/2*d*x)^4*tan(1/2*c)^6*tan(c)^2 + 3*d*tan(d*x)^2*tan(1/2*d*x)^6*tan(1/2*c)^2 + 9*d*tan(d*x)^2*tan(1/2*d*x)^4*tan(1/2*c)^4 + 3*d*tan(1/2*d*x)^6*tan(1/2*c)^4 + 3*d*tan(d*x)^2*tan(1/2*d*x)^2*tan(1/2*c)^6 + 3*d*tan(1/2*d*x)^4*tan(1/2*c)^6 + d*tan(d*x

```
)^2*tan(1/2*d*x)^6*tan(c)^2 + 9*d*tan(d*x)^2*tan(1/2*d*x)^4*tan(1/2*c)^2*ta
n(c)^2 + 3*d*tan(1/2*d*x)^6*tan(1/2*c)^2*tan(c)^2 + 9*d*tan(d*x)^2*tan(1/2*
d*x)^2*tan(1/2*c)^4*tan(c)^2 + 9*d*tan(1/2*d*x)^4*tan(1/2*c)^4*tan(c)^2 + d
*tan(d*x)^2*tan(1/2*c)^6*tan(c)^2 + 3*d*tan(1/2*d*x)^2*tan(1/2*c)^6*tan(c)^
2 + d*tan(d*x)^2*tan(1/2*d*x)^6 + 9*d*tan(d*x)^2*tan(1/2*d*x)^4*tan(1/2*c)^
2 + 3*d*tan(1/2*d*x)^6*tan(1/2*c)^2 + 9*d*tan(d*x)^2*tan(1/2*d*x)^2*tan(1/2
*c)^4 + 9*d*tan(1/2*d*x)^4*tan(1/2*c)^4 + d*tan(d*x)^2*tan(1/2*c)^6 + 3*d*t
an(1/2*d*x)^2*tan(1/2*c)^6 + 3*d*tan(d*x)^2*tan(1/2*d*x)^4*tan(c)^2 + d*tan
(1/2*d*x)^6*tan(c)^2 + 9*d*tan(d*x)^2*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(c)^2
+ 9*d*tan(1/2*d*x)^4*tan(1/2*c)^2*tan(c)^2 + 3*d*tan(d*x)^2*tan(1/2*c)^4*ta
n(c)^2 + 9*d*tan(1/2*d*x)^2*tan(1/2*c)^4*tan(c)^2 + d*tan(1/2*c)^6*tan(c)^2
 + 3*d*tan(d*x)^2*tan(1/2*d*x)^4 + d*tan(1/2*d*x)^6 + 9*d*tan(d*x)^2*tan(1/
2*d*x)^2*tan(1/2*c)^2 + 9*d*tan(1/2*d*x)^4*tan(1/2*c)^2 + 3*d*tan(d*x)^2*ta
n(1/2*c)^4 + 9*d*tan(1/2*d*x)^2*tan(1/2*c)^4 + d*tan(1/2*c)^6 + 3*d*tan(d*x
)^2*tan(1/2*d*x)^2*tan(c)^2 + 3*d*tan(1/2*d*x)^4*tan(c)^2 + 3*d*tan(d*x)^2*
tan(1/2*c)^2*tan(c)^2 + 9*d*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(c)^2 + 3*d*tan(
1/2*c)^4*tan(c)^2 + 3*d*tan(d*x)^2*tan(1/2*d*x)^2 + 3*d*tan(1/2*d*x)^4 + 3*
d*tan(d*x)^2*tan(1/2*c)^2 + 9*d*tan(1/2*d*x)^2*tan(1/2*c)^2 + 3*d*tan(1/2*c
)^4 + d*tan(d*x)^2*tan(c)^2 + 3*d*tan(1/2*d*x)^2*tan(c)^2 + 3*d*tan(1/2*c)^
2*tan(c)^2 + d*tan(d*x)^2 + 3*d*tan(1/2*d*x)^2 + 3*d*tan(1/2*c)^2 + d*tan(c
)^2 + d)
```

maple [A] time = 0.08, size = 86, normalized size = 1.00

$$\frac{a^2\left(-\frac{\sin(dx+c)\left(\cos^3(dx+c)\right)}{4}+\frac{\cos(dx+c)\sin(dx+c)}{8}+\frac{dx}{8}+\frac{c}{8}\right)+\frac{2ab\left(\sin^3(dx+c)\right)}{3}+b^2\left(-\frac{\cos(dx+c)\sin(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(a*sin(d*x+c)+b*tan(d*x+c))^2,x)
```

```
[Out] 1/d*(a^2*(-1/4*sin(d*x+c)*cos(d*x+c)^3+1/8*cos(d*x+c)*sin(d*x+c)+1/8*d*x+1/
8*c)+2/3*a*b*sin(d*x+c)^3+b^2*(-1/2*cos(d*x+c)*sin(d*x+c)+1/2*d*x+1/2*c))
```

maxima [A] time = 0.33, size = 66, normalized size = 0.77

$$\frac{64\,ab\sin\left(dx+c\right)^3+3\,\left(4\,dx+4\,c-\sin\left(4\,dx+4\,c\right)\right)a^2+24\,\left(2\,dx+2\,c-\sin\left(2\,dx+2\,c\right)\right)b^2}{96\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a*sin(d*x+c)+b*tan(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 1/96*(64*a*b*sin(d*x + c)^3 + 3*(4*d*x + 4*c - sin(4*d*x + 4*c))*a^2 + 24*(
2*d*x + 2*c - sin(2*d*x + 2*c))*b^2)/d
```

mupad [B] time = 0.71, size = 76, normalized size = 0.88

$$\frac{a^2 x}{8}+\frac{b^2 x}{2}-\frac{a^2 \sin(4 c+4 d x)}{32 d}-\frac{b^2 \sin(2 c+2 d x)}{4 d}+\frac{a b \sin(c+d x)}{2 d}-\frac{a b \sin(3 c+3 d x)}{6 d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^2*(a*sin(c + d*x) + b*tan(c + d*x))^2,x)
```

```
[Out] (a^2*x)/8 + (b^2*x)/2 - (a^2*sin(4*c + 4*d*x))/(32*d) - (b^2*sin(2*c + 2*d*
x))/(4*d) + (a*b*sin(c + d*x))/(2*d) - (a*b*sin(3*c + 3*d*x))/(6*d)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a \sin(c+dx)+b \tan(c+dx))^2 \cos^2(c+dx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(a*sin(d*x+c)+b*tan(d*x+c))**2,x)
```

```
[Out] Integral((a*sin(c + d*x) + b*tan(c + d*x))**2*cos(c + d*x)**2, x)
```

3.238 $\int \cos(c+dx)(a\sin(c+dx)+b\tan(c+dx))^2\,dx$

Optimal. Leaf size=87

$$\frac{(a^2-2b^2)\sin(c+dx)}{3d}-\frac{ab\sin(c+dx)\cos(c+dx)}{3d}-\frac{\sin(c+dx)(a\cos(c+dx)+b)^2}{3d}+abx+\frac{b^2\tanh^{-1}(\sin(c+dx$$

```
[Out] a*b*x+b^2*arctanh(sin(d*x+c))/d+1/3*(a^2-2*b^2)*sin(d*x+c)/d-1/3*a*b*cos(d*
x+c)*sin(d*x+c)/d-1/3*(b+a*cos(d*x+c))^2*sin(d*x+c)/d
```

Rubi [A] time = 0.32, antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.269, Rules used = {4397, 2889, 3050, 3033, 3023, 2735, 3770}

$$\frac{(a^2-2b^2)\sin(c+dx)}{3d}-\frac{ab\sin(c+dx)\cos(c+dx)}{3d}-\frac{\sin(c+dx)(a\cos(c+dx)+b)^2}{3d}+abx+\frac{b^2\tanh^{-1}(\sin(c+dx$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]*(a*Sin[c + d*x] + b*Tan[c + d*x])^2,x]
```

```
[Out] a*b*x + (b^2*ArcTanh[Sin[c + d*x]])/d + ((a^2 - 2*b^2)*Sin[c + d*x])/(3*d)
- (a*b*Cos[c + d*x]*Sin[c + d*x])/(3*d) - ((b + a*Cos[c + d*x])^2*Sin[c + d
*x])/(3*d)
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2889

```
Int[cos[(e_.) + (f_.)*(x_)]^2*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[(d*Sin[e + f*x])^n*(a
 + b*Sin[e + f*x])^m*(1 - Sin[e + f*x]^2), x] /; FreeQ[{a, b, d, e, f, m, n
}, x] && NeQ[a^2 - b^2, 0] && (IGtQ[m, 0] || IntegersQ[2*m, 2*n])
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
```

```
!LtQ[m, -1]
```

Rule 3033

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> -Simp[(C*d*Cos[e + f*x]*Sin[e + f*x]*(a + b*Sin[
e + f*x])^(m + 1))/(b*f*(m + 3)), x] + Dist[1/(b*(m + 3)), Int[(a + b*Sin[e
 + f*x])^m*Simp[a*C*d + A*b*c*(m + 3) + b*(B*c*(m + 3) + d*(C*(m + 2) + A*(
m + 3)))*Sin[e + f*x] - (2*a*C*d - b*(c*C + B*d)*(m + 3))*Sin[e + f*x]^2, x
], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m}, x] && NeQ[b*c - a*d, 0]
&& NeQ[a^2 - b^2, 0] &&  !LtQ[m, -1]
```

Rule 3050

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :
> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)
)/(d*f*(m + n + 2)), x] + Dist[1/(d*(m + n + 2)), Int[(a + b*Sin[e + f*x])^
(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 2) + C*(b*c*m + a*d*(n +
 1)) + (A*b*d*(m + n + 2) - C*(a*c - b*d*(m + n + 1)))*Sin[e + f*x] + C*(a*
d*m - b*c*(m + 1))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A
, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
&& GtQ[m, 0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])
))
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 4397

```
Int[u_, x_Symbol] :> Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]
```

Rubi steps

$$\int \cos(c+dx)(a\sin(c+dx)+b\tan(c+dx))^2\,dx = \int (b+a\cos(c+dx))^2\sin(c+dx)\tan(c+dx)\,dx$$
$$= \int (b+a\cos(c+dx))^2\left(1-\cos^2(c+dx)\right)\sec(c+dx)\,dx$$
$$= -\frac{(b+a\cos(c+dx))^2\sin(c+dx)}{3d}+\frac{1}{3}\int (b+a\cos(c+dx))$$
$$= -\frac{ab\cos(c+dx)\sin(c+dx)}{3d}-\frac{(b+a\cos(c+dx))^2\sin(c+d}{3d}$$
$$= \frac{\left(a^2-2b^2\right)\sin(c+dx)}{3d}-\frac{ab\cos(c+dx)\sin(c+dx)}{3d}-\frac{(b+a}{}$$
$$= abx+\frac{\left(a^2-2b^2\right)\sin(c+dx)}{3d}-\frac{ab\cos(c+dx)\sin(c+dx)}{3d}-$$
$$= abx+\frac{b^2\tanh^{-1}(\sin(c+dx))}{d}+\frac{\left(a^2-2b^2\right)\sin(c+dx)}{3d}-\frac{ab}{}$$

Mathematica [A] time = 0.15, size = 117, normalized size = 1.34

$$\frac{3\left(a^2-4b^2\right)\sin(c+dx)+a^2(-\sin(3(c+dx)))-6ab\sin(2(c+dx))+12abc+12abdx-12b^2\log\left(\cos\left(\frac{1}{2}(c+dx\right.\right.}{12d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]*(a*Sin[c + d*x] + b*Tan[c + d*x])^2,x]
```

```
[Out] (12*a*b*c + 12*a*b*d*x - 12*b^2*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] +
12*b^2*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] + 3*(a^2 - 4*b^2)*Sin[c + d
*x] - 6*a*b*Sin[2*(c + d*x)] - a^2*Sin[3*(c + d*x)])/(12*d)
```

fricas [A] time = 0.55, size = 83, normalized size = 0.95

$$\frac{6\,abdx+3\,b^2\log\left(\sin\left(dx+c\right)+1\right)-3\,b^2\log\left(-\sin\left(dx+c\right)+1\right)-2\left(a^2\cos\left(dx+c\right)^2+3\,ab\cos\left(dx+c\right)-a^2+\right.}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a*sin(d*x+c)+b*tan(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/6*(6*a*b*d*x + 3*b^2*log(sin(d*x + c) + 1) - 3*b^2*log(-sin(d*x + c) + 1)
 - 2*(a^2*cos(d*x + c)^2 + 3*a*b*cos(d*x + c) - a^2 + 3*b^2)*sin(d*x + c))/
d
```

giac [B] time = 4.22, size = 5713, normalized size = 65.67

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a*sin(d*x+c)+b*tan(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/12*a^2*sin(3*d*x + 3*c)/d + 1/4*a^2*sin(d*x + c)/d + 1/2*(2*a*b*d*x*tan(
d*x)^2*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(c)^2 - b^2*log(2*(tan(1/2*d*x)^4*tan
(1/2*c)^2 + 2*tan(1/2*d*x)^4*tan(1/2*c) + 2*tan(1/2*d*x)^3*tan(1/2*c)^2 + t
an(1/2*d*x)^4 + 2*tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^3 + 2*tan(1/
2*d*x)*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 2*
tan(1/2*c) + 1)/(tan(1/2*c)^2 + 1))*tan(d*x)^2*tan(1/2*d*x)^2*tan(1/2*c)^2*
tan(c)^2 + b^2*log(2*(tan(1/2*d*x)^4*tan(1/2*c)^2 - 2*tan(1/2*d*x)^4*tan(1/
2*c) - 2*tan(1/2*d*x)^3*tan(1/2*c)^2 + tan(1/2*d*x)^4 + 2*tan(1/2*d*x)^2*ta
n(1/2*c)^2 + 2*tan(1/2*d*x)^3 - 2*tan(1/2*d*x)*tan(1/2*c)^2 + 2*tan(1/2*d*x
)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) + 2*tan(1/2*c) + 1)/(tan(1/2*c)^2 + 1))
*tan(d*x)^2*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(c)^2 + 2*a*b*d*x*tan(d*x)^2*tan
(1/2*d*x)^2*tan(1/2*c)^2 + 2*a*b*d*x*tan(d*x)^2*tan(1/2*d*x)^2*tan(c)^2 + 2
*a*b*d*x*tan(d*x)^2*tan(1/2*c)^2*tan(c)^2 + 2*a*b*d*x*tan(1/2*d*x)^2*tan(1/
2*c)^2*tan(c)^2 - b^2*log(2*(tan(1/2*d*x)^4*tan(1/2*c)^2 + 2*tan(1/2*d*x)^4
*tan(1/2*c) + 2*tan(1/2*d*x)^3*tan(1/2*c)^2 + tan(1/2*d*x)^4 + 2*tan(1/2*d*
x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^3 + 2*tan(1/2*d*x)*tan(1/2*c)^2 + 2*tan(
1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 2*tan(1/2*c) + 1)/(tan(1/2*c)^
2 + 1))*tan(d*x)^2*tan(1/2*d*x)^2*tan(1/2*c)^2 + b^2*log(2*(tan(1/2*d*x)^4*
tan(1/2*c)^2 - 2*tan(1/2*d*x)^4*tan(1/2*c) - 2*tan(1/2*d*x)^3*tan(1/2*c)^2
+ tan(1/2*d*x)^4 + 2*tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^3 - 2*tan
(1/2*d*x)*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) +
 2*tan(1/2*c) + 1)/(tan(1/2*c)^2 + 1))*tan(d*x)^2*tan(1/2*d*x)^2*tan(1/2*c)
^2 + 2*a*b*tan(d*x)^2*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(c) - b^2*log(2*(tan(1
/2*d*x)^4*tan(1/2*c)^2 + 2*tan(1/2*d*x)^4*tan(1/2*c) + 2*tan(1/2*d*x)^3*tan
(1/2*c)^2 + tan(1/2*d*x)^4 + 2*tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)
^3 + 2*tan(1/2*d*x)*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(
1/2*d*x) - 2*tan(1/2*c) + 1)/(tan(1/2*c)^2 + 1))*tan(d*x)^2*tan(1/2*d*x)^2*
tan(c)^2 + b^2*log(2*(tan(1/2*d*x)^4*tan(1/2*c)^2 - 2*tan(1/2*d*x)^4*tan(1/
2*c) - 2*tan(1/2*d*x)^3*tan(1/2*c)^2 + tan(1/2*d*x)^4 + 2*tan(1/2*d*x)^2*ta
n(1/2*c)^2 + 2*tan(1/2*d*x)^3 - 2*tan(1/2*d*x)*tan(1/2*c)^2 + 2*tan(1/2*d*x
)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) + 2*tan(1/2*c) + 1)/(tan(1/2*c)^2 + 1))
*tan(d*x)^2*tan(1/2*d*x)^2*tan(c)^2 + 4*b^2*tan(d*x)^2*tan(1/2*d*x)^2*tan(1
/2*c)*tan(c)^2 - b^2*log(2*(tan(1/2*d*x)^4*tan(1/2*c)^2 + 2*tan(1/2*d*x)^4*
tan(1/2*c) + 2*tan(1/2*d*x)^3*tan(1/2*c)^2 + tan(1/2*d*x)^4 + 2*tan(1/2*d*x
)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^3 + 2*tan(1/2*d*x)*tan(1/2*c)^2 + 2*tan(1
/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 2*tan(1/2*c) + 1)/(tan(1/2*c)^2
 + 1))*tan(d*x)^2*tan(1/2*c)^2*tan(c)^2 + b^2*log(2*(tan(1/2*d*x)^4*tan(1/2
```

```
*c)^2 - 2*tan(1/2*d*x)^4*tan(1/2*c) - 2*tan(1/2*d*x)^3*tan(1/2*c)^2 + tan(1
/2*d*x)^4 + 2*tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^3 - 2*tan(1/2*d*
x)*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) + 2*tan(
1/2*c) + 1)/(tan(1/2*c)^2 + 1))*tan(d*x)^2*tan(1/2*c)^2*tan(c)^2 + 4*b^2*ta
n(d*x)^2*tan(1/2*d*x)*tan(1/2*c)^2*tan(c)^2 - b^2*log(2*(tan(1/2*d*x)^4*tan
(1/2*c)^2 + 2*tan(1/2*d*x)^4*tan(1/2*c) + 2*tan(1/2*d*x)^3*tan(1/2*c)^2 + t
an(1/2*d*x)^4 + 2*tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^3 + 2*tan(1/
2*d*x)*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 2*
tan(1/2*c) + 1)/(tan(1/2*c)^2 + 1))*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(c)^2 +
b^2*log(2*(tan(1/2*d*x)^4*tan(1/2*c)^2 - 2*tan(1/2*d*x)^4*tan(1/2*c) - 2*ta
n(1/2*d*x)^3*tan(1/2*c)^2 + tan(1/2*d*x)^4 + 2*tan(1/2*d*x)^2*tan(1/2*c)^2
+ 2*tan(1/2*d*x)^3 - 2*tan(1/2*d*x)*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2 + tan(1
/2*c)^2 + 2*tan(1/2*d*x) + 2*tan(1/2*c) + 1)/(tan(1/2*c)^2 + 1))*tan(1/2*d*
x)^2*tan(1/2*c)^2*tan(c)^2 + 2*a*b*tan(d*x)*tan(1/2*d*x)^2*tan(1/2*c)^2*tan
(c)^2 + 2*a*b*d*x*tan(d*x)^2*tan(1/2*d*x)^2 + 2*a*b*d*x*tan(d*x)^2*tan(1/2*
c)^2 + 2*a*b*d*x*tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*a*b*d*x*tan(d*x)^2*tan(c)^
2 + 2*a*b*d*x*tan(1/2*d*x)^2*tan(c)^2 + 2*a*b*d*x*tan(1/2*c)^2*tan(c)^2 - b
^2*log(2*(tan(1/2*d*x)^4*tan(1/2*c)^2 + 2*tan(1/2*d*x)^4*tan(1/2*c) + 2*tan
(1/2*d*x)^3*tan(1/2*c)^2 + tan(1/2*d*x)^4 + 2*tan(1/2*d*x)^2*tan(1/2*c)^2 -
 2*tan(1/2*d*x)^3 + 2*tan(1/2*d*x)*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2 + tan(1/
2*c)^2 - 2*tan(1/2*d*x) - 2*tan(1/2*c) + 1)/(tan(1/2*c)^2 + 1))*tan(d*x)^2*
tan(1/2*d*x)^2 + b^2*log(2*(tan(1/2*d*x)^4*tan(1/2*c)^2 - 2*tan(1/2*d*x)^4*
tan(1/2*c) - 2*tan(1/2*d*x)^3*tan(1/2*c)^2 + tan(1/2*d*x)^4 + 2*tan(1/2*d*x
)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^3 - 2*tan(1/2*d*x)*tan(1/2*c)^2 + 2*tan(1
/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) + 2*tan(1/2*c) + 1)/(tan(1/2*c)^2
 + 1))*tan(d*x)^2*tan(1/2*d*x)^2 + 4*b^2*tan(d*x)^2*tan(1/2*d*x)^2*tan(1/2*
c) - b^2*log(2*(tan(1/2*d*x)^4*tan(1/2*c)^2 + 2*tan(1/2*d*x)^4*tan(1/2*c) +
 2*tan(1/2*d*x)^3*tan(1/2*c)^2 + tan(1/2*d*x)^4 + 2*tan(1/2*d*x)^2*tan(1/2*
c)^2 - 2*tan(1/2*d*x)^3 + 2*tan(1/2*d*x)*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2 +
tan(1/2*c)^2 - 2*tan(1/2*d*x) - 2*tan(1/2*c) + 1)/(tan(1/2*c)^2 + 1))*tan(d
*x)^2*tan(1/2*c)^2 + b^2*log(2*(tan(1/2*d*x)^4*tan(1/2*c)^2 - 2*tan(1/2*d*x
)^4*tan(1/2*c) - 2*tan(1/2*d*x)^3*tan(1/2*c)^2 + tan(1/2*d*x)^4 + 2*tan(1/2
*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^3 - 2*tan(1/2*d*x)*tan(1/2*c)^2 + 2*t
an(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) + 2*tan(1/2*c) + 1)/(tan(1/2*
c)^2 + 1))*tan(d*x)^2*tan(1/2*c)^2 + 4*b^2*tan(d*x)^2*tan(1/2*d*x)*tan(1/2*
c)^2 - b^2*log(2*(tan(1/2*d*x)^4*tan(1/2*c)^2 + 2*tan(1/2*d*x)^4*tan(1/2*c)
 + 2*tan(1/2*d*x)^3*tan(1/2*c)^2 + tan(1/2*d*x)^4 + 2*tan(1/2*d*x)^2*tan(1/
2*c)^2 - 2*tan(1/2*d*x)^3 + 2*tan(1/2*d*x)*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2
+ tan(1/2*c)^2 - 2*tan(1/2*d*x) - 2*tan(1/2*c) + 1)/(tan(1/2*c)^2 + 1))*tan
(1/2*d*x)^2*tan(1/2*c)^2 + b^2*log(2*(tan(1/2*d*x)^4*tan(1/2*c)^2 - 2*tan(1
/2*d*x)^4*tan(1/2*c) - 2*tan(1/2*d*x)^3*tan(1/2*c)^2 + tan(1/2*d*x)^4 + 2*t
an(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^3 - 2*tan(1/2*d*x)*tan(1/2*c)^2
 + 2*tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) + 2*tan(1/2*c) + 1)/(ta
n(1/2*c)^2 + 1))*tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*a*b*tan(d*x)*tan(1/2*d*x)^
2*tan(1/2*c)^2 + 2*a*b*tan(d*x)^2*tan(1/2*d*x)^2*tan(c) + 2*a*b*tan(d*x)^2*
```

```
tan(1/2*c)^2*tan(c) - 2*a*b*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(c) - b^2*log(2*
(tan(1/2*d*x)^4*tan(1/2*c)^2 + 2*tan(1/2*d*x)^4*tan(1/2*c) + 2*tan(1/2*d*x)
^3*tan(1/2*c)^2 + tan(1/2*d*x)^4 + 2*tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/
2*d*x)^3 + 2*tan(1/2*d*x)*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2 + tan(1/2*c)^2 -
2*tan(1/2*d*x) - 2*tan(1/2*c) + 1)/(tan(1/2*c)^2 + 1))*tan(d*x)^2*tan(c)^2
+ b^2*log(2*(tan(1/2*d*x)^4*tan(1/2*c)^2 - 2*tan(1/2*d*x)^4*tan(1/2*c) - 2*
tan(1/2*d*x)^3*tan(1/2*c)^2 + tan(1/2*d*x)^4 + 2*tan(1/2*d*x)^2*tan(1/2*c)^
2 + 2*tan(1/2*d*x)^3 - 2*tan(1/2*d*x)*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2 + tan
(1/2*c)^2 + 2*tan(1/2*d*x) + 2*tan(1/2*c) + 1)/(tan(1/2*c)^2 + 1))*tan(d*x)
^2*tan(c)^2 - 4*b^2*tan(d*x)^2*tan(1/2*d*x)*tan(c)^2 - b^2*log(2*(tan(1/2*d
*x)^4*tan(1/2*c)^2 + 2*tan(1/2*d*x)^4*tan(1/2*c) + 2*tan(1/2*d*x)^3*tan(1/2
*c)^2 + tan(1/2*d*x)^4 + 2*tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^3 +
 2*tan(1/2*d*x)*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*
d*x) - 2*tan(1/2*c) + 1)/(tan(1/2*c)^2 + 1))*tan(1/2*d*x)^2*tan(c)^2 + b^2*
log(2*(tan(1/2*d*x)^4*tan(1/2*c)^2 - 2*tan(1/2*d*x)^4*tan(1/2*c) - 2*tan(1/
2*d*x)^3*tan(1/2*c)^2 + tan(1/2*d*x)^4 + 2*tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*
tan(1/2*d*x)^3 - 2*tan(1/2*d*x)*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2 + tan(1/2*c
)^2 + 2*tan(1/2*d*x) + 2*tan(1/2*c) + 1)/(tan(1/2*c)^2 + 1))*tan(1/2*d*x)^2
*tan(c)^2 + 2*a*b*tan(d*x)*tan(1/2*d*x)^2*tan(c)^2 - 4*b^2*tan(d*x)^2*tan(1
/2*c)*tan(c)^2 + 4*b^2*tan(1/2*d*x)^2*tan(1/2*c)*tan(c)^2 - b^2*log(2*(tan(
1/2*d*x)^4*tan(1/2*c)^2 + 2*tan(1/2*d*x)^4*tan(1/2*c) + 2*tan(1/2*d*x)^3*ta
n(1/2*c)^2 + tan(1/2*d*x)^4 + 2*tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x
)^3 + 2*tan(1/2*d*x)*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan
(1/2*d*x) - 2*tan(1/2*c) + 1)/(tan(1/2*c)^2 + 1))*tan(1/2*c)^2*tan(c)^2 + b
^2*log(2*(tan(1/2*d*x)^4*tan(1/2*c)^2 - 2*tan(1/2*d*x)^4*tan(1/2*c) - 2*tan
(1/2*d*x)^3*tan(1/2*c)^2 + tan(1/2*d*x)^4 + 2*tan(1/2*d*x)^2*tan(1/2*c)^2 +
 2*tan(1/2*d*x)^3 - 2*tan(1/2*d*x)*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2 + tan(1/
2*c)^2 + 2*tan(1/2*d*x) + 2*tan(1/2*c) + 1)/(tan(1/2*c)^2 + 1))*tan(1/2*c)^
2*tan(c)^2 + 2*a*b*tan(d*x)*tan(1/2*c)^2*tan(c)^2 + 4*b^2*tan(1/2*d*x)*tan(
1/2*c)^2*tan(c)^2 + 2*a*b*d*x*tan(d*x)^2 + 2*a*b*d*x*tan(1/2*d*x)^2 + 2*a*b
*d*x*tan(1/2*c)^2 + 2*a*b*d*x*tan(c)^2 - b^2*log(2*(tan(1/2*d*x)^4*tan(1/2*
c)^2 + 2*tan(1/2*d*x)^4*tan(1/2*c) + 2*tan(1/2*d*x)^3*tan(1/2*c)^2 + tan(1/
2*d*x)^4 + 2*tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^3 + 2*tan(1/2*d*x
)*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 2*tan(1
/2*c) + 1)/(tan(1/2*c)^2 + 1))*tan(d*x)^2 + b^2*log(2*(tan(1/2*d*x)^4*tan(1
/2*c)^2 - 2*tan(1/2*d*x)^4*tan(1/2*c) - 2*tan(1/2*d*x)^3*tan(1/2*c)^2 + tan
(1/2*d*x)^4 + 2*tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^3 - 2*tan(1/2*
d*x)*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) + 2*ta
n(1/2*c) + 1)/(tan(1/2*c)^2 + 1))*tan(d*x)^2 - 4*b^2*tan(d*x)^2*tan(1/2*d*x
) - b^2*log(2*(tan(1/2*d*x)^4*tan(1/2*c)^2 + 2*tan(1/2*d*x)^4*tan(1/2*c) +
2*tan(1/2*d*x)^3*tan(1/2*c)^2 + tan(1/2*d*x)^4 + 2*tan(1/2*d*x)^2*tan(1/2*c
)^2 - 2*tan(1/2*d*x)^3 + 2*tan(1/2*d*x)*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2 + t
an(1/2*c)^2 - 2*tan(1/2*d*x) - 2*tan(1/2*c) + 1)/(tan(1/2*c)^2 + 1))*tan(1/
2*d*x)^2 + b^2*log(2*(tan(1/2*d*x)^4*tan(1/2*c)^2 - 2*tan(1/2*d*x)^4*tan(1/
2*c) - 2*tan(1/2*d*x)^3*tan(1/2*c)^2 + tan(1/2*d*x)^4 + 2*tan(1/2*d*x)^2*ta
```

```
n(1/2*c)^2 + 2*tan(1/2*d*x)^3 - 2*tan(1/2*d*x)*tan(1/2*c)^2 + 2*tan(1/2*d*x
)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) + 2*tan(1/2*c) + 1)/(tan(1/2*c)^2 + 1))
*tan(1/2*d*x)^2 - 2*a*b*tan(d*x)*tan(1/2*d*x)^2 - 4*b^2*tan(d*x)^2*tan(1/2*
c) + 4*b^2*tan(1/2*d*x)^2*tan(1/2*c) - b^2*log(2*(tan(1/2*d*x)^4*tan(1/2*c)
^2 + 2*tan(1/2*d*x)^4*tan(1/2*c) + 2*tan(1/2*d*x)^3*tan(1/2*c)^2 + tan(1/2*
d*x)^4 + 2*tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^3 + 2*tan(1/2*d*x)*
tan(1/2*c)^2 + 2*tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 2*tan(1/2
*c) + 1)/(tan(1/2*c)^2 + 1))*tan(1/2*c)^2 + b^2*log(2*(tan(1/2*d*x)^4*tan(1
/2*c)^2 - 2*tan(1/2*d*x)^4*tan(1/2*c) - 2*tan(1/2*d*x)^3*tan(1/2*c)^2 + tan
(1/2*d*x)^4 + 2*tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^3 - 2*tan(1/2*
d*x)*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) + 2*ta
n(1/2*c) + 1)/(tan(1/2*c)^2 + 1))*tan(1/2*c)^2 - 2*a*b*tan(d*x)*tan(1/2*c)^
2 + 4*b^2*tan(1/2*d*x)*tan(1/2*c)^2 + 2*a*b*tan(d*x)^2*tan(c) - 2*a*b*tan(1
/2*d*x)^2*tan(c) - 2*a*b*tan(1/2*c)^2*tan(c) - b^2*log(2*(tan(1/2*d*x)^4*ta
n(1/2*c)^2 + 2*tan(1/2*d*x)^4*tan(1/2*c) + 2*tan(1/2*d*x)^3*tan(1/2*c)^2 +
tan(1/2*d*x)^4 + 2*tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^3 + 2*tan(1
/2*d*x)*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 2
*tan(1/2*c) + 1)/(tan(1/2*c)^2 + 1))*tan(c)^2 + b^2*log(2*(tan(1/2*d*x)^4*t
an(1/2*c)^2 - 2*tan(1/2*d*x)^4*tan(1/2*c) - 2*tan(1/2*d*x)^3*tan(1/2*c)^2 +
 tan(1/2*d*x)^4 + 2*tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^3 - 2*tan(
1/2*d*x)*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) +
2*tan(1/2*c) + 1)/(tan(1/2*c)^2 + 1))*tan(c)^2 + 2*a*b*tan(d*x)*tan(c)^2 -
4*b^2*tan(1/2*d*x)*tan(c)^2 - 4*b^2*tan(1/2*c)*tan(c)^2 + 2*a*b*d*x - b^2*l
og(2*(tan(1/2*d*x)^4*tan(1/2*c)^2 + 2*tan(1/2*d*x)^4*tan(1/2*c) + 2*tan(1/2
*d*x)^3*tan(1/2*c)^2 + tan(1/2*d*x)^4 + 2*tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*t
an(1/2*d*x)^3 + 2*tan(1/2*d*x)*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2 + tan(1/2*c)
^2 - 2*tan(1/2*d*x) - 2*tan(1/2*c) + 1)/(tan(1/2*c)^2 + 1)) + b^2*log(2*(ta
n(1/2*d*x)^4*tan(1/2*c)^2 - 2*tan(1/2*d*x)^4*tan(1/2*c) - 2*tan(1/2*d*x)^3*
tan(1/2*c)^2 + tan(1/2*d*x)^4 + 2*tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d
*x)^3 - 2*tan(1/2*d*x)*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*t
an(1/2*d*x) + 2*tan(1/2*c) + 1)/(tan(1/2*c)^2 + 1)) - 2*a*b*tan(d*x) - 4*b^
2*tan(1/2*d*x) - 4*b^2*tan(1/2*c) - 2*a*b*tan(c))/(d*tan(d*x)^2*tan(1/2*d*x
)^2*tan(1/2*c)^2*tan(c)^2 + d*tan(d*x)^2*tan(1/2*d*x)^2*tan(1/2*c)^2 + d*ta
n(d*x)^2*tan(1/2*d*x)^2*tan(c)^2 + d*tan(d*x)^2*tan(1/2*c)^2*tan(c)^2 + d*t
an(1/2*d*x)^2*tan(1/2*c)^2*tan(c)^2 + d*tan(d*x)^2*tan(1/2*d*x)^2 + d*tan(d
*x)^2*tan(1/2*c)^2 + d*tan(1/2*d*x)^2*tan(1/2*c)^2 + d*tan(d*x)^2*tan(c)^2
+ d*tan(1/2*d*x)^2*tan(c)^2 + d*tan(1/2*c)^2*tan(c)^2 + d*tan(d*x)^2 + d*ta
n(1/2*d*x)^2 + d*tan(1/2*c)^2 + d*tan(c)^2 + d)
```

maple [A] time = 0.07, size = 83, normalized size = 0.95

$$\frac{a^2 \left(\sin^3(dx+c)\right)}{3d} - \frac{ab\cos(dx+c)\sin(dx+c)}{d} + abx + \frac{abc}{d} - \frac{b^2\sin(dx+c)}{d} + \frac{b^2\ln(\sec(dx+c)+\tan(dx+c))}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(a*sin(d*x+c)+b*tan(d*x+c))^2,x)
```

```
[Out] 1/3*a^2*sin(d*x+c)^3/d-a*b*cos(d*x+c)*sin(d*x+c)/d+a*b*x+1/d*a*b*c-b^2*sin(
d*x+c)/d+1/d*b^2*ln(sec(d*x+c)+tan(d*x+c))
```

maxima [A] time = 0.34, size = 76, normalized size = 0.87

$$\frac{2\,a^2\sin\left(dx+c\right)^3+3\left(2\,dx+2\,c-\sin\left(2\,dx+2\,c\right)\right)ab+3\,b^2\Big(\log\left(\sin\left(dx+c\right)+1\right)-\log\left(\sin\left(dx+c\right)-1\right)-2\sin$$
$$6\,d$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a*sin(d*x+c)+b*tan(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 1/6*(2*a^2*sin(d*x + c)^3 + 3*(2*d*x + 2*c - sin(2*d*x + 2*c))*a*b + 3*b^2*
(log(sin(d*x + c) + 1) - log(sin(d*x + c) - 1) - 2*sin(d*x + c)))/d
```

mupad [B] time = 0.84, size = 121, normalized size = 1.39

$$\frac{a^2\,\sin\left(c+d\,x\right)}{4\,d}-\frac{b^2\,\sin\left(c+d\,x\right)}{d}+\frac{2\,b^2\,\mathrm{atanh}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d}-\frac{a^2\,\sin\left(3\,c+3\,d\,x\right)}{12\,d}+\frac{2\,a\,b\,\mathrm{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d}-\frac{a\,b\,\sin\left(2\right.}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)*(a*sin(c + d*x) + b*tan(c + d*x))^2,x)
```

```
[Out] (a^2*sin(c + d*x))/(4*d) - (b^2*sin(c + d*x))/d + (2*b^2*atanh(sin(c/2 + (d
*x)/2)/cos(c/2 + (d*x)/2)))/d - (a^2*sin(3*c + 3*d*x))/(12*d) + (2*a*b*atan
(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/d - (a*b*sin(2*c + 2*d*x))/(2*d)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a\sin\left(c+dx\right)+b\tan\left(c+dx\right)\right)^2\cos\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a*sin(d*x+c)+b*tan(d*x+c))**2,x)
```

```
[Out] Integral((a*sin(c + d*x) + b*tan(c + d*x))**2*cos(c + d*x), x)
```

3.239 $\int (a\sin(c+dx)+b\tan(c+dx))^2\,dx$

Optimal. Leaf size=77

$$-\frac{a^2\sin(c+dx)\cos(c+dx)}{2d}+\frac{a^2x}{2}-\frac{2ab\sin(c+dx)}{d}+\frac{2ab\tanh^{-1}(\sin(c+dx))}{d}+\frac{b^2\tan(c+dx)}{d}-b^2x$$

```
[Out] 1/2*a^2*x-b^2*x+2*a*b*arctanh(sin(d*x+c))/d-2*a*b*sin(d*x+c)/d-1/2*a^2*cos(
d*x+c)*sin(d*x+c)/d+b^2*tan(d*x+c)/d
```

Rubi [A] time = 0.12, antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 8, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.421, Rules used = {4397, 2722, 2635, 8, 2592, 321, 206, 3473}

$$-\frac{a^2\sin(c+dx)\cos(c+dx)}{2d}+\frac{a^2x}{2}-\frac{2ab\sin(c+dx)}{d}+\frac{2ab\tanh^{-1}(\sin(c+dx))}{d}+\frac{b^2\tan(c+dx)}{d}-b^2x$$

Antiderivative was successfully verified.

```
[In] Int[(a*Sin[c + d*x] + b*Tan[c + d*x])^2,x]
```

```
[Out] (a^2*x)/2 - b^2*x + (2*a*b*ArcTanh[Sin[c + d*x]])/d - (2*a*b*Sin[c + d*x])/
d - (a^2*Cos[c + d*x]*Sin[c + d*x])/(2*d) + (b^2*Tan[c + d*x])/d
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2592

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(
```

```
ff*x)^(m + n)/(a^2 - ff^2*x^2)^((n + 1)/2), x], x, (a*Sin[e + f*x])/ff], x]
] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2722

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((g_.)*tan[(e_.) + (f_.)*(
x_)])^(p_.), x_Symbol] :> Int[ExpandIntegrand[(g*Tan[e + f*x])^p, (a + b*Si
n[e + f*x])^m, x], x] /; FreeQ[{a, b, e, f, g, p}, x] && NeQ[a^2 - b^2, 0]
&& IGtQ[m, 0]
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 4397

```
Int[u_, x_Symbol] :> Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]
```

Rubi steps

$$\begin{aligned}
\int (a\sin(c+dx)+b\tan(c+dx))^2\,dx &= \int (b+a\cos(c+dx))^2\tan^2(c+dx)\,dx \\
&= \int \left(a^2\sin^2(c+dx)+2ab\sin(c+dx)\tan(c+dx)+b^2\tan^2(c+dx)\right)\,dx \\
&= a^2\int \sin^2(c+dx)\,dx+(2ab)\int \sin(c+dx)\tan(c+dx)\,dx+b^2\int \tan^2(c \\
&= -\frac{a^2\cos(c+dx)\sin(c+dx)}{2d}+\frac{b^2\tan(c+dx)}{d}+\frac{1}{2}a^2\int 1\,dx-b^2\int 1\,dx+ \\
&= \frac{a^2x}{2}-b^2x-\frac{2ab\sin(c+dx)}{d}-\frac{a^2\cos(c+dx)\sin(c+dx)}{2d}+\frac{b^2\tan(c+dx}{d} \\
&= \frac{a^2x}{2}-b^2x+\frac{2ab\tanh^{-1}(\sin(c+dx))}{d}-\frac{2ab\sin(c+dx)}{d}-\frac{a^2\cos(c+dx)\,s}{2d}
\end{aligned}$$

Mathematica [A] time = 0.58, size = 116, normalized size = 1.51

$$-\frac{-2\left(a^2-2b^2\right)(c+dx)+\tan(c+dx)\left(a^2\cos(2(c+dx))+a^2-4b^2\right)+8ab\sin(c+dx)+8ab\log\left(\cos\left(\frac{1}{2}(c+dx\right.\right.}{4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Sin[c + d*x] + b*Tan[c + d*x])^2,x]
```

```
[Out] -1/4*(-2*(a^2 - 2*b^2)*(c + d*x) + 8*a*b*Log[Cos[(c + d*x)/2] - Sin[(c + d*
x)/2]] - 8*a*b*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] + 8*a*b*Sin[c + d*x
] + (a^2 - 4*b^2 + a^2*Cos[2*(c + d*x)])*Tan[c + d*x])/d
```

fricas [A] time = 0.65, size = 108, normalized size = 1.40

$$\frac{\left(a^2-2\,b^2\right)dx\cos\left(dx+c\right)+2\,ab\cos\left(dx+c\right)\log\left(\sin\left(dx+c\right)+1\right)-2\,ab\cos\left(dx+c\right)\log\left(-\sin\left(dx+c\right)+1\right)-}{2\,d\cos\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sin(d*x+c)+b*tan(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/2*((a^2 - 2*b^2)*d*x*cos(d*x + c) + 2*a*b*cos(d*x + c)*log(sin(d*x + c) +
 1) - 2*a*b*cos(d*x + c)*log(-sin(d*x + c) + 1) - (a^2*cos(d*x + c)^2 + 4*a
*b*cos(d*x + c) - 2*b^2)*sin(d*x + c))/(d*cos(d*x + c))
```

giac [B] time = 1.28, size = 2752, normalized size = 35.74

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sin(d*x+c)+b*tan(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/2*a^2*x - 1/4*a^2*sin(2*d*x + 2*c)/d - (b^2*d*x*tan(d*x)*tan(1/2*d*x)^2*t
an(1/2*c)^2*tan(c) + a*b*log(2*(tan(1/2*d*x)^4*tan(1/2*c)^2 + 2*tan(1/2*d*x
)^4*tan(1/2*c) + 2*tan(1/2*d*x)^3*tan(1/2*c)^2 + tan(1/2*d*x)^4 + 2*tan(1/2
*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^3 + 2*tan(1/2*d*x)*tan(1/2*c)^2 + 2*t
an(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 2*tan(1/2*c) + 1)/(tan(1/2*
c)^2 + 1))*tan(d*x)*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(c) - a*b*log(2*(tan(1/2
*d*x)^4*tan(1/2*c)^2 - 2*tan(1/2*d*x)^4*tan(1/2*c) - 2*tan(1/2*d*x)^3*tan(1
/2*c)^2 + tan(1/2*d*x)^4 + 2*tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^3
 - 2*tan(1/2*d*x)*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/
2*d*x) + 2*tan(1/2*c) + 1)/(tan(1/2*c)^2 + 1))*tan(d*x)*tan(1/2*d*x)^2*tan(
1/2*c)^2*tan(c) - b^2*d*x*tan(1/2*d*x)^2*tan(1/2*c)^2 + b^2*d*x*tan(d*x)*ta
n(1/2*d*x)^2*tan(c) + b^2*d*x*tan(d*x)*tan(1/2*c)^2*tan(c) - a*b*log(2*(tan
```

(1/2*d*x)^4*tan(1/2*c)^2 + 2*tan(1/2*d*x)^4*tan(1/2*c) + 2*tan(1/2*d*x)^3*t
an(1/2*c)^2 + tan(1/2*d*x)^4 + 2*tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*
x)^3 + 2*tan(1/2*d*x)*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*ta
n(1/2*d*x) - 2*tan(1/2*c) + 1)/(tan(1/2*c)^2 + 1))*tan(1/2*d*x)^2*tan(1/2*c
)^2 + a*b*log(2*(tan(1/2*d*x)^4*tan(1/2*c)^2 - 2*tan(1/2*d*x)^4*tan(1/2*c)
- 2*tan(1/2*d*x)^3*tan(1/2*c)^2 + tan(1/2*d*x)^4 + 2*tan(1/2*d*x)^2*tan(1/2
*c)^2 + 2*tan(1/2*d*x)^3 - 2*tan(1/2*d*x)*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2 +
tan(1/2*c)^2 + 2*tan(1/2*d*x) + 2*tan(1/2*c) + 1)/(tan(1/2*c)^2 + 1))*tan(
1/2*d*x)^2*tan(1/2*c)^2 + b^2*tan(d*x)*tan(1/2*d*x)^2*tan(1/2*c)^2 + a*b*lo
g(2*(tan(1/2*d*x)^4*tan(1/2*c)^2 + 2*tan(1/2*d*x)^4*tan(1/2*c) + 2*tan(1/2*
d*x)^3*tan(1/2*c)^2 + tan(1/2*d*x)^4 + 2*tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*ta
n(1/2*d*x)^3 + 2*tan(1/2*d*x)*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2 + tan(1/2*c)^
2 - 2*tan(1/2*d*x) - 2*tan(1/2*c) + 1)/(tan(1/2*c)^2 + 1))*tan(d*x)*tan(1/2
*d*x)^2*tan(c) - a*b*log(2*(tan(1/2*d*x)^4*tan(1/2*c)^2 - 2*tan(1/2*d*x)^4*
tan(1/2*c) - 2*tan(1/2*d*x)^3*tan(1/2*c)^2 + tan(1/2*d*x)^4 + 2*tan(1/2*d*x
)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^3 - 2*tan(1/2*d*x)*tan(1/2*c)^2 + 2*tan(1
/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) + 2*tan(1/2*c) + 1)/(tan(1/2*c)^2
+ 1))*tan(d*x)*tan(1/2*d*x)^2*tan(c) - 4*a*b*tan(d*x)*tan(1/2*d*x)^2*tan(1
/2*c)*tan(c) + a*b*log(2*(tan(1/2*d*x)^4*tan(1/2*c)^2 + 2*tan(1/2*d*x)^4*ta
n(1/2*c) + 2*tan(1/2*d*x)^3*tan(1/2*c)^2 + tan(1/2*d*x)^4 + 2*tan(1/2*d*x)^
2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^3 + 2*tan(1/2*d*x)*tan(1/2*c)^2 + 2*tan(1/2
*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 2*tan(1/2*c) + 1)/(tan(1/2*c)^2 +
1))*tan(d*x)*tan(1/2*c)^2*tan(c) - a*b*log(2*(tan(1/2*d*x)^4*tan(1/2*c)^2
- 2*tan(1/2*d*x)^4*tan(1/2*c) - 2*tan(1/2*d*x)^3*tan(1/2*c)^2 + tan(1/2*d*x
)^4 + 2*tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^3 - 2*tan(1/2*d*x)*tan
(1/2*c)^2 + 2*tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) + 2*tan(1/2*c)
+ 1)/(tan(1/2*c)^2 + 1))*tan(d*x)*tan(1/2*c)^2*tan(c) - 4*a*b*tan(d*x)*tan
(1/2*d*x)*tan(1/2*c)^2*tan(c) + b^2*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(c) - b^
2*d*x*tan(1/2*d*x)^2 - b^2*d*x*tan(1/2*c)^2 + b^2*d*x*tan(d*x)*tan(c) - a*b
log(2(tan(1/2*d*x)^4*tan(1/2*c)^2 + 2*tan(1/2*d*x)^4*tan(1/2*c) + 2*tan(1
/2*d*x)^3*tan(1/2*c)^2 + tan(1/2*d*x)^4 + 2*tan(1/2*d*x)^2*tan(1/2*c)^2 - 2
*tan(1/2*d*x)^3 + 2*tan(1/2*d*x)*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2 + tan(1/2*
c)^2 - 2*tan(1/2*d*x) - 2*tan(1/2*c) + 1)/(tan(1/2*c)^2 + 1))*tan(1/2*d*x)^
2 + a*b*log(2*(tan(1/2*d*x)^4*tan(1/2*c)^2 - 2*tan(1/2*d*x)^4*tan(1/2*c) -
2*tan(1/2*d*x)^3*tan(1/2*c)^2 + tan(1/2*d*x)^4 + 2*tan(1/2*d*x)^2*tan(1/2*c
)^2 + 2*tan(1/2*d*x)^3 - 2*tan(1/2*d*x)*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2 + t
an(1/2*c)^2 + 2*tan(1/2*d*x) + 2*tan(1/2*c) + 1)/(tan(1/2*c)^2 + 1))*tan(1/
2*d*x)^2 + b^2*tan(d*x)*tan(1/2*d*x)^2 + 4*a*b*tan(1/2*d*x)^2*tan(1/2*c) -
a*b*log(2*(tan(1/2*d*x)^4*tan(1/2*c)^2 + 2*tan(1/2*d*x)^4*tan(1/2*c) + 2*ta
n(1/2*d*x)^3*tan(1/2*c)^2 + tan(1/2*d*x)^4 + 2*tan(1/2*d*x)^2*tan(1/2*c)^2
- 2*tan(1/2*d*x)^3 + 2*tan(1/2*d*x)*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2 + tan(1
/2*c)^2 - 2*tan(1/2*d*x) - 2*tan(1/2*c) + 1)/(tan(1/2*c)^2 + 1))*tan(1/2*c)
^2 + a*b*log(2*(tan(1/2*d*x)^4*tan(1/2*c)^2 - 2*tan(1/2*d*x)^4*tan(1/2*c) -
2*tan(1/2*d*x)^3*tan(1/2*c)^2 + tan(1/2*d*x)^4 + 2*tan(1/2*d*x)^2*tan(1/2*
c)^2 + 2*tan(1/2*d*x)^3 - 2*tan(1/2*d*x)*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2 +

```
tan(1/2*c)^2 + 2*tan(1/2*d*x) + 2*tan(1/2*c) + 1)/(tan(1/2*c)^2 + 1))*tan(1
/2*c)^2 + b^2*tan(d*x)*tan(1/2*c)^2 + 4*a*b*tan(1/2*d*x)*tan(1/2*c)^2 + a*b
*log(2*(tan(1/2*d*x)^4*tan(1/2*c)^2 + 2*tan(1/2*d*x)^4*tan(1/2*c) + 2*tan(1
/2*d*x)^3*tan(1/2*c)^2 + tan(1/2*d*x)^4 + 2*tan(1/2*d*x)^2*tan(1/2*c)^2 - 2
*tan(1/2*d*x)^3 + 2*tan(1/2*d*x)*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2 + tan(1/2*
c)^2 - 2*tan(1/2*d*x) - 2*tan(1/2*c) + 1)/(tan(1/2*c)^2 + 1))*tan(d*x)*tan(
c) - a*b*log(2*(tan(1/2*d*x)^4*tan(1/2*c)^2 - 2*tan(1/2*d*x)^4*tan(1/2*c) -
 2*tan(1/2*d*x)^3*tan(1/2*c)^2 + tan(1/2*d*x)^4 + 2*tan(1/2*d*x)^2*tan(1/2*
c)^2 + 2*tan(1/2*d*x)^3 - 2*tan(1/2*d*x)*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2 +
tan(1/2*c)^2 + 2*tan(1/2*d*x) + 2*tan(1/2*c) + 1)/(tan(1/2*c)^2 + 1))*tan(d
*x)*tan(c) + 4*a*b*tan(d*x)*tan(1/2*d*x)*tan(c) + b^2*tan(1/2*d*x)^2*tan(c)
 + 4*a*b*tan(d*x)*tan(1/2*c)*tan(c) + b^2*tan(1/2*c)^2*tan(c) - b^2*d*x - a
*b*log(2*(tan(1/2*d*x)^4*tan(1/2*c)^2 + 2*tan(1/2*d*x)^4*tan(1/2*c) + 2*tan
(1/2*d*x)^3*tan(1/2*c)^2 + tan(1/2*d*x)^4 + 2*tan(1/2*d*x)^2*tan(1/2*c)^2 -
 2*tan(1/2*d*x)^3 + 2*tan(1/2*d*x)*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2 + tan(1/
2*c)^2 - 2*tan(1/2*d*x) - 2*tan(1/2*c) + 1)/(tan(1/2*c)^2 + 1)) + a*b*log(2
*(tan(1/2*d*x)^4*tan(1/2*c)^2 - 2*tan(1/2*d*x)^4*tan(1/2*c) - 2*tan(1/2*d*x
)^3*tan(1/2*c)^2 + tan(1/2*d*x)^4 + 2*tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1
/2*d*x)^3 - 2*tan(1/2*d*x)*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2 + tan(1/2*c)^2 +
 2*tan(1/2*d*x) + 2*tan(1/2*c) + 1)/(tan(1/2*c)^2 + 1)) + b^2*tan(d*x) - 4*
a*b*tan(1/2*d*x) - 4*a*b*tan(1/2*c) + b^2*tan(c))/(d*tan(d*x)*tan(1/2*d*x)^
2*tan(1/2*c)^2*tan(c) - d*tan(1/2*d*x)^2*tan(1/2*c)^2 + d*tan(d*x)*tan(1/2*
d*x)^2*tan(c) + d*tan(d*x)*tan(1/2*c)^2*tan(c) - d*tan(1/2*d*x)^2 - d*tan(1
/2*c)^2 + d*tan(d*x)*tan(c) - d)
```

maple [A] time = 0.06, size = 99, normalized size = 1.29

$$-\frac{a^2\cos(dx+c)\sin(dx+c)}{2d}+\frac{a^2x}{2}+\frac{a^2c}{2d}+\frac{2ab\ln(\sec(dx+c)+\tan(dx+c))}{d}-\frac{2ab\sin(dx+c)}{d}-b^2x+\frac{b^2\tan(dx}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*sin(d*x+c)+b*tan(d*x+c))^2,x)
```

```
[Out] -1/2*a^2*cos(d*x+c)*sin(d*x+c)/d+1/2*a^2*x+1/2/d*a^2*c+2/d*a*b*ln(sec(d*x+c
)+tan(d*x+c))-2*a*b*sin(d*x+c)/d-b^2*x+b^2*tan(d*x+c)/d-1/d*b^2*c
```

maxima [A] time = 0.69, size = 84, normalized size = 1.09

$$\frac{(2dx+2c-\sin(2dx+2c))a^2}{4d}-\frac{(dx+c-\tan(dx+c))b^2}{d}+\frac{ab\Big(\log(\sin(dx+c)+1)-\log(\sin(dx+c)-1)-2}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sin(d*x+c)+b*tan(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 1/4*(2*d*x + 2*c - sin(2*d*x + 2*c))*a^2/d - (d*x + c - tan(d*x + c))*b^2/d
 + a*b*(log(sin(d*x + c) + 1) - log(sin(d*x + c) - 1) - 2*sin(d*x + c))/d
```

mupad [B] time = 0.80, size = 143, normalized size = 1.86

$$\frac{a^2 \operatorname{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{d x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d x}{2}\right)}\right)}{d} - \frac{2 b^2 \operatorname{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{d x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d x}{2}\right)}\right)}{d} + \frac{b^2 \sin(c+d x)}{d \cos(c+d x)} - \frac{2 a b \sin(c+d x)}{d} + \frac{4 a b \operatorname{atanh}\left(\frac{\sin\left(\frac{c}{2}+\frac{d x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d x}{2}\right)}\right)}{d} - \frac{a^2 \cos}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*sin(c + d*x) + b*tan(c + d*x))^2,x)
```

```
[Out] (a^2*atan(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/d - (2*b^2*atan(sin(c/2 +
 (d*x)/2)/cos(c/2 + (d*x)/2)))/d + (b^2*sin(c + d*x))/(d*cos(c + d*x)) - (2
*a*b*sin(c + d*x))/d + (4*a*b*atanh(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))
/d - (a^2*cos(c + d*x)*sin(c + d*x))/(2*d)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a \sin(c+dx) + b \tan(c+dx))^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sin(d*x+c)+b*tan(d*x+c))**2,x)
```

```
[Out] Integral((a*sin(c + d*x) + b*tan(c + d*x))**2, x)
```

3.240 $\int \sec(c+dx)(a\sin(c+dx)+b\tan(c+dx))^2\,dx$

Optimal. Leaf size=90

$$\frac{(2a^2-b^2)\tanh^{-1}(\sin(c+dx))}{2d}-\frac{3a^2\sin(c+dx)}{2d}+\frac{ab\tan(c+dx)}{d}+\frac{\tan(c+dx)\sec(c+dx)(a\cos(c+dx)+b)^2}{2d}-$$

```
[Out] -2*a*b*x+1/2*(2*a^2-b^2)*arctanh(sin(d*x+c))/d-3/2*a^2*sin(d*x+c)/d+a*b*tan
(d*x+c)/d+1/2*(b+a*cos(d*x+c))^2*sec(d*x+c)*tan(d*x+c)/d
```

Rubi [A] time = 0.43, antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.269, Rules used = {4397, 2889, 3048, 3031, 3023, 2735, 3770}

$$\frac{(2a^2-b^2)\tanh^{-1}(\sin(c+dx))}{2d}-\frac{3a^2\sin(c+dx)}{2d}+\frac{ab\tan(c+dx)}{d}+\frac{\tan(c+dx)\sec(c+dx)(a\cos(c+dx)+b)^2}{2d}-$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]*(a*Sin[c + d*x] + b*Tan[c + d*x])^2,x]
```

```
[Out] -2*a*b*x + ((2*a^2 - b^2)*ArcTanh[Sin[c + d*x]])/(2*d) - (3*a^2*Sin[c + d*x
])/(2*d) + (a*b*Tan[c + d*x])/d + ((b + a*Cos[c + d*x])^2*Sec[c + d*x]*Tan[
c + d*x])/(2*d)
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2889

```
Int[cos[(e_.) + (f_.)*(x_)]^2*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[(d*Sin[e + f*x])^n*(a
 + b*Sin[e + f*x])^m*(1 - Sin[e + f*x]^2), x] /; FreeQ[{a, b, d, e, f, m, n
}, x] && NeQ[a^2 - b^2, 0] && (IGtQ[m, 0] || IntegersQ[2*m, 2*n])
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
```

```
!LtQ[m, -1]
```

Rule 3031

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> -Simp[((b*c - a*d)*(A*b^2 - a*b*B + a^2*C)*Cos[e
 + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b^2*f*(m + 1)*(a^2 - b^2)), x] - Dis
t[1/(b^2*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[b*(m +
 1)*((b*B - a*C)*(b*c - a*d) - A*b*(a*c - b*d)) + (b*B*(a^2*d + b^2*d*(m +
1) - a*b*c*(m + 2)) + (b*c - a*d)*(A*b^2*(m + 2) + C*(a^2 + b^2*(m + 1))))*
Sin[e + f*x] - b*C*d*(m + 1)*(a^2 - b^2)*Sin[e + f*x]^2, x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
 && LtQ[m, -1]
```

Rule 3048

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
-Simp[((c^2*C + A*d^2)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f
*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)
), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(
b*d*m + a*c*(n + 1)) + c*C*(b*c*m + a*d*(n + 1)) - (A*d*(a*d*(n + 2) - b*c*
(n + 1)) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] - b*(A*d
^2*(m + n + 2) + C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]^2, x], x], x]
/; FreeQ[{a, b, c, d, e, f, A, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2
, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 4397

```
Int[u_, x_Symbol] :> Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]
```

Rubi steps

$$\int \sec(c+dx)(a\sin(c+dx)+b\tan(c+dx))^2\,dx = \int (b+a\cos(c+dx))^2\sec(c+dx)\tan^2(c+dx)\,dx$$
$$= \int (b+a\cos(c+dx))^2\left(1-\cos^2(c+dx)\right)\sec^3(c+dx)\,dx$$
$$= \frac{(b+a\cos(c+dx))^2\sec(c+dx)\tan(c+dx)}{2d} + \frac{1}{2}\int (b+a\,\mathrm{co}$$
$$= \frac{ab\tan(c+dx)}{d} + \frac{(b+a\cos(c+dx))^2\sec(c+dx)\tan(c+dx}{2d}$$
$$= -\frac{3a^2\sin(c+dx)}{2d} + \frac{ab\tan(c+dx)}{d} + \frac{(b+a\cos(c+dx))^2\,\mathrm{se}}{2d}$$
$$= -2abx - \frac{3a^2\sin(c+dx)}{2d} + \frac{ab\tan(c+dx)}{d} + \frac{(b+a\cos(c+}{}$$
$$= -2abx + \frac{\left(2a^2-b^2\right)\tanh^{-1}(\sin(c+dx))}{2d} - \frac{3a^2\sin(c+dx)}{2d} +$$

Mathematica [A] time = 0.19, size = 75, normalized size = 0.83

$$\frac{\left(2a^2-b^2\right)\tanh^{-1}(\sin(c+dx)) - 2a^2\sin(c+dx) - 4ab\tan^{-1}(\tan(c+dx)) + 4ab\tan(c+dx) + b^2\tan(c+dx)\,\mathrm{se}}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]*(a*Sin[c + d*x] + b*Tan[c + d*x])^2,x]
```

```
[Out] (-4*a*b*ArcTan[Tan[c + d*x]] + (2*a^2 - b^2)*ArcTanh[Sin[c + d*x]] - 2*a^2*
Sin[c + d*x] + 4*a*b*Tan[c + d*x] + b^2*Sec[c + d*x]*Tan[c + d*x])/(2*d)
```

fricas [A] time = 0.71, size = 126, normalized size = 1.40

$$-\frac{8\,abdx\cos(dx+c)^2 - \left(2\,a^2-b^2\right)\cos(dx+c)^2\log(\sin(dx+c)+1) + \left(2\,a^2-b^2\right)\cos(dx+c)^2\log(-\sin(dx}{4\,d\cos(dx+c)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a*sin(d*x+c)+b*tan(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -1/4*(8*a*b*d*x*cos(d*x + c)^2 - (2*a^2 - b^2)*cos(d*x + c)^2*log(sin(d*x +
 c) + 1) + (2*a^2 - b^2)*cos(d*x + c)^2*log(-sin(d*x + c) + 1) + 2*(2*a^2*c
os(d*x + c)^2 - 4*a*b*cos(d*x + c) - b^2)*sin(d*x + c))/(d*cos(d*x + c)^2)
```

giac [B] time = 1.58, size = 171, normalized size = 1.90

$$-\frac{4\,(dx+c)ab-\left(2\,a^2-b^2\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)+\left(2\,a^2-b^2\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)+\frac{4\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a*sin(d*x+c)+b*tan(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/2*(4*(d*x + c)*a*b - (2*a^2 - b^2)*log(abs(tan(1/2*d*x + 1/2*c) + 1)) +
(2*a^2 - b^2)*log(abs(tan(1/2*d*x + 1/2*c) - 1)) + 4*a^2*tan(1/2*d*x + 1/2*
c)/(tan(1/2*d*x + 1/2*c)^2 + 1) + 2*(4*a*b*tan(1/2*d*x + 1/2*c)^3 - b^2*tan
(1/2*d*x + 1/2*c)^3 - 4*a*b*tan(1/2*d*x + 1/2*c) - b^2*tan(1/2*d*x + 1/2*c)
)/(tan(1/2*d*x + 1/2*c)^2 - 1)^2)/d
```

maple [A] time = 0.09, size = 123, normalized size = 1.37

$$-\frac{a^2\sin(dx+c)}{d}+\frac{a^2\ln(\sec(dx+c)+\tan(dx+c))}{d}-2abx+\frac{2ab\tan(dx+c)}{d}-\frac{2abc}{d}+\frac{b^2\left(\sin^3(dx+c)\right)}{2d\cos(dx+c)^2}+\frac{b^2\sin(dx}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)*(a*sin(d*x+c)+b*tan(d*x+c))^2,x)
```

```
[Out] -a^2*sin(d*x+c)/d+1/d*a^2*ln(sec(d*x+c)+tan(d*x+c))-2*a*b*x+2*a*b*tan(d*x+c
)/d-2/d*a*b*c+1/2/d*b^2*sin(d*x+c)^3/cos(d*x+c)^2+1/2*b^2*sin(d*x+c)/d-1/2/
d*b^2*ln(sec(d*x+c)+tan(d*x+c))
```

maxima [A] time = 0.46, size = 102, normalized size = 1.13

$$-\frac{8\,(dx+c-\tan(dx+c))ab+b^2\left(\frac{2\,\sin(dx+c)}{\sin(dx+c)^2-1}+\log(\sin(dx+c)+1)-\log(\sin(dx+c)-1)\right)-2\,a^2\left(\log(\sin(dx\right.}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a*sin(d*x+c)+b*tan(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -1/4*(8*(d*x + c - tan(d*x + c))*a*b + b^2*(2*sin(d*x + c)/(sin(d*x + c)^2
- 1) + log(sin(d*x + c) + 1) - log(sin(d*x + c) - 1)) - 2*a^2*(log(sin(d*x
+ c) + 1) - log(sin(d*x + c) - 1) - 2*sin(d*x + c)))/d
```

mupad [B] time = 0.79, size = 147, normalized size = 1.63

$$\frac{2\,a^2\,\mathrm{atanh}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d}-\frac{a^2\,\sin\left(c+d\,x\right)}{d}-\frac{b^2\,\mathrm{atanh}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d}+\frac{b^2\,\sin\left(c+d\,x\right)}{2\,d\,{\cos\left(c+d\,x\right)}^2}-\frac{4\,a\,b\,\mathrm{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d}+\frac{2\,a\,b}{d\,\mathrm{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*sin(c + d*x) + b*tan(c + d*x))^2/cos(c + d*x),x)
```

```
[Out] (2*a^2*atanh(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/d - (a^2*sin(c + d*x))
/d - (b^2*atanh(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/d + (b^2*sin(c + d*
x))/(2*d*cos(c + d*x)^2) - (4*a*b*atan(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2
)))/d + (2*a*b*sin(c + d*x))/(d*cos(c + d*x))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a\sin(c+dx)+b\tan(c+dx))^2\sec(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a*sin(d*x+c)+b*tan(d*x+c))**2,x)
```

```
[Out] Integral((a*sin(c + d*x) + b*tan(c + d*x))**2*sec(c + d*x), x)
```

3.241 $\int \sec^2(c+dx)(a\sin(c+dx)+b\tan(c+dx))^2\,dx$

Optimal. Leaf size=99

$$\frac{(2a^2-b^2)\tan(c+dx)}{3d}+a^2(-x)-\frac{ab\tanh^{-1}(\sin(c+dx))}{d}+\frac{ab\tan(c+dx)\sec(c+dx)}{3d}+\frac{\tan(c+dx)\sec^2(c+dx)(a\,c}{3d}$$

```
[Out] -a^2*x-a*b*arctanh(sin(d*x+c))/d+1/3*(2*a^2-b^2)*tan(d*x+c)/d+1/3*a*b*sec(d
*x+c)*tan(d*x+c)/d+1/3*(b+a*cos(d*x+c))^2*sec(d*x+c)^2*tan(d*x+c)/d
```

Rubi [A] time = 0.47, antiderivative size = 99, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {4397, 2889, 3048, 3031, 3021, 2735, 3770}

$$\frac{(2a^2-b^2)\tan(c+dx)}{3d}+a^2(-x)-\frac{ab\tanh^{-1}(\sin(c+dx))}{d}+\frac{ab\tan(c+dx)\sec(c+dx)}{3d}+\frac{\tan(c+dx)\sec^2(c+dx)(a\,c}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^2*(a*Sin[c + d*x] + b*Tan[c + d*x])^2,x]
```

```
[Out] -(a^2*x) - (a*b*ArcTanh[Sin[c + d*x]])/d + ((2*a^2 - b^2)*Tan[c + d*x])/(3*
d) + (a*b*Sec[c + d*x]*Tan[c + d*x])/(3*d) + ((b + a*Cos[c + d*x])^2*Sec[c
+ d*x]^2*Tan[c + d*x])/(3*d)
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2889

```
Int[cos[(e_.) + (f_.)*(x_)]^2*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[(d*Sin[e + f*x])^n*(a
 + b*Sin[e + f*x])^m*(1 - Sin[e + f*x]^2), x] /; FreeQ[{a, b, d, e, f, m, n
}, x] && NeQ[a^2 - b^2, 0] && (IGtQ[m, 0] || IntegersQ[2*m, 2*n])
```

Rule 3021

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2
 - a*b*B + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 1)*(
a^2 - b^2)), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*b
- a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B,
```

```
C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rule 3031

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> -Simp[((b*c - a*d)*(A*b^2 - a*b*B + a^2*C)*Cos[e
 + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b^2*f*(m + 1)*(a^2 - b^2)), x] - Dis
t[1/(b^2*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[b*(m +
 1)*((b*B - a*C)*(b*c - a*d) - A*b*(a*c - b*d)) + (b*B*(a^2*d + b^2*d*(m +
1) - a*b*c*(m + 2)) + (b*c - a*d)*(A*b^2*(m + 2) + C*(a^2 + b^2*(m + 1))))*
Sin[e + f*x] - b*C*d*(m + 1)*(a^2 - b^2)*Sin[e + f*x]^2, x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
 && LtQ[m, -1]
```

Rule 3048

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
-Simp[((c^2*C + A*d^2)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f
*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)
), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(
b*d*m + a*c*(n + 1)) + c*C*(b*c*m + a*d*(n + 1)) - (A*d*(a*d*(n + 2) - b*c*
(n + 1)) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] - b*(A*d
^2*(m + n + 2) + C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]^2, x], x], x]
/; FreeQ[{a, b, c, d, e, f, A, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2
, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 4397

```
Int[u_, x_Symbol] :> Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]
```

Rubi steps

$$\int \sec^2(c+dx)(a\sin(c+dx)+b\tan(c+dx))^2\,dx = \int (b+a\cos(c+dx))^2\sec^2(c+dx)\tan^2(c+dx)\,dx$$
$$= \int (b+a\cos(c+dx))^2\left(1-\cos^2(c+dx)\right)\sec^4(c+dx)\,dx$$
$$= \frac{(b+a\cos(c+dx))^2\sec^2(c+dx)\tan(c+dx)}{3d} + \frac{1}{3}\int (b+a\,\mathrm{co}$$
$$= \frac{ab\sec(c+dx)\tan(c+dx)}{3d} + \frac{(b+a\cos(c+dx))^2\sec^2(c+d}{3d}$$
$$= \frac{\left(2a^2-b^2\right)\tan(c+dx)}{3d} + \frac{ab\sec(c+dx)\tan(c+dx)}{3d} + \frac{(b+a}{}$$
$$= -a^2x + \frac{\left(2a^2-b^2\right)\tan(c+dx)}{3d} + \frac{ab\sec(c+dx)\tan(c+dx)}{3d}$$
$$= -a^2x - \frac{ab\tanh^{-1}(\sin(c+dx))}{d} + \frac{\left(2a^2-b^2\right)\tan(c+dx)}{3d} + \frac{a}{}$$

Mathematica [B] time = 1.12, size = 201, normalized size = 2.03

$$\frac{\sec^3(c+dx)\left(2\sin(c+dx)\left(\left(3a^2-b^2\right)\cos(2(c+dx))+3a^2+6ab\cos(c+dx)+b^2\right)-9a\cos(c+dx)\left(a(c+dx)-\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^2*(a*Sin[c + d*x] + b*Tan[c + d*x])^2,x]
```

```
[Out] (Sec[c + d*x]^3*(-9*a*Cos[c + d*x]*(a*(c + d*x) - b*Log[Cos[(c + d*x)/2] -
Sin[(c + d*x)/2]] + b*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]) - 3*a*Cos[3
*(c + d*x)]*(a*(c + d*x) - b*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + b*L
og[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]) + 2*(3*a^2 + b^2 + 6*a*b*Cos[c + d
*x] + (3*a^2 - b^2)*Cos[2*(c + d*x)])*Sin[c + d*x]))/(12*d)
```

fricas [A] time = 0.65, size = 115, normalized size = 1.16

$$-\frac{6\,a^2dx\cos(dx+c)^3+3\,ab\cos(dx+c)^3\log(\sin(dx+c)+1)-3\,ab\cos(dx+c)^3\log(-\sin(dx+c)+1)-2\left(3\right.}{6\,d\cos(dx+c)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a*sin(d*x+c)+b*tan(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -1/6*(6*a^2*d*x*cos(d*x + c)^3 + 3*a*b*cos(d*x + c)^3*log(sin(d*x + c) + 1)
 - 3*a*b*cos(d*x + c)^3*log(-sin(d*x + c) + 1) - 2*(3*a*b*cos(d*x + c) + (3
*a^2 - b^2)*cos(d*x + c)^2 + b^2)*sin(d*x + c))/(d*cos(d*x + c)^3)
```

giac [A] time = 2.09, size = 158, normalized size = 1.60

$$-\frac{3\,(dx+c)a^2+3\,ab\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-3\,ab\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)+\frac{2\left(3\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-3\,ab\tan\left(\frac{1}{2}\right.}{}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a*sin(d*x+c)+b*tan(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/3*(3*(d*x + c)*a^2 + 3*a*b*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 3*a*b*lo
g(abs(tan(1/2*d*x + 1/2*c) - 1)) + 2*(3*a^2*tan(1/2*d*x + 1/2*c)^5 - 3*a*b*
tan(1/2*d*x + 1/2*c)^5 - 6*a^2*tan(1/2*d*x + 1/2*c)^3 + 4*b^2*tan(1/2*d*x +
 1/2*c)^3 + 3*a^2*tan(1/2*d*x + 1/2*c) + 3*a*b*tan(1/2*d*x + 1/2*c))/(tan(1
/2*d*x + 1/2*c)^2 - 1)^3)/d
```

maple [A] time = 0.11, size = 109, normalized size = 1.10

$$-a^2x+\frac{a^2\tan(dx+c)}{d}-\frac{a^2c}{d}+\frac{ab\left(\sin^3(dx+c)\right)}{d\cos(dx+c)^2}+\frac{ab\sin(dx+c)}{d}-\frac{ab\ln(\sec(dx+c)+\tan(dx+c))}{d}+\frac{b^2\left(\sin^3(dx\right.}{3d\cos(dx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2*(a*sin(d*x+c)+b*tan(d*x+c))^2,x)
```

```
[Out] -a^2*x+a^2*tan(d*x+c)/d-1/d*a^2*c+1/d*a*b*sin(d*x+c)^3/cos(d*x+c)^2+a*b*sin
(d*x+c)/d-1/d*a*b*ln(sec(d*x+c)+tan(d*x+c))+1/3/d*b^2*sin(d*x+c)^3/cos(d*x+
c)^3
```

maxima [A] time = 0.44, size = 82, normalized size = 0.83

$$\frac{2\,b^2\tan(dx+c)^3-6\,(dx+c-\tan(dx+c))a^2-3\,ab\left(\frac{2\,\sin(dx+c)}{\sin(dx+c)^2-1}+\log(\sin(dx+c)+1)-\log(\sin(dx+c)-1)\right.}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a*sin(d*x+c)+b*tan(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 1/6*(2*b^2*tan(d*x + c)^3 - 6*(d*x + c - tan(d*x + c))*a^2 - 3*a*b*(2*sin(d
*x + c)/(sin(d*x + c)^2 - 1) + log(sin(d*x + c) + 1) - log(sin(d*x + c) - 1
)))/d
```

mupad [B] time = 1.03, size = 227, normalized size = 2.29

$$-\frac{\frac{b^2\sin(3c+3dx)}{12}-\frac{b^2\sin(c+dx)}{4}-\frac{a^2\sin(3c+3dx)}{4}-\frac{a^2\sin(c+dx)}{4}+\frac{3a^2\cos(c+dx)\,\mathrm{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{dx}{2}\right)}{\cos\left(\frac{c}{2}+\frac{dx}{2}\right)}\right)}{2}+\frac{a^2\,\mathrm{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{dx}{2}\right)}{\cos\left(\frac{c}{2}+\frac{dx}{2}\right)}\right)\cos(3c+3dx)}{2}}{d\left(\frac{3\cos(c+dx)}{4}+\frac{\cos(3c+3dx)}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*sin(c + d*x) + b*tan(c + d*x))^2/cos(c + d*x)^2,x)
```

```
[Out] -((b^2*sin(3*c + 3*d*x))/12 - (b^2*sin(c + d*x))/4 - (a^2*sin(3*c + 3*d*x))
/4 - (a^2*sin(c + d*x))/4 + (3*a^2*cos(c + d*x)*atan(sin(c/2 + (d*x)/2)/cos
(c/2 + (d*x)/2)))/2 + (a^2*atan(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2))*cos(
3*c + 3*d*x))/2 - (a*b*sin(2*c + 2*d*x))/2 + (3*a*b*cos(c + d*x)*atanh(sin(
c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/2 + (a*b*atanh(sin(c/2 + (d*x)/2)/cos(c
/2 + (d*x)/2))*cos(3*c + 3*d*x))/2)/(d*((3*cos(c + d*x))/4 + cos(3*c + 3*d*
x)/4))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a\sin(c+dx)+b\tan(c+dx))^2\sec^2(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2*(a*sin(d*x+c)+b*tan(d*x+c))**2,x)
```

```
[Out] Integral((a*sin(c + d*x) + b*tan(c + d*x))**2*sec(c + d*x)**2, x)
```

3.242 $\int \sec^3(c+dx)(a\sin(c+dx)+b\tan(c+dx))^2\,dx$

Optimal. Leaf size=125

$$-\frac{(4a^2+b^2)\tanh^{-1}(\sin(c+dx))}{8d}+\frac{(2a^2-b^2)\tan(c+dx)\sec(c+dx)}{8d}-\frac{2ab\tan(c+dx)}{3d}+\frac{ab\tan(c+dx)\sec^2(c+}{6d}$$

```
[Out] -1/8*(4*a^2+b^2)*arctanh(sin(d*x+c))/d-2/3*a*b*tan(d*x+c)/d+1/8*(2*a^2-b^2)
*sec(d*x+c)*tan(d*x+c)/d+1/6*a*b*sec(d*x+c)^2*tan(d*x+c)/d+1/4*(b+a*cos(d*x
+c))^2*sec(d*x+c)^3*tan(d*x+c)/d
```

Rubi [A] time = 0.44, antiderivative size = 125, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.321, Rules used = {4397, 2889, 3048, 3031, 3021, 2748, 3767, 8, 3770}

$$-\frac{(4a^2+b^2)\tanh^{-1}(\sin(c+dx))}{8d}+\frac{(2a^2-b^2)\tan(c+dx)\sec(c+dx)}{8d}-\frac{2ab\tan(c+dx)}{3d}+\frac{ab\tan(c+dx)\sec^2(c+}{6d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^3*(a*Sin[c + d*x] + b*Tan[c + d*x])^2,x]
```

```
[Out] -((4*a^2 + b^2)*ArcTanh[Sin[c + d*x]])/(8*d) - (2*a*b*Tan[c + d*x])/(3*d) +
 ((2*a^2 - b^2)*Sec[c + d*x]*Tan[c + d*x])/(8*d) + (a*b*Sec[c + d*x]^2*Tan[
c + d*x])/(6*d) + ((b + a*Cos[c + d*x])^2*Sec[c + d*x]^3*Tan[c + d*x])/(4*d
)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2889

```
Int[cos[(e_.) + (f_.)*(x_)]^2*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[(d*Sin[e + f*x])^n*(a
 + b*Sin[e + f*x])^m*(1 - Sin[e + f*x]^2), x] /; FreeQ[{a, b, d, e, f, m, n
}, x] && NeQ[a^2 - b^2, 0] && (IGtQ[m, 0] || IntegersQ[2*m, 2*n])
```

Rule 3021

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2
 - a*b*B + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 1)*(
a^2 - b^2)), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*b
- a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B,
C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rule 3031

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> -Simp[((b*c - a*d)*(A*b^2 - a*b*B + a^2*C)*Cos[e
 + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b^2*f*(m + 1)*(a^2 - b^2)), x] - Dis
t[1/(b^2*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[b*(m +
 1)*((b*B - a*C)*(b*c - a*d) - A*b*(a*c - b*d)) + (b*B*(a^2*d + b^2*d*(m +
1) - a*b*c*(m + 2)) + (b*c - a*d)*(A*b^2*(m + 2) + C*(a^2 + b^2*(m + 1))))*
Sin[e + f*x] - b*C*d*(m + 1)*(a^2 - b^2)*Sin[e + f*x]^2, x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
 && LtQ[m, -1]
```

Rule 3048

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
-Simp[((c^2*C + A*d^2)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f
*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)
), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(
b*d*m + a*c*(n + 1)) + c*C*(b*c*m + a*d*(n + 1)) - (A*d*(a*d*(n + 2) - b*c*
(n + 1)) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] - b*(A*d
^2*(m + n + 2) + C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]^2, x], x], x]
/; FreeQ[{a, b, c, d, e, f, A, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2
, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 4397

```
Int[u_, x_Symbol] :> Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]
```

Rubi steps

$$\int \sec^3(c+dx)(a\sin(c+dx)+b\tan(c+dx))^2\,dx = \int (b+a\cos(c+dx))^2\sec^3(c+dx)\tan^2(c+dx)\,dx$$
$$= \int (b+a\cos(c+dx))^2\left(1-\cos^2(c+dx)\right)\sec^5(c+dx)\,dx$$
$$= \frac{(b+a\cos(c+dx))^2\sec^3(c+dx)\tan(c+dx)}{4d}+\frac{1}{4}\int (b+a$$
$$= \frac{ab\sec^2(c+dx)\tan(c+dx)}{6d}+\frac{(b+a\cos(c+dx))^2\sec^3(c+}{4d}$$
$$= \frac{\left(2a^2-b^2\right)\sec(c+dx)\tan(c+dx)}{8d}+\frac{ab\sec^2(c+dx)\tan(c}{6d}$$
$$= \frac{\left(2a^2-b^2\right)\sec(c+dx)\tan(c+dx)}{8d}+\frac{ab\sec^2(c+dx)\tan(c}{6d}$$
$$= -\frac{\left(4a^2+b^2\right)\tanh^{-1}(\sin(c+dx))}{8d}+\frac{\left(2a^2-b^2\right)\sec(c+dx)\,t}{8d}$$
$$= -\frac{\left(4a^2+b^2\right)\tanh^{-1}(\sin(c+dx))}{8d}-\frac{2ab\tan(c+dx)}{3d}+\frac{(2a^2}{}$$

Mathematica [B] time = 0.61, size = 336, normalized size = 2.69

$$\frac{\sec^4(c+dx)\left(12\left(4a^2+b^2\right)\cos(2(c+dx))\left(\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)-\log\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\right.\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^3*(a*Sin[c + d*x] + b*Tan[c + d*x])^2,x]
```

```
[Out] (Sec[c + d*x]^4*(36*a^2*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + 9*b^2*Lo
g[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + 12*(4*a^2 + b^2)*Cos[2*(c + d*x)]*
(Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] - Log[Cos[(c + d*x)/2] + Sin[(c +
 d*x)/2]]) + 3*(4*a^2 + b^2)*Cos[4*(c + d*x)]*(Log[Cos[(c + d*x)/2] - Sin[(
c + d*x)/2]] - Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]) - 36*a^2*Log[Cos[(
c + d*x)/2] + Sin[(c + d*x)/2]] - 9*b^2*Log[Cos[(c + d*x)/2] + Sin[(c + d*x
)/2]] + 24*a^2*Sin[c + d*x] + 42*b^2*Sin[c + d*x] + 32*a*b*Sin[2*(c + d*x)]
 + 24*a^2*Sin[3*(c + d*x)] - 6*b^2*Sin[3*(c + d*x)] - 16*a*b*Sin[4*(c + d*x
)]))/(192*d)
```

fricas [A] time = 0.58, size = 129, normalized size = 1.03

$$-\frac{3\left(4\,a^2+b^2\right)\cos\left(dx+c\right)^4\log\left(\sin\left(dx+c\right)+1\right)-3\left(4\,a^2+b^2\right)\cos\left(dx+c\right)^4\log\left(-\sin\left(dx+c\right)+1\right)+2\left(16\,ab\,c\right.}{48\,d\cos\left(dx+c\right)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a*sin(d*x+c)+b*tan(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -1/48*(3*(4*a^2 + b^2)*cos(d*x + c)^4*log(sin(d*x + c) + 1) - 3*(4*a^2 + b^
2)*cos(d*x + c)^4*log(-sin(d*x + c) + 1) + 2*(16*a*b*cos(d*x + c)^3 - 16*a*
b*cos(d*x + c) - 3*(4*a^2 - b^2)*cos(d*x + c)^2 - 6*b^2)*sin(d*x + c))/(d*c
os(d*x + c)^4)
```

giac [A] time = 1.89, size = 226, normalized size = 1.81

$$-\frac{3\left(4\,a^2+b^2\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-3\left(4\,a^2+b^2\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)-\frac{2\left(12\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7+3\,b^2\tan\left(\right.\right.}{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a*sin(d*x+c)+b*tan(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/24*(3*(4*a^2 + b^2)*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 3*(4*a^2 + b^2)
*log(abs(tan(1/2*d*x + 1/2*c) - 1)) - 2*(12*a^2*tan(1/2*d*x + 1/2*c)^7 + 3*
b^2*tan(1/2*d*x + 1/2*c)^7 - 12*a^2*tan(1/2*d*x + 1/2*c)^5 - 64*a*b*tan(1/2
*d*x + 1/2*c)^5 + 21*b^2*tan(1/2*d*x + 1/2*c)^5 - 12*a^2*tan(1/2*d*x + 1/2*
c)^3 + 64*a*b*tan(1/2*d*x + 1/2*c)^3 + 21*b^2*tan(1/2*d*x + 1/2*c)^3 + 12*a
^2*tan(1/2*d*x + 1/2*c) + 3*b^2*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)
^2 - 1)^4)/d
```

maple [A] time = 0.11, size = 169, normalized size = 1.35

$$\frac{a^2\left(\sin^3\left(dx+c\right)\right)}{2d\cos\left(dx+c\right)^2}+\frac{a^2\sin\left(dx+c\right)}{2d}-\frac{a^2\ln\left(\sec\left(dx+c\right)+\tan\left(dx+c\right)\right)}{2d}+\frac{2ab\left(\sin^3\left(dx+c\right)\right)}{3d\cos\left(dx+c\right)^3}+\frac{b^2\left(\sin^3\left(dx+c\right)\right)}{4d\cos\left(dx+c\right)^4}+\frac{b^2}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^3*(a*sin(d*x+c)+b*tan(d*x+c))^2,x)
```

```
[Out] 1/2/d*a^2*sin(d*x+c)^3/cos(d*x+c)^2+1/2*a^2*sin(d*x+c)/d-1/2/d*a^2*ln(sec(d
*x+c)+tan(d*x+c))+2/3/d*a*b*sin(d*x+c)^3/cos(d*x+c)^3+1/4/d*b^2*sin(d*x+c)^
3/cos(d*x+c)^4+1/8/d*b^2*sin(d*x+c)^3/cos(d*x+c)^2+1/8*b^2*sin(d*x+c)/d-1/8
/d*b^2*ln(sec(d*x+c)+tan(d*x+c))
```

maxima [A] time = 0.39, size = 129, normalized size = 1.03

$$\frac{32\,ab\tan\left(dx+c\right)^3+3\,b^2\left(\frac{2\left(\sin(dx+c)^3+\sin(dx+c)\right)}{\sin(dx+c)^4-2\,\sin(dx+c)^2+1}-\log\left(\sin\left(dx+c\right)+1\right)+\log\left(\sin\left(dx+c\right)-1\right)\right)-12\,a^2\left(\frac{2\,\sin(d}{\sin(dx+}\right.}{48\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a*sin(d*x+c)+b*tan(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 1/48*(32*a*b*tan(d*x + c)^3 + 3*b^2*(2*(sin(d*x + c)^3 + sin(d*x + c))/(sin
(d*x + c)^4 - 2*sin(d*x + c)^2 + 1) - log(sin(d*x + c) + 1) + log(sin(d*x +
 c) - 1)) - 12*a^2*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) + log(sin(d*x + c)
+ 1) - log(sin(d*x + c) - 1)))/d
```

mupad [B] time = 3.27, size = 177, normalized size = 1.42

$$\frac{\left(a^2+\frac{b^2}{4}\right)\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7+\left(-a^2-\frac{16\,a\,b}{3}+\frac{7\,b^2}{4}\right)\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5+\left(-a^2+\frac{16\,a\,b}{3}+\frac{7\,b^2}{4}\right)\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3+\left(a^2+\frac{b^2}{4}\right)\mathrm{t}}{d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8-4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6+6\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4-4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*sin(c + d*x) + b*tan(c + d*x))^2/cos(c + d*x)^3,x)
```

```
[Out] (tan(c/2 + (d*x)/2)^3*((16*a*b)/3 - a^2 + (7*b^2)/4) + tan(c/2 + (d*x)/2)*(
a^2 + b^2/4) + tan(c/2 + (d*x)/2)^7*(a^2 + b^2/4) - tan(c/2 + (d*x)/2)^5*((
16*a*b)/3 + a^2 - (7*b^2)/4))/(d*(6*tan(c/2 + (d*x)/2)^4 - 4*tan(c/2 + (d*x
)/2)^2 - 4*tan(c/2 + (d*x)/2)^6 + tan(c/2 + (d*x)/2)^8 + 1)) - (atanh(tan(c
/2 + (d*x)/2))*(a^2 + b^2/4))/d
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a\sin\left(c+dx\right)+b\tan\left(c+dx\right)\right)^2\sec^3\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**3*(a*sin(d*x+c)+b*tan(d*x+c))**2,x)
```

```
[Out] Integral((a*sin(c + d*x) + b*tan(c + d*x))**2*sec(c + d*x)**3, x)
```

3.243 $\int \cos^3(c+dx)(a\sin(c+dx)+b\tan(c+dx))^3\,dx$

Optimal. Leaf size=77

$$\frac{(a\cos(c+dx)+b)^6}{6a^3d}-\frac{2b(a\cos(c+dx)+b)^5}{5a^3d}-\frac{\left(a^2-b^2\right)(a\cos(c+dx)+b)^4}{4a^3d}$$

```
[Out] -1/4*(a^2-b^2)*(b+a*cos(d*x+c))^4/a^3/d-2/5*b*(b+a*cos(d*x+c))^5/a^3/d+1/6*
(b+a*cos(d*x+c))^6/a^3/d
```

Rubi [A] time = 0.19, antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.107, Rules used = {4397, 2668, 697}

$$-\frac{\left(a^2-b^2\right)(a\cos(c+dx)+b)^4}{4a^3d}+\frac{(a\cos(c+dx)+b)^6}{6a^3d}-\frac{2b(a\cos(c+dx)+b)^5}{5a^3d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^3*(a*Sin[c + d*x] + b*Tan[c + d*x])^3,x]
```

```
[Out] -((a^2 - b^2)*(b + a*Cos[c + d*x])^4)/(4*a^3*d) - (2*b*(b + a*Cos[c + d*x])
^5)/(5*a^3*d) + (b + a*Cos[c + d*x])^6/(6*a^3*d)
```

Rule 697

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m},
 x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0]
```

Rule 2668

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rule 4397

```
Int[u_, x_Symbol] :> Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]
```

Rubi steps

$$\int \cos^3(c+dx)(a\sin(c+dx)+b\tan(c+dx))^3\,dx = \int (b+a\cos(c+dx))^3\sin^3(c+dx)\,dx$$
$$= -\frac{\text{Subst}\left(\int (b+x)^3\left(a^2-x^2\right)\,dx, x, a\cos(c+dx)\right)}{a^3d}$$
$$= -\frac{\text{Subst}\left(\int \left(\left(a^2-b^2\right)(b+x)^3+2b(b+x)^4-(b+x)^5\right)\,dx,$$
$$= -\frac{\left(a^2-b^2\right)(b+a\cos(c+dx))^4}{4a^3d} - \frac{2b(b+a\cos(c+dx))^5}{5a^3d} +$$

Mathematica [A] time = 0.23, size = 114, normalized size = 1.48

$$\frac{-45\left(a^3+8ab^2\right)\cos(2(c+dx))+5a^3\cos(6(c+dx))-360b\left(a^2+2b^2\right)\cos(c+dx)-60a^2b\cos(3(c+dx))+36a}{960d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^3*(a*Sin[c + d*x] + b*Tan[c + d*x])^3,x]
```

```
[Out] (-360*b*(a^2 + 2*b^2)*Cos[c + d*x] - 45*(a^3 + 8*a*b^2)*Cos[2*(c + d*x)] -
60*a^2*b*Cos[3*(c + d*x)] + 80*b^3*Cos[3*(c + d*x)] + 90*a*b^2*Cos[4*(c + d
*x)] + 36*a^2*b*Cos[5*(c + d*x)] + 5*a^3*Cos[6*(c + d*x)])/(960*d)
```

fricas [A] time = 1.27, size = 100, normalized size = 1.30

$$\frac{10\,a^3\cos(dx+c)^6+36\,a^2b\cos(dx+c)^5-90\,ab^2\cos(dx+c)^2-15\left(a^3-3\,ab^2\right)\cos(dx+c)^4-60\,b^3\cos(dx+}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a*sin(d*x+c)+b*tan(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/60*(10*a^3*cos(d*x + c)^6 + 36*a^2*b*cos(d*x + c)^5 - 90*a*b^2*cos(d*x +
c)^2 - 15*(a^3 - 3*a*b^2)*cos(d*x + c)^4 - 60*b^3*cos(d*x + c) - 20*(3*a^2*
b - b^3)*cos(d*x + c)^3)/d
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a*sin(d*x+c)+b*tan(d*x+c))^3,x, algorithm="giac")
```

```
[Out] Timed out
```

maple [A] time = 0.11, size = 109, normalized size = 1.42

$$\frac{a^3\left(-\frac{(\sin^2(dx+c))(\cos^4(dx+c))}{6}-\frac{(\cos^4(dx+c))}{12}\right)+3a^2b\left(-\frac{(\sin^2(dx+c))(\cos^3(dx+c))}{5}-\frac{2(\cos^3(dx+c))}{15}\right)+\frac{3a\,b^2(\sin^4(dx+c))}{4}-\frac{b^3(2+\sin^2(}{}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3*(a*sin(d*x+c)+b*tan(d*x+c))^3,x)
```

```
[Out] 1/d*(a^3*(-1/6*sin(d*x+c)^2*cos(d*x+c)^4-1/12*cos(d*x+c)^4)+3*a^2*b*(-1/5*s
in(d*x+c)^2*cos(d*x+c)^3-2/15*cos(d*x+c)^3)+3/4*a*b^2*sin(d*x+c)^4-1/3*b^3*
(2+sin(d*x+c)^2)*cos(d*x+c))
```

maxima [A] time = 0.38, size = 95, normalized size = 1.23

$$\frac{45\,ab^2\sin(dx+c)^4-5\left(2\,\sin(dx+c)^6-3\,\sin(dx+c)^4\right)a^3+12\left(3\,\cos(dx+c)^5-5\,\cos(dx+c)^3\right)a^2b+20\,(\mathrm{co}}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a*sin(d*x+c)+b*tan(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/60*(45*a*b^2*sin(d*x + c)^4 - 5*(2*sin(d*x + c)^6 - 3*sin(d*x + c)^4)*a^3
 + 12*(3*cos(d*x + c)^5 - 5*cos(d*x + c)^3)*a^2*b + 20*(cos(d*x + c)^3 - 3*
cos(d*x + c))*b^3)/d
```

mupad [B] time = 0.80, size = 149, normalized size = 1.94

$$\frac{32\,a^3}{3\,d\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+1\right)^6}+\frac{4\,(a-b)^3}{d\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+1\right)^2}-\frac{32\,a^2\,(5\,a-3\,b)}{5\,d\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+1\right)^5}-\frac{8\,(a-b)^2\,(7\,a-b)}{3\,d\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+1\right)^3}+\frac{12\,a\,(3\,a}{d\left(\tan\left(\frac{c}{2}\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^3*(a*sin(c + d*x) + b*tan(c + d*x))^3,x)
```

```
[Out] (32*a^3)/(3*d*(tan(c/2 + (d*x)/2)^2 + 1)^6) + (4*(a - b)^3)/(d*(tan(c/2 + (
d*x)/2)^2 + 1)^2) - (32*a^2*(5*a - 3*b))/(5*d*(tan(c/2 + (d*x)/2)^2 + 1)^5)
 - (8*(a - b)^2*(7*a - b))/(3*d*(tan(c/2 + (d*x)/2)^2 + 1)^3) + (12*a*(3*a^
2 - 4*a*b + b^2))/(d*(tan(c/2 + (d*x)/2)^2 + 1)^4)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a\sin(c+dx)+b\tan(c+dx))^3\cos^3(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3*(a*sin(d*x+c)+b*tan(d*x+c))**3,x)
```

```
[Out] Integral((a*sin(c + d*x) + b*tan(c + d*x))**3*cos(c + d*x)**3, x)
```

3.244 $\int \cos^2(c+dx)(a\sin(c+dx)+b\tan(c+dx))^3\,dx$

Optimal. Leaf size=120

$$\frac{a^3\cos^5(c+dx)}{5d}-\frac{a\left(a^2-3b^2\right)\cos^3(c+dx)}{3d}-\frac{b\left(3a^2-b^2\right)\cos^2(c+dx)}{2d}+\frac{3a^2b\cos^4(c+dx)}{4d}-\frac{3ab^2\cos(c+dx)}{d}-\frac{b^3\,\mathrm{lo}}{}$$

```
[Out] -3*a*b^2*cos(d*x+c)/d-1/2*b*(3*a^2-b^2)*cos(d*x+c)^2/d-1/3*a*(a^2-3*b^2)*co
s(d*x+c)^3/d+3/4*a^2*b*cos(d*x+c)^4/d+1/5*a^3*cos(d*x+c)^5/d-b^3*ln(cos(d*x
+c))/d
```

Rubi [A] time = 0.19, antiderivative size = 120, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {4397, 2837, 12, 894}

$$-\frac{a\left(a^2-3b^2\right)\cos^3(c+dx)}{3d}-\frac{b\left(3a^2-b^2\right)\cos^2(c+dx)}{2d}+\frac{3a^2b\cos^4(c+dx)}{4d}+\frac{a^3\cos^5(c+dx)}{5d}-\frac{3ab^2\cos(c+dx)}{d}-\frac{b^3}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2*(a*Sin[c + d*x] + b*Tan[c + d*x])^3,x]
```

```
[Out] (-3*a*b^2*Cos[c + d*x])/d - (b*(3*a^2 - b^2)*Cos[c + d*x]^2)/(2*d) - (a*(a^
2 - 3*b^2)*Cos[c + d*x]^3)/(3*d) + (3*a^2*b*Cos[c + d*x]^4)/(4*d) + (a^3*Co
s[c + d*x]^5)/(5*d) - (b^3*Log[Cos[c + d*x]])/d
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 894

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ
[m, 0] && ILtQ[n, 0]))
```

Rule 2837

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^m*(c + (d*x)/b)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*S
in[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/
```

```
2] && NeQ[a^2 - b^2, 0]
```

Rule 4397

```
Int[u_, x_Symbol] :> Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]
```

Rubi steps

$$\int \cos^2(c+dx)(a\sin(c+dx)+b\tan(c+dx))^3\,dx = \int (b+a\cos(c+dx))^3\sin^2(c+dx)\tan(c+dx)\,dx$$
$$= -\frac{\text{Subst}\left(\int \frac{a(b+x)^3(a^2-x^2)}{x}\,dx, x, a\cos(c+dx)\right)}{a^3d}$$
$$= -\frac{\text{Subst}\left(\int \frac{(b+x)^3(a^2-x^2)}{x}\,dx, x, a\cos(c+dx)\right)}{a^2d}$$
$$= -\frac{\text{Subst}\left(\int \left(3a^2b^2+\frac{a^2b^3}{x}+b\left(3a^2-b^2\right)x+\left(a^2-3b^2\right)x^2 -\right.\right.}{a^2d}$$
$$= -\frac{3ab^2\cos(c+dx)}{d}-\frac{b\left(3a^2-b^2\right)\cos^2(c+dx)}{2d}-\frac{a\left(a^2-3b\right.}{}$$

Mathematica [A] time = 0.19, size = 106, normalized size = 0.88

$$-\frac{-\frac{1}{5}a^3\cos^5(c+dx)+\frac{1}{3}a\left(a^2-3b^2\right)\cos^3(c+dx)+\frac{1}{2}b\left(3a^2-b^2\right)\cos^2(c+dx)-\frac{3}{4}a^2b\cos^4(c+dx)+3ab^2\cos(c}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2*(a*Sin[c + d*x] + b*Tan[c + d*x])^3,x]
```

```
[Out] -((3*a*b^2*Cos[c + d*x] + (b*(3*a^2 - b^2)*Cos[c + d*x]^2)/2 + (a*(a^2 - 3*
b^2)*Cos[c + d*x]^3)/3 - (3*a^2*b*Cos[c + d*x]^4)/4 - (a^3*Cos[c + d*x]^5)/
5 + b^3*Log[Cos[c + d*x]])/d)
```

fricas [A] time = 0.69, size = 101, normalized size = 0.84

$$\frac{12\,a^3\cos(dx+c)^5+45\,a^2b\cos(dx+c)^4-180\,ab^2\cos(dx+c)-20\left(a^3-3\,ab^2\right)\cos(dx+c)^3-60\,b^3\log(-\cos}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a*sin(d*x+c)+b*tan(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/60*(12*a^3*cos(d*x + c)^5 + 45*a^2*b*cos(d*x + c)^4 - 180*a*b^2*cos(d*x +
 c) - 20*(a^3 - 3*a*b^2)*cos(d*x + c)^3 - 60*b^3*log(-cos(d*x + c)) - 30*(3
*a^2*b - b^3)*cos(d*x + c)^2)/d
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a*sin(d*x+c)+b*tan(d*x+c))^3,x, algorithm="giac")
```

```
[Out] Timed out
```

maple [A] time = 0.11, size = 128, normalized size = 1.07

$$-\frac{a^3\left(\sin^2(dx+c)\right)\left(\cos^3(dx+c)\right)}{5d}-\frac{2a^3\left(\cos^3(dx+c)\right)}{15d}+\frac{3a^2b\left(\sin^4(dx+c)\right)}{4d}-\frac{\cos(dx+c)\left(\sin^2(dx+c)\right)a\,b^2}{d}-2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(a*sin(d*x+c)+b*tan(d*x+c))^3,x)
```

```
[Out] -1/5/d*a^3*sin(d*x+c)^2*cos(d*x+c)^3-2/15*a^3*cos(d*x+c)^3/d+3/4/d*a^2*b*si
n(d*x+c)^4-1/d*cos(d*x+c)*sin(d*x+c)^2*a*b^2-2*a*b^2*cos(d*x+c)/d-1/2/d*b^3
*sin(d*x+c)^2-b^3*ln(cos(d*x+c))/d
```

maxima [A] time = 0.46, size = 94, normalized size = 0.78

$$\frac{45\,a^2b\sin(dx+c)^4+4\left(3\,\cos(dx+c)^5-5\,\cos(dx+c)^3\right)a^3+60\left(\cos(dx+c)^3-3\,\cos(dx+c)\right)ab^2-30\left(\sin(\right.}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a*sin(d*x+c)+b*tan(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/60*(45*a^2*b*sin(d*x + c)^4 + 4*(3*cos(d*x + c)^5 - 5*cos(d*x + c)^3)*a^3
 + 60*(cos(d*x + c)^3 - 3*cos(d*x + c))*a*b^2 - 30*(sin(d*x + c)^2 + log(si
n(d*x + c)^2 - 1))*b^3)/d
```

mupad [B] time = 0.79, size = 237, normalized size = 1.98

$$\frac{40\,a^3\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6}{3\,d}-\frac{4\,a^3\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4}{d}-\frac{16\,a^3\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8}{d}+\frac{32\,a^3\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{10}}{5\,d}-\frac{2\,b^3\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{d}+\frac{2\,b^3\cos}{}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(c + d*x)^2*(a*sin(c + d*x) + b*tan(c + d*x))^3,x)

[Out] (40*a^3*cos(c/2 + (d*x)/2)^6)/(3*d) - (4*a^3*cos(c/2 + (d*x)/2)^4)/d - (16*a^3*cos(c/2 + (d*x)/2)^8)/d + (32*a^3*cos(c/2 + (d*x)/2)^10)/(5*d) - (2*b^3*cos(c/2 + (d*x)/2)^2)/d + (2*b^3*cos(c/2 + (d*x)/2)^4)/d + (2*b^3*atanh(1/cos(c/2 + (d*x)/2)^2 - 1))/d - (12*a*b^2*cos(c/2 + (d*x)/2)^4)/d + (12*a^2*b*cos(c/2 + (d*x)/2)^4)/d + (8*a*b^2*cos(c/2 + (d*x)/2)^6)/d - (24*a^2*b*cos(c/2 + (d*x)/2)^6)/d + (12*a^2*b*cos(c/2 + (d*x)/2)^8)/d

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a\sin(c+dx) + b\tan(c+dx))^3 \cos^2(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)**2*(a*sin(d*x+c)+b*tan(d*x+c))**3,x)

[Out] Integral((a*sin(c + d*x) + b*tan(c + d*x))**3*cos(c + d*x)**2, x)

3.245 $\int \cos(c+dx)(a\sin(c+dx)+b\tan(c+dx))^3\,dx$

Optimal. Leaf size=112

$$\frac{a^3\cos^4(c+dx)}{4d}-\frac{a\left(a^2-3b^2\right)\cos^2(c+dx)}{2d}-\frac{b\left(3a^2-b^2\right)\cos(c+dx)}{d}+\frac{a^2b\cos^3(c+dx)}{d}-\frac{3ab^2\log(\cos(c+dx))}{d}+$$

```
[Out] -b*(3*a^2-b^2)*cos(d*x+c)/d-1/2*a*(a^2-3*b^2)*cos(d*x+c)^2/d+a^2*b*cos(d*x+
c)^3/d+1/4*a^3*cos(d*x+c)^4/d-3*a*b^2*ln(cos(d*x+c))/d+b^3*sec(d*x+c)/d
```

Rubi [A] time = 0.18, antiderivative size = 112, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {4397, 2837, 12, 894}

$$-\frac{a\left(a^2-3b^2\right)\cos^2(c+dx)}{2d}-\frac{b\left(3a^2-b^2\right)\cos(c+dx)}{d}+\frac{a^2b\cos^3(c+dx)}{d}+\frac{a^3\cos^4(c+dx)}{4d}-\frac{3ab^2\log(\cos(c+dx))}{d}+$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]*(a*Sin[c + d*x] + b*Tan[c + d*x])^3,x]
```

```
[Out] -((b*(3*a^2 - b^2)*Cos[c + d*x])/d) - (a*(a^2 - 3*b^2)*Cos[c + d*x]^2)/(2*d
) + (a^2*b*Cos[c + d*x]^3)/d + (a^3*Cos[c + d*x]^4)/(4*d) - (3*a*b^2*Log[Co
s[c + d*x]])/d + (b^3*Sec[c + d*x])/d
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 894

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ
[m, 0] && ILtQ[n, 0]))
```

Rule 2837

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^m*(c + (d*x)/b)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*S
in[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/
2] && NeQ[a^2 - b^2, 0]
```

Rule 4397

```
Int[u_, x_Symbol] :> Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]
```

Rubi steps

$$\int \cos(c+dx)(a\sin(c+dx)+b\tan(c+dx))^3\,dx = \int (b+a\cos(c+dx))^3\sin(c+dx)\tan^2(c+dx)\,dx$$
$$= -\frac{\text{Subst}\left(\int \frac{a^2(b+x)^3(a^2-x^2)}{x^2}\,dx, x, a\cos(c+dx)\right)}{a^3d}$$
$$= -\frac{\text{Subst}\left(\int \frac{(b+x)^3(a^2-x^2)}{x^2}\,dx, x, a\cos(c+dx)\right)}{ad}$$
$$= -\frac{\text{Subst}\left(\int \left(3a^2b\left(1-\frac{b^2}{3a^2}\right)+\frac{a^2b^3}{x^2}+\frac{3a^2b^2}{x}+\left(a^2-3b^2\right)x-3b\right.\right.}{ad}$$
$$= -\frac{b\left(3a^2-b^2\right)\cos(c+dx)}{d}-\frac{a\left(a^2-3b^2\right)\cos^2(c+dx)}{2d}+\frac{a^2b}{}$$

Mathematica [A] time = 0.27, size = 98, normalized size = 0.88

$$\frac{-4\left(a^3-6ab^2\right)\cos(2(c+dx))+a^3\cos(4(c+dx))+8b\left(4b^2-9a^2\right)\cos(c+dx)+8a^2b\cos(3(c+dx))-96ab^2\log}{32d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]*(a*Sin[c + d*x] + b*Tan[c + d*x])^3,x]
```

```
[Out] (8*b*(-9*a^2 + 4*b^2)*Cos[c + d*x] - 4*(a^3 - 6*a*b^2)*Cos[2*(c + d*x)] + 8
*a^2*b*Cos[3*(c + d*x)] + a^3*Cos[4*(c + d*x)] - 96*a*b^2*Log[Cos[c + d*x]]
 + 32*b^3*Sec[c + d*x])/(32*d)
```

fricas [A] time = 0.61, size = 128, normalized size = 1.14

$$\frac{8\,a^3\cos(dx+c)^5+32\,a^2b\cos(dx+c)^4-96\,ab^2\cos(dx+c)\log(-\cos(dx+c))-16\left(a^3-3\,ab^2\right)\cos(dx+c)^3}{32\,d\cos(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a*sin(d*x+c)+b*tan(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/32*(8*a^3*cos(d*x + c)^5 + 32*a^2*b*cos(d*x + c)^4 - 96*a*b^2*cos(d*x + c
)*log(-cos(d*x + c)) - 16*(a^3 - 3*a*b^2)*cos(d*x + c)^3 + 32*b^3 - 32*(3*a
^2*b - b^3)*cos(d*x + c)^2 + (5*a^3 - 24*a*b^2)*cos(d*x + c))/(d*cos(d*x +
c))
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a*sin(d*x+c)+b*tan(d*x+c))^3,x, algorithm="giac")
```

```
[Out] Timed out
```

maple [A] time = 0.08, size = 147, normalized size = 1.31

$$\frac{a^3\left(\sin^4\left(dx+c\right)\right)}{4d}-\frac{\cos\left(dx+c\right)\left(\sin^2\left(dx+c\right)\right)a^2b}{d}-\frac{2a^2b\cos\left(dx+c\right)}{d}-\frac{3a\,b^2\left(\sin^2\left(dx+c\right)\right)}{2d}-\frac{3a\,b^2\ln\left(\cos\left(dx+c\right.\right.}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(a*sin(d*x+c)+b*tan(d*x+c))^3,x)
```

```
[Out] 1/4/d*a^3*sin(d*x+c)^4-1/d*cos(d*x+c)*sin(d*x+c)^2*a^2*b-2*a^2*b*cos(d*x+c)
/d-3/2/d*a*b^2*sin(d*x+c)^2-3*a*b^2*ln(cos(d*x+c))/d+1/d*b^3*sin(d*x+c)^4/c
os(d*x+c)+1/d*b^3*cos(d*x+c)*sin(d*x+c)^2+2*b^3*cos(d*x+c)/d
```

maxima [A] time = 0.35, size = 87, normalized size = 0.78

$$\frac{a^3\sin\left(dx+c\right)^4+4\left(\cos\left(dx+c\right)^3-3\cos\left(dx+c\right)\right)a^2b-6\left(\sin\left(dx+c\right)^2+\log\left(\sin\left(dx+c\right)^2-1\right)\right)ab^2+4\,b^3\left(\frac{}{\cos(}\right.}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a*sin(d*x+c)+b*tan(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/4*(a^3*sin(d*x + c)^4 + 4*(cos(d*x + c)^3 - 3*cos(d*x + c))*a^2*b - 6*(si
n(d*x + c)^2 + log(sin(d*x + c)^2 - 1))*a*b^2 + 4*b^3*(1/cos(d*x + c) + cos
(d*x + c)))/d
```

mupad [B] time = 4.18, size = 225, normalized size = 2.01

$$\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4\,\left(4\,a^3+4\,a^2\,b-6\,a\,b^2+12\,b^3\right)-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\,\left(12\,a^2\,b+6\,a\,b^2-12\,b^3\right)+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6\,\left(-4\,a^3+\right.}{d\,\left(-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{10}-3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8-2\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6+2\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)*(a*sin(c + d*x) + b*tan(c + d*x))^3,x)
```

```
[Out] (tan(c/2 + (d*x)/2)^4*(4*a^2*b - 6*a*b^2 + 4*a^3 + 12*b^3) - tan(c/2 + (d*x
)/2)^2*(6*a*b^2 + 12*a^2*b - 12*b^3) + tan(c/2 + (d*x)/2)^6*(6*a*b^2 + 12*a
^2*b - 4*a^3 + 4*b^3) - 4*a^2*b + 4*b^3 + 6*a*b^2*tan(c/2 + (d*x)/2)^8)/(d*
(3*tan(c/2 + (d*x)/2)^2 + 2*tan(c/2 + (d*x)/2)^4 - 2*tan(c/2 + (d*x)/2)^6 -
 3*tan(c/2 + (d*x)/2)^8 - tan(c/2 + (d*x)/2)^10 + 1)) + (6*a*b^2*atanh(tan(
c/2 + (d*x)/2)^2))/d
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a\sin(c+dx)+b\tan(c+dx))^3\cos(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a*sin(d*x+c)+b*tan(d*x+c))**3,x)
```

```
[Out] Integral((a*sin(c + d*x) + b*tan(c + d*x))**3*cos(c + d*x), x)
```

3.246 $\int (a\sin(c+dx)+b\tan(c+dx))^3\,dx$

Optimal. Leaf size=116

$$\frac{a^3\cos^3(c+dx)}{3d}-\frac{a\left(a^2-3b^2\right)\cos(c+dx)}{d}-\frac{b\left(3a^2-b^2\right)\log(\cos(c+dx))}{d}+\frac{3a^2b\cos^2(c+dx)}{2d}+\frac{3ab^2\sec(c+dx)}{d}+$$

```
[Out] -a*(a^2-3*b^2)*cos(d*x+c)/d+3/2*a^2*b*cos(d*x+c)^2/d+1/3*a^3*cos(d*x+c)^3/d
-b*(3*a^2-b^2)*ln(cos(d*x+c))/d+3*a*b^2*sec(d*x+c)/d+1/2*b^3*sec(d*x+c)^2/d
```

Rubi [A] time = 0.10, antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {4397, 2721, 894}

$$-\frac{a\left(a^2-3b^2\right)\cos(c+dx)}{d}-\frac{b\left(3a^2-b^2\right)\log(\cos(c+dx))}{d}+\frac{3a^2b\cos^2(c+dx)}{2d}+\frac{a^3\cos^3(c+dx)}{3d}+\frac{3ab^2\sec(c+dx)}{d}+$$

Antiderivative was successfully verified.

```
[In] Int[(a*Sin[c + d*x] + b*Tan[c + d*x])^3,x]
```

```
[Out] -((a*(a^2 - 3*b^2)*Cos[c + d*x])/d) + (3*a^2*b*Cos[c + d*x]^2)/(2*d) + (a^3
*Cos[c + d*x]^3)/(3*d) - (b*(3*a^2 - b^2)*Log[Cos[c + d*x]])/d + (3*a*b^2*S
ec[c + d*x])/d + (b^3*Sec[c + d*x]^2)/(2*d)
```

Rule 894

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ
[m, 0] && ILtQ[n, 0]))
```

Rule 2721

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Dist[1/f, Subst[Int[(x^p*(a + x)^m)/(b^2 - x^2)^((p + 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b^
2, 0] && IntegerQ[(p + 1)/2]
```

Rule 4397

```
Int[u_, x_Symbol] :> Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]
```

Rubi steps

$$\int (a\sin(c+dx)+b\tan(c+dx))^3\,dx = \int (b+a\cos(c+dx))^3\tan^3(c+dx)\,dx$$
$$= -\frac{\text{Subst}\left(\int \frac{(b+x)^3(a^2-x^2)}{x^3}\,dx, x, a\cos(c+dx)\right)}{d}$$
$$= -\frac{\text{Subst}\left(\int \left(a^2\left(1-\frac{3b^2}{a^2}\right)+\frac{a^2b^3}{x^3}+\frac{3a^2b^2}{x^2}+\frac{3a^2b-b^3}{x}-3bx-x^2\right)dx, x, a\cos\right.}{d}$$
$$= -\frac{a\left(a^2-3b^2\right)\cos(c+dx)}{d}+\frac{3a^2b\cos^2(c+dx)}{2d}+\frac{a^3\cos^3(c+dx)}{3d}-\frac{b\left(3\right.}{}$$

Mathematica [A] time = 0.29, size = 102, normalized size = 0.88

$$\frac{a^3\cos(3(c+dx))-9a\left(a^2-4b^2\right)\cos(c+dx)+9a^2b\cos(2(c+dx))-36a^2b\log(\cos(c+dx))+36ab^2\sec(c+dx}{12d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Sin[c + d*x] + b*Tan[c + d*x])^3,x]
```

```
[Out] (-9*a*(a^2 - 4*b^2)*Cos[c + d*x] + 9*a^2*b*Cos[2*(c + d*x)] + a^3*Cos[3*(c
+ d*x)] - 36*a^2*b*Log[Cos[c + d*x]] + 12*b^3*Log[Cos[c + d*x]] + 36*a*b^2*
Sec[c + d*x] + 6*b^3*Sec[c + d*x]^2)/(12*d)
```

fricas [A] time = 0.61, size = 123, normalized size = 1.06

$$\frac{4\,a^3\cos(dx+c)^5+18\,a^2b\cos(dx+c)^4-9\,a^2b\cos(dx+c)^2+36\,ab^2\cos(dx+c)-12\left(a^3-3\,ab^2\right)\cos(dx+c)}{12\,d\cos(dx+c)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sin(d*x+c)+b*tan(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/12*(4*a^3*cos(d*x + c)^5 + 18*a^2*b*cos(d*x + c)^4 - 9*a^2*b*cos(d*x + c)
^2 + 36*a*b^2*cos(d*x + c) - 12*(a^3 - 3*a*b^2)*cos(d*x + c)^3 - 12*(3*a^2*
b - b^3)*cos(d*x + c)^2*log(-cos(d*x + c)) + 6*b^3)/(d*cos(d*x + c)^2)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a*sin(d*x+c)+b*tan(d*x+c))^3,x, algorithm="giac")

[Out] Exception raised: TypeError >> An error occurred running a Giac command:INPUT:sage2:=int(sage0,x):;OUTPUT:Modgcd: no suitable evaluation pointindex.cc index_m operator + Error: Bad Argument ValueUnable to check sign: (2*pi/x/2)>(-2*pi/x/2)Evaluation time: 62.4Done

maple [A] time = 0.08, size = 164, normalized size = 1.41

$$-\frac{\cos\left(dx+c\right)\left(\sin^{2}\left(dx+c\right)\right)a^{3}}{3d}-\frac{2a^{3}\cos\left(dx+c\right)}{3d}-\frac{3a^{2}b\left(\sin^{2}\left(dx+c\right)\right)}{2d}-\frac{3a^{2}b\ln\left(\cos\left(dx+c\right)\right)}{d}+\frac{3a\,b^{2}\left(\sin^{4}\left(dx+\right.\right.}{d\cos\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a*sin(d*x+c)+b*tan(d*x+c))^3,x)

[Out] -1/3/d*cos(d*x+c)*sin(d*x+c)^2*a^3-2/3*a^3*cos(d*x+c)/d-3/2/d*a^2*b*sin(d*x+c)^2-3*a^2*b*ln(cos(d*x+c))/d+3/d*a*b^2*sin(d*x+c)^4/cos(d*x+c)+3/d*cos(d*x+c)*sin(d*x+c)^2*a*b^2+6*a*b^2*cos(d*x+c)/d+1/2/d*b^3*tan(d*x+c)^2+b^3*ln(cos(d*x+c))/d

maxima [A] time = 0.42, size = 113, normalized size = 0.97

$$\frac{\left(\cos\left(dx+c\right)^{3}-3\,\cos\left(dx+c\right)\right)a^{3}}{3\,d}-\frac{3\left(\sin\left(dx+c\right)^{2}+\log\left(\sin\left(dx+c\right)^{2}-1\right)\right)a^{2}b}{2\,d}-\frac{b^{3}\left(\frac{1}{\sin(dx+c)^{2}-1}-\log\left(\sin\left(dx+\right.\right.\right.}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a*sin(d*x+c)+b*tan(d*x+c))^3,x, algorithm="maxima")

[Out] 1/3*(cos(d*x + c)^3 - 3*cos(d*x + c))*a^3/d - 3/2*(sin(d*x + c)^2 + log(sin(d*x + c)^2 - 1))*a^2*b/d - 1/2*b^3*(1/(sin(d*x + c)^2 - 1) - log(sin(d*x + c)^2 - 1))/d + 3*a*b^2*(1/cos(d*x + c) + cos(d*x + c))/d

mupad [B] time = 4.52, size = 219, normalized size = 1.89

$$\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\,\left(-\frac{4\,a^3}{3}-6\,a^2\,b+12\,a\,b^2+2\,b^3\right)-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6\,\left(4\,a^3-6\,a^2\,b+12\,a\,b^2-6\,b^3\right)+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4\,\left(\cdots\right.}{d\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2-1\right)^2\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+\cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*sin(c + d*x) + b*tan(c + d*x))^3,x)
```

```
[Out] (tan(c/2 + (d*x)/2)^2*(12*a*b^2 - 6*a^2*b - (4*a^3)/3 + 2*b^3) - tan(c/2 +
(d*x)/2)^6*(12*a*b^2 - 6*a^2*b + 4*a^3 - 6*b^3) + tan(c/2 + (d*x)/2)^4*(6*a
^2*b - 12*a*b^2 + (20*a^3)/3 + 6*b^3) + 12*a*b^2 - tan(c/2 + (d*x)/2)^8*(6*
a^2*b - 2*b^3) - (4*a^3)/3)/(d*(tan(c/2 + (d*x)/2)^2 - 1)^2*(tan(c/2 + (d*x
)/2)^2 + 1)^3) - (2*b^3*atanh(tan(c/2 + (d*x)/2)^2) - 6*a^2*b*atanh(tan(c/2
 + (d*x)/2)^2))/d
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a\sin(c+dx)+b\tan(c+dx))^3\ dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sin(d*x+c)+b*tan(d*x+c))**3,x)
```

```
[Out] Integral((a*sin(c + d*x) + b*tan(c + d*x))**3, x)
```

3.247 $\int \sec(c+dx)(a\sin(c+dx)+b\tan(c+dx))^3\,dx$

Optimal. Leaf size=115

$$\frac{a^3\cos^2(c+dx)}{2d}+\frac{b\left(3a^2-b^2\right)\sec(c+dx)}{d}-\frac{a\left(a^2-3b^2\right)\log(\cos(c+dx))}{d}+\frac{3a^2b\cos(c+dx)}{d}+\frac{3ab^2\sec^2(c+dx)}{2d}+\cdots$$

```
[Out] 3*a^2*b*cos(d*x+c)/d+1/2*a^3*cos(d*x+c)^2/d-a*(a^2-3*b^2)*ln(cos(d*x+c))/d+
b*(3*a^2-b^2)*sec(d*x+c)/d+3/2*a*b^2*sec(d*x+c)^2/d+1/3*b^3*sec(d*x+c)^3/d
```

Rubi [A] time = 0.24, antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {4397, 2837, 12, 894}

$$\frac{b\left(3a^2-b^2\right)\sec(c+dx)}{d}-\frac{a\left(a^2-3b^2\right)\log(\cos(c+dx))}{d}+\frac{3a^2b\cos(c+dx)}{d}+\frac{a^3\cos^2(c+dx)}{2d}+\frac{3ab^2\sec^2(c+dx)}{2d}+\cdots$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]*(a*Sin[c + d*x] + b*Tan[c + d*x])^3,x]
```

```
[Out] (3*a^2*b*Cos[c + d*x])/d + (a^3*Cos[c + d*x]^2)/(2*d) - (a*(a^2 - 3*b^2)*Lo
g[Cos[c + d*x]])/d + (b*(3*a^2 - b^2)*Sec[c + d*x])/d + (3*a*b^2*Sec[c + d*
x]^2)/(2*d) + (b^3*Sec[c + d*x]^3)/(3*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 894

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ
[m, 0] && ILtQ[n, 0]))
```

Rule 2837

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^m*(c + (d*x)/b)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*S
in[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/
2] && NeQ[a^2 - b^2, 0]
```

Rule 4397

```
Int[u_, x_Symbol] :> Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]
```

Rubi steps

$$\int \sec(c+dx)(a\sin(c+dx)+b\tan(c+dx))^3\,dx = \int (b+a\cos(c+dx))^3\sec(c+dx)\tan^3(c+dx)\,dx$$
$$= -\frac{\text{Subst}\left(\int \frac{a^4(b+x)^3\left(a^2-x^2\right)}{x^4}\,dx, x, a\cos(c+dx)\right)}{a^3 d}$$
$$= -\frac{a\,\text{Subst}\left(\int \frac{(b+x)^3\left(a^2-x^2\right)}{x^4}\,dx, x, a\cos(c+dx)\right)}{d}$$
$$= -\frac{a\,\text{Subst}\left(\int \left(-3b+\frac{a^2b^3}{x^4}+\frac{3a^2b^2}{x^3}+\frac{3a^2b-b^3}{x^2}+\frac{a^2-3b^2}{x}-x\right)dx, x,$$
$$= \frac{3a^2b\cos(c+dx)}{d}+\frac{a^3\cos^2(c+dx)}{2d}-\frac{a\left(a^2-3b^2\right)\log(\cos(c$$

Mathematica [A] time = 0.51, size = 100, normalized size = 0.87

$$\frac{3a^3\cos(2(c+dx))+2\left(-6b\left(b^2-3a^2\right)\sec(c+dx)-6a\left(a^2-3b^2\right)\log(\cos(c+dx))+9ab^2\sec^2(c+dx)+2b^3\text{se}}{12d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]*(a*Sin[c + d*x] + b*Tan[c + d*x])^3,x]
```

```
[Out] (36*a^2*b*Cos[c + d*x] + 3*a^3*Cos[2*(c + d*x)] + 2*(-6*a*(a^2 - 3*b^2)*Log
[Cos[c + d*x]] - 6*b*(-3*a^2 + b^2)*Sec[c + d*x] + 9*a*b^2*Sec[c + d*x]^2 +
 2*b^3*Sec[c + d*x]^3))/(12*d)
```

fricas [A] time = 0.49, size = 122, normalized size = 1.06

$$\frac{6\,a^3\cos(dx+c)^5+36\,a^2b\cos(dx+c)^4-3\,a^3\cos(dx+c)^3-12\left(a^3-3\,ab^2\right)\cos(dx+c)^3\log(-\cos(dx+c))+}{12\,d\cos(dx+c)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a*sin(d*x+c)+b*tan(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/12*(6*a^3*cos(d*x + c)^5 + 36*a^2*b*cos(d*x + c)^4 - 3*a^3*cos(d*x + c)^3
 - 12*(a^3 - 3*a*b^2)*cos(d*x + c)^3*log(-cos(d*x + c)) + 18*a*b^2*cos(d*x
+ c) + 4*b^3 + 12*(3*a^2*b - b^3)*cos(d*x + c)^2)/(d*cos(d*x + c)^3)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a*sin(d*x+c)+b*tan(d*x+c))^3,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Modgcd: no suitable evaluation pointindex.cc
 index_m operator + Error: Bad Argument ValueUnable to check sign: (2*pi/t_
nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/
t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable
 to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (
2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>
(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2
)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check
sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nos
tep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_n
ostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to
 check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*p
i/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2
*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Un
able to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sig
n: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep
/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nost
ep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to ch
eck sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t
_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi
/t_nostep/2)Evaluation time: 61.28Done
```

maple [A] time = 0.11, size = 213, normalized size = 1.85

$$-\frac{a^{3} \left(\sin^{2}\left(d x+c\right)\right)}{2 d}-\frac{a^{3} \ln \left(\cos \left(d x+c\right)\right)}{d}+\frac{3 a^{2} b \left(\sin^{4}\left(d x+c\right)\right)}{d \cos \left(d x+c\right)}+\frac{3 \cos \left(d x+c\right) \left(\sin^{2}\left(d x+c\right)\right) a^{2} b}{d}+\frac{6 a^{2} b \cos \left(d x+\right.}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)*(a*sin(d*x+c)+b*tan(d*x+c))^3,x)
```

```
[Out] -1/2/d*a^3*sin(d*x+c)^2-a^3*ln(cos(d*x+c))/d+3/d*a^2*b*sin(d*x+c)^4/cos(d*x
+c)+3/d*cos(d*x+c)*sin(d*x+c)^2*a^2*b+6*a^2*b*cos(d*x+c)/d+3/2/d*a*b^2*tan(
```

```
d*x+c)^2+3*a*b^2*ln(cos(d*x+c))/d+1/3/d*b^3*sin(d*x+c)^4/cos(d*x+c)^3-1/3/d
*b^3*sin(d*x+c)^4/cos(d*x+c)-1/3/d*b^3*cos(d*x+c)*sin(d*x+c)^2-2/3*b^3*cos(
d*x+c)/d
```

maxima [A] time = 0.35, size = 109, normalized size = 0.95

$$-\frac{3\left(\sin\left(dx+c\right)^2+\log\left(\sin\left(dx+c\right)^2-1\right)\right)a^3+9\,ab^2\left(\frac{1}{\sin(dx+c)^2-1}-\log\left(\sin\left(dx+c\right)^2-1\right)\right)-18\,a^2b\left(\frac{1}{\cos(dx+c)}+\right.}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a*sin(d*x+c)+b*tan(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -1/6*(3*(sin(d*x + c)^2 + log(sin(d*x + c)^2 - 1))*a^3 + 9*a*b^2*(1/(sin(d*
x + c)^2 - 1) - log(sin(d*x + c)^2 - 1)) - 18*a^2*b*(1/cos(d*x + c) + cos(d
*x + c)) + 2*(3*cos(d*x + c)^2 - 1)*b^3/cos(d*x + c)^3)/d
```

mupad [B] time = 4.45, size = 219, normalized size = 1.90

$$\frac{2\,a^3\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\right)-6\,a\,b^2\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\right)}{d}-\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\,\left(-2\,a^3-12\,a^2\,b+6\,a\,b^2+\frac{4\,b^3}{3}\right)-\mathrm{tan}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*sin(c + d*x) + b*tan(c + d*x))^3/cos(c + d*x),x)
```

