Computer algebra independent integration tests

4-Trig-functions/4.7-Miscellaneous/4.7.7-Trig-functions

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July 17, 2021 Compiled on July 17, 2021 at 11:35pm

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Chapter 1

Introduction

This report gives the result of running the computer algebra independent integration problems. The listing of the problems are maintained by and can be downloaded from `https://rulebasedintegration.org`

The number of integrals in this report is [950]. This is test number [141].

1.1 Listing of CAS systems tested

The following systems were tested at this time.

1. Mathematica 12.3 (64 bit) on windows 10.
2. Rubi 4.16.1 in Mathematica 12.1 on windows 10.
3. Maple 2021.1 (64 bit) on windows 10.
4. Maxima 5.44 on Linux. (via sagemath 9.3)
5. Fricas 1.3.7 on Linux (via sagemath 9.3)
6. Giac/Xcas 1.7 on Linux. (via sagemath 9.3)
7. Sympy 1.8 under Python 3.8.8 using Anaconda distribution on Ubuntu.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 under windows 10 (64 bit)

Maxima, Fricas and Giac/Xcas were called from inside SageMath. This was done using SageMath integrate command by changing the name of the algorithm to use the different CAS systems.

Sympy was called directly using Python.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed.

If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	solved	Failed
Rubi	% 99.37 (944)	% 0.63 (6)
Mathematica	% 98.74 (938)	% 1.26 (12)
Maple	% 95.58 (908)	% 4.42 (42)
Maxima	% 68.53 (651)	% 31.47 (299)
Fricas	% 89.58 (851)	% 10.42 (99)
Sympy	% 44.00 (418)	% 56.00 (532)
Giac	% 74.21 (705)	% 25.79 (245)
Mupad	% 73.68 (700)	% 26.32 (250)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

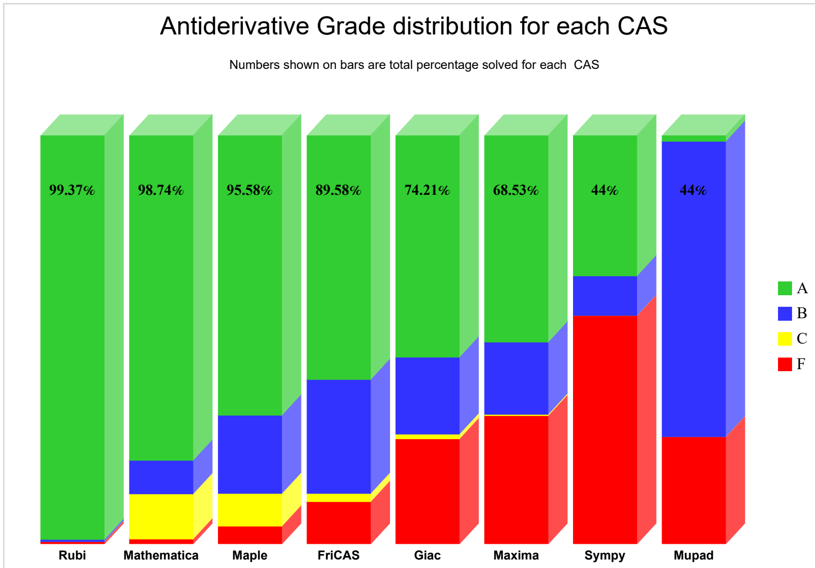
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

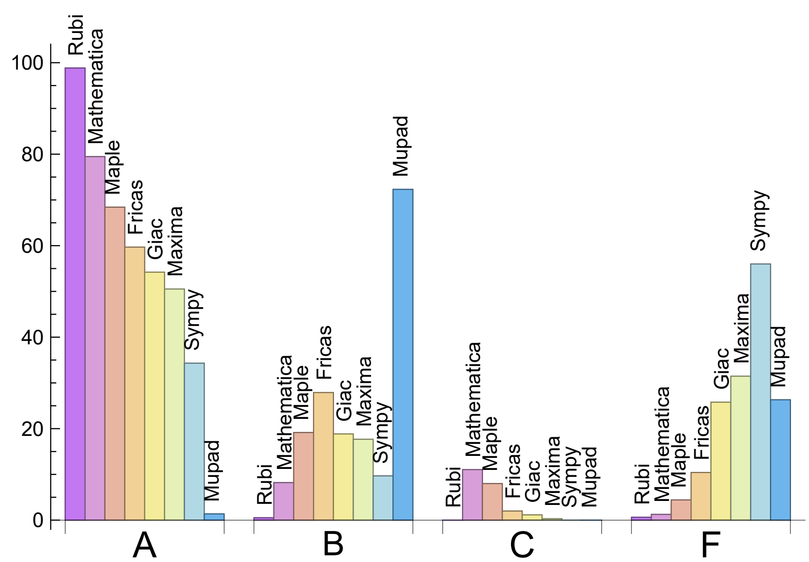
System	% A grade	% B grade	% C grade	% F grade
Rubi	98.84	0.53	0.00	0.63
Mathematica	79.47	8.21	11.05	1.26
Maple	68.42	19.16	8.00	4.42
Maxima	50.53	17.68	0.32	31.47
Fricas	59.68	27.89	2.00	10.42
Sympy	34.32	9.68	0.00	56.00
Giac	54.21	18.84	1.16	25.79
Mupad	1.37	72.32	0.00	26.32

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the CAS systems for each grade level.



The following table shows the distribution of the different types of failure for each CAS. There are 3 types of reasons why it can fail. The first is when CAS returns back the input within the time limit, which means it could not solve it. This the typical normal failure **F** .

The second is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned **F(-1)**.

The third is due to an exception generated. Assigned **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and Giac) or it could be an indication of an internal error in CAS. This type of error requires more investigations to determine the cause.

System	Number failed	Percentage normal failure	Percentage timeout failure	Percentage exception failure
Rubi	6	100.00 %	0.00 %	0.00 %
Mathematica	12	66.67 %	33.33 %	0.00 %
Maple	42	85.71 %	14.29 %	0.00 %
Maxima	299	62.54 %	5.02 %	32.44 %
Fricas	99	79.80 %	0.00 %	20.20 %
Sympy	532	65.04 %	34.59 %	0.38 %
Giac	245	71.43 %	15.10 %	13.47 %
Mupad	250	100.00 %	0.00 %	0.00 %

Table 1.4: Time and leaf size performance for each CAS

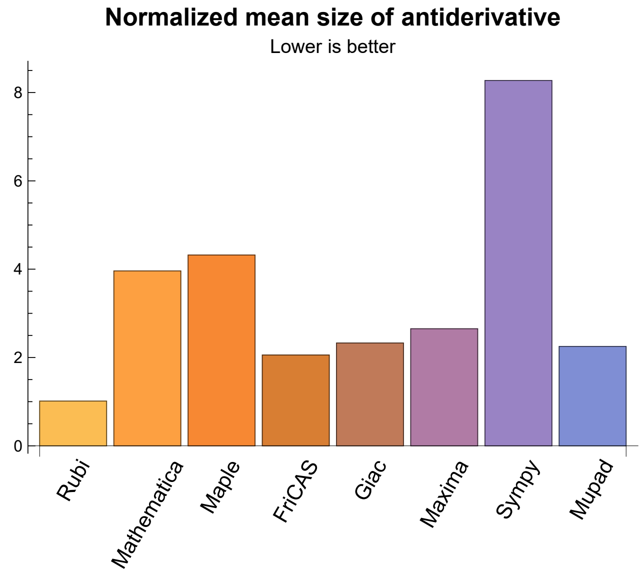
1.3 Performance

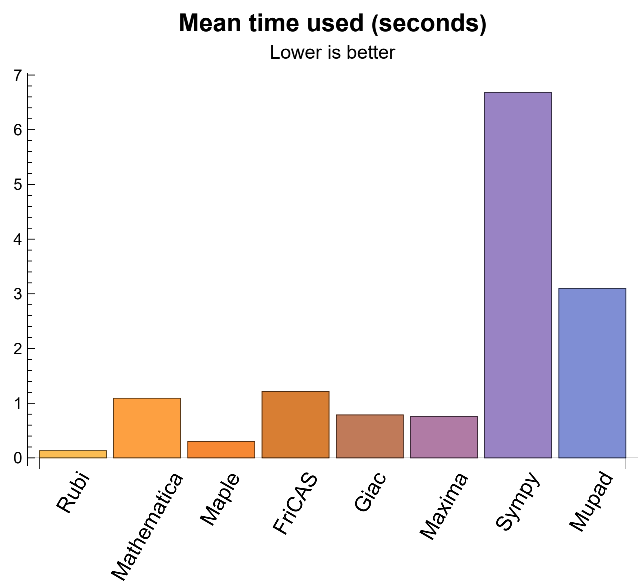
The table below summarizes the performance of each CAS system in terms of CPU time and leaf size of results.

System	Mean time (sec)	Mean size	Normalized mean	Median size	Normalized median
Rubi	0.13	76.23	1.01	43.50	1.00
Mathematica	1.09	373.14	3.96	40.00	1.00
Maple	0.30	699.55	4.32	44.50	1.16
Maxima	0.76	113.26	2.65	29.00	1.00
Fricas	1.22	191.42	2.06	43.00	1.34
Sympy	6.68	233.72	8.27	35.00	1.37
Giac	0.79	154.70	2.33	37.00	1.10
Mupad	3.10	175.14	2.25	29.00	1.03

Table 1.5: Time and leaf size performance for each CAS

The following are bar charts for the normalized leafsize and time used columns from the above table.





1.4 list of integrals that has no closed form antiderivative

{42, 43, 56, 57, 180, 582, 583, 584, 644, 645, 646, 647, 932}

1.5 list of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

Mupad {}

1.6 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not mean necessarily that the anti-derivative is wrong, as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it easier to do further investigation to determine why it was not possible to verify the result produced.

Rubi {}

Mathematica {31, 32, 36, 37, 38, 46, 47, 51, 52, 87, 89, 93, 107, 108, 109, 160, 163, 240, 241, 242, 243, 244, 245, 246, 247, 271, 393, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 430, 431, 432, 433, 434, 437, 438, 439, 440, 448, 449, 450, 451, 452, 455, 462, 463, 464, 465, 466, 469, 497, 556, 557, 558, 559, 560, 561, 566, 588, 597, 630, 859}

Maple Verification phase not implemented yet.

Maxima Verification phase not implemented yet.

Fricas Verification phase not implemented yet.

Sympy Verification phase not implemented yet.

Giac Verification phase not implemented yet.

Mupad Verification phase not implemented yet.

1.7 Timing

The command `AboluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of _int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call has completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out is not counted in the final statistics.

1.8 Verification

A verification phase was applied on the result of integration for Rubi and Mathematica. Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative produced was correct.

Verification phase has 3 minutes time out. An integral whose result was not verified could still be correct. Further investigation is needed on those integrals which failed verifications. Such integrals are marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.9 Important notes about some of the results

1.9.1 Important note about Maxima results

Since these integrals are run in a batch mode, using an automated script, and by using sagemath (SageMath uses Maxima), then any integral where Maxima needs an interactive response from the user to answer a question during evaluation of the integral in order to complete the integration, will fail and is counted as failed.

The exception raised is `ValueError`. Therefore Maxima result below is lower than what could result if Maxima was run directly and each question Maxima asks was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the Timofeev test file, there were about 14 such integrals out of total 705, or about 2 percent. This pecrentage can be higher or lower depending on the specific input test file.

Such integrals can be indentified by looking at the output of the integration in each section for Maxima. The exception message will indicate of the error is due to the interactive question being asked or not.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath loading of Maxima `abs_integrate` was found to cause some problem. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See `https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/` for reference.

1.9.2 Important note about FriCAS and Giac/XCAS results

There are Few integrals which failed due to SageMath not able to translate the result back to SageMath syntax and not because these CAS system were not able to do the integrations.

These will fail With error `Exception raised: NotImplementedError`

The number of such cases seems to be very small. About 1 or 2 percent of all integrals.

Hopefully the next version of SageMath will have complete translation of FriCAS and XCAS syntax and I will re-run all the tests again when this happens.

1.9.3 Important note about finding leaf size of antiderivative

For Mathematica, Rubi and Maple, the buildin system function `LeafSize` is used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special buildin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy and Giac antiderivatives is determined using the following function, thanks to user `slelievre` at `https://`

ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which is called directly from Python, the following code is used to obtain the leafsize of its result

```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
         leafCount =1
```

1.9.4 Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative, Maple was used to determine the leaf size of Mupad output by post processing.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

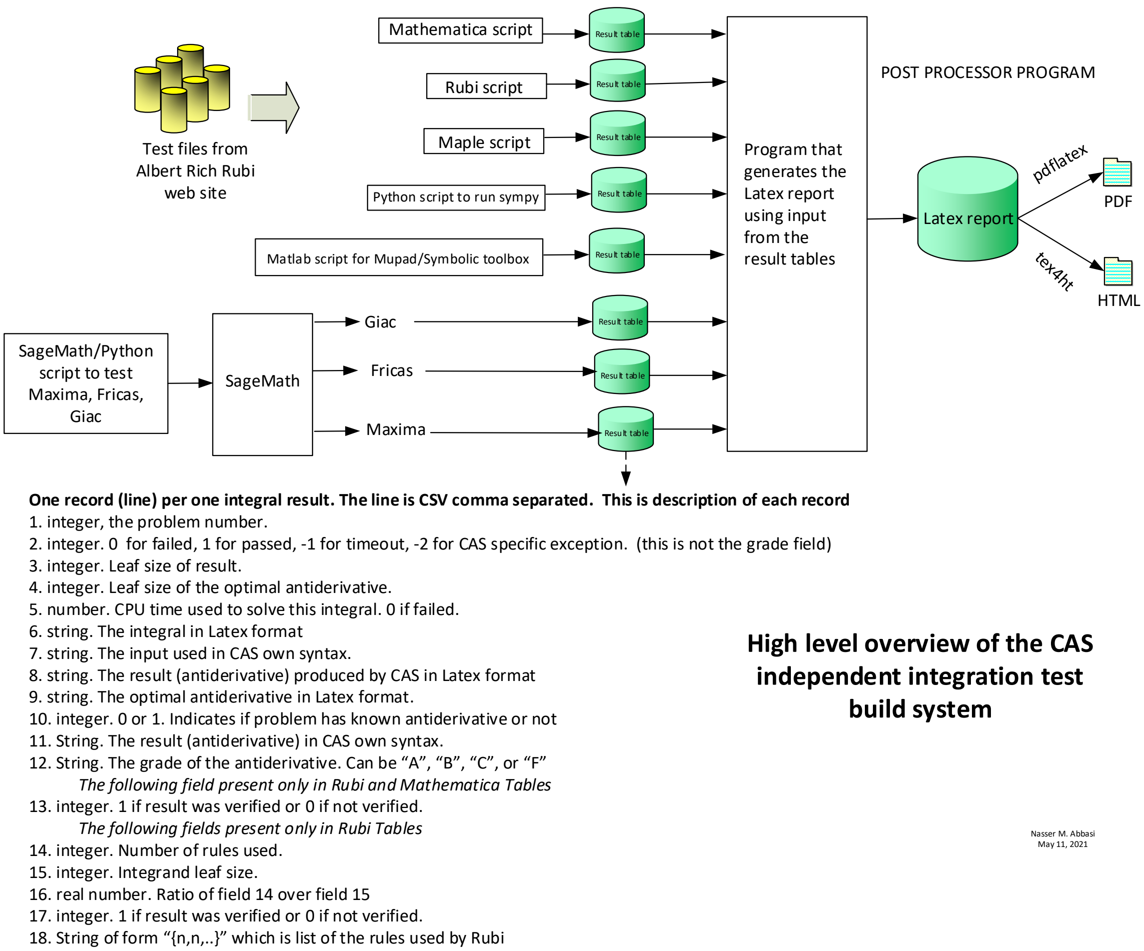
The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives `sin(x)^2/2`

1.10 Design of the test system

The following diagram gives a high level view of the current test build system.



Chapter 2

detailed summary tables of results

2.1 List of integrals sorted by grade for each CAS

2.1.1 Rubi

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 493, 494, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504, 505, 506, 507, 508, 509, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 520, 521, 522, 523, 524, 525, 526, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 549, 550, 551, 552, 553, 554, 556, 557, 558, 559, 560, 561, 562, 563, 564, 565, 566, 567, 568, 569, 570, 571, 572, 573, 574, 575, 576, 577, 578, 579, 580, 581, 582, 583, 584, 585, 586, 587, 588, 589, 590, 591, 592, 593, 594, 595, 596, 597, 598, 599, 600, 601, 602, 603, 604, 605, 606, 607, 608, 609, 610, 611, 612, 613, 614, 615, 616, 617, 618, 619, 620, 621, 622, 623, 624, 625, 626, 627, 628, 629, 630, 631, 632, 633, 634, 635, 636, 637, 638, 639, 640, 641, 642, 643, 644, 645, 646, 647, 648, 649, 650, 651, 652, 653, 654, 655, 656, 657, 658, 659, 660, 661, 662, 663, 664, 665, 666, 667, 668, 669, 670, 671, 672, 673, 674, 675, 676, 677, 678, 679, 680, 681, 682, 683, 684, 685, 686, 687, 688, 689, 690, 691, 692, 693, 694, 695, 696, 697, 698, 699, 700, 701, 702, 703, 704, 705, 706, 707, 708, 709, 710, 711, 712, 713, 714, 715, 716, 717, 718, 719, 720, 721, 722, 723, 724, 725, 726, 727, 728, 729, 730, 731, 732, 733, 734, 735, 736, 737, 738, 739, 740, 741, 742, 743, 744, 745, 746, 747, 748, 749, 750, 751, 752, 753, 754, 755, 756, 757, 758, 760, 761, 762, 763, 764, 765, 766, 767, 768, 769, 770, 771, 772, 773, 774, 775, 776, 777, 778, 779, 780, 781, 782, 783, 784, 785, 786, 787, 788, 789, 790, 791, 792, 793, 794, 795, 797, 798, 799, 800, 801, 802, 803, 804, 805, 806, 807, 808, 809, 810, 811, 812, 813, 814, 815, 816, 817, 818, 819, 820, 821, 822, 823, 824, 825, 826, 827, 828, 829, 830, 831, 832, 833, 834, 835, 836, 837, 838, 839, 840, 841, 842, 843, 844, 845, 846, 847, 848, 849, 850, 851, 852, 853, 854, 855, 856, 857, 861, 862, 863, 864, 865, 866, 867, 868, 869, 870, 871, 872, 873, 874, 875, 876, 877, 878, 879, 880, 881, 882, 883, 884, 885, 886, 887, 888, 889, 890, 891, 892, 893, 894, 895, 896, 897, 898, 899, 900, 901, 902, 903, 904, 905, 906, 907, 908, 909, 910, 911, 913, 916, 917, 918, 919, 920, 921, 922, 923, 924, 925, 926, 927, 928, 929, 930, 932, 934, 935, 936, 937, 938, 939, 940, 941, 942, 943, 944, 945, 946, 947, 948, 949, 950 }

B grade: { 555, 759, 858, 860, 912 }

C grade: { }

F grade: { 796, 859, 914, 915, 931, 933 }

2.1.2 Mathematica

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 32, 33, 39, 40, 41, 42, 43, 44, 45, 47, 48, 49, 50, 53, 54, 55, 56, 57, 58, 59, 60, 61, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 86, 88, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 111, 113, 115, 116, 117, 118, 119, 120, 121, 122, 124, 126, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 161, 162, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 177, 178, 179, 180, 181, 183, 184, 186, 187, 188, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 229, 230, 231, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 264, 265, 266, 268, 270, 272, 273, 275, 276, 279, 282, 283, 284, 285, 286, 288, 289, 290, 291, 292, 293, 294, 295, 296, 298, 299, 300, 301, 302, 303, 304, 305, 307, 308, 309, 311, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 326, 327, 328, 330, 332, 333, 334, 335, 336, 337, 338, 339, 340, 342, 344, 345, 346, 347, 348, 349, 350, 352, 353, 357, 358, 359, 360, 363, 364, 365, 367, 368, 371, 372, 373, 374, 375, 376, 377, 381, 382, 383, 385, 386, 388, 389, 390, 392, 393, 395, 396, 397, 398, 399, 400, 401, 417, 418, 419, 423, 424, 425, 426, 443, 444, 445, 446, 447, 458, 459, 460, 472, 473, 474, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 493, 494, 495, 496, 498, 499, 500, 501, 502, 504, 505, 506, 507, 508, 509, 515, 516, 518, 519, 520, 521, 522, 523, 524, 525, 526, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 550, 551, 552, 553, 554, 555, 562, 563, 564, 565, 566, 567, 568, 569, 570, 571, 572, 573, 574, 575, 576, 577, 578, 579, 580, 582, 583, 584, 585, 586, 587, 589, 590, 591, 592, 593, 594, 595, 596, 598, 599, 600, 601, 602, 603, 604, 605, 606, 607, 608, 609, 610, 611, 612, 613, 614, 615, 616, 617, 618, 619, 620, 621, 622, 623, 624, 625, 626, 627, 628, 629, 630, 631, 632, 633, 637, 639, 641, 642, 643, 644, 645, 646, 647, 648, 649, 650, 651, 652, 653, 655, 656, 658, 659, 660, 661, 662, 663, 664, 665, 666, 667, 668, 669, 670, 671, 672, 674, 675, 676, 678, 679, 680, 681, 682, 683, 684, 685, 686, 687, 688, 689, 690, 692, 693, 694, 695, 696, 697, 698, 699, 700, 701, 702, 703, 704, 706, 707, 708, 714, 715, 716, 717, 718, 719, 720, 721, 722, 723, 724, 725, 726, 727, 729, 730, 731, 732, 733, 734, 735, 736, 737, 738, 739, 740, 741, 742, 743, 744, 745, 746, 747, 748, 749, 750, 751, 752, 753, 754, 755, 756, 757, 758, 761, 762, 763, 764, 765, 766, 767, 768, 769, 770, 771, 772, 773, 774, 775, 776, 777, 778, 779, 780, 782, 783, 785, 786, 787, 788, 789, 790, 791, 792, 793, 794, 795, 796, 797, 798, 799, 800, 801, 804, 808, 809, 810, 811, 812, 813, 814, 815, 816, 817, 818, 819, 820, 821, 822, 823, 824, 825, 827, 828, 829, 830, 831, 832, 833, 835, 836, 837, 838, 839, 840, 841, 842, 843, 844, 845, 846, 847, 848, 849, 850, 851, 852, 853, 854, 855, 856, 857, 858, 859, 860, 862, 863, 864, 865, 866, 867, 868, 869, 870, 871, 872, 873, 874, 875, 876, 877, 878, 879, 880, 881, 882, 883, 884, 886, 887, 888, 889, 890, 891, 892, 893, 894, 895, 896, 897, 898, 899, 900, 901, 902, 903, 905, 906, 907, 908, 909, 911, 913, 914, 915, 916, 917, 918, 919, 920, 921, 922, 923, 924, 925, 926, 928, 929, 930, 931, 932, 933, 934, 935, 936, 937, 938, 939, 940, 941, 942, 943, 944, 946, 947, 948, 949, 950 }

B grade: { 35, 90, 106, 108, 110, 114, 123, 127, 160, 163, 185, 189, 263, 267, 269, 271, 274, 277, 278, 280, 281, 287, 297, 306, 310, 312, 325, 329, 331, 341, 343, 361, 362, 366, 369, 370, 378, 380, 384, 387, 391, 394, 402, 461, 475, 497, 548, 549, 581, 638, 640, 654, 673, 677, 691, 705, 709, 710, 711, 712, 713, 728, 759, 760, 781, 784, 802, 803, 805, 806, 807, 826, 834, 861, 885, 904, 927, 945 }

C grade: { 31, 34, 36, 37, 38, 46, 51, 52, 62, 85, 87, 89, 105, 107, 109, 112, 125, 174, 175, 176, 182, 228, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 351, 354, 355, 356, 379, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 420, 421, 422, 427, 428, 429, 430, 431, 432, 433, 434, 437, 438, 439, 440, 448, 449, 450, 451, 452, 455, 462, 463, 464, 465, 466, 469, 503, 510, 511, 512, 513, 514, 517, 556, 557, 558, 559, 560, 561, 588, 597, 634, 635, 636, 657, 910, 912 }

F grade: { 435, 436, 441, 442, 453, 454, 456, 457, 467, 468, 470, 471 }

2.1.3 Maple

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 35, 36, 37, 38, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 56, 57, 58, 59, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 77, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 143, 144, 150, 151, 152, 159, 162, 180, 184, 185, 186, 187, 188, 189, 190, 191, 192, 201, 202, 203, 204, 206, 207, 208, 209, 211, 212, 213, 216, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 252, 253, 254, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 272, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 292, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 341, 342, 343, 344, 345, 346, 347, 348, 350, 351, 353, 354, 356, 357, 358, 359, 360, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 381, 382, 383, 388, 389, 390, 395, 396, 397, 398, 399, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 436, 437, 438, 442, 444, 446, 459, 460, 461, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 490, 491, 492, 496, 499, 500, 504, 505, 506, 510, 511, 514, 516, 518, 519, 520, 522, 523, 524, 526, 527, 528, 530, 531, 533, 534, 567, 568, 569, 570, 571, 576, 582, 583, 584, 590, 591, 593, 599, 600, 602, 603, 604, 605, 606, 611, 612, 613, 614, 637, 641, 642, 643, 644, 645, 646, 647, 648, 649, 650, 651, 652, 653, 654, 655, 656, 658, 659, 660, 661, 662, 663, 664, 665, 666, 667, 668, 669, 671, 672, 673, 674, 675, 676, 677, 678, 679, 680, 681, 682, 683, 684, 685, 686, 687, 688, 689, 690, 691, 692, 693, 694, 695, 696, 697, 698, 699, 700, 701, 702, 703, 704, 705, 706, 707, 708, 714, 715, 716, 717, 718, 719, 720, 723, 724, 725, 726, 727, 728, 730, 731, 732, 733, 734, 735, 736, 737, 738, 739, 740, 749, 750, 752, 753, 754, 755, 756, 757, 758, 760, 761, 762, 763, 764, 765, 766, 767, 768, 769, 770, 771, 772, 773, 774, 775, 776, 777, 778, 779, 780, 781, 782, 783, 784, 785, 786, 787, 788, 789, 790, 791, 792, 794, 797, 798, 799, 800, 801, 802, 803, 804, 805, 806, 807, 808, 810, 811, 812, 813, 814, 815, 816, 817, 818, 819, 820, 821, 822, 823, 824, 825, 826, 827, 828, 829, 830, 831, 832, 833, 835, 837, 838, 839, 840, 841, 842, 843, 844, 845, 846, 847, 848, 849, 850, 851, 852, 853, 854, 855, 856, 857, 858, 861, 862, 863, 866, 867, 868, 869, 870, 873, 874, 875, 876, 877, 879, 880, 881, 882, 883, 884, 886, 887, 889, 890, 892, 893, 894, 895, 896, 897, 898, 899, 900, 901, 902, 903, 905, 906, 907, 908, 909, 910, 911, 913, 915, 916, 917, 919, 920, 921, 922, 923, 924, 925, 928, 929, 930, 931, 932, 934, 935, 936, 937, 938, 939, 942, 943, 944, 945, 946, 947, 948, 949, 950 }

B grade: { 76, 78, 145, 146, 147, 148, 149, 160, 161, 163, 164, 193, 194, 195, 196, 197, 198, 199, 200, 205, 210, 214, 215, 219, 230, 249, 250, 251, 255, 271, 273, 274, 275, 291, 293, 294, 295, 333, 334, 335, 336, 337, 338, 339, 340, 349, 352, 355, 361, 362, 379, 380, 384, 385, 386, 387, 391, 392, 393, 394, 400, 401, 402, 410, 411, 412, 413, 414, 415, 416, 433, 434, 435, 439, 440, 441, 443, 445, 447, 458, 497, 498, 501, 502, 503, 507, 508, 509, 512, 513, 517, 521, 525, 529, 532, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 549, 550, 551, 552, 553, 554, 555, 556, 557, 558, 559, 560, 561, 562, 563, 564, 565, 572, 573, 574, 575, 577, 578, 579, 580, 581, 589, 598, 607, 608, 609, 610, 615, 616, 617, 618, 619, 620, 621, 622, 623, 624, 625, 626, 627, 628, 629, 630, 631, 632, 638, 639, 640, 670, 721, 722, 729, 759, 793, 795, 834, 836, 864, 871, 872, 885, 888, 891, 904, 926, 933, 940, 941 }

C grade: { 34, 139, 140, 141, 142, 153, 154, 155, 156, 157, 158, 319, 320, 321, 403, 404, 405, 406, 407, 408, 409, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 487, 488, 489, 493, 494, 495, 515, 588, 597, 633, 634, 635, 636, 709, 710, 711, 712, 713, 741, 742, 743, 744, 745, 746, 747, 748, 751, 796, 809, 859, 860, 878, 912, 914, 927 }

F grade: { 39, 40, 41, 53, 54, 55, 60, 79, 115, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 181, 182, 183, 217, 218, 318, 566, 585, 586, 587, 592, 594, 595, 596, 601, 657, 865, 918 }

2.1.4 Maxima

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 33, 35, 42, 44, 45, 48, 49, 50, 56, 58, 59, 60, 62, 66, 67, 68, 69, 70, 71, 72, 73, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 121, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 159, 162, 180, 184, 185, 186, 187, 188, 191, 192, 193, 194, 195, 196, 201, 202, 203, 204, 207, 208, 209, 212, 213, 214, 220, 221, 222, 223, 224, 225, 226, 227, 229, 231, 252, 253, 254, 255, 256, 263, 264, 265, 266, 267, 274, 276, 277, 279, 283, 284, 285, 286, 287, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 308, 310, 312, 322, 323, 324, 325, 327, 329, 331, 333, 341, 342, 343, 344, 345, 346, 347, 348, 350, 355, 356, 357, 358, 359, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 378, 380, 381, 382, 383, 388, 389, 390, 395, 396, 397, 398, 461, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 493, 494, 495, 496, 499, 500, 504, 505, 510, 511, 512, 514, 515, 516, 517, 518, 519, 522, 523, 528, 567, 568, 569, 582, 583, 584, 590, 599, 637, 638, 639, 640, 641, 642, 643, 644, 645, 646, 647, 648, 649, 650, 651, 652, 653, 655, 656, 658, 659, 660, 661, 662, 663, 664, 665, 666, 667, 668, 669, 671, 672, 673, 674, 675, 676, 677, 678, 679, 680, 681, 682, 683, 684, 685, 686, 687, 688, 689, 690, 691, 692, 693, 694, 695, 696, 697, 698, 699, 700, 701, 702, 703, 704, 705, 706, 707, 708, 709, 710, 711, 712, 713, 714, 716, 717, 718, 719, 720, 721, 723, 724, 725, 726, 727, 729, 730, 731, 732, 733, 734, 735, 736, 737, 738, 739, 740, 741, 742, 745, 746, 749, 753, 754, 755, 756, 758, 759, 760, 763, 764, 765, 766, 767, 768, 769, 772, 773, 774, 775, 776, 777, 778, 779, 780, 781, 782, 783, 784, 785, 786, 787, 788, 789, 790, 791, 792, 793, 794, 795, 798, 799, 800, 801, 802, 803, 806, 807, 808, 809, 811, 812, 813, 814, 815, 816, 817, 818, 819, 820, 821, 822, 823, 824, 825, 826, 828, 830, 831, 832, 833, 834, 835, 837, 838, 839, 840, 843, 844, 845, 852, 854, 857, 862, 863, 864, 867, 868, 869, 870, 871, 872, 873, 880, 881, 882, 883, 884, 885, 889, 890, 891, 892, 893, 894, 895, 896, 897, 898, 899, 900, 901, 902, 903, 905, 908, 909, 910, 911, 913, 914, 916, 917, 920, 923, 924, 925, 926, 929, 932, 934, 935, 936, 937, 940, 941, 943, 945, 946, 948, 950 }

B grade: { 61, 64, 65, 74, 80, 81, 82, 85, 86, 90, 91, 92, 105, 110, 111, 112, 116, 117, 122, 123, 124, 125, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 157, 158, 189, 190, 219, 228, 230, 248, 249, 250, 251, 257, 258, 259, 260, 261, 262, 268, 270, 272, 273, 275, 278, 280, 281, 282, 288, 290, 292, 293, 294, 295, 307, 309, 311, 313, 314, 315, 316, 317, 326, 328, 330, 332, 334, 335, 336, 349, 351, 352, 353, 354, 377, 379, 384, 385, 386, 387, 391, 392, 393, 394, 444, 513, 529, 530, 531, 532, 533, 534, 589, 591, 593, 598, 600, 602, 607, 608, 609, 610, 615, 616, 654, 657, 670, 715, 722, 728, 750, 752, 757, 761, 770, 771, 796, 797, 804, 805, 810, 827, 829, 836, 841, 842, 850, 851, 853, 855, 856, 858, 859, 860, 866, 874, 876, 877, 878, 879, 886, 887, 888, 904, 915, 918, 919, 927, 928, 930, 933, 944, 947, 949 }

C grade: { 751, 907, 921 }

F grade: { 31, 32, 34, 36, 37, 38, 39, 40, 41, 43, 46, 47, 51, 52, 53, 54, 55, 57, 63, 75, 76, 77, 78, 79, 83, 84, 87, 88, 89, 93, 94, 106, 107, 108, 109, 113, 114, 115, 118, 119, 120, 126, 127, 153, 154, 155, 156, 160, 161, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 181, 182, 183, 197, 198, 199, 200, 205, 206, 210, 211, 215, 216, 217, 218, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 269, 271, 289, 291, 318, 319, 320, 321, 337, 338, 339, 340, 360, 361, 362, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 497, 498, 501, 502, 503, 506, 507, 508, 509, 520, 521, 524, 525, 526, 527, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 549, 550, 551, 552, 553, 554, 555, 556, 557, 558, 559, 560, 561, 562, 563, 564, 565, 566, 570, 571, 572, 573, 574, 575, 576, 577, 578, 579, 580, 581, 585, 586, 587, 588, 592, 594, 595, 596, 597, 601, 603, 604, 605, 606, 611, 612, 613, 614, 617, 618, 619, 620, 621, 622, 623, 624, 625, 626, 627, 628, 629, 630, 631, 632, 633, 634, 635, 636, 743, 744, 747, 748, 762, 846, 847, 848, 849, 861, 865, 875, 906, 912, 922, 931, 938, 939, 942 }

2.1.5 FriCAS

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 33, 35, 36, 37, 38, 42, 43, 44, 45, 48, 50, 51, 52, 56, 57, 58, 59, 61, 66, 67, 68, 69, 70, 71, 72, 73, 75, 76, 77, 78, 86, 88, 91, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 107, 109, 111, 113, 117, 124, 126, 129, 130, 132, 134, 136, 138, 147, 148, 149, 150, 151, 152, 153, 155, 157, 158, 183, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 220, 221, 222, 223, 224, 225, 227, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 268, 270, 273, 275, 276, 277, 278, 279, 280, 282, 284, 285, 286, 288, 290, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 305, 307, 309, 310, 311, 313, 314, 315, 316, 317, 321, 322, 323, 324, 326, 333, 334, 335, 336, 337, 341, 342, 344, 345, 346, 348, 355, 356, 357, 358, 359, 360, 363, 364, 365, 370, 371, 372, 373, 374, 375, 376, 381, 382, 383, 388, 389, 390, 395, 396, 397, 398, 417, 418, 419, 423, 424, 425, 426, 430, 431, 432, 433, 437, 438, 439, 444, 447, 461, 472, 473, 474, 476, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 493, 494, 495, 499, 500, 501, 504, 505, 506, 507, 508, 510, 511, 512, 514, 515, 516, 517, 518, 519, 520, 522, 523, 524, 525, 526, 527, 528, 529, 532, 538, 539, 543, 544, 548, 549, 553, 554, 555, 562, 563, 567, 568, 569, 570, 582, 583, 584, 585, 586, 587, 588, 589, 590, 591, 594, 595, 596, 597, 598, 599, 600, 603, 604, 605, 606, 607, 609, 610, 611, 612, 613, 614, 615, 617, 618, 619, 620, 621, 622, 623, 624, 625, 626, 627, 628, 629, 630, 631, 632, 635, 636, 637, 641, 642, 644, 645, 646, 647, 648, 649, 654, 656, 658, 659, 660, 661, 662, 663, 664, 665, 666, 672, 678, 679, 680, 681, 682, 683, 684, 685, 686, 687, 688, 689, 692, 693, 696, 700, 705, 706, 708, 715, 717, 723, 724, 725, 726, 727, 729, 730, 731, 734, 735, 736, 737, 739, 740, 741, 742, 743, 744, 745, 746, 747, 748, 749, 750, 753, 754, 755, 756, 757, 758, 760, 761, 762, 763, 764, 765, 766, 767, 768, 769, 770, 771, 772, 773, 774, 776, 777, 778, 779, 780, 781, 782, 783, 784, 785, 786, 787, 788, 789, 790, 792, 794, 796, 799, 800, 802, 803, 804, 806, 807, 808, 809, 811, 812, 813, 814, 815, 817, 819, 820, 821, 822, 824, 827, 828, 830, 831, 832, 833, 834, 835, 837, 838, 839, 840, 841, 842, 843, 844, 845, 848, 849, 850, 851, 852, 853, 854, 855, 856, 857, 858, 860, 861, 866, 867, 874, 880, 881, 882, 883, 884, 886, 887, 889, 890, 891, 892, 893, 894, 895, 896, 898, 900, 901, 903, 905, 907, 908, 909, 910, 911, 914, 915, 916, 917, 918, 919, 920, 921, 922, 923, 925, 926, 927, 928, 929, 930, 933, 934, 935, 936, 937, 940, 941, 942, 943, 944, 945, 946, 947, 948, 949, 950 }

B grade: { 49, 62, 64, 65, 74, 80, 81, 82, 83, 84, 85, 87, 89, 90, 92, 93, 94, 105, 106, 108, 110, 112, 114, 116, 118, 119, 120, 121, 122, 123, 125, 127, 128, 131, 133, 135, 137, 139, 140, 141, 142, 143, 144, 145, 146, 154, 156, 159, 160, 162, 163, 195, 196, 197, 198, 214, 215, 216, 219, 226, 228, 229, 230, 231, 267, 269, 271, 272, 274, 281, 283, 287, 289, 291, 304, 306, 308, 312, 318, 319, 320, 325, 327, 328, 329, 330, 331, 332, 338, 339, 340, 343, 347, 349, 350, 351, 352, 353, 354, 361, 362, 366, 367, 368, 369, 377, 378, 379, 380, 384, 385, 386, 387, 391, 392, 393, 394, 399, 400, 401, 402, 420, 421, 422, 427, 428, 429, 434, 435, 436, 440, 441, 442, 443, 445, 446, 458, 459, 460, 475, 477, 478, 479, 480, 496, 497, 502, 503, 509, 513, 521, 530, 531, 533, 534, 535, 536, 537, 540, 541, 542, 545, 546, 547, 550, 551, 552, 564, 565, 571, 572, 581, 592, 593, 601, 602, 608, 616, 633, 634, 638, 639, 640, 643, 650, 651, 652, 653, 655, 657, 667, 668, 669, 670, 671, 673, 674, 675, 676, 677, 690, 691, 694, 695, 697, 698, 699, 701, 702, 703, 704, 707, 709, 710, 711, 712, 713, 714, 716, 718, 719, 720, 721, 722, 728, 732, 733, 738, 752, 759, 775, 791, 793, 795, 797, 798, 801, 805, 810, 816, 818, 823, 825, 826, 829, 836, 846, 847, 859, 862, 863, 864, 868, 871, 877, 885, 888, 897, 899, 902, 904, 906, 912, 913, 924 }

C grade: { 31, 32, 46, 47, 161, 164, 184, 498, 579, 580, 751, 869, 870, 872, 873, 875, 876, 878, 879 }

F grade: { 34, 39, 40, 41, 53, 54, 55, 60, 63, 79, 115, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 217, 218, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 556, 557, 558, 559, 560, 561, 566, 573, 574, 575, 576, 577, 578, 865, 931, 932, 938, 939 }

2.1.6 Sympy

A grade: { 1, 8, 15, 17, 18, 19, 22, 29, 30, 33, 35, 42, 43, 44, 45, 48, 49, 50, 56, 57, 58, 59, 62, 66, 67, 68, 69, 70, 71, 72, 73, 91, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 124, 135, 136, 137, 138, 146, 180, 185, 186, 187, 188, 203, 204, 206, 219, 220, 221, 222, 223, 224, 225, 248, 249, 250, 251, 252, 253, 254, 255, 256, 264, 265, 266, 267, 268, 270, 272, 274, 276, 277, 284, 285, 286, 287, 294, 296, 303, 304, 305, 322, 323, 324, 341, 342, 343, 344, 349, 352, 355, 356, 357, 358, 363, 364, 365, 366, 370, 371, 372, 373, 374, 375, 376, 377, 381, 382, 383, 384, 388, 389, 390, 391, 395, 396, 397, 398, 399, 444, 481, 482, 483, 499, 500, 510, 511, 512, 526, 527, 528, 529, 532, 538, 539, 543, 544, 548, 549, 553, 554, 562, 567, 568, 569, 582, 589, 590, 598, 599, 637, 638, 639, 640, 641, 642, 643, 644, 645, 646, 647, 648, 649, 651, 652, 654, 655, 656, 659, 660, 661, 665, 666, 674, 675, 676, 677, 678, 679, 680, 681, 684, 685, 686, 701, 702, 703, 705, 706, 707, 714, 715, 718, 720, 721, 722, 723, 724, 725, 726, 728, 730, 731, 732, 735, 736, 737, 754, 755, 756, 758, 763, 764, 765, 766, 767, 768, 769, 770, 771, 772, 773, 776, 777, 778, 779, 780, 781, 782, 783, 784, 785, 786, 787, 788, 789, 790, 791, 792, 794, 795, 799, 800, 801, 806, 807, 808, 811, 812, 813, 814, 815, 816, 817, 818, 819, 820, 823, 824, 825, 826, 827, 830, 831, 832, 833, 834, 835, 837, 838, 839, 840, 842, 843, 844, 845, 846, 847, 852, 855, 856, 884, 887, 889, 890, 891, 892, 894, 895, 900, 901, 902, 905, 907, 908, 911, 913, 916, 920, 921, 923, 924, 925, 926, 929, 932, 933, 934, 935, 936, 937, 940, 941, 943, 944, 946 }

B grade: { 3, 4, 5, 10, 11, 12, 24, 25, 26, 80, 90, 92, 110, 121, 122, 123, 125, 128, 129, 130, 131, 132, 133, 134, 139, 140, 141, 142, 201, 202, 205, 207, 208, 210, 212, 213, 263, 273, 275, 278, 280, 282, 283, 293, 295, 297, 306, 325, 472, 473, 474, 475, 476, 477, 478, 591, 653, 672, 694, 695, 719, 727, 738, 751, 753, 759, 760, 775, 793, 798, 802, 803, 804, 805, 809, 821, 822, 828, 836, 841, 848, 849, 854, 862, 880, 881, 882, 883, 885, 888, 909, 945 }

C grade: { }

F grade: { 2, 6, 7, 9, 13, 14, 16, 20, 21, 23, 27, 28, 31, 32, 34, 36, 37, 38, 39, 40, 41, 46, 47, 51, 52, 53, 54, 55, 60, 61, 63, 64, 65, 74, 75, 76, 77, 78, 79, 81, 82, 83, 84, 85, 86, 87, 88, 89, 93, 94, 105, 106, 107, 108, 109, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 126, 127, 143, 144, 145, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 181, 182, 183, 184, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 209, 211, 214, 215, 216, 217, 218, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 257, 258, 259, 260, 261, 262, 269, 271, 279, 281, 288, 289, 290, 291, 292, 298, 299, 300, 301, 302, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 345, 346, 347, 348, 350, 351, 353, 354, 359, 360, 361, 362, 367, 368, 369, 378, 379, 380, 385, 386, 387, 392, 393, 394, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 479, 480, 484, 485, 486, 487, 488, 489, 490, 491, 492, 493, 494, 495, 496, 497, 498, 501, 502, 503, 504, 505, 506, 507, 508, 509, 513, 514, 515, 516, 517, 518, 519, 520, 521, 522, 523, 524, 525, 530, 531, 533, 534, 535, 536, 537, 540, 541, 542, 545, 546, 547, 550, 551, 552, 555, 556, 557, 558, 559, 560, 561, 563, 564, 565, 566, 570, 571, 572, 573, 574, 575, 576, 577, 578, 579, 580, 581, 583, 584, 585, 586, 587, 588, 592, 593, 594, 595, 596, 597, 600, 601, 602, 603, 604, 605, 606, 607, 608, 609, 610, 611, 612, 613, 614, 615, 616, 617, 618, 619, 620, 621, 622, 623, 624, 625, 626, 627, 628, 629, 630, 631, 632, 633, 634, 635, 636, 650, 657, 658, 662, 663, 664, 667, 668, 669, 670, 671, 673, 682, 683, 687, 688, 689, 690, 691, 692, 693, 696, 697, 698, 699, 700, 704, 708, 709, 710, 711, 712, 713, 716, 717, 729, 733, 734, 739, 740, 741, 742, 743, 744, 745, 746, 747, 748, 749, 750, 752, 757, 761, 762, 774, 796, 797, 810, 829, 850, 851, 853, 857, 858, 859, 860, 861, 863, 864, 865, 866, 867, 868, 869, 870, 871, 872, 873, 874, 875, 876, 877, 878, 879, 886, 893, 896, 897, 898, 899, 903, 904, 906, 910, 912, 914, 915, 917, 918, 919, 922, 927, 928, 930, 931, 938, 939, 942, 947, 948, 949, 950 }

2.1.7 Giac

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 33, 35, 42, 43, 44, 45, 48, 49, 50, 56, 57, 58, 59, 61, 62, 66, 67, 68, 69, 70, 71, 72, 73, 86, 88, 91, 93, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 111, 113, 117, 119, 120, 124, 126, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 150, 151, 152, 159, 180, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 220, 222, 223, 224, 225, 226, 227, 229, 231, 248, 252, 253, 254, 255, 256, 257, 258, 259, 260, 263, 264, 265, 266, 268, 269, 270, 272, 274, 276, 279, 281, 283, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 305, 306, 308, 310, 312, 322, 323, 324, 326, 327, 328, 329, 330, 331, 332, 344, 345, 346, 348, 349, 350, 351, 352, 353, 355, 356, 357, 358, 359, 360, 361, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 388, 389, 390, 395, 396, 397, 398, 399, 400, 443, 444, 445, 446, 447, 458, 459, 460, 461, 472, 473, 474, 476, 477, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 493, 494, 495, 499, 500, 504, 505, 506, 520, 522, 523, 525, 526, 527, 529, 530, 531, 532, 533, 535, 536, 540, 541, 545, 546, 550, 551, 562, 563, 567, 568, 569, 570, 571, 572, 582, 583, 584, 589, 591, 598, 600, 640, 644, 645, 646, 647, 648, 649, 651, 652, 653, 654, 655, 656, 657, 658, 659, 660, 661, 665, 666, 668, 669, 671, 672, 673, 674, 675, 677, 678, 679, 680, 681, 682, 684, 685, 686, 687, 690, 691, 692, 694, 695, 696, 697, 698, 699, 700, 701, 702, 703, 704, 705, 706, 708, 710, 711, 713, 714, 715, 716, 724, 725, 726, 727, 729, 730, 731, 734, 735, 736, 737, 738, 739, 740, 749, 753, 754, 755, 756, 758, 760, 761, 762, 763, 764, 765, 766, 767, 768, 769, 770, 771, 772, 773, 774, 776, 777, 778, 779, 780, 781, 782, 783, 784, 785, 786, 787, 788, 789, 790, 791, 792, 794, 795, 798, 799, 800, 801, 802, 803, 804, 806, 807, 808, 810, 811, 812, 813, 814, 817, 818, 819, 820, 821, 822, 823, 824, 828, 830, 831, 832, 833, 834, 835, 836, 837, 838, 839, 840, 841, 843, 844, 845, 846, 847, 851, 852, 853, 854, 855, 856, 857, 866, 867, 880, 881, 882, 883, 884, 887, 889, 890, 891, 892, 893, 894, 895, 896, 897, 898, 899, 901, 902, 903, 905, 906, 907, 908, 909, 911, 913, 916, 919, 920, 921, 922, 923, 924, 925, 926, 929, 932, 934, 935, 936, 937, 940, 941, 942, 943, 944, 945, 946, 947, 948, 949, 950 }

B grade: { 36, 37, 38, 51, 52, 64, 65, 75, 80, 81, 82, 83, 84, 85, 87, 89, 90, 92, 94, 110, 112, 114, 116, 118, 121, 122, 123, 125, 127, 143, 144, 145, 146, 157, 158, 162, 185, 196, 197, 198, 199, 200, 219, 221, 228, 230, 249, 250, 251, 261, 262, 267, 271, 273, 275, 277, 278, 280, 282, 284, 304, 307, 309, 311, 313, 325, 336, 338, 339, 340, 341, 342, 343, 347, 354, 362, 384, 385, 386, 387, 391, 392, 393, 394, 401, 402, 421, 422, 475, 478, 496, 501, 502, 507, 508, 509, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 521, 524, 528, 534, 537, 538, 539, 542, 543, 544, 547, 548, 549, 552, 553, 554, 555, 564, 565, 590, 599, 638, 642, 650, 667, 670, 676, 683, 707, 718, 719, 720, 721, 722, 728, 732, 743, 744, 747, 748, 752, 757, 759, 775, 793, 796, 797, 805, 815, 816, 825, 826, 827, 829, 842, 848, 849, 850, 860, 861, 862, 863, 864, 885, 886, 888, 904, 910, 915, 927, 928, 930, 933 }

C grade: { 428, 429, 585, 586, 587, 588, 594, 595, 596, 597, 712 }

F grade: { 31, 32, 34, 39, 40, 41, 46, 47, 53, 54, 55, 60, 63, 74, 76, 77, 78, 79, 105, 106, 107, 108, 109, 115, 139, 140, 141, 142, 147, 148, 149, 153, 154, 155, 156, 160, 161, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 181, 182, 183, 184, 217, 218, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 314, 315, 316, 317, 318, 319, 320, 321, 333, 334, 335, 337, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 423, 424, 425, 426, 427, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 497, 498, 503, 556, 557, 558, 559, 560, 561, 566, 573, 574, 575, 576, 577, 578, 579, 580, 581, 592, 593, 601, 602, 603, 604, 605, 606, 607, 608, 609, 610, 611, 612, 613, 614, 615, 616, 617, 618, 619, 620, 621, 622, 623, 624, 625, 626, 627, 628, 629, 630, 631, 632, 633, 634, 635, 636, 637, 639, 641, 643, 662, 663, 664, 688, 689, 693, 709, 717, 723, 733, 741, 742, 745, 746, 750, 751, 809, 858, 859, 865, 868, 869, 870, 871, 872, 873, 874, 875, 876, 877, 878, 879, 900, 912, 914, 917, 918, 931, 938, 939 }

2.1.8 Mupad

A grade: { 42, 43, 56, 57, 180, 582, 583, 584, 644, 645, 646, 647, 932 }

B grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 33, 35, 44, 45, 48, 49, 50, 58, 59, 61, 62, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 150, 155, 159, 162, 165, 166, 167, 171, 172, 173, 183, 185, 186, 187, 188, 189, 190, 191, 192, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 317, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 336, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 419, 426, 443, 444, 445, 446, 447, 458, 459, 460, 461, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 493, 494, 495, 496, 499, 500, 501, 502, 503, 508, 509, 510, 511, 512, 513, 518, 519, 520, 521, 526, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 549, 550, 551, 552, 553, 554, 555, 562, 563, 564, 565, 567, 568, 569, 570, 571, 572, 589, 590, 598, 599, 603, 604, 605, 606, 611, 612, 613, 614, 633, 637, 638, 639, 640, 641, 642, 643, 648, 649, 650, 651, 652, 653, 654, 655, 656, 659, 660, 661, 665, 666, 667, 668, 669, 670, 671, 672, 673, 674, 675, 676, 677, 678, 680, 681, 682, 683, 684, 685, 686, 690, 691, 692, 693, 694, 695, 696, 697, 698, 699, 700, 701, 702, 703, 704, 705, 706, 707, 708, 712, 714, 715, 716, 717, 718, 719, 720, 721, 722, 723, 724, 725, 726, 727, 728, 729, 730, 731, 732, 733, 734, 735, 736, 737, 738, 739, 740, 741, 742, 743, 744, 745, 746, 747, 748, 749, 750, 751, 752, 753, 754, 755, 756, 757, 758, 759, 760, 761, 762, 763, 764, 765, 766, 767, 768, 769, 770, 771, 772, 773, 774, 775, 776, 777, 778, 779, 780, 781, 782, 783, 784, 785, 786, 787, 788, 789, 790, 791, 792, 793, 794, 795, 796, 797, 798, 799, 800, 801, 802, 803, 804, 805, 806, 807, 808, 809, 810, 811, 812, 813, 814, 815, 816, 817, 818, 819, 820, 821, 822, 823, 824, 825, 826, 827, 828, 829, 830, 831, 832, 833, 834, 835, 836, 837, 838, 839, 840, 841, 842, 843, 844, 845, 846, 847, 848, 849, 852, 854, 855, 856, 857, 858, 861, 862, 863, 864, 865, 880, 881, 882, 883, 884, 885, 886, 887, 888, 889, 890, 891, 892, 893, 894, 895, 896, 897, 898, 899, 900, 901, 902, 903, 904, 905, 906, 907, 908, 909, 910, 911, 912, 913, 914, 915, 916, 917, 918, 919, 920, 923, 924, 925, 926, 927, 928, 929, 930, 931, 933, 934, 935, 936, 937, 938, 939, 940, 941, 942, 943, 944, 945, 946, 947, 948, 949, 950 }

C grade: { }

F grade: { 31, 32, 34, 36, 37, 38, 39, 40, 41, 46, 47, 51, 52, 53, 54, 55, 60, 63, 79, 115, 148, 149, 151, 152, 153, 154, 156, 157, 158, 160, 161, 163, 164, 168, 169, 170, 174, 175, 176, 177, 178, 179, 181, 182, 184, 193, 194, 195, 196, 197, 198, 199, 217, 218, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 259, 260, 261, 262, 314, 315, 316, 318, 319, 320, 321, 333, 334, 335, 337, 338, 339, 340, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 420, 421, 422, 423, 424, 425, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 497, 498, 504, 505, 506, 507, 514, 515, 516, 517, 522, 523, 524, 525, 556, 557, 558, 559, 560, 561, 566, 573, 574, 575, 576, 577, 578, 579, 580, 581, 585, 586, 587, 588, 591, 592, 593, 594, 595, 596, 597, 600, 601, 602, 607, 608, 609, 610, 615, 616, 617, 618, 619, 620, 621, 622, 623, 624, 625, 626, 627, 628, 629, 630, 631, 632, 634, 635, 636, 657, 658, 662, 663, 664, 679, 687, 688, 689, 709, 710, 711, 713, 850, 851, 853, 859, 860, 866, 867, 868, 869, 870, 871, 872, 873, 874, 875, 876, 877, 878, 879, 921, 922 }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by table below. The elapsed time is in seconds. For failed result it is given as F(-1) if the failure was due to timeout. It is given as F(-2) if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given just an F.

In this table,the column **normalized size** is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$

Problem 1	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	22	17	26	33	32	57	35
normalized size	1	1.00	0.50	0.39	0.59	0.75	0.73	1.30	0.80
time (sec)	N/A	0.040	0.036	0.075	0.411	0.653	0.263	0.150	2.582

Problem 2	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	22	17	26	33	0	57	16
normalized size	1	1.00	0.50	0.39	0.59	0.75	0.00	1.30	0.36
time (sec)	N/A	0.038	0.027	0.305	0.643	0.688	0.000	0.220	2.689

Problem 3	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	22	17	16	43	246	57	35
normalized size	1	1.00	0.46	0.35	0.33	0.90	5.12	1.19	0.73
time (sec)	N/A	0.022	0.046	0.168	0.890	0.647	6.818	0.128	2.486

Problem 4	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	22	17	16	43	246	57	35
normalized size	1	1.00	0.46	0.35	0.33	0.90	5.12	1.19	0.73
time (sec)	N/A	0.020	0.057	0.099	0.411	0.731	7.115	0.146	2.380

Problem 5	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	22	17	16	43	246	57	35
normalized size	1	1.00	0.46	0.35	0.33	0.90	5.12	1.19	0.73
time (sec)	N/A	0.026	0.025	0.227	0.565	0.735	6.995	0.157	2.376

Problem 6	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	22	17	16	43	0	16	16
normalized size	1	1.00	0.46	0.35	0.33	0.90	0.00	0.33	0.33
time (sec)	N/A	0.044	0.018	0.288	0.442	0.510	0.000	1.542	2.402

Problem 7	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	22	17	16	43	0	57	16
normalized size	1	1.00	0.46	0.35	0.33	0.90	0.00	1.19	0.33
time (sec)	N/A	0.041	0.019	0.286	0.614	0.644	0.000	0.283	2.398

Problem 8	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	22	17	54	74	42	39	16
normalized size	1	1.00	0.37	0.28	0.90	1.23	0.70	0.65	0.27
time (sec)	N/A	0.026	0.037	0.058	0.568	0.506	0.312	0.167	2.566

Problem 9	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	22	17	54	74	0	39	16
normalized size	1	1.00	0.37	0.28	0.90	1.23	0.00	0.65	0.27
time (sec)	N/A	0.046	0.039	0.303	0.490	0.912	0.000	0.238	2.624

Problem 10	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	22	17	34	86	1644	39	16
normalized size	1	1.00	0.37	0.28	0.57	1.43	27.40	0.65	0.27
time (sec)	N/A	0.020	0.058	0.148	0.409	0.654	18.471	0.162	2.453

Problem 11	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	22	17	34	86	1644	39	16
normalized size	1	1.00	0.37	0.28	0.57	1.43	27.40	0.65	0.27
time (sec)	N/A	0.019	0.069	0.098	0.497	0.569	14.403	0.166	2.440

Problem 12	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	22	17	34	86	1644	39	16
normalized size	1	1.00	0.37	0.28	0.57	1.43	27.40	0.65	0.27
time (sec)	N/A	0.030	0.031	0.225	0.413	0.781	17.090	0.202	2.404

Problem 13	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	22	17	34	86	0	39	16
normalized size	1	1.00	0.37	0.28	0.57	1.43	0.00	0.65	0.27
time (sec)	N/A	0.045	0.022	0.277	1.032	0.771	0.000	1.484	2.402

Problem 14	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	22	17	34	86	0	39	16
normalized size	1	1.00	0.37	0.28	0.57	1.43	0.00	0.65	0.27
time (sec)	N/A	0.046	0.031	0.263	0.413	0.684	0.000	0.357	2.410

Problem 15	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	42	22	18	27	31	34	57	36
normalized size	1	1.00	0.52	0.43	0.64	0.74	0.81	1.36	0.86
time (sec)	N/A	0.037	0.029	0.060	0.420	1.319	0.255	0.145	2.536

Problem 16	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	42	22	18	27	31	0	57	17
normalized size	1	1.00	0.52	0.43	0.64	0.74	0.00	1.36	0.40
time (sec)	N/A	0.037	0.021	0.314	0.432	0.597	0.000	0.225	2.689

Problem 17	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	22	18	17	43	76	57	36
normalized size	1	1.00	0.46	0.38	0.35	0.90	1.58	1.19	0.75
time (sec)	N/A	0.019	0.020	0.146	0.443	0.755	0.750	0.146	2.438

Problem 18	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	22	18	17	43	76	57	36
normalized size	1	1.00	0.46	0.38	0.35	0.90	1.58	1.19	0.75
time (sec)	N/A	0.016	0.040	0.095	0.443	0.685	0.648	0.145	2.367

Problem 19	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	22	18	17	43	76	57	36
normalized size	1	1.00	0.46	0.38	0.35	0.90	1.58	1.19	0.75
time (sec)	N/A	0.027	0.020	0.228	0.444	0.508	0.798	0.158	2.367

Problem 20	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	22	18	17	43	0	17	17
normalized size	1	1.00	0.46	0.38	0.35	0.90	0.00	0.35	0.35
time (sec)	N/A	0.042	0.018	0.269	1.181	0.609	0.000	1.232	2.371

Problem 21	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	22	18	17	43	0	57	17
normalized size	1	1.00	0.46	0.38	0.35	0.90	0.00	1.19	0.35
time (sec)	N/A	0.045	0.018	0.276	0.745	0.554	0.000	0.327	2.380

Problem 22	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	22	18	53	74	39	39	17
normalized size	1	1.00	0.36	0.30	0.87	1.21	0.64	0.64	0.28
time (sec)	N/A	0.030	0.026	0.059	0.433	0.541	0.308	0.164	2.501

Problem 23	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	22	18	53	74	0	39	17
normalized size	1	1.00	0.36	0.30	0.87	1.21	0.00	0.64	0.28
time (sec)	N/A	0.046	0.034	0.313	0.526	0.549	0.000	0.240	2.713

Problem 24	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	22	18	32	85	1481	39	17
normalized size	1	1.00	0.36	0.30	0.52	1.39	24.28	0.64	0.28
time (sec)	N/A	0.020	0.064	0.147	0.501	0.587	14.067	0.187	2.515

Problem 25	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	22	18	32	85	1481	39	17
normalized size	1	1.00	0.36	0.30	0.52	1.39	24.28	0.64	0.28
time (sec)	N/A	0.019	0.057	0.096	0.516	0.594	12.827	0.171	2.443

Problem 26	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	22	18	32	85	1481	39	17
normalized size	1	1.00	0.36	0.30	0.52	1.39	24.28	0.64	0.28
time (sec)	N/A	0.031	0.029	0.229	0.536	0.575	16.478	0.206	2.418

Problem 27	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	22	18	32	85	0	37	17
normalized size	1	1.00	0.36	0.30	0.52	1.39	0.00	0.61	0.28
time (sec)	N/A	0.043	0.021	0.277	1.342	0.536	0.000	1.294	2.400

Problem 28	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	22	18	32	85	0	39	17
normalized size	1	1.00	0.36	0.30	0.52	1.39	0.00	0.64	0.28
time (sec)	N/A	0.047	0.034	0.302	1.182	0.538	0.000	0.418	2.334

Problem 29	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	30	25	24	22	41	24	24
normalized size	1	1.00	1.00	0.83	0.80	0.73	1.37	0.80	0.80
time (sec)	N/A	0.035	0.061	0.037	0.326	0.508	0.163	0.137	0.057

Problem 30	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	56	56	48	57	48	46	85	46	46
normalized size	1	1.00	0.86	1.02	0.86	0.82	1.52	0.82	0.82
time (sec)	N/A	0.067	0.090	0.039	0.517	0.487	0.304	0.137	2.308

Problem 31	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	C	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	213	213	172	229	0	187	0	0	-1
normalized size	1	1.00	0.81	1.08	0.00	0.88	0.00	0.00	-0.00
time (sec)	N/A	0.536	0.322	0.053	0.000	0.559	0.000	0.000	0.000

Problem 32	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	C	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	271	271	238	320	0	434	0	0	-1
normalized size	1	1.00	0.88	1.18	0.00	1.60	0.00	0.00	-0.00
time (sec)	N/A	0.802	0.564	0.043	0.000	2.021	0.000	0.000	0.000

Problem 33	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	10	10	10	9	8	8	10	8	8
normalized size	1	1.00	1.00	0.90	0.80	0.80	1.00	0.80	0.80
time (sec)	N/A	0.018	0.023	0.007	0.309	0.588	0.258	0.125	2.419

Problem 34	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	46	72	0	0	0	0	-1
normalized size	1	1.00	1.64	2.57	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.471	0.700	0.079	0.000	1.267	0.000	0.000	0.000

Problem 35	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	26	12	17	22	10	11	11
normalized size	1	1.00	2.17	1.00	1.42	1.83	0.83	0.92	0.92
time (sec)	N/A	0.029	0.020	0.076	0.306	0.626	1.363	0.137	2.399

Problem 36	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	A	F	B	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	100	100	272	142	0	139	0	630	-1
normalized size	1	1.00	2.72	1.42	0.00	1.39	0.00	6.30	-0.01
time (sec)	N/A	0.164	5.353	0.036	0.000	1.086	0.000	9.107	0.000

Problem 37	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	A	F(-1)	B	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	107	107	330	195	0	149	0	681	-1
normalized size	1	1.00	3.08	1.82	0.00	1.39	0.00	6.36	-0.01
time (sec)	N/A	0.192	7.270	0.071	0.000	0.608	0.000	62.661	0.000

Problem 38	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	A	F(-1)	B	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	194	194	657	295	0	277	0	1239	-1
normalized size	1	1.00	3.39	1.52	0.00	1.43	0.00	6.39	-0.01
time (sec)	N/A	0.322	7.705	0.070	0.000	1.137	0.000	177.526	0.000

Problem 39	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	53	0	0	0	0	0	-1
normalized size	1	1.00	0.91	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.111	0.106	0.576	0.000	2.026	0.000	0.000	0.000

Problem 40	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	57	0	0	0	0	0	-1
normalized size	1	1.00	0.98	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.082	0.079	0.292	0.000	0.966	0.000	0.000	0.000

Problem 41	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	26	0	0	0	0	0	-1
normalized size	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.040	0.045	0.070	0.000	0.949	0.000	0.000	0.000

Problem 42	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	40	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.037	5.908	0.179	0.000	1.145	0.000	0.000	0.000

Problem 43	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-1)	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	42	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.081	19.895	0.243	0.000	1.171	0.000	0.000	0.000

Problem 44	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	26	25	24	23	41	24	24
normalized size	1	1.00	0.87	0.83	0.80	0.77	1.37	0.80	0.80
time (sec)	N/A	0.034	0.076	0.035	0.306	2.513	0.164	0.137	0.049

Problem 45	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	56	56	51	57	46	44	85	46	48
normalized size	1	1.00	0.91	1.02	0.82	0.79	1.52	0.82	0.86
time (sec)	N/A	0.069	0.117	0.030	0.314	0.851	0.307	0.145	0.076

Problem 46	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	C	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	213	213	172	229	0	189	0	0	-1
normalized size	1	1.00	0.81	1.08	0.00	0.89	0.00	0.00	-0.00
time (sec)	N/A	0.308	0.336	0.048	0.000	1.123	0.000	0.000	0.000

Problem 47	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	C	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	271	271	236	320	0	436	0	0	-1
normalized size	1	1.00	0.87	1.18	0.00	1.61	0.00	0.00	-0.00
time (sec)	N/A	0.562	0.582	0.050	0.000	1.036	0.000	0.000	0.000

Problem 48	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	10	10	10	9	8	8	8	8	8
normalized size	1	1.00	1.00	0.90	0.80	0.80	0.80	0.80	0.80
time (sec)	N/A	0.136	0.037	0.045	0.350	1.626	0.432	0.141	2.260

Problem 49	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	22	18	17	37	20	17	15
normalized size	1	1.00	1.00	0.82	0.77	1.68	0.91	0.77	0.68
time (sec)	N/A	0.189	0.055	0.079	0.411	0.941	0.684	0.119	2.316

Problem 50	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	20	16	16	15	15	19	16
normalized size	1	1.00	0.83	0.67	0.67	0.62	0.62	0.79	0.67
time (sec)	N/A	0.493	0.173	0.072	0.306	1.705	5.494	0.149	2.349

Problem 51	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	A	F	B	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	101	101	260	142	0	138	0	633	-1
normalized size	1	1.00	2.57	1.41	0.00	1.37	0.00	6.27	-0.01
time (sec)	N/A	0.133	5.510	0.095	0.000	4.118	0.000	12.269	0.000

Problem 52	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	A	F(-1)	B	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	107	107	400	195	0	144	0	683	-1
normalized size	1	1.00	3.74	1.82	0.00	1.35	0.00	6.38	-0.01
time (sec)	N/A	0.161	6.135	0.136	0.000	2.197	0.000	68.629	0.000

Problem 53	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	53	0	0	0	0	0	-1
normalized size	1	1.00	0.91	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.111	0.023	0.744	0.000	2.694	0.000	0.000	0.000

Problem 54	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	51	0	0	0	0	0	-1
normalized size	1	1.00	0.88	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.080	0.040	0.363	0.000	1.494	0.000	0.000	0.000

Problem 55	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	26	0	0	0	0	0	-1
normalized size	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.038	0.007	0.079	0.000	1.773	0.000	0.000	0.000

Problem 56	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	40	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.038	3.068	0.107	0.000	0.496	0.000	0.000	0.000

Problem 57	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-1)	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	42	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.084	11.878	0.250	0.000	1.129	0.000	0.000	0.000

Problem 58	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	9	9	9	8	7	13	10	8	19
normalized size	1	1.00	1.00	0.89	0.78	1.44	1.11	0.89	2.11
time (sec)	N/A	0.010	0.013	0.004	0.302	1.996	0.171	0.143	3.179

Problem 59	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	18	13	12	12	14	12	20
normalized size	1	1.00	1.12	0.81	0.75	0.75	0.88	0.75	1.25
time (sec)	N/A	0.018	0.037	0.008	0.537	1.972	0.178	0.122	2.584

Problem 60	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	70	70	0	80	0	0	0	-1
normalized size	1	1.00	1.00	0.00	1.14	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.091	0.023	0.105	0.414	0.858	0.000	0.000	0.000

Problem 61	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	18	18	83	23	0	18	21
normalized size	1	1.00	0.95	0.95	4.37	1.21	0.00	0.95	1.11
time (sec)	N/A	0.018	0.744	0.152	0.376	1.661	0.000	8.595	2.511

Problem 62	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	26	14	14	28	15	22	20
normalized size	1	1.00	1.62	0.88	0.88	1.75	0.94	1.38	1.25
time (sec)	N/A	0.018	0.049	0.007	0.481	1.222	0.169	0.129	2.549

Problem 63	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	92	92	85	150	0	0	0	0	-1
normalized size	1	1.00	0.92	1.63	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.164	7.219	0.621	0.000	1.199	0.000	0.000	0.000

Problem 64	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	35	48	6257	72	0	948	29
normalized size	1	1.00	1.00	1.37	178.77	2.06	0.00	27.09	0.83
time (sec)	N/A	0.033	0.037	0.169	108.038	0.942	0.000	1.153	0.089

Problem 65	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	35	48	6257	72	0	948	29
normalized size	1	1.00	1.00	1.37	178.77	2.06	0.00	27.09	0.83
time (sec)	N/A	0.033	0.025	0.000	116.999	0.980	0.000	1.194	0.002

Problem 66	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	15	7	11	10	20	6	6
normalized size	1	1.00	1.00	0.47	0.73	0.67	1.33	0.40	0.40
time (sec)	N/A	0.009	0.006	0.049	0.307	2.012	0.452	0.122	0.027

Problem 67	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	17	14	13	13	20	13	13
normalized size	1	1.00	1.00	0.82	0.76	0.76	1.18	0.76	0.76
time (sec)	N/A	0.008	0.007	0.109	0.308	0.585	0.416	0.135	0.029

Problem 68	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	17	14	13	18	20	13	13
normalized size	1	1.00	1.00	0.82	0.76	1.06	1.18	0.76	0.76
time (sec)	N/A	0.008	0.008	0.096	0.305	1.122	0.410	0.138	0.029

Problem 69	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	25	28	28	26	78	29	64
normalized size	1	1.00	0.71	0.80	0.80	0.74	2.23	0.83	1.83
time (sec)	N/A	0.031	0.048	0.079	0.306	1.083	0.795	0.136	2.323

Problem 70	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	15	12	11	9	20	11	9
normalized size	1	1.00	1.00	0.80	0.73	0.60	1.33	0.73	0.60
time (sec)	N/A	0.008	0.005	0.070	0.307	0.785	0.450	0.127	0.023

Problem 71	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	17	14	13	13	20	13	13
normalized size	1	1.00	1.00	0.82	0.76	0.76	1.18	0.76	0.76
time (sec)	N/A	0.008	0.006	0.102	0.538	0.830	0.415	0.122	0.025

Problem 72	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	17	14	13	17	20	13	17
normalized size	1	1.00	1.00	0.82	0.76	1.00	1.18	0.76	1.00
time (sec)	N/A	0.008	0.007	0.108	0.505	1.061	0.410	0.123	0.028

Problem 73	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	24	28	28	24	37	29	37
normalized size	1	1.00	0.69	0.80	0.80	0.69	1.06	0.83	1.06
time (sec)	N/A	0.027	0.044	0.033	0.317	0.805	0.930	0.133	0.100

Problem 74	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	20	18	141	38	0	0	17
normalized size	1	1.00	1.00	0.90	7.05	1.90	0.00	0.00	0.85
time (sec)	N/A	0.023	0.014	0.161	0.436	2.061	0.000	0.000	2.394

Problem 75	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	21	38	0	39	0	364	26
normalized size	1	1.00	0.45	0.81	0.00	0.83	0.00	7.74	0.55
time (sec)	N/A	0.052	0.031	0.301	0.000	1.907	0.000	0.266	2.339

Problem 76	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	71	71	69	115	0	101	0	0	103
normalized size	1	1.00	0.97	1.62	0.00	1.42	0.00	0.00	1.45
time (sec)	N/A	0.109	0.082	0.411	0.000	2.459	0.000	0.000	2.562

Problem 77	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	112	112	100	84	0	136	0	0	107
normalized size	1	1.00	0.89	0.75	0.00	1.21	0.00	0.00	0.96
time (sec)	N/A	0.170	0.176	0.411	0.000	0.835	0.000	0.000	2.885

Problem 78	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	84	256	0	134	0	0	131
normalized size	1	1.00	0.94	2.88	0.00	1.51	0.00	0.00	1.47
time (sec)	N/A	0.271	0.150	0.615	0.000	1.793	0.000	0.000	3.113

Problem 79	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	105	105	200	0	0	0	0	0	-1
normalized size	1	1.00	1.90	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.077	0.178	0.437	0.000	0.732	0.000	0.000	0.000

Problem 80	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	10	10	10	12	37	19	19	19	10
normalized size	1	1.00	1.00	1.20	3.70	1.90	1.90	1.90	1.00
time (sec)	N/A	0.022	0.011	0.076	0.408	0.680	0.815	0.138	2.319

Problem 81	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	20	17	127	36	0	34	16
normalized size	1	1.00	1.00	0.85	6.35	1.80	0.00	1.70	0.80
time (sec)	N/A	0.027	0.019	0.117	0.444	0.978	0.000	0.150	2.366

Problem 82	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	28	30	173	52	0	50	29
normalized size	1	1.00	1.00	1.07	6.18	1.86	0.00	1.79	1.04
time (sec)	N/A	0.051	0.037	0.125	0.544	0.571	0.000	0.138	2.378

Problem 83	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	82	76	70	0	127	0	111	119
normalized size	1	1.00	0.93	0.85	0.00	1.55	0.00	1.35	1.45
time (sec)	N/A	0.196	0.238	0.209	0.000	0.562	0.000	0.326	2.609

Problem 84	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	38	49	0	70	0	70	37
normalized size	1	1.00	1.00	1.29	0.00	1.84	0.00	1.84	0.97
time (sec)	N/A	0.082	0.070	0.158	0.000	0.869	0.000	0.168	2.500

Problem 85	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	174	13	129	33	0	49	12
normalized size	1	1.00	11.60	0.87	8.60	2.20	0.00	3.27	0.80
time (sec)	N/A	0.015	0.415	0.134	0.427	0.602	0.000	0.171	2.289

Problem 86	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	17	18	81	19	0	24	15
normalized size	1	1.00	0.81	0.86	3.86	0.90	0.00	1.14	0.71
time (sec)	N/A	0.027	0.009	0.188	0.418	0.876	0.000	0.142	2.274

Problem 87	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	B	F	B	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	71	71	4845	54	0	121	0	133	112
normalized size	1	1.00	68.24	0.76	0.00	1.70	0.00	1.87	1.58
time (sec)	N/A	0.062	57.934	0.206	0.000	0.677	0.000	0.142	0.088

Problem 88	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	62	62	57	43	0	72	0	67	47
normalized size	1	1.00	0.92	0.69	0.00	1.16	0.00	1.08	0.76
time (sec)	N/A	0.073	0.099	0.228	0.000	0.689	0.000	0.160	0.535

Problem 89	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	B	F	B	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	627	80	0	153	0	182	118
normalized size	1	1.00	7.38	0.94	0.00	1.80	0.00	2.14	1.39
time (sec)	N/A	0.063	9.333	0.244	0.000	1.055	0.000	0.164	2.283

Problem 90	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	7	7	37	9	35	17	15	17	5
normalized size	1	1.00	5.29	1.29	5.00	2.43	2.14	2.43	0.71
time (sec)	N/A	0.012	0.006	0.080	0.421	2.742	0.851	0.139	0.109

Problem 91	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	15	14	125	58	76	31	17
normalized size	1	1.00	0.33	0.31	2.78	1.29	1.69	0.69	0.38
time (sec)	N/A	0.040	0.019	0.205	0.441	1.090	1.869	0.180	2.793

Problem 92	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	26	28	171	50	294	48	27
normalized size	1	1.00	1.00	1.08	6.58	1.92	11.31	1.85	1.04
time (sec)	N/A	0.025	0.021	0.227	0.433	1.972	7.364	0.136	2.433

Problem 93	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	165	165	84	66	0	231	0	105	217
normalized size	1	1.00	0.51	0.40	0.00	1.40	0.00	0.64	1.32
time (sec)	N/A	0.141	0.106	0.287	0.000	0.933	0.000	0.282	2.592

Problem 94	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	30	47	0	68	0	68	35
normalized size	1	1.00	0.83	1.31	0.00	1.89	0.00	1.89	0.97
time (sec)	N/A	0.046	0.036	0.263	0.000	1.240	0.000	0.151	2.460

Problem 95	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	8	8	6	9	6	8	5	6	6
normalized size	1	1.00	0.75	1.12	0.75	1.00	0.62	0.75	0.75
time (sec)	N/A	0.030	0.011	0.102	0.309	0.829	0.981	0.133	2.252

Problem 96	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	8	8	8	9	6	6	7	6	6
normalized size	1	1.00	1.00	1.12	0.75	0.75	0.88	0.75	0.75
time (sec)	N/A	0.014	0.003	0.043	0.320	2.972	3.256	0.131	0.027

Problem 97	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	15	7	11	6	22	6	6
normalized size	1	1.00	1.00	0.47	0.73	0.40	1.47	0.40	0.40
time (sec)	N/A	0.009	0.005	0.043	0.308	1.484	0.446	0.131	0.018

Problem 98	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	17	14	13	13	22	13	13
normalized size	1	1.00	1.00	0.82	0.76	0.76	1.29	0.76	0.76
time (sec)	N/A	0.008	0.006	0.070	0.307	0.900	0.416	0.137	0.026

Problem 99	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	17	14	13	13	22	13	14
normalized size	1	1.00	1.00	0.82	0.76	0.76	1.29	0.76	0.82
time (sec)	N/A	0.008	0.005	0.058	0.313	0.554	0.409	0.123	0.024

Problem 100	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	26	28	27	25	44	29	57
normalized size	1	1.00	0.74	0.80	0.77	0.71	1.26	0.83	1.63
time (sec)	N/A	0.030	0.047	0.018	0.316	0.434	0.777	0.139	2.285

Problem 101	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	15	12	11	12	20	11	9
normalized size	1	1.00	1.00	0.80	0.73	0.80	1.33	0.73	0.60
time (sec)	N/A	0.009	0.005	0.066	0.305	1.586	0.448	0.135	0.025

Problem 102	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	17	14	13	7	20	13	7
normalized size	1	1.00	1.00	0.82	0.76	0.41	1.18	0.76	0.41
time (sec)	N/A	0.009	0.005	0.072	0.322	0.565	0.412	0.134	0.022

Problem 103	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	17	14	13	18	20	13	13
normalized size	1	1.00	1.00	0.82	0.76	1.06	1.18	0.76	0.76
time (sec)	N/A	0.008	0.005	0.100	0.314	1.081	0.409	0.136	0.025

Problem 104	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	25	28	28	25	56	29	39
normalized size	1	1.00	0.71	0.80	0.80	0.71	1.60	0.83	1.11
time (sec)	N/A	0.029	0.038	0.029	0.317	2.475	0.935	0.136	0.124

Problem 105	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	B	B	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	183	18	133	38	0	0	42
normalized size	1	1.00	9.15	0.90	6.65	1.90	0.00	0.00	2.10
time (sec)	N/A	0.025	0.234	0.040	0.434	2.030	0.000	0.000	2.358

Problem 106	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	F	B	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	48	19	0	38	0	0	42
normalized size	1	1.00	2.29	0.90	0.00	1.81	0.00	0.00	2.00
time (sec)	N/A	0.024	0.053	0.047	0.000	1.507	0.000	0.000	2.312

Problem 107	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	A	F	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	71	71	6196	68	0	101	0	0	295
normalized size	1	1.00	87.27	0.96	0.00	1.42	0.00	0.00	4.15
time (sec)	N/A	0.083	59.547	0.078	0.000	0.945	0.000	0.000	2.446

Problem 108	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	F	B	F	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	84	84	215	72	0	129	0	0	407
normalized size	1	1.00	2.56	0.86	0.00	1.54	0.00	0.00	4.85
time (sec)	N/A	0.098	0.590	0.071	0.000	1.201	0.000	0.000	2.497

Problem 109	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	A	F	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	679	104	0	134	0	0	787
normalized size	1	1.00	7.63	1.17	0.00	1.51	0.00	0.00	8.84
time (sec)	N/A	0.238	9.042	0.085	0.000	0.852	0.000	0.000	4.074

Problem 110	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	10	10	25	14	37	21	19	19	20
normalized size	1	1.00	2.50	1.40	3.70	2.10	1.90	1.90	2.00
time (sec)	N/A	0.020	0.013	0.086	0.329	1.297	0.832	0.141	2.336

Problem 111	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	47	36	131	39	0	39	39
normalized size	1	1.00	1.04	0.80	2.91	0.87	0.00	0.87	0.87
time (sec)	N/A	0.053	0.017	0.315	0.416	3.045	0.000	0.132	2.369

Problem 112	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	73	30	165	53	0	50	67
normalized size	1	1.00	2.61	1.07	5.89	1.89	0.00	1.79	2.39
time (sec)	N/A	0.049	0.066	0.408	0.425	1.007	0.000	0.140	2.349

Problem 113	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	110	110	133	82	0	137	0	117	611
normalized size	1	1.00	1.21	0.75	0.00	1.25	0.00	1.06	5.55
time (sec)	N/A	0.155	0.125	0.437	0.000	0.625	0.000	0.160	3.034

Problem 114	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	F	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	87	49	0	71	0	70	86
normalized size	1	1.00	2.29	1.29	0.00	1.87	0.00	1.84	2.26
time (sec)	N/A	0.071	0.090	0.555	0.000	0.691	0.000	0.175	2.435

Problem 115	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	92	92	179	0	0	0	0	0	-1
normalized size	1	1.00	1.95	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.091	0.183	0.553	0.000	1.066	0.000	0.000	0.000

Problem 116	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	15	13	137	33	0	31	12
normalized size	1	1.00	1.00	0.87	9.13	2.20	0.00	2.07	0.80
time (sec)	N/A	0.015	0.007	0.134	1.076	1.128	0.000	0.134	0.114

Problem 117	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	15	13	76	53	0	31	16
normalized size	1	1.00	0.34	0.30	1.73	1.20	0.00	0.70	0.36
time (sec)	N/A	0.036	0.015	0.175	0.430	1.488	0.000	0.169	2.664

Problem 118	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	71	71	67	54	0	121	0	99	95
normalized size	1	1.00	0.94	0.76	0.00	1.70	0.00	1.39	1.34
time (sec)	N/A	0.046	0.104	0.217	0.000	1.983	0.000	0.276	2.272

Problem 119	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	163	163	84	68	0	231	0	105	217
normalized size	1	1.00	0.52	0.42	0.00	1.42	0.00	0.64	1.33
time (sec)	N/A	0.129	0.099	0.259	0.000	2.070	0.000	0.284	2.667

Problem 120	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	81	80	0	154	0	132	118
normalized size	1	1.00	0.95	0.94	0.00	1.81	0.00	1.55	1.39
time (sec)	N/A	0.061	0.083	0.264	0.000	4.125	0.000	0.245	2.292

Problem 121	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	10	10	10	14	19	21	20	21	10
normalized size	1	1.00	1.00	1.40	1.90	2.10	2.00	2.10	1.00
time (sec)	N/A	0.018	0.006	0.117	0.313	0.680	1.053	0.141	2.247

Problem 122	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	14	18	129	25	427	25	12
normalized size	1	1.00	1.00	1.29	9.21	1.79	30.50	1.79	0.86
time (sec)	N/A	0.020	0.007	0.139	0.478	1.472	5.350	0.126	0.025

Problem 123	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	7	7	21	11	35	19	15	17	5
normalized size	1	1.00	3.00	1.57	5.00	2.71	2.14	2.43	0.71
time (sec)	N/A	0.011	0.003	0.080	0.315	2.121	0.875	0.123	0.027

Problem 124	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	21	34	129	19	17	24	17
normalized size	1	1.00	1.00	1.62	6.14	0.90	0.81	1.14	0.81
time (sec)	N/A	0.026	0.008	0.223	0.426	1.274	1.332	0.140	0.103

Problem 125	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	66	28	163	52	248	48	55
normalized size	1	1.00	2.54	1.08	6.27	2.00	9.54	1.85	2.12
time (sec)	N/A	0.026	0.059	0.141	0.442	0.829	6.139	0.135	2.307

Problem 126	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	62	62	57	80	0	72	0	67	51
normalized size	1	1.00	0.92	1.29	0.00	1.16	0.00	1.08	0.82
time (sec)	N/A	0.070	0.060	0.259	0.000	1.394	0.000	0.140	2.679

Problem 127	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	F	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	83	47	0	70	0	68	74
normalized size	1	1.00	2.31	1.31	0.00	1.94	0.00	1.89	2.06
time (sec)	N/A	0.041	0.077	0.167	0.000	0.707	0.000	0.151	2.368

Problem 128	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	33	26	25	57	63	25	57
normalized size	1	1.00	1.00	0.79	0.76	1.73	1.91	0.76	1.73
time (sec)	N/A	0.031	0.016	0.253	0.454	3.969	5.211	0.126	0.080

Problem 129	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	33	26	25	39	71	25	78
normalized size	1	1.00	1.00	0.79	0.76	1.18	2.15	0.76	2.36
time (sec)	N/A	0.033	0.017	0.135	0.312	0.579	5.728	0.141	2.469

Problem 130	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	25	20	19	32	48	19	25
normalized size	1	1.00	1.00	0.80	0.76	1.28	1.92	0.76	1.00
time (sec)	N/A	0.028	0.016	0.124	0.319	2.009	1.728	0.124	2.289

Problem 131	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	23	18	17	42	42	17	17
normalized size	1	1.00	1.00	0.78	0.74	1.83	1.83	0.74	0.74
time (sec)	N/A	0.026	0.012	0.129	0.311	0.603	1.637	0.125	2.487

Problem 132	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	33	26	25	49	65	49	150
normalized size	1	1.00	1.00	0.79	0.76	1.48	1.97	1.48	4.55
time (sec)	N/A	0.031	0.014	0.141	0.561	0.612	5.187	0.140	2.698

Problem 133	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	31	24	23	67	70	23	198
normalized size	1	1.00	1.00	0.77	0.74	2.16	2.26	0.74	6.39
time (sec)	N/A	0.030	0.016	0.450	0.727	1.289	5.272	0.140	3.179

Problem 134	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	41	32	31	25	228	31	25
normalized size	1	1.00	1.00	0.78	0.76	0.61	5.56	0.76	0.61
time (sec)	N/A	0.043	0.018	0.171	0.355	0.987	18.542	0.125	2.271

Problem 135	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	26	24	23	50	58	23	36
normalized size	1	1.00	0.96	0.89	0.85	1.85	2.15	0.85	1.33
time (sec)	N/A	0.024	0.047	0.053	0.307	0.623	0.742	0.124	2.476

Problem 136	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	26	24	23	44	61	23	36
normalized size	1	1.00	0.96	0.89	0.85	1.63	2.26	0.85	1.33
time (sec)	N/A	0.025	0.034	0.046	0.703	0.820	0.734	0.137	2.461

Problem 137	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	26	24	23	50	58	23	36
normalized size	1	1.00	0.96	0.89	0.85	1.85	2.15	0.85	1.33
time (sec)	N/A	0.017	0.025	0.045	0.564	0.558	0.724	0.124	2.275

Problem 138	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	26	24	23	44	58	23	36
normalized size	1	1.00	0.96	0.89	0.85	1.63	2.15	0.85	1.33
time (sec)	N/A	0.019	0.023	0.046	0.332	2.126	0.731	0.149	2.264

Problem 139	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	B	B	B	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	31	173	371	145	7713	0	207
normalized size	1	1.00	0.79	4.44	9.51	3.72	197.77	0.00	5.31
time (sec)	N/A	0.066	0.518	0.136	0.653	0.635	6.576	0.000	4.989

Problem 140	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	B	B	B	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	28	145	290	145	7720	0	196
normalized size	1	1.00	0.82	4.26	8.53	4.26	227.06	0.00	5.76
time (sec)	N/A	0.068	0.529	0.135	0.346	2.007	8.525	0.000	4.998

Problem 141	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	B	B	B	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	31	177	549	118	7485	0	207
normalized size	1	1.00	0.79	4.54	14.08	3.03	191.92	0.00	5.31
time (sec)	N/A	0.032	0.510	0.189	0.928	1.526	24.277	0.000	4.815

Problem 142	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	B	B	B	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	30	149	432	118	7499	0	200
normalized size	1	1.00	0.88	4.38	12.71	3.47	220.56	0.00	5.88
time (sec)	N/A	0.034	0.501	0.185	0.439	2.945	24.889	0.000	5.055

Problem 143	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	28	55	349	107	0	171	249
normalized size	1	1.00	0.78	1.53	9.69	2.97	0.00	4.75	6.92
time (sec)	N/A	0.019	0.229	0.475	0.342	1.988	0.000	0.233	7.837

Problem 144	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	26	53	322	93	0	169	249
normalized size	1	1.00	0.79	1.61	9.76	2.82	0.00	5.12	7.55
time (sec)	N/A	0.018	0.235	0.460	0.357	0.650	0.000	0.217	7.728

Problem 145	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	28	169	564	110	0	396	249
normalized size	1	1.00	0.78	4.69	15.67	3.06	0.00	11.00	6.92
time (sec)	N/A	0.018	0.243	0.490	0.354	0.754	0.000	0.238	7.774

Problem 146	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	29	81	536	96	1838	397	249
normalized size	1	1.00	0.88	2.45	16.24	2.91	55.70	12.03	7.55
time (sec)	N/A	0.018	0.220	0.477	0.502	0.785	129.081	0.227	7.873

Problem 147	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	13	177	57	22	0	0	20
normalized size	1	1.00	1.00	13.62	4.38	1.69	0.00	0.00	1.54
time (sec)	N/A	0.032	0.078	0.395	0.492	0.743	0.000	0.000	2.562

Problem 148	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	23	587	57	26	0	0	-1
normalized size	1	1.00	0.74	18.94	1.84	0.84	0.00	0.00	-0.03
time (sec)	N/A	0.053	0.038	0.263	0.474	0.488	0.000	0.000	0.000

Problem 149	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	50	29	324	82	38	0	0	-1
normalized size	1	1.00	0.58	6.48	1.64	0.76	0.00	0.00	-0.02
time (sec)	N/A	0.075	0.098	0.345	0.444	0.718	0.000	0.000	0.000

Problem 150	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	13	20	188	19	0	12	18
normalized size	1	1.00	1.00	1.54	14.46	1.46	0.00	0.92	1.38
time (sec)	N/A	0.038	0.068	0.254	0.624	0.664	0.000	0.156	2.653

Problem 151	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	21	26	314	23	0	19	-1
normalized size	1	1.00	0.68	0.84	10.13	0.74	0.00	0.61	-0.03
time (sec)	N/A	0.068	0.036	0.248	0.542	0.728	0.000	0.148	0.000

Problem 152	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	50	29	34	427	35	0	27	-1
normalized size	1	1.00	0.58	0.68	8.54	0.70	0.00	0.54	-0.02
time (sec)	N/A	0.094	0.098	0.303	0.627	0.599	0.000	0.143	0.000

Problem 153	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F(-2)	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	56	159	0	236	0	0	-1
normalized size	1	1.00	0.97	2.74	0.00	4.07	0.00	0.00	-0.02
time (sec)	N/A	0.078	0.139	0.524	0.000	1.557	0.000	0.000	0.000

Problem 154	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F(-2)	B	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	84	257	0	459	0	0	-1
normalized size	1	1.00	0.99	3.02	0.00	5.40	0.00	0.00	-0.01
time (sec)	N/A	0.106	0.260	0.829	0.000	0.630	0.000	0.000	0.000

Problem 155	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F(-2)	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	58	154	0	227	0	0	132
normalized size	1	1.00	0.98	2.61	0.00	3.85	0.00	0.00	2.24
time (sec)	N/A	0.059	0.102	0.264	0.000	0.571	0.000	0.000	3.292

Problem 156	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F(-2)	B	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	85	250	0	417	0	0	-1
normalized size	1	1.00	0.97	2.84	0.00	4.74	0.00	0.00	-0.01
time (sec)	N/A	0.103	0.315	0.435	0.000	0.659	0.000	0.000	0.000

Problem 157	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	B	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	50	48	86	250	80	0	322	-1
normalized size	1	1.00	0.96	1.72	5.00	1.60	0.00	6.44	-0.02
time (sec)	N/A	0.083	0.184	0.318	0.355	0.588	0.000	0.357	0.000

Problem 158	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	B	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	50	48	87	250	81	0	322	-1
normalized size	1	1.00	0.96	1.74	5.00	1.62	0.00	6.44	-0.02
time (sec)	N/A	0.082	0.187	0.308	0.329	0.743	0.000	0.345	0.000

Problem 159	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	32	24	23	205	0	40	24
normalized size	1	1.00	1.00	0.75	0.72	6.41	0.00	1.25	0.75
time (sec)	N/A	0.055	0.093	0.322	0.432	0.623	0.000	0.616	2.560

Problem 160	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F	B	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	211	211	512	1003	0	3344	0	0	-1
normalized size	1	1.00	2.43	4.75	0.00	15.85	0.00	0.00	-0.00
time (sec)	N/A	0.528	6.438	0.602	0.000	1.299	0.000	0.000	0.000

Problem 161	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	337	337	294	1251	0	4644	0	0	-1
normalized size	1	1.00	0.87	3.71	0.00	13.78	0.00	0.00	-0.00
time (sec)	N/A	0.900	1.089	0.463	0.000	2.281	0.000	0.000	0.000

Problem 162	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	40	34	43	300	0	76	45
normalized size	1	1.00	1.00	0.85	1.08	7.50	0.00	1.90	1.12
time (sec)	N/A	0.605	0.255	0.362	1.069	0.750	0.000	0.972	2.597

Problem 163	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F	B	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	267	267	751	1670	0	4160	0	0	-1
normalized size	1	1.00	2.81	6.25	0.00	15.58	0.00	0.00	-0.00
time (sec)	N/A	0.719	4.226	0.605	0.000	1.383	0.000	0.000	0.000

Problem 164	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	407	407	499	2061	0	5772	0	0	-1
normalized size	1	1.00	1.23	5.06	0.00	14.18	0.00	0.00	-0.00
time (sec)	N/A	1.075	2.214	0.547	0.000	1.285	0.000	0.000	0.000

Problem 165	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	155	155	61	0	0	0	0	0	111
normalized size	1	1.00	0.39	0.00	0.00	0.00	0.00	0.00	0.72
time (sec)	N/A	0.199	0.512	0.402	0.000	0.000	0.000	0.000	3.053

Problem 166	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	118	118	54	0	0	0	0	0	86
normalized size	1	1.00	0.46	0.00	0.00	0.00	0.00	0.00	0.73
time (sec)	N/A	0.176	0.333	0.176	0.000	0.000	0.000	0.000	2.808

Problem 167	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	44	0	0	0	0	0	61
normalized size	1	1.00	0.59	0.00	0.00	0.00	0.00	0.00	0.82
time (sec)	N/A	0.110	0.203	0.169	0.000	0.000	0.000	0.000	2.692

Problem 168	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	86	86	52	0	0	0	0	0	-1
normalized size	1	1.00	0.60	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.183	0.214	0.174	0.000	0.000	0.000	0.000	0.000

Problem 169	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	123	123	65	0	0	0	0	0	-1
normalized size	1	1.00	0.53	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.199	0.243	0.183	0.000	0.000	0.000	0.000	0.000

Problem 170	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	176	176	87	0	0	0	0	0	-1
normalized size	1	1.00	0.49	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.226	0.284	0.184	0.000	0.000	0.000	0.000	0.000

Problem 171	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F(-1)	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	393	393	113	0	0	0	0	0	216
normalized size	1	1.00	0.29	0.00	0.00	0.00	0.00	0.00	0.55
time (sec)	N/A	0.375	1.156	0.177	0.000	0.000	0.000	0.000	4.147

Problem 172	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	265	265	95	0	0	0	0	0	159
normalized size	1	1.00	0.36	0.00	0.00	0.00	0.00	0.00	0.60
time (sec)	N/A	0.274	0.808	0.170	0.000	0.000	0.000	0.000	3.765

Problem 173	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	168	168	73	0	0	0	0	0	123
normalized size	1	1.00	0.43	0.00	0.00	0.00	0.00	0.00	0.73
time (sec)	N/A	0.143	0.639	0.168	0.000	0.000	0.000	0.000	1.206

Problem 174	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F(-2)	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	186	186	150	0	0	0	0	0	-1
normalized size	1	1.00	0.81	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.662	1.223	0.171	0.000	0.000	0.000	0.000	0.000

Problem 175	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F(-2)	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	273	273	231	0	0	0	0	0	-1
normalized size	1	1.00	0.85	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.665	1.438	0.171	0.000	0.000	0.000	0.000	0.000

Problem 176	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F(-2)	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	385	385	317	0	0	0	0	0	-1
normalized size	1	1.00	0.82	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.741	1.838	0.168	0.000	0.000	0.000	0.000	0.000

Problem 177	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	767	767	247	0	0	0	0	0	-1
normalized size	1	1.00	0.32	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.351	2.658	0.208	0.000	0.000	0.000	0.000	0.000

Problem 178	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	555	555	194	0	0	0	0	0	-1
normalized size	1	1.00	0.35	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.877	1.888	0.187	0.000	0.000	0.000	0.000	0.000

Problem 179	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	355	355	154	0	0	0	0	0	-1
normalized size	1	1.00	0.43	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.511	1.220	0.354	0.000	0.000	0.000	0.000	0.000

Problem 180	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	F(-2)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	110	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.646	3.544	0.188	0.000	0.000	0.000	0.000	0.000

Problem 181	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	536	536	193	0	0	0	0	0	-1
normalized size	1	1.00	0.36	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	3.550	2.082	0.144	0.000	0.729	0.000	0.000	0.000

Problem 182	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	280	280	154	0	0	0	0	0	-1
normalized size	1	1.00	0.55	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	2.155	1.556	0.150	0.000	0.000	0.000	0.000	0.000

Problem 183	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	171	171	150	0	0	72	0	0	88
normalized size	1	1.00	0.88	0.00	0.00	0.42	0.00	0.00	0.51
time (sec)	N/A	0.973	0.603	0.146	0.000	0.556	0.000	0.000	3.316

Problem 184	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	300	300	85	154	56	75	0	0	-1
normalized size	1	1.00	0.28	0.51	0.19	0.25	0.00	0.00	-0.00
time (sec)	N/A	0.438	0.090	0.145	0.494	0.771	0.000	0.000	0.000

Problem 185	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	51	24	23	32	27	51	54
normalized size	1	1.00	2.83	1.33	1.28	1.78	1.50	2.83	3.00
time (sec)	N/A	0.098	0.015	0.099	0.307	0.665	2.168	0.175	2.476

Problem 186	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	67	52	54	60	61	100	403
normalized size	1	1.00	1.18	0.91	0.95	1.05	1.07	1.75	7.07
time (sec)	N/A	0.202	0.081	0.100	0.319	0.766	3.542	0.162	2.464

Problem 187	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	75	75	80	77	84	77	92	125	431
normalized size	1	1.00	1.07	1.03	1.12	1.03	1.23	1.67	5.75
time (sec)	N/A	0.297	0.102	0.106	0.314	1.193	7.628	0.176	2.477

Problem 188	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	104	104	97	103	118	89	116	149	460
normalized size	1	1.00	0.93	0.99	1.13	0.86	1.12	1.43	4.42
time (sec)	N/A	0.403	0.123	0.115	0.310	0.695	15.258	0.160	2.523

Problem 189	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	71	46	63	47	0	46	25
normalized size	1	1.00	2.84	1.84	2.52	1.88	0.00	1.84	1.00
time (sec)	N/A	0.093	0.084	0.088	0.423	1.141	0.000	0.152	2.364

Problem 190	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	76	71	93	85	0	77	50
normalized size	1	1.00	1.58	1.48	1.94	1.77	0.00	1.60	1.04
time (sec)	N/A	0.185	0.207	0.087	0.463	0.741	0.000	0.162	2.349

Problem 191	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	75	75	88	95	119	122	0	102	92
normalized size	1	1.00	1.17	1.27	1.59	1.63	0.00	1.36	1.23
time (sec)	N/A	0.311	0.347	0.096	0.639	0.742	0.000	0.162	2.366

Problem 192	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	96	96	104	119	143	158	0	126	140
normalized size	1	1.00	1.08	1.24	1.49	1.65	0.00	1.31	1.46
time (sec)	N/A	0.414	0.750	0.096	0.771	0.568	0.000	0.155	2.340

Problem 193	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	98	98	78	228	43	123	0	134	-1
normalized size	1	1.00	0.80	2.33	0.44	1.26	0.00	1.37	-0.01
time (sec)	N/A	0.432	0.158	0.382	1.119	0.730	0.000	0.210	0.000

Problem 194	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	62	199	26	99	0	92	-1
normalized size	1	1.00	0.86	2.76	0.36	1.38	0.00	1.28	-0.01
time (sec)	N/A	0.294	0.101	0.310	1.105	0.656	0.000	0.189	0.000

Problem 195	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	47	152	13	81	0	61	-1
normalized size	1	1.00	1.07	3.45	0.30	1.84	0.00	1.39	-0.02
time (sec)	N/A	0.160	0.036	0.303	0.470	0.779	0.000	0.180	0.000

Problem 196	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	68	52	192	58	116	0	133	-1
normalized size	1	1.00	0.76	2.82	0.85	1.71	0.00	1.96	-0.01
time (sec)	N/A	0.195	0.048	0.348	1.053	0.606	0.000	0.702	0.000

Problem 197	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-1)	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	92	92	73	270	0	187	0	168	-1
normalized size	1	1.00	0.79	2.93	0.00	2.03	0.00	1.83	-0.01
time (sec)	N/A	0.336	0.370	0.345	0.000	0.698	0.000	0.764	0.000

Problem 198	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-1)	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	120	120	95	322	0	234	0	199	-1
normalized size	1	1.00	0.79	2.68	0.00	1.95	0.00	1.66	-0.01
time (sec)	N/A	0.482	0.500	0.355	0.000	0.728	0.000	0.945	0.000

Problem 199	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	A	F(-1)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	25	80	0	35	0	283	-1
normalized size	1	1.00	1.00	3.20	0.00	1.40	0.00	11.32	-0.04
time (sec)	N/A	0.053	0.199	0.702	0.000	0.714	0.000	0.324	0.000

Problem 200	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	A	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	24	91	0	36	0	397	68
normalized size	1	1.00	1.00	3.79	0.00	1.50	0.00	16.54	2.83
time (sec)	N/A	0.056	0.126	0.408	0.000	0.741	0.000	0.323	2.745

Problem 201	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	8	8	8	9	8	15	41	8	8
normalized size	1	1.00	1.00	1.12	1.00	1.88	5.12	1.00	1.00
time (sec)	N/A	0.041	0.008	0.081	0.439	0.649	0.970	0.126	2.307

Problem 202	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	24	16	15	35	248	49	26
normalized size	1	1.00	0.67	0.44	0.42	0.97	6.89	1.36	0.72
time (sec)	N/A	0.041	0.028	0.083	0.551	0.655	48.461	0.129	2.324

Problem 203	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	8	8	8	11	10	15	12	16	8
normalized size	1	1.00	1.00	1.38	1.25	1.88	1.50	2.00	1.00
time (sec)	N/A	0.041	0.007	0.078	0.773	0.646	0.859	0.150	2.287

Problem 204	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	23	17	16	35	61	49	27
normalized size	1	1.00	0.62	0.46	0.43	0.95	1.65	1.32	0.73
time (sec)	N/A	0.038	0.031	0.075	0.423	1.143	2.660	0.130	2.286

Problem 205	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	14	32	0	13	61	26	13
normalized size	1	1.00	1.00	2.29	0.00	0.93	4.36	1.86	0.93
time (sec)	N/A	0.128	0.010	0.098	0.000	0.635	56.758	0.147	2.474

Problem 206	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	105	105	73	148	0	254	2608	110	2429
normalized size	1	1.00	0.70	1.41	0.00	2.42	24.84	1.05	23.13
time (sec)	N/A	0.260	0.152	0.108	0.000	1.017	111.367	0.138	4.043

Problem 207	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	34	44	29	45	143	62	242
normalized size	1	1.00	0.60	0.77	0.51	0.79	2.51	1.09	4.25
time (sec)	N/A	0.134	0.085	0.128	0.423	1.328	10.667	0.139	2.427

Problem 208	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	15	20	17	19	24	29	13
normalized size	1	1.00	1.00	1.33	1.13	1.27	1.60	1.93	0.87
time (sec)	N/A	0.089	0.013	0.109	0.420	0.612	1.178	0.145	2.309

Problem 209	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	47	78	40	228	0	58	1987
normalized size	1	1.00	0.96	1.59	0.82	4.65	0.00	1.18	40.55
time (sec)	N/A	0.150	0.161	0.118	1.340	0.791	0.000	0.155	2.848

Problem 210	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	13	28	0	12	56	22	13
normalized size	1	1.00	1.00	2.15	0.00	0.92	4.31	1.69	1.00
time (sec)	N/A	0.119	0.011	0.115	0.000	0.680	103.318	0.145	2.461

Problem 211	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	100	100	72	153	0	262	0	93	1646
normalized size	1	1.00	0.72	1.53	0.00	2.62	0.00	0.93	16.46
time (sec)	N/A	0.240	0.183	0.080	0.000	0.782	0.000	0.155	5.150

Problem 212	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	56	56	31	42	28	45	520	62	249
normalized size	1	1.00	0.55	0.75	0.50	0.80	9.29	1.11	4.45
time (sec)	N/A	0.197	0.081	0.121	0.433	1.011	55.654	0.151	2.387

Problem 213	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	14	17	14	18	51	23	12
normalized size	1	1.00	1.00	1.21	1.00	1.29	3.64	1.64	0.86
time (sec)	N/A	0.058	0.012	0.109	0.429	1.077	1.331	0.149	2.315

Problem 214	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	47	84	42	255	0	70	1774
normalized size	1	1.00	0.96	1.71	0.86	5.20	0.00	1.43	36.20
time (sec)	N/A	0.161	0.151	0.125	0.429	2.032	0.000	0.143	2.946

Problem 215	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	98	135	0	318	0	125	1302
normalized size	1	1.00	1.32	1.82	0.00	4.30	0.00	1.69	17.59
time (sec)	N/A	0.246	0.451	0.100	0.000	3.458	0.000	0.165	3.514

Problem 216	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	102	120	0	332	0	110	463
normalized size	1	1.00	1.42	1.67	0.00	4.61	0.00	1.53	6.43
time (sec)	N/A	0.238	0.530	0.106	0.000	3.955	0.000	0.170	2.832

Problem 217	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	136	136	94	0	0	0	0	0	-1
normalized size	1	1.00	0.69	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.058	0.238	1.001	0.000	1.855	0.000	0.000	0.000

Problem 218	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	95	95	88	0	0	0	0	0	-1
normalized size	1	1.00	0.93	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.049	0.171	1.055	0.000	1.177	0.000	0.000	0.000

Problem 219	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	127	127	246	321	257	257	461	316	422
normalized size	1	1.00	1.94	2.53	2.02	2.02	3.63	2.49	3.32
time (sec)	N/A	0.078	1.022	0.414	0.330	1.010	6.556	0.664	6.158

Problem 220	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	161	161	192	285	238	219	770	235	519
normalized size	1	1.00	1.19	1.77	1.48	1.36	4.78	1.46	3.22
time (sec)	N/A	0.079	0.691	0.335	0.330	1.184	4.623	0.315	4.117

Problem 221	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	94	94	156	175	172	155	267	187	248
normalized size	1	1.00	1.66	1.86	1.83	1.65	2.84	1.99	2.64
time (sec)	N/A	0.045	0.455	0.267	0.748	0.863	2.217	0.292	2.720

Problem 222	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	108	108	107	153	136	121	381	122	320
normalized size	1	1.00	0.99	1.42	1.26	1.12	3.53	1.13	2.96
time (sec)	N/A	0.044	0.303	0.253	0.323	1.110	1.271	0.188	3.461

Problem 223	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	81	75	84	77	117	91	104
normalized size	1	1.00	1.40	1.29	1.45	1.33	2.02	1.57	1.79
time (sec)	N/A	0.024	0.323	0.242	0.310	1.318	0.520	0.154	2.480

Problem 224	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	52	70	68	52	128	50	63
normalized size	1	1.00	0.95	1.27	1.24	0.95	2.33	0.91	1.15
time (sec)	N/A	0.019	0.102	0.239	0.474	1.096	0.277	0.154	2.418

Problem 225	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	46	25	24	23	31	24	38
normalized size	1	1.00	1.92	1.04	1.00	0.96	1.29	1.00	1.58
time (sec)	N/A	0.014	0.012	0.044	0.401	1.269	0.145	0.141	2.324

Problem 226	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-2)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	45	43	80	131	0	74	39
normalized size	1	1.00	0.96	0.91	1.70	2.79	0.00	1.57	0.83
time (sec)	N/A	0.027	0.060	0.404	0.682	1.740	0.000	0.221	2.801

Problem 227	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	32	21	21	57	0	20	47
normalized size	1	1.00	1.00	0.66	0.66	1.78	0.00	0.62	1.47
time (sec)	N/A	0.016	0.036	0.490	0.523	0.586	0.000	0.154	2.342

Problem 228	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	B	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	103	103	132	191	326	294	0	221	260
normalized size	1	1.00	1.28	1.85	3.17	2.85	0.00	2.15	2.52
time (sec)	N/A	0.057	0.294	0.534	0.463	1.375	0.000	0.230	4.555

Problem 229	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	98	98	85	64	85	217	0	50	222
normalized size	1	1.00	0.87	0.65	0.87	2.21	0.00	0.51	2.27
time (sec)	N/A	0.042	0.290	0.559	0.338	0.674	0.000	0.153	3.116

Problem 230	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	156	156	157	514	822	544	0	588	719
normalized size	1	1.00	1.01	3.29	5.27	3.49	0.00	3.77	4.61
time (sec)	N/A	0.086	1.147	0.604	1.038	1.126	0.000	0.309	6.093

Problem 231	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	151	151	182	125	174	441	0	118	470
normalized size	1	1.00	1.21	0.83	1.15	2.92	0.00	0.78	3.11
time (sec)	N/A	0.069	0.523	0.644	0.790	2.052	0.000	0.221	5.151

Problem 232	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	186	186	205	183	0	0	0	0	-1
normalized size	1	1.00	1.10	0.98	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.096	1.816	0.444	0.000	1.095	0.000	0.000	0.000

Problem 233	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	131	131	256	246	0	0	0	0	-1
normalized size	1	1.00	1.95	1.88	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.058	1.600	0.372	0.000	0.929	0.000	0.000	0.000

Problem 234	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	131	131	143	163	0	0	0	0	-1
normalized size	1	1.00	1.09	1.24	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.058	1.290	0.372	0.000	0.811	0.000	0.000	0.000

Problem 235	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	75	75	268	159	0	0	0	0	-1
normalized size	1	1.00	3.57	2.12	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.030	1.096	0.355	0.000	0.801	0.000	0.000	0.000

Problem 236	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	75	75	92	121	0	0	0	0	-1
normalized size	1	1.00	1.23	1.61	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.030	0.188	0.251	0.000	0.871	0.000	0.000	0.000

Problem 237	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	138	138	219	228	0	0	0	0	-1
normalized size	1	1.00	1.59	1.65	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.058	3.058	0.346	0.000	1.931	0.000	0.000	0.000

Problem 238	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	142	142	145	178	0	0	0	0	-1
normalized size	1	1.00	1.02	1.25	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.055	1.684	0.335	0.000	1.884	0.000	0.000	0.000

Problem 239	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	197	197	277	309	0	0	0	0	-1
normalized size	1	1.00	1.41	1.57	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.086	2.436	0.379	0.000	1.394	0.000	0.000	0.000

Problem 240	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	120	120	153	128	0	0	0	0	-1
normalized size	1	1.00	1.28	1.07	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.070	0.476	0.346	0.000	0.598	0.000	0.000	0.000

Problem 241	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	75	75	199	174	0	0	0	0	-1
normalized size	1	1.00	2.65	2.32	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.045	0.820	0.372	0.000	0.971	0.000	0.000	0.000

Problem 242	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	75	75	133	108	0	0	0	0	-1
normalized size	1	1.00	1.77	1.44	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.043	0.299	0.336	0.000	1.247	0.000	0.000	0.000

Problem 243	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	184	112	0	0	0	0	-1
normalized size	1	1.00	6.81	4.15	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.023	0.811	0.333	0.000	1.187	0.000	0.000	0.000

Problem 244	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	88	85	0	0	0	0	-1
normalized size	1	1.00	3.26	3.15	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.024	0.094	0.243	0.000	1.768	0.000	0.000	0.000

Problem 245	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	190	162	0	0	0	0	-1
normalized size	1	1.00	2.60	2.22	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.041	1.032	0.336	0.000	1.090	0.000	0.000	0.000

Problem 246	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	75	75	157	118	0	0	0	0	-1
normalized size	1	1.00	2.09	1.57	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.042	0.706	0.332	0.000	0.505	0.000	0.000	0.000

Problem 247	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	120	120	224	205	0	0	0	0	-1
normalized size	1	1.00	1.87	1.71	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.064	1.951	0.349	0.000	1.544	0.000	0.000	0.000

Problem 248	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	31	31	59	23	44	23	-1
normalized size	1	1.00	0.97	0.97	1.84	0.72	1.38	0.72	-0.03
time (sec)	N/A	0.015	0.085	0.259	0.424	1.829	0.254	0.447	0.000

Problem 249	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	31	151	132	17	37	52	84
normalized size	1	1.00	1.00	4.87	4.26	0.55	1.19	1.68	2.71
time (sec)	N/A	0.018	0.111	0.292	0.461	1.834	0.141	0.190	2.555

Problem 250	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	31	76	83	17	37	52	66
normalized size	1	1.00	1.00	2.45	2.68	0.55	1.19	1.68	2.13
time (sec)	N/A	0.016	0.077	0.276	0.311	0.852	0.138	0.183	2.465

Problem 251	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	31	73	69	17	37	52	44
normalized size	1	1.00	1.00	2.35	2.23	0.55	1.19	1.68	1.42
time (sec)	N/A	0.014	0.052	0.261	0.544	1.691	0.134	0.151	2.425

Problem 252	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	51	26	24	15	26	24	20
normalized size	1	1.00	1.96	1.00	0.92	0.58	1.00	0.92	0.77
time (sec)	N/A	0.014	0.012	0.002	1.120	1.932	0.123	0.150	2.389

Problem 253	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	29	23	29	17	31	21	25
normalized size	1	1.00	1.00	0.79	1.00	0.59	1.07	0.72	0.86
time (sec)	N/A	0.015	0.032	0.402	0.323	0.917	0.142	0.135	2.389

Problem 254	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	31	23	22	17	46	30	31
normalized size	1	1.00	1.00	0.74	0.71	0.55	1.48	0.97	1.00
time (sec)	N/A	0.016	0.040	0.398	0.328	0.935	0.144	0.153	2.412

Problem 255	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	31	57	29	17	46	36	68
normalized size	1	1.00	1.00	1.84	0.94	0.55	1.48	1.16	2.19
time (sec)	N/A	0.015	0.045	0.429	0.628	1.621	0.145	0.146	2.465

Problem 256	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	31	36	29	17	46	44	91
normalized size	1	1.00	1.00	1.16	0.94	0.55	1.48	1.42	2.94
time (sec)	N/A	0.015	0.047	0.420	0.566	1.329	0.150	0.165	2.558

Problem 257	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	32	28	51	17	0	17	35
normalized size	1	1.00	0.97	0.85	1.55	0.52	0.00	0.52	1.06
time (sec)	N/A	0.016	0.030	0.363	0.693	1.469	0.000	0.567	0.432

Problem 258	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	32	28	51	17	0	17	33
normalized size	1	1.00	0.97	0.85	1.55	0.52	0.00	0.52	1.00
time (sec)	N/A	0.016	0.029	0.220	0.903	1.080	0.000	0.457	2.378

Problem 259	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	30	28	51	17	0	25	-1
normalized size	1	1.00	0.97	0.90	1.65	0.55	0.00	0.81	-0.03
time (sec)	N/A	0.016	0.022	0.219	0.845	2.385	0.000	0.138	0.000

Problem 260	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	30	28	51	17	0	37	-1
normalized size	1	1.00	0.97	0.90	1.65	0.55	0.00	1.19	-0.03
time (sec)	N/A	0.017	0.034	0.221	0.429	1.039	0.000	0.335	0.000

Problem 261	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	32	28	51	17	0	65	-1
normalized size	1	1.00	0.97	0.85	1.55	0.52	0.00	1.97	-0.03
time (sec)	N/A	0.016	0.031	0.214	1.064	1.487	0.000	1.228	0.000

Problem 262	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	32	28	51	17	0	67	-1
normalized size	1	1.00	0.97	0.85	1.55	0.52	0.00	2.03	-0.03
time (sec)	N/A	0.016	0.034	0.211	0.662	0.969	0.000	3.299	0.000

Problem 263	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	149	149	303	199	204	166	308	178	272
normalized size	1	1.00	2.03	1.34	1.37	1.11	2.07	1.19	1.83
time (sec)	N/A	0.211	1.239	0.184	0.318	0.925	7.121	0.154	2.883

Problem 264	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	100	100	96	96	72	80	97	131	115
normalized size	1	1.00	0.96	0.96	0.72	0.80	0.97	1.31	1.15
time (sec)	N/A	0.196	0.197	0.089	1.049	0.839	4.224	0.160	2.532

Problem 265	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	75	75	123	82	95	85	122	86	126
normalized size	1	1.00	1.64	1.09	1.27	1.13	1.63	1.15	1.68
time (sec)	N/A	0.136	0.572	0.082	0.353	2.072	4.936	0.142	2.503

Problem 266	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	25	26	26	29	22	40	40
normalized size	1	1.00	0.93	0.96	0.96	1.07	0.81	1.48	1.48
time (sec)	N/A	0.054	0.047	0.049	0.410	1.622	1.254	0.161	2.380

Problem 267	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	42	16	14	25	24	34	37
normalized size	1	1.00	3.50	1.33	1.17	2.08	2.00	2.83	3.08
time (sec)	N/A	0.007	0.005	0.003	0.303	1.495	0.091	0.129	2.493

Problem 268	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	11	11	11	12	50	11	32	12	55
normalized size	1	1.00	1.00	1.09	4.55	1.00	2.91	1.09	5.00
time (sec)	N/A	0.034	0.007	0.118	0.504	1.020	0.418	0.130	3.792

Problem 269	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	F(-2)	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	344	106	0	308	0	94	604
normalized size	1	1.00	5.21	1.61	0.00	4.67	0.00	1.42	9.15
time (sec)	N/A	0.133	2.021	0.163	0.000	1.058	0.000	0.168	2.806

Problem 270	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	40	57	201	83	503	43	106
normalized size	1	1.00	0.78	1.12	3.94	1.63	9.86	0.84	2.08
time (sec)	N/A	0.075	0.173	0.174	0.513	0.947	2.759	0.138	2.698

Problem 271	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F(-2)	B	F	B	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	156	156	2661	967	0	931	0	369	2782
normalized size	1	1.00	17.06	6.20	0.00	5.97	0.00	2.37	17.83
time (sec)	N/A	0.337	6.359	0.208	0.000	1.403	0.000	0.168	6.373

Problem 272	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	101	86	130	483	217	1719	91	541
normalized size	1	1.00	0.85	1.29	4.78	2.15	17.02	0.90	5.36
time (sec)	N/A	0.116	0.338	0.211	0.507	1.058	15.190	0.139	3.837

Problem 273	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	54	106	141	38	68	62	59
normalized size	1	1.00	1.80	3.53	4.70	1.27	2.27	2.07	1.97
time (sec)	N/A	0.050	0.106	0.118	0.344	0.957	7.127	0.141	2.438

Problem 274	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	64	71	28	61	44	20	20
normalized size	1	1.00	2.13	2.37	0.93	2.03	1.47	0.67	0.67
time (sec)	N/A	0.102	0.133	0.095	0.440	0.911	4.096	0.155	2.373

Problem 275	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	31	45	52	21	44	48	43
normalized size	1	1.00	1.72	2.50	2.89	1.17	2.44	2.67	2.39
time (sec)	N/A	0.044	0.023	0.096	0.329	0.907	4.807	0.172	2.395

Problem 276	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	14	15	14	28	10	14	14
normalized size	1	1.00	0.88	0.94	0.88	1.75	0.62	0.88	0.88
time (sec)	N/A	0.070	0.011	0.046	0.451	0.890	1.108	0.130	2.357

Problem 277	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	9	38	13	10	9	20	31	19
normalized size	1	0.69	2.92	1.00	0.77	0.69	1.54	2.38	1.46
time (sec)	N/A	0.006	0.005	0.001	0.310	3.009	0.091	0.147	2.399

Problem 278	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	5	5	16	6	31	5	17	22	21
normalized size	1	1.00	3.20	1.20	6.20	1.00	3.40	4.40	4.20
time (sec)	N/A	0.024	0.013	0.116	0.316	0.610	0.138	0.138	2.776

Problem 279	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	27	15	28	25	0	14	14
normalized size	1	1.00	1.93	1.07	2.00	1.79	0.00	1.00	1.00
time (sec)	N/A	0.041	0.024	0.118	0.413	1.006	0.000	0.150	2.342

Problem 280	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	34	17	64	20	301	45	41
normalized size	1	1.00	2.12	1.06	4.00	1.25	18.81	2.81	2.56
time (sec)	N/A	0.046	0.019	0.158	0.442	1.067	0.830	0.162	2.359

Problem 281	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	62	23	64	63	0	20	19
normalized size	1	1.00	2.38	0.88	2.46	2.42	0.00	0.77	0.73
time (sec)	N/A	0.069	0.068	0.151	0.438	0.862	0.000	0.155	2.340

Problem 282	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	39	23	92	35	1059	64	61
normalized size	1	1.00	1.77	1.05	4.18	1.59	48.14	2.91	2.77
time (sec)	N/A	0.048	0.046	0.184	0.661	1.057	2.762	0.150	2.379

Problem 283	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	152	152	143	204	188	292	308	169	174
normalized size	1	1.00	0.94	1.34	1.24	1.92	2.03	1.11	1.14
time (sec)	N/A	0.220	0.696	0.136	0.309	0.872	102.278	0.128	2.580

Problem 284	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	101	95	93	80	95	97	215	127
normalized size	1	1.00	0.94	0.92	0.79	0.94	0.96	2.13	1.26
time (sec)	N/A	0.215	0.261	0.067	0.414	0.951	33.025	0.158	2.530

Problem 285	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	77	79	87	87	128	124	86	82
normalized size	1	1.00	1.03	1.13	1.13	1.66	1.61	1.12	1.06
time (sec)	N/A	0.136	0.288	0.068	0.313	0.969	14.106	0.145	2.448

Problem 286	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	24	29	29	28	31	52	30
normalized size	1	1.00	0.83	1.00	1.00	0.97	1.07	1.79	1.03
time (sec)	N/A	0.056	0.134	0.042	0.425	0.923	2.786	0.150	2.417

Problem 287	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	25	16	15	27	24	15	27
normalized size	1	1.00	2.08	1.33	1.25	2.25	2.00	1.25	2.25
time (sec)	N/A	0.007	0.007	0.003	0.314	0.923	0.095	0.141	2.408

Problem 288	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	12	13	45	12	0	13	36
normalized size	1	1.00	1.00	1.08	3.75	1.00	0.00	1.08	3.00
time (sec)	N/A	0.035	0.016	0.108	0.404	1.204	0.000	0.121	3.283

Problem 289	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	67	71	86	0	307	0	107	440
normalized size	1	1.00	1.06	1.28	0.00	4.58	0.00	1.60	6.57
time (sec)	N/A	0.117	0.269	0.122	0.000	1.045	0.000	0.137	3.044

Problem 290	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	50	77	56	177	70	0	45	311
normalized size	1	1.00	1.54	1.12	3.54	1.40	0.00	0.90	6.22
time (sec)	N/A	0.079	0.112	0.119	0.428	0.844	0.000	0.126	2.770

Problem 291	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	159	159	150	534	0	878	0	282	3068
normalized size	1	1.00	0.94	3.36	0.00	5.52	0.00	1.77	19.30
time (sec)	N/A	0.338	0.484	0.143	0.000	2.076	0.000	0.195	8.246

Problem 292	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	100	100	138	132	497	166	0	93	538
normalized size	1	1.00	1.38	1.32	4.97	1.66	0.00	0.93	5.38
time (sec)	N/A	0.123	0.335	0.128	0.480	2.030	0.000	0.128	3.680

Problem 293	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	32	105	125	37	68	22	34
normalized size	1	1.00	1.14	3.75	4.46	1.32	2.43	0.79	1.21
time (sec)	N/A	0.051	0.072	0.108	0.315	1.584	107.210	0.124	2.417

Problem 294	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	30	68	56	36	44	20	16
normalized size	1	1.00	1.00	2.27	1.87	1.20	1.47	0.67	0.53
time (sec)	N/A	0.101	0.044	0.088	0.418	1.702	35.042	0.131	2.413

Problem 295	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	20	49	46	22	46	18	25
normalized size	1	1.00	1.00	2.45	2.30	1.10	2.30	0.90	1.25
time (sec)	N/A	0.048	0.039	0.081	0.309	1.675	14.638	0.146	2.385

Problem 296	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	12	15	16	16	17	12	10
normalized size	1	1.00	0.75	0.94	1.00	1.00	1.06	0.75	0.62
time (sec)	N/A	0.069	0.022	0.043	0.406	1.157	2.703	0.158	2.404

Problem 297	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	9	9	20	13	12	7	20	11	19
normalized size	1	1.00	2.22	1.44	1.33	0.78	2.22	1.22	2.11
time (sec)	N/A	0.005	0.004	0.001	0.304	1.030	0.096	0.151	2.409

Problem 298	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	7	7	9	8	14	9	0	7	9
normalized size	1	1.00	1.29	1.14	2.00	1.29	0.00	1.00	1.29
time (sec)	N/A	0.027	0.011	0.108	0.406	0.915	0.000	0.132	2.857

Problem 299	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	12	11	23	18	0	10	10
normalized size	1	1.00	0.86	0.79	1.64	1.29	0.00	0.71	0.71
time (sec)	N/A	0.041	0.012	0.109	0.406	1.096	0.000	0.144	2.407

Problem 300	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	18	15	28	21	0	14	18
normalized size	1	1.00	1.29	1.07	2.00	1.50	0.00	1.00	1.29
time (sec)	N/A	0.047	0.013	0.151	0.410	0.863	0.000	0.141	2.345

Problem 301	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	30	17	35	40	0	16	16
normalized size	1	1.00	1.15	0.65	1.35	1.54	0.00	0.62	0.62
time (sec)	N/A	0.071	0.013	0.142	0.407	0.954	0.000	0.129	2.371

Problem 302	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	32	25	39	38	0	22	26
normalized size	1	1.00	1.33	1.04	1.62	1.58	0.00	0.92	1.08
time (sec)	N/A	0.050	0.013	0.158	0.413	1.990	0.000	0.121	2.408

Problem 303	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	38	39	36	51	44	39	59
normalized size	1	1.00	0.86	0.89	0.82	1.16	1.00	0.89	1.34
time (sec)	N/A	0.034	0.030	0.060	0.314	2.054	8.781	0.125	2.494

Problem 304	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	61	32	37	57	42	99	75
normalized size	1	1.00	1.79	0.94	1.09	1.68	1.24	2.91	2.21
time (sec)	N/A	0.046	0.020	0.058	0.311	1.771	3.469	0.161	2.488

Problem 305	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	18	15	16	20	15	23	21
normalized size	1	1.00	0.82	0.68	0.73	0.91	0.68	1.05	0.95
time (sec)	N/A	0.024	0.015	0.049	0.312	1.330	1.569	0.150	2.404

Problem 306	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	8	8	19	12	11	21	19	9	8
normalized size	1	1.00	2.38	1.50	1.38	2.62	2.38	1.12	1.00
time (sec)	N/A	0.005	0.004	0.001	0.318	0.944	0.089	0.148	0.021

Problem 307	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	2	2	2	5	17	4	0	17	12
normalized size	1	1.00	1.00	2.50	8.50	2.00	0.00	8.50	6.00
time (sec)	N/A	0.019	0.004	0.099	0.308	2.197	0.000	0.138	2.462

Problem 308	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	8	8	8	7	6	14	0	6	6
normalized size	1	1.00	1.00	0.88	0.75	1.75	0.00	0.75	0.75
time (sec)	N/A	0.014	0.003	0.106	0.326	1.696	0.000	0.130	2.406

Problem 309	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	17	14	103	14	0	59	13
normalized size	1	1.00	1.00	0.82	6.06	0.82	0.00	3.47	0.76
time (sec)	N/A	0.038	0.019	0.116	0.311	1.203	0.000	0.132	2.547

Problem 310	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	37	14	13	26	0	13	23
normalized size	1	1.00	2.18	0.82	0.76	1.53	0.00	0.76	1.35
time (sec)	N/A	0.018	0.017	0.113	0.311	1.541	0.000	0.122	2.566

Problem 311	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	25	20	187	20	0	101	19
normalized size	1	1.00	1.00	0.80	7.48	0.80	0.00	4.04	0.76
time (sec)	N/A	0.040	0.015	0.122	0.358	0.969	0.000	0.152	2.944

Problem 312	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	57	20	19	40	0	19	33
normalized size	1	1.00	2.28	0.80	0.76	1.60	0.00	0.76	1.32
time (sec)	N/A	0.022	0.017	0.125	0.346	0.859	0.000	0.127	2.915

Problem 313	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	33	26	271	26	0	143	25
normalized size	1	1.00	1.00	0.79	8.21	0.79	0.00	4.33	0.76
time (sec)	N/A	0.043	0.016	0.135	0.343	0.652	0.000	0.143	3.538

Problem 314	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	37	40	578	44	0	0	-1
normalized size	1	1.00	0.51	0.55	7.92	0.60	0.00	0.00	-0.01
time (sec)	N/A	0.148	0.080	0.313	0.572	0.998	0.000	0.000	0.000

Problem 315	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	50	29	34	427	35	0	0	-1
normalized size	1	1.00	0.58	0.68	8.54	0.70	0.00	0.00	-0.02
time (sec)	N/A	0.112	0.077	0.282	0.556	1.719	0.000	0.000	0.000

Problem 316	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	21	26	314	23	0	0	-1
normalized size	1	1.00	0.68	0.84	10.13	0.74	0.00	0.00	-0.03
time (sec)	N/A	0.081	0.036	0.238	0.524	0.911	0.000	0.000	0.000

Problem 317	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	13	20	188	19	0	0	15
normalized size	1	1.00	1.00	1.54	14.46	1.46	0.00	0.00	1.15
time (sec)	N/A	0.049	0.025	0.234	0.508	0.770	0.000	0.000	2.483

Problem 318	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	44	0	0	124	0	0	-1
normalized size	1	1.00	0.73	0.00	0.00	2.07	0.00	0.00	-0.02
time (sec)	N/A	0.091	0.267	0.257	0.000	1.058	0.000	0.000	0.000

Problem 319	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	60	450	0	152	0	0	-1
normalized size	1	1.00	0.75	5.62	0.00	1.90	0.00	0.00	-0.01
time (sec)	N/A	0.116	0.151	0.549	0.000	1.802	0.000	0.000	0.000

Problem 320	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	99	99	69	382	0	165	0	0	-1
normalized size	1	1.00	0.70	3.86	0.00	1.67	0.00	0.00	-0.01
time (sec)	N/A	0.151	0.515	0.353	0.000	1.955	0.000	0.000	0.000

Problem 321	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	118	118	74	487	0	167	0	0	-1
normalized size	1	1.00	0.63	4.13	0.00	1.42	0.00	0.00	-0.01
time (sec)	N/A	0.180	0.263	0.387	0.000	1.124	0.000	0.000	0.000

Problem 322	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	38	40	26	37	44	35	80
normalized size	1	1.00	0.86	0.91	0.59	0.84	1.00	0.80	1.82
time (sec)	N/A	0.031	0.032	0.059	0.322	1.854	8.968	0.135	2.573

Problem 323	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	38	30	37	49	42	39	68
normalized size	1	1.00	1.12	0.88	1.09	1.44	1.24	1.15	2.00
time (sec)	N/A	0.042	0.010	0.059	0.319	0.942	3.738	0.146	2.464

Problem 324	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	16	13	12	22	14	18	49
normalized size	1	1.00	0.73	0.59	0.55	1.00	0.64	0.82	2.23
time (sec)	N/A	0.021	0.017	0.046	0.307	0.952	1.622	0.143	2.410

Problem 325	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	8	8	37	12	11	21	19	29	14
normalized size	1	1.00	4.62	1.50	1.38	2.62	2.38	3.62	1.75
time (sec)	N/A	0.005	0.004	0.002	0.315	0.945	0.087	0.154	2.300

Problem 326	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	4	4	4	7	21	6	0	6	6
normalized size	1	1.00	1.00	1.75	5.25	1.50	0.00	1.50	1.50
time (sec)	N/A	0.018	0.003	0.096	0.322	0.830	0.000	0.149	2.368

Problem 327	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	8	8	8	7	6	18	0	6	6
normalized size	1	1.00	1.00	0.88	0.75	2.25	0.00	0.75	0.75
time (sec)	N/A	0.014	0.003	0.098	0.321	0.901	0.000	0.146	2.450

Problem 328	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	17	14	73	28	0	14	14
normalized size	1	1.00	1.00	0.82	4.29	1.65	0.00	0.82	0.82
time (sec)	N/A	0.038	0.009	0.105	0.319	1.018	0.000	0.132	2.382

Problem 329	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	37	14	14	39	0	14	16
normalized size	1	1.00	2.18	0.82	0.82	2.29	0.00	0.82	0.94
time (sec)	N/A	0.017	0.022	0.106	0.321	0.648	0.000	0.141	2.431

Problem 330	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	25	20	121	46	0	20	20
normalized size	1	1.00	1.00	0.80	4.84	1.84	0.00	0.80	0.80
time (sec)	N/A	0.041	0.011	0.109	0.335	1.384	0.000	0.143	2.421

Problem 331	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	57	20	20	57	0	20	46
normalized size	1	1.00	2.28	0.80	0.80	2.28	0.00	0.80	1.84
time (sec)	N/A	0.021	0.019	0.110	0.319	1.471	0.000	0.124	2.446

Problem 332	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	33	26	169	64	0	26	109
normalized size	1	1.00	1.00	0.79	5.12	1.94	0.00	0.79	3.30
time (sec)	N/A	0.042	0.013	0.113	0.334	2.268	0.000	0.149	2.506

Problem 333	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	37	603	82	44	0	0	-1
normalized size	1	1.00	0.51	8.26	1.12	0.60	0.00	0.00	-0.01
time (sec)	N/A	0.113	0.213	0.408	0.441	0.990	0.000	0.000	0.000

Problem 334	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	50	29	321	82	38	0	0	-1
normalized size	1	1.00	0.58	6.42	1.64	0.76	0.00	0.00	-0.02
time (sec)	N/A	0.086	0.080	0.330	0.428	0.863	0.000	0.000	0.000

Problem 335	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	23	584	57	26	0	0	-1
normalized size	1	1.00	0.74	18.84	1.84	0.84	0.00	0.00	-0.03
time (sec)	N/A	0.064	0.038	0.266	0.428	1.249	0.000	0.000	0.000

Problem 336	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	13	174	57	22	0	46	20
normalized size	1	1.00	1.00	13.38	4.38	1.69	0.00	3.54	1.54
time (sec)	N/A	0.041	0.026	0.313	0.434	1.260	0.000	0.205	2.424

Problem 337	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	52	52	43	105	0	72	0	0	-1
normalized size	1	1.00	0.83	2.02	0.00	1.38	0.00	0.00	-0.02
time (sec)	N/A	0.078	0.250	0.281	0.000	1.005	0.000	0.000	0.000

Problem 338	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	56	265	0	119	0	114	-1
normalized size	1	1.00	0.78	3.68	0.00	1.65	0.00	1.58	-0.01
time (sec)	N/A	0.094	0.175	0.314	0.000	0.974	0.000	0.366	0.000

Problem 339	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	91	91	73	454	0	147	0	170	-1
normalized size	1	1.00	0.80	4.99	0.00	1.62	0.00	1.87	-0.01
time (sec)	N/A	0.121	0.649	0.320	0.000	1.024	0.000	0.405	0.000

Problem 340	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F(-1)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	110	110	74	494	0	171	0	229	-1
normalized size	1	1.00	0.67	4.49	0.00	1.55	0.00	2.08	-0.01
time (sec)	N/A	0.140	0.358	0.358	0.000	1.630	0.000	0.449	0.000

Problem 341	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	129	66	68	78	90	1375	88
normalized size	1	1.00	2.35	1.20	1.24	1.42	1.64	25.00	1.60
time (sec)	N/A	0.110	0.203	0.066	0.422	1.741	4.627	5.638	2.539

Problem 342	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	40	39	42	47	46	173	65
normalized size	1	1.00	1.05	1.03	1.11	1.24	1.21	4.55	1.71
time (sec)	N/A	0.050	0.039	0.069	0.317	2.016	5.985	0.451	2.450

Problem 343	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	60	25	28	44	31	177	61
normalized size	1	1.00	2.40	1.00	1.12	1.76	1.24	7.08	2.44
time (sec)	N/A	0.063	0.095	0.046	0.409	0.971	1.663	0.202	2.423

Problem 344	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	10	10	10	11	8	12	8	11	22
normalized size	1	1.00	1.00	1.10	0.80	1.20	0.80	1.10	2.20
time (sec)	N/A	0.005	0.003	0.003	0.309	1.435	0.051	0.141	2.399

Problem 345	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	35	24	25	35	0	28	16
normalized size	1	1.00	1.46	1.00	1.04	1.46	0.00	1.17	0.67
time (sec)	N/A	0.058	0.014	0.106	0.308	1.897	0.000	0.144	2.467

Problem 346	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	57	32	45	34	0	31	40
normalized size	1	1.00	1.73	0.97	1.36	1.03	0.00	0.94	1.21
time (sec)	N/A	0.127	0.015	0.115	0.310	1.448	0.000	0.140	2.393

Problem 347	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	83	56	73	130	0	95	48
normalized size	1	1.00	1.38	0.93	1.22	2.17	0.00	1.58	0.80
time (sec)	N/A	0.072	0.017	0.126	0.314	1.821	0.000	0.156	2.386

Problem 348	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	129	64	97	78	0	65	87
normalized size	1	1.00	1.98	0.98	1.49	1.20	0.00	1.00	1.34
time (sec)	N/A	0.211	0.019	0.127	0.324	1.064	0.000	0.151	2.453

Problem 349	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	68	150	153	144	641	131	695
normalized size	1	1.00	0.92	2.03	2.07	1.95	8.66	1.77	9.39
time (sec)	N/A	0.063	0.197	0.144	0.423	0.930	36.723	0.211	7.017

Problem 350	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	75	75	82	108	146	200	0	130	105
normalized size	1	1.00	1.09	1.44	1.95	2.67	0.00	1.73	1.40
time (sec)	N/A	0.060	0.314	0.168	0.421	1.970	0.000	0.187	2.583

Problem 351	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	B	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	116	116	132	177	338	279	0	199	227
normalized size	1	1.00	1.14	1.53	2.91	2.41	0.00	1.72	1.96
time (sec)	N/A	0.110	0.387	0.193	0.436	0.920	0.000	0.238	2.856

Problem 352	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	67	150	153	143	678	131	692
normalized size	1	1.00	0.92	2.05	2.10	1.96	9.29	1.79	9.48
time (sec)	N/A	0.053	0.144	0.131	0.423	1.663	37.030	0.225	6.525

Problem 353	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	76	76	82	109	156	201	0	132	126
normalized size	1	1.00	1.08	1.43	2.05	2.64	0.00	1.74	1.66
time (sec)	N/A	0.053	0.234	0.158	0.425	0.680	0.000	0.187	2.623

Problem 354	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	B	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	116	116	118	204	366	279	0	245	251
normalized size	1	1.00	1.02	1.76	3.16	2.41	0.00	2.11	2.16
time (sec)	N/A	0.108	0.337	0.181	0.443	0.932	0.000	0.246	2.879

Problem 355	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	246	246	238	514	354	221	857	287	522
normalized size	1	1.00	0.97	2.09	1.44	0.90	3.48	1.17	2.12
time (sec)	N/A	0.169	1.419	0.302	0.328	1.064	3.038	0.347	7.319

Problem 356	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	178	178	163	250	207	145	415	199	261
normalized size	1	1.00	0.92	1.40	1.16	0.81	2.33	1.12	1.47
time (sec)	N/A	0.102	0.598	0.251	0.322	0.848	1.375	0.235	7.324

Problem 357	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	116	116	111	124	113	81	192	92	100
normalized size	1	1.00	0.96	1.07	0.97	0.70	1.66	0.79	0.86
time (sec)	N/A	0.058	0.212	0.220	0.307	0.945	0.454	0.168	3.075

Problem 358	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	36	36	35	34	42	35	48
normalized size	1	1.00	0.97	0.97	0.95	0.92	1.14	0.95	1.30
time (sec)	N/A	0.015	0.033	0.039	0.306	0.920	0.142	0.123	2.672

Problem 359	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	49	50	40	75	0	43	38
normalized size	1	1.00	1.00	1.02	0.82	1.53	0.00	0.88	0.78
time (sec)	N/A	0.036	0.099	0.289	0.307	1.640	0.000	0.160	2.824

Problem 360	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	129	129	98	233	0	192	0	160	274
normalized size	1	1.00	0.76	1.81	0.00	1.49	0.00	1.24	2.12
time (sec)	N/A	0.085	0.247	0.395	0.000	1.736	0.000	0.203	3.673

Problem 361	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F(-2)	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	191	191	420	496	0	490	0	346	592
normalized size	1	1.00	2.20	2.60	0.00	2.57	0.00	1.81	3.10
time (sec)	N/A	0.133	2.591	0.522	0.000	0.964	0.000	0.610	8.120

Problem 362	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F(-2)	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	259	259	533	823	0	739	0	599	1004
normalized size	1	1.00	2.06	3.18	0.00	2.85	0.00	2.31	3.88
time (sec)	N/A	0.189	2.030	0.710	0.000	1.850	0.000	2.525	12.306

Problem 363	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	157	157	135	177	191	134	291	151	239
normalized size	1	1.00	0.86	1.13	1.22	0.85	1.85	0.96	1.52
time (sec)	N/A	0.143	0.413	0.256	0.321	1.990	0.756	0.183	2.575

Problem 364	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	92	101	99	71	170	78	96
normalized size	1	1.00	1.14	1.25	1.22	0.88	2.10	0.96	1.19
time (sec)	N/A	0.050	0.144	0.219	0.305	1.423	0.314	0.163	3.213

Problem 365	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	53	30	29	27	39	29	29
normalized size	1	1.00	1.83	1.03	1.00	0.93	1.34	1.00	1.00
time (sec)	N/A	0.016	0.015	0.001	0.303	0.883	0.140	0.139	2.428

Problem 366	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	57	23	29	60	63	23	22
normalized size	1	1.00	2.28	0.92	1.16	2.40	2.52	0.92	0.88
time (sec)	N/A	0.022	0.050	0.385	0.313	1.340	1.099	0.126	2.820

Problem 367	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	75	75	115	91	90	154	0	86	79
normalized size	1	1.00	1.53	1.21	1.20	2.05	0.00	1.15	1.05
time (sec)	N/A	0.049	0.553	0.490	0.322	0.991	0.000	0.136	2.479

Problem 368	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	134	134	186	211	190	433	0	171	162
normalized size	1	1.00	1.39	1.57	1.42	3.23	0.00	1.28	1.21
time (sec)	N/A	0.112	2.996	0.560	0.336	0.947	0.000	0.166	2.481

Problem 369	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	207	207	492	378	307	791	0	304	260
normalized size	1	1.00	2.38	1.83	1.48	3.82	0.00	1.47	1.26
time (sec)	N/A	0.248	1.727	0.599	0.358	1.051	0.000	0.212	2.526

Problem 370	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	50	21	28	31	36	21	20
normalized size	1	1.00	2.17	0.91	1.22	1.35	1.57	0.91	0.87
time (sec)	N/A	0.021	0.030	0.396	0.309	1.170	0.609	0.157	2.488

Problem 371	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	75	75	93	60	80	100	168	68	59
normalized size	1	1.00	1.24	0.80	1.07	1.33	2.24	0.91	0.79
time (sec)	N/A	0.048	0.186	0.408	0.324	1.212	1.829	0.166	2.454

Problem 372	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	123	123	135	100	146	143	423	107	90
normalized size	1	1.00	1.10	0.81	1.19	1.16	3.44	0.87	0.73
time (sec)	N/A	0.107	0.571	0.447	0.340	1.764	6.486	0.165	2.446

Problem 373	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	168	168	247	140	208	237	792	139	161
normalized size	1	1.00	1.47	0.83	1.24	1.41	4.71	0.83	0.96
time (sec)	N/A	0.186	0.984	0.456	0.346	0.952	23.777	0.201	2.441

Problem 374	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	157	157	136	178	188	134	291	151	258
normalized size	1	1.00	0.87	1.13	1.20	0.85	1.85	0.96	1.64
time (sec)	N/A	0.134	0.434	0.250	0.329	1.947	0.764	0.187	3.214

Problem 375	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	92	100	98	71	170	78	84
normalized size	1	1.00	1.14	1.23	1.21	0.88	2.10	0.96	1.04
time (sec)	N/A	0.047	0.145	0.223	0.323	0.525	0.320	0.161	2.504

Problem 376	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	53	30	29	28	39	29	29
normalized size	1	1.00	1.83	1.03	1.00	0.97	1.34	1.00	1.00
time (sec)	N/A	0.014	0.012	0.001	0.311	0.843	0.146	0.149	2.429

Problem 377	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	50	42	54	60	95	42	26
normalized size	1	1.00	2.00	1.68	2.16	2.40	3.80	1.68	1.04
time (sec)	N/A	0.021	0.151	0.421	0.315	0.905	1.300	0.158	2.617

Problem 378	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	75	75	229	110	137	162	0	115	91
normalized size	1	1.00	3.05	1.47	1.83	2.16	0.00	1.53	1.21
time (sec)	N/A	0.053	0.417	0.470	0.332	0.956	0.000	0.168	2.546

Problem 379	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	B	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	134	134	350	272	265	438	0	239	186
normalized size	1	1.00	2.61	2.03	1.98	3.27	0.00	1.78	1.39
time (sec)	N/A	0.113	0.612	0.566	0.349	1.085	0.000	0.206	4.468

Problem 380	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	207	207	494	416	382	796	0	363	301
normalized size	1	1.00	2.39	2.01	1.85	3.85	0.00	1.75	1.45
time (sec)	N/A	0.240	1.172	0.593	0.378	2.475	0.000	0.247	6.051

Problem 381	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	157	157	135	177	191	127	291	151	292
normalized size	1	1.00	0.86	1.13	1.22	0.81	1.85	0.96	1.86
time (sec)	N/A	0.140	0.443	0.246	0.329	0.768	0.754	0.202	3.603

Problem 382	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	92	101	99	72	170	79	127
normalized size	1	1.00	1.14	1.25	1.22	0.89	2.10	0.98	1.57
time (sec)	N/A	0.047	0.144	0.227	0.319	0.765	0.314	0.165	3.718

Problem 383	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	53	30	29	27	39	29	29
normalized size	1	1.00	1.83	1.03	1.00	0.93	1.34	1.00	1.00
time (sec)	N/A	0.016	0.014	0.002	0.315	2.087	0.138	0.139	2.440

Problem 384	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	93	104	66	54	107	82	33
normalized size	1	1.00	2.82	3.15	2.00	1.64	3.24	2.48	1.00
time (sec)	N/A	0.022	0.067	0.413	0.321	1.752	1.691	0.202	2.830

Problem 385	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	83	162	166	185	148	0	196	126
normalized size	1	1.00	1.95	2.00	2.23	1.78	0.00	2.36	1.52
time (sec)	N/A	0.050	0.552	0.534	0.337	1.250	0.000	0.218	2.721

Problem 386	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	142	142	255	639	493	420	0	481	360
normalized size	1	1.00	1.80	4.50	3.47	2.96	0.00	3.39	2.54
time (sec)	N/A	0.112	2.434	0.572	0.348	0.857	0.000	0.241	6.445

Problem 387	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	215	215	632	1069	963	729	0	1006	730
normalized size	1	1.00	2.94	4.97	4.48	3.39	0.00	4.68	3.40
time (sec)	N/A	0.244	3.014	0.586	0.393	1.089	0.000	0.306	7.217

Problem 388	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	157	157	136	176	188	126	291	151	292
normalized size	1	1.00	0.87	1.12	1.20	0.80	1.85	0.96	1.86
time (sec)	N/A	0.136	0.436	0.253	0.320	0.925	0.758	0.182	3.439

Problem 389	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	92	100	98	70	170	79	128
normalized size	1	1.00	1.14	1.23	1.21	0.86	2.10	0.98	1.58
time (sec)	N/A	0.047	0.149	0.233	0.324	0.902	0.316	0.142	3.743

Problem 390	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	53	30	29	26	39	29	29
normalized size	1	1.00	1.83	1.03	1.00	0.90	1.34	1.00	1.00
time (sec)	N/A	0.014	0.012	0.001	0.328	2.899	0.142	0.144	2.445

Problem 391	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	96	61	62	57	109	82	32
normalized size	1	1.00	2.91	1.85	1.88	1.73	3.30	2.48	0.97
time (sec)	N/A	0.022	0.101	0.417	0.329	1.984	1.720	0.211	2.739

Problem 392	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	83	166	178	182	154	0	198	126
normalized size	1	1.00	2.00	2.14	2.19	1.86	0.00	2.39	1.52
time (sec)	N/A	0.053	0.617	0.507	0.338	0.841	0.000	0.216	2.738

Problem 393	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F(-1)	B	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	142	142	261	687	491	423	0	481	361
normalized size	1	1.00	1.84	4.84	3.46	2.98	0.00	3.39	2.54
time (sec)	N/A	0.108	2.771	0.532	0.359	1.512	0.000	0.241	6.482

Problem 394	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	215	215	636	1149	959	735	0	1006	731
normalized size	1	1.00	2.96	5.34	4.46	3.42	0.00	4.68	3.40
time (sec)	N/A	0.235	1.920	0.614	0.392	1.999	0.000	0.282	7.088

Problem 395	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	260	260	237	335	330	255	682	286	376
normalized size	1	1.00	0.91	1.29	1.27	0.98	2.62	1.10	1.45
time (sec)	N/A	0.400	1.052	0.268	0.333	1.069	1.856	0.301	3.374

Problem 396	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	170	170	144	177	189	147	294	167	333
normalized size	1	1.00	0.85	1.04	1.11	0.86	1.73	0.98	1.96
time (sec)	N/A	0.186	0.433	0.243	0.331	2.942	0.755	0.199	3.700

Problem 397	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	91	91	77	99	100	73	162	81	125
normalized size	1	1.00	0.85	1.09	1.10	0.80	1.78	0.89	1.37
time (sec)	N/A	0.046	0.173	0.224	0.311	1.516	0.307	0.150	3.784

Problem 398	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	49	28	27	26	34	27	40
normalized size	1	1.00	1.81	1.04	1.00	0.96	1.26	1.00	1.48
time (sec)	N/A	0.016	0.012	0.001	0.311	0.887	0.139	0.147	2.506

Problem 399	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	57	61	0	434	3179	91	75
normalized size	1	1.00	0.93	1.00	0.00	7.11	52.11	1.49	1.23
time (sec)	N/A	0.084	0.121	0.378	0.000	1.705	152.527	0.165	4.011

Problem 400	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	121	121	116	424	0	819	0	222	195
normalized size	1	1.00	0.96	3.50	0.00	6.77	0.00	1.83	1.61
time (sec)	N/A	0.108	0.359	0.499	0.000	1.038	0.000	0.164	3.057

Problem 401	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	197	197	200	3933	0	1947	0	892	700
normalized size	1	1.00	1.02	19.96	0.00	9.88	0.00	4.53	3.55
time (sec)	N/A	0.198	0.952	0.563	0.000	2.014	0.000	0.315	6.055

Problem 402	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F(-2)	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	292	292	606	16909	0	4069	0	2671	1946
normalized size	1	1.00	2.08	57.91	0.00	13.93	0.00	9.15	6.66
time (sec)	N/A	0.376	2.113	0.658	0.000	1.632	0.000	0.576	4.806

Problem 403	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	185	185	399	701	0	0	0	0	-1
normalized size	1	1.00	2.16	3.79	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.268	5.906	0.661	0.000	1.184	0.000	0.000	0.000

Problem 404	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	139	139	349	686	0	0	0	0	-1
normalized size	1	1.00	2.51	4.94	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.139	3.420	0.458	0.000	0.868	0.000	0.000	0.000

Problem 405	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	326	461	0	0	0	0	-1
normalized size	1	1.00	7.24	10.24	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.031	2.273	0.405	0.000	0.521	0.000	0.000	0.000

Problem 406	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	128	152	0	0	0	0	-1
normalized size	1	1.00	2.84	3.38	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.037	0.258	0.314	0.000	2.261	0.000	0.000	0.000

Problem 407	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	94	94	390	437	0	0	0	0	-1
normalized size	1	1.00	4.15	4.65	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.054	6.035	0.490	0.000	1.602	0.000	0.000	0.000

Problem 408	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	187	187	430	542	0	0	0	0	-1
normalized size	1	1.00	2.30	2.90	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.200	3.182	0.606	0.000	2.882	0.000	0.000	0.000

Problem 409	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	233	233	436	571	0	0	0	0	-1
normalized size	1	1.00	1.87	2.45	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.260	3.838	0.626	0.000	0.835	0.000	0.000	0.000

Problem 410	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	347	347	3767	2303	0	0	0	0	-1
normalized size	1	1.00	10.86	6.64	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.532	6.619	1.003	0.000	1.880	0.000	0.000	0.000

Problem 411	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	283	283	2190	1516	0	0	0	0	-1
normalized size	1	1.00	7.74	5.36	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.282	6.264	0.708	0.000	0.856	0.000	0.000	0.000

Problem 412	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	108	108	1408	720	0	0	0	0	-1
normalized size	1	1.00	13.04	6.67	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.071	6.231	0.460	0.000	0.751	0.000	0.000	0.000

Problem 413	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	108	108	285	303	0	0	0	0	-1
normalized size	1	1.00	2.64	2.81	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.070	0.562	0.366	0.000	3.145	0.000	0.000	0.000

Problem 414	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	186	186	1540	2388	0	0	0	0	-1
normalized size	1	1.00	8.28	12.84	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.104	6.354	0.773	0.000	0.742	0.000	0.000	0.000

Problem 415	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	382	382	2408	2967	0	0	0	0	-1
normalized size	1	1.00	6.30	7.77	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.363	6.409	2.135	0.000	2.965	0.000	0.000	0.000

Problem 416	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	490	490	4116	3876	0	0	0	0	-1
normalized size	1	1.00	8.40	7.91	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.619	6.660	6.686	0.000	1.124	0.000	0.000	0.000

Problem 417	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	139	139	130	74	0	101	0	0	-1
normalized size	1	1.00	0.94	0.53	0.00	0.73	0.00	0.00	-0.01
time (sec)	N/A	0.065	0.595	0.367	0.000	1.086	0.000	0.000	0.000

Problem 418	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	93	93	104	60	0	81	0	0	-1
normalized size	1	1.00	1.12	0.65	0.00	0.87	0.00	0.00	-0.01
time (sec)	N/A	0.040	0.333	0.346	0.000	0.918	0.000	0.000	0.000

Problem 419	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	75	50	0	61	0	0	39
normalized size	1	1.00	1.70	1.14	0.00	1.39	0.00	0.00	0.89
time (sec)	N/A	0.018	0.040	0.297	0.000	1.839	0.000	0.000	0.310

Problem 420	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	101	77	0	147	0	0	-1
normalized size	1	1.00	2.10	1.60	0.00	3.06	0.00	0.00	-0.02
time (sec)	N/A	0.065	0.105	0.254	0.000	1.171	0.000	0.000	0.000

Problem 421	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	96	96	154	117	0	268	0	284	-1
normalized size	1	1.00	1.60	1.22	0.00	2.79	0.00	2.96	-0.01
time (sec)	N/A	0.053	0.295	0.313	0.000	0.926	0.000	0.744	0.000

Problem 422	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	142	142	180	190	0	341	0	417	-1
normalized size	1	1.00	1.27	1.34	0.00	2.40	0.00	2.94	-0.01
time (sec)	N/A	0.077	0.405	0.316	0.000	2.933	0.000	0.974	0.000

Problem 423	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	185	185	151	86	0	121	0	0	-1
normalized size	1	1.00	0.82	0.46	0.00	0.65	0.00	0.00	-0.01
time (sec)	N/A	0.094	1.818	0.283	0.000	0.846	0.000	0.000	0.000

Problem 424	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	139	139	127	74	0	101	0	0	-1
normalized size	1	1.00	0.91	0.53	0.00	0.73	0.00	0.00	-0.01
time (sec)	N/A	0.074	0.489	0.306	0.000	0.652	0.000	0.000	0.000

Problem 425	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	93	93	103	60	0	80	0	0	-1
normalized size	1	1.00	1.11	0.65	0.00	0.86	0.00	0.00	-0.01
time (sec)	N/A	0.038	0.223	0.306	0.000	0.728	0.000	0.000	0.000

Problem 426	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	75	50	0	59	0	0	39
normalized size	1	1.00	1.70	1.14	0.00	1.34	0.00	0.00	0.89
time (sec)	N/A	0.017	0.044	0.296	0.000	0.646	0.000	0.000	0.423

Problem 427	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	99	77	0	88	0	0	-1
normalized size	1	1.00	2.02	1.57	0.00	1.80	0.00	0.00	-0.02
time (sec)	N/A	0.061	0.093	0.329	0.000	2.112	0.000	0.000	0.000

Problem 428	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	B	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	96	96	152	118	0	210	0	249	-1
normalized size	1	1.00	1.58	1.23	0.00	2.19	0.00	2.59	-0.01
time (sec)	N/A	0.052	0.313	0.369	0.000	1.034	0.000	0.529	0.000

Problem 429	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	B	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	142	142	178	190	0	280	0	381	-1
normalized size	1	1.00	1.25	1.34	0.00	1.97	0.00	2.68	-0.01
time (sec)	N/A	0.075	0.379	0.338	0.000	0.893	0.000	0.805	0.000

Problem 430	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F(-2)	A	F(-1)	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	258	258	11888	306	0	268	0	0	-1
normalized size	1	1.00	46.08	1.19	0.00	1.04	0.00	0.00	-0.00
time (sec)	N/A	0.179	33.456	0.362	0.000	1.522	0.000	0.000	0.000

Problem 431	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F(-2)	A	F(-1)	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	190	190	11771	200	0	189	0	0	-1
normalized size	1	1.00	61.95	1.05	0.00	0.99	0.00	0.00	-0.01
time (sec)	N/A	0.122	34.256	0.368	0.000	0.956	0.000	0.000	0.000

Problem 432	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F(-2)	A	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	126	126	11679	126	0	125	0	0	-1
normalized size	1	1.00	92.69	1.00	0.00	0.99	0.00	0.00	-0.01
time (sec)	N/A	0.075	33.015	0.369	0.000	0.680	0.000	0.000	0.000

Problem 433	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F(-2)	A	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	11586	113	0	80	0	0	-1
normalized size	1	1.00	210.65	2.05	0.00	1.45	0.00	0.00	-0.02
time (sec)	N/A	0.033	32.708	0.355	0.000	0.912	0.000	0.000	0.000

Problem 434	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F(-2)	B	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	63264	172	0	349	0	0	-1
normalized size	1	1.00	718.91	1.95	0.00	3.97	0.00	0.00	-0.01
time (sec)	N/A	0.120	33.827	0.378	0.000	2.776	0.000	0.000	0.000

Problem 435	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F(-1)	B	F(-2)	B	F	F(-2)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	160	160	0	350	0	646	0	0	-1
normalized size	1	1.00	0.00	2.19	0.00	4.04	0.00	0.00	-0.01
time (sec)	N/A	0.133	180.015	0.410	0.000	1.819	0.000	0.000	0.000

Problem 436	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F(-1)	A	F(-2)	B	F	F(-2)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	226	226	0	350	0	895	0	0	-1
normalized size	1	1.00	0.00	1.55	0.00	3.96	0.00	0.00	-0.00
time (sec)	N/A	0.186	180.032	0.402	0.000	2.526	0.000	0.000	0.000

Problem 437	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F(-2)	A	F(-1)	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	196	196	11602	204	0	192	0	0	-1
normalized size	1	1.00	59.19	1.04	0.00	0.98	0.00	0.00	-0.01
time (sec)	N/A	0.134	34.189	0.375	0.000	1.122	0.000	0.000	0.000

Problem 438	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F(-2)	A	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	130	130	11512	130	0	127	0	0	-1
normalized size	1	1.00	88.55	1.00	0.00	0.98	0.00	0.00	-0.01
time (sec)	N/A	0.081	21.611	0.379	0.000	1.476	0.000	0.000	0.000

Problem 439	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F(-2)	A	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	11415	117	0	80	0	0	-1
normalized size	1	1.00	200.26	2.05	0.00	1.40	0.00	0.00	-0.02
time (sec)	N/A	0.038	21.948	0.355	0.000	2.892	0.000	0.000	0.000

Problem 440	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F(-2)	B	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	91	91	61904	175	0	107	0	0	-1
normalized size	1	1.00	680.26	1.92	0.00	1.18	0.00	0.00	-0.01
time (sec)	N/A	0.098	34.470	0.302	0.000	1.146	0.000	0.000	0.000

Problem 441	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F(-1)	B	F(-2)	B	F	F(-2)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	164	164	0	363	0	442	0	0	-1
normalized size	1	1.00	0.00	2.21	0.00	2.70	0.00	0.00	-0.01
time (sec)	N/A	0.126	180.001	0.362	0.000	0.859	0.000	0.000	0.000

Problem 442	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F(-1)	A	F(-2)	B	F	F(-2)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	232	232	0	363	0	655	0	0	-1
normalized size	1	1.00	0.00	1.56	0.00	2.82	0.00	0.00	-0.00
time (sec)	N/A	0.170	180.061	0.454	0.000	1.197	0.000	0.000	0.000

Problem 443	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	101	80	438	0	579	0	160	950
normalized size	1	1.00	0.79	4.34	0.00	5.73	0.00	1.58	9.41
time (sec)	N/A	0.096	0.217	0.116	0.000	1.207	0.000	0.163	11.444

Problem 444	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	30	22	25	41	11	22	25	34
normalized size	1	1.36	1.00	1.14	1.86	0.50	1.00	1.14	1.55
time (sec)	N/A	0.031	0.041	0.112	0.409	2.116	0.256	0.136	2.789

Problem 445	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	97	79	414	0	553	0	158	988
normalized size	1	1.00	0.81	4.27	0.00	5.70	0.00	1.63	10.19
time (sec)	N/A	0.127	0.189	0.151	0.000	1.761	0.000	0.145	13.029

Problem 446	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	50	53	0	349	0	73	47
normalized size	1	1.00	0.98	1.04	0.00	6.84	0.00	1.43	0.92
time (sec)	N/A	0.076	0.044	0.129	0.000	0.509	0.000	0.174	2.777

Problem 447	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	142	142	120	430	0	663	0	161	977
normalized size	1	1.00	0.85	3.03	0.00	4.67	0.00	1.13	6.88
time (sec)	N/A	0.515	0.284	0.135	0.000	4.999	0.000	0.174	11.437

Problem 448	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	371	371	2490	21186	0	0	0	0	-1
normalized size	1	1.00	6.71	57.11	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.447	6.455	4.408	0.000	0.879	0.000	0.000	0.000

Problem 449	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	118	118	1580	12462	0	0	0	0	-1
normalized size	1	1.00	13.39	105.61	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.144	6.249	1.886	0.000	0.645	0.000	0.000	0.000

Problem 450	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	118	118	339	722	0	0	0	0	-1
normalized size	1	1.00	2.87	6.12	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.166	0.866	1.862	0.000	2.897	0.000	0.000	0.000

Problem 451	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	240	240	1732	12574	0	0	0	0	-1
normalized size	1	1.00	7.22	52.39	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.216	6.402	1.220	0.000	1.837	0.000	0.000	0.000

Problem 452	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	492	492	2708	64693	0	0	0	0	-1
normalized size	1	1.00	5.50	131.49	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.519	6.553	2.549	0.000	0.957	0.000	0.000	0.000

Problem 453	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	C	F	F	F(-1)	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	371	371	0	20776	0	0	0	0	-1
normalized size	1	1.00	0.00	56.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.390	150.813	1.287	0.000	0.934	0.000	0.000	0.000

Problem 454	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	C	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	118	118	0	12459	0	0	0	0	-1
normalized size	1	1.00	0.00	105.58	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.147	21.221	1.023	0.000	1.117	0.000	0.000	0.000

Problem 455	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	118	118	506	714	0	0	0	0	-1
normalized size	1	1.00	4.29	6.05	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.152	2.901	1.527	0.000	0.892	0.000	0.000	0.000

Problem 456	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	C	F	F	F(-1)	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	240	240	0	12564	0	0	0	0	-1
normalized size	1	1.00	0.00	52.35	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.211	23.993	0.950	0.000	0.895	0.000	0.000	0.000

Problem 457	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	C	F	F	F(-1)	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	492	492	0	64683	0	0	0	0	-1
normalized size	1	1.00	0.00	131.47	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.495	27.744	1.745	0.000	1.568	0.000	0.000	0.000

Problem 458	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	98	98	80	446	0	555	0	158	965
normalized size	1	1.00	0.82	4.55	0.00	5.66	0.00	1.61	9.85
time (sec)	N/A	0.103	0.224	0.132	0.000	2.053	0.000	0.145	13.762

Problem 459	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	50	53	0	349	0	73	47
normalized size	1	1.00	0.98	1.04	0.00	6.84	0.00	1.43	0.92
time (sec)	N/A	0.072	0.045	0.112	0.000	0.809	0.000	0.148	2.787

Problem 460	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	120	120	104	184	0	669	0	142	531
normalized size	1	1.00	0.87	1.53	0.00	5.58	0.00	1.18	4.42
time (sec)	N/A	0.533	0.282	0.114	0.000	6.047	0.000	0.169	8.698

Problem 461	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	51	10	14	24	0	22	9
normalized size	1	1.00	2.43	0.48	0.67	1.14	0.00	1.05	0.43
time (sec)	N/A	0.048	0.023	0.118	0.409	0.886	0.000	0.140	3.141

Problem 462	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	371	371	2490	20463	0	0	0	0	-1
normalized size	1	1.00	6.71	55.16	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.432	6.440	3.116	0.000	1.049	0.000	0.000	0.000

Problem 463	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	118	118	1580	12367	0	0	0	0	-1
normalized size	1	1.00	13.39	104.81	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.144	6.242	1.757	0.000	1.232	0.000	0.000	0.000

Problem 464	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	118	118	339	713	0	0	0	0	-1
normalized size	1	1.00	2.87	6.04	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.166	0.907	1.679	0.000	0.666	0.000	0.000	0.000

Problem 465	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	240	240	1732	12236	0	0	0	0	-1
normalized size	1	1.00	7.22	50.98	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.212	6.400	1.122	0.000	1.098	0.000	0.000	0.000

Problem 466	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	492	492	2708	62955	0	0	0	0	-1
normalized size	1	1.00	5.50	127.96	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.497	6.494	2.163	0.000	0.677	0.000	0.000	0.000

Problem 467	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	C	F	F	F(-1)	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	371	371	0	20454	0	0	0	0	-1
normalized size	1	1.00	0.00	55.13	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.383	52.856	1.098	0.000	0.627	0.000	0.000	0.000

Problem 468	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	C	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	118	118	0	12365	0	0	0	0	-1
normalized size	1	1.00	0.00	104.79	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.141	12.339	1.056	0.000	1.297	0.000	0.000	0.000

Problem 469	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	118	118	519	705	0	0	0	0	-1
normalized size	1	1.00	4.40	5.97	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.149	2.845	1.160	0.000	0.859	0.000	0.000	0.000

Problem 470	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	C	F	F	F(-1)	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	240	240	0	12231	0	0	0	0	-1
normalized size	1	1.00	0.00	50.96	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.205	20.778	0.957	0.000	1.196	0.000	0.000	0.000

Problem 471	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	C	F	F	F(-1)	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	492	492	0	62945	0	0	0	0	-1
normalized size	1	1.00	0.00	127.94	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.491	25.269	1.652	0.000	1.943	0.000	0.000	0.000

Problem 472	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	1	1	1	2	1	1	10	1	1
normalized size	1	1.00	1.00	2.00	1.00	1.00	10.00	1.00	1.00
time (sec)	N/A	0.009	0.000	0.059	0.405	0.891	0.357	0.138	2.645

Problem 473	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	1	1	1	2	1	1	22	1	1
normalized size	1	1.00	1.00	2.00	1.00	1.00	22.00	1.00	1.00
time (sec)	N/A	0.009	0.000	0.047	0.411	1.487	0.830	0.120	2.630

Problem 474	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	1	1	1	2	1	1	34	1	1
normalized size	1	1.00	1.00	2.00	1.00	1.00	34.00	1.00	1.00
time (sec)	N/A	0.009	0.000	0.049	0.411	0.986	2.096	0.127	2.594

Problem 475	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	11	11	23	4	15	23	36	33	3
normalized size	1	1.00	2.09	0.36	1.36	2.09	3.27	3.00	0.27
time (sec)	N/A	0.015	0.007	0.098	0.321	0.850	0.345	0.131	2.905

Problem 476	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	8	18	12	15	48	6	6
normalized size	1	1.00	0.62	1.38	0.92	1.15	3.69	0.46	0.46
time (sec)	N/A	0.023	0.003	0.105	0.304	1.306	1.336	0.119	2.633

Problem 477	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	22	48	38	74	765	37	32
normalized size	1	1.00	0.69	1.50	1.19	2.31	23.91	1.16	1.00
time (sec)	N/A	0.027	0.006	0.115	0.311	0.764	3.476	0.134	2.651

Problem 478	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	9	9	9	10	9	35	12007	20	9
normalized size	1	1.00	1.00	1.11	1.00	3.89	1334.11	2.22	1.00
time (sec)	N/A	0.018	0.032	0.116	0.405	0.686	22.401	0.146	2.834

Problem 479	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	11	11	11	12	11	31	0	22	11
normalized size	1	1.00	1.00	1.09	1.00	2.82	0.00	2.00	1.00
time (sec)	N/A	0.020	0.031	0.109	0.411	2.044	0.000	0.133	2.832

Problem 480	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	15	16	15	43	0	26	15
normalized size	1	1.00	1.00	1.07	1.00	2.87	0.00	1.73	1.00
time (sec)	N/A	0.026	0.042	0.125	0.404	2.388	0.000	0.127	2.845

Problem 481	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	25	18	17	43	87	61	36
normalized size	1	1.00	0.47	0.34	0.32	0.81	1.64	1.15	0.68
time (sec)	N/A	0.037	0.040	0.229	0.410	0.986	0.808	0.129	2.759

Problem 482	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	36	38	35	182	241	48	51
normalized size	1	1.00	0.84	0.88	0.81	4.23	5.60	1.12	1.19
time (sec)	N/A	0.149	0.095	0.110	0.405	0.703	1.564	0.132	2.730

Problem 483	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	36	36	35	181	267	48	48
normalized size	1	1.00	0.84	0.84	0.81	4.21	6.21	1.12	1.12
time (sec)	N/A	0.109	0.055	0.105	0.405	1.490	1.621	0.135	2.672

Problem 484	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	19	16	15	35	0	15	15
normalized size	1	1.00	0.53	0.44	0.42	0.97	0.00	0.42	0.42
time (sec)	N/A	0.028	0.046	0.118	0.407	1.079	0.000	0.124	2.652

Problem 485	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	42	27	27	68	0	27	27
normalized size	1	1.00	0.86	0.55	0.55	1.39	0.00	0.55	0.55
time (sec)	N/A	0.045	0.137	0.124	0.405	1.017	0.000	0.148	2.664

Problem 486	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	79	40	45	100	0	39	40
normalized size	1	1.00	1.07	0.54	0.61	1.35	0.00	0.53	0.54
time (sec)	N/A	0.055	0.180	0.131	0.415	2.123	0.000	0.149	2.698

Problem 487	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	1	1	1	4	1	1	0	1	1
normalized size	1	1.00	1.00	4.00	1.00	1.00	0.00	1.00	1.00
time (sec)	N/A	0.012	0.001	0.062	0.428	0.764	0.000	0.139	2.746

Problem 488	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	1	1	1	4	1	1	0	1	1
normalized size	1	1.00	1.00	4.00	1.00	1.00	0.00	1.00	1.00
time (sec)	N/A	0.013	0.001	0.062	0.408	0.842	0.000	0.149	2.581

Problem 489	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	1	1	1	4	1	1	0	1	1
normalized size	1	1.00	1.00	4.00	1.00	1.00	0.00	1.00	1.00
time (sec)	N/A	0.012	0.000	0.066	0.406	1.529	0.000	0.142	2.571

Problem 490	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	19	17	16	35	0	49	16
normalized size	1	1.00	0.51	0.46	0.43	0.95	0.00	1.32	0.43
time (sec)	N/A	0.031	0.042	0.159	0.410	1.321	0.000	0.141	2.713

Problem 491	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	64	28	27	66	0	60	27
normalized size	1	1.00	1.36	0.60	0.57	1.40	0.00	1.28	0.57
time (sec)	N/A	0.040	0.105	0.184	0.404	1.074	0.000	0.136	2.688

Problem 492	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	66	39	42	98	0	69	43
normalized size	1	1.00	0.92	0.54	0.58	1.36	0.00	0.96	0.60
time (sec)	N/A	0.076	0.156	0.219	0.424	1.625	0.000	0.132	2.683

Problem 493	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	3	3	3	6	3	3	0	3	3
normalized size	1	1.00	1.00	2.00	1.00	1.00	0.00	1.00	1.00
time (sec)	N/A	0.013	0.000	0.061	0.423	1.121	0.000	0.150	2.729

Problem 494	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	1	1	1	4	1	1	0	1	1
normalized size	1	1.00	1.00	4.00	1.00	1.00	0.00	1.00	1.00
time (sec)	N/A	0.013	0.000	0.065	0.416	0.805	0.000	0.150	2.650

Problem 495	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	3	3	3	6	3	3	0	3	3
normalized size	1	1.00	1.00	2.00	1.00	1.00	0.00	1.00	1.00
time (sec)	N/A	0.013	0.000	0.066	0.411	0.497	0.000	0.138	2.615

Problem 496	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	33	27	26	259	0	61	43
normalized size	1	1.00	1.00	0.82	0.79	7.85	0.00	1.85	1.30
time (sec)	N/A	0.050	0.060	0.122	0.431	0.963	0.000	0.142	2.855

Problem 497	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F	B	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	239	239	507	820	0	2929	0	0	-1
normalized size	1	1.00	2.12	3.43	0.00	12.26	0.00	0.00	-0.00
time (sec)	N/A	0.492	3.019	0.299	0.000	1.347	0.000	0.000	0.000

Problem 498	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	365	365	258	1161	0	4357	0	0	-1
normalized size	1	1.00	0.71	3.18	0.00	11.94	0.00	0.00	-0.00
time (sec)	N/A	0.739	3.791	0.288	0.000	2.222	0.000	0.000	0.000

Problem 499	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	195	195	149	255	246	150	566	158	456
normalized size	1	1.00	0.76	1.31	1.26	0.77	2.90	0.81	2.34
time (sec)	N/A	0.393	0.902	0.100	0.323	0.530	2.998	0.183	4.490

Problem 500	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	109	109	77	115	112	76	204	79	88
normalized size	1	1.00	0.71	1.06	1.03	0.70	1.87	0.72	0.81
time (sec)	N/A	0.099	0.286	0.079	0.320	0.740	0.674	0.148	2.898

Problem 501	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	A	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	23	52	0	23	0	52	39
normalized size	1	1.00	1.00	2.26	0.00	1.00	0.00	2.26	1.70
time (sec)	N/A	0.089	0.058	0.336	0.000	0.830	0.000	0.179	2.841

Problem 502	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	157	157	140	1297	0	795	0	454	497
normalized size	1	1.00	0.89	8.26	0.00	5.06	0.00	2.89	3.17
time (sec)	N/A	0.415	0.961	0.332	0.000	1.981	0.000	0.253	6.063

Problem 503	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	B	F(-1)	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	242	242	286	832	0	6695	0	0	10465
normalized size	1	1.00	1.18	3.44	0.00	27.67	0.00	0.00	43.24
time (sec)	N/A	0.940	0.765	0.178	0.000	18.385	0.000	0.000	17.109

Problem 504	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	331	331	140	269	556	112	0	239	-1
normalized size	1	1.00	0.42	0.81	1.68	0.34	0.00	0.72	-0.00
time (sec)	N/A	0.323	0.838	0.786	0.453	0.846	0.000	5.029	0.000

Problem 505	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	185	185	70	107	187	43	0	98	-1
normalized size	1	1.00	0.38	0.58	1.01	0.23	0.00	0.53	-0.01
time (sec)	N/A	0.109	0.185	0.586	0.430	0.870	0.000	0.206	0.000

Problem 506	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	137	137	85	176	0	204	0	208	-1
normalized size	1	1.00	0.62	1.28	0.00	1.49	0.00	1.52	-0.01
time (sec)	N/A	0.198	0.165	0.458	0.000	0.925	0.000	0.384	0.000

Problem 507	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	A	F(-1)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	239	239	144	741	0	527	0	479	-1
normalized size	1	1.00	0.60	3.10	0.00	2.21	0.00	2.00	-0.00
time (sec)	N/A	0.272	0.347	0.413	0.000	1.646	0.000	0.750	0.000

Problem 508	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	A	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	11	11	11	33	0	11	0	32	24
normalized size	1	1.00	1.00	3.00	0.00	1.00	0.00	2.91	2.18
time (sec)	N/A	0.081	0.050	0.086	0.000	0.778	0.000	0.169	2.937

Problem 509	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	246	246	241	2556	0	6697	0	5302	11781
normalized size	1	1.00	0.98	10.39	0.00	27.22	0.00	21.55	47.89
time (sec)	N/A	0.789	0.557	0.124	0.000	16.183	0.000	8.397	16.769

Problem 510	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	144	144	153	245	150	149	248	2326	205
normalized size	1	1.00	1.06	1.70	1.04	1.03	1.72	16.15	1.42
time (sec)	N/A	0.269	2.100	0.008	0.405	1.715	0.625	5.119	2.854

Problem 511	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	88	117	74	74	122	709	105
normalized size	1	1.00	1.22	1.62	1.03	1.03	1.69	9.85	1.46
time (sec)	N/A	0.076	0.326	0.007	0.409	1.125	0.250	1.018	2.770

Problem 512	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	101	187	222	161	191	1358	204	152
normalized size	1	1.00	1.85	2.20	1.59	1.89	13.45	2.02	1.50
time (sec)	N/A	0.258	2.338	0.164	0.412	0.722	1.623	0.894	3.089

Problem 513	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	B	B	F(-2)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	197	197	308	458	419	580	0	451	388
normalized size	1	1.00	1.56	2.32	2.13	2.94	0.00	2.29	1.97
time (sec)	N/A	0.535	5.409	0.176	0.432	2.021	0.000	1.938	3.261

Problem 514	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	284	284	147	158	166	102	0	1751	-1
normalized size	1	1.00	0.52	0.56	0.58	0.36	0.00	6.17	-0.00
time (sec)	N/A	0.226	1.332	0.309	0.426	2.830	0.000	2.435	0.000

Problem 515	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	122	122	58	75	65	38	0	395	-1
normalized size	1	1.00	0.48	0.61	0.53	0.31	0.00	3.24	-0.01
time (sec)	N/A	0.101	0.302	0.319	0.421	2.631	0.000	0.460	0.000

Problem 516	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	138	138	88	114	137	71	0	554	-1
normalized size	1	1.00	0.64	0.83	0.99	0.51	0.00	4.01	-0.01
time (sec)	N/A	0.188	0.684	0.227	0.424	0.725	0.000	3.068	0.000

Problem 517	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	A	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	316	316	268	622	498	355	0	1571	-1
normalized size	1	1.00	0.85	1.97	1.58	1.12	0.00	4.97	-0.00
time (sec)	N/A	0.402	3.441	0.190	0.440	2.034	0.000	10.073	0.000

Problem 518	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	184	184	130	246	299	198	0	470	323
normalized size	1	1.00	0.71	1.34	1.62	1.08	0.00	2.55	1.76
time (sec)	N/A	0.430	0.874	0.141	0.337	2.426	0.000	0.391	3.304

Problem 519	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	76	76	64	110	126	125	0	191	160
normalized size	1	1.00	0.84	1.45	1.66	1.64	0.00	2.51	2.11
time (sec)	N/A	0.078	0.284	0.092	0.337	1.496	0.000	0.252	2.908

Problem 520	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	92	92	97	163	0	279	0	145	444
normalized size	1	1.00	1.05	1.77	0.00	3.03	0.00	1.58	4.83
time (sec)	N/A	0.304	0.387	0.287	0.000	0.847	0.000	0.278	3.023

Problem 521	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	230	230	276	1118	0	1335	0	490	5469
normalized size	1	1.00	1.20	4.86	0.00	5.80	0.00	2.13	23.78
time (sec)	N/A	0.833	1.529	0.348	0.000	2.110	0.000	0.486	11.473

Problem 522	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	359	359	128	387	440	162	0	652	-1
normalized size	1	1.00	0.36	1.08	1.23	0.45	0.00	1.82	-0.00
time (sec)	N/A	0.287	0.818	0.608	0.452	1.580	0.000	0.512	0.000

Problem 523	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	173	173	67	211	164	85	0	224	-1
normalized size	1	1.00	0.39	1.22	0.95	0.49	0.00	1.29	-0.01
time (sec)	N/A	0.119	0.259	0.585	0.439	0.961	0.000	0.305	0.000

Problem 524	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	142	142	92	157	0	184	0	1504	-1
normalized size	1	1.00	0.65	1.11	0.00	1.30	0.00	10.59	-0.01
time (sec)	N/A	0.213	0.368	0.665	0.000	1.064	0.000	4.827	0.000

Problem 525	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	330	330	216	756	0	798	0	569	-1
normalized size	1	1.00	0.65	2.29	0.00	2.42	0.00	1.72	-0.00
time (sec)	N/A	0.566	1.004	0.614	0.000	1.086	0.000	5.124	0.000

Problem 526	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	19	8	0	6	8	14	16
normalized size	1	1.00	1.12	0.47	0.00	0.35	0.47	0.82	0.94
time (sec)	N/A	0.040	0.005	0.196	0.000	0.703	0.077	0.141	2.760

Problem 527	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	19	8	0	6	10	14	13
normalized size	1	1.00	1.12	0.47	0.00	0.35	0.59	0.82	0.76
time (sec)	N/A	0.037	0.006	0.174	0.000	0.652	0.076	0.140	2.745

Problem 528	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	6	6	6	7	6	11	7	16	32
normalized size	1	1.00	1.00	1.17	1.00	1.83	1.17	2.67	5.33
time (sec)	N/A	0.023	0.026	0.058	0.309	0.824	0.119	0.176	2.918

Problem 529	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	39	111	181	59	371	77	1976
normalized size	1	1.00	0.83	2.36	3.85	1.26	7.89	1.64	42.04
time (sec)	N/A	0.041	0.142	0.135	0.422	0.882	0.877	0.167	12.819

Problem 530	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	75	113	271	194	0	132	129
normalized size	1	1.00	1.01	1.53	3.66	2.62	0.00	1.78	1.74
time (sec)	N/A	0.068	0.241	0.173	0.426	1.627	0.000	0.236	3.007

Problem 531	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	64	37	199	152	0	26	95
normalized size	1	1.00	0.97	0.56	3.02	2.30	0.00	0.39	1.44
time (sec)	N/A	0.057	0.189	0.188	0.340	0.540	0.000	0.202	2.823

Problem 532	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	84	84	78	222	243	155	1042	148	1099
normalized size	1	1.00	0.93	2.64	2.89	1.85	12.40	1.76	13.08
time (sec)	N/A	0.058	0.253	0.137	0.416	0.772	38.589	0.248	9.406

Problem 533	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	92	124	286	226	0	150	141
normalized size	1	1.00	1.08	1.46	3.36	2.66	0.00	1.76	1.66
time (sec)	N/A	0.057	0.275	0.168	0.424	3.023	0.000	0.234	3.051

Problem 534	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	129	129	122	218	451	311	0	270	264
normalized size	1	1.00	0.95	1.69	3.50	2.41	0.00	2.09	2.05
time (sec)	N/A	0.124	0.661	0.195	0.453	1.691	0.000	0.258	3.439

Problem 535	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	115	115	95	544	0	625	0	178	1709
normalized size	1	1.00	0.83	4.73	0.00	5.43	0.00	1.55	14.86
time (sec)	N/A	0.130	0.286	0.124	0.000	1.784	0.000	0.180	25.561

Problem 536	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	113	113	118	254	0	1277	0	209	205
normalized size	1	1.00	1.04	2.25	0.00	11.30	0.00	1.85	1.81
time (sec)	N/A	0.106	0.326	0.175	0.000	1.690	0.000	0.190	3.253

Problem 537	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	200	200	326	1109	0	3402	0	1162	946
normalized size	1	1.00	1.63	5.54	0.00	17.01	0.00	5.81	4.73
time (sec)	N/A	0.255	0.937	0.210	0.000	2.159	0.000	0.546	6.845

Problem 538	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	84	84	147	153	0	72	95	168	99
normalized size	1	1.00	1.75	1.82	0.00	0.86	1.13	2.00	1.18
time (sec)	N/A	0.045	0.222	0.183	0.000	0.756	0.576	0.162	4.333

Problem 539	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	84	84	147	284	0	56	75	168	584
normalized size	1	1.00	1.75	3.38	0.00	0.67	0.89	2.00	6.95
time (sec)	N/A	0.042	0.191	0.185	0.000	2.144	0.578	0.155	8.386

Problem 540	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	116	116	96	542	0	625	0	177	1741
normalized size	1	1.00	0.83	4.67	0.00	5.39	0.00	1.53	15.01
time (sec)	N/A	0.108	0.288	0.129	0.000	2.587	0.000	0.185	24.863

Problem 541	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	114	114	123	255	0	1301	0	206	204
normalized size	1	1.00	1.08	2.24	0.00	11.41	0.00	1.81	1.79
time (sec)	N/A	0.101	0.386	0.183	0.000	1.240	0.000	0.201	3.191

Problem 542	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	200	200	361	1088	0	3513	0	1054	912
normalized size	1	1.00	1.80	5.44	0.00	17.56	0.00	5.27	4.56
time (sec)	N/A	0.249	0.925	0.217	0.000	1.891	0.000	0.520	6.734

Problem 543	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	152	151	0	71	104	169	96
normalized size	1	1.00	1.79	1.78	0.00	0.84	1.22	1.99	1.13
time (sec)	N/A	0.046	0.293	0.184	0.000	0.968	0.798	0.155	4.354

Problem 544	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	152	280	0	57	80	169	96
normalized size	1	1.00	1.79	3.29	0.00	0.67	0.94	1.99	1.13
time (sec)	N/A	0.046	0.258	0.175	0.000	0.550	0.744	0.176	4.343

Problem 545	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	119	119	98	824	0	687	0	187	1864
normalized size	1	1.00	0.82	6.92	0.00	5.77	0.00	1.57	15.66
time (sec)	N/A	0.113	0.385	0.123	0.000	5.064	0.000	0.150	28.555

Problem 546	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	110	110	116	255	0	1316	0	205	202
normalized size	1	1.00	1.05	2.32	0.00	11.96	0.00	1.86	1.84
time (sec)	N/A	0.096	0.406	0.177	0.000	1.024	0.000	0.184	3.238

Problem 547	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	197	197	311	881	0	3264	0	1034	923
normalized size	1	1.00	1.58	4.47	0.00	16.57	0.00	5.25	4.69
time (sec)	N/A	0.232	0.905	0.222	0.000	2.203	0.000	0.512	6.362

Problem 548	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F(-2)	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	92	87	195	212	0	78	116	178	118
normalized size	1	0.95	2.12	2.30	0.00	0.85	1.26	1.93	1.28
time (sec)	N/A	0.078	0.324	0.195	0.000	1.053	0.850	0.160	5.319

Problem 549	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F(-2)	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	90	85	195	388	0	68	102	178	118
normalized size	1	0.94	2.17	4.31	0.00	0.76	1.13	1.98	1.31
time (sec)	N/A	0.078	0.294	0.191	0.000	0.959	0.813	0.159	4.506

Problem 550	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	131	131	110	954	0	711	0	199	2711
normalized size	1	1.00	0.84	7.28	0.00	5.43	0.00	1.52	20.69
time (sec)	N/A	0.126	0.386	0.132	0.000	1.845	0.000	0.184	55.105

Problem 551	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	127	127	137	329	0	1556	0	241	227
normalized size	1	1.00	1.08	2.59	0.00	12.25	0.00	1.90	1.79
time (sec)	N/A	0.123	0.504	0.183	0.000	2.177	0.000	0.167	3.487

Problem 552	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	237	237	452	1422	0	4240	0	1506	1160
normalized size	1	1.00	1.91	6.00	0.00	17.89	0.00	6.35	4.89
time (sec)	N/A	0.277	1.281	0.226	0.000	1.620	0.000	0.543	8.215

Problem 553	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	105	105	165	257	0	89	138	203	132
normalized size	1	1.00	1.57	2.45	0.00	0.85	1.31	1.93	1.26
time (sec)	N/A	0.074	0.445	0.185	0.000	1.460	1.373	0.148	6.927

Problem 554	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	103	103	167	475	0	73	109	203	133
normalized size	1	1.00	1.62	4.61	0.00	0.71	1.06	1.97	1.29
time (sec)	N/A	0.073	0.451	0.194	0.000	1.091	1.271	0.151	6.858

Problem 555	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	B	A	B	F(-2)	A	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	68	32	70	0	24	0	68	62
normalized size	1	2.83	1.33	2.92	0.00	1.00	0.00	2.83	2.58
time (sec)	N/A	0.068	0.094	0.189	0.000	0.893	0.000	0.179	3.031

Problem 556	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	390	390	7823	3502	0	0	0	0	-1
normalized size	1	1.00	20.06	8.98	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.888	6.941	1.279	0.000	1.631	0.000	0.000	0.000

Problem 557	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	294	294	5218	2238	0	0	0	0	-1
normalized size	1	1.00	17.75	7.61	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.555	6.584	1.011	0.000	0.996	0.000	0.000	0.000

Problem 558	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	229	229	3006	1460	0	0	0	0	-1
normalized size	1	1.00	13.13	6.38	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.332	6.370	0.774	0.000	1.179	0.000	0.000	0.000

Problem 559	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	180	180	1319	777	0	0	0	0	-1
normalized size	1	1.00	7.33	4.32	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.187	6.305	0.654	0.000	0.961	0.000	0.000	0.000

Problem 560	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	250	250	3176	2596	0	0	0	0	-1
normalized size	1	1.00	12.70	10.38	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.323	6.571	1.070	0.000	2.937	0.000	0.000	0.000

Problem 561	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	378	378	5554	3164	0	0	0	0	-1
normalized size	1	1.00	14.69	8.37	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.561	6.952	3.886	0.000	0.954	0.000	0.000	0.000

Problem 562	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	84	84	80	178	0	346	1110	141	1143
normalized size	1	1.00	0.95	2.12	0.00	4.12	13.21	1.68	13.61
time (sec)	N/A	0.152	0.254	0.253	0.000	0.996	29.516	0.180	9.625

Problem 563	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	118	118	114	426	0	458	0	187	227
normalized size	1	1.00	0.97	3.61	0.00	3.88	0.00	1.58	1.92
time (sec)	N/A	0.157	0.433	0.395	0.000	1.364	0.000	0.198	3.327

Problem 564	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	185	185	174	1891	0	880	0	596	557
normalized size	1	1.00	0.94	10.22	0.00	4.76	0.00	3.22	3.01
time (sec)	N/A	0.246	0.914	0.428	0.000	1.170	0.000	0.265	5.272

Problem 565	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	258	258	244	5051	0	1411	0	1340	1085
normalized size	1	1.00	0.95	19.58	0.00	5.47	0.00	5.19	4.21
time (sec)	N/A	0.405	2.637	0.463	0.000	0.787	0.000	0.307	6.218

Problem 566	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	131	131	145	0	0	0	0	0	-1
normalized size	1	1.00	1.11	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.112	0.589	1.111	0.000	0.910	0.000	0.000	0.000

Problem 567	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	107	107	75	106	80	97	190	75	125
normalized size	1	1.00	0.70	0.99	0.75	0.91	1.78	0.70	1.17
time (sec)	N/A	0.082	0.272	0.248	0.383	1.753	3.355	0.187	3.437

Problem 568	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	48	69	48	63	129	46	78
normalized size	1	1.00	0.79	1.13	0.79	1.03	2.11	0.75	1.28
time (sec)	N/A	0.034	0.155	0.242	0.335	2.401	0.955	0.155	3.034

Problem 569	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	38	19	18	22	26	18	22
normalized size	1	1.00	1.90	0.95	0.90	1.10	1.30	0.90	1.10
time (sec)	N/A	0.016	0.007	0.003	0.483	0.754	0.178	0.135	2.942

Problem 570	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	48	45	0	290	0	61	44
normalized size	1	1.00	1.00	0.94	0.00	6.04	0.00	1.27	0.92
time (sec)	N/A	0.066	0.076	0.313	0.000	0.836	0.000	0.163	3.114

Problem 571	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	95	95	94	139	0	493	0	116	181
normalized size	1	1.00	0.99	1.46	0.00	5.19	0.00	1.22	1.91
time (sec)	N/A	0.109	0.408	0.445	0.000	1.446	0.000	0.177	3.050

Problem 572	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	149	149	120	640	0	969	0	252	396
normalized size	1	1.00	0.81	4.30	0.00	6.50	0.00	1.69	2.66
time (sec)	N/A	0.177	0.930	0.488	0.000	1.063	0.000	0.190	3.874

Problem 573	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	265	265	202	1138	0	0	0	0	-1
normalized size	1	1.00	0.76	4.29	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.366	1.848	0.521	0.000	2.714	0.000	0.000	0.000

Problem 574	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	212	212	167	844	0	0	0	0	-1
normalized size	1	1.00	0.79	3.98	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.220	1.456	0.442	0.000	2.978	0.000	0.000	0.000

Problem 575	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	76	76	75	312	0	0	0	0	-1
normalized size	1	1.00	0.99	4.11	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.061	0.108	0.407	0.000	2.013	0.000	0.000	0.000

Problem 576	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	76	76	70	165	0	0	0	0	-1
normalized size	1	1.00	0.92	2.17	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.067	0.134	0.426	0.000	2.005	0.000	0.000	0.000

Problem 577	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	143	143	101	570	0	0	0	0	-1
normalized size	1	1.00	0.71	3.99	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.094	0.419	0.501	0.000	0.549	0.000	0.000	0.000

Problem 578	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	295	295	201	1554	0	0	0	0	-1
normalized size	1	1.00	0.68	5.27	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.301	1.516	0.515	0.000	1.743	0.000	0.000	0.000

Problem 579	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	C	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	461	461	340	2282	0	3324	0	0	-1
normalized size	1	1.00	0.74	4.95	0.00	7.21	0.00	0.00	-0.00
time (sec)	N/A	0.630	0.871	0.297	0.000	1.294	0.000	0.000	0.000

Problem 580	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	C	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	340	340	256	1782	0	2506	0	0	-1
normalized size	1	1.00	0.75	5.24	0.00	7.37	0.00	0.00	-0.00
time (sec)	N/A	0.537	0.720	0.227	0.000	2.282	0.000	0.000	0.000

Problem 581	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	225	225	788	1284	0	1688	0	0	-1
normalized size	1	1.00	3.50	5.71	0.00	7.50	0.00	0.00	-0.00
time (sec)	N/A	0.319	1.406	0.217	0.000	2.255	0.000	0.000	0.000

Problem 582	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	20	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.05
time (sec)	N/A	0.085	1.581	0.246	0.000	0.690	0.000	0.000	0.000

Problem 583	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	79	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.155	5.465	2.193	0.000	0.964	0.000	0.000	0.000

Problem 584	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	79	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.146	4.968	2.187	0.000	0.912	0.000	0.000	0.000

Problem 585	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F(-1)	F(-2)	A	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	175	175	198	0	0	186	0	7347	-1
normalized size	1	1.00	1.13	0.00	0.00	1.06	0.00	41.98	-0.01
time (sec)	N/A	0.297	1.447	180.000	0.000	1.074	0.000	1.202	0.000

Problem 586	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F(-1)	F(-2)	A	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	131	131	142	0	0	142	0	4175	-1
normalized size	1	1.00	1.08	0.00	0.00	1.08	0.00	31.87	-0.01
time (sec)	N/A	0.227	0.970	180.000	0.000	1.036	0.000	0.811	0.000

Problem 587	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F(-1)	F(-2)	A	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	77	0	0	77	0	1033	-1
normalized size	1	1.00	0.96	0.00	0.00	0.96	0.00	12.91	-0.01
time (sec)	N/A	0.131	0.821	180.000	0.000	1.034	0.000	0.477	0.000

Problem 588	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F(-2)	A	F	C	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	56	56	242	108	0	45	0	496	-1
normalized size	1	1.00	4.32	1.93	0.00	0.80	0.00	8.86	-0.02
time (sec)	N/A	0.102	7.419	4.279	0.000	0.867	0.000	0.369	0.000

Problem 589	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	24	77	114	24	20	39	24
normalized size	1	1.00	0.69	2.20	3.26	0.69	0.57	1.11	0.69
time (sec)	N/A	0.024	0.277	1.471	0.317	1.955	3.415	0.193	3.028

Problem 590	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	20	21	20	21	19	42	23
normalized size	1	1.00	1.00	1.05	1.00	1.05	0.95	2.10	1.15
time (sec)	N/A	0.038	0.030	0.195	0.311	1.746	3.482	0.177	0.118

Problem 591	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	B	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	32	54	100	34	112	53	-1
normalized size	1	1.00	0.91	1.54	2.86	0.97	3.20	1.51	-0.03
time (sec)	N/A	0.039	0.456	1.226	0.312	0.823	5.046	0.148	0.000

Problem 592	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F(-2)	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	104	104	157	0	0	295	0	0	-1
normalized size	1	1.00	1.51	0.00	0.00	2.84	0.00	0.00	-0.01
time (sec)	N/A	0.091	1.003	2.473	0.000	2.955	0.000	0.000	0.000

Problem 593	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	127	127	102	172	608	406	0	0	-1
normalized size	1	1.00	0.80	1.35	4.79	3.20	0.00	0.00	-0.01
time (sec)	N/A	0.181	1.049	1.105	0.378	0.997	0.000	0.000	0.000

Problem 594	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F(-1)	F(-2)	A	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	176	176	194	0	0	162	0	7279	-1
normalized size	1	1.00	1.10	0.00	0.00	0.92	0.00	41.36	-0.01
time (sec)	N/A	0.299	1.223	180.000	0.000	0.915	0.000	1.209	0.000

Problem 595	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F(-1)	F(-2)	A	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	132	132	136	0	0	185	0	3130	-1
normalized size	1	1.00	1.03	0.00	0.00	1.40	0.00	23.71	-0.01
time (sec)	N/A	0.226	0.795	180.000	0.000	0.921	0.000	0.691	0.000

Problem 596	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F(-1)	F(-2)	A	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	71	0	0	73	0	997	-1
normalized size	1	1.00	0.89	0.00	0.00	0.91	0.00	12.46	-0.01
time (sec)	N/A	0.128	0.677	180.000	0.000	0.900	0.000	0.489	0.000

Problem 597	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F(-2)	A	F	C	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	56	56	237	106	0	62	0	366	-1
normalized size	1	1.00	4.23	1.89	0.00	1.11	0.00	6.54	-0.02
time (sec)	N/A	0.093	7.403	5.142	0.000	0.951	0.000	0.353	0.000

Problem 598	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	22	70	114	23	20	32	22
normalized size	1	1.00	0.65	2.06	3.35	0.68	0.59	0.94	0.65
time (sec)	N/A	0.022	0.214	1.787	0.328	3.572	3.003	0.167	0.163

Problem 599	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	19	20	19	22	20	40	22
normalized size	1	1.00	1.00	1.05	1.00	1.16	1.05	2.11	1.16
time (sec)	N/A	0.056	0.019	0.238	0.305	0.861	3.007	0.177	0.093

Problem 600	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	31	53	100	36	0	52	-1
normalized size	1	1.00	0.94	1.61	3.03	1.09	0.00	1.58	-0.03
time (sec)	N/A	0.038	0.406	1.187	0.313	1.846	0.000	0.153	0.000

Problem 601	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F(-2)	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	110	110	176	0	0	290	0	0	-1
normalized size	1	1.00	1.60	0.00	0.00	2.64	0.00	0.00	-0.01
time (sec)	N/A	0.093	1.112	2.540	0.000	0.993	0.000	0.000	0.000

Problem 602	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	124	124	130	141	381	378	0	0	-1
normalized size	1	1.00	1.05	1.14	3.07	3.05	0.00	0.00	-0.01
time (sec)	N/A	0.183	1.074	1.242	0.463	1.883	0.000	0.000	0.000

Problem 603	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-1)	A	F(-1)	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	157	157	64	98	0	106	0	0	463
normalized size	1	1.00	0.41	0.62	0.00	0.68	0.00	0.00	2.95
time (sec)	N/A	0.445	0.224	1.589	0.000	0.885	0.000	0.000	8.884

Problem 604	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-1)	A	F(-1)	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	110	110	62	88	0	84	0	0	148
normalized size	1	1.00	0.56	0.80	0.00	0.76	0.00	0.00	1.35
time (sec)	N/A	0.276	0.178	1.167	0.000	1.501	0.000	0.000	11.027

Problem 605	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	44	78	0	64	0	0	129
normalized size	1	1.00	0.61	1.08	0.00	0.89	0.00	0.00	1.79
time (sec)	N/A	0.199	0.176	1.095	0.000	0.906	0.000	0.000	7.343

Problem 606	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	30	52	0	40	0	0	87
normalized size	1	1.00	0.91	1.58	0.00	1.21	0.00	0.00	2.64
time (sec)	N/A	0.065	0.082	0.971	0.000	0.427	0.000	0.000	3.677

Problem 607	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	73	136	430	201	0	0	-1
normalized size	1	1.00	1.62	3.02	9.56	4.47	0.00	0.00	-0.02
time (sec)	N/A	0.040	0.128	0.944	0.496	1.465	0.000	0.000	0.000

Problem 608	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	84	84	92	391	1049	351	0	0	-1
normalized size	1	1.00	1.10	4.65	12.49	4.18	0.00	0.00	-0.01
time (sec)	N/A	0.139	0.240	1.139	0.610	2.277	0.000	0.000	0.000

Problem 609	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	129	129	105	657	1421	419	0	0	-1
normalized size	1	1.00	0.81	5.09	11.02	3.25	0.00	0.00	-0.01
time (sec)	N/A	0.214	0.249	1.218	0.721	0.953	0.000	0.000	0.000

Problem 610	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	176	176	116	933	2333	481	0	0	-1
normalized size	1	1.00	0.66	5.30	13.26	2.73	0.00	0.00	-0.01
time (sec)	N/A	0.288	0.294	1.171	1.131	2.224	0.000	0.000	0.000

Problem 611	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-1)	A	F(-1)	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	208	208	85	105	0	132	0	0	594
normalized size	1	1.00	0.41	0.50	0.00	0.63	0.00	0.00	2.86
time (sec)	N/A	0.528	0.347	1.119	0.000	1.047	0.000	0.000	10.183

Problem 612	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-1)	A	F(-1)	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	148	148	73	95	0	111	0	0	479
normalized size	1	1.00	0.49	0.64	0.00	0.75	0.00	0.00	3.24
time (sec)	N/A	0.348	0.214	0.944	0.000	0.974	0.000	0.000	9.262

Problem 613	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-1)	A	F(-1)	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	110	110	59	85	0	88	0	0	149
normalized size	1	1.00	0.54	0.77	0.00	0.80	0.00	0.00	1.35
time (sec)	N/A	0.268	0.226	0.863	0.000	0.902	0.000	0.000	11.037

Problem 614	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-1)	A	F(-1)	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	75	75	51	61	0	67	0	0	158
normalized size	1	1.00	0.68	0.81	0.00	0.89	0.00	0.00	2.11
time (sec)	N/A	0.110	0.159	0.832	0.000	0.884	0.000	0.000	7.866

Problem 615	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	86	253	1317	296	0	0	-1
normalized size	1	1.00	1.08	3.16	16.46	3.70	0.00	0.00	-0.01
time (sec)	N/A	0.060	0.153	0.950	0.894	1.033	0.000	0.000	0.000

Problem 616	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	86	86	93	518	1058	369	0	0	-1
normalized size	1	1.00	1.08	6.02	12.30	4.29	0.00	0.00	-0.01
time (sec)	N/A	0.222	0.244	1.087	1.148	2.090	0.000	0.000	0.000

Problem 617	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-1)	A	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	133	133	105	792	0	437	0	0	-1
normalized size	1	1.00	0.79	5.95	0.00	3.29	0.00	0.00	-0.01
time (sec)	N/A	0.256	0.271	1.022	0.000	1.041	0.000	0.000	0.000

Problem 618	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-1)	A	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	182	182	117	1078	0	503	0	0	-1
normalized size	1	1.00	0.64	5.92	0.00	2.76	0.00	0.00	-0.01
time (sec)	N/A	0.312	0.230	1.112	0.000	0.728	0.000	0.000	0.000

Problem 619	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	175	175	112	984	0	380	0	0	-1
normalized size	1	1.00	0.64	5.62	0.00	2.17	0.00	0.00	-0.01
time (sec)	N/A	0.600	0.673	1.357	0.000	0.508	0.000	0.000	0.000

Problem 620	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	129	129	89	677	0	294	0	0	-1
normalized size	1	1.00	0.69	5.25	0.00	2.28	0.00	0.00	-0.01
time (sec)	N/A	0.359	0.391	1.208	0.000	2.195	0.000	0.000	0.000

Problem 621	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	67	478	0	245	0	0	-1
normalized size	1	1.00	0.76	5.43	0.00	2.78	0.00	0.00	-0.01
time (sec)	N/A	0.238	0.243	1.194	0.000	1.625	0.000	0.000	0.000

Problem 622	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	64	236	0	146	0	0	-1
normalized size	1	1.00	1.16	4.29	0.00	2.65	0.00	0.00	-0.02
time (sec)	N/A	0.076	0.149	1.067	0.000	2.001	0.000	0.000	0.000

Problem 623	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	A	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	100	100	94	301	0	309	0	0	-1
normalized size	1	1.00	0.94	3.01	0.00	3.09	0.00	0.00	-0.01
time (sec)	N/A	0.090	0.313	1.007	0.000	1.904	0.000	0.000	0.000

Problem 624	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	138	138	166	1030	0	481	0	0	-1
normalized size	1	1.00	1.20	7.46	0.00	3.49	0.00	0.00	-0.01
time (sec)	N/A	0.279	2.321	1.171	0.000	0.963	0.000	0.000	0.000

Problem 625	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	182	182	186	1835	0	569	0	0	-1
normalized size	1	1.00	1.02	10.08	0.00	3.13	0.00	0.00	-0.01
time (sec)	N/A	0.370	2.902	1.095	0.000	1.947	0.000	0.000	0.000

Problem 626	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	180	180	100	1211	0	350	0	0	-1
normalized size	1	1.00	0.56	6.73	0.00	1.94	0.00	0.00	-0.01
time (sec)	N/A	0.513	1.304	1.129	0.000	1.155	0.000	0.000	0.000

Problem 627	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	128	128	94	930	0	276	0	0	-1
normalized size	1	1.00	0.73	7.27	0.00	2.16	0.00	0.00	-0.01
time (sec)	N/A	0.306	0.603	1.148	0.000	1.998	0.000	0.000	0.000

Problem 628	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	93	93	84	433	0	272	0	0	-1
normalized size	1	1.00	0.90	4.66	0.00	2.92	0.00	0.00	-0.01
time (sec)	N/A	0.236	0.588	1.041	0.000	1.026	0.000	0.000	0.000

Problem 629	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	93	93	83	599	0	269	0	0	-1
normalized size	1	1.00	0.89	6.44	0.00	2.89	0.00	0.00	-0.01
time (sec)	N/A	0.121	0.623	1.036	0.000	1.210	0.000	0.000	0.000

Problem 630	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	138	138	196	561	0	438	0	0	-1
normalized size	1	1.00	1.42	4.07	0.00	3.17	0.00	0.00	-0.01
time (sec)	N/A	0.143	3.700	1.015	0.000	0.665	0.000	0.000	0.000

Problem 631	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	178	178	342	1157	0	528	0	0	-1
normalized size	1	1.00	1.92	6.50	0.00	2.97	0.00	0.00	-0.01
time (sec)	N/A	0.320	6.202	1.073	0.000	1.051	0.000	0.000	0.000

Problem 632	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	234	234	356	1787	0	616	0	0	-1
normalized size	1	1.00	1.52	7.64	0.00	2.63	0.00	0.00	-0.00
time (sec)	N/A	0.497	6.207	1.024	0.000	0.975	0.000	0.000	0.000

Problem 633	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	B	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	16	119	0	29	0	0	14
normalized size	1	1.00	1.00	7.44	0.00	1.81	0.00	0.00	0.88
time (sec)	N/A	0.087	0.031	0.213	0.000	1.869	0.000	0.000	3.099

Problem 634	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	119	397	0	120	0	0	-1
normalized size	1	1.00	1.72	5.75	0.00	1.74	0.00	0.00	-0.01
time (sec)	N/A	0.364	5.949	0.313	0.000	2.702	0.000	0.000	0.000

Problem 635	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F(-1)	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	79	79	183	397	0	120	0	0	-1
normalized size	1	1.00	2.32	5.03	0.00	1.52	0.00	0.00	-0.01
time (sec)	N/A	0.567	4.955	0.280	0.000	1.040	0.000	0.000	0.000

Problem 636	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F(-1)	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	95	95	184	761	0	136	0	0	-1
normalized size	1	1.00	1.94	8.01	0.00	1.43	0.00	0.00	-0.01
time (sec)	N/A	0.576	15.721	0.383	0.000	0.469	0.000	0.000	0.000

Problem 637	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	51	31	30	59	138	0	63
normalized size	1	1.00	1.70	1.03	1.00	1.97	4.60	0.00	2.10
time (sec)	N/A	0.059	1.218	0.461	0.305	3.048	74.067	0.000	5.540

Problem 638	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	A	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	938	137	24	122	129	142	185
normalized size	1	1.00	36.08	5.27	0.92	4.69	4.96	5.46	7.12
time (sec)	N/A	0.044	6.538	0.608	0.308	0.686	35.718	1.954	3.566

Problem 639	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	31	118	24	92	100	0	100
normalized size	1	1.00	1.19	4.54	0.92	3.54	3.85	0.00	3.85
time (sec)	N/A	0.043	1.224	0.556	0.310	0.982	11.161	0.000	3.223

Problem 640	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	67	57	24	61	73	45	61
normalized size	1	1.00	2.58	2.19	0.92	2.35	2.81	1.73	2.35
time (sec)	N/A	0.028	0.037	0.460	0.549	0.976	3.457	2.671	3.177

Problem 641	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	29	23	22	33	63	0	133
normalized size	1	1.00	1.32	1.05	1.00	1.50	2.86	0.00	6.05
time (sec)	N/A	0.048	0.463	0.519	0.318	1.134	7.479	0.000	4.859

Problem 642	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	27	25	24	29	49	108	47
normalized size	1	1.00	1.12	1.04	1.00	1.21	2.04	4.50	1.96
time (sec)	N/A	0.044	0.305	0.631	0.426	0.903	21.183	0.726	3.236

Problem 643	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	29	25	24	63	80	0	291
normalized size	1	1.00	1.12	0.96	0.92	2.42	3.08	0.00	11.19
time (sec)	N/A	0.045	0.710	0.658	0.308	1.908	44.089	0.000	6.313

Problem 644	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	21	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.05
time (sec)	N/A	0.013	0.045	0.053	0.000	0.858	0.000	0.000	0.000

Problem 645	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	21	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.05
time (sec)	N/A	0.013	0.035	0.030	0.000	0.903	0.000	0.000	0.000

Problem 646	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	23	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.016	0.081	0.056	0.000	0.864	0.000	0.000	0.000

Problem 647	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	23	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.017	0.073	0.053	0.000	1.974	0.000	0.000	0.000

Problem 648	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	12	13	12	15	17	13	12
normalized size	1	1.00	1.00	1.08	1.00	1.25	1.42	1.08	1.00
time (sec)	N/A	0.022	0.018	0.033	0.310	0.734	0.327	0.131	0.057

Problem 649	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	19	21	20	23	63	20	20
normalized size	1	1.00	0.95	1.05	1.00	1.15	3.15	1.00	1.00
time (sec)	N/A	0.025	0.031	0.032	0.380	0.628	2.018	0.139	3.150

Problem 650	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	5	5	5	6	5	36	0	14	5
normalized size	1	1.00	1.00	1.20	1.00	7.20	0.00	2.80	1.00
time (sec)	N/A	0.023	0.021	0.059	1.414	0.958	0.000	0.125	3.078

Problem 651	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	5	5	5	6	5	20	5	5	5
normalized size	1	1.00	1.00	1.20	1.00	4.00	1.00	1.00	1.00
time (sec)	N/A	0.009	2.566	0.007	0.545	0.789	0.436	0.140	0.088

Problem 652	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	21	23	17	73	34	17	22
normalized size	1	1.00	0.75	0.82	0.61	2.61	1.21	0.61	0.79
time (sec)	N/A	0.026	1.476	0.010	0.586	0.999	5.567	0.140	3.046

Problem 653	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	26	21	20	168	54	20	20
normalized size	1	1.00	1.00	0.81	0.77	6.46	2.08	0.77	0.77
time (sec)	N/A	0.046	4.503	0.155	0.326	1.921	47.646	0.152	3.134

Problem 654	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	137	40	103	39	46	39	39
normalized size	1	1.00	3.81	1.11	2.86	1.08	1.28	1.08	1.08
time (sec)	N/A	0.087	0.281	0.013	0.392	0.923	11.654	0.130	0.090

Problem 655	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	9	9	9	8	7	22	7	7	7
normalized size	1	1.00	1.00	0.89	0.78	2.44	0.78	0.78	0.78
time (sec)	N/A	0.011	2.678	0.007	0.318	0.951	0.422	0.158	3.010

Problem 656	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	24	26	17	17	26	17	17
normalized size	1	1.00	0.77	0.84	0.55	0.55	0.84	0.55	0.55
time (sec)	N/A	0.023	0.122	0.012	0.317	0.842	0.674	0.149	0.116

Problem 657	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	B	B	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	9	9	162	0	18	29	0	7	-1
normalized size	1	1.00	18.00	0.00	2.00	3.22	0.00	0.78	-0.11
time (sec)	N/A	0.072	2.217	1.263	0.600	1.114	0.000	0.154	0.000

Problem 658	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	71	71	59	49	51	34	0	75	-1
normalized size	1	1.00	0.83	0.69	0.72	0.48	0.00	1.06	-0.01
time (sec)	N/A	0.066	0.160	0.173	0.318	1.679	0.000	0.136	0.000

Problem 659	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	18	18	17	17	39	17	17
normalized size	1	1.00	1.00	1.00	0.94	0.94	2.17	0.94	0.94
time (sec)	N/A	0.014	0.048	0.006	0.310	1.038	0.643	0.143	0.101

Problem 660	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	23	24	23	23	54	23	23
normalized size	1	1.00	1.00	1.04	1.00	1.00	2.35	1.00	1.00
time (sec)	N/A	0.015	0.235	0.057	0.371	1.402	9.836	0.152	3.160

Problem 661	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	23	24	23	23	54	23	23
normalized size	1	1.00	0.96	1.00	0.96	0.96	2.25	0.96	0.96
time (sec)	N/A	0.014	0.042	0.033	0.321	0.897	2.226	0.148	3.023

Problem 662	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	14	16	14	14	0	0	-1
normalized size	1	1.00	1.00	1.14	1.00	1.00	0.00	0.00	-0.07
time (sec)	N/A	0.022	0.040	0.033	0.374	1.490	0.000	0.000	0.000

Problem 663	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	19	22	20	20	0	0	-1
normalized size	1	1.00	1.00	1.16	1.05	1.05	0.00	0.00	-0.05
time (sec)	N/A	0.022	0.061	0.074	0.385	0.892	0.000	0.000	0.000

Problem 664	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	19	22	20	20	0	0	-1
normalized size	1	1.00	0.95	1.10	1.00	1.00	0.00	0.00	-0.05
time (sec)	N/A	0.023	0.057	0.050	0.435	0.768	0.000	0.000	0.000

Problem 665	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	11	11	11	12	11	11	14	12	11
normalized size	1	1.00	1.00	1.09	1.00	1.00	1.27	1.09	1.00
time (sec)	N/A	0.022	0.006	0.034	0.321	1.974	0.318	0.141	0.034

Problem 666	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	18	20	19	22	56	19	19
normalized size	1	1.00	0.95	1.05	1.00	1.16	2.95	1.00	1.00
time (sec)	N/A	0.022	0.020	0.026	0.321	0.571	1.877	0.148	3.135

Problem 667	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	3	3	3	4	3	39	0	16	9
normalized size	1	1.00	1.00	1.33	1.00	13.00	0.00	5.33	3.00
time (sec)	N/A	0.023	0.008	0.057	0.414	1.190	0.000	0.128	0.021

Problem 668	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	7	7	7	6	5	53	0	5	5
normalized size	1	1.00	1.00	0.86	0.71	7.57	0.00	0.71	0.71
time (sec)	N/A	0.025	0.008	0.076	0.418	3.019	0.000	0.154	2.979

Problem 669	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	13	10	9	71	0	9	9
normalized size	1	1.00	1.00	0.77	0.69	5.46	0.00	0.69	0.69
time (sec)	N/A	0.026	0.028	0.084	0.417	0.596	0.000	0.224	2.984

Problem 670	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	21	48	38	79	0	38	47
normalized size	1	1.00	1.00	2.29	1.81	3.76	0.00	1.81	2.24
time (sec)	N/A	0.024	0.015	0.079	0.328	0.988	0.000	0.150	3.095

Problem 671	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	28	23	22	61	0	22	20
normalized size	1	1.00	1.00	0.82	0.79	2.18	0.00	0.79	0.71
time (sec)	N/A	0.026	0.018	0.072	0.463	1.075	0.000	0.141	2.965

Problem 672	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	14	11	10	12	27	10	10
normalized size	1	1.00	1.00	0.79	0.71	0.86	1.93	0.71	0.71
time (sec)	N/A	0.034	0.007	0.007	0.367	0.881	0.729	0.133	0.100

Problem 673	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	40	17	20	35	0	20	14
normalized size	1	1.00	2.11	0.89	1.05	1.84	0.00	1.05	0.74
time (sec)	N/A	0.032	0.018	0.099	0.312	2.962	0.000	0.156	3.170

Problem 674	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	3	3	3	4	3	17	3	3	3
normalized size	1	1.00	1.00	1.33	1.00	5.67	1.00	1.00	1.00
time (sec)	N/A	0.008	1.466	0.009	0.319	1.642	0.412	0.142	2.953

Problem 675	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	4	4	4	5	4	41	5	4	4
normalized size	1	1.00	1.00	1.25	1.00	10.25	1.25	1.00	1.00
time (sec)	N/A	0.022	8.959	0.013	0.318	0.669	8.852	0.132	3.002

Problem 676	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	4	4	4	9	8	47	10	29	21
normalized size	1	1.00	1.00	2.25	2.00	11.75	2.50	7.25	5.25
time (sec)	N/A	0.007	0.005	0.010	0.330	0.878	1.330	0.145	3.240

Problem 677	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	128	40	39	103	44	39	73
normalized size	1	1.00	3.56	1.11	1.08	2.86	1.22	1.08	2.03
time (sec)	N/A	0.077	0.349	0.014	0.334	1.193	11.379	0.123	0.072

Problem 678	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	9	13	8	8	12	8	8
normalized size	1	1.00	0.64	0.93	0.57	0.57	0.86	0.57	0.57
time (sec)	N/A	0.016	0.010	0.004	0.330	0.861	0.636	0.138	2.915

Problem 679	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	20	14	19	28	36	13	-1
normalized size	1	1.00	0.80	0.56	0.76	1.12	1.44	0.52	-0.04
time (sec)	N/A	0.049	0.020	0.157	0.322	0.892	2.069	0.130	0.000