```
[Out] (2*a^3*atanh(tan(c/2 + (d*x)/2)^2) - 6*a*b^2*atanh(tan(c/2 + (d*x)/2)^2))/d
 - (tan(c/2 + (d*x)/2)^2*(6*a*b^2 - 12*a^2*b - 2*a^3 + (4*b^3)/3) - tan(c/2
 + (d*x)/2)^6*(6*a*b^2 - 12*a^2*b + 6*a^3 - 4*b^3) + tan(c/2 + (d*x)/2)^4*(
6*a*b^2 - 12*a^2*b + 6*a^3 + (20*b^3)/3) + 12*a^2*b - tan(c/2 + (d*x)/2)^8*
(6*a*b^2 - 2*a^3) - (4*b^3)/3)/(d*(tan(c/2 + (d*x)/2)^2 - 1)^3*(tan(c/2 + (
d*x)/2)^2 + 1)^2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a\sin\left(c+dx\right)+b\tan\left(c+dx\right)\right)^3\sec\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a*sin(d*x+c)+b*tan(d*x+c))**3,x)
```

```
[Out] Integral((a*sin(c + d*x) + b*tan(c + d*x))**3*sec(c + d*x), x)
```

3.248 $\int \sec^2(c+dx)(a\sin(c+dx)+b\tan(c+dx))^3\,dx$

Optimal. Leaf size=111

$$\frac{a^3\cos(c+dx)}{d}+\frac{b\left(3a^2-b^2\right)\sec^2(c+dx)}{2d}+\frac{a\left(a^2-3b^2\right)\sec(c+dx)}{d}+\frac{3a^2b\log(\cos(c+dx))}{d}+\frac{ab^2\sec^3(c+dx)}{d}+b^3$$

```
[Out] a^3*cos(d*x+c)/d+3*a^2*b*ln(cos(d*x+c))/d+a*(a^2-3*b^2)*sec(d*x+c)/d+1/2*b*
(3*a^2-b^2)*sec(d*x+c)^2/d+a*b^2*sec(d*x+c)^3/d+1/4*b^3*sec(d*x+c)^4/d
```

Rubi [A] time = 0.25, antiderivative size = 111, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {4397, 2837, 12, 894}

$$\frac{b\left(3a^2-b^2\right)\sec^2(c+dx)}{2d}+\frac{a\left(a^2-3b^2\right)\sec(c+dx)}{d}+\frac{3a^2b\log(\cos(c+dx))}{d}+\frac{a^3\cos(c+dx)}{d}+\frac{ab^2\sec^3(c+dx)}{d}+b^3$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^2*(a*Sin[c + d*x] + b*Tan[c + d*x])^3,x]
```

```
[Out] (a^3*Cos[c + d*x])/d + (3*a^2*b*Log[Cos[c + d*x]])/d + (a*(a^2 - 3*b^2)*Sec
[c + d*x])/d + (b*(3*a^2 - b^2)*Sec[c + d*x]^2)/(2*d) + (a*b^2*Sec[c + d*x]
^3)/d + (b^3*Sec[c + d*x]^4)/(4*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 894

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ
[m, 0] && ILtQ[n, 0]))
```

Rule 2837

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^m*(c + (d*x)/b)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*S
in[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/
2] && NeQ[a^2 - b^2, 0]
```

Rule 4397

```
Int[u_, x_Symbol] :> Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]
```

Rubi steps

$$\int \sec^2(c+dx)(a\sin(c+dx)+b\tan(c+dx))^3\,dx = \int (b+a\cos(c+dx))^3\sec^2(c+dx)\tan^3(c+dx)\,dx$$

$$= -\frac{\text{Subst}\left(\int \frac{a^5(b+x)^3(a^2-x^2)}{x^5}\,dx, x, a\cos(c+dx)\right)}{a^3d}$$

$$= -\frac{a^2\,\text{Subst}\left(\int \frac{(b+x)^3(a^2-x^2)}{x^5}\,dx, x, a\cos(c+dx)\right)}{d}$$

$$= -\frac{a^2\,\text{Subst}\left(\int \left(-1+\frac{a^2b^3}{x^5}+\frac{3a^2b^2}{x^4}+\frac{3a^2b-b^3}{x^3}+\frac{a^2-3b^2}{x^2}-\frac{3b}{x}\right) dx \ldots}{d}$$

$$= \frac{a^3\cos(c+dx)}{d}+\frac{3a^2b\log(\cos(c+dx))}{d}+\frac{a\left(a^2-3b^2\right)\sec(\ldots}{d}$$

Mathematica [A] time = 1.90, size = 97, normalized size = 0.87

$$\frac{4a^3\cos(c+dx)+\left(6a^2b-2b^3\right)\sec^2(c+dx)+4a\left(a^2-3b^2\right)\sec(c+dx)+12a^2b\log(\cos(c+dx))+4ab^2\sec^3(c \ldots}{4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^2*(a*Sin[c + d*x] + b*Tan[c + d*x])^3,x]
```

```
[Out] (4*a^3*Cos[c + d*x] + 12*a^2*b*Log[Cos[c + d*x]] + 4*a*(a^2 - 3*b^2)*Sec[c
+ d*x] + (6*a^2*b - 2*b^3)*Sec[c + d*x]^2 + 4*a*b^2*Sec[c + d*x]^3 + b^3*Se
c[c + d*x]^4)/(4*d)
```

fricas [A] time = 0.61, size = 107, normalized size = 0.96

$$\frac{4\,a^3\cos(dx+c)^5+12\,a^2b\cos(dx+c)^4\log(-\cos(dx+c))+4\,ab^2\cos(dx+c)+4\left(a^3-3\,ab^2\right)\cos(dx+c)^3+\ldots}{4\,d\cos(dx+c)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a*sin(d*x+c)+b*tan(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/4*(4*a^3*cos(d*x + c)^5 + 12*a^2*b*cos(d*x + c)^4*log(-cos(d*x + c)) + 4*
a*b^2*cos(d*x + c) + 4*(a^3 - 3*a*b^2)*cos(d*x + c)^3 + b^3 + 2*(3*a^2*b -
b^3)*cos(d*x + c)^2)/(d*cos(d*x + c)^4)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a*sin(d*x+c)+b*tan(d*x+c))^3,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Modgcd: no suitable evaluation pointindex.cc
 index_m operator + Error: Bad Argument ValueUnable to check sign: (2*pi/t_
nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/
t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable
 to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (
2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>
(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2
)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check
sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nos
tep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_n
ostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to
 check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*p
i/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2
*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Un
able to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sig
n: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep
/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nost
ep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to ch
eck sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t
_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi
/t_nostep/2)Evaluation time: 76.28Done
```

maple [A] time = 0.14, size = 204, normalized size = 1.84

$$\frac{a^3\left(\sin^4(dx+c)\right)}{d\cos(dx+c)}+\frac{\cos(dx+c)\left(\sin^2(dx+c)\right)a^3}{d}+\frac{2a^3\cos(dx+c)}{d}+\frac{3a^2b\left(\tan^2(dx+c)\right)}{2d}+\frac{3a^2b\ln(\cos(dx+c))}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2*(a*sin(d*x+c)+b*tan(d*x+c))^3,x)
```

```
[Out] 1/d*a^3*sin(d*x+c)^4/cos(d*x+c)+1/d*cos(d*x+c)*sin(d*x+c)^2*a^3+2*a^3*cos(d
*x+c)/d+3/2*a^2*b*tan(d*x+c)^2/d+3*a^2*b*ln(cos(d*x+c))/d+1/d*a*b^2*sin(d*x
```

```
+c)^4/cos(d*x+c)^3-1/d*a*b^2*sin(d*x+c)^4/cos(d*x+c)-1/d*cos(d*x+c)*sin(d*x
+c)^2*a*b^2-2*a*b^2*cos(d*x+c)/d+1/4/d*b^3*sin(d*x+c)^4/cos(d*x+c)^4
```

maxima [A] time = 0.36, size = 96, normalized size = 0.86

$$\frac{b^3 \tan(dx+c)^4 - 6a^2b\left(\frac{1}{\sin(dx+c)^2-1} - \log\left(\sin(dx+c)^2-1\right)\right) + 4a^3\left(\frac{1}{\cos(dx+c)} + \cos(dx+c)\right) - \frac{4\left(3\cos(dx+c)^2-1\right)ab}{\cos(dx+c)^3}}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a*sin(d*x+c)+b*tan(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/4*(b^3*tan(d*x + c)^4 - 6*a^2*b*(1/(sin(d*x + c)^2 - 1) - log(sin(d*x + c
)^2 - 1)) + 4*a^3*(1/cos(d*x + c) + cos(d*x + c)) - 4*(3*cos(d*x + c)^2 - 1
)*a*b^2/cos(d*x + c)^3)/d
```

mupad [B] time = 4.22, size = 223, normalized size = 2.01

$$\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2\left(-12a^3+6a^2b+12ab^2\right)+\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4\left(12a^3-6a^2b+4ab^2+4b^3\right)-\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^6\left(4a^3\right.}{d\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{10}-3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^8+2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^6+2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*sin(c + d*x) + b*tan(c + d*x))^3/cos(c + d*x)^2,x)
```

```
[Out] (tan(c/2 + (d*x)/2)^2*(12*a*b^2 + 6*a^2*b - 12*a^3) + tan(c/2 + (d*x)/2)^4*
(4*a*b^2 - 6*a^2*b + 12*a^3 + 4*b^3) - tan(c/2 + (d*x)/2)^6*(12*a*b^2 + 6*a
^2*b + 4*a^3 - 4*b^3) - 4*a*b^2 + 4*a^3 + 6*a^2*b*tan(c/2 + (d*x)/2)^8)/(d*
(2*tan(c/2 + (d*x)/2)^4 - 3*tan(c/2 + (d*x)/2)^2 + 2*tan(c/2 + (d*x)/2)^6 -
 3*tan(c/2 + (d*x)/2)^8 + tan(c/2 + (d*x)/2)^10 + 1)) - (6*a^2*b*atanh(tan(
c/2 + (d*x)/2)^2))/d
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a\sin(c+dx)+b\tan(c+dx))^3\sec^2(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2*(a*sin(d*x+c)+b*tan(d*x+c))**3,x)
```

```
[Out] Integral((a*sin(c + d*x) + b*tan(c + d*x))**3*sec(c + d*x)**2, x)
```

3.249 $\int \sec^3(c+dx)(a\sin(c+dx)+b\tan(c+dx))^3\,dx$

Optimal. Leaf size=119

$$\frac{a^3\log(\cos(c+dx))}{d}+\frac{b\left(3a^2-b^2\right)\sec^3(c+dx)}{3d}+\frac{a\left(a^2-3b^2\right)\sec^2(c+dx)}{2d}-\frac{3a^2b\sec(c+dx)}{d}+\frac{3ab^2\sec^4(c+dx)}{4d}+$$

```
[Out] a^3*ln(cos(d*x+c))/d-3*a^2*b*sec(d*x+c)/d+1/2*a*(a^2-3*b^2)*sec(d*x+c)^2/d+
1/3*b*(3*a^2-b^2)*sec(d*x+c)^3/d+3/4*a*b^2*sec(d*x+c)^4/d+1/5*b^3*sec(d*x+c
)^5/d
```

Rubi [A] time = 0.22, antiderivative size = 119, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {4397, 2837, 12, 894}

$$\frac{b\left(3a^2-b^2\right)\sec^3(c+dx)}{3d}+\frac{a\left(a^2-3b^2\right)\sec^2(c+dx)}{2d}-\frac{3a^2b\sec(c+dx)}{d}+\frac{a^3\log(\cos(c+dx))}{d}+\frac{3ab^2\sec^4(c+dx)}{4d}+$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^3*(a*Sin[c + d*x] + b*Tan[c + d*x])^3,x]
```

```
[Out] (a^3*Log[Cos[c + d*x]])/d - (3*a^2*b*Sec[c + d*x])/d + (a*(a^2 - 3*b^2)*Sec
[c + d*x]^2)/(2*d) + (b*(3*a^2 - b^2)*Sec[c + d*x]^3)/(3*d) + (3*a*b^2*Sec[
c + d*x]^4)/(4*d) + (b^3*Sec[c + d*x]^5)/(5*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 894

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ
[m, 0] && ILtQ[n, 0]))
```

Rule 2837

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^m*(c + (d*x)/b)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*S
in[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/
```

```
2] && NeQ[a^2 - b^2, 0]
```

Rule 4397

```
Int[u_, x_Symbol] :> Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]
```

Rubi steps

$$\int \sec^3(c+dx)(a\sin(c+dx)+b\tan(c+dx))^3\,dx = \int (b+a\cos(c+dx))^3\sec^3(c+dx)\tan^3(c+dx)\,dx$$
$$= -\frac{\text{Subst}\left(\int \frac{a^6(b+x)^3(a^2-x^2)}{x^6}\,dx, x, a\cos(c+dx)\right)}{a^3d}$$
$$= -\frac{a^3\,\text{Subst}\left(\int \frac{(b+x)^3(a^2-x^2)}{x^6}\,dx, x, a\cos(c+dx)\right)}{d}$$
$$= -\frac{a^3\,\text{Subst}\left(\int \left(\frac{a^2b^3}{x^6}+\frac{3a^2b^2}{x^5}+\frac{3a^2b-b^3}{x^4}+\frac{a^2-3b^2}{x^3}-\frac{3b}{x^2}-\frac{1}{x}\right)\,dx,\right.}{d}$$
$$= \frac{a^3\log(\cos(c+dx))}{d}-\frac{3a^2b\sec(c+dx)}{d}+\frac{a\left(a^2-3b^2\right)\sec^2}{2d}$$

Mathematica [A] time = 0.32, size = 99, normalized size = 0.83

$$\frac{60a^3\log(\cos(c+dx))-20b\left(b^2-3a^2\right)\sec^3(c+dx)+30a\left(a^2-3b^2\right)\sec^2(c+dx)-180a^2b\sec(c+dx)+45ab^2}{60d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^3*(a*Sin[c + d*x] + b*Tan[c + d*x])^3,x]
```

```
[Out] (60*a^3*Log[Cos[c + d*x]] - 180*a^2*b*Sec[c + d*x] + 30*a*(a^2 - 3*b^2)*Sec
[c + d*x]^2 - 20*b*(-3*a^2 + b^2)*Sec[c + d*x]^3 + 45*a*b^2*Sec[c + d*x]^4
+ 12*b^3*Sec[c + d*x]^5)/(60*d)
```

fricas [A] time = 0.65, size = 109, normalized size = 0.92

$$\frac{60\,a^3\cos\left(dx+c\right)^5\log\left(-\cos\left(dx+c\right)\right)-180\,a^2b\cos\left(dx+c\right)^4+45\,ab^2\cos\left(dx+c\right)+30\left(a^3-3\,ab^2\right)\cos\left(dx+\right.}{60\,d\cos\left(dx+c\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a*sin(d*x+c)+b*tan(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/60*(60*a^3*cos(d*x + c)^5*log(-cos(d*x + c)) - 180*a^2*b*cos(d*x + c)^4 +
 45*a*b^2*cos(d*x + c) + 30*(a^3 - 3*a*b^2)*cos(d*x + c)^3 + 12*b^3 + 20*(3
*a^2*b - b^3)*cos(d*x + c)^2)/(d*cos(d*x + c)^5)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a*sin(d*x+c)+b*tan(d*x+c))^3,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Modgcd: no suitable evaluation pointindex.cc
 index_m operator + Error: Bad Argument ValueUnable to check sign: (2*pi/t_
nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/
t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable
 to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (
2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>
(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2
)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check
sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nos
tep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_n
ostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to
 check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*p
i/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2
*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Un
able to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sig
n: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep
/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nost
ep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to ch
eck sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t
_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi
/t_nostep/2)Evaluation time: 78.52Done
```

maple [B] time = 0.14, size = 252, normalized size = 2.12

$$\frac{a^3\left(\tan^2\left(dx+c\right)\right)}{2d}+\frac{a^3\ln\left(\cos\left(dx+c\right)\right)}{d}+\frac{a^2b\left(\sin^4\left(dx+c\right)\right)}{d\cos\left(dx+c\right)^3}-\frac{a^2b\left(\sin^4\left(dx+c\right)\right)}{d\cos\left(dx+c\right)}-\frac{\cos\left(dx+c\right)\left(\sin^2\left(dx+c\right)\right)a^2b}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^3*(a*sin(d*x+c)+b*tan(d*x+c))^3,x)
```

```
[Out] 1/2/d*a^3*tan(d*x+c)^2+a^3*ln(cos(d*x+c))/d+1/d*a^2*b*sin(d*x+c)^4/cos(d*x+
c)^3-1/d*a^2*b*sin(d*x+c)^4/cos(d*x+c)-1/d*cos(d*x+c)*sin(d*x+c)^2*a^2*b-2*
a^2*b*cos(d*x+c)/d+3/4/d*a*b^2*sin(d*x+c)^4/cos(d*x+c)^4+1/5/d*b^3*sin(d*x+
c)^4/cos(d*x+c)^5+1/15/d*b^3*sin(d*x+c)^4/cos(d*x+c)^3-1/15/d*b^3*sin(d*x+c
)^4/cos(d*x+c)-1/15/d*b^3*cos(d*x+c)*sin(d*x+c)^2-2/15*b^3*cos(d*x+c)/d
```

maxima [A] time = 0.36, size = 128, normalized size = 1.08

$$-\frac{30\,a^3\left(\frac{1}{\sin(dx+c)^2-1}-\log\left(\sin(dx+c)^2-1\right)\right)-\frac{45\left(2\,\sin(dx+c)^2-1\right)ab^2}{\sin(dx+c)^4-2\,\sin(dx+c)^2+1}+\frac{60\left(3\,\cos(dx+c)^2-1\right)a^2b}{\cos(dx+c)^3}+\frac{4\left(5\,\cos(dx+c)^2-3\right)b^3}{\cos(dx+c)^5}}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a*sin(d*x+c)+b*tan(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -1/60*(30*a^3*(1/(sin(d*x + c)^2 - 1) - log(sin(d*x + c)^2 - 1)) - 45*(2*si
n(d*x + c)^2 - 1)*a*b^2/(sin(d*x + c)^4 - 2*sin(d*x + c)^2 + 1) + 60*(3*cos
(d*x + c)^2 - 1)*a^2*b/cos(d*x + c)^3 + 4*(5*cos(d*x + c)^2 - 3)*b^3/cos(d*
x + c)^5)/d
```

mupad [B] time = 4.15, size = 220, normalized size = 1.85

$$-\frac{2\,a^3\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\right)}{d}-\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6\,\left(6\,a^3+12\,a^2\,b-12\,a\,b^2+4\,b^3\right)+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4\,\left(-6\,a^3-28\,a^2\,b+\right.}{d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{10}-5\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8+10\,\tan\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*sin(c + d*x) + b*tan(c + d*x))^3/cos(c + d*x)^3,x)
```

```
[Out] - (2*a^3*atanh(tan(c/2 + (d*x)/2)^2))/d - (tan(c/2 + (d*x)/2)^6*(12*a^2*b -
 12*a*b^2 + 6*a^3 + 4*b^3) + tan(c/2 + (d*x)/2)^4*(12*a*b^2 - 28*a^2*b - 6*
a^3 + (4*b^3)/3) - 2*a^3*tan(c/2 + (d*x)/2)^8 - 4*a^2*b + tan(c/2 + (d*x)/2
)^2*(20*a^2*b + 2*a^3 + (4*b^3)/3) - (4*b^3)/15)/(d*(5*tan(c/2 + (d*x)/2)^2
 - 10*tan(c/2 + (d*x)/2)^4 + 10*tan(c/2 + (d*x)/2)^6 - 5*tan(c/2 + (d*x)/2)
^8 + tan(c/2 + (d*x)/2)^10 - 1))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**3*(a*sin(d*x+c)+b*tan(d*x+c))**3,x)
```

```
[Out] Timed out
```

3.250 $\int \frac{\cos^3(c+dx)}{a\sin(c+dx)+b\tan(c+dx)}\,dx$

Optimal. Leaf size=113

$$-\frac{b\cos(c+dx)}{a^2d}-\frac{b^4\log(a\cos(c+dx)+b)}{a^3d\left(a^2-b^2\right)}+\frac{\log(1-\cos(c+dx))}{2d(a+b)}+\frac{\log(\cos(c+dx)+1)}{2d(a-b)}+\frac{\cos^2(c+dx)}{2ad}$$

```
[Out] -b*cos(d*x+c)/a^2/d+1/2*cos(d*x+c)^2/a/d+1/2*ln(1-cos(d*x+c))/(a+b)/d+1/2*l
n(1+cos(d*x+c))/(a-b)/d-b^4*ln(b+a*cos(d*x+c))/a^3/(a^2-b^2)/d
```

Rubi [A] time = 0.34, antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {4397, 2837, 12, 1629}

$$-\frac{b^4\log(a\cos(c+dx)+b)}{a^3d\left(a^2-b^2\right)}-\frac{b\cos(c+dx)}{a^2d}+\frac{\log(1-\cos(c+dx))}{2d(a+b)}+\frac{\log(\cos(c+dx)+1)}{2d(a-b)}+\frac{\cos^2(c+dx)}{2ad}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^3/(a*Sin[c + d*x] + b*Tan[c + d*x]),x]
```

```
[Out] -((b*Cos[c + d*x])/(a^2*d)) + Cos[c + d*x]^2/(2*a*d) + Log[1 - Cos[c + d*x]
]/(2*(a + b)*d) + Log[1 + Cos[c + d*x]]/(2*(a - b)*d) - (b^4*Log[b + a*Cos[
c + d*x]])/(a^3*(a^2 - b^2)*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 1629

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol]
:> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
 d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 2837

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^m*(c + (d*x)/b)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*S
in[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/
2] && NeQ[a^2 - b^2, 0]
```

Rule 4397

```
Int[u_, x_Symbol] :> Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]
```

Rubi steps

$$\int \frac{\cos^3(c+dx)}{a\sin(c+dx)+b\tan(c+dx)}\,dx = \int \frac{\cos^3(c+dx)\cot(c+dx)}{b+a\cos(c+dx)}\,dx$$
$$= -\frac{a\,\text{Subst}\left(\int \frac{x^4}{a^4(b+x)\left(a^2-x^2\right)}\,dx, x, a\cos(c+dx)\right)}{d}$$
$$= -\frac{\text{Subst}\left(\int \frac{x^4}{(b+x)\left(a^2-x^2\right)}\,dx, x, a\cos(c+dx)\right)}{a^3d}$$
$$= -\frac{\text{Subst}\left(\int \left(b+\frac{a^3}{2(a+b)(a-x)}-x-\frac{a^3}{2(a-b)(a+x)}+\frac{b^4}{(a-b)(a+b)(b+x)}\right)dx, x, a\cos(c$$
$$= -\frac{b\cos(c+dx)}{a^2d}+\frac{\cos^2(c+dx)}{2ad}+\frac{\log(1-\cos(c+dx))}{2(a+b)d}+\frac{\log(1+\cos(c+}{2(a-b)d}$$

Mathematica [A] time = 0.37, size = 100, normalized size = 0.88

$$\frac{-\frac{4b\cos(c+dx)}{a^2}+4\left(\frac{b^4\log(a\cos(c+dx)+b)}{a^3\left(b^2-a^2\right)}+\frac{\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)}{a+b}+\frac{\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)}{a-b}\right)+\frac{\cos(2(c+dx))}{a}}{4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^3/(a*Sin[c + d*x] + b*Tan[c + d*x]),x]
```

```
[Out] ((-4*b*Cos[c + d*x])/a^2 + Cos[2*(c + d*x)]/a + 4*(Log[Cos[(c + d*x)/2]]/(a
 - b) + (b^4*Log[b + a*Cos[c + d*x]])/(a^3*(-a^2 + b^2)) + Log[Sin[(c + d*x
)/2]]/(a + b)))/(4*d)
```

fricas [A] time = 0.64, size = 123, normalized size = 1.09

$$-\frac{2\,b^4\log\left(a\cos\left(dx+c\right)+b\right)-\left(a^4-a^2b^2\right)\cos\left(dx+c\right)^2+2\left(a^3b-ab^3\right)\cos\left(dx+c\right)-\left(a^4+a^3b\right)\log\left(\frac{1}{2}\cos\left(dx\right.\right.}{2\left(a^5-a^3b^2\right)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3/(a*sin(d*x+c)+b*tan(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/2*(2*b^4*log(a*cos(d*x + c) + b) - (a^4 - a^2*b^2)*cos(d*x + c)^2 + 2*(a
^3*b - a*b^3)*cos(d*x + c) - (a^4 + a^3*b)*log(1/2*cos(d*x + c) + 1/2) - (a
^4 - a^3*b)*log(-1/2*cos(d*x + c) + 1/2))/((a^5 - a^3*b^2)*d)
```

giac [B] time = 0.40, size = 303, normalized size = 2.68

$$-\frac{\frac{2\,b^4\log\left(\left|-a-b-\frac{a(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{b(\cos(dx+c)-1)}{\cos(dx+c)+1}\right|\right)}{a^5-a^3b^2}-\frac{\log\left(\frac{|-\cos(dx+c)+1|}{|\cos(dx+c)+1|}\right)}{a+b}+\frac{2\left(a^2+b^2\right)\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right|\right)}{a^3}-\frac{3\,a^2-4\,ab+3\,b^2-\frac{2\,a^2(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{4\,ab}{c}}{}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3/(a*sin(d*x+c)+b*tan(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/2*(2*b^4*log(abs(-a - b - a*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + b*(c
os(d*x + c) - 1)/(cos(d*x + c) + 1)))/(a^5 - a^3*b^2) - log(abs(-cos(d*x +
c) + 1)/abs(cos(d*x + c) + 1))/(a + b) + 2*(a^2 + b^2)*log(abs(-(cos(d*x +
c) - 1)/(cos(d*x + c) + 1) + 1))/a^3 - (3*a^2 - 4*a*b + 3*b^2 - 2*a^2*(cos(
d*x + c) - 1)/(cos(d*x + c) + 1) + 4*a*b*(cos(d*x + c) - 1)/(cos(d*x + c) +
 1) - 6*b^2*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 3*a^2*(cos(d*x + c) - 1
)^2/(cos(d*x + c) + 1)^2 + 3*b^2*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2)
/(a^3*((cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 1)^2))/d
```

maple [A] time = 0.18, size = 111, normalized size = 0.98

$$\frac{\cos^2(dx+c)}{2ad}-\frac{b\cos(dx+c)}{a^2d}-\frac{b^4\ln(b+a\cos(dx+c))}{d\,a^3(a+b)(a-b)}+\frac{\ln(\cos(dx+c)-1)}{d(2a+2b)}+\frac{\ln(1+\cos(dx+c))}{d(2a-2b)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3/(a*sin(d*x+c)+b*tan(d*x+c)),x)
```

```
[Out] 1/2*cos(d*x+c)^2/a/d-b*cos(d*x+c)/a^2/d-1/d/a^3*b^4/(a+b)/(a-b)*ln(b+a*cos(
d*x+c))+1/d/(2*a+2*b)*ln(cos(d*x+c)-1)+1/d/(2*a-2*b)*ln(1+cos(d*x+c))
```

maxima [A] time = 0.43, size = 188, normalized size = 1.66

$$-\frac{\frac{b^4\log\left(a+b-\frac{(a-b)\sin(dx+c)^2}{(\cos(dx+c)+1)^2}\right)}{a^5-a^3b^2}+\frac{2\left(b+\frac{(a+b)\sin(dx+c)^2}{(\cos(dx+c)+1)^2}\right)}{a^2+\frac{2\,a^2\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{a^2\sin(dx+c)^4}{(\cos(dx+c)+1)^4}}-\frac{\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a+b}+\frac{\left(a^2+b^2\right)\log\left(\frac{\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+1\right)}{a^3}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3/(a*sin(d*x+c)+b*tan(d*x+c)),x, algorithm="maxima")
```

```
[Out] -(b^4*log(a + b - (a - b)*sin(d*x + c)^2/(cos(d*x + c) + 1)^2)/(a^5 - a^3*b
^2) + 2*(b + (a + b)*sin(d*x + c)^2/(cos(d*x + c) + 1)^2)/(a^2 + 2*a^2*sin(
d*x + c)^2/(cos(d*x + c) + 1)^2 + a^2*sin(d*x + c)^4/(cos(d*x + c) + 1)^4)
- log(sin(d*x + c)/(cos(d*x + c) + 1))/(a + b) + (a^2 + b^2)*log(sin(d*x +
c)^2/(cos(d*x + c) + 1)^2 + 1)/a^3)/d
```

mupad [B] time = 1.16, size = 161, normalized size = 1.42

$$\frac{\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{d\,\left(a+b\right)}-\frac{\frac{2\,b}{a^2}+\frac{2\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\,\left(a+b\right)}{a^2}}{d\,\left({\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^4+2\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2+1\right)}-\frac{b^4\,\ln\left(a+b-a\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2+b\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2\right)}{d\,\left(a^5-a^3\,b^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^3/(a*sin(c + d*x) + b*tan(c + d*x)),x)
```

```
[Out] log(tan(c/2 + (d*x)/2))/(d*(a + b)) - ((2*b)/a^2 + (2*tan(c/2 + (d*x)/2)^2*
(a + b))/a^2)/(d*(2*tan(c/2 + (d*x)/2)^2 + tan(c/2 + (d*x)/2)^4 + 1)) - (b^
4*log(a + b - a*tan(c/2 + (d*x)/2)^2 + b*tan(c/2 + (d*x)/2)^2))/(d*(a^5 - a
^3*b^2)) - (log(tan(c/2 + (d*x)/2)^2 + 1)*(a^2 + b^2))/(a^3*d)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3/(a*sin(d*x+c)+b*tan(d*x+c)),x)
```

```
[Out] Timed out
```

3.251 $\int \frac{\cos^2(c+dx)}{a\sin(c+dx)+b\tan(c+dx)}\,dx$

Optimal. Leaf size=92

$$\frac{b^3\log(a\cos(c+dx)+b)}{a^2d\left(a^2-b^2\right)}+\frac{\log(1-\cos(c+dx))}{2d(a+b)}-\frac{\log(\cos(c+dx)+1)}{2d(a-b)}+\frac{\cos(c+dx)}{ad}$$

```
[Out] cos(d*x+c)/a/d+1/2*ln(1-cos(d*x+c))/(a+b)/d-1/2*ln(1+cos(d*x+c))/(a-b)/d+b^
3*ln(b+a*cos(d*x+c))/a^2/(a^2-b^2)/d
```

Rubi [A] time = 0.28, antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {4397, 2837, 12, 1629}

$$\frac{b^3\log(a\cos(c+dx)+b)}{a^2d\left(a^2-b^2\right)}+\frac{\log(1-\cos(c+dx))}{2d(a+b)}-\frac{\log(\cos(c+dx)+1)}{2d(a-b)}+\frac{\cos(c+dx)}{ad}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2/(a*Sin[c + d*x] + b*Tan[c + d*x]),x]
```

```
[Out] Cos[c + d*x]/(a*d) + Log[1 - Cos[c + d*x]]/(2*(a + b)*d) - Log[1 + Cos[c +
d*x]]/(2*(a - b)*d) + (b^3*Log[b + a*Cos[c + d*x]])/(a^2*(a^2 - b^2)*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 1629

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol]
:> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
 d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 2837

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^m*(c + (d*x)/b)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*S
in[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/
2] && NeQ[a^2 - b^2, 0]
```

Rule 4397

```
Int[u_, x_Symbol] :> Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]
```

Rubi steps

$$\begin{aligned}
\int \frac{\cos^2(c+dx)}{a\sin(c+dx)+b\tan(c+dx)}\,dx &= \int \frac{\cos^2(c+dx)\cot(c+dx)}{b+a\cos(c+dx)}\,dx \\
&= -\frac{a\,\text{Subst}\left(\int \frac{x^3}{a^3(b+x)\left(a^2-x^2\right)}\,dx, x, a\cos(c+dx)\right)}{d} \\
&= -\frac{\text{Subst}\left(\int \frac{x^3}{(b+x)\left(a^2-x^2\right)}\,dx, x, a\cos(c+dx)\right)}{a^2 d} \\
&= -\frac{\text{Subst}\left(\int \left(-1+\frac{a^2}{2(a+b)(a-x)}+\frac{a^2}{2(a-b)(a+x)}+\frac{b^3}{(-a+b)(a+b)(b+x)}\right) dx, x, a\cos(c+\right.}{a^2 d} \\
&= \frac{\cos(c+dx)}{ad}+\frac{\log(1-\cos(c+dx))}{2(a+b)d}-\frac{\log(1+\cos(c+dx))}{2(a-b)d}+\frac{b^3\log(b+}{a^2\left(a^2\right.}
\end{aligned}$$

Mathematica [A] time = 0.23, size = 80, normalized size = 0.87

$$\frac{\frac{b^3\log(a\cos(c+dx)+b)}{a^4-a^2b^2}+\frac{\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)}{a+b}+\frac{\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)}{b-a}+\frac{\cos(c+dx)}{a}}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2/(a*Sin[c + d*x] + b*Tan[c + d*x]),x]
```

```
[Out] (Cos[c + d*x]/a + Log[Cos[(c + d*x)/2]]/(-a + b) + (b^3*Log[b + a*Cos[c + d
*x]])/(a^4 - a^2*b^2) + Log[Sin[(c + d*x)/2]]/(a + b))/d
```

fricas [A] time = 0.78, size = 98, normalized size = 1.07

$$\frac{2\,b^3\log\left(a\cos\left(dx+c\right)+b\right)+2\left(a^3-ab^2\right)\cos\left(dx+c\right)-\left(a^3+a^2b\right)\log\left(\frac{1}{2}\cos\left(dx+c\right)+\frac{1}{2}\right)+\left(a^3-a^2b\right)\log\left(-\right.}{2\left(a^4-a^2b^2\right)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(a*sin(d*x+c)+b*tan(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/2*(2*b^3*log(a*cos(d*x + c) + b) + 2*(a^3 - a*b^2)*cos(d*x + c) - (a^3 +
a^2*b)*log(1/2*cos(d*x + c) + 1/2) + (a^3 - a^2*b)*log(-1/2*cos(d*x + c) +
1/2))/((a^4 - a^2*b^2)*d)
```

giac [B] time = 0.35, size = 190, normalized size = 2.07

$$\frac{\frac{2\,b^{3}\log\left(\left|-a-b-\frac{a(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{b(\cos(dx+c)-1)}{\cos(dx+c)+1}\right|\right)}{a^{4}-a^{2}b^{2}}+\frac{\log\left(\frac{|-\cos(dx+c)+1|}{|\cos(dx+c)+1|}\right)}{a+b}+\frac{2\,b\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right|\right)}{a^{2}}-\frac{2\left(2\,a-b+\frac{b(\cos(dx+c)-1)}{\cos(dx+c)+1}\right)}{a^{2}\left(\frac{\cos(dx+c)-1}{\cos(dx+c)+1}-1\right)}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(a*sin(d*x+c)+b*tan(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/2*(2*b^3*log(abs(-a - b - a*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + b*(co
s(d*x + c) - 1)/(cos(d*x + c) + 1)))/(a^4 - a^2*b^2) + log(abs(-cos(d*x + c
) + 1)/abs(cos(d*x + c) + 1))/(a + b) + 2*b*log(abs(-(cos(d*x + c) - 1)/(co
s(d*x + c) + 1) + 1))/a^2 - 2*(2*a - b + b*(cos(d*x + c) - 1)/(cos(d*x + c)
 + 1))/(a^2*((cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 1)))/d
```

maple [A] time = 0.18, size = 93, normalized size = 1.01

$$\frac{\cos\left(dx+c\right)}{ad}+\frac{b^{3}\ln\left(b+a\cos\left(dx+c\right)\right)}{d\,a^{2}\left(a+b\right)\left(a-b\right)}+\frac{\ln\left(\cos\left(dx+c\right)-1\right)}{d\left(2a+2b\right)}-\frac{\ln\left(1+\cos\left(dx+c\right)\right)}{d\left(2a-2b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2/(a*sin(d*x+c)+b*tan(d*x+c)),x)
```

```
[Out] cos(d*x+c)/a/d+1/d/a^2*b^3/(a+b)/(a-b)*ln(b+a*cos(d*x+c))+1/d/(2*a+2*b)*ln(
cos(d*x+c)-1)-1/d/(2*a-2*b)*ln(1+cos(d*x+c))
```

maxima [A] time = 0.45, size = 129, normalized size = 1.40

$$\frac{\frac{b^{3}\log\left(a+b-\frac{(a-b)\sin(dx+c)^{2}}{(\cos(dx+c)+1)^{2}}\right)}{a^{4}-a^{2}b^{2}}+\frac{\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a+b}+\frac{b\log\left(\frac{\sin(dx+c)^{2}}{(\cos(dx+c)+1)^{2}}+1\right)}{a^{2}}+\frac{2}{a+\frac{a\sin(dx+c)^{2}}{(\cos(dx+c)+1)^{2}}}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(a*sin(d*x+c)+b*tan(d*x+c)),x, algorithm="maxima")
```

```
[Out] (b^3*log(a + b - (a - b)*sin(d*x + c)^2/(cos(d*x + c) + 1)^2)/(a^4 - a^2*b^
2) + log(sin(d*x + c)/(cos(d*x + c) + 1))/(a + b) + b*log(sin(d*x + c)^2/(c
os(d*x + c) + 1)^2 + 1)/a^2 + 2/(a + a*sin(d*x + c)^2/(cos(d*x + c) + 1)^2)
)/d
```

mupad [B] time = 0.82, size = 117, normalized size = 1.27

$$\frac{2}{a\,d\,\left({\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2+1\right)}+\frac{\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{d\,\left(a+b\right)}+\frac{b\,\ln\left({\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2+1\right)}{a^2\,d}+\frac{b^3\,\ln\left(a+b-a\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2+b\,\tan\left(\right.\right.}{a^2\,d\,\left(a^2-b^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^2/(a*sin(c + d*x) + b*tan(c + d*x)),x)
```

```
[Out] 2/(a*d*(tan(c/2 + (d*x)/2)^2 + 1)) + log(tan(c/2 + (d*x)/2))/(d*(a + b)) +
(b*log(tan(c/2 + (d*x)/2)^2 + 1))/(a^2*d) + (b^3*log(a + b - a*tan(c/2 + (d
*x)/2)^2 + b*tan(c/2 + (d*x)/2)^2))/(a^2*d*(a^2 - b^2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos^2\left(c+dx\right)}{a\sin\left(c+dx\right)+b\tan\left(c+dx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2/(a*sin(d*x+c)+b*tan(d*x+c)),x)
```

```
[Out] Integral(cos(c + d*x)**2/(a*sin(c + d*x) + b*tan(c + d*x)), x)
```

3.252 $\int \frac{\cos(c+dx)}{a\sin(c+dx)+b\tan(c+dx)}\,dx$

Optimal. Leaf size=80

$$-\frac{b^2\log(a\cos(c+dx)+b)}{ad\left(a^2-b^2\right)}+\frac{\log(1-\cos(c+dx))}{2d(a+b)}+\frac{\log(\cos(c+dx)+1)}{2d(a-b)}$$

```
[Out] 1/2*ln(1-cos(d*x+c))/(a+b)/d+1/2*ln(1+cos(d*x+c))/(a-b)/d-b^2*ln(b+a*cos(d*
x+c))/a/(a^2-b^2)/d
```

Rubi [A] time = 0.24, antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {4397, 2837, 12, 1629}

$$-\frac{b^2\log(a\cos(c+dx)+b)}{ad\left(a^2-b^2\right)}+\frac{\log(1-\cos(c+dx))}{2d(a+b)}+\frac{\log(\cos(c+dx)+1)}{2d(a-b)}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]/(a*Sin[c + d*x] + b*Tan[c + d*x]),x]
```

```
[Out] Log[1 - Cos[c + d*x]]/(2*(a + b)*d) + Log[1 + Cos[c + d*x]]/(2*(a - b)*d) -
 (b^2*Log[b + a*Cos[c + d*x]])/(a*(a^2 - b^2)*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 1629

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol]
:> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
 d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 2837

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^m*(c + (d*x)/b)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*S
in[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/
2] && NeQ[a^2 - b^2, 0]
```

Rule 4397

```
Int[u_, x_Symbol] :> Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]
```

Rubi steps

$$\begin{aligned}\int \frac{\cos(c+dx)}{a\sin(c+dx)+b\tan(c+dx)}\,dx &= \int \frac{\cos(c+dx)\cot(c+dx)}{b+a\cos(c+dx)}\,dx\\ &= -\frac{a\,\text{Subst}\left(\int \frac{x^2}{a^2(b+x)\left(a^2-x^2\right)}\,dx, x, a\cos(c+dx)\right)}{d}\\ &= -\frac{\text{Subst}\left(\int \frac{x^2}{(b+x)\left(a^2-x^2\right)}\,dx, x, a\cos(c+dx)\right)}{ad}\\ &= -\frac{\text{Subst}\left(\int \left(\frac{a}{2(a+b)(a-x)}-\frac{a}{2(a-b)(a+x)}+\frac{b^2}{(a-b)(a+b)(b+x)}\right)dx, x, a\cos(c+dx)\right)}{ad}\\ &= \frac{\log(1-\cos(c+dx))}{2(a+b)d}+\frac{\log(1+\cos(c+dx))}{2(a-b)d}-\frac{b^2\log(b+a\cos(c+dx))}{a\left(a^2-b^2\right)d}\end{aligned}$$

Mathematica [A] time = 0.10, size = 70, normalized size = 0.88

$$\frac{b^2(-\log(a\cos(c+dx)+b))+a(a-b)\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)+a(a+b)\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)}{ad(a-b)(a+b)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]/(a*Sin[c + d*x] + b*Tan[c + d*x]),x]
```

```
[Out] (a*(a + b)*Log[Cos[(c + d*x)/2]] - b^2*Log[b + a*Cos[c + d*x]] + a*(a - b)*
Log[Sin[(c + d*x)/2]])/(a*(a - b)*(a + b)*d)
```

fricas [A] time = 0.56, size = 75, normalized size = 0.94

$$-\frac{2\,b^2\log\left(a\cos\left(dx+c\right)+b\right)-\left(a^2+ab\right)\log\left(\frac{1}{2}\cos\left(dx+c\right)+\frac{1}{2}\right)-\left(a^2-ab\right)\log\left(-\frac{1}{2}\cos\left(dx+c\right)+\frac{1}{2}\right)}{2\left(a^3-ab^2\right)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a*sin(d*x+c)+b*tan(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/2*(2*b^2*log(a*cos(d*x + c) + b) - (a^2 + a*b)*log(1/2*cos(d*x + c) + 1/
2) - (a^2 - a*b)*log(-1/2*cos(d*x + c) + 1/2))/((a^3 - a*b^2)*d)
```

giac [B] time = 0.39, size = 257, normalized size = 3.21

$$-\frac{\frac{a\log\left(\left|-a-b+\frac{2\,b(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{a(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}-\frac{b(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}\right|\right)}{a^2-b^2}-\frac{\left(a^2-2\,b^2\right)\log\left(\frac{\left|-2\,b-\frac{2\,a(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{2\,b(\cos(dx+c)-1)}{\cos(dx+c)+1}-2\,|a|\right|}{\left|-2\,b-\frac{2\,a(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{2\,b(\cos(dx+c)-1)}{\cos(dx+c)+1}+2\,|a|\right|}\right)}{\left(a^2-b^2\right)|a|}-\frac{\log\left(\frac{|-\cos(dx+c)+1|}{|\cos(dx+c)+1|}\right)}{a+b}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a*sin(d*x+c)+b*tan(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/2*(a*log(abs(-a - b + 2*b*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + a*(cos
(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 - b*(cos(d*x + c) - 1)^2/(cos(d*x + c
) + 1)^2))/(a^2 - b^2) - (a^2 - 2*b^2)*log(abs(-2*b - 2*a*(cos(d*x + c) - 1
)/(cos(d*x + c) + 1) + 2*b*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 2*abs(a)
)/abs(-2*b - 2*a*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 2*b*(cos(d*x + c)
- 1)/(cos(d*x + c) + 1) + 2*abs(a)))/((a^2 - b^2)*abs(a)) - log(abs(-cos(d*
x + c) + 1)/abs(cos(d*x + c) + 1))/(a + b))/d
```

maple [A] time = 0.15, size = 80, normalized size = 1.00

$$-\frac{b^2\ln\left(b+a\cos\left(dx+c\right)\right)}{d\left(a+b\right)\left(a-b\right)a}+\frac{\ln\left(\cos\left(dx+c\right)-1\right)}{d\left(2a+2b\right)}+\frac{\ln\left(1+\cos\left(dx+c\right)\right)}{d\left(2a-2b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)/(a*sin(d*x+c)+b*tan(d*x+c)),x)
```

```
[Out] -1/d*b^2/(a+b)/(a-b)/a*ln(b+a*cos(d*x+c))+1/d/(2*a+2*b)*ln(cos(d*x+c)-1)+1/
d/(2*a-2*b)*ln(1+cos(d*x+c))
```

maxima [A] time = 0.42, size = 102, normalized size = 1.28

$$-\frac{\frac{b^2\log\left(a+b-\frac{(a-b)\sin(dx+c)^2}{(\cos(dx+c)+1)^2}\right)}{a^3-ab^2}-\frac{\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a+b}+\frac{\log\left(\frac{\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+1\right)}{a}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a*sin(d*x+c)+b*tan(d*x+c)),x, algorithm="maxima")
```

```
[Out] -(b^2*log(a + b - (a - b)*sin(d*x + c)^2/(cos(d*x + c) + 1)^2)/(a^3 - a*b^2
) - log(sin(d*x + c)/(cos(d*x + c) + 1))/(a + b) + log(sin(d*x + c)^2/(cos(
d*x + c) + 1)^2 + 1)/a)/d
```

mupad [B] time = 0.87, size = 93, normalized size = 1.16

$$\frac{\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{d\,\left(a+b\right)}-\frac{\ln\left({\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2+1\right)}{a\,d}+\frac{b^2\,\ln\left(a+b-a\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2+b\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2\right)}{d\,\left(a\,b^2-a^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)/(a*sin(c + d*x) + b*tan(c + d*x)),x)
```

```
[Out] log(tan(c/2 + (d*x)/2))/(d*(a + b)) - log(tan(c/2 + (d*x)/2)^2 + 1)/(a*d) +
 (b^2*log(a + b - a*tan(c/2 + (d*x)/2)^2 + b*tan(c/2 + (d*x)/2)^2))/(d*(a*b
^2 - a^3))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos{\left(c + dx \right)}}{a \sin{\left(c + dx \right)} + b \tan{\left(c + dx \right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a*sin(d*x+c)+b*tan(d*x+c)),x)
```

```
[Out] Integral(cos(c + d*x)/(a*sin(c + d*x) + b*tan(c + d*x)), x)
```

3.253 $\int \frac{1}{a\sin(c+dx)+b\tan(c+dx)}\,dx$

Optimal. Leaf size=74

$$\frac{b\log(a\cos(c+dx)+b)}{d\left(a^2-b^2\right)}+\frac{\log(1-\cos(c+dx))}{2d(a+b)}-\frac{\log(\cos(c+dx)+1)}{2d(a-b)}$$

```
[Out] 1/2*ln(1-cos(d*x+c))/(a+b)/d-1/2*ln(1+cos(d*x+c))/(a-b)/d+b*ln(b+a*cos(d*x+
c))/(a^2-b^2)/d
```

Rubi [A] time = 0.08, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {4397, 2721, 801}

$$\frac{b\log(a\cos(c+dx)+b)}{d\left(a^2-b^2\right)}+\frac{\log(1-\cos(c+dx))}{2d(a+b)}-\frac{\log(\cos(c+dx)+1)}{2d(a-b)}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Sin[c + d*x] + b*Tan[c + d*x])^(-1),x]
```

```
[Out] Log[1 - Cos[c + d*x]]/(2*(a + b)*d) - Log[1 + Cos[c + d*x]]/(2*(a - b)*d) +
 (b*Log[b + a*Cos[c + d*x]])/((a^2 - b^2)*d)
```

Rule 801

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a + c*x^2), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rule 2721

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Dist[1/f, Subst[Int[(x^p*(a + x)^m)/(b^2 - x^2)^((p + 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b^
2, 0] && IntegerQ[(p + 1)/2]
```

Rule 4397

```
Int[u_, x_Symbol] :> Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]
```

Rubi steps

$$\int \frac{1}{a\sin(c+dx)+b\tan(c+dx)}\,dx = \int \frac{\cot(c+dx)}{b+a\cos(c+dx)}\,dx$$
$$= -\frac{\text{Subst}\left(\int \frac{x}{(b+x)\left(a^2-x^2\right)}\,dx, x, a\cos(c+dx)\right)}{d}$$
$$= -\frac{\text{Subst}\left(\int \left(\frac{1}{2(a+b)(a-x)}+\frac{1}{2(a-b)(a+x)}+\frac{b}{(-a+b)(a+b)(b+x)}\right)dx, x, a\cos(c+dx)\right)}{d}$$
$$= \frac{\log(1-\cos(c+dx))}{2(a+b)d} - \frac{\log(1+\cos(c+dx))}{2(a-b)d} + \frac{b\log(b+a\cos(c+dx))}{\left(a^2-b^2\right)d}$$

Mathematica [A] time = 0.09, size = 63, normalized size = 0.85

$$\frac{(a-b)\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)-\left((a+b)\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)\right)+b\log(a\cos(c+dx)+b)}{d(a-b)(a+b)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Sin[c + d*x] + b*Tan[c + d*x])^(-1),x]
```

```
[Out] (-((a + b)*Log[Cos[(c + d*x)/2]]) + b*Log[b + a*Cos[c + d*x]] + (a - b)*Log
[Sin[(c + d*x)/2]])/((a - b)*(a + b)*d)
```

fricas [A] time = 0.54, size = 64, normalized size = 0.86

$$\frac{2\,b\log\left(a\cos\left(dx+c\right)+b\right)-(a+b)\log\left(\frac{1}{2}\cos\left(dx+c\right)+\frac{1}{2}\right)+(a-b)\log\left(-\frac{1}{2}\cos\left(dx+c\right)+\frac{1}{2}\right)}{2\left(a^2-b^2\right)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sin(d*x+c)+b*tan(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/2*(2*b*log(a*cos(d*x + c) + b) - (a + b)*log(1/2*cos(d*x + c) + 1/2) + (a
 - b)*log(-1/2*cos(d*x + c) + 1/2))/((a^2 - b^2)*d)
```

giac [A] time = 0.21, size = 100, normalized size = 1.35

$$\frac{\frac{2\,b\log\left(\left|-a-b-\frac{a(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{b(\cos(dx+c)-1)}{\cos(dx+c)+1}\right|\right)}{a^2-b^2}+\frac{\log\left(\frac{|-\cos(dx+c)+1|}{|\cos(dx+c)+1|}\right)}{a+b}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sin(d*x+c)+b*tan(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/2*(2*b*log(abs(-a - b - a*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + b*(cos(
d*x + c) - 1)/(cos(d*x + c) + 1)))/(a^2 - b^2) + log(abs(-cos(d*x + c) + 1)
/abs(cos(d*x + c) + 1))/(a + b))/d
```

maple [A] time = 0.15, size = 75, normalized size = 1.01

$$\frac{b\ln\left(b+a\cos\left(dx+c\right)\right)}{d\left(a+b\right)\left(a-b\right)}+\frac{\ln\left(\cos\left(dx+c\right)-1\right)}{d\left(2a+2b\right)}-\frac{\ln\left(1+\cos\left(dx+c\right)\right)}{d\left(2a-2b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*sin(d*x+c)+b*tan(d*x+c)),x)
```

```
[Out] 1/d*b/(a+b)/(a-b)*ln(b+a*cos(d*x+c))+1/d/(2*a+2*b)*ln(cos(d*x+c)-1)-1/d/(2*
a-2*b)*ln(1+cos(d*x+c))
```

maxima [A] time = 0.33, size = 71, normalized size = 0.96

$$\frac{\frac{b\log\left(a+b-\frac{(a-b)\sin(dx+c)^2}{(\cos(dx+c)+1)^2}\right)}{a^2-b^2}+\frac{\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a+b}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sin(d*x+c)+b*tan(d*x+c)),x, algorithm="maxima")
```

```
[Out] (b*log(a + b - (a - b)*sin(d*x + c)^2/(cos(d*x + c) + 1)^2)/(a^2 - b^2) + l
og(sin(d*x + c)/(cos(d*x + c) + 1))/(a + b))/d
```

mupad [B] time = 0.90, size = 67, normalized size = 0.91

$$\frac{\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{d\,\left(a+b\right)}+\frac{b\,\ln\left(a+b-a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\right)}{d\,\left(a^2-b^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*sin(c + d*x) + b*tan(c + d*x)),x)
```

```
[Out] log(tan(c/2 + (d*x)/2))/(d*(a + b)) + (b*log(a + b - a*tan(c/2 + (d*x)/2)^2
 + b*tan(c/2 + (d*x)/2)^2))/(d*(a^2 - b^2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{a\sin{(c+dx)}+b\tan{(c+dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sin(d*x+c)+b*tan(d*x+c)),x)
```

```
[Out] Integral(1/(a*sin(c + d*x) + b*tan(c + d*x)), x)
```

3.254 $\int \frac{\sec(c+dx)}{a\sin(c+dx)+b\tan(c+dx)}\,dx$

Optimal. Leaf size=75

$$-\frac{a\log(a\cos(c+dx)+b)}{d\left(a^2-b^2\right)}+\frac{\log(1-\cos(c+dx))}{2d(a+b)}+\frac{\log(\cos(c+dx)+1)}{2d(a-b)}$$

```
[Out] 1/2*ln(1-cos(d*x+c))/(a+b)/d+1/2*ln(1+cos(d*x+c))/(a-b)/d-a*ln(b+a*cos(d*x+
c))/(a^2-b^2)/d
```

Rubi [A] time = 0.19, antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {4397, 2668, 706, 31, 633}

$$-\frac{a\log(a\cos(c+dx)+b)}{d\left(a^2-b^2\right)}+\frac{\log(1-\cos(c+dx))}{2d(a+b)}+\frac{\log(\cos(c+dx)+1)}{2d(a-b)}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]/(a*Sin[c + d*x] + b*Tan[c + d*x]),x]
```

```
[Out] Log[1 - Cos[c + d*x]]/(2*(a + b)*d) + Log[1 + Cos[c + d*x]]/(2*(a - b)*d) -
 (a*Log[b + a*Cos[c + d*x]])/((a^2 - b^2)*d)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 633

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> With[{q = Rt[-(
a*c), 2]}, Dist[e/2 + (c*d)/(2*q), Int[1/(-q + c*x), x], x] + Dist[e/2 - (c
*d)/(2*q), Int[1/(q + c*x), x], x]] /; FreeQ[{a, c, d, e}, x] && NiceSqrtQ[
-(a*c)]
```

Rule 706

```
Int[1/(((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)), x_Symbol] :> Dist[e^2/(c
*d^2 + a*e^2), Int[1/(d + e*x), x], x] + Dist[1/(c*d^2 + a*e^2), Int[(c*d -
 c*e*x)/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2,
 0]
```

Rule 2668

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rule 4397

```
Int[u_, x_Symbol] :> Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sec(c+dx)}{a\sin(c+dx)+b\tan(c+dx)}\,dx &= \int \frac{\csc(c+dx)}{b+a\cos(c+dx)}\,dx \\
&= -\frac{a\,\text{Subst}\left(\int \frac{1}{(b+x)\left(a^2-x^2\right)}\,dx, x, a\cos(c+dx)\right)}{d} \\
&= -\frac{a\,\text{Subst}\left(\int \frac{1}{b+x}\,dx, x, a\cos(c+dx)\right)}{\left(a^2-b^2\right)d} - \frac{a\,\text{Subst}\left(\int \frac{-b+x}{a^2-x^2}\,dx, x, a\cos(c+d
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{a\log(b+a\cos(c+dx))}{\left(a^2-b^2\right)d} - \frac{\text{Subst}\left(\int \frac{1}{-a-x}\,dx, x, a\cos(c+dx)\right)}{2(a-b)d} - \frac{\text{Subst}(}{} \\
&= \frac{\log(1-\cos(c+dx))}{2(a+b)d} + \frac{\log(1+\cos(c+dx))}{2(a-b)d} - \frac{a\log(b+a\cos(c+dx))}{\left(a^2-b^2\right)d}
\end{aligned}
$$

Mathematica [A] time = 0.06, size = 64, normalized size = 0.85

$$
\frac{(a-b)\log(1-\cos(c+dx))+(a+b)\log(\cos(c+dx)+1)-2a\log(a\cos(c+dx)+b)}{2d(a-b)(a+b)}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]/(a*Sin[c + d*x] + b*Tan[c + d*x]),x]
```

```
[Out] ((a - b)*Log[1 - Cos[c + d*x]] + (a + b)*Log[1 + Cos[c + d*x]] - 2*a*Log[b
+ a*Cos[c + d*x]])/(2*(a - b)*(a + b)*d)
```

fricas [A] time = 0.77, size = 65, normalized size = 0.87

$$
-\frac{2\,a\log\left(a\cos\left(dx+c\right)+b\right)-(a+b)\log\left(\frac{1}{2}\,\cos\left(dx+c\right)+\frac{1}{2}\right)-(a-b)\log\left(-\frac{1}{2}\,\cos\left(dx+c\right)+\frac{1}{2}\right)}{2\left(a^2-b^2\right)d}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a*sin(d*x+c)+b*tan(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/2*(2*a*log(a*cos(d*x + c) + b) - (a + b)*log(1/2*cos(d*x + c) + 1/2) - (
a - b)*log(-1/2*cos(d*x + c) + 1/2))/((a^2 - b^2)*d)
```

giac [A] time = 0.32, size = 101, normalized size = 1.35

$$-\frac{\frac{2\,a\log\left(\left|-a-b-\frac{a(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{b(\cos(dx+c)-1)}{\cos(dx+c)+1}\right|\right)}{a^2-b^2}-\frac{\log\left(\frac{|-\cos(dx+c)+1|}{|\cos(dx+c)+1|}\right)}{a+b}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a*sin(d*x+c)+b*tan(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/2*(2*a*log(abs(-a - b - a*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + b*(cos
(d*x + c) - 1)/(cos(d*x + c) + 1)))/(a^2 - b^2) - log(abs(-cos(d*x + c) + 1
)/abs(cos(d*x + c) + 1))/(a + b))/d
```

maple [A] time = 0.19, size = 75, normalized size = 1.00

$$-\frac{a\ln\left(b+a\cos\left(dx+c\right)\right)}{d\left(a+b\right)\left(a-b\right)}+\frac{\ln\left(\cos\left(dx+c\right)-1\right)}{d\left(2a+2b\right)}+\frac{\ln\left(1+\cos\left(dx+c\right)\right)}{d\left(2a-2b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)/(a*sin(d*x+c)+b*tan(d*x+c)),x)
```

```
[Out] -1/d*a/(a+b)/(a-b)*ln(b+a*cos(d*x+c))+1/d/(2*a+2*b)*ln(cos(d*x+c)-1)+1/d/(2
*a-2*b)*ln(1+cos(d*x+c))
```

maxima [A] time = 0.36, size = 73, normalized size = 0.97

$$-\frac{\frac{a\log\left(a+b-\frac{(a-b)\sin(dx+c)^2}{(\cos(dx+c)+1)^2}\right)}{a^2-b^2}-\frac{\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a+b}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a*sin(d*x+c)+b*tan(d*x+c)),x, algorithm="maxima")
```

```
[Out] -(a*log(a + b - (a - b)*sin(d*x + c)^2/(cos(d*x + c) + 1)^2)/(a^2 - b^2) -
log(sin(d*x + c)/(cos(d*x + c) + 1))/(a + b))/d
```

mupad [B] time = 0.71, size = 68, normalized size = 0.91

$$\frac{\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{d\,\left(a+b\right)}-\frac{a\,\ln\left(a+b-a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\right)}{d\,\left(a^2-b^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)*(a*sin(c + d*x) + b*tan(c + d*x))),x)
```

```
[Out] log(tan(c/2 + (d*x)/2))/(d*(a + b)) - (a*log(a + b - a*tan(c/2 + (d*x)/2)^2
 + b*tan(c/2 + (d*x)/2)^2))/(d*(a^2 - b^2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sec\left(c+dx\right)}{a\sin\left(c+dx\right)+b\tan\left(c+dx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a*sin(d*x+c)+b*tan(d*x+c)),x)
```

```
[Out] Integral(sec(c + d*x)/(a*sin(c + d*x) + b*tan(c + d*x)), x)
```

3.255 $\int \frac{\sec^2(c+dx)}{a\sin(c+dx)+b\tan(c+dx)}\,dx$

Optimal. Leaf size=94

$$\frac{a^2\log(a\cos(c+dx)+b)}{bd\left(a^2-b^2\right)}+\frac{\log(1-\cos(c+dx))}{2d(a+b)}-\frac{\log(\cos(c+dx)+1)}{2d(a-b)}-\frac{\log(\cos(c+dx))}{bd}$$

```
[Out] 1/2*ln(1-cos(d*x+c))/(a+b)/d-ln(cos(d*x+c))/b/d-1/2*ln(1+cos(d*x+c))/(a-b)/
d+a^2*ln(b+a*cos(d*x+c))/b/(a^2-b^2)/d
```

Rubi [A] time = 0.27, antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {4397, 2837, 12, 894}

$$\frac{a^2\log(a\cos(c+dx)+b)}{bd\left(a^2-b^2\right)}+\frac{\log(1-\cos(c+dx))}{2d(a+b)}-\frac{\log(\cos(c+dx)+1)}{2d(a-b)}-\frac{\log(\cos(c+dx))}{bd}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^2/(a*Sin[c + d*x] + b*Tan[c + d*x]),x]
```

```
[Out] Log[1 - Cos[c + d*x]]/(2*(a + b)*d) - Log[Cos[c + d*x]]/(b*d) - Log[1 + Cos
[c + d*x]]/(2*(a - b)*d) + (a^2*Log[b + a*Cos[c + d*x]])/(b*(a^2 - b^2)*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 894

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ
[m, 0] && ILtQ[n, 0]))
```

Rule 2837

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^m*(c + (d*x)/b)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*S
in[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/
2] && NeQ[a^2 - b^2, 0]
```

Rule 4397

```
Int[u_, x_Symbol] :> Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]
```

Rubi steps

$$\int \frac{\sec^2(c+dx)}{a\sin(c+dx)+b\tan(c+dx)}\,dx = \int \frac{\csc(c+dx)\sec(c+dx)}{b+a\cos(c+dx)}\,dx$$

$$= -\frac{a\,\text{Subst}\left(\int \frac{a}{x(b+x)\left(a^2-x^2\right)}\,dx, x, a\cos(c+dx)\right)}{d}$$

$$= -\frac{a^2\,\text{Subst}\left(\int \frac{1}{x(b+x)\left(a^2-x^2\right)}\,dx, x, a\cos(c+dx)\right)}{d}$$

$$= -\frac{a^2\,\text{Subst}\left(\int \left(\frac{1}{2a^2(a+b)(a-x)}+\frac{1}{a^2bx}+\frac{1}{2a^2(a-b)(a+x)}+\frac{1}{b(-a+b)(a+b)(b+x)}\right) dx, x, a \cdots\right.}{d}$$

$$= \frac{\log(1-\cos(c+dx))}{2(a+b)d}-\frac{\log(\cos(c+dx))}{bd}-\frac{\log(1+\cos(c+dx))}{2(a-b)d}+\frac{a^2\log\cdots}{\cdots}$$

Mathematica [A] time = 0.15, size = 103, normalized size = 1.10

$$2\left(-\frac{a^2\log(a\cos(c+dx)+b)}{2bd\left(b^2-a^2\right)}+\frac{\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)}{2d(a+b)}+\frac{\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)}{2d(b-a)}-\frac{\log(\cos(c+dx))}{2bd}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^2/(a*Sin[c + d*x] + b*Tan[c + d*x]),x]
```

```
[Out] 2*(Log[Cos[(c + d*x)/2]]/(2*(-a + b)*d) - Log[Cos[c + d*x]]/(2*b*d) - (a^2*
Log[b + a*Cos[c + d*x]])/(2*b*(-a^2 + b^2)*d) + Log[Sin[(c + d*x)/2]]/(2*(a
 + b)*d))
```

fricas [A] time = 0.90, size = 96, normalized size = 1.02

$$\frac{2\,a^2\log\left(a\cos\left(dx+c\right)+b\right)-2\left(a^2-b^2\right)\log\left(-\cos\left(dx+c\right)\right)-\left(ab+b^2\right)\log\left(\frac{1}{2}\cos\left(dx+c\right)+\frac{1}{2}\right)+\left(ab-b^2\right)\text{lo}\cdots}{2\left(a^2b-b^3\right)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a*sin(d*x+c)+b*tan(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/2*(2*a^2*log(a*cos(d*x + c) + b) - 2*(a^2 - b^2)*log(-cos(d*x + c)) - (a*
b + b^2)*log(1/2*cos(d*x + c) + 1/2) + (a*b - b^2)*log(-1/2*cos(d*x + c) +
1/2))/((a^2*b - b^3)*d)
```

giac [B] time = 0.39, size = 253, normalized size = 2.69

$$\frac{\frac{b\log\left(\left|a+b+\frac{2\,a(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{a(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}-\frac{b(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}\right|\right)}{a^2-b^2}+\frac{\left(2\,a^2-b^2\right)\log\left(\frac{\left|-2\,a-\frac{2\,a(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{2\,b(\cos(dx+c)-1)}{\cos(dx+c)+1}-2\,|b|\right|}{\left|-2\,a-\frac{2\,a(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{2\,b(\cos(dx+c)-1)}{\cos(dx+c)+1}+2\,|b|\right|}\right)}{\left(a^2-b^2\right)|b|}+\frac{\log\left(\frac{|-\cos(dx+c)+1|}{|\cos(dx+c)+1|}\right)}{a+b}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a*sin(d*x+c)+b*tan(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/2*(b*log(abs(a + b + 2*a*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + a*(cos(d
*x + c) - 1)^2/(cos(d*x + c) + 1)^2 - b*(cos(d*x + c) - 1)^2/(cos(d*x + c)
+ 1)^2))/(a^2 - b^2) + (2*a^2 - b^2)*log(abs(-2*a - 2*a*(cos(d*x + c) - 1)/
(cos(d*x + c) + 1) + 2*b*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 2*abs(b))/
abs(-2*a - 2*a*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 2*b*(cos(d*x + c) -
1)/(cos(d*x + c) + 1) + 2*abs(b)))/((a^2 - b^2)*abs(b)) + log(abs(-cos(d*x
+ c) + 1)/abs(cos(d*x + c) + 1))/(a + b))/d
```

maple [A] time = 0.20, size = 95, normalized size = 1.01

$$\frac{a^2\ln\left(b+a\cos\left(dx+c\right)\right)}{d\left(a+b\right)\left(a-b\right)b}+\frac{\ln\left(\cos\left(dx+c\right)-1\right)}{d\left(2a+2b\right)}-\frac{\ln\left(1+\cos\left(dx+c\right)\right)}{d\left(2a-2b\right)}-\frac{\ln\left(\cos\left(dx+c\right)\right)}{bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2/(a*sin(d*x+c)+b*tan(d*x+c)),x)
```

```
[Out] 1/d*a^2/(a+b)/(a-b)/b*ln(b+a*cos(d*x+c))+1/d/(2*a+2*b)*ln(cos(d*x+c)-1)-1/d
/(2*a-2*b)*ln(1+cos(d*x+c))-ln(cos(d*x+c))/b/d
```

maxima [A] time = 0.37, size = 125, normalized size = 1.33

$$\frac{\frac{a^2\log\left(a+b-\frac{(a-b)\sin(dx+c)^2}{(\cos(dx+c)+1)^2}\right)}{a^2b-b^3}-\frac{\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)}{b}-\frac{\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}-1\right)}{b}+\frac{\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a+b}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a*sin(d*x+c)+b*tan(d*x+c)),x, algorithm="maxima")
```

```
[Out] (a^2*log(a + b - (a - b)*sin(d*x + c)^2/(cos(d*x + c) + 1)^2)/(a^2*b - b^3)
 - log(sin(d*x + c)/(cos(d*x + c) + 1) + 1)/b - log(sin(d*x + c)/(cos(d*x +
 c) + 1) - 1)/b + log(sin(d*x + c)/(cos(d*x + c) + 1))/(a + b))/d
```

mupad [B] time = 0.84, size = 93, normalized size = 0.99

$$\frac{\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{d\,\left(a+b\right)}-\frac{\ln\left({\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2-1\right)}{b\,d}+\frac{a^2\,\ln\left(a+b-a\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2+b\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2\right)}{d\,\left(a^2\,b-b^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^2*(a*sin(c + d*x) + b*tan(c + d*x))),x)
```

```
[Out] log(tan(c/2 + (d*x)/2))/(d*(a + b)) - log(tan(c/2 + (d*x)/2)^2 - 1)/(b*d) +
 (a^2*log(a + b - a*tan(c/2 + (d*x)/2)^2 + b*tan(c/2 + (d*x)/2)^2))/(d*(a^2
*b - b^3))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec^2(c+dx)}{a\sin(c+dx)+b\tan(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2/(a*sin(d*x+c)+b*tan(d*x+c)),x)
```

```
[Out] Integral(sec(c + d*x)**2/(a*sin(c + d*x) + b*tan(c + d*x)), x)
```

3.256 $\int \frac{\sec^3(c+dx)}{a\sin(c+dx)+b\tan(c+dx)}\,dx$

Optimal. Leaf size=108

$$-\frac{a^3\log(a\cos(c+dx)+b)}{b^2d\left(a^2-b^2\right)}+\frac{a\log(\cos(c+dx))}{b^2d}+\frac{\log(1-\cos(c+dx))}{2d(a+b)}+\frac{\log(\cos(c+dx)+1)}{2d(a-b)}+\frac{\sec(c+dx)}{bd}$$

```
[Out] 1/2*ln(1-cos(d*x+c))/(a+b)/d+a*ln(cos(d*x+c))/b^2/d+1/2*ln(1+cos(d*x+c))/(a
-b)/d-a^3*ln(b+a*cos(d*x+c))/b^2/(a^2-b^2)/d+sec(d*x+c)/b/d
```

Rubi [A] time = 0.28, antiderivative size = 108, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {4397, 2837, 12, 894}

$$-\frac{a^3\log(a\cos(c+dx)+b)}{b^2d\left(a^2-b^2\right)}+\frac{a\log(\cos(c+dx))}{b^2d}+\frac{\log(1-\cos(c+dx))}{2d(a+b)}+\frac{\log(\cos(c+dx)+1)}{2d(a-b)}+\frac{\sec(c+dx)}{bd}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^3/(a*Sin[c + d*x] + b*Tan[c + d*x]),x]
```

```
[Out] Log[1 - Cos[c + d*x]]/(2*(a + b)*d) + (a*Log[Cos[c + d*x]])/(b^2*d) + Log[1
 + Cos[c + d*x]]/(2*(a - b)*d) - (a^3*Log[b + a*Cos[c + d*x]])/(b^2*(a^2 -
b^2)*d) + Sec[c + d*x]/(b*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 894

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ
[m, 0] && ILtQ[n, 0]))
```

Rule 2837

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^m*(c + (d*x)/b)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*S
in[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/
```

```
2] && NeQ[a^2 - b^2, 0]
```

Rule 4397

```
Int[u_, x_Symbol] :> Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]
```

Rubi steps

$$\int \frac{\sec^3(c+dx)}{a\sin(c+dx)+b\tan(c+dx)}\,dx = \int \frac{\csc(c+dx)\sec^2(c+dx)}{b+a\cos(c+dx)}\,dx$$
$$= -\frac{a\,\text{Subst}\left(\int \frac{a^2}{x^2(b+x)\left(a^2-x^2\right)}\,dx, x, a\cos(c+dx)\right)}{d}$$
$$= -\frac{a^3\,\text{Subst}\left(\int \frac{1}{x^2(b+x)\left(a^2-x^2\right)}\,dx, x, a\cos(c+dx)\right)}{d}$$
$$= -\frac{a^3\,\text{Subst}\left(\int \left(\frac{1}{2a^3(a+b)(a-x)}+\frac{1}{a^2bx^2}-\frac{1}{a^2b^2x}-\frac{1}{2a^3(a-b)(a+x)}-\frac{1}{b^2(-a+b)(a+b)(b+x)}\right.\right.}{d}$$
$$= \frac{\log(1-\cos(c+dx))}{2(a+b)d}+\frac{a\log(\cos(c+dx))}{b^2d}+\frac{\log(1+\cos(c+dx))}{2(a-b)d}-\frac{a^3\,\text{lc}}{}$$

Mathematica [A] time = 0.29, size = 92, normalized size = 0.85

$$\frac{\frac{a^3\log(a\cos(c+dx)+b)}{b^4-a^2b^2}+\frac{a\log(\cos(c+dx))}{b^2}+\frac{\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)}{a+b}+\frac{\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)}{a-b}+\frac{\sec(c+dx)}{b}}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^3/(a*Sin[c + d*x] + b*Tan[c + d*x]),x]
[Out] (Log[Cos[(c + d*x)/2]]/(a - b) + (a*Log[Cos[c + d*x]])/b^2 + (a^3*Log[b + a
*Cos[c + d*x]])/(-(a^2*b^2) + b^4) + Log[Sin[(c + d*x)/2]]/(a + b) + Sec[c
+ d*x]/b)/d
```

fricas [A] time = 0.67, size = 147, normalized size = 1.36

$$-\frac{2\,a^3\cos(dx+c)\log(a\cos(dx+c)+b)-2\,a^2b+2\,b^3-2\left(a^3-ab^2\right)\cos(dx+c)\log(-\cos(dx+c))-\left(ab^2+\right.}{2\left(a^2b^2-b^4\right)d\cos(dx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a*sin(d*x+c)+b*tan(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/2*(2*a^3*cos(d*x + c)*log(a*cos(d*x + c) + b) - 2*a^2*b + 2*b^3 - 2*(a^3
 - a*b^2)*cos(d*x + c)*log(-cos(d*x + c)) - (a*b^2 + b^3)*cos(d*x + c)*log(
1/2*cos(d*x + c) + 1/2) - (a*b^2 - b^3)*cos(d*x + c)*log(-1/2*cos(d*x + c)
+ 1/2))/((a^2*b^2 - b^4)*d*cos(d*x + c))
```

giac [A] time = 0.39, size = 190, normalized size = 1.76

$$-\frac{\frac{2\,a^3\log\left(\left|-a-b-\frac{a(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{b(\cos(dx+c)-1)}{\cos(dx+c)+1}\right|\right)}{a^2b^2-b^4}-\frac{\log\left(\frac{|-\cos(dx+c)+1|}{|\cos(dx+c)+1|}\right)}{a+b}-\frac{2\,a\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}-1\right|\right)}{b^2}+\frac{2\left(a-2\,b+\frac{a(\cos(dx+c)-1)}{\cos(dx+c)+1}\right)}{b^2\left(\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right)}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a*sin(d*x+c)+b*tan(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/2*(2*a^3*log(abs(-a - b - a*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + b*(c
os(d*x + c) - 1)/(cos(d*x + c) + 1)))/(a^2*b^2 - b^4) - log(abs(-cos(d*x +
c) + 1)/abs(cos(d*x + c) + 1))/(a + b) - 2*a*log(abs(-(cos(d*x + c) - 1)/(c
os(d*x + c) + 1) - 1))/b^2 + 2*(a - 2*b + a*(cos(d*x + c) - 1)/(cos(d*x + c
) + 1))/(b^2*((cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1)))/d
```

maple [A] time = 0.20, size = 110, normalized size = 1.02

$$-\frac{a^3\ln(b+a\cos(dx+c))}{d\,(a+b)\,(a-b)\,b^2}+\frac{\ln(\cos(dx+c)-1)}{d\,(2a+2b)}+\frac{\ln(1+\cos(dx+c))}{d\,(2a-2b)}+\frac{a\ln(\cos(dx+c))}{b^2d}+\frac{1}{db\cos(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^3/(a*sin(d*x+c)+b*tan(d*x+c)),x)
```

```
[Out] -1/d*a^3/(a+b)/(a-b)/b^2*ln(b+a*cos(d*x+c))+1/d/(2*a+2*b)*ln(cos(d*x+c)-1)+
1/d/(2*a-2*b)*ln(1+cos(d*x+c))+a*ln(cos(d*x+c))/b^2/d+1/d/b/cos(d*x+c)
```

maxima [A] time = 0.44, size = 158, normalized size = 1.46

$$-\frac{\frac{a^3\log\left(a+b-\frac{(a-b)\sin(dx+c)^2}{(\cos(dx+c)+1)^2}\right)}{a^2b^2-b^4}-\frac{a\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)}{b^2}-\frac{a\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}-1\right)}{b^2}-\frac{\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a+b}-\frac{2}{b-\frac{b\sin(dx+c)^2}{(\cos(dx+c)+1)^2}}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a*sin(d*x+c)+b*tan(d*x+c)),x, algorithm="maxima")
```

```
[Out] -(a^3*log(a + b - (a - b)*sin(d*x + c)^2/(cos(d*x + c) + 1)^2)/(a^2*b^2 - b
^4) - a*log(sin(d*x + c)/(cos(d*x + c) + 1) + 1)/b^2 - a*log(sin(d*x + c)/(
cos(d*x + c) + 1) - 1)/b^2 - log(sin(d*x + c)/(cos(d*x + c) + 1))/(a + b) -
 2/(b - b*sin(d*x + c)^2/(cos(d*x + c) + 1)^2))/d
```

mupad [B] time = 0.83, size = 118, normalized size = 1.09

$$\frac{\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{d\,\left(a+b\right)}-\frac{2}{b\,d\,\left({\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2-1\right)}+\frac{a\,\ln\left({\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2-1\right)}{b^2\,d}-\frac{a^3\,\ln\left(a+b-a\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2+b\,\tan\left(\right.\right.}{b^2\,d\,\left(a^2-b^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^3*(a*sin(c + d*x) + b*tan(c + d*x))),x)
```

```
[Out] log(tan(c/2 + (d*x)/2))/(d*(a + b)) - 2/(b*d*(tan(c/2 + (d*x)/2)^2 - 1)) +
(a*log(tan(c/2 + (d*x)/2)^2 - 1))/(b^2*d) - (a^3*log(a + b - a*tan(c/2 + (d
*x)/2)^2 + b*tan(c/2 + (d*x)/2)^2))/(b^2*d*(a^2 - b^2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec^3(c+dx)}{a\sin(c+dx)+b\tan(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**3/(a*sin(d*x+c)+b*tan(d*x+c)),x)
```

```
[Out] Integral(sec(c + d*x)**3/(a*sin(c + d*x) + b*tan(c + d*x)), x)
```

3.257 $\int \frac{\cos^3(c+dx)}{(a\sin(c+dx)+b\tan(c+dx))^2}\,dx$

Optimal. Leaf size=243

$$\frac{2b^6\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^3d(a-b)^{5/2}(a+b)^{5/2}}+\frac{2bx}{a^3}-\frac{b^5\sin(c+dx)}{a^2d\left(a^2-b^2\right)^2(a\cos(c+dx)+b)}-\frac{\sin(c+dx)}{a^2d}+\frac{2b^4\left(5a^2-3b^2\right)\tanh^{-1}\left(\frac{\sqrt{a-}\cdots}{}\right.}{a^3d(a-b)^{5/2}(a+\cdots}$$

```
[Out] 2*b*x/a^3+2*b^6*arctanh((a-b)^(1/2)*tan(1/2*d*x+1/2*c)/(a+b)^(1/2))/a^3/(a-
b)^(5/2)/(a+b)^(5/2)/d+2*b^4*(5*a^2-3*b^2)*arctanh((a-b)^(1/2)*tan(1/2*d*x+
1/2*c)/(a+b)^(1/2))/a^3/(a-b)^(5/2)/(a+b)^(5/2)/d-sin(d*x+c)/a^2/d-1/2*sin(
d*x+c)/(a+b)^2/d/(1-cos(d*x+c))-1/2*sin(d*x+c)/(a-b)^2/d/(1+cos(d*x+c))-b^5
*sin(d*x+c)/a^2/(a^2-b^2)^2/d/(b+a*cos(d*x+c))
```

Rubi [A] time = 0.61, antiderivative size = 243, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 8, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {4397, 2897, 2648, 2637, 2664, 12, 2659, 208}

$$-\frac{b^5\sin(c+dx)}{a^2d\left(a^2-b^2\right)^2(a\cos(c+dx)+b)}+\frac{2b^6\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^3d(a-b)^{5/2}(a+b)^{5/2}}+\frac{2b^4\left(5a^2-3b^2\right)\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^3d(a-b)^{5/2}(a+b)^{5/2}}+\frac{2\cdots}{a\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^3/(a*Sin[c + d*x] + b*Tan[c + d*x])^2,x]
```

```
[Out] (2*b*x)/a^3 + (2*b^6*ArcTanh[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/(
a^3*(a - b)^(5/2)*(a + b)^(5/2)*d) + (2*b^4*(5*a^2 - 3*b^2)*ArcTanh[(Sqrt[a
 - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/(a^3*(a - b)^(5/2)*(a + b)^(5/2)*d) -
 Sin[c + d*x]/(a^2*d) - Sin[c + d*x]/(2*(a + b)^2*d*(1 - Cos[c + d*x])) - S
in[c + d*x]/(2*(a - b)^2*d*(1 + Cos[c + d*x])) - (b^5*Sin[c + d*x])/(a^2*(a
^2 - b^2)^2*d*(b + a*Cos[c + d*x]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 2648

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> -Simp[Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 2664

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[
c + d*x]*(a + b*Sin[c + d*x])^(n + 1))/(d*(n + 1)*(a^2 - b^2)), x] + Dist[1
/((n + 1)*(a^2 - b^2)), Int[(a + b*Sin[c + d*x])^(n + 1)*Simp[a*(n + 1) - b
*(n + 2)*Sin[c + d*x], x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^
2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2897

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_)
 + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig[(d*sin[
e + f*x])^n*(a + b*sin[e + f*x])^m*(1 - sin[e + f*x]^2)^(p/2), x], x] /; Fr
eeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && IntegersQ[m, 2*n, p/2] && (
LtQ[m, -1] || (EqQ[m, -1] && GtQ[p, 0]))
```

Rule 4397

```
Int[u_, x_Symbol] :> Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]
```

Rubi steps

$$\int \frac{\cos^3(c+dx)}{(a\sin(c+dx)+b\tan(c+dx))^2}\,dx = \int \frac{\cos^3(c+dx)\cot^2(c+dx)}{(b+a\cos(c+dx))^2}\,dx$$

$$= -\int \left(-\frac{2b}{a^3} - \frac{1}{2(a-b)^2(-1-\cos(c+dx))} - \frac{1}{2(a+b)^2(1-\cos(c+dx))} + \right.$$

$$= \frac{2bx}{a^3} - \frac{\int \cos(c+dx)\,dx}{a^2} + \frac{\int \frac{1}{-1-\cos(c+dx)}\,dx}{2(a-b)^2} + \frac{\int \frac{1}{1-\cos(c+dx)}\,dx}{2(a+b)^2} - \frac{\left(b^4\left(5a^2\right.\right.}{}$$

$$= \frac{2bx}{a^3} - \frac{\sin(c+dx)}{a^2d} - \frac{\sin(c+dx)}{2(a+b)^2d(1-\cos(c+dx))} - \frac{\sin(c+dx)}{2(a-b)^2d(1+\cos(c}$$

$$= \frac{2bx}{a^3} + \frac{2b^4\left(5a^2-3b^2\right)\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^3(a-b)^{5/2}(a+b)^{5/2}d} - \frac{\sin(c+dx)}{a^2d} - \frac{}{2(a+b}$$

$$= \frac{2bx}{a^3} + \frac{2b^4\left(5a^2-3b^2\right)\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^3(a-b)^{5/2}(a+b)^{5/2}d} - \frac{\sin(c+dx)}{a^2d} - \frac{}{2(a+b}$$

$$= \frac{2bx}{a^3} + \frac{2b^6\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^3(a-b)^{5/2}(a+b)^{5/2}d} + \frac{2b^4\left(5a^2-3b^2\right)\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\right.\right.}{\sqrt{a-}}\right)}{a^3(a-b)^{5/2}(a+b)^{5/2}d}$$

Mathematica [A] time = 2.95, size = 164, normalized size = 0.67

$$-\frac{-\frac{4b(c+dx)}{a^3}+\frac{2b^5\sin(c+dx)}{a^2(a-b)^2(a+b)^2(a\cos(c+dx)+b)}+\frac{2\sin(c+dx)}{a^2}+\frac{4b^4\left(5a^2-2b^2\right)\tanh^{-1}\left(\frac{(b-a)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{a^3\left(a^2-b^2\right)^{5/2}}+\frac{\tan\left(\frac{1}{2}(c+dx)\right)}{(a-b)^2}+\frac{\cot\left(\frac{1}{2}(c+dx)\right)}{(a+b)^2}}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^3/(a*Sin[c + d*x] + b*Tan[c + d*x])^2,x]
```

```
[Out] -1/2*((-4*b*(c + d*x))/a^3 + (4*b^4*(5*a^2 - 2*b^2)*ArcTanh[((-a + b)*Tan[(
c + d*x)/2])/Sqrt[a^2 - b^2]])/(a^3*(a^2 - b^2)^(5/2)) + Cot[(c + d*x)/2]/(
a + b)^2 + (2*Sin[c + d*x])/a^2 + (2*b^5*Sin[c + d*x])/(a^2*(a - b)^2*(a +
b)^2*(b + a*Cos[c + d*x])) + Tan[(c + d*x)/2]/(a - b)^2)/d
```

fricas [A] time = 0.59, size = 857, normalized size = 3.53

$$\left[-\frac{4\,a^{7}b - 6\,a^{5}b^{3} + 6\,a^{3}b^{5} - 4\,ab^{7} - 2\left(a^{8} - 3\,a^{6}b^{2} + 3\,a^{4}b^{4} - a^{2}b^{6}\right)\cos\left(dx + c\right)^{3} + \left(5\,a^{2}b^{5} - 2\,b^{7} + \left(5\,a^{3}b^{4} - 2\,ab\right.\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3/(a*sin(d*x+c)+b*tan(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] [-1/2*(4*a^7*b - 6*a^5*b^3 + 6*a^3*b^5 - 4*a*b^7 - 2*(a^8 - 3*a^6*b^2 + 3*a
^4*b^4 - a^2*b^6)*cos(d*x + c)^3 + (5*a^2*b^5 - 2*b^7 + (5*a^3*b^4 - 2*a*b^
6)*cos(d*x + c))*sqrt(a^2 - b^2)*log((2*a*b*cos(d*x + c) - (a^2 - 2*b^2)*co
s(d*x + c)^2 - 2*sqrt(a^2 - b^2)*(b*cos(d*x + c) + a)*sin(d*x + c) + 2*a^2
- b^2)/(a^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + b^2))*sin(d*x + c) - 2*(3
*a^7*b - 5*a^5*b^3 + 4*a^3*b^5 - 2*a*b^7)*cos(d*x + c)^2 + 2*(2*a^8 - 5*a^6
*b^2 + 4*a^4*b^4 - a^2*b^6)*cos(d*x + c) - 4*((a^7*b - 3*a^5*b^3 + 3*a^3*b^
5 - a*b^7)*d*x*cos(d*x + c) + (a^6*b^2 - 3*a^4*b^4 + 3*a^2*b^6 - b^8)*d*x)*
sin(d*x + c))/(((a^10 - 3*a^8*b^2 + 3*a^6*b^4 - a^4*b^6)*d*cos(d*x + c) + (
a^9*b - 3*a^7*b^3 + 3*a^5*b^5 - a^3*b^7)*d)*sin(d*x + c)), -(2*a^7*b - 3*a^
5*b^3 + 3*a^3*b^5 - 2*a*b^7 - (a^8 - 3*a^6*b^2 + 3*a^4*b^4 - a^2*b^6)*cos(d
*x + c)^3 - (5*a^2*b^5 - 2*b^7 + (5*a^3*b^4 - 2*a*b^6)*cos(d*x + c))*sqrt(-
a^2 + b^2)*arctan(-sqrt(-a^2 + b^2)*(b*cos(d*x + c) + a)/((a^2 - b^2)*sin(d
*x + c)))*sin(d*x + c) - (3*a^7*b - 5*a^5*b^3 + 4*a^3*b^5 - 2*a*b^7)*cos(d*
x + c)^2 + (2*a^8 - 5*a^6*b^2 + 4*a^4*b^4 - a^2*b^6)*cos(d*x + c) - 2*((a^7
*b - 3*a^5*b^3 + 3*a^3*b^5 - a*b^7)*d*x*cos(d*x + c) + (a^6*b^2 - 3*a^4*b^4
 + 3*a^2*b^6 - b^8)*d*x)*sin(d*x + c))/(((a^10 - 3*a^8*b^2 + 3*a^6*b^4 - a^
4*b^6)*d*cos(d*x + c) + (a^9*b - 3*a^7*b^3 + 3*a^5*b^5 - a^3*b^7)*d)*sin(d*
x + c))]
```

giac [B] time = 1.13, size = 1362, normalized size = 5.60

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3/(a*sin(d*x+c)+b*tan(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/2*(2*((2*a^4*b - 2*a^3*b^2 - 4*a^2*b^3 - a*b^4 + 2*b^5)*sqrt(-a^2 + b^2)
*abs(a^7 - 2*a^5*b^2 + a^3*b^4)*abs(a - b) - (2*a^11*b - 2*a^10*b^2 - 8*a^9
*b^3 + 13*a^8*b^4 + 12*a^7*b^5 - 24*a^6*b^6 - 8*a^5*b^7 + 17*a^4*b^8 + 2*a^
3*b^9 - 4*a^2*b^10)*sqrt(-a^2 + b^2)*abs(a - b))*(pi*floor(1/2*(d*x + c)/pi
 + 1/2) + arctan(tan(1/2*d*x + 1/2*c)/sqrt(-(a^6*b - 2*a^4*b^3 + a^2*b^5 +
sqrt((a^7 + a^6*b - 2*a^5*b^2 - 2*a^4*b^3 + a^3*b^4 + a^2*b^5)*(a^7 - a^6*b
 - 2*a^5*b^2 + 2*a^4*b^3 + a^3*b^4 - a^2*b^5) + (a^6*b - 2*a^4*b^3 + a^2*b^
```

```
5)^2))/(a^7 - a^6*b - 2*a^5*b^2 + 2*a^4*b^3 + a^3*b^4 - a^2*b^5))))/((a^7 -
 2*a^5*b^2 + a^3*b^4)^2*(a^2 - 2*a*b + b^2) + (a^8*b - 2*a^7*b^2 - a^6*b^3
+ 4*a^5*b^4 - a^4*b^5 - 2*a^3*b^6 + a^2*b^7)*abs(a^7 - 2*a^5*b^2 + a^3*b^4)
) + 2*(2*a^11*b - 2*a^10*b^2 - 8*a^9*b^3 + 13*a^8*b^4 + 12*a^7*b^5 - 24*a^6
*b^6 - 8*a^5*b^7 + 17*a^4*b^8 + 2*a^3*b^9 - 4*a^2*b^10 + 2*a^4*b*abs(a^7 -
2*a^5*b^2 + a^3*b^4) - 2*a^3*b^2*abs(a^7 - 2*a^5*b^2 + a^3*b^4) - 4*a^2*b^3
*abs(a^7 - 2*a^5*b^2 + a^3*b^4) - a*b^4*abs(a^7 - 2*a^5*b^2 + a^3*b^4) + 2*
b^5*abs(a^7 - 2*a^5*b^2 + a^3*b^4))*(pi*floor(1/2*(d*x + c)/pi + 1/2) + arc
tan(tan(1/2*d*x + 1/2*c)/sqrt(-(a^6*b - 2*a^4*b^3 + a^2*b^5 - sqrt((a^7 + a
^6*b - 2*a^5*b^2 - 2*a^4*b^3 + a^3*b^4 + a^2*b^5)*(a^7 - a^6*b - 2*a^5*b^2
+ 2*a^4*b^3 + a^3*b^4 - a^2*b^5) + (a^6*b - 2*a^4*b^3 + a^2*b^5)^2))/(a^7 -
 a^6*b - 2*a^5*b^2 + 2*a^4*b^3 + a^3*b^4 - a^2*b^5))))/(a^6*b*abs(a^7 - 2*a
^5*b^2 + a^3*b^4) - 2*a^4*b^3*abs(a^7 - 2*a^5*b^2 + a^3*b^4) + a^2*b^5*abs(
a^7 - 2*a^5*b^2 + a^3*b^4) - (a^7 - 2*a^5*b^2 + a^3*b^4)^2) + tan(1/2*d*x +
 1/2*c)/(a^2 - 2*a*b + b^2) + (5*a^5*tan(1/2*d*x + 1/2*c)^4 - 7*a^4*b*tan(1
/2*d*x + 1/2*c)^4 - 5*a^3*b^2*tan(1/2*d*x + 1/2*c)^4 + 7*a^2*b^3*tan(1/2*d*
x + 1/2*c)^4 + 4*a*b^4*tan(1/2*d*x + 1/2*c)^4 - 8*b^5*tan(1/2*d*x + 1/2*c)^
4 - 4*a^5*tan(1/2*d*x + 1/2*c)^2 - 6*a^4*b*tan(1/2*d*x + 1/2*c)^2 + 12*a^3*
b^2*tan(1/2*d*x + 1/2*c)^2 + 6*a^2*b^3*tan(1/2*d*x + 1/2*c)^2 - 4*a*b^4*tan
(1/2*d*x + 1/2*c)^2 - 8*b^5*tan(1/2*d*x + 1/2*c)^2 - a^5 + a^4*b + a^3*b^2
- a^2*b^3)/((a^6 - 2*a^4*b^2 + a^2*b^4)*(a*tan(1/2*d*x + 1/2*c)^5 - b*tan(1
/2*d*x + 1/2*c)^5 - 2*b*tan(1/2*d*x + 1/2*c)^3 - a*tan(1/2*d*x + 1/2*c) - b
*tan(1/2*d*x + 1/2*c))))/d
```

maple [A] time = 0.24, size = 291, normalized size = 1.20

$$-\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2d\left(a^{2}-2ab+b^{2}\right)}+\frac{2b^{5}\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d\left(a+b\right)^{2}\left(a-b\right)^{2}a^{2}\left(\left(\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a-b\left(\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-a-b\right)}+\frac{10b^{4}\operatorname{arctanh}\left(\frac{\tan(}{\sqrt{}}\right.}{d\left(a+b\right)^{2}\left(a-b\right)^{2}a\sqrt{}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3/(a*sin(d*x+c)+b*tan(d*x+c))^2,x)
```

```
[Out] -1/2/d/(a^2-2*a*b+b^2)*tan(1/2*d*x+1/2*c)+2/d*b^5/(a+b)^2/(a-b)^2/a^2*tan(1
/2*d*x+1/2*c)/(tan(1/2*d*x+1/2*c)^2*a-b*tan(1/2*d*x+1/2*c)^2-a-b)+10/d*b^4/
(a+b)^2/(a-b)^2/a/((a+b)*(a-b))^(1/2)*arctanh(tan(1/2*d*x+1/2*c)*(a-b)/((a+
b)*(a-b))^(1/2))-4/d*b^6/(a+b)^2/(a-b)^2/a^3/((a+b)*(a-b))^(1/2)*arctanh(ta
n(1/2*d*x+1/2*c)*(a-b)/((a+b)*(a-b))^(1/2))-2/d/a^2*tan(1/2*d*x+1/2*c)/(tan
(1/2*d*x+1/2*c)^2+1)+4/d/a^3*b*arctan(tan(1/2*d*x+1/2*c))-1/2/d/(a+b)^2/tan
(1/2*d*x+1/2*c)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3/(a*sin(d*x+c)+b*tan(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` for
more details)Is 4*a^2-4*b^2 positive or negative?
```

mupad [B] time = 5.44, size = 7329, normalized size = 30.16

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^3/(a*sin(c + d*x) + b*tan(c + d*x))^2,x)
```

```
[Out] ((a^2 - 2*a*b + b^2)/(a + b) + (2*tan(c/2 + (d*x)/2)^2*(2*a*b^4 + 3*a^4*b +
 2*a^5 + 4*b^5 - 3*a^2*b^3 - 6*a^3*b^2))/(a^2*(a + b)^2) - (tan(c/2 + (d*x)
/2)^4*(4*a*b^4 - 7*a^4*b + 5*a^5 - 8*b^5 + 7*a^2*b^3 - 5*a^3*b^2))/(a^2*(a
+ b)^2))/(d*(tan(c/2 + (d*x)/2)^5*(6*a*b^2 - 6*a^2*b + 2*a^3 - 2*b^3) - tan
(c/2 + (d*x)/2)^3*(4*a^2*b - 8*a*b^2 + 4*b^3) + tan(c/2 + (d*x)/2)*(2*a*b^2
 + 2*a^2*b - 2*a^3 - 2*b^3))) - tan(c/2 + (d*x)/2)/(2*d*(a - b)^2) + (4*b*a
tan((1920*a^7*b^22*tan(c/2 + (d*x)/2))/(1920*a^7*b^22 - 1920*a^8*b^21 - 166
40*a^9*b^20 + 16640*a^10*b^19 + 62080*a^11*b^18 - 62080*a^12*b^17 - 131072*
a^13*b^16 + 131072*a^14*b^15 + 172672*a^15*b^14 - 172672*a^16*b^13 - 147200
*a^17*b^12 + 147200*a^18*b^11 + 81280*a^19*b^10 - 81280*a^20*b^9 - 28160*a^
21*b^8 + 28160*a^22*b^7 + 5632*a^23*b^6 - 5632*a^24*b^5 - 512*a^25*b^4 + 51
2*a^26*b^3) - (1920*a^8*b^21*tan(c/2 + (d*x)/2))/(1920*a^7*b^22 - 1920*a^8*
b^21 - 16640*a^9*b^20 + 16640*a^10*b^19 + 62080*a^11*b^18 - 62080*a^12*b^17
 - 131072*a^13*b^16 + 131072*a^14*b^15 + 172672*a^15*b^14 - 172672*a^16*b^1
3 - 147200*a^17*b^12 + 147200*a^18*b^11 + 81280*a^19*b^10 - 81280*a^20*b^9
- 28160*a^21*b^8 + 28160*a^22*b^7 + 5632*a^23*b^6 - 5632*a^24*b^5 - 512*a^2
5*b^4 + 512*a^26*b^3) - (16640*a^9*b^20*tan(c/2 + (d*x)/2))/(1920*a^7*b^22
- 1920*a^8*b^21 - 16640*a^9*b^20 + 16640*a^10*b^19 + 62080*a^11*b^18 - 6208
0*a^12*b^17 - 131072*a^13*b^16 + 131072*a^14*b^15 + 172672*a^15*b^14 - 1726
72*a^16*b^13 - 147200*a^17*b^12 + 147200*a^18*b^11 + 81280*a^19*b^10 - 8128
0*a^20*b^9 - 28160*a^21*b^8 + 28160*a^22*b^7 + 5632*a^23*b^6 - 5632*a^24*b^
5 - 512*a^25*b^4 + 512*a^26*b^3) + (16640*a^10*b^19*tan(c/2 + (d*x)/2))/(19
20*a^7*b^22 - 1920*a^8*b^21 - 16640*a^9*b^20 + 16640*a^10*b^19 + 62080*a^11
*b^18 - 62080*a^12*b^17 - 131072*a^13*b^16 + 131072*a^14*b^15 + 172672*a^15
*b^14 - 172672*a^16*b^13 - 147200*a^17*b^12 + 147200*a^18*b^11 + 81280*a^19
*b^10 - 81280*a^20*b^9 - 28160*a^21*b^8 + 28160*a^22*b^7 + 5632*a^23*b^6 -
5632*a^24*b^5 - 512*a^25*b^4 + 512*a^26*b^3) + (62080*a^11*b^18*tan(c/2 + (
d*x)/2))/(1920*a^7*b^22 - 1920*a^8*b^21 - 16640*a^9*b^20 + 16640*a^10*b^19
+ 62080*a^11*b^18 - 62080*a^12*b^17 - 131072*a^13*b^16 + 131072*a^14*b^15 +
```

172672*a^15*b^14 - 172672*a^16*b^13 - 147200*a^17*b^12 + 147200*a^18*b^11
+ 81280*a^19*b^10 - 81280*a^20*b^9 - 28160*a^21*b^8 + 28160*a^22*b^7 + 5632
*a^23*b^6 - 5632*a^24*b^5 - 512*a^25*b^4 + 512*a^26*b^3) - (62080*a^12*b^17
*tan(c/2 + (d*x)/2))/(1920*a^7*b^22 - 1920*a^8*b^21 - 16640*a^9*b^20 + 1664
0*a^10*b^19 + 62080*a^11*b^18 - 62080*a^12*b^17 - 131072*a^13*b^16 + 131072
*a^14*b^15 + 172672*a^15*b^14 - 172672*a^16*b^13 - 147200*a^17*b^12 + 14720
0*a^18*b^11 + 81280*a^19*b^10 - 81280*a^20*b^9 - 28160*a^21*b^8 + 28160*a^2
2*b^7 + 5632*a^23*b^6 - 5632*a^24*b^5 - 512*a^25*b^4 + 512*a^26*b^3) - (131
072*a^13*b^16*tan(c/2 + (d*x)/2))/(1920*a^7*b^22 - 1920*a^8*b^21 - 16640*a^
9*b^20 + 16640*a^10*b^19 + 62080*a^11*b^18 - 62080*a^12*b^17 - 131072*a^13*
b^16 + 131072*a^14*b^15 + 172672*a^15*b^14 - 172672*a^16*b^13 - 147200*a^17
*b^12 + 147200*a^18*b^11 + 81280*a^19*b^10 - 81280*a^20*b^9 - 28160*a^21*b^
8 + 28160*a^22*b^7 + 5632*a^23*b^6 - 5632*a^24*b^5 - 512*a^25*b^4 + 512*a^2
6*b^3) + (131072*a^14*b^15*tan(c/2 + (d*x)/2))/(1920*a^7*b^22 - 1920*a^8*b^
21 - 16640*a^9*b^20 + 16640*a^10*b^19 + 62080*a^11*b^18 - 62080*a^12*b^17 -
131072*a^13*b^16 + 131072*a^14*b^15 + 172672*a^15*b^14 - 172672*a^16*b^13
- 147200*a^17*b^12 + 147200*a^18*b^11 + 81280*a^19*b^10 - 81280*a^20*b^9 -
28160*a^21*b^8 + 28160*a^22*b^7 + 5632*a^23*b^6 - 5632*a^24*b^5 - 512*a^25*
b^4 + 512*a^26*b^3) + (172672*a^15*b^14*tan(c/2 + (d*x)/2))/(1920*a^7*b^22
- 1920*a^8*b^21 - 16640*a^9*b^20 + 16640*a^10*b^19 + 62080*a^11*b^18 - 6208
0*a^12*b^17 - 131072*a^13*b^16 + 131072*a^14*b^15 + 172672*a^15*b^14 - 1726
72*a^16*b^13 - 147200*a^17*b^12 + 147200*a^18*b^11 + 81280*a^19*b^10 - 8128
0*a^20*b^9 - 28160*a^21*b^8 + 28160*a^22*b^7 + 5632*a^23*b^6 - 5632*a^24*b^
5 - 512*a^25*b^4 + 512*a^26*b^3) - (172672*a^16*b^13*tan(c/2 + (d*x)/2))/(1
920*a^7*b^22 - 1920*a^8*b^21 - 16640*a^9*b^20 + 16640*a^10*b^19 + 62080*a^1
1*b^18 - 62080*a^12*b^17 - 131072*a^13*b^16 + 131072*a^14*b^15 + 172672*a^1
5*b^14 - 172672*a^16*b^13 - 147200*a^17*b^12 + 147200*a^18*b^11 + 81280*a^1
9*b^10 - 81280*a^20*b^9 - 28160*a^21*b^8 + 28160*a^22*b^7 + 5632*a^23*b^6 -
5632*a^24*b^5 - 512*a^25*b^4 + 512*a^26*b^3) - (147200*a^17*b^12*tan(c/2 +
(d*x)/2))/(1920*a^7*b^22 - 1920*a^8*b^21 - 16640*a^9*b^20 + 16640*a^10*b^1
9 + 62080*a^11*b^18 - 62080*a^12*b^17 - 131072*a^13*b^16 + 131072*a^14*b^15
+ 172672*a^15*b^14 - 172672*a^16*b^13 - 147200*a^17*b^12 + 147200*a^18*b^1
1 + 81280*a^19*b^10 - 81280*a^20*b^9 - 28160*a^21*b^8 + 28160*a^22*b^7 + 56
32*a^23*b^6 - 5632*a^24*b^5 - 512*a^25*b^4 + 512*a^26*b^3) + (147200*a^18*b
^11*tan(c/2 + (d*x)/2))/(1920*a^7*b^22 - 1920*a^8*b^21 - 16640*a^9*b^20 + 1
6640*a^10*b^19 + 62080*a^11*b^18 - 62080*a^12*b^17 - 131072*a^13*b^16 + 131
072*a^14*b^15 + 172672*a^15*b^14 - 172672*a^16*b^13 - 147200*a^17*b^12 + 14
7200*a^18*b^11 + 81280*a^19*b^10 - 81280*a^20*b^9 - 28160*a^21*b^8 + 28160*
a^22*b^7 + 5632*a^23*b^6 - 5632*a^24*b^5 - 512*a^25*b^4 + 512*a^26*b^3) + (
81280*a^19*b^10*tan(c/2 + (d*x)/2))/(1920*a^7*b^22 - 1920*a^8*b^21 - 16640*
a^9*b^20 + 16640*a^10*b^19 + 62080*a^11*b^18 - 62080*a^12*b^17 - 131072*a^1
3*b^16 + 131072*a^14*b^15 + 172672*a^15*b^14 - 172672*a^16*b^13 - 147200*a^
17*b^12 + 147200*a^18*b^11 + 81280*a^19*b^10 - 81280*a^20*b^9 - 28160*a^21*
b^8 + 28160*a^22*b^7 + 5632*a^23*b^6 - 5632*a^24*b^5 - 512*a^25*b^4 + 512*a
^26*b^3) - (81280*a^20*b^9*tan(c/2 + (d*x)/2))/(1920*a^7*b^22 - 1920*a^8*b^

21 - 16640*a^9*b^20 + 16640*a^10*b^19 + 62080*a^11*b^18 - 62080*a^12*b^17 -
131072*a^13*b^16 + 131072*a^14*b^15 + 172672*a^15*b^14 - 172672*a^16*b^13
- 147200*a^17*b^12 + 147200*a^18*b^11 + 81280*a^19*b^10 - 81280*a^20*b^9 -
28160*a^21*b^8 + 28160*a^22*b^7 + 5632*a^23*b^6 - 5632*a^24*b^5 - 512*a^25*
b^4 + 512*a^26*b^3) - (28160*a^21*b^8*tan(c/2 + (d*x)/2))/(1920*a^7*b^22 -
1920*a^8*b^21 - 16640*a^9*b^20 + 16640*a^10*b^19 + 62080*a^11*b^18 - 62080*
a^12*b^17 - 131072*a^13*b^16 + 131072*a^14*b^15 + 172672*a^15*b^14 - 172672
*a^16*b^13 - 147200*a^17*b^12 + 147200*a^18*b^11 + 81280*a^19*b^10 - 81280*
a^20*b^9 - 28160*a^21*b^8 + 28160*a^22*b^7 + 5632*a^23*b^6 - 5632*a^24*b^5
- 512*a^25*b^4 + 512*a^26*b^3) + (28160*a^22*b^7*tan(c/2 + (d*x)/2))/(1920*
a^7*b^22 - 1920*a^8*b^21 - 16640*a^9*b^20 + 16640*a^10*b^19 + 62080*a^11*b^
18 - 62080*a^12*b^17 - 131072*a^13*b^16 + 131072*a^14*b^15 + 172672*a^15*b^
14 - 172672*a^16*b^13 - 147200*a^17*b^12 + 147200*a^18*b^11 + 81280*a^19*b^
10 - 81280*a^20*b^9 - 28160*a^21*b^8 + 28160*a^22*b^7 + 5632*a^23*b^6 - 563
2*a^24*b^5 - 512*a^25*b^4 + 512*a^26*b^3) + (5632*a^23*b^6*tan(c/2 + (d*x)/
2))/(1920*a^7*b^22 - 1920*a^8*b^21 - 16640*a^9*b^20 + 16640*a^10*b^19 + 620
80*a^11*b^18 - 62080*a^12*b^17 - 131072*a^13*b^16 + 131072*a^14*b^15 + 1726
72*a^15*b^14 - 172672*a^16*b^13 - 147200*a^17*b^12 + 147200*a^18*b^11 + 812
80*a^19*b^10 - 81280*a^20*b^9 - 28160*a^21*b^8 + 28160*a^22*b^7 + 5632*a^23
*b^6 - 5632*a^24*b^5 - 512*a^25*b^4 + 512*a^26*b^3) - (5632*a^24*b^5*tan(c/
2 + (d*x)/2))/(1920*a^7*b^22 - 1920*a^8*b^21 - 16640*a^9*b^20 + 16640*a^10*
b^19 + 62080*a^11*b^18 - 62080*a^12*b^17 - 131072*a^13*b^16 + 131072*a^14*b
^15 + 172672*a^15*b^14 - 172672*a^16*b^13 - 147200*a^17*b^12 + 147200*a^18*
b^11 + 81280*a^19*b^10 - 81280*a^20*b^9 - 28160*a^21*b^8 + 28160*a^22*b^7 +
5632*a^23*b^6 - 5632*a^24*b^5 - 512*a^25*b^4 + 512*a^26*b^3) - (512*a^25*b
^4*tan(c/2 + (d*x)/2))/(1920*a^7*b^22 - 1920*a^8*b^21 - 16640*a^9*b^20 + 16
640*a^10*b^19 + 62080*a^11*b^18 - 62080*a^12*b^17 - 131072*a^13*b^16 + 1310
72*a^14*b^15 + 172672*a^15*b^14 - 172672*a^16*b^13 - 147200*a^17*b^12 + 147
200*a^18*b^11 + 81280*a^19*b^10 - 81280*a^20*b^9 - 28160*a^21*b^8 + 28160*a
^22*b^7 + 5632*a^23*b^6 - 5632*a^24*b^5 - 512*a^25*b^4 + 512*a^26*b^3) + (5
12*a^26*b^3*tan(c/2 + (d*x)/2))/(1920*a^7*b^22 - 1920*a^8*b^21 - 16640*a^9*
b^20 + 16640*a^10*b^19 + 62080*a^11*b^18 - 62080*a^12*b^17 - 131072*a^13*b^
16 + 131072*a^14*b^15 + 172672*a^15*b^14 - 172672*a^16*b^13 - 147200*a^17*b
^12 + 147200*a^18*b^11 + 81280*a^19*b^10 - 81280*a^20*b^9 - 28160*a^21*b^8
+ 28160*a^22*b^7 + 5632*a^23*b^6 - 5632*a^24*b^5 - 512*a^25*b^4 + 512*a^26*
b^3)))/(a^3*d) + (b^4*atan(((b^4*(tan(c/2 + (d*x)/2)*(256*a^6*b^25 - 512*a^
7*b^24 - 2304*a^8*b^23 + 5120*a^9*b^22 + 8480*a^10*b^21 - 22560*a^11*b^20 -
15040*a^12*b^19 + 57280*a^13*b^18 + 7520*a^14*b^17 - 92000*a^15*b^16 + 220
16*a^16*b^15 + 96256*a^17*b^14 - 53920*a^18*b^13 - 64352*a^19*b^12 + 59840*
a^20*b^11 + 24640*a^21*b^10 - 39520*a^22*b^9 - 2720*a^23*b^8 + 16000*a^24*b
^7 - 1920*a^25*b^6 - 3712*a^26*b^5 + 896*a^27*b^4 + 384*a^28*b^3 - 128*a^29
b^2) - (b^4(5*a^2 - 2*b^2)*((a + b)^5*(a - b)^5)^(1/2)*(96*a^11*b^22 - 64
*a^10*b^23 - 64*a^32*b + 640*a^12*b^21 - 1120*a^13*b^20 - 2624*a^14*b^19 +
5568*a^15*b^18 + 5568*a^16*b^17 - 15744*a^17*b^16 - 5760*a^18*b^15 + 28224*
a^19*b^14 - 33600*a^21*b^12 + 8064*a^22*b^11 + 26880*a^23*b^10 - 11136*a^24

*b^9 - 14208*a^25*b^8 + 7872*a^26*b^7 + 4704*a^27*b^6 - 3200*a^28*b^5 - 864 *a^29*b^4 + 704*a^30*b^3 + 64*a^31*b^2 + (b^4*tan(c/2 + (d*x)/2)*(5*a^2 - 2 *b^2)*((a + b)^5*(a - b)^5)^(1/2)*(128*a^13*b^22 - 64*a^12*b^23 - 64*a^34*b + 576*a^14*b^21 - 1280*a^15*b^20 - 2240*a^16*b^19 + 5760*a^17*b^18 + 4800* a^18*b^17 - 15360*a^19*b^16 - 5760*a^20*b^15 + 26880*a^21*b^14 + 2688*a^22* b^13 - 32256*a^23*b^12 + 2688*a^24*b^11 + 26880*a^25*b^10 - 5760*a^26*b^9 - 15360*a^27*b^8 + 4800*a^28*b^7 + 5760*a^29*b^6 - 2240*a^30*b^5 - 1280*a^31 *b^4 + 576*a^32*b^3 + 128*a^33*b^2))/(a^13 - a^3*b^10 + 5*a^5*b^8 - 10*a^7* b^6 + 10*a^9*b^4 - 5*a^11*b^2)))/(a^13 - a^3*b^10 + 5*a^5*b^8 - 10*a^7*b^6 + 10*a^9*b^4 - 5*a^11*b^2))*(5*a^2 - 2*b^2)*((a + b)^5*(a - b)^5)^(1/2)*1i) /(a^13 - a^3*b^10 + 5*a^5*b^8 - 10*a^7*b^6 + 10*a^9*b^4 - 5*a^11*b^2) + (b^ 4*(tan(c/2 + (d*x)/2)*(256*a^6*b^25 - 512*a^7*b^24 - 2304*a^8*b^23 + 5120*a ^9*b^22 + 8480*a^10*b^21 - 22560*a^11*b^20 - 15040*a^12*b^19 + 57280*a^13*b ^18 + 7520*a^14*b^17 - 92000*a^15*b^16 + 22016*a^16*b^15 + 96256*a^17*b^14 - 53920*a^18*b^13 - 64352*a^19*b^12 + 59840*a^20*b^11 + 24640*a^21*b^10 - 3 9520*a^22*b^9 - 2720*a^23*b^8 + 16000*a^24*b^7 - 1920*a^25*b^6 - 3712*a^26* b^5 + 896*a^27*b^4 + 384*a^28*b^3 - 128*a^29*b^2) - (b^4*(5*a^2 - 2*b^2)*((a + b)^5*(a - b)^5)^(1/2)*(64*a^32*b + 64*a^10*b^23 - 96*a^11*b^22 - 640*a^ 12*b^21 + 1120*a^13*b^20 + 2624*a^14*b^19 - 5568*a^15*b^18 - 5568*a^16*b^17 + 15744*a^17*b^16 + 5760*a^18*b^15 - 28224*a^19*b^14 + 33600*a^21*b^12 - 8 064*a^22*b^11 - 26880*a^23*b^10 + 11136*a^24*b^9 + 14208*a^25*b^8 - 7872*a^ 26*b^7 - 4704*a^27*b^6 + 3200*a^28*b^5 + 864*a^29*b^4 - 704*a^30*b^3 - 64*a ^31*b^2 + (b^4*tan(c/2 + (d*x)/2)*(5*a^2 - 2*b^2)*((a + b)^5*(a - b)^5)^(1/ 2)*(128*a^13*b^22 - 64*a^12*b^23 - 64*a^34*b + 576*a^14*b^21 - 1280*a^15*b^ 20 - 2240*a^16*b^19 + 5760*a^17*b^18 + 4800*a^18*b^17 - 15360*a^19*b^16 - 5 760*a^20*b^15 + 26880*a^21*b^14 + 2688*a^22*b^13 - 32256*a^23*b^12 + 2688*a ^24*b^11 + 26880*a^25*b^10 - 5760*a^26*b^9 - 15360*a^27*b^8 + 4800*a^28*b^7 + 5760*a^29*b^6 - 2240*a^30*b^5 - 1280*a^31*b^4 + 576*a^32*b^3 + 128*a^33* b^2))/(a^13 - a^3*b^10 + 5*a^5*b^8 - 10*a^7*b^6 + 10*a^9*b^4 - 5*a^11*b^2)))/(a^13 - a^3*b^10 + 5*a^5*b^8 - 10*a^7*b^6 + 10*a^9*b^4 - 5*a^11*b^2))*(5* a^2 - 2*b^2)*((a + b)^5*(a - b)^5)^(1/2)*1i)/(a^13 - a^3*b^10 + 5*a^5*b^8 - 10*a^7*b^6 + 10*a^9*b^4 - 5*a^11*b^2))/(512*a^4*b^25 - 768*a^5*b^24 - 5120 *a^6*b^23 + 7040*a^7*b^22 + 22912*a^8*b^21 - 27904*a^9*b^20 - 61184*a^10*b^ 19 + 63872*a^11*b^18 + 108160*a^12*b^17 - 94720*a^13*b^16 - 131072*a^14*b^1 5 + 96128*a^15*b^14 + 108160*a^16*b^13 - 67840*a^17*b^12 - 58112*a^18*b^11 + 32384*a^19*b^10 + 18304*a^20*b^9 - 9472*a^21*b^8 - 2560*a^22*b^7 + 1280*a ^23*b^6 + (b^4*(tan(c/2 + (d*x)/2)*(256*a^6*b^25 - 512*a^7*b^24 - 2304*a^8* b^23 + 5120*a^9*b^22 + 8480*a^10*b^21 - 22560*a^11*b^20 - 15040*a^12*b^19 + 57280*a^13*b^18 + 7520*a^14*b^17 - 92000*a^15*b^16 + 22016*a^16*b^15 + 962 56*a^17*b^14 - 53920*a^18*b^13 - 64352*a^19*b^12 + 59840*a^20*b^11 + 24640* a^21*b^10 - 39520*a^22*b^9 - 2720*a^23*b^8 + 16000*a^24*b^7 - 1920*a^25*b^6 - 3712*a^26*b^5 + 896*a^27*b^4 + 384*a^28*b^3 - 128*a^29*b^2) - (b^4*(5*a^ 2 - 2*b^2)*((a + b)^5*(a - b)^5)^(1/2)*(96*a^11*b^22 - 64*a^10*b^23 - 64*a^ 32*b + 640*a^12*b^21 - 1120*a^13*b^20 - 2624*a^14*b^19 + 5568*a^15*b^18 + 5 568*a^16*b^17 - 15744*a^17*b^16 - 5760*a^18*b^15 + 28224*a^19*b^14 - 33600*

```
a^21*b^12 + 8064*a^22*b^11 + 26880*a^23*b^10 - 11136*a^24*b^9 - 14208*a^25*
b^8 + 7872*a^26*b^7 + 4704*a^27*b^6 - 3200*a^28*b^5 - 864*a^29*b^4 + 704*a^
30*b^3 + 64*a^31*b^2 + (b^4*tan(c/2 + (d*x)/2)*(5*a^2 - 2*b^2)*((a + b)^5*(
a - b)^5)^(1/2)*(128*a^13*b^22 - 64*a^12*b^23 - 64*a^34*b + 576*a^14*b^21 -
 1280*a^15*b^20 - 2240*a^16*b^19 + 5760*a^17*b^18 + 4800*a^18*b^17 - 15360*
a^19*b^16 - 5760*a^20*b^15 + 26880*a^21*b^14 + 2688*a^22*b^13 - 32256*a^23*
b^12 + 2688*a^24*b^11 + 26880*a^25*b^10 - 5760*a^26*b^9 - 15360*a^27*b^8 +
4800*a^28*b^7 + 5760*a^29*b^6 - 2240*a^30*b^5 - 1280*a^31*b^4 + 576*a^32*b^
3 + 128*a^33*b^2))/(a^13 - a^3*b^10 + 5*a^5*b^8 - 10*a^7*b^6 + 10*a^9*b^4 -
 5*a^11*b^2)))/(a^13 - a^3*b^10 + 5*a^5*b^8 - 10*a^7*b^6 + 10*a^9*b^4 - 5*a
^11*b^2))*(5*a^2 - 2*b^2)*((a + b)^5*(a - b)^5)^(1/2))/(a^13 - a^3*b^10 + 5
*a^5*b^8 - 10*a^7*b^6 + 10*a^9*b^4 - 5*a^11*b^2) - (b^4*(tan(c/2 + (d*x)/2)
*(256*a^6*b^25 - 512*a^7*b^24 - 2304*a^8*b^23 + 5120*a^9*b^22 + 8480*a^10*b
^21 - 22560*a^11*b^20 - 15040*a^12*b^19 + 57280*a^13*b^18 + 7520*a^14*b^17
- 92000*a^15*b^16 + 22016*a^16*b^15 + 96256*a^17*b^14 - 53920*a^18*b^13 - 6
4352*a^19*b^12 + 59840*a^20*b^11 + 24640*a^21*b^10 - 39520*a^22*b^9 - 2720*
a^23*b^8 + 16000*a^24*b^7 - 1920*a^25*b^6 - 3712*a^26*b^5 + 896*a^27*b^4 +
384*a^28*b^3 - 128*a^29*b^2) - (b^4*(5*a^2 - 2*b^2)*((a + b)^5*(a - b)^5)^(
1/2)*(64*a^32*b + 64*a^10*b^23 - 96*a^11*b^22 - 640*a^12*b^21 + 1120*a^13*b
^20 + 2624*a^14*b^19 - 5568*a^15*b^18 - 5568*a^16*b^17 + 15744*a^17*b^16 +
5760*a^18*b^15 - 28224*a^19*b^14 + 33600*a^21*b^12 - 8064*a^22*b^11 - 26880
*a^23*b^10 + 11136*a^24*b^9 + 14208*a^25*b^8 - 7872*a^26*b^7 - 4704*a^27*b^
6 + 3200*a^28*b^5 + 864*a^29*b^4 - 704*a^30*b^3 - 64*a^31*b^2 + (b^4*tan(c/
2 + (d*x)/2)*(5*a^2 - 2*b^2)*((a + b)^5*(a - b)^5)^(1/2)*(128*a^13*b^22 - 6
4*a^12*b^23 - 64*a^34*b + 576*a^14*b^21 - 1280*a^15*b^20 - 2240*a^16*b^19 +
 5760*a^17*b^18 + 4800*a^18*b^17 - 15360*a^19*b^16 - 5760*a^20*b^15 + 26880
*a^21*b^14 + 2688*a^22*b^13 - 32256*a^23*b^12 + 2688*a^24*b^11 + 26880*a^25
*b^10 - 5760*a^26*b^9 - 15360*a^27*b^8 + 4800*a^28*b^7 + 5760*a^29*b^6 - 22
40*a^30*b^5 - 1280*a^31*b^4 + 576*a^32*b^3 + 128*a^33*b^2))/(a^13 - a^3*b^1
0 + 5*a^5*b^8 - 10*a^7*b^6 + 10*a^9*b^4 - 5*a^11*b^2)))/(a^13 - a^3*b^10 +
5*a^5*b^8 - 10*a^7*b^6 + 10*a^9*b^4 - 5*a^11*b^2))*(5*a^2 - 2*b^2)*((a + b)
^5*(a - b)^5)^(1/2))/(a^13 - a^3*b^10 + 5*a^5*b^8 - 10*a^7*b^6 + 10*a^9*b^4
 - 5*a^11*b^2)))*(5*a^2 - 2*b^2)*((a + b)^5*(a - b)^5)^(1/2)*2i)/(d*(a^13 -
 a^3*b^10 + 5*a^5*b^8 - 10*a^7*b^6 + 10*a^9*b^4 - 5*a^11*b^2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3/(a*sin(d*x+c)+b*tan(d*x+c))**2,x)
```

```
[Out] Timed out
```

3.258 $\int \frac{\cos^2(c+dx)}{(a\sin(c+dx)+b\tan(c+dx))^2}\,dx$

Optimal. Leaf size=227

$$-\frac{2b^5\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^2d(a-b)^{5/2}(a+b)^{5/2}}+\frac{b^4\sin(c+dx)}{ad\left(a^2-b^2\right)^2(a\cos(c+dx)+b)}-\frac{4b^3\left(2a^2-b^2\right)\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^2d(a-b)^{5/2}(a+b)^{5/2}}-\frac{x}{a^2}-$$

```
[Out] -x/a^2-2*b^5*arctanh((a-b)^(1/2)*tan(1/2*d*x+1/2*c)/(a+b)^(1/2))/a^2/(a-b)^
(5/2)/(a+b)^(5/2)/d-4*b^3*(2*a^2-b^2)*arctanh((a-b)^(1/2)*tan(1/2*d*x+1/2*c
)/(a+b)^(1/2))/a^2/(a-b)^(5/2)/(a+b)^(5/2)/d-1/2*sin(d*x+c)/(a+b)^2/d/(1-co
s(d*x+c))+1/2*sin(d*x+c)/(a-b)^2/d/(1+cos(d*x+c))+b^4*sin(d*x+c)/a/(a^2-b^2
)^2/d/(b+a*cos(d*x+c))
```

Rubi [A] time = 0.49, antiderivative size = 227, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 7, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {4397, 2897, 2648, 2664, 12, 2659, 208}

$$\frac{b^4\sin(c+dx)}{ad\left(a^2-b^2\right)^2(a\cos(c+dx)+b)}-\frac{2b^5\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^2d(a-b)^{5/2}(a+b)^{5/2}}-\frac{4b^3\left(2a^2-b^2\right)\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^2d(a-b)^{5/2}(a+b)^{5/2}}-\frac{x}{a^2}-\frac{}{2}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2/(a*Sin[c + d*x] + b*Tan[c + d*x])^2,x]
```

```
[Out] -(x/a^2) - (2*b^5*ArcTanh[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/(a^2
*(a - b)^(5/2)*(a + b)^(5/2)*d) - (4*b^3*(2*a^2 - b^2)*ArcTanh[(Sqrt[a - b]
*Tan[(c + d*x)/2])/Sqrt[a + b]])/(a^2*(a - b)^(5/2)*(a + b)^(5/2)*d) - Sin[
c + d*x]/(2*(a + b)^2*d*(1 - Cos[c + d*x])) + Sin[c + d*x]/(2*(a - b)^2*d*(
1 + Cos[c + d*x])) + (b^4*Sin[c + d*x])/(a*(a^2 - b^2)^2*d*(b + a*Cos[c + d
*x]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2648

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> -Simp[Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 2664

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[
c + d*x]*(a + b*Sin[c + d*x])^(n + 1))/(d*(n + 1)*(a^2 - b^2)), x] + Dist[1
/((n + 1)*(a^2 - b^2)), Int[(a + b*Sin[c + d*x])^(n + 1)*Simp[a*(n + 1) - b
*(n + 2)*Sin[c + d*x], x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^
2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2897

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_)
 + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig[(d*sin[
e + f*x])^n*(a + b*sin[e + f*x])^m*(1 - sin[e + f*x]^2)^(p/2), x], x] /; Fr
eeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && IntegersQ[m, 2*n, p/2] && (
LtQ[m, -1] || (EqQ[m, -1] && GtQ[p, 0]))
```

Rule 4397