Problem 680	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	10	10	10	8	7	7	8	7	7
normalized size	1	1.00	1.00	0.80	0.70	0.70	0.80	0.70	0.70
time (sec)	N/A	0.027	0.010	0.019	0.319	1.184	0.461	0.149	3.002

Problem 681	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	6	6	6	6	5	5	7	5	6
normalized size	1	1.00	1.00	1.00	0.83	0.83	1.17	0.83	1.00
time (sec)	N/A	0.010	0.009	0.028	0.315	0.940	0.540	0.121	2.908

Problem 682	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	10	10	7	7	7	6	0	12	7
normalized size	1	1.00	0.70	0.70	0.70	0.60	0.00	1.20	0.70
time (sec)	N/A	0.012	0.030	0.079	0.950	0.787	0.000	0.148	2.993

Problem 683	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	10	10	10	13	7	12	0	18	7
normalized size	1	1.00	1.00	1.30	0.70	1.20	0.00	1.80	0.70
time (sec)	N/A	0.011	0.009	0.079	0.328	0.718	0.000	0.145	2.952

Problem 684	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	17	17	16	16	36	16	16
normalized size	1	1.00	1.00	1.00	0.94	0.94	2.12	0.94	0.94
time (sec)	N/A	0.013	0.017	0.008	0.310	0.777	0.425	0.131	0.107

Problem 685	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	23	23	22	22	51	22	22
normalized size	1	1.00	1.05	1.05	1.00	1.00	2.32	1.00	1.00
time (sec)	N/A	0.013	0.136	0.058	0.317	1.046	9.316	0.132	3.080

Problem 686	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	23	23	22	22	51	22	22
normalized size	1	1.00	1.00	1.00	0.96	0.96	2.22	0.96	0.96
time (sec)	N/A	0.013	0.043	0.033	0.311	0.800	2.274	0.147	2.985

Problem 687	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	13	17	13	13	0	13	-1
normalized size	1	1.00	1.00	1.31	1.00	1.00	0.00	1.00	-0.08
time (sec)	N/A	0.020	0.039	0.032	0.372	0.997	0.000	0.142	0.000

Problem 688	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	18	23	19	19	0	0	-1
normalized size	1	1.00	1.00	1.28	1.06	1.06	0.00	0.00	-0.06
time (sec)	N/A	0.021	0.066	0.104	0.380	0.668	0.000	0.000	0.000

Problem 689	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	18	23	19	19	0	0	-1
normalized size	1	1.00	0.95	1.21	1.00	1.00	0.00	0.00	-0.05
time (sec)	N/A	0.020	0.060	0.066	0.410	0.921	0.000	0.000	0.000

Problem 690	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	11	11	20	12	11	40	0	12	11
normalized size	1	1.00	1.82	1.09	1.00	3.64	0.00	1.09	1.00
time (sec)	N/A	0.034	0.056	0.073	0.311	1.825	0.000	0.122	3.026

Problem 691	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	11	11	23	4	15	23	0	17	3
normalized size	1	1.00	2.09	0.36	1.36	2.09	0.00	1.55	0.27
time (sec)	N/A	0.031	0.006	0.099	0.313	0.569	0.000	0.156	3.082

Problem 692	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	9	8	7	21	0	7	7
normalized size	1	1.00	0.33	0.30	0.26	0.78	0.00	0.26	0.26
time (sec)	N/A	0.029	0.024	0.102	0.409	0.548	0.000	0.152	2.882

Problem 693	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	18	20	19	37	0	0	37
normalized size	1	1.00	0.95	1.05	1.00	1.95	0.00	0.00	1.95
time (sec)	N/A	0.035	0.189	0.079	0.314	1.058	0.000	0.000	3.563

Problem 694	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	4	4	4	5	4	12	27	4	4
normalized size	1	1.00	1.00	1.25	1.00	3.00	6.75	1.00	1.00
time (sec)	N/A	0.043	0.005	0.115	0.413	0.891	0.743	0.127	2.937

Problem 695	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	4	4	4	5	4	12	27	4	4
normalized size	1	1.00	1.00	1.25	1.00	3.00	6.75	1.00	1.00
time (sec)	N/A	0.063	0.003	0.114	0.499	0.888	0.736	0.124	2.997

Problem 696	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	31	6	5	35	0	5	5
normalized size	1	1.00	0.94	0.18	0.15	1.06	0.00	0.15	0.15
time (sec)	N/A	0.042	0.034	0.135	0.411	0.860	0.000	0.126	3.122

Problem 697	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	10	15	16	18	17	36	0	19	16
normalized size	1	1.50	1.60	1.80	1.70	3.60	0.00	1.90	1.60
time (sec)	N/A	0.048	0.035	0.129	0.312	1.294	0.000	0.139	3.098

Problem 698	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	10	15	16	16	15	36	0	17	14
normalized size	1	1.50	1.60	1.60	1.50	3.60	0.00	1.70	1.40
time (sec)	N/A	0.053	0.034	0.136	0.311	0.866	0.000	0.159	2.984

Problem 699	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	176	176	74	80	89	441	0	61	75
normalized size	1	1.00	0.42	0.45	0.51	2.51	0.00	0.35	0.43
time (sec)	N/A	0.140	0.115	0.123	0.435	1.104	0.000	0.167	3.308

Problem 700	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	22	18	17	48	0	17	17
normalized size	1	1.00	0.42	0.34	0.32	0.91	0.00	0.32	0.32
time (sec)	N/A	0.066	0.054	0.148	0.448	0.981	0.000	0.126	3.115

Problem 701	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	54	35	28	71	29	29	27
normalized size	1	1.00	1.93	1.25	1.00	2.54	1.04	1.04	0.96
time (sec)	N/A	0.087	0.384	0.119	0.318	0.912	4.907	0.145	3.093

Problem 702	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	62	80	63	122	56	64	65
normalized size	1	1.00	1.17	1.51	1.19	2.30	1.06	1.21	1.23
time (sec)	N/A	0.138	0.592	0.133	0.353	1.627	7.628	0.149	2.984

Problem 703	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	78	78	133	143	118	201	95	123	122
normalized size	1	1.00	1.71	1.83	1.51	2.58	1.22	1.58	1.56
time (sec)	N/A	0.150	0.960	0.162	0.524	2.111	10.567	0.148	2.955

Problem 704	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	12	11	10	36	0	10	12
normalized size	1	1.00	1.00	0.92	0.83	3.00	0.00	0.83	1.00
time (sec)	N/A	0.076	0.035	0.129	0.318	0.851	0.000	0.149	2.930

Problem 705	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	67	42	25	46	27	25	25
normalized size	1	1.00	2.03	1.27	0.76	1.39	0.82	0.76	0.76
time (sec)	N/A	0.092	0.025	0.064	0.394	0.861	19.284	0.139	2.922

Problem 706	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	32	24	23	52	41	24	30
normalized size	1	1.00	0.70	0.52	0.50	1.13	0.89	0.52	0.65
time (sec)	N/A	0.089	0.221	0.204	0.410	2.513	9.252	0.158	2.965

Problem 707	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	4	4	4	5	4	12	3	15	4
normalized size	1	1.00	1.00	1.25	1.00	3.00	0.75	3.75	1.00
time (sec)	N/A	0.018	0.002	0.076	0.459	0.408	5.384	0.193	2.882

Problem 708	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	29	14	13	29	0	15	9
normalized size	1	1.00	1.38	0.67	0.62	1.38	0.00	0.71	0.43
time (sec)	N/A	0.115	0.030	0.131	0.318	0.578	0.000	0.156	3.508

Problem 709	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	C	A	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	9	9	43	103	8	25	0	0	-1
normalized size	1	1.00	4.78	11.44	0.89	2.78	0.00	0.00	-0.11
time (sec)	N/A	0.047	0.042	0.282	0.425	0.832	0.000	0.000	0.000

Problem 710	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	C	A	B	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	9	9	47	171	7	45	0	7	-1
normalized size	1	1.00	5.22	19.00	0.78	5.00	0.00	0.78	-0.11
time (sec)	N/A	0.047	0.060	0.687	0.423	0.833	0.000	0.172	0.000

Problem 711	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	C	A	B	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	46	171	16	67	0	17	-1
normalized size	1	1.00	3.29	12.21	1.14	4.79	0.00	1.21	-0.07
time (sec)	N/A	0.044	0.047	0.720	0.314	1.005	0.000	0.177	0.000

Problem 712	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	C	A	B	F	C	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	52	223	30	78	0	142	19
normalized size	1	1.00	2.74	11.74	1.58	4.11	0.00	7.47	1.00
time (sec)	N/A	0.049	0.493	0.612	0.415	0.968	0.000	0.202	3.061

Problem 713	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	C	A	B	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	63	492	20	72	0	20	-1
normalized size	1	1.00	2.42	18.92	0.77	2.77	0.00	0.77	-0.04
time (sec)	N/A	0.046	0.115	0.427	0.408	1.045	0.000	0.147	0.000

Problem 714	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	4	4	4	4	3	8	3	3	3
normalized size	1	1.00	1.00	1.00	0.75	2.00	0.75	0.75	0.75
time (sec)	N/A	0.012	0.059	0.048	0.317	0.518	0.952	0.132	3.102

Problem 715	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	25	20	42	20	19	20	14
normalized size	1	1.00	1.47	1.18	2.47	1.18	1.12	1.18	0.82
time (sec)	N/A	0.067	0.017	0.046	0.312	0.806	6.969	0.150	2.923

Problem 716	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	20	13	12	45	0	22	16
normalized size	1	1.00	1.67	1.08	1.00	3.75	0.00	1.83	1.33
time (sec)	N/A	0.041	0.059	0.073	0.332	1.439	0.000	0.167	3.012

Problem 717	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	19	21	20	38	0	0	43
normalized size	1	1.00	0.95	1.05	1.00	1.90	0.00	0.00	2.15
time (sec)	N/A	0.041	0.205	0.080	0.317	0.653	0.000	0.000	3.186

Problem 718	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	6	6	6	7	8	14	3	16	6
normalized size	1	1.00	1.00	1.17	1.33	2.33	0.50	2.67	1.00
time (sec)	N/A	0.016	0.002	0.064	0.409	0.672	4.214	0.147	2.935

Problem 719	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	6	6	6	7	8	14	27	16	6
normalized size	1	1.00	1.00	1.17	1.33	2.33	4.50	2.67	1.00
time (sec)	N/A	0.047	0.005	0.088	0.409	0.660	0.699	0.141	2.942

Problem 720	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	56	56	46	76	31	68	35
normalized size	1	1.00	2.00	2.00	1.64	2.71	1.11	2.43	1.25
time (sec)	N/A	0.082	0.377	0.103	0.319	1.170	26.515	0.152	3.069

Problem 721	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	62	119	92	182	58	139	92
normalized size	1	1.00	1.17	2.25	1.74	3.43	1.09	2.62	1.74
time (sec)	N/A	0.136	0.561	0.131	0.319	1.473	55.639	0.165	3.072

Problem 722	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	78	78	135	202	161	320	97	232	141
normalized size	1	1.00	1.73	2.59	2.06	4.10	1.24	2.97	1.81
time (sec)	N/A	0.139	1.304	0.164	0.317	1.624	69.993	0.172	3.080

Problem 723	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	6	6	6	6	5	9	5	0	7
normalized size	1	1.00	1.00	1.00	0.83	1.50	0.83	0.00	1.17
time (sec)	N/A	0.015	0.070	0.049	0.316	0.640	18.865	0.000	2.938

Problem 724	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	11	20	20	12	11	19	14	22	48
normalized size	1	1.82	1.82	1.09	1.00	1.73	1.27	2.00	4.36
time (sec)	N/A	0.046	0.016	0.040	0.309	0.671	0.463	0.142	3.246

Problem 725	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	5	5	5	4	5	5	3	5	7
normalized size	1	1.00	1.00	0.80	1.00	1.00	0.60	1.00	1.40
time (sec)	N/A	0.033	0.022	0.070	0.415	4.939	0.209	0.142	2.950

Problem 726	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	11	11	11	8	7	7	8	7	13
normalized size	1	1.00	1.00	0.73	0.64	0.64	0.73	0.64	1.18
time (sec)	N/A	0.035	0.026	0.059	0.438	0.967	0.233	0.132	3.044

Problem 727	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	7	7	9	12	7	9	15	7	9
normalized size	1	1.00	1.29	1.71	1.00	1.29	2.14	1.00	1.29
time (sec)	N/A	0.032	0.005	0.082	0.310	0.845	0.216	0.145	3.072

Problem 728	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	5	5	38	6	33	27	5	36	7
normalized size	1	1.00	7.60	1.20	6.60	5.40	1.00	7.20	1.40
time (sec)	N/A	0.045	0.028	0.099	0.313	0.946	0.727	0.150	3.102

Problem 729	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	13	25	13	16	0	21	13
normalized size	1	1.00	1.00	1.92	1.00	1.23	0.00	1.62	1.00
time (sec)	N/A	0.079	0.017	0.109	0.420	0.668	0.000	0.150	0.106

Problem 730	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	4	4	4	4	3	5	3	5	5
normalized size	1	1.00	1.00	1.00	0.75	1.25	0.75	1.25	1.25
time (sec)	N/A	0.022	0.008	0.020	0.331	0.922	0.614	0.140	3.093

Problem 731	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	9	9	9	10	9	11	7	11	11
normalized size	1	1.00	1.00	1.11	1.00	1.22	0.78	1.22	1.22
time (sec)	N/A	0.022	0.007	0.022	0.313	0.473	0.651	0.146	3.123

Problem 732	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	20	11	10	29	12	37	18
normalized size	1	1.00	1.67	0.92	0.83	2.42	1.00	3.08	1.50
time (sec)	N/A	0.055	0.075	0.076	0.316	0.760	1.015	0.166	3.114

Problem 733	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	43	65	35	122	0	0	36
normalized size	1	1.00	1.00	1.51	0.81	2.84	0.00	0.00	0.84
time (sec)	N/A	0.095	0.052	0.116	0.426	0.954	0.000	0.000	3.257

Problem 734	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	22	34	20	20	0	34	18
normalized size	1	1.00	1.00	1.55	0.91	0.91	0.00	1.55	0.82
time (sec)	N/A	0.091	0.025	0.114	0.422	0.666	0.000	0.154	3.026

Problem 735	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	20	20	13	12	20	17	22	31
normalized size	1	1.67	1.67	1.08	1.00	1.67	1.42	1.83	2.58
time (sec)	N/A	0.042	0.016	0.049	0.310	0.689	0.387	0.130	3.180

Problem 736	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	14	13	12	14	12	14	14
normalized size	1	1.00	1.00	0.93	0.86	1.00	0.86	1.00	1.00
time (sec)	N/A	0.025	0.018	0.036	0.302	1.140	0.612	0.250	2.968

Problem 737	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	3	3	3	6	3	3	5	3	26
normalized size	1	1.00	1.00	2.00	1.00	1.00	1.67	1.00	8.67
time (sec)	N/A	0.032	0.012	0.053	0.402	0.709	0.192	0.148	3.215

Problem 738	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	41	35	49	73	151	48	57
normalized size	1	1.00	0.95	0.81	1.14	1.70	3.51	1.12	1.33
time (sec)	N/A	0.057	0.659	0.080	0.402	0.641	1.715	0.244	3.148

Problem 739	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	14	15	14	21	0	21	34
normalized size	1	1.00	1.00	1.07	1.00	1.50	0.00	1.50	2.43
time (sec)	N/A	0.077	0.018	0.164	0.400	1.069	0.000	0.151	3.114

Problem 740	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	28	38	37	57	0	48	28
normalized size	1	1.00	0.65	0.88	0.86	1.33	0.00	1.12	0.65
time (sec)	N/A	0.106	0.055	0.237	0.401	0.684	0.000	0.507	3.144

Problem 741	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	28	104	37	27	0	0	27
normalized size	1	1.00	0.65	2.42	0.86	0.63	0.00	0.00	0.63
time (sec)	N/A	0.037	0.058	0.144	0.339	0.600	0.000	0.000	3.178

Problem 742	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	28	104	37	27	0	0	27
normalized size	1	1.00	0.65	2.42	0.86	0.63	0.00	0.00	0.63
time (sec)	N/A	0.035	0.028	0.000	0.352	0.649	0.000	0.000	0.002

Problem 743	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	36	122	0	33	0	138	33
normalized size	1	1.00	0.56	1.91	0.00	0.52	0.00	2.16	0.52
time (sec)	N/A	0.036	0.064	0.130	0.000	0.982	0.000	0.231	3.177

Problem 744	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	36	122	0	33	0	138	33
normalized size	1	1.00	0.56	1.91	0.00	0.52	0.00	2.16	0.52
time (sec)	N/A	0.038	0.029	0.001	0.000	0.692	0.000	0.235	0.002

Problem 745	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	28	105	37	27	0	0	27
normalized size	1	1.00	0.65	2.44	0.86	0.63	0.00	0.00	0.63
time (sec)	N/A	0.041	0.141	0.109	0.339	0.537	0.000	0.000	3.214

Problem 746	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	28	105	37	27	0	0	27
normalized size	1	1.00	0.65	2.44	0.86	0.63	0.00	0.00	0.63
time (sec)	N/A	0.035	0.034	0.000	0.341	0.720	0.000	0.000	0.002

Problem 747	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	36	123	0	33	0	195	33
normalized size	1	1.00	0.56	1.92	0.00	0.52	0.00	3.05	0.52
time (sec)	N/A	0.037	0.173	0.122	0.000	2.004	0.000	0.216	3.174

Problem 748	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	36	123	0	33	0	195	33
normalized size	1	1.00	0.56	1.92	0.00	0.52	0.00	3.05	0.52
time (sec)	N/A	0.037	0.033	0.000	0.000	0.503	0.000	0.220	0.002

Problem 749	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	9	9	9	8	7	12	0	7	27
normalized size	1	1.00	1.00	0.89	0.78	1.33	0.00	0.78	3.00
time (sec)	N/A	0.022	0.005	0.079	0.321	0.612	0.000	0.144	5.268

Problem 750	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	9	9	9	8	265	7	0	0	27
normalized size	1	1.00	1.00	0.89	29.44	0.78	0.00	0.00	3.00
time (sec)	N/A	0.020	0.010	0.062	0.456	0.510	0.000	0.000	3.527

Problem 751	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	C	C	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	3	3	3	5	4	4	14	0	4
normalized size	1	1.00	1.00	1.67	1.33	1.33	4.67	0.00	1.33
time (sec)	N/A	0.009	0.000	0.060	0.426	0.549	0.142	0.000	0.026

Problem 752	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	8	8	8	9	74	18	0	103	8
normalized size	1	1.00	1.00	1.12	9.25	2.25	0.00	12.88	1.00
time (sec)	N/A	0.018	0.004	0.024	0.423	0.528	0.000	0.157	0.024

Problem 753	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	28	26	22	27	76	22	22
normalized size	1	1.00	0.82	0.76	0.65	0.79	2.24	0.65	0.65
time (sec)	N/A	0.022	0.017	0.069	0.318	2.298	0.956	0.133	2.973

Problem 754	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	10	10	10	7	6	6	10	6	6
normalized size	1	1.00	1.00	0.70	0.60	0.60	1.00	0.60	0.60
time (sec)	N/A	0.012	0.003	0.017	0.318	0.588	0.250	0.137	0.067

Problem 755	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	11	11	11	9	8	14	15	9	12
normalized size	1	1.00	1.00	0.82	0.73	1.27	1.36	0.82	1.09
time (sec)	N/A	0.012	0.008	0.049	0.314	1.376	0.312	0.132	3.536

Problem 756	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	7	7	9	8	7	9	8	7	7
normalized size	1	1.00	1.29	1.14	1.00	1.29	1.14	1.00	1.00
time (sec)	N/A	0.044	0.006	0.078	0.316	0.709	2.166	0.140	2.924

Problem 757	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	19	16	74	28	0	103	15
normalized size	1	1.00	1.00	0.84	3.89	1.47	0.00	5.42	0.79
time (sec)	N/A	0.019	0.009	0.025	0.414	0.708	0.000	0.176	2.897

Problem 758	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	26	31	26	29	32	27	25
normalized size	1	1.00	0.70	0.84	0.70	0.78	0.86	0.73	0.68
time (sec)	N/A	0.014	0.028	0.058	0.317	0.702	0.423	0.125	2.901

Problem 759	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	B	B	B	A	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	129	49	176	8	39	236	37	49
normalized size	1	10.75	4.08	14.67	0.67	3.25	19.67	3.08	4.08
time (sec)	N/A	0.324	0.027	0.205	0.337	1.533	0.080	0.145	2.986

Problem 760	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	9	9	19	8	7	13	19	8	19
normalized size	1	1.00	2.11	0.89	0.78	1.44	2.11	0.89	2.11
time (sec)	N/A	0.007	0.009	0.003	0.323	0.654	0.139	0.151	0.117

Problem 761	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	8	8	8	7	35	12	0	6	14
normalized size	1	1.00	1.00	0.88	4.38	1.50	0.00	0.75	1.75
time (sec)	N/A	0.013	0.017	0.030	0.324	1.040	0.000	0.118	0.099

Problem 762	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	15	12	0	13	0	15	13
normalized size	1	1.00	1.00	0.80	0.00	0.87	0.00	1.00	0.87
time (sec)	N/A	0.029	0.015	0.172	0.000	0.657	0.000	0.657	2.965

Problem 763	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	20	16	15	15	19	15	15
normalized size	1	1.00	0.87	0.70	0.65	0.65	0.83	0.65	0.65
time (sec)	N/A	0.022	0.012	0.040	0.429	1.577	0.266	0.148	0.074

Problem 764	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	37	30	29	29	42	29	33
normalized size	1	1.00	1.00	0.81	0.78	0.78	1.14	0.78	0.89
time (sec)	N/A	0.034	0.023	0.171	0.328	1.963	0.516	0.125	2.973

Problem 765	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	10	10	12	9	8	10	8	10	8
normalized size	1	1.00	1.20	0.90	0.80	1.00	0.80	1.00	0.80
time (sec)	N/A	0.033	0.019	0.040	0.321	1.375	0.168	0.145	2.918

Problem 766	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	8	8	8	7	6	6	5	6	6
normalized size	1	1.00	1.00	0.88	0.75	0.75	0.62	0.75	0.75
time (sec)	N/A	0.007	0.001	0.039	0.315	0.531	0.150	0.127	0.056

Problem 767	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	10	10	10	9	8	8	7	8	8
normalized size	1	1.00	1.00	0.90	0.80	0.80	0.70	0.80	0.80
time (sec)	N/A	0.012	0.003	0.039	0.317	0.765	0.279	0.123	0.059

Problem 768	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	8	8	8	7	6	6	5	6	6
normalized size	1	1.00	1.00	0.88	0.75	0.75	0.62	0.75	0.75
time (sec)	N/A	0.009	0.002	0.039	0.314	0.674	0.477	0.138	0.067

Problem 769	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	10	10	10	9	8	8	7	8	8
normalized size	1	1.00	1.00	0.90	0.80	0.80	0.70	0.80	0.80
time (sec)	N/A	0.008	0.010	0.004	0.326	0.487	0.149	0.118	2.924

Problem 770	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	8	8	8	7	56	8	5	8	8
normalized size	1	1.00	1.00	0.88	7.00	1.00	0.62	1.00	1.00
time (sec)	N/A	0.063	0.006	0.026	0.329	0.768	0.285	0.119	0.075

Problem 771	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	10	10	12	11	67	13	7	10	10
normalized size	1	1.00	1.20	1.10	6.70	1.30	0.70	1.00	1.00
time (sec)	N/A	0.018	0.023	0.006	0.330	0.722	0.222	0.140	2.916

Problem 772	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	11	11	11	10	9	15	12	10	13
normalized size	1	1.00	1.00	0.91	0.82	1.36	1.09	0.91	1.18
time (sec)	N/A	0.010	0.018	0.002	0.327	0.791	0.119	0.172	0.283

Problem 773	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	12	11	10	12	12	10	13
normalized size	1	1.00	1.00	0.92	0.83	1.00	1.00	0.83	1.08
time (sec)	N/A	0.005	0.006	0.039	0.322	0.566	0.790	0.140	2.923

Problem 774	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	25	20	18	29	0	18	16
normalized size	1	1.00	1.19	0.95	0.86	1.38	0.00	0.86	0.76
time (sec)	N/A	0.060	0.017	0.150	0.326	0.675	0.000	0.153	3.001

Problem 775	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	19	16	15	40	32	39	15
normalized size	1	1.00	1.00	0.84	0.79	2.11	1.68	2.05	0.79
time (sec)	N/A	0.037	0.009	0.230	0.331	0.502	5.251	0.141	3.027

Problem 776	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	35	33	32	32	48	32	43
normalized size	1	1.00	0.74	0.70	0.68	0.68	1.02	0.68	0.91
time (sec)	N/A	0.064	0.031	0.029	0.327	0.711	71.975	0.124	3.004

Problem 777	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	28	36	25	29	32	25	25
normalized size	1	1.00	0.80	1.03	0.71	0.83	0.91	0.71	0.71
time (sec)	N/A	0.158	0.044	0.026	0.334	0.753	1.817	0.148	2.966

Problem 778	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	4	4	4	5	4	4	3	4	4
normalized size	1	1.00	1.00	1.25	1.00	1.00	0.75	1.00	1.00
time (sec)	N/A	0.007	0.001	0.001	0.325	0.786	0.152	0.136	2.924

Problem 779	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	6	6	6	7	6	6	5	6	6
normalized size	1	1.00	1.00	1.17	1.00	1.00	0.83	1.00	1.00
time (sec)	N/A	0.013	0.003	0.040	0.322	0.646	0.267	0.119	0.057

Problem 780	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	7	7	7	8	7	7	5	7	7
normalized size	1	1.00	1.00	1.14	1.00	1.00	0.71	1.00	1.00
time (sec)	N/A	0.004	0.007	0.001	0.427	0.841	0.080	0.120	0.055

Problem 781	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	10	10	21	9	8	8	8	8	8
normalized size	1	1.00	2.10	0.90	0.80	0.80	0.80	0.80	0.80
time (sec)	N/A	0.009	0.013	0.004	0.324	0.713	0.153	0.123	0.042

Problem 782	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	10	10	10	9	8	8	7	8	8
normalized size	1	1.00	1.00	0.90	0.80	0.80	0.70	0.80	0.80
time (sec)	N/A	0.009	0.003	0.040	0.316	0.580	0.149	0.136	2.981

Problem 783	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	10	10	10	9	8	8	7	8	8
normalized size	1	1.00	1.00	0.90	0.80	0.80	0.70	0.80	0.80
time (sec)	N/A	0.010	0.003	0.001	0.325	0.623	0.264	0.142	0.046

Problem 784	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	10	10	21	9	8	8	8	8	8
normalized size	1	1.00	2.10	0.90	0.80	0.80	0.80	0.80	0.80
time (sec)	N/A	0.012	0.013	0.004	0.324	0.958	0.261	0.139	0.047

Problem 785	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	6	6	6	7	6	6	5	6	6
normalized size	1	1.00	1.00	1.17	1.00	1.00	0.83	1.00	1.00
time (sec)	N/A	0.009	0.002	0.000	0.323	0.659	0.267	0.137	0.051

Problem 786	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	14	15	14	14	15	14	14
normalized size	1	1.00	1.00	1.07	1.00	1.00	1.07	1.00	1.00
time (sec)	N/A	0.011	0.028	0.028	0.324	0.527	0.154	0.138	2.945

Problem 787	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	20	17	16	16	15	16	16
normalized size	1	1.00	1.00	0.85	0.80	0.80	0.75	0.80	0.80
time (sec)	N/A	0.017	0.007	0.053	0.327	0.587	1.515	0.137	2.965

Problem 788	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	16	18	15	17	20	15	15
normalized size	1	1.00	0.70	0.78	0.65	0.74	0.87	0.65	0.65
time (sec)	N/A	0.009	0.013	0.025	0.327	0.693	0.402	0.135	0.021

Problem 789	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	20	17	16	16	15	16	16
normalized size	1	1.00	1.00	0.85	0.80	0.80	0.75	0.80	0.80
time (sec)	N/A	0.017	0.002	0.005	0.327	0.529	0.483	0.140	2.959

Problem 790	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	20	17	16	16	15	16	16
normalized size	1	1.00	1.00	0.85	0.80	0.80	0.75	0.80	0.80
time (sec)	N/A	0.017	0.006	0.039	0.327	1.524	0.480	0.137	0.050

Problem 791	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	9	9	9	8	7	19	7	7	7
normalized size	1	1.00	1.00	0.89	0.78	2.11	0.78	0.78	0.78
time (sec)	N/A	0.010	1.384	0.033	0.328	0.793	0.419	0.136	0.072

Problem 792	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	11	11	11	10	9	11	10	9	17
normalized size	1	1.00	1.00	0.91	0.82	1.00	0.91	0.82	1.55
time (sec)	N/A	0.032	0.026	0.032	0.325	0.685	0.171	0.119	3.008

Problem 793	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	10	10	10	65	8	45	36	61	8
normalized size	1	1.00	1.00	6.50	0.80	4.50	3.60	6.10	0.80
time (sec)	N/A	0.038	0.018	0.089	0.326	0.620	0.522	0.150	3.153

Problem 794	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	9	9	9	12	13	10	8	8	6
normalized size	1	1.00	1.00	1.33	1.44	1.11	0.89	0.89	0.67
time (sec)	N/A	0.029	0.004	0.065	0.330	0.625	1.795	0.144	2.926

Problem 795	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	5	5	5	20	7	11	7	7	7
normalized size	1	1.00	1.00	4.00	1.40	2.20	1.40	1.40	1.40
time (sec)	N/A	0.016	0.010	0.038	0.314	0.699	0.067	0.141	3.001

Problem 796	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	F	A	C	B	A	F(-1)	B	B
verified	N/A	N/A	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	0	13	30	88	14	0	794	14
normalized size	1	0.00	1.00	2.31	6.77	1.08	0.00	61.08	1.08
time (sec)	N/A	0.640	0.293	0.393	0.771	0.677	0.000	0.201	3.121

Problem 797	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	9	9	9	10	104	20	0	52	9
normalized size	1	1.00	1.00	1.11	11.56	2.22	0.00	5.78	1.00
time (sec)	N/A	0.016	0.016	0.026	0.318	0.624	0.000	0.189	0.028

Problem 798	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	20	15	14	31	37	14	18
normalized size	1	1.00	1.00	0.75	0.70	1.55	1.85	0.70	0.90
time (sec)	N/A	0.017	0.011	0.146	0.306	1.736	0.460	0.156	0.028

Problem 799	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	19	15	15	15	22	15	14
normalized size	1	1.00	1.00	0.79	0.79	0.79	1.16	0.79	0.74
time (sec)	N/A	0.015	0.010	0.023	0.315	0.676	0.480	0.136	2.952

Problem 800	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	14	13	16	14	10	18	16
normalized size	1	1.00	1.00	0.93	1.14	1.00	0.71	1.29	1.14
time (sec)	N/A	0.015	0.005	0.039	0.305	0.799	0.072	0.125	2.950

Problem 801	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	20	29	20	37	20	28	32
normalized size	1	1.00	0.91	1.32	0.91	1.68	0.91	1.27	1.45
time (sec)	N/A	0.033	0.023	0.059	0.318	0.603	0.083	0.128	2.971

Problem 802	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	5	5	36	6	5	5	12	5	5
normalized size	1	1.00	7.20	1.20	1.00	1.00	2.40	1.00	1.00
time (sec)	N/A	0.015	0.007	0.053	0.314	1.437	2.044	0.143	2.938

Problem 803	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	7	7	20	6	5	7	12	7	5
normalized size	1	1.00	2.86	0.86	0.71	1.00	1.71	1.00	0.71
time (sec)	N/A	0.016	0.006	0.050	0.314	0.741	1.888	0.130	2.907

Problem 804	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	5	5	8	6	11	5	14	9	6
normalized size	1	1.00	1.60	1.20	2.20	1.00	2.80	1.80	1.20
time (sec)	N/A	0.022	0.002	0.047	0.309	0.530	3.113	0.151	2.888

Problem 805	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	61	20	69	35	32	29	24
normalized size	1	1.00	4.07	1.33	4.60	2.33	2.13	1.93	1.60
time (sec)	N/A	0.045	0.009	0.263	0.422	0.752	1.865	0.148	3.110

Problem 806	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	11	11	26	12	11	15	12	15	11
normalized size	1	1.00	2.36	1.09	1.00	1.36	1.09	1.36	1.00
time (sec)	N/A	0.046	0.092	0.053	0.307	0.702	0.190	0.133	0.085

Problem 807	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	82	18	17	17	19	17	17
normalized size	1	1.00	4.10	0.90	0.85	0.85	0.95	0.85	0.85
time (sec)	N/A	0.053	0.172	0.036	0.428	0.620	0.426	0.125	2.905

Problem 808	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	11	11	11	12	11	13	10	12	9
normalized size	1	1.00	1.00	1.09	1.00	1.18	0.91	1.09	0.82
time (sec)	N/A	0.021	0.008	0.084	0.313	0.601	0.178	0.152	2.979

Problem 809	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	26	1856	34	27	82	0	29
normalized size	1	1.00	1.00	71.38	1.31	1.04	3.15	0.00	1.12
time (sec)	N/A	0.049	0.043	0.750	0.425	1.390	1.067	0.000	3.075

Problem 810	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	39	24	220	35	0	28	16
normalized size	1	1.00	1.62	1.00	9.17	1.46	0.00	1.17	0.67
time (sec)	N/A	0.028	0.033	0.242	0.329	0.602	0.000	0.138	3.073

Problem 811	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	29	35	38	26	39	26	34
normalized size	1	1.00	0.72	0.88	0.95	0.65	0.98	0.65	0.85
time (sec)	N/A	0.065	0.041	0.043	0.330	0.553	0.292	0.125	2.900

Problem 812	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	22	22	19	21	26	19	19
normalized size	1	1.00	0.81	0.81	0.70	0.78	0.96	0.70	0.70
time (sec)	N/A	0.010	0.029	0.028	0.326	0.603	0.403	0.136	0.031

Problem 813	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	31	18	17	12	26	17	12
normalized size	1	1.00	1.35	0.78	0.74	0.52	1.13	0.74	0.52
time (sec)	N/A	0.040	0.025	0.026	0.323	0.710	0.286	0.133	0.102

Problem 814	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	46	41	24	36	27	24	24
normalized size	1	1.00	1.53	1.37	0.80	1.20	0.90	0.80	0.80
time (sec)	N/A	0.030	0.017	0.008	0.327	0.587	0.060	0.141	2.998

Problem 815	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	6	6	6	7	6	6	3	14	6
normalized size	1	1.00	1.00	1.17	1.00	1.00	0.50	2.33	1.00
time (sec)	N/A	0.011	0.001	0.082	0.331	0.534	1.132	0.130	2.945

Problem 816	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	7	7	7	8	7	15	8	30	12
normalized size	1	1.00	1.00	1.14	1.00	2.14	1.14	4.29	1.71
time (sec)	N/A	0.038	0.004	0.095	0.328	0.736	4.809	0.135	2.967

Problem 817	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	10	10	13	10	14	13	6
normalized size	1	1.00	0.77	0.77	1.00	0.77	1.08	1.00	0.46
time (sec)	N/A	0.014	0.004	0.036	0.321	1.083	0.062	0.149	2.930

Problem 818	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	11	11	11	10	11	22	12	11	14
normalized size	1	1.00	1.00	0.91	1.00	2.00	1.09	1.00	1.27
time (sec)	N/A	0.008	0.006	0.032	0.326	0.587	0.056	0.127	2.918

Problem 819	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	10	10	10	17	10	18	10	11	24
normalized size	1	1.00	1.00	1.70	1.00	1.80	1.00	1.10	2.40
time (sec)	N/A	0.008	0.010	0.027	0.320	0.915	0.056	0.146	3.083

Problem 820	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	10	10	10	9	8	8	8	8	8
normalized size	1	1.00	1.00	0.90	0.80	0.80	0.80	0.80	0.80
time (sec)	N/A	0.008	0.009	0.004	0.321	0.698	0.154	0.140	0.050

Problem 821	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	18	13	12	14	32	12	12
normalized size	1	1.00	1.00	0.72	0.67	0.78	1.78	0.67	0.67
time (sec)	N/A	0.041	0.031	0.066	0.327	0.625	0.746	0.128	2.993

Problem 822	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	10	10	10	9	8	8	31	8	8
normalized size	1	1.00	1.00	0.90	0.80	0.80	3.10	0.80	0.80
time (sec)	N/A	0.012	0.006	0.003	0.326	0.602	1.285	0.135	2.919

Problem 823	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	25	11	10	21	12	10	14
normalized size	1	1.00	1.56	0.69	0.62	1.31	0.75	0.62	0.88
time (sec)	N/A	0.016	0.021	0.037	0.328	0.750	0.463	0.205	0.079

Problem 824	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	7	7	7	8	7	13	8	9	9
normalized size	1	1.00	1.00	1.14	1.00	1.86	1.14	1.29	1.29
time (sec)	N/A	0.007	0.006	0.003	0.326	0.607	0.119	0.138	0.074

Problem 825	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	13	17	16	25	15	41	15
normalized size	1	1.00	1.00	1.31	1.23	1.92	1.15	3.15	1.15
time (sec)	N/A	0.012	0.017	0.003	0.324	0.686	1.032	0.159	3.526

Problem 826	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	5	5	21	11	10	23	10	43	31
normalized size	1	1.00	4.20	2.20	2.00	4.60	2.00	8.60	6.20
time (sec)	N/A	0.009	0.014	0.003	0.326	0.647	1.257	0.137	3.681

Problem 827	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	7	9	7	8	15	7	8	16	26
normalized size	1	1.29	1.00	1.14	2.14	1.00	1.14	2.29	3.71
time (sec)	N/A	0.045	0.007	0.118	0.327	0.617	2.311	0.152	3.254

Problem 828	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	4	4	4	5	4	4	20	4	4
normalized size	1	1.00	1.00	1.25	1.00	1.00	5.00	1.00	1.00
time (sec)	N/A	0.017	0.001	0.161	0.325	0.690	0.687	0.143	3.074

Problem 829	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	21	14	74	27	0	81	13
normalized size	1	1.00	1.62	1.08	5.69	2.08	0.00	6.23	1.00
time (sec)	N/A	0.020	0.006	0.027	0.436	1.317	0.000	0.171	2.994

Problem 830	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	18	28	20	22	20	20	18
normalized size	1	1.00	1.12	1.75	1.25	1.38	1.25	1.25	1.12
time (sec)	N/A	0.029	0.024	0.039	0.431	0.592	0.054	0.141	2.961

Problem 831	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	26	34	35	28	36	31	26
normalized size	1	1.00	0.81	1.06	1.09	0.88	1.12	0.97	0.81
time (sec)	N/A	0.034	0.021	0.041	0.428	1.629	0.055	0.132	2.995

Problem 832	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	16	19	10	19	12	10	18
normalized size	1	1.00	0.89	1.06	0.56	1.06	0.67	0.56	1.00
time (sec)	N/A	0.028	0.007	0.009	0.330	1.560	0.058	0.137	0.047

Problem 833	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	32	29	18	25	32	22	24
normalized size	1	1.00	0.94	0.85	0.53	0.74	0.94	0.65	0.71
time (sec)	N/A	0.050	0.009	0.009	0.326	2.391	0.053	0.121	0.050

Problem 834	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	33	29	13	21	12	13	13
normalized size	1	1.00	2.54	2.23	1.00	1.62	0.92	1.00	1.00
time (sec)	N/A	0.024	0.010	0.009	0.324	0.468	0.055	0.129	0.039

Problem 835	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	24	36	16	31	31	16	33
normalized size	1	1.00	0.52	0.78	0.35	0.67	0.67	0.35	0.72
time (sec)	N/A	0.056	0.007	0.011	0.326	1.967	0.064	0.139	0.041

Problem 836	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	9	9	9	21	20	19	20	18	13
normalized size	1	1.00	1.00	2.33	2.22	2.11	2.22	2.00	1.44
time (sec)	N/A	0.028	0.005	0.076	0.323	0.573	0.324	0.125	3.008

Problem 837	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	17	13	13	12	12	13	11
normalized size	1	1.00	1.21	0.93	0.93	0.86	0.86	0.93	0.79
time (sec)	N/A	0.017	0.005	0.024	0.324	0.712	0.054	0.143	2.970

Problem 838	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	1	1	1	2	1	1	0	1	1
normalized size	1	1.00	1.00	2.00	1.00	1.00	0.00	1.00	1.00
time (sec)	N/A	0.010	0.000	0.016	0.332	1.351	0.055	0.137	2.934

Problem 839	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	6	6	8	7	6	6	7	6	6
normalized size	1	1.00	1.33	1.17	1.00	1.00	1.17	1.00	1.00
time (sec)	N/A	0.011	0.002	0.001	0.323	0.412	0.056	0.120	2.936

Problem 840	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	9	9	9	10	9	9	7	9	9
normalized size	1	1.00	1.00	1.11	1.00	1.00	0.78	1.00	1.00
time (sec)	N/A	0.009	0.006	0.026	0.310	0.555	0.249	0.125	0.072

Problem 841	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	8	8	8	7	35	6	22	6	6
normalized size	1	1.00	1.00	0.88	4.38	0.75	2.75	0.75	0.75
time (sec)	N/A	0.017	0.004	0.003	0.316	1.469	0.116	0.141	2.945

Problem 842	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	6	6	6	9	51	8	5	26	8
normalized size	1	1.00	1.00	1.50	8.50	1.33	0.83	4.33	1.33
time (sec)	N/A	0.178	0.024	0.021	0.483	2.142	0.481	0.153	0.085

Problem 843	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	8	8	8	7	8	8	8	8	8
normalized size	1	1.00	1.00	0.88	1.00	1.00	1.00	1.00	1.00
time (sec)	N/A	0.197	0.018	0.085	0.314	1.437	0.282	0.119	3.069

Problem 844	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	8	8	12	9	8	8	8	6	8
normalized size	1	1.00	1.50	1.12	1.00	1.00	1.00	0.75	1.00
time (sec)	N/A	0.011	0.012	0.039	0.309	0.733	0.266	0.120	3.051

Problem 845	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	8	8	8	7	8	8	7	8	8
normalized size	1	1.00	1.00	0.88	1.00	1.00	0.88	1.00	1.00
time (sec)	N/A	0.184	0.017	0.058	0.311	0.567	0.286	0.136	3.005

Problem 846	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	70	55	69	0	320	155	77	1108
normalized size	1	1.27	1.00	1.25	0.00	5.82	2.82	1.40	20.15
time (sec)	N/A	0.170	0.078	0.191	0.000	1.906	9.312	0.157	4.178

Problem 847	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	70	54	69	0	322	155	77	1374
normalized size	1	1.27	0.98	1.25	0.00	5.85	2.82	1.40	24.98
time (sec)	N/A	0.133	0.060	0.172	0.000	1.277	9.453	0.158	3.438

Problem 848	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	52	52	48	80	0	225	432	141	108
normalized size	1	1.00	0.92	1.54	0.00	4.33	8.31	2.71	2.08
time (sec)	N/A	0.125	0.087	0.134	0.000	1.220	35.268	0.172	3.434

Problem 849	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	52	52	50	80	0	224	432	159	684
normalized size	1	1.00	0.96	1.54	0.00	4.31	8.31	3.06	13.15
time (sec)	N/A	0.094	0.054	0.131	0.000	1.110	35.327	0.148	3.319

Problem 850	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	31	30	76	91	0	61	-1
normalized size	1	1.00	1.03	1.00	2.53	3.03	0.00	2.03	-0.03
time (sec)	N/A	0.035	0.038	0.146	0.426	0.894	0.000	0.345	0.000

Problem 851	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	49	31	51	84	0	31	-1
normalized size	1	1.00	1.58	1.00	1.65	2.71	0.00	1.00	-0.03
time (sec)	N/A	0.032	0.062	0.159	0.322	1.213	0.000	0.247	0.000

Problem 852	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	8	8	8	7	6	6	7	6	6
normalized size	1	1.00	1.00	0.88	0.75	0.75	0.88	0.75	0.75
time (sec)	N/A	0.013	0.003	0.020	0.310	2.081	0.292	0.138	3.185

Problem 853	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	18	25	41	21	0	14	-1
normalized size	1	1.00	0.95	1.32	2.16	1.11	0.00	0.74	-0.05
time (sec)	N/A	0.030	0.014	0.448	0.425	1.326	0.000	0.158	0.000

Problem 854	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	16	24	12	23	476	12	12
normalized size	1	1.00	0.55	0.83	0.41	0.79	16.41	0.41	0.41
time (sec)	N/A	0.056	0.017	0.042	0.302	0.415	20.216	0.127	3.201

Problem 855	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	37	29	34	103	26	32	34	45
normalized size	1	1.28	1.00	1.17	3.55	0.90	1.10	1.17	1.55
time (sec)	N/A	0.133	0.096	0.140	0.411	2.014	0.366	0.167	3.309

Problem 856	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	37	29	34	103	26	32	34	45
normalized size	1	1.28	1.00	1.17	3.55	0.90	1.10	1.17	1.55
time (sec)	N/A	0.091	0.073	0.136	0.420	1.010	0.370	0.165	3.192

Problem 857	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	38	31	30	46	0	34	32
normalized size	1	1.00	0.86	0.70	0.68	1.05	0.00	0.77	0.73
time (sec)	N/A	0.056	0.077	0.161	0.314	1.168	0.000	0.121	0.066

Problem 858	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	B	A	A	B	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	88	19	16	145	15	0	0	15
normalized size	1	4.63	1.00	0.84	7.63	0.79	0.00	0.00	0.79
time (sec)	N/A	2.243	0.063	0.381	0.467	0.875	0.000	0.000	3.786

Problem 859	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	F	A	C	B	B	F	F	F
verified	N/A	N/A	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	44	0	68	917	518	96	0	0	-1
normalized size	1	0.00	1.55	20.84	11.77	2.18	0.00	0.00	-0.02
time (sec)	N/A	2.569	0.395	0.767	0.764	0.887	0.000	0.000	0.000

Problem 860	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	B	A	C	B	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	72	17	12372	329	3	0	42	-1
normalized size	1	3.79	0.89	651.16	17.32	0.16	0.00	2.21	-0.05
time (sec)	N/A	1.707	0.013	0.417	0.495	1.389	0.000	0.168	0.000

Problem 861	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	F	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	37	10	0	21	0	55	14
normalized size	1	1.00	2.85	0.77	0.00	1.62	0.00	4.23	1.08
time (sec)	N/A	0.146	0.042	0.144	0.000	0.893	0.000	0.323	0.292

Problem 862	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	14	11	10	25	29	68	29
normalized size	1	1.00	1.00	0.79	0.71	1.79	2.07	4.86	2.07
time (sec)	N/A	0.044	0.052	0.063	0.324	1.256	0.679	0.160	3.231

Problem 863	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	30	34	21	35	0	128	24
normalized size	1	1.00	1.20	1.36	0.84	1.40	0.00	5.12	0.96
time (sec)	N/A	0.085	0.191	0.168	0.333	0.868	0.000	0.169	3.334

Problem 864	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	18	38	21	44	0	128	24
normalized size	1	1.00	0.72	1.52	0.84	1.76	0.00	5.12	0.96
time (sec)	N/A	0.081	0.038	0.181	0.321	0.406	0.000	0.143	3.418

Problem 865	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	17	0	0	0	0	0	77
normalized size	1	1.00	0.85	0.00	0.00	0.00	0.00	0.00	3.85
time (sec)	N/A	0.151	0.443	0.501	0.000	0.000	0.000	0.000	3.460

Problem 866	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	76	84	40	54	136	34	0	97	-1
normalized size	1	1.11	0.53	0.71	1.79	0.45	0.00	1.28	-0.01
time (sec)	N/A	0.162	0.089	0.375	0.713	1.922	0.000	0.153	0.000

Problem 867	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	55	65	86	41	0	44	-1
normalized size	1	1.00	0.68	0.80	1.06	0.51	0.00	0.54	-0.01
time (sec)	N/A	0.159	0.061	0.270	0.930	0.934	0.000	0.151	0.000

Problem 868	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	76	76	69	98	79	124	0	0	-1
normalized size	1	1.00	0.91	1.29	1.04	1.63	0.00	0.00	-0.01
time (sec)	N/A	0.535	0.058	0.228	0.847	1.271	0.000	0.000	0.000

Problem 869	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	128	128	99	132	107	227	0	0	-1
normalized size	1	1.00	0.77	1.03	0.84	1.77	0.00	0.00	-0.01
time (sec)	N/A	0.592	0.066	0.205	0.677	1.622	0.000	0.000	0.000

Problem 870	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	186	186	147	172	131	327	0	0	-1
normalized size	1	1.00	0.79	0.92	0.70	1.76	0.00	0.00	-0.01
time (sec)	N/A	0.570	0.096	0.188	0.822	1.309	0.000	0.000	0.000

Problem 871	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	50	147	83	138	0	0	-1
normalized size	1	1.00	0.62	1.81	1.02	1.70	0.00	0.00	-0.01
time (sec)	N/A	0.487	0.040	0.204	1.922	2.100	0.000	0.000	0.000

Problem 872	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	109	109	75	183	113	248	0	0	-1
normalized size	1	1.00	0.69	1.68	1.04	2.28	0.00	0.00	-0.01
time (sec)	N/A	0.574	0.060	0.185	1.098	2.498	0.000	0.000	0.000

Problem 873	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	143	143	87	221	137	356	0	0	-1
normalized size	1	1.00	0.61	1.55	0.96	2.49	0.00	0.00	-0.01
time (sec)	N/A	0.610	0.068	0.190	0.554	2.051	0.000	0.000	0.000

Problem 874	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	105	105	108	86	299	140	0	0	-1
normalized size	1	1.00	1.03	0.82	2.85	1.33	0.00	0.00	-0.01
time (sec)	N/A	0.343	0.077	0.253	0.704	0.853	0.000	0.000	0.000

Problem 875	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	225	225	174	200	0	337	0	0	-1
normalized size	1	1.00	0.77	0.89	0.00	1.50	0.00	0.00	-0.00
time (sec)	N/A	0.531	0.122	0.302	0.000	0.808	0.000	0.000	0.000

Problem 876	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	341	341	290	250	568	539	0	0	-1
normalized size	1	1.00	0.85	0.73	1.67	1.58	0.00	0.00	-0.00
time (sec)	N/A	0.629	0.412	0.418	0.476	1.147	0.000	0.000	0.000

Problem 877	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	142	142	85	165	432	270	0	0	-1
normalized size	1	1.00	0.60	1.16	3.04	1.90	0.00	0.00	-0.01
time (sec)	N/A	0.399	0.233	0.195	0.504	0.760	0.000	0.000	0.000

Problem 878	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	B	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	220	220	138	254	653	550	0	0	-1
normalized size	1	1.00	0.63	1.15	2.97	2.50	0.00	0.00	-0.00
time (sec)	N/A	0.537	0.634	0.231	0.521	2.048	0.000	0.000	0.000

Problem 879	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	356	356	191	324	870	736	0	0	-1
normalized size	1	1.00	0.54	0.91	2.44	2.07	0.00	0.00	-0.00
time (sec)	N/A	0.637	1.069	0.213	1.397	1.079	0.000	0.000	0.000

Problem 880	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	25	20	19	17	114	13	14
normalized size	1	1.00	1.00	0.80	0.76	0.68	4.56	0.52	0.56
time (sec)	N/A	0.031	0.010	0.129	0.319	0.914	10.546	0.122	2.936

Problem 881	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	30	23	22	25	116	22	22
normalized size	1	1.00	1.00	0.77	0.73	0.83	3.87	0.73	0.73
time (sec)	N/A	0.033	0.010	0.141	0.308	0.868	10.539	0.147	3.037

Problem 882	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	30	23	22	25	114	22	22
normalized size	1	1.00	1.00	0.77	0.73	0.83	3.80	0.73	0.73
time (sec)	N/A	0.033	0.008	0.093	0.316	0.711	10.409	0.132	3.008

Problem 883	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	25	20	19	19	112	19	19
normalized size	1	1.00	1.00	0.80	0.76	0.76	4.48	0.76	0.76
time (sec)	N/A	0.031	0.008	0.085	0.312	0.880	10.452	0.131	3.178

Problem 884	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	8	8	8	7	6	6	7	6	6
normalized size	1	1.00	1.00	0.88	0.75	0.75	0.88	0.75	0.75
time (sec)	N/A	0.006	0.006	0.004	0.313	0.830	0.155	0.135	0.046

Problem 885	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	A	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	11	11	25	97	9	34	46	19	20
normalized size	1	1.00	2.27	8.82	0.82	3.09	4.18	1.73	1.82
time (sec)	N/A	0.021	0.077	0.082	0.327	0.635	1.664	0.127	3.190

Problem 886	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	11	11	18	12	133	15	0	52	16
normalized size	1	1.00	1.64	1.09	12.09	1.36	0.00	4.73	1.45
time (sec)	N/A	0.019	0.011	0.044	0.320	0.806	0.000	0.133	3.089

Problem 887	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	12	9	18	13	7	8	8
normalized size	1	1.00	1.00	0.75	1.50	1.08	0.58	0.67	0.67
time (sec)	N/A	0.047	0.016	0.124	0.314	1.093	1.411	0.153	2.928

Problem 888	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	17	17	27	26	33	29	24	19
normalized size	1	1.42	1.42	2.25	2.17	2.75	2.42	2.00	1.58
time (sec)	N/A	0.039	0.014	0.076	0.306	0.930	1.492	0.143	0.087

Problem 889	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	19	18	20	20	19	20	19
normalized size	1	1.00	1.00	0.95	1.05	1.05	1.00	1.05	1.00
time (sec)	N/A	0.040	0.013	0.049	0.307	1.067	24.474	0.139	3.672

Problem 890	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	52	50	49	65	51	51	69
normalized size	1	1.00	1.18	1.14	1.11	1.48	1.16	1.16	1.57
time (sec)	N/A	0.038	0.044	0.108	0.311	0.920	0.112	0.299	3.112

Problem 891	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	35	60	33	32	34	33	30
normalized size	1	1.00	0.95	1.62	0.89	0.86	0.92	0.89	0.81
time (sec)	N/A	0.033	0.029	0.111	0.307	1.847	0.086	0.757	3.099

Problem 892	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	68	58	57	73	61	59	85
normalized size	1	1.00	1.26	1.07	1.06	1.35	1.13	1.09	1.57
time (sec)	N/A	0.041	0.096	0.121	0.308	0.933	0.109	0.423	7.214

Problem 893	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	37	32	31	34	0	33	33
normalized size	1	1.00	1.00	0.86	0.84	0.92	0.00	0.89	0.89
time (sec)	N/A	0.039	0.027	0.096	0.305	0.410	0.000	0.145	2.944

Problem 894	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	42	46	46	49	49	42	69
normalized size	1	1.00	1.24	1.35	1.35	1.44	1.44	1.24	2.03
time (sec)	N/A	0.025	0.009	0.056	0.310	0.971	0.147	0.147	3.038

Problem 895	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	43	68	41	52	42	41	42
normalized size	1	1.00	1.00	1.58	0.95	1.21	0.98	0.95	0.98
time (sec)	N/A	0.036	0.030	0.169	0.306	0.964	0.089	0.160	3.056

Problem 896	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	87	87	87	72	73	92	0	73	74
normalized size	1	1.00	1.00	0.83	0.84	1.06	0.00	0.84	0.85
time (sec)	N/A	0.130	0.062	0.156	0.308	1.047	0.000	0.489	2.970

Problem 897	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	42	42	37	44	79	0	52	71
normalized size	1	1.00	1.00	0.88	1.05	1.88	0.00	1.24	1.69
time (sec)	N/A	0.118	0.041	0.155	0.304	0.898	0.000	0.144	3.161

Problem 898	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	63	56	57	84	0	57	57
normalized size	1	1.00	1.00	0.89	0.90	1.33	0.00	0.90	0.90
time (sec)	N/A	0.124	0.034	0.151	0.311	2.934	0.000	0.165	2.967

Problem 899	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	52	49	52	103	0	60	84
normalized size	1	1.00	0.87	0.82	0.87	1.72	0.00	1.00	1.40
time (sec)	N/A	0.126	0.127	0.158	0.318	0.837	0.000	0.249	4.674

Problem 900	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	59	38	41	42	44	0	42
normalized size	1	1.00	1.26	0.81	0.87	0.89	0.94	0.00	0.89
time (sec)	N/A	0.102	0.049	0.137	0.326	0.626	5.196	0.000	5.378

Problem 901	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	71	38	37	41	44	39	37
normalized size	1	1.00	1.65	0.88	0.86	0.95	1.02	0.91	0.86
time (sec)	N/A	0.116	0.022	0.062	0.321	1.946	2.946	6.001	0.077

Problem 902	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	42	42	69	44	79	41	52	71
normalized size	1	1.00	1.00	1.64	1.05	1.88	0.98	1.24	1.69
time (sec)	N/A	0.041	0.029	0.109	0.335	1.956	0.105	0.147	3.051

Problem 903	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	63	55	91	82	0	72	41
normalized size	1	1.00	0.85	0.74	1.23	1.11	0.00	0.97	0.55
time (sec)	N/A	0.246	0.083	0.072	1.075	0.834	0.000	0.228	3.137

Problem 904	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	47	32	176	39	0	1179	34
normalized size	1	1.00	3.36	2.29	12.57	2.79	0.00	84.21	2.43
time (sec)	N/A	0.011	0.040	0.039	1.316	0.673	0.000	0.522	3.173

Problem 905	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	14	13	12	10	12	12	10
normalized size	1	1.00	1.00	0.93	0.86	0.71	0.86	0.86	0.71
time (sec)	N/A	0.038	0.015	0.079	0.331	0.765	1.428	0.137	0.166

Problem 906	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	11	11	11	10	0	24	0	9	18
normalized size	1	1.00	1.00	0.91	0.00	2.18	0.00	0.82	1.64
time (sec)	N/A	0.054	0.016	0.113	0.000	1.313	0.000	0.148	3.142

Problem 907	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	C	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	32	35	19	32	32	34	33
normalized size	1	1.00	0.94	1.03	0.56	0.94	0.94	1.00	0.97
time (sec)	N/A	0.049	0.028	0.059	0.367	0.870	3.681	0.144	2.998

Problem 908	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	25	24	17	17	24	17	18
normalized size	1	1.00	1.19	1.14	0.81	0.81	1.14	0.81	0.86
time (sec)	N/A	0.030	0.013	0.075	0.559	0.817	1.780	0.145	0.069

Problem 909	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	99	99	55	97	51	66	189	58	69
normalized size	1	1.00	0.56	0.98	0.52	0.67	1.91	0.59	0.70
time (sec)	N/A	0.060	0.229	0.042	0.398	1.090	1.298	0.127	3.063

Problem 910	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	40	51	59	51	0	70	64
normalized size	1	1.00	1.08	1.38	1.59	1.38	0.00	1.89	1.73
time (sec)	N/A	0.090	0.059	0.108	0.583	1.409	0.000	0.205	3.128

Problem 911	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	49	46	41	73	90	41	45
normalized size	1	1.00	0.86	0.81	0.72	1.28	1.58	0.72	0.79
time (sec)	N/A	0.073	0.095	0.140	0.325	1.130	6.019	0.120	3.048

Problem 912	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	B	C	C	F	B	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	243	68	134	0	85	0	0	51
normalized size	1	4.26	1.19	2.35	0.00	1.49	0.00	0.00	0.89
time (sec)	N/A	0.211	0.058	0.256	0.000	0.983	0.000	0.000	4.619

Problem 913	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	5	5	5	8	7	17	7	10	10
normalized size	1	1.00	1.00	1.60	1.40	3.40	1.40	2.00	2.00
time (sec)	N/A	0.023	0.004	0.063	0.322	0.847	2.184	0.126	2.966

Problem 914	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	F	A	C	A	A	F	F(-1)	B
verified	N/A	N/A	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	11	0	11	30	11	11	0	0	11
normalized size	1	0.00	1.00	2.73	1.00	1.00	0.00	0.00	1.00
time (sec)	N/A	0.276	0.308	0.194	0.744	0.744	0.000	0.000	3.158

Problem 915	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	F	A	A	B	A	F	B	B
verified	N/A	N/A	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	0	24	22	74	27	0	93	38
normalized size	1	0.00	0.89	0.81	2.74	1.00	0.00	3.44	1.41
time (sec)	N/A	0.063	0.167	0.360	0.642	1.585	0.000	3.119	0.482

Problem 916	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	50	62	66	50	100	51	65
normalized size	1	1.00	0.77	0.95	1.02	0.77	1.54	0.78	1.00
time (sec)	N/A	0.068	0.090	0.026	0.332	1.651	1.218	1.979	3.029

Problem 917	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	8	8	8	11	10	14	0	0	10
normalized size	1	1.00	1.00	1.38	1.25	1.75	0.00	0.00	1.25
time (sec)	N/A	0.023	0.015	0.085	0.313	0.916	0.000	0.000	0.145

Problem 918	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	B	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	32	0	125	41	0	0	45
normalized size	1	1.00	1.00	0.00	3.91	1.28	0.00	0.00	1.41
time (sec)	N/A	0.040	0.079	0.368	0.444	0.954	0.000	0.000	3.430

Problem 919	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	3	3	3	4	45	3	0	3	26
normalized size	1	1.00	1.00	1.33	15.00	1.00	0.00	1.00	8.67
time (sec)	N/A	0.030	0.018	0.152	0.334	1.419	0.000	0.118	3.190

Problem 920	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	32	36	35	27	39	27	34
normalized size	1	1.00	0.91	1.03	1.00	0.77	1.11	0.77	0.97
time (sec)	N/A	0.065	0.042	0.032	0.323	0.588	0.325	0.124	0.061

Problem 921	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	C	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	8	8	8	7	17	6	5	6	-1
normalized size	1	1.00	1.00	0.88	2.12	0.75	0.62	0.75	-0.12
time (sec)	N/A	0.007	0.002	0.026	0.361	0.731	0.638	0.131	0.000

Problem 922	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	29	38	0	43	0	29	-1
normalized size	1	1.00	0.78	1.03	0.00	1.16	0.00	0.78	-0.03
time (sec)	N/A	0.173	0.071	0.034	0.000	0.851	0.000	0.126	0.000

Problem 923	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	14	11	10	10	10	10	41
normalized size	1	1.00	1.00	0.79	0.71	0.71	0.71	0.71	2.93
time (sec)	N/A	0.013	0.006	0.019	0.325	0.872	84.149	0.163	3.143

Problem 924	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	10	10	10	15	8	20	7	8	19
normalized size	1	1.00	1.00	1.50	0.80	2.00	0.70	0.80	1.90
time (sec)	N/A	0.037	0.003	0.098	0.321	0.426	1.037	0.139	3.094

Problem 925	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	17	16	15	15	27	15	56
normalized size	1	1.00	1.00	0.94	0.88	0.88	1.59	0.88	3.29
time (sec)	N/A	0.024	0.022	0.049	0.343	1.700	20.860	0.179	3.261

Problem 926	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	129	129	120	403	126	85	241	126	147
normalized size	1	1.00	0.93	3.12	0.98	0.66	1.87	0.98	1.14
time (sec)	N/A	0.144	0.565	0.950	0.339	1.091	74.974	0.441	3.455

Problem 927	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	C	B	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	110	110	352	160	3830	115	0	1455	730
normalized size	1	1.00	3.20	1.45	34.82	1.05	0.00	13.23	6.64
time (sec)	N/A	0.112	0.871	0.309	0.859	0.960	0.000	1.641	13.421

Problem 928	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	6	6	6	7	36	12	0	20	14
normalized size	1	1.00	1.00	1.17	6.00	2.00	0.00	3.33	2.33
time (sec)	N/A	0.020	0.018	0.035	0.381	1.042	0.000	0.127	3.028

Problem 929	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	4	4	4	5	4	4	3	4	4
normalized size	1	1.00	1.00	1.25	1.00	1.00	0.75	1.00	1.00
time (sec)	N/A	0.010	0.002	0.001	0.319	0.916	0.267	0.132	2.952

Problem 930	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	30	24	98	39	0	943	23
normalized size	1	1.00	1.11	0.89	3.63	1.44	0.00	34.93	0.85
time (sec)	N/A	0.023	0.015	0.041	0.451	0.827	0.000	0.374	0.095

Problem 931	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	F	A	A	F	F(-2)	F	F	B
verified	N/A	N/A	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	26	0	26	28	0	0	0	0	22
normalized size	1	0.00	1.00	1.08	0.00	0.00	0.00	0.00	0.85
time (sec)	N/A	0.811	0.448	1.042	0.000	0.000	0.000	0.000	3.475

Problem 932	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	F(-2)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	29	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.111	7.568	1.096	0.000	0.000	0.000	0.000	0.000

Problem 933	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	F	A	B	B	A	A	B	B
verified	N/A	N/A	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	9	0	9	57	82	10	10	83	10
normalized size	1	0.00	1.00	6.33	9.11	1.11	1.11	9.22	1.11
time (sec)	N/A	0.381	0.117	0.188	0.563	0.882	0.292	0.150	2.955

Problem 934	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	77	76	60	57	60	150	62	111
normalized size	1	1.00	0.99	0.78	0.74	0.78	1.95	0.81	1.44
time (sec)	N/A	0.126	0.152	0.051	0.493	1.131	0.970	0.130	6.559

Problem 935	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	161	161	134	125	131	123	326	143	210
normalized size	1	1.00	0.83	0.78	0.81	0.76	2.02	0.89	1.30
time (sec)	N/A	0.270	0.210	0.076	0.326	0.912	6.008	0.192	6.744

Problem 936	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	105	84	79	72	201	70	73
normalized size	1	1.00	1.18	0.94	0.89	0.81	2.26	0.79	0.82
time (sec)	N/A	0.109	0.151	0.076	0.460	0.879	1.034	0.153	3.183

Problem 937	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	288	288	167	213	218	162	541	186	456
normalized size	1	1.00	0.58	0.74	0.76	0.56	1.88	0.65	1.58
time (sec)	N/A	0.400	0.449	0.086	0.404	1.962	6.813	0.178	4.517

Problem 938	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	55	136	0	0	0	0	51
normalized size	1	1.00	0.90	2.23	0.00	0.00	0.00	0.00	0.84
time (sec)	N/A	0.289	0.179	0.338	0.000	1.780	0.000	0.000	3.247

Problem 939	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-1)	F	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	148	148	137	266	0	0	0	0	129
normalized size	1	1.00	0.93	1.80	0.00	0.00	0.00	0.00	0.87
time (sec)	N/A	0.239	0.281	0.325	0.000	1.460	0.000	0.000	6.684

Problem 940	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	43	86	50	30	107	31	35
normalized size	1	1.00	1.26	2.53	1.47	0.88	3.15	0.91	1.03
time (sec)	N/A	0.097	0.100	0.437	0.807	0.630	6.708	0.616	3.456

Problem 941	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	43	86	62	30	107	31	35
normalized size	1	1.00	1.19	2.39	1.72	0.83	2.97	0.86	0.97
time (sec)	N/A	0.097	0.089	0.393	0.492	0.932	6.665	0.834	3.351

Problem 942	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	120	120	73	184	0	150	0	128	226
normalized size	1	1.00	0.61	1.53	0.00	1.25	0.00	1.07	1.88
time (sec)	N/A	0.701	0.570	0.736	0.000	0.894	0.000	0.529	4.216

Problem 943	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	25	108	58	74	122	48	23
normalized size	1	1.00	0.35	1.50	0.81	1.03	1.69	0.67	0.32
time (sec)	N/A	0.151	0.032	0.385	0.457	0.968	5.594	0.261	3.166

Problem 944	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	42	56	154	42	76	52	105
normalized size	1	1.00	0.76	1.02	2.80	0.76	1.38	0.95	1.91
time (sec)	N/A	0.409	0.200	0.648	0.434	0.895	1.023	0.287	3.305

Problem 945	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	33	17	22	16	32	14	14
normalized size	1	1.00	2.06	1.06	1.38	1.00	2.00	0.88	0.88
time (sec)	N/A	0.054	0.012	0.230	0.331	0.546	0.255	0.176	3.034

Problem 946	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	18	19	18	22	31	29	50
normalized size	1	1.00	1.00	1.06	1.00	1.22	1.72	1.61	2.78
time (sec)	N/A	0.028	0.044	0.246	0.312	1.165	0.391	0.175	3.136

Problem 947	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	19	32	70	22	0	29	50
normalized size	1	1.00	1.00	1.68	3.68	1.16	0.00	1.53	2.63
time (sec)	N/A	0.314	0.059	0.746	0.414	1.348	0.000	0.294	3.371

Problem 948	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	33	18	23	17	0	14	14
normalized size	1	1.00	1.94	1.06	1.35	1.00	0.00	0.82	0.82
time (sec)	N/A	0.175	0.015	0.342	0.311	1.407	0.000	0.243	3.054

Problem 949	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	42	56	154	42	0	52	106
normalized size	1	1.00	0.78	1.04	2.85	0.78	0.00	0.96	1.96
time (sec)	N/A	0.532	0.235	0.827	0.438	0.935	0.000	0.390	3.233

Problem 950	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	26	108	58	74	0	48	23
normalized size	1	1.00	0.36	1.50	0.81	1.03	0.00	0.67	0.32
time (sec)	N/A	1.396	0.024	0.511	0.428	1.155	0.000	0.386	3.089

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is given. The larger this ratio is, the harder the integral was to solve. In this test, problem number [341] had the largest ratio of [1.286]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	2	2	1.00	14	0.143
2	A	3	3	1.00	27	0.111
3	A	2	2	1.00	12	0.167

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
4	A	2	2	1.00	14	0.143
5	A	2	1	1.00	21	0.048
6	A	2	2	1.00	23	0.087
7	A	2	2	1.00	21	0.095
8	A	3	3	1.00	14	0.214
9	A	4	4	1.00	25	0.160
10	A	2	2	1.00	14	0.143
11	A	2	2	1.00	14	0.143
12	A	2	1	1.00	23	0.043
13	A	2	2	1.00	23	0.087
14	A	2	2	1.00	23	0.087
15	A	2	2	1.00	12	0.167
16	A	3	3	1.00	25	0.120
17	A	2	2	1.00	14	0.143
18	A	2	2	1.00	12	0.167
19	A	2	1	1.00	21	0.048
20	A	2	2	1.00	21	0.095
21	A	2	2	1.00	23	0.087
22	A	3	3	1.00	14	0.214
23	A	4	4	1.00	25	0.160
24	A	2	2	1.00	14	0.143
25	A	2	2	1.00	14	0.143
26	A	2	1	1.00	21	0.048
27	A	2	2	1.00	21	0.095
28	A	2	2	1.00	23	0.087
29	A	6	5	1.00	6	0.833

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
30	A	9	6	1.00	6	1.000
31	A	8	4	1.00	16	0.250
32	A	8	4	1.00	19	0.210
33	A	3	3	1.00	16	0.188
34	A	3	3	1.00	27	0.111
35	A	2	1	1.00	11	0.091
36	A	5	5	1.00	14	0.357
37	A	6	6	1.00	16	0.375
38	A	9	5	1.00	16	0.312
39	A	5	3	1.00	36	0.083
40	A	4	3	1.00	36	0.083
41	A	2	2	1.00	34	0.059
42	A	0	0	0.00	0	0.000
43	A	0	0	0.00	0	0.000
44	A	6	5	1.00	6	0.833
45	A	9	6	1.00	6	1.000
46	A	8	4	1.00	16	0.250
47	A	8	4	1.00	19	0.210
48	A	4	4	1.00	21	0.190
49	A	4	4	1.00	27	0.148
50	A	5	4	1.00	37	0.108
51	A	5	5	1.00	14	0.357
52	A	6	6	1.00	16	0.375
53	A	5	3	1.00	36	0.083
54	A	4	3	1.00	36	0.083
55	A	2	2	1.00	34	0.059

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
56	A	0	0	0.00	0	0.000
57	A	0	0	0.00	0	0.000
58	A	2	2	1.00	12	0.167
59	A	3	3	1.00	14	0.214
60	A	6	6	1.00	12	0.500
61	A	2	1	1.00	33	0.030
62	A	3	3	1.00	14	0.214
63	A	2	2	1.00	25	0.080
64	A	4	3	1.00	16	0.188
65	A	4	3	1.00	15	0.200
66	A	1	1	1.00	7	0.143
67	A	1	1	1.00	7	0.143
68	A	1	1	1.00	7	0.143
69	A	4	2	1.00	7	0.286
70	A	1	1	1.00	7	0.143
71	A	1	1	1.00	7	0.143
72	A	1	1	1.00	7	0.143
73	A	4	2	1.00	7	0.286
74	A	4	3	1.00	7	0.429
75	A	9	4	1.00	7	0.571
76	A	5	3	1.00	7	0.429
77	A	10	4	1.00	7	0.571
78	A	10	5	1.00	7	0.714
79	A	6	3	1.00	7	0.429
80	A	3	2	1.00	7	0.286
81	A	3	2	1.00	7	0.286

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
82	A	6	3	1.00	7	0.429
83	A	6	3	1.00	7	0.429
84	A	7	3	1.00	7	0.429
85	A	2	2	1.00	7	0.286
86	A	5	5	1.00	7	0.714
87	A	4	3	1.00	7	0.429
88	A	7	6	1.00	7	0.857
89	A	7	4	1.00	7	0.571
90	A	2	2	1.00	7	0.286
91	A	2	1	1.00	7	0.143
92	A	4	2	1.00	7	0.286
93	A	4	2	1.00	7	0.286
94	A	7	3	1.00	7	0.429
95	A	3	2	1.00	7	0.286
96	A	2	2	1.00	9	0.222
97	A	1	1	1.00	7	0.143
98	A	1	1	1.00	7	0.143
99	A	1	1	1.00	7	0.143
100	A	4	2	1.00	7	0.286
101	A	1	1	1.00	7	0.143
102	A	1	1	1.00	7	0.143
103	A	1	1	1.00	7	0.143
104	A	4	2	1.00	7	0.286
105	A	4	3	1.00	7	0.429
106	A	3	2	1.00	7	0.286
107	A	6	4	1.00	7	0.571

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
108	A	6	3	1.00	7	0.429
109	A	10	5	1.00	7	0.714
110	A	4	3	1.00	7	0.429
111	A	9	4	1.00	7	0.571
112	A	6	3	1.00	7	0.429
113	A	10	4	1.00	7	0.571
114	A	7	3	1.00	7	0.429
115	A	6	3	1.00	7	0.429
116	A	2	2	1.00	7	0.286
117	A	2	1	1.00	7	0.143
118	A	4	3	1.00	7	0.429
119	A	4	2	1.00	7	0.286
120	A	7	4	1.00	7	0.571
121	A	3	3	1.00	7	0.429
122	A	3	3	1.00	9	0.333
123	A	2	2	1.00	7	0.286
124	A	5	5	1.00	7	0.714
125	A	4	2	1.00	7	0.286
126	A	7	6	1.00	7	0.857
127	A	7	3	1.00	7	0.429
128	A	6	2	1.00	9	0.222
129	A	6	2	1.00	11	0.182
130	A	5	2	1.00	11	0.182
131	A	5	2	1.00	9	0.222
132	A	6	2	1.00	9	0.222
133	A	6	2	1.00	11	0.182

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
134	A	7	2	1.00	13	0.154
135	A	3	2	1.00	13	0.154
136	A	3	2	1.00	14	0.143
137	A	3	2	1.00	13	0.154
138	A	3	2	1.00	14	0.143
139	A	4	3	1.00	13	0.231
140	A	4	3	1.00	14	0.214
141	A	4	3	1.00	13	0.231
142	A	4	3	1.00	14	0.214
143	A	3	2	1.00	13	0.154
144	A	3	2	1.00	14	0.143
145	A	3	2	1.00	13	0.154
146	A	3	2	1.00	14	0.143
147	A	2	2	1.00	9	0.222
148	A	3	3	1.00	9	0.333
149	A	4	4	1.00	9	0.444
150	A	2	2	1.00	9	0.222
151	A	3	3	1.00	9	0.333
152	A	4	4	1.00	9	0.444
153	A	4	4	1.00	12	0.333
154	A	6	6	1.00	12	0.500
155	A	3	3	1.00	12	0.250
156	A	5	5	1.00	12	0.417
157	A	3	3	1.00	14	0.214
158	A	3	3	1.00	14	0.214
159	A	2	2	1.00	23	0.087

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
160	A	9	6	1.00	24	0.250
161	A	11	7	1.00	26	0.269
162	A	2	1	1.00	33	0.030
163	A	9	6	1.00	34	0.176
164	A	11	7	1.00	36	0.194
165	A	5	3	1.00	33	0.091
166	A	4	3	1.00	33	0.091
167	A	3	3	1.00	31	0.097
168	A	4	4	1.00	33	0.121
169	A	5	5	1.00	33	0.152
170	A	6	5	1.00	33	0.152
171	A	11	9	1.00	33	0.273
172	A	8	7	1.00	33	0.212
173	A	3	3	1.00	31	0.097
174	A	11	7	1.00	33	0.212
175	A	13	8	1.00	33	0.242
176	A	15	8	1.00	33	0.242
177	A	20	11	1.00	37	0.297
178	A	17	10	1.00	37	0.270
179	A	14	9	1.00	35	0.257
180	A	0	0	0.00	0	0.000
181	A	51	17	1.00	33	0.515
182	A	34	14	1.00	33	0.424
183	A	26	12	1.00	31	0.387
184	A	15	8	1.00	22	0.364
185	A	5	5	1.00	13	0.385

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
186	A	6	6	1.00	15	0.400
187	A	7	6	1.00	15	0.400
188	A	8	6	1.00	15	0.400
189	A	4	4	1.00	15	0.267
190	A	5	4	1.00	15	0.267
191	A	6	4	1.00	15	0.267
192	A	7	4	1.00	15	0.267
193	A	6	5	1.00	17	0.294
194	A	5	5	1.00	17	0.294
195	A	4	4	1.00	17	0.235
196	A	6	5	1.00	17	0.294
197	A	7	6	1.00	17	0.353
198	A	8	6	1.00	17	0.353
199	A	3	3	1.00	16	0.188
200	A	3	3	1.00	16	0.188
201	A	4	4	1.00	17	0.235
202	A	3	3	1.00	17	0.176
203	A	4	4	1.00	17	0.235
204	A	3	3	1.00	17	0.176
205	A	4	3	1.00	22	0.136
206	A	8	5	1.00	17	0.294
207	A	5	3	1.00	19	0.158
208	A	3	2	1.00	20	0.100
209	A	4	3	1.00	19	0.158
210	A	4	3	1.00	22	0.136
211	A	10	6	1.00	17	0.353

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
212	A	4	2	1.00	19	0.105
213	A	3	3	1.00	20	0.150
214	A	4	3	1.00	19	0.158
215	A	6	6	1.00	17	0.353
216	A	7	7	1.00	17	0.412
217	A	2	2	1.00	19	0.105
218	A	2	2	1.00	19	0.105
219	A	3	2	1.00	19	0.105
220	A	4	2	1.00	19	0.105
221	A	3	2	1.00	19	0.105
222	A	3	2	1.00	19	0.105
223	A	2	1	1.00	19	0.053
224	A	2	2	1.00	19	0.105
225	A	3	2	1.00	17	0.118
226	A	2	2	1.00	19	0.105
227	A	1	1	1.00	19	0.053
228	A	3	3	1.00	19	0.158
229	A	2	2	1.00	19	0.105
230	A	4	3	1.00	19	0.158
231	A	3	2	1.00	19	0.105
232	A	4	3	1.00	21	0.143
233	A	3	3	1.00	21	0.143
234	A	3	3	1.00	21	0.143
235	A	2	2	1.00	21	0.095
236	A	2	2	1.00	21	0.095
237	A	3	3	1.00	21	0.143

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
238	A	3	3	1.00	21	0.143
239	A	4	3	1.00	21	0.143
240	A	4	3	1.00	21	0.143
241	A	3	3	1.00	21	0.143
242	A	3	3	1.00	21	0.143
243	A	2	2	1.00	21	0.095
244	A	2	2	1.00	21	0.095
245	A	3	3	1.00	21	0.143
246	A	3	3	1.00	21	0.143
247	A	4	3	1.00	21	0.143
248	A	1	1	1.00	22	0.045
249	A	1	1	1.00	22	0.045
250	A	1	1	1.00	22	0.045
251	A	1	1	1.00	22	0.045
252	A	3	2	1.00	20	0.100
253	A	1	1	1.00	22	0.045
254	A	1	1	1.00	22	0.045
255	A	1	1	1.00	22	0.045
256	A	1	1	1.00	22	0.045
257	A	1	1	1.00	24	0.042
258	A	1	1	1.00	24	0.042
259	A	1	1	1.00	24	0.042
260	A	1	1	1.00	24	0.042
261	A	1	1	1.00	24	0.042
262	A	1	1	1.00	24	0.042
263	A	8	7	1.00	11	0.636

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
264	A	4	4	1.00	11	0.364
265	A	7	6	1.00	11	0.546
266	A	4	3	1.00	11	0.273
267	A	3	2	1.00	9	0.222
268	A	3	3	1.00	11	0.273
269	A	6	6	1.00	11	0.546
270	A	4	3	1.00	11	0.273
271	A	8	8	1.00	11	0.727
272	A	4	3	1.00	11	0.273
273	A	4	3	1.00	7	0.429
274	A	5	4	1.00	7	0.571
275	A	4	3	1.00	7	0.429
276	A	4	4	1.00	7	0.571
277	A	3	2	0.69	5	0.400
278	A	3	3	1.00	7	0.429
279	A	3	3	1.00	7	0.429
280	A	4	3	1.00	7	0.429
281	A	4	3	1.00	7	0.429
282	A	4	3	1.00	7	0.429
283	A	8	7	1.00	11	0.636
284	A	4	4	1.00	11	0.364
285	A	7	6	1.00	11	0.546
286	A	4	3	1.00	11	0.273
287	A	3	2	1.00	9	0.222
288	A	3	3	1.00	11	0.273
289	A	5	5	1.00	11	0.454

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
290	A	4	3	1.00	11	0.273
291	A	7	7	1.00	11	0.636
292	A	4	3	1.00	11	0.273
293	A	4	3	1.00	7	0.429
294	A	5	4	1.00	7	0.571
295	A	4	3	1.00	7	0.429
296	A	4	4	1.00	7	0.571
297	A	3	2	1.00	5	0.400
298	A	3	3	1.00	7	0.429
299	A	3	3	1.00	7	0.429
300	A	4	3	1.00	7	0.429
301	A	4	3	1.00	7	0.429
302	A	4	3	1.00	7	0.429
303	A	6	3	1.00	9	0.333
304	A	6	5	1.00	9	0.556
305	A	4	3	1.00	9	0.333
306	A	3	2	1.00	7	0.286
307	A	3	3	1.00	9	0.333
308	A	2	1	1.00	9	0.111
309	A	4	3	1.00	9	0.333
310	A	2	0	1.00	9	0.000
311	A	4	3	1.00	9	0.333
312	A	3	1	1.00	9	0.111
313	A	4	3	1.00	9	0.333
314	A	6	5	1.00	11	0.454
315	A	5	5	1.00	11	0.454

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
316	A	4	4	1.00	11	0.364
317	A	3	3	1.00	11	0.273
318	A	8	8	1.00	11	0.727
319	A	9	9	1.00	11	0.818
320	A	10	10	1.00	11	0.909
321	A	11	10	1.00	11	0.909
322	A	6	3	1.00	9	0.333
323	A	6	5	1.00	9	0.556
324	A	4	3	1.00	9	0.333
325	A	3	2	1.00	7	0.286
326	A	3	3	1.00	9	0.333
327	A	2	1	1.00	9	0.111
328	A	4	3	1.00	9	0.333
329	A	2	0	1.00	9	0.000
330	A	4	3	1.00	9	0.333
331	A	3	1	1.00	9	0.111
332	A	4	3	1.00	9	0.333
333	A	6	5	1.00	11	0.454
334	A	5	5	1.00	11	0.454
335	A	4	4	1.00	11	0.364
336	A	3	3	1.00	11	0.273
337	A	8	8	1.00	11	0.727
338	A	9	9	1.00	11	0.818
339	A	10	10	1.00	11	0.909
340	A	11	10	1.00	11	0.909
341	A	18	9	1.00	7	1.286

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
342	A	4	3	1.00	7	0.429
343	A	9	7	1.00	7	1.000
344	A	3	2	1.00	5	0.400
345	A	6	6	1.00	7	0.857
346	A	11	6	1.00	7	0.857
347	A	5	4	1.00	7	0.571
348	A	18	6	1.00	7	0.857
349	A	3	3	1.00	18	0.167
350	A	3	3	1.00	18	0.167
351	A	4	4	1.00	18	0.222
352	A	3	3	1.00	18	0.167
353	A	3	3	1.00	18	0.167
354	A	4	4	1.00	18	0.222
355	A	6	3	1.00	30	0.100
356	A	5	3	1.00	30	0.100
357	A	4	3	1.00	30	0.100
358	A	3	2	1.00	28	0.071
359	A	1	1	1.00	30	0.033
360	A	2	2	1.00	30	0.067
361	A	3	2	1.00	30	0.067
362	A	4	2	1.00	30	0.067
363	A	5	4	1.00	24	0.167
364	A	4	3	1.00	24	0.125
365	A	3	2	1.00	22	0.091
366	A	2	2	1.00	24	0.083
367	A	4	4	1.00	24	0.167

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
368	A	4	4	1.00	24	0.167
369	A	5	5	1.00	24	0.208
370	A	2	2	1.00	24	0.083
371	A	4	4	1.00	24	0.167
372	A	4	4	1.00	24	0.167
373	A	5	5	1.00	24	0.208
374	A	5	4	1.00	24	0.167
375	A	4	3	1.00	24	0.125
376	A	3	2	1.00	22	0.091
377	A	2	2	1.00	24	0.083
378	A	4	4	1.00	24	0.167
379	A	4	4	1.00	24	0.167
380	A	5	5	1.00	24	0.208
381	A	5	4	1.00	24	0.167
382	A	4	3	1.00	24	0.125
383	A	3	2	1.00	22	0.091
384	A	2	2	1.00	24	0.083
385	A	4	4	1.00	24	0.167
386	A	4	4	1.00	24	0.167
387	A	5	5	1.00	24	0.208
388	A	5	4	1.00	24	0.167
389	A	4	3	1.00	24	0.125
390	A	3	2	1.00	22	0.091
391	A	2	2	1.00	24	0.083
392	A	4	4	1.00	24	0.167
393	A	4	4	1.00	24	0.167

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
394	A	5	5	1.00	24	0.208
395	A	6	4	1.00	20	0.200
396	A	5	4	1.00	20	0.200
397	A	4	3	1.00	20	0.150
398	A	3	2	1.00	18	0.111
399	A	3	3	1.00	20	0.150
400	A	5	5	1.00	20	0.250
401	A	5	5	1.00	20	0.250
402	A	6	6	1.00	20	0.300
403	A	7	7	1.00	22	0.318
404	A	6	6	1.00	22	0.273
405	A	2	2	1.00	22	0.091
406	A	2	2	1.00	22	0.091
407	A	3	3	1.00	22	0.136
408	A	7	7	1.00	22	0.318
409	A	8	7	1.00	22	0.318
410	A	7	7	1.00	22	0.318
411	A	6	6	1.00	22	0.273
412	A	2	2	1.00	22	0.091
413	A	2	2	1.00	22	0.091
414	A	3	3	1.00	22	0.136
415	A	7	7	1.00	22	0.318
416	A	8	7	1.00	22	0.318
417	A	3	2	1.00	22	0.091
418	A	2	2	1.00	22	0.091
419	A	1	1	1.00	22	0.045

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
420	A	3	3	1.00	22	0.136
421	A	4	4	1.00	22	0.182
422	A	5	4	1.00	22	0.182
423	A	4	2	1.00	22	0.091
424	A	3	2	1.00	22	0.091
425	A	2	2	1.00	22	0.091
426	A	1	1	1.00	22	0.045
427	A	3	3	1.00	22	0.136
428	A	4	4	1.00	22	0.182
429	A	5	4	1.00	22	0.182
430	A	4	2	1.00	32	0.062
431	A	3	2	1.00	32	0.062
432	A	2	2	1.00	32	0.062
433	A	1	1	1.00	32	0.031
434	A	3	3	1.00	32	0.094
435	A	4	4	1.00	32	0.125
436	A	5	4	1.00	32	0.125
437	A	3	2	1.00	34	0.059
438	A	2	2	1.00	34	0.059
439	A	1	1	1.00	34	0.029
440	A	3	3	1.00	34	0.088
441	A	4	4	1.00	34	0.118
442	A	5	4	1.00	34	0.118
443	A	4	4	1.00	15	0.267
444	A	3	3	1.36	11	0.273
445	A	5	5	1.00	12	0.417

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
446	A	4	4	1.00	15	0.267
447	A	10	8	1.00	17	0.471
448	A	7	7	1.00	33	0.212
449	A	3	3	1.00	33	0.091
450	A	3	3	1.00	33	0.091
451	A	4	4	1.00	33	0.121
452	A	8	8	1.00	33	0.242
453	A	7	7	1.00	33	0.212
454	A	3	3	1.00	33	0.091
455	A	3	3	1.00	33	0.091
456	A	4	4	1.00	33	0.121
457	A	8	8	1.00	33	0.242
458	A	5	5	1.00	12	0.417
459	A	4	4	1.00	15	0.267
460	A	9	7	1.00	17	0.412
461	A	4	4	1.00	15	0.267
462	A	7	7	1.00	33	0.212
463	A	3	3	1.00	33	0.091
464	A	3	3	1.00	33	0.091
465	A	4	4	1.00	33	0.121
466	A	8	8	1.00	33	0.242
467	A	7	7	1.00	33	0.212
468	A	3	3	1.00	33	0.091
469	A	3	3	1.00	33	0.091
470	A	4	4	1.00	33	0.121
471	A	8	8	1.00	33	0.242

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
472	A	2	2	1.00	11	0.182
473	A	2	2	1.00	11	0.182
474	A	2	2	1.00	11	0.182
475	A	2	1	1.00	13	0.077
476	A	2	1	1.00	13	0.077
477	A	4	3	1.00	13	0.231
478	A	2	1	1.00	15	0.067
479	A	2	1	1.00	15	0.067
480	A	2	1	1.00	19	0.053
481	A	2	1	1.00	23	0.043
482	A	4	3	1.00	20	0.150
483	A	4	3	1.00	20	0.150
484	A	4	2	1.00	11	0.182
485	A	6	4	1.00	11	0.364
486	A	6	4	1.00	11	0.364
487	A	2	2	1.00	13	0.154
488	A	2	2	1.00	13	0.154
489	A	2	2	1.00	13	0.154
490	A	4	2	1.00	11	0.182
491	A	6	4	1.00	11	0.364
492	A	6	4	1.00	11	0.364
493	A	2	2	1.00	13	0.154
494	A	2	2	1.00	13	0.154
495	A	2	2	1.00	13	0.154
496	A	2	1	1.00	16	0.062
497	A	9	6	1.00	18	0.333

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
498	A	11	7	1.00	20	0.350
499	A	5	3	1.00	39	0.077
500	A	2	2	1.00	37	0.054
501	A	3	3	1.00	39	0.077
502	A	9	6	1.00	39	0.154
503	A	7	4	1.00	21	0.190
504	A	4	3	1.00	41	0.073
505	A	2	2	1.00	41	0.049
506	A	5	5	1.00	41	0.122
507	A	8	6	1.00	41	0.146
508	A	3	3	1.00	27	0.111
509	A	5	3	1.00	21	0.143
510	A	7	5	1.00	39	0.128
511	A	3	3	1.00	37	0.081
512	A	4	4	1.00	39	0.103
513	A	6	4	1.00	39	0.103
514	A	6	5	1.00	41	0.122
515	A	3	3	1.00	41	0.073
516	A	3	3	1.00	41	0.073
517	A	5	4	1.00	41	0.098
518	A	8	7	1.00	39	0.180
519	A	5	4	1.00	37	0.108
520	A	6	6	1.00	39	0.154
521	A	8	7	1.00	39	0.180
522	A	7	6	1.00	41	0.146
523	A	5	5	1.00	41	0.122

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
524	A	5	5	1.00	41	0.122
525	A	7	7	1.00	41	0.171
526	A	1	1	1.00	21	0.048
527	A	1	1	1.00	21	0.048
528	A	1	1	1.00	15	0.067
529	A	1	1	1.00	21	0.048
530	A	3	3	1.00	21	0.143
531	A	3	3	1.00	21	0.143
532	A	3	3	1.00	22	0.136
533	A	3	3	1.00	22	0.136
534	A	4	4	1.00	22	0.182
535	A	4	4	1.00	19	0.210
536	A	4	4	1.00	19	0.210
537	A	5	5	1.00	19	0.263
538	A	1	1	1.00	22	0.045
539	A	1	1	1.00	22	0.045
540	A	4	4	1.00	19	0.210
541	A	4	4	1.00	19	0.210
542	A	5	5	1.00	19	0.263
543	A	1	1	1.00	22	0.045
544	A	1	1	1.00	22	0.045
545	A	4	4	1.00	22	0.182
546	A	4	4	1.00	22	0.182
547	A	5	5	1.00	22	0.227
548	A	1	1	0.95	25	0.040
549	A	1	1	0.94	25	0.040

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
550	A	4	4	1.00	23	0.174
551	A	4	4	1.00	23	0.174
552	A	5	5	1.00	23	0.217
553	A	1	1	1.00	26	0.038
554	A	1	1	1.00	26	0.038
555	B	1	1	2.83	30	0.033
556	A	8	6	1.00	27	0.222
557	A	7	6	1.00	27	0.222
558	A	6	6	1.00	27	0.222
559	A	5	5	1.00	27	0.185
560	A	6	6	1.00	27	0.222
561	A	7	6	1.00	27	0.222
562	A	7	7	1.00	31	0.226
563	A	8	8	1.00	31	0.258
564	A	9	8	1.00	31	0.258
565	A	10	8	1.00	31	0.258
566	A	4	4	1.00	18	0.222
567	A	3	3	1.00	18	0.167
568	A	2	2	1.00	18	0.111
569	A	3	2	1.00	16	0.125
570	A	4	4	1.00	18	0.222
571	A	6	6	1.00	18	0.333
572	A	7	7	1.00	18	0.389
573	A	8	8	1.00	20	0.400
574	A	7	7	1.00	20	0.350
575	A	3	3	1.00	20	0.150

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
576	A	3	3	1.00	20	0.150
577	A	5	5	1.00	20	0.250
578	A	8	8	1.00	20	0.400
579	A	13	8	1.00	14	0.571
580	A	11	7	1.00	14	0.500
581	A	9	6	1.00	12	0.500
582	A	0	0	0.00	0	0.000
583	A	0	0	0.00	0	0.000
584	A	0	0	0.00	0	0.000
585	A	15	6	1.00	26	0.231
586	A	11	5	1.00	26	0.192
587	A	6	6	1.00	26	0.231
588	A	4	3	1.00	26	0.115
589	A	1	1	1.00	23	0.043
590	A	1	1	1.00	22	0.045
591	A	3	3	1.00	20	0.150
592	A	7	5	1.00	24	0.208
593	A	9	9	1.00	26	0.346
594	A	15	6	1.00	24	0.250
595	A	11	5	1.00	24	0.208
596	A	6	6	1.00	24	0.250
597	A	4	3	1.00	24	0.125
598	A	1	1	1.00	21	0.048
599	A	1	1	1.00	20	0.050
600	A	3	3	1.00	18	0.167
601	A	7	5	1.00	22	0.227

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
602	A	9	9	1.00	24	0.375
603	A	5	5	1.00	31	0.161
604	A	4	4	1.00	31	0.129
605	A	3	3	1.00	31	0.097
606	A	2	2	1.00	29	0.069
607	A	3	3	1.00	20	0.150
608	A	4	4	1.00	29	0.138
609	A	5	4	1.00	31	0.129
610	A	6	4	1.00	31	0.129
611	A	7	7	1.00	31	0.226
612	A	5	5	1.00	31	0.161
613	A	4	4	1.00	31	0.129
614	A	3	3	1.00	29	0.103
615	A	5	5	1.00	20	0.250
616	A	6	6	1.00	29	0.207
617	A	6	6	1.00	31	0.194
618	A	7	6	1.00	31	0.194
619	A	6	6	1.00	31	0.194
620	A	5	5	1.00	31	0.161
621	A	4	4	1.00	31	0.129
622	A	3	3	1.00	29	0.103
623	A	6	5	1.00	20	0.250
624	A	7	6	1.00	29	0.207
625	A	8	7	1.00	31	0.226
626	A	6	6	1.00	31	0.194
627	A	5	5	1.00	31	0.161

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
628	A	4	4	1.00	31	0.129
629	A	4	4	1.00	29	0.138
630	A	7	6	1.00	20	0.300
631	A	8	7	1.00	29	0.241
632	A	9	7	1.00	31	0.226
633	A	3	2	1.00	13	0.154
634	A	6	4	1.00	21	0.190
635	A	6	4	1.00	28	0.143
636	A	6	4	1.00	30	0.133
637	A	1	1	1.00	43	0.023
638	A	1	1	1.00	43	0.023
639	A	1	1	1.00	43	0.023
640	A	1	1	1.00	41	0.024
641	A	1	1	1.00	43	0.023
642	A	1	1	1.00	43	0.023
643	A	1	1	1.00	43	0.023
644	A	0	0	0.00	0	0.000
645	A	0	0	0.00	0	0.000
646	A	0	0	0.00	0	0.000
647	A	0	0	0.00	0	0.000
648	A	2	2	1.00	11	0.182
649	A	2	2	1.00	11	0.182
650	A	2	2	1.00	13	0.154
651	A	2	2	1.00	6	0.333
652	A	4	4	1.00	11	0.364
653	A	6	3	1.00	13	0.231

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
654	A	4	3	1.00	17	0.176
655	A	2	2	1.00	10	0.200
656	A	3	3	1.00	21	0.143
657	A	3	3	1.00	19	0.158
658	A	3	2	1.00	15	0.133
659	A	2	2	1.00	17	0.118
660	A	2	2	1.00	22	0.091
661	A	2	2	1.00	22	0.091
662	A	2	2	1.00	17	0.118
663	A	2	2	1.00	22	0.091
664	A	2	2	1.00	22	0.091
665	A	2	2	1.00	11	0.182
666	A	2	2	1.00	11	0.182
667	A	2	2	1.00	13	0.154
668	A	2	2	1.00	15	0.133
669	A	2	2	1.00	19	0.105
670	A	4	4	1.00	11	0.364
671	A	3	3	1.00	15	0.200
672	A	2	2	1.00	15	0.133
673	A	3	3	1.00	16	0.188
674	A	2	2	1.00	6	0.333
675	A	3	2	1.00	10	0.200
676	A	2	2	1.00	6	0.333
677	A	4	3	1.00	17	0.176
678	A	3	3	1.00	9	0.333
679	A	4	3	1.00	13	0.231

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
680	A	2	2	1.00	17	0.118
681	A	2	2	1.00	9	0.222
682	A	2	2	1.00	12	0.167
683	A	2	2	1.00	18	0.111
684	A	2	2	1.00	17	0.118
685	A	2	2	1.00	22	0.091
686	A	2	2	1.00	22	0.091
687	A	2	2	1.00	17	0.118
688	A	2	2	1.00	22	0.091
689	A	2	2	1.00	22	0.091
690	A	2	2	1.00	13	0.154
691	A	2	2	1.00	15	0.133
692	A	2	2	1.00	13	0.154
693	A	2	2	1.00	13	0.154
694	A	3	1	1.00	15	0.067
695	A	4	3	1.00	19	0.158
696	A	3	3	1.00	17	0.176
697	A	3	2	1.50	16	0.125
698	A	3	2	1.50	18	0.111
699	A	7	7	1.00	15	0.467
700	A	3	3	1.00	19	0.158
701	A	3	2	1.00	19	0.105
702	A	3	2	1.00	21	0.095
703	A	3	2	1.00	21	0.095
704	A	2	2	1.00	17	0.118
705	A	4	3	1.00	17	0.176

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
706	A	5	5	1.00	19	0.263
707	A	2	2	1.00	11	0.182
708	A	4	2	1.00	17	0.118
709	A	2	2	1.00	17	0.118
710	A	2	2	1.00	17	0.118
711	A	3	3	1.00	15	0.200
712	A	3	3	1.00	17	0.176
713	A	3	3	1.00	17	0.176
714	A	2	2	1.00	9	0.222
715	A	4	3	1.00	15	0.200
716	A	2	2	1.00	13	0.154
717	A	2	2	1.00	13	0.154
718	A	2	2	1.00	11	0.182
719	A	4	2	1.00	15	0.133
720	A	3	2	1.00	19	0.105
721	A	3	2	1.00	21	0.095
722	A	3	2	1.00	21	0.095
723	A	2	2	1.00	11	0.182
724	A	4	4	1.82	13	0.308
725	A	2	2	1.00	13	0.154
726	A	2	2	1.00	15	0.133
727	A	2	2	1.00	14	0.143
728	A	3	3	1.00	15	0.200
729	A	2	1	1.00	15	0.067
730	A	2	2	1.00	9	0.222
731	A	2	2	1.00	9	0.222

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
732	A	2	2	1.00	19	0.105
733	A	3	2	1.00	23	0.087
734	A	2	1	1.00	23	0.043
735	A	4	4	1.67	13	0.308
736	A	2	2	1.00	15	0.133
737	A	2	2	1.00	13	0.154
738	A	3	3	1.00	21	0.143
739	A	2	1	1.00	15	0.067
740	A	3	2	1.00	23	0.087
741	A	4	3	1.00	20	0.150
742	A	4	3	1.00	19	0.158
743	A	4	3	1.00	24	0.125
744	A	4	3	1.00	21	0.143
745	A	4	3	1.00	20	0.150
746	A	4	3	1.00	19	0.158
747	A	4	3	1.00	24	0.125
748	A	4	3	1.00	21	0.143
749	A	1	3	1.00	8	0.375
750	A	1	2	1.00	8	0.250
751	A	3	2	1.00	11	0.182
752	A	2	2	1.00	6	0.333
753	A	4	3	1.00	8	0.375
754	A	2	2	1.00	9	0.222
755	A	2	2	1.00	12	0.167
756	A	3	2	1.00	13	0.154
757	A	2	2	1.00	8	0.250

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
758	A	1	1	1.00	12	0.083
759	B	25	3	10.75	20	0.150
760	A	2	2	1.00	6	0.333
761	A	3	3	1.00	8	0.375
762	A	4	3	1.00	15	0.200
763	A	2	2	1.00	15	0.133
764	A	8	4	1.00	17	0.235
765	A	3	2	1.00	13	0.154
766	A	2	2	1.00	6	0.333
767	A	2	2	1.00	10	0.200
768	A	2	2	1.00	8	0.250
769	A	2	2	1.00	10	0.200
770	A	3	3	1.00	10	0.300
771	A	3	3	1.00	10	0.300
772	A	2	2	1.00	8	0.250
773	A	1	1	1.00	8	0.125
774	A	3	1	1.00	18	0.056
775	A	6	3	1.00	16	0.188
776	A	6	3	1.00	8	0.375
777	A	2	2	1.00	17	0.118
778	A	3	3	1.00	7	0.429
779	A	3	3	1.00	11	0.273
780	A	3	2	1.00	10	0.200
781	A	2	2	1.00	8	0.250
782	A	2	2	1.00	8	0.250
783	A	3	2	1.00	10	0.200

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
784	A	2	2	1.00	10	0.200
785	A	3	3	1.00	9	0.333
786	A	2	2	1.00	8	0.250
787	A	3	3	1.00	8	0.375
788	A	1	1	1.00	8	0.125
789	A	3	3	1.00	8	0.375
790	A	3	3	1.00	8	0.375
791	A	2	2	1.00	8	0.250
792	A	2	2	1.00	13	0.154
793	A	1	1	1.00	11	0.091
794	A	3	3	1.00	9	0.333
795	A	3	2	1.00	7	0.286
796	F	0	0	N/A	0	N/A
797	A	2	2	1.00	6	0.333
798	A	3	2	1.00	11	0.182
799	A	3	2	1.00	8	0.250
800	A	3	2	1.00	7	0.286
801	A	4	3	1.00	9	0.333
802	A	2	2	1.00	9	0.222
803	A	2	2	1.00	7	0.286
804	A	2	2	1.00	13	0.154
805	A	6	4	1.00	10	0.400
806	A	2	2	1.00	21	0.095
807	A	3	3	1.00	15	0.200
808	A	2	2	1.00	12	0.167
809	A	5	5	1.00	16	0.312

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
810	A	4	2	1.00	11	0.182
811	A	8	4	1.00	13	0.308
812	A	1	1	1.00	10	0.100
813	A	3	2	1.00	13	0.154
814	A	4	2	1.00	12	0.167
815	A	3	2	1.00	8	0.250
816	A	5	4	1.00	10	0.400
817	A	6	4	1.00	15	0.267
818	A	4	3	1.00	11	0.273
819	A	3	2	1.00	13	0.154
820	A	2	2	1.00	8	0.250
821	A	2	2	1.00	28	0.071
822	A	1	1	1.00	12	0.083
823	A	2	2	1.00	23	0.087
824	A	3	3	1.00	7	0.429
825	A	2	2	1.00	10	0.200
826	A	2	2	1.00	8	0.250
827	A	4	2	1.29	13	0.154
828	A	3	1	1.00	20	0.050
829	A	3	3	1.00	9	0.333
830	A	5	5	1.00	10	0.500
831	A	5	4	1.00	9	0.444
832	A	4	4	1.00	10	0.400
833	A	5	4	1.00	10	0.400
834	A	4	3	1.00	10	0.300
835	A	6	4	1.00	10	0.400

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
836	A	5	5	1.00	14	0.357
837	A	5	4	1.00	13	0.308
838	A	5	2	1.00	9	0.222
839	A	5	2	1.00	11	0.182
840	A	2	2	1.00	7	0.286
841	A	6	2	1.00	9	0.222
842	A	13	5	1.00	10	0.500
843	A	3	3	1.00	18	0.167
844	A	1	1	1.00	18	0.056
845	A	3	3	1.00	18	0.167
846	A	9	7	1.27	15	0.467
847	A	8	7	1.27	15	0.467
848	A	4	2	1.00	15	0.133
849	A	4	2	1.00	15	0.133
850	A	3	3	1.00	21	0.143
851	A	3	3	1.00	21	0.143
852	A	1	1	1.00	14	0.071
853	A	4	4	1.00	15	0.267
854	A	4	3	1.00	14	0.214
855	A	7	5	1.28	16	0.312
856	A	7	5	1.28	16	0.312
857	A	4	2	1.00	15	0.133
858	B	5	3	4.63	18	0.167
859	F	0	0	N/A	0	N/A
860	B	17	9	3.79	16	0.562
861	A	4	4	1.00	12	0.333

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
862	A	2	2	1.00	15	0.133
863	A	6	5	1.00	15	0.333
864	A	6	5	1.00	15	0.333
865	A	8	5	1.00	18	0.278
866	A	10	8	1.11	23	0.348
867	A	7	6	1.00	23	0.261
868	A	6	4	1.00	16	0.250
869	A	8	5	1.00	18	0.278
870	A	10	6	1.00	18	0.333
871	A	5	5	1.00	16	0.312
872	A	6	6	1.00	18	0.333
873	A	7	7	1.00	18	0.389
874	A	10	10	1.00	16	0.625
875	A	17	14	1.00	18	0.778
876	A	21	13	1.00	18	0.722
877	A	12	11	1.00	16	0.688
878	A	16	13	1.00	18	0.722
879	A	21	17	1.00	18	0.944
880	A	5	2	1.00	11	0.182
881	A	5	2	1.00	11	0.182
882	A	5	2	1.00	11	0.182
883	A	5	2	1.00	11	0.182
884	A	2	2	1.00	6	0.333
885	A	1	1	1.00	15	0.067
886	A	4	4	1.00	9	0.444
887	A	3	2	1.00	15	0.133

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
888	A	4	3	1.42	16	0.188
889	A	3	2	1.00	15	0.133
890	A	5	4	1.00	13	0.308
891	A	3	2	1.00	13	0.154
892	A	5	4	1.00	13	0.308
893	A	4	3	1.00	15	0.200
894	A	5	4	1.00	7	0.571
895	A	3	2	1.00	13	0.154
896	A	5	4	1.00	27	0.148
897	A	5	4	1.00	27	0.148
898	A	5	4	1.00	27	0.148
899	A	5	4	1.00	27	0.148
900	A	5	3	1.00	21	0.143
901	A	7	5	1.00	23	0.217
902	A	4	3	1.00	13	0.231
903	A	7	7	1.00	15	0.467
904	A	2	2	1.00	10	0.200
905	A	3	2	1.00	17	0.118
906	A	5	4	1.00	17	0.235
907	A	5	3	1.00	8	0.375
908	A	3	2	1.00	13	0.154
909	A	4	3	1.00	16	0.188
910	A	6	5	1.00	13	0.385
911	A	8	3	1.00	12	0.250
912	B	22	9	4.26	18	0.500
913	A	3	3	1.00	9	0.333

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
914	F	0	0	N/A	0	N/A
915	F	0	0	N/A	0	N/A
916	A	6	4	1.00	12	0.333
917	A	2	2	1.00	9	0.222
918	A	1	1	1.00	21	0.048
919	A	3	3	1.00	10	0.300
920	A	8	4	1.00	13	0.308
921	A	1	1	1.00	8	0.125
922	A	7	5	1.00	12	0.417
923	A	1	1	1.00	18	0.056
924	A	1	1	1.00	14	0.071
925	A	1	1	1.00	22	0.045
926	A	7	4	1.00	22	0.182
927	A	6	4	1.00	22	0.182
928	A	3	3	1.00	10	0.300
929	A	3	3	1.00	9	0.333
930	A	2	2	1.00	14	0.143
931	F	0	0	N/A	0	N/A
932	A	0	0	0.00	0	0.000
933	F	0	0	N/A	0	N/A
934	A	7	5	1.00	28	0.179
935	A	9	6	1.00	30	0.200
936	A	7	6	1.00	36	0.167
937	A	16	5	1.00	38	0.132
938	A	7	6	1.00	29	0.207

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
939	A	11	8	1.00	31	0.258
940	A	4	4	1.00	27	0.148
941	A	4	4	1.00	27	0.148
942	A	7	4	1.00	39	0.103
943	A	4	2	1.00	39	0.051
944	A	5	3	1.00	39	0.077
945	A	6	3	1.00	39	0.077
946	A	1	1	1.00	31	0.032
947	A	4	2	1.00	31	0.065
948	A	2	1	1.00	39	0.026
949	A	5	3	1.00	39	0.077
950	A	4	2	1.00	39	0.051

Chapter 3

Listing of integrals

3.1 $\int \frac{2}{3-\cos(4+6x)}\,dx$

Optimal. Leaf size=44

$$\frac{x}{\sqrt{2}}+\frac{\tan^{-1}\left(\frac{\sin(6x+4)}{-\cos(6x+4)+2\sqrt{2}+3}\right)}{3\sqrt{2}}$$

```
[Out] 1/2*x*2^(1/2)+1/6*arctan(sin(4+6*x)/(3-cos(4+6*x)+2*2^(1/2)))*2^(1/2)
```

Rubi [A] time = 0.04, antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {12, 2657}

$$\frac{x}{\sqrt{2}}+\frac{\tan^{-1}\left(\frac{\sin(6x+4)}{-\cos(6x+4)+2\sqrt{2}+3}\right)}{3\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[2/(3 - Cos[4 + 6*x]),x]
```

```
[Out] x/Sqrt[2] + ArcTan[Sin[4 + 6*x]/(3 + 2*Sqrt[2] - Cos[4 + 6*x])]/(3*Sqrt[2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2657

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{q = Rt[
a^2 - b^2, 2]}, Simp[x/q, x] + Simp[(2*ArcTan[(b*Cos[c + d*x])/(a + q + b*S
in[c + d*x])])/(d*q), x]] /; FreeQ[{a, b, c, d}, x] && GtQ[a^2 - b^2, 0] &&
 PosQ[a]
```

Rubi steps

$$\int \frac{2}{3-\cos(4+6x)}\,dx = 2\int \frac{1}{3-\cos(4+6x)}\,dx$$
$$= \frac{x}{\sqrt{2}} + \frac{\tan^{-1}\left(\frac{\sin(4+6x)}{3+2\sqrt{2}-\cos(4+6x)}\right)}{3\sqrt{2}}$$

Mathematica [A] time = 0.04, size = 22, normalized size = 0.50

$$\frac{\tan^{-1}\left(\sqrt{2}\tan(3x+2)\right)}{3\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[2/(3 - Cos[4 + 6*x]),x]
```

```
[Out] ArcTan[Sqrt[2]*Tan[2 + 3*x]]/(3*Sqrt[2])
```

fricas [A] time = 0.65, size = 33, normalized size = 0.75

$$-\frac{1}{12}\sqrt{2}\arctan\left(\frac{3\sqrt{2}\cos(6x+4)-\sqrt{2}}{4\sin(6x+4)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(2/(3-cos(4+6*x)),x, algorithm="fricas")
```

```
[Out] -1/12*sqrt(2)*arctan(1/4*(3*sqrt(2)*cos(6*x + 4) - sqrt(2))/sin(6*x + 4))
```

giac [A] time = 0.15, size = 57, normalized size = 1.30

$$\frac{1}{6}\sqrt{2}\left(3x+\arctan\left(-\frac{\sqrt{2}\sin(6x+4)-2\sin(6x+4)}{\sqrt{2}\cos(6x+4)+\sqrt{2}-2\cos(6x+4)+2}\right)+2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(2/(3-cos(4+6*x)),x, algorithm="giac")
```

```
[Out] 1/6*sqrt(2)*(3*x + arctan(-(sqrt(2)*sin(6*x + 4) - 2*sin(6*x + 4))/(sqrt(2)
*cos(6*x + 4) + sqrt(2) - 2*cos(6*x + 4) + 2)) + 2)
```

maple [A] time = 0.08, size = 17, normalized size = 0.39

$$\frac{\sqrt{2}\ \arctan\left(\sqrt{2}\ \tan\left(2+3x\right)\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(2/(3-cos(4+6*x)),x)
```

```
[Out] 1/6*2^(1/2)*arctan(2^(1/2)*tan(2+3*x))
```

maxima [A] time = 0.41, size = 26, normalized size = 0.59

$$\frac{1}{6}\sqrt{2}\arctan\left(\frac{\sqrt{2}\sin\left(6x+4\right)}{\cos\left(6x+4\right)+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(2/(3-cos(4+6*x)),x, algorithm="maxima")
```

```
[Out] 1/6*sqrt(2)*arctan(sqrt(2)*sin(6*x + 4)/(cos(6*x + 4) + 1))
```

mupad [B] time = 2.58, size = 35, normalized size = 0.80

$$\frac{\sqrt{2}\ \left(3x-\mathrm{atan}\left(\tan\left(3x+2\right)\right)\right)}{6}+\frac{\sqrt{2}\ \mathrm{atan}\left(\sqrt{2}\ \tan\left(3x+2\right)\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-2/(cos(6*x + 4) - 3),x)
```

```
[Out] (2^(1/2)*(3*x - atan(tan(3*x + 2))))/6 + (2^(1/2)*atan(2^(1/2)*tan(3*x + 2)
))/6
```

sympy [A] time = 0.26, size = 32, normalized size = 0.73

$$\frac{\sqrt{2}\left(\mathrm{atan}\left(\sqrt{2}\ \tan\left(3x+2\right)\right)+\pi\left\lfloor\frac{3x-\frac{\pi}{2}+2}{\pi}\right\rfloor\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(2/(3-cos(4+6*x)),x)
```

```
[Out] sqrt(2)*(atan(sqrt(2)*tan(3*x + 2)) + pi*floor((3*x - pi/2 + 2)/pi))/6
```

3.2 $\int \frac{2\csc(4+6x)}{-\cot(4+6x)+3\csc(4+6x)}\,dx$

Optimal. Leaf size=44

$$\frac{x}{\sqrt{2}}+\frac{\tan^{-1}\left(\frac{\sin(6x+4)}{-\cos(6x+4)+2\sqrt{2}+3}\right)}{3\sqrt{2}}$$

```
[Out] 1/2*x*2^(1/2)+1/6*arctan(sin(4+6*x)/(3-cos(4+6*x)+2*2^(1/2)))*2^(1/2)
```

Rubi [A] time = 0.04, antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {12, 3166, 2657}

$$\frac{x}{\sqrt{2}}+\frac{\tan^{-1}\left(\frac{\sin(6x+4)}{-\cos(6x+4)+2\sqrt{2}+3}\right)}{3\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[(2*Csc[4 + 6*x])/(-Cot[4 + 6*x] + 3*Csc[4 + 6*x]),x]
```

```
[Out] x/Sqrt[2] + ArcTan[Sin[4 + 6*x]/(3 + 2*Sqrt[2] - Cos[4 + 6*x])]/(3*Sqrt[2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2657

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{q = Rt[
a^2 - b^2, 2]}, Simp[x/q, x] + Simp[(2*ArcTan[(b*Cos[c + d*x])/(a + q + b*S
in[c + d*x])])/(d*q), x]] /; FreeQ[{a, b, c, d}, x] && GtQ[a^2 - b^2, 0] &&
 PosQ[a]
```

Rule 3166

```
Int[csc[(d_.) + (e_.)*(x_)]^(n_.)*((a_.) + csc[(d_.) + (e_.)*(x_)]*(b_.) +
cot[(d_.) + (e_.)*(x_)]*(c_.))^(m_), x_Symbol] :> Int[1/(b + a*Sin[d + e*x]
 + c*Cos[d + e*x])^n, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[m + n, 0] && I
ntegerQ[n]
```

Rubi steps

$$\int \frac{2\csc(4+6x)}{-\cot(4+6x)+3\csc(4+6x)}\,dx = 2\int \frac{\csc(4+6x)}{-\cot(4+6x)+3\csc(4+6x)}\,dx$$
$$= 2\int \frac{1}{3-\cos(4+6x)}\,dx$$
$$= \frac{x}{\sqrt{2}} + \frac{\tan^{-1}\left(\frac{\sin(4+6x)}{3+2\sqrt{2}-\cos(4+6x)}\right)}{3\sqrt{2}}$$

Mathematica [A] time = 0.03, size = 22, normalized size = 0.50

$$\frac{\tan^{-1}\left(\sqrt{2}\tan(3x+2)\right)}{3\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2*Csc[4 + 6*x])/(-Cot[4 + 6*x] + 3*Csc[4 + 6*x]),x]
[Out] ArcTan[Sqrt[2]*Tan[2 + 3*x]]/(3*Sqrt[2])
```

fricas [A] time = 0.69, size = 33, normalized size = 0.75

$$-\frac{1}{12}\sqrt{2}\arctan\left(\frac{3\sqrt{2}\cos(6x+4)-\sqrt{2}}{4\sin(6x+4)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(2*csc(4+6*x)/(-cot(4+6*x)+3*csc(4+6*x)),x, algorithm="fricas")
[Out] -1/12*sqrt(2)*arctan(1/4*(3*sqrt(2)*cos(6*x + 4) - sqrt(2))/sin(6*x + 4))
```

giac [A] time = 0.22, size = 57, normalized size = 1.30

$$\frac{1}{6}\sqrt{2}\left(3x+\arctan\left(-\frac{\sqrt{2}\sin(6x+4)-2\sin(6x+4)}{\sqrt{2}\cos(6x+4)+\sqrt{2}-2\cos(6x+4)+2}\right)+2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(2*csc(4+6*x)/(-cot(4+6*x)+3*csc(4+6*x)),x, algorithm="giac")
[Out] 1/6*sqrt(2)*(3*x + arctan(-(sqrt(2)*sin(6*x + 4) - 2*sin(6*x + 4))/(sqrt(2)
*cos(6*x + 4) + sqrt(2) - 2*cos(6*x + 4) + 2)) + 2)
```

maple [A] time = 0.30, size = 17, normalized size = 0.39

$$\frac{\sqrt{2}\ \arctan\left(\sqrt{2}\ \tan\left(2+3x\right)\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(2*csc(4+6*x)/(-cot(4+6*x)+3*csc(4+6*x)),x)
[Out] 1/6*2^(1/2)*arctan(2^(1/2)*tan(2+3*x))
```

maxima [A] time = 0.64, size = 26, normalized size = 0.59

$$\frac{1}{6}\sqrt{2}\arctan\left(\frac{\sqrt{2}\sin\left(6\,x+4\right)}{\cos\left(6\,x+4\right)+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(2*csc(4+6*x)/(-cot(4+6*x)+3*csc(4+6*x)),x, algorithm="maxima")
[Out] 1/6*sqrt(2)*arctan(sqrt(2)*sin(6*x + 4)/(cos(6*x + 4) + 1))
```

mupad [B] time = 2.69, size = 16, normalized size = 0.36

$$\frac{\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\,\tan\left(3\,x+2\right)\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-2/(sin(6*x + 4)*(cot(6*x + 4) - 3/sin(6*x + 4))),x)
[Out] (2^(1/2)*atan(2^(1/2)*tan(3*x + 2)))/6
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-2\int\frac{\csc\left(6x+4\right)}{\cot\left(6x+4\right)-3\csc\left(6x+4\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(2*csc(4+6*x)/(-cot(4+6*x)+3*csc(4+6*x)),x)
[Out] -2*Integral(csc(6*x + 4)/(cot(6*x + 4) - 3*csc(6*x + 4)), x)
```

3.3 $\int \frac{1}{1+\sin^2(2+3x)}\,dx$

Optimal. Leaf size=48

$$\frac{x}{\sqrt{2}}+\frac{\tan^{-1}\left(\frac{\sin(3x+2)\cos(3x+2)}{\sin^2(3x+2)+\sqrt{2}+1}\right)}{3\sqrt{2}}$$

```
[Out] 1/2*x*2^(1/2)+1/6*arctan(cos(2+3*x)*sin(2+3*x)/(1+sin(2+3*x)^2+2^(1/2)))*2^
(1/2)
```

Rubi [A] time = 0.02, antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3181, 203}

$$\frac{x}{\sqrt{2}}+\frac{\tan^{-1}\left(\frac{\sin(3x+2)\cos(3x+2)}{\sin^2(3x+2)+\sqrt{2}+1}\right)}{3\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + Sin[2 + 3*x]^2)^(-1),x]
```

```
[Out] x/Sqrt[2] + ArcTan[(Cos[2 + 3*x]*Sin[2 + 3*x])/(1 + Sqrt[2] + Sin[2 + 3*x]^
2)]/(3*Sqrt[2])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 3181

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(-1), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[1/(a + (a + b)*ff^2*x^2
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x]
```

Rubi steps

$$\int \frac{1}{1+\sin^2(2+3x)}\,dx = \frac{1}{3}\operatorname{Subst}\left(\int \frac{1}{1+2x^2}\,dx, x, \tan(2+3x)\right)$$
$$= \frac{x}{\sqrt{2}} + \frac{\tan^{-1}\left(\frac{\cos(2+3x)\sin(2+3x)}{1+\sqrt{2}+\sin^2(2+3x)}\right)}{3\sqrt{2}}$$

Mathematica [A] time = 0.05, size = 22, normalized size = 0.46

$$\frac{\tan^{-1}\left(\sqrt{2}\tan(3x+2)\right)}{3\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + Sin[2 + 3*x]^2)^(-1),x]
[Out] ArcTan[Sqrt[2]*Tan[2 + 3*x]]/(3*Sqrt[2])
```

fricas [A] time = 0.65, size = 43, normalized size = 0.90

$$-\frac{1}{12}\sqrt{2}\arctan\left(\frac{3\sqrt{2}\cos(3x+2)^2 - 2\sqrt{2}}{4\cos(3x+2)\sin(3x+2)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+sin(2+3*x)^2),x, algorithm="fricas")
[Out] -1/12*sqrt(2)*arctan(1/4*(3*sqrt(2)*cos(3*x + 2)^2 - 2*sqrt(2))/(cos(3*x +
2)*sin(3*x + 2)))
```

giac [A] time = 0.13, size = 57, normalized size = 1.19

$$\frac{1}{6}\sqrt{2}\left(3x + \arctan\left(-\frac{\sqrt{2}\sin(6x+4) - 2\sin(6x+4)}{\sqrt{2}\cos(6x+4) + \sqrt{2} - 2\cos(6x+4) + 2}\right) + 2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+sin(2+3*x)^2),x, algorithm="giac")
[Out] 1/6*sqrt(2)*(3*x + arctan(-(sqrt(2)*sin(6*x + 4) - 2*sin(6*x + 4))/(sqrt(2)
*cos(6*x + 4) + sqrt(2) - 2*cos(6*x + 4) + 2)) + 2)
```

maple [A] time = 0.17, size = 17, normalized size = 0.35

$$\frac{\sqrt{2}\ \arctan\left(\sqrt{2}\ \tan\left(2+3x\right)\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1+sin(2+3*x)^2),x)
```

```
[Out] 1/6*2^(1/2)*arctan(2^(1/2)*tan(2+3*x))
```

maxima [A] time = 0.89, size = 16, normalized size = 0.33

$$\frac{1}{6}\sqrt{2}\arctan\left(\sqrt{2}\tan\left(3\,x+2\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+sin(2+3*x)^2),x, algorithm="maxima")
```

```
[Out] 1/6*sqrt(2)*arctan(sqrt(2)*tan(3*x + 2))
```

mupad [B] time = 2.49, size = 35, normalized size = 0.73

$$\frac{\sqrt{2}\ \left(3\,x-\mathrm{atan}\left(\tan\left(3\,x+2\right)\right)\right)}{6}+\frac{\sqrt{2}\ \mathrm{atan}\left(\sqrt{2}\ \tan\left(3\,x+2\right)\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(3*x + 2)^2 + 1),x)
```

```
[Out] (2^(1/2)*(3*x - atan(tan(3*x + 2))))/6 + (2^(1/2)*atan(2^(1/2)*tan(3*x + 2)
))/6
```

sympy [B] time = 6.82, size = 246, normalized size = 5.12

$$\frac{47321\sqrt{2}\sqrt{3-2\sqrt{2}}\ \left(\mathrm{atan}\left(\frac{\tan\left(\frac{3x}{2}+1\right)}{\sqrt{3-2\sqrt{2}}}\right)+\pi\left\lfloor\frac{\frac{3x}{2}-\frac{\pi}{2}+1}{\pi}\right\rfloor\right)}{83160\sqrt{2}\ +117606}+\frac{66922\sqrt{3-2\sqrt{2}}\ \left(\mathrm{atan}\left(\frac{\tan\left(\frac{3x}{2}+1\right)}{\sqrt{3-2\sqrt{2}}}\right)+\pi\left\lfloor\frac{\frac{3x}{2}-\frac{\pi}{2}+1}{\pi}\right\rfloor\right)}{83160\sqrt{2}\ +117606}+\frac{81\cdots}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+sin(2+3*x)**2),x)
```

```
[Out] 47321*sqrt(2)*sqrt(3 - 2*sqrt(2))*(atan(tan(3*x/2 + 1)/sqrt(3 - 2*sqrt(2)))
 + pi*floor((3*x/2 - pi/2 + 1)/pi))/(83160*sqrt(2) + 117606) + 66922*sqrt(3
```

```
 - 2*sqrt(2))*(atan(tan(3*x/2 + 1)/sqrt(3 - 2*sqrt(2))) + pi*floor((3*x/2 -
 pi/2 + 1)/pi))/(83160*sqrt(2) + 117606) + 8119*sqrt(2)*sqrt(2*sqrt(2) + 3)
*(atan(tan(3*x/2 + 1)/sqrt(2*sqrt(2) + 3)) + pi*floor((3*x/2 - pi/2 + 1)/pi
))/(83160*sqrt(2) + 117606) + 11482*sqrt(2*sqrt(2) + 3)*(atan(tan(3*x/2 + 1
)/sqrt(2*sqrt(2) + 3)) + pi*floor((3*x/2 - pi/2 + 1)/pi))/(83160*sqrt(2) +
117606)
```

3.4 $\int \frac{1}{2-\cos^2(2+3x)}\,dx$

Optimal. Leaf size=48

$$\frac{x}{\sqrt{2}}+\frac{\tan^{-1}\left(\frac{\sin(3x+2)\cos(3x+2)}{\sin^2(3x+2)+\sqrt{2}+1}\right)}{3\sqrt{2}}$$

```
[Out] 1/2*x*2^(1/2)+1/6*arctan(cos(2+3*x)*sin(2+3*x)/(1+sin(2+3*x)^2+2^(1/2)))*2^
(1/2)
```

Rubi [A] time = 0.02, antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3181, 203}

$$\frac{x}{\sqrt{2}}+\frac{\tan^{-1}\left(\frac{\sin(3x+2)\cos(3x+2)}{\sin^2(3x+2)+\sqrt{2}+1}\right)}{3\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[(2 - Cos[2 + 3*x]^2)^(-1),x]
```

```
[Out] x/Sqrt[2] + ArcTan[(Cos[2 + 3*x]*Sin[2 + 3*x])/(1 + Sqrt[2] + Sin[2 + 3*x]^
2)]/(3*Sqrt[2])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 3181

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(-1), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[1/(a + (a + b)*ff^2*x^2
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x]
```

Rubi steps

$$\int \frac{1}{2-\cos^2(2+3x)}\,dx = -\left(\frac{1}{3}\,\mathrm{Subst}\left(\int \frac{1}{2+x^2}\,dx, x, \cot(2+3x)\right)\right)$$
$$= \frac{x}{\sqrt{2}} + \frac{\tan^{-1}\left(\frac{\cos(2+3x)\sin(2+3x)}{1+\sqrt{2}+\sin^2(2+3x)}\right)}{3\sqrt{2}}$$

Mathematica [A] time = 0.06, size = 22, normalized size = 0.46

$$\frac{\tan^{-1}\left(\sqrt{2}\tan(3x+2)\right)}{3\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 - Cos[2 + 3*x]^2)^(-1),x]
```

```
[Out] ArcTan[Sqrt[2]*Tan[2 + 3*x]]/(3*Sqrt[2])
```

fricas [A] time = 0.73, size = 43, normalized size = 0.90

$$-\frac{1}{12}\,\sqrt{2}\,\arctan\left(\frac{3\,\sqrt{2}\,\cos\left(3\,x+2\right)^2 - 2\,\sqrt{2}}{4\,\cos\left(3\,x+2\right)\sin\left(3\,x+2\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2-cos(2+3*x)^2),x, algorithm="fricas")
```

```
[Out] -1/12*sqrt(2)*arctan(1/4*(3*sqrt(2)*cos(3*x + 2)^2 - 2*sqrt(2))/(cos(3*x +
2)*sin(3*x + 2)))
```

giac [A] time = 0.15, size = 57, normalized size = 1.19

$$\frac{1}{6}\,\sqrt{2}\left(3\,x + \arctan\left(-\frac{\sqrt{2}\,\sin\left(6\,x+4\right) - 2\,\sin\left(6\,x+4\right)}{\sqrt{2}\,\cos\left(6\,x+4\right) + \sqrt{2} - 2\,\cos\left(6\,x+4\right) + 2}\right) + 2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2-cos(2+3*x)^2),x, algorithm="giac")
```

```
[Out] 1/6*sqrt(2)*(3*x + arctan(-(sqrt(2)*sin(6*x + 4) - 2*sin(6*x + 4))/(sqrt(2)
*cos(6*x + 4) + sqrt(2) - 2*cos(6*x + 4) + 2)) + 2)
```

maple [A] time = 0.10, size = 17, normalized size = 0.35

$$\frac{\sqrt{2}\ \arctan\left(\sqrt{2}\ \tan\left(2+3x\right)\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2-cos(2+3*x)^2),x)
```

```
[Out] 1/6*2^(1/2)*arctan(2^(1/2)*tan(2+3*x))
```

maxima [A] time = 0.41, size = 16, normalized size = 0.33

$$\frac{1}{6}\sqrt{2}\arctan\left(\sqrt{2}\tan\left(3x+2\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2-cos(2+3*x)^2),x, algorithm="maxima")
```

```
[Out] 1/6*sqrt(2)*arctan(sqrt(2)*tan(3*x + 2))
```

mupad [B] time = 2.38, size = 35, normalized size = 0.73

$$\frac{\sqrt{2}\ \left(3x-\mathrm{atan}\left(\tan\left(3x+2\right)\right)\right)}{6}+\frac{\sqrt{2}\ \mathrm{atan}\left(\sqrt{2}\ \tan\left(3x+2\right)\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-1/(cos(3*x + 2)^2 - 2),x)
```

```
[Out] (2^(1/2)*(3*x - atan(tan(3*x + 2))))/6 + (2^(1/2)*atan(2^(1/2)*tan(3*x + 2)
))/6
```

sympy [B] time = 7.12, size = 246, normalized size = 5.12

$$\frac{47321\sqrt{2}\sqrt{3-2\sqrt{2}}\left(\mathrm{atan}\left(\frac{\tan\left(\frac{3x}{2}+1\right)}{\sqrt{3-2\sqrt{2}}}\right)+\pi\left\lfloor\frac{\frac{3x}{2}-\frac{\pi}{2}+1}{\pi}\right\rfloor\right)}{83160\sqrt{2}+117606}+\frac{66922\sqrt{3-2\sqrt{2}}\left(\mathrm{atan}\left(\frac{\tan\left(\frac{3x}{2}+1\right)}{\sqrt{3-2\sqrt{2}}}\right)+\pi\left\lfloor\frac{\frac{3x}{2}-\frac{\pi}{2}+1}{\pi}\right\rfloor\right)}{83160\sqrt{2}+117606}+\frac{81\cdots}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2-cos(2+3*x)**2),x)
```

```
[Out] 47321*sqrt(2)*sqrt(3 - 2*sqrt(2))*(atan(tan(3*x/2 + 1)/sqrt(3 - 2*sqrt(2)))
 + pi*floor((3*x/2 - pi/2 + 1)/pi))/(83160*sqrt(2) + 117606) + 66922*sqrt(3
```

```
 - 2*sqrt(2))*(atan(tan(3*x/2 + 1)/sqrt(3 - 2*sqrt(2))) + pi*floor((3*x/2 -
 pi/2 + 1)/pi))/(83160*sqrt(2) + 117606) + 8119*sqrt(2)*sqrt(2*sqrt(2) + 3)
*(atan(tan(3*x/2 + 1)/sqrt(2*sqrt(2) + 3)) + pi*floor((3*x/2 - pi/2 + 1)/pi
))/(83160*sqrt(2) + 117606) + 11482*sqrt(2*sqrt(2) + 3)*(atan(tan(3*x/2 + 1
)/sqrt(2*sqrt(2) + 3)) + pi*floor((3*x/2 - pi/2 + 1)/pi))/(83160*sqrt(2) +
117606)
```

3.5 $\int \frac{1}{\cos^2(2+3x)+2\sin^2(2+3x)}\,dx$

Optimal. Leaf size=48

$$\frac{x}{\sqrt{2}}+\frac{\tan^{-1}\left(\frac{\sin(3x+2)\cos(3x+2)}{\sin^2(3x+2)+\sqrt{2}+1}\right)}{3\sqrt{2}}$$

```
[Out] 1/2*x*2^(1/2)+1/6*arctan(cos(2+3*x)*sin(2+3*x)/(1+sin(2+3*x)^2+2^(1/2)))*2^
(1/2)
```

Rubi [A] time = 0.03, antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.048, Rules used = {203}

$$\frac{x}{\sqrt{2}}+\frac{\tan^{-1}\left(\frac{\sin(3x+2)\cos(3x+2)}{\sin^2(3x+2)+\sqrt{2}+1}\right)}{3\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[2 + 3*x]^2 + 2*Sin[2 + 3*x]^2)^(-1),x]
```

```
[Out] x/Sqrt[2] + ArcTan[(Cos[2 + 3*x]*Sin[2 + 3*x])/(1 + Sqrt[2] + Sin[2 + 3*x]^
2)]/(3*Sqrt[2])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\cos^2(2+3x)+2\sin^2(2+3x)}\,dx &= \frac{1}{3}\operatorname{Subst}\left(\int \frac{1}{1+2x^2}\,dx,x,\tan(2+3x)\right)\\ &= \frac{x}{\sqrt{2}}+\frac{\tan^{-1}\left(\frac{\cos(2+3x)\sin(2+3x)}{1+\sqrt{2}+\sin^2(2+3x)}\right)}{3\sqrt{2}}\end{aligned}$$

Mathematica [A] time = 0.02, size = 22, normalized size = 0.46

$$\frac{\tan^{-1}\left(\sqrt{2}\tan(3x+2)\right)}{3\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[2 + 3*x]^2 + 2*Sin[2 + 3*x]^2)^(-1),x]
```

```
[Out] ArcTan[Sqrt[2]*Tan[2 + 3*x]]/(3*Sqrt[2])
```

fricas [A] time = 0.74, size = 43, normalized size = 0.90

$$-\frac{1}{12}\sqrt{2}\arctan\left(\frac{3\sqrt{2}\cos(3x+2)^2 - 2\sqrt{2}}{4\cos(3x+2)\sin(3x+2)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(cos(2+3*x)^2+2*sin(2+3*x)^2),x, algorithm="fricas")
```

```
[Out] -1/12*sqrt(2)*arctan(1/4*(3*sqrt(2)*cos(3*x + 2)^2 - 2*sqrt(2))/(cos(3*x +
2)*sin(3*x + 2)))
```

giac [A] time = 0.16, size = 57, normalized size = 1.19

$$\frac{1}{6}\sqrt{2}\left(3x + \arctan\left(-\frac{\sqrt{2}\sin(6x+4) - 2\sin(6x+4)}{\sqrt{2}\cos(6x+4) + \sqrt{2} - 2\cos(6x+4) + 2}\right) + 2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(cos(2+3*x)^2+2*sin(2+3*x)^2),x, algorithm="giac")
```

```
[Out] 1/6*sqrt(2)*(3*x + arctan(-(sqrt(2)*sin(6*x + 4) - 2*sin(6*x + 4))/(sqrt(2)
*cos(6*x + 4) + sqrt(2) - 2*cos(6*x + 4) + 2)) + 2)
```

maple [A] time = 0.23, size = 17, normalized size = 0.35

$$\frac{\sqrt{2}\arctan\left(\sqrt{2}\tan(2+3x)\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(2+3*x)^2+2*sin(2+3*x)^2),x)
```

```
[Out] 1/6*2^(1/2)*arctan(2^(1/2)*tan(2+3*x))
```

maxima [A] time = 0.57, size = 16, normalized size = 0.33

$$\frac{1}{6}\sqrt{2}\arctan\left(\sqrt{2}\tan(3x+2)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(cos(2+3*x)^2+2*sin(2+3*x)^2),x, algorithm="maxima")
```

```
[Out] 1/6*sqrt(2)*arctan(sqrt(2)*tan(3*x + 2))
```

mupad [B] time = 2.38, size = 35, normalized size = 0.73

$$\frac{\sqrt{2}\,\left(3\,x-\mathrm{atan}\left(\tan\left(3\,x+2\right)\right)\right)}{6}+\frac{\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\,\tan\left(3\,x+2\right)\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2*sin(3*x + 2)^2 + cos(3*x + 2)^2),x)
```

```
[Out] (2^(1/2)*(3*x - atan(tan(3*x + 2))))/6 + (2^(1/2)*atan(2^(1/2)*tan(3*x + 2)
))/6
```

sympy [B] time = 6.99, size = 246, normalized size = 5.12

$$\frac{47321\sqrt{2}\sqrt{3-2\sqrt{2}}\left(\operatorname{atan}\left(\frac{\tan\left(\frac{3x}{2}+1\right)}{\sqrt{3-2\sqrt{2}}}\right)+\pi\left\lfloor\frac{\frac{3x}{2}-\frac{\pi}{2}+1}{\pi}\right\rfloor\right)}{83160\sqrt{2}+117606}+\frac{66922\sqrt{3-2\sqrt{2}}\left(\operatorname{atan}\left(\frac{\tan\left(\frac{3x}{2}+1\right)}{\sqrt{3-2\sqrt{2}}}\right)+\pi\left\lfloor\frac{\frac{3x}{2}-\frac{\pi}{2}+1}{\pi}\right\rfloor\right)}{83160\sqrt{2}+117606}+\frac{81\cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(cos(2+3*x)**2+2*sin(2+3*x)**2),x)
```

```
[Out] 47321*sqrt(2)*sqrt(3 - 2*sqrt(2))*(atan(tan(3*x/2 + 1)/sqrt(3 - 2*sqrt(2)))
 + pi*floor((3*x/2 - pi/2 + 1)/pi))/(83160*sqrt(2) + 117606) + 66922*sqrt(3
 - 2*sqrt(2))*(atan(tan(3*x/2 + 1)/sqrt(3 - 2*sqrt(2))) + pi*floor((3*x/2 -
 pi/2 + 1)/pi))/(83160*sqrt(2) + 117606) + 8119*sqrt(2)*sqrt(2*sqrt(2) + 3)
*(atan(tan(3*x/2 + 1)/sqrt(2*sqrt(2) + 3)) + pi*floor((3*x/2 - pi/2 + 1)/pi
))/(83160*sqrt(2) + 117606) + 11482*sqrt(2*sqrt(2) + 3)*(atan(tan(3*x/2 + 1
)/sqrt(2*sqrt(2) + 3)) + pi*floor((3*x/2 - pi/2 + 1)/pi))/(83160*sqrt(2) +
117606)
```

3.6 $\int \frac{\sec^2(2+3x)}{1+2\tan^2(2+3x)}\,dx$

Optimal. Leaf size=48

$$\frac{x}{\sqrt{2}}+\frac{\tan^{-1}\left(\frac{\sin(3x+2)\cos(3x+2)}{\sin^2(3x+2)+\sqrt{2}+1}\right)}{3\sqrt{2}}$$

```
[Out] 1/2*x*2^(1/2)+1/6*arctan(cos(2+3*x)*sin(2+3*x)/(1+sin(2+3*x)^2+2^(1/2)))*2^
(1/2)
```

Rubi [A] time = 0.04, antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {3675, 203}

$$\frac{x}{\sqrt{2}}+\frac{\tan^{-1}\left(\frac{\sin(3x+2)\cos(3x+2)}{\sin^2(3x+2)+\sqrt{2}+1}\right)}{3\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[2 + 3*x]^2/(1 + 2*Tan[2 + 3*x]^2),x]
```

```
[Out] x/Sqrt[2] + ArcTan[(Cos[2 + 3*x]*Sin[2 + 3*x])/(1 + Sqrt[2] + Sin[2 + 3*x]^
2)]/(3*Sqrt[2])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 3675

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/(c^(m - 1)*f), Subst[Int[(c^2 + ff^2*x^2)^(m/2 - 1)*(a + b*(ff*x)^n)^p
, x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && In
tegerQ[m/2] && (IntegersQ[n, p] || IGtQ[m, 0] || IGtQ[p, 0] || EqQ[n^2, 4]
|| EqQ[n^2, 16])
```

Rubi steps

$$\int \frac{\sec^2(2+3x)}{1+2\tan^2(2+3x)}\,dx = \frac{1}{3}\operatorname{Subst}\left(\int \frac{1}{1+2x^2}\,dx, x, \tan(2+3x)\right)$$
$$= \frac{x}{\sqrt{2}} + \frac{\tan^{-1}\left(\frac{\cos(2+3x)\sin(2+3x)}{1+\sqrt{2}+\sin^2(2+3x)}\right)}{3\sqrt{2}}$$

Mathematica [A] time = 0.02, size = 22, normalized size = 0.46

$$\frac{\tan^{-1}\left(\sqrt{2}\tan(3x+2)\right)}{3\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[2 + 3*x]^2/(1 + 2*Tan[2 + 3*x]^2),x]
[Out] ArcTan[Sqrt[2]*Tan[2 + 3*x]]/(3*Sqrt[2])
```

fricas [A] time = 0.51, size = 43, normalized size = 0.90

$$-\frac{1}{12}\sqrt{2}\arctan\left(\frac{3\sqrt{2}\cos(3x+2)^2 - 2\sqrt{2}}{4\cos(3x+2)\sin(3x+2)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(2+3*x)^2/(1+2*tan(2+3*x)^2),x, algorithm="fricas")
[Out] -1/12*sqrt(2)*arctan(1/4*(3*sqrt(2)*cos(3*x + 2)^2 - 2*sqrt(2))/(cos(3*x +
2)*sin(3*x + 2)))
```

giac [A] time = 1.54, size = 16, normalized size = 0.33

$$\frac{1}{6}\sqrt{2}\arctan\left(\sqrt{2}\tan(3x+2)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(2+3*x)^2/(1+2*tan(2+3*x)^2),x, algorithm="giac")
[Out] 1/6*sqrt(2)*arctan(sqrt(2)*tan(3*x + 2))
```

maple [A] time = 0.29, size = 17, normalized size = 0.35

$$\frac{\sqrt{2}\arctan\left(\sqrt{2}\tan(2+3x)\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(2+3*x)^2/(1+2*tan(2+3*x)^2),x)
```

```
[Out] 1/6*2^(1/2)*arctan(2^(1/2)*tan(2+3*x))
```

maxima [A] time = 0.44, size = 16, normalized size = 0.33

$$\frac{1}{6}\sqrt{2}\arctan\left(\sqrt{2}\tan\left(3x+2\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(2+3*x)^2/(1+2*tan(2+3*x)^2),x, algorithm="maxima")
```

```
[Out] 1/6*sqrt(2)*arctan(sqrt(2)*tan(3*x + 2))
```

mupad [B] time = 2.40, size = 16, normalized size = 0.33

$$\frac{\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\tan\left(3x+2\right)\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(3*x + 2)^2*(2*tan(3*x + 2)^2 + 1)),x)
```

```
[Out] (2^(1/2)*atan(2^(1/2)*tan(3*x + 2)))/6
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec^{2}\left(3x+2\right)}{2\tan^{2}\left(3x+2\right)+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(2+3*x)**2/(1+2*tan(2+3*x)**2),x)
```

```
[Out] Integral(sec(3*x + 2)**2/(2*tan(3*x + 2)**2 + 1), x)
```

3.7 $\int \frac{\csc^2(2+3x)}{2+\cot^2(2+3x)}\,dx$

Optimal. Leaf size=48

$$\frac{x}{\sqrt{2}}+\frac{\tan^{-1}\left(\frac{\sin(3x+2)\cos(3x+2)}{\sin^2(3x+2)+\sqrt{2}+1}\right)}{3\sqrt{2}}$$

```
[Out] 1/2*x*2^(1/2)+1/6*arctan(cos(2+3*x)*sin(2+3*x)/(1+sin(2+3*x)^2+2^(1/2)))*2^
(1/2)
```

Rubi [A] time = 0.04, antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {3675, 203}

$$\frac{x}{\sqrt{2}}+\frac{\tan^{-1}\left(\frac{\sin(3x+2)\cos(3x+2)}{\sin^2(3x+2)+\sqrt{2}+1}\right)}{3\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[2 + 3*x]^2/(2 + Cot[2 + 3*x]^2),x]
```

```
[Out] x/Sqrt[2] + ArcTan[(Cos[2 + 3*x]*Sin[2 + 3*x])/(1 + Sqrt[2] + Sin[2 + 3*x]^
2)]/(3*Sqrt[2])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 3675

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/(c^(m - 1)*f), Subst[Int[(c^2 + ff^2*x^2)^(m/2 - 1)*(a + b*(ff*x)^n)^p
, x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && In
tegerQ[m/2] && (IntegersQ[n, p] || IGtQ[m, 0] || IGtQ[p, 0] || EqQ[n^2, 4]
|| EqQ[n^2, 16])
```

Rubi steps

$$\int \frac{\csc^2(2+3x)}{2+\cot^2(2+3x)}\,dx = -\left(\frac{1}{3}\operatorname{Subst}\left(\int \frac{1}{2+x^2}\,dx, x, \cot(2+3x)\right)\right)$$
$$= \frac{x}{\sqrt{2}} + \frac{\tan^{-1}\left(\frac{\cos(2+3x)\sin(2+3x)}{1+\sqrt{2}+\sin^2(2+3x)}\right)}{3\sqrt{2}}$$

Mathematica [**A**] time = 0.02, size = 22, normalized size = 0.46

$$\frac{\tan^{-1}\left(\sqrt{2}\tan(3x+2)\right)}{3\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[2 + 3*x]^2/(2 + Cot[2 + 3*x]^2),x]
```

```
[Out] ArcTan[Sqrt[2]*Tan[2 + 3*x]]/(3*Sqrt[2])
```

fricas [**A**] time = 0.64, size = 43, normalized size = 0.90

$$-\frac{1}{12}\sqrt{2}\arctan\left(\frac{3\sqrt{2}\cos(3x+2)^2 - 2\sqrt{2}}{4\cos(3x+2)\sin(3x+2)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(2+3*x)^2/(2+cot(2+3*x)^2),x, algorithm="fricas")
```

```
[Out] -1/12*sqrt(2)*arctan(1/4*(3*sqrt(2)*cos(3*x + 2)^2 - 2*sqrt(2))/(cos(3*x +
2)*sin(3*x + 2)))
```

giac [**A**] time = 0.28, size = 57, normalized size = 1.19

$$\frac{1}{6}\sqrt{2}\left(3x + \arctan\left(-\frac{\sqrt{2}\sin(6x+4) - 2\sin(6x+4)}{\sqrt{2}\cos(6x+4) + \sqrt{2} - 2\cos(6x+4) + 2}\right) + 2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(2+3*x)^2/(2+cot(2+3*x)^2),x, algorithm="giac")
```

```
[Out] 1/6*sqrt(2)*(3*x + arctan(-(sqrt(2)*sin(6*x + 4) - 2*sin(6*x + 4))/(sqrt(2)
*cos(6*x + 4) + sqrt(2) - 2*cos(6*x + 4) + 2)) + 2)
```

maple [A] time = 0.29, size = 17, normalized size = 0.35

$$\frac{\sqrt{2}\,\arctan\left(\sqrt{2}\,\tan\left(2+3x\right)\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(2+3*x)^2/(2+cot(2+3*x)^2),x)
[Out] 1/6*2^(1/2)*arctan(2^(1/2)*tan(2+3*x))
```

maxima [A] time = 0.61, size = 16, normalized size = 0.33

$$\frac{1}{6}\sqrt{2}\arctan\left(\sqrt{2}\tan\left(3x+2\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(2+3*x)^2/(2+cot(2+3*x)^2),x, algorithm="maxima")
[Out] 1/6*sqrt(2)*arctan(sqrt(2)*tan(3*x + 2))
```

mupad [B] time = 2.40, size = 16, normalized size = 0.33

$$\frac{\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\tan\left(3x+2\right)\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(3*x + 2)^2*(cot(3*x + 2)^2 + 2)),x)
[Out] (2^(1/2)*atan(2^(1/2)*tan(3*x + 2)))/6
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\csc^2\left(3x+2\right)}{\cot^2\left(3x+2\right)+2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(2+3*x)**2/(2+cot(2+3*x)**2),x)
[Out] Integral(csc(3*x + 2)**2/(cot(3*x + 2)**2 + 2), x)
```

3.8 $\int \frac{2}{1-3\cos(4+6x)}\,dx$

Optimal. Leaf size=60

$$\frac{\log\left(\cos(3x+2)-\sqrt{2}\sin(3x+2)\right)}{6\sqrt{2}}-\frac{\log\left(\sqrt{2}\sin(3x+2)+\cos(3x+2)\right)}{6\sqrt{2}}$$

```
[Out] 1/12*ln(cos(2+3*x)-sin(2+3*x)*2^(1/2))*2^(1/2)-1/12*ln(cos(2+3*x)+sin(2+3*x
)*2^(1/2))*2^(1/2)
```

Rubi [A] time = 0.03, antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {12, 2659, 207}

$$\frac{\log\left(\cos(3x+2)-\sqrt{2}\sin(3x+2)\right)}{6\sqrt{2}}-\frac{\log\left(\sqrt{2}\sin(3x+2)+\cos(3x+2)\right)}{6\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[2/(1 - 3*Cos[4 + 6*x]),x]
```

```
[Out] Log[Cos[2 + 3*x] - Sqrt[2]*Sin[2 + 3*x]]/(6*Sqrt[2]) - Log[Cos[2 + 3*x] + S
qrt[2]*Sin[2 + 3*x]]/(6*Sqrt[2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{2}{1-3\cos(4+6x)}\,dx = 2\int \frac{1}{1-3\cos(4+6x)}\,dx$$
$$= \frac{2}{3}\,\text{Subst}\left(\int \frac{1}{-2+4x^2}\,dx, x, \tan\left(\frac{1}{2}(4+6x)\right)\right)$$
$$= \frac{\log\left(\cos(2+3x)-\sqrt{2}\sin(2+3x)\right)}{6\sqrt{2}} - \frac{\log\left(\cos(2+3x)+\sqrt{2}\sin(2+3x)\right)}{6\sqrt{2}}$$

Mathematica [A] time = 0.04, size = 22, normalized size = 0.37

$$-\frac{\tanh^{-1}\left(\sqrt{2}\tan(3x+2)\right)}{3\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[2/(1 - 3*Cos[4 + 6*x]),x]
```

```
[Out] -1/3*ArcTanh[Sqrt[2]*Tan[2 + 3*x]]/Sqrt[2]
```

fricas [A] time = 0.51, size = 74, normalized size = 1.23

$$\frac{1}{24}\sqrt{2}\log\left(-\frac{7\cos\left(6x+4\right)^2-4\left(\sqrt{2}\cos\left(6x+4\right)-3\sqrt{2}\right)\sin\left(6x+4\right)+6\cos\left(6x+4\right)-17}{9\cos\left(6x+4\right)^2-6\cos\left(6x+4\right)+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(2/(1-3*cos(4+6*x)),x, algorithm="fricas")
```

```
[Out] 1/24*sqrt(2)*log(-(7*cos(6*x + 4)^2 - 4*(sqrt(2)*cos(6*x + 4) - 3*sqrt(2))*
sin(6*x + 4) + 6*cos(6*x + 4) - 17)/(9*cos(6*x + 4)^2 - 6*cos(6*x + 4) + 1)
)
```

giac [A] time = 0.17, size = 39, normalized size = 0.65

$$\frac{1}{12}\sqrt{2}\log\left(\frac{\left|-2\sqrt{2}+4\tan\left(3x+2\right)\right|}{\left|2\sqrt{2}+4\tan\left(3x+2\right)\right|}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(2/(1-3*cos(4+6*x)),x, algorithm="giac")
```

```
[Out] 1/12*sqrt(2)*log(abs(-2*sqrt(2) + 4*tan(3*x + 2))/abs(2*sqrt(2) + 4*tan(3*x
 + 2)))
```

maple [A] time = 0.06, size = 17, normalized size = 0.28

$$-\frac{\sqrt{2}\ \operatorname{arctanh}\left(\sqrt{2}\ \tan\left(2+3x\right)\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(2/(1-3*cos(4+6*x)),x)
```

```
[Out] -1/6*2^(1/2)*arctanh(2^(1/2)*tan(2+3*x))
```

maxima [A] time = 0.57, size = 54, normalized size = 0.90

$$\frac{1}{12}\sqrt{2}\log\left(-\frac{\sqrt{2}-\frac{2\sin\left(6x+4\right)}{\cos\left(6x+4\right)+1}}{\sqrt{2}+\frac{2\sin\left(6x+4\right)}{\cos\left(6x+4\right)+1}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(2/(1-3*cos(4+6*x)),x, algorithm="maxima")
```

```
[Out] 1/12*sqrt(2)*log(-(sqrt(2) - 2*sin(6*x + 4)/(cos(6*x + 4) + 1))/(sqrt(2) +
2*sin(6*x + 4)/(cos(6*x + 4) + 1)))
```

mupad [B] time = 2.57, size = 16, normalized size = 0.27

$$-\frac{\sqrt{2}\ \operatorname{atanh}\left(\sqrt{2}\ \tan\left(3\,x+2\right)\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-2/(3*cos(6*x + 4) - 1),x)
```

```
[Out] -(2^(1/2)*atanh(2^(1/2)*tan(3*x + 2)))/6
```

sympy [A] time = 0.31, size = 42, normalized size = 0.70

$$\frac{\sqrt{2}\log\left(\tan\left(3x+2\right)-\frac{\sqrt{2}}{2}\right)}{12}-\frac{\sqrt{2}\log\left(\tan\left(3x+2\right)+\frac{\sqrt{2}}{2}\right)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(2/(1-3*cos(4+6*x)),x)
```

```
[Out] sqrt(2)*log(tan(3*x + 2) - sqrt(2)/2)/12 - sqrt(2)*log(tan(3*x + 2) + sqrt(
2)/2)/12
```

3.9 $\int \frac{2\csc(4+6x)}{-3\cot(4+6x)+\csc(4+6x)}\,dx$

Optimal. Leaf size=60

$$\frac{\log\left(\cos(3x+2)-\sqrt{2}\sin(3x+2)\right)}{6\sqrt{2}}-\frac{\log\left(\sqrt{2}\sin(3x+2)+\cos(3x+2)\right)}{6\sqrt{2}}$$

```
[Out] 1/12*ln(cos(2+3*x)-sin(2+3*x)*2^(1/2))*2^(1/2)-1/12*ln(cos(2+3*x)+sin(2+3*x
)*2^(1/2))*2^(1/2)
```

Rubi [A] time = 0.05, antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {12, 3166, 2659, 207}

$$\frac{\log\left(\cos(3x+2)-\sqrt{2}\sin(3x+2)\right)}{6\sqrt{2}}-\frac{\log\left(\sqrt{2}\sin(3x+2)+\cos(3x+2)\right)}{6\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[(2*Csc[4 + 6*x])/(-3*Cot[4 + 6*x] + Csc[4 + 6*x]),x]
```

```
[Out] Log[Cos[2 + 3*x] - Sqrt[2]*Sin[2 + 3*x]]/(6*Sqrt[2]) - Log[Cos[2 + 3*x] + S
qrt[2]*Sin[2 + 3*x]]/(6*Sqrt[2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 3166

```
Int[csc[(d_.) + (e_.)*(x_)]^(n_.)*((a_.) + csc[(d_.) + (e_.)*(x_)]*(b_.) +
cot[(d_.) + (e_.)*(x_)]*(c_.))^(m_), x_Symbol] :> Int[1/(b + a*Sin[d + e*x]
 + c*Cos[d + e*x])^n, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[m + n, 0] && I
ntegerQ[n]
```

Rubi steps

$$\begin{aligned}
\int \frac{2\csc(4+6x)}{-3\cot(4+6x)+\csc(4+6x)}\,dx &= 2\int \frac{\csc(4+6x)}{-3\cot(4+6x)+\csc(4+6x)}\,dx \\
&= 2\int \frac{1}{1-3\cos(4+6x)}\,dx \\
&= \frac{2}{3}\,\text{Subst}\left(\int \frac{1}{-2+4x^2}\,dx, x, \tan\left(\frac{1}{2}(4+6x)\right)\right) \\
&= \frac{\log\left(\cos(2+3x)-\sqrt{2}\sin(2+3x)\right)}{6\sqrt{2}} - \frac{\log\left(\cos(2+3x)+\sqrt{2}\sin(2+3x)\right)}{6\sqrt{2}}
\end{aligned}$$

Mathematica [A] time = 0.04, size = 22, normalized size = 0.37

$$-\frac{\tanh^{-1}\left(\sqrt{2}\tan(3x+2)\right)}{3\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2*Csc[4 + 6*x])/(-3*Cot[4 + 6*x] + Csc[4 + 6*x]),x]
```

```
[Out] -1/3*ArcTanh[Sqrt[2]*Tan[2 + 3*x]]/Sqrt[2]
```

fricas [A] time = 0.91, size = 74, normalized size = 1.23

$$\frac{1}{24}\sqrt{2}\log\left(-\frac{7\cos\left(6x+4\right)^{2}-4\left(\sqrt{2}\cos\left(6x+4\right)-3\sqrt{2}\right)\sin\left(6x+4\right)+6\cos\left(6x+4\right)-17}{9\cos\left(6x+4\right)^{2}-6\cos\left(6x+4\right)+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(2*csc(4+6*x)/(-3*cot(4+6*x)+csc(4+6*x)),x, algorithm="fricas")
```

```
[Out] 1/24*sqrt(2)*log(-(7*cos(6*x + 4)^2 - 4*(sqrt(2)*cos(6*x + 4) - 3*sqrt(2))*
sin(6*x + 4) + 6*cos(6*x + 4) - 17)/(9*cos(6*x + 4)^2 - 6*cos(6*x + 4) + 1)
)
```

giac [A] time = 0.24, size = 39, normalized size = 0.65

$$\frac{1}{12}\,\sqrt{2}\,\log\left(\frac{\left|-2\,\sqrt{2}+4\,\tan\left(3\,x+2\right)\right|}{\left|2\,\sqrt{2}+4\,\tan\left(3\,x+2\right)\right|}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(2*csc(4+6*x)/(-3*cot(4+6*x)+csc(4+6*x)),x, algorithm="giac")
```

```
[Out] 1/12*sqrt(2)*log(abs(-2*sqrt(2) + 4*tan(3*x + 2))/abs(2*sqrt(2) + 4*tan(3*x
 + 2)))
```

maple [A] time = 0.30, size = 17, normalized size = 0.28

$$-\frac{\sqrt{2}\,\operatorname{arctanh}\left(\sqrt{2}\,\tan\left(2+3x\right)\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(2*csc(4+6*x)/(-3*cot(4+6*x)+csc(4+6*x)),x)
```

```
[Out] -1/6*2^(1/2)*arctanh(2^(1/2)*tan(2+3*x))
```

maxima [A] time = 0.49, size = 54, normalized size = 0.90

$$\frac{1}{12}\,\sqrt{2}\,\log\left(-\frac{\sqrt{2}-\frac{2\,\sin\left(6\,x+4\right)}{\cos\left(6\,x+4\right)+1}}{\sqrt{2}+\frac{2\,\sin\left(6\,x+4\right)}{\cos\left(6\,x+4\right)+1}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(2*csc(4+6*x)/(-3*cot(4+6*x)+csc(4+6*x)),x, algorithm="maxima")
```

```
[Out] 1/12*sqrt(2)*log(-(sqrt(2) - 2*sin(6*x + 4)/(cos(6*x + 4) + 1))/(sqrt(2) +
2*sin(6*x + 4)/(cos(6*x + 4) + 1)))
```

mupad [B] time = 2.62, size = 16, normalized size = 0.27

$$-\frac{\sqrt{2}\,\mathrm{atanh}\left(\sqrt{2}\,\tan\left(3\,x+2\right)\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-2/(sin(6*x + 4)*(3*cot(6*x + 4) - 1/sin(6*x + 4))),x)
```

```
[Out] -(2^(1/2)*atanh(2^(1/2)*tan(3*x + 2)))/6
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-2\int \frac{\csc(6x+4)}{3\cot(6x+4)-\csc(6x+4)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(2*csc(4+6*x)/(-3*cot(4+6*x)+csc(4+6*x)),x)
```

```
[Out] -2*Integral(csc(6*x + 4)/(3*cot(6*x + 4) - csc(6*x + 4)), x)
```

3.10 $\int \frac{1}{-1+3\sin^2(2+3x)}\,dx$

Optimal. Leaf size=60

$$\frac{\log\left(\cos(3x+2)-\sqrt{2}\sin(3x+2)\right)}{6\sqrt{2}}-\frac{\log\left(\sqrt{2}\sin(3x+2)+\cos(3x+2)\right)}{6\sqrt{2}}$$

```
[Out] 1/12*ln(cos(2+3*x)-sin(2+3*x)*2^(1/2))*2^(1/2)-1/12*ln(cos(2+3*x)+sin(2+3*x
)*2^(1/2))*2^(1/2)
```

Rubi [A] time = 0.02, antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3181, 207}

$$\frac{\log\left(\cos(3x+2)-\sqrt{2}\sin(3x+2)\right)}{6\sqrt{2}}-\frac{\log\left(\sqrt{2}\sin(3x+2)+\cos(3x+2)\right)}{6\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[(-1 + 3*Sin[2 + 3*x]^2)^(-1),x]
```

```
[Out] Log[Cos[2 + 3*x] - Sqrt[2]*Sin[2 + 3*x]]/(6*Sqrt[2]) - Log[Cos[2 + 3*x] + S
qrt[2]*Sin[2 + 3*x]]/(6*Sqrt[2])
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 3181

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(-1), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[1/(a + (a + b)*ff^2*x^2
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{-1+3\sin^2(2+3x)}\,dx &= \frac{1}{3}\text{Subst}\left(\int \frac{1}{-1+2x^2}\,dx, x, \tan(2+3x)\right)\\ &= \frac{\log\left(\cos(2+3x)-\sqrt{2}\sin(2+3x)\right)}{6\sqrt{2}}-\frac{\log\left(\cos(2+3x)+\sqrt{2}\sin(2+3x)\right)}{6\sqrt{2}}\end{aligned}$$

Mathematica [A] time = 0.06, size = 22, normalized size = 0.37

$$-\frac{\tanh^{-1}\left(\sqrt{2}\tan(3x+2)\right)}{3\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-1 + 3*Sin[2 + 3*x]^2)^(-1),x]
```

```
[Out] -1/3*ArcTanh[Sqrt[2]*Tan[2 + 3*x]]/Sqrt[2]
```

fricas [A] time = 0.65, size = 86, normalized size = 1.43

$$\frac{1}{24}\sqrt{2}\log\left(-\frac{7\cos(3x+2)^4-4\cos(3x+2)^2-4\left(\sqrt{2}\cos(3x+2)^3-2\sqrt{2}\cos(3x+2)\right)\sin(3x+2)-4}{9\cos(3x+2)^4-12\cos(3x+2)^2+4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-1+3*sin(2+3*x)^2),x, algorithm="fricas")
```

```
[Out] 1/24*sqrt(2)*log(-(7*cos(3*x + 2)^4 - 4*cos(3*x + 2)^2 - 4*(sqrt(2)*cos(3*x
 + 2)^3 - 2*sqrt(2)*cos(3*x + 2))*sin(3*x + 2) - 4)/(9*cos(3*x + 2)^4 - 12*
cos(3*x + 2)^2 + 4))
```

giac [A] time = 0.16, size = 39, normalized size = 0.65

$$\frac{1}{12}\sqrt{2}\log\left(\frac{\left|-2\sqrt{2}+4\tan(3x+2)\right|}{\left|2\sqrt{2}+4\tan(3x+2)\right|}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-1+3*sin(2+3*x)^2),x, algorithm="giac")
```

```
[Out] 1/12*sqrt(2)*log(abs(-2*sqrt(2) + 4*tan(3*x + 2))/abs(2*sqrt(2) + 4*tan(3*x
 + 2)))
```

maple [A] time = 0.15, size = 17, normalized size = 0.28

$$-\frac{\sqrt{2}\operatorname{arctanh}\left(\sqrt{2}\tan(2+3x)\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-1+3*sin(2+3*x)^2),x)
```

```
[Out] -1/6*2^(1/2)*arctanh(2^(1/2)*tan(2+3*x))
```

maxima [A] time = 0.41, size = 34, normalized size = 0.57

$$\frac{1}{12}\,\sqrt{2}\log\left(-\frac{\sqrt{2}-2\,\tan\left(3\,x+2\right)}{\sqrt{2}+2\,\tan\left(3\,x+2\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-1+3*sin(2+3*x)^2),x, algorithm="maxima")
```

```
[Out] 1/12*sqrt(2)*log(-(sqrt(2) - 2*tan(3*x + 2))/(sqrt(2) + 2*tan(3*x + 2)))
```

mupad [B] time = 2.45, size = 16, normalized size = 0.27

$$-\frac{\sqrt{2}\,\mathrm{atanh}\left(\sqrt{2}\,\tan\left(3\,x+2\right)\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(3*sin(3*x + 2)^2 - 1),x)
```

```
[Out] -(2^(1/2)*atanh(2^(1/2)*tan(3*x + 2)))/6
```

sympy [B] time = 18.47, size = 1644, normalized size = 27.40

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-1+3*sin(2+3*x)**2),x)
```

```
[Out] -1387702511766624*sqrt(5 - 2*sqrt(6))*log(tan(3*x/2 + 1) - sqrt(5 - 2*sqrt(
6)))/(-467972363532675 - 191048917396548*sqrt(6) + 13665597568857156*sqrt(6
)*sqrt(5 - 2*sqrt(6))*sqrt(2*sqrt(6) + 5) + 33473741073918339*sqrt(5 - 2*sq
rt(6))*sqrt(2*sqrt(6) + 5)) - 566527178101133*sqrt(6)*sqrt(5 - 2*sqrt(6))*l
og(tan(3*x/2 + 1) - sqrt(5 - 2*sqrt(6)))/(-467972363532675 - 19104891739654
8*sqrt(6) + 13665597568857156*sqrt(6)*sqrt(5 - 2*sqrt(6))*sqrt(2*sqrt(6) +
5) + 33473741073918339*sqrt(5 - 2*sqrt(6))*sqrt(2*sqrt(6) + 5)) + 137649929
5618884*sqrt(2*sqrt(6) + 5)*log(tan(3*x/2 + 1) - sqrt(5 - 2*sqrt(6)))/(-467
972363532675 - 191048917396548*sqrt(6) + 13665597568857156*sqrt(6)*sqrt(5 -
 2*sqrt(6))*sqrt(2*sqrt(6) + 5) + 33473741073918339*sqrt(5 - 2*sqrt(6))*sqr
t(2*sqrt(6) + 5)) + 561953484261121*sqrt(6)*sqrt(2*sqrt(6) + 5)*log(tan(3*x
/2 + 1) - sqrt(5 - 2*sqrt(6)))/(-467972363532675 - 191048917396548*sqrt(6)
+ 13665597568857156*sqrt(6)*sqrt(5 - 2*sqrt(6))*sqrt(2*sqrt(6) + 5) + 33473
741073918339*sqrt(5 - 2*sqrt(6))*sqrt(2*sqrt(6) + 5)) - 1247944371758796*sq
rt(2*sqrt(6) + 5)*log(tan(3*x/2 + 1) + sqrt(5 - 2*sqrt(6)))/(-4679723635326
75 - 191048917396548*sqrt(6) + 13665597568857156*sqrt(6)*sqrt(5 - 2*sqrt(6)
```

```
)*sqrt(2*sqrt(6) + 5) + 33473741073918339*sqrt(5 - 2*sqrt(6))*sqrt(2*sqrt(6
) + 5)) - 509471156364528*sqrt(6)*sqrt(2*sqrt(6) + 5)*log(tan(3*x/2 + 1) +
sqrt(5 - 2*sqrt(6)))/(-467972363532675 - 191048917396548*sqrt(6) + 13665597
568857156*sqrt(6)*sqrt(5 - 2*sqrt(6))*sqrt(2*sqrt(6) + 5) + 334737410739183
39*sqrt(5 - 2*sqrt(6))*sqrt(2*sqrt(6) + 5)) + 47005690897992*sqrt(6)*sqrt(5
 - 2*sqrt(6))*log(tan(3*x/2 + 1) + sqrt(5 - 2*sqrt(6)))/(-467972363532675 -
 191048917396548*sqrt(6) + 13665597568857156*sqrt(6)*sqrt(5 - 2*sqrt(6))*sq
rt(2*sqrt(6) + 5) + 33473741073918339*sqrt(5 - 2*sqrt(6))*sqrt(2*sqrt(6) +
5)) + 115139957707068*sqrt(5 - 2*sqrt(6))*log(tan(3*x/2 + 1) + sqrt(5 - 2*s
qrt(6)))/(-467972363532675 - 191048917396548*sqrt(6) + 13665597568857156*sq
rt(6)*sqrt(5 - 2*sqrt(6))*sqrt(2*sqrt(6) + 5) + 33473741073918339*sqrt(5 -
2*sqrt(6))*sqrt(2*sqrt(6) + 5)) - 12353375735168316*sqrt(5 - 2*sqrt(6))*log
(tan(3*x/2 + 1) - sqrt(2*sqrt(6) + 5))/(-467972363532675 - 191048917396548*
sqrt(6) + 13665597568857156*sqrt(6)*sqrt(5 - 2*sqrt(6))*sqrt(2*sqrt(6) + 5)
 + 33473741073918339*sqrt(5 - 2*sqrt(6))*sqrt(2*sqrt(6) + 5)) - 50432445253
40232*sqrt(6)*sqrt(5 - 2*sqrt(6))*log(tan(3*x/2 + 1) - sqrt(2*sqrt(6) + 5))
/(-467972363532675 - 191048917396548*sqrt(6) + 13665597568857156*sqrt(6)*sq
rt(5 - 2*sqrt(6))*sqrt(2*sqrt(6) + 5) + 33473741073918339*sqrt(5 - 2*sqrt(6
))*sqrt(2*sqrt(6) + 5)) + 4748539075824*sqrt(6)*sqrt(2*sqrt(6) + 5)*log(tan
(3*x/2 + 1) - sqrt(2*sqrt(6) + 5))/(-467972363532675 - 191048917396548*sqrt
(6) + 13665597568857156*sqrt(6)*sqrt(5 - 2*sqrt(6))*sqrt(2*sqrt(6) + 5) + 3
3473741073918339*sqrt(5 - 2*sqrt(6))*sqrt(2*sqrt(6) + 5)) + 11631497759436*
sqrt(2*sqrt(6) + 5)*log(tan(3*x/2 + 1) - sqrt(2*sqrt(6) + 5))/(-46797236353
2675 - 191048917396548*sqrt(6) + 13665597568857156*sqrt(6)*sqrt(5 - 2*sqrt(
6))*sqrt(2*sqrt(6) + 5) + 33473741073918339*sqrt(5 - 2*sqrt(6))*sqrt(2*sqrt
(6) + 5)) - 140186421619524*sqrt(2*sqrt(6) + 5)*log(tan(3*x/2 + 1) + sqrt(2
*sqrt(6) + 5))/(-467972363532675 - 191048917396548*sqrt(6) + 13665597568857
156*sqrt(6)*sqrt(5 - 2*sqrt(6))*sqrt(2*sqrt(6) + 5) + 33473741073918339*sqr
t(5 - 2*sqrt(6))*sqrt(2*sqrt(6) + 5)) - 57230866972417*sqrt(6)*sqrt(2*sqrt(
6) + 5)*log(tan(3*x/2 + 1) + sqrt(2*sqrt(6) + 5))/(-467972363532675 - 19104
8917396548*sqrt(6) + 13665597568857156*sqrt(6)*sqrt(5 - 2*sqrt(6))*sqrt(2*s
qrt(6) + 5) + 33473741073918339*sqrt(5 - 2*sqrt(6))*sqrt(2*sqrt(6) + 5)) +
13625938289227872*sqrt(5 - 2*sqrt(6))*log(tan(3*x/2 + 1) + sqrt(2*sqrt(6) +
 5))/(-467972363532675 - 191048917396548*sqrt(6) + 13665597568857156*sqrt(6
)*sqrt(5 - 2*sqrt(6))*sqrt(2*sqrt(6) + 5) + 33473741073918339*sqrt(5 - 2*sq
rt(6))*sqrt(2*sqrt(6) + 5)) + 5562766012543373*sqrt(6)*sqrt(5 - 2*sqrt(6))*
log(tan(3*x/2 + 1) + sqrt(2*sqrt(6) + 5))/(-467972363532675 - 1910489173965
48*sqrt(6) + 13665597568857156*sqrt(6)*sqrt(5 - 2*sqrt(6))*sqrt(2*sqrt(6) +
 5) + 33473741073918339*sqrt(5 - 2*sqrt(6))*sqrt(2*sqrt(6) + 5))
```

3.11 $\int \frac{1}{2-3\cos^2(2+3x)} dx$

Optimal. Leaf size=60

$$\frac{\log\left(\cos(3x+2)-\sqrt{2}\sin(3x+2)\right)}{6\sqrt{2}}-\frac{\log\left(\sqrt{2}\sin(3x+2)+\cos(3x+2)\right)}{6\sqrt{2}}$$

```
[Out] 1/12*ln(cos(2+3*x)-sin(2+3*x)*2^(1/2))*2^(1/2)-1/12*ln(cos(2+3*x)+sin(2+3*x
)*2^(1/2))*2^(1/2)
```

Rubi [A] time = 0.02, antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3181, 206}

$$\frac{\log\left(\cos(3x+2)-\sqrt{2}\sin(3x+2)\right)}{6\sqrt{2}}-\frac{\log\left(\sqrt{2}\sin(3x+2)+\cos(3x+2)\right)}{6\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[(2 - 3*Cos[2 + 3*x]^2)^(-1),x]
```

```
[Out] Log[Cos[2 + 3*x] - Sqrt[2]*Sin[2 + 3*x]]/(6*Sqrt[2]) - Log[Cos[2 + 3*x] + S
qrt[2]*Sin[2 + 3*x]]/(6*Sqrt[2])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3181

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(-1), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[1/(a + (a + b)*ff^2*x^2
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{2-3\cos^2(2+3x)}\,dx &= -\left(\frac{1}{3}\operatorname{Subst}\left(\int \frac{1}{2-x^2}\,dx, x, \cot(2+3x)\right)\right)\\ &= \frac{\log\left(\cos(2+3x)-\sqrt{2}\sin(2+3x)\right)}{6\sqrt{2}}-\frac{\log\left(\cos(2+3x)+\sqrt{2}\sin(2+3x)\right)}{6\sqrt{2}}\end{aligned}$$

Mathematica [A] time = 0.07, size = 22, normalized size = 0.37

$$-\frac{\tanh^{-1}\left(\sqrt{2}\tan(3x+2)\right)}{3\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 - 3*Cos[2 + 3*x]^2)^(-1),x]
```

```
[Out] -1/3*ArcTanh[Sqrt[2]*Tan[2 + 3*x]]/Sqrt[2]
```

fricas [A] time = 0.57, size = 86, normalized size = 1.43

$$\frac{1}{24}\sqrt{2}\log\left(-\frac{7\cos(3x+2)^4 - 4\cos(3x+2)^2 - 4\left(\sqrt{2}\cos(3x+2)^3 - 2\sqrt{2}\cos(3x+2)\right)\sin(3x+2) - 4}{9\cos(3x+2)^4 - 12\cos(3x+2)^2 + 4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2-3*cos(2+3*x)^2),x, algorithm="fricas")
```

```
[Out] 1/24*sqrt(2)*log(-(7*cos(3*x + 2)^4 - 4*cos(3*x + 2)^2 - 4*(sqrt(2)*cos(3*x
 + 2)^3 - 2*sqrt(2)*cos(3*x + 2))*sin(3*x + 2) - 4)/(9*cos(3*x + 2)^4 - 12*
cos(3*x + 2)^2 + 4))
```

giac [A] time = 0.17, size = 39, normalized size = 0.65

$$\frac{1}{12}\sqrt{2}\log\left(\frac{\left|-2\sqrt{2} + 4\tan(3x+2)\right|}{\left|2\sqrt{2} + 4\tan(3x+2)\right|}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2-3*cos(2+3*x)^2),x, algorithm="giac")
```

```
[Out] 1/12*sqrt(2)*log(abs(-2*sqrt(2) + 4*tan(3*x + 2))/abs(2*sqrt(2) + 4*tan(3*x
 + 2)))
```

maple [A] time = 0.10, size = 17, normalized size = 0.28

$$-\frac{\sqrt{2}\,\operatorname{arctanh}\left(\sqrt{2}\tan(2+3x)\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2-3*cos(2+3*x)^2),x)
```

```
[Out] -1/6*2^(1/2)*arctanh(2^(1/2)*tan(2+3*x))
```

maxima [A] time = 0.50, size = 34, normalized size = 0.57

$$\frac{1}{12}\sqrt{2}\log\left(-\frac{\sqrt{2}-2\tan\left(3x+2\right)}{\sqrt{2}+2\tan\left(3x+2\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2-3*cos(2+3*x)^2),x, algorithm="maxima")
```

```
[Out] 1/12*sqrt(2)*log(-(sqrt(2) - 2*tan(3*x + 2))/(sqrt(2) + 2*tan(3*x + 2)))
```

mupad [B] time = 2.44, size = 16, normalized size = 0.27

$$-\frac{\sqrt{2}\,\mathrm{atanh}\left(\sqrt{2}\tan\left(3x+2\right)\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-1/(3*cos(3*x + 2)^2 - 2),x)
```

```
[Out] -(2^(1/2)*atanh(2^(1/2)*tan(3*x + 2)))/6
```

sympy [B] time = 14.40, size = 1644, normalized size = 27.40

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2-3*cos(2+3*x)**2),x)
```

```
[Out] -1387702511766624*sqrt(5 - 2*sqrt(6))*log(tan(3*x/2 + 1) - sqrt(5 - 2*sqrt(
6)))/(-467972363532675 - 191048917396548*sqrt(6) + 13665597568857156*sqrt(6
)*sqrt(5 - 2*sqrt(6))*sqrt(2*sqrt(6) + 5) + 33473741073918339*sqrt(5 - 2*sq
rt(6))*sqrt(2*sqrt(6) + 5)) - 566527178101133*sqrt(6)*sqrt(5 - 2*sqrt(6))*l
og(tan(3*x/2 + 1) - sqrt(5 - 2*sqrt(6)))/(-467972363532675 - 19104891739654
8*sqrt(6) + 13665597568857156*sqrt(6)*sqrt(5 - 2*sqrt(6))*sqrt(2*sqrt(6) +
5) + 33473741073918339*sqrt(5 - 2*sqrt(6))*sqrt(2*sqrt(6) + 5)) + 137649929
5618884*sqrt(2*sqrt(6) + 5)*log(tan(3*x/2 + 1) - sqrt(5 - 2*sqrt(6)))/(-467
972363532675 - 191048917396548*sqrt(6) + 13665597568857156*sqrt(6)*sqrt(5 -
 2*sqrt(6))*sqrt(2*sqrt(6) + 5) + 33473741073918339*sqrt(5 - 2*sqrt(6))*sqr
t(2*sqrt(6) + 5)) + 561953484261121*sqrt(6)*sqrt(2*sqrt(6) + 5)*log(tan(3*x
/2 + 1) - sqrt(5 - 2*sqrt(6)))/(-467972363532675 - 191048917396548*sqrt(6)
+ 13665597568857156*sqrt(6)*sqrt(5 - 2*sqrt(6))*sqrt(2*sqrt(6) + 5) + 33473
741073918339*sqrt(5 - 2*sqrt(6))*sqrt(2*sqrt(6) + 5)) - 1247944371758796*sq
rt(2*sqrt(6) + 5)*log(tan(3*x/2 + 1) + sqrt(5 - 2*sqrt(6)))/(-4679723635326
75 - 191048917396548*sqrt(6) + 13665597568857156*sqrt(6)*sqrt(5 - 2*sqrt(6)
```

```
)*sqrt(2*sqrt(6) + 5) + 33473741073918339*sqrt(5 - 2*sqrt(6))*sqrt(2*sqrt(6
) + 5)) - 509471156364528*sqrt(6)*sqrt(2*sqrt(6) + 5)*log(tan(3*x/2 + 1) +
sqrt(5 - 2*sqrt(6)))/(-467972363532675 - 191048917396548*sqrt(6) + 13665597
568857156*sqrt(6)*sqrt(5 - 2*sqrt(6))*sqrt(2*sqrt(6) + 5) + 334737410739183
39*sqrt(5 - 2*sqrt(6))*sqrt(2*sqrt(6) + 5)) + 47005690897992*sqrt(6)*sqrt(5
 - 2*sqrt(6))*log(tan(3*x/2 + 1) + sqrt(5 - 2*sqrt(6)))/(-467972363532675 -
 191048917396548*sqrt(6) + 13665597568857156*sqrt(6)*sqrt(5 - 2*sqrt(6))*sq
rt(2*sqrt(6) + 5) + 33473741073918339*sqrt(5 - 2*sqrt(6))*sqrt(2*sqrt(6) +
5)) + 115139957707068*sqrt(5 - 2*sqrt(6))*log(tan(3*x/2 + 1) + sqrt(5 - 2*s
qrt(6)))/(-467972363532675 - 191048917396548*sqrt(6) + 13665597568857156*sq
rt(6)*sqrt(5 - 2*sqrt(6))*sqrt(2*sqrt(6) + 5) + 33473741073918339*sqrt(5 -
2*sqrt(6))*sqrt(2*sqrt(6) + 5)) - 12353375735168316*sqrt(5 - 2*sqrt(6))*log
(tan(3*x/2 + 1) - sqrt(2*sqrt(6) + 5))/(-467972363532675 - 191048917396548*
sqrt(6) + 13665597568857156*sqrt(6)*sqrt(5 - 2*sqrt(6))*sqrt(2*sqrt(6) + 5)
 + 33473741073918339*sqrt(5 - 2*sqrt(6))*sqrt(2*sqrt(6) + 5)) - 50432445253
40232*sqrt(6)*sqrt(5 - 2*sqrt(6))*log(tan(3*x/2 + 1) - sqrt(2*sqrt(6) + 5))
/(-467972363532675 - 191048917396548*sqrt(6) + 13665597568857156*sqrt(6)*sq
rt(5 - 2*sqrt(6))*sqrt(2*sqrt(6) + 5) + 33473741073918339*sqrt(5 - 2*sqrt(6
))*sqrt(2*sqrt(6) + 5)) + 4748539075824*sqrt(6)*sqrt(2*sqrt(6) + 5)*log(tan
(3*x/2 + 1) - sqrt(2*sqrt(6) + 5))/(-467972363532675 - 191048917396548*sqrt
(6) + 13665597568857156*sqrt(6)*sqrt(5 - 2*sqrt(6))*sqrt(2*sqrt(6) + 5) + 3
3473741073918339*sqrt(5 - 2*sqrt(6))*sqrt(2*sqrt(6) + 5)) + 11631497759436*
sqrt(2*sqrt(6) + 5)*log(tan(3*x/2 + 1) - sqrt(2*sqrt(6) + 5))/(-46797236353
2675 - 191048917396548*sqrt(6) + 13665597568857156*sqrt(6)*sqrt(5 - 2*sqrt(
6))*sqrt(2*sqrt(6) + 5) + 33473741073918339*sqrt(5 - 2*sqrt(6))*sqrt(2*sqrt
(6) + 5)) - 140186421619524*sqrt(2*sqrt(6) + 5)*log(tan(3*x/2 + 1) + sqrt(2
*sqrt(6) + 5))/(-467972363532675 - 191048917396548*sqrt(6) + 13665597568857
156*sqrt(6)*sqrt(5 - 2*sqrt(6))*sqrt(2*sqrt(6) + 5) + 33473741073918339*sqr
t(5 - 2*sqrt(6))*sqrt(2*sqrt(6) + 5)) - 57230866972417*sqrt(6)*sqrt(2*sqrt(
6) + 5)*log(tan(3*x/2 + 1) + sqrt(2*sqrt(6) + 5))/(-467972363532675 - 19104
8917396548*sqrt(6) + 13665597568857156*sqrt(6)*sqrt(5 - 2*sqrt(6))*sqrt(2*s
qrt(6) + 5) + 33473741073918339*sqrt(5 - 2*sqrt(6))*sqrt(2*sqrt(6) + 5)) +
13625938289227872*sqrt(5 - 2*sqrt(6))*log(tan(3*x/2 + 1) + sqrt(2*sqrt(6) +
 5))/(-467972363532675 - 191048917396548*sqrt(6) + 13665597568857156*sqrt(6
)*sqrt(5 - 2*sqrt(6))*sqrt(2*sqrt(6) + 5) + 33473741073918339*sqrt(5 - 2*sq
rt(6))*sqrt(2*sqrt(6) + 5)) + 5562766012543373*sqrt(6)*sqrt(5 - 2*sqrt(6))*
log(tan(3*x/2 + 1) + sqrt(2*sqrt(6) + 5))/(-467972363532675 - 1910489173965
48*sqrt(6) + 13665597568857156*sqrt(6)*sqrt(5 - 2*sqrt(6))*sqrt(2*sqrt(6) +
 5) + 33473741073918339*sqrt(5 - 2*sqrt(6))*sqrt(2*sqrt(6) + 5))
```

3.12 $\int \frac{1}{-\cos^2(2+3x)+2\sin^2(2+3x)}\,dx$

Optimal. Leaf size=60

$$\frac{\log\left(\cos(3x+2)-\sqrt{2}\sin(3x+2)\right)}{6\sqrt{2}}-\frac{\log\left(\sqrt{2}\sin(3x+2)+\cos(3x+2)\right)}{6\sqrt{2}}$$

```
[Out] 1/12*ln(cos(2+3*x)-sin(2+3*x)*2^(1/2))*2^(1/2)-1/12*ln(cos(2+3*x)+sin(2+3*x
)*2^(1/2))*2^(1/2)
```

Rubi [A] time = 0.03, antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.043, Rules used = {207}

$$\frac{\log\left(\cos(3x+2)-\sqrt{2}\sin(3x+2)\right)}{6\sqrt{2}}-\frac{\log\left(\sqrt{2}\sin(3x+2)+\cos(3x+2)\right)}{6\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[(-Cos[2 + 3*x]^2 + 2*Sin[2 + 3*x]^2)^(-1),x]
```

```
[Out] Log[Cos[2 + 3*x] - Sqrt[2]*Sin[2 + 3*x]]/(6*Sqrt[2]) - Log[Cos[2 + 3*x] + S
qrt[2]*Sin[2 + 3*x]]/(6*Sqrt[2])
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{-\cos^2(2+3x)+2\sin^2(2+3x)}\,dx &= \frac{1}{3}\text{Subst}\left(\int \frac{1}{-1+2x^2}\,dx, x, \tan(2+3x)\right)\\ &= \frac{\log\left(\cos(2+3x)-\sqrt{2}\sin(2+3x)\right)}{6\sqrt{2}}-\frac{\log\left(\cos(2+3x)+\sqrt{2}\sin(2+\right.}{6\sqrt{2}}\end{aligned}$$

Mathematica [A] time = 0.03, size = 22, normalized size = 0.37

$$-\frac{\tanh^{-1}\left(\sqrt{2}\tan(3x+2)\right)}{3\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-Cos[2 + 3*x]^2 + 2*Sin[2 + 3*x]^2)^(-1),x]
```

```
[Out] -1/3*ArcTanh[Sqrt[2]*Tan[2 + 3*x]]/Sqrt[2]
```

fricas [A] time = 0.78, size = 86, normalized size = 1.43

$$\frac{1}{24}\sqrt{2}\log\left(-\frac{7\cos\left(3x+2\right)^{4}-4\cos\left(3x+2\right)^{2}-4\left(\sqrt{2}\cos\left(3x+2\right)^{3}-2\sqrt{2}\cos\left(3x+2\right)\right)\sin\left(3x+2\right)-4}{9\cos\left(3x+2\right)^{4}-12\cos\left(3x+2\right)^{2}+4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-cos(2+3*x)^2+2*sin(2+3*x)^2),x, algorithm="fricas")
```

```
[Out] 1/24*sqrt(2)*log(-(7*cos(3*x + 2)^4 - 4*cos(3*x + 2)^2 - 4*(sqrt(2)*cos(3*x
 + 2)^3 - 2*sqrt(2)*cos(3*x + 2))*sin(3*x + 2) - 4)/(9*cos(3*x + 2)^4 - 12*
cos(3*x + 2)^2 + 4))
```

giac [A] time = 0.20, size = 39, normalized size = 0.65

$$\frac{1}{12}\sqrt{2}\log\left(\frac{\left|-2\sqrt{2}+4\tan\left(3x+2\right)\right|}{\left|2\sqrt{2}+4\tan\left(3x+2\right)\right|}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-cos(2+3*x)^2+2*sin(2+3*x)^2),x, algorithm="giac")
```

```
[Out] 1/12*sqrt(2)*log(abs(-2*sqrt(2) + 4*tan(3*x + 2))/abs(2*sqrt(2) + 4*tan(3*x
 + 2)))
```

maple [A] time = 0.22, size = 17, normalized size = 0.28

$$-\frac{\sqrt{2}\,\operatorname{arctanh}\left(\sqrt{2}\,\tan\left(2+3x\right)\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-cos(2+3*x)^2+2*sin(2+3*x)^2),x)
```

```
[Out] -1/6*2^(1/2)*arctanh(2^(1/2)*tan(2+3*x))
```

maxima [A] time = 0.41, size = 34, normalized size = 0.57

$$\frac{1}{12}\sqrt{2}\log\left(-\frac{\sqrt{2}-2\tan\left(3x+2\right)}{\sqrt{2}+2\tan\left(3x+2\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-cos(2+3*x)^2+2*sin(2+3*x)^2),x, algorithm="maxima")
```

```
[Out] 1/12*sqrt(2)*log(-(sqrt(2) - 2*tan(3*x + 2))/(sqrt(2) + 2*tan(3*x + 2)))
```

mupad [B] time = 2.40, size = 16, normalized size = 0.27

$$-\frac{\sqrt{2}\,\mathrm{atanh}\left(\sqrt{2}\,\tan\left(3\,x+2\right)\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2*sin(3*x + 2)^2 - cos(3*x + 2)^2),x)
```

```
[Out] -(2^(1/2)*atanh(2^(1/2)*tan(3*x + 2)))/6
```

sympy [B] time = 17.09, size = 1644, normalized size = 27.40

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-cos(2+3*x)**2+2*sin(2+3*x)**2),x)
```

```
[Out] -1387702511766624*sqrt(5 - 2*sqrt(6))*log(tan(3*x/2 + 1) - sqrt(5 - 2*sqrt(
6)))/(-467972363532675 - 191048917396548*sqrt(6) + 13665597568857156*sqrt(6
)*sqrt(5 - 2*sqrt(6))*sqrt(2*sqrt(6) + 5) + 33473741073918339*sqrt(5 - 2*sq
rt(6))*sqrt(2*sqrt(6) + 5)) - 566527178101133*sqrt(6)*sqrt(5 - 2*sqrt(6))*l
og(tan(3*x/2 + 1) - sqrt(5 - 2*sqrt(6)))/(-467972363532675 - 19104891739654
8*sqrt(6) + 13665597568857156*sqrt(6)*sqrt(5 - 2*sqrt(6))*sqrt(2*sqrt(6) +
5) + 33473741073918339*sqrt(5 - 2*sqrt(6))*sqrt(2*sqrt(6) + 5)) + 137649929
5618884*sqrt(2*sqrt(6) + 5)*log(tan(3*x/2 + 1) - sqrt(5 - 2*sqrt(6)))/(-467
972363532675 - 191048917396548*sqrt(6) + 13665597568857156*sqrt(6)*sqrt(5 -
 2*sqrt(6))*sqrt(2*sqrt(6) + 5) + 33473741073918339*sqrt(5 - 2*sqrt(6))*sqr
t(2*sqrt(6) + 5)) + 561953484261121*sqrt(6)*sqrt(2*sqrt(6) + 5)*log(tan(3*x
/2 + 1) - sqrt(5 - 2*sqrt(6)))/(-467972363532675 - 191048917396548*sqrt(6)
+ 13665597568857156*sqrt(6)*sqrt(5 - 2*sqrt(6))*sqrt(2*sqrt(6) + 5) + 33473
741073918339*sqrt(5 - 2*sqrt(6))*sqrt(2*sqrt(6) + 5)) - 1247944371758796*sq
rt(2*sqrt(6) + 5)*log(tan(3*x/2 + 1) + sqrt(5 - 2*sqrt(6)))/(-4679723635326
75 - 191048917396548*sqrt(6) + 13665597568857156*sqrt(6)*sqrt(5 - 2*sqrt(6)
)*sqrt(2*sqrt(6) + 5) + 33473741073918339*sqrt(5 - 2*sqrt(6))*sqrt(2*sqrt(6
) + 5)) - 509471156364528*sqrt(6)*sqrt(2*sqrt(6) + 5)*log(tan(3*x/2 + 1) +
sqrt(5 - 2*sqrt(6)))/(-467972363532675 - 191048917396548*sqrt(6) + 13665597
568857156*sqrt(6)*sqrt(5 - 2*sqrt(6))*sqrt(2*sqrt(6) + 5) + 334737410739183
39*sqrt(5 - 2*sqrt(6))*sqrt(2*sqrt(6) + 5)) + 47005690897992*sqrt(6)*sqrt(5
 - 2*sqrt(6))*log(tan(3*x/2 + 1) + sqrt(5 - 2*sqrt(6)))/(-467972363532675 -
```

```
 191048917396548*sqrt(6) + 13665597568857156*sqrt(6)*sqrt(5 - 2*sqrt(6))*sq
rt(2*sqrt(6) + 5) + 33473741073918339*sqrt(5 - 2*sqrt(6))*sqrt(2*sqrt(6) +
5)) + 115139957707068*sqrt(5 - 2*sqrt(6))*log(tan(3*x/2 + 1) + sqrt(5 - 2*s
qrt(6)))/(-467972363532675 - 191048917396548*sqrt(6) + 13665597568857156*sq
rt(6)*sqrt(5 - 2*sqrt(6))*sqrt(2*sqrt(6) + 5) + 33473741073918339*sqrt(5 -
2*sqrt(6))*sqrt(2*sqrt(6) + 5)) - 12353375735168316*sqrt(5 - 2*sqrt(6))*log
(tan(3*x/2 + 1) - sqrt(2*sqrt(6) + 5))/(-467972363532675 - 191048917396548*
sqrt(6) + 13665597568857156*sqrt(6)*sqrt(5 - 2*sqrt(6))*sqrt(2*sqrt(6) + 5)
 + 33473741073918339*sqrt(5 - 2*sqrt(6))*sqrt(2*sqrt(6) + 5)) - 50432445253
40232*sqrt(6)*sqrt(5 - 2*sqrt(6))*log(tan(3*x/2 + 1) - sqrt(2*sqrt(6) + 5))
/(-467972363532675 - 191048917396548*sqrt(6) + 13665597568857156*sqrt(6)*sq
rt(5 - 2*sqrt(6))*sqrt(2*sqrt(6) + 5) + 33473741073918339*sqrt(5 - 2*sqrt(6
))*sqrt(2*sqrt(6) + 5)) + 4748539075824*sqrt(6)*sqrt(2*sqrt(6) + 5)*log(tan
(3*x/2 + 1) - sqrt(2*sqrt(6) + 5))/(-467972363532675 - 191048917396548*sqrt
(6) + 13665597568857156*sqrt(6)*sqrt(5 - 2*sqrt(6))*sqrt(2*sqrt(6) + 5) + 3
3473741073918339*sqrt(5 - 2*sqrt(6))*sqrt(2*sqrt(6) + 5)) + 11631497759436*
sqrt(2*sqrt(6) + 5)*log(tan(3*x/2 + 1) - sqrt(2*sqrt(6) + 5))/(-46797236353
2675 - 191048917396548*sqrt(6) + 13665597568857156*sqrt(6)*sqrt(5 - 2*sqrt(
6))*sqrt(2*sqrt(6) + 5) + 33473741073918339*sqrt(5 - 2*sqrt(6))*sqrt(2*sqrt
(6) + 5)) - 140186421619524*sqrt(2*sqrt(6) + 5)*log(tan(3*x/2 + 1) + sqrt(2
*sqrt(6) + 5))/(-467972363532675 - 191048917396548*sqrt(6) + 13665597568857
156*sqrt(6)*sqrt(5 - 2*sqrt(6))*sqrt(2*sqrt(6) + 5) + 33473741073918339*sqr
t(5 - 2*sqrt(6))*sqrt(2*sqrt(6) + 5)) - 57230866972417*sqrt(6)*sqrt(2*sqrt(
6) + 5)*log(tan(3*x/2 + 1) + sqrt(2*sqrt(6) + 5))/(-467972363532675 - 19104
8917396548*sqrt(6) + 13665597568857156*sqrt(6)*sqrt(5 - 2*sqrt(6))*sqrt(2*s
qrt(6) + 5) + 33473741073918339*sqrt(5 - 2*sqrt(6))*sqrt(2*sqrt(6) + 5)) +
13625938289227872*sqrt(5 - 2*sqrt(6))*log(tan(3*x/2 + 1) + sqrt(2*sqrt(6) +
 5))/(-467972363532675 - 191048917396548*sqrt(6) + 13665597568857156*sqrt(6
)*sqrt(5 - 2*sqrt(6))*sqrt(2*sqrt(6) + 5) + 33473741073918339*sqrt(5 - 2*sq
rt(6))*sqrt(2*sqrt(6) + 5)) + 5562766012543373*sqrt(6)*sqrt(5 - 2*sqrt(6))*
log(tan(3*x/2 + 1) + sqrt(2*sqrt(6) + 5))/(-467972363532675 - 1910489173965
48*sqrt(6) + 13665597568857156*sqrt(6)*sqrt(5 - 2*sqrt(6))*sqrt(2*sqrt(6) +
 5) + 33473741073918339*sqrt(5 - 2*sqrt(6))*sqrt(2*sqrt(6) + 5))
```

3.13 $\int \frac{\sec^2(2+3x)}{-1+2\tan^2(2+3x)}\,dx$

Optimal. Leaf size=60

$$\frac{\log\left(\cos(3x+2)-\sqrt{2}\sin(3x+2)\right)}{6\sqrt{2}}-\frac{\log\left(\sqrt{2}\sin(3x+2)+\cos(3x+2)\right)}{6\sqrt{2}}$$

```
[Out] 1/12*ln(cos(2+3*x)-sin(2+3*x)*2^(1/2))*2^(1/2)-1/12*ln(cos(2+3*x)+sin(2+3*x
)*2^(1/2))*2^(1/2)
```

Rubi [A] time = 0.05, antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {3675, 207}

$$\frac{\log\left(\cos(3x+2)-\sqrt{2}\sin(3x+2)\right)}{6\sqrt{2}}-\frac{\log\left(\sqrt{2}\sin(3x+2)+\cos(3x+2)\right)}{6\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[2 + 3*x]^2/(-1 + 2*Tan[2 + 3*x]^2),x]
```

```
[Out] Log[Cos[2 + 3*x] - Sqrt[2]*Sin[2 + 3*x]]/(6*Sqrt[2]) - Log[Cos[2 + 3*x] + S
qrt[2]*Sin[2 + 3*x]]/(6*Sqrt[2])
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 3675

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/(c^(m - 1)*f), Subst[Int[(c^2 + ff^2*x^2)^(m/2 - 1)*(a + b*(ff*x)^n)^p
, x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && In
tegerQ[m/2] && (IntegersQ[n, p] || IGtQ[m, 0] || IGtQ[p, 0] || EqQ[n^2, 4]
|| EqQ[n^2, 16])
```

Rubi steps

$$\int \frac{\sec^2(2+3x)}{-1+2\tan^2(2+3x)}\,dx = \frac{1}{3}\,\text{Subst}\left(\int \frac{1}{-1+2x^2}\,dx, x, \tan(2+3x)\right)$$
$$= \frac{\log\left(\cos(2+3x)-\sqrt{2}\sin(2+3x)\right)}{6\sqrt{2}} - \frac{\log\left(\cos(2+3x)+\sqrt{2}\sin(2+3x)\right)}{6\sqrt{2}}$$

Mathematica [A] time = 0.02, size = 22, normalized size = 0.37

$$-\frac{\tanh^{-1}\left(\sqrt{2}\tan(3x+2)\right)}{3\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[2 + 3*x]^2/(-1 + 2*Tan[2 + 3*x]^2),x]
```

```
[Out] -1/3*ArcTanh[Sqrt[2]*Tan[2 + 3*x]]/Sqrt[2]
```

fricas [A] time = 0.77, size = 86, normalized size = 1.43

$$\frac{1}{24}\sqrt{2}\log\left(-\frac{7\cos(3x+2)^4 - 4\cos(3x+2)^2 - 4\left(\sqrt{2}\cos(3x+2)^3 - 2\sqrt{2}\cos(3x+2)\right)\sin(3x+2) - 4}{9\cos(3x+2)^4 - 12\cos(3x+2)^2 + 4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(2+3*x)^2/(-1+2*tan(2+3*x)^2),x, algorithm="fricas")
```

```
[Out] 1/24*sqrt(2)*log(-(7*cos(3*x + 2)^4 - 4*cos(3*x + 2)^2 - 4*(sqrt(2)*cos(3*x
 + 2)^3 - 2*sqrt(2)*cos(3*x + 2))*sin(3*x + 2) - 4)/(9*cos(3*x + 2)^4 - 12*
cos(3*x + 2)^2 + 4))
```

giac [A] time = 1.48, size = 39, normalized size = 0.65

$$-\frac{1}{12}\sqrt{2}\log\left(\left|\frac{1}{2}\sqrt{2} + \tan(3x+2)\right|\right) + \frac{1}{12}\sqrt{2}\log\left(\left|-\frac{1}{2}\sqrt{2} + \tan(3x+2)\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(2+3*x)^2/(-1+2*tan(2+3*x)^2),x, algorithm="giac")
```

```
[Out] -1/12*sqrt(2)*log(abs(1/2*sqrt(2) + tan(3*x + 2))) + 1/12*sqrt(2)*log(abs(-
1/2*sqrt(2) + tan(3*x + 2)))
```

maple [A] time = 0.28, size = 17, normalized size = 0.28

$$-\frac{\sqrt{2}\ \operatorname{arctanh}\left(\sqrt{2}\ \tan\left(2+3x\right)\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(2+3*x)^2/(-1+2*tan(2+3*x)^2),x)
```

```
[Out] -1/6*2^(1/2)*arctanh(2^(1/2)*tan(2+3*x))
```

maxima [A] time = 1.03, size = 34, normalized size = 0.57

$$\frac{1}{12}\sqrt{2}\log\left(-\frac{\sqrt{2}-2\tan\left(3x+2\right)}{\sqrt{2}+2\tan\left(3x+2\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(2+3*x)^2/(-1+2*tan(2+3*x)^2),x, algorithm="maxima")
```

```
[Out] 1/12*sqrt(2)*log(-(sqrt(2) - 2*tan(3*x + 2))/(sqrt(2) + 2*tan(3*x + 2)))
```

mupad [B] time = 2.40, size = 16, normalized size = 0.27

$$-\frac{\sqrt{2}\operatorname{atanh}\left(\sqrt{2}\tan\left(3x+2\right)\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(3*x + 2)^2*(2*tan(3*x + 2)^2 - 1)),x)
```

```
[Out] -(2^(1/2)*atanh(2^(1/2)*tan(3*x + 2)))/6
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sec^{2}\left(3x+2\right)}{2\tan^{2}\left(3x+2\right)-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(2+3*x)**2/(-1+2*tan(2+3*x)**2),x)
```

```
[Out] Integral(sec(3*x + 2)**2/(2*tan(3*x + 2)**2 - 1), x)
```

3.14 $\int \frac{\csc^2(2+3x)}{2-\cot^2(2+3x)}\,dx$

Optimal. Leaf size=60

$$\frac{\log\left(\cos(3x+2)-\sqrt{2}\sin(3x+2)\right)}{6\sqrt{2}}-\frac{\log\left(\sqrt{2}\sin(3x+2)+\cos(3x+2)\right)}{6\sqrt{2}}$$

```
[Out] 1/12*ln(cos(2+3*x)-sin(2+3*x)*2^(1/2))*2^(1/2)-1/12*ln(cos(2+3*x)+sin(2+3*x
)*2^(1/2))*2^(1/2)
```

Rubi [A] time = 0.05, antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {3675, 206}

$$\frac{\log\left(\cos(3x+2)-\sqrt{2}\sin(3x+2)\right)}{6\sqrt{2}}-\frac{\log\left(\sqrt{2}\sin(3x+2)+\cos(3x+2)\right)}{6\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[2 + 3*x]^2/(2 - Cot[2 + 3*x]^2),x]
```

```
[Out] Log[Cos[2 + 3*x] - Sqrt[2]*Sin[2 + 3*x]]/(6*Sqrt[2]) - Log[Cos[2 + 3*x] + S
qrt[2]*Sin[2 + 3*x]]/(6*Sqrt[2])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3675

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/(c^(m - 1)*f), Subst[Int[(c^2 + ff^2*x^2)^(m/2 - 1)*(a + b*(ff*x)^n)^p
, x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && In
tegerQ[m/2] && (IntegersQ[n, p] || IGtQ[m, 0] || IGtQ[p, 0] || EqQ[n^2, 4]
|| EqQ[n^2, 16])
```

Rubi steps

$$\int \frac{\csc^2(2+3x)}{2-\cot^2(2+3x)}\,dx = -\left(\frac{1}{3}\,\text{Subst}\left(\int \frac{1}{2-x^2}\,dx, x, \cot(2+3x)\right)\right)$$
$$= \frac{\log\left(\cos(2+3x)-\sqrt{2}\sin(2+3x)\right)}{6\sqrt{2}} - \frac{\log\left(\cos(2+3x)+\sqrt{2}\sin(2+3x)\right)}{6\sqrt{2}}$$

Mathematica [A] time = 0.03, size = 22, normalized size = 0.37

$$-\frac{\tanh^{-1}\left(\sqrt{2}\tan(3x+2)\right)}{3\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[2 + 3*x]^2/(2 - Cot[2 + 3*x]^2),x]
```

```
[Out] -1/3*ArcTanh[Sqrt[2]*Tan[2 + 3*x]]/Sqrt[2]
```

fricas [A] time = 0.68, size = 86, normalized size = 1.43

$$\frac{1}{24}\sqrt{2}\log\left(-\frac{7\cos\left(3x+2\right)^4-4\cos\left(3x+2\right)^2-4\left(\sqrt{2}\cos\left(3x+2\right)^3-2\sqrt{2}\cos\left(3x+2\right)\right)\sin\left(3x+2\right)-4}{9\cos\left(3x+2\right)^4-12\cos\left(3x+2\right)^2+4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(2+3*x)^2/(2-cot(2+3*x)^2),x, algorithm="fricas")
```

```
[Out] 1/24*sqrt(2)*log(-(7*cos(3*x + 2)^4 - 4*cos(3*x + 2)^2 - 4*(sqrt(2)*cos(3*x
 + 2)^3 - 2*sqrt(2)*cos(3*x + 2))*sin(3*x + 2) - 4)/(9*cos(3*x + 2)^4 - 12*
cos(3*x + 2)^2 + 4))
```

giac [A] time = 0.36, size = 39, normalized size = 0.65

$$\frac{1}{12}\sqrt{2}\log\left(\frac{\left|-2\sqrt{2}+4\tan\left(3x+2\right)\right|}{\left|2\sqrt{2}+4\tan\left(3x+2\right)\right|}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(2+3*x)^2/(2-cot(2+3*x)^2),x, algorithm="giac")
```

```
[Out] 1/12*sqrt(2)*log(abs(-2*sqrt(2) + 4*tan(3*x + 2))/abs(2*sqrt(2) + 4*tan(3*x
 + 2)))
```

maple [A] time = 0.26, size = 17, normalized size = 0.28

$$-\frac{\sqrt{2}\ \operatorname{arctanh}\left(\sqrt{2}\ \tan\left(2+3x\right)\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(2+3*x)^2/(2-cot(2+3*x)^2),x)
```

```
[Out] -1/6*2^(1/2)*arctanh(2^(1/2)*tan(2+3*x))
```

maxima [A] time = 0.41, size = 34, normalized size = 0.57

$$\frac{1}{12}\sqrt{2}\log\left(-\frac{\sqrt{2}-2\tan\left(3x+2\right)}{\sqrt{2}+2\tan\left(3x+2\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(2+3*x)^2/(2-cot(2+3*x)^2),x, algorithm="maxima")
```

```
[Out] 1/12*sqrt(2)*log(-(sqrt(2) - 2*tan(3*x + 2))/(sqrt(2) + 2*tan(3*x + 2)))
```

mupad [B] time = 2.41, size = 16, normalized size = 0.27

$$-\frac{\sqrt{2}\operatorname{atanh}\left(\sqrt{2}\tan\left(3x+2\right)\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-1/(sin(3*x + 2)^2*(cot(3*x + 2)^2 - 2)),x)
```

```
[Out] -(2^(1/2)*atanh(2^(1/2)*tan(3*x + 2)))/6
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int\frac{\csc^{2}\left(3x+2\right)}{\cot^{2}\left(3x+2\right)-2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(2+3*x)**2/(2-cot(2+3*x)**2),x)
```

```
[Out] -Integral(csc(3*x + 2)**2/(cot(3*x + 2)**2 - 2), x)
```

3.15 $\int \frac{2}{3+\cos(4+6x)}\,dx$

Optimal. Leaf size=42

$$\frac{x}{\sqrt{2}}-\frac{\tan^{-1}\left(\frac{\sin(6x+4)}{\cos(6x+4)+2\sqrt{2}+3}\right)}{3\sqrt{2}}$$

[Out] 1/2*x*2^(1/2)-1/6*arctan(sin(4+6*x)/(3+cos(4+6*x)+2*2^(1/2)))*2^(1/2)

Rubi [A] time = 0.04, antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {12, 2657}

$$\frac{x}{\sqrt{2}}-\frac{\tan^{-1}\left(\frac{\sin(6x+4)}{\cos(6x+4)+2\sqrt{2}+3}\right)}{3\sqrt{2}}$$

Antiderivative was successfully verified.