```
Int[u_, x_Symbol] :> Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]
```

Rubi steps

$$\int \frac{\cos^2(c+dx)}{(a\sin(c+dx)+b\tan(c+dx))^2}\,dx = \int \frac{\cos^2(c+dx)\cot^2(c+dx)}{(b+a\cos(c+dx))^2}\,dx$$

$$= \int \left(-\frac{1}{a^2} - \frac{1}{2(a-b)^2(-1-\cos(c+dx))} + \frac{1}{2(a+b)^2(1-\cos(c+dx))} + \frac{}{a^2\cdots}\right.$$

$$= -\frac{x}{a^2} - \frac{\int \frac{1}{-1-\cos(c+dx)}\,dx}{2(a-b)^2} + \frac{\int \frac{1}{1-\cos(c+dx)}\,dx}{2(a+b)^2} + \frac{b^4 \int \frac{1}{(-b-a\cos(c+dx))^2}\,dx}{a^2\left(a^2-b^2\right)} + \frac{(2\cdots}{}$$

$$= -\frac{x}{a^2} - \frac{\sin(c+dx)}{2(a+b)^2 d(1-\cos(c+dx))} + \frac{\sin(c+dx)}{2(a-b)^2 d(1+\cos(c+dx))} + \frac{}{a\left(a^2\cdots\right.}$$

$$= -\frac{x}{a^2} - \frac{4b^3\left(2a^2-b^2\right)\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^2(a-b)^{5/2}(a+b)^{5/2}d} - \frac{\sin(c+dx)}{2(a+b)^2 d(1-\cos(c+\cdots}$$

$$= -\frac{x}{a^2} - \frac{4b^3\left(2a^2-b^2\right)\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^2(a-b)^{5/2}(a+b)^{5/2}d} - \frac{\sin(c+dx)}{2(a+b)^2 d(1-\cos(c+\cdots}$$

$$= -\frac{x}{a^2} - \frac{2b^5\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^2(a-b)^{5/2}(a+b)^{5/2}d} - \frac{4b^3\left(2a^2-b^2\right)\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}\cdots\right.}{\sqrt{a+b}}\right)}{a^2(a-b)^{5/2}(a+b)^{5/2}d}$$

Mathematica [A] time = 2.08, size = 151, normalized size = 0.67

$$\frac{-\frac{4b^3\left(b^2-4a^2\right)\tanh^{-1}\left(\frac{(b-a)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{a^2\left(a^2-b^2\right)^{5/2}} - \frac{2(c+dx)}{a^2} + \frac{2b^4\sin(c+dx)}{a(a-b)^2(a+b)^2(a\cos(c+dx)+b)} + \frac{\tan\left(\frac{1}{2}(c+dx)\right)}{(a-b)^2} - \frac{\cot\left(\frac{1}{2}(c+dx)\right)}{(a+b)^2}}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2/(a*Sin[c + d*x] + b*Tan[c + d*x])^2,x]
```

```
[Out] ((-2*(c + d*x))/a^2 - (4*b^3*(-4*a^2 + b^2)*ArcTanh[((-a + b)*Tan[(c + d*x)
/2])/Sqrt[a^2 - b^2]])/(a^2*(a^2 - b^2)^(5/2)) - Cot[(c + d*x)/2]/(a + b)^2
 + (2*b^4*Sin[c + d*x])/(a*(a - b)^2*(a + b)^2*(b + a*Cos[c + d*x])) + Tan[
(c + d*x)/2]/(a - b)^2)/(2*d)
```

fricas [A] time = 0.74, size = 705, normalized size = 3.11

$$\left[\frac{4\,a^5b^2-2\,a^3b^4-2\,ab^6-\left(4\,a^2b^4-b^6+\left(4\,a^3b^3-ab^5\right)\cos\left(dx+c\right)\right)\sqrt{a^2-b^2}\,\log\left(\frac{2\,ab\cos(dx+c)-\left(a^2-2\,b^2\right)\cos(dx+c)^2+}{a^2\cos(dx+c)}\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(a*sin(d*x+c)+b*tan(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] [1/2*(4*a^5*b^2 - 2*a^3*b^4 - 2*a*b^6 - (4*a^2*b^4 - b^6 + (4*a^3*b^3 - a*b
^5)*cos(d*x + c))*sqrt(a^2 - b^2)*log((2*a*b*cos(d*x + c) - (a^2 - 2*b^2)*c
os(d*x + c)^2 + 2*sqrt(a^2 - b^2)*(b*cos(d*x + c) + a)*sin(d*x + c) + 2*a^2
 - b^2)/(a^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + b^2))*sin(d*x + c) - 2*(
a^7 - a*b^6)*cos(d*x + c)^2 + 2*(a^6*b - 2*a^4*b^3 + a^2*b^5)*cos(d*x + c)
- 2*((a^7 - 3*a^5*b^2 + 3*a^3*b^4 - a*b^6)*d*x*cos(d*x + c) + (a^6*b - 3*a^
4*b^3 + 3*a^2*b^5 - b^7)*d*x)*sin(d*x + c))/(((a^9 - 3*a^7*b^2 + 3*a^5*b^4
- a^3*b^6)*d*cos(d*x + c) + (a^8*b - 3*a^6*b^3 + 3*a^4*b^5 - a^2*b^7)*d)*si
n(d*x + c)), (2*a^5*b^2 - a^3*b^4 - a*b^6 - (4*a^2*b^4 - b^6 + (4*a^3*b^3 -
 a*b^5)*cos(d*x + c))*sqrt(-a^2 + b^2)*arctan(-sqrt(-a^2 + b^2)*(b*cos(d*x
+ c) + a)/((a^2 - b^2)*sin(d*x + c)))*sin(d*x + c) - (a^7 - a*b^6)*cos(d*x
+ c)^2 + (a^6*b - 2*a^4*b^3 + a^2*b^5)*cos(d*x + c) - ((a^7 - 3*a^5*b^2 + 3
*a^3*b^4 - a*b^6)*d*x*cos(d*x + c) + (a^6*b - 3*a^4*b^3 + 3*a^2*b^5 - b^7)*
d*x)*sin(d*x + c))/(((a^9 - 3*a^7*b^2 + 3*a^5*b^4 - a^3*b^6)*d*cos(d*x + c)
 + (a^8*b - 3*a^6*b^3 + 3*a^4*b^5 - a^2*b^7)*d)*sin(d*x + c))]
```

giac [A] time = 0.74, size = 331, normalized size = 1.46

$$\frac{\frac{4\left(4\,a^2b^3-b^5\right)\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(2\,a-2\,b)+\arctan\left(\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\sqrt{-a^2+b^2}}\right)\right)}{\left(a^6-2\,a^4b^2+a^2b^4\right)\sqrt{-a^2+b^2}}+\frac{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{a^2-2\,ab+b^2}-\frac{a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-3\,a^3b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+3\,a}{\left(a^5-2\,a^3b^2+ab^4\right)\left(a\tan\left(\frac{1}{2}\,a\right.\right.}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(a*sin(d*x+c)+b*tan(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/2*(4*(4*a^2*b^3 - b^5)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(2*a - 2*b) +
 arctan((a*tan(1/2*d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqrt(-a^2 + b^2))
)/((a^6 - 2*a^4*b^2 + a^2*b^4)*sqrt(-a^2 + b^2)) + tan(1/2*d*x + 1/2*c)/(a^
2 - 2*a*b + b^2) - (a^4*tan(1/2*d*x + 1/2*c)^2 - 3*a^3*b*tan(1/2*d*x + 1/2*
c)^2 + 3*a^2*b^2*tan(1/2*d*x + 1/2*c)^2 - a*b^3*tan(1/2*d*x + 1/2*c)^2 + 4*
b^4*tan(1/2*d*x + 1/2*c)^2 - a^4 + a^3*b + a^2*b^2 - a*b^3)/((a^5 - 2*a^3*b
```

```
^2 + a*b^4)*(a*tan(1/2*d*x + 1/2*c)^3 - b*tan(1/2*d*x + 1/2*c)^3 - a*tan(1/
2*d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))) - 2*(d*x + c)/a^2)/d
```

maple [A] time = 0.21, size = 255, normalized size = 1.12

$$\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2d\left(a^2-2ab+b^2\right)}-\frac{2b^4\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d\,(a+b)^2\,(a-b)^2\,a\left(\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a-b\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-a-b\right)}-\frac{8b^3\,\mathrm{arctanh}\left(\frac{\tan\left(\frac{dx}{2}+\right.}{\sqrt{(a+b)}}\right.}{d\,(a+b)^2\,(a-b)^2\,\sqrt{(a+}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2/(a*sin(d*x+c)+b*tan(d*x+c))^2,x)
```

```
[Out] 1/2/d/(a^2-2*a*b+b^2)*tan(1/2*d*x+1/2*c)-2/d*b^4/(a+b)^2/(a-b)^2/a*tan(1/2*
d*x+1/2*c)/(tan(1/2*d*x+1/2*c)^2*a-b*tan(1/2*d*x+1/2*c)^2-a-b)-8/d*b^3/(a+b
)^2/(a-b)^2/((a+b)*(a-b))^(1/2)*arctanh(tan(1/2*d*x+1/2*c)*(a-b)/((a+b)*(a-
b))^(1/2))+2/d*b^5/(a+b)^2/(a-b)^2/a^2/((a+b)*(a-b))^(1/2)*arctanh(tan(1/2*
d*x+1/2*c)*(a-b)/((a+b)*(a-b))^(1/2))-2/d/a^2*arctan(tan(1/2*d*x+1/2*c))-1/
2/d/(a+b)^2/tan(1/2*d*x+1/2*c)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(a*sin(d*x+c)+b*tan(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` for
more details)Is 4*a^2-4*b^2 positive or negative?
```

mupad [B] time = 5.25, size = 6093, normalized size = 26.84

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^2/(a*sin(c + d*x) + b*tan(c + d*x))^2,x)
```

```
[Out] ((a^2 - 2*a*b + b^2)/(a + b) - (tan(c/2 + (d*x)/2)^2*(a^4 - 3*a^3*b - a*b^3
 + 4*b^4 + 3*a^2*b^2))/(a*(a + b)^2))/(d*(tan(c/2 + (d*x)/2)^3*(6*a*b^2 - 6
*a^2*b + 2*a^3 - 2*b^3) + tan(c/2 + (d*x)/2)*(2*a*b^2 + 2*a^2*b - 2*a^3 - 2
*b^3))) - (2*atan(-((tan(c/2 + (d*x)/2)*(32*a^26 - 96*a^25*b - 64*a^3*b^23
+ 128*a^4*b^22 + 672*a^5*b^21 - 1376*a^6*b^20 - 3008*a^7*b^19 + 6528*a^8*b^
```

18 + 7072*a^9*b^17 - 17632*a^10*b^16 - 8480*a^11*b^15 + 29600*a^12*b^14 + 2
176*a^13*b^13 - 31744*a^14*b^12 + 8224*a^15*b^11 + 21344*a^16*b^10 - 12992*
a^17*b^9 - 8128*a^18*b^8 + 9568*a^19*b^7 + 992*a^20*b^6 - 4000*a^21*b^5 + 4
80*a^22*b^4 + 928*a^23*b^3 - 224*a^24*b^2) - ((32*a^28 - 32*a^27*b + 32*a^6
*b^22 - 416*a^8*b^20 + 224*a^9*b^19 + 2080*a^10*b^18 - 1824*a^11*b^17 - 547
2*a^12*b^16 + 6528*a^13*b^15 + 8256*a^14*b^14 - 13440*a^15*b^13 - 6720*a^16
*b^12 + 17472*a^17*b^11 + 1344*a^18*b^10 - 14784*a^19*b^9 + 2880*a^20*b^8 +
8064*a^21*b^7 - 3168*a^22*b^6 - 2688*a^23*b^5 + 1504*a^24*b^4 + 480*a^25*b
^3 - 352*a^26*b^2 - (tan(c/2 + (d*x)/2)*(128*a^8*b^22 - 64*a^7*b^23 - 64*a^
29*b + 576*a^9*b^21 - 1280*a^10*b^20 - 2240*a^11*b^19 + 5760*a^12*b^18 + 48
00*a^13*b^17 - 15360*a^14*b^16 - 5760*a^15*b^15 + 26880*a^16*b^14 + 2688*a^
17*b^13 - 32256*a^18*b^12 + 2688*a^19*b^11 + 26880*a^20*b^10 - 5760*a^21*b^
9 - 15360*a^22*b^8 + 4800*a^23*b^7 + 5760*a^24*b^6 - 2240*a^25*b^5 - 1280*a
^26*b^4 + 576*a^27*b^3 + 128*a^28*b^2)*1i)/a^2)*1i)/a^2)/a^2 + (tan(c/2 + (
d*x)/2)*(32*a^26 - 96*a^25*b - 64*a^3*b^23 + 128*a^4*b^22 + 672*a^5*b^21 -
1376*a^6*b^20 - 3008*a^7*b^19 + 6528*a^8*b^18 + 7072*a^9*b^17 - 17632*a^10*
b^16 - 8480*a^11*b^15 + 29600*a^12*b^14 + 2176*a^13*b^13 - 31744*a^14*b^12
+ 8224*a^15*b^11 + 21344*a^16*b^10 - 12992*a^17*b^9 - 8128*a^18*b^8 + 9568*
a^19*b^7 + 992*a^20*b^6 - 4000*a^21*b^5 + 480*a^22*b^4 + 928*a^23*b^3 - 224
*a^24*b^2) + ((32*a^28 - 32*a^27*b + 32*a^6*b^22 - 416*a^8*b^20 + 224*a^9*b
^19 + 2080*a^10*b^18 - 1824*a^11*b^17 - 5472*a^12*b^16 + 6528*a^13*b^15 + 8
256*a^14*b^14 - 13440*a^15*b^13 - 6720*a^16*b^12 + 17472*a^17*b^11 + 1344*a
^18*b^10 - 14784*a^19*b^9 + 2880*a^20*b^8 + 8064*a^21*b^7 - 3168*a^22*b^6 -
2688*a^23*b^5 + 1504*a^24*b^4 + 480*a^25*b^3 - 352*a^26*b^2 + (tan(c/2 + (
d*x)/2)*(128*a^8*b^22 - 64*a^7*b^23 - 64*a^29*b + 576*a^9*b^21 - 1280*a^10*
b^20 - 2240*a^11*b^19 + 5760*a^12*b^18 + 4800*a^13*b^17 - 15360*a^14*b^16 -
5760*a^15*b^15 + 26880*a^16*b^14 + 2688*a^17*b^13 - 32256*a^18*b^12 + 2688
*a^19*b^11 + 26880*a^20*b^10 - 5760*a^21*b^9 - 15360*a^22*b^8 + 4800*a^23*b
^7 + 5760*a^24*b^6 - 2240*a^25*b^5 - 1280*a^26*b^4 + 576*a^27*b^3 + 128*a^2
8*b^2)*1i)/a^2)*1i)/a^2)/a^2)/(64*a^2*b^22 - 192*a^3*b^21 - 640*a^4*b^20 +
1984*a^5*b^19 + 2624*a^6*b^18 - 8192*a^7*b^17 - 6400*a^8*b^16 + 18496*a^9*b
^15 + 11072*a^10*b^14 - 25856*a^11*b^13 - 14464*a^12*b^12 + 23872*a^13*b^11
+ 13760*a^14*b^10 - 15104*a^15*b^9 - 8704*a^16*b^8 + 6592*a^17*b^7 + 3200*
a^18*b^6 - 1856*a^19*b^5 - 512*a^20*b^4 + 256*a^21*b^3 - ((tan(c/2 + (d*x)/
2)*(32*a^26 - 96*a^25*b - 64*a^3*b^23 + 128*a^4*b^22 + 672*a^5*b^21 - 1376*
a^6*b^20 - 3008*a^7*b^19 + 6528*a^8*b^18 + 7072*a^9*b^17 - 17632*a^10*b^16
- 8480*a^11*b^15 + 29600*a^12*b^14 + 2176*a^13*b^13 - 31744*a^14*b^12 + 822
4*a^15*b^11 + 21344*a^16*b^10 - 12992*a^17*b^9 - 8128*a^18*b^8 + 9568*a^19*
b^7 + 992*a^20*b^6 - 4000*a^21*b^5 + 480*a^22*b^4 + 928*a^23*b^3 - 224*a^24
*b^2) - ((32*a^28 - 32*a^27*b + 32*a^6*b^22 - 416*a^8*b^20 + 224*a^9*b^19 +
2080*a^10*b^18 - 1824*a^11*b^17 - 5472*a^12*b^16 + 6528*a^13*b^15 + 8256*a
^14*b^14 - 13440*a^15*b^13 - 6720*a^16*b^12 + 17472*a^17*b^11 + 1344*a^18*b
^10 - 14784*a^19*b^9 + 2880*a^20*b^8 + 8064*a^21*b^7 - 3168*a^22*b^6 - 2688
*a^23*b^5 + 1504*a^24*b^4 + 480*a^25*b^3 - 352*a^26*b^2 - (tan(c/2 + (d*x)/
2)*(128*a^8*b^22 - 64*a^7*b^23 - 64*a^29*b + 576*a^9*b^21 - 1280*a^10*b^20

- 2240*a^11*b^19 + 5760*a^12*b^18 + 4800*a^13*b^17 - 15360*a^14*b^16 - 5760
*a^15*b^15 + 26880*a^16*b^14 + 2688*a^17*b^13 - 32256*a^18*b^12 + 2688*a^19
*b^11 + 26880*a^20*b^10 - 5760*a^21*b^9 - 15360*a^22*b^8 + 4800*a^23*b^7 +
5760*a^24*b^6 - 2240*a^25*b^5 - 1280*a^26*b^4 + 576*a^27*b^3 + 128*a^28*b^2
)*1i)/a^2)*1i)/a^2)*1i)/a^2 + ((tan(c/2 + (d*x)/2)*(32*a^26 - 96*a^25*b - 6
4*a^3*b^23 + 128*a^4*b^22 + 672*a^5*b^21 - 1376*a^6*b^20 - 3008*a^7*b^19 +
6528*a^8*b^18 + 7072*a^9*b^17 - 17632*a^10*b^16 - 8480*a^11*b^15 + 29600*a^
12*b^14 + 2176*a^13*b^13 - 31744*a^14*b^12 + 8224*a^15*b^11 + 21344*a^16*b^
10 - 12992*a^17*b^9 - 8128*a^18*b^8 + 9568*a^19*b^7 + 992*a^20*b^6 - 4000*a
^21*b^5 + 480*a^22*b^4 + 928*a^23*b^3 - 224*a^24*b^2) + ((32*a^28 - 32*a^27
*b + 32*a^6*b^22 - 416*a^8*b^20 + 224*a^9*b^19 + 2080*a^10*b^18 - 1824*a^11
*b^17 - 5472*a^12*b^16 + 6528*a^13*b^15 + 8256*a^14*b^14 - 13440*a^15*b^13
- 6720*a^16*b^12 + 17472*a^17*b^11 + 1344*a^18*b^10 - 14784*a^19*b^9 + 2880
*a^20*b^8 + 8064*a^21*b^7 - 3168*a^22*b^6 - 2688*a^23*b^5 + 1504*a^24*b^4 +
480*a^25*b^3 - 352*a^26*b^2 + (tan(c/2 + (d*x)/2)*(128*a^8*b^22 - 64*a^7*b
^23 - 64*a^29*b + 576*a^9*b^21 - 1280*a^10*b^20 - 2240*a^11*b^19 + 5760*a^1
2*b^18 + 4800*a^13*b^17 - 15360*a^14*b^16 - 5760*a^15*b^15 + 26880*a^16*b^1
4 + 2688*a^17*b^13 - 32256*a^18*b^12 + 2688*a^19*b^11 + 26880*a^20*b^10 - 5
760*a^21*b^9 - 15360*a^22*b^8 + 4800*a^23*b^7 + 5760*a^24*b^6 - 2240*a^25*b
^5 - 1280*a^26*b^4 + 576*a^27*b^3 + 128*a^28*b^2)*1i)/a^2)*1i)/a^2)*1i)/a^2
)))/(a^2*d) + tan(c/2 + (d*x)/2)/(2*d*(a - b)^2) + (b^3*atan(((b^3*(2*a + b
)*(tan(c/2 + (d*x)/2)*(32*a^26 - 96*a^25*b - 64*a^3*b^23 + 128*a^4*b^22 + 6
72*a^5*b^21 - 1376*a^6*b^20 - 3008*a^7*b^19 + 6528*a^8*b^18 + 7072*a^9*b^17
- 17632*a^10*b^16 - 8480*a^11*b^15 + 29600*a^12*b^14 + 2176*a^13*b^13 - 31
744*a^14*b^12 + 8224*a^15*b^11 + 21344*a^16*b^10 - 12992*a^17*b^9 - 8128*a^
18*b^8 + 9568*a^19*b^7 + 992*a^20*b^6 - 4000*a^21*b^5 + 480*a^22*b^4 + 928*
a^23*b^3 - 224*a^24*b^2) + (b^3*(2*a + b)*((a + b)^5*(a - b)^5)^(1/2)*(2*a
- b)*(32*a^28 - 32*a^27*b + 32*a^6*b^22 - 416*a^8*b^20 + 224*a^9*b^19 + 208
0*a^10*b^18 - 1824*a^11*b^17 - 5472*a^12*b^16 + 6528*a^13*b^15 + 8256*a^14*
b^14 - 13440*a^15*b^13 - 6720*a^16*b^12 + 17472*a^17*b^11 + 1344*a^18*b^10
- 14784*a^19*b^9 + 2880*a^20*b^8 + 8064*a^21*b^7 - 3168*a^22*b^6 - 2688*a^2
3*b^5 + 1504*a^24*b^4 + 480*a^25*b^3 - 352*a^26*b^2 + (b^3*tan(c/2 + (d*x)/
2)*(2*a + b)*((a + b)^5*(a - b)^5)^(1/2)*(2*a - b)*(128*a^8*b^22 - 64*a^7*b
^23 - 64*a^29*b + 576*a^9*b^21 - 1280*a^10*b^20 - 2240*a^11*b^19 + 5760*a^1
2*b^18 + 4800*a^13*b^17 - 15360*a^14*b^16 - 5760*a^15*b^15 + 26880*a^16*b^1
4 + 2688*a^17*b^13 - 32256*a^18*b^12 + 2688*a^19*b^11 + 26880*a^20*b^10 - 5
760*a^21*b^9 - 15360*a^22*b^8 + 4800*a^23*b^7 + 5760*a^24*b^6 - 2240*a^25*b
^5 - 1280*a^26*b^4 + 576*a^27*b^3 + 128*a^28*b^2))/(a^12 - a^2*b^10 + 5*a^4
*b^8 - 10*a^6*b^6 + 10*a^8*b^4 - 5*a^10*b^2)))/(a^12 - a^2*b^10 + 5*a^4*b^8
- 10*a^6*b^6 + 10*a^8*b^4 - 5*a^10*b^2))*((a + b)^5*(a - b)^5)^(1/2)*(2*a
- b)*1i)/(a^12 - a^2*b^10 + 5*a^4*b^8 - 10*a^6*b^6 + 10*a^8*b^4 - 5*a^10*b^
2) + (b^3*(2*a + b)*(tan(c/2 + (d*x)/2)*(32*a^26 - 96*a^25*b - 64*a^3*b^23
+ 128*a^4*b^22 + 672*a^5*b^21 - 1376*a^6*b^20 - 3008*a^7*b^19 + 6528*a^8*b^
18 + 7072*a^9*b^17 - 17632*a^10*b^16 - 8480*a^11*b^15 + 29600*a^12*b^14 + 2
176*a^13*b^13 - 31744*a^14*b^12 + 8224*a^15*b^11 + 21344*a^16*b^10 - 12992*

a^17*b^9 - 8128*a^18*b^8 + 9568*a^19*b^7 + 992*a^20*b^6 - 4000*a^21*b^5 + 480*a^22*b^4 + 928*a^23*b^3 - 224*a^24*b^2) - (b^3*(2*a + b)*((a + b)^5*(a - b)^5)^(1/2)*(2*a - b)*(32*a^28 - 32*a^27*b + 32*a^6*b^22 - 416*a^8*b^20 + 224*a^9*b^19 + 2080*a^10*b^18 - 1824*a^11*b^17 - 5472*a^12*b^16 + 6528*a^13*b^15 + 8256*a^14*b^14 - 13440*a^15*b^13 - 6720*a^16*b^12 + 17472*a^17*b^11 + 1344*a^18*b^10 - 14784*a^19*b^9 + 2880*a^20*b^8 + 8064*a^21*b^7 - 3168*a^22*b^6 - 2688*a^23*b^5 + 1504*a^24*b^4 + 480*a^25*b^3 - 352*a^26*b^2 - (b^3*tan(c/2 + (d*x)/2)*(2*a + b)*((a + b)^5*(a - b)^5)^(1/2)*(2*a - b)*(128*a^8*b^22 - 64*a^7*b^23 - 64*a^29*b + 576*a^9*b^21 - 1280*a^10*b^20 - 2240*a^11*b^19 + 5760*a^12*b^18 + 4800*a^13*b^17 - 15360*a^14*b^16 - 5760*a^15*b^15 + 26880*a^16*b^14 + 2688*a^17*b^13 - 32256*a^18*b^12 + 2688*a^19*b^11 + 26880*a^20*b^10 - 5760*a^21*b^9 - 15360*a^22*b^8 + 4800*a^23*b^7 + 5760*a^24*b^6 - 2240*a^25*b^5 - 1280*a^26*b^4 + 576*a^27*b^3 + 128*a^28*b^2))/(a^12 - a^2*b^10 + 5*a^4*b^8 - 10*a^6*b^6 + 10*a^8*b^4 - 5*a^10*b^2)))/(a^12 - a^2*b^10 + 5*a^4*b^8 - 10*a^6*b^6 + 10*a^8*b^4 - 5*a^10*b^2))*((a + b)^5*(a - b)^5)^(1/2)*(2*a - b)*1i)/(a^12 - a^2*b^10 + 5*a^4*b^8 - 10*a^6*b^6 + 10*a^8*b^4 - 5*a^10*b^2))/(64*a^2*b^22 - 192*a^3*b^21 - 640*a^4*b^20 + 1984*a^5*b^19 + 2624*a^6*b^18 - 8192*a^7*b^17 - 6400*a^8*b^16 + 18496*a^9*b^15 + 11072*a^10*b^14 - 25856*a^11*b^13 - 14464*a^12*b^12 + 23872*a^13*b^11 + 13760*a^14*b^10 - 15104*a^15*b^9 - 8704*a^16*b^8 + 6592*a^17*b^7 + 3200*a^18*b^6 - 1856*a^19*b^5 - 512*a^20*b^4 + 256*a^21*b^3 + (b^3*(2*a + b)*(tan(c/2 + (d*x)/2)*(32*a^26 - 96*a^25*b - 64*a^3*b^23 + 128*a^4*b^22 + 672*a^5*b^21 - 1376*a^6*b^20 - 3008*a^7*b^19 + 6528*a^8*b^18 + 7072*a^9*b^17 - 17632*a^10*b^16 - 8480*a^11*b^15 + 29600*a^12*b^14 + 2176*a^13*b^13 - 31744*a^14*b^12 + 8224*a^15*b^11 + 21344*a^16*b^10 - 12992*a^17*b^9 - 8128*a^18*b^8 + 9568*a^19*b^7 + 992*a^20*b^6 - 4000*a^21*b^5 + 480*a^22*b^4 + 928*a^23*b^3 - 224*a^24*b^2) + (b^3*(2*a + b)*((a + b)^5*(a - b)^5)^(1/2)*(2*a - b)*(32*a^28 - 32*a^27*b + 32*a^6*b^22 - 416*a^8*b^20 + 224*a^9*b^19 + 2080*a^10*b^18 - 1824*a^11*b^17 - 5472*a^12*b^16 + 6528*a^13*b^15 + 8256*a^14*b^14 - 13440*a^15*b^13 - 6720*a^16*b^12 + 17472*a^17*b^11 + 1344*a^18*b^10 - 14784*a^19*b^9 + 2880*a^20*b^8 + 8064*a^21*b^7 - 3168*a^22*b^6 - 2688*a^23*b^5 + 1504*a^24*b^4 + 480*a^25*b^3 - 352*a^26*b^2 + (b^3*tan(c/2 + (d*x)/2)*(2*a + b)*((a + b)^5*(a - b)^5)^(1/2)*(2*a - b)*(128*a^8*b^22 - 64*a^7*b^23 - 64*a^29*b + 576*a^9*b^21 - 1280*a^10*b^20 - 2240*a^11*b^19 + 5760*a^12*b^18 + 4800*a^13*b^17 - 15360*a^14*b^16 - 5760*a^15*b^15 + 26880*a^16*b^14 + 2688*a^17*b^13 - 32256*a^18*b^12 + 2688*a^19*b^11 + 26880*a^20*b^10 - 5760*a^21*b^9 - 15360*a^22*b^8 + 4800*a^23*b^7 + 5760*a^24*b^6 - 2240*a^25*b^5 - 1280*a^26*b^4 + 576*a^27*b^3 + 128*a^28*b^2))/(a^12 - a^2*b^10 + 5*a^4*b^8 - 10*a^6*b^6 + 10*a^8*b^4 - 5*a^10*b^2)))/(a^12 - a^2*b^10 + 5*a^4*b^8 - 10*a^6*b^6 + 10*a^8*b^4 - 5*a^10*b^2))*((a + b)^5*(a - b)^5)^(1/2)*(2*a - b))/(a^12 - a^2*b^10 + 5*a^4*b^8 - 10*a^6*b^6 + 10*a^8*b^4 - 5*a^10*b^2) - (b^3*(2*a + b)*(tan(c/2 + (d*x)/2)*(32*a^26 - 96*a^25*b - 64*a^3*b^23 + 128*a^4*b^22 + 672*a^5*b^21 - 1376*a^6*b^20 - 3008*a^7*b^19 + 6528*a^8*b^18 + 7072*a^9*b^17 - 17632*a^10*b^16 - 8480*a^11*b^15 + 29600*a^12*b^14 + 2176*a^13*b^13 - 31744*a^14*b^12 + 8224*a^15*b^11 + 21344*a^16*b^10 - 12992*a^17*b^9 - 8128*

```
a^18*b^8 + 9568*a^19*b^7 + 992*a^20*b^6 - 4000*a^21*b^5 + 480*a^22*b^4 + 92
8*a^23*b^3 - 224*a^24*b^2) - (b^3*(2*a + b)*((a + b)^5*(a - b)^5)^(1/2)*(2*
a - b)*(32*a^28 - 32*a^27*b + 32*a^6*b^22 - 416*a^8*b^20 + 224*a^9*b^19 + 2
080*a^10*b^18 - 1824*a^11*b^17 - 5472*a^12*b^16 + 6528*a^13*b^15 + 8256*a^1
4*b^14 - 13440*a^15*b^13 - 6720*a^16*b^12 + 17472*a^17*b^11 + 1344*a^18*b^1
0 - 14784*a^19*b^9 + 2880*a^20*b^8 + 8064*a^21*b^7 - 3168*a^22*b^6 - 2688*a
^23*b^5 + 1504*a^24*b^4 + 480*a^25*b^3 - 352*a^26*b^2 - (b^3*tan(c/2 + (d*x
)/2)*(2*a + b)*((a + b)^5*(a - b)^5)^(1/2)*(2*a - b)*(128*a^8*b^22 - 64*a^7
*b^23 - 64*a^29*b + 576*a^9*b^21 - 1280*a^10*b^20 - 2240*a^11*b^19 + 5760*a
^12*b^18 + 4800*a^13*b^17 - 15360*a^14*b^16 - 5760*a^15*b^15 + 26880*a^16*b
^14 + 2688*a^17*b^13 - 32256*a^18*b^12 + 2688*a^19*b^11 + 26880*a^20*b^10 -
 5760*a^21*b^9 - 15360*a^22*b^8 + 4800*a^23*b^7 + 5760*a^24*b^6 - 2240*a^25
*b^5 - 1280*a^26*b^4 + 576*a^27*b^3 + 128*a^28*b^2))/(a^12 - a^2*b^10 + 5*a
^4*b^8 - 10*a^6*b^6 + 10*a^8*b^4 - 5*a^10*b^2)))/(a^12 - a^2*b^10 + 5*a^4*b
^8 - 10*a^6*b^6 + 10*a^8*b^4 - 5*a^10*b^2))*((a + b)^5*(a - b)^5)^(1/2)*(2*
a - b))/(a^12 - a^2*b^10 + 5*a^4*b^8 - 10*a^6*b^6 + 10*a^8*b^4 - 5*a^10*b^2
)))*(2*a + b)*((a + b)^5*(a - b)^5)^(1/2)*(2*a - b)*2i)/(d*(a^12 - a^2*b^10
 + 5*a^4*b^8 - 10*a^6*b^6 + 10*a^8*b^4 - 5*a^10*b^2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos^2(c+dx)}{(a\sin(c+dx)+b\tan(c+dx))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2/(a*sin(d*x+c)+b*tan(d*x+c))**2,x)
```

```
[Out] Integral(cos(c + d*x)**2/(a*sin(c + d*x) + b*tan(c + d*x))**2, x)
```

3.259 $\int \frac{\cos(c+dx)}{(a\sin(c+dx)+b\tan(c+dx))^2}\,dx$

Optimal. Leaf size=219

$$\frac{2b^2\left(3a^2-b^2\right)\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{ad(a-b)^{5/2}(a+b)^{5/2}}-\frac{b^3\sin(c+dx)}{d\left(a^2-b^2\right)^2(a\cos(c+dx)+b)}+\frac{2b^4\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{ad(a-b)^{5/2}(a+b)^{5/2}}-\frac{}{2d(a}$$

```
[Out] 2*b^4*arctanh((a-b)^(1/2)*tan(1/2*d*x+1/2*c)/(a+b)^(1/2))/a/(a-b)^(5/2)/(a+
b)^(5/2)/d+2*b^2*(3*a^2-b^2)*arctanh((a-b)^(1/2)*tan(1/2*d*x+1/2*c)/(a+b)^(
1/2))/a/(a-b)^(5/2)/(a+b)^(5/2)/d-1/2*sin(d*x+c)/(a+b)^2/d/(1-cos(d*x+c))-1
/2*sin(d*x+c)/(a-b)^2/d/(1+cos(d*x+c))-b^3*sin(d*x+c)/(a^2-b^2)^2/d/(b+a*co
s(d*x+c))
```

Rubi [A] time = 0.38, antiderivative size = 219, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 7, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.269, Rules used = {4397, 2897, 2648, 2659, 208, 2664, 12}

$$-\frac{b^3\sin(c+dx)}{d\left(a^2-b^2\right)^2(a\cos(c+dx)+b)}+\frac{2b^2\left(3a^2-b^2\right)\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{ad(a-b)^{5/2}(a+b)^{5/2}}+\frac{2b^4\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{ad(a-b)^{5/2}(a+b)^{5/2}}-\frac{}{2d(}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]/(a*Sin[c + d*x] + b*Tan[c + d*x])^2,x]
```

```
[Out] (2*b^4*ArcTanh[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/(a*(a - b)^(5/2
)*(a + b)^(5/2)*d) + (2*b^2*(3*a^2 - b^2)*ArcTanh[(Sqrt[a - b]*Tan[(c + d*x
)/2])/Sqrt[a + b]])/(a*(a - b)^(5/2)*(a + b)^(5/2)*d) - Sin[c + d*x]/(2*(a
+ b)^2*d*(1 - Cos[c + d*x])) - Sin[c + d*x]/(2*(a - b)^2*d*(1 + Cos[c + d*x
])) - (b^3*Sin[c + d*x])/((a^2 - b^2)^2*d*(b + a*Cos[c + d*x]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2648

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> -Simp[Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 2664

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[
c + d*x]*(a + b*Sin[c + d*x])^(n + 1))/(d*(n + 1)*(a^2 - b^2)), x] + Dist[1
/((n + 1)*(a^2 - b^2)), Int[(a + b*Sin[c + d*x])^(n + 1)*Simp[a*(n + 1) - b
*(n + 2)*Sin[c + d*x], x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^
2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2897

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_)
 + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig[(d*sin[
e + f*x])^n*(a + b*sin[e + f*x])^m*(1 - sin[e + f*x]^2)^(p/2), x], x] /; Fr
eeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && IntegersQ[m, 2*n, p/2] && (
LtQ[m, -1] || (EqQ[m, -1] && GtQ[p, 0]))
```

Rule 4397

```
Int[u_, x_Symbol] :> Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]
```

Rubi steps

$$\int \frac{\cos(c+dx)}{(a\sin(c+dx)+b\tan(c+dx))^2}\,dx = \int \frac{\cos(c+dx)\cot^2(c+dx)}{(b+a\cos(c+dx))^2}\,dx$$

$$= -\int\left(-\frac{1}{2(a-b)^2(-1-\cos(c+dx))} - \frac{1}{2(a+b)^2(1-\cos(c+dx))} + \frac{\ }{a\,(a}\right.$$

$$= \frac{\int \frac{1}{-1-\cos(c+dx)}\,dx}{2(a-b)^2} + \frac{\int \frac{1}{1-\cos(c+dx)}\,dx}{2(a+b)^2} - \frac{b^3\int \frac{1}{(b+a\cos(c+dx))^2}\,dx}{a\left(a^2-b^2\right)} - \frac{\left(b^2\left(3a^2\right.\right.}{}$$

$$= -\frac{\sin(c+dx)}{2(a+b)^2d(1-\cos(c+dx))} - \frac{\sin(c+dx)}{2(a-b)^2d(1+\cos(c+dx))} - \frac{}{\left(a^2-b^2\right)}$$

$$= \frac{2b^2\left(3a^2-b^2\right)\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a(a-b)^{5/2}(a+b)^{5/2}d} - \frac{\sin(c+dx)}{2(a+b)^2d(1-\cos(c+dx))}$$

$$= \frac{2b^2\left(3a^2-b^2\right)\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a(a-b)^{5/2}(a+b)^{5/2}d} - \frac{\sin(c+dx)}{2(a+b)^2d(1-\cos(c+dx))}$$

$$= \frac{2b^4\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a(a-b)^{5/2}(a+b)^{5/2}d} + \frac{2b^2\left(3a^2-b^2\right)\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tan\left(\frac{1}{2}(c+dx)\right.}{\sqrt{a+b}}\right.}{a(a-b)^{5/2}(a+b)^{5/2}d}$$

Mathematica [A] time = 1.29, size = 131, normalized size = 0.60

$$\frac{\frac{\csc(c+dx)\left(\left(2a^2b+b^3\right)\cos(2(c+dx))-2a\left(a^2-b^2\right)\cos(c+dx)-3b^3\right)}{a\cos(c+dx)+b} - \frac{12ab^2\tanh^{-1}\left(\frac{(b-a)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}}}{2d(a-b)^2(a+b)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]/(a*Sin[c + d*x] + b*Tan[c + d*x])^2,x]
```

```
[Out] ((-12*a*b^2*ArcTanh[((-a + b)*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/Sqrt[a^2
- b^2] + ((-3*b^3 - 2*a*(a^2 - b^2)*Cos[c + d*x] + (2*a^2*b + b^3)*Cos[2*(c
 + d*x)])*Csc[c + d*x])/(b + a*Cos[c + d*x]))/(2*(a - b)^2*(a + b)^2*d)
```

fricas [A] time = 0.54, size = 518, normalized size = 2.37

$$\left[-\frac{2\,a^4b+2\,a^2b^3-4\,b^5-3\left(a^2b^2\cos\left(dx+c\right)+ab^3\right)\sqrt{a^2-b^2}\,\log\left(\frac{2\,ab\cos(dx+c)-\left(a^2-2\,b^2\right)\cos(dx+c)^2+2\,\sqrt{a^2-b^2}\,(b\cos(dx+c)}{a^2\cos(dx+c)^2+2\,ab\cos(dx+c)+b^2}\right.}{2\left(\left(a^7-3\,a^5b^2+3\,a^3b^4-ab^6\right)d\cos\left(dx+c\right)+\left(\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a*sin(d*x+c)+b*tan(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] [-1/2*(2*a^4*b + 2*a^2*b^3 - 4*b^5 - 3*(a^2*b^2*cos(d*x + c) + a*b^3)*sqrt(
a^2 - b^2)*log((2*a*b*cos(d*x + c) - (a^2 - 2*b^2)*cos(d*x + c)^2 + 2*sqrt(
a^2 - b^2)*(b*cos(d*x + c) + a)*sin(d*x + c) + 2*a^2 - b^2)/(a^2*cos(d*x +
c)^2 + 2*a*b*cos(d*x + c) + b^2))*sin(d*x + c) - 2*(2*a^4*b - a^2*b^3 - b^5
)*cos(d*x + c)^2 + 2*(a^5 - 2*a^3*b^2 + a*b^4)*cos(d*x + c))/(((a^7 - 3*a^5
*b^2 + 3*a^3*b^4 - a*b^6)*d*cos(d*x + c) + (a^6*b - 3*a^4*b^3 + 3*a^2*b^5 -
 b^7)*d)*sin(d*x + c)), -(a^4*b + a^2*b^3 - 2*b^5 - 3*(a^2*b^2*cos(d*x + c)
 + a*b^3)*sqrt(-a^2 + b^2)*arctan(-sqrt(-a^2 + b^2)*(b*cos(d*x + c) + a)/((
a^2 - b^2)*sin(d*x + c)))*sin(d*x + c) - (2*a^4*b - a^2*b^3 - b^5)*cos(d*x
+ c)^2 + (a^5 - 2*a^3*b^2 + a*b^4)*cos(d*x + c))/(((a^7 - 3*a^5*b^2 + 3*a^3
*b^4 - a*b^6)*d*cos(d*x + c) + (a^6*b - 3*a^4*b^3 + 3*a^2*b^5 - b^7)*d)*sin
(d*x + c))]
```

giac [A] time = 0.67, size = 282, normalized size = 1.29

$$-\frac{\frac{12\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(2\,a-2\,b)+\arctan\left(\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\sqrt{-a^2+b^2}}\right)\right)ab^2}{\left(a^4-2\,a^2b^2+b^4\right)\sqrt{-a^2+b^2}}+\frac{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{a^2-2\,ab+b^2}+\frac{a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-3\,a^2b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+3\,ab^2\tan(}{\left(a^4-2\,a^2b^2+b^4\right)\left(a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-b\tan\left(\frac{1}{2}\,dx\right.\right.}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a*sin(d*x+c)+b*tan(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/2*(12*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(2*a - 2*b) + arctan((a*tan(1
/2*d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqrt(-a^2 + b^2)))*a*b^2/((a^4 -
2*a^2*b^2 + b^4)*sqrt(-a^2 + b^2)) + tan(1/2*d*x + 1/2*c)/(a^2 - 2*a*b + b^
2) + (a^3*tan(1/2*d*x + 1/2*c)^2 - 3*a^2*b*tan(1/2*d*x + 1/2*c)^2 + 3*a*b^2
*tan(1/2*d*x + 1/2*c)^2 - 5*b^3*tan(1/2*d*x + 1/2*c)^2 - a^3 + a^2*b + a*b^
2 - b^3)/((a^4 - 2*a^2*b^2 + b^4)*(a*tan(1/2*d*x + 1/2*c)^3 - b*tan(1/2*d*x
 + 1/2*c)^3 - a*tan(1/2*d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))))/d
```

maple [A] time = 0.18, size = 155, normalized size = 0.71

$$\frac{-\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2\left(a^2-2ab+b^2\right)}-\frac{2b^2\left(-\frac{b\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a-b\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-a-b}-\frac{3a\,\mathrm{arctanh}\left(\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)(a-b)}{\sqrt{(a+b)(a-b)}}\right)}{\sqrt{(a+b)(a-b)}}\right)}{(a-b)^2(a+b)^2}-\frac{1}{2(a+b)^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)/(a*sin(d*x+c)+b*tan(d*x+c))^2,x)
```

```
[Out] 1/d*(-1/2/(a^2-2*a*b+b^2)*tan(1/2*d*x+1/2*c)-2*b^2/(a-b)^2/(a+b)^2*(-b*tan(
1/2*d*x+1/2*c)/(tan(1/2*d*x+1/2*c)^2*a-b*tan(1/2*d*x+1/2*c)^2-a-b)-3*a/((a+
b)*(a-b))^(1/2)*arctanh(tan(1/2*d*x+1/2*c)*(a-b)/((a+b)*(a-b))^(1/2)))-1/2/
(a+b)^2/tan(1/2*d*x+1/2*c))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a*sin(d*x+c)+b*tan(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` for
more details)Is 4*a^2-4*b^2 positive or negative?
```

mupad [B] time = 1.13, size = 213, normalized size = 0.97

$$\frac{\frac{a^2-2\,a\,b+b^2}{a+b}-\frac{{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2\,\left(a^3-3\,a^2\,b+3\,a\,b^2-5\,b^3\right)}{{(a+b)}^2}}{d\,\left(\left(2\,a^3-6\,a^2\,b+6\,a\,b^2-2\,b^3\right)\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^3+\left(-2\,a^3+2\,a^2\,b+2\,a\,b^2-2\,b^3\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}-\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{2\,d\,{(a-b)}^2}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)/(a*sin(c + d*x) + b*tan(c + d*x))^2,x)
```

```
[Out] ((a^2 - 2*a*b + b^2)/(a + b) - (tan(c/2 + (d*x)/2)^2*(3*a*b^2 - 3*a^2*b + a
^3 - 5*b^3))/(a + b)^2)/(d*(tan(c/2 + (d*x)/2)^3*(6*a*b^2 - 6*a^2*b + 2*a^3
 - 2*b^3) + tan(c/2 + (d*x)/2)*(2*a*b^2 + 2*a^2*b - 2*a^3 - 2*b^3))) - tan(
```

```
c/2 + (d*x)/2)/(2*d*(a - b)^2) + (6*a*b^2*atanh((tan(c/2 + (d*x)/2)*(a^4 +
b^4 - 2*a^2*b^2))/((a + b)^(5/2)*(a - b)^(3/2))))/(d*(a + b)^(5/2)*(a - b)^
(5/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos(c+dx)}{(a\sin(c+dx)+b\tan(c+dx))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a*sin(d*x+c)+b*tan(d*x+c))**2,x)
```

```
[Out] Integral(cos(c + d*x)/(a*sin(c + d*x) + b*tan(c + d*x))**2, x)
```

3.260 $\int \frac{1}{(a\sin(c+dx)+b\tan(c+dx))^2}\,dx$

Optimal. Leaf size=203

$$\frac{ab^2\sin(c+dx)}{d\left(a^2-b^2\right)^2(a\cos(c+dx)+b)}-\frac{4a^2b\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{d(a-b)^{5/2}(a+b)^{5/2}}-\frac{2b^3\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{d(a-b)^{5/2}(a+b)^{5/2}}-\frac{\sin(c+}{2d(a+b)^2(1-}$$

```
[Out] -4*a^2*b*arctanh((a-b)^(1/2)*tan(1/2*d*x+1/2*c)/(a+b)^(1/2))/(a-b)^(5/2)/(a
+b)^(5/2)/d-2*b^3*arctanh((a-b)^(1/2)*tan(1/2*d*x+1/2*c)/(a+b)^(1/2))/(a-b)
^(5/2)/(a+b)^(5/2)/d-1/2*sin(d*x+c)/(a+b)^2/d/(1-cos(d*x+c))+1/2*sin(d*x+c)
/(a-b)^2/d/(1+cos(d*x+c))+a*b^2*sin(d*x+c)/(a^2-b^2)^2/d/(b+a*cos(d*x+c))
```

Rubi [A] time = 0.40, antiderivative size = 203, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 7, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.368, Rules used = {4397, 2731, 2648, 2664, 12, 2659, 208}

$$\frac{ab^2\sin(c+dx)}{d\left(a^2-b^2\right)^2(a\cos(c+dx)+b)}-\frac{4a^2b\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{d(a-b)^{5/2}(a+b)^{5/2}}-\frac{2b^3\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{d(a-b)^{5/2}(a+b)^{5/2}}-\frac{\sin(c+}{2d(a+b)^2(1-}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Sin[c + d*x] + b*Tan[c + d*x])^(-2),x]
```

```
[Out] (-4*a^2*b*ArcTanh[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/((a - b)^(5/
2)*(a + b)^(5/2)*d) - (2*b^3*ArcTanh[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a
+ b]])/((a - b)^(5/2)*(a + b)^(5/2)*d) - Sin[c + d*x]/(2*(a + b)^2*d*(1 - C
os[c + d*x])) + Sin[c + d*x]/(2*(a - b)^2*d*(1 + Cos[c + d*x])) + (a*b^2*Si
n[c + d*x])/((a^2 - b^2)^2*d*(b + a*Cos[c + d*x]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2648

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> -Simp[Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 2664

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[
c + d*x]*(a + b*Sin[c + d*x])^(n + 1))/(d*(n + 1)*(a^2 - b^2)), x] + Dist[1
/((n + 1)*(a^2 - b^2)), Int[(a + b*Sin[c + d*x])^(n + 1)*Simp[a*(n + 1) - b
*(n + 2)*Sin[c + d*x], x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^
2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2731

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*tan[(e_.) + (f_.)*(x_)]^(p_
), x_Symbol] :> Int[ExpandIntegrand[(Sin[e + f*x]^p*(a + b*Sin[e + f*x])^m)
/(1 - Sin[e + f*x]^2)^(p/2), x], x] /; FreeQ[{a, b, e, f}, x] && NeQ[a^2 -
b^2, 0] && IntegersQ[m, p/2]
```

Rule 4397

```
Int[u_, x_Symbol] :> Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]
```

Rubi steps

$$\int \frac{1}{(a\sin(c+dx)+b\tan(c+dx))^2}\,dx = \int \frac{\cot^2(c+dx)}{(b+a\cos(c+dx))^2}\,dx$$

$$= \int \left(-\frac{1}{2(a+b)^2(-1+\cos(c+dx))} + \frac{1}{2(a-b)^2(1+\cos(c+dx))} - \frac{}{(-a^2}\right.$$

$$= \frac{\int \frac{1}{1+\cos(c+dx)}\,dx}{2(a-b)^2} - \frac{\int \frac{1}{-1+\cos(c+dx)}\,dx}{2(a+b)^2} - \frac{(2a^2b)\int \frac{1}{b+a\cos(c+dx)}\,dx}{(a^2-b^2)^2} + \frac{b^2\int}{}$$

$$= -\frac{\sin(c+dx)}{2(a+b)^2d(1-\cos(c+dx))} + \frac{\sin(c+dx)}{2(a-b)^2d(1+\cos(c+dx))} + \frac{}{(a^2-b^2}$$

$$= -\frac{4a^2b\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{(a-b)^{5/2}(a+b)^{5/2}d} - \frac{\sin(c+dx)}{2(a+b)^2d(1-\cos(c+dx))} + \frac{}{2(a-}$$

$$= -\frac{4a^2b\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{(a-b)^{5/2}(a+b)^{5/2}d} - \frac{\sin(c+dx)}{2(a+b)^2d(1-\cos(c+dx))} + \frac{}{2(a-}$$

$$= -\frac{4a^2b\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{(a-b)^{5/2}(a+b)^{5/2}d} - \frac{2b^3\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{(a-b)^{5/2}(a+b)^{5/2}d} - \frac{}{2(a}$$

Mathematica [A] time = 1.26, size = 128, normalized size = 0.63

$$\frac{\frac{4b(2a^2+b^2)\tanh^{-1}\left(\frac{(b-a)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{5/2}} + \frac{\frac{2ab^2\sin(c+dx)}{(a+b)^2(a\cos(c+dx)+b)}+\tan\left(\frac{1}{2}(c+dx)\right)}{(a-b)^2} - \frac{\cot\left(\frac{1}{2}(c+dx)\right)}{(a+b)^2}}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Sin[c + d*x] + b*Tan[c + d*x])^(-2),x]
```

```
[Out] ((4*b*(2*a^2 + b^2)*ArcTanh[((-a + b)*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(
a^2 - b^2)^(5/2) - Cot[(c + d*x)/2]/(a + b)^2 + ((2*a*b^2*Sin[c + d*x])/((a
 + b)^2*(b + a*Cos[c + d*x])) + Tan[(c + d*x)/2])/(a - b)^2)/(2*d)
```

fricas [A] time = 0.70, size = 526, normalized size = 2.59

$$\left[\frac{6\,a^3b^2-6\,ab^4+\left(2\,a^2b^2+b^4+\left(2\,a^3b+ab^3\right)\cos\left(dx+c\right)\right)\sqrt{a^2-b^2}\,\log\left(\frac{2\,ab\cos(dx+c)-\left(a^2-2\,b^2\right)\cos(dx+c)^2-2\,\sqrt{a^2-b^2}\,(b\,c\cdots}{a^2\cos(dx+c)^2+2\,ab\cos(dx\cdots}\right.}{2\left(\left(a^7-3\,a^5b^2+3\,a^3b^4-ab^6\right)d\cos\left(dx+c\right)+\cdots\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sin(d*x+c)+b*tan(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] [1/2*(6*a^3*b^2 - 6*a*b^4 + (2*a^2*b^2 + b^4 + (2*a^3*b + a*b^3)*cos(d*x +
c))*sqrt(a^2 - b^2)*log((2*a*b*cos(d*x + c) - (a^2 - 2*b^2)*cos(d*x + c)^2
- 2*sqrt(a^2 - b^2)*(b*cos(d*x + c) + a)*sin(d*x + c) + 2*a^2 - b^2)/(a^2*c
os(d*x + c)^2 + 2*a*b*cos(d*x + c) + b^2))*sin(d*x + c) - 2*(a^5 + a^3*b^2
- 2*a*b^4)*cos(d*x + c)^2 + 2*(a^4*b - 2*a^2*b^3 + b^5)*cos(d*x + c))/(((a^
7 - 3*a^5*b^2 + 3*a^3*b^4 - a*b^6)*d*cos(d*x + c) + (a^6*b - 3*a^4*b^3 + 3*
a^2*b^5 - b^7)*d)*sin(d*x + c)), (3*a^3*b^2 - 3*a*b^4 - (2*a^2*b^2 + b^4 +
(2*a^3*b + a*b^3)*cos(d*x + c))*sqrt(-a^2 + b^2)*arctan(-sqrt(-a^2 + b^2)*(
b*cos(d*x + c) + a)/((a^2 - b^2)*sin(d*x + c)))*sin(d*x + c) - (a^5 + a^3*b
^2 - 2*a*b^4)*cos(d*x + c)^2 + (a^4*b - 2*a^2*b^3 + b^5)*cos(d*x + c))/(((a
^7 - 3*a^5*b^2 + 3*a^3*b^4 - a*b^6)*d*cos(d*x + c) + (a^6*b - 3*a^4*b^3 + 3
*a^2*b^5 - b^7)*d)*sin(d*x + c))]
```

giac [A] time = 0.24, size = 289, normalized size = 1.42

$$\frac{\frac{4\left(2\,a^2b+b^3\right)\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(2\,a-2\,b)+\arctan\left(\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\sqrt{-a^2+b^2}}\right)\right)}{\left(a^4-2\,a^2b^2+b^4\right)\sqrt{-a^2+b^2}}+\frac{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{a^2-2\,ab+b^2}-\frac{a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-3\,a^2b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+7\,ab^2\cdots}{\left(a^4-2\,a^2b^2+b^4\right)\left(a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-b\tan\left(\cdots\right.\right.}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sin(d*x+c)+b*tan(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/2*(4*(2*a^2*b + b^3)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(2*a - 2*b) + a
rctan((a*tan(1/2*d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqrt(-a^2 + b^2)))/
((a^4 - 2*a^2*b^2 + b^4)*sqrt(-a^2 + b^2)) + tan(1/2*d*x + 1/2*c)/(a^2 - 2*
a*b + b^2) - (a^3*tan(1/2*d*x + 1/2*c)^2 - 3*a^2*b*tan(1/2*d*x + 1/2*c)^2 +
 7*a*b^2*tan(1/2*d*x + 1/2*c)^2 - b^3*tan(1/2*d*x + 1/2*c)^2 - a^3 + a^2*b
+ a*b^2 - b^3)/((a^4 - 2*a^2*b^2 + b^4)*(a*tan(1/2*d*x + 1/2*c)^3 - b*tan(1
/2*d*x + 1/2*c)^3 - a*tan(1/2*d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))))/d
```

maple [A] time = 0.18, size = 162, normalized size = 0.80

$$\frac{\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2a^2-4ab+2b^2}+\frac{2b\left(-\frac{ab\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a-b\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-a-b}-\frac{(2a^2+b^2)\operatorname{arctanh}\left(\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)(a-b)}{\sqrt{(a+b)(a-b)}}\right)}{\sqrt{(a+b)(a-b)}}\right)}{(a-b)^2(a+b)^2}-\frac{1}{2(a+b)^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*sin(d*x+c)+b*tan(d*x+c))^2,x)
```

```
[Out] 1/d*(1/2/(a^2-2*a*b+b^2)*tan(1/2*d*x+1/2*c)+2*b/(a-b)^2/(a+b)^2*(-a*b*tan(1
/2*d*x+1/2*c)/(tan(1/2*d*x+1/2*c)^2*a-b*tan(1/2*d*x+1/2*c)^2-a-b)-(2*a^2+b^
2)/((a+b)*(a-b))^(1/2)*arctanh(tan(1/2*d*x+1/2*c)*(a-b)/((a+b)*(a-b))^(1/2)
))-1/2/(a+b)^2/tan(1/2*d*x+1/2*c))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sin(d*x+c)+b*tan(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` for
more details)Is 4*a^2-4*b^2 positive or negative?
```

mupad [B] time = 1.19, size = 245, normalized size = 1.21

$$\frac{\frac{a^2-2\,a\,b+b^2}{a+b}-\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\,\left(a^3-3\,a^2\,b+7\,a\,b^2-b^3\right)}{(a+b)^2}}{d\,\left(\left(2\,a^3-6\,a^2\,b+6\,a\,b^2-2\,b^3\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3+\left(-2\,a^3+2\,a^2\,b+2\,a\,b^2-2\,b^3\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}+\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{2\,d\,(a-b)^2}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*sin(c + d*x) + b*tan(c + d*x))^2,x)
```

```
[Out] ((a^2 - 2*a*b + b^2)/(a + b) - (tan(c/2 + (d*x)/2)^2*(7*a*b^2 - 3*a^2*b + a
^3 - b^3))/(a + b)^2)/(d*(tan(c/2 + (d*x)/2)^3*(6*a*b^2 - 6*a^2*b + 2*a^3 -
 2*b^3) + tan(c/2 + (d*x)/2)*(2*a*b^2 + 2*a^2*b - 2*a^3 - 2*b^3))) + tan(c/
```

```
2 + (d*x)/2)/(2*d*(a - b)^2) + (b*atan((a^4*tan(c/2 + (d*x)/2)*1i + b^4*tan
(c/2 + (d*x)/2)*1i - a^2*b^2*tan(c/2 + (d*x)/2)*2i)/((a + b)^(5/2)*(a - b)^
(3/2)))*(2*a^2 + b^2)*2i)/(d*(a + b)^(5/2)*(a - b)^(5/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a\sin(c+dx)+b\tan(c+dx))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sin(d*x+c)+b*tan(d*x+c))**2,x)
```

```
[Out] Integral((a*sin(c + d*x) + b*tan(c + d*x))**(-2), x)
```

3.261 $\int \frac{\sec(c+dx)}{(a\sin(c+dx)+b\tan(c+dx))^2}\,dx$

Optimal. Leaf size=136

$$-\frac{\csc(c+dx)\left(a^2-3ab\cos(c+dx)+2b^2\right)}{d\left(a^2-b^2\right)^2}-\frac{b\csc(c+dx)}{d\left(a^2-b^2\right)(a\cos(c+dx)+b)}+\frac{2a\left(a^2+2b^2\right)\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tan\left(\frac{1}{2}(c+d\right.}{\sqrt{a+b}}\right.}{d(a-b)^{5/2}(a+b)^{5/2}}$$

```
[Out] 2*a*(a^2+2*b^2)*arctanh((a-b)^(1/2)*tan(1/2*d*x+1/2*c)/(a+b)^(1/2))/(a-b)^(
5/2)/(a+b)^(5/2)/d-b*csc(d*x+c)/(a^2-b^2)/d/(b+a*cos(d*x+c))-(a^2+2*b^2-3*a
*b*cos(d*x+c))*csc(d*x+c)/(a^2-b^2)^2/d
```

Rubi [A] time = 0.32, antiderivative size = 136, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {4397, 2864, 2866, 12, 2659, 208}

$$-\frac{\csc(c+dx)\left(a^2-3ab\cos(c+dx)+2b^2\right)}{d\left(a^2-b^2\right)^2}-\frac{b\csc(c+dx)}{d\left(a^2-b^2\right)(a\cos(c+dx)+b)}+\frac{2a\left(a^2+2b^2\right)\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tan\left(\frac{1}{2}(c+d\right.}{\sqrt{a+b}}\right.}{d(a-b)^{5/2}(a+b)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]/(a*Sin[c + d*x] + b*Tan[c + d*x])^2,x]
```

```
[Out] (2*a*(a^2 + 2*b^2)*ArcTanh[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/((a
 - b)^(5/2)*(a + b)^(5/2)*d) - (b*Csc[c + d*x])/((a^2 - b^2)*d*(b + a*Cos[c
 + d*x])) - ((a^2 + 2*b^2 - 3*a*b*Cos[c + d*x])*Csc[c + d*x])/((a^2 - b^2)^
2*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
```

```
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 2864

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> -Simp[((b*c
 - a*d)*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^(m + 1))/(f*g*(a^2 -
b^2)*(m + 1)), x] + Dist[1/((a^2 - b^2)*(m + 1)), Int[(g*Cos[e + f*x])^p*(a
 + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m + p +
2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[a^
2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rule 2866

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[((g*C
os[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^(m + 1)*(b*c - a*d - (a*c - b*d)*
Sin[e + f*x]))/(f*g*(a^2 - b^2)*(p + 1)), x] + Dist[1/(g^2*(a^2 - b^2)*(p +
 1)), Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^m*Simp[c*(a^2*(p +
2) - b^2*(m + p + 2)) + a*b*d*m + b*(a*c - b*d)*(m + p + 3)*Sin[e + f*x], x
], x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[a^2 - b^2, 0] && LtQ
[p, -1] && IntegerQ[2*m]
```

Rule 4397

```
Int[u_, x_Symbol] :> Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]
```

Rubi steps

$$\int \frac{\sec(c+dx)}{(a\sin(c+dx)+b\tan(c+dx))^2}\,dx = \int \frac{\cot(c+dx)\csc(c+dx)}{(b+a\cos(c+dx))^2}\,dx$$

$$= -\frac{b\csc(c+dx)}{(a^2-b^2)\,d(b+a\cos(c+dx))} - \frac{\int \frac{(-a+2b\cos(c+dx))\csc^2(c+dx)}{b+a\cos(c+dx)}dx}{a^2-b^2}$$

$$= -\frac{b\csc(c+dx)}{(a^2-b^2)\,d(b+a\cos(c+dx))} - \frac{(a^2+2b^2-3ab\cos(c+dx))\csc(c+}{(a^2-b^2)^2\,d}$$

$$= -\frac{b\csc(c+dx)}{(a^2-b^2)\,d(b+a\cos(c+dx))} - \frac{(a^2+2b^2-3ab\cos(c+dx))\csc(c+}{(a^2-b^2)^2\,d}$$

$$= -\frac{b\csc(c+dx)}{(a^2-b^2)\,d(b+a\cos(c+dx))} - \frac{(a^2+2b^2-3ab\cos(c+dx))\csc(c+}{(a^2-b^2)^2\,d}$$

$$= \frac{2a(a^2+2b^2)\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{(a-b)^{5/2}(a+b)^{5/2}d} - \frac{b\csc(c+dx)}{(a^2-b^2)\,d(b+a\cos(c+dx)}$$

Mathematica [A] time = 1.13, size = 127, normalized size = 0.93

$$-\frac{\frac{4a(a^2+2b^2)\tanh^{-1}\left(\frac{(b-a)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{5/2}} + \frac{\frac{2a^2b\sin(c+dx)}{(a+b)^2(a\cos(c+dx)+b)}+\tan\left(\frac{1}{2}(c+dx)\right)}{(a-b)^2} + \frac{\cot\left(\frac{1}{2}(c+dx)\right)}{(a+b)^2}}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]/(a*Sin[c + d*x] + b*Tan[c + d*x])^2,x]
```

```
[Out] -1/2*((4*a*(a^2 + 2*b^2)*ArcTanh[((-a + b)*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2
]])/(a^2 - b^2)^(5/2) + Cot[(c + d*x)/2]/(a + b)^2 + ((2*a^2*b*Sin[c + d*x]
)/((a + b)^2*(b + a*Cos[c + d*x])) + Tan[(c + d*x)/2])/(a - b)^2)/d
```

fricas [A] time = 0.88, size = 532, normalized size = 3.91

$$\left[-\frac{4a^4b-2a^2b^3-2b^5-\left(a^3b+2ab^3+\left(a^4+2a^2b^2\right)\cos(dx+c)\right)\sqrt{a^2-b^2}\,\log\left(\frac{2ab\cos(dx+c)-\left(a^2-2b^2\right)\cos(dx+c)^2+2}{a^2\cos(dx+c)^2-}\right.}{2\left(\left(a^7-3a^5b^2+3a^3b^4-ab^6\right)d\cos(dx+c\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a*sin(d*x+c)+b*tan(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] [-1/2*(4*a^4*b - 2*a^2*b^3 - 2*b^5 - (a^3*b + 2*a*b^3 + (a^4 + 2*a^2*b^2)*c
os(d*x + c))*sqrt(a^2 - b^2)*log((2*a*b*cos(d*x + c) - (a^2 - 2*b^2)*cos(d*
x + c)^2 + 2*sqrt(a^2 - b^2)*(b*cos(d*x + c) + a)*sin(d*x + c) + 2*a^2 - b^
2)/(a^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + b^2))*sin(d*x + c) - 6*(a^4*b
 - a^2*b^3)*cos(d*x + c)^2 + 2*(a^5 - 2*a^3*b^2 + a*b^4)*cos(d*x + c))/(((a
^7 - 3*a^5*b^2 + 3*a^3*b^4 - a*b^6)*d*cos(d*x + c) + (a^6*b - 3*a^4*b^3 + 3
*a^2*b^5 - b^7)*d)*sin(d*x + c)), -(2*a^4*b - a^2*b^3 - b^5 - (a^3*b + 2*a*
b^3 + (a^4 + 2*a^2*b^2)*cos(d*x + c))*sqrt(-a^2 + b^2)*arctan(-sqrt(-a^2 +
b^2)*(b*cos(d*x + c) + a)/((a^2 - b^2)*sin(d*x + c)))*sin(d*x + c) - 3*(a^4
*b - a^2*b^3)*cos(d*x + c)^2 + (a^5 - 2*a^3*b^2 + a*b^4)*cos(d*x + c))/(((a
^7 - 3*a^5*b^2 + 3*a^3*b^4 - a*b^6)*d*cos(d*x + c) + (a^6*b - 3*a^4*b^3 + 3
*a^2*b^5 - b^7)*d)*sin(d*x + c))]
```

giac [B] time = 0.50, size = 288, normalized size = 2.12

$$-\frac{\frac{4\left(a^3+2ab^2\right)\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(2a-2b)+\arctan\left(\frac{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{\sqrt{-a^2+b^2}}\right)\right)}{\left(a^4-2a^2b^2+b^4\right)\sqrt{-a^2+b^2}}+\frac{\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{a^2-2ab+b^2}+\frac{a^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-7a^2b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+3a\cdots}{\left(a^4-2a^2b^2+b^4\right)\left(a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3-b\tan\cdots\right.}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a*sin(d*x+c)+b*tan(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/2*(4*(a^3 + 2*a*b^2)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(2*a - 2*b) +
arctan((a*tan(1/2*d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqrt(-a^2 + b^2)))
/((a^4 - 2*a^2*b^2 + b^4)*sqrt(-a^2 + b^2)) + tan(1/2*d*x + 1/2*c)/(a^2 - 2
*a*b + b^2) + (a^3*tan(1/2*d*x + 1/2*c)^2 - 7*a^2*b*tan(1/2*d*x + 1/2*c)^2
+ 3*a*b^2*tan(1/2*d*x + 1/2*c)^2 - b^3*tan(1/2*d*x + 1/2*c)^2 - a^3 + a^2*b
 + a*b^2 - b^3)/((a^4 - 2*a^2*b^2 + b^4)*(a*tan(1/2*d*x + 1/2*c)^3 - b*tan(
1/2*d*x + 1/2*c)^3 - a*tan(1/2*d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))))/d
```

maple [A] time = 0.22, size = 162, normalized size = 1.19

$$\frac{-\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2\left(a^2-2ab+b^2\right)}-\frac{2a\left(-\frac{ab\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a-b\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-a-b}-\frac{\left(a^2+2b^2\right)\operatorname{arctanh}\left(\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)(a-b)}{\sqrt{(a+b)(a-b)}}\right)}{\sqrt{(a+b)(a-b)}}\right)}{(a-b)^2(a+b)^2}-\frac{1}{2(a+b)^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)/(a*sin(d*x+c)+b*tan(d*x+c))^2,x)
```

```
[Out] 1/d*(-1/2/(a^2-2*a*b+b^2)*tan(1/2*d*x+1/2*c)-2*a/(a-b)^2/(a+b)^2*(-a*b*tan(
1/2*d*x+1/2*c)/(tan(1/2*d*x+1/2*c)^2*a-b*tan(1/2*d*x+1/2*c)^2-a-b)-(a^2+2*b
^2)/((a+b)*(a-b))^(1/2)*arctanh(tan(1/2*d*x+1/2*c)*(a-b)/((a+b)*(a-b))^(1/2
)))-1/2/(a+b)^2/tan(1/2*d*x+1/2*c))
```

maxima [**F(-2)**] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a*sin(d*x+c)+b*tan(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` for
more details)Is 4*a^2-4*b^2 positive or negative?
```

mupad [**B**] time = 1.21, size = 245, normalized size = 1.80

$$\frac{\frac{a^2-2\,a\,b+b^2}{a+b}-\frac{{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2\,\left(a^3-7\,a^2\,b+3\,a\,b^2-b^3\right)}{{\left(a+b\right)}^2}}{d\,\left(\left(2\,a^3-6\,a^2\,b+6\,a\,b^2-2\,b^3\right)\,{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^3+\left(-2\,a^3+2\,a^2\,b+2\,a\,b^2-2\,b^3\right)\,\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}-\frac{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{2\,d\,{\left(a-b\right)}^2}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)*(a*sin(c + d*x) + b*tan(c + d*x))^2),x)
```

```
[Out] ((a^2 - 2*a*b + b^2)/(a + b) - (tan(c/2 + (d*x)/2)^2*(3*a*b^2 - 7*a^2*b + a
^3 - b^3))/(a + b)^2)/(d*(tan(c/2 + (d*x)/2)^3*(6*a*b^2 - 6*a^2*b + 2*a^3 -
 2*b^3) + tan(c/2 + (d*x)/2)*(2*a*b^2 + 2*a^2*b - 2*a^3 - 2*b^3))) - tan(c/
2 + (d*x)/2)/(2*d*(a - b)^2) - (a*atan((a^4*tan(c/2 + (d*x)/2)*1i + b^4*tan
(c/2 + (d*x)/2)*1i - a^2*b^2*tan(c/2 + (d*x)/2)*2i)/((a + b)^(5/2)*(a - b)^
(3/2)))*(a^2 + 2*b^2)*2i)/(d*(a + b)^(5/2)*(a - b)^(5/2))
```

sympy [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec{\left(c + dx \right)}}{\left(a \sin{\left(c + dx \right)} + b \tan{\left(c + dx \right)}\right)^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a*sin(d*x+c)+b*tan(d*x+c))**2,x)
[Out] Integral(sec(c + d*x)/(a*sin(c + d*x) + b*tan(c + d*x))**2, x)
```

3.262 $\int \frac{\sec^2(c+dx)}{(a\sin(c+dx)+b\tan(c+dx))^2}\,dx$

Optimal. Leaf size=131

$$\frac{a\csc(c+dx)}{d\left(a^2-b^2\right)(a\cos(c+dx)+b)}+\frac{\csc(c+dx)\left(3ab-\left(2a^2+b^2\right)\cos(c+dx)\right)}{d\left(a^2-b^2\right)^2}-\frac{6a^2b\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{d(a-b)^{5/2}(a+b)^{5/2}}$$

```
[Out] -6*a^2*b*arctanh((a-b)^(1/2)*tan(1/2*d*x+1/2*c)/(a+b)^(1/2))/(a-b)^(5/2)/(a
+b)^(5/2)/d+a*csc(d*x+c)/(a^2-b^2)/d/(b+a*cos(d*x+c))+(3*a*b-(2*a^2+b^2)*co
s(d*x+c))*csc(d*x+c)/(a^2-b^2)^2/d
```

Rubi [A] time = 0.33, antiderivative size = 131, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {4397, 2694, 2866, 12, 2659, 208}

$$\frac{a\csc(c+dx)}{d\left(a^2-b^2\right)(a\cos(c+dx)+b)}+\frac{\csc(c+dx)\left(3ab-\left(2a^2+b^2\right)\cos(c+dx)\right)}{d\left(a^2-b^2\right)^2}-\frac{6a^2b\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{d(a-b)^{5/2}(a+b)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^2/(a*Sin[c + d*x] + b*Tan[c + d*x])^2,x]
```

```
[Out] (-6*a^2*b*ArcTanh[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/((a - b)^(5/
2)*(a + b)^(5/2)*d) + (a*Csc[c + d*x])/((a^2 - b^2)*d*(b + a*Cos[c + d*x]))
 + ((3*a*b - (2*a^2 + b^2)*Cos[c + d*x])*Csc[c + d*x])/((a^2 - b^2)^2*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
```

```
&& NeQ[a^2 - b^2, 0]
```

Rule 2694

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*
x])^(m + 1))/(f*g*(a^2 - b^2)*(m + 1)), x] + Dist[1/((a^2 - b^2)*(m + 1)),
Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1)*(a*(m + 1) - b*(m + p +
 2)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f, g, p}, x] && NeQ[a^2 - b^2,
 0] && LtQ[m, -1] && IntegersQ[2*m, 2*p]
```

Rule 2866

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[((g*C
os[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^(m + 1)*(b*c - a*d - (a*c - b*d)*
Sin[e + f*x]))/(f*g*(a^2 - b^2)*(p + 1)), x] + Dist[1/(g^2*(a^2 - b^2)*(p +
 1)), Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^m*Simp[c*(a^2*(p +
2) - b^2*(m + p + 2)) + a*b*d*m + b*(a*c - b*d)*(m + p + 3)*Sin[e + f*x], x
], x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[a^2 - b^2, 0] && LtQ
[p, -1] && IntegerQ[2*m]
```

Rule 4397

```
Int[u_, x_Symbol] :> Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]
```

Rubi steps

$$\int \frac{\sec^2(c+dx)}{(a\sin(c+dx)+b\tan(c+dx))^2}\,dx = \int \frac{\csc^2(c+dx)}{(b+a\cos(c+dx))^2}\,dx$$

$$= \frac{a\csc(c+dx)}{\left(a^2-b^2\right)d(b+a\cos(c+dx))} + \frac{\int \frac{(-b+2a\cos(c+dx))\csc^2(c+dx)}{b+a\cos(c+dx)}\,dx}{a^2-b^2}$$

$$= \frac{a\csc(c+dx)}{\left(a^2-b^2\right)d(b+a\cos(c+dx))} + \frac{\left(3ab-\left(2a^2+b^2\right)\cos(c+dx)\right)\csc(c+}{\left(a^2-b^2\right)^2 d}$$

$$= \frac{a\csc(c+dx)}{\left(a^2-b^2\right)d(b+a\cos(c+dx))} + \frac{\left(3ab-\left(2a^2+b^2\right)\cos(c+dx)\right)\csc(c+}{\left(a^2-b^2\right)^2 d}$$

$$= \frac{a\csc(c+dx)}{\left(a^2-b^2\right)d(b+a\cos(c+dx))} + \frac{\left(3ab-\left(2a^2+b^2\right)\cos(c+dx)\right)\csc(c+}{\left(a^2-b^2\right)^2 d}$$

$$= -\frac{6a^2b\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{(a-b)^{5/2}(a+b)^{5/2}d} + \frac{a\csc(c+dx)}{\left(a^2-b^2\right)d(b+a\cos(c+dx))} + \frac{(3a}{}$$

Mathematica [A] time = 0.78, size = 121, normalized size = 0.92

$$\frac{\frac{\frac{2a^3\sin(c+dx)}{(a+b)^2(a\cos(c+dx)+b)}+\tan\left(\frac{1}{2}(c+dx)\right)}{(a-b)^2}+\frac{12a^2b\tanh^{-1}\left(\frac{(b-a)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{5/2}}-\frac{\cot\left(\frac{1}{2}(c+dx)\right)}{(a+b)^2}}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^2/(a*Sin[c + d*x] + b*Tan[c + d*x])^2,x]
```

```
[Out] ((12*a^2*b*ArcTanh[((-a + b)*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(a^2 - b^2
)^(5/2) - Cot[(c + d*x)/2]/(a + b)^2 + ((2*a^3*Sin[c + d*x])/((a + b)^2*(b
+ a*Cos[c + d*x])) + Tan[(c + d*x)/2])/(a - b)^2)/(2*d)
```

fricas [A] time = 0.71, size = 516, normalized size = 3.94

$$\left[\frac{2a^5+2a^3b^2-4ab^4+3\left(a^3b\cos(dx+c)+a^2b^2\right)\sqrt{a^2-b^2}\,\log\left(\frac{2ab\cos(dx+c)-\left(a^2-2b^2\right)\cos(dx+c)^2-2\sqrt{a^2-b^2}\,(b\cos(dx+c)}{a^2\cos(dx+c)^2+2ab\cos(dx+c)+b^2}\right.}{2\left(\left(a^7-3a^5b^2+3a^3b^4-ab^6\right)d\cos(dx+c)+\right(}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a*sin(d*x+c)+b*tan(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] [1/2*(2*a^5 + 2*a^3*b^2 - 4*a*b^4 + 3*(a^3*b*cos(d*x + c) + a^2*b^2)*sqrt(a
^2 - b^2)*log((2*a*b*cos(d*x + c) - (a^2 - 2*b^2)*cos(d*x + c)^2 - 2*sqrt(a
^2 - b^2)*(b*cos(d*x + c) + a)*sin(d*x + c) + 2*a^2 - b^2)/(a^2*cos(d*x + c
)^2 + 2*a*b*cos(d*x + c) + b^2))*sin(d*x + c) - 2*(2*a^5 - a^3*b^2 - a*b^4)
*cos(d*x + c)^2 + 2*(a^4*b - 2*a^2*b^3 + b^5)*cos(d*x + c))/(((a^7 - 3*a^5*
b^2 + 3*a^3*b^4 - a*b^6)*d*cos(d*x + c) + (a^6*b - 3*a^4*b^3 + 3*a^2*b^5 -
b^7)*d)*sin(d*x + c)), (a^5 + a^3*b^2 - 2*a*b^4 - 3*(a^3*b*cos(d*x + c) + a
^2*b^2)*sqrt(-a^2 + b^2)*arctan(-sqrt(-a^2 + b^2)*(b*cos(d*x + c) + a)/((a^
2 - b^2)*sin(d*x + c)))*sin(d*x + c) - (2*a^5 - a^3*b^2 - a*b^4)*cos(d*x +
c)^2 + (a^4*b - 2*a^2*b^3 + b^5)*cos(d*x + c))/(((a^7 - 3*a^5*b^2 + 3*a^3*b
^4 - a*b^6)*d*cos(d*x + c) + (a^6*b - 3*a^4*b^3 + 3*a^2*b^5 - b^7)*d)*sin(d
*x + c))]
```

giac [B] time = 0.56, size = 284, normalized size = 2.17

$$\frac{\frac{12\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(2a-2b)+\arctan\left(\frac{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{\sqrt{-a^2+b^2}}\right)\right)a^2b}{\left(a^4-2a^2b^2+b^4\right)\sqrt{-a^2+b^2}}+\frac{\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{a^2-2ab+b^2}-\frac{5a^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-3a^2b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+3ab^2\tan\left(\cdots\right.}{\left(a^4-2a^2b^2+b^4\right)\left(a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3-b\tan\left(\frac{1}{2}dx+\cdots\right.\right.}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a*sin(d*x+c)+b*tan(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/2*(12*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(2*a - 2*b) + arctan((a*tan(1/
2*d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqrt(-a^2 + b^2)))*a^2*b/((a^4 - 2
*a^2*b^2 + b^4)*sqrt(-a^2 + b^2)) + tan(1/2*d*x + 1/2*c)/(a^2 - 2*a*b + b^2
) - (5*a^3*tan(1/2*d*x + 1/2*c)^2 - 3*a^2*b*tan(1/2*d*x + 1/2*c)^2 + 3*a*b^
2*tan(1/2*d*x + 1/2*c)^2 - b^3*tan(1/2*d*x + 1/2*c)^2 - a^3 + a^2*b + a*b^2
 - b^3)/((a^4 - 2*a^2*b^2 + b^4)*(a*tan(1/2*d*x + 1/2*c)^3 - b*tan(1/2*d*x
+ 1/2*c)^3 - a*tan(1/2*d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))))/d
```

maple [A] time = 0.22, size = 155, normalized size = 1.18

$$\frac{\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2a^2-4ab+2b^2}+\frac{2a^2\left(-\frac{a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a-b\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-a-b}-\frac{3b\,\mathrm{arctanh}\left(\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)(a-b)}{\sqrt{(a+b)(a-b)}}\right)}{\sqrt{(a+b)(a-b)}}\right)}{(a-b)^2(a+b)^2}-\frac{1}{2(a+b)^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2/(a*sin(d*x+c)+b*tan(d*x+c))^2,x)
```

```
[Out] 1/d*(1/2/(a^2-2*a*b+b^2)*tan(1/2*d*x+1/2*c)+2*a^2/(a-b)^2/(a+b)^2*(-a*tan(1
/2*d*x+1/2*c)/(tan(1/2*d*x+1/2*c)^2*a-b*tan(1/2*d*x+1/2*c)^2-a-b)-3*b/((a+b
)*(a-b))^(1/2)*arctanh(tan(1/2*d*x+1/2*c)*(a-b)/((a+b)*(a-b))^(1/2)))-1/2/(
a+b)^2/tan(1/2*d*x+1/2*c))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a*sin(d*x+c)+b*tan(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` for
more details)Is 4*a^2-4*b^2 positive or negative?
```

mupad [B] time = 1.08, size = 215, normalized size = 1.64

$$\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{2\,d\,{\left(a-b\right)}^2}+\frac{\frac{a^2-2\,a\,b+b^2}{a+b}-\frac{{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2\,\left(5\,a^3-3\,a^2\,b+3\,a\,b^2-b^3\right)}{{\left(a+b\right)}^2}}{d\,\left(\left(2\,a^3-6\,a^2\,b+6\,a\,b^2-2\,b^3\right)\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^3+\left(-2\,a^3+2\,a^2\,b+2\,a\,b^2-2\,b^3\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^2*(a*sin(c + d*x) + b*tan(c + d*x))^2),x)
```

```
[Out] tan(c/2 + (d*x)/2)/(2*d*(a - b)^2) + ((a^2 - 2*a*b + b^2)/(a + b) - (tan(c/
2 + (d*x)/2)^2*(3*a*b^2 - 3*a^2*b + 5*a^3 - b^3))/(a + b)^2)/(d*(tan(c/2 +
(d*x)/2)^3*(6*a*b^2 - 6*a^2*b + 2*a^3 - 2*b^3) + tan(c/2 + (d*x)/2)*(2*a*b^
2 + 2*a^2*b - 2*a^3 - 2*b^3))) - (6*a^2*b*atanh((tan(c/2 + (d*x)/2)*(a^4 +
b^4 - 2*a^2*b^2))/((a + b)^(5/2)*(a - b)^(3/2))))/(d*(a + b)^(5/2)*(a - b)^
(5/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec^{2}{\left(c + dx \right)}}{\left(a \sin{\left(c + dx \right)} + b \tan{\left(c + dx \right)}\right)^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2/(a*sin(d*x+c)+b*tan(d*x+c))**2,x)
```

```
[Out] Integral(sec(c + d*x)**2/(a*sin(c + d*x) + b*tan(c + d*x))**2, x)
```

3.263 $\int \frac{\sec^3(c+dx)}{(a\sin(c+dx)+b\tan(c+dx))^2}\,dx$

Optimal. Leaf size=231

$$\frac{2a^3\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{d(a-b)^{5/2}(a+b)^{5/2}}-\frac{a^4\sin(c+dx)}{bd\left(a^2-b^2\right)^2(a\cos(c+dx)+b)}-\frac{2a^3\left(a^2-3b^2\right)\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{b^2d(a-b)^{5/2}(a+b)^{5/2}}-\frac{}{2d(a}$$

```
[Out] arctanh(sin(d*x+c))/b^2/d+2*a^3*arctanh((a-b)^(1/2)*tan(1/2*d*x+1/2*c)/(a+b
)^(1/2))/(a-b)^(5/2)/(a+b)^(5/2)/d-2*a^3*(a^2-3*b^2)*arctanh((a-b)^(1/2)*ta
n(1/2*d*x+1/2*c)/(a+b)^(1/2))/(a-b)^(5/2)/b^2/(a+b)^(5/2)/d-1/2*sin(d*x+c)/
(a+b)^2/d/(1-cos(d*x+c))-1/2*sin(d*x+c)/(a-b)^2/d/(1+cos(d*x+c))-a^4*sin(d*
x+c)/b/(a^2-b^2)^2/d/(b+a*cos(d*x+c))
```

Rubi [A] time = 0.44, antiderivative size = 231, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 8, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {4397, 2897, 2648, 2664, 12, 2659, 208, 3770}

$$-\frac{a^4\sin(c+dx)}{bd\left(a^2-b^2\right)^2(a\cos(c+dx)+b)}-\frac{2a^3\left(a^2-3b^2\right)\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{b^2d(a-b)^{5/2}(a+b)^{5/2}}+\frac{2a^3\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{d(a-b)^{5/2}(a+b)^{5/2}}-\frac{}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^3/(a*Sin[c + d*x] + b*Tan[c + d*x])^2,x]
```

```
[Out] ArcTanh[Sin[c + d*x]]/(b^2*d) + (2*a^3*ArcTanh[(Sqrt[a - b]*Tan[(c + d*x)/2
])/Sqrt[a + b]])/((a - b)^(5/2)*(a + b)^(5/2)*d) - (2*a^3*(a^2 - 3*b^2)*Arc
Tanh[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/((a - b)^(5/2)*b^2*(a + b
)^(5/2)*d) - Sin[c + d*x]/(2*(a + b)^2*d*(1 - Cos[c + d*x])) - Sin[c + d*x]
/(2*(a - b)^2*d*(1 + Cos[c + d*x])) - (a^4*Sin[c + d*x])/(b*(a^2 - b^2)^2*d
*(b + a*Cos[c + d*x]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2648

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> -Simp[Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 2664

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[
c + d*x]*(a + b*Sin[c + d*x])^(n + 1))/(d*(n + 1)*(a^2 - b^2)), x] + Dist[1
/((n + 1)*(a^2 - b^2)), Int[(a + b*Sin[c + d*x])^(n + 1)*Simp[a*(n + 1) - b
*(n + 2)*Sin[c + d*x], x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^
2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2897

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_)
 + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig[(d*sin[
e + f*x])^n*(a + b*sin[e + f*x])^m*(1 - sin[e + f*x]^2)^(p/2), x], x] /; Fr
eeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && IntegersQ[m, 2*n, p/2] && (
LtQ[m, -1] || (EqQ[m, -1] && GtQ[p, 0]))
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 4397

```
Int[u_, x_Symbol] :> Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]
```

Rubi steps

$$\int \frac{\sec^3(c+dx)}{(a\sin(c+dx)+b\tan(c+dx))^2}\,dx = \int \frac{\csc^2(c+dx)\sec(c+dx)}{(b+a\cos(c+dx))^2}\,dx$$

$$= -\int \left(-\frac{1}{2(a-b)^2(-1-\cos(c+dx))} - \frac{1}{2(a+b)^2(1-\cos(c+dx))} + \frac{}{b(a}\right.$$

$$= \frac{\int \frac{1}{-1-\cos(c+dx)}\,dx}{2(a-b)^2} + \frac{\int \sec(c+dx)\,dx}{b^2} + \frac{\int \frac{1}{1-\cos(c+dx)}\,dx}{2(a+b)^2} + \frac{\left(a^3\left(a^2-3b^2\right.\right.}{b^2}$$

$$= \frac{\tanh^{-1}(\sin(c+dx))}{b^2 d} - \frac{\sin(c+dx)}{2(a+b)^2 d(1-\cos(c+dx))} - \frac{\sin(c+d}{2(a-b)^2 d(1+\mathrm{co}}$$

$$= \frac{\tanh^{-1}(\sin(c+dx))}{b^2 d} - \frac{2a^3\left(a^2-3b^2\right)\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{(a-b)^{5/2}b^2(a+b)^{5/2}d} - \frac{}{2(a}$$

$$= \frac{\tanh^{-1}(\sin(c+dx))}{b^2 d} - \frac{2a^3\left(a^2-3b^2\right)\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{(a-b)^{5/2}b^2(a+b)^{5/2}d} - \frac{}{2(a}$$

$$= \frac{\tanh^{-1}(\sin(c+dx))}{b^2 d} + \frac{2a^3\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{(a-b)^{5/2}(a+b)^{5/2}d} - \frac{2a^3\left(a^2-3b^2\right)}{(a-b}$$

Mathematica [A] time = 1.97, size = 196, normalized size = 0.85

$$-\frac{\frac{2a^4\sin(c+dx)}{b(a-b)^2(a+b)^2(a\cos(c+dx)+b)} - \frac{4\left(a^5-4a^3b^2\right)\tanh^{-1}\left(\frac{(b-a)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{b^2\left(a^2-b^2\right)^{5/2}} + \frac{\tan\left(\frac{1}{2}(c+dx)\right)}{(a-b)^2} + \frac{\cot\left(\frac{1}{2}(c+dx)\right)}{(a+b)^2} + \frac{2\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+d\right.\right.}{b^2}}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^3/(a*Sin[c + d*x] + b*Tan[c + d*x])^2,x]
```

```
[Out] -1/2*((-4*(a^5 - 4*a^3*b^2)*ArcTanh[((-a + b)*Tan[(c + d*x)/2])/Sqrt[a^2 -
b^2]])/(b^2*(a^2 - b^2)^(5/2)) + Cot[(c + d*x)/2]/(a + b)^2 + (2*Log[Cos[(c
 + d*x)/2] - Sin[(c + d*x)/2]])/b^2 - (2*Log[Cos[(c + d*x)/2] + Sin[(c + d*
```

```
x)/2]])/b^2 + (2*a^4*Sin[c + d*x])/((a - b)^2*b*(a + b)^2*(b + a*Cos[c + d*
x])) + Tan[(c + d*x)/2]/(a - b)^2)/d
```

fricas [A] time = 1.66, size = 864, normalized size = 3.74

$$\left[-\frac{2\,a^{6}b-2\,b^{7}+\left(a^{5}b-4\,a^{3}b^{3}+\left(a^{6}-4\,a^{4}b^{2}\right)\cos\left(dx+c\right)\right)\sqrt{a^{2}-b^{2}}\,\log\left(\frac{2\,ab\cos(dx+c)-\left(a^{2}-2\,b^{2}\right)\cos(dx+c)^{2}+2\,\sqrt{a^{2}-b^{2}}\,(b\,c}{a^{2}\cos(dx+c)^{2}+2\,ab\cos(dx}\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a*sin(d*x+c)+b*tan(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] [-1/2*(2*a^6*b - 2*b^7 + (a^5*b - 4*a^3*b^3 + (a^6 - 4*a^4*b^2)*cos(d*x + c
))*sqrt(a^2 - b^2)*log((2*a*b*cos(d*x + c) - (a^2 - 2*b^2)*cos(d*x + c)^2 +
 2*sqrt(a^2 - b^2)*(b*cos(d*x + c) + a)*sin(d*x + c) + 2*a^2 - b^2)/(a^2*co
s(d*x + c)^2 + 2*a*b*cos(d*x + c) + b^2))*sin(d*x + c) - 2*(a^6*b + a^4*b^3
 - 2*a^2*b^5)*cos(d*x + c)^2 - (a^6*b - 3*a^4*b^3 + 3*a^2*b^5 - b^7 + (a^7
- 3*a^5*b^2 + 3*a^3*b^4 - a*b^6)*cos(d*x + c))*log(sin(d*x + c) + 1)*sin(d*
x + c) + (a^6*b - 3*a^4*b^3 + 3*a^2*b^5 - b^7 + (a^7 - 3*a^5*b^2 + 3*a^3*b^
4 - a*b^6)*cos(d*x + c))*log(-sin(d*x + c) + 1)*sin(d*x + c) + 2*(a^5*b^2 -
 2*a^3*b^4 + a*b^6)*cos(d*x + c))/(((a^7*b^2 - 3*a^5*b^4 + 3*a^3*b^6 - a*b^
8)*d*cos(d*x + c) + (a^6*b^3 - 3*a^4*b^5 + 3*a^2*b^7 - b^9)*d)*sin(d*x + c)
), -1/2*(2*a^6*b - 2*b^7 + 2*(a^5*b - 4*a^3*b^3 + (a^6 - 4*a^4*b^2)*cos(d*x
 + c))*sqrt(-a^2 + b^2)*arctan(-sqrt(-a^2 + b^2)*(b*cos(d*x + c) + a)/((a^2
 - b^2)*sin(d*x + c)))*sin(d*x + c) - 2*(a^6*b + a^4*b^3 - 2*a^2*b^5)*cos(d
*x + c)^2 - (a^6*b - 3*a^4*b^3 + 3*a^2*b^5 - b^7 + (a^7 - 3*a^5*b^2 + 3*a^3
*b^4 - a*b^6)*cos(d*x + c))*log(sin(d*x + c) + 1)*sin(d*x + c) + (a^6*b - 3
*a^4*b^3 + 3*a^2*b^5 - b^7 + (a^7 - 3*a^5*b^2 + 3*a^3*b^4 - a*b^6)*cos(d*x
+ c))*log(-sin(d*x + c) + 1)*sin(d*x + c) + 2*(a^5*b^2 - 2*a^3*b^4 + a*b^6)
*cos(d*x + c))/(((a^7*b^2 - 3*a^5*b^4 + 3*a^3*b^6 - a*b^8)*d*cos(d*x + c) +
 (a^6*b^3 - 3*a^4*b^5 + 3*a^2*b^7 - b^9)*d)*sin(d*x + c))]
```

giac [A] time = 0.68, size = 354, normalized size = 1.53

$$\frac{\frac{4\left(a^{5}-4\,a^{3}b^{2}\right)\left(\pi\left\lfloor\frac{dx+c}{2\,\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(2\,a-2\,b)+\arctan\left(\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\sqrt{-a^{2}+b^{2}}}\right)\right)}{\left(a^{4}b^{2}-2\,a^{2}b^{4}+b^{6}\right)\sqrt{-a^{2}+b^{2}}}-\frac{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{a^{2}-2\,ab+b^{2}}+\frac{4\,a^{4}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-a^{3}b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+3\,a^{2}b}{\left(a^{4}b-2\,a^{2}b^{3}+b^{5}\right)\left(a\tan\left(\frac{1}{2}\,dx\right.\right.}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a*sin(d*x+c)+b*tan(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/2*(4*(a^5 - 4*a^3*b^2)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(2*a - 2*b) +
 arctan((a*tan(1/2*d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqrt(-a^2 + b^2))
)/((a^4*b^2 - 2*a^2*b^4 + b^6)*sqrt(-a^2 + b^2)) - tan(1/2*d*x + 1/2*c)/(a^
2 - 2*a*b + b^2) + (4*a^4*tan(1/2*d*x + 1/2*c)^2 - a^3*b*tan(1/2*d*x + 1/2*
c)^2 + 3*a^2*b^2*tan(1/2*d*x + 1/2*c)^2 - 3*a*b^3*tan(1/2*d*x + 1/2*c)^2 +
b^4*tan(1/2*d*x + 1/2*c)^2 + a^3*b - a^2*b^2 - a*b^3 + b^4)/((a^4*b - 2*a^2
*b^3 + b^5)*(a*tan(1/2*d*x + 1/2*c)^3 - b*tan(1/2*d*x + 1/2*c)^3 - a*tan(1/
2*d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))) + 2*log(abs(tan(1/2*d*x + 1/2*c)
+ 1))/b^2 - 2*log(abs(tan(1/2*d*x + 1/2*c) - 1))/b^2)/d
```

maple [A] time = 0.24, size = 276, normalized size = 1.19

$$-\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2d\left(a^2-2ab+b^2\right)}-\frac{\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{d\,b^2}+\frac{2a^4\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d\,(a-b)^2\,b\,(a+b)^2\left(\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a-b\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-a-b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^3/(a*sin(d*x+c)+b*tan(d*x+c))^2,x)
```

```
[Out] -1/2/d/(a^2-2*a*b+b^2)*tan(1/2*d*x+1/2*c)-1/d/b^2*ln(tan(1/2*d*x+1/2*c)-1)+
2/d*a^4/(a-b)^2/b/(a+b)^2*tan(1/2*d*x+1/2*c)/(tan(1/2*d*x+1/2*c)^2*a-b*tan(
1/2*d*x+1/2*c)^2-a-b)-2/d*a^5/(a-b)^2/b^2/(a+b)^2/((a+b)*(a-b))^(1/2)*arcta
nh(tan(1/2*d*x+1/2*c)*(a-b)/((a+b)*(a-b))^(1/2))+8/d*a^3/(a-b)^2/(a+b)^2/((
a+b)*(a-b))^(1/2)*arctanh(tan(1/2*d*x+1/2*c)*(a-b)/((a+b)*(a-b))^(1/2))+1/d
/b^2*ln(tan(1/2*d*x+1/2*c)+1)-1/2/d/(a+b)^2/tan(1/2*d*x+1/2*c)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a*sin(d*x+c)+b*tan(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` for
more details)Is 4*a^2-4*b^2 positive or negative?
```

mupad [B] time = 5.21, size = 6056, normalized size = 26.22

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^3*(a*sin(c + d*x) + b*tan(c + d*x))^2),x)

[Out] ((a^2 - 2*a*b + b^2)/(a + b) + (tan(c/2 + (d*x)/2)^2*(4*a^4 - a^3*b - 3*a*b
^3 + b^4 + 3*a^2*b^2))/(b*(a + b)^2))/(d*(tan(c/2 + (d*x)/2)^3*(6*a*b^2 - 6
*a^2*b + 2*a^3 - 2*b^3) + tan(c/2 + (d*x)/2)*(2*a*b^2 + 2*a^2*b - 2*a^3 - 2
*b^3))) - (atan(-(((tan(c/2 + (d*x)/2)*(32*b^26 - 96*a*b^25 - 224*a^2*b^24
+ 928*a^3*b^23 + 480*a^4*b^22 - 4000*a^5*b^21 + 992*a^6*b^20 + 9568*a^7*b^1
9 - 8128*a^8*b^18 - 12992*a^9*b^17 + 21344*a^10*b^16 + 8224*a^11*b^15 - 317
44*a^12*b^14 + 2176*a^13*b^13 + 29600*a^14*b^12 - 8480*a^15*b^11 - 17632*a^
16*b^10 + 7072*a^17*b^9 + 6528*a^18*b^8 - 3008*a^19*b^7 - 1376*a^20*b^6 + 6
72*a^21*b^5 + 128*a^22*b^4 - 64*a^23*b^3) + (32*b^28 - 32*a*b^27 - 352*a^2*
b^26 + 480*a^3*b^25 + 1504*a^4*b^24 - 2688*a^5*b^23 - 3168*a^6*b^22 + 8064*
a^7*b^21 + 2880*a^8*b^20 - 14784*a^9*b^19 + 1344*a^10*b^18 + 17472*a^11*b^1
7 - 6720*a^12*b^16 - 13440*a^13*b^15 + 8256*a^14*b^14 + 6528*a^15*b^13 - 54
72*a^16*b^12 - 1824*a^17*b^11 + 2080*a^18*b^10 + 224*a^19*b^9 - 416*a^20*b^
8 + 32*a^22*b^6 - (tan(c/2 + (d*x)/2)*(128*a^2*b^28 - 64*a*b^29 + 576*a^3*b
^27 - 1280*a^4*b^26 - 2240*a^5*b^25 + 5760*a^6*b^24 + 4800*a^7*b^23 - 15360
*a^8*b^22 - 5760*a^9*b^21 + 26880*a^10*b^20 + 2688*a^11*b^19 - 32256*a^12*b
^18 + 2688*a^13*b^17 + 26880*a^14*b^16 - 5760*a^15*b^15 - 15360*a^16*b^14 +
 4800*a^17*b^13 + 5760*a^18*b^12 - 2240*a^19*b^11 - 1280*a^20*b^10 + 576*a^
21*b^9 + 128*a^22*b^8 - 64*a^23*b^7))/b^2)/b^2)*1i)/b^2 + ((tan(c/2 + (d*x)
/2)*(32*b^26 - 96*a*b^25 - 224*a^2*b^24 + 928*a^3*b^23 + 480*a^4*b^22 - 400
0*a^5*b^21 + 992*a^6*b^20 + 9568*a^7*b^19 - 8128*a^8*b^18 - 12992*a^9*b^17
+ 21344*a^10*b^16 + 8224*a^11*b^15 - 31744*a^12*b^14 + 2176*a^13*b^13 + 296
00*a^14*b^12 - 8480*a^15*b^11 - 17632*a^16*b^10 + 7072*a^17*b^9 + 6528*a^18
*b^8 - 3008*a^19*b^7 - 1376*a^20*b^6 + 672*a^21*b^5 + 128*a^22*b^4 - 64*a^2
3*b^3) - (32*b^28 - 32*a*b^27 - 352*a^2*b^26 + 480*a^3*b^25 + 1504*a^4*b^24
 - 2688*a^5*b^23 - 3168*a^6*b^22 + 8064*a^7*b^21 + 2880*a^8*b^20 - 14784*a^
9*b^19 + 1344*a^10*b^18 + 17472*a^11*b^17 - 6720*a^12*b^16 - 13440*a^13*b^1
5 + 8256*a^14*b^14 + 6528*a^15*b^13 - 5472*a^16*b^12 - 1824*a^17*b^11 + 208
0*a^18*b^10 + 224*a^19*b^9 - 416*a^20*b^8 + 32*a^22*b^6 + (tan(c/2 + (d*x)/
2)*(128*a^2*b^28 - 64*a*b^29 + 576*a^3*b^27 - 1280*a^4*b^26 - 2240*a^5*b^25
 + 5760*a^6*b^24 + 4800*a^7*b^23 - 15360*a^8*b^22 - 5760*a^9*b^21 + 26880*a
^10*b^20 + 2688*a^11*b^19 - 32256*a^12*b^18 + 2688*a^13*b^17 + 26880*a^14*b
^16 - 5760*a^15*b^15 - 15360*a^16*b^14 + 4800*a^17*b^13 + 5760*a^18*b^12 -
2240*a^19*b^11 - 1280*a^20*b^10 + 576*a^21*b^9 + 128*a^22*b^8 - 64*a^23*b^7
))/b^2)/b^2)*1i)/b^2)/(256*a^3*b^21 - 512*a^4*b^20 - 1856*a^5*b^19 + 3200*a
^6*b^18 + 6592*a^7*b^17 - 8704*a^8*b^16 - 15104*a^9*b^15 + 13760*a^10*b^14
+ 23872*a^11*b^13 - 14464*a^12*b^12 - 25856*a^13*b^11 + 11072*a^14*b^10 + 1
8496*a^15*b^9 - 6400*a^16*b^8 - 8192*a^17*b^7 + 2624*a^18*b^6 + 1984*a^19*b
^5 - 640*a^20*b^4 - 192*a^21*b^3 + 64*a^22*b^2 - (tan(c/2 + (d*x)/2)*(32*b^
26 - 96*a*b^25 - 224*a^2*b^24 + 928*a^3*b^23 + 480*a^4*b^22 - 4000*a^5*b^21
 + 992*a^6*b^20 + 9568*a^7*b^19 - 8128*a^8*b^18 - 12992*a^9*b^17 + 21344*a^
10*b^16 + 8224*a^11*b^15 - 31744*a^12*b^14 + 2176*a^13*b^13 + 29600*a^14*b^
12 - 8480*a^15*b^11 - 17632*a^16*b^10 + 7072*a^17*b^9 + 6528*a^18*b^8 - 300
```

8*a^19*b^7 - 1376*a^20*b^6 + 672*a^21*b^5 + 128*a^22*b^4 - 64*a^23*b^3) + (
32*b^28 - 32*a*b^27 - 352*a^2*b^26 + 480*a^3*b^25 + 1504*a^4*b^24 - 2688*a^
5*b^23 - 3168*a^6*b^22 + 8064*a^7*b^21 + 2880*a^8*b^20 - 14784*a^9*b^19 + 1
344*a^10*b^18 + 17472*a^11*b^17 - 6720*a^12*b^16 - 13440*a^13*b^15 + 8256*a
^14*b^14 + 6528*a^15*b^13 - 5472*a^16*b^12 - 1824*a^17*b^11 + 2080*a^18*b^1
0 + 224*a^19*b^9 - 416*a^20*b^8 + 32*a^22*b^6 - (tan(c/2 + (d*x)/2)*(128*a^
2*b^28 - 64*a*b^29 + 576*a^3*b^27 - 1280*a^4*b^26 - 2240*a^5*b^25 + 5760*a^
6*b^24 + 4800*a^7*b^23 - 15360*a^8*b^22 - 5760*a^9*b^21 + 26880*a^10*b^20 +
2688*a^11*b^19 - 32256*a^12*b^18 + 2688*a^13*b^17 + 26880*a^14*b^16 - 5760
*a^15*b^15 - 15360*a^16*b^14 + 4800*a^17*b^13 + 5760*a^18*b^12 - 2240*a^19*
b^11 - 1280*a^20*b^10 + 576*a^21*b^9 + 128*a^22*b^8 - 64*a^23*b^7))/b^2)/b^
2)/b^2 + (tan(c/2 + (d*x)/2)*(32*b^26 - 96*a*b^25 - 224*a^2*b^24 + 928*a^3*
b^23 + 480*a^4*b^22 - 4000*a^5*b^21 + 992*a^6*b^20 + 9568*a^7*b^19 - 8128*a
^8*b^18 - 12992*a^9*b^17 + 21344*a^10*b^16 + 8224*a^11*b^15 - 31744*a^12*b^
14 + 2176*a^13*b^13 + 29600*a^14*b^12 - 8480*a^15*b^11 - 17632*a^16*b^10 +
7072*a^17*b^9 + 6528*a^18*b^8 - 3008*a^19*b^7 - 1376*a^20*b^6 + 672*a^21*b^
5 + 128*a^22*b^4 - 64*a^23*b^3) - (32*b^28 - 32*a*b^27 - 352*a^2*b^26 + 480
*a^3*b^25 + 1504*a^4*b^24 - 2688*a^5*b^23 - 3168*a^6*b^22 + 8064*a^7*b^21 +
2880*a^8*b^20 - 14784*a^9*b^19 + 1344*a^10*b^18 + 17472*a^11*b^17 - 6720*a
^12*b^16 - 13440*a^13*b^15 + 8256*a^14*b^14 + 6528*a^15*b^13 - 5472*a^16*b^
12 - 1824*a^17*b^11 + 2080*a^18*b^10 + 224*a^19*b^9 - 416*a^20*b^8 + 32*a^2
2*b^6 + (tan(c/2 + (d*x)/2)*(128*a^2*b^28 - 64*a*b^29 + 576*a^3*b^27 - 1280
*a^4*b^26 - 2240*a^5*b^25 + 5760*a^6*b^24 + 4800*a^7*b^23 - 15360*a^8*b^22
- 5760*a^9*b^21 + 26880*a^10*b^20 + 2688*a^11*b^19 - 32256*a^12*b^18 + 2688
*a^13*b^17 + 26880*a^14*b^16 - 5760*a^15*b^15 - 15360*a^16*b^14 + 4800*a^17
*b^13 + 5760*a^18*b^12 - 2240*a^19*b^11 - 1280*a^20*b^10 + 576*a^21*b^9 + 1
28*a^22*b^8 - 64*a^23*b^7))/b^2)/b^2)/b^2))*2i)/(b^2*d) - tan(c/2 + (d*x)/2
)/(2*d*(a - b)^2) + (a^3*atan(((a^3*(a - 2*b)*(a + 2*b)*((a + b)^5*(a - b)^
5)^(1/2)*(tan(c/2 + (d*x)/2)*(32*b^26 - 96*a*b^25 - 224*a^2*b^24 + 928*a^3*
b^23 + 480*a^4*b^22 - 4000*a^5*b^21 + 992*a^6*b^20 + 9568*a^7*b^19 - 8128*a
^8*b^18 - 12992*a^9*b^17 + 21344*a^10*b^16 + 8224*a^11*b^15 - 31744*a^12*b^
14 + 2176*a^13*b^13 + 29600*a^14*b^12 - 8480*a^15*b^11 - 17632*a^16*b^10 +
7072*a^17*b^9 + 6528*a^18*b^8 - 3008*a^19*b^7 - 1376*a^20*b^6 + 672*a^21*b^
5 + 128*a^22*b^4 - 64*a^23*b^3) + (a^3*(a - 2*b)*(a + 2*b)*((a + b)^5*(a -
b)^5)^(1/2)*(32*b^28 - 32*a*b^27 - 352*a^2*b^26 + 480*a^3*b^25 + 1504*a^4*b
^24 - 2688*a^5*b^23 - 3168*a^6*b^22 + 8064*a^7*b^21 + 2880*a^8*b^20 - 14784
*a^9*b^19 + 1344*a^10*b^18 + 17472*a^11*b^17 - 6720*a^12*b^16 - 13440*a^13*
b^15 + 8256*a^14*b^14 + 6528*a^15*b^13 - 5472*a^16*b^12 - 1824*a^17*b^11 +
2080*a^18*b^10 + 224*a^19*b^9 - 416*a^20*b^8 + 32*a^22*b^6 - (a^3*tan(c/2 +
(d*x)/2)*(a - 2*b)*(a + 2*b)*((a + b)^5*(a - b)^5)^(1/2)*(128*a^2*b^28 - 6
4*a*b^29 + 576*a^3*b^27 - 1280*a^4*b^26 - 2240*a^5*b^25 + 5760*a^6*b^24 + 4
800*a^7*b^23 - 15360*a^8*b^22 - 5760*a^9*b^21 + 26880*a^10*b^20 + 2688*a^11
*b^19 - 32256*a^12*b^18 + 2688*a^13*b^17 + 26880*a^14*b^16 - 5760*a^15*b^15
- 15360*a^16*b^14 + 4800*a^17*b^13 + 5760*a^18*b^12 - 2240*a^19*b^11 - 128
0*a^20*b^10 + 576*a^21*b^9 + 128*a^22*b^8 - 64*a^23*b^7))/(b^12 - 5*a^2*b^1

0 + 10*a^4*b^8 - 10*a^6*b^6 + 5*a^8*b^4 - a^10*b^2)))/(b^12 - 5*a^2*b^10 + 10*a^4*b^8 - 10*a^6*b^6 + 5*a^8*b^4 - a^10*b^2))*1i)/(b^12 - 5*a^2*b^10 + 10*a^4*b^8 - 10*a^6*b^6 + 5*a^8*b^4 - a^10*b^2) + (a^3*(a - 2*b)*(a + 2*b)*((a + b)^5*(a - b)^5)^(1/2)*(tan(c/2 + (d*x)/2)*(32*b^26 - 96*a*b^25 - 224*a^2*b^24 + 928*a^3*b^23 + 480*a^4*b^22 - 4000*a^5*b^21 + 992*a^6*b^20 + 9568*a^7*b^19 - 8128*a^8*b^18 - 12992*a^9*b^17 + 21344*a^10*b^16 + 8224*a^11*b^15 - 31744*a^12*b^14 + 2176*a^13*b^13 + 29600*a^14*b^12 - 8480*a^15*b^11 - 17632*a^16*b^10 + 7072*a^17*b^9 + 6528*a^18*b^8 - 3008*a^19*b^7 - 1376*a^20*b^6 + 672*a^21*b^5 + 128*a^22*b^4 - 64*a^23*b^3) - (a^3*(a - 2*b)*(a + 2*b)*((a + b)^5*(a - b)^5)^(1/2)*(32*b^28 - 32*a*b^27 - 352*a^2*b^26 + 480*a^3*b^25 + 1504*a^4*b^24 - 2688*a^5*b^23 - 3168*a^6*b^22 + 8064*a^7*b^21 + 2880*a^8*b^20 - 14784*a^9*b^19 + 1344*a^10*b^18 + 17472*a^11*b^17 - 6720*a^12*b^16 - 13440*a^13*b^15 + 8256*a^14*b^14 + 6528*a^15*b^13 - 5472*a^16*b^12 - 1824*a^17*b^11 + 2080*a^18*b^10 + 224*a^19*b^9 - 416*a^20*b^8 + 32*a^22*b^6 + (a^3*tan(c/2 + (d*x)/2)*(a - 2*b)*(a + 2*b)*((a + b)^5*(a - b)^5)^(1/2)*(128*a^2*b^28 - 64*a*b^29 + 576*a^3*b^27 - 1280*a^4*b^26 - 2240*a^5*b^25 + 5760*a^6*b^24 + 4800*a^7*b^23 - 15360*a^8*b^22 - 5760*a^9*b^21 + 26880*a^10*b^20 + 2688*a^11*b^19 - 32256*a^12*b^18 + 2688*a^13*b^17 + 26880*a^14*b^16 - 5760*a^15*b^15 - 15360*a^16*b^14 + 4800*a^17*b^13 + 5760*a^18*b^12 - 2240*a^19*b^11 - 1280*a^20*b^10 + 576*a^21*b^9 + 128*a^22*b^8 - 64*a^23*b^7))/(b^12 - 5*a^2*b^10 + 10*a^4*b^8 - 10*a^6*b^6 + 5*a^8*b^4 - a^10*b^2)))/(b^12 - 5*a^2*b^10 + 10*a^4*b^8 - 10*a^6*b^6 + 5*a^8*b^4 - a^10*b^2))*1i)/(b^12 - 5*a^2*b^10 + 10*a^4*b^8 - 10*a^6*b^6 + 5*a^8*b^4 - a^10*b^2))/(256*a^3*b^21 - 512*a^4*b^20 - 1856*a^5*b^19 + 3200*a^6*b^18 + 6592*a^7*b^17 - 8704*a^8*b^16 - 15104*a^9*b^15 + 13760*a^10*b^14 + 23872*a^11*b^13 - 14464*a^12*b^12 - 25856*a^13*b^11 + 11072*a^14*b^10 + 18496*a^15*b^9 - 6400*a^16*b^8 - 8192*a^17*b^7 + 2624*a^18*b^6 + 1984*a^19*b^5 - 640*a^20*b^4 - 192*a^21*b^3 + 64*a^22*b^2 - (a^3*(a - 2*b)*(a + 2*b)*((a + b)^5*(a - b)^5)^(1/2)*(tan(c/2 + (d*x)/2)*(32*b^26 - 96*a*b^25 - 224*a^2*b^24 + 928*a^3*b^23 + 480*a^4*b^22 - 4000*a^5*b^21 + 992*a^6*b^20 + 9568*a^7*b^19 - 8128*a^8*b^18 - 12992*a^9*b^17 + 21344*a^10*b^16 + 8224*a^11*b^15 - 31744*a^12*b^14 + 2176*a^13*b^13 + 29600*a^14*b^12 - 8480*a^15*b^11 - 17632*a^16*b^10 + 7072*a^17*b^9 + 6528*a^18*b^8 - 3008*a^19*b^7 - 1376*a^20*b^6 + 672*a^21*b^5 + 128*a^22*b^4 - 64*a^23*b^3) + (a^3*(a - 2*b)*(a + 2*b)*((a + b)^5*(a - b)^5)^(1/2)*(32*b^28 - 32*a*b^27 - 352*a^2*b^26 + 480*a^3*b^25 + 1504*a^4*b^24 - 2688*a^5*b^23 - 3168*a^6*b^22 + 8064*a^7*b^21 + 2880*a^8*b^20 - 14784*a^9*b^19 + 1344*a^10*b^18 + 17472*a^11*b^17 - 6720*a^12*b^16 - 13440*a^13*b^15 + 8256*a^14*b^14 + 6528*a^15*b^13 - 5472*a^16*b^12 - 1824*a^17*b^11 + 2080*a^18*b^10 + 224*a^19*b^9 - 416*a^20*b^8 + 32*a^22*b^6 - (a^3*tan(c/2 + (d*x)/2)*(a - 2*b)*(a + 2*b)*((a + b)^5*(a - b)^5)^(1/2)*(128*a^2*b^28 - 64*a*b^29 + 576*a^3*b^27 - 1280*a^4*b^26 - 2240*a^5*b^25 + 5760*a^6*b^24 + 4800*a^7*b^23 - 15360*a^8*b^22 - 5760*a^9*b^21 + 26880*a^10*b^20 + 2688*a^11*b^19 - 32256*a^12*b^18 + 2688*a^13*b^17 + 26880*a^14*b^16 - 5760*a^15*b^15 - 15360*a^16*b^14 + 4800*a^17*b^13 + 5760*a^18*b^12 - 2240*a^19*b^11 - 1280*a^20*b^10 + 576*a^21*b^9 + 128*a^22*b^8 - 64*a^23*b^7))/(b^12 - 5*a^2*b^10 + 10*a^4*b^

```
8 - 10*a^6*b^6 + 5*a^8*b^4 - a^10*b^2)))/(b^12 - 5*a^2*b^10 + 10*a^4*b^8 -
10*a^6*b^6 + 5*a^8*b^4 - a^10*b^2)))/(b^12 - 5*a^2*b^10 + 10*a^4*b^8 - 10*a
^6*b^6 + 5*a^8*b^4 - a^10*b^2) + (a^3*(a - 2*b)*(a + 2*b)*((a + b)^5*(a - b
)^5)^(1/2)*(tan(c/2 + (d*x)/2)*(32*b^26 - 96*a*b^25 - 224*a^2*b^24 + 928*a^
3*b^23 + 480*a^4*b^22 - 4000*a^5*b^21 + 992*a^6*b^20 + 9568*a^7*b^19 - 8128
*a^8*b^18 - 12992*a^9*b^17 + 21344*a^10*b^16 + 8224*a^11*b^15 - 31744*a^12*
b^14 + 2176*a^13*b^13 + 29600*a^14*b^12 - 8480*a^15*b^11 - 17632*a^16*b^10
+ 7072*a^17*b^9 + 6528*a^18*b^8 - 3008*a^19*b^7 - 1376*a^20*b^6 + 672*a^21*
b^5 + 128*a^22*b^4 - 64*a^23*b^3) - (a^3*(a - 2*b)*(a + 2*b)*((a + b)^5*(a
- b)^5)^(1/2)*(32*b^28 - 32*a*b^27 - 352*a^2*b^26 + 480*a^3*b^25 + 1504*a^4
*b^24 - 2688*a^5*b^23 - 3168*a^6*b^22 + 8064*a^7*b^21 + 2880*a^8*b^20 - 147
84*a^9*b^19 + 1344*a^10*b^18 + 17472*a^11*b^17 - 6720*a^12*b^16 - 13440*a^1
3*b^15 + 8256*a^14*b^14 + 6528*a^15*b^13 - 5472*a^16*b^12 - 1824*a^17*b^11
+ 2080*a^18*b^10 + 224*a^19*b^9 - 416*a^20*b^8 + 32*a^22*b^6 + (a^3*tan(c/2
 + (d*x)/2)*(a - 2*b)*(a + 2*b)*((a + b)^5*(a - b)^5)^(1/2)*(128*a^2*b^28 -
 64*a*b^29 + 576*a^3*b^27 - 1280*a^4*b^26 - 2240*a^5*b^25 + 5760*a^6*b^24 +
 4800*a^7*b^23 - 15360*a^8*b^22 - 5760*a^9*b^21 + 26880*a^10*b^20 + 2688*a^
11*b^19 - 32256*a^12*b^18 + 2688*a^13*b^17 + 26880*a^14*b^16 - 5760*a^15*b^
15 - 15360*a^16*b^14 + 4800*a^17*b^13 + 5760*a^18*b^12 - 2240*a^19*b^11 - 1
280*a^20*b^10 + 576*a^21*b^9 + 128*a^22*b^8 - 64*a^23*b^7))/(b^12 - 5*a^2*b
^10 + 10*a^4*b^8 - 10*a^6*b^6 + 5*a^8*b^4 - a^10*b^2)))/(b^12 - 5*a^2*b^10
+ 10*a^4*b^8 - 10*a^6*b^6 + 5*a^8*b^4 - a^10*b^2)))/(b^12 - 5*a^2*b^10 + 10
*a^4*b^8 - 10*a^6*b^6 + 5*a^8*b^4 - a^10*b^2)))*(a - 2*b)*(a + 2*b)*((a + b
)^5*(a - b)^5)^(1/2)*2i)/(d*(b^12 - 5*a^2*b^10 + 10*a^4*b^8 - 10*a^6*b^6 +
5*a^8*b^4 - a^10*b^2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec^3(c+dx)}{(a\sin(c+dx)+b\tan(c+dx))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**3/(a*sin(d*x+c)+b*tan(d*x+c))**2,x)
```

```
[Out] Integral(sec(c + d*x)**3/(a*sin(c + d*x) + b*tan(c + d*x))**2, x)
```

3.264 $\int \frac{\cos^3(c+dx)}{(a\sin(c+dx)+b\tan(c+dx))^3}\,dx$

Optimal. Leaf size=248

$$-\frac{\csc^2(c+dx)\left(a\left(a^2+3b^2\right)-b\left(3a^2+b^2\right)\cos(c+dx)\right)}{2d\left(a^2-b^2\right)^3}+\frac{b^6}{2a^3d\left(a^2-b^2\right)^2(a\cos(c+dx)+b)^2}-\frac{2b^5\left(3a^2-\right.}{a^3d\left(a^2-b^2\right)^3(a\cos\right.}$$

```
[Out] 1/2*b^6/a^3/(a^2-b^2)^2/d/(b+a*cos(d*x+c))^2-2*b^5*(3*a^2-b^2)/a^3/(a^2-b^2
)^3/d/(b+a*cos(d*x+c))-1/2*(a*(a^2+3*b^2)-b*(3*a^2+b^2)*cos(d*x+c))*csc(d*x
+c)^2/(a^2-b^2)^3/d-1/4*(2*a+5*b)*ln(1-cos(d*x+c))/(a+b)^4/d-1/4*(2*a-5*b)*
ln(1+cos(d*x+c))/(a-b)^4/d-b^4*(15*a^4-4*a^2*b^2+b^4)*ln(b+a*cos(d*x+c))/a^
3/(a^2-b^2)^4/d
```

Rubi [A] time = 0.90, antiderivative size = 248, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.179, Rules used = {4397, 2837, 12, 1647, 1629}

$$\frac{b^6}{2a^3d\left(a^2-b^2\right)^2(a\cos(c+dx)+b)^2}-\frac{2b^5\left(3a^2-b^2\right)}{a^3d\left(a^2-b^2\right)^3(a\cos(c+dx)+b)}-\frac{b^4\left(-4a^2b^2+15a^4+b^4\right)\log(a\cos(c+d\right.}{a^3d\left(a^2-b^2\right)^4}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^3/(a*Sin[c + d*x] + b*Tan[c + d*x])^3,x]
```

```
[Out] b^6/(2*a^3*(a^2 - b^2)^2*d*(b + a*Cos[c + d*x])^2) - (2*b^5*(3*a^2 - b^2))/
(a^3*(a^2 - b^2)^3*d*(b + a*Cos[c + d*x])) - ((a*(a^2 + 3*b^2) - b*(3*a^2 +
 b^2)*Cos[c + d*x])*Csc[c + d*x]^2)/(2*(a^2 - b^2)^3*d) - ((2*a + 5*b)*Log[
1 - Cos[c + d*x]])/(4*(a + b)^4*d) - ((2*a - 5*b)*Log[1 + Cos[c + d*x]])/(4
*(a - b)^4*d) - (b^4*(15*a^4 - 4*a^2*b^2 + b^4)*Log[b + a*Cos[c + d*x]])/(a
^3*(a^2 - b^2)^4*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 1629

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol]
:> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
 d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 1647

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[(d + e*x)^m*Pq, a + c*x^2, x], f = Coeff[Pol
ynomialRemainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 0], g = Coeff[Polynomial
Remainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 1]}, Simp[((a*g - c*f*x)*(a + c
*x^2)^(p + 1))/(2*a*c*(p + 1)), x] + Dist[1/(2*a*c*(p + 1)), Int[(d + e*x)^
m*(a + c*x^2)^(p + 1)*ExpandToSum[(2*a*c*(p + 1)*Q)/(d + e*x)^m + (c*f*(2*p
 + 3))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, c, d, e}, x] && PolyQ[Pq, x] &
& NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 2837

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^m*(c + (d*x)/b)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*S
in[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/
2] && NeQ[a^2 - b^2, 0]
```

Rule 4397

```
Int[u_, x_Symbol] :> Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]
```

Rubi steps

$$\int \frac{\cos^3(c+dx)}{(a\sin(c+dx)+b\tan(c+dx))^3}\,dx = \int \frac{\cos^3(c+dx)\cot^3(c+dx)}{(b+a\cos(c+dx))^3}\,dx$$

$$= -\frac{a^3\,\mathrm{Subst}\left(\int \frac{x^6}{a^6(b+x)^3(a^2-x^2)^2}\,dx, x, a\cos(c+dx)\right)}{d}$$

$$= -\frac{\mathrm{Subst}\left(\int \frac{x^6}{(b+x)^3(a^2-x^2)^2}\,dx, x, a\cos(c+dx)\right)}{a^3 d}$$

$$= -\frac{\left(a\left(a^2+3b^2\right)-b\left(3a^2+b^2\right)\cos(c+dx)\right)\csc^2(c+dx)}{2\left(a^2-b^2\right)^3 d} - \frac{\mathrm{Subst}\left(\int \frac{\frac{a^6b^4(3}{(a^2-}}{} \right.}{}$$

$$= -\frac{\left(a\left(a^2+3b^2\right)-b\left(3a^2+b^2\right)\cos(c+dx)\right)\csc^2(c+dx)}{2\left(a^2-b^2\right)^3 d} - \frac{\mathrm{Subst}\left(\int\left(-\frac{a}{2(}\right.\right.}{}$$

$$= \frac{b^6}{2a^3\left(a^2-b^2\right)^2 d(b+a\cos(c+dx))^2} - \frac{2b^5\left(3a^2-b^2\right)}{a^3\left(a^2-b^2\right)^3 d(b+a\cos(c+dx))}$$

Mathematica [C] time = 6.36, size = 713, normalized size = 2.88

$$\frac{b^6\tan^3(c+dx)(a\cos(c+dx)+b)}{2a^3d(b-a)^2(a+b)^2(a\sin(c+dx)+b\tan(c+dx))^3} - \frac{2i\left(a^5-4a^3b^2-9ab^4\right)(c+dx)\tan^3(c+dx)(a\cos(c+dx)+}{d(a-b)^4(a+b)^4(a\sin(c+dx)+b\tan(c+dx))^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^3/(a*Sin[c + d*x] + b*Tan[c + d*x])^3,x]
```

```
[Out] (b^6*(b + a*Cos[c + d*x])*Tan[c + d*x]^3)/(2*a^3*(-a + b)^2*(a + b)^2*d*(a*
Sin[c + d*x] + b*Tan[c + d*x])^3) - (2*b^5*(-3*a^2 + b^2)*(b + a*Cos[c + d*
x])^2*Tan[c + d*x]^3)/(a^3*(-a + b)^3*(a + b)^3*d*(a*Sin[c + d*x] + b*Tan[c
 + d*x])^3) - ((2*I)*(a^5 - 4*a^3*b^2 - 9*a*b^4)*(c + d*x)*(b + a*Cos[c + d
*x])^3*Tan[c + d*x]^3)/((a - b)^4*(a + b)^4*d*(a*Sin[c + d*x] + b*Tan[c + d
*x])^3) - ((I/2)*(-2*a - 5*b)*ArcTan[Tan[c + d*x]]*(b + a*Cos[c + d*x])^3*T
an[c + d*x]^3)/((a + b)^4*d*(a*Sin[c + d*x] + b*Tan[c + d*x])^3) - ((I/2)*(
-2*a + 5*b)*ArcTan[Tan[c + d*x]]*(b + a*Cos[c + d*x])^3*Tan[c + d*x]^3)/((-
a + b)^4*d*(a*Sin[c + d*x] + b*Tan[c + d*x])^3) - ((b + a*Cos[c + d*x])^3*C
```

```
sc[(c + d*x)/2]^2*Tan[c + d*x]^3)/(8*(a + b)^3*d*(a*Sin[c + d*x] + b*Tan[c
+ d*x])^3) + ((-2*a + 5*b)*(b + a*Cos[c + d*x])^3*Log[Cos[(c + d*x)/2]^2]*T
an[c + d*x]^3)/(4*(-a + b)^4*d*(a*Sin[c + d*x] + b*Tan[c + d*x])^3) + ((-15
*a^4*b^4 + 4*a^2*b^6 - b^8)*(b + a*Cos[c + d*x])^3*Log[b + a*Cos[c + d*x]]*
Tan[c + d*x]^3)/(a^3*(-a^2 + b^2)^4*d*(a*Sin[c + d*x] + b*Tan[c + d*x])^3)
+ ((-2*a - 5*b)*(b + a*Cos[c + d*x])^3*Log[Sin[(c + d*x)/2]^2]*Tan[c + d*x]
^3)/(4*(a + b)^4*d*(a*Sin[c + d*x] + b*Tan[c + d*x])^3) + ((b + a*Cos[c + d
*x])^3*Sec[(c + d*x)/2]^2*Tan[c + d*x]^3)/(8*(-a + b)^3*d*(a*Sin[c + d*x] +
 b*Tan[c + d*x])^3)
```

fricas [B] time = 1.08, size = 1180, normalized size = 4.76

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3/(a*sin(d*x+c)+b*tan(d*x+c))^3,x, algorithm="fricas")

[Out] 1/4*(2*a^8*b^2 + 4*a^6*b^4 + 16*a^4*b^6 - 28*a^2*b^8 + 6*b^10 - 2*(3*a^9*b
- 2*a^7*b^3 + 11*a^5*b^5 - 16*a^3*b^7 + 4*a*b^9)*cos(d*x + c)^3 + 2*(a^10 -
 4*a^8*b^2 + a^6*b^4 - 9*a^4*b^6 + 14*a^2*b^8 - 3*b^10)*cos(d*x + c)^2 + 2*
(2*a^9*b + a^7*b^3 + 8*a^5*b^5 - 15*a^3*b^7 + 4*a*b^9)*cos(d*x + c) + 4*(15
*a^4*b^6 - 4*a^2*b^8 + b^10 - (15*a^6*b^4 - 4*a^4*b^6 + a^2*b^8)*cos(d*x +
c)^4 - 2*(15*a^5*b^5 - 4*a^3*b^7 + a*b^9)*cos(d*x + c)^3 + (15*a^6*b^4 - 19
*a^4*b^6 + 5*a^2*b^8 - b^10)*cos(d*x + c)^2 + 2*(15*a^5*b^5 - 4*a^3*b^7 + a
*b^9)*cos(d*x + c))*log(a*cos(d*x + c) + b) + (2*a^8*b^2 + 3*a^7*b^3 - 8*a^
6*b^4 - 22*a^5*b^5 - 18*a^4*b^6 - 5*a^3*b^7 - (2*a^10 + 3*a^9*b - 8*a^8*b^2
 - 22*a^7*b^3 - 18*a^6*b^4 - 5*a^5*b^5)*cos(d*x + c)^4 - 2*(2*a^9*b + 3*a^8
*b^2 - 8*a^7*b^3 - 22*a^6*b^4 - 18*a^5*b^5 - 5*a^4*b^6)*cos(d*x + c)^3 + (2
*a^10 + 3*a^9*b - 10*a^8*b^2 - 25*a^7*b^3 - 10*a^6*b^4 + 17*a^5*b^5 + 18*a^
4*b^6 + 5*a^3*b^7)*cos(d*x + c)^2 + 2*(2*a^9*b + 3*a^8*b^2 - 8*a^7*b^3 - 22
*a^6*b^4 - 18*a^5*b^5 - 5*a^4*b^6)*cos(d*x + c))*log(1/2*cos(d*x + c) + 1/2
) + (2*a^8*b^2 - 3*a^7*b^3 - 8*a^6*b^4 + 22*a^5*b^5 - 18*a^4*b^6 + 5*a^3*b^
7 - (2*a^10 - 3*a^9*b - 8*a^8*b^2 + 22*a^7*b^3 - 18*a^6*b^4 + 5*a^5*b^5)*co
s(d*x + c)^4 - 2*(2*a^9*b - 3*a^8*b^2 - 8*a^7*b^3 + 22*a^6*b^4 - 18*a^5*b^5
 + 5*a^4*b^6)*cos(d*x + c)^3 + (2*a^10 - 3*a^9*b - 10*a^8*b^2 + 25*a^7*b^3
- 10*a^6*b^4 - 17*a^5*b^5 + 18*a^4*b^6 - 5*a^3*b^7)*cos(d*x + c)^2 + 2*(2*a
^9*b - 3*a^8*b^2 - 8*a^7*b^3 + 22*a^6*b^4 - 18*a^5*b^5 + 5*a^4*b^6)*cos(d*x
 + c))*log(-1/2*cos(d*x + c) + 1/2))/((a^13 - 4*a^11*b^2 + 6*a^9*b^4 - 4*a^
7*b^6 + a^5*b^8)*d*cos(d*x + c)^4 + 2*(a^12*b - 4*a^10*b^3 + 6*a^8*b^5 - 4*
a^6*b^7 + a^4*b^9)*d*cos(d*x + c)^3 - (a^13 - 5*a^11*b^2 + 10*a^9*b^4 - 10*
a^7*b^6 + 5*a^5*b^8 - a^3*b^10)*d*cos(d*x + c)^2 - 2*(a^12*b - 4*a^10*b^3 +
 6*a^8*b^5 - 4*a^6*b^7 + a^4*b^9)*d*cos(d*x + c) - (a^11*b^2 - 4*a^9*b^4 +
6*a^7*b^6 - 4*a^5*b^8 + a^3*b^10)*d)
```

giac [B] time = 1.64, size = 848, normalized size = 3.42

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3/(a*sin(d*x+c)+b*tan(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -1/8*(2*(2*a + 5*b)*log(abs(-cos(d*x + c) + 1)/abs(cos(d*x + c) + 1))/(a^4
+ 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4) + 8*(15*a^4*b^4 - 4*a^2*b^6 + b^8)*l
og(abs(-a - b - a*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + b*(cos(d*x + c) -
 1)/(cos(d*x + c) + 1)))/(a^11 - 4*a^9*b^2 + 6*a^7*b^4 - 4*a^5*b^6 + a^3*b^
8) - (a + b + 4*a*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 10*b*(cos(d*x + c
) - 1)/(cos(d*x + c) + 1))*(cos(d*x + c) + 1)/((a^4 + 4*a^3*b + 6*a^2*b^2 +
 4*a*b^3 + b^4)*(cos(d*x + c) - 1)) - (cos(d*x + c) - 1)/((a^3 - 3*a^2*b +
3*a*b^2 - b^3)*(cos(d*x + c) + 1)) - 4*(45*a^6*b^4 + 66*a^5*b^5 - 15*a^4*b^
6 - 44*a^3*b^7 - a^2*b^8 + 10*a*b^9 + 3*b^10 + 90*a^6*b^4*(cos(d*x + c) - 1
)/(cos(d*x + c) + 1) - 24*a^5*b^5*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 1
18*a^4*b^6*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 28*a^3*b^7*(cos(d*x + c)
 - 1)/(cos(d*x + c) + 1) + 34*a^2*b^8*(cos(d*x + c) - 1)/(cos(d*x + c) + 1)
 - 4*a*b^9*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 6*b^10*(cos(d*x + c) - 1
)/(cos(d*x + c) + 1) + 45*a^6*b^4*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2
 - 90*a^5*b^5*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 + 33*a^4*b^6*(cos(d
*x + c) - 1)^2/(cos(d*x + c) + 1)^2 + 24*a^3*b^7*(cos(d*x + c) - 1)^2/(cos(
d*x + c) + 1)^2 - 9*a^2*b^8*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 - 6*a
*b^9*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 + 3*b^10*(cos(d*x + c) - 1)^
2/(cos(d*x + c) + 1)^2)/((a^11 - 4*a^9*b^2 + 6*a^7*b^4 - 4*a^5*b^6 + a^3*b^
8)*(a + b + a*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - b*(cos(d*x + c) - 1)/
(cos(d*x + c) + 1))^2) - 8*log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) +
 1))/a^3)/d
```

maple [A] time = 0.23, size = 333, normalized size = 1.34

$$\frac{b^6}{2d\,a^3\,(a+b)^2\,(a-b)^2\,(b+a\cos(dx+c))^2}-\frac{15b^4a\ln(b+a\cos(dx+c))}{d\,(a+b)^4\,(a-b)^4}+\frac{4b^6\ln(b+a\cos(dx+c))}{d\,(a+b)^4\,(a-b)^4\,a}-\frac{b^8\ln(b+a}{d\,(a+b)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3/(a*sin(d*x+c)+b*tan(d*x+c))^3,x)
```

```
[Out] 1/2/d*b^6/a^3/(a+b)^2/(a-b)^2/(b+a*cos(d*x+c))^2-15/d*b^4/(a+b)^4/(a-b)^4*a
*ln(b+a*cos(d*x+c))+4/d*b^6/(a+b)^4/(a-b)^4/a*ln(b+a*cos(d*x+c))-1/d*b^8/(a
+b)^4/(a-b)^4/a^3*ln(b+a*cos(d*x+c))-6/d*b^5/a/(a+b)^3/(a-b)^3/(b+a*cos(d*x
+c))+2/d*b^7/a^3/(a+b)^3/(a-b)^3/(b+a*cos(d*x+c))+1/4/d/(a+b)^3/(cos(d*x+c)
-1)-1/2/d/(a+b)^4*ln(cos(d*x+c)-1)*a-5/4/d/(a+b)^4*ln(cos(d*x+c)-1)*b-1/4/d
```

```
/(a-b)^3/(1+cos(d*x+c))-1/2*a*ln(1+cos(d*x+c))/(a-b)^4/d+5/4*b*ln(1+cos(d*x
+c))/(a-b)^4/d
```

maxima [B] time = 0.48, size = 684, normalized size = 2.76

$$-\frac{\frac{8\left(15\,a^4b^4-4\,a^2b^6+b^8\right)\log\left(a+b-\frac{(a-b)\sin(dx+c)^2}{(\cos(dx+c)+1)^2}\right)}{a^{11}-4\,a^9b^2+6\,a^7b^4-4\,a^5b^6+a^3b^8}+\frac{4\,(2\,a+5\,b)\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^4+4\,a^3b+6\,a^2b^2+4\,ab^3+b^4}+\frac{a^8-2\,a^7b-a^6b^2+4\,a^5b^3-a^4b^4-2\,a^3b^5+a^2b^6-\frac{2\,(}{}}{\frac{\left(a^{11}+a^{10}b-4\,a^9b^2-4\,a^8b^3+6\,a^7b^4+6\,a^6b^5-4\,a^5b^6-4\,a^4b^7+a^3b^8+a^2b^9\right)s}{(\cos(dx+c)+1)^2}}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3/(a*sin(d*x+c)+b*tan(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -1/8*(8*(15*a^4*b^4 - 4*a^2*b^6 + b^8)*log(a + b - (a - b)*sin(d*x + c)^2/(
cos(d*x + c) + 1)^2)/(a^11 - 4*a^9*b^2 + 6*a^7*b^4 - 4*a^5*b^6 + a^3*b^8) +
 4*(2*a + 5*b)*log(sin(d*x + c)/(cos(d*x + c) + 1))/(a^4 + 4*a^3*b + 6*a^2*
b^2 + 4*a*b^3 + b^4) + (a^8 - 2*a^7*b - a^6*b^2 + 4*a^5*b^3 - a^4*b^4 - 2*a
^3*b^5 + a^2*b^6 - 2*(a^8 - 4*a^7*b + 5*a^6*b^2 - 5*a^4*b^4 - 44*a^3*b^5 -
49*a^2*b^6 + 8*a*b^7 + 8*b^8)*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + (a^8 -
6*a^7*b + 15*a^6*b^2 - 20*a^5*b^3 + 15*a^4*b^4 - 102*a^3*b^5 + 81*a^2*b^6 +
 32*a*b^7 - 16*b^8)*sin(d*x + c)^4/(cos(d*x + c) + 1)^4)/((a^11 + a^10*b -
4*a^9*b^2 - 4*a^8*b^3 + 6*a^7*b^4 + 6*a^6*b^5 - 4*a^5*b^6 - 4*a^4*b^7 + a^3
*b^8 + a^2*b^9)*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 - 2*(a^11 - a^10*b - 4*
a^9*b^2 + 4*a^8*b^3 + 6*a^7*b^4 - 6*a^6*b^5 - 4*a^5*b^6 + 4*a^4*b^7 + a^3*b
^8 - a^2*b^9)*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + (a^11 - 3*a^10*b + 8*a^
8*b^3 - 6*a^7*b^4 - 6*a^6*b^5 + 8*a^5*b^6 - 3*a^3*b^8 + a^2*b^9)*sin(d*x +
c)^6/(cos(d*x + c) + 1)^6) + sin(d*x + c)^2/((a^3 - 3*a^2*b + 3*a*b^2 - b^3
)*(cos(d*x + c) + 1)^2) - 8*log(sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 1)/a^
3)/d
```

mupad [B] time = 2.05, size = 527, normalized size = 2.12

$$\frac{\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4\left(-a^7+5\,a^6\,b-10\,a^5\,b^2+10\,a^4\,b^3-5\,a^3\,b^4+97\,a^2\,b^5+16\,a\,b^6-16\,b^7\right)}{2\,a^2\,(a+b)\left(a^2+2\,a\,b+b^2\right)}-\frac{a^3-3\,a^2\,b+3\,a\,b^2}{2\,(a+b)}}{d\left(\left(4\,a^5-20\,a^4\,b+40\,a^3\,b^2-40\,a^2\,b^3+20\,a\,b^4-4\,b^5\right)\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6+\left(-8\,a^5+24\,a^4\,b-16\,a^3\,b^2-16\,a^2\,b^3+\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^3/(a*sin(c + d*x) + b*tan(c + d*x))^3,x)
```

```
[Out] ((tan(c/2 + (d*x)/2)^4*(16*a*b^6 + 5*a^6*b - a^7 - 16*b^7 + 97*a^2*b^5 - 5*
a^3*b^4 + 10*a^4*b^3 - 10*a^5*b^2))/(2*a^2*(a + b)*(2*a*b + a^2 + b^2)) - (
3*a*b^2 - 3*a^2*b + a^3 - b^3)/(2*(a + b)) + (tan(c/2 + (d*x)/2)^2*(a^7 - 5
```

```
*a^6*b + 8*b^7 - 49*a^2*b^5 + 5*a^3*b^4 - 10*a^4*b^3 + 10*a^5*b^2))/(a^2*(a
 + b)^2*(a - b)))/(d*(tan(c/2 + (d*x)/2)^2*(4*a*b^4 - 4*a^4*b + 4*a^5 - 4*b
^5 + 8*a^2*b^3 - 8*a^3*b^2) - tan(c/2 + (d*x)/2)^4*(8*a^5 - 24*a^4*b - 24*a
*b^4 + 8*b^5 + 16*a^2*b^3 + 16*a^3*b^2) + tan(c/2 + (d*x)/2)^6*(20*a*b^4 -
20*a^4*b + 4*a^5 - 4*b^5 - 40*a^2*b^3 + 40*a^3*b^2))) - tan(c/2 + (d*x)/2)^
2/(8*d*(a - b)^3) + log(tan(c/2 + (d*x)/2)^2 + 1)/(a^3*d) - (log(tan(c/2 +
(d*x)/2))*(2*a + 5*b))/(d*(8*a*b^3 + 8*a^3*b + 2*a^4 + 2*b^4 + 12*a^2*b^2))
 - (b^4*log(a + b - a*tan(c/2 + (d*x)/2)^2 + b*tan(c/2 + (d*x)/2)^2)*(15*a^
4 + b^4 - 4*a^2*b^2))/(a^3*d*(a^2 - b^2)^4)
```

sympy **[F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3/(a*sin(d*x+c)+b*tan(d*x+c))**3,x)
```

```
[Out] Timed out
```

3.265 $\int \frac{\cos^2(c+dx)}{(a\sin(c+dx)+b\tan(c+dx))^3}\,dx$

Optimal. Leaf size=232

$$\frac{\csc^2(c+dx)\left(b\left(3a^2+b^2\right)-a\left(a^2+3b^2\right)\cos(c+dx)\right)}{2d\left(a^2-b^2\right)^3}-\frac{b^5}{2a^2d\left(a^2-b^2\right)^2(a\cos(c+dx)+b)^2}+\frac{b^4\left(5a^2-\right.}{a^2d\left(a^2-b^2\right)^3(a\,\mathrm{co}}$$

```
[Out] -1/2*b^5/a^2/(a^2-b^2)^2/d/(b+a*cos(d*x+c))^2+b^4*(5*a^2-b^2)/a^2/(a^2-b^2)
^3/d/(b+a*cos(d*x+c))+1/2*(b*(3*a^2+b^2)-a*(a^2+3*b^2)*cos(d*x+c))*csc(d*x+
c)^2/(a^2-b^2)^3/d-1/4*(a+4*b)*ln(1-cos(d*x+c))/(a+b)^4/d+1/4*(a-4*b)*ln(1+
cos(d*x+c))/(a-b)^4/d+2*b^3*(5*a^2+b^2)*ln(b+a*cos(d*x+c))/(a^2-b^2)^4/d
```

Rubi [A] time = 0.76, antiderivative size = 232, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.179, Rules used = {4397, 2837, 12, 1647, 1629}

$$-\frac{b^5}{2a^2d\left(a^2-b^2\right)^2(a\cos(c+dx)+b)^2}+\frac{b^4\left(5a^2-b^2\right)}{a^2d\left(a^2-b^2\right)^3(a\cos(c+dx)+b)}+\frac{2b^3\left(5a^2+b^2\right)\log(a\cos(c+dx)+b)}{d\left(a^2-b^2\right)^4}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2/(a*Sin[c + d*x] + b*Tan[c + d*x])^3,x]
```

```
[Out] -b^5/(2*a^2*(a^2 - b^2)^2*d*(b + a*Cos[c + d*x])^2) + (b^4*(5*a^2 - b^2))/(
a^2*(a^2 - b^2)^3*d*(b + a*Cos[c + d*x])) + ((b*(3*a^2 + b^2) - a*(a^2 + 3*
b^2)*Cos[c + d*x])*Csc[c + d*x]^2)/(2*(a^2 - b^2)^3*d) - ((a + 4*b)*Log[1 -
 Cos[c + d*x]])/(4*(a + b)^4*d) + ((a - 4*b)*Log[1 + Cos[c + d*x]])/(4*(a -
 b)^4*d) + (2*b^3*(5*a^2 + b^2)*Log[b + a*Cos[c + d*x]])/((a^2 - b^2)^4*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 1629

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol]
:> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
 d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 1647

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[(d + e*x)^m*Pq, a + c*x^2, x], f = Coeff[Pol
```

```
ynomialRemainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 0], g = Coeff[Polynomial
Remainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 1]}, Simp[((a*g - c*f*x)*(a + c
*x^2)^(p + 1))/(2*a*c*(p + 1)), x] + Dist[1/(2*a*c*(p + 1)), Int[(d + e*x)^
m*(a + c*x^2)^(p + 1)*ExpandToSum[(2*a*c*(p + 1)*Q)/(d + e*x)^m + (c*f*(2*p
 + 3))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, c, d, e}, x] && PolyQ[Pq, x] &
& NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 2837

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^m*(c + (d*x)/b)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*S
in[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/
2] && NeQ[a^2 - b^2, 0]
```

Rule 4397

```
Int[u_, x_Symbol] :> Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]
```

Rubi steps

$$\int \frac{\cos^2(c+dx)}{(a\sin(c+dx)+b\tan(c+dx))^3}\,dx = \int \frac{\cos^2(c+dx)\cot^3(c+dx)}{(b+a\cos(c+dx))^3}\,dx$$
$$= -\frac{a^3\,\text{Subst}\left(\int \frac{x^5}{a^5(b+x)^3\left(a^2-x^2\right)^2}\,dx, x, a\cos(c+dx)\right)}{d}$$
$$= -\frac{\text{Subst}\left(\int \frac{x^5}{(b+x)^3\left(a^2-x^2\right)^2}\,dx, x, a\cos(c+dx)\right)}{a^2 d}$$
$$= \frac{\left(b\left(3a^2+b^2\right)-a\left(a^2+3b^2\right)\cos(c+dx)\right)\csc^2(c+dx)}{2\left(a^2-b^2\right)^3 d} - \frac{\text{Subst}\left(\int \frac{-\frac{a^6b^3}{(a\cdots}}{\cdots}\right)}{\cdots}$$
$$= \frac{\left(b\left(3a^2+b^2\right)-a\left(a^2+3b^2\right)\cos(c+dx)\right)\csc^2(c+dx)}{2\left(a^2-b^2\right)^3 d} - \frac{\text{Subst}\left(\int\left(-\frac{\cdots}{2(\cdots}\right.\right.}{\cdots}$$
$$= -\frac{b^5}{2a^2\left(a^2-b^2\right)^2 d(b+a\cos(c+dx))^2} + \frac{b^4\left(5a^2-b^2\right)}{a^2\left(a^2-b^2\right)^3 d(b+a\cos(c+d\cdots}$$

Mathematica [A] time = 6.11, size = 204, normalized size = 0.88

$$\frac{-\frac{4b^5}{a^2(a-b)^2(a+b)^2(a\cos(c+dx)+b)^2}+\frac{8b^4\left(b^2-5a^2\right)}{a^2(b-a)^3(a+b)^3(a\cos(c+dx)+b)}+\frac{16b^3\left(5a^2+b^2\right)\log(a\cos(c+dx)+b)}{\left(a^2-b^2\right)^4}-\frac{\csc^2\left(\frac{1}{2}(c+dx)\right)}{(a+b)^3}+\frac{\sec^2\left(\frac{1}{2}(c+dx)\right)}{(a-b)^3}\cdots}{8d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2/(a*Sin[c + d*x] + b*Tan[c + d*x])^3,x]
```

```
[Out] ((-4*b^5)/(a^2*(a - b)^2*(a + b)^2*(b + a*Cos[c + d*x])^2) + (8*b^4*(-5*a^2
 + b^2))/(a^2*(-a + b)^3*(a + b)^3*(b + a*Cos[c + d*x])) - Csc[(c + d*x)/2]
^2/(a + b)^3 + (4*(a - 4*b)*Log[Cos[(c + d*x)/2]])/(a - b)^4 + (16*b^3*(5*a
^2 + b^2)*Log[b + a*Cos[c + d*x]])/(a^2 - b^2)^4 - (4*(a + 4*b)*Log[Sin[(c
+ d*x)/2]])/(a + b)^4 + Sec[(c + d*x)/2]^2/(a - b)^3)/(8*d)
```

fricas [B] time = 1.87, size = 1045, normalized size = 4.50

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(a*sin(d*x+c)+b*tan(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] -1/4*(6*a^6*b^3 + 14*a^4*b^5 - 22*a^2*b^7 + 2*b^9 - 2*(a^9 + 2*a^7*b^2 + 7*
a^5*b^4 - 12*a^3*b^6 + 2*a*b^8)*cos(d*x + c)^3 + 2*(a^8*b - 6*a^6*b^3 - 4*a
^4*b^5 + 10*a^2*b^7 - b^9)*cos(d*x + c)^2 + 2*(5*a^7*b^2 + 4*a^5*b^4 - 11*a
^3*b^6 + 2*a*b^8)*cos(d*x + c) + 8*(5*a^4*b^5 + a^2*b^7 - (5*a^6*b^3 + a^4*
b^5)*cos(d*x + c)^4 - 2*(5*a^5*b^4 + a^3*b^6)*cos(d*x + c)^3 + (5*a^6*b^3 -
 4*a^4*b^5 - a^2*b^7)*cos(d*x + c)^2 + 2*(5*a^5*b^4 + a^3*b^6)*cos(d*x + c)
)*log(a*cos(d*x + c) + b) + (a^7*b^2 - 10*a^5*b^4 - 20*a^4*b^5 - 15*a^3*b^6
 - 4*a^2*b^7 - (a^9 - 10*a^7*b^2 - 20*a^6*b^3 - 15*a^5*b^4 - 4*a^4*b^5)*cos
(d*x + c)^4 - 2*(a^8*b - 10*a^6*b^3 - 20*a^5*b^4 - 15*a^4*b^5 - 4*a^3*b^6)*
cos(d*x + c)^3 + (a^9 - 11*a^7*b^2 - 20*a^6*b^3 - 5*a^5*b^4 + 16*a^4*b^5 +
15*a^3*b^6 + 4*a^2*b^7)*cos(d*x + c)^2 + 2*(a^8*b - 10*a^6*b^3 - 20*a^5*b^4
 - 15*a^4*b^5 - 4*a^3*b^6)*cos(d*x + c))*log(1/2*cos(d*x + c) + 1/2) - (a^7
*b^2 - 10*a^5*b^4 + 20*a^4*b^5 - 15*a^3*b^6 + 4*a^2*b^7 - (a^9 - 10*a^7*b^2
 + 20*a^6*b^3 - 15*a^5*b^4 + 4*a^4*b^5)*cos(d*x + c)^4 - 2*(a^8*b - 10*a^6*
b^3 + 20*a^5*b^4 - 15*a^4*b^5 + 4*a^3*b^6)*cos(d*x + c)^3 + (a^9 - 11*a^7*b
^2 + 20*a^6*b^3 - 5*a^5*b^4 - 16*a^4*b^5 + 15*a^3*b^6 - 4*a^2*b^7)*cos(d*x
+ c)^2 + 2*(a^8*b - 10*a^6*b^3 + 20*a^5*b^4 - 15*a^4*b^5 + 4*a^3*b^6)*cos(d
*x + c))*log(-1/2*cos(d*x + c) + 1/2))/((a^12 - 4*a^10*b^2 + 6*a^8*b^4 - 4*
a^6*b^6 + a^4*b^8)*d*cos(d*x + c)^4 + 2*(a^11*b - 4*a^9*b^3 + 6*a^7*b^5 - 4
*a^5*b^7 + a^3*b^9)*d*cos(d*x + c)^3 - (a^12 - 5*a^10*b^2 + 10*a^8*b^4 - 10
*a^6*b^6 + 5*a^4*b^8 - a^2*b^10)*d*cos(d*x + c)^2 - 2*(a^11*b - 4*a^9*b^3 +
 6*a^7*b^5 - 4*a^5*b^7 + a^3*b^9)*d*cos(d*x + c) - (a^10*b^2 - 4*a^8*b^4 +
6*a^6*b^6 - 4*a^4*b^8 + a^2*b^10)*d)
```

giac [B] time = 1.39, size = 676, normalized size = 2.91

$$-\frac{\frac{2\,(a+4\,b)\log\left(\frac{|-\cos(dx+c)+1|}{|\cos(dx+c)+1|}\right)}{a^4+4\,a^3b+6\,a^2b^2+4\,ab^3+b^4}-\frac{16\left(5\,a^2b^3+b^5\right)\log\left(\left|-a-b-\frac{a(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{b(\cos(dx+c)-1)}{\cos(dx+c)+1}\right|\right)}{a^8-4\,a^6b^2+6\,a^4b^4-4\,a^2b^6+b^8}-\frac{\left(a+b+\frac{2\,a(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{8\,b(\cos(dx+c)-1)}{\cos(dx+c)+1}\right)(\cos(dx+c)+1)}{\left(a^4+4\,a^3b+6\,a^2b^2+4\,ab^3+b^4\right)(\cos(dx+c)-1)}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(a*sin(d*x+c)+b*tan(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -1/8*(2*(a + 4*b)*log(abs(-cos(d*x + c) + 1)/abs(cos(d*x + c) + 1))/(a^4 +
4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4) - 16*(5*a^2*b^3 + b^5)*log(abs(-a - b
- a*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + b*(cos(d*x + c) - 1)/(cos(d*x +
 c) + 1)))/(a^8 - 4*a^6*b^2 + 6*a^4*b^4 - 4*a^2*b^6 + b^8) - (a + b + 2*a*(
cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 8*b*(cos(d*x + c) - 1)/(cos(d*x + c)
 + 1))*(cos(d*x + c) + 1)/((a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4)*(cos
(d*x + c) - 1)) + (cos(d*x + c) - 1)/((a^3 - 3*a^2*b + 3*a*b^2 - b^3)*(cos(
```

```
d*x + c) + 1)) + 8*(15*a^4*b^3 + 20*a^3*b^4 - 2*a^2*b^5 - 4*a*b^6 + 3*b^7 +
 30*a^4*b^3*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 10*a^3*b^4*(cos(d*x + c
) - 1)/(cos(d*x + c) + 1) - 26*a^2*b^5*(cos(d*x + c) - 1)/(cos(d*x + c) + 1
) + 10*a*b^6*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 4*b^7*(cos(d*x + c) -
1)/(cos(d*x + c) + 1) + 15*a^4*b^3*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^
2 - 30*a^3*b^4*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 + 18*a^2*b^5*(cos(
d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 - 6*a*b^6*(cos(d*x + c) - 1)^2/(cos(d*
x + c) + 1)^2 + 3*b^7*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2)/((a^8 - 4*
a^6*b^2 + 6*a^4*b^4 - 4*a^2*b^6 + b^8)*(a + b + a*(cos(d*x + c) - 1)/(cos(d
*x + c) + 1) - b*(cos(d*x + c) - 1)/(cos(d*x + c) + 1))^2))/d
```

maple [A] time = 0.23, size = 295, normalized size = 1.27

$$-\frac{b^5}{2d\,a^2\,(a+b)^2\,(a-b)^2\,(b+a\cos(dx+c))^2}+\frac{10b^3\ln(b+a\cos(dx+c))\,a^2}{d\,(a+b)^4\,(a-b)^4}+\frac{2b^5\ln(b+a\cos(dx+c))}{d\,(a+b)^4\,(a-b)^4}+\frac{\cdots}{d\,(a+\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2/(a*sin(d*x+c)+b*tan(d*x+c))^3,x)
```

```
[Out] -1/2/d*b^5/a^2/(a+b)^2/(a-b)^2/(b+a*cos(d*x+c))^2+10/d*b^3/(a+b)^4/(a-b)^4*
ln(b+a*cos(d*x+c))*a^2+2/d*b^5/(a+b)^4/(a-b)^4*ln(b+a*cos(d*x+c))+5/d*b^4/(
a+b)^3/(a-b)^3/(b+a*cos(d*x+c))-1/d*b^6/(a+b)^3/(a-b)^3/a^2/(b+a*cos(d*x+c)
)+1/4/d/(a+b)^3/(cos(d*x+c)-1)-1/4/d/(a+b)^4*ln(cos(d*x+c)-1)*a-1/d/(a+b)^4
*ln(cos(d*x+c)-1)*b+1/4/d/(a-b)^3/(1+cos(d*x+c))+1/4*a*ln(1+cos(d*x+c))/(a-
b)^4/d-b*ln(1+cos(d*x+c))/(a-b)^4/d
```

maxima [B] time = 0.88, size = 589, normalized size = 2.54

$$\frac{\frac{16\left(5a^2b^3+b^5\right)\log\left(a+b-\frac{(a-b)\sin(dx+c)^2}{(\cos(dx+c)+1)^2}\right)}{a^8-4a^6b^2+6a^4b^4-4a^2b^6+b^8}-\frac{4(a+4b)\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^4+4a^3b+6a^2b^2+4ab^3+b^4}-\frac{a^6-2a^5b-a^4b^2+4a^3b^3-a^2b^4-2ab^5+b^6-\frac{2\left(a^6-4\cdots\right.}{\cdots}}{\frac{\left(a^9+a^8b-4a^7b^2-4a^6b^3+6a^5b^4+6a^4b^5-4a^3b^6-4a^2b^7+ab^8+b^9\right)\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-\frac{2\left(a^9\cdots\right.}{\cdots}}}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(a*sin(d*x+c)+b*tan(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/8*(16*(5*a^2*b^3 + b^5)*log(a + b - (a - b)*sin(d*x + c)^2/(cos(d*x + c)
+ 1)^2)/(a^8 - 4*a^6*b^2 + 6*a^4*b^4 - 4*a^2*b^6 + b^8) - 4*(a + 4*b)*log(s
in(d*x + c)/(cos(d*x + c) + 1))/(a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4)
 - (a^6 - 2*a^5*b - a^4*b^2 + 4*a^3*b^3 - a^2*b^4 - 2*a*b^5 + b^6 - 2*(a^6
- 4*a^5*b + 5*a^4*b^2 + 35*a^2*b^4 + 44*a*b^5 - b^6)*sin(d*x + c)^2/(cos(d*
x + c) + 1)^2 + (a^6 - 6*a^5*b + 15*a^4*b^2 - 20*a^3*b^3 + 95*a^2*b^4 - 70*
a*b^5 - 15*b^6)*sin(d*x + c)^4/(cos(d*x + c) + 1)^4)/((a^9 + a^8*b - 4*a^7*
b^2 - 4*a^6*b^3 + 6*a^5*b^4 + 6*a^4*b^5 - 4*a^3*b^6 - 4*a^2*b^7 + a*b^8 + b
```

```
^9)*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 - 2*(a^9 - a^8*b - 4*a^7*b^2 + 4*a^
6*b^3 + 6*a^5*b^4 - 6*a^4*b^5 - 4*a^3*b^6 + 4*a^2*b^7 + a*b^8 - b^9)*sin(d*
x + c)^4/(cos(d*x + c) + 1)^4 + (a^9 - 3*a^8*b + 8*a^6*b^3 - 6*a^5*b^4 - 6*
a^4*b^5 + 8*a^3*b^6 - 3*a*b^8 + b^9)*sin(d*x + c)^6/(cos(d*x + c) + 1)^6) +
 sin(d*x + c)^2/((a^3 - 3*a^2*b + 3*a*b^2 - b^3)*(cos(d*x + c) + 1)^2))/d
```

mupad [B] time = 1.21, size = 491, normalized size = 2.12

$$\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2}{8d(a-b)^3}-\frac{\frac{a^3-3a^2b+3ab^2-b^3}{2(a+b)}+\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4\big(a^5-5a^4b+10a^3b^2-10a\cdots}{2(a+b)\big(a^2+2ab+b^2\cdots}}{d\Big(\left(4a^5-20a^4b+40a^3b^2-40a^2b^3+20ab^4-4b^5\right)\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^6+\big(-8a^5+24a^4b-16a^3\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^2/(a*sin(c + d*x) + b*tan(c + d*x))^3,x)
```

```
[Out] tan(c/2 + (d*x)/2)^2/(8*d*(a - b)^3) - ((3*a*b^2 - 3*a^2*b + a^3 - b^3)/(2*
(a + b)) + (tan(c/2 + (d*x)/2)^4*(85*a*b^4 - 5*a^4*b + a^5 + 15*b^5 - 10*a^
2*b^3 + 10*a^3*b^2))/(2*(a + b)*(2*a*b + a^2 + b^2)) - (tan(c/2 + (d*x)/2)^
2*(45*a*b^4 - 5*a^4*b + a^5 - b^5 - 10*a^2*b^3 + 10*a^3*b^2))/((a - b)*(2*a
*b + a^2 + b^2)))/(d*(tan(c/2 + (d*x)/2)^2*(4*a*b^4 - 4*a^4*b + 4*a^5 - 4*b
^5 + 8*a^2*b^3 - 8*a^3*b^2) - tan(c/2 + (d*x)/2)^4*(8*a^5 - 24*a^4*b - 24*a
*b^4 + 8*b^5 + 16*a^2*b^3 + 16*a^3*b^2) + tan(c/2 + (d*x)/2)^6*(20*a*b^4 -
20*a^4*b + 4*a^5 - 4*b^5 - 40*a^2*b^3 + 40*a^3*b^2))) - (log(tan(c/2 + (d*x
)/2))*(a + 4*b))/(d*(8*a*b^3 + 8*a^3*b + 2*a^4 + 2*b^4 + 12*a^2*b^2)) + (lo
g(a + b - a*tan(c/2 + (d*x)/2)^2 + b*tan(c/2 + (d*x)/2)^2)*(2*b^5 + 10*a^2*
b^3))/(d*(a^8 + b^8 - 4*a^2*b^6 + 6*a^4*b^4 - 4*a^6*b^2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos^2(c+dx)}{(a\sin(c+dx)+b\tan(c+dx))^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2/(a*sin(d*x+c)+b*tan(d*x+c))**3,x)
```

```
[Out] Integral(cos(c + d*x)**2/(a*sin(c + d*x) + b*tan(c + d*x))**3, x)
```

3.266 $\int \frac{\cos(c+dx)}{(a\sin(c+dx)+b\tan(c+dx))^3}\,dx$

Optimal. Leaf size=211

$$-\frac{6ab^2\left(a^2+b^2\right)\log(a\cos(c+dx)+b)}{d\left(a^2-b^2\right)^4}-\frac{\csc^2(c+dx)\left(a\left(a^2+3b^2\right)-b\left(3a^2+b^2\right)\cos(c+dx)\right)}{2d\left(a^2-b^2\right)^3}+\frac{}{2ad\left(a^2-b^2\right)^2(}$$

```
[Out] 1/2*b^4/a/(a^2-b^2)^2/d/(b+a*cos(d*x+c))^2-4*a*b^3/(a^2-b^2)^3/d/(b+a*cos(d
*x+c))-1/2*(a*(a^2+3*b^2)-b*(3*a^2+b^2)*cos(d*x+c))*csc(d*x+c)^2/(a^2-b^2)^
3/d-3/4*b*ln(1-cos(d*x+c))/(a+b)^4/d+3/4*b*ln(1+cos(d*x+c))/(a-b)^4/d-6*a*b
^2*(a^2+b^2)*ln(b+a*cos(d*x+c))/(a^2-b^2)^4/d
```

Rubi [A] time = 0.67, antiderivative size = 211, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {4397, 2837, 12, 1647, 1629}

$$\frac{b^4}{2ad\left(a^2-b^2\right)^2(a\cos(c+dx)+b)^2}-\frac{4ab^3}{d\left(a^2-b^2\right)^3(a\cos(c+dx)+b)}-\frac{6ab^2\left(a^2+b^2\right)\log(a\cos(c+dx)+b)}{d\left(a^2-b^2\right)^4}-\csc$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]/(a*Sin[c + d*x] + b*Tan[c + d*x])^3,x]
```

```
[Out] b^4/(2*a*(a^2 - b^2)^2*d*(b + a*Cos[c + d*x])^2) - (4*a*b^3)/((a^2 - b^2)^3
*d*(b + a*Cos[c + d*x])) - ((a*(a^2 + 3*b^2) - b*(3*a^2 + b^2)*Cos[c + d*x]
)*Csc[c + d*x]^2)/(2*(a^2 - b^2)^3*d) - (3*b*Log[1 - Cos[c + d*x]])/(4*(a +
 b)^4*d) + (3*b*Log[1 + Cos[c + d*x]])/(4*(a - b)^4*d) - (6*a*b^2*(a^2 + b^
2)*Log[b + a*Cos[c + d*x]])/((a^2 - b^2)^4*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 1629

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol]
:> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
 d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 1647

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[(d + e*x)^m*Pq, a + c*x^2, x], f = Coeff[Pol
```

```
ynomialRemainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 0], g = Coeff[Polynomial
Remainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 1]}, Simp[((a*g - c*f*x)*(a + c
*x^2)^(p + 1))/(2*a*c*(p + 1)), x] + Dist[1/(2*a*c*(p + 1)), Int[(d + e*x)^
m*(a + c*x^2)^(p + 1)*ExpandToSum[(2*a*c*(p + 1)*Q)/(d + e*x)^m + (c*f*(2*p
 + 3))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, c, d, e}, x] && PolyQ[Pq, x] &
& NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 2837

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^m*(c + (d*x)/b)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*S
in[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/
2] && NeQ[a^2 - b^2, 0]
```

Rule 4397

```
Int[u_, x_Symbol] :> Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]
```

Rubi steps

$$\int \frac{\cos(c+dx)}{(a\sin(c+dx)+b\tan(c+dx))^3}\,dx = \int \frac{\cos(c+dx)\cot^3(c+dx)}{(b+a\cos(c+dx))^3}\,dx$$

$$= -\frac{a^3\,\text{Subst}\left(\int \frac{x^4}{a^4(b+x)^3\left(a^2-x^2\right)^2}\,dx, x, a\cos(c+dx)\right)}{d}$$

$$= -\frac{\text{Subst}\left(\int \frac{x^4}{(b+x)^3\left(a^2-x^2\right)^2}\,dx, x, a\cos(c+dx)\right)}{ad}$$

$$= -\frac{\left(a\left(a^2+3b^2\right)-b\left(3a^2+b^2\right)\cos(c+dx)\right)\csc^2(c+dx)}{2\left(a^2-b^2\right)^3 d} - \frac{\text{Subst}\left(\int \frac{a^4b^4}{(a}\right.}{}$$

$$= -\frac{\left(a\left(a^2+3b^2\right)-b\left(3a^2+b^2\right)\cos(c+dx)\right)\csc^2(c+dx)}{2\left(a^2-b^2\right)^3 d} - \frac{\text{Subst}\left(\int\left(-\right.\right.}{}$$

$$= \frac{b^4}{2a\left(a^2-b^2\right)^2 d(b+a\cos(c+dx))^2} - \frac{4ab^3}{\left(a^2-b^2\right)^3 d(b+a\cos(c+dx))} -$$

Mathematica [A] time = 5.33, size = 184, normalized size = 0.87

$$\frac{-\frac{48ab^2\left(a^2+b^2\right)\log(a\cos(c+dx)+b)}{\left(a^2-b^2\right)^4} + \frac{4b^4}{a(a-b)^2(a+b)^2(a\cos(c+dx)+b)^2} + \frac{32ab^3}{(b-a)^3(a+b)^3(a\cos(c+dx)+b)} - \frac{\csc^2\left(\frac{1}{2}(c+dx)\right)}{(a+b)^3} + \frac{\sec^2\left(\frac{1}{2}(c+dx)\right)}{(b-a)^3} - \cdots}{8d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]/(a*Sin[c + d*x] + b*Tan[c + d*x])^3,x]
```

```
[Out] ((4*b^4)/(a*(a - b)^2*(a + b)^2*(b + a*Cos[c + d*x])^2) + (32*a*b^3)/((-a +
 b)^3*(a + b)^3*(b + a*Cos[c + d*x])) - Csc[(c + d*x)/2]^2/(a + b)^3 + (12*
b*Log[Cos[(c + d*x)/2]])/(a - b)^4 - (48*a*b^2*(a^2 + b^2)*Log[b + a*Cos[c
+ d*x]])/(a^2 - b^2)^4 - (12*b*Log[Sin[(c + d*x)/2]])/(a + b)^4 + Sec[(c +
d*x)/2]^2/(-a + b)^3)/(8*d)
```

fricas [B] time = 0.74, size = 994, normalized size = 4.71

$$\frac{2a^6b^2 + 18a^4b^4 - 18a^2b^6 - 2b^8 - 6\left(a^7b + 2a^5b^3 - 3a^3b^5\right)\cos(dx+c)^3 + 2\left(a^8 - 4a^6b^2 - 6a^4b^4 + 8a^2b^6 + b^8\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a*sin(d*x+c)+b*tan(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/4*(2*a^6*b^2 + 18*a^4*b^4 - 18*a^2*b^6 - 2*b^8 - 6*(a^7*b + 2*a^5*b^3 - 3
*a^3*b^5)*cos(d*x + c)^3 + 2*(a^8 - 4*a^6*b^2 - 6*a^4*b^4 + 8*a^2*b^6 + b^8
)*cos(d*x + c)^2 + 2*(2*a^7*b + 9*a^5*b^3 - 12*a^3*b^5 + a*b^7)*cos(d*x + c
) + 24*(a^4*b^4 + a^2*b^6 - (a^6*b^2 + a^4*b^4)*cos(d*x + c)^4 - 2*(a^5*b^3
 + a^3*b^5)*cos(d*x + c)^3 + (a^6*b^2 - a^2*b^6)*cos(d*x + c)^2 + 2*(a^5*b^
3 + a^3*b^5)*cos(d*x + c))*log(a*cos(d*x + c) + b) - 3*(a^5*b^3 + 4*a^4*b^4
 + 6*a^3*b^5 + 4*a^2*b^6 + a*b^7 - (a^7*b + 4*a^6*b^2 + 6*a^5*b^3 + 4*a^4*b
^4 + a^3*b^5)*cos(d*x + c)^4 - 2*(a^6*b^2 + 4*a^5*b^3 + 6*a^4*b^4 + 4*a^3*b
^5 + a^2*b^6)*cos(d*x + c)^3 + (a^7*b + 4*a^6*b^2 + 5*a^5*b^3 - 5*a^3*b^5 -
 4*a^2*b^6 - a*b^7)*cos(d*x + c)^2 + 2*(a^6*b^2 + 4*a^5*b^3 + 6*a^4*b^4 + 4
*a^3*b^5 + a^2*b^6)*cos(d*x + c))*log(1/2*cos(d*x + c) + 1/2) + 3*(a^5*b^3
- 4*a^4*b^4 + 6*a^3*b^5 - 4*a^2*b^6 + a*b^7 - (a^7*b - 4*a^6*b^2 + 6*a^5*b^
3 - 4*a^4*b^4 + a^3*b^5)*cos(d*x + c)^4 - 2*(a^6*b^2 - 4*a^5*b^3 + 6*a^4*b^
4 - 4*a^3*b^5 + a^2*b^6)*cos(d*x + c)^3 + (a^7*b - 4*a^6*b^2 + 5*a^5*b^3 -
5*a^3*b^5 + 4*a^2*b^6 - a*b^7)*cos(d*x + c)^2 + 2*(a^6*b^2 - 4*a^5*b^3 + 6*
a^4*b^4 - 4*a^3*b^5 + a^2*b^6)*cos(d*x + c))*log(-1/2*cos(d*x + c) + 1/2))/
((a^11 - 4*a^9*b^2 + 6*a^7*b^4 - 4*a^5*b^6 + a^3*b^8)*d*cos(d*x + c)^4 + 2*
(a^10*b - 4*a^8*b^3 + 6*a^6*b^5 - 4*a^4*b^7 + a^2*b^9)*d*cos(d*x + c)^3 - (
a^11 - 5*a^9*b^2 + 10*a^7*b^4 - 10*a^5*b^6 + 5*a^3*b^8 - a*b^10)*d*cos(d*x
+ c)^2 - 2*(a^10*b - 4*a^8*b^3 + 6*a^6*b^5 - 4*a^4*b^7 + a^2*b^9)*d*cos(d*x
 + c) - (a^9*b^2 - 4*a^7*b^4 + 6*a^5*b^6 - 4*a^3*b^8 + a*b^10)*d)
```

giac [B] time = 1.44, size = 690, normalized size = 3.27

$$-\frac{\frac{6\,b\log\left(\frac{|-\cos(dx+c)+1|}{|\cos(dx+c)+1|}\right)}{a^4+4\,a^3b+6\,a^2b^2+4\,ab^3+b^4}+\frac{48\left(a^3b^2+ab^4\right)\log\left(\left|-a-b-\frac{a(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{b(\cos(dx+c)-1)}{\cos(dx+c)+1}\right|\right)}{a^8-4\,a^6b^2+6\,a^4b^4-4\,a^2b^6+b^8}-\frac{\left(a+b+\frac{6\,b(\cos(dx+c)-1)}{\cos(dx+c)+1}\right)(\cos(dx+c)+1)}{\left(a^4+4\,a^3b+6\,a^2b^2+4\,ab^3+b^4\right)(\cos(dx+c)-1)}-\frac{\cdots}{\left(a^3-3\,a^2\cdots\right.}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a*sin(d*x+c)+b*tan(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -1/8*(6*b*log(abs(-cos(d*x + c) + 1)/abs(cos(d*x + c) + 1))/(a^4 + 4*a^3*b
+ 6*a^2*b^2 + 4*a*b^3 + b^4) + 48*(a^3*b^2 + a*b^4)*log(abs(-a - b - a*(cos
(d*x + c) - 1)/(cos(d*x + c) + 1) + b*(cos(d*x + c) - 1)/(cos(d*x + c) + 1)
))/(a^8 - 4*a^6*b^2 + 6*a^4*b^4 - 4*a^2*b^6 + b^8) - (a + b + 6*b*(cos(d*x
+ c) - 1)/(cos(d*x + c) + 1))*(cos(d*x + c) + 1)/((a^4 + 4*a^3*b + 6*a^2*b^
2 + 4*a*b^3 + b^4)*(cos(d*x + c) - 1)) - (cos(d*x + c) - 1)/((a^3 - 3*a^2*b
 + 3*a*b^2 - b^3)*(cos(d*x + c) + 1)) - 8*(9*a^5*b^2 + 10*a^4*b^3 + 2*a^3*b
^4 + 8*a^2*b^5 + 5*a*b^6 - 2*b^7 + 18*a^5*b^2*(cos(d*x + c) - 1)/(cos(d*x +
 c) + 1) - 8*a^4*b^3*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 2*a^3*b^4*(cos
```

```
(d*x + c) - 1)/(cos(d*x + c) + 1) + 6*a^2*b^5*(cos(d*x + c) - 1)/(cos(d*x +
 c) + 1) - 16*a*b^6*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 2*b^7*(cos(d*x
+ c) - 1)/(cos(d*x + c) + 1) + 9*a^5*b^2*(cos(d*x + c) - 1)^2/(cos(d*x + c)
 + 1)^2 - 18*a^4*b^3*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 + 18*a^3*b^4
*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 - 18*a^2*b^5*(cos(d*x + c) - 1)^
2/(cos(d*x + c) + 1)^2 + 9*a*b^6*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2)
/((a^8 - 4*a^6*b^2 + 6*a^4*b^4 - 4*a^2*b^6 + b^8)*(a + b + a*(cos(d*x + c)
- 1)/(cos(d*x + c) + 1) - b*(cos(d*x + c) - 1)/(cos(d*x + c) + 1))^2))/d
```

maple [A] time = 0.19, size = 220, normalized size = 1.04

$$\frac{b^4}{2d\left(a+b\right)^2\left(a-b\right)^2 a\left(b+a\cos\left(dx+c\right)\right)^2}-\frac{4a\,b^3}{d\left(a+b\right)^3\left(a-b\right)^3\left(b+a\cos\left(dx+c\right)\right)}-\frac{6a^3b^2\ln\left(b+a\cos\left(dx+c\right)\right)}{d\left(a+b\right)^4\left(a-b\right)^4}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)/(a*sin(d*x+c)+b*tan(d*x+c))^3,x)
```

```
[Out] 1/2/d*b^4/(a+b)^2/(a-b)^2/a/(b+a*cos(d*x+c))^2-4/d*a*b^3/(a+b)^3/(a-b)^3/(b
+a*cos(d*x+c))-6/d*a^3*b^2/(a+b)^4/(a-b)^4*ln(b+a*cos(d*x+c))-6/d*b^4/(a+b)
^4/(a-b)^4*a*ln(b+a*cos(d*x+c))+1/4/d/(a+b)^3/(cos(d*x+c)-1)-3/4/d/(a+b)^4*
ln(cos(d*x+c)-1)*b-1/4/d/(a-b)^3/(1+cos(d*x+c))+3/4*b*ln(1+cos(d*x+c))/(a-b
)^4/d
```

maxima [B] time = 0.62, size = 593, normalized size = 2.81

$$-\frac{\frac{48\left(a^3b^2+ab^4\right)\log\left(a+b-\frac{(a-b)\sin(dx+c)^2}{(\cos(dx+c)+1)^2}\right)}{a^8-4a^6b^2+6a^4b^4-4a^2b^6+b^8}+\frac{12\,b\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^4+4a^3b+6a^2b^2+4ab^3+b^4}+\frac{a^6-2a^5b-a^4b^2+4a^3b^3-a^2b^4-2ab^5+b^6-\frac{2\left(a^6-4a^5\right.}{}}{\frac{\left(a^9+a^8b-4a^7b^2-4a^6b^3+6a^5b^4+6a^4b^5-4a^3b^6-4a^2b^7+ab^8+b^9\right)\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-\frac{2\left(a\right.}{}}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a*sin(d*x+c)+b*tan(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -1/8*(48*(a^3*b^2 + a*b^4)*log(a + b - (a - b)*sin(d*x + c)^2/(cos(d*x + c)
 + 1)^2)/(a^8 - 4*a^6*b^2 + 6*a^4*b^4 - 4*a^2*b^6 + b^8) + 12*b*log(sin(d*x
 + c)/(cos(d*x + c) + 1))/(a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4) + (a^
6 - 2*a^5*b - a^4*b^2 + 4*a^3*b^3 - a^2*b^4 - 2*a*b^5 + b^6 - 2*(a^6 - 4*a^
5*b + 5*a^4*b^2 - 32*a^3*b^3 - 37*a^2*b^4 - 4*a*b^5 - 9*b^6)*sin(d*x + c)^2
/(cos(d*x + c) + 1)^2 + (a^6 - 6*a^5*b + 15*a^4*b^2 - 84*a^3*b^3 + 63*a^2*b
^4 - 6*a*b^5 + 17*b^6)*sin(d*x + c)^4/(cos(d*x + c) + 1)^4)/((a^9 + a^8*b -
 4*a^7*b^2 - 4*a^6*b^3 + 6*a^5*b^4 + 6*a^4*b^5 - 4*a^3*b^6 - 4*a^2*b^7 + a*
b^8 + b^9)*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 - 2*(a^9 - a^8*b - 4*a^7*b^2
 + 4*a^6*b^3 + 6*a^5*b^4 - 6*a^4*b^5 - 4*a^3*b^6 + 4*a^2*b^7 + a*b^8 - b^9)
*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + (a^9 - 3*a^8*b + 8*a^6*b^3 - 6*a^5*b
```

```
^4 - 6*a^4*b^5 + 8*a^3*b^6 - 3*a*b^8 + b^9)*sin(d*x + c)^6/(cos(d*x + c) +
1)^6) + sin(d*x + c)^2/((a^3 - 3*a^2*b + 3*a*b^2 - b^3)*(cos(d*x + c) + 1)^
2))/d
```

mupad [B] time = 1.11, size = 490, normalized size = 2.32

$$\frac{\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\,\left(a^5-5\,a^4\,b+10\,a^3\,b^2-42\,a^2\,b^3+5\,a\,b^4-9\,b^5\right)}{(a-b)\,\left(a^2+2\,a\,b+b^2\right)}-\frac{a^3-3\,a^2\,b+3\,a\,b^2-b^3}{2\,(a+b)}+\cdots}{d\,\left(\left(4\,a^5-20\,a^4\,b+40\,a^3\,b^2-40\,a^2\,b^3+20\,a\,b^4-4\,b^5\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6+\left(-8\,a^5+24\,a^4\,b-16\,a^3\,b^2-16\,a^2\,b^3+2\cdots\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)/(a*sin(c + d*x) + b*tan(c + d*x))^3,x)
```

```
[Out] ((tan(c/2 + (d*x)/2)^2*(5*a*b^4 - 5*a^4*b + a^5 - 9*b^5 - 42*a^2*b^3 + 10*a
^3*b^2))/((a - b)*(2*a*b + a^2 + b^2)) - (3*a*b^2 - 3*a^2*b + a^3 - b^3)/(2
*(a + b)) + (tan(c/2 + (d*x)/2)^4*(11*a*b^4 + 5*a^4*b - a^5 + 17*b^5 + 74*a
^2*b^3 - 10*a^3*b^2))/(2*(a + b)*(2*a*b + a^2 + b^2)))/(d*(tan(c/2 + (d*x)/
2)^2*(4*a*b^4 - 4*a^4*b + 4*a^5 - 4*b^5 + 8*a^2*b^3 - 8*a^3*b^2) - tan(c/2
+ (d*x)/2)^4*(8*a^5 - 24*a^4*b - 24*a*b^4 + 8*b^5 + 16*a^2*b^3 + 16*a^3*b^2
) + tan(c/2 + (d*x)/2)^6*(20*a*b^4 - 20*a^4*b + 4*a^5 - 4*b^5 - 40*a^2*b^3
+ 40*a^3*b^2))) - tan(c/2 + (d*x)/2)^2/(8*d*(a - b)^3) - (log(a + b - a*tan
(c/2 + (d*x)/2)^2 + b*tan(c/2 + (d*x)/2)^2)*(6*a*b^4 + 6*a^3*b^2))/(d*(a^8
+ b^8 - 4*a^2*b^6 + 6*a^4*b^4 - 4*a^6*b^2)) - (3*b*log(tan(c/2 + (d*x)/2)))
/(d*(8*a*b^3 + 8*a^3*b + 2*a^4 + 2*b^4 + 12*a^2*b^2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos(c + dx)}{(a\sin(c + dx) + b\tan(c + dx))^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a*sin(d*x+c)+b*tan(d*x+c))**3,x)
```

```
[Out] Integral(cos(c + d*x)/(a*sin(c + d*x) + b*tan(c + d*x))**3, x)
```

3.267 $\int \frac{1}{(a\sin(c+dx)+b\tan(c+dx))^3}\,dx$

Optimal. Leaf size=229

$$\frac{b^2\left(3a^2+b^2\right)}{d\left(a^2-b^2\right)^3(a\cos(c+dx)+b)}+\frac{\csc^2(c+dx)\left(b\left(3a^2+b^2\right)-a\left(a^2+3b^2\right)\cos(c+dx)\right)}{2d\left(a^2-b^2\right)^3}-\frac{b^3}{2d\left(a^2-b^2\right)^2(a\cos(c+\right.}$$

```
[Out] -1/2*b^3/(a^2-b^2)^2/d/(b+a*cos(d*x+c))^2+b^2*(3*a^2+b^2)/(a^2-b^2)^3/d/(b+
a*cos(d*x+c))+1/2*(b*(3*a^2+b^2)-a*(a^2+3*b^2)*cos(d*x+c))*csc(d*x+c)^2/(a^
2-b^2)^3/d+1/4*(a-2*b)*ln(1-cos(d*x+c))/(a+b)^4/d-1/4*(a+2*b)*ln(1+cos(d*x+
c))/(a-b)^4/d+b*(3*a^4+8*a^2*b^2+b^4)*ln(b+a*cos(d*x+c))/(a^2-b^2)^4/d
```

Rubi [A] time = 0.48, antiderivative size = 229, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.210, Rules used = {4397, 2721, 1647, 1629}

$$-\frac{b^3}{2d\left(a^2-b^2\right)^2(a\cos(c+dx)+b)^2}+\frac{b^2\left(3a^2+b^2\right)}{d\left(a^2-b^2\right)^3(a\cos(c+dx)+b)}+\frac{b\left(8a^2b^2+3a^4+b^4\right)\log(a\cos(c+dx)+b}{d\left(a^2-b^2\right)^4}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Sin[c + d*x] + b*Tan[c + d*x])^(-3),x]
```

```
[Out] -b^3/(2*(a^2 - b^2)^2*d*(b + a*Cos[c + d*x])^2) + (b^2*(3*a^2 + b^2))/((a^2
 - b^2)^3*d*(b + a*Cos[c + d*x])) + ((b*(3*a^2 + b^2) - a*(a^2 + 3*b^2)*Cos
[c + d*x])*Csc[c + d*x]^2)/(2*(a^2 - b^2)^3*d) + ((a - 2*b)*Log[1 - Cos[c +
 d*x]])/(4*(a + b)^4*d) - ((a + 2*b)*Log[1 + Cos[c + d*x]])/(4*(a - b)^4*d)
 + (b*(3*a^4 + 8*a^2*b^2 + b^4)*Log[b + a*Cos[c + d*x]])/((a^2 - b^2)^4*d)
```

Rule 1629

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol]
:> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
 d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 1647

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[(d + e*x)^m*Pq, a + c*x^2, x], f = Coeff[Pol
ynomialRemainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 0], g = Coeff[Polynomial
Remainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 1]}, Simp[((a*g - c*f*x)*(a + c
*x^2)^(p + 1))/(2*a*c*(p + 1)), x] + Dist[1/(2*a*c*(p + 1)), Int[(d + e*x)^
m*(a + c*x^2)^(p + 1)*ExpandToSum[(2*a*c*(p + 1)*Q)/(d + e*x)^m + (c*f*(2*p
 + 3))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, c, d, e}, x] && PolyQ[Pq, x] &
```

```
& NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 2721

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Dist[1/f, Subst[Int[(x^p*(a + x)^m)/(b^2 - x^2)^((p + 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b^
2, 0] && IntegerQ[(p + 1)/2]
```

Rule 4397

```
Int[u_, x_Symbol] :> Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]
```

Rubi steps

$$\int \frac{1}{(a\sin(c+dx)+b\tan(c+dx))^3}\,dx = \int \frac{\cot^3(c+dx)}{(b+a\cos(c+dx))^3}\,dx$$

$$= -\frac{\text{Subst}\left(\int \frac{x^3}{(b+x)^3\left(a^2-x^2\right)^2}\,dx, x, a\cos(c+dx)\right)}{d}$$

$$= \frac{\left(b\left(3a^2+b^2\right)-a\left(a^2+3b^2\right)\cos(c+dx)\right)\csc^2(c+dx)}{2\left(a^2-b^2\right)^3 d} - \frac{\text{Subst}\left(\int \frac{-\frac{a^4b^3(a\cdots}{(a^2-\cdots}}{\cdots}\right.}{\cdots}$$

$$= \frac{\left(b\left(3a^2+b^2\right)-a\left(a^2+3b^2\right)\cos(c+dx)\right)\csc^2(c+dx)}{2\left(a^2-b^2\right)^3 d} - \frac{\text{Subst}\left(\int\left(\frac{a^2(a\cdots}{2(a+b)\cdots}\right.\right.}{\cdots}$$

$$= -\frac{b^3}{2\left(a^2-b^2\right)^2 d(b+a\cos(c+dx))^2} + \frac{b^2\left(3a^2+b^2\right)}{\left(a^2-b^2\right)^3 d(b+a\cos(c+dx))} + \cdots$$

Mathematica [C] time = 6.32, size = 696, normalized size = 3.04

$$-\frac{b^2\left(3a^2+b^2\right)\tan^3(c+dx)(a\cos(c+dx)+b)^2}{d(b-a)^3(a+b)^3(a\sin(c+dx)+b\tan(c+dx))^3} - \frac{2i\left(3a^4b+8a^2b^3+b^5\right)(c+dx)\tan^3(c+dx)(a\cos(c+dx)+b\cdots}{d(a-b)^4(a+b)^4(a\sin(c+dx)+b\tan(c+dx))^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Sin[c + d*x] + b*Tan[c + d*x])^(-3),x]
```

```
[Out] -1/2*(b^3*(b + a*Cos[c + d*x])*Tan[c + d*x]^3)/((-a + b)^2*(a + b)^2*d*(a*S
in[c + d*x] + b*Tan[c + d*x])^3) - (b^2*(3*a^2 + b^2)*(b + a*Cos[c + d*x])^
2*Tan[c + d*x]^3)/((-a + b)^3*(a + b)^3*d*(a*Sin[c + d*x] + b*Tan[c + d*x])
^3) - ((2*I)*(3*a^4*b + 8*a^2*b^3 + b^5)*(c + d*x)*(b + a*Cos[c + d*x])^3*T
an[c + d*x]^3)/((a - b)^4*(a + b)^4*d*(a*Sin[c + d*x] + b*Tan[c + d*x])^3)
- ((I/2)*(-a - 2*b)*ArcTan[Tan[c + d*x]]*(b + a*Cos[c + d*x])^3*Tan[c + d*x
]^3)/((-a + b)^4*d*(a*Sin[c + d*x] + b*Tan[c + d*x])^3) - ((I/2)*(a - 2*b)*
ArcTan[Tan[c + d*x]]*(b + a*Cos[c + d*x])^3*Tan[c + d*x]^3)/((a + b)^4*d*(a
*Sin[c + d*x] + b*Tan[c + d*x])^3) - ((b + a*Cos[c + d*x])^3*Csc[(c + d*x)/
2]^2*Tan[c + d*x]^3)/(8*(a + b)^3*d*(a*Sin[c + d*x] + b*Tan[c + d*x])^3) +
((-a - 2*b)*(b + a*Cos[c + d*x])^3*Log[Cos[(c + d*x)/2]^2]*Tan[c + d*x]^3)/
(4*(-a + b)^4*d*(a*Sin[c + d*x] + b*Tan[c + d*x])^3) + ((3*a^4*b + 8*a^2*b^
3 + b^5)*(b + a*Cos[c + d*x])^3*Log[b + a*Cos[c + d*x]]*Tan[c + d*x]^3)/((-
a^2 + b^2)^4*d*(a*Sin[c + d*x] + b*Tan[c + d*x])^3) + ((a - 2*b)*(b + a*Cos
[c + d*x])^3*Log[Sin[(c + d*x)/2]^2]*Tan[c + d*x]^3)/(4*(a + b)^4*d*(a*Sin[
c + d*x] + b*Tan[c + d*x])^3) - ((b + a*Cos[c + d*x])^3*Sec[(c + d*x)/2]^2*
Tan[c + d*x]^3)/(8*(-a + b)^3*d*(a*Sin[c + d*x] + b*Tan[c + d*x])^3)
```

fricas [B] time = 0.97, size = 1071, normalized size = 4.68

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sin(d*x+c)+b*tan(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] -1/4*(16*a^4*b^3 - 8*a^2*b^5 - 8*b^7 - 2*(a^7 + 8*a^5*b^2 - 7*a^3*b^4 - 2*a
*b^6)*cos(d*x + c)^3 + 2*(a^6*b - 11*a^4*b^3 + 7*a^2*b^5 + 3*b^7)*cos(d*x +
 c)^2 + 2*(11*a^5*b^2 - 10*a^3*b^4 - a*b^6)*cos(d*x + c) + 4*(3*a^4*b^3 + 8
*a^2*b^5 + b^7 - (3*a^6*b + 8*a^4*b^3 + a^2*b^5)*cos(d*x + c)^4 - 2*(3*a^5*
b^2 + 8*a^3*b^4 + a*b^6)*cos(d*x + c)^3 + (3*a^6*b + 5*a^4*b^3 - 7*a^2*b^5
- b^7)*cos(d*x + c)^2 + 2*(3*a^5*b^2 + 8*a^3*b^4 + a*b^6)*cos(d*x + c))*log
(a*cos(d*x + c) + b) - (a^5*b^2 + 6*a^4*b^3 + 14*a^3*b^4 + 16*a^2*b^5 + 9*a
*b^6 + 2*b^7 - (a^7 + 6*a^6*b + 14*a^5*b^2 + 16*a^4*b^3 + 9*a^3*b^4 + 2*a^2
*b^5)*cos(d*x + c)^4 - 2*(a^6*b + 6*a^5*b^2 + 14*a^4*b^3 + 16*a^3*b^4 + 9*a
^2*b^5 + 2*a*b^6)*cos(d*x + c)^3 + (a^7 + 6*a^6*b + 13*a^5*b^2 + 10*a^4*b^3
 - 5*a^3*b^4 - 14*a^2*b^5 - 9*a*b^6 - 2*b^7)*cos(d*x + c)^2 + 2*(a^6*b + 6*
a^5*b^2 + 14*a^4*b^3 + 16*a^3*b^4 + 9*a^2*b^5 + 2*a*b^6)*cos(d*x + c))*log(
1/2*cos(d*x + c) + 1/2) + (a^5*b^2 - 6*a^4*b^3 + 14*a^3*b^4 - 16*a^2*b^5 +
9*a*b^6 - 2*b^7 - (a^7 - 6*a^6*b + 14*a^5*b^2 - 16*a^4*b^3 + 9*a^3*b^4 - 2*
a^2*b^5)*cos(d*x + c)^4 - 2*(a^6*b - 6*a^5*b^2 + 14*a^4*b^3 - 16*a^3*b^4 +
9*a^2*b^5 - 2*a*b^6)*cos(d*x + c)^3 + (a^7 - 6*a^6*b + 13*a^5*b^2 - 10*a^4*
b^3 - 5*a^3*b^4 + 14*a^2*b^5 - 9*a*b^6 + 2*b^7)*cos(d*x + c)^2 + 2*(a^6*b -
 6*a^5*b^2 + 14*a^4*b^3 - 16*a^3*b^4 + 9*a^2*b^5 - 2*a*b^6)*cos(d*x + c))*l
```

```
og(-1/2*cos(d*x + c) + 1/2))/((a^10 - 4*a^8*b^2 + 6*a^6*b^4 - 4*a^4*b^6 + a
^2*b^8)*d*cos(d*x + c)^4 + 2*(a^9*b - 4*a^7*b^3 + 6*a^5*b^5 - 4*a^3*b^7 + a
*b^9)*d*cos(d*x + c)^3 - (a^10 - 5*a^8*b^2 + 10*a^6*b^4 - 10*a^4*b^6 + 5*a^
2*b^8 - b^10)*d*cos(d*x + c)^2 - 2*(a^9*b - 4*a^7*b^3 + 6*a^5*b^5 - 4*a^3*b
^7 + a*b^9)*d*cos(d*x + c) - (a^8*b^2 - 4*a^6*b^4 + 6*a^4*b^6 - 4*a^2*b^8 +
 b^10)*d)
```

giac [B] time = 0.42, size = 800, normalized size = 3.49

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sin(d*x+c)+b*tan(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/8*(2*(a - 2*b)*log(abs(-cos(d*x + c) + 1)/abs(cos(d*x + c) + 1))/(a^4 + 4
*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4) + 8*(3*a^4*b + 8*a^2*b^3 + b^5)*log(abs
(-a - b - a*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + b*(cos(d*x + c) - 1)/(c
os(d*x + c) + 1)))/(a^8 - 4*a^6*b^2 + 6*a^4*b^4 - 4*a^2*b^6 + b^8) + (a + b
 - 2*a*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 4*b*(cos(d*x + c) - 1)/(cos(
d*x + c) + 1))*(cos(d*x + c) + 1)/((a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b
^4)*(cos(d*x + c) - 1)) - (cos(d*x + c) - 1)/((a^3 - 3*a^2*b + 3*a*b^2 - b^
3)*(cos(d*x + c) + 1)) - 4*(9*a^6*b + 6*a^5*b^2 + 9*a^4*b^3 + 28*a^3*b^4 +
11*a^2*b^5 - 2*a*b^6 + 3*b^7 + 18*a^6*b*(cos(d*x + c) - 1)/(cos(d*x + c) +
1) - 12*a^5*b^2*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 26*a^4*b^3*(cos(d*x
 + c) - 1)/(cos(d*x + c) + 1) + 4*a^3*b^4*(cos(d*x + c) - 1)/(cos(d*x + c)
+ 1) - 38*a^2*b^5*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 8*a*b^6*(cos(d*x
+ c) - 1)/(cos(d*x + c) + 1) - 6*b^7*(cos(d*x + c) - 1)/(cos(d*x + c) + 1)
+ 9*a^6*b*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 - 18*a^5*b^2*(cos(d*x +
 c) - 1)^2/(cos(d*x + c) + 1)^2 + 33*a^4*b^3*(cos(d*x + c) - 1)^2/(cos(d*x
+ c) + 1)^2 - 48*a^3*b^4*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 + 27*a^2
*b^5*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 - 6*a*b^6*(cos(d*x + c) - 1)
^2/(cos(d*x + c) + 1)^2 + 3*b^7*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2)/
((a^8 - 4*a^6*b^2 + 6*a^4*b^4 - 4*a^2*b^6 + b^8)*(a + b + a*(cos(d*x + c) -
 1)/(cos(d*x + c) + 1) - b*(cos(d*x + c) - 1)/(cos(d*x + c) + 1))^2))/d
```

maple [A] time = 0.20, size = 322, normalized size = 1.41

$$-\frac{b^3}{2d\left(a+b\right)^2\left(a-b\right)^2\left(b+a\cos\left(dx+c\right)\right)^2}+\frac{3b\ln\left(b+a\cos\left(dx+c\right)\right)a^4}{d\left(a+b\right)^4\left(a-b\right)^4}+\frac{8b^3\ln\left(b+a\cos\left(dx+c\right)\right)a^2}{d\left(a+b\right)^4\left(a-b\right)^4}+\frac{b^5\ln\left(b+\right.}{d\left(a+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*sin(d*x+c)+b*tan(d*x+c))^3,x)
```

```
[Out] -1/2/d*b^3/(a+b)^2/(a-b)^2/(b+a*cos(d*x+c))^2+3/d*b/(a+b)^4/(a-b)^4*ln(b+a*
cos(d*x+c))*a^4+8/d*b^3/(a+b)^4/(a-b)^4*ln(b+a*cos(d*x+c))*a^2+1/d*b^5/(a+b
```

```
)^4/(a-b)^4*ln(b+a*cos(d*x+c))+3/d*b^2/(a+b)^3/(a-b)^3/(b+a*cos(d*x+c))*a^2
+1/d*b^4/(a+b)^3/(a-b)^3/(b+a*cos(d*x+c))+1/4/d/(a+b)^3/(cos(d*x+c)-1)+1/4/
d/(a+b)^4*ln(cos(d*x+c)-1)*a-1/2/d/(a+b)^4*ln(cos(d*x+c)-1)*b+1/4/d/(a-b)^3
/(1+cos(d*x+c))-1/4*a*ln(1+cos(d*x+c))/(a-b)^4/d-1/2*b*ln(1+cos(d*x+c))/(a-
b)^4/d
```

maxima [B] time = 0.42, size = 601, normalized size = 2.62

$$\frac{\frac{8\left(3\,a^4b+8\,a^2b^3+b^5\right)\log\left(a+b-\frac{(a-b)\sin(dx+c)^2}{(\cos(dx+c)+1)^2}\right)}{a^8-4\,a^6b^2+6\,a^4b^4-4\,a^2b^6+b^8}+\frac{4\,(a-2\,b)\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^4+4\,a^3b+6\,a^2b^2+4\,ab^3+b^4}-\frac{a^6-2\,a^5b-a^4b^2+4\,a^3b^3-a^2b^4-2\,ab^5+b^6-\frac{2\,(a^6\cdots}{}}{\frac{\left(a^9+a^8b-4\,a^7b^2-4\,a^6b^3+6\,a^5b^4+6\,a^4b^5-4\,a^3b^6-4\,a^2b^7+ab^8+b^9\right)\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-\cdots}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sin(d*x+c)+b*tan(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/8*(8*(3*a^4*b + 8*a^2*b^3 + b^5)*log(a + b - (a - b)*sin(d*x + c)^2/(cos(
d*x + c) + 1)^2)/(a^8 - 4*a^6*b^2 + 6*a^4*b^4 - 4*a^2*b^6 + b^8) + 4*(a - 2
*b)*log(sin(d*x + c)/(cos(d*x + c) + 1))/(a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b
^3 + b^4) - (a^6 - 2*a^5*b - a^4*b^2 + 4*a^3*b^3 - a^2*b^4 - 2*a*b^5 + b^6
- 2*(a^6 - 4*a^5*b + 29*a^4*b^2 + 24*a^3*b^3 + 11*a^2*b^4 + 20*a*b^5 - b^6)
*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + (a^6 - 6*a^5*b + 63*a^4*b^2 - 52*a^3
*b^3 + 31*a^2*b^4 - 38*a*b^5 + b^6)*sin(d*x + c)^4/(cos(d*x + c) + 1)^4)/((
a^9 + a^8*b - 4*a^7*b^2 - 4*a^6*b^3 + 6*a^5*b^4 + 6*a^4*b^5 - 4*a^3*b^6 - 4
*a^2*b^7 + a*b^8 + b^9)*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 - 2*(a^9 - a^8*
b - 4*a^7*b^2 + 4*a^6*b^3 + 6*a^5*b^4 - 6*a^4*b^5 - 4*a^3*b^6 + 4*a^2*b^7 +
 a*b^8 - b^9)*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + (a^9 - 3*a^8*b + 8*a^6*
b^3 - 6*a^5*b^4 - 6*a^4*b^5 + 8*a^3*b^6 - 3*a*b^8 + b^9)*sin(d*x + c)^6/(co
s(d*x + c) + 1)^6) + sin(d*x + c)^2/((a^3 - 3*a^2*b + 3*a*b^2 - b^3)*(cos(d
*x + c) + 1)^2))/d
```

mupad [B] time = 1.13, size = 494, normalized size = 2.16

$$\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{8\,d\,(a-b)^3}-\frac{\frac{a^3-3\,a^2\,b+3\,a\,b^2-b^3}{2\,(a+b)}+\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4\left(a^5-5\,a^4\,b+58\,a^3\,b^2+\cdots\right.}{2\,(a+b)\left(a^2+2\,a\,b+\cdots\right.}}{d\left(\left(4\,a^5-20\,a^4\,b+40\,a^3\,b^2-40\,a^2\,b^3+20\,a\,b^4-4\,b^5\right)\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6+\left(-8\,a^5+24\,a^4\,b-16\,a^{\cdots}\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*sin(c + d*x) + b*tan(c + d*x))^3,x)
```

```
[Out] tan(c/2 + (d*x)/2)^2/(8*d*(a - b)^3) - ((3*a*b^2 - 3*a^2*b + a^3 - b^3)/(2*
(a + b)) + (tan(c/2 + (d*x)/2)^4*(37*a*b^4 - 5*a^4*b + a^5 - b^5 + 6*a^2*b^
```

```
3 + 58*a^3*b^2))/(2*(a + b)*(2*a*b + a^2 + b^2)) - (tan(c/2 + (d*x)/2)^2*(2
1*a*b^4 - 5*a^4*b + a^5 - b^5 - 10*a^2*b^3 + 34*a^3*b^2))/((a - b)*(2*a*b +
 a^2 + b^2)))/(d*(tan(c/2 + (d*x)/2)^2*(4*a*b^4 - 4*a^4*b + 4*a^5 - 4*b^5 +
 8*a^2*b^3 - 8*a^3*b^2) - tan(c/2 + (d*x)/2)^4*(8*a^5 - 24*a^4*b - 24*a*b^4
 + 8*b^5 + 16*a^2*b^3 + 16*a^3*b^2) + tan(c/2 + (d*x)/2)^6*(20*a*b^4 - 20*a
^4*b + 4*a^5 - 4*b^5 - 40*a^2*b^3 + 40*a^3*b^2))) + (log(tan(c/2 + (d*x)/2)
)*(a - 2*b))/(d*(8*a*b^3 + 8*a^3*b + 2*a^4 + 2*b^4 + 12*a^2*b^2)) + (log(a
+ b - a*tan(c/2 + (d*x)/2)^2 + b*tan(c/2 + (d*x)/2)^2)*(3*a^4*b + b^5 + 8*a
^2*b^3))/(d*(a^8 + b^8 - 4*a^2*b^6 + 6*a^4*b^4 - 4*a^6*b^2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a\sin\left(c+dx\right)+b\tan\left(c+dx\right)\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sin(d*x+c)+b*tan(d*x+c))**3,x)
```

```
[Out] Integral((a*sin(c + d*x) + b*tan(c + d*x))**(-3), x)
```

3.268 $\int \frac{\sec(c+dx)}{(a\sin(c+dx)+b\tan(c+dx))^3}\,dx$

Optimal. Leaf size=231

$$\frac{ab^2}{2d\left(a^2-b^2\right)^2(a\cos(c+dx)+b)^2}-\frac{2ab\left(a^2+b^2\right)}{d\left(a^2-b^2\right)^3(a\cos(c+dx)+b)}-\frac{\csc^2(c+dx)\left(a\left(a^2+3b^2\right)-b\left(3a^2+b^2\right)\cos\right.}{2d\left(a^2-b^2\right)^3}$$

```
[Out] 1/2*a*b^2/(a^2-b^2)^2/d/(b+a*cos(d*x+c))^2-2*a*b*(a^2+b^2)/(a^2-b^2)^3/d/(b
+a*cos(d*x+c))-1/2*(a*(a^2+3*b^2)-b*(3*a^2+b^2)*cos(d*x+c))*csc(d*x+c)^2/(a
^2-b^2)^3/d+1/4*(2*a-b)*ln(1-cos(d*x+c))/(a+b)^4/d+1/4*(2*a+b)*ln(1+cos(d*x
+c))/(a-b)^4/d-a*(a^4+8*a^2*b^2+3*b^4)*ln(b+a*cos(d*x+c))/(a^2-b^2)^4/d
```

Rubi [A] time = 0.62, antiderivative size = 231, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {4397, 2837, 12, 1647, 1629}

$$\frac{ab^2}{2d\left(a^2-b^2\right)^2(a\cos(c+dx)+b)^2}-\frac{2ab\left(a^2+b^2\right)}{d\left(a^2-b^2\right)^3(a\cos(c+dx)+b)}-\frac{a\left(8a^2b^2+a^4+3b^4\right)\log(a\cos(c+dx)+b)}{d\left(a^2-b^2\right)^4}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]/(a*Sin[c + d*x] + b*Tan[c + d*x])^3,x]
```

```
[Out] (a*b^2)/(2*(a^2 - b^2)^2*d*(b + a*Cos[c + d*x])^2) - (2*a*b*(a^2 + b^2))/((
a^2 - b^2)^3*d*(b + a*Cos[c + d*x])) - ((a*(a^2 + 3*b^2) - b*(3*a^2 + b^2)*
Cos[c + d*x])*Csc[c + d*x]^2)/(2*(a^2 - b^2)^3*d) + ((2*a - b)*Log[1 - Cos[
c + d*x]])/(4*(a + b)^4*d) + ((2*a + b)*Log[1 + Cos[c + d*x]])/(4*(a - b)^4
*d) - (a*(a^4 + 8*a^2*b^2 + 3*b^4)*Log[b + a*Cos[c + d*x]])/((a^2 - b^2)^4*
d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 1629

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol]
:> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
 d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 1647

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[(d + e*x)^m*Pq, a + c*x^2, x], f = Coeff[Pol
ynomialRemainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 0], g = Coeff[Polynomial
Remainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 1]}, Simp[((a*g - c*f*x)*(a + c
*x^2)^(p + 1))/(2*a*c*(p + 1)), x] + Dist[1/(2*a*c*(p + 1)), Int[(d + e*x)^
m*(a + c*x^2)^(p + 1)*ExpandToSum[(2*a*c*(p + 1)*Q)/(d + e*x)^m + (c*f*(2*p
 + 3))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, c, d, e}, x] && PolyQ[Pq, x] &
& NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 2837

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^m*(c + (d*x)/b)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*S
in[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/
2] && NeQ[a^2 - b^2, 0]
```

Rule 4397

```
Int[u_, x_Symbol] :> Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]
```

Rubi steps

$$\int \frac{\sec(c+dx)}{(a\sin(c+dx)+b\tan(c+dx))^3}\,dx = \int \frac{\cot^2(c+dx)\csc(c+dx)}{(b+a\cos(c+dx))^3}\,dx$$

$$= -\frac{a^3\,\text{Subst}\left(\int \frac{x^2}{a^2(b+x)^3(a^2-x^2)^2}\,dx, x, a\cos(c+dx)\right)}{d}$$

$$= -\frac{a\,\text{Subst}\left(\int \frac{x^2}{(b+x)^3(a^2-x^2)^2}\,dx, x, a\cos(c+dx)\right)}{d}$$

$$= -\frac{\left(a\left(a^2+3b^2\right)-b\left(3a^2+b^2\right)\cos(c+dx)\right)\csc^2(c+dx)}{2\left(a^2-b^2\right)^3 d} - \frac{\text{Subst}\left(\int \frac{a^2b^4 \cdots}{(\cdots}\right.}{}$$

$$= -\frac{\left(a\left(a^2+3b^2\right)-b\left(3a^2+b^2\right)\cos(c+dx)\right)\csc^2(c+dx)}{2\left(a^2-b^2\right)^3 d} - \frac{\text{Subst}\left(\int\left(\frac{\cdots}{2(\cdots}\right.\right.}{}$$

$$= \frac{ab^2}{2\left(a^2-b^2\right)^2 d(b+a\cos(c+dx))^2} - \frac{2ab\left(a^2+b^2\right)}{\left(a^2-b^2\right)^3 d(b+a\cos(c+dx))} - \cdots$$

Mathematica [C] time = 6.38, size = 703, normalized size = 3.04

$$\frac{2i\left(a^5+8a^3b^2+3ab^4\right)(c+dx)\tan^3(c+dx)(a\cos(c+dx)+b)^3}{d(a-b)^4(a+b)^4(a\sin(c+dx)+b\tan(c+dx))^3} + \frac{\left(-a^5-8a^3b^2-3ab^4\right)\tan^3(c+dx)(a\cos(c+d\cdots}{d\left(b^2-a^2\right)^4(a\sin(c+dx)+b\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]/(a*Sin[c + d*x] + b*Tan[c + d*x])^3,x]
```

```
[Out] (a*b^2*(b + a*Cos[c + d*x])*Tan[c + d*x]^3)/(2*(-a + b)^2*(a + b)^2*d*(a*Si
n[c + d*x] + b*Tan[c + d*x])^3) + (2*a*b*((-I)*a + b)*(I*a + b)*(b + a*Cos[
c + d*x])^2*Tan[c + d*x]^3)/((-a + b)^3*(a + b)^3*d*(a*Sin[c + d*x] + b*Tan
[c + d*x])^3) + ((2*I)*(a^5 + 8*a^3*b^2 + 3*a*b^4)*(c + d*x)*(b + a*Cos[c +
 d*x])^3*Tan[c + d*x]^3)/((a - b)^4*(a + b)^4*d*(a*Sin[c + d*x] + b*Tan[c +
 d*x])^3) - ((I/2)*(2*a - b)*ArcTan[Tan[c + d*x]]*(b + a*Cos[c + d*x])^3*Ta
n[c + d*x]^3)/((a + b)^4*d*(a*Sin[c + d*x] + b*Tan[c + d*x])^3) - ((I/2)*(2
*a + b)*ArcTan[Tan[c + d*x]]*(b + a*Cos[c + d*x])^3*Tan[c + d*x]^3)/((-a +
```

```
b)^4*d*(a*Sin[c + d*x] + b*Tan[c + d*x])^3) - ((b + a*Cos[c + d*x])^3*Csc[(
c + d*x)/2]^2*Tan[c + d*x]^3)/(8*(a + b)^3*d*(a*Sin[c + d*x] + b*Tan[c + d*
x])^3) + ((2*a + b)*(b + a*Cos[c + d*x])^3*Log[Cos[(c + d*x)/2]^2]*Tan[c +
d*x]^3)/(4*(-a + b)^4*d*(a*Sin[c + d*x] + b*Tan[c + d*x])^3) + ((-a^5 - 8*a
^3*b^2 - 3*a*b^4)*(b + a*Cos[c + d*x])^3*Log[b + a*Cos[c + d*x]]*Tan[c + d*
x]^3)/((-a^2 + b^2)^4*d*(a*Sin[c + d*x] + b*Tan[c + d*x])^3) + ((2*a - b)*(
b + a*Cos[c + d*x])^3*Log[Sin[(c + d*x)/2]^2]*Tan[c + d*x]^3)/(4*(a + b)^4*
d*(a*Sin[c + d*x] + b*Tan[c + d*x])^3) + ((b + a*Cos[c + d*x])^3*Sec[(c + d
*x)/2]^2*Tan[c + d*x]^3)/(8*(-a + b)^3*d*(a*Sin[c + d*x] + b*Tan[c + d*x])^
3)
```

fricas [B] time = 0.84, size = 1076, normalized size = 4.66

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a*sin(d*x+c)+b*tan(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/4*(8*a^5*b^2 + 8*a^3*b^4 - 16*a*b^6 - 2*(7*a^6*b - 2*a^4*b^3 - 5*a^2*b^5)
*cos(d*x + c)^3 + 2*(a^7 - 7*a^5*b^2 - a^3*b^4 + 7*a*b^6)*cos(d*x + c)^2 +
2*(6*a^6*b + a^4*b^3 - 8*a^2*b^5 + b^7)*cos(d*x + c) + 4*(a^5*b^2 + 8*a^3*b
^4 + 3*a*b^6 - (a^7 + 8*a^5*b^2 + 3*a^3*b^4)*cos(d*x + c)^4 - 2*(a^6*b + 8*
a^4*b^3 + 3*a^2*b^5)*cos(d*x + c)^3 + (a^7 + 7*a^5*b^2 - 5*a^3*b^4 - 3*a*b^
6)*cos(d*x + c)^2 + 2*(a^6*b + 8*a^4*b^3 + 3*a^2*b^5)*cos(d*x + c))*log(a*c
os(d*x + c) + b) - (2*a^5*b^2 + 9*a^4*b^3 + 16*a^3*b^4 + 14*a^2*b^5 + 6*a*b
^6 + b^7 - (2*a^7 + 9*a^6*b + 16*a^5*b^2 + 14*a^4*b^3 + 6*a^3*b^4 + a^2*b^5
)*cos(d*x + c)^4 - 2*(2*a^6*b + 9*a^5*b^2 + 16*a^4*b^3 + 14*a^3*b^4 + 6*a^2
*b^5 + a*b^6)*cos(d*x + c)^3 + (2*a^7 + 9*a^6*b + 14*a^5*b^2 + 5*a^4*b^3 -
10*a^3*b^4 - 13*a^2*b^5 - 6*a*b^6 - b^7)*cos(d*x + c)^2 + 2*(2*a^6*b + 9*a^
5*b^2 + 16*a^4*b^3 + 14*a^3*b^4 + 6*a^2*b^5 + a*b^6)*cos(d*x + c))*log(1/2*
cos(d*x + c) + 1/2) - (2*a^5*b^2 - 9*a^4*b^3 + 16*a^3*b^4 - 14*a^2*b^5 + 6*
a*b^6 - b^7 - (2*a^7 - 9*a^6*b + 16*a^5*b^2 - 14*a^4*b^3 + 6*a^3*b^4 - a^2*
b^5)*cos(d*x + c)^4 - 2*(2*a^6*b - 9*a^5*b^2 + 16*a^4*b^3 - 14*a^3*b^4 + 6*
a^2*b^5 - a*b^6)*cos(d*x + c)^3 + (2*a^7 - 9*a^6*b + 14*a^5*b^2 - 5*a^4*b^3
 - 10*a^3*b^4 + 13*a^2*b^5 - 6*a*b^6 + b^7)*cos(d*x + c)^2 + 2*(2*a^6*b - 9
*a^5*b^2 + 16*a^4*b^3 - 14*a^3*b^4 + 6*a^2*b^5 - a*b^6)*cos(d*x + c))*log(-
1/2*cos(d*x + c) + 1/2))/((a^10 - 4*a^8*b^2 + 6*a^6*b^4 - 4*a^4*b^6 + a^2*b
^8)*d*cos(d*x + c)^4 + 2*(a^9*b - 4*a^7*b^3 + 6*a^5*b^5 - 4*a^3*b^7 + a*b^9
)*d*cos(d*x + c)^3 - (a^10 - 5*a^8*b^2 + 10*a^6*b^4 - 10*a^4*b^6 + 5*a^2*b^
8 - b^10)*d*cos(d*x + c)^2 - 2*(a^9*b - 4*a^7*b^3 + 6*a^5*b^5 - 4*a^3*b^7 +
 a*b^9)*d*cos(d*x + c) - (a^8*b^2 - 4*a^6*b^4 + 6*a^4*b^6 - 4*a^2*b^8 + b^1
0)*d)
```

giac [B] time = 0.94, size = 801, normalized size = 3.47

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a*sin(d*x+c)+b*tan(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/8*(2*(2*a - b)*log(abs(-cos(d*x + c) + 1)/abs(cos(d*x + c) + 1))/(a^4 + 4
*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4) - 8*(a^5 + 8*a^3*b^2 + 3*a*b^4)*log(abs
(-a - b - a*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + b*(cos(d*x + c) - 1)/(c
os(d*x + c) + 1)))/(a^8 - 4*a^6*b^2 + 6*a^4*b^4 - 4*a^2*b^6 + b^8) + (a + b
 - 4*a*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 2*b*(cos(d*x + c) - 1)/(cos(
d*x + c) + 1))*(cos(d*x + c) + 1)/((a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b
^4)*(cos(d*x + c) - 1)) + (cos(d*x + c) - 1)/((a^3 - 3*a^2*b + 3*a*b^2 - b^
3)*(cos(d*x + c) + 1)) + 4*(3*a^7 - 2*a^6*b + 11*a^5*b^2 + 28*a^4*b^3 + 9*a
^3*b^4 + 6*a^2*b^5 + 9*a*b^6 + 6*a^7*(cos(d*x + c) - 1)/(cos(d*x + c) + 1)
- 8*a^6*b*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 38*a^5*b^2*(cos(d*x + c)
- 1)/(cos(d*x + c) + 1) - 4*a^4*b^3*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) -
 26*a^3*b^4*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 12*a^2*b^5*(cos(d*x + c
) - 1)/(cos(d*x + c) + 1) - 18*a*b^6*(cos(d*x + c) - 1)/(cos(d*x + c) + 1)
+ 3*a^7*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 - 6*a^6*b*(cos(d*x + c) -
 1)^2/(cos(d*x + c) + 1)^2 + 27*a^5*b^2*(cos(d*x + c) - 1)^2/(cos(d*x + c)
+ 1)^2 - 48*a^4*b^3*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 + 33*a^3*b^4*
(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 - 18*a^2*b^5*(cos(d*x + c) - 1)^2
/(cos(d*x + c) + 1)^2 + 9*a*b^6*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2)/
((a^8 - 4*a^6*b^2 + 6*a^4*b^4 - 4*a^2*b^6 + b^8)*(a + b + a*(cos(d*x + c) -
 1)/(cos(d*x + c) + 1) - b*(cos(d*x + c) - 1)/(cos(d*x + c) + 1))^2))/d
```

maple [A] time = 0.23, size = 324, normalized size = 1.40

$$-\frac{a^5 \ln(b + a\cos(dx+c))}{d(a+b)^4(a-b)^4} - \frac{8a^3b^2 \ln(b + a\cos(dx+c))}{d(a+b)^4(a-b)^4} - \frac{3b^4 a \ln(b + a\cos(dx+c))}{d(a+b)^4(a-b)^4} + \frac{b^2 a}{2d(a+b)^2(a-b)^2(b+a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)/(a*sin(d*x+c)+b*tan(d*x+c))^3,x)
```

```
[Out] -1/d*a^5/(a+b)^4/(a-b)^4*ln(b+a*cos(d*x+c))-8/d*a^3*b^2/(a+b)^4/(a-b)^4*ln(
b+a*cos(d*x+c))-3/d*b^4/(a+b)^4/(a-b)^4*a*ln(b+a*cos(d*x+c))+1/2/d*b^2/(a+b
)^2*a/(a-b)^2/(b+a*cos(d*x+c))^2-2/d*a^3*b/(a+b)^3/(a-b)^3/(b+a*cos(d*x+c))
-2/d*a*b^3/(a+b)^3/(a-b)^3/(b+a*cos(d*x+c))+1/4/d/(a+b)^3/(cos(d*x+c)-1)+1/
2/d/(a+b)^4*ln(cos(d*x+c)-1)*a-1/4/d/(a+b)^4*ln(cos(d*x+c)-1)*b-1/4/d/(a-b)
^3/(1+cos(d*x+c))+1/2*a*ln(1+cos(d*x+c))/(a-b)^4/d+1/4*b*ln(1+cos(d*x+c))/(
a-b)^4/d
```

maxima [B] time = 0.38, size = 602, normalized size = 2.61

$$-\frac{\frac{8\left(a^5+8a^3b^2+3ab^4\right)\log\left(a+b-\frac{(a-b)\sin(dx+c)^2}{(\cos(dx+c)+1)^2}\right)}{a^8-4a^6b^2+6a^4b^4-4a^2b^6+b^8}-\frac{4(2a-b)\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^4+4a^3b+6a^2b^2+4ab^3+b^4}+\frac{a^6-2a^5b-a^4b^2+4a^3b^3-a^2b^4-2ab^5+b^6-\frac{2(a^6 \cdots}{\cdots}}{\frac{\left(a^9+a^8b-4a^7b^2-4a^6b^3+6a^5b^4+6a^4b^5-4a^3b^6-4a^2b^7+ab^8+b^9\right)\sin(dx+c)^2}{(\cos(dx+c)+1)^2}\cdots}}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a*sin(d*x+c)+b*tan(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -1/8*(8*(a^5 + 8*a^3*b^2 + 3*a*b^4)*log(a + b - (a - b)*sin(d*x + c)^2/(cos
(d*x + c) + 1)^2)/(a^8 - 4*a^6*b^2 + 6*a^4*b^4 - 4*a^2*b^6 + b^8) - 4*(2*a
- b)*log(sin(d*x + c)/(cos(d*x + c) + 1))/(a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*
b^3 + b^4) + (a^6 - 2*a^5*b - a^4*b^2 + 4*a^3*b^3 - a^2*b^4 - 2*a*b^5 + b^6
 - 2*(a^6 - 20*a^5*b - 11*a^4*b^2 - 24*a^3*b^3 - 29*a^2*b^4 + 4*a*b^5 - b^6
)*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + (a^6 - 38*a^5*b + 31*a^4*b^2 - 52*a
^3*b^3 + 63*a^2*b^4 - 6*a*b^5 + b^6)*sin(d*x + c)^4/(cos(d*x + c) + 1)^4)/(
(a^9 + a^8*b - 4*a^7*b^2 - 4*a^6*b^3 + 6*a^5*b^4 + 6*a^4*b^5 - 4*a^3*b^6 -
4*a^2*b^7 + a*b^8 + b^9)*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 - 2*(a^9 - a^8
*b - 4*a^7*b^2 + 4*a^6*b^3 + 6*a^5*b^4 - 6*a^4*b^5 - 4*a^3*b^6 + 4*a^2*b^7
+ a*b^8 - b^9)*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + (a^9 - 3*a^8*b + 8*a^6
*b^3 - 6*a^5*b^4 - 6*a^4*b^5 + 8*a^3*b^6 - 3*a*b^8 + b^9)*sin(d*x + c)^6/(c
os(d*x + c) + 1)^6) + sin(d*x + c)^2/((a^3 - 3*a^2*b + 3*a*b^2 - b^3)*(cos(
d*x + c) + 1)^2))/d
```

mupad [B] time = 1.12, size = 496, normalized size = 2.15

$$\frac{\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4\left(-a^5+37\,a^4\,b+6\,a^3\,b^2+58\,a^2\,b^3-5\,a\,b^4+b^5\right)}{2\,(a+b)\left(a^2+2\,a\,b+b^2\right)}-\frac{a^3-3\,a^2\,b+3\,a\,b^2-b^3}{2\,(a+b)}+\cdots}{d\left(\left(4\,a^5-20\,a^4\,b+40\,a^3\,b^2-40\,a^2\,b^3+20\,a\,b^4-4\,b^5\right)\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6+\left(-8\,a^5+24\,a^4\,b-16\,a^3\,b^2-16\,a^2\,b^3+2\cdots\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)*(a*sin(c + d*x) + b*tan(c + d*x))^3),x)
```

```
[Out] ((tan(c/2 + (d*x)/2)^4*(37*a^4*b - 5*a*b^4 - a^5 + b^5 + 58*a^2*b^3 + 6*a^3
*b^2))/(2*(a + b)*(2*a*b + a^2 + b^2)) - (3*a*b^2 - 3*a^2*b + a^3 - b^3)/(2
*(a + b)) + (tan(c/2 + (d*x)/2)^2*(5*a*b^4 - 21*a^4*b + a^5 - b^5 - 34*a^2*
b^3 + 10*a^3*b^2))/((a - b)*(2*a*b + a^2 + b^2)))/(d*(tan(c/2 + (d*x)/2)^2*
(4*a*b^4 - 4*a^4*b + 4*a^5 - 4*b^5 + 8*a^2*b^3 - 8*a^3*b^2) - tan(c/2 + (d*
x)/2)^4*(8*a^5 - 24*a^4*b - 24*a*b^4 + 8*b^5 + 16*a^2*b^3 + 16*a^3*b^2) + t
an(c/2 + (d*x)/2)^6*(20*a*b^4 - 20*a^4*b + 4*a^5 - 4*b^5 - 40*a^2*b^3 + 40*
a^3*b^2))) - tan(c/2 + (d*x)/2)^2/(8*d*(a - b)^3) + (log(tan(c/2 + (d*x)/2)
```

```
)*(2*a - b))/(d*(8*a*b^3 + 8*a^3*b + 2*a^4 + 2*b^4 + 12*a^2*b^2)) - (log(a
+ b - a*tan(c/2 + (d*x)/2)^2 + b*tan(c/2 + (d*x)/2)^2)*(3*a*b^4 + a^5 + 8*a
^3*b^2))/(d*(a^8 + b^8 - 4*a^2*b^6 + 6*a^4*b^4 - 4*a^6*b^2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec (c + dx)}{(a \sin (c + dx) + b \tan (c + dx))^3} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a*sin(d*x+c)+b*tan(d*x+c))**3,x)
```

```
[Out] Integral(sec(c + d*x)/(a*sin(c + d*x) + b*tan(c + d*x))**3, x)
```

3.269 $\int \frac{\sec^2(c+dx)}{(a\sin(c+dx)+b\tan(c+dx))^3}\,dx$

Optimal. Leaf size=212

$$\frac{3a^2(a^2+3b^2)}{2d(a^2-b^2)^3(a\cos(c+dx)+b)}-\frac{3a^2b}{2d(a^2-b^2)^2(a\cos(c+dx)+b)^2}+\frac{6a^2b(a^2+b^2)\log(a\cos(c+dx)+b)}{d(a^2-b^2)^4}+\frac{\csc^2}{2d\,(}$$

```
[Out] -3/2*a^2*b/(a^2-b^2)^2/d/(b+a*cos(d*x+c))^2+3/2*a^2*(a^2+3*b^2)/(a^2-b^2)^3
/d/(b+a*cos(d*x+c))+1/2*(b-a*cos(d*x+c))*csc(d*x+c)^2/(a^2-b^2)/d/(b+a*cos(
d*x+c))^2+3/4*a*ln(1-cos(d*x+c))/(a+b)^4/d-3/4*a*ln(1+cos(d*x+c))/(a-b)^4/d
+6*a^2*b*(a^2+b^2)*ln(b+a*cos(d*x+c))/(a^2-b^2)^4/d
```

Rubi [A] time = 0.36, antiderivative size = 212, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.179, Rules used = {4397, 2837, 12, 823, 801}

$$\frac{3a^2(a^2+3b^2)}{2d(a^2-b^2)^3(a\cos(c+dx)+b)}-\frac{3a^2b}{2d(a^2-b^2)^2(a\cos(c+dx)+b)^2}+\frac{6a^2b(a^2+b^2)\log(a\cos(c+dx)+b)}{d(a^2-b^2)^4}+\frac{\csc^2}{2d\,(}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^2/(a*Sin[c + d*x] + b*Tan[c + d*x])^3,x]
```

```
[Out] (-3*a^2*b)/(2*(a^2 - b^2)^2*d*(b + a*Cos[c + d*x])^2) + (3*a^2*(a^2 + 3*b^2
))/(2*(a^2 - b^2)^3*d*(b + a*Cos[c + d*x])) + ((b - a*Cos[c + d*x])*Csc[c +
 d*x]^2)/(2*(a^2 - b^2)*d*(b + a*Cos[c + d*x])^2) + (3*a*Log[1 - Cos[c + d*
x]])/(4*(a + b)^4*d) - (3*a*Log[1 + Cos[c + d*x]])/(4*(a - b)^4*d) + (6*a^2
*b*(a^2 + b^2)*Log[b + a*Cos[c + d*x]])/((a^2 - b^2)^4*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 801

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a + c*x^2), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rule 823

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> -Simp[((d + e*x)^(m + 1)*(f*a*c*e - a*g*c*d + c*(c*d*f + a
```

```
*e*g)*x)*(a + c*x^2)^(p + 1))/(2*a*c*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[1/
(2*a*c*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*(a + c*x^2)^(p + 1)*Simp[f
*(c^2*d^2*(2*p + 3) + a*c*e^2*(m + 2*p + 3)) - a*c*d*e*g*m + c*e*(c*d*f + a
*e*g)*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*
d^2 + a*e^2, 0] && LtQ[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2
*m, 2*p])
```

Rule 2837

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^m*(c + (d*x)/b)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*S
in[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/
2] && NeQ[a^2 - b^2, 0]
```

Rule 4397

```
Int[u_, x_Symbol] :> Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]
```

Rubi steps

$$\int \frac{\sec^2(c+dx)}{(a\sin(c+dx)+b\tan(c+dx))^3}\,dx = \int \frac{\cot(c+dx)\csc^2(c+dx)}{(b+a\cos(c+dx))^3}\,dx$$

$$= -\frac{a^3\,\text{Subst}\left(\int \frac{x}{a(b+x)^3\left(a^2-x^2\right)^2}\,dx, x, a\cos(c+dx)\right)}{d}$$

$$= -\frac{a^2\,\text{Subst}\left(\int \frac{x}{(b+x)^3\left(a^2-x^2\right)^2}\,dx, x, a\cos(c+dx)\right)}{d}$$

$$= \frac{(b-a\cos(c+dx))\csc^2(c+dx)}{2\left(a^2-b^2\right)d(b+a\cos(c+dx))^2} - \frac{\text{Subst}\left(\int \frac{-3a^2b+3a^2x}{(b+x)^3\left(a^2-x^2\right)}\,dx, x, a\cos(c\right.}{2\left(a^2-b^2\right)d}$$

$$= \frac{(b-a\cos(c+dx))\csc^2(c+dx)}{2\left(a^2-b^2\right)d(b+a\cos(c+dx))^2} - \frac{\text{Subst}\left(\int \left(\frac{3a(a-b)}{2(a+b)^3(a-x)} + \frac{3a(a+b)}{2(a-b)^3(a+x)}\right.\right.}{}$$

$$= -\frac{3a^2b}{2\left(a^2-b^2\right)^2 d(b+a\cos(c+dx))^2} + \frac{3a^2\left(a^2+3b^2\right)}{2\left(a^2-b^2\right)^3 d(b+a\cos(c+dx))}$$

Mathematica [A] time = 6.27, size = 217, normalized size = 1.02

$$-\frac{a^2\left(a^2+3b^2\right)}{d(b-a)^3(a+b)^3(a\cos(c+dx)+b)}-\frac{a^2b}{2d(b-a)^2(a+b)^2(a\cos(c+dx)+b)^2}+\frac{6\left(a^4b+a^2b^3\right)\log(a\cos(c+dx)+}{d\left(b^2-a^2\right)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^2/(a*Sin[c + d*x] + b*Tan[c + d*x])^3,x]
```

```
[Out] -1/2*(a^2*b)/((-a + b)^2*(a + b)^2*d*(b + a*Cos[c + d*x])^2) - (a^2*(a^2 +
3*b^2))/((-a + b)^3*(a + b)^3*d*(b + a*Cos[c + d*x])) - Csc[(c + d*x)/2]^2/
(8*(a + b)^3*d) - (3*a*Log[Cos[(c + d*x)/2]])/(2*(-a + b)^4*d) + (6*(a^4*b
+ a^2*b^3)*Log[b + a*Cos[c + d*x]])/((-a^2 + b^2)^4*d) + (3*a*Log[Sin[(c +
d*x)/2]])/(2*(a + b)^4*d) - Sec[(c + d*x)/2]^2/(8*(-a + b)^3*d)
```

fricas [B] time = 1.01, size = 939, normalized size = 4.43

$$-\frac{2\,a^6b+18\,a^4b^3-18\,a^2b^5-2\,b^7-6\left(a^7+2\,a^5b^2-3\,a^3b^4\right)\cos\left(dx+c\right)^3-24\left(a^4b^3-a^2b^5\right)\cos\left(dx+c\right)^2+2\left(2\,a\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a*sin(d*x+c)+b*tan(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] -1/4*(2*a^6*b + 18*a^4*b^3 - 18*a^2*b^5 - 2*b^7 - 6*(a^7 + 2*a^5*b^2 - 3*a^
3*b^4)*cos(d*x + c)^3 - 24*(a^4*b^3 - a^2*b^5)*cos(d*x + c)^2 + 2*(2*a^7 +
9*a^5*b^2 - 12*a^3*b^4 + a*b^6)*cos(d*x + c) + 24*(a^4*b^3 + a^2*b^5 - (a^6
*b + a^4*b^3)*cos(d*x + c)^4 - 2*(a^5*b^2 + a^3*b^4)*cos(d*x + c)^3 + (a^6*
b - a^2*b^5)*cos(d*x + c)^2 + 2*(a^5*b^2 + a^3*b^4)*cos(d*x + c))*log(a*cos
(d*x + c) + b) - 3*(a^5*b^2 + 4*a^4*b^3 + 6*a^3*b^4 + 4*a^2*b^5 + a*b^6 - (
a^7 + 4*a^6*b + 6*a^5*b^2 + 4*a^4*b^3 + a^3*b^4)*cos(d*x + c)^4 - 2*(a^6*b
+ 4*a^5*b^2 + 6*a^4*b^3 + 4*a^3*b^4 + a^2*b^5)*cos(d*x + c)^3 + (a^7 + 4*a^
6*b + 5*a^5*b^2 - 5*a^3*b^4 - 4*a^2*b^5 - a*b^6)*cos(d*x + c)^2 + 2*(a^6*b
+ 4*a^5*b^2 + 6*a^4*b^3 + 4*a^3*b^4 + a^2*b^5)*cos(d*x + c))*log(1/2*cos(d*
x + c) + 1/2) + 3*(a^5*b^2 - 4*a^4*b^3 + 6*a^3*b^4 - 4*a^2*b^5 + a*b^6 - (a
^7 - 4*a^6*b + 6*a^5*b^2 - 4*a^4*b^3 + a^3*b^4)*cos(d*x + c)^4 - 2*(a^6*b -
 4*a^5*b^2 + 6*a^4*b^3 - 4*a^3*b^4 + a^2*b^5)*cos(d*x + c)^3 + (a^7 - 4*a^6
*b + 5*a^5*b^2 - 5*a^3*b^4 + 4*a^2*b^5 - a*b^6)*cos(d*x + c)^2 + 2*(a^6*b -
 4*a^5*b^2 + 6*a^4*b^3 - 4*a^3*b^4 + a^2*b^5)*cos(d*x + c))*log(-1/2*cos(d*
x + c) + 1/2))/((a^10 - 4*a^8*b^2 + 6*a^6*b^4 - 4*a^4*b^6 + a^2*b^8)*d*cos(
d*x + c)^4 + 2*(a^9*b - 4*a^7*b^3 + 6*a^5*b^5 - 4*a^3*b^7 + a*b^9)*d*cos(d*
x + c)^3 - (a^10 - 5*a^8*b^2 + 10*a^6*b^4 - 10*a^4*b^6 + 5*a^2*b^8 - b^10)*
d*cos(d*x + c)^2 - 2*(a^9*b - 4*a^7*b^3 + 6*a^5*b^5 - 4*a^3*b^7 + a*b^9)*d*
cos(d*x + c) - (a^8*b^2 - 4*a^6*b^4 + 6*a^4*b^6 - 4*a^2*b^8 + b^10)*d)
```

giac [B] time = 1.11, size = 689, normalized size = 3.25

$$\frac{6\,a\log\left(\frac{|-\cos(dx+c)+1|}{|\cos(dx+c)+1|}\right)}{a^4+4\,a^3b+6\,a^2b^2+4\,ab^3+b^4} + \frac{48\left(a^4b+a^2b^3\right)\log\left(\left|-a-b-\frac{a(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{b(\cos(dx+c)-1)}{\cos(dx+c)+1}\right|\right)}{a^8-4\,a^6b^2+6\,a^4b^4-4\,a^2b^6+b^8} + \frac{\left(a+b-\frac{6\,a(\cos(dx+c)-1)}{\cos(dx+c)+1}\right)(\cos(dx+c)+1)}{\left(a^4+4\,a^3b+6\,a^2b^2+4\,ab^3+b^4\right)(\cos(dx+c)-1)} - \frac{}{\left(a^3-3\,a^2\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a*sin(d*x+c)+b*tan(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/8*(6*a*log(abs(-cos(d*x + c) + 1)/abs(cos(d*x + c) + 1))/(a^4 + 4*a^3*b +
 6*a^2*b^2 + 4*a*b^3 + b^4) + 48*(a^4*b + a^2*b^3)*log(abs(-a - b - a*(cos(
d*x + c) - 1)/(cos(d*x + c) + 1) + b*(cos(d*x + c) - 1)/(cos(d*x + c) + 1))
)/(a^8 - 4*a^6*b^2 + 6*a^4*b^4 - 4*a^2*b^6 + b^8) + (a + b - 6*a*(cos(d*x +
 c) - 1)/(cos(d*x + c) + 1))*(cos(d*x + c) + 1)/((a^4 + 4*a^3*b + 6*a^2*b^2
 + 4*a*b^3 + b^4)*(cos(d*x + c) - 1)) - (cos(d*x + c) - 1)/((a^3 - 3*a^2*b
+ 3*a*b^2 - b^3)*(cos(d*x + c) + 1)) + 8*(2*a^7 - 5*a^6*b - 8*a^5*b^2 - 2*a
^4*b^3 - 10*a^3*b^4 - 9*a^2*b^5 + 2*a^7*(cos(d*x + c) - 1)/(cos(d*x + c) +
1) - 16*a^6*b*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 6*a^5*b^2*(cos(d*x +
c) - 1)/(cos(d*x + c) + 1) - 2*a^4*b^3*(cos(d*x + c) - 1)/(cos(d*x + c) + 1
) - 8*a^3*b^4*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 18*a^2*b^5*(cos(d*x +
 c) - 1)/(cos(d*x + c) + 1) - 9*a^6*b*(cos(d*x + c) - 1)^2/(cos(d*x + c) +
1)^2 + 18*a^5*b^2*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 - 18*a^4*b^3*(c
os(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 + 18*a^3*b^4*(cos(d*x + c) - 1)^2/(
cos(d*x + c) + 1)^2 - 9*a^2*b^5*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2)/
((a^8 - 4*a^6*b^2 + 6*a^4*b^4 - 4*a^2*b^6 + b^8)*(a + b + a*(cos(d*x + c) -
 1)/(cos(d*x + c) + 1) - b*(cos(d*x + c) - 1)/(cos(d*x + c) + 1))^2))/d
```

maple [A] time = 0.25, size = 251, normalized size = 1.18

$$-\frac{b\,a^2}{2d\,(a+b)^2\,(a-b)^2\,(b+a\cos(dx+c))^2}+\frac{a^4}{d\,(a+b)^3\,(a-b)^3\,(b+a\cos(dx+c))}+\frac{3b^2a^2}{d\,(a+b)^3\,(a-b)^3\,(b+a\cos(\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2/(a*sin(d*x+c)+b*tan(d*x+c))^3,x)
```

```
[Out] -1/2/d*b*a^2/(a+b)^2/(a-b)^2/(b+a*cos(d*x+c))^2+1/d*a^4/(a+b)^3/(a-b)^3/(b+
a*cos(d*x+c))+3/d*b^2/(a+b)^3/(a-b)^3/(b+a*cos(d*x+c))*a^2+6/d*b/(a+b)^4/(a
-b)^4*ln(b+a*cos(d*x+c))*a^4+6/d*b^3/(a+b)^4/(a-b)^4*ln(b+a*cos(d*x+c))*a^2
+1/4/d/(a+b)^3/(cos(d*x+c)-1)+3/4/d/(a+b)^4*ln(cos(d*x+c)-1)*a+1/4/d/(a-b)^
3/(1+cos(d*x+c))-3/4*a*ln(1+cos(d*x+c))/(a-b)^4/d
```

maxima [B] time = 0.39, size = 596, normalized size = 2.81

$$\frac{48\left(a^4b+a^2b^3\right)\log\left(a+b-\frac{(a-b)\sin(dx+c)^2}{(\cos(dx+c)+1)^2}\right)}{a^8-4a^6b^2+6a^4b^4-4a^2b^6+b^8}+\frac{12a\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^4+4a^3b+6a^2b^2+4ab^3+b^4}-\frac{a^6-2a^5b-a^4b^2+4a^3b^3-a^2b^4-2ab^5+b^6-\frac{2\left(9a^6+4a^5b+3\cdots\right.}{\cdots}}{\frac{\left(a^9+a^8b-4a^7b^2-4a^6b^3+6a^5b^4+6a^4b^5-4a^3b^6-4a^2b^7+ab^8+b^9\right)\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-\frac{2\left(a^9-a^8\cdots\right.}{\cdots}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a*sin(d*x+c)+b*tan(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/8*(48*(a^4*b + a^2*b^3)*log(a + b - (a - b)*sin(d*x + c)^2/(cos(d*x + c)
+ 1)^2)/(a^8 - 4*a^6*b^2 + 6*a^4*b^4 - 4*a^2*b^6 + b^8) + 12*a*log(sin(d*x
+ c)/(cos(d*x + c) + 1))/(a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4) - (a^6
 - 2*a^5*b - a^4*b^2 + 4*a^3*b^3 - a^2*b^4 - 2*a*b^5 + b^6 - 2*(9*a^6 + 4*a
^5*b + 37*a^4*b^2 + 32*a^3*b^3 - 5*a^2*b^4 + 4*a*b^5 - b^6)*sin(d*x + c)^2/
(cos(d*x + c) + 1)^2 + (17*a^6 - 6*a^5*b + 63*a^4*b^2 - 84*a^3*b^3 + 15*a^2
*b^4 - 6*a*b^5 + b^6)*sin(d*x + c)^4/(cos(d*x + c) + 1)^4)/((a^9 + a^8*b -
4*a^7*b^2 - 4*a^6*b^3 + 6*a^5*b^4 + 6*a^4*b^5 - 4*a^3*b^6 - 4*a^2*b^7 + a*b
^8 + b^9)*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 - 2*(a^9 - a^8*b - 4*a^7*b^2
+ 4*a^6*b^3 + 6*a^5*b^4 - 6*a^4*b^5 - 4*a^3*b^6 + 4*a^2*b^7 + a*b^8 - b^9)*
sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + (a^9 - 3*a^8*b + 8*a^6*b^3 - 6*a^5*b^
4 - 6*a^4*b^5 + 8*a^3*b^6 - 3*a*b^8 + b^9)*sin(d*x + c)^6/(cos(d*x + c) + 1
)^6) + sin(d*x + c)^2/((a^3 - 3*a^2*b + 3*a*b^2 - b^3)*(cos(d*x + c) + 1)^2
))/d
```

mupad [B] time = 1.13, size = 492, normalized size = 2.32

$$\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{8\,d\,(a-b)^3}-\frac{\frac{a^3-3\,a^2\,b+3\,a\,b^2-b^3}{2\,(a+b)}-\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\left(9\,a^5-5\,a^4\,b+42\,a^3\,b^2-10\cdots\right.}{(a-b)\left(a^2+2\,a\,b+b^2\right)}\cdots}{d\,\left(\left(4\,a^5-20\,a^4\,b+40\,a^3\,b^2-40\,a^2\,b^3+20\,a\,b^4-4\,b^5\right)\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6+\left(-8\,a^5+24\,a^4\,b-16\,a^3\cdots\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^2*(a*sin(c + d*x) + b*tan(c + d*x))^3),x)
```

```
[Out] tan(c/2 + (d*x)/2)^2/(8*d*(a - b)^3) - ((3*a*b^2 - 3*a^2*b + a^3 - b^3)/(2*
(a + b)) - (tan(c/2 + (d*x)/2)^2*(5*a*b^4 - 5*a^4*b + 9*a^5 - b^5 - 10*a^2*
b^3 + 42*a^3*b^2))/((a - b)*(2*a*b + a^2 + b^2)) + (tan(c/2 + (d*x)/2)^4*(5
*a*b^4 + 11*a^4*b + 17*a^5 - b^5 - 10*a^2*b^3 + 74*a^3*b^2))/(2*(a + b)*(2*
a*b + a^2 + b^2)))/(d*(tan(c/2 + (d*x)/2)^2*(4*a*b^4 - 4*a^4*b + 4*a^5 - 4*
b^5 + 8*a^2*b^3 - 8*a^3*b^2) - tan(c/2 + (d*x)/2)^4*(8*a^5 - 24*a^4*b - 24*
a*b^4 + 8*b^5 + 16*a^2*b^3 + 16*a^3*b^2) + tan(c/2 + (d*x)/2)^6*(20*a*b^4 -
 20*a^4*b + 4*a^5 - 4*b^5 - 40*a^2*b^3 + 40*a^3*b^2))) + (log(a + b - a*tan
```

```
(c/2 + (d*x)/2)^2 + b*tan(c/2 + (d*x)/2)^2)*(6*a^4*b + 6*a^2*b^3))/(d*(a^8
+ b^8 - 4*a^2*b^6 + 6*a^4*b^4 - 4*a^6*b^2)) + (3*a*log(tan(c/2 + (d*x)/2)))
/(d*(8*a*b^3 + 8*a^3*b + 2*a^4 + 2*b^4 + 12*a^2*b^2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec^2(c+dx)}{(a\sin(c+dx)+b\tan(c+dx))^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2/(a*sin(d*x+c)+b*tan(d*x+c))**3,x)
```

```
[Out] Integral(sec(c + d*x)**2/(a*sin(c + d*x) + b*tan(c + d*x))**3, x)
```

3.270 $\int \frac{\sec^3(c+dx)}{(a\sin(c+dx)+b\tan(c+dx))^3}\,dx$

Optimal. Leaf size=228

$$-\frac{ab\left(11a^2+b^2\right)}{2d\left(a^2-b^2\right)^3(a\cos(c+dx)+b)}+\frac{a\left(2a^2+b^2\right)}{2d\left(a^2-b^2\right)^2(a\cos(c+dx)+b)^2}-\frac{\csc^2(c+dx)(a-b\cos(c+dx))}{2d\left(a^2-b^2\right)(a\cos(c+dx)+b)^2}-\frac{2a^3\left(a^2\right.}{}$$

```
[Out] 1/2*a*(2*a^2+b^2)/(a^2-b^2)^2/d/(b+a*cos(d*x+c))^2-1/2*a*b*(11*a^2+b^2)/(a^
2-b^2)^3/d/(b+a*cos(d*x+c))-1/2*(a-b*cos(d*x+c))*csc(d*x+c)^2/(a^2-b^2)/d/(
b+a*cos(d*x+c))^2+1/4*(4*a+b)*ln(1-cos(d*x+c))/(a+b)^4/d+1/4*(4*a-b)*ln(1+c
os(d*x+c))/(a-b)^4/d-2*a^3*(a^2+5*b^2)*ln(b+a*cos(d*x+c))/(a^2-b^2)^4/d
```

Rubi [A] time = 0.41, antiderivative size = 228, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {4397, 2668, 741, 801}

$$-\frac{ab\left(11a^2+b^2\right)}{2d\left(a^2-b^2\right)^3(a\cos(c+dx)+b)}+\frac{a\left(2a^2+b^2\right)}{2d\left(a^2-b^2\right)^2(a\cos(c+dx)+b)^2}-\frac{2a^3\left(a^2+5b^2\right)\log(a\cos(c+dx)+b)}{d\left(a^2-b^2\right)^4}-\frac{\text{cs}}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^3/(a*Sin[c + d*x] + b*Tan[c + d*x])^3,x]
```

```
[Out] (a*(2*a^2 + b^2))/(2*(a^2 - b^2)^2*d*(b + a*Cos[c + d*x])^2) - (a*b*(11*a^2
 + b^2))/(2*(a^2 - b^2)^3*d*(b + a*Cos[c + d*x])) - ((a - b*Cos[c + d*x])*C
sc[c + d*x]^2)/(2*(a^2 - b^2)*d*(b + a*Cos[c + d*x])^2) + ((4*a + b)*Log[1
- Cos[c + d*x]])/(4*(a + b)^4*d) + ((4*a - b)*Log[1 + Cos[c + d*x]])/(4*(a
- b)^4*d) - (2*a^3*(a^2 + 5*b^2)*Log[b + a*Cos[c + d*x]])/((a^2 - b^2)^4*d)
```

Rule 741

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp
[((d + e*x)^(m + 1)*(a*e + c*d*x)*(a + c*x^2)^(p + 1))/(2*a*(p + 1)*(c*d^2
+ a*e^2)), x] + Dist[1/(2*a*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*Simp[
c*d^2*(2*p + 3) + a*e^2*(m + 2*p + 3) + c*e*d*(m + 2*p + 4)*x, x]*(a + c*x^
2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, m}, x] && NeQ[c*d^2 + a*e^2, 0] &&
 LtQ[p, -1] && IntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 801

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a + c*x^2), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rule 2668

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rule 4397

```
Int[u_, x_Symbol] :> Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]
```

Rubi steps

$$\int \frac{\sec^3(c+dx)}{(a\sin(c+dx)+b\tan(c+dx))^3}\,dx = \int \frac{\csc^3(c+dx)}{(b+a\cos(c+dx))^3}\,dx$$

$$= -\frac{a^3\,\text{Subst}\left(\int \frac{1}{(b+x)^3\left(a^2-x^2\right)^2}\,dx, x, a\cos(c+dx)\right)}{d}$$

$$= -\frac{(a-b\cos(c+dx))\csc^2(c+dx)}{2\left(a^2-b^2\right)d(b+a\cos(c+dx))^2} + \frac{a\,\text{Subst}\left(\int \frac{-4a^2+b^2+3bx}{(b+x)^3\left(a^2-x^2\right)}\,dx, x, a\cos\right.}{2\left(a^2-b^2\right)d}$$

$$= -\frac{(a-b\cos(c+dx))\csc^2(c+dx)}{2\left(a^2-b^2\right)d(b+a\cos(c+dx))^2} + \frac{a\,\text{Subst}\left(\int\left(\frac{(-4a-b)(a-b)}{2a(a+b)^3(a-x)} + \frac{(4a-b)(a}{2a(a-b)^3(}\right.\right.}{}$$

$$= \frac{a\left(2a^2+b^2\right)}{2\left(a^2-b^2\right)^2 d(b+a\cos(c+dx))^2} - \frac{ab\left(11a^2+b^2\right)}{2\left(a^2-b^2\right)^3 d(b+a\cos(c+dx))} -$$

Mathematica [A] time = 6.23, size = 217, normalized size = 0.95

$$\frac{4a^3b}{d(b-a)^3(a+b)^3(a\cos(c+dx)+b)} + \frac{a^3}{2d(b-a)^2(a+b)^2(a\cos(c+dx)+b)^2} - \frac{2\left(a^5+5a^3b^2\right)\log(a\cos(c+dx)+}{d\left(b^2-a^2\right)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^3/(a*Sin[c + d*x] + b*Tan[c + d*x])^3,x]
```

```
[Out] a^3/(2*(-a + b)^2*(a + b)^2*d*(b + a*Cos[c + d*x])^2) + (4*a^3*b)/((-a + b)
^3*(a + b)^3*d*(b + a*Cos[c + d*x])) - Csc[(c + d*x)/2]^2/(8*(a + b)^3*d) +
```

```
((4*a - b)*Log[Cos[(c + d*x)/2]])/(2*(-a + b)^4*d) - (2*(a^5 + 5*a^3*b^2)*
Log[b + a*Cos[c + d*x]])/((-a^2 + b^2)^4*d) + ((4*a + b)*Log[Sin[(c + d*x)/
2]])/(2*(a + b)^4*d) + Sec[(c + d*x)/2]^2/(8*(-a + b)^3*d)
```

fricas [B] time = 0.78, size = 971, normalized size = 4.26

$$-\frac{2\,a^7 - 22\,a^5b^2 + 14\,a^3b^4 + 6\,ab^6 + 2\left(11\,a^6b - 10\,a^4b^3 - a^2b^5\right)\cos\left(dx+c\right)^3 - 4\left(a^7 - 7\,a^5b^2 + 5\,a^3b^4 + ab^6\right)\cos(}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a*sin(d*x+c)+b*tan(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] -1/4*(2*a^7 - 22*a^5*b^2 + 14*a^3*b^4 + 6*a*b^6 + 2*(11*a^6*b - 10*a^4*b^3
- a^2*b^5)*cos(d*x + c)^3 - 4*(a^7 - 7*a^5*b^2 + 5*a^3*b^4 + a*b^6)*cos(d*x
 + c)^2 - 2*(10*a^6*b - 7*a^4*b^3 - 4*a^2*b^5 + b^7)*cos(d*x + c) - 8*(a^5*
b^2 + 5*a^3*b^4 - (a^7 + 5*a^5*b^2)*cos(d*x + c)^4 - 2*(a^6*b + 5*a^4*b^3)*
cos(d*x + c)^3 + (a^7 + 4*a^5*b^2 - 5*a^3*b^4)*cos(d*x + c)^2 + 2*(a^6*b +
5*a^4*b^3)*cos(d*x + c))*log(a*cos(d*x + c) + b) + (4*a^5*b^2 + 15*a^4*b^3
+ 20*a^3*b^4 + 10*a^2*b^5 - b^7 - (4*a^7 + 15*a^6*b + 20*a^5*b^2 + 10*a^4*b
^3 - a^2*b^5)*cos(d*x + c)^4 - 2*(4*a^6*b + 15*a^5*b^2 + 20*a^4*b^3 + 10*a^
3*b^4 - a*b^6)*cos(d*x + c)^3 + (4*a^7 + 15*a^6*b + 16*a^5*b^2 - 5*a^4*b^3
- 20*a^3*b^4 - 11*a^2*b^5 + b^7)*cos(d*x + c)^2 + 2*(4*a^6*b + 15*a^5*b^2 +
 20*a^4*b^3 + 10*a^3*b^4 - a*b^6)*cos(d*x + c))*log(1/2*cos(d*x + c) + 1/2)
 + (4*a^5*b^2 - 15*a^4*b^3 + 20*a^3*b^4 - 10*a^2*b^5 + b^7 - (4*a^7 - 15*a^
6*b + 20*a^5*b^2 - 10*a^4*b^3 + a^2*b^5)*cos(d*x + c)^4 - 2*(4*a^6*b - 15*a
^5*b^2 + 20*a^4*b^3 - 10*a^3*b^4 + a*b^6)*cos(d*x + c)^3 + (4*a^7 - 15*a^6*
b + 16*a^5*b^2 + 5*a^4*b^3 - 20*a^3*b^4 + 11*a^2*b^5 - b^7)*cos(d*x + c)^2
+ 2*(4*a^6*b - 15*a^5*b^2 + 20*a^4*b^3 - 10*a^3*b^4 + a*b^6)*cos(d*x + c))*
log(-1/2*cos(d*x + c) + 1/2))/((a^10 - 4*a^8*b^2 + 6*a^6*b^4 - 4*a^4*b^6 +
a^2*b^8)*d*cos(d*x + c)^4 + 2*(a^9*b - 4*a^7*b^3 + 6*a^5*b^5 - 4*a^3*b^7 +
a*b^9)*d*cos(d*x + c)^3 - (a^10 - 5*a^8*b^2 + 10*a^6*b^4 - 10*a^4*b^6 + 5*a
^2*b^8 - b^10)*d*cos(d*x + c)^2 - 2*(a^9*b - 4*a^7*b^3 + 6*a^5*b^5 - 4*a^3*
b^7 + a*b^9)*d*cos(d*x + c) - (a^8*b^2 - 4*a^6*b^4 + 6*a^4*b^6 - 4*a^2*b^8
+ b^10)*d)
```

giac [B] time = 1.25, size = 675, normalized size = 2.96

$$\frac{\frac{2\,(4\,a+b)\log\left(\frac{|-\cos(dx+c)+1|}{|\cos(dx+c)+1|}\right)}{a^4+4\,a^3b+6\,a^2b^2+4\,ab^3+b^4} - \frac{16\left(a^5+5\,a^3b^2\right)\log\left(\left|-a-b-\frac{a(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{b(\cos(dx+c)-1)}{\cos(dx+c)+1}\right|\right)}{a^8-4\,a^6b^2+6\,a^4b^4-4\,a^2b^6+b^8} + \frac{\left(a+b-\frac{8\,a(\cos(dx+c)-1)}{\cos(dx+c)+1}-\frac{2\,b(\cos(dx+c)-1)}{\cos(dx+c)+1}\right)(\cos(dx+c)+1)}{\left(a^4+4\,a^3b+6\,a^2b^2+4\,ab^3+b^4\right)(\cos(dx+c)-1)} +}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a*sin(d*x+c)+b*tan(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/8*(2*(4*a + b)*log(abs(-cos(d*x + c) + 1)/abs(cos(d*x + c) + 1))/(a^4 + 4
*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4) - 16*(a^5 + 5*a^3*b^2)*log(abs(-a - b -
 a*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + b*(cos(d*x + c) - 1)/(cos(d*x +
c) + 1)))/(a^8 - 4*a^6*b^2 + 6*a^4*b^4 - 4*a^2*b^6 + b^8) + (a + b - 8*a*(c
os(d*x + c) - 1)/(cos(d*x + c) + 1) - 2*b*(cos(d*x + c) - 1)/(cos(d*x + c)
+ 1))*(cos(d*x + c) + 1)/((a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4)*(cos(
d*x + c) - 1)) + (cos(d*x + c) - 1)/((a^3 - 3*a^2*b + 3*a*b^2 - b^3)*(cos(d
*x + c) + 1)) + 8*(3*a^7 - 4*a^6*b - 2*a^5*b^2 + 20*a^4*b^3 + 15*a^3*b^4 +
4*a^7*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 10*a^6*b*(cos(d*x + c) - 1)/(
cos(d*x + c) + 1) + 26*a^5*b^2*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 10*a
^4*b^3*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 30*a^3*b^4*(cos(d*x + c) - 1
)/(cos(d*x + c) + 1) + 3*a^7*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 - 6*
a^6*b*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 + 18*a^5*b^2*(cos(d*x + c)
- 1)^2/(cos(d*x + c) + 1)^2 - 30*a^4*b^3*(cos(d*x + c) - 1)^2/(cos(d*x + c)
 + 1)^2 + 15*a^3*b^4*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2)/((a^8 - 4*a
^6*b^2 + 6*a^4*b^4 - 4*a^2*b^6 + b^8)*(a + b + a*(cos(d*x + c) - 1)/(cos(d*
x + c) + 1) - b*(cos(d*x + c) - 1)/(cos(d*x + c) + 1))^2))/d
```

maple [A] time = 0.25, size = 256, normalized size = 1.12

$$\frac{a^3}{2d(a+b)^2(a-b)^2(b+a\cos(dx+c))^2}-\frac{4a^3b}{d(a+b)^3(a-b)^3(b+a\cos(dx+c))}-\frac{2a^5\ln(b+a\cos(dx+c))}{d(a+b)^4(a-b)^4}-\frac{10a}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^3/(a*sin(d*x+c)+b*tan(d*x+c))^3,x)
```