[In] Int[2/(3 + Cos[4 + 6*x]),x]

[Out] x/Sqrt[2] - ArcTan[Sin[4 + 6*x]/(3 + 2*Sqrt[2] + Cos[4 + 6*x])]/(3*Sqrt[2])

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2657

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{q = Rt[
a^2 - b^2, 2]}, Simp[x/q, x] + Simp[(2*ArcTan[(b*Cos[c + d*x])/(a + q + b*S
in[c + d*x])])/(d*q), x]] /; FreeQ[{a, b, c, d}, x] && GtQ[a^2 - b^2, 0] &&
 PosQ[a]
```

Rubi steps

$$\begin{aligned}\int \frac{2}{3+\cos(4+6x)}\,dx &= 2\int \frac{1}{3+\cos(4+6x)}\,dx\\ &= \frac{x}{\sqrt{2}}-\frac{\tan^{-1}\left(\frac{\sin(4+6x)}{3+2\sqrt{2}+\cos(4+6x)}\right)}{3\sqrt{2}}\end{aligned}$$

Mathematica [A] time = 0.03, size = 22, normalized size = 0.52

$$\frac{\tan^{-1}\left(\frac{\tan(3x+2)}{\sqrt{2}}\right)}{3\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[2/(3 + Cos[4 + 6*x]),x]
[Out] ArcTan[Tan[2 + 3*x]/Sqrt[2]]/(3*Sqrt[2])
```

fricas [A] time = 1.32, size = 31, normalized size = 0.74

$$-\frac{1}{12}\sqrt{2}\arctan\left(\frac{3\sqrt{2}\cos\left(6x+4\right)+\sqrt{2}}{4\sin\left(6x+4\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(2/(3+cos(4+6*x)),x, algorithm="fricas")
[Out] -1/12*sqrt(2)*arctan(1/4*(3*sqrt(2)*cos(6*x + 4) + sqrt(2))/sin(6*x + 4))
```

giac [A] time = 0.14, size = 57, normalized size = 1.36

$$\frac{1}{6}\sqrt{2}\left(3x+\arctan\left(-\frac{\sqrt{2}\sin\left(6x+4\right)-\sin\left(6x+4\right)}{\sqrt{2}\cos\left(6x+4\right)+\sqrt{2}-\cos\left(6x+4\right)+1}\right)+2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(2/(3+cos(4+6*x)),x, algorithm="giac")
[Out] 1/6*sqrt(2)*(3*x + arctan(-(sqrt(2)*sin(6*x + 4) - sin(6*x + 4))/(sqrt(2)*c
os(6*x + 4) + sqrt(2) - cos(6*x + 4) + 1)) + 2)
```

maple [A] time = 0.06, size = 18, normalized size = 0.43

$$\frac{\sqrt{2}\arctan\left(\frac{\sqrt{2}\tan(2+3x)}{2}\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(2/(3+cos(4+6*x)),x)
[Out] 1/6*2^(1/2)*arctan(1/2*2^(1/2)*tan(2+3*x))
```

maxima [A] time = 0.42, size = 27, normalized size = 0.64

$$\frac{1}{6}\sqrt{2}\arctan\left(\frac{\sqrt{2}\sin(6x+4)}{2(\cos(6x+4)+1)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(2/(3+cos(4+6*x)),x, algorithm="maxima")
```

```
[Out] 1/6*sqrt(2)*arctan(1/2*sqrt(2)*sin(6*x + 4)/(cos(6*x + 4) + 1))
```

mupad [B] time = 2.54, size = 36, normalized size = 0.86

$$\frac{\sqrt{2}\ (3x-\operatorname{atan}(\tan(3x+2)))}{6}+\frac{\sqrt{2}\operatorname{atan}\left(\frac{\sqrt{2}\tan(3x+2)}{2}\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(2/(cos(6*x + 4) + 3),x)
```

```
[Out] (2^(1/2)*(3*x - atan(tan(3*x + 2))))/6 + (2^(1/2)*atan((2^(1/2)*tan(3*x + 2
))/2))/6
```

sympy [A] time = 0.26, size = 34, normalized size = 0.81

$$\frac{\sqrt{2}\left(\operatorname{atan}\left(\frac{\sqrt{2}\tan(3x+2)}{2}\right)+\pi\left\lfloor\frac{3x-\frac{\pi}{2}+2}{\pi}\right\rfloor\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(2/(3+cos(4+6*x)),x)
```

```
[Out] sqrt(2)*(atan(sqrt(2)*tan(3*x + 2)/2) + pi*floor((3*x - pi/2 + 2)/pi))/6
```

3.16 $\int \frac{2\csc(4+6x)}{\cot(4+6x)+3\csc(4+6x)}\,dx$

Optimal. Leaf size=42

$$\frac{x}{\sqrt{2}}-\frac{\tan^{-1}\left(\frac{\sin(6x+4)}{\cos(6x+4)+2\sqrt{2}+3}\right)}{3\sqrt{2}}$$

```
[Out] 1/2*x*2^(1/2)-1/6*arctan(sin(4+6*x)/(3+cos(4+6*x)+2*2^(1/2)))*2^(1/2)
```

Rubi [A] time = 0.04, antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {12, 3166, 2657}

$$\frac{x}{\sqrt{2}}-\frac{\tan^{-1}\left(\frac{\sin(6x+4)}{\cos(6x+4)+2\sqrt{2}+3}\right)}{3\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[(2*Csc[4 + 6*x])/(Cot[4 + 6*x] + 3*Csc[4 + 6*x]),x]
```

```
[Out] x/Sqrt[2] - ArcTan[Sin[4 + 6*x]/(3 + 2*Sqrt[2] + Cos[4 + 6*x])]/(3*Sqrt[2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2657

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{q = Rt[
a^2 - b^2, 2]}, Simp[x/q, x] + Simp[(2*ArcTan[(b*Cos[c + d*x])/(a + q + b*S
in[c + d*x])])/(d*q), x]] /; FreeQ[{a, b, c, d}, x] && GtQ[a^2 - b^2, 0] &&
 PosQ[a]
```

Rule 3166

```
Int[csc[(d_.) + (e_.)*(x_)]^(n_.)*((a_.) + csc[(d_.) + (e_.)*(x_)]*(b_.) +
cot[(d_.) + (e_.)*(x_)]*(c_.))^(m_), x_Symbol] :> Int[1/(b + a*Sin[d + e*x]
 + c*Cos[d + e*x])^n, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[m + n, 0] && I
ntegerQ[n]
```

Rubi steps

$$\int \frac{2\csc(4+6x)}{\cot(4+6x)+3\csc(4+6x)}\,dx = 2\int \frac{\csc(4+6x)}{\cot(4+6x)+3\csc(4+6x)}\,dx$$
$$= 2\int \frac{1}{3+\cos(4+6x)}\,dx$$
$$= \frac{x}{\sqrt{2}} - \frac{\tan^{-1}\left(\frac{\sin(4+6x)}{3+2\sqrt{2}+\cos(4+6x)}\right)}{3\sqrt{2}}$$

Mathematica [A] time = 0.02, size = 22, normalized size = 0.52

$$\frac{\tan^{-1}\left(\frac{\tan(3x+2)}{\sqrt{2}}\right)}{3\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2*Csc[4 + 6*x])/(Cot[4 + 6*x] + 3*Csc[4 + 6*x]),x]
[Out] ArcTan[Tan[2 + 3*x]/Sqrt[2]]/(3*Sqrt[2])
```

fricas [A] time = 0.60, size = 31, normalized size = 0.74

$$-\frac{1}{12}\sqrt{2}\arctan\left(\frac{3\sqrt{2}\cos\left(6x+4\right)+\sqrt{2}}{4\sin\left(6x+4\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(2*csc(4+6*x)/(cot(4+6*x)+3*csc(4+6*x)),x, algorithm="fricas")
[Out] -1/12*sqrt(2)*arctan(1/4*(3*sqrt(2)*cos(6*x + 4) + sqrt(2))/sin(6*x + 4))
```

giac [A] time = 0.23, size = 57, normalized size = 1.36

$$\frac{1}{6}\sqrt{2}\left(3x+\arctan\left(-\frac{\sqrt{2}\sin\left(6x+4\right)-\sin\left(6x+4\right)}{\sqrt{2}\cos\left(6x+4\right)+\sqrt{2}-\cos\left(6x+4\right)+1}\right)+2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(2*csc(4+6*x)/(cot(4+6*x)+3*csc(4+6*x)),x, algorithm="giac")
[Out] 1/6*sqrt(2)*(3*x + arctan(-(sqrt(2)*sin(6*x + 4) - sin(6*x + 4))/(sqrt(2)*c
os(6*x + 4) + sqrt(2) - cos(6*x + 4) + 1)) + 2)
```

maple [A] time = 0.31, size = 18, normalized size = 0.43

$$\frac{\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\,\tan(2+3x)}{2}\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(2*csc(4+6*x)/(cot(4+6*x)+3*csc(4+6*x)),x)
[Out] 1/6*2^(1/2)*arctan(1/2*2^(1/2)*tan(2+3*x))
```

maxima [A] time = 0.43, size = 27, normalized size = 0.64

$$\frac{1}{6}\sqrt{2}\arctan\left(\frac{\sqrt{2}\sin\left(6x+4\right)}{2\left(\cos\left(6x+4\right)+1\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(2*csc(4+6*x)/(cot(4+6*x)+3*csc(4+6*x)),x, algorithm="maxima")
[Out] 1/6*sqrt(2)*arctan(1/2*sqrt(2)*sin(6*x + 4)/(cos(6*x + 4) + 1))
```

mupad [B] time = 2.69, size = 17, normalized size = 0.40

$$\frac{\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}\,\tan(3x+2)}{2}\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(2/(sin(6*x + 4)*(cot(6*x + 4) + 3/sin(6*x + 4))),x)
[Out] (2^(1/2)*atan((2^(1/2)*tan(3*x + 2))/2))/6
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$2\int\frac{\csc\left(6x+4\right)}{\cot\left(6x+4\right)+3\csc\left(6x+4\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(2*csc(4+6*x)/(cot(4+6*x)+3*csc(4+6*x)),x)
[Out] 2*Integral(csc(6*x + 4)/(cot(6*x + 4) + 3*csc(6*x + 4)), x)
```

3.17 $\int \frac{1}{2-\sin^2(2+3x)}\,dx$

Optimal. Leaf size=48

$$\frac{x}{\sqrt{2}}-\frac{\tan^{-1}\left(\frac{\sin(3x+2)\cos(3x+2)}{\cos^2(3x+2)+\sqrt{2}+1}\right)}{3\sqrt{2}}$$

```
[Out] 1/2*x*2^(1/2)-1/6*arctan(cos(2+3*x)*sin(2+3*x)/(1+cos(2+3*x)^2+2^(1/2)))*2^
(1/2)
```

Rubi [A] time = 0.02, antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3181, 203}

$$\frac{x}{\sqrt{2}}-\frac{\tan^{-1}\left(\frac{\sin(3x+2)\cos(3x+2)}{\cos^2(3x+2)+\sqrt{2}+1}\right)}{3\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[(2 - Sin[2 + 3*x]^2)^(-1),x]
```

```
[Out] x/Sqrt[2] - ArcTan[(Cos[2 + 3*x]*Sin[2 + 3*x])/(1 + Sqrt[2] + Cos[2 + 3*x]^
2)]/(3*Sqrt[2])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 3181

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(-1), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[1/(a + (a + b)*ff^2*x^2
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x]
```

Rubi steps

$$\int \frac{1}{2-\sin^2(2+3x)}\,dx = \frac{1}{3}\operatorname{Subst}\left(\int \frac{1}{2+x^2}\,dx, x, \tan(2+3x)\right)$$
$$= \frac{x}{\sqrt{2}} - \frac{\tan^{-1}\left(\frac{\cos(2+3x)\sin(2+3x)}{1+\sqrt{2}+\cos^2(2+3x)}\right)}{3\sqrt{2}}$$

Mathematica [A] time = 0.02, size = 22, normalized size = 0.46

$$\frac{\tan^{-1}\left(\frac{\tan(3x+2)}{\sqrt{2}}\right)}{3\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 - Sin[2 + 3*x]^2)^(-1),x]
```

```
[Out] ArcTan[Tan[2 + 3*x]/Sqrt[2]]/(3*Sqrt[2])
```

fricas [A] time = 0.76, size = 43, normalized size = 0.90

$$-\frac{1}{12}\sqrt{2}\arctan\left(\frac{3\sqrt{2}\cos\left(3x+2\right)^2-\sqrt{2}}{4\cos\left(3x+2\right)\sin\left(3x+2\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2-sin(2+3*x)^2),x, algorithm="fricas")
```

```
[Out] -1/12*sqrt(2)*arctan(1/4*(3*sqrt(2)*cos(3*x + 2)^2 - sqrt(2))/(cos(3*x + 2)
*sin(3*x + 2)))
```

giac [A] time = 0.15, size = 57, normalized size = 1.19

$$\frac{1}{6}\sqrt{2}\left(3x+\arctan\left(-\frac{\sqrt{2}\sin\left(6x+4\right)-\sin\left(6x+4\right)}{\sqrt{2}\cos\left(6x+4\right)+\sqrt{2}-\cos\left(6x+4\right)+1}\right)+2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2-sin(2+3*x)^2),x, algorithm="giac")
```

```
[Out] 1/6*sqrt(2)*(3*x + arctan(-(sqrt(2)*sin(6*x + 4) - sin(6*x + 4))/(sqrt(2)*c
os(6*x + 4) + sqrt(2) - cos(6*x + 4) + 1)) + 2)
```

maple [A] time = 0.15, size = 18, normalized size = 0.38

$$\frac{\sqrt{2}\ \arctan\left(\frac{\sqrt{2}\ \tan(2+3x)}{2}\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2-sin(2+3*x)^2),x)
[Out] 1/6*2^(1/2)*arctan(1/2*2^(1/2)*tan(2+3*x))
```

maxima [A] time = 0.44, size = 17, normalized size = 0.35

$$\frac{1}{6}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\tan\left(3x+2\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2-sin(2+3*x)^2),x, algorithm="maxima")
[Out] 1/6*sqrt(2)*arctan(1/2*sqrt(2)*tan(3*x + 2))
```

mupad [B] time = 2.44, size = 36, normalized size = 0.75

$$\frac{\sqrt{2}\ \left(3x-\mathrm{atan}\left(\tan\left(3x+2\right)\right)\right)}{6}+\frac{\sqrt{2}\ \mathrm{atan}\left(\frac{\sqrt{2}\ \tan(3x+2)}{2}\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-1/(sin(3*x + 2)^2 - 2),x)
[Out] (2^(1/2)*(3*x - atan(tan(3*x + 2))))/6 + (2^(1/2)*atan((2^(1/2)*tan(3*x + 2
))/2))/6
```

sympy [A] time = 0.75, size = 76, normalized size = 1.58

$$\frac{\sqrt{2}\left(\mathrm{atan}\left(\sqrt{2}\tan\left(\frac{3x}{2}+1\right)-1\right)+\pi\left\lfloor\frac{\frac{3x}{2}-\frac{\pi}{2}+1}{\pi}\right\rfloor\right)}{6}+\frac{\sqrt{2}\left(\mathrm{atan}\left(\sqrt{2}\tan\left(\frac{3x}{2}+1\right)+1\right)+\pi\left\lfloor\frac{\frac{3x}{2}-\frac{\pi}{2}+1}{\pi}\right\rfloor\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2-sin(2+3*x)**2),x)
[Out] sqrt(2)*(atan(sqrt(2)*tan(3*x/2 + 1) - 1) + pi*floor((3*x/2 - pi/2 + 1)/pi)
)/6 + sqrt(2)*(atan(sqrt(2)*tan(3*x/2 + 1) + 1) + pi*floor((3*x/2 - pi/2 +
1)/pi))/6
```

3.18 $\int \frac{1}{1+\cos^2(2+3x)}\,dx$

Optimal. Leaf size=48

$$\frac{x}{\sqrt{2}} - \frac{\tan^{-1}\left(\frac{\sin(3x+2)\cos(3x+2)}{\cos^2(3x+2)+\sqrt{2}+1}\right)}{3\sqrt{2}}$$

```
[Out] 1/2*x*2^(1/2)-1/6*arctan(cos(2+3*x)*sin(2+3*x)/(1+cos(2+3*x)^2+2^(1/2)))*2^
(1/2)
```

Rubi [A] time = 0.02, antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3181, 203}

$$\frac{x}{\sqrt{2}} - \frac{\tan^{-1}\left(\frac{\sin(3x+2)\cos(3x+2)}{\cos^2(3x+2)+\sqrt{2}+1}\right)}{3\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + Cos[2 + 3*x]^2)^(-1),x]
```

```
[Out] x/Sqrt[2] - ArcTan[(Cos[2 + 3*x]*Sin[2 + 3*x])/(1 + Sqrt[2] + Cos[2 + 3*x]^
2)]/(3*Sqrt[2])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 3181

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(-1), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[1/(a + (a + b)*ff^2*x^2
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x]
```

Rubi steps

$$\int \frac{1}{1+\cos^2(2+3x)}\,dx = -\left(\frac{1}{3}\operatorname{Subst}\left(\int \frac{1}{1+2x^2}\,dx, x, \cot(2+3x)\right)\right)$$
$$= \frac{x}{\sqrt{2}} - \frac{\tan^{-1}\left(\frac{\cos(2+3x)\sin(2+3x)}{1+\sqrt{2}+\cos^2(2+3x)}\right)}{3\sqrt{2}}$$

Mathematica [A] time = 0.04, size = 22, normalized size = 0.46

$$\frac{\tan^{-1}\left(\frac{\tan(3x+2)}{\sqrt{2}}\right)}{3\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + Cos[2 + 3*x]^2)^(-1),x]
[Out] ArcTan[Tan[2 + 3*x]/Sqrt[2]]/(3*Sqrt[2])
```

fricas [A] time = 0.68, size = 43, normalized size = 0.90

$$-\frac{1}{12}\sqrt{2}\arctan\left(\frac{3\sqrt{2}\cos(3x+2)^2-\sqrt{2}}{4\cos(3x+2)\sin(3x+2)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+cos(2+3*x)^2),x, algorithm="fricas")
[Out] -1/12*sqrt(2)*arctan(1/4*(3*sqrt(2)*cos(3*x + 2)^2 - sqrt(2))/(cos(3*x + 2)
*sin(3*x + 2)))
```

giac [A] time = 0.15, size = 57, normalized size = 1.19

$$\frac{1}{6}\sqrt{2}\left(3x+\arctan\left(-\frac{\sqrt{2}\sin(6x+4)-\sin(6x+4)}{\sqrt{2}\cos(6x+4)+\sqrt{2}-\cos(6x+4)+1}\right)+2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+cos(2+3*x)^2),x, algorithm="giac")
[Out] 1/6*sqrt(2)*(3*x + arctan(-(sqrt(2)*sin(6*x + 4) - sin(6*x + 4))/(sqrt(2)*c
os(6*x + 4) + sqrt(2) - cos(6*x + 4) + 1)) + 2)
```

maple [A] time = 0.10, size = 18, normalized size = 0.38

$$\frac{\sqrt{2}\ \arctan\left(\frac{\sqrt{2}\ \tan(2+3x)}{2}\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1+cos(2+3*x)^2),x)
```

```
[Out] 1/6*2^(1/2)*arctan(1/2*2^(1/2)*tan(2+3*x))
```

maxima [A] time = 0.44, size = 17, normalized size = 0.35

$$\frac{1}{6}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\tan\left(3x+2\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+cos(2+3*x)^2),x, algorithm="maxima")
```

```
[Out] 1/6*sqrt(2)*arctan(1/2*sqrt(2)*tan(3*x + 2))
```

mupad [B] time = 2.37, size = 36, normalized size = 0.75

$$\frac{\sqrt{2}\ \left(3x-\mathrm{atan}\left(\tan\left(3x+2\right)\right)\right)}{6}+\frac{\sqrt{2}\ \mathrm{atan}\left(\frac{\sqrt{2}\ \tan(3x+2)}{2}\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(3*x + 2)^2 + 1),x)
```

```
[Out] (2^(1/2)*(3*x - atan(tan(3*x + 2))))/6 + (2^(1/2)*atan((2^(1/2)*tan(3*x + 2
))/2))/6
```

sympy [A] time = 0.65, size = 76, normalized size = 1.58

$$\frac{\sqrt{2}\left(\mathrm{atan}\left(\sqrt{2}\ \tan\left(\frac{3x}{2}+1\right)-1\right)+\pi\left\lfloor\frac{\frac{3x}{2}-\frac{\pi}{2}+1}{\pi}\right\rfloor\right)}{6}+\frac{\sqrt{2}\left(\mathrm{atan}\left(\sqrt{2}\ \tan\left(\frac{3x}{2}+1\right)+1\right)+\pi\left\lfloor\frac{\frac{3x}{2}-\frac{\pi}{2}+1}{\pi}\right\rfloor\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+cos(2+3*x)**2),x)
```

```
[Out] sqrt(2)*(atan(sqrt(2)*tan(3*x/2 + 1) - 1) + pi*floor((3*x/2 - pi/2 + 1)/pi)
)/6 + sqrt(2)*(atan(sqrt(2)*tan(3*x/2 + 1) + 1) + pi*floor((3*x/2 - pi/2 +
1)/pi))/6
```

3.19 $\int \frac{1}{2\cos^2(2+3x)+\sin^2(2+3x)}\,dx$

Optimal. Leaf size=48

$$\frac{x}{\sqrt{2}}-\frac{\tan^{-1}\left(\frac{\sin(3x+2)\cos(3x+2)}{\cos^2(3x+2)+\sqrt{2}+1}\right)}{3\sqrt{2}}$$

```
[Out] 1/2*x*2^(1/2)-1/6*arctan(cos(2+3*x)*sin(2+3*x)/(1+cos(2+3*x)^2+2^(1/2)))*2^
(1/2)
```

Rubi [A] time = 0.03, antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.048, Rules used = {203}

$$\frac{x}{\sqrt{2}}-\frac{\tan^{-1}\left(\frac{\sin(3x+2)\cos(3x+2)}{\cos^2(3x+2)+\sqrt{2}+1}\right)}{3\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[(2*Cos[2 + 3*x]^2 + Sin[2 + 3*x]^2)^(-1),x]
```

```
[Out] x/Sqrt[2] - ArcTan[(Cos[2 + 3*x]*Sin[2 + 3*x])/(1 + Sqrt[2] + Cos[2 + 3*x]^
2)]/(3*Sqrt[2])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{2\cos^2(2+3x)+\sin^2(2+3x)}\,dx &= \frac{1}{3}\operatorname{Subst}\left(\int \frac{1}{2+x^2}\,dx, x, \tan(2+3x)\right)\\ &= \frac{x}{\sqrt{2}}-\frac{\tan^{-1}\left(\frac{\cos(2+3x)\sin(2+3x)}{1+\sqrt{2}+\cos^2(2+3x)}\right)}{3\sqrt{2}}\end{aligned}$$

Mathematica [A] time = 0.02, size = 22, normalized size = 0.46

$$\frac{\tan^{-1}\left(\frac{\tan(3x+2)}{\sqrt{2}}\right)}{3\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2*Cos[2 + 3*x]^2 + Sin[2 + 3*x]^2)^(-1),x]
```

```
[Out] ArcTan[Tan[2 + 3*x]/Sqrt[2]]/(3*Sqrt[2])
```

fricas [A] time = 0.51, size = 43, normalized size = 0.90

$$-\frac{1}{12}\sqrt{2}\arctan\left(\frac{3\sqrt{2}\cos\left(3x+2\right)^{2}-\sqrt{2}}{4\cos\left(3x+2\right)\sin\left(3x+2\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*cos(2+3*x)^2+sin(2+3*x)^2),x, algorithm="fricas")
```

```
[Out] -1/12*sqrt(2)*arctan(1/4*(3*sqrt(2)*cos(3*x + 2)^2 - sqrt(2))/(cos(3*x + 2)
*sin(3*x + 2)))
```

giac [A] time = 0.16, size = 57, normalized size = 1.19

$$\frac{1}{6}\sqrt{2}\left(3x+\arctan\left(-\frac{\sqrt{2}\sin\left(6x+4\right)-\sin\left(6x+4\right)}{\sqrt{2}\cos\left(6x+4\right)+\sqrt{2}-\cos\left(6x+4\right)+1}\right)+2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*cos(2+3*x)^2+sin(2+3*x)^2),x, algorithm="giac")
```

```
[Out] 1/6*sqrt(2)*(3*x + arctan(-(sqrt(2)*sin(6*x + 4) - sin(6*x + 4))/(sqrt(2)*c
os(6*x + 4) + sqrt(2) - cos(6*x + 4) + 1)) + 2)
```

maple [A] time = 0.23, size = 18, normalized size = 0.38

$$\frac{\sqrt{2}\arctan\left(\frac{\sqrt{2}\tan(2+3x)}{2}\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2*cos(2+3*x)^2+sin(2+3*x)^2),x)
```

```
[Out] 1/6*2^(1/2)*arctan(1/2*2^(1/2)*tan(2+3*x))
```

maxima [A] time = 0.44, size = 17, normalized size = 0.35

$$\frac{1}{6}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\tan\left(3x+2\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*cos(2+3*x)^2+sin(2+3*x)^2),x, algorithm="maxima")
```

```
[Out] 1/6*sqrt(2)*arctan(1/2*sqrt(2)*tan(3*x + 2))
```

mupad [B] time = 2.37, size = 36, normalized size = 0.75

$$\frac{\sqrt{2}\ (3\,x-\mathrm{atan}\,(\tan\,(3\,x+2)))}{6}+\frac{\sqrt{2}\ \mathrm{atan}\left(\frac{\sqrt{2}\ \tan(3\,x+2)}{2}\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(3*x + 2)^2 + 2*cos(3*x + 2)^2),x)
```

```
[Out] (2^(1/2)*(3*x - atan(tan(3*x + 2))))/6 + (2^(1/2)*atan((2^(1/2)*tan(3*x + 2
))/2))/6
```

sympy [A] time = 0.80, size = 76, normalized size = 1.58

$$\frac{\sqrt{2}\left(\mathrm{atan}\left(\sqrt{2}\ \tan\left(\frac{3x}{2}+1\right)-1\right)+\pi\left\lfloor\frac{\frac{3x}{2}-\frac{\pi}{2}+1}{\pi}\right\rfloor\right)}{6}+\frac{\sqrt{2}\left(\mathrm{atan}\left(\sqrt{2}\ \tan\left(\frac{3x}{2}+1\right)+1\right)+\pi\left\lfloor\frac{\frac{3x}{2}-\frac{\pi}{2}+1}{\pi}\right\rfloor\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*cos(2+3*x)**2+sin(2+3*x)**2),x)
```

```
[Out] sqrt(2)*(atan(sqrt(2)*tan(3*x/2 + 1) - 1) + pi*floor((3*x/2 - pi/2 + 1)/pi)
)/6 + sqrt(2)*(atan(sqrt(2)*tan(3*x/2 + 1) + 1) + pi*floor((3*x/2 - pi/2 +
1)/pi))/6
```

3.20 $\int \frac{\sec^2(2+3x)}{2+\tan^2(2+3x)}\,dx$

Optimal. Leaf size=48

$$\frac{x}{\sqrt{2}}-\frac{\tan^{-1}\left(\frac{\sin(3x+2)\cos(3x+2)}{\cos^2(3x+2)+\sqrt{2}+1}\right)}{3\sqrt{2}}$$

```
[Out] 1/2*x*2^(1/2)-1/6*arctan(cos(2+3*x)*sin(2+3*x)/(1+cos(2+3*x)^2+2^(1/2)))*2^
(1/2)
```

Rubi [A] time = 0.04, antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {3675, 203}

$$\frac{x}{\sqrt{2}}-\frac{\tan^{-1}\left(\frac{\sin(3x+2)\cos(3x+2)}{\cos^2(3x+2)+\sqrt{2}+1}\right)}{3\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[2 + 3*x]^2/(2 + Tan[2 + 3*x]^2),x]
```

```
[Out] x/Sqrt[2] - ArcTan[(Cos[2 + 3*x]*Sin[2 + 3*x])/(1 + Sqrt[2] + Cos[2 + 3*x]^
2)]/(3*Sqrt[2])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 3675

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/(c^(m - 1)*f), Subst[Int[(c^2 + ff^2*x^2)^(m/2 - 1)*(a + b*(ff*x)^n)^p
, x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && In
tegerQ[m/2] && (IntegersQ[n, p] || IGtQ[m, 0] || IGtQ[p, 0] || EqQ[n^2, 4]
|| EqQ[n^2, 16])
```

Rubi steps

$$\int \frac{\sec^2(2+3x)}{2+\tan^2(2+3x)}\,dx = \frac{1}{3}\,\text{Subst}\left(\int \frac{1}{2+x^2}\,dx, x, \tan(2+3x)\right)$$
$$= \frac{x}{\sqrt{2}} - \frac{\tan^{-1}\left(\frac{\cos(2+3x)\sin(2+3x)}{1+\sqrt{2}+\cos^2(2+3x)}\right)}{3\sqrt{2}}$$

Mathematica [A] time = 0.02, size = 22, normalized size = 0.46

$$\frac{\tan^{-1}\left(\frac{\tan(3x+2)}{\sqrt{2}}\right)}{3\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[2 + 3*x]^2/(2 + Tan[2 + 3*x]^2),x]
[Out] ArcTan[Tan[2 + 3*x]/Sqrt[2]]/(3*Sqrt[2])
```

fricas [A] time = 0.61, size = 43, normalized size = 0.90

$$-\frac{1}{12}\sqrt{2}\arctan\left(\frac{3\sqrt{2}\cos(3x+2)^2-\sqrt{2}}{4\cos(3x+2)\sin(3x+2)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(2+3*x)^2/(2+tan(2+3*x)^2),x, algorithm="fricas")
[Out] -1/12*sqrt(2)*arctan(1/4*(3*sqrt(2)*cos(3*x + 2)^2 - sqrt(2))/(cos(3*x + 2)
*sin(3*x + 2)))
```

giac [A] time = 1.23, size = 17, normalized size = 0.35

$$\frac{1}{6}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\tan(3x+2)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(2+3*x)^2/(2+tan(2+3*x)^2),x, algorithm="giac")
[Out] 1/6*sqrt(2)*arctan(1/2*sqrt(2)*tan(3*x + 2))
```

maple [A] time = 0.27, size = 18, normalized size = 0.38

$$\frac{\sqrt{2}\arctan\left(\frac{\sqrt{2}\tan(2+3x)}{2}\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(2+3*x)^2/(2+tan(2+3*x)^2),x)
```

```
[Out] 1/6*2^(1/2)*arctan(1/2*2^(1/2)*tan(2+3*x))
```

maxima [A] time = 1.18, size = 17, normalized size = 0.35

$$\frac{1}{6}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\tan\left(3x+2\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(2+3*x)^2/(2+tan(2+3*x)^2),x, algorithm="maxima")
```

```
[Out] 1/6*sqrt(2)*arctan(1/2*sqrt(2)*tan(3*x + 2))
```

mupad [B] time = 2.37, size = 17, normalized size = 0.35

$$\frac{\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}\tan(3x+2)}{2}\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(3*x + 2)^2*(tan(3*x + 2)^2 + 2)),x)
```

```
[Out] (2^(1/2)*atan((2^(1/2)*tan(3*x + 2))/2))/6
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sec^{2}\left(3x+2\right)}{\tan^{2}\left(3x+2\right)+2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(2+3*x)**2/(2+tan(2+3*x)**2),x)
```

```
[Out] Integral(sec(3*x + 2)**2/(tan(3*x + 2)**2 + 2), x)
```

3.21 $\int \frac{\csc^2(2+3x)}{1+2\cot^2(2+3x)}\,dx$

Optimal. Leaf size=48

$$\frac{x}{\sqrt{2}}-\frac{\tan^{-1}\left(\frac{\sin(3x+2)\cos(3x+2)}{\cos^2(3x+2)+\sqrt{2}+1}\right)}{3\sqrt{2}}$$

```
[Out] 1/2*x*2^(1/2)-1/6*arctan(cos(2+3*x)*sin(2+3*x)/(1+cos(2+3*x)^2+2^(1/2)))*2^
(1/2)
```

Rubi [A] time = 0.04, antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {3675, 203}

$$\frac{x}{\sqrt{2}}-\frac{\tan^{-1}\left(\frac{\sin(3x+2)\cos(3x+2)}{\cos^2(3x+2)+\sqrt{2}+1}\right)}{3\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[2 + 3*x]^2/(1 + 2*Cot[2 + 3*x]^2),x]
```

```
[Out] x/Sqrt[2] - ArcTan[(Cos[2 + 3*x]*Sin[2 + 3*x])/(1 + Sqrt[2] + Cos[2 + 3*x]^
2)]/(3*Sqrt[2])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 3675

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/(c^(m - 1)*f), Subst[Int[(c^2 + ff^2*x^2)^(m/2 - 1)*(a + b*(ff*x)^n)^p
, x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && In
tegerQ[m/2] && (IntegersQ[n, p] || IGtQ[m, 0] || IGtQ[p, 0] || EqQ[n^2, 4]
|| EqQ[n^2, 16])
```

Rubi steps

$$\int \frac{\csc^2(2+3x)}{1+2\cot^2(2+3x)}\,dx = -\left(\frac{1}{3}\,\text{Subst}\left(\int \frac{1}{1+2x^2}\,dx, x, \cot(2+3x)\right)\right)$$
$$= \frac{x}{\sqrt{2}} - \frac{\tan^{-1}\left(\frac{\cos(2+3x)\sin(2+3x)}{1+\sqrt{2}+\cos^2(2+3x)}\right)}{3\sqrt{2}}$$

Mathematica [A] time = 0.02, size = 22, normalized size = 0.46

$$\frac{\tan^{-1}\left(\frac{\tan(3x+2)}{\sqrt{2}}\right)}{3\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[2 + 3*x]^2/(1 + 2*Cot[2 + 3*x]^2),x]
```

```
[Out] ArcTan[Tan[2 + 3*x]/Sqrt[2]]/(3*Sqrt[2])
```

fricas [A] time = 0.55, size = 43, normalized size = 0.90

$$-\frac{1}{12}\,\sqrt{2}\,\arctan\left(\frac{3\,\sqrt{2}\,\cos\left(3\,x+2\right)^2-\sqrt{2}}{4\,\cos\left(3\,x+2\right)\sin\left(3\,x+2\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(2+3*x)^2/(1+2*cot(2+3*x)^2),x, algorithm="fricas")
```

```
[Out] -1/12*sqrt(2)*arctan(1/4*(3*sqrt(2)*cos(3*x + 2)^2 - sqrt(2))/(cos(3*x + 2)
*sin(3*x + 2)))
```

giac [A] time = 0.33, size = 57, normalized size = 1.19

$$\frac{1}{6}\,\sqrt{2}\left(3\,x+\arctan\left(-\frac{\sqrt{2}\,\sin\left(6\,x+4\right)-\sin\left(6\,x+4\right)}{\sqrt{2}\,\cos\left(6\,x+4\right)+\sqrt{2}\,-\cos\left(6\,x+4\right)+1}\right)+2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(2+3*x)^2/(1+2*cot(2+3*x)^2),x, algorithm="giac")
```

```
[Out] 1/6*sqrt(2)*(3*x + arctan(-(sqrt(2)*sin(6*x + 4) - sin(6*x + 4))/(sqrt(2)*c
os(6*x + 4) + sqrt(2) - cos(6*x + 4) + 1)) + 2)
```

maple [A] time = 0.28, size = 18, normalized size = 0.38

$$\frac{\sqrt{2}\ \arctan\left(\frac{\sqrt{2}\ \tan(2+3x)}{2}\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(2+3*x)^2/(1+2*cot(2+3*x)^2),x)
[Out] 1/6*2^(1/2)*arctan(1/2*2^(1/2)*tan(2+3*x))
```

maxima [A] time = 0.74, size = 17, normalized size = 0.35

$$\frac{1}{6}\,\sqrt{2}\,\arctan\left(\frac{1}{2}\,\sqrt{2}\,\tan\left(3\,x+2\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(2+3*x)^2/(1+2*cot(2+3*x)^2),x, algorithm="maxima")
[Out] 1/6*sqrt(2)*arctan(1/2*sqrt(2)*tan(3*x + 2))
```

mupad [B] time = 2.38, size = 17, normalized size = 0.35

$$\frac{\sqrt{2}\ \mathrm{atan}\left(\frac{\sqrt{2}\ \tan(3\,x+2)}{2}\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(3*x + 2)^2*(2*cot(3*x + 2)^2 + 1)),x)
[Out] (2^(1/2)*atan((2^(1/2)*tan(3*x + 2))/2))/6
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc^{2}\left(3x+2\right)}{2\cot^{2}\left(3x+2\right)+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(2+3*x)**2/(1+2*cot(2+3*x)**2),x)
[Out] Integral(csc(3*x + 2)**2/(2*cot(3*x + 2)**2 + 1), x)
```

3.22 $\int -\frac{2}{1+3\cos(4+6x)}\,dx$

Optimal. Leaf size=61

$$\frac{\log\left(\sqrt{2}\cos(3x+2)-\sin(3x+2)\right)}{6\sqrt{2}}-\frac{\log\left(\sin(3x+2)+\sqrt{2}\cos(3x+2)\right)}{6\sqrt{2}}$$

```
[Out] 1/12*ln(-sin(2+3*x)+cos(2+3*x)*2^(1/2))*2^(1/2)-1/12*ln(sin(2+3*x)+cos(2+3*
x)*2^(1/2))*2^(1/2)
```

Rubi [A] time = 0.03, antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {12, 2659, 206}

$$\frac{\log\left(\sqrt{2}\cos(3x+2)-\sin(3x+2)\right)}{6\sqrt{2}}-\frac{\log\left(\sin(3x+2)+\sqrt{2}\cos(3x+2)\right)}{6\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[-2/(1 + 3*Cos[4 + 6*x]),x]
```

```
[Out] Log[Sqrt[2]*Cos[2 + 3*x] - Sin[2 + 3*x]]/(6*Sqrt[2]) - Log[Sqrt[2]*Cos[2 +
3*x] + Sin[2 + 3*x]]/(6*Sqrt[2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int -\frac{2}{1+3\cos(4+6x)}\,dx = -\left(2\int \frac{1}{1+3\cos(4+6x)}\,dx\right)$$
$$= -\left(\frac{2}{3}\operatorname{Subst}\left(\int \frac{1}{4-2x^2}\,dx, x, \tan\left(\frac{1}{2}(4+6x)\right)\right)\right)$$
$$= \frac{\log\left(\sqrt{2}\cos(2+3x)-\sin(2+3x)\right)}{6\sqrt{2}} - \frac{\log\left(\sqrt{2}\cos(2+3x)+\sin(2+3x)\right)}{6\sqrt{2}}$$

Mathematica [A] time = 0.03, size = 22, normalized size = 0.36

$$-\frac{\tanh^{-1}\left(\frac{\tan(3x+2)}{\sqrt{2}}\right)}{3\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[-2/(1 + 3*Cos[4 + 6*x]),x]
[Out] -1/3*ArcTanh[Tan[2 + 3*x]/Sqrt[2]]/Sqrt[2]
```

fricas [A] time = 0.54, size = 74, normalized size = 1.21

$$\frac{1}{24}\sqrt{2}\log\left(-\frac{7\cos\left(6x+4\right)^2+4\left(\sqrt{2}\cos\left(6x+4\right)+3\sqrt{2}\right)\sin\left(6x+4\right)-6\cos\left(6x+4\right)-17}{9\cos\left(6x+4\right)^2+6\cos\left(6x+4\right)+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-2/(1+3*cos(4+6*x)),x, algorithm="fricas")
[Out] 1/24*sqrt(2)*log(-(7*cos(6*x + 4)^2 + 4*(sqrt(2)*cos(6*x + 4) + 3*sqrt(2))*
sin(6*x + 4) - 6*cos(6*x + 4) - 17)/(9*cos(6*x + 4)^2 + 6*cos(6*x + 4) + 1)
)
```

giac [A] time = 0.16, size = 39, normalized size = 0.64

$$\frac{1}{12}\sqrt{2}\log\left(\frac{\left|-2\sqrt{2}+2\tan\left(3x+2\right)\right|}{\left|2\sqrt{2}+2\tan\left(3x+2\right)\right|}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-2/(1+3*cos(4+6*x)),x, algorithm="giac")
```

```
[Out] 1/12*sqrt(2)*log(abs(-2*sqrt(2) + 2*tan(3*x + 2))/abs(2*sqrt(2) + 2*tan(3*x
 + 2)))
```

maple [A] time = 0.06, size = 18, normalized size = 0.30

$$-\frac{\sqrt{2}\ \operatorname{arctanh}\left(\frac{\sqrt{2}\ \tan(2+3x)}{2}\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-2/(1+3*cos(4+6*x)),x)
```

```
[Out] -1/6*2^(1/2)*arctanh(1/2*2^(1/2)*tan(2+3*x))
```

maxima [A] time = 0.43, size = 53, normalized size = 0.87

$$\frac{1}{12}\sqrt{2}\log\left(-\frac{\sqrt{2}-\frac{\sin(6x+4)}{\cos(6x+4)+1}}{\sqrt{2}+\frac{\sin(6x+4)}{\cos(6x+4)+1}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-2/(1+3*cos(4+6*x)),x, algorithm="maxima")
```

```
[Out] 1/12*sqrt(2)*log(-(sqrt(2) - sin(6*x + 4)/(cos(6*x + 4) + 1))/(sqrt(2) + si
n(6*x + 4)/(cos(6*x + 4) + 1)))
```

mupad [B] time = 2.50, size = 17, normalized size = 0.28

$$-\frac{\sqrt{2}\ \operatorname{atanh}\left(\frac{\sqrt{2}\ \tan(3x+2)}{2}\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-2/(3*cos(6*x + 4) + 1),x)
```

```
[Out] -(2^(1/2)*atanh((2^(1/2)*tan(3*x + 2))/2))/6
```

sympy [A] time = 0.31, size = 39, normalized size = 0.64

$$\frac{\sqrt{2}\log\left(\tan(3x+2)-\sqrt{2}\right)}{12}-\frac{\sqrt{2}\log\left(\tan(3x+2)+\sqrt{2}\right)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-2/(1+3*cos(4+6*x)),x)
```

```
[Out] sqrt(2)*log(tan(3*x + 2) - sqrt(2))/12 - sqrt(2)*log(tan(3*x + 2) + sqrt(2)
)/12
```

3.23 $\int -\frac{2\csc(4+6x)}{3\cot(4+6x)+\csc(4+6x)}\,dx$

Optimal. Leaf size=61

$$\frac{\log\left(\sqrt{2}\cos(3x+2)-\sin(3x+2)\right)}{6\sqrt{2}}-\frac{\log\left(\sin(3x+2)+\sqrt{2}\cos(3x+2)\right)}{6\sqrt{2}}$$

```
[Out] 1/12*ln(-sin(2+3*x)+cos(2+3*x)*2^(1/2))*2^(1/2)-1/12*ln(sin(2+3*x)+cos(2+3*
x)*2^(1/2))*2^(1/2)
```

Rubi [A] time = 0.05, antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {12, 3166, 2659, 206}

$$\frac{\log\left(\sqrt{2}\cos(3x+2)-\sin(3x+2)\right)}{6\sqrt{2}}-\frac{\log\left(\sin(3x+2)+\sqrt{2}\cos(3x+2)\right)}{6\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[(-2*Csc[4 + 6*x])/(3*Cot[4 + 6*x] + Csc[4 + 6*x]),x]
```

```
[Out] Log[Sqrt[2]*Cos[2 + 3*x] - Sin[2 + 3*x]]/(6*Sqrt[2]) - Log[Sqrt[2]*Cos[2 +
3*x] + Sin[2 + 3*x]]/(6*Sqrt[2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 3166

```
Int[csc[(d_.) + (e_.)*(x_)]^(n_.)*((a_.) + csc[(d_.) + (e_.)*(x_)]*(b_.) +
cot[(d_.) + (e_.)*(x_)]*(c_.))^(m_), x_Symbol] :> Int[1/(b + a*Sin[d + e*x]
 + c*Cos[d + e*x])^n, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[m + n, 0] && I
ntegerQ[n]
```

Rubi steps

$$\begin{aligned}\int -\frac{2\csc(4+6x)}{3\cot(4+6x)+\csc(4+6x)}\,dx &= -\left(2\int \frac{\csc(4+6x)}{3\cot(4+6x)+\csc(4+6x)}\,dx\right)\\ &= -\left(2\int \frac{1}{1+3\cos(4+6x)}\,dx\right)\\ &= -\left(\frac{2}{3}\operatorname{Subst}\left(\int \frac{1}{4-2x^2}\,dx, x, \tan\left(\frac{1}{2}(4+6x)\right)\right)\right)\\ &= \frac{\log\left(\sqrt{2}\cos(2+3x)-\sin(2+3x)\right)}{6\sqrt{2}}-\frac{\log\left(\sqrt{2}\cos(2+3x)+\sin(2+3x)\right)}{6\sqrt{2}}\end{aligned}$$

Mathematica [A] time = 0.03, size = 22, normalized size = 0.36

$$-\frac{\tanh^{-1}\left(\frac{\tan(3x+2)}{\sqrt{2}}\right)}{3\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-2*Csc[4 + 6*x])/(3*Cot[4 + 6*x] + Csc[4 + 6*x]),x]
[Out] -1/3*ArcTanh[Tan[2 + 3*x]/Sqrt[2]]/Sqrt[2]
```

fricas [A] time = 0.55, size = 74, normalized size = 1.21

$$\frac{1}{24}\sqrt{2}\log\left(-\frac{7\cos\left(6x+4\right)^2+4\left(\sqrt{2}\cos\left(6x+4\right)+3\sqrt{2}\right)\sin\left(6x+4\right)-6\cos\left(6x+4\right)-17}{9\cos\left(6x+4\right)^2+6\cos\left(6x+4\right)+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-2*csc(4+6*x)/(3*cot(4+6*x)+csc(4+6*x)),x, algorithm="fricas")
[Out] 1/24*sqrt(2)*log(-(7*cos(6*x + 4)^2 + 4*(sqrt(2)*cos(6*x + 4) + 3*sqrt(2))*
sin(6*x + 4) - 6*cos(6*x + 4) - 17)/(9*cos(6*x + 4)^2 + 6*cos(6*x + 4) + 1)
)
```

giac [A] time = 0.24, size = 39, normalized size = 0.64

$$\frac{1}{12}\,\sqrt{2}\log\left(\frac{\left|-2\,\sqrt{2}+2\,\tan\left(3\,x+2\right)\right|}{\left|2\,\sqrt{2}+2\,\tan\left(3\,x+2\right)\right|}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-2*csc(4+6*x)/(3*cot(4+6*x)+csc(4+6*x)),x, algorithm="giac")
```

```
[Out] 1/12*sqrt(2)*log(abs(-2*sqrt(2) + 2*tan(3*x + 2))/abs(2*sqrt(2) + 2*tan(3*x
 + 2)))
```

maple [A] time = 0.31, size = 18, normalized size = 0.30

$$-\frac{\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{2}\,\tan(2+3x)}{2}\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-2*csc(4+6*x)/(3*cot(4+6*x)+csc(4+6*x)),x)
```

```
[Out] -1/6*2^(1/2)*arctanh(1/2*2^(1/2)*tan(2+3*x))
```

maxima [A] time = 0.53, size = 53, normalized size = 0.87

$$\frac{1}{12}\,\sqrt{2}\log\left(-\frac{\sqrt{2}-\frac{\sin(6\,x+4)}{\cos(6\,x+4)+1}}{\sqrt{2}+\frac{\sin(6\,x+4)}{\cos(6\,x+4)+1}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-2*csc(4+6*x)/(3*cot(4+6*x)+csc(4+6*x)),x, algorithm="maxima")
```

```
[Out] 1/12*sqrt(2)*log(-(sqrt(2) - sin(6*x + 4)/(cos(6*x + 4) + 1))/(sqrt(2) + si
n(6*x + 4)/(cos(6*x + 4) + 1)))
```

mupad [B] time = 2.71, size = 17, normalized size = 0.28

$$-\frac{\sqrt{2}\,\operatorname{atanh}\left(\frac{\sqrt{2}\,\tan(3\,x+2)}{2}\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-2/(sin(6*x + 4)*(3*cot(6*x + 4) + 1/sin(6*x + 4))),x)
```

```
[Out] -(2^(1/2)*atanh((2^(1/2)*tan(3*x + 2))/2))/6
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-2\int \frac{\csc(6x+4)}{3\cot(6x+4)+\csc(6x+4)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-2*csc(4+6*x)/(3*cot(4+6*x)+csc(4+6*x)),x)
[Out] -2*Integral(csc(6*x + 4)/(3*cot(6*x + 4) + csc(6*x + 4)), x)
```

3.24 $\int \frac{1}{-2+3\sin^2(2+3x)} \, dx$

Optimal. Leaf size=61

$$\frac{\log\left(\sqrt{2}\cos(3x+2)-\sin(3x+2)\right)}{6\sqrt{2}}-\frac{\log\left(\sin(3x+2)+\sqrt{2}\cos(3x+2)\right)}{6\sqrt{2}}$$

```
[Out] 1/12*ln(-sin(2+3*x)+cos(2+3*x)*2^(1/2))*2^(1/2)-1/12*ln(sin(2+3*x)+cos(2+3*
x)*2^(1/2))*2^(1/2)
```

Rubi [A] time = 0.02, antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3181, 207}

$$\frac{\log\left(\sqrt{2}\cos(3x+2)-\sin(3x+2)\right)}{6\sqrt{2}}-\frac{\log\left(\sin(3x+2)+\sqrt{2}\cos(3x+2)\right)}{6\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[(-2 + 3*Sin[2 + 3*x]^2)^(-1),x]
```

```
[Out] Log[Sqrt[2]*Cos[2 + 3*x] - Sin[2 + 3*x]]/(6*Sqrt[2]) - Log[Sqrt[2]*Cos[2 +
3*x] + Sin[2 + 3*x]]/(6*Sqrt[2])
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 3181

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(-1), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[1/(a + (a + b)*ff^2*x^2
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{-2+3\sin^2(2+3x)} \, dx &= \frac{1}{3}\text{Subst}\left(\int \frac{1}{-2+x^2} \, dx, x, \tan(2+3x)\right)\\ &= \frac{\log\left(\sqrt{2}\cos(2+3x)-\sin(2+3x)\right)}{6\sqrt{2}}-\frac{\log\left(\sqrt{2}\cos(2+3x)+\sin(2+3x)\right)}{6\sqrt{2}}\end{aligned}$$

Mathematica [A] time = 0.06, size = 22, normalized size = 0.36

$$-\frac{\tanh^{-1}\left(\frac{\tan(3x+2)}{\sqrt{2}}\right)}{3\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-2 + 3*Sin[2 + 3*x]^2)^(-1),x]
```

```
[Out] -1/3*ArcTanh[Tan[2 + 3*x]/Sqrt[2]]/Sqrt[2]
```

fricas [A] time = 0.59, size = 85, normalized size = 1.39

$$\frac{1}{24}\sqrt{2}\log\left(-\frac{7\cos(3x+2)^{4}-10\cos(3x+2)^{2}+4\left(\sqrt{2}\cos(3x+2)^{3}+\sqrt{2}\cos(3x+2)\right)\sin(3x+2)-1}{9\cos(3x+2)^{4}-6\cos(3x+2)^{2}+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-2+3*sin(2+3*x)^2),x, algorithm="fricas")
```

```
[Out] 1/24*sqrt(2)*log(-(7*cos(3*x + 2)^4 - 10*cos(3*x + 2)^2 + 4*(sqrt(2)*cos(3*
x + 2)^3 + sqrt(2)*cos(3*x + 2))*sin(3*x + 2) - 1)/(9*cos(3*x + 2)^4 - 6*co
s(3*x + 2)^2 + 1))
```

giac [A] time = 0.19, size = 39, normalized size = 0.64

$$\frac{1}{12}\sqrt{2}\log\left(\frac{\left|-2\sqrt{2}+2\tan(3x+2)\right|}{\left|2\sqrt{2}+2\tan(3x+2)\right|}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-2+3*sin(2+3*x)^2),x, algorithm="giac")
```

```
[Out] 1/12*sqrt(2)*log(abs(-2*sqrt(2) + 2*tan(3*x + 2))/abs(2*sqrt(2) + 2*tan(3*x
 + 2)))
```

maple [A] time = 0.15, size = 18, normalized size = 0.30

$$-\frac{\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{2}\,\tan(2+3x)}{2}\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-2+3*sin(2+3*x)^2),x)
```

```
[Out] -1/6*2^(1/2)*arctanh(1/2*2^(1/2)*tan(2+3*x))
```

maxima [A] time = 0.50, size = 32, normalized size = 0.52

$$\frac{1}{12}\sqrt{2}\log\left(-\frac{\sqrt{2}-\tan(3x+2)}{\sqrt{2}+\tan(3x+2)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-2+3*sin(2+3*x)^2),x, algorithm="maxima")
[Out] 1/12*sqrt(2)*log(-(sqrt(2) - tan(3*x + 2))/(sqrt(2) + tan(3*x + 2)))
```

mupad [B] time = 2.51, size = 17, normalized size = 0.28

$$-\frac{\sqrt{2}\,\mathrm{atanh}\left(\frac{\sqrt{2}\tan(3x+2)}{2}\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(3*sin(3*x + 2)^2 - 2),x)
[Out] -(2^(1/2)*atanh((2^(1/2)*tan(3*x + 2))/2))/6
```

sympy [B] time = 14.07, size = 1481, normalized size = 24.28

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-2+3*sin(2+3*x)**2),x)
[Out] -4988289*sqrt(3)*sqrt(sqrt(3) + 2)*log(tan(3*x/2 + 1) - sqrt(2 - sqrt(3)))/
(-39175383*sqrt(2 - sqrt(3))*sqrt(sqrt(3) + 2) - 2003742*sqrt(3) + 3470583
+ 22617918*sqrt(3)*sqrt(2 - sqrt(3))*sqrt(sqrt(3) + 2)) - 136929*sqrt(2 - s
qrt(3))*log(tan(3*x/2 + 1) - sqrt(2 - sqrt(3)))/(-39175383*sqrt(2 - sqrt(3)
)*sqrt(sqrt(3) + 2) - 2003742*sqrt(3) + 3470583 + 22617918*sqrt(3)*sqrt(2 -
 sqrt(3))*sqrt(sqrt(3) + 2)) + 79056*sqrt(3)*sqrt(2 - sqrt(3))*log(tan(3*x/
2 + 1) - sqrt(2 - sqrt(3)))/(-39175383*sqrt(2 - sqrt(3))*sqrt(sqrt(3) + 2)
- 2003742*sqrt(3) + 3470583 + 22617918*sqrt(3)*sqrt(2 - sqrt(3))*sqrt(sqrt(
3) + 2)) + 8639970*sqrt(sqrt(3) + 2)*log(tan(3*x/2 + 1) - sqrt(2 - sqrt(3))
)/(-39175383*sqrt(2 - sqrt(3))*sqrt(sqrt(3) + 2) - 2003742*sqrt(3) + 347058
3 + 22617918*sqrt(3)*sqrt(2 - sqrt(3))*sqrt(sqrt(3) + 2)) - 11281635*sqrt(s
qrt(3) + 2)*log(tan(3*x/2 + 1) + sqrt(2 - sqrt(3)))/(-39175383*sqrt(2 - sqr
t(3))*sqrt(sqrt(3) + 2) - 2003742*sqrt(3) + 3470583 + 22617918*sqrt(3)*sqrt
(2 - sqrt(3))*sqrt(sqrt(3) + 2)) - 487723*sqrt(3)*sqrt(2 - sqrt(3))*log(tan
```

```
(3*x/2 + 1) + sqrt(2 - sqrt(3)))/(-39175383*sqrt(2 - sqrt(3))*sqrt(sqrt(3)
+ 2) - 2003742*sqrt(3) + 3470583 + 22617918*sqrt(3)*sqrt(2 - sqrt(3))*sqrt(
sqrt(3) + 2)) + 844761*sqrt(2 - sqrt(3))*log(tan(3*x/2 + 1) + sqrt(2 - sqrt
(3)))/(-39175383*sqrt(2 - sqrt(3))*sqrt(sqrt(3) + 2) - 2003742*sqrt(3) + 34
70583 + 22617918*sqrt(3)*sqrt(2 - sqrt(3))*sqrt(sqrt(3) + 2)) + 6513455*sqr
t(3)*sqrt(sqrt(3) + 2)*log(tan(3*x/2 + 1) + sqrt(2 - sqrt(3)))/(-39175383*s
qrt(2 - sqrt(3))*sqrt(sqrt(3) + 2) - 2003742*sqrt(3) + 3470583 + 22617918*s
qrt(3)*sqrt(2 - sqrt(3))*sqrt(sqrt(3) + 2)) - 1820207*sqrt(3)*sqrt(sqrt(3)
+ 2)*log(tan(3*x/2 + 1) - sqrt(sqrt(3) + 2))/(-39175383*sqrt(2 - sqrt(3))*s
qrt(sqrt(3) + 2) - 2003742*sqrt(3) + 3470583 + 22617918*sqrt(3)*sqrt(2 - sq
rt(3))*sqrt(sqrt(3) + 2)) - 3022905*sqrt(2 - sqrt(3))*log(tan(3*x/2 + 1) -
sqrt(sqrt(3) + 2))/(-39175383*sqrt(2 - sqrt(3))*sqrt(sqrt(3) + 2) - 2003742
*sqrt(3) + 3470583 + 22617918*sqrt(3)*sqrt(2 - sqrt(3))*sqrt(sqrt(3) + 2))
+ 1745275*sqrt(3)*sqrt(2 - sqrt(3))*log(tan(3*x/2 + 1) - sqrt(sqrt(3) + 2))
/(-39175383*sqrt(2 - sqrt(3))*sqrt(sqrt(3) + 2) - 2003742*sqrt(3) + 3470583
 + 22617918*sqrt(3)*sqrt(2 - sqrt(3))*sqrt(sqrt(3) + 2)) + 3152691*sqrt(sqr
t(3) + 2)*log(tan(3*x/2 + 1) - sqrt(sqrt(3) + 2))/(-39175383*sqrt(2 - sqrt(
3))*sqrt(sqrt(3) + 2) - 2003742*sqrt(3) + 3470583 + 22617918*sqrt(3)*sqrt(2
 - sqrt(3))*sqrt(sqrt(3) + 2)) - 1336608*sqrt(3)*sqrt(2 - sqrt(3))*log(tan(
3*x/2 + 1) + sqrt(sqrt(3) + 2))/(-39175383*sqrt(2 - sqrt(3))*sqrt(sqrt(3) +
 2) - 2003742*sqrt(3) + 3470583 + 22617918*sqrt(3)*sqrt(2 - sqrt(3))*sqrt(s
qrt(3) + 2)) - 511026*sqrt(sqrt(3) + 2)*log(tan(3*x/2 + 1) + sqrt(sqrt(3) +
 2))/(-39175383*sqrt(2 - sqrt(3))*sqrt(sqrt(3) + 2) - 2003742*sqrt(3) + 347
0583 + 22617918*sqrt(3)*sqrt(2 - sqrt(3))*sqrt(sqrt(3) + 2)) + 295041*sqrt(
3)*sqrt(sqrt(3) + 2)*log(tan(3*x/2 + 1) + sqrt(sqrt(3) + 2))/(-39175383*sqr
t(2 - sqrt(3))*sqrt(sqrt(3) + 2) - 2003742*sqrt(3) + 3470583 + 22617918*sqr
t(3)*sqrt(2 - sqrt(3))*sqrt(sqrt(3) + 2)) + 2315073*sqrt(2 - sqrt(3))*log(t
an(3*x/2 + 1) + sqrt(sqrt(3) + 2))/(-39175383*sqrt(2 - sqrt(3))*sqrt(sqrt(3
) + 2) - 2003742*sqrt(3) + 3470583 + 22617918*sqrt(3)*sqrt(2 - sqrt(3))*sqr
t(sqrt(3) + 2))
```

3.25 $\int \frac{1}{1-3\cos^2(2+3x)}\,dx$

Optimal. Leaf size=61

$$\frac{\log\left(\sqrt{2}\cos(3x+2)-\sin(3x+2)\right)}{6\sqrt{2}}-\frac{\log\left(\sin(3x+2)+\sqrt{2}\cos(3x+2)\right)}{6\sqrt{2}}$$

```
[Out] 1/12*ln(-sin(2+3*x)+cos(2+3*x)*2^(1/2))*2^(1/2)-1/12*ln(sin(2+3*x)+cos(2+3*
x)*2^(1/2))*2^(1/2)
```

Rubi [A] time = 0.02, antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3181, 206}

$$\frac{\log\left(\sqrt{2}\cos(3x+2)-\sin(3x+2)\right)}{6\sqrt{2}}-\frac{\log\left(\sin(3x+2)+\sqrt{2}\cos(3x+2)\right)}{6\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 - 3*Cos[2 + 3*x]^2)^(-1),x]
```

```
[Out] Log[Sqrt[2]*Cos[2 + 3*x] - Sin[2 + 3*x]]/(6*Sqrt[2]) - Log[Sqrt[2]*Cos[2 +
3*x] + Sin[2 + 3*x]]/(6*Sqrt[2])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3181

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(-1), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[1/(a + (a + b)*ff^2*x^2
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{1-3\cos^2(2+3x)}\,dx &= -\left(\frac{1}{3}\text{Subst}\left(\int \frac{1}{1-2x^2}\,dx, x, \cot(2+3x)\right)\right)\\ &= \frac{\log\left(\sqrt{2}\cos(2+3x)-\sin(2+3x)\right)}{6\sqrt{2}}-\frac{\log\left(\sqrt{2}\cos(2+3x)+\sin(2+3x)\right)}{6\sqrt{2}}\end{aligned}$$

Mathematica [A] time = 0.06, size = 22, normalized size = 0.36

$$-\frac{\tanh^{-1}\left(\frac{\tan(3x+2)}{\sqrt{2}}\right)}{3\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - 3*Cos[2 + 3*x]^2)^(-1),x]
```

```
[Out] -1/3*ArcTanh[Tan[2 + 3*x]/Sqrt[2]]/Sqrt[2]
```

fricas [A] time = 0.59, size = 85, normalized size = 1.39

$$\frac{1}{24}\sqrt{2}\log\left(-\frac{7\cos(3x+2)^4-10\cos(3x+2)^2+4\left(\sqrt{2}\cos(3x+2)^3+\sqrt{2}\cos(3x+2)\right)\sin(3x+2)-1}{9\cos(3x+2)^4-6\cos(3x+2)^2+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-3*cos(2+3*x)^2),x, algorithm="fricas")
```

```
[Out] 1/24*sqrt(2)*log(-(7*cos(3*x + 2)^4 - 10*cos(3*x + 2)^2 + 4*(sqrt(2)*cos(3*
x + 2)^3 + sqrt(2)*cos(3*x + 2))*sin(3*x + 2) - 1)/(9*cos(3*x + 2)^4 - 6*co
s(3*x + 2)^2 + 1))
```

giac [A] time = 0.17, size = 39, normalized size = 0.64

$$\frac{1}{12}\sqrt{2}\log\left(\frac{\left|-2\sqrt{2}+2\tan(3x+2)\right|}{\left|2\sqrt{2}+2\tan(3x+2)\right|}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-3*cos(2+3*x)^2),x, algorithm="giac")
```

```
[Out] 1/12*sqrt(2)*log(abs(-2*sqrt(2) + 2*tan(3*x + 2))/abs(2*sqrt(2) + 2*tan(3*x
 + 2)))
```

maple [A] time = 0.10, size = 18, normalized size = 0.30

$$-\frac{\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{2}\,\tan(2+3x)}{2}\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1-3*cos(2+3*x)^2),x)
```

[Out] -1/6*2^(1/2)*arctanh(1/2*2^(1/2)*tan(2+3*x))

maxima [A] time = 0.52, size = 32, normalized size = 0.52

$$\frac{1}{12}\,\sqrt{2}\,\log\left(-\frac{\sqrt{2}-\tan\left(3\,x+2\right)}{\sqrt{2}+\tan\left(3\,x+2\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(1-3*cos(2+3*x)^2),x, algorithm="maxima")

[Out] 1/12*sqrt(2)*log(-(sqrt(2) - tan(3*x + 2))/(sqrt(2) + tan(3*x + 2)))

mupad [B] time = 2.44, size = 17, normalized size = 0.28

$$-\frac{\sqrt{2}\,\mathrm{atanh}\left(\frac{\sqrt{2}\,\tan\left(3\,x+2\right)}{2}\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(-1/(3*cos(3*x + 2)^2 - 1),x)

[Out] -(2^(1/2)*atanh((2^(1/2)*tan(3*x + 2))/2))/6

sympy [B] time = 12.83, size = 1481, normalized size = 24.28

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(1-3*cos(2+3*x)**2),x)

[Out] -4988289*sqrt(3)*sqrt(sqrt(3) + 2)*log(tan(3*x/2 + 1) - sqrt(2 - sqrt(3)))/(-39175383*sqrt(2 - sqrt(3))*sqrt(sqrt(3) + 2) - 2003742*sqrt(3) + 3470583 + 22617918*sqrt(3)*sqrt(2 - sqrt(3))*sqrt(sqrt(3) + 2)) - 136929*sqrt(2 - sqrt(3))*log(tan(3*x/2 + 1) - sqrt(2 - sqrt(3)))/(-39175383*sqrt(2 - sqrt(3))*sqrt(sqrt(3) + 2) - 2003742*sqrt(3) + 3470583 + 22617918*sqrt(3)*sqrt(2 - sqrt(3))*sqrt(sqrt(3) + 2)) + 79056*sqrt(3)*sqrt(2 - sqrt(3))*log(tan(3*x/2 + 1) - sqrt(2 - sqrt(3)))/(-39175383*sqrt(2 - sqrt(3))*sqrt(sqrt(3) + 2) - 2003742*sqrt(3) + 3470583 + 22617918*sqrt(3)*sqrt(2 - sqrt(3))*sqrt(sqrt(3) + 2)) + 8639970*sqrt(sqrt(3) + 2)*log(tan(3*x/2 + 1) - sqrt(2 - sqrt(3)))/(-39175383*sqrt(2 - sqrt(3))*sqrt(sqrt(3) + 2) - 2003742*sqrt(3) + 3470583 + 22617918*sqrt(3)*sqrt(2 - sqrt(3))*sqrt(sqrt(3) + 2)) - 11281635*sqrt(sqrt(3) + 2)*log(tan(3*x/2 + 1) + sqrt(2 - sqrt(3)))/(-39175383*sqrt(2 - sqrt(3))*sqrt(sqrt(3) + 2) - 2003742*sqrt(3) + 3470583 + 22617918*sqrt(3)*sqrt(2 - sqrt(3))*sqrt(sqrt(3) + 2)) - 487723*sqrt(3)*sqrt(2 - sqrt(3))*log(tan

```
(3*x/2 + 1) + sqrt(2 - sqrt(3)))/(-39175383*sqrt(2 - sqrt(3))*sqrt(sqrt(3)
+ 2) - 2003742*sqrt(3) + 3470583 + 22617918*sqrt(3)*sqrt(2 - sqrt(3))*sqrt(
sqrt(3) + 2)) + 844761*sqrt(2 - sqrt(3))*log(tan(3*x/2 + 1) + sqrt(2 - sqrt
(3)))/(-39175383*sqrt(2 - sqrt(3))*sqrt(sqrt(3) + 2) - 2003742*sqrt(3) + 34
70583 + 22617918*sqrt(3)*sqrt(2 - sqrt(3))*sqrt(sqrt(3) + 2)) + 6513455*sqr
t(3)*sqrt(sqrt(3) + 2)*log(tan(3*x/2 + 1) + sqrt(2 - sqrt(3)))/(-39175383*s
qrt(2 - sqrt(3))*sqrt(sqrt(3) + 2) - 2003742*sqrt(3) + 3470583 + 22617918*s
qrt(3)*sqrt(2 - sqrt(3))*sqrt(sqrt(3) + 2)) - 1820207*sqrt(3)*sqrt(sqrt(3)
+ 2)*log(tan(3*x/2 + 1) - sqrt(sqrt(3) + 2))/(-39175383*sqrt(2 - sqrt(3))*s
qrt(sqrt(3) + 2) - 2003742*sqrt(3) + 3470583 + 22617918*sqrt(3)*sqrt(2 - sq
rt(3))*sqrt(sqrt(3) + 2)) - 3022905*sqrt(2 - sqrt(3))*log(tan(3*x/2 + 1) -
sqrt(sqrt(3) + 2))/(-39175383*sqrt(2 - sqrt(3))*sqrt(sqrt(3) + 2) - 2003742
*sqrt(3) + 3470583 + 22617918*sqrt(3)*sqrt(2 - sqrt(3))*sqrt(sqrt(3) + 2))
+ 1745275*sqrt(3)*sqrt(2 - sqrt(3))*log(tan(3*x/2 + 1) - sqrt(sqrt(3) + 2))
/(-39175383*sqrt(2 - sqrt(3))*sqrt(sqrt(3) + 2) - 2003742*sqrt(3) + 3470583
 + 22617918*sqrt(3)*sqrt(2 - sqrt(3))*sqrt(sqrt(3) + 2)) + 3152691*sqrt(sqr
t(3) + 2)*log(tan(3*x/2 + 1) - sqrt(sqrt(3) + 2))/(-39175383*sqrt(2 - sqrt(
3))*sqrt(sqrt(3) + 2) - 2003742*sqrt(3) + 3470583 + 22617918*sqrt(3)*sqrt(2
 - sqrt(3))*sqrt(sqrt(3) + 2)) - 1336608*sqrt(3)*sqrt(2 - sqrt(3))*log(tan(
3*x/2 + 1) + sqrt(sqrt(3) + 2))/(-39175383*sqrt(2 - sqrt(3))*sqrt(sqrt(3) +
 2) - 2003742*sqrt(3) + 3470583 + 22617918*sqrt(3)*sqrt(2 - sqrt(3))*sqrt(s
qrt(3) + 2)) - 511026*sqrt(sqrt(3) + 2)*log(tan(3*x/2 + 1) + sqrt(sqrt(3) +
 2))/(-39175383*sqrt(2 - sqrt(3))*sqrt(sqrt(3) + 2) - 2003742*sqrt(3) + 347
0583 + 22617918*sqrt(3)*sqrt(2 - sqrt(3))*sqrt(sqrt(3) + 2)) + 295041*sqrt(
3)*sqrt(sqrt(3) + 2)*log(tan(3*x/2 + 1) + sqrt(sqrt(3) + 2))/(-39175383*sqr
t(2 - sqrt(3))*sqrt(sqrt(3) + 2) - 2003742*sqrt(3) + 3470583 + 22617918*sqr
t(3)*sqrt(2 - sqrt(3))*sqrt(sqrt(3) + 2)) + 2315073*sqrt(2 - sqrt(3))*log(t
an(3*x/2 + 1) + sqrt(sqrt(3) + 2))/(-39175383*sqrt(2 - sqrt(3))*sqrt(sqrt(3
) + 2) - 2003742*sqrt(3) + 3470583 + 22617918*sqrt(3)*sqrt(2 - sqrt(3))*sqr
t(sqrt(3) + 2))
```

3.26 $\int \frac{1}{-2\cos^2(2+3x)+\sin^2(2+3x)}\,dx$

Optimal. Leaf size=61

$$\frac{\log\left(\sqrt{2}\cos(3x+2)-\sin(3x+2)\right)}{6\sqrt{2}}-\frac{\log\left(\sin(3x+2)+\sqrt{2}\cos(3x+2)\right)}{6\sqrt{2}}$$

```
[Out] 1/12*ln(-sin(2+3*x)+cos(2+3*x)*2^(1/2))*2^(1/2)-1/12*ln(sin(2+3*x)+cos(2+3*
x)*2^(1/2))*2^(1/2)
```

Rubi [A] time = 0.03, antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.048, Rules used = {207}

$$\frac{\log\left(\sqrt{2}\cos(3x+2)-\sin(3x+2)\right)}{6\sqrt{2}}-\frac{\log\left(\sin(3x+2)+\sqrt{2}\cos(3x+2)\right)}{6\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[(-2*Cos[2 + 3*x]^2 + Sin[2 + 3*x]^2)^(-1),x]
```

```
[Out] Log[Sqrt[2]*Cos[2 + 3*x] - Sin[2 + 3*x]]/(6*Sqrt[2]) - Log[Sqrt[2]*Cos[2 +
3*x] + Sin[2 + 3*x]]/(6*Sqrt[2])
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{-2\cos^2(2+3x)+\sin^2(2+3x)}\,dx &= \frac{1}{3}\operatorname{Subst}\left(\int \frac{1}{-2+x^2}\,dx, x, \tan(2+3x)\right)\\ &= \frac{\log\left(\sqrt{2}\cos(2+3x)-\sin(2+3x)\right)}{6\sqrt{2}}-\frac{\log\left(\sqrt{2}\cos(2+3x)+\sin(2+3x)\right)}{6\sqrt{2}}\end{aligned}$$

Mathematica [A] time = 0.03, size = 22, normalized size = 0.36

$$-\frac{\tanh^{-1}\left(\frac{\tan(3x+2)}{\sqrt{2}}\right)}{3\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-2*Cos[2 + 3*x]^2 + Sin[2 + 3*x]^2)^(-1),x]
```

```
[Out] -1/3*ArcTanh[Tan[2 + 3*x]/Sqrt[2]]/Sqrt[2]
```

fricas [A] time = 0.57, size = 85, normalized size = 1.39

$$\frac{1}{24}\sqrt{2}\log\left(-\frac{7\cos(3x+2)^4-10\cos(3x+2)^2+4\left(\sqrt{2}\cos(3x+2)^3+\sqrt{2}\cos(3x+2)\right)\sin(3x+2)-1}{9\cos(3x+2)^4-6\cos(3x+2)^2+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-2*cos(2+3*x)^2+sin(2+3*x)^2),x, algorithm="fricas")
```

```
[Out] 1/24*sqrt(2)*log(-(7*cos(3*x + 2)^4 - 10*cos(3*x + 2)^2 + 4*(sqrt(2)*cos(3*
x + 2)^3 + sqrt(2)*cos(3*x + 2))*sin(3*x + 2) - 1)/(9*cos(3*x + 2)^4 - 6*co
s(3*x + 2)^2 + 1))
```

giac [A] time = 0.21, size = 39, normalized size = 0.64

$$\frac{1}{12}\sqrt{2}\log\left(\frac{\left|-2\sqrt{2}+2\tan(3x+2)\right|}{\left|2\sqrt{2}+2\tan(3x+2)\right|}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-2*cos(2+3*x)^2+sin(2+3*x)^2),x, algorithm="giac")
```

```
[Out] 1/12*sqrt(2)*log(abs(-2*sqrt(2) + 2*tan(3*x + 2))/abs(2*sqrt(2) + 2*tan(3*x
 + 2)))
```

maple [A] time = 0.23, size = 18, normalized size = 0.30

$$-\frac{\sqrt{2}\ \operatorname{arctanh}\left(\frac{\sqrt{2}\ \tan(2+3x)}{2}\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-2*cos(2+3*x)^2+sin(2+3*x)^2),x)
```

```
[Out] -1/6*2^(1/2)*arctanh(1/2*2^(1/2)*tan(2+3*x))
```

maxima [A] time = 0.54, size = 32, normalized size = 0.52

$$\frac{1}{12}\sqrt{2}\log\left(-\frac{\sqrt{2}-\tan(3x+2)}{\sqrt{2}+\tan(3x+2)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-2*cos(2+3*x)^2+sin(2+3*x)^2),x, algorithm="maxima")
```

```
[Out] 1/12*sqrt(2)*log(-(sqrt(2) - tan(3*x + 2))/(sqrt(2) + tan(3*x + 2)))
```

mupad [B] time = 2.42, size = 17, normalized size = 0.28

$$-\frac{\sqrt{2}\,\mathrm{atanh}\left(\frac{\sqrt{2}\,\tan(3\,x+2)}{2}\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(3*x + 2)^2 - 2*cos(3*x + 2)^2),x)
```

```
[Out] -(2^(1/2)*atanh((2^(1/2)*tan(3*x + 2))/2))/6
```

sympy [B] time = 16.48, size = 1481, normalized size = 24.28

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-2*cos(2+3*x)**2+sin(2+3*x)**2),x)
```

```
[Out] -4988289*sqrt(3)*sqrt(sqrt(3) + 2)*log(tan(3*x/2 + 1) - sqrt(2 - sqrt(3)))/
(-39175383*sqrt(2 - sqrt(3))*sqrt(sqrt(3) + 2) - 2003742*sqrt(3) + 3470583
+ 22617918*sqrt(3)*sqrt(2 - sqrt(3))*sqrt(sqrt(3) + 2)) - 136929*sqrt(2 - s
qrt(3))*log(tan(3*x/2 + 1) - sqrt(2 - sqrt(3)))/(-39175383*sqrt(2 - sqrt(3)
)*sqrt(sqrt(3) + 2) - 2003742*sqrt(3) + 3470583 + 22617918*sqrt(3)*sqrt(2 -
 sqrt(3))*sqrt(sqrt(3) + 2)) + 79056*sqrt(3)*sqrt(2 - sqrt(3))*log(tan(3*x/
2 + 1) - sqrt(2 - sqrt(3)))/(-39175383*sqrt(2 - sqrt(3))*sqrt(sqrt(3) + 2)
- 2003742*sqrt(3) + 3470583 + 22617918*sqrt(3)*sqrt(2 - sqrt(3))*sqrt(sqrt(
3) + 2)) + 8639970*sqrt(sqrt(3) + 2)*log(tan(3*x/2 + 1) - sqrt(2 - sqrt(3))
)/(-39175383*sqrt(2 - sqrt(3))*sqrt(sqrt(3) + 2) - 2003742*sqrt(3) + 347058
3 + 22617918*sqrt(3)*sqrt(2 - sqrt(3))*sqrt(sqrt(3) + 2)) - 11281635*sqrt(s
qrt(3) + 2)*log(tan(3*x/2 + 1) + sqrt(2 - sqrt(3)))/(-39175383*sqrt(2 - sqr
t(3))*sqrt(sqrt(3) + 2) - 2003742*sqrt(3) + 3470583 + 22617918*sqrt(3)*sqrt
(2 - sqrt(3))*sqrt(sqrt(3) + 2)) - 487723*sqrt(3)*sqrt(2 - sqrt(3))*log(tan
(3*x/2 + 1) + sqrt(2 - sqrt(3)))/(-39175383*sqrt(2 - sqrt(3))*sqrt(sqrt(3)
+ 2) - 2003742*sqrt(3) + 3470583 + 22617918*sqrt(3)*sqrt(2 - sqrt(3))*sqrt(
sqrt(3) + 2)) + 844761*sqrt(2 - sqrt(3))*log(tan(3*x/2 + 1) + sqrt(2 - sqrt
(3)))/(-39175383*sqrt(2 - sqrt(3))*sqrt(sqrt(3) + 2) - 2003742*sqrt(3) + 34
70583 + 22617918*sqrt(3)*sqrt(2 - sqrt(3))*sqrt(sqrt(3) + 2)) + 6513455*sqr
t(3)*sqrt(sqrt(3) + 2)*log(tan(3*x/2 + 1) + sqrt(2 - sqrt(3)))/(-39175383*s
qrt(2 - sqrt(3))*sqrt(sqrt(3) + 2) - 2003742*sqrt(3) + 3470583 + 22617918*s
```

```
qrt(3)*sqrt(2 - sqrt(3))*sqrt(sqrt(3) + 2)) - 1820207*sqrt(3)*sqrt(sqrt(3)
+ 2)*log(tan(3*x/2 + 1) - sqrt(sqrt(3) + 2))/(-39175383*sqrt(2 - sqrt(3))*s
qrt(sqrt(3) + 2) - 2003742*sqrt(3) + 3470583 + 22617918*sqrt(3)*sqrt(2 - sq
rt(3))*sqrt(sqrt(3) + 2)) - 3022905*sqrt(2 - sqrt(3))*log(tan(3*x/2 + 1) -
sqrt(sqrt(3) + 2))/(-39175383*sqrt(2 - sqrt(3))*sqrt(sqrt(3) + 2) - 2003742
*sqrt(3) + 3470583 + 22617918*sqrt(3)*sqrt(2 - sqrt(3))*sqrt(sqrt(3) + 2))
+ 1745275*sqrt(3)*sqrt(2 - sqrt(3))*log(tan(3*x/2 + 1) - sqrt(sqrt(3) + 2))
/(-39175383*sqrt(2 - sqrt(3))*sqrt(sqrt(3) + 2) - 2003742*sqrt(3) + 3470583
 + 22617918*sqrt(3)*sqrt(2 - sqrt(3))*sqrt(sqrt(3) + 2)) + 3152691*sqrt(sqr
t(3) + 2)*log(tan(3*x/2 + 1) - sqrt(sqrt(3) + 2))/(-39175383*sqrt(2 - sqrt(
3))*sqrt(sqrt(3) + 2) - 2003742*sqrt(3) + 3470583 + 22617918*sqrt(3)*sqrt(2
 - sqrt(3))*sqrt(sqrt(3) + 2)) - 1336608*sqrt(3)*sqrt(2 - sqrt(3))*log(tan(
3*x/2 + 1) + sqrt(sqrt(3) + 2))/(-39175383*sqrt(2 - sqrt(3))*sqrt(sqrt(3) +
 2) - 2003742*sqrt(3) + 3470583 + 22617918*sqrt(3)*sqrt(2 - sqrt(3))*sqrt(s
qrt(3) + 2)) - 511026*sqrt(sqrt(3) + 2)*log(tan(3*x/2 + 1) + sqrt(sqrt(3) +
 2))/(-39175383*sqrt(2 - sqrt(3))*sqrt(sqrt(3) + 2) - 2003742*sqrt(3) + 347
0583 + 22617918*sqrt(3)*sqrt(2 - sqrt(3))*sqrt(sqrt(3) + 2)) + 295041*sqrt(
3)*sqrt(sqrt(3) + 2)*log(tan(3*x/2 + 1) + sqrt(sqrt(3) + 2))/(-39175383*sqr
t(2 - sqrt(3))*sqrt(sqrt(3) + 2) - 2003742*sqrt(3) + 3470583 + 22617918*sqr
t(3)*sqrt(2 - sqrt(3))*sqrt(sqrt(3) + 2)) + 2315073*sqrt(2 - sqrt(3))*log(t
an(3*x/2 + 1) + sqrt(sqrt(3) + 2))/(-39175383*sqrt(2 - sqrt(3))*sqrt(sqrt(3
) + 2) - 2003742*sqrt(3) + 3470583 + 22617918*sqrt(3)*sqrt(2 - sqrt(3))*sqr
t(sqrt(3) + 2))
```

3.27 $\int \frac{\sec^2(2+3x)}{-2+\tan^2(2+3x)}\,dx$

Optimal. Leaf size=61

$$\frac{\log\left(\sqrt{2}\cos(3x+2)-\sin(3x+2)\right)}{6\sqrt{2}}-\frac{\log\left(\sin(3x+2)+\sqrt{2}\cos(3x+2)\right)}{6\sqrt{2}}$$

```
[Out] 1/12*ln(-sin(2+3*x)+cos(2+3*x)*2^(1/2))*2^(1/2)-1/12*ln(sin(2+3*x)+cos(2+3*
x)*2^(1/2))*2^(1/2)
```

Rubi [A] time = 0.04, antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {3675, 207}

$$\frac{\log\left(\sqrt{2}\cos(3x+2)-\sin(3x+2)\right)}{6\sqrt{2}}-\frac{\log\left(\sin(3x+2)+\sqrt{2}\cos(3x+2)\right)}{6\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[2 + 3*x]^2/(-2 + Tan[2 + 3*x]^2),x]
```

```
[Out] Log[Sqrt[2]*Cos[2 + 3*x] - Sin[2 + 3*x]]/(6*Sqrt[2]) - Log[Sqrt[2]*Cos[2 +
3*x] + Sin[2 + 3*x]]/(6*Sqrt[2])
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 3675

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/(c^(m - 1)*f), Subst[Int[(c^2 + ff^2*x^2)^(m/2 - 1)*(a + b*(ff*x)^n)^p
, x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && In
tegerQ[m/2] && (IntegersQ[n, p] || IGtQ[m, 0] || IGtQ[p, 0] || EqQ[n^2, 4]
|| EqQ[n^2, 16])
```

Rubi steps

$$\int \frac{\sec^2(2+3x)}{-2+\tan^2(2+3x)}\,dx = \frac{1}{3}\,\text{Subst}\left(\int \frac{1}{-2+x^2}\,dx, x, \tan(2+3x)\right)$$
$$= \frac{\log\left(\sqrt{2}\cos(2+3x)-\sin(2+3x)\right)}{6\sqrt{2}} - \frac{\log\left(\sqrt{2}\cos(2+3x)+\sin(2+3x)\right)}{6\sqrt{2}}$$

Mathematica [A] time = 0.02, size = 22, normalized size = 0.36

$$-\frac{\tanh^{-1}\left(\frac{\tan(3x+2)}{\sqrt{2}}\right)}{3\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[2 + 3*x]^2/(-2 + Tan[2 + 3*x]^2),x]
```

```
[Out] -1/3*ArcTanh[Tan[2 + 3*x]/Sqrt[2]]/Sqrt[2]
```

fricas [A] time = 0.54, size = 85, normalized size = 1.39

$$\frac{1}{24}\sqrt{2}\log\left(-\frac{7\cos(3x+2)^4-10\cos(3x+2)^2+4\left(\sqrt{2}\cos(3x+2)^3+\sqrt{2}\cos(3x+2)\right)\sin(3x+2)-1}{9\cos(3x+2)^4-6\cos(3x+2)^2+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(2+3*x)^2/(-2+tan(2+3*x)^2),x, algorithm="fricas")
```

```
[Out] 1/24*sqrt(2)*log(-(7*cos(3*x + 2)^4 - 10*cos(3*x + 2)^2 + 4*(sqrt(2)*cos(3*
x + 2)^3 + sqrt(2)*cos(3*x + 2))*sin(3*x + 2) - 1)/(9*cos(3*x + 2)^4 - 6*co
s(3*x + 2)^2 + 1))
```

giac [A] time = 1.29, size = 37, normalized size = 0.61

$$-\frac{1}{12}\sqrt{2}\log\left(\left|\sqrt{2}+\tan(3x+2)\right|\right)+\frac{1}{12}\sqrt{2}\log\left(\left|-\sqrt{2}+\tan(3x+2)\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(2+3*x)^2/(-2+tan(2+3*x)^2),x, algorithm="giac")
```

```
[Out] -1/12*sqrt(2)*log(abs(sqrt(2) + tan(3*x + 2))) + 1/12*sqrt(2)*log(abs(-sqrt
(2) + tan(3*x + 2)))
```

maple [A] time = 0.28, size = 18, normalized size = 0.30

$$-\frac{\sqrt{2}\ \operatorname{arctanh}\left(\frac{\sqrt{2}\ \tan(2+3x)}{2}\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(2+3*x)^2/(-2+tan(2+3*x)^2),x)
[Out] -1/6*2^(1/2)*arctanh(1/2*2^(1/2)*tan(2+3*x))
```

maxima [A] time = 1.34, size = 32, normalized size = 0.52

$$\frac{1}{12}\sqrt{2}\log\left(-\frac{\sqrt{2}-\tan\left(3x+2\right)}{\sqrt{2}+\tan\left(3x+2\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(2+3*x)^2/(-2+tan(2+3*x)^2),x, algorithm="maxima")
[Out] 1/12*sqrt(2)*log(-(sqrt(2) - tan(3*x + 2))/(sqrt(2) + tan(3*x + 2)))
```

mupad [B] time = 2.40, size = 17, normalized size = 0.28

$$-\frac{\sqrt{2}\ \operatorname{atanh}\left(\frac{\sqrt{2}\ \tan(3x+2)}{2}\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(3*x + 2)^2*(tan(3*x + 2)^2 - 2)),x)
[Out] -(2^(1/2)*atanh((2^(1/2)*tan(3*x + 2))/2))/6
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sec^{2}\left(3x+2\right)}{\tan^{2}\left(3x+2\right)-2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(2+3*x)**2/(-2+tan(2+3*x)**2),x)
[Out] Integral(sec(3*x + 2)**2/(tan(3*x + 2)**2 - 2), x)
```

3.28 $\int \frac{\csc^2(2+3x)}{1-2\cot^2(2+3x)}\,dx$

Optimal. Leaf size=61

$$\frac{\log\left(\sqrt{2}\cos(3x+2)-\sin(3x+2)\right)}{6\sqrt{2}}-\frac{\log\left(\sin(3x+2)+\sqrt{2}\cos(3x+2)\right)}{6\sqrt{2}}$$

```
[Out] 1/12*ln(-sin(2+3*x)+cos(2+3*x)*2^(1/2))*2^(1/2)-1/12*ln(sin(2+3*x)+cos(2+3*
x)*2^(1/2))*2^(1/2)
```

Rubi [A] time = 0.05, antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {3675, 206}

$$\frac{\log\left(\sqrt{2}\cos(3x+2)-\sin(3x+2)\right)}{6\sqrt{2}}-\frac{\log\left(\sin(3x+2)+\sqrt{2}\cos(3x+2)\right)}{6\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[2 + 3*x]^2/(1 - 2*Cot[2 + 3*x]^2),x]
```

```
[Out] Log[Sqrt[2]*Cos[2 + 3*x] - Sin[2 + 3*x]]/(6*Sqrt[2]) - Log[Sqrt[2]*Cos[2 +
3*x] + Sin[2 + 3*x]]/(6*Sqrt[2])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3675

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/(c^(m - 1)*f), Subst[Int[(c^2 + ff^2*x^2)^(m/2 - 1)*(a + b*(ff*x)^n)^p
, x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && In
tegerQ[m/2] && (IntegersQ[n, p] || IGtQ[m, 0] || IGtQ[p, 0] || EqQ[n^2, 4]
|| EqQ[n^2, 16])
```

Rubi steps

$$\int \frac{\csc^2(2+3x)}{1-2\cot^2(2+3x)}\,dx = -\left(\frac{1}{3}\operatorname{Subst}\left(\int \frac{1}{1-2x^2}\,dx, x, \cot(2+3x)\right)\right)$$
$$= \frac{\log\left(\sqrt{2}\cos(2+3x)-\sin(2+3x)\right)}{6\sqrt{2}} - \frac{\log\left(\sqrt{2}\cos(2+3x)+\sin(2+3x)\right)}{6\sqrt{2}}$$

Mathematica [A] time = 0.03, size = 22, normalized size = 0.36

$$-\frac{\tanh^{-1}\left(\frac{\tan(3x+2)}{\sqrt{2}}\right)}{3\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[2 + 3*x]^2/(1 - 2*Cot[2 + 3*x]^2),x]
```

```
[Out] -1/3*ArcTanh[Tan[2 + 3*x]/Sqrt[2]]/Sqrt[2]
```

fricas [A] time = 0.54, size = 85, normalized size = 1.39

$$\frac{1}{24}\sqrt{2}\log\left(-\frac{7\cos(3x+2)^4-10\cos(3x+2)^2+4\left(\sqrt{2}\cos(3x+2)^3+\sqrt{2}\cos(3x+2)\right)\sin(3x+2)-1}{9\cos(3x+2)^4-6\cos(3x+2)^2+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(2+3*x)^2/(1-2*cot(2+3*x)^2),x, algorithm="fricas")
```

```
[Out] 1/24*sqrt(2)*log(-(7*cos(3*x + 2)^4 - 10*cos(3*x + 2)^2 + 4*(sqrt(2)*cos(3*
x + 2)^3 + sqrt(2)*cos(3*x + 2))*sin(3*x + 2) - 1)/(9*cos(3*x + 2)^4 - 6*co
s(3*x + 2)^2 + 1))
```

giac [A] time = 0.42, size = 39, normalized size = 0.64

$$\frac{1}{12}\sqrt{2}\log\left(\frac{\left|-2\sqrt{2}+2\tan(3x+2)\right|}{\left|2\sqrt{2}+2\tan(3x+2)\right|}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(2+3*x)^2/(1-2*cot(2+3*x)^2),x, algorithm="giac")
```

```
[Out] 1/12*sqrt(2)*log(abs(-2*sqrt(2) + 2*tan(3*x + 2))/abs(2*sqrt(2) + 2*tan(3*x
 + 2)))
```

maple [A] time = 0.30, size = 18, normalized size = 0.30

$$-\frac{\sqrt{2}\ \operatorname{arctanh}\left(\frac{\sqrt{2}\ \tan(2+3x)}{2}\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(2+3*x)^2/(1-2*cot(2+3*x)^2),x)
[Out] -1/6*2^(1/2)*arctanh(1/2*2^(1/2)*tan(2+3*x))
```

maxima [A] time = 1.18, size = 32, normalized size = 0.52

$$\frac{1}{12}\sqrt{2}\log\left(-\frac{\sqrt{2}-\tan\left(3x+2\right)}{\sqrt{2}+\tan\left(3x+2\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(2+3*x)^2/(1-2*cot(2+3*x)^2),x, algorithm="maxima")
[Out] 1/12*sqrt(2)*log(-(sqrt(2) - tan(3*x + 2))/(sqrt(2) + tan(3*x + 2)))
```

mupad [B] time = 2.33, size = 17, normalized size = 0.28

$$-\frac{\sqrt{2}\ \operatorname{atanh}\left(\frac{\sqrt{2}\ \tan(3x+2)}{2}\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-1/(sin(3*x + 2)^2*(2*cot(3*x + 2)^2 - 1)),x)
[Out] -(2^(1/2)*atanh((2^(1/2)*tan(3*x + 2))/2))/6
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int\frac{\csc^{2}\left(3x+2\right)}{2\cot^{2}\left(3x+2\right)-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(2+3*x)**2/(1-2*cot(2+3*x)**2),x)
[Out] -Integral(csc(3*x + 2)**2/(2*cot(3*x + 2)**2 - 1), x)
```

3.29 $\int (x+\sin(x))^2\,dx$

Optimal. Leaf size=30

$$\frac{x^3}{3}+\frac{x}{2}+2\sin(x)-2x\cos(x)-\frac{1}{2}\sin(x)\cos(x)$$

```
[Out] 1/2*x+1/3*x^3-2*x*cos(x)+2*sin(x)-1/2*cos(x)*sin(x)
```

Rubi [A] time = 0.03, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.833, Rules used = {6742, 3296, 2637, 2635, 8}

$$\frac{x^3}{3}+\frac{x}{2}+2\sin(x)-2x\cos(x)-\frac{1}{2}\sin(x)\cos(x)$$

Antiderivative was successfully verified.

```
[In] Int[(x + Sin[x])^2,x]
```

```
[Out] x/2 + x^3/3 - 2*x*Cos[x] + 2*Sin[x] - (Cos[x]*Sin[x])/2
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\begin{aligned}\int (x+\sin(x))^2\,dx &= \int \left(x^2+2x\sin(x)+\sin^2(x)\right)\,dx\\ &= \frac{x^3}{3}+2\int x\sin(x)\,dx+\int \sin^2(x)\,dx\\ &= \frac{x^3}{3}-2x\cos(x)-\frac{1}{2}\cos(x)\sin(x)+\frac{\int 1\,dx}{2}+2\int \cos(x)\,dx\\ &= \frac{x}{2}+\frac{x^3}{3}-2x\cos(x)+2\sin(x)-\frac{1}{2}\cos(x)\sin(x)\end{aligned}$$

Mathematica [A] time = 0.06, size = 30, normalized size = 1.00

$$\frac{1}{6}x\left(2x^2+3\right)+2\sin(x)-\frac{1}{4}\sin(2x)-2x\cos(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(x + Sin[x])^2,x]
[Out] (x*(3 + 2*x^2))/6 - 2*x*Cos[x] + 2*Sin[x] - Sin[2*x]/4
```

fricas [A] time = 0.51, size = 22, normalized size = 0.73

$$\frac{1}{3}\,x^3-2\,x\cos(x)-\frac{1}{2}\,(\cos(x)-4)\sin(x)+\frac{1}{2}\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+sin(x))^2,x, algorithm="fricas")
[Out] 1/3*x^3 - 2*x*cos(x) - 1/2*(cos(x) - 4)*sin(x) + 1/2*x
```

giac [A] time = 0.14, size = 24, normalized size = 0.80

$$\frac{1}{3}\,x^3-2\,x\cos(x)+\frac{1}{2}\,x-\frac{1}{4}\,\sin\left(2\,x\right)+2\,\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+sin(x))^2,x, algorithm="giac")
```

```
[Out] 1/3*x^3 - 2*x*cos(x) + 1/2*x - 1/4*sin(2*x) + 2*sin(x)
```

maple [A] time = 0.04, size = 25, normalized size = 0.83

$$\frac{x}{2}+\frac{x^3}{3}-2x\cos(x)+2\sin(x)-\frac{\cos(x)\sin(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x+sin(x))^2,x)
[Out] 1/2*x+1/3*x^3-2*x*cos(x)+2*sin(x)-1/2*cos(x)*sin(x)
```

maxima [A] time = 0.33, size = 24, normalized size = 0.80

$$\frac{1}{3}x^3-2x\cos(x)+\frac{1}{2}x-\frac{1}{4}\sin(2x)+2\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+sin(x))^2,x, algorithm="maxima")
[Out] 1/3*x^3 - 2*x*cos(x) + 1/2*x - 1/4*sin(2*x) + 2*sin(x)
```

mupad [B] time = 0.06, size = 24, normalized size = 0.80

$$\frac{x}{2}+2\sin(x)-\frac{\cos(x)\sin(x)}{2}-2x\cos(x)+\frac{x^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + sin(x))^2,x)
[Out] x/2 + 2*sin(x) - (cos(x)*sin(x))/2 - 2*x*cos(x) + x^3/3
```

sympy [A] time = 0.16, size = 41, normalized size = 1.37

$$\frac{x^3}{3}+\frac{x\sin^2(x)}{2}+\frac{x\cos^2(x)}{2}-2x\cos(x)-\frac{\sin(x)\cos(x)}{2}+2\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+sin(x))**2,x)
[Out] x**3/3 + x*sin(x)**2/2 + x*cos(x)**2/2 - 2*x*cos(x) - sin(x)*cos(x)/2 + 2*s
in(x)
```

3.30 $\int (x+\sin(x))^3\,dx$

Optimal. Leaf size=56

$$\frac{x^4}{4}+\frac{3x^2}{4}-3x^2\cos(x)+\frac{3\sin^2(x)}{4}+6x\sin(x)+\frac{\cos^3(x)}{3}+5\cos(x)-\frac{3}{2}x\sin(x)\cos(x)$$

```
[Out] 3/4*x^2+1/4*x^4+5*cos(x)-3*x^2*cos(x)+1/3*cos(x)^3+6*x*sin(x)-3/2*x*cos(x)*
sin(x)+3/4*sin(x)^2
```

Rubi [A] time = 0.07, antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.000, Rules used = {6742, 3296, 2638, 3310, 30, 2633}

$$\frac{x^4}{4}+\frac{3x^2}{4}-3x^2\cos(x)+\frac{3\sin^2(x)}{4}+6x\sin(x)+\frac{\cos^3(x)}{3}+5\cos(x)-\frac{3}{2}x\sin(x)\cos(x)$$

Antiderivative was successfully verified.

```
[In] Int[(x + Sin[x])^3,x]
```

```
[Out] (3*x^2)/4 + x^4/4 + 5*Cos[x] - 3*x^2*Cos[x] + Cos[x]^3/3 + 6*x*Sin[x] - (3*
x*Cos[x]*Sin[x])/2 + (3*Sin[x]^2)/4
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3310

```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[(d*(b*Sin[e + f*x])^n)/(f^2*n^2), x] + (Dist[(b^2*(n - 1))/n, Int[(c
+ d*x)*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[(b*(c + d*x)*Cos[e + f*x]*(b
*Sin[e + f*x])^(n - 1))/(f*n), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1
]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$
\begin{aligned}
\int (x+\sin(x))^3\,dx &= \int \left(x^3+3x^2\sin(x)+3x\sin^2(x)+\sin^3(x)\right)\,dx\\
&= \frac{x^4}{4}+3\int x^2\sin(x)\,dx+3\int x\sin^2(x)\,dx+\int \sin^3(x)\,dx\\
&= \frac{x^4}{4}-3x^2\cos(x)-\frac{3}{2}x\cos(x)\sin(x)+\frac{3\sin^2(x)}{4}+\frac{3\int x\,dx}{2}+6\int x\cos(x)\,dx-\text{Subst}\left(\int \left(\right.\right.\\
&= \frac{3x^2}{4}+\frac{x^4}{4}-\cos(x)-3x^2\cos(x)+\frac{\cos^3(x)}{3}+6x\sin(x)-\frac{3}{2}x\cos(x)\sin(x)+\frac{3\sin^2(x)}{4}-6\\
&= \frac{3x^2}{4}+\frac{x^4}{4}+5\cos(x)-3x^2\cos(x)+\frac{\cos^3(x)}{3}+6x\sin(x)-\frac{3}{2}x\cos(x)\sin(x)+\frac{3\sin^2(x)}{4}
\end{aligned}
$$

Mathematica [A] time = 0.09, size = 48, normalized size = 0.86

$$\frac{1}{24}\left(6x\left(x^3+3x+24\sin(x)-3\sin(2x)\right)-18\left(4x^2-7\right)\cos(x)-9\cos(2x)+2\cos(3x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(x + Sin[x])^3,x]
```

```
[Out] (-18*(-7 + 4*x^2)*Cos[x] - 9*Cos[2*x] + 2*Cos[3*x] + 6*x*(3*x + x^3 + 24*Si
n[x] - 3*Sin[2*x]))/24
```

fricas [A] time = 0.49, size = 46, normalized size = 0.82

$$\frac{1}{4}x^4+\frac{1}{3}\cos(x)^3+\frac{3}{4}x^2-\left(3x^2-5\right)\cos(x)-\frac{3}{4}\cos(x)^2-\frac{3}{2}\left(x\cos(x)-4x\right)\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+sin(x))^3,x, algorithm="fricas")
```

```
[Out] 1/4*x^4 + 1/3*cos(x)^3 + 3/4*x^2 - (3*x^2 - 5)*cos(x) - 3/4*cos(x)^2 - 3/2*
(x*cos(x) - 4*x)*sin(x)
```

giac [A] time = 0.14, size = 46, normalized size = 0.82

$$\frac{1}{4}x^4 + \frac{3}{4}x^2 - \frac{3}{4}\left(4x^2 - 7\right)\cos(x) - \frac{3}{4}x\sin(2x) + 6x\sin(x) + \frac{1}{12}\cos(3x) - \frac{3}{8}\cos(2x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+sin(x))^3,x, algorithm="giac")
```

```
[Out] 1/4*x^4 + 3/4*x^2 - 3/4*(4*x^2 - 7)*cos(x) - 3/4*x*sin(2*x) + 6*x*sin(x) +
1/12*cos(3*x) - 3/8*cos(2*x)
```

maple [A] time = 0.04, size = 57, normalized size = 1.02

$$-\frac{\left(2+\sin^2(x)\right)\cos(x)}{3}+3x\left(-\frac{\cos(x)\sin(x)}{2}+\frac{x}{2}\right)-\frac{3x^2}{4}+\frac{3\left(\sin^2(x)\right)}{4}-3x^2\cos(x)+6\cos(x)+6x\sin(x)+\frac{x^4}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x+sin(x))^3,x)
```

```
[Out] -1/3*(2+sin(x)^2)*cos(x)+3*x*(-1/2*cos(x)*sin(x)+1/2*x)-3/4*x^2+3/4*sin(x)^
2-3*x^2*cos(x)+6*cos(x)+6*x*sin(x)+1/4*x^4
```

maxima [A] time = 0.52, size = 48, normalized size = 0.86

$$\frac{1}{4}x^4 + \frac{1}{3}\cos(x)^3 + \frac{3}{4}x^2 - 3\left(x^2 - 2\right)\cos(x) - \frac{3}{4}x\sin(2x) + 6x\sin(x) - \frac{3}{8}\cos(2x) - \cos(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+sin(x))^3,x, algorithm="maxima")
```

```
[Out] 1/4*x^4 + 1/3*cos(x)^3 + 3/4*x^2 - 3*(x^2 - 2)*cos(x) - 3/4*x*sin(2*x) + 6*
x*sin(x) - 3/8*cos(2*x) - cos(x)
```

mupad [B] time = 2.31, size = 46, normalized size = 0.82

$$5\cos(x) - 3x^2\cos(x) - \frac{3\cos(x)^2}{4} + \frac{\cos(x)^3}{3} + 6x\sin(x) + \frac{3x^2}{4} + \frac{x^4}{4} - \frac{3x\cos(x)\sin(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + sin(x))^3,x)
```

```
[Out] 5*cos(x) - 3*x^2*cos(x) - (3*cos(x)^2)/4 + cos(x)^3/3 + 6*x*sin(x) + (3*x^2
)/4 + x^4/4 - (3*x*cos(x)*sin(x))/2
```

sympy [A] time = 0.30, size = 85, normalized size = 1.52

$$\frac{x^4}{4}+\frac{3x^2\sin^2(x)}{4}+\frac{3x^2\cos^2(x)}{4}-3x^2\cos(x)-\frac{3x\sin(x)\cos(x)}{2}+6x\sin(x)-\sin^2(x)\cos(x)+\frac{3\sin^2(x)}{4}-\frac{2\cos^3(x)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+sin(x))**3,x)
```

```
[Out] x**4/4 + 3*x**2*sin(x)**2/4 + 3*x**2*cos(x)**2/4 - 3*x**2*cos(x) - 3*x*sin(
x)*cos(x)/2 + 6*x*sin(x) - sin(x)**2*cos(x) + 3*sin(x)**2/4 - 2*cos(x)**3/3
 + 6*cos(x)
```

3.31 $\int \frac{\sin(a+bx)}{c+dx^2}\,dx$

Optimal. Leaf size=213

$$-\frac{\sin\left(a-\frac{b\sqrt{-c}}{\sqrt{d}}\right)\text{Ci}\left(xb+\frac{\sqrt{-c}\,b}{\sqrt{d}}\right)}{2\sqrt{-c}\,\sqrt{d}}+\frac{\sin\left(a+\frac{b\sqrt{-c}}{\sqrt{d}}\right)\text{Ci}\left(\frac{b\sqrt{-c}}{\sqrt{d}}-bx\right)}{2\sqrt{-c}\,\sqrt{d}}-\frac{\cos\left(a+\frac{b\sqrt{-c}}{\sqrt{d}}\right)\text{Si}\left(\frac{b\sqrt{-c}}{\sqrt{d}}-bx\right)}{2\sqrt{-c}\,\sqrt{d}}-\frac{\cos\left(a-\frac{b\sqrt{-c}}{\sqrt{d}}\right.}{2\sqrt{-}}$$

```
[Out] 1/2*cos(a+b*(-c)^(1/2)/d^(1/2))*Si(b*x-b*(-c)^(1/2)/d^(1/2))/(-c)^(1/2)/d^(
1/2)-1/2*cos(a-b*(-c)^(1/2)/d^(1/2))*Si(b*x+b*(-c)^(1/2)/d^(1/2))/(-c)^(1/2
)/d^(1/2)-1/2*Ci(b*x+b*(-c)^(1/2)/d^(1/2))*sin(a-b*(-c)^(1/2)/d^(1/2))/(-c)
^(1/2)/d^(1/2)+1/2*Ci(-b*x+b*(-c)^(1/2)/d^(1/2))*sin(a+b*(-c)^(1/2)/d^(1/2)
)/(-c)^(1/2)/d^(1/2)
```

Rubi [A] time = 0.54, antiderivative size = 213, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {3333, 3303, 3299, 3302}

$$-\frac{\sin\left(a-\frac{b\sqrt{-c}}{\sqrt{d}}\right)\text{CosIntegral}\left(\frac{b\sqrt{-c}}{\sqrt{d}}+bx\right)}{2\sqrt{-c}\,\sqrt{d}}+\frac{\sin\left(a+\frac{b\sqrt{-c}}{\sqrt{d}}\right)\text{CosIntegral}\left(\frac{b\sqrt{-c}}{\sqrt{d}}-bx\right)}{2\sqrt{-c}\,\sqrt{d}}-\frac{\cos\left(a+\frac{b\sqrt{-c}}{\sqrt{d}}\right)\text{Si}\left(\frac{b\sqrt{-c}}{\sqrt{d}}-\right.}{2\sqrt{-c}\,\sqrt{d}}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b*x]/(c + d*x^2),x]
```

```
[Out] -(CosIntegral[(b*Sqrt[-c])/Sqrt[d] + b*x]*Sin[a - (b*Sqrt[-c])/Sqrt[d]])/(2
*Sqrt[-c]*Sqrt[d]) + (CosIntegral[(b*Sqrt[-c])/Sqrt[d] - b*x]*Sin[a + (b*Sq
rt[-c])/Sqrt[d]])/(2*Sqrt[-c]*Sqrt[d]) - (Cos[a + (b*Sqrt[-c])/Sqrt[d]]*Sin
Integral[(b*Sqrt[-c])/Sqrt[d] - b*x])/(2*Sqrt[-c]*Sqrt[d]) - (Cos[a - (b*Sq
rt[-c])/Sqrt[d]]*SinIntegral[(b*Sqrt[-c])/Sqrt[d] + b*x])/(2*Sqrt[-c]*Sqrt[
d])
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3302

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3333

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*Sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Int
[ExpandIntegrand[Sin[c + d*x], (a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d},
 x] && ILtQ[p, 0] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -1])
```

Rubi steps

$$\int \frac{\sin(a+bx)}{c+dx^2}\,dx = \int \left(\frac{\sqrt{-c}\,\sin(a+bx)}{2c\left(\sqrt{-c}-\sqrt{d}\,x\right)}+\frac{\sqrt{-c}\,\sin(a+bx)}{2c\left(\sqrt{-c}+\sqrt{d}\,x\right)}\right)dx$$

$$= -\frac{\int \frac{\sin(a+bx)}{\sqrt{-c}-\sqrt{d}\,x}\,dx}{2\sqrt{-c}}-\frac{\int \frac{\sin(a+bx)}{\sqrt{-c}+\sqrt{d}\,x}\,dx}{2\sqrt{-c}}$$

$$= -\frac{\cos\left(a-\frac{b\sqrt{-c}}{\sqrt{d}}\right)\int \frac{\sin\left(\frac{b\sqrt{-c}}{\sqrt{d}}+bx\right)}{\sqrt{-c}+\sqrt{d}\,x}\,dx}{2\sqrt{-c}}+\frac{\cos\left(a+\frac{b\sqrt{-c}}{\sqrt{d}}\right)\int \frac{\sin\left(\frac{b\sqrt{-c}}{\sqrt{d}}-bx\right)}{\sqrt{-c}-\sqrt{d}\,x}\,dx}{2\sqrt{-c}}-\frac{\sin\left(a-\frac{b\sqrt{-c}}{\sqrt{d}}\right)\int -}{2\sqrt{-c}}$$

$$= -\frac{\text{Ci}\left(\frac{b\sqrt{-c}}{\sqrt{d}}+bx\right)\sin\left(a-\frac{b\sqrt{-c}}{\sqrt{d}}\right)}{2\sqrt{-c}\,\sqrt{d}}+\frac{\text{Ci}\left(\frac{b\sqrt{-c}}{\sqrt{d}}-bx\right)\sin\left(a+\frac{b\sqrt{-c}}{\sqrt{d}}\right)}{2\sqrt{-c}\,\sqrt{d}}-\frac{\cos\left(a+\frac{b\sqrt{-c}}{\sqrt{d}}\right)\text{Si}\left(\frac{b\sqrt{-}}{\sqrt{d}}\right.}{2\sqrt{-c}\,\sqrt{d}}$$

Mathematica [C] time = 0.32, size = 172, normalized size = 0.81

$$\frac{i\left(\sin\left(a-\frac{ib\sqrt{c}}{\sqrt{d}}\right)\text{Ci}\left(b\left(x+\frac{i\sqrt{c}}{\sqrt{d}}\right)\right)-\sin\left(a+\frac{ib\sqrt{c}}{\sqrt{d}}\right)\text{Ci}\left(b\left(x-\frac{i\sqrt{c}}{\sqrt{d}}\right)\right)+\cos\left(a-\frac{ib\sqrt{c}}{\sqrt{d}}\right)\text{Si}\left(b\left(x+\frac{i\sqrt{c}}{\sqrt{d}}\right)\right)+\cos\left(a+\right.\right.}{2\sqrt{c}\,\sqrt{d}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sin[a + b*x]/(c + d*x^2),x]
```

```
[Out] ((I/2)*(CosIntegral[b*((I*Sqrt[c])/Sqrt[d] + x)]*Sin[a - (I*b*Sqrt[c])/Sqrt
[d]] - CosIntegral[b*(((-I)*Sqrt[c])/Sqrt[d] + x)]*Sin[a + (I*b*Sqrt[c])/Sq
rt[d]] + Cos[a - (I*b*Sqrt[c])/Sqrt[d]]*SinIntegral[b*((I*Sqrt[c])/Sqrt[d]
+ x)] + Cos[a + (I*b*Sqrt[c])/Sqrt[d]]*SinIntegral[(I*b*Sqrt[c])/Sqrt[d] -
b*x]))/(Sqrt[c]*Sqrt[d])
```

fricas [C] time = 0.56, size = 187, normalized size = 0.88

$$\frac{\sqrt{\frac{b^2c}{d}}\,\mathrm{Ei}\left(i\,bx-\sqrt{\frac{b^2c}{d}}\right)e^{\left(i\,a+\sqrt{\frac{b^2c}{d}}\right)}-\sqrt{\frac{b^2c}{d}}\,\mathrm{Ei}\left(i\,bx+\sqrt{\frac{b^2c}{d}}\right)e^{\left(i\,a-\sqrt{\frac{b^2c}{d}}\right)}+\sqrt{\frac{b^2c}{d}}\,\mathrm{Ei}\left(-i\,bx-\sqrt{\frac{b^2c}{d}}\right)e^{\left(-i\,a+\sqrt{\frac{b^2c}{d}}\right)}-\sqrt{\frac{b^2}{d}}}{4\,bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)/(d*x^2+c),x, algorithm="fricas")
```

```
[Out] 1/4*(sqrt(b^2*c/d)*Ei(I*b*x - sqrt(b^2*c/d))*e^(I*a + sqrt(b^2*c/d)) - sqrt
(b^2*c/d)*Ei(I*b*x + sqrt(b^2*c/d))*e^(I*a - sqrt(b^2*c/d)) + sqrt(b^2*c/d)
*Ei(-I*b*x - sqrt(b^2*c/d))*e^(-I*a + sqrt(b^2*c/d)) - sqrt(b^2*c/d)*Ei(-I*
b*x + sqrt(b^2*c/d))*e^(-I*a - sqrt(b^2*c/d)))/(b*c)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin(bx+a)}{dx^2+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)/(d*x^2+c),x, algorithm="giac")
```

```
[Out] integrate(sin(b*x + a)/(d*x^2 + c), x)
```

maple [A] time = 0.05, size = 229, normalized size = 1.08

$$b\left(\frac{\mathrm{Si}\left(bx+a-\frac{b\sqrt{-cd}+da}{d}\right)\cos\left(\frac{b\sqrt{-cd}+da}{d}\right)+\mathrm{Ci}\left(bx+a-\frac{b\sqrt{-cd}+da}{d}\right)\sin\left(\frac{b\sqrt{-cd}+da}{d}\right)}{2\left(\frac{b\sqrt{-cd}+da}{d}-a\right)d}+\frac{\mathrm{Si}\left(bx+a+\frac{b\sqrt{-cd}-da}{d}\right)\cos\left(\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(b*x+a)/(d*x^2+c),x)
```

```
[Out] b*(1/2/((b*(-c*d)^(1/2)+d*a)/d-a)/d*(Si(b*x+a-(b*(-c*d)^(1/2)+d*a)/d)*cos((
b*(-c*d)^(1/2)+d*a)/d)+Ci(b*x+a-(b*(-c*d)^(1/2)+d*a)/d)*sin((b*(-c*d)^(1/2)
+d*a)/d))+1/2/(-(b*(-c*d)^(1/2)-d*a)/d-a)/d*(Si(b*x+a+(b*(-c*d)^(1/2)-d*a)/
d)*cos((b*(-c*d)^(1/2)-d*a)/d)-Ci(b*x+a+(b*(-c*d)^(1/2)-d*a)/d)*sin((b*(-c*
d)^(1/2)-d*a)/d)))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin(bx+a)}{dx^2+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)/(d*x^2+c),x, algorithm="maxima")
```

```
[Out] integrate(sin(b*x + a)/(d*x^2 + c), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sin(a+bx)}{dx^2+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b*x)/(c + d*x^2),x)
```

```
[Out] int(sin(a + b*x)/(c + d*x^2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin(a+bx)}{c+dx^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)/(d*x**2+c),x)
```

```
[Out] Integral(sin(a + b*x)/(c + d*x**2), x)
```

3.32 $\int \frac{\sin(a+bx)}{c+dx+ex^2}\,dx$

Optimal. Leaf size=271

$$\frac{\sin\left(a-\frac{b\left(d-\sqrt{d^2-4ce}\right)}{2e}\right)\text{Ci}\left(\frac{b\left(d-\sqrt{d^2-4ce}\right)}{2e}+bx\right)}{\sqrt{d^2-4ce}}-\frac{\sin\left(a-\frac{b\left(\sqrt{d^2-4ce}+d\right)}{2e}\right)\text{Ci}\left(\frac{b\left(d+\sqrt{d^2-4ce}\right)}{2e}+bx\right)}{\sqrt{d^2-4ce}}+\frac{\cos\left(a-\frac{b\left(d-\sqrt{d^2-4ce}\right)}{2e}\right.}{\sqrt{d^2}}$$

```
[Out] cos(a-1/2*b*(d-(-4*c*e+d^2)^(1/2))/e)*Si(b*x+1/2*b*(d-(-4*c*e+d^2)^(1/2))/e
)/(-4*c*e+d^2)^(1/2)-cos(a-1/2*b*(d+(-4*c*e+d^2)^(1/2))/e)*Si(b*x+1/2*b*(d+
(-4*c*e+d^2)^(1/2))/e)/(-4*c*e+d^2)^(1/2)+Ci(b*x+1/2*b*(d-(-4*c*e+d^2)^(1/2
))/e)*sin(a-1/2*b*(d-(-4*c*e+d^2)^(1/2))/e)/(-4*c*e+d^2)^(1/2)-Ci(b*x+1/2*b
*(d+(-4*c*e+d^2)^(1/2))/e)*sin(a-1/2*b*(d+(-4*c*e+d^2)^(1/2))/e)/(-4*c*e+d^
2)^(1/2)
```

Rubi [A] time = 0.80, antiderivative size = 271, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.210, Rules used = {6728, 3303, 3299, 3302}

$$\frac{\sin\left(a-\frac{b\left(d-\sqrt{d^2-4ce}\right)}{2e}\right)\text{CosIntegral}\left(\frac{b\left(d-\sqrt{d^2-4ce}\right)}{2e}+bx\right)}{\sqrt{d^2-4ce}}-\frac{\sin\left(a-\frac{b\left(\sqrt{d^2-4ce}+d\right)}{2e}\right)\text{CosIntegral}\left(\frac{b\left(\sqrt{d^2-4ce}+d\right)}{2e}+bx\right)}{\sqrt{d^2-4ce}}+$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b*x]/(c + d*x + e*x^2),x]
```

```
[Out] (CosIntegral[(b*(d - Sqrt[d^2 - 4*c*e]))/(2*e) + b*x]*Sin[a - (b*(d - Sqrt[
d^2 - 4*c*e]))/(2*e)])/Sqrt[d^2 - 4*c*e] - (CosIntegral[(b*(d + Sqrt[d^2 -
4*c*e]))/(2*e) + b*x]*Sin[a - (b*(d + Sqrt[d^2 - 4*c*e]))/(2*e)])/Sqrt[d^2
- 4*c*e] + (Cos[a - (b*(d - Sqrt[d^2 - 4*c*e]))/(2*e)]*SinIntegral[(b*(d -
Sqrt[d^2 - 4*c*e]))/(2*e) + b*x])/Sqrt[d^2 - 4*c*e] - (Cos[a - (b*(d + Sqrt
[d^2 - 4*c*e]))/(2*e)]*SinIntegral[(b*(d + Sqrt[d^2 - 4*c*e]))/(2*e) + b*x]
)/Sqrt[d^2 - 4*c*e]
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3302

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
```

```
c*f, 0]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 6728

```
Int[(u_)/((a_.) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.)), x_Symbol] :> With[
{v = RationalFunctionExpand[u/(a + b*x^n + c*x^(2*n)), x]}, Int[v, x] /; Su
mQ[v]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{\sin(a+bx)}{c+dx+ex^2}\,dx = \int \left(\frac{2e\sin(a+bx)}{\sqrt{d^2-4ce}\left(d-\sqrt{d^2-4ce}+2ex\right)} - \frac{2e\sin(a+bx)}{\sqrt{d^2-4ce}\left(d+\sqrt{d^2-4ce}+2ex\right)}\right)dx$$

$$= \frac{(2e)\int \frac{\sin(a+bx)}{d-\sqrt{d^2-4ce}+2ex}\,dx}{\sqrt{d^2-4ce}} - \frac{(2e)\int \frac{\sin(a+bx)}{d+\sqrt{d^2-4ce}+2ex}\,dx}{\sqrt{d^2-4ce}}$$

$$= \frac{\left(2e\cos\left(a-\frac{b\left(d-\sqrt{d^2-4ce}\right)}{2e}\right)\right)\int \frac{\sin\left(\frac{b\left(d-\sqrt{d^2-4ce}\right)}{2e}+bx\right)}{d-\sqrt{d^2-4ce}+2ex}\,dx}{\sqrt{d^2-4ce}} - \frac{\left(2e\cos\left(a-\frac{b\left(d+\sqrt{d^2-4ce}\right)}{2e}\right)\right)\int \frac{\sin\left(\frac{b(d+\sqrt{}}{}\right.}{d+\sqrt{d^2}}}{\sqrt{d^2-4ce}}$$

$$= \frac{\text{Ci}\left(\frac{b\left(d-\sqrt{d^2-4ce}\right)}{2e}+bx\right)\sin\left(a-\frac{b\left(d-\sqrt{d^2-4ce}\right)}{2e}\right)}{\sqrt{d^2-4ce}} - \frac{\text{Ci}\left(\frac{b\left(d+\sqrt{d^2-4ce}\right)}{2e}+bx\right)\sin\left(a-\frac{b\left(d+\sqrt{d^2-4ce}\right.}{2e}\right.}{\sqrt{d^2-4ce}}$$

Mathematica [A] time = 0.56, size = 238, normalized size = 0.88

$$\frac{\sin\left(a+\frac{b\left(\sqrt{d^2-4ce}-d\right)}{2e}\right)\text{Ci}\left(\frac{b\left(d+2ex-\sqrt{d^2-4ce}\right)}{2e}\right)-\sin\left(a-\frac{b\left(\sqrt{d^2-4ce}+d\right)}{2e}\right)\text{Ci}\left(\frac{b\left(d+2ex+\sqrt{d^2-4ce}\right)}{2e}\right)-\cos\left(a+\frac{b\left(\sqrt{d^2-4ce}-d\right)}{2e}\right.}{\sqrt{d^2-4ce}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sin[a + b*x]/(c + d*x + e*x^2),x]
```

```
[Out] (CosIntegral[(b*(d - Sqrt[d^2 - 4*c*e] + 2*e*x))/(2*e)]*Sin[a + (b*(-d + Sq
rt[d^2 - 4*c*e]))/(2*e)] - CosIntegral[(b*(d + Sqrt[d^2 - 4*c*e] + 2*e*x))/
(2*e)]*Sin[a - (b*(d + Sqrt[d^2 - 4*c*e]))/(2*e)] - Cos[a + (b*(-d + Sqrt[d
^2 - 4*c*e]))/(2*e)]*SinIntegral[(b*(-d + Sqrt[d^2 - 4*c*e]))/(2*e) - b*x]
- Cos[a - (b*(d + Sqrt[d^2 - 4*c*e]))/(2*e)]*SinIntegral[(b*(d + Sqrt[d^2 -
 4*c*e] + 2*e*x))/(2*e)])/Sqrt[d^2 - 4*c*e]
```

fricas [C] time = 2.02, size = 434, normalized size = 1.60

$$-\frac{e\sqrt{-\frac{b^2d^2-4\,b^2ce}{e^2}}\,\mathrm{Ei}\left(\frac{-2i\,bex-i\,bd-e\sqrt{-\frac{b^2d^2-4\,b^2ce}{e^2}}}{2\,e}\right)e^{\left(\frac{i\,bd-2i\,ae+e\sqrt{-\frac{b^2d^2-4\,b^2ce}{e^2}}}{2\,e}\right)}-e\sqrt{-\frac{b^2d^2-4\,b^2ce}{e^2}}\,\mathrm{Ei}\left(\frac{-2i\,bex-i\,bd+e\sqrt{-\frac{b^2d^2-4\,b^2ce}{e^2}}}{2\,e}\right)e^{\left(\cdots\right.}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)/(e*x^2+d*x+c),x, algorithm="fricas")
```

```
[Out] -1/2*(e*sqrt(-(b^2*d^2 - 4*b^2*c*e)/e^2)*Ei(1/2*(-2*I*b*e*x - I*b*d - e*sqr
t(-(b^2*d^2 - 4*b^2*c*e)/e^2))/e)*e^(1/2*(I*b*d - 2*I*a*e + e*sqrt(-(b^2*d^
2 - 4*b^2*c*e)/e^2))/e) - e*sqrt(-(b^2*d^2 - 4*b^2*c*e)/e^2)*Ei(1/2*(-2*I*b
*e*x - I*b*d + e*sqrt(-(b^2*d^2 - 4*b^2*c*e)/e^2))/e)*e^(1/2*(I*b*d - 2*I*a
*e - e*sqrt(-(b^2*d^2 - 4*b^2*c*e)/e^2))/e) + e*sqrt(-(b^2*d^2 - 4*b^2*c*e)
/e^2)*Ei(1/2*(2*I*b*e*x + I*b*d - e*sqrt(-(b^2*d^2 - 4*b^2*c*e)/e^2))/e)*e^
(1/2*(-I*b*d + 2*I*a*e + e*sqrt(-(b^2*d^2 - 4*b^2*c*e)/e^2))/e) - e*sqrt(-(
b^2*d^2 - 4*b^2*c*e)/e^2)*Ei(1/2*(2*I*b*e*x + I*b*d + e*sqrt(-(b^2*d^2 - 4*
b^2*c*e)/e^2))/e)*e^(1/2*(-I*b*d + 2*I*a*e - e*sqrt(-(b^2*d^2 - 4*b^2*c*e)/
e^2))/e))/(b*d^2 - 4*b*c*e)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin\left(bx+a\right)}{ex^2+dx+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)/(e*x^2+d*x+c),x, algorithm="giac")
```

```
[Out] integrate(sin(b*x + a)/(e*x^2 + d*x + c), x)
```

maple [A] time = 0.04, size = 320, normalized size = 1.18

$$b\left(\frac{\mathrm{Si}\left(bx+a-\frac{2ae-db+\sqrt{-4b^2ce+b^2d^2}}{2e}\right)\cos\left(\frac{2ae-db+\sqrt{-4b^2ce+b^2d^2}}{2e}\right)+\mathrm{Ci}\left(bx+a-\frac{2ae-db+\sqrt{-4b^2ce+b^2d^2}}{2e}\right)\sin\left(\frac{2ae-db+\sqrt{-4}}{2e}\right.}{\sqrt{-4b^2ce+b^2d^2}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(b*x+a)/(e*x^2+d*x+c),x)
```

```
[Out] b*(1/(-4*b^2*c*e+b^2*d^2)^(1/2)*(Si(b*x+a-1/2/e*(2*a*e-d*b+(-4*b^2*c*e+b^2*
d^2)^(1/2)))*cos(1/2/e*(2*a*e-d*b+(-4*b^2*c*e+b^2*d^2)^(1/2)))+Ci(b*x+a-1/2
/e*(2*a*e-d*b+(-4*b^2*c*e+b^2*d^2)^(1/2)))*sin(1/2/e*(2*a*e-d*b+(-4*b^2*c*e
+b^2*d^2)^(1/2))))-1/(-4*b^2*c*e+b^2*d^2)^(1/2)*(Si(b*x+a+1/2*(-2*a*e+d*b+(
-4*b^2*c*e+b^2*d^2)^(1/2))/e)*cos(1/2*(-2*a*e+d*b+(-4*b^2*c*e+b^2*d^2)^(1/2
))/e)-Ci(b*x+a+1/2*(-2*a*e+d*b+(-4*b^2*c*e+b^2*d^2)^(1/2))/e)*sin(1/2*(-2*a
*e+d*b+(-4*b^2*c*e+b^2*d^2)^(1/2))/e)))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin(bx+a)}{ex^2+dx+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)/(e*x^2+d*x+c),x, algorithm="maxima")
```

```
[Out] integrate(sin(b*x + a)/(e*x^2 + d*x + c), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sin(a+b\,x)}{e\,x^2+d\,x+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b*x)/(c + d*x + e*x^2),x)
```

```
[Out] int(sin(a + b*x)/(c + d*x + e*x^2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin(a+bx)}{c+dx+ex^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)/(e*x**2+d*x+c),x)
```

```
[Out] Integral(sin(a + b*x)/(c + d*x + e*x**2), x)
```

3.33 $\int \frac{\sin\left(\sqrt{-7+x}\right)}{\sqrt{-7+x}}\,dx$

Optimal. Leaf size=10

$$-2\cos\left(\sqrt{x-7}\right)$$

```
[Out] -2*cos((-7+x)^(1/2))
```

Rubi [A] time = 0.02, antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {3431, 15, 2638}

$$-2\cos\left(\sqrt{x-7}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sin[Sqrt[-7 + x]]/Sqrt[-7 + x],x]
```

```
[Out] -2*Cos[Sqrt[-7 + x]]
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3431

```
Int[((g_.) + (h_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f
_.)*(x_))^(n_)])^(p_.), x_Symbol] :> Dist[1/(n*f), Subst[Int[ExpandIntegran
d[(a + b*Sin[c + d*x])^p, x^(1/n - 1)*(g - (e*h)/f + (h*x^(1/n))/f)^m, x],
x], x, (e + f*x)^n], x] /; FreeQ[{a, b, c, d, e, f, g, h, m}, x] && IGtQ[p,
 0] && IntegerQ[1/n]
```

Rubi steps

$$\int \frac{\sin\left(\sqrt{-7+x}\right)}{\sqrt{-7+x}}\,dx = 2\,\text{Subst}\left(\int \frac{x\sin(x)}{\sqrt{x^2}}\,dx, x, \sqrt{-7+x}\right)$$
$$= 2\,\text{Subst}\left(\int \sin(x)\,dx, x, \sqrt{-7+x}\right)$$
$$= -2\cos\left(\sqrt{-7+x}\right)$$

Mathematica [A] time = 0.02, size = 10, normalized size = 1.00

$$-2\cos\left(\sqrt{x-7}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[Sqrt[-7 + x]]/Sqrt[-7 + x],x]
[Out] -2*Cos[Sqrt[-7 + x]]
```

fricas [A] time = 0.59, size = 8, normalized size = 0.80

$$-2\cos\left(\sqrt{x-7}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin((-7+x)^(1/2))/(-7+x)^(1/2),x, algorithm="fricas")
[Out] -2*cos(sqrt(x - 7))
```

giac [A] time = 0.12, size = 8, normalized size = 0.80

$$-2\cos\left(\sqrt{x-7}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin((-7+x)^(1/2))/(-7+x)^(1/2),x, algorithm="giac")
[Out] -2*cos(sqrt(x - 7))
```

maple [A] time = 0.01, size = 9, normalized size = 0.90

$$-2\cos\left(\sqrt{-7+x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin((-7+x)^(1/2))/(-7+x)^(1/2),x)
```

```
[Out] -2*cos((-7+x)^(1/2))
```

maxima [A] time = 0.31, size = 8, normalized size = 0.80

$$-2\cos\left(\sqrt{x-7}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin((-7+x)^(1/2))/(-7+x)^(1/2),x, algorithm="maxima")
[Out] -2*cos(sqrt(x - 7))
```

mupad [B] time = 2.42, size = 8, normalized size = 0.80

$$-2\cos\left(\sqrt{x-7}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin((x - 7)^(1/2))/(x - 7)^(1/2),x)
[Out] -2*cos((x - 7)^(1/2))
```

sympy [A] time = 0.26, size = 10, normalized size = 1.00

$$-2\cos\left(\sqrt{x-7}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin((-7+x)**(1/2))/(-7+x)**(1/2),x)
[Out] -2*cos(sqrt(x - 7))
```

3.34 $\int \frac{\sqrt{b-\frac{a}{x^2}} \sin(x)}{\sqrt{a-bx^2}}\, dx$

Optimal. Leaf size=28

$$\frac{x\text{Si}(x)\sqrt{b-\frac{a}{x^2}}}{\sqrt{a-bx^2}}$$

[Out] x*Si(x)*(b-a/x^2)^(1/2)/(-b*x^2+a)^(1/2)

Rubi [A] time = 0.47, antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {6721, 23, 3299}

$$\frac{x\text{Si}(x)\sqrt{b-\frac{a}{x^2}}}{\sqrt{a-bx^2}}$$

Antiderivative was successfully verified.

[In] Int[(Sqrt[b - a/x^2]*Sin[x])/Sqrt[a - b*x^2],x]

[Out] (Sqrt[b - a/x^2]*x*SinIntegral[x])/Sqrt[a - b*x^2]

Rule 23

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_)*((c_) + (d_.)*(v_))^(n_), x_Symbol] :> D
ist[(a + b*v)^m/(c + d*v)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b
, c, d, m, n}, x] && EqQ[b*c - a*d, 0] &&  !(IntegerQ[m] || IntegerQ[n] ||
GtQ[b/d, 0])
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 6721

```
Int[(u_.)*((a_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(b^IntPart[p]*(
a + b*x^n)^FracPart[p])/(x^(n*FracPart[p])*(1 + a/(x^n*b))^FracPart[p]), In
t[u*x^(n*p)*(1 + a/(x^n*b))^p, x], x] /; FreeQ[{a, b, p}, x] &&  !IntegerQ[
p] && ILtQ[n, 0] &&  !RationalFunctionQ[u, x] && IntegerQ[p + 1/2]
```

Rubi steps

$$\int \frac{\sqrt{b-\frac{a}{x^2}}\sin(x)}{\sqrt{a-bx^2}}\,dx = \frac{\left(\sqrt{b-\frac{a}{x^2}}\,x\right)\int \frac{\sqrt{1-\frac{bx^2}{a}}\sin(x)}{x\sqrt{a-bx^2}}\,dx}{\sqrt{1-\frac{bx^2}{a}}}$$
$$= \frac{\left(\sqrt{b-\frac{a}{x^2}}\,x\right)\int \frac{\sin(x)}{x}\,dx}{\sqrt{a-bx^2}}$$
$$= \frac{\sqrt{b-\frac{a}{x^2}}\,x\mathrm{Si}(x)}{\sqrt{a-bx^2}}$$

Mathematica [C] time = 0.70, size = 46, normalized size = 1.64

$$\frac{ix(\mathrm{Ei}(-ix)-\mathrm{Ei}(ix))\sqrt{b-\frac{a}{x^2}}}{2\sqrt{a-bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[b - a/x^2]*Sin[x])/Sqrt[a - b*x^2],x]
[Out] ((I/2)*Sqrt[b - a/x^2]*x*(ExpIntegralEi[(-I)*x] - ExpIntegralEi[I*x]))/Sqrt
[a - b*x^2]
```

fricas [F] time = 1.27, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(-\frac{\sqrt{-bx^2+a}\sqrt{\frac{bx^2-a}{x^2}}\sin(x)}{bx^2-a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)*(b-a/x^2)^(1/2)/(-b*x^2+a)^(1/2),x, algorithm="fricas")
[Out] integral(-sqrt(-b*x^2 + a)*sqrt((b*x^2 - a)/x^2)*sin(x)/(b*x^2 - a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{b-\frac{a}{x^2}}\sin(x)}{\sqrt{-bx^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)*(b-a/x^2)^(1/2)/(-b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b - a/x^2)*sin(x)/sqrt(-b*x^2 + a), x)
```

maple [C] time = 0.08, size = 72, normalized size = 2.57

$$-\frac{\sqrt{-\frac{-b\,x^{2}+a}{x^{2}}}\,\left(b\,x^{2}-a\right)x\sqrt{\frac{-b\,x^{2}+a}{b\,x^{2}-a}}\,\left(-i\,\mathrm{Si}(x)+\frac{i\pi\,\mathrm{csgn}(x)}{2}\right)}{\left(-b\,x^{2}+a\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)*(b-a/x^2)^(1/2)/(-b*x^2+a)^(1/2),x)
```

```
[Out] -(-(-b*x^2+a)/x^2)^(1/2)*(b*x^2-a)/(-b*x^2+a)^(3/2)*x*(1/(b*x^2-a)*(-b*x^2+
a))^(1/2)*(-I*Si(x)+1/2*I*Pi*csgn(x))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{b-\frac{a}{x^{2}}}\,\sin(x)}{\sqrt{-bx^{2}+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)*(b-a/x^2)^(1/2)/(-b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b - a/x^2)*sin(x)/sqrt(-b*x^2 + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\sin(x)\,\sqrt{b-\frac{a}{x^{2}}}}{\sqrt{a-b\,x^{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((sin(x)*(b - a/x^2)^(1/2))/(a - b*x^2)^(1/2),x)
```

```
[Out] int((sin(x)*(b - a/x^2)^(1/2))/(a - b*x^2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-\frac{a}{x^{2}}+b}\,\sin{(x)}}{\sqrt{a-bx^{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)*(b-a/x**2)**(1/2)/(-b*x**2+a)**(1/2),x)
[Out] Integral(sqrt(-a/x**2 + b)*sin(x)/sqrt(a - b*x**2), x)
```

3.35 $\int \frac{1}{x(1+\sin(\log(x)))} \, dx$

Optimal. Leaf size=12

$$-\frac{\cos(\log(x))}{\sin(\log(x))+1}$$

```
[Out] -cos(ln(x))/(1+sin(ln(x)))
```

Rubi [A] time = 0.03, antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {2648}

$$-\frac{\cos(\log(x))}{\sin(\log(x))+1}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(1 + Sin[Log[x]])),x]
```

```
[Out] -(Cos[Log[x]]/(1 + Sin[Log[x]]))
```

Rule 2648

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> -Simp[Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rubi steps

$$\begin{aligned} \int \frac{1}{x(1+\sin(\log(x)))} \, dx &= \text{Subst}\left(\int \frac{1}{1+\sin(x)} \, dx, x, \log(x)\right) \\ &= -\frac{\cos(\log(x))}{1+\sin(\log(x))} \end{aligned}$$

Mathematica [B] time = 0.02, size = 26, normalized size = 2.17

$$\frac{2\sin\left(\frac{\log(x)}{2}\right)}{\sin\left(\frac{\log(x)}{2}\right)+\cos\left(\frac{\log(x)}{2}\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(1 + Sin[Log[x]])),x]
```

```
[Out] (2*Sin[Log[x]/2])/(Cos[Log[x]/2] + Sin[Log[x]/2])
```

fricas [A] time = 0.63, size = 22, normalized size = 1.83

$$-\frac{\cos\left(\log(x)\right)-\sin\left(\log(x)\right)+1}{\cos\left(\log(x)\right)+\sin\left(\log(x)\right)+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(1+sin(log(x))),x, algorithm="fricas")
[Out] -(cos(log(x)) - sin(log(x)) + 1)/(cos(log(x)) + sin(log(x)) + 1)
```

giac [A] time = 0.14, size = 11, normalized size = 0.92

$$-\frac{2}{\tan\left(\frac{1}{2}\log(x)\right)+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(1+sin(log(x))),x, algorithm="giac")
[Out] -2/(tan(1/2*log(x)) + 1)
```

maple [A] time = 0.08, size = 12, normalized size = 1.00

$$-\frac{2}{\tan\left(\frac{\ln(x)}{2}\right)+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(1+sin(ln(x))),x)
[Out] -2/(tan(1/2*ln(x))+1)
```

maxima [A] time = 0.31, size = 17, normalized size = 1.42

$$-\frac{2}{\frac{\sin\left(\log(x)\right)}{\cos\left(\log(x)\right)+1}+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(1+sin(log(x))),x, algorithm="maxima")
[Out] -2/(sin(log(x))/(cos(log(x)) + 1) + 1)
```

mupad [B] time = 2.40, size = 11, normalized size = 0.92

$$-\frac{2}{\tan\left(\frac{\ln(x)}{2}\right)+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(sin(log(x)) + 1)),x)
[Out] -2/(tan(log(x)/2) + 1)
```

sympy [A] time = 1.36, size = 10, normalized size = 0.83

$$-\frac{2}{\tan\left(\frac{\log{(x)}}{2}\right)+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(1+sin(ln(x))),x)
[Out] -2/(tan(log(x)/2) + 1)
```

3.36 $\int \sin\left(\frac{a+bx}{c+dx}\right) dx$

Optimal. Leaf size=100

$$\frac{\cos\left(\frac{b}{d}\right)(bc-ad)\text{Ci}\left(\frac{bc-ad}{d(c+dx)}\right)}{d^2}+\frac{\sin\left(\frac{b}{d}\right)(bc-ad)\text{Si}\left(\frac{bc-ad}{d(c+dx)}\right)}{d^2}+\frac{(c+dx)\sin\left(\frac{a+bx}{c+dx}\right)}{d}$$

```
[Out] (-a*d+b*c)*Ci((-a*d+b*c)/d/(d*x+c))*cos(b/d)/d^2+(-a*d+b*c)*Si((-a*d+b*c)/d
/(d*x+c))*sin(b/d)/d^2+(d*x+c)*sin((b*x+a)/(d*x+c))/d
```

Rubi [A] time = 0.16, antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {4563, 3297, 3303, 3299, 3302}

$$\frac{\cos\left(\frac{b}{d}\right)(bc-ad)\text{CosIntegral}\left(\frac{bc-ad}{d(c+dx)}\right)}{d^2}+\frac{\sin\left(\frac{b}{d}\right)(bc-ad)\text{Si}\left(\frac{bc-ad}{d(c+dx)}\right)}{d^2}+\frac{(c+dx)\sin\left(\frac{a+bx}{c+dx}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[(a + b*x)/(c + d*x)],x]
```

```
[Out] ((b*c - a*d)*Cos[b/d]*CosIntegral[(b*c - a*d)/(d*(c + d*x))])/d^2 + ((c + d
*x)*Sin[(a + b*x)/(c + d*x)])/d + ((b*c - a*d)*Sin[b/d]*SinIntegral[(b*c -
a*d)/(d*(c + d*x))])/d^2
```

Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[((
c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3302

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 4563

```
Int[Sin[((e_.)*((a_.) + (b_.)*(x_)))/((c_.) + (d_.)*(x_))]^(n_.), x_Symbol]
 :> -Dist[d^(-1), Subst[Int[Sin[(b*e)/d - (e*(b*c - a*d)*x)/d]^n/x^2, x], x
, 1/(c + d*x)], x] /; FreeQ[{a, b, c, d}, x] && IGtQ[n, 0] && NeQ[b*c - a*d
, 0]
```

Rubi steps

$$\int \sin\left(\frac{a+bx}{c+dx}\right)dx = -\frac{\text{Subst}\left(\int \frac{\sin\left(\frac{b}{d}-\frac{(bc-ad)x}{d}\right)}{x^2}dx, x, \frac{1}{c+dx}\right)}{d}$$

$$= \frac{(c+dx)\sin\left(\frac{a+bx}{c+dx}\right)}{d} + \frac{(bc-ad)\,\text{Subst}\left(\int \frac{\cos\left(\frac{b}{d}-\frac{(bc-ad)x}{d}\right)}{x}dx, x, \frac{1}{c+dx}\right)}{d^2}$$

$$= \frac{(c+dx)\sin\left(\frac{a+bx}{c+dx}\right)}{d} + \frac{\left((bc-ad)\cos\left(\frac{b}{d}\right)\right)\text{Subst}\left(\int \frac{\cos\left(\frac{(bc-ad)x}{d}\right)}{x}dx, x, \frac{1}{c+dx}\right)}{d^2} + \frac{\left((bc-ad)\sin\right.}{}$$

$$= \frac{(bc-ad)\cos\left(\frac{b}{d}\right)\text{Ci}\left(\frac{bc-ad}{d(c+dx)}\right)}{d^2} + \frac{(c+dx)\sin\left(\frac{a+bx}{c+dx}\right)}{d} + \frac{(bc-ad)\sin\left(\frac{b}{d}\right)\text{Si}\left(\frac{bc-ad}{d(c+dx)}\right)}{d^2}$$

Mathematica [C] time = 5.35, size = 272, normalized size = 2.72

$$\frac{2\cos\left(\frac{b}{d}\right)(bc-ad)\text{Ci}\left(\frac{ad-bc}{d(c+dx)}\right) + d\exp\left(-\frac{i(ad+2bc+bdx)}{d(c+dx)}\right)\left(ic\left(e^{\frac{2ibc}{d(c+dx)}} - e^{2i\left(\frac{a}{c+dx}+\frac{b}{d}\right)}\right) + dx\sin\left(\frac{b}{d}\right)\left(e^{i\left(\frac{2a}{c+dx}+\frac{b}{d}\right)} + e^{\frac{ib(3c+d}{d(c+d}}\right.\right.}{2d^2}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sin[(a + b*x)/(c + d*x)],x]
```

```
[Out] (2*(b*c - a*d)*Cos[b/d]*CosIntegral[(-(b*c) + a*d)/(d*(c + d*x))] + (d*(I*c
*(E^(((2*I)*b*c)/(d*(c + d*x))) - E^((2*I)*(b/d + a/(c + d*x)))) + d*(E^((I
```

```
*b*(3*c + d*x))/(d*(c + d*x))) + E^(I*(b/d + (2*a)/(c + d*x))))*x*Sin[b/d]
+ 2*d*E^((I*(2*b*c + a*d + b*d*x))/(d*(c + d*x)))*x*Cos[b/d]*Sin[(-(b*c) +
a*d)/(d*(c + d*x))]))/E^((I*(2*b*c + a*d + b*d*x))/(d*(c + d*x))) - 2*(b*c
- a*d)*Sin[b/d]*SinIntegral[(-(b*c) + a*d)/(d*(c + d*x))])/(2*d^2)
```

fricas [A] time = 1.09, size = 139, normalized size = 1.39

$$-\frac{2\,(bc-ad)\sin\left(\frac{b}{d}\right)\operatorname{Si}\left(-\frac{bc-ad}{d^2x+cd}\right)-\left((bc-ad)\operatorname{Ci}\left(\frac{bc-ad}{d^2x+cd}\right)+(bc-ad)\operatorname{Ci}\left(-\frac{bc-ad}{d^2x+cd}\right)\right)\cos\left(\frac{b}{d}\right)-2\left(d^2x+cd\right)\sin\left(\frac{bx+}{dx+}\right.}{2\,d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin((b*x+a)/(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/2*(2*(b*c - a*d)*sin(b/d)*sin_integral(-(b*c - a*d)/(d^2*x + c*d)) - ((b
*c - a*d)*cos_integral((b*c - a*d)/(d^2*x + c*d)) + (b*c - a*d)*cos_integra
l(-(b*c - a*d)/(d^2*x + c*d)))*cos(b/d) - 2*(d^2*x + c*d)*sin((b*x + a)/(d*
x + c)))/d^2
```

giac [B] time = 9.11, size = 630, normalized size = 6.30

$$\frac{\left(b^3c^2\cos\left(\frac{b}{d}\right)\operatorname{Ci}\left(-\frac{b-\frac{(bx+a)d}{dx+c}}{d}\right)-2\,ab^2cd\cos\left(\frac{b}{d}\right)\operatorname{Ci}\left(-\frac{b-\frac{(bx+a)d}{dx+c}}{d}\right)-\frac{(bx+a)b^2c^2d\cos\left(\frac{b}{d}\right)\operatorname{Ci}\left(-\frac{b-\frac{(bx+a)d}{dx+c}}{d}\right)}{dx+c}+a^2bd^2\cos\left(\frac{b}{d}\right)\operatorname{Ci}\left(-\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin((b*x+a)/(d*x+c)),x, algorithm="giac")
```

```
[Out] (b^3*c^2*cos(b/d)*cos_integral(-(b - (b*x + a)*d/(d*x + c))/d) - 2*a*b^2*c*
d*cos(b/d)*cos_integral(-(b - (b*x + a)*d/(d*x + c))/d) - (b*x + a)*b^2*c^2
*d*cos(b/d)*cos_integral(-(b - (b*x + a)*d/(d*x + c))/d)/(d*x + c) + a^2*b*
d^2*cos(b/d)*cos_integral(-(b - (b*x + a)*d/(d*x + c))/d) + 2*(b*x + a)*a*b
*c*d^2*cos(b/d)*cos_integral(-(b - (b*x + a)*d/(d*x + c))/d)/(d*x + c) - (b
*x + a)*a^2*d^3*cos(b/d)*cos_integral(-(b - (b*x + a)*d/(d*x + c))/d)/(d*x
+ c) + b^3*c^2*sin(b/d)*sin_integral((b - (b*x + a)*d/(d*x + c))/d) - 2*a*b
^2*c*d*sin(b/d)*sin_integral((b - (b*x + a)*d/(d*x + c))/d) - (b*x + a)*b^2
*c^2*d*sin(b/d)*sin_integral((b - (b*x + a)*d/(d*x + c))/d)/(d*x + c) + a^2
*b*d^2*sin(b/d)*sin_integral((b - (b*x + a)*d/(d*x + c))/d) + 2*(b*x + a)*a
*b*c*d^2*sin(b/d)*sin_integral((b - (b*x + a)*d/(d*x + c))/d)/(d*x + c) - (
b*x + a)*a^2*d^3*sin(b/d)*sin_integral((b - (b*x + a)*d/(d*x + c))/d)/(d*x
+ c) + b^2*c^2*d*sin((b*x + a)/(d*x + c)) - 2*a*b*c*d^2*sin((b*x + a)/(d*x
```

```
+ c)) + a^2*d^3*sin((b*x + a)/(d*x + c)))*(b*c/(b*c - a*d)^2 - a*d/(b*c - a
*d)^2)/(b*d^2 - (b*x + a)*d^3/(d*x + c))
```

maple [A] time = 0.04, size = 142, normalized size = 1.42

$$-(da-cb)\left(-\frac{\sin\left(\frac{b}{d}+\frac{da-cb}{d(dx+c)}\right)}{\left(\left(\frac{b}{d}+\frac{da-cb}{d(dx+c)}\right)d-b\right)d}+\frac{-\frac{\operatorname{Si}\left(\frac{da-cb}{d(dx+c)}\right)\sin\left(\frac{b}{d}\right)}{d}+\frac{\operatorname{Ci}\left(\frac{da-cb}{d(dx+c)}\right)\cos\left(\frac{b}{d}\right)}{d}}{d}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin((b*x+a)/(d*x+c)),x)
```

```
[Out] -(a*d-b*c)*(-sin(b/d+(a*d-b*c)/d/(d*x+c))/((b/d+(a*d-b*c)/d/(d*x+c))*d-b)/d
+(-Si((a*d-b*c)/d/(d*x+c))*sin(b/d)/d+Ci((a*d-b*c)/d/(d*x+c))*cos(b/d)/d)/d
)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sin\left(\frac{bx+a}{dx+c}\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin((b*x+a)/(d*x+c)),x, algorithm="maxima")
```

```
[Out] integrate(sin((b*x + a)/(d*x + c)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \sin\left(\frac{a+b\,x}{c+d\,x}\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin((a + b*x)/(c + d*x)),x)
```

```
[Out] int(sin((a + b*x)/(c + d*x)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sin\left(\frac{a+bx}{c+dx}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin((b*x+a)/(d*x+c)),x)
```

```
[Out] Integral(sin((a + b*x)/(c + d*x)), x)
```

3.37 $\int \sin^2\left(\frac{a+bx}{c+dx}\right) dx$

Optimal. Leaf size=107

$$\frac{\sin\left(\frac{2b}{d}\right)(bc-ad)\text{Ci}\left(\frac{2(bc-ad)}{d(c+dx)}\right)}{d^2}-\frac{\cos\left(\frac{2b}{d}\right)(bc-ad)\text{Si}\left(\frac{2(bc-ad)}{d(c+dx)}\right)}{d^2}+\frac{(c+dx)\sin^2\left(\frac{a+bx}{c+dx}\right)}{d}$$

```
[Out] -(-a*d+b*c)*cos(2*b/d)*Si(2*(-a*d+b*c)/d/(d*x+c))/d^2+(-a*d+b*c)*Ci(2*(-a*d
+b*c)/d/(d*x+c))*sin(2*b/d)/d^2+(d*x+c)*sin((b*x+a)/(d*x+c))^2/d
```

Rubi [A] time = 0.19, antiderivative size = 107, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {4563, 3313, 12, 3303, 3299, 3302}

$$\frac{\sin\left(\frac{2b}{d}\right)(bc-ad)\text{CosIntegral}\left(\frac{2(bc-ad)}{d(c+dx)}\right)}{d^2}-\frac{\cos\left(\frac{2b}{d}\right)(bc-ad)\text{Si}\left(\frac{2(bc-ad)}{d(c+dx)}\right)}{d^2}+\frac{(c+dx)\sin^2\left(\frac{a+bx}{c+dx}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[(a + b*x)/(c + d*x)]^2,x]
```

```
[Out] ((b*c - a*d)*CosIntegral[(2*(b*c - a*d))/(d*(c + d*x))]*Sin[(2*b)/d])/d^2 +
 ((c + d*x)*Sin[(a + b*x)/(c + d*x)]^2)/d - ((b*c - a*d)*Cos[(2*b)/d]*SinIn
tegral[(2*(b*c - a*d))/(d*(c + d*x))])/d^2
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3302

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3313

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> Si
mp[((c + d*x)^(m + 1)*Sin[e + f*x]^n)/(d*(m + 1)), x] - Dist[(f*n)/(d*(m +
1)), Int[ExpandTrigReduce[(c + d*x)^(m + 1), Cos[e + f*x]*Sin[e + f*x]^(n -
 1), x], x], x] /; FreeQ[{c, d, e, f, m}, x] && IGtQ[n, 1] && GeQ[m, -2] &&
 LtQ[m, -1]
```

Rule 4563

```
Int[Sin[((e_.)*((a_.) + (b_.)*(x_)))/((c_.) + (d_.)*(x_))]^(n_.), x_Symbol]
 :> -Dist[d^(-1), Subst[Int[Sin[(b*e)/d - (e*(b*c - a*d)*x)/d]^n/x^2, x], x
, 1/(c + d*x)], x] /; FreeQ[{a, b, c, d}, x] && IGtQ[n, 0] && NeQ[b*c - a*d
, 0]
```

Rubi steps

$$\int \sin^2\left(\frac{a+bx}{c+dx}\right)dx = -\frac{\text{Subst}\left(\int \frac{\sin^2\left(\frac{b}{d}-\frac{(bc-ad)x}{d}\right)}{x^2}\,dx, x, \frac{1}{c+dx}\right)}{d}$$

$$= \frac{(c+dx)\sin^2\left(\frac{a+bx}{c+dx}\right)}{d} + \frac{(2(bc-ad))\,\text{Subst}\left(\int \frac{\sin\left(\frac{2b}{d}-\frac{2(bc-ad)x}{d}\right)}{2x}\,dx, x, \frac{1}{c+dx}\right)}{d^2}$$

$$= \frac{(c+dx)\sin^2\left(\frac{a+bx}{c+dx}\right)}{d} + \frac{(bc-ad)\,\text{Subst}\left(\int \frac{\sin\left(\frac{2b}{d}-\frac{2(bc-ad)x}{d}\right)}{x}\,dx, x, \frac{1}{c+dx}\right)}{d^2}$$

$$= \frac{(c+dx)\sin^2\left(\frac{a+bx}{c+dx}\right)}{d} - \frac{\left((bc-ad)\cos\left(\frac{2b}{d}\right)\right)\text{Subst}\left(\int \frac{\sin\left(\frac{2(bc-ad)x}{d}\right)}{x}\,dx, x, \frac{1}{c+dx}\right)}{d^2} + \frac{\left((bc-ad)\right.}{}$$

$$= \frac{(bc-ad)\text{Ci}\left(\frac{2(bc-ad)}{d(c+dx)}\right)\sin\left(\frac{2b}{d}\right)}{d^2} + \frac{(c+dx)\sin^2\left(\frac{a+bx}{c+dx}\right)}{d} - \frac{(bc-ad)\cos\left(\frac{2b}{d}\right)\text{Si}\left(\frac{2(bc-ad)}{d(c+dx)}\right)}{d^2}$$

Mathematica [C] time = 7.27, size = 330, normalized size = 3.08

$$\frac{8\sin\left(\frac{2b}{d}\right)(bc-ad)\text{Ci}\left(\frac{2(ad-bc)}{d(c+dx)}\right)-d\exp\left(-\frac{2i(ad+2bc+bdx)}{d(c+dx)}\right)\Bigg(dx\Bigg(-4\exp\left(\frac{2i(ad+2bc+bdx)}{d(c+dx)}\right)+e^{4i\left(\frac{a}{c+dx}+\frac{b}{d}\right)}+e^{\frac{4ia}{c+dx}}+e^{\frac{4ibc}{d(c+dx)}}}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sin[(a + b*x)/(c + d*x)]^2,x]
```

```
[Out] (8*(b*c - a*d)*CosIntegral[(2*(-(b*c) + a*d))/(d*(c + d*x))]*Sin[(2*b)/d] -
 (d*(2*c*(E^(((4*I)*b*c)/(d*(c + d*x))) + E^((4*I)*(b/d + a/(c + d*x)))) +
d*(E^(((4*I)*a)/(c + d*x)) + E^(((4*I)*b*c)/(d*(c + d*x))) + E^(((4*I)*b*(2
*c + d*x))/(d*(c + d*x))) - 4*E^(((2*I)*(2*b*c + a*d + b*d*x))/(d*(c + d*x)
)) + E^((4*I)*(b/d + a/(c + d*x))))*x - 4*d*E^(((2*I)*(2*b*c + a*d + b*d*x)
)/(d*(c + d*x)))*x*Sin[(2*b)/d]*Sin[(2*(-(b*c) + a*d))/(d*(c + d*x))]))/E^(
((2*I)*(2*b*c + a*d + b*d*x))/(d*(c + d*x))) + 8*(b*c - a*d)*Cos[(2*b)/d]*S
inIntegral[(2*(-(b*c) + a*d))/(d*(c + d*x))])/(8*d^2)
```

fricas [A] time = 0.61, size = 149, normalized size = 1.39

$$\frac{2\,d^2x-2\left(d^2x+cd\right)\cos\left(\frac{bx+a}{dx+c}\right)^2+2\,(bc-ad)\cos\left(\frac{2\,b}{d}\right)\text{Si}\left(-\frac{2\,(bc-ad)}{d^2x+cd}\right)+\Bigg((bc-ad)\,\text{Ci}\left(\frac{2\,(bc-ad)}{d^2x+cd}\right)+(bc-ad)\,\text{Ci}\Bigg(-\frac{2}{}}{2\,d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin((b*x+a)/(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/2*(2*d^2*x - 2*(d^2*x + c*d)*cos((b*x + a)/(d*x + c))^2 + 2*(b*c - a*d)*c
os(2*b/d)*sin_integral(-2*(b*c - a*d)/(d^2*x + c*d)) + ((b*c - a*d)*cos_int
egral(2*(b*c - a*d)/(d^2*x + c*d)) + (b*c - a*d)*cos_integral(-2*(b*c - a*d
)/(d^2*x + c*d)))*sin(2*b/d))/d^2
```

giac [B] time = 62.66, size = 681, normalized size = 6.36

$$\frac{\Bigg(2\,b^3c^2\,\text{Ci}\left(-\frac{2\left(b-\frac{(bx+a)d}{dx+c}\right)}{d}\right)\sin\left(\frac{2\,b}{d}\right)-4\,ab^2cd\,\text{Ci}\left(-\frac{2\left(b-\frac{(bx+a)d}{dx+c}\right)}{d}\right)\sin\left(\frac{2\,b}{d}\right)-\frac{2\,(bx+a)b^2c^2d\,\text{Ci}\left(-\frac{2\left(b-\frac{(bx+a)d}{dx+c}\right)}{d}\right)\sin\left(\frac{2\,b}{d}\right)}{dx+c}+2\,a^2bd}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin((b*x+a)/(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/2*(2*b^3*c^2*cos_integral(-2*(b - (b*x + a)*d/(d*x + c))/d)*sin(2*b/d) -
4*a*b^2*c*d*cos_integral(-2*(b - (b*x + a)*d/(d*x + c))/d)*sin(2*b/d) - 2*(
b*x + a)*b^2*c^2*d*cos_integral(-2*(b - (b*x + a)*d/(d*x + c))/d)*sin(2*b/d
)/(d*x + c) + 2*a^2*b*d^2*cos_integral(-2*(b - (b*x + a)*d/(d*x + c))/d)*si
n(2*b/d) + 4*(b*x + a)*a*b*c*d^2*cos_integral(-2*(b - (b*x + a)*d/(d*x + c)
)/d)*sin(2*b/d)/(d*x + c) - 2*(b*x + a)*a^2*d^3*cos_integral(-2*(b - (b*x +
 a)*d/(d*x + c))/d)*sin(2*b/d)/(d*x + c) - 2*b^3*c^2*cos(2*b/d)*sin_integra
l(2*(b - (b*x + a)*d/(d*x + c))/d) + 4*a*b^2*c*d*cos(2*b/d)*sin_integral(2*
(b - (b*x + a)*d/(d*x + c))/d) + 2*(b*x + a)*b^2*c^2*d*cos(2*b/d)*sin_integ
ral(2*(b - (b*x + a)*d/(d*x + c))/d)/(d*x + c) - 2*a^2*b*d^2*cos(2*b/d)*sin
_integral(2*(b - (b*x + a)*d/(d*x + c))/d) - 4*(b*x + a)*a*b*c*d^2*cos(2*b/
d)*sin_integral(2*(b - (b*x + a)*d/(d*x + c))/d)/(d*x + c) + 2*(b*x + a)*a^
2*d^3*cos(2*b/d)*sin_integral(2*(b - (b*x + a)*d/(d*x + c))/d)/(d*x + c) -
b^2*c^2*d*cos(2*(b*x + a)/(d*x + c)) + 2*a*b*c*d^2*cos(2*(b*x + a)/(d*x + c
)) - a^2*d^3*cos(2*(b*x + a)/(d*x + c)) + b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3
)*(b*c/(b*c - a*d)^2 - a*d/(b*c - a*d)^2)/(b*d^2 - (b*x + a)*d^3/(d*x + c))
```

maple [A] time = 0.07, size = 195, normalized size = 1.82

$$-\frac{(da-cb)\left(-\frac{d}{2\left(\left(\frac{b}{d}+\frac{da-cb}{d(dx+c)}\right)d-b\right)}-\frac{d^{2}\left(-\frac{2\cos\left(\frac{2da-2cb}{d(dx+c)}+\frac{2b}{d}\right)}{\left(\left(\frac{b}{d}+\frac{da-cb}{d(dx+c)}\right)d-b\right)d}-\frac{2\left(\frac{2\,\mathrm{Si}\left(\frac{2da-2cb}{d(dx+c)}\right)\cos\left(\frac{2b}{d}\right)}{d}+\frac{2\,\mathrm{Ci}\left(\frac{2da-2cb}{d(dx+c)}\right)\sin\left(\frac{2b}{d}\right)}{d}\right)}{d}\right)}{4}\right)}{d^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin((b*x+a)/(d*x+c))^2,x)
```

```
[Out] -1/d^2*(a*d-b*c)*(-1/2*d/((b/d+(a*d-b*c)/d/(d*x+c))*d-b)-1/4*d^2*(-2*cos(2*
(a*d-b*c)/d/(d*x+c)+2*b/d)/((b/d+(a*d-b*c)/d/(d*x+c))*d-b)/d-2*(2*Si(2*(a*d
-b*c)/d/(d*x+c))*cos(2*b/d)/d+2*Ci(2*(a*d-b*c)/d/(d*x+c))*sin(2*b/d)/d)/d))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{2}\,x-\frac{1}{2}\,\int \cos\left(\frac{2\,(bx+a)}{dx+c}\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin((b*x+a)/(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 1/2*x - 1/2*integrate(cos(2*(b*x + a)/(d*x + c)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \sin\left(\frac{a+b\,x}{c+d\,x}\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin((a + b*x)/(c + d*x))^2,x)
[Out] int(sin((a + b*x)/(c + d*x))^2, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin((b*x+a)/(d*x+c))**2,x)
[Out] Timed out
```

3.38 $\int \sin^3\left(\frac{a+bx}{c+dx}\right) dx$

Optimal. Leaf size=194

$$\frac{3\cos\left(\frac{b}{d}\right)(bc-ad)\text{Ci}\left(\frac{bc-ad}{d(c+dx)}\right)}{4d^2}-\frac{3\cos\left(\frac{3b}{d}\right)(bc-ad)\text{Ci}\left(\frac{3(bc-ad)}{d(c+dx)}\right)}{4d^2}+\frac{3\sin\left(\frac{b}{d}\right)(bc-ad)\text{Si}\left(\frac{bc-ad}{d(c+dx)}\right)}{4d^2}-\frac{3\sin\left(\frac{3b}{d}\right)(bc-\ldots}{4\ldots}$$

```
[Out] 3/4*(-a*d+b*c)*Ci((-a*d+b*c)/d/(d*x+c))*cos(b/d)/d^2-3/4*(-a*d+b*c)*Ci(3*(-
a*d+b*c)/d/(d*x+c))*cos(3*b/d)/d^2+3/4*(-a*d+b*c)*Si((-a*d+b*c)/d/(d*x+c))*
sin(b/d)/d^2-3/4*(-a*d+b*c)*Si(3*(-a*d+b*c)/d/(d*x+c))*sin(3*b/d)/d^2+(d*x+
c)*sin((b*x+a)/(d*x+c))^3/d
```

Rubi [A] time = 0.32, antiderivative size = 194, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312, Rules used = {4563, 3313, 3303, 3299, 3302}

$$\frac{3\cos\left(\frac{b}{d}\right)(bc-ad)\text{CosIntegral}\left(\frac{bc-ad}{d(c+dx)}\right)}{4d^2}-\frac{3\cos\left(\frac{3b}{d}\right)(bc-ad)\text{CosIntegral}\left(\frac{3(bc-ad)}{d(c+dx)}\right)}{4d^2}+\frac{3\sin\left(\frac{b}{d}\right)(bc-ad)\text{Si}\left(\frac{bc-\ldots}{d(c+\ldots}\right)}{4d^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[(a + b*x)/(c + d*x)]^3,x]
```

```
[Out] (3*(b*c - a*d)*Cos[b/d]*CosIntegral[(b*c - a*d)/(d*(c + d*x))])/(4*d^2) - (
3*(b*c - a*d)*Cos[(3*b)/d]*CosIntegral[(3*(b*c - a*d))/(d*(c + d*x))])/(4*d
^2) + ((c + d*x)*Sin[(a + b*x)/(c + d*x)]^3)/d + (3*(b*c - a*d)*Sin[b/d]*Si
nIntegral[(b*c - a*d)/(d*(c + d*x))])/(4*d^2) - (3*(b*c - a*d)*Sin[(3*b)/d]
*SinIntegral[(3*(b*c - a*d))/(d*(c + d*x))])/(4*d^2)
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3302

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
```

```
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3313

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> Si
mp[((c + d*x)^(m + 1)*Sin[e + f*x]^n)/(d*(m + 1)), x] - Dist[(f*n)/(d*(m +
1)), Int[ExpandTrigReduce[(c + d*x)^(m + 1), Cos[e + f*x]*Sin[e + f*x]^(n -
 1), x], x], x] /; FreeQ[{c, d, e, f, m}, x] && IGtQ[n, 1] && GeQ[m, -2] &&
 LtQ[m, -1]
```

Rule 4563

```
Int[Sin[((e_.)*((a_.) + (b_.)*(x_)))/((c_.) + (d_.)*(x_))]^(n_.), x_Symbol]
 :> -Dist[d^(-1), Subst[Int[Sin[(b*e)/d - (e*(b*c - a*d)*x)/d]^n/x^2, x], x
, 1/(c + d*x)], x] /; FreeQ[{a, b, c, d}, x] && IGtQ[n, 0] && NeQ[b*c - a*d
, 0]
```

Rubi steps

$$\int \sin^3\left(\frac{a+bx}{c+dx}\right)dx = -\frac{\text{Subst}\left(\int \frac{\sin^3\left(\frac{b}{d}-\frac{(bc-ad)x}{d}\right)}{x^2}dx, x, \frac{1}{c+dx}\right)}{d}$$

$$= \frac{(c+dx)\sin^3\left(\frac{a+bx}{c+dx}\right)}{d} + \frac{(3(bc-ad))\,\text{Subst}\left(\int\left(-\frac{\cos\left(\frac{3b}{d}-\frac{3(bc-ad)x}{d}\right)}{4x}+\frac{\cos\left(\frac{b}{d}-\frac{(bc-ad)x}{d}\right)}{4x}\right)dx, x, \frac{1}{c+dx}\right)}{d^2}$$

$$= \frac{(c+dx)\sin^3\left(\frac{a+bx}{c+dx}\right)}{d} - \frac{(3(bc-ad))\,\text{Subst}\left(\int \frac{\cos\left(\frac{3b}{d}-\frac{3(bc-ad)x}{d}\right)}{x}dx, x, \frac{1}{c+dx}\right)}{4d^2} + \frac{(3(bc-ad))\,\text{Sub}}{}$$

$$= \frac{(c+dx)\sin^3\left(\frac{a+bx}{c+dx}\right)}{d} + \frac{\left(3(bc-ad)\cos\left(\frac{b}{d}\right)\right)\text{Subst}\left(\int \frac{\cos\left(\frac{(bc-ad)x}{d}\right)}{x}dx, x, \frac{1}{c+dx}\right)}{4d^2} - \frac{\left(3(bc-ad)\right.}{}$$

$$= \frac{3(bc-ad)\cos\left(\frac{b}{d}\right)\text{Ci}\left(\frac{bc-ad}{d(c+dx)}\right)}{4d^2} - \frac{3(bc-ad)\cos\left(\frac{3b}{d}\right)\text{Ci}\left(\frac{3(bc-ad)}{d(c+dx)}\right)}{4d^2} + \frac{(c+dx)\sin^3\left(\frac{a+bx}{c+dx}\right)}{d} +$$

Mathematica [C] time = 7.71, size = 657, normalized size = 3.39

$$-\frac{3\left(acd-bc^2\right)\left(\frac{i\left(1+e^{\frac{2ib}{d}}\right)\left(e^{\frac{2ibc}{d(c+dx)}}-e^{\frac{2ia}{c+dx}}\right)\exp\left(-\frac{i(ad+2bc+bdx)}{d(c+dx)}\right)}{4(bc-ad)}-\frac{i\left(-1+e^{\frac{2ib}{d}}\right)\left(e^{\frac{2ia}{c+dx}}+e^{\frac{2ibc}{d(c+dx)}}\right)\exp\left(-\frac{i(ad+2bc+bdx)}{d(c+dx)}\right)}{4(bc-ad)}\right)}{4d}+\frac{3\left(acd-bc^2\right)\left(\cdots\right.}{\cdots}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sin[(a + b*x)/(c + d*x)]^3,x]
```

```
[Out] (-3*(-(b*c^2) + a*c*d)*(((I/4)*(1 + E^(((2*I)*b)/d))*(-E^(((2*I)*a)/(c + d*
x)) + E^(((2*I)*b*c)/(d*(c + d*x)))))/((b*c - a*d)*E^((I*(2*b*c + a*d + b*d
*x))/(d*(c + d*x)))) - ((I/4)*(-1 + E^(((2*I)*b)/d))*(E^(((2*I)*a)/(c + d*x
)) + E^(((2*I)*b*c)/(d*(c + d*x)))))/((b*c - a*d)*E^((I*(2*b*c + a*d + b*d*
x))/(d*(c + d*x))))))/(4*d) + (3*(-(b*c^2) + a*c*d)*(((I/12)*(1 + E^(((6*I)
*b)/d))*(-E^(((6*I)*a)/(c + d*x)) + E^(((6*I)*b*c)/(d*(c + d*x)))))/((b*c -
 a*d)*E^(((3*I)*(2*b*c + a*d + b*d*x))/(d*(c + d*x)))) - ((I/12)*(-1 + E^((
(6*I)*b)/d))*(E^(((6*I)*a)/(c + d*x)) + E^(((6*I)*b*c)/(d*(c + d*x)))))/((b
*c - a*d)*E^(((3*I)*(2*b*c + a*d + b*d*x))/(d*(c + d*x))))))/(4*d) + (3*x*C
os[(-(b*c) + a*d)/(d*(c + d*x))]*Sin[b/d])/4 - (x*Cos[(3*(-(b*c) + a*d))/(d
*(c + d*x))]*Sin[(3*b)/d])/4 + (3*x*Cos[b/d]*Sin[(-(b*c) + a*d)/(d*(c + d*x
))])/4 - (x*Cos[(3*b)/d]*Sin[(3*(-(b*c) + a*d))/(d*(c + d*x))])/4 + (3*(-(b
*c) + a*d)*(-(Cos[b/d]*CosIntegral[(-(b*c) + a*d)/(d*(c + d*x))]) + Cos[(3*
b)/d]*CosIntegral[(3*(-(b*c) + a*d))/(d*(c + d*x))] + Sin[b/d]*SinIntegral[
(-(b*c) + a*d)/(d*(c + d*x))] - Sin[(3*b)/d]*SinIntegral[(3*(-(b*c) + a*d))
/(d*(c + d*x))]))/(4*d^2)
```

fricas [A] time = 1.14, size = 277, normalized size = 1.43

$$-\frac{6\,(bc-ad)\sin\left(\frac{b}{d}\right)\operatorname{Si}\left(-\frac{bc-ad}{d^2x+cd}\right)-6\,(bc-ad)\sin\left(\frac{3\,b}{d}\right)\operatorname{Si}\left(-\frac{3\,(bc-ad)}{d^2x+cd}\right)+3\left((bc-ad)\operatorname{Ci}\left(\frac{3\,(bc-ad)}{d^2x+cd}\right)+(bc-ad)\operatorname{Ci}\left(\cdots\right.\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin((b*x+a)/(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] -1/8*(6*(b*c - a*d)*sin(b/d)*sin_integral(-(b*c - a*d)/(d^2*x + c*d)) - 6*(
b*c - a*d)*sin(3*b/d)*sin_integral(-3*(b*c - a*d)/(d^2*x + c*d)) + 3*((b*c
- a*d)*cos_integral(3*(b*c - a*d)/(d^2*x + c*d)) + (b*c - a*d)*cos_integral
(-3*(b*c - a*d)/(d^2*x + c*d)))*cos(3*b/d) - 3*((b*c - a*d)*cos_integral((b
*c - a*d)/(d^2*x + c*d)) + (b*c - a*d)*cos_integral(-(b*c - a*d)/(d^2*x + c
*d)))*cos(b/d) - 8*(d^2*x - (d^2*x + c*d)*cos((b*x + a)/(d*x + c))^2 + c*d)
*sin((b*x + a)/(d*x + c)))/d^2
```

giac [B] time = 177.53, size = 1239, normalized size = 6.39

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin((b*x+a)/(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/4*(3*b^3*c^2*cos(b/d)*cos_integral(-(b - (b*x + a)*d/(d*x + c))/d) - 6*a*
b^2*c*d*cos(b/d)*cos_integral(-(b - (b*x + a)*d/(d*x + c))/d) - 3*(b*x + a)
*b^2*c^2*d*cos(b/d)*cos_integral(-(b - (b*x + a)*d/(d*x + c))/d)/(d*x + c)
+ 3*a^2*b*d^2*cos(b/d)*cos_integral(-(b - (b*x + a)*d/(d*x + c))/d) + 6*(b*
x + a)*a*b*c*d^2*cos(b/d)*cos_integral(-(b - (b*x + a)*d/(d*x + c))/d)/(d*x
 + c) - 3*(b*x + a)*a^2*d^3*cos(b/d)*cos_integral(-(b - (b*x + a)*d/(d*x +
c))/d)/(d*x + c) - 3*b^3*c^2*cos(3*b/d)*cos_integral(-3*(b - (b*x + a)*d/(d
*x + c))/d) + 6*a*b^2*c*d*cos(3*b/d)*cos_integral(-3*(b - (b*x + a)*d/(d*x
+ c))/d) + 3*(b*x + a)*b^2*c^2*d*cos(3*b/d)*cos_integral(-3*(b - (b*x + a)*
d/(d*x + c))/d)/(d*x + c) - 3*a^2*b*d^2*cos(3*b/d)*cos_integral(-3*(b - (b*
x + a)*d/(d*x + c))/d) - 6*(b*x + a)*a*b*c*d^2*cos(3*b/d)*cos_integral(-3*(
b - (b*x + a)*d/(d*x + c))/d)/(d*x + c) + 3*(b*x + a)*a^2*d^3*cos(3*b/d)*co
s_integral(-3*(b - (b*x + a)*d/(d*x + c))/d)/(d*x + c) - 3*b^3*c^2*sin(3*b/
d)*sin_integral(3*(b - (b*x + a)*d/(d*x + c))/d) + 6*a*b^2*c*d*sin(3*b/d)*s
in_integral(3*(b - (b*x + a)*d/(d*x + c))/d) + 3*(b*x + a)*b^2*c^2*d*sin(3*
b/d)*sin_integral(3*(b - (b*x + a)*d/(d*x + c))/d)/(d*x + c) - 3*a^2*b*d^2*
sin(3*b/d)*sin_integral(3*(b - (b*x + a)*d/(d*x + c))/d) - 6*(b*x + a)*a*b*
c*d^2*sin(3*b/d)*sin_integral(3*(b - (b*x + a)*d/(d*x + c))/d)/(d*x + c) +
3*(b*x + a)*a^2*d^3*sin(3*b/d)*sin_integral(3*(b - (b*x + a)*d/(d*x + c))/d
)/(d*x + c) + 3*b^3*c^2*sin(b/d)*sin_integral((b - (b*x + a)*d/(d*x + c))/d
) - 6*a*b^2*c*d*sin(b/d)*sin_integral((b - (b*x + a)*d/(d*x + c))/d) - 3*(b
*x + a)*b^2*c^2*d*sin(b/d)*sin_integral((b - (b*x + a)*d/(d*x + c))/d)/(d*x
 + c) + 3*a^2*b*d^2*sin(b/d)*sin_integral((b - (b*x + a)*d/(d*x + c))/d) +
6*(b*x + a)*a*b*c*d^2*sin(b/d)*sin_integral((b - (b*x + a)*d/(d*x + c))/d)/
(d*x + c) - 3*(b*x + a)*a^2*d^3*sin(b/d)*sin_integral((b - (b*x + a)*d/(d*x
 + c))/d)/(d*x + c) - b^2*c^2*d*sin(3*(b*x + a)/(d*x + c)) + 2*a*b*c*d^2*si
n(3*(b*x + a)/(d*x + c)) - a^2*d^3*sin(3*(b*x + a)/(d*x + c)) + 3*b^2*c^2*d
*sin((b*x + a)/(d*x + c)) - 6*a*b*c*d^2*sin((b*x + a)/(d*x + c)) + 3*a^2*d^
3*sin((b*x + a)/(d*x + c)))*(b*c/(b*c - a*d)^2 - a*d/(b*c - a*d)^2)/(b*d^2
- (b*x + a)*d^3/(d*x + c))
```

maple [A] time = 0.07, size = 295, normalized size = 1.52

$$-\frac{(da-cb)\left(-\frac{d^2\left(-\frac{3\sin\left(\frac{3da-3cb}{d(dx+c)}+\frac{3b}{d}\right)}{\left(\left(\frac{b}{d}+\frac{da-cb}{d(dx+c)}\right)d-b\right)d}+\frac{-\frac{9\,\mathrm{Si}\left(\frac{3da-3cb}{d(dx+c)}\right)\sin\left(\frac{3b}{d}\right)}{d}+\frac{9\,\mathrm{Ci}\left(\frac{3da-3cb}{d(dx+c)}\right)\cos\left(\frac{3b}{d}\right)}{d}}{d}\right)}{12}+\frac{3d^2\left(-\frac{\sin\left(\frac{b}{d}+\frac{da-cb}{d(dx+c)}\right)}{\left(\left(\frac{b}{d}+\frac{da-cb}{d(dx+c)}\right)d-b\right)d}+\frac{-\frac{\mathrm{Si}\left(\frac{da-cb}{d(dx+c)}\right)\sin\left(\frac{b}{d}\right)}{d}+\frac{\mathrm{Ci}\left(\frac{da-cb}{d(dx+c)}\right)\cos\left(\frac{b}{d}\right)}{d}}{d}\right)}{4}\right)}{d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin((b*x+a)/(d*x+c))^3,x)
```

```
[Out] -1/d^2*(a*d-b*c)*(-1/12*d^2*(-3*sin(3*(a*d-b*c)/d/(d*x+c)+3*b/d)/((b/d+(a*d
-b*c)/d/(d*x+c))*d-b)/d+3*(-3*Si(3*(a*d-b*c)/d/(d*x+c))*sin(3*b/d)/d+3*Ci(3
*(a*d-b*c)/d/(d*x+c))*cos(3*b/d)/d)/d)+3/4*d^2*(-sin(b/d+(a*d-b*c)/d/(d*x+c
))/((b/d+(a*d-b*c)/d/(d*x+c))*d-b)/d+(-Si((a*d-b*c)/d/(d*x+c))*sin(b/d)/d+C
i((a*d-b*c)/d/(d*x+c))*cos(b/d)/d)/d))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sin\left(\frac{bx+a}{dx+c}\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin((b*x+a)/(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] integrate(sin((b*x + a)/(d*x + c))^3, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \sin\left(\frac{a+b\,x}{c+d\,x}\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin((a + b*x)/(c + d*x))^3,x)
```

```
[Out] int(sin((a + b*x)/(c + d*x))^3, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin((b*x+a)/(d*x+c))**3,x)
```

```
[Out] Timed out
```

3.39 $\int \frac{\sin^3\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{1-a^2x^2}\,dx$

Optimal. Leaf size=58

$$\frac{\mathrm{Si}\left(\frac{3\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{4a}-\frac{3\mathrm{Si}\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{4a}$$

```
[Out] -3/4*Si((-a*x+1)^(1/2)/(a*x+1)^(1/2))/a+1/4*Si(3*(-a*x+1)^(1/2)/(a*x+1)^(1/
2))/a
```

Rubi [A] time = 0.11, antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {6681, 3312, 3299}

$$\frac{\mathrm{Si}\left(\frac{3\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{4a}-\frac{3\mathrm{Si}\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{4a}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[Sqrt[1 - a*x]/Sqrt[1 + a*x]]^3/(1 - a^2*x^2),x]
```

```
[Out] (-3*SinIntegral[Sqrt[1 - a*x]/Sqrt[1 + a*x]])/(4*a) + SinIntegral[(3*Sqrt[1
 - a*x])/Sqrt[1 + a*x]]/(4*a)
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 6681

```
Int[((a_.) + (b_.)*(F_)[((c_.)*Sqrt[(d_.) + (e_.)*(x_)])/Sqrt[(f_.) + (g_.)
*(x_)]])^(n_.)/((A_.) + (C_.)*(x_)^2), x_Symbol] :> Dist[(2*e*g)/(C*(e*f -
d*g)), Subst[Int[(a + b*F[c*x])^n/x, x], x, Sqrt[d + e*x]/Sqrt[f + g*x]], x
] /; FreeQ[{a, b, c, d, e, f, g, A, C, F}, x] && EqQ[C*d*f - A*e*g, 0] && E
qQ[e*f + d*g, 0] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{\sin^3\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{1-a^2x^2}\,dx = -\frac{\text{Subst}\left(\int \frac{\sin^3(x)}{x}\,dx, x, \frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{a}$$
$$= -\frac{\text{Subst}\left(\int \left(\frac{3\sin(x)}{4x} - \frac{\sin(3x)}{4x}\right)dx, x, \frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{a}$$
$$= \frac{\text{Subst}\left(\int \frac{\sin(3x)}{x}\,dx, x, \frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{4a} - \frac{3\,\text{Subst}\left(\int \frac{\sin(x)}{x}\,dx, x, \frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{4a}$$
$$= -\frac{3\text{Si}\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{4a} + \frac{\text{Si}\left(\frac{3\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{4a}$$

Mathematica [A] time = 0.11, size = 53, normalized size = 0.91

$$\frac{\text{Si}\left(\frac{3\sqrt{1-ax}}{\sqrt{ax+1}}\right) - 3\text{Si}\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{4a}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[Sqrt[1 - a*x]/Sqrt[1 + a*x]]^3/(1 - a^2*x^2),x]
```

```
[Out] (-3*SinIntegral[Sqrt[1 - a*x]/Sqrt[1 + a*x]] + SinIntegral[(3*Sqrt[1 - a*x]
)/Sqrt[1 + a*x]])/(4*a)
```

fricas [F] time = 2.03, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(\cos\left(\frac{\sqrt{-ax+1}}{\sqrt{ax+1}}\right)^2 - 1\right)\sin\left(\frac{\sqrt{-ax+1}}{\sqrt{ax+1}}\right)}{a^2x^2-1}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin((-a*x+1)^(1/2)/(a*x+1)^(1/2))^3/(-a^2*x^2+1),x, algorithm="fr
icas")
```

```
[Out] integral((cos(sqrt(-a*x + 1)/sqrt(a*x + 1))^2 - 1)*sin(sqrt(-a*x + 1)/sqrt(
a*x + 1))/(a^2*x^2 - 1), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{\sin\left(\frac{\sqrt{-ax+1}}{\sqrt{ax+1}}\right)^3}{a^2x^2-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin((-a*x+1)^(1/2)/(a*x+1)^(1/2))^3/(-a^2*x^2+1),x, algorithm="gi
ac")
```

```
[Out] integrate(-sin(sqrt(-a*x + 1)/sqrt(a*x + 1))^3/(a^2*x^2 - 1), x)
```

maple [F] time = 0.58, size = 0, normalized size = 0.00

$$\int \frac{\sin^3\left(\frac{\sqrt{-ax+1}}{\sqrt{ax+1}}\right)}{-a^2x^2+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin((-a*x+1)^(1/2)/(a*x+1)^(1/2))^3/(-a^2*x^2+1),x)
```

```
[Out] int(sin((-a*x+1)^(1/2)/(a*x+1)^(1/2))^3/(-a^2*x^2+1),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{\sin\left(\frac{\sqrt{-ax+1}}{\sqrt{ax+1}}\right)^3}{a^2x^2-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin((-a*x+1)^(1/2)/(a*x+1)^(1/2))^3/(-a^2*x^2+1),x, algorithm="ma
xima")
```

```
[Out] -integrate(sin(sqrt(-a*x + 1)/sqrt(a*x + 1))^3/(a^2*x^2 - 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$-\int \frac{\sin\left(\frac{\sqrt{1-a\,x}}{\sqrt{a\,x+1}}\right)^3}{a^2\,x^2-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-sin((1 - a*x)^(1/2)/(a*x + 1)^(1/2))^3/(a^2*x^2 - 1),x)
```

```
[Out] -int(sin((1 - a*x)^(1/2)/(a*x + 1)^(1/2))^3/(a^2*x^2 - 1), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{\sin^3\left(\frac{\sqrt{-ax+1}}{\sqrt{ax+1}}\right)}{a^2x^2-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin((-a*x+1)**(1/2)/(a*x+1)**(1/2))**3/(-a**2*x**2+1),x)
[Out] -Integral(sin(sqrt(-a*x + 1)/sqrt(a*x + 1))**3/(a**2*x**2 - 1), x)
```

3.40 $\int \frac{\sin^2\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{1-a^2x^2}\,dx$

Optimal. Leaf size=58

$$\frac{\text{Ci}\left(\frac{2\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{2a}-\frac{\log\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{2a}$$

```
[Out] 1/2*Ci(2*(-a*x+1)^(1/2)/(a*x+1)^(1/2))/a-1/2*ln((-a*x+1)^(1/2)/(a*x+1)^(1/2
))/a
```

Rubi [A] time = 0.08, antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {6681, 3312, 3302}

$$\frac{\text{CosIntegral}\left(\frac{2\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{2a}-\frac{\log\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{2a}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[Sqrt[1 - a*x]/Sqrt[1 + a*x]]^2/(1 - a^2*x^2),x]
```

```
[Out] CosIntegral[(2*Sqrt[1 - a*x])/Sqrt[1 + a*x]]/(2*a) - Log[Sqrt[1 - a*x]/Sqrt
[1 + a*x]]/(2*a)
```

Rule 3302

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 6681

```
Int[((a_.) + (b_.)*(F_)[((c_.)*Sqrt[(d_.) + (e_.)*(x_)])/Sqrt[(f_.) + (g_.)
*(x_)]])^(n_.)/((A_.) + (C_.)*(x_)^2), x_Symbol] :> Dist[(2*e*g)/(C*(e*f -
d*g)), Subst[Int[(a + b*F[c*x])^n/x, x], x, Sqrt[d + e*x]/Sqrt[f + g*x]], x
] /; FreeQ[{a, b, c, d, e, f, g, A, C, F}, x] && EqQ[C*d*f - A*e*g, 0] && E
qQ[e*f + d*g, 0] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{\sin^2\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{1-a^2x^2}\,dx = -\frac{\text{Subst}\left(\int \frac{\sin^2(x)}{x}\,dx, x, \frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{a}$$
$$= -\frac{\text{Subst}\left(\int \left(\frac{1}{2x}-\frac{\cos(2x)}{2x}\right)dx, x, \frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{a}$$
$$= -\frac{\log\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{2a} + \frac{\text{Subst}\left(\int \frac{\cos(2x)}{x}\,dx, x, \frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{2a}$$
$$= \frac{\text{Ci}\left(\frac{2\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{2a} - \frac{\log\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{2a}$$

Mathematica [A] time = 0.08, size = 57, normalized size = 0.98

$$\frac{\text{Ci}\left(\frac{2\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{2a} - \frac{\log(1-ax)}{4a} + \frac{\log(ax+1)}{4a}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[Sqrt[1 - a*x]/Sqrt[1 + a*x]]^2/(1 - a^2*x^2),x]
```

```
[Out] CosIntegral[(2*Sqrt[1 - a*x])/Sqrt[1 + a*x]]/(2*a) - Log[1 - a*x]/(4*a) + L
og[1 + a*x]/(4*a)
```

fricas [F] time = 0.97, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\cos\left(\frac{\sqrt{-ax+1}}{\sqrt{ax+1}}\right)^2 - 1}{a^2x^2-1}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin((-a*x+1)^(1/2)/(a*x+1)^(1/2))^2/(-a^2*x^2+1),x, algorithm="fr
icas")
```

```
[Out] integral((cos(sqrt(-a*x + 1)/sqrt(a*x + 1))^2 - 1)/(a^2*x^2 - 1), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{\sin\left(\frac{\sqrt{-ax+1}}{\sqrt{ax+1}}\right)^2}{a^2x^2-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin((-a*x+1)^(1/2)/(a*x+1)^(1/2))^2/(-a^2*x^2+1),x, algorithm="gi
ac")
```

```
[Out] integrate(-sin(sqrt(-a*x + 1)/sqrt(a*x + 1))^2/(a^2*x^2 - 1), x)
```

maple [F] time = 0.29, size = 0, normalized size = 0.00

$$\int \frac{\sin^2\left(\frac{\sqrt{-ax+1}}{\sqrt{ax+1}}\right)}{-a^2x^2+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin((-a*x+1)^(1/2)/(a*x+1)^(1/2))^2/(-a^2*x^2+1),x)
```

```
[Out] int(sin((-a*x+1)^(1/2)/(a*x+1)^(1/2))^2/(-a^2*x^2+1),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{a\int \frac{\cos\left(\frac{2\sqrt{-ax+1}}{\sqrt{ax+1}}\right)}{a^2x^2-1}\,dx + a\int \frac{\cos\left(\frac{2\sqrt{-ax+1}}{\sqrt{ax+1}}\right)}{\left(a^2x^2-1\right)\cos\left(\frac{2\sqrt{-ax+1}}{\sqrt{ax+1}}\right)^2+\left(a^2x^2-1\right)\sin\left(\frac{2\sqrt{-ax+1}}{\sqrt{ax+1}}\right)^2}\,dx + \log\left(ax+1\right) - \log\left(ax-1\right)}{4\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin((-a*x+1)^(1/2)/(a*x+1)^(1/2))^2/(-a^2*x^2+1),x, algorithm="ma
xima")
```

```
[Out] 1/4*(4*a*integrate(1/4*cos(2*sqrt(-a*x + 1)/sqrt(a*x + 1))/(a^2*x^2 - 1), x
) + 4*a*integrate(1/4*cos(2*sqrt(-a*x + 1)/sqrt(a*x + 1))/((a^2*x^2 - 1)*co
s(2*sqrt(-a*x + 1)/sqrt(a*x + 1))^2 + (a^2*x^2 - 1)*sin(2*sqrt(-a*x + 1)/sq
rt(a*x + 1))^2), x) + log(a*x + 1) - log(a*x - 1))/a
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$-\int \frac{\sin\left(\frac{\sqrt{1-a\,x}}{\sqrt{a\,x+1}}\right)^2}{a^2\,x^2-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-sin((1 - a*x)^(1/2)/(a*x + 1)^(1/2))^2/(a^2*x^2 - 1),x)
```

```
[Out] -int(sin((1 - a*x)^(1/2)/(a*x + 1)^(1/2))^2/(a^2*x^2 - 1), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{\sin^2\left(\frac{\sqrt{-ax+1}}{\sqrt{ax+1}}\right)}{a^2x^2-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin((-a*x+1)**(1/2)/(a*x+1)**(1/2))**2/(-a**2*x**2+1),x)
[Out] -Integral(sin(sqrt(-a*x + 1)/sqrt(a*x + 1))**2/(a**2*x**2 - 1), x)
```

3.41 $\int \frac{\sin\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{1-a^2x^2}\,dx$

Optimal. Leaf size=26

$$-\frac{\mathrm{Si}\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{a}$$

```
[Out] -Si((-a*x+1)^(1/2)/(a*x+1)^(1/2))/a
```

Rubi [A] time = 0.04, antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {6681, 3299}

$$-\frac{\mathrm{Si}\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[Sqrt[1 - a*x]/Sqrt[1 + a*x]]/(1 - a^2*x^2),x]
```

```
[Out] -(SinIntegral[Sqrt[1 - a*x]/Sqrt[1 + a*x]]/a)
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 6681

```
Int[((a_.) + (b_.)*(F_)[((c_.)*Sqrt[(d_.) + (e_.)*(x_)])/Sqrt[(f_.) + (g_.)
*(x_)]])^(n_.)/((A_.) + (C_.)*(x_)^2), x_Symbol] :> Dist[(2*e*g)/(C*(e*f -
d*g)), Subst[Int[(a + b*F[c*x])^n/x, x], x, Sqrt[d + e*x]/Sqrt[f + g*x]], x
] /; FreeQ[{a, b, c, d, e, f, g, A, C, F}, x] && EqQ[C*d*f - A*e*g, 0] && E
qQ[e*f + d*g, 0] && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\sin\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{1-a^2x^2}\,dx &= -\frac{\mathrm{Subst}\left(\int \frac{\sin(x)}{x}\,dx, x, \frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{a}\\ &= -\frac{\mathrm{Si}\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{a}\end{aligned}$$

Mathematica [A] time = 0.05, size = 26, normalized size = 1.00

$$-\frac{\text{Si}\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{a}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[Sqrt[1 - a*x]/Sqrt[1 + a*x]]/(1 - a^2*x^2),x]
```

```
[Out] -(SinIntegral[Sqrt[1 - a*x]/Sqrt[1 + a*x]]/a)
```

fricas [F] time = 0.95, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\sin\left(\frac{\sqrt{-ax+1}}{\sqrt{ax+1}}\right)}{a^2x^2-1}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin((-a*x+1)^(1/2)/(a*x+1)^(1/2))/(-a^2*x^2+1),x, algorithm="fric
as")
```

```
[Out] integral(-sin(sqrt(-a*x + 1)/sqrt(a*x + 1))/(a^2*x^2 - 1), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{\sin\left(\frac{\sqrt{-ax+1}}{\sqrt{ax+1}}\right)}{a^2x^2-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin((-a*x+1)^(1/2)/(a*x+1)^(1/2))/(-a^2*x^2+1),x, algorithm="giac
")
```

```
[Out] integrate(-sin(sqrt(-a*x + 1)/sqrt(a*x + 1))/(a^2*x^2 - 1), x)
```

maple [F] time = 0.07, size = 0, normalized size = 0.00

$$\int \frac{\sin\left(\frac{\sqrt{-ax+1}}{\sqrt{ax+1}}\right)}{-a^2x^2+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin((-a*x+1)^(1/2)/(a*x+1)^(1/2))/(-a^2*x^2+1),x)
```

```
[Out] int(sin((-a*x+1)^(1/2)/(a*x+1)^(1/2))/(-a^2*x^2+1),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{\sin\left(\frac{\sqrt{-ax+1}}{\sqrt{ax+1}}\right)}{a^2x^2-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin((-a*x+1)^(1/2)/(a*x+1)^(1/2))/(-a^2*x^2+1),x, algorithm="maxi
ma")
[Out] -integrate(sin(sqrt(-a*x + 1)/sqrt(a*x + 1))/(a^2*x^2 - 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.04

$$-\int \frac{\sin\left(\frac{\sqrt{1-a\,x}}{\sqrt{a\,x+1}}\right)}{a^2\,x^2-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-sin((1 - a*x)^(1/2)/(a*x + 1)^(1/2))/(a^2*x^2 - 1),x)
[Out] -int(sin((1 - a*x)^(1/2)/(a*x + 1)^(1/2))/(a^2*x^2 - 1), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{\sin\left(\frac{\sqrt{-ax+1}}{\sqrt{ax+1}}\right)}{a^2x^2-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin((-a*x+1)**(1/2)/(a*x+1)**(1/2))/(-a**2*x**2+1),x)
[Out] -Integral(sin(sqrt(-a*x + 1)/sqrt(a*x + 1))/(a**2*x**2 - 1), x)
```

3.42 $\int \frac{\csc\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{1-a^2x^2}\,dx$

Optimal. Leaf size=40

$$\text{Int}\left(\frac{\csc\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{(1-ax)(ax+1)},x\right)$$

```
[Out] Unintegrable(csc((-a*x+1)^(1/2)/(a*x+1)^(1/2))/(-a*x+1)/(a*x+1),x)
```

Rubi [A] time = 0.04, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\csc\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{1-a^2x^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Csc[Sqrt[1 - a*x]/Sqrt[1 + a*x]]/(1 - a^2*x^2),x]
```

```
[Out] -(Defer[Subst][Defer[Int][Csc[x]/x, x], x, Sqrt[1 - a*x]/Sqrt[1 + a*x]]/a)
```

Rubi steps

$$\int \frac{\csc\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{1-a^2x^2}\,dx = -\frac{\text{Subst}\left(\int \frac{\csc(x)}{x}\,dx,x,\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{a}$$

Mathematica [A] time = 5.91, size = 0, normalized size = 0.00

$$\int \frac{\csc\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{1-a^2x^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Csc[Sqrt[1 - a*x]/Sqrt[1 + a*x]]/(1 - a^2*x^2),x]
```

```
[Out] Integrate[Csc[Sqrt[1 - a*x]/Sqrt[1 + a*x]]/(1 - a^2*x^2), x]
```

fricas [A] time = 1.15, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{1}{\left(a^2x^2-1\right)\sin\left(\frac{\sqrt{-ax+1}}{\sqrt{ax+1}}\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a^2*x^2+1)/sin((-a*x+1)^(1/2)/(a*x+1)^(1/2)),x, algorithm="fr
icas")
```

```
[Out] integral(-1/((a^2*x^2 - 1)*sin(sqrt(-a*x + 1)/sqrt(a*x + 1))), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{1}{\left(a^2x^2-1\right)\sin\left(\frac{\sqrt{-ax+1}}{\sqrt{ax+1}}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a^2*x^2+1)/sin((-a*x+1)^(1/2)/(a*x+1)^(1/2)),x, algorithm="gi
ac")
```

```
[Out] integrate(-1/((a^2*x^2 - 1)*sin(sqrt(-a*x + 1)/sqrt(a*x + 1))), x)
```

maple [A] time = 0.18, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-a^2x^2+1\right)\sin\left(\frac{\sqrt{-ax+1}}{\sqrt{ax+1}}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-a^2*x^2+1)/sin((-a*x+1)^(1/2)/(a*x+1)^(1/2)),x)
```

```
[Out] int(1/(-a^2*x^2+1)/sin((-a*x+1)^(1/2)/(a*x+1)^(1/2)),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{1}{\left(a^2x^2-1\right)\sin\left(\frac{\sqrt{-ax+1}}{\sqrt{ax+1}}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a^2*x^2+1)/sin((-a*x+1)^(1/2)/(a*x+1)^(1/2)),x, algorithm="ma
xima")
```

```
[Out] -integrate(1/((a^2*x^2 - 1)*sin(sqrt(-a*x + 1)/sqrt(a*x + 1))), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.02

$$-\int \frac{1}{\sin\left(\frac{\sqrt{1-a\,x}}{\sqrt{a\,x+1}}\right)\,\left(a^2\,x^2-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-1/(sin((1 - a*x)^(1/2)/(a*x + 1)^(1/2))*(a^2*x^2 - 1)),x)
[Out] -int(1/(sin((1 - a*x)^(1/2)/(a*x + 1)^(1/2))*(a^2*x^2 - 1)), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{1}{a^2x^2\sin\left(\frac{\sqrt{-ax+1}}{\sqrt{ax+1}}\right)-\sin\left(\frac{\sqrt{-ax+1}}{\sqrt{ax+1}}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a**2*x**2+1)/sin((-a*x+1)**(1/2)/(a*x+1)**(1/2)),x)
[Out] -Integral(1/(a**2*x**2*sin(sqrt(-a*x + 1)/sqrt(a*x + 1)) - sin(sqrt(-a*x +
1)/sqrt(a*x + 1))), x)
```

3.43 $\int \frac{\csc^2\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{1-a^2x^2}\,dx$

Optimal. Leaf size=42

$$\operatorname{Int}\left(\frac{\csc^2\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{(1-ax)(ax+1)},x\right)$$

```
[Out] Unintegrable(csc((-a*x+1)^(1/2)/(a*x+1)^(1/2))^2/(-a*x+1)/(a*x+1),x)
```

Rubi [A] time = 0.08, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\csc^2\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{1-a^2x^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Csc[Sqrt[1 - a*x]/Sqrt[1 + a*x]]^2/(1 - a^2*x^2),x]
```

```
[Out] -(Defer[Subst][Defer[Int][Csc[x]^2/x, x], x, Sqrt[1 - a*x]/Sqrt[1 + a*x]]/a
)
```

Rubi steps

$$\int \frac{\csc^2\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{1-a^2x^2}\,dx = -\frac{\operatorname{Subst}\left(\int \frac{\csc^2(x)}{x}\,dx, x, \frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{a}$$

Mathematica [A] time = 19.89, size = 0, normalized size = 0.00

$$\int \frac{\csc^2\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{1-a^2x^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Csc[Sqrt[1 - a*x]/Sqrt[1 + a*x]]^2/(1 - a^2*x^2),x]
```

```
[Out] Integrate[Csc[Sqrt[1 - a*x]/Sqrt[1 + a*x]]^2/(1 - a^2*x^2), x]
```

fricas [A] time = 1.17, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{1}{a^2x^2-\left(a^2x^2-1\right)\cos\left(\frac{\sqrt{-ax+1}}{\sqrt{ax+1}}\right)^2-1},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a^2*x^2+1)/sin((-a*x+1)^(1/2)/(a*x+1)^(1/2))^2,x, algorithm="
fricas")
```

```
[Out] integral(-1/(a^2*x^2 - (a^2*x^2 - 1)*cos(sqrt(-a*x + 1)/sqrt(a*x + 1))^2 -
1), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{1}{\left(a^2x^2-1\right)\sin\left(\frac{\sqrt{-ax+1}}{\sqrt{ax+1}}\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a^2*x^2+1)/sin((-a*x+1)^(1/2)/(a*x+1)^(1/2))^2,x, algorithm="
giac")
```

```
[Out] integrate(-1/((a^2*x^2 - 1)*sin(sqrt(-a*x + 1)/sqrt(a*x + 1))^2), x)
```

maple [A] time = 0.24, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-a^2x^2+1\right)\sin\left(\frac{\sqrt{-ax+1}}{\sqrt{ax+1}}\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-a^2*x^2+1)/sin((-a*x+1)^(1/2)/(a*x+1)^(1/2))^2,x)
```

```
[Out] int(1/(-a^2*x^2+1)/sin((-a*x+1)^(1/2)/(a*x+1)^(1/2))^2,x)
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a^2*x^2+1)/sin((-a*x+1)^(1/2)/(a*x+1)^(1/2))^2,x, algorithm="
maxima")
```

```
[Out] Timed out
```

mupad [A] time = 0.00, size = -1, normalized size = -0.02

$$-\int \frac{1}{\sin\left(\frac{\sqrt{1-a\,x}}{\sqrt{a\,x+1}}\right)^2\,\left(a^2\,x^2-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-1/(sin((1 - a*x)^(1/2)/(a*x + 1)^(1/2))^2*(a^2*x^2 - 1)),x)
[Out] -int(1/(sin((1 - a*x)^(1/2)/(a*x + 1)^(1/2))^2*(a^2*x^2 - 1)), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{1}{a^2x^2\sin^2\left(\frac{\sqrt{-ax+1}}{\sqrt{ax+1}}\right)-\sin^2\left(\frac{\sqrt{-ax+1}}{\sqrt{ax+1}}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a**2*x**2+1)/sin((-a*x+1)**(1/2)/(a*x+1)**(1/2))**2,x)
[Out] -Integral(1/(a**2*x**2*sin(sqrt(-a*x + 1)/sqrt(a*x + 1))**2 - sin(sqrt(-a*x
 + 1)/sqrt(a*x + 1))**2), x)
```

3.44 $\int (x+\cos(x))^2\,dx$

Optimal. Leaf size=30

$$\frac{x^3}{3}+\frac{x}{2}+2x\sin(x)+2\cos(x)+\frac{1}{2}\sin(x)\cos(x)$$

```
[Out] 1/2*x+1/3*x^3+2*cos(x)+2*x*sin(x)+1/2*cos(x)*sin(x)
```

Rubi [A] time = 0.03, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.833, Rules used = {6742, 3296, 2638, 2635, 8}

$$\frac{x^3}{3}+\frac{x}{2}+2x\sin(x)+2\cos(x)+\frac{1}{2}\sin(x)\cos(x)$$

Antiderivative was successfully verified.

```
[In] Int[(x + Cos[x])^2,x]
```

```
[Out] x/2 + x^3/3 + 2*Cos[x] + 2*x*Sin[x] + (Cos[x]*Sin[x])/2
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\begin{aligned}\int (x+\cos(x))^2\,dx &= \int \left(x^2+2x\cos(x)+\cos^2(x)\right)\,dx\\ &= \frac{x^3}{3}+2\int x\cos(x)\,dx+\int \cos^2(x)\,dx\\ &= \frac{x^3}{3}+2x\sin(x)+\frac{1}{2}\cos(x)\sin(x)+\frac{\int 1\,dx}{2}-2\int \sin(x)\,dx\\ &= \frac{x}{2}+\frac{x^3}{3}+2\cos(x)+2x\sin(x)+\frac{1}{2}\cos(x)\sin(x)\end{aligned}$$

Mathematica [A] time = 0.08, size = 26, normalized size = 0.87

$$\frac{1}{6}\left(x\left(2x^2+12\sin(x)+3\right)+3(\sin(x)+4)\cos(x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(x + Cos[x])^2,x]
[Out] (3*Cos[x]*(4 + Sin[x]) + x*(3 + 2*x^2 + 12*Sin[x]))/6
```

fricas [A] time = 2.51, size = 23, normalized size = 0.77

$$\frac{1}{3}\,x^3+\frac{1}{2}\,(4\,x+\cos(x))\sin(x)+\frac{1}{2}\,x+2\,\cos(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+cos(x))^2,x, algorithm="fricas")
[Out] 1/3*x^3 + 1/2*(4*x + cos(x))*sin(x) + 1/2*x + 2*cos(x)
```

giac [A] time = 0.14, size = 24, normalized size = 0.80

$$\frac{1}{3}\,x^3+2\,x\sin(x)+\frac{1}{2}\,x+2\,\cos(x)+\frac{1}{4}\,\sin(2\,x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+cos(x))^2,x, algorithm="giac")
```

[Out] 1/3*x^3 + 2*x*sin(x) + 1/2*x + 2*cos(x) + 1/4*sin(2*x)

maple [A] time = 0.04, size = 25, normalized size = 0.83

$$\frac{x}{2}+\frac{x^3}{3}+2\cos(x)+2x\sin(x)+\frac{\cos(x)\sin(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x+cos(x))^2,x)

[Out] 1/2*x+1/3*x^3+2*cos(x)+2*x*sin(x)+1/2*cos(x)*sin(x)

maxima [A] time = 0.31, size = 24, normalized size = 0.80

$$\frac{1}{3}x^3+2x\sin(x)+\frac{1}{2}x+2\cos(x)+\frac{1}{4}\sin(2x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x+cos(x))^2,x, algorithm="maxima")

[Out] 1/3*x^3 + 2*x*sin(x) + 1/2*x + 2*cos(x) + 1/4*sin(2*x)

mupad [B] time = 0.05, size = 24, normalized size = 0.80

$$\frac{x}{2}+2\cos(x)+\frac{\cos(x)\sin(x)}{2}+2x\sin(x)+\frac{x^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x + cos(x))^2,x)

[Out] x/2 + 2*cos(x) + (cos(x)*sin(x))/2 + 2*x*sin(x) + x^3/3

sympy [A] time = 0.16, size = 41, normalized size = 1.37

$$\frac{x^3}{3}+\frac{x\sin^2(x)}{2}+2x\sin(x)+\frac{x\cos^2(x)}{2}+\frac{\sin(x)\cos(x)}{2}+2\cos(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x+cos(x))**2,x)

[Out] x**3/3 + x*sin(x)**2/2 + 2*x*sin(x) + x*cos(x)**2/2 + sin(x)*cos(x)/2 + 2*c
os(x)

3.45 $\int (x+\cos(x))^3\,dx$

Optimal. Leaf size=56

$$\frac{x^4}{4}+\frac{3x^2}{4}+3x^2\sin(x)-\frac{\sin^3(x)}{3}-5\sin(x)+\frac{3\cos^2(x)}{4}+6x\cos(x)+\frac{3}{2}x\sin(x)\cos(x)$$

```
[Out] 3/4*x^2+1/4*x^4+6*x*cos(x)+3/4*cos(x)^2-5*sin(x)+3*x^2*sin(x)+3/2*x*cos(x)*
sin(x)-1/3*sin(x)^3
```

Rubi [A] time = 0.07, antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.000, Rules used = {6742, 3296, 2637, 3310, 30, 2633}

$$\frac{x^4}{4}+\frac{3x^2}{4}+3x^2\sin(x)-\frac{\sin^3(x)}{3}-5\sin(x)+\frac{3\cos^2(x)}{4}+6x\cos(x)+\frac{3}{2}x\sin(x)\cos(x)$$

Antiderivative was successfully verified.