```
[Out] 1/2/d*a^3/(a+b)^2/(a-b)^2/(b+a*cos(d*x+c))^2-4/d*a^3*b/(a+b)^3/(a-b)^3/(b+a
*cos(d*x+c))-2/d*a^5/(a+b)^4/(a-b)^4*ln(b+a*cos(d*x+c))-10/d*a^3*b^2/(a+b)^
4/(a-b)^4*ln(b+a*cos(d*x+c))+1/4/d/(a+b)^3/(cos(d*x+c)-1)+1/d/(a+b)^4*ln(co
s(d*x+c)-1)*a+1/4/d/(a+b)^4*ln(cos(d*x+c)-1)*b-1/4/d/(a-b)^3/(1+cos(d*x+c))
+a*ln(1+cos(d*x+c))/(a-b)^4/d-1/4*b*ln(1+cos(d*x+c))/(a-b)^4/d
```

maxima [B] time = 0.37, size = 591, normalized size = 2.59

$$-\frac{\frac{16\left(a^5+5a^3b^2\right)\log\left(a+b-\frac{(a-b)\sin(dx+c)^2}{(\cos(dx+c)+1)^2}\right)}{a^8-4a^6b^2+6a^4b^4-4a^2b^6+b^8}-\frac{4(4a+b)\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^4+4a^3b+6a^2b^2+4ab^3+b^4}+\frac{a^6-2a^5b-a^4b^2+4a^3b^3-a^2b^4-2ab^5+b^6-\frac{2(a^6\ldots}{}}{\frac{\left(a^9+a^8b-4a^7b^2-4a^6b^3+6a^5b^4+6a^4b^5-4a^3b^6-4a^2b^7+ab^8+b^9\right)\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-\frac{2(\ldots}{}}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a*sin(d*x+c)+b*tan(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -1/8*(16*(a^5 + 5*a^3*b^2)*log(a + b - (a - b)*sin(d*x + c)^2/(cos(d*x + c)
 + 1)^2)/(a^8 - 4*a^6*b^2 + 6*a^4*b^4 - 4*a^2*b^6 + b^8) - 4*(4*a + b)*log(
sin(d*x + c)/(cos(d*x + c) + 1))/(a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4
) + (a^6 - 2*a^5*b - a^4*b^2 + 4*a^3*b^3 - a^2*b^4 - 2*a*b^5 + b^6 - 2*(a^6
 - 44*a^5*b - 35*a^4*b^2 - 5*a^2*b^4 + 4*a*b^5 - b^6)*sin(d*x + c)^2/(cos(d
*x + c) + 1)^2 - (15*a^6 + 70*a^5*b - 95*a^4*b^2 + 20*a^3*b^3 - 15*a^2*b^4
+ 6*a*b^5 - b^6)*sin(d*x + c)^4/(cos(d*x + c) + 1)^4)/((a^9 + a^8*b - 4*a^7
*b^2 - 4*a^6*b^3 + 6*a^5*b^4 + 6*a^4*b^5 - 4*a^3*b^6 - 4*a^2*b^7 + a*b^8 +
b^9)*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 - 2*(a^9 - a^8*b - 4*a^7*b^2 + 4*a
^6*b^3 + 6*a^5*b^4 - 6*a^4*b^5 - 4*a^3*b^6 + 4*a^2*b^7 + a*b^8 - b^9)*sin(d
*x + c)^4/(cos(d*x + c) + 1)^4 + (a^9 - 3*a^8*b + 8*a^6*b^3 - 6*a^5*b^4 - 6
*a^4*b^5 + 8*a^3*b^6 - 3*a*b^8 + b^9)*sin(d*x + c)^6/(cos(d*x + c) + 1)^6)
+ sin(d*x + c)^2/((a^3 - 3*a^2*b + 3*a*b^2 - b^3)*(cos(d*x + c) + 1)^2))/d
```

mupad [B] time = 1.13, size = 490, normalized size = 2.15

$$\frac{\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4\,\left(15\,a^5+85\,a^4\,b-10\,a^3\,b^2+10\,a^2\,b^3-5\,a\,b^4+b^5\right)}{2\,(a+b)\,\left(a^2+2\,a\,b+b^2\right)}-\frac{a^3-3\,a^2\,b+3\,a\,b^2-b^3}{2\,(a+b)}}{d\,\left(\left(4\,a^5-20\,a^4\,b+40\,a^3\,b^2-40\,a^2\,b^3+20\,a\,b^4-4\,b^5\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6+\left(-8\,a^5+24\,a^4\,b-16\,a^3\,b^2-16\,a^2\,b^3+2\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^3*(a*sin(c + d*x) + b*tan(c + d*x))^3),x)
```

```
[Out] ((tan(c/2 + (d*x)/2)^4*(85*a^4*b - 5*a*b^4 + 15*a^5 + b^5 + 10*a^2*b^3 - 10
*a^3*b^2))/(2*(a + b)*(2*a*b + a^2 + b^2)) - (3*a*b^2 - 3*a^2*b + a^3 - b^3
)/(2*(a + b)) + (tan(c/2 + (d*x)/2)^2*(5*a*b^4 - 45*a^4*b + a^5 - b^5 - 10*
a^2*b^3 + 10*a^3*b^2))/((a - b)*(2*a*b + a^2 + b^2)))/(d*(tan(c/2 + (d*x)/2
)^2*(4*a*b^4 - 4*a^4*b + 4*a^5 - 4*b^5 + 8*a^2*b^3 - 8*a^3*b^2) - tan(c/2 +
 (d*x)/2)^4*(8*a^5 - 24*a^4*b - 24*a*b^4 + 8*b^5 + 16*a^2*b^3 + 16*a^3*b^2)
 + tan(c/2 + (d*x)/2)^6*(20*a*b^4 - 20*a^4*b + 4*a^5 - 4*b^5 - 40*a^2*b^3 +
 40*a^3*b^2))) - tan(c/2 + (d*x)/2)^2/(8*d*(a - b)^3) + (log(tan(c/2 + (d*x
)/2))*(4*a + b))/(d*(8*a*b^3 + 8*a^3*b + 2*a^4 + 2*b^4 + 12*a^2*b^2)) - (lo
g(a + b - a*tan(c/2 + (d*x)/2)^2 + b*tan(c/2 + (d*x)/2)^2)*(2*a^5 + 10*a^3*
b^2))/(d*(a^8 + b^8 - 4*a^2*b^6 + 6*a^4*b^4 - 4*a^6*b^2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec^3(c+dx)}{(a\sin(c+dx)+b\tan(c+dx))^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**3/(a*sin(d*x+c)+b*tan(d*x+c))**3,x)
```

```
[Out] Integral(sec(c + d*x)**3/(a*sin(c + d*x) + b*tan(c + d*x))**3, x)
```

3.271 $\int \cos^m(c+dx)(a\sin(c+dx)+b\tan(c+dx))^3\,dx$

Optimal. Leaf size=155

$$\frac{a^3\cos^{m+3}(c+dx)}{d(m+3)}-\frac{a\left(a^2-3b^2\right)\cos^{m+1}(c+dx)}{d(m+1)}-\frac{b\left(3a^2-b^2\right)\cos^m(c+dx)}{dm}+\frac{3a^2b\cos^{m+2}(c+dx)}{d(m+2)}+\frac{3ab^2\cos^{m-1}}{d(1-}$$

```
[Out] b^3*cos(d*x+c)^(-2+m)/d/(2-m)+3*a*b^2*cos(d*x+c)^(-1+m)/d/(1-m)-b*(3*a^2-b^
2)*cos(d*x+c)^m/d/m-a*(a^2-3*b^2)*cos(d*x+c)^(1+m)/d/(1+m)+3*a^2*b*cos(d*x+
c)^(2+m)/d/(2+m)+a^3*cos(d*x+c)^(3+m)/d/(3+m)
```

Rubi [A] time = 0.38, antiderivative size = 155, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.107, Rules used = {4397, 2837, 948}

$$-\frac{b\left(3a^2-b^2\right)\cos^m(c+dx)}{dm}-\frac{a\left(a^2-3b^2\right)\cos^{m+1}(c+dx)}{d(m+1)}+\frac{3a^2b\cos^{m+2}(c+dx)}{d(m+2)}+\frac{a^3\cos^{m+3}(c+dx)}{d(m+3)}+\frac{3ab^2\cos^{m-}}{d(1-}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^m*(a*Sin[c + d*x] + b*Tan[c + d*x])^3,x]
```

```
[Out] (b^3*Cos[c + d*x]^(-2 + m))/(d*(2 - m)) + (3*a*b^2*Cos[c + d*x]^(-1 + m))/(
d*(1 - m)) - (b*(3*a^2 - b^2)*Cos[c + d*x]^m)/(d*m) - (a*(a^2 - 3*b^2)*Cos[
c + d*x]^(1 + m))/(d*(1 + m)) + (3*a^2*b*Cos[c + d*x]^(2 + m))/(d*(2 + m))
+ (a^3*Cos[c + d*x]^(3 + m))/(d*(3 + m))
```

Rule 948

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IGtQ[p, 0] && (IGtQ[m, 0] || (EqQ[m, -2] && EqQ[p, 1] &
& EqQ[d, 0]))
```

Rule 2837

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^m*(c + (d*x)/b)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*S
in[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/
2] && NeQ[a^2 - b^2, 0]
```

Rule 4397

```
Int[u_, x_Symbol] :> Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]
```

Rubi steps

$$\int \cos^m(c+dx)(a\sin(c+dx)+b\tan(c+dx))^3\,dx = \int \cos^{-3+m}(c+dx)(b+a\cos(c+dx))^3\sin^3(c+dx)\,dx$$

$$= -\frac{\text{Subst}\left(\int \left(\frac{x}{a}\right)^{-3+m}(b+x)^3\left(a^2-x^2\right)\,dx, x, a\cos(c+dx)\right)}{a^3d}$$

$$= -\frac{\text{Subst}\left(\int \left(a^2b^3\left(\frac{x}{a}\right)^{-3+m}+3a^3b^2\left(\frac{x}{a}\right)^{-2+m}+a^2b\left(3a^2-b^2\right)\right.\right.}{}$$

$$= \frac{b^3\cos^{-2+m}(c+dx)}{d(2-m)}+\frac{3ab^2\cos^{-1+m}(c+dx)}{d(1-m)}-\frac{b\left(3a^2-b^2\right)c}{dr}$$

Mathematica [A] time = 1.45, size = 246, normalized size = 1.59

$$\frac{\cos^{m+1}(c+dx)(a+b\sec(c+dx))^3\left(-am\left(m^3-m^2-4m+4\right)\left(a^2(m+9)-12b^2(m+3)\right)\cos^3(c+dx)+\left(m^3-2r\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^m*(a*Sin[c + d*x] + b*Tan[c + d*x])^3,x]
```

```
[Out] (Cos[c + d*x]^(1 + m)*(-4*b^3*m*(-6 - 5*m + 5*m^2 + 5*m^3 + m^4) - 12*a*b^2
*m*(-12 - 16*m - m^2 + 4*m^3 + m^4)*Cos[c + d*x] - a*m*(4 - 4*m - m^2 + m^3
)*(-12*b^2*(3 + m) + a^2*(9 + m))*Cos[c + d*x]^3 + (2 - m - 2*m^2 + m^3)*Co
s[c + d*x]^2*(2*b*(3 + m)*(2*b^2*(2 + m) - 3*a^2*(4 + m)) + 6*a^2*b*m*(3 +
m)*Cos[2*(c + d*x)] + a^3*m*(2 + m)*Cos[3*(c + d*x)]))*(a + b*Sec[c + d*x])
^3)/(4*d*(-2 + m)*(-1 + m)*m*(1 + m)*(2 + m)*(3 + m)*(b + a*Cos[c + d*x])^3
)
```

fricas [B] time = 0.53, size = 412, normalized size = 2.66

$$-\frac{\left(b^3m^5+5\,b^3m^4+5\,b^3m^3-\left(a^3m^5-5\,a^3m^3+4\,a^3m\right)\cos\left(dx+c\right)^5-5\,b^3m^2-3\left(a^2bm^5+a^2bm^4-7\,a^2bm^3-a\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^m*(a*sin(d*x+c)+b*tan(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] -(b^3*m^5 + 5*b^3*m^4 + 5*b^3*m^3 - (a^3*m^5 - 5*a^3*m^3 + 4*a^3*m)*cos(d*x
 + c)^5 - 5*b^3*m^2 - 3*(a^2*b*m^5 + a^2*b*m^4 - 7*a^2*b*m^3 - a^2*b*m^2 +
```

```
6*a^2*b*m)*cos(d*x + c)^4 - 6*b^3*m + ((a^3 - 3*a*b^2)*m^5 + 2*(a^3 - 3*a*b
^2)*m^4 - 7*(a^3 - 3*a*b^2)*m^3 - 8*(a^3 - 3*a*b^2)*m^2 + 12*(a^3 - 3*a*b^2
)*m)*cos(d*x + c)^3 + ((3*a^2*b - b^3)*m^5 + 3*(3*a^2*b - b^3)*m^4 - 5*(3*a
^2*b - b^3)*m^3 + 36*a^2*b - 12*b^3 - 15*(3*a^2*b - b^3)*m^2 + 4*(3*a^2*b -
 b^3)*m)*cos(d*x + c)^2 + 3*(a*b^2*m^5 + 4*a*b^2*m^4 - a*b^2*m^3 - 16*a*b^2
*m^2 - 12*a*b^2*m)*cos(d*x + c))*cos(d*x + c)^m/((d*m^6 + 3*d*m^5 - 5*d*m^4
 - 15*d*m^3 + 4*d*m^2 + 12*d*m)*cos(d*x + c)^2)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^m*(a*sin(d*x+c)+b*tan(d*x+c))^3,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Modgcd: no suitable evaluation pointindex.cc
 index_m operator + Error: Bad Argument ValueUnable to check sign: (2*pi/t_
nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/
t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable
 to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (
2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>
(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2
)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check
sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nos
tep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_n
ostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to
 check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*p
i/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2
*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Un
able to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sig
n: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep
/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nost
ep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to ch
eck sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t
_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi
/t_nostep/2)Evaluation time: 61.5Done
```

maple [F] time = 4.64, size = 0, normalized size = 0.00

$$\int (\cos^{m}(dx+c))\,(a\sin(dx+c)+b\tan(dx+c))^{3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^m*(a*sin(d*x+c)+b*tan(d*x+c))^3,x)
```

[Out] int(cos(d*x+c)^m*(a*sin(d*x+c)+b*tan(d*x+c))^3,x)

maxima [A] time = 0.36, size = 180, normalized size = 1.16

$$\frac{\frac{\left((m+1)\cos(dx+c)^3-(m+3)\cos(dx+c)\right)a^3\cos(dx+c)^m}{m^2+4m+3}+\frac{3\left(m\cos(dx+c)^2-m-2\right)a^2b\cos(dx+c)^m}{m^2+2m}+\frac{3\left((m-1)\cos(dx+c)^2-m-1\right)ab^2\cos(dx+c)^m}{\left(m^2-1\right)\cos(dx+c)}+\frac{((}{}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^m*(a*sin(d*x+c)+b*tan(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] (((m + 1)*cos(d*x + c)^3 - (m + 3)*cos(d*x + c))*a^3*cos(d*x + c)^m/(m^2 +
4*m + 3) + 3*(m*cos(d*x + c)^2 - m - 2)*a^2*b*cos(d*x + c)^m/(m^2 + 2*m) +
3*((m - 1)*cos(d*x + c)^2 - m - 1)*a*b^2*cos(d*x + c)^m/((m^2 - 1)*cos(d*x
+ c)) + ((m - 2)*cos(d*x + c)^2 - m)*b^3*cos(d*x + c)^m/((m^2 - 2*m)*cos(d*
x + c)^2))/d
```

mupad [B] time = 7.51, size = 861, normalized size = 5.55

$$\left(\frac{1}{2}\right)^m\left(e^{-c\,1i-d\,x\,1i}+e^{c\,1i+d\,x\,1i}\right)^m\left(\frac{a^3\left(\frac{m^4}{8}-\frac{5\,m^2}{8}+\frac{1}{2}\right)}{d\left(m^5+3\,m^4-5\,m^3-15\,m^2+4\,m+12\right)}+\frac{a^3\,e^{c\,10i+d\,x\,10i}\left(m^4-5\,m^2+4\right)}{8\,d\left(m^5+3\,m^4-5\,m^3-15\,m^2+4\,m+12\right)}-\frac{a\,e^{c\,2i+d\,x\,2i}\left(-m^3+m^2+4\,m\right.}{8\,d\left(m^5+3\,m^4-\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^m*(a*sin(c + d*x) + b*tan(c + d*x))^3,x)
```

```
[Out] ((1/2)^m*(exp(- c*1i - d*x*1i) + exp(c*1i + d*x*1i))^m*((a^3*(m^4/8 - (5*m^
2)/8 + 1/2))/(d*(4*m - 15*m^2 - 5*m^3 + 3*m^4 + m^5 + 12)) + (a^3*exp(c*10i
 + d*x*10i)*(m^4 - 5*m^2 + 4))/(8*d*(4*m - 15*m^2 - 5*m^3 + 3*m^4 + m^5 + 1
2)) - (a*exp(c*2i + d*x*2i)*(4*m + m^2 - m^3 - 4)*(a^2*m + 12*b^2*m - 7*a^2
 + 36*b^2))/(8*d*(4*m - 15*m^2 - 5*m^3 + 3*m^4 + m^5 + 12)) - (a*exp(c*8i +
 d*x*8i)*(4*m + m^2 - m^3 - 4)*(a^2*m + 12*b^2*m - 7*a^2 + 36*b^2))/(8*d*(4
*m - 15*m^2 - 5*m^3 + 3*m^4 + m^5 + 12)) + (3*a^2*b*exp(c*1i + d*x*1i)*(m^3
 - 7*m^2 - m + m^4 + 6))/(4*d*(4*m - 15*m^2 - 5*m^3 + 3*m^4 + m^5 + 12)) +
(3*a^2*b*exp(c*9i + d*x*9i)*(m^3 - 7*m^2 - m + m^4 + 6))/(4*d*(4*m - 15*m^2
 - 5*m^3 + 3*m^4 + m^5 + 12)) - (a*exp(c*4i + d*x*4i)*(m^2 - 4)*(12*a^2*m +
 60*b^2*m - 13*a^2 + 126*b^2 + a^2*m^2 + 6*b^2*m^2))/(4*d*(4*m - 15*m^2 - 5
*m^3 + 3*m^4 + m^5 + 12)) - (a*exp(c*6i + d*x*6i)*(m^2 - 4)*(12*a^2*m + 60*
b^2*m - 13*a^2 + 126*b^2 + a^2*m^2 + 6*b^2*m^2))/(4*d*(4*m - 15*m^2 - 5*m^3
 + 3*m^4 + m^5 + 12)) + (b*exp(c*3i + d*x*3i)*(b^2*m - 6*a^2 + 2*b^2)*(m^3
- 7*m^2 - m + m^4 + 6))/(d*m*(4*m - 15*m^2 - 5*m^3 + 3*m^4 + m^5 + 12)) + (
b*exp(c*7i + d*x*7i)*(b^2*m - 6*a^2 + 2*b^2)*(m^3 - 7*m^2 - m + m^4 + 6))/(
d*m*(4*m - 15*m^2 - 5*m^3 + 3*m^4 + m^5 + 12)) + (b*exp(c*5i + d*x*5i)*(m -
```

```
 3*m^2 - m^3 + 3)*(18*a^2*m + 16*b^2*m - 48*a^2 + 16*b^2 + 3*a^2*m^2 + 4*b^
2*m^2))/(2*d*m*(4*m - 15*m^2 - 5*m^3 + 3*m^4 + m^5 + 12))))/(exp(c*3i + d*x
*3i) + 2*exp(c*5i + d*x*5i) + exp(c*7i + d*x*7i))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a\sin(c+dx)+b\tan(c+dx))^3\cos^m(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**m*(a*sin(d*x+c)+b*tan(d*x+c))**3,x)
```

```
[Out] Integral((a*sin(c + d*x) + b*tan(c + d*x))**3*cos(c + d*x)**m, x)
```

3.272 $\int \cos^m(c+dx)(a\sin(c+dx)+b\tan(c+dx))^2\,dx$

Optimal. Leaf size=264

$$-\frac{\left(a^2(1-m)-b^2(m+2)\right)\sin(c+dx)\cos^{m-1}(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{m-1}{2};\frac{m+1}{2};\cos^2(c+dx)\right)}{d(1-m)m(m+2)\sqrt{\sin^2(c+dx)}}+\frac{\left(a^2-2b^2\right)\sin(c+dx)\cos^n}{dm(m+2)}$$

```
[Out] (a^2-2*b^2)*cos(d*x+c)^(-1+m)*sin(d*x+c)/d/m/(2+m)-2*a*b*cos(d*x+c)^m*sin(d
*x+c)/d/(m^2+3*m+2)-cos(d*x+c)^(-1+m)*(b+a*cos(d*x+c))^2*sin(d*x+c)/d/(2+m)
-(a^2*(1-m)-b^2*(2+m))*cos(d*x+c)^(-1+m)*hypergeom([1/2, -1/2+1/2*m],[1/2+1
/2*m],cos(d*x+c)^2)*sin(d*x+c)/d/(1-m)/m/(2+m)/(sin(d*x+c)^2)^(1/2)-2*a*b*c
os(d*x+c)^m*hypergeom([1/2, 1/2*m],[1+1/2*m],cos(d*x+c)^2)*sin(d*x+c)/d/m/(
1+m)/(sin(d*x+c)^2)^(1/2)
```

Rubi [A] time = 0.76, antiderivative size = 264, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {4397, 2889, 3050, 3033, 3023, 2748, 2643}

$$-\frac{\left(a^2(1-m)-b^2(m+2)\right)\sin(c+dx)\cos^{m-1}(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{m-1}{2};\frac{m+1}{2};\cos^2(c+dx)\right)}{d(1-m)m(m+2)\sqrt{\sin^2(c+dx)}}+\frac{\left(a^2-2b^2\right)\sin(c+dx)\cos^n}{dm(m+2)}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^m*(a*Sin[c + d*x] + b*Tan[c + d*x])^2,x]
```

```
[Out] ((a^2 - 2*b^2)*Cos[c + d*x]^(-1 + m)*Sin[c + d*x])/(d*m*(2 + m)) - (2*a*b*C
os[c + d*x]^m*Sin[c + d*x])/(d*(2 + 3*m + m^2)) - (Cos[c + d*x]^(-1 + m)*(b
 + a*Cos[c + d*x])^2*Sin[c + d*x])/(d*(2 + m)) - ((a^2*(1 - m) - b^2*(2 + m
))*Cos[c + d*x]^(-1 + m)*Hypergeometric2F1[1/2, (-1 + m)/2, (1 + m)/2, Cos[
c + d*x]^2]*Sin[c + d*x])/(d*(1 - m)*m*(2 + m)*Sqrt[Sin[c + d*x]^2]) - (2*a
*b*Cos[c + d*x]^m*Hypergeometric2F1[1/2, m/2, (2 + m)/2, Cos[c + d*x]^2]*Si
n[c + d*x])/(d*m*(1 + m)*Sqrt[Sin[c + d*x]^2])
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
```

```
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2889

```
Int[cos[(e_.) + (f_.)*(x_)]^2*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[(d*Sin[e + f*x])^n*(a
 + b*Sin[e + f*x])^m*(1 - Sin[e + f*x]^2), x] /; FreeQ[{a, b, d, e, f, m, n
}, x] && NeQ[a^2 - b^2, 0] && (IGtQ[m, 0] || IntegersQ[2*m, 2*n])
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 3033

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> -Simp[(C*d*Cos[e + f*x]*Sin[e + f*x]*(a + b*Sin[
e + f*x])^(m + 1))/(b*f*(m + 3)), x] + Dist[1/(b*(m + 3)), Int[(a + b*Sin[e
 + f*x])^m*Simp[a*C*d + A*b*c*(m + 3) + b*(B*c*(m + 3) + d*(C*(m + 2) + A*(
m + 3)))*Sin[e + f*x] - (2*a*C*d - b*(c*C + B*d)*(m + 3))*Sin[e + f*x]^2, x
], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m}, x] && NeQ[b*c - a*d, 0]
&& NeQ[a^2 - b^2, 0] &&  !LtQ[m, -1]
```

Rule 3050

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :
> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)
)/(d*f*(m + n + 2)), x] + Dist[1/(d*(m + n + 2)), Int[(a + b*Sin[e + f*x])^
(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 2) + C*(b*c*m + a*d*(n +
 1)) + (A*b*d*(m + n + 2) - C*(a*c - b*d*(m + n + 1)))*Sin[e + f*x] + C*(a*
d*m - b*c*(m + 1))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A
, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
&& GtQ[m, 0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])
))
```

Rule 4397

```
Int[u_, x_Symbol] :> Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]
```

Rubi steps

$$\int \cos^m(c+dx)(a\sin(c+dx)+b\tan(c+dx))^2\,dx = \int \cos^{-2+m}(c+dx)(b+a\cos(c+dx))^2\sin^2(c+dx)\,dx$$
$$= \int \cos^{-2+m}(c+dx)(b+a\cos(c+dx))^2\left(1-\cos^2(c+dx)\right)d$$
$$= -\frac{\cos^{-1+m}(c+dx)(b+a\cos(c+dx))^2\sin(c+dx)}{d(2+m)} + \frac{\int \cos^{-2}}{}$$
$$= -\frac{2ab\cos^m(c+dx)\sin(c+dx)}{d\left(2+3m+m^2\right)} - \frac{\cos^{-1+m}(c+dx)(b+a\cos}{d(2+n}$$
$$= \frac{\left(a^2-2b^2\right)\cos^{-1+m}(c+dx)\sin(c+dx)}{dm(2+m)} - \frac{2ab\cos^m(c+dx)\,\mathrm{s}}{d\left(2+3m+\right.}$$
$$= \frac{\left(a^2-2b^2\right)\cos^{-1+m}(c+dx)\sin(c+dx)}{dm(2+m)} - \frac{2ab\cos^m(c+dx)\,\mathrm{s}}{d\left(2+3m+\right.}$$
$$= \frac{\left(a^2-2b^2\right)\cos^{-1+m}(c+dx)\sin(c+dx)}{dm(2+m)} - \frac{2ab\cos^m(c+dx)\,\mathrm{s}}{d\left(2+3m+\right.}$$

Mathematica [C] time = 30.60, size = 6669, normalized size = 25.26

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[c + d*x]^m*(a*Sin[c + d*x] + b*Tan[c + d*x])^2,x]
[Out] Result too large to show
```

fricas [F] time = 1.47, size = 0, normalized size = 0.00

$$\text{integral}\left(-\left(a^2\cos(dx+c)^2 - 2\,ab\sin(dx+c)\tan(dx+c) - b^2\tan(dx+c)^2 - a^2\right)\cos(dx+c)^m, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^m*(a*sin(d*x+c)+b*tan(d*x+c))^2,x, algorithm="fricas")
[Out] integral(-(a^2*cos(d*x + c)^2 - 2*a*b*sin(d*x + c)*tan(d*x + c) - b^2*tan(d
*x + c)^2 - a^2)*cos(d*x + c)^m, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a\sin(dx+c)+b\tan(dx+c))^2\cos(dx+c)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^m*(a*sin(d*x+c)+b*tan(d*x+c))^2,x, algorithm="giac")
[Out] integrate((a*sin(d*x + c) + b*tan(d*x + c))^2*cos(d*x + c)^m, x)
```

maple [F] time = 2.25, size = 0, normalized size = 0.00

$$\int (\cos^m(dx+c))(a\sin(dx+c)+b\tan(dx+c))^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^m*(a*sin(d*x+c)+b*tan(d*x+c))^2,x)
[Out] int(cos(d*x+c)^m*(a*sin(d*x+c)+b*tan(d*x+c))^2,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a\sin(dx+c)+b\tan(dx+c))^2\cos(dx+c)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^m*(a*sin(d*x+c)+b*tan(d*x+c))^2,x, algorithm="maxima")
[Out] integrate((a*sin(d*x + c) + b*tan(d*x + c))^2*cos(d*x + c)^m, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \cos(c+d\,x)^m\,(a\,\sin(c+d\,x)+b\tan(c+d\,x))^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^m*(a*sin(c + d*x) + b*tan(c + d*x))^2,x)
[Out] int(cos(c + d*x)^m*(a*sin(c + d*x) + b*tan(c + d*x))^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a\sin(c+dx)+b\tan(c+dx))^2\cos^m(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**m*(a*sin(d*x+c)+b*tan(d*x+c))**2,x)
[Out] Integral((a*sin(c + d*x) + b*tan(c + d*x))**2*cos(c + d*x)**m, x)
```

3.273 $\int \cos^m(c+dx)(a\sin(c+dx)+b\tan(c+dx))\,dx$

Optimal. Leaf size=39

$$-\frac{a\cos^{m+1}(c+dx)}{d(m+1)}-\frac{b\cos^m(c+dx)}{dm}$$

```
[Out] -b*cos(d*x+c)^m/d/m-a*cos(d*x+c)^(1+m)/d/(1+m)
```

Rubi [A] time = 0.07, antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {4377, 12, 2565, 30}

$$-\frac{a\cos^{m+1}(c+dx)}{d(m+1)}-\frac{b\cos^m(c+dx)}{dm}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^m*(a*Sin[c + d*x] + b*Tan[c + d*x]),x]
```

```
[Out] -((b*Cos[c + d*x]^m)/(d*m)) - (a*Cos[c + d*x]^(1 + m))/(d*(1 + m))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2565

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> -Dist[(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 4377

```
Int[(u_)*((v_) + (d_.)*(F_)[(c_.)*((a_.) + (b_.)*(x_))]^(n_.)), x_Symbol] :
> With[{e = FreeFactors[Cos[c*(a + b*x)], x]}, Int[ActivateTrig[u*v], x] +
Dist[d, Int[ActivateTrig[u]*Sin[c*(a + b*x)]^n, x], x] /; FunctionOfQ[Cos[c
*(a + b*x)]/e, u, x]] /; FreeQ[{a, b, c, d}, x] &&  !FreeQ[v, x] && Integer
Q[(n - 1)/2] && NonsumQ[u] && (EqQ[F, Sin] || EqQ[F, sin])
```

Rubi steps

$$\int \cos^m(c+dx)(a\sin(c+dx)+b\tan(c+dx))\,dx = a\int \cos^m(c+dx)\sin(c+dx)\,dx + \int b\cos^{-1+m}(c+dx)\sin$$

$$= b\int \cos^{-1+m}(c+dx)\sin(c+dx)\,dx - \frac{a\,\mathrm{Subst}\left(\int x^m\,dx,x,c\right.}{d}$$

$$= -\frac{a\cos^{1+m}(c+dx)}{d(1+m)} - \frac{b\,\mathrm{Subst}\left(\int x^{-1+m}\,dx,x,\cos(c+dx)\right)}{d}$$

$$= -\frac{b\cos^m(c+dx)}{dm} - \frac{a\cos^{1+m}(c+dx)}{d(1+m)}$$

Mathematica [A] time = 0.09, size = 35, normalized size = 0.90

$$-\frac{\cos^m(c+dx)(am\cos(c+dx)+bm+b)}{dm(m+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^m*(a*Sin[c + d*x] + b*Tan[c + d*x]),x]
[Out] -((Cos[c + d*x]^m*(b + b*m + a*m*Cos[c + d*x]))/(d*m*(1 + m)))
```

fricas [A] time = 0.52, size = 35, normalized size = 0.90

$$-\frac{(am\cos(dx+c)+bm+b)\cos(dx+c)^m}{dm^2+dm}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^m*(a*sin(d*x+c)+b*tan(d*x+c)),x, algorithm="fricas")
[Out] -(a*m*cos(d*x + c) + b*m + b)*cos(d*x + c)^m/(d*m^2 + d*m)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a\sin(dx+c)+b\tan(dx+c))\cos(dx+c)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^m*(a*sin(d*x+c)+b*tan(d*x+c)),x, algorithm="giac")
[Out] integrate((a*sin(d*x + c) + b*tan(d*x + c))*cos(d*x + c)^m, x)
```

maple [A] time = 0.09, size = 40, normalized size = 1.03

$$-\frac{b\left(\cos^{m}\left(dx+c\right)\right)}{dm}-\frac{a\left(\cos^{1+m}\left(dx+c\right)\right)}{d\left(1+m\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^m*(a*sin(d*x+c)+b*tan(d*x+c)),x)
[Out] -b*cos(d*x+c)^m/d/m-a*cos(d*x+c)^(1+m)/d/(1+m)
```

maxima [A] time = 0.34, size = 36, normalized size = 0.92

$$-\frac{\frac{a\cos(dx+c)^{m+1}}{m+1}+\frac{b\cos(dx+c)^{m}}{m}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^m*(a*sin(d*x+c)+b*tan(d*x+c)),x, algorithm="maxima")
[Out] -(a*cos(d*x + c)^(m + 1)/(m + 1) + b*cos(d*x + c)^m/m)/d
```

mupad [B] time = 0.98, size = 35, normalized size = 0.90

$$-\frac{\cos\left(c+d\,x\right)^m\,\left(b+b\,m+a\,m\,\cos\left(c+d\,x\right)\right)}{d\,m\,\left(m+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^m*(a*sin(c + d*x) + b*tan(c + d*x)),x)
[Out] -(cos(c + d*x)^m*(b + b*m + a*m*cos(c + d*x)))/(d*m*(m + 1))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a\sin\left(c+dx\right)+b\tan\left(c+dx\right)\right)\cos^{m}\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**m*(a*sin(d*x+c)+b*tan(d*x+c)),x)
[Out] Integral((a*sin(c + d*x) + b*tan(c + d*x))*cos(c + d*x)**m, x)
```

3.274 $\int \frac{\cos^m(c+dx)}{a\sin(c+dx)+b\tan(c+dx)}\,dx$

Optimal. Leaf size=144

$$-\frac{a^2\cos^{m+2}(c+dx)\,{}_2F_1\left(1,m+2;m+3;-\frac{a\cos(c+dx)}{b}\right)}{bd(m+2)\left(a^2-b^2\right)}+\frac{\cos^{m+2}(c+dx)\,{}_2F_1(1,m+2;m+3;-\cos(c+dx))}{2d(m+2)(a-b)}-\frac{\cos^{n}}{}$$

```
[Out] 1/2*cos(d*x+c)^(2+m)*hypergeom([1, 2+m],[3+m],-cos(d*x+c))/(a-b)/d/(2+m)-1/
2*cos(d*x+c)^(2+m)*hypergeom([1, 2+m],[3+m],cos(d*x+c))/(a+b)/d/(2+m)-a^2*c
os(d*x+c)^(2+m)*hypergeom([1, 2+m],[3+m],-a*cos(d*x+c)/b)/b/(a^2-b^2)/d/(2+
m)
```

Rubi [A] time = 0.43, antiderivative size = 144, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {4397, 2837, 961, 64}

$$-\frac{a^2\cos^{m+2}(c+dx)\,{}_2F_1\left(1,m+2;m+3;-\frac{a\cos(c+dx)}{b}\right)}{bd(m+2)\left(a^2-b^2\right)}+\frac{\cos^{m+2}(c+dx)\,{}_2F_1(1,m+2;m+3;-\cos(c+dx))}{2d(m+2)(a-b)}-\frac{\cos^{n}}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^m/(a*Sin[c + d*x] + b*Tan[c + d*x]),x]
```

```
[Out] (Cos[c + d*x]^(2 + m)*Hypergeometric2F1[1, 2 + m, 3 + m, -Cos[c + d*x]])/(2
*(a - b)*d*(2 + m)) - (Cos[c + d*x]^(2 + m)*Hypergeometric2F1[1, 2 + m, 3 +
 m, Cos[c + d*x]])/(2*(a + b)*d*(2 + m)) - (a^2*Cos[c + d*x]^(2 + m)*Hyperg
eometric2F1[1, 2 + m, 3 + m, -((a*Cos[c + d*x])/b)])/(b*(a^2 - b^2)*d*(2 +
m))
```

Rule 64

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(c^n*(b*x
)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*x)/c)])/(b*(m + 1)), x]
/; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 &&  !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-(d/(b*c)), 0])))
```

Rule 961

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && (IntegerQ[p] || (ILtQ[m, 0] && ILtQ[n, 0])) &&  !(IGtQ[
```

```
m, 0] || IGtQ[n, 0])
```

Rule 2837

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^m*(c + (d*x)/b)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*S
in[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/
2] && NeQ[a^2 - b^2, 0]
```

Rule 4397

```
Int[u_, x_Symbol] :> Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]
```

Rubi steps

$$\int \frac{\cos^m(c+dx)}{a\sin(c+dx)+b\tan(c+dx)}\,dx = \int \frac{\cos^{1+m}(c+dx)\csc(c+dx)}{b+a\cos(c+dx)}\,dx$$

$$= -\frac{a\,\mathrm{Subst}\left(\int \frac{\left(\frac{x}{a}\right)^{1+m}}{(b+x)\left(a^2-x^2\right)}\,dx, x, a\cos(c+dx)\right)}{d}$$

$$= -\frac{a\,\mathrm{Subst}\left(\int \left(\frac{\left(\frac{x}{a}\right)^{1+m}}{2a(a+b)(a-x)} - \frac{\left(\frac{x}{a}\right)^{1+m}}{2a(a-b)(a+x)} + \frac{\left(\frac{x}{a}\right)^{1+m}}{(a-b)(a+b)(b+x)}\right) dx, x, a\cos(c+dx)\right)}{d}$$

$$= \frac{\mathrm{Subst}\left(\int \frac{\left(\frac{x}{a}\right)^{1+m}}{a+x}\,dx, x, a\cos(c+dx)\right)}{2(a-b)d} - \frac{\mathrm{Subst}\left(\int \frac{\left(\frac{x}{a}\right)^{1+m}}{a-x}\,dx, x, a\cos(c+dx)\right)}{2(a+b)d}$$

$$= \frac{\cos^{2+m}(c+dx)\,{}_2F_1(1,2+m;3+m;-\cos(c+dx))}{2(a-b)d(2+m)} - \frac{\cos^{2+m}(c+dx)\,{}_2F_1(1,}{2(a+}$$

Mathematica [A] time = 0.37, size = 106, normalized size = 0.74

$$\frac{\cos^{m+2}(c+dx)\left(-2a^2\,{}_2F_1\left(1,m+2;m+3;-\frac{a\cos(c+dx)}{b}\right)+b(a+b)\,{}_2F_1(1,m+2;m+3;-\cos(c+dx))-b(a-b)\,{}_2}{2bd(m+2)(a-b)(a+b)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^m/(a*Sin[c + d*x] + b*Tan[c + d*x]),x]
```

```
[Out] (Cos[c + d*x]^(2 + m)*(b*(a + b)*Hypergeometric2F1[1, 2 + m, 3 + m, -Cos[c
+ d*x]] - (a - b)*b*Hypergeometric2F1[1, 2 + m, 3 + m, Cos[c + d*x]] - 2*a^
2*Hypergeometric2F1[1, 2 + m, 3 + m, -((a*Cos[c + d*x])/b)]))/(2*(a - b)*b*
(a + b)*d*(2 + m))
```

fricas [F] time = 0.47, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\cos\left(dx+c\right)^{m}}{a\sin\left(dx+c\right)+b\tan\left(dx+c\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^m/(a*sin(d*x+c)+b*tan(d*x+c)),x, algorithm="fricas")
```

```
[Out] integral(cos(d*x + c)^m/(a*sin(d*x + c) + b*tan(d*x + c)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\cos\left(dx+c\right)^{m}}{a\sin\left(dx+c\right)+b\tan\left(dx+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^m/(a*sin(d*x+c)+b*tan(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate(cos(d*x + c)^m/(a*sin(d*x + c) + b*tan(d*x + c)), x)
```

maple [F] time = 0.65, size = 0, normalized size = 0.00

$$\int\frac{\cos^{m}\left(dx+c\right)}{a\sin\left(dx+c\right)+b\tan\left(dx+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^m/(a*sin(d*x+c)+b*tan(d*x+c)),x)
```

```
[Out] int(cos(d*x+c)^m/(a*sin(d*x+c)+b*tan(d*x+c)),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\cos\left(dx+c\right)^{m}}{a\sin\left(dx+c\right)+b\tan\left(dx+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^m/(a*sin(d*x+c)+b*tan(d*x+c)),x, algorithm="maxima")
```

```
[Out] integrate(cos(d*x + c)^m/(a*sin(d*x + c) + b*tan(d*x + c)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos(c + d\,x)\,\cos(c + d\,x)^m}{\sin(c + d\,x)\,\left(b + a\,\cos(c + d\,x)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^m/(a*sin(c + d*x) + b*tan(c + d*x)),x)
[Out] int((cos(c + d*x)*cos(c + d*x)^m)/(sin(c + d*x)*(b + a*cos(c + d*x))), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos^m(c + dx)}{a\sin(c + dx) + b\tan(c + dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**m/(a*sin(d*x+c)+b*tan(d*x+c)),x)
[Out] Integral(cos(c + d*x)**m/(a*sin(c + d*x) + b*tan(c + d*x)), x)
```

3.275 $\int \frac{\cos(x)\sin(x)}{a\cos(x)+b\sin(x)}\,dx$

Optimal. Leaf size=65

$$\frac{b\sin(x)}{a^2+b^2}-\frac{a\cos(x)}{a^2+b^2}+\frac{ab\tanh^{-1}\left(\frac{b\cos(x)-a\sin(x)}{\sqrt{a^2+b^2}}\right)}{\left(a^2+b^2\right)^{3/2}}$$

```
[Out] a*b*arctanh((b*cos(x)-a*sin(x))/(a^2+b^2)^(1/2))/(a^2+b^2)^(3/2)-a*cos(x)/(
a^2+b^2)+b*sin(x)/(a^2+b^2)
```

Rubi [A] time = 0.07, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312, Rules used = {3109, 2637, 2638, 3074, 206}

$$\frac{b\sin(x)}{a^2+b^2}-\frac{a\cos(x)}{a^2+b^2}+\frac{ab\tanh^{-1}\left(\frac{b\cos(x)-a\sin(x)}{\sqrt{a^2+b^2}}\right)}{\left(a^2+b^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[x]*Sin[x])/(a*Cos[x] + b*Sin[x]),x]
```

```
[Out] (a*b*ArcTanh[(b*Cos[x] - a*Sin[x])/Sqrt[a^2 + b^2]])/(a^2 + b^2)^(3/2) - (a
*Cos[x])/(a^2 + b^2) + (b*Sin[x])/(a^2 + b^2)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3074

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x
_Symbol] :> -Dist[d^(-1), Subst[Int[1/(a^2 + b^2 - x^2), x], x, b*Cos[c + d
*x] - a*Sin[c + d*x]], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

Rule 3109

```
Int[(cos[(c_.) + (d_.)*(x_)]^(m_.)*sin[(c_.) + (d_.)*(x_)]^(n_.))/(cos[(c_.
) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[b
/(a^2 + b^2), Int[Cos[c + d*x]^m*Sin[c + d*x]^(n - 1), x], x] + (Dist[a/(a^
2 + b^2), Int[Cos[c + d*x]^(m - 1)*Sin[c + d*x]^n, x], x] - Dist[(a*b)/(a^2
 + b^2), Int[(Cos[c + d*x]^(m - 1)*Sin[c + d*x]^(n - 1))/(a*Cos[c + d*x] +
b*Sin[c + d*x]), x], x]) /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0] &&
IGtQ[m, 0] && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\cos(x)\sin(x)}{a\cos(x)+b\sin(x)}\,dx &= \frac{a\int \sin(x)\,dx}{a^2+b^2}+\frac{b\int \cos(x)\,dx}{a^2+b^2}-\frac{(ab)\int \frac{1}{a\cos(x)+b\sin(x)}\,dx}{a^2+b^2}\\
&= -\frac{a\cos(x)}{a^2+b^2}+\frac{b\sin(x)}{a^2+b^2}+\frac{(ab)\,\mathrm{Subst}\left(\int \frac{1}{a^2+b^2-x^2}\,dx, x, b\cos(x)-a\sin(x)\right)}{a^2+b^2}\\
&= \frac{ab\tanh^{-1}\left(\frac{b\cos(x)-a\sin(x)}{\sqrt{a^2+b^2}}\right)}{\left(a^2+b^2\right)^{3/2}}-\frac{a\cos(x)}{a^2+b^2}+\frac{b\sin(x)}{a^2+b^2}
\end{aligned}$$

Mathematica [A] time = 0.14, size = 61, normalized size = 0.94

$$\frac{b\sin(x)-a\cos(x)}{a^2+b^2}-\frac{2ab\tanh^{-1}\left(\frac{a\tan\left(\frac{x}{2}\right)-b}{\sqrt{a^2+b^2}}\right)}{\left(a^2+b^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[x]*Sin[x])/(a*Cos[x] + b*Sin[x]),x]
```

```
[Out] (-2*a*b*ArcTanh[(-b + a*Tan[x/2])/Sqrt[a^2 + b^2]])/(a^2 + b^2)^(3/2) + (-(
a*Cos[x]) + b*Sin[x])/(a^2 + b^2)
```

fricas [B] time = 0.65, size = 142, normalized size = 2.18

$$\frac{\sqrt{a^2+b^2}\,ab\log\left(\frac{2\,ab\cos(x)\sin(x)+\left(a^2-b^2\right)\cos(x)^2-2\,a^2-b^2-2\,\sqrt{a^2+b^2}\,(b\cos(x)-a\sin(x))}{2\,ab\cos(x)\sin(x)+\left(a^2-b^2\right)\cos(x)^2+b^2}\right)-2\left(a^3+ab^2\right)\cos(x)+2\left(a^2b+b^3\right)\mathrm{s}}{2\left(a^4+2\,a^2b^2+b^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*sin(x)/(a*cos(x)+b*sin(x)),x, algorithm="fricas")
```

```
[Out] 1/2*(sqrt(a^2 + b^2)*a*b*log((2*a*b*cos(x)*sin(x) + (a^2 - b^2)*cos(x)^2 -
2*a^2 - b^2 - 2*sqrt(a^2 + b^2)*(b*cos(x) - a*sin(x)))/(2*a*b*cos(x)*sin(x)
 + (a^2 - b^2)*cos(x)^2 + b^2)) - 2*(a^3 + a*b^2)*cos(x) + 2*(a^2*b + b^3)*
sin(x))/(a^4 + 2*a^2*b^2 + b^4)
```

giac [A] time = 5.98, size = 94, normalized size = 1.45

$$\frac{ab\log\left(\frac{\left|2\,a\tan\left(\frac{1}{2}\,x\right)-2\,b-2\,\sqrt{a^2+b^2}\right|}{\left|2\,a\tan\left(\frac{1}{2}\,x\right)-2\,b+2\,\sqrt{a^2+b^2}\right|}\right)}{\left(a^2+b^2\right)^{\frac{3}{2}}}+\frac{2\left(b\tan\left(\frac{1}{2}\,x\right)-a\right)}{\left(a^2+b^2\right)\left(\tan\left(\frac{1}{2}\,x\right)^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*sin(x)/(a*cos(x)+b*sin(x)),x, algorithm="giac")
```

```
[Out] a*b*log(abs(2*a*tan(1/2*x) - 2*b - 2*sqrt(a^2 + b^2))/abs(2*a*tan(1/2*x) -
2*b + 2*sqrt(a^2 + b^2)))/(a^2 + b^2)^(3/2) + 2*(b*tan(1/2*x) - a)/((a^2 +
b^2)*(tan(1/2*x)^2 + 1))
```

maple [A] time = 0.08, size = 81, normalized size = 1.25

$$-\frac{4ab\,\mathrm{arctanh}\left(\frac{2a\tan\left(\frac{x}{2}\right)-2b}{2\sqrt{a^2+b^2}}\right)}{\left(2a^2+2b^2\right)\sqrt{a^2+b^2}}-\frac{2\left(-b\tan\left(\frac{x}{2}\right)+a\right)}{\left(a^2+b^2\right)\left(\tan^2\left(\frac{x}{2}\right)+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)*sin(x)/(a*cos(x)+b*sin(x)),x)
```

```
[Out] -4*a*b/(2*a^2+2*b^2)/(a^2+b^2)^(1/2)*arctanh(1/2*(2*a*tan(1/2*x)-2*b)/(a^2+
b^2)^(1/2))-2/(a^2+b^2)*(-b*tan(1/2*x)+a)/(tan(1/2*x)^2+1)
```

maxima [A] time = 0.45, size = 105, normalized size = 1.62

$$\frac{ab\log\left(\frac{b-\frac{a\sin(x)}{\cos(x)+1}+\sqrt{a^2+b^2}}{b-\frac{a\sin(x)}{\cos(x)+1}-\sqrt{a^2+b^2}}\right)}{\left(a^2+b^2\right)^{\frac{3}{2}}}-\frac{2\left(a-\frac{b\sin(x)}{\cos(x)+1}\right)}{a^2+b^2+\frac{\left(a^2+b^2\right)\sin(x)^2}{(\cos(x)+1)^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*sin(x)/(a*cos(x)+b*sin(x)),x, algorithm="maxima")
```

```
[Out] a*b*log((b - a*sin(x)/(cos(x) + 1) + sqrt(a^2 + b^2))/(b - a*sin(x)/(cos(x)
 + 1) - sqrt(a^2 + b^2)))/(a^2 + b^2)^(3/2) - 2*(a - b*sin(x)/(cos(x) + 1))
/(a^2 + b^2 + (a^2 + b^2)*sin(x)^2/(cos(x) + 1)^2)
```

mupad [B] time = 1.38, size = 93, normalized size = 1.43

$$\frac{2\,a\,b\,\mathrm{atanh}\left(\frac{2\,a^2\,b+2\,b^3-2\,a\,\tan\left(\frac{x}{2}\right)\,\left(a^2+b^2\right)}{2\,\left(a^2+b^2\right)^{3/2}}\right)}{\left(a^2+b^2\right)^{3/2}}-\frac{\frac{2\,a}{a^2+b^2}-\frac{2\,b\,\tan\left(\frac{x}{2}\right)}{a^2+b^2}}{\tan\left(\frac{x}{2}\right)^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(x)*sin(x))/(a*cos(x) + b*sin(x)),x)
```

```
[Out] (2*a*b*atanh((2*a^2*b + 2*b^3 - 2*a*tan(x/2)*(a^2 + b^2))/(2*(a^2 + b^2)^(3
/2))))/(a^2 + b^2)^(3/2) - ((2*a)/(a^2 + b^2) - (2*b*tan(x/2))/(a^2 + b^2))
/(tan(x/2)^2 + 1)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*sin(x)/(a*cos(x)+b*sin(x)),x)
```

```
[Out] Timed out
```

3.276 $\int \frac{\cos(x)\sin^2(x)}{a\cos(x)+b\sin(x)}\,dx$

Optimal. Leaf size=92

$$\frac{ax}{2\left(a^2+b^2\right)}-\frac{ab^2x}{\left(a^2+b^2\right)^2}+\frac{b\sin^2(x)}{2\left(a^2+b^2\right)}-\frac{a\sin(x)\cos(x)}{2\left(a^2+b^2\right)}+\frac{a^2b\log(a\cos(x)+b\sin(x))}{\left(a^2+b^2\right)^2}$$

```
[Out] -a*b^2*x/(a^2+b^2)^2+1/2*a*x/(a^2+b^2)+a^2*b*ln(a*cos(x)+b*sin(x))/(a^2+b^2
)^2-1/2*a*cos(x)*sin(x)/(a^2+b^2)+1/2*b*sin(x)^2/(a^2+b^2)
```

Rubi [A] time = 0.14, antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.389, Rules used = {3109, 2564, 30, 2635, 8, 3097, 3133}

$$\frac{ax}{2\left(a^2+b^2\right)}-\frac{ab^2x}{\left(a^2+b^2\right)^2}+\frac{b\sin^2(x)}{2\left(a^2+b^2\right)}-\frac{a\sin(x)\cos(x)}{2\left(a^2+b^2\right)}+\frac{a^2b\log(a\cos(x)+b\sin(x))}{\left(a^2+b^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[x]*Sin[x]^2)/(a*Cos[x] + b*Sin[x]),x]
```

```
[Out] -((a*b^2*x)/(a^2 + b^2)^2) + (a*x)/(2*(a^2 + b^2)) + (a^2*b*Log[a*Cos[x] +
b*Sin[x]])/(a^2 + b^2)^2 - (a*Cos[x]*Sin[x])/(2*(a^2 + b^2)) + (b*Sin[x]^2)
/(2*(a^2 + b^2))
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2564

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 3097

```
Int[sin[(c_.) + (d_.)*(x_)]/(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.
) + (d_.)*(x_)]), x_Symbol] :> Simp[(b*x)/(a^2 + b^2), x] - Dist[a/(a^2 + b
^2), Int[(b*Cos[c + d*x] - a*Sin[c + d*x])/(a*Cos[c + d*x] + b*Sin[c + d*x]
), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

Rule 3109

```
Int[(cos[(c_.) + (d_.)*(x_)]^(m_.)*sin[(c_.) + (d_.)*(x_)]^(n_.))/(cos[(c_.
) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[b
/(a^2 + b^2), Int[Cos[c + d*x]^m*Sin[c + d*x]^(n - 1), x], x] + (Dist[a/(a^
2 + b^2), Int[Cos[c + d*x]^(m - 1)*Sin[c + d*x]^n, x], x] - Dist[(a*b)/(a^2
 + b^2), Int[(Cos[c + d*x]^(m - 1)*Sin[c + d*x]^(n - 1))/(a*Cos[c + d*x] +
b*Sin[c + d*x]), x], x]) /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0] &&
IGtQ[m, 0] && IGtQ[n, 0]
```

Rule 3133

```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])
/((a_.) + cos[(d_.) + (e_.)*(x_)]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)]), x
_Symbol] :> Simp[((b*B + c*C)*x)/(b^2 + c^2), x] + Simp[((c*B - b*C)*Log[a
+ b*Cos[d + e*x] + c*Sin[d + e*x]])/(e*(b^2 + c^2)), x] /; FreeQ[{a, b, c,
d, e, A, B, C}, x] && NeQ[b^2 + c^2, 0] && EqQ[A*(b^2 + c^2) - a*(b*B + c*C
), 0]
```

Rubi steps

$$\int \frac{\cos(x)\sin^2(x)}{a\cos(x)+b\sin(x)}\,dx = \frac{a\int \sin^2(x)\,dx}{a^2+b^2} + \frac{b\int \cos(x)\sin(x)\,dx}{a^2+b^2} - \frac{(ab)\int \frac{\sin(x)}{a\cos(x)+b\sin(x)}\,dx}{a^2+b^2}$$

$$= -\frac{ab^2x}{\left(a^2+b^2\right)^2} - \frac{a\cos(x)\sin(x)}{2\left(a^2+b^2\right)} + \frac{\left(a^2b\right)\int \frac{b\cos(x)-a\sin(x)}{a\cos(x)+b\sin(x)}\,dx}{\left(a^2+b^2\right)^2} + \frac{a\int 1\,dx}{2\left(a^2+b^2\right)} + \frac{b\,\mathrm{Subst}(\int$$

$$= -\frac{ab^2x}{\left(a^2+b^2\right)^2} + \frac{ax}{2\left(a^2+b^2\right)} + \frac{a^2b\log(a\cos(x)+b\sin(x))}{\left(a^2+b^2\right)^2} - \frac{a\cos(x)\sin(x)}{2\left(a^2+b^2\right)} + \frac{b\sin^2(}{2\left(a^2+\right.}$$

Mathematica [C] time = 0.34, size = 153, normalized size = 1.66

$$-\frac{-2a^3x+2a^3\sin(2x)+2b\left(a^2+b^2\right)\cos(2x)-2ib\left(b^2-3a^2\right)\tan^{-1}(\tan(x))-2\left(a^2+b^2\right)(b\log(a\cos(x)+b\sin}{8}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[x]*Sin[x]^2)/(a*Cos[x] + b*Sin[x]),x]
```

```
[Out] -1/8*(-2*a^3*x - (6*I)*a^2*b*x + 6*a*b^2*x + (2*I)*b^3*x - (2*I)*b*(-3*a^2
+ b^2)*ArcTan[Tan[x]] + 2*b*(a^2 + b^2)*Cos[2*x] - 2*(a^2 + b^2)*(a*x + b*L
og[a*Cos[x] + b*Sin[x]]) - 3*a^2*b*Log[(a*Cos[x] + b*Sin[x])^2] + b^3*Log[(
a*Cos[x] + b*Sin[x])^2] + 2*a^3*Sin[2*x] + 2*a*b^2*Sin[2*x])/(a^2 + b^2)^2
```

fricas [A] time = 0.47, size = 94, normalized size = 1.02

$$\frac{a^2b\log\left(2\,ab\cos(x)\sin(x)+\left(a^2-b^2\right)\cos(x)^2+b^2\right)-\left(a^2b+b^3\right)\cos(x)^2-\left(a^3+ab^2\right)\cos(x)\sin(x)+\left(a^3-ab^2\right.}{2\left(a^4+2\,a^2b^2+b^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*sin(x)^2/(a*cos(x)+b*sin(x)),x, algorithm="fricas")
```

```
[Out] 1/2*(a^2*b*log(2*a*b*cos(x)*sin(x) + (a^2 - b^2)*cos(x)^2 + b^2) - (a^2*b +
 b^3)*cos(x)^2 - (a^3 + a*b^2)*cos(x)*sin(x) + (a^3 - a*b^2)*x)/(a^4 + 2*a^
2*b^2 + b^4)
```

giac [A] time = 2.01, size = 152, normalized size = 1.65

$$\frac{a^2b^2\log\left(|b\tan(x)+a|\right)}{a^4b+2\,a^2b^3+b^5}-\frac{a^2b\log\left(\tan(x)^2+1\right)}{2\left(a^4+2\,a^2b^2+b^4\right)}+\frac{\left(a^3-ab^2\right)x}{2\left(a^4+2\,a^2b^2+b^4\right)}+\frac{a^2b\tan(x)^2-a^3\tan(x)-ab^2\tan(x)-b^3}{2\left(a^4+2\,a^2b^2+b^4\right)\left(\tan(x)^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*sin(x)^2/(a*cos(x)+b*sin(x)),x, algorithm="giac")
```

```
[Out] a^2*b^2*log(abs(b*tan(x) + a))/(a^4*b + 2*a^2*b^3 + b^5) - 1/2*a^2*b*log(ta
n(x)^2 + 1)/(a^4 + 2*a^2*b^2 + b^4) + 1/2*(a^3 - a*b^2)*x/(a^4 + 2*a^2*b^2
+ b^4) + 1/2*(a^2*b*tan(x)^2 - a^3*tan(x) - a*b^2*tan(x) - b^3)/((a^4 + 2*a
^2*b^2 + b^4)*(tan(x)^2 + 1))
```

maple [B] time = 0.08, size = 174, normalized size = 1.89

$$-\frac{\tan(x)a^3}{2\left(a^2+b^2\right)^2\left(\tan^2(x)+1\right)}-\frac{\tan(x)a\,b^2}{2\left(a^2+b^2\right)^2\left(\tan^2(x)+1\right)}-\frac{a^2b}{2\left(a^2+b^2\right)^2\left(\tan^2(x)+1\right)}-\frac{b^3}{2\left(a^2+b^2\right)^2\left(\tan^2(x)+1\right)}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)*sin(x)^2/(a*cos(x)+b*sin(x)),x)
```

```
[Out] -1/2/(a^2+b^2)^2/(tan(x)^2+1)*tan(x)*a^3-1/2/(a^2+b^2)^2/(tan(x)^2+1)*tan(x
)*a*b^2-1/2/(a^2+b^2)^2/(tan(x)^2+1)*a^2*b-1/2/(a^2+b^2)^2/(tan(x)^2+1)*b^3
-1/2/(a^2+b^2)^2*ln(tan(x)^2+1)*a^2*b+1/2/(a^2+b^2)^2*arctan(tan(x))*a^3-1/
2/(a^2+b^2)^2*arctan(tan(x))*a*b^2+b*a^2/(a^2+b^2)^2*ln(a+b*tan(x))
```

maxima [B] time = 0.46, size = 211, normalized size = 2.29

$$\frac{a^2 b \log\left(-a - \frac{2\,b\sin(x)}{\cos(x)+1} + \frac{a\sin(x)^2}{(\cos(x)+1)^2}\right)}{a^4 + 2\,a^2 b^2 + b^4} - \frac{a^2 b \log\left(\frac{\sin(x)^2}{(\cos(x)+1)^2} + 1\right)}{a^4 + 2\,a^2 b^2 + b^4} + \frac{\left(a^3 - ab^2\right)\arctan\left(\frac{\sin(x)}{\cos(x)+1}\right)}{a^4 + 2\,a^2 b^2 + b^4} - \frac{\frac{a\sin(x)}{\cos(x)+1} - \frac{2\,b\sin(x)^2}{(\cos(x)+1)}}{a^2 + b^2 + \frac{2\left(a^2+b^2\right)\sin(x}{(\cos(x)+1)^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*sin(x)^2/(a*cos(x)+b*sin(x)),x, algorithm="maxima")
```

```
[Out] a^2*b*log(-a - 2*b*sin(x)/(cos(x) + 1) + a*sin(x)^2/(cos(x) + 1)^2)/(a^4 +
2*a^2*b^2 + b^4) - a^2*b*log(sin(x)^2/(cos(x) + 1)^2 + 1)/(a^4 + 2*a^2*b^2
+ b^4) + (a^3 - a*b^2)*arctan(sin(x)/(cos(x) + 1))/(a^4 + 2*a^2*b^2 + b^4)
- (a*sin(x)/(cos(x) + 1) - 2*b*sin(x)^2/(cos(x) + 1)^2 - a*sin(x)^3/(cos(x)
 + 1)^3)/(a^2 + b^2 + 2*(a^2 + b^2)*sin(x)^2/(cos(x) + 1)^2 + (a^2 + b^2)*s
in(x)^4/(cos(x) + 1)^4)
```

mupad [B] time = 6.23, size = 3401, normalized size = 36.97

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(x)*sin(x)^2)/(a*cos(x) + b*sin(x)),x)
```

```
[Out] ((a*tan(x/2)^3)/(a^2 + b^2) - (a*tan(x/2))/(a^2 + b^2) + (2*b*tan(x/2)^2)/(
a^2 + b^2))/(2*tan(x/2)^2 + tan(x/2)^4 + 1) + (a^2*b*log(a + 2*b*tan(x/2) -
 a*tan(x/2)^2))/(a^4 + b^4 + 2*a^2*b^2) - (4*a^2*b*log(1/(cos(x) + 1)))/(4*
a^4 + 4*b^4 + 8*a^2*b^2) - (a*atan((tan(x/2)*((((4*a^2*b*((a*(a + b)*((8*(1
2*a^9*b + 12*a^5*b^5 + 24*a^7*b^3))/(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2) - (
32*a^2*b*(12*a*b^10 + 48*a^3*b^8 + 72*a^5*b^6 + 48*a^7*b^4 + 12*a^9*b^2))/(
(4*a^4 + 4*b^4 + 8*a^2*b^2)*(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2)))*(a - b))/
(2*(a^4 + b^4 + 2*a^2*b^2)) - (16*a^3*b*(a + b)*(a - b)*(12*a*b^10 + 48*a^3
*b^8 + 72*a^5*b^6 + 48*a^7*b^4 + 12*a^9*b^2))/((4*a^4 + 4*b^4 + 8*a^2*b^2)*
(a^4 + b^4 + 2*a^2*b^2)*(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2))))/(4*a^4 + 4*b
^4 + 8*a^2*b^2) - (a*((8*(a^9 + 2*a^3*b^6 - 7*a^5*b^4 - 8*a^7*b^2))/(a^6 +
b^6 + 3*a^2*b^4 + 3*a^4*b^2) - (4*a^2*b*((8*(12*a^9*b + 12*a^5*b^5 + 24*a^7
```

```
*b^3))/(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2) - (32*a^2*b*(12*a*b^10 + 48*a^3*
b^8 + 72*a^5*b^6 + 48*a^7*b^4 + 12*a^9*b^2))/((4*a^4 + 4*b^4 + 8*a^2*b^2)*(
a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2))))/(4*a^4 + 4*b^4 + 8*a^2*b^2))*(a + b)*
(a - b))/(2*(a^4 + b^4 + 2*a^2*b^2)) + (a^3*(a + b)^3*(a - b)^3*(12*a*b^10
+ 48*a^3*b^8 + 72*a^5*b^6 + 48*a^7*b^4 + 12*a^9*b^2))/((a^4 + b^4 + 2*a^2*b
^2)^3*(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2)))*(a^6 - b^6 + 35*a^2*b^4 - 35*a^
4*b^2))/(a^6 + b^6 + 15*a^2*b^4 + 15*a^4*b^2)^2 - (2*a*b*(5*a^4 + 5*b^4 - 2
6*a^2*b^2)*((8*(a^7*b + 2*a^5*b^3))/(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2) + (
4*a^2*b*((8*(a^9 + 2*a^3*b^6 - 7*a^5*b^4 - 8*a^7*b^2))/(a^6 + b^6 + 3*a^2*b
^4 + 3*a^4*b^2) - (4*a^2*b*((8*(12*a^9*b + 12*a^5*b^5 + 24*a^7*b^3))/(a^6 +
 b^6 + 3*a^2*b^4 + 3*a^4*b^2) - (32*a^2*b*(12*a*b^10 + 48*a^3*b^8 + 72*a^5*
b^6 + 48*a^7*b^4 + 12*a^9*b^2))/((4*a^4 + 4*b^4 + 8*a^2*b^2)*(a^6 + b^6 + 3
*a^2*b^4 + 3*a^4*b^2))))/(4*a^4 + 4*b^4 + 8*a^2*b^2)))/(4*a^4 + 4*b^4 + 8*a
^2*b^2) + (a*(a + b)*(a - b)*((a*(a + b)*((8*(12*a^9*b + 12*a^5*b^5 + 24*a^
7*b^3))/(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2) - (32*a^2*b*(12*a*b^10 + 48*a^3
*b^8 + 72*a^5*b^6 + 48*a^7*b^4 + 12*a^9*b^2))/((4*a^4 + 4*b^4 + 8*a^2*b^2)*
(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2)))*(a - b))/(2*(a^4 + b^4 + 2*a^2*b^2))
- (16*a^3*b*(a + b)*(a - b)*(12*a*b^10 + 48*a^3*b^8 + 72*a^5*b^6 + 48*a^7*b
^4 + 12*a^9*b^2))/((4*a^4 + 4*b^4 + 8*a^2*b^2)*(a^4 + b^4 + 2*a^2*b^2)*(a^6
 + b^6 + 3*a^2*b^4 + 3*a^4*b^2))))/(2*(a^4 + b^4 + 2*a^2*b^2)) - (8*a^4*b*(
a + b)^2*(a - b)^2*(12*a*b^10 + 48*a^3*b^8 + 72*a^5*b^6 + 48*a^7*b^4 + 12*a
^9*b^2))/((4*a^4 + 4*b^4 + 8*a^2*b^2)*(a^4 + b^4 + 2*a^2*b^2)^2*(a^6 + b^6
+ 3*a^2*b^4 + 3*a^4*b^2))))/(a^6 + b^6 + 15*a^2*b^4 + 15*a^4*b^2)^2)*(a^10
+ b^10 + 5*a^2*b^8 + 10*a^4*b^6 + 10*a^6*b^4 + 5*a^8*b^2))/(4*a^6 - 4*a^4*b
^2) + (((a*(a + b)*((8*(3*a^8*b + 3*a^4*b^5 + 6*a^6*b^3))/(a^6 + b^6 + 3*a^
2*b^4 + 3*a^4*b^2) - (4*a^2*b*((8*(4*a^4*b^6 - 2*a^2*b^8 - 2*a^10 + 12*a^6*
b^4 + 4*a^8*b^2))/(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2) + (32*a^2*b*(12*a^10*
b + 12*a^2*b^9 + 48*a^4*b^7 + 72*a^6*b^5 + 48*a^8*b^3))/((4*a^4 + 4*b^4 + 8
*a^2*b^2)*(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2))))/(4*a^4 + 4*b^4 + 8*a^2*b^2
))*(a - b))/(2*(a^4 + b^4 + 2*a^2*b^2)) - (4*a^2*b*((a*(a + b)*(a - b)*((8*
(4*a^4*b^6 - 2*a^2*b^8 - 2*a^10 + 12*a^6*b^4 + 4*a^8*b^2))/(a^6 + b^6 + 3*a
^2*b^4 + 3*a^4*b^2) + (32*a^2*b*(12*a^10*b + 12*a^2*b^9 + 48*a^4*b^7 + 72*a
^6*b^5 + 48*a^8*b^3))/((4*a^4 + 4*b^4 + 8*a^2*b^2)*(a^6 + b^6 + 3*a^2*b^4 +
 3*a^4*b^2))))/(2*(a^4 + b^4 + 2*a^2*b^2)) + (16*a^3*b*(a + b)*(a - b)*(12*
a^10*b + 12*a^2*b^9 + 48*a^4*b^7 + 72*a^6*b^5 + 48*a^8*b^3))/((4*a^4 + 4*b^
4 + 8*a^2*b^2)*(a^4 + b^4 + 2*a^2*b^2)*(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2))
))/(4*a^4 + 4*b^4 + 8*a^2*b^2) + (a^3*(a + b)^3*(a - b)^3*(12*a^10*b + 12*a
^2*b^9 + 48*a^4*b^7 + 72*a^6*b^5 + 48*a^8*b^3))/((a^4 + b^4 + 2*a^2*b^2)^3*
(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2)))*(a^6 - b^6 + 35*a^2*b^4 - 35*a^4*b^2)
*(a^10 + b^10 + 5*a^2*b^8 + 10*a^4*b^6 + 10*a^6*b^4 + 5*a^8*b^2))/((4*a^6 -
 4*a^4*b^2)*(a^6 + b^6 + 15*a^2*b^4 + 15*a^4*b^2)^2) + (2*a*b*(5*a^4 + 5*b^
4 - 26*a^2*b^2)*((8*a^6*b^2)/(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2) + (4*a^2*b
*((8*(3*a^8*b + 3*a^4*b^5 + 6*a^6*b^3))/(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2)
 - (4*a^2*b*((8*(4*a^4*b^6 - 2*a^2*b^8 - 2*a^10 + 12*a^6*b^4 + 4*a^8*b^2))/
(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2) + (32*a^2*b*(12*a^10*b + 12*a^2*b^9 + 4
```

```
8*a^4*b^7 + 72*a^6*b^5 + 48*a^8*b^3))/((4*a^4 + 4*b^4 + 8*a^2*b^2)*(a^6 + b
^6 + 3*a^2*b^4 + 3*a^4*b^2))))/(4*a^4 + 4*b^4 + 8*a^2*b^2)))/(4*a^4 + 4*b^4
 + 8*a^2*b^2) + (a*((a*(a + b)*(a - b)*((8*(4*a^4*b^6 - 2*a^2*b^8 - 2*a^10
+ 12*a^6*b^4 + 4*a^8*b^2))/(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2) + (32*a^2*b*
(12*a^10*b + 12*a^2*b^9 + 48*a^4*b^7 + 72*a^6*b^5 + 48*a^8*b^3))/((4*a^4 +
4*b^4 + 8*a^2*b^2)*(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2))))/(2*(a^4 + b^4 + 2
*a^2*b^2)) + (16*a^3*b*(a + b)*(a - b)*(12*a^10*b + 12*a^2*b^9 + 48*a^4*b^7
 + 72*a^6*b^5 + 48*a^8*b^3))/((4*a^4 + 4*b^4 + 8*a^2*b^2)*(a^4 + b^4 + 2*a^
2*b^2)*(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2)))*(a + b)*(a - b))/(2*(a^4 + b^4
 + 2*a^2*b^2)) + (8*a^4*b*(a + b)^2*(a - b)^2*(12*a^10*b + 12*a^2*b^9 + 48*
a^4*b^7 + 72*a^6*b^5 + 48*a^8*b^3))/((4*a^4 + 4*b^4 + 8*a^2*b^2)*(a^4 + b^4
 + 2*a^2*b^2)^2*(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2)))*(a^10 + b^10 + 5*a^2*
b^8 + 10*a^4*b^6 + 10*a^6*b^4 + 5*a^8*b^2))/((4*a^6 - 4*a^4*b^2)*(a^6 + b^6
 + 15*a^2*b^4 + 15*a^4*b^2)^2))*(a + b)*(a - b))/(a^4 + b^4 + 2*a^2*b^2)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*sin(x)**2/(a*cos(x)+b*sin(x)),x)
```

```
[Out] Timed out
```

3.277 $\int \frac{\cos(x)\sin^3(x)}{a\cos(x)+b\sin(x)}\,dx$

Optimal. Leaf size=122

$$\frac{b\sin^3(x)}{3\left(a^2+b^2\right)}+\frac{a^2b\sin(x)}{\left(a^2+b^2\right)^2}+\frac{a\cos^3(x)}{3\left(a^2+b^2\right)}-\frac{a\cos(x)}{a^2+b^2}+\frac{ab^2\cos(x)}{\left(a^2+b^2\right)^2}+\frac{a^3b\tanh^{-1}\left(\frac{b\cos(x)-a\sin(x)}{\sqrt{a^2+b^2}}\right)}{\left(a^2+b^2\right)^{5/2}}$$

```
[Out] a^3*b*arctanh((b*cos(x)-a*sin(x))/(a^2+b^2)^(1/2))/(a^2+b^2)^(5/2)+a*b^2*co
s(x)/(a^2+b^2)^2-a*cos(x)/(a^2+b^2)+1/3*a*cos(x)^3/(a^2+b^2)+a^2*b*sin(x)/(
a^2+b^2)^2+1/3*b*sin(x)^3/(a^2+b^2)
```

Rubi [A] time = 0.17, antiderivative size = 122, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.444, Rules used = {3109, 2564, 30, 2633, 3099, 3074, 206, 2638}

$$\frac{b\sin^3(x)}{3\left(a^2+b^2\right)}+\frac{a^2b\sin(x)}{\left(a^2+b^2\right)^2}+\frac{a\cos^3(x)}{3\left(a^2+b^2\right)}-\frac{a\cos(x)}{a^2+b^2}+\frac{ab^2\cos(x)}{\left(a^2+b^2\right)^2}+\frac{a^3b\tanh^{-1}\left(\frac{b\cos(x)-a\sin(x)}{\sqrt{a^2+b^2}}\right)}{\left(a^2+b^2\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[x]*Sin[x]^3)/(a*Cos[x] + b*Sin[x]),x]
```

```
[Out] (a^3*b*ArcTanh[(b*Cos[x] - a*Sin[x])/Sqrt[a^2 + b^2]])/(a^2 + b^2)^(5/2) +
(a*b^2*Cos[x])/(a^2 + b^2)^2 - (a*Cos[x])/(a^2 + b^2) + (a*Cos[x]^3)/(3*(a^
2 + b^2)) + (a^2*b*Sin[x])/(a^2 + b^2)^2 + (b*Sin[x]^3)/(3*(a^2 + b^2))
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2564

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
```

```
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3074

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x
_Symbol] :> -Dist[d^(-1), Subst[Int[1/(a^2 + b^2 - x^2), x], x, b*Cos[c + d
*x] - a*Sin[c + d*x]], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

Rule 3099

```
Int[sin[(c_.) + (d_.)*(x_)]^(m_)/(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin
[(c_.) + (d_.)*(x_)]), x_Symbol] :> -Simp[(a*Sin[c + d*x]^(m - 1))/(d*(a^2
+ b^2)*(m - 1)), x] + (Dist[a^2/(a^2 + b^2), Int[Sin[c + d*x]^(m - 2)/(a*Co
s[c + d*x] + b*Sin[c + d*x]), x], x] + Dist[b/(a^2 + b^2), Int[Sin[c + d*x]
^(m - 1), x], x]) /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0] && GtQ[m,
1]
```

Rule 3109

```
Int[(cos[(c_.) + (d_.)*(x_)]^(m_.)*sin[(c_.) + (d_.)*(x_)]^(n_.))/(cos[(c_.
) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[b
/(a^2 + b^2), Int[Cos[c + d*x]^m*Sin[c + d*x]^(n - 1), x], x] + (Dist[a/(a^
2 + b^2), Int[Cos[c + d*x]^(m - 1)*Sin[c + d*x]^n, x], x] - Dist[(a*b)/(a^2
 + b^2), Int[(Cos[c + d*x]^(m - 1)*Sin[c + d*x]^(n - 1))/(a*Cos[c + d*x] +
b*Sin[c + d*x]), x], x]) /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0] &&
IGtQ[m, 0] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{\cos(x)\sin^3(x)}{a\cos(x)+b\sin(x)}\,dx = \frac{a\int \sin^3(x)\,dx}{a^2+b^2} + \frac{b\int \cos(x)\sin^2(x)\,dx}{a^2+b^2} - \frac{(ab)\int \frac{\sin^2(x)}{a\cos(x)+b\sin(x)}\,dx}{a^2+b^2}$$

$$= \frac{a^2b\sin(x)}{\left(a^2+b^2\right)^2} - \frac{\left(a^3b\right)\int \frac{1}{a\cos(x)+b\sin(x)}\,dx}{\left(a^2+b^2\right)^2} - \frac{\left(ab^2\right)\int \sin(x)\,dx}{\left(a^2+b^2\right)^2} - \frac{a\,\text{Subst}\left(\int \left(1-x^2\right)\,dx\right.}{a^2+b^2}$$

$$= \frac{ab^2\cos(x)}{\left(a^2+b^2\right)^2} - \frac{a\cos(x)}{a^2+b^2} + \frac{a\cos^3(x)}{3\left(a^2+b^2\right)} + \frac{a^2b\sin(x)}{\left(a^2+b^2\right)^2} + \frac{b\sin^3(x)}{3\left(a^2+b^2\right)} + \frac{\left(a^3b\right)\text{Subst}\left(\int \right.}{}$$

$$= \frac{a^3b\tanh^{-1}\left(\frac{b\cos(x)-a\sin(x)}{\sqrt{a^2+b^2}}\right)}{\left(a^2+b^2\right)^{5/2}} + \frac{ab^2\cos(x)}{\left(a^2+b^2\right)^2} - \frac{a\cos(x)}{a^2+b^2} + \frac{a\cos^3(x)}{3\left(a^2+b^2\right)} + \frac{a^2b\sin(x)}{\left(a^2+b^2\right)^2} + \frac{}{3}$$

Mathematica [A] time = 0.98, size = 113, normalized size = 0.93

$$\frac{\left(3ab^2-9a^3\right)\cos(x)+a\left(a^2+b^2\right)\cos(3x)-2b\sin(x)\left(\left(a^2+b^2\right)\cos(2x)-7a^2-b^2\right)}{12\left(a^2+b^2\right)^2} - \frac{2a^3b\tanh^{-1}\left(\frac{a\tan\left(\frac{x}{2}\right)-b}{\sqrt{a^2+b^2}}\right)}{\left(a^2+b^2\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[x]*Sin[x]^3)/(a*Cos[x] + b*Sin[x]),x]
```

```
[Out] (-2*a^3*b*ArcTanh[(-b + a*Tan[x/2])/Sqrt[a^2 + b^2]])/(a^2 + b^2)^(5/2) + (
(-9*a^3 + 3*a*b^2)*Cos[x] + a*(a^2 + b^2)*Cos[3*x] - 2*b*(-7*a^2 - b^2 + (a
^2 + b^2)*Cos[2*x])*Sin[x])/(12*(a^2 + b^2)^2)
```

fricas [A] time = 0.44, size = 210, normalized size = 1.72

$$\frac{3\sqrt{a^2+b^2}\,a^3b\log\left(\frac{2\,ab\cos(x)\sin(x)+\left(a^2-b^2\right)\cos(x)^2-2\,a^2-b^2-2\,\sqrt{a^2+b^2}\,(b\cos(x)-a\sin(x))}{2\,ab\cos(x)\sin(x)+\left(a^2-b^2\right)\cos(x)^2+b^2}\right)+2\left(a^5+2\,a^3b^2+ab^4\right)\cos(x)^3-6}{6\left(a^6+3\,a^4b^2+3\,a^2b^4+b^6\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*sin(x)^3/(a*cos(x)+b*sin(x)),x, algorithm="fricas")
```

```
[Out] 1/6*(3*sqrt(a^2 + b^2)*a^3*b*log((2*a*b*cos(x)*sin(x) + (a^2 - b^2)*cos(x)^
2 - 2*a^2 - b^2 - 2*sqrt(a^2 + b^2)*(b*cos(x) - a*sin(x)))/(2*a*b*cos(x)*si
n(x) + (a^2 - b^2)*cos(x)^2 + b^2)) + 2*(a^5 + 2*a^3*b^2 + a*b^4)*cos(x)^3
```

```
- 6*(a^5 + a^3*b^2)*cos(x) + 2*(4*a^4*b + 5*a^2*b^3 + b^5 - (a^4*b + 2*a^2*
b^3 + b^5)*cos(x)^2)*sin(x))/(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)
```

giac [A] time = 4.00, size = 190, normalized size = 1.56

$$\frac{a^3 b \log\left(\frac{\left|2\,a\tan\left(\frac{1}{2}\,x\right)-2\,b-2\,\sqrt{a^2+b^2}\right|}{\left|2\,a\tan\left(\frac{1}{2}\,x\right)-2\,b+2\,\sqrt{a^2+b^2}\right|}\right)}{\left(a^4+2\,a^2b^2+b^4\right)\sqrt{a^2+b^2}}+\frac{2\left(3\,a^2b\tan\left(\frac{1}{2}\,x\right)^5+3\,ab^2\tan\left(\frac{1}{2}\,x\right)^4+10\,a^2b\tan\left(\frac{1}{2}\,x\right)^3+4\,b^3\tan\left(\frac{1}{2}\,x\right)^3-6\,a\right.}{3\left(a^4+2\,a^2b^2+b^4\right)\left(\tan\left(\frac{1}{2}\,x\right)^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*sin(x)^3/(a*cos(x)+b*sin(x)),x, algorithm="giac")
```

```
[Out] a^3*b*log(abs(2*a*tan(1/2*x) - 2*b - 2*sqrt(a^2 + b^2))/abs(2*a*tan(1/2*x)
- 2*b + 2*sqrt(a^2 + b^2)))/((a^4 + 2*a^2*b^2 + b^4)*sqrt(a^2 + b^2)) + 2/3
*(3*a^2*b*tan(1/2*x)^5 + 3*a*b^2*tan(1/2*x)^4 + 10*a^2*b*tan(1/2*x)^3 + 4*b
^3*tan(1/2*x)^3 - 6*a^3*tan(1/2*x)^2 + 3*a^2*b*tan(1/2*x) - 2*a^3 + a*b^2)/
((a^4 + 2*a^2*b^2 + b^4)*(tan(1/2*x)^2 + 1)^3)
```

maple [A] time = 0.09, size = 166, normalized size = 1.36

$$-\frac{16a^3b\,\mathrm{arctanh}\left(\frac{2a\tan\left(\frac{x}{2}\right)-2b}{2\sqrt{a^2+b^2}}\right)}{\left(8a^4+16a^2b^2+8b^4\right)\sqrt{a^2+b^2}}-\frac{2\left(-a^2b\left(\tan^5\left(\frac{x}{2}\right)\right)-a\,b^2\left(\tan^4\left(\frac{x}{2}\right)\right)+\left(-\frac{10}{3}a^2b-\frac{4}{3}b^3\right)\left(\tan^3\left(\frac{x}{2}\right)\right)+2a^3\left(\tan^2\left(\right.\right.\right.}{\left(a^4+2a^2b^2+b^4\right)\left(\tan^2\left(\frac{x}{2}\right)+1\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)*sin(x)^3/(a*cos(x)+b*sin(x)),x)
```

```
[Out] -16*a^3*b/(8*a^4+16*a^2*b^2+8*b^4)/(a^2+b^2)^(1/2)*arctanh(1/2*(2*a*tan(1/2
*x)-2*b)/(a^2+b^2)^(1/2))-2/(a^4+2*a^2*b^2+b^4)*(-a^2*b*tan(1/2*x)^5-a*b^2*
tan(1/2*x)^4+(-10/3*a^2*b-4/3*b^3)*tan(1/2*x)^3+2*a^3*tan(1/2*x)^2-a^2*b*ta
n(1/2*x)+2/3*a^3-1/3*a*b^2)/(tan(1/2*x)^2+1)^3
```

maxima [B] time = 0.45, size = 278, normalized size = 2.28

$$\frac{a^3b\log\left(\frac{b-\frac{a\sin(x)}{\cos(x)+1}+\sqrt{a^2+b^2}}{b-\frac{a\sin(x)}{\cos(x)+1}-\sqrt{a^2+b^2}}\right)}{\left(a^4+2\,a^2b^2+b^4\right)\sqrt{a^2+b^2}}-\frac{2\left(2\,a^3-ab^2-\frac{3\,a^2b\sin(x)}{\cos(x)+1}+\frac{6\,a^3\sin(x)^2}{(\cos(x)+1)^2}-\frac{3\,ab^2\sin(x)^4}{(\cos(x)+1)^4}-\frac{3\,a^2b\sin(x)^5}{(\cos(x)+1)^5}-\frac{2\left(5\,a^2b+2\,b^3\right)\sin(x)^3}{(\cos(x)+1)^3}\right)}{3\left(a^4+2\,a^2b^2+b^4+\frac{3\left(a^4+2\,a^2b^2+b^4\right)\sin(x)^2}{(\cos(x)+1)^2}+\frac{3\left(a^4+2\,a^2b^2+b^4\right)\sin(x)^4}{(\cos(x)+1)^4}+\frac{\left(a^4+2\,a^2b^2+b^4\right)\sin(x)^6}{(\cos(x)+1)^6}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*sin(x)^3/(a*cos(x)+b*sin(x)),x, algorithm="maxima")
```

```
[Out] a^3*b*log((b - a*sin(x)/(cos(x) + 1) + sqrt(a^2 + b^2))/(b - a*sin(x)/(cos(
x) + 1) - sqrt(a^2 + b^2)))/((a^4 + 2*a^2*b^2 + b^4)*sqrt(a^2 + b^2)) - 2/3
*(2*a^3 - a*b^2 - 3*a^2*b*sin(x)/(cos(x) + 1) + 6*a^3*sin(x)^2/(cos(x) + 1)
^2 - 3*a*b^2*sin(x)^4/(cos(x) + 1)^4 - 3*a^2*b*sin(x)^5/(cos(x) + 1)^5 - 2*
(5*a^2*b + 2*b^3)*sin(x)^3/(cos(x) + 1)^3)/(a^4 + 2*a^2*b^2 + b^4 + 3*(a^4
+ 2*a^2*b^2 + b^4)*sin(x)^2/(cos(x) + 1)^2 + 3*(a^4 + 2*a^2*b^2 + b^4)*sin(
x)^4/(cos(x) + 1)^4 + (a^4 + 2*a^2*b^2 + b^4)*sin(x)^6/(cos(x) + 1)^6)
```

mupad [B] time = 1.20, size = 286, normalized size = 2.34

$$\frac{\frac{2\left(a\,b^2-2\,a^3\right)}{3\left(a^4+2\,a^2\,b^2+b^4\right)}+\frac{4\tan\left(\frac{x}{2}\right)^3\left(5\,a^2\,b+2\,b^3\right)}{3\left(a^4+2\,a^2\,b^2+b^4\right)}-\frac{4\,a^3\tan\left(\frac{x}{2}\right)^2}{a^4+2\,a^2\,b^2+b^4}+\frac{2\,a\,b^2\tan\left(\frac{x}{2}\right)^4}{a^4+2\,a^2\,b^2+b^4}+\frac{2\,a^2\,b\tan\left(\frac{x}{2}\right)^5}{a^4+2\,a^2\,b^2+b^4}+\frac{2\,a^2\,b\tan\left(\frac{x}{2}\right)}{a^4+2\,a^2\,b^2+b^4}}{\tan\left(\frac{x}{2}\right)^6+3\tan\left(\frac{x}{2}\right)^4+3\tan\left(\frac{x}{2}\right)^2+1}+\frac{2\,a^3\,b\,\mathrm{atanh}\left(\frac{2\,a^4\,b+\cdots}{\cdots}\right)}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(x)*sin(x)^3)/(a*cos(x) + b*sin(x)),x)
```

```
[Out] ((2*(a*b^2 - 2*a^3))/(3*(a^4 + b^4 + 2*a^2*b^2)) + (4*tan(x/2)^3*(5*a^2*b +
 2*b^3))/(3*(a^4 + b^4 + 2*a^2*b^2)) - (4*a^3*tan(x/2)^2)/(a^4 + b^4 + 2*a^
2*b^2) + (2*a*b^2*tan(x/2)^4)/(a^4 + b^4 + 2*a^2*b^2) + (2*a^2*b*tan(x/2)^5
)/(a^4 + b^4 + 2*a^2*b^2) + (2*a^2*b*tan(x/2))/(a^4 + b^4 + 2*a^2*b^2))/(3*
tan(x/2)^2 + 3*tan(x/2)^4 + tan(x/2)^6 + 1) + (2*a^3*b*atanh((2*a^4*b + 2*b
^5 + 4*a^2*b^3 - 2*a*tan(x/2)*(a^4 + b^4 + 2*a^2*b^2))/(2*(a^2 + b^2)^(5/2)
)))/(a^2 + b^2)^(5/2)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*sin(x)**3/(a*cos(x)+b*sin(x)),x)
```

```
[Out] Timed out
```

3.278 $\int \frac{\cos^2(x)\sin(x)}{a\cos(x)+b\sin(x)}\,dx$

Optimal. Leaf size=93

$$-\frac{a^2bx}{(a^2+b^2)^2}+\frac{bx}{2(a^2+b^2)}+\frac{a\sin^2(x)}{2(a^2+b^2)}+\frac{b\sin(x)\cos(x)}{2(a^2+b^2)}-\frac{ab^2\log(a\cos(x)+b\sin(x))}{(a^2+b^2)^2}$$

```
[Out] -a^2*b*x/(a^2+b^2)^2+1/2*b*x/(a^2+b^2)-a*b^2*ln(a*cos(x)+b*sin(x))/(a^2+b^2
)^2+1/2*b*cos(x)*sin(x)/(a^2+b^2)+1/2*a*sin(x)^2/(a^2+b^2)
```

Rubi [A] time = 0.13, antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.389, Rules used = {3109, 2635, 8, 2564, 30, 3098, 3133}

$$-\frac{a^2bx}{(a^2+b^2)^2}+\frac{bx}{2(a^2+b^2)}+\frac{a\sin^2(x)}{2(a^2+b^2)}+\frac{b\sin(x)\cos(x)}{2(a^2+b^2)}-\frac{ab^2\log(a\cos(x)+b\sin(x))}{(a^2+b^2)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[x]^2*Sin[x])/(a*Cos[x] + b*Sin[x]),x]
```

```
[Out] -((a^2*b*x)/(a^2 + b^2)^2) + (b*x)/(2*(a^2 + b^2)) - (a*b^2*Log[a*Cos[x] +
b*Sin[x]])/(a^2 + b^2)^2 + (b*Cos[x]*Sin[x])/(2*(a^2 + b^2)) + (a*Sin[x]^2)
/(2*(a^2 + b^2))
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2564

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] && !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 3098

```
Int[cos[(c_.) + (d_.)*(x_)]/(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.
) + (d_.)*(x_)]), x_Symbol] :> Simp[(a*x)/(a^2 + b^2), x] + Dist[b/(a^2 + b
^2), Int[(b*Cos[c + d*x] - a*Sin[c + d*x])/(a*Cos[c + d*x] + b*Sin[c + d*x]
), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

Rule 3109

```
Int[(cos[(c_.) + (d_.)*(x_)]^(m_.)*sin[(c_.) + (d_.)*(x_)]^(n_.))/(cos[(c_.
) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[b
/(a^2 + b^2), Int[Cos[c + d*x]^m*Sin[c + d*x]^(n - 1), x], x] + (Dist[a/(a^
2 + b^2), Int[Cos[c + d*x]^(m - 1)*Sin[c + d*x]^n, x], x] - Dist[(a*b)/(a^2
 + b^2), Int[(Cos[c + d*x]^(m - 1)*Sin[c + d*x]^(n - 1))/(a*Cos[c + d*x] +
b*Sin[c + d*x]), x], x]) /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0] &&
IGtQ[m, 0] && IGtQ[n, 0]
```

Rule 3133

```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])
/((a_.) + cos[(d_.) + (e_.)*(x_)]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)]), x
_Symbol] :> Simp[((b*B + c*C)*x)/(b^2 + c^2), x] + Simp[((c*B - b*C)*Log[a
+ b*Cos[d + e*x] + c*Sin[d + e*x]])/(e*(b^2 + c^2)), x] /; FreeQ[{a, b, c,
d, e, A, B, C}, x] && NeQ[b^2 + c^2, 0] && EqQ[A*(b^2 + c^2) - a*(b*B + c*C
), 0]
```

Rubi steps

$$\int \frac{\cos^2(x)\sin(x)}{a\cos(x)+b\sin(x)}\,dx = \frac{a\int \cos(x)\sin(x)\,dx}{a^2+b^2}+\frac{b\int \cos^2(x)\,dx}{a^2+b^2}-\frac{(ab)\int \frac{\cos(x)}{a\cos(x)+b\sin(x)}\,dx}{a^2+b^2}$$

$$= -\frac{a^2bx}{\left(a^2+b^2\right)^2}+\frac{b\cos(x)\sin(x)}{2\left(a^2+b^2\right)}-\frac{\left(ab^2\right)\int \frac{b\cos(x)-a\sin(x)}{a\cos(x)+b\sin(x)}\,dx}{\left(a^2+b^2\right)^2}+\frac{a\,\mathrm{Subst}(\int x\,dx, x, \sin(x}{a^2+b^2}$$

$$= -\frac{a^2bx}{\left(a^2+b^2\right)^2}+\frac{bx}{2\left(a^2+b^2\right)}-\frac{ab^2\log(a\cos(x)+b\sin(x))}{\left(a^2+b^2\right)^2}+\frac{b\cos(x)\sin(x)}{2\left(a^2+b^2\right)}+\frac{a\sin}{2\left(a^2\right.}$$

Mathematica [C] time = 0.31, size = 82, normalized size = 0.88

$$\frac{b\left(a^2+b^2\right)\sin(2x)-a\left(a^2+b^2\right)\cos(2x)+4iab^2\tan^{-1}(\tan(x))-2b\left(ab\log\left((a\cos(x)+b\sin(x))^2\right)+x(a+ib)^2\right)}{4\left(a^2+b^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[x]^2*Sin[x])/(a*Cos[x] + b*Sin[x]),x]
```

```
[Out] ((4*I)*a*b^2*ArcTan[Tan[x]] - a*(a^2 + b^2)*Cos[2*x] - 2*b*((a + I*b)^2*x +
 a*b*Log[(a*Cos[x] + b*Sin[x])^2]) + b*(a^2 + b^2)*Sin[2*x])/(4*(a^2 + b^2)
^2)
```

fricas [A] time = 0.48, size = 94, normalized size = 1.01

$$-\frac{ab^2\log\left(2\,ab\cos(x)\sin(x)+\left(a^2-b^2\right)\cos(x)^2+b^2\right)+\left(a^3+ab^2\right)\cos(x)^2-\left(a^2b+b^3\right)\cos(x)\sin(x)+\left(a^2b-b^3\right.}{2\left(a^4+2\,a^2b^2+b^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^2*sin(x)/(a*cos(x)+b*sin(x)),x, algorithm="fricas")
```

```
[Out] -1/2*(a*b^2*log(2*a*b*cos(x)*sin(x) + (a^2 - b^2)*cos(x)^2 + b^2) + (a^3 +
a*b^2)*cos(x)^2 - (a^2*b + b^3)*cos(x)*sin(x) + (a^2*b - b^3)*x)/(a^4 + 2*a
^2*b^2 + b^4)
```

giac [A] time = 1.92, size = 156, normalized size = 1.68

$$-\frac{ab^3\log\left(|b\tan(x)+a|\right)}{a^4b+2\,a^2b^3+b^5}+\frac{ab^2\log\left(\tan(x)^2+1\right)}{2\left(a^4+2\,a^2b^2+b^4\right)}-\frac{\left(a^2b-b^3\right)x}{2\left(a^4+2\,a^2b^2+b^4\right)}-\frac{ab^2\tan(x)^2-a^2b\tan(x)-b^3\tan(x)+a^3}{2\left(a^4+2\,a^2b^2+b^4\right)\left(\tan(x)^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^2*sin(x)/(a*cos(x)+b*sin(x)),x, algorithm="giac")
```

```
[Out] -a*b^3*log(abs(b*tan(x) + a))/(a^4*b + 2*a^2*b^3 + b^5) + 1/2*a*b^2*log(tan
(x)^2 + 1)/(a^4 + 2*a^2*b^2 + b^4) - 1/2*(a^2*b - b^3)*x/(a^4 + 2*a^2*b^2 +
 b^4) - 1/2*(a*b^2*tan(x)^2 - a^2*b*tan(x) - b^3*tan(x) + a^3 + 2*a*b^2)/((
a^4 + 2*a^2*b^2 + b^4)*(tan(x)^2 + 1))
```

maple [A] time = 0.08, size = 175, normalized size = 1.88

$$\frac{\tan(x)a^2b}{2\left(a^2+b^2\right)^2\left(\tan^2(x)+1\right)}+\frac{\tan(x)b^3}{2\left(a^2+b^2\right)^2\left(\tan^2(x)+1\right)}-\frac{a^3}{2\left(a^2+b^2\right)^2\left(\tan^2(x)+1\right)}-\frac{a\,b^2}{2\left(a^2+b^2\right)^2\left(\tan^2(x)+1\right)}+\frac{\ln}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)^2*sin(x)/(a*cos(x)+b*sin(x)),x)
```

```
[Out] 1/2/(a^2+b^2)^2/(tan(x)^2+1)*tan(x)*a^2*b+1/2/(a^2+b^2)^2/(tan(x)^2+1)*tan(
x)*b^3-1/2/(a^2+b^2)^2/(tan(x)^2+1)*a^3-1/2/(a^2+b^2)^2/(tan(x)^2+1)*a*b^2+
1/2/(a^2+b^2)^2*ln(tan(x)^2+1)*a*b^2-1/2/(a^2+b^2)^2*arctan(tan(x))*a^2*b+1
/2/(a^2+b^2)^2*arctan(tan(x))*b^3-b^2*a/(a^2+b^2)^2*ln(a+b*tan(x))
```

maxima [B]   time = 0.45, size = 212, normalized size = 2.28

$$-\frac{ab^2\log\left(-a-\frac{2\,b\sin(x)}{\cos(x)+1}+\frac{a\sin(x)^2}{(\cos(x)+1)^2}\right)}{a^4+2\,a^2b^2+b^4}+\frac{ab^2\log\left(\frac{\sin(x)^2}{(\cos(x)+1)^2}+1\right)}{a^4+2\,a^2b^2+b^4}-\frac{\left(a^2b-b^3\right)\arctan\left(\frac{\sin(x)}{\cos(x)+1}\right)}{a^4+2\,a^2b^2+b^4}+\frac{\frac{b\sin(x)}{\cos(x)+1}+\frac{2\,a\sin}{(\cos(x}}{a^2+b^2+\frac{2\left(a^2+b^2\right)s}{(\cos(x)+}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^2*sin(x)/(a*cos(x)+b*sin(x)),x, algorithm="maxima")
```

```
[Out] -a*b^2*log(-a - 2*b*sin(x)/(cos(x) + 1) + a*sin(x)^2/(cos(x) + 1)^2)/(a^4 +
 2*a^2*b^2 + b^4) + a*b^2*log(sin(x)^2/(cos(x) + 1)^2 + 1)/(a^4 + 2*a^2*b^2
 + b^4) - (a^2*b - b^3)*arctan(sin(x)/(cos(x) + 1))/(a^4 + 2*a^2*b^2 + b^4)
 + (b*sin(x)/(cos(x) + 1) + 2*a*sin(x)^2/(cos(x) + 1)^2 - b*sin(x)^3/(cos(x
) + 1)^3)/(a^2 + b^2 + 2*(a^2 + b^2)*sin(x)^2/(cos(x) + 1)^2 + (a^2 + b^2)*
sin(x)^4/(cos(x) + 1)^4)
```

mupad [B]   time = 6.19, size = 3419, normalized size = 36.76

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(x)^2*sin(x))/(a*cos(x) + b*sin(x)),x)
```

```
[Out] ((b*tan(x/2))/(a^2 + b^2) + (2*a*tan(x/2)^2)/(a^2 + b^2) - (b*tan(x/2)^3)/(
a^2 + b^2))/(2*tan(x/2)^2 + tan(x/2)^4 + 1) - (a*b^2*log(a + 2*b*tan(x/2) -
 a*tan(x/2)^2))/(a^4 + b^4 + 2*a^2*b^2) + (4*a*b^2*log(1/(cos(x) + 1)))/(4*
a^4 + 4*b^4 + 8*a^2*b^2) - (b*atan((tan(x/2)*((((4*a*b^2*((b*(a + b)*(a - b
)*((8*(12*a^4*b^6 + 24*a^6*b^4 + 12*a^8*b^2))/(a^6 + b^6 + 3*a^2*b^4 + 3*a^
4*b^2) - (32*a*b^2*(12*a*b^10 + 48*a^3*b^8 + 72*a^5*b^6 + 48*a^7*b^4 + 12*a
^9*b^2))/((4*a^4 + 4*b^4 + 8*a^2*b^2)*(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2)))
)/(2*(a^4 + b^4 + 2*a^2*b^2)) - (16*a*b^3*(a + b)*(a - b)*(12*a*b^10 + 48*a
^3*b^8 + 72*a^5*b^6 + 48*a^7*b^4 + 12*a^9*b^2))/((4*a^4 + 4*b^4 + 8*a^2*b^2
)*(a^4 + b^4 + 2*a^2*b^2)*(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2))))/(4*a^4 + 4
*b^4 + 8*a^2*b^2) - (b*(a + b)*((8*(2*a*b^8 - 7*a^3*b^6 - 8*a^5*b^4 + a^7*b
^2))/(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2) - (4*a*b^2*((8*(12*a^4*b^6 + 24*a^
```

```
6*b^4 + 12*a^8*b^2))/(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2) - (32*a*b^2*(12*a*
b^10 + 48*a^3*b^8 + 72*a^5*b^6 + 48*a^7*b^4 + 12*a^9*b^2))/((4*a^4 + 4*b^4
+ 8*a^2*b^2)*(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2))))/(4*a^4 + 4*b^4 + 8*a^2*
b^2))*(a - b))/(2*(a^4 + b^4 + 2*a^2*b^2)) + (b^3*(a + b)^3*(a - b)^3*(12*a
*b^10 + 48*a^3*b^8 + 72*a^5*b^6 + 48*a^7*b^4 + 12*a^9*b^2))/((a^4 + b^4 + 2
*a^2*b^2)^3*(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2)))*(a^6 - b^6 + 35*a^2*b^4 -
 35*a^4*b^2))/(a^6 + b^6 + 15*a^2*b^4 + 15*a^4*b^2)^2 - (2*a*b*(5*a^4 + 5*b
^4 - 26*a^2*b^2)*((8*(2*a^2*b^6 + a^4*b^4))/(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*
b^2) + (4*a*b^2*((8*(2*a*b^8 - 7*a^3*b^6 - 8*a^5*b^4 + a^7*b^2))/(a^6 + b^6
 + 3*a^2*b^4 + 3*a^4*b^2) - (4*a*b^2*((8*(12*a^4*b^6 + 24*a^6*b^4 + 12*a^8*
b^2))/(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2) - (32*a*b^2*(12*a*b^10 + 48*a^3*b
^8 + 72*a^5*b^6 + 48*a^7*b^4 + 12*a^9*b^2))/((4*a^4 + 4*b^4 + 8*a^2*b^2)*(a
^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2))))/(4*a^4 + 4*b^4 + 8*a^2*b^2)))/(4*a^4 +
 4*b^4 + 8*a^2*b^2) + (b*((b*(a + b)*(a - b)*((8*(12*a^4*b^6 + 24*a^6*b^4 +
 12*a^8*b^2))/(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2) - (32*a*b^2*(12*a*b^10 +
48*a^3*b^8 + 72*a^5*b^6 + 48*a^7*b^4 + 12*a^9*b^2))/((4*a^4 + 4*b^4 + 8*a^2
*b^2)*(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2))))/(2*(a^4 + b^4 + 2*a^2*b^2)) -
(16*a*b^3*(a + b)*(a - b)*(12*a*b^10 + 48*a^3*b^8 + 72*a^5*b^6 + 48*a^7*b^4
 + 12*a^9*b^2))/((4*a^4 + 4*b^4 + 8*a^2*b^2)*(a^4 + b^4 + 2*a^2*b^2)*(a^6 +
 b^6 + 3*a^2*b^4 + 3*a^4*b^2)))*(a + b)*(a - b))/(2*(a^4 + b^4 + 2*a^2*b^2)
) - (8*a*b^4*(a + b)^2*(a - b)^2*(12*a*b^10 + 48*a^3*b^8 + 72*a^5*b^6 + 48*
a^7*b^4 + 12*a^9*b^2))/((4*a^4 + 4*b^4 + 8*a^2*b^2)*(a^4 + b^4 + 2*a^2*b^2)
^2*(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2))))/(a^6 + b^6 + 15*a^2*b^4 + 15*a^4*
b^2)^2)*(a^10 + b^10 + 5*a^2*b^8 + 10*a^4*b^6 + 10*a^6*b^4 + 5*a^8*b^2))/(4
*a*b^5 - 4*a^3*b^3) + (((b*(a + b)*(a - b)*((8*(3*a^2*b^7 + 6*a^4*b^5 + 3*a
^6*b^3))/(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2) - (4*a*b^2*((8*(4*a^3*b^7 - 2*
a^9*b - 2*a*b^9 + 12*a^5*b^5 + 4*a^7*b^3))/(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b
^2) + (32*a*b^2*(12*a^10*b + 12*a^2*b^9 + 48*a^4*b^7 + 72*a^6*b^5 + 48*a^8*
b^3))/((4*a^4 + 4*b^4 + 8*a^2*b^2)*(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2))))/(
4*a^4 + 4*b^4 + 8*a^2*b^2)))/(2*(a^4 + b^4 + 2*a^2*b^2)) - (4*a*b^2*((b*(a
+ b)*(a - b)*((8*(4*a^3*b^7 - 2*a^9*b - 2*a*b^9 + 12*a^5*b^5 + 4*a^7*b^3))/
(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2) + (32*a*b^2*(12*a^10*b + 12*a^2*b^9 + 4
8*a^4*b^7 + 72*a^6*b^5 + 48*a^8*b^3))/((4*a^4 + 4*b^4 + 8*a^2*b^2)*(a^6 + b
^6 + 3*a^2*b^4 + 3*a^4*b^2))))/(2*(a^4 + b^4 + 2*a^2*b^2)) + (16*a*b^3*(a +
 b)*(a - b)*(12*a^10*b + 12*a^2*b^9 + 48*a^4*b^7 + 72*a^6*b^5 + 48*a^8*b^3)
)/((4*a^4 + 4*b^4 + 8*a^2*b^2)*(a^4 + b^4 + 2*a^2*b^2)*(a^6 + b^6 + 3*a^2*b
^4 + 3*a^4*b^2))))/(4*a^4 + 4*b^4 + 8*a^2*b^2) + (b^3*(a + b)^3*(a - b)^3*(
12*a^10*b + 12*a^2*b^9 + 48*a^4*b^7 + 72*a^6*b^5 + 48*a^8*b^3))/((a^4 + b^4
 + 2*a^2*b^2)^3*(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2)))*(a^6 - b^6 + 35*a^2*b
^4 - 35*a^4*b^2)*(a^10 + b^10 + 5*a^2*b^8 + 10*a^4*b^6 + 10*a^6*b^4 + 5*a^8
*b^2))/((4*a*b^5 - 4*a^3*b^3)*(a^6 + b^6 + 15*a^2*b^4 + 15*a^4*b^2)^2) + (2
*a*b*(5*a^4 + 5*b^4 - 26*a^2*b^2)*((8*a^3*b^5)/(a^6 + b^6 + 3*a^2*b^4 + 3*a
^4*b^2) + (4*a*b^2*((8*(3*a^2*b^7 + 6*a^4*b^5 + 3*a^6*b^3))/(a^6 + b^6 + 3*
a^2*b^4 + 3*a^4*b^2) - (4*a*b^2*((8*(4*a^3*b^7 - 2*a^9*b - 2*a*b^9 + 12*a^5
*b^5 + 4*a^7*b^3))/(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2) + (32*a*b^2*(12*a^10
```

```
*b + 12*a^2*b^9 + 48*a^4*b^7 + 72*a^6*b^5 + 48*a^8*b^3))/((4*a^4 + 4*b^4 +
8*a^2*b^2)*(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2))))/(4*a^4 + 4*b^4 + 8*a^2*b^
2)))/(4*a^4 + 4*b^4 + 8*a^2*b^2) + (b*((b*(a + b)*(a - b)*((8*(4*a^3*b^7 -
2*a^9*b - 2*a*b^9 + 12*a^5*b^5 + 4*a^7*b^3))/(a^6 + b^6 + 3*a^2*b^4 + 3*a^4
*b^2) + (32*a*b^2*(12*a^10*b + 12*a^2*b^9 + 48*a^4*b^7 + 72*a^6*b^5 + 48*a^
8*b^3))/((4*a^4 + 4*b^4 + 8*a^2*b^2)*(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2))))
/(2*(a^4 + b^4 + 2*a^2*b^2)) + (16*a*b^3*(a + b)*(a - b)*(12*a^10*b + 12*a^
2*b^9 + 48*a^4*b^7 + 72*a^6*b^5 + 48*a^8*b^3))/((4*a^4 + 4*b^4 + 8*a^2*b^2)
*(a^4 + b^4 + 2*a^2*b^2)*(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2)))*(a + b)*(a -
 b))/(2*(a^4 + b^4 + 2*a^2*b^2)) + (8*a*b^4*(a + b)^2*(a - b)^2*(12*a^10*b
+ 12*a^2*b^9 + 48*a^4*b^7 + 72*a^6*b^5 + 48*a^8*b^3))/((4*a^4 + 4*b^4 + 8*a
^2*b^2)*(a^4 + b^4 + 2*a^2*b^2)^2*(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2)))*(a^
10 + b^10 + 5*a^2*b^8 + 10*a^4*b^6 + 10*a^6*b^4 + 5*a^8*b^2))/((4*a*b^5 - 4
*a^3*b^3)*(a^6 + b^6 + 15*a^2*b^4 + 15*a^4*b^2)^2))*(a + b)*(a - b))/(a^4 +
 b^4 + 2*a^2*b^2)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)**2*sin(x)/(a*cos(x)+b*sin(x)),x)
```

```
[Out] Timed out
```

3.279 $\int \frac{\cos^2(x)\sin^2(x)}{a\cos(x)+b\sin(x)}\,dx$

Optimal. Leaf size=112

$$\frac{a\sin^3(x)}{3(a^2+b^2)}-\frac{ab^2\sin(x)}{(a^2+b^2)^2}-\frac{b\cos^3(x)}{3(a^2+b^2)}+\frac{a^2b\cos(x)}{(a^2+b^2)^2}-\frac{a^2b^2\tanh^{-1}\left(\frac{b\cos(x)-a\sin(x)}{\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{5/2}}$$

```
[Out] -a^2*b^2*arctanh((b*cos(x)-a*sin(x))/(a^2+b^2)^(1/2))/(a^2+b^2)^(5/2)+a^2*b
*cos(x)/(a^2+b^2)^2-1/3*b*cos(x)^3/(a^2+b^2)-a*b^2*sin(x)/(a^2+b^2)^2+1/3*a
*sin(x)^3/(a^2+b^2)
```

Rubi [A] time = 0.20, antiderivative size = 112, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 8, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {3109, 2565, 30, 2564, 2637, 2638, 3074, 206}

$$\frac{a\sin^3(x)}{3(a^2+b^2)}-\frac{ab^2\sin(x)}{(a^2+b^2)^2}-\frac{b\cos^3(x)}{3(a^2+b^2)}+\frac{a^2b\cos(x)}{(a^2+b^2)^2}-\frac{a^2b^2\tanh^{-1}\left(\frac{b\cos(x)-a\sin(x)}{\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[x]^2*Sin[x]^2)/(a*Cos[x] + b*Sin[x]),x]
```

```
[Out] -((a^2*b^2*ArcTanh[(b*Cos[x] - a*Sin[x])/Sqrt[a^2 + b^2]])/(a^2 + b^2)^(5/2
)) + (a^2*b*Cos[x])/(a^2 + b^2)^2 - (b*Cos[x]^3)/(3*(a^2 + b^2)) - (a*b^2*S
in[x])/(a^2 + b^2)^2 + (a*Sin[x]^3)/(3*(a^2 + b^2))
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2564

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
```

```
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 2565

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> -Dist[(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3074

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x
_Symbol] :> -Dist[d^(-1), Subst[Int[1/(a^2 + b^2 - x^2), x], x, b*Cos[c + d
*x] - a*Sin[c + d*x]], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

Rule 3109

```
Int[(cos[(c_.) + (d_.)*(x_)]^(m_.)*sin[(c_.) + (d_.)*(x_)]^(n_.))/(cos[(c_.
) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[b
/(a^2 + b^2), Int[Cos[c + d*x]^m*Sin[c + d*x]^(n - 1), x], x] + (Dist[a/(a^
2 + b^2), Int[Cos[c + d*x]^(m - 1)*Sin[c + d*x]^n, x], x] - Dist[(a*b)/(a^2
 + b^2), Int[(Cos[c + d*x]^(m - 1)*Sin[c + d*x]^(n - 1))/(a*Cos[c + d*x] +
b*Sin[c + d*x]), x], x]) /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0] &&
IGtQ[m, 0] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{\cos^2(x)\sin^2(x)}{a\cos(x)+b\sin(x)}\,dx = \frac{a\int \cos(x)\sin^2(x)\,dx}{a^2+b^2} + \frac{b\int \cos^2(x)\sin(x)\,dx}{a^2+b^2} - \frac{(ab)\int \frac{\cos(x)\sin(x)}{a\cos(x)+b\sin(x)}\,dx}{a^2+b^2}$$

$$= -\frac{(a^2b)\int \sin(x)\,dx}{(a^2+b^2)^2} - \frac{(ab^2)\int \cos(x)\,dx}{(a^2+b^2)^2} + \frac{(a^2b^2)\int \frac{1}{a\cos(x)+b\sin(x)}\,dx}{(a^2+b^2)^2} + \frac{a\,\mathrm{Subst}\left(\int x^2 \ldots\right.}{a^2+\ldots}$$

$$= \frac{a^2b\cos(x)}{(a^2+b^2)^2} - \frac{b\cos^3(x)}{3(a^2+b^2)} - \frac{ab^2\sin(x)}{(a^2+b^2)^2} + \frac{a\sin^3(x)}{3(a^2+b^2)} - \frac{(a^2b^2)\,\mathrm{Subst}\left(\int \frac{1}{a^2+b^2-x^2}\,dx, x, \ldots\right.}{(a^2+b^2)^2}$$

$$= -\frac{a^2b^2\tanh^{-1}\left(\frac{b\cos(x)-a\sin(x)}{\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{5/2}} + \frac{a^2b\cos(x)}{(a^2+b^2)^2} - \frac{b\cos^3(x)}{3(a^2+b^2)} - \frac{ab^2\sin(x)}{(a^2+b^2)^2} + \frac{a\sin^3(x)}{3(a^2+b^2)}$$

Mathematica [A] time = 0.63, size = 115, normalized size = 1.03

$$\frac{2a^2b^2\tanh^{-1}\left(\frac{a\tan\left(\frac{x}{2}\right)-b}{\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{5/2}} - \frac{(3b^3-9a^2b)\cos(x)+b(a^2+b^2)\cos(3x)+2a\sin(x)\left((a^2+b^2)\cos(2x)-a^2+5b^2\right)}{12(a^2+b^2)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[x]^2*Sin[x]^2)/(a*Cos[x] + b*Sin[x]),x]
```

```
[Out] (2*a^2*b^2*ArcTanh[(-b + a*Tan[x/2])/Sqrt[a^2 + b^2]])/(a^2 + b^2)^(5/2) -
((-9*a^2*b + 3*b^3)*Cos[x] + b*(a^2 + b^2)*Cos[3*x] + 2*a*(-a^2 + 5*b^2 + (
a^2 + b^2)*Cos[2*x])*Sin[x])/(12*(a^2 + b^2)^2)
```

fricas [B] time = 0.77, size = 215, normalized size = 1.92

$$\frac{3\sqrt{a^2+b^2}\,a^2b^2\log\left(-\frac{2ab\cos(x)\sin(x)+(a^2-b^2)\cos(x)^2-2a^2-b^2+2\sqrt{a^2+b^2}(b\cos(x)-a\sin(x))}{2ab\cos(x)\sin(x)+(a^2-b^2)\cos(x)^2+b^2}\right) - 2(a^4b+2a^2b^3+b^5)\cos(x)^3 + \ldots}{6(a^6+3a^4b^2+3a^2b^4+b^6)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^2*sin(x)^2/(a*cos(x)+b*sin(x)),x, algorithm="fricas")
```

```
[Out] 1/6*(3*sqrt(a^2 + b^2)*a^2*b^2*log(-(2*a*b*cos(x)*sin(x) + (a^2 - b^2)*cos(
x)^2 - 2*a^2 - b^2 + 2*sqrt(a^2 + b^2)*(b*cos(x) - a*sin(x)))/(2*a*b*cos(x)
*sin(x) + (a^2 - b^2)*cos(x)^2 + b^2)) - 2*(a^4*b + 2*a^2*b^3 + b^5)*cos(x)
```

```
^3 + 6*(a^4*b + a^2*b^3)*cos(x) + 2*(a^5 - a^3*b^2 - 2*a*b^4 - (a^5 + 2*a^3
*b^2 + a*b^4)*cos(x)^2)*sin(x))/(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)
```

giac [A] time = 8.00, size = 192, normalized size = 1.71

$$-\frac{a^2b^2\log\left(\frac{\left|2\,a\tan\left(\frac{1}{2}\,x\right)-2\,b-2\,\sqrt{a^2+b^2}\right|}{\left|2\,a\tan\left(\frac{1}{2}\,x\right)-2\,b+2\,\sqrt{a^2+b^2}\right|}\right)}{\left(a^4+2\,a^2b^2+b^4\right)\sqrt{a^2+b^2}}-\frac{2\left(3\,ab^2\tan\left(\frac{1}{2}\,x\right)^5+3\,b^3\tan\left(\frac{1}{2}\,x\right)^4-4\,a^3\tan\left(\frac{1}{2}\,x\right)^3+2\,ab^2\tan\left(\frac{1}{2}\,x\right)^3-6\right.}{3\left(a^4+2\,a^2b^2+b^4\right)\left(\tan\left(\frac{1}{2}\,x\right)^2+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^2*sin(x)^2/(a*cos(x)+b*sin(x)),x, algorithm="giac")
```

```
[Out] -a^2*b^2*log(abs(2*a*tan(1/2*x) - 2*b - 2*sqrt(a^2 + b^2))/abs(2*a*tan(1/2*
x) - 2*b + 2*sqrt(a^2 + b^2)))/((a^4 + 2*a^2*b^2 + b^4)*sqrt(a^2 + b^2)) -
2/3*(3*a*b^2*tan(1/2*x)^5 + 3*b^3*tan(1/2*x)^4 - 4*a^3*tan(1/2*x)^3 + 2*a*b
^2*tan(1/2*x)^3 - 6*a^2*b*tan(1/2*x)^2 + 3*a*b^2*tan(1/2*x) - 2*a^2*b + b^3
)/((a^4 + 2*a^2*b^2 + b^4)*(tan(1/2*x)^2 + 1)^3)
```

maple [A] time = 0.09, size = 168, normalized size = 1.50

$$\frac{8a^2b^2\operatorname{arctanh}\left(\frac{2a\tan\left(\frac{x}{2}\right)-2b}{2\sqrt{a^2+b^2}}\right)}{\left(4a^4+8a^2b^2+4b^4\right)\sqrt{a^2+b^2}}+\frac{-2a\,b^2\left(\tan^5\left(\frac{x}{2}\right)\right)-2b^3\left(\tan^4\left(\frac{x}{2}\right)\right)+2\left(\frac{4}{3}a^3-\frac{2}{3}a\,b^2\right)\left(\tan^3\left(\frac{x}{2}\right)\right)+4a^2b\left(\tan^2\left(\frac{x}{2}\right.\right.}{\left(a^4+2a^2b^2+b^4\right)\left(\tan^2\left(\frac{x}{2}\right)+1\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)^2*sin(x)^2/(a*cos(x)+b*sin(x)),x)
```

```
[Out] 8*a^2*b^2/(4*a^4+8*a^2*b^2+4*b^4)/(a^2+b^2)^(1/2)*arctanh(1/2*(2*a*tan(1/2*
x)-2*b)/(a^2+b^2)^(1/2))+2/(a^4+2*a^2*b^2+b^4)*(-a*b^2*tan(1/2*x)^5-b^3*tan
(1/2*x)^4+(4/3*a^3-2/3*a*b^2)*tan(1/2*x)^3+2*a^2*b*tan(1/2*x)^2-a*b^2*tan(1
/2*x)+2/3*a^2*b-1/3*b^3)/(tan(1/2*x)^2+1)^3
```

maxima [B] time = 0.46, size = 281, normalized size = 2.51

$$-\frac{a^2b^2\log\left(\frac{b-\frac{a\sin(x)}{\cos(x)+1}+\sqrt{a^2+b^2}}{b-\frac{a\sin(x)}{\cos(x)+1}-\sqrt{a^2+b^2}}\right)}{\left(a^4+2\,a^2b^2+b^4\right)\sqrt{a^2+b^2}}+\frac{2\left(2\,a^2b-b^3-\frac{3\,ab^2\sin(x)}{\cos(x)+1}+\frac{6\,a^2b\sin(x)^2}{(\cos(x)+1)^2}-\frac{3\,b^3\sin(x)^4}{(\cos(x)+1)^4}-\frac{3\,ab^2\sin(x)^5}{(\cos(x)+1)^5}+\frac{2\left(2\,a^3-ab^2\right)\sin(x)}{(\cos(x)+1)^3}\right.}{3\left(a^4+2\,a^2b^2+b^4+\frac{3\left(a^4+2\,a^2b^2+b^4\right)\sin(x)^2}{(\cos(x)+1)^2}+\frac{3\left(a^4+2\,a^2b^2+b^4\right)\sin(x)^4}{(\cos(x)+1)^4}+\frac{\left(a^4+2\,a^2b^2+b^4\right)\sin}{(\cos(x)+1)^6}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^2*sin(x)^2/(a*cos(x)+b*sin(x)),x, algorithm="maxima")
```

```
[Out] -a^2*b^2*log((b - a*sin(x)/(cos(x) + 1) + sqrt(a^2 + b^2))/(b - a*sin(x)/(c
os(x) + 1) - sqrt(a^2 + b^2)))/((a^4 + 2*a^2*b^2 + b^4)*sqrt(a^2 + b^2)) +
2/3*(2*a^2*b - b^3 - 3*a*b^2*sin(x)/(cos(x) + 1) + 6*a^2*b*sin(x)^2/(cos(x)
 + 1)^2 - 3*b^3*sin(x)^4/(cos(x) + 1)^4 - 3*a*b^2*sin(x)^5/(cos(x) + 1)^5 +
 2*(2*a^3 - a*b^2)*sin(x)^3/(cos(x) + 1)^3)/(a^4 + 2*a^2*b^2 + b^4 + 3*(a^4
 + 2*a^2*b^2 + b^4)*sin(x)^2/(cos(x) + 1)^2 + 3*(a^4 + 2*a^2*b^2 + b^4)*sin
(x)^4/(cos(x) + 1)^4 + (a^4 + 2*a^2*b^2 + b^4)*sin(x)^6/(cos(x) + 1)^6)
```

mupad [B] time = 1.00, size = 277, normalized size = 2.47

$$-\frac{\frac{4\tan\left(\frac{x}{2}\right)^3\left(a\,b^2-2\,a^3\right)}{3\left(a^4+2\,a^2\,b^2+b^4\right)}-\frac{2\,b\left(2\,a^2-b^2\right)}{3\left(a^2+b^2\right)^2}+\frac{2\,b^3\tan\left(\frac{x}{2}\right)^4}{a^4+2\,a^2\,b^2+b^4}-\frac{4\,a^2\,b\tan\left(\frac{x}{2}\right)^2}{a^4+2\,a^2\,b^2+b^4}+\frac{2\,a\,b^2\tan\left(\frac{x}{2}\right)^5}{a^4+2\,a^2\,b^2+b^4}+\frac{2\,a\,b^2\tan\left(\frac{x}{2}\right)}{a^4+2\,a^2\,b^2+b^4}}{\tan\left(\frac{x}{2}\right)^6+3\tan\left(\frac{x}{2}\right)^4+3\tan\left(\frac{x}{2}\right)^2+1}-\frac{2\,a^2\,b^2\,\mathrm{atanh}\left(\frac{2\,a^4\,b+2\,b^5+\cdots}{\cdots}\right)}{\left(a^2\cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(x)^2*sin(x)^2)/(a*cos(x) + b*sin(x)),x)
```

```
[Out] - ((4*tan(x/2)^3*(a*b^2 - 2*a^3))/(3*(a^4 + b^4 + 2*a^2*b^2)) - (2*b*(2*a^2
 - b^2))/(3*(a^2 + b^2)^2) + (2*b^3*tan(x/2)^4)/(a^4 + b^4 + 2*a^2*b^2) - (
4*a^2*b*tan(x/2)^2)/(a^4 + b^4 + 2*a^2*b^2) + (2*a*b^2*tan(x/2)^5)/(a^4 + b
^4 + 2*a^2*b^2) + (2*a*b^2*tan(x/2))/(a^4 + b^4 + 2*a^2*b^2))/(3*tan(x/2)^2
 + 3*tan(x/2)^4 + tan(x/2)^6 + 1) - (2*a^2*b^2*atanh((2*a^4*b + 2*b^5 + 4*a
^2*b^3 - 2*a*tan(x/2)*(a^4 + b^4 + 2*a^2*b^2))/(2*(a^2 + b^2)^(5/2))))/(a^2
 + b^2)^(5/2)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)**2*sin(x)**2/(a*cos(x)+b*sin(x)),x)
```

```
[Out] Timed out
```

3.280 $\int \frac{\cos^2(x)\sin^3(x)}{a\cos(x)+b\sin(x)}\,dx$

Optimal. Leaf size=176

$$\frac{bx}{8(a^2+b^2)}-\frac{a^2bx}{2(a^2+b^2)^2}+\frac{a\sin^4(x)}{4(a^2+b^2)}-\frac{ab^2\sin^2(x)}{2(a^2+b^2)^2}-\frac{b\sin(x)\cos^3(x)}{4(a^2+b^2)}+\frac{b\sin(x)\cos(x)}{8(a^2+b^2)}+\frac{a^2b\sin(x)\cos(x)}{2(a^2+b^2)^2}+\frac{a^2}{(a^2}$$

```
[Out] a^2*b^3*x/(a^2+b^2)^3-1/2*a^2*b*x/(a^2+b^2)^2+1/8*b*x/(a^2+b^2)-a^3*b^2*ln(
a*cos(x)+b*sin(x))/(a^2+b^2)^3+1/2*a^2*b*cos(x)*sin(x)/(a^2+b^2)^2+1/8*b*co
s(x)*sin(x)/(a^2+b^2)-1/4*b*cos(x)^3*sin(x)/(a^2+b^2)-1/2*a*b^2*sin(x)^2/(a
^2+b^2)^2+1/4*a*sin(x)^4/(a^2+b^2)
```

Rubi [A] time = 0.28, antiderivative size = 176, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 8, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {3109, 2568, 2635, 8, 2564, 30, 3097, 3133}

$$\frac{bx}{8(a^2+b^2)}-\frac{a^2bx}{2(a^2+b^2)^2}+\frac{a^2b^3x}{(a^2+b^2)^3}+\frac{a\sin^4(x)}{4(a^2+b^2)}-\frac{ab^2\sin^2(x)}{2(a^2+b^2)^2}-\frac{b\sin(x)\cos^3(x)}{4(a^2+b^2)}+\frac{b\sin(x)\cos(x)}{8(a^2+b^2)}+\frac{a^2b\sin(x)}{2(a^2+b}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[x]^2*Sin[x]^3)/(a*Cos[x] + b*Sin[x]),x]
```

```
[Out] (a^2*b^3*x)/(a^2 + b^2)^3 - (a^2*b*x)/(2*(a^2 + b^2)^2) + (b*x)/(8*(a^2 + b
^2)) - (a^3*b^2*Log[a*Cos[x] + b*Sin[x]])/(a^2 + b^2)^3 + (a^2*b*Cos[x]*Sin
[x])/(2*(a^2 + b^2)^2) + (b*Cos[x]*Sin[x])/(8*(a^2 + b^2)) - (b*Cos[x]^3*Si
n[x])/(4*(a^2 + b^2)) - (a*b^2*Sin[x]^2)/(2*(a^2 + b^2)^2) + (a*Sin[x]^4)/(
4*(a^2 + b^2))
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2564

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
```

```
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 2568

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> -Simp[(a*(b*Cos[e + f*x])^(n + 1)*(a*Sin[e + f*x])^(m - 1)
)/(b*f*(m + n)), x] + Dist[(a^2*(m - 1))/(m + n), Int[(b*Cos[e + f*x])^n*(a
*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1] &&
 NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 3097

```
Int[sin[(c_.) + (d_.)*(x_)]/(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.
) + (d_.)*(x_)]), x_Symbol] :> Simp[(b*x)/(a^2 + b^2), x] - Dist[a/(a^2 + b
^2), Int[(b*Cos[c + d*x] - a*Sin[c + d*x])/(a*Cos[c + d*x] + b*Sin[c + d*x]
), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

Rule 3109

```
Int[(cos[(c_.) + (d_.)*(x_)]^(m_.)*sin[(c_.) + (d_.)*(x_)]^(n_.))/(cos[(c_.
) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[b
/(a^2 + b^2), Int[Cos[c + d*x]^m*Sin[c + d*x]^(n - 1), x], x] + (Dist[a/(a^
2 + b^2), Int[Cos[c + d*x]^(m - 1)*Sin[c + d*x]^n, x], x] - Dist[(a*b)/(a^2
 + b^2), Int[(Cos[c + d*x]^(m - 1)*Sin[c + d*x]^(n - 1))/(a*Cos[c + d*x] +
b*Sin[c + d*x]), x], x]) /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0] &&
IGtQ[m, 0] && IGtQ[n, 0]
```

Rule 3133