```
[In] Int[(x + Cos[x])^3,x]
```

```
[Out] (3*x^2)/4 + x^4/4 + 6*x*Cos[x] + (3*Cos[x]^2)/4 - 5*Sin[x] + 3*x^2*Sin[x] +
 (3*x*Cos[x]*Sin[x])/2 - Sin[x]^3/3
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3310

```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[(d*(b*Sin[e + f*x])^n)/(f^2*n^2), x] + (Dist[(b^2*(n - 1))/n, Int[(c
+ d*x)*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[(b*(c + d*x)*Cos[e + f*x]*(b
*Sin[e + f*x])^(n - 1))/(f*n), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1
]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\begin{aligned}\int (x+\cos(x))^3\,dx &= \int \left(x^3+3x^2\cos(x)+3x\cos^2(x)+\cos^3(x)\right)\,dx\\ &= \frac{x^4}{4}+3\int x^2\cos(x)\,dx+3\int x\cos^2(x)\,dx+\int \cos^3(x)\,dx\\ &= \frac{x^4}{4}+\frac{3\cos^2(x)}{4}+3x^2\sin(x)+\frac{3}{2}x\cos(x)\sin(x)+\frac{3\int x\,dx}{2}-6\int x\sin(x)\,dx-\text{Subst}\left(\int (1\right.\\ &= \frac{3x^2}{4}+\frac{x^4}{4}+6x\cos(x)+\frac{3\cos^2(x)}{4}+\sin(x)+3x^2\sin(x)+\frac{3}{2}x\cos(x)\sin(x)-\frac{\sin^3(x)}{3}-6\int\\ &= \frac{3x^2}{4}+\frac{x^4}{4}+6x\cos(x)+\frac{3\cos^2(x)}{4}-5\sin(x)+3x^2\sin(x)+\frac{3}{2}x\cos(x)\sin(x)-\frac{\sin^3(x)}{3}\end{aligned}$$

Mathematica [A] time = 0.12, size = 51, normalized size = 0.91

$$\frac{1}{12}\left(3x^4+9x^2+9\left(4x^2-7\right)\sin(x)+9x\sin(2x)+\sin(3x)\right)+6x\cos(x)+\frac{3}{8}\cos(2x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(x + Cos[x])^3,x]
```

```
[Out] 6*x*Cos[x] + (3*Cos[2*x])/8 + (9*x^2 + 3*x^4 + 9*(-7 + 4*x^2)*Sin[x] + 9*x*
Sin[2*x] + Sin[3*x])/12
```

fricas [A] time = 0.85, size = 44, normalized size = 0.79

$$\frac{1}{4}\,x^4+\frac{3}{4}\,x^2+6\,x\cos(x)+\frac{3}{4}\,\cos(x)^2+\frac{1}{6}\left(18\,x^2+9\,x\cos(x)+2\,\cos(x)^2-32\right)\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+cos(x))^3,x, algorithm="fricas")
```

```
[Out] 1/4*x^4 + 3/4*x^2 + 6*x*cos(x) + 3/4*cos(x)^2 + 1/6*(18*x^2 + 9*x*cos(x) +
2*cos(x)^2 - 32)*sin(x)
```

giac [A] time = 0.15, size = 46, normalized size = 0.82

$$\frac{1}{4}x^4 + \frac{3}{4}x^2 + 6x\cos(x) + \frac{3}{4}x\sin(2x) + \frac{3}{4}\left(4x^2 - 7\right)\sin(x) + \frac{3}{8}\cos(2x) + \frac{1}{12}\sin(3x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+cos(x))^3,x, algorithm="giac")
```

```
[Out] 1/4*x^4 + 3/4*x^2 + 6*x*cos(x) + 3/4*x*sin(2*x) + 3/4*(4*x^2 - 7)*sin(x) +
3/8*cos(2*x) + 1/12*sin(3*x)
```

maple [A] time = 0.03, size = 57, normalized size = 1.02

$$\frac{\left(2+\cos^2(x)\right)\sin(x)}{3}+3x\left(\frac{\cos(x)\sin(x)}{2}+\frac{x}{2}\right)-\frac{3x^2}{4}-\frac{3\left(\sin^2(x)\right)}{4}+3x^2\sin(x)-6\sin(x)+6x\cos(x)+\frac{x^4}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x+cos(x))^3,x)
```

```
[Out] 1/3*(2+cos(x)^2)*sin(x)+3*x*(1/2*cos(x)*sin(x)+1/2*x)-3/4*x^2-3/4*sin(x)^2+
3*x^2*sin(x)-6*sin(x)+6*x*cos(x)+1/4*x^4
```

maxima [A] time = 0.31, size = 46, normalized size = 0.82

$$\frac{1}{4}x^4 - \frac{1}{3}\sin(x)^3 + \frac{3}{4}x^2 + 6x\cos(x) + \frac{3}{4}x\sin(2x) + 3\left(x^2 - 2\right)\sin(x) + \frac{3}{8}\cos(2x) + \sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+cos(x))^3,x, algorithm="maxima")
```

```
[Out] 1/4*x^4 - 1/3*sin(x)^3 + 3/4*x^2 + 6*x*cos(x) + 3/4*x*sin(2*x) + 3*(x^2 - 2
)*sin(x) + 3/8*cos(2*x) + sin(x)
```

mupad [B] time = 0.08, size = 48, normalized size = 0.86

$$3x^2\sin(x)-\frac{16\sin(x)}{3}+\frac{3\cos(x)^2}{4}+\frac{\cos(x)^2\sin(x)}{3}+6x\cos(x)+\frac{3x^2}{4}+\frac{x^4}{4}+\frac{3x\cos(x)\sin(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + cos(x))^3,x)
```

```
[Out] 3*x^2*sin(x) - (16*sin(x))/3 + (3*cos(x)^2)/4 + (cos(x)^2*sin(x))/3 + 6*x*c
os(x) + (3*x^2)/4 + x^4/4 + (3*x*cos(x)*sin(x))/2
```

sympy [A] time = 0.31, size = 85, normalized size = 1.52

$$\frac{x^4}{4}+\frac{3x^2\sin^2(x)}{4}+3x^2\sin(x)+\frac{3x^2\cos^2(x)}{4}+\frac{3x\sin(x)\cos(x)}{2}+6x\cos(x)+\frac{2\sin^3(x)}{3}-\frac{3\sin^2(x)}{4}+\sin(x)\cos^2(x)-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+cos(x))**3,x)
```

```
[Out] x**4/4 + 3*x**2*sin(x)**2/4 + 3*x**2*sin(x) + 3*x**2*cos(x)**2/4 + 3*x*sin(
x)*cos(x)/2 + 6*x*cos(x) + 2*sin(x)**3/3 - 3*sin(x)**2/4 + sin(x)*cos(x)**2
 - 6*sin(x)
```

3.46 $\int \frac{\cos(a+bx)}{c+dx^2}\,dx$

Optimal. Leaf size=213

$$\frac{\cos\left(a+\frac{b\sqrt{-c}}{\sqrt{d}}\right)\operatorname{Ci}\left(\frac{b\sqrt{-c}}{\sqrt{d}}-bx\right)}{2\sqrt{-c}\sqrt{d}}-\frac{\cos\left(a-\frac{b\sqrt{-c}}{\sqrt{d}}\right)\operatorname{Ci}\left(xb+\frac{\sqrt{-c}\,b}{\sqrt{d}}\right)}{2\sqrt{-c}\sqrt{d}}+\frac{\sin\left(a+\frac{b\sqrt{-c}}{\sqrt{d}}\right)\operatorname{Si}\left(\frac{b\sqrt{-c}}{\sqrt{d}}-bx\right)}{2\sqrt{-c}\sqrt{d}}+\frac{\sin\left(a-\frac{b\sqrt{-c}}{\sqrt{d}}\right.}{2\sqrt{-}}$$

```
[Out] -1/2*Ci(b*x+b*(-c)^(1/2)/d^(1/2))*cos(a-b*(-c)^(1/2)/d^(1/2))/(-c)^(1/2)/d^
(1/2)+1/2*Ci(-b*x+b*(-c)^(1/2)/d^(1/2))*cos(a+b*(-c)^(1/2)/d^(1/2))/(-c)^(1
/2)/d^(1/2)+1/2*Si(b*x+b*(-c)^(1/2)/d^(1/2))*sin(a-b*(-c)^(1/2)/d^(1/2))/(-
c)^(1/2)/d^(1/2)-1/2*Si(b*x-b*(-c)^(1/2)/d^(1/2))*sin(a+b*(-c)^(1/2)/d^(1/2
))/(-c)^(1/2)/d^(1/2)
```

Rubi [**A**] time = 0.31, antiderivative size = 213, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {3334, 3303, 3299, 3302}

$$\frac{\cos\left(a+\frac{b\sqrt{-c}}{\sqrt{d}}\right)\text{CosIntegral}\left(\frac{b\sqrt{-c}}{\sqrt{d}}-bx\right)}{2\sqrt{-c}\sqrt{d}}-\frac{\cos\left(a-\frac{b\sqrt{-c}}{\sqrt{d}}\right)\text{CosIntegral}\left(\frac{b\sqrt{-c}}{\sqrt{d}}+bx\right)}{2\sqrt{-c}\sqrt{d}}+\frac{\sin\left(a+\frac{b\sqrt{-c}}{\sqrt{d}}\right)\operatorname{Si}\left(\frac{b\sqrt{-c}}{\sqrt{d}}-\right.}{2\sqrt{-c}\sqrt{d}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x]/(c + d*x^2),x]
```

```
[Out] (Cos[a + (b*Sqrt[-c])/Sqrt[d]]*CosIntegral[(b*Sqrt[-c])/Sqrt[d] - b*x])/(2*
Sqrt[-c]*Sqrt[d]) - (Cos[a - (b*Sqrt[-c])/Sqrt[d]]*CosIntegral[(b*Sqrt[-c])
/Sqrt[d] + b*x])/(2*Sqrt[-c]*Sqrt[d]) + (Sin[a + (b*Sqrt[-c])/Sqrt[d]]*SinI
ntegral[(b*Sqrt[-c])/Sqrt[d] - b*x])/(2*Sqrt[-c]*Sqrt[d]) + (Sin[a - (b*Sqr
t[-c])/Sqrt[d]]*SinIntegral[(b*Sqrt[-c])/Sqrt[d] + b*x])/(2*Sqrt[-c]*Sqrt[d
])
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3302

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3334

```
Int[Cos[(c_.) + (d_.)*(x_)]*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int
[ExpandIntegrand[Cos[c + d*x], (a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d},
 x] && ILtQ[p, 0] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -1])
```

Rubi steps

$$\int \frac{\cos(a+bx)}{c+dx^2}\,dx = \int \left(\frac{\sqrt{-c}\,\cos(a+bx)}{2c\left(\sqrt{-c}-\sqrt{d}\,x\right)}+\frac{\sqrt{-c}\,\cos(a+bx)}{2c\left(\sqrt{-c}+\sqrt{d}\,x\right)}\right)dx$$

$$= -\frac{\int \frac{\cos(a+bx)}{\sqrt{-c}-\sqrt{d}\,x}\,dx}{2\sqrt{-c}}-\frac{\int \frac{\cos(a+bx)}{\sqrt{-c}+\sqrt{d}\,x}\,dx}{2\sqrt{-c}}$$

$$= -\frac{\cos\left(a-\frac{b\sqrt{-c}}{\sqrt{d}}\right)\int \frac{\cos\left(\frac{b\sqrt{-c}}{\sqrt{d}}+bx\right)}{\sqrt{-c}+\sqrt{d}\,x}\,dx}{2\sqrt{-c}}-\frac{\cos\left(a+\frac{b\sqrt{-c}}{\sqrt{d}}\right)\int \frac{\cos\left(\frac{b\sqrt{-c}}{\sqrt{d}}-bx\right)}{\sqrt{-c}-\sqrt{d}\,x}\,dx}{2\sqrt{-c}}+\frac{\sin\left(a-\frac{b\sqrt{-c}}{\sqrt{d}}\right)\int \frac{\sin}{}}{2\sqrt{-c}}$$

$$= \frac{\cos\left(a+\frac{b\sqrt{-c}}{\sqrt{d}}\right)\mathrm{Ci}\left(\frac{b\sqrt{-c}}{\sqrt{d}}-bx\right)}{2\sqrt{-c}\,\sqrt{d}}-\frac{\cos\left(a-\frac{b\sqrt{-c}}{\sqrt{d}}\right)\mathrm{Ci}\left(\frac{b\sqrt{-c}}{\sqrt{d}}+bx\right)}{2\sqrt{-c}\,\sqrt{d}}+\frac{\sin\left(a+\frac{b\sqrt{-c}}{\sqrt{d}}\right)\mathrm{Si}\left(\frac{b\sqrt{-c}}{\sqrt{d}}\right.}{2\sqrt{-c}\,\sqrt{d}}$$

Mathematica [C] time = 0.34, size = 172, normalized size = 0.81

$$-\frac{i\left(\cos\left(a+\frac{ib\sqrt{c}}{\sqrt{d}}\right)\mathrm{Ci}\left(b\left(x-\frac{i\sqrt{c}}{\sqrt{d}}\right)\right)-\cos\left(a-\frac{ib\sqrt{c}}{\sqrt{d}}\right)\mathrm{Ci}\left(b\left(x+\frac{i\sqrt{c}}{\sqrt{d}}\right)\right)+\sin\left(a-\frac{ib\sqrt{c}}{\sqrt{d}}\right)\mathrm{Si}\left(b\left(x+\frac{i\sqrt{c}}{\sqrt{d}}\right)\right)+\sin\left(a+\right.\right.}{2\sqrt{c}\,\sqrt{d}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[a + b*x]/(c + d*x^2),x]
```

```
[Out] ((-1/2*I)*(Cos[a + (I*b*Sqrt[c])/Sqrt[d]]*CosIntegral[b*(((-I)*Sqrt[c])/Sqr
t[d] + x)] - Cos[a - (I*b*Sqrt[c])/Sqrt[d]]*CosIntegral[b*((I*Sqrt[c])/Sqrt
[d] + x)] + Sin[a - (I*b*Sqrt[c])/Sqrt[d]]*SinIntegral[b*((I*Sqrt[c])/Sqrt[
d] + x)] + Sin[a + (I*b*Sqrt[c])/Sqrt[d]]*SinIntegral[(I*b*Sqrt[c])/Sqrt[d]
 - b*x]))/(Sqrt[c]*Sqrt[d])
```

fricas [C] time = 1.12, size = 189, normalized size = 0.89

$$\frac{2i\sqrt{\frac{b^2c}{d}}\,\mathrm{Ei}\left(i\,bx-\sqrt{\frac{b^2c}{d}}\right)e^{\left(i\,a+\sqrt{\frac{b^2c}{d}}\right)}-2i\sqrt{\frac{b^2c}{d}}\,\mathrm{Ei}\left(i\,bx+\sqrt{\frac{b^2c}{d}}\right)e^{\left(i\,a-\sqrt{\frac{b^2c}{d}}\right)}-2i\sqrt{\frac{b^2c}{d}}\,\mathrm{Ei}\left(-i\,bx-\sqrt{\frac{b^2c}{d}}\right)e^{\left(-i\,a+\sqrt{\frac{b^2}{d}}\right.}}{8\,bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)/(d*x^2+c),x, algorithm="fricas")
```

```
[Out] 1/8*(2*I*sqrt(b^2*c/d)*Ei(I*b*x - sqrt(b^2*c/d))*e^(I*a + sqrt(b^2*c/d)) -
2*I*sqrt(b^2*c/d)*Ei(I*b*x + sqrt(b^2*c/d))*e^(I*a - sqrt(b^2*c/d)) - 2*I*s
qrt(b^2*c/d)*Ei(-I*b*x - sqrt(b^2*c/d))*e^(-I*a + sqrt(b^2*c/d)) + 2*I*sqrt
(b^2*c/d)*Ei(-I*b*x + sqrt(b^2*c/d))*e^(-I*a - sqrt(b^2*c/d)))/(b*c)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos(bx+a)}{dx^2+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)/(d*x^2+c),x, algorithm="giac")
```

```
[Out] integrate(cos(b*x + a)/(d*x^2 + c), x)
```

maple [A] time = 0.05, size = 229, normalized size = 1.08

$$b\left(\frac{-\mathrm{Si}\left(bx+a-\frac{b\sqrt{-cd}+da}{d}\right)\sin\left(\frac{b\sqrt{-cd}+da}{d}\right)+\mathrm{Ci}\left(bx+a-\frac{b\sqrt{-cd}+da}{d}\right)\cos\left(\frac{b\sqrt{-cd}+da}{d}\right)}{2d\left(\frac{b\sqrt{-cd}+da}{d}-a\right)}+\frac{\mathrm{Si}\left(bx+a+\frac{b\sqrt{-cd}-da}{d}\right)\mathrm{si}}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)/(d*x^2+c),x)
```

```
[Out] b*(1/2/d/((b*(-c*d)^(1/2)+d*a)/d-a)*(-Si(b*x+a-(b*(-c*d)^(1/2)+d*a)/d)*sin(
(b*(-c*d)^(1/2)+d*a)/d)+Ci(b*x+a-(b*(-c*d)^(1/2)+d*a)/d)*cos((b*(-c*d)^(1/2
)+d*a)/d))+1/2/d/(-(b*(-c*d)^(1/2)-d*a)/d-a)*(Si(b*x+a+(b*(-c*d)^(1/2)-d*a)
/d)*sin((b*(-c*d)^(1/2)-d*a)/d)+Ci(b*x+a+(b*(-c*d)^(1/2)-d*a)/d)*cos((b*(-c
*d)^(1/2)-d*a)/d)))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos(bx+a)}{dx^2+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)/(d*x^2+c),x, algorithm="maxima")
```

```
[Out] integrate(cos(b*x + a)/(d*x^2 + c), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cos(a + b\,x)}{d\,x^2 + c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)/(c + d*x^2),x)
```

```
[Out] int(cos(a + b*x)/(c + d*x^2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos(a + bx)}{c + dx^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)/(d*x**2+c),x)
```

```
[Out] Integral(cos(a + b*x)/(c + d*x**2), x)
```

3.47 $\int \frac{\cos(a+bx)}{c+dx+ex^2}\,dx$

Optimal. Leaf size=271

$$\frac{\cos\left(a-\frac{b\left(d-\sqrt{d^2-4ce}\right)}{2e}\right)\text{Ci}\left(\frac{b\left(d-\sqrt{d^2-4ce}\right)}{2e}+bx\right)}{\sqrt{d^2-4ce}}-\frac{\cos\left(a-\frac{b\left(\sqrt{d^2-4ce}+d\right)}{2e}\right)\text{Ci}\left(\frac{b\left(d+\sqrt{d^2-4ce}\right)}{2e}+bx\right)}{\sqrt{d^2-4ce}}-\frac{\sin\left(a-\frac{b\left(d-\sqrt{d^2-4c\right.}}{2e}\right.}{\sqrt{}}$$

```
[Out] Ci(b*x+1/2*b*(d-(-4*c*e+d^2)^(1/2))/e)*cos(a-1/2*b*(d-(-4*c*e+d^2)^(1/2))/e
)/(-4*c*e+d^2)^(1/2)-Ci(b*x+1/2*b*(d+(-4*c*e+d^2)^(1/2))/e)*cos(a-1/2*b*(d+
(-4*c*e+d^2)^(1/2))/e)/(-4*c*e+d^2)^(1/2)-Si(b*x+1/2*b*(d-(-4*c*e+d^2)^(1/2
))/e)*sin(a-1/2*b*(d-(-4*c*e+d^2)^(1/2))/e)/(-4*c*e+d^2)^(1/2)+Si(b*x+1/2*b
*(d+(-4*c*e+d^2)^(1/2))/e)*sin(a-1/2*b*(d+(-4*c*e+d^2)^(1/2))/e)/(-4*c*e+d^
2)^(1/2)
```

Rubi [A] time = 0.56, antiderivative size = 271, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.210, Rules used = {6728, 3303, 3299, 3302}

$$\frac{\cos\left(a-\frac{b\left(d-\sqrt{d^2-4ce}\right)}{2e}\right)\text{CosIntegral}\left(\frac{b\left(d-\sqrt{d^2-4ce}\right)}{2e}+bx\right)}{\sqrt{d^2-4ce}}-\frac{\cos\left(a-\frac{b\left(\sqrt{d^2-4ce}+d\right)}{2e}\right)\text{CosIntegral}\left(\frac{b\left(\sqrt{d^2-4ce}+d\right)}{2e}+bx\right.}{\sqrt{d^2-4ce}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x]/(c + d*x + e*x^2),x]
```

```
[Out] (Cos[a - (b*(d - Sqrt[d^2 - 4*c*e]))/(2*e)]*CosIntegral[(b*(d - Sqrt[d^2 -
4*c*e]))/(2*e) + b*x])/Sqrt[d^2 - 4*c*e] - (Cos[a - (b*(d + Sqrt[d^2 - 4*c*
e]))/(2*e)]*CosIntegral[(b*(d + Sqrt[d^2 - 4*c*e]))/(2*e) + b*x])/Sqrt[d^2
- 4*c*e] - (Sin[a - (b*(d - Sqrt[d^2 - 4*c*e]))/(2*e)]*SinIntegral[(b*(d -
Sqrt[d^2 - 4*c*e]))/(2*e) + b*x])/Sqrt[d^2 - 4*c*e] + (Sin[a - (b*(d + Sqrt
[d^2 - 4*c*e]))/(2*e)]*SinIntegral[(b*(d + Sqrt[d^2 - 4*c*e]))/(2*e) + b*x]
)/Sqrt[d^2 - 4*c*e]
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3302

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
```

```
c*f, 0]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 6728

```
Int[(u_)/((a_.) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.)), x_Symbol] :> With[
{v = RationalFunctionExpand[u/(a + b*x^n + c*x^(2*n)), x]}, Int[v, x] /; Su
mQ[v]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{\cos(a+bx)}{c+dx+ex^2}\,dx = \int \left(\frac{2e\cos(a+bx)}{\sqrt{d^2-4ce}\left(d-\sqrt{d^2-4ce}+2ex\right)}-\frac{2e\cos(a+bx)}{\sqrt{d^2-4ce}\left(d+\sqrt{d^2-4ce}+2ex\right)}\right)dx$$

$$=\frac{(2e)\int \frac{\cos(a+bx)}{d-\sqrt{d^2-4ce}+2ex}\,dx}{\sqrt{d^2-4ce}}-\frac{(2e)\int \frac{\cos(a+bx)}{d+\sqrt{d^2-4ce}+2ex}\,dx}{\sqrt{d^2-4ce}}$$

$$=\frac{\left(2e\cos\left(a-\frac{b\left(d-\sqrt{d^2-4ce}\right)}{2e}\right)\right)\int \frac{\cos\left(\frac{b\left(d-\sqrt{d^2-4ce}\right)}{2e}+bx\right)}{d-\sqrt{d^2-4ce}+2ex}\,dx}{\sqrt{d^2-4ce}}-\frac{\left(2e\cos\left(a-\frac{b\left(d+\sqrt{d^2-4ce}\right)}{2e}\right)\right)\int \frac{\cos\left(\frac{b\left(d+\sqrt{d^2-4ce}\right)}{2e}\right.}{d+\sqrt{d^2-\right.}}{\sqrt{d^2-4ce}}$$

$$=\frac{\cos\left(a-\frac{b\left(d-\sqrt{d^2-4ce}\right)}{2e}\right)\text{Ci}\left(\frac{b\left(d-\sqrt{d^2-4ce}\right)}{2e}+bx\right)}{\sqrt{d^2-4ce}}-\frac{\cos\left(a-\frac{b\left(d+\sqrt{d^2-4ce}\right)}{2e}\right)\text{Ci}\left(\frac{b\left(d+\sqrt{d^2-4ce}\right)}{2e}+bx\right.}{\sqrt{d^2-4ce}}$$

Mathematica [A] time = 0.58, size = 236, normalized size = 0.87

$$\frac{\cos\left(a+\frac{b\left(\sqrt{d^2-4ce}-d\right)}{2e}\right)\text{Ci}\left(\frac{b\left(d+2ex-\sqrt{d^2-4ce}\right)}{2e}\right)-\cos\left(a-\frac{b\left(\sqrt{d^2-4ce}+d\right)}{2e}\right)\text{Ci}\left(\frac{b\left(d+2ex+\sqrt{d^2-4ce}\right)}{2e}\right)+\sin\left(a+\frac{b\left(\sqrt{d^2-4ce}-d\right)}{2e}\right.}{\sqrt{d^2-4ce}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[a + b*x]/(c + d*x + e*x^2),x]
```

```
[Out] (Cos[a + (b*(-d + Sqrt[d^2 - 4*c*e]))/(2*e)]*CosIntegral[(b*(d - Sqrt[d^2 -
 4*c*e] + 2*e*x))/(2*e)] - Cos[a - (b*(d + Sqrt[d^2 - 4*c*e]))/(2*e)]*CosIn
tegral[(b*(d + Sqrt[d^2 - 4*c*e] + 2*e*x))/(2*e)] + Sin[a + (b*(-d + Sqrt[d
^2 - 4*c*e]))/(2*e)]*SinIntegral[(b*(-d + Sqrt[d^2 - 4*c*e]))/(2*e) - b*x]
+ Sin[a - (b*(d + Sqrt[d^2 - 4*c*e]))/(2*e)]*SinIntegral[(b*(d + Sqrt[d^2 -
 4*c*e] + 2*e*x))/(2*e)])/Sqrt[d^2 - 4*c*e]
```

fricas [C] time = 1.04, size = 436, normalized size = 1.61

$$-\frac{-ie\sqrt{-\frac{b^2d^2-4\,b^2ce}{e^2}}\,\mathrm{Ei}\left(\frac{-2i\,bex-i\,bd-e\sqrt{-\frac{b^2d^2-4\,b^2ce}{e^2}}}{2\,e}\right)e^{\left(\frac{i\,bd-2i\,ae+e\sqrt{-\frac{b^2d^2-4\,b^2ce}{e^2}}}{2\,e}\right)}+i\,e\sqrt{-\frac{b^2d^2-4\,b^2ce}{e^2}}\,\mathrm{Ei}\left(\frac{-2i\,bex-i\,bd+e\sqrt{-\frac{b^2d^2-4\,b^2c}{e^2}}}{2\,e}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)/(e*x^2+d*x+c),x, algorithm="fricas")
```

```
[Out] -1/2*(-I*e*sqrt(-(b^2*d^2 - 4*b^2*c*e)/e^2)*Ei(1/2*(-2*I*b*e*x - I*b*d - e*
sqrt(-(b^2*d^2 - 4*b^2*c*e)/e^2))/e)*e^(1/2*(I*b*d - 2*I*a*e + e*sqrt(-(b^2
*d^2 - 4*b^2*c*e)/e^2))/e) + I*e*sqrt(-(b^2*d^2 - 4*b^2*c*e)/e^2)*Ei(1/2*(-
2*I*b*e*x - I*b*d + e*sqrt(-(b^2*d^2 - 4*b^2*c*e)/e^2))/e)*e^(1/2*(I*b*d -
2*I*a*e - e*sqrt(-(b^2*d^2 - 4*b^2*c*e)/e^2))/e) + I*e*sqrt(-(b^2*d^2 - 4*b
^2*c*e)/e^2)*Ei(1/2*(2*I*b*e*x + I*b*d - e*sqrt(-(b^2*d^2 - 4*b^2*c*e)/e^2)
)/e)*e^(1/2*(-I*b*d + 2*I*a*e + e*sqrt(-(b^2*d^2 - 4*b^2*c*e)/e^2))/e) - I*
e*sqrt(-(b^2*d^2 - 4*b^2*c*e)/e^2)*Ei(1/2*(2*I*b*e*x + I*b*d + e*sqrt(-(b^2
*d^2 - 4*b^2*c*e)/e^2))/e)*e^(1/2*(-I*b*d + 2*I*a*e - e*sqrt(-(b^2*d^2 - 4*
b^2*c*e)/e^2))/e))/(b*d^2 - 4*b*c*e)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos\left(bx+a\right)}{ex^2+dx+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)/(e*x^2+d*x+c),x, algorithm="giac")
```

```
[Out] integrate(cos(b*x + a)/(e*x^2 + d*x + c), x)
```

maple [A] time = 0.05, size = 320, normalized size = 1.18

$$b\left(\frac{-\operatorname{Si}\left(bx+a-\frac{2ae-db+\sqrt{-4b^2ce+b^2d^2}}{2e}\right)\sin\left(\frac{2ae-db+\sqrt{-4b^2ce+b^2d^2}}{2e}\right)+\operatorname{Ci}\left(bx+a-\frac{2ae-db+\sqrt{-4b^2ce+b^2d^2}}{2e}\right)\cos\left(\frac{2ae-db+\sqrt{-4}}{2e}\right.}{\sqrt{-4b^2ce+b^2d^2}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)/(e*x^2+d*x+c),x)
```

```
[Out] b*(1/(-4*b^2*c*e+b^2*d^2)^(1/2)*(-Si(b*x+a-1/2/e*(2*a*e-d*b+(-4*b^2*c*e+b^2
*d^2)^(1/2)))*sin(1/2/e*(2*a*e-d*b+(-4*b^2*c*e+b^2*d^2)^(1/2)))+Ci(b*x+a-1/
2/e*(2*a*e-d*b+(-4*b^2*c*e+b^2*d^2)^(1/2)))*cos(1/2/e*(2*a*e-d*b+(-4*b^2*c*
e+b^2*d^2)^(1/2))))-1/(-4*b^2*c*e+b^2*d^2)^(1/2)*(Si(b*x+a+1/2*(-2*a*e+d*b+
(-4*b^2*c*e+b^2*d^2)^(1/2))/e)*sin(1/2*(-2*a*e+d*b+(-4*b^2*c*e+b^2*d^2)^(1/
2))/e)+Ci(b*x+a+1/2*(-2*a*e+d*b+(-4*b^2*c*e+b^2*d^2)^(1/2))/e)*cos(1/2*(-2*
a*e+d*b+(-4*b^2*c*e+b^2*d^2)^(1/2))/e)))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos(bx+a)}{ex^2+dx+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)/(e*x^2+d*x+c),x, algorithm="maxima")
```

```
[Out] integrate(cos(b*x + a)/(e*x^2 + d*x + c), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cos(a+b\,x)}{e\,x^2+d\,x+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)/(c + d*x + e*x^2),x)
```

```
[Out] int(cos(a + b*x)/(c + d*x + e*x^2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos(a+bx)}{c+dx+ex^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)/(e*x**2+d*x+c),x)
```

```
[Out] Integral(cos(a + b*x)/(c + d*x + e*x**2), x)
```

3.48 $\int \frac{x\cos\left(\sqrt{1+x^2}\right)}{\sqrt{1+x^2}}\,dx$

Optimal. Leaf size=10

$$\sin\left(\sqrt{x^2+1}\right)$$

[Out] sin((x^2+1)^(1/2))

Rubi [A] time = 0.14, antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {6715, 3432, 15, 2637}

$$\sin\left(\sqrt{x^2+1}\right)$$

Antiderivative was successfully verified.

[In] Int[(x*Cos[Sqrt[1 + x^2]])/Sqrt[1 + x^2],x]

[Out] Sin[Sqrt[1 + x^2]]

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3432

```
Int[((a_.) + Cos[(c_.) + (d_.)*((e_.) + (f_.)*(x_))^(n_)]*(b_.))^(p_.)*((g_
.) + (h_.)*(x_))^(m_.), x_Symbol] :> Dist[1/(n*f), Subst[Int[ExpandIntegran
d[(a + b*Cos[c + d*x])^p, x^(1/n - 1)*(g - (e*h)/f + (h*x^(1/n))/f)^m, x],
x], x, (e + f*x)^n], x] /; FreeQ[{a, b, c, d, e, f, g, h, m}, x] && IGtQ[p,
 0] && IntegerQ[1/n]
```

Rule 6715

```
Int[(u_)*(x_)^(m_.), x_Symbol] :> Dist[1/(m + 1), Subst[Int[SubstFor[x^(m +
 1), u, x], x], x, x^(m + 1)], x] /; FreeQ[m, x] && NeQ[m, -1] && FunctionO
```

```
fQ[x^(m + 1), u, x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x\cos\left(\sqrt{1+x^2}\right)}{\sqrt{1+x^2}}\,dx &= \frac{1}{2}\,\text{Subst}\left(\int \frac{\cos\left(\sqrt{1+x}\right)}{\sqrt{1+x}}\,dx, x, x^2\right)\\
&= \text{Subst}\left(\int \frac{x\cos(x)}{\sqrt{x^2}}\,dx, x, \sqrt{1+x^2}\right)\\
&= 1\,\text{Subst}\left(\int \cos(x)\,dx, x, \sqrt{1+x^2}\right)\\
&= \sin\left(\sqrt{1+x^2}\right)
\end{aligned}
$$

Mathematica [A] time = 0.04, size = 10, normalized size = 1.00

$$\sin\left(\sqrt{x^2+1}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*Cos[Sqrt[1 + x^2]])/Sqrt[1 + x^2],x]
[Out] Sin[Sqrt[1 + x^2]]
```

fricas [A] time = 1.63, size = 8, normalized size = 0.80

$$\sin\left(\sqrt{x^2+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos((x^2+1)^(1/2))/(x^2+1)^(1/2),x, algorithm="fricas")
[Out] sin(sqrt(x^2 + 1))
```

giac [A] time = 0.14, size = 8, normalized size = 0.80

$$\sin\left(\sqrt{x^2+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos((x^2+1)^(1/2))/(x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] sin(sqrt(x^2 + 1))
```

maple [A] time = 0.04, size = 9, normalized size = 0.90

$$\sin\left(\sqrt{x^2+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*cos((x^2+1)^(1/2))/(x^2+1)^(1/2),x)
[Out] sin((x^2+1)^(1/2))
```

maxima [A] time = 0.35, size = 8, normalized size = 0.80

$$\sin\left(\sqrt{x^2+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos((x^2+1)^(1/2))/(x^2+1)^(1/2),x, algorithm="maxima")
[Out] sin(sqrt(x^2 + 1))
```

mupad [B] time = 2.26, size = 8, normalized size = 0.80

$$\sin\left(\sqrt{x^2+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*cos((x^2 + 1)^(1/2)))/(x^2 + 1)^(1/2),x)
[Out] sin((x^2 + 1)^(1/2))
```

sympy [A] time = 0.43, size = 8, normalized size = 0.80

$$\sin\left(\sqrt{x^2+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos((x**2+1)**(1/2))/(x**2+1)**(1/2),x)
[Out] sin(sqrt(x**2 + 1))
```

3.49 $\int \frac{x\cos\left(\sqrt{3}\sqrt{2+x^2}\right)}{\sqrt{2+x^2}}\,dx$

Optimal. Leaf size=22

$$\frac{\sin\left(\sqrt{3}\sqrt{x^2+2}\right)}{\sqrt{3}}$$

[Out] 1/3*sin(3^(1/2)*(x^2+2)^(1/2))*3^(1/2)

Rubi [A] time = 0.19, antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {6715, 3432, 15, 2637}

$$\frac{\sin\left(\sqrt{3}\sqrt{x^2+2}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

[In] Int[(x*Cos[Sqrt[3]*Sqrt[2 + x^2]])/Sqrt[2 + x^2],x]

[Out] Sin[Sqrt[3]*Sqrt[2 + x^2]]/Sqrt[3]

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3432

```
Int[((a_.) + Cos[(c_.) + (d_.)*((e_.) + (f_.)*(x_))^(n_)]*(b_.))^(p_.)*((g_
.) + (h_.)*(x_))^(m_.), x_Symbol] :> Dist[1/(n*f), Subst[Int[ExpandIntegran
d[(a + b*Cos[c + d*x])^p, x^(1/n - 1)*(g - (e*h)/f + (h*x^(1/n))/f)^m, x],
x], x, (e + f*x)^n], x] /; FreeQ[{a, b, c, d, e, f, g, h, m}, x] && IGtQ[p,
 0] && IntegerQ[1/n]
```

Rule 6715

```
Int[(u_)*(x_)^(m_.), x_Symbol] :> Dist[1/(m + 1), Subst[Int[SubstFor[x^(m +
 1), u, x], x], x, x^(m + 1)], x] /; FreeQ[m, x] && NeQ[m, -1] && FunctionO
fQ[x^(m + 1), u, x]
```

Rubi steps

$$\begin{aligned}\int \frac{x\cos\left(\sqrt{3}\sqrt{2+x^2}\right)}{\sqrt{2+x^2}}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{\cos\left(\sqrt{3}\sqrt{2+x}\right)}{\sqrt{2+x}}\,dx, x, x^2\right)\\ &= \operatorname{Subst}\left(\int \frac{x\cos\left(\sqrt{3}x\right)}{\sqrt{x^2}}\,dx, x, \sqrt{2+x^2}\right)\\ &= 1\operatorname{Subst}\left(\int \cos\left(\sqrt{3}x\right)\,dx, x, \sqrt{2+x^2}\right)\\ &= \frac{\sin\left(\sqrt{3}\sqrt{2+x^2}\right)}{\sqrt{3}}\end{aligned}$$

Mathematica [A] time = 0.06, size = 22, normalized size = 1.00

$$\frac{\sin\left(\sqrt{3}\sqrt{x^2+2}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*Cos[Sqrt[3]*Sqrt[2 + x^2]])/Sqrt[2 + x^2],x]
[Out] Sin[Sqrt[3]*Sqrt[2 + x^2]]/Sqrt[3]
```

fricas [B] time = 0.94, size = 37, normalized size = 1.68

$$\frac{2\sqrt{3}\tan\left(\frac{1}{2}\sqrt{3}\sqrt{x^2+2}\right)}{3\left(\tan\left(\frac{1}{2}\sqrt{3}\sqrt{x^2+2}\right)^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(3^(1/2)*(x^2+2)^(1/2))/(x^2+2)^(1/2),x, algorithm="fricas")
[Out] 2/3*sqrt(3)*tan(1/2*sqrt(3)*sqrt(x^2 + 2))/(tan(1/2*sqrt(3)*sqrt(x^2 + 2))^
2 + 1)
```

giac [A] time = 0.12, size = 17, normalized size = 0.77

$$\frac{1}{3}\sqrt{3}\sin\left(\sqrt{3}\sqrt{x^2+2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(3^(1/2)*(x^2+2)^(1/2))/(x^2+2)^(1/2),x, algorithm="giac")
[Out] 1/3*sqrt(3)*sin(sqrt(3)*sqrt(x^2 + 2))
```

maple [A] time = 0.08, size = 18, normalized size = 0.82

$$\frac{\sin\left(\sqrt{3}\,\sqrt{x^2+2}\right)\sqrt{3}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*cos(3^(1/2)*(x^2+2)^(1/2))/(x^2+2)^(1/2),x)
[Out] 1/3*sin(3^(1/2)*(x^2+2)^(1/2))*3^(1/2)
```

maxima [A] time = 0.41, size = 17, normalized size = 0.77

$$\frac{1}{3}\sqrt{3}\sin\left(\sqrt{3}\sqrt{x^2+2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(3^(1/2)*(x^2+2)^(1/2))/(x^2+2)^(1/2),x, algorithm="maxima")
[Out] 1/3*sqrt(3)*sin(sqrt(3)*sqrt(x^2 + 2))
```

mupad [B] time = 2.32, size = 15, normalized size = 0.68

$$\frac{\sqrt{3}\,\sin\left(\sqrt{3\,x^2+6}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*cos(3^(1/2)*(x^2 + 2)^(1/2)))/(x^2 + 2)^(1/2),x)
[Out] (3^(1/2)*sin((3*x^2 + 6)^(1/2)))/3
```

sympy [A] time = 0.68, size = 20, normalized size = 0.91

$$\frac{\sqrt{3}\sin\left(\sqrt{3}\sqrt{x^2+2}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(3**(1/2)*(x**2+2)**(1/2))/(x**2+2)**(1/2),x)
```

```
[Out] sqrt(3)*sin(sqrt(3)*sqrt(x**2 + 2))/3
```

3.50 $\int \frac{(-1+2x)\cos\left(\sqrt{6+3(-1+2x)^2}\right)}{\sqrt{6+3(-1+2x)^2}}\,dx$

Optimal. Leaf size=24

$$\frac{1}{6}\sin\left(\sqrt{3}\sqrt{(2x-1)^2+2}\right)$$

[Out] 1/6*sin(3^(1/2)*(2+(-1+2*x)^2)^(1/2))

Rubi [A] time = 0.49, antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.108, Rules used = {6715, 3432, 15, 2637}

$$\frac{1}{6}\sin\left(\sqrt{3}\sqrt{(2x-1)^2+2}\right)$$

Antiderivative was successfully verified.

[In] Int[((-1 + 2*x)*Cos[Sqrt[6 + 3*(-1 + 2*x)^2]])/Sqrt[6 + 3*(-1 + 2*x)^2],x]

[Out] Sin[Sqrt[3]*Sqrt[2 + (-1 + 2*x)^2]]/6

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3432

```
Int[((a_.) + Cos[(c_.) + (d_.)*((e_.) + (f_.)*(x_))^(n_)]*(b_.))^(p_.)*((g_
.) + (h_.)*(x_))^(m_.), x_Symbol] :> Dist[1/(n*f), Subst[Int[ExpandIntegran
d[(a + b*Cos[c + d*x])^p, x^(1/n - 1)*(g - (e*h)/f + (h*x^(1/n))/f)^m, x],
x], x, (e + f*x)^n], x] /; FreeQ[{a, b, c, d, e, f, g, h, m}, x] && IGtQ[p,
 0] && IntegerQ[1/n]
```

Rule 6715

```
Int[(u_)*(x_)^(m_.), x_Symbol] :> Dist[1/(m + 1), Subst[Int[SubstFor[x^(m +
 1), u, x], x], x, x^(m + 1)], x] /; FreeQ[m, x] && NeQ[m, -1] && FunctionO
```

```
fQ[x^(m + 1), u, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{(-1+2x)\cos\left(\sqrt{6+3(-1+2x)^2}\right)}{\sqrt{6+3(-1+2x)^2}}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{x\cos\left(\sqrt{6+3x^2}\right)}{\sqrt{6+3x^2}}\,dx, x, -1+2x\right)\\
&= \frac{1}{4}\operatorname{Subst}\left(\int \frac{\cos\left(\sqrt{6+3x}\right)}{\sqrt{6+3x}}\,dx, x, (-1+2x)^2\right)\\
&= \frac{1}{6}\operatorname{Subst}\left(\int \frac{x\cos(x)}{\sqrt{x^2}}\,dx, x, \sqrt{3}\sqrt{2+(-1+2x)^2}\right)\\
&= \frac{1}{6}\operatorname{Subst}\left(\int \cos(x)\,dx, x, \sqrt{3}\sqrt{2+(-1+2x)^2}\right)\\
&= \frac{1}{6}\sin\left(\sqrt{3}\sqrt{2+(-1+2x)^2}\right)
\end{aligned}$$

Mathematica [A] time = 0.17, size = 20, normalized size = 0.83

$$\frac{1}{6}\sin\left(\sqrt{3(1-2x)^2+6}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((-1 + 2*x)*Cos[Sqrt[6 + 3*(-1 + 2*x)^2]])/Sqrt[6 + 3*(-1 + 2*x)^
2],x]
```

```
[Out] Sin[Sqrt[6 + 3*(1 - 2*x)^2]]/6
```

fricas [A] time = 1.70, size = 15, normalized size = 0.62

$$\frac{1}{6}\sin\left(\sqrt{12x^2-12x+9}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+2*x)*cos((6+3*(-1+2*x)^2)^(1/2))/(6+3*(-1+2*x)^2)^(1/2),x, al
gorithm="fricas")
```

```
[Out] 1/6*sin(sqrt(12*x^2 - 12*x + 9))
```

giac [A] time = 0.15, size = 19, normalized size = 0.79

$$\frac{1}{6}\sin\left(\sqrt{3}\sqrt{4x^2-4x+3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+2*x)*cos((6+3*(-1+2*x)^2)^(1/2))/(6+3*(-1+2*x)^2)^(1/2),x, al
gorithm="giac")
```

```
[Out] 1/6*sin(sqrt(3)*sqrt(4*x^2 - 4*x + 3))
```

maple [A] time = 0.07, size = 16, normalized size = 0.67

$$\frac{\sin\left(\sqrt{12x^2-12x+9}\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-1+2*x)*cos((6+3*(-1+2*x)^2)^(1/2))/(6+3*(-1+2*x)^2)^(1/2),x)
```

```
[Out] 1/6*sin((12*x^2-12*x+9)^(1/2))
```

maxima [A] time = 0.31, size = 16, normalized size = 0.67

$$\frac{1}{6}\sin\left(\sqrt{3\,(2\,x-1)^2+6}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+2*x)*cos((6+3*(-1+2*x)^2)^(1/2))/(6+3*(-1+2*x)^2)^(1/2),x, al
gorithm="maxima")
```

```
[Out] 1/6*sin(sqrt(3*(2*x - 1)^2 + 6))
```

mupad [B] time = 2.35, size = 16, normalized size = 0.67

$$\frac{\sin\left(\sqrt{3\,(2\,x-1)^2+6}\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos((3*(2*x - 1)^2 + 6)^(1/2))*(2*x - 1))/(3*(2*x - 1)^2 + 6)^(1/2),x)
```

```
[Out] sin((3*(2*x - 1)^2 + 6)^(1/2))/6
```

sympy [A] time = 5.49, size = 15, normalized size = 0.62

$$\frac{\sin\left(\sqrt{3\,(2x-1)^2+6}\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+2*x)*cos((6+3*(-1+2*x)**2)**(1/2))/(6+3*(-1+2*x)**2)**(1/2),x
)
```

```
[Out] sin(sqrt(3*(2*x - 1)**2 + 6))/6
```

3.51 $\int \cos\left(\frac{a+bx}{c+dx}\right)\,dx$

Optimal. Leaf size=101

$$-\frac{\sin\left(\frac{b}{d}\right)(bc-ad)\mathrm{Ci}\left(\frac{bc-ad}{d(c+dx)}\right)}{d^2}+\frac{\cos\left(\frac{b}{d}\right)(bc-ad)\mathrm{Si}\left(\frac{bc-ad}{d(c+dx)}\right)}{d^2}+\frac{(c+dx)\cos\left(\frac{a+bx}{c+dx}\right)}{d}$$

```
[Out] (d*x+c)*cos((b*x+a)/(d*x+c))/d+(-a*d+b*c)*cos(b/d)*Si((-a*d+b*c)/d/(d*x+c))
/d^2-(-a*d+b*c)*Ci((-a*d+b*c)/d/(d*x+c))*sin(b/d)/d^2
```

Rubi [A] time = 0.13, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {4564, 3297, 3303, 3299, 3302}

$$-\frac{\sin\left(\frac{b}{d}\right)(bc-ad)\mathrm{CosIntegral}\left(\frac{bc-ad}{d(c+dx)}\right)}{d^2}+\frac{\cos\left(\frac{b}{d}\right)(bc-ad)\mathrm{Si}\left(\frac{bc-ad}{d(c+dx)}\right)}{d^2}+\frac{(c+dx)\cos\left(\frac{a+bx}{c+dx}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[(a + b*x)/(c + d*x)],x]
```

```
[Out] ((c + d*x)*Cos[(a + b*x)/(c + d*x)])/d - ((b*c - a*d)*CosIntegral[(b*c - a*
d)/(d*(c + d*x))]*Sin[b/d])/d^2 + ((b*c - a*d)*Cos[b/d]*SinIntegral[(b*c -
a*d)/(d*(c + d*x))])/d^2
```

Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[((
c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3302

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 4564

```
Int[Cos[((e_.)*((a_.) + (b_.)*(x_)))/((c_.) + (d_.)*(x_))]^(n_.), x_Symbol]
 :> -Dist[d^(-1), Subst[Int[Cos[(b*e)/d - (e*(b*c - a*d)*x)/d]^n/x^2, x], x
, 1/(c + d*x)], x] /; FreeQ[{a, b, c, d}, x] && IGtQ[n, 0] && NeQ[b*c - a*d
, 0]
```

Rubi steps

$$\int \cos\left(\frac{a+bx}{c+dx}\right)dx = -\frac{\text{Subst}\left(\int \frac{\cos\left(\frac{b}{d}-\frac{(bc-ad)x}{d}\right)}{x^2}\,dx, x, \frac{1}{c+dx}\right)}{d}$$

$$= \frac{(c+dx)\cos\left(\frac{a+bx}{c+dx}\right)}{d} - \frac{(bc-ad)\,\text{Subst}\left(\int \frac{\sin\left(\frac{b}{d}-\frac{(bc-ad)x}{d}\right)}{x}\,dx, x, \frac{1}{c+dx}\right)}{d^2}$$

$$= \frac{(c+dx)\cos\left(\frac{a+bx}{c+dx}\right)}{d} + \frac{\left((bc-ad)\cos\left(\frac{b}{d}\right)\right)\text{Subst}\left(\int \frac{\sin\left(\frac{(bc-ad)x}{d}\right)}{x}\,dx, x, \frac{1}{c+dx}\right)}{d^2} - \frac{\left((bc-ad)\sin\right.}{}$$

$$= \frac{(c+dx)\cos\left(\frac{a+bx}{c+dx}\right)}{d} - \frac{(bc-ad)\text{Ci}\left(\frac{bc-ad}{d(c+dx)}\right)\sin\left(\frac{b}{d}\right)}{d^2} + \frac{(bc-ad)\cos\left(\frac{b}{d}\right)\text{Si}\left(\frac{bc-ad}{d(c+dx)}\right)}{d^2}$$

Mathematica [C] time = 5.51, size = 260, normalized size = 2.57

$$\frac{-4\sin\left(\frac{b}{d}\right)(bc-ad)\text{Ci}\left(\frac{ad-bc}{d(c+dx)}\right) + d\exp\left(-\frac{i(ad+2bc+bdx)}{d(c+dx)}\right)\left(2c\left(e^{2i\left(\frac{a}{c+dx}+\frac{b}{d}\right)} + e^{\frac{2ibc}{d(c+dx)}}\right) + dx\left(1+e^{\frac{2ib}{d}}\right)\left(e^{\frac{2ia}{c+dx}} + e^{\frac{2ibc}{d(c+dx)}}\right.\right.}{4d^2}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[(a + b*x)/(c + d*x)],x]
```

```
[Out] (-4*(b*c - a*d)*CosIntegral[(-(b*c) + a*d)/(d*(c + d*x))]*Sin[b/d] + (d*(2*
c*(E^(((2*I)*b*c)/(d*(c + d*x))) + E^((2*I)*(b/d + a/(c + d*x)))) + d*(1 +
```

```
E^(((2*I)*b)/d))*(E^(((2*I)*a)/(c + d*x)) + E^(((2*I)*b*c)/(d*(c + d*x))))*
x - 4*d*E^((I*(2*b*c + a*d + b*d*x))/(d*(c + d*x)))*x*Sin[b/d]*Sin[(-(b*c)
+ a*d)/(d*(c + d*x))]))/E^((I*(2*b*c + a*d + b*d*x))/(d*(c + d*x))) - 4*(b*
c - a*d)*Cos[b/d]*SinIntegral[(-(b*c) + a*d)/(d*(c + d*x))])/(4*d^2)
```

fricas [A] time = 4.12, size = 138, normalized size = 1.37

$$-\frac{2\,(bc-ad)\cos\left(\frac{b}{d}\right)\mathrm{Si}\left(-\frac{bc-ad}{d^2x+cd}\right)-2\left(d^2x+cd\right)\cos\left(\frac{bx+a}{dx+c}\right)+\left((bc-ad)\,\mathrm{Ci}\left(\frac{bc-ad}{d^2x+cd}\right)+(bc-ad)\,\mathrm{Ci}\left(-\frac{bc-ad}{d^2x+cd}\right)\right)\sin}{2\,d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos((b*x+a)/(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/2*(2*(b*c - a*d)*cos(b/d)*sin_integral(-(b*c - a*d)/(d^2*x + c*d)) - 2*(
d^2*x + c*d)*cos((b*x + a)/(d*x + c)) + ((b*c - a*d)*cos_integral((b*c - a*
d)/(d^2*x + c*d)) + (b*c - a*d)*cos_integral(-(b*c - a*d)/(d^2*x + c*d)))*s
in(b/d))/d^2
```

giac [B] time = 12.27, size = 633, normalized size = 6.27

$$-\frac{\left(b^3c^2\,\mathrm{Ci}\left(-\frac{b-\frac{(bx+a)d}{dx+c}}{d}\right)\sin\left(\frac{b}{d}\right)-2\,ab^2cd\,\mathrm{Ci}\left(-\frac{b-\frac{(bx+a)d}{dx+c}}{d}\right)\sin\left(\frac{b}{d}\right)-\frac{(bx+a)b^2c^2d\,\mathrm{Ci}\left(-\frac{b-\frac{(bx+a)d}{dx+c}}{d}\right)\sin\left(\frac{b}{d}\right)}{dx+c}+a^2bd^2\,\mathrm{Ci}\left(-\frac{b-\frac{(bx+a)}{dx+c}}{d}\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos((b*x+a)/(d*x+c)),x, algorithm="giac")
```

```
[Out] -(b^3*c^2*cos_integral(-(b - (b*x + a)*d/(d*x + c))/d)*sin(b/d) - 2*a*b^2*c
*d*cos_integral(-(b - (b*x + a)*d/(d*x + c))/d)*sin(b/d) - (b*x + a)*b^2*c^
2*d*cos_integral(-(b - (b*x + a)*d/(d*x + c))/d)*sin(b/d)/(d*x + c) + a^2*b
*d^2*cos_integral(-(b - (b*x + a)*d/(d*x + c))/d)*sin(b/d) + 2*(b*x + a)*a*
b*c*d^2*cos_integral(-(b - (b*x + a)*d/(d*x + c))/d)*sin(b/d)/(d*x + c) - (
b*x + a)*a^2*d^3*cos_integral(-(b - (b*x + a)*d/(d*x + c))/d)*sin(b/d)/(d*x
 + c) - b^3*c^2*cos(b/d)*sin_integral((b - (b*x + a)*d/(d*x + c))/d) + 2*a*
b^2*c*d*cos(b/d)*sin_integral((b - (b*x + a)*d/(d*x + c))/d) + (b*x + a)*b^
2*c^2*d*cos(b/d)*sin_integral((b - (b*x + a)*d/(d*x + c))/d)/(d*x + c) - a^
2*b*d^2*cos(b/d)*sin_integral((b - (b*x + a)*d/(d*x + c))/d) - 2*(b*x + a)*
a*b*c*d^2*cos(b/d)*sin_integral((b - (b*x + a)*d/(d*x + c))/d)/(d*x + c) +
(b*x + a)*a^2*d^3*cos(b/d)*sin_integral((b - (b*x + a)*d/(d*x + c))/d)/(d*x
 + c) - b^2*c^2*d*cos((b*x + a)/(d*x + c)) + 2*a*b*c*d^2*cos((b*x + a)/(d*x
```

```
 + c)) - a^2*d^3*cos((b*x + a)/(d*x + c)))*(b*c/(b*c - a*d)^2 - a*d/(b*c -
a*d)^2)/(b*d^2 - (b*x + a)*d^3/(d*x + c))
```

maple [A] time = 0.10, size = 142, normalized size = 1.41

$$-(da-cb)\left(-\frac{\cos\left(\frac{b}{d}+\frac{da-cb}{d(dx+c)}\right)}{\left(\left(\frac{b}{d}+\frac{da-cb}{d(dx+c)}\right)d-b\right)d}-\frac{\frac{\mathrm{Si}\left(\frac{da-cb}{d(dx+c)}\right)\cos\left(\frac{b}{d}\right)}{d}+\frac{\mathrm{Ci}\left(\frac{da-cb}{d(dx+c)}\right)\sin\left(\frac{b}{d}\right)}{d}}{d}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos((b*x+a)/(d*x+c)),x)
```

```
[Out] -(a*d-b*c)*(-cos(b/d+(a*d-b*c)/d/(d*x+c))/((b/d+(a*d-b*c)/d/(d*x+c))*d-b)/d
-(Si((a*d-b*c)/d/(d*x+c))*cos(b/d)/d+Ci((a*d-b*c)/d/(d*x+c))*sin(b/d)/d)/d)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \cos\left(\frac{bx+a}{dx+c}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos((b*x+a)/(d*x+c)),x, algorithm="maxima")
```

```
[Out] integrate(cos((b*x + a)/(d*x + c)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \cos\left(\frac{a+b\,x}{c+d\,x}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos((a + b*x)/(c + d*x)),x)
```

```
[Out] int(cos((a + b*x)/(c + d*x)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \cos\left(\frac{a+bx}{c+dx}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos((b*x+a)/(d*x+c)),x)
```

```
[Out] Integral(cos((a + b*x)/(c + d*x)), x)
```

3.52 $\int \cos^2\left(\frac{a+bx}{c+dx}\right) dx$

Optimal. Leaf size=107

$$-\frac{\sin\left(\frac{2b}{d}\right)(bc-ad)\text{Ci}\left(\frac{2(bc-ad)}{d(c+dx)}\right)}{d^2}+\frac{\cos\left(\frac{2b}{d}\right)(bc-ad)\text{Si}\left(\frac{2(bc-ad)}{d(c+dx)}\right)}{d^2}+\frac{(c+dx)\cos^2\left(\frac{a+bx}{c+dx}\right)}{d}$$

```
[Out] (d*x+c)*cos((b*x+a)/(d*x+c))^2/d+(-a*d+b*c)*cos(2*b/d)*Si(2*(-a*d+b*c)/d/(d
*x+c))/d^2-(-a*d+b*c)*Ci(2*(-a*d+b*c)/d/(d*x+c))*sin(2*b/d)/d^2
```

Rubi [A] time = 0.16, antiderivative size = 107, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {4564, 3313, 12, 3303, 3299, 3302}

$$-\frac{\sin\left(\frac{2b}{d}\right)(bc-ad)\text{CosIntegral}\left(\frac{2(bc-ad)}{d(c+dx)}\right)}{d^2}+\frac{\cos\left(\frac{2b}{d}\right)(bc-ad)\text{Si}\left(\frac{2(bc-ad)}{d(c+dx)}\right)}{d^2}+\frac{(c+dx)\cos^2\left(\frac{a+bx}{c+dx}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[(a + b*x)/(c + d*x)]^2,x]
```

```
[Out] ((c + d*x)*Cos[(a + b*x)/(c + d*x)]^2)/d - ((b*c - a*d)*CosIntegral[(2*(b*c
 - a*d))/(d*(c + d*x))]*Sin[(2*b)/d])/d^2 + ((b*c - a*d)*Cos[(2*b)/d]*SinIn
tegral[(2*(b*c - a*d))/(d*(c + d*x))])/d^2
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3302

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3313

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> Si
mp[((c + d*x)^(m + 1)*Sin[e + f*x]^n)/(d*(m + 1)), x] - Dist[(f*n)/(d*(m +
1)), Int[ExpandTrigReduce[(c + d*x)^(m + 1), Cos[e + f*x]*Sin[e + f*x]^(n -
 1), x], x], x] /; FreeQ[{c, d, e, f, m}, x] && IGtQ[n, 1] && GeQ[m, -2] &&
 LtQ[m, -1]
```

Rule 4564

```
Int[Cos[((e_.)*((a_.) + (b_.)*(x_)))/((c_.) + (d_.)*(x_))]^(n_.), x_Symbol]
 :> -Dist[d^(-1), Subst[Int[Cos[(b*e)/d - (e*(b*c - a*d)*x)/d]^n/x^2, x], x
, 1/(c + d*x)], x] /; FreeQ[{a, b, c, d}, x] && IGtQ[n, 0] && NeQ[b*c - a*d
, 0]
```

Rubi steps

$$\int \cos^2\left(\frac{a+bx}{c+dx}\right)dx = -\frac{\text{Subst}\left(\int \frac{\cos^2\left(\frac{b}{d}-\frac{(bc-ad)x}{d}\right)}{x^2}dx, x, \frac{1}{c+dx}\right)}{d}$$

$$= \frac{(c+dx)\cos^2\left(\frac{a+bx}{c+dx}\right)}{d} + \frac{(2(bc-ad))\,\text{Subst}\left(\int -\frac{\sin\left(\frac{2b}{d}-\frac{2(bc-ad)x}{d}\right)}{2x}dx, x, \frac{1}{c+dx}\right)}{d^2}$$

$$= \frac{(c+dx)\cos^2\left(\frac{a+bx}{c+dx}\right)}{d} - \frac{(bc-ad)\,\text{Subst}\left(\int \frac{\sin\left(\frac{2b}{d}-\frac{2(bc-ad)x}{d}\right)}{x}dx, x, \frac{1}{c+dx}\right)}{d^2}$$

$$= \frac{(c+dx)\cos^2\left(\frac{a+bx}{c+dx}\right)}{d} + \frac{\left((bc-ad)\cos\left(\frac{2b}{d}\right)\right)\text{Subst}\left(\int \frac{\sin\left(\frac{2(bc-ad)x}{d}\right)}{x}dx, x, \frac{1}{c+dx}\right)}{d^2} - \frac{\left((bc-ad\right.}{}$$

$$= \frac{(c+dx)\cos^2\left(\frac{a+bx}{c+dx}\right)}{d} - \frac{(bc-ad)\text{Ci}\left(\frac{2(bc-ad)}{d(c+dx)}\right)\sin\left(\frac{2b}{d}\right)}{d^2} + \frac{(bc-ad)\cos\left(\frac{2b}{d}\right)\text{Si}\left(\frac{2(bc-ad)}{d(c+dx)}\right)}{d^2}$$

Mathematica [C] time = 6.14, size = 400, normalized size = 3.74

$$\frac{\left(acd-bc^2\right)\left(\frac{\left(-1+e^{\frac{4ib}{d}}\right)\left(e^{\frac{4ibc}{d(c+dx)}}-e^{\frac{4ia}{c+dx}}\right)\exp\left(-\frac{2i(ad+2bc+bdx)}{d(c+dx)}\right)}{8(bc-ad)}-\frac{\left(1+e^{\frac{4ib}{d}}\right)\left(e^{\frac{4ia}{c+dx}}+e^{\frac{4ibc}{d(c+dx)}}\right)\exp\left(-\frac{2i(ad+2bc+bdx)}{d(c+dx)}\right)}{8(bc-ad)}\right)}{d}+\frac{2ad\sin\left(\frac{2b}{d}\right)\mathrm{Ci}\left(\frac{2(a\ldots}{d(c\ldots}\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[(a + b*x)/(c + d*x)]^2,x]
```

```
[Out] ((-(b*c^2) + a*c*d)*(((-1 + E^(((4*I)*b)/d))*(-E^(((4*I)*a)/(c + d*x)) + E^
(((4*I)*b*c)/(d*(c + d*x)))))/(8*(b*c - a*d)*E^(((2*I)*(2*b*c + a*d + b*d*x
))/(d*(c + d*x)))) - ((1 + E^(((4*I)*b)/d))*(E^(((4*I)*a)/(c + d*x)) + E^((
(4*I)*b*c)/(d*(c + d*x)))))/(8*(b*c - a*d)*E^(((2*I)*(2*b*c + a*d + b*d*x))
/(d*(c + d*x))))))/d + (x*Cos[(2*b)/d]*Cos[(2*(-(b*c) + a*d))/(d*(c + d*x))
])/2 - (x*Sin[(2*b)/d]*Sin[(2*(-(b*c) + a*d))/(d*(c + d*x))])/2 + (d^2*x -
2*b*c*CosIntegral[(2*(-(b*c) + a*d))/(d*(c + d*x))]*Sin[(2*b)/d] + 2*a*d*Co
sIntegral[(2*(-(b*c) + a*d))/(d*(c + d*x))]*Sin[(2*b)/d] - 2*b*c*Cos[(2*b)/
d]*SinIntegral[(2*(-(b*c) + a*d))/(d*(c + d*x))] + 2*a*d*Cos[(2*b)/d]*SinIn
tegral[(2*(-(b*c) + a*d))/(d*(c + d*x))])/(2*d^2)
```

fricas [A] time = 2.20, size = 144, normalized size = 1.35

$$\frac{2\left(d^2x+cd\right)\cos\left(\frac{bx+a}{dx+c}\right)^2-2\left(bc-ad\right)\cos\left(\frac{2b}{d}\right)\mathrm{Si}\left(-\frac{2\left(bc-ad\right)}{d^2x+cd}\right)-\left(\left(bc-ad\right)\mathrm{Ci}\left(\frac{2\left(bc-ad\right)}{d^2x+cd}\right)+\left(bc-ad\right)\mathrm{Ci}\left(-\frac{2\left(bc-ad\right)}{d^2x+cd}\right)\right)\ldots}{2d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos((b*x+a)/(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/2*(2*(d^2*x + c*d)*cos((b*x + a)/(d*x + c))^2 - 2*(b*c - a*d)*cos(2*b/d)*
sin_integral(-2*(b*c - a*d)/(d^2*x + c*d)) - ((b*c - a*d)*cos_integral(2*(b
*c - a*d)/(d^2*x + c*d)) + (b*c - a*d)*cos_integral(-2*(b*c - a*d)/(d^2*x +
 c*d)))*sin(2*b/d))/d^2
```

giac [B] time = 68.63, size = 683, normalized size = 6.38

$$-\frac{\left(2b^3c^2\,\mathrm{Ci}\left(-\frac{2\left(b-\frac{(bx+a)d}{dx+c}\right)}{d}\right)\sin\left(\frac{2b}{d}\right)-4ab^2cd\,\mathrm{Ci}\left(-\frac{2\left(b-\frac{(bx+a)d}{dx+c}\right)}{d}\right)\sin\left(\frac{2b}{d}\right)-\frac{2(bx+a)b^2c^2d\,\mathrm{Ci}\left(-\frac{2\left(b-\frac{(bx+a)d}{dx+c}\right)}{d}\right)\sin\left(\frac{2b}{d}\right)}{dx+c}+2a^2b\ldots\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos((b*x+a)/(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/2*(2*b^3*c^2*cos_integral(-2*(b - (b*x + a)*d/(d*x + c))/d)*sin(2*b/d) -
 4*a*b^2*c*d*cos_integral(-2*(b - (b*x + a)*d/(d*x + c))/d)*sin(2*b/d) - 2*
(b*x + a)*b^2*c^2*d*cos_integral(-2*(b - (b*x + a)*d/(d*x + c))/d)*sin(2*b/
d)/(d*x + c) + 2*a^2*b*d^2*cos_integral(-2*(b - (b*x + a)*d/(d*x + c))/d)*s
in(2*b/d) + 4*(b*x + a)*a*b*c*d^2*cos_integral(-2*(b - (b*x + a)*d/(d*x + c
))/d)*sin(2*b/d)/(d*x + c) - 2*(b*x + a)*a^2*d^3*cos_integral(-2*(b - (b*x
+ a)*d/(d*x + c))/d)*sin(2*b/d)/(d*x + c) - 2*b^3*c^2*cos(2*b/d)*sin_integr
al(2*(b - (b*x + a)*d/(d*x + c))/d) + 4*a*b^2*c*d*cos(2*b/d)*sin_integral(2
*(b - (b*x + a)*d/(d*x + c))/d) + 2*(b*x + a)*b^2*c^2*d*cos(2*b/d)*sin_inte
gral(2*(b - (b*x + a)*d/(d*x + c))/d)/(d*x + c) - 2*a^2*b*d^2*cos(2*b/d)*si
n_integral(2*(b - (b*x + a)*d/(d*x + c))/d) - 4*(b*x + a)*a*b*c*d^2*cos(2*b
/d)*sin_integral(2*(b - (b*x + a)*d/(d*x + c))/d)/(d*x + c) + 2*(b*x + a)*a
^2*d^3*cos(2*b/d)*sin_integral(2*(b - (b*x + a)*d/(d*x + c))/d)/(d*x + c) -
 b^2*c^2*d*cos(2*(b*x + a)/(d*x + c)) + 2*a*b*c*d^2*cos(2*(b*x + a)/(d*x +
c)) - a^2*d^3*cos(2*(b*x + a)/(d*x + c)) - b^2*c^2*d + 2*a*b*c*d^2 - a^2*d^
3)*(b*c/(b*c - a*d)^2 - a*d/(b*c - a*d)^2)/(b*d^2 - (b*x + a)*d^3/(d*x + c)
)
```

maple [A] time = 0.14, size = 195, normalized size = 1.82

$$-\frac{(da-cb)\left(\frac{d^2\left(-\frac{2\cos\left(\frac{2da-2cb}{d(dx+c)}+\frac{2b}{d}\right)}{\left(\left(\frac{b}{d}+\frac{da-cb}{d(dx+c)}\right)d-b\right)d}-\frac{2\left(\frac{2\,\mathrm{Si}\left(\frac{2da-2cb}{d(dx+c)}\right)\cos\left(\frac{2b}{d}\right)}{d}+\frac{2\,\mathrm{Ci}\left(\frac{2da-2cb}{d(dx+c)}\right)\sin\left(\frac{2b}{d}\right)}{d}\right)}{d}\right)}{4}-\frac{d}{2\left(\left(\frac{b}{d}+\frac{da-cb}{d(dx+c)}\right)d-b\right)}\right)}{d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos((b*x+a)/(d*x+c))^2,x)
```

```
[Out] -1/d^2*(a*d-b*c)*(1/4*d^2*(-2*cos(2*(a*d-b*c)/d/(d*x+c)+2*b/d)/((b/d+(a*d-b
*c)/d/(d*x+c))*d-b)/d-2*(2*Si(2*(a*d-b*c)/d/(d*x+c))*cos(2*b/d)/d+2*Ci(2*(a
*d-b*c)/d/(d*x+c))*sin(2*b/d)/d)/d)-1/2*d/((b/d+(a*d-b*c)/d/(d*x+c))*d-b))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{2}\,x + \frac{1}{2}\,\int \cos\left(\frac{2\,(bx + a)}{dx + c}\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos((b*x+a)/(d*x+c))^2,x, algorithm="maxima")
[Out] 1/2*x + 1/2*integrate(cos(2*(b*x + a)/(d*x + c)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \cos\left(\frac{a+b\,x}{c+d\,x}\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos((a + b*x)/(c + d*x))^2,x)
[Out] int(cos((a + b*x)/(c + d*x))^2, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos((b*x+a)/(d*x+c))**2,x)
[Out] Timed out
```

3.53 $\int \frac{\cos^3\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{1-a^2x^2}\,dx$

Optimal. Leaf size=58

$$-\frac{3\text{Ci}\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{4a}-\frac{\text{Ci}\left(\frac{3\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{4a}$$

```
[Out] -3/4*Ci((-a*x+1)^(1/2)/(a*x+1)^(1/2))/a-1/4*Ci(3*(-a*x+1)^(1/2)/(a*x+1)^(1/
2))/a
```

Rubi [A] time = 0.11, antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {6681, 3312, 3302}

$$-\frac{3\text{CosIntegral}\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{4a}-\frac{\text{CosIntegral}\left(\frac{3\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{4a}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[Sqrt[1 - a*x]/Sqrt[1 + a*x]]^3/(1 - a^2*x^2),x]
```

```
[Out] (-3*CosIntegral[Sqrt[1 - a*x]/Sqrt[1 + a*x]])/(4*a) - CosIntegral[(3*Sqrt[1
 - a*x])/Sqrt[1 + a*x]]/(4*a)
```

Rule 3302

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 6681

```
Int[((a_.) + (b_.)*(F_)[((c_.)*Sqrt[(d_.) + (e_.)*(x_)])/Sqrt[(f_.) + (g_.)
*(x_)]])^(n_.)/((A_.) + (C_.)*(x_)^2), x_Symbol] :> Dist[(2*e*g)/(C*(e*f -
d*g)), Subst[Int[(a + b*F[c*x])^n/x, x], x, Sqrt[d + e*x]/Sqrt[f + g*x]], x
] /; FreeQ[{a, b, c, d, e, f, g, A, C, F}, x] && EqQ[C*d*f - A*e*g, 0] && E
qQ[e*f + d*g, 0] && IGtQ[n, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\cos^3\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{1-a^2x^2}\,dx &= -\frac{\text{Subst}\left(\int \frac{\cos^3(x)}{x}\,dx, x, \frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{a}\\
&= -\frac{\text{Subst}\left(\int \left(\frac{3\cos(x)}{4x}+\frac{\cos(3x)}{4x}\right)dx, x, \frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{a}\\
&= -\frac{\text{Subst}\left(\int \frac{\cos(3x)}{x}\,dx, x, \frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{4a}-\frac{3\,\text{Subst}\left(\int \frac{\cos(x)}{x}\,dx, x, \frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{4a}\\
&= -\frac{3\text{Ci}\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{4a}-\frac{\text{Ci}\left(\frac{3\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{4a}
\end{aligned}
$$

Mathematica [A] time = 0.02, size = 53, normalized size = 0.91

$$
-\frac{3\text{Ci}\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)+\text{Ci}\left(\frac{3\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{4a}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[Sqrt[1 - a*x]/Sqrt[1 + a*x]]^3/(1 - a^2*x^2),x]
```

```
[Out] -1/4*(3*CosIntegral[Sqrt[1 - a*x]/Sqrt[1 + a*x]] + CosIntegral[(3*Sqrt[1 - a*x])/Sqrt[1 + a*x]])/a
```

fricas [F] time = 2.69, size = 0, normalized size = 0.00

$$
\text{integral}\left(-\frac{\cos\left(\frac{\sqrt{-ax+1}}{\sqrt{ax+1}}\right)^3}{a^2x^2-1}, x\right)
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos((-a*x+1)^(1/2)/(a*x+1)^(1/2))^3/(-a^2*x^2+1),x, algorithm="fricas")
```

```
[Out] integral(-cos(sqrt(-a*x + 1)/sqrt(a*x + 1))^3/(a^2*x^2 - 1), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$
\int -\frac{\cos\left(\frac{\sqrt{-ax+1}}{\sqrt{ax+1}}\right)^3}{a^2x^2-1}\,dx
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos((-a*x+1)^(1/2)/(a*x+1)^(1/2))^3/(-a^2*x^2+1),x, algorithm="gi
ac")
```

```
[Out] integrate(-cos(sqrt(-a*x + 1)/sqrt(a*x + 1))^3/(a^2*x^2 - 1), x)
```

maple [F] time = 0.74, size = 0, normalized size = 0.00

$$\int \frac{\cos^3\left(\frac{\sqrt{-ax+1}}{\sqrt{ax+1}}\right)}{-a^2x^2+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos((-a*x+1)^(1/2)/(a*x+1)^(1/2))^3/(-a^2*x^2+1),x)
```

```
[Out] int(cos((-a*x+1)^(1/2)/(a*x+1)^(1/2))^3/(-a^2*x^2+1),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{\cos\left(\frac{\sqrt{-ax+1}}{\sqrt{ax+1}}\right)^3}{a^2x^2-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos((-a*x+1)^(1/2)/(a*x+1)^(1/2))^3/(-a^2*x^2+1),x, algorithm="ma
xima")
```

```
[Out] -integrate(cos(sqrt(-a*x + 1)/sqrt(a*x + 1))^3/(a^2*x^2 - 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$-\int \frac{\cos\left(\frac{\sqrt{1-a\,x}}{\sqrt{a\,x+1}}\right)^3}{a^2\,x^2-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-cos((1 - a*x)^(1/2)/(a*x + 1)^(1/2))^3/(a^2*x^2 - 1),x)
```

```
[Out] -int(cos((1 - a*x)^(1/2)/(a*x + 1)^(1/2))^3/(a^2*x^2 - 1), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{\cos^3\left(\frac{\sqrt{-ax+1}}{\sqrt{ax+1}}\right)}{a^2x^2-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos((-a*x+1)**(1/2)/(a*x+1)**(1/2))**3/(-a**2*x**2+1),x)
[Out] -Integral(cos(sqrt(-a*x + 1)/sqrt(a*x + 1))**3/(a**2*x**2 - 1), x)
```

3.54 $\int \frac{\cos^2\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{1-a^2x^2}\,dx$

Optimal. Leaf size=58

$$-\frac{\text{Ci}\left(\frac{2\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{2a}-\frac{\log\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{2a}$$

```
[Out] -1/2*Ci(2*(-a*x+1)^(1/2)/(a*x+1)^(1/2))/a-1/2*ln((-a*x+1)^(1/2)/(a*x+1)^(1/
2))/a
```

Rubi [A] time = 0.08, antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {6681, 3312, 3302}

$$-\frac{\text{CosIntegral}\left(\frac{2\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{2a}-\frac{\log\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{2a}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[Sqrt[1 - a*x]/Sqrt[1 + a*x]]^2/(1 - a^2*x^2),x]
```

```
[Out] -CosIntegral[(2*Sqrt[1 - a*x])/Sqrt[1 + a*x]]/(2*a) - Log[Sqrt[1 - a*x]/Sqr
t[1 + a*x]]/(2*a)
```

Rule 3302

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 6681

```
Int[((a_.) + (b_.)*(F_)[((c_.)*Sqrt[(d_.) + (e_.)*(x_)])/Sqrt[(f_.) + (g_.)
*(x_)]])^(n_.)/((A_.) + (C_.)*(x_)^2), x_Symbol] :> Dist[(2*e*g)/(C*(e*f -
d*g)), Subst[Int[(a + b*F[c*x])^n/x, x], x, Sqrt[d + e*x]/Sqrt[f + g*x]], x
] /; FreeQ[{a, b, c, d, e, f, g, A, C, F}, x] && EqQ[C*d*f - A*e*g, 0] && E
qQ[e*f + d*g, 0] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{\cos^2\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{1-a^2x^2}\,dx = -\frac{\text{Subst}\left(\int \frac{\cos^2(x)}{x}\,dx, x, \frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{a}$$
$$= -\frac{\text{Subst}\left(\int \left(\frac{1}{2x}+\frac{\cos(2x)}{2x}\right)dx, x, \frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{a}$$
$$= -\frac{\log\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{2a} - \frac{\text{Subst}\left(\int \frac{\cos(2x)}{x}\,dx, x, \frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{2a}$$
$$= -\frac{\text{Ci}\left(\frac{2\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{2a} - \frac{\log\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{2a}$$

Mathematica [A] time = 0.04, size = 51, normalized size = 0.88

$$-\frac{\text{Ci}\left(\frac{2\sqrt{1-ax}}{\sqrt{ax+1}}\right)+\log\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{2a}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[Sqrt[1 - a*x]/Sqrt[1 + a*x]]^2/(1 - a^2*x^2),x]
[Out] -1/2*(CosIntegral[(2*Sqrt[1 - a*x])/Sqrt[1 + a*x]] + Log[Sqrt[1 - a*x]/Sqrt
[1 + a*x]])/a
```

fricas [F] time = 1.49, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\cos\left(\frac{\sqrt{-ax+1}}{\sqrt{ax+1}}\right)^2}{a^2x^2-1}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos((-a*x+1)^(1/2)/(a*x+1)^(1/2))^2/(-a^2*x^2+1),x, algorithm="fr
icas")
[Out] integral(-cos(sqrt(-a*x + 1)/sqrt(a*x + 1))^2/(a^2*x^2 - 1), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{\cos\left(\frac{\sqrt{-ax+1}}{\sqrt{ax+1}}\right)^2}{a^2x^2-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos((-a*x+1)^(1/2)/(a*x+1)^(1/2))^2/(-a^2*x^2+1),x, algorithm="gi
ac")
```

```
[Out] integrate(-cos(sqrt(-a*x + 1)/sqrt(a*x + 1))^2/(a^2*x^2 - 1), x)
```

maple [F] time = 0.36, size = 0, normalized size = 0.00

$$\int \frac{\cos^2\left(\frac{\sqrt{-ax+1}}{\sqrt{ax+1}}\right)}{-a^2x^2+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos((-a*x+1)^(1/2)/(a*x+1)^(1/2))^2/(-a^2*x^2+1),x)
```

```
[Out] int(cos((-a*x+1)^(1/2)/(a*x+1)^(1/2))^2/(-a^2*x^2+1),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{a\int \frac{\cos\left(\frac{2\sqrt{-ax+1}}{\sqrt{ax+1}}\right)}{a^2x^2-1}\,dx + a\int \frac{\cos\left(\frac{2\sqrt{-ax+1}}{\sqrt{ax+1}}\right)}{\left(a^2x^2-1\right)\cos\left(\frac{2\sqrt{-ax+1}}{\sqrt{ax+1}}\right)^2+\left(a^2x^2-1\right)\sin\left(\frac{2\sqrt{-ax+1}}{\sqrt{ax+1}}\right)^2}\,dx - \log\left(ax+1\right)+\log\left(ax-1\right)}{4a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos((-a*x+1)^(1/2)/(a*x+1)^(1/2))^2/(-a^2*x^2+1),x, algorithm="ma
xima")
```

```
[Out] -1/4*(4*a*integrate(1/4*cos(2*sqrt(-a*x + 1)/sqrt(a*x + 1))/(a^2*x^2 - 1),
x) + 4*a*integrate(1/4*cos(2*sqrt(-a*x + 1)/sqrt(a*x + 1))/((a^2*x^2 - 1)*c
os(2*sqrt(-a*x + 1)/sqrt(a*x + 1))^2 + (a^2*x^2 - 1)*sin(2*sqrt(-a*x + 1)/s
qrt(a*x + 1))^2), x) - log(a*x + 1) + log(a*x - 1))/a
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$-\int \frac{\cos\left(\frac{\sqrt{1-a\,x}}{\sqrt{a\,x+1}}\right)^2}{a^2\,x^2-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-cos((1 - a*x)^(1/2)/(a*x + 1)^(1/2))^2/(a^2*x^2 - 1),x)
```

```
[Out] -int(cos((1 - a*x)^(1/2)/(a*x + 1)^(1/2))^2/(a^2*x^2 - 1), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{\cos^2\left(\frac{\sqrt{-ax+1}}{\sqrt{ax+1}}\right)}{a^2x^2-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos((-a*x+1)**(1/2)/(a*x+1)**(1/2))**2/(-a**2*x**2+1),x)
[Out] -Integral(cos(sqrt(-a*x + 1)/sqrt(a*x + 1))**2/(a**2*x**2 - 1), x)
```

3.55 $\int \frac{\cos\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{1-a^2x^2} \, dx$

Optimal. Leaf size=26

$$-\frac{\text{Ci}\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{a}$$

```
[Out] -Ci((-a*x+1)^(1/2)/(a*x+1)^(1/2))/a
```

Rubi [A] time = 0.04, antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {6681, 3302}

$$-\frac{\text{CosIntegral}\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[Sqrt[1 - a*x]/Sqrt[1 + a*x]]/(1 - a^2*x^2),x]
```

```
[Out] -(CosIntegral[Sqrt[1 - a*x]/Sqrt[1 + a*x]]/a)
```

Rule 3302

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 6681

```
Int[((a_.) + (b_.)*(F_)[((c_.)*Sqrt[(d_.) + (e_.)*(x_)])/Sqrt[(f_.) + (g_.)
*(x_)]])^(n_.)/((A_.) + (C_.)*(x_)^2), x_Symbol] :> Dist[(2*e*g)/(C*(e*f -
d*g)), Subst[Int[(a + b*F[c*x])^n/x, x], x, Sqrt[d + e*x]/Sqrt[f + g*x]], x
] /; FreeQ[{a, b, c, d, e, f, g, A, C, F}, x] && EqQ[C*d*f - A*e*g, 0] && E
qQ[e*f + d*g, 0] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{\cos\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{1-a^2x^2}\,dx = -\frac{\text{Subst}\left(\int \frac{\cos(x)}{x}\,dx, x, \frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{a}$$
$$= -\frac{\text{Ci}\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{a}$$

Mathematica [**A**] time = 0.01, size = 26, normalized size = 1.00

$$-\frac{\text{Ci}\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{a}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[Sqrt[1 - a*x]/Sqrt[1 + a*x]]/(1 - a^2*x^2),x]
```

```
[Out] -(CosIntegral[Sqrt[1 - a*x]/Sqrt[1 + a*x]]/a)
```

fricas [**F**] time = 1.77, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\cos\left(\frac{\sqrt{-ax+1}}{\sqrt{ax+1}}\right)}{a^2x^2-1}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos((-a*x+1)^(1/2)/(a*x+1)^(1/2))/(-a^2*x^2+1),x, algorithm="fric
as")
```

```
[Out] integral(-cos(sqrt(-a*x + 1)/sqrt(a*x + 1))/(a^2*x^2 - 1), x)
```

giac [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{\cos\left(\frac{\sqrt{-ax+1}}{\sqrt{ax+1}}\right)}{a^2x^2-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos((-a*x+1)^(1/2)/(a*x+1)^(1/2))/(-a^2*x^2+1),x, algorithm="giac
")
```

```
[Out] integrate(-cos(sqrt(-a*x + 1)/sqrt(a*x + 1))/(a^2*x^2 - 1), x)
```

maple [F] time = 0.08, size = 0, normalized size = 0.00

$$\int \frac{\cos\left(\frac{\sqrt{-ax+1}}{\sqrt{ax+1}}\right)}{-a^2x^2+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos((-a*x+1)^(1/2)/(a*x+1)^(1/2))/(-a^2*x^2+1),x)
```

```
[Out] int(cos((-a*x+1)^(1/2)/(a*x+1)^(1/2))/(-a^2*x^2+1),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{\cos\left(\frac{\sqrt{-ax+1}}{\sqrt{ax+1}}\right)}{a^2x^2-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos((-a*x+1)^(1/2)/(a*x+1)^(1/2))/(-a^2*x^2+1),x, algorithm="maxi
ma")
```

```
[Out] -integrate(cos(sqrt(-a*x + 1)/sqrt(a*x + 1))/(a^2*x^2 - 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.04

$$-\int \frac{\cos\left(\frac{\sqrt{1-a\,x}}{\sqrt{a\,x+1}}\right)}{a^2\,x^2-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-cos((1 - a*x)^(1/2)/(a*x + 1)^(1/2))/(a^2*x^2 - 1),x)
```

```
[Out] -int(cos((1 - a*x)^(1/2)/(a*x + 1)^(1/2))/(a^2*x^2 - 1), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{\cos\left(\frac{\sqrt{-ax+1}}{\sqrt{ax+1}}\right)}{a^2x^2-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos((-a*x+1)**(1/2)/(a*x+1)**(1/2))/(-a**2*x**2+1),x)
```

```
[Out] -Integral(cos(sqrt(-a*x + 1)/sqrt(a*x + 1))/(a**2*x**2 - 1), x)
```

3.56 $\int \frac{\sec\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{1-a^2x^2}\,dx$

Optimal. Leaf size=40

$$\text{Int}\left(\frac{\sec\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{(1-ax)(ax+1)},x\right)$$

```
[Out] Unintegrable(sec((-a*x+1)^(1/2)/(a*x+1)^(1/2))/(-a*x+1)/(a*x+1),x)
```

Rubi [A] time = 0.04, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\sec\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{1-a^2x^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Sec[Sqrt[1 - a*x]/Sqrt[1 + a*x]]/(1 - a^2*x^2),x]
```

```
[Out] -(Defer[Subst][Defer[Int][Sec[x]/x, x], x, Sqrt[1 - a*x]/Sqrt[1 + a*x]]/a)
```

Rubi steps

$$\int \frac{\sec\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{1-a^2x^2}\,dx = -\frac{\text{Subst}\left(\int \frac{\sec(x)}{x}\,dx,x,\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{a}$$

Mathematica [A] time = 3.07, size = 0, normalized size = 0.00

$$\int \frac{\sec\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{1-a^2x^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Sec[Sqrt[1 - a*x]/Sqrt[1 + a*x]]/(1 - a^2*x^2),x]
```

```
[Out] Integrate[Sec[Sqrt[1 - a*x]/Sqrt[1 + a*x]]/(1 - a^2*x^2), x]
```

fricas [A] time = 0.50, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{1}{\left(a^2x^2-1\right)\cos\left(\frac{\sqrt{-ax+1}}{\sqrt{ax+1}}\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a^2*x^2+1)/cos((-a*x+1)^(1/2)/(a*x+1)^(1/2)),x, algorithm="fr
icas")
[Out] integral(-1/((a^2*x^2 - 1)*cos(sqrt(-a*x + 1)/sqrt(a*x + 1))), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{1}{\left(a^2x^2-1\right)\cos\left(\frac{\sqrt{-ax+1}}{\sqrt{ax+1}}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a^2*x^2+1)/cos((-a*x+1)^(1/2)/(a*x+1)^(1/2)),x, algorithm="gi
ac")
[Out] integrate(-1/((a^2*x^2 - 1)*cos(sqrt(-a*x + 1)/sqrt(a*x + 1))), x)
```

maple [A] time = 0.11, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-a^2x^2+1\right)\cos\left(\frac{\sqrt{-ax+1}}{\sqrt{ax+1}}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-a^2*x^2+1)/cos((-a*x+1)^(1/2)/(a*x+1)^(1/2)),x)
[Out] int(1/(-a^2*x^2+1)/cos((-a*x+1)^(1/2)/(a*x+1)^(1/2)),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{1}{\left(a^2x^2-1\right)\cos\left(\frac{\sqrt{-ax+1}}{\sqrt{ax+1}}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a^2*x^2+1)/cos((-a*x+1)^(1/2)/(a*x+1)^(1/2)),x, algorithm="ma
xima")
```

```
[Out] -integrate(1/((a^2*x^2 - 1)*cos(sqrt(-a*x + 1)/sqrt(a*x + 1))), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.02

$$-\int \frac{1}{\cos\left(\frac{\sqrt{1-a\,x}}{\sqrt{a\,x+1}}\right)\,\left(a^2\,x^2-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-1/(cos((1 - a*x)^(1/2)/(a*x + 1)^(1/2))*(a^2*x^2 - 1)),x)
```

```
[Out] -int(1/(cos((1 - a*x)^(1/2)/(a*x + 1)^(1/2))*(a^2*x^2 - 1)), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{1}{a^{2}x^{2}\cos\left(\frac{\sqrt{-ax+1}}{\sqrt{ax+1}}\right)-\cos\left(\frac{\sqrt{-ax+1}}{\sqrt{ax+1}}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a**2*x**2+1)/cos((-a*x+1)**(1/2)/(a*x+1)**(1/2)),x)
```

```
[Out] -Integral(1/(a**2*x**2*cos(sqrt(-a*x + 1)/sqrt(a*x + 1)) - cos(sqrt(-a*x +
1)/sqrt(a*x + 1))), x)
```

3.57 $\int \frac{\sec^2\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{1-a^2x^2}\,dx$

Optimal. Leaf size=42

$$\text{Int}\left(\frac{\sec^2\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{(1-ax)(ax+1)},x\right)$$

```
[Out] Unintegrable(sec((-a*x+1)^(1/2)/(a*x+1)^(1/2))^2/(-a*x+1)/(a*x+1),x)
```

Rubi [A] time = 0.08, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\sec^2\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{1-a^2x^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Sec[Sqrt[1 - a*x]/Sqrt[1 + a*x]]^2/(1 - a^2*x^2),x]
```

```
[Out] -(Defer[Subst][Defer[Int][Sec[x]^2/x, x], x, Sqrt[1 - a*x]/Sqrt[1 + a*x]]/a
)
```

Rubi steps

$$\int \frac{\sec^2\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{1-a^2x^2}\,dx = -\frac{\text{Subst}\left(\int \frac{\sec^2(x)}{x}\,dx, x, \frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{a}$$

Mathematica [A] time = 11.88, size = 0, normalized size = 0.00

$$\int \frac{\sec^2\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{1-a^2x^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Sec[Sqrt[1 - a*x]/Sqrt[1 + a*x]]^2/(1 - a^2*x^2),x]
```

```
[Out] Integrate[Sec[Sqrt[1 - a*x]/Sqrt[1 + a*x]]^2/(1 - a^2*x^2), x]
```

fricas [A]   time = 1.13, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{1}{\left(a^{2}x^{2}-1\right)\cos\left(\frac{\sqrt{-ax+1}}{\sqrt{ax+1}}\right)^{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a^2*x^2+1)/cos((-a*x+1)^(1/2)/(a*x+1)^(1/2))^2,x, algorithm="
fricas")
```

```
[Out] integral(-1/((a^2*x^2 - 1)*cos(sqrt(-a*x + 1)/sqrt(a*x + 1))^2), x)
```

giac [A]   time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{1}{\left(a^{2}x^{2}-1\right)\cos\left(\frac{\sqrt{-ax+1}}{\sqrt{ax+1}}\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a^2*x^2+1)/cos((-a*x+1)^(1/2)/(a*x+1)^(1/2))^2,x, algorithm="
giac")
```

```
[Out] integrate(-1/((a^2*x^2 - 1)*cos(sqrt(-a*x + 1)/sqrt(a*x + 1))^2), x)
```

maple [A]   time = 0.25, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-a^{2}x^{2}+1\right)\cos\left(\frac{\sqrt{-ax+1}}{\sqrt{ax+1}}\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-a^2*x^2+1)/cos((-a*x+1)^(1/2)/(a*x+1)^(1/2))^2,x)
```

```
[Out] int(1/(-a^2*x^2+1)/cos((-a*x+1)^(1/2)/(a*x+1)^(1/2))^2,x)
```

maxima [F(-1)]   time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a^2*x^2+1)/cos((-a*x+1)^(1/2)/(a*x+1)^(1/2))^2,x, algorithm="
maxima")
```

```
[Out] Timed out
```

mupad [A] time = 0.00, size = -1, normalized size = -0.02

$$-\int \frac{1}{\cos\left(\frac{\sqrt{1-a\,x}}{\sqrt{a\,x+1}}\right)^2\,\left(a^2\,x^2-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-1/(cos((1 - a*x)^(1/2)/(a*x + 1)^(1/2))^2*(a^2*x^2 - 1)),x)
[Out] -int(1/(cos((1 - a*x)^(1/2)/(a*x + 1)^(1/2))^2*(a^2*x^2 - 1)), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{1}{a^2x^2\cos^2\left(\frac{\sqrt{-ax+1}}{\sqrt{ax+1}}\right)-\cos^2\left(\frac{\sqrt{-ax+1}}{\sqrt{ax+1}}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a**2*x**2+1)/cos((-a*x+1)**(1/2)/(a*x+1)**(1/2))**2,x)
[Out] -Integral(1/(a**2*x**2*cos(sqrt(-a*x + 1)/sqrt(a*x + 1))**2 - cos(sqrt(-a*x
 + 1)/sqrt(a*x + 1))**2), x)
```

3.58 $\int \frac{\tan(\sqrt{x})}{\sqrt{x}}\,dx$

Optimal. Leaf size=9

$$-2\log\left(\cos\left(\sqrt{x}\right)\right)$$

```
[Out] -2*ln(cos(x^(1/2)))
```

Rubi [A] time = 0.01, antiderivative size = 9, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3747, 3475}

$$-2\log\left(\cos\left(\sqrt{x}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Int[Tan[Sqrt[x]]/Sqrt[x],x]
```

```
[Out] -2*Log[Cos[Sqrt[x]]]
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3747

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Tan[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Tan[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IGtQ[Simplify[(m +
1)/n], 0] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}\int \frac{\tan\left(\sqrt{x}\right)}{\sqrt{x}}\,dx &= 2\,\text{Subst}\left(\int \tan(x)\,dx, x, \sqrt{x}\right)\\ &= -2\log\left(\cos\left(\sqrt{x}\right)\right)\end{aligned}$$

Mathematica [A] time = 0.01, size = 9, normalized size = 1.00

$$-2\log\left(\cos\left(\sqrt{x}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[Sqrt[x]]/Sqrt[x],x]
[Out] -2*Log[Cos[Sqrt[x]]]
```

fricas [A] time = 2.00, size = 13, normalized size = 1.44

$$-\log\left(\frac{1}{\tan\left(\sqrt{x}\right)^2+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x^(1/2))/x^(1/2),x, algorithm="fricas")
[Out] -log(1/(tan(sqrt(x))^2 + 1))
```

giac [A] time = 0.14, size = 8, normalized size = 0.89

$$-2\,\log\left(\left|\cos\left(\sqrt{x}\right)\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x^(1/2))/x^(1/2),x, algorithm="giac")
[Out] -2*log(abs(cos(sqrt(x))))
```

maple [A] time = 0.00, size = 8, normalized size = 0.89

$$-2\ln\left(\cos\left(\sqrt{x}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(x^(1/2))/x^(1/2),x)
[Out] -2*ln(cos(x^(1/2)))
```

maxima [A] time = 0.30, size = 7, normalized size = 0.78

$$2\,\log\left(\sec\left(\sqrt{x}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x^(1/2))/x^(1/2),x, algorithm="maxima")
[Out] 2*log(sec(sqrt(x)))
```

mupad [B] time = 3.18, size = 19, normalized size = 2.11

$$-2\ln\left(e^{\sqrt{x}\,2i}+1\right)+\sqrt{x}\,2i$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(x^(1/2))/x^(1/2),x)
[Out] x^(1/2)*2i - 2*log(exp(x^(1/2)*2i) + 1)
```

sympy [A] time = 0.17, size = 10, normalized size = 1.11

$$\log\left(\tan^2\left(\sqrt{x}\right)+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x**(1/2))/x**(1/2),x)
[Out] log(tan(sqrt(x))**2 + 1)
```

3.59 $\int \frac{\tan^2(\sqrt{x})}{\sqrt{x}} dx$

Optimal. Leaf size=16

$$2\tan\left(\sqrt{x}\right) - 2\sqrt{x}$$

```
[Out] -2*x^(1/2)+2*tan(x^(1/2))
```

Rubi [A] time = 0.02, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {3747, 3473, 8}

$$2\tan\left(\sqrt{x}\right) - 2\sqrt{x}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[Sqrt[x]]^2/Sqrt[x],x]
```

```
[Out] -2*Sqrt[x] + 2*Tan[Sqrt[x]]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3747

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Tan[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Tan[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IGtQ[Simplify[(m +
1)/n], 0] && IntegerQ[p]
```

Rubi steps

$$\int \frac{\tan^2\left(\sqrt{x}\right)}{\sqrt{x}}\,dx = 2\,\text{Subst}\left(\int \tan^2(x)\,dx, x, \sqrt{x}\right)$$
$$= 2\tan\left(\sqrt{x}\right) - 2\,\text{Subst}\left(\int 1\,dx, x, \sqrt{x}\right)$$
$$= -2\sqrt{x} + 2\tan\left(\sqrt{x}\right)$$

Mathematica [**A**] time = 0.04, size = 18, normalized size = 1.12

$$2\tan\left(\sqrt{x}\right) - 2\tan^{-1}\left(\tan\left(\sqrt{x}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[Sqrt[x]]^2/Sqrt[x],x]
[Out] -2*ArcTan[Tan[Sqrt[x]]] + 2*Tan[Sqrt[x]]
```

fricas [**A**] time = 1.97, size = 12, normalized size = 0.75

$$-2\sqrt{x} + 2\tan\left(\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x^(1/2))^2/x^(1/2),x, algorithm="fricas")
[Out] -2*sqrt(x) + 2*tan(sqrt(x))
```

giac [**A**] time = 0.12, size = 12, normalized size = 0.75

$$-2\sqrt{x} + 2\tan\left(\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x^(1/2))^2/x^(1/2),x, algorithm="giac")
[Out] -2*sqrt(x) + 2*tan(sqrt(x))
```

maple [**A**] time = 0.01, size = 13, normalized size = 0.81

$$-2\sqrt{x} + 2\tan\left(\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(x^(1/2))^2/x^(1/2),x)
```

```
[Out] -2*x^(1/2)+2*tan(x^(1/2))
```

maxima [A] time = 0.54, size = 12, normalized size = 0.75

$$-2\sqrt{x} + 2\tan\left(\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x^(1/2))^2/x^(1/2),x, algorithm="maxima")
[Out] -2*sqrt(x) + 2*tan(sqrt(x))
```

mupad [B] time = 2.58, size = 20, normalized size = 1.25

$$-2\sqrt{x} + \frac{4i}{e^{\sqrt{x}\,2i} + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(x^(1/2))^2/x^(1/2),x)
[Out] 4i/(exp(x^(1/2)*2i) + 1) - 2*x^(1/2)
```

sympy [A] time = 0.18, size = 14, normalized size = 0.88

$$-2\sqrt{x} + 2\tan\left(\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x**(1/2))**2/x**(1/2),x)
[Out] -2*sqrt(x) + 2*tan(sqrt(x))
```

3.60 $\int \sqrt{x} \tan\left(\sqrt{x}\right) dx$

Optimal. Leaf size=70

$$2i\sqrt{x}\,\text{Li}_2\left(-e^{2i\sqrt{x}}\right)-\text{Li}_3\left(-e^{2i\sqrt{x}}\right)+\frac{2}{3}ix^{3/2}-2x\log\left(1+e^{2i\sqrt{x}}\right)$$

```
[Out] 2/3*I*x^(3/2)-2*x*ln(1+exp(2*I*x^(1/2)))-polylog(3,-exp(2*I*x^(1/2)))+2*I*p
olylog(2,-exp(2*I*x^(1/2)))*x^(1/2)
```

Rubi [A] time = 0.09, antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {3747, 3719, 2190, 2531, 2282, 6589}

$$2i\sqrt{x}\,\text{PolyLog}\left(2,-e^{2i\sqrt{x}}\right)-\text{PolyLog}\left(3,-e^{2i\sqrt{x}}\right)+\frac{2}{3}ix^{3/2}-2x\log\left(1+e^{2i\sqrt{x}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x]*Tan[Sqrt[x]],x]
```

```
[Out] ((2*I)/3)*x^(3/2) - 2*x*Log[1 + E^((2*I)*Sqrt[x])] + (2*I)*Sqrt[x]*PolyLog[
2, -E^((2*I)*Sqrt[x])] - PolyLog[3, -E^((2*I)*Sqrt[x])]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
```

```
, g, n}, x] && GtQ[m, 0]
```

Rule 3719

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
I*(c + d*x)^(m + 1))/(d*(m + 1)), x] - Dist[2*I, Int[((c + d*x)^m*E^(2*I*(e
 + f*x)))/(1 + E^(2*I*(e + f*x))), x], x] /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 3747

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Tan[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Tan[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IGtQ[Simplify[(m +
1)/n], 0] && IntegerQ[p]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$
\begin{aligned}
\int \sqrt{x}\tan\left(\sqrt{x}\right)\,dx &= 2\,\text{Subst}\left(\int x^2\tan(x)\,dx,x,\sqrt{x}\right)\\
&= \frac{2}{3}ix^{3/2}-4i\,\text{Subst}\left(\int \frac{e^{2ix}x^2}{1+e^{2ix}}\,dx,x,\sqrt{x}\right)\\
&= \frac{2}{3}ix^{3/2}-2x\log\left(1+e^{2i\sqrt{x}}\right)+4\,\text{Subst}\left(\int x\log\left(1+e^{2ix}\right)\,dx,x,\sqrt{x}\right)\\
&= \frac{2}{3}ix^{3/2}-2x\log\left(1+e^{2i\sqrt{x}}\right)+2i\sqrt{x}\,\text{Li}_2\left(-e^{2i\sqrt{x}}\right)-2i\,\text{Subst}\left(\int \text{Li}_2\left(-e^{2ix}\right)\,dx,x,\sqrt{x}\right)\\
&= \frac{2}{3}ix^{3/2}-2x\log\left(1+e^{2i\sqrt{x}}\right)+2i\sqrt{x}\,\text{Li}_2\left(-e^{2i\sqrt{x}}\right)-\text{Subst}\left(\int \frac{\text{Li}_2(-x)}{x}\,dx,x,e^{2i\sqrt{x}}\right)\\
&= \frac{2}{3}ix^{3/2}-2x\log\left(1+e^{2i\sqrt{x}}\right)+2i\sqrt{x}\,\text{Li}_2\left(-e^{2i\sqrt{x}}\right)-\text{Li}_3\left(-e^{2i\sqrt{x}}\right)
\end{aligned}
$$

Mathematica [A] time = 0.02, size = 70, normalized size = 1.00

$$2i\sqrt{x}\,\text{Li}_2\left(-e^{2i\sqrt{x}}\right)-\text{Li}_3\left(-e^{2i\sqrt{x}}\right)+\frac{2}{3}ix^{3/2}-2x\log\left(1+e^{2i\sqrt{x}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x]*Tan[Sqrt[x]],x]
```

```
[Out] ((2*I)/3)*x^(3/2) - 2*x*Log[1 + E^((2*I)*Sqrt[x])] + (2*I)*Sqrt[x]*PolyLog[
2, -E^((2*I)*Sqrt[x])] - PolyLog[3, -E^((2*I)*Sqrt[x])]
```

fricas [F] time = 0.86, size = 0, normalized size = 0.00

$$\text{integral}\left(\sqrt{x}\tan\left(\sqrt{x}\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)*tan(x^(1/2)),x, algorithm="fricas")
```

```
[Out] integral(sqrt(x)*tan(sqrt(x)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{x}\tan\left(\sqrt{x}\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)*tan(x^(1/2)),x, algorithm="giac")
```

```
[Out] integrate(sqrt(x)*tan(sqrt(x)), x)
```

maple [F] time = 0.10, size = 0, normalized size = 0.00

$$\int \sqrt{x}\,\tan\left(\sqrt{x}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)*tan(x^(1/2)),x)
```

```
[Out] int(x^(1/2)*tan(x^(1/2)),x)
```

maxima [A] time = 0.41, size = 80, normalized size = 1.14

$$-2ix\arctan\left(\sin\left(2\sqrt{x}\right),\cos\left(2\sqrt{x}\right)+1\right)-x\log\left(\cos\left(2\sqrt{x}\right)^{2}+\sin\left(2\sqrt{x}\right)^{2}+2\cos\left(2\sqrt{x}\right)+1\right)+\frac{2}{3}ix^{\frac{3}{2}}+2i\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)*tan(x^(1/2)),x, algorithm="maxima")
```

```
[Out] -2*I*x*arctan2(sin(2*sqrt(x)), cos(2*sqrt(x)) + 1) - x*log(cos(2*sqrt(x))^2
 + sin(2*sqrt(x))^2 + 2*cos(2*sqrt(x)) + 1) + 2/3*I*x^(3/2) + 2*I*sqrt(x)*d
ilog(-e^(2*I*sqrt(x))) - polylog(3, -e^(2*I*sqrt(x)))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \sqrt{x}\,\tan\left(\sqrt{x}\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)*tan(x^(1/2)),x)
[Out] int(x^(1/2)*tan(x^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{x}\,\tan\left(\sqrt{x}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(1/2)*tan(x**(1/2)),x)
[Out] Integral(sqrt(x)*tan(sqrt(x)), x)
```

3.61 $\int \left(\frac{b\tan\left(a+bx+cx^2\right)}{2c}+x\tan\left(a+bx+cx^2\right)\right)dx$

Optimal. Leaf size=19

$$-\frac{\log\left(\cos\left(a+bx+cx^2\right)\right)}{2c}$$

```
[Out] -1/2*ln(cos(c*x^2+b*x+a))/c
```

Rubi [A] time = 0.02, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.030, Rules used = {3763}

$$-\frac{\log\left(\cos\left(a+bx+cx^2\right)\right)}{2c}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Tan[a + b*x + c*x^2])/(2*c) + x*Tan[a + b*x + c*x^2],x]
```

```
[Out] -Log[Cos[a + b*x + c*x^2]]/(2*c)
```

Rule 3763

```
Int[((d_.) + (e_.)*(x_))*Tan[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol]
:> -Simp[(e*Log[Cos[a + b*x + c*x^2]])/(2*c), x] + Dist[(2*c*d - b*e)/(2*c)
, Int[Tan[a + b*x + c*x^2], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*
d - b*e, 0]
```

Rubi steps

$$\begin{aligned}\int\left(\frac{b\tan\left(a+bx+cx^2\right)}{2c}+x\tan\left(a+bx+cx^2\right)\right)dx&=\frac{b\int\tan\left(a+bx+cx^2\right)\,dx}{2c}+\int x\tan\left(a+bx+cx^2\right)\,dx\\&=-\frac{\log\left(\cos\left(a+bx+cx^2\right)\right)}{2c}\end{aligned}$$

Mathematica [A] time = 0.74, size = 18, normalized size = 0.95

$$-\frac{\log(\cos(a+x(b+cx)))}{2c}$$

Antiderivative was successfully verified.

[In] Integrate[(b*Tan[a + b*x + c*x^2])/(2*c) + x*Tan[a + b*x + c*x^2],x]

[Out] -1/2*Log[Cos[a + x*(b + c*x)]]/c

fricas [A] time = 1.66, size = 23, normalized size = 1.21

$$-\frac{\log\left(\frac{1}{\tan\left(cx^2+bx+a\right)^2+1}\right)}{4c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/2*b*tan(c*x^2+b*x+a)/c+x*tan(c*x^2+b*x+a),x, algorithm="fricas")

[Out] -1/4*log(1/(tan(c*x^2 + b*x + a)^2 + 1))/c

giac [A] time = 8.60, size = 18, normalized size = 0.95

$$-\frac{\log\left(\left|\cos\left(cx^2+bx+a\right)\right|\right)}{2c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/2*b*tan(c*x^2+b*x+a)/c+x*tan(c*x^2+b*x+a),x, algorithm="giac")

[Out] -1/2*log(abs(cos(c*x^2 + b*x + a)))/c

maple [A] time = 0.15, size = 18, normalized size = 0.95

$$-\frac{\ln\left(\cos\left(c\,x^2+bx+a\right)\right)}{2c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/2*b*tan(c*x^2+b*x+a)/c+x*tan(c*x^2+b*x+a),x)

[Out] -1/2*ln(cos(c*x^2+b*x+a))/c

maxima [B] time = 0.38, size = 83, normalized size = 4.37

$$-\frac{\log\left(\cos\left(2\,cx^2\right)^2+2\,\cos\left(2\,cx^2\right)\cos\left(2\,bx+2\,a\right)+\cos\left(2\,bx+2\,a\right)^2+\sin\left(2\,cx^2\right)^2-2\,\sin\left(2\,cx^2\right)\sin\left(2\,bx+2\right.\right.}{4c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/2*b*tan(c*x^2+b*x+a)/c+x*tan(c*x^2+b*x+a),x, algorithm="maxima")

```
[Out] -1/4*log(cos(2*c*x^2)^2 + 2*cos(2*c*x^2)*cos(2*b*x + 2*a) + cos(2*b*x + 2*a
)^2 + sin(2*c*x^2)^2 - 2*sin(2*c*x^2)*sin(2*b*x + 2*a) + sin(2*b*x + 2*a)^2
)/c
```

mupad [B] time = 2.51, size = 21, normalized size = 1.11

$$\frac{\ln\left(\tan\left(c\,x^2+b\,x+a\right)^2+1\right)}{4\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*tan(a + b*x + c*x^2) + (b*tan(a + b*x + c*x^2))/(2*c),x)
```

```
[Out] log(tan(a + b*x + c*x^2)^2 + 1)/(4*c)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int b\tan\left(a+bx+cx^2\right)dx+\int 2cx\tan\left(a+bx+cx^2\right)dx}{2c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/2*b*tan(c*x**2+b*x+a)/c+x*tan(c*x**2+b*x+a),x)
```

```
[Out] (Integral(b*tan(a + b*x + c*x**2), x) + Integral(2*c*x*tan(a + b*x + c*x**2
), x))/(2*c)
```

3.62 $\int \frac{\cot^2\left(\sqrt{x}\right)}{\sqrt{x}}\,dx$

Optimal. Leaf size=16

$$-2\sqrt{x}-2\cot\left(\sqrt{x}\right)$$

```
[Out] -2*cot(x^(1/2))-2*x^(1/2)
```

Rubi [A] time = 0.02, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {3748, 3473, 8}

$$-2\sqrt{x}-2\cot\left(\sqrt{x}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Cot[Sqrt[x]]^2/Sqrt[x],x]
```

```
[Out] -2*Sqrt[x] - 2*Cot[Sqrt[x]]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3748

```
Int[((a_.) + Cot[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Cot[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IGtQ[Simplify[(m +
1)/n], 0] && IntegerQ[p]
```

Rubi steps

$$\int \frac{\cot^2\left(\sqrt{x}\right)}{\sqrt{x}}\,dx = 2\,\text{Subst}\left(\int \cot^2(x)\,dx, x, \sqrt{x}\right)$$
$$= -2\cot\left(\sqrt{x}\right) - 2\,\text{Subst}\left(\int 1\,dx, x, \sqrt{x}\right)$$
$$= -2\sqrt{x} - 2\cot\left(\sqrt{x}\right)$$

Mathematica [**C**] time = 0.05, size = 26, normalized size = 1.62

$$-2\cot\left(\sqrt{x}\right)\,{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};-\tan^2\left(\sqrt{x}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[Sqrt[x]]^2/Sqrt[x],x]
[Out] -2*Cot[Sqrt[x]]*Hypergeometric2F1[-1/2, 1, 1/2, -Tan[Sqrt[x]]^2]
```

fricas [**B**] time = 1.22, size = 28, normalized size = 1.75

$$-\frac{2\left(\sqrt{x}\,\sin\left(2\sqrt{x}\right) + \cos\left(2\sqrt{x}\right) + 1\right)}{\sin\left(2\sqrt{x}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x^(1/2))^2/x^(1/2),x, algorithm="fricas")
[Out] -2*(sqrt(x)*sin(2*sqrt(x)) + cos(2*sqrt(x)) + 1)/sin(2*sqrt(x))
```

giac [**A**] time = 0.13, size = 22, normalized size = 1.38

$$-2\sqrt{x} - \frac{1}{\tan\left(\frac{1}{2}\sqrt{x}\right)} + \tan\left(\frac{1}{2}\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x^(1/2))^2/x^(1/2),x, algorithm="giac")
[Out] -2*sqrt(x) - 1/tan(1/2*sqrt(x)) + tan(1/2*sqrt(x))
```

maple [**A**] time = 0.01, size = 14, normalized size = 0.88

$$-2\cot\left(\sqrt{x}\right) + \pi - 2\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(x^(1/2))^2/x^(1/2),x)
```

```
[Out] -2*cot(x^(1/2))+Pi-2*x^(1/2)
```

maxima [A] time = 0.48, size = 14, normalized size = 0.88

$$-2\sqrt{x} - \frac{2}{\tan\left(\sqrt{x}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x^(1/2))^2/x^(1/2),x, algorithm="maxima")
```

```
[Out] -2*sqrt(x) - 2/tan(sqrt(x))
```

mupad [B] time = 2.55, size = 20, normalized size = 1.25

$$-2\sqrt{x} - \frac{4i}{e^{\sqrt{x}\,2i} - 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(x^(1/2))^2/x^(1/2),x)
```

```
[Out] - 4i/(exp(x^(1/2)*2i) - 1) - 2*x^(1/2)
```

sympy [A] time = 0.17, size = 15, normalized size = 0.94

$$-2\sqrt{x} - 2\cot\left(\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x**(1/2))**2/x**(1/2),x)
```

```
[Out] -2*sqrt(x) - 2*cot(sqrt(x))
```

3.63 $\int \frac{\sqrt{a+b\sec(c+dx)}}{1+\cos(c+dx)}\,dx$

Optimal. Leaf size=92

$$\frac{\sqrt{\frac{1}{\sec(c+dx)+1}}\,\sqrt{a+b\sec(c+dx)}\,E\left(\sin^{-1}\left(\frac{\tan(c+dx)}{\sec(c+dx)+1}\right)|\frac{a-b}{a+b}\right)}{d\sqrt{\frac{a+b\sec(c+dx)}{(a+b)(\sec(c+dx)+1)}}}$$

```
[Out] EllipticE(tan(d*x+c)/(1+sec(d*x+c)),((a-b)/(a+b))^(1/2))*(1/(1+sec(d*x+c)))
^(1/2)*(a+b*sec(d*x+c))^(1/2)/d/((a+b*sec(d*x+c))/(a+b)/(1+sec(d*x+c)))^(1/
2)
```

Rubi [A] time = 0.16, antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {2829, 3968}

$$\frac{\sqrt{\frac{1}{\sec(c+dx)+1}}\,\sqrt{a+b\sec(c+dx)}\,E\left(\sin^{-1}\left(\frac{\tan(c+dx)}{\sec(c+dx)+1}\right)|\frac{a-b}{a+b}\right)}{d\sqrt{\frac{a+b\sec(c+dx)}{(a+b)(\sec(c+dx)+1)}}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*Sec[c + d*x]]/(1 + Cos[c + d*x]),x]
```

```
[Out] (EllipticE[ArcSin[Tan[c + d*x]/(1 + Sec[c + d*x])], (a - b)/(a + b)]*Sqrt[(
1 + Sec[c + d*x])^(-1)]*Sqrt[a + b*Sec[c + d*x]])/(d*Sqrt[(a + b*Sec[c + d*
x])/((a + b)*(1 + Sec[c + d*x]))])
```

Rule 2829

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_)*((a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)])^(m_.), x_Symbol] :> Int[((b + a*Csc[e + f*x])^m*(c + d*Csc[e +
f*x])^n)/Csc[e + f*x]^m, x] /; FreeQ[{a, b, c, d, e, f, n}, x] &&  !Integer
Q[n] && IntegerQ[m]
```

Rule 3968

```
Int[(csc[(e_.) + (f_.)*(x_)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)])/(c
sc[(e_.) + (f_.)*(x_)]*(d_.) + (c_)), x_Symbol] :> -Simp[(Sqrt[a + b*Csc[e
+ f*x]]*Sqrt[c/(c + d*Csc[e + f*x])]*EllipticE[ArcSin[(c*Cot[e + f*x])/(c +
 d*Csc[e + f*x])], -((b*c - a*d)/(b*c + a*d))])/(d*f*Sqrt[(c*d*(a + b*Csc[e
 + f*x]))/((b*c + a*d)*(c + d*Csc[e + f*x]))])), x] /; FreeQ[{a, b, c, d, e,
```

```
f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && EqQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{\sqrt{a+b\sec(c+dx)}}{1+\cos(c+dx)}\,dx = \int \frac{\sec(c+dx)\sqrt{a+b\sec(c+dx)}}{1+\sec(c+dx)}\,dx$$
$$= \frac{E\left(\sin^{-1}\left(\frac{\tan(c+dx)}{1+\sec(c+dx)}\right)|\frac{a-b}{a+b}\right)\sqrt{\frac{1}{1+\sec(c+dx)}}\sqrt{a+b\sec(c+dx)}}{d\sqrt{\frac{a+b\sec(c+dx)}{(a+b)(1+\sec(c+dx))}}}$$

Mathematica [A] time = 7.22, size = 85, normalized size = 0.92

$$\frac{\sqrt{\frac{1}{\sec(c+dx)+1}}\sqrt{a+b\sec(c+dx)}\,E\left(\sin^{-1}\left(\tan\left(\frac{1}{2}(c+dx)\right)\right)|\frac{a-b}{a+b}\right)}{d\sqrt{\frac{a\cos(c+dx)+b}{(a+b)(\cos(c+dx)+1)}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*Sec[c + d*x]]/(1 + Cos[c + d*x]),x]
```

```
[Out] (EllipticE[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*Sqrt[(1 + Sec[c + d*x
])^(-1)]*Sqrt[a + b*Sec[c + d*x]])/(d*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1
 + Cos[c + d*x]))])
```

fricas [F] time = 1.20, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{b\sec(dx+c)+a}}{\cos(dx+c)+1}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(1/2)/(1+cos(d*x+c)),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*sec(d*x + c) + a)/(cos(d*x + c) + 1), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{b\sec(dx+c)+a}}{\cos(dx+c)+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(1/2)/(1+cos(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*sec(d*x + c) + a)/(cos(d*x + c) + 1), x)
```

maple [A] time = 0.62, size = 150, normalized size = 1.63

$$-\frac{\mathrm{EllipticE}\left(\frac{\cos(dx+c)-1}{\sin(dx+c)}, \sqrt{\frac{a-b}{a+b}}\right)\sqrt{\frac{a\cos(dx+c)+b}{(1+\cos(dx+c))(a+b)}}\,\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\,\left(\cos\left(dx+c\right)-1\right)\sqrt{\frac{a\cos(dx+c)+b}{\cos(dx+c)}}\,\left(1+\cos\left(dx+c\right.\right.}{d\left(a\cos\left(dx+c\right)+b\right)\sin\left(dx+c\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))^(1/2)/(1+cos(d*x+c)),x)
```

```
[Out] -1/d*EllipticE((cos(d*x+c)-1)/sin(d*x+c),((a-b)/(a+b))^(1/2))*((a*cos(d*x+c
)+b)/(1+cos(d*x+c))/(a+b))^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(cos(d*x
+c)-1)*((a*cos(d*x+c)+b)/cos(d*x+c))^(1/2)*(1+cos(d*x+c))^2/(a*cos(d*x+c)+b
)/sin(d*x+c)^2*(-a-b)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{b\sec\left(dx+c\right)+a}}{\cos\left(dx+c\right)+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(1/2)/(1+cos(d*x+c)),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*sec(d*x + c) + a)/(cos(d*x + c) + 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{a+\frac{b}{\cos(c+d\,x)}}}{\cos\left(c+d\,x\right)+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(c + d*x))^(1/2)/(cos(c + d*x) + 1),x)
```

```
[Out] int((a + b/cos(c + d*x))^(1/2)/(cos(c + d*x) + 1), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a+b\sec\left(c+dx\right)}}{\cos\left(c+dx\right)+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))**(1/2)/(1+cos(d*x+c)),x)
[Out] Integral(sqrt(a + b*sec(c + d*x))/(cos(c + d*x) + 1), x)
```

3.64 $\int \sec(a+bx)\sec(2a+2bx)\,dx$

Optimal. Leaf size=35

$$\frac{\sqrt{2}\tanh^{-1}\left(\sqrt{2}\sin(a+bx)\right)}{b}-\frac{\tanh^{-1}(\sin(a+bx))}{b}$$

```
[Out] -arctanh(sin(b*x+a))/b+arctanh(sin(b*x+a)*2^(1/2))*2^(1/2)/b
```

Rubi [A] time = 0.03, antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {4364, 1093, 207}

$$\frac{\sqrt{2}\tanh^{-1}\left(\sqrt{2}\sin(a+bx)\right)}{b}-\frac{\tanh^{-1}(\sin(a+bx))}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[a + b*x]*Sec[2*a + 2*b*x],x]
```

```
[Out] -(ArcTanh[Sin[a + b*x]]/b) + (Sqrt[2]*ArcTanh[Sqrt[2]*Sin[a + b*x]])/b
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 1093

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(-1), x_Symbol] :> With[{q = Rt[b^
2 - 4*a*c, 2]}, Dist[c/q, Int[1/(b/2 - q/2 + c*x^2), x], x] - Dist[c/q, Int
[1/(b/2 + q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c,
0] && PosQ[b^2 - 4*a*c]
```

Rule 4364

```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))]^(n_), x_Symbol] :> With[{d = Free
Factors[Sin[c*(a + b*x)], x]}, Dist[d/(b*c), Subst[Int[SubstFor[(1 - d^2*x^
2)^((n - 1)/2), Sin[c*(a + b*x)]/d, u, x], x], x, Sin[c*(a + b*x)]/d], x] /
; FunctionOfQ[Sin[c*(a + b*x)]/d, u, x]] /; FreeQ[{a, b, c}, x] && IntegerQ
[(n - 1)/2] && NonsumQ[u] && (EqQ[F, Cos] || EqQ[F, cos])
```

Rubi steps

$$\int \sec(a+bx)\sec(2a+2bx)\,dx = \frac{\text{Subst}\left(\int \frac{1}{1-3x^2+2x^4}\,dx, x, \sin(a+bx)\right)}{b}$$
$$= \frac{2\,\text{Subst}\left(\int \frac{1}{-2+2x^2}\,dx, x, \sin(a+bx)\right)}{b} - \frac{2\,\text{Subst}\left(\int \frac{1}{-1+2x^2}\,dx, x, \sin(a+bx)\right)}{b}$$
$$= -\frac{\tanh^{-1}(\sin(a+bx))}{b} + \frac{\sqrt{2}\tanh^{-1}\left(\sqrt{2}\sin(a+bx)\right)}{b}$$

Mathematica [A] time = 0.04, size = 35, normalized size = 1.00

$$\frac{\sqrt{2}\tanh^{-1}\left(\sqrt{2}\sin(a+bx)\right)}{b} - \frac{\tanh^{-1}(\sin(a+bx))}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[a + b*x]*Sec[2*a + 2*b*x],x]
[Out] -(ArcTanh[Sin[a + b*x]]/b) + (Sqrt[2]*ArcTanh[Sqrt[2]*Sin[a + b*x]])/b
```

fricas [B] time = 0.94, size = 72, normalized size = 2.06

$$\frac{\sqrt{2}\log\left(-\frac{2\cos(bx+a)^2-2\sqrt{2}\sin(bx+a)-3}{2\cos(bx+a)^2-1}\right) - \log(\sin(bx+a)+1) + \log(-\sin(bx+a)+1)}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)*sec(2*b*x+2*a),x, algorithm="fricas")
[Out] 1/2*(sqrt(2)*log(-(2*cos(b*x + a)^2 - 2*sqrt(2)*sin(b*x + a) - 3)/(2*cos(b*
x + a)^2 - 1)) - log(sin(b*x + a) + 1) + log(-sin(b*x + a) + 1))/b
```

giac [B] time = 1.15, size = 948, normalized size = 27.09

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)*sec(2*b*x+2*a),x, algorithm="giac")
[Out] 1/2*(sqrt(2)*log(abs(2*tan(1/2*b*x + 2*a)*tan(1/2*a)^6 + 12*tan(1/2*b*x + 2
*a)*tan(1/2*a)^5 - 2*tan(1/2*a)^6 - 30*tan(1/2*b*x + 2*a)*tan(1/2*a)^4 + 12
*tan(1/2*a)^5 - 40*tan(1/2*b*x + 2*a)*tan(1/2*a)^3 + 30*tan(1/2*a)^4 + 30*t
```

```
an(1/2*b*x + 2*a)*tan(1/2*a)^2 - 40*tan(1/2*a)^3 + 12*tan(1/2*b*x + 2*a)*ta
n(1/2*a) - 30*tan(1/2*a)^2 - 2*sqrt(2)*(tan(1/2*a)^6 + 3*tan(1/2*a)^4 + 3*t
an(1/2*a)^2 + 1) - 2*tan(1/2*b*x + 2*a) + 12*tan(1/2*a) + 2)/abs(2*tan(1/2*
b*x + 2*a)*tan(1/2*a)^6 + 12*tan(1/2*b*x + 2*a)*tan(1/2*a)^5 - 2*tan(1/2*a)
^6 - 30*tan(1/2*b*x + 2*a)*tan(1/2*a)^4 + 12*tan(1/2*a)^5 - 40*tan(1/2*b*x
+ 2*a)*tan(1/2*a)^3 + 30*tan(1/2*a)^4 + 30*tan(1/2*b*x + 2*a)*tan(1/2*a)^2
- 40*tan(1/2*a)^3 + 12*tan(1/2*b*x + 2*a)*tan(1/2*a) - 30*tan(1/2*a)^2 + 2*
sqrt(2)*(tan(1/2*a)^6 + 3*tan(1/2*a)^4 + 3*tan(1/2*a)^2 + 1) - 2*tan(1/2*b*
x + 2*a) + 12*tan(1/2*a) + 2)) + sqrt(2)*log(abs(2*tan(1/2*b*x + 2*a)*tan(1
/2*a)^6 - 12*tan(1/2*b*x + 2*a)*tan(1/2*a)^5 + 2*tan(1/2*a)^6 - 30*tan(1/2*
b*x + 2*a)*tan(1/2*a)^4 + 12*tan(1/2*a)^5 + 40*tan(1/2*b*x + 2*a)*tan(1/2*a
)^3 - 30*tan(1/2*a)^4 + 30*tan(1/2*b*x + 2*a)*tan(1/2*a)^2 - 40*tan(1/2*a)^
3 - 12*tan(1/2*b*x + 2*a)*tan(1/2*a) + 30*tan(1/2*a)^2 - 2*sqrt(2)*(tan(1/2
*a)^6 + 3*tan(1/2*a)^4 + 3*tan(1/2*a)^2 + 1) - 2*tan(1/2*b*x + 2*a) + 12*ta
n(1/2*a) - 2)/abs(2*tan(1/2*b*x + 2*a)*tan(1/2*a)^6 - 12*tan(1/2*b*x + 2*a)
*tan(1/2*a)^5 + 2*tan(1/2*a)^6 - 30*tan(1/2*b*x + 2*a)*tan(1/2*a)^4 + 12*ta
n(1/2*a)^5 + 40*tan(1/2*b*x + 2*a)*tan(1/2*a)^3 - 30*tan(1/2*a)^4 + 30*tan(
1/2*b*x + 2*a)*tan(1/2*a)^2 - 40*tan(1/2*a)^3 - 12*tan(1/2*b*x + 2*a)*tan(1
/2*a) + 30*tan(1/2*a)^2 + 2*sqrt(2)*(tan(1/2*a)^6 + 3*tan(1/2*a)^4 + 3*tan(
1/2*a)^2 + 1) - 2*tan(1/2*b*x + 2*a) + 12*tan(1/2*a) - 2)) - 2*log(abs(tan(
1/2*b*x + 2*a)*tan(1/2*a)^3 + 3*tan(1/2*b*x + 2*a)*tan(1/2*a)^2 - tan(1/2*a
)^3 - 3*tan(1/2*b*x + 2*a)*tan(1/2*a) + 3*tan(1/2*a)^2 - tan(1/2*b*x + 2*a)
 + 3*tan(1/2*a) - 1)) + 2*log(abs(tan(1/2*b*x + 2*a)*tan(1/2*a)^3 - 3*tan(1
/2*b*x + 2*a)*tan(1/2*a)^2 + tan(1/2*a)^3 - 3*tan(1/2*b*x + 2*a)*tan(1/2*a)
 + 3*tan(1/2*a)^2 + tan(1/2*b*x + 2*a) - 3*tan(1/2*a) - 1)))/b
```

maple [A] time = 0.17, size = 48, normalized size = 1.37

$$\frac{\ln(\sin(bx+a)-1)}{2b}+\frac{\operatorname{arctanh}\left(\sin(bx+a)\sqrt{2}\right)\sqrt{2}}{b}-\frac{\ln(1+\sin(bx+a))}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(b*x+a)*sec(2*b*x+2*a),x)
```

```
[Out] 1/2/b*ln(sin(b*x+a)-1)+arctanh(sin(b*x+a)*2^(1/2))*2^(1/2)/b-1/2/b*ln(1+sin
(b*x+a))
```

maxima [B] time = 108.04, size = 6257, normalized size = 178.77

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)*sec(2*b*x+2*a),x, algorithm="maxima")
```

```
[Out] -1/8*(2*(sqrt(2)*cos(3*a)*cos(3/4*arctan2(sin(4*a), cos(4*a))) - sqrt(2)*co
s(a)*cos(1/4*arctan2(sin(4*a), cos(4*a))) + sqrt(2)*sin(3*a)*sin(3/4*arctan
2(sin(4*a), cos(4*a))) - sqrt(2)*sin(a)*sin(1/4*arctan2(sin(4*a), cos(4*a))
))*arctan2(sqrt(2)*cos(1/4*arctan2(sin(4*a), cos(4*a)))*sin(b*x) + sqrt(2)*
cos(b*x)*sin(1/4*arctan2(sin(4*a), cos(4*a))) + cos(1/2*arctan2(sin(4*a), c
os(4*a)))*sin(2*b*x) + cos(2*b*x)*sin(1/2*arctan2(sin(4*a), cos(4*a))), sqr
t(2)*cos(b*x)*cos(1/4*arctan2(sin(4*a), cos(4*a))) - sqrt(2)*sin(b*x)*sin(1
/4*arctan2(sin(4*a), cos(4*a))) + cos(2*b*x)*cos(1/2*arctan2(sin(4*a), cos(
4*a))) - sin(2*b*x)*sin(1/2*arctan2(sin(4*a), cos(4*a))) + 1) - 2*(sqrt(2)*
cos(3*a)*cos(3/4*arctan2(sin(4*a), cos(4*a))) - sqrt(2)*cos(a)*cos(1/4*arct
an2(sin(4*a), cos(4*a))) + sqrt(2)*sin(3*a)*sin(3/4*arctan2(sin(4*a), cos(4
*a))) - sqrt(2)*sin(a)*sin(1/4*arctan2(sin(4*a), cos(4*a))))*arctan2(-sqrt(
2)*cos(1/4*arctan2(sin(4*a), cos(4*a)))*sin(b*x) - sqrt(2)*cos(b*x)*sin(1/4
*arctan2(sin(4*a), cos(4*a))) + cos(1/2*arctan2(sin(4*a), cos(4*a)))*sin(2*
b*x) + cos(2*b*x)*sin(1/2*arctan2(sin(4*a), cos(4*a))), -sqrt(2)*cos(b*x)*c
os(1/4*arctan2(sin(4*a), cos(4*a))) + sqrt(2)*sin(b*x)*sin(1/4*arctan2(sin(
4*a), cos(4*a))) + cos(2*b*x)*cos(1/2*arctan2(sin(4*a), cos(4*a))) - sin(2*
b*x)*sin(1/2*arctan2(sin(4*a), cos(4*a))) + 1) - 2*((sqrt(2)*cos(3*a)*cos(1
/2*arctan2(sin(4*a), cos(4*a))) + sqrt(2)*sin(3*a)*sin(1/2*arctan2(sin(4*a)
, cos(4*a))) + sqrt(2)*cos(a))*cos(1/2*pi + 1/4*arctan2(sin(4*a), cos(4*a))
) + (sqrt(2)*cos(1/2*arctan2(sin(4*a), cos(4*a)))*sin(3*a) - sqrt(2)*cos(3*
a)*sin(1/2*arctan2(sin(4*a), cos(4*a))) + sqrt(2)*sin(a))*sin(1/2*pi + 1/4*
arctan2(sin(4*a), cos(4*a))))*arctan2(-2*((sqrt(2)*cos(1/2*arctan2(sin(4*a)
, cos(4*a)))*sin(1/2*pi + 1/4*arctan2(sin(4*a), cos(4*a))) - sqrt(2)*cos(1/
2*pi + 1/4*arctan2(sin(4*a), cos(4*a)))*sin(1/2*arctan2(sin(4*a), cos(4*a))
))*cos(b*x) + cos(1/4*arctan2(sin(4*a), cos(4*a)))*sin(1/2*pi + 1/4*arctan2
(sin(4*a), cos(4*a))) - (sqrt(2)*cos(1/2*pi + 1/4*arctan2(sin(4*a), cos(4*a
)))*cos(1/2*arctan2(sin(4*a), cos(4*a))) + sqrt(2)*sin(1/2*pi + 1/4*arctan2
(sin(4*a), cos(4*a)))*sin(1/2*arctan2(sin(4*a), cos(4*a))))*sin(b*x) - cos(
1/2*pi + 1/4*arctan2(sin(4*a), cos(4*a)))*sin(1/4*arctan2(sin(4*a), cos(4*a
))))/(2*(cos(1/2*arctan2(sin(4*a), cos(4*a)))^2 + sin(1/2*arctan2(sin(4*a),
 cos(4*a)))^2)*cos(b*x)^2 + 2*(cos(1/2*arctan2(sin(4*a), cos(4*a)))^2 + sin
(1/2*arctan2(sin(4*a), cos(4*a)))^2)*sin(b*x)^2 + cos(1/2*pi + 1/4*arctan2(
sin(4*a), cos(4*a)))^2 + 2*(sqrt(2)*cos(1/2*pi + 1/4*arctan2(sin(4*a), cos(
4*a)))*cos(1/2*arctan2(sin(4*a), cos(4*a))) + sqrt(2)*cos(1/2*arctan2(sin(4
*a), cos(4*a)))*cos(1/4*arctan2(sin(4*a), cos(4*a))) + sqrt(2)*sin(1/2*pi +
 1/4*arctan2(sin(4*a), cos(4*a)))*sin(1/2*arctan2(sin(4*a), cos(4*a))) + sq
rt(2)*sin(1/2*arctan2(sin(4*a), cos(4*a)))*sin(1/4*arctan2(sin(4*a), cos(4*
a))))*cos(b*x) + 2*cos(1/2*pi + 1/4*arctan2(sin(4*a), cos(4*a)))*cos(1/4*ar
ctan2(sin(4*a), cos(4*a))) + cos(1/4*arctan2(sin(4*a), cos(4*a)))^2 + sin(1
/2*pi + 1/4*arctan2(sin(4*a), cos(4*a)))^2 + 2*(sqrt(2)*cos(1/2*arctan2(sin
(4*a), cos(4*a)))*sin(1/2*pi + 1/4*arctan2(sin(4*a), cos(4*a))) - sqrt(2)*c
os(1/2*pi + 1/4*arctan2(sin(4*a), cos(4*a)))*sin(1/2*arctan2(sin(4*a), cos(
4*a))) - sqrt(2)*cos(1/4*arctan2(sin(4*a), cos(4*a)))*sin(1/2*arctan2(sin(4
*a), cos(4*a))) + sqrt(2)*cos(1/2*arctan2(sin(4*a), cos(4*a)))*sin(1/4*arct
```

```
an2(sin(4*a), cos(4*a))))*sin(b*x) + 2*sin(1/2*pi + 1/4*arctan2(sin(4*a), c
os(4*a)))*sin(1/4*arctan2(sin(4*a), cos(4*a))) + sin(1/4*arctan2(sin(4*a),
cos(4*a)))^2), (2*(cos(1/2*arctan2(sin(4*a), cos(4*a)))^2 + sin(1/2*arctan2
(sin(4*a), cos(4*a)))^2)*cos(b*x)^2 + 2*(cos(1/2*arctan2(sin(4*a), cos(4*a)
))^2 + sin(1/2*arctan2(sin(4*a), cos(4*a)))^2)*sin(b*x)^2 - cos(1/2*pi + 1/
4*arctan2(sin(4*a), cos(4*a)))^2 + 2*(sqrt(2)*cos(1/2*arctan2(sin(4*a), cos
(4*a)))*cos(1/4*arctan2(sin(4*a), cos(4*a))) + sqrt(2)*sin(1/2*arctan2(sin(
4*a), cos(4*a)))*sin(1/4*arctan2(sin(4*a), cos(4*a))))*cos(b*x) + cos(1/4*a
rctan2(sin(4*a), cos(4*a)))^2 - sin(1/2*pi + 1/4*arctan2(sin(4*a), cos(4*a)
))^2 - 2*(sqrt(2)*cos(1/4*arctan2(sin(4*a), cos(4*a)))*sin(1/2*arctan2(sin(
4*a), cos(4*a))) - sqrt(2)*cos(1/2*arctan2(sin(4*a), cos(4*a)))*sin(1/4*arc
tan2(sin(4*a), cos(4*a))))*sin(b*x) + sin(1/4*arctan2(sin(4*a), cos(4*a)))^
2)/(2*(cos(1/2*arctan2(sin(4*a), cos(4*a)))^2 + sin(1/2*arctan2(sin(4*a), c
os(4*a)))^2)*cos(b*x)^2 + 2*(cos(1/2*arctan2(sin(4*a), cos(4*a)))^2 + sin(1
/2*arctan2(sin(4*a), cos(4*a)))^2)*sin(b*x)^2 + cos(1/2*pi + 1/4*arctan2(si
n(4*a), cos(4*a)))^2 + 2*(sqrt(2)*cos(1/2*pi + 1/4*arctan2(sin(4*a), cos(4*
a)))*cos(1/2*arctan2(sin(4*a), cos(4*a))) + sqrt(2)*cos(1/2*arctan2(sin(4*a
), cos(4*a)))*cos(1/4*arctan2(sin(4*a), cos(4*a))) + sqrt(2)*sin(1/2*pi + 1
/4*arctan2(sin(4*a), cos(4*a)))*sin(1/2*arctan2(sin(4*a), cos(4*a))) + sqrt
(2)*sin(1/2*arctan2(sin(4*a), cos(4*a)))*sin(1/4*arctan2(sin(4*a), cos(4*a)
)))*cos(b*x) + 2*cos(1/2*pi + 1/4*arctan2(sin(4*a), cos(4*a)))*cos(1/4*arct
an2(sin(4*a), cos(4*a))) + cos(1/4*arctan2(sin(4*a), cos(4*a)))^2 + sin(1/2
*pi + 1/4*arctan2(sin(4*a), cos(4*a)))^2 + 2*(sqrt(2)*cos(1/2*arctan2(sin(4
*a), cos(4*a)))*sin(1/2*pi + 1/4*arctan2(sin(4*a), cos(4*a))) - sqrt(2)*cos
(1/2*pi + 1/4*arctan2(sin(4*a), cos(4*a)))*sin(1/2*arctan2(sin(4*a), cos(4*
a))) - sqrt(2)*cos(1/4*arctan2(sin(4*a), cos(4*a)))*sin(1/2*arctan2(sin(4*a
), cos(4*a))) + sqrt(2)*cos(1/2*arctan2(sin(4*a), cos(4*a)))*sin(1/4*arctan
2(sin(4*a), cos(4*a))))*sin(b*x) + 2*sin(1/2*pi + 1/4*arctan2(sin(4*a), cos
(4*a)))*sin(1/4*arctan2(sin(4*a), cos(4*a))) + sin(1/4*arctan2(sin(4*a), co
s(4*a)))^2)) - 2*((sqrt(2)*cos(3*a)*cos(1/2*arctan2(sin(4*a), cos(4*a))) +
sqrt(2)*sin(3*a)*sin(1/2*arctan2(sin(4*a), cos(4*a))) + sqrt(2)*cos(a))*cos
(1/2*pi + 1/4*arctan2(sin(4*a), cos(4*a))) + (sqrt(2)*cos(1/2*arctan2(sin(4
*a), cos(4*a)))*sin(3*a) - sqrt(2)*cos(3*a)*sin(1/2*arctan2(sin(4*a), cos(4
*a))) + sqrt(2)*sin(a))*sin(1/2*pi + 1/4*arctan2(sin(4*a), cos(4*a))))*arct
an2(-2*((sqrt(2)*cos(1/2*arctan2(sin(4*a), cos(4*a)))*sin(1/2*pi + 1/4*arct
an2(sin(4*a), cos(4*a))) - sqrt(2)*cos(1/2*pi + 1/4*arctan2(sin(4*a), cos(4
*a)))*sin(1/2*arctan2(sin(4*a), cos(4*a))))*cos(b*x) - cos(1/4*arctan2(sin(
4*a), cos(4*a)))*sin(1/2*pi + 1/4*arctan2(sin(4*a), cos(4*a))) - (sqrt(2)*c
os(1/2*pi + 1/4*arctan2(sin(4*a), cos(4*a)))*cos(1/2*arctan2(sin(4*a), cos(
4*a))) + sqrt(2)*sin(1/2*pi + 1/4*arctan2(sin(4*a), cos(4*a)))*sin(1/2*arct
an2(sin(4*a), cos(4*a))))*sin(b*x) + cos(1/2*pi + 1/4*arctan2(sin(4*a), cos
(4*a)))*sin(1/4*arctan2(sin(4*a), cos(4*a))))/(2*(cos(1/2*arctan2(sin(4*a),
 cos(4*a)))^2 + sin(1/2*arctan2(sin(4*a), cos(4*a)))^2)*cos(b*x)^2 + 2*(cos
(1/2*arctan2(sin(4*a), cos(4*a)))^2 + sin(1/2*arctan2(sin(4*a), cos(4*a)))^
2)*sin(b*x)^2 + cos(1/2*pi + 1/4*arctan2(sin(4*a), cos(4*a)))^2 + 2*(sqrt(2
```

```
)*cos(1/2*pi + 1/4*arctan2(sin(4*a), cos(4*a)))*cos(1/2*arctan2(sin(4*a), c
os(4*a))) - sqrt(2)*cos(1/2*arctan2(sin(4*a), cos(4*a)))*cos(1/4*arctan2(si
n(4*a), cos(4*a))) + sqrt(2)*sin(1/2*pi + 1/4*arctan2(sin(4*a), cos(4*a)))*
sin(1/2*arctan2(sin(4*a), cos(4*a))) - sqrt(2)*sin(1/2*arctan2(sin(4*a), co
s(4*a)))*sin(1/4*arctan2(sin(4*a), cos(4*a))))*cos(b*x) - 2*cos(1/2*pi + 1/
4*arctan2(sin(4*a), cos(4*a)))*cos(1/4*arctan2(sin(4*a), cos(4*a))) + cos(1
/4*arctan2(sin(4*a), cos(4*a)))^2 + sin(1/2*pi + 1/4*arctan2(sin(4*a), cos(
4*a)))^2 + 2*(sqrt(2)*cos(1/2*arctan2(sin(4*a), cos(4*a)))*sin(1/2*pi + 1/4
*arctan2(sin(4*a), cos(4*a))) - sqrt(2)*cos(1/2*pi + 1/4*arctan2(sin(4*a),
cos(4*a)))*sin(1/2*arctan2(sin(4*a), cos(4*a))) + sqrt(2)*cos(1/4*arctan2(s
in(4*a), cos(4*a)))*sin(1/2*arctan2(sin(4*a), cos(4*a))) - sqrt(2)*cos(1/2*
arctan2(sin(4*a), cos(4*a)))*sin(1/4*arctan2(sin(4*a), cos(4*a))))*sin(b*x)
 - 2*sin(1/2*pi + 1/4*arctan2(sin(4*a), cos(4*a)))*sin(1/4*arctan2(sin(4*a)
, cos(4*a))) + sin(1/4*arctan2(sin(4*a), cos(4*a)))^2), (2*(cos(1/2*arctan2
(sin(4*a), cos(4*a)))^2 + sin(1/2*arctan2(sin(4*a), cos(4*a)))^2)*cos(b*x)^
2 + 2*(cos(1/2*arctan2(sin(4*a), cos(4*a)))^2 + sin(1/2*arctan2(sin(4*a), c
os(4*a)))^2)*sin(b*x)^2 - cos(1/2*pi + 1/4*arctan2(sin(4*a), cos(4*a)))^2 -
 2*(sqrt(2)*cos(1/2*arctan2(sin(4*a), cos(4*a)))*cos(1/4*arctan2(sin(4*a),
cos(4*a))) + sqrt(2)*sin(1/2*arctan2(sin(4*a), cos(4*a)))*sin(1/4*arctan2(s
in(4*a), cos(4*a))))*cos(b*x) + cos(1/4*arctan2(sin(4*a), cos(4*a)))^2 - si
n(1/2*pi + 1/4*arctan2(sin(4*a), cos(4*a)))^2 + 2*(sqrt(2)*cos(1/4*arctan2(
sin(4*a), cos(4*a)))*sin(1/2*arctan2(sin(4*a), cos(4*a))) - sqrt(2)*cos(1/2
*arctan2(sin(4*a), cos(4*a)))*sin(1/4*arctan2(sin(4*a), cos(4*a))))*sin(b*x
) + sin(1/4*arctan2(sin(4*a), cos(4*a)))^2)/(2*(cos(1/2*arctan2(sin(4*a), c
os(4*a)))^2 + sin(1/2*arctan2(sin(4*a), cos(4*a)))^2)*cos(b*x)^2 + 2*(cos(1
/2*arctan2(sin(4*a), cos(4*a)))^2 + sin(1/2*arctan2(sin(4*a), cos(4*a)))^2)
*sin(b*x)^2 + cos(1/2*pi + 1/4*arctan2(sin(4*a), cos(4*a)))^2 + 2*(sqrt(2)*
cos(1/2*pi + 1/4*arctan2(sin(4*a), cos(4*a)))*cos(1/2*arctan2(sin(4*a), cos
(4*a))) - sqrt(2)*cos(1/2*arctan2(sin(4*a), cos(4*a)))*cos(1/4*arctan2(sin(
4*a), cos(4*a))) + sqrt(2)*sin(1/2*pi + 1/4*arctan2(sin(4*a), cos(4*a)))*si
n(1/2*arctan2(sin(4*a), cos(4*a))) - sqrt(2)*sin(1/2*arctan2(sin(4*a), cos(
4*a)))*sin(1/4*arctan2(sin(4*a), cos(4*a))))*cos(b*x) - 2*cos(1/2*pi + 1/4*
arctan2(sin(4*a), cos(4*a)))*cos(1/4*arctan2(sin(4*a), cos(4*a))) + cos(1/4
*arctan2(sin(4*a), cos(4*a)))^2 + sin(1/2*pi + 1/4*arctan2(sin(4*a), cos(4*
a)))^2 + 2*(sqrt(2)*cos(1/2*arctan2(sin(4*a), cos(4*a)))*sin(1/2*pi + 1/4*a
rctan2(sin(4*a), cos(4*a))) - sqrt(2)*cos(1/2*pi + 1/4*arctan2(sin(4*a), co
s(4*a)))*sin(1/2*arctan2(sin(4*a), cos(4*a))) + sqrt(2)*cos(1/4*arctan2(sin
(4*a), cos(4*a)))*sin(1/2*arctan2(sin(4*a), cos(4*a))) - sqrt(2)*cos(1/2*ar
ctan2(sin(4*a), cos(4*a)))*sin(1/4*arctan2(sin(4*a), cos(4*a))))*sin(b*x) -
 2*sin(1/2*pi + 1/4*arctan2(sin(4*a), cos(4*a)))*sin(1/4*arctan2(sin(4*a),
cos(4*a))) + sin(1/4*arctan2(sin(4*a), cos(4*a)))^2)) + (sqrt(2)*cos(3/4*ar
ctan2(sin(4*a), cos(4*a)))*sin(3*a) - sqrt(2)*cos(1/4*arctan2(sin(4*a), cos
(4*a)))*sin(a) - sqrt(2)*cos(3*a)*sin(3/4*arctan2(sin(4*a), cos(4*a))) + sq
rt(2)*cos(a)*sin(1/4*arctan2(sin(4*a), cos(4*a))))*log((cos(1/2*arctan2(sin
(4*a), cos(4*a)))^2 + sin(1/2*arctan2(sin(4*a), cos(4*a)))^2)*cos(2*b*x)^2
```

```
+ 2*(cos(1/4*arctan2(sin(4*a), cos(4*a)))^2 + sin(1/4*arctan2(sin(4*a), cos
(4*a)))^2)*cos(b*x)^2 + (cos(1/2*arctan2(sin(4*a), cos(4*a)))^2 + sin(1/2*a
rctan2(sin(4*a), cos(4*a)))^2)*sin(2*b*x)^2 + 2*(cos(1/4*arctan2(sin(4*a),
cos(4*a)))^2 + sin(1/4*arctan2(sin(4*a), cos(4*a)))^2)*sin(b*x)^2 + 2*sqrt(
2)*cos(b*x)*cos(1/4*arctan2(sin(4*a), cos(4*a))) - 2*sqrt(2)*sin(b*x)*sin(1
/4*arctan2(sin(4*a), cos(4*a))) + 2*((sqrt(2)*cos(1/2*arctan2(sin(4*a), cos
(4*a)))*cos(1/4*arctan2(sin(4*a), cos(4*a))) + sqrt(2)*sin(1/2*arctan2(sin(
4*a), cos(4*a)))*sin(1/4*arctan2(sin(4*a), cos(4*a))))*cos(b*x) + (sqrt(2)*
cos(1/4*arctan2(sin(4*a), cos(4*a)))*sin(1/2*arctan2(sin(4*a), cos(4*a))) -
 sqrt(2)*cos(1/2*arctan2(sin(4*a), cos(4*a)))*sin(1/4*arctan2(sin(4*a), cos
(4*a))))*sin(b*x) + cos(1/2*arctan2(sin(4*a), cos(4*a))))*cos(2*b*x) - 2*((
sqrt(2)*cos(1/4*arctan2(sin(4*a), cos(4*a)))*sin(1/2*arctan2(sin(4*a), cos(
4*a))) - sqrt(2)*cos(1/2*arctan2(sin(4*a), cos(4*a)))*sin(1/4*arctan2(sin(4
*a), cos(4*a))))*cos(b*x) - (sqrt(2)*cos(1/2*arctan2(sin(4*a), cos(4*a)))*c
os(1/4*arctan2(sin(4*a), cos(4*a))) + sqrt(2)*sin(1/2*arctan2(sin(4*a), cos
(4*a)))*sin(1/4*arctan2(sin(4*a), cos(4*a))))*sin(b*x) + sin(1/2*arctan2(si
n(4*a), cos(4*a))))*sin(2*b*x) + 1) - (sqrt(2)*cos(3/4*arctan2(sin(4*a), co
s(4*a)))*sin(3*a) - sqrt(2)*cos(1/4*arctan2(sin(4*a), cos(4*a)))*sin(a) - s
qrt(2)*cos(3*a)*sin(3/4*arctan2(sin(4*a), cos(4*a))) + sqrt(2)*cos(a)*sin(1
/4*arctan2(sin(4*a), cos(4*a))))*log((cos(1/2*arctan2(sin(4*a), cos(4*a)))^
2 + sin(1/2*arctan2(sin(4*a), cos(4*a)))^2)*cos(2*b*x)^2 + 2*(cos(1/4*arcta
n2(sin(4*a), cos(4*a)))^2 + sin(1/4*arctan2(sin(4*a), cos(4*a)))^2)*cos(b*x
)^2 + (cos(1/2*arctan2(sin(4*a), cos(4*a)))^2 + sin(1/2*arctan2(sin(4*a), c
os(4*a)))^2)*sin(2*b*x)^2 + 2*(cos(1/4*arctan2(sin(4*a), cos(4*a)))^2 + sin
(1/4*arctan2(sin(4*a), cos(4*a)))^2)*sin(b*x)^2 - 2*sqrt(2)*cos(b*x)*cos(1/
4*arctan2(sin(4*a), cos(4*a))) + 2*sqrt(2)*sin(b*x)*sin(1/4*arctan2(sin(4*a
), cos(4*a))) - 2*((sqrt(2)*cos(1/2*arctan2(sin(4*a), cos(4*a)))*cos(1/4*ar
ctan2(sin(4*a), cos(4*a))) + sqrt(2)*sin(1/2*arctan2(sin(4*a), cos(4*a)))*s
in(1/4*arctan2(sin(4*a), cos(4*a))))*cos(b*x) + (sqrt(2)*cos(1/4*arctan2(si
n(4*a), cos(4*a)))*sin(1/2*arctan2(sin(4*a), cos(4*a))) - sqrt(2)*cos(1/2*a
rctan2(sin(4*a), cos(4*a)))*sin(1/4*arctan2(sin(4*a), cos(4*a))))*sin(b*x)
- cos(1/2*arctan2(sin(4*a), cos(4*a))))*cos(2*b*x) + 2*((sqrt(2)*cos(1/4*ar
ctan2(sin(4*a), cos(4*a)))*sin(1/2*arctan2(sin(4*a), cos(4*a))) - sqrt(2)*c
os(1/2*arctan2(sin(4*a), cos(4*a)))*sin(1/4*arctan2(sin(4*a), cos(4*a))))*c
os(b*x) - (sqrt(2)*cos(1/2*arctan2(sin(4*a), cos(4*a)))*cos(1/4*arctan2(sin
(4*a), cos(4*a))) + sqrt(2)*sin(1/2*arctan2(sin(4*a), cos(4*a)))*sin(1/4*ar
ctan2(sin(4*a), cos(4*a))))*sin(b*x) - sin(1/2*arctan2(sin(4*a), cos(4*a)))
)*sin(2*b*x) + 1) - ((sqrt(2)*cos(1/2*arctan2(sin(4*a), cos(4*a)))*sin(3*a)
 - sqrt(2)*cos(3*a)*sin(1/2*arctan2(sin(4*a), cos(4*a))) + sqrt(2)*sin(a))*
cos(1/2*pi + 1/4*arctan2(sin(4*a), cos(4*a))) - (sqrt(2)*cos(3*a)*cos(1/2*a
rctan2(sin(4*a), cos(4*a))) + sqrt(2)*sin(3*a)*sin(1/2*arctan2(sin(4*a), co
s(4*a))) + sqrt(2)*cos(a))*sin(1/2*pi + 1/4*arctan2(sin(4*a), cos(4*a))))*l
og(((cos(1/2*arctan2(sin(4*a), cos(4*a)))^2 + sin(1/2*arctan2(sin(4*a), cos
(4*a)))^2)*cos(b*x)^2 + (cos(1/2*arctan2(sin(4*a), cos(4*a)))^2 + sin(1/2*a
rctan2(sin(4*a), cos(4*a)))^2)*sin(b*x)^2 + ((sqrt(2)*cos(1/4*arctan2(sin(4
```

```
*a), cos(4*a))) + sqrt(2)*sin(1/4*arctan2(sin(4*a), cos(4*a))))*cos(1/2*arc
tan2(sin(4*a), cos(4*a))) - (sqrt(2)*cos(1/4*arctan2(sin(4*a), cos(4*a))) -
 sqrt(2)*sin(1/4*arctan2(sin(4*a), cos(4*a))))*sin(1/2*arctan2(sin(4*a), co
s(4*a))))*cos(b*x) + cos(1/4*arctan2(sin(4*a), cos(4*a)))^2 - ((sqrt(2)*cos
(1/4*arctan2(sin(4*a), cos(4*a))) - sqrt(2)*sin(1/4*arctan2(sin(4*a), cos(4
*a))))*cos(1/2*arctan2(sin(4*a), cos(4*a))) + (sqrt(2)*cos(1/4*arctan2(sin(
4*a), cos(4*a))) + sqrt(2)*sin(1/4*arctan2(sin(4*a), cos(4*a))))*sin(1/2*ar
ctan2(sin(4*a), cos(4*a))))*sin(b*x) + sin(1/4*arctan2(sin(4*a), cos(4*a)))
^2)/((cos(1/2*arctan2(sin(4*a), cos(4*a)))^2 + sin(1/2*arctan2(sin(4*a), co
s(4*a)))^2)*cos(b*x)^2 + (cos(1/2*arctan2(sin(4*a), cos(4*a)))^2 + sin(1/2*
arctan2(sin(4*a), cos(4*a)))^2)*sin(b*x)^2 + ((sqrt(2)*cos(1/4*arctan2(sin(
4*a), cos(4*a))) - sqrt(2)*sin(1/4*arctan2(sin(4*a), cos(4*a))))*cos(1/2*ar
ctan2(sin(4*a), cos(4*a))) + (sqrt(2)*cos(1/4*arctan2(sin(4*a), cos(4*a)))
+ sqrt(2)*sin(1/4*arctan2(sin(4*a), cos(4*a))))*sin(1/2*arctan2(sin(4*a), c
os(4*a))))*cos(b*x) + cos(1/4*arctan2(sin(4*a), cos(4*a)))^2 + ((sqrt(2)*co
s(1/4*arctan2(sin(4*a), cos(4*a))) + sqrt(2)*sin(1/4*arctan2(sin(4*a), cos(
4*a))))*cos(1/2*arctan2(sin(4*a), cos(4*a))) - (sqrt(2)*cos(1/4*arctan2(sin
(4*a), cos(4*a))) - sqrt(2)*sin(1/4*arctan2(sin(4*a), cos(4*a))))*sin(1/2*a
rctan2(sin(4*a), cos(4*a))))*sin(b*x) + sin(1/4*arctan2(sin(4*a), cos(4*a))
)^2)) - ((sqrt(2)*cos(1/2*arctan2(sin(4*a), cos(4*a)))*sin(3*a) - sqrt(2)*c
os(3*a)*sin(1/2*arctan2(sin(4*a), cos(4*a))) + sqrt(2)*sin(a))*cos(1/2*pi +
 1/4*arctan2(sin(4*a), cos(4*a))) - (sqrt(2)*cos(3*a)*cos(1/2*arctan2(sin(4
*a), cos(4*a))) + sqrt(2)*sin(3*a)*sin(1/2*arctan2(sin(4*a), cos(4*a))) + s
qrt(2)*cos(a))*sin(1/2*pi + 1/4*arctan2(sin(4*a), cos(4*a))))*log(((cos(1/2
*arctan2(sin(4*a), cos(4*a)))^2 + sin(1/2*arctan2(sin(4*a), cos(4*a)))^2)*c
os(b*x)^2 + (cos(1/2*arctan2(sin(4*a), cos(4*a)))^2 + sin(1/2*arctan2(sin(4
*a), cos(4*a)))^2)*sin(b*x)^2 - ((sqrt(2)*cos(1/4*arctan2(sin(4*a), cos(4*a
))) - sqrt(2)*sin(1/4*arctan2(sin(4*a), cos(4*a))))*cos(1/2*arctan2(sin(4*a
), cos(4*a))) + (sqrt(2)*cos(1/4*arctan2(sin(4*a), cos(4*a))) + sqrt(2)*sin
(1/4*arctan2(sin(4*a), cos(4*a))))*sin(1/2*arctan2(sin(4*a), cos(4*a))))*co
s(b*x) + cos(1/4*arctan2(sin(4*a), cos(4*a)))^2 - ((sqrt(2)*cos(1/4*arctan2
(sin(4*a), cos(4*a))) + sqrt(2)*sin(1/4*arctan2(sin(4*a), cos(4*a))))*cos(1
/2*arctan2(sin(4*a), cos(4*a))) - (sqrt(2)*cos(1/4*arctan2(sin(4*a), cos(4*
a))) - sqrt(2)*sin(1/4*arctan2(sin(4*a), cos(4*a))))*sin(1/2*arctan2(sin(4*
a), cos(4*a))))*sin(b*x) + sin(1/4*arctan2(sin(4*a), cos(4*a)))^2)/((cos(1/
2*arctan2(sin(4*a), cos(4*a)))^2 + sin(1/2*arctan2(sin(4*a), cos(4*a)))^2)*
cos(b*x)^2 + (cos(1/2*arctan2(sin(4*a), cos(4*a)))^2 + sin(1/2*arctan2(sin(
4*a), cos(4*a)))^2)*sin(b*x)^2 - ((sqrt(2)*cos(1/4*arctan2(sin(4*a), cos(4*
a))) + sqrt(2)*sin(1/4*arctan2(sin(4*a), cos(4*a))))*cos(1/2*arctan2(sin(4*
a), cos(4*a))) - (sqrt(2)*cos(1/4*arctan2(sin(4*a), cos(4*a))) - sqrt(2)*si
n(1/4*arctan2(sin(4*a), cos(4*a))))*sin(1/2*arctan2(sin(4*a), cos(4*a))))*c
os(b*x) + cos(1/4*arctan2(sin(4*a), cos(4*a)))^2 + ((sqrt(2)*cos(1/4*arctan
2(sin(4*a), cos(4*a))) - sqrt(2)*sin(1/4*arctan2(sin(4*a), cos(4*a))))*cos(
1/2*arctan2(sin(4*a), cos(4*a))) + (sqrt(2)*cos(1/4*arctan2(sin(4*a), cos(4
*a))) + sqrt(2)*sin(1/4*arctan2(sin(4*a), cos(4*a))))*sin(1/2*arctan2(sin(4
```

```
*a), cos(4*a))))*sin(b*x) + sin(1/4*arctan2(sin(4*a), cos(4*a)))^2)) - 4*lo
g((cos(b*x + 2*a)^2 + cos(a)^2 - 2*cos(a)*sin(b*x + 2*a) + sin(b*x + 2*a)^2
 + 2*cos(b*x + 2*a)*sin(a) + sin(a)^2)/(cos(b*x + 2*a)^2 + cos(a)^2 + 2*cos
(a)*sin(b*x + 2*a) + sin(b*x + 2*a)^2 - 2*cos(b*x + 2*a)*sin(a) + sin(a)^2)
))/b
```

mupad [B] time = 0.09, size = 29, normalized size = 0.83

$$-\frac{\operatorname{atanh}\left(\sin\left(a+b\,x\right)\right)-\sqrt{2}\,\operatorname{atanh}\left(\sqrt{2}\,\sin\left(a+b\,x\right)\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(a + b*x)*cos(2*a + 2*b*x)),x)
[Out] -(atanh(sin(a + b*x)) - 2^(1/2)*atanh(2^(1/2)*sin(a + b*x)))/b
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sec\left(a+bx\right)\sec\left(2a+2bx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)*sec(2*b*x+2*a),x)
[Out] Integral(sec(a + b*x)*sec(2*a + 2*b*x), x)
```

3.65 $\int \sec(a+bx)\sec(2(a+bx))\,dx$

Optimal. Leaf size=35

$$\frac{\sqrt{2}\tanh^{-1}\left(\sqrt{2}\sin(a+bx)\right)}{b}-\frac{\tanh^{-1}(\sin(a+bx))}{b}$$

```
[Out] -arctanh(sin(b*x+a))/b+arctanh(sin(b*x+a)*2^(1/2))*2^(1/2)/b
```

Rubi [A] time = 0.03, antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {4364, 1093, 207}

$$\frac{\sqrt{2}\tanh^{-1}\left(\sqrt{2}\sin(a+bx)\right)}{b}-\frac{\tanh^{-1}(\sin(a+bx))}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[a + b*x]*Sec[2*(a + b*x)],x]
```

```
[Out] -(ArcTanh[Sin[a + b*x]]/b) + (Sqrt[2]*ArcTanh[Sqrt[2]*Sin[a + b*x]])/b
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 1093

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(-1), x_Symbol] :> With[{q = Rt[b^
2 - 4*a*c, 2]}, Dist[c/q, Int[1/(b/2 - q/2 + c*x^2), x], x] - Dist[c/q, Int
[1/(b/2 + q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c,
0] && PosQ[b^2 - 4*a*c]
```

Rule 4364

```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))]^(n_), x_Symbol] :> With[{d = Free
Factors[Sin[c*(a + b*x)], x]}, Dist[d/(b*c), Subst[Int[SubstFor[(1 - d^2*x^
2)^((n - 1)/2), Sin[c*(a + b*x)]/d, u, x], x], x, Sin[c*(a + b*x)]/d], x] /
; FunctionOfQ[Sin[c*(a + b*x)]/d, u, x]] /; FreeQ[{a, b, c}, x] && IntegerQ
[(n - 1)/2] && NonsumQ[u] && (EqQ[F, Cos] || EqQ[F, cos])
```

Rubi steps

$$\int \sec(a+bx)\sec(2(a+bx))\,dx = \frac{\text{Subst}\left(\int \frac{1}{1-3x^2+2x^4}\,dx, x, \sin(a+bx)\right)}{b}$$
$$= \frac{2\,\text{Subst}\left(\int \frac{1}{-2+2x^2}\,dx, x, \sin(a+bx)\right)}{b} - \frac{2\,\text{Subst}\left(\int \frac{1}{-1+2x^2}\,dx, x, \sin(a+bx)\right)}{b}$$
$$= -\frac{\tanh^{-1}(\sin(a+bx))}{b} + \frac{\sqrt{2}\tanh^{-1}\left(\sqrt{2}\sin(a+bx)\right)}{b}$$

Mathematica [A] time = 0.03, size = 35, normalized size = 1.00

$$\frac{\sqrt{2}\tanh^{-1}\left(\sqrt{2}\sin(a+bx)\right)}{b} - \frac{\tanh^{-1}(\sin(a+bx))}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[a + b*x]*Sec[2*(a + b*x)],x]
[Out] -(ArcTanh[Sin[a + b*x]]/b) + (Sqrt[2]*ArcTanh[Sqrt[2]*Sin[a + b*x]])/b
```

fricas [B] time = 0.98, size = 72, normalized size = 2.06

$$\frac{\sqrt{2}\log\left(-\frac{2\cos(bx+a)^2-2\sqrt{2}\sin(bx+a)-3}{2\cos(bx+a)^2-1}\right) - \log(\sin(bx+a)+1) + \log(-\sin(bx+a)+1)}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)*sec(2*b*x+2*a),x, algorithm="fricas")
[Out] 1/2*(sqrt(2)*log(-(2*cos(b*x + a)^2 - 2*sqrt(2)*sin(b*x + a) - 3)/(2*cos(b*
x + a)^2 - 1)) - log(sin(b*x + a) + 1) + log(-sin(b*x + a) + 1))/b
```

giac [B] time = 1.19, size = 948, normalized size = 27.09

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)*sec(2*b*x+2*a),x, algorithm="giac")
[Out] 1/2*(sqrt(2)*log(abs(2*tan(1/2*b*x + 2*a)*tan(1/2*a)^6 + 12*tan(1/2*b*x + 2
*a)*tan(1/2*a)^5 - 2*tan(1/2*a)^6 - 30*tan(1/2*b*x + 2*a)*tan(1/2*a)^4 + 12
*tan(1/2*a)^5 - 40*tan(1/2*b*x + 2*a)*tan(1/2*a)^3 + 30*tan(1/2*a)^4 + 30*t
```

```
an(1/2*b*x + 2*a)*tan(1/2*a)^2 - 40*tan(1/2*a)^3 + 12*tan(1/2*b*x + 2*a)*ta
n(1/2*a) - 30*tan(1/2*a)^2 - 2*sqrt(2)*(tan(1/2*a)^6 + 3*tan(1/2*a)^4 + 3*t
an(1/2*a)^2 + 1) - 2*tan(1/2*b*x + 2*a) + 12*tan(1/2*a) + 2)/abs(2*tan(1/2*
b*x + 2*a)*tan(1/2*a)^6 + 12*tan(1/2*b*x + 2*a)*tan(1/2*a)^5 - 2*tan(1/2*a)
^6 - 30*tan(1/2*b*x + 2*a)*tan(1/2*a)^4 + 12*tan(1/2*a)^5 - 40*tan(1/2*b*x
+ 2*a)*tan(1/2*a)^3 + 30*tan(1/2*a)^4 + 30*tan(1/2*b*x + 2*a)*tan(1/2*a)^2
- 40*tan(1/2*a)^3 + 12*tan(1/2*b*x + 2*a)*tan(1/2*a) - 30*tan(1/2*a)^2 + 2*
sqrt(2)*(tan(1/2*a)^6 + 3*tan(1/2*a)^4 + 3*tan(1/2*a)^2 + 1) - 2*tan(1/2*b*
x + 2*a) + 12*tan(1/2*a) + 2)) + sqrt(2)*log(abs(2*tan(1/2*b*x + 2*a)*tan(1
/2*a)^6 - 12*tan(1/2*b*x + 2*a)*tan(1/2*a)^5 + 2*tan(1/2*a)^6 - 30*tan(1/2*
b*x + 2*a)*tan(1/2*a)^4 + 12*tan(1/2*a)^5 + 40*tan(1/2*b*x + 2*a)*tan(1/2*a
)^3 - 30*tan(1/2*a)^4 + 30*tan(1/2*b*x + 2*a)*tan(1/2*a)^2 - 40*tan(1/2*a)^
3 - 12*tan(1/2*b*x + 2*a)*tan(1/2*a) + 30*tan(1/2*a)^2 - 2*sqrt(2)*(tan(1/2
*a)^6 + 3*tan(1/2*a)^4 + 3*tan(1/2*a)^2 + 1) - 2*tan(1/2*b*x + 2*a) + 12*ta
n(1/2*a) - 2)/abs(2*tan(1/2*b*x + 2*a)*tan(1/2*a)^6 - 12*tan(1/2*b*x + 2*a)
*tan(1/2*a)^5 + 2*tan(1/2*a)^6 - 30*tan(1/2*b*x + 2*a)*tan(1/2*a)^4 + 12*ta
n(1/2*a)^5 + 40*tan(1/2*b*x + 2*a)*tan(1/2*a)^3 - 30*tan(1/2*a)^4 + 30*tan(
1/2*b*x + 2*a)*tan(1/2*a)^2 - 40*tan(1/2*a)^3 - 12*tan(1/2*b*x + 2*a)*tan(1
/2*a) + 30*tan(1/2*a)^2 + 2*sqrt(2)*(tan(1/2*a)^6 + 3*tan(1/2*a)^4 + 3*tan(
1/2*a)^2 + 1) - 2*tan(1/2*b*x + 2*a) + 12*tan(1/2*a) - 2)) - 2*log(abs(tan(
1/2*b*x + 2*a)*tan(1/2*a)^3 + 3*tan(1/2*b*x + 2*a)*tan(1/2*a)^2 - tan(1/2*a
)^3 - 3*tan(1/2*b*x + 2*a)*tan(1/2*a) + 3*tan(1/2*a)^2 - tan(1/2*b*x + 2*a)
 + 3*tan(1/2*a) - 1)) + 2*log(abs(tan(1/2*b*x + 2*a)*tan(1/2*a)^3 - 3*tan(1
/2*b*x + 2*a)*tan(1/2*a)^2 + tan(1/2*a)^3 - 3*tan(1/2*b*x + 2*a)*tan(1/2*a)
 + 3*tan(1/2*a)^2 + tan(1/2*b*x + 2*a) - 3*tan(1/2*a) - 1)))/b
```

maple [A] time = 0.00, size = 48, normalized size = 1.37

$$\frac{\ln\left(\sin\left(bx+a\right)-1\right)}{2b}+\frac{\operatorname{arctanh}\left(\sin\left(bx+a\right)\sqrt{2}\right)\sqrt{2}}{b}-\frac{\ln\left(1+\sin\left(bx+a\right)\right)}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(b*x+a)*sec(2*b*x+2*a),x)
```

```
[Out] 1/2/b*ln(sin(b*x+a)-1)+arctanh(sin(b*x+a)*2^(1/2))*2^(1/2)/b-1/2/b*ln(1+sin
(b*x+a))
```

maxima [B] time = 117.00, size = 6257, normalized size = 178.77

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)*sec(2*b*x+2*a),x, algorithm="maxima")
```

```
[Out] -1/8*(2*(sqrt(2)*cos(3*a)*cos(3/4*arctan2(sin(4*a), cos(4*a))) - sqrt(2)*co
s(a)*cos(1/4*arctan2(sin(4*a), cos(4*a))) + sqrt(2)*sin(3*a)*sin(3/4*arctan
2(sin(4*a), cos(4*a))) - sqrt(2)*sin(a)*sin(1/4*arctan2(sin(4*a), cos(4*a))
))*arctan2(sqrt(2)*cos(1/4*arctan2(sin(4*a), cos(4*a)))*sin(b*x) + sqrt(2)*
cos(b*x)*sin(1/4*arctan2(sin(4*a), cos(4*a))) + cos(1/2*arctan2(sin(4*a), c
os(4*a)))*sin(2*b*x) + cos(2*b*x)*sin(1/2*arctan2(sin(4*a), cos(4*a))), sqr
t(2)*cos(b*x)*cos(1/4*arctan2(sin(4*a), cos(4*a))) - sqrt(2)*sin(b*x)*sin(1
/4*arctan2(sin(4*a), cos(4*a))) + cos(2*b*x)*cos(1/2*arctan2(sin(4*a), cos(
4*a))) - sin(2*b*x)*sin(1/2*arctan2(sin(4*a), cos(4*a))) + 1) - 2*(sqrt(2)*
cos(3*a)*cos(3/4*arctan2(sin(4*a), cos(4*a))) - sqrt(2)*cos(a)*cos(1/4*arct
an2(sin(4*a), cos(4*a))) + sqrt(2)*sin(3*a)*sin(3/4*arctan2(sin(4*a), cos(4
*a))) - sqrt(2)*sin(a)*sin(1/4*arctan2(sin(4*a), cos(4*a))))*arctan2(-sqrt(
2)*cos(1/4*arctan2(sin(4*a), cos(4*a)))*sin(b*x) - sqrt(2)*cos(b*x)*sin(1/4
*arctan2(sin(4*a), cos(4*a))) + cos(1/2*arctan2(sin(4*a), cos(4*a)))*sin(2*
b*x) + cos(2*b*x)*sin(1/2*arctan2(sin(4*a), cos(4*a))), -sqrt(2)*cos(b*x)*c
os(1/4*arctan2(sin(4*a), cos(4*a))) + sqrt(2)*sin(b*x)*sin(1/4*arctan2(sin(
4*a), cos(4*a))) + cos(2*b*x)*cos(1/2*arctan2(sin(4*a), cos(4*a))) - sin(2*
b*x)*sin(1/2*arctan2(sin(4*a), cos(4*a))) + 1) - 2*((sqrt(2)*cos(3*a)*cos(1
/2*arctan2(sin(4*a), cos(4*a))) + sqrt(2)*sin(3*a)*sin(1/2*arctan2(sin(4*a)
, cos(4*a))) + sqrt(2)*cos(a))*cos(1/2*pi + 1/4*arctan2(sin(4*a), cos(4*a))
) + (sqrt(2)*cos(1/2*arctan2(sin(4*a), cos(4*a)))*sin(3*a) - sqrt(2)*cos(3*
a)*sin(1/2*arctan2(sin(4*a), cos(4*a))) + sqrt(2)*sin(a))*sin(1/2*pi + 1/4*
arctan2(sin(4*a), cos(4*a))))*arctan2(-2*((sqrt(2)*cos(1/2*arctan2(sin(4*a)
, cos(4*a)))*sin(1/2*pi + 1/4*arctan2(sin(4*a), cos(4*a))) - sqrt(2)*cos(1/
2*pi + 1/4*arctan2(sin(4*a), cos(4*a)))*sin(1/2*arctan2(sin(4*a), cos(4*a))
))*cos(b*x) + cos(1/4*arctan2(sin(4*a), cos(4*a)))*sin(1/2*pi + 1/4*arctan2
(sin(4*a), cos(4*a))) - (sqrt(2)*cos(1/2*pi + 1/4*arctan2(sin(4*a), cos(4*a
)))*cos(1/2*arctan2(sin(4*a), cos(4*a))) + sqrt(2)*sin(1/2*pi + 1/4*arctan2
(sin(4*a), cos(4*a)))*sin(1/2*arctan2(sin(4*a), cos(4*a))))*sin(b*x) - cos(
1/2*pi + 1/4*arctan2(sin(4*a), cos(4*a)))*sin(1/4*arctan2(sin(4*a), cos(4*a
))))/(2*(cos(1/2*arctan2(sin(4*a), cos(4*a)))^2 + sin(1/2*arctan2(sin(4*a),
 cos(4*a)))^2)*cos(b*x)^2 + 2*(cos(1/2*arctan2(sin(4*a), cos(4*a)))^2 + sin
(1/2*arctan2(sin(4*a), cos(4*a)))^2)*sin(b*x)^2 + cos(1/2*pi + 1/4*arctan2(
sin(4*a), cos(4*a)))^2 + 2*(sqrt(2)*cos(1/2*pi + 1/4*arctan2(sin(4*a), cos(
4*a)))*cos(1/2*arctan2(sin(4*a), cos(4*a))) + sqrt(2)*cos(1/2*arctan2(sin(4
*a), cos(4*a)))*cos(1/4*arctan2(sin(4*a), cos(4*a))) + sqrt(2)*sin(1/2*pi +
 1/4*arctan2(sin(4*a), cos(4*a)))*sin(1/2*arctan2(sin(4*a), cos(4*a))) + sq
rt(2)*sin(1/2*arctan2(sin(4*a), cos(4*a)))*sin(1/4*arctan2(sin(4*a), cos(4*
a))))*cos(b*x) + 2*cos(1/2*pi + 1/4*arctan2(sin(4*a), cos(4*a)))*cos(1/4*ar
ctan2(sin(4*a), cos(4*a))) + cos(1/4*arctan2(sin(4*a), cos(4*a)))^2 + sin(1
/2*pi + 1/4*arctan2(sin(4*a), cos(4*a)))^2 + 2*(sqrt(2)*cos(1/2*arctan2(sin
(4*a), cos(4*a)))*sin(1/2*pi + 1/4*arctan2(sin(4*a), cos(4*a))) - sqrt(2)*c
os(1/2*pi + 1/4*arctan2(sin(4*a), cos(4*a)))*sin(1/2*arctan2(sin(4*a), cos(
4*a))) - sqrt(2)*cos(1/4*arctan2(sin(4*a), cos(4*a)))*sin(1/2*arctan2(sin(4
*a), cos(4*a))) + sqrt(2)*cos(1/2*arctan2(sin(4*a), cos(4*a)))*sin(1/4*arct
```

```
an2(sin(4*a), cos(4*a))))*sin(b*x) + 2*sin(1/2*pi + 1/4*arctan2(sin(4*a), c
os(4*a)))*sin(1/4*arctan2(sin(4*a), cos(4*a))) + sin(1/4*arctan2(sin(4*a),
cos(4*a)))^2), (2*(cos(1/2*arctan2(sin(4*a), cos(4*a)))^2 + sin(1/2*arctan2
(sin(4*a), cos(4*a)))^2)*cos(b*x)^2 + 2*(cos(1/2*arctan2(sin(4*a), cos(4*a)
))^2 + sin(1/2*arctan2(sin(4*a), cos(4*a)))^2)*sin(b*x)^2 - cos(1/2*pi + 1/
4*arctan2(sin(4*a), cos(4*a)))^2 + 2*(sqrt(2)*cos(1/2*arctan2(sin(4*a), cos
(4*a)))*cos(1/4*arctan2(sin(4*a), cos(4*a))) + sqrt(2)*sin(1/2*arctan2(sin(
4*a), cos(4*a)))*sin(1/4*arctan2(sin(4*a), cos(4*a))))*cos(b*x) + cos(1/4*a
rctan2(sin(4*a), cos(4*a)))^2 - sin(1/2*pi + 1/4*arctan2(sin(4*a), cos(4*a)
))^2 - 2*(sqrt(2)*cos(1/4*arctan2(sin(4*a), cos(4*a)))*sin(1/2*arctan2(sin(
4*a), cos(4*a))) - sqrt(2)*cos(1/2*arctan2(sin(4*a), cos(4*a)))*sin(1/4*arc
tan2(sin(4*a), cos(4*a))))*sin(b*x) + sin(1/4*arctan2(sin(4*a), cos(4*a)))^
2)/(2*(cos(1/2*arctan2(sin(4*a), cos(4*a)))^2 + sin(1/2*arctan2(sin(4*a), c
os(4*a)))^2)*cos(b*x)^2 + 2*(cos(1/2*arctan2(sin(4*a), cos(4*a)))^2 + sin(1
/2*arctan2(sin(4*a), cos(4*a)))^2)*sin(b*x)^2 + cos(1/2*pi + 1/4*arctan2(si
n(4*a), cos(4*a)))^2 + 2*(sqrt(2)*cos(1/2*pi + 1/4*arctan2(sin(4*a), cos(4*
a)))*cos(1/2*arctan2(sin(4*a), cos(4*a))) + sqrt(2)*cos(1/2*arctan2(sin(4*a
), cos(4*a)))*cos(1/4*arctan2(sin(4*a), cos(4*a))) + sqrt(2)*sin(1/2*pi + 1
/4*arctan2(sin(4*a), cos(4*a)))*sin(1/2*arctan2(sin(4*a), cos(4*a))) + sqrt
(2)*sin(1/2*arctan2(sin(4*a), cos(4*a)))*sin(1/4*arctan2(sin(4*a), cos(4*a)
)))*cos(b*x) + 2*cos(1/2*pi + 1/4*arctan2(sin(4*a), cos(4*a)))*cos(1/4*arct
an2(sin(4*a), cos(4*a))) + cos(1/4*arctan2(sin(4*a), cos(4*a)))^2 + sin(1/2
*pi + 1/4*arctan2(sin(4*a), cos(4*a)))^2 + 2*(sqrt(2)*cos(1/2*arctan2(sin(4
*a), cos(4*a)))*sin(1/2*pi + 1/4*arctan2(sin(4*a), cos(4*a))) - sqrt(2)*cos
(1/2*pi + 1/4*arctan2(sin(4*a), cos(4*a)))*sin(1/2*arctan2(sin(4*a), cos(4*
a))) - sqrt(2)*cos(1/4*arctan2(sin(4*a), cos(4*a)))*sin(1/2*arctan2(sin(4*a
), cos(4*a))) + sqrt(2)*cos(1/2*arctan2(sin(4*a), cos(4*a)))*sin(1/4*arctan
2(sin(4*a), cos(4*a))))*sin(b*x) + 2*sin(1/2*pi + 1/4*arctan2(sin(4*a), cos
(4*a)))*sin(1/4*arctan2(sin(4*a), cos(4*a))) + sin(1/4*arctan2(sin(4*a), co
s(4*a)))^2)) - 2*((sqrt(2)*cos(3*a)*cos(1/2*arctan2(sin(4*a), cos(4*a))) +
sqrt(2)*sin(3*a)*sin(1/2*arctan2(sin(4*a), cos(4*a))) + sqrt(2)*cos(a))*cos
(1/2*pi + 1/4*arctan2(sin(4*a), cos(4*a))) + (sqrt(2)*cos(1/2*arctan2(sin(4
*a), cos(4*a)))*sin(3*a) - sqrt(2)*cos(3*a)*sin(1/2*arctan2(sin(4*a), cos(4
*a))) + sqrt(2)*sin(a))*sin(1/2*pi + 1/4*arctan2(sin(4*a), cos(4*a))))*arct
an2(-2*((sqrt(2)*cos(1/2*arctan2(sin(4*a), cos(4*a)))*sin(1/2*pi + 1/4*arct
an2(sin(4*a), cos(4*a))) - sqrt(2)*cos(1/2*pi + 1/4*arctan2(sin(4*a), cos(4
*a)))*sin(1/2*arctan2(sin(4*a), cos(4*a))))*cos(b*x) - cos(1/4*arctan2(sin(
4*a), cos(4*a)))*sin(1/2*pi + 1/4*arctan2(sin(4*a), cos(4*a))) - (sqrt(2)*c
os(1/2*pi + 1/4*arctan2(sin(4*a), cos(4*a)))*cos(1/2*arctan2(sin(4*a), cos(
4*a))) + sqrt(2)*sin(1/2*pi + 1/4*arctan2(sin(4*a), cos(4*a)))*sin(1/2*arct
an2(sin(4*a), cos(4*a))))*sin(b*x) + cos(1/2*pi + 1/4*arctan2(sin(4*a), cos
(4*a)))*sin(1/4*arctan2(sin(4*a), cos(4*a))))/(2*(cos(1/2*arctan2(sin(4*a),
 cos(4*a)))^2 + sin(1/2*arctan2(sin(4*a), cos(4*a)))^2)*cos(b*x)^2 + 2*(cos
(1/2*arctan2(sin(4*a), cos(4*a)))^2 + sin(1/2*arctan2(sin(4*a), cos(4*a)))^
2)*sin(b*x)^2 + cos(1/2*pi + 1/4*arctan2(sin(4*a), cos(4*a)))^2 + 2*(sqrt(2
```

)*cos(1/2*pi + 1/4*arctan2(sin(4*a), cos(4*a)))*cos(1/2*arctan2(sin(4*a), c
os(4*a))) - sqrt(2)*cos(1/2*arctan2(sin(4*a), cos(4*a)))*cos(1/4*arctan2(si
n(4*a), cos(4*a))) + sqrt(2)*sin(1/2*pi + 1/4*arctan2(sin(4*a), cos(4*a)))*
sin(1/2*arctan2(sin(4*a), cos(4*a))) - sqrt(2)*sin(1/2*arctan2(sin(4*a), co
s(4*a)))*sin(1/4*arctan2(sin(4*a), cos(4*a))))*cos(b*x) - 2*cos(1/2*pi + 1/
4*arctan2(sin(4*a), cos(4*a)))*cos(1/4*arctan2(sin(4*a), cos(4*a))) + cos(1
/4*arctan2(sin(4*a), cos(4*a)))^2 + sin(1/2*pi + 1/4*arctan2(sin(4*a), cos(
4*a)))^2 + 2*(sqrt(2)*cos(1/2*arctan2(sin(4*a), cos(4*a)))*sin(1/2*pi + 1/4
*arctan2(sin(4*a), cos(4*a))) - sqrt(2)*cos(1/2*pi + 1/4*arctan2(sin(4*a),
cos(4*a)))*sin(1/2*arctan2(sin(4*a), cos(4*a))) + sqrt(2)*cos(1/4*arctan2(s
in(4*a), cos(4*a)))*sin(1/2*arctan2(sin(4*a), cos(4*a))) - sqrt(2)*cos(1/2*
arctan2(sin(4*a), cos(4*a)))*sin(1/4*arctan2(sin(4*a), cos(4*a))))*sin(b*x)
 - 2*sin(1/2*pi + 1/4*arctan2(sin(4*a), cos(4*a)))*sin(1/4*arctan2(sin(4*a)
, cos(4*a))) + sin(1/4*arctan2(sin(4*a), cos(4*a)))^2), (2*(cos(1/2*arctan2
(sin(4*a), cos(4*a)))^2 + sin(1/2*arctan2(sin(4*a), cos(4*a)))^2)*cos(b*x)^
2 + 2*(cos(1/2*arctan2(sin(4*a), cos(4*a)))^2 + sin(1/2*arctan2(sin(4*a), c
os(4*a)))^2)*sin(b*x)^2 - cos(1/2*pi + 1/4*arctan2(sin(4*a), cos(4*a)))^2 -
 2*(sqrt(2)*cos(1/2*arctan2(sin(4*a), cos(4*a)))*cos(1/4*arctan2(sin(4*a),
cos(4*a))) + sqrt(2)*sin(1/2*arctan2(sin(4*a), cos(4*a)))*sin(1/4*arctan2(s
in(4*a), cos(4*a))))*cos(b*x) + cos(1/4*arctan2(sin(4*a), cos(4*a)))^2 - si
n(1/2*pi + 1/4*arctan2(sin(4*a), cos(4*a)))^2 + 2*(sqrt(2)*cos(1/4*arctan2(
sin(4*a), cos(4*a)))*sin(1/2*arctan2(sin(4*a), cos(4*a))) - sqrt(2)*cos(1/2
*arctan2(sin(4*a), cos(4*a)))*sin(1/4*arctan2(sin(4*a), cos(4*a))))*sin(b*x
) + sin(1/4*arctan2(sin(4*a), cos(4*a)))^2)/(2*(cos(1/2*arctan2(sin(4*a), c
os(4*a)))^2 + sin(1/2*arctan2(sin(4*a), cos(4*a)))^2)*cos(b*x)^2 + 2*(cos(1
/2*arctan2(sin(4*a), cos(4*a)))^2 + sin(1/2*arctan2(sin(4*a), cos(4*a)))^2)
*sin(b*x)^2 + cos(1/2*pi + 1/4*arctan2(sin(4*a), cos(4*a)))^2 + 2*(sqrt(2)*
cos(1/2*pi + 1/4*arctan2(sin(4*a), cos(4*a)))*cos(1/2*arctan2(sin(4*a), cos
(4*a))) - sqrt(2)*cos(1/2*arctan2(sin(4*a), cos(4*a)))*cos(1/4*arctan2(sin(
4*a), cos(4*a))) + sqrt(2)*sin(1/2*pi + 1/4*arctan2(sin(4*a), cos(4*a)))*si
n(1/2*arctan2(sin(4*a), cos(4*a))) - sqrt(2)*sin(1/2*arctan2(sin(4*a), cos(
4*a)))*sin(1/4*arctan2(sin(4*a), cos(4*a))))*cos(b*x) - 2*cos(1/2*pi + 1/4*
arctan2(sin(4*a), cos(4*a)))*cos(1/4*arctan2(sin(4*a), cos(4*a))) + cos(1/4
*arctan2(sin(4*a), cos(4*a)))^2 + sin(1/2*pi + 1/4*arctan2(sin(4*a), cos(4*
a)))^2 + 2*(sqrt(2)*cos(1/2*arctan2(sin(4*a), cos(4*a)))*sin(1/2*pi + 1/4*a
rctan2(sin(4*a), cos(4*a))) - sqrt(2)*cos(1/2*pi + 1/4*arctan2(sin(4*a), co
s(4*a)))*sin(1/2*arctan2(sin(4*a), cos(4*a))) + sqrt(2)*cos(1/4*arctan2(sin
(4*a), cos(4*a)))*sin(1/2*arctan2(sin(4*a), cos(4*a))) - sqrt(2)*cos(1/2*ar
ctan2(sin(4*a), cos(4*a)))*sin(1/4*arctan2(sin(4*a), cos(4*a))))*sin(b*x) -
 2*sin(1/2*pi + 1/4*arctan2(sin(4*a), cos(4*a)))*sin(1/4*arctan2(sin(4*a),
cos(4*a))) + sin(1/4*arctan2(sin(4*a), cos(4*a)))^2)) + (sqrt(2)*cos(3/4*ar
ctan2(sin(4*a), cos(4*a)))*sin(3*a) - sqrt(2)*cos(1/4*arctan2(sin(4*a), cos
(4*a)))*sin(a) - sqrt(2)*cos(3*a)*sin(3/4*arctan2(sin(4*a), cos(4*a))) + sq
rt(2)*cos(a)*sin(1/4*arctan2(sin(4*a), cos(4*a))))*log((cos(1/2*arctan2(sin
(4*a), cos(4*a)))^2 + sin(1/2*arctan2(sin(4*a), cos(4*a)))^2)*cos(2*b*x)^2

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+ 2*(cos(1/4*arctan2(sin(4*a), cos(4*a)))^2 + sin(1/4*arctan2(sin(4*a), cos
(4*a)))^2)*cos(b*x)^2 + (cos(1/2*arctan2(sin(4*a), cos(4*a)))^2 + sin(1/2*a
rctan2(sin(4*a), cos(4*a)))^2)*sin(2*b*x)^2 + 2*(cos(1/4*arctan2(sin(4*a),
cos(4*a)))^2 + sin(1/4*arctan2(sin(4*a), cos(4*a)))^2)*sin(b*x)^2 + 2*sqrt(
2)*cos(b*x)*cos(1/4*arctan2(sin(4*a), cos(4*a))) - 2*sqrt(2)*sin(b*x)*sin(1
/4*arctan2(sin(4*a), cos(4*a))) + 2*((sqrt(2)*cos(1/2*arctan2(sin(4*a), cos
(4*a)))*cos(1/4*arctan2(sin(4*a), cos(4*a))) + sqrt(2)*sin(1/2*arctan2(sin(
4*a), cos(4*a)))*sin(1/4*arctan2(sin(4*a), cos(4*a))))*cos(b*x) + (sqrt(2)*
cos(1/4*arctan2(sin(4*a), cos(4*a)))*sin(1/2*arctan2(sin(4*a), cos(4*a))) -
 sqrt(2)*cos(1/2*arctan2(sin(4*a), cos(4*a)))*sin(1/4*arctan2(sin(4*a), cos
(4*a))))*sin(b*x) + cos(1/2*arctan2(sin(4*a), cos(4*a))))*cos(2*b*x) - 2*((
sqrt(2)*cos(1/4*arctan2(sin(4*a), cos(4*a)))*sin(1/2*arctan2(sin(4*a), cos(
4*a))) - sqrt(2)*cos(1/2*arctan2(sin(4*a), cos(4*a)))*sin(1/4*arctan2(sin(4
*a), cos(4*a))))*cos(b*x) - (sqrt(2)*cos(1/2*arctan2(sin(4*a), cos(4*a)))*c
os(1/4*arctan2(sin(4*a), cos(4*a))) + sqrt(2)*sin(1/2*arctan2(sin(4*a), cos
(4*a)))*sin(1/4*arctan2(sin(4*a), cos(4*a))))*sin(b*x) + sin(1/2*arctan2(si
n(4*a), cos(4*a))))*sin(2*b*x) + 1) - (sqrt(2)*cos(3/4*arctan2(sin(4*a), co
s(4*a)))*sin(3*a) - sqrt(2)*cos(1/4*arctan2(sin(4*a), cos(4*a)))*sin(a) - s
qrt(2)*cos(3*a)*sin(3/4*arctan2(sin(4*a), cos(4*a))) + sqrt(2)*cos(a)*sin(1
/4*arctan2(sin(4*a), cos(4*a))))*log((cos(1/2*arctan2(sin(4*a), cos(4*a)))^
2 + sin(1/2*arctan2(sin(4*a), cos(4*a)))^2)*cos(2*b*x)^2 + 2*(cos(1/4*arcta
n2(sin(4*a), cos(4*a)))^2 + sin(1/4*arctan2(sin(4*a), cos(4*a)))^2)*cos(b*x
)^2 + (cos(1/2*arctan2(sin(4*a), cos(4*a)))^2 + sin(1/2*arctan2(sin(4*a), c
os(4*a)))^2)*sin(2*b*x)^2 + 2*(cos(1/4*arctan2(sin(4*a), cos(4*a)))^2 + sin
(1/4*arctan2(sin(4*a), cos(4*a)))^2)*sin(b*x)^2 - 2*sqrt(2)*cos(b*x)*cos(1/
4*arctan2(sin(4*a), cos(4*a))) + 2*sqrt(2)*sin(b*x)*sin(1/4*arctan2(sin(4*a
), cos(4*a))) - 2*((sqrt(2)*cos(1/2*arctan2(sin(4*a), cos(4*a)))*cos(1/4*ar
ctan2(sin(4*a), cos(4*a))) + sqrt(2)*sin(1/2*arctan2(sin(4*a), cos(4*a)))*s
in(1/4*arctan2(sin(4*a), cos(4*a))))*cos(b*x) + (sqrt(2)*cos(1/4*arctan2(si
n(4*a), cos(4*a)))*sin(1/2*arctan2(sin(4*a), cos(4*a))) - sqrt(2)*cos(1/2*a
rctan2(sin(4*a), cos(4*a)))*sin(1/4*arctan2(sin(4*a), cos(4*a))))*sin(b*x)
- cos(1/2*arctan2(sin(4*a), cos(4*a))))*cos(2*b*x) + 2*((sqrt(2)*cos(1/4*ar
ctan2(sin(4*a), cos(4*a)))*sin(1/2*arctan2(sin(4*a), cos(4*a))) - sqrt(2)*c
os(1/2*arctan2(sin(4*a), cos(4*a)))*sin(1/4*arctan2(sin(4*a), cos(4*a))))*c
os(b*x) - (sqrt(2)*cos(1/2*arctan2(sin(4*a), cos(4*a)))*cos(1/4*arctan2(sin
(4*a), cos(4*a))) + sqrt(2)*sin(1/2*arctan2(sin(4*a), cos(4*a)))*sin(1/4*ar
ctan2(sin(4*a), cos(4*a))))*sin(b*x) - sin(1/2*arctan2(sin(4*a), cos(4*a)))
)*sin(2*b*x) + 1) - ((sqrt(2)*cos(1/2*arctan2(sin(4*a), cos(4*a)))*sin(3*a)
 - sqrt(2)*cos(3*a)*sin(1/2*arctan2(sin(4*a), cos(4*a))) + sqrt(2)*sin(a))*
cos(1/2*pi + 1/4*arctan2(sin(4*a), cos(4*a))) - (sqrt(2)*cos(3*a)*cos(1/2*a
rctan2(sin(4*a), cos(4*a))) + sqrt(2)*sin(3*a)*sin(1/2*arctan2(sin(4*a), co
s(4*a))) + sqrt(2)*cos(a))*sin(1/2*pi + 1/4*arctan2(sin(4*a), cos(4*a))))*l
og(((cos(1/2*arctan2(sin(4*a), cos(4*a)))^2 + sin(1/2*arctan2(sin(4*a), cos
(4*a)))^2)*cos(b*x)^2 + (cos(1/2*arctan2(sin(4*a), cos(4*a)))^2 + sin(1/2*a
rctan2(sin(4*a), cos(4*a)))^2)*sin(b*x)^2 + ((sqrt(2)*cos(1/4*arctan2(sin(4
```

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*a), cos(4*a))) + sqrt(2)*sin(1/4*arctan2(sin(4*a), cos(4*a))))*cos(1/2*arc
tan2(sin(4*a), cos(4*a))) - (sqrt(2)*cos(1/4*arctan2(sin(4*a), cos(4*a))) -
 sqrt(2)*sin(1/4*arctan2(sin(4*a), cos(4*a))))*sin(1/2*arctan2(sin(4*a), co
s(4*a))))*cos(b*x) + cos(1/4*arctan2(sin(4*a), cos(4*a)))^2 - ((sqrt(2)*cos
(1/4*arctan2(sin(4*a), cos(4*a))) - sqrt(2)*sin(1/4*arctan2(sin(4*a), cos(4
*a))))*cos(1/2*arctan2(sin(4*a), cos(4*a))) + (sqrt(2)*cos(1/4*arctan2(sin(
4*a), cos(4*a))) + sqrt(2)*sin(1/4*arctan2(sin(4*a), cos(4*a))))*sin(1/2*ar
ctan2(sin(4*a), cos(4*a))))*sin(b*x) + sin(1/4*arctan2(sin(4*a), cos(4*a)))
^2)/((cos(1/2*arctan2(sin(4*a), cos(4*a)))^2 + sin(1/2*arctan2(sin(4*a), co
s(4*a)))^2)*cos(b*x)^2 + (cos(1/2*arctan2(sin(4*a), cos(4*a)))^2 + sin(1/2*
arctan2(sin(4*a), cos(4*a)))^2)*sin(b*x)^2 + ((sqrt(2)*cos(1/4*arctan2(sin(
4*a), cos(4*a))) - sqrt(2)*sin(1/4*arctan2(sin(4*a), cos(4*a))))*cos(1/2*ar
ctan2(sin(4*a), cos(4*a))) + (sqrt(2)*cos(1/4*arctan2(sin(4*a), cos(4*a)))
 + sqrt(2)*sin(1/4*arctan2(sin(4*a), cos(4*a))))*sin(1/2*arctan2(sin(4*a), c
os(4*a))))*cos(b*x) + cos(1/4*arctan2(sin(4*a), cos(4*a)))^2 + ((sqrt(2)*co
s(1/4*arctan2(sin(4*a), cos(4*a))) + sqrt(2)*sin(1/4*arctan2(sin(4*a), cos(
4*a))))*cos(1/2*arctan2(sin(4*a), cos(4*a))) - (sqrt(2)*cos(1/4*arctan2(sin
(4*a), cos(4*a))) - sqrt(2)*sin(1/4*arctan2(sin(4*a), cos(4*a))))*sin(1/2*a
rctan2(sin(4*a), cos(4*a))))*sin(b*x) + sin(1/4*arctan2(sin(4*a), cos(4*a))
)^2)) - ((sqrt(2)*cos(1/2*arctan2(sin(4*a), cos(4*a)))*sin(3*a) - sqrt(2)*c
os(3*a)*sin(1/2*arctan2(sin(4*a), cos(4*a))) + sqrt(2)*sin(a))*cos(1/2*pi +
 1/4*arctan2(sin(4*a), cos(4*a))) - (sqrt(2)*cos(3*a)*cos(1/2*arctan2(sin(4
*a), cos(4*a))) + sqrt(2)*sin(3*a)*sin(1/2*arctan2(sin(4*a), cos(4*a))) + s
qrt(2)*cos(a))*sin(1/2*pi + 1/4*arctan2(sin(4*a), cos(4*a))))*log(((cos(1/2
*arctan2(sin(4*a), cos(4*a)))^2 + sin(1/2*arctan2(sin(4*a), cos(4*a)))^2)*c
os(b*x)^2 + (cos(1/2*arctan2(sin(4*a), cos(4*a)))^2 + sin(1/2*arctan2(sin(4
*a), cos(4*a)))^2)*sin(b*x)^2 - ((sqrt(2)*cos(1/4*arctan2(sin(4*a), cos(4*a
))) - sqrt(2)*sin(1/4*arctan2(sin(4*a), cos(4*a))))*cos(1/2*arctan2(sin(4*a
), cos(4*a))) + (sqrt(2)*cos(1/4*arctan2(sin(4*a), cos(4*a))) + sqrt(2)*sin
(1/4*arctan2(sin(4*a), cos(4*a))))*sin(1/2*arctan2(sin(4*a), cos(4*a))))*co
s(b*x) + cos(1/4*arctan2(sin(4*a), cos(4*a)))^2 - ((sqrt(2)*cos(1/4*arctan2
(sin(4*a), cos(4*a))) + sqrt(2)*sin(1/4*arctan2(sin(4*a), cos(4*a))))*cos(1
/2*arctan2(sin(4*a), cos(4*a))) - (sqrt(2)*cos(1/4*arctan2(sin(4*a), cos(4*
a))) - sqrt(2)*sin(1/4*arctan2(sin(4*a), cos(4*a))))*sin(1/2*arctan2(sin(4*
a), cos(4*a))))*sin(b*x) + sin(1/4*arctan2(sin(4*a), cos(4*a)))^2)/((cos(1/
2*arctan2(sin(4*a), cos(4*a)))^2 + sin(1/2*arctan2(sin(4*a), cos(4*a)))^2)*
cos(b*x)^2 + (cos(1/2*arctan2(sin(4*a), cos(4*a)))^2 + sin(1/2*arctan2(sin(
4*a), cos(4*a)))^2)*sin(b*x)^2 - ((sqrt(2)*cos(1/4*arctan2(sin(4*a), cos(4*
a))) + sqrt(2)*sin(1/4*arctan2(sin(4*a), cos(4*a))))*cos(1/2*arctan2(sin(4*
a), cos(4*a))) - (sqrt(2)*cos(1/4*arctan2(sin(4*a), cos(4*a))) - sqrt(2)*si
n(1/4*arctan2(sin(4*a), cos(4*a))))*sin(1/2*arctan2(sin(4*a), cos(4*a))))*c
os(b*x) + cos(1/4*arctan2(sin(4*a), cos(4*a)))^2 + ((sqrt(2)*cos(1/4*arctan
2(sin(4*a), cos(4*a))) - sqrt(2)*sin(1/4*arctan2(sin(4*a), cos(4*a))))*cos(
1/2*arctan2(sin(4*a), cos(4*a))) + (sqrt(2)*cos(1/4*arctan2(sin(4*a), cos(4
*a))) + sqrt(2)*sin(1/4*arctan2(sin(4*a), cos(4*a))))*sin(1/2*arctan2(sin(4
```

```
*a), cos(4*a))))*sin(b*x) + sin(1/4*arctan2(sin(4*a), cos(4*a)))^2)) - 4*lo
g((cos(b*x + 2*a)^2 + cos(a)^2 - 2*cos(a)*sin(b*x + 2*a) + sin(b*x + 2*a)^2
 + 2*cos(b*x + 2*a)*sin(a) + sin(a)^2)/(cos(b*x + 2*a)^2 + cos(a)^2 + 2*cos
(a)*sin(b*x + 2*a) + sin(b*x + 2*a)^2 - 2*cos(b*x + 2*a)*sin(a) + sin(a)^2)
))/b
```

mupad [B] time = 0.00, size = 29, normalized size = 0.83

$$-\frac{\operatorname{atanh}\left(\sin\left(a+b\,x\right)\right)-\sqrt{2}\,\operatorname{atanh}\left(\sqrt{2}\,\sin\left(a+b\,x\right)\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(a + b*x)*cos(2*a + 2*b*x)),x)
[Out] -(atanh(sin(a + b*x)) - 2^(1/2)*atanh(2^(1/2)*sin(a + b*x)))/b
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sec\left(a+bx\right)\sec\left(2a+2bx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)*sec(2*b*x+2*a),x)
[Out] Integral(sec(a + b*x)*sec(2*a + 2*b*x), x)
```

3.66 $\int \sin(x)\sin(2x)\,dx$

Optimal. Leaf size=15

$$\frac{\sin(x)}{2} - \frac{1}{6}\sin(3x)$$

```
[Out] 1/2*sin(x)-1/6*sin(3*x)
```

Rubi [A] time = 0.01, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {4282}

$$\frac{\sin(x)}{2} - \frac{1}{6}\sin(3x)$$

Antiderivative was successfully verified.

```
[In] Int[Sin[x]*Sin[2*x],x]
```

```
[Out] Sin[x]/2 - Sin[3*x]/6
```

Rule 4282

```
Int[sin[(a_.) + (b_.)*(x_)]*sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[
a - c + (b - d)*x]/(2*(b - d)), x] - Simp[Sin[a + c + (b + d)*x]/(2*(b + d)
), x] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - d^2, 0]
```

Rubi steps

$$\int \sin(x)\sin(2x)\,dx = \frac{\sin(x)}{2} - \frac{1}{6}\sin(3x)$$

Mathematica [A] time = 0.01, size = 15, normalized size = 1.00

$$\frac{\sin(x)}{2} - \frac{1}{6}\sin(3x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[x]*Sin[2*x],x]
```

```
[Out] Sin[x]/2 - Sin[3*x]/6
```

fricas [A] time = 2.01, size = 10, normalized size = 0.67

$$-\frac{2}{3}\left(\cos(x)^2 - 1\right)\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)*sin(2*x),x, algorithm="fricas")
[Out] -2/3*(cos(x)^2 - 1)*sin(x)
```

giac [A] time = 0.12, size = 6, normalized size = 0.40

$$\frac{2}{3}\,\sin(x)^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)*sin(2*x),x, algorithm="giac")
[Out] 2/3*sin(x)^3
```

maple [A] time = 0.05, size = 7, normalized size = 0.47

$$\frac{2\left(\sin^3(x)\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)*sin(2*x),x)
[Out] 2/3*sin(x)^3
```

maxima [A] time = 0.31, size = 11, normalized size = 0.73

$$-\frac{1}{6}\,\sin\left(3\,x\right)+\frac{1}{2}\,\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)*sin(2*x),x, algorithm="maxima")
[Out] -1/6*sin(3*x) + 1/2*sin(x)
```

mupad [B] time = 0.03, size = 6, normalized size = 0.40

$$\frac{2\sin(x)^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(2*x)*sin(x),x)
[Out] (2*sin(x)^3)/3
```

sympy [A] time = 0.45, size = 20, normalized size = 1.33

$$-\frac{2\sin(x)\cos(2x)}{3}+\frac{\sin(2x)\cos(x)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)*sin(2*x),x)
```

```
[Out] -2*sin(x)*cos(2*x)/3 + sin(2*x)*cos(x)/3
```

3.67 $\int \sin(x)\sin(3x)\,dx$

Optimal. Leaf size=17

$$\frac{1}{4}\sin(2x) - \frac{1}{8}\sin(4x)$$

```
[Out] 1/4*sin(2*x)-1/8*sin(4*x)
```

Rubi [A] time = 0.01, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {4282}

$$\frac{1}{4}\sin(2x) - \frac{1}{8}\sin(4x)$$

Antiderivative was successfully verified.

```
[In] Int[Sin[x]*Sin[3*x],x]
```

```
[Out] Sin[2*x]/4 - Sin[4*x]/8
```

Rule 4282

```
Int[sin[(a_.) + (b_.)*(x_)]*sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[
a - c + (b - d)*x]/(2*(b - d)), x] - Simp[Sin[a + c + (b + d)*x]/(2*(b + d)
), x] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - d^2, 0]
```

Rubi steps

$$\int \sin(x)\sin(3x)\,dx = \frac{1}{4}\sin(2x) - \frac{1}{8}\sin(4x)$$

Mathematica [A] time = 0.01, size = 17, normalized size = 1.00

$$\frac{1}{4}\sin(2x) - \frac{1}{8}\sin(4x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[x]*Sin[3*x],x]
```

```
[Out] Sin[2*x]/4 - Sin[4*x]/8
```

fricas [A] time = 0.59, size = 13, normalized size = 0.76

$$-\left(\cos(x)^3 - \cos(x)\right)\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)*sin(3*x),x, algorithm="fricas")
```

```
[Out] -(cos(x)^3 - cos(x))*sin(x)
```

giac [A] time = 0.14, size = 13, normalized size = 0.76

$$-\frac{1}{8}\,\sin\left(4\,x\right) + \frac{1}{4}\,\sin\left(2\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)*sin(3*x),x, algorithm="giac")
```

```
[Out] -1/8*sin(4*x) + 1/4*sin(2*x)
```

maple [A] time = 0.11, size = 14, normalized size = 0.82

$$\frac{\sin\left(2x\right)}{4} - \frac{\sin\left(4x\right)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)*sin(3*x),x)
```

```
[Out] 1/4*sin(2*x)-1/8*sin(4*x)
```

maxima [A] time = 0.31, size = 13, normalized size = 0.76

$$-\frac{1}{8}\,\sin\left(4\,x\right) + \frac{1}{4}\,\sin\left(2\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)*sin(3*x),x, algorithm="maxima")
```

```
[Out] -1/8*sin(4*x) + 1/4*sin(2*x)
```

mupad [B] time = 0.03, size = 13, normalized size = 0.76

$$\frac{\sin\left(2\,x\right)}{4} - \frac{\sin\left(4\,x\right)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(3*x)*sin(x),x)
```

```
[Out] sin(2*x)/4 - sin(4*x)/8
```

sympy [A] time = 0.42, size = 20, normalized size = 1.18

$$-\frac{3\sin(x)\cos(3x)}{8}+\frac{\sin(3x)\cos(x)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)*sin(3*x),x)
```

```
[Out] -3*sin(x)*cos(3*x)/8 + sin(3*x)*cos(x)/8
```

3.68 $\int \sin(x)\sin(4x)\,dx$

Optimal. Leaf size=17

$$\frac{1}{6}\sin(3x) - \frac{1}{10}\sin(5x)$$

```
[Out] 1/6*sin(3*x)-1/10*sin(5*x)
```

Rubi [A] time = 0.01, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {4282}

$$\frac{1}{6}\sin(3x) - \frac{1}{10}\sin(5x)$$

Antiderivative was successfully verified.

```
[In] Int[Sin[x]*Sin[4*x],x]
```

```
[Out] Sin[3*x]/6 - Sin[5*x]/10
```

Rule 4282

```
Int[sin[(a_.) + (b_.)*(x_)]*sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[
a - c + (b - d)*x]/(2*(b - d)), x] - Simp[Sin[a + c + (b + d)*x]/(2*(b + d)
), x] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - d^2, 0]
```

Rubi steps

$$\int \sin(x)\sin(4x)\,dx = \frac{1}{6}\sin(3x) - \frac{1}{10}\sin(5x)$$

Mathematica [A] time = 0.01, size = 17, normalized size = 1.00

$$\frac{1}{6}\sin(3x) - \frac{1}{10}\sin(5x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[x]*Sin[4*x],x]
```

```
[Out] Sin[3*x]/6 - Sin[5*x]/10
```

fricas [A] time = 1.12, size = 18, normalized size = 1.06

$$-\frac{4}{15}\left(6\cos(x)^4 - 7\cos(x)^2 + 1\right)\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)*sin(4*x),x, algorithm="fricas")
[Out] -4/15*(6*cos(x)^4 - 7*cos(x)^2 + 1)*sin(x)
```

giac [A] time = 0.14, size = 13, normalized size = 0.76

$$-\frac{8}{5}\sin(x)^5 + \frac{4}{3}\sin(x)^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)*sin(4*x),x, algorithm="giac")
[Out] -8/5*sin(x)^5 + 4/3*sin(x)^3
```

maple [A] time = 0.10, size = 14, normalized size = 0.82

$$\frac{\sin(3x)}{6} - \frac{\sin(5x)}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)*sin(4*x),x)
[Out] 1/6*sin(3*x)-1/10*sin(5*x)
```

maxima [A] time = 0.31, size = 13, normalized size = 0.76

$$-\frac{1}{10}\sin(5x) + \frac{1}{6}\sin(3x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)*sin(4*x),x, algorithm="maxima")
[Out] -1/10*sin(5*x) + 1/6*sin(3*x)
```

mupad [B] time = 0.03, size = 13, normalized size = 0.76

$$\frac{\sin(3x)}{6} - \frac{\sin(5x)}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(4*x)*sin(x),x)
[Out] sin(3*x)/6 - sin(5*x)/10
```

sympy [A] time = 0.41, size = 20, normalized size = 1.18

$$-\frac{4\sin(x)\cos(4x)}{15} + \frac{\sin(4x)\cos(x)}{15}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)*sin(4*x),x)
```

```
[Out] -4*sin(x)*cos(4*x)/15 + sin(4*x)*cos(x)/15
```

3.69 $\int \sin(x)\sin(mx)\,dx$

Optimal. Leaf size=35

$$\frac{\sin((1-m)x)}{2(1-m)}-\frac{\sin((m+1)x)}{2(m+1)}$$

```
[Out] 1/2*sin((1-m)*x)/(1-m)-1/2*sin((1+m)*x)/(1+m)
```

Rubi [A] time = 0.03, antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {4569, 2637}

$$\frac{\sin((1-m)x)}{2(1-m)}-\frac{\sin((m+1)x)}{2(m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[x]*Sin[m*x],x]
```

```
[Out] Sin[(1 - m)*x]/(2*(1 - m)) - Sin[(1 + m)*x]/(2*(1 + m))
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 4569

```
Int[Sin[v_]^(p_.)*Sin[w_]^(q_.), x_Symbol] :> Int[ExpandTrigReduce[Sin[v]^p
*Sin[w]^q, x], x] /; ((PolynomialQ[v, x] && PolynomialQ[w, x]) || (Binomial
Q[{v, w}, x] && IndependentQ[Cancel[v/w], x])) && IGtQ[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int \sin(x)\sin(mx)\,dx &= \int \left(\frac{1}{2}\cos((1-m)x)-\frac{1}{2}\cos((1+m)x)\right)dx\\ &= \frac{1}{2}\int \cos((1-m)x)\,dx-\frac{1}{2}\int \cos((1+m)x)\,dx\\ &= \frac{\sin((1-m)x)}{2(1-m)}-\frac{\sin((1+m)x)}{2(1+m)}\end{aligned}$$

Mathematica [A] time = 0.05, size = 25, normalized size = 0.71

$$\frac{\cos(x)\sin(mx)-m\sin(x)\cos(mx)}{m^2-1}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[x]*Sin[m*x],x]
[Out] (-(m*Cos[m*x]*Sin[x]) + Cos[x]*Sin[m*x])/(-1 + m^2)
```

fricas [A] time = 1.08, size = 26, normalized size = 0.74

$$-\frac{m\cos(mx)\sin(x)-\cos(x)\sin(mx)}{m^2-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)*sin(m*x),x, algorithm="fricas")
[Out] -(m*cos(m*x)*sin(x) - cos(x)*sin(m*x))/(m^2 - 1)
```

giac [A] time = 0.14, size = 29, normalized size = 0.83

$$-\frac{\sin(mx+x)}{2(m+1)}+\frac{\sin(mx-x)}{2(m-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)*sin(m*x),x, algorithm="giac")
[Out] -1/2*sin(m*x + x)/(m + 1) + 1/2*sin(m*x - x)/(m - 1)
```

maple [A] time = 0.08, size = 28, normalized size = 0.80

$$\frac{\sin((-1+m)x)}{-2+2m}-\frac{\sin((1+m)x)}{2(1+m)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)*sin(m*x),x)
[Out] 1/2/(-1+m)*sin((-1+m)*x)-1/2*sin((1+m)*x)/(1+m)
```

maxima [A] time = 0.31, size = 28, normalized size = 0.80

$$-\frac{\sin((m+1)x)}{2(m+1)}-\frac{\sin(-(m-1)x)}{2(m-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)*sin(m*x),x, algorithm="maxima")
[Out] -1/2*sin((m + 1)*x)/(m + 1) - 1/2*sin(-(m - 1)*x)/(m - 1)
```

mupad [B] time = 2.32, size = 64, normalized size = 1.83

$$\begin{cases} \frac{x}{2} - \frac{\sin(2x)}{4} & \text{if } m = 1 \\ \frac{\sin(2x)}{4} - \frac{x}{2} & \text{if } m = -1 \\ \frac{\sin(x(m-1))}{2m-2} - \frac{\sin(x(m+1))}{2m+2} & \text{if } m \neq -1 \land m \neq 1 \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(m*x)*sin(x),x)
```

```
[Out] piecewise(m == 1, x/2 - sin(2*x)/4, m == -1, - x/2 + sin(2*x)/4, m ~= -1 &
m ~= 1, sin(x*(m - 1))/(2*m - 2) - sin(x*(m + 1))/(2*m + 2))
```

sympy [A] time = 0.79, size = 78, normalized size = 2.23

$$\begin{cases} -\frac{x\sin^2(x)}{2} - \frac{x\cos^2(x)}{2} + \frac{\sin(x)\cos(x)}{2} & \text{for } m = -1 \\ \frac{x\sin^2(x)}{2} + \frac{x\cos^2(x)}{2} - \frac{\sin(x)\cos(x)}{2} & \text{for } m = 1 \\ -\frac{m\sin(x)\cos(mx)}{m^2-1} + \frac{\sin(mx)\cos(x)}{m^2-1} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)*sin(m*x),x)
```

```
[Out] Piecewise((-x*sin(x)**2/2 - x*cos(x)**2/2 + sin(x)*cos(x)/2, Eq(m, -1)), (x
*sin(x)**2/2 + x*cos(x)**2/2 - sin(x)*cos(x)/2, Eq(m, 1)), (-m*sin(x)*cos(m
*x)/(m**2 - 1) + sin(m*x)*cos(x)/(m**2 - 1), True))
```

3.70 $\int \cos(2x)\sin(x)\,dx$

Optimal. Leaf size=15

$$\frac{\cos(x)}{2}-\frac{1}{6}\cos(3x)$$

```
[Out] 1/2*cos(x)-1/6*cos(3*x)
```

Rubi [A] time = 0.01, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {4284}

$$\frac{\cos(x)}{2}-\frac{1}{6}\cos(3x)$$

Antiderivative was successfully verified.