```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])
/((a_.) + cos[(d_.) + (e_.)*(x_)]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)]), x
_Symbol] :> Simp[((b*B + c*C)*x)/(b^2 + c^2), x] + Simp[((c*B - b*C)*Log[a
+ b*Cos[d + e*x] + c*Sin[d + e*x]])/(e*(b^2 + c^2)), x] /; FreeQ[{a, b, c,
d, e, A, B, C}, x] && NeQ[b^2 + c^2, 0] && EqQ[A*(b^2 + c^2) - a*(b*B + c*C
), 0]
```

Rubi steps

$$\int \frac{\cos^2(x)\sin^3(x)}{a\cos(x)+b\sin(x)}\,dx = \frac{a\int\cos(x)\sin^3(x)\,dx}{a^2+b^2} + \frac{b\int\cos^2(x)\sin^2(x)\,dx}{a^2+b^2} - \frac{(ab)\int\frac{\cos(x)\sin^2(x)}{a\cos(x)+b\sin(x)}\,dx}{a^2+b^2}$$

$$= -\frac{b\cos^3(x)\sin(x)}{4(a^2+b^2)} - \frac{(a^2b)\int\sin^2(x)\,dx}{(a^2+b^2)^2} - \frac{(ab^2)\int\cos(x)\sin(x)\,dx}{(a^2+b^2)^2} + \frac{(a^2b^2)\int\frac{\sin}{a\cos(x)}}{(a^2+b}$$

$$= \frac{a^2b^3x}{(a^2+b^2)^3} + \frac{a^2b\cos(x)\sin(x)}{2(a^2+b^2)^2} + \frac{b\cos(x)\sin(x)}{8(a^2+b^2)} - \frac{b\cos^3(x)\sin(x)}{4(a^2+b^2)} + \frac{a\sin^4(x)}{4(a^2+b^2)} -$$

$$= \frac{a^2b^3x}{(a^2+b^2)^3} - \frac{a^2bx}{2(a^2+b^2)^2} + \frac{bx}{8(a^2+b^2)} - \frac{a^3b^2\log(a\cos(x)+b\sin(x))}{(a^2+b^2)^3} + \frac{a^2b\cos(x}{2(a^2+}$$

Mathematica [C] time = 0.52, size = 178, normalized size = 1.01

$$\frac{a^5\cos(4x) - 4a(a^4-b^4)\cos(2x) - 12a^4bx + 8a^4b\sin(2x) - a^4b\sin(4x) - 32ia^3b^2x + 2a^3b^2\cos(4x) + 32ia^3b^2\,\mathrm{t}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[x]^2*Sin[x]^3)/(a*Cos[x] + b*Sin[x]),x]
```

```
[Out] (-12*a^4*b*x - (32*I)*a^3*b^2*x + 24*a^2*b^3*x + 4*b^5*x + (32*I)*a^3*b^2*A
rcTan[Tan[x]] - 4*a*(a^4 - b^4)*Cos[2*x] + a^5*Cos[4*x] + 2*a^3*b^2*Cos[4*x
] + a*b^4*Cos[4*x] - 16*a^3*b^2*Log[(a*Cos[x] + b*Sin[x])^2] + 8*a^4*b*Sin[
2*x] + 8*a^2*b^3*Sin[2*x] - a^4*b*Sin[4*x] - 2*a^2*b^3*Sin[4*x] - b^5*Sin[4
*x])/(32*(a^2 + b^2)^3)
```

fricas [A] time = 0.73, size = 174, normalized size = 0.99

$$-\frac{4\,a^3b^2\log\left(2\,ab\cos(x)\sin(x)+(a^2-b^2)\cos(x)^2+b^2\right) - 2\left(a^5+2\,a^3b^2+ab^4\right)\cos(x)^4 + 4\left(a^5+a^3b^2\right)\cos(x)^2}{8\left(a^6+3\,a^4b^2+3\,a^2b^4\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^2*sin(x)^3/(a*cos(x)+b*sin(x)),x, algorithm="fricas")
```

```
[Out] -1/8*(4*a^3*b^2*log(2*a*b*cos(x)*sin(x) + (a^2 - b^2)*cos(x)^2 + b^2) - 2*(
a^5 + 2*a^3*b^2 + a*b^4)*cos(x)^4 + 4*(a^5 + a^3*b^2)*cos(x)^2 + (3*a^4*b -
 6*a^2*b^3 - b^5)*x + (2*(a^4*b + 2*a^2*b^3 + b^5)*cos(x)^3 - (5*a^4*b + 6*
a^2*b^3 + b^5)*cos(x))*sin(x))/(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)
```

giac [A] time = 1.92, size = 275, normalized size = 1.56

$$-\frac{a^3b^3\log(|b\tan(x)+a|)}{a^6b+3a^4b^3+3a^2b^5+b^7}+\frac{a^3b^2\log\left(\tan(x)^2+1\right)}{2\left(a^6+3a^4b^2+3a^2b^4+b^6\right)}-\frac{\left(3a^4b-6a^2b^3-b^5\right)x}{8\left(a^6+3a^4b^2+3a^2b^4+b^6\right)}-\frac{6a^3b^2\tan(x)^4-5a^4b}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^2*sin(x)^3/(a*cos(x)+b*sin(x)),x, algorithm="giac")
```

```
[Out] -a^3*b^3*log(abs(b*tan(x) + a))/(a^6*b + 3*a^4*b^3 + 3*a^2*b^5 + b^7) + 1/2
*a^3*b^2*log(tan(x)^2 + 1)/(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6) - 1/8*(3*a^4
*b - 6*a^2*b^3 - b^5)*x/(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6) - 1/8*(6*a^3*b^
2*tan(x)^4 - 5*a^4*b*tan(x)^3 - 6*a^2*b^3*tan(x)^3 - b^5*tan(x)^3 + 4*a^5*t
an(x)^2 + 16*a^3*b^2*tan(x)^2 - 3*a^4*b*tan(x) - 2*a^2*b^3*tan(x) + b^5*tan
(x) + 2*a^5 + 6*a^3*b^2 - 2*a*b^4)/((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*(ta
n(x)^2 + 1)^2)
```

maple [B] time = 0.09, size = 363, normalized size = 2.06

$$\frac{3\left(\tan^3(x)\right)a^2b^3}{4\left(a^2+b^2\right)^3\left(\tan^2(x)+1\right)^2}+\frac{\left(\tan^3(x)\right)b^5}{8\left(a^2+b^2\right)^3\left(\tan^2(x)+1\right)^2}+\frac{5\left(\tan^3(x)\right)a^4b}{8\left(a^2+b^2\right)^3\left(\tan^2(x)+1\right)^2}-\frac{\left(\tan^2(x)\right)a^5}{2\left(a^2+b^2\right)^3\left(\tan^2(x)+1\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)^2*sin(x)^3/(a*cos(x)+b*sin(x)),x)
```

```
[Out] 3/4/(a^2+b^2)^3/(tan(x)^2+1)^2*tan(x)^3*a^2*b^3+1/8/(a^2+b^2)^3/(tan(x)^2+1
)^2*tan(x)^3*b^5+5/8/(a^2+b^2)^3/(tan(x)^2+1)^2*tan(x)^3*a^4*b-1/2/(a^2+b^2
)^3/(tan(x)^2+1)^2*tan(x)^2*a^5-1/2/(a^2+b^2)^3/(tan(x)^2+1)^2*tan(x)^2*a^3
*b^2+3/8/(a^2+b^2)^3/(tan(x)^2+1)^2*tan(x)*a^4*b+1/4/(a^2+b^2)^3/(tan(x)^2+
1)^2*tan(x)*a^2*b^3-1/8/(a^2+b^2)^3/(tan(x)^2+1)^2*tan(x)*b^5-1/4/(a^2+b^2)
^3/(tan(x)^2+1)^2*a^5+1/4/(a^2+b^2)^3/(tan(x)^2+1)^2*a*b^4+1/2/(a^2+b^2)^3*
ln(tan(x)^2+1)*a^3*b^2-3/8/(a^2+b^2)^3*arctan(tan(x))*a^4*b+3/4/(a^2+b^2)^3
*arctan(tan(x))*a^2*b^3+1/8/(a^2+b^2)^3*arctan(tan(x))*b^5-a^3*b^2/(a^2+b^2
)^3*ln(a+b*tan(x))
```

maxima [B] time = 0.45, size = 431, normalized size = 2.45

$$-\frac{a^3b^2\log\left(-a-\frac{2b\sin(x)}{\cos(x)+1}+\frac{a\sin(x)^2}{(\cos(x)+1)^2}\right)}{a^6+3a^4b^2+3a^2b^4+b^6}+\frac{a^3b^2\log\left(\frac{\sin(x)^2}{(\cos(x)+1)^2}+1\right)}{a^6+3a^4b^2+3a^2b^4+b^6}-\frac{\left(3a^4b-6a^2b^3-b^5\right)\arctan\left(\frac{\sin(x)}{\cos(x)+1}\right)}{4\left(a^6+3a^4b^2+3a^2b^4+b^6\right)}-\frac{\frac{8ab^2\sin}{(\cos(x)+}}{4\left(a^4+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^2*sin(x)^3/(a*cos(x)+b*sin(x)),x, algorithm="maxima")
```

```
[Out] -a^3*b^2*log(-a - 2*b*sin(x)/(cos(x) + 1) + a*sin(x)^2/(cos(x) + 1)^2)/(a^6
 + 3*a^4*b^2 + 3*a^2*b^4 + b^6) + a^3*b^2*log(sin(x)^2/(cos(x) + 1)^2 + 1)/
(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6) - 1/4*(3*a^4*b - 6*a^2*b^3 - b^5)*arcta
n(sin(x)/(cos(x) + 1))/(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6) - 1/4*(8*a*b^2*s
in(x)^2/(cos(x) + 1)^2 - 16*a^3*sin(x)^4/(cos(x) + 1)^4 + 8*a*b^2*sin(x)^6/
(cos(x) + 1)^6 - (3*a^2*b - b^3)*sin(x)/(cos(x) + 1) - (11*a^2*b + 7*b^3)*s
in(x)^3/(cos(x) + 1)^3 + (11*a^2*b + 7*b^3)*sin(x)^5/(cos(x) + 1)^5 + (3*a^
2*b - b^3)*sin(x)^7/(cos(x) + 1)^7)/(a^4 + 2*a^2*b^2 + b^4 + 4*(a^4 + 2*a^2
*b^2 + b^4)*sin(x)^2/(cos(x) + 1)^2 + 6*(a^4 + 2*a^2*b^2 + b^4)*sin(x)^4/(c
os(x) + 1)^4 + 4*(a^4 + 2*a^2*b^2 + b^4)*sin(x)^6/(cos(x) + 1)^6 + (a^4 + 2
*a^2*b^2 + b^4)*sin(x)^8/(cos(x) + 1)^8)
```

mupad [B] time = 11.95, size = 5902, normalized size = 33.53

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(x)^2*sin(x)^3)/(a*cos(x) + b*sin(x)),x)
```

```
[Out] (64*a^3*b^2*log(1/(cos(x) + 1)))/(64*a^6 + 64*b^6 + 192*a^2*b^4 + 192*a^4*b
^2) - (b*atan((tan(x/2)*(((((64*a^3*b^2*((b*((448*a^8*b^8 - 96*a^4*b^12 - 48
*a^6*b^10 - 16*a^2*b^14 + 912*a^10*b^6 + 672*a^12*b^4 + 176*a^14*b^2)/(2*(a
^12 + b^12 + 6*a^2*b^10 + 15*a^4*b^8 + 20*a^6*b^6 + 15*a^8*b^4 + 6*a^10*b^2
)) - (32*a^3*b^2*(192*a*b^16 + 1344*a^3*b^14 + 4032*a^5*b^12 + 6720*a^7*b^1
0 + 6720*a^9*b^8 + 4032*a^11*b^6 + 1344*a^13*b^4 + 192*a^15*b^2))/((64*a^6
+ 64*b^6 + 192*a^2*b^4 + 192*a^4*b^2)*(a^12 + b^12 + 6*a^2*b^10 + 15*a^4*b^
8 + 20*a^6*b^6 + 15*a^8*b^4 + 6*a^10*b^2)))*(b^4 - 3*a^4 + 6*a^2*b^2))/(8*(
a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2)) - (4*a^3*b^3*(b^4 - 3*a^4 + 6*a^2*b^2)*
(192*a*b^16 + 1344*a^3*b^14 + 4032*a^5*b^12 + 6720*a^7*b^10 + 6720*a^9*b^8
+ 4032*a^11*b^6 + 1344*a^13*b^4 + 192*a^15*b^2))/((64*a^6 + 64*b^6 + 192*a^
2*b^4 + 192*a^4*b^2)*(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2)*(a^12 + b^12 + 6*a
^2*b^10 + 15*a^4*b^8 + 20*a^6*b^6 + 15*a^8*b^4 + 6*a^10*b^2))))/(64*a^6 + 6
4*b^6 + 192*a^2*b^4 + 192*a^4*b^2) - (b*((2*a*b^14 + 27*a^3*b^12 + 129*a^5*
b^10 + 62*a^7*b^8 - 156*a^9*b^6 - 105*a^11*b^4 + 9*a^13*b^2)/(2*(a^12 + b^1
2 + 6*a^2*b^10 + 15*a^4*b^8 + 20*a^6*b^6 + 15*a^8*b^4 + 6*a^10*b^2)) - (64*
a^3*b^2*((448*a^8*b^8 - 96*a^4*b^12 - 48*a^6*b^10 - 16*a^2*b^14 + 912*a^10*
b^6 + 672*a^12*b^4 + 176*a^14*b^2)/(2*(a^12 + b^12 + 6*a^2*b^10 + 15*a^4*b^
8 + 20*a^6*b^6 + 15*a^8*b^4 + 6*a^10*b^2)) - (32*a^3*b^2*(192*a*b^16 + 1344
*a^3*b^14 + 4032*a^5*b^12 + 6720*a^7*b^10 + 6720*a^9*b^8 + 4032*a^11*b^6 +
1344*a^13*b^4 + 192*a^15*b^2))/((64*a^6 + 64*b^6 + 192*a^2*b^4 + 192*a^4*b^
2)*(a^12 + b^12 + 6*a^2*b^10 + 15*a^4*b^8 + 20*a^6*b^6 + 15*a^8*b^4 + 6*a^1
0*b^2))))/(64*a^6 + 64*b^6 + 192*a^2*b^4 + 192*a^4*b^2))*(b^4 - 3*a^4 + 6*a
^2*b^2))/(8*(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2)) + (b^3*(b^4 - 3*a^4 + 6*a^
```

2*b^2)^3*(192*a*b^16 + 1344*a^3*b^14 + 4032*a^5*b^12 + 6720*a^7*b^10 + 6720
*a^9*b^8 + 4032*a^11*b^6 + 1344*a^13*b^4 + 192*a^15*b^2))/(1024*(a^6 + b^6
+ 3*a^2*b^4 + 3*a^4*b^2)^3*(a^12 + b^12 + 6*a^2*b^10 + 15*a^4*b^8 + 20*a^6*
b^6 + 15*a^8*b^4 + 6*a^10*b^2)))*(9*a^10 - b^10 - 11*a^2*b^8 + 46*a^4*b^6 +
706*a^6*b^4 - 493*a^8*b^2))/(9*a^10 + b^10 + 13*a^2*b^8 + 42*a^4*b^6 + 250
*a^6*b^4 + 229*a^8*b^2)^2 + (2*a*b*((2*a^4*b^10 + 21*a^6*b^8 + 44*a^8*b^6 +
9*a^10*b^4)/(2*(a^12 + b^12 + 6*a^2*b^10 + 15*a^4*b^8 + 20*a^6*b^6 + 15*a^
8*b^4 + 6*a^10*b^2)) + (64*a^3*b^2*((2*a*b^14 + 27*a^3*b^12 + 129*a^5*b^10
+ 62*a^7*b^8 - 156*a^9*b^6 - 105*a^11*b^4 + 9*a^13*b^2)/(2*(a^12 + b^12 + 6
*a^2*b^10 + 15*a^4*b^8 + 20*a^6*b^6 + 15*a^8*b^4 + 6*a^10*b^2)) - (64*a^3*b
^2*((448*a^8*b^8 - 96*a^4*b^12 - 48*a^6*b^10 - 16*a^2*b^14 + 912*a^10*b^6 +
672*a^12*b^4 + 176*a^14*b^2)/(2*(a^12 + b^12 + 6*a^2*b^10 + 15*a^4*b^8 + 2
0*a^6*b^6 + 15*a^8*b^4 + 6*a^10*b^2)) - (32*a^3*b^2*(192*a*b^16 + 1344*a^3*
b^14 + 4032*a^5*b^12 + 6720*a^7*b^10 + 6720*a^9*b^8 + 4032*a^11*b^6 + 1344*
a^13*b^4 + 192*a^15*b^2))/((64*a^6 + 64*b^6 + 192*a^2*b^4 + 192*a^4*b^2)*(a
^12 + b^12 + 6*a^2*b^10 + 15*a^4*b^8 + 20*a^6*b^6 + 15*a^8*b^4 + 6*a^10*b^2
))))/(64*a^6 + 64*b^6 + 192*a^2*b^4 + 192*a^4*b^2)))/(64*a^6 + 64*b^6 + 192
*a^2*b^4 + 192*a^4*b^2) + (b*((b*((448*a^8*b^8 - 96*a^4*b^12 - 48*a^6*b^10
- 16*a^2*b^14 + 912*a^10*b^6 + 672*a^12*b^4 + 176*a^14*b^2)/(2*(a^12 + b^12
+ 6*a^2*b^10 + 15*a^4*b^8 + 20*a^6*b^6 + 15*a^8*b^4 + 6*a^10*b^2)) - (32*a
^3*b^2*(192*a*b^16 + 1344*a^3*b^14 + 4032*a^5*b^12 + 6720*a^7*b^10 + 6720*a
^9*b^8 + 4032*a^11*b^6 + 1344*a^13*b^4 + 192*a^15*b^2))/((64*a^6 + 64*b^6 +
192*a^2*b^4 + 192*a^4*b^2)*(a^12 + b^12 + 6*a^2*b^10 + 15*a^4*b^8 + 20*a^6
*b^6 + 15*a^8*b^4 + 6*a^10*b^2)))*(b^4 - 3*a^4 + 6*a^2*b^2))/(8*(a^6 + b^6
+ 3*a^2*b^4 + 3*a^4*b^2)) - (4*a^3*b^3*(b^4 - 3*a^4 + 6*a^2*b^2)*(192*a*b^1
6 + 1344*a^3*b^14 + 4032*a^5*b^12 + 6720*a^7*b^10 + 6720*a^9*b^8 + 4032*a^1
1*b^6 + 1344*a^13*b^4 + 192*a^15*b^2))/((64*a^6 + 64*b^6 + 192*a^2*b^4 + 19
2*a^4*b^2)*(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2)*(a^12 + b^12 + 6*a^2*b^10 +
15*a^4*b^8 + 20*a^6*b^6 + 15*a^8*b^4 + 6*a^10*b^2)))*(b^4 - 3*a^4 + 6*a^2*b
^2))/(8*(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2)) - (a^3*b^4*(b^4 - 3*a^4 + 6*a^
2*b^2)^2*(192*a*b^16 + 1344*a^3*b^14 + 4032*a^5*b^12 + 6720*a^7*b^10 + 6720
*a^9*b^8 + 4032*a^11*b^6 + 1344*a^13*b^4 + 192*a^15*b^2))/(2*(64*a^6 + 64*b
^6 + 192*a^2*b^4 + 192*a^4*b^2)*(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2)^2*(a^12
+ b^12 + 6*a^2*b^10 + 15*a^4*b^8 + 20*a^6*b^6 + 15*a^8*b^4 + 6*a^10*b^2)))
*(57*a^8 + b^8 + 28*a^2*b^6 + 110*a^4*b^4 - 436*a^6*b^2))/(9*a^10 + b^10 +
13*a^2*b^8 + 42*a^4*b^6 + 250*a^6*b^4 + 229*a^8*b^2)^2)*(16*a^16 + 16*b^16
+ 128*a^2*b^14 + 448*a^4*b^12 + 896*a^6*b^10 + 1120*a^8*b^8 + 896*a^10*b^6
+ 448*a^12*b^4 + 128*a^14*b^2))/(a*b^7 + 6*a^3*b^5 - 3*a^5*b^3) + (((64*a^3
b^2((b*((8*a*b^15 + 24*a^15*b + 72*a^3*b^13 + 72*a^5*b^11 - 248*a^7*b^9 -
552*a^9*b^7 - 360*a^11*b^5 - 40*a^13*b^3)/(2*(a^12 + b^12 + 6*a^2*b^10 + 1
5*a^4*b^8 + 20*a^6*b^6 + 15*a^8*b^4 + 6*a^10*b^2)) - (32*a^3*b^2*(192*a^16*
b + 192*a^2*b^15 + 1344*a^4*b^13 + 4032*a^6*b^11 + 6720*a^8*b^9 + 6720*a^10
*b^7 + 4032*a^12*b^5 + 1344*a^14*b^3))/((64*a^6 + 64*b^6 + 192*a^2*b^4 + 19
2*a^4*b^2)*(a^12 + b^12 + 6*a^2*b^10 + 15*a^4*b^8 + 20*a^6*b^6 + 15*a^8*b^4
+ 6*a^10*b^2)))*(b^4 - 3*a^4 + 6*a^2*b^2))/(8*(a^6 + b^6 + 3*a^2*b^4 + 3*a

```
^4*b^2)) - (4*a^3*b^3*(b^4 - 3*a^4 + 6*a^2*b^2)*(192*a^16*b + 192*a^2*b^15
+ 1344*a^4*b^13 + 4032*a^6*b^11 + 6720*a^8*b^9 + 6720*a^10*b^7 + 4032*a^12*
b^5 + 1344*a^14*b^3))/((64*a^6 + 64*b^6 + 192*a^2*b^4 + 192*a^4*b^2)*(a^6 +
 b^6 + 3*a^2*b^4 + 3*a^4*b^2)*(a^12 + b^12 + 6*a^2*b^10 + 15*a^4*b^8 + 20*a
^6*b^6 + 15*a^8*b^4 + 6*a^10*b^2))))/(64*a^6 + 64*b^6 + 192*a^2*b^4 + 192*a
^4*b^2) + (b*((3*a^4*b^11 - a^2*b^13 + 54*a^6*b^9 + 134*a^8*b^7 + 123*a^10*
b^5 + 39*a^12*b^3)/(2*(a^12 + b^12 + 6*a^2*b^10 + 15*a^4*b^8 + 20*a^6*b^6 +
 15*a^8*b^4 + 6*a^10*b^2)) + (64*a^3*b^2*((8*a*b^15 + 24*a^15*b + 72*a^3*b^
13 + 72*a^5*b^11 - 248*a^7*b^9 - 552*a^9*b^7 - 360*a^11*b^5 - 40*a^13*b^3)/
(2*(a^12 + b^12 + 6*a^2*b^10 + 15*a^4*b^8 + 20*a^6*b^6 + 15*a^8*b^4 + 6*a^1
0*b^2)) - (32*a^3*b^2*(192*a^16*b + 192*a^2*b^15 + 1344*a^4*b^13 + 4032*a^6
*b^11 + 6720*a^8*b^9 + 6720*a^10*b^7 + 4032*a^12*b^5 + 1344*a^14*b^3))/((64
*a^6 + 64*b^6 + 192*a^2*b^4 + 192*a^4*b^2)*(a^12 + b^12 + 6*a^2*b^10 + 15*a
^4*b^8 + 20*a^6*b^6 + 15*a^8*b^4 + 6*a^10*b^2))))/(64*a^6 + 64*b^6 + 192*a^
2*b^4 + 192*a^4*b^2))*(b^4 - 3*a^4 + 6*a^2*b^2))/(8*(a^6 + b^6 + 3*a^2*b^4
+ 3*a^4*b^2)) + (b^3*(b^4 - 3*a^4 + 6*a^2*b^2)^3*(192*a^16*b + 192*a^2*b^15
 + 1344*a^4*b^13 + 4032*a^6*b^11 + 6720*a^8*b^9 + 6720*a^10*b^7 + 4032*a^12
*b^5 + 1344*a^14*b^3))/(1024*(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2)^3*(a^12 +
b^12 + 6*a^2*b^10 + 15*a^4*b^8 + 20*a^6*b^6 + 15*a^8*b^4 + 6*a^10*b^2)))*(9
*a^10 - b^10 - 11*a^2*b^8 + 46*a^4*b^6 + 706*a^6*b^4 - 493*a^8*b^2)*(16*a^1
6 + 16*b^16 + 128*a^2*b^14 + 448*a^4*b^12 + 896*a^6*b^10 + 1120*a^8*b^8 + 8
96*a^10*b^6 + 448*a^12*b^4 + 128*a^14*b^2))/((a*b^7 + 6*a^3*b^5 - 3*a^5*b^3
)*(9*a^10 + b^10 + 13*a^2*b^8 + 42*a^4*b^6 + 250*a^6*b^4 + 229*a^8*b^2)^2)
+ (2*a*b*((a^5*b^9 + 2*a^7*b^7 - 15*a^9*b^5)/(2*(a^12 + b^12 + 6*a^2*b^10 +
 15*a^4*b^8 + 20*a^6*b^6 + 15*a^8*b^4 + 6*a^10*b^2)) + (b*((b*((8*a*b^15 +
24*a^15*b + 72*a^3*b^13 + 72*a^5*b^11 - 248*a^7*b^9 - 552*a^9*b^7 - 360*a^1
1*b^5 - 40*a^13*b^3)/(2*(a^12 + b^12 + 6*a^2*b^10 + 15*a^4*b^8 + 20*a^6*b^6
 + 15*a^8*b^4 + 6*a^10*b^2)) - (32*a^3*b^2*(192*a^16*b + 192*a^2*b^15 + 134
4*a^4*b^13 + 4032*a^6*b^11 + 6720*a^8*b^9 + 6720*a^10*b^7 + 4032*a^12*b^5 +
 1344*a^14*b^3))/((64*a^6 + 64*b^6 + 192*a^2*b^4 + 192*a^4*b^2)*(a^12 + b^1
2 + 6*a^2*b^10 + 15*a^4*b^8 + 20*a^6*b^6 + 15*a^8*b^4 + 6*a^10*b^2)))*(b^4
- 3*a^4 + 6*a^2*b^2))/(8*(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2)) - (4*a^3*b^3*
(b^4 - 3*a^4 + 6*a^2*b^2)*(192*a^16*b + 192*a^2*b^15 + 1344*a^4*b^13 + 4032
*a^6*b^11 + 6720*a^8*b^9 + 6720*a^10*b^7 + 4032*a^12*b^5 + 1344*a^14*b^3))/
((64*a^6 + 64*b^6 + 192*a^2*b^4 + 192*a^4*b^2)*(a^6 + b^6 + 3*a^2*b^4 + 3*a
^4*b^2)*(a^12 + b^12 + 6*a^2*b^10 + 15*a^4*b^8 + 20*a^6*b^6 + 15*a^8*b^4 +
6*a^10*b^2)))*(b^4 - 3*a^4 + 6*a^2*b^2))/(8*(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*
b^2)) - (64*a^3*b^2*((3*a^4*b^11 - a^2*b^13 + 54*a^6*b^9 + 134*a^8*b^7 + 12
3*a^10*b^5 + 39*a^12*b^3)/(2*(a^12 + b^12 + 6*a^2*b^10 + 15*a^4*b^8 + 20*a^
6*b^6 + 15*a^8*b^4 + 6*a^10*b^2)) + (64*a^3*b^2*((8*a*b^15 + 24*a^15*b + 72
*a^3*b^13 + 72*a^5*b^11 - 248*a^7*b^9 - 552*a^9*b^7 - 360*a^11*b^5 - 40*a^1
3*b^3)/(2*(a^12 + b^12 + 6*a^2*b^10 + 15*a^4*b^8 + 20*a^6*b^6 + 15*a^8*b^4
+ 6*a^10*b^2)) - (32*a^3*b^2*(192*a^16*b + 192*a^2*b^15 + 1344*a^4*b^13 + 4
032*a^6*b^11 + 6720*a^8*b^9 + 6720*a^10*b^7 + 4032*a^12*b^5 + 1344*a^14*b^3
))/((64*a^6 + 64*b^6 + 192*a^2*b^4 + 192*a^4*b^2)*(a^12 + b^12 + 6*a^2*b^10
```

```
 + 15*a^4*b^8 + 20*a^6*b^6 + 15*a^8*b^4 + 6*a^10*b^2))))/(64*a^6 + 64*b^6 +
 192*a^2*b^4 + 192*a^4*b^2)))/(64*a^6 + 64*b^6 + 192*a^2*b^4 + 192*a^4*b^2)
 - (a^3*b^4*(b^4 - 3*a^4 + 6*a^2*b^2)^2*(192*a^16*b + 192*a^2*b^15 + 1344*a
^4*b^13 + 4032*a^6*b^11 + 6720*a^8*b^9 + 6720*a^10*b^7 + 4032*a^12*b^5 + 13
44*a^14*b^3))/(2*(64*a^6 + 64*b^6 + 192*a^2*b^4 + 192*a^4*b^2)*(a^6 + b^6 +
 3*a^2*b^4 + 3*a^4*b^2)^2*(a^12 + b^12 + 6*a^2*b^10 + 15*a^4*b^8 + 20*a^6*b
^6 + 15*a^8*b^4 + 6*a^10*b^2)))*(57*a^8 + b^8 + 28*a^2*b^6 + 110*a^4*b^4 -
436*a^6*b^2)*(16*a^16 + 16*b^16 + 128*a^2*b^14 + 448*a^4*b^12 + 896*a^6*b^1
0 + 1120*a^8*b^8 + 896*a^10*b^6 + 448*a^12*b^4 + 128*a^14*b^2))/((a*b^7 + 6
*a^3*b^5 - 3*a^5*b^3)*(9*a^10 + b^10 + 13*a^2*b^8 + 42*a^4*b^6 + 250*a^6*b^
4 + 229*a^8*b^2)^2))*(b^4 - 3*a^4 + 6*a^2*b^2))/(4*(a^6 + b^6 + 3*a^2*b^4 +
 3*a^4*b^2)) - (a^3*b^2*log(a + 2*b*tan(x/2) - a*tan(x/2)^2))/(a^6 + b^6 +
3*a^2*b^4 + 3*a^4*b^2) - ((tan(x/2)^7*(3*a^2*b - b^3))/(4*(a^4 + b^4 + 2*a^
2*b^2)) - (tan(x/2)^3*(11*a^2*b + 7*b^3))/(4*(a^4 + b^4 + 2*a^2*b^2)) + (ta
n(x/2)^5*(11*a^2*b + 7*b^3))/(4*(a^4 + b^4 + 2*a^2*b^2)) - (tan(x/2)*(3*a^2
*b - b^3))/(4*(a^4 + b^4 + 2*a^2*b^2)) - (4*a^3*tan(x/2)^4)/(a^4 + b^4 + 2*
a^2*b^2) + (2*a*b^2*tan(x/2)^2)/(a^4 + b^4 + 2*a^2*b^2) + (2*a*b^2*tan(x/2)
^6)/(a^4 + b^4 + 2*a^2*b^2))/(4*tan(x/2)^2 + 6*tan(x/2)^4 + 4*tan(x/2)^6 +
tan(x/2)^8 + 1)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)**2*sin(x)**3/(a*cos(x)+b*sin(x)),x)
```

```
[Out] Timed out
```

3.281 $\int \frac{\cos^3(x)\sin(x)}{a\cos(x)+b\sin(x)}\,dx$

Optimal. Leaf size=123

$$-\frac{b\sin^3(x)}{3\left(a^2+b^2\right)}+\frac{b\sin(x)}{a^2+b^2}-\frac{a^2b\sin(x)}{\left(a^2+b^2\right)^2}-\frac{a\cos^3(x)}{3\left(a^2+b^2\right)}-\frac{ab^2\cos(x)}{\left(a^2+b^2\right)^2}+\frac{ab^3\tanh^{-1}\left(\frac{b\cos(x)-a\sin(x)}{\sqrt{a^2+b^2}}\right)}{\left(a^2+b^2\right)^{5/2}}$$

```
[Out] a*b^3*arctanh((b*cos(x)-a*sin(x))/(a^2+b^2)^(1/2))/(a^2+b^2)^(5/2)-a*b^2*co
s(x)/(a^2+b^2)^2-1/3*a*cos(x)^3/(a^2+b^2)-a^2*b*sin(x)/(a^2+b^2)^2+b*sin(x)
/(a^2+b^2)-1/3*b*sin(x)^3/(a^2+b^2)
```

Rubi [A] time = 0.16, antiderivative size = 123, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.444, Rules used = {3109, 2633, 2565, 30, 3100, 2637, 3074, 206}

$$-\frac{b\sin^3(x)}{3\left(a^2+b^2\right)}+\frac{b\sin(x)}{a^2+b^2}-\frac{a^2b\sin(x)}{\left(a^2+b^2\right)^2}-\frac{a\cos^3(x)}{3\left(a^2+b^2\right)}-\frac{ab^2\cos(x)}{\left(a^2+b^2\right)^2}+\frac{ab^3\tanh^{-1}\left(\frac{b\cos(x)-a\sin(x)}{\sqrt{a^2+b^2}}\right)}{\left(a^2+b^2\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[x]^3*Sin[x])/(a*Cos[x] + b*Sin[x]),x]
```

```
[Out] (a*b^3*ArcTanh[(b*Cos[x] - a*Sin[x])/Sqrt[a^2 + b^2]])/(a^2 + b^2)^(5/2) -
(a*b^2*Cos[x])/(a^2 + b^2)^2 - (a*Cos[x]^3)/(3*(a^2 + b^2)) - (a^2*b*Sin[x]
)/(a^2 + b^2)^2 + (b*Sin[x])/(a^2 + b^2) - (b*Sin[x]^3)/(3*(a^2 + b^2))
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2565

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> -Dist[(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
```

```
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3074

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x
_Symbol] :> -Dist[d^(-1), Subst[Int[1/(a^2 + b^2 - x^2), x], x, b*Cos[c + d
*x] - a*Sin[c + d*x]], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

Rule 3100

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_)/(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin
[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[(b*Cos[c + d*x]^(m - 1))/(d*(a^2 +
 b^2)*(m - 1)), x] + (Dist[a/(a^2 + b^2), Int[Cos[c + d*x]^(m - 1), x], x]
+ Dist[b^2/(a^2 + b^2), Int[Cos[c + d*x]^(m - 2)/(a*Cos[c + d*x] + b*Sin[c
+ d*x]), x], x]) /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0] && GtQ[m, 1
]
```

Rule 3109

```
Int[(cos[(c_.) + (d_.)*(x_)]^(m_.)*sin[(c_.) + (d_.)*(x_)]^(n_.))/(cos[(c_.
) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[b
/(a^2 + b^2), Int[Cos[c + d*x]^m*Sin[c + d*x]^(n - 1), x], x] + (Dist[a/(a^
2 + b^2), Int[Cos[c + d*x]^(m - 1)*Sin[c + d*x]^n, x], x] - Dist[(a*b)/(a^2
 + b^2), Int[(Cos[c + d*x]^(m - 1)*Sin[c + d*x]^(n - 1))/(a*Cos[c + d*x] +
b*Sin[c + d*x]), x], x]) /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0] &&
IGtQ[m, 0] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{\cos^3(x)\sin(x)}{a\cos(x)+b\sin(x)}\,dx = \frac{a\int \cos^2(x)\sin(x)\,dx}{a^2+b^2} + \frac{b\int \cos^3(x)\,dx}{a^2+b^2} - \frac{(ab)\int \frac{\cos^2(x)}{a\cos(x)+b\sin(x)}\,dx}{a^2+b^2}$$

$$= -\frac{ab^2\cos(x)}{\left(a^2+b^2\right)^2} - \frac{\left(a^2b\right)\int \cos(x)\,dx}{\left(a^2+b^2\right)^2} - \frac{\left(ab^3\right)\int \frac{1}{a\cos(x)+b\sin(x)}\,dx}{\left(a^2+b^2\right)^2} - \frac{a\,\mathrm{Subst}\left(\int x^2\,dx, x, \mathrm{c}\right.}{a^2+b^2}$$

$$= -\frac{ab^2\cos(x)}{\left(a^2+b^2\right)^2} - \frac{a\cos^3(x)}{3\left(a^2+b^2\right)} - \frac{a^2b\sin(x)}{\left(a^2+b^2\right)^2} + \frac{b\sin(x)}{a^2+b^2} - \frac{b\sin^3(x)}{3\left(a^2+b^2\right)} + \frac{\left(ab^3\right)\mathrm{Subst}\left(\int \right.}{}$$

$$= \frac{ab^3\tanh^{-1}\left(\frac{b\cos(x)-a\sin(x)}{\sqrt{a^2+b^2}}\right)}{\left(a^2+b^2\right)^{5/2}} - \frac{ab^2\cos(x)}{\left(a^2+b^2\right)^2} - \frac{a\cos^3(x)}{3\left(a^2+b^2\right)} - \frac{a^2b\sin(x)}{\left(a^2+b^2\right)^2} + \frac{b\sin(x)}{a^2+b^2} - \frac{b}{3(}$$

Mathematica [A] time = 1.02, size = 112, normalized size = 0.91

$$-\frac{3a\left(a^2+5b^2\right)\cos(x)+a\left(a^2+b^2\right)\cos(3x)-2b\sin(x)\left(\left(a^2+b^2\right)\cos(2x)-a^2+5b^2\right)}{12\left(a^2+b^2\right)^2} - \frac{2ab^3\tanh^{-1}\left(\frac{a\tan\left(\frac{x}{2}\right)-b}{\sqrt{a^2+b^2}}\right)}{\left(a^2+b^2\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[x]^3*Sin[x])/(a*Cos[x] + b*Sin[x]),x]
```

```
[Out] (-2*a*b^3*ArcTanh[(-b + a*Tan[x/2])/Sqrt[a^2 + b^2]])/(a^2 + b^2)^(5/2) - (
3*a*(a^2 + 5*b^2)*Cos[x] + a*(a^2 + b^2)*Cos[3*x] - 2*b*(-a^2 + 5*b^2 + (a^
2 + b^2)*Cos[2*x])*Sin[x])/(12*(a^2 + b^2)^2)
```

fricas [A] time = 0.52, size = 213, normalized size = 1.73

$$\frac{3\sqrt{a^2+b^2}\,ab^3\log\left(\frac{2\,ab\cos(x)\sin(x)+\left(a^2-b^2\right)\cos(x)^2-2\,a^2-b^2-2\,\sqrt{a^2+b^2}\,(b\cos(x)-a\sin(x))}{2\,ab\cos(x)\sin(x)+\left(a^2-b^2\right)\cos(x)^2+b^2}\right)-2\left(a^5+2\,a^3b^2+ab^4\right)\cos(x)^3-6}{6\left(a^6+3\,a^4b^2+3\,a^2b^4+b^6\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^3*sin(x)/(a*cos(x)+b*sin(x)),x, algorithm="fricas")
```

```
[Out] 1/6*(3*sqrt(a^2 + b^2)*a*b^3*log((2*a*b*cos(x)*sin(x) + (a^2 - b^2)*cos(x)^
2 - 2*a^2 - b^2 - 2*sqrt(a^2 + b^2)*(b*cos(x) - a*sin(x)))/(2*a*b*cos(x)*si
n(x) + (a^2 - b^2)*cos(x)^2 + b^2)) - 2*(a^5 + 2*a^3*b^2 + a*b^4)*cos(x)^3
```

```
- 6*(a^3*b^2 + a*b^4)*cos(x) - 2*(a^4*b - a^2*b^3 - 2*b^5 - (a^4*b + 2*a^2*
b^3 + b^5)*cos(x)^2)*sin(x))/(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)
```

giac [A] time = 8.01, size = 201, normalized size = 1.63

$$\frac{ab^3\log\left(\frac{\left|2\,a\tan\left(\frac{1}{2}\,x\right)-2\,b-2\,\sqrt{a^2+b^2}\right|}{\left|2\,a\tan\left(\frac{1}{2}\,x\right)-2\,b+2\,\sqrt{a^2+b^2}\right|}\right)}{\left(a^4+2\,a^2b^2+b^4\right)\sqrt{a^2+b^2}}+\frac{2\left(3\,b^3\tan\left(\frac{1}{2}\,x\right)^5-3\,a^3\tan\left(\frac{1}{2}\,x\right)^4-6\,ab^2\tan\left(\frac{1}{2}\,x\right)^4-4\,a^2b\tan\left(\frac{1}{2}\,x\right)^3+2\,b^3\tan\cdots\right.}{3\left(a^4+2\,a^2b^2+b^4\right)\left(\tan\left(\frac{1}{2}\,x\cdots\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^3*sin(x)/(a*cos(x)+b*sin(x)),x, algorithm="giac")
```

```
[Out] a*b^3*log(abs(2*a*tan(1/2*x) - 2*b - 2*sqrt(a^2 + b^2))/abs(2*a*tan(1/2*x)
- 2*b + 2*sqrt(a^2 + b^2)))/((a^4 + 2*a^2*b^2 + b^4)*sqrt(a^2 + b^2)) + 2/3
*(3*b^3*tan(1/2*x)^5 - 3*a^3*tan(1/2*x)^4 - 6*a*b^2*tan(1/2*x)^4 - 4*a^2*b*
tan(1/2*x)^3 + 2*b^3*tan(1/2*x)^3 - 6*a*b^2*tan(1/2*x)^2 + 3*b^3*tan(1/2*x)
 - a^3 - 4*a*b^2)/((a^4 + 2*a^2*b^2 + b^4)*(tan(1/2*x)^2 + 1)^3)
```

maple [A] time = 0.09, size = 170, normalized size = 1.38

$$-\frac{4b^3a\operatorname{arctanh}\left(\frac{2a\tan\left(\frac{x}{2}\right)-2b}{2\sqrt{a^2+b^2}}\right)}{\left(2a^4+4a^2b^2+2b^4\right)\sqrt{a^2+b^2}}-\frac{2\left(-b^3\left(\tan^5\left(\frac{x}{2}\right)\right)+\left(a^3+2a\,b^2\right)\left(\tan^4\left(\frac{x}{2}\right)\right)+\left(\frac{4}{3}a^2b-\frac{2}{3}b^3\right)\left(\tan^3\left(\frac{x}{2}\right)\right)+2a\,b^2\left(\cdots\right.\right.}{\left(a^4+2a^2b^2+b^4\right)\left(\tan^2\left(\frac{x}{2}\right)+1\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)^3*sin(x)/(a*cos(x)+b*sin(x)),x)
```

```
[Out] -4*b^3*a/(2*a^4+4*a^2*b^2+2*b^4)/(a^2+b^2)^(1/2)*arctanh(1/2*(2*a*tan(1/2*x
)-2*b)/(a^2+b^2)^(1/2))-2/(a^4+2*a^2*b^2+b^4)*(-b^3*tan(1/2*x)^5+(a^3+2*a*b
^2)*tan(1/2*x)^4+(4/3*a^2*b-2/3*b^3)*tan(1/2*x)^3+2*a*b^2*tan(1/2*x)^2-b^3*
tan(1/2*x)+1/3*a^3+4/3*a*b^2)/(tan(1/2*x)^2+1)^3
```

maxima [B] time = 0.43, size = 281, normalized size = 2.28

$$\frac{ab^3\log\left(\frac{b-\frac{a\sin(x)}{\cos(x)+1}+\sqrt{a^2+b^2}}{b-\frac{a\sin(x)}{\cos(x)+1}-\sqrt{a^2+b^2}}\right)}{\left(a^4+2\,a^2b^2+b^4\right)\sqrt{a^2+b^2}}-\frac{2\left(a^3+4\,ab^2-\frac{3\,b^3\sin(x)}{\cos(x)+1}+\frac{6\,ab^2\sin(x)^2}{(\cos(x)+1)^2}-\frac{3\,b^3\sin(x)^5}{(\cos(x)+1)^5}+\frac{2\left(2\,a^2b-b^3\right)\sin(x)^3}{(\cos(x)+1)^3}+\frac{3\left(a^3+2\,ab^2\right)\sin(x\cdots}{(\cos(x)+1)^4}\right.}{3\left(a^4+2\,a^2b^2+b^4+\frac{3\left(a^4+2\,a^2b^2+b^4\right)\sin(x)^2}{(\cos(x)+1)^2}+\frac{3\left(a^4+2\,a^2b^2+b^4\right)\sin(x)^4}{(\cos(x)+1)^4}+\frac{\left(a^4+2\,a^2b^2+b^4\right)\sin(x)^6}{(\cos(x)+1)^6}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^3*sin(x)/(a*cos(x)+b*sin(x)),x, algorithm="maxima")
```

```
[Out] a*b^3*log((b - a*sin(x)/(cos(x) + 1) + sqrt(a^2 + b^2))/(b - a*sin(x)/(cos(
x) + 1) - sqrt(a^2 + b^2)))/((a^4 + 2*a^2*b^2 + b^4)*sqrt(a^2 + b^2)) - 2/3
*(a^3 + 4*a*b^2 - 3*b^3*sin(x)/(cos(x) + 1) + 6*a*b^2*sin(x)^2/(cos(x) + 1)
^2 - 3*b^3*sin(x)^5/(cos(x) + 1)^5 + 2*(2*a^2*b - b^3)*sin(x)^3/(cos(x) + 1
)^3 + 3*(a^3 + 2*a*b^2)*sin(x)^4/(cos(x) + 1)^4)/(a^4 + 2*a^2*b^2 + b^4 + 3
*(a^4 + 2*a^2*b^2 + b^4)*sin(x)^2/(cos(x) + 1)^2 + 3*(a^4 + 2*a^2*b^2 + b^4
)*sin(x)^4/(cos(x) + 1)^4 + (a^4 + 2*a^2*b^2 + b^4)*sin(x)^6/(cos(x) + 1)^6
)
```

mupad [B] time = 1.26, size = 291, normalized size = 2.37

$$\frac{2\,a\,b^3\,\mathrm{atanh}\left(\frac{2\,a^4\,b+2\,b^5+4\,a^2\,b^3-2\,a\,\tan\left(\frac{x}{2}\right)\left(a^4+2\,a^2\,b^2+b^4\right)}{2\left(a^2+b^2\right)^{5/2}}\right)}{\left(a^2+b^2\right)^{5/2}}-\frac{\frac{2\left(a^3+4\,a\,b^2\right)}{3\left(a^4+2\,a^2\,b^2+b^4\right)}+\frac{4\tan\left(\frac{x}{2}\right)^3\left(2\,a^2\,b-b^3\right)}{3\left(a^4+2\,a^2\,b^2+b^4\right)}-\frac{2\,b^3\tan\left(\frac{x}{2}\right)}{a^4+2\,a^2\,b^2+b^4}+\frac{2\tan\left(\frac{x}{2}\right)^4\,(a}{a^4+2\,a^2}}{\tan\left(\frac{x}{2}\right)^6+3\tan\left(\frac{x}{2}\right)^4+3\tan\left(\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(x)^3*sin(x))/(a*cos(x) + b*sin(x)),x)
```

```
[Out] (2*a*b^3*atanh((2*a^4*b + 2*b^5 + 4*a^2*b^3 - 2*a*tan(x/2)*(a^4 + b^4 + 2*a
^2*b^2))/(2*(a^2 + b^2)^(5/2))))/(a^2 + b^2)^(5/2) - ((2*(4*a*b^2 + a^3))/(
3*(a^4 + b^4 + 2*a^2*b^2)) + (4*tan(x/2)^3*(2*a^2*b - b^3))/(3*(a^4 + b^4 +
 2*a^2*b^2)) - (2*b^3*tan(x/2))/(a^4 + b^4 + 2*a^2*b^2) + (2*tan(x/2)^4*(2*
a*b^2 + a^3))/(a^4 + b^4 + 2*a^2*b^2) - (2*b^3*tan(x/2)^5)/(a^4 + b^4 + 2*a
^2*b^2) + (4*a*b^2*tan(x/2)^2)/(a^4 + b^4 + 2*a^2*b^2))/(3*tan(x/2)^2 + 3*t
an(x/2)^4 + tan(x/2)^6 + 1)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)**3*sin(x)/(a*cos(x)+b*sin(x)),x)
```

```
[Out] Timed out
```

3.282 $\int \frac{\cos^3(x)\sin^2(x)}{a\cos(x)+b\sin(x)}\,dx$

Optimal. Leaf size=175

$$\frac{ax}{8\left(a^2+b^2\right)}-\frac{ab^2x}{2\left(a^2+b^2\right)^2}-\frac{a^2b\sin^2(x)}{2\left(a^2+b^2\right)^2}-\frac{b\cos^4(x)}{4\left(a^2+b^2\right)}-\frac{a\sin(x)\cos^3(x)}{4\left(a^2+b^2\right)}+\frac{a\sin(x)\cos(x)}{8\left(a^2+b^2\right)}-\frac{ab^2\sin(x)\cos(x)}{2\left(a^2+b^2\right)^2}+\frac{a^2b^3\,\mathrm{lo}}{}$$

```
[Out] a^3*b^2*x/(a^2+b^2)^3-1/2*a*b^2*x/(a^2+b^2)^2+1/8*a*x/(a^2+b^2)-1/4*b*cos(x
)^4/(a^2+b^2)+a^2*b^3*ln(a*cos(x)+b*sin(x))/(a^2+b^2)^3-1/2*a*b^2*cos(x)*si
n(x)/(a^2+b^2)^2+1/8*a*cos(x)*sin(x)/(a^2+b^2)-1/4*a*cos(x)^3*sin(x)/(a^2+b
^2)-1/2*a^2*b*sin(x)^2/(a^2+b^2)^2
```

Rubi [A] time = 0.28, antiderivative size = 175, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 9, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.450, Rules used = {3109, 2565, 30, 2568, 2635, 8, 2564, 3098, 3133}

$$\frac{ax}{8\left(a^2+b^2\right)}-\frac{ab^2x}{2\left(a^2+b^2\right)^2}+\frac{a^3b^2x}{\left(a^2+b^2\right)^3}-\frac{a^2b\sin^2(x)}{2\left(a^2+b^2\right)^2}-\frac{b\cos^4(x)}{4\left(a^2+b^2\right)}-\frac{a\sin(x)\cos^3(x)}{4\left(a^2+b^2\right)}+\frac{a\sin(x)\cos(x)}{8\left(a^2+b^2\right)}-\frac{ab^2\sin(x)\,\mathrm{co}}{2\left(a^2+b^2\right.}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[x]^3*Sin[x]^2)/(a*Cos[x] + b*Sin[x]),x]
```

```
[Out] (a^3*b^2*x)/(a^2 + b^2)^3 - (a*b^2*x)/(2*(a^2 + b^2)^2) + (a*x)/(8*(a^2 + b
^2)) - (b*Cos[x]^4)/(4*(a^2 + b^2)) + (a^2*b^3*Log[a*Cos[x] + b*Sin[x]])/(a
^2 + b^2)^3 - (a*b^2*Cos[x]*Sin[x])/(2*(a^2 + b^2)^2) + (a*Cos[x]*Sin[x])/(
8*(a^2 + b^2)) - (a*Cos[x]^3*Sin[x])/(4*(a^2 + b^2)) - (a^2*b*Sin[x]^2)/(2*
(a^2 + b^2)^2)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2564

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
```

```
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 2565

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> -Dist[(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 2568

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> -Simp[(a*(b*Cos[e + f*x])^(n + 1)*(a*Sin[e + f*x])^(m - 1)
)/(b*f*(m + n)), x] + Dist[(a^2*(m - 1))/(m + n), Int[(b*Cos[e + f*x])^n*(a
*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1] &&
 NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 3098

```
Int[cos[(c_.) + (d_.)*(x_)]/(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.
) + (d_.)*(x_)]), x_Symbol] :> Simp[(a*x)/(a^2 + b^2), x] + Dist[b/(a^2 + b
^2), Int[(b*Cos[c + d*x] - a*Sin[c + d*x])/(a*Cos[c + d*x] + b*Sin[c + d*x]
), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

Rule 3109

```
Int[(cos[(c_.) + (d_.)*(x_)]^(m_.)*sin[(c_.) + (d_.)*(x_)]^(n_.))/(cos[(c_.
) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[b
/(a^2 + b^2), Int[Cos[c + d*x]^m*Sin[c + d*x]^(n - 1), x], x] + (Dist[a/(a^
2 + b^2), Int[Cos[c + d*x]^(m - 1)*Sin[c + d*x]^n, x], x] - Dist[(a*b)/(a^2
 + b^2), Int[(Cos[c + d*x]^(m - 1)*Sin[c + d*x]^(n - 1))/(a*Cos[c + d*x] +
b*Sin[c + d*x]), x], x]) /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0] &&
IGtQ[m, 0] && IGtQ[n, 0]
```

Rule 3133

```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])
/((a_.) + cos[(d_.) + (e_.)*(x_)]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)]), x
```

```
_Symbol] :> Simp[((b*B + c*C)*x)/(b^2 + c^2), x] + Simp[((c*B - b*C)*Log[a
+ b*Cos[d + e*x] + c*Sin[d + e*x]])/(e*(b^2 + c^2)), x] /; FreeQ[{a, b, c,
d, e, A, B, C}, x] && NeQ[b^2 + c^2, 0] && EqQ[A*(b^2 + c^2) - a*(b*B + c*C
), 0]
```

Rubi steps

$$\int \frac{\cos^3(x)\sin^2(x)}{a\cos(x)+b\sin(x)}\,dx = \frac{a\int \cos^2(x)\sin^2(x)\,dx}{a^2+b^2} + \frac{b\int \cos^3(x)\sin(x)\,dx}{a^2+b^2} - \frac{(ab)\int \frac{\cos^2(x)\sin(x)}{a\cos(x)+b\sin(x)}\,dx}{a^2+b^2}$$

$$= -\frac{a\cos^3(x)\sin(x)}{4\left(a^2+b^2\right)} - \frac{\left(a^2b\right)\int \cos(x)\sin(x)\,dx}{\left(a^2+b^2\right)^2} - \frac{\left(ab^2\right)\int \cos^2(x)\,dx}{\left(a^2+b^2\right)^2} + \frac{\left(a^2b^2\right)\int \frac{\cos}{a\cos(x)+}}{\left(a^2+b^2\right.}$$

$$= \frac{a^3b^2x}{\left(a^2+b^2\right)^3} - \frac{b\cos^4(x)}{4\left(a^2+b^2\right)} - \frac{ab^2\cos(x)\sin(x)}{2\left(a^2+b^2\right)^2} + \frac{a\cos(x)\sin(x)}{8\left(a^2+b^2\right)} - \frac{a\cos^3(x)\sin(x)}{4\left(a^2+b^2\right)} + \cdots$$

$$= \frac{a^3b^2x}{\left(a^2+b^2\right)^3} - \frac{ab^2x}{2\left(a^2+b^2\right)^2} + \frac{ax}{8\left(a^2+b^2\right)} - \frac{b\cos^4(x)}{4\left(a^2+b^2\right)} + \frac{a^2b^3\log(a\cos(x)+b\sin(x))}{\left(a^2+b^2\right)^3}$$

Mathematica [C] time = 0.79, size = 287, normalized size = 1.64

$$-\frac{-4a^5x + a^5\sin(4x) + 4b\left(b^4-a^4\right)\cos(2x) + 4ia^4bx + a^4b\cos(4x) - 4a^4b\log(a\cos(x)+b\sin(x)) + 2a^4b\log\left((a\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[x]^3*Sin[x]^2)/(a*Cos[x] + b*Sin[x]),x]
```

```
[Out] -1/32*(-4*a^5*x + (4*I)*a^4*b*x - 24*a^3*b^2*x - (24*I)*a^2*b^3*x + 12*a*b^
4*x + (4*I)*b^5*x - (4*I)*b*(a^4 - 6*a^2*b^2 + b^4)*ArcTan[Tan[x]] + 4*b*(-
a^4 + b^4)*Cos[2*x] + a^4*b*Cos[4*x] + 2*a^2*b^3*Cos[4*x] + b^5*Cos[4*x] -
4*a^4*b*Log[a*Cos[x] + b*Sin[x]] - 8*a^2*b^3*Log[a*Cos[x] + b*Sin[x]] - 4*b
^5*Log[a*Cos[x] + b*Sin[x]] + 2*a^4*b*Log[(a*Cos[x] + b*Sin[x])^2] - 12*a^2
*b^3*Log[(a*Cos[x] + b*Sin[x])^2] + 2*b^5*Log[(a*Cos[x] + b*Sin[x])^2] + 8*
a^3*b^2*Sin[2*x] + 8*a*b^4*Sin[2*x] + a^5*Sin[4*x] + 2*a^3*b^2*Sin[4*x] + a
*b^4*Sin[4*x])/(a^2 + b^2)^3
```

fricas [A] time = 0.64, size = 175, normalized size = 1.00

$$\frac{4\,a^2b^3\log\left(2\,ab\cos(x)\sin(x) + \left(a^2-b^2\right)\cos(x)^2 + b^2\right) - 2\left(a^4b + 2\,a^2b^3 + b^5\right)\cos(x)^4 + 4\left(a^4b + a^2b^3\right)\cos(x)^2 + \cdots}{8\left(a^6 + 3\,a^4b^2 + 3\,a^2b^4 + \cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^3*sin(x)^2/(a*cos(x)+b*sin(x)),x, algorithm="fricas")
```

```
[Out] 1/8*(4*a^2*b^3*log(2*a*b*cos(x)*sin(x) + (a^2 - b^2)*cos(x)^2 + b^2) - 2*(a
^4*b + 2*a^2*b^3 + b^5)*cos(x)^4 + 4*(a^4*b + a^2*b^3)*cos(x)^2 + (a^5 + 6*
a^3*b^2 - 3*a*b^4)*x - (2*(a^5 + 2*a^3*b^2 + a*b^4)*cos(x)^3 - (a^5 - 2*a^3
*b^2 - 3*a*b^4)*cos(x))*sin(x))/(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)
```

giac [A] time = 0.21, size = 273, normalized size = 1.56

$$\frac{a^2b^4\log\left(|b\tan(x)+a|\right)}{a^6b+3\,a^4b^3+3\,a^2b^5+b^7}-\frac{a^2b^3\log\left(\tan(x)^2+1\right)}{2\left(a^6+3\,a^4b^2+3\,a^2b^4+b^6\right)}+\frac{\left(a^5+6\,a^3b^2-3\,ab^4\right)x}{8\left(a^6+3\,a^4b^2+3\,a^2b^4+b^6\right)}+\frac{6\,a^2b^3\tan(x)^4+a^5\tan}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^3*sin(x)^2/(a*cos(x)+b*sin(x)),x, algorithm="giac")
```

```
[Out] a^2*b^4*log(abs(b*tan(x) + a))/(a^6*b + 3*a^4*b^3 + 3*a^2*b^5 + b^7) - 1/2*
a^2*b^3*log(tan(x)^2 + 1)/(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6) + 1/8*(a^5 +
6*a^3*b^2 - 3*a*b^4)*x/(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6) + 1/8*(6*a^2*b^3
*tan(x)^4 + a^5*tan(x)^3 - 2*a^3*b^2*tan(x)^3 - 3*a*b^4*tan(x)^3 + 4*a^4*b*
tan(x)^2 + 16*a^2*b^3*tan(x)^2 - a^5*tan(x) - 6*a^3*b^2*tan(x) - 5*a*b^4*ta
n(x) + 2*a^4*b + 6*a^2*b^3 - 2*b^5)/((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*(t
an(x)^2 + 1)^2)
```

maple [B] time = 0.09, size = 363, normalized size = 2.07

$$\frac{\left(\tan^3(x)\right)a^5}{8\left(a^2+b^2\right)^3\left(\tan^2(x)+1\right)^2}-\frac{\left(\tan^3(x)\right)a^3b^2}{4\left(a^2+b^2\right)^3\left(\tan^2(x)+1\right)^2}-\frac{3\left(\tan^3(x)\right)a\,b^4}{8\left(a^2+b^2\right)^3\left(\tan^2(x)+1\right)^2}+\frac{\left(\tan^2(x)\right)a^4b}{2\left(a^2+b^2\right)^3\left(\tan^2(x)+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)^3*sin(x)^2/(a*cos(x)+b*sin(x)),x)
```

```
[Out] 1/8/(a^2+b^2)^3/(tan(x)^2+1)^2*tan(x)^3*a^5-1/4/(a^2+b^2)^3/(tan(x)^2+1)^2*
tan(x)^3*a^3*b^2-3/8/(a^2+b^2)^3/(tan(x)^2+1)^2*tan(x)^3*a*b^4+1/2/(a^2+b^2
)^3/(tan(x)^2+1)^2*tan(x)^2*a^4*b+1/2/(a^2+b^2)^3/(tan(x)^2+1)^2*tan(x)^2*a
^2*b^3-3/4/(a^2+b^2)^3/(tan(x)^2+1)^2*tan(x)*a^3*b^2-5/8/(a^2+b^2)^3/(tan(x
)^2+1)^2*tan(x)*a*b^4-1/8/(a^2+b^2)^3/(tan(x)^2+1)^2*tan(x)*a^5+1/4/(a^2+b^
2)^3/(tan(x)^2+1)^2*a^4*b-1/4/(a^2+b^2)^3/(tan(x)^2+1)^2*b^5-1/2/(a^2+b^2)^
3*ln(tan(x)^2+1)*a^2*b^3+1/8/(a^2+b^2)^3*arctan(tan(x))*a^5+3/4/(a^2+b^2)^3
*arctan(tan(x))*a^3*b^2-3/8/(a^2+b^2)^3*arctan(tan(x))*a*b^4+a^2*b^3/(a^2+b
^2)^3*ln(a+b*tan(x))
```

maxima [B] time = 0.45, size = 424, normalized size = 2.42

$$\frac{a^2b^3\log\left(-a-\frac{2\,b\sin(x)}{\cos(x)+1}+\frac{a\sin(x)^2}{(\cos(x)+1)^2}\right)}{a^6+3\,a^4b^2+3\,a^2b^4+b^6}-\frac{a^2b^3\log\left(\frac{\sin(x)^2}{(\cos(x)+1)^2}+1\right)}{a^6+3\,a^4b^2+3\,a^2b^4+b^6}+\frac{\left(a^5+6\,a^3b^2-3\,ab^4\right)\arctan\left(\frac{\sin(x)}{\cos(x)+1}\right)}{4\left(a^6+3\,a^4b^2+3\,a^2b^4+b^6\right)}+\frac{\frac{8\,b^3\sin(x}{(\cos(x)+}}{4\left(a^4+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^3*sin(x)^2/(a*cos(x)+b*sin(x)),x, algorithm="maxima")
```

```
[Out] a^2*b^3*log(-a - 2*b*sin(x)/(cos(x) + 1) + a*sin(x)^2/(cos(x) + 1)^2)/(a^6
+ 3*a^4*b^2 + 3*a^2*b^4 + b^6) - a^2*b^3*log(sin(x)^2/(cos(x) + 1)^2 + 1)/(
a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6) + 1/4*(a^5 + 6*a^3*b^2 - 3*a*b^4)*arctan
(sin(x)/(cos(x) + 1))/(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6) + 1/4*(8*b^3*sin(
x)^2/(cos(x) + 1)^2 - 16*a^2*b*sin(x)^4/(cos(x) + 1)^4 + 8*b^3*sin(x)^6/(co
s(x) + 1)^6 - (a^3 + 5*a*b^2)*sin(x)/(cos(x) + 1) + (7*a^3 + 3*a*b^2)*sin(x
)^3/(cos(x) + 1)^3 - (7*a^3 + 3*a*b^2)*sin(x)^5/(cos(x) + 1)^5 + (a^3 + 5*a
*b^2)*sin(x)^7/(cos(x) + 1)^7)/(a^4 + 2*a^2*b^2 + b^4 + 4*(a^4 + 2*a^2*b^2
+ b^4)*sin(x)^2/(cos(x) + 1)^2 + 6*(a^4 + 2*a^2*b^2 + b^4)*sin(x)^4/(cos(x)
 + 1)^4 + 4*(a^4 + 2*a^2*b^2 + b^4)*sin(x)^6/(cos(x) + 1)^6 + (a^4 + 2*a^2*
b^2 + b^4)*sin(x)^8/(cos(x) + 1)^8)
```

mupad [B] time = 11.14, size = 5870, normalized size = 33.54

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(x)^3*sin(x)^2)/(a*cos(x) + b*sin(x)),x)
```

```
[Out] ((tan(x/2)^3*(3*a*b^2 + 7*a^3))/(4*(a^4 + b^4 + 2*a^2*b^2)) - (tan(x/2)^5*(
3*a*b^2 + 7*a^3))/(4*(a^4 + b^4 + 2*a^2*b^2)) - (tan(x/2)*(5*a*b^2 + a^3))/
(4*(a^4 + b^4 + 2*a^2*b^2)) + (tan(x/2)^7*(5*a*b^2 + a^3))/(4*(a^4 + b^4 +
2*a^2*b^2)) + (2*b^3*tan(x/2)^2)/(a^4 + b^4 + 2*a^2*b^2) + (2*b^3*tan(x/2)^
6)/(a^4 + b^4 + 2*a^2*b^2) - (4*a^2*b*tan(x/2)^4)/(a^4 + b^4 + 2*a^2*b^2))/
(4*tan(x/2)^2 + 6*tan(x/2)^4 + 4*tan(x/2)^6 + tan(x/2)^8 + 1) - (a*atan((ta
n(x/2)*((((64*a^2*b^3*((a*((16*a^15*b + 16*a^3*b^13 + 288*a^5*b^11 + 1008*a
^7*b^9 + 1472*a^9*b^7 + 1008*a^11*b^5 + 288*a^13*b^3)/(2*(a^12 + b^12 + 6*a
^2*b^10 + 15*a^4*b^8 + 20*a^6*b^6 + 15*a^8*b^4 + 6*a^10*b^2)) - (32*a^2*b^3
*(192*a*b^16 + 1344*a^3*b^14 + 4032*a^5*b^12 + 6720*a^7*b^10 + 6720*a^9*b^8
 + 4032*a^11*b^6 + 1344*a^13*b^4 + 192*a^15*b^2))/((64*a^6 + 64*b^6 + 192*a
^2*b^4 + 192*a^4*b^2)*(a^12 + b^12 + 6*a^2*b^10 + 15*a^4*b^8 + 20*a^6*b^6 +
 15*a^8*b^4 + 6*a^10*b^2)))*(a^4 - 3*b^4 + 6*a^2*b^2))/(8*(a^6 + b^6 + 3*a^
2*b^4 + 3*a^4*b^2)) - (4*a^3*b^3*(a^4 - 3*b^4 + 6*a^2*b^2)*(192*a*b^16 + 13
44*a^3*b^14 + 4032*a^5*b^12 + 6720*a^7*b^10 + 6720*a^9*b^8 + 4032*a^11*b^6
```

+ 1344*a^13*b^4 + 192*a^15*b^2))/((64*a^6 + 64*b^6 + 192*a^2*b^4 + 192*a^4*
b^2)*(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2)*(a^12 + b^12 + 6*a^2*b^10 + 15*a^4
*b^8 + 20*a^6*b^6 + 15*a^8*b^4 + 6*a^10*b^2))))/(64*a^6 + 64*b^6 + 192*a^2*
b^4 + 192*a^4*b^2) - (a*((a^15 + 18*a^3*b^12 - 141*a^5*b^10 - 327*a^7*b^8 -
146*a^9*b^6 + 36*a^11*b^4 + 15*a^13*b^2)/(2*(a^12 + b^12 + 6*a^2*b^10 + 15
*a^4*b^8 + 20*a^6*b^6 + 15*a^8*b^4 + 6*a^10*b^2)) - (64*a^2*b^3*((16*a^15*b
+ 16*a^3*b^13 + 288*a^5*b^11 + 1008*a^7*b^9 + 1472*a^9*b^7 + 1008*a^11*b^5
+ 288*a^13*b^3)/(2*(a^12 + b^12 + 6*a^2*b^10 + 15*a^4*b^8 + 20*a^6*b^6 + 1
5*a^8*b^4 + 6*a^10*b^2)) - (32*a^2*b^3*(192*a*b^16 + 1344*a^3*b^14 + 4032*a
^5*b^12 + 6720*a^7*b^10 + 6720*a^9*b^8 + 4032*a^11*b^6 + 1344*a^13*b^4 + 19
2*a^15*b^2))/((64*a^6 + 64*b^6 + 192*a^2*b^4 + 192*a^4*b^2)*(a^12 + b^12 +
6*a^2*b^10 + 15*a^4*b^8 + 20*a^6*b^6 + 15*a^8*b^4 + 6*a^10*b^2))))/(64*a^6
+ 64*b^6 + 192*a^2*b^4 + 192*a^4*b^2))*(a^4 - 3*b^4 + 6*a^2*b^2))/(8*(a^6 +
b^6 + 3*a^2*b^4 + 3*a^4*b^2)) + (a^3*(a^4 - 3*b^4 + 6*a^2*b^2)^3*(192*a*b^
16 + 1344*a^3*b^14 + 4032*a^5*b^12 + 6720*a^7*b^10 + 6720*a^9*b^8 + 4032*a^
11*b^6 + 1344*a^13*b^4 + 192*a^15*b^2))/(1024*(a^6 + b^6 + 3*a^2*b^4 + 3*a^
4*b^2)^3*(a^12 + b^12 + 6*a^2*b^10 + 15*a^4*b^8 + 20*a^6*b^6 + 15*a^8*b^4 +
6*a^10*b^2)))*(a^10 - 9*b^10 + 493*a^2*b^8 - 706*a^4*b^6 - 46*a^6*b^4 + 11
*a^8*b^2))/(a^10 + 9*b^10 + 229*a^2*b^8 + 250*a^4*b^6 + 42*a^6*b^4 + 13*a^8
*b^2)^2 - (2*a*b*((18*a^5*b^9 + 13*a^7*b^7 + 12*a^9*b^5 + a^11*b^3)/(2*(a^1
2 + b^12 + 6*a^2*b^10 + 15*a^4*b^8 + 20*a^6*b^6 + 15*a^8*b^4 + 6*a^10*b^2))
+ (a*((a*((16*a^15*b + 16*a^3*b^13 + 288*a^5*b^11 + 1008*a^7*b^9 + 1472*a^
9*b^7 + 1008*a^11*b^5 + 288*a^13*b^3)/(2*(a^12 + b^12 + 6*a^2*b^10 + 15*a^4
*b^8 + 20*a^6*b^6 + 15*a^8*b^4 + 6*a^10*b^2)) - (32*a^2*b^3*(192*a*b^16 + 1
344*a^3*b^14 + 4032*a^5*b^12 + 6720*a^7*b^10 + 6720*a^9*b^8 + 4032*a^11*b^6
+ 1344*a^13*b^4 + 192*a^15*b^2))/((64*a^6 + 64*b^6 + 192*a^2*b^4 + 192*a^4
b^2)(a^12 + b^12 + 6*a^2*b^10 + 15*a^4*b^8 + 20*a^6*b^6 + 15*a^8*b^4 + 6*
a^10*b^2)))*(a^4 - 3*b^4 + 6*a^2*b^2))/(8*(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^
2)) - (4*a^3*b^3*(a^4 - 3*b^4 + 6*a^2*b^2)*(192*a*b^16 + 1344*a^3*b^14 + 40
32*a^5*b^12 + 6720*a^7*b^10 + 6720*a^9*b^8 + 4032*a^11*b^6 + 1344*a^13*b^4
+ 192*a^15*b^2))/((64*a^6 + 64*b^6 + 192*a^2*b^4 + 192*a^4*b^2)*(a^6 + b^6
+ 3*a^2*b^4 + 3*a^4*b^2)*(a^12 + b^12 + 6*a^2*b^10 + 15*a^4*b^8 + 20*a^6*b^
6 + 15*a^8*b^4 + 6*a^10*b^2)))*(a^4 - 3*b^4 + 6*a^2*b^2))/(8*(a^6 + b^6 + 3
*a^2*b^4 + 3*a^4*b^2)) + (64*a^2*b^3*((a^15 + 18*a^3*b^12 - 141*a^5*b^10 -
327*a^7*b^8 - 146*a^9*b^6 + 36*a^11*b^4 + 15*a^13*b^2)/(2*(a^12 + b^12 + 6*
a^2*b^10 + 15*a^4*b^8 + 20*a^6*b^6 + 15*a^8*b^4 + 6*a^10*b^2)) - (64*a^2*b^
3*((16*a^15*b + 16*a^3*b^13 + 288*a^5*b^11 + 1008*a^7*b^9 + 1472*a^9*b^7 +
1008*a^11*b^5 + 288*a^13*b^3)/(2*(a^12 + b^12 + 6*a^2*b^10 + 15*a^4*b^8 + 2
0*a^6*b^6 + 15*a^8*b^4 + 6*a^10*b^2)) - (32*a^2*b^3*(192*a*b^16 + 1344*a^3*
b^14 + 4032*a^5*b^12 + 6720*a^7*b^10 + 6720*a^9*b^8 + 4032*a^11*b^6 + 1344*
a^13*b^4 + 192*a^15*b^2))/((64*a^6 + 64*b^6 + 192*a^2*b^4 + 192*a^4*b^2)*(a
^12 + b^12 + 6*a^2*b^10 + 15*a^4*b^8 + 20*a^6*b^6 + 15*a^8*b^4 + 6*a^10*b^2
))))/(64*a^6 + 64*b^6 + 192*a^2*b^4 + 192*a^4*b^2)))/(64*a^6 + 64*b^6 + 192
*a^2*b^4 + 192*a^4*b^2) - (a^4*b^3*(a^4 - 3*b^4 + 6*a^2*b^2)^2*(192*a*b^16
+ 1344*a^3*b^14 + 4032*a^5*b^12 + 6720*a^7*b^10 + 6720*a^9*b^8 + 4032*a^11*

b^6 + 1344*a^13*b^4 + 192*a^15*b^2))/(2*(64*a^6 + 64*b^6 + 192*a^2*b^4 + 19
2*a^4*b^2)*(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2)^2*(a^12 + b^12 + 6*a^2*b^10
+ 15*a^4*b^8 + 20*a^6*b^6 + 15*a^8*b^4 + 6*a^10*b^2)))*(a^8 + 57*b^8 - 436*
a^2*b^6 + 110*a^4*b^4 + 28*a^6*b^2))/(a^10 + 9*b^10 + 229*a^2*b^8 + 250*a^4
*b^6 + 42*a^6*b^4 + 13*a^8*b^2)^2)*(16*a^16 + 16*b^16 + 128*a^2*b^14 + 448*
a^4*b^12 + 896*a^6*b^10 + 1120*a^8*b^8 + 896*a^10*b^6 + 448*a^12*b^4 + 128*
a^14*b^2))/(a^8 - 3*a^4*b^4 + 6*a^6*b^2) + (((a*((39*a^4*b^11 - a^14*b + 12
3*a^6*b^9 + 134*a^8*b^7 + 54*a^10*b^5 + 3*a^12*b^3)/(2*(a^12 + b^12 + 6*a^2
*b^10 + 15*a^4*b^8 + 20*a^6*b^6 + 15*a^8*b^4 + 6*a^10*b^2)) + (64*a^2*b^3*(
(8*a^16 + 24*a^2*b^14 - 40*a^4*b^12 - 360*a^6*b^10 - 552*a^8*b^8 - 248*a^10
*b^6 + 72*a^12*b^4 + 72*a^14*b^2)/(2*(a^12 + b^12 + 6*a^2*b^10 + 15*a^4*b^8
+ 20*a^6*b^6 + 15*a^8*b^4 + 6*a^10*b^2)) - (32*a^2*b^3*(192*a^16*b + 192*a
^2*b^15 + 1344*a^4*b^13 + 4032*a^6*b^11 + 6720*a^8*b^9 + 6720*a^10*b^7 + 40
32*a^12*b^5 + 1344*a^14*b^3))/((64*a^6 + 64*b^6 + 192*a^2*b^4 + 192*a^4*b^2
)*(a^12 + b^12 + 6*a^2*b^10 + 15*a^4*b^8 + 20*a^6*b^6 + 15*a^8*b^4 + 6*a^10
*b^2))))/(64*a^6 + 64*b^6 + 192*a^2*b^4 + 192*a^4*b^2))*(a^4 - 3*b^4 + 6*a^
2*b^2))/(8*(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2)) + (64*a^2*b^3*((a*((8*a^16
+ 24*a^2*b^14 - 40*a^4*b^12 - 360*a^6*b^10 - 552*a^8*b^8 - 248*a^10*b^6 + 7
2*a^12*b^4 + 72*a^14*b^2)/(2*(a^12 + b^12 + 6*a^2*b^10 + 15*a^4*b^8 + 20*a^
6*b^6 + 15*a^8*b^4 + 6*a^10*b^2)) - (32*a^2*b^3*(192*a^16*b + 192*a^2*b^15
+ 1344*a^4*b^13 + 4032*a^6*b^11 + 6720*a^8*b^9 + 6720*a^10*b^7 + 4032*a^12*
b^5 + 1344*a^14*b^3))/((64*a^6 + 64*b^6 + 192*a^2*b^4 + 192*a^4*b^2)*(a^12
+ b^12 + 6*a^2*b^10 + 15*a^4*b^8 + 20*a^6*b^6 + 15*a^8*b^4 + 6*a^10*b^2)))*
(a^4 - 3*b^4 + 6*a^2*b^2))/(8*(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2)) - (4*a^3
b^3(a^4 - 3*b^4 + 6*a^2*b^2)*(192*a^16*b + 192*a^2*b^15 + 1344*a^4*b^13 +
4032*a^6*b^11 + 6720*a^8*b^9 + 6720*a^10*b^7 + 4032*a^12*b^5 + 1344*a^14*b
^3))/((64*a^6 + 64*b^6 + 192*a^2*b^4 + 192*a^4*b^2)*(a^6 + b^6 + 3*a^2*b^4
+ 3*a^4*b^2)*(a^12 + b^12 + 6*a^2*b^10 + 15*a^4*b^8 + 20*a^6*b^6 + 15*a^8*b
^4 + 6*a^10*b^2))))/(64*a^6 + 64*b^6 + 192*a^2*b^4 + 192*a^4*b^2) + (a^3*(a
^4 - 3*b^4 + 6*a^2*b^2)^3*(192*a^16*b + 192*a^2*b^15 + 1344*a^4*b^13 + 4032
*a^6*b^11 + 6720*a^8*b^9 + 6720*a^10*b^7 + 4032*a^12*b^5 + 1344*a^14*b^3))/
(1024*(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2)^3*(a^12 + b^12 + 6*a^2*b^10 + 15*
a^4*b^8 + 20*a^6*b^6 + 15*a^8*b^4 + 6*a^10*b^2)))*(a^10 - 9*b^10 + 493*a^2*
b^8 - 706*a^4*b^6 - 46*a^6*b^4 + 11*a^8*b^2)*(16*a^16 + 16*b^16 + 128*a^2*b
^14 + 448*a^4*b^12 + 896*a^6*b^10 + 1120*a^8*b^8 + 896*a^10*b^6 + 448*a^12*
b^4 + 128*a^14*b^2))/((a^8 - 3*a^4*b^4 + 6*a^6*b^2)*(a^10 + 9*b^10 + 229*a^
2*b^8 + 250*a^4*b^6 + 42*a^6*b^4 + 13*a^8*b^2)^2) - (2*a*b*((2*a^8*b^6 - 15
*a^6*b^8 + a^10*b^4)/(2*(a^12 + b^12 + 6*a^2*b^10 + 15*a^4*b^8 + 20*a^6*b^6
+ 15*a^8*b^4 + 6*a^10*b^2)) + (a*((a*((8*a^16 + 24*a^2*b^14 - 40*a^4*b^12
- 360*a^6*b^10 - 552*a^8*b^8 - 248*a^10*b^6 + 72*a^12*b^4 + 72*a^14*b^2)/(2
*(a^12 + b^12 + 6*a^2*b^10 + 15*a^4*b^8 + 20*a^6*b^6 + 15*a^8*b^4 + 6*a^10*
b^2)) - (32*a^2*b^3*(192*a^16*b + 192*a^2*b^15 + 1344*a^4*b^13 + 4032*a^6*b
^11 + 6720*a^8*b^9 + 6720*a^10*b^7 + 4032*a^12*b^5 + 1344*a^14*b^3))/((64*a
^6 + 64*b^6 + 192*a^2*b^4 + 192*a^4*b^2)*(a^12 + b^12 + 6*a^2*b^10 + 15*a^4
*b^8 + 20*a^6*b^6 + 15*a^8*b^4 + 6*a^10*b^2)))*(a^4 - 3*b^4 + 6*a^2*b^2))/(

```
8*(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2)) - (4*a^3*b^3*(a^4 - 3*b^4 + 6*a^2*b^
2)*(192*a^16*b + 192*a^2*b^15 + 1344*a^4*b^13 + 4032*a^6*b^11 + 6720*a^8*b^
9 + 6720*a^10*b^7 + 4032*a^12*b^5 + 1344*a^14*b^3))/((64*a^6 + 64*b^6 + 192
*a^2*b^4 + 192*a^4*b^2)*(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2)*(a^12 + b^12 +
6*a^2*b^10 + 15*a^4*b^8 + 20*a^6*b^6 + 15*a^8*b^4 + 6*a^10*b^2)))*(a^4 - 3*
b^4 + 6*a^2*b^2))/(8*(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2)) - (64*a^2*b^3*((3
9*a^4*b^11 - a^14*b + 123*a^6*b^9 + 134*a^8*b^7 + 54*a^10*b^5 + 3*a^12*b^3)
/(2*(a^12 + b^12 + 6*a^2*b^10 + 15*a^4*b^8 + 20*a^6*b^6 + 15*a^8*b^4 + 6*a^
10*b^2)) + (64*a^2*b^3*((8*a^16 + 24*a^2*b^14 - 40*a^4*b^12 - 360*a^6*b^10
- 552*a^8*b^8 - 248*a^10*b^6 + 72*a^12*b^4 + 72*a^14*b^2)/(2*(a^12 + b^12 +
 6*a^2*b^10 + 15*a^4*b^8 + 20*a^6*b^6 + 15*a^8*b^4 + 6*a^10*b^2)) - (32*a^2
*b^3*(192*a^16*b + 192*a^2*b^15 + 1344*a^4*b^13 + 4032*a^6*b^11 + 6720*a^8*
b^9 + 6720*a^10*b^7 + 4032*a^12*b^5 + 1344*a^14*b^3))/((64*a^6 + 64*b^6 + 1
92*a^2*b^4 + 192*a^4*b^2)*(a^12 + b^12 + 6*a^2*b^10 + 15*a^4*b^8 + 20*a^6*b
^6 + 15*a^8*b^4 + 6*a^10*b^2))))/(64*a^6 + 64*b^6 + 192*a^2*b^4 + 192*a^4*b
^2)))/(64*a^6 + 64*b^6 + 192*a^2*b^4 + 192*a^4*b^2) - (a^4*b^3*(a^4 - 3*b^4
 + 6*a^2*b^2)^2*(192*a^16*b + 192*a^2*b^15 + 1344*a^4*b^13 + 4032*a^6*b^11
+ 6720*a^8*b^9 + 6720*a^10*b^7 + 4032*a^12*b^5 + 1344*a^14*b^3))/(2*(64*a^6
 + 64*b^6 + 192*a^2*b^4 + 192*a^4*b^2)*(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2)^
2*(a^12 + b^12 + 6*a^2*b^10 + 15*a^4*b^8 + 20*a^6*b^6 + 15*a^8*b^4 + 6*a^10
*b^2)))*(a^8 + 57*b^8 - 436*a^2*b^6 + 110*a^4*b^4 + 28*a^6*b^2)*(16*a^16 +
16*b^16 + 128*a^2*b^14 + 448*a^4*b^12 + 896*a^6*b^10 + 1120*a^8*b^8 + 896*a
^10*b^6 + 448*a^12*b^4 + 128*a^14*b^2))/((a^8 - 3*a^4*b^4 + 6*a^6*b^2)*(a^1
0 + 9*b^10 + 229*a^2*b^8 + 250*a^4*b^6 + 42*a^6*b^4 + 13*a^8*b^2)^2))*(a^4
- 3*b^4 + 6*a^2*b^2))/(4*(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2)) + (a^2*b^3*lo
g(a + 2*b*tan(x/2) - a*tan(x/2)^2))/(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2) - (
64*a^2*b^3*log(1/(cos(x) + 1)))/(64*a^6 + 64*b^6 + 192*a^2*b^4 + 192*a^4*b^
2)
```

sympy **[F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)**3*sin(x)**2/(a*cos(x)+b*sin(x)),x)
```

```
[Out] Timed out
```

3.283 $\int \frac{\cos^3(x)\sin^3(x)}{a\cos(x)+b\sin(x)}\,dx$

Optimal. Leaf size=193

$$-\frac{b\sin^5(x)}{5\left(a^2+b^2\right)}+\frac{b\sin^3(x)}{3\left(a^2+b^2\right)}-\frac{a^2b\sin^3(x)}{3\left(a^2+b^2\right)^2}+\frac{a\cos^5(x)}{5\left(a^2+b^2\right)}-\frac{a\cos^3(x)}{3\left(a^2+b^2\right)}+\frac{ab^2\cos^3(x)}{3\left(a^2+b^2\right)^2}+\frac{a^2b^3\sin(x)}{\left(a^2+b^2\right)^3}-\frac{a^3b^2\cos(x)}{\left(a^2+b^2\right)^3}+\frac{a^3b^3\,\mathrm{t}}{}$$

```
[Out] a^3*b^3*arctanh((b*cos(x)-a*sin(x))/(a^2+b^2)^(1/2))/(a^2+b^2)^(7/2)-a^3*b^
2*cos(x)/(a^2+b^2)^3+1/3*a*b^2*cos(x)^3/(a^2+b^2)^2-1/3*a*cos(x)^3/(a^2+b^2
)+1/5*a*cos(x)^5/(a^2+b^2)+a^2*b^3*sin(x)/(a^2+b^2)^3-1/3*a^2*b*sin(x)^3/(a
^2+b^2)^2+1/3*b*sin(x)^3/(a^2+b^2)-1/5*b*sin(x)^5/(a^2+b^2)
```

Rubi [A] time = 0.36, antiderivative size = 193, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 9, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.450, Rules used = {3109, 2564, 14, 2565, 30, 2637, 2638, 3074, 206}

$$-\frac{b\sin^5(x)}{5\left(a^2+b^2\right)}+\frac{b\sin^3(x)}{3\left(a^2+b^2\right)}-\frac{a^2b\sin^3(x)}{3\left(a^2+b^2\right)^2}+\frac{a^2b^3\sin(x)}{\left(a^2+b^2\right)^3}+\frac{a\cos^5(x)}{5\left(a^2+b^2\right)}-\frac{a\cos^3(x)}{3\left(a^2+b^2\right)}+\frac{ab^2\cos^3(x)}{3\left(a^2+b^2\right)^2}-\frac{a^3b^2\cos(x)}{\left(a^2+b^2\right)^3}+\frac{a^3b^3\,\mathrm{t}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[x]^3*Sin[x]^3)/(a*Cos[x] + b*Sin[x]),x]
```

```
[Out] (a^3*b^3*ArcTanh[(b*Cos[x] - a*Sin[x])/Sqrt[a^2 + b^2]])/(a^2 + b^2)^(7/2)
- (a^3*b^2*Cos[x])/(a^2 + b^2)^3 + (a*b^2*Cos[x]^3)/(3*(a^2 + b^2)^2) - (a*
Cos[x]^3)/(3*(a^2 + b^2)) + (a*Cos[x]^5)/(5*(a^2 + b^2)) + (a^2*b^3*Sin[x])
/(a^2 + b^2)^3 - (a^2*b*Sin[x]^3)/(3*(a^2 + b^2)^2) + (b*Sin[x]^3)/(3*(a^2
+ b^2)) - (b*Sin[x]^5)/(5*(a^2 + b^2))
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2564

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 2565

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> -Dist[(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3074

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x
_Symbol] :> -Dist[d^(-1), Subst[Int[1/(a^2 + b^2 - x^2), x], x, b*Cos[c + d
*x] - a*Sin[c + d*x]], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

Rule 3109

```
Int[(cos[(c_.) + (d_.)*(x_)]^(m_.)*sin[(c_.) + (d_.)*(x_)]^(n_.))/(cos[(c_.
) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[b
/(a^2 + b^2), Int[Cos[c + d*x]^m*Sin[c + d*x]^(n - 1), x], x] + (Dist[a/(a^
2 + b^2), Int[Cos[c + d*x]^(m - 1)*Sin[c + d*x]^n, x], x] - Dist[(a*b)/(a^2
 + b^2), Int[(Cos[c + d*x]^(m - 1)*Sin[c + d*x]^(n - 1))/(a*Cos[c + d*x] +
b*Sin[c + d*x]), x], x]) /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0] &&
IGtQ[m, 0] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{\cos^3(x)\sin^3(x)}{a\cos(x)+b\sin(x)}\,dx = \frac{a\int \cos^2(x)\sin^3(x)\,dx}{a^2+b^2} + \frac{b\int \cos^3(x)\sin^2(x)\,dx}{a^2+b^2} - \frac{(ab)\int \frac{\cos^2(x)\sin^2(x)}{a\cos(x)+b\sin(x)}\,dx}{a^2+b^2}$$

$$= -\frac{\left(a^2b\right)\int \cos(x)\sin^2(x)\,dx}{\left(a^2+b^2\right)^2} - \frac{\left(ab^2\right)\int \cos^2(x)\sin(x)\,dx}{\left(a^2+b^2\right)^2} + \frac{\left(a^2b^2\right)\int \frac{\cos(x)\sin(x)}{a\cos(x)+b\sin(x)}\,dx}{\left(a^2+b^2\right)^2} -$$

$$= \frac{\left(a^3b^2\right)\int \sin(x)\,dx}{\left(a^2+b^2\right)^3} + \frac{\left(a^2b^3\right)\int \cos(x)\,dx}{\left(a^2+b^2\right)^3} - \frac{\left(a^3b^3\right)\int \frac{1}{a\cos(x)+b\sin(x)}\,dx}{\left(a^2+b^2\right)^3} - \frac{\left(a^2b\right)\text{Subst}\left(\right.}{\left(a^2\right.}$$

$$= -\frac{a^3b^2\cos(x)}{\left(a^2+b^2\right)^3} + \frac{ab^2\cos^3(x)}{3\left(a^2+b^2\right)^2} - \frac{a\cos^3(x)}{3\left(a^2+b^2\right)} + \frac{a\cos^5(x)}{5\left(a^2+b^2\right)} + \frac{a^2b^3\sin(x)}{\left(a^2+b^2\right)^3} - \frac{a^2b\sin^3(x)}{3\left(a^2+b^2\right)^2}$$

$$= \frac{a^3b^3\tanh^{-1}\left(\frac{b\cos(x)-a\sin(x)}{\sqrt{a^2+b^2}}\right)}{\left(a^2+b^2\right)^{7/2}} - \frac{a^3b^2\cos(x)}{\left(a^2+b^2\right)^3} + \frac{ab^2\cos^3(x)}{3\left(a^2+b^2\right)^2} - \frac{a\cos^3(x)}{3\left(a^2+b^2\right)} + \frac{a\cos^5(x)}{5\left(a^2+b^2\right)}$$

Mathematica [A] time = 1.48, size = 223, normalized size = 1.16

$$\frac{3a^5\cos(5x) - 30a^4b\sin(x) + 15a^4b\sin(3x) - 3a^4b\sin(5x) + 6a^3b^2\cos(5x) + 240a^2b^3\sin(x) + 10a^2b^3\sin(3x) - 6}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[x]^3*Sin[x]^3)/(a*Cos[x] + b*Sin[x]),x]
```

```
[Out] (-2*a^3*b^3*ArcTanh[(-b + a*Tan[x/2])/Sqrt[a^2 + b^2]])/(a^2 + b^2)^(7/2) +
 (-30*a*(a^4 + 8*a^2*b^2 - b^4)*Cos[x] - 5*a*(a^4 - 2*a^2*b^2 - 3*b^4)*Cos[
3*x] + 3*a^5*Cos[5*x] + 6*a^3*b^2*Cos[5*x] + 3*a*b^4*Cos[5*x] - 30*a^4*b*Si
n[x] + 240*a^2*b^3*Sin[x] + 30*b^5*Sin[x] + 15*a^4*b*Sin[3*x] + 10*a^2*b^3*
Sin[3*x] - 5*b^5*Sin[3*x] - 3*a^4*b*Sin[5*x] - 6*a^2*b^3*Sin[5*x] - 3*b^5*S
in[5*x])/(240*(a^2 + b^2)^3)
```

fricas [A] time = 0.60, size = 307, normalized size = 1.59

$$\frac{15\sqrt{a^2+b^2}\,a^3b^3\log\left(\frac{2ab\cos(x)\sin(x)+\left(a^2-b^2\right)\cos(x)^2-2a^2-b^2-2\sqrt{a^2+b^2}\left(b\cos(x)-a\sin(x)\right)}{2ab\cos(x)\sin(x)+\left(a^2-b^2\right)\cos(x)^2+b^2}\right) + 6\left(a^7+3a^5b^2+3a^3b^4+ab^6\right)\text{c}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^3*sin(x)^3/(a*cos(x)+b*sin(x)),x, algorithm="fricas")
```

```
[Out] 1/30*(15*sqrt(a^2 + b^2)*a^3*b^3*log((2*a*b*cos(x)*sin(x) + (a^2 - b^2)*cos
(x)^2 - 2*a^2 - b^2 - 2*sqrt(a^2 + b^2)*(b*cos(x) - a*sin(x)))/(2*a*b*cos(x
)*sin(x) + (a^2 - b^2)*cos(x)^2 + b^2)) + 6*(a^7 + 3*a^5*b^2 + 3*a^3*b^4 +
a*b^6)*cos(x)^5 - 10*(a^7 + 2*a^5*b^2 + a^3*b^4)*cos(x)^3 - 30*(a^5*b^2 + a
^3*b^4)*cos(x) - 2*(3*a^6*b - 11*a^4*b^3 - 16*a^2*b^5 - 2*b^7 + 3*(a^6*b +
3*a^4*b^3 + 3*a^2*b^5 + b^7)*cos(x)^4 - (6*a^6*b + 13*a^4*b^3 + 8*a^2*b^5 +
 b^7)*cos(x)^2)*sin(x))/(a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)
```

giac [B] time = 1.69, size = 361, normalized size = 1.87

$$\frac{a^3b^3\log\left(\frac{\left|2\,a\tan\left(\frac{1}{2}\,x\right)-2\,b-2\,\sqrt{a^2+b^2}\right|}{\left|2\,a\tan\left(\frac{1}{2}\,x\right)-2\,b+2\,\sqrt{a^2+b^2}\right|}\right)}{\left(a^6+3\,a^4b^2+3\,a^2b^4+b^6\right)\sqrt{a^2+b^2}}+\frac{2\left(15\,a^2b^3\tan\left(\frac{1}{2}\,x\right)^9+15\,ab^4\tan\left(\frac{1}{2}\,x\right)^8+80\,a^2b^3\tan\left(\frac{1}{2}\,x\right)^7+20\,b^5\tan\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^3*sin(x)^3/(a*cos(x)+b*sin(x)),x, algorithm="giac")
```

```
[Out] a^3*b^3*log(abs(2*a*tan(1/2*x) - 2*b - 2*sqrt(a^2 + b^2))/abs(2*a*tan(1/2*x
) - 2*b + 2*sqrt(a^2 + b^2)))/((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*sqrt(a^2
 + b^2)) + 2/15*(15*a^2*b^3*tan(1/2*x)^9 + 15*a*b^4*tan(1/2*x)^8 + 80*a^2*b
^3*tan(1/2*x)^7 + 20*b^5*tan(1/2*x)^7 - 30*a^5*tan(1/2*x)^6 - 90*a^3*b^2*ta
n(1/2*x)^6 - 48*a^4*b*tan(1/2*x)^5 + 34*a^2*b^3*tan(1/2*x)^5 - 8*b^5*tan(1/
2*x)^5 + 10*a^5*tan(1/2*x)^4 - 50*a^3*b^2*tan(1/2*x)^4 + 30*a*b^4*tan(1/2*x
)^4 + 80*a^2*b^3*tan(1/2*x)^3 + 20*b^5*tan(1/2*x)^3 - 10*a^5*tan(1/2*x)^2 -
 70*a^3*b^2*tan(1/2*x)^2 + 15*a^2*b^3*tan(1/2*x) - 2*a^5 - 14*a^3*b^2 + 3*a
*b^4)/((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*(tan(1/2*x)^2 + 1)^5)
```

maple [A] time = 0.11, size = 305, normalized size = 1.58

$$-\frac{16a^3b^3\operatorname{arctanh}\left(\frac{2a\tan\left(\frac{x}{2}\right)-2b}{2\sqrt{a^2+b^2}}\right)}{\left(8a^6+24a^4b^2+24a^2b^4+8b^6\right)\sqrt{a^2+b^2}}-\frac{2\left(-a^2b^3\left(\tan^9\left(\frac{x}{2}\right)\right)-a\,b^4\left(\tan^8\left(\frac{x}{2}\right)\right)+\left(-\frac{16}{3}a^2b^3-\frac{4}{3}b^5\right)\left(\tan^7\left(\frac{x}{2}\right)\right)\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)^3*sin(x)^3/(a*cos(x)+b*sin(x)),x)
```

```
[Out] -16*a^3*b^3/(8*a^6+24*a^4*b^2+24*a^2*b^4+8*b^6)/(a^2+b^2)^(1/2)*arctanh(1/2
*(2*a*tan(1/2*x)-2*b)/(a^2+b^2)^(1/2))-2/(a^4+2*a^2*b^2+b^4)/(a^2+b^2)*(-a^
```

```
2*b^3*tan(1/2*x)^9-a*b^4*tan(1/2*x)^8+(-16/3*a^2*b^3-4/3*b^5)*tan(1/2*x)^7+
(2*a^5+6*a^3*b^2)*tan(1/2*x)^6+(16/5*a^4*b-34/15*a^2*b^3+8/15*b^5)*tan(1/2*
x)^5+(-2/3*a^5+10/3*a^3*b^2-2*a*b^4)*tan(1/2*x)^4+(-16/3*a^2*b^3-4/3*b^5)*t
an(1/2*x)^3+(2/3*a^5+14/3*a^3*b^2)*tan(1/2*x)^2-a^2*b^3*tan(1/2*x)+2/15*a^5
+14/15*a^3*b^2-1/5*a*b^4)/(tan(1/2*x)^2+1)^5
```

maxima [B] time = 0.45, size = 521, normalized size = 2.70

$$\frac{a^3b^3\log\left(\frac{b-\frac{a\sin(x)}{\cos(x)+1}+\sqrt{a^2+b^2}}{b-\frac{a\sin(x)}{\cos(x)+1}-\sqrt{a^2+b^2}}\right)}{\left(a^6+3\,a^4b^2+3\,a^2b^4+b^6\right)\sqrt{a^2+b^2}}-\frac{2\left(2\,a^5+14\,a^3b^2-3\,ab^4-\frac{15\,a^2b^3\sin(x)}{\cos(x)+1}-\frac{15\,ab^4\sin(x)^8}{(\cos(x)+1)^8}-\frac{15\,a^2b^3\sin(x)^9}{(\cos(x)+1)^9}+\frac{10\,(a^5+7\cdots}{(\mathrm{co}\cdots}\right.}{15\left(a^6+3\,a^4b^2+3\,a^2b^4+b^6+\frac{5\left(a^6+3\,a^4b^2+3\,a^2b^4+b^6\right)\sin(x)^2}{(\cos(x)+1)^2}+\cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^3*sin(x)^3/(a*cos(x)+b*sin(x)),x, algorithm="maxima")
```

```
[Out] a^3*b^3*log((b - a*sin(x)/(cos(x) + 1) + sqrt(a^2 + b^2))/(b - a*sin(x)/(co
s(x) + 1) - sqrt(a^2 + b^2)))/((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*sqrt(a^2
 + b^2)) - 2/15*(2*a^5 + 14*a^3*b^2 - 3*a*b^4 - 15*a^2*b^3*sin(x)/(cos(x) +
 1) - 15*a*b^4*sin(x)^8/(cos(x) + 1)^8 - 15*a^2*b^3*sin(x)^9/(cos(x) + 1)^9
 + 10*(a^5 + 7*a^3*b^2)*sin(x)^2/(cos(x) + 1)^2 - 20*(4*a^2*b^3 + b^5)*sin(
x)^3/(cos(x) + 1)^3 - 10*(a^5 - 5*a^3*b^2 + 3*a*b^4)*sin(x)^4/(cos(x) + 1)^
4 + 2*(24*a^4*b - 17*a^2*b^3 + 4*b^5)*sin(x)^5/(cos(x) + 1)^5 + 30*(a^5 + 3
*a^3*b^2)*sin(x)^6/(cos(x) + 1)^6 - 20*(4*a^2*b^3 + b^5)*sin(x)^7/(cos(x) +
 1)^7)/(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6 + 5*(a^6 + 3*a^4*b^2 + 3*a^2*b^4
+ b^6)*sin(x)^2/(cos(x) + 1)^2 + 10*(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*sin
(x)^4/(cos(x) + 1)^4 + 10*(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*sin(x)^6/(cos
(x) + 1)^6 + 5*(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*sin(x)^8/(cos(x) + 1)^8
+ (a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*sin(x)^10/(cos(x) + 1)^10)
```

mupad [B] time = 1.64, size = 600, normalized size = 3.11

$$\frac{\frac{8\tan\left(\frac{x}{2}\right)^3\left(4\,a^2\,b^3+b^5\right)}{3\left(a^6+3\,a^4\,b^2+3\,a^2\,b^4+b^6\right)}-\frac{4\tan\left(\frac{x}{2}\right)^2\left(a^5+7\,a^3\,b^2\right)}{3\left(a^6+3\,a^4\,b^2+3\,a^2\,b^4+b^6\right)}-\frac{4\tan\left(\frac{x}{2}\right)^6\left(a^5+3\,a^3\,b^2\right)}{a^6+3\,a^4\,b^2+3\,a^2\,b^4+b^6}-\frac{2\left(2\,a^5+14\,a^3\,b^2-3\,a\,b^4\right)}{15\left(a^6+3\,a^4\,b^2+3\,a^2\,b^4+b^6\right)}+\frac{4\tan\left(\frac{x}{2}\right)^4\left(a^5-5\,a^3\,b^2+3\,a\,b^4\right)}{3\left(a^6+3\,a^4\,b^2+3\,a^2\,b^4+b^6\right)}+\cdots}{\tan\left(\frac{x}{2}\right)^{10}+5\tan\left(\frac{x}{2}\right)^8+10\tan\left(\frac{x}{2}\right)\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(x)^3*sin(x)^3)/(a*cos(x) + b*sin(x)),x)
```

```
[Out] ((8*tan(x/2)^3*(b^5 + 4*a^2*b^3))/(3*(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2)) -
 (4*tan(x/2)^2*(a^5 + 7*a^3*b^2))/(3*(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2)) -
 (4*tan(x/2)^6*(a^5 + 3*a^3*b^2))/(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2) - (2*
(2*a^5 - 3*a*b^4 + 14*a^3*b^2))/(15*(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2)) +
```

```
(4*tan(x/2)^4*(3*a*b^4 + a^5 - 5*a^3*b^2))/(3*(a^6 + b^6 + 3*a^2*b^4 + 3*a^
4*b^2)) + (8*b^3*tan(x/2)^7*(4*a^2 + b^2))/(3*(a^6 + b^6 + 3*a^2*b^4 + 3*a^
4*b^2)) + (2*a^2*b^3*tan(x/2))/(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2) + (2*a*b
^4*tan(x/2)^8)/(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2) + (2*a^2*b^3*tan(x/2)^9)
/(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2) - (4*b*tan(x/2)^5*(24*a^4 + 4*b^4 - 17
*a^2*b^2))/(15*(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2)))/(5*tan(x/2)^2 + 10*tan
(x/2)^4 + 10*tan(x/2)^6 + 5*tan(x/2)^8 + tan(x/2)^10 + 1) + (2*a^3*b^3*atan
h((2*a^6*b + 2*b^7 + 6*a^2*b^5 + 6*a^4*b^3 - 2*a*tan(x/2)*(a^6 + b^6 + 3*a^
2*b^4 + 3*a^4*b^2))/(2*(a^2 + b^2)^(7/2))))/(a^2 + b^2)^(7/2)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)**3*sin(x)**3/(a*cos(x)+b*sin(x)),x)
```

```
[Out] Timed out
```

3.284 $\int \frac{\cos(x)\sin(x)}{(a\cos(x)+b\sin(x))^2}\,dx$

Optimal. Leaf size=70

$$\frac{2abx}{\left(a^2+b^2\right)^2}-\frac{b\sin(x)}{\left(a^2+b^2\right)(a\cos(x)+b\sin(x))}-\frac{\left(a^2-b^2\right)\log(a\cos(x)+b\sin(x))}{\left(a^2+b^2\right)^2}$$

```
[Out] 2*a*b*x/(a^2+b^2)^2-(a^2-b^2)*ln(a*cos(x)+b*sin(x))/(a^2+b^2)^2-b*sin(x)/(a
^2+b^2)/(a*cos(x)+b*sin(x))
```

Rubi [A] time = 0.17, antiderivative size = 87, normalized size of antiderivative = 1.24, number of steps used = 6, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312, Rules used = {3111, 3098, 3133, 3097, 3075}

$$\frac{2abx}{\left(a^2+b^2\right)^2}-\frac{b\sin(x)}{\left(a^2+b^2\right)(a\cos(x)+b\sin(x))}-\frac{a^2\log(a\cos(x)+b\sin(x))}{\left(a^2+b^2\right)^2}+\frac{b^2\log(a\cos(x)+b\sin(x))}{\left(a^2+b^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[x]*Sin[x])/(a*Cos[x] + b*Sin[x])^2,x]
```

```
[Out] (2*a*b*x)/(a^2 + b^2)^2 - (a^2*Log[a*Cos[x] + b*Sin[x]])/(a^2 + b^2)^2 + (b
^2*Log[a*Cos[x] + b*Sin[x]])/(a^2 + b^2)^2 - (b*Sin[x])/((a^2 + b^2)*(a*Cos
[x] + b*Sin[x]))
```

Rule 3075

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-2), x
_Symbol] :> Simp[Sin[c + d*x]/(a*d*(a*Cos[c + d*x] + b*Sin[c + d*x])), x] /
; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

Rule 3097

```
Int[sin[(c_.) + (d_.)*(x_)]/(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.
) + (d_.)*(x_)]), x_Symbol] :> Simp[(b*x)/(a^2 + b^2), x] - Dist[a/(a^2 + b
^2), Int[(b*Cos[c + d*x] - a*Sin[c + d*x])/(a*Cos[c + d*x] + b*Sin[c + d*x]
), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

Rule 3098

```
Int[cos[(c_.) + (d_.)*(x_)]/(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.
) + (d_.)*(x_)]), x_Symbol] :> Simp[(a*x)/(a^2 + b^2), x] + Dist[b/(a^2 + b
^2), Int[(b*Cos[c + d*x] - a*Sin[c + d*x])/(a*Cos[c + d*x] + b*Sin[c + d*x]
```

```
), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

Rule 3111

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*sin[(c_.) + (d_.)*(x_)]^(n_.)*(cos[(c_.)
+ (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(p_), x_Symbol] :> Dis
t[b/(a^2 + b^2), Int[Cos[c + d*x]^m*Sin[c + d*x]^(n - 1)*(a*Cos[c + d*x] +
b*Sin[c + d*x])^(p + 1), x], x] + (Dist[a/(a^2 + b^2), Int[Cos[c + d*x]^(m
- 1)*Sin[c + d*x]^n*(a*Cos[c + d*x] + b*Sin[c + d*x])^(p + 1), x], x] - Dis
t[(a*b)/(a^2 + b^2), Int[Cos[c + d*x]^(m - 1)*Sin[c + d*x]^(n - 1)*(a*Cos[c
 + d*x] + b*Sin[c + d*x])^p, x], x]) /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 +
 b^2, 0] && IGtQ[m, 0] && IGtQ[n, 0] && ILtQ[p, 0]
```

Rule 3133

```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])
/((a_.) + cos[(d_.) + (e_.)*(x_)]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)]), x
_Symbol] :> Simp[((b*B + c*C)*x)/(b^2 + c^2), x] + Simp[((c*B - b*C)*Log[a
+ b*Cos[d + e*x] + c*Sin[d + e*x]])/(e*(b^2 + c^2)), x] /; FreeQ[{a, b, c,
d, e, A, B, C}, x] && NeQ[b^2 + c^2, 0] && EqQ[A*(b^2 + c^2) - a*(b*B + c*C
), 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\cos(x)\sin(x)}{(a\cos(x)+b\sin(x))^2}\,dx &= \frac{a\int \frac{\sin(x)}{a\cos(x)+b\sin(x)}\,dx}{a^2+b^2} + \frac{b\int \frac{\cos(x)}{a\cos(x)+b\sin(x)}\,dx}{a^2+b^2} - \frac{(ab)\int \frac{1}{(a\cos(x)+b\sin(x))^2}\,dx}{a^2+b^2} \\
&= \frac{2abx}{\left(a^2+b^2\right)^2} - \frac{b\sin(x)}{\left(a^2+b^2\right)(a\cos(x)+b\sin(x))} - \frac{a^2\int \frac{b\cos(x)-a\sin(x)}{a\cos(x)+b\sin(x)}\,dx}{\left(a^2+b^2\right)^2} + \frac{b^2\int \frac{b\cos(x}{a\cos(x}}{\left(a^2+\right.} \\
&= \frac{2abx}{\left(a^2+b^2\right)^2} - \frac{a^2\log(a\cos(x)+b\sin(x))}{\left(a^2+b^2\right)^2} + \frac{b^2\log(a\cos(x)+b\sin(x))}{\left(a^2+b^2\right)^2} - \frac{}{\left(a^2+b^2\right.}
\end{aligned}
$$

Mathematica [C] time = 0.26, size = 144, normalized size = 2.06

$$
\frac{a\cos(x)\left(\left(b^2-a^2\right)\log\left((a\cos(x)+b\sin(x))^2\right)-2ix(a+ib)^2\right)+b\sin(x)\left(\left(b^2-a^2\right)\log\left((a\cos(x)+b\sin(x))^2\right)+\right.}{2\left(a^2+b^2\right)^2(a\cos(x)+b\sin(x}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[x]*Sin[x])/(a*Cos[x] + b*Sin[x])^2,x]
```

```
[Out] (a*Cos[x]*((-2*I)*(a + I*b)^2*x + (-a^2 + b^2)*Log[(a*Cos[x] + b*Sin[x])^2]
) + b*(2*(a + I*b)*(a*(-1 - I*x) + b*(I + x)) + (-a^2 + b^2)*Log[(a*Cos[x]
+ b*Sin[x])^2])*Sin[x] + (2*I)*(a^2 - b^2)*ArcTan[Tan[x]]*(a*Cos[x] + b*Sin
[x]))/(2*(a^2 + b^2)^2*(a*Cos[x] + b*Sin[x]))
```

fricas [A] time = 0.51, size = 138, normalized size = 1.97

$$\frac{2\left(2\,a^{2}bx+ab^{2}\right)\cos(x)-\left(\left(a^{3}-ab^{2}\right)\cos(x)+\left(a^{2}b-b^{3}\right)\sin(x)\right)\log\left(2\,ab\cos(x)\sin(x)+\left(a^{2}-b^{2}\right)\cos(x)^{2}+b^{2}\right)}{2\left(\left(a^{5}+2\,a^{3}b^{2}+ab^{4}\right)\cos(x)+\left(a^{4}b+2\,a^{2}b^{3}+b^{5}\right)\sin(x)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*sin(x)/(a*cos(x)+b*sin(x))^2,x, algorithm="fricas")
```

```
[Out] 1/2*(2*(2*a^2*b*x + a*b^2)*cos(x) - ((a^3 - a*b^2)*cos(x) + (a^2*b - b^3)*s
in(x))*log(2*a*b*cos(x)*sin(x) + (a^2 - b^2)*cos(x)^2 + b^2) + 2*(2*a*b^2*x
 - a^2*b)*sin(x))/((a^5 + 2*a^3*b^2 + a*b^4)*cos(x) + (a^4*b + 2*a^2*b^3 +
b^5)*sin(x))
```

giac [B] time = 0.19, size = 144, normalized size = 2.06

$$\frac{2\,abx}{a^{4}+2\,a^{2}b^{2}+b^{4}}+\frac{\left(a^{2}-b^{2}\right)\log\left(\tan(x)^{2}+1\right)}{2\left(a^{4}+2\,a^{2}b^{2}+b^{4}\right)}-\frac{\left(a^{2}b-b^{3}\right)\log\left(|b\tan(x)+a|\right)}{a^{4}b+2\,a^{2}b^{3}+b^{5}}+\frac{a^{2}b\tan(x)-b^{3}\tan(x)+2\,a^{3}}{\left(a^{4}+2\,a^{2}b^{2}+b^{4}\right)\left(b\tan(x)+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*sin(x)/(a*cos(x)+b*sin(x))^2,x, algorithm="giac")
```

```
[Out] 2*a*b*x/(a^4 + 2*a^2*b^2 + b^4) + 1/2*(a^2 - b^2)*log(tan(x)^2 + 1)/(a^4 +
2*a^2*b^2 + b^4) - (a^2*b - b^3)*log(abs(b*tan(x) + a))/(a^4*b + 2*a^2*b^3
+ b^5) + (a^2*b*tan(x) - b^3*tan(x) + 2*a^3)/((a^4 + 2*a^2*b^2 + b^4)*(b*ta
n(x) + a))
```

maple [A] time = 0.10, size = 120, normalized size = 1.71

$$\frac{\ln\left(\tan^{2}(x)+1\right)a^{2}}{2\left(a^{2}+b^{2}\right)^{2}}-\frac{\ln\left(\tan^{2}(x)+1\right)b^{2}}{2\left(a^{2}+b^{2}\right)^{2}}+\frac{2ab\arctan\left(\tan(x)\right)}{\left(a^{2}+b^{2}\right)^{2}}+\frac{a}{\left(a^{2}+b^{2}\right)\left(a+b\tan(x)\right)}-\frac{\ln\left(a+b\tan(x)\right)a^{2}}{\left(a^{2}+b^{2}\right)^{2}}+\frac{\ln(}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)*sin(x)/(a*cos(x)+b*sin(x))^2,x)
```

```
[Out] 1/2/(a^2+b^2)^2*ln(tan(x)^2+1)*a^2-1/2/(a^2+b^2)^2*ln(tan(x)^2+1)*b^2+2/(a^
2+b^2)^2*a*b*arctan(tan(x))+a/(a^2+b^2)/(a+b*tan(x))-1/(a^2+b^2)^2*ln(a+b*t
an(x))*a^2+1/(a^2+b^2)^2*ln(a+b*tan(x))*b^2
```

maxima [A] time = 0.45, size = 118, normalized size = 1.69

$$\frac{2\,abx}{a^4+2\,a^2b^2+b^4}-\frac{\left(a^2-b^2\right)\log\left(b\tan(x)+a\right)}{a^4+2\,a^2b^2+b^4}+\frac{\left(a^2-b^2\right)\log\left(\tan(x)^2+1\right)}{2\left(a^4+2\,a^2b^2+b^4\right)}+\frac{a}{a^3+ab^2+\left(a^2b+b^3\right)\tan(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*sin(x)/(a*cos(x)+b*sin(x))^2,x, algorithm="maxima")
```

```
[Out] 2*a*b*x/(a^4 + 2*a^2*b^2 + b^4) - (a^2 - b^2)*log(b*tan(x) + a)/(a^4 + 2*a^
2*b^2 + b^4) + 1/2*(a^2 - b^2)*log(tan(x)^2 + 1)/(a^4 + 2*a^2*b^2 + b^4) +
a/(a^3 + a*b^2 + (a^2*b + b^3)*tan(x))
```

mupad [B] time = 5.16, size = 1017, normalized size = 14.53

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(x)*sin(x))/(a*cos(x) + b*sin(x))^2,x)
```

```
[Out] -(b^3*sin(x) + a^3*log((a*cos(x) + b*sin(x))/cos(x/2)^2)*cos(x) - b^3*log((
a*cos(x) + b*sin(x))/cos(x/2)^2)*sin(x) + a^2*b*sin(x) - a^3*log(-(65536*a^
4*b^10 - 131072*a^6*b^8 + 196608*a^8*b^6 - 131072*a^10*b^4 + 65536*a^12*b^2
)/(a^16/2 + b^16/2 + 4*a^2*b^14 + 14*a^4*b^12 + 28*a^6*b^10 + 35*a^8*b^8 +
28*a^10*b^6 + 14*a^12*b^4 + 4*a^14*b^2 + (a^16*cos(x))/2 + (b^16*cos(x))/2
+ 4*a^2*b^14*cos(x) + 14*a^4*b^12*cos(x) + 28*a^6*b^10*cos(x) + 35*a^8*b^8*
cos(x) + 28*a^10*b^6*cos(x) + 14*a^12*b^4*cos(x) + 4*a^14*b^2*cos(x)))*cos(
x) + b^3*log(-(65536*a^4*b^10 - 131072*a^6*b^8 + 196608*a^8*b^6 - 131072*a^
10*b^4 + 65536*a^12*b^2)/(a^16/2 + b^16/2 + 4*a^2*b^14 + 14*a^4*b^12 + 28*a
^6*b^10 + 35*a^8*b^8 + 28*a^10*b^6 + 14*a^12*b^4 + 4*a^14*b^2 + (a^16*cos(x
))/2 + (b^16*cos(x))/2 + 4*a^2*b^14*cos(x) + 14*a^4*b^12*cos(x) + 28*a^6*b^
10*cos(x) + 35*a^8*b^8*cos(x) + 28*a^10*b^6*cos(x) + 14*a^12*b^4*cos(x) + 4
*a^14*b^2*cos(x)))*sin(x) - 4*a^2*b*atan(sin(x/2)/cos(x/2))*cos(x) - 4*a*b^
2*atan(sin(x/2)/cos(x/2))*sin(x) + a*b^2*log(-(65536*a^4*b^10 - 131072*a^6*
b^8 + 196608*a^8*b^6 - 131072*a^10*b^4 + 65536*a^12*b^2)/(a^16/2 + b^16/2 +
 4*a^2*b^14 + 14*a^4*b^12 + 28*a^6*b^10 + 35*a^8*b^8 + 28*a^10*b^6 + 14*a^1
2*b^4 + 4*a^14*b^2 + (a^16*cos(x))/2 + (b^16*cos(x))/2 + 4*a^2*b^14*cos(x)
+ 14*a^4*b^12*cos(x) + 28*a^6*b^10*cos(x) + 35*a^8*b^8*cos(x) + 28*a^10*b^6
*cos(x) + 14*a^12*b^4*cos(x) + 4*a^14*b^2*cos(x)))*cos(x) - a^2*b*log(-(655
36*a^4*b^10 - 131072*a^6*b^8 + 196608*a^8*b^6 - 131072*a^10*b^4 + 65536*a^1
2*b^2)/(a^16/2 + b^16/2 + 4*a^2*b^14 + 14*a^4*b^12 + 28*a^6*b^10 + 35*a^8*b
^8 + 28*a^10*b^6 + 14*a^12*b^4 + 4*a^14*b^2 + (a^16*cos(x))/2 + (b^16*cos(x
))/2 + 4*a^2*b^14*cos(x) + 14*a^4*b^12*cos(x) + 28*a^6*b^10*cos(x) + 35*a^8
*b^8*cos(x) + 28*a^10*b^6*cos(x) + 14*a^12*b^4*cos(x) + 4*a^14*b^2*cos(x)))
```

```
*sin(x) - a*b^2*log((a*cos(x) + b*sin(x))/cos(x/2)^2)*cos(x) + a^2*b*log((a
*cos(x) + b*sin(x))/cos(x/2)^2)*sin(x))/(b^5*sin(x) + a^5*cos(x) + a*b^4*co
s(x) + a^4*b*sin(x) + 2*a^3*b^2*cos(x) + 2*a^2*b^3*sin(x))
```

sympy [A] time = 2.10, size = 991, normalized size = 14.16

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*sin(x)/(a*cos(x)+b*sin(x))**2,x)
```

```
[Out] Piecewise((zoo*log(sin(x)), Eq(a, 0) & Eq(b, 0)), (-log(cos(x))/a**2, Eq(b,
 0)), (2*I*x*sin(x)**2/(8*b**2*sin(x)**2 - 16*I*b**2*sin(x)*cos(x) - 8*b**2
*cos(x)**2) + 4*x*sin(x)*cos(x)/(8*b**2*sin(x)**2 - 16*I*b**2*sin(x)*cos(x)
 - 8*b**2*cos(x)**2) - 2*I*x*cos(x)**2/(8*b**2*sin(x)**2 - 16*I*b**2*sin(x)
*cos(x) - 8*b**2*cos(x)**2) + sin(x)**2/(8*b**2*sin(x)**2 - 16*I*b**2*sin(x
)*cos(x) - 8*b**2*cos(x)**2) - cos(x)**2/(8*b**2*sin(x)**2 - 16*I*b**2*sin(
x)*cos(x) - 8*b**2*cos(x)**2), Eq(a, -I*b)), (-2*I*x*sin(x)**2/(8*b**2*sin(
x)**2 + 16*I*b**2*sin(x)*cos(x) - 8*b**2*cos(x)**2) + 4*x*sin(x)*cos(x)/(8*
b**2*sin(x)**2 + 16*I*b**2*sin(x)*cos(x) - 8*b**2*cos(x)**2) + 2*I*x*cos(x)
**2/(8*b**2*sin(x)**2 + 16*I*b**2*sin(x)*cos(x) - 8*b**2*cos(x)**2) + sin(x
)**2/(8*b**2*sin(x)**2 + 16*I*b**2*sin(x)*cos(x) - 8*b**2*cos(x)**2) - cos(
x)**2/(8*b**2*sin(x)**2 + 16*I*b**2*sin(x)*cos(x) - 8*b**2*cos(x)**2), Eq(a
, I*b)), (-a**3*log(a*cos(x)/b + sin(x))*cos(x)/(a**5*cos(x) + a**4*b*sin(x
) + 2*a**3*b**2*cos(x) + 2*a**2*b**3*sin(x) + a*b**4*cos(x) + b**5*sin(x))
+ a**3*cos(x)/(a**5*cos(x) + a**4*b*sin(x) + 2*a**3*b**2*cos(x) + 2*a**2*b*
*3*sin(x) + a*b**4*cos(x) + b**5*sin(x)) + 2*a**2*b*x*cos(x)/(a**5*cos(x) +
 a**4*b*sin(x) + 2*a**3*b**2*cos(x) + 2*a**2*b**3*sin(x) + a*b**4*cos(x) +
b**5*sin(x)) - a**2*b*log(a*cos(x)/b + sin(x))*sin(x)/(a**5*cos(x) + a**4*b
*sin(x) + 2*a**3*b**2*cos(x) + 2*a**2*b**3*sin(x) + a*b**4*cos(x) + b**5*si
n(x)) + 2*a*b**2*x*sin(x)/(a**5*cos(x) + a**4*b*sin(x) + 2*a**3*b**2*cos(x)
 + 2*a**2*b**3*sin(x) + a*b**4*cos(x) + b**5*sin(x)) + a*b**2*log(a*cos(x)/
b + sin(x))*cos(x)/(a**5*cos(x) + a**4*b*sin(x) + 2*a**3*b**2*cos(x) + 2*a*
*2*b**3*sin(x) + a*b**4*cos(x) + b**5*sin(x)) + a*b**2*cos(x)/(a**5*cos(x)
+ a**4*b*sin(x) + 2*a**3*b**2*cos(x) + 2*a**2*b**3*sin(x) + a*b**4*cos(x) +
 b**5*sin(x)) + b**3*log(a*cos(x)/b + sin(x))*sin(x)/(a**5*cos(x) + a**4*b*
sin(x) + 2*a**3*b**2*cos(x) + 2*a**2*b**3*sin(x) + a*b**4*cos(x) + b**5*sin
(x)), True))
```

3.285 $\int \frac{\cos(x)\sin^2(x)}{(a\cos(x)+b\sin(x))^2}\,dx$

Optimal. Leaf size=110

$$-\frac{(a^2-b^2)\sin(x)}{(a^2+b^2)^2}-\frac{2ab\cos(x)}{(a^2+b^2)^2}-\frac{a^2b}{(a^2+b^2)^2(a\cos(x)+b\sin(x))}-\frac{a(a^2-2b^2)\tanh^{-1}\left(\frac{b\cos(x)-a\sin(x)}{\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{5/2}}$$

```
[Out] -a*(a^2-2*b^2)*arctanh((b*cos(x)-a*sin(x))/(a^2+b^2)^(1/2))/(a^2+b^2)^(5/2)
-2*a*b*cos(x)/(a^2+b^2)^2-(a^2-b^2)*sin(x)/(a^2+b^2)^2-a^2*b/(a^2+b^2)^2/(a
*cos(x)+b*sin(x))
```

Rubi [A] time = 0.24, antiderivative size = 152, normalized size of antiderivative = 1.38, number of steps used = 13, number of rules used = 8, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.444, Rules used = {3111, 3109, 2637, 2638, 3074, 206, 3099, 3154}

$$-\frac{a^2\sin(x)}{(a^2+b^2)^2}+\frac{b^2\sin(x)}{(a^2+b^2)^2}-\frac{2ab\cos(x)}{(a^2+b^2)^2}-\frac{a^2b}{(a^2+b^2)^2(a\cos(x)+b\sin(x))}-\frac{a^3\tanh^{-1}\left(\frac{b\cos(x)-a\sin(x)}{\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{5/2}}+\frac{2ab^2\tanh^{-1}}{(a^2}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[x]*Sin[x]^2)/(a*Cos[x] + b*Sin[x])^2,x]
```

```
[Out] -((a^3*ArcTanh[(b*Cos[x] - a*Sin[x])/Sqrt[a^2 + b^2]])/(a^2 + b^2)^(5/2)) +
 (2*a*b^2*ArcTanh[(b*Cos[x] - a*Sin[x])/Sqrt[a^2 + b^2]])/(a^2 + b^2)^(5/2)
 - (2*a*b*Cos[x])/(a^2 + b^2)^2 - (a^2*Sin[x])/(a^2 + b^2)^2 + (b^2*Sin[x])
/(a^2 + b^2)^2 - (a^2*b)/((a^2 + b^2)^2*(a*Cos[x] + b*Sin[x]))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3074

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x
_Symbol] :> -Dist[d^(-1), Subst[Int[1/(a^2 + b^2 - x^2), x], x, b*Cos[c + d
*x] - a*Sin[c + d*x]], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

Rule 3099

```
Int[sin[(c_.) + (d_.)*(x_)]^(m_)/(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin
[(c_.) + (d_.)*(x_)]), x_Symbol] :> -Simp[(a*Sin[c + d*x]^(m - 1))/(d*(a^2
+ b^2)*(m - 1)), x] + (Dist[a^2/(a^2 + b^2), Int[Sin[c + d*x]^(m - 2)/(a*Co
s[c + d*x] + b*Sin[c + d*x]), x], x] + Dist[b/(a^2 + b^2), Int[Sin[c + d*x]
^(m - 1), x], x]) /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0] && GtQ[m,
1]
```

Rule 3109

```
Int[(cos[(c_.) + (d_.)*(x_)]^(m_.)*sin[(c_.) + (d_.)*(x_)]^(n_.))/(cos[(c_.
) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[b
/(a^2 + b^2), Int[Cos[c + d*x]^m*Sin[c + d*x]^(n - 1), x], x] + (Dist[a/(a^
2 + b^2), Int[Cos[c + d*x]^(m - 1)*Sin[c + d*x]^n, x], x] - Dist[(a*b)/(a^2
 + b^2), Int[(Cos[c + d*x]^(m - 1)*Sin[c + d*x]^(n - 1))/(a*Cos[c + d*x] +
b*Sin[c + d*x]), x], x]) /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0] &&
IGtQ[m, 0] && IGtQ[n, 0]
```

Rule 3111

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*sin[(c_.) + (d_.)*(x_)]^(n_.)*(cos[(c_.)
+ (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(p_), x_Symbol] :> Dis
t[b/(a^2 + b^2), Int[Cos[c + d*x]^m*Sin[c + d*x]^(n - 1)*(a*Cos[c + d*x] +
b*Sin[c + d*x])^(p + 1), x], x] + (Dist[a/(a^2 + b^2), Int[Cos[c + d*x]^(m
- 1)*Sin[c + d*x]^n*(a*Cos[c + d*x] + b*Sin[c + d*x])^(p + 1), x], x] - Dis
t[(a*b)/(a^2 + b^2), Int[Cos[c + d*x]^(m - 1)*Sin[c + d*x]^(n - 1)*(a*Cos[c
 + d*x] + b*Sin[c + d*x])^p, x], x]) /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 +
 b^2, 0] && IGtQ[m, 0] && IGtQ[n, 0] && ILtQ[p, 0]
```

Rule 3154

```
Int[((A_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])/((a_.) + cos[(d_.) + (e_.)*(x_)
]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)])^2, x_Symbol] :> -Simp[(b*C + (a*C
- c*A)*Cos[d + e*x] + b*A*Sin[d + e*x])/(e*(a^2 - b^2 - c^2)*(a + b*Cos[d +
 e*x] + c*Sin[d + e*x])), x] + Dist[(a*A - c*C)/(a^2 - b^2 - c^2), Int[1/(a
 + b*Cos[d + e*x] + c*Sin[d + e*x]), x], x] /; FreeQ[{a, b, c, d, e, A, C},
 x] && NeQ[a^2 - b^2 - c^2, 0] && NeQ[a*A - c*C, 0]
```

Rubi steps

$$\int \frac{\cos(x)\sin^2(x)}{(a\cos(x)+b\sin(x))^2}\,dx = \frac{a\int \frac{\sin^2(x)}{a\cos(x)+b\sin(x)}\,dx}{a^2+b^2} + \frac{b\int \frac{\cos(x)\sin(x)}{a\cos(x)+b\sin(x)}\,dx}{a^2+b^2} - \frac{(ab)\int \frac{\sin(x)}{(a\cos(x)+b\sin(x))^2}\,dx}{a^2+b^2}$$

$$= -\frac{a^2\sin(x)}{\left(a^2+b^2\right)^2} - \frac{a^2b}{\left(a^2+b^2\right)^2(a\cos(x)+b\sin(x))} + \frac{a^3\int \frac{1}{a\cos(x)+b\sin(x)}\,dx}{\left(a^2+b^2\right)^2} + 2\frac{(ab)\int}{\left(a^2\right.}$$

$$= -\frac{2ab\cos(x)}{\left(a^2+b^2\right)^2} - \frac{a^2\sin(x)}{\left(a^2+b^2\right)^2} + \frac{b^2\sin(x)}{\left(a^2+b^2\right)^2} - \frac{a^2b}{\left(a^2+b^2\right)^2(a\cos(x)+b\sin(x))} - \frac{a^3\,\text{Su}}{}$$

$$= -\frac{a^3\tanh^{-1}\left(\frac{b\cos(x)-a\sin(x)}{\sqrt{a^2+b^2}}\right)}{\left(a^2+b^2\right)^{5/2}} + \frac{2ab^2\tanh^{-1}\left(\frac{b\cos(x)-a\sin(x)}{\sqrt{a^2+b^2}}\right)}{\left(a^2+b^2\right)^{5/2}} - \frac{2ab\cos(x)}{\left(a^2+b^2\right)^2} - \frac{a^2\sin}{\left(a^2+\right.}$$

Mathematica [A] time = 0.59, size = 111, normalized size = 1.01

$$\frac{2a\left(a^2-2b^2\right)\tanh^{-1}\left(\frac{a\tan\left(\frac{x}{2}\right)-b}{\sqrt{a^2+b^2}}\right)}{\left(a^2+b^2\right)^{5/2}} - \frac{a\left(a^2+b^2\right)\sin(2x)+b\left(a^2+b^2\right)\cos(2x)+5a^2b-b^3}{2\left(a^2+b^2\right)^2(a\cos(x)+b\sin(x))}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[x]*Sin[x]^2)/(a*Cos[x] + b*Sin[x])^2,x]
```

```
[Out] (2*a*(a^2 - 2*b^2)*ArcTanh[(-b + a*Tan[x/2])/Sqrt[a^2 + b^2]])/(a^2 + b^2)^
(5/2) - (5*a^2*b - b^3 + b*(a^2 + b^2)*Cos[2*x] + a*(a^2 + b^2)*Sin[2*x])/(
2*(a^2 + b^2)^2*(a*Cos[x] + b*Sin[x]))
```

fricas [B] time = 0.51, size = 252, normalized size = 2.29

$$-\frac{4a^4b+2a^2b^3-2b^5+2\left(a^4b+2a^2b^3+b^5\right)\cos(x)^2+2\left(a^5+2a^3b^2+ab^4\right)\cos(x)\sin(x)+\sqrt{a^2+b^2}\left(\left(a^4-2\right.\right.}{2\left(\left(a^7+3a^5b^2+3a^3b^4+ab^6\right)\cos(x)+\left(a^6\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*sin(x)^2/(a*cos(x)+b*sin(x))^2,x, algorithm="fricas")
```

```
[Out] -1/2*(4*a^4*b + 2*a^2*b^3 - 2*b^5 + 2*(a^4*b + 2*a^2*b^3 + b^5)*cos(x)^2 +
2*(a^5 + 2*a^3*b^2 + a*b^4)*cos(x)*sin(x) + sqrt(a^2 + b^2)*((a^4 - 2*a^2*b
```

```
^2)*cos(x) + (a^3*b - 2*a*b^3)*sin(x))*log((2*a*b*cos(x)*sin(x) + (a^2 - b^
2)*cos(x)^2 - 2*a^2 - b^2 - 2*sqrt(a^2 + b^2)*(b*cos(x) - a*sin(x)))/(2*a*b
*cos(x)*sin(x) + (a^2 - b^2)*cos(x)^2 + b^2)))/((a^7 + 3*a^5*b^2 + 3*a^3*b^
4 + a*b^6)*cos(x) + (a^6*b + 3*a^4*b^3 + 3*a^2*b^5 + b^7)*sin(x))
```

giac [A] time = 0.20, size = 209, normalized size = 1.90

$$-\frac{\left(a^{3}-2\,ab^{2}\right)\log\left(\frac{\left|2\,a\tan\left(\frac{1}{2}\,x\right)-2\,b-2\,\sqrt{a^{2}+b^{2}}\right|}{\left|2\,a\tan\left(\frac{1}{2}\,x\right)-2\,b+2\,\sqrt{a^{2}+b^{2}}\right|}\right)}{\left(a^{4}+2\,a^{2}b^{2}+b^{4}\right)\sqrt{a^{2}+b^{2}}}-\frac{2\left(a^{3}\tan\left(\frac{1}{2}\,x\right)^{3}-2\,ab^{2}\tan\left(\frac{1}{2}\,x\right)^{3}-a^{2}b\tan\left(\frac{1}{2}\,x\right)^{2}+2\,b^{3}\tan\left(\frac{1}{2}\,x\right)^{2}\right.}{\left(a\tan\left(\frac{1}{2}\,x\right)^{4}-2\,b\tan\left(\frac{1}{2}\,x\right)^{3}-2\,b\tan\left(\frac{1}{2}\,x\right)-a\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*sin(x)^2/(a*cos(x)+b*sin(x))^2,x, algorithm="giac")
```

```
[Out] -(a^3 - 2*a*b^2)*log(abs(2*a*tan(1/2*x) - 2*b - 2*sqrt(a^2 + b^2))/abs(2*a*
tan(1/2*x) - 2*b + 2*sqrt(a^2 + b^2)))/((a^4 + 2*a^2*b^2 + b^4)*sqrt(a^2 +
b^2)) - 2*(a^3*tan(1/2*x)^3 - 2*a*b^2*tan(1/2*x)^3 - a^2*b*tan(1/2*x)^2 + 2
*b^3*tan(1/2*x)^2 - a^3*tan(1/2*x) - 4*a*b^2*tan(1/2*x) - 3*a^2*b)/((a*tan(
1/2*x)^4 - 2*b*tan(1/2*x)^3 - 2*b*tan(1/2*x) - a)*(a^4 + 2*a^2*b^2 + b^4))
```

maple [A] time = 0.12, size = 142, normalized size = 1.29

$$-\frac{2a\left(\frac{-\tan\left(\frac{x}{2}\right)b^{2}-ab}{\left(\tan^{2}\left(\frac{x}{2}\right)\right)a-2b\tan\left(\frac{x}{2}\right)-a}-\frac{\left(a^{2}-2b^{2}\right)\operatorname{arctanh}\left(\frac{2a\tan\left(\frac{x}{2}\right)-2b}{2\sqrt{a^{2}+b^{2}}}\right)}{\sqrt{a^{2}+b^{2}}}\right)}{\left(a^{2}+b^{2}\right)^{2}}+\frac{2\left(-a^{2}+b^{2}\right)\tan\left(\frac{x}{2}\right)-4ab}{\left(a^{4}+2a^{2}b^{2}+b^{4}\right)\left(\tan^{2}\left(\frac{x}{2}\right)+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)*sin(x)^2/(a*cos(x)+b*sin(x))^2,x)
```

```
[Out] -2*a/(a^2+b^2)^2*((-tan(1/2*x)*b^2-a*b)/(tan(1/2*x)^2*a-2*b*tan(1/2*x)-a)-(
a^2-2*b^2)/(a^2+b^2)^(1/2)*arctanh(1/2*(2*a*tan(1/2*x)-2*b)/(a^2+b^2)^(1/2)
))+2/(a^4+2*a^2*b^2+b^4)*((-a^2+b^2)*tan(1/2*x)-2*a*b)/(tan(1/2*x)^2+1)
```

maxima [B] time = 0.46, size = 265, normalized size = 2.41

$$-\frac{\left(a^{2}-2\,b^{2}\right)a\log\left(\frac{b-\frac{a\sin(x)}{\cos(x)+1}+\sqrt{a^{2}+b^{2}}}{b-\frac{a\sin(x)}{\cos(x)+1}-\sqrt{a^{2}+b^{2}}}\right)}{\left(a^{4}+2\,a^{2}b^{2}+b^{4}\right)\sqrt{a^{2}+b^{2}}}-\frac{2\left(3\,a^{2}b+\frac{\left(a^{3}+4\,ab^{2}\right)\sin(x)}{\cos(x)+1}+\frac{\left(a^{2}b-2\,b^{3}\right)\sin(x)^{2}}{\left(\cos(x)+1\right)^{2}}-\frac{\left(a^{3}-2\,ab^{2}\right)\sin(x)^{3}}{\left(\cos(x)+1\right)^{3}}\right)}{a^{5}+2\,a^{3}b^{2}+ab^{4}+\frac{2\left(a^{4}b+2\,a^{2}b^{3}+b^{5}\right)\sin(x)}{\cos(x)+1}+\frac{2\left(a^{4}b+2\,a^{2}b^{3}+b^{5}\right)\sin(x)^{3}}{\left(\cos(x)+1\right)^{3}}-\frac{\left(a^{5}+2\,a^{3}b^{2}+a\right.}{\left(\cos(x\right.}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*sin(x)^2/(a*cos(x)+b*sin(x))^2,x, algorithm="maxima")
```

```
[Out] -(a^2 - 2*b^2)*a*log((b - a*sin(x)/(cos(x) + 1) + sqrt(a^2 + b^2))/(b - a*s
in(x)/(cos(x) + 1) - sqrt(a^2 + b^2)))/((a^4 + 2*a^2*b^2 + b^4)*sqrt(a^2 +
b^2)) - 2*(3*a^2*b + (a^3 + 4*a*b^2)*sin(x)/(cos(x) + 1) + (a^2*b - 2*b^3)*
sin(x)^2/(cos(x) + 1)^2 - (a^3 - 2*a*b^2)*sin(x)^3/(cos(x) + 1)^3)/(a^5 + 2
*a^3*b^2 + a*b^4 + 2*(a^4*b + 2*a^2*b^3 + b^5)*sin(x)/(cos(x) + 1) + 2*(a^4
*b + 2*a^2*b^3 + b^5)*sin(x)^3/(cos(x) + 1)^3 - (a^5 + 2*a^3*b^2 + a*b^4)*s
in(x)^4/(cos(x) + 1)^4)
```

mupad [**B**] time = 1.23, size = 249, normalized size = 2.26

$$-\frac{\frac{2\tan\left(\frac{x}{2}\right)\left(a^3+4\,a\,b^2\right)}{a^4+2\,a^2\,b^2+b^4}+\frac{6\,a^2\,b}{a^4+2\,a^2\,b^2+b^4}-\frac{2\,a\tan\left(\frac{x}{2}\right)^3\left(a^2-2\,b^2\right)}{a^4+2\,a^2\,b^2+b^4}+\frac{2\,b\tan\left(\frac{x}{2}\right)^2\left(a^2-2\,b^2\right)}{a^4+2\,a^2\,b^2+b^4}}{-a\tan\left(\frac{x}{2}\right)^4+2\,b\tan\left(\frac{x}{2}\right)^3+2\,b\tan\left(\frac{x}{2}\right)+a}-\frac{a\,\mathrm{atan}\left(\frac{1\mathrm{i}\tan\left(\frac{x}{2}\right)a^5-a^4\,b\,1\mathrm{i}+2\mathrm{i}\tan\left(\frac{x}{2}\right)a^3\,b^2-a^2\,b^3}{\left(a^2+b^2\right)^{5/2}}\right.}{\left(a^2+b^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(x)*sin(x)^2)/(a*cos(x) + b*sin(x))^2,x)
```

```
[Out] - ((2*tan(x/2)*(4*a*b^2 + a^3))/(a^4 + b^4 + 2*a^2*b^2) + (6*a^2*b)/(a^4 +
b^4 + 2*a^2*b^2) - (2*a*tan(x/2)^3*(a^2 - 2*b^2))/(a^4 + b^4 + 2*a^2*b^2) +
 (2*b*tan(x/2)^2*(a^2 - 2*b^2))/(a^4 + b^4 + 2*a^2*b^2))/(a + 2*b*tan(x/2)
- a*tan(x/2)^4 + 2*b*tan(x/2)^3) - (a*atan((a^5*tan(x/2)*1i - a^4*b*1i - b^
5*1i - a^2*b^3*2i + a^3*b^2*tan(x/2)*2i + a*b^4*tan(x/2)*1i)/(a^2 + b^2)^(5
/2))*(a^2 - 2*b^2)*2i)/(a^2 + b^2)^(5/2)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*sin(x)**2/(a*cos(x)+b*sin(x))**2,x)
```

```
[Out] Timed out
```

3.286 $\int \frac{\cos(x)\sin^3(x)}{(a\cos(x)+b\sin(x))^2}\,dx$

Optimal. Leaf size=129

$$-\frac{(a^2-b^2)\sin^2(x)}{2(a^2+b^2)^2}-\frac{a^2b\sin(x)}{(a^2+b^2)^2(a\cos(x)+b\sin(x))}-\frac{ab\sin(x)\cos(x)}{(a^2+b^2)^2}-\frac{a^2(a^2-3b^2)\log(a\cos(x)+b\sin(x))}{(a^2+b^2)^3}+\frac{bx}{(}$$

```
[Out] b*(3*a^3-a*b^2)*x/(a^2+b^2)^3-a^2*(a^2-3*b^2)*ln(a*cos(x)+b*sin(x))/(a^2+b^
2)^3-a*b*cos(x)*sin(x)/(a^2+b^2)^2-1/2*(a^2-b^2)*sin(x)^2/(a^2+b^2)^2-a^2*b
*sin(x)/(a^2+b^2)^2/(a*cos(x)+b*sin(x))
```

Rubi [A] time = 0.51, antiderivative size = 198, normalized size of antiderivative = 1.53, number of steps used = 17, number of rules used = 13, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.722, Rules used = {3111, 3109, 2564, 30, 2635, 8, 3097, 3133, 3099, 3085, 3483, 3531, 3530}

$$\frac{a^3bx}{(a^2+b^2)^3}+\frac{abx}{(a^2+b^2)^2}-\frac{ab^3x}{(a^2+b^2)^3}+\frac{abx(a^2-b^2)}{(a^2+b^2)^3}-\frac{a^2\sin^2(x)}{2(a^2+b^2)^2}+\frac{b^2\sin^2(x)}{2(a^2+b^2)^2}-\frac{a^2b}{(a^2+b^2)^2(a\cot(x)+b)}-\frac{ab\sin(}{(a^2 -}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[x]*Sin[x]^3)/(a*Cos[x] + b*Sin[x])^2,x]
```

```
[Out] (a^3*b*x)/(a^2 + b^2)^3 - (a*b^3*x)/(a^2 + b^2)^3 + (a*b*(a^2 - b^2)*x)/(a^
2 + b^2)^3 + (a*b*x)/(a^2 + b^2)^2 - (a^2*b)/((a^2 + b^2)^2*(b + a*Cot[x]))
 - (a^4*Log[a*Cos[x] + b*Sin[x]])/(a^2 + b^2)^3 + (3*a^2*b^2*Log[a*Cos[x] +
 b*Sin[x]])/(a^2 + b^2)^3 - (a*b*Cos[x]*Sin[x])/(a^2 + b^2)^2 - (a^2*Sin[x]
^2)/(2*(a^2 + b^2)^2) + (b^2*Sin[x]^2)/(2*(a^2 + b^2)^2)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2564

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
```

```
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 3085

```
Int[sin[(c_.) + (d_.)*(x_)]^(m_)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin
[(c_.) + (d_.)*(x_)])^(n_.), x_Symbol] :> Int[(b + a*Cot[c + d*x])^n, x] /;
 FreeQ[{a, b, c, d}, x] && EqQ[m + n, 0] && IntegerQ[n] && NeQ[a^2 + b^2, 0
]
```

Rule 3097

```
Int[sin[(c_.) + (d_.)*(x_)]/(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.
) + (d_.)*(x_)]), x_Symbol] :> Simp[(b*x)/(a^2 + b^2), x] - Dist[a/(a^2 + b
^2), Int[(b*Cos[c + d*x] - a*Sin[c + d*x])/(a*Cos[c + d*x] + b*Sin[c + d*x]
), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

Rule 3099

```
Int[sin[(c_.) + (d_.)*(x_)]^(m_)/(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin
[(c_.) + (d_.)*(x_)]), x_Symbol] :> -Simp[(a*Sin[c + d*x]^(m - 1))/(d*(a^2
+ b^2)*(m - 1)), x] + (Dist[a^2/(a^2 + b^2), Int[Sin[c + d*x]^(m - 2)/(a*Co
s[c + d*x] + b*Sin[c + d*x]), x], x] + Dist[b/(a^2 + b^2), Int[Sin[c + d*x]
^(m - 1), x], x]) /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0] && GtQ[m,
1]
```

Rule 3109

```
Int[(cos[(c_.) + (d_.)*(x_)]^(m_.)*sin[(c_.) + (d_.)*(x_)]^(n_.))/(cos[(c_.
) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[b
/(a^2 + b^2), Int[Cos[c + d*x]^m*Sin[c + d*x]^(n - 1), x], x] + (Dist[a/(a^
2 + b^2), Int[Cos[c + d*x]^(m - 1)*Sin[c + d*x]^n, x], x] - Dist[(a*b)/(a^2
 + b^2), Int[(Cos[c + d*x]^(m - 1)*Sin[c + d*x]^(n - 1))/(a*Cos[c + d*x] +
b*Sin[c + d*x]), x], x]) /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0] &&
IGtQ[m, 0] && IGtQ[n, 0]
```

Rule 3111

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*sin[(c_.) + (d_.)*(x_)]^(n_.)*(cos[(c_.)
+ (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(p_), x_Symbol] :> Dis
t[b/(a^2 + b^2), Int[Cos[c + d*x]^m*Sin[c + d*x]^(n - 1)*(a*Cos[c + d*x] +
b*Sin[c + d*x])^(p + 1), x], x] + (Dist[a/(a^2 + b^2), Int[Cos[c + d*x]^(m
- 1)*Sin[c + d*x]^n*(a*Cos[c + d*x] + b*Sin[c + d*x])^(p + 1), x], x] - Dis
t[(a*b)/(a^2 + b^2), Int[Cos[c + d*x]^(m - 1)*Sin[c + d*x]^(n - 1)*(a*Cos[c
 + d*x] + b*Sin[c + d*x])^p, x], x]) /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 +
 b^2, 0] && IGtQ[m, 0] && IGtQ[n, 0] && ILtQ[p, 0]
```

Rule 3133

```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])
/((a_.) + cos[(d_.) + (e_.)*(x_)]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)]), x
_Symbol] :> Simp[((b*B + c*C)*x)/(b^2 + c^2), x] + Simp[((c*B - b*C)*Log[a
+ b*Cos[d + e*x] + c*Sin[d + e*x]])/(e*(b^2 + c^2)), x] /; FreeQ[{a, b, c,
d, e, A, B, C}, x] && NeQ[b^2 + c^2, 0] && EqQ[A*(b^2 + c^2) - a*(b*B + c*C
), 0]
```

Rule 3483

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(a +
b*Tan[c + d*x])^(n + 1))/(d*(n + 1)*(a^2 + b^2)), x] + Dist[1/(a^2 + b^2),
Int[(a - b*Tan[c + d*x])*(a + b*Tan[c + d*x])^(n + 1), x], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 + b^2, 0] && LtQ[n, -1]
```

Rule 3530

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_) + (b_.)*tan[(e_.) + (f_.)*
(x_)]), x_Symbol] :> Simp[(c*Log[RemoveContent[a*Cos[e + f*x] + b*Sin[e + f
*x], x]])/(b*f), x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
 NeQ[a^2 + b^2, 0] && EqQ[a*c + b*d, 0]
```

Rule 3531

```
Int[((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_.) + (b_.)*tan[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[((a*c + b*d)*x)/(a^2 + b^2), x] + Dist[(b*c - a
*d)/(a^2 + b^2), Int[(b - a*Tan[e + f*x])/(a + b*Tan[e + f*x]), x], x] /; F
reeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && Ne
Q[a*c + b*d, 0]
```

Rubi steps

$$\int \frac{\cos(x)\sin^3(x)}{(a\cos(x)+b\sin(x))^2}\,dx = \frac{a\int \frac{\sin^3(x)}{a\cos(x)+b\sin(x)}\,dx}{a^2+b^2} + \frac{b\int \frac{\cos(x)\sin^2(x)}{a\cos(x)+b\sin(x)}\,dx}{a^2+b^2} - \frac{(ab)\int \frac{\sin^2(x)}{(a\cos(x)+b\sin(x))^2}\,dx}{a^2+b^2}$$

$$= -\frac{a^2\sin^2(x)}{2\left(a^2+b^2\right)^2} + \frac{a^3\int \frac{\sin(x)}{a\cos(x)+b\sin(x)}\,dx}{\left(a^2+b^2\right)^2} + 2\frac{(ab)\int \sin^2(x)\,dx}{\left(a^2+b^2\right)^2} + \frac{b^2\int \cos(x)\sin(x)\,\ldots}{\left(a^2+b^2\right)^2}$$

$$= \frac{a^3bx}{\left(a^2+b^2\right)^3} - \frac{ab^3x}{\left(a^2+b^2\right)^3} - \frac{a^2b}{\left(a^2+b^2\right)^2(b+a\cot(x))} - \frac{a^2\sin^2(x)}{2\left(a^2+b^2\right)^2} - \frac{a^4\int \frac{b\cos(x)-\ldots}{a\cos(x)+\ldots}}{\left(a^2+\ldots\right.}$$

$$= \frac{a^3bx}{\left(a^2+b^2\right)^3} - \frac{ab^3x}{\left(a^2+b^2\right)^3} + \frac{ab\left(a^2-b^2\right)x}{\left(a^2+b^2\right)^3} - \frac{a^2b}{\left(a^2+b^2\right)^2(b+a\cot(x))} - \frac{a^4\log(a\,\mathrm{c}\ldots}{(a\ldots}$$

$$= \frac{a^3bx}{\left(a^2+b^2\right)^3} - \frac{ab^3x}{\left(a^2+b^2\right)^3} + \frac{ab\left(a^2-b^2\right)x}{\left(a^2+b^2\right)^3} - \frac{a^2b}{\left(a^2+b^2\right)^2(b+a\cot(x))} - \frac{a^4\log(a\,\mathrm{c}\ldots}{(a\ldots}$$

Mathematica [C] time = 1.48, size = 226, normalized size = 1.75

$$\frac{4ia^2\left(a^2-3b^2\right)\tan^{-1}(\tan(x))(a\cos(x)+b\sin(x)) + a\cos(x)\left(\left(a^4-b^4\right)\cos(2x)+2a\left(-b\left(a^2+b^2\right)\sin(2x)-a\left(a\ldots\right.\right.\right.}{\ldots}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[x]*Sin[x]^3)/(a*Cos[x] + b*Sin[x])^2,x]
```

```
[Out] ((4*I)*a^2*(a^2 - 3*b^2)*ArcTan[Tan[x]]*(a*Cos[x] + b*Sin[x]) + a*Cos[x]*((
a^4 - b^4)*Cos[2*x] + 2*a*(2*(I*a - b)^3*x - a*(a^2 - 3*b^2)*Log[(a*Cos[x]
+ b*Sin[x])^2] - b*(a^2 + b^2)*Sin[2*x])) - b*Sin[x]*((-a^4 + b^4)*Cos[2*x]
 + 2*a*(2*(a^3*(1 + I*x) + a*b^2*(1 - (3*I)*x) - 3*a^2*b*x + b^3*x) + a*(a^
2 - 3*b^2)*Log[(a*Cos[x] + b*Sin[x])^2] + b*(a^2 + b^2)*Sin[2*x])))/(4*(a^2
 + b^2)^3*(a*Cos[x] + b*Sin[x]))
```

fricas [A] time = 0.74, size = 236, normalized size = 1.83

$$\frac{2\left(a^5+2\,a^3b^2+ab^4\right)\cos(x)^3-\left(a^5+3\,ab^4-4\left(3\,a^4b-a^2b^3\right)x\right)\cos(x)-2\left(\left(a^5-3\,a^3b^2\right)\cos(x)+\left(a^4b-3\,a^2b^3\right)\ldots\right.}{4\left(\left(a^7+3\,a^5b^2+3\,a^3b^4+ab^6\ldots\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*sin(x)^3/(a*cos(x)+b*sin(x))^2,x, algorithm="fricas")
```

```
[Out] 1/4*(2*(a^5 + 2*a^3*b^2 + a*b^4)*cos(x)^3 - (a^5 + 3*a*b^4 - 4*(3*a^4*b - a
^2*b^3)*x)*cos(x) - 2*((a^5 - 3*a^3*b^2)*cos(x) + (a^4*b - 3*a^2*b^3)*sin(x
))*log(2*a*b*cos(x)*sin(x) + (a^2 - b^2)*cos(x)^2 + b^2) - (5*a^4*b - b^5 +
 2*(a^4*b + 2*a^2*b^3 + b^5)*cos(x)^2 - 4*(3*a^3*b^2 - a*b^4)*x)*sin(x))/((
a^7 + 3*a^5*b^2 + 3*a^3*b^4 + a*b^6)*cos(x) + (a^6*b + 3*a^4*b^3 + 3*a^2*b^
5 + b^7)*sin(x))
```

giac [A] time = 0.45, size = 223, normalized size = 1.73

$$\frac{\left(3\,a^3b-ab^3\right)x}{a^6+3\,a^4b^2+3\,a^2b^4+b^6}+\frac{\left(a^4-3\,a^2b^2\right)\log\left(\tan(x)^2+1\right)}{2\left(a^6+3\,a^4b^2+3\,a^2b^4+b^6\right)}-\frac{\left(a^4b-3\,a^2b^3\right)\log\left(|b\tan(x)+a|\right)}{a^6b+3\,a^4b^3+3\,a^2b^5+b^7}+\frac{2\,a^3\tan(x)^2-2\,ab}{2\left(a^4+2\,a^2b^2\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*sin(x)^3/(a*cos(x)+b*sin(x))^2,x, algorithm="giac")
```

```
[Out] (3*a^3*b - a*b^3)*x/(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6) + 1/2*(a^4 - 3*a^2*
b^2)*log(tan(x)^2 + 1)/(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6) - (a^4*b - 3*a^2
*b^3)*log(abs(b*tan(x) + a))/(a^6*b + 3*a^4*b^3 + 3*a^2*b^5 + b^7) + 1/2*(2
*a^3*tan(x)^2 - 2*a*b^2*tan(x)^2 - a^2*b*tan(x) - b^3*tan(x) + 3*a^3 - a*b^
2)/((a^4 + 2*a^2*b^2 + b^4)*(b*tan(x)^3 + a*tan(x)^2 + b*tan(x) + a))
```

maple [A] time = 0.11, size = 243, normalized size = 1.88

$$-\frac{\tan(x)a^3b}{\left(a^2+b^2\right)^3\left(\tan^2(x)+1\right)}-\frac{\tan(x)a\,b^3}{\left(a^2+b^2\right)^3\left(\tan^2(x)+1\right)}+\frac{a^4}{2\left(a^2+b^2\right)^3\left(\tan^2(x)+1\right)}-\frac{b^4}{2\left(a^2+b^2\right)^3\left(\tan^2(x)+1\right)}+\frac{\ln\left(\right.}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)*sin(x)^3/(a*cos(x)+b*sin(x))^2,x)
```

```
[Out] -1/(a^2+b^2)^3/(tan(x)^2+1)*tan(x)*a^3*b-1/(a^2+b^2)^3/(tan(x)^2+1)*tan(x)*
a*b^3+1/2/(a^2+b^2)^3/(tan(x)^2+1)*a^4-1/2/(a^2+b^2)^3/(tan(x)^2+1)*b^4+1/2
/(a^2+b^2)^3*ln(tan(x)^2+1)*a^4-3/2/(a^2+b^2)^3*ln(tan(x)^2+1)*a^2*b^2+3/(a
^2+b^2)^3*arctan(tan(x))*a^3*b-1/(a^2+b^2)^3*a*arctan(tan(x))*b^3+a^3/(a^2+
b^2)^2/(a+b*tan(x))-a^4/(a^2+b^2)^3*ln(a+b*tan(x))+3*a^2/(a^2+b^2)^3*ln(a+b
*tan(x))*b^2
```

maxima [B] time = 0.46, size = 259, normalized size = 2.01

$$\frac{\left(3\,a^3b-ab^3\right)x}{a^6+3\,a^4b^2+3\,a^2b^4+b^6}-\frac{\left(a^4-3\,a^2b^2\right)\log\left(b\tan(x)+a\right)}{a^6+3\,a^4b^2+3\,a^2b^4+b^6}+\frac{\left(a^4-3\,a^2b^2\right)\log\left(\tan(x)^2+1\right)}{2\left(a^6+3\,a^4b^2+3\,a^2b^4+b^6\right)}+\frac{}{2\left(a^5+2\,a^3b^2+ab^4\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*sin(x)^3/(a*cos(x)+b*sin(x))^2,x, algorithm="maxima")
```

```
[Out] (3*a^3*b - a*b^3)*x/(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6) - (a^4 - 3*a^2*b^2)
*log(b*tan(x) + a)/(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6) + 1/2*(a^4 - 3*a^2*b
^2)*log(tan(x)^2 + 1)/(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6) + 1/2*(3*a^3 - a*
b^2 + 2*(a^3 - a*b^2)*tan(x)^2 - (a^2*b + b^3)*tan(x))/(a^5 + 2*a^3*b^2 + a
*b^4 + (a^4*b + 2*a^2*b^3 + b^5)*tan(x)^3 + (a^5 + 2*a^3*b^2 + a*b^4)*tan(x
)^2 + (a^4*b + 2*a^2*b^3 + b^5)*tan(x))
```

mupad [B] time = 7.68, size = 5431, normalized size = 42.10

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(x)*sin(x)^3)/(a*cos(x) + b*sin(x))^2,x)
```

```
[Out] (log(1/(cos(x) + 1))*(2*a^4 - 6*a^2*b^2))/(2*(a^6 + b^6 + 3*a^2*b^4 + 3*a^4
*b^2)) - (log(a + 2*b*tan(x/2) - a*tan(x/2)^2)*(a^4 - 3*a^2*b^2))/(a^6 + b^
6 + 3*a^2*b^4 + 3*a^4*b^2) - ((2*a*tan(x/2)^2)/(a^2 + b^2) - (2*a*tan(x/2)^
4)/(a^2 + b^2) + (4*a^2*b*tan(x/2))/(a^2 + b^2)^2 + (4*a^2*b*tan(x/2)^5)/(a
^4 + b^4 + 2*a^2*b^2) + (4*b*tan(x/2)^3*(a^2 - b^2))/(a^2 + b^2)^2)/(a + 2*
b*tan(x/2) + a*tan(x/2)^2 - a*tan(x/2)^4 - a*tan(x/2)^6 + 4*b*tan(x/2)^3 +
2*b*tan(x/2)^5) + (2*a*b*atan((((((a*b*((32*(3*a^4*b^11 - a^2*b^13 - 4*a^14
*b + 18*a^6*b^9 + 22*a^8*b^7 + 3*a^10*b^5 - 9*a^12*b^3))/(a^12 + b^12 + 6*a
^2*b^10 + 15*a^4*b^8 + 20*a^6*b^6 + 15*a^8*b^4 + 6*a^10*b^2) - (16*(2*a^4 -
 6*a^2*b^2)*(3*a^16*b + 3*a^2*b^15 + 21*a^4*b^13 + 63*a^6*b^11 + 105*a^8*b^
9 + 105*a^10*b^7 + 63*a^12*b^5 + 21*a^14*b^3))/((a^6 + b^6 + 3*a^2*b^4 + 3*
a^4*b^2)*(a^12 + b^12 + 6*a^2*b^10 + 15*a^4*b^8 + 20*a^6*b^6 + 15*a^8*b^4 +
 6*a^10*b^2)))*(3*a^2 - b^2))/(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2) - (16*a*b
*(2*a^4 - 6*a^2*b^2)*(3*a^2 - b^2)*(3*a^16*b + 3*a^2*b^15 + 21*a^4*b^13 + 6
3*a^6*b^11 + 105*a^8*b^9 + 105*a^10*b^7 + 63*a^12*b^5 + 21*a^14*b^3))/((a^6
 + b^6 + 3*a^2*b^4 + 3*a^4*b^2)^2*(a^12 + b^12 + 6*a^2*b^10 + 15*a^4*b^8 +
20*a^6*b^6 + 15*a^8*b^4 + 6*a^10*b^2)))*(2*a^4 - 6*a^2*b^2))/(2*(a^6 + b^6
+ 3*a^2*b^4 + 3*a^4*b^2)) + (a*b*((32*(5*a^4*b^9 - 3*a^12*b + 12*a^6*b^7 +
6*a^8*b^5 - 4*a^10*b^3))/(a^12 + b^12 + 6*a^2*b^10 + 15*a^4*b^8 + 20*a^6*b^
6 + 15*a^8*b^4 + 6*a^10*b^2) + (((32*(3*a^4*b^11 - a^2*b^13 - 4*a^14*b + 18
*a^6*b^9 + 22*a^8*b^7 + 3*a^10*b^5 - 9*a^12*b^3))/(a^12 + b^12 + 6*a^2*b^10
 + 15*a^4*b^8 + 20*a^6*b^6 + 15*a^8*b^4 + 6*a^10*b^2) - (16*(2*a^4 - 6*a^2*
b^2)*(3*a^16*b + 3*a^2*b^15 + 21*a^4*b^13 + 63*a^6*b^11 + 105*a^8*b^9 + 105
*a^10*b^7 + 63*a^12*b^5 + 21*a^14*b^3))/((a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2
)*(a^12 + b^12 + 6*a^2*b^10 + 15*a^4*b^8 + 20*a^6*b^6 + 15*a^8*b^4 + 6*a^10
*b^2)))*(2*a^4 - 6*a^2*b^2))/(2*(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2)))*(3*a^
2 - b^2))/(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2) + (32*a^3*b^3*(3*a^2 - b^2)^3
*(3*a^16*b + 3*a^2*b^15 + 21*a^4*b^13 + 63*a^6*b^11 + 105*a^8*b^9 + 105*a^1
0*b^7 + 63*a^12*b^5 + 21*a^14*b^3))/((a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2)^3*
```

(a^12 + b^12 + 6*a^2*b^10 + 15*a^4*b^8 + 20*a^6*b^6 + 15*a^8*b^4 + 6*a^10*b
^2)))*(4*a^8 + b^8 - 67*a^2*b^6 + 155*a^4*b^4 - 61*a^6*b^2)*(a^16 + b^16 +
8*a^2*b^14 + 28*a^4*b^12 + 56*a^6*b^10 + 70*a^8*b^8 + 56*a^10*b^6 + 28*a^12
*b^4 + 8*a^14*b^2))/((96*a^6*b - 32*a^4*b^3)*(4*a^8 + b^8 + 31*a^2*b^6 + 15
*a^4*b^4 - 11*a^6*b^2)^2) - (tan(x/2)*(((((2*a^4 - 6*a^2*b^2)*((a*b*(3*a^2
- b^2)*((32*(a^3*b^12 - 2*a^15 + 15*a^5*b^10 + 48*a^7*b^8 + 62*a^9*b^6 + 33
*a^11*b^4 + 3*a^13*b^2))/(a^12 + b^12 + 6*a^2*b^10 + 15*a^4*b^8 + 20*a^6*b^
6 + 15*a^8*b^4 + 6*a^10*b^2) + (16*(2*a^4 - 6*a^2*b^2)*(3*a*b^16 + 21*a^3*b
^14 + 63*a^5*b^12 + 105*a^7*b^10 + 105*a^9*b^8 + 63*a^11*b^6 + 21*a^13*b^4
+ 3*a^15*b^2))/((a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2)*(a^12 + b^12 + 6*a^2*b^
10 + 15*a^4*b^8 + 20*a^6*b^6 + 15*a^8*b^4 + 6*a^10*b^2))))/(a^6 + b^6 + 3*a
^2*b^4 + 3*a^4*b^2) + (16*a*b*(2*a^4 - 6*a^2*b^2)*(3*a^2 - b^2)*(3*a*b^16 +
21*a^3*b^14 + 63*a^5*b^12 + 105*a^7*b^10 + 105*a^9*b^8 + 63*a^11*b^6 + 21*
a^13*b^4 + 3*a^15*b^2))/((a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2)^2*(a^12 + b^12
+ 6*a^2*b^10 + 15*a^4*b^8 + 20*a^6*b^6 + 15*a^8*b^4 + 6*a^10*b^2))))/(2*(a
^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2)) + (a*b*((32*(2*a^3*b^10 - 24*a^5*b^8 - 3
6*a^7*b^6 + 8*a^9*b^4 + 18*a^11*b^2))/(a^12 + b^12 + 6*a^2*b^10 + 15*a^4*b^
8 + 20*a^6*b^6 + 15*a^8*b^4 + 6*a^10*b^2) + ((2*a^4 - 6*a^2*b^2)*((32*(a^3*
b^12 - 2*a^15 + 15*a^5*b^10 + 48*a^7*b^8 + 62*a^9*b^6 + 33*a^11*b^4 + 3*a^1
3*b^2))/(a^12 + b^12 + 6*a^2*b^10 + 15*a^4*b^8 + 20*a^6*b^6 + 15*a^8*b^4 +
6*a^10*b^2) + (16*(2*a^4 - 6*a^2*b^2)*(3*a*b^16 + 21*a^3*b^14 + 63*a^5*b^12
+ 105*a^7*b^10 + 105*a^9*b^8 + 63*a^11*b^6 + 21*a^13*b^4 + 3*a^15*b^2))/((
a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2)*(a^12 + b^12 + 6*a^2*b^10 + 15*a^4*b^8 +
20*a^6*b^6 + 15*a^8*b^4 + 6*a^10*b^2))))/(2*(a^6 + b^6 + 3*a^2*b^4 + 3*a^4
b^2)))(3*a^2 - b^2))/(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2) - (32*a^3*b^3*(3
a^2 - b^2)^3(3*a*b^16 + 21*a^3*b^14 + 63*a^5*b^12 + 105*a^7*b^10 + 105*a^
9*b^8 + 63*a^11*b^6 + 21*a^13*b^4 + 3*a^15*b^2))/((a^6 + b^6 + 3*a^2*b^4 +
3*a^4*b^2)^3*(a^12 + b^12 + 6*a^2*b^10 + 15*a^4*b^8 + 20*a^6*b^6 + 15*a^8*b
^4 + 6*a^10*b^2)))*(4*a^8 + b^8 - 67*a^2*b^6 + 155*a^4*b^4 - 61*a^6*b^2))/(
4*a^8 + b^8 + 31*a^2*b^6 + 15*a^4*b^4 - 11*a^6*b^2)^2 - (2*a*b*((32*(2*a^11
- 6*a^5*b^6 + 2*a^7*b^4 - 6*a^9*b^2))/(a^12 + b^12 + 6*a^2*b^10 + 15*a^4*b
^8 + 20*a^6*b^6 + 15*a^8*b^4 + 6*a^10*b^2) + ((2*a^4 - 6*a^2*b^2)*((32*(2*a
^3*b^10 - 24*a^5*b^8 - 36*a^7*b^6 + 8*a^9*b^4 + 18*a^11*b^2))/(a^12 + b^12
+ 6*a^2*b^10 + 15*a^4*b^8 + 20*a^6*b^6 + 15*a^8*b^4 + 6*a^10*b^2) + ((2*a^4
- 6*a^2*b^2)*((32*(a^3*b^12 - 2*a^15 + 15*a^5*b^10 + 48*a^7*b^8 + 62*a^9*b
^6 + 33*a^11*b^4 + 3*a^13*b^2))/(a^12 + b^12 + 6*a^2*b^10 + 15*a^4*b^8 + 20
*a^6*b^6 + 15*a^8*b^4 + 6*a^10*b^2) + (16*(2*a^4 - 6*a^2*b^2)*(3*a*b^16 + 2
1*a^3*b^14 + 63*a^5*b^12 + 105*a^7*b^10 + 105*a^9*b^8 + 63*a^11*b^6 + 21*a^
13*b^4 + 3*a^15*b^2))/((a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2)*(a^12 + b^12 + 6
*a^2*b^10 + 15*a^4*b^8 + 20*a^6*b^6 + 15*a^8*b^4 + 6*a^10*b^2))))/(2*(a^6 +
b^6 + 3*a^2*b^4 + 3*a^4*b^2))))/(2*(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2)) -
(a*b*((a*b*(3*a^2 - b^2)*((32*(a^3*b^12 - 2*a^15 + 15*a^5*b^10 + 48*a^7*b^8
+ 62*a^9*b^6 + 33*a^11*b^4 + 3*a^13*b^2))/(a^12 + b^12 + 6*a^2*b^10 + 15*a
^4*b^8 + 20*a^6*b^6 + 15*a^8*b^4 + 6*a^10*b^2) + (16*(2*a^4 - 6*a^2*b^2)*(3
*a*b^16 + 21*a^3*b^14 + 63*a^5*b^12 + 105*a^7*b^10 + 105*a^9*b^8 + 63*a^11*

```
b^6 + 21*a^13*b^4 + 3*a^15*b^2))/((a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2)*(a^12
 + b^12 + 6*a^2*b^10 + 15*a^4*b^8 + 20*a^6*b^6 + 15*a^8*b^4 + 6*a^10*b^2)))
)/(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2) + (16*a*b*(2*a^4 - 6*a^2*b^2)*(3*a^2
- b^2)*(3*a*b^16 + 21*a^3*b^14 + 63*a^5*b^12 + 105*a^7*b^10 + 105*a^9*b^8 +
 63*a^11*b^6 + 21*a^13*b^4 + 3*a^15*b^2))/((a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b
^2)^2*(a^12 + b^12 + 6*a^2*b^10 + 15*a^4*b^8 + 20*a^6*b^6 + 15*a^8*b^4 + 6*
a^10*b^2)))*(3*a^2 - b^2))/(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2) - (16*a^2*b^
2*(2*a^4 - 6*a^2*b^2)*(3*a^2 - b^2)^2*(3*a*b^16 + 21*a^3*b^14 + 63*a^5*b^12
 + 105*a^7*b^10 + 105*a^9*b^8 + 63*a^11*b^6 + 21*a^13*b^4 + 3*a^15*b^2))/((
a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2)^3*(a^12 + b^12 + 6*a^2*b^10 + 15*a^4*b^8
 + 20*a^6*b^6 + 15*a^8*b^4 + 6*a^10*b^2)))*(10*a^6 - 7*b^6 + 68*a^2*b^4 - 5
9*a^4*b^2))/(4*a^8 + b^8 + 31*a^2*b^6 + 15*a^4*b^4 - 11*a^6*b^2)^2)*(a^16 +
 b^16 + 8*a^2*b^14 + 28*a^4*b^12 + 56*a^6*b^10 + 70*a^8*b^8 + 56*a^10*b^6 +
 28*a^12*b^4 + 8*a^14*b^2))/(96*a^6*b - 32*a^4*b^3) + (2*a*b*((32*(2*a^10*b
 + 6*a^6*b^5 - 8*a^8*b^3))/(a^12 + b^12 + 6*a^2*b^10 + 15*a^4*b^8 + 20*a^6*
b^6 + 15*a^8*b^4 + 6*a^10*b^2) - ((2*a^4 - 6*a^2*b^2)*((32*(5*a^4*b^9 - 3*a
^12*b + 12*a^6*b^7 + 6*a^8*b^5 - 4*a^10*b^3))/(a^12 + b^12 + 6*a^2*b^10 + 1
5*a^4*b^8 + 20*a^6*b^6 + 15*a^8*b^4 + 6*a^10*b^2) + (((32*(3*a^4*b^11 - a^2
*b^13 - 4*a^14*b + 18*a^6*b^9 + 22*a^8*b^7 + 3*a^10*b^5 - 9*a^12*b^3))/(a^1
2 + b^12 + 6*a^2*b^10 + 15*a^4*b^8 + 20*a^6*b^6 + 15*a^8*b^4 + 6*a^10*b^2)
- (16*(2*a^4 - 6*a^2*b^2)*(3*a^16*b + 3*a^2*b^15 + 21*a^4*b^13 + 63*a^6*b^1
1 + 105*a^8*b^9 + 105*a^10*b^7 + 63*a^12*b^5 + 21*a^14*b^3))/((a^6 + b^6 +
3*a^2*b^4 + 3*a^4*b^2)*(a^12 + b^12 + 6*a^2*b^10 + 15*a^4*b^8 + 20*a^6*b^6
+ 15*a^8*b^4 + 6*a^10*b^2)))*(2*a^4 - 6*a^2*b^2))/(2*(a^6 + b^6 + 3*a^2*b^4
 + 3*a^4*b^2))))/(2*(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2)) + (a*b*((a*b*((32*
(3*a^4*b^11 - a^2*b^13 - 4*a^14*b + 18*a^6*b^9 + 22*a^8*b^7 + 3*a^10*b^5 -
9*a^12*b^3))/(a^12 + b^12 + 6*a^2*b^10 + 15*a^4*b^8 + 20*a^6*b^6 + 15*a^8*b
^4 + 6*a^10*b^2) - (16*(2*a^4 - 6*a^2*b^2)*(3*a^16*b + 3*a^2*b^15 + 21*a^4*
b^13 + 63*a^6*b^11 + 105*a^8*b^9 + 105*a^10*b^7 + 63*a^12*b^5 + 21*a^14*b^3
))/((a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2)*(a^12 + b^12 + 6*a^2*b^10 + 15*a^4*
b^8 + 20*a^6*b^6 + 15*a^8*b^4 + 6*a^10*b^2)))*(3*a^2 - b^2))/(a^6 + b^6 + 3
*a^2*b^4 + 3*a^4*b^2) - (16*a*b*(2*a^4 - 6*a^2*b^2)*(3*a^2 - b^2)*(3*a^16*b
 + 3*a^2*b^15 + 21*a^4*b^13 + 63*a^6*b^11 + 105*a^8*b^9 + 105*a^10*b^7 + 63
*a^12*b^5 + 21*a^14*b^3))/((a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2)^2*(a^12 + b^
12 + 6*a^2*b^10 + 15*a^4*b^8 + 20*a^6*b^6 + 15*a^8*b^4 + 6*a^10*b^2)))*(3*a
^2 - b^2))/(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2) - (16*a^2*b^2*(2*a^4 - 6*a^2
*b^2)*(3*a^2 - b^2)^2*(3*a^16*b + 3*a^2*b^15 + 21*a^4*b^13 + 63*a^6*b^11 +
105*a^8*b^9 + 105*a^10*b^7 + 63*a^12*b^5 + 21*a^14*b^3))/((a^6 + b^6 + 3*a^
2*b^4 + 3*a^4*b^2)^3*(a^12 + b^12 + 6*a^2*b^10 + 15*a^4*b^8 + 20*a^6*b^6 +
15*a^8*b^4 + 6*a^10*b^2)))*(10*a^6 - 7*b^6 + 68*a^2*b^4 - 59*a^4*b^2)*(a^16
 + b^16 + 8*a^2*b^14 + 28*a^4*b^12 + 56*a^6*b^10 + 70*a^8*b^8 + 56*a^10*b^6
 + 28*a^12*b^4 + 8*a^14*b^2))/((96*a^6*b - 32*a^4*b^3)*(4*a^8 + b^8 + 31*a^
2*b^6 + 15*a^4*b^4 - 11*a^6*b^2)^2))*(3*a^2 - b^2))/(a^6 + b^6 + 3*a^2*b^4
+ 3*a^4*b^2)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*sin(x)**3/(a*cos(x)+b*sin(x))**2,x)
```

```
[Out] Timed out
```

3.287 $\int \frac{\cos^2(x)\sin(x)}{(a\cos(x)+b\sin(x))^2}\,dx$

Optimal. Leaf size=109

$$\frac{2ab\sin(x)}{\left(a^2+b^2\right)^2}-\frac{\left(a^2-b^2\right)\cos(x)}{\left(a^2+b^2\right)^2}+\frac{ab^2}{\left(a^2+b^2\right)^2(a\cos(x)+b\sin(x))}-\frac{b\left(b^2-2a^2\right)\tanh^{-1}\left(\frac{b\cos(x)-a\sin(x)}{\sqrt{a^2+b^2}}\right)}{\left(a^2+b^2\right)^{5/2}}$$

```
[Out] -b*(-2*a^2+b^2)*arctanh((b*cos(x)-a*sin(x))/(a^2+b^2)^(1/2))/(a^2+b^2)^(5/2
)-(a^2-b^2)*cos(x)/(a^2+b^2)^2+2*a*b*sin(x)/(a^2+b^2)^2+a*b^2/(a^2+b^2)^2/(
a*cos(x)+b*sin(x))
```

Rubi [A] time = 0.26, antiderivative size = 151, normalized size of antiderivative = 1.39, number of steps used = 13, number of rules used = 8, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.444, Rules used = {3111, 3100, 2637, 3074, 206, 3109, 2638, 3155}

$$\frac{2ab\sin(x)}{\left(a^2+b^2\right)^2}+\frac{b^2\cos(x)}{\left(a^2+b^2\right)^2}-\frac{a^2\cos(x)}{\left(a^2+b^2\right)^2}+\frac{ab^2}{\left(a^2+b^2\right)^2(a\cos(x)+b\sin(x))}-\frac{b^3\tanh^{-1}\left(\frac{b\cos(x)-a\sin(x)}{\sqrt{a^2+b^2}}\right)}{\left(a^2+b^2\right)^{5/2}}+\frac{2a^2b\tanh^{-1}\left(\right.}{\left(a^2+\right.}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[x]^2*Sin[x])/(a*Cos[x] + b*Sin[x])^2,x]
```

```
[Out] (2*a^2*b*ArcTanh[(b*Cos[x] - a*Sin[x])/Sqrt[a^2 + b^2]])/(a^2 + b^2)^(5/2)
- (b^3*ArcTanh[(b*Cos[x] - a*Sin[x])/Sqrt[a^2 + b^2]])/(a^2 + b^2)^(5/2) -
(a^2*Cos[x])/(a^2 + b^2)^2 + (b^2*Cos[x])/(a^2 + b^2)^2 + (2*a*b*Sin[x])/(a
^2 + b^2)^2 + (a*b^2)/((a^2 + b^2)^2*(a*Cos[x] + b*Sin[x]))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3074

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x
_Symbol] :> -Dist[d^(-1), Subst[Int[1/(a^2 + b^2 - x^2), x], x, b*Cos[c + d
*x] - a*Sin[c + d*x]], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

Rule 3100

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_)/(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin
[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[(b*Cos[c + d*x]^(m - 1))/(d*(a^2 +
 b^2)*(m - 1)), x] + (Dist[a/(a^2 + b^2), Int[Cos[c + d*x]^(m - 1), x], x]
+ Dist[b^2/(a^2 + b^2), Int[Cos[c + d*x]^(m - 2)/(a*Cos[c + d*x] + b*Sin[c
+ d*x]), x], x]) /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0] && GtQ[m, 1
]
```

Rule 3109

```
Int[(cos[(c_.) + (d_.)*(x_)]^(m_.)*sin[(c_.) + (d_.)*(x_)]^(n_.))/(cos[(c_.
) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[b
/(a^2 + b^2), Int[Cos[c + d*x]^m*Sin[c + d*x]^(n - 1), x], x] + (Dist[a/(a^
2 + b^2), Int[Cos[c + d*x]^(m - 1)*Sin[c + d*x]^n, x], x] - Dist[(a*b)/(a^2
 + b^2), Int[(Cos[c + d*x]^(m - 1)*Sin[c + d*x]^(n - 1))/(a*Cos[c + d*x] +
b*Sin[c + d*x]), x], x]) /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0] &&
IGtQ[m, 0] && IGtQ[n, 0]
```

Rule 3111

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*sin[(c_.) + (d_.)*(x_)]^(n_.)*(cos[(c_.)
+ (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(p_), x_Symbol] :> Dis
t[b/(a^2 + b^2), Int[Cos[c + d*x]^m*Sin[c + d*x]^(n - 1)*(a*Cos[c + d*x] +
b*Sin[c + d*x])^(p + 1), x], x] + (Dist[a/(a^2 + b^2), Int[Cos[c + d*x]^(m
- 1)*Sin[c + d*x]^n*(a*Cos[c + d*x] + b*Sin[c + d*x])^(p + 1), x], x] - Dis
t[(a*b)/(a^2 + b^2), Int[Cos[c + d*x]^(m - 1)*Sin[c + d*x]^(n - 1)*(a*Cos[c
 + d*x] + b*Sin[c + d*x])^p, x], x]) /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 +
 b^2, 0] && IGtQ[m, 0] && IGtQ[n, 0] && ILtQ[p, 0]
```

Rule 3155

```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.))/((a_.) + cos[(d_.) + (e_.)*(x_)
]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)])^2, x_Symbol] :> Simp[(c*B + c*A*Co
s[d + e*x] + (a*B - b*A)*Sin[d + e*x])/(e*(a^2 - b^2 - c^2)*(a + b*Cos[d +
e*x] + c*Sin[d + e*x])), x] + Dist[(a*A - b*B)/(a^2 - b^2 - c^2), Int[1/(a
+ b*Cos[d + e*x] + c*Sin[d + e*x]), x], x] /; FreeQ[{a, b, c, d, e, A, B},
x] && NeQ[a^2 - b^2 - c^2, 0] && NeQ[a*A - b*B, 0]
```

Rubi steps

$$\int \frac{\cos^2(x)\sin(x)}{(a\cos(x)+b\sin(x))^2}\,dx = \frac{a\int \frac{\cos(x)\sin(x)}{a\cos(x)+b\sin(x)}\,dx}{a^2+b^2} + \frac{b\int \frac{\cos^2(x)}{a\cos(x)+b\sin(x)}\,dx}{a^2+b^2} - \frac{(ab)\int \frac{\cos(x)}{(a\cos(x)+b\sin(x))^2}\,dx}{a^2+b^2}$$

$$= \frac{b^2\cos(x)}{\left(a^2+b^2\right)^2} + \frac{ab^2}{\left(a^2+b^2\right)^2(a\cos(x)+b\sin(x))} + \frac{a^2\int \sin(x)\,dx}{\left(a^2+b^2\right)^2} + 2\frac{(ab)\int \cos(x)\,d}{\left(a^2+b^2\right)^2}$$

$$= -\frac{a^2\cos(x)}{\left(a^2+b^2\right)^2} + \frac{b^2\cos(x)}{\left(a^2+b^2\right)^2} + \frac{2ab\sin(x)}{\left(a^2+b^2\right)^2} + \frac{ab^2}{\left(a^2+b^2\right)^2(a\cos(x)+b\sin(x))} + 2\frac{\left(a^2\right.}{}$$

$$= \frac{2a^2b\tanh^{-1}\left(\frac{b\cos(x)-a\sin(x)}{\sqrt{a^2+b^2}}\right)}{\left(a^2+b^2\right)^{5/2}} - \frac{b^3\tanh^{-1}\left(\frac{b\cos(x)-a\sin(x)}{\sqrt{a^2+b^2}}\right)}{\left(a^2+b^2\right)^{5/2}} - \frac{a^2\cos(x)}{\left(a^2+b^2\right)^2} + \frac{b^2\cos(}{\left(a^2+b^2\right.}$$

Mathematica [A] time = 0.68, size = 110, normalized size = 1.01

$$\frac{2b\left(b^2-2a^2\right)\tanh^{-1}\left(\frac{a\tan\left(\frac{x}{2}\right)-b}{\sqrt{a^2+b^2}}\right)}{\left(a^2+b^2\right)^{5/2}} - \frac{a^3-b\left(a^2+b^2\right)\sin(2x)+a\left(a^2+b^2\right)\cos(2x)-5ab^2}{2\left(a^2+b^2\right)^2(a\cos(x)+b\sin(x))}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[x]^2*Sin[x])/(a*Cos[x] + b*Sin[x])^2,x]
```

```
[Out] (2*b*(-2*a^2 + b^2)*ArcTanh[(-b + a*Tan[x/2])/Sqrt[a^2 + b^2]])/(a^2 + b^2)
^(5/2) - (a^3 - 5*a*b^2 + a*(a^2 + b^2)*Cos[2*x] - b*(a^2 + b^2)*Sin[2*x])/
(2*(a^2 + b^2)^2*(a*Cos[x] + b*Sin[x]))
```

fricas [B] time = 0.66, size = 252, normalized size = 2.31

$$\frac{6\,a^3b^2+6\,ab^4-2\left(a^5+2\,a^3b^2+ab^4\right)\cos(x)^2+2\left(a^4b+2\,a^2b^3+b^5\right)\cos(x)\sin(x)-\sqrt{a^2+b^2}\left(\left(2\,a^3b-ab^3\right)\mathrm{co}\right.}{2\left(\left(a^7+3\,a^5b^2+3\,a^3b^4+ab^6\right)\cos(x)+\left(a^6b+3\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^2*sin(x)/(a*cos(x)+b*sin(x))^2,x, algorithm="fricas")
```

```
[Out] 1/2*(6*a^3*b^2 + 6*a*b^4 - 2*(a^5 + 2*a^3*b^2 + a*b^4)*cos(x)^2 + 2*(a^4*b
+ 2*a^2*b^3 + b^5)*cos(x)*sin(x) - sqrt(a^2 + b^2)*((2*a^3*b - a*b^3)*cos(x
```

```
) + (2*a^2*b^2 - b^4)*sin(x))*log(-(2*a*b*cos(x)*sin(x) + (a^2 - b^2)*cos(x
)^2 - 2*a^2 - b^2 + 2*sqrt(a^2 + b^2)*(b*cos(x) - a*sin(x)))/(2*a*b*cos(x)*
sin(x) + (a^2 - b^2)*cos(x)^2 + b^2)))/((a^7 + 3*a^5*b^2 + 3*a^3*b^4 + a*b^
6)*cos(x) + (a^6*b + 3*a^4*b^3 + 3*a^2*b^5 + b^7)*sin(x))
```

giac [A] time = 0.99, size = 204, normalized size = 1.87

$$\frac{\left(2\,a^2b-b^3\right)\log\left(\frac{\left|2\,a\tan\left(\frac{1}{2}\,x\right)-2\,b-2\,\sqrt{a^2+b^2}\right|}{\left|2\,a\tan\left(\frac{1}{2}\,x\right)-2\,b+2\,\sqrt{a^2+b^2}\right|}\right)}{\left(a^4+2\,a^2b^2+b^4\right)\sqrt{a^2+b^2}}+\frac{2\left(2\,a^2b\tan\left(\frac{1}{2}\,x\right)^3-b^3\tan\left(\frac{1}{2}\,x\right)^3-a^3\tan\left(\frac{1}{2}\,x\right)^2-4\,ab^2\tan\left(\frac{1}{2}\,x\right)^2-\right.}{\left(a\tan\left(\frac{1}{2}\,x\right)^4-2\,b\tan\left(\frac{1}{2}\,x\right)^3-2\,b\tan\left(\frac{1}{2}\,x\right)-a\right)\left(a^4+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^2*sin(x)/(a*cos(x)+b*sin(x))^2,x, algorithm="giac")
```

```
[Out] (2*a^2*b - b^3)*log(abs(2*a*tan(1/2*x) - 2*b - 2*sqrt(a^2 + b^2))/abs(2*a*t
an(1/2*x) - 2*b + 2*sqrt(a^2 + b^2)))/((a^4 + 2*a^2*b^2 + b^4)*sqrt(a^2 + b
^2)) + 2*(2*a^2*b*tan(1/2*x)^3 - b^3*tan(1/2*x)^3 - a^3*tan(1/2*x)^2 - 4*a*
b^2*tan(1/2*x)^2 - 3*b^3*tan(1/2*x) + a^3 - 2*a*b^2)/((a*tan(1/2*x)^4 - 2*b
*tan(1/2*x)^3 - 2*b*tan(1/2*x) - a)*(a^4 + 2*a^2*b^2 + b^4))
```

maple [A] time = 0.11, size = 152, normalized size = 1.39

$$\frac{4b\left(\frac{-\frac{\tan\left(\frac{x}{2}\right)b^2}{2}-\frac{ab}{2}}{\left(\tan^2\left(\frac{x}{2}\right)\right)a-2b\tan\left(\frac{x}{2}\right)-a}-\frac{\left(2a^2-b^2\right)\operatorname{arctanh}\left(\frac{2a\tan\left(\frac{x}{2}\right)-2b}{2\sqrt{a^2+b^2}}\right)}{2\sqrt{a^2+b^2}}\right)}{a^4+2a^2b^2+b^4}-\frac{4\left(-ab\tan\left(\frac{x}{2}\right)+\frac{a^2}{2}-\frac{b^2}{2}\right)}{\left(a^4+2a^2b^2+b^4\right)\left(\tan^2\left(\frac{x}{2}\right)+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)^2*sin(x)/(a*cos(x)+b*sin(x))^2,x)
```

```
[Out] 4*b/(a^4+2*a^2*b^2+b^4)*((-1/2*tan(1/2*x)*b^2-1/2*a*b)/(tan(1/2*x)^2*a-2*b*
tan(1/2*x)-a)-1/2*(2*a^2-b^2)/(a^2+b^2)^(1/2)*arctanh(1/2*(2*a*tan(1/2*x)-2
*b)/(a^2+b^2)^(1/2)))-4/(a^4+2*a^2*b^2+b^4)*(-a*b*tan(1/2*x)+1/2*a^2-1/2*b^
2)/(tan(1/2*x)^2+1)
```

maxima [B] time = 0.44, size = 264, normalized size = 2.42

$$\frac{\left(2\,a^2b-b^3\right)\log\left(\frac{b-\frac{a\sin(x)}{\cos(x)+1}+\sqrt{a^2+b^2}}{b-\frac{a\sin(x)}{\cos(x)+1}-\sqrt{a^2+b^2}}\right)}{\left(a^4+2\,a^2b^2+b^4\right)\sqrt{a^2+b^2}}-\frac{2\left(a^3-2\,ab^2-\frac{3\,b^3\sin(x)}{\cos(x)+1}-\frac{\left(a^3+4\,ab^2\right)\sin(x)^2}{(\cos(x)+1)^2}+\frac{\left(2\,a^2b-b^3\right)\sin(x)^3}{(\cos(x)+1)^3}\right)}{a^5+2\,a^3b^2+ab^4+\frac{2\left(a^4b+2\,a^2b^3+b^5\right)\sin(x)}{\cos(x)+1}+\frac{2\left(a^4b+2\,a^2b^3+b^5\right)\sin(x)^3}{(\cos(x)+1)^3}-\frac{\left(a^5+2\,a^3b^2+ab^4\right.}{(\cos(x)+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^2*sin(x)/(a*cos(x)+b*sin(x))^2,x, algorithm="maxima")
```

```
[Out] (2*a^2*b - b^3)*log((b - a*sin(x)/(cos(x) + 1) + sqrt(a^2 + b^2))/(b - a*si
n(x)/(cos(x) + 1) - sqrt(a^2 + b^2)))/((a^4 + 2*a^2*b^2 + b^4)*sqrt(a^2 + b
^2)) - 2*(a^3 - 2*a*b^2 - 3*b^3*sin(x)/(cos(x) + 1) - (a^3 + 4*a*b^2)*sin(x
)^2/(cos(x) + 1)^2 + (2*a^2*b - b^3)*sin(x)^3/(cos(x) + 1)^3)/(a^5 + 2*a^3*
b^2 + a*b^4 + 2*(a^4*b + 2*a^2*b^3 + b^5)*sin(x)/(cos(x) + 1) + 2*(a^4*b +
2*a^2*b^3 + b^5)*sin(x)^3/(cos(x) + 1)^3 - (a^5 + 2*a^3*b^2 + a*b^4)*sin(x)
^4/(cos(x) + 1)^4)
```

mupad [B] time = 1.16, size = 253, normalized size = 2.32

$$\frac{\frac{2\left(2\,a\,b^2-a^3\right)}{a^4+2\,a^2\,b^2+b^4}+\frac{6\,b^3\,\tan\left(\frac{x}{2}\right)}{a^4+2\,a^2\,b^2+b^4}+\frac{2\,\tan\left(\frac{x}{2}\right)^2\,\left(a^3+4\,a\,b^2\right)}{a^4+2\,a^2\,b^2+b^4}-\frac{2\,b\,\tan\left(\frac{x}{2}\right)^3\,\left(2\,a^2-b^2\right)}{a^4+2\,a^2\,b^2+b^4}}{-a\,\tan\left(\frac{x}{2}\right)^4+2\,b\,\tan\left(\frac{x}{2}\right)^3+2\,b\,\tan\left(\frac{x}{2}\right)+a}+\frac{b\,\mathrm{atan}\left(\frac{1\mathrm{i}\,\tan\left(\frac{x}{2}\right)\,a^5-a^4\,b\,1\mathrm{i}+2\mathrm{i}\,\tan\left(\frac{x}{2}\right)\,a^3\,b^2-a^2\,b^3\,2\mathrm{i}+1\mathrm{i}\,\mathrm{ta}}{\left(a^2+b^2\right)^{5/2}}\right.}{\left(a^2+b^2\right)^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(x)^2*sin(x))/(a*cos(x) + b*sin(x))^2,x)
```

```
[Out] ((2*(2*a*b^2 - a^3))/(a^4 + b^4 + 2*a^2*b^2) + (6*b^3*tan(x/2))/(a^4 + b^4
+ 2*a^2*b^2) + (2*tan(x/2)^2*(4*a*b^2 + a^3))/(a^4 + b^4 + 2*a^2*b^2) - (2*
b*tan(x/2)^3*(2*a^2 - b^2))/(a^4 + b^4 + 2*a^2*b^2))/(a + 2*b*tan(x/2) - a*
tan(x/2)^4 + 2*b*tan(x/2)^3) + (b*atan((a^5*tan(x/2)*1i - a^4*b*1i - b^5*1i
 - a^2*b^3*2i + a^3*b^2*tan(x/2)*2i + a*b^4*tan(x/2)*1i)/(a^2 + b^2)^(5/2))
*(2*a^2 - b^2)*2i)/(a^2 + b^2)^(5/2)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)**2*sin(x)/(a*cos(x)+b*sin(x))**2,x)
```

```
[Out] Timed out
```

3.288 $\int \frac{\cos^2(x)\sin^2(x)}{(a\cos(x)+b\sin(x))^2}\,dx$

Optimal. Leaf size=131

$$\frac{ab\sin^2(x)}{\left(a^2+b^2\right)^2}+\frac{ab^2\sin(x)}{\left(a^2+b^2\right)^2(a\cos(x)+b\sin(x))}+\frac{\left(b^2-a^2\right)\sin(x)\cos(x)}{2\left(a^2+b^2\right)^2}+\frac{2ab\left(a^2-b^2\right)\log(a\cos(x)+b\sin(x))}{\left(a^2+b^2\right)^3}+\frac{x\left(a\right.}{}$$

```
[Out] 1/2*(a^4-6*a^2*b^2+b^4)*x/(a^2+b^2)^3+2*a*b*(a^2-b^2)*ln(a*cos(x)+b*sin(x))
/(a^2+b^2)^3+1/2*(-a^2+b^2)*cos(x)*sin(x)/(a^2+b^2)^2+a*b*sin(x)^2/(a^2+b^2
)^2+a*b^2*sin(x)/(a^2+b^2)^2/(a*cos(x)+b*sin(x))
```

Rubi [A] time = 0.54, antiderivative size = 186, normalized size of antiderivative = 1.42, number of steps used = 21, number of rules used = 10, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {3111, 3109, 2635, 8, 2564, 30, 3098, 3133, 3097, 3075}

$$\frac{a^2x}{2\left(a^2+b^2\right)^2}-\frac{4a^2b^2x}{\left(a^2+b^2\right)^3}+\frac{b^2x}{2\left(a^2+b^2\right)^2}+\frac{ab\sin^2(x)}{\left(a^2+b^2\right)^2}-\frac{a^2\sin(x)\cos(x)}{2\left(a^2+b^2\right)^2}+\frac{ab^2\sin(x)}{\left(a^2+b^2\right)^2(a\cos(x)+b\sin(x))}+\frac{b^2\sin(x)\,c}{2\left(a^2+\right.}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[x]^2*Sin[x]^2)/(a*Cos[x] + b*Sin[x])^2,x]
```

```
[Out] (-4*a^2*b^2*x)/(a^2 + b^2)^3 + (a^2*x)/(2*(a^2 + b^2)^2) + (b^2*x)/(2*(a^2
+ b^2)^2) + (2*a^3*b*Log[a*Cos[x] + b*Sin[x]])/(a^2 + b^2)^3 - (2*a*b^3*Log
[a*Cos[x] + b*Sin[x]])/(a^2 + b^2)^3 - (a^2*Cos[x]*Sin[x])/(2*(a^2 + b^2)^2
) + (b^2*Cos[x]*Sin[x])/(2*(a^2 + b^2)^2) + (a*b*Sin[x]^2)/(a^2 + b^2)^2 +
(a*b^2*Sin[x])/((a^2 + b^2)^2*(a*Cos[x] + b*Sin[x]))
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2564

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] && !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 3075

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-2), x
_Symbol] :> Simp[Sin[c + d*x]/(a*d*(a*Cos[c + d*x] + b*Sin[c + d*x])), x] /
; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

Rule 3097

```
Int[sin[(c_.) + (d_.)*(x_)]/(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.
) + (d_.)*(x_)]), x_Symbol] :> Simp[(b*x)/(a^2 + b^2), x] - Dist[a/(a^2 + b
^2), Int[(b*Cos[c + d*x] - a*Sin[c + d*x])/(a*Cos[c + d*x] + b*Sin[c + d*x]
), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

Rule 3098

```
Int[cos[(c_.) + (d_.)*(x_)]/(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.
) + (d_.)*(x_)]), x_Symbol] :> Simp[(a*x)/(a^2 + b^2), x] + Dist[b/(a^2 + b
^2), Int[(b*Cos[c + d*x] - a*Sin[c + d*x])/(a*Cos[c + d*x] + b*Sin[c + d*x]
), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

Rule 3109

```
Int[(cos[(c_.) + (d_.)*(x_)]^(m_.)*sin[(c_.) + (d_.)*(x_)]^(n_.))/(cos[(c_.
) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[b
/(a^2 + b^2), Int[Cos[c + d*x]^m*Sin[c + d*x]^(n - 1), x], x] + (Dist[a/(a^
2 + b^2), Int[Cos[c + d*x]^(m - 1)*Sin[c + d*x]^n, x], x] - Dist[(a*b)/(a^2
 + b^2), Int[(Cos[c + d*x]^(m - 1)*Sin[c + d*x]^(n - 1))/(a*Cos[c + d*x] +
b*Sin[c + d*x]), x], x]) /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0] &&
IGtQ[m, 0] && IGtQ[n, 0]
```

Rule 3111

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*sin[(c_.) + (d_.)*(x_)]^(n_.)*(cos[(c_.)
+ (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(p_), x_Symbol] :> Dis
t[b/(a^2 + b^2), Int[Cos[c + d*x]^m*Sin[c + d*x]^(n - 1)*(a*Cos[c + d*x] +
b*Sin[c + d*x])^(p + 1), x], x] + (Dist[a/(a^2 + b^2), Int[Cos[c + d*x]^(m
- 1)*Sin[c + d*x]^n*(a*Cos[c + d*x] + b*Sin[c + d*x])^(p + 1), x], x] - Dis
t[(a*b)/(a^2 + b^2), Int[Cos[c + d*x]^(m - 1)*Sin[c + d*x]^(n - 1)*(a*Cos[c
```

```
+ d*x] + b*Sin[c + d*x])^p, x], x]) /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 +
b^2, 0] && IGtQ[m, 0] && IGtQ[n, 0] && ILtQ[p, 0]
```

Rule 3133

```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])
/((a_.) + cos[(d_.) + (e_.)*(x_)]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)]), x
_Symbol] :> Simp[((b*B + c*C)*x)/(b^2 + c^2), x] + Simp[((c*B - b*C)*Log[a
+ b*Cos[d + e*x] + c*Sin[d + e*x]])/(e*(b^2 + c^2)), x] /; FreeQ[{a, b, c,
d, e, A, B, C}, x] && NeQ[b^2 + c^2, 0] && EqQ[A*(b^2 + c^2) - a*(b*B + c*C
), 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\cos^2(x)\sin^2(x)}{(a\cos(x)+b\sin(x))^2}\,dx &= \frac{a\int \frac{\cos(x)\sin^2(x)}{a\cos(x)+b\sin(x)}\,dx}{a^2+b^2}+\frac{b\int \frac{\cos^2(x)\sin(x)}{a\cos(x)+b\sin(x)}\,dx}{a^2+b^2}-\frac{(ab)\int \frac{\cos(x)\sin(x)}{(a\cos(x)+b\sin(x))^2}\,dx}{a^2+b^2}\\
&= \frac{a^2\int \sin^2(x)\,dx}{\left(a^2+b^2\right)^2}+2\frac{(ab)\int \cos(x)\sin(x)\,dx}{\left(a^2+b^2\right)^2}-2\frac{\left(a^2b\right)\int \frac{\sin(x)}{a\cos(x)+b\sin(x)}\,dx}{\left(a^2+b^2\right)^2}+\frac{b^2\int \cos\cdots}{\left(a^2+\cdots\right.}\\
&= -\frac{a^2\cos(x)\sin(x)}{2\left(a^2+b^2\right)^2}+\frac{b^2\cos(x)\sin(x)}{2\left(a^2+b^2\right)^2}+\frac{ab^2\sin(x)}{\left(a^2+b^2\right)^2(a\cos(x)+b\sin(x))}-2\left(\frac{a^2b^2\cdots}{\left(a^2+\cdots\right.}\right.\\
&= \frac{a^2x}{2\left(a^2+b^2\right)^2}+\frac{b^2x}{2\left(a^2+b^2\right)^2}-2\left(\frac{a^2b^2x}{\left(a^2+b^2\right)^3}-\frac{a^3b\log(a\cos(x)+b\sin(x))}{\left(a^2+b^2\right)^3}\right)-2\left(\cdots\right.
\end{aligned}$$

Mathematica [A] time = 1.66, size = 145, normalized size = 1.11

$$\frac{\sin(x)}{8a(a\cos(x)+b\sin(x))}-\frac{2\left(a^4-b^4\right)\sin(2x)+4ab\left(a^2+b^2\right)\cos(2x)-16ab\left(a^2-b^2\right)\log(a\cos(x)+b\sin(x))-4x\cdots}{8\left(a^2+b^2\right)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[x]^2*Sin[x]^2)/(a*Cos[x] + b*Sin[x])^2,x]
```

```
[Out] Sin[x]/(8*a*(a*Cos[x] + b*Sin[x])) - (-4*(a^4 - 6*a^2*b^2 + b^4)*x + 4*a*b*
(a^2 + b^2)*Cos[2*x] - 16*a*b*(a^2 - b^2)*Log[a*Cos[x] + b*Sin[x]] + ((a^2
+ b^2)*(a^4 - 6*a^2*b^2 + b^4)*Sin[x])/(a*(a*Cos[x] + b*Sin[x])) + 2*(a^4 -
b^4)*Sin[2*x])/(8*(a^2 + b^2)^3)
```

fricas [A] time = 0.65, size = 244, normalized size = 1.86

$$-\frac{\left(a^4b+2\,a^2b^3+b^5\right)\cos(x)^3+\left(a^2b^3-b^5-\left(a^5-6\,a^3b^2+ab^4\right)x\right)\cos(x)-2\left(\left(a^4b-a^2b^3\right)\cos(x)+\left(a^3b^2-ab^4\right.\right.}{2\left(\left(a^7+3\,a^5b^2+3\,a^3b^4+\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^2*sin(x)^2/(a*cos(x)+b*sin(x))^2,x, algorithm="fricas")
```

```
[Out] -1/2*((a^4*b + 2*a^2*b^3 + b^5)*cos(x)^3 + (a^2*b^3 - b^5 - (a^5 - 6*a^3*b^
2 + a*b^4)*x)*cos(x) - 2*((a^4*b - a^2*b^3)*cos(x) + (a^3*b^2 - a*b^4)*sin(
x))*log(2*a*b*cos(x)*sin(x) + (a^2 - b^2)*cos(x)^2 + b^2) - (3*a^3*b^2 + a*
b^4 - (a^5 + 2*a^3*b^2 + a*b^4)*cos(x)^2 + (a^4*b - 6*a^2*b^3 + b^5)*x)*sin
(x))/((a^7 + 3*a^5*b^2 + 3*a^3*b^4 + a*b^6)*cos(x) + (a^6*b + 3*a^4*b^3 + 3
*a^2*b^5 + b^7)*sin(x))
```

giac [A] time = 0.15, size = 219, normalized size = 1.67

$$\frac{\left(a^4-6\,a^2b^2+b^4\right)x}{2\left(a^6+3\,a^4b^2+3\,a^2b^4+b^6\right)}-\frac{\left(a^3b-ab^3\right)\log\left(\tan(x)^2+1\right)}{a^6+3\,a^4b^2+3\,a^2b^4+b^6}+\frac{2\left(a^3b^2-ab^4\right)\log\left(|b\tan(x)+a|\right)}{a^6b+3\,a^4b^3+3\,a^2b^5+b^7}-\frac{3\,a^2b\tan(x)^2-}{2\left(a^4+2\,a^2b^2\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^2*sin(x)^2/(a*cos(x)+b*sin(x))^2,x, algorithm="giac")
```

```
[Out] 1/2*(a^4 - 6*a^2*b^2 + b^4)*x/(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6) - (a^3*b
- a*b^3)*log(tan(x)^2 + 1)/(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6) + 2*(a^3*b^2
 - a*b^4)*log(abs(b*tan(x) + a))/(a^6*b + 3*a^4*b^3 + 3*a^2*b^5 + b^7) - 1/
2*(3*a^2*b*tan(x)^2 - b^3*tan(x)^2 + a^3*tan(x) + a*b^2*tan(x) + 4*a^2*b)/(
(a^4 + 2*a^2*b^2 + b^4)*(b*tan(x)^3 + a*tan(x)^2 + b*tan(x) + a))
```

maple [B] time = 0.12, size = 260, normalized size = 1.98

$$-\frac{\tan(x)a^4}{2\left(a^2+b^2\right)^3\left(\tan^2(x)+1\right)}+\frac{\tan(x)b^4}{2\left(a^2+b^2\right)^3\left(\tan^2(x)+1\right)}-\frac{a^3b}{\left(a^2+b^2\right)^3\left(\tan^2(x)+1\right)}-\frac{b^3a}{\left(a^2+b^2\right)^3\left(\tan^2(x)+1\right)}-\frac{\ln}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)^2*sin(x)^2/(a*cos(x)+b*sin(x))^2,x)
```

```
[Out] -1/2/(a^2+b^2)^3/(tan(x)^2+1)*tan(x)*a^4+1/2/(a^2+b^2)^3/(tan(x)^2+1)*tan(x
)*b^4-1/(a^2+b^2)^3/(tan(x)^2+1)*a^3*b-1/(a^2+b^2)^3/(tan(x)^2+1)*b^3*a-1/(
a^2+b^2)^3*ln(tan(x)^2+1)*a^3*b+1/(a^2+b^2)^3*ln(tan(x)^2+1)*a*b^3-3/(a^2+b
^2)^3*arctan(tan(x))*a^2*b^2+1/2/(a^2+b^2)^3*arctan(tan(x))*b^4+1/2/(a^2+b^
```

```
2)^3*arctan(tan(x))*a^4-b*a^2/(a^2+b^2)^2/(a+b*tan(x))+2*b*a^3/(a^2+b^2)^3*
ln(a+b*tan(x))-2*b^3*a/(a^2+b^2)^3*ln(a+b*tan(x))
```

maxima [B] time = 0.46, size = 257, normalized size = 1.96

$$\frac{\left(a^4 - 6\,a^2b^2 + b^4\right)x}{2\left(a^6 + 3\,a^4b^2 + 3\,a^2b^4 + b^6\right)} + \frac{2\left(a^3b - ab^3\right)\log\left(b\tan(x) + a\right)}{a^6 + 3\,a^4b^2 + 3\,a^2b^4 + b^6} - \frac{\left(a^3b - ab^3\right)\log\left(\tan(x)^2 + 1\right)}{a^6 + 3\,a^4b^2 + 3\,a^2b^4 + b^6} - \frac{}{2\left(a^5 + 2\,a^3b^2 + ab\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^2*sin(x)^2/(a*cos(x)+b*sin(x))^2,x, algorithm="maxima")
```

```
[Out] 1/2*(a^4 - 6*a^2*b^2 + b^4)*x/(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6) + 2*(a^3*
b - a*b^3)*log(b*tan(x) + a)/(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6) - (a^3*b -
 a*b^3)*log(tan(x)^2 + 1)/(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6) - 1/2*(4*a^2*
b + (3*a^2*b - b^3)*tan(x)^2 + (a^3 + a*b^2)*tan(x))/(a^5 + 2*a^3*b^2 + a*b
^4 + (a^4*b + 2*a^2*b^3 + b^5)*tan(x)^3 + (a^5 + 2*a^3*b^2 + a*b^4)*tan(x)^
2 + (a^4*b + 2*a^2*b^3 + b^5)*tan(x))
```

mupad [B] time = 12.13, size = 6012, normalized size = 45.89

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(x)^2*sin(x)^2)/(a*cos(x) + b*sin(x))^2,x)
```

```
[Out] ((tan(x/2)^5*(3*a*b^2 - a^3))/(a^4 + b^4 + 2*a^2*b^2) + (2*b*tan(x/2)^2)/(a
^2 + b^2) - (2*b*tan(x/2)^4)/(a^2 + b^2) + (tan(x/2)*(3*a*b^2 - a^3))/(a^2
+ b^2)^2 + (2*tan(x/2)^3*(5*a*b^2 + a^3))/(a^2 + b^2)^2)/(a + 2*b*tan(x/2)
+ a*tan(x/2)^2 - a*tan(x/2)^4 - a*tan(x/2)^6 + 4*b*tan(x/2)^3 + 2*b*tan(x/2
)^5) - (log(a + 2*b*tan(x/2) - a*tan(x/2)^2)*(2*a*b^3 - 2*a^3*b))/(a^6 + b^
6 + 3*a^2*b^4 + 3*a^4*b^2) + (log(1/(cos(x) + 1))*(16*a*b^3 - 16*a^3*b))/(2
*(4*a^6 + 4*b^6 + 12*a^2*b^4 + 12*a^4*b^2)) - (atan((tan(x/2)*((((((((8*(4*
a^2*b^13 - 20*a^14*b + 48*a^4*b^11 + 132*a^6*b^9 + 128*a^8*b^7 + 12*a^10*b^
5 - 48*a^12*b^3))/(a^12 + b^12 + 6*a^2*b^10 + 15*a^4*b^8 + 20*a^6*b^6 + 15*
a^8*b^4 + 6*a^10*b^2) - (4*(16*a*b^3 - 16*a^3*b)*(12*a*b^16 + 84*a^3*b^14 +
 252*a^5*b^12 + 420*a^7*b^10 + 420*a^9*b^8 + 252*a^11*b^6 + 84*a^13*b^4 + 1
2*a^15*b^2))/((4*a^6 + 4*b^6 + 12*a^2*b^4 + 12*a^4*b^2)*(a^12 + b^12 + 6*a^
2*b^10 + 15*a^4*b^8 + 20*a^6*b^6 + 15*a^8*b^4 + 6*a^10*b^2)))*(2*a*b - a^2
+ b^2)*(2*a*b + a^2 - b^2))/(2*(a^2 + b^2)*(a^4 + b^4 + 2*a^2*b^2)) - (2*(1
6*a*b^3 - 16*a^3*b)*(2*a*b - a^2 + b^2)*(2*a*b + a^2 - b^2)*(12*a*b^16 + 84
*a^3*b^14 + 252*a^5*b^12 + 420*a^7*b^10 + 420*a^9*b^8 + 252*a^11*b^6 + 84*a
^13*b^4 + 12*a^15*b^2))/((a^2 + b^2)*(a^4 + b^4 + 2*a^2*b^2)*(4*a^6 + 4*b^6
 + 12*a^2*b^4 + 12*a^4*b^2)*(a^12 + b^12 + 6*a^2*b^10 + 15*a^4*b^8 + 20*a^6
*b^6 + 15*a^8*b^4 + 6*a^10*b^2)))*(16*a*b^3 - 16*a^3*b))/(2*(4*a^6 + 4*b^6
```

```
+ 12*a^2*b^4 + 12*a^4*b^2)) - (((8*(2*a*b^12 + a^13 - 53*a^3*b^10 - 7*a^5*b
^8 + 126*a^7*b^6 + 52*a^9*b^4 - 25*a^11*b^2))/(a^12 + b^12 + 6*a^2*b^10 + 1
5*a^4*b^8 + 20*a^6*b^6 + 15*a^8*b^4 + 6*a^10*b^2) - ((16*a*b^3 - 16*a^3*b)*
((8*(4*a^2*b^13 - 20*a^14*b + 48*a^4*b^11 + 132*a^6*b^9 + 128*a^8*b^7 + 12*
a^10*b^5 - 48*a^12*b^3))/(a^12 + b^12 + 6*a^2*b^10 + 15*a^4*b^8 + 20*a^6*b^
6 + 15*a^8*b^4 + 6*a^10*b^2) - (4*(16*a*b^3 - 16*a^3*b)*(12*a*b^16 + 84*a^3
*b^14 + 252*a^5*b^12 + 420*a^7*b^10 + 420*a^9*b^8 + 252*a^11*b^6 + 84*a^13*
b^4 + 12*a^15*b^2))/((4*a^6 + 4*b^6 + 12*a^2*b^4 + 12*a^4*b^2)*(a^12 + b^12
 + 6*a^2*b^10 + 15*a^4*b^8 + 20*a^6*b^6 + 15*a^8*b^4 + 6*a^10*b^2))))/(2*(4
*a^6 + 4*b^6 + 12*a^2*b^4 + 12*a^4*b^2)))*(2*a*b - a^2 + b^2)*(2*a*b + a^2
- b^2))/(2*(a^2 + b^2)*(a^4 + b^4 + 2*a^2*b^2)) + ((2*a*b - a^2 + b^2)^3*(2
*a*b + a^2 - b^2)^3*(12*a*b^16 + 84*a^3*b^14 + 252*a^5*b^12 + 420*a^7*b^10
+ 420*a^9*b^8 + 252*a^11*b^6 + 84*a^13*b^4 + 12*a^15*b^2))/((a^2 + b^2)^3*(
a^4 + b^4 + 2*a^2*b^2)^3*(a^12 + b^12 + 6*a^2*b^10 + 15*a^4*b^8 + 20*a^6*b^
6 + 15*a^8*b^4 + 6*a^10*b^2)))*(a^10 - b^10 + 109*a^2*b^8 - 466*a^4*b^6 + 4
66*a^6*b^4 - 109*a^8*b^2))/(a^10 + b^10 + 53*a^2*b^8 - 38*a^4*b^6 - 38*a^6*
b^4 + 53*a^8*b^2)^2 + (((8*(2*a^10*b - 4*a^2*b^9 + 10*a^4*b^7 - 30*a^6*b^5
+ 22*a^8*b^3))/(a^12 + b^12 + 6*a^2*b^10 + 15*a^4*b^8 + 20*a^6*b^6 + 15*a^8
*b^4 + 6*a^10*b^2) - ((16*a*b^3 - 16*a^3*b)*((8*(2*a*b^12 + a^13 - 53*a^3*b
^10 - 7*a^5*b^8 + 126*a^7*b^6 + 52*a^9*b^4 - 25*a^11*b^2))/(a^12 + b^12 + 6
*a^2*b^10 + 15*a^4*b^8 + 20*a^6*b^6 + 15*a^8*b^4 + 6*a^10*b^2) - ((16*a*b^3
 - 16*a^3*b)*((8*(4*a^2*b^13 - 20*a^14*b + 48*a^4*b^11 + 132*a^6*b^9 + 128*
a^8*b^7 + 12*a^10*b^5 - 48*a^12*b^3))/(a^12 + b^12 + 6*a^2*b^10 + 15*a^4*b^
8 + 20*a^6*b^6 + 15*a^8*b^4 + 6*a^10*b^2) - (4*(16*a*b^3 - 16*a^3*b)*(12*a*
b^16 + 84*a^3*b^14 + 252*a^5*b^12 + 420*a^7*b^10 + 420*a^9*b^8 + 252*a^11*b
^6 + 84*a^13*b^4 + 12*a^15*b^2))/((4*a^6 + 4*b^6 + 12*a^2*b^4 + 12*a^4*b^2)
*(a^12 + b^12 + 6*a^2*b^10 + 15*a^4*b^8 + 20*a^6*b^6 + 15*a^8*b^4 + 6*a^10*
b^2))))/(2*(4*a^6 + 4*b^6 + 12*a^2*b^4 + 12*a^4*b^2))))/(2*(4*a^6 + 4*b^6 +
 12*a^2*b^4 + 12*a^4*b^2)) - (((((8*(4*a^2*b^13 - 20*a^14*b + 48*a^4*b^11 +
 132*a^6*b^9 + 128*a^8*b^7 + 12*a^10*b^5 - 48*a^12*b^3))/(a^12 + b^12 + 6*a
^2*b^10 + 15*a^4*b^8 + 20*a^6*b^6 + 15*a^8*b^4 + 6*a^10*b^2) - (4*(16*a*b^3
 - 16*a^3*b)*(12*a*b^16 + 84*a^3*b^14 + 252*a^5*b^12 + 420*a^7*b^10 + 420*a
^9*b^8 + 252*a^11*b^6 + 84*a^13*b^4 + 12*a^15*b^2))/((4*a^6 + 4*b^6 + 12*a^
2*b^4 + 12*a^4*b^2)*(a^12 + b^12 + 6*a^2*b^10 + 15*a^4*b^8 + 20*a^6*b^6 + 1
5*a^8*b^4 + 6*a^10*b^2)))*(2*a*b - a^2 + b^2)*(2*a*b + a^2 - b^2))/(2*(a^2
+ b^2)*(a^4 + b^4 + 2*a^2*b^2)) - (2*(16*a*b^3 - 16*a^3*b)*(2*a*b - a^2 + b
^2)*(2*a*b + a^2 - b^2)*(12*a*b^16 + 84*a^3*b^14 + 252*a^5*b^12 + 420*a^7*b
^10 + 420*a^9*b^8 + 252*a^11*b^6 + 84*a^13*b^4 + 12*a^15*b^2))/((a^2 + b^2)
*(a^4 + b^4 + 2*a^2*b^2)*(4*a^6 + 4*b^6 + 12*a^2*b^4 + 12*a^4*b^2)*(a^12 +
b^12 + 6*a^2*b^10 + 15*a^4*b^8 + 20*a^6*b^6 + 15*a^8*b^4 + 6*a^10*b^2)))*(2
*a*b - a^2 + b^2)*(2*a*b + a^2 - b^2))/(2*(a^2 + b^2)*(a^4 + b^4 + 2*a^2*b^
2)) + ((16*a*b^3 - 16*a^3*b)*(2*a*b - a^2 + b^2)^2*(2*a*b + a^2 - b^2)^2*(1
2*a*b^16 + 84*a^3*b^14 + 252*a^5*b^12 + 420*a^7*b^10 + 420*a^9*b^8 + 252*a^
11*b^6 + 84*a^13*b^4 + 12*a^15*b^2))/((a^2 + b^2)^2*(a^4 + b^4 + 2*a^2*b^2)
^2*(4*a^6 + 4*b^6 + 12*a^2*b^4 + 12*a^4*b^2)*(a^12 + b^12 + 6*a^2*b^10 + 15
```

```
*a^4*b^8 + 20*a^6*b^6 + 15*a^8*b^4 + 6*a^10*b^2)))*(18*a*b^9 + 18*a^9*b - 2
80*a^3*b^7 + 556*a^5*b^5 - 280*a^7*b^3))/(a^10 + b^10 + 53*a^2*b^8 - 38*a^4
*b^6 - 38*a^6*b^4 + 53*a^8*b^2)^2)*(a^16 + b^16 + 8*a^2*b^14 + 28*a^4*b^12
+ 56*a^6*b^10 + 70*a^8*b^8 + 56*a^10*b^6 + 28*a^12*b^4 + 8*a^14*b^2))/(4*a*
b^4 + 4*a^5 - 24*a^3*b^2) + (((8*(6*a^3*b^8 - 26*a^5*b^6 + 26*a^7*b^4 - 6*a
^9*b^2))/(a^12 + b^12 + 6*a^2*b^10 + 15*a^4*b^8 + 20*a^6*b^6 + 15*a^8*b^4 +
 6*a^10*b^2) + (((8*(7*a^12*b + 7*a^2*b^11 + 3*a^4*b^9 - 26*a^6*b^7 - 26*a^
8*b^5 + 3*a^10*b^3))/(a^12 + b^12 + 6*a^2*b^10 + 15*a^4*b^8 + 20*a^6*b^6 +
15*a^8*b^4 + 6*a^10*b^2) + ((16*a*b^3 - 16*a^3*b)*((8*(2*a*b^14 - 2*a^15 -
14*a^3*b^12 - 54*a^5*b^10 - 38*a^7*b^8 + 38*a^9*b^6 + 54*a^11*b^4 + 14*a^13
*b^2))/(a^12 + b^12 + 6*a^2*b^10 + 15*a^4*b^8 + 20*a^6*b^6 + 15*a^8*b^4 + 6
*a^10*b^2) - (4*(16*a*b^3 - 16*a^3*b)*(12*a^16*b + 12*a^2*b^15 + 84*a^4*b^1
3 + 252*a^6*b^11 + 420*a^8*b^9 + 420*a^10*b^7 + 252*a^12*b^5 + 84*a^14*b^3)
)/((4*a^6 + 4*b^6 + 12*a^2*b^4 + 12*a^4*b^2)*(a^12 + b^12 + 6*a^2*b^10 + 15
*a^4*b^8 + 20*a^6*b^6 + 15*a^8*b^4 + 6*a^10*b^2))))/(2*(4*a^6 + 4*b^6 + 12*
a^2*b^4 + 12*a^4*b^2)))*(16*a*b^3 - 16*a^3*b))/(2*(4*a^6 + 4*b^6 + 12*a^2*b
^4 + 12*a^4*b^2)) - (((((8*(2*a*b^14 - 2*a^15 - 14*a^3*b^12 - 54*a^5*b^10 -
 38*a^7*b^8 + 38*a^9*b^6 + 54*a^11*b^4 + 14*a^13*b^2))/(a^12 + b^12 + 6*a^2
*b^10 + 15*a^4*b^8 + 20*a^6*b^6 + 15*a^8*b^4 + 6*a^10*b^2) - (4*(16*a*b^3 -
 16*a^3*b)*(12*a^16*b + 12*a^2*b^15 + 84*a^4*b^13 + 252*a^6*b^11 + 420*a^8*
b^9 + 420*a^10*b^7 + 252*a^12*b^5 + 84*a^14*b^3))/((4*a^6 + 4*b^6 + 12*a^2*
b^4 + 12*a^4*b^2)*(a^12 + b^12 + 6*a^2*b^10 + 15*a^4*b^8 + 20*a^6*b^6 + 15*
a^8*b^4 + 6*a^10*b^2)))*(2*a*b - a^2 + b^2)*(2*a*b + a^2 - b^2))/(2*(a^2 +
b^2)*(a^4 + b^4 + 2*a^2*b^2)) - (2*(16*a*b^3 - 16*a^3*b)*(2*a*b - a^2 + b^2
)*(2*a*b + a^2 - b^2)*(12*a^16*b + 12*a^2*b^15 + 84*a^4*b^13 + 252*a^6*b^11
 + 420*a^8*b^9 + 420*a^10*b^7 + 252*a^12*b^5 + 84*a^14*b^3))/((a^2 + b^2)*(
a^4 + b^4 + 2*a^2*b^2)*(4*a^6 + 4*b^6 + 12*a^2*b^4 + 12*a^4*b^2)*(a^12 + b^
12 + 6*a^2*b^10 + 15*a^4*b^8 + 20*a^6*b^6 + 15*a^8*b^4 + 6*a^10*b^2)))*(2*a
*b - a^2 + b^2)*(2*a*b + a^2 - b^2))/(2*(a^2 + b^2)*(a^4 + b^4 + 2*a^2*b^2)
) + ((16*a*b^3 - 16*a^3*b)*(2*a*b - a^2 + b^2)^2*(2*a*b + a^2 - b^2)^2*(12*
a^16*b + 12*a^2*b^15 + 84*a^4*b^13 + 252*a^6*b^11 + 420*a^8*b^9 + 420*a^10*
b^7 + 252*a^12*b^5 + 84*a^14*b^3))/((a^2 + b^2)^2*(a^4 + b^4 + 2*a^2*b^2)^2
*(4*a^6 + 4*b^6 + 12*a^2*b^4 + 12*a^4*b^2)*(a^12 + b^12 + 6*a^2*b^10 + 15*a
^4*b^8 + 20*a^6*b^6 + 15*a^8*b^4 + 6*a^10*b^2)))*(18*a*b^9 + 18*a^9*b - 280
*a^3*b^7 + 556*a^5*b^5 - 280*a^7*b^3)*(a^16 + b^16 + 8*a^2*b^14 + 28*a^4*b^
12 + 56*a^6*b^10 + 70*a^8*b^8 + 56*a^10*b^6 + 28*a^12*b^4 + 8*a^14*b^2))/((
4*a*b^4 + 4*a^5 - 24*a^3*b^2)*(a^10 + b^10 + 53*a^2*b^8 - 38*a^4*b^6 - 38*a
^6*b^4 + 53*a^8*b^2)^2) + (((((((8*(2*a*b^14 - 2*a^15 - 14*a^3*b^12 - 54*a^
5*b^10 - 38*a^7*b^8 + 38*a^9*b^6 + 54*a^11*b^4 + 14*a^13*b^2))/(a^12 + b^12
 + 6*a^2*b^10 + 15*a^4*b^8 + 20*a^6*b^6 + 15*a^8*b^4 + 6*a^10*b^2) - (4*(16
*a*b^3 - 16*a^3*b)*(12*a^16*b + 12*a^2*b^15 + 84*a^4*b^13 + 252*a^6*b^11 +
420*a^8*b^9 + 420*a^10*b^7 + 252*a^12*b^5 + 84*a^14*b^3))/((4*a^6 + 4*b^6 +
 12*a^2*b^4 + 12*a^4*b^2)*(a^12 + b^12 + 6*a^2*b^10 + 15*a^4*b^8 + 20*a^6*b
^6 + 15*a^8*b^4 + 6*a^10*b^2)))*(2*a*b - a^2 + b^2)*(2*a*b + a^2 - b^2))/(2
*(a^2 + b^2)*(a^4 + b^4 + 2*a^2*b^2)) - (2*(16*a*b^3 - 16*a^3*b)*(2*a*b - a
```

```
^2 + b^2)*(2*a*b + a^2 - b^2)*(12*a^16*b + 12*a^2*b^15 + 84*a^4*b^13 + 252*
a^6*b^11 + 420*a^8*b^9 + 420*a^10*b^7 + 252*a^12*b^5 + 84*a^14*b^3))/((a^2
+ b^2)*(a^4 + b^4 + 2*a^2*b^2)*(4*a^6 + 4*b^6 + 12*a^2*b^4 + 12*a^4*b^2)*(a
^12 + b^12 + 6*a^2*b^10 + 15*a^4*b^8 + 20*a^6*b^6 + 15*a^8*b^4 + 6*a^10*b^2
)))*(16*a*b^3 - 16*a^3*b))/(2*(4*a^6 + 4*b^6 + 12*a^2*b^4 + 12*a^4*b^2)) +
(((8*(7*a^12*b + 7*a^2*b^11 + 3*a^4*b^9 - 26*a^6*b^7 - 26*a^8*b^5 + 3*a^10*
b^3))/(a^12 + b^12 + 6*a^2*b^10 + 15*a^4*b^8 + 20*a^6*b^6 + 15*a^8*b^4 + 6*
a^10*b^2) + ((16*a*b^3 - 16*a^3*b)*((8*(2*a*b^14 - 2*a^15 - 14*a^3*b^12 - 5
4*a^5*b^10 - 38*a^7*b^8 + 38*a^9*b^6 + 54*a^11*b^4 + 14*a^13*b^2))/(a^12 +
b^12 + 6*a^2*b^10 + 15*a^4*b^8 + 20*a^6*b^6 + 15*a^8*b^4 + 6*a^10*b^2) - (4
*(16*a*b^3 - 16*a^3*b)*(12*a^16*b + 12*a^2*b^15 + 84*a^4*b^13 + 252*a^6*b^1
1 + 420*a^8*b^9 + 420*a^10*b^7 + 252*a^12*b^5 + 84*a^14*b^3))/((4*a^6 + 4*b
^6 + 12*a^2*b^4 + 12*a^4*b^2)*(a^12 + b^12 + 6*a^2*b^10 + 15*a^4*b^8 + 20*a
^6*b^6 + 15*a^8*b^4 + 6*a^10*b^2))))/(2*(4*a^6 + 4*b^6 + 12*a^2*b^4 + 12*a^
4*b^2)))*(2*a*b - a^2 + b^2)*(2*a*b + a^2 - b^2))/(2*(a^2 + b^2)*(a^4 + b^4
 + 2*a^2*b^2)) + ((2*a*b - a^2 + b^2)^3*(2*a*b + a^2 - b^2)^3*(12*a^16*b +
12*a^2*b^15 + 84*a^4*b^13 + 252*a^6*b^11 + 420*a^8*b^9 + 420*a^10*b^7 + 252
*a^12*b^5 + 84*a^14*b^3))/((a^2 + b^2)^3*(a^4 + b^4 + 2*a^2*b^2)^3*(a^12 +
b^12 + 6*a^2*b^10 + 15*a^4*b^8 + 20*a^6*b^6 + 15*a^8*b^4 + 6*a^10*b^2)))*(a
^10 - b^10 + 109*a^2*b^8 - 466*a^4*b^6 + 466*a^6*b^4 - 109*a^8*b^2)*(a^16 +
 b^16 + 8*a^2*b^14 + 28*a^4*b^12 + 56*a^6*b^10 + 70*a^8*b^8 + 56*a^10*b^6 +
 28*a^12*b^4 + 8*a^14*b^2))/((4*a*b^4 + 4*a^5 - 24*a^3*b^2)*(a^10 + b^10 +
53*a^2*b^8 - 38*a^4*b^6 - 38*a^6*b^4 + 53*a^8*b^2)^2))*(2*a*b - a^2 + b^2)*
(2*a*b + a^2 - b^2))/((a^2 + b^2)*(a^4 + b^4 + 2*a^2*b^2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)**2*sin(x)**2/(a*cos(x)+b*sin(x))**2,x)
```

```
[Out] Timed out
```

3.289 $\int \frac{\cos^2(x)\sin^3(x)}{(a\cos(x)+b\sin(x))^2}\,dx$

Optimal. Leaf size=172

$$\frac{2ab\sin^3(x)}{3\left(a^2+b^2\right)^2}+\frac{2ab\left(a^2-b^2\right)\sin(x)}{\left(a^2+b^2\right)^3}+\frac{\left(a^2-b^2\right)\cos^3(x)}{3\left(a^2+b^2\right)^2}-\frac{a^2\left(a^2-3b^2\right)\cos(x)}{\left(a^2+b^2\right)^3}+\frac{a^2b\left(2a^2-3b^2\right)\tanh^{-1}\left(\frac{b\cos(x)-a\sin(x)}{\sqrt{a^2+b^2}}\right)}{\left(a^2+b^2\right)^{7/2}}$$

```
[Out] a^2*b*(2*a^2-3*b^2)*arctanh((b*cos(x)-a*sin(x))/(a^2+b^2)^(1/2))/(a^2+b^2)^
(7/2)-a^2*(a^2-3*b^2)*cos(x)/(a^2+b^2)^3+1/3*(a^2-b^2)*cos(x)^3/(a^2+b^2)^2
+2*a*b*(a^2-b^2)*sin(x)/(a^2+b^2)^3+2/3*a*b*sin(x)^3/(a^2+b^2)^2+a^3*b^2/(a
^2+b^2)^3/(a*cos(x)+b*sin(x))
```

Rubi [A] time = 0.68, antiderivative size = 238, normalized size of antiderivative = 1.38, number of steps used = 33, number of rules used = 12, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.600, Rules used = {3111, 3109, 2565, 30, 2564, 2637, 2638, 3074, 206, 2633, 3099, 3154}

$$\frac{2ab\sin^3(x)}{3\left(a^2+b^2\right)^2}+\frac{2a^3b\sin(x)}{\left(a^2+b^2\right)^3}-\frac{2ab^3\sin(x)}{\left(a^2+b^2\right)^3}+\frac{a^2\cos^3(x)}{3\left(a^2+b^2\right)^2}-\frac{b^2\cos^3(x)}{3\left(a^2+b^2\right)^2}-\frac{a^2\cos(x)}{\left(a^2+b^2\right)^2}+\frac{4a^2b^2\cos(x)}{\left(a^2+b^2\right)^3}+\frac{a^3b^2}{\left(a^2+b^2\right)^3(a\cos(x)+b\sin(x))}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[x]^2*Sin[x]^3)/(a*Cos[x] + b*Sin[x])^2,x]
```

```
[Out] (2*a^4*b*ArcTanh[(b*Cos[x] - a*Sin[x])/Sqrt[a^2 + b^2]])/(a^2 + b^2)^(7/2)
- (3*a^2*b^3*ArcTanh[(b*Cos[x] - a*Sin[x])/Sqrt[a^2 + b^2]])/(a^2 + b^2)^(7
/2) + (4*a^2*b^2*Cos[x])/(a^2 + b^2)^3 - (a^2*Cos[x])/(a^2 + b^2)^2 + (a^2*
Cos[x]^3)/(3*(a^2 + b^2)^2) - (b^2*Cos[x]^3)/(3*(a^2 + b^2)^2) + (2*a^3*b*S
in[x])/(a^2 + b^2)^3 - (2*a*b^3*Sin[x])/(a^2 + b^2)^3 + (2*a*b*Sin[x]^3)/(3
*(a^2 + b^2)^2) + (a^3*b^2)/((a^2 + b^2)^3*(a*Cos[x] + b*Sin[x]))
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2564

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 2565

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> -Dist[(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3074

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x
_Symbol] :> -Dist[d^(-1), Subst[Int[1/(a^2 + b^2 - x^2), x], x, b*Cos[c + d
*x] - a*Sin[c + d*x]], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

Rule 3099

```
Int[sin[(c_.) + (d_.)*(x_)]^(m_)/(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin
[(c_.) + (d_.)*(x_)]), x_Symbol] :> -Simp[(a*Sin[c + d*x]^(m - 1))/(d*(a^2
+ b^2)*(m - 1)), x] + (Dist[a^2/(a^2 + b^2), Int[Sin[c + d*x]^(m - 2)/(a*Co
s[c + d*x] + b*Sin[c + d*x]), x], x] + Dist[b/(a^2 + b^2), Int[Sin[c + d*x]
^(m - 1), x], x]) /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0] && GtQ[m,
1]
```

Rule 3109

```
Int[(cos[(c_.) + (d_.)*(x_)]^(m_.)*sin[(c_.) + (d_.)*(x_)]^(n_.))/(cos[(c_.
) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[b
/(a^2 + b^2), Int[Cos[c + d*x]^m*Sin[c + d*x]^(n - 1), x], x] + (Dist[a/(a^
2 + b^2), Int[Cos[c + d*x]^(m - 1)*Sin[c + d*x]^n, x], x] - Dist[(a*b)/(a^2
 + b^2), Int[(Cos[c + d*x]^(m - 1)*Sin[c + d*x]^(n - 1))/(a*Cos[c + d*x] +
b*Sin[c + d*x]), x], x]) /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0] &&
IGtQ[m, 0] && IGtQ[n, 0]
```

Rule 3111

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*sin[(c_.) + (d_.)*(x_)]^(n_.)*(cos[(c_.)
+ (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(p_), x_Symbol] :> Dis
t[b/(a^2 + b^2), Int[Cos[c + d*x]^m*Sin[c + d*x]^(n - 1)*(a*Cos[c + d*x] +
b*Sin[c + d*x])^(p + 1), x], x] + (Dist[a/(a^2 + b^2), Int[Cos[c + d*x]^(m
- 1)*Sin[c + d*x]^n*(a*Cos[c + d*x] + b*Sin[c + d*x])^(p + 1), x], x] - Dis
t[(a*b)/(a^2 + b^2), Int[Cos[c + d*x]^(m - 1)*Sin[c + d*x]^(n - 1)*(a*Cos[c
 + d*x] + b*Sin[c + d*x])^p, x], x]) /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 +
 b^2, 0] && IGtQ[m, 0] && IGtQ[n, 0] && ILtQ[p, 0]
```

Rule 3154

```
Int[((A_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])/((a_.) + cos[(d_.) + (e_.)*(x_)
]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)])^2, x_Symbol] :> -Simp[(b*C + (a*C
- c*A)*Cos[d + e*x] + b*A*Sin[d + e*x])/(e*(a^2 - b^2 - c^2)*(a + b*Cos[d +
 e*x] + c*Sin[d + e*x])), x] + Dist[(a*A - c*C)/(a^2 - b^2 - c^2), Int[1/(a
 + b*Cos[d + e*x] + c*Sin[d + e*x]), x], x] /; FreeQ[{a, b, c, d, e, A, C},
 x] && NeQ[a^2 - b^2 - c^2, 0] && NeQ[a*A - c*C, 0]
```

Rubi steps

$$\int \frac{\cos^2(x)\sin^3(x)}{(a\cos(x)+b\sin(x))^2}\,dx = \frac{a\int \frac{\cos(x)\sin^3(x)}{a\cos(x)+b\sin(x)}\,dx}{a^2+b^2} + \frac{b\int \frac{\cos^2(x)\sin^2(x)}{a\cos(x)+b\sin(x)}\,dx}{a^2+b^2} - \frac{(ab)\int \frac{\cos(x)\sin^2(x)}{(a\cos(x)+b\sin(x))^2}\,dx}{a^2+b^2}$$

$$= \frac{a^2\int \sin^3(x)\,dx}{\left(a^2+b^2\right)^2} + 2\frac{(ab)\int \cos(x)\sin^2(x)\,dx}{\left(a^2+b^2\right)^2} - 2\frac{\left(a^2b\right)\int \frac{\sin^2(x)}{a\cos(x)+b\sin(x)}\,dx}{\left(a^2+b^2\right)^2} + \frac{b^2\int c}{}$$

$$= \frac{a^3b^2}{\left(a^2+b^2\right)^3(a\cos(x)+b\sin(x))} - 2\left(-\frac{a^3b\sin(x)}{\left(a^2+b^2\right)^3} + \frac{\left(a^4b\right)\int \frac{1}{a\cos(x)+b\sin(x)}\,dx}{\left(a^2+b^2\right)^3} + \frac{(a}{}\right.$$

$$= -\frac{a^2\cos(x)}{\left(a^2+b^2\right)^2} + \frac{a^2\cos^3(x)}{3\left(a^2+b^2\right)^2} - \frac{b^2\cos^3(x)}{3\left(a^2+b^2\right)^2} + \frac{2ab\sin^3(x)}{3\left(a^2+b^2\right)^2} + \frac{a^3b^2}{\left(a^2+b^2\right)^3(a\cos(x)}$$

$$= -\frac{a^2b^3\tanh^{-1}\left(\frac{b\cos(x)-a\sin(x)}{\sqrt{a^2+b^2}}\right)}{\left(a^2+b^2\right)^{7/2}} - \frac{a^2\cos(x)}{\left(a^2+b^2\right)^2} + \frac{a^2\cos^3(x)}{3\left(a^2+b^2\right)^2} - \frac{b^2\cos^3(x)}{3\left(a^2+b^2\right)^2} + \frac{2ab\,s}{3\left(a^2\right.}$$

Mathematica [A] time = 1.21, size = 200, normalized size = 1.16

$$\frac{-9a^5+18a^4b\sin(2x)-a^4b\sin(4x)+90a^3b^2+16a^2b^3\sin(2x)-2a^2b^3\sin(4x)+a\left(a^2+b^2\right)^2\cos(4x)+\left(-8a^5+\right.}{24\left(a^2+b^2\right)^3(a\cos(x)+b\sin(x))}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[x]^2*Sin[x]^3)/(a*Cos[x] + b*Sin[x])^2,x]
```

```
[Out] (-2*a^2*b*(2*a^2 - 3*b^2)*ArcTanh[(-b + a*Tan[x/2])/Sqrt[a^2 + b^2]])/(a^2
+ b^2)^(7/2) + (-9*a^5 + 90*a^3*b^2 - 21*a*b^4 + (-8*a^5 + 4*a^3*b^2 + 12*a
*b^4)*Cos[2*x] + a*(a^2 + b^2)^2*Cos[4*x] + 18*a^4*b*Sin[2*x] + 16*a^2*b^3*
Sin[2*x] - 2*b^5*Sin[2*x] - a^4*b*Sin[4*x] - 2*a^2*b^3*Sin[4*x] - b^5*Sin[4
*x])/(24*(a^2 + b^2)^3*(a*Cos[x] + b*Sin[x]))
```

fricas [B] time = 0.60, size = 360, normalized size = 2.09

$$\frac{22\,a^5b^2+14\,a^3b^4-8\,ab^6+2\left(a^7+3\,a^5b^2+3\,a^3b^4+ab^6\right)\cos(x)^4-2\left(3\,a^7+4\,a^5b^2-a^3b^4-2\,ab^6\right)\cos(x)^2-3}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^2*sin(x)^3/(a*cos(x)+b*sin(x))^2,x, algorithm="fricas")
```

```
[Out] 1/6*(22*a^5*b^2 + 14*a^3*b^4 - 8*a*b^6 + 2*(a^7 + 3*a^5*b^2 + 3*a^3*b^4 + a
*b^6)*cos(x)^4 - 2*(3*a^7 + 4*a^5*b^2 - a^3*b^4 - 2*a*b^6)*cos(x)^2 - 3*sqr
t(a^2 + b^2)*((2*a^5*b - 3*a^3*b^3)*cos(x) + (2*a^4*b^2 - 3*a^2*b^4)*sin(x)
)*log(-(2*a*b*cos(x)*sin(x) + (a^2 - b^2)*cos(x)^2 - 2*a^2 - b^2 + 2*sqrt(a
^2 + b^2)*(b*cos(x) - a*sin(x)))/(2*a*b*cos(x)*sin(x) + (a^2 - b^2)*cos(x)^
2 + b^2)) - 2*((a^6*b + 3*a^4*b^3 + 3*a^2*b^5 + b^7)*cos(x)^3 - 5*(a^6*b +
2*a^4*b^3 + a^2*b^5)*cos(x))*sin(x))/((a^9 + 4*a^7*b^2 + 6*a^5*b^4 + 4*a^3*
b^6 + a*b^8)*cos(x) + (a^8*b + 4*a^6*b^3 + 6*a^4*b^5 + 4*a^2*b^7 + b^9)*sin
(x))
```

giac [B] time = 0.25, size = 342, normalized size = 1.99

$$\frac{\left(2\,a^4b-3\,a^2b^3\right)\log\left(\frac{\left|2\,a\tan\left(\frac{1}{2}\,x\right)-2\,b-2\,\sqrt{a^2+b^2}\right|}{\left|2\,a\tan\left(\frac{1}{2}\,x\right)-2\,b+2\,\sqrt{a^2+b^2}\right|}\right)}{\left(a^6+3\,a^4b^2+3\,a^2b^4+b^6\right)\sqrt{a^2+b^2}}-\frac{2\left(a^2b^3\tan\left(\frac{1}{2}\,x\right)+a^3b^2\right)}{\left(a^6+3\,a^4b^2+3\,a^2b^4+b^6\right)\left(a\tan\left(\frac{1}{2}\,x\right)^2-2\,b\tan\left(\frac{1}{2}\,x\right)-a\right)}+\frac{2\left(6\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^2*sin(x)^3/(a*cos(x)+b*sin(x))^2,x, algorithm="giac")
```

```
[Out] (2*a^4*b - 3*a^2*b^3)*log(abs(2*a*tan(1/2*x) - 2*b - 2*sqrt(a^2 + b^2))/abs
(2*a*tan(1/2*x) - 2*b + 2*sqrt(a^2 + b^2)))/((a^6 + 3*a^4*b^2 + 3*a^2*b^4 +
 b^6)*sqrt(a^2 + b^2)) - 2*(a^2*b^3*tan(1/2*x) + a^3*b^2)/((a^6 + 3*a^4*b^2
 + 3*a^2*b^4 + b^6)*(a*tan(1/2*x)^2 - 2*b*tan(1/2*x) - a)) + 2/3*(6*a^3*b*t
an(1/2*x)^5 - 6*a*b^3*tan(1/2*x)^5 + 9*a^2*b^2*tan(1/2*x)^4 - 3*b^4*tan(1/2
*x)^4 + 20*a^3*b*tan(1/2*x)^3 - 4*a*b^3*tan(1/2*x)^3 - 6*a^4*tan(1/2*x)^2 +
 18*a^2*b^2*tan(1/2*x)^2 + 6*a^3*b*tan(1/2*x) - 6*a*b^3*tan(1/2*x) - 2*a^4
+ 9*a^2*b^2 - b^4)/((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*(tan(1/2*x)^2 + 1)^
3)
```

maple [A] time = 0.14, size = 269, normalized size = 1.56

$$\frac{4a^2b\left(\frac{-\frac{\tan\left(\frac{x}{2}\right)b^2}{2}-\frac{ab}{2}}{\left(\tan^2\left(\frac{x}{2}\right)\right)a-2b\tan\left(\frac{x}{2}\right)-a}-\frac{\left(2a^2-3b^2\right)\operatorname{arctanh}\left(\frac{2a\tan\left(\frac{x}{2}\right)-2b}{2\sqrt{a^2+b^2}}\right)}{2\sqrt{a^2+b^2}}\right)}{\left(a^4+2a^2b^2+b^4\right)\left(a^2+b^2\right)}-\frac{4\left(\left(-a^3b+b^3a\right)\left(\tan^5\left(\frac{x}{2}\right)\right)+\left(-\frac{3}{2}a^2b^2+\frac{1}{2}b^4\right)\left(\tan^4\left(\frac{x}{2}\right)\right)\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)^2*sin(x)^3/(a*cos(x)+b*sin(x))^2,x)
```

```
[Out] 4*a^2*b/(a^4+2*a^2*b^2+b^4)/(a^2+b^2)*((-1/2*tan(1/2*x)*b^2-1/2*a*b)/(tan(1
/2*x)^2*a-2*b*tan(1/2*x)-a)-1/2*(2*a^2-3*b^2)/(a^2+b^2)^(1/2)*arctanh(1/2*(
2*a*tan(1/2*x)-2*b)/(a^2+b^2)^(1/2)))-4/(a^2+b^2)/(a^4+2*a^2*b^2+b^4)*((-a^
3*b+a*b^3)*tan(1/2*x)^5+(-3/2*a^2*b^2+1/2*b^4)*tan(1/2*x)^4+(-10/3*a^3*b+2/
3*b^3*a)*tan(1/2*x)^3+(a^4-3*a^2*b^2)*tan(1/2*x)^2+(-a^3*b+a*b^3)*tan(1/2*x
)+1/3*a^4-3/2*a^2*b^2+1/6*b^4)/(tan(1/2*x)^2+1)^3
```

maxima [B] time = 0.48, size = 611, normalized size = 3.55

$$\frac{\left(2\,a^2b-3\,b^3\right)a^2\log\left(\frac{b-\frac{a\sin(x)}{\cos(x)+1}+\sqrt{a^2+b^2}}{b-\frac{a\sin(x)}{\cos(x)+1}-\sqrt{a^2+b^2}}\right)}{\left(a^6+3\,a^4b^2+3\,a^2b^4+b^6\right)\sqrt{a^2+b^2}}-\frac{2\left(2\,a^5-12\,a^3b^2+ab^4-\frac{\left(2\,a^4b+15\,a^2b^3-2\,b^5\right)\sin(x)}{\cos(x)+1}+\frac{(4}{}\right.}{3\left(a^7+3\,a^5b^2+3\,a^3b^4+ab^6+\frac{2\left(a^6b+3\,a^4b^3+3\,a^2b^5+b^7\right)\sin(x)}{\cos(x)+1}+\frac{2\left(a^7+3\,a^5b^2+3\right.}{(\cos}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^2*sin(x)^3/(a*cos(x)+b*sin(x))^2,x, algorithm="maxima")
```

```
[Out] (2*a^2*b - 3*b^3)*a^2*log((b - a*sin(x)/(cos(x) + 1) + sqrt(a^2 + b^2))/(b
- a*sin(x)/(cos(x) + 1) - sqrt(a^2 + b^2)))/((a^6 + 3*a^4*b^2 + 3*a^2*b^4 +
 b^6)*sqrt(a^2 + b^2)) - 2/3*(2*a^5 - 12*a^3*b^2 + a*b^4 - (2*a^4*b + 15*a^
2*b^3 - 2*b^5)*sin(x)/(cos(x) + 1) + (4*a^5 - 30*a^3*b^2 + 11*a*b^4)*sin(x)
^2/(cos(x) + 1)^2 - (2*a^4*b + 47*a^2*b^3)*sin(x)^3/(cos(x) + 1)^3 - (6*a^5
 + 40*a^3*b^2 - 11*a*b^4)*sin(x)^4/(cos(x) + 1)^4 + (14*a^4*b - 25*a^2*b^3
+ 6*b^5)*sin(x)^5/(cos(x) + 1)^5 - 3*(2*a^3*b^2 - 3*a*b^4)*sin(x)^6/(cos(x)
 + 1)^6 + 3*(2*a^4*b - 3*a^2*b^3)*sin(x)^7/(cos(x) + 1)^7)/(a^7 + 3*a^5*b^2
 + 3*a^3*b^4 + a*b^6 + 2*(a^6*b + 3*a^4*b^3 + 3*a^2*b^5 + b^7)*sin(x)/(cos(
x) + 1) + 2*(a^7 + 3*a^5*b^2 + 3*a^3*b^4 + a*b^6)*sin(x)^2/(cos(x) + 1)^2 +
 6*(a^6*b + 3*a^4*b^3 + 3*a^2*b^5 + b^7)*sin(x)^3/(cos(x) + 1)^3 + 6*(a^6*b
 + 3*a^4*b^3 + 3*a^2*b^5 + b^7)*sin(x)^5/(cos(x) + 1)^5 - 2*(a^7 + 3*a^5*b^
2 + 3*a^3*b^4 + a*b^6)*sin(x)^6/(cos(x) + 1)^6 + 2*(a^6*b + 3*a^4*b^3 + 3*a
^2*b^5 + b^7)*sin(x)^7/(cos(x) + 1)^7 - (a^7 + 3*a^5*b^2 + 3*a^3*b^4 + a*b^
6)*sin(x)^8/(cos(x) + 1)^8)
```

mupad [B] time = 2.92, size = 594, normalized size = 3.45

$$-\frac{\frac{2\tan\left(\frac{x}{2}\right)^6\left(3\,a\,b^4-2\,a^3\,b^2\right)}{a^6+3\,a^4\,b^2+3\,a^2\,b^4+b^6}-\frac{2\tan\left(\frac{x}{2}\right)^4\left(6\,a^5+40\,a^3\,b^2-11\,a\,b^4\right)}{3\left(a^6+3\,a^4\,b^2+3\,a^2\,b^4+b^6\right)}-\frac{2\tan\left(\frac{x}{2}\right)^3\left(2\,a^4\,b+47\,a^2\,b^3\right)}{3\left(a^2+b^2\right)\left(a^4+2\,a^2\,b^2+b^4\right)}+\frac{2\,a\left(2\,a^4-12\,a^2\,b^2+b^4\right)}{3\left(a^2+b^2\right)\left(a^4+2\,a^2\,b^2+b^4\right)}+\frac{2\,a\tan\left(\frac{x}{2}\right)^2\left(4\,a^4\right.}{3\left(a^2+b^2\right)\left(a^4\right.}}{-a\tan\left(\frac{x}{2}\right)^8+2\,b\tan\left(\frac{x}{2}\right)^7-2\,a\tan\left(\frac{x}{2}\right)^6+6\,b\tan\left(\frac{x}{2}\right)^5+6\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(x)^2*sin(x)^3)/(a*cos(x) + b*sin(x))^2,x)
```

```
[Out] (a^2*b*atan((a^7*tan(x/2)*1i - a^6*b*1i - b^7*1i - a^2*b^5*3i - a^4*b^3*3i
+ a^3*b^4*tan(x/2)*3i + a^5*b^2*tan(x/2)*3i + a*b^6*tan(x/2)*1i)/(a^2 + b^2
)^(7/2))*(2*a^2 - 3*b^2)*2i)/(a^2 + b^2)^(7/2) - ((2*tan(x/2)^6*(3*a*b^4 -
2*a^3*b^2))/(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2) - (2*tan(x/2)^4*(6*a^5 - 11
*a*b^4 + 40*a^3*b^2))/(3*(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2)) - (2*tan(x/2)
^3*(2*a^4*b + 47*a^2*b^3))/(3*(a^2 + b^2)*(a^4 + b^4 + 2*a^2*b^2)) + (2*a*(
2*a^4 + b^4 - 12*a^2*b^2))/(3*(a^2 + b^2)*(a^4 + b^4 + 2*a^2*b^2)) + (2*a*t
an(x/2)^2*(4*a^4 + 11*b^4 - 30*a^2*b^2))/(3*(a^2 + b^2)*(a^4 + b^4 + 2*a^2*
b^2)) + (2*b*tan(x/2)^5*(14*a^4 + 6*b^4 - 25*a^2*b^2))/(3*(a^2 + b^2)*(a^4
+ b^4 + 2*a^2*b^2)) + (2*a^2*b*tan(x/2)^7*(2*a^2 - 3*b^2))/(a^6 + b^6 + 3*a
^2*b^4 + 3*a^4*b^2) - (2*b*tan(x/2)*(2*a^4 - 2*b^4 + 15*a^2*b^2))/(3*(a^2 +
 b^2)*(a^4 + b^4 + 2*a^2*b^2)))/(a + 2*b*tan(x/2) + 2*a*tan(x/2)^2 - 2*a*ta
n(x/2)^6 - a*tan(x/2)^8 + 6*b*tan(x/2)^3 + 6*b*tan(x/2)^5 + 2*b*tan(x/2)^7)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)**2*sin(x)**3/(a*cos(x)+b*sin(x))**2,x)
```

```
[Out] Timed out
```

3.290 $\int \frac{\cos^3(x)\sin(x)}{(a\cos(x)+b\sin(x))^2}\,dx$

Optimal. Leaf size=128

$$-\frac{abx\left(a^2-3b^2\right)}{\left(a^2+b^2\right)^3}+\frac{\left(a^2-b^2\right)\sin^2(x)}{2\left(a^2+b^2\right)^2}+\frac{ab^2\cos(x)}{\left(a^2+b^2\right)^2(a\cos(x)+b\sin(x))}+\frac{ab\sin(x)\cos(x)}{\left(a^2+b^2\right)^2}-\frac{b^2\left(3a^2-b^2\right)\log(a\cos(\ldots}{\left(a^2+b^2\right)^3}$$

```
[Out] -a*b*(a^2-3*b^2)*x/(a^2+b^2)^3-b^2*(3*a^2-b^2)*ln(a*cos(x)+b*sin(x))/(a^2+b
^2)^3+a*b*cos(x)*sin(x)/(a^2+b^2)^2+1/2*(a^2-b^2)*sin(x)^2/(a^2+b^2)^2+a*b^
2*cos(x)/(a^2+b^2)^2/(a*cos(x)+b*sin(x))
```

Rubi [A] time = 0.41, antiderivative size = 196, normalized size of antiderivative = 1.53, number of steps used = 17, number of rules used = 13, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.722, Rules used = {3111, 3100, 2635, 8, 3098, 3133, 3109, 2564, 30, 3086, 3483, 3531, 3530}

$$\frac{ab^3x}{\left(a^2+b^2\right)^3}+\frac{abx}{\left(a^2+b^2\right)^2}-\frac{a^3bx}{\left(a^2+b^2\right)^3}-\frac{abx\left(a^2-b^2\right)}{\left(a^2+b^2\right)^3}+\frac{a^2\sin^2(x)}{2\left(a^2+b^2\right)^2}+\frac{b^2\cos^2(x)}{2\left(a^2+b^2\right)^2}+\frac{ab^2}{\left(a^2+b^2\right)^2(a+b\tan(x))}+\frac{ab\sin\ldots}{(a\ldots}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[x]^3*Sin[x])/(a*Cos[x] + b*Sin[x])^2,x]
```

```
[Out] -((a^3*b*x)/(a^2 + b^2)^3) + (a*b^3*x)/(a^2 + b^2)^3 - (a*b*(a^2 - b^2)*x)/
(a^2 + b^2)^3 + (a*b*x)/(a^2 + b^2)^2 + (b^2*Cos[x]^2)/(2*(a^2 + b^2)^2) -
(3*a^2*b^2*Log[a*Cos[x] + b*Sin[x]])/(a^2 + b^2)^3 + (b^4*Log[a*Cos[x] + b*
Sin[x]])/(a^2 + b^2)^3 + (a*b*Cos[x]*Sin[x])/(a^2 + b^2)^2 + (a^2*Sin[x]^2)
/(2*(a^2 + b^2)^2) + (a*b^2)/((a^2 + b^2)^2*(a + b*Tan[x]))
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2564

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
```

```
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 3086

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin
[(c_.) + (d_.)*(x_)])^(n_.), x_Symbol] :> Int[(a + b*Tan[c + d*x])^n, x] /;
 FreeQ[{a, b, c, d}, x] && EqQ[m + n, 0] && IntegerQ[n] && NeQ[a^2 + b^2, 0
]
```

Rule 3098

```
Int[cos[(c_.) + (d_.)*(x_)]/(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.
) + (d_.)*(x_)]), x_Symbol] :> Simp[(a*x)/(a^2 + b^2), x] + Dist[b/(a^2 + b
^2), Int[(b*Cos[c + d*x] - a*Sin[c + d*x])/(a*Cos[c + d*x] + b*Sin[c + d*x]
), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

Rule 3100

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_)/(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin
[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[(b*Cos[c + d*x]^(m - 1))/(d*(a^2 +
 b^2)*(m - 1)), x] + (Dist[a/(a^2 + b^2), Int[Cos[c + d*x]^(m - 1), x], x]
+ Dist[b^2/(a^2 + b^2), Int[Cos[c + d*x]^(m - 2)/(a*Cos[c + d*x] + b*Sin[c
+ d*x]), x], x]) /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0] && GtQ[m, 1
]
```

Rule 3109

```
Int[(cos[(c_.) + (d_.)*(x_)]^(m_.)*sin[(c_.) + (d_.)*(x_)]^(n_.))/(cos[(c_.
) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[b
/(a^2 + b^2), Int[Cos[c + d*x]^m*Sin[c + d*x]^(n - 1), x], x] + (Dist[a/(a^
2 + b^2), Int[Cos[c + d*x]^(m - 1)*Sin[c + d*x]^n, x], x] - Dist[(a*b)/(a^2
 + b^2), Int[(Cos[c + d*x]^(m - 1)*Sin[c + d*x]^(n - 1))/(a*Cos[c + d*x] +
b*Sin[c + d*x]), x], x]) /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0] &&
IGtQ[m, 0] && IGtQ[n, 0]
```

Rule 3111

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*sin[(c_.) + (d_.)*(x_)]^(n_.)*(cos[(c_.)
+ (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(p_), x_Symbol] :> Dis
t[b/(a^2 + b^2), Int[Cos[c + d*x]^m*Sin[c + d*x]^(n - 1)*(a*Cos[c + d*x] +
b*Sin[c + d*x])^(p + 1), x], x] + (Dist[a/(a^2 + b^2), Int[Cos[c + d*x]^(m
- 1)*Sin[c + d*x]^n*(a*Cos[c + d*x] + b*Sin[c + d*x])^(p + 1), x], x] - Dis
t[(a*b)/(a^2 + b^2), Int[Cos[c + d*x]^(m - 1)*Sin[c + d*x]^(n - 1)*(a*Cos[c
 + d*x] + b*Sin[c + d*x])^p, x], x]) /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 +
 b^2, 0] && IGtQ[m, 0] && IGtQ[n, 0] && ILtQ[p, 0]
```