```
[In] Int[Cos[2*x]*Sin[x],x]
[Out] Cos[x]/2 - Cos[3*x]/6
```

Rule 4284

```
Int[cos[(c_.) + (d_.)*(x_)]*sin[(a_.) + (b_.)*(x_)], x_Symbol] :> -Simp[Cos
[a - c + (b - d)*x]/(2*(b - d)), x] - Simp[Cos[a + c + (b + d)*x]/(2*(b + d
)), x] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - d^2, 0]
```

Rubi steps

$$\int \cos(2x)\sin(x)\,dx=\frac{\cos(x)}{2}-\frac{1}{6}\cos(3x)$$

Mathematica [A] time = 0.01, size = 15, normalized size = 1.00

$$\frac{\cos(x)}{2}-\frac{1}{6}\cos(3x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[2*x]*Sin[x],x]
[Out] Cos[x]/2 - Cos[3*x]/6
```

fricas [A] time = 0.78, size = 9, normalized size = 0.60

$$-\frac{2}{3}\cos(x)^3+\cos(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(2*x)*sin(x),x, algorithm="fricas")
[Out] -2/3*cos(x)^3 + cos(x)
```

giac [A] time = 0.13, size = 11, normalized size = 0.73

$$-\frac{1}{6}\cos\left(3\,x\right)+\frac{1}{2}\cos\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(2*x)*sin(x),x, algorithm="giac")
[Out] -1/6*cos(3*x) + 1/2*cos(x)
```

maple [A] time = 0.07, size = 12, normalized size = 0.80

$$\frac{\cos(x)}{2}-\frac{\cos(3x)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(2*x)*sin(x),x)
[Out] 1/2*cos(x)-1/6*cos(3*x)
```

maxima [A] time = 0.31, size = 11, normalized size = 0.73

$$-\frac{1}{6}\cos\left(3\,x\right)+\frac{1}{2}\cos\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(2*x)*sin(x),x, algorithm="maxima")
[Out] -1/6*cos(3*x) + 1/2*cos(x)
```

mupad [B] time = 0.02, size = 9, normalized size = 0.60

$$\cos(x)-\frac{2\cos(x)^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(2*x)*sin(x),x)
[Out] cos(x) - (2*cos(x)^3)/3
```

sympy [**A**] time = 0.45, size = 20, normalized size = 1.33

$$\frac{2\sin(x)\sin(2x)}{3} + \frac{\cos(x)\cos(2x)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(2*x)*sin(x),x)
```

```
[Out] 2*sin(x)*sin(2*x)/3 + cos(x)*cos(2*x)/3
```

3.71 $\int \cos(3x)\sin(x)\,dx$

Optimal. Leaf size=17

$$\frac{1}{4}\cos(2x)-\frac{1}{8}\cos(4x)$$

```
[Out] 1/4*cos(2*x)-1/8*cos(4*x)
```

Rubi [A] time = 0.01, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {4284}

$$\frac{1}{4}\cos(2x)-\frac{1}{8}\cos(4x)$$

Antiderivative was successfully verified.

```
[In] Int[Cos[3*x]*Sin[x],x]
```

```
[Out] Cos[2*x]/4 - Cos[4*x]/8
```

Rule 4284

```
Int[cos[(c_.) + (d_.)*(x_)]*sin[(a_.) + (b_.)*(x_)], x_Symbol] :> -Simp[Cos
[a - c + (b - d)*x]/(2*(b - d)), x] - Simp[Cos[a + c + (b + d)*x]/(2*(b + d
)), x] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - d^2, 0]
```

Rubi steps

$$\int \cos(3x)\sin(x)\,dx=\frac{1}{4}\cos(2x)-\frac{1}{8}\cos(4x)$$

Mathematica [A] time = 0.01, size = 17, normalized size = 1.00

$$\frac{\cos^2(x)}{2}-\frac{1}{8}\cos(4x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[3*x]*Sin[x],x]
```

```
[Out] Cos[x]^2/2 - Cos[4*x]/8
```

fricas [A] time = 0.83, size = 13, normalized size = 0.76

$$-\cos(x)^4+\frac{3}{2}\cos(x)^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(3*x)*sin(x),x, algorithm="fricas")
```

```
[Out] -cos(x)^4 + 3/2*cos(x)^2
```

giac [A] time = 0.12, size = 13, normalized size = 0.76

$$-\sin(x)^4 + \frac{1}{2}\sin(x)^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(3*x)*sin(x),x, algorithm="giac")
```

```
[Out] -sin(x)^4 + 1/2*sin(x)^2
```

maple [A] time = 0.10, size = 14, normalized size = 0.82

$$\frac{\cos(2x)}{4} - \frac{\cos(4x)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(3*x)*sin(x),x)
```

```
[Out] 1/4*cos(2*x)-1/8*cos(4*x)
```

maxima [A] time = 0.54, size = 13, normalized size = 0.76

$$-\frac{1}{8}\cos(4x) + \frac{1}{4}\cos(2x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(3*x)*sin(x),x, algorithm="maxima")
```

```
[Out] -1/8*cos(4*x) + 1/4*cos(2*x)
```

mupad [B] time = 0.02, size = 13, normalized size = 0.76

$$\frac{3\cos(x)^2}{2} - \cos(x)^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(3*x)*sin(x),x)
```

```
[Out] (3*cos(x)^2)/2 - cos(x)^4
```

sympy [A] time = 0.41, size = 20, normalized size = 1.18

$$\frac{3\sin(x)\sin(3x)}{8} + \frac{\cos(x)\cos(3x)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(3*x)*sin(x),x)
```

```
[Out] 3*sin(x)*sin(3*x)/8 + cos(x)*cos(3*x)/8
```

3.72 $\int \cos(4x)\sin(x)\,dx$

Optimal. Leaf size=17

$$\frac{1}{6}\cos(3x)-\frac{1}{10}\cos(5x)$$

[Out] 1/6*cos(3*x)-1/10*cos(5*x)

Rubi [A] time = 0.01, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {4284}

$$\frac{1}{6}\cos(3x)-\frac{1}{10}\cos(5x)$$

Antiderivative was successfully verified.

[In] Int[Cos[4*x]*Sin[x],x]

[Out] Cos[3*x]/6 - Cos[5*x]/10

Rule 4284

```
Int[cos[(c_.) + (d_.)*(x_)]*sin[(a_.) + (b_.)*(x_)], x_Symbol] :> -Simp[Cos
[a - c + (b - d)*x]/(2*(b - d)), x] - Simp[Cos[a + c + (b + d)*x]/(2*(b + d
)), x] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - d^2, 0]
```

Rubi steps

$$\int \cos(4x)\sin(x)\,dx=\frac{1}{6}\cos(3x)-\frac{1}{10}\cos(5x)$$

Mathematica [A] time = 0.01, size = 17, normalized size = 1.00

$$\frac{1}{6}\cos(3x)-\frac{1}{10}\cos(5x)$$

Antiderivative was successfully verified.

[In] Integrate[Cos[4*x]*Sin[x],x]

[Out] Cos[3*x]/6 - Cos[5*x]/10

fricas [A] time = 1.06, size = 17, normalized size = 1.00

$$-\frac{8}{5}\cos(x)^5+\frac{8}{3}\cos(x)^3-\cos(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(4*x)*sin(x),x, algorithm="fricas")
```

```
[Out] -8/5*cos(x)^5 + 8/3*cos(x)^3 - cos(x)
```

giac [A] time = 0.12, size = 13, normalized size = 0.76

$$-\frac{1}{10}\cos\left(5\,x\right)+\frac{1}{6}\cos\left(3\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(4*x)*sin(x),x, algorithm="giac")
```

```
[Out] -1/10*cos(5*x) + 1/6*cos(3*x)
```

maple [A] time = 0.11, size = 14, normalized size = 0.82

$$\frac{\cos\left(3x\right)}{6}-\frac{\cos\left(5x\right)}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(4*x)*sin(x),x)
```

```
[Out] 1/6*cos(3*x)-1/10*cos(5*x)
```

maxima [A] time = 0.51, size = 13, normalized size = 0.76

$$-\frac{1}{10}\cos\left(5\,x\right)+\frac{1}{6}\cos\left(3\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(4*x)*sin(x),x, algorithm="maxima")
```

```
[Out] -1/10*cos(5*x) + 1/6*cos(3*x)
```

mupad [B] time = 0.03, size = 17, normalized size = 1.00

$$-\frac{8\cos(x)^5}{5}+\frac{8\cos(x)^3}{3}-\cos(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(4*x)*sin(x),x)
```

```
[Out] (8*cos(x)^3)/3 - cos(x) - (8*cos(x)^5)/5
```

sympy [A] time = 0.41, size = 20, normalized size = 1.18

$$\frac{4\sin(x)\sin(4x)}{15}+\frac{\cos(x)\cos(4x)}{15}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(4*x)*sin(x),x)
```

```
[Out] 4*sin(x)*sin(4*x)/15 + cos(x)*cos(4*x)/15
```

3.73 $\int \cos(mx)\sin(x)\,dx$

Optimal. Leaf size=35

$$-\frac{\cos((1-m)x)}{2(1-m)}-\frac{\cos((m+1)x)}{2(m+1)}$$

```
[Out] -1/2*cos((1-m)*x)/(1-m)-1/2*cos((1+m)*x)/(1+m)
```

Rubi [A] time = 0.03, antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {4574, 2638}

$$-\frac{\cos((1-m)x)}{2(1-m)}-\frac{\cos((m+1)x)}{2(m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[m*x]*Sin[x],x]
```

```
[Out] -Cos[(1 - m)*x]/(2*(1 - m)) - Cos[(1 + m)*x]/(2*(1 + m))
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 4574

```
Int[Cos[w_]^(q_.)*Sin[v_]^(p_.), x_Symbol] :> Int[ExpandTrigReduce[Sin[v]^p
*Cos[w]^q, x], x] /; IGtQ[p, 0] && IGtQ[q, 0] && ((PolynomialQ[v, x] && Pol
ynomialQ[w, x]) || (BinomialQ[{v, w}, x] && IndependentQ[Cancel[v/w], x]))
```

Rubi steps

$$\begin{aligned}\int \cos(mx)\sin(x)\,dx &= \int \left(\frac{1}{2}\sin((1-m)x)+\frac{1}{2}\sin((1+m)x)\right)dx\\ &= \frac{1}{2}\int \sin((1-m)x)\,dx+\frac{1}{2}\int \sin((1+m)x)\,dx\\ &= -\frac{\cos((1-m)x)}{2(1-m)}-\frac{\cos((1+m)x)}{2(1+m)}\end{aligned}$$

Mathematica [A] time = 0.04, size = 24, normalized size = 0.69

$$\frac{m\sin(x)\sin(mx)+\cos(x)\cos(mx)}{m^2-1}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[m*x]*Sin[x],x]
```

```
[Out] (Cos[x]*Cos[m*x] + m*Sin[x]*Sin[m*x])/(-1 + m^2)
```

fricas [A] time = 0.80, size = 24, normalized size = 0.69

$$\frac{m\sin(mx)\sin(x)+\cos(mx)\cos(x)}{m^2-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(m*x)*sin(x),x, algorithm="fricas")
```

```
[Out] (m*sin(m*x)*sin(x) + cos(m*x)*cos(x))/(m^2 - 1)
```

giac [A] time = 0.13, size = 29, normalized size = 0.83

$$-\frac{\cos(mx+x)}{2(m+1)}+\frac{\cos(mx-x)}{2(m-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(m*x)*sin(x),x, algorithm="giac")
```

```
[Out] -1/2*cos(m*x + x)/(m + 1) + 1/2*cos(m*x - x)/(m - 1)
```

maple [A] time = 0.03, size = 28, normalized size = 0.80

$$\frac{\cos((-1+m)x)}{-2+2m}-\frac{\cos((1+m)x)}{2(1+m)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(m*x)*sin(x),x)
```

```
[Out] 1/2*cos((-1+m)*x)/(-1+m)-1/2*cos((1+m)*x)/(1+m)
```

maxima [A] time = 0.32, size = 28, normalized size = 0.80

$$-\frac{\cos((m+1)x)}{2(m+1)}+\frac{\cos(-(m-1)x)}{2(m-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(m*x)*sin(x),x, algorithm="maxima")
```

```
[Out] -1/2*cos((m + 1)*x)/(m + 1) + 1/2*cos(-(m - 1)*x)/(m - 1)
```

mupad [B] time = 0.10, size = 37, normalized size = 1.06

$$\begin{cases} \frac{\sin(x)^2}{2} & \text{if } m = -1 \vee m = 1 \\ \frac{\cos(x\,(m-1))}{2\,m-2} - \frac{\cos(x\,(m+1))}{2\,m+2} & \text{if } m \neq -1 \wedge m \neq 1 \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(m*x)*sin(x),x)
```

```
[Out] piecewise(m == -1 | m == 1, sin(x)^2/2, m ~= -1 & m ~= 1, cos(x*(m - 1))/(2
*m - 2) - cos(x*(m + 1))/(2*m + 2))
```

sympy [A] time = 0.93, size = 37, normalized size = 1.06

$$\begin{cases} \frac{\sin^{2}{\left(x \right)}}{2} & \text{for}\: m = -1 \vee m = 1 \\ \frac{m \sin{\left(x \right)} \sin{\left(m x \right)}}{m^{2} - 1} + \frac{\cos{\left(x \right)} \cos{\left(m x \right)}}{m^{2} - 1} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(m*x)*sin(x),x)
```

```
[Out] Piecewise((sin(x)**2/2, Eq(m, -1) | Eq(m, 1)), (m*sin(x)*sin(m*x)/(m**2 - 1
) + cos(x)*cos(m*x)/(m**2 - 1), True))
```

3.74 $\int \sin(x) \tan(2x) \, dx$

Optimal. Leaf size=20

$$\frac{\tanh^{-1}\left(\sqrt{2}\sin(x)\right)}{\sqrt{2}} - \sin(x)$$

```
[Out] -sin(x)+1/2*arctanh(sin(x)*2^(1/2))*2^(1/2)
```

Rubi [A] time = 0.02, antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {12, 321, 206}

$$\frac{\tanh^{-1}\left(\sqrt{2}\sin(x)\right)}{\sqrt{2}} - \sin(x)$$

Antiderivative was successfully verified.

```
[In] Int[Sin[x]*Tan[2*x],x]
```

```
[Out] ArcTanh[Sqrt[2]*Sin[x]]/Sqrt[2] - Sin[x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\int \sin(x)\tan(2x)\,dx = \text{Subst}\left(\int \frac{2x^2}{1-2x^2}\,dx, x, \sin(x)\right)$$
$$= 2\,\text{Subst}\left(\int \frac{x^2}{1-2x^2}\,dx, x, \sin(x)\right)$$
$$= -\sin(x) + \text{Subst}\left(\int \frac{1}{1-2x^2}\,dx, x, \sin(x)\right)$$
$$= \frac{\tanh^{-1}\left(\sqrt{2}\sin(x)\right)}{\sqrt{2}} - \sin(x)$$

Mathematica [A] time = 0.01, size = 20, normalized size = 1.00

$$\frac{\tanh^{-1}\left(\sqrt{2}\sin(x)\right)}{\sqrt{2}} - \sin(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[x]*Tan[2*x],x]
```

```
[Out] ArcTanh[Sqrt[2]*Sin[x]]/Sqrt[2] - Sin[x]
```

fricas [B] time = 2.06, size = 38, normalized size = 1.90

$$\frac{1}{4}\sqrt{2}\log\left(-\frac{2\cos(x)^2 - 2\sqrt{2}\sin(x) - 3}{2\cos(x)^2 - 1}\right) - \sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)*tan(2*x),x, algorithm="fricas")
```

```
[Out] 1/4*sqrt(2)*log(-(2*cos(x)^2 - 2*sqrt(2)*sin(x) - 3)/(2*cos(x)^2 - 1)) - si
n(x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sin(x)\tan(2x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)*tan(2*x),x, algorithm="giac")
```

```
[Out] integrate(sin(x)*tan(2*x), x)
```

maple [A] time = 0.16, size = 18, normalized size = 0.90

$$-\sin(x) + \frac{\operatorname{arctanh}\left(\sin(x)\sqrt{2}\right)\sqrt{2}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)*tan(2*x),x)
```

```
[Out] -sin(x)+1/2*arctanh(sin(x)*2^(1/2))*2^(1/2)
```

maxima [B] time = 0.44, size = 141, normalized size = 7.05

$$\frac{1}{8}\sqrt{2}\log\left(2\cos(x)^2 + 2\sin(x)^2 + 2\sqrt{2}\cos(x) + 2\sqrt{2}\sin(x) + 2\right) - \frac{1}{8}\sqrt{2}\log\left(2\cos(x)^2 + 2\sin(x)^2 + 2\sqrt{2}\cos\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)*tan(2*x),x, algorithm="maxima")
```

```
[Out] 1/8*sqrt(2)*log(2*cos(x)^2 + 2*sin(x)^2 + 2*sqrt(2)*cos(x) + 2*sqrt(2)*sin(
x) + 2) - 1/8*sqrt(2)*log(2*cos(x)^2 + 2*sin(x)^2 + 2*sqrt(2)*cos(x) - 2*sq
rt(2)*sin(x) + 2) + 1/8*sqrt(2)*log(2*cos(x)^2 + 2*sin(x)^2 - 2*sqrt(2)*cos
(x) + 2*sqrt(2)*sin(x) + 2) - 1/8*sqrt(2)*log(2*cos(x)^2 + 2*sin(x)^2 - 2*s
qrt(2)*cos(x) - 2*sqrt(2)*sin(x) + 2) - sin(x)
```

mupad [B] time = 2.39, size = 17, normalized size = 0.85

$$\frac{\sqrt{2}\operatorname{atanh}\left(\sqrt{2}\sin(x)\right)}{2} - \sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(2*x)*sin(x),x)
```

```
[Out] (2^(1/2)*atanh(2^(1/2)*sin(x)))/2 - sin(x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sin(x)\tan(2x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)*tan(2*x),x)
```

```
[Out] Integral(sin(x)*tan(2*x), x)
```

3.75 $\int \sin(x)\tan(3x)\,dx$

Optimal. Leaf size=47

$$-\sin(x)-\frac{1}{6}\log(1-2\sin(x))-\frac{1}{6}\log(1-\sin(x))+\frac{1}{6}\log(\sin(x)+1)+\frac{1}{6}\log(2\sin(x)+1)$$

```
[Out] -1/6*ln(1-2*sin(x))-1/6*ln(1-sin(x))+1/6*ln(1+sin(x))+1/6*ln(1+2*sin(x))-si
n(x)
```

Rubi [A] time = 0.05, antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 4, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.571, Rules used = {1279, 1161, 616, 31}

$$-\sin(x)-\frac{1}{6}\log(1-2\sin(x))-\frac{1}{6}\log(1-\sin(x))+\frac{1}{6}\log(\sin(x)+1)+\frac{1}{6}\log(2\sin(x)+1)$$

Antiderivative was successfully verified.

```
[In] Int[Sin[x]*Tan[3*x],x]
```

```
[Out] -Log[1 - 2*Sin[x]]/6 - Log[1 - Sin[x]]/6 + Log[1 + Sin[x]]/6 + Log[1 + 2*Si
n[x]]/6 - Sin[x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 616

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = Rt[b^2
 - 4*a*c, 2]}, Dist[c/q, Int[1/Simp[b/2 - q/2 + c*x, x], x], x] - Dist[c/q,
 Int[1/Simp[b/2 + q/2 + c*x, x], x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[b^2 - 4*a*c] && PerfectSquareQ[b^2 - 4*a*c]
```

Rule 1161

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[(2*d)/e - b/c, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2
, x], x], x] + Dist[e/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; Fre
eQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] && (
GtQ[(2*d)/e - b/c, 0] || ( !LtQ[(2*d)/e - b/c, 0] && EqQ[d - e*Rt[a/c, 2],
0]))
```

Rule 1279

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[(e*f*(f*x)^(m - 1)*(a + b*x^2 + c*x^4)^(p +
1))/(c*(m + 4*p + 3)), x] - Dist[f^2/(c*(m + 4*p + 3)), Int[(f*x)^(m - 2)*(
a + b*x^2 + c*x^4)^p*Simp[a*e*(m - 1) + (b*e*(m + 2*p + 1) - c*d*(m + 4*p +
 3))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c,
 0] && GtQ[m, 1] && NeQ[m + 4*p + 3, 0] && IntegerQ[2*p] && (IntegerQ[p] ||
 IntegerQ[m])
```

Rubi steps

$$\begin{aligned}\int \sin(x)\tan(3x)\,dx &= \text{Subst}\left(\int \frac{x^2\left(3-4x^2\right)}{1-5x^2+4x^4}\,dx, x, \sin(x)\right)\\ &= -\sin(x)-\frac{1}{4}\text{Subst}\left(\int \frac{-4+8x^2}{1-5x^2+4x^4}\,dx, x, \sin(x)\right)\\ &= -\sin(x)-\frac{1}{4}\text{Subst}\left(\int \frac{1}{-\frac{1}{2}-\frac{x}{2}+x^2}\,dx, x, \sin(x)\right)-\frac{1}{4}\text{Subst}\left(\int \frac{1}{-\frac{1}{2}+\frac{x}{2}+x^2}\,dx, x, \sin(x)\right)\\ &= -\sin(x)-\frac{1}{6}\text{Subst}\left(\int \frac{1}{-1+x}\,dx, x, \sin(x)\right)-\frac{1}{6}\text{Subst}\left(\int \frac{1}{-\frac{1}{2}+x}\,dx, x, \sin(x)\right)+\frac{1}{6}\text{Subs}\\ &= -\frac{1}{6}\log(1-2\sin(x))-\frac{1}{6}\log(1-\sin(x))+\frac{1}{6}\log(1+\sin(x))+\frac{1}{6}\log(1+2\sin(x))-\sin(x\end{aligned}$$

Mathematica [A] time = 0.03, size = 21, normalized size = 0.45

$$-\sin(x)+\frac{1}{3}\tanh^{-1}(\sin(x))+\frac{1}{3}\tanh^{-1}(2\sin(x))$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[x]*Tan[3*x],x]
```

```
[Out] ArcTanh[Sin[x]]/3 + ArcTanh[2*Sin[x]]/3 - Sin[x]
```

fricas [A] time = 1.91, size = 39, normalized size = 0.83

$$\frac{1}{6}\log\left(2\sin(x)+1\right)+\frac{1}{6}\log\left(\sin(x)+1\right)-\frac{1}{6}\log\left(-\sin(x)+1\right)-\frac{1}{6}\log\left(-2\sin(x)+1\right)-\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)*tan(3*x),x, algorithm="fricas")
```

```
[Out] 1/6*log(2*sin(x) + 1) + 1/6*log(sin(x) + 1) - 1/6*log(-sin(x) + 1) - 1/6*lo
g(-2*sin(x) + 1) - sin(x)
```

giac [B] time = 0.27, size = 364, normalized size = 7.74

$$\log\left(\frac{\tan\left(\frac{1}{2}x\right)^4+8\tan\left(\frac{1}{2}x\right)^3+18\tan\left(\frac{1}{2}x\right)^2+8\tan\left(\frac{1}{2}x\right)+1}{\tan\left(\frac{1}{2}x\right)^4+2\tan\left(\frac{1}{2}x\right)^2+1}\right)\tan\left(\frac{1}{2}x\right)^2-\log\left(\frac{\tan\left(\frac{1}{2}x\right)^4-8\tan\left(\frac{1}{2}x\right)^3+18\tan\left(\frac{1}{2}x\right)^2-8\tan\left(\frac{1}{2}x\right)+1}{\tan\left(\frac{1}{2}x\right)^4+2\tan\left(\frac{1}{2}x\right)^2+1}\right)\tan\left(\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)*tan(3*x),x, algorithm="giac")
```

```
[Out] 1/12*(log((tan(1/2*x)^4 + 8*tan(1/2*x)^3 + 18*tan(1/2*x)^2 + 8*tan(1/2*x) +
 1)/(tan(1/2*x)^4 + 2*tan(1/2*x)^2 + 1))*tan(1/2*x)^2 - log((tan(1/2*x)^4 -
 8*tan(1/2*x)^3 + 18*tan(1/2*x)^2 - 8*tan(1/2*x) + 1)/(tan(1/2*x)^4 + 2*tan
(1/2*x)^2 + 1))*tan(1/2*x)^2 + 2*log(2*(tan(1/2*x)^2 + 2*tan(1/2*x) + 1)/(t
an(1/2*x)^2 + 1))*tan(1/2*x)^2 - 2*log(2*(tan(1/2*x)^2 - 2*tan(1/2*x) + 1)/
(tan(1/2*x)^2 + 1))*tan(1/2*x)^2 + log((tan(1/2*x)^4 + 8*tan(1/2*x)^3 + 18*
tan(1/2*x)^2 + 8*tan(1/2*x) + 1)/(tan(1/2*x)^4 + 2*tan(1/2*x)^2 + 1)) - log
((tan(1/2*x)^4 - 8*tan(1/2*x)^3 + 18*tan(1/2*x)^2 - 8*tan(1/2*x) + 1)/(tan(
1/2*x)^4 + 2*tan(1/2*x)^2 + 1)) + 2*log(2*(tan(1/2*x)^2 + 2*tan(1/2*x) + 1)
/(tan(1/2*x)^2 + 1)) - 2*log(2*(tan(1/2*x)^2 - 2*tan(1/2*x) + 1)/(tan(1/2*x
)^2 + 1)) - 24*tan(1/2*x))/(tan(1/2*x)^2 + 1)
```

maple [A] time = 0.30, size = 38, normalized size = 0.81

$$-\frac{\ln(-1+2\sin(x))}{6}+\frac{\ln(1+2\sin(x))}{6}-\frac{\ln(\sin(x)-1)}{6}+\frac{\ln(1+\sin(x))}{6}-\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)*tan(3*x),x)
```

```
[Out] -1/6*ln(-1+2*sin(x))+1/6*ln(1+2*sin(x))-1/6*ln(sin(x)-1)+1/6*ln(1+sin(x))-s
in(x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{(\cos(3x)+\cos(x))\cos(4x)-(\cos(2x)-1)\cos(3x)-\cos(2x)\cos(x)+(\sin(3x)+\sin(x))\sin(4x)-\mathrm{si}}{3\left(2(\cos(2x)-1)\cos(4x)-\cos(4x)^2-\cos(2x)^2-\sin(4x)^2+2\sin(4x)\sin(2x)-\mathrm{sin}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)*tan(3*x),x, algorithm="maxima")
```

```
[Out] integrate(-1/3*((cos(3*x) + cos(x))*cos(4*x) - (cos(2*x) - 1)*cos(3*x) - co
s(2*x)*cos(x) + (sin(3*x) + sin(x))*sin(4*x) - sin(3*x)*sin(2*x) - sin(2*x)
*sin(x) + cos(x))/(2*(cos(2*x) - 1)*cos(4*x) - cos(4*x)^2 - cos(2*x)^2 - si
n(4*x)^2 + 2*sin(4*x)*sin(2*x) - sin(2*x)^2 + 2*cos(2*x) - 1), x) + 1/6*log
(cos(x)^2 + sin(x)^2 + 2*sin(x) + 1) - 1/6*log(cos(x)^2 + sin(x)^2 - 2*sin(
x) + 1) - sin(x)
```

mupad [B] time = 2.34, size = 26, normalized size = 0.55

$$\frac{2\,\mathrm{atanh}\left(\frac{\sin\left(\frac{x}{2}\right)}{\cos\left(\frac{x}{2}\right)}\right)}{3}+\frac{\mathrm{atanh}\left(2\,\sin(x)\right)}{3}-\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(3*x)*sin(x),x)
```

```
[Out] (2*atanh(sin(x/2)/cos(x/2)))/3 + atanh(2*sin(x))/3 - sin(x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sin\left(x\right)\tan\left(3x\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)*tan(3*x),x)
```

```
[Out] Integral(sin(x)*tan(3*x), x)
```

3.76 $\int \sin(x)\tan(4x)\,dx$

Optimal. Leaf size=71

$$-\sin(x)+\frac{1}{4}\sqrt{2-\sqrt{2}}\,\tanh^{-1}\left(\frac{2\sin(x)}{\sqrt{2-\sqrt{2}}}\right)+\frac{1}{4}\sqrt{2+\sqrt{2}}\,\tanh^{-1}\left(\frac{2\sin(x)}{\sqrt{2+\sqrt{2}}}\right)$$

```
[Out] -sin(x)+1/4*arctanh(2*sin(x)/(2-2^(1/2))^(1/2))*(2-2^(1/2))^(1/2)+1/4*arcta
nh(2*sin(x)/(2+2^(1/2))^(1/2))*(2+2^(1/2))^(1/2)
```

Rubi [A] time = 0.11, antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {1279, 1166, 207}

$$-\sin(x)+\frac{1}{4}\sqrt{2-\sqrt{2}}\,\tanh^{-1}\left(\frac{2\sin(x)}{\sqrt{2-\sqrt{2}}}\right)+\frac{1}{4}\sqrt{2+\sqrt{2}}\,\tanh^{-1}\left(\frac{2\sin(x)}{\sqrt{2+\sqrt{2}}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sin[x]*Tan[4*x],x]
```

```
[Out] (Sqrt[2 - Sqrt[2]]*ArcTanh[(2*Sin[x])/Sqrt[2 - Sqrt[2]]])/4 + (Sqrt[2 + Sqr
t[2]]*ArcTanh[(2*Sin[x])/Sqrt[2 + Sqrt[2]]])/4 - Sin[x]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1279

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[(e*f*(f*x)^(m - 1)*(a + b*x^2 + c*x^4)^(p +
1))/(c*(m + 4*p + 3)), x] - Dist[f^2/(c*(m + 4*p + 3)), Int[(f*x)^(m - 2)*(
```

```
a + b*x^2 + c*x^4)^p*Simp[a*e*(m - 1) + (b*e*(m + 2*p + 1) - c*d*(m + 4*p +
 3))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c,
 0] && GtQ[m, 1] && NeQ[m + 4*p + 3, 0] && IntegerQ[2*p] && (IntegerQ[p] ||
 IntegerQ[m])
```

Rubi steps

$$\begin{aligned}\int \sin(x)\tan(4x)\,dx &= \text{Subst}\left(\int \frac{x^2\left(4-8x^2\right)}{1-8x^2+8x^4}\,dx, x, \sin(x)\right)\\ &= -\sin(x)-\frac{1}{8}\text{Subst}\left(\int \frac{-8+32x^2}{1-8x^2+8x^4}\,dx, x, \sin(x)\right)\\ &= -\sin(x)-\left(2-\sqrt{2}\right)\text{Subst}\left(\int \frac{1}{-4+2\sqrt{2}+8x^2}\,dx, x, \sin(x)\right)-\left(2+\sqrt{2}\right)\text{Subst}\left(\int \frac{}{-4-}\right.\\ &= \frac{1}{4}\sqrt{2-\sqrt{2}}\tanh^{-1}\left(\frac{2\sin(x)}{\sqrt{2-\sqrt{2}}}\right)+\frac{1}{4}\sqrt{2+\sqrt{2}}\tanh^{-1}\left(\frac{2\sin(x)}{\sqrt{2+\sqrt{2}}}\right)-\sin(x)\end{aligned}$$

Mathematica [A] time = 0.08, size = 69, normalized size = 0.97

$$\frac{1}{4}\left(-4\sin(x)+\sqrt{2-\sqrt{2}}\tanh^{-1}\left(\frac{2\sin(x)}{\sqrt{2-\sqrt{2}}}\right)+\sqrt{2+\sqrt{2}}\tanh^{-1}\left(\frac{2\sin(x)}{\sqrt{2+\sqrt{2}}}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[x]*Tan[4*x],x]
```

```
[Out] (Sqrt[2 - Sqrt[2]]*ArcTanh[(2*Sin[x])/Sqrt[2 - Sqrt[2]]] + Sqrt[2 + Sqrt[2]
]*ArcTanh[(2*Sin[x])/Sqrt[2 + Sqrt[2]]] - 4*Sin[x])/4
```

fricas [A] time = 2.46, size = 101, normalized size = 1.42

$$\frac{1}{8}\sqrt{\sqrt{2}+2}\log\left(\sqrt{\sqrt{2}+2}+2\sin(x)\right)-\frac{1}{8}\sqrt{\sqrt{2}+2}\log\left(\sqrt{\sqrt{2}+2}-2\sin(x)\right)+\frac{1}{8}\sqrt{-\sqrt{2}+2}\log\left(\sqrt{-\sqrt{2}+2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)*tan(4*x),x, algorithm="fricas")
```

```
[Out] 1/8*sqrt(sqrt(2) + 2)*log(sqrt(sqrt(2) + 2) + 2*sin(x)) - 1/8*sqrt(sqrt(2)
+ 2)*log(sqrt(sqrt(2) + 2) - 2*sin(x)) + 1/8*sqrt(-sqrt(2) + 2)*log(sqrt(-s
```

```
qrt(2) + 2) + 2*sin(x)) - 1/8*sqrt(-sqrt(2) + 2)*log(sqrt(-sqrt(2) + 2) - 2
*sin(x)) - sin(x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sin(x)\tan(4x)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)*tan(4*x),x, algorithm="giac")
```

```
[Out] integrate(sin(x)*tan(4*x), x)
```

maple [B] time = 0.41, size = 115, normalized size = 1.62

$$\frac{\left(\sqrt{2}-2\right)\sqrt{2}\ \mathrm{arctanh}\left(\frac{2\sin(x)}{\sqrt{2-\sqrt{2}}}\right)}{4\sqrt{2-\sqrt{2}}}+\frac{\sqrt{2+\sqrt{2}}\ \sqrt{2}\ \mathrm{arctanh}\left(\frac{2\sin(x)}{\sqrt{2+\sqrt{2}}}\right)}{4}-\sin(x)+\frac{\sqrt{2}\ \mathrm{arctanh}\left(\frac{2\sin(x)}{\sqrt{2-\sqrt{2}}}\right)}{4\sqrt{2-\sqrt{2}}}-\frac{\sqrt{2}\ \mathrm{arcta}}{4\sqrt{}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)*tan(4*x),x)
```

```
[Out] 1/4*(2^(1/2)-2)*2^(1/2)/(2-2^(1/2))^(1/2)*arctanh(2*sin(x)/(2-2^(1/2))^(1/2
))+1/4*(2+2^(1/2))^(1/2)*2^(1/2)*arctanh(2*sin(x)/(2+2^(1/2))^(1/2))-sin(x)
+1/4*2^(1/2)/(2-2^(1/2))^(1/2)*arctanh(2*sin(x)/(2-2^(1/2))^(1/2))-1/4*2^(1
/2)/(2+2^(1/2))^(1/2)*arctanh(2*sin(x)/(2+2^(1/2))^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(\cos(7x)+\cos(x))\cos(8x)+(\sin(7x)+\sin(x))\sin(8x)+\cos(7x)+\cos(x)}{\cos(8x)^2+\sin(8x)^2+2\cos(8x)+1}\, dx-\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)*tan(4*x),x, algorithm="maxima")
```

```
[Out] integrate(((cos(7*x) + cos(x))*cos(8*x) + (sin(7*x) + sin(x))*sin(8*x) + co
s(7*x) + cos(x))/(cos(8*x)^2 + sin(8*x)^2 + 2*cos(8*x) + 1), x) - sin(x)
```

mupad [B] time = 2.56, size = 103, normalized size = 1.45

$$\frac{\mathrm{atanh}\left(\frac{34\sin(x)\sqrt{\sqrt{2}+2}+24\sqrt{2}\sin(x)\sqrt{\sqrt{2}+2}}{41\sqrt{2}+58}\right)\sqrt{\sqrt{2}+2}}{4}-\sin(x)-\frac{\mathrm{atanh}\left(\frac{34\sin(x)\sqrt{2-\sqrt{2}}-24\sqrt{2}\sin(x)\sqrt{2-\sqrt{2}}}{41\sqrt{2}-58}\right)\sqrt{2-}}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(4*x)*sin(x),x)
```

```
[Out] (atanh((34*sin(x)*(2^(1/2) + 2)^(1/2) + 24*2^(1/2)*sin(x)*(2^(1/2) + 2)^(1/
2))/(41*2^(1/2) + 58))*(2^(1/2) + 2)^(1/2))/4 - sin(x) - (atanh((34*sin(x)*
(2 - 2^(1/2))^(1/2) - 24*2^(1/2)*sin(x)*(2 - 2^(1/2))^(1/2))/(41*2^(1/2) -
58))*(2 - 2^(1/2))^(1/2))/4
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sin(x)\tan(4x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)*tan(4*x),x)
```

```
[Out] Integral(sin(x)*tan(4*x), x)
```

3.77 $\int \sin(x)\tan(5x)\,dx$

Optimal. Leaf size=112

$$-\sin(x)-\frac{1}{20}\left(1-\sqrt{5}\right)\log\left(-4\sin(x)-\sqrt{5}+1\right)-\frac{1}{20}\left(1+\sqrt{5}\right)\log\left(-4\sin(x)+\sqrt{5}+1\right)+\frac{1}{20}\left(1-\sqrt{5}\right)\log\left(4\,\mathrm{si}\right.$$

```
[Out] 1/5*arctanh(sin(x))-sin(x)-1/20*ln(1-4*sin(x)-5^(1/2))*(-5^(1/2)+1)+1/20*ln
(1+4*sin(x)-5^(1/2))*(-5^(1/2)+1)-1/20*ln(1-4*sin(x)+5^(1/2))*(5^(1/2)+1)+1
/20*ln(1+4*sin(x)+5^(1/2))*(5^(1/2)+1)
```

Rubi [A] time = 0.17, antiderivative size = 112, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 4, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.571, Rules used = {2075, 207, 632, 31}

$$-\sin(x)-\frac{1}{20}\left(1-\sqrt{5}\right)\log\left(-4\sin(x)-\sqrt{5}+1\right)-\frac{1}{20}\left(1+\sqrt{5}\right)\log\left(-4\sin(x)+\sqrt{5}+1\right)+\frac{1}{20}\left(1-\sqrt{5}\right)\log\left(4\,\mathrm{si}\right.$$

Antiderivative was successfully verified.

```
[In] Int[Sin[x]*Tan[5*x],x]
```

```
[Out] ArcTanh[Sin[x]]/5 - ((1 - Sqrt[5])*Log[1 - Sqrt[5] - 4*Sin[x]])/20 - ((1 +
Sqrt[5])*Log[1 + Sqrt[5] - 4*Sin[x]])/20 + ((1 - Sqrt[5])*Log[1 - Sqrt[5] +
 4*Sin[x]])/20 + ((1 + Sqrt[5])*Log[1 + Sqrt[5] + 4*Sin[x]])/20 - Sin[x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 632

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> W
ith[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(c*d - e*(b/2 - q/2))/q, Int[1/(b/2 - q/
2 + c*x), x], x] - Dist[(c*d - e*(b/2 + q/2))/q, Int[1/(b/2 + q/2 + c*x), x
], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a
*c, 0] && NiceSqrtQ[b^2 - 4*a*c]
```

Rule 2075

```
Int[(P_)^(p_)*(Qm_), x_Symbol] :> With[{PP = Factor[P]}, Int[ExpandIntegran
d[PP^p*Qm, x], x] /; QuadraticProductQ[PP, x]] /; PolyQ[Qm, x] && PolyQ[P,
x] && ILtQ[p, 0]
```

Rubi steps

$$\int \sin(x)\tan(5x)\,dx = \text{Subst}\left(\int \frac{x^2\left(5-20x^2+16x^4\right)}{1-13x^2+28x^4-16x^6}\,dx, x, \sin(x)\right)$$
$$= \text{Subst}\left(\int \left(-1-\frac{1}{5\left(-1+x^2\right)}-\frac{2(1+x)}{5\left(-1-2x+4x^2\right)}+\frac{2(-1+x)}{5\left(-1+2x+4x^2\right)}\right)dx, x, \sin(x)\right)$$
$$= -\sin(x)-\frac{1}{5}\text{Subst}\left(\int \frac{1}{-1+x^2}\,dx, x, \sin(x)\right)-\frac{2}{5}\text{Subst}\left(\int \frac{1+x}{-1-2x+4x^2}\,dx, x, \sin(x)\right)+$$
$$= \frac{1}{5}\tanh^{-1}(\sin(x))-\sin(x)+\frac{1}{5}\left(1-\sqrt{5}\right)\text{Subst}\left(\int \frac{1}{1-\sqrt{5}+4x}\,dx, x, \sin(x)\right)-\frac{1}{5}\left(1-\sqrt{5}\right.$$
$$= \frac{1}{5}\tanh^{-1}(\sin(x))-\frac{1}{20}\left(1-\sqrt{5}\right)\log\left(1-\sqrt{5}-4\sin(x)\right)-\frac{1}{20}\left(1+\sqrt{5}\right)\log\left(1+\sqrt{5}-4\,\text{s}\right.$$

Mathematica [A] time = 0.18, size = 100, normalized size = 0.89

$$\frac{1}{20}\left(-20\sin(x)+\left(\sqrt{5}-1\right)\log\left(-4\sin(x)-\sqrt{5}+1\right)-\left(1+\sqrt{5}\right)\log\left(-4\sin(x)+\sqrt{5}+1\right)-\left(\sqrt{5}-1\right)\log\left(4\,\text{sin}\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[x]*Tan[5*x],x]
```

```
[Out] (4*ArcTanh[Sin[x]] + (-1 + Sqrt[5])*Log[1 - Sqrt[5] - 4*Sin[x]] - (1 + Sqrt
[5])*Log[1 + Sqrt[5] - 4*Sin[x]] - (-1 + Sqrt[5])*Log[1 - Sqrt[5] + 4*Sin[x
]] + (1 + Sqrt[5])*Log[1 + Sqrt[5] + 4*Sin[x]] - 20*Sin[x])/20
```

fricas [A] time = 0.84, size = 136, normalized size = 1.21

$$\frac{1}{20}\sqrt{5}\log\left(\frac{8\cos(x)^2-4\left(\sqrt{5}-1\right)\sin(x)+\sqrt{5}-11}{4\cos(x)^2+2\sin(x)-3}\right)+\frac{1}{20}\sqrt{5}\log\left(-\frac{8\cos(x)^2-4\left(\sqrt{5}+1\right)\sin(x)-\sqrt{5}-11}{4\cos(x)^2-2\sin(x)-3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)*tan(5*x),x, algorithm="fricas")
```

```
[Out] 1/20*sqrt(5)*log((8*cos(x)^2 - 4*(sqrt(5) - 1)*sin(x) + sqrt(5) - 11)/(4*co
s(x)^2 + 2*sin(x) - 3)) + 1/20*sqrt(5)*log(-(8*cos(x)^2 - 4*(sqrt(5) + 1)*s
```

```
in(x) - sqrt(5) - 11)/(4*cos(x)^2 - 2*sin(x) - 3)) - 1/20*log(4*cos(x)^2 +
2*sin(x) - 3) + 1/20*log(4*cos(x)^2 - 2*sin(x) - 3) + 1/10*log(sin(x) + 1)
- 1/10*log(-sin(x) + 1) - sin(x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sin(x)\tan(5x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)*tan(5*x),x, algorithm="giac")
```

```
[Out] integrate(sin(x)*tan(5*x), x)
```

maple [A] time = 0.41, size = 84, normalized size = 0.75

$$\frac{\ln\left(4\left(\sin^2(x)\right)+2\sin(x)-1\right)}{20}+\frac{\sqrt{5}\ \operatorname{arctanh}\left(\frac{(8\sin(x)+2)\sqrt{5}}{10}\right)}{10}-\frac{\ln(\sin(x)-1)}{10}-\frac{\ln\left(4\left(\sin^2(x)\right)-2\sin(x)-1\right)}{20}+\frac{\sqrt{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)*tan(5*x),x)
```

```
[Out] 1/20*ln(4*sin(x)^2+2*sin(x)-1)+1/10*5^(1/2)*arctanh(1/10*(8*sin(x)+2)*5^(1/
2))-1/10*ln(sin(x)-1)-1/20*ln(4*sin(x)^2-2*sin(x)-1)+1/10*5^(1/2)*arctanh(1
/10*(8*sin(x)-2)*5^(1/2))+1/10*ln(1+sin(x))-sin(x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{(3\cos(7x)-\cos(5x)-\cos(3x)+3\cos(x))\cos(8x)-3(\cos(6x)-\cos(4x)+\cos(2x)-1)\cos(7x)+}{5\left(2(\cos(6\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)*tan(5*x),x, algorithm="maxima")
```

```
[Out] integrate(-1/5*((3*cos(7*x) - cos(5*x) - cos(3*x) + 3*cos(x))*cos(8*x) - 3*
(cos(6*x) - cos(4*x) + cos(2*x) - 1)*cos(7*x) + (cos(5*x) + cos(3*x) - 3*co
s(x))*cos(6*x) - (cos(4*x) - cos(2*x) + 1)*cos(5*x) - (cos(3*x) - 3*cos(x))
*cos(4*x) + (cos(2*x) - 1)*cos(3*x) - 3*cos(2*x)*cos(x) + (3*sin(7*x) - sin
(5*x) - sin(3*x) + 3*sin(x))*sin(8*x) - 3*(sin(6*x) - sin(4*x) + sin(2*x))*
sin(7*x) + (sin(5*x) + sin(3*x) - 3*sin(x))*sin(6*x) - (sin(4*x) - sin(2*x)
)*sin(5*x) - (sin(3*x) - 3*sin(x))*sin(4*x) + sin(3*x)*sin(2*x) - 3*sin(2*x
)*sin(x) + 3*cos(x))/(2*(cos(6*x) - cos(4*x) + cos(2*x) - 1)*cos(8*x) - cos
(8*x)^2 + 2*(cos(4*x) - cos(2*x) + 1)*cos(6*x) - cos(6*x)^2 + 2*(cos(2*x) -
```

```
 1)*cos(4*x) - cos(4*x)^2 - cos(2*x)^2 + 2*(sin(6*x) - sin(4*x) + sin(2*x))
*sin(8*x) - sin(8*x)^2 + 2*(sin(4*x) - sin(2*x))*sin(6*x) - sin(6*x)^2 - si
n(4*x)^2 + 2*sin(4*x)*sin(2*x) - sin(2*x)^2 + 2*cos(2*x) - 1), x) + 1/10*lo
g(cos(x)^2 + sin(x)^2 + 2*sin(x) + 1) - 1/10*log(cos(x)^2 + sin(x)^2 - 2*si
n(x) + 1) - sin(x)
```

mupad [B] time = 2.89, size = 107, normalized size = 0.96

$$\frac{2\,\mathrm{atanh}\left(\frac{\sin\left(\frac{x}{2}\right)}{\cos\left(\frac{x}{2}\right)}\right)}{5}+\frac{\mathrm{atan}\left(\frac{\sin(x)\,1042\mathrm{i}-\sqrt{5}\,\sin(x)\,466\mathrm{i}}{377\,\sqrt{5}-843}\right)\,1\mathrm{i}}{10}-\frac{\mathrm{atanh}\left(\sin(x)-\sqrt{5}\,\sin(x)\right)}{10}-\sin(x)-\frac{\sqrt{5}\,\mathrm{atanh}\left(\sin(x)-\sqrt{5}\right.}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(5*x)*sin(x),x)
```

```
[Out] (atan((sin(x)*1042i - 5^(1/2)*sin(x)*466i)/(377*5^(1/2) - 843))*1i)/10 - at
anh(sin(x) - 5^(1/2)*sin(x))/10 + (2*atanh(sin(x/2)/cos(x/2)))/5 - sin(x) -
 (5^(1/2)*atanh(sin(x) - 5^(1/2)*sin(x)))/10 - (5^(1/2)*atan((sin(x)*1042i
- 5^(1/2)*sin(x)*466i)/(377*5^(1/2) - 843))*1i)/10
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sin(x)\tan(5x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)*tan(5*x),x)
```

```
[Out] Integral(sin(x)*tan(5*x), x)
```

3.78 $\int \sin(x)\tan(6x)\,dx$

Optimal. Leaf size=89

$$-\sin(x)+\frac{\tanh^{-1}\left(\sqrt{2}\sin(x)\right)}{3\sqrt{2}}+\frac{1}{6}\sqrt{2-\sqrt{3}}\ \tanh^{-1}\left(\frac{2\sin(x)}{\sqrt{2-\sqrt{3}}}\right)+\frac{1}{6}\sqrt{2+\sqrt{3}}\ \tanh^{-1}\left(\frac{2\sin(x)}{\sqrt{2+\sqrt{3}}}\right)$$

```
[Out] -sin(x)+1/6*arctanh(sin(x)*2^(1/2))*2^(1/2)+1/6*arctanh(2*sin(x)/(1/2*6^(1/
2)-1/2*2^(1/2)))*(1/2*6^(1/2)-1/2*2^(1/2))+1/6*arctanh(2*sin(x)/(1/2*6^(1/2
)+1/2*2^(1/2)))*(1/2*6^(1/2)+1/2*2^(1/2))
```

Rubi [A] time = 0.27, antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 5, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.714, Rules used = {12, 6742, 2073, 207, 1166}

$$-\sin(x)+\frac{\tanh^{-1}\left(\sqrt{2}\sin(x)\right)}{3\sqrt{2}}+\frac{1}{6}\sqrt{2-\sqrt{3}}\ \tanh^{-1}\left(\frac{2\sin(x)}{\sqrt{2-\sqrt{3}}}\right)+\frac{1}{6}\sqrt{2+\sqrt{3}}\ \tanh^{-1}\left(\frac{2\sin(x)}{\sqrt{2+\sqrt{3}}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sin[x]*Tan[6*x],x]
```

```
[Out] ArcTanh[Sqrt[2]*Sin[x]]/(3*Sqrt[2]) + (Sqrt[2 - Sqrt[3]]*ArcTanh[(2*Sin[x])
/Sqrt[2 - Sqrt[3]]])/6 + (Sqrt[2 + Sqrt[3]]*ArcTanh[(2*Sin[x])/Sqrt[2 + Sqr
t[3]]])/6 - Sin[x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
```

```
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 2073

```
Int[(P_)^(p_)*(Q_)^(q_.), x_Symbol] :> With[{PP = Factor[P /. x -> Sqrt[x]]
}, Int[ExpandIntegrand[(PP /. x -> x^2)^p*Q^q, x], x] /;  !SumQ[NonfreeFact
ors[PP, x]]] /; FreeQ[q, x] && PolyQ[P, x^2] && PolyQ[Q, x] && ILtQ[p, 0]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$
\begin{aligned}
\int \sin(x)\tan(6x)\,dx &= \text{Subst}\left(\int \frac{2x^2\left(3-16x^2+16x^4\right)}{1-18x^2+48x^4-32x^6}\,dx, x, \sin(x)\right)\\
&= 2\,\text{Subst}\left(\int \frac{x^2\left(3-16x^2+16x^4\right)}{1-18x^2+48x^4-32x^6}\,dx, x, \sin(x)\right)\\
&= 2\,\text{Subst}\left(\int \left(-\frac{1}{2}+\frac{1-12x^2+16x^4}{2\left(1-18x^2+48x^4-32x^6\right)}\right)dx, x, \sin(x)\right)\\
&= -\sin(x)+\text{Subst}\left(\int \frac{1-12x^2+16x^4}{1-18x^2+48x^4-32x^6}\,dx, x, \sin(x)\right)\\
&= -\sin(x)+\text{Subst}\left(\int \left(-\frac{1}{3\left(-1+2x^2\right)}-\frac{2\left(-1+8x^2\right)}{3\left(1-16x^2+16x^4\right)}\right)dx, x, \sin(x)\right)\\
&= -\sin(x)-\frac{1}{3}\text{Subst}\left(\int \frac{1}{-1+2x^2}\,dx, x, \sin(x)\right)-\frac{2}{3}\text{Subst}\left(\int \frac{-1+8x^2}{1-16x^2+16x^4}\,dx, x, \sin(x\right)\\
&= \frac{\tanh^{-1}\left(\sqrt{2}\sin(x)\right)}{3\sqrt{2}}-\sin(x)-\frac{1}{3}\left(4\left(2-\sqrt{3}\right)\right)\text{Subst}\left(\int \frac{1}{-8+4\sqrt{3}+16x^2}\,dx, x, \sin(x)\right)\\
&= \frac{\tanh^{-1}\left(\sqrt{2}\sin(x)\right)}{3\sqrt{2}}+\frac{1}{6}\sqrt{2-\sqrt{3}}\tanh^{-1}\left(\frac{2\sin(x)}{\sqrt{2-\sqrt{3}}}\right)+\frac{1}{6}\sqrt{2+\sqrt{3}}\tanh^{-1}\left(\frac{2\sin(x)}{\sqrt{2+\sqrt{3}}}\right.
\end{aligned}
$$

Mathematica [A] time = 0.15, size = 84, normalized size = 0.94

$$\frac{1}{6}\left(-6\sin(x)+\sqrt{2}\tanh^{-1}\left(\sqrt{2}\sin(x)\right)+\sqrt{2-\sqrt{3}}\tanh^{-1}\left(\frac{2\sin(x)}{\sqrt{2-\sqrt{3}}}\right)+\sqrt{2+\sqrt{3}}\tanh^{-1}\left(\frac{2\sin(x)}{\sqrt{2+\sqrt{3}}}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[x]*Tan[6*x],x]
```

```
[Out] (Sqrt[2]*ArcTanh[Sqrt[2]*Sin[x]] + Sqrt[2 - Sqrt[3]]*ArcTanh[(2*Sin[x])/Sqr
t[2 - Sqrt[3]]] + Sqrt[2 + Sqrt[3]]*ArcTanh[(2*Sin[x])/Sqrt[2 + Sqrt[3]]] -
 6*Sin[x])/6
```

fricas [A] time = 1.79, size = 134, normalized size = 1.51

$$\frac{1}{12}\sqrt{\sqrt{3}+2}\log\left(\sqrt{\sqrt{3}+2}+2\sin(x)\right)-\frac{1}{12}\sqrt{\sqrt{3}+2}\log\left(\sqrt{\sqrt{3}+2}-2\sin(x)\right)+\frac{1}{12}\sqrt{-\sqrt{3}+2}\log\left(\sqrt{-\sqrt{3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)*tan(6*x),x, algorithm="fricas")
```

```
[Out] 1/12*sqrt(sqrt(3) + 2)*log(sqrt(sqrt(3) + 2) + 2*sin(x)) - 1/12*sqrt(sqrt(3
) + 2)*log(sqrt(sqrt(3) + 2) - 2*sin(x)) + 1/12*sqrt(-sqrt(3) + 2)*log(sqrt
(-sqrt(3) + 2) + 2*sin(x)) - 1/12*sqrt(-sqrt(3) + 2)*log(sqrt(-sqrt(3) + 2)
 - 2*sin(x)) + 1/12*sqrt(2)*log(-(2*cos(x)^2 - 2*sqrt(2)*sin(x) - 3)/(2*cos
(x)^2 - 1)) - sin(x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\sin(x)\tan(6x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)*tan(6*x),x, algorithm="giac")
```

```
[Out] integrate(sin(x)*tan(6*x), x)
```

maple [B] time = 0.62, size = 256, normalized size = 2.88

$$\frac{\left(-3+2\sqrt{3}\right)\sqrt{3}\ \mathrm{arctanh}\left(\frac{8\sin(x)}{2\sqrt{6}-2\sqrt{2}}\right)}{6\sqrt{6}-6\sqrt{2}}+\frac{\left(3+2\sqrt{3}\right)\sqrt{3}\ \mathrm{arctanh}\left(\frac{8\sin(x)}{2\sqrt{6}+2\sqrt{2}}\right)}{6\sqrt{6}+6\sqrt{2}}+\frac{\mathrm{arctanh}\left(\sin(x)\sqrt{2}\right)\sqrt{2}}{6}-\frac{4\,\mathrm{arctan}}{3\left(2\sqrt{\right.}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)*tan(6*x),x)
```

```
[Out] 1/3*(-3+2*3^(1/2))*3^(1/2)/(2*6^(1/2)-2*2^(1/2))*arctanh(8*sin(x)/(2*6^(1/2
)-2*2^(1/2)))+1/3*(3+2*3^(1/2))*3^(1/2)/(2*6^(1/2)+2*2^(1/2))*arctanh(8*sin
(x)/(2*6^(1/2)+2*2^(1/2)))+1/6*arctanh(sin(x)*2^(1/2))*2^(1/2)-4/3/(2*6^(1/
2)-2*2^(1/2))*arctanh(8*sin(x)/(2*6^(1/2)-2*2^(1/2)))-4/3/(2*6^(1/2)+2*2^(1
/2))*arctanh(8*sin(x)/(2*6^(1/2)+2*2^(1/2)))-sin(x)+1/9*(3+2*3^(1/2))*3^(1/
2)/(2*6^(1/2)-2*2^(1/2))*arctanh(8*sin(x)/(2*6^(1/2)-2*2^(1/2)))+1/9*(-3+2*
3^(1/2))*3^(1/2)/(2*6^(1/2)+2*2^(1/2))*arctanh(8*sin(x)/(2*6^(1/2)+2*2^(1/2
)))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{24}\sqrt{2}\log\left(2\cos(x)^2+2\sin(x)^2+2\sqrt{2}\cos(x)+2\sqrt{2}\sin(x)+2\right)-\frac{1}{24}\sqrt{2}\log\left(2\cos(x)^2+2\sin(x)^2+2\sqrt{2}\,\mathrm{c}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)*tan(6*x),x, algorithm="maxima")
```

```
[Out] 1/24*sqrt(2)*log(2*cos(x)^2 + 2*sin(x)^2 + 2*sqrt(2)*cos(x) + 2*sqrt(2)*sin
(x) + 2) - 1/24*sqrt(2)*log(2*cos(x)^2 + 2*sin(x)^2 + 2*sqrt(2)*cos(x) - 2*
sqrt(2)*sin(x) + 2) + 1/24*sqrt(2)*log(2*cos(x)^2 + 2*sin(x)^2 - 2*sqrt(2)*
cos(x) + 2*sqrt(2)*sin(x) + 2) - 1/24*sqrt(2)*log(2*cos(x)^2 + 2*sin(x)^2 -
 2*sqrt(2)*cos(x) - 2*sqrt(2)*sin(x) + 2) + integrate(-1/3*((2*cos(7*x) - c
os(5*x) - cos(3*x) + 2*cos(x))*cos(8*x) - 2*(cos(4*x) - 1)*cos(7*x) + (cos(
4*x) - 1)*cos(5*x) + (cos(3*x) - 2*cos(x))*cos(4*x) + (2*sin(7*x) - sin(5*x
) - sin(3*x) + 2*sin(x))*sin(8*x) + (sin(3*x) - 2*sin(x))*sin(4*x) - 2*sin(
7*x)*sin(4*x) + sin(5*x)*sin(4*x) - cos(3*x) + 2*cos(x))/(2*(cos(4*x) - 1)*
cos(8*x) - cos(8*x)^2 - cos(4*x)^2 - sin(8*x)^2 + 2*sin(8*x)*sin(4*x) - sin
(4*x)^2 + 2*cos(4*x) - 1), x) - sin(x)
```

mupad [B] time = 3.11, size = 131, normalized size = 1.47

$$\frac{\sqrt{2}\,\mathrm{atanh}\left(\sqrt{2}\,\sin(x)\right)}{6}-\sin(x)-\frac{\sqrt{2}\,\mathrm{atanh}\left(\sqrt{2}\,\sin(x)-\sqrt{6}\,\sin(x)\right)}{12}-\frac{\sqrt{6}\,\mathrm{atanh}\left(\sqrt{2}\,\sin(x)-\sqrt{6}\,\sin(x)\right)}{12}+\frac{\sqrt{2}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(6*x)*sin(x),x)
```

```
[Out] (2^(1/2)*atan((2^(1/2)*sin(x)*102818i - 6^(1/2)*sin(x)*59362i)/(40545*2^(1/
2)*6^(1/2) - 140452))*1i)/12 - sin(x) - (6^(1/2)*atan((2^(1/2)*sin(x)*10281
8i - 6^(1/2)*sin(x)*59362i)/(40545*2^(1/2)*6^(1/2) - 140452))*1i)/12 + (2^(
1/2)*atanh(2^(1/2)*sin(x)))/6 - (2^(1/2)*atanh(2^(1/2)*sin(x) - 6^(1/2)*sin
(x)))/12 - (6^(1/2)*atanh(2^(1/2)*sin(x) - 6^(1/2)*sin(x)))/12
```

sympy [F]   time = 0.00, size = 0, normalized size = 0.00

$$\int \sin(x)\tan(6x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)*tan(6*x),x)
```

```
[Out] Integral(sin(x)*tan(6*x), x)
```

3.79 $\int \sin(x)\tan(nx)\,dx$

Optimal. Leaf size=105

$$-ie^{-ix}\,{}_2F_1\left(1,-\frac{1}{2n};1-\frac{1}{2n};-e^{2inx}\right)-ie^{ix}\,{}_2F_1\left(1,\frac{1}{2n};\frac{1}{2}\left(2+\frac{1}{n}\right);-e^{2inx}\right)+\frac{1}{2}ie^{-ix}+\frac{1}{2}ie^{ix}$$

```
[Out] 1/2*I/exp(I*x)+1/2*I*exp(I*x)-I*hypergeom([1, -1/2/n],[1-1/2/n],-exp(2*I*n*
x))/exp(I*x)-I*exp(I*x)*hypergeom([1, 1/2/n],[1+1/2/n],-exp(2*I*n*x))
```

Rubi [A] time = 0.08, antiderivative size = 105, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {4557, 2194, 2251}

$$-ie^{-ix}\,{}_2F_1\left(1,-\frac{1}{2n};1-\frac{1}{2n};-e^{2inx}\right)-ie^{ix}\,{}_2F_1\left(1,\frac{1}{2n};\frac{1}{2}\left(2+\frac{1}{n}\right);-e^{2inx}\right)+\frac{1}{2}ie^{-ix}+\frac{1}{2}ie^{ix}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[x]*Tan[n*x],x]
```

```
[Out] (I/2)/E^(I*x) + (I/2)*E^(I*x) - (I*Hypergeometric2F1[1, -1/(2*n), 1 - 1/(2*
n), -E^((2*I)*n*x)])/E^(I*x) - I*E^(I*x)*Hypergeometric2F1[1, 1/(2*n), (2 +
 n^(-1))/2, -E^((2*I)*n*x)]
```

Rule 2194

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rule 2251

```
Int[((a_) + (b_.)*(F_)^((e_.)*((c_.) + (d_.)*(x_))))^(p_)*(G_)^((h_.)*((f_.
) + (g_.)*(x_))), x_Symbol] :> Simp[(a^p*G^(h*(f + g*x))*Hypergeometric2F1[
-p, (g*h*Log[G])/(d*e*Log[F]), (g*h*Log[G])/(d*e*Log[F]) + 1, Simplify[-((b
*F^(e*(c + d*x)))/a)]])/(g*h*Log[G]), x] /; FreeQ[{F, G, a, b, c, d, e, f,
g, h, p}, x] && (ILtQ[p, 0] || GtQ[a, 0])
```

Rule 4557

```
Int[Sin[(a_.) + (b_.)*(x_)]*Tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Int[1/(E^
(I*(a + b*x))*2) - E^(I*(a + b*x))/2 - 1/(E^(I*(a + b*x))*(1 + E^(2*I*(c +
d*x)))) + E^(I*(a + b*x))/(1 + E^(2*I*(c + d*x))), x] /; FreeQ[{a, b, c, d}
, x] && NeQ[b^2 - d^2, 0]
```

Rubi steps

$$
\begin{aligned}
\int \sin(x)\tan(nx)\,dx &= \int \left(\frac{e^{-ix}}{2} - \frac{e^{ix}}{2} - \frac{e^{-ix}}{1+e^{2inx}} + \frac{e^{ix}}{1+e^{2inx}}\right) dx \\
&= \frac{1}{2}\int e^{-ix}\,dx - \frac{1}{2}\int e^{ix}\,dx - \int \frac{e^{-ix}}{1+e^{2inx}}\,dx + \int \frac{e^{ix}}{1+e^{2inx}}\,dx \\
&= \frac{1}{2}ie^{-ix} + \frac{1}{2}ie^{ix} - ie^{-ix}\,{}_2F_1\left(1,-\frac{1}{2n};1-\frac{1}{2n};-e^{2inx}\right) - ie^{ix}\,{}_2F_1\left(1,\frac{1}{2n};\frac{1}{2}\left(2+\frac{1}{n}\right);-e^{2inx}\right)
\end{aligned}
$$

Mathematica [A] time = 0.18, size = 200, normalized size = 1.90

$$
-\frac{ie^{-2ix}\left((2n+1)e^{i(2nx+x)}\,{}_2F_1\left(1,1-\frac{1}{2n};2-\frac{1}{2n};-e^{2inx}\right)+(2n-1)\left((2n+1)e^{ix}\left({}_2F_1\left(1,-\frac{1}{2n};1-\frac{1}{2n};-e^{2inx}\right)+e^{2ix}\,{}_2\right.\right.\right.}{2\left(4n^2-1\right)}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[x]*Tan[n*x],x]
```

```
[Out] ((-1/2*I)*(E^(I*(x + 2*n*x))*(1 + 2*n)*Hypergeometric2F1[1, 1 - 1/(2*n), 2
- 1/(2*n), -E^((2*I)*n*x)] + (-1 + 2*n)*(-(E^(I*(3 + 2*n)*x)*Hypergeometric
2F1[1, 1 + 1/(2*n), 2 + 1/(2*n), -E^((2*I)*n*x)]) + E^(I*x)*(1 + 2*n)*(Hype
rgeometric2F1[1, -1/2*1/n, 1 - 1/(2*n), -E^((2*I)*n*x)] + E^((2*I)*x)*Hyper
geometric2F1[1, 1/(2*n), 1 + 1/(2*n), -E^((2*I)*n*x)]))))/(E^((2*I)*x)*(-1
+ 4*n^2))
```

fricas [F] time = 0.73, size = 0, normalized size = 0.00

$$
\mathrm{integral}\left(\sin(x)\tan\left(nx\right),x\right)
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)*tan(n*x),x, algorithm="fricas")
```

```
[Out] integral(sin(x)*tan(n*x), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$
\int \sin(x)\tan\left(nx\right)\,dx
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)*tan(n*x),x, algorithm="giac")
```

[Out] integrate(sin(x)*tan(n*x), x)

maple [F] time = 0.44, size = 0, normalized size = 0.00

$$\int \sin(x)\tan(nx)\ dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sin(x)*tan(n*x),x)

[Out] int(sin(x)*tan(n*x),x)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sin(x)\tan(nx)\ dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(x)*tan(n*x),x, algorithm="maxima")

[Out] integrate(sin(x)*tan(n*x), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \tan(n\,x)\ \sin(x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(tan(n*x)*sin(x),x)

[Out] int(tan(n*x)*sin(x), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sin(x)\tan(nx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(x)*tan(n*x),x)

[Out] Integral(sin(x)*tan(n*x), x)

3.80 $\int \cot(2x)\sin(x)\,dx$

Optimal. Leaf size=10

$$\sin(x) - \frac{1}{2}\tanh^{-1}(\sin(x))$$

```
[Out] -1/2*arctanh(sin(x))+sin(x)
```

Rubi [A] time = 0.02, antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {388, 206}

$$\sin(x) - \frac{1}{2}\tanh^{-1}(\sin(x))$$

Antiderivative was successfully verified.

```
[In] Int[Cot[2*x]*Sin[x],x]
```

```
[Out] -ArcTanh[Sin[x]]/2 + Sin[x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rubi steps

$$\begin{aligned}
\int \cot(2x)\sin(x)\,dx &= \text{Subst}\left(\int \frac{1-2x^2}{2-2x^2}\,dx, x, \sin(x)\right)\\
&= \sin(x) - \text{Subst}\left(\int \frac{1}{2-2x^2}\,dx, x, \sin(x)\right)\\
&= -\frac{1}{2}\tanh^{-1}(\sin(x)) + \sin(x)
\end{aligned}$$

Mathematica [**A**] time = 0.01, size = 10, normalized size = 1.00

$$\sin(x) - \frac{1}{2}\tanh^{-1}(\sin(x))$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[2*x]*Sin[x],x]
[Out] -1/2*ArcTanh[Sin[x]] + Sin[x]
```

fricas [**B**] time = 0.68, size = 19, normalized size = 1.90

$$-\frac{1}{4}\log(\sin(x)+1) + \frac{1}{4}\log(-\sin(x)+1) + \sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(2*x)*sin(x),x, algorithm="fricas")
[Out] -1/4*log(sin(x) + 1) + 1/4*log(-sin(x) + 1) + sin(x)
```

giac [**B**] time = 0.14, size = 19, normalized size = 1.90

$$-\frac{1}{4}\log(\sin(x)+1) + \frac{1}{4}\log(-\sin(x)+1) + \sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(2*x)*sin(x),x, algorithm="giac")
[Out] -1/4*log(sin(x) + 1) + 1/4*log(-sin(x) + 1) + sin(x)
```

maple [**A**] time = 0.08, size = 12, normalized size = 1.20

$$\sin(x) - \frac{\ln(\sec(x)+\tan(x))}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(2*x)*sin(x),x)
[Out] sin(x)-1/2*ln(sec(x)+tan(x))
```

maxima [**B**] time = 0.41, size = 37, normalized size = 3.70

$$-\frac{1}{4}\log\left(\cos(x)^2+\sin(x)^2+2\sin(x)+1\right) + \frac{1}{4}\log\left(\cos(x)^2+\sin(x)^2-2\sin(x)+1\right) + \sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(2*x)*sin(x),x, algorithm="maxima")
```

```
[Out] -1/4*log(cos(x)^2 + sin(x)^2 + 2*sin(x) + 1) + 1/4*log(cos(x)^2 + sin(x)^2
- 2*sin(x) + 1) + sin(x)
```

mupad [B] time = 2.32, size = 10, normalized size = 1.00

$$\sin(x) - \operatorname{atanh}\left(\tan\left(\frac{x}{2}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(2*x)*sin(x),x)
```

```
[Out] sin(x) - atanh(tan(x/2))
```

sympy [B] time = 0.82, size = 19, normalized size = 1.90

$$\frac{\log\left(\sin\left(x\right)-1\right)}{4} - \frac{\log\left(\sin\left(x\right)+1\right)}{4} + \sin\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(2*x)*sin(x),x)
```

```
[Out] log(sin(x) - 1)/4 - log(sin(x) + 1)/4 + sin(x)
```

3.81 $\int \cot(3x)\sin(x)\,dx$

Optimal. Leaf size=20

$$\sin(x)-\frac{\tanh^{-1}\left(\frac{2\sin(x)}{\sqrt{3}}\right)}{\sqrt{3}}$$

```
[Out] sin(x)-1/3*arctanh(2/3*sin(x)*3^(1/2))*3^(1/2)
```

Rubi [A] time = 0.03, antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {388, 206}

$$\sin(x)-\frac{\tanh^{-1}\left(\frac{2\sin(x)}{\sqrt{3}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[3*x]*Sin[x],x]
```

```
[Out] -(ArcTanh[(2*Sin[x])/Sqrt[3]]/Sqrt[3]) + Sin[x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rubi steps

$$\int \cot(3x)\sin(x)\,dx = \text{Subst}\left(\int \frac{1-4x^2}{3-4x^2}\,dx, x, \sin(x)\right)$$
$$= \sin(x) - 2\,\text{Subst}\left(\int \frac{1}{3-4x^2}\,dx, x, \sin(x)\right)$$
$$= -\frac{\tanh^{-1}\left(\frac{2\sin(x)}{\sqrt{3}}\right)}{\sqrt{3}} + \sin(x)$$

Mathematica [A] time = 0.02, size = 20, normalized size = 1.00

$$\sin(x) - \frac{\tanh^{-1}\left(\frac{2\sin(x)}{\sqrt{3}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[3*x]*Sin[x],x]
[Out] -(ArcTanh[(2*Sin[x])/Sqrt[3]]/Sqrt[3]) + Sin[x]
```

fricas [B] time = 0.98, size = 36, normalized size = 1.80

$$\frac{1}{6}\sqrt{3}\,\log\left(-\frac{4\,\cos(x)^2 + 4\sqrt{3}\,\sin(x) - 7}{4\,\cos(x)^2 - 1}\right) + \sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(3*x)*sin(x),x, algorithm="fricas")
[Out] 1/6*sqrt(3)*log(-(4*cos(x)^2 + 4*sqrt(3)*sin(x) - 7)/(4*cos(x)^2 - 1)) + si
n(x)
```

giac [B] time = 0.15, size = 34, normalized size = 1.70

$$\frac{1}{6}\sqrt{3}\,\log\left(\frac{\left|-4\sqrt{3} + 8\,\sin(x)\right|}{\left|4\sqrt{3} + 8\,\sin(x)\right|}\right) + \sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(3*x)*sin(x),x, algorithm="giac")
[Out] 1/6*sqrt(3)*log(abs(-4*sqrt(3) + 8*sin(x))/abs(4*sqrt(3) + 8*sin(x))) + sin
(x)
```

maple [A] time = 0.12, size = 17, normalized size = 0.85

$$\sin(x) - \frac{\operatorname{arctanh}\left(\frac{2\sin(x)\sqrt{3}}{3}\right)\sqrt{3}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(3*x)*sin(x),x)
```

```
[Out] sin(x)-1/3*arctanh(2/3*sin(x)*3^(1/2))*3^(1/2)
```

maxima [B] time = 0.44, size = 127, normalized size = 6.35

$$-\frac{1}{12}\sqrt{3}\log\left(\frac{4}{3}\cos(x)^2+\frac{4}{3}\sin(x)^2+\frac{4}{3}\sqrt{3}\sin(x)+\frac{4}{3}\cos(x)+\frac{4}{3}\right)-\frac{1}{12}\sqrt{3}\log\left(\frac{4}{3}\cos(x)^2+\frac{4}{3}\sin(x)^2+\frac{4}{3}\sqrt{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(3*x)*sin(x),x, algorithm="maxima")
```

```
[Out] -1/12*sqrt(3)*log(4/3*cos(x)^2 + 4/3*sin(x)^2 + 4/3*sqrt(3)*sin(x) + 4/3*co
s(x) + 4/3) - 1/12*sqrt(3)*log(4/3*cos(x)^2 + 4/3*sin(x)^2 + 4/3*sqrt(3)*si
n(x) - 4/3*cos(x) + 4/3) + 1/12*sqrt(3)*log(4/3*cos(x)^2 + 4/3*sin(x)^2 - 4
/3*sqrt(3)*sin(x) + 4/3*cos(x) + 4/3) + 1/12*sqrt(3)*log(4/3*cos(x)^2 + 4/3
*sin(x)^2 - 4/3*sqrt(3)*sin(x) - 4/3*cos(x) + 4/3) + sin(x)
```

mupad [B] time = 2.37, size = 16, normalized size = 0.80

$$\sin(x) - \frac{\sqrt{3}\,\operatorname{atanh}\left(\frac{2\sqrt{3}\,\sin(x)}{3}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(3*x)*sin(x),x)
```

```
[Out] sin(x) - (3^(1/2)*atanh((2*3^(1/2)*sin(x))/3))/3
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sin(x)\cot(3x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(3*x)*sin(x),x)
```

```
[Out] Integral(sin(x)*cot(3*x), x)
```

3.82 $\int \cot(4x)\sin(x)\,dx$

Optimal. Leaf size=28

$$\sin(x)-\frac{1}{4}\tanh^{-1}(\sin(x))-\frac{\tanh^{-1}\left(\sqrt{2}\sin(x)\right)}{2\sqrt{2}}$$

```
[Out] -1/4*arctanh(sin(x))+sin(x)-1/4*arctanh(sin(x)*2^(1/2))*2^(1/2)
```

Rubi [A] time = 0.05, antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {1676, 1166, 207}

$$\sin(x)-\frac{1}{4}\tanh^{-1}(\sin(x))-\frac{\tanh^{-1}\left(\sqrt{2}\sin(x)\right)}{2\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[4*x]*Sin[x],x]
```

```
[Out] -ArcTanh[Sin[x]]/4 - ArcTanh[Sqrt[2]*Sin[x]]/(2*Sqrt[2]) + Sin[x]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1676

```
Int[(Pq_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> Int[ExpandInte
grand[Pq/(a + b*x^2 + c*x^4), x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x^
2] && Expon[Pq, x^2] > 1
```

Rubi steps

$$\begin{aligned}\int \cot(4x)\sin(x)\,dx &= \text{Subst}\left(\int \frac{1-8x^2+8x^4}{4-12x^2+8x^4}\,dx, x, \sin(x)\right)\\ &= \text{Subst}\left(\int \left(1-\frac{3-4x^2}{4-12x^2+8x^4}\right)dx, x, \sin(x)\right)\\ &= \sin(x) - \text{Subst}\left(\int \frac{3-4x^2}{4-12x^2+8x^4}\,dx, x, \sin(x)\right)\\ &= \sin(x) + 2\,\text{Subst}\left(\int \frac{1}{-8+8x^2}\,dx, x, \sin(x)\right) + 2\,\text{Subst}\left(\int \frac{1}{-4+8x^2}\,dx, x, \sin(x)\right)\\ &= -\frac{1}{4}\tanh^{-1}(\sin(x)) - \frac{\tanh^{-1}\left(\sqrt{2}\sin(x)\right)}{2\sqrt{2}} + \sin(x)\end{aligned}$$

Mathematica [**A**] time = 0.04, size = 28, normalized size = 1.00

$$\sin(x) - \frac{1}{4}\tanh^{-1}(\sin(x)) - \frac{\tanh^{-1}\left(\sqrt{2}\sin(x)\right)}{2\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[4*x]*Sin[x],x]
[Out] -1/4*ArcTanh[Sin[x]] - ArcTanh[Sqrt[2]*Sin[x]]/(2*Sqrt[2]) + Sin[x]
```

fricas [**B**] time = 0.57, size = 52, normalized size = 1.86

$$\frac{1}{8}\sqrt{2}\log\left(-\frac{2\cos(x)^2 + 2\sqrt{2}\sin(x) - 3}{2\cos(x)^2 - 1}\right) - \frac{1}{8}\log(\sin(x)+1) + \frac{1}{8}\log(-\sin(x)+1) + \sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(4*x)*sin(x),x, algorithm="fricas")
[Out] 1/8*sqrt(2)*log(-(2*cos(x)^2 + 2*sqrt(2)*sin(x) - 3)/(2*cos(x)^2 - 1)) - 1/
8*log(sin(x) + 1) + 1/8*log(-sin(x) + 1) + sin(x)
```

giac [**B**] time = 0.14, size = 50, normalized size = 1.79

$$\frac{1}{8}\sqrt{2}\log\left(\frac{\left|-2\sqrt{2} + 4\sin(x)\right|}{\left|2\sqrt{2} + 4\sin(x)\right|}\right) - \frac{1}{8}\log(\sin(x)+1) + \frac{1}{8}\log(-\sin(x)+1) + \sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(4*x)*sin(x),x, algorithm="giac")
```

```
[Out] 1/8*sqrt(2)*log(abs(-2*sqrt(2) + 4*sin(x))/abs(2*sqrt(2) + 4*sin(x))) - 1/8
*log(sin(x) + 1) + 1/8*log(-sin(x) + 1) + sin(x)
```

maple [A] time = 0.12, size = 30, normalized size = 1.07

$$\sin(x) + \frac{\ln(\sin(x) - 1)}{8} - \frac{\operatorname{arctanh}\left(\sin(x)\sqrt{2}\right)\sqrt{2}}{4} - \frac{\ln(1 + \sin(x))}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(4*x)*sin(x),x)
```

```
[Out] sin(x)+1/8*ln(sin(x)-1)-1/4*arctanh(sin(x)*2^(1/2))*2^(1/2)-1/8*ln(1+sin(x)
)
```

maxima [B] time = 0.54, size = 173, normalized size = 6.18

$$-\frac{1}{16}\sqrt{2}\log\left(2\cos(x)^2 + 2\sin(x)^2 + 2\sqrt{2}\cos(x) + 2\sqrt{2}\sin(x) + 2\right) + \frac{1}{16}\sqrt{2}\log\left(2\cos(x)^2 + 2\sin(x)^2 + 2\sqrt{\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(4*x)*sin(x),x, algorithm="maxima")
```

```
[Out] -1/16*sqrt(2)*log(2*cos(x)^2 + 2*sin(x)^2 + 2*sqrt(2)*cos(x) + 2*sqrt(2)*si
n(x) + 2) + 1/16*sqrt(2)*log(2*cos(x)^2 + 2*sin(x)^2 + 2*sqrt(2)*cos(x) - 2
*sqrt(2)*sin(x) + 2) - 1/16*sqrt(2)*log(2*cos(x)^2 + 2*sin(x)^2 - 2*sqrt(2)
*cos(x) + 2*sqrt(2)*sin(x) + 2) + 1/16*sqrt(2)*log(2*cos(x)^2 + 2*sin(x)^2
- 2*sqrt(2)*cos(x) - 2*sqrt(2)*sin(x) + 2) - 1/8*log(cos(x)^2 + sin(x)^2 +
2*sin(x) + 1) + 1/8*log(cos(x)^2 + sin(x)^2 - 2*sin(x) + 1) + sin(x)
```

mupad [B] time = 2.38, size = 29, normalized size = 1.04

$$\sin(x) - \frac{\operatorname{atanh}\left(\frac{\sin\left(\frac{x}{2}\right)}{\cos\left(\frac{x}{2}\right)}\right)}{2} - \frac{\sqrt{2}\operatorname{atanh}\left(\sqrt{2}\sin(x)\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(4*x)*sin(x),x)
```

```
[Out] sin(x) - atanh(sin(x/2)/cos(x/2))/2 - (2^(1/2)*atanh(2^(1/2)*sin(x)))/4
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sin(x)\cot(4x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(4*x)*sin(x),x)
```

```
[Out] Integral(sin(x)*cot(4*x), x)
```

3.83 $\int \cot(5x)\sin(x)\,dx$

Optimal. Leaf size=82

$$\sin(x)-\frac{1}{5}\sqrt{\frac{1}{2}\left(5+\sqrt{5}\right)}\tanh^{-1}\left(2\sqrt{\frac{2}{5+\sqrt{5}}}\sin(x)\right)-\frac{1}{5}\sqrt{\frac{1}{2}\left(5-\sqrt{5}\right)}\tanh^{-1}\left(\sqrt{\frac{2}{5}\left(5+\sqrt{5}\right)}\sin(x)\right)$$

```
[Out] sin(x)-1/10*arctanh(1/5*sin(x)*(50+10*5^(1/2))^(1/2))*(10-2*5^(1/2))^(1/2)-
1/10*arctanh(2*sin(x)*2^(1/2)/(5+5^(1/2))^(1/2))*(10+2*5^(1/2))^(1/2)
```

Rubi [A] time = 0.20, antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {1676, 1166, 207}

$$\sin(x)-\frac{1}{5}\sqrt{\frac{1}{2}\left(5+\sqrt{5}\right)}\tanh^{-1}\left(2\sqrt{\frac{2}{5+\sqrt{5}}}\sin(x)\right)-\frac{1}{5}\sqrt{\frac{1}{2}\left(5-\sqrt{5}\right)}\tanh^{-1}\left(\sqrt{\frac{2}{5}\left(5+\sqrt{5}\right)}\sin(x)\right)$$

Antiderivative was successfully verified.

```
[In] Int[Cot[5*x]*Sin[x],x]
```

```
[Out] -(Sqrt[(5 + Sqrt[5])/2]*ArcTanh[2*Sqrt[2/(5 + Sqrt[5])]*Sin[x]])/5 - (Sqrt[
(5 - Sqrt[5])/2]*ArcTanh[Sqrt[(2*(5 + Sqrt[5]))/5]*Sin[x]])/5 + Sin[x]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1676

```
Int[(Pq_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> Int[ExpandInte
grand[Pq/(a + b*x^2 + c*x^4), x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x^
2] && Expon[Pq, x^2] > 1
```

Rubi steps

$$\int \cot(5x)\sin(x)\,dx = \text{Subst}\left(\int \frac{1-12x^2+16x^4}{5-20x^2+16x^4}\,dx, x, \sin(x)\right)$$
$$= \text{Subst}\left(\int \left(1-\frac{4\left(1-2x^2\right)}{5-20x^2+16x^4}\right)dx, x, \sin(x)\right)$$
$$= \sin(x) - 4\,\text{Subst}\left(\int \frac{1-2x^2}{5-20x^2+16x^4}\,dx, x, \sin(x)\right)$$
$$= \sin(x) + \frac{1}{5}\left(4\left(5-\sqrt{5}\right)\right)\text{Subst}\left(\int \frac{1}{-10+2\sqrt{5}+16x^2}\,dx, x, \sin(x)\right) + \frac{1}{5}\left(4\left(5+\sqrt{5}\right)\right)\text{Sub}$$
$$= -\frac{1}{5}\sqrt{\frac{1}{2}\left(5+\sqrt{5}\right)}\tanh^{-1}\left(2\sqrt{\frac{2}{5+\sqrt{5}}}\sin(x)\right) - \frac{1}{5}\sqrt{\frac{1}{2}\left(5-\sqrt{5}\right)}\tanh^{-1}\left(\sqrt{\frac{2}{5}\left(5+\sqrt{5}\right)}\,\text{s}\right.$$

Mathematica [A] time = 0.24, size = 76, normalized size = 0.93

$$\frac{1}{10}\left(10\sin(x) - \sqrt{10-2\sqrt{5}}\tanh^{-1}\left(\sqrt{2+\frac{2}{\sqrt{5}}}\sin(x)\right) - \sqrt{2\left(5+\sqrt{5}\right)}\tanh^{-1}\left(2\sqrt{\frac{2}{5+\sqrt{5}}}\sin(x)\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[5*x]*Sin[x],x]
```

```
[Out] (-(Sqrt[10 - 2*Sqrt[5]]*ArcTanh[Sqrt[2 + 2/Sqrt[5]]*Sin[x]]) - Sqrt[2*(5 +
Sqrt[5])]*ArcTanh[2*Sqrt[2/(5 + Sqrt[5])]*Sin[x]] + 10*Sin[x])/10
```

fricas [B] time = 0.56, size = 127, normalized size = 1.55

$$-\frac{1}{20}\sqrt{2}\sqrt{\sqrt{5}+5}\log\left(\sqrt{2}\sqrt{\sqrt{5}+5}+4\sin(x)\right)+\frac{1}{20}\sqrt{2}\sqrt{\sqrt{5}+5}\log\left(\sqrt{2}\sqrt{\sqrt{5}+5}-4\sin(x)\right)-\frac{1}{20}\sqrt{2}\sqrt{-\ldots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(5*x)*sin(x),x, algorithm="fricas")
```

```
[Out] -1/20*sqrt(2)*sqrt(sqrt(5) + 5)*log(sqrt(2)*sqrt(sqrt(5) + 5) + 4*sin(x)) +
 1/20*sqrt(2)*sqrt(sqrt(5) + 5)*log(sqrt(2)*sqrt(sqrt(5) + 5) - 4*sin(x)) -
 1/20*sqrt(2)*sqrt(-sqrt(5) + 5)*log(sqrt(2)*sqrt(-sqrt(5) + 5) + 4*sin(x))
 + 1/20*sqrt(2)*sqrt(-sqrt(5) + 5)*log(sqrt(2)*sqrt(-sqrt(5) + 5) - 4*sin(x
)) + sin(x)
```

giac [B] time = 0.33, size = 111, normalized size = 1.35

$$-\frac{1}{20}\sqrt{2\sqrt{5}+10}\log\left(\left|\frac{1}{2}\sqrt{\frac{1}{2}}\sqrt{\sqrt{5}+5}+\sin(x)\right|\right)+\frac{1}{20}\sqrt{2\sqrt{5}+10}\log\left(\left|-\frac{1}{2}\sqrt{\frac{1}{2}}\sqrt{\sqrt{5}+5}+\sin(x)\right|\right)-\frac{1}{20}\sqrt{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(5*x)*sin(x),x, algorithm="giac")
```

```
[Out] -1/20*sqrt(2*sqrt(5) + 10)*log(abs(1/2*sqrt(1/2)*sqrt(sqrt(5) + 5) + sin(x)
)) + 1/20*sqrt(2*sqrt(5) + 10)*log(abs(-1/2*sqrt(1/2)*sqrt(sqrt(5) + 5) + s
in(x))) - 1/20*sqrt(-2*sqrt(5) + 10)*log(abs(sqrt(-1/8*sqrt(5) + 5/8) + sin
(x))) + 1/20*sqrt(-2*sqrt(5) + 10)*log(abs(-sqrt(-1/8*sqrt(5) + 5/8) + sin(
x))) + sin(x)
```

maple [A] time = 0.21, size = 70, normalized size = 0.85

$$\sin(x)-\frac{\left(\sqrt{5}-1\right)\sqrt{5}\ \operatorname{arctanh}\left(\frac{4\sin(x)}{\sqrt{10-2\sqrt{5}}}\right)}{5\sqrt{10-2\sqrt{5}}}-\frac{\left(\sqrt{5}+1\right)\sqrt{5}\ \operatorname{arctanh}\left(\frac{4\sin(x)}{\sqrt{10+2\sqrt{5}}}\right)}{5\sqrt{10+2\sqrt{5}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(5*x)*sin(x),x)
```

```
[Out] sin(x)-1/5*(5^(1/2)-1)*5^(1/2)/(10-2*5^(1/2))^(1/2)*arctanh(4*sin(x)/(10-2*
5^(1/2))^(1/2))-1/5*(5^(1/2)+1)*5^(1/2)/(10+2*5^(1/2))^(1/2)*arctanh(4*sin(
x)/(10+2*5^(1/2))^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(5*x)*sin(x),x, algorithm="maxima")
```

```
[Out] -integrate(1/2*((cos(3*x) + cos(2*x) + cos(x))*cos(4*x) + (2*cos(2*x) + 2*c
os(x) + 1)*cos(3*x) + cos(3*x)^2 + (2*cos(x) + 1)*cos(2*x) + cos(2*x)^2 + c
os(x)^2 + (sin(3*x) + sin(2*x) + sin(x))*sin(4*x) + 2*(sin(2*x) + sin(x))*s
in(3*x) + sin(3*x)^2 + sin(2*x)^2 + 2*sin(2*x)*sin(x) + sin(x)^2 + cos(x))/
(2*(cos(3*x) + cos(2*x) + cos(x) + 1)*cos(4*x) + cos(4*x)^2 + 2*(cos(2*x) +
 cos(x) + 1)*cos(3*x) + cos(3*x)^2 + 2*(cos(x) + 1)*cos(2*x) + cos(2*x)^2 +
 cos(x)^2 + 2*(sin(3*x) + sin(2*x) + sin(x))*sin(4*x) + sin(4*x)^2 + 2*(sin
(2*x) + sin(x))*sin(3*x) + sin(3*x)^2 + sin(2*x)^2 + 2*sin(2*x)*sin(x) + si
```

```
n(x)^2 + 2*cos(x) + 1), x) - integrate(-1/2*((cos(3*x) - cos(2*x) + cos(x))
*cos(4*x) + (2*cos(2*x) - 2*cos(x) + 1)*cos(3*x) - cos(3*x)^2 + (2*cos(x) -
 1)*cos(2*x) - cos(2*x)^2 - cos(x)^2 + (sin(3*x) - sin(2*x) + sin(x))*sin(4
*x) + 2*(sin(2*x) - sin(x))*sin(3*x) - sin(3*x)^2 - sin(2*x)^2 + 2*sin(2*x)
*sin(x) - sin(x)^2 + cos(x))/(2*(cos(3*x) - cos(2*x) + cos(x) - 1)*cos(4*x)
 - cos(4*x)^2 + 2*(cos(2*x) - cos(x) + 1)*cos(3*x) - cos(3*x)^2 + 2*(cos(x)
 - 1)*cos(2*x) - cos(2*x)^2 - cos(x)^2 + 2*(sin(3*x) - sin(2*x) + sin(x))*s
in(4*x) - sin(4*x)^2 + 2*(sin(2*x) - sin(x))*sin(3*x) - sin(3*x)^2 - sin(2*
x)^2 + 2*sin(2*x)*sin(x) - sin(x)^2 + 2*cos(x) - 1), x) + sin(x)
```

mupad [B] time = 2.61, size = 119, normalized size = 1.45

$$\sin(x)-\frac{\sqrt{2}\,\mathrm{atanh}\left(\frac{\frac{25\,\sqrt{2}\,\sin(x)\,\sqrt{\sqrt{5}+5}}{2}+\frac{11\,\sqrt{2}\,\sqrt{5}\,\sin(x)\,\sqrt{\sqrt{5}+5}}{2}}{20\,\sqrt{5}+45}\right)\sqrt{\sqrt{5}+5}}{10}+\frac{\sqrt{2}\,\mathrm{atanh}\left(\frac{\frac{25\,\sqrt{2}\,\sin(x)\,\sqrt{5-\sqrt{5}}}{2}-\frac{11\,\sqrt{2}\,\sqrt{5}\,\sin(x)\,\sqrt{5-\sqrt{5}}}{2}}{20\,\sqrt{5}-45}\right.}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(5*x)*sin(x),x)
```

```
[Out] sin(x) - (2^(1/2)*atanh(((25*2^(1/2)*sin(x)*(5^(1/2) + 5)^(1/2))/2 + (11*2^
(1/2)*5^(1/2)*sin(x)*(5^(1/2) + 5)^(1/2))/2)/(20*5^(1/2) + 45))*(5^(1/2) +
5)^(1/2))/10 + (2^(1/2)*atanh(((25*2^(1/2)*sin(x)*(5 - 5^(1/2))^(1/2))/2 -
(11*2^(1/2)*5^(1/2)*sin(x)*(5 - 5^(1/2))^(1/2))/2)/(20*5^(1/2) - 45))*(5 -
5^(1/2))^(1/2))/10
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sin(x)\cot(5x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(5*x)*sin(x),x)
```

```
[Out] Integral(sin(x)*cot(5*x), x)
```

3.84 $\int \cot(6x)\sin(x)\,dx$

Optimal. Leaf size=38

$$\sin(x)-\frac{1}{6}\tanh^{-1}(\sin(x))-\frac{1}{6}\tanh^{-1}(2\sin(x))-\frac{\tanh^{-1}\left(\frac{2\sin(x)}{\sqrt{3}}\right)}{2\sqrt{3}}$$

```
[Out] -1/6*arctanh(sin(x))-1/6*arctanh(2*sin(x))+sin(x)-1/6*arctanh(2/3*sin(x)*3^
(1/2))*3^(1/2)
```

Rubi [A] time = 0.08, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 3, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {12, 2073, 207}

$$\sin(x)-\frac{1}{6}\tanh^{-1}(\sin(x))-\frac{1}{6}\tanh^{-1}(2\sin(x))-\frac{\tanh^{-1}\left(\frac{2\sin(x)}{\sqrt{3}}\right)}{2\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[6*x]*Sin[x],x]
```

```
[Out] -ArcTanh[Sin[x]]/6 - ArcTanh[2*Sin[x]]/6 - ArcTanh[(2*Sin[x])/Sqrt[3]]/(2*S
qrt[3]) + Sin[x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 2073

```
Int[(P_)^(p_)*(Q_)^(q_.), x_Symbol] :> With[{PP = Factor[P /. x -> Sqrt[x]]
}, Int[ExpandIntegrand[(PP /. x -> x^2)^p*Q^q, x], x] /;  !SumQ[NonfreeFact
ors[PP, x]]] /; FreeQ[q, x] && PolyQ[P, x^2] && PolyQ[Q, x] && ILtQ[p, 0]
```

Rubi steps

$$\int \cot(6x)\sin(x)\,dx = \text{Subst}\left(\int \frac{1-18x^2+48x^4-32x^6}{2\left(3-19x^2+32x^4-16x^6\right)}\,dx, x, \sin(x)\right)$$
$$= \frac{1}{2}\text{Subst}\left(\int \frac{1-18x^2+48x^4-32x^6}{3-19x^2+32x^4-16x^6}\,dx, x, \sin(x)\right)$$
$$= \frac{1}{2}\text{Subst}\left(\int \left(2+\frac{1}{3\left(-1+x^2\right)}+\frac{2}{-3+4x^2}+\frac{2}{3\left(-1+4x^2\right)}\right)dx, x, \sin(x)\right)$$
$$= \sin(x)+\frac{1}{6}\text{Subst}\left(\int \frac{1}{-1+x^2}\,dx, x, \sin(x)\right)+\frac{1}{3}\text{Subst}\left(\int \frac{1}{-1+4x^2}\,dx, x, \sin(x)\right)+\text{Subst}$$
$$= -\frac{1}{6}\tanh^{-1}(\sin(x))-\frac{1}{6}\tanh^{-1}(2\sin(x))-\frac{\tanh^{-1}\left(\frac{2\sin(x)}{\sqrt{3}}\right)}{2\sqrt{3}}+\sin(x)$$

Mathematica [A] time = 0.07, size = 38, normalized size = 1.00

$$\sin(x)-\frac{1}{6}\tanh^{-1}(\sin(x))-\frac{1}{6}\tanh^{-1}(2\sin(x))-\frac{\tanh^{-1}\left(\frac{2\sin(x)}{\sqrt{3}}\right)}{2\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[6*x]*Sin[x],x]
```

```
[Out] -1/6*ArcTanh[Sin[x]] - ArcTanh[2*Sin[x]]/6 - ArcTanh[(2*Sin[x])/Sqrt[3]]/(2
*Sqrt[3]) + Sin[x]
```

fricas [B] time = 0.87, size = 70, normalized size = 1.84

$$\frac{1}{12}\sqrt{3}\log\left(-\frac{4\cos(x)^2+4\sqrt{3}\sin(x)-7}{4\cos(x)^2-1}\right)-\frac{1}{12}\log(2\sin(x)+1)-\frac{1}{12}\log(\sin(x)+1)+\frac{1}{12}\log(-\sin(x)+1)+\frac{1}{1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(6*x)*sin(x),x, algorithm="fricas")
```

```
[Out] 1/12*sqrt(3)*log(-(4*cos(x)^2 + 4*sqrt(3)*sin(x) - 7)/(4*cos(x)^2 - 1)) - 1
/12*log(2*sin(x) + 1) - 1/12*log(sin(x) + 1) + 1/12*log(-sin(x) + 1) + 1/12
*log(-2*sin(x) + 1) + sin(x)
```

giac [B] time = 0.17, size = 70, normalized size = 1.84

$$\frac{1}{12}\sqrt{3}\log\left(\frac{\left|-4\sqrt{3}+8\sin(x)\right|}{\left|4\sqrt{3}+8\sin(x)\right|}\right)-\frac{1}{12}\log(\sin(x)+1)+\frac{1}{12}\log(-\sin(x)+1)-\frac{1}{12}\log(|2\sin(x)+1|)+\frac{1}{12}\log(|2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(6*x)*sin(x),x, algorithm="giac")
```

```
[Out] 1/12*sqrt(3)*log(abs(-4*sqrt(3) + 8*sin(x))/abs(4*sqrt(3) + 8*sin(x))) - 1/
12*log(sin(x) + 1) + 1/12*log(-sin(x) + 1) - 1/12*log(abs(2*sin(x) + 1)) +
1/12*log(abs(2*sin(x) - 1)) + sin(x)
```

maple [A] time = 0.16, size = 49, normalized size = 1.29

$$\sin(x)+\frac{\ln(-1+2\sin(x))}{12}-\frac{\ln(1+2\sin(x))}{12}-\frac{\operatorname{arctanh}\left(\frac{2\sin(x)\sqrt{3}}{3}\right)\sqrt{3}}{6}+\frac{\ln(\sin(x)-1)}{12}-\frac{\ln(1+\sin(x))}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(6*x)*sin(x),x)
```

```
[Out] sin(x)+1/12*ln(-1+2*sin(x))-1/12*ln(1+2*sin(x))-1/6*arctanh(2/3*sin(x)*3^(1
/2))*3^(1/2)+1/12*ln(sin(x)-1)-1/12*ln(1+sin(x))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{1}{24}\sqrt{3}\log\left(\frac{4}{3}\cos(x)^2+\frac{4}{3}\sin(x)^2+\frac{4}{3}\sqrt{3}\sin(x)+\frac{4}{3}\cos(x)+\frac{4}{3}\right)-\frac{1}{24}\sqrt{3}\log\left(\frac{4}{3}\cos(x)^2+\frac{4}{3}\sin(x)^2+\frac{4}{3}\sqrt{\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(6*x)*sin(x),x, algorithm="maxima")
```

```
[Out] -1/24*sqrt(3)*log(4/3*cos(x)^2 + 4/3*sin(x)^2 + 4/3*sqrt(3)*sin(x) + 4/3*co
s(x) + 4/3) - 1/24*sqrt(3)*log(4/3*cos(x)^2 + 4/3*sin(x)^2 + 4/3*sqrt(3)*si
n(x) - 4/3*cos(x) + 4/3) + 1/24*sqrt(3)*log(4/3*cos(x)^2 + 4/3*sin(x)^2 - 4
/3*sqrt(3)*sin(x) + 4/3*cos(x) + 4/3) + 1/24*sqrt(3)*log(4/3*cos(x)^2 + 4/3
*sin(x)^2 - 4/3*sqrt(3)*sin(x) - 4/3*cos(x) + 4/3) - integrate(-1/6*((cos(3
*x) + cos(x))*cos(4*x) - (cos(2*x) - 1)*cos(3*x) - cos(2*x)*cos(x) + (sin(3
*x) + sin(x))*sin(4*x) - sin(3*x)*sin(2*x) - sin(2*x)*sin(x) + cos(x))/(2*(
cos(2*x) - 1)*cos(4*x) - cos(4*x)^2 - cos(2*x)^2 - sin(4*x)^2 + 2*sin(4*x)*
sin(2*x) - sin(2*x)^2 + 2*cos(2*x) - 1), x) - 1/12*log(cos(x)^2 + sin(x)^2
+ 2*sin(x) + 1) + 1/12*log(cos(x)^2 + sin(x)^2 - 2*sin(x) + 1) + sin(x)
```

mupad [B] time = 2.50, size = 37, normalized size = 0.97

$$\sin(x)-\frac{\operatorname{atanh}(2\sin(x))}{6}-\frac{\operatorname{atanh}\left(\frac{\sin\left(\frac{x}{2}\right)}{\cos\left(\frac{x}{2}\right)}\right)}{3}-\frac{\sqrt{3}\operatorname{atanh}\left(\frac{2\sqrt{3}\sin(x)}{3}\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(6*x)*sin(x),x)
```

```
[Out] sin(x) - atanh(2*sin(x))/6 - atanh(sin(x/2)/cos(x/2))/3 - (3^(1/2)*atanh((2
*3^(1/2)*sin(x))/3))/6
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sin(x)\cot(6x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(6*x)*sin(x),x)
```

```
[Out] Integral(sin(x)*cot(6*x), x)
```

3.85 $\int \sec(2x)\sin(x)\,dx$

Optimal. Leaf size=15

$$\frac{\tanh^{-1}\left(\sqrt{2}\cos(x)\right)}{\sqrt{2}}$$

```
[Out] 1/2*arctanh(cos(x)*2^(1/2))*2^(1/2)
```

Rubi [A] time = 0.02, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {4357, 207}

$$\frac{\tanh^{-1}\left(\sqrt{2}\cos(x)\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[2*x]*Sin[x],x]
```

```
[Out] ArcTanh[Sqrt[2]*Cos[x]]/Sqrt[2]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 4357

```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{d = FreeFacto
rs[Cos[c*(a + b*x)], x]}, -Dist[d/(b*c), Subst[Int[SubstFor[1, Cos[c*(a + b
*x)]/d, u, x], x], x, Cos[c*(a + b*x)]/d], x] /; FunctionOfQ[Cos[c*(a + b*x
)]/d, u, x]] /; FreeQ[{a, b, c}, x] && (EqQ[F, Sin] || EqQ[F, sin])
```

Rubi steps

$$\begin{aligned}\int \sec(2x)\sin(x)\,dx &= -\text{Subst}\left(\int \frac{1}{-1+2x^2}\,dx, x, \cos(x)\right)\\ &= \frac{\tanh^{-1}\left(\sqrt{2}\cos(x)\right)}{\sqrt{2}}\end{aligned}$$

Mathematica [C] time = 0.41, size = 174, normalized size = 11.60

$$\frac{4\tanh^{-1}\left(\tan\left(\frac{x}{2}\right)+\sqrt{2}\right)-\log\left(-\sqrt{2}\sin(x)-\sqrt{2}\cos(x)+2\right)+\log\left(-\sqrt{2}\sin(x)+\sqrt{2}\cos(x)+2\right)+2i\tan^{-1}\left(\ldots\right.}{4\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[2*x]*Sin[x],x]
```

```
[Out] ((2*I)*ArcTan[(Cos[x/2] - (-1 + Sqrt[2])*Sin[x/2])/((1 + Sqrt[2])*Cos[x/2]
- Sin[x/2])] - (2*I)*ArcTan[(Cos[x/2] - (1 + Sqrt[2])*Sin[x/2])/((-1 + Sqrt
[2])*Cos[x/2] - Sin[x/2])] + 4*ArcTanh[Sqrt[2] + Tan[x/2]] - Log[2 - Sqrt[2
]*Cos[x] - Sqrt[2]*Sin[x]] + Log[2 + Sqrt[2]*Cos[x] - Sqrt[2]*Sin[x]])/(4*S
qrt[2])
```

fricas [B] time = 0.60, size = 33, normalized size = 2.20

$$\frac{1}{4}\sqrt{2}\log\left(-\frac{2\cos(x)^2+2\sqrt{2}\cos(x)+1}{2\cos(x)^2-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(2*x)*sin(x),x, algorithm="fricas")
```

```
[Out] 1/4*sqrt(2)*log(-(2*cos(x)^2 + 2*sqrt(2)*cos(x) + 1)/(2*cos(x)^2 - 1))
```

giac [B] time = 0.17, size = 49, normalized size = 3.27

$$\frac{1}{4}\sqrt{2}\log\left(\frac{\left|-4\sqrt{2}-\frac{2(\cos(x)-1)}{\cos(x)+1}-6\right|}{\left|4\sqrt{2}-\frac{2(\cos(x)-1)}{\cos(x)+1}-6\right|}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(2*x)*sin(x),x, algorithm="giac")
```

```
[Out] 1/4*sqrt(2)*log(abs(-4*sqrt(2) - 2*(cos(x) - 1)/(cos(x) + 1) - 6)/abs(4*sqr
t(2) - 2*(cos(x) - 1)/(cos(x) + 1) - 6))
```

maple [A] time = 0.13, size = 13, normalized size = 0.87

$$\frac{\operatorname{arctanh}\left(\cos(x)\sqrt{2}\right)\sqrt{2}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(2*x)*sin(x),x)
```

```
[Out] 1/2*arctanh(cos(x)*2^(1/2))*2^(1/2)
```

maxima [B] time = 0.43, size = 129, normalized size = 8.60

$$\frac{1}{8}\sqrt{2}\log\left(2\sqrt{2}\sin(2x)\sin(x)+2\left(\sqrt{2}\cos(x)+1\right)\cos(2x)+\cos(2x)^2+2\cos(x)^2+\sin(2x)^2+2\sin(x)^2+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(2*x)*sin(x),x, algorithm="maxima")
```

```
[Out] 1/8*sqrt(2)*log(2*sqrt(2)*sin(2*x)*sin(x) + 2*(sqrt(2)*cos(x) + 1)*cos(2*x)
 + cos(2*x)^2 + 2*cos(x)^2 + sin(2*x)^2 + 2*sin(x)^2 + 2*sqrt(2)*cos(x) + 1
) - 1/8*sqrt(2)*log(-2*sqrt(2)*sin(2*x)*sin(x) - 2*(sqrt(2)*cos(x) - 1)*cos
(2*x) + cos(2*x)^2 + 2*cos(x)^2 + sin(2*x)^2 + 2*sin(x)^2 - 2*sqrt(2)*cos(x
) + 1)
```

mupad [B] time = 2.29, size = 12, normalized size = 0.80

$$\frac{\sqrt{2}\,\mathrm{atanh}\left(\sqrt{2}\,\cos(x)\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)/cos(2*x),x)
```

```
[Out] (2^(1/2)*atanh(2^(1/2)*cos(x)))/2
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sin(x)\sec(2x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(2*x)*sin(x),x)
```

```
[Out] Integral(sin(x)*sec(2*x), x)
```

3.86 $\int \sec(3x)\sin(x)\,dx$

Optimal. Leaf size=21

$$\frac{1}{3}\log(\cos(x)) - \frac{1}{6}\log\left(3 - 4\cos^2(x)\right)$$

```
[Out] 1/3*ln(cos(x))-1/6*ln(3-4*cos(x)^2)
```

Rubi [A] time = 0.03, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.714, Rules used = {4357, 266, 36, 29, 31}

$$\frac{1}{3}\log(\cos(x)) - \frac{1}{6}\log\left(3 - 4\cos^2(x)\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sec[3*x]*Sin[x],x]
```

```
[Out] Log[Cos[x]]/3 - Log[3 - 4*Cos[x]^2]/6
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 4357

```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{d = FreeFacto
rs[Cos[c*(a + b*x)], x]}, -Dist[d/(b*c), Subst[Int[SubstFor[1, Cos[c*(a + b
```

```
*x)]/d, u, x], x], x, Cos[c*(a + b*x)]/d], x] /; FunctionOfQ[Cos[c*(a + b*x
)]/d, u, x]] /; FreeQ[{a, b, c}, x] && (EqQ[F, Sin] || EqQ[F, sin])
```

Rubi steps

$$
\begin{aligned}
\int \sec(3x)\sin(x)\,dx &= -\operatorname{Subst}\left(\int \frac{1}{x\left(-3+4x^2\right)}\,dx, x, \cos(x)\right)\\
&= -\left(\frac{1}{2}\operatorname{Subst}\left(\int \frac{1}{x(-3+4x)}\,dx, x, \cos^2(x)\right)\right)\\
&= \frac{1}{6}\operatorname{Subst}\left(\int \frac{1}{x}\,dx, x, \cos^2(x)\right) - \frac{2}{3}\operatorname{Subst}\left(\int \frac{1}{-3+4x}\,dx, x, \cos^2(x)\right)\\
&= \frac{1}{3}\log(\cos(x)) - \frac{1}{6}\log\left(3-4\cos^2(x)\right)
\end{aligned}
$$

Mathematica [A] time = 0.01, size = 17, normalized size = 0.81

$$-\frac{1}{3}\tanh^{-1}\left(\frac{1}{3}\left(8\sin^2(x)-5\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[3*x]*Sin[x],x]
[Out] -1/3*ArcTanh[(-5 + 8*Sin[x]^2)/3]
```

fricas [A] time = 0.88, size = 19, normalized size = 0.90

$$-\frac{1}{6}\log\left(4\cos(x)^2-3\right)+\frac{1}{3}\log\left(-\cos(x)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(3*x)*sin(x),x, algorithm="fricas")
[Out] -1/6*log(4*cos(x)^2 - 3) + 1/3*log(-cos(x))
```

giac [A] time = 0.14, size = 24, normalized size = 1.14

$$\frac{1}{6}\log\left(-\sin(x)^2+1\right)-\frac{1}{6}\log\left(\left|4\sin(x)^2-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(3*x)*sin(x),x, algorithm="giac")
[Out] 1/6*log(-sin(x)^2 + 1) - 1/6*log(abs(4*sin(x)^2 - 1))
```

maple [A] time = 0.19, size = 18, normalized size = 0.86

$$-\frac{\ln\left(4\left(\cos^2(x)\right)-3\right)}{6}+\frac{\ln(\cos(x))}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(3*x)*sin(x),x)
[Out] -1/6*ln(4*cos(x)^2-3)+1/3*ln(cos(x))
```

maxima [B] time = 0.42, size = 81, normalized size = 3.86

$$-\frac{1}{12}\log\left(-2(\cos(2x)-1)\cos(4x)+\cos(4x)^2+\cos(2x)^2+\sin(4x)^2-2\sin(4x)\sin(2x)+\sin(2x)^2-2\,\text{co}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(3*x)*sin(x),x, algorithm="maxima")
[Out] -1/12*log(-2*(cos(2*x) - 1)*cos(4*x) + cos(4*x)^2 + cos(2*x)^2 + sin(4*x)^2
 - 2*sin(4*x)*sin(2*x) + sin(2*x)^2 - 2*cos(2*x) + 1) + 1/6*log(cos(2*x)^2
+ sin(2*x)^2 + 2*cos(2*x) + 1)
```

mupad [B] time = 2.27, size = 15, normalized size = 0.71

$$\frac{\ln(\cos(x))}{3}-\frac{\ln\left(\cos(x)^2-\frac{3}{4}\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)/cos(3*x),x)
[Out] log(cos(x))/3 - log(cos(x)^2 - 3/4)/6
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sin(x)\sec(3x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(3*x)*sin(x),x)
[Out] Integral(sin(x)*sec(3*x), x)
```

3.87 $\int \sec(4x)\sin(x)\,dx$

Optimal. Leaf size=71

$$\frac{\tanh^{-1}\left(\frac{2\cos(x)}{\sqrt{2+\sqrt{2}}}\right)}{2\sqrt{2\left(2+\sqrt{2}\right)}}-\frac{\tanh^{-1}\left(\frac{2\cos(x)}{\sqrt{2-\sqrt{2}}}\right)}{2\sqrt{2\left(2-\sqrt{2}\right)}}$$

```
[Out] -1/2*arctanh(2*cos(x)/(2-2^(1/2))^(1/2))/(4-2*2^(1/2))^(1/2)+1/2*arctanh(2*
cos(x)/(2+2^(1/2))^(1/2))/(4+2*2^(1/2))^(1/2)
```

Rubi [A] time = 0.06, antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {4357, 1093, 207}

$$\frac{\tanh^{-1}\left(\frac{2\cos(x)}{\sqrt{2+\sqrt{2}}}\right)}{2\sqrt{2\left(2+\sqrt{2}\right)}}-\frac{\tanh^{-1}\left(\frac{2\cos(x)}{\sqrt{2-\sqrt{2}}}\right)}{2\sqrt{2\left(2-\sqrt{2}\right)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[4*x]*Sin[x],x]
```

```
[Out] -ArcTanh[(2*Cos[x])/Sqrt[2 - Sqrt[2]]]/(2*Sqrt[2*(2 - Sqrt[2])]) + ArcTanh[
(2*Cos[x])/Sqrt[2 + Sqrt[2]]]/(2*Sqrt[2*(2 + Sqrt[2])])
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 1093

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(-1), x_Symbol] :> With[{q = Rt[b^
2 - 4*a*c, 2]}, Dist[c/q, Int[1/(b/2 - q/2 + c*x^2), x], x] - Dist[c/q, Int
[1/(b/2 + q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c,
0] && PosQ[b^2 - 4*a*c]
```

Rule 4357

```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{d = FreeFacto
rs[Cos[c*(a + b*x)], x]}, -Dist[d/(b*c), Subst[Int[SubstFor[1, Cos[c*(a + b
```

```
*x)]/d, u, x], x], x, Cos[c*(a + b*x)]/d], x] /; FunctionOfQ[Cos[c*(a + b*x
)]/d, u, x]] /; FreeQ[{a, b, c}, x] && (EqQ[F, Sin] || EqQ[F, sin])
```

Rubi steps

$$\begin{aligned}\int \sec(4x)\sin(x)\,dx &= -\text{Subst}\left(\int \frac{1}{1-8x^2+8x^4}\,dx, x, \cos(x)\right)\\ &= -\left(\sqrt{2}\,\text{Subst}\left(\int \frac{1}{-4-2\sqrt{2}+8x^2}\,dx, x, \cos(x)\right)\right)+\sqrt{2}\,\text{Subst}\left(\int \frac{1}{-4+2\sqrt{2}+8x^2}\,dx, x,\right.\\ &= -\frac{\tanh^{-1}\left(\frac{2\cos(x)}{\sqrt{2-\sqrt{2}}}\right)}{2\sqrt{2\left(2-\sqrt{2}\right)}}+\frac{\tanh^{-1}\left(\frac{2\cos(x)}{\sqrt{2+\sqrt{2}}}\right)}{2\sqrt{2\left(2+\sqrt{2}\right)}}\end{aligned}$$

Mathematica [C] time = 57.93, size = 4845, normalized size = 68.24

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sec[4*x]*Sin[x],x]
```

```
[Out] ((-2*(-1)^(3/8)*(1 + Sqrt[2])*x - (2*(-1)^(1/4)*(-2 - (1 - I)*(-1)^(5/8) +
(-1)^(5/8)*Sqrt[2])*ArcTan[(-Cos[x] + (1 + Sqrt[2])*Sin[x])/(2*(-1)^(3/8) +
 Cos[x] - Sqrt[2]*Cos[x] + Sin[x])])/((-1 + I) + 2*(-1)^(3/8) + Sqrt[2]) -
(2*(1 - I)^(3/2)*2^(1/4)*((-3 - I) + 2*(-1)^(5/8) + (2 + I)*Sqrt[2] - (2 +
2*I)*(-1)^(3/8)*Sqrt[2] + 2*(-1)^(5/8)*Sqrt[2])*ArcTan[((1 + I) + I*Sqrt[2]
 + ((-1 + I) + 2*(-1)^(3/8) + Sqrt[2])*Tan[x/2])/(Sqrt[1 - I]*2^(3/4))])/((
-1 + I) + 2*(-1)^(3/8) + Sqrt[2]) + 2*(-1)^(3/8)*Log[Sec[x/2]^2] + ((-1)^(3
/4)*(-2 - (1 - I)*(-1)^(5/8) + (-1)^(5/8)*Sqrt[2])*Log[-(Sec[x/2]^4*(-2 + (
1 - I)*Sqrt[2] + 2*(-1)^(3/8)*(-1 + Sqrt[2])*Cos[x] + Sqrt[2]*Cos[2*x] - 2*
(-1)^(3/8)*Sin[x] + Sqrt[2]*Sin[2*x]))])/((-1 + I) + 2*(-1)^(3/8) + Sqrt[2]
))*((-1/2 - I/2)/(((-1 + I) + Sqrt[1 - I]*Sqrt[1 + I])*(-((-1 - I)^(3/2)*(1
 - I)^(1/4)*(1 + I)^(1/4)) - (1 + I)*Cos[x] + I*Sqrt[1 - I]*Sqrt[1 + I]*Cos
[x] + (1 - I)*Sin[x] + Sqrt[1 - I]*Sqrt[1 + I]*Sin[x])) - Sin[x]/(Sqrt[-1 -
 I]*(1 - I)^(1/4)*(1 + I)^(1/4)*((-1 + I) + Sqrt[1 - I]*Sqrt[1 + I])*(-((-1
 - I)^(3/2)*(1 - I)^(1/4)*(1 + I)^(1/4)) - (1 + I)*Cos[x] + I*Sqrt[1 - I]*S
qrt[1 + I]*Cos[x] + (1 - I)*Sin[x] + Sqrt[1 - I]*Sqrt[1 + I]*Sin[x])) - ((I
/2)*Sqrt[-1 - I]*(1 - I)^(1/4)*(1 + I)^(1/4)*Sin[x])/(((-1 + I) + Sqrt[1 -
I]*Sqrt[1 + I])*(-((-1 - I)^(3/2)*(1 - I)^(1/4)*(1 + I)^(1/4)) - (1 + I)*Co
s[x] + I*Sqrt[1 - I]*Sqrt[1 + I]*Cos[x] + (1 - I)*Sin[x] + Sqrt[1 - I]*Sqrt
```

```
[1 + I]*Sin[x]))))/(-2*(-1)^(3/8)*(1 + Sqrt[2]) - (2*(-1)^(1/4)*(-2 - (1 -
I)*(-1)^(5/8) + (-1)^(5/8)*Sqrt[2])*(((1 + Sqrt[2])*Cos[x] + Sin[x])/(2*(-1
)^(3/8) + Cos[x] - Sqrt[2]*Cos[x] + Sin[x]) - ((Cos[x] - Sin[x] + Sqrt[2]*S
in[x])*(-Cos[x] + (1 + Sqrt[2])*Sin[x]))/(2*(-1)^(3/8) + Cos[x] - Sqrt[2]*C
os[x] + Sin[x])^2))/(((-1 + I) + 2*(-1)^(3/8) + Sqrt[2])*(1 + (-Cos[x] + (1
 + Sqrt[2])*Sin[x])^2/(2*(-1)^(3/8) + Cos[x] - Sqrt[2]*Cos[x] + Sin[x])^2))
 + 2*(-1)^(3/8)*Tan[x/2] - ((-1)^(3/4)*(-2 - (1 - I)*(-1)^(5/8) + (-1)^(5/8
)*Sqrt[2])*Cos[x/2]^4*(-(Sec[x/2]^4*(-2*(-1)^(3/8)*Cos[x] + 2*Sqrt[2]*Cos[2
*x] - 2*(-1)^(3/8)*(-1 + Sqrt[2])*Sin[x] - 2*Sqrt[2]*Sin[2*x])) - 2*Sec[x/2
]^4*(-2 + (1 - I)*Sqrt[2] + 2*(-1)^(3/8)*(-1 + Sqrt[2])*Cos[x] + Sqrt[2]*Co
s[2*x] - 2*(-1)^(3/8)*Sin[x] + Sqrt[2]*Sin[2*x])*Tan[x/2]))/(((-1 + I) + 2*
(-1)^(3/8) + Sqrt[2])*(-2 + (1 - I)*Sqrt[2] + 2*(-1)^(3/8)*(-1 + Sqrt[2])*C
os[x] + Sqrt[2]*Cos[2*x] - 2*(-1)^(3/8)*Sin[x] + Sqrt[2]*Sin[2*x])) - ((1 -
 I)*((-3 - I) + 2*(-1)^(5/8) + (2 + I)*Sqrt[2] - (2 + 2*I)*(-1)^(3/8)*Sqrt[
2] + 2*(-1)^(5/8)*Sqrt[2])*Sec[x/2]^2)/(Sqrt[2]*(1 + ((1/4 + I/4)*((1 + I)
+ I*Sqrt[2] + ((-1 + I) + 2*(-1)^(3/8) + Sqrt[2])*Tan[x/2])^2)/Sqrt[2]))) +
 ((-4*Sqrt[-1 - I]*(-1 + Sqrt[2])*ArcTanh[((-I)*((1 + I) + Sqrt[2]) + ((1 +
 I) + 2*(-1)^(5/8) - Sqrt[2])*Tan[x/2])/(Sqrt[-1 - I]*2^(3/4))] + (-1)^(1/8
)*2^(1/4)*(2*ArcTan[(Cos[x] + (1 + Sqrt[2])*Sin[x])/(2*(-1)^(5/8) + (-1 + S
qrt[2])*Cos[x] + Sin[x])] - I*(2*(1 + Sqrt[2])*x + 2*Log[Sec[x/2]^2] - Log[
Sec[x/2]^4*(2 - (1 + I)*Sqrt[2] + 2*(-1)^(5/8)*(-1 + Sqrt[2])*Cos[x] - Sqrt
[2]*Cos[2*x] + 2*(-1)^(5/8)*Sin[x] + Sqrt[2]*Sin[2*x])])))*(2 + I*Sqrt[-1 +
 I]*2^(1/4)*((1 + I) + Sqrt[2])*Sin[x]))/(2^(1/4)*(4*Sqrt[-1 + I]*2^(1/4)*(
(-1 - I) + Sqrt[2]) - 8*(-1 + Sqrt[2])*Cos[x] - 8*Sin[x])*((2*(-1)^(1/8)*(-
2 - (1 + I)*Sqrt[2] + (-1)^(1/8)*((1 + I) + I*Sqrt[2])*Cos[x] + (2*I)*(1 +
Sqrt[2])*Cos[2*x] + (-1)^(1/8)*Sin[x] - (-1)^(5/8)*Sin[x] + 3*(-1)^(1/8)*Sq
rt[2]*Sin[x] - (2*I)*Sin[2*x]))/(2 - (1 + I)*Sqrt[2] + 2*(-1)^(5/8)*(-1 + S
qrt[2])*Cos[x] - Sqrt[2]*Cos[2*x] + 2*(-1)^(5/8)*Sin[x] + Sqrt[2]*Sin[2*x])
 - (((1 + I) + 2*(-1)^(5/8) - Sqrt[2])*(-1 + Sqrt[2])*Sec[x/2]^2)/(1 + ((1/
4 - I/4)*(I*((1 + I) + Sqrt[2]) + ((-1 - I) - 2*(-1)^(5/8) + Sqrt[2])*Tan[x
/2])^2)/Sqrt[2]))) + ((-2*(-1)^(3/8)*Sqrt[2]*(1 + (-1)^(1/4))*x + (2*(-2*I
+ 2*(-1)^(3/4) + 2*(-1)^(1/8)*Sqrt[2] - (-1)^(3/8)*Sqrt[2] + (-1)^(7/8)*Sqr
t[2])*ArcTan[Cos[x]/(-((-1)^(1/8)*Sqrt[2]) + (-1)^(3/4)*Cos[x] + (1 + (-1)^
(1/4))*Sin[x])])/(-I + (-1)^(3/4) + (-1)^(1/8)*Sqrt[2]) - ((4 + 4*I)*(-1)^(
5/8)*((3 - 3*I) - (2 - 2*I)*Sqrt[2] + (-1)^(1/8)*Sqrt[2] - (-1)^(3/8)*Sqrt[
2] + (1 - I)*(-1)^(5/8)*Sqrt[2] + (1 + I)*(-1)^(7/8)*Sqrt[2])*ArcTanh[(1/2
+ I/2)*(-1)^(5/8)*(-1 - (-1)^(1/4) + (-I + (-1)^(3/4) + (-1)^(1/8)*Sqrt[2])
*Tan[x/2])])/(-I + (-1)^(3/4) + (-1)^(1/8)*Sqrt[2]) - 2*(-1)^(7/8)*Sqrt[2]*
(-1 + (-1)^(1/4))*Log[Sec[x/2]^2] - ((-1 + (-1)^(1/4))*(2 - (-1)^(3/8)*Sqrt
[2] + (-1)^(5/8)*Sqrt[2])*Log[(1/4 + I/4)*Sec[x/2]^4*((2 - 2*I) + 6*Sqrt[2]
 - (4 - 4*I)*(-1)^(7/8)*Sqrt[2]*Cos[x] - 2*((1 + I) + Sqrt[2])*Cos[2*x] - (
4 - 4*I)*(-1)^(1/8)*Sqrt[2]*Sin[x] - (4 - 4*I)*(-1)^(3/8)*Sqrt[2]*Sin[x] -
(2 - 2*I)*Sin[2*x] + (2*I)*Sqrt[2]*Sin[2*x])])/(-I + (-1)^(3/4) + (-1)^(1/8
)*Sqrt[2]))*(I/(Sqrt[1 - I]*((-1 + I) + Sqrt[1 - I]*Sqrt[1 + I])^2*(Sqrt[-1
 - I]*(1 - I)^(3/4)*(1 + I)^(1/4) + Sqrt[1 - I]*Cos[x] - Sqrt[1 + I]*Cos[x]
```

```
 + I*Sqrt[1 - I]*Sin[x] + I*Sqrt[1 + I]*Sin[x])) + 1/(Sqrt[1 + I]*((-1 + I)
 + Sqrt[1 - I]*Sqrt[1 + I])^2*(Sqrt[-1 - I]*(1 - I)^(3/4)*(1 + I)^(1/4) + S
qrt[1 - I]*Cos[x] - Sqrt[1 + I]*Cos[x] + I*Sqrt[1 - I]*Sin[x] + I*Sqrt[1 +
I]*Sin[x])) - (2*Sin[x])/(Sqrt[-1 - I]*(1 - I)^(1/4)*(1 + I)^(3/4)*((-1 + I
) + Sqrt[1 - I]*Sqrt[1 + I])^2*(Sqrt[-1 - I]*(1 - I)^(3/4)*(1 + I)^(1/4) +
Sqrt[1 - I]*Cos[x] - Sqrt[1 + I]*Cos[x] + I*Sqrt[1 - I]*Sin[x] + I*Sqrt[1 +
 I]*Sin[x]))))/(-2*(-1)^(3/8)*Sqrt[2]*(1 + (-1)^(1/4)) + (2*(-2*I + 2*(-1)^
(3/4) + 2*(-1)^(1/8)*Sqrt[2] - (-1)^(3/8)*Sqrt[2] + (-1)^(7/8)*Sqrt[2])*(-(
(Cos[x]*((1 + (-1)^(1/4))*Cos[x] - (-1)^(3/4)*Sin[x]))/(-((-1)^(1/8)*Sqrt[2
]) + (-1)^(3/4)*Cos[x] + (1 + (-1)^(1/4))*Sin[x])^2) - Sin[x]/(-((-1)^(1/8)
*Sqrt[2]) + (-1)^(3/4)*Cos[x] + (1 + (-1)^(1/4))*Sin[x])))/((-I + (-1)^(3/4
) + (-1)^(1/8)*Sqrt[2])*(1 + Cos[x]^2/(-((-1)^(1/8)*Sqrt[2]) + (-1)^(3/4)*C
os[x] + (1 + (-1)^(1/4))*Sin[x])^2)) - 2*(-1)^(7/8)*Sqrt[2]*(-1 + (-1)^(1/4
))*Tan[x/2] - ((2 - 2*I)*(-1 + (-1)^(1/4))*(2 - (-1)^(3/8)*Sqrt[2] + (-1)^(
5/8)*Sqrt[2])*Cos[x/2]^4*((1/4 + I/4)*Sec[x/2]^4*((-4 + 4*I)*(-1)^(1/8)*Sqr
t[2]*Cos[x] - (4 - 4*I)*(-1)^(3/8)*Sqrt[2]*Cos[x] - (4 - 4*I)*Cos[2*x] + (4
*I)*Sqrt[2]*Cos[2*x] + (4 - 4*I)*(-1)^(7/8)*Sqrt[2]*Sin[x] + 4*((1 + I) + S
qrt[2])*Sin[2*x]) + (1/2 + I/2)*Sec[x/2]^4*((2 - 2*I) + 6*Sqrt[2] - (4 - 4*
I)*(-1)^(7/8)*Sqrt[2]*Cos[x] - 2*((1 + I) + Sqrt[2])*Cos[2*x] - (4 - 4*I)*(
-1)^(1/8)*Sqrt[2]*Sin[x] - (4 - 4*I)*(-1)^(3/8)*Sqrt[2]*Sin[x] - (2 - 2*I)*
Sin[2*x] + (2*I)*Sqrt[2]*Sin[2*x])*Tan[x/2]))/((-I + (-1)^(3/4) + (-1)^(1/8
)*Sqrt[2])*((2 - 2*I) + 6*Sqrt[2] - (4 - 4*I)*(-1)^(7/8)*Sqrt[2]*Cos[x] - 2
*((1 + I) + Sqrt[2])*Cos[2*x] - (4 - 4*I)*(-1)^(1/8)*Sqrt[2]*Sin[x] - (4 -
4*I)*(-1)^(3/8)*Sqrt[2]*Sin[x] - (2 - 2*I)*Sin[2*x] + (2*I)*Sqrt[2]*Sin[2*x
])) + (2*(-1)^(3/4)*((3 - 3*I) - (2 - 2*I)*Sqrt[2] + (-1)^(1/8)*Sqrt[2] - (
-1)^(3/8)*Sqrt[2] + (1 - I)*(-1)^(5/8)*Sqrt[2] + (1 + I)*(-1)^(7/8)*Sqrt[2]
)*Sec[x/2]^2)/(1 + ((-1)^(3/4)*(-1 - (-1)^(1/4) + (-I + (-1)^(3/4) + (-1)^(
1/8)*Sqrt[2])*Tan[x/2])^2)/2)) + ((2*((-1)^(1/8) + (-1)^(3/8))*x - (2*(-1)^
(7/8)*(2 - Sqrt[2] - (-1)^(3/8)*Sqrt[2] + (-1)^(5/8)*Sqrt[2])*ArcTan[Cos[x]
/(-((-1)^(1/8)*Sqrt[2]) + (-1)^(3/4)*Cos[x] - (1 + (-1)^(1/4))*Sin[x])])/(-
I + (-1)^(3/4) + (-1)^(1/8)*Sqrt[2]) - ((4 + 4*I)*(-1)^(5/8)*(3*I + (-1)^(1
/8) - (-1)^(3/8) - (1 + I)*(-1)^(5/8) - (2*I)*Sqrt[2] + (1 + I)*(-1)^(5/8)*
Sqrt[2])*ArcTanh[(1/2 + I/2)*(-1)^(5/8)*(1 + (-1)^(1/4) + (-I + (-1)^(3/4)
+ (-1)^(1/8)*Sqrt[2])*Tan[x/2])])/(-I + (-1)^(3/4) + (-1)^(1/8)*Sqrt[2]) +
2*(-1)^(3/8)*(-I + (-1)^(1/4))*Log[Sec[x/2]^2] + ((-1)^(3/8)*(2 - Sqrt[2] -
 (-1)^(3/8)*Sqrt[2] + (-1)^(5/8)*Sqrt[2])*Log[(1/4 + I/4)*Sec[x/2]^4*((2 -
2*I) + 6*Sqrt[2] - (4 - 4*I)*(-1)^(7/8)*Sqrt[2]*Cos[x] - 2*((1 + I) + Sqrt[
2])*Cos[2*x] + (4 - 4*I)*(-1)^(1/8)*((1 + I) + Sqrt[2])*Sin[x] + (2 - 2*I)*
Sin[2*x] - (2*I)*Sqrt[2]*Sin[2*x])])/(-I + (-1)^(3/4) + (-1)^(1/8)*Sqrt[2])
)*(1/(Sqrt[1 - I]*((-1 - I) + Sqrt[1 - I]*Sqrt[1 + I])^2*(-(Sqrt[-1 + I]*(1
 - I)^(1/4)*(1 + I)^(3/4)) + Sqrt[1 - I]*Cos[x] - Sqrt[1 + I]*Cos[x] - I*Sq
rt[1 - I]*Sin[x] - I*Sqrt[1 + I]*Sin[x])) - I/(Sqrt[1 + I]*((-1 - I) + Sqrt
[1 - I]*Sqrt[1 + I])^2*(-(Sqrt[-1 + I]*(1 - I)^(1/4)*(1 + I)^(3/4)) + Sqrt[
1 - I]*Cos[x] - Sqrt[1 + I]*Cos[x] - I*Sqrt[1 - I]*Sin[x] - I*Sqrt[1 + I]*S
in[x])) + (2*Sin[x])/(Sqrt[-1 + I]*(1 - I)^(3/4)*(1 + I)^(1/4)*((-1 - I) +
```

```
Sqrt[1 - I]*Sqrt[1 + I])^2*(-(Sqrt[-1 + I]*(1 - I)^(1/4)*(1 + I)^(3/4)) + S
qrt[1 - I]*Cos[x] - Sqrt[1 + I]*Cos[x] - I*Sqrt[1 - I]*Sin[x] - I*Sqrt[1 +
I]*Sin[x]))))/(2*((-1)^(1/8) + (-1)^(3/8)) - (2*(-1)^(7/8)*(2 - Sqrt[2] - (
-1)^(3/8)*Sqrt[2] + (-1)^(5/8)*Sqrt[2])*(-((Cos[x]*(-((1 + (-1)^(1/4))*Cos[
x]) - (-1)^(3/4)*Sin[x]))/(-((-1)^(1/8)*Sqrt[2]) + (-1)^(3/4)*Cos[x] - (1 +
 (-1)^(1/4))*Sin[x])^2) - Sin[x]/(-((-1)^(1/8)*Sqrt[2]) + (-1)^(3/4)*Cos[x]
 - (1 + (-1)^(1/4))*Sin[x])))/((-I + (-1)^(3/4) + (-1)^(1/8)*Sqrt[2])*(1 +
Cos[x]^2/(-((-1)^(1/8)*Sqrt[2]) + (-1)^(3/4)*Cos[x] - (1 + (-1)^(1/4))*Sin[
x])^2)) + 2*(-1)^(3/8)*(-I + (-1)^(1/4))*Tan[x/2] + ((2 - 2*I)*(-1)^(3/8)*(
2 - Sqrt[2] - (-1)^(3/8)*Sqrt[2] + (-1)^(5/8)*Sqrt[2])*Cos[x/2]^4*((1/4 + I
/4)*Sec[x/2]^4*((4 - 4*I)*(-1)^(1/8)*((1 + I) + Sqrt[2])*Cos[x] + (4 - 4*I)
*Cos[2*x] - (4*I)*Sqrt[2]*Cos[2*x] + (4 - 4*I)*(-1)^(7/8)*Sqrt[2]*Sin[x] +
4*((1 + I) + Sqrt[2])*Sin[2*x]) + (1/2 + I/2)*Sec[x/2]^4*((2 - 2*I) + 6*Sqr
t[2] - (4 - 4*I)*(-1)^(7/8)*Sqrt[2]*Cos[x] - 2*((1 + I) + Sqrt[2])*Cos[2*x]
 + (4 - 4*I)*(-1)^(1/8)*((1 + I) + Sqrt[2])*Sin[x] + (2 - 2*I)*Sin[2*x] - (
2*I)*Sqrt[2]*Sin[2*x])*Tan[x/2]))/((-I + (-1)^(3/4) + (-1)^(1/8)*Sqrt[2])*(
(2 - 2*I) + 6*Sqrt[2] - (4 - 4*I)*(-1)^(7/8)*Sqrt[2]*Cos[x] - 2*((1 + I) +
Sqrt[2])*Cos[2*x] + (4 - 4*I)*(-1)^(1/8)*((1 + I) + Sqrt[2])*Sin[x] + (2 -
2*I)*Sin[2*x] - (2*I)*Sqrt[2]*Sin[2*x])) + (2*(-1)^(3/4)*(3*I + (-1)^(1/8)
- (-1)^(3/8) - (1 + I)*(-1)^(5/8) - (2*I)*Sqrt[2] + (1 + I)*(-1)^(5/8)*Sqrt
[2])*Sec[x/2]^2)/(1 + ((-1)^(3/4)*(1 + (-1)^(1/4) + (-I + (-1)^(3/4) + (-1)
^(1/8)*Sqrt[2])*Tan[x/2])^2)/2))
```

fricas [B] time = 0.68, size = 121, normalized size = 1.70

$$-\frac{1}{8}\sqrt{\sqrt{2}+2}\log\left(\sqrt{\sqrt{2}+2}\left(\sqrt{2}-1\right)+2\cos(x)\right)+\frac{1}{8}\sqrt{\sqrt{2}+2}\log\left(\sqrt{\sqrt{2}+2}\left(\sqrt{2}-1\right)-2\cos(x)\right)+\frac{1}{8}\sqrt{-\sqrt{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(4*x)*sin(x),x, algorithm="fricas")
```

```
[Out] -1/8*sqrt(sqrt(2) + 2)*log(sqrt(sqrt(2) + 2)*(sqrt(2) - 1) + 2*cos(x)) + 1/
8*sqrt(sqrt(2) + 2)*log(sqrt(sqrt(2) + 2)*(sqrt(2) - 1) - 2*cos(x)) + 1/8*s
qrt(-sqrt(2) + 2)*log((sqrt(2) + 1)*sqrt(-sqrt(2) + 2) + 2*cos(x)) - 1/8*sq
rt(-sqrt(2) + 2)*log((sqrt(2) + 1)*sqrt(-sqrt(2) + 2) - 2*cos(x))
```

giac [B] time = 0.14, size = 133, normalized size = 1.87

$$-\frac{2.16139547686000\log\left(-\frac{\cos(x)-1}{\cos(x)+1}-0.0395661298966000\right)}{\frac{140(\cos(x)-1)}{\cos(x)+1}+28.1312524456150}-\frac{4.18450863968000\log\left(-\frac{\cos(x)-1}{\cos(x)+1}-0.44646269\right.}{\frac{140(\cos(x)-1)}{\cos(x)+1}+44.3876588494000}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(4*x)*sin(x),x, algorithm="giac")
```

```
[Out] -2.16139547686000*log(-(cos(x) - 1)/(cos(x) + 1) - 0.0395661298966000)/(140
*(cos(x) - 1)/(cos(x) + 1) + 28.1312524456150) - 4.18450863968000*log(-(cos
(x) - 1)/(cos(x) + 1) - 0.446462692172000)/(140*(cos(x) - 1)/(cos(x) + 1) +
 44.3876588494000) - 20.9929814212000*log(-(cos(x) - 1)/(cos(x) + 1) - 2.23
982880884000)/(140*(cos(x) - 1)/(cos(x) + 1) + 404.466590643000) - 1380.661
11446200*log(-(cos(x) - 1)/(cos(x) + 1) - 25.2741423691000)/(140*(cos(x) -
1)/(cos(x) + 1) - 10892.9855019000)
```

maple [A] time = 0.21, size = 54, normalized size = 0.76

$$-\frac{\sqrt{2}\ \operatorname{arctanh}\left(\frac{2\cos(x)}{\sqrt{2-\sqrt{2}}}\right)}{4\sqrt{2-\sqrt{2}}}+\frac{\sqrt{2}\ \operatorname{arctanh}\left(\frac{2\cos(x)}{\sqrt{2+\sqrt{2}}}\right)}{4\sqrt{2+\sqrt{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(4*x)*sin(x),x)
```

```
[Out] -1/4*2^(1/2)/(2-2^(1/2))^(1/2)*arctanh(2*cos(x)/(2-2^(1/2))^(1/2))+1/4*2^(1
/2)/(2+2^(1/2))^(1/2)*arctanh(2*cos(x)/(2+2^(1/2))^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sec(4x)\sin(x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(4*x)*sin(x),x, algorithm="maxima")
```

```
[Out] integrate(sec(4*x)*sin(x), x)
```

mupad [B] time = 0.09, size = 112, normalized size = 1.58

$$\frac{\operatorname{atanh}\left(\frac{\cos(x)\sqrt{2-\sqrt{2}}}{64\left(\frac{\sqrt{2}}{128}-\frac{1}{64}\right)}-\frac{\sqrt{2}\ \cos(x)\sqrt{2-\sqrt{2}}}{64\left(\frac{\sqrt{2}}{128}-\frac{1}{64}\right)}\right)\sqrt{2-\sqrt{2}}}{4}-\frac{\operatorname{atanh}\left(\frac{\cos(x)\sqrt{\sqrt{2}+2}}{64\left(\frac{\sqrt{2}}{128}+\frac{1}{64}\right)}+\frac{\sqrt{2}\ \cos(x)\sqrt{\sqrt{2}+2}}{64\left(\frac{\sqrt{2}}{128}+\frac{1}{64}\right)}\right)\sqrt{\sqrt{2}+2}}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)/cos(4*x),x)
```

```
[Out] (atanh((cos(x)*(2 - 2^(1/2))^(1/2))/(64*(2^(1/2)/128 - 1/64)) - (2^(1/2)*co
s(x)*(2 - 2^(1/2))^(1/2))/(64*(2^(1/2)/128 - 1/64)))*(2 - 2^(1/2))^(1/2))/4
 - (atanh((cos(x)*(2^(1/2) + 2)^(1/2))/(64*(2^(1/2)/128 + 1/64)) + (2^(1/2)
```

```
*cos(x)*(2^(1/2) + 2)^(1/2))/(64*(2^(1/2)/128 + 1/64)))*(2^(1/2) + 2)^(1/2)
)/4
```

sympy **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \sin(x)\sec(4x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(4*x)*sin(x),x)
```

```
[Out] Integral(sin(x)*sec(4*x), x)
```

3.88 $\int \sec(5x)\sin(x)\,dx$

Optimal. Leaf size=62

$$\frac{1}{20}\left(1+\sqrt{5}\right)\log\left(-8\cos^2(x)-\sqrt{5}+5\right)+\frac{1}{20}\left(1-\sqrt{5}\right)\log\left(-8\cos^2(x)+\sqrt{5}+5\right)-\frac{1}{5}\log(\cos(x))$$

```
[Out] -1/5*ln(cos(x))+1/20*ln(5-8*cos(x)^2+5^(1/2))*(-5^(1/2)+1)+1/20*ln(5-8*cos(
x)^2-5^(1/2))*(5^(1/2)+1)
```

Rubi [A] time = 0.07, antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.857, Rules used = {4357, 1114, 705, 29, 632, 31}

$$\frac{1}{20}\left(1+\sqrt{5}\right)\log\left(-8\cos^2(x)-\sqrt{5}+5\right)+\frac{1}{20}\left(1-\sqrt{5}\right)\log\left(-8\cos^2(x)+\sqrt{5}+5\right)-\frac{1}{5}\log(\cos(x))$$

Antiderivative was successfully verified.

```
[In] Int[Sec[5*x]*Sin[x],x]
```

```
[Out] -Log[Cos[x]]/5 + ((1 + Sqrt[5])*Log[5 - Sqrt[5] - 8*Cos[x]^2])/20 + ((1 - S
qrt[5])*Log[5 + Sqrt[5] - 8*Cos[x]^2])/20
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 632

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> W
ith[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(c*d - e*(b/2 - q/2))/q, Int[1/(b/2 - q/
2 + c*x), x], x] - Dist[(c*d - e*(b/2 + q/2))/q, Int[1/(b/2 + q/2 + c*x), x
], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a
*c, 0] && NiceSqrtQ[b^2 - 4*a*c]
```

Rule 705

```
Int[1/(((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)), x_Symbol]
 :> Dist[e^2/(c*d^2 - b*d*e + a*e^2), Int[1/(d + e*x), x], x] + Dist[1/(c*d
^2 - b*d*e + a*e^2), Int[(c*d - b*e - c*e*x)/(a + b*x + c*x^2), x], x] /; F
```

```
reeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^
2, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 1114

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 4357

```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{d = FreeFacto
rs[Cos[c*(a + b*x)], x]}, -Dist[d/(b*c), Subst[Int[SubstFor[1, Cos[c*(a + b
*x)]/d, u, x], x], x, Cos[c*(a + b*x)]/d], x] /; FunctionOfQ[Cos[c*(a + b*x
)]/d, u, x]] /; FreeQ[{a, b, c}, x] && (EqQ[F, Sin] || EqQ[F, sin])
```

Rubi steps

$$
\begin{aligned}
\int \sec(5x)\sin(x)\,dx &= -\text{Subst}\left(\int \frac{1}{x\left(5-20x^2+16x^4\right)}\,dx, x, \cos(x)\right)\\
&= -\left(\frac{1}{2}\text{Subst}\left(\int \frac{1}{x\left(5-20x+16x^2\right)}\,dx, x, \cos^2(x)\right)\right)\\
&= -\left(\frac{1}{10}\text{Subst}\left(\int \frac{1}{x}\,dx, x, \cos^2(x)\right)\right)-\frac{1}{10}\text{Subst}\left(\int \frac{20-16x}{5-20x+16x^2}\,dx, x, \cos^2(x)\right)\\
&= -\frac{1}{5}\log(\cos(x))+\frac{1}{5}\left(4\left(1-\sqrt{5}\right)\right)\text{Subst}\left(\int \frac{1}{-10-2\sqrt{5}+16x}\,dx, x, \cos^2(x)\right)+\frac{1}{5}\left(4\left(1+\right.\right.\\
&= -\frac{1}{5}\log(\cos(x))+\frac{1}{20}\left(1+\sqrt{5}\right)\log\left(5-\sqrt{5}-8\cos^2(x)\right)+\frac{1}{20}\left(1-\sqrt{5}\right)\log\left(5+\sqrt{5}-8\right.
\end{aligned}
$$

Mathematica [A] time = 0.10, size = 57, normalized size = 0.92

$$\frac{1}{20}\left(-4\log(\cos(x))-\left(\sqrt{5}-1\right)\log\left(4\cos(2x)-\sqrt{5}-1\right)+\left(1+\sqrt{5}\right)\log\left(4\cos(2x)+\sqrt{5}-1\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[5*x]*Sin[x],x]
```

```
[Out] (-4*Log[Cos[x]] - (-1 + Sqrt[5])*Log[-1 - Sqrt[5] + 4*Cos[2*x]] + (1 + Sqrt
[5])*Log[-1 + Sqrt[5] + 4*Cos[2*x]])/20
```

fricas [A] time = 0.69, size = 72, normalized size = 1.16

$$\frac{1}{20}\sqrt{5}\log\left(\frac{32\cos(x)^4+8\left(\sqrt{5}-5\right)\cos(x)^2-5\sqrt{5}+15}{16\cos(x)^4-20\cos(x)^2+5}\right)+\frac{1}{20}\log\left(16\cos(x)^4-20\cos(x)^2+5\right)-\frac{1}{5}\log\left(-\cos\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(5*x)*sin(x),x, algorithm="fricas")
```

```
[Out] 1/20*sqrt(5)*log((32*cos(x)^4 + 8*(sqrt(5) - 5)*cos(x)^2 - 5*sqrt(5) + 15)/
(16*cos(x)^4 - 20*cos(x)^2 + 5)) + 1/20*log(16*cos(x)^4 - 20*cos(x)^2 + 5)
- 1/5*log(-cos(x))
```

giac [A] time = 0.16, size = 67, normalized size = 1.08

$$\frac{1}{20}\sqrt{5}\log\left(\frac{\left|32\sin(x)^2-4\sqrt{5}-12\right|}{\left|32\sin(x)^2+4\sqrt{5}-12\right|}\right)-\frac{1}{10}\log\left(-\sin(x)^2+1\right)+\frac{1}{20}\log\left(\left|16\sin(x)^4-12\sin(x)^2+1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(5*x)*sin(x),x, algorithm="giac")
```

```
[Out] 1/20*sqrt(5)*log(abs(32*sin(x)^2 - 4*sqrt(5) - 12)/abs(32*sin(x)^2 + 4*sqrt
(5) - 12)) - 1/10*log(-sin(x)^2 + 1) + 1/20*log(abs(16*sin(x)^4 - 12*sin(x)
^2 + 1))
```

maple [A] time = 0.23, size = 43, normalized size = 0.69

$$\frac{\ln\left(16\left(\cos^4(x)\right)-20\left(\cos^2(x)\right)+5\right)}{20}+\frac{\sqrt{5}\ \operatorname{arctanh}\left(\frac{\left(32\left(\cos^2(x)\right)-20\right)\sqrt{5}}{20}\right)}{10}-\frac{\ln(\cos(x))}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(5*x)*sin(x),x)
```

```
[Out] 1/20*ln(16*cos(x)^4-20*cos(x)^2+5)+1/10*5^(1/2)*arctanh(1/20*(32*cos(x)^2-2
0)*5^(1/2))-1/5*ln(cos(x))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(5*x)*sin(x),x, algorithm="maxima")
```

```
[Out] 1/5*integrate(-(cos(4*x)*sin(8*x) - cos(8*x)*sin(4*x) + cos(3/2*arctan2(sin
(4*x), cos(4*x)))*sin(4*x) + cos(1/2*arctan2(sin(4*x), cos(4*x)))*sin(4*x)
- cos(4*x)*sin(3/2*arctan2(sin(4*x), cos(4*x))) - cos(4*x)*sin(1/2*arctan2(
sin(4*x), cos(4*x))) - sin(4*x))/(2*(cos(4*x) + 1)*cos(8*x) + cos(8*x)^2 +
cos(4*x)^2 - 2*(cos(8*x) + cos(4*x) - cos(1/2*arctan2(sin(4*x), cos(4*x)))
+ 1)*cos(3/2*arctan2(sin(4*x), cos(4*x))) + cos(3/2*arctan2(sin(4*x), cos(4
*x)))^2 - 2*(cos(8*x) + cos(4*x) + 1)*cos(1/2*arctan2(sin(4*x), cos(4*x)))
+ cos(1/2*arctan2(sin(4*x), cos(4*x)))^2 + sin(8*x)^2 + 2*sin(8*x)*sin(4*x)
 + sin(4*x)^2 - 2*(sin(8*x) + sin(4*x) - sin(1/2*arctan2(sin(4*x), cos(4*x)
)))*sin(3/2*arctan2(sin(4*x), cos(4*x))) + sin(3/2*arctan2(sin(4*x), cos(4*
x)))^2 - 2*(sin(8*x) + sin(4*x))*sin(1/2*arctan2(sin(4*x), cos(4*x))) + sin
(1/2*arctan2(sin(4*x), cos(4*x)))^2 + 2*cos(4*x) + 1), x) + 4/5*integrate(-
(cos(2*x)*sin(8*x) - cos(2*x)*sin(6*x) + cos(2*x)*sin(4*x) - cos(8*x)*sin(2
*x) + cos(6*x)*sin(2*x) - cos(4*x)*sin(2*x) - sin(2*x))/(2*(cos(6*x) - cos(
4*x) + cos(2*x) - 1)*cos(8*x) - cos(8*x)^2 + 2*(cos(4*x) - cos(2*x) + 1)*co
s(6*x) - cos(6*x)^2 + 2*(cos(2*x) - 1)*cos(4*x) - cos(4*x)^2 - cos(2*x)^2 +
 2*(sin(6*x) - sin(4*x) + sin(2*x))*sin(8*x) - sin(8*x)^2 + 2*(sin(4*x) - s
in(2*x))*sin(6*x) - sin(6*x)^2 - sin(4*x)^2 + 2*sin(4*x)*sin(2*x) - sin(2*x
)^2 + 2*cos(2*x) - 1), x) - 2/5*integrate(-(cos(4/3*arctan2(sin(6*x), cos(6
*x)))*sin(6*x) + cos(2/3*arctan2(sin(6*x), cos(6*x)))*sin(6*x) - cos(1/3*ar
ctan2(sin(6*x), cos(6*x)))*sin(6*x) - cos(6*x)*sin(4/3*arctan2(sin(6*x), co
s(6*x))) - cos(6*x)*sin(2/3*arctan2(sin(6*x), cos(6*x))) + cos(6*x)*sin(1/3
*arctan2(sin(6*x), cos(6*x))) + sin(6*x))/(cos(6*x)^2 - 2*(cos(6*x) - cos(2
/3*arctan2(sin(6*x), cos(6*x))) + cos(1/3*arctan2(sin(6*x), cos(6*x))) - 1)
*cos(4/3*arctan2(sin(6*x), cos(6*x))) + cos(4/3*arctan2(sin(6*x), cos(6*x))
)^2 - 2*(cos(6*x) + cos(1/3*arctan2(sin(6*x), cos(6*x))) - 1)*cos(2/3*arcta
n2(sin(6*x), cos(6*x))) + cos(2/3*arctan2(sin(6*x), cos(6*x)))^2 + 2*(cos(6
*x) - 1)*cos(1/3*arctan2(sin(6*x), cos(6*x))) + cos(1/3*arctan2(sin(6*x), c
os(6*x)))^2 + sin(6*x)^2 - 2*(sin(6*x) - sin(2/3*arctan2(sin(6*x), cos(6*x)
)) + sin(1/3*arctan2(sin(6*x), cos(6*x))))*sin(4/3*arctan2(sin(6*x), cos(6*
x))) + sin(4/3*arctan2(sin(6*x), cos(6*x)))^2 - 2*(sin(6*x) + sin(1/3*arcta
n2(sin(6*x), cos(6*x))))*sin(2/3*arctan2(sin(6*x), cos(6*x))) + sin(2/3*arc
tan2(sin(6*x), cos(6*x)))^2 + 2*sin(6*x)*sin(1/3*arctan2(sin(6*x), cos(6*x)
)) + sin(1/3*arctan2(sin(6*x), cos(6*x)))^2 - 2*cos(6*x) + 1), x) - 2/5*int
egrate(-(sin(8*x) - sin(6*x) + sin(4*x) - sin(2*x))/(2*(cos(6*x) - cos(4*x)
 + cos(2*x) - 1)*cos(8*x) - cos(8*x)^2 + 2*(cos(4*x) - cos(2*x) + 1)*cos(6*
x) - cos(6*x)^2 + 2*(cos(2*x) - 1)*cos(4*x) - cos(4*x)^2 - cos(2*x)^2 + 2*(
sin(6*x) - sin(4*x) + sin(2*x))*sin(8*x) - sin(8*x)^2 + 2*(sin(4*x) - sin(2
*x))*sin(6*x) - sin(6*x)^2 - sin(4*x)^2 + 2*sin(4*x)*sin(2*x) - sin(2*x)^2
+ 2*cos(2*x) - 1), x) - 1/10*log(cos(2*x)^2 + sin(2*x)^2 + 2*cos(2*x) + 1)
```

mupad [**B**] time = 0.54, size = 47, normalized size = 0.76

$$\ln\left(\cos(x)^2+\frac{\sqrt{5}}{8}-\frac{5}{8}\right)\left(\frac{\sqrt{5}}{20}+\frac{1}{20}\right)-\ln\left(\cos(x)^2-\frac{\sqrt{5}}{8}-\frac{5}{8}\right)\left(\frac{\sqrt{5}}{20}-\frac{1}{20}\right)-\frac{\ln(\cos(x))}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)/cos(5*x),x)
```

```
[Out] log(cos(x)^2 + 5^(1/2)/8 - 5/8)*(5^(1/2)/20 + 1/20) - log(cos(x)^2 - 5^(1/2
)/8 - 5/8)*(5^(1/2)/20 - 1/20) - log(cos(x))/5
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sin(x)\sec(5x)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(5*x)*sin(x),x)
```

```
[Out] Integral(sin(x)*sec(5*x), x)
```

3.89 $\int \sec(6x)\sin(x)\,dx$

Optimal. Leaf size=85

$$-\frac{\tanh^{-1}\left(\sqrt{2}\cos(x)\right)}{3\sqrt{2}}+\frac{\tanh^{-1}\left(\frac{2\cos(x)}{\sqrt{2-\sqrt{3}}}\right)}{6\sqrt{2-\sqrt{3}}}+\frac{\tanh^{-1}\left(\frac{2\cos(x)}{\sqrt{2+\sqrt{3}}}\right)}{6\sqrt{2+\sqrt{3}}}$$

```
[Out] -1/6*arctanh(cos(x)*2^(1/2))*2^(1/2)+1/6*arctanh(2*cos(x)/(1/2*6^(1/2)-1/2*
2^(1/2)))/(1/2*6^(1/2)-1/2*2^(1/2))+1/6*arctanh(2*cos(x)/(1/2*6^(1/2)+1/2*2
^(1/2)))/(1/2*6^(1/2)+1/2*2^(1/2))
```

Rubi [A] time = 0.06, antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.571, Rules used = {4357, 2057, 207, 1166}

$$-\frac{\tanh^{-1}\left(\sqrt{2}\cos(x)\right)}{3\sqrt{2}}+\frac{\tanh^{-1}\left(\frac{2\cos(x)}{\sqrt{2-\sqrt{3}}}\right)}{6\sqrt{2-\sqrt{3}}}+\frac{\tanh^{-1}\left(\frac{2\cos(x)}{\sqrt{2+\sqrt{3}}}\right)}{6\sqrt{2+\sqrt{3}}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[6*x]*Sin[x],x]
```

```
[Out] -ArcTanh[Sqrt[2]*Cos[x]]/(3*Sqrt[2]) + ArcTanh[(2*Cos[x])/Sqrt[2 - Sqrt[3]]
]/(6*Sqrt[2 - Sqrt[3]]) + ArcTanh[(2*Cos[x])/Sqrt[2 + Sqrt[3]]]/(6*Sqrt[2 +
 Sqrt[3]])
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 2057

```
Int[(P_)^(p_), x_Symbol] :> With[{u = Factor[P /. x -> Sqrt[x]]}, Int[Expan
dIntegrand[(u /. x -> x^2)^p, x], x] /;  !SumQ[NonfreeFactors[u, x]]] /; Po
lyQ[P, x^2] && ILtQ[p, 0]
```

Rule 4357

```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{d = FreeFacto
rs[Cos[c*(a + b*x)], x]}, -Dist[d/(b*c), Subst[Int[SubstFor[1, Cos[c*(a + b
*x)]/d, u, x], x], x, Cos[c*(a + b*x)]/d], x] /; FunctionOfQ[Cos[c*(a + b*x
)]/d, u, x]] /; FreeQ[{a, b, c}, x] && (EqQ[F, Sin] || EqQ[F, sin])
```

Rubi steps

$$\begin{aligned}
\int \sec(6x)\sin(x)\,dx &= -\text{Subst}\left(\int \frac{1}{-1+18x^2-48x^4+32x^6}\,dx, x, \cos(x)\right)\\
&= -\text{Subst}\left(\int \left(-\frac{1}{3\left(-1+2x^2\right)}+\frac{4\left(-1+2x^2\right)}{3\left(1-16x^2+16x^4\right)}\right)dx, x, \cos(x)\right)\\
&= \frac{1}{3}\text{Subst}\left(\int \frac{1}{-1+2x^2}\,dx, x, \cos(x)\right)-\frac{4}{3}\text{Subst}\left(\int \frac{-1+2x^2}{1-16x^2+16x^4}\,dx, x, \cos(x)\right)\\
&= -\frac{\tanh^{-1}\left(\sqrt{2}\cos(x)\right)}{3\sqrt{2}}-\frac{4}{3}\text{Subst}\left(\int \frac{1}{-8-4\sqrt{3}+16x^2}\,dx, x, \cos(x)\right)-\frac{4}{3}\text{Subst}\left(\int \frac{}{-8+
\right.\\
&= -\frac{\tanh^{-1}\left(\sqrt{2}\cos(x)\right)}{3\sqrt{2}}+\frac{\tanh^{-1}\left(\frac{2\cos(x)}{\sqrt{2-\sqrt{3}}}\right)}{6\sqrt{2-\sqrt{3}}}+\frac{\tanh^{-1}\left(\frac{2\cos(x)}{\sqrt{2+\sqrt{3}}}\right)}{6\sqrt{2+\sqrt{3}}}
\end{aligned}$$

Mathematica [C] time = 9.33, size = 627, normalized size = 7.38

$$\frac{1}{24}\left((-4-4i)(-1)^{3/4}\tanh^{-1}\left(\frac{\tan\left(\frac{x}{2}\right)-1}{\sqrt{2}}\right)-(4-4i)\sqrt[4]{-1}\tanh^{-1}\left(\frac{\tan\left(\frac{x}{2}\right)+1}{\sqrt{2}}\right)+\frac{2\left(1+\sqrt{2}\right)\left(x-\log\left(\sec^2\left(\frac{x}{2}\right)\right)\right.}{}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sec[6*x]*Sin[x],x]
```

```
[Out] ((-4 - 4*I)*(-1)^(3/4)*ArcTanh[(-1 + Tan[x/2])/Sqrt[2]] - (4 - 4*I)*(-1)^(1
/4)*ArcTanh[(1 + Tan[x/2])/Sqrt[2]] + (2*(1 + Sqrt[2])*(x + 2*Sqrt[3]*ArcTa
```

```
nh[(2 + (2 + Sqrt[2])*Tan[x/2])/Sqrt[6]] - Log[Sec[x/2]^2] + Log[-(Sec[x/2]
^2*(Sqrt[2] - 2*Cos[x] + 2*Sin[x]))]))/(2 + Sqrt[2]) - Sqrt[2]*(x - 2*Sqrt[
3]*ArcTanh[(Sqrt[2] + (-1 + Sqrt[2])*Tan[x/2])/Sqrt[3]] - Log[Sec[x/2]^2] +
 Log[Sec[x/2]^2*(1 + Sqrt[2]*Cos[x] - Sqrt[2]*Sin[x])]) + (2*(2*(Sqrt[2] +
Sqrt[3])*ArcTanh[(2 + (2 + Sqrt[6])*Tan[x/2])/Sqrt[2]] + (3 + Sqrt[6])*(x -
 Log[Sec[x/2]^2] + Log[-(Sec[x/2]^2*(Sqrt[6] - 2*Cos[x] + 2*Sin[x]))]))*(1
+ Sqrt[6]*Sin[x])*(3 + Sqrt[6] - (2 + Sqrt[6])*Cos[x] + (2 + Sqrt[6])*Sin[x
]))/((12 + 5*Sqrt[6])*Cos[2*x] + 2*Cos[x]*(5 + 2*Sqrt[6] + 5*Sqrt[6]*Sin[x]
) - 2*(12 + 5*Sqrt[6] + 4*(5 + 2*Sqrt[6])*Sin[x] - 6*Sin[2*x])) + ((-2*(-2
+ Sqrt[6])*ArcTanh[Sqrt[2] + (Sqrt[2] - Sqrt[3])*Tan[x/2]] + (3*Sqrt[2] - 2
*Sqrt[3])*(x - Log[Sec[x/2]^2] + Log[-(Sec[x/2]^2*(Sqrt[3] + Sqrt[2]*Cos[x]
 - Sqrt[2]*Sin[x]))]))*(Sqrt[2] - 2*Sqrt[3]*Sin[x])*(-3 + Sqrt[6] - (-2 + S
qrt[6])*Cos[x] + (-2 + Sqrt[6])*Sin[x]))/((-12 + 5*Sqrt[6])*Cos[2*x] + 2*Co
s[x]*(-5 + 2*Sqrt[6] + 5*Sqrt[6]*Sin[x]) - 2*(-12 + 5*Sqrt[6] + 4*(-5 + 2*S
qrt[6])*Sin[x] + 6*Sin[2*x])))/24
```

fricas [B] time = 1.05, size = 153, normalized size = 1.80

$$-\frac{1}{12}\sqrt{\sqrt{3}+2}\log\left(\sqrt{\sqrt{3}+2}\left(\sqrt{3}-2\right)+2\cos(x)\right)+\frac{1}{12}\sqrt{\sqrt{3}+2}\log\left(\sqrt{\sqrt{3}+2}\left(\sqrt{3}-2\right)-2\cos(x)\right)+\frac{1}{12}\sqrt{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(6*x)*sin(x),x, algorithm="fricas")
```

```
[Out] -1/12*sqrt(sqrt(3) + 2)*log(sqrt(sqrt(3) + 2)*(sqrt(3) - 2) + 2*cos(x)) + 1
/12*sqrt(sqrt(3) + 2)*log(sqrt(sqrt(3) + 2)*(sqrt(3) - 2) - 2*cos(x)) + 1/1
2*sqrt(-sqrt(3) + 2)*log((sqrt(3) + 2)*sqrt(-sqrt(3) + 2) + 2*cos(x)) - 1/1
2*sqrt(-sqrt(3) + 2)*log((sqrt(3) + 2)*sqrt(-sqrt(3) + 2) - 2*cos(x)) + 1/1
2*sqrt(2)*log((2*cos(x)^2 - 2*sqrt(2)*cos(x) + 1)/(2*cos(x)^2 - 1))
```

giac [B] time = 0.16, size = 182, normalized size = 2.14

$$-\frac{1}{12}\sqrt{2}\log\left(\frac{\left|-4\sqrt{2}-\frac{2(\cos(x)-1)}{\cos(x)+1}-6\right|}{\left|4\sqrt{2}-\frac{2(\cos(x)-1)}{\cos(x)+1}-6\right|}\right)-\frac{2.39014968180000\log\left(-\frac{\cos(x)-1}{\cos(x)+1}-0.0173323801210000\right)}{\frac{268(\cos(x)-1)}{\cos(x)+1}+60.0540532247402}+\frac{5.829519}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(6*x)*sin(x),x, algorithm="giac")
```

```
[Out] -1/12*sqrt(2)*log(abs(-4*sqrt(2) - 2*(cos(x) - 1)/(cos(x) + 1) - 6)/abs(4*s
qrt(2) - 2*(cos(x) - 1)/(cos(x) + 1) - 6)) - 2.39014968180000*log(-(cos(x)
- 1)/(cos(x) + 1) - 0.0173323801210000)/(268*(cos(x) - 1)/(cos(x) + 1) + 60
.0540532247402) + 5.82951931426000*log(-(cos(x) - 1)/(cos(x) + 1) - 0.58879
0706481000)/(268*(cos(x) - 1)/(cos(x) + 1) + 121.584934401100) + 16.8155413
```

```
244667*log(-(cos(x) - 1)/(cos(x) + 1) - 1.69839637242000)/(268*(cos(x) - 1)
/(cos(x) + 1) + 559.622604171000) - 7956.25491093333*log(-(cos(x) - 1)/(cos
(x) + 1) - 57.6954805410000)/(268*(cos(x) - 1)/(cos(x) + 1) - 168981.261592
000)
```

maple [A] time = 0.24, size = 80, normalized size = 0.94

$$\frac{2\,\mathrm{arctanh}\left(\frac{8\cos(x)}{2\sqrt{6}-2\sqrt{2}}\right)}{3\left(2\sqrt{6}-2\sqrt{2}\right)}+\frac{2\,\mathrm{arctanh}\left(\frac{8\cos(x)}{2\sqrt{6}+2\sqrt{2}}\right)}{3\left(2\sqrt{6}+2\sqrt{2}\right)}-\frac{\mathrm{arctanh}\left(\cos(x)\sqrt{2}\right)\sqrt{2}}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(6*x)*sin(x),x)
```

```
[Out] 2/3/(2*6^(1/2)-2*2^(1/2))*arctanh(8*cos(x)/(2*6^(1/2)-2*2^(1/2)))+2/3/(2*6^
(1/2)+2*2^(1/2))*arctanh(8*cos(x)/(2*6^(1/2)+2*2^(1/2)))-1/6*arctanh(cos(x)
*2^(1/2))*2^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{1}{24}\sqrt{2}\,\log\left(2\sqrt{2}\,\sin\left(2\,x\right)\sin(x)+2\left(\sqrt{2}\,\cos(x)+1\right)\cos\left(2\,x\right)+\cos\left(2\,x\right)^{2}+2\,\cos(x)^{2}+\sin\left(2\,x\right)^{2}+2\,\sin(x)^{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(6*x)*sin(x),x, algorithm="maxima")
```

```
[Out] -1/24*sqrt(2)*log(2*sqrt(2)*sin(2*x)*sin(x) + 2*(sqrt(2)*cos(x) + 1)*cos(2*
x) + cos(2*x)^2 + 2*cos(x)^2 + sin(2*x)^2 + 2*sin(x)^2 + 2*sqrt(2)*cos(x) +
 1) + 1/24*sqrt(2)*log(-2*sqrt(2)*sin(2*x)*sin(x) - 2*(sqrt(2)*cos(x) - 1)*
cos(2*x) + cos(2*x)^2 + 2*cos(x)^2 + sin(2*x)^2 + 2*sin(x)^2 - 2*sqrt(2)*co
s(x) + 1) - integrate(1/3*((sin(7*x) - sin(5*x) + sin(3*x) - sin(x))*cos(8*
x) - (sin(3*x) - sin(x))*cos(4*x) - (cos(7*x) - cos(5*x) + cos(3*x) - cos(x
))*sin(8*x) - (cos(4*x) - 1)*sin(7*x) + (cos(4*x) - 1)*sin(5*x) + (cos(3*x)
 - cos(x))*sin(4*x) + cos(7*x)*sin(4*x) - cos(5*x)*sin(4*x) + sin(3*x) - si
n(x))/(2*(cos(4*x) - 1)*cos(8*x) - cos(8*x)^2 - cos(4*x)^2 - sin(8*x)^2 + 2
*sin(8*x)*sin(4*x) - sin(4*x)^2 + 2*cos(4*x) - 1), x)
```

mupad [B] time = 2.28, size = 118, normalized size = 1.39

$$\mathrm{atanh}\left(\frac{5\sqrt{2}\,\cos(x)}{2097152\left(\frac{\sqrt{2}\,\sqrt{6}}{4194304}+\frac{1}{1048576}\right)}+\frac{3\sqrt{6}\,\cos(x)}{2097152\left(\frac{\sqrt{2}\,\sqrt{6}}{4194304}+\frac{1}{1048576}\right)}\right)\left(\frac{\sqrt{2}}{12}+\frac{\sqrt{6}}{12}\right)-\mathrm{atanh}\left(\frac{5\sqrt{2}\,\cos(x)}{2097152\left(\frac{\sqrt{2}\,\sqrt{6}}{4194304}-\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)/cos(6*x),x)
```

```
[Out] atanh((5*2^(1/2)*cos(x))/(2097152*((2^(1/2)*6^(1/2))/4194304 + 1/1048576))
+ (3*6^(1/2)*cos(x))/(2097152*((2^(1/2)*6^(1/2))/4194304 + 1/1048576)))*(2^
(1/2)/12 + 6^(1/2)/12) - atanh((5*2^(1/2)*cos(x))/(2097152*((2^(1/2)*6^(1/2
))/4194304 - 1/1048576)) - (3*6^(1/2)*cos(x))/(2097152*((2^(1/2)*6^(1/2))/4
194304 - 1/1048576)))*(2^(1/2)/12 - 6^(1/2)/12) - (2^(1/2)*atanh(2^(1/2)*co
s(x)))/6
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sin(x)\sec(6x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(6*x)*sin(x),x)
```

```
[Out] Integral(sin(x)*sec(6*x), x)
```

3.90 $\int \csc(2x)\sin(x)\,dx$

Optimal. Leaf size=7

$$\frac{1}{2}\tanh^{-1}(\sin(x))$$

```
[Out] 1/2*arctanh(sin(x))
```

Rubi [A] time = 0.01, antiderivative size = 7, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {4288, 3770}

$$\frac{1}{2}\tanh^{-1}(\sin(x))$$

Antiderivative was successfully verified.

```
[In] Int[Csc[2*x]*Sin[x],x]
```

```
[Out] ArcTanh[Sin[x]]/2
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 4288

```
Int[((f_.)*sin[(a_.) + (b_.)*(x_)])^(n_.)*sin[(c_.) + (d_.)*(x_)]^(p_.), x_
Symbol] :> Dist[2^p/f^p, Int[Cos[a + b*x]^p*(f*Sin[a + b*x])^(n + p), x], x
] /; FreeQ[{a, b, c, d, f, n}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && In
tegerQ[p]
```

Rubi steps

$$\begin{aligned}\int \csc(2x)\sin(x)\,dx &= \frac{1}{2}\int \sec(x)\,dx\\ &= \frac{1}{2}\tanh^{-1}(\sin(x))\end{aligned}$$

Mathematica [B] time = 0.01, size = 37, normalized size = 5.29

$$\frac{1}{2}\left(\log\left(\sin\left(\frac{x}{2}\right)+\cos\left(\frac{x}{2}\right)\right)-\log\left(\cos\left(\frac{x}{2}\right)-\sin\left(\frac{x}{2}\right)\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[2*x]*Sin[x],x]
```

```
[Out] (-Log[Cos[x/2] - Sin[x/2]] + Log[Cos[x/2] + Sin[x/2]])/2
```

fricas [B] time = 2.74, size = 17, normalized size = 2.43

$$\frac{1}{4}\log\left(\sin(x)+1\right)-\frac{1}{4}\log\left(-\sin(x)+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(2*x)*sin(x),x, algorithm="fricas")
```

```
[Out] 1/4*log(sin(x) + 1) - 1/4*log(-sin(x) + 1)
```

giac [B] time = 0.14, size = 17, normalized size = 2.43

$$\frac{1}{4}\log\left(\sin(x)+1\right)-\frac{1}{4}\log\left(-\sin(x)+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(2*x)*sin(x),x, algorithm="giac")
```

```
[Out] 1/4*log(sin(x) + 1) - 1/4*log(-sin(x) + 1)
```

maple [A] time = 0.08, size = 9, normalized size = 1.29

$$\frac{\ln\left(\sec(x)+\tan(x)\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(2*x)*sin(x),x)
```

```
[Out] 1/2*ln(sec(x)+tan(x))
```

maxima [B] time = 0.42, size = 35, normalized size = 5.00

$$\frac{1}{4}\log\left(\cos(x)^2+\sin(x)^2+2\sin(x)+1\right)-\frac{1}{4}\log\left(\cos(x)^2+\sin(x)^2-2\sin(x)+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(2*x)*sin(x),x, algorithm="maxima")
```

```
[Out] 1/4*log(cos(x)^2 + sin(x)^2 + 2*sin(x) + 1) - 1/4*log(cos(x)^2 + sin(x)^2 -
 2*sin(x) + 1)
```

mupad [B] time = 0.11, size = 5, normalized size = 0.71

$$\frac{\operatorname{atanh}(\sin(x))}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)/sin(2*x),x)
```

```
[Out] atanh(sin(x))/2
```

sympy [B] time = 0.85, size = 15, normalized size = 2.14

$$-\frac{\log(\sin(x)-1)}{4}+\frac{\log(\sin(x)+1)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(2*x)*sin(x),x)
```

```
[Out] -log(sin(x) - 1)/4 + log(sin(x) + 1)/4
```

3.91 $\int \csc(3x)\sin(x)\,dx$

Optimal. Leaf size=45

$$\frac{\log\left(\sin(x)+\sqrt{3}\cos(x)\right)}{2\sqrt{3}}-\frac{\log\left(\sqrt{3}\cos(x)-\sin(x)\right)}{2\sqrt{3}}$$

```
[Out] -1/6*ln(-sin(x)+cos(x)*3^(1/2))*3^(1/2)+1/6*ln(sin(x)+cos(x)*3^(1/2))*3^(1/
2)
```

Rubi [A] time = 0.04, antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {206}

$$\frac{\log\left(\sin(x)+\sqrt{3}\cos(x)\right)}{2\sqrt{3}}-\frac{\log\left(\sqrt{3}\cos(x)-\sin(x)\right)}{2\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[3*x]*Sin[x],x]
```

```
[Out] -Log[Sqrt[3]*Cos[x] - Sin[x]]/(2*Sqrt[3]) + Log[Sqrt[3]*Cos[x] + Sin[x]]/(2
*Sqrt[3])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \csc(3x)\sin(x)\,dx &= \text{Subst}\left(\int \frac{1}{3-x^2}\,dx, x, \tan(x)\right)\\ &= -\frac{\log\left(\sqrt{3}\cos(x)-\sin(x)\right)}{2\sqrt{3}}+\frac{\log\left(\sqrt{3}\cos(x)+\sin(x)\right)}{2\sqrt{3}}\end{aligned}$$

Mathematica [A] time = 0.02, size = 15, normalized size = 0.33

$$\frac{\tanh^{-1}\left(\frac{\tan(x)}{\sqrt{3}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[3*x]*Sin[x],x]
```

```
[Out] ArcTanh[Tan[x]/Sqrt[3]]/Sqrt[3]
```

fricas [A] time = 1.09, size = 58, normalized size = 1.29

$$\frac{1}{12}\sqrt{3}\log\left(-\frac{8\cos(x)^4-16\cos(x)^2-4\left(2\sqrt{3}\cos(x)^3+\sqrt{3}\cos(x)\right)\sin(x)-1}{16\cos(x)^4-8\cos(x)^2+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(3*x)*sin(x),x, algorithm="fricas")
```

```
[Out] 1/12*sqrt(3)*log(-(8*cos(x)^4 - 16*cos(x)^2 - 4*(2*sqrt(3)*cos(x)^3 + sqrt(
3)*cos(x))*sin(x) - 1)/(16*cos(x)^4 - 8*cos(x)^2 + 1))
```

giac [A] time = 0.18, size = 31, normalized size = 0.69

$$-\frac{1}{6}\sqrt{3}\log\left(\frac{\left|-2\sqrt{3}+2\tan(x)\right|}{\left|2\sqrt{3}+2\tan(x)\right|}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(3*x)*sin(x),x, algorithm="giac")
```

```
[Out] -1/6*sqrt(3)*log(abs(-2*sqrt(3) + 2*tan(x))/abs(2*sqrt(3) + 2*tan(x)))
```

maple [A] time = 0.20, size = 14, normalized size = 0.31

$$\frac{\sqrt{3}\,\operatorname{arctanh}\left(\frac{\tan(x)\sqrt{3}}{3}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(3*x)*sin(x),x)
```

```
[Out] 1/3*3^(1/2)*arctanh(1/3*tan(x)*3^(1/2))
```

maxima [B] time = 0.44, size = 125, normalized size = 2.78

$$-\frac{1}{12}\sqrt{3}\log\left(\frac{4}{3}\cos(x)^2+\frac{4}{3}\sin(x)^2+\frac{4}{3}\sqrt{3}\sin(x)+\frac{4}{3}\cos(x)+\frac{4}{3}\right)+\frac{1}{12}\sqrt{3}\log\left(\frac{4}{3}\cos(x)^2+\frac{4}{3}\sin(x)^2+\frac{4}{3}\sqrt{3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(3*x)*sin(x),x, algorithm="maxima")
```

```
[Out] -1/12*sqrt(3)*log(4/3*cos(x)^2 + 4/3*sin(x)^2 + 4/3*sqrt(3)*sin(x) + 4/3*co
s(x) + 4/3) + 1/12*sqrt(3)*log(4/3*cos(x)^2 + 4/3*sin(x)^2 + 4/3*sqrt(3)*si
n(x) - 4/3*cos(x) + 4/3) + 1/12*sqrt(3)*log(4/3*cos(x)^2 + 4/3*sin(x)^2 - 4
/3*sqrt(3)*sin(x) + 4/3*cos(x) + 4/3) - 1/12*sqrt(3)*log(4/3*cos(x)^2 + 4/3
*sin(x)^2 - 4/3*sqrt(3)*sin(x) - 4/3*cos(x) + 4/3)
```

mupad [B] time = 2.79, size = 17, normalized size = 0.38

$$\frac{\sqrt{3}\ \mathrm{atanh}\left(\frac{\sqrt{3}\ \sin(x)}{3\ \cos(x)}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)/sin(3*x),x)
```

```
[Out] (3^(1/2)*atanh((3^(1/2)*sin(x))/(3*cos(x))))/3
```

sympy [A] time = 1.87, size = 76, normalized size = 1.69

$$\frac{\sqrt{3}\ \log\left(\tan\left(\frac{x}{2}\right)-\sqrt{3}\right)}{6}-\frac{\sqrt{3}\ \log\left(\tan\left(\frac{x}{2}\right)-\frac{\sqrt{3}}{3}\right)}{6}+\frac{\sqrt{3}\ \log\left(\tan\left(\frac{x}{2}\right)+\frac{\sqrt{3}}{3}\right)}{6}-\frac{\sqrt{3}\ \log\left(\tan\left(\frac{x}{2}\right)+\sqrt{3}\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(3*x)*sin(x),x)
```

```
[Out] sqrt(3)*log(tan(x/2) - sqrt(3))/6 - sqrt(3)*log(tan(x/2) - sqrt(3)/3)/6 + s
qrt(3)*log(tan(x/2) + sqrt(3)/3)/6 - sqrt(3)*log(tan(x/2) + sqrt(3))/6
```

3.92 $\int \csc(4x)\sin(x)\,dx$

Optimal. Leaf size=26

$$\frac{\tanh^{-1}\left(\sqrt{2}\sin(x)\right)}{2\sqrt{2}}-\frac{1}{4}\tanh^{-1}(\sin(x))$$

```
[Out] -1/4*arctanh(sin(x))+1/4*arctanh(sin(x)*2^(1/2))*2^(1/2)
```

Rubi [A] time = 0.03, antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {1093, 207}

$$\frac{\tanh^{-1}\left(\sqrt{2}\sin(x)\right)}{2\sqrt{2}}-\frac{1}{4}\tanh^{-1}(\sin(x))$$

Antiderivative was successfully verified.

```
[In] Int[Csc[4*x]*Sin[x],x]
```

```
[Out] -ArcTanh[Sin[x]]/4 + ArcTanh[Sqrt[2]*Sin[x]]/(2*Sqrt[2])
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 1093

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(-1), x_Symbol] :> With[{q = Rt[b^
2 - 4*a*c, 2]}, Dist[c/q, Int[1/(b/2 - q/2 + c*x^2), x], x] - Dist[c/q, Int
[1/(b/2 + q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c,
0] && PosQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \csc(4x)\sin(x)\,dx = \text{Subst}\left(\int \frac{1}{4-12x^2+8x^4}\,dx, x, \sin(x)\right)$$
$$= 2\,\text{Subst}\left(\int \frac{1}{-8+8x^2}\,dx, x, \sin(x)\right) - 2\,\text{Subst}\left(\int \frac{1}{-4+8x^2}\,dx, x, \sin(x)\right)$$
$$= -\frac{1}{4}\tanh^{-1}(\sin(x)) + \frac{\tanh^{-1}\left(\sqrt{2}\sin(x)\right)}{2\sqrt{2}}$$

Mathematica [A] time = 0.02, size = 26, normalized size = 1.00

$$\frac{\tanh^{-1}\left(\sqrt{2}\sin(x)\right)}{2\sqrt{2}} - \frac{1}{4}\tanh^{-1}(\sin(x))$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[4*x]*Sin[x],x]
[Out] -1/4*ArcTanh[Sin[x]] + ArcTanh[Sqrt[2]*Sin[x]]/(2*Sqrt[2])
```

fricas [B] time = 1.97, size = 50, normalized size = 1.92

$$\frac{1}{8}\sqrt{2}\log\left(-\frac{2\cos(x)^2 - 2\sqrt{2}\sin(x) - 3}{2\cos(x)^2 - 1}\right) - \frac{1}{8}\log(\sin(x)+1) + \frac{1}{8}\log(-\sin(x)+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(4*x)*sin(x),x, algorithm="fricas")
[Out] 1/8*sqrt(2)*log(-(2*cos(x)^2 - 2*sqrt(2)*sin(x) - 3)/(2*cos(x)^2 - 1)) - 1/
8*log(sin(x) + 1) + 1/8*log(-sin(x) + 1)
```

giac [B] time = 0.14, size = 48, normalized size = 1.85

$$-\frac{1}{8}\sqrt{2}\log\left(\frac{\left|-2\sqrt{2} + 4\sin(x)\right|}{\left|2\sqrt{2} + 4\sin(x)\right|}\right) - \frac{1}{8}\log(\sin(x)+1) + \frac{1}{8}\log(-\sin(x)+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(4*x)*sin(x),x, algorithm="giac")
[Out] -1/8*sqrt(2)*log(abs(-2*sqrt(2) + 4*sin(x))/abs(2*sqrt(2) + 4*sin(x))) - 1/
8*log(sin(x) + 1) + 1/8*log(-sin(x) + 1)
```

maple [A] time = 0.23, size = 28, normalized size = 1.08

$$\frac{\ln\left(\sin(x)-1\right)}{8}+\frac{\operatorname{arctanh}\left(\sin(x)\sqrt{2}\right)\sqrt{2}}{4}-\frac{\ln\left(1+\sin(x)\right)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(4*x)*sin(x),x)
```

```
[Out] 1/8*ln(sin(x)-1)+1/4*arctanh(sin(x)*2^(1/2))*2^(1/2)-1/8*ln(1+sin(x))
```

maxima [B] time = 0.43, size = 171, normalized size = 6.58

$$\frac{1}{16}\sqrt{2}\log\left(2\cos(x)^2+2\sin(x)^2+2\sqrt{2}\cos(x)+2\sqrt{2}\sin(x)+2\right)-\frac{1}{16}\sqrt{2}\log\left(2\cos(x)^2+2\sin(x)^2+2\sqrt{2}\,c\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(4*x)*sin(x),x, algorithm="maxima")
```

```
[Out] 1/16*sqrt(2)*log(2*cos(x)^2 + 2*sin(x)^2 + 2*sqrt(2)*cos(x) + 2*sqrt(2)*sin
(x) + 2) - 1/16*sqrt(2)*log(2*cos(x)^2 + 2*sin(x)^2 + 2*sqrt(2)*cos(x) - 2*
sqrt(2)*sin(x) + 2) + 1/16*sqrt(2)*log(2*cos(x)^2 + 2*sin(x)^2 - 2*sqrt(2)*
cos(x) + 2*sqrt(2)*sin(x) + 2) - 1/16*sqrt(2)*log(2*cos(x)^2 + 2*sin(x)^2 -
 2*sqrt(2)*cos(x) - 2*sqrt(2)*sin(x) + 2) - 1/8*log(cos(x)^2 + sin(x)^2 + 2
*sin(x) + 1) + 1/8*log(cos(x)^2 + sin(x)^2 - 2*sin(x) + 1)
```

mupad [B] time = 2.43, size = 27, normalized size = 1.04

$$\frac{\sqrt{2}\,\operatorname{atanh}\left(\sqrt{2}\,\sin(x)\right)}{4}-\frac{\operatorname{atanh}\left(\frac{\sin\left(\frac{x}{2}\right)}{\cos\left(\frac{x}{2}\right)}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)/sin(4*x),x)
```

```
[Out] (2^(1/2)*atanh(2^(1/2)*sin(x)))/4 - atanh(sin(x/2)/cos(x/2))/2
```

sympy [B] time = 7.36, size = 294, normalized size = 11.31

$$\frac{27720\sqrt{2}\log\left(\tan\left(\frac{x}{2}\right)-1\right)}{110880\sqrt{2}+156808}+\frac{39202\log\left(\tan\left(\frac{x}{2}\right)-1\right)}{110880\sqrt{2}+156808}-\frac{39202\log\left(\tan\left(\frac{x}{2}\right)+1\right)}{110880\sqrt{2}+156808}-\frac{27720\sqrt{2}\log\left(\tan\left(\frac{x}{2}\right)+1\right)}{110880\sqrt{2}+156808}+\frac{27}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(4*x)*sin(x),x)
```

```
[Out] 27720*sqrt(2)*log(tan(x/2) - 1)/(110880*sqrt(2) + 156808) + 39202*log(tan(x
/2) - 1)/(110880*sqrt(2) + 156808) - 39202*log(tan(x/2) + 1)/(110880*sqrt(2
) + 156808) - 27720*sqrt(2)*log(tan(x/2) + 1)/(110880*sqrt(2) + 156808) + 2
7720*log(tan(x/2) - 1 + sqrt(2))/(110880*sqrt(2) + 156808) + 19601*sqrt(2)*
log(tan(x/2) - 1 + sqrt(2))/(110880*sqrt(2) + 156808) + 27720*log(tan(x/2)
+ 1 + sqrt(2))/(110880*sqrt(2) + 156808) + 19601*sqrt(2)*log(tan(x/2) + 1 +
 sqrt(2))/(110880*sqrt(2) + 156808) - 19601*sqrt(2)*log(tan(x/2) - sqrt(2)
- 1)/(110880*sqrt(2) + 156808) - 27720*log(tan(x/2) - sqrt(2) - 1)/(110880*
sqrt(2) + 156808) - 19601*sqrt(2)*log(tan(x/2) - sqrt(2) + 1)/(110880*sqrt(
2) + 156808) - 27720*log(tan(x/2) - sqrt(2) + 1)/(110880*sqrt(2) + 156808)
```

3.93 $\int \csc(5x)\sin(x)\,dx$

Optimal. Leaf size=165

$$-\frac{1}{10}\sqrt{\frac{1}{2}\left(5-\sqrt{5}\right)}\log\left(\sqrt{5-2\sqrt{5}}\cos(x)-\sin(x)\right)+\frac{1}{10}\sqrt{\frac{1}{2}\left(5+\sqrt{5}\right)}\log\left(\sqrt{5+2\sqrt{5}}\cos(x)-\sin(x)\right)+\frac{1}{10}\sqrt{\frac{1}{2}}$$

```
[Out] -1/20*ln(-sin(x)+cos(x)*(5-2*5^(1/2))^(1/2))*(10-2*5^(1/2))^(1/2)+1/20*ln(s
in(x)+cos(x)*(5-2*5^(1/2))^(1/2))*(10-2*5^(1/2))^(1/2)+1/20*ln(-sin(x)+cos(
x)*(5+2*5^(1/2))^(1/2))*(10+2*5^(1/2))^(1/2)-1/20*ln(sin(x)+cos(x)*(5+2*5^(
1/2))^(1/2))*(10+2*5^(1/2))^(1/2)
```

Rubi [A] time = 0.14, antiderivative size = 165, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {1166, 207}

$$-\frac{1}{10}\sqrt{\frac{1}{2}\left(5-\sqrt{5}\right)}\log\left(\sqrt{5-2\sqrt{5}}\cos(x)-\sin(x)\right)+\frac{1}{10}\sqrt{\frac{1}{2}\left(5+\sqrt{5}\right)}\log\left(\sqrt{5+2\sqrt{5}}\cos(x)-\sin(x)\right)+\frac{1}{10}\sqrt{\frac{1}{2}}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[5*x]*Sin[x],x]
```

```
[Out] -(Sqrt[(5 - Sqrt[5])/2]*Log[Sqrt[5 - 2*Sqrt[5]]*Cos[x] - Sin[x]])/10 + (Sqr
t[(5 + Sqrt[5])/2]*Log[Sqrt[5 + 2*Sqrt[5]]*Cos[x] - Sin[x]])/10 + (Sqrt[(5
- Sqrt[5])/2]*Log[Sqrt[5 - 2*Sqrt[5]]*Cos[x] + Sin[x]])/10 - (Sqrt[(5 + Sqr
t[5])/2]*Log[Sqrt[5 + 2*Sqrt[5]]*Cos[x] + Sin[x]])/10
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \csc(5x)\sin(x)\,dx = \text{Subst}\left(\int \frac{1+x^2}{5-10x^2+x^4}\,dx, x, \tan(x)\right)$$
$$= \frac{1}{10}\left(5-3\sqrt{5}\right)\text{Subst}\left(\int \frac{1}{-5+2\sqrt{5}+x^2}\,dx, x, \tan(x)\right) + \frac{1}{10}\left(5+3\sqrt{5}\right)\text{Subst}\left(\int \frac{}{-5-2}\right.$$
$$= -\frac{1}{10}\sqrt{\frac{1}{2}\left(5-\sqrt{5}\right)}\log\left(\sqrt{5-2\sqrt{5}}\cos(x)-\sin(x)\right) + \frac{1}{10}\sqrt{\frac{1}{2}\left(5+\sqrt{5}\right)}\log\left(\sqrt{5+2\sqrt{5}}\right.$$

Mathematica [A] time = 0.11, size = 84, normalized size = 0.51

$$\frac{\sqrt{5+\sqrt{5}}\tanh^{-1}\left(\frac{\left(\sqrt{5}-3\right)\tan(x)}{\sqrt{10-2\sqrt{5}}}\right)+\sqrt{5-\sqrt{5}}\tanh^{-1}\left(\frac{\left(3+\sqrt{5}\right)\tan(x)}{\sqrt{2\left(5+\sqrt{5}\right)}}\right)}{5\sqrt{2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Csc[5*x]*Sin[x],x]
```

```
[Out] (Sqrt[5 + Sqrt[5]]*ArcTanh[((-3 + Sqrt[5])*Tan[x])/Sqrt[10 - 2*Sqrt[5]]] +
Sqrt[5 - Sqrt[5]]*ArcTanh[((3 + Sqrt[5])*Tan[x])/Sqrt[2*(5 + Sqrt[5])]])/(5
*Sqrt[2])
```

fricas [B] time = 0.93, size = 231, normalized size = 1.40

$$-\frac{1}{40}\sqrt{2}\sqrt{\sqrt{5}+5}\log\left(\left(\sqrt{5}\sqrt{2}-\sqrt{2}\right)\sqrt{\sqrt{5}+5}\cos(x)\sin(x)+2\left(\sqrt{5}+1\right)\cos(x)^2-\sqrt{5}+3\right)+\frac{1}{40}\sqrt{2}\sqrt{\sqrt{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(5*x)*sin(x),x, algorithm="fricas")
```

```
[Out] -1/40*sqrt(2)*sqrt(sqrt(5) + 5)*log((sqrt(5)*sqrt(2) - sqrt(2))*sqrt(sqrt(5
) + 5)*cos(x)*sin(x) + 2*(sqrt(5) + 1)*cos(x)^2 - sqrt(5) + 3) + 1/40*sqrt(
2)*sqrt(sqrt(5) + 5)*log(-(sqrt(5)*sqrt(2) - sqrt(2))*sqrt(sqrt(5) + 5)*cos
(x)*sin(x) + 2*(sqrt(5) + 1)*cos(x)^2 - sqrt(5) + 3) - 1/40*sqrt(2)*sqrt(-s
qrt(5) + 5)*log((sqrt(5)*sqrt(2) + sqrt(2))*sqrt(-sqrt(5) + 5)*cos(x)*sin(x
) + 2*(sqrt(5) - 1)*cos(x)^2 - sqrt(5) - 3) + 1/40*sqrt(2)*sqrt(-sqrt(5) +
5)*log(-(sqrt(5)*sqrt(2) + sqrt(2))*sqrt(-sqrt(5) + 5)*cos(x)*sin(x) + 2*(s
qrt(5) - 1)*cos(x)^2 - sqrt(5) - 3)
```

giac [A] time = 0.28, size = 105, normalized size = 0.64

$$-\frac{1}{20}\sqrt{2\sqrt{5}+10}\log\left(\left|\sqrt{2\sqrt{5}+5}+\tan(x)\right|\right)+\frac{1}{20}\sqrt{2\sqrt{5}+10}\log\left(\left|-\sqrt{2\sqrt{5}+5}+\tan(x)\right|\right)+\frac{1}{20}\sqrt{-2\sqrt{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(5*x)*sin(x),x, algorithm="giac")
```

```
[Out] -1/20*sqrt(2*sqrt(5) + 10)*log(abs(sqrt(2*sqrt(5) + 5) + tan(x))) + 1/20*sq
rt(2*sqrt(5) + 10)*log(abs(-sqrt(2*sqrt(5) + 5) + tan(x))) + 1/20*sqrt(-2*s
qrt(5) + 10)*log(abs(sqrt(-2*sqrt(5) + 5) + tan(x))) - 1/20*sqrt(-2*sqrt(5)
 + 10)*log(abs(-sqrt(-2*sqrt(5) + 5) + tan(x)))
```

maple [A] time = 0.29, size = 66, normalized size = 0.40

$$-\frac{\left(3+\sqrt{5}\right)\sqrt{5}\ \operatorname{arctanh}\left(\frac{\tan(x)}{\sqrt{5+2\sqrt{5}}}\right)}{10\sqrt{5+2\sqrt{5}}}-\frac{\left(\sqrt{5}-3\right)\sqrt{5}\ \operatorname{arctanh}\left(\frac{\tan(x)}{\sqrt{5-2\sqrt{5}}}\right)}{10\sqrt{5-2\sqrt{5}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(5*x)*sin(x),x)
```

```
[Out] -1/10*(3+5^(1/2))*5^(1/2)/(5+2*5^(1/2))^(1/2)*arctanh(tan(x)/(5+2*5^(1/2))^
(1/2))-1/10*(5^(1/2)-3)*5^(1/2)/(5-2*5^(1/2))^(1/2)*arctanh(tan(x)/(5-2*5^(
1/2))^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \csc(5x)\sin(x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(5*x)*sin(x),x, algorithm="maxima")
```

```
[Out] integrate(csc(5*x)*sin(x), x)
```

mupad [B] time = 2.59, size = 217, normalized size = 1.32

$$\frac{\sqrt{2}\ \operatorname{atanh}\left(-\frac{34359738368\sqrt{2}\tan\left(\frac{x}{2}\right)\sqrt{\sqrt{5}+5}}{1953125\left(\frac{90194313216\sqrt{5}}{1953125}-\frac{90194313216\sqrt{5}\tan\left(\frac{x}{2}\right)^2}{1953125}-\frac{201863462912\tan\left(\frac{x}{2}\right)^2}{1953125}+\frac{201863462912}{1953125}\right)}-\frac{77309411328\sqrt{2}}{9765625\left(\frac{90194313216\sqrt{5}}{1953125}-\frac{90194313216\sqrt{5}}{1953125}\right.}\right.}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)/sin(5*x),x)
```

```
[Out] (2^(1/2)*atanh(- (34359738368*2^(1/2)*tan(x/2)*(5^(1/2) + 5)^(1/2))/(195312
5*((90194313216*5^(1/2))/1953125 - (90194313216*5^(1/2)*tan(x/2)^2)/1953125
 - (201863462912*tan(x/2)^2)/1953125 + 201863462912/1953125)) - (7730941132
8*2^(1/2)*5^(1/2)*tan(x/2)*(5^(1/2) + 5)^(1/2))/(9765625*((90194313216*5^(1
/2))/1953125 - (90194313216*5^(1/2)*tan(x/2)^2)/1953125 - (201863462912*tan
(x/2)^2)/1953125 + 201863462912/1953125)))*(5^(1/2) + 5)^(1/2))/10 - (2^(1/
2)*atanh((77309411328*2^(1/2)*5^(1/2)*tan(x/2)*(5 - 5^(1/2))^(1/2))/(976562
5*((90194313216*5^(1/2))/1953125 - (90194313216*5^(1/2)*tan(x/2)^2)/1953125
 + (201863462912*tan(x/2)^2)/1953125 - 201863462912/1953125)) - (3435973836
8*2^(1/2)*tan(x/2)*(5 - 5^(1/2))^(1/2))/(1953125*((90194313216*5^(1/2))/195
3125 - (90194313216*5^(1/2)*tan(x/2)^2)/1953125 + (201863462912*tan(x/2)^2)
/1953125 - 201863462912/1953125)))*(5 - 5^(1/2))^(1/2))/10
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sin(x)\csc(5x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(5*x)*sin(x),x)
```

```
[Out] Integral(sin(x)*csc(5*x), x)
```

3.94 $\int \csc(6x)\sin(x)\,dx$

Optimal. Leaf size=36

$$\frac{1}{6}\tanh^{-1}(\sin(x))+\frac{1}{6}\tanh^{-1}(2\sin(x))-\frac{\tanh^{-1}\left(\frac{2\sin(x)}{\sqrt{3}}\right)}{2\sqrt{3}}$$

```
[Out] 1/6*arctanh(sin(x))+1/6*arctanh(2*sin(x))-1/6*arctanh(2/3*sin(x)*3^(1/2))*3
^(1/2)
```

Rubi [A] time = 0.05, antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 3, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {12, 2057, 207}

$$\frac{1}{6}\tanh^{-1}(\sin(x))+\frac{1}{6}\tanh^{-1}(2\sin(x))-\frac{\tanh^{-1}\left(\frac{2\sin(x)}{\sqrt{3}}\right)}{2\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[6*x]*Sin[x],x]
```

```
[Out] ArcTanh[Sin[x]]/6 + ArcTanh[2*Sin[x]]/6 - ArcTanh[(2*Sin[x])/Sqrt[3]]/(2*Sq
rt[3])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 2057

```
Int[(P_)^(p_), x_Symbol] :> With[{u = Factor[P /. x -> Sqrt[x]]}, Int[Expan
dIntegrand[(u /. x -> x^2)^p, x], x] /;  !SumQ[NonfreeFactors[u, x]]] /; Po
lyQ[P, x^2] && ILtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \csc(6x)\sin(x)\,dx &= \text{Subst}\left(\int \frac{1}{2\left(3-19x^2+32x^4-16x^6\right)}\,dx, x, \sin(x)\right)\\ &= \frac{1}{2}\text{Subst}\left(\int \frac{1}{3-19x^2+32x^4-16x^6}\,dx, x, \sin(x)\right)\\ &= \frac{1}{2}\text{Subst}\left(\int \left(-\frac{1}{3\left(-1+x^2\right)}+\frac{2}{-3+4x^2}-\frac{2}{3\left(-1+4x^2\right)}\right)dx, x, \sin(x)\right)\\ &= -\left(\frac{1}{6}\text{Subst}\left(\int \frac{1}{-1+x^2}\,dx, x, \sin(x)\right)\right)-\frac{1}{3}\text{Subst}\left(\int \frac{1}{-1+4x^2}\,dx, x, \sin(x)\right)+\text{Subst}\left(\int\right.\\ &= \frac{1}{6}\tanh^{-1}(\sin(x))+\frac{1}{6}\tanh^{-1}(2\sin(x))-\frac{\tanh^{-1}\left(\frac{2\sin(x)}{\sqrt{3}}\right)}{2\sqrt{3}}\end{aligned}$$

Mathematica [A] time = 0.04, size = 30, normalized size = 0.83

$$\frac{1}{6}\left(\tanh^{-1}(\sin(x))+\tanh^{-1}(2\sin(x))-\sqrt{3}\tanh^{-1}\left(\frac{2\sin(x)}{\sqrt{3}}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[6*x]*Sin[x],x]
```

```
[Out] (ArcTanh[Sin[x]] + ArcTanh[2*Sin[x]] - Sqrt[3]*ArcTanh[(2*Sin[x])/Sqrt[3]])
/6
```

fricas [B] time = 1.24, size = 68, normalized size = 1.89

$$\frac{1}{12}\sqrt{3}\log\left(-\frac{4\cos(x)^2+4\sqrt{3}\sin(x)-7}{4\cos(x)^2-1}\right)+\frac{1}{12}\log(2\sin(x)+1)+\frac{1}{12}\log(\sin(x)+1)-\frac{1}{12}\log(-\sin(x)+1)-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(6*x)*sin(x),x, algorithm="fricas")
```

```
[Out] 1/12*sqrt(3)*log(-(4*cos(x)^2 + 4*sqrt(3)*sin(x) - 7)/(4*cos(x)^2 - 1)) + 1
/12*log(2*sin(x) + 1) + 1/12*log(sin(x) + 1) - 1/12*log(-sin(x) + 1) - 1/12
*log(-2*sin(x) + 1)
```

giac [B] time = 0.15, size = 68, normalized size = 1.89

$$\frac{1}{12}\sqrt{3}\log\left(\frac{\left|-4\sqrt{3}+8\sin(x)\right|}{\left|4\sqrt{3}+8\sin(x)\right|}\right)+\frac{1}{12}\log(\sin(x)+1)-\frac{1}{12}\log(-\sin(x)+1)+\frac{1}{12}\log(|2\sin(x)+1|)-\frac{1}{12}\log(|2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(6*x)*sin(x),x, algorithm="giac")
```

```
[Out] 1/12*sqrt(3)*log(abs(-4*sqrt(3) + 8*sin(x))/abs(4*sqrt(3) + 8*sin(x))) + 1/
12*log(sin(x) + 1) - 1/12*log(-sin(x) + 1) + 1/12*log(abs(2*sin(x) + 1)) -
1/12*log(abs(2*sin(x) - 1))
```

maple [A] time = 0.26, size = 47, normalized size = 1.31

$$-\frac{\ln\left(-1+2\sin(x)\right)}{12}+\frac{\ln\left(1+2\sin(x)\right)}{12}-\frac{\operatorname{arctanh}\left(\frac{2\sin(x)\sqrt{3}}{3}\right)\sqrt{3}}{6}-\frac{\ln\left(\sin(x)-1\right)}{12}+\frac{\ln\left(1+\sin(x)\right)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(6*x)*sin(x),x)
```

```
[Out] -1/12*ln(-1+2*sin(x))+1/12*ln(1+2*sin(x))-1/6*arctanh(2/3*sin(x)*3^(1/2))*3
^(1/2)-1/12*ln(sin(x)-1)+1/12*ln(1+sin(x))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{1}{24}\sqrt{3}\log\left(\frac{4}{3}\cos(x)^2+\frac{4}{3}\sin(x)^2+\frac{4}{3}\sqrt{3}\sin(x)+\frac{4}{3}\cos(x)+\frac{4}{3}\right)-\frac{1}{24}\sqrt{3}\log\left(\frac{4}{3}\cos(x)^2+\frac{4}{3}\sin(x)^2+\frac{4}{3}\sqrt{3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(6*x)*sin(x),x, algorithm="maxima")
```

```
[Out] -1/24*sqrt(3)*log(4/3*cos(x)^2 + 4/3*sin(x)^2 + 4/3*sqrt(3)*sin(x) + 4/3*co
s(x) + 4/3) - 1/24*sqrt(3)*log(4/3*cos(x)^2 + 4/3*sin(x)^2 + 4/3*sqrt(3)*si
n(x) - 4/3*cos(x) + 4/3) + 1/24*sqrt(3)*log(4/3*cos(x)^2 + 4/3*sin(x)^2 - 4
/3*sqrt(3)*sin(x) + 4/3*cos(x) + 4/3) + 1/24*sqrt(3)*log(4/3*cos(x)^2 + 4/3
*sin(x)^2 - 4/3*sqrt(3)*sin(x) - 4/3*cos(x) + 4/3) + integrate(-1/6*((cos(3
*x) + cos(x))*cos(4*x) - (cos(2*x) - 1)*cos(3*x) - cos(2*x)*cos(x) + (sin(3
*x) + sin(x))*sin(4*x) - sin(3*x)*sin(2*x) - sin(2*x)*sin(x) + cos(x))/(2*(
cos(2*x) - 1)*cos(4*x) - cos(4*x)^2 - cos(2*x)^2 - sin(4*x)^2 + 2*sin(4*x)*
sin(2*x) - sin(2*x)^2 + 2*cos(2*x) - 1), x) + 1/12*log(cos(x)^2 + sin(x)^2
+ 2*sin(x) + 1) - 1/12*log(cos(x)^2 + sin(x)^2 - 2*sin(x) + 1)
```

mupad [B] time = 2.46, size = 35, normalized size = 0.97

$$\frac{\operatorname{atanh}\left(\frac{\sin\left(\frac{x}{2}\right)}{\cos\left(\frac{x}{2}\right)}\right)}{3}+\frac{\operatorname{atanh}\left(2\ \sin(x)\right)}{6}-\frac{\sqrt{3}\ \operatorname{atanh}\left(\frac{2\ \sqrt{3}\ \sin(x)}{3}\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)/sin(6*x),x)
```

```
[Out] atanh(sin(x/2)/cos(x/2))/3 + atanh(2*sin(x))/6 - (3^(1/2)*atanh((2*3^(1/2)*
sin(x))/3))/6
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sin(x)\csc(6x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(6*x)*sin(x),x)
```

```
[Out] Integral(sin(x)*csc(6*x), x)
```

3.95 $\int \csc(x)\sin(3x)\,dx$

Optimal. Leaf size=8

$$x + 2\sin(x)\cos(x)$$

```
[Out] x+2*cos(x)*sin(x)
```

Rubi [A] time = 0.03, antiderivative size = 8, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {385, 203}

$$x + 2\sin(x)\cos(x)$$

Antiderivative was successfully verified.

```
[In] Int[Csc[x]*Sin[3*x],x]
```

```
[Out] x + 2*Cos[x]*Sin[x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rubi steps

$$\begin{aligned}\int \csc(x)\sin(3x)\,dx &= \text{Subst}\left(\int \frac{3-x^2}{\left(1+x^2\right)^2}\,dx, x, \tan(x)\right)\\ &= 2\cos(x)\sin(x) + \text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \tan(x)\right)\\ &= x + 2\cos(x)\sin(x)\end{aligned}$$

Mathematica [A] time = 0.01, size = 6, normalized size = 0.75

$$x + \sin(2x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[x]*Sin[3*x],x]
[Out] x + Sin[2*x]
```

fricas [A] time = 0.83, size = 8, normalized size = 1.00

$$2\cos(x)\sin(x) + x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)*sin(3*x),x, algorithm="fricas")
[Out] 2*cos(x)*sin(x) + x
```

giac [A] time = 0.13, size = 6, normalized size = 0.75

$$x + \sin(2x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)*sin(3*x),x, algorithm="giac")
[Out] x + sin(2*x)
```

maple [A] time = 0.10, size = 9, normalized size = 1.12

$$x + 2\cos(x)\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(x)*sin(3*x),x)
[Out] x+2*cos(x)*sin(x)
```

maxima [A] time = 0.31, size = 6, normalized size = 0.75

$$x + \sin(2x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)*sin(3*x),x, algorithm="maxima")
[Out] x + sin(2*x)
```

mupad [B] time = 2.25, size = 6, normalized size = 0.75

$$x + \sin(2x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(3*x)/sin(x),x)
```

```
[Out] x + sin(2*x)
```

sympy **[A]** time = 0.98, size = 5, normalized size = 0.62

$$x + \sin(2x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)*sin(3*x),x)
```

```
[Out] x + sin(2*x)
```

3.96 $\int \csc(3x)\sin(6x)\,dx$

Optimal. Leaf size=8

$$\frac{2}{3}\sin(3x)$$

[Out] 2/3*sin(3*x)

Rubi [A] time = 0.01, antiderivative size = 8, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {4288, 2637}

$$\frac{2}{3}\sin(3x)$$

Antiderivative was successfully verified.

[In] Int[Csc[3*x]*Sin[6*x],x]

[Out] (2*Sin[3*x])/3

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 4288

```
Int[((f_.)*sin[(a_.) + (b_.)*(x_)])^(n_.)*sin[(c_.) + (d_.)*(x_)]^(p_.), x_
Symbol] :> Dist[2^p/f^p, Int[Cos[a + b*x]^p*(f*Sin[a + b*x])^(n + p), x], x
] /; FreeQ[{a, b, c, d, f, n}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && In
tegerQ[p]
```

Rubi steps

$$\begin{aligned}\int \csc(3x)\sin(6x)\,dx &= 2\int \cos(3x)\,dx\\ &= \frac{2}{3}\sin(3x)\end{aligned}$$

Mathematica [A] time = 0.00, size = 8, normalized size = 1.00

$$\frac{2}{3}\sin(3x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[3*x]*Sin[6*x],x]
[Out] (2*Sin[3*x])/3
```

fricas [A] time = 2.97, size = 6, normalized size = 0.75

$$\frac{2}{3} \sin(3x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(3*x)*sin(6*x),x, algorithm="fricas")
[Out] 2/3*sin(3*x)
```

giac [A] time = 0.13, size = 6, normalized size = 0.75

$$\frac{2}{3} \sin(3x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(3*x)*sin(6*x),x, algorithm="giac")
[Out] 2/3*sin(3*x)
```

maple [A] time = 0.04, size = 9, normalized size = 1.12

$$\frac{2}{3\csc(3x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(3*x)*sin(6*x),x)
[Out] 2/3/csc(3*x)
```

maxima [A] time = 0.32, size = 6, normalized size = 0.75

$$\frac{2}{3} \sin(3x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(3*x)*sin(6*x),x, algorithm="maxima")
[Out] 2/3*sin(3*x)
```

mupad [B] time = 0.03, size = 6, normalized size = 0.75

$$\frac{2 \sin(3x)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(6*x)/sin(3*x),x)
```

```
[Out] (2*sin(3*x))/3
```

sympy [A] time = 3.26, size = 7, normalized size = 0.88

$$\frac{2\sin(3x)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(3*x)*sin(6*x),x)
```

```
[Out] 2*sin(3*x)/3
```

3.97 $\int \cos(x)\sin(2x)\,dx$

Optimal. Leaf size=15

$$-\frac{\cos(x)}{2}-\frac{1}{6}\cos(3x)$$

```
[Out] -1/2*cos(x)-1/6*cos(3*x)
```

Rubi [A] time = 0.01, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {4284}

$$-\frac{\cos(x)}{2}-\frac{1}{6}\cos(3x)$$

Antiderivative was successfully verified.

```
[In] Int[Cos[x]*Sin[2*x],x]
[Out] -Cos[x]/2 - Cos[3*x]/6
```

Rule 4284

```
Int[cos[(c_.) + (d_.)*(x_)]*sin[(a_.) + (b_.)*(x_)], x_Symbol] :> -Simp[Cos
[a - c + (b - d)*x]/(2*(b - d)), x] - Simp[Cos[a + c + (b + d)*x]/(2*(b + d
)), x] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - d^2, 0]
```

Rubi steps

$$\int \cos(x)\sin(2x)\,dx=-\frac{\cos(x)}{2}-\frac{1}{6}\cos(3x)$$

Mathematica [A] time = 0.00, size = 15, normalized size = 1.00

$$-\frac{\cos(x)}{2}-\frac{1}{6}\cos(3x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[x]*Sin[2*x],x]
[Out] -1/2*Cos[x] - Cos[3*x]/6
```

fricas [A] time = 1.48, size = 6, normalized size = 0.40

$$-\frac{2}{3}\cos(x)^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*sin(2*x),x, algorithm="fricas")
```

```
[Out] -2/3*cos(x)^3
```

giac [A] time = 0.13, size = 6, normalized size = 0.40

$$-\frac{2}{3}\cos(x)^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*sin(2*x),x, algorithm="giac")
```

```
[Out] -2/3*cos(x)^3
```

maple [A] time = 0.04, size = 7, normalized size = 0.47

$$-\frac{2\left(\cos^3(x)\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)*sin(2*x),x)
```

```
[Out] -2/3*cos(x)^3
```

maxima [A] time = 0.31, size = 11, normalized size = 0.73

$$-\frac{1}{6}\cos(3x)-\frac{1}{2}\cos(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*sin(2*x),x, algorithm="maxima")
```

```
[Out] -1/6*cos(3*x) - 1/2*cos(x)
```

mupad [B] time = 0.02, size = 6, normalized size = 0.40

$$-\frac{2\cos(x)^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(2*x)*cos(x),x)
```

```
[Out] -(2*cos(x)^3)/3
```

sympy [A] time = 0.45, size = 22, normalized size = 1.47

$$-\frac{\sin(x)\sin(2x)}{3}-\frac{2\cos(x)\cos(2x)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*sin(2*x),x)
```

```
[Out] -sin(x)*sin(2*x)/3 - 2*cos(x)*cos(2*x)/3
```

3.98 $\int \cos(x)\sin(3x)\,dx$

Optimal. Leaf size=17

$$-\frac{1}{4}\cos(2x)-\frac{1}{8}\cos(4x)$$

```
[Out] -1/4*cos(2*x)-1/8*cos(4*x)
```

Rubi [A] time = 0.01, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {4284}

$$-\frac{1}{4}\cos(2x)-\frac{1}{8}\cos(4x)$$

Antiderivative was successfully verified.

```
[In] Int[Cos[x]*Sin[3*x],x]
[Out] -Cos[2*x]/4 - Cos[4*x]/8
```

Rule 4284

```
Int[cos[(c_.) + (d_.)*(x_)]*sin[(a_.) + (b_.)*(x_)], x_Symbol] :> -Simp[Cos
[a - c + (b - d)*x]/(2*(b - d)), x] - Simp[Cos[a + c + (b + d)*x]/(2*(b + d
)), x] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - d^2, 0]
```

Rubi steps

$$\int \cos(x)\sin(3x)\,dx=-\frac{1}{4}\cos(2x)-\frac{1}{8}\cos(4x)$$

Mathematica [A] time = 0.01, size = 17, normalized size = 1.00

$$-\frac{1}{2}\cos^2(x)-\frac{1}{8}\cos(4x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[x]*Sin[3*x],x]
[Out] -1/2*Cos[x]^2 - Cos[4*x]/8
```

fricas [A] time = 0.90, size = 13, normalized size = 0.76

$$-\cos(x)^4+\frac{1}{2}\cos(x)^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*sin(3*x),x, algorithm="fricas")
```

```
[Out] -cos(x)^4 + 1/2*cos(x)^2
```

giac [A] time = 0.14, size = 13, normalized size = 0.76

$$-\cos(x)^4 + \frac{1}{2}\cos(x)^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*sin(3*x),x, algorithm="giac")
```

```
[Out] -cos(x)^4 + 1/2*cos(x)^2
```

maple [A] time = 0.07, size = 14, normalized size = 0.82

$$-\left(\cos^4(x)\right) + \frac{\left(\cos^2(x)\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)*sin(3*x),x)
```

```
[Out] -cos(x)^4+1/2*cos(x)^2
```

maxima [A] time = 0.31, size = 13, normalized size = 0.76

$$-\frac{1}{8}\cos(4x) - \frac{1}{4}\cos(2x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*sin(3*x),x, algorithm="maxima")
```

```
[Out] -1/8*cos(4*x) - 1/4*cos(2*x)
```

mupad [B] time = 0.03, size = 13, normalized size = 0.76

$$\frac{\cos(x)^2}{2} - \cos(x)^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(3*x)*cos(x),x)
```

```
[Out] cos(x)^2/2 - cos(x)^4
```

sympy [A] time = 0.42, size = 22, normalized size = 1.29

$$-\frac{\sin(x)\sin(3x)}{8} - \frac{3\cos(x)\cos(3x)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*sin(3*x),x)
```

```
[Out] -sin(x)*sin(3*x)/8 - 3*cos(x)*cos(3*x)/8
```

3.99 $\int \cos(x)\sin(4x)\,dx$

Optimal. Leaf size=17

$$-\frac{1}{6}\cos(3x)-\frac{1}{10}\cos(5x)$$

```
[Out] -1/6*cos(3*x)-1/10*cos(5*x)
```

Rubi [A] time = 0.01, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {4284}

$$-\frac{1}{6}\cos(3x)-\frac{1}{10}\cos(5x)$$

Antiderivative was successfully verified.

```
[In] Int[Cos[x]*Sin[4*x],x]
```

```
[Out] -Cos[3*x]/6 - Cos[5*x]/10
```

Rule 4284

```
Int[cos[(c_.) + (d_.)*(x_)]*sin[(a_.) + (b_.)*(x_)], x_Symbol] :> -Simp[Cos
[a - c + (b - d)*x]/(2*(b - d)), x] - Simp[Cos[a + c + (b + d)*x]/(2*(b + d
)), x] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - d^2, 0]
```

Rubi steps

$$\int \cos(x)\sin(4x)\,dx=-\frac{1}{6}\cos(3x)-\frac{1}{10}\cos(5x)$$

Mathematica [A] time = 0.01, size = 17, normalized size = 1.00

$$-\frac{1}{6}\cos(3x)-\frac{1}{10}\cos(5x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[x]*Sin[4*x],x]
```

```
[Out] -1/6*Cos[3*x] - Cos[5*x]/10
```

fricas [A] time = 0.55, size = 13, normalized size = 0.76

$$-\frac{8}{5}\cos(x)^5+\frac{4}{3}\cos(x)^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*sin(4*x),x, algorithm="fricas")
[Out] -8/5*cos(x)^5 + 4/3*cos(x)^3
```

giac [A] time = 0.12, size = 13, normalized size = 0.76

$$-\frac{8}{5}\cos(x)^5 + \frac{4}{3}\cos(x)^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*sin(4*x),x, algorithm="giac")
[Out] -8/5*cos(x)^5 + 4/3*cos(x)^3
```

maple [A] time = 0.06, size = 14, normalized size = 0.82

$$-\frac{8\left(\cos^5(x)\right)}{5} + \frac{4\left(\cos^3(x)\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)*sin(4*x),x)
[Out] -8/5*cos(x)^5+4/3*cos(x)^3
```

maxima [A] time = 0.31, size = 13, normalized size = 0.76

$$-\frac{1}{10}\cos(5x) - \frac{1}{6}\cos(3x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*sin(4*x),x, algorithm="maxima")
[Out] -1/10*cos(5*x) - 1/6*cos(3*x)
```

mupad [B] time = 0.02, size = 14, normalized size = 0.82

$$-\frac{4\cos(x)^3\left(6\cos(x)^2 - 5\right)}{15}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(4*x)*cos(x),x)
[Out] -(4*cos(x)^3*(6*cos(x)^2 - 5))/15
```

sympy [A] time = 0.41, size = 22, normalized size = 1.29

$$-\frac{\sin(x)\sin(4x)}{15}-\frac{4\cos(x)\cos(4x)}{15}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*sin(4*x),x)
```

```
[Out] -sin(x)*sin(4*x)/15 - 4*cos(x)*cos(4*x)/15
```

3.100 $\int \cos(x)\sin(mx)\,dx$

Optimal. Leaf size=35

$$\frac{\cos((1-m)x)}{2(1-m)}-\frac{\cos((m+1)x)}{2(m+1)}$$

```
[Out] 1/2*cos((1-m)*x)/(1-m)-1/2*cos((1+m)*x)/(1+m)
```

Rubi [A] time = 0.03, antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {4574, 2638}

$$\frac{\cos((1-m)x)}{2(1-m)}-\frac{\cos((m+1)x)}{2(m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[x]*Sin[m*x],x]
```

```
[Out] Cos[(1 - m)*x]/(2*(1 - m)) - Cos[(1 + m)*x]/(2*(1 + m))
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 4574

```
Int[Cos[w_]^(q_.)*Sin[v_]^(p_.), x_Symbol] :> Int[ExpandTrigReduce[Sin[v]^p
*Cos[w]^q, x], x] /; IGtQ[p, 0] && IGtQ[q, 0] && ((PolynomialQ[v, x] && Pol
ynomialQ[w, x]) || (BinomialQ[{v, w}, x] && IndependentQ[Cancel[v/w], x]))
```

Rubi steps

$$\begin{aligned}\int \cos(x)\sin(mx)\,dx &= \int \left(-\frac{1}{2}\sin((1-m)x)+\frac{1}{2}\sin((1+m)x)\right)dx\\ &= -\left(\frac{1}{2}\int \sin((1-m)x)\,dx\right)+\frac{1}{2}\int \sin((1+m)x)\,dx\\ &= \frac{\cos((1-m)x)}{2(1-m)}-\frac{\cos((1+m)x)}{2(1+m)}\end{aligned}$$

Mathematica [A] time = 0.05, size = 26, normalized size = 0.74

$$\frac{\sin(x)\sin(mx)+m\cos(x)\cos(mx)}{1-m^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[x]*Sin[m*x],x]
[Out] (m*Cos[x]*Cos[m*x] + Sin[x]*Sin[m*x])/(1 - m^2)
```

fricas [A] time = 0.43, size = 25, normalized size = 0.71

$$-\frac{m\cos(mx)\cos(x)+\sin(mx)\sin(x)}{m^2-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*sin(m*x),x, algorithm="fricas")
[Out] -(m*cos(m*x)*cos(x) + sin(m*x)*sin(x))/(m^2 - 1)
```

giac [A] time = 0.14, size = 29, normalized size = 0.83

$$-\frac{\cos(mx+x)}{2(m+1)}-\frac{\cos(mx-x)}{2(m-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*sin(m*x),x, algorithm="giac")
[Out] -1/2*cos(m*x + x)/(m + 1) - 1/2*cos(m*x - x)/(m - 1)
```

maple [A] time = 0.02, size = 28, normalized size = 0.80

$$-\frac{\cos((-1+m)x)}{2(-1+m)}-\frac{\cos((1+m)x)}{2(1+m)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)*sin(m*x),x)
[Out] -1/2*cos((-1+m)*x)/(-1+m)-1/2*cos((1+m)*x)/(1+m)
```

maxima [A] time = 0.32, size = 27, normalized size = 0.77

$$-\frac{\cos((m+1)x)}{2(m+1)}-\frac{\cos((m-1)x)}{2(m-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*sin(m*x),x, algorithm="maxima")
[Out] -1/2*cos((m + 1)*x)/(m + 1) - 1/2*cos((m - 1)*x)/(m - 1)
```

mupad [B] time = 2.29, size = 57, normalized size = 1.63

$$\begin{cases} \frac{\sin(x)^2}{2} & \text{if } m = 1 \\ \frac{\cos(x)^2}{2} & \text{if } m = -1 \\ -\frac{\cos(x(m-1))}{2m-2} - \frac{\cos(x(m+1))}{2m+2} & \text{if } m \neq -1 \land m \neq 1 \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(m*x)*cos(x),x)
```

```
[Out] piecewise(m == 1, sin(x)^2/2, m == -1, cos(x)^2/2, m ~= -1 & m ~= 1, - cos(
x*(m - 1))/(2*m - 2) - cos(x*(m + 1))/(2*m + 2))
```

sympy [A] time = 0.78, size = 44, normalized size = 1.26

$$\begin{cases} -\frac{\sin^2(x)}{2} & \text{for } m = -1 \\ \frac{\sin^2(x)}{2} & \text{for } m = 1 \\ -\frac{m\cos(x)\cos(mx)}{m^2-1} - \frac{\sin(x)\sin(mx)}{m^2-1} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*sin(m*x),x)
```

```
[Out] Piecewise((-sin(x)**2/2, Eq(m, -1)), (sin(x)**2/2, Eq(m, 1)), (-m*cos(x)*co
s(m*x)/(m**2 - 1) - sin(x)*sin(m*x)/(m**2 - 1), True))
```

3.101 $\int \cos(x)\cos(2x)\,dx$

Optimal. Leaf size=15

$$\frac{\sin(x)}{2}+\frac{1}{6}\sin(3x)$$

```
[Out] 1/2*sin(x)+1/6*sin(3*x)
```

Rubi [A] time = 0.01, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {4283}

$$\frac{\sin(x)}{2}+\frac{1}{6}\sin(3x)$$

Antiderivative was successfully verified.

```
[In] Int[Cos[x]*Cos[2*x],x]
```

```
[Out] Sin[x]/2 + Sin[3*x]/6
```

Rule 4283

```
Int[cos[(a_.) + (b_.)*(x_)]*cos[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[
a - c + (b - d)*x]/(2*(b - d)), x] + Simp[Sin[a + c + (b + d)*x]/(2*(b + d)
), x] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - d^2, 0]
```

Rubi steps

$$\int \cos(x)\cos(2x)\,dx=\frac{\sin(x)}{2}+\frac{1}{6}\sin(3x)$$

Mathematica [A] time = 0.01, size = 15, normalized size = 1.00

$$\frac{\sin(x)}{2}+\frac{1}{6}\sin(3x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[x]*Cos[2*x],x]
```

```
[Out] Sin[x]/2 + Sin[3*x]/6
```

fricas [A] time = 1.59, size = 12, normalized size = 0.80

$$\frac{1}{3}\left(2\cos(x)^2+1\right)\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*cos(2*x),x, algorithm="fricas")
```

```
[Out] 1/3*(2*cos(x)^2 + 1)*sin(x)
```

giac [A] time = 0.13, size = 11, normalized size = 0.73

$$\frac{1}{6}\sin(3x) + \frac{1}{2}\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*cos(2*x),x, algorithm="giac")
```

```
[Out] 1/6*sin(3*x) + 1/2*sin(x)
```

maple [A] time = 0.07, size = 12, normalized size = 0.80

$$\frac{\sin(x)}{2} + \frac{\sin(3x)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)*cos(2*x),x)
```

```
[Out] 1/2*sin(x)+1/6*sin(3*x)
```

maxima [A] time = 0.30, size = 11, normalized size = 0.73

$$\frac{1}{6}\sin(3x) + \frac{1}{2}\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*cos(2*x),x, algorithm="maxima")
```

```
[Out] 1/6*sin(3*x) + 1/2*sin(x)
```

mupad [B] time = 0.02, size = 9, normalized size = 0.60

$$\sin(x) - \frac{2\sin(x)^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(2*x)*cos(x),x)
```

```
[Out] sin(x) - (2*sin(x)^3)/3
```

sympy [A] time = 0.45, size = 20, normalized size = 1.33

$$-\frac{\sin(x)\cos(2x)}{3} + \frac{2\sin(2x)\cos(x)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*cos(2*x),x)
```

```
[Out] -sin(x)*cos(2*x)/3 + 2*sin(2*x)*cos(x)/3
```

3.102 $\int \cos(x)\cos(3x)\,dx$

Optimal. Leaf size=17

$$\frac{1}{4}\sin(2x)+\frac{1}{8}\sin(4x)$$

```
[Out] 1/4*sin(2*x)+1/8*sin(4*x)
```

Rubi [A] time = 0.01, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {4283}

$$\frac{1}{4}\sin(2x)+\frac{1}{8}\sin(4x)$$

Antiderivative was successfully verified.

```
[In] Int[Cos[x]*Cos[3*x],x]
```

```
[Out] Sin[2*x]/4 + Sin[4*x]/8
```

Rule 4283

```
Int[cos[(a_.) + (b_.)*(x_)]*cos[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[
a - c + (b - d)*x]/(2*(b - d)), x] + Simp[Sin[a + c + (b + d)*x]/(2*(b + d)
), x] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - d^2, 0]
```

Rubi steps

$$\int \cos(x)\cos(3x)\,dx=\frac{1}{4}\sin(2x)+\frac{1}{8}\sin(4x)$$

Mathematica [A] time = 0.01, size = 17, normalized size = 1.00

$$\frac{1}{4}\sin(2x)+\frac{1}{8}\sin(4x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[x]*Cos[3*x],x]
```

```
[Out] Sin[2*x]/4 + Sin[4*x]/8
```

fricas [A] time = 0.57, size = 7, normalized size = 0.41

$$\cos(x)^3\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*cos(3*x),x, algorithm="fricas")
```

```
[Out] cos(x)^3*sin(x)
```

giac [A] time = 0.13, size = 13, normalized size = 0.76

$$\frac{1}{8}\sin(4x)+\frac{1}{4}\sin(2x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*cos(3*x),x, algorithm="giac")
```

```
[Out] 1/8*sin(4*x) + 1/4*sin(2*x)
```

maple [A] time = 0.07, size = 14, normalized size = 0.82

$$\frac{\sin(2x)}{4}+\frac{\sin(4x)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)*cos(3*x),x)
```

```
[Out] 1/4*sin(2*x)+1/8*sin(4*x)
```

maxima [A] time = 0.32, size = 13, normalized size = 0.76

$$\frac{1}{8}\sin(4x)+\frac{1}{4}\sin(2x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*cos(3*x),x, algorithm="maxima")
```

```
[Out] 1/8*sin(4*x) + 1/4*sin(2*x)
```

mupad [B] time = 0.02, size = 7, normalized size = 0.41

$$\cos(x)^3\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(3*x)*cos(x),x)
```

```
[Out] cos(x)^3*sin(x)
```

sympy [A] time = 0.41, size = 20, normalized size = 1.18

$$-\frac{\sin(x)\cos(3x)}{8}+\frac{3\sin(3x)\cos(x)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*cos(3*x),x)
```

```
[Out] -sin(x)*cos(3*x)/8 + 3*sin(3*x)*cos(x)/8
```

3.103 $\int \cos(x)\cos(4x)\,dx$

Optimal. Leaf size=17

$$\frac{1}{6}\sin(3x) + \frac{1}{10}\sin(5x)$$

```
[Out] 1/6*sin(3*x)+1/10*sin(5*x)
```

Rubi [A] time = 0.01, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {4283}

$$\frac{1}{6}\sin(3x) + \frac{1}{10}\sin(5x)$$

Antiderivative was successfully verified.

```
[In] Int[Cos[x]*Cos[4*x],x]
```

```
[Out] Sin[3*x]/6 + Sin[5*x]/10
```

Rule 4283

```
Int[cos[(a_.) + (b_.)*(x_)]*cos[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[
a - c + (b - d)*x]/(2*(b - d)), x] + Simp[Sin[a + c + (b + d)*x]/(2*(b + d)
), x] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - d^2, 0]
```

Rubi steps

$$\int \cos(x)\cos(4x)\,dx = \frac{1}{6}\sin(3x) + \frac{1}{10}\sin(5x)$$

Mathematica [A] time = 0.01, size = 17, normalized size = 1.00

$$\frac{1}{6}\sin(3x) + \frac{1}{10}\sin(5x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[x]*Cos[4*x],x]
```

```
[Out] Sin[3*x]/6 + Sin[5*x]/10
```

fricas [A] time = 1.08, size = 18, normalized size = 1.06

$$\frac{1}{15}\left(24\cos(x)^4 - 8\cos(x)^2 - 1\right)\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*cos(4*x),x, algorithm="fricas")
```

```
[Out] 1/15*(24*cos(x)^4 - 8*cos(x)^2 - 1)*sin(x)
```

giac [A] time = 0.14, size = 13, normalized size = 0.76

$$\frac{1}{10}\sin\left(5\,x\right) + \frac{1}{6}\sin\left(3\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*cos(4*x),x, algorithm="giac")
```

```
[Out] 1/10*sin(5*x) + 1/6*sin(3*x)
```

maple [A] time = 0.10, size = 14, normalized size = 0.82

$$\frac{\sin\left(3x\right)}{6} + \frac{\sin\left(5x\right)}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)*cos(4*x),x)
```

```
[Out] 1/6*sin(3*x)+1/10*sin(5*x)
```

maxima [A] time = 0.31, size = 13, normalized size = 0.76

$$\frac{1}{10}\sin\left(5\,x\right) + \frac{1}{6}\sin\left(3\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*cos(4*x),x, algorithm="maxima")
```

```
[Out] 1/10*sin(5*x) + 1/6*sin(3*x)
```

mupad [B] time = 0.02, size = 13, normalized size = 0.76

$$\frac{\sin\left(3\,x\right)}{6} + \frac{\sin\left(5\,x\right)}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(4*x)*cos(x),x)
```

```
[Out] sin(3*x)/6 + sin(5*x)/10
```

sympy [A] time = 0.41, size = 20, normalized size = 1.18

$$-\frac{\sin(x)\cos(4x)}{15}+\frac{4\sin(4x)\cos(x)}{15}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*cos(4*x),x)
```

```
[Out] -sin(x)*cos(4*x)/15 + 4*sin(4*x)*cos(x)/15
```

3.104 $\int \cos(x)\cos(mx)\,dx$

Optimal. Leaf size=35

$$\frac{\sin((1-m)x)}{2(1-m)}+\frac{\sin((m+1)x)}{2(m+1)}$$

```
[Out] 1/2*sin((1-m)*x)/(1-m)+1/2*sin((1+m)*x)/(1+m)
```

Rubi [A] time = 0.03, antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {4570, 2637}

$$\frac{\sin((1-m)x)}{2(1-m)}+\frac{\sin((m+1)x)}{2(m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[x]*Cos[m*x],x]
```

```
[Out] Sin[(1 - m)*x]/(2*(1 - m)) + Sin[(1 + m)*x]/(2*(1 + m))
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 4570

```
Int[Cos[v_]^(p_.)*Cos[w_]^(q_.), x_Symbol] :> Int[ExpandTrigReduce[Cos[v]^p
*Cos[w]^q, x], x] /; ((PolynomialQ[v, x] && PolynomialQ[w, x]) || (Binomial
Q[{v, w}, x] && IndependentQ[Cancel[v/w], x])) && IGtQ[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int \cos(x)\cos(mx)\,dx &= \int \left(\frac{1}{2}\cos((1-m)x)+\frac{1}{2}\cos((1+m)x)\right)dx\\ &= \frac{1}{2}\int \cos((1-m)x)\,dx+\frac{1}{2}\int \cos((1+m)x)\,dx\\ &= \frac{\sin((1-m)x)}{2(1-m)}+\frac{\sin((1+m)x)}{2(1+m)}\end{aligned}$$

Mathematica [A] time = 0.04, size = 25, normalized size = 0.71

$$\frac{m\cos(x)\sin(mx)-\sin(x)\cos(mx)}{m^2-1}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[x]*Cos[m*x],x]
[Out] (-(Cos[m*x]*Sin[x]) + m*Cos[x]*Sin[m*x])/(-1 + m^2)
```

fricas [A] time = 2.47, size = 25, normalized size = 0.71

$$\frac{m\cos(x)\sin(mx) - \cos(mx)\sin(x)}{m^2 - 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*cos(m*x),x, algorithm="fricas")
[Out] (m*cos(x)*sin(m*x) - cos(m*x)*sin(x))/(m^2 - 1)
```

giac [A] time = 0.14, size = 29, normalized size = 0.83

$$\frac{\sin(mx + x)}{2(m + 1)} + \frac{\sin(mx - x)}{2(m - 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*cos(m*x),x, algorithm="giac")
[Out] 1/2*sin(m*x + x)/(m + 1) + 1/2*sin(m*x - x)/(m - 1)
```

maple [A] time = 0.03, size = 28, normalized size = 0.80

$$\frac{\sin((-1 + m)x)}{-2 + 2m} + \frac{\sin((1 + m)x)}{2 + 2m}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)*cos(m*x),x)
[Out] 1/2/(-1+m)*sin((-1+m)*x)+1/2*sin((1+m)*x)/(1+m)
```

maxima [A] time = 0.32, size = 28, normalized size = 0.80

$$\frac{\sin((m + 1)x)}{2(m + 1)} - \frac{\sin(-(m - 1)x)}{2(m - 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*cos(m*x),x, algorithm="maxima")
[Out] 1/2*sin((m + 1)*x)/(m + 1) - 1/2*sin(-(m - 1)*x)/(m - 1)
```

mupad [B] time = 0.12, size = 39, normalized size = 1.11

$$\begin{cases} \frac{x}{2}+\frac{\sin(2\,x)}{4} & \text{if } m=-1\vee m=1\\ \frac{\sin(x\,(m-1))}{2\,m-2}+\frac{\sin(x\,(m+1))}{2\,m+2} & \text{if } m\neq -1\wedge m\neq 1 \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(m*x)*cos(x),x)
```

```
[Out] piecewise(m == -1 | m == 1, x/2 + sin(2*x)/4, m ~= -1 & m ~= 1, sin(x*(m -
1))/(2*m - 2) + sin(x*(m + 1))/(2*m + 2))
```

sympy [A] time = 0.94, size = 56, normalized size = 1.60

$$\begin{cases} \frac{x\sin^2(x)}{2}+\frac{x\cos^2(x)}{2}+\frac{\sin(x)\cos(x)}{2} & \text{for } m=-1\vee m=1\\ \frac{m\sin(mx)\cos(x)}{m^2-1}-\frac{\sin(x)\cos(mx)}{m^2-1} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*cos(m*x),x)
```

```
[Out] Piecewise((x*sin(x)**2/2 + x*cos(x)**2/2 + sin(x)*cos(x)/2, Eq(m, -1) | Eq(
m, 1)), (m*sin(m*x)*cos(x)/(m**2 - 1) - sin(x)*cos(m*x)/(m**2 - 1), True))
```

3.105 $\int \cos(x)\tan(2x)\,dx$

Optimal. Leaf size=20

$$\frac{\tanh^{-1}\left(\sqrt{2}\cos(x)\right)}{\sqrt{2}}-\cos(x)$$

```
[Out] -cos(x)+1/2*arctanh(cos(x)*2^(1/2))*2^(1/2)
```

Rubi [A] time = 0.03, antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {12, 321, 207}

$$\frac{\tanh^{-1}\left(\sqrt{2}\cos(x)\right)}{\sqrt{2}}-\cos(x)$$

Antiderivative was successfully verified.

```
[In] Int[Cos[x]*Tan[2*x],x]
```

```
[Out] ArcTanh[Sqrt[2]*Cos[x]]/Sqrt[2] - Cos[x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\begin{aligned}\int \cos(x)\tan(2x)\,dx &= -\operatorname{Subst}\left(\int \frac{2x^2}{-1+2x^2}\,dx, x, \cos(x)\right)\\ &= -\left(2\operatorname{Subst}\left(\int \frac{x^2}{-1+2x^2}\,dx, x, \cos(x)\right)\right)\\ &= -\cos(x) - \operatorname{Subst}\left(\int \frac{1}{-1+2x^2}\,dx, x, \cos(x)\right)\\ &= \frac{\tanh^{-1}\left(\sqrt{2}\cos(x)\right)}{\sqrt{2}} - \cos(x)\end{aligned}$$

Mathematica [C] time = 0.23, size = 183, normalized size = 9.15

$$\frac{-4\sqrt{2}\cos(x) + 4\tanh^{-1}\left(\tan\left(\frac{x}{2}\right) + \sqrt{2}\right) - \log\left(-\sqrt{2}\sin(x) - \sqrt{2}\cos(x) + 2\right) + \log\left(-\sqrt{2}\sin(x) + \sqrt{2}\cos(x)\right.}{4\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[x]*Tan[2*x],x]
```

```
[Out] ((2*I)*ArcTan[(Cos[x/2] - (-1 + Sqrt[2])*Sin[x/2])/((1 + Sqrt[2])*Cos[x/2]
- Sin[x/2])] - (2*I)*ArcTan[(Cos[x/2] - (1 + Sqrt[2])*Sin[x/2])/((-1 + Sqrt
[2])*Cos[x/2] - Sin[x/2])] + 4*ArcTanh[Sqrt[2] + Tan[x/2]] - 4*Sqrt[2]*Cos[
x] - Log[2 - Sqrt[2]*Cos[x] - Sqrt[2]*Sin[x]] + Log[2 + Sqrt[2]*Cos[x] - Sq
rt[2]*Sin[x]])/(4*Sqrt[2])
```

fricas [B] time = 2.03, size = 38, normalized size = 1.90

$$\frac{1}{4}\sqrt{2}\log\left(-\frac{2\cos(x)^2 + 2\sqrt{2}\cos(x) + 1}{2\cos(x)^2 - 1}\right) - \cos(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*tan(2*x),x, algorithm="fricas")
```

```
[Out] 1/4*sqrt(2)*log(-(2*cos(x)^2 + 2*sqrt(2)*cos(x) + 1)/(2*cos(x)^2 - 1)) - co
s(x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \cos(x)\tan(2x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*tan(2*x),x, algorithm="giac")
```

```
[Out] integrate(cos(x)*tan(2*x), x)
```

maple [A] time = 0.04, size = 18, normalized size = 0.90

$$-\cos(x)+\frac{\operatorname{arctanh}\left(\cos(x)\sqrt{2}\right)\sqrt{2}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)*tan(2*x),x)
```

```
[Out] -cos(x)+1/2*arctanh(cos(x)*2^(1/2))*2^(1/2)
```

maxima [B] time = 0.43, size = 133, normalized size = 6.65

$$\frac{1}{8}\sqrt{2}\log\left(2\sqrt{2}\sin(2x)\sin(x)+2\left(\sqrt{2}\cos(x)+1\right)\cos(2x)+\cos(2x)^2+2\cos(x)^2+\sin(2x)^2+2\sin(x)^2+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*tan(2*x),x, algorithm="maxima")
```

```
[Out] 1/8*sqrt(2)*log(2*sqrt(2)*sin(2*x)*sin(x) + 2*(sqrt(2)*cos(x) + 1)*cos(2*x)
 + cos(2*x)^2 + 2*cos(x)^2 + sin(2*x)^2 + 2*sin(x)^2 + 2*sqrt(2)*cos(x) + 1
) - 1/8*sqrt(2)*log(-2*sqrt(2)*sin(2*x)*sin(x) - 2*(sqrt(2)*cos(x) - 1)*cos
(2*x) + cos(2*x)^2 + 2*cos(x)^2 + sin(2*x)^2 + 2*sin(x)^2 - 2*sqrt(2)*cos(x
) + 1) - cos(x)
```

mupad [B] time = 2.36, size = 42, normalized size = 2.10

$$-\frac{\sqrt{2}\operatorname{atanh}\left(\frac{8\sqrt{2}\tan\left(\frac{x}{2}\right)^2}{12\tan\left(\frac{x}{2}\right)^2-4}\right)}{2}-\frac{2}{\tan\left(\frac{x}{2}\right)^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(2*x)*cos(x),x)
```

```
[Out] - (2^(1/2)*atanh((8*2^(1/2)*tan(x/2)^2)/(12*tan(x/2)^2 - 4)))/2 - 2/(tan(x/
2)^2 + 1)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\cos(x)\tan(2x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*tan(2*x),x)
```

```
[Out] Integral(cos(x)*tan(2*x), x)
```

3.106 $\int \cos(x)\tan(3x)\,dx$

Optimal. Leaf size=21

$$\frac{\tanh^{-1}\left(\frac{2\cos(x)}{\sqrt{3}}\right)}{\sqrt{3}}-\cos(x)$$

[Out] -cos(x)+1/3*arctanh(2/3*cos(x)*3^(1/2))*3^(1/2)

Rubi [A] time = 0.02, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {388, 206}

$$\frac{\tanh^{-1}\left(\frac{2\cos(x)}{\sqrt{3}}\right)}{\sqrt{3}}-\cos(x)$$

Antiderivative was successfully verified.

[In] Int[Cos[x]*Tan[3*x],x]

[Out] ArcTanh[(2*Cos[x])/Sqrt[3]]/Sqrt[3] - Cos[x]

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rubi steps

$$\int \cos(x)\tan(3x)\,dx = -\text{Subst}\left(\int \frac{1-4x^2}{3-4x^2}\,dx, x, \cos(x)\right)$$
$$= -\cos(x) + 2\,\text{Subst}\left(\int \frac{1}{3-4x^2}\,dx, x, \cos(x)\right)$$
$$= \frac{\tanh^{-1}\left(\frac{2\cos(x)}{\sqrt{3}}\right)}{\sqrt{3}} - \cos(x)$$

Mathematica [B] time = 0.05, size = 48, normalized size = 2.29

$$-\cos(x) - \frac{\tanh^{-1}\left(\frac{\tan\left(\frac{x}{2}\right)-2}{\sqrt{3}}\right)}{\sqrt{3}} + \frac{\tanh^{-1}\left(\frac{\tan\left(\frac{x}{2}\right)+2}{\sqrt{3}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[x]*Tan[3*x],x]
```

```
[Out] -(ArcTanh[(-2 + Tan[x/2])/Sqrt[3]]/Sqrt[3]) + ArcTanh[(2 + Tan[x/2])/Sqrt[3
]]/Sqrt[3] - Cos[x]
```

fricas [B] time = 1.51, size = 38, normalized size = 1.81

$$\frac{1}{6}\sqrt{3}\,\log\left(-\frac{4\,\cos(x)^2 + 4\,\sqrt{3}\,\cos(x) + 3}{4\,\cos(x)^2 - 3}\right) - \cos(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*tan(3*x),x, algorithm="fricas")
```

```
[Out] 1/6*sqrt(3)*log(-(4*cos(x)^2 + 4*sqrt(3)*cos(x) + 3)/(4*cos(x)^2 - 3)) - co
s(x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \cos(x)\tan(3\,x)\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*tan(3*x),x, algorithm="giac")
```

```
[Out] integrate(cos(x)*tan(3*x), x)
```

maple [A] time = 0.05, size = 19, normalized size = 0.90

$$-\cos(x)+\frac{\operatorname{arctanh}\left(\frac{2\cos(x)\sqrt{3}}{3}\right)\sqrt{3}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)*tan(3*x),x)
```

```
[Out] -cos(x)+1/3*arctanh(2/3*cos(x)*3^(1/2))*3^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\cos(x)-\int\frac{(\sin(3x)-\sin(x))\cos(4x)-(\cos(3x)-\cos(x))\sin(4x)-(\cos(2x)-1)\sin(3x)+\cos(3x)\sin(2x)}{2(\cos(2x)-1)\cos(4x)-\cos(4x)^2-\cos(2x)^2-\sin(4x)^2+2\sin(4x)\sin(2x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*tan(3*x),x, algorithm="maxima")
```

```
[Out] -cos(x) - integrate(((sin(3*x) - sin(x))*cos(4*x) - (cos(3*x) - cos(x))*sin
(4*x) - (cos(2*x) - 1)*sin(3*x) + cos(3*x)*sin(2*x) - cos(x)*sin(2*x) + cos
(2*x)*sin(x) - sin(x))/(2*(cos(2*x) - 1)*cos(4*x) - cos(4*x)^2 - cos(2*x)^2
 - sin(4*x)^2 + 2*sin(4*x)*sin(2*x) - sin(2*x)^2 + 2*cos(2*x) - 1), x)
```

mupad [B] time = 2.31, size = 42, normalized size = 2.00

$$-\frac{\sqrt{3}\operatorname{atanh}\left(\frac{32\sqrt{3}\tan\left(\frac{x}{2}\right)^2}{3\left(\frac{56\tan\left(\frac{x}{2}\right)^2}{3}-\frac{8}{3}\right)}\right)}{3}-\frac{2}{\tan\left(\frac{x}{2}\right)^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(3*x)*cos(x),x)
```

```
[Out] - (3^(1/2)*atanh((32*3^(1/2)*tan(x/2)^2)/(3*((56*tan(x/2)^2)/3 - 8/3))))/3
- 2/(tan(x/2)^2 + 1)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\cos(x)\tan(3x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*tan(3*x),x)
```

```
[Out] Integral(cos(x)*tan(3*x), x)
```

3.107 $\int \cos(x)\tan(4x)\,dx$

Optimal. Leaf size=71

$$-\cos(x)+\frac{1}{4}\sqrt{2-\sqrt{2}}\,\tanh^{-1}\left(\frac{2\cos(x)}{\sqrt{2-\sqrt{2}}}\right)+\frac{1}{4}\sqrt{2+\sqrt{2}}\,\tanh^{-1}\left(\frac{2\cos(x)}{\sqrt{2+\sqrt{2}}}\right)$$

```
[Out] -cos(x)+1/4*arctanh(2*cos(x)/(2-2^(1/2))^(1/2))*(2-2^(1/2))^(1/2)+1/4*arcta
nh(2*cos(x)/(2+2^(1/2))^(1/2))*(2+2^(1/2))^(1/2)
```

Rubi [A] time = 0.08, antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.571, Rules used = {12, 1279, 1166, 207}

$$-\cos(x)+\frac{1}{4}\sqrt{2-\sqrt{2}}\,\tanh^{-1}\left(\frac{2\cos(x)}{\sqrt{2-\sqrt{2}}}\right)+\frac{1}{4}\sqrt{2+\sqrt{2}}\,\tanh^{-1}\left(\frac{2\cos(x)}{\sqrt{2+\sqrt{2}}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Cos[x]*Tan[4*x],x]
```

```
[Out] (Sqrt[2 - Sqrt[2]]*ArcTanh[(2*Cos[x])/Sqrt[2 - Sqrt[2]]])/4 + (Sqrt[2 + Sqr
t[2]]*ArcTanh[(2*Cos[x])/Sqrt[2 + Sqrt[2]]])/4 - Cos[x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1279

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[(e*f*(f*x)^(m - 1)*(a + b*x^2 + c*x^4)^(p +
1))/(c*(m + 4*p + 3)), x] - Dist[f^2/(c*(m + 4*p + 3)), Int[(f*x)^(m - 2)*(
a + b*x^2 + c*x^4)^p*Simp[a*e*(m - 1) + (b*e*(m + 2*p + 1) - c*d*(m + 4*p +
 3))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c,
 0] && GtQ[m, 1] && NeQ[m + 4*p + 3, 0] && IntegerQ[2*p] && (IntegerQ[p] ||
 IntegerQ[m])
```

Rubi steps

$$\begin{aligned}
\int \cos(x)\tan(4x)\,dx &= -\text{Subst}\left(\int \frac{4x^2\left(-1+2x^2\right)}{1-8x^2+8x^4}\,dx, x, \cos(x)\right) \\
&= -\left(4\,\text{Subst}\left(\int \frac{x^2\left(-1+2x^2\right)}{1-8x^2+8x^4}\,dx, x, \cos(x)\right)\right) \\
&= -\cos(x) + \frac{1}{2}\,\text{Subst}\left(\int \frac{2-8x^2}{1-8x^2+8x^4}\,dx, x, \cos(x)\right) \\
&= -\cos(x) + \left(-2+\sqrt{2}\right)\text{Subst}\left(\int \frac{1}{-4+2\sqrt{2}+8x^2}\,dx, x, \cos(x)\right) - \left(2+\sqrt{2}\right)\text{Subst}\left(\int \right. \\
&= \frac{1}{4}\sqrt{2-\sqrt{2}}\,\tanh^{-1}\left(\frac{2\cos(x)}{\sqrt{2-\sqrt{2}}}\right) + \frac{1}{4}\sqrt{2+\sqrt{2}}\,\tanh^{-1}\left(\frac{2\cos(x)}{\sqrt{2+\sqrt{2}}}\right) - \cos(x)
\end{aligned}$$

Mathematica [C] time = 59.55, size = 6196, normalized size = 87.27

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[x]*Tan[4*x],x]
```

```
[Out] Result too large to show
```

fricas [A] time = 0.94, size = 101, normalized size = 1.42

$$\frac{1}{8}\sqrt{\sqrt{2}+2}\,\log\left(\sqrt{\sqrt{2}+2}+2\,\cos(x)\right)-\frac{1}{8}\sqrt{\sqrt{2}+2}\,\log\left(\sqrt{\sqrt{2}+2}-2\,\cos(x)\right)+\frac{1}{8}\sqrt{-\sqrt{2}+2}\,\log\left(\sqrt{-\sqrt{2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*tan(4*x),x, algorithm="fricas")
```

```
[Out] 1/8*sqrt(sqrt(2) + 2)*log(sqrt(sqrt(2) + 2) + 2*cos(x)) - 1/8*sqrt(sqrt(2)
+ 2)*log(sqrt(sqrt(2) + 2) - 2*cos(x)) + 1/8*sqrt(-sqrt(2) + 2)*log(sqrt(-s
qrt(2) + 2) + 2*cos(x)) - 1/8*sqrt(-sqrt(2) + 2)*log(sqrt(-sqrt(2) + 2) - 2
*cos(x)) - cos(x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \cos(x)\tan(4x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*tan(4*x),x, algorithm="giac")
```

```
[Out] integrate(cos(x)*tan(4*x), x)
```

maple [A] time = 0.08, size = 68, normalized size = 0.96

$$-\cos(x)+\frac{\left(\sqrt{2}-1\right)\sqrt{2}\ \operatorname{arctanh}\left(\frac{2\cos(x)}{\sqrt{2-\sqrt{2}}}\right)}{4\sqrt{2-\sqrt{2}}}+\frac{\left(1+\sqrt{2}\right)\sqrt{2}\ \operatorname{arctanh}\left(\frac{2\cos(x)}{\sqrt{2+\sqrt{2}}}\right)}{4\sqrt{2+\sqrt{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)*tan(4*x),x)
```

```
[Out] -cos(x)+1/4*(2^(1/2)-1)*2^(1/2)/(2-2^(1/2))^(1/2)*arctanh(2*cos(x)/(2-2^(1/
2))^(1/2))+1/4*(1+2^(1/2))*2^(1/2)/(2+2^(1/2))^(1/2)*arctanh(2*cos(x)/(2+2^
(1/2))^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\cos(x)-\int -\frac{(\sin(7x)-\sin(x))\cos(8x)-(\cos(7x)-\cos(x))\sin(8x)+\sin(7x)-\sin(x)}{\cos(8x)^2+\sin(8x)^2+2\,\cos(8x)+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*tan(4*x),x, algorithm="maxima")
```

```
[Out] -cos(x) - integrate(-((sin(7*x) - sin(x))*cos(8*x) - (cos(7*x) - cos(x))*si
n(8*x) + sin(7*x) - sin(x))/(cos(8*x)^2 + sin(8*x)^2 + 2*cos(8*x) + 1), x)
```

mupad [B] time = 2.45, size = 295, normalized size = 4.15

$$-\frac{\operatorname{atanh}\left(\frac{219747975168\tan\left(\frac{x}{2}\right)^2\sqrt{2-\sqrt{2}}}{6098518016\sqrt{2}-254015438848\sqrt{2}\tan\left(\frac{x}{2}\right)^2+386664497152\tan\left(\frac{x}{2}\right)^2-20887633920}-\frac{15971909632\sqrt{2-}}{6098518016\sqrt{2}-254015438848\sqrt{2}\tan\left(\frac{x}{2}\right)^2+38}\right.}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(4*x)*cos(x),x)
```

```
[Out] - (atanh((219747975168*tan(x/2)^2*(2 - 2^(1/2))^(1/2))/(6098518016*2^(1/2)
- 254015438848*2^(1/2)*tan(x/2)^2 + 386664497152*tan(x/2)^2 - 20887633920)
- (15971909632*(2 - 2^(1/2))^(1/2))/(6098518016*2^(1/2) - 254015438848*2^(1
/2)*tan(x/2)^2 + 386664497152*tan(x/2)^2 - 20887633920) - (130056978432*2^(
1/2)*tan(x/2)^2*(2 - 2^(1/2))^(1/2))/(6098518016*2^(1/2) - 254015438848*2^(
1/2)*tan(x/2)^2 + 386664497152*tan(x/2)^2 - 20887633920))*(2 - 2^(1/2))^(1/
2))/4 - 2/(tan(x/2)^2 + 1) - (atanh((15971909632*(2^(1/2) + 2)^(1/2))/(6098
518016*2^(1/2) - 254015438848*2^(1/2)*tan(x/2)^2 - 386664497152*tan(x/2)^2
+ 20887633920) - (219747975168*tan(x/2)^2*(2^(1/2) + 2)^(1/2))/(6098518016*
2^(1/2) - 254015438848*2^(1/2)*tan(x/2)^2 - 386664497152*tan(x/2)^2 + 20887
633920) - (130056978432*2^(1/2)*tan(x/2)^2*(2^(1/2) + 2)^(1/2))/(6098518016
*2^(1/2) - 254015438848*2^(1/2)*tan(x/2)^2 - 386664497152*tan(x/2)^2 + 2088
7633920))*(2^(1/2) + 2)^(1/2))/4
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \cos(x)\tan(4x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*tan(4*x),x)
```

```
[Out] Integral(cos(x)*tan(4*x), x)
```

3.108 $\int \cos(x)\tan(5x)\,dx$

Optimal. Leaf size=84

$$-\cos(x)+\frac{1}{5}\sqrt{\frac{1}{2}\left(5+\sqrt{5}\right)}\tanh^{-1}\left(2\sqrt{\frac{2}{5+\sqrt{5}}}\cos(x)\right)+\frac{1}{5}\sqrt{\frac{1}{2}\left(5-\sqrt{5}\right)}\tanh^{-1}\left(\sqrt{\frac{2}{5}\left(5+\sqrt{5}\right)}\cos(x)\right)$$

```
[Out] -cos(x)+1/10*arctanh(1/5*cos(x)*(50+10*5^(1/2))^(1/2))*(10-2*5^(1/2))^(1/2)
+1/10*arctanh(2*cos(x)*2^(1/2)/(5+5^(1/2))^(1/2))*(10+2*5^(1/2))^(1/2)
```

Rubi [A] time = 0.10, antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {1676, 1166, 207}

$$-\cos(x)+\frac{1}{5}\sqrt{\frac{1}{2}\left(5+\sqrt{5}\right)}\tanh^{-1}\left(2\sqrt{\frac{2}{5+\sqrt{5}}}\cos(x)\right)+\frac{1}{5}\sqrt{\frac{1}{2}\left(5-\sqrt{5}\right)}\tanh^{-1}\left(\sqrt{\frac{2}{5}\left(5+\sqrt{5}\right)}\cos(x)\right)$$

Antiderivative was successfully verified.

```
[In] Int[Cos[x]*Tan[5*x],x]
```

```
[Out] (Sqrt[(5 + Sqrt[5])/2]*ArcTanh[2*Sqrt[2/(5 + Sqrt[5])]*Cos[x]])/5 + (Sqrt[(
5 - Sqrt[5])/2]*ArcTanh[Sqrt[(2*(5 + Sqrt[5]))/5]*Cos[x]])/5 - Cos[x]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1676

```
Int[(Pq_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> Int[ExpandInte
grand[Pq/(a + b*x^2 + c*x^4), x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x^
2] && Expon[Pq, x^2] > 1
```

Rubi steps

$$
\begin{aligned}
\int \cos(x)\tan(5x)\,dx &= -\text{Subst}\left(\int \frac{1-12x^2+16x^4}{5-20x^2+16x^4}\,dx, x, \cos(x)\right)\\
&= -\text{Subst}\left(\int \left(1-\frac{4\left(1-2x^2\right)}{5-20x^2+16x^4}\right)dx, x, \cos(x)\right)\\
&= -\cos(x)+4\,\text{Subst}\left(\int \frac{1-2x^2}{5-20x^2+16x^4}\,dx, x, \cos(x)\right)\\
&= -\cos(x)-\frac{1}{5}\left(4\left(5-\sqrt{5}\right)\right)\text{Subst}\left(\int \frac{1}{-10+2\sqrt{5}+16x^2}\,dx, x, \cos(x)\right)-\frac{1}{5}\left(4\left(5+\sqrt{5}\right)\right.\\
&= \frac{1}{5}\sqrt{\frac{1}{2}\left(5+\sqrt{5}\right)}\tanh^{-1}\left(2\sqrt{\frac{2}{5+\sqrt{5}}}\cos(x)\right)+\frac{1}{5}\sqrt{\frac{1}{2}\left(5-\sqrt{5}\right)}\tanh^{-1}\left(\sqrt{\frac{2}{5}\left(5+\sqrt{5}\right)}\right.
\end{aligned}
$$

Mathematica [B] time = 0.59, size = 215, normalized size = 2.56

$$
-\cos(x)+\frac{\left(1+\sqrt{5}\right)\tanh^{-1}\left(\frac{4-\left(\sqrt{5}-1\right)\tan\left(\frac{x}{2}\right)}{\sqrt{2\left(5+\sqrt{5}\right)}}\right)}{\sqrt{10\left(5+\sqrt{5}\right)}}+\frac{\left(1+\sqrt{5}\right)\tanh^{-1}\left(\frac{\left(\sqrt{5}-1\right)\tan\left(\frac{x}{2}\right)+4}{\sqrt{2\left(5+\sqrt{5}\right)}}\right)}{\sqrt{10\left(5+\sqrt{5}\right)}}+\frac{\left(\sqrt{5}-1\right)\tanh^{-1}\left(\frac{4-\left(1+\sqrt{5}\right)\tan}{\sqrt{10-2\sqrt{5}}}\right.}{\sqrt{50-10\sqrt{5}}}
$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[x]*Tan[5*x],x]
```

```
[Out] ((1 + Sqrt[5])*ArcTanh[(4 - (-1 + Sqrt[5])*Tan[x/2])/Sqrt[2*(5 + Sqrt[5])]]
)/Sqrt[10*(5 + Sqrt[5])] + ((1 + Sqrt[5])*ArcTanh[(4 + (-1 + Sqrt[5])*Tan[x
/2])/Sqrt[2*(5 + Sqrt[5])]])/Sqrt[10*(5 + Sqrt[5])] + ((-1 + Sqrt[5])*ArcTa
nh[(4 - (1 + Sqrt[5])*Tan[x/2])/Sqrt[10 - 2*Sqrt[5]]])/Sqrt[50 - 10*Sqrt[5]
] + ((-1 + Sqrt[5])*ArcTanh[(4 + (1 + Sqrt[5])*Tan[x/2])/Sqrt[10 - 2*Sqrt[5
]]])/Sqrt[50 - 10*Sqrt[5]] - Cos[x]
```

fricas [B] time = 1.20, size = 129, normalized size = 1.54

$$
\frac{1}{20}\sqrt{2}\sqrt{\sqrt{5}+5}\log\left(\sqrt{2}\sqrt{\sqrt{5}+5}+4\cos(x)\right)-\frac{1}{20}\sqrt{2}\sqrt{\sqrt{5}+5}\log\left(\sqrt{2}\sqrt{\sqrt{5}+5}-4\cos(x)\right)+\frac{1}{20}\sqrt{2}\sqrt{-}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*tan(5*x),x, algorithm="fricas")
```

```
[Out] 1/20*sqrt(2)*sqrt(sqrt(5) + 5)*log(sqrt(2)*sqrt(sqrt(5) + 5) + 4*cos(x)) -
1/20*sqrt(2)*sqrt(sqrt(5) + 5)*log(sqrt(2)*sqrt(sqrt(5) + 5) - 4*cos(x)) +
1/20*sqrt(2)*sqrt(-sqrt(5) + 5)*log(sqrt(2)*sqrt(-sqrt(5) + 5) + 4*cos(x))
- 1/20*sqrt(2)*sqrt(-sqrt(5) + 5)*log(sqrt(2)*sqrt(-sqrt(5) + 5) - 4*cos(x)
) - cos(x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \cos(x) \tan(5x) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*tan(5*x),x, algorithm="giac")
```

```
[Out] integrate(cos(x)*tan(5*x), x)
```

maple [A] time = 0.07, size = 72, normalized size = 0.86

$$-\cos(x) + \frac{\left(\sqrt{5}-1\right)\sqrt{5}\ \operatorname{arctanh}\left(\frac{4\cos(x)}{\sqrt{10-2\sqrt{5}}}\right)}{5\sqrt{10-2\sqrt{5}}} + \frac{\left(\sqrt{5}+1\right)\sqrt{5}\ \operatorname{arctanh}\left(\frac{4\cos(x)}{\sqrt{10+2\sqrt{5}}}\right)}{5\sqrt{10+2\sqrt{5}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)*tan(5*x),x)
```

```
[Out] -cos(x)+1/5*(5^(1/2)-1)*5^(1/2)/(10-2*5^(1/2))^(1/2)*arctanh(4*cos(x)/(10-2
*5^(1/2))^(1/2))+1/5*(5^(1/2)+1)*5^(1/2)/(10+2*5^(1/2))^(1/2)*arctanh(4*cos
(x)/(10+2*5^(1/2))^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\cos(x) - \int \frac{(\sin(7x) - \sin(5x) + \sin(3x) - \sin(x))\cos(8x) + (\sin(6x) - \sin(4x) + \sin(2x))\cos(7x) + (\sin}{2(\cos(6x) - \mathrm{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*tan(5*x),x, algorithm="maxima")
```

```
[Out] -cos(x) - integrate(((sin(7*x) - sin(5*x) + sin(3*x) - sin(x))*cos(8*x) + (
sin(6*x) - sin(4*x) + sin(2*x))*cos(7*x) + (sin(5*x) - sin(3*x) + sin(x))*c
os(6*x) + (sin(4*x) - sin(2*x))*cos(5*x) + (sin(3*x) - sin(x))*cos(4*x) - (
cos(7*x) - cos(5*x) + cos(3*x) - cos(x))*sin(8*x) - (cos(6*x) - cos(4*x) +
cos(2*x) - 1)*sin(7*x) - (cos(5*x) - cos(3*x) + cos(x))*sin(6*x) - (cos(4*x
) - cos(2*x) + 1)*sin(5*x) - (cos(3*x) - cos(x))*sin(4*x) - (cos(2*x) - 1)*
```

```
sin(3*x) + cos(3*x)*sin(2*x) - cos(x)*sin(2*x) + cos(2*x)*sin(x) - sin(x))/
(2*(cos(6*x) - cos(4*x) + cos(2*x) - 1)*cos(8*x) - cos(8*x)^2 + 2*(cos(4*x)
 - cos(2*x) + 1)*cos(6*x) - cos(6*x)^2 + 2*(cos(2*x) - 1)*cos(4*x) - cos(4*
x)^2 - cos(2*x)^2 + 2*(sin(6*x) - sin(4*x) + sin(2*x))*sin(8*x) - sin(8*x)^
2 + 2*(sin(4*x) - sin(2*x))*sin(6*x) - sin(6*x)^2 - sin(4*x)^2 + 2*sin(4*x)
*sin(2*x) - sin(2*x)^2 + 2*cos(2*x) - 1), x)
```

mupad [B] time = 2.50, size = 407, normalized size = 4.85

$$\sqrt{2}\,\operatorname{atanh}\left(\frac{18032420192256\,\sqrt{2}\,\tan\left(\frac{x}{2}\right)^2\sqrt{\sqrt{5}+5}}{\frac{8851927597056\,\sqrt{5}}{25}-\frac{676375744741376\,\sqrt{5}\,\tan\left(\frac{x}{2}\right)^2}{25}-\frac{333433343574016\tan\left(\frac{x}{2}\right)^2}{5}+2398739234816}-\frac{867583393}{25\left(\frac{8851927597056\,\sqrt{5}}{25}-\frac{676375744741376\,\sqrt{5}}{25}\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(5*x)*cos(x),x)
```

```
[Out] (2^(1/2)*atanh((18032420192256*2^(1/2)*tan(x/2)^2*(5^(1/2) + 5)^(1/2))/((88
51927597056*5^(1/2))/25 - (676375744741376*5^(1/2)*tan(x/2)^2)/25 - (333433
343574016*tan(x/2)^2)/5 + 2398739234816) - (867583393792*2^(1/2)*5^(1/2)*(5
^(1/2) + 5)^(1/2))/(25*((8851927597056*5^(1/2))/25 - (676375744741376*5^(1/
2)*tan(x/2)^2)/25 - (333433343574016*tan(x/2)^2)/5 + 2398739234816)) - (380
5341024256*2^(1/2)*(5^(1/2) + 5)^(1/2))/(5*((8851927597056*5^(1/2))/25 - (6
76375744741376*5^(1/2)*tan(x/2)^2)/25 - (333433343574016*tan(x/2)^2)/5 + 23
98739234816)) + (6886980059136*2^(1/2)*5^(1/2)*tan(x/2)^2*(5^(1/2) + 5)^(1/
2))/((8851927597056*5^(1/2))/25 - (676375744741376*5^(1/2)*tan(x/2)^2)/25 -
 (333433343574016*tan(x/2)^2)/5 + 2398739234816))*(5^(1/2) + 5)^(1/2))/10 -
 (2^(1/2)*atanh((867583393792*2^(1/2)*5^(1/2)*(5 - 5^(1/2))^(1/2))/(25*((88
51927597056*5^(1/2))/25 - (676375744741376*5^(1/2)*tan(x/2)^2)/25 + (333433
343574016*tan(x/2)^2)/5 - 2398739234816)) - (3805341024256*2^(1/2)*(5 - 5^(
1/2))^(1/2))/(5*((8851927597056*5^(1/2))/25 - (676375744741376*5^(1/2)*tan(
x/2)^2)/25 + (333433343574016*tan(x/2)^2)/5 - 2398739234816)) + (1803242019
2256*2^(1/2)*tan(x/2)^2*(5 - 5^(1/2))^(1/2))/((8851927597056*5^(1/2))/25 -
(676375744741376*5^(1/2)*tan(x/2)^2)/25 + (333433343574016*tan(x/2)^2)/5 -
2398739234816) - (6886980059136*2^(1/2)*5^(1/2)*tan(x/2)^2*(5 - 5^(1/2))^(1
/2))/((8851927597056*5^(1/2))/25 - (676375744741376*5^(1/2)*tan(x/2)^2)/25
+ (333433343574016*tan(x/2)^2)/5 - 2398739234816))*(5 - 5^(1/2))^(1/2))/10
- 2/(tan(x/2)^2 + 1)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \cos(x)\tan(5x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*tan(5*x),x)
```

```
[Out] Integral(cos(x)*tan(5*x), x)
```

3.109 $\int \cos(x) \tan(6x) \, dx$

Optimal. Leaf size=89

$$-\cos(x)+\frac{\tanh^{-1}\left(\sqrt{2}\cos(x)\right)}{3\sqrt{2}}+\frac{1}{6}\sqrt{2-\sqrt{3}}\tanh^{-1}\left(\frac{2\cos(x)}{\sqrt{2-\sqrt{3}}}\right)+\frac{1}{6}\sqrt{2+\sqrt{3}}\tanh^{-1}\left(\frac{2\cos(x)}{\sqrt{2+\sqrt{3}}}\right)$$

```
[Out] -cos(x)+1/6*arctanh(cos(x)*2^(1/2))*2^(1/2)+1/6*arctanh(2*cos(x)/(1/2*6^(1/
2)-1/2*2^(1/2)))*(1/2*6^(1/2)-1/2*2^(1/2))+1/6*arctanh(2*cos(x)/(1/2*6^(1/2
)+1/2*2^(1/2)))*(1/2*6^(1/2)+1/2*2^(1/2))
```

Rubi [A] time = 0.24, antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 5, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.714, Rules used = {12, 6742, 2073, 207, 1166}

$$-\cos(x)+\frac{\tanh^{-1}\left(\sqrt{2}\cos(x)\right)}{3\sqrt{2}}+\frac{1}{6}\sqrt{2-\sqrt{3}}\tanh^{-1}\left(\frac{2\cos(x)}{\sqrt{2-\sqrt{3}}}\right)+\frac{1}{6}\sqrt{2+\sqrt{3}}\tanh^{-1}\left(\frac{2\cos(x)}{\sqrt{2+\sqrt{3}}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Cos[x]*Tan[6*x],x]
```

```
[Out] ArcTanh[Sqrt[2]*Cos[x]]/(3*Sqrt[2]) + (Sqrt[2 - Sqrt[3]]*ArcTanh[(2*Cos[x])
/Sqrt[2 - Sqrt[3]]])/6 + (Sqrt[2 + Sqrt[3]]*ArcTanh[(2*Cos[x])/Sqrt[2 + Sqr
t[3]]])/6 - Cos[x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
```

```
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 2073

```
Int[(P_)^(p_)*(Q_)^(q_.), x_Symbol] :> With[{PP = Factor[P /. x -> Sqrt[x]]
}, Int[ExpandIntegrand[(PP /. x -> x^2)^p*Q^q, x], x] /;  !SumQ[NonfreeFact
ors[PP, x]]] /; FreeQ[q, x] && PolyQ[P, x^2] && PolyQ[Q, x] && ILtQ[p, 0]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int \cos(x)\tan(6x)\,dx = -\text{Subst}\left(\int \frac{2x^2\left(-3+16x^2-16x^4\right)}{1-18x^2+48x^4-32x^6}\,dx, x, \cos(x)\right)$$

$$= -\left(2\,\text{Subst}\left(\int \frac{x^2\left(-3+16x^2-16x^4\right)}{1-18x^2+48x^4-32x^6}\,dx, x, \cos(x)\right)\right)$$

$$= -\left(2\,\text{Subst}\left(\int \left(\frac{1}{2}-\frac{1-12x^2+16x^4}{2\left(1-18x^2+48x^4-32x^6\right)}\right)dx, x, \cos(x)\right)\right)$$

$$= -\cos(x)+\text{Subst}\left(\int \frac{1-12x^2+16x^4}{1-18x^2+48x^4-32x^6}\,dx, x, \cos(x)\right)$$

$$= -\cos(x)+\text{Subst}\left(\int \left(-\frac{1}{3\left(-1+2x^2\right)}-\frac{2\left(-1+8x^2\right)}{3\left(1-16x^2+16x^4\right)}\right)dx, x, \cos(x)\right)$$

$$= -\cos(x)-\frac{1}{3}\text{Subst}\left(\int \frac{1}{-1+2x^2}\,dx, x, \cos(x)\right)-\frac{2}{3}\text{Subst}\left(\int \frac{-1+8x^2}{1-16x^2+16x^4}\,dx, x, \cos(x\right.$$

$$= \frac{\tanh^{-1}\left(\sqrt{2}\cos(x)\right)}{3\sqrt{2}}-\cos(x)-\frac{1}{3}\left(4\left(2-\sqrt{3}\right)\right)\text{Subst}\left(\int \frac{1}{-8+4\sqrt{3}+16x^2}\,dx, x, \cos(x)\right.$$

$$= \frac{\tanh^{-1}\left(\sqrt{2}\cos(x)\right)}{3\sqrt{2}}+\frac{1}{6}\sqrt{2-\sqrt{3}}\,\tanh^{-1}\left(\frac{2\cos(x)}{\sqrt{2-\sqrt{3}}}\right)+\frac{1}{6}\sqrt{2+\sqrt{3}}\,\tanh^{-1}\left(\frac{2\cos(x)}{\sqrt{2+\sqrt{3}}}\right.$$

Mathematica [C] time = 9.04, size = 679, normalized size = 7.63

$$-\cos(x)+\left(-\frac{1}{6}-\frac{i}{6}\right)\sqrt[4]{-1}\tan^{-1}\left(\left(\frac{1}{2}+\frac{i}{2}\right)\sqrt[4]{-1}\sec\left(\frac{x}{2}\right)\left(\sin\left(\frac{x}{2}\right)+\cos\left(\frac{x}{2}\right)\right)\right)+\left(\frac{1}{6}+\frac{i}{6}\right)(-1)^{3/4}\tanh^{-1}\left(\left(\frac{1}{2}+\frac{i}{2}\right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[x]*Tan[6*x],x]
```

```
[Out] (-1/6 - I/6)*(-1)^(1/4)*ArcTan[(1/2 + I/2)*(-1)^(1/4)*Sec[x/2]*(Cos[x/2] +
Sin[x/2])] + (1/6 + I/6)*(-1)^(3/4)*ArcTanh[(1/2 + I/2)*(-1)^(3/4)*Sec[x/2]
*(Cos[x/2] - Sin[x/2])] - Cos[x] - ((1 + Sqrt[2])*(x - 2*Sqrt[3]*ArcTanh[(2
 + (2 + Sqrt[2])*Tan[x/2])/Sqrt[6]] - Log[Sec[x/2]^2] + Log[-(Sec[x/2]^2*(S
qrt[2] - 2*Cos[x] + 2*Sin[x]))]))/(12*(2 + Sqrt[2])) + (x + 2*Sqrt[3]*ArcTa
nh[(Sqrt[2] + (-1 + Sqrt[2])*Tan[x/2])/Sqrt[3]] - Log[Sec[x/2]^2] + Log[Sec
[x/2]^2*(1 + Sqrt[2]*Cos[x] - Sqrt[2]*Sin[x])])/(12*Sqrt[2]) - ((2*(-2 + Sq
rt[6])*ArcTanh[Sqrt[2] + (Sqrt[2] - Sqrt[3])*Tan[x/2]] + (3*Sqrt[2] - 2*Sqr
t[3])*(x - Log[Sec[x/2]^2] + Log[-(Sec[x/2]^2*(Sqrt[3] + Sqrt[2]*Cos[x] - S
qrt[2]*Sin[x]))]))*(Sqrt[2] - Sqrt[3]*Sin[x])*(-3 + Sqrt[6] - (-2 + Sqrt[6]
)*Cos[x] + (-2 + Sqrt[6])*Sin[x]))/(12*(-36 + 15*Sqrt[6] + (20 - 8*Sqrt[6])
*Cos[x] + (12 - 5*Sqrt[6])*Cos[2*x] - 50*Sin[x] + 20*Sqrt[6]*Sin[x] + 12*Si
n[2*x] - 5*Sqrt[6]*Sin[2*x])) + ((-2*(Sqrt[2] + Sqrt[3])*ArcTanh[(2 + (2 +
Sqrt[6])*Tan[x/2])/Sqrt[2]] + (3 + Sqrt[6])*(x - Log[Sec[x/2]^2] + Log[-(Se
c[x/2]^2*(Sqrt[6] - 2*Cos[x] + 2*Sin[x]))]))*(2 + Sqrt[6]*Sin[x])*(3 + Sqrt
[6] - (2 + Sqrt[6])*Cos[x] + (2 + Sqrt[6])*Sin[x]))/(12*(-36 - 15*Sqrt[6] +
 4*(5 + 2*Sqrt[6])*Cos[x] + (12 + 5*Sqrt[6])*Cos[2*x] - 50*Sin[x] - 20*Sqrt
[6]*Sin[x] + 12*Sin[2*x] + 5*Sqrt[6]*Sin[2*x]))
```

fricas [A] time = 0.85, size = 134, normalized size = 1.51

$$\frac{1}{12}\sqrt{\sqrt{3}+2}\log\left(\sqrt{\sqrt{3}+2}+2\cos(x)\right)-\frac{1}{12}\sqrt{\sqrt{3}+2}\log\left(\sqrt{\sqrt{3}+2}-2\cos(x)\right)+\frac{1}{12}\sqrt{-\sqrt{3}+2}\log\left(\sqrt{-\sqrt{}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*tan(6*x),x, algorithm="fricas")
```

```
[Out] 1/12*sqrt(sqrt(3) + 2)*log(sqrt(sqrt(3) + 2) + 2*cos(x)) - 1/12*sqrt(sqrt(3
) + 2)*log(sqrt(sqrt(3) + 2) - 2*cos(x)) + 1/12*sqrt(-sqrt(3) + 2)*log(sqrt
(-sqrt(3) + 2) + 2*cos(x)) - 1/12*sqrt(-sqrt(3) + 2)*log(sqrt(-sqrt(3) + 2)
 - 2*cos(x)) + 1/12*sqrt(2)*log(-(2*cos(x)^2 + 2*sqrt(2)*cos(x) + 1)/(2*cos
(x)^2 - 1)) - cos(x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \cos(x)\tan(6x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*tan(6*x),x, algorithm="giac")
```

```
[Out] integrate(cos(x)*tan(6*x), x)
```

maple [A] time = 0.08, size = 104, normalized size = 1.17

$$-\cos(x)+\frac{2\left(-3+2\sqrt{3}\right)\sqrt{3}\ \operatorname{arctanh}\left(\frac{8\cos(x)}{2\sqrt{6}-2\sqrt{2}}\right)}{9\left(2\sqrt{6}-2\sqrt{2}\right)}+\frac{2\left(3+2\sqrt{3}\right)\sqrt{3}\ \operatorname{arctanh}\left(\frac{8\cos(x)}{2\sqrt{6}+2\sqrt{2}}\right)}{9\left(2\sqrt{6}+2\sqrt{2}\right)}+\frac{\operatorname{arctanh}\left(\cos(x)\sqrt{2}\right)\sqrt{}}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)*tan(6*x),x)
```

```
[Out] -cos(x)+2/9*(-3+2*3^(1/2))*3^(1/2)/(2*6^(1/2)-2*2^(1/2))*arctanh(8*cos(x)/(
2*6^(1/2)-2*2^(1/2)))+2/9*(3+2*3^(1/2))*3^(1/2)/(2*6^(1/2)+2*2^(1/2))*arcta
nh(8*cos(x)/(2*6^(1/2)+2*2^(1/2)))+1/6*arctanh(cos(x)*2^(1/2))*2^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{24}\sqrt{2}\log\left(2\sqrt{2}\sin\left(2x\right)\sin(x)+2\left(\sqrt{2}\cos(x)+1\right)\cos\left(2x\right)+\cos\left(2x\right)^{2}+2\cos(x)^{2}+\sin\left(2x\right)^{2}+2\sin(x)^{2}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*tan(6*x),x, algorithm="maxima")
```

```
[Out] 1/24*sqrt(2)*log(2*sqrt(2)*sin(2*x)*sin(x) + 2*(sqrt(2)*cos(x) + 1)*cos(2*x
) + cos(2*x)^2 + 2*cos(x)^2 + sin(2*x)^2 + 2*sin(x)^2 + 2*sqrt(2)*cos(x) +
1) - 1/24*sqrt(2)*log(-2*sqrt(2)*sin(2*x)*sin(x) - 2*(sqrt(2)*cos(x) - 1)*c
os(2*x) + cos(2*x)^2 + 2*cos(x)^2 + sin(2*x)^2 + 2*sin(x)^2 - 2*sqrt(2)*cos
(x) + 1) - cos(x) - integrate(1/3*((2*sin(7*x) + sin(5*x) - sin(3*x) - 2*si
n(x))*cos(8*x) + (sin(3*x) + 2*sin(x))*cos(4*x) - (2*cos(7*x) + cos(5*x) -
cos(3*x) - 2*cos(x))*sin(8*x) - 2*(cos(4*x) - 1)*sin(7*x) - (cos(4*x) - 1)*
sin(5*x) - (cos(3*x) + 2*cos(x))*sin(4*x) + 2*cos(7*x)*sin(4*x) + cos(5*x)*
sin(4*x) - sin(3*x) - 2*sin(x))/(2*(cos(4*x) - 1)*cos(8*x) - cos(8*x)^2 - c
os(4*x)^2 - sin(8*x)^2 + 2*sin(8*x)*sin(4*x) - sin(4*x)^2 + 2*cos(4*x) - 1)
, x)
```

mupad [B] time = 4.07, size = 787, normalized size = 8.84

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(6*x)*cos(x),x)
```

```
[Out] (6^(1/2)*(atan((2^(1/2)*321030945816576i)/(213254896304333030400*tan(x/2)^4
 - 129275829262795438080*tan(x/2)^2 + 2176593611144037376) + (6^(1/2)*88840
5273481134080i)/(213254896304333030400*tan(x/2)^4 - 129275829262795438080*t
an(x/2)^2 + 2176593611144037376) - (2^(1/2)*tan(x/2)^2*18711054724802560i)/
(213254896304333030400*tan(x/2)^4 - 129275829262795438080*tan(x/2)^2 + 2176
593611144037376) + (2^(1/2)*tan(x/2)^4*10905601889064960i)/(213254896304333
030400*tan(x/2)^4 - 129275829262795438080*tan(x/2)^2 + 2176593611144037376)
 - (6^(1/2)*tan(x/2)^2*52765833462352287744i)/(213254896304333030400*tan(x/
2)^4 - 129275829262795438080*tan(x/2)^2 + 2176593611144037376) + (6^(1/2)*t
an(x/2)^4*87054650497106012160i)/(213254896304333030400*tan(x/2)^4 - 129275
829262795438080*tan(x/2)^2 + 2176593611144037376)) + atan((2^(1/2)*14433255
04589801788190484332544i)/(589232404262260650654553866240*2^(1/2)*6^(1/2) +
 119129717169909888440949339586560*tan(x/2)^2 - 343672717269879599464668620
39040*2^(1/2)*6^(1/2)*tan(x/2)^2 - 2087090309450798997834557292544) - (6^(1
/2)*852047139771204346616741888000i)/(589232404262260650654553866240*2^(1/2
)*6^(1/2) + 119129717169909888440949339586560*tan(x/2)^2 - 3436727172698795
9946466862039040*2^(1/2)*6^(1/2)*tan(x/2)^2 - 20870903094507989978345572925
44) - (2^(1/2)*tan(x/2)^2*84182283571305304543568582410240i)/(5892324042622
60650654553866240*2^(1/2)*6^(1/2) + 119129717169909888440949339586560*tan(x
/2)^2 - 34367271726987959946466862039040*2^(1/2)*6^(1/2)*tan(x/2)^2 - 20870
90309450798997834557292544) + (6^(1/2)*tan(x/2)^2*4863450107523648650487342
4060416i)/(589232404262260650654553866240*2^(1/2)*6^(1/2) + 119129717169909
888440949339586560*tan(x/2)^2 - 34367271726987959946466862039040*2^(1/2)*6^
(1/2)*tan(x/2)^2 - 2087090309450798997834557292544)))*1i)/12 - 2/(tan(x/2)^
2 + 1) - (2^(1/2)*(2*atan((2^(1/2)*2276803846003180334341033033728i)/(18766
876017666378997952094928896*tan(x/2)^2 - 3219886877884552553529320931328) -
 (2^(1/2)*tan(x/2)^2*13270185293778646110081740963840i)/(187668760176663789
97952094928896*tan(x/2)^2 - 3219886877884552553529320931328)) - atan((2^(1/
2)*321030945816576i)/(213254896304333030400*tan(x/2)^4 - 129275829262795438
080*tan(x/2)^2 + 2176593611144037376) + (6^(1/2)*888405273481134080i)/(2132
54896304333030400*tan(x/2)^4 - 129275829262795438080*tan(x/2)^2 + 217659361
1144037376) - (2^(1/2)*tan(x/2)^2*18711054724802560i)/(21325489630433303040
0*tan(x/2)^4 - 129275829262795438080*tan(x/2)^2 + 2176593611144037376) + (2
^(1/2)*tan(x/2)^4*10905601889064960i)/(213254896304333030400*tan(x/2)^4 - 1
29275829262795438080*tan(x/2)^2 + 2176593611144037376) - (6^(1/2)*tan(x/2)^
2*52765833462352287744i)/(213254896304333030400*tan(x/2)^4 - 12927582926279
5438080*tan(x/2)^2 + 2176593611144037376) + (6^(1/2)*tan(x/2)^4*87054650497
106012160i)/(213254896304333030400*tan(x/2)^4 - 129275829262795438080*tan(x
/2)^2 + 2176593611144037376)) + atan((2^(1/2)*14433255045898017881904843325
44i)/(589232404262260650654553866240*2^(1/2)*6^(1/2) + 11912971716990988844
0949339586560*tan(x/2)^2 - 34367271726987959946466862039040*2^(1/2)*6^(1/2)
*tan(x/2)^2 - 2087090309450798997834557292544) - (6^(1/2)*85204713977120434
6616741888000i)/(589232404262260650654553866240*2^(1/2)*6^(1/2) + 119129717
```

```
169909888440949339586560*tan(x/2)^2 - 34367271726987959946466862039040*2^(1
/2)*6^(1/2)*tan(x/2)^2 - 2087090309450798997834557292544) - (2^(1/2)*tan(x/
2)^2*84182283571305304543568582410240i)/(589232404262260650654553866240*2^(
1/2)*6^(1/2) + 119129717169909888440949339586560*tan(x/2)^2 - 3436727172698
7959946466862039040*2^(1/2)*6^(1/2)*tan(x/2)^2 - 20870903094507989978345572
92544) + (6^(1/2)*tan(x/2)^2*48634501075236486504873424060416i)/(5892324042
62260650654553866240*2^(1/2)*6^(1/2) + 119129717169909888440949339586560*ta
n(x/2)^2 - 34367271726987959946466862039040*2^(1/2)*6^(1/2)*tan(x/2)^2 - 20
87090309450798997834557292544)))*1i)/12
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \cos(x)\tan(6x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*tan(6*x),x)
```

```
[Out] Integral(cos(x)*tan(6*x), x)
```

3.110 $\int \cos(x)\cot(2x)\,dx$

Optimal. Leaf size=10

$$\cos(x) - \frac{1}{2}\tanh^{-1}(\cos(x))$$

```
[Out] -1/2*arctanh(cos(x))+cos(x)
```

Rubi [A] time = 0.02, antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {12, 388, 206}

$$\cos(x) - \frac{1}{2}\tanh^{-1}(\cos(x))$$

Antiderivative was successfully verified.

```
[In] Int[Cos[x]*Cot[2*x],x]
```

```
[Out] -ArcTanh[Cos[x]]/2 + Cos[x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rubi steps

$$\int \cos(x)\cot(2x)\,dx = -\text{Subst}\left(\int \frac{-1+2x^2}{2(1-x^2)}\,dx, x, \cos(x)\right)$$
$$= -\left(\frac{1}{2}\text{Subst}\left(\int \frac{-1+2x^2}{1-x^2}\,dx, x, \cos(x)\right)\right)$$
$$= \cos(x) - \frac{1}{2}\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \cos(x)\right)$$
$$= -\frac{1}{2}\tanh^{-1}(\cos(x)) + \cos(x)$$

Mathematica [B] time = 0.01, size = 25, normalized size = 2.50

$$\cos(x) + \frac{1}{2}\log\left(\sin\left(\frac{x}{2}\right)\right) - \frac{1}{2}\log\left(\cos\left(\frac{x}{2}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[x]*Cot[2*x],x]
[Out] Cos[x] - Log[Cos[x/2]]/2 + Log[Sin[x/2]]/2
```

fricas [B] time = 1.30, size = 21, normalized size = 2.10

$$\cos(x) - \frac{1}{4}\log\left(\frac{1}{2}\cos(x) + \frac{1}{2}\right) + \frac{1}{4}\log\left(-\frac{1}{2}\cos(x) + \frac{1}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*cot(2*x),x, algorithm="fricas")
[Out] cos(x) - 1/4*log(1/2*cos(x) + 1/2) + 1/4*log(-1/2*cos(x) + 1/2)
```

giac [B] time = 0.14, size = 19, normalized size = 1.90

$$\cos(x) - \frac{1}{4}\log(\cos(x) + 1) + \frac{1}{4}\log(-\cos(x) + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*cot(2*x),x, algorithm="giac")
[Out] cos(x) - 1/4*log(cos(x) + 1) + 1/4*log(-cos(x) + 1)
```

maple [A] time = 0.09, size = 14, normalized size = 1.40

$$\cos(x) + \frac{\ln(\csc(x) - \cot(x))}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)*cot(2*x),x)
[Out] cos(x)+1/2*ln(csc(x)-cot(x))
```

maxima [B] time = 0.33, size = 37, normalized size = 3.70

$$\cos(x) - \frac{1}{4}\log\left(\cos(x)^2 + \sin(x)^2 + 2\cos(x) + 1\right) + \frac{1}{4}\log\left(\cos(x)^2 + \sin(x)^2 - 2\cos(x) + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*cot(2*x),x, algorithm="maxima")
[Out] cos(x) - 1/4*log(cos(x)^2 + sin(x)^2 + 2*cos(x) + 1) + 1/4*log(cos(x)^2 + s
in(x)^2 - 2*cos(x) + 1)
```

mupad [B] time = 2.34, size = 20, normalized size = 2.00

$$\frac{\ln\left(\tan\left(\frac{x}{2}\right)\right)}{2} + \frac{2}{\tan\left(\frac{x}{2}\right)^2 + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(2*x)*cos(x),x)
[Out] log(tan(x/2))/2 + 2/(tan(x/2)^2 + 1)
```

sympy [B] time = 0.83, size = 19, normalized size = 1.90

$$\frac{\log(\cos(x) - 1)}{4} - \frac{\log(\cos(x) + 1)}{4} + \cos(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*cot(2*x),x)
[Out] log(cos(x) - 1)/4 - log(cos(x) + 1)/4 + cos(x)
```

3.111 $\int \cos(x)\cot(3x)\,dx$

Optimal. Leaf size=45

$$\cos(x)+\frac{1}{6}\log(1-2\cos(x))+\frac{1}{6}\log(1-\cos(x))-\frac{1}{6}\log(\cos(x)+1)-\frac{1}{6}\log(2\cos(x)+1)$$

```
[Out] cos(x)+1/6*ln(1-2*cos(x))+1/6*ln(1-cos(x))-1/6*ln(1+cos(x))-1/6*ln(1+2*cos(
x))
```

Rubi [A] time = 0.05, antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 4, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.571, Rules used = {1279, 1161, 616, 31}

$$\cos(x)+\frac{1}{6}\log(1-2\cos(x))+\frac{1}{6}\log(1-\cos(x))-\frac{1}{6}\log(\cos(x)+1)-\frac{1}{6}\log(2\cos(x)+1)$$

Antiderivative was successfully verified.

```
[In] Int[Cos[x]*Cot[3*x],x]
```

```
[Out] Cos[x] + Log[1 - 2*Cos[x]]/6 + Log[1 - Cos[x]]/6 - Log[1 + Cos[x]]/6 - Log[
1 + 2*Cos[x]]/6
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 616

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = Rt[b^2
 - 4*a*c, 2]}, Dist[c/q, Int[1/Simp[b/2 - q/2 + c*x, x], x], x] - Dist[c/q,
 Int[1/Simp[b/2 + q/2 + c*x, x], x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[b^2 - 4*a*c] && PerfectSquareQ[b^2 - 4*a*c]
```

Rule 1161

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[(2*d)/e - b/c, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2
, x], x], x] + Dist[e/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; Fre
eQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] && (
GtQ[(2*d)/e - b/c, 0] || ( !LtQ[(2*d)/e - b/c, 0] && EqQ[d - e*Rt[a/c, 2],
0]))
```

Rule 1279

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[(e*f*(f*x)^(m - 1)*(a + b*x^2 + c*x^4)^(p +
1))/(c*(m + 4*p + 3)), x] - Dist[f^2/(c*(m + 4*p + 3)), Int[(f*x)^(m - 2)*(
a + b*x^2 + c*x^4)^p*Simp[a*e*(m - 1) + (b*e*(m + 2*p + 1) - c*d*(m + 4*p +
 3))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c,
 0] && GtQ[m, 1] && NeQ[m + 4*p + 3, 0] && IntegerQ[2*p] && (IntegerQ[p] ||
 IntegerQ[m])
```

Rubi steps

$$\begin{aligned}\int \cos(x)\cot(3x)\,dx &= -\text{Subst}\left(\int \frac{x^2\left(3-4x^2\right)}{1-5x^2+4x^4}\,dx,x,\cos(x)\right)\\ &= \cos(x)+\frac{1}{4}\text{Subst}\left(\int \frac{-4+8x^2}{1-5x^2+4x^4}\,dx,x,\cos(x)\right)\\ &= \cos(x)+\frac{1}{4}\text{Subst}\left(\int \frac{1}{-\frac{1}{2}-\frac{x}{2}+x^2}\,dx,x,\cos(x)\right)+\frac{1}{4}\text{Subst}\left(\int \frac{1}{-\frac{1}{2}+\frac{x}{2}+x^2}\,dx,x,\cos(x\right.\\ &= \cos(x)+\frac{1}{6}\text{Subst}\left(\int \frac{1}{-1+x}\,dx,x,\cos(x)\right)+\frac{1}{6}\text{Subst}\left(\int \frac{1}{-\frac{1}{2}+x}\,dx,x,\cos(x)\right)-\frac{1}{6}\text{Sub}\\ &= \cos(x)+\frac{1}{6}\log(1-2\cos(x))+\frac{1}{6}\log(1-\cos(x))-\frac{1}{6}\log(1+\cos(x))-\frac{1}{6}\log(1+2\cos(\end{aligned}$$

Mathematica [A] time = 0.02, size = 47, normalized size = 1.04

$$\cos(x)+\frac{1}{3}\log\left(\sin\left(\frac{x}{2}\right)\right)-\frac{1}{3}\log\left(\cos\left(\frac{x}{2}\right)\right)+\frac{1}{6}\log(1-2\cos(x))-\frac{1}{6}\log(2\cos(x)+1)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[x]*Cot[3*x],x]
```

```
[Out] Cos[x] - Log[Cos[x/2]]/3 + Log[1 - 2*Cos[x]]/6 - Log[1 + 2*Cos[x]]/6 + Log[
Sin[x/2]]/3
```

fricas [A] time = 3.04, size = 39, normalized size = 0.87

$$\cos(x)-\frac{1}{6}\log\left(\frac{1}{2}\cos(x)+\frac{1}{2}\right)+\frac{1}{6}\log\left(-\frac{1}{2}\cos(x)+\frac{1}{2}\right)+\frac{1}{6}\log\left(-2\cos(x)+1\right)-\frac{1}{6}\log\left(-2\cos(x)-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*cot(3*x),x, algorithm="fricas")
```

```
[Out] cos(x) - 1/6*log(1/2*cos(x) + 1/2) + 1/6*log(-1/2*cos(x) + 1/2) + 1/6*log(-
2*cos(x) + 1) - 1/6*log(-2*cos(x) - 1)
```

giac [A] time = 0.13, size = 39, normalized size = 0.87

$$\cos(x) - \frac{1}{6}\log(\cos(x)+1) + \frac{1}{6}\log(-\cos(x)+1) - \frac{1}{6}\log(|2\cos(x)+1|) + \frac{1}{6}\log(|2\cos(x)-1|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*cot(3*x),x, algorithm="giac")
```

```
[Out] cos(x) - 1/6*log(cos(x) + 1) + 1/6*log(-cos(x) + 1) - 1/6*log(abs(2*cos(x)
+ 1)) + 1/6*log(abs(2*cos(x) - 1))
```

maple [A] time = 0.32, size = 36, normalized size = 0.80

$$\frac{\ln(2\cos(x)-1)}{6} - \frac{\ln(1+2\cos(x))}{6} + \frac{\ln(-1+\cos(x))}{6} - \frac{\ln(1+\cos(x))}{6} + \cos(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)*cot(3*x),x)
```

```
[Out] 1/6*ln(2*cos(x)-1)-1/6*ln(1+2*cos(x))+1/6*ln(-1+cos(x))-1/6*ln(1+cos(x))+co
s(x)
```

maxima [B] time = 0.42, size = 131, normalized size = 2.91

$$\cos(x) - \frac{1}{12}\log\left(2(\cos(x)+1)\cos(2x) + \cos(2x)^2 + \cos(x)^2 + \sin(2x)^2 + 2\sin(2x)\sin(x) + \sin(x)^2 + 2\cos(x)\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*cot(3*x),x, algorithm="maxima")
```

```
[Out] cos(x) - 1/12*log(2*(cos(x) + 1)*cos(2*x) + cos(2*x)^2 + cos(x)^2 + sin(2*x
)^2 + 2*sin(2*x)*sin(x) + sin(x)^2 + 2*cos(x) + 1) + 1/12*log(-2*(cos(x) -
1)*cos(2*x) + cos(2*x)^2 + cos(x)^2 + sin(2*x)^2 - 2*sin(2*x)*sin(x) + sin(
x)^2 - 2*cos(x) + 1) - 1/6*log(cos(x)^2 + sin(x)^2 + 2*cos(x) + 1) + 1/6*lo
g(cos(x)^2 + sin(x)^2 - 2*cos(x) + 1)
```

mupad [B] time = 2.37, size = 39, normalized size = 0.87

$$\frac{\ln\left(\tan\left(\frac{x}{2}\right)\right)}{3}+\frac{\operatorname{atanh}\left(\frac{8}{183\left(\frac{488\tan\left(\frac{x}{2}\right)^2}{243}-\frac{56}{81}\right)}+\frac{121}{122}\right)}{3}+\frac{2}{\tan\left(\frac{x}{2}\right)^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(3*x)*cos(x),x)
```

```
[Out] log(tan(x/2))/3 + atanh(8/(183*((488*tan(x/2)^2)/243 - 56/81)) + 121/122)/3
 + 2/(tan(x/2)^2 + 1)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \cos(x)\cot(3x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*cot(3*x),x)
```

```
[Out] Integral(cos(x)*cot(3*x), x)
```

3.112 $\int \cos(x)\cot(4x)\,dx$

Optimal. Leaf size=28

$$\cos(x) - \frac{1}{4}\tanh^{-1}(\cos(x)) - \frac{\tanh^{-1}\left(\sqrt{2}\cos(x)\right)}{2\sqrt{2}}$$

```
[Out] -1/4*arctanh(cos(x))+cos(x)-1/4*arctanh(cos(x)*2^(1/2))*2^(1/2)
```

Rubi [A] time = 0.05, antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {1676, 1166, 207}

$$\cos(x) - \frac{1}{4}\tanh^{-1}(\cos(x)) - \frac{\tanh^{-1}\left(\sqrt{2}\cos(x)\right)}{2\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[x]*Cot[4*x],x]
```

```
[Out] -ArcTanh[Cos[x]]/4 - ArcTanh[Sqrt[2]*Cos[x]]/(2*Sqrt[2]) + Cos[x]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1676

```
Int[(Pq_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> Int[ExpandInte
grand[Pq/(a + b*x^2 + c*x^4), x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x^
2] && Expon[Pq, x^2] > 1
```

Rubi steps

$$\int \cos(x)\cot(4x)\,dx = -\text{Subst}\left(\int \frac{-1+8x^2-8x^4}{4-12x^2+8x^4}\,dx, x, \cos(x)\right)$$
$$= -\text{Subst}\left(\int \left(-1+\frac{3-4x^2}{4-12x^2+8x^4}\right)dx, x, \cos(x)\right)$$
$$= \cos(x) - \text{Subst}\left(\int \frac{3-4x^2}{4-12x^2+8x^4}\,dx, x, \cos(x)\right)$$
$$= \cos(x) + 2\,\text{Subst}\left(\int \frac{1}{-8+8x^2}\,dx, x, \cos(x)\right) + 2\,\text{Subst}\left(\int \frac{1}{-4+8x^2}\,dx, x, \cos(x)\right)$$
$$= -\frac{1}{4}\tanh^{-1}(\cos(x)) - \frac{\tanh^{-1}\left(\sqrt{2}\cos(x)\right)}{2\sqrt{2}} + \cos(x)$$

Mathematica [C] time = 0.07, size = 73, normalized size = 2.61

$$\frac{1}{4}\left(4\cos(x)+\log\left(\sin\left(\frac{x}{2}\right)\right)-\log\left(\cos\left(\frac{x}{2}\right)\right)+(-1-i)(-1)^{3/4}\tanh^{-1}\left(\frac{\tan\left(\frac{x}{2}\right)-1}{\sqrt{2}}\right)-(1-i)\sqrt[4]{-1}\,\tanh^{-1}\left(\frac{\tan\left(\frac{x}{2}\right.}{\sqrt{}}\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[x]*Cot[4*x],x]
```

```
[Out] ((-1 - I)*(-1)^(3/4)*ArcTanh[(-1 + Tan[x/2])/Sqrt[2]] - (1 - I)*(-1)^(1/4)*
ArcTanh[(1 + Tan[x/2])/Sqrt[2]] + 4*Cos[x] - Log[Cos[x/2]] + Log[Sin[x/2]])
/4
```

fricas [B] time = 1.01, size = 53, normalized size = 1.89

$$\frac{1}{8}\sqrt{2}\log\left(\frac{2\cos(x)^2-2\sqrt{2}\cos(x)+1}{2\cos(x)^2-1}\right)+\cos(x)-\frac{1}{8}\log\left(\frac{1}{2}\cos(x)+\frac{1}{2}\right)+\frac{1}{8}\log\left(-\frac{1}{2}\cos(x)+\frac{1}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*cot(4*x),x, algorithm="fricas")
```

```
[Out] 1/8*sqrt(2)*log((2*cos(x)^2 - 2*sqrt(2)*cos(x) + 1)/(2*cos(x)^2 - 1)) + cos
(x) - 1/8*log(1/2*cos(x) + 1/2) + 1/8*log(-1/2*cos(x) + 1/2)
```

giac [B] time = 0.14, size = 50, normalized size = 1.79

$$\frac{1}{8}\sqrt{2}\log\left(\frac{\left|-2\sqrt{2}+4\cos(x)\right|}{\left|2\sqrt{2}+4\cos(x)\right|}\right)+\cos(x)-\frac{1}{8}\log\left(\cos(x)+1\right)+\frac{1}{8}\log\left(-\cos(x)+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*cot(4*x),x, algorithm="giac")
```

```
[Out] 1/8*sqrt(2)*log(abs(-2*sqrt(2) + 4*cos(x))/abs(2*sqrt(2) + 4*cos(x))) + cos
(x) - 1/8*log(cos(x) + 1) + 1/8*log(-cos(x) + 1)
```

maple [A] time = 0.41, size = 30, normalized size = 1.07

$$\frac{\ln\left(-1+\cos(x)\right)}{8}-\frac{\operatorname{arctanh}\left(\cos(x)\sqrt{2}\right)\sqrt{2}}{4}-\frac{\ln\left(1+\cos(x)\right)}{8}+\cos(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)*cot(4*x),x)
```

```
[Out] 1/8*ln(-1+cos(x))-1/4*arctanh(cos(x)*2^(1/2))*2^(1/2)-1/8*ln(1+cos(x))+cos(
x)
```

maxima [B] time = 0.42, size = 165, normalized size = 5.89

$$-\frac{1}{16}\sqrt{2}\log\left(2\sqrt{2}\sin\left(2x\right)\sin(x)+2\left(\sqrt{2}\cos(x)+1\right)\cos\left(2x\right)+\cos\left(2x\right)^{2}+2\cos(x)^{2}+\sin\left(2x\right)^{2}+2\sin(x)^{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*cot(4*x),x, algorithm="maxima")
```

```
[Out] -1/16*sqrt(2)*log(2*sqrt(2)*sin(2*x)*sin(x) + 2*(sqrt(2)*cos(x) + 1)*cos(2*
x) + cos(2*x)^2 + 2*cos(x)^2 + sin(2*x)^2 + 2*sin(x)^2 + 2*sqrt(2)*cos(x) +
 1) + 1/16*sqrt(2)*log(-2*sqrt(2)*sin(2*x)*sin(x) - 2*(sqrt(2)*cos(x) - 1)*
cos(2*x) + cos(2*x)^2 + 2*cos(x)^2 + sin(2*x)^2 + 2*sin(x)^2 - 2*sqrt(2)*co
s(x) + 1) + cos(x) - 1/8*log(cos(x)^2 + sin(x)^2 + 2*cos(x) + 1) + 1/8*log(
cos(x)^2 + sin(x)^2 - 2*cos(x) + 1)
```

mupad [B] time = 2.35, size = 67, normalized size = 2.39

$$\frac{\ln\left(\tan\left(\frac{x}{2}\right)\right)}{4}-\frac{\sqrt{2}\operatorname{atanh}\left(\frac{7\sqrt{2}}{8\left(\frac{29\tan\left(\frac{x}{2}\right)^2}{4}-\frac{5}{4}\right)}-\frac{41\sqrt{2}\tan\left(\frac{x}{2}\right)^2}{8\left(\frac{29\tan\left(\frac{x}{2}\right)^2}{4}-\frac{5}{4}\right)}\right)}{4}+\frac{2}{\tan\left(\frac{x}{2}\right)^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(4*x)*cos(x),x)
```

```
[Out] log(tan(x/2))/4 - (2^(1/2)*atanh((7*2^(1/2))/(8*((29*tan(x/2)^2)/4 - 5/4))
- (41*2^(1/2)*tan(x/2)^2)/(8*((29*tan(x/2)^2)/4 - 5/4))))/4 + 2/(tan(x/2)^2
 + 1)
```

sympy [F]   time = 0.00, size = 0, normalized size = 0.00

$$\int \cos(x)\cot(4x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*cot(4*x),x)
```

```
[Out] Integral(cos(x)*cot(4*x), x)
```

3.113 $\int \cos(x)\cot(5x)\,dx$

Optimal. Leaf size=110

$$\cos(x)+\frac{1}{20}\left(1-\sqrt{5}\right)\log\left(-4\cos(x)-\sqrt{5}+1\right)+\frac{1}{20}\left(1+\sqrt{5}\right)\log\left(-4\cos(x)+\sqrt{5}+1\right)-\frac{1}{20}\left(1-\sqrt{5}\right)\log\left(4\cos(\right.$$

```
[Out] -1/5*arctanh(cos(x))+cos(x)+1/20*ln(1-4*cos(x)-5^(1/2))*(-5^(1/2)+1)-1/20*l
n(1+4*cos(x)-5^(1/2))*(-5^(1/2)+1)+1/20*ln(1-4*cos(x)+5^(1/2))*(5^(1/2)+1)-
1/20*ln(1+4*cos(x)+5^(1/2))*(5^(1/2)+1)
```

Rubi [A] time = 0.16, antiderivative size = 110, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 4, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.571, Rules used = {2075, 207, 632, 31}

$$\cos(x)+\frac{1}{20}\left(1-\sqrt{5}\right)\log\left(-4\cos(x)-\sqrt{5}+1\right)+\frac{1}{20}\left(1+\sqrt{5}\right)\log\left(-4\cos(x)+\sqrt{5}+1\right)-\frac{1}{20}\left(1-\sqrt{5}\right)\log\left(4\cos(\right.$$

Antiderivative was successfully verified.

```
[In] Int[Cos[x]*Cot[5*x],x]
```

```
[Out] -ArcTanh[Cos[x]]/5 + Cos[x] + ((1 - Sqrt[5])*Log[1 - Sqrt[5] - 4*Cos[x]])/2
0 + ((1 + Sqrt[5])*Log[1 + Sqrt[5] - 4*Cos[x]])/20 - ((1 - Sqrt[5])*Log[1 -
 Sqrt[5] + 4*Cos[x]])/20 - ((1 + Sqrt[5])*Log[1 + Sqrt[5] + 4*Cos[x]])/20
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 632

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> W
ith[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(c*d - e*(b/2 - q/2))/q, Int[1/(b/2 - q/
2 + c*x), x], x] - Dist[(c*d - e*(b/2 + q/2))/q, Int[1/(b/2 + q/2 + c*x), x
], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a
*c, 0] && NiceSqrtQ[b^2 - 4*a*c]
```

Rule 2075

```
Int[(P_)^(p_)*(Qm_), x_Symbol] :> With[{PP = Factor[P]}, Int[ExpandIntegran
d[PP^p*Qm, x], x] /; QuadraticProductQ[PP, x]] /; PolyQ[Qm, x] && PolyQ[P,
x] && ILtQ[p, 0]
```

Rubi steps

$$\int \cos(x)\cot(5x)\,dx = -\text{Subst}\left(\int \frac{x^2\left(5-20x^2+16x^4\right)}{1-13x^2+28x^4-16x^6}\,dx, x, \cos(x)\right)$$
$$= -\text{Subst}\left(\int \left(-1-\frac{1}{5\left(-1+x^2\right)}-\frac{2(1+x)}{5\left(-1-2x+4x^2\right)}+\frac{2(-1+x)}{5\left(-1+2x+4x^2\right)}\right)dx, x, \cos(x)\right)$$
$$= \cos(x)+\frac{1}{5}\text{Subst}\left(\int \frac{1}{-1+x^2}\,dx, x, \cos(x)\right)+\frac{2}{5}\text{Subst}\left(\int \frac{1+x}{-1-2x+4x^2}\,dx, x, \cos(x)\right)$$
$$= -\frac{1}{5}\tanh^{-1}(\cos(x))+\cos(x)-\frac{1}{5}\left(1-\sqrt{5}\right)\text{Subst}\left(\int \frac{1}{1-\sqrt{5}+4x}\,dx, x, \cos(x)\right)+\frac{1}{5}\left(1-\right.$$
$$= -\frac{1}{5}\tanh^{-1}(\cos(x))+\cos(x)+\frac{1}{20}\left(1-\sqrt{5}\right)\log\left(1-\sqrt{5}-4\cos(x)\right)+\frac{1}{20}\left(1+\sqrt{5}\right)\log\left(\right.$$

Mathematica [A] time = 0.12, size = 133, normalized size = 1.21

$$\frac{1}{100}\left(100\cos(x)+20\log\left(\sin\left(\frac{x}{2}\right)\right)-20\log\left(\cos\left(\frac{x}{2}\right)\right)+\sqrt{5}\left(\sqrt{5}-5\right)\log\left(-4\cos(x)-\sqrt{5}+1\right)+\sqrt{5}\left(5+\sqrt{5}\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[x]*Cot[5*x],x]
```

```
[Out] (100*Cos[x] - 20*Log[Cos[x/2]] + Sqrt[5]*(-5 + Sqrt[5])*Log[1 - Sqrt[5] - 4
*Cos[x]] + Sqrt[5]*(5 + Sqrt[5])*Log[1 + Sqrt[5] - 4*Cos[x]] - Sqrt[5]*(-5
+ Sqrt[5])*Log[1 - Sqrt[5] + 4*Cos[x]] - Sqrt[5]*(5 + Sqrt[5])*Log[1 + Sqrt
[5] + 4*Cos[x]] + 20*Log[Sin[x/2]])/100
```

fricas [A] time = 0.62, size = 137, normalized size = 1.25

$$\frac{1}{20}\sqrt{5}\log\left(-\frac{4\left(\sqrt{5}-1\right)\cos(x)-8\cos(x)^2+\sqrt{5}-3}{4\cos(x)^2+2\cos(x)-1}\right)+\frac{1}{20}\sqrt{5}\log\left(-\frac{4\left(\sqrt{5}+1\right)\cos(x)-8\cos(x)^2-\sqrt{5}-3}{4\cos(x)^2-2\cos(x)-1}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*cot(5*x),x, algorithm="fricas")
```

```
[Out] 1/20*sqrt(5)*log(-(4*(sqrt(5) - 1)*cos(x) - 8*cos(x)^2 + sqrt(5) - 3)/(4*co
s(x)^2 + 2*cos(x) - 1)) + 1/20*sqrt(5)*log(-(4*(sqrt(5) + 1)*cos(x) - 8*cos
(x)^2 - sqrt(5) - 3)/(4*cos(x)^2 - 2*cos(x) - 1)) + cos(x) - 1/20*log(4*cos
(x)^2 + 2*cos(x) - 1) + 1/20*log(4*cos(x)^2 - 2*cos(x) - 1) - 1/10*log(1/2*
cos(x) + 1/2) + 1/10*log(-1/2*cos(x) + 1/2)
```

giac [A] time = 0.16, size = 117, normalized size = 1.06

$$\frac{1}{20}\sqrt{5}\log\left(\frac{\left|-2\sqrt{5}+8\cos(x)+2\right|}{\left|2\sqrt{5}+8\cos(x)+2\right|}\right)+\frac{1}{20}\sqrt{5}\log\left(\frac{\left|-2\sqrt{5}+8\cos(x)-2\right|}{\left|2\sqrt{5}+8\cos(x)-2\right|}\right)+\cos(x)-\frac{1}{10}\log\left(\cos(x)+1\right)+\frac{1}{10}\log$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*cot(5*x),x, algorithm="giac")
```

```
[Out] 1/20*sqrt(5)*log(abs(-2*sqrt(5) + 8*cos(x) + 2)/abs(2*sqrt(5) + 8*cos(x) +
2)) + 1/20*sqrt(5)*log(abs(-2*sqrt(5) + 8*cos(x) - 2)/abs(2*sqrt(5) + 8*cos
(x) - 2)) + cos(x) - 1/10*log(cos(x) + 1) + 1/10*log(-cos(x) + 1) - 1/20*lo
g(abs(4*cos(x)^2 + 2*cos(x) - 1)) + 1/20*log(abs(4*cos(x)^2 - 2*cos(x) - 1)
)
```

maple [A] time = 0.44, size = 82, normalized size = 0.75

$$-\frac{\ln\left(4\left(\cos^{2}(x)\right)+2\cos(x)-1\right)}{20}-\frac{\sqrt{5}\ \operatorname{arctanh}\left(\frac{(8\cos(x)+2)\sqrt{5}}{10}\right)}{10}+\frac{\ln\left(-1+\cos(x)\right)}{10}+\frac{\ln\left(4\left(\cos^{2}(x)\right)-2\cos(x)-1\right)}{20}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)*cot(5*x),x)
```

```
[Out] -1/20*ln(4*cos(x)^2+2*cos(x)-1)-1/10*5^(1/2)*arctanh(1/10*(8*cos(x)+2)*5^(1
/2))+1/10*ln(-1+cos(x))+1/20*ln(4*cos(x)^2-2*cos(x)-1)-1/10*5^(1/2)*arctanh
(1/10*(8*cos(x)-2)*5^(1/2))-1/10*ln(1+cos(x))+cos(x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*cot(5*x),x, algorithm="maxima")
```

```
[Out] cos(x) + 1/10*integrate(-(cos(2*x)*sin(4*x) - cos(4*x)*sin(2*x) + cos(3/2*a
rctan2(sin(2*x), cos(2*x)))*sin(2*x) + cos(1/2*arctan2(sin(2*x), cos(2*x)))
*sin(2*x) - cos(2*x)*sin(3/2*arctan2(sin(2*x), cos(2*x))) - cos(2*x)*sin(1/
2*arctan2(sin(2*x), cos(2*x))) - sin(2*x))/(2*(cos(2*x) + 1)*cos(4*x) + cos
```

```
(4*x)^2 + cos(2*x)^2 - 2*(cos(4*x) + cos(2*x) - cos(1/2*arctan2(sin(2*x), c
os(2*x))) + 1)*cos(3/2*arctan2(sin(2*x), cos(2*x))) + cos(3/2*arctan2(sin(2
*x), cos(2*x)))^2 - 2*(cos(4*x) + cos(2*x) + 1)*cos(1/2*arctan2(sin(2*x), c
os(2*x))) + cos(1/2*arctan2(sin(2*x), cos(2*x)))^2 + sin(4*x)^2 + 2*sin(4*x
)*sin(2*x) + sin(2*x)^2 - 2*(sin(4*x) + sin(2*x) - sin(1/2*arctan2(sin(2*x)
, cos(2*x))))*sin(3/2*arctan2(sin(2*x), cos(2*x))) + sin(3/2*arctan2(sin(2*
x), cos(2*x)))^2 - 2*(sin(4*x) + sin(2*x))*sin(1/2*arctan2(sin(2*x), cos(2*
x))) + sin(1/2*arctan2(sin(2*x), cos(2*x)))^2 + 2*cos(2*x) + 1), x) + 1/10*
integrate((cos(2*x)*sin(4*x) - cos(4*x)*sin(2*x) - cos(3/2*arctan2(sin(2*x)
, cos(2*x)))*sin(2*x) - cos(1/2*arctan2(sin(2*x), cos(2*x)))*sin(2*x) + cos
(2*x)*sin(3/2*arctan2(sin(2*x), cos(2*x))) + cos(2*x)*sin(1/2*arctan2(sin(2
*x), cos(2*x))) - sin(2*x))/(2*(cos(2*x) + 1)*cos(4*x) + cos(4*x)^2 + cos(2
*x)^2 + 2*(cos(4*x) + cos(2*x) + cos(1/2*arctan2(sin(2*x), cos(2*x))) + 1)*
cos(3/2*arctan2(sin(2*x), cos(2*x))) + cos(3/2*arctan2(sin(2*x), cos(2*x)))
^2 + 2*(cos(4*x) + cos(2*x) + 1)*cos(1/2*arctan2(sin(2*x), cos(2*x))) + cos
(1/2*arctan2(sin(2*x), cos(2*x)))^2 + sin(4*x)^2 + 2*sin(4*x)*sin(2*x) + si
n(2*x)^2 + 2*(sin(4*x) + sin(2*x) + sin(1/2*arctan2(sin(2*x), cos(2*x))))*s
in(3/2*arctan2(sin(2*x), cos(2*x))) + sin(3/2*arctan2(sin(2*x), cos(2*x)))^
2 + 2*(sin(4*x) + sin(2*x))*sin(1/2*arctan2(sin(2*x), cos(2*x))) + sin(1/2*
arctan2(sin(2*x), cos(2*x)))^2 + 2*cos(2*x) + 1), x) - 1/10*integrate((cos(
x)*sin(4*x) + cos(x)*sin(3*x) + cos(x)*sin(2*x) - cos(4*x)*sin(x) - cos(3*x
)*sin(x) - cos(2*x)*sin(x) - sin(x))/(2*(cos(3*x) + cos(2*x) + cos(x) + 1)*
cos(4*x) + cos(4*x)^2 + 2*(cos(2*x) + cos(x) + 1)*cos(3*x) + cos(3*x)^2 + 2
*(cos(x) + 1)*cos(2*x) + cos(2*x)^2 + cos(x)^2 + 2*(sin(3*x) + sin(2*x) + s
in(x))*sin(4*x) + sin(4*x)^2 + 2*(sin(2*x) + sin(x))*sin(3*x) + sin(3*x)^2
+ sin(2*x)^2 + 2*sin(2*x)*sin(x) + sin(x)^2 + 2*cos(x) + 1), x) - 1/10*inte
grate(-(cos(x)*sin(4*x) - cos(x)*sin(3*x) + cos(x)*sin(2*x) - cos(4*x)*sin(
x) + cos(3*x)*sin(x) - cos(2*x)*sin(x) - sin(x))/(2*(cos(3*x) - cos(2*x) +
cos(x) - 1)*cos(4*x) - cos(4*x)^2 + 2*(cos(2*x) - cos(x) + 1)*cos(3*x) - co
s(3*x)^2 + 2*(cos(x) - 1)*cos(2*x) - cos(2*x)^2 - cos(x)^2 + 2*(sin(3*x) -
sin(2*x) + sin(x))*sin(4*x) - sin(4*x)^2 + 2*(sin(2*x) - sin(x))*sin(3*x) -
 sin(3*x)^2 - sin(2*x)^2 + 2*sin(2*x)*sin(x) - sin(x)^2 + 2*cos(x) - 1), x)
 + 3/10*integrate(-(cos(4/3*arctan2(sin(3*x), cos(3*x)))*sin(3*x) + cos(2/3
*arctan2(sin(3*x), cos(3*x)))*sin(3*x) + cos(1/3*arctan2(sin(3*x), cos(3*x)
))*sin(3*x) - cos(3*x)*sin(4/3*arctan2(sin(3*x), cos(3*x))) - cos(3*x)*sin(
2/3*arctan2(sin(3*x), cos(3*x))) - cos(3*x)*sin(1/3*arctan2(sin(3*x), cos(3
*x))) + sin(3*x))/(cos(3*x)^2 + 2*(cos(3*x) + cos(2/3*arctan2(sin(3*x), cos
(3*x))) + cos(1/3*arctan2(sin(3*x), cos(3*x))) + 1)*cos(4/3*arctan2(sin(3*x
), cos(3*x))) + cos(4/3*arctan2(sin(3*x), cos(3*x)))^2 + 2*(cos(3*x) + cos(
1/3*arctan2(sin(3*x), cos(3*x))) + 1)*cos(2/3*arctan2(sin(3*x), cos(3*x)))
+ cos(2/3*arctan2(sin(3*x), cos(3*x)))^2 + 2*(cos(3*x) + 1)*cos(1/3*arctan2
(sin(3*x), cos(3*x))) + cos(1/3*arctan2(sin(3*x), cos(3*x)))^2 + sin(3*x)^2
 + 2*(sin(3*x) + sin(2/3*arctan2(sin(3*x), cos(3*x))) + sin(1/3*arctan2(sin
(3*x), cos(3*x))))*sin(4/3*arctan2(sin(3*x), cos(3*x))) + sin(4/3*arctan2(s
in(3*x), cos(3*x)))^2 + 2*(sin(3*x) + sin(1/3*arctan2(sin(3*x), cos(3*x))))
```

```
*sin(2/3*arctan2(sin(3*x), cos(3*x))) + sin(2/3*arctan2(sin(3*x), cos(3*x))
)^2 + 2*sin(3*x)*sin(1/3*arctan2(sin(3*x), cos(3*x))) + sin(1/3*arctan2(sin
(3*x), cos(3*x)))^2 + 2*cos(3*x) + 1), x) + 3/10*integrate(-(cos(4/3*arctan
2(sin(3*x), cos(3*x)))*sin(3*x) + cos(2/3*arctan2(sin(3*x), cos(3*x)))*sin(
3*x) - cos(1/3*arctan2(sin(3*x), cos(3*x)))*sin(3*x) - cos(3*x)*sin(4/3*arc
tan2(sin(3*x), cos(3*x))) - cos(3*x)*sin(2/3*arctan2(sin(3*x), cos(3*x))) +
 cos(3*x)*sin(1/3*arctan2(sin(3*x), cos(3*x))) + sin(3*x))/(cos(3*x)^2 - 2*
(cos(3*x) - cos(2/3*arctan2(sin(3*x), cos(3*x))) + cos(1/3*arctan2(sin(3*x)
, cos(3*x))) - 1)*cos(4/3*arctan2(sin(3*x), cos(3*x))) + cos(4/3*arctan2(si
n(3*x), cos(3*x)))^2 - 2*(cos(3*x) + cos(1/3*arctan2(sin(3*x), cos(3*x))) -
 1)*cos(2/3*arctan2(sin(3*x), cos(3*x))) + cos(2/3*arctan2(sin(3*x), cos(3*
x)))^2 + 2*(cos(3*x) - 1)*cos(1/3*arctan2(sin(3*x), cos(3*x))) + cos(1/3*ar
ctan2(sin(3*x), cos(3*x)))^2 + sin(3*x)^2 - 2*(sin(3*x) - sin(2/3*arctan2(s
in(3*x), cos(3*x))) + sin(1/3*arctan2(sin(3*x), cos(3*x))))*sin(4/3*arctan2
(sin(3*x), cos(3*x))) + sin(4/3*arctan2(sin(3*x), cos(3*x)))^2 - 2*(sin(3*x
) + sin(1/3*arctan2(sin(3*x), cos(3*x))))*sin(2/3*arctan2(sin(3*x), cos(3*x
))) + sin(2/3*arctan2(sin(3*x), cos(3*x)))^2 + 2*sin(3*x)*sin(1/3*arctan2(s
in(3*x), cos(3*x))) + sin(1/3*arctan2(sin(3*x), cos(3*x)))^2 - 2*cos(3*x) +
 1), x) + 1/5*integrate((sin(4*x) + sin(3*x) + sin(2*x) + sin(x))/(2*(cos(3
*x) + cos(2*x) + cos(x) + 1)*cos(4*x) + cos(4*x)^2 + 2*(cos(2*x) + cos(x) +
 1)*cos(3*x) + cos(3*x)^2 + 2*(cos(x) + 1)*cos(2*x) + cos(2*x)^2 + cos(x)^2
 + 2*(sin(3*x) + sin(2*x) + sin(x))*sin(4*x) + sin(4*x)^2 + 2*(sin(2*x) + s
in(x))*sin(3*x) + sin(3*x)^2 + sin(2*x)^2 + 2*sin(2*x)*sin(x) + sin(x)^2 +
2*cos(x) + 1), x) - 1/5*integrate(-(sin(4*x) - sin(3*x) + sin(2*x) - sin(x)
)/(2*(cos(3*x) - cos(2*x) + cos(x) - 1)*cos(4*x) - cos(4*x)^2 + 2*(cos(2*x)
 - cos(x) + 1)*cos(3*x) - cos(3*x)^2 + 2*(cos(x) - 1)*cos(2*x) - cos(2*x)^2
 - cos(x)^2 + 2*(sin(3*x) - sin(2*x) + sin(x))*sin(4*x) - sin(4*x)^2 + 2*(s
in(2*x) - sin(x))*sin(3*x) - sin(3*x)^2 - sin(2*x)^2 + 2*sin(2*x)*sin(x) -
sin(x)^2 + 2*cos(x) - 1), x) - 1/10*log(cos(x)^2 + sin(x)^2 + 2*cos(x) + 1)
 + 1/10*log(cos(x)^2 + sin(x)^2 - 2*cos(x) + 1)
```

mupad [B] time = 3.03, size = 611, normalized size = 5.55

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(5*x)*cos(x),x)
```

```
[Out] (atan((tan(x/2)^2*4813499234516992i)/(1220703125*((213485644414976*5^(1/2))
/1220703125 - (2152646198689792*5^(1/2)*tan(x/2)^2)/1220703125 - (495922908
5483008*tan(x/2)^2)/1220703125 + 110872433262592/244140625)) - 954873235374
08i/(244140625*((213485644414976*5^(1/2))/1220703125 - (2152646198689792*5^
(1/2)*tan(x/2)^2)/1220703125 - (4959229085483008*tan(x/2)^2)/1220703125 + 1
10872433262592/244140625)) - (5^(1/2)*247887795585024i)/(1220703125*((21348
5644414976*5^(1/2))/1220703125 - (2152646198689792*5^(1/2)*tan(x/2)^2)/1220
```

```
703125 - (4959229085483008*tan(x/2)^2)/1220703125 + 110872433262592/2441406
25)) + (5^(1/2)*tan(x/2)^2*2217818569310208i)/(1220703125*((213485644414976
*5^(1/2))/1220703125 - (2152646198689792*5^(1/2)*tan(x/2)^2)/1220703125 - (
4959229085483008*tan(x/2)^2)/1220703125 + 110872433262592/244140625)))*1i)/
10 + (atan(95487323537408i/(244140625*((213485644414976*5^(1/2))/1220703125
 - (2152646198689792*5^(1/2)*tan(x/2)^2)/1220703125 + (4959229085483008*tan
(x/2)^2)/1220703125 - 110872433262592/244140625)) - (5^(1/2)*24788779558502
4i)/(1220703125*((213485644414976*5^(1/2))/1220703125 - (2152646198689792*5
^(1/2)*tan(x/2)^2)/1220703125 + (4959229085483008*tan(x/2)^2)/1220703125 -
110872433262592/244140625)) - (tan(x/2)^2*4813499234516992i)/(1220703125*((
213485644414976*5^(1/2))/1220703125 - (2152646198689792*5^(1/2)*tan(x/2)^2)
/1220703125 + (4959229085483008*tan(x/2)^2)/1220703125 - 110872433262592/24
4140625)) + (5^(1/2)*tan(x/2)^2*2217818569310208i)/(1220703125*((2134856444
14976*5^(1/2))/1220703125 - (2152646198689792*5^(1/2)*tan(x/2)^2)/122070312
5 + (4959229085483008*tan(x/2)^2)/1220703125 - 110872433262592/244140625)))
*1i)/10 + log(tan(x/2))/5 + 2/(tan(x/2)^2 + 1) + (5^(1/2)*(atan((tan(x/2)^2
*4813499234516992i)/(1220703125*((213485644414976*5^(1/2))/1220703125 - (21
52646198689792*5^(1/2)*tan(x/2)^2)/1220703125 - (4959229085483008*tan(x/2)^
2)/1220703125 + 110872433262592/244140625)) - 95487323537408i/(244140625*((
213485644414976*5^(1/2))/1220703125 - (2152646198689792*5^(1/2)*tan(x/2)^2)
/1220703125 - (4959229085483008*tan(x/2)^2)/1220703125 + 110872433262592/24
4140625)) - (5^(1/2)*247887795585024i)/(1220703125*((213485644414976*5^(1/2
))/1220703125 - (2152646198689792*5^(1/2)*tan(x/2)^2)/1220703125 - (4959229
085483008*tan(x/2)^2)/1220703125 + 110872433262592/244140625)) + (5^(1/2)*t
an(x/2)^2*2217818569310208i)/(1220703125*((213485644414976*5^(1/2))/1220703
125 - (2152646198689792*5^(1/2)*tan(x/2)^2)/1220703125 - (4959229085483008*
tan(x/2)^2)/1220703125 + 110872433262592/244140625))) - atan(95487323537408
i/(244140625*((213485644414976*5^(1/2))/1220703125 - (2152646198689792*5^(1
/2)*tan(x/2)^2)/1220703125 + (4959229085483008*tan(x/2)^2)/1220703125 - 110
872433262592/244140625)) - (5^(1/2)*247887795585024i)/(1220703125*((2134856
44414976*5^(1/2))/1220703125 - (2152646198689792*5^(1/2)*tan(x/2)^2)/122070
3125 + (4959229085483008*tan(x/2)^2)/1220703125 - 110872433262592/244140625
)) - (tan(x/2)^2*4813499234516992i)/(1220703125*((213485644414976*5^(1/2))/
1220703125 - (2152646198689792*5^(1/2)*tan(x/2)^2)/1220703125 + (4959229085
483008*tan(x/2)^2)/1220703125 - 110872433262592/244140625)) + (5^(1/2)*tan(
x/2)^2*2217818569310208i)/(1220703125*((213485644414976*5^(1/2))/1220703125
 - (2152646198689792*5^(1/2)*tan(x/2)^2)/1220703125 + (4959229085483008*tan
(x/2)^2)/1220703125 - 110872433262592/244140625))))*1i)/10
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \cos(x)\cot(5x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*cot(5*x),x)
[Out] Integral(cos(x)*cot(5*x), x)
```

3.114 $\int \cos(x)\cot(6x)\,dx$

Optimal. Leaf size=38

$$\cos(x)-\frac{1}{6}\tanh^{-1}(\cos(x))-\frac{1}{6}\tanh^{-1}(2\cos(x))-\frac{\tanh^{-1}\left(\frac{2\cos(x)}{\sqrt{3}}\right)}{2\sqrt{3}}$$

```
[Out] -1/6*arctanh(cos(x))-1/6*arctanh(2*cos(x))+cos(x)-1/6*arctanh(2/3*cos(x)*3^
(1/2))*3^(1/2)
```

Rubi [A] time = 0.07, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 3, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {12, 2073, 207}

$$\cos(x)-\frac{1}{6}\tanh^{-1}(\cos(x))-\frac{1}{6}\tanh^{-1}(2\cos(x))-\frac{\tanh^{-1}\left(\frac{2\cos(x)}{\sqrt{3}}\right)}{2\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[x]*Cot[6*x],x]
```

```
[Out] -ArcTanh[Cos[x]]/6 - ArcTanh[2*Cos[x]]/6 - ArcTanh[(2*Cos[x])/Sqrt[3]]/(2*S
qrt[3]) + Cos[x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 2073

```
Int[(P_)^(p_)*(Q_)^(q_.), x_Symbol] :> With[{PP = Factor[P /. x -> Sqrt[x]]
}, Int[ExpandIntegrand[(PP /. x -> x^2)^p*Q^q, x], x] /;  !SumQ[NonfreeFact
ors[PP, x]]] /; FreeQ[q, x] && PolyQ[P, x^2] && PolyQ[Q, x] && ILtQ[p, 0]
```

Rubi steps

$$\int \cos(x)\cot(6x)\,dx = -\text{Subst}\left(\int \frac{-1+18x^2-48x^4+32x^6}{2\left(3-19x^2+32x^4-16x^6\right)}\,dx, x, \cos(x)\right)$$
$$= -\left(\frac{1}{2}\text{Subst}\left(\int \frac{-1+18x^2-48x^4+32x^6}{3-19x^2+32x^4-16x^6}\,dx, x, \cos(x)\right)\right)$$
$$= -\left(\frac{1}{2}\text{Subst}\left(\int \left(-2-\frac{1}{3\left(-1+x^2\right)}-\frac{2}{-3+4x^2}-\frac{2}{3\left(-1+4x^2\right)}\right)dx, x, \cos(x)\right)\right)$$
$$= \cos(x)+\frac{1}{6}\text{Subst}\left(\int \frac{1}{-1+x^2}\,dx, x, \cos(x)\right)+\frac{1}{3}\text{Subst}\left(\int \frac{1}{-1+4x^2}\,dx, x, \cos(x)\right)+\text{Subs}$$
$$= -\frac{1}{6}\tanh^{-1}(\cos(x))-\frac{1}{6}\tanh^{-1}(2\cos(x))-\frac{\tanh^{-1}\left(\frac{2\cos(x)}{\sqrt{3}}\right)}{2\sqrt{3}}+\cos(x)$$

Mathematica [B] time = 0.09, size = 87, normalized size = 2.29

$$\frac{1}{12}\left(12\cos(x)+2\log\left(\sin\left(\frac{x}{2}\right)\right)-2\log\left(\cos\left(\frac{x}{2}\right)\right)+\log(1-2\cos(x))-\log(2\cos(x)+1)+2\sqrt{3}\tanh^{-1}\left(\frac{\tan\left(\frac{x}{2}\right)}{\sqrt{3}}\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[x]*Cot[6*x],x]
```

```
[Out] (2*Sqrt[3]*ArcTanh[(-2 + Tan[x/2])/Sqrt[3]] - 2*Sqrt[3]*ArcTanh[(2 + Tan[x/
2])/Sqrt[3]] + 12*Cos[x] - 2*Log[Cos[x/2]] + Log[1 - 2*Cos[x]] - Log[1 + 2*
Cos[x]] + 2*Log[Sin[x/2]])/12
```

fricas [B] time = 0.69, size = 71, normalized size = 1.87

$$\frac{1}{12}\sqrt{3}\log\left(\frac{4\cos(x)^2-4\sqrt{3}\cos(x)+3}{4\cos(x)^2-3}\right)+\cos(x)-\frac{1}{12}\log\left(\frac{1}{2}\cos(x)+\frac{1}{2}\right)+\frac{1}{12}\log\left(-\frac{1}{2}\cos(x)+\frac{1}{2}\right)+\frac{1}{12}\log(-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*cot(6*x),x, algorithm="fricas")
```

```
[Out] 1/12*sqrt(3)*log((4*cos(x)^2 - 4*sqrt(3)*cos(x) + 3)/(4*cos(x)^2 - 3)) + co
s(x) - 1/12*log(1/2*cos(x) + 1/2) + 1/12*log(-1/2*cos(x) + 1/2) + 1/12*log(
-2*cos(x) + 1) - 1/12*log(-2*cos(x) - 1)
```

giac [B] time = 0.17, size = 70, normalized size = 1.84

$$\frac{1}{12}\sqrt{3}\log\left(\frac{\left|-4\sqrt{3}+8\cos(x)\right|}{\left|4\sqrt{3}+8\cos(x)\right|}\right)+\cos(x)-\frac{1}{12}\log(\cos(x)+1)+\frac{1}{12}\log(-\cos(x)+1)-\frac{1}{12}\log(|2\cos(x)+1|)+\frac{1}{1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*cot(6*x),x, algorithm="giac")
```

```
[Out] 1/12*sqrt(3)*log(abs(-4*sqrt(3) + 8*cos(x))/abs(4*sqrt(3) + 8*cos(x))) + co
s(x) - 1/12*log(cos(x) + 1) + 1/12*log(-cos(x) + 1) - 1/12*log(abs(2*cos(x)
 + 1)) + 1/12*log(abs(2*cos(x) - 1))
```

maple [A] time = 0.56, size = 49, normalized size = 1.29

$$\frac{\ln(2\cos(x)-1)}{12}-\frac{\ln(1+2\cos(x))}{12}-\frac{\operatorname{arctanh}\left(\frac{2\cos(x)\sqrt{3}}{3}\right)\sqrt{3}}{6}+\frac{\ln(-1+\cos(x))}{12}-\frac{\ln(1+\cos(x))}{12}+\cos(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)*cot(6*x),x)
```

```
[Out] 1/12*ln(2*cos(x)-1)-1/12*ln(1+2*cos(x))-1/6*arctanh(2/3*cos(x)*3^(1/2))*3^(
1/2)+1/12*ln(-1+cos(x))-1/12*ln(1+cos(x))+cos(x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\cos(x)+\int\frac{(\sin(3x)-\sin(x))\cos(4x)-(\cos(3x)-\cos(x))\sin(4x)-(\cos(2x)-1)\sin(3x)+\cos(3x)\sin(2}{2\left(2(\cos(2x)-1)\cos(4x)-\cos(4x)^2-\cos(2x)^2-\sin(4x)^2+2\sin(4x)\sin(2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*cot(6*x),x, algorithm="maxima")
```

```
[Out] cos(x) + integrate(1/2*((sin(3*x) - sin(x))*cos(4*x) - (cos(3*x) - cos(x))*
sin(4*x) - (cos(2*x) - 1)*sin(3*x) + cos(3*x)*sin(2*x) - cos(x)*sin(2*x) +
cos(2*x)*sin(x) - sin(x))/(2*(cos(2*x) - 1)*cos(4*x) - cos(4*x)^2 - cos(2*x
)^2 - sin(4*x)^2 + 2*sin(4*x)*sin(2*x) - sin(2*x)^2 + 2*cos(2*x) - 1), x) -
 1/24*log(2*(cos(x) + 1)*cos(2*x) + cos(2*x)^2 + cos(x)^2 + sin(2*x)^2 + 2*
sin(2*x)*sin(x) + sin(x)^2 + 2*cos(x) + 1) + 1/24*log(-2*(cos(x) - 1)*cos(2
*x) + cos(2*x)^2 + cos(x)^2 + sin(2*x)^2 - 2*sin(2*x)*sin(x) + sin(x)^2 - 2
*cos(x) + 1) - 1/12*log(cos(x)^2 + sin(x)^2 + 2*cos(x) + 1) + 1/12*log(cos(
x)^2 + sin(x)^2 - 2*cos(x) + 1)
```

mupad [B] time = 2.44, size = 86, normalized size = 2.26

$$\frac{\operatorname{atanh}\left(\frac{1073741824}{10761687\left(\frac{427973089951744\tan\left(\frac{x}{2}\right)^2}{14348907}-\frac{47552804159488}{4782969}\right)}+\frac{797161}{797162}\right)}{6}+\frac{\ln\left(\tan\left(\frac{x}{2}\right)\right)}{6}-\frac{\sqrt{3}\operatorname{atanh}\left(\frac{303181204553728\sqrt{}}{4782969\left(\frac{7314051205955584\tan\left(\frac{x}{2}\right)^2}{4782969}-\right.}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(6*x)*cos(x),x)
```

```
[Out] atanh(1073741824/(10761687*((427973089951744*tan(x/2)^2)/14348907 - 4755280
4159488/4782969)) + 797161/797162)/6 + log(tan(x/2))/6 - (3^(1/2)*atanh((30
3181204553728*3^(1/2))/(4782969*((7314051205955584*tan(x/2)^2)/4782969 - 52
5125250187264/4782969)) - (4222769432625152*3^(1/2)*tan(x/2)^2)/(4782969*((
7314051205955584*tan(x/2)^2)/4782969 - 525125250187264/4782969))))/6 + 2/(t
an(x/2)^2 + 1)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \cos(x)\cot(6x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*cot(6*x),x)
```

```
[Out] Integral(cos(x)*cot(6*x), x)
```

3.115 $\int \cos(x)\cot(nx)\,dx$

Optimal. Leaf size=92

$$e^{-ix}\,{}_2F_1\left(1,-\frac{1}{2n};1-\frac{1}{2n};e^{2inx}\right)-e^{ix}\,{}_2F_1\left(1,\frac{1}{2n};\frac{1}{2}\left(2+\frac{1}{n}\right);e^{2inx}\right)-\frac{e^{-ix}}{2}+\frac{e^{ix}}{2}$$

```
[Out] -1/2/exp(I*x)+1/2*exp(I*x)+hypergeom([1, -1/2/n],[1-1/2/n],exp(2*I*n*x))/ex
p(I*x)-exp(I*x)*hypergeom([1, 1/2/n],[1+1/2/n],exp(2*I*n*x))
```

Rubi [A] time = 0.09, antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {4558, 2194, 2251}

$$e^{-ix}\,{}_2F_1\left(1,-\frac{1}{2n};1-\frac{1}{2n};e^{2inx}\right)-e^{ix}\,{}_2F_1\left(1,\frac{1}{2n};\frac{1}{2}\left(2+\frac{1}{n}\right);e^{2inx}\right)-\frac{e^{-ix}}{2}+\frac{e^{ix}}{2}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[x]*Cot[n*x],x]
```

```
[Out] -1/(2*E^(I*x)) + E^(I*x)/2 + Hypergeometric2F1[1, -1/(2*n), 1 - 1/(2*n), E^
((2*I)*n*x)]/E^(I*x) - E^(I*x)*Hypergeometric2F1[1, 1/(2*n), (2 + n^(-1))/2
, E^((2*I)*n*x)]
```

Rule 2194

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rule 2251

```
Int[((a_) + (b_.)*(F_)^((e_.)*((c_.) + (d_.)*(x_))))^(p_)*(G_)^((h_.)*((f_.
) + (g_.)*(x_))), x_Symbol] :> Simp[(a^p*G^(h*(f + g*x))*Hypergeometric2F1[
-p, (g*h*Log[G])/(d*e*Log[F]), (g*h*Log[G])/(d*e*Log[F]) + 1, Simplify[-((b
*F^(e*(c + d*x)))/a)]])/(g*h*Log[G]), x] /; FreeQ[{F, G, a, b, c, d, e, f,
g, h, p}, x] && (ILtQ[p, 0] || GtQ[a, 0])
```

Rule 4558

```
Int[Cos[(a_.) + (b_.)*(x_)]*Cot[(c_.) + (d_.)*(x_)], x_Symbol] :> Int[I/(E^
(I*(a + b*x))*2) + (I*E^(I*(a + b*x)))/2 - I/(E^(I*(a + b*x))*(1 - E^(2*I*(
c + d*x)))) - (I*E^(I*(a + b*x)))/(1 - E^(2*I*(c + d*x))), x] /; FreeQ[{a,
b, c, d}, x] && NeQ[b^2 - d^2, 0]
```

Rubi steps

$$\int \cos(x)\cot(nx)\,dx = \int \left(\frac{1}{2}ie^{-ix} + \frac{1}{2}ie^{ix} - \frac{ie^{-ix}}{1-e^{2inx}} - \frac{ie^{ix}}{1-e^{2inx}}\right)dx$$
$$= \frac{1}{2}i\int e^{-ix}\,dx + \frac{1}{2}i\int e^{ix}\,dx - i\int \frac{e^{-ix}}{1-e^{2inx}}\,dx - i\int \frac{e^{ix}}{1-e^{2inx}}\,dx$$
$$= -\frac{1}{2}e^{-ix} + \frac{e^{ix}}{2} + e^{-ix}\,{}_2F_1\left(1,-\frac{1}{2n};1-\frac{1}{2n};e^{2inx}\right) - e^{ix}\,{}_2F_1\left(1,\frac{1}{2n};\frac{1}{2}\left(2+\frac{1}{n}\right);e^{2inx}\right)$$

Mathematica [A] time = 0.18, size = 179, normalized size = 1.95

$$\frac{1}{2}e^{-2ix}\left(-\frac{e^{i(2nx+x)}\,{}_2F_1\left(1,1-\frac{1}{2n};2-\frac{1}{2n};e^{2inx}\right)}{2n-1} - \frac{e^{i(2n+3)x}\,{}_2F_1\left(1,1+\frac{1}{2n};2+\frac{1}{2n};e^{2inx}\right)}{2n+1} + e^{ix}\,{}_2F_1\left(1,-\frac{1}{2n};1-\frac{1}{2n};e^{2i\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[x]*Cot[n*x],x]
```

```
[Out] (-((E^(I*(x + 2*n*x))*Hypergeometric2F1[1, 1 - 1/(2*n), 2 - 1/(2*n), E^((2*
I)*n*x)])/(-1 + 2*n)) - (E^(I*(3 + 2*n)*x)*Hypergeometric2F1[1, 1 + 1/(2*n)
, 2 + 1/(2*n), E^((2*I)*n*x)])/(1 + 2*n) + E^(I*x)*Hypergeometric2F1[1, -1/
2*1/n, 1 - 1/(2*n), E^((2*I)*n*x)] - E^((3*I)*x)*Hypergeometric2F1[1, 1/(2*
n), 1 + 1/(2*n), E^((2*I)*n*x)])/(2*E^((2*I)*x))
```

fricas [F] time = 1.07, size = 0, normalized size = 0.00

$$\text{integral}\left(\cos(x)\cot(nx), x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*cot(n*x),x, algorithm="fricas")
```

```
[Out] integral(cos(x)*cot(n*x), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \cos(x)\cot(nx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*cot(n*x),x, algorithm="giac")
```

```
[Out] integrate(cos(x)*cot(n*x), x)
```

maple [F] time = 0.55, size = 0, normalized size = 0.00

$$\int \cos(x)\cot(nx)\ dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)*cot(n*x),x)
[Out] int(cos(x)*cot(n*x),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \cos(x)\cot(nx)\ dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*cot(n*x),x, algorithm="maxima")
[Out] integrate(cos(x)*cot(n*x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \cot(n\,x)\ \cos(x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(n*x)*cos(x),x)
[Out] int(cot(n*x)*cos(x), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \cos(x)\cot(nx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*cot(n*x),x)
[Out] Integral(cos(x)*cot(n*x), x)
```

3.116 $\int \cos(x)\sec(2x)\,dx$

Optimal. Leaf size=15

$$\frac{\tanh^{-1}\left(\sqrt{2}\,\sin(x)\right)}{\sqrt{2}}$$

```
[Out] 1/2*arctanh(sin(x)*2^(1/2))*2^(1/2)
```

Rubi [A] time = 0.01, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {4356, 206}

$$\frac{\tanh^{-1}\left(\sqrt{2}\,\sin(x)\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[x]*Sec[2*x],x]
```

```
[Out] ArcTanh[Sqrt[2]*Sin[x]]/Sqrt[2]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 4356

```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{d = FreeFacto
rs[Sin[c*(a + b*x)], x]}, Dist[d/(b*c), Subst[Int[SubstFor[1, Sin[c*(a + b*
x)]/d, u, x], x], x, Sin[c*(a + b*x)]/d], x] /; FunctionOfQ[Sin[c*(a + b*x)
]/d, u, x]] /; FreeQ[{a, b, c}, x] && (EqQ[F, Cos] || EqQ[F, cos])
```

Rubi steps

$$\begin{aligned}\int \cos(x)\sec(2x)\,dx &= \text{Subst}\left(\int \frac{1}{1-2x^2}\,dx, x, \sin(x)\right)\\ &= \frac{\tanh^{-1}\left(\sqrt{2}\,\sin(x)\right)}{\sqrt{2}}\end{aligned}$$

Mathematica [A] time = 0.01, size = 15, normalized size = 1.00

$$\frac{\tanh^{-1}\left(\sqrt{2}\sin(x)\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[x]*Sec[2*x],x]
[Out] ArcTanh[Sqrt[2]*Sin[x]]/Sqrt[2]
```

fricas [B] time = 1.13, size = 33, normalized size = 2.20

$$\frac{1}{4}\sqrt{2}\log\left(-\frac{2\cos(x)^2-2\sqrt{2}\sin(x)-3}{2\cos(x)^2-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*sec(2*x),x, algorithm="fricas")
[Out] 1/4*sqrt(2)*log(-(2*cos(x)^2 - 2*sqrt(2)*sin(x) - 3)/(2*cos(x)^2 - 1))
```

giac [B] time = 0.13, size = 31, normalized size = 2.07

$$\frac{1}{4}\sqrt{2}\log\left(\left|\frac{1}{2}\sqrt{2}+\sin(x)\right|\right)-\frac{1}{4}\sqrt{2}\log\left(\left|-\frac{1}{2}\sqrt{2}+\sin(x)\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*sec(2*x),x, algorithm="giac")
[Out] 1/4*sqrt(2)*log(abs(1/2*sqrt(2) + sin(x))) - 1/4*sqrt(2)*log(abs(-1/2*sqrt(
2) + sin(x)))
```

maple [A] time = 0.13, size = 13, normalized size = 0.87

$$\frac{\operatorname{arctanh}\left(\sin(x)\sqrt{2}\right)\sqrt{2}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)*sec(2*x),x)
[Out] 1/2*arctanh(sin(x)*2^(1/2))*2^(1/2)
```

maxima [B] time = 1.08, size = 137, normalized size = 9.13

$$\frac{1}{8}\sqrt{2}\log\left(2\cos(x)^2+2\sin(x)^2+2\sqrt{2}\cos(x)+2\sqrt{2}\sin(x)+2\right)-\frac{1}{8}\sqrt{2}\log\left(2\cos(x)^2+2\sin(x)^2+2\sqrt{2}\,\mathrm{co}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*sec(2*x),x, algorithm="maxima")
```

```
[Out] 1/8*sqrt(2)*log(2*cos(x)^2 + 2*sin(x)^2 + 2*sqrt(2)*cos(x) + 2*sqrt(2)*sin(
x) + 2) - 1/8*sqrt(2)*log(2*cos(x)^2 + 2*sin(x)^2 + 2*sqrt(2)*cos(x) - 2*sq
rt(2)*sin(x) + 2) + 1/8*sqrt(2)*log(2*cos(x)^2 + 2*sin(x)^2 - 2*sqrt(2)*cos
(x) + 2*sqrt(2)*sin(x) + 2) - 1/8*sqrt(2)*log(2*cos(x)^2 + 2*sin(x)^2 - 2*s
qrt(2)*cos(x) - 2*sqrt(2)*sin(x) + 2)
```

mupad [B] time = 0.11, size = 12, normalized size = 0.80

$$\frac{\sqrt{2}\ \mathrm{atanh}\left(\sqrt{2}\ \sin(x)\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)/cos(2*x),x)
```

```
[Out] (2^(1/2)*atanh(2^(1/2)*sin(x)))/2
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \cos(x)\sec(2x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*sec(2*x),x)
```

```
[Out] Integral(cos(x)*sec(2*x), x)
```

3.117 $\int \cos(x)\sec(3x)\,dx$

Optimal. Leaf size=44

$$\frac{\log\left(\sqrt{3}\sin(x)+\cos(x)\right)}{2\sqrt{3}}-\frac{\log\left(\cos(x)-\sqrt{3}\sin(x)\right)}{2\sqrt{3}}$$

```
[Out] -1/6*ln(cos(x)-sin(x)*3^(1/2))*3^(1/2)+1/6*ln(cos(x)+sin(x)*3^(1/2))*3^(1/2
)
```

Rubi [A] time = 0.04, antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {206}

$$\frac{\log\left(\sqrt{3}\sin(x)+\cos(x)\right)}{2\sqrt{3}}-\frac{\log\left(\cos(x)-\sqrt{3}\sin(x)\right)}{2\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[x]*Sec[3*x],x]
```

```
[Out] -Log[Cos[x] - Sqrt[3]*Sin[x]]/(2*Sqrt[3]) + Log[Cos[x] + Sqrt[3]*Sin[x]]/(2
*Sqrt[3])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \cos(x)\sec(3x)\,dx &= \text{Subst}\left(\int \frac{1}{1-3x^2}\,dx, x, \tan(x)\right)\\ &= -\frac{\log\left(\cos(x)-\sqrt{3}\sin(x)\right)}{2\sqrt{3}}+\frac{\log\left(\cos(x)+\sqrt{3}\sin(x)\right)}{2\sqrt{3}}\end{aligned}$$

Mathematica [A] time = 0.02, size = 15, normalized size = 0.34

$$\frac{\tanh^{-1}\left(\sqrt{3}\tan(x)\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[x]*Sec[3*x],x]
```

```
[Out] ArcTanh[Sqrt[3]*Tan[x]]/Sqrt[3]
```

fricas [A] time = 1.49, size = 53, normalized size = 1.20

$$\frac{1}{12}\sqrt{3}\log\left(-\frac{8\cos(x)^4+4\left(2\sqrt{3}\cos(x)^3-3\sqrt{3}\cos(x)\right)\sin(x)-9}{16\cos(x)^4-24\cos(x)^2+9}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*sec(3*x),x, algorithm="fricas")
```

```
[Out] 1/12*sqrt(3)*log(-(8*cos(x)^4 + 4*(2*sqrt(3)*cos(x)^3 - 3*sqrt(3)*cos(x))*s
in(x) - 9)/(16*cos(x)^4 - 24*cos(x)^2 + 9))
```

giac [A] time = 0.17, size = 31, normalized size = 0.70

$$-\frac{1}{6}\sqrt{3}\log\left(\frac{\left|-2\sqrt{3}+6\tan(x)\right|}{\left|2\sqrt{3}+6\tan(x)\right|}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*sec(3*x),x, algorithm="giac")
```

```
[Out] -1/6*sqrt(3)*log(abs(-2*sqrt(3) + 6*tan(x))/abs(2*sqrt(3) + 6*tan(x)))
```

maple [A] time = 0.18, size = 13, normalized size = 0.30

$$\frac{\sqrt{3}\,\operatorname{arctanh}\left(\tan(x)\sqrt{3}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)*sec(3*x),x)
```

```
[Out] 1/3*3^(1/2)*arctanh(tan(x)*3^(1/2))
```

maxima [B] time = 0.43, size = 76, normalized size = 1.73

$$\frac{1}{12}\sqrt{3}\left(\log\left(\frac{4}{3}\cos(2x)^2+\frac{4}{3}\sin(2x)^2+\frac{4}{3}\sqrt{3}\sin(2x)-\frac{4}{3}\cos(2x)+\frac{4}{3}\right)-\log\left(\frac{4}{3}\cos(2x)^2+\frac{4}{3}\sin(2x)^2-\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*sec(3*x),x, algorithm="maxima")
```

```
[Out] 1/12*sqrt(3)*(log(4/3*cos(2*x)^2 + 4/3*sin(2*x)^2 + 4/3*sqrt(3)*sin(2*x) -
4/3*cos(2*x) + 4/3) - log(4/3*cos(2*x)^2 + 4/3*sin(2*x)^2 - 4/3*sqrt(3)*sin
(2*x) - 4/3*cos(2*x) + 4/3))
```

mupad [B] time = 2.66, size = 16, normalized size = 0.36

$$\frac{\sqrt{3}\,\mathrm{atanh}\left(\frac{\sqrt{3}\,\sin(x)}{\cos(x)}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)/cos(3*x),x)
```

```
[Out] (3^(1/2)*atanh((3^(1/2)*sin(x))/cos(x)))/3
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \cos(x)\sec(3x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*sec(3*x),x)
```

```
[Out] Integral(cos(x)*sec(3*x), x)
```

3.118 $\int \cos(x)\sec(4x)\,dx$

Optimal. Leaf size=71

$$\frac{\tanh^{-1}\left(\frac{2\sin(x)}{\sqrt{2-\sqrt{2}}}\right)}{2\sqrt{2\left(2-\sqrt{2}\right)}}-\frac{\tanh^{-1}\left(\frac{2\sin(x)}{\sqrt{2+\sqrt{2}}}\right)}{2\sqrt{2\left(2+\sqrt{2}\right)}}$$

```
[Out] 1/2*arctanh(2*sin(x)/(2-2^(1/2))^(1/2))/(4-2*2^(1/2))^(1/2)-1/2*arctanh(2*s
in(x)/(2+2^(1/2))^(1/2))/(4+2*2^(1/2))^(1/2)
```

Rubi [A] time = 0.05, antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {4356, 1093, 207}

$$\frac{\tanh^{-1}\left(\frac{2\sin(x)}{\sqrt{2-\sqrt{2}}}\right)}{2\sqrt{2\left(2-\sqrt{2}\right)}}-\frac{\tanh^{-1}\left(\frac{2\sin(x)}{\sqrt{2+\sqrt{2}}}\right)}{2\sqrt{2\left(2+\sqrt{2}\right)}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[x]*Sec[4*x],x]
```

```
[Out] ArcTanh[(2*Sin[x])/Sqrt[2 - Sqrt[2]]]/(2*Sqrt[2*(2 - Sqrt[2])]) - ArcTanh[(
2*Sin[x])/Sqrt[2 + Sqrt[2]]]/(2*Sqrt[2*(2 + Sqrt[2])])
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 1093

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(-1), x_Symbol] :> With[{q = Rt[b^
2 - 4*a*c, 2]}, Dist[c/q, Int[1/(b/2 - q/2 + c*x^2), x], x] - Dist[c/q, Int
[1/(b/2 + q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c,
0] && PosQ[b^2 - 4*a*c]
```

Rule 4356

```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{d = FreeFacto
rs[Sin[c*(a + b*x)], x]}, Dist[d/(b*c), Subst[Int[SubstFor[1, Sin[c*(a + b*
```

```
x)]/d, u, x], x], x, Sin[c*(a + b*x)]/d], x] /; FunctionOfQ[Sin[c*(a + b*x)
]/d, u, x]] /; FreeQ[{a, b, c}, x] && (EqQ[F, Cos] || EqQ[F, cos])
```

Rubi steps

$$
\begin{aligned}
\int \cos(x)\sec(4x)\,dx &= \text{Subst}\left(\int \frac{1}{1-8x^2+8x^4}\,dx, x, \sin(x)\right)\\
&= \sqrt{2}\,\text{Subst}\left(\int \frac{1}{-4-2\sqrt{2}+8x^2}\,dx, x, \sin(x)\right)-\sqrt{2}\,\text{Subst}\left(\int \frac{1}{-4+2\sqrt{2}+8x^2}\,dx, x, \sin(x)\right)\\
&= \frac{\tanh^{-1}\left(\frac{2\sin(x)}{\sqrt{2-\sqrt{2}}}\right)}{2\sqrt{2\left(2-\sqrt{2}\right)}}-\frac{\tanh^{-1}\left(\frac{2\sin(x)}{\sqrt{2+\sqrt{2}}}\right)}{2\sqrt{2\left(2+\sqrt{2}\right)}}
\end{aligned}
$$

Mathematica [A] time = 0.10, size = 67, normalized size = 0.94

$$
\frac{1}{4}\sqrt{2+\sqrt{2}}\,\tanh^{-1}\left(\frac{2\sin(x)}{\sqrt{2-\sqrt{2}}}\right)-\frac{\tanh^{-1}\left(\frac{2\sin(x)}{\sqrt{2+\sqrt{2}}}\right)}{2\sqrt{2\left(2+\sqrt{2}\right)}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[x]*Sec[4*x],x]
```

```
[Out] (Sqrt[2 + Sqrt[2]]*ArcTanh[(2*Sin[x])/Sqrt[2 - Sqrt[2]]])/4 - ArcTanh[(2*Si
n[x])/Sqrt[2 + Sqrt[2]]]/(2*Sqrt[2*(2 + Sqrt[2])])
```

fricas [B] time = 1.98, size = 121, normalized size = 1.70

$$
\frac{1}{8}\sqrt{\sqrt{2}+2}\,\log\left(\sqrt{\sqrt{2}+2}\left(\sqrt{2}-1\right)+2\sin(x)\right)-\frac{1}{8}\sqrt{\sqrt{2}+2}\,\log\left(\sqrt{\sqrt{2}+2}\left(\sqrt{2}-1\right)-2\sin(x)\right)-\frac{1}{8}\sqrt{-\sqrt{2}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*sec(4*x),x, algorithm="fricas")
```

```
[Out] 1/8*sqrt(sqrt(2) + 2)*log(sqrt(sqrt(2) + 2)*(sqrt(2) - 1) + 2*sin(x)) - 1/8
*sqrt(sqrt(2) + 2)*log(sqrt(sqrt(2) + 2)*(sqrt(2) - 1) - 2*sin(x)) - 1/8*sq
rt(-sqrt(2) + 2)*log((sqrt(2) + 1)*sqrt(-sqrt(2) + 2) + 2*sin(x)) + 1/8*sqr
t(-sqrt(2) + 2)*log((sqrt(2) + 1)*sqrt(-sqrt(2) + 2) - 2*sin(x))
```

giac [B] time = 0.28, size = 99, normalized size = 1.39

$$-\frac{1}{8}\sqrt{-\sqrt{2}+2}\log\left(\left|\frac{1}{2}\sqrt{\sqrt{2}+2}+\sin(x)\right|\right)+\frac{1}{8}\sqrt{-\sqrt{2}+2}\log\left(\left|-\frac{1}{2}\sqrt{\sqrt{2}+2}+\sin(x)\right|\right)+\frac{1}{8}\sqrt{\sqrt{2}+2}\log\left(\left|\sqrt{\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*sec(4*x),x, algorithm="giac")
```

```
[Out] -1/8*sqrt(-sqrt(2) + 2)*log(abs(1/2*sqrt(sqrt(2) + 2) + sin(x))) + 1/8*sqrt
(-sqrt(2) + 2)*log(abs(-1/2*sqrt(sqrt(2) + 2) + sin(x))) + 1/8*sqrt(sqrt(2)
 + 2)*log(abs(sqrt(-1/4*sqrt(2) + 1/2) + sin(x))) - 1/8*sqrt(sqrt(2) + 2)*l
og(abs(-sqrt(-1/4*sqrt(2) + 1/2) + sin(x)))
```

maple [A] time = 0.22, size = 54, normalized size = 0.76

$$\frac{\sqrt{2}\,\operatorname{arctanh}\left(\frac{2\sin(x)}{\sqrt{2-\sqrt{2}}}\right)}{4\sqrt{2-\sqrt{2}}}-\frac{\sqrt{2}\,\operatorname{arctanh}\left(\frac{2\sin(x)}{\sqrt{2+\sqrt{2}}}\right)}{4\sqrt{2+\sqrt{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)*sec(4*x),x)
```

```
[Out] 1/4*2^(1/2)/(2-2^(1/2))^(1/2)*arctanh(2*sin(x)/(2-2^(1/2))^(1/2))-1/4*2^(1/
2)/(2+2^(1/2))^(1/2)*arctanh(2*sin(x)/(2+2^(1/2))^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \cos(x)\sec(4\,x)\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*sec(4*x),x, algorithm="maxima")
```

```
[Out] integrate(cos(x)*sec(4*x), x)
```

mupad [B] time = 2.27, size = 95, normalized size = 1.34

$$\frac{\operatorname{atanh}\left(\frac{2\,\sin(x)\,\sqrt{\sqrt{2}+2}+2\,\sqrt{2}\,\sin(x)\,\sqrt{\sqrt{2}+2}}{\sqrt{2}+2}\right)\sqrt{\sqrt{2}+2}}{4}-\frac{\operatorname{atanh}\left(\frac{2\,\sin(x)\,\sqrt{2-\sqrt{2}}-2\,\sqrt{2}\,\sin(x)\,\sqrt{2-\sqrt{2}}}{\sqrt{2}-2}\right)\sqrt{2-\sqrt{2}}}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)/cos(4*x),x)
```

```
[Out] (atanh((2*sin(x)*(2^(1/2) + 2)^(1/2) + 2*2^(1/2)*sin(x)*(2^(1/2) + 2)^(1/2)
)/(2^(1/2) + 2))*(2^(1/2) + 2)^(1/2))/4 - (atanh((2*sin(x)*(2 - 2^(1/2))^(1
/2) - 2*2^(1/2)*sin(x)*(2 - 2^(1/2))^(1/2))/(2^(1/2) - 2))*(2 - 2^(1/2))^(1
/2))/4
```

sympy **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \cos(x)\sec(4x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*sec(4*x),x)
```

```
[Out] Integral(cos(x)*sec(4*x), x)
```

3.119 $\int \cos(x)\sec(5x)\,dx$

Optimal. Leaf size=163

$$\frac{1}{10}\sqrt{\frac{1}{2}\left(5-\sqrt{5}\right)}\log\left(\cos(x)-\sqrt{5-2\sqrt{5}}\sin(x)\right)-\frac{1}{10}\sqrt{\frac{1}{2}\left(5-\sqrt{5}\right)}\log\left(\sqrt{5-2\sqrt{5}}\sin(x)+\cos(x)\right)-\frac{1}{10}\sqrt{\frac{1}{2}(}$$

```
[Out] 1/20*ln(cos(x)-sin(x)*(5-2*5^(1/2))^(1/2))*(10-2*5^(1/2))^(1/2)-1/20*ln(cos
(x)+sin(x)*(5-2*5^(1/2))^(1/2))*(10-2*5^(1/2))^(1/2)-1/20*ln(cos(x)-sin(x)*
(5+2*5^(1/2))^(1/2))*(10+2*5^(1/2))^(1/2)+1/20*ln(cos(x)+sin(x)*(5+2*5^(1/2
))^(1/2))*(10+2*5^(1/2))^(1/2)
```

Rubi [A] time = 0.13, antiderivative size = 163, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {1166, 207}

$$\frac{1}{10}\sqrt{\frac{1}{2}\left(5-\sqrt{5}\right)}\log\left(\cos(x)-\sqrt{5-2\sqrt{5}}\sin(x)\right)-\frac{1}{10}\sqrt{\frac{1}{2}\left(5-\sqrt{5}\right)}\log\left(\sqrt{5-2\sqrt{5}}\sin(x)+\cos(x)\right)-\frac{1}{10}\sqrt{\frac{1}{2}(}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[x]*Sec[5*x],x]
```

```
[Out] (Sqrt[(5 - Sqrt[5])/2]*Log[Cos[x] - Sqrt[5 - 2*Sqrt[5]]*Sin[x]])/10 - (Sqrt
[(5 - Sqrt[5])/2]*Log[Cos[x] + Sqrt[5 - 2*Sqrt[5]]*Sin[x]])/10 - (Sqrt[(5 +
 Sqrt[5])/2]*Log[Cos[x] - Sqrt[5 + 2*Sqrt[5]]*Sin[x]])/10 + (Sqrt[(5 + Sqrt
[5])/2]*Log[Cos[x] + Sqrt[5 + 2*Sqrt[5]]*Sin[x]])/10
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \cos(x)\sec(5x)\,dx = \text{Subst}\left(\int \frac{1+x^2}{1-10x^2+5x^4}\,dx, x, \tan(x)\right)$$
$$= \frac{1}{2}\left(1-\sqrt{5}\right)\text{Subst}\left(\int \frac{1}{-5+2\sqrt{5}+5x^2}\,dx, x, \tan(x)\right) + \frac{1}{2}\left(1+\sqrt{5}\right)\text{Subst}\left(\int \frac{1}{-5-2\sqrt{5}\right.$$
$$= \frac{1}{10}\sqrt{\frac{1}{2}\left(5-\sqrt{5}\right)}\,\log\left(\cos(x)-\sqrt{5-2\sqrt{5}}\,\sin(x)\right) - \frac{1}{10}\sqrt{\frac{1}{2}\left(5-\sqrt{5}\right)}\,\log\left(\cos(x)+\sqrt{5}\right.$$

Mathematica [A] time = 0.10, size = 84, normalized size = 0.52

$$\frac{\sqrt{5+\sqrt{5}}\,\tanh^{-1}\left(\frac{\left(5+\sqrt{5}\right)\tan(x)}{\sqrt{10-2\sqrt{5}}}\right)+\sqrt{5-\sqrt{5}}\,\tanh^{-1}\left(\frac{\left(\sqrt{5}-5\right)\tan(x)}{\sqrt{2\left(5+\sqrt{5}\right)}}\right)}{5\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[x]*Sec[5*x],x]
```

```
[Out] (Sqrt[5 + Sqrt[5]]*ArcTanh[((5 + Sqrt[5])*Tan[x])/Sqrt[10 - 2*Sqrt[5]]] + S
qrt[5 - Sqrt[5]]*ArcTanh[((-5 + Sqrt[5])*Tan[x])/Sqrt[2*(5 + Sqrt[5])]])/(5
*Sqrt[2])
```

fricas [B] time = 2.07, size = 231, normalized size = 1.42

$$-\frac{1}{40}\sqrt{2}\sqrt{\sqrt{5}+5}\,\log\left(\left(\sqrt{5}\sqrt{2}-\sqrt{2}\right)\sqrt{\sqrt{5}+5}\,\cos(x)\sin(x)+2\left(\sqrt{5}+1\right)\cos(x)^2-\sqrt{5}-5\right)+\frac{1}{40}\sqrt{2}\sqrt{\sqrt{5}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*sec(5*x),x, algorithm="fricas")
```

```
[Out] -1/40*sqrt(2)*sqrt(sqrt(5) + 5)*log((sqrt(5)*sqrt(2) - sqrt(2))*sqrt(sqrt(5
) + 5)*cos(x)*sin(x) + 2*(sqrt(5) + 1)*cos(x)^2 - sqrt(5) - 5) + 1/40*sqrt(
2)*sqrt(sqrt(5) + 5)*log(-(sqrt(5)*sqrt(2) - sqrt(2))*sqrt(sqrt(5) + 5)*cos
(x)*sin(x) + 2*(sqrt(5) + 1)*cos(x)^2 - sqrt(5) - 5) - 1/40*sqrt(2)*sqrt(-s
qrt(5) + 5)*log((sqrt(5)*sqrt(2) + sqrt(2))*sqrt(-sqrt(5) + 5)*cos(x)*sin(x
) + 2*(sqrt(5) - 1)*cos(x)^2 - sqrt(5) + 5) + 1/40*sqrt(2)*sqrt(-sqrt(5) +
5)*log(-(sqrt(5)*sqrt(2) + sqrt(2))*sqrt(-sqrt(5) + 5)*cos(x)*sin(x) + 2*(s
qrt(5) - 1)*cos(x)^2 - sqrt(5) + 5)
```

giac [A] time = 0.28, size = 105, normalized size = 0.64

$$-\frac{1}{20}\sqrt{-2\sqrt{5}+10}\,\log\left(\left|\sqrt{\frac{2}{5}\sqrt{5}+1}+\tan(x)\right|\right)+\frac{1}{20}\sqrt{-2\sqrt{5}+10}\,\log\left(\left|-\sqrt{\frac{2}{5}\sqrt{5}+1}+\tan(x)\right|\right)+\frac{1}{20}\sqrt{2\sqrt{5}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*sec(5*x),x, algorithm="giac")
```

```
[Out] -1/20*sqrt(-2*sqrt(5) + 10)*log(abs(sqrt(2/5*sqrt(5) + 1) + tan(x))) + 1/20
*sqrt(-2*sqrt(5) + 10)*log(abs(-sqrt(2/5*sqrt(5) + 1) + tan(x))) + 1/20*sqr
t(2*sqrt(5) + 10)*log(abs(sqrt(-2/5*sqrt(5) + 1) + tan(x))) - 1/20*sqrt(2*s
qrt(5) + 10)*log(abs(-sqrt(-2/5*sqrt(5) + 1) + tan(x)))
```

maple [A] time = 0.26, size = 68, normalized size = 0.42

$$-\frac{\left(5+\sqrt{5}\right)\sqrt{5}\ \operatorname{arctanh}\left(\frac{5\tan(x)}{\sqrt{25+10\sqrt{5}}}\right)}{10\sqrt{25+10\sqrt{5}}}-\frac{\sqrt{5}\ \left(\sqrt{5}-5\right)\operatorname{arctanh}\left(\frac{5\tan(x)}{\sqrt{25-10\sqrt{5}}}\right)}{10\sqrt{25-10\sqrt{5}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)*sec(5*x),x)
```

```
[Out] -1/10*(5+5^(1/2))*5^(1/2)/(25+10*5^(1/2))^(1/2)*arctanh(5*tan(x)/(25+10*5^(
1/2))^(1/2))-1/10*5^(1/2)*(5^(1/2)-5)/(25-10*5^(1/2))^(1/2)*arctanh(5*tan(x
)/(25-10*5^(1/2))^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \cos(x)\sec\left(5\,x\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*sec(5*x),x, algorithm="maxima")
```

```
[Out] integrate(cos(x)*sec(5*x), x)
```

mupad [B] time = 2.67, size = 217, normalized size = 1.33

$$\frac{\sqrt{2}\ \operatorname{atanh}\left(-\frac{34359738368\,\sqrt{2}\ \tan\left(\frac{x}{2}\right)\sqrt{5-\sqrt{5}}}{5\left(\frac{124554051584\,\sqrt{5}}{25}-\frac{124554051584\,\sqrt{5}\ \tan\left(\frac{x}{2}\right)^2}{25}-\frac{55834574848\tan\left(\frac{x}{2}\right)^2}{5}+\frac{55834574848}{5}\right)}-\frac{77309411328\,\sqrt{2}\ \sqrt{5}\ \tan\left(\frac{x}{2}\right)\cdots}{25\left(\frac{124554051584\,\sqrt{5}}{25}-\frac{124554051584\,\sqrt{5}\ \tan\left(\frac{x}{2}\right)^2}{25}-\cdots\right)}\cdots\right)\cdots}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)/cos(5*x),x)
```

```
[Out] (2^(1/2)*atanh(- (34359738368*2^(1/2)*tan(x/2)*(5 - 5^(1/2))^(1/2))/(5*((12
4554051584*5^(1/2))/25 - (124554051584*5^(1/2)*tan(x/2)^2)/25 - (5583457484
8*tan(x/2)^2)/5 + 55834574848/5)) - (77309411328*2^(1/2)*5^(1/2)*tan(x/2)*(
5 - 5^(1/2))^(1/2))/(25*((124554051584*5^(1/2))/25 - (124554051584*5^(1/2)*
tan(x/2)^2)/25 - (55834574848*tan(x/2)^2)/5 + 55834574848/5)))*(5 - 5^(1/2)
)^(1/2))/10 - (2^(1/2)*atanh((77309411328*2^(1/2)*5^(1/2)*tan(x/2)*(5^(1/2)
 + 5)^(1/2))/(25*((124554051584*5^(1/2))/25 - (124554051584*5^(1/2)*tan(x/2
)^2)/25 + (55834574848*tan(x/2)^2)/5 - 55834574848/5)) - (34359738368*2^(1/
2)*tan(x/2)*(5^(1/2) + 5)^(1/2))/(5*((124554051584*5^(1/2))/25 - (124554051
584*5^(1/2)*tan(x/2)^2)/25 + (55834574848*tan(x/2)^2)/5 - 55834574848/5)))*
(5^(1/2) + 5)^(1/2))/10
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \cos(x) \sec(5x) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*sec(5*x),x)
```

```
[Out] Integral(cos(x)*sec(5*x), x)
```

3.120 $\int \cos(x)\sec(6x)\,dx$

Optimal. Leaf size=85

$$-\frac{\tanh^{-1}\left(\sqrt{2}\sin(x)\right)}{3\sqrt{2}}+\frac{\tanh^{-1}\left(\frac{2\sin(x)}{\sqrt{2-\sqrt{3}}}\right)}{6\sqrt{2-\sqrt{3}}}+\frac{\tanh^{-1}\left(\frac{2\sin(x)}{\sqrt{2+\sqrt{3}}}\right)}{6\sqrt{2+\sqrt{3}}}$$

```
[Out] -1/6*arctanh(sin(x)*2^(1/2))*2^(1/2)+1/6*arctanh(2*sin(x)/(1/2*6^(1/2)-1/2*
2^(1/2)))/(1/2*6^(1/2)-1/2*2^(1/2))+1/6*arctanh(2*sin(x)/(1/2*6^(1/2)+1/2*2
^(1/2)))/(1/2*6^(1/2)+1/2*2^(1/2))
```

Rubi [A] time = 0.06, antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.571, Rules used = {4356, 2057, 207, 1166}

$$-\frac{\tanh^{-1}\left(\sqrt{2}\sin(x)\right)}{3\sqrt{2}}+\frac{\tanh^{-1}\left(\frac{2\sin(x)}{\sqrt{2-\sqrt{3}}}\right)}{6\sqrt{2-\sqrt{3}}}+\frac{\tanh^{-1}\left(\frac{2\sin(x)}{\sqrt{2+\sqrt{3}}}\right)}{6\sqrt{2+\sqrt{3}}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[x]*Sec[6*x],x]
```

```
[Out] -ArcTanh[Sqrt[2]*Sin[x]]/(3*Sqrt[2]) + ArcTanh[(2*Sin[x])/Sqrt[2 - Sqrt[3]]
]/(6*Sqrt[2 - Sqrt[3]]) + ArcTanh[(2*Sin[x])/Sqrt[2 + Sqrt[3]]]/(6*Sqrt[2 +
 Sqrt[3]])
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 2057

```
Int[(P_)^(p_), x_Symbol] :> With[{u = Factor[P /. x -> Sqrt[x]]}, Int[Expan
dIntegrand[(u /. x -> x^2)^p, x], x] /;  !SumQ[NonfreeFactors[u, x]]] /; Po
lyQ[P, x^2] && ILtQ[p, 0]
```

Rule 4356

```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{d = FreeFacto
rs[Sin[c*(a + b*x)], x]}, Dist[d/(b*c), Subst[Int[SubstFor[1, Sin[c*(a + b*
x)]/d, u, x], x], x, Sin[c*(a + b*x)]/d], x] /; FunctionOfQ[Sin[c*(a + b*x)
]/d, u, x]] /; FreeQ[{a, b, c}, x] && (EqQ[F, Cos] || EqQ[F, cos])
```

Rubi steps

$$
\begin{aligned}
\int \cos(x)\sec(6x)\,dx &= \text{Subst}\left(\int \frac{1}{1-18x^2+48x^4-32x^6}\,dx, x, \sin(x)\right)\\
&= \text{Subst}\left(\int \left(\frac{1}{3\left(-1+2x^2\right)}-\frac{4\left(-1+2x^2\right)}{3\left(1-16x^2+16x^4\right)}\right)dx, x, \sin(x)\right)\\
&= \frac{1}{3}\text{Subst}\left(\int \frac{1}{-1+2x^2}\,dx, x, \sin(x)\right)-\frac{4}{3}\text{Subst}\left(\int \frac{-1+2x^2}{1-16x^2+16x^4}\,dx, x, \sin(x)\right)\\
&= -\frac{\tanh^{-1}\left(\sqrt{2}\sin(x)\right)}{3\sqrt{2}}-\frac{4}{3}\text{Subst}\left(\int \frac{1}{-8-4\sqrt{3}+16x^2}\,dx, x, \sin(x)\right)-\frac{4}{3}\text{Subst}\left(\int \frac{}{-8+}\right.\\
&= -\frac{\tanh^{-1}\left(\sqrt{2}\sin(x)\right)}{3\sqrt{2}}+\frac{\tanh^{-1}\left(\frac{2\sin(x)}{\sqrt{2-\sqrt{3}}}\right)}{6\sqrt{2-\sqrt{3}}}+\frac{\tanh^{-1}\left(\frac{2\sin(x)}{\sqrt{2+\sqrt{3}}}\right)}{6\sqrt{2+\sqrt{3}}}
\end{aligned}
$$

Mathematica [A] time = 0.08, size = 81, normalized size = 0.95

$$
\frac{1}{6}\left(-\sqrt{2}\tanh^{-1}\left(\sqrt{2}\sin(x)\right)+\sqrt{2+\sqrt{3}}\tanh^{-1}\left(\frac{2\sin(x)}{\sqrt{2-\sqrt{3}}}\right)+\sqrt{2-\sqrt{3}}\tanh^{-1}\left(\frac{2\sin(x)}{\sqrt{2+\sqrt{3}}}\right)\right)
$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[x]*Sec[6*x],x]
```

```
[Out] (-(Sqrt[2]*ArcTanh[Sqrt[2]*Sin[x]]) + Sqrt[2 + Sqrt[3]]*ArcTanh[(2*Sin[x])/
Sqrt[2 - Sqrt[3]]] + Sqrt[2 - Sqrt[3]]*ArcTanh[(2*Sin[x])/Sqrt[2 + Sqrt[3]]
])/6
```

fricas [B] time = 4.13, size = 154, normalized size = 1.81

$$-\frac{1}{12}\sqrt{\sqrt{3}+2}\log\left(\sqrt{\sqrt{3}+2}\left(\sqrt{3}-2\right)+2\sin(x)\right)+\frac{1}{12}\sqrt{\sqrt{3}+2}\log\left(\sqrt{\sqrt{3}+2}\left(\sqrt{3}-2\right)-2\sin(x)\right)+\frac{1}{12}\sqrt{-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*sec(6*x),x, algorithm="fricas")
```

```
[Out] -1/12*sqrt(sqrt(3) + 2)*log(sqrt(sqrt(3) + 2)*(sqrt(3) - 2) + 2*sin(x)) + 1
/12*sqrt(sqrt(3) + 2)*log(sqrt(sqrt(3) + 2)*(sqrt(3) - 2) - 2*sin(x)) + 1/1
2*sqrt(-sqrt(3) + 2)*log((sqrt(3) + 2)*sqrt(-sqrt(3) + 2) + 2*sin(x)) - 1/1
2*sqrt(-sqrt(3) + 2)*log((sqrt(3) + 2)*sqrt(-sqrt(3) + 2) - 2*sin(x)) + 1/1
2*sqrt(2)*log(-(2*cos(x)^2 + 2*sqrt(2)*sin(x) - 3)/(2*cos(x)^2 - 1))
```

giac [A] time = 0.25, size = 132, normalized size = 1.55

$$\frac{1}{24}\left(\sqrt{6}-\sqrt{2}\right)\log\left(\left|\frac{1}{4}\sqrt{6}+\frac{1}{4}\sqrt{2}+\sin(x)\right|\right)+\frac{1}{24}\left(\sqrt{6}+\sqrt{2}\right)\log\left(\left|\frac{1}{4}\sqrt{6}-\frac{1}{4}\sqrt{2}+\sin(x)\right|\right)-\frac{1}{24}\left(\sqrt{6}+\sqrt{2}\right)\mathrm{lo}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*sec(6*x),x, algorithm="giac")
```

```
[Out] 1/24*(sqrt(6) - sqrt(2))*log(abs(1/4*sqrt(6) + 1/4*sqrt(2) + sin(x))) + 1/2
4*(sqrt(6) + sqrt(2))*log(abs(1/4*sqrt(6) - 1/4*sqrt(2) + sin(x))) - 1/24*(
sqrt(6) + sqrt(2))*log(abs(-1/4*sqrt(6) + 1/4*sqrt(2) + sin(x))) - 1/24*(sq
rt(6) - sqrt(2))*log(abs(-1/4*sqrt(6) - 1/4*sqrt(2) + sin(x))) + 1/12*sqrt(
2)*log(abs(-2*sqrt(2) + 4*sin(x))/abs(2*sqrt(2) + 4*sin(x)))
```

maple [A] time = 0.26, size = 80, normalized size = 0.94

$$\frac{2\,\mathrm{arctanh}\left(\frac{8\sin(x)}{2\sqrt{6}-2\sqrt{2}}\right)}{3\left(2\sqrt{6}-2\sqrt{2}\right)}+\frac{2\,\mathrm{arctanh}\left(\frac{8\sin(x)}{2\sqrt{6}+2\sqrt{2}}\right)}{3\left(2\sqrt{6}+2\sqrt{2}\right)}-\frac{\mathrm{arctanh}\left(\sin(x)\sqrt{2}\right)\sqrt{2}}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)*sec(6*x),x)
```

```
[Out] 2/3/(2*6^(1/2)-2*2^(1/2))*arctanh(8*sin(x)/(2*6^(1/2)-2*2^(1/2)))+2/3/(2*6^
(1/2)+2*2^(1/2))*arctanh(8*sin(x)/(2*6^(1/2)+2*2^(1/2)))-1/6*arctanh(sin(x)
*2^(1/2))*2^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{1}{24}\sqrt{2}\log\left(2\cos(x)^2+2\sin(x)^2+2\sqrt{2}\cos(x)+2\sqrt{2}\sin(x)+2\right)+\frac{1}{24}\sqrt{2}\log\left(2\cos(x)^2+2\sin(x)^2+2\sqrt{2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*sec(6*x),x, algorithm="maxima")
```

```
[Out] -1/24*sqrt(2)*log(2*cos(x)^2 + 2*sin(x)^2 + 2*sqrt(2)*cos(x) + 2*sqrt(2)*si
n(x) + 2) + 1/24*sqrt(2)*log(2*cos(x)^2 + 2*sin(x)^2 + 2*sqrt(2)*cos(x) - 2
*sqrt(2)*sin(x) + 2) - 1/24*sqrt(2)*log(2*cos(x)^2 + 2*sin(x)^2 - 2*sqrt(2)
*cos(x) + 2*sqrt(2)*sin(x) + 2) + 1/24*sqrt(2)*log(2*cos(x)^2 + 2*sin(x)^2
- 2*sqrt(2)*cos(x) - 2*sqrt(2)*sin(x) + 2) + integrate(-1/3*((cos(7*x) + co
s(5*x) + cos(3*x) + cos(x))*cos(8*x) - (cos(4*x) - 1)*cos(7*x) - (cos(4*x)
- 1)*cos(5*x) - (cos(3*x) + cos(x))*cos(4*x) + (sin(7*x) + sin(5*x) + sin(3
*x) + sin(x))*sin(8*x) - (sin(3*x) + sin(x))*sin(4*x) - sin(7*x)*sin(4*x) -
 sin(5*x)*sin(4*x) + cos(3*x) + cos(x))/(2*(cos(4*x) - 1)*cos(8*x) - cos(8*
x)^2 - cos(4*x)^2 - sin(8*x)^2 + 2*sin(8*x)*sin(4*x) - sin(4*x)^2 + 2*cos(4
*x) - 1), x)
```

mupad [B] time = 2.29, size = 118, normalized size = 1.39

$$\mathrm{atanh}\left(\frac{5\sqrt{2}\ \sin(x)}{2097152\left(\frac{\sqrt{2}\ \sqrt{6}}{4194304}+\frac{1}{1048576}\right)}+\frac{3\sqrt{6}\ \sin(x)}{2097152\left(\frac{\sqrt{2}\ \sqrt{6}}{4194304}+\frac{1}{1048576}\right)}\right)\left(\frac{\sqrt{2}}{12}+\frac{\sqrt{6}}{12}\right)-\mathrm{atanh}\left(\frac{5\sqrt{2}\ \sin(x)}{2097152\left(\frac{\sqrt{2}\ \sqrt{6}}{4194304}-\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)/cos(6*x),x)
```

```
[Out] atanh((5*2^(1/2)*sin(x))/(2097152*((2^(1/2)*6^(1/2))/4194304 + 1/1048576))
+ (3*6^(1/2)*sin(x))/(2097152*((2^(1/2)*6^(1/2))/4194304 + 1/1048576)))*(2^
(1/2)/12 + 6^(1/2)/12) - atanh((5*2^(1/2)*sin(x))/(2097152*((2^(1/2)*6^(1/2
))/4194304 - 1/1048576)) - (3*6^(1/2)*sin(x))/(2097152*((2^(1/2)*6^(1/2))/4
194304 - 1/1048576)))*(2^(1/2)/12 - 6^(1/2)/12) - (2^(1/2)*atanh(2^(1/2)*si
n(x)))/6
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \cos(x)\sec(6x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*sec(6*x),x)
```

```
[Out] Integral(cos(x)*sec(6*x), x)
```

3.121 $\int \cos(2x)\sec(x)\,dx$

Optimal. Leaf size=10

$$2\sin(x) - \tanh^{-1}(\sin(x))$$

```
[Out] -arctanh(sin(x))+2*sin(x)
```

Rubi [A] time = 0.02, antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {4364, 388, 206}

$$2\sin(x) - \tanh^{-1}(\sin(x))$$

Antiderivative was successfully verified.

```
[In] Int[Cos[2*x]*Sec[x],x]
```

```
[Out] -ArcTanh[Sin[x]] + 2*Sin[x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 4364

```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))]^(n_), x_Symbol] :> With[{d = Free
Factors[Sin[c*(a + b*x)], x]}, Dist[d/(b*c), Subst[Int[SubstFor[(1 - d^2*x^
2)^((n - 1)/2), Sin[c*(a + b*x)]/d, u, x], x], x, Sin[c*(a + b*x)]/d], x] /
; FunctionOfQ[Sin[c*(a + b*x)]/d, u, x]] /; FreeQ[{a, b, c}, x] && IntegerQ
[(n - 1)/2] && NonsumQ[u] && (EqQ[F, Cos] || EqQ[F, cos])
```

Rubi steps

$$\int \cos(2x)\sec(x)\,dx = \text{Subst}\left(\int \frac{1-2x^2}{1-x^2}\,dx, x, \sin(x)\right)$$
$$= 2\sin(x) - \text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \sin(x)\right)$$
$$= -\tanh^{-1}(\sin(x)) + 2\sin(x)$$

Mathematica [A] time = 0.01, size = 10, normalized size = 1.00

$$2\sin(x) - \tanh^{-1}(\sin(x))$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[2*x]*Sec[x],x]
[Out] -ArcTanh[Sin[x]] + 2*Sin[x]
```

fricas [B] time = 0.68, size = 21, normalized size = 2.10

$$-\frac{1}{2}\,\log\left(\sin(x)+1\right) + \frac{1}{2}\,\log\left(-\sin(x)+1\right) + 2\,\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(2*x)*sec(x),x, algorithm="fricas")
[Out] -1/2*log(sin(x) + 1) + 1/2*log(-sin(x) + 1) + 2*sin(x)
```

giac [B] time = 0.14, size = 21, normalized size = 2.10

$$-\frac{1}{2}\,\log\left(\sin(x)+1\right) + \frac{1}{2}\,\log\left(-\sin(x)+1\right) + 2\,\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(2*x)*sec(x),x, algorithm="giac")
[Out] -1/2*log(sin(x) + 1) + 1/2*log(-sin(x) + 1) + 2*sin(x)
```

maple [A] time = 0.12, size = 14, normalized size = 1.40

$$2\sin(x) - \ln\left(\sec(x) + \tan(x)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(2*x)*sec(x),x)
[Out] 2*sin(x)-ln(sec(x)+tan(x))
```

maxima [A] time = 0.31, size = 19, normalized size = 1.90

$$-\frac{1}{2}\log\left(\sin(x)+1\right)+\frac{1}{2}\log\left(\sin(x)-1\right)+2\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(2*x)*sec(x),x, algorithm="maxima")
[Out] -1/2*log(sin(x) + 1) + 1/2*log(sin(x) - 1) + 2*sin(x)
```

mupad [B] time = 2.25, size = 10, normalized size = 1.00

$$2\sin(x)-\operatorname{atanh}\left(\sin(x)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(2*x)/cos(x),x)
[Out] 2*sin(x) - atanh(sin(x))
```

sympy [B] time = 1.05, size = 20, normalized size = 2.00

$$\frac{\log\left(\sin\left(x\right)-1\right)}{2}-\frac{\log\left(\sin\left(x\right)+1\right)}{2}+2\sin\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(2*x)*sec(x),x)
[Out] log(sin(x) - 1)/2 - log(sin(x) + 1)/2 + 2*sin(x)
```

3.122 $\int \cos(4x)\sec(2x)\,dx$

Optimal. Leaf size=14

$$\sin(2x) - \frac{1}{2}\tanh^{-1}(\sin(2x))$$

```
[Out] -1/2*arctanh(sin(2*x))+sin(2*x)
```

Rubi [A] time = 0.02, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {4364, 388, 206}

$$\sin(2x) - \frac{1}{2}\tanh^{-1}(\sin(2x))$$

Antiderivative was successfully verified.

```
[In] Int[Cos[4*x]*Sec[2*x],x]
```

```
[Out] -ArcTanh[Sin[2*x]]/2 + Sin[2*x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 4364

```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))]^(n_), x_Symbol] :> With[{d = Free
Factors[Sin[c*(a + b*x)], x]}, Dist[d/(b*c), Subst[Int[SubstFor[(1 - d^2*x^
2)^((n - 1)/2), Sin[c*(a + b*x)]/d, u, x], x], x, Sin[c*(a + b*x)]/d], x] /
; FunctionOfQ[Sin[c*(a + b*x)]/d, u, x]] /; FreeQ[{a, b, c}, x] && IntegerQ
[(n - 1)/2] && NonsumQ[u] && (EqQ[F, Cos] || EqQ[F, cos])
```

Rubi steps

$$\int \cos(4x)\sec(2x)\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{1-2x^2}{1-x^2}\,dx, x, \sin(2x)\right)$$
$$= \sin(2x) - \frac{1}{2}\,\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \sin(2x)\right)$$
$$= -\frac{1}{2}\tanh^{-1}(\sin(2x)) + \sin(2x)$$

Mathematica [**A**] time = 0.01, size = 14, normalized size = 1.00

$$\sin(2x) - \frac{1}{2}\tanh^{-1}(\sin(2x))$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[4*x]*Sec[2*x],x]
[Out] -1/2*ArcTanh[Sin[2*x]] + Sin[2*x]
```

fricas [**B**] time = 1.47, size = 25, normalized size = 1.79

$$-\frac{1}{4}\log\left(\sin\left(2\,x\right)+1\right)+\frac{1}{4}\log\left(-\sin\left(2\,x\right)+1\right)+\sin\left(2\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(4*x)*sec(2*x),x, algorithm="fricas")
[Out] -1/4*log(sin(2*x) + 1) + 1/4*log(-sin(2*x) + 1) + sin(2*x)
```

giac [**B**] time = 0.13, size = 25, normalized size = 1.79

$$-\frac{1}{4}\log\left(\sin\left(2\,x\right)+1\right)+\frac{1}{4}\log\left(-\sin\left(2\,x\right)+1\right)+\sin\left(2\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(4*x)*sec(2*x),x, algorithm="giac")
[Out] -1/4*log(sin(2*x) + 1) + 1/4*log(-sin(2*x) + 1) + sin(2*x)
```

maple [**A**] time = 0.14, size = 18, normalized size = 1.29

$$-\frac{\ln\left(\sec\left(2x\right)+\tan\left(2x\right)\right)}{2}+\sin\left(2x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(4*x)*sec(2*x),x)
```

```
[Out] -1/2*ln(sec(2*x)+tan(2*x))+sin(2*x)
```

maxima [B] time = 0.48, size = 129, normalized size = 9.21

$$\frac{1}{4}\log\left(2\cos(x)^2+2\sin(x)^2+2\sqrt{2}\cos(x)+2\sqrt{2}\sin(x)+2\right)-\frac{1}{4}\log\left(2\cos(x)^2+2\sin(x)^2+2\sqrt{2}\cos(x)-\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(4*x)*sec(2*x),x, algorithm="maxima")
```

```
[Out] 1/4*log(2*cos(x)^2 + 2*sin(x)^2 + 2*sqrt(2)*cos(x) + 2*sqrt(2)*sin(x) + 2)
- 1/4*log(2*cos(x)^2 + 2*sin(x)^2 + 2*sqrt(2)*cos(x) - 2*sqrt(2)*sin(x) + 2
) - 1/4*log(2*cos(x)^2 + 2*sin(x)^2 - 2*sqrt(2)*cos(x) + 2*sqrt(2)*sin(x) +
 2) + 1/4*log(2*cos(x)^2 + 2*sin(x)^2 - 2*sqrt(2)*cos(x) - 2*sqrt(2)*sin(x)
 + 2) + sin(2*x)
```

mupad [B] time = 0.02, size = 12, normalized size = 0.86

$$\sin(2x)-\frac{\operatorname{atanh}(\sin(2x))}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(4*x)/cos(2*x),x)
```

```
[Out] sin(2*x) - atanh(sin(2*x))/2
```

sympy [B] time = 5.35, size = 427, normalized size = 30.50

$$-4x+\frac{32x\tan^4\left(\frac{x}{2}\right)}{8\tan^4\left(\frac{x}{2}\right)+16\tan^2\left(\frac{x}{2}\right)+8}+\frac{64x\tan^2\left(\frac{x}{2}\right)}{8\tan^4\left(\frac{x}{2}\right)+16\tan^2\left(\frac{x}{2}\right)+8}+\frac{32x}{8\tan^4\left(\frac{x}{2}\right)+16\tan^2\left(\frac{x}{2}\right)+8}-\frac{3\log\left(\tan^2\left(\frac{x}{2}\right)\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(4*x)*sec(2*x),x)
```

```
[Out] -4*x + 32*x*tan(x/2)**4/(8*tan(x/2)**4 + 16*tan(x/2)**2 + 8) + 64*x*tan(x/2
)**2/(8*tan(x/2)**4 + 16*tan(x/2)**2 + 8) + 32*x/(8*tan(x/2)**4 + 16*tan(x/
2)**2 + 8) - 3*log(tan(x/2)**2 - 2*tan(x/2) - 1)/2 + 3*log(tan(x/2)**2 + 2*
tan(x/2) - 1)/2 + 8*log(tan(x/2)**2 - 2*tan(x/2) - 1)*tan(x/2)**4/(8*tan(x/
2)**4 + 16*tan(x/2)**2 + 8) + 16*log(tan(x/2)**2 - 2*tan(x/2) - 1)*tan(x/2)
**2/(8*tan(x/2)**4 + 16*tan(x/2)**2 + 8) + 8*log(tan(x/2)**2 - 2*tan(x/2) -
```

```
 1)/(8*tan(x/2)**4 + 16*tan(x/2)**2 + 8) - 8*log(tan(x/2)**2 + 2*tan(x/2) -
 1)*tan(x/2)**4/(8*tan(x/2)**4 + 16*tan(x/2)**2 + 8) - 16*log(tan(x/2)**2 +
 2*tan(x/2) - 1)*tan(x/2)**2/(8*tan(x/2)**4 + 16*tan(x/2)**2 + 8) - 8*log(t
an(x/2)**2 + 2*tan(x/2) - 1)/(8*tan(x/2)**4 + 16*tan(x/2)**2 + 8) - 32*tan(
x/2)**3/(8*tan(x/2)**4 + 16*tan(x/2)**2 + 8) + 32*tan(x/2)/(8*tan(x/2)**4 +
 16*tan(x/2)**2 + 8)
```

3.123 $\int \cos(x)\csc(2x)\,dx$

Optimal. Leaf size=7

$$-\frac{1}{2}\tanh^{-1}(\cos(x))$$

```
[Out] -1/2*arctanh(cos(x))
```

Rubi [A] time = 0.01, antiderivative size = 7, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {4287, 3770}

$$-\frac{1}{2}\tanh^{-1}(\cos(x))$$

Antiderivative was successfully verified.

```
[In] Int[Cos[x]*Csc[2*x],x]
```

```
[Out] -ArcTanh[Cos[x]]/2
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 4287

```
Int[(cos[(a_.) + (b_.)*(x_)]*(e_.))^(m_.)*sin[(c_.) + (d_.)*(x_)]^(p_.), x_
Symbol] :> Dist[2^p/e^p, Int[(e*Cos[a + b*x])^(m + p)*Sin[a + b*x]^p, x], x
] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && In
tegerQ[p]
```

Rubi steps

$$\begin{aligned}\int \cos(x)\csc(2x)\,dx &= \frac{1}{2}\int \csc(x)\,dx\\ &= -\frac{1}{2}\tanh^{-1}(\cos(x))\end{aligned}$$

Mathematica [B] time = 0.00, size = 21, normalized size = 3.00

$$\frac{1}{2}\left(\log\left(\sin\left(\frac{x}{2}\right)\right)-\log\left(\cos\left(\frac{x}{2}\right)\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[x]*Csc[2*x],x]
[Out] (-Log[Cos[x/2]] + Log[Sin[x/2]])/2
```

fricas [B] time = 2.12, size = 19, normalized size = 2.71

$$-\frac{1}{4}\log\left(\frac{1}{2}\cos(x)+\frac{1}{2}\right)+\frac{1}{4}\log\left(-\frac{1}{2}\cos(x)+\frac{1}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*csc(2*x),x, algorithm="fricas")
[Out] -1/4*log(1/2*cos(x) + 1/2) + 1/4*log(-1/2*cos(x) + 1/2)
```

giac [B] time = 0.12, size = 17, normalized size = 2.43

$$-\frac{1}{4}\log\left(\cos(x)+1\right)+\frac{1}{4}\log\left(-\cos(x)+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*csc(2*x),x, algorithm="giac")
[Out] -1/4*log(cos(x) + 1) + 1/4*log(-cos(x) + 1)
```

maple [A] time = 0.08, size = 11, normalized size = 1.57

$$\frac{\ln\left(\csc(x)-\cot(x)\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)*csc(2*x),x)
[Out] 1/2*ln(csc(x)-cot(x))
```

maxima [B] time = 0.32, size = 35, normalized size = 5.00

$$-\frac{1}{4}\log\left(\cos(x)^2+\sin(x)^2+2\cos(x)+1\right)+\frac{1}{4}\log\left(\cos(x)^2+\sin(x)^2-2\cos(x)+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*csc(2*x),x, algorithm="maxima")
[Out] -1/4*log(cos(x)^2 + sin(x)^2 + 2*cos(x) + 1) + 1/4*log(cos(x)^2 + sin(x)^2
- 2*cos(x) + 1)
```

mupad [B] time = 0.03, size = 5, normalized size = 0.71

$$-\frac{\operatorname{atanh}\left(\cos(x)\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)/sin(2*x),x)
```

```
[Out] -atanh(cos(x))/2
```

sympy [B] time = 0.87, size = 15, normalized size = 2.14

$$\frac{\log\left(\cos\left(x\right)-1\right)}{4}-\frac{\log\left(\cos\left(x\right)+1\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*csc(2*x),x)
```

```
[Out] log(cos(x) - 1)/4 - log(cos(x) + 1)/4
```

3.124 $\int \cos(x)\csc(3x)\,dx$

Optimal. Leaf size=21

$$\frac{1}{3}\log(\sin(x))-\frac{1}{6}\log\left(3-4\sin^2(x)\right)$$

```
[Out] 1/3*ln(sin(x))-1/6*ln(3-4*sin(x)^2)
```

Rubi [A] time = 0.03, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.714, Rules used = {4356, 266, 36, 31, 29}

$$\frac{1}{3}\log(\sin(x))-\frac{1}{6}\log\left(3-4\sin^2(x)\right)$$

Antiderivative was successfully verified.

```
[In] Int[Cos[x]*Csc[3*x],x]
```

```
[Out] Log[Sin[x]]/3 - Log[3 - 4*Sin[x]^2]/6
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 4356

```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{d = FreeFacto
rs[Sin[c*(a + b*x)], x]}, Dist[d/(b*c), Subst[Int[SubstFor[1, Sin[c*(a + b*
```

```
x)]/d, u, x], x], x, Sin[c*(a + b*x)]/d], x] /; FunctionOfQ[Sin[c*(a + b*x)
]/d, u, x]] /; FreeQ[{a, b, c}, x] && (EqQ[F, Cos] || EqQ[F, cos])
```

Rubi steps

$$\begin{aligned}\int \cos(x)\csc(3x)\,dx &= \text{Subst}\left(\int \frac{1}{x\left(3-4x^2\right)}\,dx, x, \sin(x)\right)\\ &= \frac{1}{2}\text{Subst}\left(\int \frac{1}{(3-4x)x}\,dx, x, \sin^2(x)\right)\\ &= \frac{1}{6}\text{Subst}\left(\int \frac{1}{x}\,dx, x, \sin^2(x)\right)+\frac{2}{3}\text{Subst}\left(\int \frac{1}{3-4x}\,dx, x, \sin^2(x)\right)\\ &= \frac{1}{3}\log(\sin(x))-\frac{1}{6}\log\left(3-4\sin^2(x)\right)\end{aligned}$$

Mathematica [A] time = 0.01, size = 21, normalized size = 1.00

$$\frac{1}{3}\log(\sin(x))-\frac{1}{6}\log\left(3-4\sin^2(x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[x]*Csc[3*x],x]
[Out] Log[Sin[x]]/3 - Log[3 - 4*Sin[x]^2]/6
```

fricas [A] time = 1.27, size = 19, normalized size = 0.90

$$-\frac{1}{6}\log\left(4\cos(x)^2-1\right)+\frac{1}{3}\log\left(\frac{1}{2}\sin(x)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*csc(3*x),x, algorithm="fricas")
[Out] -1/6*log(4*cos(x)^2 - 1) + 1/3*log(1/2*sin(x))
```

giac [A] time = 0.14, size = 24, normalized size = 1.14

$$\frac{1}{6}\log\left(-\cos(x)^2+1\right)-\frac{1}{6}\log\left(\left|4\cos(x)^2-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*csc(3*x),x, algorithm="giac")
```

```
[Out] 1/6*log(-cos(x)^2 + 1) - 1/6*log(abs(4*cos(x)^2 - 1))
```

maple [A] time = 0.22, size = 34, normalized size = 1.62

$$-\frac{\ln(2\cos(x)-1)}{6}-\frac{\ln(1+2\cos(x))}{6}+\frac{\ln(-1+\cos(x))}{6}+\frac{\ln(1+\cos(x))}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)*csc(3*x),x)
```

```
[Out] -1/6*ln(2*cos(x)-1)-1/6*ln(1+2*cos(x))+1/6*ln(-1+cos(x))+1/6*ln(1+cos(x))
```

maxima [B] time = 0.43, size = 129, normalized size = 6.14

$$-\frac{1}{12}\log\left(2(\cos(x)+1)\cos(2x)+\cos(2x)^2+\cos(x)^2+\sin(2x)^2+2\sin(2x)\sin(x)+\sin(x)^2+2\cos(x)+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*csc(3*x),x, algorithm="maxima")
```

```
[Out] -1/12*log(2*(cos(x) + 1)*cos(2*x) + cos(2*x)^2 + cos(x)^2 + sin(2*x)^2 + 2*
sin(2*x)*sin(x) + sin(x)^2 + 2*cos(x) + 1) - 1/12*log(-2*(cos(x) - 1)*cos(2
*x) + cos(2*x)^2 + cos(x)^2 + sin(2*x)^2 - 2*sin(2*x)*sin(x) + sin(x)^2 - 2
*cos(x) + 1) + 1/6*log(cos(x)^2 + sin(x)^2 + 2*cos(x) + 1) + 1/6*log(cos(x)
^2 + sin(x)^2 - 2*cos(x) + 1)
```

mupad [B] time = 0.10, size = 17, normalized size = 0.81

$$\frac{\ln(\sin(x))}{3}-\frac{\ln\left(\frac{1}{4}-\cos(x)^2\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)/sin(3*x),x)
```

```
[Out] log(sin(x))/3 - log(1/4 - cos(x)^2)/6
```

sympy [A] time = 1.33, size = 17, normalized size = 0.81

$$-\frac{\log\left(4\sin^2(x)-3\right)}{6}+\frac{\log(\sin(x))}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*csc(3*x),x)
```

```
[Out] -log(4*sin(x)**2 - 3)/6 + log(sin(x))/3
```

3.125 $\int \cos(x)\csc(4x)\,dx$

Optimal. Leaf size=26

$$\frac{\tanh^{-1}\left(\sqrt{2}\cos(x)\right)}{2\sqrt{2}}-\frac{1}{4}\tanh^{-1}(\cos(x))$$

```
[Out] -1/4*arctanh(cos(x))+1/4*arctanh(cos(x)*2^(1/2))*2^(1/2)
```

Rubi [A] time = 0.03, antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {1093, 206}

$$\frac{\tanh^{-1}\left(\sqrt{2}\cos(x)\right)}{2\sqrt{2}}-\frac{1}{4}\tanh^{-1}(\cos(x))$$

Antiderivative was successfully verified.

```
[In] Int[Cos[x]*Csc[4*x],x]
```

```
[Out] -ArcTanh[Cos[x]]/4 + ArcTanh[Sqrt[2]*Cos[x]]/(2*Sqrt[2])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 1093

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(-1), x_Symbol] :> With[{q = Rt[b^
2 - 4*a*c, 2]}, Dist[c/q, Int[1/(b/2 - q/2 + c*x^2), x], x] - Dist[c/q, Int
[1/(b/2 + q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c,
0] && PosQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \cos(x)\csc(4x)\,dx = -\text{Subst}\left(\int \frac{1}{-4+12x^2-8x^4}\,dx, x, \cos(x)\right)$$
$$= 2\,\text{Subst}\left(\int \frac{1}{4-8x^2}\,dx, x, \cos(x)\right) - 2\,\text{Subst}\left(\int \frac{1}{8-8x^2}\,dx, x, \cos(x)\right)$$
$$= -\frac{1}{4}\tanh^{-1}(\cos(x)) + \frac{\tanh^{-1}\left(\sqrt{2}\cos(x)\right)}{2\sqrt{2}}$$

Mathematica [C] time = 0.06, size = 66, normalized size = 2.54

$$\frac{1}{4}\left(\log\left(\sin\left(\frac{x}{2}\right)\right) - \log\left(\cos\left(\frac{x}{2}\right)\right) + (1+i)(-1)^{3/4}\tanh^{-1}\left(\frac{\tan\left(\frac{x}{2}\right)-1}{\sqrt{2}}\right) + \sqrt{2}\tanh^{-1}\left(\frac{\tan\left(\frac{x}{2}\right)+1}{\sqrt{2}}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[x]*Csc[4*x],x]
```

```
[Out] ((1 + I)*(-1)^(3/4)*ArcTanh[(-1 + Tan[x/2])/Sqrt[2]] + Sqrt[2]*ArcTanh[(1 +
 Tan[x/2])/Sqrt[2]] - Log[Cos[x/2]] + Log[Sin[x/2]])/4
```

fricas [B] time = 0.83, size = 52, normalized size = 2.00

$$\frac{1}{8}\sqrt{2}\log\left(-\frac{2\cos(x)^2 + 2\sqrt{2}\cos(x) + 1}{2\cos(x)^2 - 1}\right) - \frac{1}{8}\log\left(\frac{1}{2}\cos(x) + \frac{1}{2}\right) + \frac{1}{8}\log\left(-\frac{1}{2}\cos(x) + \frac{1}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*csc(4*x),x, algorithm="fricas")
```

```
[Out] 1/8*sqrt(2)*log(-(2*cos(x)^2 + 2*sqrt(2)*cos(x) + 1)/(2*cos(x)^2 - 1)) - 1/
8*log(1/2*cos(x) + 1/2) + 1/8*log(-1/2*cos(x) + 1/2)
```

giac [B] time = 0.13, size = 48, normalized size = 1.85

$$-\frac{1}{8}\sqrt{2}\log\left(\frac{\left|-2\sqrt{2} + 4\cos(x)\right|}{\left|2\sqrt{2} + 4\cos(x)\right|}\right) - \frac{1}{8}\log\left(\cos(x) + 1\right) + \frac{1}{8}\log\left(-\cos(x) + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*csc(4*x),x, algorithm="giac")
```

```
[Out] -1/8*sqrt(2)*log(abs(-2*sqrt(2) + 4*cos(x))/abs(2*sqrt(2) + 4*cos(x))) - 1/
8*log(cos(x) + 1) + 1/8*log(-cos(x) + 1)
```

maple [A] time = 0.14, size = 28, normalized size = 1.08

$$\frac{\ln(-1+\cos(x))}{8}+\frac{\operatorname{arctanh}\left(\cos(x)\sqrt{2}\right)\sqrt{2}}{4}-\frac{\ln(1+\cos(x))}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)*csc(4*x),x)
```

```
[Out] 1/8*ln(-1+cos(x))+1/4*arctanh(cos(x)*2^(1/2))*2^(1/2)-1/8*ln(1+cos(x))
```

maxima [B] time = 0.44, size = 163, normalized size = 6.27

$$\frac{1}{16}\sqrt{2}\log\left(2\sqrt{2}\sin(2x)\sin(x)+2\left(\sqrt{2}\cos(x)+1\right)\cos(2x)+\cos(2x)^2+2\cos(x)^2+\sin(2x)^2+2\sin(x)^2\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*csc(4*x),x, algorithm="maxima")
```

```
[Out] 1/16*sqrt(2)*log(2*sqrt(2)*sin(2*x)*sin(x) + 2*(sqrt(2)*cos(x) + 1)*cos(2*x
) + cos(2*x)^2 + 2*cos(x)^2 + sin(2*x)^2 + 2*sin(x)^2 + 2*sqrt(2)*cos(x) +
1) - 1/16*sqrt(2)*log(-2*sqrt(2)*sin(2*x)*sin(x) - 2*(sqrt(2)*cos(x) - 1)*c
os(2*x) + cos(2*x)^2 + 2*cos(x)^2 + sin(2*x)^2 + 2*sin(x)^2 - 2*sqrt(2)*cos
(x) + 1) - 1/8*log(cos(x)^2 + sin(x)^2 + 2*cos(x) + 1) + 1/8*log(cos(x)^2 +
 sin(x)^2 - 2*cos(x) + 1)
```

mupad [B] time = 2.31, size = 55, normalized size = 2.12

$$\frac{\ln\left(\tan\left(\frac{x}{2}\right)\right)}{4}+\frac{\sqrt{2}\operatorname{atanh}\left(\frac{41\sqrt{2}}{8\left(\frac{169\tan\left(\frac{x}{2}\right)^2}{4}-\frac{29}{4}\right)}-\frac{239\sqrt{2}\tan\left(\frac{x}{2}\right)^2}{8\left(\frac{169\tan\left(\frac{x}{2}\right)^2}{4}-\frac{29}{4}\right)}\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)/sin(4*x),x)
```

```
[Out] log(tan(x/2))/4 + (2^(1/2)*atanh((41*2^(1/2))/(8*((169*tan(x/2)^2)/4 - 29/4
)) - (239*2^(1/2)*tan(x/2)^2)/(8*((169*tan(x/2)^2)/4 - 29/4))))/4
```

sympy [B] time = 6.14, size = 248, normalized size = 9.54

$$-\frac{19601\sqrt{2}\log\left(\tan\left(\frac{x}{2}\right)-1+\sqrt{2}\right)}{110880\sqrt{2}+156808}-\frac{27720\log\left(\tan\left(\frac{x}{2}\right)-1+\sqrt{2}\right)}{110880\sqrt{2}+156808}+\frac{27720\log\left(\tan\left(\frac{x}{2}\right)+1+\sqrt{2}\right)}{110880\sqrt{2}+156808}+\frac{19601\sqrt{2}\,\mathrm{lo}}{1108}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*csc(4*x),x)
```

```
[Out] -19601*sqrt(2)*log(tan(x/2) - 1 + sqrt(2))/(110880*sqrt(2) + 156808) - 2772
0*log(tan(x/2) - 1 + sqrt(2))/(110880*sqrt(2) + 156808) + 27720*log(tan(x/2
) + 1 + sqrt(2))/(110880*sqrt(2) + 156808) + 19601*sqrt(2)*log(tan(x/2) + 1
 + sqrt(2))/(110880*sqrt(2) + 156808) + 27720*log(tan(x/2) - sqrt(2) - 1)/(
110880*sqrt(2) + 156808) + 19601*sqrt(2)*log(tan(x/2) - sqrt(2) - 1)/(11088
0*sqrt(2) + 156808) - 19601*sqrt(2)*log(tan(x/2) - sqrt(2) + 1)/(110880*sqr
t(2) + 156808) - 27720*log(tan(x/2) - sqrt(2) + 1)/(110880*sqrt(2) + 156808
) + 27720*sqrt(2)*log(tan(x/2))/(110880*sqrt(2) + 156808) + 39202*log(tan(x
/2))/(110880*sqrt(2) + 156808)
```

3.126 $\int \cos(x)\csc(5x)\,dx$

Optimal. Leaf size=62

$$-\frac{1}{20}\left(1+\sqrt{5}\right)\log\left(-8\sin^2(x)-\sqrt{5}+5\right)-\frac{1}{20}\left(1-\sqrt{5}\right)\log\left(-8\sin^2(x)+\sqrt{5}+5\right)+\frac{1}{5}\log(\sin(x))$$

```
[Out] 1/5*ln(sin(x))-1/20*ln(5-8*sin(x)^2+5^(1/2))*(-5^(1/2)+1)-1/20*ln(5-8*sin(x
)^2-5^(1/2))*(5^(1/2)+1)
```

Rubi [A] time = 0.07, antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.857, Rules used = {4356, 1114, 705, 29, 632, 31}

$$-\frac{1}{20}\left(1+\sqrt{5}\right)\log\left(-8\sin^2(x)-\sqrt{5}+5\right)-\frac{1}{20}\left(1-\sqrt{5}\right)\log\left(-8\sin^2(x)+\sqrt{5}+5\right)+\frac{1}{5}\log(\sin(x))$$

Antiderivative was successfully verified.

```
[In] Int[Cos[x]*Csc[5*x],x]
```

```
[Out] Log[Sin[x]]/5 - ((1 + Sqrt[5])*Log[5 - Sqrt[5] - 8*Sin[x]^2])/20 - ((1 - Sq
rt[5])*Log[5 + Sqrt[5] - 8*Sin[x]^2])/20
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 632

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> W
ith[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(c*d - e*(b/2 - q/2))/q, Int[1/(b/2 - q/
2 + c*x), x], x] - Dist[(c*d - e*(b/2 + q/2))/q, Int[1/(b/2 + q/2 + c*x), x
], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a
*c, 0] && NiceSqrtQ[b^2 - 4*a*c]
```

Rule 705

```
Int[1/(((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)), x_Symbol]
 :> Dist[e^2/(c*d^2 - b*d*e + a*e^2), Int[1/(d + e*x), x], x] + Dist[1/(c*d
^2 - b*d*e + a*e^2), Int[(c*d - b*e - c*e*x)/(a + b*x + c*x^2), x], x] /; F
```

```
reeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^
2, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 1114

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 4356

```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{d = FreeFacto
rs[Sin[c*(a + b*x)], x]}, Dist[d/(b*c), Subst[Int[SubstFor[1, Sin[c*(a + b*
x)]/d, u, x], x], x, Sin[c*(a + b*x)]/d], x] /; FunctionOfQ[Sin[c*(a + b*x)
]/d, u, x]] /; FreeQ[{a, b, c}, x] && (EqQ[F, Cos] || EqQ[F, cos])
```

Rubi steps

$$\begin{aligned}\int \cos(x)\csc(5x)\,dx &= \text{Subst}\left(\int \frac{1}{x\left(5-20x^2+16x^4\right)}\,dx, x, \sin(x)\right)\\ &= \frac{1}{2}\text{Subst}\left(\int \frac{1}{x\left(5-20x+16x^2\right)}\,dx, x, \sin^2(x)\right)\\ &= \frac{1}{10}\text{Subst}\left(\int \frac{1}{x}\,dx, x, \sin^2(x)\right)+\frac{1}{10}\text{Subst}\left(\int \frac{20-16x}{5-20x+16x^2}\,dx, x, \sin^2(x)\right)\\ &= \frac{1}{5}\log(\sin(x))-\frac{1}{5}\left(4\left(1-\sqrt{5}\right)\right)\text{Subst}\left(\int \frac{1}{-10-2\sqrt{5}+16x}\,dx, x, \sin^2(x)\right)-\frac{1}{5}\left(4\left(1+\sqrt{5}\right.\right.\\ &= \frac{1}{5}\log(\sin(x))-\frac{1}{20}\left(1+\sqrt{5}\right)\log\left(5-\sqrt{5}-8\sin^2(x)\right)-\frac{1}{20}\left(1-\sqrt{5}\right)\log\left(5+\sqrt{5}-8\sin\right.\end{aligned}$$

Mathematica [A] time = 0.06, size = 57, normalized size = 0.92

$$\frac{1}{20}\left(4\log(\sin(x))-\left(\left(1+\sqrt{5}\right)\log\left(4\cos(2x)-\sqrt{5}+1\right)\right)+\left(\sqrt{5}-1\right)\log\left(4\cos(2x)+\sqrt{5}+1\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[x]*Csc[5*x],x]
```

```
[Out] (-((1 + Sqrt[5])*Log[1 - Sqrt[5] + 4*Cos[2*x]]) + (-1 + Sqrt[5])*Log[1 + Sq
rt[5] + 4*Cos[2*x]] + 4*Log[Sin[x]])/20
```

fricas [A] time = 1.39, size = 72, normalized size = 1.16

$$\frac{1}{20}\sqrt{5}\log\left(\frac{32\cos(x)^4+8\left(\sqrt{5}-3\right)\cos(x)^2-3\sqrt{5}+7}{16\cos(x)^4-12\cos(x)^2+1}\right)-\frac{1}{20}\log\left(16\cos(x)^4-12\cos(x)^2+1\right)+\frac{1}{5}\log\left(\frac{1}{2}\sin\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*csc(5*x),x, algorithm="fricas")
```

```
[Out] 1/20*sqrt(5)*log((32*cos(x)^4 + 8*(sqrt(5) - 3)*cos(x)^2 - 3*sqrt(5) + 7)/(
16*cos(x)^4 - 12*cos(x)^2 + 1)) - 1/20*log(16*cos(x)^4 - 12*cos(x)^2 + 1) +
 1/5*log(1/2*sin(x))
```

giac [A] time = 0.14, size = 67, normalized size = 1.08

$$-\frac{1}{20}\sqrt{5}\log\left(\frac{\left|32\cos(x)^2-4\sqrt{5}-12\right|}{\left|32\cos(x)^2+4\sqrt{5}-12\right|}\right)+\frac{1}{10}\log\left(-\cos(x)^2+1\right)-\frac{1}{20}\log\left(\left|16\cos(x)^4-12\cos(x)^2+1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*csc(5*x),x, algorithm="giac")
```

```
[Out] -1/20*sqrt(5)*log(abs(32*cos(x)^2 - 4*sqrt(5) - 12)/abs(32*cos(x)^2 + 4*sqr
t(5) - 12)) + 1/10*log(-cos(x)^2 + 1) - 1/20*log(abs(16*cos(x)^4 - 12*cos(x
)^2 + 1))
```

maple [A] time = 0.26, size = 80, normalized size = 1.29

$$-\frac{\ln\left(4\left(\cos^2(x)\right)+2\cos(x)-1\right)}{20}-\frac{\sqrt{5}\ \operatorname{arctanh}\left(\frac{(8\cos(x)+2)\sqrt{5}}{10}\right)}{10}+\frac{\ln\left(-1+\cos(x)\right)}{10}-\frac{\ln\left(4\left(\cos^2(x)\right)-2\cos(x)-1\right.}{20}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)*csc(5*x),x)
```

```
[Out] -1/20*ln(4*cos(x)^2+2*cos(x)-1)-1/10*5^(1/2)*arctanh(1/10*(8*cos(x)+2)*5^(1
/2))+1/10*ln(-1+cos(x))-1/20*ln(4*cos(x)^2-2*cos(x)-1)+1/10*5^(1/2)*arctanh
(1/10*(8*cos(x)-2)*5^(1/2))+1/10*ln(1+cos(x))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*csc(5*x),x, algorithm="maxima")
```

```
[Out] -1/10*integrate(-(cos(2*x)*sin(4*x) - cos(4*x)*sin(2*x) + cos(3/2*arctan2(s
in(2*x), cos(2*x)))*sin(2*x) + cos(1/2*arctan2(sin(2*x), cos(2*x)))*sin(2*x
) - cos(2*x)*sin(3/2*arctan2(sin(2*x), cos(2*x))) - cos(2*x)*sin(1/2*arctan
2(sin(2*x), cos(2*x))) - sin(2*x))/(2*(cos(2*x) + 1)*cos(4*x) + cos(4*x)^2
+ cos(2*x)^2 - 2*(cos(4*x) + cos(2*x) - cos(1/2*arctan2(sin(2*x), cos(2*x))
) + 1)*cos(3/2*arctan2(sin(2*x), cos(2*x))) + cos(3/2*arctan2(sin(2*x), cos
(2*x)))^2 - 2*(cos(4*x) + cos(2*x) + 1)*cos(1/2*arctan2(sin(2*x), cos(2*x))
) + cos(1/2*arctan2(sin(2*x), cos(2*x)))^2 + sin(4*x)^2 + 2*sin(4*x)*sin(2*
x) + sin(2*x)^2 - 2*(sin(4*x) + sin(2*x) - sin(1/2*arctan2(sin(2*x), cos(2*
x))))*sin(3/2*arctan2(sin(2*x), cos(2*x))) + sin(3/2*arctan2(sin(2*x), cos(
2*x)))^2 - 2*(sin(4*x) + sin(2*x))*sin(1/2*arctan2(sin(2*x), cos(2*x))) + s
in(1/2*arctan2(sin(2*x), cos(2*x)))^2 + 2*cos(2*x) + 1), x) + 1/10*integrat
e((cos(2*x)*sin(4*x) - cos(4*x)*sin(2*x) - cos(3/2*arctan2(sin(2*x), cos(2*
x)))*sin(2*x) - cos(1/2*arctan2(sin(2*x), cos(2*x)))*sin(2*x) + cos(2*x)*si
n(3/2*arctan2(sin(2*x), cos(2*x))) + cos(2*x)*sin(1/2*arctan2(sin(2*x), cos
(2*x))) - sin(2*x))/(2*(cos(2*x) + 1)*cos(4*x) + cos(4*x)^2 + cos(2*x)^2 +
2*(cos(4*x) + cos(2*x) + cos(1/2*arctan2(sin(2*x), cos(2*x))) + 1)*cos(3/2*
arctan2(sin(2*x), cos(2*x))) + cos(3/2*arctan2(sin(2*x), cos(2*x)))^2 + 2*(
cos(4*x) + cos(2*x) + 1)*cos(1/2*arctan2(sin(2*x), cos(2*x))) + cos(1/2*arc
tan2(sin(2*x), cos(2*x)))^2 + sin(4*x)^2 + 2*sin(4*x)*sin(2*x) + sin(2*x)^2
 + 2*(sin(4*x) + sin(2*x) + sin(1/2*arctan2(sin(2*x), cos(2*x))))*sin(3/2*a
rctan2(sin(2*x), cos(2*x))) + sin(3/2*arctan2(sin(2*x), cos(2*x)))^2 + 2*(s
in(4*x) + sin(2*x))*sin(1/2*arctan2(sin(2*x), cos(2*x))) + sin(1/2*arctan2(
sin(2*x), cos(2*x)))^2 + 2*cos(2*x) + 1), x) - 1/10*integrate((cos(x)*sin(4
*x) + cos(x)*sin(3*x) + cos(x)*sin(2*x) - cos(4*x)*sin(x) - cos(3*x)*sin(x)
 - cos(2*x)*sin(x) - sin(x))/(2*(cos(3*x) + cos(2*x) + cos(x) + 1)*cos(4*x)
 + cos(4*x)^2 + 2*(cos(2*x) + cos(x) + 1)*cos(3*x) + cos(3*x)^2 + 2*(cos(x)
 + 1)*cos(2*x) + cos(2*x)^2 + cos(x)^2 + 2*(sin(3*x) + sin(2*x) + sin(x))*s
in(4*x) + sin(4*x)^2 + 2*(sin(2*x) + sin(x))*sin(3*x) + sin(3*x)^2 + sin(2*
x)^2 + 2*sin(2*x)*sin(x) + sin(x)^2 + 2*cos(x) + 1), x) + 1/10*integrate(-(
cos(x)*sin(4*x) - cos(x)*sin(3*x) + cos(x)*sin(2*x) - cos(4*x)*sin(x) + cos
(3*x)*sin(x) - cos(2*x)*sin(x) - sin(x))/(2*(cos(3*x) - cos(2*x) + cos(x) -
 1)*cos(4*x) - cos(4*x)^2 + 2*(cos(2*x) - cos(x) + 1)*cos(3*x) - cos(3*x)^2
 + 2*(cos(x) - 1)*cos(2*x) - cos(2*x)^2 - cos(x)^2 + 2*(sin(3*x) - sin(2*x)
 + sin(x))*sin(4*x) - sin(4*x)^2 + 2*(sin(2*x) - sin(x))*sin(3*x) - sin(3*x
)^2 - sin(2*x)^2 + 2*sin(2*x)*sin(x) - sin(x)^2 + 2*cos(x) - 1), x) + 3/10*
integrate(-(cos(4/3*arctan2(sin(3*x), cos(3*x)))*sin(3*x) + cos(2/3*arctan2
(sin(3*x), cos(3*x)))*sin(3*x) + cos(1/3*arctan2(sin(3*x), cos(3*x)))*sin(3
*x) - cos(3*x)*sin(4/3*arctan2(sin(3*x), cos(3*x))) - cos(3*x)*sin(2/3*arct
an2(sin(3*x), cos(3*x))) - cos(3*x)*sin(1/3*arctan2(sin(3*x), cos(3*x))) +
sin(3*x))/(cos(3*x)^2 + 2*(cos(3*x) + cos(2/3*arctan2(sin(3*x), cos(3*x)))
+ cos(1/3*arctan2(sin(3*x), cos(3*x))) + 1)*cos(4/3*arctan2(sin(3*x), cos(3
*x))) + cos(4/3*arctan2(sin(3*x), cos(3*x)))^2 + 2*(cos(3*x) + cos(1/3*arct
```

```
an2(sin(3*x), cos(3*x))) + 1)*cos(2/3*arctan2(sin(3*x), cos(3*x))) + cos(2/
3*arctan2(sin(3*x), cos(3*x)))^2 + 2*(cos(3*x) + 1)*cos(1/3*arctan2(sin(3*x
), cos(3*x))) + cos(1/3*arctan2(sin(3*x), cos(3*x)))^2 + sin(3*x)^2 + 2*(si
n(3*x) + sin(2/3*arctan2(sin(3*x), cos(3*x))) + sin(1/3*arctan2(sin(3*x), c
os(3*x))))*sin(4/3*arctan2(sin(3*x), cos(3*x))) + sin(4/3*arctan2(sin(3*x),
 cos(3*x)))^2 + 2*(sin(3*x) + sin(1/3*arctan2(sin(3*x), cos(3*x))))*sin(2/3
*arctan2(sin(3*x), cos(3*x))) + sin(2/3*arctan2(sin(3*x), cos(3*x)))^2 + 2*
sin(3*x)*sin(1/3*arctan2(sin(3*x), cos(3*x))) + sin(1/3*arctan2(sin(3*x), c
os(3*x)))^2 + 2*cos(3*x) + 1), x) - 3/10*integrate(-(cos(4/3*arctan2(sin(3*
x), cos(3*x)))*sin(3*x) + cos(2/3*arctan2(sin(3*x), cos(3*x)))*sin(3*x) - c
os(1/3*arctan2(sin(3*x), cos(3*x)))*sin(3*x) - cos(3*x)*sin(4/3*arctan2(sin
(3*x), cos(3*x))) - cos(3*x)*sin(2/3*arctan2(sin(3*x), cos(3*x))) + cos(3*x
)*sin(1/3*arctan2(sin(3*x), cos(3*x))) + sin(3*x))/(cos(3*x)^2 - 2*(cos(3*x
) - cos(2/3*arctan2(sin(3*x), cos(3*x))) + cos(1/3*arctan2(sin(3*x), cos(3*
x))) - 1)*cos(4/3*arctan2(sin(3*x), cos(3*x))) + cos(4/3*arctan2(sin(3*x),
cos(3*x)))^2 - 2*(cos(3*x) + cos(1/3*arctan2(sin(3*x), cos(3*x))) - 1)*cos(
2/3*arctan2(sin(3*x), cos(3*x))) + cos(2/3*arctan2(sin(3*x), cos(3*x)))^2 +
 2*(cos(3*x) - 1)*cos(1/3*arctan2(sin(3*x), cos(3*x))) + cos(1/3*arctan2(si
n(3*x), cos(3*x)))^2 + sin(3*x)^2 - 2*(sin(3*x) - sin(2/3*arctan2(sin(3*x),
 cos(3*x))) + sin(1/3*arctan2(sin(3*x), cos(3*x))))*sin(4/3*arctan2(sin(3*x
), cos(3*x))) + sin(4/3*arctan2(sin(3*x), cos(3*x)))^2 - 2*(sin(3*x) + sin(
1/3*arctan2(sin(3*x), cos(3*x))))*sin(2/3*arctan2(sin(3*x), cos(3*x))) + si
n(2/3*arctan2(sin(3*x), cos(3*x)))^2 + 2*sin(3*x)*sin(1/3*arctan2(sin(3*x),
 cos(3*x))) + sin(1/3*arctan2(sin(3*x), cos(3*x)))^2 - 2*cos(3*x) + 1), x)
+ 1/5*integrate((sin(4*x) + sin(3*x) + sin(2*x) + sin(x))/(2*(cos(3*x) + co
s(2*x) + cos(x) + 1)*cos(4*x) + cos(4*x)^2 + 2*(cos(2*x) + cos(x) + 1)*cos(
3*x) + cos(3*x)^2 + 2*(cos(x) + 1)*cos(2*x) + cos(2*x)^2 + cos(x)^2 + 2*(si
n(3*x) + sin(2*x) + sin(x))*sin(4*x) + sin(4*x)^2 + 2*(sin(2*x) + sin(x))*s
in(3*x) + sin(3*x)^2 + sin(2*x)^2 + 2*sin(2*x)*sin(x) + sin(x)^2 + 2*cos(x)
 + 1), x) + 1/5*integrate(-(sin(4*x) - sin(3*x) + sin(2*x) - sin(x))/(2*(co
s(3*x) - cos(2*x) + cos(x) - 1)*cos(4*x) - cos(4*x)^2 + 2*(cos(2*x) - cos(x
) + 1)*cos(3*x) - cos(3*x)^2 + 2*(cos(x) - 1)*cos(2*x) - cos(2*x)^2 - cos(x
)^2 + 2*(sin(3*x) - sin(2*x) + sin(x))*sin(4*x) - sin(4*x)^2 + 2*(sin(2*x)
- sin(x))*sin(3*x) - sin(3*x)^2 - sin(2*x)^2 + 2*sin(2*x)*sin(x) - sin(x)^2
 + 2*cos(x) - 1), x) + 1/10*log(cos(x)^2 + sin(x)^2 + 2*cos(x) + 1) + 1/10*
log(cos(x)^2 + sin(x)^2 - 2*cos(x) + 1)
```

mupad [B] time = 2.68, size = 51, normalized size = 0.82

$$\frac{\ln(\sin(x))}{5}+\ln\left(-\cos(x)^2-\frac{\sqrt{5}}{8}+\frac{3}{8}\right)\left(\frac{\sqrt{5}}{20}-\frac{1}{20}\right)-\ln\left(-\cos(x)^2+\frac{\sqrt{5}}{8}+\frac{3}{8}\right)\left(\frac{\sqrt{5}}{20}+\frac{1}{20}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)/sin(5*x),x)
```

```
[Out] log(sin(x))/5 + log(3/8 - 5^(1/2)/8 - cos(x)^2)*(5^(1/2)/20 - 1/20) - log(5
^(1/2)/8 - cos(x)^2 + 3/8)*(5^(1/2)/20 + 1/20)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \cos(x)\csc(5x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*csc(5*x),x)
```

```
[Out] Integral(cos(x)*csc(5*x), x)
```

3.127 $\int \cos(x)\csc(6x)\,dx$

Optimal. Leaf size=36

$$-\frac{1}{6}\tanh^{-1}(\cos(x))-\frac{1}{6}\tanh^{-1}(2\cos(x))+\frac{\tanh^{-1}\left(\frac{2\cos(x)}{\sqrt{3}}\right)}{2\sqrt{3}}$$

```
[Out] -1/6*arctanh(cos(x))-1/6*arctanh(2*cos(x))+1/6*arctanh(2/3*cos(x)*3^(1/2))*
3^(1/2)
```

Rubi [A] time = 0.04, antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 3, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {12, 2057, 207}

$$-\frac{1}{6}\tanh^{-1}(\cos(x))-\frac{1}{6}\tanh^{-1}(2\cos(x))+\frac{\tanh^{-1}\left(\frac{2\cos(x)}{\sqrt{3}}\right)}{2\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[x]*Csc[6*x],x]
```

```
[Out] -ArcTanh[Cos[x]]/6 - ArcTanh[2*Cos[x]]/6 + ArcTanh[(2*Cos[x])/Sqrt[3]]/(2*S
qrt[3])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 2057

```
Int[(P_)^(p_), x_Symbol] :> With[{u = Factor[P /. x -> Sqrt[x]]}, Int[Expan
dIntegrand[(u /. x -> x^2)^p, x], x] /;  !SumQ[NonfreeFactors[u, x]]] /; Po
lyQ[P, x^2] && ILtQ[p, 0]
```

Rubi steps

$$\int \cos(x)\csc(6x)\,dx = -\text{Subst}\left(\int \frac{1}{2\left(3-19x^2+32x^4-16x^6\right)}\,dx, x, \cos(x)\right)$$
$$= -\left(\frac{1}{2}\text{Subst}\left(\int \frac{1}{3-19x^2+32x^4-16x^6}\,dx, x, \cos(x)\right)\right)$$
$$= -\left(\frac{1}{2}\text{Subst}\left(\int \left(-\frac{1}{3\left(-1+x^2\right)}+\frac{2}{-3+4x^2}-\frac{2}{3\left(-1+4x^2\right)}\right)dx, x, \cos(x)\right)\right)$$
$$= \frac{1}{6}\text{Subst}\left(\int \frac{1}{-1+x^2}\,dx, x, \cos(x)\right)+\frac{1}{3}\text{Subst}\left(\int \frac{1}{-1+4x^2}\,dx, x, \cos(x)\right)-\text{Subst}\left(\int \frac{}{-3}\right.$$
$$= -\frac{1}{6}\tanh^{-1}(\cos(x))-\frac{1}{6}\tanh^{-1}(2\cos(x))+\frac{\tanh^{-1}\left(\frac{2\cos(x)}{\sqrt{3}}\right)}{2\sqrt{3}}$$

Mathematica [B] time = 0.08, size = 83, normalized size = 2.31

$$\frac{1}{12}\left(2\log\left(\sin\left(\frac{x}{2}\right)\right)-2\log\left(\cos\left(\frac{x}{2}\right)\right)+\log(1-2\cos(x))-\log(2\cos(x)+1)-2\sqrt{3}\tanh^{-1}\left(\frac{\tan\left(\frac{x}{2}\right)-2}{\sqrt{3}}\right)+2\sqrt{3}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[x]*Csc[6*x],x]
```

```
[Out] (-2*Sqrt[3]*ArcTanh[(-2 + Tan[x/2])/Sqrt[3]] + 2*Sqrt[3]*ArcTanh[(2 + Tan[x
/2])/Sqrt[3]] - 2*Log[Cos[x/2]] + Log[1 - 2*Cos[x]] - Log[1 + 2*Cos[x]] + 2
*Log[Sin[x/2]])/12
```

fricas [B] time = 0.71, size = 70, normalized size = 1.94

$$\frac{1}{12}\sqrt{3}\log\left(-\frac{4\cos(x)^2+4\sqrt{3}\cos(x)+3}{4\cos(x)^2-3}\right)-\frac{1}{12}\log\left(\frac{1}{2}\cos(x)+\frac{1}{2}\right)+\frac{1}{12}\log\left(-\frac{1}{2}\cos(x)+\frac{1}{2}\right)+\frac{1}{12}\log\left(-2\cos(\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*csc(6*x),x, algorithm="fricas")
```

```
[Out] 1/12*sqrt(3)*log(-(4*cos(x)^2 + 4*sqrt(3)*cos(x) + 3)/(4*cos(x)^2 - 3)) - 1
/12*log(1/2*cos(x) + 1/2) + 1/12*log(-1/2*cos(x) + 1/2) + 1/12*log(-2*cos(x
) + 1) - 1/12*log(-2*cos(x) - 1)
```

giac [B] time = 0.15, size = 68, normalized size = 1.89

$$-\frac{1}{12}\sqrt{3}\log\left(\frac{\left|-4\sqrt{3}+8\cos(x)\right|}{\left|4\sqrt{3}+8\cos(x)\right|}\right)-\frac{1}{12}\log\left(\cos(x)+1\right)+\frac{1}{12}\log\left(-\cos(x)+1\right)-\frac{1}{12}\log\left(\left|2\cos(x)+1\right|\right)+\frac{1}{12}\log\left(\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*csc(6*x),x, algorithm="giac")
```

```
[Out] -1/12*sqrt(3)*log(abs(-4*sqrt(3) + 8*cos(x))/abs(4*sqrt(3) + 8*cos(x))) - 1
/12*log(cos(x) + 1) + 1/12*log(-cos(x) + 1) - 1/12*log(abs(2*cos(x) + 1)) +
 1/12*log(abs(2*cos(x) - 1))
```

maple [A] time = 0.17, size = 47, normalized size = 1.31

$$\frac{\ln(2\cos(x)-1)}{12}-\frac{\ln(1+2\cos(x))}{12}+\frac{\operatorname{arctanh}\left(\frac{2\cos(x)\sqrt{3}}{3}\right)\sqrt{3}}{6}+\frac{\ln(-1+\cos(x))}{12}-\frac{\ln(1+\cos(x))}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)*csc(6*x),x)
```

```
[Out] 1/12*ln(2*cos(x)-1)-1/12*ln(1+2*cos(x))+1/6*arctanh(2/3*cos(x)*3^(1/2))*3^(
1/2)+1/12*ln(-1+cos(x))-1/12*ln(1+cos(x))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{(\sin(3x)-\sin(x))\cos(4x)-(\cos(3x)-\cos(x))\sin(4x)-(\cos(2x)-1)\sin(3x)+\cos(3x)\sin(2x)-c}{2\left(2(\cos(2x)-1)\cos(4x)-\cos(4x)^2-\cos(2x)^2-\sin(4x)^2+2\sin(4x)\sin(2x)-\sin\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*csc(6*x),x, algorithm="maxima")
```

```
[Out] -integrate(1/2*((sin(3*x) - sin(x))*cos(4*x) - (cos(3*x) - cos(x))*sin(4*x)
 - (cos(2*x) - 1)*sin(3*x) + cos(3*x)*sin(2*x) - cos(x)*sin(2*x) + cos(2*x)
*sin(x) - sin(x))/(2*(cos(2*x) - 1)*cos(4*x) - cos(4*x)^2 - cos(2*x)^2 - si
n(4*x)^2 + 2*sin(4*x)*sin(2*x) - sin(2*x)^2 + 2*cos(2*x) - 1), x) - 1/24*lo
g(2*(cos(x) + 1)*cos(2*x) + cos(2*x)^2 + cos(x)^2 + sin(2*x)^2 + 2*sin(2*x)
*sin(x) + sin(x)^2 + 2*cos(x) + 1) + 1/24*log(-2*(cos(x) - 1)*cos(2*x) + co
s(2*x)^2 + cos(x)^2 + sin(2*x)^2 - 2*sin(2*x)*sin(x) + sin(x)^2 - 2*cos(x)
+ 1) - 1/12*log(cos(x)^2 + sin(x)^2 + 2*cos(x) + 1) + 1/12*log(cos(x)^2 + s
in(x)^2 - 2*cos(x) + 1)
```

mupad [B] time = 2.37, size = 74, normalized size = 2.06

$$\frac{\operatorname{atanh}\left(\frac{10737418 24}{10761687\left(\frac{427973089951744\tan\left(\frac{x}{2}\right)^2}{14348907}-\frac{47552804159488}{4782969}\right)}+\frac{797161}{797162}\right)}{6}+\frac{\ln\left(\tan\left(\frac{x}{2}\right)\right)}{6}+\frac{\sqrt{3}\operatorname{atanh}\left(\frac{4222769432625152}{4782969\left(\frac{101871591633190912\tan\left(\frac{x}{2}\right)^2}{4782969}\right.}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)/sin(6*x),x)
```

```
[Out] atanh(1073741824/(10761687*((427973089951744*tan(x/2)^2)/14348907 - 4755280
4159488/4782969)) + 797161/797162)/6 + log(tan(x/2))/6 + (3^(1/2)*atanh((42
22769432625152*3^(1/2))/(4782969*((101871591633190912*tan(x/2)^2)/4782969 -
 7314051205955584/4782969)) - (19605196950732800*3^(1/2)*tan(x/2)^2)/(15943
23*((101871591633190912*tan(x/2)^2)/4782969 - 7314051205955584/4782969))))/
6
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \cos(x)\csc(6x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*csc(6*x),x)
```

```
[Out] Integral(cos(x)*csc(6*x), x)
```

3.128 $\int \cos^3(6x)\sin(x)\,dx$

Optimal. Leaf size=33

$$\frac{3}{40}\cos(5x)-\frac{3}{56}\cos(7x)+\frac{1}{136}\cos(17x)-\frac{1}{152}\cos(19x)$$

```
[Out] 3/40*cos(5*x)-3/56*cos(7*x)+1/136*cos(17*x)-1/152*cos(19*x)
```

Rubi [A] time = 0.03, antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {4354, 2638}

$$\frac{3}{40}\cos(5x)-\frac{3}{56}\cos(7x)+\frac{1}{136}\cos(17x)-\frac{1}{152}\cos(19x)$$

Antiderivative was successfully verified.

```
[In] Int[Cos[6*x]^3*Sin[x],x]
```

```
[Out] (3*Cos[5*x])/40 - (3*Cos[7*x])/56 + Cos[17*x]/136 - Cos[19*x]/152
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 4354

```
Int[(F_)[(a_.) + (b_.)*(x_)]^(p_.)*(G_)[(c_.) + (d_.)*(x_)]^(q_.), x_Symbol
] :> Int[ExpandTrigReduce[ActivateTrig[F[a + b*x]^p*G[c + d*x]^q], x], x] /
; FreeQ[{a, b, c, d}, x] && (EqQ[F, sin] || EqQ[F, cos]) && (EqQ[G, sin] ||
 EqQ[G, cos]) && IGtQ[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int \cos^3(6x)\sin(x)\,dx &= \int\left(-\frac{3}{8}\sin(5x)+\frac{3}{8}\sin(7x)-\frac{1}{8}\sin(17x)+\frac{1}{8}\sin(19x)\right)dx\\ &= -\left(\frac{1}{8}\int\sin(17x)\,dx\right)+\frac{1}{8}\int\sin(19x)\,dx-\frac{3}{8}\int\sin(5x)\,dx+\frac{3}{8}\int\sin(7x)\,dx\\ &= \frac{3}{40}\cos(5x)-\frac{3}{56}\cos(7x)+\frac{1}{136}\cos(17x)-\frac{1}{152}\cos(19x)\end{aligned}$$

Mathematica [A] time = 0.02, size = 33, normalized size = 1.00

$$\frac{3}{40}\cos(5x)-\frac{3}{56}\cos(7x)+\frac{1}{136}\cos(17x)-\frac{1}{152}\cos(19x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[6*x]^3*Sin[x],x]
[Out] (3*Cos[5*x])/40 - (3*Cos[7*x])/56 + Cos[17*x]/136 - Cos[19*x]/152
```

fricas [B] time = 3.97, size = 57, normalized size = 1.73

$$-\frac{32768}{19}\cos(x)^{19}+\frac{147456}{17}\cos(x)^{17}-18432\cos(x)^{15}+21504\cos(x)^{13}-14976\cos(x)^{11}+6336\cos(x)^{9}-\frac{11112}{7}\cos$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(6*x)^3*sin(x),x, algorithm="fricas")
[Out] -32768/19*cos(x)^19 + 147456/17*cos(x)^17 - 18432*cos(x)^15 + 21504*cos(x)^
13 - 14976*cos(x)^11 + 6336*cos(x)^9 - 11112/7*cos(x)^7 + 1116/5*cos(x)^5 -
 18*cos(x)^3 + cos(x)
```

giac [A] time = 0.13, size = 25, normalized size = 0.76

$$-\frac{1}{152}\cos(19x)+\frac{1}{136}\cos(17x)-\frac{3}{56}\cos(7x)+\frac{3}{40}\cos(5x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(6*x)^3*sin(x),x, algorithm="giac")
[Out] -1/152*cos(19*x) + 1/136*cos(17*x) - 3/56*cos(7*x) + 3/40*cos(5*x)
```

maple [A] time = 0.25, size = 26, normalized size = 0.79

$$\frac{3\cos(5x)}{40}-\frac{3\cos(7x)}{56}+\frac{\cos(17x)}{136}-\frac{\cos(19x)}{152}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(6*x)^3*sin(x),x)
[Out] 3/40*cos(5*x)-3/56*cos(7*x)+1/136*cos(17*x)-1/152*cos(19*x)
```

maxima [A] time = 0.45, size = 25, normalized size = 0.76

$$-\frac{1}{152}\cos(19x)+\frac{1}{136}\cos(17x)-\frac{3}{56}\cos(7x)+\frac{3}{40}\cos(5x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(6*x)^3*sin(x),x, algorithm="maxima")
```

```
[Out] -1/152*cos(19*x) + 1/136*cos(17*x) - 3/56*cos(7*x) + 3/40*cos(5*x)
```

mupad [B] time = 0.08, size = 57, normalized size = 1.73

$$-\frac{32768\cos(x)^{19}}{19}+\frac{147456\cos(x)^{17}}{17}-18432\cos(x)^{15}+21504\cos(x)^{13}-14976\cos(x)^{11}+6336\cos(x)^{9}-\frac{11112\cos(}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(6*x)^3*sin(x),x)
```

```
[Out] cos(x) - 18*cos(x)^3 + (1116*cos(x)^5)/5 - (11112*cos(x)^7)/7 + 6336*cos(x)
^9 - 14976*cos(x)^11 + 21504*cos(x)^13 - 18432*cos(x)^15 + (147456*cos(x)^1
7)/17 - (32768*cos(x)^19)/19
```

sympy [B] time = 5.21, size = 63, normalized size = 1.91

$$\frac{1296\sin(x)\sin^3(6x)}{11305}+\frac{1926\sin(x)\sin(6x)\cos^2(6x)}{11305}+\frac{216\sin^2(6x)\cos(x)\cos(6x)}{11305}+\frac{251\cos(x)\cos^3(6x)}{11305}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(6*x)**3*sin(x),x)
```

```
[Out] 1296*sin(x)*sin(6*x)**3/11305 + 1926*sin(x)*sin(6*x)*cos(6*x)**2/11305 + 21
6*sin(6*x)**2*cos(x)*cos(6*x)/11305 + 251*cos(x)*cos(6*x)**3/11305
```

3.129 $\int \cos^3(6x)\sin(9x)\,dx$

Optimal. Leaf size=33

$$-\frac{1}{8}\cos(3x)+\frac{1}{72}\cos(9x)-\frac{1}{40}\cos(15x)-\frac{1}{216}\cos(27x)$$

```
[Out] -1/8*cos(3*x)+1/72*cos(9*x)-1/40*cos(15*x)-1/216*cos(27*x)
```

Rubi [A] time = 0.03, antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {4354, 2638}

$$-\frac{1}{8}\cos(3x)+\frac{1}{72}\cos(9x)-\frac{1}{40}\cos(15x)-\frac{1}{216}\cos(27x)$$

Antiderivative was successfully verified.

```
[In] Int[Cos[6*x]^3*Sin[9*x],x]
```

```
[Out] -Cos[3*x]/8 + Cos[9*x]/72 - Cos[15*x]/40 - Cos[27*x]/216
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 4354

```
Int[(F_)[(a_.) + (b_.)*(x_)]^(p_.)*(G_)[(c_.) + (d_.)*(x_)]^(q_.), x_Symbol
] :> Int[ExpandTrigReduce[ActivateTrig[F[a + b*x]^p*G[c + d*x]^q], x], x] /
; FreeQ[{a, b, c, d}, x] && (EqQ[F, sin] || EqQ[F, cos]) && (EqQ[G, sin] ||
 EqQ[G, cos]) && IGtQ[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int \cos^3(6x)\sin(9x)\,dx &= \int \left(\frac{3}{8}\sin(3x)-\frac{1}{8}\sin(9x)+\frac{3}{8}\sin(15x)+\frac{1}{8}\sin(27x)\right)dx\\ &= -\left(\frac{1}{8}\int \sin(9x)\,dx\right)+\frac{1}{8}\int \sin(27x)\,dx+\frac{3}{8}\int \sin(3x)\,dx+\frac{3}{8}\int \sin(15x)\,dx\\ &= -\frac{1}{8}\cos(3x)+\frac{1}{72}\cos(9x)-\frac{1}{40}\cos(15x)-\frac{1}{216}\cos(27x)\end{aligned}$$

Mathematica [A] time = 0.02, size = 33, normalized size = 1.00

$$-\frac{1}{8}\cos(3x)+\frac{1}{72}\cos(9x)-\frac{1}{40}\cos(15x)-\frac{1}{216}\cos(27x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[6*x]^3*Sin[9*x],x]
```

```
[Out] -1/8*Cos[3*x] + Cos[9*x]/72 - Cos[15*x]/40 - Cos[27*x]/216
```

fricas [A] time = 0.58, size = 39, normalized size = 1.18

$$-\frac{32}{27}\cos\left(3\,x\right)^{9}+\frac{8}{3}\cos\left(3\,x\right)^{7}-\frac{12}{5}\cos\left(3\,x\right)^{5}+\frac{10}{9}\cos\left(3\,x\right)^{3}-\frac{1}{3}\cos\left(3\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(6*x)^3*sin(9*x),x, algorithm="fricas")
```

```
[Out] -32/27*cos(3*x)^9 + 8/3*cos(3*x)^7 - 12/5*cos(3*x)^5 + 10/9*cos(3*x)^3 - 1/
3*cos(3*x)
```

giac [A] time = 0.14, size = 25, normalized size = 0.76

$$-\frac{1}{216}\cos\left(27\,x\right)-\frac{1}{40}\cos\left(15\,x\right)+\frac{1}{72}\cos\left(9\,x\right)-\frac{1}{8}\cos\left(3\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(6*x)^3*sin(9*x),x, algorithm="giac")
```

```
[Out] -1/216*cos(27*x) - 1/40*cos(15*x) + 1/72*cos(9*x) - 1/8*cos(3*x)
```

maple [A] time = 0.14, size = 26, normalized size = 0.79

$$-\frac{\cos\left(3x\right)}{8}+\frac{\cos\left(9x\right)}{72}-\frac{\cos\left(15x\right)}{40}-\frac{\cos\left(27x\right)}{216}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(6*x)^3*sin(9*x),x)
```

```
[Out] -1/8*cos(3*x)+1/72*cos(9*x)-1/40*cos(15*x)-1/216*cos(27*x)
```

maxima [A] time = 0.31, size = 25, normalized size = 0.76

$$-\frac{1}{216}\cos\left(27\,x\right)-\frac{1}{40}\cos\left(15\,x\right)+\frac{1}{72}\cos\left(9\,x\right)-\frac{1}{8}\cos\left(3\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(6*x)^3*sin(9*x),x, algorithm="maxima")
```

```
[Out] -1/216*cos(27*x) - 1/40*cos(15*x) + 1/72*cos(9*x) - 1/8*cos(3*x)
```

mupad [B] time = 2.47, size = 78, normalized size = 2.36

$$-\frac{2\left(135\tan\left(\frac{3x}{2}\right)^{16}-900\tan\left(\frac{3x}{2}\right)^{14}+5640\tan\left(\frac{3x}{2}\right)^{12}-13140\tan\left(\frac{3x}{2}\right)^{10}+15534\tan\left(\frac{3x}{2}\right)^{8}-4044\tan\left(\frac{3x}{2}\right)^{6}\right.}{135\left(\tan\left(\frac{3x}{2}\right)^{2}+1\right)^{9}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(6*x)^3*sin(9*x),x)
```

```
[Out] -(2*(36*tan((3*x)/2)^2 + 1584*tan((3*x)/2)^4 - 4044*tan((3*x)/2)^6 + 15534*
tan((3*x)/2)^8 - 13140*tan((3*x)/2)^10 + 5640*tan((3*x)/2)^12 - 900*tan((3*
x)/2)^14 + 135*tan((3*x)/2)^16 + 19))/(135*(tan((3*x)/2)^2 + 1)^9)
```

sympy [B] time = 5.73, size = 71, normalized size = 2.15

$$-\frac{16\sin^{3}(6x)\sin(9x)}{135}-\frac{8\sin^{2}(6x)\cos(6x)\cos(9x)}{45}-\frac{2\sin(6x)\sin(9x)\cos^{2}(6x)}{45}-\frac{19\cos^{3}(6x)\cos(9x)}{135}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(6*x)**3*sin(9*x),x)
```

```
[Out] -16*sin(6*x)**3*sin(9*x)/135 - 8*sin(6*x)**2*cos(6*x)*cos(9*x)/45 - 2*sin(6
*x)*sin(9*x)*cos(6*x)**2/45 - 19*cos(6*x)**3*cos(9*x)/135
```

3.130 $\int \cos(2x)\sin^2(6x)\,dx$

Optimal. Leaf size=25

$$\frac{1}{4}\sin(2x)-\frac{1}{40}\sin(10x)-\frac{1}{56}\sin(14x)$$

```
[Out] 1/4*sin(2*x)-1/40*sin(10*x)-1/56*sin(14*x)
```

Rubi [A] time = 0.03, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {4354, 2637}

$$\frac{1}{4}\sin(2x)-\frac{1}{40}\sin(10x)-\frac{1}{56}\sin(14x)$$

Antiderivative was successfully verified.

```
[In] Int[Cos[2*x]*Sin[6*x]^2,x]
```

```
[Out] Sin[2*x]/4 - Sin[10*x]/40 - Sin[14*x]/56
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 4354

```
Int[(F_)[(a_.) + (b_.)*(x_)]^(p_.)*(G_)[(c_.) + (d_.)*(x_)]^(q_.), x_Symbol
] :> Int[ExpandTrigReduce[ActivateTrig[F[a + b*x]^p*G[c + d*x]^q], x], x] /
; FreeQ[{a, b, c, d}, x] && (EqQ[F, sin] || EqQ[F, cos]) && (EqQ[G, sin] ||
 EqQ[G, cos]) && IGtQ[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int \cos(2x)\sin^2(6x)\,dx &= \int \left(\frac{1}{2}\cos(2x)-\frac{1}{4}\cos(10x)-\frac{1}{4}\cos(14x)\right)dx\\ &= -\left(\frac{1}{4}\int \cos(10x)\,dx\right)-\frac{1}{4}\int \cos(14x)\,dx+\frac{1}{2}\int \cos(2x)\,dx\\ &= \frac{1}{4}\sin(2x)-\frac{1}{40}\sin(10x)-\frac{1}{56}\sin(14x)\end{aligned}$$

Mathematica [A] time = 0.02, size = 25, normalized size = 1.00

$$\frac{1}{4}\sin(2x) - \frac{1}{40}\sin(10x) - \frac{1}{56}\sin(14x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[2*x]*Sin[6*x]^2,x]
[Out] Sin[2*x]/4 - Sin[10*x]/40 - Sin[14*x]/56
```

fricas [A] time = 2.01, size = 32, normalized size = 1.28

$$-\frac{1}{70}\left(80\cos(2x)^6 - 72\cos(2x)^4 + 9\cos(2x)^2 - 17\right)\sin(2x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(2*x)*sin(6*x)^2,x, algorithm="fricas")
[Out] -1/70*(80*cos(2*x)^6 - 72*cos(2*x)^4 + 9*cos(2*x)^2 - 17)*sin(2*x)
```

giac [A] time = 0.12, size = 19, normalized size = 0.76

$$-\frac{1}{56}\sin(14x) - \frac{1}{40}\sin(10x) + \frac{1}{4}\sin(2x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(2*x)*sin(6*x)^2,x, algorithm="giac")
[Out] -1/56*sin(14*x) - 1/40*sin(10*x) + 1/4*sin(2*x)
```

maple [A] time = 0.12, size = 20, normalized size = 0.80

$$\frac{\sin(2x)}{4} - \frac{\sin(10x)}{40} - \frac{\sin(14x)}{56}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(2*x)*sin(6*x)^2,x)
[Out] 1/4*sin(2*x)-1/40*sin(10*x)-1/56*sin(14*x)
```

maxima [A] time = 0.32, size = 19, normalized size = 0.76

$$-\frac{1}{56}\sin(14x) - \frac{1}{40}\sin(10x) + \frac{1}{4}\sin(2x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(2*x)*sin(6*x)^2,x, algorithm="maxima")
```

```
[Out] -1/56*sin(14*x) - 1/40*sin(10*x) + 1/4*sin(2*x)
```

mupad [B] time = 2.29, size = 25, normalized size = 1.00

$$\frac{8\sin\left(2\,x\right)^7}{7}-\frac{12\sin\left(2\,x\right)^5}{5}+\frac{3\sin\left(2\,x\right)^3}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(2*x)*sin(6*x)^2,x)
```

```
[Out] (3*sin(2*x)^3)/2 - (12*sin(2*x)^5)/5 + (8*sin(2*x)^7)/7
```

sympy [B] time = 1.73, size = 48, normalized size = 1.92

$$\frac{17\sin\left(2x\right)\sin^{2}\left(6x\right)}{70}+\frac{9\sin\left(2x\right)\cos^{2}\left(6x\right)}{35}-\frac{3\sin\left(6x\right)\cos\left(2x\right)\cos\left(6x\right)}{35}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(2*x)*sin(6*x)**2,x)
```

```
[Out] 17*sin(2*x)*sin(6*x)**2/70 + 9*sin(2*x)*cos(6*x)**2/35 - 3*sin(6*x)*cos(2*x
)*cos(6*x)/35
```

3.131 $\int \cos(x)\sin^2(6x)\,dx$

Optimal. Leaf size=23

$$\frac{\sin(x)}{2}-\frac{1}{44}\sin(11x)-\frac{1}{52}\sin(13x)$$

```
[Out] 1/2*sin(x)-1/44*sin(11*x)-1/52*sin(13*x)
```

Rubi [A] time = 0.03, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {4354, 2637}

$$\frac{\sin(x)}{2}-\frac{1}{44}\sin(11x)-\frac{1}{52}\sin(13x)$$

Antiderivative was successfully verified.