Rule 3133

```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])
/((a_.) + cos[(d_.) + (e_.)*(x_)]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)]), x
_Symbol] :> Simp[((b*B + c*C)*x)/(b^2 + c^2), x] + Simp[((c*B - b*C)*Log[a
+ b*Cos[d + e*x] + c*Sin[d + e*x]])/(e*(b^2 + c^2)), x] /; FreeQ[{a, b, c,
d, e, A, B, C}, x] && NeQ[b^2 + c^2, 0] && EqQ[A*(b^2 + c^2) - a*(b*B + c*C
), 0]
```

Rule 3483

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(a +
b*Tan[c + d*x])^(n + 1))/(d*(n + 1)*(a^2 + b^2)), x] + Dist[1/(a^2 + b^2),
Int[(a - b*Tan[c + d*x])*(a + b*Tan[c + d*x])^(n + 1), x], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 + b^2, 0] && LtQ[n, -1]
```

Rule 3530

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_) + (b_.)*tan[(e_.) + (f_.)*
(x_)]), x_Symbol] :> Simp[(c*Log[RemoveContent[a*Cos[e + f*x] + b*Sin[e + f
*x], x]])/(b*f), x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
 NeQ[a^2 + b^2, 0] && EqQ[a*c + b*d, 0]
```

Rule 3531

```
Int[((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_.) + (b_.)*tan[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[((a*c + b*d)*x)/(a^2 + b^2), x] + Dist[(b*c - a
*d)/(a^2 + b^2), Int[(b - a*Tan[e + f*x])/(a + b*Tan[e + f*x]), x], x] /; F
reeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && Ne
Q[a*c + b*d, 0]
```

Rubi steps

$$\int \frac{\cos^3(x)\sin(x)}{(a\cos(x)+b\sin(x))^2}\,dx = \frac{a\int \frac{\cos^2(x)\sin(x)}{a\cos(x)+b\sin(x)}\,dx}{a^2+b^2} + \frac{b\int \frac{\cos^3(x)}{a\cos(x)+b\sin(x)}\,dx}{a^2+b^2} - \frac{(ab)\int \frac{\cos^2(x)}{(a\cos(x)+b\sin(x))^2}\,dx}{a^2+b^2}$$

$$= \frac{b^2\cos^2(x)}{2\left(a^2+b^2\right)^2} + \frac{a^2\int \cos(x)\sin(x)\,dx}{\left(a^2+b^2\right)^2} + 2\frac{(ab)\int \cos^2(x)\,dx}{\left(a^2+b^2\right)^2} - \frac{\left(a^2b\right)\int \frac{\cos(x)}{a\cos(x)+b\sin(x)}}{\left(a^2+b^2\right)^2}$$

$$= -\frac{a^3bx}{\left(a^2+b^2\right)^3} + \frac{ab^3x}{\left(a^2+b^2\right)^3} + \frac{b^2\cos^2(x)}{2\left(a^2+b^2\right)^2} + \frac{ab^2}{\left(a^2+b^2\right)^2(a+b\tan(x))} - \frac{\left(a^2b^2\right)\int \frac{b\,\mathrm{c}}{a\,\mathrm{c}}}{\left(a^2\right.}$$

$$= -\frac{a^3bx}{\left(a^2+b^2\right)^3} + \frac{ab^3x}{\left(a^2+b^2\right)^3} - \frac{ab\left(a^2-b^2\right)x}{\left(a^2+b^2\right)^3} + \frac{b^2\cos^2(x)}{2\left(a^2+b^2\right)^2} - \frac{a^2b^2\log(a\cos(x)+b\,\mathrm{s}}{\left(a^2+b^2\right)^3}$$

$$= -\frac{a^3bx}{\left(a^2+b^2\right)^3} + \frac{ab^3x}{\left(a^2+b^2\right)^3} - \frac{ab\left(a^2-b^2\right)x}{\left(a^2+b^2\right)^3} + \frac{b^2\cos^2(x)}{2\left(a^2+b^2\right)^2} - \frac{3a^2b^2\log(a\cos(x)+b}{\left(a^2+b^2\right)^3}$$

Mathematica [C] time = 1.33, size = 221, normalized size = 1.73

$$\frac{-4ib^2\left(b^2-3a^2\right)\tan^{-1}(\tan(x))(a\cos(x)+b\sin(x)) - a\cos(x)\left(\left(a^4-b^4\right)\cos(2x)+2b\left(-a\left(a^2+b^2\right)\sin(2x)-b\left(b\right.\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[x]^3*Sin[x])/(a*Cos[x] + b*Sin[x])^2,x]
```

```
[Out] ((-4*I)*b^2*(-3*a^2 + b^2)*ArcTan[Tan[x]]*(a*Cos[x] + b*Sin[x]) - a*Cos[x]*
((a^4 - b^4)*Cos[2*x] + 2*b*(2*(a + I*b)^3*x - b*(-3*a^2 + b^2)*Log[(a*Cos[
x] + b*Sin[x])^2] - a*(a^2 + b^2)*Sin[2*x])) + b*Sin[x]*((-a^4 + b^4)*Cos[2
*x] + 2*b*(-2*(a + I*b)*(a^2*x - b^2*(I + x) + a*(b + (2*I)*b*x)) + (-3*a^2
*b + b^3)*Log[(a*Cos[x] + b*Sin[x])^2] + a*(a^2 + b^2)*Sin[2*x])))/(4*(a^2
+ b^2)^3*(a*Cos[x] + b*Sin[x]))
```

fricas [A] time = 0.65, size = 252, normalized size = 1.97

$$-\frac{2\left(a^5+2\,a^3b^2+ab^4\right)\cos(x)^3-\left(a^5+4\,a^3b^2+7\,ab^4-4\left(a^4b-3\,a^2b^3\right)x\right)\cos(x)+2\left(\left(3\,a^3b^2-ab^4\right)\cos(x)+\left(3\,a\right.\right.}{4\left(\left(a^7+3\,a^5b^2+3\,a^3\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^3*sin(x)/(a*cos(x)+b*sin(x))^2,x, algorithm="fricas")
```

```
[Out] -1/4*(2*(a^5 + 2*a^3*b^2 + a*b^4)*cos(x)^3 - (a^5 + 4*a^3*b^2 + 7*a*b^4 - 4
*(a^4*b - 3*a^2*b^3)*x)*cos(x) + 2*((3*a^3*b^2 - a*b^4)*cos(x) + (3*a^2*b^3
 - b^5)*sin(x))*log(2*a*b*cos(x)*sin(x) + (a^2 - b^2)*cos(x)^2 + b^2) - (a^
4*b - 4*a^2*b^3 - b^5 + 2*(a^4*b + 2*a^2*b^3 + b^5)*cos(x)^2 - 4*(a^3*b^2 -
 3*a*b^4)*x)*sin(x))/((a^7 + 3*a^5*b^2 + 3*a^3*b^4 + a*b^6)*cos(x) + (a^6*b
 + 3*a^4*b^3 + 3*a^2*b^5 + b^7)*sin(x))
```

giac [A] time = 0.19, size = 214, normalized size = 1.67

$$-\frac{\left(a^{3}b-3\,ab^{3}\right)x}{a^{6}+3\,a^{4}b^{2}+3\,a^{2}b^{4}+b^{6}}+\frac{\left(3\,a^{2}b^{2}-b^{4}\right)\log\left(\tan(x)^{2}+1\right)}{2\left(a^{6}+3\,a^{4}b^{2}+3\,a^{2}b^{4}+b^{6}\right)}-\frac{\left(3\,a^{2}b^{3}-b^{5}\right)\log\left(|b\tan(x)+a|\right)}{a^{6}b+3\,a^{4}b^{3}+3\,a^{2}b^{5}+b^{7}}+\frac{4\,ab^{2}\tan(x)^{2}}{2\left(a^{4}+2\,a^{2}b^{2}+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^3*sin(x)/(a*cos(x)+b*sin(x))^2,x, algorithm="giac")
```

```
[Out] -(a^3*b - 3*a*b^3)*x/(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6) + 1/2*(3*a^2*b^2 -
 b^4)*log(tan(x)^2 + 1)/(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6) - (3*a^2*b^3 -
b^5)*log(abs(b*tan(x) + a))/(a^6*b + 3*a^4*b^3 + 3*a^2*b^5 + b^7) + 1/2*(4*
a*b^2*tan(x)^2 + a^2*b*tan(x) + b^3*tan(x) - a^3 + 3*a*b^2)/((a^4 + 2*a^2*b
^2 + b^4)*(b*tan(x)^3 + a*tan(x)^2 + b*tan(x) + a))
```

maple [A] time = 0.12, size = 241, normalized size = 1.88

$$\frac{\tan(x)a^{3}b}{\left(a^{2}+b^{2}\right)^{3}\left(\tan^{2}(x)+1\right)}+\frac{\tan(x)a\,b^{3}}{\left(a^{2}+b^{2}\right)^{3}\left(\tan^{2}(x)+1\right)}-\frac{a^{4}}{2\left(a^{2}+b^{2}\right)^{3}\left(\tan^{2}(x)+1\right)}+\frac{b^{4}}{2\left(a^{2}+b^{2}\right)^{3}\left(\tan^{2}(x)+1\right)}+\frac{3\ln}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)^3*sin(x)/(a*cos(x)+b*sin(x))^2,x)
```

```
[Out] 1/(a^2+b^2)^3/(tan(x)^2+1)*tan(x)*a^3*b+1/(a^2+b^2)^3/(tan(x)^2+1)*tan(x)*a
*b^3-1/2/(a^2+b^2)^3/(tan(x)^2+1)*a^4+1/2/(a^2+b^2)^3/(tan(x)^2+1)*b^4+3/2/
(a^2+b^2)^3*ln(tan(x)^2+1)*a^2*b^2-1/2/(a^2+b^2)^3*ln(tan(x)^2+1)*b^4-1/(a^
2+b^2)^3*arctan(tan(x))*a^3*b+3/(a^2+b^2)^3*a*arctan(tan(x))*b^3+a*b^2/(a^2
+b^2)^2/(a+b*tan(x))-3*a^2/(a^2+b^2)^3*ln(a+b*tan(x))*b^2+b^4/(a^2+b^2)^3*l
n(a+b*tan(x))
```

maxima [B] time = 0.46, size = 256, normalized size = 2.00

$$-\frac{\left(a^{3}b-3\,ab^{3}\right)x}{a^{6}+3\,a^{4}b^{2}+3\,a^{2}b^{4}+b^{6}}-\frac{\left(3\,a^{2}b^{2}-b^{4}\right)\log\left(b\tan(x)+a\right)}{a^{6}+3\,a^{4}b^{2}+3\,a^{2}b^{4}+b^{6}}+\frac{\left(3\,a^{2}b^{2}-b^{4}\right)\log\left(\tan(x)^{2}+1\right)}{2\left(a^{6}+3\,a^{4}b^{2}+3\,a^{2}b^{4}+b^{6}\right)}+\frac{}{2\left(a^{5}+2\,a^{3}b^{2}+a\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^3*sin(x)/(a*cos(x)+b*sin(x))^2,x, algorithm="maxima")
```

```
[Out] -(a^3*b - 3*a*b^3)*x/(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6) - (3*a^2*b^2 - b^4
)*log(b*tan(x) + a)/(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6) + 1/2*(3*a^2*b^2 -
b^4)*log(tan(x)^2 + 1)/(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6) + 1/2*(4*a*b^2*t
an(x)^2 - a^3 + 3*a*b^2 + (a^2*b + b^3)*tan(x))/(a^5 + 2*a^3*b^2 + a*b^4 +
(a^4*b + 2*a^2*b^3 + b^5)*tan(x)^3 + (a^5 + 2*a^3*b^2 + a*b^4)*tan(x)^2 + (
a^4*b + 2*a^2*b^3 + b^5)*tan(x))
```

mupad [B] time = 8.16, size = 5428, normalized size = 42.41

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(x)^3*sin(x))/(a*cos(x) + b*sin(x))^2,x)
```

```
[Out] ((2*a*tan(x/2)^2)/(a^2 + b^2) - (8*b^3*tan(x/2)^3)/(a^2 + b^2)^2 - (2*a*tan
(x/2)^4)/(a^2 + b^2) + (2*b*tan(x/2)*(a^2 - b^2))/(a^2 + b^2)^2 + (2*b*tan(
x/2)^5*(a^2 - b^2))/(a^4 + b^4 + 2*a^2*b^2))/(a + 2*b*tan(x/2) + a*tan(x/2)
^2 - a*tan(x/2)^4 - a*tan(x/2)^6 + 4*b*tan(x/2)^3 + 2*b*tan(x/2)^5) + (log(
a + 2*b*tan(x/2) - a*tan(x/2)^2)*(b^4 - 3*a^2*b^2))/(a^6 + b^6 + 3*a^2*b^4
+ 3*a^4*b^2) - (log(1/(cos(x) + 1))*(2*b^4 - 6*a^2*b^2))/(2*(a^6 + b^6 + 3*
a^2*b^4 + 3*a^4*b^2)) - (2*a*b*atan((tan(x/2)*((((((a*b*((32*(a*b^14 + 9*a^
3*b^12 + 18*a^5*b^10 + 2*a^7*b^8 - 27*a^9*b^6 - 27*a^11*b^4 - 8*a^13*b^2))/
(a^12 + b^12 + 6*a^2*b^10 + 15*a^4*b^8 + 20*a^6*b^6 + 15*a^8*b^4 + 6*a^10*b
^2) - (16*(2*b^4 - 6*a^2*b^2)*(3*a*b^16 + 21*a^3*b^14 + 63*a^5*b^12 + 105*a
^7*b^10 + 105*a^9*b^8 + 63*a^11*b^6 + 21*a^13*b^4 + 3*a^15*b^2))/((a^6 + b^
6 + 3*a^2*b^4 + 3*a^4*b^2)*(a^12 + b^12 + 6*a^2*b^10 + 15*a^4*b^8 + 20*a^6*
b^6 + 15*a^8*b^4 + 6*a^10*b^2)))*(a^2 - 3*b^2))/(a^6 + b^6 + 3*a^2*b^4 + 3*
a^4*b^2) - (16*a*b*(a^2 - 3*b^2)*(2*b^4 - 6*a^2*b^2)*(3*a*b^16 + 21*a^3*b^1
4 + 63*a^5*b^12 + 105*a^7*b^10 + 105*a^9*b^8 + 63*a^11*b^6 + 21*a^13*b^4 +
3*a^15*b^2))/((a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2)^2*(a^12 + b^12 + 6*a^2*b^
10 + 15*a^4*b^8 + 20*a^6*b^6 + 15*a^8*b^4 + 6*a^10*b^2)))*(2*b^4 - 6*a^2*b^
2))/(2*(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2)) + (a*b*(a^2 - 3*b^2)*((32*(3*a*
b^12 - 21*a^3*b^10 - 34*a^5*b^8 + 6*a^7*b^6 + 15*a^9*b^4 - a^11*b^2))/(a^12
 + b^12 + 6*a^2*b^10 + 15*a^4*b^8 + 20*a^6*b^6 + 15*a^8*b^4 + 6*a^10*b^2) +
 (((32*(a*b^14 + 9*a^3*b^12 + 18*a^5*b^10 + 2*a^7*b^8 - 27*a^9*b^6 - 27*a^1
1*b^4 - 8*a^13*b^2))/(a^12 + b^12 + 6*a^2*b^10 + 15*a^4*b^8 + 20*a^6*b^6 +
15*a^8*b^4 + 6*a^10*b^2) - (16*(2*b^4 - 6*a^2*b^2)*(3*a*b^16 + 21*a^3*b^14
+ 63*a^5*b^12 + 105*a^7*b^10 + 105*a^9*b^8 + 63*a^11*b^6 + 21*a^13*b^4 + 3*
a^15*b^2))/((a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2)*(a^12 + b^12 + 6*a^2*b^10 +
 15*a^4*b^8 + 20*a^6*b^6 + 15*a^8*b^4 + 6*a^10*b^2)))*(2*b^4 - 6*a^2*b^2))/
(2*(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2))))/(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^
2) + (32*a^3*b^3*(a^2 - 3*b^2)^3*(3*a*b^16 + 21*a^3*b^14 + 63*a^5*b^12 + 10
5*a^7*b^10 + 105*a^9*b^8 + 63*a^11*b^6 + 21*a^13*b^4 + 3*a^15*b^2))/((a^6 +
```

b^6 + 3*a^2*b^4 + 3*a^4*b^2)^3*(a^12 + b^12 + 6*a^2*b^10 + 15*a^4*b^8 + 20
*a^6*b^6 + 15*a^8*b^4 + 6*a^10*b^2)))*(a^8 + 4*b^8 - 61*a^2*b^6 + 155*a^4*b
^4 - 67*a^6*b^2))/(a^8 + 4*b^8 - 11*a^2*b^6 + 15*a^4*b^4 + 31*a^6*b^2)^2 +
(2*a*b*(7*a^6 - 10*b^6 + 59*a^2*b^4 - 68*a^4*b^2)*((32*(a*b^10 + 3*a^3*b^8
- 17*a^5*b^6 - 3*a^7*b^4))/(a^12 + b^12 + 6*a^2*b^10 + 15*a^4*b^8 + 20*a^6*
b^6 + 15*a^8*b^4 + 6*a^10*b^2) - ((2*b^4 - 6*a^2*b^2)*((32*(3*a*b^12 - 21*a
^3*b^10 - 34*a^5*b^8 + 6*a^7*b^6 + 15*a^9*b^4 - a^11*b^2))/(a^12 + b^12 + 6
*a^2*b^10 + 15*a^4*b^8 + 20*a^6*b^6 + 15*a^8*b^4 + 6*a^10*b^2) + (((32*(a*b
^14 + 9*a^3*b^12 + 18*a^5*b^10 + 2*a^7*b^8 - 27*a^9*b^6 - 27*a^11*b^4 - 8*a
^13*b^2))/(a^12 + b^12 + 6*a^2*b^10 + 15*a^4*b^8 + 20*a^6*b^6 + 15*a^8*b^4
+ 6*a^10*b^2) - (16*(2*b^4 - 6*a^2*b^2)*(3*a*b^16 + 21*a^3*b^14 + 63*a^5*b^
12 + 105*a^7*b^10 + 105*a^9*b^8 + 63*a^11*b^6 + 21*a^13*b^4 + 3*a^15*b^2))/
((a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2)*(a^12 + b^12 + 6*a^2*b^10 + 15*a^4*b^8
 + 20*a^6*b^6 + 15*a^8*b^4 + 6*a^10*b^2)))*(2*b^4 - 6*a^2*b^2))/(2*(a^6 + b
^6 + 3*a^2*b^4 + 3*a^4*b^2))))/(2*(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2)) + (a
b(a^2 - 3*b^2)*((a*b*((32*(a*b^14 + 9*a^3*b^12 + 18*a^5*b^10 + 2*a^7*b^8
- 27*a^9*b^6 - 27*a^11*b^4 - 8*a^13*b^2))/(a^12 + b^12 + 6*a^2*b^10 + 15*a^
4*b^8 + 20*a^6*b^6 + 15*a^8*b^4 + 6*a^10*b^2) - (16*(2*b^4 - 6*a^2*b^2)*(3*
a*b^16 + 21*a^3*b^14 + 63*a^5*b^12 + 105*a^7*b^10 + 105*a^9*b^8 + 63*a^11*b
^6 + 21*a^13*b^4 + 3*a^15*b^2))/((a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2)*(a^12
+ b^12 + 6*a^2*b^10 + 15*a^4*b^8 + 20*a^6*b^6 + 15*a^8*b^4 + 6*a^10*b^2)))*
(a^2 - 3*b^2))/(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2) - (16*a*b*(a^2 - 3*b^2)*
(2*b^4 - 6*a^2*b^2)*(3*a*b^16 + 21*a^3*b^14 + 63*a^5*b^12 + 105*a^7*b^10 +
105*a^9*b^8 + 63*a^11*b^6 + 21*a^13*b^4 + 3*a^15*b^2))/((a^6 + b^6 + 3*a^2*
b^4 + 3*a^4*b^2)^2*(a^12 + b^12 + 6*a^2*b^10 + 15*a^4*b^8 + 20*a^6*b^6 + 15
*a^8*b^4 + 6*a^10*b^2))))/(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2) - (16*a^2*b^2
*(a^2 - 3*b^2)^2*(2*b^4 - 6*a^2*b^2)*(3*a*b^16 + 21*a^3*b^14 + 63*a^5*b^12
+ 105*a^7*b^10 + 105*a^9*b^8 + 63*a^11*b^6 + 21*a^13*b^4 + 3*a^15*b^2))/((a
^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2)^3*(a^12 + b^12 + 6*a^2*b^10 + 15*a^4*b^8
+ 20*a^6*b^6 + 15*a^8*b^4 + 6*a^10*b^2))))/(a^8 + 4*b^8 - 11*a^2*b^6 + 15*a
^4*b^4 + 31*a^6*b^2)^2)*(a^16 + b^16 + 8*a^2*b^14 + 28*a^4*b^12 + 56*a^6*b^
10 + 70*a^8*b^8 + 56*a^10*b^6 + 28*a^12*b^4 + 8*a^14*b^2))/(96*a^2*b^5 - 32
*a^4*b^3) + (((((a*b*((32*(3*a^6*b^9 - 4*a^2*b^13 - 9*a^4*b^11 - a^14*b + 2
2*a^8*b^7 + 18*a^10*b^5 + 3*a^12*b^3))/(a^12 + b^12 + 6*a^2*b^10 + 15*a^4*b
^8 + 20*a^6*b^6 + 15*a^8*b^4 + 6*a^10*b^2) - (16*(2*b^4 - 6*a^2*b^2)*(3*a^1
6*b + 3*a^2*b^15 + 21*a^4*b^13 + 63*a^6*b^11 + 105*a^8*b^9 + 105*a^10*b^7 +
 63*a^12*b^5 + 21*a^14*b^3))/((a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2)*(a^12 + b
^12 + 6*a^2*b^10 + 15*a^4*b^8 + 20*a^6*b^6 + 15*a^8*b^4 + 6*a^10*b^2)))*(a^
2 - 3*b^2))/(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2) - (16*a*b*(a^2 - 3*b^2)*(2*
b^4 - 6*a^2*b^2)*(3*a^16*b + 3*a^2*b^15 + 21*a^4*b^13 + 63*a^6*b^11 + 105*a
^8*b^9 + 105*a^10*b^7 + 63*a^12*b^5 + 21*a^14*b^3))/((a^6 + b^6 + 3*a^2*b^4
 + 3*a^4*b^2)^2*(a^12 + b^12 + 6*a^2*b^10 + 15*a^4*b^8 + 20*a^6*b^6 + 15*a^
8*b^4 + 6*a^10*b^2)))*(2*b^4 - 6*a^2*b^2))/(2*(a^6 + b^6 + 3*a^2*b^4 + 3*a^
4*b^2)) + (a*b*(a^2 - 3*b^2)*((32*(6*a^6*b^7 - 4*a^4*b^9 - 3*a^2*b^11 + 12*
a^8*b^5 + 5*a^10*b^3))/(a^12 + b^12 + 6*a^2*b^10 + 15*a^4*b^8 + 20*a^6*b^6

```
+ 15*a^8*b^4 + 6*a^10*b^2) + (((32*(3*a^6*b^9 - 4*a^2*b^13 - 9*a^4*b^11 - a
^14*b + 22*a^8*b^7 + 18*a^10*b^5 + 3*a^12*b^3))/(a^12 + b^12 + 6*a^2*b^10 +
 15*a^4*b^8 + 20*a^6*b^6 + 15*a^8*b^4 + 6*a^10*b^2) - (16*(2*b^4 - 6*a^2*b^
2)*(3*a^16*b + 3*a^2*b^15 + 21*a^4*b^13 + 63*a^6*b^11 + 105*a^8*b^9 + 105*a
^10*b^7 + 63*a^12*b^5 + 21*a^14*b^3))/((a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2)*
(a^12 + b^12 + 6*a^2*b^10 + 15*a^4*b^8 + 20*a^6*b^6 + 15*a^8*b^4 + 6*a^10*b
^2)))*(2*b^4 - 6*a^2*b^2))/(2*(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2))))/(a^6 +
 b^6 + 3*a^2*b^4 + 3*a^4*b^2) + (32*a^3*b^3*(a^2 - 3*b^2)^3*(3*a^16*b + 3*a
^2*b^15 + 21*a^4*b^13 + 63*a^6*b^11 + 105*a^8*b^9 + 105*a^10*b^7 + 63*a^12*
b^5 + 21*a^14*b^3))/((a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2)^3*(a^12 + b^12 + 6
*a^2*b^10 + 15*a^4*b^8 + 20*a^6*b^6 + 15*a^8*b^4 + 6*a^10*b^2)))*(a^8 + 4*b
^8 - 61*a^2*b^6 + 155*a^4*b^4 - 67*a^6*b^2)*(a^16 + b^16 + 8*a^2*b^14 + 28*
a^4*b^12 + 56*a^6*b^10 + 70*a^8*b^8 + 56*a^10*b^6 + 28*a^12*b^4 + 8*a^14*b^
2))/((96*a^2*b^5 - 32*a^4*b^3)*(a^8 + 4*b^8 - 11*a^2*b^6 + 15*a^4*b^4 + 31*
a^6*b^2)^2) + (2*a*b*(7*a^6 - 10*b^6 + 59*a^2*b^4 - 68*a^4*b^2)*((32*(2*a^2
*b^9 - 8*a^4*b^7 + 6*a^6*b^5))/(a^12 + b^12 + 6*a^2*b^10 + 15*a^4*b^8 + 20*
a^6*b^6 + 15*a^8*b^4 + 6*a^10*b^2) - ((2*b^4 - 6*a^2*b^2)*((32*(6*a^6*b^7 -
 4*a^4*b^9 - 3*a^2*b^11 + 12*a^8*b^5 + 5*a^10*b^3))/(a^12 + b^12 + 6*a^2*b^
10 + 15*a^4*b^8 + 20*a^6*b^6 + 15*a^8*b^4 + 6*a^10*b^2) + (((32*(3*a^6*b^9
- 4*a^2*b^13 - 9*a^4*b^11 - a^14*b + 22*a^8*b^7 + 18*a^10*b^5 + 3*a^12*b^3)
)/(a^12 + b^12 + 6*a^2*b^10 + 15*a^4*b^8 + 20*a^6*b^6 + 15*a^8*b^4 + 6*a^10
*b^2) - (16*(2*b^4 - 6*a^2*b^2)*(3*a^16*b + 3*a^2*b^15 + 21*a^4*b^13 + 63*a
^6*b^11 + 105*a^8*b^9 + 105*a^10*b^7 + 63*a^12*b^5 + 21*a^14*b^3))/((a^6 +
b^6 + 3*a^2*b^4 + 3*a^4*b^2)*(a^12 + b^12 + 6*a^2*b^10 + 15*a^4*b^8 + 20*a^
6*b^6 + 15*a^8*b^4 + 6*a^10*b^2)))*(2*b^4 - 6*a^2*b^2))/(2*(a^6 + b^6 + 3*a
^2*b^4 + 3*a^4*b^2))))/(2*(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2)) + (a*b*(a^2
- 3*b^2)*((a*b*((32*(3*a^6*b^9 - 4*a^2*b^13 - 9*a^4*b^11 - a^14*b + 22*a^8*
b^7 + 18*a^10*b^5 + 3*a^12*b^3))/(a^12 + b^12 + 6*a^2*b^10 + 15*a^4*b^8 + 2
0*a^6*b^6 + 15*a^8*b^4 + 6*a^10*b^2) - (16*(2*b^4 - 6*a^2*b^2)*(3*a^16*b +
3*a^2*b^15 + 21*a^4*b^13 + 63*a^6*b^11 + 105*a^8*b^9 + 105*a^10*b^7 + 63*a^
12*b^5 + 21*a^14*b^3))/((a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2)*(a^12 + b^12 +
6*a^2*b^10 + 15*a^4*b^8 + 20*a^6*b^6 + 15*a^8*b^4 + 6*a^10*b^2)))*(a^2 - 3*
b^2))/(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2) - (16*a*b*(a^2 - 3*b^2)*(2*b^4 -
6*a^2*b^2)*(3*a^16*b + 3*a^2*b^15 + 21*a^4*b^13 + 63*a^6*b^11 + 105*a^8*b^9
 + 105*a^10*b^7 + 63*a^12*b^5 + 21*a^14*b^3))/((a^6 + b^6 + 3*a^2*b^4 + 3*a
^4*b^2)^2*(a^12 + b^12 + 6*a^2*b^10 + 15*a^4*b^8 + 20*a^6*b^6 + 15*a^8*b^4
+ 6*a^10*b^2))))/(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2) - (16*a^2*b^2*(a^2 - 3
*b^2)^2*(2*b^4 - 6*a^2*b^2)*(3*a^16*b + 3*a^2*b^15 + 21*a^4*b^13 + 63*a^6*b
^11 + 105*a^8*b^9 + 105*a^10*b^7 + 63*a^12*b^5 + 21*a^14*b^3))/((a^6 + b^6
+ 3*a^2*b^4 + 3*a^4*b^2)^3*(a^12 + b^12 + 6*a^2*b^10 + 15*a^4*b^8 + 20*a^6*
b^6 + 15*a^8*b^4 + 6*a^10*b^2)))*(a^16 + b^16 + 8*a^2*b^14 + 28*a^4*b^12 +
56*a^6*b^10 + 70*a^8*b^8 + 56*a^10*b^6 + 28*a^12*b^4 + 8*a^14*b^2))/((96*a^
2*b^5 - 32*a^4*b^3)*(a^8 + 4*b^8 - 11*a^2*b^6 + 15*a^4*b^4 + 31*a^6*b^2)^2)
)*(a^2 - 3*b^2))/(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)**3*sin(x)/(a*cos(x)+b*sin(x))**2,x)
```

```
[Out] Timed out
```

3.291 $\int \frac{\cos^3(x)\sin^2(x)}{(a\cos(x)+b\sin(x))^2}\,dx$

Optimal. Leaf size=176

$$\frac{(a^2-b^2)\sin^3(x)}{3(a^2+b^2)^2}-\frac{b^2(3a^2-b^2)\sin(x)}{(a^2+b^2)^3}-\frac{2ab\cos^3(x)}{3(a^2+b^2)^2}+\frac{2ab(a^2-b^2)\cos(x)}{(a^2+b^2)^3}-\frac{ab^2(3a^2-2b^2)\tanh^{-1}\left(\frac{b\cos(x)-a\sin(x)}{\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{7/2}}$$

```
[Out] -a*b^2*(3*a^2-2*b^2)*arctanh((b*cos(x)-a*sin(x))/(a^2+b^2)^(1/2))/(a^2+b^2)
^(7/2)+2*a*b*(a^2-b^2)*cos(x)/(a^2+b^2)^3-2/3*a*b*cos(x)^3/(a^2+b^2)^2-b^2*
(3*a^2-b^2)*sin(x)/(a^2+b^2)^3+1/3*(a^2-b^2)*sin(x)^3/(a^2+b^2)^2-a^2*b^3/(
a^2+b^2)^3/(a*cos(x)+b*sin(x))
```

Rubi [A] time = 0.70, antiderivative size = 238, normalized size of antiderivative = 1.35, number of steps used = 33, number of rules used = 12, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.600, Rules used = {3111, 3109, 2633, 2565, 30, 3100, 2637, 3074, 206, 2564, 2638, 3155}

$$-\frac{b^2\sin^3(x)}{3(a^2+b^2)^2}+\frac{a^2\sin^3(x)}{3(a^2+b^2)^2}+\frac{b^2\sin(x)}{(a^2+b^2)^2}-\frac{4a^2b^2\sin(x)}{(a^2+b^2)^3}-\frac{2ab\cos^3(x)}{3(a^2+b^2)^2}-\frac{2ab^3\cos(x)}{(a^2+b^2)^3}+\frac{2a^3b\cos(x)}{(a^2+b^2)^3}-\frac{a^2b^3}{(a^2+b^2)^3(a\cos(x)+b\sin(x))}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[x]^3*Sin[x]^2)/(a*Cos[x] + b*Sin[x])^2,x]
```

```
[Out] (-3*a^3*b^2*ArcTanh[(b*Cos[x] - a*Sin[x])/Sqrt[a^2 + b^2]])/(a^2 + b^2)^(7/
2) + (2*a*b^4*ArcTanh[(b*Cos[x] - a*Sin[x])/Sqrt[a^2 + b^2]])/(a^2 + b^2)^(
7/2) + (2*a^3*b*Cos[x])/(a^2 + b^2)^3 - (2*a*b^3*Cos[x])/(a^2 + b^2)^3 - (2
*a*b*Cos[x]^3)/(3*(a^2 + b^2)^2) - (4*a^2*b^2*Sin[x])/(a^2 + b^2)^3 + (b^2*
Sin[x])/(a^2 + b^2)^2 + (a^2*Sin[x]^3)/(3*(a^2 + b^2)^2) - (b^2*Sin[x]^3)/(
3*(a^2 + b^2)^2) - (a^2*b^3)/((a^2 + b^2)^3*(a*Cos[x] + b*Sin[x]))
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2564

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 2565

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> -Dist[(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3074

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x
_Symbol] :> -Dist[d^(-1), Subst[Int[1/(a^2 + b^2 - x^2), x], x, b*Cos[c + d
*x] - a*Sin[c + d*x]], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

Rule 3100

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_)/(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin
[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[(b*Cos[c + d*x]^(m - 1))/(d*(a^2 +
 b^2)*(m - 1)), x] + (Dist[a/(a^2 + b^2), Int[Cos[c + d*x]^(m - 1), x], x]
+ Dist[b^2/(a^2 + b^2), Int[Cos[c + d*x]^(m - 2)/(a*Cos[c + d*x] + b*Sin[c
+ d*x]), x], x]) /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0] && GtQ[m, 1
]
```

Rule 3109

```
Int[(cos[(c_.) + (d_.)*(x_)]^(m_.)*sin[(c_.) + (d_.)*(x_)]^(n_.))/(cos[(c_.
) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[b
/(a^2 + b^2), Int[Cos[c + d*x]^m*Sin[c + d*x]^(n - 1), x], x] + (Dist[a/(a^
2 + b^2), Int[Cos[c + d*x]^(m - 1)*Sin[c + d*x]^n, x], x] - Dist[(a*b)/(a^2
 + b^2), Int[(Cos[c + d*x]^(m - 1)*Sin[c + d*x]^(n - 1))/(a*Cos[c + d*x] +
b*Sin[c + d*x]), x], x]) /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0] &&
IGtQ[m, 0] && IGtQ[n, 0]
```

Rule 3111

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*sin[(c_.) + (d_.)*(x_)]^(n_.)*(cos[(c_.)
+ (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(p_), x_Symbol] :> Dis
t[b/(a^2 + b^2), Int[Cos[c + d*x]^m*Sin[c + d*x]^(n - 1)*(a*Cos[c + d*x] +
b*Sin[c + d*x])^(p + 1), x], x] + (Dist[a/(a^2 + b^2), Int[Cos[c + d*x]^(m
- 1)*Sin[c + d*x]^n*(a*Cos[c + d*x] + b*Sin[c + d*x])^(p + 1), x], x] - Dis
t[(a*b)/(a^2 + b^2), Int[Cos[c + d*x]^(m - 1)*Sin[c + d*x]^(n - 1)*(a*Cos[c
 + d*x] + b*Sin[c + d*x])^p, x], x]) /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 +
 b^2, 0] && IGtQ[m, 0] && IGtQ[n, 0] && ILtQ[p, 0]
```

Rule 3155

```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.))/((a_.) + cos[(d_.) + (e_.)*(x_)
]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)])^2, x_Symbol] :> Simp[(c*B + c*A*Co
s[d + e*x] + (a*B - b*A)*Sin[d + e*x])/(e*(a^2 - b^2 - c^2)*(a + b*Cos[d +
e*x] + c*Sin[d + e*x])), x] + Dist[(a*A - b*B)/(a^2 - b^2 - c^2), Int[1/(a
+ b*Cos[d + e*x] + c*Sin[d + e*x]), x], x] /; FreeQ[{a, b, c, d, e, A, B},
x] && NeQ[a^2 - b^2 - c^2, 0] && NeQ[a*A - b*B, 0]
```

Rubi steps

$$\int \frac{\cos^3(x)\sin^2(x)}{(a\cos(x)+b\sin(x))^2}\,dx = \frac{a\int \frac{\cos^2(x)\sin^2(x)}{a\cos(x)+b\sin(x)}\,dx}{a^2+b^2} + \frac{b\int \frac{\cos^3(x)\sin(x)}{a\cos(x)+b\sin(x)}\,dx}{a^2+b^2} - \frac{(ab)\int \frac{\cos^2(x)\sin(x)}{(a\cos(x)+b\sin(x))^2}\,dx}{a^2+b^2}$$

$$= \frac{a^2\int \cos(x)\sin^2(x)\,dx}{\left(a^2+b^2\right)^2} + 2\frac{(ab)\int \cos^2(x)\sin(x)\,dx}{\left(a^2+b^2\right)^2} - 2\frac{\left(a^2b\right)\int \frac{\cos(x)\sin(x)}{a\cos(x)+b\sin(x)}\,dx}{\left(a^2+b^2\right)^2} +$$

$$= -\frac{a^2b^3}{\left(a^2+b^2\right)^3(a\cos(x)+b\sin(x))} - 2\left(\frac{\left(a^3b\right)\int \sin(x)\,dx}{\left(a^2+b^2\right)^3} + \frac{\left(a^2b^2\right)\int \cos(x)\,dx}{\left(a^2+b^2\right)^3} - \right.$$

$$= -\frac{2ab\cos^3(x)}{3\left(a^2+b^2\right)^2} + \frac{b^2\sin(x)}{\left(a^2+b^2\right)^2} + \frac{a^2\sin^3(x)}{3\left(a^2+b^2\right)^2} - \frac{b^2\sin^3(x)}{3\left(a^2+b^2\right)^2} - \frac{a^2b^3}{\left(a^2+b^2\right)^3(a\cos(x)}$$

$$= -\frac{a^3b^2\tanh^{-1}\left(\frac{b\cos(x)-a\sin(x)}{\sqrt{a^2+b^2}}\right)}{\left(a^2+b^2\right)^{7/2}} - \frac{2ab\cos^3(x)}{3\left(a^2+b^2\right)^2} + \frac{b^2\sin(x)}{\left(a^2+b^2\right)^2} + \frac{a^2\sin^3(x)}{3\left(a^2+b^2\right)^2} - \frac{b^2 s}{3\left(a^2\right.}$$

Mathematica [A] time = 1.27, size = 198, normalized size = 1.12

$$\frac{2ab^2\left(3a^2-2b^2\right)\tanh^{-1}\left(\frac{a\tan\left(\frac{x}{2}\right)-b}{\sqrt{a^2+b^2}}\right)}{\left(a^2+b^2\right)^{7/2}} - \frac{-2a^5\sin(2x)+a^5\sin(4x)-21a^4b+16a^3b^2\sin(2x)+2a^3b^2\sin(4x)+90a}{24\left(a^2\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[x]^3*Sin[x]^2)/(a*Cos[x] + b*Sin[x])^2,x]
```

```
[Out] (2*a*b^2*(3*a^2 - 2*b^2)*ArcTanh[(-b + a*Tan[x/2])/Sqrt[a^2 + b^2]])/(a^2 +
 b^2)^(7/2) - (-21*a^4*b + 90*a^2*b^3 - 9*b^5 - 4*b*(3*a^4 + a^2*b^2 - 2*b^
4)*Cos[2*x] + b*(a^2 + b^2)^2*Cos[4*x] - 2*a^5*Sin[2*x] + 16*a^3*b^2*Sin[2*
x] + 18*a*b^4*Sin[2*x] + a^5*Sin[4*x] + 2*a^3*b^2*Sin[4*x] + a*b^4*Sin[4*x]
)/(24*(a^2 + b^2)^3*(a*Cos[x] + b*Sin[x]))
```

fricas [B] time = 0.56, size = 369, normalized size = 2.10

$$\frac{2a^6b - 22a^4b^3 - 20a^2b^5 + 4b^7 - 2\left(a^6b + 3a^4b^3 + 3a^2b^5 + b^7\right)\cos(x)^4 + 2\left(4a^6b + 7a^4b^3 + 2a^2b^5 - b^7\right)\cos(x}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^3*sin(x)^2/(a*cos(x)+b*sin(x))^2,x, algorithm="fricas")
```

```
[Out] 1/6*(2*a^6*b - 22*a^4*b^3 - 20*a^2*b^5 + 4*b^7 - 2*(a^6*b + 3*a^4*b^3 + 3*a
^2*b^5 + b^7)*cos(x)^4 + 2*(4*a^6*b + 7*a^4*b^3 + 2*a^2*b^5 - b^7)*cos(x)^2
 - 3*sqrt(a^2 + b^2)*((3*a^4*b^2 - 2*a^2*b^4)*cos(x) + (3*a^3*b^3 - 2*a*b^5
)*sin(x))*log((2*a*b*cos(x)*sin(x) + (a^2 - b^2)*cos(x)^2 - 2*a^2 - b^2 - 2
*sqrt(a^2 + b^2)*(b*cos(x) - a*sin(x)))/(2*a*b*cos(x)*sin(x) + (a^2 - b^2)*
cos(x)^2 + b^2)) - 2*((a^7 + 3*a^5*b^2 + 3*a^3*b^4 + a*b^6)*cos(x)^3 - (a^7
 - 2*a^5*b^2 - 7*a^3*b^4 - 4*a*b^6)*cos(x))*sin(x))/((a^9 + 4*a^7*b^2 + 6*a
^5*b^4 + 4*a^3*b^6 + a*b^8)*cos(x) + (a^8*b + 4*a^6*b^3 + 6*a^4*b^5 + 4*a^2
*b^7 + b^9)*sin(x))
```

giac [A] time = 0.26, size = 335, normalized size = 1.90

$$-\frac{\left(3\,a^3b^2-2\,ab^4\right)\log\left(\frac{\left|2\,a\tan\left(\frac{1}{2}\,x\right)-2\,b-2\,\sqrt{a^2+b^2}\right|}{\left|2\,a\tan\left(\frac{1}{2}\,x\right)-2\,b+2\,\sqrt{a^2+b^2}\right|}\right)}{\left(a^6+3\,a^4b^2+3\,a^2b^4+b^6\right)\sqrt{a^2+b^2}}+\frac{2\left(ab^4\tan\left(\frac{1}{2}\,x\right)+a^2b^3\right)}{\left(a^6+3\,a^4b^2+3\,a^2b^4+b^6\right)\left(a\tan\left(\frac{1}{2}\,x\right)^2-2\,b\tan\left(\frac{1}{2}\,x\right)-a\right)}-\frac{2\left(\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^3*sin(x)^2/(a*cos(x)+b*sin(x))^2,x, algorithm="giac")
```

```
[Out] -(3*a^3*b^2 - 2*a*b^4)*log(abs(2*a*tan(1/2*x) - 2*b - 2*sqrt(a^2 + b^2))/ab
s(2*a*tan(1/2*x) - 2*b + 2*sqrt(a^2 + b^2)))/((a^6 + 3*a^4*b^2 + 3*a^2*b^4
+ b^6)*sqrt(a^2 + b^2)) + 2*(a*b^4*tan(1/2*x) + a^2*b^3)/((a^6 + 3*a^4*b^2
+ 3*a^2*b^4 + b^6)*(a*tan(1/2*x)^2 - 2*b*tan(1/2*x) - a)) - 2/3*(9*a^2*b^2*
tan(1/2*x)^5 - 3*b^4*tan(1/2*x)^5 + 12*a*b^3*tan(1/2*x)^4 - 4*a^4*tan(1/2*x
)^3 + 18*a^2*b^2*tan(1/2*x)^3 - 2*b^4*tan(1/2*x)^3 - 12*a^3*b*tan(1/2*x)^2
+ 12*a*b^3*tan(1/2*x)^2 + 9*a^2*b^2*tan(1/2*x) - 3*b^4*tan(1/2*x) - 4*a^3*b
 + 8*a*b^3)/((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*(tan(1/2*x)^2 + 1)^3)
```

maple [A] time = 0.14, size = 261, normalized size = 1.48

$$-\frac{2a\,b^2\left(\frac{-\tan\left(\frac{x}{2}\right)b^2-ab}{\left(\tan^2\left(\frac{x}{2}\right)\right)a-2b\tan\left(\frac{x}{2}\right)-a}-\frac{\left(3a^2-2b^2\right)\operatorname{arctanh}\left(\frac{2a\tan\left(\frac{x}{2}\right)-2b}{2\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}}\right)}{\left(a^4+2a^2b^2+b^4\right)\left(a^2+b^2\right)}+\frac{2\left(-3a^2b^2+b^4\right)\left(\tan^5\left(\frac{x}{2}\right)\right)-8b^3a\left(\tan^4\left(\frac{x}{2}\right)\right)+2\left(\frac{4}{3}a^4\right.}{\left(\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)^3*sin(x)^2/(a*cos(x)+b*sin(x))^2,x)
```

```
[Out] -2*a*b^2/(a^4+2*a^2*b^2+b^4)/(a^2+b^2)*((-tan(1/2*x)*b^2-a*b)/(tan(1/2*x)^2
*a-2*b*tan(1/2*x)-a)-(3*a^2-2*b^2)/(a^2+b^2)^(1/2)*arctanh(1/2*(2*a*tan(1/2
*x)-2*b)/(a^2+b^2)^(1/2)))+2/(a^2+b^2)/(a^4+2*a^2*b^2+b^4)*((-3*a^2*b^2+b^4
)*tan(1/2*x)^5-4*b^3*a*tan(1/2*x)^4+(4/3*a^4-6*a^2*b^2+2/3*b^4)*tan(1/2*x)^
3+(4*a^3*b-4*a*b^3)*tan(1/2*x)^2+(-3*a^2*b^2+b^4)*tan(1/2*x)+4/3*a^3*b-8/3*
b^3*a)/(tan(1/2*x)^2+1)^3
```

maxima [B] time = 0.48, size = 606, normalized size = 3.44

$$-\frac{\left(3\,a^2b^2-2\,b^4\right)a\log\left(\frac{b-\frac{a\sin(x)}{\cos(x)+1}+\sqrt{a^2+b^2}}{b-\frac{a\sin(x)}{\cos(x)+1}-\sqrt{a^2+b^2}}\right)}{\left(a^6+3\,a^4b^2+3\,a^2b^4+b^6\right)\sqrt{a^2+b^2}}+\frac{2\left(4\,a^4b-11\,a^2b^3-\frac{\left(a^3b^2+16\,ab^4\right)\sin(x)}{\cos(x)+1}+\frac{(8\,a^4b-}{}\right.}{3\left(a^7+3\,a^5b^2+3\,a^3b^4+ab^6+\frac{2\left(a^6b+3\,a^4b^3+3\,a^2b^5+b^7\right)\sin(x)}{\cos(x)+1}+\frac{2\left(a^7+3\,a^5b^2\right.}{(c}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^3*sin(x)^2/(a*cos(x)+b*sin(x))^2,x, algorithm="maxima")
```

```
[Out] -(3*a^2*b^2 - 2*b^4)*a*log((b - a*sin(x)/(cos(x) + 1) + sqrt(a^2 + b^2))/(b
 - a*sin(x)/(cos(x) + 1) - sqrt(a^2 + b^2)))/((a^6 + 3*a^4*b^2 + 3*a^2*b^4
+ b^6)*sqrt(a^2 + b^2)) + 2/3*(4*a^4*b - 11*a^2*b^3 - (a^3*b^2 + 16*a*b^4)*
sin(x)/(cos(x) + 1) + (8*a^4*b - 31*a^2*b^3 + 6*b^5)*sin(x)^2/(cos(x) + 1)^
2 + (4*a^5 + 15*a^3*b^2 - 34*a*b^4)*sin(x)^3/(cos(x) + 1)^3 - (4*a^4*b + 45
*a^2*b^3 - 4*b^5)*sin(x)^4/(cos(x) + 1)^4 - (4*a^5 - 9*a^3*b^2 + 32*a*b^4)*
sin(x)^5/(cos(x) + 1)^5 - 3*(3*a^2*b^3 - 2*b^5)*sin(x)^6/(cos(x) + 1)^6 + 3
*(3*a^3*b^2 - 2*a*b^4)*sin(x)^7/(cos(x) + 1)^7)/(a^7 + 3*a^5*b^2 + 3*a^3*b^
4 + a*b^6 + 2*(a^6*b + 3*a^4*b^3 + 3*a^2*b^5 + b^7)*sin(x)/(cos(x) + 1) + 2
*(a^7 + 3*a^5*b^2 + 3*a^3*b^4 + a*b^6)*sin(x)^2/(cos(x) + 1)^2 + 6*(a^6*b +
 3*a^4*b^3 + 3*a^2*b^5 + b^7)*sin(x)^3/(cos(x) + 1)^3 + 6*(a^6*b + 3*a^4*b^
3 + 3*a^2*b^5 + b^7)*sin(x)^5/(cos(x) + 1)^5 - 2*(a^7 + 3*a^5*b^2 + 3*a^3*b
^4 + a*b^6)*sin(x)^6/(cos(x) + 1)^6 + 2*(a^6*b + 3*a^4*b^3 + 3*a^2*b^5 + b^
7)*sin(x)^7/(cos(x) + 1)^7 - (a^7 + 3*a^5*b^2 + 3*a^3*b^4 + a*b^6)*sin(x)^8
/(cos(x) + 1)^8)
```

mupad [B] time = 2.87, size = 586, normalized size = 3.33

$$-\frac{\frac{2\tan\left(\frac{x}{2}\right)^4\left(4\,a^4\,b+45\,a^2\,b^3-4\,b^5\right)}{3\left(a^6+3\,a^4\,b^2+3\,a^2\,b^4+b^6\right)}-\frac{2\tan\left(\frac{x}{2}\right)^6\left(2\,b^5-3\,a^2\,b^3\right)}{a^6+3\,a^4\,b^2+3\,a^2\,b^4+b^6}+\frac{2\tan\left(\frac{x}{2}\right)^5\left(4\,a^5-9\,a^3\,b^2+32\,a\,b^4\right)}{3\left(a^2+b^2\right)\left(a^4+2\,a^2\,b^2+b^4\right)}-\frac{2\tan\left(\frac{x}{2}\right)^3\left(4\,a^5+15\,a^3\,b^2-34\,a\,b^4\right)}{3\left(a^2+b^2\right)\left(a^4+2\,a^2\,b^2+b^4\right)}-\frac{2\tan\left(\frac{x}{2}\right.}{3\left(a^2\right.}}{-a\tan\left(\frac{x}{2}\right)^8+2\,b\tan\left(\frac{x}{2}\right)^7-2\,a\tan\left(\frac{x}{2}\right)^6+6\,b\tan\left(\frac{x}{2}\right)^5+6\,b\,t}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(x)^3*sin(x)^2)/(a*cos(x) + b*sin(x))^2,x)
```

```
[Out] - ((2*tan(x/2)^4*(4*a^4*b - 4*b^5 + 45*a^2*b^3))/(3*(a^6 + b^6 + 3*a^2*b^4
+ 3*a^4*b^2)) - (2*tan(x/2)^6*(2*b^5 - 3*a^2*b^3))/(a^6 + b^6 + 3*a^2*b^4 +
 3*a^4*b^2) + (2*tan(x/2)^5*(32*a*b^4 + 4*a^5 - 9*a^3*b^2))/(3*(a^2 + b^2)*
(a^4 + b^4 + 2*a^2*b^2)) - (2*tan(x/2)^3*(4*a^5 - 34*a*b^4 + 15*a^3*b^2))/(
3*(a^2 + b^2)*(a^4 + b^4 + 2*a^2*b^2)) - (2*tan(x/2)^2*(8*a^4*b + 6*b^5 - 3
1*a^2*b^3))/(3*(a^2 + b^2)*(a^4 + b^4 + 2*a^2*b^2)) + (2*a*(11*a*b^3 - 4*a^
3*b))/(3*(a^2 + b^2)*(a^4 + b^4 + 2*a^2*b^2)) + (2*b*tan(x/2)^7*(2*a*b^3 -
3*a^3*b))/(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2) + (2*b*tan(x/2)*(16*a*b^3 + a
^3*b))/(3*(a^2 + b^2)*(a^4 + b^4 + 2*a^2*b^2)))/(a + 2*b*tan(x/2) + 2*a*tan
(x/2)^2 - 2*a*tan(x/2)^6 - a*tan(x/2)^8 + 6*b*tan(x/2)^3 + 6*b*tan(x/2)^5 +
 2*b*tan(x/2)^7) - (a*b^2*atan((a^7*tan(x/2)*1i - a^6*b*1i - b^7*1i - a^2*b
^5*3i - a^4*b^3*3i + a^3*b^4*tan(x/2)*3i + a^5*b^2*tan(x/2)*3i + a*b^6*tan(
x/2)*1i)/(a^2 + b^2)^(7/2))*(3*a^2 - 2*b^2)*2i)/(a^2 + b^2)^(7/2)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)**3*sin(x)**2/(a*cos(x)+b*sin(x))**2,x)
```

```
[Out] Timed out
```

3.292 $\int \frac{\cos^3(x)\sin^3(x)}{(a\cos(x)+b\sin(x))^2}\,dx$

Optimal. Leaf size=210

$$\frac{a^2\sin^4(x)}{4\left(a^2+b^2\right)^2}-\frac{2a^2b^2\sin^2(x)}{\left(a^2+b^2\right)^3}-\frac{b^2\cos^4(x)}{4\left(a^2+b^2\right)^2}-\frac{ab\sin(x)\cos^3(x)}{2\left(a^2+b^2\right)^2}+\frac{ab\left(5a^2-3b^2\right)\sin(x)\cos(x)}{4\left(a^2+b^2\right)^3}-\frac{3a^2b^2\left(a^2-b^2\right)\log(}{\left(a^2+\right.}$$

```
[Out] -3/4*a*b*(a^4-6*a^2*b^2+b^4)*x/(a^2+b^2)^4-1/4*b^2*cos(x)^4/(a^2+b^2)^2-3*a
^2*b^2*(a^2-b^2)*ln(a*cos(x)+b*sin(x))/(a^2+b^2)^4+1/4*a*b*(5*a^2-3*b^2)*co
s(x)*sin(x)/(a^2+b^2)^3-1/2*a*b*cos(x)^3*sin(x)/(a^2+b^2)^2-2*a^2*b^2*sin(x
)^2/(a^2+b^2)^3+1/4*a^2*sin(x)^4/(a^2+b^2)^2-a^2*b^3*sin(x)/(a^2+b^2)^3/(a*
cos(x)+b*sin(x))
```

Rubi [A] time = 1.25, antiderivative size = 289, normalized size of antiderivative = 1.38, number of steps used = 48, number of rules used = 12, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.600, Rules used = {3111, 3109, 2565, 30, 2568, 2635, 8, 2564, 3098, 3133, 3097, 3075}

$$-\frac{a^3bx}{\left(a^2+b^2\right)^3}+\frac{6a^3b^3x}{\left(a^2+b^2\right)^4}+\frac{abx}{4\left(a^2+b^2\right)^2}-\frac{ab^3x}{\left(a^2+b^2\right)^3}+\frac{a^2\sin^4(x)}{4\left(a^2+b^2\right)^2}-\frac{2a^2b^2\sin^2(x)}{\left(a^2+b^2\right)^3}-\frac{b^2\cos^4(x)}{4\left(a^2+b^2\right)^2}-\frac{ab\sin(x)\cos^3(x}{2\left(a^2+b^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[x]^3*Sin[x]^3)/(a*Cos[x] + b*Sin[x])^2,x]
```

```
[Out] (6*a^3*b^3*x)/(a^2 + b^2)^4 - (a^3*b*x)/(a^2 + b^2)^3 - (a*b^3*x)/(a^2 + b^
2)^3 + (a*b*x)/(4*(a^2 + b^2)^2) - (b^2*Cos[x]^4)/(4*(a^2 + b^2)^2) - (3*a^
4*b^2*Log[a*Cos[x] + b*Sin[x]])/(a^2 + b^2)^4 + (3*a^2*b^4*Log[a*Cos[x] + b
*Sin[x]])/(a^2 + b^2)^4 + (a^3*b*Cos[x]*Sin[x])/(a^2 + b^2)^3 - (a*b^3*Cos[
x]*Sin[x])/(a^2 + b^2)^3 + (a*b*Cos[x]*Sin[x])/(4*(a^2 + b^2)^2) - (a*b*Cos
[x]^3*Sin[x])/(2*(a^2 + b^2)^2) - (2*a^2*b^2*Sin[x]^2)/(a^2 + b^2)^3 + (a^2
*Sin[x]^4)/(4*(a^2 + b^2)^2) - (a^2*b^3*Sin[x])/((a^2 + b^2)^3*(a*Cos[x] +
b*Sin[x]))
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2564

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 2565

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> -Dist[(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 2568

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> -Simp[(a*(b*Cos[e + f*x])^(n + 1)*(a*Sin[e + f*x])^(m - 1)
)/(b*f*(m + n)), x] + Dist[(a^2*(m - 1))/(m + n), Int[(b*Cos[e + f*x])^n*(a
*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1] &&
 NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 3075

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-2), x
_Symbol] :> Simp[Sin[c + d*x]/(a*d*(a*Cos[c + d*x] + b*Sin[c + d*x])), x] /
; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

Rule 3097

```
Int[sin[(c_.) + (d_.)*(x_)]/(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.
) + (d_.)*(x_)]), x_Symbol] :> Simp[(b*x)/(a^2 + b^2), x] - Dist[a/(a^2 + b
^2), Int[(b*Cos[c + d*x] - a*Sin[c + d*x])/(a*Cos[c + d*x] + b*Sin[c + d*x]
), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

Rule 3098

```
Int[cos[(c_.) + (d_.)*(x_)]/(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.
```

```
) + (d_.)*(x_)]), x_Symbol] :> Simp[(a*x)/(a^2 + b^2), x] + Dist[b/(a^2 + b
^2), Int[(b*Cos[c + d*x] - a*Sin[c + d*x])/(a*Cos[c + d*x] + b*Sin[c + d*x]
), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

Rule 3109

```
Int[(cos[(c_.) + (d_.)*(x_)]^(m_.)*sin[(c_.) + (d_.)*(x_)]^(n_.))/(cos[(c_.
) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[b
/(a^2 + b^2), Int[Cos[c + d*x]^m*Sin[c + d*x]^(n - 1), x], x] + (Dist[a/(a^
2 + b^2), Int[Cos[c + d*x]^(m - 1)*Sin[c + d*x]^n, x], x] - Dist[(a*b)/(a^2
 + b^2), Int[(Cos[c + d*x]^(m - 1)*Sin[c + d*x]^(n - 1))/(a*Cos[c + d*x] +
b*Sin[c + d*x]), x], x]) /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0] &&
IGtQ[m, 0] && IGtQ[n, 0]
```

Rule 3111

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*sin[(c_.) + (d_.)*(x_)]^(n_.)*(cos[(c_.)
+ (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(p_), x_Symbol] :> Dis
t[b/(a^2 + b^2), Int[Cos[c + d*x]^m*Sin[c + d*x]^(n - 1)*(a*Cos[c + d*x] +
b*Sin[c + d*x])^(p + 1), x], x] + (Dist[a/(a^2 + b^2), Int[Cos[c + d*x]^(m
- 1)*Sin[c + d*x]^n*(a*Cos[c + d*x] + b*Sin[c + d*x])^(p + 1), x], x] - Dis
t[(a*b)/(a^2 + b^2), Int[Cos[c + d*x]^(m - 1)*Sin[c + d*x]^(n - 1)*(a*Cos[c
 + d*x] + b*Sin[c + d*x])^p, x], x]) /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 +
 b^2, 0] && IGtQ[m, 0] && IGtQ[n, 0] && ILtQ[p, 0]
```

Rule 3133

```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])
/((a_.) + cos[(d_.) + (e_.)*(x_)]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)]), x
_Symbol] :> Simp[((b*B + c*C)*x)/(b^2 + c^2), x] + Simp[((c*B - b*C)*Log[a
+ b*Cos[d + e*x] + c*Sin[d + e*x]])/(e*(b^2 + c^2)), x] /; FreeQ[{a, b, c,
d, e, A, B, C}, x] && NeQ[b^2 + c^2, 0] && EqQ[A*(b^2 + c^2) - a*(b*B + c*C
), 0]
```

Rubi steps

$$\int \frac{\cos^3(x)\sin^3(x)}{(a\cos(x)+b\sin(x))^2}\,dx = \frac{a\int \frac{\cos^2(x)\sin^3(x)}{a\cos(x)+b\sin(x)}\,dx}{a^2+b^2} + \frac{b\int \frac{\cos^3(x)\sin^2(x)}{a\cos(x)+b\sin(x)}\,dx}{a^2+b^2} - \frac{(ab)\int \frac{\cos^2(x)\sin^2(x)}{(a\cos(x)+b\sin(x))^2}\,dx}{a^2+b^2}$$

$$= \frac{a^2\int \cos(x)\sin^3(x)\,dx}{\left(a^2+b^2\right)^2} + 2\frac{(ab)\int \cos^2(x)\sin^2(x)\,dx}{\left(a^2+b^2\right)^2} - 2\frac{\left(a^2b\right)\int \frac{\cos(x)\sin^2(x)}{a\cos(x)+b\sin(x)}\,dx}{\left(a^2+b^2\right)^2} +$$

$$= -2\left(\frac{\left(a^3b\right)\int \sin^2(x)\,dx}{\left(a^2+b^2\right)^3} + \frac{\left(a^2b^2\right)\int \cos(x)\sin(x)\,dx}{\left(a^2+b^2\right)^3} - \frac{\left(a^3b^2\right)\int \frac{\sin(x)}{a\cos(x)+b\sin(x)}\,dx}{\left(a^2+b^2\right)^3}\right) +$$

$$= \frac{2a^3b^3x}{\left(a^2+b^2\right)^4} - \frac{b^2\cos^4(x)}{4\left(a^2+b^2\right)^2} + \frac{a^2\sin^4(x)}{4\left(a^2+b^2\right)^2} - \frac{a^2b^3\sin(x)}{\left(a^2+b^2\right)^3(a\cos(x)+b\sin(x))} - \frac{\left(a^4b^2\right.}{}$$

$$= \frac{2a^3b^3x}{\left(a^2+b^2\right)^4} - \frac{b^2\cos^4(x)}{4\left(a^2+b^2\right)^2} - \frac{a^4b^2\log(a\cos(x)+b\sin(x))}{\left(a^2+b^2\right)^4} + \frac{a^2b^4\log(a\cos(x)+b\,\mathrm{s}}{\left(a^2+b^2\right)^4}$$

Mathematica [C] time = 2.76, size = 409, normalized size = 1.95

$$\frac{16ab\left(a^4-b^4\right)\sin(2x) - 12abx\left(a^2-3b^2\right)\left(3a^2-b^2\right) - 2ab\left(a^2+b^2\right)^2\sin(4x) + \left(a^2-b^2\right)\left(a^2+b^2\right)^2\cos(4x) + \frac{3(a^2}{}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[x]^3*Sin[x]^3)/(a*Cos[x] + b*Sin[x])^2,x]
```

```
[Out] (-12*a*b*(a^2 - 3*b^2)*(3*a^2 - b^2)*x + (6*I)*(a^6 - 15*a^4*b^2 + 15*a^2*b
^4 - b^6)*x - (6*I)*(a^6 - 15*a^4*b^2 + 15*a^2*b^4 - b^6)*ArcTan[Tan[x]] -
4*(a^2 + b^2)*(a^4 - 6*a^2*b^2 + b^4)*Cos[2*x] + (a^2 - b^2)*(a^2 + b^2)^2*
Cos[4*x] + 3*(a^6 - 15*a^4*b^2 + 15*a^2*b^4 - b^6)*Log[(a*Cos[x] + b*Sin[x]
)^2] + (2*b*(a^2 + b^2)*(3*a^4 - 10*a^2*b^2 + 3*b^4)*Sin[x])/(a*Cos[x] + b*
Sin[x]) + (3*(a^2 + b^2)^2*(a*Cos[x]*((-2*I)*(a + I*b)^2*x + (-a^2 + b^2)*L
og[(a*Cos[x] + b*Sin[x])^2]) + b*(2*(a + I*b)*(a*(-1 - I*x) + b*(I + x)) +
(-a^2 + b^2)*Log[(a*Cos[x] + b*Sin[x])^2])*Sin[x] + (2*I)*(a^2 - b^2)*ArcTa
n[Tan[x]]*(a*Cos[x] + b*Sin[x])))/(a*Cos[x] + b*Sin[x]) + 16*a*b*(a^4 - b^4
)*Sin[2*x] - 2*a*b*(a^2 + b^2)^2*Sin[4*x])/(32*(a^2 + b^2)^4)
```

fricas [A] time = 0.50, size = 371, normalized size = 1.77

$$\frac{8\left(a^7+3\,a^5b^2+3\,a^3b^4+ab^6\right)\cos(x)^5 - 8\left(2\,a^7+3\,a^5b^2-ab^6\right)\cos(x)^3 + \left(5\,a^7+21\,a^5b^2+27\,a^3b^4-21\,ab^6-24\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^3*sin(x)^3/(a*cos(x)+b*sin(x))^2,x, algorithm="fricas")
```

```
[Out] 1/32*(8*(a^7 + 3*a^5*b^2 + 3*a^3*b^4 + a*b^6)*cos(x)^5 - 8*(2*a^7 + 3*a^5*b
^2 - a*b^6)*cos(x)^3 + (5*a^7 + 21*a^5*b^2 + 27*a^3*b^4 - 21*a*b^6 - 24*(a^
6*b - 6*a^4*b^3 + a^2*b^5)*x)*cos(x) - 48*((a^5*b^2 - a^3*b^4)*cos(x) + (a^
4*b^3 - a^2*b^5)*sin(x))*log(2*a*b*cos(x)*sin(x) + (a^2 - b^2)*cos(x)^2 + b
^2) + (5*a^6*b - 51*a^4*b^3 - 21*a^2*b^5 + 3*b^7 - 8*(a^6*b + 3*a^4*b^3 + 3
*a^2*b^5 + b^7)*cos(x)^4 + 24*(a^6*b + 2*a^4*b^3 + a^2*b^5)*cos(x)^2 - 24*(
a^5*b^2 - 6*a^3*b^4 + a*b^6)*x)*sin(x))/((a^9 + 4*a^7*b^2 + 6*a^5*b^4 + 4*a
^3*b^6 + a*b^8)*cos(x) + (a^8*b + 4*a^6*b^3 + 6*a^4*b^5 + 4*a^2*b^7 + b^9)*
sin(x))
```

giac [B] time = 0.16, size = 435, normalized size = 2.07

$$-\frac{3\left(a^{5}b-6\,a^{3}b^{3}+ab^{5}\right)x}{4\left(a^{8}+4\,a^{6}b^{2}+6\,a^{4}b^{4}+4\,a^{2}b^{6}+b^{8}\right)}+\frac{3\left(a^{4}b^{2}-a^{2}b^{4}\right)\log\left(\tan(x)^{2}+1\right)}{2\left(a^{8}+4\,a^{6}b^{2}+6\,a^{4}b^{4}+4\,a^{2}b^{6}+b^{8}\right)}-\frac{3\left(a^{4}b^{3}-a^{2}b^{5}\right)\log\left(|b\tan(x)+\right.}{a^{8}b+4\,a^{6}b^{3}+6\,a^{4}b^{5}+4\,a^{2}b^{7}\ -}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^3*sin(x)^3/(a*cos(x)+b*sin(x))^2,x, algorithm="giac")
```

```
[Out] -3/4*(a^5*b - 6*a^3*b^3 + a*b^5)*x/(a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6
 + b^8) + 3/2*(a^4*b^2 - a^2*b^4)*log(tan(x)^2 + 1)/(a^8 + 4*a^6*b^2 + 6*a^
4*b^4 + 4*a^2*b^6 + b^8) - 3*(a^4*b^3 - a^2*b^5)*log(abs(b*tan(x) + a))/(a^
8*b + 4*a^6*b^3 + 6*a^4*b^5 + 4*a^2*b^7 + b^9) + (3*a^4*b^3*tan(x) - 3*a^2*
b^5*tan(x) + 4*a^5*b^2 - 2*a^3*b^4)/((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b
^6 + b^8)*(b*tan(x) + a)) - 1/4*(9*a^4*b^2*tan(x)^4 - 9*a^2*b^4*tan(x)^4 -
5*a^5*b*tan(x)^3 - 2*a^3*b^3*tan(x)^3 + 3*a*b^5*tan(x)^3 + 2*a^6*tan(x)^2 +
 14*a^4*b^2*tan(x)^2 - 24*a^2*b^4*tan(x)^2 - 3*a^5*b*tan(x) + 2*a^3*b^3*tan
(x) + 5*a*b^5*tan(x) + a^6 + 4*a^4*b^2 - 14*a^2*b^4 + b^6)/((a^8 + 4*a^6*b^
2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)*(tan(x)^2 + 1)^2)
```

maple [B] time = 0.13, size = 515, normalized size = 2.45

$$\frac{\left(\tan^{3}(x)\right)a^{3}b^{3}}{2\left(a^{2}+b^{2}\right)^{4}\left(\tan^{2}(x)+1\right)^{2}}-\frac{3\left(\tan^{3}(x)\right)a\,b^{5}}{4\left(a^{2}+b^{2}\right)^{4}\left(\tan^{2}(x)+1\right)^{2}}+\frac{5\left(\tan^{3}(x)\right)a^{5}b}{4\left(a^{2}+b^{2}\right)^{4}\left(\tan^{2}(x)+1\right)^{2}}-\frac{\left(\tan^{2}(x)\right)a^{6}}{2\left(a^{2}+b^{2}\right)^{4}\left(\tan^{2}(x)+1\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)^3*sin(x)^3/(a*cos(x)+b*sin(x))^2,x)
```

```
[Out] 1/2/(a^2+b^2)^4/(tan(x)^2+1)^2*tan(x)^3*a^3*b^3-3/4/(a^2+b^2)^4/(tan(x)^2+1
)^2*tan(x)^3*a*b^5+5/4/(a^2+b^2)^4/(tan(x)^2+1)^2*tan(x)^3*a^5*b-1/2/(a^2+b
```

```
^2)^4/(tan(x)^2+1)^2*tan(x)^2*a^6+1/(a^2+b^2)^4/(tan(x)^2+1)^2*tan(x)^2*a^4
*b^2+3/2/(a^2+b^2)^4/(tan(x)^2+1)^2*tan(x)^2*a^2*b^4+3/4/(a^2+b^2)^4/(tan(x
)^2+1)^2*tan(x)*a^5*b-1/2/(a^2+b^2)^4/(tan(x)^2+1)^2*tan(x)*a^3*b^3-5/4/(a^
2+b^2)^4/(tan(x)^2+1)^2*tan(x)*a*b^5-1/4/(a^2+b^2)^4/(tan(x)^2+1)^2*a^6+5/4
/(a^2+b^2)^4/(tan(x)^2+1)^2*a^4*b^2+5/4/(a^2+b^2)^4/(tan(x)^2+1)^2*a^2*b^4-
1/4/(a^2+b^2)^4/(tan(x)^2+1)^2*b^6+3/2/(a^2+b^2)^4*ln(tan(x)^2+1)*a^4*b^2-3
/2/(a^2+b^2)^4*ln(tan(x)^2+1)*a^2*b^4-3/4/(a^2+b^2)^4*arctan(tan(x))*a^5*b+
9/2/(a^2+b^2)^4*arctan(tan(x))*a^3*b^3-3/4/(a^2+b^2)^4*arctan(tan(x))*a*b^5
+a^3*b^2/(a^2+b^2)^3/(a+b*tan(x))-3*a^4*b^2/(a^2+b^2)^4*ln(a+b*tan(x))+3*a^
2*b^4/(a^2+b^2)^4*ln(a+b*tan(x))
```

maxima [B] time = 0.47, size = 456, normalized size = 2.17

$$-\frac{3\left(a^5b-6\,a^3b^3+ab^5\right)x}{4\left(a^8+4\,a^6b^2+6\,a^4b^4+4\,a^2b^6+b^8\right)}-\frac{3\left(a^4b^2-a^2b^4\right)\log\left(b\tan(x)+a\right)}{a^8+4\,a^6b^2+6\,a^4b^4+4\,a^2b^6+b^8}+\frac{3\left(a^4b^2-a^2b^4\right)\log\left(\tan(x)^2+1\right)}{2\left(a^8+4\,a^6b^2+6\,a^4b^4+4\,a^2b^6+b^8\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^3*sin(x)^3/(a*cos(x)+b*sin(x))^2,x, algorithm="maxima")
```

```
[Out] -3/4*(a^5*b - 6*a^3*b^3 + a*b^5)*x/(a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6
 + b^8) - 3*(a^4*b^2 - a^2*b^4)*log(b*tan(x) + a)/(a^8 + 4*a^6*b^2 + 6*a^4*
b^4 + 4*a^2*b^6 + b^8) + 3/2*(a^4*b^2 - a^2*b^4)*log(tan(x)^2 + 1)/(a^8 + 4
*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8) - 1/4*(a^5 - 10*a^3*b^2 + a*b^4 - 3
*(3*a^3*b^2 - a*b^4)*tan(x)^4 - 3*(a^4*b + a^2*b^3)*tan(x)^3 + (2*a^5 - 17*
a^3*b^2 + 5*a*b^4)*tan(x)^2 - (2*a^4*b + a^2*b^3 - b^5)*tan(x))/(a^7 + 3*a^
5*b^2 + 3*a^3*b^4 + a*b^6 + (a^6*b + 3*a^4*b^3 + 3*a^2*b^5 + b^7)*tan(x)^5
+ (a^7 + 3*a^5*b^2 + 3*a^3*b^4 + a*b^6)*tan(x)^4 + 2*(a^6*b + 3*a^4*b^3 + 3
*a^2*b^5 + b^7)*tan(x)^3 + 2*(a^7 + 3*a^5*b^2 + 3*a^3*b^4 + a*b^6)*tan(x)^2
 + (a^6*b + 3*a^4*b^3 + 3*a^2*b^5 + b^7)*tan(x))
```

mupad [B] time = 13.92, size = 8198, normalized size = 39.04

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(x)^3*sin(x)^3)/(a*cos(x) + b*sin(x))^2,x)
```

```
[Out] ((tan(x/2)^4*(a*b^2 + 4*a^3))/(a^4 + b^4 + 2*a^2*b^2) - (tan(x/2)^6*(a*b^2
+ 4*a^3))/(a^4 + b^4 + 2*a^2*b^2) - (3*a*b^2*tan(x/2)^2)/(a^4 + b^4 + 2*a^2
*b^2) + (3*a*b^2*tan(x/2)^8)/(a^4 + b^4 + 2*a^2*b^2) + (3*b*tan(x/2)^9*(a^4
 - 3*a^2*b^2))/(2*(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2)) + (3*b*tan(x/2)*(a^4
 - 3*a^2*b^2))/(2*(a^2 + b^2)*(a^4 + b^4 + 2*a^2*b^2)) + (4*b*tan(x/2)^3*(a
^4 + b^4 - 4*a^2*b^2))/((a^2 + b^2)*(a^4 + b^4 + 2*a^2*b^2)) + (4*b*tan(x/2
)^7*(a^4 + b^4 - 4*a^2*b^2))/((a^2 + b^2)*(a^4 + b^4 + 2*a^2*b^2)) - (3*b*t
```

an(x/2)^5*(a^4 + 13*a^2*b^2))/((a^2 + b^2)*(a^4 + b^4 + 2*a^2*b^2)))/(a + 2*b*tan(x/2) + 3*a*tan(x/2)^2 + 2*a*tan(x/2)^4 - 2*a*tan(x/2)^6 - 3*a*tan(x/2)^8 - a*tan(x/2)^10 + 8*b*tan(x/2)^3 + 12*b*tan(x/2)^5 + 8*b*tan(x/2)^7 + 2*b*tan(x/2)^9) + (log(a + 2*b*tan(x/2) - a*tan(x/2)^2)*(3*a^2*b^4 - 3*a^4*b^2))/(a^8 + b^8 + 4*a^2*b^6 + 6*a^4*b^4 + 4*a^6*b^2) - (log(1/(cos(x) + 1))*(96*a^2*b^4 - 96*a^4*b^2))/(2*(16*a^8 + 16*b^8 + 64*a^2*b^6 + 96*a^4*b^4 + 64*a^6*b^2)) + (3*a*b*atan((tan(x/2)*((((6*(45*a^7*b^10 - 18*a^5*b^12 - 135*a^9*b^8 + 99*a^11*b^6 + 9*a^13*b^4))/(a^18 + b^18 + 9*a^2*b^16 + 36*a^4*b^14 + 84*a^6*b^12 + 126*a^8*b^10 + 126*a^10*b^8 + 84*a^12*b^6 + 36*a^14*b^4 + 9*a^16*b^2) - (((6*(6*a^3*b^16 - 153*a^5*b^14 - 180*a^7*b^12 + 357*a^9*b^10 + 534*a^11*b^8 + 81*a^13*b^6 - 72*a^15*b^4 + 3*a^17*b^2))/(a^18 + b^18 + 9*a^2*b^16 + 36*a^4*b^14 + 84*a^6*b^12 + 126*a^8*b^10 + 126*a^10*b^8 + 84*a^12*b^6 + 36*a^14*b^4 + 9*a^16*b^2) - ((96*a^2*b^4 - 96*a^4*b^2)*((6*(8*a^3*b^18 + 112*a^5*b^16 + 464*a^7*b^14 + 880*a^9*b^12 + 800*a^11*b^10 + 208*a^13*b^8 - 208*a^15*b^6 - 176*a^17*b^4 - 40*a^19*b^2))/(a^18 + b^18 + 9*a^2*b^16 + 36*a^4*b^14 + 84*a^6*b^12 + 126*a^8*b^10 + 126*a^10*b^8 + 84*a^12*b^6 + 36*a^14*b^4 + 9*a^16*b^2) - (3*(96*a^2*b^4 - 96*a^4*b^2)*(16*a*b^22 + 160*a^3*b^20 + 720*a^5*b^18 + 1920*a^7*b^16 + 3360*a^9*b^14 + 4032*a^11*b^12 + 3360*a^13*b^10 + 1920*a^15*b^8 + 720*a^17*b^6 + 160*a^19*b^4 + 16*a^21*b^2))/((16*a^8 + 16*b^8 + 64*a^2*b^6 + 96*a^4*b^4 + 64*a^6*b^2)*(a^18 + b^18 + 9*a^2*b^16 + 36*a^4*b^14 + 84*a^6*b^12 + 126*a^8*b^10 + 126*a^10*b^8 + 84*a^12*b^6 + 36*a^14*b^4 + 9*a^16*b^2))))/(2*(16*a^8 + 16*b^8 + 64*a^2*b^6 + 96*a^4*b^4 + 64*a^6*b^2)))*(96*a^2*b^4 - 96*a^4*b^2))/(2*(16*a^8 + 16*b^8 + 64*a^2*b^6 + 96*a^4*b^4 + 64*a^6*b^2)) - (3*a*b*((3*a*b*((6*(8*a^3*b^18 + 112*a^5*b^16 + 464*a^7*b^14 + 880*a^9*b^12 + 800*a^11*b^10 + 208*a^13*b^8 - 208*a^15*b^6 - 176*a^17*b^4 - 40*a^19*b^2))/(a^18 + b^18 + 9*a^2*b^16 + 36*a^4*b^14 + 84*a^6*b^12 + 126*a^8*b^10 + 126*a^10*b^8 + 84*a^12*b^6 + 36*a^14*b^4 + 9*a^16*b^2) - (3*(96*a^2*b^4 - 96*a^4*b^2)*(16*a*b^22 + 160*a^3*b^20 + 720*a^5*b^18 + 1920*a^7*b^16 + 3360*a^9*b^14 + 4032*a^11*b^12 + 3360*a^13*b^10 + 1920*a^15*b^8 + 720*a^17*b^6 + 160*a^19*b^4 + 16*a^21*b^2))/((16*a^8 + 16*b^8 + 64*a^2*b^6 + 96*a^4*b^4 + 64*a^6*b^2)*(a^18 + b^18 + 9*a^2*b^16 + 36*a^4*b^14 + 84*a^6*b^12 + 126*a^8*b^10 + 126*a^10*b^8 + 84*a^12*b^6 + 36*a^14*b^4 + 9*a^16*b^2)))*(2*a*b - a^2 + b^2)*(2*a*b + a^2 - b^2))/(4*(a^8 + b^8 + 4*a^2*b^6 + 6*a^4*b^4 + 4*a^6*b^2)) - (9*a*b*(96*a^2*b^4 - 96*a^4*b^2)*(2*a*b - a^2 + b^2)*(2*a*b + a^2 - b^2)*(16*a*b^22 + 160*a^3*b^20 + 720*a^5*b^18 + 1920*a^7*b^16 + 3360*a^9*b^14 + 4032*a^11*b^12 + 3360*a^13*b^10 + 1920*a^15*b^8 + 720*a^17*b^6 + 160*a^19*b^4 + 16*a^21*b^2))/(4*(16*a^8 + 16*b^8 + 64*a^2*b^6 + 96*a^4*b^4 + 64*a^6*b^2)*(a^8 + b^8 + 4*a^2*b^6 + 6*a^4*b^4 + 4*a^6*b^2)*(a^18 + b^18 + 9*a^2*b^16 + 36*a^4*b^14 + 84*a^6*b^12 + 126*a^8*b^10 + 126*a^10*b^8 + 84*a^12*b^6 + 36*a^14*b^4 + 9*a^16*b^2)))*(2*a*b - a^2 + b^2)*(2*a*b + a^2 - b^2))/(4*(a^8 + b^8 + 4*a^2*b^6 + 6*a^4*b^4 + 4*a^6*b^2)) + (27*a^2*b^2*(96*a^2*b^4 - 96*a^4*b^2)*(2*a*b - a^2 + b^2)^2*(2*a*b + a^2 - b^2)^2*(16*a*b^22 + 160*a^3*b^20 + 720*a^5*b^18 + 1920*a^7*b^16 + 3360*a^9*b^14 + 4032*a^11*b^12 + 3360*a^13*b^10 + 1920*a^15*b^8 + 720*a^17*b^6 + 160*a^19*b^4 + 16*a^21*b^2))/(16*(16*a^8 + 16*b^8

```
 + 64*a^2*b^6 + 96*a^4*b^4 + 64*a^6*b^2)*(a^8 + b^8 + 4*a^2*b^6 + 6*a^4*b^4
 + 4*a^6*b^2)^2*(a^18 + b^18 + 9*a^2*b^16 + 36*a^4*b^14 + 84*a^6*b^12 + 126
*a^8*b^10 + 126*a^10*b^8 + 84*a^12*b^6 + 36*a^14*b^4 + 9*a^16*b^2)))*(18*a*
b^9 + 18*a^9*b - 280*a^3*b^7 + 556*a^5*b^5 - 280*a^7*b^3))/(a^10 + b^10 + 5
3*a^2*b^8 - 38*a^4*b^6 - 38*a^6*b^4 + 53*a^8*b^2)^2 + ((((96*a^2*b^4 - 96*a
^4*b^2)*((3*a*b*((6*(8*a^3*b^18 + 112*a^5*b^16 + 464*a^7*b^14 + 880*a^9*b^1
2 + 800*a^11*b^10 + 208*a^13*b^8 - 208*a^15*b^6 - 176*a^17*b^4 - 40*a^19*b^
2))/(a^18 + b^18 + 9*a^2*b^16 + 36*a^4*b^14 + 84*a^6*b^12 + 126*a^8*b^10 +
126*a^10*b^8 + 84*a^12*b^6 + 36*a^14*b^4 + 9*a^16*b^2) - (3*(96*a^2*b^4 - 9
6*a^4*b^2)*(16*a*b^22 + 160*a^3*b^20 + 720*a^5*b^18 + 1920*a^7*b^16 + 3360*
a^9*b^14 + 4032*a^11*b^12 + 3360*a^13*b^10 + 1920*a^15*b^8 + 720*a^17*b^6 +
 160*a^19*b^4 + 16*a^21*b^2))/((16*a^8 + 16*b^8 + 64*a^2*b^6 + 96*a^4*b^4 +
 64*a^6*b^2)*(a^18 + b^18 + 9*a^2*b^16 + 36*a^4*b^14 + 84*a^6*b^12 + 126*a^
8*b^10 + 126*a^10*b^8 + 84*a^12*b^6 + 36*a^14*b^4 + 9*a^16*b^2)))*(2*a*b -
a^2 + b^2)*(2*a*b + a^2 - b^2))/(4*(a^8 + b^8 + 4*a^2*b^6 + 6*a^4*b^4 + 4*a
^6*b^2)) - (9*a*b*(96*a^2*b^4 - 96*a^4*b^2)*(2*a*b - a^2 + b^2)*(2*a*b + a^
2 - b^2)*(16*a*b^22 + 160*a^3*b^20 + 720*a^5*b^18 + 1920*a^7*b^16 + 3360*a^
9*b^14 + 4032*a^11*b^12 + 3360*a^13*b^10 + 1920*a^15*b^8 + 720*a^17*b^6 + 1
60*a^19*b^4 + 16*a^21*b^2))/(4*(16*a^8 + 16*b^8 + 64*a^2*b^6 + 96*a^4*b^4 +
 64*a^6*b^2)*(a^8 + b^8 + 4*a^2*b^6 + 6*a^4*b^4 + 4*a^6*b^2)*(a^18 + b^18 +
 9*a^2*b^16 + 36*a^4*b^14 + 84*a^6*b^12 + 126*a^8*b^10 + 126*a^10*b^8 + 84*
a^12*b^6 + 36*a^14*b^4 + 9*a^16*b^2))))/(2*(16*a^8 + 16*b^8 + 64*a^2*b^6 +
96*a^4*b^4 + 64*a^6*b^2)) - (3*a*b*((6*(6*a^3*b^16 - 153*a^5*b^14 - 180*a^7
*b^12 + 357*a^9*b^10 + 534*a^11*b^8 + 81*a^13*b^6 - 72*a^15*b^4 + 3*a^17*b^
2))/(a^18 + b^18 + 9*a^2*b^16 + 36*a^4*b^14 + 84*a^6*b^12 + 126*a^8*b^10 +
126*a^10*b^8 + 84*a^12*b^6 + 36*a^14*b^4 + 9*a^16*b^2) - ((96*a^2*b^4 - 96*
a^4*b^2)*((6*(8*a^3*b^18 + 112*a^5*b^16 + 464*a^7*b^14 + 880*a^9*b^12 + 800
*a^11*b^10 + 208*a^13*b^8 - 208*a^15*b^6 - 176*a^17*b^4 - 40*a^19*b^2))/(a^
18 + b^18 + 9*a^2*b^16 + 36*a^4*b^14 + 84*a^6*b^12 + 126*a^8*b^10 + 126*a^1
0*b^8 + 84*a^12*b^6 + 36*a^14*b^4 + 9*a^16*b^2) - (3*(96*a^2*b^4 - 96*a^4*b
^2)*(16*a*b^22 + 160*a^3*b^20 + 720*a^5*b^18 + 1920*a^7*b^16 + 3360*a^9*b^1
4 + 4032*a^11*b^12 + 3360*a^13*b^10 + 1920*a^15*b^8 + 720*a^17*b^6 + 160*a^
19*b^4 + 16*a^21*b^2))/((16*a^8 + 16*b^8 + 64*a^2*b^6 + 96*a^4*b^4 + 64*a^6
*b^2)*(a^18 + b^18 + 9*a^2*b^16 + 36*a^4*b^14 + 84*a^6*b^12 + 126*a^8*b^10
+ 126*a^10*b^8 + 84*a^12*b^6 + 36*a^14*b^4 + 9*a^16*b^2))))/(2*(16*a^8 + 16
*b^8 + 64*a^2*b^6 + 96*a^4*b^4 + 64*a^6*b^2)))*(2*a*b - a^2 + b^2)*(2*a*b +
 a^2 - b^2))/(4*(a^8 + b^8 + 4*a^2*b^6 + 6*a^4*b^4 + 4*a^6*b^2)) + (81*a^3*
b^3*(2*a*b - a^2 + b^2)^3*(2*a*b + a^2 - b^2)^3*(16*a*b^22 + 160*a^3*b^20 +
 720*a^5*b^18 + 1920*a^7*b^16 + 3360*a^9*b^14 + 4032*a^11*b^12 + 3360*a^13*
b^10 + 1920*a^15*b^8 + 720*a^17*b^6 + 160*a^19*b^4 + 16*a^21*b^2))/(32*(a^8
 + b^8 + 4*a^2*b^6 + 6*a^4*b^4 + 4*a^6*b^2)^3*(a^18 + b^18 + 9*a^2*b^16 + 3
6*a^4*b^14 + 84*a^6*b^12 + 126*a^8*b^10 + 126*a^10*b^8 + 84*a^12*b^6 + 36*a
^14*b^4 + 9*a^16*b^2)))*(a^10 - b^10 + 109*a^2*b^8 - 466*a^4*b^6 + 466*a^6*
b^4 - 109*a^8*b^2))/(a^10 + b^10 + 53*a^2*b^8 - 38*a^4*b^6 - 38*a^6*b^4 + 5
3*a^8*b^2)^2)*(2*a^22 + 2*b^22 + 22*a^2*b^20 + 110*a^4*b^18 + 330*a^6*b^16
```

+ 660*a^8*b^14 + 924*a^10*b^12 + 924*a^12*b^10 + 660*a^14*b^8 + 330*a^16*b^
6 + 110*a^18*b^4 + 22*a^20*b^2))/(27*a^4*b^7 - 162*a^6*b^5 + 27*a^8*b^3) +
(((6*(27*a^6*b^11 - 117*a^8*b^9 + 117*a^10*b^7 - 27*a^12*b^5))/(a^18 + b^18
 + 9*a^2*b^16 + 36*a^4*b^14 + 84*a^6*b^12 + 126*a^8*b^10 + 126*a^10*b^8 + 8
4*a^12*b^6 + 36*a^14*b^4 + 9*a^16*b^2) + ((96*a^2*b^4 - 96*a^4*b^2)*((6*(21
*a^4*b^15 + 30*a^6*b^13 - 69*a^8*b^11 - 156*a^10*b^9 - 69*a^12*b^7 + 30*a^1
4*b^5 + 21*a^16*b^3))/(a^18 + b^18 + 9*a^2*b^16 + 36*a^4*b^14 + 84*a^6*b^12
 + 126*a^8*b^10 + 126*a^10*b^8 + 84*a^12*b^6 + 36*a^14*b^4 + 9*a^16*b^2) -
((96*a^2*b^4 - 96*a^4*b^2)*((6*(4*a^20*b - 4*a^2*b^19 + 20*a^4*b^17 + 160*a
^6*b^15 + 320*a^8*b^13 + 184*a^10*b^11 - 184*a^12*b^9 - 320*a^14*b^7 - 160*
a^16*b^5 - 20*a^18*b^3))/(a^18 + b^18 + 9*a^2*b^16 + 36*a^4*b^14 + 84*a^6*b
^12 + 126*a^8*b^10 + 126*a^10*b^8 + 84*a^12*b^6 + 36*a^14*b^4 + 9*a^16*b^2)
 + (3*(96*a^2*b^4 - 96*a^4*b^2)*(16*a^22*b + 16*a^2*b^21 + 160*a^4*b^19 + 7
20*a^6*b^17 + 1920*a^8*b^15 + 3360*a^10*b^13 + 4032*a^12*b^11 + 3360*a^14*b
^9 + 1920*a^16*b^7 + 720*a^18*b^5 + 160*a^20*b^3))/((16*a^8 + 16*b^8 + 64*a
^2*b^6 + 96*a^4*b^4 + 64*a^6*b^2)*(a^18 + b^18 + 9*a^2*b^16 + 36*a^4*b^14 +
 84*a^6*b^12 + 126*a^8*b^10 + 126*a^10*b^8 + 84*a^12*b^6 + 36*a^14*b^4 + 9*
a^16*b^2))))/(2*(16*a^8 + 16*b^8 + 64*a^2*b^6 + 96*a^4*b^4 + 64*a^6*b^2))))
/(2*(16*a^8 + 16*b^8 + 64*a^2*b^6 + 96*a^4*b^4 + 64*a^6*b^2)) + (3*a*b*((3*
a*b*((6*(4*a^20*b - 4*a^2*b^19 + 20*a^4*b^17 + 160*a^6*b^15 + 320*a^8*b^13
+ 184*a^10*b^11 - 184*a^12*b^9 - 320*a^14*b^7 - 160*a^16*b^5 - 20*a^18*b^3)
)/(a^18 + b^18 + 9*a^2*b^16 + 36*a^4*b^14 + 84*a^6*b^12 + 126*a^8*b^10 + 12
6*a^10*b^8 + 84*a^12*b^6 + 36*a^14*b^4 + 9*a^16*b^2) + (3*(96*a^2*b^4 - 96*
a^4*b^2)*(16*a^22*b + 16*a^2*b^21 + 160*a^4*b^19 + 720*a^6*b^17 + 1920*a^8*
b^15 + 3360*a^10*b^13 + 4032*a^12*b^11 + 3360*a^14*b^9 + 1920*a^16*b^7 + 72
0*a^18*b^5 + 160*a^20*b^3))/((16*a^8 + 16*b^8 + 64*a^2*b^6 + 96*a^4*b^4 + 6
4*a^6*b^2)*(a^18 + b^18 + 9*a^2*b^16 + 36*a^4*b^14 + 84*a^6*b^12 + 126*a^8*
b^10 + 126*a^10*b^8 + 84*a^12*b^6 + 36*a^14*b^4 + 9*a^16*b^2)))*(2*a*b - a^
2 + b^2)*(2*a*b + a^2 - b^2))/(4*(a^8 + b^8 + 4*a^2*b^6 + 6*a^4*b^4 + 4*a^6
*b^2)) + (9*a*b*(96*a^2*b^4 - 96*a^4*b^2)*(2*a*b - a^2 + b^2)*(2*a*b + a^2
- b^2)*(16*a^22*b + 16*a^2*b^21 + 160*a^4*b^19 + 720*a^6*b^17 + 1920*a^8*b^
15 + 3360*a^10*b^13 + 4032*a^12*b^11 + 3360*a^14*b^9 + 1920*a^16*b^7 + 720*
a^18*b^5 + 160*a^20*b^3))/(4*(16*a^8 + 16*b^8 + 64*a^2*b^6 + 96*a^4*b^4 + 6
4*a^6*b^2)*(a^8 + b^8 + 4*a^2*b^6 + 6*a^4*b^4 + 4*a^6*b^2)*(a^18 + b^18 + 9
*a^2*b^16 + 36*a^4*b^14 + 84*a^6*b^12 + 126*a^8*b^10 + 126*a^10*b^8 + 84*a^
12*b^6 + 36*a^14*b^4 + 9*a^16*b^2)))*(2*a*b - a^2 + b^2)*(2*a*b + a^2 - b^2
))/(4*(a^8 + b^8 + 4*a^2*b^6 + 6*a^4*b^4 + 4*a^6*b^2)) + (27*a^2*b^2*(96*a^
2*b^4 - 96*a^4*b^2)*(2*a*b - a^2 + b^2)^2*(2*a*b + a^2 - b^2)^2*(16*a^22*b
+ 16*a^2*b^21 + 160*a^4*b^19 + 720*a^6*b^17 + 1920*a^8*b^15 + 3360*a^10*b^1
3 + 4032*a^12*b^11 + 3360*a^14*b^9 + 1920*a^16*b^7 + 720*a^18*b^5 + 160*a^2
0*b^3))/(16*(16*a^8 + 16*b^8 + 64*a^2*b^6 + 96*a^4*b^4 + 64*a^6*b^2)*(a^8 +
 b^8 + 4*a^2*b^6 + 6*a^4*b^4 + 4*a^6*b^2)^2*(a^18 + b^18 + 9*a^2*b^16 + 36*
a^4*b^14 + 84*a^6*b^12 + 126*a^8*b^10 + 126*a^10*b^8 + 84*a^12*b^6 + 36*a^1
4*b^4 + 9*a^16*b^2)))*(18*a*b^9 + 18*a^9*b - 280*a^3*b^7 + 556*a^5*b^5 - 28
0*a^7*b^3)*(2*a^22 + 2*b^22 + 22*a^2*b^20 + 110*a^4*b^18 + 330*a^6*b^16 + 6

```
60*a^8*b^14 + 924*a^10*b^12 + 924*a^12*b^10 + 660*a^14*b^8 + 330*a^16*b^6 +
 110*a^18*b^4 + 22*a^20*b^2))/((27*a^4*b^7 - 162*a^6*b^5 + 27*a^8*b^3)*(a^1
0 + b^10 + 53*a^2*b^8 - 38*a^4*b^6 - 38*a^6*b^4 + 53*a^8*b^2)^2) + (((3*a*b
*((6*(21*a^4*b^15 + 30*a^6*b^13 - 69*a^8*b^11 - 156*a^10*b^9 - 69*a^12*b^7
+ 30*a^14*b^5 + 21*a^16*b^3))/(a^18 + b^18 + 9*a^2*b^16 + 36*a^4*b^14 + 84*
a^6*b^12 + 126*a^8*b^10 + 126*a^10*b^8 + 84*a^12*b^6 + 36*a^14*b^4 + 9*a^16
*b^2) - ((96*a^2*b^4 - 96*a^4*b^2)*((6*(4*a^20*b - 4*a^2*b^19 + 20*a^4*b^17
 + 160*a^6*b^15 + 320*a^8*b^13 + 184*a^10*b^11 - 184*a^12*b^9 - 320*a^14*b^
7 - 160*a^16*b^5 - 20*a^18*b^3))/(a^18 + b^18 + 9*a^2*b^16 + 36*a^4*b^14 +
84*a^6*b^12 + 126*a^8*b^10 + 126*a^10*b^8 + 84*a^12*b^6 + 36*a^14*b^4 + 9*a
^16*b^2) + (3*(96*a^2*b^4 - 96*a^4*b^2)*(16*a^22*b + 16*a^2*b^21 + 160*a^4*
b^19 + 720*a^6*b^17 + 1920*a^8*b^15 + 3360*a^10*b^13 + 4032*a^12*b^11 + 336
0*a^14*b^9 + 1920*a^16*b^7 + 720*a^18*b^5 + 160*a^20*b^3))/((16*a^8 + 16*b^
8 + 64*a^2*b^6 + 96*a^4*b^4 + 64*a^6*b^2)*(a^18 + b^18 + 9*a^2*b^16 + 36*a^
4*b^14 + 84*a^6*b^12 + 126*a^8*b^10 + 126*a^10*b^8 + 84*a^12*b^6 + 36*a^14*
b^4 + 9*a^16*b^2))))/(2*(16*a^8 + 16*b^8 + 64*a^2*b^6 + 96*a^4*b^4 + 64*a^6
*b^2)))*(2*a*b - a^2 + b^2)*(2*a*b + a^2 - b^2))/(4*(a^8 + b^8 + 4*a^2*b^6
+ 6*a^4*b^4 + 4*a^6*b^2)) - ((96*a^2*b^4 - 96*a^4*b^2)*((3*a*b*((6*(4*a^20*
b - 4*a^2*b^19 + 20*a^4*b^17 + 160*a^6*b^15 + 320*a^8*b^13 + 184*a^10*b^11
- 184*a^12*b^9 - 320*a^14*b^7 - 160*a^16*b^5 - 20*a^18*b^3))/(a^18 + b^18 +
 9*a^2*b^16 + 36*a^4*b^14 + 84*a^6*b^12 + 126*a^8*b^10 + 126*a^10*b^8 + 84*
a^12*b^6 + 36*a^14*b^4 + 9*a^16*b^2) + (3*(96*a^2*b^4 - 96*a^4*b^2)*(16*a^2
2*b + 16*a^2*b^21 + 160*a^4*b^19 + 720*a^6*b^17 + 1920*a^8*b^15 + 3360*a^10
*b^13 + 4032*a^12*b^11 + 3360*a^14*b^9 + 1920*a^16*b^7 + 720*a^18*b^5 + 160
*a^20*b^3))/((16*a^8 + 16*b^8 + 64*a^2*b^6 + 96*a^4*b^4 + 64*a^6*b^2)*(a^18
 + b^18 + 9*a^2*b^16 + 36*a^4*b^14 + 84*a^6*b^12 + 126*a^8*b^10 + 126*a^10*
b^8 + 84*a^12*b^6 + 36*a^14*b^4 + 9*a^16*b^2)))*(2*a*b - a^2 + b^2)*(2*a*b
+ a^2 - b^2))/(4*(a^8 + b^8 + 4*a^2*b^6 + 6*a^4*b^4 + 4*a^6*b^2)) + (9*a*b*
(96*a^2*b^4 - 96*a^4*b^2)*(2*a*b - a^2 + b^2)*(2*a*b + a^2 - b^2)*(16*a^22*
b + 16*a^2*b^21 + 160*a^4*b^19 + 720*a^6*b^17 + 1920*a^8*b^15 + 3360*a^10*b
^13 + 4032*a^12*b^11 + 3360*a^14*b^9 + 1920*a^16*b^7 + 720*a^18*b^5 + 160*a
^20*b^3))/(4*(16*a^8 + 16*b^8 + 64*a^2*b^6 + 96*a^4*b^4 + 64*a^6*b^2)*(a^8
+ b^8 + 4*a^2*b^6 + 6*a^4*b^4 + 4*a^6*b^2)*(a^18 + b^18 + 9*a^2*b^16 + 36*a
^4*b^14 + 84*a^6*b^12 + 126*a^8*b^10 + 126*a^10*b^8 + 84*a^12*b^6 + 36*a^14
*b^4 + 9*a^16*b^2))))/(2*(16*a^8 + 16*b^8 + 64*a^2*b^6 + 96*a^4*b^4 + 64*a^
6*b^2)) + (81*a^3*b^3*(2*a*b - a^2 + b^2)^3*(2*a*b + a^2 - b^2)^3*(16*a^22*
b + 16*a^2*b^21 + 160*a^4*b^19 + 720*a^6*b^17 + 1920*a^8*b^15 + 3360*a^10*b
^13 + 4032*a^12*b^11 + 3360*a^14*b^9 + 1920*a^16*b^7 + 720*a^18*b^5 + 160*a
^20*b^3))/(32*(a^8 + b^8 + 4*a^2*b^6 + 6*a^4*b^4 + 4*a^6*b^2)^3*(a^18 + b^1
8 + 9*a^2*b^16 + 36*a^4*b^14 + 84*a^6*b^12 + 126*a^8*b^10 + 126*a^10*b^8 +
84*a^12*b^6 + 36*a^14*b^4 + 9*a^16*b^2)))*(a^10 - b^10 + 109*a^2*b^8 - 466*
a^4*b^6 + 466*a^6*b^4 - 109*a^8*b^2)*(2*a^22 + 2*b^22 + 22*a^2*b^20 + 110*a
^4*b^18 + 330*a^6*b^16 + 660*a^8*b^14 + 924*a^10*b^12 + 924*a^12*b^10 + 660
*a^14*b^8 + 330*a^16*b^6 + 110*a^18*b^4 + 22*a^20*b^2))/((27*a^4*b^7 - 162*
a^6*b^5 + 27*a^8*b^3)*(a^10 + b^10 + 53*a^2*b^8 - 38*a^4*b^6 - 38*a^6*b^4 +
```

```
53*a^8*b^2)^2))*(2*a*b - a^2 + b^2)*(2*a*b + a^2 - b^2))/(2*(a^8 + b^8 + 4
*a^2*b^6 + 6*a^4*b^4 + 4*a^6*b^2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)**3*sin(x)**3/(a*cos(x)+b*sin(x))**2,x)
```

```
[Out] Timed out
```

3.293 $\int \frac{\tan(x)}{b\cos(x)+a\sin(x)}\,dx$

Optimal. Leaf size=47

$$\frac{b\tanh^{-1}\left(\frac{a\cos(x)-b\sin(x)}{\sqrt{a^2+b^2}}\right)}{a\sqrt{a^2+b^2}}+\frac{\tanh^{-1}(\sin(x))}{a}$$

```
[Out] arctanh(sin(x))/a+b*arctanh((a*cos(x)-b*sin(x))/(a^2+b^2)^(1/2))/a/(a^2+b^2
)^(1/2)
```

Rubi [A] time = 0.08, antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3110, 3770, 3074, 206}

$$\frac{b\tanh^{-1}\left(\frac{a\cos(x)-b\sin(x)}{\sqrt{a^2+b^2}}\right)}{a\sqrt{a^2+b^2}}+\frac{\tanh^{-1}(\sin(x))}{a}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[x]/(b*Cos[x] + a*Sin[x]),x]
```

```
[Out] ArcTanh[Sin[x]]/a + (b*ArcTanh[(a*Cos[x] - b*Sin[x])/Sqrt[a^2 + b^2]])/(a*S
qrt[a^2 + b^2])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3074

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x
_Symbol] :> -Dist[d^(-1), Subst[Int[1/(a^2 + b^2 - x^2), x], x, b*Cos[c + d
*x] - a*Sin[c + d*x]], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

Rule 3110

```
Int[(cos[(c_.) + (d_.)*(x_)]^(m_.)*sin[(c_.) + (d_.)*(x_)]^(n_.))/(cos[(c_.
) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> Int[Ex
pandTrig[(cos[c + d*x]^m*sin[c + d*x]^n)/(a*cos[c + d*x] + b*sin[c + d*x]),
 x], x] /; FreeQ[{a, b, c, d, m, n}, x] && IntegersQ[m, n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\tan(x)}{b\cos(x)+a\sin(x)}\,dx &= \int \left(\frac{\sec(x)}{a}-\frac{b}{a(b\cos(x)+a\sin(x))}\right)dx\\ &= \frac{\int \sec(x)\,dx}{a}-\frac{b\int \frac{1}{b\cos(x)+a\sin(x)}\,dx}{a}\\ &= \frac{\tanh^{-1}(\sin(x))}{a}+\frac{b\,\text{Subst}\left(\int \frac{1}{a^2+b^2-x^2}\,dx,x,a\cos(x)-b\sin(x)\right)}{a}\\ &= \frac{\tanh^{-1}(\sin(x))}{a}+\frac{b\tanh^{-1}\left(\frac{a\cos(x)-b\sin(x)}{\sqrt{a^2+b^2}}\right)}{a\sqrt{a^2+b^2}}\end{aligned}$$

Mathematica [A] time = 0.11, size = 76, normalized size = 1.62

$$\frac{-\frac{2b\tanh^{-1}\left(\frac{b\tan\left(\frac{x}{2}\right)-a}{\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}}-\log\left(\cos\left(\frac{x}{2}\right)-\sin\left(\frac{x}{2}\right)\right)+\log\left(\sin\left(\frac{x}{2}\right)+\cos\left(\frac{x}{2}\right)\right)}{a}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[x]/(b*Cos[x] + a*Sin[x]),x]
```

```
[Out] ((-2*b*ArcTanh[(-a + b*Tan[x/2])/Sqrt[a^2 + b^2]])/Sqrt[a^2 + b^2] - Log[Co
s[x/2] - Sin[x/2]] + Log[Cos[x/2] + Sin[x/2]])/a
```

fricas [B] time = 0.70, size = 140, normalized size = 2.98

$$\frac{\sqrt{a^2+b^2}\,b\log\left(\frac{2ab\cos(x)\sin(x)-\left(a^2-b^2\right)\cos(x)^2-a^2-2b^2-2\sqrt{a^2+b^2}\,(a\cos(x)-b\sin(x))}{2ab\cos(x)\sin(x)-\left(a^2-b^2\right)\cos(x)^2+a^2}\right)+\left(a^2+b^2\right)\log(\sin(x)+1)-\left(a^2+b^2\right.}{2\left(a^3+ab^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)/(b*cos(x)+a*sin(x)),x, algorithm="fricas")
```

```
[Out] 1/2*(sqrt(a^2 + b^2)*b*log((2*a*b*cos(x)*sin(x) - (a^2 - b^2)*cos(x)^2 - a^
2 - 2*b^2 - 2*sqrt(a^2 + b^2)*(a*cos(x) - b*sin(x)))/(2*a*b*cos(x)*sin(x) -
 (a^2 - b^2)*cos(x)^2 + a^2)) + (a^2 + b^2)*log(sin(x) + 1) - (a^2 + b^2)*l
og(-sin(x) + 1))/(a^3 + a*b^2)
```

giac [**B**] time = 1.78, size = 90, normalized size = 1.91

$$\frac{b\log\left(\frac{\left|2\,b\tan\left(\frac{1}{2}\,x\right)-2\,a-2\,\sqrt{a^2+b^2}\right|}{\left|2\,b\tan\left(\frac{1}{2}\,x\right)-2\,a+2\,\sqrt{a^2+b^2}\right|}\right)}{\sqrt{a^2+b^2}\,a}+\frac{\log\left(\left|\tan\left(\frac{1}{2}\,x\right)+1\right|\right)}{a}-\frac{\log\left(\left|\tan\left(\frac{1}{2}\,x\right)-1\right|\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)/(b*cos(x)+a*sin(x)),x, algorithm="giac")
```

```
[Out] b*log(abs(2*b*tan(1/2*x) - 2*a - 2*sqrt(a^2 + b^2))/abs(2*b*tan(1/2*x) - 2*
a + 2*sqrt(a^2 + b^2)))/(sqrt(a^2 + b^2)*a) + log(abs(tan(1/2*x) + 1))/a -
log(abs(tan(1/2*x) - 1))/a
```

maple [**A**] time = 0.11, size = 63, normalized size = 1.34

$$-\frac{\ln\left(\tan\left(\frac{x}{2}\right)-1\right)}{a}-\frac{2b\,\mathrm{arctanh}\left(\frac{2b\tan\left(\frac{x}{2}\right)-2a}{2\sqrt{a^2+b^2}}\right)}{a\sqrt{a^2+b^2}}+\frac{\ln\left(\tan\left(\frac{x}{2}\right)+1\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(x)/(b*cos(x)+a*sin(x)),x)
```

```
[Out] -1/a*ln(tan(1/2*x)-1)-2*b/a/(a^2+b^2)^(1/2)*arctanh(1/2*(2*b*tan(1/2*x)-2*a
)/(a^2+b^2)^(1/2))+1/a*ln(tan(1/2*x)+1)
```

maxima [**B**] time = 0.45, size = 98, normalized size = 2.09

$$\frac{b\log\left(\frac{a-\frac{b\sin(x)}{\cos(x)+1}+\sqrt{a^2+b^2}}{a-\frac{b\sin(x)}{\cos(x)+1}-\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}\,a}+\frac{\log\left(\frac{\sin(x)}{\cos(x)+1}+1\right)}{a}-\frac{\log\left(\frac{\sin(x)}{\cos(x)+1}-1\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)/(b*cos(x)+a*sin(x)),x, algorithm="maxima")
```

```
[Out] b*log((a - b*sin(x)/(cos(x) + 1) + sqrt(a^2 + b^2))/(a - b*sin(x)/(cos(x) +
 1) - sqrt(a^2 + b^2)))/(sqrt(a^2 + b^2)*a) + log(sin(x)/(cos(x) + 1) + 1)/
a - log(sin(x)/(cos(x) + 1) - 1)/a
```

mupad [B] time = 1.48, size = 408, normalized size = 8.68

$$\frac{2\,\mathrm{atanh}\left(\tan\left(\frac{x}{2}\right)\right)}{a}-\frac{2\,b\,\mathrm{atanh}\left(\frac{64\,b^3}{\sqrt{a^2+b^2}\left(128\,b^2\tan\left(\frac{x}{2}\right)-\frac{128\,b^4\tan\left(\frac{x}{2}\right)}{a^2+b^2}+\frac{64\,a\,b^3}{a^2+b^2}\right)}-\frac{64\,b^5}{\left(a^2+b^2\right)^{3/2}\left(128\,b^2\tan\left(\frac{x}{2}\right)-\frac{128\,b^4\tan\left(\frac{x}{2}\right)}{a^2+b^2}+\frac{64\,a\,b^3}{a^2+b^2}\right)}+ \cdots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(x)/(b*cos(x) + a*sin(x)),x)
```

```
[Out] (2*atanh(tan(x/2)))/a - (2*b*atanh((64*b^3)/((a^2 + b^2)^(1/2)*(128*b^2*tan
(x/2) - (128*b^4*tan(x/2))/(a^2 + b^2) + (64*a*b^3)/(a^2 + b^2))) - (64*b^5
)/((a^2 + b^2)^(3/2)*(128*b^2*tan(x/2) - (128*b^4*tan(x/2))/(a^2 + b^2) + (
64*a*b^3)/(a^2 + b^2))) + (128*b^4*tan(x/2))/((a^2 + b^2)^(1/2)*((64*a^2*b^
3)/(a^2 + b^2) + 128*a*b^2*tan(x/2) - (128*a*b^4*tan(x/2))/(a^2 + b^2))) -
(128*b^6*tan(x/2))/((a^2 + b^2)^(3/2)*((64*a^2*b^3)/(a^2 + b^2) + 128*a*b^2
*tan(x/2) - (128*a*b^4*tan(x/2))/(a^2 + b^2))) + (128*a*b^2*tan(x/2))/((a^2
 + b^2)^(1/2)*(128*b^2*tan(x/2) - (128*b^4*tan(x/2))/(a^2 + b^2) + (64*a*b^
3)/(a^2 + b^2))) - (192*a*b^4*tan(x/2))/((a^2 + b^2)^(3/2)*(128*b^2*tan(x/2
) - (128*b^4*tan(x/2))/(a^2 + b^2) + (64*a*b^3)/(a^2 + b^2)))))/(a*(a^2 + b
^2)^(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan(x)}{a\sin(x)+b\cos(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)/(b*cos(x)+a*sin(x)),x)
```

```
[Out] Integral(tan(x)/(a*sin(x) + b*cos(x)), x)
```

3.294 $\int \frac{\cot(x)}{b\cos(x)+a\sin(x)}\,dx$

Optimal. Leaf size=48

$$\frac{a\tanh^{-1}\left(\frac{a\cos(x)-b\sin(x)}{\sqrt{a^2+b^2}}\right)}{b\sqrt{a^2+b^2}}-\frac{\tanh^{-1}(\cos(x))}{b}$$

```
[Out] -arctanh(cos(x))/b+a*arctanh((a*cos(x)-b*sin(x))/(a^2+b^2)^(1/2))/b/(a^2+b^
2)^(1/2)
```

Rubi [A] time = 0.08, antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3110, 3770, 3074, 206}

$$\frac{a\tanh^{-1}\left(\frac{a\cos(x)-b\sin(x)}{\sqrt{a^2+b^2}}\right)}{b\sqrt{a^2+b^2}}-\frac{\tanh^{-1}(\cos(x))}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[x]/(b*Cos[x] + a*Sin[x]),x]
```

```
[Out] -(ArcTanh[Cos[x]]/b) + (a*ArcTanh[(a*Cos[x] - b*Sin[x])/Sqrt[a^2 + b^2]])/(
b*Sqrt[a^2 + b^2])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3074

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x
_Symbol] :> -Dist[d^(-1), Subst[Int[1/(a^2 + b^2 - x^2), x], x, b*Cos[c + d
*x] - a*Sin[c + d*x]], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

Rule 3110

```
Int[(cos[(c_.) + (d_.)*(x_)]^(m_.)*sin[(c_.) + (d_.)*(x_)]^(n_.))/(cos[(c_.
) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> Int[Ex
pandTrig[(cos[c + d*x]^m*sin[c + d*x]^n)/(a*cos[c + d*x] + b*sin[c + d*x]),
 x], x] /; FreeQ[{a, b, c, d, m, n}, x] && IntegersQ[m, n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\cot(x)}{b\cos(x)+a\sin(x)}\,dx &= \int \left(\frac{\csc(x)}{b}-\frac{a}{b(b\cos(x)+a\sin(x))}\right)dx\\ &= \frac{\int \csc(x)\,dx}{b}-\frac{a\int \frac{1}{b\cos(x)+a\sin(x)}\,dx}{b}\\ &= -\frac{\tanh^{-1}(\cos(x))}{b}+\frac{a\,\text{Subst}\left(\int \frac{1}{a^2+b^2-x^2}\,dx,x,a\cos(x)-b\sin(x)\right)}{b}\\ &= -\frac{\tanh^{-1}(\cos(x))}{b}+\frac{a\tanh^{-1}\left(\frac{a\cos(x)-b\sin(x)}{\sqrt{a^2+b^2}}\right)}{b\sqrt{a^2+b^2}}\end{aligned}$$

Mathematica [A] time = 0.08, size = 60, normalized size = 1.25

$$\frac{-\frac{2a\tanh^{-1}\left(\frac{b\tan\left(\frac{x}{2}\right)-a}{\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}}+\log\left(\sin\left(\frac{x}{2}\right)\right)-\log\left(\cos\left(\frac{x}{2}\right)\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[x]/(b*Cos[x] + a*Sin[x]),x]
```

```
[Out] ((-2*a*ArcTanh[(-a + b*Tan[x/2])/Sqrt[a^2 + b^2]])/Sqrt[a^2 + b^2] - Log[Co
s[x/2]] + Log[Sin[x/2]])/b
```

fricas [B] time = 0.47, size = 142, normalized size = 2.96

$$\frac{\sqrt{a^2+b^2}\,a\log\left(\frac{2\,ab\cos(x)\sin(x)-\left(a^2-b^2\right)\cos(x)^2-a^2-2\,b^2-2\,\sqrt{a^2+b^2}\,(a\cos(x)-b\sin(x))}{2\,ab\cos(x)\sin(x)-\left(a^2-b^2\right)\cos(x)^2+a^2}\right)-\left(a^2+b^2\right)\log\left(\frac{1}{2}\,\cos(x)+\frac{1}{2}\right)+\left(a^2+\right.}{2\left(a^2b+b^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)/(b*cos(x)+a*sin(x)),x, algorithm="fricas")
```

```
[Out] 1/2*(sqrt(a^2 + b^2)*a*log((2*a*b*cos(x)*sin(x) - (a^2 - b^2)*cos(x)^2 - a^
2 - 2*b^2 - 2*sqrt(a^2 + b^2)*(a*cos(x) - b*sin(x)))/(2*a*b*cos(x)*sin(x) -
 (a^2 - b^2)*cos(x)^2 + a^2)) - (a^2 + b^2)*log(1/2*cos(x) + 1/2) + (a^2 +
b^2)*log(-1/2*cos(x) + 1/2))/(a^2*b + b^3)
```

giac [A] time = 0.21, size = 75, normalized size = 1.56

$$\frac{a\log\left(\frac{\left|2\,b\tan\left(\frac{1}{2}\,x\right)-2\,a-2\,\sqrt{a^2+b^2}\right|}{\left|2\,b\tan\left(\frac{1}{2}\,x\right)-2\,a+2\,\sqrt{a^2+b^2}\right|}\right)}{\sqrt{a^2+b^2}\,b}+\frac{\log\left(\left|\tan\left(\frac{1}{2}\,x\right)\right|\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)/(b*cos(x)+a*sin(x)),x, algorithm="giac")
```

```
[Out] a*log(abs(2*b*tan(1/2*x) - 2*a - 2*sqrt(a^2 + b^2))/abs(2*b*tan(1/2*x) - 2*
a + 2*sqrt(a^2 + b^2)))/(sqrt(a^2 + b^2)*b) + log(abs(tan(1/2*x)))/b
```

maple [A] time = 0.10, size = 49, normalized size = 1.02

$$-\frac{2a\,\mathrm{arctanh}\left(\frac{2b\tan\left(\frac{x}{2}\right)-2a}{2\sqrt{a^2+b^2}}\right)}{b\sqrt{a^2+b^2}}+\frac{\ln\left(\tan\left(\frac{x}{2}\right)\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(x)/(b*cos(x)+a*sin(x)),x)
```

```
[Out] -2*a/b/(a^2+b^2)^(1/2)*arctanh(1/2*(2*b*tan(1/2*x)-2*a)/(a^2+b^2)^(1/2))+1/
b*ln(tan(1/2*x))
```

maxima [A] time = 0.46, size = 79, normalized size = 1.65

$$\frac{a\log\left(\frac{a-\frac{b\sin(x)}{\cos(x)+1}+\sqrt{a^2+b^2}}{a-\frac{b\sin(x)}{\cos(x)+1}-\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}\,b}+\frac{\log\left(\frac{\sin(x)}{\cos(x)+1}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)/(b*cos(x)+a*sin(x)),x, algorithm="maxima")
```

```
[Out] a*log((a - b*sin(x)/(cos(x) + 1) + sqrt(a^2 + b^2))/(a - b*sin(x)/(cos(x) +
 1) - sqrt(a^2 + b^2)))/(sqrt(a^2 + b^2)*b) + log(sin(x)/(cos(x) + 1))/b
```

mupad [B] time = 0.98, size = 123, normalized size = 2.56

$$\frac{\ln\left(\frac{\sin\left(\frac{x}{2}\right)}{\cos\left(\frac{x}{2}\right)}\right)}{b}-\frac{2\,a\,\mathrm{atanh}\left(\frac{\sqrt{a^2+b^2}\,\left(4\mathrm{i}\,\sin\left(\frac{x}{2}\right)\,a^2+2\mathrm{i}\,\cos\left(\frac{x}{2}\right)\,a\,b+1\mathrm{i}\,\sin\left(\frac{x}{2}\right)\,b^2\right)}{a^3\,\sin\left(\frac{x}{2}\right)\,4\mathrm{i}+a^2\,b\,\cos\left(\frac{x}{2}\right)\,1\mathrm{i}+a\,b^2\,\sin\left(\frac{x}{2}\right)\,3\mathrm{i}+b\,\cos\left(\frac{x}{2}\right)\,\left(a^2+b^2\right)\,1\mathrm{i}}\right)}{b\,\sqrt{a^2+b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(x)/(b*cos(x) + a*sin(x)),x)
```

```
[Out] log(sin(x/2)/cos(x/2))/b - (2*a*atanh(((a^2 + b^2)^(1/2)*(a^2*sin(x/2)*4i +
 b^2*sin(x/2)*1i + a*b*cos(x/2)*2i))/(a^3*sin(x/2)*4i + a^2*b*cos(x/2)*1i +
 a*b^2*sin(x/2)*3i + b*cos(x/2)*(a^2 + b^2)*1i)))/(b*(a^2 + b^2)^(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cot(x)}{a\sin(x)+b\cos(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)/(b*cos(x)+a*sin(x)),x)
```

```
[Out] Integral(cot(x)/(a*sin(x) + b*cos(x)), x)
```

Chapter 4

Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

4.0.1 Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)


(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*          GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)


GradeAntiderivative[result_,optimal_] :=
  If[ExpnType[result]<=ExpnType[optimal],
```

```
    If[FreeQ[result,Complex] || Not[FreeQ[optimal,Complex]],
      If[LeafCount[result]<=2*LeafCount[optimal],
        "A",
      "B"],
    "C"],
  If[FreeQ[result,Integrate] && FreeQ[result,Int],
    "C",
  "F"]]


(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
```

```
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
        Exp,Log,
        Sin,Cos,Tan,Cot,Sec,Csc,
        ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
        Sinh,Cosh,Tanh,Coth,Sech,Csch,
        ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
        Erf, Erfc, Erfi,
        FresnelS, FresnelC,
        ExpIntegralE, ExpIntegralEi, LogIntegral,
        SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
        Gamma, LogGamma, PolyGamma,
        Zeta, PolyLog, ProductLog,
        EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]


AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

4.0.2 Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
```

```
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems

GradeAntiderivative := proc(result,optimal)
local leaf_count_result, leaf_count_optimal,ExpnType_result,ExpnType_optimal,
   debug:=false;

      leaf_count_result:=leafcount(result);
      #do NOT call ExpnType() if leaf size is too large. Recursion problem
      if leaf_count_result > 500000 then
          return "B";
      fi;

      leaf_count_optimal:=leafcount(optimal);

      ExpnType_result:=ExpnType(result);
      ExpnType_optimal:=ExpnType(optimal);

      if debug then
         print("ExpnType_result",ExpnType_result," ExpnType_optimal=",
   ExpnType_optimal);
      fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
#       antiderivative
#   "A" if result can be considered optimal

  #This check below actually is not needed, since I only
  #call this grading only for passed integrals. i.e. I check
  #for "F" before calling this. But no harm of keeping it here.
  #just in case.


  if not type(result,freeof('int')) then
     return "F";
  end if;


  if ExpnType_result<=ExpnType_optimal then
      if debug then
           print("ExpnType_result<=ExpnType_optimal");
      fi;
```

```
      if is_contains_complex(result) then
         if is_contains_complex(optimal) then
            if debug then
               print("both result and optimal complex");
            fi;
            #both result and optimal complex
            if leaf_count_result<=2*leaf_count_optimal then
               return "A";
            else
               return "B";
            end if
         else #result contains complex but optimal is not
            if debug then
               print("result contains complex but optimal is not");
            fi;
            return "C";
         end if
      else # result do not contain complex
           # this assumes optimal do not as well
           if debug then
               print("result do not contain complex, this assumes optimal do not
    as well");
           fi;
           if leaf_count_result<=2*leaf_count_optimal then
              if debug then
                 print("leaf_count_result<=2*leaf_count_optimal");
              fi;
              return "A";
           else
              if debug then
                 print("leaf_count_result>2*leaf_count_optimal");
              fi;
              return "B";
           end if
      end if
  else #ExpnType(result) > ExpnType(optimal)
      if debug then
         print("ExpnType(result) > ExpnType(optimal)");
      fi;
      return "C";
  end if

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
```

```
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
```

```
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:
```

```
#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

4.0.3 Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#              Port of original Maple grading function by
#              Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#              added 'exp_polar'
from sympy import *

def leaf_count(expr):
    #sympy do not have leaf count function. This is approximation
    return round(1.7*count_ops(expr))

def is_sqrt(expr):
    if isinstance(expr,Pow):
        if expr.args[1] == Rational(1,2):
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
            asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
            asinh,acosh,atanh,acoth,asech,acsch
        ]

def is_special_function(func):
    return func in [ erf,erfc,erfi,
             fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
             gamma,loggamma,digamma,zeta,polylog,LambertW,
             elliptic_f,elliptic_e,elliptic_pi,exp_polar
         ]

def is_hypergeometric_function(func):
    return func in [hyper]

def is_appell_function(func):
    return func in [appellf1]
```

```
def is_atom(expn):
    try:
        if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
           return True
        else:
           return False

    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
       return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow):   #type(expn,'`^`')
        if isinstance(expn.args[1],Integer):  #type(op(2,expn),'integer')
            return expnType(expn.args[0])   #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational):  #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)
   ))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,
   ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type
   (expn,'`*`')
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):  #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))  #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,apply(max,map(ExpnType,[op(expn)])))
```

```
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,
    expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,
    Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        return "F"

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    return "A"
                else:
                    return "B"
            else: #result contains complex but optimal is not
                return "C"
        else: # result do not contain complex, this assumes optimal do not as
    well
            if leaf_count_result <= 2*leaf_count_optimal:
                return "A"
            else:
                return "B"
    else:
        return "C"
```

4.0.4 SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#              Albert Rich to use with Sagemath. This is used to
#              grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#              'arctan2','floor','abs','log_integral'

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    debug=False
    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
        if m:
```

```
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    debug=False
    if debug: print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
           'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','
    sinh_integral'
           'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
           'polylog','lambert_w','elliptic_f','elliptic_e',
           'elliptic_pi','exp_integral_e','log_integral']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m


def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U
    ']

def is_appell_function(func):
    return func.name() in ['hypergeometric']   #[appellf1] can't find this in
    sagemath

def is_atom(expn):

    debug=False
    if debug: print ("Enter is_atom")


    #thanks to answer at https://ask.sagemath.org/question/49179/what-is-
    sagemath-equivalent-to-atomic-type-in-maple/
    try:
        if expn.parent() is SR:
```

```
            return expn.operator() is None
        if expn.parent() in (ZZ, QQ, AA, QQbar):
            return expn in expn.parent() # Should always return True
        if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
            return expn in expn.parent().base_ring() or expn in expn.parent().
    gens()
        return False

    except AttributeError as error:
        return False


def expnType(expn):

    if debug:
        print (">>>>>Enter expnType, expn=", expn)
        print (">>>>>is_atom(expn)=", is_atom(expn))

    if is_atom(expn):
        return 1
    elif type(expn)==list:   #isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if  type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0],
    Rational):
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.
    args[0]))
    elif expn.operator() == operator.pow:   #isinstance(expn,Pow)
        if type(expn.operands()[1])==Integer:  #isinstance(expn.args[1],Integer)
            return expnType(expn.operands()[0])   #expnType(expn.args[0])
        elif type(expn.operands()[1])==Rational:  #isinstance(expn.args[1],
    Rational)
            if type(expn.operands()[0])==Rational: #isinstance(expn.args[0],
    Rational)
                return 1
            else:
                return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.
    args[0]))
        else:
            return max(3,expnType(expn.operands()[0]),expnType(expn.operands()
    [1])) #max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1]))
    elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #
    isinstance(expn,Add) or isinstance(expn,Mul)
        m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
        m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
```

```
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.operator()):  #is_elementary_function(expn.
   func)
        return max(3,expnType(expn.operands()[0]))
    elif is_special_function(expn.operator()): #is_special_function(expn.func)
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(
   expn.args)))
        return max(4,m1)   #max(4,m1)
    elif is_hypergeometric_function(expn.operator()): #
   is_hypergeometric_function(expn.func)
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(
   expn.args)))
        return max(5,m1)   #max(5,m1)
    elif is_appell_function(expn.operator()):
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(
   expn.args)))
        return max(6,m1)   #max(6,m1)
    elif str(expn).find("Integral") != -1: #this will never happen, since it
                #is checked before calling the grading function that is passed.
                #but kept it here.
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(
   expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    if debug: print ("Enter grade_antiderivative for sagemath")

    leaf_count_result  = tree_size(result) #leaf_count(result)
    leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

    if debug: print ("leaf_count_result=", leaf_count_result, "
   leaf_count_optimal=",leaf_count_optimal)


    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",
   expnType_optimal)

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
```

```
            if leaf_count_result <= 2*leaf_count_optimal:
                return "A"
            else:
                return "B"
        else: #result contains complex but optimal is not
            return "C"
    else: # result do not contain complex, this assumes optimal do not as
well
        if leaf_count_result <= 2*leaf_count_optimal:
            return "A"
        else:
            return "B"
else:
    return "C"
```