```
[In] Int[Cos[x]*Sin[6*x]^2,x]
```

```
[Out] Sin[x]/2 - Sin[11*x]/44 - Sin[13*x]/52
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 4354

```
Int[(F_)[(a_.) + (b_.)*(x_)]^(p_.)*(G_)[(c_.) + (d_.)*(x_)]^(q_.), x_Symbol
] :> Int[ExpandTrigReduce[ActivateTrig[F[a + b*x]^p*G[c + d*x]^q], x], x] /
; FreeQ[{a, b, c, d}, x] && (EqQ[F, sin] || EqQ[F, cos]) && (EqQ[G, sin] ||
 EqQ[G, cos]) && IGtQ[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int \cos(x)\sin^2(6x)\,dx &= \int\left(\frac{\cos(x)}{2}-\frac{1}{4}\cos(11x)-\frac{1}{4}\cos(13x)\right)dx\\ &= -\left(\frac{1}{4}\int\cos(11x)\,dx\right)-\frac{1}{4}\int\cos(13x)\,dx+\frac{1}{2}\int\cos(x)\,dx\\ &= \frac{\sin(x)}{2}-\frac{1}{44}\sin(11x)-\frac{1}{52}\sin(13x)\end{aligned}$$

Mathematica [A] time = 0.01, size = 23, normalized size = 1.00

$$\frac{\sin(x)}{2}-\frac{1}{44}\sin(11x)-\frac{1}{52}\sin(13x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[x]*Sin[6*x]^2,x]
[Out] Sin[x]/2 - Sin[11*x]/44 - Sin[13*x]/52
```

fricas [B] time = 0.60, size = 42, normalized size = 1.83

$$-\frac{4}{143}\left(2816\cos(x)^{12}-6912\cos(x)^{10}+6048\cos(x)^{8}-2240\cos(x)^{6}+315\cos(x)^{4}-9\cos(x)^{2}-18\right)\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*sin(6*x)^2,x, algorithm="fricas")
[Out] -4/143*(2816*cos(x)^12 - 6912*cos(x)^10 + 6048*cos(x)^8 - 2240*cos(x)^6 + 3
15*cos(x)^4 - 9*cos(x)^2 - 18)*sin(x)
```

giac [A] time = 0.13, size = 17, normalized size = 0.74

$$-\frac{1}{52}\sin(13x)-\frac{1}{44}\sin(11x)+\frac{1}{2}\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*sin(6*x)^2,x, algorithm="giac")
[Out] -1/52*sin(13*x) - 1/44*sin(11*x) + 1/2*sin(x)
```

maple [A] time = 0.13, size = 18, normalized size = 0.78

$$\frac{\sin(x)}{2}-\frac{\sin(11x)}{44}-\frac{\sin(13x)}{52}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)*sin(6*x)^2,x)
[Out] 1/2*sin(x)-1/44*sin(11*x)-1/52*sin(13*x)
```

maxima [A] time = 0.31, size = 17, normalized size = 0.74

$$-\frac{1}{52}\sin(13x)-\frac{1}{44}\sin(11x)+\frac{1}{2}\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*sin(6*x)^2,x, algorithm="maxima")
```

```
[Out] -1/52*sin(13*x) - 1/44*sin(11*x) + 1/2*sin(x)
```

mupad [B] time = 2.49, size = 17, normalized size = 0.74

$$\frac{\sin(x)}{2} - \frac{\sin\left(13\,x\right)}{52} - \frac{\sin\left(11\,x\right)}{44}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(6*x)^2*cos(x),x)
```

```
[Out] sin(x)/2 - sin(13*x)/52 - sin(11*x)/44
```

sympy [B] time = 1.64, size = 42, normalized size = 1.83

$$\frac{71\sin\left(x\right)\sin^{2}\left(6x\right)}{143} + \frac{72\sin\left(x\right)\cos^{2}\left(6x\right)}{143} - \frac{12\sin\left(6x\right)\cos\left(x\right)\cos\left(6x\right)}{143}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*sin(6*x)**2,x)
```

```
[Out] 71*sin(x)*sin(6*x)**2/143 + 72*sin(x)*cos(6*x)**2/143 - 12*sin(6*x)*cos(x)*
cos(6*x)/143
```

3.132 $\int \cos(x)\sin^3(6x)\,dx$

Optimal. Leaf size=33

$$-\frac{3}{40}\cos(5x)-\frac{3}{56}\cos(7x)+\frac{1}{136}\cos(17x)+\frac{1}{152}\cos(19x)$$

```
[Out] -3/40*cos(5*x)-3/56*cos(7*x)+1/136*cos(17*x)+1/152*cos(19*x)
```

Rubi [A] time = 0.03, antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {4354, 2638}

$$-\frac{3}{40}\cos(5x)-\frac{3}{56}\cos(7x)+\frac{1}{136}\cos(17x)+\frac{1}{152}\cos(19x)$$

Antiderivative was successfully verified.

```
[In] Int[Cos[x]*Sin[6*x]^3,x]
```

```
[Out] (-3*Cos[5*x])/40 - (3*Cos[7*x])/56 + Cos[17*x]/136 + Cos[19*x]/152
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 4354

```
Int[(F_)[(a_.) + (b_.)*(x_)]^(p_.)*(G_)[(c_.) + (d_.)*(x_)]^(q_.), x_Symbol
] :> Int[ExpandTrigReduce[ActivateTrig[F[a + b*x]^p*G[c + d*x]^q], x], x] /
; FreeQ[{a, b, c, d}, x] && (EqQ[F, sin] || EqQ[F, cos]) && (EqQ[G, sin] ||
 EqQ[G, cos]) && IGtQ[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int \cos(x)\sin^3(6x)\,dx &= \int \left(\frac{3}{8}\sin(5x)+\frac{3}{8}\sin(7x)-\frac{1}{8}\sin(17x)-\frac{1}{8}\sin(19x)\right)dx\\ &= -\left(\frac{1}{8}\int \sin(17x)\,dx\right)-\frac{1}{8}\int \sin(19x)\,dx+\frac{3}{8}\int \sin(5x)\,dx+\frac{3}{8}\int \sin(7x)\,dx\\ &= -\frac{3}{40}\cos(5x)-\frac{3}{56}\cos(7x)+\frac{1}{136}\cos(17x)+\frac{1}{152}\cos(19x)\end{aligned}$$

Mathematica [A] time = 0.01, size = 33, normalized size = 1.00

$$-\frac{3}{40}\cos(5x)-\frac{3}{56}\cos(7x)+\frac{1}{136}\cos(17x)+\frac{1}{152}\cos(19x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[x]*Sin[6*x]^3,x]
[Out] (-3*Cos[5*x])/40 - (3*Cos[7*x])/56 + Cos[17*x]/136 + Cos[19*x]/152
```

fricas [A] time = 0.61, size = 49, normalized size = 1.48

$$\frac{32768}{19}\cos(x)^{19}-\frac{131072}{17}\cos(x)^{17}+14336\cos(x)^{15}-14336\cos(x)^{13}+8320\cos(x)^{11}-2816\cos(x)^{9}+\frac{3672}{7}\cos(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*sin(6*x)^3,x, algorithm="fricas")
[Out] 32768/19*cos(x)^19 - 131072/17*cos(x)^17 + 14336*cos(x)^15 - 14336*cos(x)^1
3 + 8320*cos(x)^11 - 2816*cos(x)^9 + 3672/7*cos(x)^7 - 216/5*cos(x)^5
```

giac [A] time = 0.14, size = 49, normalized size = 1.48

$$\frac{32768}{19}\cos(x)^{19}-\frac{131072}{17}\cos(x)^{17}+14336\cos(x)^{15}-14336\cos(x)^{13}+8320\cos(x)^{11}-2816\cos(x)^{9}+\frac{3672}{7}\cos(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*sin(6*x)^3,x, algorithm="giac")
[Out] 32768/19*cos(x)^19 - 131072/17*cos(x)^17 + 14336*cos(x)^15 - 14336*cos(x)^1
3 + 8320*cos(x)^11 - 2816*cos(x)^9 + 3672/7*cos(x)^7 - 216/5*cos(x)^5
```

maple [A] time = 0.14, size = 26, normalized size = 0.79

$$-\frac{3\cos(5x)}{40}-\frac{3\cos(7x)}{56}+\frac{\cos(17x)}{136}+\frac{\cos(19x)}{152}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)*sin(6*x)^3,x)
[Out] -3/40*cos(5*x)-3/56*cos(7*x)+1/136*cos(17*x)+1/152*cos(19*x)
```

maxima [A] time = 0.56, size = 25, normalized size = 0.76

$$\frac{1}{152}\cos(19x)+\frac{1}{136}\cos(17x)-\frac{3}{56}\cos(7x)-\frac{3}{40}\cos(5x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*sin(6*x)^3,x, algorithm="maxima")
```

```
[Out] 1/152*cos(19*x) + 1/136*cos(17*x) - 3/56*cos(7*x) - 3/40*cos(5*x)
```

mupad [B] time = 2.70, size = 150, normalized size = 4.55

$$-\frac{32\left(305235\tan\left(\frac{x}{2}\right)^{34}-9665775\tan\left(\frac{x}{2}\right)^{32}+153838440\tan\left(\frac{x}{2}\right)^{30}-1348695544\tan\left(\frac{x}{2}\right)^{28}+7083812484\tan\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(6*x)^3*cos(x),x)
```

```
[Out] -(32*(1539*tan(x/2)^2 - 291384*tan(x/2)^4 + 9744264*tan(x/2)^6 - 153524484*
tan(x/2)^8 + 1349637412*tan(x/2)^10 - 7081614792*tan(x/2)^12 + 23582909592*
tan(x/2)^14 - 51607368282*tan(x/2)^16 + 75935973762*tan(x/2)^18 - 759284911
44*tan(x/2)^20 + 51613490424*tan(x/2)^22 - 23578828164*tan(x/2)^24 + 708381
2484*tan(x/2)^26 - 1348695544*tan(x/2)^28 + 153838440*tan(x/2)^30 - 9665775
*tan(x/2)^32 + 305235*tan(x/2)^34 + 81))/(11305*(tan(x/2)^2 + 1)^19)
```

sympy [B] time = 5.19, size = 65, normalized size = 1.97

$$-\frac{251\sin(x)\sin^3(6x)}{11305}-\frac{216\sin(x)\sin(6x)\cos^2(6x)}{11305}-\frac{1926\sin^2(6x)\cos(x)\cos(6x)}{11305}-\frac{1296\cos(x)\cos^3(6x)}{11305}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*sin(6*x)**3,x)
```

```
[Out] -251*sin(x)*sin(6*x)**3/11305 - 216*sin(x)*sin(6*x)*cos(6*x)**2/11305 - 192
6*sin(6*x)**2*cos(x)*cos(6*x)/11305 - 1296*cos(x)*cos(6*x)**3/11305
```

3.133 $\int \cos(7x)\sin^3(6x)\,dx$

Optimal. Leaf size=31

$$\frac{3\cos(x)}{8}+\frac{1}{88}\cos(11x)-\frac{3}{104}\cos(13x)+\frac{1}{200}\cos(25x)$$

```
[Out] 3/8*cos(x)+1/88*cos(11*x)-3/104*cos(13*x)+1/200*cos(25*x)
```

Rubi [A] time = 0.03, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {4354, 2638}

$$\frac{3\cos(x)}{8}+\frac{1}{88}\cos(11x)-\frac{3}{104}\cos(13x)+\frac{1}{200}\cos(25x)$$

Antiderivative was successfully verified.

```
[In] Int[Cos[7*x]*Sin[6*x]^3,x]
```

```
[Out] (3*Cos[x])/8 + Cos[11*x]/88 - (3*Cos[13*x])/104 + Cos[25*x]/200
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 4354

```
Int[(F_)[(a_.) + (b_.)*(x_)]^(p_.)*(G_)[(c_.) + (d_.)*(x_)]^(q_.), x_Symbol
] :> Int[ExpandTrigReduce[ActivateTrig[F[a + b*x]^p*G[c + d*x]^q], x], x] /
; FreeQ[{a, b, c, d}, x] && (EqQ[F, sin] || EqQ[F, cos]) && (EqQ[G, sin] ||
 EqQ[G, cos]) && IGtQ[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int \cos(7x)\sin^3(6x)\,dx &= \int \left(-\frac{3\sin(x)}{8}-\frac{1}{8}\sin(11x)+\frac{3}{8}\sin(13x)-\frac{1}{8}\sin(25x)\right)dx\\ &= -\left(\frac{1}{8}\int \sin(11x)\,dx\right)-\frac{1}{8}\int \sin(25x)\,dx-\frac{3}{8}\int \sin(x)\,dx+\frac{3}{8}\int \sin(13x)\,dx\\ &= \frac{3\cos(x)}{8}+\frac{1}{88}\cos(11x)-\frac{3}{104}\cos(13x)+\frac{1}{200}\cos(25x)\end{aligned}$$

Mathematica [A] time = 0.02, size = 31, normalized size = 1.00

$$\frac{3\cos(x)}{8}+\frac{1}{88}\cos(11x)-\frac{3}{104}\cos(13x)+\frac{1}{200}\cos(25x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[7*x]*Sin[6*x]^3,x]
[Out] (3*Cos[x])/8 + Cos[11*x]/88 - (3*Cos[13*x])/104 + Cos[25*x]/200
```

fricas [B] time = 1.29, size = 67, normalized size = 2.16

$$\frac{2097152}{25}\cos(x)^{25}-524288\cos(x)^{23}+1441792\cos(x)^{21}-2293760\cos(x)^{19}+2334720\cos(x)^{17}-\frac{7938048}{5}\cos(x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(7*x)*sin(6*x)^3,x, algorithm="fricas")
[Out] 2097152/25*cos(x)^25 - 524288*cos(x)^23 + 1441792*cos(x)^21 - 2293760*cos(x
)^19 + 2334720*cos(x)^17 - 7938048/5*cos(x)^15 + 9503232/13*cos(x)^13 - 248
4992/11*cos(x)^11 + 45248*cos(x)^9 - 5400*cos(x)^7 + 1512/5*cos(x)^5
```

giac [A] time = 0.14, size = 23, normalized size = 0.74

$$\frac{1}{200}\cos\left(25\,x\right)-\frac{3}{104}\cos\left(13\,x\right)+\frac{1}{88}\cos\left(11\,x\right)+\frac{3}{8}\cos(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(7*x)*sin(6*x)^3,x, algorithm="giac")
[Out] 1/200*cos(25*x) - 3/104*cos(13*x) + 1/88*cos(11*x) + 3/8*cos(x)
```

maple [A] time = 0.45, size = 24, normalized size = 0.77

$$\frac{3\cos(x)}{8}+\frac{\cos\left(11x\right)}{88}-\frac{3\cos\left(13x\right)}{104}+\frac{\cos\left(25x\right)}{200}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(7*x)*sin(6*x)^3,x)
[Out] 3/8*cos(x)+1/88*cos(11*x)-3/104*cos(13*x)+1/200*cos(25*x)
```

maxima [A] time = 0.73, size = 23, normalized size = 0.74

$$\frac{1}{200}\cos\left(25\,x\right)-\frac{3}{104}\cos\left(13\,x\right)+\frac{1}{88}\cos\left(11\,x\right)+\frac{3}{8}\cos(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(7*x)*sin(6*x)^3,x, algorithm="maxima")
```

```
[Out] 1/200*cos(25*x) - 3/104*cos(13*x) + 1/88*cos(11*x) + 3/8*cos(x)
```

mupad [B] time = 3.18, size = 198, normalized size = 6.39

$$\frac{32\left(-96525\tan\left(\frac{x}{2}\right)^{46}+8655075\tan\left(\frac{x}{2}\right)^{44}-300482325\tan\left(\frac{x}{2}\right)^{42}+5743927475\tan\left(\frac{x}{2}\right)^{40}-67792485475\tan\left(\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(7*x)*sin(6*x)^3,x)
```

```
[Out] (32*(2025*tan(x/2)^2 + 120825*tan(x/2)^4 - 8468775*tan(x/2)^6 + 301506975*t
an(x/2)^8 - 5739623945*tan(x/2)^10 + 67806830575*tan(x/2)^12 - 523829476225
*tan(x/2)^14 + 2750536240650*tan(x/2)^16 - 10084340561350*tan(x/2)^18 + 263
26043727610*tan(x/2)^20 - 49575456537350*tan(x/2)^22 + 67896209197950*tan(x
/2)^24 - 67895787973650*tan(x/2)^26 + 49575817586750*tan(x/2)^28 - 26325778
958050*tan(x/2)^30 + 10084506042325*tan(x/2)^32 - 2750448633075*tan(x/2)^34
 + 523868412925*tan(x/2)^36 - 67792485475*tan(x/2)^38 + 5743927475*tan(x/2)
^40 - 300482325*tan(x/2)^42 + 8655075*tan(x/2)^44 - 96525*tan(x/2)^46 + 81)
)/(3575*(tan(x/2)^2 + 1)^25)
```

sympy [B] time = 5.27, size = 70, normalized size = 2.26

$$\frac{1421\sin^{3}(6x)\sin(7x)}{3575}+\frac{1062\sin^{2}(6x)\cos(6x)\cos(7x)}{3575}+\frac{1512\sin(6x)\sin(7x)\cos^{2}(6x)}{3575}+\frac{1296\cos^{3}(6x)\cos(7x}{3575}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(7*x)*sin(6*x)**3,x)
```

```
[Out] 1421*sin(6*x)**3*sin(7*x)/3575 + 1062*sin(6*x)**2*cos(6*x)*cos(7*x)/3575 +
1512*sin(6*x)*sin(7*x)*cos(6*x)**2/3575 + 1296*cos(6*x)**3*cos(7*x)/3575
```

3.134 $\int \cos^2(3x)\sin^3(2x)\,dx$

Optimal. Leaf size=41

$$-\frac{3}{16}\cos(2x)+\frac{3}{64}\cos(4x)+\frac{1}{48}\cos(6x)-\frac{3}{128}\cos(8x)+\frac{1}{192}\cos(12x)$$

```
[Out] -3/16*cos(2*x)+3/64*cos(4*x)+1/48*cos(6*x)-3/128*cos(8*x)+1/192*cos(12*x)
```

Rubi [A] time = 0.04, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {4354, 2638}

$$-\frac{3}{16}\cos(2x)+\frac{3}{64}\cos(4x)+\frac{1}{48}\cos(6x)-\frac{3}{128}\cos(8x)+\frac{1}{192}\cos(12x)$$

Antiderivative was successfully verified.

```
[In] Int[Cos[3*x]^2*Sin[2*x]^3,x]
```

```
[Out] (-3*Cos[2*x])/16 + (3*Cos[4*x])/64 + Cos[6*x]/48 - (3*Cos[8*x])/128 + Cos[1
2*x]/192
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 4354

```
Int[(F_)[(a_.) + (b_.)*(x_)]^(p_.)*(G_)[(c_.) + (d_.)*(x_)]^(q_.), x_Symbol
] :> Int[ExpandTrigReduce[ActivateTrig[F[a + b*x]^p*G[c + d*x]^q], x], x] /
; FreeQ[{a, b, c, d}, x] && (EqQ[F, sin] || EqQ[F, cos]) && (EqQ[G, sin] ||
 EqQ[G, cos]) && IGtQ[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int \cos^2(3x)\sin^3(2x)\,dx &= \int \left(\frac{3}{8}\sin(2x)-\frac{3}{16}\sin(4x)-\frac{1}{8}\sin(6x)+\frac{3}{16}\sin(8x)-\frac{1}{16}\sin(12x)\right)dx\\ &= -\left(\frac{1}{16}\int \sin(12x)\,dx\right)-\frac{1}{8}\int \sin(6x)\,dx-\frac{3}{16}\int \sin(4x)\,dx+\frac{3}{16}\int \sin(8x)\,dx+\frac{3}{8}\int\\ &= -\frac{3}{16}\cos(2x)+\frac{3}{64}\cos(4x)+\frac{1}{48}\cos(6x)-\frac{3}{128}\cos(8x)+\frac{1}{192}\cos(12x)\end{aligned}$$

Mathematica [A]   time = 0.02, size = 41, normalized size = 1.00

$$-\frac{3}{16}\cos(2x)+\frac{3}{64}\cos(4x)+\frac{1}{48}\cos(6x)-\frac{3}{128}\cos(8x)+\frac{1}{192}\cos(12x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[3*x]^2*Sin[2*x]^3,x]
```

```
[Out] (-3*Cos[2*x])/16 + (3*Cos[4*x])/64 + Cos[6*x]/48 - (3*Cos[8*x])/128 + Cos[1
2*x]/192
```

fricas [A]   time = 0.99, size = 25, normalized size = 0.61

$$\frac{32}{3}\cos(x)^{12}-32\cos(x)^{10}+33\cos(x)^{8}-12\cos(x)^{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(3*x)^2*sin(2*x)^3,x, algorithm="fricas")
```

```
[Out] 32/3*cos(x)^12 - 32*cos(x)^10 + 33*cos(x)^8 - 12*cos(x)^6
```

giac [A]   time = 0.12, size = 31, normalized size = 0.76

$$\frac{1}{192}\cos(12x)-\frac{3}{128}\cos(8x)+\frac{1}{48}\cos(6x)+\frac{3}{64}\cos(4x)-\frac{3}{16}\cos(2x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(3*x)^2*sin(2*x)^3,x, algorithm="giac")
```

```
[Out] 1/192*cos(12*x) - 3/128*cos(8*x) + 1/48*cos(6*x) + 3/64*cos(4*x) - 3/16*cos
(2*x)
```

maple [A]   time = 0.17, size = 32, normalized size = 0.78

$$-\frac{3\cos(2x)}{16}+\frac{3\cos(4x)}{64}+\frac{\cos(6x)}{48}-\frac{3\cos(8x)}{128}+\frac{\cos(12x)}{192}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(3*x)^2*sin(2*x)^3,x)
```

```
[Out] -3/16*cos(2*x)+3/64*cos(4*x)+1/48*cos(6*x)-3/128*cos(8*x)+1/192*cos(12*x)
```

maxima [A]   time = 0.35, size = 31, normalized size = 0.76

$$\frac{1}{192}\cos(12x)-\frac{3}{128}\cos(8x)+\frac{1}{48}\cos(6x)+\frac{3}{64}\cos(4x)-\frac{3}{16}\cos(2x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(3*x)^2*sin(2*x)^3,x, algorithm="maxima")
```

```
[Out] 1/192*cos(12*x) - 3/128*cos(8*x) + 1/48*cos(6*x) + 3/64*cos(4*x) - 3/16*cos
(2*x)
```

mupad [B] time = 2.27, size = 25, normalized size = 0.61

$$\frac{32\cos(x)^{12}}{3} - 32\cos(x)^{10} + 33\cos(x)^{8} - 12\cos(x)^{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(3*x)^2*sin(2*x)^3,x)
```

```
[Out] 33*cos(x)^8 - 12*cos(x)^6 - 32*cos(x)^10 + (32*cos(x)^12)/3
```

sympy [B] time = 18.54, size = 228, normalized size = 5.56

$$-\frac{x\sin^{3}(2x)\sin^{2}(3x)}{16}+\frac{x\sin^{3}(2x)\cos^{2}(3x)}{16}-\frac{3x\sin^{2}(2x)\sin(3x)\cos(2x)\cos(3x)}{8}+\frac{3x\sin(2x)\sin^{2}(3x)\cos^{2}(\ldots}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(3*x)**2*sin(2*x)**3,x)
```

```
[Out] -x*sin(2*x)**3*sin(3*x)**2/16 + x*sin(2*x)**3*cos(3*x)**2/16 - 3*x*sin(2*x)
**2*sin(3*x)*cos(2*x)*cos(3*x)/8 + 3*x*sin(2*x)*sin(3*x)**2*cos(2*x)**2/16
- 3*x*sin(2*x)*cos(2*x)**2*cos(3*x)**2/16 + x*sin(3*x)*cos(2*x)**3*cos(3*x)
/8 + 5*sin(2*x)**3*sin(3*x)*cos(3*x)/16 - sin(2*x)**2*sin(3*x)**2*cos(2*x)/
2 - 3*sin(2*x)*sin(3*x)*cos(2*x)**2*cos(3*x)/8 - 11*sin(3*x)**2*cos(2*x)**3
/96 - 7*cos(2*x)**3*cos(3*x)**2/32
```

3.135 $\int \sin(a+bx)\sin(c+bx)\,dx$

Optimal. Leaf size=27

$$\frac{1}{2}x\cos(a-c)-\frac{\sin(a+2bx+c)}{4b}$$

```
[Out] 1/2*x*cos(a-c)-1/4*sin(2*b*x+a+c)/b
```

Rubi [A] time = 0.02, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {4569, 2637}

$$\frac{1}{2}x\cos(a-c)-\frac{\sin(a+2bx+c)}{4b}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b*x]*Sin[c + b*x],x]
```

```
[Out] (x*Cos[a - c])/2 - Sin[a + c + 2*b*x]/(4*b)
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 4569

```
Int[Sin[v_]^(p_.)*Sin[w_]^(q_.), x_Symbol] :> Int[ExpandTrigReduce[Sin[v]^p
*Sin[w]^q, x], x] /; ((PolynomialQ[v, x] && PolynomialQ[w, x]) || (Binomial
Q[{v, w}, x] && IndependentQ[Cancel[v/w], x])) && IGtQ[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int \sin(a+bx)\sin(c+bx)\,dx &= \int \left(\frac{1}{2}\cos(a-c)-\frac{1}{2}\cos(a+c+2bx)\right)dx\\ &= \frac{1}{2}x\cos(a-c)-\frac{1}{2}\int \cos(a+c+2bx)\,dx\\ &= \frac{1}{2}x\cos(a-c)-\frac{\sin(a+c+2bx)}{4b}\end{aligned}$$

Mathematica [A] time = 0.05, size = 26, normalized size = 0.96

$$-\frac{\sin(a+2bx+c)-2bx\cos(a-c)}{4b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + b*x]*Sin[c + b*x],x]
```

```
[Out] -1/4*(-2*b*x*Cos[a - c] + Sin[a + c + 2*b*x])/b
```

fricas [B] time = 0.62, size = 50, normalized size = 1.85

$$\frac{bx\cos\left(-a+c\right)-\cos\left(bx+c\right)\cos\left(-a+c\right)\sin\left(bx+c\right)+\cos\left(bx+c\right)^{2}\sin\left(-a+c\right)}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)*sin(b*x+c),x, algorithm="fricas")
```

```
[Out] 1/2*(b*x*cos(-a + c) - cos(b*x + c)*cos(-a + c)*sin(b*x + c) + cos(b*x + c)
^2*sin(-a + c))/b
```

giac [A] time = 0.12, size = 23, normalized size = 0.85

$$\frac{1}{2}\,x\cos\left(a-c\right)-\frac{\sin\left(2\,bx+a+c\right)}{4\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)*sin(b*x+c),x, algorithm="giac")
```

```
[Out] 1/2*x*cos(a - c) - 1/4*sin(2*b*x + a + c)/b
```

maple [A] time = 0.05, size = 24, normalized size = 0.89

$$\frac{x\cos\left(a-c\right)}{2}-\frac{\sin\left(2bx+a+c\right)}{4b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(b*x+a)*sin(b*x+c),x)
```

```
[Out] 1/2*x*cos(a-c)-1/4*sin(2*b*x+a+c)/b
```

maxima [A] time = 0.31, size = 23, normalized size = 0.85

$$\frac{1}{2}\,x\cos\left(-a+c\right)-\frac{\sin\left(2\,bx+a+c\right)}{4\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)*sin(b*x+c),x, algorithm="maxima")
```

```
[Out] 1/2*x*cos(-a + c) - 1/4*sin(2*b*x + a + c)/b
```

mupad [B] time = 2.48, size = 36, normalized size = 1.33

$$\begin{cases} x\,\sin(a)\,\sin(c) & \text{if } b=0 \\ \frac{x\,\cos(a-c)}{2}-\frac{\sin(a+c+2\,b\,x)}{4\,b} & \text{if } b\neq 0 \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b*x)*sin(c + b*x),x)
```

```
[Out] piecewise(b == 0, x*sin(a)*sin(c), b ~= 0, (x*cos(a - c))/2 - sin(a + c + 2
*b*x)/(4*b))
```

sympy [A] time = 0.74, size = 58, normalized size = 2.15

$$\begin{cases} \frac{x\sin(a+bx)\sin(bx+c)}{2}+\frac{x\cos(a+bx)\cos(bx+c)}{2}-\frac{\sin(bx+c)\cos(a+bx)}{2b} & \text{for } b\neq 0 \\ x\sin(a)\sin(c) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)*sin(b*x+c),x)
```

```
[Out] Piecewise((x*sin(a + b*x)*sin(b*x + c)/2 + x*cos(a + b*x)*cos(b*x + c)/2 -
sin(b*x + c)*cos(a + b*x)/(2*b), Ne(b, 0)), (x*sin(a)*sin(c), True))
```

3.136 $\int \sin(c-bx)\sin(a+bx)\,dx$

Optimal. Leaf size=27

$$\frac{\sin(a+2bx-c)}{4b}-\frac{1}{2}x\cos(a+c)$$

```
[Out] -1/2*x*cos(a+c)+1/4*sin(2*b*x+a-c)/b
```

Rubi [A] time = 0.03, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {4569, 2637}

$$\frac{\sin(a+2bx-c)}{4b}-\frac{1}{2}x\cos(a+c)$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c - b*x]*Sin[a + b*x],x]
```

```
[Out] -(x*Cos[a + c])/2 + Sin[a - c + 2*b*x]/(4*b)
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 4569

```
Int[Sin[v_]^(p_.)*Sin[w_]^(q_.), x_Symbol] :> Int[ExpandTrigReduce[Sin[v]^p
*Sin[w]^q, x], x] /; ((PolynomialQ[v, x] && PolynomialQ[w, x]) || (Binomial
Q[{v, w}, x] && IndependentQ[Cancel[v/w], x])) && IGtQ[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int \sin(c-bx)\sin(a+bx)\,dx &= \int \left(-\frac{1}{2}\cos(a+c)+\frac{1}{2}\cos(a-c+2bx)\right)dx\\ &= -\frac{1}{2}x\cos(a+c)+\frac{1}{2}\int \cos(a-c+2bx)\,dx\\ &= -\frac{1}{2}x\cos(a+c)+\frac{\sin(a-c+2bx)}{4b}\end{aligned}$$

Mathematica [A] time = 0.03, size = 26, normalized size = 0.96

$$\frac{\sin(a+2bx-c)-2bx\cos(a+c)}{4b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c - b*x]*Sin[a + b*x],x]
[Out] (-2*b*x*Cos[a + c] + Sin[a - c + 2*b*x])/(4*b)
```

fricas [A] time = 0.82, size = 44, normalized size = 1.63

$$-\frac{bx\cos(a+c)-\cos(bx+a)\cos(a+c)\sin(bx+a)+\cos(bx+a)^2\sin(a+c)}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-sin(b*x-c)*sin(b*x+a),x, algorithm="fricas")
[Out] -1/2*(b*x*cos(a + c) - cos(b*x + a)*cos(a + c)*sin(b*x + a) + cos(b*x + a)^
2*sin(a + c))/b
```

giac [A] time = 0.14, size = 23, normalized size = 0.85

$$-\frac{1}{2}\,x\cos(a+c)+\frac{\sin(2\,bx+a-c)}{4\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-sin(b*x-c)*sin(b*x+a),x, algorithm="giac")
[Out] -1/2*x*cos(a + c) + 1/4*sin(2*b*x + a - c)/b
```

maple [A] time = 0.05, size = 24, normalized size = 0.89

$$-\frac{x\cos(a+c)}{2}+\frac{\sin(2bx+a-c)}{4b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-sin(b*x-c)*sin(b*x+a),x)
[Out] -1/2*x*cos(a+c)+1/4*sin(2*b*x+a-c)/b
```

maxima [A] time = 0.70, size = 23, normalized size = 0.85

$$-\frac{1}{2}\,x\cos(a+c)+\frac{\sin(2\,bx+a-c)}{4\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-sin(b*x-c)*sin(b*x+a),x, algorithm="maxima")
```

```
[Out] -1/2*x*cos(a + c) + 1/4*sin(2*b*x + a - c)/b
```

mupad [B] time = 2.46, size = 36, normalized size = 1.33

$$\begin{cases} x\sin(a)\sin(c) & \text{if } b = 0 \\ \frac{\sin(a-c+2bx)}{4b} - \frac{x\cos(a+c)}{2} & \text{if } b \neq 0 \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b*x)*sin(c - b*x),x)
```

```
[Out] piecewise(b == 0, x*sin(a)*sin(c), b ~= 0, sin(a - c + 2*b*x)/(4*b) - (x*co
s(a + c))/2)
```

sympy [A] time = 0.73, size = 61, normalized size = 2.26

$$-\begin{cases} \frac{x\sin(a+bx)\sin(bx-c)}{2} + \frac{x\cos(a+bx)\cos(bx-c)}{2} - \frac{\sin(bx-c)\cos(a+bx)}{2b} & \text{for } b \neq 0 \\ -x\sin(a)\sin(c) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-sin(b*x-c)*sin(b*x+a),x)
```

```
[Out] -Piecewise((x*sin(a + b*x)*sin(b*x - c)/2 + x*cos(a + b*x)*cos(b*x - c)/2 -
 sin(b*x - c)*cos(a + b*x)/(2*b), Ne(b, 0)), (-x*sin(a)*sin(c), True))
```

3.137 $\int \cos(a+bx)\cos(c+bx)\,dx$

Optimal. Leaf size=27

$$\frac{\sin(a+2bx+c)}{4b}+\frac{1}{2}x\cos(a-c)$$

```
[Out] 1/2*x*cos(a-c)+1/4*sin(2*b*x+a+c)/b
```

Rubi [A] time = 0.02, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {4570, 2637}

$$\frac{\sin(a+2bx+c)}{4b}+\frac{1}{2}x\cos(a-c)$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x]*Cos[c + b*x],x]
```

```
[Out] (x*Cos[a - c])/2 + Sin[a + c + 2*b*x]/(4*b)
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 4570

```
Int[Cos[v_]^(p_.)*Cos[w_]^(q_.), x_Symbol] :> Int[ExpandTrigReduce[Cos[v]^p
*Cos[w]^q, x], x] /; ((PolynomialQ[v, x] && PolynomialQ[w, x]) || (Binomial
Q[{v, w}, x] && IndependentQ[Cancel[v/w], x])) && IGtQ[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int \cos(a+bx)\cos(c+bx)\,dx &= \int \left(\frac{1}{2}\cos(a-c)+\frac{1}{2}\cos(a+c+2bx)\right)dx\\ &= \frac{1}{2}x\cos(a-c)+\frac{1}{2}\int \cos(a+c+2bx)\,dx\\ &= \frac{1}{2}x\cos(a-c)+\frac{\sin(a+c+2bx)}{4b}\end{aligned}$$

Mathematica [A] time = 0.02, size = 26, normalized size = 0.96

$$\frac{\sin(a+2bx+c)+2bx\cos(a-c)}{4b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x]*Cos[c + b*x],x]
```

```
[Out] (2*b*x*Cos[a - c] + Sin[a + c + 2*b*x])/(4*b)
```

fricas [B] time = 0.56, size = 50, normalized size = 1.85

$$\frac{bx\cos\left(-a+c\right)+\cos\left(bx+c\right)\cos\left(-a+c\right)\sin\left(bx+c\right)-\cos\left(bx+c\right)^{2}\sin\left(-a+c\right)}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*cos(b*x+c),x, algorithm="fricas")
```

```
[Out] 1/2*(b*x*cos(-a + c) + cos(b*x + c)*cos(-a + c)*sin(b*x + c) - cos(b*x + c)
^2*sin(-a + c))/b
```

giac [A] time = 0.12, size = 23, normalized size = 0.85

$$\frac{1}{2}\,x\cos\left(a-c\right)+\frac{\sin\left(2\,bx+a+c\right)}{4\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*cos(b*x+c),x, algorithm="giac")
```

```
[Out] 1/2*x*cos(a - c) + 1/4*sin(2*b*x + a + c)/b
```

maple [A] time = 0.04, size = 24, normalized size = 0.89

$$\frac{x\cos\left(a-c\right)}{2}+\frac{\sin\left(2bx+a+c\right)}{4b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)*cos(b*x+c),x)
```

```
[Out] 1/2*x*cos(a-c)+1/4*sin(2*b*x+a+c)/b
```

maxima [A] time = 0.56, size = 23, normalized size = 0.85

$$\frac{1}{2}\,x\cos\left(-a+c\right)+\frac{\sin\left(2\,bx+a+c\right)}{4\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*cos(b*x+c),x, algorithm="maxima")
```

```
[Out] 1/2*x*cos(-a + c) + 1/4*sin(2*b*x + a + c)/b
```

mupad [B] time = 2.27, size = 36, normalized size = 1.33

$$\begin{cases} x\cos(a)\cos(c) & \text{if } b = 0 \\ \frac{x\cos(a-c)}{2} + \frac{\sin(a+c+2\,b\,x)}{4\,b} & \text{if } b \neq 0 \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)*cos(c + b*x),x)
```

```
[Out] piecewise(b == 0, x*cos(a)*cos(c), b ~= 0, (x*cos(a - c))/2 + sin(a + c + 2
*b*x)/(4*b))
```

sympy [A] time = 0.72, size = 58, normalized size = 2.15

$$\begin{cases} \frac{x\sin(a+bx)\sin(bx+c)}{2} + \frac{x\cos(a+bx)\cos(bx+c)}{2} + \frac{\sin(a+bx)\cos(bx+c)}{2b} & \text{for } b \neq 0 \\ x\cos(a)\cos(c) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*cos(b*x+c),x)
```

```
[Out] Piecewise((x*sin(a + b*x)*sin(b*x + c)/2 + x*cos(a + b*x)*cos(b*x + c)/2 +
sin(a + b*x)*cos(b*x + c)/(2*b), Ne(b, 0)), (x*cos(a)*cos(c), True))
```

3.138 $\int \cos(c-bx)\cos(a+bx)\,dx$

Optimal. Leaf size=27

$$\frac{\sin(a+2bx-c)}{4b}+\frac{1}{2}x\cos(a+c)$$

[Out] 1/2*x*cos(a+c)+1/4*sin(2*b*x+a-c)/b

Rubi [A] time = 0.02, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {4570, 2637}

$$\frac{\sin(a+2bx-c)}{4b}+\frac{1}{2}x\cos(a+c)$$

Antiderivative was successfully verified.

[In] Int[Cos[c - b*x]*Cos[a + b*x],x]

[Out] (x*Cos[a + c])/2 + Sin[a - c + 2*b*x]/(4*b)

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 4570

```
Int[Cos[v_]^(p_.)*Cos[w_]^(q_.), x_Symbol] :> Int[ExpandTrigReduce[Cos[v]^p
*Cos[w]^q, x], x] /; ((PolynomialQ[v, x] && PolynomialQ[w, x]) || (Binomial
Q[{v, w}, x] && IndependentQ[Cancel[v/w], x])) && IGtQ[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int \cos(c-bx)\cos(a+bx)\,dx &= \int \left(\frac{1}{2}\cos(a+c)+\frac{1}{2}\cos(a-c+2bx)\right)dx\\ &= \frac{1}{2}x\cos(a+c)+\frac{1}{2}\int \cos(a-c+2bx)\,dx\\ &= \frac{1}{2}x\cos(a+c)+\frac{\sin(a-c+2bx)}{4b}\end{aligned}$$

Mathematica [A] time = 0.02, size = 26, normalized size = 0.96

$$\frac{\sin(a+2bx-c)+2bx\cos(a+c)}{4b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c - b*x]*Cos[a + b*x],x]
```

```
[Out] (2*b*x*Cos[a + c] + Sin[a - c + 2*b*x])/(4*b)
```

fricas [A] time = 2.13, size = 44, normalized size = 1.63

$$\frac{bx\cos(a+c)+\cos(bx+a)\cos(a+c)\sin(bx+a)-\cos(bx+a)^{2}\sin(a+c)}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x-c)*cos(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/2*(b*x*cos(a + c) + cos(b*x + a)*cos(a + c)*sin(b*x + a) - cos(b*x + a)^2
*sin(a + c))/b
```

giac [A] time = 0.15, size = 23, normalized size = 0.85

$$\frac{1}{2}\,x\cos(a+c)+\frac{\sin(2\,bx+a-c)}{4\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x-c)*cos(b*x+a),x, algorithm="giac")
```

```
[Out] 1/2*x*cos(a + c) + 1/4*sin(2*b*x + a - c)/b
```

maple [A] time = 0.05, size = 24, normalized size = 0.89

$$\frac{x\cos(a+c)}{2}+\frac{\sin(2bx+a-c)}{4b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x-c)*cos(b*x+a),x)
```

```
[Out] 1/2*x*cos(a+c)+1/4*sin(2*b*x+a-c)/b
```

maxima [A] time = 0.33, size = 23, normalized size = 0.85

$$\frac{1}{2}\,x\cos(a+c)+\frac{\sin(2\,bx+a-c)}{4\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x-c)*cos(b*x+a),x, algorithm="maxima")
```

```
[Out] 1/2*x*cos(a + c) + 1/4*sin(2*b*x + a - c)/b
```

mupad [B] time = 2.26, size = 36, normalized size = 1.33

$$\begin{cases} x\cos(a)\cos(c) & \text{if } b=0 \\ \frac{\sin(a-c+2\,b\,x)}{4\,b}+\frac{x\cos(a+c)}{2} & \text{if } b\neq 0 \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)*cos(c - b*x),x)
```

```
[Out] piecewise(b == 0, x*cos(a)*cos(c), b ~= 0, sin(a - c + 2*b*x)/(4*b) + (x*co
s(a + c))/2)
```

sympy [A] time = 0.73, size = 58, normalized size = 2.15

$$\begin{cases} \frac{x\sin(a+bx)\sin(bx-c)}{2}+\frac{x\cos(a+bx)\cos(bx-c)}{2}+\frac{\sin(bx-c)\cos(a+bx)}{2b} & \text{for } b\neq 0 \\ x\cos(a)\cos(c) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x-c)*cos(b*x+a),x)
```

```
[Out] Piecewise((x*sin(a + b*x)*sin(b*x - c)/2 + x*cos(a + b*x)*cos(b*x - c)/2 +
sin(b*x - c)*cos(a + b*x)/(2*b), Ne(b, 0)), (x*cos(a)*cos(c), True))
```

3.139 $\int \tan(a+bx)\tan(c+bx)\,dx$

Optimal. Leaf size=39

$$-\frac{\cot(a-c)\log(\cos(a+bx))}{b}+\frac{\cot(a-c)\log(\cos(bx+c))}{b}-x$$

```
[Out] -x-cot(a-c)*ln(cos(b*x+a))/b+cot(a-c)*ln(cos(b*x+c))/b
```

Rubi [A] time = 0.07, antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {4612, 4610, 3475}

$$-\frac{\cot(a-c)\log(\cos(a+bx))}{b}+\frac{\cot(a-c)\log(\cos(bx+c))}{b}-x$$

Antiderivative was successfully verified.

```
[In] Int[Tan[a + b*x]*Tan[c + b*x],x]
```

```
[Out] -x - (Cot[a - c]*Log[Cos[a + b*x]])/b + (Cot[a - c]*Log[Cos[c + b*x]])/b
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 4610

```
Int[Sec[(a_.) + (b_.)*(x_)]*Sec[(c_) + (d_.)*(x_)], x_Symbol] :> -Dist[Csc[
(b*c - a*d)/d], Int[Tan[a + b*x], x], x] + Dist[Csc[(b*c - a*d)/b], Int[Tan
[c + d*x], x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b^2 - d^2, 0] && NeQ[b*c
 - a*d, 0]
```

Rule 4612

```
Int[Tan[(a_.) + (b_.)*(x_)]*Tan[(c_) + (d_.)*(x_)], x_Symbol] :> -Simp[(b*x
)/d, x] + Dist[(b*Cos[(b*c - a*d)/d])/d, Int[Sec[a + b*x]*Sec[c + d*x], x],
 x] /; FreeQ[{a, b, c, d}, x] && EqQ[b^2 - d^2, 0] && NeQ[b*c - a*d, 0]
```

Rubi steps

$$
\begin{aligned}
\int \tan(a+bx)\tan(c+bx)\,dx &= -x+\cos(a-c)\int \sec(a+bx)\sec(c+bx)\,dx \\
&= -x+\cot(a-c)\int \tan(a+bx)\,dx-\cot(a-c)\int \tan(c+bx)\,dx \\
&= -x-\frac{\cot(a-c)\log(\cos(a+bx))}{b}+\frac{\cot(a-c)\log(\cos(c+bx))}{b}
\end{aligned}
$$

Mathematica [A] time = 0.52, size = 31, normalized size = 0.79

$$\frac{\cot(a-c)(\log(\cos(bx+c))-\log(\cos(a+bx)))}{b}-x$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[a + b*x]*Tan[c + b*x],x]
```

```
[Out] -x + (Cot[a - c]*(-Log[Cos[a + b*x]] + Log[Cos[c + b*x]]))/b
```

fricas [B] time = 0.63, size = 145, normalized size = 3.72

$$-\frac{2\,bx\sin\left(-2\,a+2\,c\right)-\left(\cos\left(-2\,a+2\,c\right)+1\right)\log\left(-\frac{\left(\cos\left(-2\,a+2\,c\right)-1\right)\tan\left(bx+c\right)^{2}-2\,\sin\left(-2\,a+2\,c\right)\tan\left(bx+c\right)-\cos\left(-2\,a+2\,c\right)-1}{\left(\cos\left(-2\,a+2\,c\right)+1\right)\tan\left(bx+c\right)^{2}+\cos\left(-2\,a+2\,c\right)+1}\right)+}{2\,b\sin\left(-2\,a+2\,c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(b*x+a)*tan(b*x+c),x, algorithm="fricas")
```

```
[Out] -1/2*(2*b*x*sin(-2*a + 2*c) - (cos(-2*a + 2*c) + 1)*log(-((cos(-2*a + 2*c)
- 1)*tan(b*x + c)^2 - 2*sin(-2*a + 2*c)*tan(b*x + c) - cos(-2*a + 2*c) - 1)
/((cos(-2*a + 2*c) + 1)*tan(b*x + c)^2 + cos(-2*a + 2*c) + 1)) + (cos(-2*a
+ 2*c) + 1)*log(1/(tan(b*x + c)^2 + 1)))/(b*sin(-2*a + 2*c))
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(b*x+a)*tan(b*x+c),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (2*pi/x/2)>(-2*pi/x/2)2/b*(-1/2*b*x+(2*tan(a/2)^2*tan(c/2)
^3-2*tan(a/2)^2*tan(c/2)+8*tan(a/2)*tan(c/2)^2-2*tan(c/2)^3+2*tan(c/2))/(8*
tan(a/2)^2*tan(c/2)^2-8*tan(a/2)*tan(c/2)^3+8*tan(a/2)*tan(c/2)-8*tan(c/2)^
```

```
2)*ln(abs(2*tan(b*x)*tan(c/2)+tan(c/2)^2-1))+(2*tan(a/2)^3*tan(c/2)^2-2*tan
(a/2)^3+8*tan(a/2)^2*tan(c/2)-2*tan(a/2)*tan(c/2)^2+2*tan(a/2))/(-8*tan(a/2
)^3*tan(c/2)+8*tan(a/2)^2*tan(c/2)^2-8*tan(a/2)^2+8*tan(a/2)*tan(c/2))*ln(a
bs(2*tan(b*x)*tan(a/2)+tan(a/2)^2-1)))
```

maple [C] time = 0.14, size = 173, normalized size = 4.44

$$-x-\frac{i\ln\left(1+e^{2i(bx+a)}\right)e^{2ia}}{b\left(e^{2ia}-e^{2ic}\right)}-\frac{i\ln\left(1+e^{2i(bx+a)}\right)e^{2ic}}{b\left(e^{2ia}-e^{2ic}\right)}+\frac{i\ln\left(e^{2i(bx+a)}+e^{2i(a-c)}\right)e^{2ia}}{b\left(e^{2ia}-e^{2ic}\right)}+\frac{i\ln\left(e^{2i(bx+a)}+e^{2i(a-c)}\right)e^{2ic}}{b\left(e^{2ia}-e^{2ic}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(b*x+a)*tan(b*x+c),x)
```

```
[Out] -x-I/b/(exp(2*I*a)-exp(2*I*c))*ln(1+exp(2*I*(b*x+a)))*exp(2*I*a)-I/b/(exp(2
*I*a)-exp(2*I*c))*ln(1+exp(2*I*(b*x+a)))*exp(2*I*c)+I/b/(exp(2*I*a)-exp(2*I
*c))*ln(exp(2*I*(b*x+a))+exp(2*I*(a-c)))*exp(2*I*a)+I/b/(exp(2*I*a)-exp(2*I
*c))*ln(exp(2*I*(b*x+a))+exp(2*I*(a-c)))*exp(2*I*c)
```

maxima [B] time = 0.65, size = 371, normalized size = 9.51

$$-\frac{\left(2\,b\cos\left(2\,a\right)\cos\left(2\,c\right)-b\cos\left(2\,c\right)^{2}+2\,b\sin\left(2\,a\right)\sin\left(2\,c\right)-b\sin\left(2\,c\right)^{2}-\left(\cos\left(2\,a\right)^{2}+\sin\left(2\,a\right)^{2}\right)b\right)x+\left(\cos\left(2\,a\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(b*x+a)*tan(b*x+c),x, algorithm="maxima")
```

```
[Out] -((2*b*cos(2*a)*cos(2*c) - b*cos(2*c)^2 + 2*b*sin(2*a)*sin(2*c) - b*sin(2*c
)^2 - (cos(2*a)^2 + sin(2*a)^2)*b)*x + (cos(2*a)^2 - cos(2*c)^2 + sin(2*a)^
2 - sin(2*c)^2)*arctan2(sin(2*b*x) - sin(2*a), cos(2*b*x) + cos(2*a)) - (co
s(2*a)^2 - cos(2*c)^2 + sin(2*a)^2 - sin(2*c)^2)*arctan2(sin(2*b*x) - sin(2
*c), cos(2*b*x) + cos(2*c)) - (cos(2*c)*sin(2*a) - cos(2*a)*sin(2*c))*log(c
os(2*b*x)^2 + 2*cos(2*b*x)*cos(2*a) + cos(2*a)^2 + sin(2*b*x)^2 - 2*sin(2*b
*x)*sin(2*a) + sin(2*a)^2) + (cos(2*c)*sin(2*a) - cos(2*a)*sin(2*c))*log(co
s(2*b*x)^2 + 2*cos(2*b*x)*cos(2*c) + cos(2*c)^2 + sin(2*b*x)^2 - 2*sin(2*b*
x)*sin(2*c) + sin(2*c)^2))/(2*b*cos(2*a)*cos(2*c) - b*cos(2*c)^2 + 2*b*sin(
2*a)*sin(2*c) - b*sin(2*c)^2 - (cos(2*a)^2 + sin(2*a)^2)*b)
```

mupad [B] time = 4.99, size = 207, normalized size = 5.31

$$-\frac{\frac{x}{2}+x\left(\sin\left(a-c\right)^{2}-\frac{1}{2}\right)}{\sin\left(a-c\right)^{2}}-\frac{\frac{\sin(2\,a-2\,c)\,\ln\left(\sin(2\,a-2\,c)^{2}\,2\mathrm{i}-\sin(a+b\,x)^{2}\,2\mathrm{i}+\sin(3\,a-2\,c+b\,x)^{2}\,2\mathrm{i}+\sin(4\,a-4\,c)+\sin(6\,a-4\,c+2\,b\,x)-\sin(2\,a+2\,b\,x)\right.}{2}}{b\sin}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(a + b*x)*tan(c + b*x),x)
```

```
[Out] - (x/2 + x*(sin(a - c)^2 - 1/2))/sin(a - c)^2 - ((sin(2*a - 2*c)*log(sin(4*
a - 4*c) + sin(6*a - 4*c + 2*b*x) - sin(2*a + 2*b*x) + sin(2*a - 2*c)^2*2i
- sin(a + b*x)^2*2i + sin(3*a - 2*c + b*x)^2*2i))/2 - (sin(2*a - 2*c)*log(s
in(4*a - 4*c) + sin(4*a - 2*c + 2*b*x) - sin(2*c + 2*b*x) + sin(2*a - 2*c)^
2*2i - sin(c + b*x)^2*2i + sin(2*a - c + b*x)^2*2i))/2)/(b*sin(a - c)^2)
```

sympy [B] time = 6.58, size = 7713, normalized size = 197.77

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(b*x+a)*tan(b*x+c),x)
```

```
[Out] Piecewise((0, Eq(a, 0) & Eq(b, 0) & Eq(c, 0)), (b*x*tan(c)**4*tan(b*x)/(b*t
an(c)**6*tan(b*x) - b*tan(c)**5 + 2*b*tan(c)**4*tan(b*x) - 2*b*tan(c)**3 +
b*tan(c)**2*tan(b*x) - b*tan(c)) - b*x*tan(c)**3/(b*tan(c)**6*tan(b*x) - b*
tan(c)**5 + 2*b*tan(c)**4*tan(b*x) - 2*b*tan(c)**3 + b*tan(c)**2*tan(b*x) -
 b*tan(c)) - b*x*tan(c)**2*tan(b*x)/(b*tan(c)**6*tan(b*x) - b*tan(c)**5 + 2
*b*tan(c)**4*tan(b*x) - 2*b*tan(c)**3 + b*tan(c)**2*tan(b*x) - b*tan(c)) +
b*x*tan(c)/(b*tan(c)**6*tan(b*x) - b*tan(c)**5 + 2*b*tan(c)**4*tan(b*x) - 2
*b*tan(c)**3 + b*tan(c)**2*tan(b*x) - b*tan(c)) + 2*log(tan(b*x) - 1/tan(c)
)*tan(c)**3*tan(b*x)/(b*tan(c)**6*tan(b*x) - b*tan(c)**5 + 2*b*tan(c)**4*ta
n(b*x) - 2*b*tan(c)**3 + b*tan(c)**2*tan(b*x) - b*tan(c)) - 2*log(tan(b*x)
- 1/tan(c))*tan(c)**2/(b*tan(c)**6*tan(b*x) - b*tan(c)**5 + 2*b*tan(c)**4*t
an(b*x) - 2*b*tan(c)**3 + b*tan(c)**2*tan(b*x) - b*tan(c)) - log(tan(b*x)**
2 + 1)*tan(c)**3*tan(b*x)/(b*tan(c)**6*tan(b*x) - b*tan(c)**5 + 2*b*tan(c)*
*4*tan(b*x) - 2*b*tan(c)**3 + b*tan(c)**2*tan(b*x) - b*tan(c)) + log(tan(b*
x)**2 + 1)*tan(c)**2/(b*tan(c)**6*tan(b*x) - b*tan(c)**5 + 2*b*tan(c)**4*ta
n(b*x) - 2*b*tan(c)**3 + b*tan(c)**2*tan(b*x) - b*tan(c)) - tan(c)**2/(b*ta
n(c)**6*tan(b*x) - b*tan(c)**5 + 2*b*tan(c)**4*tan(b*x) - 2*b*tan(c)**3 + b
*tan(c)**2*tan(b*x) - b*tan(c)) - 1/(b*tan(c)**6*tan(b*x) - b*tan(c)**5 + 2
*b*tan(c)**4*tan(b*x) - 2*b*tan(c)**3 + b*tan(c)**2*tan(b*x) - b*tan(c)), E
q(a, atan(tan(c)) + pi*floor((c - pi/2)/pi) + pi*floor(c/pi - 1/2))), (0, E
q(b, 0)), (-2*b*x*tan(a)/(2*b*tan(a)**3 + 2*b*tan(a)) - 2*log(tan(b*x) - 1/
tan(a))/(2*b*tan(a)**3 + 2*b*tan(a)) - log(tan(b*x)**2 + 1)*tan(a)**2/(2*b*
tan(a)**3 + 2*b*tan(a)), Eq(c, 0)), (-2*b*x*tan(c)/(2*b*tan(c)**3 + 2*b*tan
(c)) - 2*log(tan(b*x) - 1/tan(c))/(2*b*tan(c)**3 + 2*b*tan(c)) - log(tan(b*
x)**2 + 1)*tan(c)**2/(2*b*tan(c)**3 + 2*b*tan(c)), Eq(a, 0)), (2*b*x*tan(a)
**2*tan(c)/(2*b*tan(a)**3*tan(c)**2 + 2*b*tan(a)**3 - 2*b*tan(a)**2*tan(c)*
*3 - 2*b*tan(a)**2*tan(c) + 2*b*tan(a)*tan(c)**2 + 2*b*tan(a) - 2*b*tan(c)*
*3 - 2*b*tan(c)) - 2*b*x*tan(a)*tan(c)**2/(2*b*tan(a)**3*tan(c)**2 + 2*b*ta
n(a)**3 - 2*b*tan(a)**2*tan(c)**3 - 2*b*tan(a)**2*tan(c) + 2*b*tan(a)*tan(c
)**2 + 2*b*tan(a) - 2*b*tan(c)**3 - 2*b*tan(c)) - 2*b*x*tan(a)/(2*b*tan(a)*
```

```
*3*tan(c)**2 + 2*b*tan(a)**3 - 2*b*tan(a)**2*tan(c)**3 - 2*b*tan(a)**2*tan(
c) + 2*b*tan(a)*tan(c)**2 + 2*b*tan(a) - 2*b*tan(c)**3 - 2*b*tan(c)) + 2*b*
x*tan(c)/(2*b*tan(a)**3*tan(c)**2 + 2*b*tan(a)**3 - 2*b*tan(a)**2*tan(c)**3
 - 2*b*tan(a)**2*tan(c) + 2*b*tan(a)*tan(c)**2 + 2*b*tan(a) - 2*b*tan(c)**3
 - 2*b*tan(c)) - 2*log(tan(b*x) - 1/tan(a))*tan(c)**2/(2*b*tan(a)**3*tan(c)
**2 + 2*b*tan(a)**3 - 2*b*tan(a)**2*tan(c)**3 - 2*b*tan(a)**2*tan(c) + 2*b*
tan(a)*tan(c)**2 + 2*b*tan(a) - 2*b*tan(c)**3 - 2*b*tan(c)) - 2*log(tan(b*x
) - 1/tan(a))/(2*b*tan(a)**3*tan(c)**2 + 2*b*tan(a)**3 - 2*b*tan(a)**2*tan(
c)**3 - 2*b*tan(a)**2*tan(c) + 2*b*tan(a)*tan(c)**2 + 2*b*tan(a) - 2*b*tan(
c)**3 - 2*b*tan(c)) + 2*log(tan(b*x) - 1/tan(c))*tan(a)**2/(2*b*tan(a)**3*t
an(c)**2 + 2*b*tan(a)**3 - 2*b*tan(a)**2*tan(c)**3 - 2*b*tan(a)**2*tan(c) +
 2*b*tan(a)*tan(c)**2 + 2*b*tan(a) - 2*b*tan(c)**3 - 2*b*tan(c)) + 2*log(ta
n(b*x) - 1/tan(c))/(2*b*tan(a)**3*tan(c)**2 + 2*b*tan(a)**3 - 2*b*tan(a)**2
*tan(c)**3 - 2*b*tan(a)**2*tan(c) + 2*b*tan(a)*tan(c)**2 + 2*b*tan(a) - 2*b
*tan(c)**3 - 2*b*tan(c)) - log(tan(b*x)**2 + 1)*tan(a)**2/(2*b*tan(a)**3*ta
n(c)**2 + 2*b*tan(a)**3 - 2*b*tan(a)**2*tan(c)**3 - 2*b*tan(a)**2*tan(c) +
2*b*tan(a)*tan(c)**2 + 2*b*tan(a) - 2*b*tan(c)**3 - 2*b*tan(c)) + log(tan(b
*x)**2 + 1)*tan(c)**2/(2*b*tan(a)**3*tan(c)**2 + 2*b*tan(a)**3 - 2*b*tan(a)
**2*tan(c)**3 - 2*b*tan(a)**2*tan(c) + 2*b*tan(a)*tan(c)**2 + 2*b*tan(a) -
2*b*tan(c)**3 - 2*b*tan(c)), True)) + Piecewise((0, Eq(a, 0) & Eq(b, 0) & E
q(c, 0)), (-4*b*x*tan(c)**2*tan(b*x)/(2*b*tan(c)**5*tan(b*x) - 2*b*tan(c)**
4 + 4*b*tan(c)**3*tan(b*x) - 4*b*tan(c)**2 + 2*b*tan(c)*tan(b*x) - 2*b) + 4
*b*x*tan(c)/(2*b*tan(c)**5*tan(b*x) - 2*b*tan(c)**4 + 4*b*tan(c)**3*tan(b*x
) - 4*b*tan(c)**2 + 2*b*tan(c)*tan(b*x) - 2*b) + 2*log(tan(b*x) - 1/tan(c))
*tan(c)**3*tan(b*x)/(2*b*tan(c)**5*tan(b*x) - 2*b*tan(c)**4 + 4*b*tan(c)**3
*tan(b*x) - 4*b*tan(c)**2 + 2*b*tan(c)*tan(b*x) - 2*b) - 2*log(tan(b*x) - 1
/tan(c))*tan(c)**2/(2*b*tan(c)**5*tan(b*x) - 2*b*tan(c)**4 + 4*b*tan(c)**3*
tan(b*x) - 4*b*tan(c)**2 + 2*b*tan(c)*tan(b*x) - 2*b) - 2*log(tan(b*x) - 1/
tan(c))*tan(c)*tan(b*x)/(2*b*tan(c)**5*tan(b*x) - 2*b*tan(c)**4 + 4*b*tan(c
)**3*tan(b*x) - 4*b*tan(c)**2 + 2*b*tan(c)*tan(b*x) - 2*b) + 2*log(tan(b*x)
 - 1/tan(c))/(2*b*tan(c)**5*tan(b*x) - 2*b*tan(c)**4 + 4*b*tan(c)**3*tan(b*
x) - 4*b*tan(c)**2 + 2*b*tan(c)*tan(b*x) - 2*b) - log(tan(b*x)**2 + 1)*tan(
c)**3*tan(b*x)/(2*b*tan(c)**5*tan(b*x) - 2*b*tan(c)**4 + 4*b*tan(c)**3*tan(
b*x) - 4*b*tan(c)**2 + 2*b*tan(c)*tan(b*x) - 2*b) + log(tan(b*x)**2 + 1)*ta
n(c)**2/(2*b*tan(c)**5*tan(b*x) - 2*b*tan(c)**4 + 4*b*tan(c)**3*tan(b*x) -
4*b*tan(c)**2 + 2*b*tan(c)*tan(b*x) - 2*b) + log(tan(b*x)**2 + 1)*tan(c)*ta
n(b*x)/(2*b*tan(c)**5*tan(b*x) - 2*b*tan(c)**4 + 4*b*tan(c)**3*tan(b*x) - 4
*b*tan(c)**2 + 2*b*tan(c)*tan(b*x) - 2*b) - log(tan(b*x)**2 + 1)/(2*b*tan(c
)**5*tan(b*x) - 2*b*tan(c)**4 + 4*b*tan(c)**3*tan(b*x) - 4*b*tan(c)**2 + 2*
b*tan(c)*tan(b*x) - 2*b) - 2*tan(c)**2/(2*b*tan(c)**5*tan(b*x) - 2*b*tan(c)
**4 + 4*b*tan(c)**3*tan(b*x) - 4*b*tan(c)**2 + 2*b*tan(c)*tan(b*x) - 2*b) -
 2/(2*b*tan(c)**5*tan(b*x) - 2*b*tan(c)**4 + 4*b*tan(c)**3*tan(b*x) - 4*b*t
an(c)**2 + 2*b*tan(c)*tan(b*x) - 2*b), Eq(a, atan(tan(c)) + pi*floor((c - p
i/2)/pi) + pi*floor(c/pi - 1/2))), (0, Eq(b, 0)), (-2*b*x*tan(a)/(2*b*tan(a
)**2 + 2*b) - 2*log(tan(b*x) - 1/tan(a))/(2*b*tan(a)**2 + 2*b) + log(tan(b*
```

```
x)**2 + 1)/(2*b*tan(a)**2 + 2*b), Eq(c, 0)), (-2*b*x*tan(c)/(2*b*tan(c)**2
+ 2*b) - 2*log(tan(b*x) - 1/tan(c))/(2*b*tan(c)**2 + 2*b) + log(tan(b*x)**2
 + 1)/(2*b*tan(c)**2 + 2*b), Eq(a, 0)), (-2*b*x*tan(a)**2/(2*b*tan(a)**3*ta
n(c)**2 + 2*b*tan(a)**3 - 2*b*tan(a)**2*tan(c)**3 - 2*b*tan(a)**2*tan(c) +
2*b*tan(a)*tan(c)**2 + 2*b*tan(a) - 2*b*tan(c)**3 - 2*b*tan(c)) + 2*b*x*tan
(c)**2/(2*b*tan(a)**3*tan(c)**2 + 2*b*tan(a)**3 - 2*b*tan(a)**2*tan(c)**3 -
 2*b*tan(a)**2*tan(c) + 2*b*tan(a)*tan(c)**2 + 2*b*tan(a) - 2*b*tan(c)**3 -
 2*b*tan(c)) - 2*log(tan(b*x) - 1/tan(a))*tan(a)*tan(c)**2/(2*b*tan(a)**3*t
an(c)**2 + 2*b*tan(a)**3 - 2*b*tan(a)**2*tan(c)**3 - 2*b*tan(a)**2*tan(c) +
 2*b*tan(a)*tan(c)**2 + 2*b*tan(a) - 2*b*tan(c)**3 - 2*b*tan(c)) - 2*log(ta
n(b*x) - 1/tan(a))*tan(a)/(2*b*tan(a)**3*tan(c)**2 + 2*b*tan(a)**3 - 2*b*ta
n(a)**2*tan(c)**3 - 2*b*tan(a)**2*tan(c) + 2*b*tan(a)*tan(c)**2 + 2*b*tan(a
) - 2*b*tan(c)**3 - 2*b*tan(c)) + 2*log(tan(b*x) - 1/tan(c))*tan(a)**2*tan(
c)/(2*b*tan(a)**3*tan(c)**2 + 2*b*tan(a)**3 - 2*b*tan(a)**2*tan(c)**3 - 2*b
*tan(a)**2*tan(c) + 2*b*tan(a)*tan(c)**2 + 2*b*tan(a) - 2*b*tan(c)**3 - 2*b
*tan(c)) + 2*log(tan(b*x) - 1/tan(c))*tan(c)/(2*b*tan(a)**3*tan(c)**2 + 2*b
*tan(a)**3 - 2*b*tan(a)**2*tan(c)**3 - 2*b*tan(a)**2*tan(c) + 2*b*tan(a)*ta
n(c)**2 + 2*b*tan(a) - 2*b*tan(c)**3 - 2*b*tan(c)) - log(tan(b*x)**2 + 1)*t
an(a)**2*tan(c)/(2*b*tan(a)**3*tan(c)**2 + 2*b*tan(a)**3 - 2*b*tan(a)**2*ta
n(c)**3 - 2*b*tan(a)**2*tan(c) + 2*b*tan(a)*tan(c)**2 + 2*b*tan(a) - 2*b*ta
n(c)**3 - 2*b*tan(c)) + log(tan(b*x)**2 + 1)*tan(a)*tan(c)**2/(2*b*tan(a)**
3*tan(c)**2 + 2*b*tan(a)**3 - 2*b*tan(a)**2*tan(c)**3 - 2*b*tan(a)**2*tan(c
) + 2*b*tan(a)*tan(c)**2 + 2*b*tan(a) - 2*b*tan(c)**3 - 2*b*tan(c)) + log(t
an(b*x)**2 + 1)*tan(a)/(2*b*tan(a)**3*tan(c)**2 + 2*b*tan(a)**3 - 2*b*tan(a
)**2*tan(c)**3 - 2*b*tan(a)**2*tan(c) + 2*b*tan(a)*tan(c)**2 + 2*b*tan(a) -
 2*b*tan(c)**3 - 2*b*tan(c)) - log(tan(b*x)**2 + 1)*tan(c)/(2*b*tan(a)**3*t
an(c)**2 + 2*b*tan(a)**3 - 2*b*tan(a)**2*tan(c)**3 - 2*b*tan(a)**2*tan(c) +
 2*b*tan(a)*tan(c)**2 + 2*b*tan(a) - 2*b*tan(c)**3 - 2*b*tan(c)), True))*ta
n(a) + Piecewise((0, Eq(a, 0) & Eq(b, 0) & Eq(c, 0)), (-4*b*x*tan(c)**2*tan
(b*x)/(2*b*tan(c)**5*tan(b*x) - 2*b*tan(c)**4 + 4*b*tan(c)**3*tan(b*x) - 4*
b*tan(c)**2 + 2*b*tan(c)*tan(b*x) - 2*b) + 4*b*x*tan(c)/(2*b*tan(c)**5*tan(
b*x) - 2*b*tan(c)**4 + 4*b*tan(c)**3*tan(b*x) - 4*b*tan(c)**2 + 2*b*tan(c)*
tan(b*x) - 2*b) + 2*log(tan(b*x) - 1/tan(c))*tan(c)**3*tan(b*x)/(2*b*tan(c)
**5*tan(b*x) - 2*b*tan(c)**4 + 4*b*tan(c)**3*tan(b*x) - 4*b*tan(c)**2 + 2*b
*tan(c)*tan(b*x) - 2*b) - 2*log(tan(b*x) - 1/tan(c))*tan(c)**2/(2*b*tan(c)*
*5*tan(b*x) - 2*b*tan(c)**4 + 4*b*tan(c)**3*tan(b*x) - 4*b*tan(c)**2 + 2*b*
tan(c)*tan(b*x) - 2*b) - 2*log(tan(b*x) - 1/tan(c))*tan(c)*tan(b*x)/(2*b*ta
n(c)**5*tan(b*x) - 2*b*tan(c)**4 + 4*b*tan(c)**3*tan(b*x) - 4*b*tan(c)**2 +
 2*b*tan(c)*tan(b*x) - 2*b) + 2*log(tan(b*x) - 1/tan(c))/(2*b*tan(c)**5*tan
(b*x) - 2*b*tan(c)**4 + 4*b*tan(c)**3*tan(b*x) - 4*b*tan(c)**2 + 2*b*tan(c)
*tan(b*x) - 2*b) - log(tan(b*x)**2 + 1)*tan(c)**3*tan(b*x)/(2*b*tan(c)**5*t
an(b*x) - 2*b*tan(c)**4 + 4*b*tan(c)**3*tan(b*x) - 4*b*tan(c)**2 + 2*b*tan(
c)*tan(b*x) - 2*b) + log(tan(b*x)**2 + 1)*tan(c)**2/(2*b*tan(c)**5*tan(b*x)
 - 2*b*tan(c)**4 + 4*b*tan(c)**3*tan(b*x) - 4*b*tan(c)**2 + 2*b*tan(c)*tan(
b*x) - 2*b) + log(tan(b*x)**2 + 1)*tan(c)*tan(b*x)/(2*b*tan(c)**5*tan(b*x)
```

```
- 2*b*tan(c)**4 + 4*b*tan(c)**3*tan(b*x) - 4*b*tan(c)**2 + 2*b*tan(c)*tan(b
*x) - 2*b) - log(tan(b*x)**2 + 1)/(2*b*tan(c)**5*tan(b*x) - 2*b*tan(c)**4 +
 4*b*tan(c)**3*tan(b*x) - 4*b*tan(c)**2 + 2*b*tan(c)*tan(b*x) - 2*b) - 2*ta
n(c)**2/(2*b*tan(c)**5*tan(b*x) - 2*b*tan(c)**4 + 4*b*tan(c)**3*tan(b*x) -
4*b*tan(c)**2 + 2*b*tan(c)*tan(b*x) - 2*b) - 2/(2*b*tan(c)**5*tan(b*x) - 2*
b*tan(c)**4 + 4*b*tan(c)**3*tan(b*x) - 4*b*tan(c)**2 + 2*b*tan(c)*tan(b*x)
- 2*b), Eq(a, atan(tan(c)) + pi*floor((c - pi/2)/pi) + pi*floor(c/pi - 1/2)
)), (0, Eq(b, 0)), (-2*b*x*tan(a)/(2*b*tan(a)**2 + 2*b) - 2*log(tan(b*x) -
1/tan(a))/(2*b*tan(a)**2 + 2*b) + log(tan(b*x)**2 + 1)/(2*b*tan(a)**2 + 2*b
), Eq(c, 0)), (-2*b*x*tan(c)/(2*b*tan(c)**2 + 2*b) - 2*log(tan(b*x) - 1/tan
(c))/(2*b*tan(c)**2 + 2*b) + log(tan(b*x)**2 + 1)/(2*b*tan(c)**2 + 2*b), Eq
(a, 0)), (-2*b*x*tan(a)**2/(2*b*tan(a)**3*tan(c)**2 + 2*b*tan(a)**3 - 2*b*t
an(a)**2*tan(c)**3 - 2*b*tan(a)**2*tan(c) + 2*b*tan(a)*tan(c)**2 + 2*b*tan(
a) - 2*b*tan(c)**3 - 2*b*tan(c)) + 2*b*x*tan(c)**2/(2*b*tan(a)**3*tan(c)**2
 + 2*b*tan(a)**3 - 2*b*tan(a)**2*tan(c)**3 - 2*b*tan(a)**2*tan(c) + 2*b*tan
(a)*tan(c)**2 + 2*b*tan(a) - 2*b*tan(c)**3 - 2*b*tan(c)) - 2*log(tan(b*x) -
 1/tan(a))*tan(a)*tan(c)**2/(2*b*tan(a)**3*tan(c)**2 + 2*b*tan(a)**3 - 2*b*
tan(a)**2*tan(c)**3 - 2*b*tan(a)**2*tan(c) + 2*b*tan(a)*tan(c)**2 + 2*b*tan
(a) - 2*b*tan(c)**3 - 2*b*tan(c)) - 2*log(tan(b*x) - 1/tan(a))*tan(a)/(2*b*
tan(a)**3*tan(c)**2 + 2*b*tan(a)**3 - 2*b*tan(a)**2*tan(c)**3 - 2*b*tan(a)*
*2*tan(c) + 2*b*tan(a)*tan(c)**2 + 2*b*tan(a) - 2*b*tan(c)**3 - 2*b*tan(c))
 + 2*log(tan(b*x) - 1/tan(c))*tan(a)**2*tan(c)/(2*b*tan(a)**3*tan(c)**2 + 2
*b*tan(a)**3 - 2*b*tan(a)**2*tan(c)**3 - 2*b*tan(a)**2*tan(c) + 2*b*tan(a)*
tan(c)**2 + 2*b*tan(a) - 2*b*tan(c)**3 - 2*b*tan(c)) + 2*log(tan(b*x) - 1/t
an(c))*tan(c)/(2*b*tan(a)**3*tan(c)**2 + 2*b*tan(a)**3 - 2*b*tan(a)**2*tan(
c)**3 - 2*b*tan(a)**2*tan(c) + 2*b*tan(a)*tan(c)**2 + 2*b*tan(a) - 2*b*tan(
c)**3 - 2*b*tan(c)) - log(tan(b*x)**2 + 1)*tan(a)**2*tan(c)/(2*b*tan(a)**3*
tan(c)**2 + 2*b*tan(a)**3 - 2*b*tan(a)**2*tan(c)**3 - 2*b*tan(a)**2*tan(c)
+ 2*b*tan(a)*tan(c)**2 + 2*b*tan(a) - 2*b*tan(c)**3 - 2*b*tan(c)) + log(tan
(b*x)**2 + 1)*tan(a)*tan(c)**2/(2*b*tan(a)**3*tan(c)**2 + 2*b*tan(a)**3 - 2
*b*tan(a)**2*tan(c)**3 - 2*b*tan(a)**2*tan(c) + 2*b*tan(a)*tan(c)**2 + 2*b*
tan(a) - 2*b*tan(c)**3 - 2*b*tan(c)) + log(tan(b*x)**2 + 1)*tan(a)/(2*b*tan
(a)**3*tan(c)**2 + 2*b*tan(a)**3 - 2*b*tan(a)**2*tan(c)**3 - 2*b*tan(a)**2*
tan(c) + 2*b*tan(a)*tan(c)**2 + 2*b*tan(a) - 2*b*tan(c)**3 - 2*b*tan(c)) -
log(tan(b*x)**2 + 1)*tan(c)/(2*b*tan(a)**3*tan(c)**2 + 2*b*tan(a)**3 - 2*b*
tan(a)**2*tan(c)**3 - 2*b*tan(a)**2*tan(c) + 2*b*tan(a)*tan(c)**2 + 2*b*tan
(a) - 2*b*tan(c)**3 - 2*b*tan(c)), True))*tan(c) + Piecewise((x, Eq(a, 0) &
 Eq(b, 0) & Eq(c, 0)), (-b*x*tan(c)**3*tan(b*x)/(b*tan(c)**5*tan(b*x) - b*t
an(c)**4 + 2*b*tan(c)**3*tan(b*x) - 2*b*tan(c)**2 + b*tan(c)*tan(b*x) - b)
+ b*x*tan(c)**2/(b*tan(c)**5*tan(b*x) - b*tan(c)**4 + 2*b*tan(c)**3*tan(b*x
) - 2*b*tan(c)**2 + b*tan(c)*tan(b*x) - b) + b*x*tan(c)*tan(b*x)/(b*tan(c)*
*5*tan(b*x) - b*tan(c)**4 + 2*b*tan(c)**3*tan(b*x) - 2*b*tan(c)**2 + b*tan(
c)*tan(b*x) - b) - b*x/(b*tan(c)**5*tan(b*x) - b*tan(c)**4 + 2*b*tan(c)**3*
tan(b*x) - 2*b*tan(c)**2 + b*tan(c)*tan(b*x) - b) - 2*log(tan(b*x) - 1/tan(
c))*tan(c)**2*tan(b*x)/(b*tan(c)**5*tan(b*x) - b*tan(c)**4 + 2*b*tan(c)**3*
```

```
tan(b*x) - 2*b*tan(c)**2 + b*tan(c)*tan(b*x) - b) + 2*log(tan(b*x) - 1/tan(
c))*tan(c)/(b*tan(c)**5*tan(b*x) - b*tan(c)**4 + 2*b*tan(c)**3*tan(b*x) - 2
*b*tan(c)**2 + b*tan(c)*tan(b*x) - b) + log(tan(b*x)**2 + 1)*tan(c)**2*tan(
b*x)/(b*tan(c)**5*tan(b*x) - b*tan(c)**4 + 2*b*tan(c)**3*tan(b*x) - 2*b*tan
(c)**2 + b*tan(c)*tan(b*x) - b) - log(tan(b*x)**2 + 1)*tan(c)/(b*tan(c)**5*
tan(b*x) - b*tan(c)**4 + 2*b*tan(c)**3*tan(b*x) - 2*b*tan(c)**2 + b*tan(c)*
tan(b*x) - b) - tan(c)**3/(b*tan(c)**5*tan(b*x) - b*tan(c)**4 + 2*b*tan(c)*
*3*tan(b*x) - 2*b*tan(c)**2 + b*tan(c)*tan(b*x) - b) - tan(c)/(b*tan(c)**5*
tan(b*x) - b*tan(c)**4 + 2*b*tan(c)**3*tan(b*x) - 2*b*tan(c)**2 + b*tan(c)*
tan(b*x) - b), Eq(a, atan(tan(c)) + pi*floor((c - pi/2)/pi) + pi*floor(c/pi
 - 1/2))), (x, Eq(b, 0)), (2*b*x/(2*b*tan(a)**2 + 2*b) - 2*log(tan(b*x) - 1
/tan(a))*tan(a)/(2*b*tan(a)**2 + 2*b) + log(tan(b*x)**2 + 1)*tan(a)/(2*b*ta
n(a)**2 + 2*b), Eq(c, 0)), (2*b*x/(2*b*tan(c)**2 + 2*b) - 2*log(tan(b*x) -
1/tan(c))*tan(c)/(2*b*tan(c)**2 + 2*b) + log(tan(b*x)**2 + 1)*tan(c)/(2*b*t
an(c)**2 + 2*b), Eq(a, 0)), (-2*b*x*tan(a)**2*tan(c)/(2*b*tan(a)**3*tan(c)*
*2 + 2*b*tan(a)**3 - 2*b*tan(a)**2*tan(c)**3 - 2*b*tan(a)**2*tan(c) + 2*b*t
an(a)*tan(c)**2 + 2*b*tan(a) - 2*b*tan(c)**3 - 2*b*tan(c)) + 2*b*x*tan(a)*t
an(c)**2/(2*b*tan(a)**3*tan(c)**2 + 2*b*tan(a)**3 - 2*b*tan(a)**2*tan(c)**3
 - 2*b*tan(a)**2*tan(c) + 2*b*tan(a)*tan(c)**2 + 2*b*tan(a) - 2*b*tan(c)**3
 - 2*b*tan(c)) + 2*b*x*tan(a)/(2*b*tan(a)**3*tan(c)**2 + 2*b*tan(a)**3 - 2*
b*tan(a)**2*tan(c)**3 - 2*b*tan(a)**2*tan(c) + 2*b*tan(a)*tan(c)**2 + 2*b*t
an(a) - 2*b*tan(c)**3 - 2*b*tan(c)) - 2*b*x*tan(c)/(2*b*tan(a)**3*tan(c)**2
 + 2*b*tan(a)**3 - 2*b*tan(a)**2*tan(c)**3 - 2*b*tan(a)**2*tan(c) + 2*b*tan
(a)*tan(c)**2 + 2*b*tan(a) - 2*b*tan(c)**3 - 2*b*tan(c)) - 2*log(tan(b*x) -
 1/tan(a))*tan(a)**2*tan(c)**2/(2*b*tan(a)**3*tan(c)**2 + 2*b*tan(a)**3 - 2
*b*tan(a)**2*tan(c)**3 - 2*b*tan(a)**2*tan(c) + 2*b*tan(a)*tan(c)**2 + 2*b*
tan(a) - 2*b*tan(c)**3 - 2*b*tan(c)) - 2*log(tan(b*x) - 1/tan(a))*tan(a)**2
/(2*b*tan(a)**3*tan(c)**2 + 2*b*tan(a)**3 - 2*b*tan(a)**2*tan(c)**3 - 2*b*t
an(a)**2*tan(c) + 2*b*tan(a)*tan(c)**2 + 2*b*tan(a) - 2*b*tan(c)**3 - 2*b*t
an(c)) + 2*log(tan(b*x) - 1/tan(c))*tan(a)**2*tan(c)**2/(2*b*tan(a)**3*tan(
c)**2 + 2*b*tan(a)**3 - 2*b*tan(a)**2*tan(c)**3 - 2*b*tan(a)**2*tan(c) + 2*
b*tan(a)*tan(c)**2 + 2*b*tan(a) - 2*b*tan(c)**3 - 2*b*tan(c)) + 2*log(tan(b
*x) - 1/tan(c))*tan(c)**2/(2*b*tan(a)**3*tan(c)**2 + 2*b*tan(a)**3 - 2*b*ta
n(a)**2*tan(c)**3 - 2*b*tan(a)**2*tan(c) + 2*b*tan(a)*tan(c)**2 + 2*b*tan(a
) - 2*b*tan(c)**3 - 2*b*tan(c)) + log(tan(b*x)**2 + 1)*tan(a)**2/(2*b*tan(a
)**3*tan(c)**2 + 2*b*tan(a)**3 - 2*b*tan(a)**2*tan(c)**3 - 2*b*tan(a)**2*ta
n(c) + 2*b*tan(a)*tan(c)**2 + 2*b*tan(a) - 2*b*tan(c)**3 - 2*b*tan(c)) - lo
g(tan(b*x)**2 + 1)*tan(c)**2/(2*b*tan(a)**3*tan(c)**2 + 2*b*tan(a)**3 - 2*b
*tan(a)**2*tan(c)**3 - 2*b*tan(a)**2*tan(c) + 2*b*tan(a)*tan(c)**2 + 2*b*ta
n(a) - 2*b*tan(c)**3 - 2*b*tan(c)), True))*tan(a)*tan(c)
```

3.140 $\int \tan(c - bx)\tan(a + bx)\,dx$

Optimal. Leaf size=34

$$-\frac{\cot(a+c)\log(\cos(c-bx))}{b}+\frac{\cot(a+c)\log(\cos(a+bx))}{b}+x$$

[Out] x-cot(a+c)*ln(cos(b*x-c))/b+cot(a+c)*ln(cos(b*x+a))/b

Rubi [A] time = 0.07, antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {4612, 4610, 3475}

$$-\frac{\cot(a+c)\log(\cos(c-bx))}{b}+\frac{\cot(a+c)\log(\cos(a+bx))}{b}+x$$

Antiderivative was successfully verified.

[In] Int[Tan[c - b*x]*Tan[a + b*x],x]

[Out] x - (Cot[a + c]*Log[Cos[c - b*x]])/b + (Cot[a + c]*Log[Cos[a + b*x]])/b

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 4610

```
Int[Sec[(a_.) + (b_.)*(x_)]*Sec[(c_) + (d_.)*(x_)], x_Symbol] :> -Dist[Csc[
(b*c - a*d)/d], Int[Tan[a + b*x], x], x] + Dist[Csc[(b*c - a*d)/b], Int[Tan
[c + d*x], x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b^2 - d^2, 0] && NeQ[b*c
 - a*d, 0]
```

Rule 4612

```
Int[Tan[(a_.) + (b_.)*(x_)]*Tan[(c_) + (d_.)*(x_)], x_Symbol] :> -Simp[(b*x
)/d, x] + Dist[(b*Cos[(b*c - a*d)/d])/d, Int[Sec[a + b*x]*Sec[c + d*x], x],
 x] /; FreeQ[{a, b, c, d}, x] && EqQ[b^2 - d^2, 0] && NeQ[b*c - a*d, 0]
```

Rubi steps

$$\int \tan(c-bx)\tan(a+bx)\,dx = x-\cos(a+c)\int \sec(c-bx)\sec(a+bx)\,dx$$
$$= x-\cot(a+c)\int \tan(c-bx)\,dx-\cot(a+c)\int \tan(a+bx)\,dx$$
$$= x-\frac{\cot(a+c)\log(\cos(c-bx))}{b}+\frac{\cot(a+c)\log(\cos(a+bx))}{b}$$

Mathematica [A] time = 0.53, size = 28, normalized size = 0.82

$$\frac{\cot(a+c)(\log(\cos(a+bx))-\log(\cos(c-bx)))}{b}+x$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c - b*x]*Tan[a + b*x],x]
[Out] x + (Cot[a + c]*(-Log[Cos[c - b*x]] + Log[Cos[a + b*x]]))/b
```

fricas [B] time = 2.01, size = 145, normalized size = 4.26

$$\frac{2\,bx\sin\left(2\,a+2\,c\right)-\left(\cos\left(2\,a+2\,c\right)+1\right)\log\left(-\frac{(\cos(2\,a+2\,c)-1)\tan(bx+a)^2-2\,\sin(2\,a+2\,c)\tan(bx+a)-\cos(2\,a+2\,c)-1}{(\cos(2\,a+2\,c)+1)\tan(bx+a)^2+\cos(2\,a+2\,c)+1}\right)+(\cos (2}{2\,b\sin\left(2\,a+2\,c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-tan(b*x-c)*tan(b*x+a),x, algorithm="fricas")
[Out] 1/2*(2*b*x*sin(2*a + 2*c) - (cos(2*a + 2*c) + 1)*log(-((cos(2*a + 2*c) - 1)
*tan(b*x + a)^2 - 2*sin(2*a + 2*c)*tan(b*x + a) - cos(2*a + 2*c) - 1)/((cos
(2*a + 2*c) + 1)*tan(b*x + a)^2 + cos(2*a + 2*c) + 1)) + (cos(2*a + 2*c) +
1)*log(1/(tan(b*x + a)^2 + 1)))/(b*sin(2*a + 2*c))
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-tan(b*x-c)*tan(b*x+a),x, algorithm="giac")
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (2*pi/x/2)>(-2*pi/x/2)-2/b*(-1/2*b*x+(-2*tan(a/2)^2*tan(c/
2)^3+2*tan(a/2)^2*tan(c/2)+8*tan(a/2)*tan(c/2)^2+2*tan(c/2)^3-2*tan(c/2))/(
8*tan(a/2)^2*tan(c/2)^2+8*tan(a/2)*tan(c/2)^3-8*tan(a/2)*tan(c/2)-8*tan(c/2
```

```
)^2)*ln(abs(2*tan(b*x)*tan(c/2)-tan(c/2)^2+1))+(-2*tan(a/2)^3*tan(c/2)^2+2*
tan(a/2)^3+8*tan(a/2)^2*tan(c/2)+2*tan(a/2)*tan(c/2)^2-2*tan(a/2))/(-8*tan(
a/2)^3*tan(c/2)-8*tan(a/2)^2*tan(c/2)^2+8*tan(a/2)^2+8*tan(a/2)*tan(c/2))*l
n(abs(2*tan(b*x)*tan(a/2)+tan(a/2)^2-1)))
```

maple [C] time = 0.14, size = 145, normalized size = 4.26

$$x+\frac{i\ln\left(1+e^{2i(bx+a)}\right)e^{2i(a+c)}}{b\left(e^{2i(a+c)}-1\right)}+\frac{i\ln\left(1+e^{2i(bx+a)}\right)}{b\left(e^{2i(a+c)}-1\right)}-\frac{i\ln\left(e^{2i(a+c)}+e^{2i(bx+a)}\right)e^{2i(a+c)}}{b\left(e^{2i(a+c)}-1\right)}-\frac{i\ln\left(e^{2i(a+c)}+e^{2i(bx+a)}\right)}{b\left(e^{2i(a+c)}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-tan(b*x-c)*tan(b*x+a),x)
```

```
[Out] x+I/b/(exp(2*I*(a+c))-1)*ln(1+exp(2*I*(b*x+a)))*exp(2*I*(a+c))+I/b/(exp(2*I
*(a+c))-1)*ln(1+exp(2*I*(b*x+a)))-I/b/(exp(2*I*(a+c))-1)*ln(exp(2*I*(a+c))+
exp(2*I*(b*x+a)))*exp(2*I*(a+c))-I/b/(exp(2*I*(a+c))-1)*ln(exp(2*I*(a+c))+e
xp(2*I*(b*x+a)))
```

maxima [B] time = 0.35, size = 290, normalized size = 8.53

$$\frac{\left(b\cos\left(2a+2c\right)^2+b\sin\left(2a+2c\right)^2-2b\cos\left(2a+2c\right)+b\right)x-\left(\cos\left(2a+2c\right)^2+\sin\left(2a+2c\right)^2-1\right)\arctan(\mathrm{s}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-tan(b*x-c)*tan(b*x+a),x, algorithm="maxima")
```

```
[Out] ((b*cos(2*a + 2*c)^2 + b*sin(2*a + 2*c)^2 - 2*b*cos(2*a + 2*c) + b)*x - (co
s(2*a + 2*c)^2 + sin(2*a + 2*c)^2 - 1)*arctan2(sin(2*b*x) - sin(2*a), cos(2
*b*x) + cos(2*a)) + (cos(2*a + 2*c)^2 + sin(2*a + 2*c)^2 - 1)*arctan2(sin(2
*b*x) + sin(2*c), cos(2*b*x) + cos(2*c)) + log(cos(2*b*x)^2 + 2*cos(2*b*x)*
cos(2*a) + cos(2*a)^2 + sin(2*b*x)^2 - 2*sin(2*b*x)*sin(2*a) + sin(2*a)^2)*
sin(2*a + 2*c) - log(cos(2*b*x)^2 + 2*cos(2*b*x)*cos(2*c) + cos(2*c)^2 + si
n(2*b*x)^2 + 2*sin(2*b*x)*sin(2*c) + sin(2*c)^2)*sin(2*a + 2*c))/(b*cos(2*a
 + 2*c)^2 + b*sin(2*a + 2*c)^2 - 2*b*cos(2*a + 2*c) + b)
```

mupad [B] time = 5.00, size = 196, normalized size = 5.76

$$\frac{\frac{x}{2}+x\left(\sin\left(a+c\right)^2-\frac{1}{2}\right)}{\sin\left(a+c\right)^2}+\frac{\frac{\sin(2a+2c)\,\ln\left(\sin(2a+2c)^2\,2\mathrm{i}-\sin(a+b\,x)^2\,2\mathrm{i}+\sin(3a+2c+b\,x)^2\,2\mathrm{i}+\sin(4a+4c)+\sin(6a+4c+2b\,x)-\sin(2a+2b\,x)\right.}{2}}{b\sin}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(a + b*x)*tan(c - b*x),x)
```

```
[Out] (x/2 + x*(sin(a + c)^2 - 1/2))/sin(a + c)^2 + ((sin(2*a + 2*c)*log(sin(4*a
+ 4*c) + sin(6*a + 4*c + 2*b*x) - sin(2*a + 2*b*x) + sin(2*a + 2*c)^2*2i -
sin(a + b*x)^2*2i + sin(3*a + 2*c + b*x)^2*2i))/2 - (sin(2*a + 2*c)*log(sin
(4*a + 4*c) + sin(4*a + 2*c + 2*b*x) + sin(2*c - 2*b*x) + sin(2*a + c + b*x
)^2*2i + sin(2*a + 2*c)^2*2i - sin(c - b*x)^2*2i))/2)/(b*sin(a + c)^2)
```

sympy [B] time = 8.52, size = 7720, normalized size = 227.06

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-tan(b*x-c)*tan(b*x+a),x)
```

```
[Out] Piecewise((0, Eq(a, 0) & Eq(b, 0) & Eq(c, 0)), (2*b*x*tan(a)/(2*b*tan(a)**2
 + 2*b) + 2*log(tan(b*x) - 1/tan(a))/(2*b*tan(a)**2 + 2*b) - log(tan(b*x)**
2 + 1)/(2*b*tan(a)**2 + 2*b), Eq(c, 0)), (-4*b*x*tan(c)**2*tan(b*x)/(2*b*ta
n(c)**5*tan(b*x) + 2*b*tan(c)**4 + 4*b*tan(c)**3*tan(b*x) + 4*b*tan(c)**2 +
 2*b*tan(c)*tan(b*x) + 2*b) - 4*b*x*tan(c)/(2*b*tan(c)**5*tan(b*x) + 2*b*ta
n(c)**4 + 4*b*tan(c)**3*tan(b*x) + 4*b*tan(c)**2 + 2*b*tan(c)*tan(b*x) + 2*
b) - 2*log(tan(b*x) + 1/tan(c))*tan(c)**3*tan(b*x)/(2*b*tan(c)**5*tan(b*x)
+ 2*b*tan(c)**4 + 4*b*tan(c)**3*tan(b*x) + 4*b*tan(c)**2 + 2*b*tan(c)*tan(b
*x) + 2*b) - 2*log(tan(b*x) + 1/tan(c))*tan(c)**2/(2*b*tan(c)**5*tan(b*x) +
 2*b*tan(c)**4 + 4*b*tan(c)**3*tan(b*x) + 4*b*tan(c)**2 + 2*b*tan(c)*tan(b*
x) + 2*b) + 2*log(tan(b*x) + 1/tan(c))*tan(c)*tan(b*x)/(2*b*tan(c)**5*tan(b
*x) + 2*b*tan(c)**4 + 4*b*tan(c)**3*tan(b*x) + 4*b*tan(c)**2 + 2*b*tan(c)*t
an(b*x) + 2*b) + 2*log(tan(b*x) + 1/tan(c))/(2*b*tan(c)**5*tan(b*x) + 2*b*t
an(c)**4 + 4*b*tan(c)**3*tan(b*x) + 4*b*tan(c)**2 + 2*b*tan(c)*tan(b*x) + 2
*b) + log(tan(b*x)**2 + 1)*tan(c)**3*tan(b*x)/(2*b*tan(c)**5*tan(b*x) + 2*b
*tan(c)**4 + 4*b*tan(c)**3*tan(b*x) + 4*b*tan(c)**2 + 2*b*tan(c)*tan(b*x) +
 2*b) + log(tan(b*x)**2 + 1)*tan(c)**2/(2*b*tan(c)**5*tan(b*x) + 2*b*tan(c)
**4 + 4*b*tan(c)**3*tan(b*x) + 4*b*tan(c)**2 + 2*b*tan(c)*tan(b*x) + 2*b) -
 log(tan(b*x)**2 + 1)*tan(c)*tan(b*x)/(2*b*tan(c)**5*tan(b*x) + 2*b*tan(c)*
*4 + 4*b*tan(c)**3*tan(b*x) + 4*b*tan(c)**2 + 2*b*tan(c)*tan(b*x) + 2*b) -
log(tan(b*x)**2 + 1)/(2*b*tan(c)**5*tan(b*x) + 2*b*tan(c)**4 + 4*b*tan(c)**
3*tan(b*x) + 4*b*tan(c)**2 + 2*b*tan(c)*tan(b*x) + 2*b) - 2*tan(c)**2/(2*b*
tan(c)**5*tan(b*x) + 2*b*tan(c)**4 + 4*b*tan(c)**3*tan(b*x) + 4*b*tan(c)**2
 + 2*b*tan(c)*tan(b*x) + 2*b) - 2/(2*b*tan(c)**5*tan(b*x) + 2*b*tan(c)**4 +
 4*b*tan(c)**3*tan(b*x) + 4*b*tan(c)**2 + 2*b*tan(c)*tan(b*x) + 2*b), Eq(a,
 -atan(tan(c)) - pi*floor((c - pi/2)/pi) - pi*floor(c/pi - 1/2))), (0, Eq(b
, 0)), (-2*b*x*tan(c)/(2*b*tan(c)**2 + 2*b) + 2*log(tan(b*x) + 1/tan(c))/(2
*b*tan(c)**2 + 2*b) - log(tan(b*x)**2 + 1)/(2*b*tan(c)**2 + 2*b), Eq(a, 0))
, (2*b*x*tan(a)**2/(2*b*tan(a)**3*tan(c)**2 + 2*b*tan(a)**3 + 2*b*tan(a)**2
*tan(c)**3 + 2*b*tan(a)**2*tan(c) + 2*b*tan(a)*tan(c)**2 + 2*b*tan(a) + 2*b
*tan(c)**3 + 2*b*tan(c)) - 2*b*x*tan(c)**2/(2*b*tan(a)**3*tan(c)**2 + 2*b*t
an(a)**3 + 2*b*tan(a)**2*tan(c)**3 + 2*b*tan(a)**2*tan(c) + 2*b*tan(a)*tan(
```

```
c)**2 + 2*b*tan(a) + 2*b*tan(c)**3 + 2*b*tan(c)) + 2*log(tan(b*x) - 1/tan(a
))*tan(a)*tan(c)**2/(2*b*tan(a)**3*tan(c)**2 + 2*b*tan(a)**3 + 2*b*tan(a)**
2*tan(c)**3 + 2*b*tan(a)**2*tan(c) + 2*b*tan(a)*tan(c)**2 + 2*b*tan(a) + 2*
b*tan(c)**3 + 2*b*tan(c)) + 2*log(tan(b*x) - 1/tan(a))*tan(a)/(2*b*tan(a)**
3*tan(c)**2 + 2*b*tan(a)**3 + 2*b*tan(a)**2*tan(c)**3 + 2*b*tan(a)**2*tan(c
) + 2*b*tan(a)*tan(c)**2 + 2*b*tan(a) + 2*b*tan(c)**3 + 2*b*tan(c)) + 2*log
(tan(b*x) + 1/tan(c))*tan(a)**2*tan(c)/(2*b*tan(a)**3*tan(c)**2 + 2*b*tan(a
)**3 + 2*b*tan(a)**2*tan(c)**3 + 2*b*tan(a)**2*tan(c) + 2*b*tan(a)*tan(c)**
2 + 2*b*tan(a) + 2*b*tan(c)**3 + 2*b*tan(c)) + 2*log(tan(b*x) + 1/tan(c))*t
an(c)/(2*b*tan(a)**3*tan(c)**2 + 2*b*tan(a)**3 + 2*b*tan(a)**2*tan(c)**3 +
2*b*tan(a)**2*tan(c) + 2*b*tan(a)*tan(c)**2 + 2*b*tan(a) + 2*b*tan(c)**3 +
2*b*tan(c)) - log(tan(b*x)**2 + 1)*tan(a)**2*tan(c)/(2*b*tan(a)**3*tan(c)**
2 + 2*b*tan(a)**3 + 2*b*tan(a)**2*tan(c)**3 + 2*b*tan(a)**2*tan(c) + 2*b*ta
n(a)*tan(c)**2 + 2*b*tan(a) + 2*b*tan(c)**3 + 2*b*tan(c)) - log(tan(b*x)**2
 + 1)*tan(a)*tan(c)**2/(2*b*tan(a)**3*tan(c)**2 + 2*b*tan(a)**3 + 2*b*tan(a
)**2*tan(c)**3 + 2*b*tan(a)**2*tan(c) + 2*b*tan(a)*tan(c)**2 + 2*b*tan(a) +
 2*b*tan(c)**3 + 2*b*tan(c)) - log(tan(b*x)**2 + 1)*tan(a)/(2*b*tan(a)**3*t
an(c)**2 + 2*b*tan(a)**3 + 2*b*tan(a)**2*tan(c)**3 + 2*b*tan(a)**2*tan(c) +
 2*b*tan(a)*tan(c)**2 + 2*b*tan(a) + 2*b*tan(c)**3 + 2*b*tan(c)) - log(tan(
b*x)**2 + 1)*tan(c)/(2*b*tan(a)**3*tan(c)**2 + 2*b*tan(a)**3 + 2*b*tan(a)**
2*tan(c)**3 + 2*b*tan(a)**2*tan(c) + 2*b*tan(a)*tan(c)**2 + 2*b*tan(a) + 2*
b*tan(c)**3 + 2*b*tan(c)), True))*tan(a) - Piecewise((0, Eq(a, 0) & Eq(b, 0
) & Eq(c, 0)), (2*b*x*tan(a)/(2*b*tan(a)**2 + 2*b) + 2*log(tan(b*x) - 1/tan
(a))/(2*b*tan(a)**2 + 2*b) - log(tan(b*x)**2 + 1)/(2*b*tan(a)**2 + 2*b), Eq
(c, 0)), (-4*b*x*tan(c)**2*tan(b*x)/(2*b*tan(c)**5*tan(b*x) + 2*b*tan(c)**4
 + 4*b*tan(c)**3*tan(b*x) + 4*b*tan(c)**2 + 2*b*tan(c)*tan(b*x) + 2*b) - 4*
b*x*tan(c)/(2*b*tan(c)**5*tan(b*x) + 2*b*tan(c)**4 + 4*b*tan(c)**3*tan(b*x)
 + 4*b*tan(c)**2 + 2*b*tan(c)*tan(b*x) + 2*b) - 2*log(tan(b*x) + 1/tan(c))*
tan(c)**3*tan(b*x)/(2*b*tan(c)**5*tan(b*x) + 2*b*tan(c)**4 + 4*b*tan(c)**3*
tan(b*x) + 4*b*tan(c)**2 + 2*b*tan(c)*tan(b*x) + 2*b) - 2*log(tan(b*x) + 1/
tan(c))*tan(c)**2/(2*b*tan(c)**5*tan(b*x) + 2*b*tan(c)**4 + 4*b*tan(c)**3*t
an(b*x) + 4*b*tan(c)**2 + 2*b*tan(c)*tan(b*x) + 2*b) + 2*log(tan(b*x) + 1/t
an(c))*tan(c)*tan(b*x)/(2*b*tan(c)**5*tan(b*x) + 2*b*tan(c)**4 + 4*b*tan(c)
**3*tan(b*x) + 4*b*tan(c)**2 + 2*b*tan(c)*tan(b*x) + 2*b) + 2*log(tan(b*x)
+ 1/tan(c))/(2*b*tan(c)**5*tan(b*x) + 2*b*tan(c)**4 + 4*b*tan(c)**3*tan(b*x
) + 4*b*tan(c)**2 + 2*b*tan(c)*tan(b*x) + 2*b) + log(tan(b*x)**2 + 1)*tan(c
)**3*tan(b*x)/(2*b*tan(c)**5*tan(b*x) + 2*b*tan(c)**4 + 4*b*tan(c)**3*tan(b
*x) + 4*b*tan(c)**2 + 2*b*tan(c)*tan(b*x) + 2*b) + log(tan(b*x)**2 + 1)*tan
(c)**2/(2*b*tan(c)**5*tan(b*x) + 2*b*tan(c)**4 + 4*b*tan(c)**3*tan(b*x) + 4
*b*tan(c)**2 + 2*b*tan(c)*tan(b*x) + 2*b) - log(tan(b*x)**2 + 1)*tan(c)*tan
(b*x)/(2*b*tan(c)**5*tan(b*x) + 2*b*tan(c)**4 + 4*b*tan(c)**3*tan(b*x) + 4*
b*tan(c)**2 + 2*b*tan(c)*tan(b*x) + 2*b) - log(tan(b*x)**2 + 1)/(2*b*tan(c)
**5*tan(b*x) + 2*b*tan(c)**4 + 4*b*tan(c)**3*tan(b*x) + 4*b*tan(c)**2 + 2*b
*tan(c)*tan(b*x) + 2*b) - 2*tan(c)**2/(2*b*tan(c)**5*tan(b*x) + 2*b*tan(c)*
*4 + 4*b*tan(c)**3*tan(b*x) + 4*b*tan(c)**2 + 2*b*tan(c)*tan(b*x) + 2*b) -
```

```
2/(2*b*tan(c)**5*tan(b*x) + 2*b*tan(c)**4 + 4*b*tan(c)**3*tan(b*x) + 4*b*ta
n(c)**2 + 2*b*tan(c)*tan(b*x) + 2*b), Eq(a, -atan(tan(c)) - pi*floor((c - p
i/2)/pi) - pi*floor(c/pi - 1/2))), (0, Eq(b, 0)), (-2*b*x*tan(c)/(2*b*tan(c
)**2 + 2*b) + 2*log(tan(b*x) + 1/tan(c))/(2*b*tan(c)**2 + 2*b) - log(tan(b*
x)**2 + 1)/(2*b*tan(c)**2 + 2*b), Eq(a, 0)), (2*b*x*tan(a)**2/(2*b*tan(a)**
3*tan(c)**2 + 2*b*tan(a)**3 + 2*b*tan(a)**2*tan(c)**3 + 2*b*tan(a)**2*tan(c
) + 2*b*tan(a)*tan(c)**2 + 2*b*tan(a) + 2*b*tan(c)**3 + 2*b*tan(c)) - 2*b*x
*tan(c)**2/(2*b*tan(a)**3*tan(c)**2 + 2*b*tan(a)**3 + 2*b*tan(a)**2*tan(c)*
*3 + 2*b*tan(a)**2*tan(c) + 2*b*tan(a)*tan(c)**2 + 2*b*tan(a) + 2*b*tan(c)*
*3 + 2*b*tan(c)) + 2*log(tan(b*x) - 1/tan(a))*tan(a)*tan(c)**2/(2*b*tan(a)*
*3*tan(c)**2 + 2*b*tan(a)**3 + 2*b*tan(a)**2*tan(c)**3 + 2*b*tan(a)**2*tan(
c) + 2*b*tan(a)*tan(c)**2 + 2*b*tan(a) + 2*b*tan(c)**3 + 2*b*tan(c)) + 2*lo
g(tan(b*x) - 1/tan(a))*tan(a)/(2*b*tan(a)**3*tan(c)**2 + 2*b*tan(a)**3 + 2*
b*tan(a)**2*tan(c)**3 + 2*b*tan(a)**2*tan(c) + 2*b*tan(a)*tan(c)**2 + 2*b*t
an(a) + 2*b*tan(c)**3 + 2*b*tan(c)) + 2*log(tan(b*x) + 1/tan(c))*tan(a)**2*
tan(c)/(2*b*tan(a)**3*tan(c)**2 + 2*b*tan(a)**3 + 2*b*tan(a)**2*tan(c)**3 +
 2*b*tan(a)**2*tan(c) + 2*b*tan(a)*tan(c)**2 + 2*b*tan(a) + 2*b*tan(c)**3 +
 2*b*tan(c)) + 2*log(tan(b*x) + 1/tan(c))*tan(c)/(2*b*tan(a)**3*tan(c)**2 +
 2*b*tan(a)**3 + 2*b*tan(a)**2*tan(c)**3 + 2*b*tan(a)**2*tan(c) + 2*b*tan(a
)*tan(c)**2 + 2*b*tan(a) + 2*b*tan(c)**3 + 2*b*tan(c)) - log(tan(b*x)**2 +
1)*tan(a)**2*tan(c)/(2*b*tan(a)**3*tan(c)**2 + 2*b*tan(a)**3 + 2*b*tan(a)**
2*tan(c)**3 + 2*b*tan(a)**2*tan(c) + 2*b*tan(a)*tan(c)**2 + 2*b*tan(a) + 2*
b*tan(c)**3 + 2*b*tan(c)) - log(tan(b*x)**2 + 1)*tan(a)*tan(c)**2/(2*b*tan(
a)**3*tan(c)**2 + 2*b*tan(a)**3 + 2*b*tan(a)**2*tan(c)**3 + 2*b*tan(a)**2*t
an(c) + 2*b*tan(a)*tan(c)**2 + 2*b*tan(a) + 2*b*tan(c)**3 + 2*b*tan(c)) - l
og(tan(b*x)**2 + 1)*tan(a)/(2*b*tan(a)**3*tan(c)**2 + 2*b*tan(a)**3 + 2*b*t
an(a)**2*tan(c)**3 + 2*b*tan(a)**2*tan(c) + 2*b*tan(a)*tan(c)**2 + 2*b*tan(
a) + 2*b*tan(c)**3 + 2*b*tan(c)) - log(tan(b*x)**2 + 1)*tan(c)/(2*b*tan(a)*
*3*tan(c)**2 + 2*b*tan(a)**3 + 2*b*tan(a)**2*tan(c)**3 + 2*b*tan(a)**2*tan(
c) + 2*b*tan(a)*tan(c)**2 + 2*b*tan(a) + 2*b*tan(c)**3 + 2*b*tan(c)), True)
)*tan(c) + Piecewise((0, Eq(a, 0) & Eq(b, 0) & Eq(c, 0)), (2*b*x*tan(a)/(2*
b*tan(a)**3 + 2*b*tan(a)) + 2*log(tan(b*x) - 1/tan(a))/(2*b*tan(a)**3 + 2*b
*tan(a)) + log(tan(b*x)**2 + 1)*tan(a)**2/(2*b*tan(a)**3 + 2*b*tan(a)), Eq(
c, 0)), (-b*x*tan(c)**4*tan(b*x)/(b*tan(c)**6*tan(b*x) + b*tan(c)**5 + 2*b*
tan(c)**4*tan(b*x) + 2*b*tan(c)**3 + b*tan(c)**2*tan(b*x) + b*tan(c)) - b*x
*tan(c)**3/(b*tan(c)**6*tan(b*x) + b*tan(c)**5 + 2*b*tan(c)**4*tan(b*x) + 2
*b*tan(c)**3 + b*tan(c)**2*tan(b*x) + b*tan(c)) + b*x*tan(c)**2*tan(b*x)/(b
*tan(c)**6*tan(b*x) + b*tan(c)**5 + 2*b*tan(c)**4*tan(b*x) + 2*b*tan(c)**3
+ b*tan(c)**2*tan(b*x) + b*tan(c)) + b*x*tan(c)/(b*tan(c)**6*tan(b*x) + b*t
an(c)**5 + 2*b*tan(c)**4*tan(b*x) + 2*b*tan(c)**3 + b*tan(c)**2*tan(b*x) +
b*tan(c)) + 2*log(tan(b*x) + 1/tan(c))*tan(c)**3*tan(b*x)/(b*tan(c)**6*tan(
b*x) + b*tan(c)**5 + 2*b*tan(c)**4*tan(b*x) + 2*b*tan(c)**3 + b*tan(c)**2*t
an(b*x) + b*tan(c)) + 2*log(tan(b*x) + 1/tan(c))*tan(c)**2/(b*tan(c)**6*tan
(b*x) + b*tan(c)**5 + 2*b*tan(c)**4*tan(b*x) + 2*b*tan(c)**3 + b*tan(c)**2*
tan(b*x) + b*tan(c)) - log(tan(b*x)**2 + 1)*tan(c)**3*tan(b*x)/(b*tan(c)**6
```

```
*tan(b*x) + b*tan(c)**5 + 2*b*tan(c)**4*tan(b*x) + 2*b*tan(c)**3 + b*tan(c)
**2*tan(b*x) + b*tan(c)) - log(tan(b*x)**2 + 1)*tan(c)**2/(b*tan(c)**6*tan(
b*x) + b*tan(c)**5 + 2*b*tan(c)**4*tan(b*x) + 2*b*tan(c)**3 + b*tan(c)**2*t
an(b*x) + b*tan(c)) + tan(c)**2/(b*tan(c)**6*tan(b*x) + b*tan(c)**5 + 2*b*t
an(c)**4*tan(b*x) + 2*b*tan(c)**3 + b*tan(c)**2*tan(b*x) + b*tan(c)) + 1/(b
*tan(c)**6*tan(b*x) + b*tan(c)**5 + 2*b*tan(c)**4*tan(b*x) + 2*b*tan(c)**3
+ b*tan(c)**2*tan(b*x) + b*tan(c)), Eq(a, -atan(tan(c)) - pi*floor((c - pi/
2)/pi) - pi*floor(c/pi - 1/2))), (0, Eq(b, 0)), (2*b*x*tan(c)/(2*b*tan(c)**
3 + 2*b*tan(c)) - 2*log(tan(b*x) + 1/tan(c))/(2*b*tan(c)**3 + 2*b*tan(c)) -
 log(tan(b*x)**2 + 1)*tan(c)**2/(2*b*tan(c)**3 + 2*b*tan(c)), Eq(a, 0)), (2
*b*x*tan(a)**2*tan(c)/(2*b*tan(a)**3*tan(c)**2 + 2*b*tan(a)**3 + 2*b*tan(a)
**2*tan(c)**3 + 2*b*tan(a)**2*tan(c) + 2*b*tan(a)*tan(c)**2 + 2*b*tan(a) +
2*b*tan(c)**3 + 2*b*tan(c)) + 2*b*x*tan(a)*tan(c)**2/(2*b*tan(a)**3*tan(c)*
*2 + 2*b*tan(a)**3 + 2*b*tan(a)**2*tan(c)**3 + 2*b*tan(a)**2*tan(c) + 2*b*t
an(a)*tan(c)**2 + 2*b*tan(a) + 2*b*tan(c)**3 + 2*b*tan(c)) + 2*b*x*tan(a)/(
2*b*tan(a)**3*tan(c)**2 + 2*b*tan(a)**3 + 2*b*tan(a)**2*tan(c)**3 + 2*b*tan
(a)**2*tan(c) + 2*b*tan(a)*tan(c)**2 + 2*b*tan(a) + 2*b*tan(c)**3 + 2*b*tan
(c)) + 2*b*x*tan(c)/(2*b*tan(a)**3*tan(c)**2 + 2*b*tan(a)**3 + 2*b*tan(a)**
2*tan(c)**3 + 2*b*tan(a)**2*tan(c) + 2*b*tan(a)*tan(c)**2 + 2*b*tan(a) + 2*
b*tan(c)**3 + 2*b*tan(c)) + 2*log(tan(b*x) - 1/tan(a))*tan(c)**2/(2*b*tan(a
)**3*tan(c)**2 + 2*b*tan(a)**3 + 2*b*tan(a)**2*tan(c)**3 + 2*b*tan(a)**2*ta
n(c) + 2*b*tan(a)*tan(c)**2 + 2*b*tan(a) + 2*b*tan(c)**3 + 2*b*tan(c)) + 2*
log(tan(b*x) - 1/tan(a))/(2*b*tan(a)**3*tan(c)**2 + 2*b*tan(a)**3 + 2*b*tan
(a)**2*tan(c)**3 + 2*b*tan(a)**2*tan(c) + 2*b*tan(a)*tan(c)**2 + 2*b*tan(a)
 + 2*b*tan(c)**3 + 2*b*tan(c)) - 2*log(tan(b*x) + 1/tan(c))*tan(a)**2/(2*b*
tan(a)**3*tan(c)**2 + 2*b*tan(a)**3 + 2*b*tan(a)**2*tan(c)**3 + 2*b*tan(a)*
*2*tan(c) + 2*b*tan(a)*tan(c)**2 + 2*b*tan(a) + 2*b*tan(c)**3 + 2*b*tan(c))
 - 2*log(tan(b*x) + 1/tan(c))/(2*b*tan(a)**3*tan(c)**2 + 2*b*tan(a)**3 + 2*
b*tan(a)**2*tan(c)**3 + 2*b*tan(a)**2*tan(c) + 2*b*tan(a)*tan(c)**2 + 2*b*t
an(a) + 2*b*tan(c)**3 + 2*b*tan(c)) + log(tan(b*x)**2 + 1)*tan(a)**2/(2*b*t
an(a)**3*tan(c)**2 + 2*b*tan(a)**3 + 2*b*tan(a)**2*tan(c)**3 + 2*b*tan(a)**
2*tan(c) + 2*b*tan(a)*tan(c)**2 + 2*b*tan(a) + 2*b*tan(c)**3 + 2*b*tan(c))
- log(tan(b*x)**2 + 1)*tan(c)**2/(2*b*tan(a)**3*tan(c)**2 + 2*b*tan(a)**3 +
 2*b*tan(a)**2*tan(c)**3 + 2*b*tan(a)**2*tan(c) + 2*b*tan(a)*tan(c)**2 + 2*
b*tan(a) + 2*b*tan(c)**3 + 2*b*tan(c)), True)) - Piecewise((-x, Eq(a, 0) &
Eq(b, 0) & Eq(c, 0)), (-2*b*x/(2*b*tan(a)**2 + 2*b) + 2*log(tan(b*x) - 1/ta
n(a))*tan(a)/(2*b*tan(a)**2 + 2*b) - log(tan(b*x)**2 + 1)*tan(a)/(2*b*tan(a
)**2 + 2*b), Eq(c, 0)), (b*x*tan(c)**3*tan(b*x)/(b*tan(c)**5*tan(b*x) + b*t
an(c)**4 + 2*b*tan(c)**3*tan(b*x) + 2*b*tan(c)**2 + b*tan(c)*tan(b*x) + b)
+ b*x*tan(c)**2/(b*tan(c)**5*tan(b*x) + b*tan(c)**4 + 2*b*tan(c)**3*tan(b*x
) + 2*b*tan(c)**2 + b*tan(c)*tan(b*x) + b) - b*x*tan(c)*tan(b*x)/(b*tan(c)*
*5*tan(b*x) + b*tan(c)**4 + 2*b*tan(c)**3*tan(b*x) + 2*b*tan(c)**2 + b*tan(
c)*tan(b*x) + b) - b*x/(b*tan(c)**5*tan(b*x) + b*tan(c)**4 + 2*b*tan(c)**3*
tan(b*x) + 2*b*tan(c)**2 + b*tan(c)*tan(b*x) + b) - 2*log(tan(b*x) + 1/tan(
c))*tan(c)**2*tan(b*x)/(b*tan(c)**5*tan(b*x) + b*tan(c)**4 + 2*b*tan(c)**3*
```

```
tan(b*x) + 2*b*tan(c)**2 + b*tan(c)*tan(b*x) + b) - 2*log(tan(b*x) + 1/tan(
c))*tan(c)/(b*tan(c)**5*tan(b*x) + b*tan(c)**4 + 2*b*tan(c)**3*tan(b*x) + 2
*b*tan(c)**2 + b*tan(c)*tan(b*x) + b) + log(tan(b*x)**2 + 1)*tan(c)**2*tan(
b*x)/(b*tan(c)**5*tan(b*x) + b*tan(c)**4 + 2*b*tan(c)**3*tan(b*x) + 2*b*tan
(c)**2 + b*tan(c)*tan(b*x) + b) + log(tan(b*x)**2 + 1)*tan(c)/(b*tan(c)**5*
tan(b*x) + b*tan(c)**4 + 2*b*tan(c)**3*tan(b*x) + 2*b*tan(c)**2 + b*tan(c)*
tan(b*x) + b) + tan(c)**3/(b*tan(c)**5*tan(b*x) + b*tan(c)**4 + 2*b*tan(c)*
*3*tan(b*x) + 2*b*tan(c)**2 + b*tan(c)*tan(b*x) + b) + tan(c)/(b*tan(c)**5*
tan(b*x) + b*tan(c)**4 + 2*b*tan(c)**3*tan(b*x) + 2*b*tan(c)**2 + b*tan(c)*
tan(b*x) + b), Eq(a, -atan(tan(c)) - pi*floor((c - pi/2)/pi) - pi*floor(c/p
i - 1/2))), (-x, Eq(b, 0)), (-2*b*x/(2*b*tan(c)**2 + 2*b) - 2*log(tan(b*x)
+ 1/tan(c))*tan(c)/(2*b*tan(c)**2 + 2*b) + log(tan(b*x)**2 + 1)*tan(c)/(2*b
*tan(c)**2 + 2*b), Eq(a, 0)), (-2*b*x*tan(a)**2*tan(c)/(2*b*tan(a)**3*tan(c
)**2 + 2*b*tan(a)**3 + 2*b*tan(a)**2*tan(c)**3 + 2*b*tan(a)**2*tan(c) + 2*b
*tan(a)*tan(c)**2 + 2*b*tan(a) + 2*b*tan(c)**3 + 2*b*tan(c)) - 2*b*x*tan(a)
*tan(c)**2/(2*b*tan(a)**3*tan(c)**2 + 2*b*tan(a)**3 + 2*b*tan(a)**2*tan(c)*
*3 + 2*b*tan(a)**2*tan(c) + 2*b*tan(a)*tan(c)**2 + 2*b*tan(a) + 2*b*tan(c)*
*3 + 2*b*tan(c)) - 2*b*x*tan(a)/(2*b*tan(a)**3*tan(c)**2 + 2*b*tan(a)**3 +
2*b*tan(a)**2*tan(c)**3 + 2*b*tan(a)**2*tan(c) + 2*b*tan(a)*tan(c)**2 + 2*b
*tan(a) + 2*b*tan(c)**3 + 2*b*tan(c)) - 2*b*x*tan(c)/(2*b*tan(a)**3*tan(c)*
*2 + 2*b*tan(a)**3 + 2*b*tan(a)**2*tan(c)**3 + 2*b*tan(a)**2*tan(c) + 2*b*t
an(a)*tan(c)**2 + 2*b*tan(a) + 2*b*tan(c)**3 + 2*b*tan(c)) + 2*log(tan(b*x)
 - 1/tan(a))*tan(a)**2*tan(c)**2/(2*b*tan(a)**3*tan(c)**2 + 2*b*tan(a)**3 +
 2*b*tan(a)**2*tan(c)**3 + 2*b*tan(a)**2*tan(c) + 2*b*tan(a)*tan(c)**2 + 2*
b*tan(a) + 2*b*tan(c)**3 + 2*b*tan(c)) + 2*log(tan(b*x) - 1/tan(a))*tan(a)*
*2/(2*b*tan(a)**3*tan(c)**2 + 2*b*tan(a)**3 + 2*b*tan(a)**2*tan(c)**3 + 2*b
*tan(a)**2*tan(c) + 2*b*tan(a)*tan(c)**2 + 2*b*tan(a) + 2*b*tan(c)**3 + 2*b
*tan(c)) - 2*log(tan(b*x) + 1/tan(c))*tan(a)**2*tan(c)**2/(2*b*tan(a)**3*ta
n(c)**2 + 2*b*tan(a)**3 + 2*b*tan(a)**2*tan(c)**3 + 2*b*tan(a)**2*tan(c) +
2*b*tan(a)*tan(c)**2 + 2*b*tan(a) + 2*b*tan(c)**3 + 2*b*tan(c)) - 2*log(tan
(b*x) + 1/tan(c))*tan(c)**2/(2*b*tan(a)**3*tan(c)**2 + 2*b*tan(a)**3 + 2*b*
tan(a)**2*tan(c)**3 + 2*b*tan(a)**2*tan(c) + 2*b*tan(a)*tan(c)**2 + 2*b*tan
(a) + 2*b*tan(c)**3 + 2*b*tan(c)) - log(tan(b*x)**2 + 1)*tan(a)**2/(2*b*tan
(a)**3*tan(c)**2 + 2*b*tan(a)**3 + 2*b*tan(a)**2*tan(c)**3 + 2*b*tan(a)**2*
tan(c) + 2*b*tan(a)*tan(c)**2 + 2*b*tan(a) + 2*b*tan(c)**3 + 2*b*tan(c)) +
log(tan(b*x)**2 + 1)*tan(c)**2/(2*b*tan(a)**3*tan(c)**2 + 2*b*tan(a)**3 + 2
*b*tan(a)**2*tan(c)**3 + 2*b*tan(a)**2*tan(c) + 2*b*tan(a)*tan(c)**2 + 2*b*
tan(a) + 2*b*tan(c)**3 + 2*b*tan(c)), True))*tan(a)*tan(c)
```

3.141 $\int \cot(a+bx)\cot(c+bx)\,dx$

Optimal. Leaf size=39

$$-\frac{\cot(a-c)\log(\sin(a+bx))}{b}+\frac{\cot(a-c)\log(\sin(bx+c))}{b}-x$$

```
[Out] -x-cot(a-c)*ln(sin(b*x+a))/b+cot(a-c)*ln(sin(b*x+c))/b
```

Rubi [A] time = 0.03, antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {4613, 4611, 3475}

$$-\frac{\cot(a-c)\log(\sin(a+bx))}{b}+\frac{\cot(a-c)\log(\sin(bx+c))}{b}-x$$

Antiderivative was successfully verified.

```
[In] Int[Cot[a + b*x]*Cot[c + b*x],x]
```

```
[Out] -x - (Cot[a - c]*Log[Sin[a + b*x]])/b + (Cot[a - c]*Log[Sin[c + b*x]])/b
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 4611

```
Int[Csc[(a_.) + (b_.)*(x_)]*Csc[(c_) + (d_.)*(x_)], x_Symbol] :> Dist[Csc[(
b*c - a*d)/b], Int[Cot[a + b*x], x], x] - Dist[Csc[(b*c - a*d)/d], Int[Cot[
c + d*x], x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b^2 - d^2, 0] && NeQ[b*c
- a*d, 0]
```

Rule 4613

```
Int[Cot[(a_.) + (b_.)*(x_)]*Cot[(c_) + (d_.)*(x_)], x_Symbol] :> -Simp[(b*x
)/d, x] + Dist[Cos[(b*c - a*d)/d], Int[Csc[a + b*x]*Csc[c + d*x], x], x] /;
 FreeQ[{a, b, c, d}, x] && EqQ[b^2 - d^2, 0] && NeQ[b*c - a*d, 0]
```

Rubi steps

$$\begin{aligned}\int \cot(a+bx)\cot(c+bx)\,dx &= -x+\cos(a-c)\int \csc(a+bx)\csc(c+bx)\,dx\\ &= -x-\cot(a-c)\int \cot(a+bx)\,dx+\cot(a-c)\int \cot(c+bx)\,dx\\ &= -x-\frac{\cot(a-c)\log(\sin(a+bx))}{b}+\frac{\cot(a-c)\log(\sin(c+bx))}{b}\end{aligned}$$

Mathematica [A] time = 0.51, size = 31, normalized size = 0.79

$$\frac{\cot(a-c)(\log(\sin(bx+c))-\log(\sin(a+bx)))}{b}-x$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[a + b*x]*Cot[c + b*x],x]
```

```
[Out] -x + (Cot[a - c]*(-Log[Sin[a + b*x]] + Log[Sin[c + b*x]]))/b
```

fricas [B] time = 1.53, size = 118, normalized size = 3.03

$$-\frac{2\,bx\sin\left(-2\,a+2\,c\right)-\left(\cos\left(-2\,a+2\,c\right)+1\right)\log\left(-\frac{\cos(2\,bx+2\,c)\cos(-2\,a+2\,c)+\sin(2\,bx+2\,c)\sin(-2\,a+2\,c)-1}{\cos(-2\,a+2\,c)+1}\right)+(\cos\left(-2\,a+\right.}{2\,b\sin\left(-2\,a+2\,c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(b*x+a)*cot(b*x+c),x, algorithm="fricas")
```

```
[Out] -1/2*(2*b*x*sin(-2*a + 2*c) - (cos(-2*a + 2*c) + 1)*log(-(cos(2*b*x + 2*c)*
cos(-2*a + 2*c) + sin(2*b*x + 2*c)*sin(-2*a + 2*c) - 1)/(cos(-2*a + 2*c) +
1)) + (cos(-2*a + 2*c) + 1)*log(-1/2*cos(2*b*x + 2*c) + 1/2))/(b*sin(-2*a +
 2*c))
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(b*x+a)*cot(b*x+c),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2
*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)2/b*(-1/2*b*x+(tan(a/2)
^2*tan(c/2)^4-2*tan(a/2)^2*tan(c/2)^2+tan(a/2)^2+4*tan(a/2)*tan(c/2)^3-4*ta
```

```
n(a/2)*tan(c/2)-tan(c/2)^4+2*tan(c/2)^2-1)/(4*tan(a/2)^2*tan(c/2)^3-4*tan(a
/2)^2*tan(c/2)-4*tan(a/2)*tan(c/2)^4+8*tan(a/2)*tan(c/2)^2-4*tan(a/2)-4*tan
(c/2)^3+4*tan(c/2))*ln(abs(tan(b*x)*tan(c/2)^2-tan(b*x)-2*tan(c/2)))+(tan(a
/2)^4*tan(c/2)^2-tan(a/2)^4+4*tan(a/2)^3*tan(c/2)-2*tan(a/2)^2*tan(c/2)^2+2
*tan(a/2)^2-4*tan(a/2)*tan(c/2)+tan(c/2)^2-1)/(-4*tan(a/2)^4*tan(c/2)+4*tan
(a/2)^3*tan(c/2)^2-4*tan(a/2)^3+8*tan(a/2)^2*tan(c/2)-4*tan(a/2)*tan(c/2)^2
+4*tan(a/2)-4*tan(c/2))*ln(abs(tan(b*x)*tan(a/2)^2-tan(b*x)-2*tan(a/2))))
```

maple [C] time = 0.19, size = 177, normalized size = 4.54

$$-x-\frac{i\ln\left(e^{2i(bx+a)}-1\right)e^{2ia}}{b\left(e^{2ia}-e^{2ic}\right)}-\frac{i\ln\left(e^{2i(bx+a)}-1\right)e^{2ic}}{b\left(e^{2ia}-e^{2ic}\right)}+\frac{i\ln\left(e^{2i(bx+a)}-e^{2i(a-c)}\right)e^{2ia}}{b\left(e^{2ia}-e^{2ic}\right)}+\frac{i\ln\left(e^{2i(bx+a)}-e^{2i(a-c)}\right)e^{2ic}}{b\left(e^{2ia}-e^{2ic}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(b*x+a)*cot(b*x+c),x)
```

```
[Out] -x-I/b/(exp(2*I*a)-exp(2*I*c))*ln(exp(2*I*(b*x+a))-1)*exp(2*I*a)-I/b/(exp(2
*I*a)-exp(2*I*c))*ln(exp(2*I*(b*x+a))-1)*exp(2*I*c)+I/b/(exp(2*I*a)-exp(2*I
*c))*ln(exp(2*I*(b*x+a))-exp(2*I*(a-c)))*exp(2*I*a)+I/b/(exp(2*I*a)-exp(2*I
*c))*ln(exp(2*I*(b*x+a))-exp(2*I*(a-c)))*exp(2*I*c)
```

maxima [B] time = 0.93, size = 549, normalized size = 14.08

$$-\frac{\left(2\,b\cos\left(2\,a\right)\cos\left(2\,c\right)-b\cos\left(2\,c\right)^{2}+2\,b\sin\left(2\,a\right)\sin\left(2\,c\right)-b\sin\left(2\,c\right)^{2}-\left(\cos\left(2\,a\right)^{2}+\sin\left(2\,a\right)^{2}\right)b\right)x+\left(\cos\left(2\,a\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(b*x+a)*cot(b*x+c),x, algorithm="maxima")
```

```
[Out] -((2*b*cos(2*a)*cos(2*c) - b*cos(2*c)^2 + 2*b*sin(2*a)*sin(2*c) - b*sin(2*c
)^2 - (cos(2*a)^2 + sin(2*a)^2)*b)*x + (cos(2*a)^2 - cos(2*c)^2 + sin(2*a)^
2 - sin(2*c)^2)*arctan2(sin(b*x) + sin(a), cos(b*x) - cos(a)) + (cos(2*a)^2
 - cos(2*c)^2 + sin(2*a)^2 - sin(2*c)^2)*arctan2(sin(b*x) - sin(a), cos(b*x
) + cos(a)) - (cos(2*a)^2 - cos(2*c)^2 + sin(2*a)^2 - sin(2*c)^2)*arctan2(s
in(b*x) + sin(c), cos(b*x) - cos(c)) - (cos(2*a)^2 - cos(2*c)^2 + sin(2*a)^
2 - sin(2*c)^2)*arctan2(sin(b*x) - sin(c), cos(b*x) + cos(c)) - (cos(2*c)*s
in(2*a) - cos(2*a)*sin(2*c))*log(cos(b*x)^2 + 2*cos(b*x)*cos(a) + cos(a)^2
+ sin(b*x)^2 - 2*sin(b*x)*sin(a) + sin(a)^2) - (cos(2*c)*sin(2*a) - cos(2*a
)*sin(2*c))*log(cos(b*x)^2 - 2*cos(b*x)*cos(a) + cos(a)^2 + sin(b*x)^2 + 2*
sin(b*x)*sin(a) + sin(a)^2) + (cos(2*c)*sin(2*a) - cos(2*a)*sin(2*c))*log(c
os(b*x)^2 + 2*cos(b*x)*cos(c) + cos(c)^2 + sin(b*x)^2 - 2*sin(b*x)*sin(c) +
 sin(c)^2) + (cos(2*c)*sin(2*a) - cos(2*a)*sin(2*c))*log(cos(b*x)^2 - 2*cos
(b*x)*cos(c) + cos(c)^2 + sin(b*x)^2 + 2*sin(b*x)*sin(c) + sin(c)^2))/(2*b*
```

```
cos(2*a)*cos(2*c) - b*cos(2*c)^2 + 2*b*sin(2*a)*sin(2*c) - b*sin(2*c)^2 - (
cos(2*a)^2 + sin(2*a)^2)*b)
```

mupad [B] time = 4.82, size = 207, normalized size = 5.31

$$-\frac{\frac{x}{2}+x\left(\sin\left(a-c\right)^2-\frac{1}{2}\right)}{\sin\left(a-c\right)^2}-\frac{\frac{\sin(2\,a-2\,c)\,\ln\left(\sin(2\,a-2\,c)^2\,2\mathrm{i}+\sin(a+b\,x)^2\,2\mathrm{i}-\sin(3\,a-2\,c+b\,x)^2\,2\mathrm{i}+\sin(4\,a-4\,c)-\sin(6\,a-4\,c+2\,b\,x)+\sin(2\,a+2\,b}{2}}{b\,\mathrm{si}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(a + b*x)*cot(c + b*x),x)
```

```
[Out] - (x/2 + x*(sin(a - c)^2 - 1/2))/sin(a - c)^2 - ((sin(2*a - 2*c)*log(sin(4*
a - 4*c) - sin(6*a - 4*c + 2*b*x) + sin(2*a + 2*b*x) + sin(2*a - 2*c)^2*2i
+ sin(a + b*x)^2*2i - sin(3*a - 2*c + b*x)^2*2i))/2 - (sin(2*a - 2*c)*log(s
in(4*a - 4*c) - sin(4*a - 2*c + 2*b*x) + sin(2*c + 2*b*x) + sin(2*a - 2*c)^
2*2i + sin(c + b*x)^2*2i - sin(2*a - c + b*x)^2*2i))/2)/(b*sin(a - c)^2)
```

sympy [B] time = 24.28, size = 7485, normalized size = 191.92

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(b*x+a)*cot(b*x+c),x)
```

```
[Out] Piecewise((x/(zoo*cot(c) + zoo + cot(c)/tan(c) + zoo/tan(c)), Eq(b, 0) & Eq
(a, atan(tan(c)) + pi*floor((c - pi/2)/pi) + pi*floor(c/pi - 1/2))), (-b*x*
tan(c)**5/(b*tan(c)**5 + b*tan(c)**4*tan(b*x) + 2*b*tan(c)**3 + 2*b*tan(c)*
*2*tan(b*x) + b*tan(c) + b*tan(b*x)) - b*x*tan(c)**4*tan(b*x)/(b*tan(c)**5
+ b*tan(c)**4*tan(b*x) + 2*b*tan(c)**3 + 2*b*tan(c)**2*tan(b*x) + b*tan(c)
+ b*tan(b*x)) + b*x*tan(c)**3/(b*tan(c)**5 + b*tan(c)**4*tan(b*x) + 2*b*tan
(c)**3 + 2*b*tan(c)**2*tan(b*x) + b*tan(c) + b*tan(b*x)) + b*x*tan(c)**2*ta
n(b*x)/(b*tan(c)**5 + b*tan(c)**4*tan(b*x) + 2*b*tan(c)**3 + 2*b*tan(c)**2*
tan(b*x) + b*tan(c) + b*tan(b*x)) - 2*log(tan(c) + tan(b*x))*tan(c)**4/(b*t
an(c)**5 + b*tan(c)**4*tan(b*x) + 2*b*tan(c)**3 + 2*b*tan(c)**2*tan(b*x) +
b*tan(c) + b*tan(b*x)) - 2*log(tan(c) + tan(b*x))*tan(c)**3*tan(b*x)/(b*tan
(c)**5 + b*tan(c)**4*tan(b*x) + 2*b*tan(c)**3 + 2*b*tan(c)**2*tan(b*x) + b*
tan(c) + b*tan(b*x)) + log(tan(b*x)**2 + 1)*tan(c)**4/(b*tan(c)**5 + b*tan(
c)**4*tan(b*x) + 2*b*tan(c)**3 + 2*b*tan(c)**2*tan(b*x) + b*tan(c) + b*tan(
b*x)) + log(tan(b*x)**2 + 1)*tan(c)**3*tan(b*x)/(b*tan(c)**5 + b*tan(c)**4*
tan(b*x) + 2*b*tan(c)**3 + 2*b*tan(c)**2*tan(b*x) + b*tan(c) + b*tan(b*x))
- tan(c)**6/(b*tan(c)**5 + b*tan(c)**4*tan(b*x) + 2*b*tan(c)**3 + 2*b*tan(c
)**2*tan(b*x) + b*tan(c) + b*tan(b*x)) - tan(c)**4/(b*tan(c)**5 + b*tan(c)*
*4*tan(b*x) + 2*b*tan(c)**3 + 2*b*tan(c)**2*tan(b*x) + b*tan(c) + b*tan(b*x
```

```
)), Eq(a, atan(tan(c)) + pi*floor((c - pi/2)/pi) + pi*floor(c/pi - 1/2))),
(x/(cot(a)*cot(c) + zoo*cot(a) + zoo*cot(c) + zoo), Eq(b, 0)), (-2*b*x*tan(
a)**3*tan(c)**2/(2*b*tan(a)**3*tan(c)**2 + 2*b*tan(a)**3 - 2*b*tan(a)**2*ta
n(c)**3 - 2*b*tan(a)**2*tan(c) + 2*b*tan(a)*tan(c)**2 + 2*b*tan(a) - 2*b*ta
n(c)**3 - 2*b*tan(c)) + 2*b*x*tan(a)**2*tan(c)**3/(2*b*tan(a)**3*tan(c)**2
+ 2*b*tan(a)**3 - 2*b*tan(a)**2*tan(c)**3 - 2*b*tan(a)**2*tan(c) + 2*b*tan(
a)*tan(c)**2 + 2*b*tan(a) - 2*b*tan(c)**3 - 2*b*tan(c)) + 2*b*x*tan(a)**2*t
an(c)/(2*b*tan(a)**3*tan(c)**2 + 2*b*tan(a)**3 - 2*b*tan(a)**2*tan(c)**3 -
2*b*tan(a)**2*tan(c) + 2*b*tan(a)*tan(c)**2 + 2*b*tan(a) - 2*b*tan(c)**3 -
2*b*tan(c)) - 2*b*x*tan(a)*tan(c)**2/(2*b*tan(a)**3*tan(c)**2 + 2*b*tan(a)*
*3 - 2*b*tan(a)**2*tan(c)**3 - 2*b*tan(a)**2*tan(c) + 2*b*tan(a)*tan(c)**2
+ 2*b*tan(a) - 2*b*tan(c)**3 - 2*b*tan(c)) - 2*log(tan(a) + tan(b*x))*tan(a
)**3*tan(c)**3/(2*b*tan(a)**3*tan(c)**2 + 2*b*tan(a)**3 - 2*b*tan(a)**2*tan
(c)**3 - 2*b*tan(a)**2*tan(c) + 2*b*tan(a)*tan(c)**2 + 2*b*tan(a) - 2*b*tan
(c)**3 - 2*b*tan(c)) - 2*log(tan(a) + tan(b*x))*tan(a)**3*tan(c)/(2*b*tan(a
)**3*tan(c)**2 + 2*b*tan(a)**3 - 2*b*tan(a)**2*tan(c)**3 - 2*b*tan(a)**2*ta
n(c) + 2*b*tan(a)*tan(c)**2 + 2*b*tan(a) - 2*b*tan(c)**3 - 2*b*tan(c)) + 2*
log(tan(c) + tan(b*x))*tan(a)**3*tan(c)**3/(2*b*tan(a)**3*tan(c)**2 + 2*b*t
an(a)**3 - 2*b*tan(a)**2*tan(c)**3 - 2*b*tan(a)**2*tan(c) + 2*b*tan(a)*tan(
c)**2 + 2*b*tan(a) - 2*b*tan(c)**3 - 2*b*tan(c)) + 2*log(tan(c) + tan(b*x))
*tan(a)*tan(c)**3/(2*b*tan(a)**3*tan(c)**2 + 2*b*tan(a)**3 - 2*b*tan(a)**2*
tan(c)**3 - 2*b*tan(a)**2*tan(c) + 2*b*tan(a)*tan(c)**2 + 2*b*tan(a) - 2*b*
tan(c)**3 - 2*b*tan(c)) + log(tan(b*x)**2 + 1)*tan(a)**3*tan(c)/(2*b*tan(a)
**3*tan(c)**2 + 2*b*tan(a)**3 - 2*b*tan(a)**2*tan(c)**3 - 2*b*tan(a)**2*tan
(c) + 2*b*tan(a)*tan(c)**2 + 2*b*tan(a) - 2*b*tan(c)**3 - 2*b*tan(c)) - log
(tan(b*x)**2 + 1)*tan(a)*tan(c)**3/(2*b*tan(a)**3*tan(c)**2 + 2*b*tan(a)**3
 - 2*b*tan(a)**2*tan(c)**3 - 2*b*tan(a)**2*tan(c) + 2*b*tan(a)*tan(c)**2 +
2*b*tan(a) - 2*b*tan(c)**3 - 2*b*tan(c)), True)) + Piecewise((zoo*x/(zoo*co
t(c) + zoo + cot(c)/tan(c) + zoo/tan(c)), Eq(b, 0) & Eq(a, atan(tan(c)) + p
i*floor((c - pi/2)/pi) + pi*floor(c/pi - 1/2))), (b*x*tan(c)**5/(b*tan(c)**
5 + b*tan(c)**4*tan(b*x) + 2*b*tan(c)**3 + 2*b*tan(c)**2*tan(b*x) + b*tan(c
) + b*tan(b*x)) + b*x*tan(c)**4*tan(b*x)/(b*tan(c)**5 + b*tan(c)**4*tan(b*x
) + 2*b*tan(c)**3 + 2*b*tan(c)**2*tan(b*x) + b*tan(c) + b*tan(b*x)) - b*x*t
an(c)**3/(b*tan(c)**5 + b*tan(c)**4*tan(b*x) + 2*b*tan(c)**3 + 2*b*tan(c)**
2*tan(b*x) + b*tan(c) + b*tan(b*x)) - b*x*tan(c)**2*tan(b*x)/(b*tan(c)**5 +
 b*tan(c)**4*tan(b*x) + 2*b*tan(c)**3 + 2*b*tan(c)**2*tan(b*x) + b*tan(c) +
 b*tan(b*x)) + 2*log(tan(c) + tan(b*x))*tan(c)**4/(b*tan(c)**5 + b*tan(c)**
4*tan(b*x) + 2*b*tan(c)**3 + 2*b*tan(c)**2*tan(b*x) + b*tan(c) + b*tan(b*x)
) + 2*log(tan(c) + tan(b*x))*tan(c)**3*tan(b*x)/(b*tan(c)**5 + b*tan(c)**4*
tan(b*x) + 2*b*tan(c)**3 + 2*b*tan(c)**2*tan(b*x) + b*tan(c) + b*tan(b*x))
- log(tan(b*x)**2 + 1)*tan(c)**4/(b*tan(c)**5 + b*tan(c)**4*tan(b*x) + 2*b*
tan(c)**3 + 2*b*tan(c)**2*tan(b*x) + b*tan(c) + b*tan(b*x)) - log(tan(b*x)*
*2 + 1)*tan(c)**3*tan(b*x)/(b*tan(c)**5 + b*tan(c)**4*tan(b*x) + 2*b*tan(c)
**3 + 2*b*tan(c)**2*tan(b*x) + b*tan(c) + b*tan(b*x)) - tan(c)**4/(b*tan(c)
**5 + b*tan(c)**4*tan(b*x) + 2*b*tan(c)**3 + 2*b*tan(c)**2*tan(b*x) + b*tan
```

```
(c) + b*tan(b*x)) - tan(c)**2/(b*tan(c)**5 + b*tan(c)**4*tan(b*x) + 2*b*tan
(c)**3 + 2*b*tan(c)**2*tan(b*x) + b*tan(c) + b*tan(b*x)), Eq(a, atan(tan(c)
) + pi*floor((c - pi/2)/pi) + pi*floor(c/pi - 1/2))), (zoo*x/(cot(a)*cot(c)
 + zoo*cot(a) + zoo*cot(c) + zoo), Eq(b, 0)), (2*b*x*tan(a)**3*tan(c)**2/(2
*b*tan(a)**3*tan(c)**2 + 2*b*tan(a)**3 - 2*b*tan(a)**2*tan(c)**3 - 2*b*tan(
a)**2*tan(c) + 2*b*tan(a)*tan(c)**2 + 2*b*tan(a) - 2*b*tan(c)**3 - 2*b*tan(
c)) - 2*b*x*tan(a)**2*tan(c)**3/(2*b*tan(a)**3*tan(c)**2 + 2*b*tan(a)**3 -
2*b*tan(a)**2*tan(c)**3 - 2*b*tan(a)**2*tan(c) + 2*b*tan(a)*tan(c)**2 + 2*b
*tan(a) - 2*b*tan(c)**3 - 2*b*tan(c)) - 2*b*x*tan(a)**2*tan(c)/(2*b*tan(a)*
*3*tan(c)**2 + 2*b*tan(a)**3 - 2*b*tan(a)**2*tan(c)**3 - 2*b*tan(a)**2*tan(
c) + 2*b*tan(a)*tan(c)**2 + 2*b*tan(a) - 2*b*tan(c)**3 - 2*b*tan(c)) + 2*b*
x*tan(a)*tan(c)**2/(2*b*tan(a)**3*tan(c)**2 + 2*b*tan(a)**3 - 2*b*tan(a)**2
*tan(c)**3 - 2*b*tan(a)**2*tan(c) + 2*b*tan(a)*tan(c)**2 + 2*b*tan(a) - 2*b
*tan(c)**3 - 2*b*tan(c)) - 2*log(tan(a) + tan(b*x))*tan(a)*tan(c)**3/(2*b*t
an(a)**3*tan(c)**2 + 2*b*tan(a)**3 - 2*b*tan(a)**2*tan(c)**3 - 2*b*tan(a)**
2*tan(c) + 2*b*tan(a)*tan(c)**2 + 2*b*tan(a) - 2*b*tan(c)**3 - 2*b*tan(c))
 - 2*log(tan(a) + tan(b*x))*tan(a)*tan(c)/(2*b*tan(a)**3*tan(c)**2 + 2*b*tan
(a)**3 - 2*b*tan(a)**2*tan(c)**3 - 2*b*tan(a)**2*tan(c) + 2*b*tan(a)*tan(c)
**2 + 2*b*tan(a) - 2*b*tan(c)**3 - 2*b*tan(c)) + 2*log(tan(c) + tan(b*x))*t
an(a)**3*tan(c)/(2*b*tan(a)**3*tan(c)**2 + 2*b*tan(a)**3 - 2*b*tan(a)**2*ta
n(c)**3 - 2*b*tan(a)**2*tan(c) + 2*b*tan(a)*tan(c)**2 + 2*b*tan(a) - 2*b*ta
n(c)**3 - 2*b*tan(c)) + 2*log(tan(c) + tan(b*x))*tan(a)*tan(c)/(2*b*tan(a)*
*3*tan(c)**2 + 2*b*tan(a)**3 - 2*b*tan(a)**2*tan(c)**3 - 2*b*tan(a)**2*tan(
c) + 2*b*tan(a)*tan(c)**2 + 2*b*tan(a) - 2*b*tan(c)**3 - 2*b*tan(c)) - log(
tan(b*x)**2 + 1)*tan(a)**3*tan(c)/(2*b*tan(a)**3*tan(c)**2 + 2*b*tan(a)**3
 - 2*b*tan(a)**2*tan(c)**3 - 2*b*tan(a)**2*tan(c) + 2*b*tan(a)*tan(c)**2 + 2
*b*tan(a) - 2*b*tan(c)**3 - 2*b*tan(c)) + log(tan(b*x)**2 + 1)*tan(a)*tan(c
)**3/(2*b*tan(a)**3*tan(c)**2 + 2*b*tan(a)**3 - 2*b*tan(a)**2*tan(c)**3 - 2
*b*tan(a)**2*tan(c) + 2*b*tan(a)*tan(c)**2 + 2*b*tan(a) - 2*b*tan(c)**3 - 2
*b*tan(c)), True))*cot(a)*cot(c) - Piecewise((zoo*x/(zoo*cot(c) + zoo + cot
(c)/tan(c) + zoo/tan(c)), Eq(b, 0) & Eq(a, atan(tan(c)) + pi*floor((c - pi/
2)/pi) + pi*floor(c/pi - 1/2))), (4*b*x*tan(c)**4/(2*b*tan(c)**5 + 2*b*tan(
c)**4*tan(b*x) + 4*b*tan(c)**3 + 4*b*tan(c)**2*tan(b*x) + 2*b*tan(c) + 2*b*
tan(b*x)) + 4*b*x*tan(c)**3*tan(b*x)/(2*b*tan(c)**5 + 2*b*tan(c)**4*tan(b*x
) + 4*b*tan(c)**3 + 4*b*tan(c)**2*tan(b*x) + 2*b*tan(c) + 2*b*tan(b*x)) - 2
*log(tan(c) + tan(b*x))*tan(c)**5/(2*b*tan(c)**5 + 2*b*tan(c)**4*tan(b*x) +
 4*b*tan(c)**3 + 4*b*tan(c)**2*tan(b*x) + 2*b*tan(c) + 2*b*tan(b*x)) - 2*lo
g(tan(c) + tan(b*x))*tan(c)**4*tan(b*x)/(2*b*tan(c)**5 + 2*b*tan(c)**4*tan(
b*x) + 4*b*tan(c)**3 + 4*b*tan(c)**2*tan(b*x) + 2*b*tan(c) + 2*b*tan(b*x))
 + 2*log(tan(c) + tan(b*x))*tan(c)**3/(2*b*tan(c)**5 + 2*b*tan(c)**4*tan(b*x
) + 4*b*tan(c)**3 + 4*b*tan(c)**2*tan(b*x) + 2*b*tan(c) + 2*b*tan(b*x)) + 2
*log(tan(c) + tan(b*x))*tan(c)**2*tan(b*x)/(2*b*tan(c)**5 + 2*b*tan(c)**4*t
an(b*x) + 4*b*tan(c)**3 + 4*b*tan(c)**2*tan(b*x) + 2*b*tan(c) + 2*b*tan(b*x
)) + log(tan(b*x)**2 + 1)*tan(c)**5/(2*b*tan(c)**5 + 2*b*tan(c)**4*tan(b*x)
 + 4*b*tan(c)**3 + 4*b*tan(c)**2*tan(b*x) + 2*b*tan(c) + 2*b*tan(b*x)) + lo
```

```
g(tan(b*x)**2 + 1)*tan(c)**4*tan(b*x)/(2*b*tan(c)**5 + 2*b*tan(c)**4*tan(b*
x) + 4*b*tan(c)**3 + 4*b*tan(c)**2*tan(b*x) + 2*b*tan(c) + 2*b*tan(b*x)) -
log(tan(b*x)**2 + 1)*tan(c)**3/(2*b*tan(c)**5 + 2*b*tan(c)**4*tan(b*x) + 4*
b*tan(c)**3 + 4*b*tan(c)**2*tan(b*x) + 2*b*tan(c) + 2*b*tan(b*x)) - log(tan
(b*x)**2 + 1)*tan(c)**2*tan(b*x)/(2*b*tan(c)**5 + 2*b*tan(c)**4*tan(b*x) +
4*b*tan(c)**3 + 4*b*tan(c)**2*tan(b*x) + 2*b*tan(c) + 2*b*tan(b*x)) + 2*tan
(c)**5/(2*b*tan(c)**5 + 2*b*tan(c)**4*tan(b*x) + 4*b*tan(c)**3 + 4*b*tan(c)
**2*tan(b*x) + 2*b*tan(c) + 2*b*tan(b*x)) + 2*tan(c)**3/(2*b*tan(c)**5 + 2*
b*tan(c)**4*tan(b*x) + 4*b*tan(c)**3 + 4*b*tan(c)**2*tan(b*x) + 2*b*tan(c)
+ 2*b*tan(b*x)), Eq(a, atan(tan(c)) + pi*floor((c - pi/2)/pi) + pi*floor(c/
pi - 1/2))), (zoo*x/(cot(a)*cot(c) + zoo*cot(a) + zoo*cot(c) + zoo), Eq(b,
0)), (2*b*x*tan(a)**3*tan(c)/(2*b*tan(a)**3*tan(c)**2 + 2*b*tan(a)**3 - 2*b
*tan(a)**2*tan(c)**3 - 2*b*tan(a)**2*tan(c) + 2*b*tan(a)*tan(c)**2 + 2*b*ta
n(a) - 2*b*tan(c)**3 - 2*b*tan(c)) - 2*b*x*tan(a)*tan(c)**3/(2*b*tan(a)**3*
tan(c)**2 + 2*b*tan(a)**3 - 2*b*tan(a)**2*tan(c)**3 - 2*b*tan(a)**2*tan(c)
+ 2*b*tan(a)*tan(c)**2 + 2*b*tan(a) - 2*b*tan(c)**3 - 2*b*tan(c)) + 2*log(t
an(a) + tan(b*x))*tan(a)**2*tan(c)**3/(2*b*tan(a)**3*tan(c)**2 + 2*b*tan(a)
**3 - 2*b*tan(a)**2*tan(c)**3 - 2*b*tan(a)**2*tan(c) + 2*b*tan(a)*tan(c)**2
 + 2*b*tan(a) - 2*b*tan(c)**3 - 2*b*tan(c)) + 2*log(tan(a) + tan(b*x))*tan(
a)**2*tan(c)/(2*b*tan(a)**3*tan(c)**2 + 2*b*tan(a)**3 - 2*b*tan(a)**2*tan(c
)**3 - 2*b*tan(a)**2*tan(c) + 2*b*tan(a)*tan(c)**2 + 2*b*tan(a) - 2*b*tan(c
)**3 - 2*b*tan(c)) - 2*log(tan(c) + tan(b*x))*tan(a)**3*tan(c)**2/(2*b*tan(
a)**3*tan(c)**2 + 2*b*tan(a)**3 - 2*b*tan(a)**2*tan(c)**3 - 2*b*tan(a)**2*t
an(c) + 2*b*tan(a)*tan(c)**2 + 2*b*tan(a) - 2*b*tan(c)**3 - 2*b*tan(c)) - 2
*log(tan(c) + tan(b*x))*tan(a)*tan(c)**2/(2*b*tan(a)**3*tan(c)**2 + 2*b*tan
(a)**3 - 2*b*tan(a)**2*tan(c)**3 - 2*b*tan(a)**2*tan(c) + 2*b*tan(a)*tan(c)
**2 + 2*b*tan(a) - 2*b*tan(c)**3 - 2*b*tan(c)) + log(tan(b*x)**2 + 1)*tan(a
)**3*tan(c)**2/(2*b*tan(a)**3*tan(c)**2 + 2*b*tan(a)**3 - 2*b*tan(a)**2*tan
(c)**3 - 2*b*tan(a)**2*tan(c) + 2*b*tan(a)*tan(c)**2 + 2*b*tan(a) - 2*b*tan
(c)**3 - 2*b*tan(c)) - log(tan(b*x)**2 + 1)*tan(a)**2*tan(c)**3/(2*b*tan(a)
**3*tan(c)**2 + 2*b*tan(a)**3 - 2*b*tan(a)**2*tan(c)**3 - 2*b*tan(a)**2*tan
(c) + 2*b*tan(a)*tan(c)**2 + 2*b*tan(a) - 2*b*tan(c)**3 - 2*b*tan(c)) - log
(tan(b*x)**2 + 1)*tan(a)**2*tan(c)/(2*b*tan(a)**3*tan(c)**2 + 2*b*tan(a)**3
 - 2*b*tan(a)**2*tan(c)**3 - 2*b*tan(a)**2*tan(c) + 2*b*tan(a)*tan(c)**2 +
2*b*tan(a) - 2*b*tan(c)**3 - 2*b*tan(c)) + log(tan(b*x)**2 + 1)*tan(a)*tan(
c)**2/(2*b*tan(a)**3*tan(c)**2 + 2*b*tan(a)**3 - 2*b*tan(a)**2*tan(c)**3 -
2*b*tan(a)**2*tan(c) + 2*b*tan(a)*tan(c)**2 + 2*b*tan(a) - 2*b*tan(c)**3 -
2*b*tan(c)), True))*cot(a) - Piecewise((zoo*x/(zoo*cot(c) + zoo + cot(c)/ta
n(c) + zoo/tan(c)), Eq(b, 0) & Eq(a, atan(tan(c)) + pi*floor((c - pi/2)/pi)
 + pi*floor(c/pi - 1/2))), (4*b*x*tan(c)**4/(2*b*tan(c)**5 + 2*b*tan(c)**4*
tan(b*x) + 4*b*tan(c)**3 + 4*b*tan(c)**2*tan(b*x) + 2*b*tan(c) + 2*b*tan(b*
x)) + 4*b*x*tan(c)**3*tan(b*x)/(2*b*tan(c)**5 + 2*b*tan(c)**4*tan(b*x) + 4*
b*tan(c)**3 + 4*b*tan(c)**2*tan(b*x) + 2*b*tan(c) + 2*b*tan(b*x)) - 2*log(t
an(c) + tan(b*x))*tan(c)**5/(2*b*tan(c)**5 + 2*b*tan(c)**4*tan(b*x) + 4*b*t
an(c)**3 + 4*b*tan(c)**2*tan(b*x) + 2*b*tan(c) + 2*b*tan(b*x)) - 2*log(tan(
```

```
c) + tan(b*x))*tan(c)**4*tan(b*x)/(2*b*tan(c)**5 + 2*b*tan(c)**4*tan(b*x) +
 4*b*tan(c)**3 + 4*b*tan(c)**2*tan(b*x) + 2*b*tan(c) + 2*b*tan(b*x)) + 2*lo
g(tan(c) + tan(b*x))*tan(c)**3/(2*b*tan(c)**5 + 2*b*tan(c)**4*tan(b*x) + 4*
b*tan(c)**3 + 4*b*tan(c)**2*tan(b*x) + 2*b*tan(c) + 2*b*tan(b*x)) + 2*log(t
an(c) + tan(b*x))*tan(c)**2*tan(b*x)/(2*b*tan(c)**5 + 2*b*tan(c)**4*tan(b*x
) + 4*b*tan(c)**3 + 4*b*tan(c)**2*tan(b*x) + 2*b*tan(c) + 2*b*tan(b*x)) + l
og(tan(b*x)**2 + 1)*tan(c)**5/(2*b*tan(c)**5 + 2*b*tan(c)**4*tan(b*x) + 4*b
*tan(c)**3 + 4*b*tan(c)**2*tan(b*x) + 2*b*tan(c) + 2*b*tan(b*x)) + log(tan(
b*x)**2 + 1)*tan(c)**4*tan(b*x)/(2*b*tan(c)**5 + 2*b*tan(c)**4*tan(b*x) + 4
*b*tan(c)**3 + 4*b*tan(c)**2*tan(b*x) + 2*b*tan(c) + 2*b*tan(b*x)) - log(ta
n(b*x)**2 + 1)*tan(c)**3/(2*b*tan(c)**5 + 2*b*tan(c)**4*tan(b*x) + 4*b*tan(
c)**3 + 4*b*tan(c)**2*tan(b*x) + 2*b*tan(c) + 2*b*tan(b*x)) - log(tan(b*x)*
*2 + 1)*tan(c)**2*tan(b*x)/(2*b*tan(c)**5 + 2*b*tan(c)**4*tan(b*x) + 4*b*ta
n(c)**3 + 4*b*tan(c)**2*tan(b*x) + 2*b*tan(c) + 2*b*tan(b*x)) + 2*tan(c)**5
/(2*b*tan(c)**5 + 2*b*tan(c)**4*tan(b*x) + 4*b*tan(c)**3 + 4*b*tan(c)**2*ta
n(b*x) + 2*b*tan(c) + 2*b*tan(b*x)) + 2*tan(c)**3/(2*b*tan(c)**5 + 2*b*tan(
c)**4*tan(b*x) + 4*b*tan(c)**3 + 4*b*tan(c)**2*tan(b*x) + 2*b*tan(c) + 2*b*
tan(b*x)), Eq(a, atan(tan(c)) + pi*floor((c - pi/2)/pi) + pi*floor(c/pi - 1
/2))), (zoo*x/(cot(a)*cot(c) + zoo*cot(a) + zoo*cot(c) + zoo), Eq(b, 0)), (
2*b*x*tan(a)**3*tan(c)/(2*b*tan(a)**3*tan(c)**2 + 2*b*tan(a)**3 - 2*b*tan(a
)**2*tan(c)**3 - 2*b*tan(a)**2*tan(c) + 2*b*tan(a)*tan(c)**2 + 2*b*tan(a) -
 2*b*tan(c)**3 - 2*b*tan(c)) - 2*b*x*tan(a)*tan(c)**3/(2*b*tan(a)**3*tan(c)
**2 + 2*b*tan(a)**3 - 2*b*tan(a)**2*tan(c)**3 - 2*b*tan(a)**2*tan(c) + 2*b*
tan(a)*tan(c)**2 + 2*b*tan(a) - 2*b*tan(c)**3 - 2*b*tan(c)) + 2*log(tan(a)
+ tan(b*x))*tan(a)**2*tan(c)**3/(2*b*tan(a)**3*tan(c)**2 + 2*b*tan(a)**3 -
2*b*tan(a)**2*tan(c)**3 - 2*b*tan(a)**2*tan(c) + 2*b*tan(a)*tan(c)**2 + 2*b
*tan(a) - 2*b*tan(c)**3 - 2*b*tan(c)) + 2*log(tan(a) + tan(b*x))*tan(a)**2*
tan(c)/(2*b*tan(a)**3*tan(c)**2 + 2*b*tan(a)**3 - 2*b*tan(a)**2*tan(c)**3 -
 2*b*tan(a)**2*tan(c) + 2*b*tan(a)*tan(c)**2 + 2*b*tan(a) - 2*b*tan(c)**3 -
 2*b*tan(c)) - 2*log(tan(c) + tan(b*x))*tan(a)**3*tan(c)**2/(2*b*tan(a)**3*
tan(c)**2 + 2*b*tan(a)**3 - 2*b*tan(a)**2*tan(c)**3 - 2*b*tan(a)**2*tan(c)
+ 2*b*tan(a)*tan(c)**2 + 2*b*tan(a) - 2*b*tan(c)**3 - 2*b*tan(c)) - 2*log(t
an(c) + tan(b*x))*tan(a)*tan(c)**2/(2*b*tan(a)**3*tan(c)**2 + 2*b*tan(a)**3
 - 2*b*tan(a)**2*tan(c)**3 - 2*b*tan(a)**2*tan(c) + 2*b*tan(a)*tan(c)**2 +
2*b*tan(a) - 2*b*tan(c)**3 - 2*b*tan(c)) + log(tan(b*x)**2 + 1)*tan(a)**3*t
an(c)**2/(2*b*tan(a)**3*tan(c)**2 + 2*b*tan(a)**3 - 2*b*tan(a)**2*tan(c)**3
 - 2*b*tan(a)**2*tan(c) + 2*b*tan(a)*tan(c)**2 + 2*b*tan(a) - 2*b*tan(c)**3
 - 2*b*tan(c)) - log(tan(b*x)**2 + 1)*tan(a)**2*tan(c)**3/(2*b*tan(a)**3*ta
n(c)**2 + 2*b*tan(a)**3 - 2*b*tan(a)**2*tan(c)**3 - 2*b*tan(a)**2*tan(c) +
2*b*tan(a)*tan(c)**2 + 2*b*tan(a) - 2*b*tan(c)**3 - 2*b*tan(c)) - log(tan(b
*x)**2 + 1)*tan(a)**2*tan(c)/(2*b*tan(a)**3*tan(c)**2 + 2*b*tan(a)**3 - 2*b
*tan(a)**2*tan(c)**3 - 2*b*tan(a)**2*tan(c) + 2*b*tan(a)*tan(c)**2 + 2*b*ta
n(a) - 2*b*tan(c)**3 - 2*b*tan(c)) + log(tan(b*x)**2 + 1)*tan(a)*tan(c)**2/
(2*b*tan(a)**3*tan(c)**2 + 2*b*tan(a)**3 - 2*b*tan(a)**2*tan(c)**3 - 2*b*ta
n(a)**2*tan(c) + 2*b*tan(a)*tan(c)**2 + 2*b*tan(a) - 2*b*tan(c)**3 - 2*b*ta
```

```
n(c)), True))*cot(c)
```

3.142 $\int \cot(c-bx)\cot(a+bx)\,dx$

Optimal. Leaf size=34

$$-\frac{\cot(a+c)\log(\sin(c-bx))}{b}+\frac{\cot(a+c)\log(\sin(a+bx))}{b}+x$$

```
[Out] x-cot(a+c)*ln(-sin(b*x-c))/b+cot(a+c)*ln(sin(b*x+a))/b
```

Rubi [A] time = 0.03, antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {4613, 4611, 3475}

$$-\frac{\cot(a+c)\log(\sin(c-bx))}{b}+\frac{\cot(a+c)\log(\sin(a+bx))}{b}+x$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c - b*x]*Cot[a + b*x],x]
```

```
[Out] x - (Cot[a + c]*Log[Sin[c - b*x]])/b + (Cot[a + c]*Log[Sin[a + b*x]])/b
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 4611

```
Int[Csc[(a_.) + (b_.)*(x_)]*Csc[(c_) + (d_.)*(x_)], x_Symbol] :> Dist[Csc[(
b*c - a*d)/b], Int[Cot[a + b*x], x], x] - Dist[Csc[(b*c - a*d)/d], Int[Cot[
c + d*x], x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b^2 - d^2, 0] && NeQ[b*c
- a*d, 0]
```

Rule 4613

```
Int[Cot[(a_.) + (b_.)*(x_)]*Cot[(c_) + (d_.)*(x_)], x_Symbol] :> -Simp[(b*x
)/d, x] + Dist[Cos[(b*c - a*d)/d], Int[Csc[a + b*x]*Csc[c + d*x], x], x] /;
 FreeQ[{a, b, c, d}, x] && EqQ[b^2 - d^2, 0] && NeQ[b*c - a*d, 0]
```

Rubi steps

$$\int \cot(c-bx)\cot(a+bx)\,dx = x+\cos(a+c)\int \csc(c-bx)\csc(a+bx)\,dx$$
$$= x+\cot(a+c)\int \cot(c-bx)\,dx+\cot(a+c)\int \cot(a+bx)\,dx$$
$$= x-\frac{\cot(a+c)\log(\sin(c-bx))}{b}+\frac{\cot(a+c)\log(\sin(a+bx))}{b}$$

Mathematica [**A**] time = 0.50, size = 30, normalized size = 0.88

$$\frac{\cot(a+c)(\log(-\sin(a+bx))-\log(\sin(c-bx)))}{b}+x$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c - b*x]*Cot[a + b*x],x]
```

```
[Out] x + (Cot[a + c]*(-Log[Sin[c - b*x]] + Log[-Sin[a + b*x]]))/b
```

fricas [**B**] time = 2.94, size = 118, normalized size = 3.47

$$\frac{2\,bx\sin\left(2\,a+2\,c\right)-\left(\cos\left(2\,a+2\,c\right)+1\right)\log\left(-\frac{\cos(2\,bx+2\,a)\cos(2\,a+2\,c)+\sin(2\,bx+2\,a)\sin(2\,a+2\,c)-1}{\cos(2\,a+2\,c)+1}\right)+\left(\cos\left(2\,a+2\,c\right)+1\right)}{2\,b\sin\left(2\,a+2\,c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-cot(b*x-c)*cot(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/2*(2*b*x*sin(2*a + 2*c) - (cos(2*a + 2*c) + 1)*log(-(cos(2*b*x + 2*a)*cos
(2*a + 2*c) + sin(2*b*x + 2*a)*sin(2*a + 2*c) - 1)/(cos(2*a + 2*c) + 1)) +
(cos(2*a + 2*c) + 1)*log(-1/2*cos(2*b*x + 2*a) + 1/2))/(b*sin(2*a + 2*c))
```

giac [**F(-2)**] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-cot(b*x-c)*cot(b*x+a),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2
*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)-2/b*(-1/2*b*x+(tan(a/2
)^2*tan(c/2)^4-2*tan(a/2)^2*tan(c/2)^2+tan(a/2)^2-4*tan(a/2)*tan(c/2)^3+4*t
an(a/2)*tan(c/2)-tan(c/2)^4+2*tan(c/2)^2-1)/(-4*tan(a/2)^2*tan(c/2)^3+4*tan
```

```
(a/2)^2*tan(c/2)-4*tan(a/2)*tan(c/2)^4+8*tan(a/2)*tan(c/2)^2-4*tan(a/2)+4*t
an(c/2)^3-4*tan(c/2))*ln(abs(tan(b*x)*tan(c/2)^2-tan(b*x)+2*tan(c/2)))+(tan
(a/2)^4*tan(c/2)^2-tan(a/2)^4-4*tan(a/2)^3*tan(c/2)-2*tan(a/2)^2*tan(c/2)^2
+2*tan(a/2)^2+4*tan(a/2)*tan(c/2)+tan(c/2)^2-1)/(4*tan(a/2)^4*tan(c/2)+4*ta
n(a/2)^3*tan(c/2)^2-4*tan(a/2)^3-8*tan(a/2)^2*tan(c/2)-4*tan(a/2)*tan(c/2)^
2+4*tan(a/2)+4*tan(c/2))*ln(abs(tan(b*x)*tan(a/2)^2-tan(b*x)-2*tan(a/2))))
```

maple [C] time = 0.18, size = 149, normalized size = 4.38

$$x-\frac{i \ln\left(-e^{2i(a+c)}+e^{2i(bx+a)}\right) e^{2i(a+c)}}{b\left(e^{2i(a+c)}-1\right)}-\frac{i \ln\left(-e^{2i(a+c)}+e^{2i(bx+a)}\right)}{b\left(e^{2i(a+c)}-1\right)}+\frac{i \ln\left(e^{2i(bx+a)}-1\right) e^{2i(a+c)}}{b\left(e^{2i(a+c)}-1\right)}+\frac{i \ln\left(e^{2i(bx+a)}-1\right)}{b\left(e^{2i(a+c)}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-cot(b*x-c)*cot(b*x+a),x)
```

```
[Out] x-I/b/(exp(2*I*(a+c))-1)*ln(-exp(2*I*(a+c))+exp(2*I*(b*x+a)))*exp(2*I*(a+c)
)-I/b/(exp(2*I*(a+c))-1)*ln(-exp(2*I*(a+c))+exp(2*I*(b*x+a)))+I/b/(exp(2*I*
(a+c))-1)*ln(exp(2*I*(b*x+a))-1)*exp(2*I*(a+c))+I/b/(exp(2*I*(a+c))-1)*ln(e
xp(2*I*(b*x+a))-1)
```

maxima [B] time = 0.44, size = 432, normalized size = 12.71

$$\left(b\cos\left(2\,a+2\,c\right)^2+b\sin\left(2\,a+2\,c\right)^2-2\,b\cos\left(2\,a+2\,c\right)+b\right)x-\left(\cos\left(2\,a+2\,c\right)^2+\sin\left(2\,a+2\,c\right)^2-1\right)\arctan$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-cot(b*x-c)*cot(b*x+a),x, algorithm="maxima")
```

```
[Out] ((b*cos(2*a + 2*c)^2 + b*sin(2*a + 2*c)^2 - 2*b*cos(2*a + 2*c) + b)*x - (co
s(2*a + 2*c)^2 + sin(2*a + 2*c)^2 - 1)*arctan2(sin(b*x) + sin(a), cos(b*x)
- cos(a)) - (cos(2*a + 2*c)^2 + sin(2*a + 2*c)^2 - 1)*arctan2(sin(b*x) - si
n(a), cos(b*x) + cos(a)) + (cos(2*a + 2*c)^2 + sin(2*a + 2*c)^2 - 1)*arctan
2(sin(b*x) + sin(c), cos(b*x) + cos(c)) + (cos(2*a + 2*c)^2 + sin(2*a + 2*c
)^2 - 1)*arctan2(sin(b*x) - sin(c), cos(b*x) - cos(c)) + log(cos(b*x)^2 + 2
*cos(b*x)*cos(a) + cos(a)^2 + sin(b*x)^2 - 2*sin(b*x)*sin(a) + sin(a)^2)*si
n(2*a + 2*c) + log(cos(b*x)^2 - 2*cos(b*x)*cos(a) + cos(a)^2 + sin(b*x)^2 +
 2*sin(b*x)*sin(a) + sin(a)^2)*sin(2*a + 2*c) - log(cos(b*x)^2 + 2*cos(b*x)
*cos(c) + cos(c)^2 + sin(b*x)^2 + 2*sin(b*x)*sin(c) + sin(c)^2)*sin(2*a + 2
*c) - log(cos(b*x)^2 - 2*cos(b*x)*cos(c) + cos(c)^2 + sin(b*x)^2 - 2*sin(b*
x)*sin(c) + sin(c)^2)*sin(2*a + 2*c))/(b*cos(2*a + 2*c)^2 + b*sin(2*a + 2*c
)^2 - 2*b*cos(2*a + 2*c) + b)
```

mupad [B] time = 5.05, size = 200, normalized size = 5.88

$$\frac{\frac{x}{2}+x\left(\sin\left(a+c\right)^2-\frac{1}{2}\right)}{\sin\left(a+c\right)^2}+\frac{\frac{\sin(2\,a+2\,c)\,\ln\left(\sin(2\,a+2\,c)^2\,2\mathrm{i}+\sin(a+b\,x)^2\,2\mathrm{i}-\sin(3\,a+2\,c+b\,x)^2\,2\mathrm{i}+\sin(4\,a+4\,c)-\sin(6\,a+4\,c+2\,b\,x)+\sin(2\,a+2\,b\,x)\right.}{2}}{b\sin}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(a + b*x)*cot(c - b*x),x)
```

```
[Out] (x/2 + x*(sin(a + c)^2 - 1/2))/sin(a + c)^2 + ((sin(2*a + 2*c)*log(sin(4*a
+ 4*c) - sin(6*a + 4*c + 2*b*x) + sin(2*a + 2*b*x) + sin(2*a + 2*c)^2*2i +
sin(a + b*x)^2*2i - sin(3*a + 2*c + b*x)^2*2i))/2 - (sin(2*a + 2*c)*log(sin
(4*a + 4*c) - sin(4*a + 2*c + 2*b*x) - sin(2*c - 2*b*x) - sin(2*a + c + b*x
)^2*2i + sin(2*a + 2*c)^2*2i + sin(c - b*x)^2*2i))/2)/(b*sin(a + c)^2)
```

sympy [B] time = 24.89, size = 7499, normalized size = 220.56

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-cot(b*x-c)*cot(b*x+a),x)
```

```
[Out] -Piecewise((x/(zoo*cot(c) + zoo + cot(c)/tan(c) + zoo/tan(c)), Eq(b, 0) & E
q(a, -atan(tan(c)) - pi*floor((c - pi/2)/pi) - pi*floor(c/pi - 1/2))), (b*x
*tan(c)**5/(-b*tan(c)**5 + b*tan(c)**4*tan(b*x) - 2*b*tan(c)**3 + 2*b*tan(c
)**2*tan(b*x) - b*tan(c) + b*tan(b*x)) - b*x*tan(c)**4*tan(b*x)/(-b*tan(c)*
*5 + b*tan(c)**4*tan(b*x) - 2*b*tan(c)**3 + 2*b*tan(c)**2*tan(b*x) - b*tan(
c) + b*tan(b*x)) - b*x*tan(c)**3/(-b*tan(c)**5 + b*tan(c)**4*tan(b*x) - 2*b
*tan(c)**3 + 2*b*tan(c)**2*tan(b*x) - b*tan(c) + b*tan(b*x)) + b*x*tan(c)**
2*tan(b*x)/(-b*tan(c)**5 + b*tan(c)**4*tan(b*x) - 2*b*tan(c)**3 + 2*b*tan(c
)**2*tan(b*x) - b*tan(c) + b*tan(b*x)) - 2*log(-tan(c) + tan(b*x))*tan(c)**
4/(-b*tan(c)**5 + b*tan(c)**4*tan(b*x) - 2*b*tan(c)**3 + 2*b*tan(c)**2*tan(
b*x) - b*tan(c) + b*tan(b*x)) + 2*log(-tan(c) + tan(b*x))*tan(c)**3*tan(b*x
)/(-b*tan(c)**5 + b*tan(c)**4*tan(b*x) - 2*b*tan(c)**3 + 2*b*tan(c)**2*tan(
b*x) - b*tan(c) + b*tan(b*x)) + log(tan(b*x)**2 + 1)*tan(c)**4/(-b*tan(c)**
5 + b*tan(c)**4*tan(b*x) - 2*b*tan(c)**3 + 2*b*tan(c)**2*tan(b*x) - b*tan(c
) + b*tan(b*x)) - log(tan(b*x)**2 + 1)*tan(c)**3*tan(b*x)/(-b*tan(c)**5 + b
*tan(c)**4*tan(b*x) - 2*b*tan(c)**3 + 2*b*tan(c)**2*tan(b*x) - b*tan(c) + b
*tan(b*x)) - tan(c)**6/(-b*tan(c)**5 + b*tan(c)**4*tan(b*x) - 2*b*tan(c)**3
 + 2*b*tan(c)**2*tan(b*x) - b*tan(c) + b*tan(b*x)) - tan(c)**4/(-b*tan(c)**
5 + b*tan(c)**4*tan(b*x) - 2*b*tan(c)**3 + 2*b*tan(c)**2*tan(b*x) - b*tan(c
) + b*tan(b*x)), Eq(a, -atan(tan(c)) - pi*floor((c - pi/2)/pi) - pi*floor(c
/pi - 1/2))), (x/(-cot(a)*cot(c) + zoo*cot(a) + zoo*cot(c) + zoo), Eq(b, 0)
), (-2*b*x*tan(a)**3*tan(c)**2/(2*b*tan(a)**3*tan(c)**2 + 2*b*tan(a)**3 + 2
```

```
*b*tan(a)**2*tan(c)**3 + 2*b*tan(a)**2*tan(c) + 2*b*tan(a)*tan(c)**2 + 2*b*
tan(a) + 2*b*tan(c)**3 + 2*b*tan(c)) - 2*b*x*tan(a)**2*tan(c)**3/(2*b*tan(a
)**3*tan(c)**2 + 2*b*tan(a)**3 + 2*b*tan(a)**2*tan(c)**3 + 2*b*tan(a)**2*ta
n(c) + 2*b*tan(a)*tan(c)**2 + 2*b*tan(a) + 2*b*tan(c)**3 + 2*b*tan(c)) - 2*
b*x*tan(a)**2*tan(c)/(2*b*tan(a)**3*tan(c)**2 + 2*b*tan(a)**3 + 2*b*tan(a)*
*2*tan(c)**3 + 2*b*tan(a)**2*tan(c) + 2*b*tan(a)*tan(c)**2 + 2*b*tan(a) + 2
*b*tan(c)**3 + 2*b*tan(c)) - 2*b*x*tan(a)*tan(c)**2/(2*b*tan(a)**3*tan(c)**
2 + 2*b*tan(a)**3 + 2*b*tan(a)**2*tan(c)**3 + 2*b*tan(a)**2*tan(c) + 2*b*ta
n(a)*tan(c)**2 + 2*b*tan(a) + 2*b*tan(c)**3 + 2*b*tan(c)) + 2*log(tan(a) +
tan(b*x))*tan(a)**3*tan(c)**3/(2*b*tan(a)**3*tan(c)**2 + 2*b*tan(a)**3 + 2*
b*tan(a)**2*tan(c)**3 + 2*b*tan(a)**2*tan(c) + 2*b*tan(a)*tan(c)**2 + 2*b*t
an(a) + 2*b*tan(c)**3 + 2*b*tan(c)) + 2*log(tan(a) + tan(b*x))*tan(a)**3*ta
n(c)/(2*b*tan(a)**3*tan(c)**2 + 2*b*tan(a)**3 + 2*b*tan(a)**2*tan(c)**3 + 2
*b*tan(a)**2*tan(c) + 2*b*tan(a)*tan(c)**2 + 2*b*tan(a) + 2*b*tan(c)**3 + 2
*b*tan(c)) - 2*log(-tan(c) + tan(b*x))*tan(a)**3*tan(c)**3/(2*b*tan(a)**3*t
an(c)**2 + 2*b*tan(a)**3 + 2*b*tan(a)**2*tan(c)**3 + 2*b*tan(a)**2*tan(c) +
 2*b*tan(a)*tan(c)**2 + 2*b*tan(a) + 2*b*tan(c)**3 + 2*b*tan(c)) - 2*log(-t
an(c) + tan(b*x))*tan(a)*tan(c)**3/(2*b*tan(a)**3*tan(c)**2 + 2*b*tan(a)**3
 + 2*b*tan(a)**2*tan(c)**3 + 2*b*tan(a)**2*tan(c) + 2*b*tan(a)*tan(c)**2 +
2*b*tan(a) + 2*b*tan(c)**3 + 2*b*tan(c)) - log(tan(b*x)**2 + 1)*tan(a)**3*t
an(c)/(2*b*tan(a)**3*tan(c)**2 + 2*b*tan(a)**3 + 2*b*tan(a)**2*tan(c)**3 +
2*b*tan(a)**2*tan(c) + 2*b*tan(a)*tan(c)**2 + 2*b*tan(a) + 2*b*tan(c)**3 +
2*b*tan(c)) + log(tan(b*x)**2 + 1)*tan(a)*tan(c)**3/(2*b*tan(a)**3*tan(c)**
2 + 2*b*tan(a)**3 + 2*b*tan(a)**2*tan(c)**3 + 2*b*tan(a)**2*tan(c) + 2*b*ta
n(a)*tan(c)**2 + 2*b*tan(a) + 2*b*tan(c)**3 + 2*b*tan(c)), True)) + Piecewi
se((zoo*x/(zoo*cot(c) + zoo + cot(c)/tan(c) + zoo/tan(c)), Eq(b, 0) & Eq(a,
 -atan(tan(c)) - pi*floor((c - pi/2)/pi) - pi*floor(c/pi - 1/2))), (-b*x*ta
n(c)**5/(-b*tan(c)**5 + b*tan(c)**4*tan(b*x) - 2*b*tan(c)**3 + 2*b*tan(c)**
2*tan(b*x) - b*tan(c) + b*tan(b*x)) + b*x*tan(c)**4*tan(b*x)/(-b*tan(c)**5
+ b*tan(c)**4*tan(b*x) - 2*b*tan(c)**3 + 2*b*tan(c)**2*tan(b*x) - b*tan(c)
+ b*tan(b*x)) + b*x*tan(c)**3/(-b*tan(c)**5 + b*tan(c)**4*tan(b*x) - 2*b*ta
n(c)**3 + 2*b*tan(c)**2*tan(b*x) - b*tan(c) + b*tan(b*x)) - b*x*tan(c)**2*t
an(b*x)/(-b*tan(c)**5 + b*tan(c)**4*tan(b*x) - 2*b*tan(c)**3 + 2*b*tan(c)**
2*tan(b*x) - b*tan(c) + b*tan(b*x)) + 2*log(-tan(c) + tan(b*x))*tan(c)**4/(
-b*tan(c)**5 + b*tan(c)**4*tan(b*x) - 2*b*tan(c)**3 + 2*b*tan(c)**2*tan(b*x
) - b*tan(c) + b*tan(b*x)) - 2*log(-tan(c) + tan(b*x))*tan(c)**3*tan(b*x)/(
-b*tan(c)**5 + b*tan(c)**4*tan(b*x) - 2*b*tan(c)**3 + 2*b*tan(c)**2*tan(b*x
) - b*tan(c) + b*tan(b*x)) - log(tan(b*x)**2 + 1)*tan(c)**4/(-b*tan(c)**5 +
 b*tan(c)**4*tan(b*x) - 2*b*tan(c)**3 + 2*b*tan(c)**2*tan(b*x) - b*tan(c) +
 b*tan(b*x)) + log(tan(b*x)**2 + 1)*tan(c)**3*tan(b*x)/(-b*tan(c)**5 + b*ta
n(c)**4*tan(b*x) - 2*b*tan(c)**3 + 2*b*tan(c)**2*tan(b*x) - b*tan(c) + b*ta
n(b*x)) - tan(c)**4/(-b*tan(c)**5 + b*tan(c)**4*tan(b*x) - 2*b*tan(c)**3 +
2*b*tan(c)**2*tan(b*x) - b*tan(c) + b*tan(b*x)) - tan(c)**2/(-b*tan(c)**5 +
 b*tan(c)**4*tan(b*x) - 2*b*tan(c)**3 + 2*b*tan(c)**2*tan(b*x) - b*tan(c) +
 b*tan(b*x)), Eq(a, -atan(tan(c)) - pi*floor((c - pi/2)/pi) - pi*floor(c/pi
```

```
 - 1/2))), (zoo*x/(-cot(a)*cot(c) + zoo*cot(a) + zoo*cot(c) + zoo), Eq(b, 0
)), (2*b*x*tan(a)**3*tan(c)**2/(2*b*tan(a)**3*tan(c)**2 + 2*b*tan(a)**3 + 2
*b*tan(a)**2*tan(c)**3 + 2*b*tan(a)**2*tan(c) + 2*b*tan(a)*tan(c)**2 + 2*b*
tan(a) + 2*b*tan(c)**3 + 2*b*tan(c)) + 2*b*x*tan(a)**2*tan(c)**3/(2*b*tan(a
)**3*tan(c)**2 + 2*b*tan(a)**3 + 2*b*tan(a)**2*tan(c)**3 + 2*b*tan(a)**2*ta
n(c) + 2*b*tan(a)*tan(c)**2 + 2*b*tan(a) + 2*b*tan(c)**3 + 2*b*tan(c)) + 2*
b*x*tan(a)**2*tan(c)/(2*b*tan(a)**3*tan(c)**2 + 2*b*tan(a)**3 + 2*b*tan(a)*
*2*tan(c)**3 + 2*b*tan(a)**2*tan(c) + 2*b*tan(a)*tan(c)**2 + 2*b*tan(a) + 2
*b*tan(c)**3 + 2*b*tan(c)) + 2*b*x*tan(a)*tan(c)**2/(2*b*tan(a)**3*tan(c)**
2 + 2*b*tan(a)**3 + 2*b*tan(a)**2*tan(c)**3 + 2*b*tan(a)**2*tan(c) + 2*b*ta
n(a)*tan(c)**2 + 2*b*tan(a) + 2*b*tan(c)**3 + 2*b*tan(c)) + 2*log(tan(a) +
tan(b*x))*tan(a)*tan(c)**3/(2*b*tan(a)**3*tan(c)**2 + 2*b*tan(a)**3 + 2*b*t
an(a)**2*tan(c)**3 + 2*b*tan(a)**2*tan(c) + 2*b*tan(a)*tan(c)**2 + 2*b*tan(
a) + 2*b*tan(c)**3 + 2*b*tan(c)) + 2*log(tan(a) + tan(b*x))*tan(a)*tan(c)/(
2*b*tan(a)**3*tan(c)**2 + 2*b*tan(a)**3 + 2*b*tan(a)**2*tan(c)**3 + 2*b*tan
(a)**2*tan(c) + 2*b*tan(a)*tan(c)**2 + 2*b*tan(a) + 2*b*tan(c)**3 + 2*b*tan
(c)) - 2*log(-tan(c) + tan(b*x))*tan(a)**3*tan(c)/(2*b*tan(a)**3*tan(c)**2
+ 2*b*tan(a)**3 + 2*b*tan(a)**2*tan(c)**3 + 2*b*tan(a)**2*tan(c) + 2*b*tan(
a)*tan(c)**2 + 2*b*tan(a) + 2*b*tan(c)**3 + 2*b*tan(c)) - 2*log(-tan(c) + t
an(b*x))*tan(a)*tan(c)/(2*b*tan(a)**3*tan(c)**2 + 2*b*tan(a)**3 + 2*b*tan(a
)**2*tan(c)**3 + 2*b*tan(a)**2*tan(c) + 2*b*tan(a)*tan(c)**2 + 2*b*tan(a) +
 2*b*tan(c)**3 + 2*b*tan(c)) + log(tan(b*x)**2 + 1)*tan(a)**3*tan(c)/(2*b*t
an(a)**3*tan(c)**2 + 2*b*tan(a)**3 + 2*b*tan(a)**2*tan(c)**3 + 2*b*tan(a)**
2*tan(c) + 2*b*tan(a)*tan(c)**2 + 2*b*tan(a) + 2*b*tan(c)**3 + 2*b*tan(c))
- log(tan(b*x)**2 + 1)*tan(a)*tan(c)**3/(2*b*tan(a)**3*tan(c)**2 + 2*b*tan(
a)**3 + 2*b*tan(a)**2*tan(c)**3 + 2*b*tan(a)**2*tan(c) + 2*b*tan(a)*tan(c)*
*2 + 2*b*tan(a) + 2*b*tan(c)**3 + 2*b*tan(c)), True))*cot(a)*cot(c) + Piece
wise((zoo*x/(zoo*cot(c) + zoo + cot(c)/tan(c) + zoo/tan(c)), Eq(b, 0) & Eq(
a, -atan(tan(c)) - pi*floor((c - pi/2)/pi) - pi*floor(c/pi - 1/2))), (4*b*x
*tan(c)**4/(-2*b*tan(c)**5 + 2*b*tan(c)**4*tan(b*x) - 4*b*tan(c)**3 + 4*b*t
an(c)**2*tan(b*x) - 2*b*tan(c) + 2*b*tan(b*x)) - 4*b*x*tan(c)**3*tan(b*x)/(
-2*b*tan(c)**5 + 2*b*tan(c)**4*tan(b*x) - 4*b*tan(c)**3 + 4*b*tan(c)**2*tan
(b*x) - 2*b*tan(c) + 2*b*tan(b*x)) + 2*log(-tan(c) + tan(b*x))*tan(c)**5/(-
2*b*tan(c)**5 + 2*b*tan(c)**4*tan(b*x) - 4*b*tan(c)**3 + 4*b*tan(c)**2*tan(
b*x) - 2*b*tan(c) + 2*b*tan(b*x)) - 2*log(-tan(c) + tan(b*x))*tan(c)**4*tan
(b*x)/(-2*b*tan(c)**5 + 2*b*tan(c)**4*tan(b*x) - 4*b*tan(c)**3 + 4*b*tan(c)
**2*tan(b*x) - 2*b*tan(c) + 2*b*tan(b*x)) - 2*log(-tan(c) + tan(b*x))*tan(c
)**3/(-2*b*tan(c)**5 + 2*b*tan(c)**4*tan(b*x) - 4*b*tan(c)**3 + 4*b*tan(c)*
*2*tan(b*x) - 2*b*tan(c) + 2*b*tan(b*x)) + 2*log(-tan(c) + tan(b*x))*tan(c)
**2*tan(b*x)/(-2*b*tan(c)**5 + 2*b*tan(c)**4*tan(b*x) - 4*b*tan(c)**3 + 4*b
*tan(c)**2*tan(b*x) - 2*b*tan(c) + 2*b*tan(b*x)) - log(tan(b*x)**2 + 1)*tan
(c)**5/(-2*b*tan(c)**5 + 2*b*tan(c)**4*tan(b*x) - 4*b*tan(c)**3 + 4*b*tan(c
)**2*tan(b*x) - 2*b*tan(c) + 2*b*tan(b*x)) + log(tan(b*x)**2 + 1)*tan(c)**4
*tan(b*x)/(-2*b*tan(c)**5 + 2*b*tan(c)**4*tan(b*x) - 4*b*tan(c)**3 + 4*b*ta
n(c)**2*tan(b*x) - 2*b*tan(c) + 2*b*tan(b*x)) + log(tan(b*x)**2 + 1)*tan(c)
```

```
**3/(-2*b*tan(c)**5 + 2*b*tan(c)**4*tan(b*x) - 4*b*tan(c)**3 + 4*b*tan(c)**
2*tan(b*x) - 2*b*tan(c) + 2*b*tan(b*x)) - log(tan(b*x)**2 + 1)*tan(c)**2*ta
n(b*x)/(-2*b*tan(c)**5 + 2*b*tan(c)**4*tan(b*x) - 4*b*tan(c)**3 + 4*b*tan(c
)**2*tan(b*x) - 2*b*tan(c) + 2*b*tan(b*x)) - 2*tan(c)**5/(-2*b*tan(c)**5 +
2*b*tan(c)**4*tan(b*x) - 4*b*tan(c)**3 + 4*b*tan(c)**2*tan(b*x) - 2*b*tan(c
) + 2*b*tan(b*x)) - 2*tan(c)**3/(-2*b*tan(c)**5 + 2*b*tan(c)**4*tan(b*x) -
4*b*tan(c)**3 + 4*b*tan(c)**2*tan(b*x) - 2*b*tan(c) + 2*b*tan(b*x)), Eq(a,
-atan(tan(c)) - pi*floor((c - pi/2)/pi) - pi*floor(c/pi - 1/2))), (zoo*x/(-
cot(a)*cot(c) + zoo*cot(a) + zoo*cot(c) + zoo), Eq(b, 0)), (-2*b*x*tan(a)**
3*tan(c)/(2*b*tan(a)**3*tan(c)**2 + 2*b*tan(a)**3 + 2*b*tan(a)**2*tan(c)**3
 + 2*b*tan(a)**2*tan(c) + 2*b*tan(a)*tan(c)**2 + 2*b*tan(a) + 2*b*tan(c)**3
 + 2*b*tan(c)) + 2*b*x*tan(a)*tan(c)**3/(2*b*tan(a)**3*tan(c)**2 + 2*b*tan(
a)**3 + 2*b*tan(a)**2*tan(c)**3 + 2*b*tan(a)**2*tan(c) + 2*b*tan(a)*tan(c)*
*2 + 2*b*tan(a) + 2*b*tan(c)**3 + 2*b*tan(c)) - 2*log(tan(a) + tan(b*x))*ta
n(a)**2*tan(c)**3/(2*b*tan(a)**3*tan(c)**2 + 2*b*tan(a)**3 + 2*b*tan(a)**2*
tan(c)**3 + 2*b*tan(a)**2*tan(c) + 2*b*tan(a)*tan(c)**2 + 2*b*tan(a) + 2*b*
tan(c)**3 + 2*b*tan(c)) - 2*log(tan(a) + tan(b*x))*tan(a)**2*tan(c)/(2*b*ta
n(a)**3*tan(c)**2 + 2*b*tan(a)**3 + 2*b*tan(a)**2*tan(c)**3 + 2*b*tan(a)**2
*tan(c) + 2*b*tan(a)*tan(c)**2 + 2*b*tan(a) + 2*b*tan(c)**3 + 2*b*tan(c)) -
 2*log(-tan(c) + tan(b*x))*tan(a)**3*tan(c)**2/(2*b*tan(a)**3*tan(c)**2 + 2
*b*tan(a)**3 + 2*b*tan(a)**2*tan(c)**3 + 2*b*tan(a)**2*tan(c) + 2*b*tan(a)*
tan(c)**2 + 2*b*tan(a) + 2*b*tan(c)**3 + 2*b*tan(c)) - 2*log(-tan(c) + tan(
b*x))*tan(a)*tan(c)**2/(2*b*tan(a)**3*tan(c)**2 + 2*b*tan(a)**3 + 2*b*tan(a
)**2*tan(c)**3 + 2*b*tan(a)**2*tan(c) + 2*b*tan(a)*tan(c)**2 + 2*b*tan(a) +
 2*b*tan(c)**3 + 2*b*tan(c)) + log(tan(b*x)**2 + 1)*tan(a)**3*tan(c)**2/(2*
b*tan(a)**3*tan(c)**2 + 2*b*tan(a)**3 + 2*b*tan(a)**2*tan(c)**3 + 2*b*tan(a
)**2*tan(c) + 2*b*tan(a)*tan(c)**2 + 2*b*tan(a) + 2*b*tan(c)**3 + 2*b*tan(c
)) + log(tan(b*x)**2 + 1)*tan(a)**2*tan(c)**3/(2*b*tan(a)**3*tan(c)**2 + 2*
b*tan(a)**3 + 2*b*tan(a)**2*tan(c)**3 + 2*b*tan(a)**2*tan(c) + 2*b*tan(a)*t
an(c)**2 + 2*b*tan(a) + 2*b*tan(c)**3 + 2*b*tan(c)) + log(tan(b*x)**2 + 1)*
tan(a)**2*tan(c)/(2*b*tan(a)**3*tan(c)**2 + 2*b*tan(a)**3 + 2*b*tan(a)**2*t
an(c)**3 + 2*b*tan(a)**2*tan(c) + 2*b*tan(a)*tan(c)**2 + 2*b*tan(a) + 2*b*t
an(c)**3 + 2*b*tan(c)) + log(tan(b*x)**2 + 1)*tan(a)*tan(c)**2/(2*b*tan(a)*
*3*tan(c)**2 + 2*b*tan(a)**3 + 2*b*tan(a)**2*tan(c)**3 + 2*b*tan(a)**2*tan(
c) + 2*b*tan(a)*tan(c)**2 + 2*b*tan(a) + 2*b*tan(c)**3 + 2*b*tan(c)), True)
)*cot(a) - Piecewise((zoo*x/(zoo*cot(c) + zoo + cot(c)/tan(c) + zoo/tan(c))
, Eq(b, 0) & Eq(a, -atan(tan(c)) - pi*floor((c - pi/2)/pi) - pi*floor(c/pi
- 1/2))), (4*b*x*tan(c)**4/(-2*b*tan(c)**5 + 2*b*tan(c)**4*tan(b*x) - 4*b*t
an(c)**3 + 4*b*tan(c)**2*tan(b*x) - 2*b*tan(c) + 2*b*tan(b*x)) - 4*b*x*tan(
c)**3*tan(b*x)/(-2*b*tan(c)**5 + 2*b*tan(c)**4*tan(b*x) - 4*b*tan(c)**3 + 4
*b*tan(c)**2*tan(b*x) - 2*b*tan(c) + 2*b*tan(b*x)) + 2*log(-tan(c) + tan(b*
x))*tan(c)**5/(-2*b*tan(c)**5 + 2*b*tan(c)**4*tan(b*x) - 4*b*tan(c)**3 + 4*
b*tan(c)**2*tan(b*x) - 2*b*tan(c) + 2*b*tan(b*x)) - 2*log(-tan(c) + tan(b*x
))*tan(c)**4*tan(b*x)/(-2*b*tan(c)**5 + 2*b*tan(c)**4*tan(b*x) - 4*b*tan(c)
**3 + 4*b*tan(c)**2*tan(b*x) - 2*b*tan(c) + 2*b*tan(b*x)) - 2*log(-tan(c) +
```

```
 tan(b*x))*tan(c)**3/(-2*b*tan(c)**5 + 2*b*tan(c)**4*tan(b*x) - 4*b*tan(c)*
*3 + 4*b*tan(c)**2*tan(b*x) - 2*b*tan(c) + 2*b*tan(b*x)) + 2*log(-tan(c) +
tan(b*x))*tan(c)**2*tan(b*x)/(-2*b*tan(c)**5 + 2*b*tan(c)**4*tan(b*x) - 4*b
*tan(c)**3 + 4*b*tan(c)**2*tan(b*x) - 2*b*tan(c) + 2*b*tan(b*x)) - log(tan(
b*x)**2 + 1)*tan(c)**5/(-2*b*tan(c)**5 + 2*b*tan(c)**4*tan(b*x) - 4*b*tan(c
)**3 + 4*b*tan(c)**2*tan(b*x) - 2*b*tan(c) + 2*b*tan(b*x)) + log(tan(b*x)**
2 + 1)*tan(c)**4*tan(b*x)/(-2*b*tan(c)**5 + 2*b*tan(c)**4*tan(b*x) - 4*b*ta
n(c)**3 + 4*b*tan(c)**2*tan(b*x) - 2*b*tan(c) + 2*b*tan(b*x)) + log(tan(b*x
)**2 + 1)*tan(c)**3/(-2*b*tan(c)**5 + 2*b*tan(c)**4*tan(b*x) - 4*b*tan(c)**
3 + 4*b*tan(c)**2*tan(b*x) - 2*b*tan(c) + 2*b*tan(b*x)) - log(tan(b*x)**2 +
 1)*tan(c)**2*tan(b*x)/(-2*b*tan(c)**5 + 2*b*tan(c)**4*tan(b*x) - 4*b*tan(c
)**3 + 4*b*tan(c)**2*tan(b*x) - 2*b*tan(c) + 2*b*tan(b*x)) - 2*tan(c)**5/(-
2*b*tan(c)**5 + 2*b*tan(c)**4*tan(b*x) - 4*b*tan(c)**3 + 4*b*tan(c)**2*tan(
b*x) - 2*b*tan(c) + 2*b*tan(b*x)) - 2*tan(c)**3/(-2*b*tan(c)**5 + 2*b*tan(c
)**4*tan(b*x) - 4*b*tan(c)**3 + 4*b*tan(c)**2*tan(b*x) - 2*b*tan(c) + 2*b*t
an(b*x)), Eq(a, -atan(tan(c)) - pi*floor((c - pi/2)/pi) - pi*floor(c/pi - 1
/2))), (zoo*x/(-cot(a)*cot(c) + zoo*cot(a) + zoo*cot(c) + zoo), Eq(b, 0)),
(-2*b*x*tan(a)**3*tan(c)/(2*b*tan(a)**3*tan(c)**2 + 2*b*tan(a)**3 + 2*b*tan
(a)**2*tan(c)**3 + 2*b*tan(a)**2*tan(c) + 2*b*tan(a)*tan(c)**2 + 2*b*tan(a)
 + 2*b*tan(c)**3 + 2*b*tan(c)) + 2*b*x*tan(a)*tan(c)**3/(2*b*tan(a)**3*tan(
c)**2 + 2*b*tan(a)**3 + 2*b*tan(a)**2*tan(c)**3 + 2*b*tan(a)**2*tan(c) + 2*
b*tan(a)*tan(c)**2 + 2*b*tan(a) + 2*b*tan(c)**3 + 2*b*tan(c)) - 2*log(tan(a
) + tan(b*x))*tan(a)**2*tan(c)**3/(2*b*tan(a)**3*tan(c)**2 + 2*b*tan(a)**3
+ 2*b*tan(a)**2*tan(c)**3 + 2*b*tan(a)**2*tan(c) + 2*b*tan(a)*tan(c)**2 + 2
*b*tan(a) + 2*b*tan(c)**3 + 2*b*tan(c)) - 2*log(tan(a) + tan(b*x))*tan(a)**
2*tan(c)/(2*b*tan(a)**3*tan(c)**2 + 2*b*tan(a)**3 + 2*b*tan(a)**2*tan(c)**3
 + 2*b*tan(a)**2*tan(c) + 2*b*tan(a)*tan(c)**2 + 2*b*tan(a) + 2*b*tan(c)**3
 + 2*b*tan(c)) - 2*log(-tan(c) + tan(b*x))*tan(a)**3*tan(c)**2/(2*b*tan(a)*
*3*tan(c)**2 + 2*b*tan(a)**3 + 2*b*tan(a)**2*tan(c)**3 + 2*b*tan(a)**2*tan(
c) + 2*b*tan(a)*tan(c)**2 + 2*b*tan(a) + 2*b*tan(c)**3 + 2*b*tan(c)) - 2*lo
g(-tan(c) + tan(b*x))*tan(a)*tan(c)**2/(2*b*tan(a)**3*tan(c)**2 + 2*b*tan(a
)**3 + 2*b*tan(a)**2*tan(c)**3 + 2*b*tan(a)**2*tan(c) + 2*b*tan(a)*tan(c)**
2 + 2*b*tan(a) + 2*b*tan(c)**3 + 2*b*tan(c)) + log(tan(b*x)**2 + 1)*tan(a)*
*3*tan(c)**2/(2*b*tan(a)**3*tan(c)**2 + 2*b*tan(a)**3 + 2*b*tan(a)**2*tan(c
)**3 + 2*b*tan(a)**2*tan(c) + 2*b*tan(a)*tan(c)**2 + 2*b*tan(a) + 2*b*tan(c
)**3 + 2*b*tan(c)) + log(tan(b*x)**2 + 1)*tan(a)**2*tan(c)**3/(2*b*tan(a)**
3*tan(c)**2 + 2*b*tan(a)**3 + 2*b*tan(a)**2*tan(c)**3 + 2*b*tan(a)**2*tan(c
) + 2*b*tan(a)*tan(c)**2 + 2*b*tan(a) + 2*b*tan(c)**3 + 2*b*tan(c)) + log(t
an(b*x)**2 + 1)*tan(a)**2*tan(c)/(2*b*tan(a)**3*tan(c)**2 + 2*b*tan(a)**3 +
 2*b*tan(a)**2*tan(c)**3 + 2*b*tan(a)**2*tan(c) + 2*b*tan(a)*tan(c)**2 + 2*
b*tan(a) + 2*b*tan(c)**3 + 2*b*tan(c)) + log(tan(b*x)**2 + 1)*tan(a)*tan(c)
**2/(2*b*tan(a)**3*tan(c)**2 + 2*b*tan(a)**3 + 2*b*tan(a)**2*tan(c)**3 + 2*
b*tan(a)**2*tan(c) + 2*b*tan(a)*tan(c)**2 + 2*b*tan(a) + 2*b*tan(c)**3 + 2*
b*tan(c)), True))*cot(c)
```

3.143 $\int \sec(a+bx)\sec(c+bx)\,dx$

Optimal. Leaf size=36

$$\frac{\csc(a-c)\log(\cos(bx+c))}{b}-\frac{\csc(a-c)\log(\cos(a+bx))}{b}$$

```
[Out] -csc(a-c)*ln(cos(b*x+a))/b+csc(a-c)*ln(cos(b*x+c))/b
```

Rubi [A] time = 0.02, antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {4610, 3475}

$$\frac{\csc(a-c)\log(\cos(bx+c))}{b}-\frac{\csc(a-c)\log(\cos(a+bx))}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[a + b*x]*Sec[c + b*x],x]
```

```
[Out] -((Csc[a - c]*Log[Cos[a + b*x]])/b) + (Csc[a - c]*Log[Cos[c + b*x]])/b
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 4610

```
Int[Sec[(a_.) + (b_.)*(x_)]*Sec[(c_) + (d_.)*(x_)], x_Symbol] :> -Dist[Csc[
(b*c - a*d)/d], Int[Tan[a + b*x], x], x] + Dist[Csc[(b*c - a*d)/b], Int[Tan
[c + d*x], x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b^2 - d^2, 0] && NeQ[b*c
 - a*d, 0]
```

Rubi steps

$$\begin{aligned}\int \sec(a+bx)\sec(c+bx)\,dx &= \csc(a-c)\int \tan(a+bx)\,dx-\csc(a-c)\int \tan(c+bx)\,dx\\ &= -\frac{\csc(a-c)\log(\cos(a+bx))}{b}+\frac{\csc(a-c)\log(\cos(c+bx))}{b}\end{aligned}$$

Mathematica [A] time = 0.23, size = 28, normalized size = 0.78

$$-\frac{\csc(a-c)(\log(\cos(a+bx))-\log(\cos(bx+c)))}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[a + b*x]*Sec[c + b*x],x]
```

```
[Out] -((Csc[a - c]*(Log[Cos[a + b*x]] - Log[Cos[c + b*x]]))/b)
```

fricas [B] time = 1.99, size = 107, normalized size = 2.97

$$-\frac{\log\left(\cos\left(bx+c\right)^{2}\right)-\log\left(\frac{4\left(2\cos(bx+c)\cos(-a+c)\sin(bx+c)\sin(-a+c)+\left(2\cos(-a+c)^{2}-1\right)\cos(bx+c)^{2}-\cos(-a+c)^{2}+1\right)}{\cos(-a+c)^{2}+2\cos(-a+c)+1}\right)}{2\,b\sin\left(-a+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)*sec(b*x+c),x, algorithm="fricas")
```

```
[Out] -1/2*(log(cos(b*x + c)^2) - log(4*(2*cos(b*x + c)*cos(-a + c)*sin(b*x + c)*
sin(-a + c) + (2*cos(-a + c)^2 - 1)*cos(b*x + c)^2 - cos(-a + c)^2 + 1)/(co
s(-a + c)^2 + 2*cos(-a + c) + 1)))/(b*sin(-a + c))
```

giac [B] time = 0.23, size = 171, normalized size = 4.75

$$\frac{\left(\tan\left(\frac{1}{2}a\right)^{2}\tan\left(\frac{1}{2}c\right)^{2}+\tan\left(\frac{1}{2}a\right)^{2}+\tan\left(\frac{1}{2}c\right)^{2}+1\right)\log\left(\left|2\tan\left(bx+a\right)\tan\left(\frac{1}{2}a\right)^{2}\tan\left(\frac{1}{2}c\right)-2\tan\left(bx+a\right)\tan\right.\right.}{2\left(\tan\left(\frac{1}{2}a\right)^{2}\tan\left(\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)*sec(b*x+c),x, algorithm="giac")
```

```
[Out] 1/2*(tan(1/2*a)^2*tan(1/2*c)^2 + tan(1/2*a)^2 + tan(1/2*c)^2 + 1)*log(abs(2
*tan(b*x + a)*tan(1/2*a)^2*tan(1/2*c) - 2*tan(b*x + a)*tan(1/2*a)*tan(1/2*c
)^2 + tan(1/2*a)^2*tan(1/2*c)^2 + 2*tan(b*x + a)*tan(1/2*a) - tan(1/2*a)^2
- 2*tan(b*x + a)*tan(1/2*c) + 4*tan(1/2*a)*tan(1/2*c) - tan(1/2*c)^2 + 1))/
((tan(1/2*a)^2*tan(1/2*c) - tan(1/2*a)*tan(1/2*c)^2 + tan(1/2*a) - tan(1/2*
c))*b)
```

maple [A] time = 0.48, size = 55, normalized size = 1.53

$$-\frac{\ln\left(-\tan\left(bx+a\right)\cos(a)\sin(c)+\tan\left(bx+a\right)\sin(a)\cos(c)+\cos(a)\cos(c)+\sin(a)\sin(c)\right)}{b\left(\cos(a)\sin(c)-\sin(a)\cos(c)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(b*x+a)*sec(b*x+c),x)
```

```
[Out] -1/b/(cos(a)*sin(c)-sin(a)*cos(c))*ln(-tan(b*x+a)*cos(a)*sin(c)+tan(b*x+a)*
sin(a)*cos(c)+cos(a)*cos(c)+sin(a)*sin(c))
```

maxima [B] time = 0.34, size = 349, normalized size = 9.69

$$-\frac{2\left(\left(\cos\left(2\,a\right)-\cos\left(2\,c\right)\right)\cos\left(a+c\right)+\left(\sin\left(2\,a\right)-\sin\left(2\,c\right)\right)\sin\left(a+c\right)\right)\arctan\left(\sin\left(2\,bx\right)-\sin\left(2\,a\right),\cos\left(2\,bx\right)\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)*sec(b*x+c),x, algorithm="maxima")
```

```
[Out] -(2*((cos(2*a) - cos(2*c))*cos(a + c) + (sin(2*a) - sin(2*c))*sin(a + c))*a
rctan2(sin(2*b*x) - sin(2*a), cos(2*b*x) + cos(2*a)) - 2*((cos(2*a) - cos(2
*c))*cos(a + c) + (sin(2*a) - sin(2*c))*sin(a + c))*arctan2(sin(2*b*x) - si
n(2*c), cos(2*b*x) + cos(2*c)) - ((sin(2*a) - sin(2*c))*cos(a + c) - (cos(2
*a) - cos(2*c))*sin(a + c))*log(cos(2*b*x)^2 + 2*cos(2*b*x)*cos(2*a) + cos(
2*a)^2 + sin(2*b*x)^2 - 2*sin(2*b*x)*sin(2*a) + sin(2*a)^2) + ((sin(2*a) -
sin(2*c))*cos(a + c) - (cos(2*a) - cos(2*c))*sin(a + c))*log(cos(2*b*x)^2 +
 2*cos(2*b*x)*cos(2*c) + cos(2*c)^2 + sin(2*b*x)^2 - 2*sin(2*b*x)*sin(2*c)
+ sin(2*c)^2))/(2*b*cos(2*a)*cos(2*c) - b*cos(2*c)^2 + 2*b*sin(2*a)*sin(2*c
) - b*sin(2*c)^2 - (cos(2*a)^2 + sin(2*a)^2)*b)
```

mupad [B] time = 7.84, size = 249, normalized size = 6.92

$$\frac{2\,\sqrt{-\mathrm{e}^{a\,2\mathrm{i}-c\,2\mathrm{i}}}\,\left(\ln\left(-\frac{2\,\sqrt{-\mathrm{e}^{a\,2\mathrm{i}}\,\mathrm{e}^{-c\,2\mathrm{i}}}\,\left(4\,b\,\mathrm{e}^{a\,2\mathrm{i}}\,\mathrm{e}^{-c\,2\mathrm{i}}+2\,b\,\mathrm{e}^{a\,2\mathrm{i}}\,\mathrm{e}^{b\,x\,2\mathrm{i}}+2\,b\,\mathrm{e}^{a\,4\mathrm{i}}\,\mathrm{e}^{-c\,2\mathrm{i}}\,\mathrm{e}^{b\,x\,2\mathrm{i}}\right)}{b\,\left(\mathrm{e}^{a\,2\mathrm{i}}\,\mathrm{e}^{-c\,2\mathrm{i}}-1\right)}+\mathrm{e}^{a\,1\mathrm{i}}\,\mathrm{e}^{a\,2\mathrm{i}}\,\mathrm{e}^{-c\,1\mathrm{i}}\,\mathrm{e}^{b\,x\,2\mathrm{i}}\,4\mathrm{i}\right)-\ln\left(-\frac{2\,\sqrt{-\mathrm{e}^{a\,2\mathrm{i}}\,\mathrm{e}}}{}\right.\right.}{b\,\left(\mathrm{e}^{a\,2\mathrm{i}-c\,2\mathrm{i}}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(a + b*x)*cos(c + b*x)),x)
```

```
[Out] (2*(-exp(a*2i - c*2i))^(1/2)*(log(exp(a*1i)*exp(a*2i)*exp(-c*1i)*exp(b*x*2i
)*4i - (2*(-exp(a*2i)*exp(-c*2i))^(1/2)*(4*b*exp(a*2i)*exp(-c*2i) + 2*b*exp
(a*2i)*exp(b*x*2i) + 2*b*exp(a*4i)*exp(-c*2i)*exp(b*x*2i)))/(b*(exp(a*2i)*e
xp(-c*2i) - 1))) - log(exp(a*1i)*exp(a*2i)*exp(-c*1i)*exp(b*x*2i)*4i - (2*(
-exp(a*2i)*exp(-c*2i))^(1/2)*(4*b*exp(a*2i)*exp(-c*2i) + 2*b*exp(a*2i)*exp(
b*x*2i) + 2*b*exp(a*4i)*exp(-c*2i)*exp(b*x*2i)))/(b - b*exp(a*2i)*exp(-c*2i
)))))/(b*(exp(a*2i - c*2i) - 1))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sec\left(a+bx\right)\sec\left(bx+c\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)*sec(b*x+c),x)
[Out] Integral(sec(a + b*x)*sec(b*x + c), x)
```

3.144 $\int \sec(c-bx)\sec(a+bx)\,dx$

Optimal. Leaf size=33

$$\frac{\csc(a+c)\log(\cos(c-bx))}{b}-\frac{\csc(a+c)\log(\cos(a+bx))}{b}$$

```
[Out] csc(a+c)*ln(cos(b*x-c))/b-csc(a+c)*ln(cos(b*x+a))/b
```

Rubi [A] time = 0.02, antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {4610, 3475}

$$\frac{\csc(a+c)\log(\cos(c-bx))}{b}-\frac{\csc(a+c)\log(\cos(a+bx))}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c - b*x]*Sec[a + b*x],x]
```

```
[Out] (Csc[a + c]*Log[Cos[c - b*x]])/b - (Csc[a + c]*Log[Cos[a + b*x]])/b
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 4610

```
Int[Sec[(a_.) + (b_.)*(x_)]*Sec[(c_) + (d_.)*(x_)], x_Symbol] :> -Dist[Csc[
(b*c - a*d)/d], Int[Tan[a + b*x], x], x] + Dist[Csc[(b*c - a*d)/b], Int[Tan
[c + d*x], x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b^2 - d^2, 0] && NeQ[b*c
 - a*d, 0]
```

Rubi steps

$$\begin{aligned}\int \sec(c-bx)\sec(a+bx)\,dx &= \csc(a+c)\int \tan(c-bx)\,dx+\csc(a+c)\int \tan(a+bx)\,dx\\ &= \frac{\csc(a+c)\log(\cos(c-bx))}{b}-\frac{\csc(a+c)\log(\cos(a+bx))}{b}\end{aligned}$$

Mathematica [A] time = 0.23, size = 26, normalized size = 0.79

$$\frac{\csc(a+c)(\log(\cos(c-bx))-\log(\cos(a+bx)))}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c - b*x]*Sec[a + b*x],x]
```

```
[Out] (Csc[a + c]*(Log[Cos[c - b*x]] - Log[Cos[a + b*x]]))/b
```

fricas [B] time = 0.65, size = 93, normalized size = 2.82

$$-\frac{\log\left(\cos\left(bx+a\right)^{2}\right)-\log\left(\frac{4\left(2\,\cos(bx+a)\cos(a+c)\sin(bx+a)\sin(a+c)+\left(2\,\cos(a+c)^{2}-1\right)\cos(bx+a)^{2}-\cos(a+c)^{2}+1\right)}{\cos(a+c)^{2}+2\,\cos(a+c)+1}\right)}{2\,b\sin\left(a+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x-c)*sec(b*x+a),x, algorithm="fricas")
```

```
[Out] -1/2*(log(cos(b*x + a)^2) - log(4*(2*cos(b*x + a)*cos(a + c)*sin(b*x + a)*s
in(a + c) + (2*cos(a + c)^2 - 1)*cos(b*x + a)^2 - cos(a + c)^2 + 1)/(cos(a
+ c)^2 + 2*cos(a + c) + 1)))/(b*sin(a + c))
```

giac [B] time = 0.22, size = 169, normalized size = 5.12

$$-\frac{\left(\tan\left(\frac{1}{2}\,a\right)^{2}\tan\left(\frac{1}{2}\,c\right)^{2}+\tan\left(\frac{1}{2}\,a\right)^{2}+\tan\left(\frac{1}{2}\,c\right)^{2}+1\right)\log\left(\left|2\,\tan\left(bx+a\right)\tan\left(\frac{1}{2}\,a\right)^{2}\tan\left(\frac{1}{2}\,c\right)+2\,\tan\left(bx+a\right)\mathrm{ta}\right.\right.}{2\left(\tan\left(\frac{1}{2}\,a\right)^{2}\mathrm{tan}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x-c)*sec(b*x+a),x, algorithm="giac")
```

```
[Out] -1/2*(tan(1/2*a)^2*tan(1/2*c)^2 + tan(1/2*a)^2 + tan(1/2*c)^2 + 1)*log(abs(
2*tan(b*x + a)*tan(1/2*a)^2*tan(1/2*c) + 2*tan(b*x + a)*tan(1/2*a)*tan(1/2*
c)^2 - tan(1/2*a)^2*tan(1/2*c)^2 - 2*tan(b*x + a)*tan(1/2*a) + tan(1/2*a)^2
 - 2*tan(b*x + a)*tan(1/2*c) + 4*tan(1/2*a)*tan(1/2*c) + tan(1/2*c)^2 - 1))
/((tan(1/2*a)^2*tan(1/2*c) + tan(1/2*a)*tan(1/2*c)^2 - tan(1/2*a) - tan(1/2
*c))*b)
```

maple [A] time = 0.46, size = 53, normalized size = 1.61

$$\frac{\ln\left(\tan\left(bx+a\right)\cos(a)\sin(c)+\tan\left(bx+a\right)\sin(a)\cos(c)+\cos(a)\cos(c)-\sin(a)\sin(c)\right)}{b\left(\sin(a)\cos(c)+\cos(a)\sin(c)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(b*x-c)*sec(b*x+a),x)
```

```
[Out] 1/b/(sin(a)*cos(c)+cos(a)*sin(c))*ln(tan(b*x+a)*cos(a)*sin(c)+tan(b*x+a)*si
n(a)*cos(c)+cos(a)*cos(c)-sin(a)*sin(c))
```

maxima [B] time = 0.36, size = 322, normalized size = 9.76

$$\frac{2\left(\cos\left(2\,a+2\,c\right)\cos\left(a+c\right)+\sin\left(2\,a+2\,c\right)\sin\left(a+c\right)-\cos\left(a+c\right)\right)\arctan\left(\sin\left(2\,bx\right)-\sin\left(2\,a\right),\cos\left(2\,bx\right)+\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x-c)*sec(b*x+a),x, algorithm="maxima")
```

```
[Out] (2*(cos(2*a + 2*c)*cos(a + c) + sin(2*a + 2*c)*sin(a + c) - cos(a + c))*arc
tan2(sin(2*b*x) - sin(2*a), cos(2*b*x) + cos(2*a)) - 2*(cos(2*a + 2*c)*cos(
a + c) + sin(2*a + 2*c)*sin(a + c) - cos(a + c))*arctan2(sin(2*b*x) + sin(2
*c), cos(2*b*x) + cos(2*c)) - (cos(a + c)*sin(2*a + 2*c) - cos(2*a + 2*c)*s
in(a + c) + sin(a + c))*log(cos(2*b*x)^2 + 2*cos(2*b*x)*cos(2*a) + cos(2*a)
^2 + sin(2*b*x)^2 - 2*sin(2*b*x)*sin(2*a) + sin(2*a)^2) + (cos(a + c)*sin(2
*a + 2*c) - cos(2*a + 2*c)*sin(a + c) + sin(a + c))*log(cos(2*b*x)^2 + 2*co
s(2*b*x)*cos(2*c) + cos(2*c)^2 + sin(2*b*x)^2 + 2*sin(2*b*x)*sin(2*c) + sin
(2*c)^2))/(b*cos(2*a + 2*c)^2 + b*sin(2*a + 2*c)^2 - 2*b*cos(2*a + 2*c) + b
)
```

mupad [B] time = 7.73, size = 249, normalized size = 7.55

$$\frac{2\sqrt{-e^{a\,2i+c\,2i}}\left(\ln\left(-\frac{2\sqrt{-e^{a\,2i}\,e^{c\,2i}}\left(4\,b\,e^{a\,2i}\,e^{c\,2i}+2\,b\,e^{a\,2i}\,e^{b\,x\,2i}+2\,b\,e^{a\,4i}\,e^{c\,2i}\,e^{b\,x\,2i}\right)}{b\left(e^{a\,2i}\,e^{c\,2i}-1\right)}+e^{a\,1i}\,e^{a\,2i}\,e^{c\,1i}\,e^{b\,x\,2i}\,4i\right)-\ln\left(-\frac{2\sqrt{-e^{a\,2i}\,e^{c\,2i}}\left(\right.}{}\right.\right.}{b\left(e^{a\,2i+c\,2i}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(a + b*x)*cos(c - b*x)),x)
```

```
[Out] (2*(-exp(a*2i + c*2i))^(1/2)*(log(exp(a*1i)*exp(a*2i)*exp(c*1i)*exp(b*x*2i)
*4i - (2*(-exp(a*2i)*exp(c*2i))^(1/2)*(4*b*exp(a*2i)*exp(c*2i) + 2*b*exp(a*
2i)*exp(b*x*2i) + 2*b*exp(a*4i)*exp(c*2i)*exp(b*x*2i)))/(b*(exp(a*2i)*exp(c
*2i) - 1))) - log(exp(a*1i)*exp(a*2i)*exp(c*1i)*exp(b*x*2i)*4i - (2*(-exp(a
*2i)*exp(c*2i))^(1/2)*(4*b*exp(a*2i)*exp(c*2i) + 2*b*exp(a*2i)*exp(b*x*2i)
+ 2*b*exp(a*4i)*exp(c*2i)*exp(b*x*2i)))/(b - b*exp(a*2i)*exp(c*2i)))))/(b*(
exp(a*2i + c*2i) - 1))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\sec\left(a+bx\right)\sec\left(bx-c\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x-c)*sec(b*x+a),x)
[Out] Integral(sec(a + b*x)*sec(b*x - c), x)
```

3.145 $\int \csc(a+bx)\csc(c+bx)\,dx$

Optimal. Leaf size=36

$$\frac{\csc(a-c)\log(\sin(bx+c))}{b}-\frac{\csc(a-c)\log(\sin(a+bx))}{b}$$

```
[Out] -csc(a-c)*ln(sin(b*x+a))/b+csc(a-c)*ln(sin(b*x+c))/b
```

Rubi [A] time = 0.02, antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {4611, 3475}

$$\frac{\csc(a-c)\log(\sin(bx+c))}{b}-\frac{\csc(a-c)\log(\sin(a+bx))}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[a + b*x]*Csc[c + b*x],x]
```

```
[Out] -((Csc[a - c]*Log[Sin[a + b*x]])/b) + (Csc[a - c]*Log[Sin[c + b*x]])/b
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 4611

```
Int[Csc[(a_.) + (b_.)*(x_)]*Csc[(c_) + (d_.)*(x_)], x_Symbol] :> Dist[Csc[(
b*c - a*d)/b], Int[Cot[a + b*x], x], x] - Dist[Csc[(b*c - a*d)/d], Int[Cot[
c + d*x], x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b^2 - d^2, 0] && NeQ[b*c
- a*d, 0]
```

Rubi steps

$$\begin{aligned}\int \csc(a+bx)\csc(c+bx)\,dx &= -\left(\csc(a-c)\int \cot(a+bx)\,dx\right)+\csc(a-c)\int \cot(c+bx)\,dx\\ &= -\frac{\csc(a-c)\log(\sin(a+bx))}{b}+\frac{\csc(a-c)\log(\sin(c+bx))}{b}\end{aligned}$$

Mathematica [A] time = 0.24, size = 28, normalized size = 0.78

$$-\frac{\csc(a-c)(\log(\sin(a+bx))-\log(\sin(bx+c)))}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[a + b*x]*Csc[c + b*x],x]
```

```
[Out] -((Csc[a - c]*(Log[Sin[a + b*x]] - Log[Sin[c + b*x]]))/b)
```

fricas [B] time = 0.75, size = 110, normalized size = 3.06

$$-\frac{\log\left(-\frac{1}{4}\cos\left(bx+c\right)^{2}+\frac{1}{4}\right)-\log\left(-\frac{2\cos(bx+c)\cos(-a+c)\sin(bx+c)\sin(-a+c)+\left(2\cos(-a+c)^{2}-1\right)\cos(bx+c)^{2}-\cos(-a+c)^{2}}{\cos(-a+c)^{2}+2\cos(-a+c)+1}\right)}{2\,b\sin\left(-a+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)*csc(b*x+c),x, algorithm="fricas")
```

```
[Out] -1/2*(log(-1/4*cos(b*x + c)^2 + 1/4) - log(-(2*cos(b*x + c)*cos(-a + c)*sin
(b*x + c)*sin(-a + c) + (2*cos(-a + c)^2 - 1)*cos(b*x + c)^2 - cos(-a + c)^
2)/(cos(-a + c)^2 + 2*cos(-a + c) + 1)))/(b*sin(-a + c))
```

giac [B] time = 0.24, size = 396, normalized size = 11.00

$$\frac{\left(\tan\left(\frac{1}{2}a\right)^{4}\tan\left(\frac{1}{2}c\right)^{4}+4\tan\left(\frac{1}{2}a\right)^{3}\tan\left(\frac{1}{2}c\right)^{3}-\tan\left(\frac{1}{2}a\right)^{4}+4\tan\left(\frac{1}{2}a\right)^{3}\tan\left(\frac{1}{2}c\right)+4\tan\left(\frac{1}{2}a\right)\tan\left(\frac{1}{2}c\right)^{3}-\tan\left(\frac{1}{2}c\right)^{4}+4\tan\left(\frac{1}{2}a\right)\tan\left(\frac{1}{2}c\right)+1\right)\log\left(\left|\tan(bx\right.\right.}{\tan\left(\frac{1}{2}a\right)^{4}\tan\left(\frac{1}{2}c\right)^{3}-\tan\left(\frac{1}{2}a\right)^{3}\tan\left(\frac{1}{2}c\right)^{4}-\tan\left(\frac{1}{2}a\right)^{4}\tan\left(\frac{1}{2}c\right)+6\tan\left(\frac{1}{2}a\right)^{3}\tan\left(\frac{1}{2}c\right)^{2}-6\tan\left(\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)*csc(b*x+c),x, algorithm="giac")
```

```
[Out] 1/2*((tan(1/2*a)^4*tan(1/2*c)^4 + 4*tan(1/2*a)^3*tan(1/2*c)^3 - tan(1/2*a)^
4 + 4*tan(1/2*a)^3*tan(1/2*c) + 4*tan(1/2*a)*tan(1/2*c)^3 - tan(1/2*c)^4 +
4*tan(1/2*a)*tan(1/2*c) + 1)*log(abs(tan(b*x + a)*tan(1/2*a)^2*tan(1/2*c)^2
 - tan(b*x + a)*tan(1/2*a)^2 + 4*tan(b*x + a)*tan(1/2*a)*tan(1/2*c) - 2*tan
(1/2*a)^2*tan(1/2*c) - tan(b*x + a)*tan(1/2*c)^2 + 2*tan(1/2*a)*tan(1/2*c)^
2 + tan(b*x + a) - 2*tan(1/2*a) + 2*tan(1/2*c)))/(tan(1/2*a)^4*tan(1/2*c)^3
 - tan(1/2*a)^3*tan(1/2*c)^4 - tan(1/2*a)^4*tan(1/2*c) + 6*tan(1/2*a)^3*tan
(1/2*c)^2 - 6*tan(1/2*a)^2*tan(1/2*c)^3 + tan(1/2*a)*tan(1/2*c)^4 - tan(1/2
*a)^3 + 6*tan(1/2*a)^2*tan(1/2*c) - 6*tan(1/2*a)*tan(1/2*c)^2 + tan(1/2*c)^
3 + tan(1/2*a) - tan(1/2*c)) - (tan(1/2*a)^2*tan(1/2*c)^2 + tan(1/2*a)^2 +
tan(1/2*c)^2 + 1)*log(abs(tan(b*x + a)))/(tan(1/2*a)^2*tan(1/2*c) - tan(1/2
*a)*tan(1/2*c)^2 + tan(1/2*a) - tan(1/2*c)))/b
```

maple [B] time = 0.49, size = 169, normalized size = 4.69

$$-\frac{\ln\left(\tan\left(bx+a\right)\cos(a)\cos(c)+\tan\left(bx+a\right)\sin(a)\sin(c)+\cos(a)\sin(c)-\sin(a)\cos(c)\right)\cos(a)\cos(c)}{b\left(\cos(a)\sin(c)-\sin(a)\cos(c)\right)\left(\cos(a)\cos(c)+\sin(a)\sin(c)\right)}-\frac{\ln\left(\tan\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(b*x+a)*csc(b*x+c),x)
```

```
[Out] -1/b/(cos(a)*sin(c)-sin(a)*cos(c))/(cos(a)*cos(c)+sin(a)*sin(c))*ln(tan(b*x
+a)*cos(a)*cos(c)+tan(b*x+a)*sin(a)*sin(c)+cos(a)*sin(c)-sin(a)*cos(c))*cos
(a)*cos(c)-1/b/(cos(a)*sin(c)-sin(a)*cos(c))/(cos(a)*cos(c)+sin(a)*sin(c))*
ln(tan(b*x+a)*cos(a)*cos(c)+tan(b*x+a)*sin(a)*sin(c)+cos(a)*sin(c)-sin(a)*c
os(c))*sin(a)*sin(c)+1/b/(cos(a)*sin(c)-sin(a)*cos(c))*ln(tan(b*x+a))
```

maxima [B] time = 0.35, size = 564, normalized size = 15.67

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)*csc(b*x+c),x, algorithm="maxima")
```

```
[Out] -(2*((cos(2*a) - cos(2*c))*cos(a + c) + (sin(2*a) - sin(2*c))*sin(a + c))*a
rctan2(sin(b*x) + sin(a), cos(b*x) - cos(a)) + 2*((cos(2*a) - cos(2*c))*cos
(a + c) + (sin(2*a) - sin(2*c))*sin(a + c))*arctan2(sin(b*x) - sin(a), cos(
b*x) + cos(a)) - 2*((cos(2*a) - cos(2*c))*cos(a + c) + (sin(2*a) - sin(2*c)
)*sin(a + c))*arctan2(sin(b*x) + sin(c), cos(b*x) - cos(c)) - 2*((cos(2*a)
- cos(2*c))*cos(a + c) + (sin(2*a) - sin(2*c))*sin(a + c))*arctan2(sin(b*x)
 - sin(c), cos(b*x) + cos(c)) - ((sin(2*a) - sin(2*c))*cos(a + c) - (cos(2*
a) - cos(2*c))*sin(a + c))*log(cos(b*x)^2 + 2*cos(b*x)*cos(a) + cos(a)^2 +
sin(b*x)^2 - 2*sin(b*x)*sin(a) + sin(a)^2) - ((sin(2*a) - sin(2*c))*cos(a +
 c) - (cos(2*a) - cos(2*c))*sin(a + c))*log(cos(b*x)^2 - 2*cos(b*x)*cos(a)
+ cos(a)^2 + sin(b*x)^2 + 2*sin(b*x)*sin(a) + sin(a)^2) + ((sin(2*a) - sin(
2*c))*cos(a + c) - (cos(2*a) - cos(2*c))*sin(a + c))*log(cos(b*x)^2 + 2*cos
(b*x)*cos(c) + cos(c)^2 + sin(b*x)^2 - 2*sin(b*x)*sin(c) + sin(c)^2) + ((si
n(2*a) - sin(2*c))*cos(a + c) - (cos(2*a) - cos(2*c))*sin(a + c))*log(cos(b
*x)^2 - 2*cos(b*x)*cos(c) + cos(c)^2 + sin(b*x)^2 + 2*sin(b*x)*sin(c) + sin
(c)^2))/(2*b*cos(2*a)*cos(2*c) - b*cos(2*c)^2 + 2*b*sin(2*a)*sin(2*c) - b*s
in(2*c)^2 - (cos(2*a)^2 + sin(2*a)^2)*b)
```

mupad [B] time = 7.77, size = 249, normalized size = 6.92

$$\frac{2\sqrt{-\mathrm{e}^{a\,2\mathrm{i}-c\,2\mathrm{i}}}\,\left(\ln\left(\frac{2\sqrt{-\mathrm{e}^{a\,2\mathrm{i}}\,\mathrm{e}^{-c\,2\mathrm{i}}}\,\left(-4\,b\,\mathrm{e}^{a\,2\mathrm{i}}\,\mathrm{e}^{-c\,2\mathrm{i}}+2\,b\,\mathrm{e}^{a\,2\mathrm{i}}\,\mathrm{e}^{b\,x\,2\mathrm{i}}+2\,b\,\mathrm{e}^{a\,4\mathrm{i}}\,\mathrm{e}^{-c\,2\mathrm{i}}\,\mathrm{e}^{b\,x\,2\mathrm{i}}\right)}{b\left(\mathrm{e}^{a\,2\mathrm{i}}\,\mathrm{e}^{-c\,2\mathrm{i}}-1\right)}-\mathrm{e}^{a\,1\mathrm{i}}\,\mathrm{e}^{a\,2\mathrm{i}}\,\mathrm{e}^{-c\,1\mathrm{i}}\,\mathrm{e}^{b\,x\,2\mathrm{i}}\,4\mathrm{i}\right)-\ln\left(\frac{2\sqrt{-\mathrm{e}^{a\,2\mathrm{i}}\,\mathrm{e}^{-c\,2\mathrm{i}}}\cdots}{\cdots}\right.\cdots}{b\,\left(\mathrm{e}^{a\,2\mathrm{i}-c\,2\mathrm{i}}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(a + b*x)*sin(c + b*x)),x)
```

```
[Out] (2*(-exp(a*2i - c*2i))^(1/2)*(log((2*(-exp(a*2i)*exp(-c*2i))^(1/2)*(2*b*exp
(a*2i)*exp(b*x*2i) - 4*b*exp(a*2i)*exp(-c*2i) + 2*b*exp(a*4i)*exp(-c*2i)*ex
p(b*x*2i)))/(b*(exp(a*2i)*exp(-c*2i) - 1)) - exp(a*1i)*exp(a*2i)*exp(-c*1i)
*exp(b*x*2i)*4i) - log((2*(-exp(a*2i)*exp(-c*2i))^(1/2)*(2*b*exp(a*2i)*exp(
b*x*2i) - 4*b*exp(a*2i)*exp(-c*2i) + 2*b*exp(a*4i)*exp(-c*2i)*exp(b*x*2i)))
/(b - b*exp(a*2i)*exp(-c*2i)) - exp(a*1i)*exp(a*2i)*exp(-c*1i)*exp(b*x*2i)*
4i)))/(b*(exp(a*2i - c*2i) - 1))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \csc(a+bx)\csc(bx+c)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)*csc(b*x+c),x)
```

```
[Out] Integral(csc(a + b*x)*csc(b*x + c), x)
```

3.146 $\int \csc(c-bx)\csc(a+bx)\,dx$

Optimal. Leaf size=33

$$\frac{\csc(a+c)\log(\sin(a+bx))}{b}-\frac{\csc(a+c)\log(\sin(c-bx))}{b}$$

```
[Out] -csc(a+c)*ln(-sin(b*x-c))/b+csc(a+c)*ln(sin(b*x+a))/b
```

Rubi [A] time = 0.02, antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {4611, 3475}

$$\frac{\csc(a+c)\log(\sin(a+bx))}{b}-\frac{\csc(a+c)\log(\sin(c-bx))}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c - b*x]*Csc[a + b*x],x]
```

```
[Out] -((Csc[a + c]*Log[Sin[c - b*x]])/b) + (Csc[a + c]*Log[Sin[a + b*x]])/b
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 4611

```
Int[Csc[(a_.) + (b_.)*(x_)]*Csc[(c_) + (d_.)*(x_)], x_Symbol] :> Dist[Csc[(
b*c - a*d)/b], Int[Cot[a + b*x], x], x] - Dist[Csc[(b*c - a*d)/d], Int[Cot[
c + d*x], x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b^2 - d^2, 0] && NeQ[b*c
- a*d, 0]
```

Rubi steps

$$\begin{aligned}\int \csc(c-bx)\csc(a+bx)\,dx &= \csc(a+c)\int \cot(c-bx)\,dx+\csc(a+c)\int \cot(a+bx)\,dx\\ &= -\frac{\csc(a+c)\log(\sin(c-bx))}{b}+\frac{\csc(a+c)\log(\sin(a+bx))}{b}\end{aligned}$$

Mathematica [A] time = 0.22, size = 29, normalized size = 0.88

$$-\frac{\csc(a+c)(\log(\sin(c-bx))-\log(-\sin(a+bx)))}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c - b*x]*Csc[a + b*x],x]
```

```
[Out] -((Csc[a + c]*(Log[Sin[c - b*x]] - Log[-Sin[a + b*x]]))/b)
```

fricas [B] time = 0.78, size = 96, normalized size = 2.91

$$\frac{\log\left(-\frac{1}{4}\cos\left(bx+a\right)^{2}+\frac{1}{4}\right)-\log\left(-\frac{2\cos(bx+a)\cos(a+c)\sin(bx+a)\sin(a+c)+\left(2\cos(a+c)^{2}-1\right)\cos(bx+a)^{2}-\cos(a+c)^{2}}{\cos(a+c)^{2}+2\cos(a+c)+1}\right)}{2\,b\sin\left(a+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-csc(b*x-c)*csc(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/2*(log(-1/4*cos(b*x + a)^2 + 1/4) - log(-(2*cos(b*x + a)*cos(a + c)*sin(b
*x + a)*sin(a + c) + (2*cos(a + c)^2 - 1)*cos(b*x + a)^2 - cos(a + c)^2)/(c
os(a + c)^2 + 2*cos(a + c) + 1)))/(b*sin(a + c))
```

giac [B] time = 0.23, size = 397, normalized size = 12.03

$$\frac{\left(\tan\left(\frac{1}{2}a\right)^{4}\tan\left(\frac{1}{2}c\right)^{4}-4\tan\left(\frac{1}{2}a\right)^{3}\tan\left(\frac{1}{2}c\right)^{3}-\tan\left(\frac{1}{2}a\right)^{4}-4\tan\left(\frac{1}{2}a\right)^{3}\tan\left(\frac{1}{2}c\right)-4\tan\left(\frac{1}{2}a\right)\tan\left(\frac{1}{2}c\right)^{3}-\tan\left(\frac{1}{2}c\right)^{4}-4\tan\left(\frac{1}{2}a\right)\tan\left(\frac{1}{2}c\right)+1\right)\log\left(\left|\tan(bx+\right.\right.}{\tan\left(\frac{1}{2}a\right)^{4}\tan\left(\frac{1}{2}c\right)^{3}+\tan\left(\frac{1}{2}a\right)^{3}\tan\left(\frac{1}{2}c\right)^{4}-\tan\left(\frac{1}{2}a\right)^{4}\tan\left(\frac{1}{2}c\right)-6\tan\left(\frac{1}{2}a\right)^{3}\tan\left(\frac{1}{2}c\right)^{2}-6\tan(}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-csc(b*x-c)*csc(b*x+a),x, algorithm="giac")
```

```
[Out] 1/2*((tan(1/2*a)^4*tan(1/2*c)^4 - 4*tan(1/2*a)^3*tan(1/2*c)^3 - tan(1/2*a)^
4 - 4*tan(1/2*a)^3*tan(1/2*c) - 4*tan(1/2*a)*tan(1/2*c)^3 - tan(1/2*c)^4 -
4*tan(1/2*a)*tan(1/2*c) + 1)*log(abs(tan(b*x + a)*tan(1/2*a)^2*tan(1/2*c)^2
 - tan(b*x + a)*tan(1/2*a)^2 - 4*tan(b*x + a)*tan(1/2*a)*tan(1/2*c) + 2*tan
(1/2*a)^2*tan(1/2*c) - tan(b*x + a)*tan(1/2*c)^2 + 2*tan(1/2*a)*tan(1/2*c)^
2 + tan(b*x + a) - 2*tan(1/2*a) - 2*tan(1/2*c)))/(tan(1/2*a)^4*tan(1/2*c)^3
 + tan(1/2*a)^3*tan(1/2*c)^4 - tan(1/2*a)^4*tan(1/2*c) - 6*tan(1/2*a)^3*tan
(1/2*c)^2 - 6*tan(1/2*a)^2*tan(1/2*c)^3 - tan(1/2*a)*tan(1/2*c)^4 + tan(1/2
*a)^3 + 6*tan(1/2*a)^2*tan(1/2*c) + 6*tan(1/2*a)*tan(1/2*c)^2 + tan(1/2*c)^
3 - tan(1/2*a) - tan(1/2*c)) - (tan(1/2*a)^2*tan(1/2*c)^2 + tan(1/2*a)^2 +
tan(1/2*c)^2 + 1)*log(abs(tan(b*x + a)))/(tan(1/2*a)^2*tan(1/2*c) + tan(1/2
*a)*tan(1/2*c)^2 - tan(1/2*a) - tan(1/2*c)))/b
```

maple [B] time = 0.48, size = 81, normalized size = 2.45

$$-\frac{\ln\left(\tan\left(bx+a\right)\cos(a)\cos(c)-\tan\left(bx+a\right)\sin(a)\sin(c)-\cos(a)\sin(c)-\sin(a)\cos(c)\right)}{b\left(\sin(a)\cos(c)+\cos(a)\sin(c)\right)}+\frac{\ln\left(\tan\left(bx+\right.\right.}{b\left(\sin(a)\cos(c)+\cos\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-csc(b*x-c)*csc(b*x+a),x)
```

```
[Out] -1/b/(sin(a)*cos(c)+cos(a)*sin(c))*ln(tan(b*x+a)*cos(a)*cos(c)-tan(b*x+a)*s
in(a)*sin(c)-cos(a)*sin(c)-sin(a)*cos(c))+1/b/(sin(a)*cos(c)+cos(a)*sin(c))
*ln(tan(b*x+a))
```

maxima [B] time = 0.50, size = 536, normalized size = 16.24

$$-\frac{2\left(\cos\left(2\,a+2\,c\right)\cos\left(a+c\right)+\sin\left(2\,a+2\,c\right)\sin\left(a+c\right)-\cos\left(a+c\right)\right)\arctan\left(\sin\left(bx\right)+\sin(a),\cos\left(bx\right)-\cos(\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-csc(b*x-c)*csc(b*x+a),x, algorithm="maxima")
```

```
[Out] -(2*(cos(2*a + 2*c)*cos(a + c) + sin(2*a + 2*c)*sin(a + c) - cos(a + c))*ar
ctan2(sin(b*x) + sin(a), cos(b*x) - cos(a)) + 2*(cos(2*a + 2*c)*cos(a + c)
+ sin(2*a + 2*c)*sin(a + c) - cos(a + c))*arctan2(sin(b*x) - sin(a), cos(b*
x) + cos(a)) - 2*(cos(2*a + 2*c)*cos(a + c) + sin(2*a + 2*c)*sin(a + c) - c
os(a + c))*arctan2(sin(b*x) + sin(c), cos(b*x) + cos(c)) - 2*(cos(2*a + 2*c
)*cos(a + c) + sin(2*a + 2*c)*sin(a + c) - cos(a + c))*arctan2(sin(b*x) - s
in(c), cos(b*x) - cos(c)) - (cos(a + c)*sin(2*a + 2*c) - cos(2*a + 2*c)*sin
(a + c) + sin(a + c))*log(cos(b*x)^2 + 2*cos(b*x)*cos(a) + cos(a)^2 + sin(b
*x)^2 - 2*sin(b*x)*sin(a) + sin(a)^2) - (cos(a + c)*sin(2*a + 2*c) - cos(2*
a + 2*c)*sin(a + c) + sin(a + c))*log(cos(b*x)^2 - 2*cos(b*x)*cos(a) + cos(
a)^2 + sin(b*x)^2 + 2*sin(b*x)*sin(a) + sin(a)^2) + (cos(a + c)*sin(2*a + 2
*c) - cos(2*a + 2*c)*sin(a + c) + sin(a + c))*log(cos(b*x)^2 + 2*cos(b*x)*c
os(c) + cos(c)^2 + sin(b*x)^2 + 2*sin(b*x)*sin(c) + sin(c)^2) + (cos(a + c)
*sin(2*a + 2*c) - cos(2*a + 2*c)*sin(a + c) + sin(a + c))*log(cos(b*x)^2 -
2*cos(b*x)*cos(c) + cos(c)^2 + sin(b*x)^2 - 2*sin(b*x)*sin(c) + sin(c)^2))/
(b*cos(2*a + 2*c)^2 + b*sin(2*a + 2*c)^2 - 2*b*cos(2*a + 2*c) + b)
```

mupad [B] time = 7.87, size = 249, normalized size = 7.55

$$\frac{2\,\sqrt{-{\mathrm{e}}^{a\,2{}\mathrm{i}+c\,2{}\mathrm{i}}}\,\left(\ln\left(\frac{2\,\sqrt{-{\mathrm{e}}^{a\,2{}\mathrm{i}}\,{\mathrm{e}}^{c\,2{}\mathrm{i}}}\,\left(-4\,b\,{\mathrm{e}}^{a\,2{}\mathrm{i}}\,{\mathrm{e}}^{c\,2{}\mathrm{i}}+2\,b\,{\mathrm{e}}^{a\,2{}\mathrm{i}}\,{\mathrm{e}}^{b\,x\,2{}\mathrm{i}}+2\,b\,{\mathrm{e}}^{a\,4{}\mathrm{i}}\,{\mathrm{e}}^{c\,2{}\mathrm{i}}\,{\mathrm{e}}^{b\,x\,2{}\mathrm{i}}\right)}{b\,\left({\mathrm{e}}^{a\,2{}\mathrm{i}}\,{\mathrm{e}}^{c\,2{}\mathrm{i}}-1\right)}+{\mathrm{e}}^{a\,1{}\mathrm{i}}\,{\mathrm{e}}^{a\,2{}\mathrm{i}}\,{\mathrm{e}}^{c\,1{}\mathrm{i}}\,{\mathrm{e}}^{b\,x\,2{}\mathrm{i}}\,4{}\mathrm{i}\right)-\ln\left(\frac{2\,\sqrt{-{\mathrm{e}}^{a\,2{}\mathrm{i}}\,{\mathrm{e}}^{c\,2{}\mathrm{i}}}\,\left(-4\right.}{}\right.\right.}{b\,\left({\mathrm{e}}^{a\,2{}\mathrm{i}+c\,2{}\mathrm{i}}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(a + b*x)*sin(c - b*x)),x)
```

```
[Out] (2*(-exp(a*2i + c*2i))^(1/2)*(log((2*(-exp(a*2i)*exp(c*2i))^(1/2)*(2*b*exp(
a*2i)*exp(b*x*2i) - 4*b*exp(a*2i)*exp(c*2i) + 2*b*exp(a*4i)*exp(c*2i)*exp(b
```

```
*x*2i)))/(b*(exp(a*2i)*exp(c*2i) - 1)) + exp(a*1i)*exp(a*2i)*exp(c*1i)*exp(
b*x*2i)*4i) - log((2*(-exp(a*2i)*exp(c*2i))^(1/2)*(2*b*exp(a*2i)*exp(b*x*2i
) - 4*b*exp(a*2i)*exp(c*2i) + 2*b*exp(a*4i)*exp(c*2i)*exp(b*x*2i)))/(b - b*
exp(a*2i)*exp(c*2i)) + exp(a*1i)*exp(a*2i)*exp(c*1i)*exp(b*x*2i)*4i)))/(b*(
exp(a*2i + c*2i) - 1))
```

sympy [A] time = 129.08, size = 1838, normalized size = 55.70

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-csc(b*x-c)*csc(b*x+a),x)
```

```
[Out] Piecewise((-tan(c/2)**4*tan(b*x/2)/(-2*b*tan(c/2)**3*tan(b*x/2) + 2*b*tan(c
/2)**2*tan(b*x/2)**2 - 2*b*tan(c/2)**2 + 2*b*tan(c/2)*tan(b*x/2)) - 2*tan(c
/2)**2*tan(b*x/2)/(-2*b*tan(c/2)**3*tan(b*x/2) + 2*b*tan(c/2)**2*tan(b*x/2)
**2 - 2*b*tan(c/2)**2 + 2*b*tan(c/2)*tan(b*x/2)) - tan(b*x/2)/(-2*b*tan(c/2
)**3*tan(b*x/2) + 2*b*tan(c/2)**2*tan(b*x/2)**2 - 2*b*tan(c/2)**2 + 2*b*tan
(c/2)*tan(b*x/2)), Eq(a, 2*atan(1/tan(c/2)))), (tan(c/2)**4*tan(b*x/2)/(-2*
b*tan(c/2)**3*tan(b*x/2) + 2*b*tan(c/2)**2*tan(b*x/2)**2 - 2*b*tan(c/2)**2
+ 2*b*tan(c/2)*tan(b*x/2)) + 2*tan(c/2)**2*tan(b*x/2)/(-2*b*tan(c/2)**3*tan
(b*x/2) + 2*b*tan(c/2)**2*tan(b*x/2)**2 - 2*b*tan(c/2)**2 + 2*b*tan(c/2)*ta
n(b*x/2)) + tan(b*x/2)/(-2*b*tan(c/2)**3*tan(b*x/2) + 2*b*tan(c/2)**2*tan(b
*x/2)**2 - 2*b*tan(c/2)**2 + 2*b*tan(c/2)*tan(b*x/2)), Eq(a, -2*atan(tan(c/
2)) - 2*pi*floor((c/2 - pi/2)/pi) - 2*pi*floor(c/(2*pi) - 1/2))), (x/(sin(a
)*sin(c)), Eq(b, 0)), (log(tan(a/2) + tan(b*x/2))*tan(a/2)/(2*b) + log(tan(
a/2) + tan(b*x/2))/(2*b*tan(a/2)) + log(tan(b*x/2) - 1/tan(a/2))*tan(a/2)/(
2*b) + log(tan(b*x/2) - 1/tan(a/2))/(2*b*tan(a/2)) - log(tan(b*x/2))*tan(a/
2)/(2*b) - log(tan(b*x/2))/(2*b*tan(a/2)), Eq(c, 0)), (-log(-tan(c/2) + tan
(b*x/2))*tan(c/2)/(2*b) - log(-tan(c/2) + tan(b*x/2))/(2*b*tan(c/2)) - log(
tan(b*x/2) + 1/tan(c/2))*tan(c/2)/(2*b) - log(tan(b*x/2) + 1/tan(c/2))/(2*b
*tan(c/2)) + log(tan(b*x/2))*tan(c/2)/(2*b) + log(tan(b*x/2))/(2*b*tan(c/2)
), Eq(a, 0)), (-log(tan(a/2) + tan(b*x/2))*tan(a/2)**2*tan(c/2)**2/(2*b*tan
(a/2)**2*tan(c/2) + 2*b*tan(a/2)*tan(c/2)**2 - 2*b*tan(a/2) - 2*b*tan(c/2))
 - log(tan(a/2) + tan(b*x/2))*tan(a/2)**2/(2*b*tan(a/2)**2*tan(c/2) + 2*b*t
an(a/2)*tan(c/2)**2 - 2*b*tan(a/2) - 2*b*tan(c/2)) - log(tan(a/2) + tan(b*x
/2))*tan(c/2)**2/(2*b*tan(a/2)**2*tan(c/2) + 2*b*tan(a/2)*tan(c/2)**2 - 2*b
*tan(a/2) - 2*b*tan(c/2)) - log(tan(a/2) + tan(b*x/2))/(2*b*tan(a/2)**2*tan
(c/2) + 2*b*tan(a/2)*tan(c/2)**2 - 2*b*tan(a/2) - 2*b*tan(c/2)) + log(-tan(
c/2) + tan(b*x/2))*tan(a/2)**2*tan(c/2)**2/(2*b*tan(a/2)**2*tan(c/2) + 2*b*
tan(a/2)*tan(c/2)**2 - 2*b*tan(a/2) - 2*b*tan(c/2)) + log(-tan(c/2) + tan(b
*x/2))*tan(a/2)**2/(2*b*tan(a/2)**2*tan(c/2) + 2*b*tan(a/2)*tan(c/2)**2 - 2
*b*tan(a/2) - 2*b*tan(c/2)) + log(-tan(c/2) + tan(b*x/2))*tan(c/2)**2/(2*b*
tan(a/2)**2*tan(c/2) + 2*b*tan(a/2)*tan(c/2)**2 - 2*b*tan(a/2) - 2*b*tan(c/
2)) + log(-tan(c/2) + tan(b*x/2))/(2*b*tan(a/2)**2*tan(c/2) + 2*b*tan(a/2)*
```

```
tan(c/2)**2 - 2*b*tan(a/2) - 2*b*tan(c/2)) - log(tan(b*x/2) - 1/tan(a/2))*t
an(a/2)**2*tan(c/2)**2/(2*b*tan(a/2)**2*tan(c/2) + 2*b*tan(a/2)*tan(c/2)**2
 - 2*b*tan(a/2) - 2*b*tan(c/2)) - log(tan(b*x/2) - 1/tan(a/2))*tan(a/2)**2/
(2*b*tan(a/2)**2*tan(c/2) + 2*b*tan(a/2)*tan(c/2)**2 - 2*b*tan(a/2) - 2*b*t
an(c/2)) - log(tan(b*x/2) - 1/tan(a/2))*tan(c/2)**2/(2*b*tan(a/2)**2*tan(c/
2) + 2*b*tan(a/2)*tan(c/2)**2 - 2*b*tan(a/2) - 2*b*tan(c/2)) - log(tan(b*x/
2) - 1/tan(a/2))/(2*b*tan(a/2)**2*tan(c/2) + 2*b*tan(a/2)*tan(c/2)**2 - 2*b
*tan(a/2) - 2*b*tan(c/2)) + log(tan(b*x/2) + 1/tan(c/2))*tan(a/2)**2*tan(c/
2)**2/(2*b*tan(a/2)**2*tan(c/2) + 2*b*tan(a/2)*tan(c/2)**2 - 2*b*tan(a/2) -
 2*b*tan(c/2)) + log(tan(b*x/2) + 1/tan(c/2))*tan(a/2)**2/(2*b*tan(a/2)**2*
tan(c/2) + 2*b*tan(a/2)*tan(c/2)**2 - 2*b*tan(a/2) - 2*b*tan(c/2)) + log(ta
n(b*x/2) + 1/tan(c/2))*tan(c/2)**2/(2*b*tan(a/2)**2*tan(c/2) + 2*b*tan(a/2)
*tan(c/2)**2 - 2*b*tan(a/2) - 2*b*tan(c/2)) + log(tan(b*x/2) + 1/tan(c/2))/
(2*b*tan(a/2)**2*tan(c/2) + 2*b*tan(a/2)*tan(c/2)**2 - 2*b*tan(a/2) - 2*b*t
an(c/2)), True))
```

3.147 $\int \sqrt{\sin(x)\tan(x)}\,dx$

Optimal. Leaf size=13

$$-2\cot(x)\sqrt{\sin(x)\tan(x)}$$

```
[Out] -2*cot(x)*(sin(x)*tan(x))^(1/2)
```

Rubi [A] time = 0.03, antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {4400, 2589}

$$-2\cot(x)\sqrt{\sin(x)\tan(x)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Sin[x]*Tan[x]],x]
```

```
[Out] -2*Cot[x]*Sqrt[Sin[x]*Tan[x]]
```

Rule 2589

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> -Simp[(b*(a*Sin[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*m
), x] /; FreeQ[{a, b, e, f, m, n}, x] && EqQ[m + n - 1, 0]
```

Rule 4400

```
Int[(u_.)*((v_)^(m_.)*(w_)^(n_.))^(p_), x_Symbol] :> With[{uu = ActivateTri
g[u], vv = ActivateTrig[v], ww = ActivateTrig[w]}, Dist[(vv^m*ww^n)^FracPar
t[p]/(vv^(m*FracPart[p])*ww^(n*FracPart[p])), Int[uu*vv^(m*p)*ww^(n*p), x],
 x]] /; FreeQ[{m, n, p}, x] && !IntegerQ[p] && ( !InertTrigFreeQ[v] || !I
nertTrigFreeQ[w])
```

Rubi steps

$$\begin{aligned}\int \sqrt{\sin(x)\tan(x)}\,dx &= \frac{\sqrt{\sin(x)\tan(x)}\int \sqrt{\sin(x)}\sqrt{\tan(x)}\,dx}{\sqrt{\sin(x)}\sqrt{\tan(x)}}\\ &= -2\cot(x)\sqrt{\sin(x)\tan(x)}\end{aligned}$$

Mathematica [A] time = 0.08, size = 13, normalized size = 1.00

$$-2\cot(x)\sqrt{\sin(x)\tan(x)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Sin[x]*Tan[x]],x]
```

```
[Out] -2*Cot[x]*Sqrt[Sin[x]*Tan[x]]
```

fricas [A] time = 0.74, size = 22, normalized size = 1.69

$$-\frac{2\sqrt{-\frac{\cos(x)^2-1}{\cos(x)}}\cos(x)}{\sin(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((sin(x)*tan(x))^(1/2),x, algorithm="fricas")
```

```
[Out] -2*sqrt(-(cos(x)^2 - 1)/cos(x))*cos(x)/sin(x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{\sin(x)\tan(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((sin(x)*tan(x))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(sin(x)*tan(x)), x)
```

maple [B] time = 0.40, size = 177, normalized size = 13.62

$$\frac{(-1+\cos(x))\left(4\cos(x)\sqrt{-\frac{\cos(x)}{(1+\cos(x))^2}}+4\sqrt{-\frac{\cos(x)}{(1+\cos(x))^2}}+\ln\left(-\frac{2\left(2\left(\cos^2(x)\right)\sqrt{-\frac{\cos(x)}{(1+\cos(x))^2}}-\left(\cos^2(x)\right)+2\cos(x)-2\sqrt{-\frac{\cos(x)}{(1+\cos(x))^2}}\right.}{\sin(x)^2}\right.\right.}{4\sin(x)^3\sqrt{-\frac{\cos(x)}{(1+\cos(x))^2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((sin(x)*tan(x))^(1/2),x)
```

```
[Out] 1/4*(-1+cos(x))*(4*cos(x)*(-cos(x)/(1+cos(x))^2)^(1/2)+4*(-cos(x)/(1+cos(x)
)^2)^(1/2)+ln(-2*(2*cos(x)^2*(-cos(x)/(1+cos(x))^2)^(1/2)-cos(x)^2+2*cos(x)
-2*(-cos(x)/(1+cos(x))^2)^(1/2)-1)/sin(x)^2)-ln(-(2*cos(x)^2*(-cos(x)/(1+co
s(x))^2)^(1/2)-cos(x)^2+2*cos(x)-2*(-cos(x)/(1+cos(x))^2)^(1/2)-1)/sin(x)^2
))*cos(x)*(-(-1+cos(x)^2)/cos(x))^(1/2)/sin(x)^3/(-cos(x)/(1+cos(x))^2)^(1/
2)*4^(1/2)
```

maxima [B] time = 0.49, size = 57, normalized size = 4.38

$$\frac{2\left(\frac{\sin(x)^2}{(\cos(x)+1)^2}-1\right)}{\sqrt{\frac{\sin(x)}{\cos(x)+1}+1}\sqrt{-\frac{\sin(x)}{\cos(x)+1}+1}\sqrt{\frac{\sin(x)^2}{(\cos(x)+1)^2}+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((sin(x)*tan(x))^(1/2),x, algorithm="maxima")
```

```
[Out] 2*(sin(x)^2/(cos(x) + 1)^2 - 1)/(sqrt(sin(x)/(cos(x) + 1) + 1)*sqrt(-sin(x)
/(cos(x) + 1) + 1)*sqrt(sin(x)^2/(cos(x) + 1)^2 + 1))
```

mupad [B] time = 2.56, size = 20, normalized size = 1.54

$$-\frac{2\sin(x)}{\sqrt{\frac{1}{\cos(x)}}\sqrt{1-\cos(x)^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((sin(x)*tan(x))^(1/2),x)
```

```
[Out] -(2*sin(x))/((1/cos(x))^(1/2)*(1 - cos(x)^2)^(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{\sin(x)\tan(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((sin(x)*tan(x))**(1/2),x)
```

```
[Out] Integral(sqrt(sin(x)*tan(x)), x)
```

3.148 $\int (\sin(x)\tan(x))^{3/2}\,dx$

Optimal. Leaf size=31

$$\frac{8}{3}\csc(x)\sqrt{\sin(x)\tan(x)}-\frac{2}{3}\sin(x)\sqrt{\sin(x)\tan(x)}$$

```
[Out] 8/3*csc(x)*(sin(x)*tan(x))^(1/2)-2/3*sin(x)*(sin(x)*tan(x))^(1/2)
```

Rubi [A] time = 0.05, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {4400, 2598, 2589}

$$\frac{8}{3}\csc(x)\sqrt{\sin(x)\tan(x)}-\frac{2}{3}\sin(x)\sqrt{\sin(x)\tan(x)}$$

Antiderivative was successfully verified.

```
[In] Int[(Sin[x]*Tan[x])^(3/2),x]
```

```
[Out] (8*Csc[x]*Sqrt[Sin[x]*Tan[x]])/3 - (2*Sin[x]*Sqrt[Sin[x]*Tan[x]])/3
```

Rule 2589

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> -Simp[(b*(a*Sin[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*m
), x] /; FreeQ[{a, b, e, f, m, n}, x] && EqQ[m + n - 1, 0]
```

Rule 2598

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> -Simp[(b*(a*Sin[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f
*m), x] + Dist[(a^2*(m + n - 1))/m, Int[(a*Sin[e + f*x])^(m - 2)*(b*Tan[e +
 f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && (GtQ[m, 1] || (EqQ[m, 1] &
& EqQ[n, 1/2])) && IntegersQ[2*m, 2*n]
```

Rule 4400

```
Int[(u_.)*((v_)^(m_.)*(w_)^(n_.))^(p_), x_Symbol] :> With[{uu = ActivateTri
g[u], vv = ActivateTrig[v], ww = ActivateTrig[w]}, Dist[(vv^m*ww^n)^FracPar
t[p]/(vv^(m*FracPart[p])*ww^(n*FracPart[p])), Int[uu*vv^(m*p)*ww^(n*p), x],
 x]] /; FreeQ[{m, n, p}, x] && !IntegerQ[p] && ( !InertTrigFreeQ[v] || !I
nertTrigFreeQ[w])
```

Rubi steps

$$\int (\sin(x)\tan(x))^{3/2}\,dx = \frac{\sqrt{\sin(x)\tan(x)}\int \sin^{\frac{3}{2}}(x)\tan^{\frac{3}{2}}(x)\,dx}{\sqrt{\sin(x)}\sqrt{\tan(x)}}$$

$$= -\frac{2}{3}\sin(x)\sqrt{\sin(x)\tan(x)} + \frac{\left(4\sqrt{\sin(x)\tan(x)}\right)\int \frac{\tan^{\frac{3}{2}}(x)}{\sqrt{\sin(x)}}\,dx}{3\sqrt{\sin(x)}\sqrt{\tan(x)}}$$

$$= \frac{8}{3}\csc(x)\sqrt{\sin(x)\tan(x)} - \frac{2}{3}\sin(x)\sqrt{\sin(x)\tan(x)}$$

Mathematica [A] time = 0.04, size = 23, normalized size = 0.74

$$\frac{2}{3}\sin(x)\left(4\csc^2(x)-1\right)\sqrt{\sin(x)\tan(x)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sin[x]*Tan[x])^(3/2),x]
[Out] (2*(-1 + 4*Csc[x]^2)*Sin[x]*Sqrt[Sin[x]*Tan[x]])/3
```

fricas [A] time = 0.49, size = 26, normalized size = 0.84

$$\frac{2\left(\cos(x)^2+3\right)\sqrt{-\frac{\cos(x)^2-1}{\cos(x)}}}{3\sin(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((sin(x)*tan(x))^(3/2),x, algorithm="fricas")
[Out] 2/3*(cos(x)^2 + 3)*sqrt(-(cos(x)^2 - 1)/cos(x))/sin(x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (\sin(x)\tan(x))^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((sin(x)*tan(x))^(3/2),x, algorithm="giac")
[Out] integrate((sin(x)*tan(x))^(3/2), x)
```

maple [B] time = 0.26, size = 587, normalized size = 18.94

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((sin(x)*tan(x))^(3/2),x)
```

```
[Out] -1/12*(-1+cos(x))^2*(3*cos(x)^3*ln(-(2*cos(x)^2*(-cos(x)/(1+cos(x))^2)^(1/2
)-cos(x)^2+2*cos(x)-2*(-cos(x)/(1+cos(x))^2)^(1/2)-1)/sin(x)^2)*(-cos(x)/(1
+cos(x))^2)^(3/2)-3*cos(x)^3*ln(-2*(2*cos(x)^2*(-cos(x)/(1+cos(x))^2)^(1/2)
-cos(x)^2+2*cos(x)-2*(-cos(x)/(1+cos(x))^2)^(1/2)-1)/sin(x)^2)*(-cos(x)/(1+
cos(x))^2)^(3/2)+9*ln(-(2*cos(x)^2*(-cos(x)/(1+cos(x))^2)^(1/2)-cos(x)^2+2*
cos(x)-2*(-cos(x)/(1+cos(x))^2)^(1/2)-1)/sin(x)^2)*cos(x)^2*(-cos(x)/(1+cos
(x))^2)^(3/2)-9*ln(-2*(2*cos(x)^2*(-cos(x)/(1+cos(x))^2)^(1/2)-cos(x)^2+2*c
os(x)-2*(-cos(x)/(1+cos(x))^2)^(1/2)-1)/sin(x)^2)*cos(x)^2*(-cos(x)/(1+cos(
x))^2)^(3/2)+9*cos(x)*ln(-(2*cos(x)^2*(-cos(x)/(1+cos(x))^2)^(1/2)-cos(x)^2
+2*cos(x)-2*(-cos(x)/(1+cos(x))^2)^(1/2)-1)/sin(x)^2)*(-cos(x)/(1+cos(x))^2
)^(3/2)-9*cos(x)*ln(-2*(2*cos(x)^2*(-cos(x)/(1+cos(x))^2)^(1/2)-cos(x)^2+2*
cos(x)-2*(-cos(x)/(1+cos(x))^2)^(1/2)-1)/sin(x)^2)*(-cos(x)/(1+cos(x))^2)^(
3/2)+3*ln(-(2*cos(x)^2*(-cos(x)/(1+cos(x))^2)^(1/2)-cos(x)^2+2*cos(x)-2*(-c
os(x)/(1+cos(x))^2)^(1/2)-1)/sin(x)^2)*(-cos(x)/(1+cos(x))^2)^(3/2)-3*ln(-2
*(2*cos(x)^2*(-cos(x)/(1+cos(x))^2)^(1/2)-cos(x)^2+2*cos(x)-2*(-cos(x)/(1+c
os(x))^2)^(1/2)-1)/sin(x)^2)*(-cos(x)/(1+cos(x))^2)^(3/2)-4*cos(x)^3-12*cos
(x))*(1+cos(x))^2*(-(-1+cos(x)^2)/cos(x))^(3/2)/sin(x)^7*4^(1/2)
```

maxima [B] time = 0.47, size = 57, normalized size = 1.84

$$-\frac{8\left(\frac{\sin(x)^6}{(\cos(x)+1)^6}-1\right)}{3\left(\frac{\sin(x)}{\cos(x)+1}+1\right)^{\frac{3}{2}}\left(-\frac{\sin(x)}{\cos(x)+1}+1\right)^{\frac{3}{2}}\left(\frac{\sin(x)^2}{(\cos(x)+1)^2}+1\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((sin(x)*tan(x))^(3/2),x, algorithm="maxima")
```

```
[Out] -8/3*(sin(x)^6/(cos(x) + 1)^6 - 1)/((sin(x)/(cos(x) + 1) + 1)^(3/2)*(-sin(x
)/(cos(x) + 1) + 1)^(3/2)*(sin(x)^2/(cos(x) + 1)^2 + 1)^(3/2))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int (\sin(x)\tan(x))^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((sin(x)*tan(x))^(3/2),x)
[Out] int((sin(x)*tan(x))^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (\sin(x)\tan(x))^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((sin(x)*tan(x))**(3/2),x)
[Out] Integral((sin(x)*tan(x))**(3/2), x)
```

3.149 $\int (\sin(x)\tan(x))^{5/2}\,dx$

Optimal. Leaf size=50

$$-\frac{2}{5}\sin^2(x)\tan(x)\sqrt{\sin(x)\tan(x)}+\frac{16}{15}\tan(x)\sqrt{\sin(x)\tan(x)}+\frac{64}{15}\cot(x)\sqrt{\sin(x)\tan(x)}$$

```
[Out] 64/15*cot(x)*(sin(x)*tan(x))^(1/2)+16/15*(sin(x)*tan(x))^(1/2)*tan(x)-2/5*s
in(x)^2*(sin(x)*tan(x))^(1/2)*tan(x)
```

Rubi [A] time = 0.08, antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.444, Rules used = {4400, 2598, 2594, 2589}

$$-\frac{2}{5}\sin^2(x)\tan(x)\sqrt{\sin(x)\tan(x)}+\frac{16}{15}\tan(x)\sqrt{\sin(x)\tan(x)}+\frac{64}{15}\cot(x)\sqrt{\sin(x)\tan(x)}$$

Antiderivative was successfully verified.

```
[In] Int[(Sin[x]*Tan[x])^(5/2),x]
```

```
[Out] (64*Cot[x]*Sqrt[Sin[x]*Tan[x]])/15 + (16*Tan[x]*Sqrt[Sin[x]*Tan[x]])/15 - (
2*Sin[x]^2*Tan[x]*Sqrt[Sin[x]*Tan[x]])/5
```

Rule 2589

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> -Simp[(b*(a*Sin[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*m
), x] /; FreeQ[{a, b, e, f, m, n}, x] && EqQ[m + n - 1, 0]
```

Rule 2594

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(b*(a*Sin[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*(
n - 1)), x] - Dist[(b^2*(m + n - 1))/(n - 1), Int[(a*Sin[e + f*x])^m*(b*Tan
[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] && Int
egersQ[2*m, 2*n] &&  !(GtQ[m, 1] &&  !IntegerQ[(m - 1)/2])
```

Rule 2598

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> -Simp[(b*(a*Sin[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f
*m), x] + Dist[(a^2*(m + n - 1))/m, Int[(a*Sin[e + f*x])^(m - 2)*(b*Tan[e +
 f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && (GtQ[m, 1] || (EqQ[m, 1] &
& EqQ[n, 1/2])) && IntegersQ[2*m, 2*n]
```

Rule 4400

```
Int[(u_.)*((v_)^(m_.)*(w_)^(n_.))^(p_), x_Symbol] :> With[{uu = ActivateTri
g[u], vv = ActivateTrig[v], ww = ActivateTrig[w]}, Dist[(vv^m*ww^n)^FracPar
t[p]/(vv^(m*FracPart[p])*ww^(n*FracPart[p])), Int[uu*vv^(m*p)*ww^(n*p), x],
 x]] /; FreeQ[{m, n, p}, x] &&  !IntegerQ[p] && ( !InertTrigFreeQ[v] ||  !I
nertTrigFreeQ[w])
```

Rubi steps

$$\begin{aligned}\int (\sin(x)\tan(x))^{5/2}\,dx &= \frac{\sqrt{\sin(x)\tan(x)}\int \sin^{\frac{5}{2}}(x)\tan^{\frac{5}{2}}(x)\,dx}{\sqrt{\sin(x)}\sqrt{\tan(x)}}\\ &= -\frac{2}{5}\sin^2(x)\tan(x)\sqrt{\sin(x)\tan(x)} + \frac{\left(8\sqrt{\sin(x)\tan(x)}\right)\int \sqrt{\sin(x)}\tan^{\frac{5}{2}}(x)\,dx}{5\sqrt{\sin(x)}\sqrt{\tan(x)}}\\ &= \frac{16}{15}\tan(x)\sqrt{\sin(x)\tan(x)} - \frac{2}{5}\sin^2(x)\tan(x)\sqrt{\sin(x)\tan(x)} - \frac{\left(32\sqrt{\sin(x)\tan(x)}\right)\int \sqrt{s}}{15\sqrt{\sin(x)}\sqrt{ta}}\\ &= \frac{64}{15}\cot(x)\sqrt{\sin(x)\tan(x)} + \frac{16}{15}\tan(x)\sqrt{\sin(x)\tan(x)} - \frac{2}{5}\sin^2(x)\tan(x)\sqrt{\sin(x)\tan(x)}\end{aligned}$$

Mathematica [A] time = 0.10, size = 29, normalized size = 0.58

$$\frac{2}{15}\tan(x)\sqrt{\sin(x)\tan(x)}\left(3\cos^2(x) + 32\cot^2(x) + 5\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sin[x]*Tan[x])^(5/2),x]
[Out] (2*(5 + 3*Cos[x]^2 + 32*Cot[x]^2)*Tan[x]*Sqrt[Sin[x]*Tan[x]])/15
```

fricas [A] time = 0.72, size = 38, normalized size = 0.76

$$-\frac{2\left(3\cos(x)^4 - 30\cos(x)^2 - 5\right)\sqrt{-\frac{\cos(x)^2-1}{\cos(x)}}}{15\cos(x)\sin(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((sin(x)*tan(x))^(5/2),x, algorithm="fricas")
[Out] -2/15*(3*cos(x)^4 - 30*cos(x)^2 - 5)*sqrt(-(cos(x)^2 - 1)/cos(x))/(cos(x)*s
in(x))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (\sin(x)\tan(x))^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((sin(x)*tan(x))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((sin(x)*tan(x))^(5/2), x)
```

maple [B] time = 0.34, size = 324, normalized size = 6.48

$$-\frac{(-1+\cos(x))^2\left(6\left(\cos^4(x)\right)-15\left(\cos^2(x)\right)\sqrt{-\frac{\cos(x)}{(1+\cos(x))^2}}\ \ln\left(-\frac{2\left(\cos^2(x)\right)\sqrt{-\frac{\cos(x)}{(1+\cos(x))^2}}-\left(\cos^2(x)\right)+2\cos(x)-2\sqrt{-\frac{\cos(x)}{(1+\cos(x))^2}}}{\sin(x)^2}\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((sin(x)*tan(x))^(5/2),x)
```

```
[Out] -1/30*(-1+cos(x))^2*(6*cos(x)^4-15*cos(x)^2*(-cos(x)/(1+cos(x))^2)^(1/2)*ln
(-(2*cos(x)^2*(-cos(x)/(1+cos(x))^2)^(1/2)-cos(x)^2+2*cos(x)-2*(-cos(x)/(1+
cos(x))^2)^(1/2)-1)/sin(x)^2)+15*cos(x)^2*(-cos(x)/(1+cos(x))^2)^(1/2)*ln(-
2*(2*cos(x)^2*(-cos(x)/(1+cos(x))^2)^(1/2)-cos(x)^2+2*cos(x)-2*(-cos(x)/(1+
cos(x))^2)^(1/2)-1)/sin(x)^2)-15*cos(x)*(-cos(x)/(1+cos(x))^2)^(1/2)*ln(-(2
*cos(x)^2*(-cos(x)/(1+cos(x))^2)^(1/2)-cos(x)^2+2*cos(x)-2*(-cos(x)/(1+cos(
x))^2)^(1/2)-1)/sin(x)^2)+15*cos(x)*(-cos(x)/(1+cos(x))^2)^(1/2)*ln(-2*(2*c
os(x)^2*(-cos(x)/(1+cos(x))^2)^(1/2)-cos(x)^2+2*cos(x)-2*(-cos(x)/(1+cos(x)
)^2)^(1/2)-1)/sin(x)^2)-60*cos(x)^2-10)*cos(x)*(1+cos(x))^2*(-(-1+cos(x)^2)
/cos(x))^(5/2)/sin(x)^9*4^(1/2)
```

maxima [B] time = 0.44, size = 82, normalized size = 1.64

$$-\frac{32\left(\frac{5\sin(x)^4}{(\cos(x)+1)^4}-\frac{5\sin(x)^6}{(\cos(x)+1)^6}+\frac{2\sin(x)^{10}}{(\cos(x)+1)^{10}}-2\right)}{15\left(\frac{\sin(x)}{\cos(x)+1}+1\right)^{\frac{5}{2}}\left(-\frac{\sin(x)}{\cos(x)+1}+1\right)^{\frac{5}{2}}\left(\frac{\sin(x)^2}{(\cos(x)+1)^2}+1\right)^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((sin(x)*tan(x))^(5/2),x, algorithm="maxima")
```

```
[Out] -32/15*(5*sin(x)^4/(cos(x) + 1)^4 - 5*sin(x)^6/(cos(x) + 1)^6 + 2*sin(x)^10
/(cos(x) + 1)^10 - 2)/((sin(x)/(cos(x) + 1) + 1)^(5/2)*(-sin(x)/(cos(x) + 1
) + 1)^(5/2)*(sin(x)^2/(cos(x) + 1)^2 + 1)^(5/2))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int (\sin(x)\tan(x))^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((sin(x)*tan(x))^(5/2),x)
```

```
[Out] int((sin(x)*tan(x))^(5/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((sin(x)*tan(x))**(5/2),x)
```

```
[Out] Timed out
```

3.150 $\int \sqrt{\cos(x)\cot(x)}\,dx$

Optimal. Leaf size=13

$$2\tan(x)\sqrt{\cos(x)\cot(x)}$$

```
[Out] 2*(cos(x)*cot(x))^(1/2)*tan(x)
```

Rubi [A] time = 0.04, antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {4400, 2589}

$$2\tan(x)\sqrt{\cos(x)\cot(x)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Cos[x]*Cot[x]],x]
```

```
[Out] 2*Sqrt[Cos[x]*Cot[x]]*Tan[x]
```

Rule 2589

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> -Simp[(b*(a*Sin[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*m
), x] /; FreeQ[{a, b, e, f, m, n}, x] && EqQ[m + n - 1, 0]
```

Rule 4400

```
Int[(u_.)*((v_)^(m_.)*(w_)^(n_.))^(p_), x_Symbol] :> With[{uu = ActivateTri
g[u], vv = ActivateTrig[v], ww = ActivateTrig[w]}, Dist[(vv^m*ww^n)^FracPar
t[p]/(vv^(m*FracPart[p])*ww^(n*FracPart[p])), Int[uu*vv^(m*p)*ww^(n*p), x],
 x]] /; FreeQ[{m, n, p}, x] && !IntegerQ[p] && ( !InertTrigFreeQ[v] ||  !I
nertTrigFreeQ[w])
```

Rubi steps

$$\begin{aligned}\int \sqrt{\cos(x)\cot(x)}\,dx &= \frac{\sqrt{\cos(x)\cot(x)}\int \sqrt{\cos(x)}\sqrt{\cot(x)}\,dx}{\sqrt{\cos(x)}\sqrt{\cot(x)}}\\ &= 2\sqrt{\cos(x)\cot(x)}\,\tan(x)\end{aligned}$$

Mathematica [A] time = 0.07, size = 13, normalized size = 1.00

$$2\tan(x)\sqrt{\cos(x)\cot(x)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Cos[x]*Cot[x]],x]
[Out] 2*Sqrt[Cos[x]*Cot[x]]*Tan[x]
```

fricas [A] time = 0.66, size = 19, normalized size = 1.46

$$\frac{2\sqrt{\frac{\cos(x)^2}{\sin(x)}}\sin(x)}{\cos(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((cos(x)*cot(x))^(1/2),x, algorithm="fricas")
[Out] 2*sqrt(cos(x)^2/sin(x))*sin(x)/cos(x)
```

giac [A] time = 0.16, size = 12, normalized size = 0.92

$$2\,\mathrm{sgn}\left(\cos(x)\right)\mathrm{sgn}\left(\sin(x)\right)\sqrt{\sin(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((cos(x)*cot(x))^(1/2),x, algorithm="giac")
[Out] 2*sgn(cos(x))*sgn(sin(x))*sqrt(sin(x))
```

maple [A] time = 0.25, size = 20, normalized size = 1.54

$$\frac{2\sin(x)\sqrt{\frac{\cos^2(x)}{\sin(x)}}}{\cos(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(x)*cot(x))^(1/2),x)
[Out] 2*sin(x)*(cos(x)^2/sin(x))^(1/2)/cos(x)
```

maxima [B] time = 0.62, size = 188, normalized size = 14.46

$$\left(\left(\cos\left(\frac{3}{2}\,x\right)-\cos\left(\frac{1}{2}\,x\right)+\sin\left(\frac{3}{2}\,x\right)+\sin\left(\frac{1}{2}\,x\right)\right)\cos\left(\frac{1}{2}\,\arctan\left(\sin(x),\cos(x)-1\right)\right)-\left(\cos\left(\frac{3}{2}\,x\right)-\cos\left(\frac{1}{2}\,x\right)-\sin\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((cos(x)*cot(x))^(1/2),x, algorithm="maxima")
```

```
[Out] (((cos(3/2*x) - cos(1/2*x) + sin(3/2*x) + sin(1/2*x))*cos(1/2*arctan2(sin(x
), cos(x) - 1)) - (cos(3/2*x) - cos(1/2*x) - sin(3/2*x) - sin(1/2*x))*sin(1
/2*arctan2(sin(x), cos(x) - 1)))*cos(1/2*arctan2(sin(x), cos(x) + 1)) - ((c
os(3/2*x) - cos(1/2*x) - sin(3/2*x) - sin(1/2*x))*cos(1/2*arctan2(sin(x), c
os(x) - 1)) + (cos(3/2*x) - cos(1/2*x) + sin(3/2*x) + sin(1/2*x))*sin(1/2*a
rctan2(sin(x), cos(x) - 1)))*sin(1/2*arctan2(sin(x), cos(x) + 1)))/((cos(x)
^2 + sin(x)^2 + 2*cos(x) + 1)^(1/4)*(cos(x)^2 + sin(x)^2 - 2*cos(x) + 1)^(1
/4))
```

mupad [B] time = 2.65, size = 18, normalized size = 1.38

$$\frac{2\;|\cos(x)|\;\sin(x)^{3/2}}{|\sin(x)|\;\cos(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(x)*cot(x))^(1/2),x)
```

```
[Out] (2*abs(cos(x))*sin(x)^(3/2))/(abs(sin(x))*cos(x))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{\cos\left(x\right)\cot\left(x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((cos(x)*cot(x))**(1/2),x)
```

```
[Out] Integral(sqrt(cos(x)*cot(x)), x)
```

3.151 $\int (\cos(x)\cot(x))^{3/2}\,dx$

Optimal. Leaf size=31

$$\frac{2}{3}\cos(x)\sqrt{\cos(x)\cot(x)}-\frac{8}{3}\sec(x)\sqrt{\cos(x)\cot(x)}$$

[Out] 2/3*cos(x)*(cos(x)*cot(x))^(1/2)-8/3*sec(x)*(cos(x)*cot(x))^(1/2)

Rubi [A] time = 0.07, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {4400, 2598, 2589}

$$\frac{2}{3}\cos(x)\sqrt{\cos(x)\cot(x)}-\frac{8}{3}\sec(x)\sqrt{\cos(x)\cot(x)}$$

Antiderivative was successfully verified.

[In] Int[(Cos[x]*Cot[x])^(3/2),x]

[Out] (2*Cos[x]*Sqrt[Cos[x]*Cot[x]])/3 - (8*Sqrt[Cos[x]*Cot[x]]*Sec[x])/3

Rule 2589

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> -Simp[(b*(a*Sin[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*m
), x] /; FreeQ[{a, b, e, f, m, n}, x] && EqQ[m + n - 1, 0]
```

Rule 2598

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> -Simp[(b*(a*Sin[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f
*m), x] + Dist[(a^2*(m + n - 1))/m, Int[(a*Sin[e + f*x])^(m - 2)*(b*Tan[e +
 f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && (GtQ[m, 1] || (EqQ[m, 1] &
& EqQ[n, 1/2])) && IntegersQ[2*m, 2*n]
```

Rule 4400

```
Int[(u_.)*((v_)^(m_.)*(w_)^(n_.))^(p_), x_Symbol] :> With[{uu = ActivateTri
g[u], vv = ActivateTrig[v], ww = ActivateTrig[w]}, Dist[(vv^m*ww^n)^FracPar
t[p]/(vv^(m*FracPart[p])*ww^(n*FracPart[p])), Int[uu*vv^(m*p)*ww^(n*p), x],
 x]] /; FreeQ[{m, n, p}, x] && !IntegerQ[p] && ( !InertTrigFreeQ[v] ||  !I
nertTrigFreeQ[w])
```

Rubi steps

$$\int (\cos(x)\cot(x))^{3/2}\,dx = \frac{\sqrt{\cos(x)\cot(x)}\int \cos^{\frac{3}{2}}(x)\cot^{\frac{3}{2}}(x)\,dx}{\sqrt{\cos(x)}\sqrt{\cot(x)}}$$
$$= \frac{2}{3}\cos(x)\sqrt{\cos(x)\cot(x)} + \frac{\left(4\sqrt{\cos(x)\cot(x)}\right)\int \frac{\cot^{\frac{3}{2}}(x)}{\sqrt{\cos(x)}}\,dx}{3\sqrt{\cos(x)}\sqrt{\cot(x)}}$$
$$= \frac{2}{3}\cos(x)\sqrt{\cos(x)\cot(x)} - \frac{8}{3}\sqrt{\cos(x)\cot(x)}\sec(x)$$

Mathematica [A] time = 0.04, size = 21, normalized size = 0.68

$$\frac{2}{3}\left(\cos^2(x)-4\right)\sec(x)\sqrt{\cos(x)\cot(x)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[x]*Cot[x])^(3/2),x]
[Out] (2*(-4 + Cos[x]^2)*Sqrt[Cos[x]*Cot[x]]*Sec[x])/3
```

fricas [A] time = 0.73, size = 23, normalized size = 0.74

$$\frac{2\left(\cos(x)^2-4\right)\sqrt{\frac{\cos(x)^2}{\sin(x)}}}{3\cos(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((cos(x)*cot(x))^(3/2),x, algorithm="fricas")
[Out] 2/3*(cos(x)^2 - 4)*sqrt(cos(x)^2/sin(x))/cos(x)
```

giac [A] time = 0.15, size = 19, normalized size = 0.61

$$-\frac{2}{3}\left(\sin(x)^{\frac{3}{2}}+\frac{3}{\sqrt{\sin(x)}}\right)\mathrm{sgn}\left(\cos(x)\right)\mathrm{sgn}\left(\sin(x)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((cos(x)*cot(x))^(3/2),x, algorithm="giac")
[Out] -2/3*(sin(x)^(3/2) + 3/sqrt(sin(x)))*sgn(cos(x))*sgn(sin(x))
```

maple [A] time = 0.25, size = 26, normalized size = 0.84

$$\frac{2\left(\cos^{2}(x)-4\right)\left(\frac{\cos^{2}(x)}{\sin(x)}\right)^{\frac{3}{2}}\sin(x)}{3\cos(x)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(x)*cot(x))^(3/2),x)
```

```
[Out] 2/3*(cos(x)^2-4)*(cos(x)^2/sin(x))^(3/2)*sin(x)/cos(x)^3
```

maxima [B] time = 0.54, size = 314, normalized size = 10.13

$$\left(\cos(x)^{2}+\sin(x)^{2}+2\,\cos(x)+1\right)^{\frac{1}{4}}\left(\cos(x)^{2}+\sin(x)^{2}-2\,\cos(x)+1\right)^{\frac{1}{4}}\left(\left(\left(\cos\left(\frac{9}{2}\,x\right)-15\,\cos\left(\frac{5}{2}\,x\right)-\cos\left(\frac{3}{2}\,x\right)+\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((cos(x)*cot(x))^(3/2),x, algorithm="maxima")
```

```
[Out] 1/6*(cos(x)^2 + sin(x)^2 + 2*cos(x) + 1)^(1/4)*(cos(x)^2 + sin(x)^2 - 2*cos
(x) + 1)^(1/4)*(((cos(9/2*x) - 15*cos(5/2*x) - cos(3/2*x) + 15*cos(1/2*x) -
 sin(9/2*x) + 15*sin(5/2*x) - sin(3/2*x) - 15*sin(1/2*x))*cos(3/2*arctan2(s
in(x), cos(x) - 1)) + (cos(9/2*x) - 15*cos(5/2*x) - cos(3/2*x) + 15*cos(1/2
*x) + sin(9/2*x) - 15*sin(5/2*x) + sin(3/2*x) + 15*sin(1/2*x))*sin(3/2*arct
an2(sin(x), cos(x) - 1)))*cos(3/2*arctan2(sin(x), cos(x) + 1)) + ((cos(9/2*
x) - 15*cos(5/2*x) - cos(3/2*x) + 15*cos(1/2*x) + sin(9/2*x) - 15*sin(5/2*x
) + sin(3/2*x) + 15*sin(1/2*x))*cos(3/2*arctan2(sin(x), cos(x) - 1)) - (cos
(9/2*x) - 15*cos(5/2*x) - cos(3/2*x) + 15*cos(1/2*x) - sin(9/2*x) + 15*sin(
5/2*x) - sin(3/2*x) - 15*sin(1/2*x))*sin(3/2*arctan2(sin(x), cos(x) - 1)))*
sin(3/2*arctan2(sin(x), cos(x) + 1)))/(cos(x)^4 + sin(x)^4 + 2*(cos(x)^2 +
1)*sin(x)^2 - 2*cos(x)^2 + 1)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int (\cos(x)\cot(x))^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(x)*cot(x))^(3/2),x)
```

```
[Out] int((cos(x)*cot(x))^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (\cos(x)\cot(x))^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((cos(x)*cot(x))**(3/2),x)
```

```
[Out] Integral((cos(x)*cot(x))**(3/2), x)
```

3.152 $\int (\cos(x)\cot(x))^{5/2}\,dx$

Optimal. Leaf size=50

$$\frac{2}{5}\cos^2(x)\cot(x)\sqrt{\cos(x)\cot(x)}-\frac{16}{15}\cot(x)\sqrt{\cos(x)\cot(x)}-\frac{64}{15}\tan(x)\sqrt{\cos(x)\cot(x)}$$

```
[Out] -16/15*cot(x)*(cos(x)*cot(x))^(1/2)+2/5*cos(x)^2*cot(x)*(cos(x)*cot(x))^(1/
2)-64/15*(cos(x)*cot(x))^(1/2)*tan(x)
```

Rubi [A] time = 0.09, antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.444, Rules used = {4400, 2598, 2594, 2589}

$$\frac{2}{5}\cos^2(x)\cot(x)\sqrt{\cos(x)\cot(x)}-\frac{16}{15}\cot(x)\sqrt{\cos(x)\cot(x)}-\frac{64}{15}\tan(x)\sqrt{\cos(x)\cot(x)}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[x]*Cot[x])^(5/2),x]
```

```
[Out] (-16*Cot[x]*Sqrt[Cos[x]*Cot[x]])/15 + (2*Cos[x]^2*Cot[x]*Sqrt[Cos[x]*Cot[x]
])/5 - (64*Sqrt[Cos[x]*Cot[x]]*Tan[x])/15
```

Rule 2589

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> -Simp[(b*(a*Sin[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*m
), x] /; FreeQ[{a, b, e, f, m, n}, x] && EqQ[m + n - 1, 0]
```

Rule 2594

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(b*(a*Sin[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*(
n - 1)), x] - Dist[(b^2*(m + n - 1))/(n - 1), Int[(a*Sin[e + f*x])^m*(b*Tan
[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] && Int
egersQ[2*m, 2*n] &&  !(GtQ[m, 1] &&  !IntegerQ[(m - 1)/2])
```

Rule 2598

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> -Simp[(b*(a*Sin[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f
*m), x] + Dist[(a^2*(m + n - 1))/m, Int[(a*Sin[e + f*x])^(m - 2)*(b*Tan[e +
 f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && (GtQ[m, 1] || (EqQ[m, 1] &
& EqQ[n, 1/2])) && IntegersQ[2*m, 2*n]
```

Rule 4400

```
Int[(u_.)*((v_)^(m_.)*(w_)^(n_.))^(p_), x_Symbol] :> With[{uu = ActivateTri
g[u], vv = ActivateTrig[v], ww = ActivateTrig[w]}, Dist[(vv^m*ww^n)^FracPar
t[p]/(vv^(m*FracPart[p])*ww^(n*FracPart[p])), Int[uu*vv^(m*p)*ww^(n*p), x],
 x]] /; FreeQ[{m, n, p}, x] &&  !IntegerQ[p] && ( !InertTrigFreeQ[v] ||  !I
nertTrigFreeQ[w])
```

Rubi steps

$$\int (\cos(x)\cot(x))^{5/2}\,dx = \frac{\sqrt{\cos(x)\cot(x)}\int \cos^{\frac{5}{2}}(x)\cot^{\frac{5}{2}}(x)\,dx}{\sqrt{\cos(x)}\sqrt{\cot(x)}}$$
$$= \frac{2}{5}\cos^2(x)\cot(x)\sqrt{\cos(x)\cot(x)} + \frac{\left(8\sqrt{\cos(x)\cot(x)}\right)\int \sqrt{\cos(x)}\cot^{\frac{5}{2}}(x)\,dx}{5\sqrt{\cos(x)}\sqrt{\cot(x)}}$$
$$= -\frac{16}{15}\cot(x)\sqrt{\cos(x)\cot(x)} + \frac{2}{5}\cos^2(x)\cot(x)\sqrt{\cos(x)\cot(x)} - \frac{\left(32\sqrt{\cos(x)\cot(x)}\right)\int}{15\sqrt{\cos(x)}}$$
$$= -\frac{16}{15}\cot(x)\sqrt{\cos(x)\cot(x)} + \frac{2}{5}\cos^2(x)\cot(x)\sqrt{\cos(x)\cot(x)} - \frac{64}{15}\sqrt{\cos(x)\cot(x)}\tan$$

Mathematica [A] time = 0.10, size = 29, normalized size = 0.58

$$-\frac{2}{15}\tan(x)\sqrt{\cos(x)\cot(x)}\left(3\cos^2(x)+5\cot^2(x)+32\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[x]*Cot[x])^(5/2),x]
[Out] (-2*Sqrt[Cos[x]*Cot[x]]*(32 + 3*Cos[x]^2 + 5*Cot[x]^2)*Tan[x])/15
```

fricas [A] time = 0.60, size = 35, normalized size = 0.70

$$\frac{2\left(3\cos(x)^4+24\cos(x)^2-32\right)\sqrt{\frac{\cos(x)^2}{\sin(x)}}}{15\cos(x)\sin(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((cos(x)*cot(x))^(5/2),x, algorithm="fricas")
[Out] 2/15*(3*cos(x)^4 + 24*cos(x)^2 - 32)*sqrt(cos(x)^2/sin(x))/(cos(x)*sin(x))
```

giac [A] time = 0.14, size = 27, normalized size = 0.54

$$\frac{2}{15}\left(3\sin(x)^{\frac{5}{2}} - 30\sqrt{\sin(x)} - \frac{5}{\sin(x)^{\frac{3}{2}}}\right)\operatorname{sgn}(\cos(x))\operatorname{sgn}(\sin(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((cos(x)*cot(x))^(5/2),x, algorithm="giac")
```

```
[Out] 2/15*(3*sin(x)^(5/2) - 30*sqrt(sin(x)) - 5/sin(x)^(3/2))*sgn(cos(x))*sgn(si
n(x))
```

maple [A] time = 0.30, size = 34, normalized size = 0.68

$$\frac{2\left(3\left(\cos^4(x)\right) + 24\left(\cos^2(x)\right) - 32\right)\left(\frac{\cos^2(x)}{\sin(x)}\right)^{\frac{5}{2}}\sin(x)}{15\cos(x)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(x)*cot(x))^(5/2),x)
```

```
[Out] 2/15*(3*cos(x)^4+24*cos(x)^2-32)*(cos(x)^2/sin(x))^(5/2)*sin(x)/cos(x)^5
```

maxima [B] time = 0.63, size = 427, normalized size = 8.54

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((cos(x)*cot(x))^(5/2),x, algorithm="maxima")
```

```
[Out] -1/60*(((3*cos(15/2*x) + 105*cos(11/2*x) - 410*cos(7/2*x) - 3*cos(5/2*x) +
410*cos(3/2*x) - 105*cos(1/2*x) + 3*sin(15/2*x) + 105*sin(11/2*x) - 410*sin
(7/2*x) + 3*sin(5/2*x) + 410*sin(3/2*x) + 105*sin(1/2*x))*cos(5/2*arctan2(s
in(x), cos(x) - 1)) - (3*cos(15/2*x) + 105*cos(11/2*x) - 410*cos(7/2*x) - 3
*cos(5/2*x) + 410*cos(3/2*x) - 105*cos(1/2*x) - 3*sin(15/2*x) - 105*sin(11/
2*x) + 410*sin(7/2*x) - 3*sin(5/2*x) - 410*sin(3/2*x) - 105*sin(1/2*x))*sin
(5/2*arctan2(sin(x), cos(x) - 1)))*cos(5/2*arctan2(sin(x), cos(x) + 1)) - (
(3*cos(15/2*x) + 105*cos(11/2*x) - 410*cos(7/2*x) - 3*cos(5/2*x) + 410*cos(
3/2*x) - 105*cos(1/2*x) - 3*sin(15/2*x) - 105*sin(11/2*x) + 410*sin(7/2*x)
- 3*sin(5/2*x) - 410*sin(3/2*x) - 105*sin(1/2*x))*cos(5/2*arctan2(sin(x), c
os(x) - 1)) + (3*cos(15/2*x) + 105*cos(11/2*x) - 410*cos(7/2*x) - 3*cos(5/2
*x) + 410*cos(3/2*x) - 105*cos(1/2*x) + 3*sin(15/2*x) + 105*sin(11/2*x) - 4
10*sin(7/2*x) + 3*sin(5/2*x) + 410*sin(3/2*x) + 105*sin(1/2*x))*sin(5/2*arc
tan2(sin(x), cos(x) - 1)))*sin(5/2*arctan2(sin(x), cos(x) + 1)))/((cos(x)^4
```

```
 + sin(x)^4 + 2*(cos(x)^2 + 1)*sin(x)^2 - 2*cos(x)^2 + 1)*(cos(x)^2 + sin(x
)^2 + 2*cos(x) + 1)^(1/4)*(cos(x)^2 + sin(x)^2 - 2*cos(x) + 1)^(1/4))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int (\cos(x)\cot(x))^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(x)*cot(x))^(5/2),x)
```

```
[Out] int((cos(x)*cot(x))^(5/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((cos(x)*cot(x))**(5/2),x)
```

```
[Out] Timed out
```

3.153 $\int \frac{x\cos(x)}{(a+b\sin(x))^2}\,dx$

Optimal. Leaf size=58

$$\frac{2\tan^{-1}\left(\frac{a\tan\left(\frac{x}{2}\right)+b}{\sqrt{a^2-b^2}}\right)}{b\sqrt{a^2-b^2}}-\frac{x}{b(a+b\sin(x))}$$

```
[Out] -x/b/(a+b*sin(x))+2*arctan((b+a*tan(1/2*x))/(a^2-b^2)^(1/2))/b/(a^2-b^2)^(1
/2)
```

Rubi [A] time = 0.08, antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {4422, 2660, 618, 204}

$$\frac{2\tan^{-1}\left(\frac{a\tan\left(\frac{x}{2}\right)+b}{\sqrt{a^2-b^2}}\right)}{b\sqrt{a^2-b^2}}-\frac{x}{b(a+b\sin(x))}$$

Antiderivative was successfully verified.

```
[In] Int[(x*Cos[x])/(a + b*Sin[x])^2,x]
```

```
[Out] (2*ArcTan[(b + a*Tan[x/2])/Sqrt[a^2 - b^2]])/(b*Sqrt[a^2 - b^2]) - x/(b*(a
+ b*Sin[x]))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 4422

```
Int[Cos[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.)*((a_) + (b_.)*Sin[(c
_.) + (d_.)*(x_)])^(n_.), x_Symbol] :> Simp[((e + f*x)^m*(a + b*Sin[c + d*x
])^(n + 1))/(b*d*(n + 1)), x] - Dist[(f*m)/(b*d*(n + 1)), Int[(e + f*x)^(m
- 1)*(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x
] && IGtQ[m, 0] && NeQ[n, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{x\cos(x)}{(a+b\sin(x))^2}\,dx &= -\frac{x}{b(a+b\sin(x))}+\frac{\int \frac{1}{a+b\sin(x)}\,dx}{b}\\ &= -\frac{x}{b(a+b\sin(x))}+\frac{2\,\text{Subst}\left(\int \frac{1}{a+2bx+ax^2}\,dx,x,\tan\left(\frac{x}{2}\right)\right)}{b}\\ &= -\frac{x}{b(a+b\sin(x))}-\frac{4\,\text{Subst}\left(\int \frac{1}{-4\left(a^2-b^2\right)-x^2}\,dx,x,2b+2a\tan\left(\frac{x}{2}\right)\right)}{b}\\ &= \frac{2\tan^{-1}\left(\frac{b+a\tan\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2}}\right)}{b\sqrt{a^2-b^2}}-\frac{x}{b(a+b\sin(x))}\end{aligned}$$

Mathematica [A] time = 0.14, size = 56, normalized size = 0.97

$$\frac{\frac{2\tan^{-1}\left(\frac{a\tan\left(\frac{x}{2}\right)+b}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}}-\frac{x}{a+b\sin(x)}}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*Cos[x])/(a + b*Sin[x])^2,x]
```

```
[Out] ((2*ArcTan[(b + a*Tan[x/2])/Sqrt[a^2 - b^2]])/Sqrt[a^2 - b^2] - x/(a + b*Si
n[x]))/b
```

fricas [A] time = 1.56, size = 236, normalized size = 4.07

$$\left[-\frac{\sqrt{-a^2+b^2}\,(b\sin(x)+a)\log\left(\frac{\left(2a^2-b^2\right)\cos(x)^2-2ab\sin(x)-a^2-b^2+2\left(a\cos(x)\sin(x)+b\cos(x)\right)\sqrt{-a^2+b^2}}{b^2\cos(x)^2-2ab\sin(x)-a^2-b^2}\right)+2\left(a^2-b^2\right)x}{2\left(a^3b-ab^3+\left(a^2b^2-b^4\right)\sin(x)\right)},-\frac{\sqrt{a^2}\cdots}{\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(x)/(a+b*sin(x))^2,x, algorithm="fricas")
```

```
[Out] [-1/2*(sqrt(-a^2 + b^2)*(b*sin(x) + a)*log(((2*a^2 - b^2)*cos(x)^2 - 2*a*b*
sin(x) - a^2 - b^2 + 2*(a*cos(x)*sin(x) + b*cos(x))*sqrt(-a^2 + b^2))/(b^2*
cos(x)^2 - 2*a*b*sin(x) - a^2 - b^2)) + 2*(a^2 - b^2)*x)/(a^3*b - a*b^3 + (
a^2*b^2 - b^4)*sin(x)), -(sqrt(a^2 - b^2)*(b*sin(x) + a)*arctan(-(a*sin(x)
+ b)/(sqrt(a^2 - b^2)*cos(x))) + (a^2 - b^2)*x)/(a^3*b - a*b^3 + (a^2*b^2 -
 b^4)*sin(x))]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x\cos(x)}{(b\sin(x)+a)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(x)/(a+b*sin(x))^2,x, algorithm="giac")
```

```
[Out] integrate(x*cos(x)/(b*sin(x) + a)^2, x)
```

maple [C] time = 0.52, size = 159, normalized size = 2.74

$$-\frac{2ix\,e^{ix}}{b\left(b\,e^{2ix}-b+2ia\,e^{ix}\right)}-\frac{\ln\left(e^{ix}+\frac{ia\sqrt{-a^2+b^2}-a^2+b^2}{\sqrt{-a^2+b^2}\,b}\right)}{\sqrt{-a^2+b^2}\,b}+\frac{\ln\left(e^{ix}+\frac{ia\sqrt{-a^2+b^2}+a^2-b^2}{\sqrt{-a^2+b^2}\,b}\right)}{\sqrt{-a^2+b^2}\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*cos(x)/(a+b*sin(x))^2,x)
```

```
[Out] -2*I*x*exp(I*x)/b/(b*exp(2*I*x)-b+2*I*a*exp(I*x))-1/(-a^2+b^2)^(1/2)/b*ln(e
xp(I*x)+(I*a*(-a^2+b^2)^(1/2)-a^2+b^2)/(-a^2+b^2)^(1/2)/b)+1/(-a^2+b^2)^(1/
2)/b*ln(exp(I*x)+(I*a*(-a^2+b^2)^(1/2)+a^2-b^2)/(-a^2+b^2)^(1/2)/b)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(x)/(a+b*sin(x))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
```

```
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more details)Is 4*b^2-4*a^2 positive or negative?
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x\,\cos(x)}{(a+b\,\sin(x))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*cos(x))/(a + b*sin(x))^2,x)
```

```
[Out] int((x*cos(x))/(a + b*sin(x))^2, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(x)/(a+b*sin(x))**2,x)
```

```
[Out] Timed out
```

3.154 $\int \frac{x\cos(x)}{(a+b\sin(x))^3}\,dx$

Optimal. Leaf size=85

$$\frac{a\tan^{-1}\left(\frac{a\tan\left(\frac{x}{2}\right)+b}{\sqrt{a^2-b^2}}\right)}{b\left(a^2-b^2\right)^{3/2}}+\frac{\cos(x)}{2\left(a^2-b^2\right)(a+b\sin(x))}-\frac{x}{2b(a+b\sin(x))^2}$$

```
[Out] a*arctan((b+a*tan(1/2*x))/(a^2-b^2)^(1/2))/b/(a^2-b^2)^(3/2)-1/2*x/b/(a+b*s
in(x))^2+1/2*cos(x)/(a^2-b^2)/(a+b*sin(x))
```

Rubi [A] time = 0.11, antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {4422, 2664, 12, 2660, 618, 204}

$$\frac{a\tan^{-1}\left(\frac{a\tan\left(\frac{x}{2}\right)+b}{\sqrt{a^2-b^2}}\right)}{b\left(a^2-b^2\right)^{3/2}}+\frac{\cos(x)}{2\left(a^2-b^2\right)(a+b\sin(x))}-\frac{x}{2b(a+b\sin(x))^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x*Cos[x])/(a + b*Sin[x])^3,x]
```

```
[Out] (a*ArcTan[(b + a*Tan[x/2])/Sqrt[a^2 - b^2]])/(b*(a^2 - b^2)^(3/2)) - x/(2*b
*(a + b*Sin[x])^2) + Cos[x]/(2*(a^2 - b^2)*(a + b*Sin[x]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2664

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[
c + d*x]*(a + b*Sin[c + d*x])^(n + 1))/(d*(n + 1)*(a^2 - b^2)), x] + Dist[1
/((n + 1)*(a^2 - b^2)), Int[(a + b*Sin[c + d*x])^(n + 1)*Simp[a*(n + 1) - b
*(n + 2)*Sin[c + d*x], x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^
2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 4422

```
Int[Cos[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.)*((a_) + (b_.)*Sin[(c
_.) + (d_.)*(x_)])^(n_.), x_Symbol] :> Simp[((e + f*x)^m*(a + b*Sin[c + d*x
])^(n + 1))/(b*d*(n + 1)), x] - Dist[(f*m)/(b*d*(n + 1)), Int[(e + f*x)^(m
- 1)*(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x
] && IGtQ[m, 0] && NeQ[n, -1]
```

Rubi steps

$$\int \frac{x\cos(x)}{(a+b\sin(x))^3}\,dx = -\frac{x}{2b(a+b\sin(x))^2} + \frac{\int \frac{1}{(a+b\sin(x))^2}\,dx}{2b}$$
$$= -\frac{x}{2b(a+b\sin(x))^2} + \frac{\cos(x)}{2\left(a^2-b^2\right)(a+b\sin(x))} + \frac{\int \frac{a}{a+b\sin(x)}\,dx}{2b\left(a^2-b^2\right)}$$
$$= -\frac{x}{2b(a+b\sin(x))^2} + \frac{\cos(x)}{2\left(a^2-b^2\right)(a+b\sin(x))} + \frac{a\int \frac{1}{a+b\sin(x)}\,dx}{2b\left(a^2-b^2\right)}$$
$$= -\frac{x}{2b(a+b\sin(x))^2} + \frac{\cos(x)}{2\left(a^2-b^2\right)(a+b\sin(x))} + \frac{a\,\text{Subst}\left(\int \frac{1}{a+2bx+ax^2}\,dx, x, \tan\left(\frac{x}{2}\right)\right)}{b\left(a^2-b^2\right)}$$
$$= -\frac{x}{2b(a+b\sin(x))^2} + \frac{\cos(x)}{2\left(a^2-b^2\right)(a+b\sin(x))} - \frac{(2a)\,\text{Subst}\left(\int \frac{1}{-4\left(a^2-b^2\right)-x^2}\,dx, x, 2b+2a}{b\left(a^2-b^2\right)}$$
$$= \frac{a\tan^{-1}\left(\frac{b+a\tan\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2}}\right)}{b\left(a^2-b^2\right)^{3/2}} - \frac{x}{2b(a+b\sin(x))^2} + \frac{\cos(x)}{2\left(a^2-b^2\right)(a+b\sin(x))}$$

Mathematica [A] time = 0.26, size = 84, normalized size = 0.99

$$\frac{a\tan^{-1}\left(\frac{a\tan\left(\frac{x}{2}\right)+b}{\sqrt{a^2-b^2}}\right)}{b\left(a^2-b^2\right)^{3/2}} + \frac{\frac{\cos(x)(a+b\sin(x))}{(a-b)(a+b)} - \frac{x}{b}}{2(a+b\sin(x))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*Cos[x])/(a + b*Sin[x])^3,x]
```

```
[Out] (a*ArcTan[(b + a*Tan[x/2])/Sqrt[a^2 - b^2]])/(b*(a^2 - b^2)^(3/2)) + (-(x/b
) + (Cos[x]*(a + b*Sin[x]))/((a - b)*(a + b)))/(2*(a + b*Sin[x])^2)
```

fricas [B] time = 0.63, size = 459, normalized size = 5.40

$$\left[\frac{2\left(a^2b^2-b^4\right)\cos(x)\sin(x) - \left(ab^2\cos(x)^2 - 2a^2b\sin(x) - a^3 - ab^2\right)\sqrt{-a^2+b^2}\,\log\left(-\frac{\left(2a^2-b^2\right)\cos(x)^2-2ab\sin(x)-a^2-b}{b^2\cos(x)^2-2}\right.}{4\left(a^6b-a^4b^3-a^2b^5+b^7-\left(a^4b^3-2a^2b^5+b^7\right)\cos(x)^2+2\left(a\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(x)/(a+b*sin(x))^3,x, algorithm="fricas")
```

```
[Out] [1/4*(2*(a^2*b^2 - b^4)*cos(x)*sin(x) - (a*b^2*cos(x)^2 - 2*a^2*b*sin(x) -
a^3 - a*b^2)*sqrt(-a^2 + b^2)*log(-((2*a^2 - b^2)*cos(x)^2 - 2*a*b*sin(x) -
 a^2 - b^2 - 2*(a*cos(x)*sin(x) + b*cos(x))*sqrt(-a^2 + b^2))/(b^2*cos(x)^2
 - 2*a*b*sin(x) - a^2 - b^2)) - 2*(a^4 - 2*a^2*b^2 + b^4)*x + 2*(a^3*b - a*
b^3)*cos(x))/(a^6*b - a^4*b^3 - a^2*b^5 + b^7 - (a^4*b^3 - 2*a^2*b^5 + b^7)
*cos(x)^2 + 2*(a^5*b^2 - 2*a^3*b^4 + a*b^6)*sin(x)), 1/2*((a^2*b^2 - b^4)*c
os(x)*sin(x) + (a*b^2*cos(x)^2 - 2*a^2*b*sin(x) - a^3 - a*b^2)*sqrt(a^2 - b
^2)*arctan(-(a*sin(x) + b)/(sqrt(a^2 - b^2)*cos(x))) - (a^4 - 2*a^2*b^2 + b
^4)*x + (a^3*b - a*b^3)*cos(x))/(a^6*b - a^4*b^3 - a^2*b^5 + b^7 - (a^4*b^3
 - 2*a^2*b^5 + b^7)*cos(x)^2 + 2*(a^5*b^2 - 2*a^3*b^4 + a*b^6)*sin(x))]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x\cos(x)}{(b\sin(x)+a)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(x)/(a+b*sin(x))^3,x, algorithm="giac")
```

```
[Out] integrate(x*cos(x)/(b*sin(x) + a)^3, x)
```

maple [C] time = 0.83, size = 257, normalized size = 3.02

$$\frac{2ia^2e^{2ix}+ib^2e^{2ix}+2x\,a^2e^{2ix}+ba\,e^{3ix}-2b^2x\,e^{2ix}-ib^2-3ab\,e^{ix}}{\left(b\,e^{2ix}-b+2ia\,e^{ix}\right)^2\left(a^2-b^2\right)b}-\frac{a\ln\left(e^{ix}+\frac{ia\sqrt{-a^2+b^2}-a^2+b^2}{\sqrt{-a^2+b^2}\,b}\right)}{2\sqrt{-a^2+b^2}\,(a+b)(a-b)\,b}+\frac{a\ln\left(e^{ix}+\frac{ia\sqrt{}}{}\right.}{2\sqrt{-a^2+b^2}\,(a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*cos(x)/(a+b*sin(x))^3,x)
```

```
[Out] (2*I*a^2*exp(2*I*x)+I*b^2*exp(2*I*x)+2*x*a^2*exp(2*I*x)+b*a*exp(3*I*x)-2*b^
2*x*exp(2*I*x)-I*b^2-3*a*b*exp(I*x))/(b*exp(2*I*x)-b+2*I*a*exp(I*x))^2/(a^2
-b^2)/b-1/2/(-a^2+b^2)^(1/2)*a/(a+b)/(a-b)/b*ln(exp(I*x)+(I*a*(-a^2+b^2)^(1
/2)-a^2+b^2)/(-a^2+b^2)^(1/2)/b)+1/2/(-a^2+b^2)^(1/2)*a/(a+b)/(a-b)/b*ln(ex
p(I*x)+(I*a*(-a^2+b^2)^(1/2)+a^2-b^2)/(-a^2+b^2)^(1/2)/b)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(x)/(a+b*sin(x))^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more details)Is 4*b^2-4*a^2 positive or negative?
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x\cos(x)}{(a+b\sin(x))^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*cos(x))/(a + b*sin(x))^3,x)
```

```
[Out] int((x*cos(x))/(a + b*sin(x))^3, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(x)/(a+b*sin(x))**3,x)
```

```
[Out] Timed out
```

3.155 $\int \frac{x\sin(x)}{(a+b\cos(x))^2}\,dx$

Optimal. Leaf size=59

$$\frac{x}{b(a+b\cos(x))}-\frac{2\tan^{-1}\left(\frac{\sqrt{a-b}\,\tan\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{b\sqrt{a-b}\,\sqrt{a+b}}$$

```
[Out] x/b/(a+b*cos(x))-2*arctan((a-b)^(1/2)*tan(1/2*x)/(a+b)^(1/2))/b/(a-b)^(1/2)
/(a+b)^(1/2)
```

Rubi [A] time = 0.06, antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {4423, 2659, 205}

$$\frac{x}{b(a+b\cos(x))}-\frac{2\tan^{-1}\left(\frac{\sqrt{a-b}\,\tan\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{b\sqrt{a-b}\,\sqrt{a+b}}$$

Antiderivative was successfully verified.

```
[In] Int[(x*Sin[x])/(a + b*Cos[x])^2,x]
```

```
[Out] (-2*ArcTan[(Sqrt[a - b]*Tan[x/2])/Sqrt[a + b]])/(Sqrt[a - b]*b*Sqrt[a + b])
 + x/(b*(a + b*Cos[x]))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 4423

```
Int[(Cos[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_.)*((e_.) + (f_.)*(x_))^(m_.)
*Sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[((e + f*x)^m*(a + b*Cos[c + d*
x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(f*m)/(b*d*(n + 1)), Int[(e + f*x)^(m
 - 1)*(a + b*Cos[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, n},
```

```
x] && IGtQ[m, 0] && NeQ[n, -1]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x\sin(x)}{(a+b\cos(x))^2}\,dx &= \frac{x}{b(a+b\cos(x))}-\frac{\int \frac{1}{a+b\cos(x)}\,dx}{b}\\
&= \frac{x}{b(a+b\cos(x))}-\frac{2\,\text{Subst}\left(\int \frac{1}{a+b+(a-b)x^2}\,dx,x,\tan\left(\frac{x}{2}\right)\right)}{b}\\
&= -\frac{2\tan^{-1}\left(\frac{\sqrt{a-b}\,\tan\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{\sqrt{a-b}\,b\sqrt{a+b}}+\frac{x}{b(a+b\cos(x))}
\end{aligned}
$$

Mathematica [A] time = 0.10, size = 58, normalized size = 0.98

$$
\frac{2\tanh^{-1}\left(\frac{(a-b)\tan\left(\frac{x}{2}\right)}{\sqrt{b^2-a^2}}\right)}{b\sqrt{b^2-a^2}}+\frac{x}{b(a+b\cos(x))}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*Sin[x])/(a + b*Cos[x])^2,x]
```

```
[Out] (2*ArcTanh[((a - b)*Tan[x/2])/Sqrt[-a^2 + b^2]])/(b*Sqrt[-a^2 + b^2]) + x/(
b*(a + b*Cos[x]))
```

fricas [A] time = 0.57, size = 227, normalized size = 3.85

$$
\left[-\frac{\sqrt{-a^2+b^2}\,(b\cos(x)+a)\log\left(\frac{2\,ab\cos(x)+\left(2\,a^2-b^2\right)\cos(x)^2-2\,\sqrt{-a^2+b^2}\,(a\cos(x)+b)\sin(x)-a^2+2\,b^2}{b^2\cos(x)^2+2\,ab\cos(x)+a^2}\right)-2\left(a^2-b^2\right)x}{2\left(a^3b-ab^3+\left(a^2b^2-b^4\right)\cos(x)\right)},-\frac{\sqrt{a^2-b^2}\cdots}{\cdots}\right.
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(x)/(a+b*cos(x))^2,x, algorithm="fricas")
```

```
[Out] [-1/2*(sqrt(-a^2 + b^2)*(b*cos(x) + a)*log((2*a*b*cos(x) + (2*a^2 - b^2)*co
s(x)^2 - 2*sqrt(-a^2 + b^2)*(a*cos(x) + b)*sin(x) - a^2 + 2*b^2)/(b^2*cos(x
)^2 + 2*a*b*cos(x) + a^2)) - 2*(a^2 - b^2)*x)/(a^3*b - a*b^3 + (a^2*b^2 - b
^4)*cos(x)), -(sqrt(a^2 - b^2)*(b*cos(x) + a)*arctan(-(a*cos(x) + b)/(sqrt(
```

```
a^2 - b^2)*sin(x))) - (a^2 - b^2)*x)/(a^3*b - a*b^3 + (a^2*b^2 - b^4)*cos(x
))]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x\sin(x)}{(b\cos(x)+a)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(x)/(a+b*cos(x))^2,x, algorithm="giac")
```

```
[Out] integrate(x*sin(x)/(b*cos(x) + a)^2, x)
```

maple [C] time = 0.26, size = 154, normalized size = 2.61

$$\frac{2x\,\mathrm{e}^{ix}}{b\left(b\,\mathrm{e}^{2ix}+2a\,\mathrm{e}^{ix}+b\right)}-\frac{i\ln\left(\mathrm{e}^{ix}+\frac{a\sqrt{a^2-b^2}+a^2-b^2}{\sqrt{a^2-b^2}\,b}\right)}{\sqrt{a^2-b^2}\,b}+\frac{i\ln\left(\mathrm{e}^{ix}+\frac{a\sqrt{a^2-b^2}-a^2+b^2}{\sqrt{a^2-b^2}\,b}\right)}{\sqrt{a^2-b^2}\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*sin(x)/(a+b*cos(x))^2,x)
```

```
[Out] 2*x*exp(I*x)/b/(b*exp(2*I*x)+2*a*exp(I*x)+b)-I/(a^2-b^2)^(1/2)/b*ln(exp(I*x
)+(a*(a^2-b^2)^(1/2)+a^2-b^2)/(a^2-b^2)^(1/2)/b)+I/(a^2-b^2)^(1/2)/b*ln(exp
(I*x)+(a*(a^2-b^2)^(1/2)-a^2+b^2)/(a^2-b^2)^(1/2)/b)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(x)/(a+b*cos(x))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` for
more details)Is 4*a^2-4*b^2 positive or negative?
```

mupad [B] time = 3.29, size = 132, normalized size = 2.24

$$\frac{2\,x\,\mathrm{e}^{x\,1\mathrm{i}}}{b\,\left(2\,a\,\mathrm{e}^{x\,1\mathrm{i}}+2\,b\,\mathrm{e}^{x\,1\mathrm{i}}\,\cos(x)\right)}+\frac{\ln\left(2\,\mathrm{e}^{x\,1\mathrm{i}}-\frac{\left(b+a\,\mathrm{e}^{x\,1\mathrm{i}}\right)2\mathrm{i}}{\sqrt{a+b}\,\sqrt{b-a}}\right)}{b\,\sqrt{a+b}\,\sqrt{b-a}}-\frac{\ln\left(2\,\mathrm{e}^{x\,1\mathrm{i}}+\frac{\left(b+a\,\mathrm{e}^{x\,1\mathrm{i}}\right)2\mathrm{i}}{\sqrt{a+b}\,\sqrt{b-a}}\right)}{b\,\sqrt{a+b}\,\sqrt{b-a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*sin(x))/(a + b*cos(x))^2,x)
```

```
[Out] (2*x*exp(x*1i))/(b*(2*a*exp(x*1i) + 2*b*exp(x*1i)*cos(x))) + log(2*exp(x*1i
) - ((b + a*exp(x*1i))*2i)/((a + b)^(1/2)*(b - a)^(1/2)))/(b*(a + b)^(1/2)*
(b - a)^(1/2)) - log(2*exp(x*1i) + ((b + a*exp(x*1i))*2i)/((a + b)^(1/2)*(b
 - a)^(1/2)))/(b*(a + b)^(1/2)*(b - a)^(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x \sin(x)}{(a + b\cos(x))^2} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(x)/(a+b*cos(x))**2,x)
```

```
[Out] Integral(x*sin(x)/(a + b*cos(x))**2, x)
```

3.156 $\int \frac{x \sin(x)}{(a+b\cos(x))^3}\, dx$

Optimal. Leaf size=88

$$\frac{\sin(x)}{2\left(a^2-b^2\right)(a+b\cos(x))}+\frac{x}{2b(a+b\cos(x))^2}-\frac{a\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{b(a-b)^{3/2}(a+b)^{3/2}}$$

```
[Out] -a*arctan((a-b)^(1/2)*tan(1/2*x)/(a+b)^(1/2))/(a-b)^(3/2)/b/(a+b)^(3/2)+1/2
*x/b/(a+b*cos(x))^2+1/2*sin(x)/(a^2-b^2)/(a+b*cos(x))
```

Rubi [A] time = 0.10, antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {4423, 2664, 12, 2659, 205}

$$\frac{\sin(x)}{2\left(a^2-b^2\right)(a+b\cos(x))}+\frac{x}{2b(a+b\cos(x))^2}-\frac{a\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{b(a-b)^{3/2}(a+b)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x*Sin[x])/(a + b*Cos[x])^3,x]
```

```
[Out] -((a*ArcTan[(Sqrt[a - b]*Tan[x/2])/Sqrt[a + b]])/((a - b)^(3/2)*b*(a + b)^(
3/2))) + x/(2*b*(a + b*Cos[x])^2) + Sin[x]/(2*(a^2 - b^2)*(a + b*Cos[x]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 2664

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[
c + d*x]*(a + b*Sin[c + d*x])^(n + 1))/(d*(n + 1)*(a^2 - b^2)), x] + Dist[1
/((n + 1)*(a^2 - b^2)), Int[(a + b*Sin[c + d*x])^(n + 1)*Simp[a*(n + 1) - b
*(n + 2)*Sin[c + d*x], x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^
2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 4423

```
Int[(Cos[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_.)*((e_.) + (f_.)*(x_))^(m_.)
*Sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[((e + f*x)^m*(a + b*Cos[c + d*
x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(f*m)/(b*d*(n + 1)), Int[(e + f*x)^(m
 - 1)*(a + b*Cos[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, n},
x] && IGtQ[m, 0] && NeQ[n, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{x\sin(x)}{(a+b\cos(x))^3}\,dx &= \frac{x}{2b(a+b\cos(x))^2}-\frac{\int \frac{1}{(a+b\cos(x))^2}\,dx}{2b}\\
&= \frac{x}{2b(a+b\cos(x))^2}+\frac{\sin(x)}{2\left(a^2-b^2\right)(a+b\cos(x))}-\frac{\int \frac{a}{a+b\cos(x)}\,dx}{2b\left(a^2-b^2\right)}\\
&= \frac{x}{2b(a+b\cos(x))^2}+\frac{\sin(x)}{2\left(a^2-b^2\right)(a+b\cos(x))}-\frac{a\int \frac{1}{a+b\cos(x)}\,dx}{2b\left(a^2-b^2\right)}\\
&= \frac{x}{2b(a+b\cos(x))^2}+\frac{\sin(x)}{2\left(a^2-b^2\right)(a+b\cos(x))}-\frac{a\,\mathrm{Subst}\left(\int \frac{1}{a+b+(a-b)x^2}\,dx,x,\tan\left(\frac{x}{2}\right)\right)}{b\left(a^2-b^2\right)}\\
&= -\frac{a\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{(a-b)^{3/2}b(a+b)^{3/2}}+\frac{x}{2b(a+b\cos(x))^2}+\frac{\sin(x)}{2\left(a^2-b^2\right)(a+b\cos(x))}
\end{aligned}$$

Mathematica [A] time = 0.32, size = 85, normalized size = 0.97

$$\frac{\frac{\sin(x)(a+b\cos(x))}{(a-b)(a+b)}+\frac{x}{b}}{2(a+b\cos(x))^2}-\frac{a\tanh^{-1}\left(\frac{(a-b)\tan\left(\frac{x}{2}\right)}{\sqrt{b^2-a^2}}\right)}{b\left(b^2-a^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*Sin[x])/(a + b*Cos[x])^3,x]
```

```
[Out] -((a*ArcTanh[((a - b)*Tan[x/2])/Sqrt[-a^2 + b^2]])/(b*(-a^2 + b^2)^(3/2)))
+ (x/b + ((a + b*Cos[x])*Sin[x])/((a - b)*(a + b)))/(2*(a + b*Cos[x])^2)
```

fricas [B] time = 0.66, size = 417, normalized size = 4.74

$$\left[\frac{\left(ab^2\cos(x)^2+2\,a^2b\cos(x)+a^3\right)\sqrt{-a^2+b^2}\,\log\left(\frac{2\,ab\cos(x)+\left(2\,a^2-b^2\right)\cos(x)^2+2\,\sqrt{-a^2+b^2}\,(a\cos(x)+b)\sin(x)-a^2+2\,b^2}{b^2\cos(x)^2+2\,ab\cos(x)+a^2}\right)+2\,(a}{4\left(a^6b-2\,a^4b^3+a^2b^5+\left(a^4b^3-2\,a^2b^5+b^7\right)\cos(x)^2+2\left(a^5b^2-2\,a\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(x)/(a+b*cos(x))^3,x, algorithm="fricas")
```

```
[Out] [1/4*((a*b^2*cos(x)^2 + 2*a^2*b*cos(x) + a^3)*sqrt(-a^2 + b^2)*log((2*a*b*c
os(x) + (2*a^2 - b^2)*cos(x)^2 + 2*sqrt(-a^2 + b^2)*(a*cos(x) + b)*sin(x) -
 a^2 + 2*b^2)/(b^2*cos(x)^2 + 2*a*b*cos(x) + a^2)) + 2*(a^4 - 2*a^2*b^2 + b
^4)*x + 2*(a^3*b - a*b^3 + (a^2*b^2 - b^4)*cos(x))*sin(x))/(a^6*b - 2*a^4*b
^3 + a^2*b^5 + (a^4*b^3 - 2*a^2*b^5 + b^7)*cos(x)^2 + 2*(a^5*b^2 - 2*a^3*b^
4 + a*b^6)*cos(x)), -1/2*((a*b^2*cos(x)^2 + 2*a^2*b*cos(x) + a^3)*sqrt(a^2
- b^2)*arctan(-(a*cos(x) + b)/(sqrt(a^2 - b^2)*sin(x))) - (a^4 - 2*a^2*b^2
+ b^4)*x - (a^3*b - a*b^3 + (a^2*b^2 - b^4)*cos(x))*sin(x))/(a^6*b - 2*a^4*
b^3 + a^2*b^5 + (a^4*b^3 - 2*a^2*b^5 + b^7)*cos(x)^2 + 2*(a^5*b^2 - 2*a^3*b
^4 + a*b^6)*cos(x))]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x\sin(x)}{(b\cos(x)+a)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(x)/(a+b*cos(x))^3,x, algorithm="giac")
```

```
[Out] integrate(x*sin(x)/(b*cos(x) + a)^3, x)
```

maple [C] time = 0.44, size = 250, normalized size = 2.84

$$\frac{i\left(-2ia^2x\,e^{2ix}+2ib^2x\,e^{2ix}+ba\,e^{3ix}+2a^2e^{2ix}+b^2e^{2ix}+3ab\,e^{ix}+b^2\right)}{b\left(b\,e^{2ix}+2a\,e^{ix}+b\right)^2\left(a^2-b^2\right)}-\frac{ia\ln\left(e^{ix}+\frac{a\sqrt{a^2-b^2}+a^2-b^2}{\sqrt{a^2-b^2}\,b}\right)}{2\sqrt{a^2-b^2}\,(a+b)\,(a-b)\,b}+\frac{ia\ln\left(e^{ix}+\right.}{2\sqrt{a^2-b^2}\,\left(\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*sin(x)/(a+b*cos(x))^3,x)
```

```
[Out] I*(-2*I*a^2*x*exp(2*I*x)+2*I*b^2*x*exp(2*I*x)+b*a*exp(3*I*x)+2*a^2*exp(2*I*
x)+b^2*exp(2*I*x)+3*a*b*exp(I*x)+b^2)/b/(b*exp(2*I*x)+2*a*exp(I*x)+b)^2/(a^
2-b^2)-1/2*I/(a^2-b^2)^(1/2)*a/(a+b)/(a-b)/b*ln(exp(I*x)+(a*(a^2-b^2)^(1/2)
+a^2-b^2)/(a^2-b^2)^(1/2)/b)+1/2*I/(a^2-b^2)^(1/2)*a/(a+b)/(a-b)/b*ln(exp(I
*x)+(a*(a^2-b^2)^(1/2)-a^2+b^2)/(a^2-b^2)^(1/2)/b)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(x)/(a+b*cos(x))^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` for
more details)Is 4*a^2-4*b^2 positive or negative?
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x\,\sin(x)}{(a+b\,\cos(x))^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*sin(x))/(a + b*cos(x))^3,x)
```

```
[Out] int((x*sin(x))/(a + b*cos(x))^3, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(x)/(a+b*cos(x))**3,x)
```

```
[Out] Timed out
```

3.157 $\int \frac{x\sec^2(x)}{(a+b\tan(x))^2}\,dx$

Optimal. Leaf size=50

$$\frac{ax}{b\left(a^2+b^2\right)}+\frac{\log(a\cos(x)+b\sin(x))}{a^2+b^2}-\frac{x}{b(a+b\tan(x))}$$

```
[Out] a*x/b/(a^2+b^2)+ln(a*cos(x)+b*sin(x))/(a^2+b^2)-x/b/(a+b*tan(x))
```

Rubi [A] time = 0.08, antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {4424, 3484, 3530}

$$\frac{ax}{b\left(a^2+b^2\right)}+\frac{\log(a\cos(x)+b\sin(x))}{a^2+b^2}-\frac{x}{b(a+b\tan(x))}$$

Antiderivative was successfully verified.

```
[In] Int[(x*Sec[x]^2)/(a + b*Tan[x])^2,x]
```

```
[Out] (a*x)/(b*(a^2 + b^2)) + Log[a*Cos[x] + b*Sin[x]]/(a^2 + b^2) - x/(b*(a + b*
Tan[x]))
```

Rule 3484

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> Simp[(a*x)/(a
^2 + b^2), x] + Dist[b/(a^2 + b^2), Int[(b - a*Tan[c + d*x])/(a + b*Tan[c +
 d*x]), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

Rule 3530

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_) + (b_.)*tan[(e_.) + (f_.)*
(x_)]), x_Symbol] :> Simp[(c*Log[RemoveContent[a*Cos[e + f*x] + b*Sin[e + f
*x], x]])/(b*f), x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
 NeQ[a^2 + b^2, 0] && EqQ[a*c + b*d, 0]
```

Rule 4424

```
Int[((e_.) + (f_.)*(x_))^(m_.)*Sec[(c_.) + (d_.)*(x_)]^2*((a_) + (b_.)*Tan[
(c_.) + (d_.)*(x_)])^(n_.), x_Symbol] :> Simp[((e + f*x)^m*(a + b*Tan[c + d
*x])^(n + 1))/(b*d*(n + 1)), x] - Dist[(f*m)/(b*d*(n + 1)), Int[(e + f*x)^(
m - 1)*(a + b*Tan[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, n},
 x] && IGtQ[m, 0] && NeQ[n, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{x\sec^2(x)}{(a+b\tan(x))^2}\,dx &= -\frac{x}{b(a+b\tan(x))}+\frac{\int \frac{1}{a+b\tan(x)}\,dx}{b}\\ &= \frac{ax}{b\left(a^2+b^2\right)}-\frac{x}{b(a+b\tan(x))}+\frac{\int \frac{b-a\tan(x)}{a+b\tan(x)}\,dx}{a^2+b^2}\\ &= \frac{ax}{b\left(a^2+b^2\right)}+\frac{\log(a\cos(x)+b\sin(x))}{a^2+b^2}-\frac{x}{b(a+b\tan(x))}\end{aligned}$$

Mathematica [A] time = 0.18, size = 48, normalized size = 0.96

$$\frac{a\log(a\cos(x)+b\sin(x))-bx}{a^3+ab^2}+\frac{x\sin(x)}{a^2\cos(x)+ab\sin(x)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*Sec[x]^2)/(a + b*Tan[x])^2,x]
```

```
[Out] (-(b*x) + a*Log[a*Cos[x] + b*Sin[x]])/(a^3 + a*b^2) + (x*Sin[x])/(a^2*Cos[x
] + a*b*Sin[x])
```

fricas [A] time = 0.59, size = 80, normalized size = 1.60

$$-\frac{2\,bx\cos(x)-2\,ax\sin(x)-(a\cos(x)+b\sin(x))\log\left(2\,ab\cos(x)\sin(x)+\left(a^2-b^2\right)\cos(x)^2+b^2\right)}{2\left(\left(a^3+ab^2\right)\cos(x)+\left(a^2b+b^3\right)\sin(x)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sec(x)^2/(a+b*tan(x))^2,x, algorithm="fricas")
```

```
[Out] -1/2*(2*b*x*cos(x) - 2*a*x*sin(x) - (a*cos(x) + b*sin(x))*log(2*a*b*cos(x)*
sin(x) + (a^2 - b^2)*cos(x)^2 + b^2))/((a^3 + a*b^2)*cos(x) + (a^2*b + b^3)
*sin(x))
```

giac [B] time = 0.36, size = 322, normalized size = 6.44

$$-\frac{2\,bx\tan\left(\frac{1}{2}\,x\right)^2-a\log\left(\frac{4\left(a^2\tan\left(\frac{1}{2}\,x\right)^4-4\,ab\tan\left(\frac{1}{2}\,x\right)^3-2\,a^2\tan\left(\frac{1}{2}\,x\right)^2+4\,b^2\tan\left(\frac{1}{2}\,x\right)^2+4\,ab\tan\left(\frac{1}{2}\,x\right)+a^2\right)}{\tan\left(\frac{1}{2}\,x\right)^4+2\,\tan\left(\frac{1}{2}\,x\right)^2+1}\right)\tan\left(\frac{1}{2}\,x\right)^2+4\,ax\tan\left(\frac{1}{2}\cdots\right.}{2\left(a^3\tan\left(\frac{1}{2}\cdots\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sec(x)^2/(a+b*tan(x))^2,x, algorithm="giac")
```

```
[Out] -1/2*(2*b*x*tan(1/2*x)^2 - a*log(4*(a^2*tan(1/2*x)^4 - 4*a*b*tan(1/2*x)^3 -
 2*a^2*tan(1/2*x)^2 + 4*b^2*tan(1/2*x)^2 + 4*a*b*tan(1/2*x) + a^2)/(tan(1/2
*x)^4 + 2*tan(1/2*x)^2 + 1))*tan(1/2*x)^2 + 4*a*x*tan(1/2*x) + 2*b*log(4*(a
^2*tan(1/2*x)^4 - 4*a*b*tan(1/2*x)^3 - 2*a^2*tan(1/2*x)^2 + 4*b^2*tan(1/2*x
)^2 + 4*a*b*tan(1/2*x) + a^2)/(tan(1/2*x)^4 + 2*tan(1/2*x)^2 + 1))*tan(1/2*
x) - 2*b*x + a*log(4*(a^2*tan(1/2*x)^4 - 4*a*b*tan(1/2*x)^3 - 2*a^2*tan(1/2
*x)^2 + 4*b^2*tan(1/2*x)^2 + 4*a*b*tan(1/2*x) + a^2)/(tan(1/2*x)^4 + 2*tan(
1/2*x)^2 + 1)))/(a^3*tan(1/2*x)^2 + a*b^2*tan(1/2*x)^2 - 2*a^2*b*tan(1/2*x)
 - 2*b^3*tan(1/2*x) - a^3 - a*b^2)
```

maple [C] time = 0.32, size = 86, normalized size = 1.72

$$-\frac{2ix}{a^2+b^2}+\frac{2ix}{\left(-ib\,e^{2ix}+a\,e^{2ix}+ib+a\right)(-ib+a)}+\frac{\ln\left(e^{2ix}-\frac{ib+a}{ib-a}\right)}{a^2+b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*sec(x)^2/(a+b*tan(x))^2,x)
```

```
[Out] -2*I/(a^2+b^2)*x+2*I*x/(-I*b*exp(2*I*x)+a*exp(2*I*x)+I*b+a)/(-I*b+a)+1/(a^2
+b^2)*ln(exp(2*I*x)-(I*b+a)/(I*b-a))
```

maxima [B] time = 0.35, size = 250, normalized size = 5.00

$$-\frac{8\,abx\cos(2\,x)-4\left(a^2-b^2\right)x\sin(2\,x)-\left(\left(a^2+b^2\right)\cos(2\,x)^2+4\,ab\sin(2\,x)+\left(a^2+b^2\right)\sin(2\,x)^2+a^2+b^2+\right.}{2\left(a^4+2\,a^2b^2+b^4+\left(a^4+2\,a^2b^2+b^4\right)\cos(2\,x)^2+\left(a^4+2\,a^2b^2+b^4\right)\sin(2\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sec(x)^2/(a+b*tan(x))^2,x, algorithm="maxima")
```

```
[Out] -1/2*(8*a*b*x*cos(2*x) - 4*(a^2 - b^2)*x*sin(2*x) - ((a^2 + b^2)*cos(2*x)^2
 + 4*a*b*sin(2*x) + (a^2 + b^2)*sin(2*x)^2 + a^2 + b^2 + 2*(a^2 - b^2)*cos(
2*x))*log(((a^2 + b^2)*cos(2*x)^2 + 4*a*b*sin(2*x) + (a^2 + b^2)*sin(2*x)^2
 + a^2 + b^2 + 2*(a^2 - b^2)*cos(2*x))/(a^2 + b^2)))/(a^4 + 2*a^2*b^2 + b^4
 + (a^4 + 2*a^2*b^2 + b^4)*cos(2*x)^2 + (a^4 + 2*a^2*b^2 + b^4)*sin(2*x)^2
+ 2*(a^4 - b^4)*cos(2*x) + 4*(a^3*b + a*b^3)*sin(2*x))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x}{\cos(x)^2\,(a+b\tan(x))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(cos(x)^2*(a + b*tan(x))^2),x)
```

```
[Out] int(x/(cos(x)^2*(a + b*tan(x))^2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x \sec^2(x)}{(a + b\tan(x))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sec(x)**2/(a+b*tan(x))**2,x)
```

```
[Out] Integral(x*sec(x)**2/(a + b*tan(x))**2, x)
```

3.158 $\int \frac{x \csc^2(x)}{(a+b \cot(x))^2} \, dx$

Optimal. Leaf size=50

$$-\frac{ax}{b\left(a^2+b^2\right)}+\frac{\log(a\sin(x)+b\cos(x))}{a^2+b^2}+\frac{x}{b(a+b\cot(x))}$$

```
[Out] -a*x/b/(a^2+b^2)+x/b/(a+b*cot(x))+ln(b*cos(x)+a*sin(x))/(a^2+b^2)
```

Rubi [A] time = 0.08, antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {4425, 3484, 3530}

$$-\frac{ax}{b\left(a^2+b^2\right)}+\frac{\log(a\sin(x)+b\cos(x))}{a^2+b^2}+\frac{x}{b(a+b\cot(x))}$$

Antiderivative was successfully verified.

```
[In] Int[(x*Csc[x]^2)/(a + b*Cot[x])^2,x]
```

```
[Out] -((a*x)/(b*(a^2 + b^2))) + x/(b*(a + b*Cot[x])) + Log[b*Cos[x] + a*Sin[x]]/
(a^2 + b^2)
```

Rule 3484

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> Simp[(a*x)/(a
^2 + b^2), x] + Dist[b/(a^2 + b^2), Int[(b - a*Tan[c + d*x])/(a + b*Tan[c +
 d*x]), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

Rule 3530

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_) + (b_.)*tan[(e_.) + (f_.)*
(x_)]), x_Symbol] :> Simp[(c*Log[RemoveContent[a*Cos[e + f*x] + b*Sin[e + f
*x], x]])/(b*f), x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
 NeQ[a^2 + b^2, 0] && EqQ[a*c + b*d, 0]
```

Rule 4425

```
Int[Csc[(c_.) + (d_.)*(x_)]^2*(Cot[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_.)*
((e_.) + (f_.)*(x_))^(m_.), x_Symbol] :> -Simp[((e + f*x)^m*(a + b*Cot[c +
d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(f*m)/(b*d*(n + 1)), Int[(e + f*x)^
(m - 1)*(a + b*Cot[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, n}
, x] && IGtQ[m, 0] && NeQ[n, -1]
```

Rubi steps

$$\int \frac{x\csc^2(x)}{(a+b\cot(x))^2}\,dx = \frac{x}{b(a+b\cot(x))} - \frac{\int \frac{1}{a+b\cot(x)}\,dx}{b}$$
$$= -\frac{ax}{b\left(a^2+b^2\right)} + \frac{x}{b(a+b\cot(x))} + \frac{\int \frac{-b+a\cot(x)}{a+b\cot(x)}\,dx}{a^2+b^2}$$
$$= -\frac{ax}{b\left(a^2+b^2\right)} + \frac{x}{b(a+b\cot(x))} + \frac{\log(b\cos(x)+a\sin(x))}{a^2+b^2}$$

Mathematica [A] time = 0.19, size = 48, normalized size = 0.96

$$\frac{b\log(a\sin(x)+b\cos(x))-ax}{a^2b+b^3} + \frac{x\sin(x)}{ab\sin(x)+b^2\cos(x)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*Csc[x]^2)/(a + b*Cot[x])^2,x]
```

```
[Out] (-(a*x) + b*Log[b*Cos[x] + a*Sin[x]])/(a^2*b + b^3) + (x*Sin[x])/(b^2*Cos[x
] + a*b*Sin[x])
```

fricas [A] time = 0.74, size = 81, normalized size = 1.62

$$-\frac{2\,ax\cos(x) - 2\,bx\sin(x) - (b\cos(x)+a\sin(x))\log\left(2\,ab\cos(x)\sin(x) - \left(a^2-b^2\right)\cos(x)^2 + a^2\right)}{2\left(\left(a^2b+b^3\right)\cos(x) + \left(a^3+ab^2\right)\sin(x)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*csc(x)^2/(a+b*cot(x))^2,x, algorithm="fricas")
```

```
[Out] -1/2*(2*a*x*cos(x) - 2*b*x*sin(x) - (b*cos(x) + a*sin(x))*log(2*a*b*cos(x)*
sin(x) - (a^2 - b^2)*cos(x)^2 + a^2))/((a^2*b + b^3)*cos(x) + (a^3 + a*b^2)
*sin(x))
```

giac [B] time = 0.34, size = 322, normalized size = 6.44

$$-\frac{2\,ax\tan\left(\frac{1}{2}\,x\right)^2 - b\log\left(\frac{4\left(b^2\tan\left(\frac{1}{2}\,x\right)^4 - 4\,ab\tan\left(\frac{1}{2}\,x\right)^3 + 4\,a^2\tan\left(\frac{1}{2}\,x\right)^2 - 2\,b^2\tan\left(\frac{1}{2}\,x\right)^2 + 4\,ab\tan\left(\frac{1}{2}\,x\right) + b^2\right)}{\tan\left(\frac{1}{2}\,x\right)^4 + 2\,\tan\left(\frac{1}{2}\,x\right)^2 + 1}\right)\tan\left(\frac{1}{2}\,x\right)^2 + 4\,bx\tan\left(\frac{1}{2}\right.}{2\left(a^2b\tan\left(\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*csc(x)^2/(a+b*cot(x))^2,x, algorithm="giac")
```

```
[Out] -1/2*(2*a*x*tan(1/2*x)^2 - b*log(4*(b^2*tan(1/2*x)^4 - 4*a*b*tan(1/2*x)^3 +
 4*a^2*tan(1/2*x)^2 - 2*b^2*tan(1/2*x)^2 + 4*a*b*tan(1/2*x) + b^2)/(tan(1/2
*x)^4 + 2*tan(1/2*x)^2 + 1))*tan(1/2*x)^2 + 4*b*x*tan(1/2*x) + 2*a*log(4*(b
^2*tan(1/2*x)^4 - 4*a*b*tan(1/2*x)^3 + 4*a^2*tan(1/2*x)^2 - 2*b^2*tan(1/2*x
)^2 + 4*a*b*tan(1/2*x) + b^2)/(tan(1/2*x)^4 + 2*tan(1/2*x)^2 + 1))*tan(1/2*
x) - 2*a*x + b*log(4*(b^2*tan(1/2*x)^4 - 4*a*b*tan(1/2*x)^3 + 4*a^2*tan(1/2
*x)^2 - 2*b^2*tan(1/2*x)^2 + 4*a*b*tan(1/2*x) + b^2)/(tan(1/2*x)^4 + 2*tan(
1/2*x)^2 + 1)))/(a^2*b*tan(1/2*x)^2 + b^3*tan(1/2*x)^2 - 2*a^3*tan(1/2*x) -
 2*a*b^2*tan(1/2*x) - a^2*b - b^3)
```

maple [C] time = 0.31, size = 87, normalized size = 1.74

$$-\frac{2ix}{a^2+b^2}-\frac{2ix}{\left(ib\,e^{2ix}+a\,e^{2ix}+ib-a\right)(ib+a)}+\frac{\ln\left(e^{2ix}+\frac{ib-a}{ib+a}\right)}{a^2+b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*csc(x)^2/(a+b*cot(x))^2,x)
```

```
[Out] -2*I/(a^2+b^2)*x-2*I*x/(I*b*exp(2*I*x)+a*exp(2*I*x)+I*b-a)/(I*b+a)+1/(a^2+b
^2)*ln(exp(2*I*x)+(I*b-a)/(I*b+a))
```

maxima [B] time = 0.33, size = 250, normalized size = 5.00

$$-\frac{8\,abx\cos(2\,x)+4\left(a^2-b^2\right)x\sin(2\,x)-\left(\left(a^2+b^2\right)\cos(2\,x)^2+4\,ab\sin(2\,x)+\left(a^2+b^2\right)\sin(2\,x)^2+a^2+b^2-\right.}{2\left(a^4+2\,a^2b^2+b^4+\left(a^4+2\,a^2b^2+b^4\right)\cos(2\,x)^2+\left(a^4+2\,a^2b^2+b^4\right)\sin(2\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*csc(x)^2/(a+b*cot(x))^2,x, algorithm="maxima")
```

```
[Out] -1/2*(8*a*b*x*cos(2*x) + 4*(a^2 - b^2)*x*sin(2*x) - ((a^2 + b^2)*cos(2*x)^2
 + 4*a*b*sin(2*x) + (a^2 + b^2)*sin(2*x)^2 + a^2 + b^2 - 2*(a^2 - b^2)*cos(
2*x))*log(((a^2 + b^2)*cos(2*x)^2 + 4*a*b*sin(2*x) + (a^2 + b^2)*sin(2*x)^2
 + a^2 + b^2 - 2*(a^2 - b^2)*cos(2*x))/(a^2 + b^2)))/(a^4 + 2*a^2*b^2 + b^4
 + (a^4 + 2*a^2*b^2 + b^4)*cos(2*x)^2 + (a^4 + 2*a^2*b^2 + b^4)*sin(2*x)^2
- 2*(a^4 - b^4)*cos(2*x) + 4*(a^3*b + a*b^3)*sin(2*x))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x}{\sin(x)^2\,(a+b\cot(x))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(sin(x)^2*(a + b*cot(x))^2),x)
```

```
[Out] int(x/(sin(x)^2*(a + b*cot(x))^2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x\csc^2(x)}{(a+b\cot(x))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*csc(x)**2/(a+b*cot(x))**2,x)
```

```
[Out] Integral(x*csc(x)**2/(a + b*cot(x))**2, x)
```

3.159 $\int \frac{\sec^2(c+dx)}{a+b\tan^2(c+dx)}\,dx$

Optimal. Leaf size=32

$$\frac{\tan^{-1}\left(\frac{\sqrt{b}\tan(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b}\,d}$$

```
[Out] arctan(b^(1/2)*tan(d*x+c)/a^(1/2))/d/a^(1/2)/b^(1/2)
```

Rubi [A] time = 0.05, antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {3675, 205}

$$\frac{\tan^{-1}\left(\frac{\sqrt{b}\tan(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b}\,d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^2/(a + b*Tan[c + d*x]^2),x]
```

```
[Out] ArcTan[(Sqrt[b]*Tan[c + d*x])/Sqrt[a]]/(Sqrt[a]*Sqrt[b]*d)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 3675

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/(c^(m - 1)*f), Subst[Int[(c^2 + ff^2*x^2)^(m/2 - 1)*(a + b*(ff*x)^n)^p
, x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && In
tegerQ[m/2] && (IntegersQ[n, p] || IGtQ[m, 0] || IGtQ[p, 0] || EqQ[n^2, 4]
|| EqQ[n^2, 16])
```

Rubi steps

$$\int \frac{\sec^2(c+dx)}{a+b\tan^2(c+dx)}\,dx = \frac{\text{Subst}\left(\int \frac{1}{a+bx^2}\,dx, x, \tan(c+dx)\right)}{d}$$
$$= \frac{\tan^{-1}\left(\frac{\sqrt{b}\tan(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b}\,d}$$

Mathematica [A] time = 0.09, size = 32, normalized size = 1.00

$$\frac{\tan^{-1}\left(\frac{\sqrt{b}\tan(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b}\,d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^2/(a + b*Tan[c + d*x]^2),x]
```

```
[Out] ArcTan[(Sqrt[b]*Tan[c + d*x])/Sqrt[a]]/(Sqrt[a]*Sqrt[b]*d)
```

fricas [B] time = 0.62, size = 205, normalized size = 6.41

$$\left[-\frac{\sqrt{-ab}\log\left(\frac{\left(a^2+6ab+b^2\right)\cos(dx+c)^4-2\left(3ab+b^2\right)\cos(dx+c)^2+4\left((a+b)\cos(dx+c)^3-b\cos(dx+c)\right)\sqrt{-ab}\sin(dx+c)+b^2}{\left(a^2-2ab+b^2\right)\cos(dx+c)^4+2\left(ab-b^2\right)\cos(dx+c)^2+b^2}\right)}{4abd}, -\frac{\sqrt{ab}\arctan\left(\frac{((}{2}\right.}{2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a+b*tan(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] [-1/4*sqrt(-a*b)*log(((a^2 + 6*a*b + b^2)*cos(d*x + c)^4 - 2*(3*a*b + b^2)*
cos(d*x + c)^2 + 4*((a + b)*cos(d*x + c)^3 - b*cos(d*x + c))*sqrt(-a*b)*sin
(d*x + c) + b^2)/((a^2 - 2*a*b + b^2)*cos(d*x + c)^4 + 2*(a*b - b^2)*cos(d*
x + c)^2 + b^2))/(a*b*d), -1/2*sqrt(a*b)*arctan(1/2*((a + b)*cos(d*x + c)^2
 - b)*sqrt(a*b)/(a*b*cos(d*x + c)*sin(d*x + c)))/(a*b*d)]
```

giac [A] time = 0.62, size = 40, normalized size = 1.25

$$\frac{\pi\left\lfloor\frac{dx+c}{\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(b)+\arctan\left(\frac{b\tan(dx+c)}{\sqrt{ab}}\right)}{\sqrt{ab}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a+b*tan(d*x+c)^2),x, algorithm="giac")
```

```
[Out] (pi*floor((d*x + c)/pi + 1/2)*sgn(b) + arctan(b*tan(d*x + c)/sqrt(a*b)))/(s
qrt(a*b)*d)
```

maple [A] time = 0.32, size = 24, normalized size = 0.75

$$\frac{\arctan\left(\frac{\tan(dx+c)b}{\sqrt{ab}}\right)}{d\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2/(a+b*tan(d*x+c)^2),x)
```

```
[Out] 1/d/(a*b)^(1/2)*arctan(tan(d*x+c)*b/(a*b)^(1/2))
```

maxima [A] time = 0.43, size = 23, normalized size = 0.72

$$\frac{\arctan\left(\frac{b\tan(dx+c)}{\sqrt{ab}}\right)}{\sqrt{ab}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a+b*tan(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] arctan(b*tan(d*x + c)/sqrt(a*b))/(sqrt(a*b)*d)
```

mupad [B] time = 2.56, size = 24, normalized size = 0.75

$$\frac{\operatorname{atan}\left(\frac{\sqrt{b}\,\tan(c+d\,x)}{\sqrt{a}}\right)}{\sqrt{a}\,\sqrt{b}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^2*(a + b*tan(c + d*x)^2)),x)
```

```
[Out] atan((b^(1/2)*tan(c + d*x))/a^(1/2))/(a^(1/2)*b^(1/2)*d)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec^2(c+dx)}{a+b\tan^2(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2/(a+b*tan(d*x+c)**2),x)
```

```
[Out] Integral(sec(c + d*x)**2/(a + b*tan(c + d*x)**2), x)
```

3.160 $\int \frac{x\sec^2(c+dx)}{a+b\tan^2(c+dx)}\,dx$

Optimal. Leaf size=211

$$-\frac{\text{Li}_2\left(-\frac{(a-b)e^{2i(c+dx)}}{\left(\sqrt{a}-\sqrt{b}\right)^2}\right)}{4\sqrt{a}\,\sqrt{b}\,d^2}+\frac{\text{Li}_2\left(-\frac{(a-b)e^{2i(c+dx)}}{\left(\sqrt{a}+\sqrt{b}\right)^2}\right)}{4\sqrt{a}\,\sqrt{b}\,d^2}-\frac{ix\log\left(1+\frac{(a-b)e^{2i(c+dx)}}{\left(\sqrt{a}-\sqrt{b}\right)^2}\right)}{2\sqrt{a}\,\sqrt{b}\,d}+\frac{ix\log\left(1+\frac{(a-b)e^{2i(c+dx)}}{\left(\sqrt{a}+\sqrt{b}\right)^2}\right)}{2\sqrt{a}\,\sqrt{b}\,d}$$

```
[Out] -1/2*I*x*ln(1+(a-b)*exp(2*I*(d*x+c))/(a^(1/2)-b^(1/2))^2)/d/a^(1/2)/b^(1/2)
+1/2*I*x*ln(1+(a-b)*exp(2*I*(d*x+c))/(a^(1/2)+b^(1/2))^2)/d/a^(1/2)/b^(1/2)
-1/4*polylog(2,-(a-b)*exp(2*I*(d*x+c))/(a^(1/2)-b^(1/2))^2)/d^2/a^(1/2)/b^(
1/2)+1/4*polylog(2,-(a-b)*exp(2*I*(d*x+c))/(a^(1/2)+b^(1/2))^2)/d^2/a^(1/2)
/b^(1/2)
```

Rubi [A] time = 0.53, antiderivative size = 211, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {4588, 3321, 2264, 2190, 2279, 2391}

$$-\frac{\text{PolyLog}\left(2,-\frac{(a-b)e^{2i(c+dx)}}{\left(\sqrt{a}-\sqrt{b}\right)^2}\right)}{4\sqrt{a}\,\sqrt{b}\,d^2}+\frac{\text{PolyLog}\left(2,-\frac{(a-b)e^{2i(c+dx)}}{\left(\sqrt{a}+\sqrt{b}\right)^2}\right)}{4\sqrt{a}\,\sqrt{b}\,d^2}-\frac{ix\log\left(1+\frac{(a-b)e^{2i(c+dx)}}{\left(\sqrt{a}-\sqrt{b}\right)^2}\right)}{2\sqrt{a}\,\sqrt{b}\,d}+\frac{ix\log\left(1+\frac{(a-b)e^{2i(c+dx)}}{\left(\sqrt{a}+\sqrt{b}\right)^2}\right)}{2\sqrt{a}\,\sqrt{b}\,d}$$

Antiderivative was successfully verified.

```
[In] Int[(x*Sec[c + d*x]^2)/(a + b*Tan[c + d*x]^2),x]
```

```
[Out] ((-I/2)*x*Log[1 + ((a - b)*E^((2*I)*(c + d*x)))/(Sqrt[a] - Sqrt[b])^2])/(Sq
rt[a]*Sqrt[b]*d) + ((I/2)*x*Log[1 + ((a - b)*E^((2*I)*(c + d*x)))/(Sqrt[a]
+ Sqrt[b])^2])/(Sqrt[a]*Sqrt[b]*d) - PolyLog[2, -(((a - b)*E^((2*I)*(c + d*
x)))/(Sqrt[a] - Sqrt[b])^2)]/(4*Sqrt[a]*Sqrt[b]*d^2) + PolyLog[2, -(((a - b
)*E^((2*I)*(c + d*x)))/(Sqrt[a] + Sqrt[b])^2)]/(4*Sqrt[a]*Sqrt[b]*d^2)
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2264

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c)/q, Int[
```

```
((f + g*x)^m*F^u)/(b - q + 2*c*F^u), x], x] - Dist[(2*c)/q, Int[((f + g*x)^
m*F^u)/(b + q + 2*c*F^u), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3321

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(
x_)]), x_Symbol] :> Dist[2, Int[((c + d*x)^m*E^(I*Pi*(k - 1/2))*E^(I*(e + f
*x)))/(b + 2*a*E^(I*Pi*(k - 1/2))*E^(I*(e + f*x)) - b*E^(2*I*k*Pi)*E^(2*I*(
e + f*x))), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[2*k] && NeQ[
a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 4588

```
Int[(((f_.) + (g_.)*(x_))^(m_.)*Sec[(d_.) + (e_.)*(x_)]^2)/((b_) + (c_.)*Ta
n[(d_.) + (e_.)*(x_)]^2), x_Symbol] :> Dist[2, Int[(f + g*x)^m/(b + c + (b
- c)*Cos[2*d + 2*e*x]), x], x] /; FreeQ[{b, c, d, e, f, g}, x] && IGtQ[m, 0
]
```

Rubi steps

$$\int \frac{x\sec^2(c+dx)}{a+b\tan^2(c+dx)}\,dx = 2\int \frac{x}{a+b+(a-b)\cos(2c+2dx)}\,dx$$
$$= 4\int \frac{e^{i(2c+2dx)}x}{a-b+2(a+b)e^{i(2c+2dx)}+(a-b)e^{2i(2c+2dx)}}\,dx$$
$$= \frac{(2(a-b))\int \frac{e^{i(2c+2dx)}x}{-4\sqrt{a}\sqrt{b}+2(a+b)+2(a-b)e^{i(2c+2dx)}}\,dx}{\sqrt{a}\sqrt{b}} - \frac{(2(a-b))\int \frac{e^{i(2c+2dx)}x}{4\sqrt{a}\sqrt{b}+2(a+b)+2(a-b)e^{i(2c+2dx)}}\,d}{\sqrt{a}\sqrt{b}}$$
$$= -\frac{ix\log\left(1+\frac{(a-b)e^{2i(c+dx)}}{\left(\sqrt{a}-\sqrt{b}\right)^2}\right)}{2\sqrt{a}\sqrt{b}d} + \frac{ix\log\left(1+\frac{(a-b)e^{2i(c+dx)}}{\left(\sqrt{a}+\sqrt{b}\right)^2}\right)}{2\sqrt{a}\sqrt{b}d} + \frac{i\int \log\left(1+\frac{2(a-b)e^{i(2c+2dx)}}{-4\sqrt{a}\sqrt{b}+2(a+b)}\right)dx}{2\sqrt{a}\sqrt{b}d}$$
$$= -\frac{ix\log\left(1+\frac{(a-b)e^{2i(c+dx)}}{\left(\sqrt{a}-\sqrt{b}\right)^2}\right)}{2\sqrt{a}\sqrt{b}d} + \frac{ix\log\left(1+\frac{(a-b)e^{2i(c+dx)}}{\left(\sqrt{a}+\sqrt{b}\right)^2}\right)}{2\sqrt{a}\sqrt{b}d} + \frac{\text{Subst}\left(\int \frac{\log\left(1+\frac{2(a-b)x}{-4\sqrt{a}\sqrt{b}+2(a+b)}\right)}{x}\,dx,\right.}{4\sqrt{a}\sqrt{b}d^2}$$
$$= -\frac{ix\log\left(1+\frac{(a-b)e^{2i(c+dx)}}{\left(\sqrt{a}-\sqrt{b}\right)^2}\right)}{2\sqrt{a}\sqrt{b}d} + \frac{ix\log\left(1+\frac{(a-b)e^{2i(c+dx)}}{\left(\sqrt{a}+\sqrt{b}\right)^2}\right)}{2\sqrt{a}\sqrt{b}d} - \frac{\text{Li}_2\left(-\frac{(a-b)e^{2i(c+dx)}}{\left(\sqrt{a}-\sqrt{b}\right)^2}\right)}{4\sqrt{a}\sqrt{b}d^2} + \frac{\text{Li}_2\left(-\frac{(a-b)}{(\sqrt{a}\right.}}{4\sqrt{a}\sqrt{}}$$

Mathematica [B] time = 6.44, size = 512, normalized size = 2.43

$$\frac{x\left(-\sqrt{a}\,\text{Li}_2\left(\frac{\sqrt{b}(1-i\tan(c+dx))}{i\sqrt{-a}+\sqrt{b}}\right)-\sqrt{a}\,\text{Li}_2\left(\frac{\sqrt{b}(i\tan(c+dx)+1)}{i\sqrt{-a}+\sqrt{b}}\right)+\sqrt{a}\,\text{Li}_2\left(-\frac{\sqrt{b}(\tan(c+dx)-i)}{\sqrt{-a}+i\sqrt{b}}\right)+\sqrt{a}\,\text{Li}_2\left(\frac{\sqrt{b}(\tan(c+dx)+i)}{\sqrt{-a}+i\sqrt{b}}\right)+4i\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x*Sec[c + d*x]^2)/(a + b*Tan[c + d*x]^2),x]
```

```
[Out] (x*((4*I)*Sqrt[-a]*c*ArcTan[(Sqrt[b]*Tan[c + d*x])/Sqrt[a]] - Sqrt[a]*Log[1
 + I*Tan[c + d*x]]*Log[(Sqrt[-a] - Sqrt[b]*Tan[c + d*x])/(Sqrt[-a] - I*Sqrt
[b])] + Sqrt[a]*Log[1 - I*Tan[c + d*x]]*Log[(Sqrt[-a] - Sqrt[b]*Tan[c + d*x
])/(Sqrt[-a] + I*Sqrt[b])] - Sqrt[a]*Log[1 - I*Tan[c + d*x]]*Log[(Sqrt[-a]
+ Sqrt[b]*Tan[c + d*x])/(Sqrt[-a] - I*Sqrt[b])] + Sqrt[a]*Log[1 + I*Tan[c +
 d*x]]*Log[(Sqrt[-a] + Sqrt[b]*Tan[c + d*x])/(Sqrt[-a] + I*Sqrt[b])] - Sqrt
[a]*PolyLog[2, (Sqrt[b]*(1 - I*Tan[c + d*x]))/(I*Sqrt[-a] + Sqrt[b])] - Sqr
t[a]*PolyLog[2, (Sqrt[b]*(1 + I*Tan[c + d*x]))/(I*Sqrt[-a] + Sqrt[b])] + Sq
rt[a]*PolyLog[2, -((Sqrt[b]*(-I + Tan[c + d*x]))/(Sqrt[-a] + I*Sqrt[b]))] +
 Sqrt[a]*PolyLog[2, (Sqrt[b]*(I + Tan[c + d*x]))/(Sqrt[-a] + I*Sqrt[b])]))/
```

```
(2*Sqrt[-a^2]*Sqrt[b]*d*((2*I)*c + Log[1 - I*Tan[c + d*x]] - Log[1 + I*Tan[
c + d*x]]))
```

fricas [B] time = 1.30, size = 3344, normalized size = 15.85

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sec(d*x+c)^2/(a+b*tan(d*x+c)^2),x, algorithm="fricas")

[Out] 1/16*(-4*I*(a - b)*sqrt(a*b/(a^2 - 2*a*b + b^2))*c*log(2*sqrt(-(2*(a - b)*s
qrt(a*b/(a^2 - 2*a*b + b^2)) + a + b)/(a - b)) + 2*cos(d*x + c) + 2*I*sin(d
*x + c)) + 4*I*(a - b)*sqrt(a*b/(a^2 - 2*a*b + b^2))*c*log(2*sqrt(-(2*(a -
b)*sqrt(a*b/(a^2 - 2*a*b + b^2)) + a + b)/(a - b)) + 2*cos(d*x + c) - 2*I*s
in(d*x + c)) + 4*I*(a - b)*sqrt(a*b/(a^2 - 2*a*b + b^2))*c*log(2*sqrt(-(2*(
a - b)*sqrt(a*b/(a^2 - 2*a*b + b^2)) + a + b)/(a - b)) - 2*cos(d*x + c) + 2
*I*sin(d*x + c)) - 4*I*(a - b)*sqrt(a*b/(a^2 - 2*a*b + b^2))*c*log(2*sqrt(-
(2*(a - b)*sqrt(a*b/(a^2 - 2*a*b + b^2)) + a + b)/(a - b)) - 2*cos(d*x + c)
 - 2*I*sin(d*x + c)) + 4*I*(a - b)*sqrt(a*b/(a^2 - 2*a*b + b^2))*c*log(2*sq
rt((2*(a - b)*sqrt(a*b/(a^2 - 2*a*b + b^2)) - a - b)/(a - b)) + 2*cos(d*x +
 c) + 2*I*sin(d*x + c)) - 4*I*(a - b)*sqrt(a*b/(a^2 - 2*a*b + b^2))*c*log(2
*sqrt((2*(a - b)*sqrt(a*b/(a^2 - 2*a*b + b^2)) - a - b)/(a - b)) + 2*cos(d*
x + c) - 2*I*sin(d*x + c)) - 4*I*(a - b)*sqrt(a*b/(a^2 - 2*a*b + b^2))*c*lo
g(2*sqrt((2*(a - b)*sqrt(a*b/(a^2 - 2*a*b + b^2)) - a - b)/(a - b)) - 2*cos
(d*x + c) + 2*I*sin(d*x + c)) + 4*I*(a - b)*sqrt(a*b/(a^2 - 2*a*b + b^2))*c
*log(2*sqrt((2*(a - b)*sqrt(a*b/(a^2 - 2*a*b + b^2)) - a - b)/(a - b)) - 2*
cos(d*x + c) - 2*I*sin(d*x + c)) + 4*(a - b)*sqrt(a*b/(a^2 - 2*a*b + b^2))*
dilog(1/2*((2*(a + b)*cos(d*x + c) + (2*I*a + 2*I*b)*sin(d*x + c) - 4*((a -
 b)*cos(d*x + c) - (-I*a + I*b)*sin(d*x + c))*sqrt(a*b/(a^2 - 2*a*b + b^2))
)*sqrt(-(2*(a - b)*sqrt(a*b/(a^2 - 2*a*b + b^2)) + a + b)/(a - b)) - 2*a +
2*b)/(a - b) + 1) + 4*(a - b)*sqrt(a*b/(a^2 - 2*a*b + b^2))*dilog(-1/2*((2*
(a + b)*cos(d*x + c) - (2*I*a + 2*I*b)*sin(d*x + c) - 4*((a - b)*cos(d*x +
c) + (-I*a + I*b)*sin(d*x + c))*sqrt(a*b/(a^2 - 2*a*b + b^2)))*sqrt(-(2*(a
- b)*sqrt(a*b/(a^2 - 2*a*b + b^2)) + a + b)/(a - b)) + 2*a - 2*b)/(a - b) +
 1) + 4*(a - b)*sqrt(a*b/(a^2 - 2*a*b + b^2))*dilog(1/2*((2*(a + b)*cos(d*x
 + c) + (-2*I*a - 2*I*b)*sin(d*x + c) - 4*((a - b)*cos(d*x + c) - (I*a - I*
b)*sin(d*x + c))*sqrt(a*b/(a^2 - 2*a*b + b^2)))*sqrt(-(2*(a - b)*sqrt(a*b/(
a^2 - 2*a*b + b^2)) + a + b)/(a - b)) - 2*a + 2*b)/(a - b) + 1) + 4*(a - b)
*sqrt(a*b/(a^2 - 2*a*b + b^2))*dilog(-1/2*((2*(a + b)*cos(d*x + c) - (-2*I*
a - 2*I*b)*sin(d*x + c) - 4*((a - b)*cos(d*x + c) + (I*a - I*b)*sin(d*x + c
))*sqrt(a*b/(a^2 - 2*a*b + b^2)))*sqrt(-(2*(a - b)*sqrt(a*b/(a^2 - 2*a*b +
b^2)) + a + b)/(a - b)) + 2*a - 2*b)/(a - b) + 1) - 4*(a - b)*sqrt(a*b/(a^2
 - 2*a*b + b^2))*dilog(1/2*((2*(a + b)*cos(d*x + c) + (2*I*a + 2*I*b)*sin(d
*x + c) + 4*((a - b)*cos(d*x + c) + (I*a - I*b)*sin(d*x + c))*sqrt(a*b/(a^2
 - 2*a*b + b^2)))*sqrt((2*(a - b)*sqrt(a*b/(a^2 - 2*a*b + b^2)) - a - b)/(a
```

```
 - b)) - 2*a + 2*b)/(a - b) + 1) - 4*(a - b)*sqrt(a*b/(a^2 - 2*a*b + b^2))*
dilog(-1/2*((2*(a + b)*cos(d*x + c) - (2*I*a + 2*I*b)*sin(d*x + c) + 4*((a
 - b)*cos(d*x + c) - (I*a - I*b)*sin(d*x + c))*sqrt(a*b/(a^2 - 2*a*b + b^2))
)*sqrt((2*(a - b)*sqrt(a*b/(a^2 - 2*a*b + b^2)) - a - b)/(a - b)) + 2*a - 2
*b)/(a - b) + 1) - 4*(a - b)*sqrt(a*b/(a^2 - 2*a*b + b^2))*dilog(1/2*((2*(a
 + b)*cos(d*x + c) + (-2*I*a - 2*I*b)*sin(d*x + c) + 4*((a - b)*cos(d*x + c
) + (-I*a + I*b)*sin(d*x + c))*sqrt(a*b/(a^2 - 2*a*b + b^2)))*sqrt((2*(a -
b)*sqrt(a*b/(a^2 - 2*a*b + b^2)) - a - b)/(a - b)) - 2*a + 2*b)/(a - b) + 1
) - 4*(a - b)*sqrt(a*b/(a^2 - 2*a*b + b^2))*dilog(-1/2*((2*(a + b)*cos(d*x
+ c) - (-2*I*a - 2*I*b)*sin(d*x + c) + 4*((a - b)*cos(d*x + c) - (-I*a + I*
b)*sin(d*x + c))*sqrt(a*b/(a^2 - 2*a*b + b^2)))*sqrt((2*(a - b)*sqrt(a*b/(a
^2 - 2*a*b + b^2)) - a - b)/(a - b)) + 2*a - 2*b)/(a - b) + 1) + 4*(I*(a -
b)*d*x + I*(a - b)*c)*sqrt(a*b/(a^2 - 2*a*b + b^2))*log(-1/2*((2*(a + b)*co
s(d*x + c) + (2*I*a + 2*I*b)*sin(d*x + c) - 4*((a - b)*cos(d*x + c) - (-I*a
 + I*b)*sin(d*x + c))*sqrt(a*b/(a^2 - 2*a*b + b^2)))*sqrt(-(2*(a - b)*sqrt(
a*b/(a^2 - 2*a*b + b^2)) + a + b)/(a - b)) - 2*a + 2*b)/(a - b)) + 4*(-I*(a
 - b)*d*x - I*(a - b)*c)*sqrt(a*b/(a^2 - 2*a*b + b^2))*log(1/2*((2*(a + b)*
cos(d*x + c) - (2*I*a + 2*I*b)*sin(d*x + c) - 4*((a - b)*cos(d*x + c) + (-I
*a + I*b)*sin(d*x + c))*sqrt(a*b/(a^2 - 2*a*b + b^2)))*sqrt(-(2*(a - b)*sqr
t(a*b/(a^2 - 2*a*b + b^2)) + a + b)/(a - b)) + 2*a - 2*b)/(a - b)) + 4*(-I*
(a - b)*d*x - I*(a - b)*c)*sqrt(a*b/(a^2 - 2*a*b + b^2))*log(-1/2*((2*(a +
b)*cos(d*x + c) + (-2*I*a - 2*I*b)*sin(d*x + c) - 4*((a - b)*cos(d*x + c) -
 (I*a - I*b)*sin(d*x + c))*sqrt(a*b/(a^2 - 2*a*b + b^2)))*sqrt(-(2*(a - b)*
sqrt(a*b/(a^2 - 2*a*b + b^2)) + a + b)/(a - b)) - 2*a + 2*b)/(a - b)) + 4*(
I*(a - b)*d*x + I*(a - b)*c)*sqrt(a*b/(a^2 - 2*a*b + b^2))*log(1/2*((2*(a +
 b)*cos(d*x + c) - (-2*I*a - 2*I*b)*sin(d*x + c) - 4*((a - b)*cos(d*x + c)
+ (I*a - I*b)*sin(d*x + c))*sqrt(a*b/(a^2 - 2*a*b + b^2)))*sqrt(-(2*(a - b)
*sqrt(a*b/(a^2 - 2*a*b + b^2)) + a + b)/(a - b)) + 2*a - 2*b)/(a - b)) + 4*
(-I*(a - b)*d*x - I*(a - b)*c)*sqrt(a*b/(a^2 - 2*a*b + b^2))*log(-1/2*((2*(
a + b)*cos(d*x + c) + (2*I*a + 2*I*b)*sin(d*x + c) + 4*((a - b)*cos(d*x + c
) + (I*a - I*b)*sin(d*x + c))*sqrt(a*b/(a^2 - 2*a*b + b^2)))*sqrt((2*(a - b
)*sqrt(a*b/(a^2 - 2*a*b + b^2)) - a - b)/(a - b)) - 2*a + 2*b)/(a - b)) + 4
*(I*(a - b)*d*x + I*(a - b)*c)*sqrt(a*b/(a^2 - 2*a*b + b^2))*log(1/2*((2*(a
 + b)*cos(d*x + c) - (2*I*a + 2*I*b)*sin(d*x + c) + 4*((a - b)*cos(d*x + c)
 - (I*a - I*b)*sin(d*x + c))*sqrt(a*b/(a^2 - 2*a*b + b^2)))*sqrt((2*(a - b)
*sqrt(a*b/(a^2 - 2*a*b + b^2)) - a - b)/(a - b)) + 2*a - 2*b)/(a - b)) + 4*
(I*(a - b)*d*x + I*(a - b)*c)*sqrt(a*b/(a^2 - 2*a*b + b^2))*log(-1/2*((2*(a
 + b)*cos(d*x + c) + (-2*I*a - 2*I*b)*sin(d*x + c) + 4*((a - b)*cos(d*x + c
) + (-I*a + I*b)*sin(d*x + c))*sqrt(a*b/(a^2 - 2*a*b + b^2)))*sqrt((2*(a -
b)*sqrt(a*b/(a^2 - 2*a*b + b^2)) - a - b)/(a - b)) - 2*a + 2*b)/(a - b)) +
4*(-I*(a - b)*d*x - I*(a - b)*c)*sqrt(a*b/(a^2 - 2*a*b + b^2))*log(1/2*((2*
(a + b)*cos(d*x + c) - (-2*I*a - 2*I*b)*sin(d*x + c) + 4*((a - b)*cos(d*x +
 c) - (-I*a + I*b)*sin(d*x + c))*sqrt(a*b/(a^2 - 2*a*b + b^2)))*sqrt((2*(a
- b)*sqrt(a*b/(a^2 - 2*a*b + b^2)) - a - b)/(a - b)) + 2*a - 2*b)/(a - b)))
/(a*b*d^2)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x \sec(dx+c)^2}{b \tan(dx+c)^2 + a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sec(d*x+c)^2/(a+b*tan(d*x+c)^2),x, algorithm="giac")
```

```
[Out] integrate(x*sec(d*x + c)^2/(b*tan(d*x + c)^2 + a), x)
```

maple [B] time = 0.60, size = 1003, normalized size = 4.75

$$-\frac{acx}{d\sqrt{ab}\left(-2\sqrt{ab}-a-b\right)}-\frac{bcx}{d\sqrt{ab}\left(-2\sqrt{ab}-a-b\right)}-\frac{i\ln\left(1-\frac{(a-b)e^{2i(dx+c)}}{-2\sqrt{ab}-a-b}\right)ax}{2d\sqrt{ab}\left(-2\sqrt{ab}-a-b\right)}-\frac{\operatorname{polylog}\left(2,\frac{(a-b)e^{2i(dx+c)}}{-2\sqrt{ab}-a-b}\right)}{2d^2\left(-2\sqrt{ab}-a-b\right)}-\text{polyl}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*sec(d*x+c)^2/(a+b*tan(d*x+c)^2),x)
```

```
[Out] -1/d/(a*b)^(1/2)/(-2*(a*b)^(1/2)-a-b)*a*c*x-1/d/(a*b)^(1/2)/(-2*(a*b)^(1/2)
-a-b)*b*c*x-1/2*I/d^2/(a*b)^(1/2)/(-2*(a*b)^(1/2)-a-b)*ln(1-(a-b)*exp(2*I*(
d*x+c))/(-2*(a*b)^(1/2)-a-b))*a*c-1/4/d^2/(a*b)^(1/2)/(-2*(a*b)^(1/2)-a-b)*
polylog(2,(a-b)*exp(2*I*(d*x+c))/(-2*(a*b)^(1/2)-a-b))*b-1/2/d^2/(a*b)^(1/2
)/(-2*(a*b)^(1/2)-a-b)*a*c^2-1/2/d^2/(a*b)^(1/2)/(-2*(a*b)^(1/2)-a-b)*b*c^2
-1/4/d^2/(a*b)^(1/2)/(-2*(a*b)^(1/2)-a-b)*polylog(2,(a-b)*exp(2*I*(d*x+c))/
(-2*(a*b)^(1/2)-a-b))*a-I/d^2*c/(a*b)^(1/2)*arctanh(1/4*(2*(a-b)*exp(2*I*(d
*x+c))+2*a+2*b)/(a*b)^(1/2))-1/2*I/d^2/(a*b)^(1/2)*ln(1-(a-b)*exp(2*I*(d*x+
c))/(2*(a*b)^(1/2)-a-b))*c-I/d^2/(-2*(a*b)^(1/2)-a-b)*ln(1-(a-b)*exp(2*I*(d
*x+c))/(-2*(a*b)^(1/2)-a-b))*c-1/2*I/d/(a*b)^(1/2)*ln(1-(a-b)*exp(2*I*(d*x+
c))/(2*(a*b)^(1/2)-a-b))*x-I/d/(-2*(a*b)^(1/2)-a-b)*ln(1-(a-b)*exp(2*I*(d*x
+c))/(-2*(a*b)^(1/2)-a-b))*x-1/2/d^2/(a*b)^(1/2)*c^2-1/d^2/(-2*(a*b)^(1/2)-
a-b)*c^2-1/2/d^2/(-2*(a*b)^(1/2)-a-b)*polylog(2,(a-b)*exp(2*I*(d*x+c))/(-2*
(a*b)^(1/2)-a-b))-1/4/d^2/(a*b)^(1/2)*polylog(2,(a-b)*exp(2*I*(d*x+c))/(2*(
a*b)^(1/2)-a-b))-1/2/(a*b)^(1/2)/(-2*(a*b)^(1/2)-a-b)*a*x^2-1/2/(a*b)^(1/2)
/(-2*(a*b)^(1/2)-a-b)*b*x^2-1/d/(a*b)^(1/2)*c*x-2/d/(-2*(a*b)^(1/2)-a-b)*c*
x-1/2*I/d/(a*b)^(1/2)/(-2*(a*b)^(1/2)-a-b)*ln(1-(a-b)*exp(2*I*(d*x+c))/(-2*
(a*b)^(1/2)-a-b))*b*x-1/2*I/d^2/(a*b)^(1/2)/(-2*(a*b)^(1/2)-a-b)*ln(1-(a-b)
*exp(2*I*(d*x+c))/(-2*(a*b)^(1/2)-a-b))*b*c-1/2*I/d/(a*b)^(1/2)/(-2*(a*b)^(
1/2)-a-b)*ln(1-(a-b)*exp(2*I*(d*x+c))/(-2*(a*b)^(1/2)-a-b))*a*x-1/2/(a*b)^(
1/2)*x^2-1/(-2*(a*b)^(1/2)-a-b)*x^2
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x \sec(dx+c)^2}{b \tan(dx+c)^2 + a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sec(d*x+c)^2/(a+b*tan(d*x+c)^2),x, algorithm="maxima")
[Out] integrate(x*sec(d*x + c)^2/(b*tan(d*x + c)^2 + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x}{\cos\left(c+d\,x\right)^2\,\left(b\,\tan\left(c+d\,x\right)^2+a\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(cos(c + d*x)^2*(a + b*tan(c + d*x)^2)),x)
[Out] int(x/(cos(c + d*x)^2*(a + b*tan(c + d*x)^2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x\sec^2\left(c+dx\right)}{a+b\tan^2\left(c+dx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sec(d*x+c)**2/(a+b*tan(d*x+c)**2),x)
[Out] Integral(x*sec(c + d*x)**2/(a + b*tan(c + d*x)**2), x)
```

3.161 $\int \frac{x^2 \sec^2(c+dx)}{a+b\tan^2(c+dx)}\,dx$

Optimal. Leaf size=337

$$\frac{i\text{Li}_3\left(-\frac{\left(\sqrt{a}-\sqrt{b}\right)e^{2i(c+dx)}}{\sqrt{a}+\sqrt{b}}\right)}{4\sqrt{a}\,\sqrt{b}\,d^3}-\frac{i\text{Li}_3\left(-\frac{\left(\sqrt{a}+\sqrt{b}\right)e^{2i(c+dx)}}{\sqrt{a}-\sqrt{b}}\right)}{4\sqrt{a}\,\sqrt{b}\,d^3}-\frac{x\text{Li}_2\left(-\frac{(a-b)e^{2i(c+dx)}}{\left(\sqrt{a}-\sqrt{b}\right)^2}\right)}{2\sqrt{a}\,\sqrt{b}\,d^2}+\frac{x\text{Li}_2\left(-\frac{(a-b)e^{2i(c+dx)}}{\left(\sqrt{a}+\sqrt{b}\right)^2}\right)}{2\sqrt{a}\,\sqrt{b}\,d^2}-\frac{ix^2\log\left(1+\frac{(a-b)e^{2i(c}}{\left(\sqrt{a}-\sqrt{b}\right.}\right.}{2\sqrt{a}\,\sqrt{b}\,d}$$

```
[Out] -1/2*I*x^2*ln(1+(a-b)*exp(2*I*(d*x+c))/(a^(1/2)-b^(1/2))^2)/d/a^(1/2)/b^(1/
2)+1/2*I*x^2*ln(1+(a-b)*exp(2*I*(d*x+c))/(a^(1/2)+b^(1/2))^2)/d/a^(1/2)/b^(
1/2)-1/2*x*polylog(2,-(a-b)*exp(2*I*(d*x+c))/(a^(1/2)-b^(1/2))^2)/d^2/a^(1/
2)/b^(1/2)+1/2*x*polylog(2,-(a-b)*exp(2*I*(d*x+c))/(a^(1/2)+b^(1/2))^2)/d^2
/a^(1/2)/b^(1/2)+1/4*I*polylog(3,-exp(2*I*(d*x+c))*(a^(1/2)-b^(1/2))/(a^(1/
2)+b^(1/2)))/d^3/a^(1/2)/b^(1/2)-1/4*I*polylog(3,-exp(2*I*(d*x+c))*(a^(1/2)
+b^(1/2))/(a^(1/2)-b^(1/2)))/d^3/a^(1/2)/b^(1/2)
```

Rubi [A] time = 0.90, antiderivative size = 337, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 7, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.269, Rules used = {4588, 3321, 2264, 2190, 2531, 2282, 6589}

$$-\frac{x\text{PolyLog}\left(2,-\frac{(a-b)e^{2i(c+dx)}}{\left(\sqrt{a}-\sqrt{b}\right)^2}\right)}{2\sqrt{a}\,\sqrt{b}\,d^2}+\frac{x\text{PolyLog}\left(2,-\frac{(a-b)e^{2i(c+dx)}}{\left(\sqrt{a}+\sqrt{b}\right)^2}\right)}{2\sqrt{a}\,\sqrt{b}\,d^2}+\frac{i\text{PolyLog}\left(3,-\frac{\left(\sqrt{a}-\sqrt{b}\right)e^{2i(c+dx)}}{\sqrt{a}+\sqrt{b}}\right)}{4\sqrt{a}\,\sqrt{b}\,d^3}-\frac{i\text{PolyLog}\left(3,-\frac{\left(\sqrt{a}\right.}{}\right.}{4\sqrt{a}\,\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*Sec[c + d*x]^2)/(a + b*Tan[c + d*x]^2),x]
```

```
[Out] ((-I/2)*x^2*Log[1 + ((a - b)*E^((2*I)*(c + d*x)))/(Sqrt[a] - Sqrt[b])^2])/(
Sqrt[a]*Sqrt[b]*d) + ((I/2)*x^2*Log[1 + ((a - b)*E^((2*I)*(c + d*x)))/(Sqrt
[a] + Sqrt[b])^2])/(Sqrt[a]*Sqrt[b]*d) - (x*PolyLog[2, -(((a - b)*E^((2*I)*
(c + d*x)))/(Sqrt[a] - Sqrt[b])^2)])/(2*Sqrt[a]*Sqrt[b]*d^2) + (x*PolyLog[2
, -(((a - b)*E^((2*I)*(c + d*x)))/(Sqrt[a] + Sqrt[b])^2)])/(2*Sqrt[a]*Sqrt[
b]*d^2) + ((I/4)*PolyLog[3, -(((Sqrt[a] - Sqrt[b])*E^((2*I)*(c + d*x)))/(Sq
rt[a] + Sqrt[b]))])/(Sqrt[a]*Sqrt[b]*d^3) - ((I/4)*PolyLog[3, -(((Sqrt[a] +
 Sqrt[b])*E^((2*I)*(c + d*x)))/(Sqrt[a] - Sqrt[b]))])/(Sqrt[a]*Sqrt[b]*d^3)
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
```

```
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2264

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c)/q, Int[
((f + g*x)^m*F^u)/(b - q + 2*c*F^u), x], x] - Dist[(2*c)/q, Int[((f + g*x)^
m*F^u)/(b + q + 2*c*F^u), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 3321

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(
x_)]), x_Symbol] :> Dist[2, Int[((c + d*x)^m*E^(I*Pi*(k - 1/2))*E^(I*(e + f
*x)))/(b + 2*a*E^(I*Pi*(k - 1/2))*E^(I*(e + f*x)) - b*E^(2*I*k*Pi)*E^(2*I*(
e + f*x))), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[2*k] && NeQ[
a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 4588

```
Int[(((f_.) + (g_.)*(x_))^(m_.)*Sec[(d_.) + (e_.)*(x_)]^2)/((b_) + (c_.)*Ta
n[(d_.) + (e_.)*(x_)]^2), x_Symbol] :> Dist[2, Int[(f + g*x)^m/(b + c + (b
- c)*Cos[2*d + 2*e*x]), x], x] /; FreeQ[{b, c, d, e, f, g}, x] && IGtQ[m, 0
]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
```

```
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{x^2 \sec^2(c+dx)}{a+b\tan^2(c+dx)}\,dx = 2\int \frac{x^2}{a+b+(a-b)\cos(2c+2dx)}\,dx$$

$$= 4\int \frac{e^{i(2c+2dx)}x^2}{a-b+2(a+b)e^{i(2c+2dx)}+(a-b)e^{2i(2c+2dx)}}\,dx$$

$$= \frac{(2(a-b))\int \frac{e^{i(2c+2dx)}x^2}{-4\sqrt{a}\sqrt{b}+2(a+b)+2(a-b)e^{i(2c+2dx)}}\,dx}{\sqrt{a}\sqrt{b}} - \frac{(2(a-b))\int \frac{e^{i(2c+2dx)}x^2}{4\sqrt{a}\sqrt{b}+2(a+b)+2(a-b)e^{i(2c+2dx)}}}{\sqrt{a}\sqrt{b}}$$

$$= -\frac{ix^2\log\left(1+\frac{(a-b)e^{2i(c+dx)}}{\left(\sqrt{a}-\sqrt{b}\right)^2}\right)}{2\sqrt{a}\sqrt{b}\,d} + \frac{ix^2\log\left(1+\frac{(a-b)e^{2i(c+dx)}}{\left(\sqrt{a}+\sqrt{b}\right)^2}\right)}{2\sqrt{a}\sqrt{b}\,d} + \frac{i\int x\log\left(1+\frac{2(a-b)e^{i(2c+2dx)}}{-4\sqrt{a}\sqrt{b}+2(a+b)}\right.}{\sqrt{a}\sqrt{b}\,d}$$

$$= -\frac{ix^2\log\left(1+\frac{(a-b)e^{2i(c+dx)}}{\left(\sqrt{a}-\sqrt{b}\right)^2}\right)}{2\sqrt{a}\sqrt{b}\,d} + \frac{ix^2\log\left(1+\frac{(a-b)e^{2i(c+dx)}}{\left(\sqrt{a}+\sqrt{b}\right)^2}\right)}{2\sqrt{a}\sqrt{b}\,d} - \frac{x\mathrm{Li}_2\left(-\frac{(a-b)e^{2i(c+dx)}}{\left(\sqrt{a}-\sqrt{b}\right)^2}\right)}{2\sqrt{a}\sqrt{b}\,d^2} + \frac{x\mathrm{Li}_2\left(\right.}{2}$$

$$= -\frac{ix^2\log\left(1+\frac{(a-b)e^{2i(c+dx)}}{\left(\sqrt{a}-\sqrt{b}\right)^2}\right)}{2\sqrt{a}\sqrt{b}\,d} + \frac{ix^2\log\left(1+\frac{(a-b)e^{2i(c+dx)}}{\left(\sqrt{a}+\sqrt{b}\right)^2}\right)}{2\sqrt{a}\sqrt{b}\,d} - \frac{x\mathrm{Li}_2\left(-\frac{(a-b)e^{2i(c+dx)}}{\left(\sqrt{a}-\sqrt{b}\right)^2}\right)}{2\sqrt{a}\sqrt{b}\,d^2} + \frac{x\mathrm{Li}_2\left(\right.}{2}$$

$$= -\frac{ix^2\log\left(1+\frac{(a-b)e^{2i(c+dx)}}{\left(\sqrt{a}-\sqrt{b}\right)^2}\right)}{2\sqrt{a}\sqrt{b}\,d} + \frac{ix^2\log\left(1+\frac{(a-b)e^{2i(c+dx)}}{\left(\sqrt{a}+\sqrt{b}\right)^2}\right)}{2\sqrt{a}\sqrt{b}\,d} - \frac{x\mathrm{Li}_2\left(-\frac{(a-b)e^{2i(c+dx)}}{\left(\sqrt{a}-\sqrt{b}\right)^2}\right)}{2\sqrt{a}\sqrt{b}\,d^2} + \frac{x\mathrm{Li}_2\left(\right.}{2}$$

Mathematica [A] time = 1.09, size = 294, normalized size = 0.87

$$\frac{i\left(2d^2x^2\log\left(1+\frac{\left(\sqrt{a}-\sqrt{b}\right)e^{2i(c+dx)}}{\sqrt{a}+\sqrt{b}}\right)-2d^2x^2\log\left(1+\frac{\left(\sqrt{a}+\sqrt{b}\right)e^{2i(c+dx)}}{\sqrt{a}-\sqrt{b}}\right)-2idx\mathrm{Li}_2\left(\frac{\left(\sqrt{b}-\sqrt{a}\right)e^{2i(c+dx)}}{\sqrt{a}+\sqrt{b}}\right)+2idx\mathrm{Li}_2\left(-\frac{\left(\sqrt{a}\right.}{}\right.\right.}{4\sqrt{a}\sqrt{b}\,d^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*Sec[c + d*x]^2)/(a + b*Tan[c + d*x]^2),x]
```

```
[Out] ((I/4)*(2*d^2*x^2*Log[1 + ((Sqrt[a] - Sqrt[b])*E^((2*I)*(c + d*x)))/(Sqrt[a
] + Sqrt[b])] - 2*d^2*x^2*Log[1 + ((Sqrt[a] + Sqrt[b])*E^((2*I)*(c + d*x)))
```

```
/(Sqrt[a] - Sqrt[b])] - (2*I)*d*x*PolyLog[2, ((-Sqrt[a] + Sqrt[b])*E^((2*I)
*(c + d*x)))/(Sqrt[a] + Sqrt[b])] + (2*I)*d*x*PolyLog[2, -(((Sqrt[a] + Sqrt
[b])*E^((2*I)*(c + d*x)))/(Sqrt[a] - Sqrt[b]))] + PolyLog[3, ((-Sqrt[a] + S
qrt[b])*E^((2*I)*(c + d*x)))/(Sqrt[a] + Sqrt[b])] - PolyLog[3, -(((Sqrt[a]
+ Sqrt[b])*E^((2*I)*(c + d*x)))/(Sqrt[a] - Sqrt[b]))]))/(Sqrt[a]*Sqrt[b]*d^
3)
```

fricas [C] time = 2.28, size = 4644, normalized size = 13.78

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*sec(d*x+c)^2/(a+b*tan(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] 1/16*(8*(a - b)*sqrt(a*b/(a^2 - 2*a*b + b^2))*d*x*dilog(1/2*((2*(a + b)*cos
(d*x + c) + (2*I*a + 2*I*b)*sin(d*x + c) - 4*((a - b)*cos(d*x + c) - (-I*a
+ I*b)*sin(d*x + c))*sqrt(a*b/(a^2 - 2*a*b + b^2)))*sqrt(-(2*(a - b)*sqrt(a
*b/(a^2 - 2*a*b + b^2)) + a + b)/(a - b)) - 2*a + 2*b)/(a - b) + 1) + 8*(a
- b)*sqrt(a*b/(a^2 - 2*a*b + b^2))*d*x*dilog(-1/2*((2*(a + b)*cos(d*x + c)
- (2*I*a + 2*I*b)*sin(d*x + c) - 4*((a - b)*cos(d*x + c) + (-I*a + I*b)*sin
(d*x + c))*sqrt(a*b/(a^2 - 2*a*b + b^2)))*sqrt(-(2*(a - b)*sqrt(a*b/(a^2 -
2*a*b + b^2)) + a + b)/(a - b)) + 2*a - 2*b)/(a - b) + 1) + 8*(a - b)*sqrt(
a*b/(a^2 - 2*a*b + b^2))*d*x*dilog(1/2*((2*(a + b)*cos(d*x + c) + (-2*I*a -
 2*I*b)*sin(d*x + c) - 4*((a - b)*cos(d*x + c) - (I*a - I*b)*sin(d*x + c))*
sqrt(a*b/(a^2 - 2*a*b + b^2)))*sqrt(-(2*(a - b)*sqrt(a*b/(a^2 - 2*a*b + b^2
)) + a + b)/(a - b)) - 2*a + 2*b)/(a - b) + 1) + 8*(a - b)*sqrt(a*b/(a^2 -
2*a*b + b^2))*d*x*dilog(-1/2*((2*(a + b)*cos(d*x + c) - (-2*I*a - 2*I*b)*si
n(d*x + c) - 4*((a - b)*cos(d*x + c) + (I*a - I*b)*sin(d*x + c))*sqrt(a*b/(
a^2 - 2*a*b + b^2)))*sqrt(-(2*(a - b)*sqrt(a*b/(a^2 - 2*a*b + b^2)) + a + b
)/(a - b)) + 2*a - 2*b)/(a - b) + 1) - 8*(a - b)*sqrt(a*b/(a^2 - 2*a*b + b^
2))*d*x*dilog(1/2*((2*(a + b)*cos(d*x + c) + (2*I*a + 2*I*b)*sin(d*x + c) +
 4*((a - b)*cos(d*x + c) + (I*a - I*b)*sin(d*x + c))*sqrt(a*b/(a^2 - 2*a*b
+ b^2)))*sqrt((2*(a - b)*sqrt(a*b/(a^2 - 2*a*b + b^2)) - a - b)/(a - b)) -
2*a + 2*b)/(a - b) + 1) - 8*(a - b)*sqrt(a*b/(a^2 - 2*a*b + b^2))*d*x*dilog
(-1/2*((2*(a + b)*cos(d*x + c) - (2*I*a + 2*I*b)*sin(d*x + c) + 4*((a - b)*
cos(d*x + c) - (I*a - I*b)*sin(d*x + c))*sqrt(a*b/(a^2 - 2*a*b + b^2)))*sqr
t((2*(a - b)*sqrt(a*b/(a^2 - 2*a*b + b^2)) - a - b)/(a - b)) + 2*a - 2*b)/(
a - b) + 1) - 8*(a - b)*sqrt(a*b/(a^2 - 2*a*b + b^2))*d*x*dilog(1/2*((2*(a
+ b)*cos(d*x + c) + (-2*I*a - 2*I*b)*sin(d*x + c) + 4*((a - b)*cos(d*x + c)
 + (-I*a + I*b)*sin(d*x + c))*sqrt(a*b/(a^2 - 2*a*b + b^2)))*sqrt((2*(a - b
)*sqrt(a*b/(a^2 - 2*a*b + b^2)) - a - b)/(a - b)) - 2*a + 2*b)/(a - b) + 1)
 - 8*(a - b)*sqrt(a*b/(a^2 - 2*a*b + b^2))*d*x*dilog(-1/2*((2*(a + b)*cos(d
*x + c) - (-2*I*a - 2*I*b)*sin(d*x + c) + 4*((a - b)*cos(d*x + c) - (-I*a +
 I*b)*sin(d*x + c))*sqrt(a*b/(a^2 - 2*a*b + b^2)))*sqrt((2*(a - b)*sqrt(a*b
/(a^2 - 2*a*b + b^2)) - a - b)/(a - b)) + 2*a - 2*b)/(a - b) + 1) + 4*I*(a
```

```
- b)*sqrt(a*b/(a^2 - 2*a*b + b^2))*c^2*log(2*sqrt(-(2*(a - b)*sqrt(a*b/(a^2
 - 2*a*b + b^2)) + a + b)/(a - b)) + 2*cos(d*x + c) + 2*I*sin(d*x + c)) - 4
*I*(a - b)*sqrt(a*b/(a^2 - 2*a*b + b^2))*c^2*log(2*sqrt(-(2*(a - b)*sqrt(a*
b/(a^2 - 2*a*b + b^2)) + a + b)/(a - b)) + 2*cos(d*x + c) - 2*I*sin(d*x + c
)) - 4*I*(a - b)*sqrt(a*b/(a^2 - 2*a*b + b^2))*c^2*log(2*sqrt(-(2*(a - b)*s
qrt(a*b/(a^2 - 2*a*b + b^2)) + a + b)/(a - b)) - 2*cos(d*x + c) + 2*I*sin(d
*x + c)) + 4*I*(a - b)*sqrt(a*b/(a^2 - 2*a*b + b^2))*c^2*log(2*sqrt(-(2*(a
- b)*sqrt(a*b/(a^2 - 2*a*b + b^2)) + a + b)/(a - b)) - 2*cos(d*x + c) - 2*I
*sin(d*x + c)) - 4*I*(a - b)*sqrt(a*b/(a^2 - 2*a*b + b^2))*c^2*log(2*sqrt((
2*(a - b)*sqrt(a*b/(a^2 - 2*a*b + b^2)) - a - b)/(a - b)) + 2*cos(d*x + c)
+ 2*I*sin(d*x + c)) + 4*I*(a - b)*sqrt(a*b/(a^2 - 2*a*b + b^2))*c^2*log(2*s
qrt((2*(a - b)*sqrt(a*b/(a^2 - 2*a*b + b^2)) - a - b)/(a - b)) + 2*cos(d*x
+ c) - 2*I*sin(d*x + c)) + 4*I*(a - b)*sqrt(a*b/(a^2 - 2*a*b + b^2))*c^2*lo
g(2*sqrt((2*(a - b)*sqrt(a*b/(a^2 - 2*a*b + b^2)) - a - b)/(a - b)) - 2*cos
(d*x + c) + 2*I*sin(d*x + c)) - 4*I*(a - b)*sqrt(a*b/(a^2 - 2*a*b + b^2))*c
^2*log(2*sqrt((2*(a - b)*sqrt(a*b/(a^2 - 2*a*b + b^2)) - a - b)/(a - b)) -
2*cos(d*x + c) - 2*I*sin(d*x + c)) + 4*(I*(a - b)*d^2*x^2 - I*(a - b)*c^2)*
sqrt(a*b/(a^2 - 2*a*b + b^2))*log(-1/2*((2*(a + b)*cos(d*x + c) + (2*I*a +
2*I*b)*sin(d*x + c) - 4*((a - b)*cos(d*x + c) - (-I*a + I*b)*sin(d*x + c))*
sqrt(a*b/(a^2 - 2*a*b + b^2)))*sqrt(-(2*(a - b)*sqrt(a*b/(a^2 - 2*a*b + b^2
)) + a + b)/(a - b)) - 2*a + 2*b)/(a - b)) + 4*(-I*(a - b)*d^2*x^2 + I*(a -
 b)*c^2)*sqrt(a*b/(a^2 - 2*a*b + b^2))*log(1/2*((2*(a + b)*cos(d*x + c) - (
2*I*a + 2*I*b)*sin(d*x + c) - 4*((a - b)*cos(d*x + c) + (-I*a + I*b)*sin(d*
x + c))*sqrt(a*b/(a^2 - 2*a*b + b^2)))*sqrt(-(2*(a - b)*sqrt(a*b/(a^2 - 2*a
*b + b^2)) + a + b)/(a - b)) + 2*a - 2*b)/(a - b)) + 4*(-I*(a - b)*d^2*x^2
+ I*(a - b)*c^2)*sqrt(a*b/(a^2 - 2*a*b + b^2))*log(-1/2*((2*(a + b)*cos(d*x
 + c) + (-2*I*a - 2*I*b)*sin(d*x + c) - 4*((a - b)*cos(d*x + c) - (I*a - I*
b)*sin(d*x + c))*sqrt(a*b/(a^2 - 2*a*b + b^2)))*sqrt(-(2*(a - b)*sqrt(a*b/(
a^2 - 2*a*b + b^2)) + a + b)/(a - b)) - 2*a + 2*b)/(a - b)) + 4*(I*(a - b)*
d^2*x^2 - I*(a - b)*c^2)*sqrt(a*b/(a^2 - 2*a*b + b^2))*log(1/2*((2*(a + b)*
cos(d*x + c) - (-2*I*a - 2*I*b)*sin(d*x + c) - 4*((a - b)*cos(d*x + c) + (I
*a - I*b)*sin(d*x + c))*sqrt(a*b/(a^2 - 2*a*b + b^2)))*sqrt(-(2*(a - b)*sqr
t(a*b/(a^2 - 2*a*b + b^2)) + a + b)/(a - b)) + 2*a - 2*b)/(a - b)) + 4*(-I*
(a - b)*d^2*x^2 + I*(a - b)*c^2)*sqrt(a*b/(a^2 - 2*a*b + b^2))*log(-1/2*((2
*(a + b)*cos(d*x + c) + (2*I*a + 2*I*b)*sin(d*x + c) + 4*((a - b)*cos(d*x +
 c) + (I*a - I*b)*sin(d*x + c))*sqrt(a*b/(a^2 - 2*a*b + b^2)))*sqrt((2*(a -
 b)*sqrt(a*b/(a^2 - 2*a*b + b^2)) - a - b)/(a - b)) - 2*a + 2*b)/(a - b)) +
 4*(I*(a - b)*d^2*x^2 - I*(a - b)*c^2)*sqrt(a*b/(a^2 - 2*a*b + b^2))*log(1/
2*((2*(a + b)*cos(d*x + c) - (2*I*a + 2*I*b)*sin(d*x + c) + 4*((a - b)*cos(
d*x + c) - (I*a - I*b)*sin(d*x + c))*sqrt(a*b/(a^2 - 2*a*b + b^2)))*sqrt((2
*(a - b)*sqrt(a*b/(a^2 - 2*a*b + b^2)) - a - b)/(a - b)) + 2*a - 2*b)/(a -
b)) + 4*(I*(a - b)*d^2*x^2 - I*(a - b)*c^2)*sqrt(a*b/(a^2 - 2*a*b + b^2))*l
og(-1/2*((2*(a + b)*cos(d*x + c) + (-2*I*a - 2*I*b)*sin(d*x + c) + 4*((a -
b)*cos(d*x + c) + (-I*a + I*b)*sin(d*x + c))*sqrt(a*b/(a^2 - 2*a*b + b^2)))
*sqrt((2*(a - b)*sqrt(a*b/(a^2 - 2*a*b + b^2)) - a - b)/(a - b)) - 2*a + 2*
```

```
b)/(a - b)) + 4*(-I*(a - b)*d^2*x^2 + I*(a - b)*c^2)*sqrt(a*b/(a^2 - 2*a*b
+ b^2))*log(1/2*((2*(a + b)*cos(d*x + c) - (-2*I*a - 2*I*b)*sin(d*x + c) +
4*((a - b)*cos(d*x + c) - (-I*a + I*b)*sin(d*x + c))*sqrt(a*b/(a^2 - 2*a*b
+ b^2)))*sqrt((2*(a - b)*sqrt(a*b/(a^2 - 2*a*b + b^2)) - a - b)/(a - b)) +
2*a - 2*b)/(a - b)) + 4*(2*I*a - 2*I*b)*sqrt(a*b/(a^2 - 2*a*b + b^2))*polyl
og(3, -1/2*(2*(a + b)*cos(d*x + c) + (2*I*a + 2*I*b)*sin(d*x + c) - 4*((a -
 b)*cos(d*x + c) - (-I*a + I*b)*sin(d*x + c))*sqrt(a*b/(a^2 - 2*a*b + b^2))
)*sqrt(-(2*(a - b)*sqrt(a*b/(a^2 - 2*a*b + b^2)) + a + b)/(a - b))/(a - b))
 + 4*(-2*I*a + 2*I*b)*sqrt(a*b/(a^2 - 2*a*b + b^2))*polylog(3, 1/2*(2*(a +
b)*cos(d*x + c) - (2*I*a + 2*I*b)*sin(d*x + c) - 4*((a - b)*cos(d*x + c) +
(-I*a + I*b)*sin(d*x + c))*sqrt(a*b/(a^2 - 2*a*b + b^2)))*sqrt(-(2*(a - b)*
sqrt(a*b/(a^2 - 2*a*b + b^2)) + a + b)/(a - b))/(a - b)) + 4*(-2*I*a + 2*I*
b)*sqrt(a*b/(a^2 - 2*a*b + b^2))*polylog(3, -1/2*(2*(a + b)*cos(d*x + c) +
(-2*I*a - 2*I*b)*sin(d*x + c) - 4*((a - b)*cos(d*x + c) - (I*a - I*b)*sin(d
*x + c))*sqrt(a*b/(a^2 - 2*a*b + b^2)))*sqrt(-(2*(a - b)*sqrt(a*b/(a^2 - 2*
a*b + b^2)) + a + b)/(a - b))/(a - b)) + 4*(2*I*a - 2*I*b)*sqrt(a*b/(a^2 -
2*a*b + b^2))*polylog(3, 1/2*(2*(a + b)*cos(d*x + c) - (-2*I*a - 2*I*b)*sin
(d*x + c) - 4*((a - b)*cos(d*x + c) + (I*a - I*b)*sin(d*x + c))*sqrt(a*b/(a
^2 - 2*a*b + b^2)))*sqrt(-(2*(a - b)*sqrt(a*b/(a^2 - 2*a*b + b^2)) + a + b)
/(a - b))/(a - b)) + 4*(-2*I*a + 2*I*b)*sqrt(a*b/(a^2 - 2*a*b + b^2))*polyl
og(3, -1/2*(2*(a + b)*cos(d*x + c) + (2*I*a + 2*I*b)*sin(d*x + c) + 4*((a -
 b)*cos(d*x + c) + (I*a - I*b)*sin(d*x + c))*sqrt(a*b/(a^2 - 2*a*b + b^2)))
*sqrt((2*(a - b)*sqrt(a*b/(a^2 - 2*a*b + b^2)) - a - b)/(a - b))/(a - b)) +
 4*(2*I*a - 2*I*b)*sqrt(a*b/(a^2 - 2*a*b + b^2))*polylog(3, 1/2*(2*(a + b)*
cos(d*x + c) - (2*I*a + 2*I*b)*sin(d*x + c) + 4*((a - b)*cos(d*x + c) - (I*
a - I*b)*sin(d*x + c))*sqrt(a*b/(a^2 - 2*a*b + b^2)))*sqrt((2*(a - b)*sqrt(
a*b/(a^2 - 2*a*b + b^2)) - a - b)/(a - b))/(a - b)) + 4*(2*I*a - 2*I*b)*sqr
t(a*b/(a^2 - 2*a*b + b^2))*polylog(3, -1/2*(2*(a + b)*cos(d*x + c) + (-2*I*
a - 2*I*b)*sin(d*x + c) + 4*((a - b)*cos(d*x + c) + (-I*a + I*b)*sin(d*x +
c))*sqrt(a*b/(a^2 - 2*a*b + b^2)))*sqrt((2*(a - b)*sqrt(a*b/(a^2 - 2*a*b +
b^2)) - a - b)/(a - b))/(a - b)) + 4*(-2*I*a + 2*I*b)*sqrt(a*b/(a^2 - 2*a*b
 + b^2))*polylog(3, 1/2*(2*(a + b)*cos(d*x + c) - (-2*I*a - 2*I*b)*sin(d*x
+ c) + 4*((a - b)*cos(d*x + c) - (-I*a + I*b)*sin(d*x + c))*sqrt(a*b/(a^2 -
 2*a*b + b^2)))*sqrt((2*(a - b)*sqrt(a*b/(a^2 - 2*a*b + b^2)) - a - b)/(a -
 b))/(a - b)))/(a*b*d^3)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2 \sec(dx+c)^2}{b\tan(dx+c)^2 + a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*sec(d*x+c)^2/(a+b*tan(d*x+c)^2),x, algorithm="giac")
```

```
[Out] integrate(x^2*sec(d*x + c)^2/(b*tan(d*x + c)^2 + a), x)
```

maple [B] time = 0.46, size = 1251, normalized size = 3.71

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*sec(d*x+c)^2/(a+b*tan(d*x+c)^2),x)
```

```
[Out] I/d^3*c^2/(a*b)^(1/2)*arctanh(1/4*(2*(a-b)*exp(2*I*(d*x+c))+2*a+2*b)/(a*b)^
(1/2))-I/d/(-2*(a*b)^(1/2)-a-b)*ln(1-(a-b)*exp(2*I*(d*x+c))/(-2*(a*b)^(1/2)
-a-b))*x^2+2/3/d^3/(a*b)^(1/2)/(-2*(a*b)^(1/2)-a-b)*a*c^3+2/3/d^3/(a*b)^(1/
2)/(-2*(a*b)^(1/2)-a-b)*b*c^3+I/d^3/(-2*(a*b)^(1/2)-a-b)*ln(1-(a-b)*exp(2*I
*(d*x+c))/(-2*(a*b)^(1/2)-a-b))*c^2+1/2*I/d^3/(a*b)^(1/2)*ln(1-(a-b)*exp(2*
I*(d*x+c))/(2*(a*b)^(1/2)-a-b))*c^2-1/2*I/d/(a*b)^(1/2)*ln(1-(a-b)*exp(2*I*
(d*x+c))/(2*(a*b)^(1/2)-a-b))*x^2+2/3/d^3/(a*b)^(1/2)*c^3+4/3/d^3/(-2*(a*b)
^(1/2)-a-b)*c^3+1/d^2/(a*b)^(1/2)*c^2*x+2/d^2/(-2*(a*b)^(1/2)-a-b)*c^2*x-1/
2/d^2/(a*b)^(1/2)*polylog(2,(a-b)*exp(2*I*(d*x+c))/(2*(a*b)^(1/2)-a-b))*x-1
/d^2/(-2*(a*b)^(1/2)-a-b)*polylog(2,(a-b)*exp(2*I*(d*x+c))/(-2*(a*b)^(1/2)-
a-b))*x-1/3/(a*b)^(1/2)/(-2*(a*b)^(1/2)-a-b)*b*x^3-1/3/(a*b)^(1/2)/(-2*(a*b
)^(1/2)-a-b)*a*x^3-1/4*I/d^3/(a*b)^(1/2)*polylog(3,(a-b)*exp(2*I*(d*x+c))/(
2*(a*b)^(1/2)-a-b))-1/2*I/d^3/(-2*(a*b)^(1/2)-a-b)*polylog(3,(a-b)*exp(2*I*
(d*x+c))/(-2*(a*b)^(1/2)-a-b))+1/2*I/d^3/(a*b)^(1/2)/(-2*(a*b)^(1/2)-a-b)*b
*ln(1-(a-b)*exp(2*I*(d*x+c))/(-2*(a*b)^(1/2)-a-b))*c^2-1/2*I/d/(a*b)^(1/2)/
(-2*(a*b)^(1/2)-a-b)*a*ln(1-(a-b)*exp(2*I*(d*x+c))/(-2*(a*b)^(1/2)-a-b))*x^
2-1/2*I/d/(a*b)^(1/2)/(-2*(a*b)^(1/2)-a-b)*b*ln(1-(a-b)*exp(2*I*(d*x+c))/(-
2*(a*b)^(1/2)-a-b))*x^2-1/3/(a*b)^(1/2)*x^3-2/3/(-2*(a*b)^(1/2)-a-b)*x^3+1/
d^2/(a*b)^(1/2)/(-2*(a*b)^(1/2)-a-b)*a*c^2*x+1/d^2/(a*b)^(1/2)/(-2*(a*b)^(1
/2)-a-b)*b*c^2*x-1/2/d^2/(a*b)^(1/2)/(-2*(a*b)^(1/2)-a-b)*b*polylog(2,(a-b)
*exp(2*I*(d*x+c))/(-2*(a*b)^(1/2)-a-b))*x-1/2/d^2/(a*b)^(1/2)/(-2*(a*b)^(1/
2)-a-b)*a*polylog(2,(a-b)*exp(2*I*(d*x+c))/(-2*(a*b)^(1/2)-a-b))*x-1/4*I/d^
3/(a*b)^(1/2)/(-2*(a*b)^(1/2)-a-b)*a*polylog(3,(a-b)*exp(2*I*(d*x+c))/(-2*(
a*b)^(1/2)-a-b))-1/4*I/d^3/(a*b)^(1/2)/(-2*(a*b)^(1/2)-a-b)*b*polylog(3,(a-
b)*exp(2*I*(d*x+c))/(-2*(a*b)^(1/2)-a-b))+1/2*I/d^3/(a*b)^(1/2)/(-2*(a*b)^(
1/2)-a-b)*a*ln(1-(a-b)*exp(2*I*(d*x+c))/(-2*(a*b)^(1/2)-a-b))*c^2
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2 \sec(dx+c)^2}{b \tan(dx+c)^2 + a}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*sec(d*x+c)^2/(a+b*tan(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] integrate(x^2*sec(d*x + c)^2/(b*tan(d*x + c)^2 + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^2}{\cos(c+d\,x)^2\,\left(b\tan(c+d\,x)^2+a\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(cos(c + d*x)^2*(a + b*tan(c + d*x)^2)),x)
[Out] int(x^2/(cos(c + d*x)^2*(a + b*tan(c + d*x)^2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2\sec^2(c+dx)}{a+b\tan^2(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*sec(d*x+c)**2/(a+b*tan(d*x+c)**2),x)
[Out] Integral(x**2*sec(c + d*x)**2/(a + b*tan(c + d*x)**2), x)
```

3.162 $\int \frac{\sec^2(c+dx)}{a+c\sec^2(c+dx)+b\tan^2(c+dx)}\,dx$

Optimal. Leaf size=40

$$\frac{\tan^{-1}\left(\frac{\sqrt{b+c}\tan(c+dx)}{\sqrt{a+c}}\right)}{d\sqrt{a+c}\sqrt{b+c}}$$

```
[Out] arctan((b+c)^(1/2)*tan(d*x+c)/(a+c)^(1/2))/d/(a+c)^(1/2)/(b+c)^(1/2)
```

Rubi [A] time = 0.61, antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.030, Rules used = {205}

$$\frac{\tan^{-1}\left(\frac{\sqrt{b+c}\tan(c+dx)}{\sqrt{a+c}}\right)}{d\sqrt{a+c}\sqrt{b+c}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^2/(a + c*Sec[c + d*x]^2 + b*Tan[c + d*x]^2),x]
```

```
[Out] ArcTan[(Sqrt[b + c]*Tan[c + d*x])/Sqrt[a + c]]/(Sqrt[a + c]*Sqrt[b + c]*d)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\begin{aligned}\int \frac{\sec^2(c+dx)}{a+c\sec^2(c+dx)+b\tan^2(c+dx)}\,dx &= \frac{\text{Subst}\left(\int \frac{1}{a+c+(b+c)x^2}\,dx, x, \tan(c+dx)\right)}{d}\\ &= \frac{\tan^{-1}\left(\frac{\sqrt{b+c}\tan(c+dx)}{\sqrt{a+c}}\right)}{\sqrt{a+c}\sqrt{b+c}\,d}\end{aligned}$$

Mathematica [A] time = 0.25, size = 40, normalized size = 1.00

$$\frac{\tan^{-1}\left(\frac{\sqrt{b+c}\tan(c+dx)}{\sqrt{a+c}}\right)}{d\sqrt{a+c}\sqrt{b+c}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^2/(a + c*Sec[c + d*x]^2 + b*Tan[c + d*x]^2),x]
```

```
[Out] ArcTan[(Sqrt[b + c]*Tan[c + d*x])/Sqrt[a + c]]/(Sqrt[a + c]*Sqrt[b + c]*d)
```

fricas [B] time = 0.75, size = 300, normalized size = 7.50

$$\left[-\frac{\sqrt{-ab-(a+b)c-c^2}\,\log\left(\frac{\left(a^2+6\,ab+b^2+8\,(a+b)c+8\,c^2\right)\cos(dx+c)^4-2\left(3\,ab+b^2+(3\,a+5\,b)c+4\,c^2\right)\cos(dx+c)^2+4\left((a+b+2\,c)\cos(dx+c)^3-(b-\right.}{\left(a^2-2\,ab+b^2\right)\cos(dx+c)^4+2\left(ab-b^2+(a-b)c\right)\cos(dx+c)^2+b^2+2}\right.}{4\left(ab+(a+b)c+c^2\right)d}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a+c*sec(d*x+c)^2+b*tan(d*x+c)^2),x, algorithm="fric
as")
```

```
[Out] [-1/4*sqrt(-a*b - (a + b)*c - c^2)*log(((a^2 + 6*a*b + b^2 + 8*(a + b)*c +
8*c^2)*cos(d*x + c)^4 - 2*(3*a*b + b^2 + (3*a + 5*b)*c + 4*c^2)*cos(d*x + c
)^2 + 4*((a + b + 2*c)*cos(d*x + c)^3 - (b + c)*cos(d*x + c))*sqrt(-a*b - (
a + b)*c - c^2)*sin(d*x + c) + b^2 + 2*b*c + c^2)/((a^2 - 2*a*b + b^2)*cos(
d*x + c)^4 + 2*(a*b - b^2 + (a - b)*c)*cos(d*x + c)^2 + b^2 + 2*b*c + c^2))
/((a*b + (a + b)*c + c^2)*d), -1/2*arctan(1/2*((a + b + 2*c)*cos(d*x + c)^2
 - b - c)/(sqrt(a*b + (a + b)*c + c^2)*cos(d*x + c)*sin(d*x + c)))/(sqrt(a*
b + (a + b)*c + c^2)*d)]
```

giac [B] time = 0.97, size = 76, normalized size = 1.90

$$\frac{\pi\left\lfloor\frac{dx+c}{\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}\left(2\,b+2\,c\right)+\arctan\left(\frac{b\tan(dx+c)+c\tan(dx+c)}{\sqrt{ab+ac+bc+c^2}}\right)}{\sqrt{ab+ac+bc+c^2}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a+c*sec(d*x+c)^2+b*tan(d*x+c)^2),x, algorithm="giac
")
```

```
[Out] (pi*floor((d*x + c)/pi + 1/2)*sgn(2*b + 2*c) + arctan((b*tan(d*x + c) + c*t
an(d*x + c))/sqrt(a*b + a*c + b*c + c^2)))/(sqrt(a*b + a*c + b*c + c^2)*d)
```

maple [A] time = 0.36, size = 34, normalized size = 0.85

$$\frac{\arctan\left(\frac{(b+c)\tan(dx+c)}{\sqrt{(a+c)(b+c)}}\right)}{d\sqrt{(a+c)\,(b+c)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2/(a+c*sec(d*x+c)^2+b*tan(d*x+c)^2),x)
```

```
[Out] 1/d/((a+c)*(b+c))^(1/2)*arctan((b+c)*tan(d*x+c)/((a+c)*(b+c))^(1/2))
```

maxima [A] time = 1.07, size = 43, normalized size = 1.08

$$\frac{\arctan\left(\frac{(b+c)\tan(dx+c)}{\sqrt{ab+(a+b)c+c^2}}\right)}{\sqrt{ab+(a+b)c+c^2}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a+c*sec(d*x+c)^2+b*tan(d*x+c)^2),x, algorithm="maxi
ma")
```

```
[Out] arctan((b + c)*tan(d*x + c)/sqrt(a*b + (a + b)*c + c^2))/(sqrt(a*b + (a + b
)*c + c^2)*d)
```

mupad [B] time = 2.60, size = 45, normalized size = 1.12

$$\frac{\operatorname{atan}\left(\frac{\tan(c+d\,x)\,(b+c)}{\sqrt{a\,b+a\,c+b\,c+c^2}}\right)}{d\,\sqrt{a\,b+a\,c+b\,c+c^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^2*(a + c/cos(c + d*x)^2 + b*tan(c + d*x)^2)),x)
```

```
[Out] atan((tan(c + d*x)*(b + c))/(a*b + a*c + b*c + c^2)^(1/2))/(d*(a*b + a*c +
b*c + c^2)^(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec^2(c+dx)}{a+b\tan^2(c+dx)+c\sec^2(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2/(a+c*sec(d*x+c)**2+b*tan(d*x+c)**2),x)
```

```
[Out] Integral(sec(c + d*x)**2/(a + b*tan(c + d*x)**2 + c*sec(c + d*x)**2), x)
```

3.163 $\int \frac{x\sec^2(c+dx)}{a+c\sec^2(c+dx)+b\tan^2(c+dx)}\,dx$

Optimal. Leaf size=267

$$-\frac{\text{Li}_2\left(-\frac{(a-b)e^{2i(c+dx)}}{a+b+2c-2\sqrt{a+c}\sqrt{b+c}}\right)}{4d^2\sqrt{a+c}\sqrt{b+c}}+\frac{\text{Li}_2\left(-\frac{(a-b)e^{2i(c+dx)}}{a+b+2\left(c+\sqrt{a+c}\sqrt{b+c}\right)}\right)}{4d^2\sqrt{a+c}\sqrt{b+c}}-\frac{ix\log\left(1+\frac{(a-b)e^{2i(c+dx)}}{-2\sqrt{a+c}\sqrt{b+c}+a+b+2c}\right)}{2d\sqrt{a+c}\sqrt{b+c}}+\frac{ix\log\left(1+\frac{(a-b)e^{2i(c+\cdots}}{2\left(\sqrt{a+c}\sqrt{b+c}+\cdots\right.}\right.}{2d\sqrt{a+c}\sqrt{b+c}}$$

```
[Out] -1/2*I*x*ln(1+(a-b)*exp(2*I*(d*x+c))/(a+b+2*c-2*(a+c)^(1/2)*(b+c)^(1/2)))/d
/(a+c)^(1/2)/(b+c)^(1/2)+1/2*I*x*ln(1+(a-b)*exp(2*I*(d*x+c))/(a+b+2*c+2*(a+
c)^(1/2)*(b+c)^(1/2)))/d/(a+c)^(1/2)/(b+c)^(1/2)-1/4*polylog(2,-(a-b)*exp(2
*I*(d*x+c))/(a+b+2*c-2*(a+c)^(1/2)*(b+c)^(1/2)))/d^2/(a+c)^(1/2)/(b+c)^(1/2
)+1/4*polylog(2,-(a-b)*exp(2*I*(d*x+c))/(a+b+2*c+2*(a+c)^(1/2)*(b+c)^(1/2))
)/d^2/(a+c)^(1/2)/(b+c)^(1/2)
```

Rubi [A] time = 0.72, antiderivative size = 267, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {4589, 3321, 2264, 2190, 2279, 2391}

$$-\frac{\text{PolyLog}\left(2,-\frac{(a-b)e^{2i(c+dx)}}{-2\sqrt{a+c}\sqrt{b+c}+a+b+2c}\right)}{4d^2\sqrt{a+c}\sqrt{b+c}}+\frac{\text{PolyLog}\left(2,-\frac{(a-b)e^{2i(c+dx)}}{2\left(\sqrt{a+c}\sqrt{b+c}+c\right)+a+b}\right)}{4d^2\sqrt{a+c}\sqrt{b+c}}-\frac{ix\log\left(1+\frac{(a-b)e^{2i(c+dx)}}{-2\sqrt{a+c}\sqrt{b+c}+a+b+2c}\right)}{2d\sqrt{a+c}\sqrt{b+c}}+\frac{ix\log\cdots}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[(x*Sec[c + d*x]^2)/(a + c*Sec[c + d*x]^2 + b*Tan[c + d*x]^2),x]
```

```
[Out] ((-I/2)*x*Log[1 + ((a - b)*E^((2*I)*(c + d*x)))/(a + b + 2*c - 2*Sqrt[a + c
]*Sqrt[b + c])])/(Sqrt[a + c]*Sqrt[b + c]*d) + ((I/2)*x*Log[1 + ((a - b)*E^
((2*I)*(c + d*x)))/(a + b + 2*(c + Sqrt[a + c]*Sqrt[b + c]))])/(Sqrt[a + c]
*Sqrt[b + c]*d) - PolyLog[2, -(((a - b)*E^((2*I)*(c + d*x)))/(a + b + 2*c -
 2*Sqrt[a + c]*Sqrt[b + c]))]/(4*Sqrt[a + c]*Sqrt[b + c]*d^2) + PolyLog[2,
-(((a - b)*E^((2*I)*(c + d*x)))/(a + b + 2*(c + Sqrt[a + c]*Sqrt[b + c])))]
/(4*Sqrt[a + c]*Sqrt[b + c]*d^2)
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2264

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c)/q, Int[
((f + g*x)^m*F^u)/(b - q + 2*c*F^u), x], x] - Dist[(2*c)/q, Int[((f + g*x)^
m*F^u)/(b + q + 2*c*F^u), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3321

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(
x_)]), x_Symbol] :> Dist[2, Int[((c + d*x)^m*E^(I*Pi*(k - 1/2))*E^(I*(e + f
*x)))/(b + 2*a*E^(I*Pi*(k - 1/2))*E^(I*(e + f*x)) - b*E^(2*I*k*Pi)*E^(2*I*(
e + f*x))), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[2*k] && NeQ[
a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 4589

```
Int[(((f_.) + (g_.)*(x_))^(m_.)*Sec[(d_.) + (e_.)*(x_)]^2)/((b_.) + (a_.)*S
ec[(d_.) + (e_.)*(x_)]^2 + (c_.)*Tan[(d_.) + (e_.)*(x_)]^2), x_Symbol] :> D
ist[2, Int[(f + g*x)^m/(2*a + b + c + (b - c)*Cos[2*d + 2*e*x]), x], x] /;
FreeQ[{a, b, c, d, e, f, g}, x] && IGtQ[m, 0] && NeQ[a + b, 0] && NeQ[a + c
, 0]
```

Rubi steps

$$\int \frac{x\sec^2(c+dx)}{a+c\sec^2(c+dx)+b\tan^2(c+dx)}\,dx = 2\int \frac{x}{a+b+2c+(a-b)\cos(2c+2dx)}\,dx$$

$$= 4\int \frac{e^{i(2c+2dx)}x}{a-b+2(a+b+2c)e^{i(2c+2dx)}+(a-b)e^{2i(2c+2dx)}}\,dx$$

$$= \frac{(2(a-b))\int \frac{e^{i(2c+2dx)}x}{-4\sqrt{a+c}\sqrt{b+c}+2(a+b+2c)+2(a-b)e^{i(2c+2dx)}}\,dx}{\sqrt{a+c}\sqrt{b+c}} - \frac{(2(a-b))\int \frac{}{4\sqrt{a+}}}{}$$

$$= -\frac{ix\log\left(1+\frac{(a-b)e^{2i(c+dx)}}{a+b+2c-2\sqrt{a+c}\sqrt{b+c}}\right)}{2\sqrt{a+c}\sqrt{b+c}\,d} + \frac{ix\log\left(1+\frac{(a-b)e^{2i(c+dx)}}{a+b+2\left(c+\sqrt{a+c}\sqrt{b+c}\right)}\right)}{2\sqrt{a+c}\sqrt{b+c}\,d} + \cdots$$

$$= -\frac{ix\log\left(1+\frac{(a-b)e^{2i(c+dx)}}{a+b+2c-2\sqrt{a+c}\sqrt{b+c}}\right)}{2\sqrt{a+c}\sqrt{b+c}\,d} + \frac{ix\log\left(1+\frac{(a-b)e^{2i(c+dx)}}{a+b+2\left(c+\sqrt{a+c}\sqrt{b+c}\right)}\right)}{2\sqrt{a+c}\sqrt{b+c}\,d} + \cdots$$

$$= -\frac{ix\log\left(1+\frac{(a-b)e^{2i(c+dx)}}{a+b+2c-2\sqrt{a+c}\sqrt{b+c}}\right)}{2\sqrt{a+c}\sqrt{b+c}\,d} + \frac{ix\log\left(1+\frac{(a-b)e^{2i(c+dx)}}{a+b+2\left(c+\sqrt{a+c}\sqrt{b+c}\right)}\right)}{2\sqrt{a+c}\sqrt{b+c}\,d} - \cdots$$

Mathematica [B] time = 4.23, size = 751, normalized size = 2.81

$$\frac{x\left(\sqrt{a+c}-\sqrt{-b-c}\tan(c+dx)\right)\left(\sqrt{a+c}+\sqrt{-b-c}\tan(c+dx)\right)\left(i\sqrt{b+c}\,\mathrm{Li}_2\left(\frac{\sqrt{-b-c}(1-i\tan(c+dx))}{\sqrt{-b-c}-i\sqrt{a+c}}\right)-i\sqrt{b+c}\,\mathrm{Li}\cdots\right.}{\cdots}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x*Sec[c + d*x]^2)/(a + c*Sec[c + d*x]^2 + b*Tan[c + d*x]^2),x]
```

```
[Out] (x*(4*Sqrt[-b - c]*c*ArcTan[(Sqrt[b + c]*Tan[c + d*x])/Sqrt[a + c]] - I*Sqr
t[b + c]*Log[1 + I*Tan[c + d*x]]*Log[(I*(Sqrt[a + c] - Sqrt[-b - c]*Tan[c +
 d*x]))/(Sqrt[-b - c] + I*Sqrt[a + c])] + I*Sqrt[b + c]*Log[1 - I*Tan[c + d
*x]]*Log[(I*(-Sqrt[a + c] + Sqrt[-b - c]*Tan[c + d*x]))/(Sqrt[-b - c] - I*S
qrt[a + c])] + I*Sqrt[b + c]*Log[1 + I*Tan[c + d*x]]*Log[((-I)*(Sqrt[a + c]
 + Sqrt[-b - c]*Tan[c + d*x]))/(Sqrt[-b - c] - I*Sqrt[a + c])] - I*Sqrt[b +
 c]*Log[1 - I*Tan[c + d*x]]*Log[(I*(Sqrt[a + c] + Sqrt[-b - c]*Tan[c + d*x]
))/(Sqrt[-b - c] + I*Sqrt[a + c])] + I*Sqrt[b + c]*PolyLog[2, (Sqrt[-b - c]
*(1 - I*Tan[c + d*x]))/(Sqrt[-b - c] - I*Sqrt[a + c])] - I*Sqrt[b + c]*Poly
Log[2, (Sqrt[-b - c]*(1 - I*Tan[c + d*x]))/(Sqrt[-b - c] + I*Sqrt[a + c])]
```

```
+ I*Sqrt[b + c]*PolyLog[2, (Sqrt[-b - c]*(1 + I*Tan[c + d*x]))/(Sqrt[-b - c
] - I*Sqrt[a + c])] - I*Sqrt[b + c]*PolyLog[2, (Sqrt[-b - c]*(1 + I*Tan[c +
 d*x]))/(Sqrt[-b - c] + I*Sqrt[a + c])])*(Sqrt[a + c] - Sqrt[-b - c]*Tan[c
+ d*x])*(Sqrt[a + c] + Sqrt[-b - c]*Tan[c + d*x]))/(2*Sqrt[a + c]*Sqrt[-(b
+ c)^2]*d*(2*c - I*Log[1 - I*Tan[c + d*x]] + I*Log[1 + I*Tan[c + d*x]])*(a
+ c*Sec[c + d*x]^2 + b*Tan[c + d*x]^2))
```

fricas [B] time = 1.38, size = 4160, normalized size = 15.58

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sec(d*x+c)^2/(a+c*sec(d*x+c)^2+b*tan(d*x+c)^2),x, algorithm="fr
icas")
```

```
[Out] 1/16*(-4*I*(a - b)*c*sqrt((a*b + (a + b)*c + c^2)/(a^2 - 2*a*b + b^2))*log(
2*sqrt(-(2*(a - b)*sqrt((a*b + (a + b)*c + c^2)/(a^2 - 2*a*b + b^2)) + a +
b + 2*c)/(a - b)) + 2*cos(d*x + c) + 2*I*sin(d*x + c)) + 4*I*(a - b)*c*sqrt
((a*b + (a + b)*c + c^2)/(a^2 - 2*a*b + b^2))*log(2*sqrt(-(2*(a - b)*sqrt((
a*b + (a + b)*c + c^2)/(a^2 - 2*a*b + b^2)) + a + b + 2*c)/(a - b)) + 2*cos
(d*x + c) - 2*I*sin(d*x + c)) + 4*I*(a - b)*c*sqrt((a*b + (a + b)*c + c^2)/
(a^2 - 2*a*b + b^2))*log(2*sqrt(-(2*(a - b)*sqrt((a*b + (a + b)*c + c^2)/(a
^2 - 2*a*b + b^2)) + a + b + 2*c)/(a - b)) - 2*cos(d*x + c) + 2*I*sin(d*x +
 c)) - 4*I*(a - b)*c*sqrt((a*b + (a + b)*c + c^2)/(a^2 - 2*a*b + b^2))*log(
2*sqrt(-(2*(a - b)*sqrt((a*b + (a + b)*c + c^2)/(a^2 - 2*a*b + b^2)) + a +
b + 2*c)/(a - b)) - 2*cos(d*x + c) - 2*I*sin(d*x + c)) + 4*I*(a - b)*c*sqrt
((a*b + (a + b)*c + c^2)/(a^2 - 2*a*b + b^2))*log(2*sqrt((2*(a - b)*sqrt((a
*b + (a + b)*c + c^2)/(a^2 - 2*a*b + b^2)) - a - b - 2*c)/(a - b)) + 2*cos(
d*x + c) + 2*I*sin(d*x + c)) - 4*I*(a - b)*c*sqrt((a*b + (a + b)*c + c^2)/(
a^2 - 2*a*b + b^2))*log(2*sqrt((2*(a - b)*sqrt((a*b + (a + b)*c + c^2)/(a^2
 - 2*a*b + b^2)) - a - b - 2*c)/(a - b)) + 2*cos(d*x + c) - 2*I*sin(d*x + c
)) - 4*I*(a - b)*c*sqrt((a*b + (a + b)*c + c^2)/(a^2 - 2*a*b + b^2))*log(2*
sqrt((2*(a - b)*sqrt((a*b + (a + b)*c + c^2)/(a^2 - 2*a*b + b^2)) - a - b -
 2*c)/(a - b)) - 2*cos(d*x + c) + 2*I*sin(d*x + c)) + 4*I*(a - b)*c*sqrt((a
*b + (a + b)*c + c^2)/(a^2 - 2*a*b + b^2))*log(2*sqrt((2*(a - b)*sqrt((a*b
+ (a + b)*c + c^2)/(a^2 - 2*a*b + b^2)) - a - b - 2*c)/(a - b)) - 2*cos(d*x
 + c) - 2*I*sin(d*x + c)) + 4*(a - b)*sqrt((a*b + (a + b)*c + c^2)/(a^2 - 2
*a*b + b^2))*dilog(1/2*((2*(a + b + 2*c)*cos(d*x + c) + (2*I*a + 2*I*b + 4*
I*c)*sin(d*x + c) - 4*((a - b)*cos(d*x + c) - (-I*a + I*b)*sin(d*x + c))*sq
rt((a*b + (a + b)*c + c^2)/(a^2 - 2*a*b + b^2)))*sqrt(-(2*(a - b)*sqrt((a*b
 + (a + b)*c + c^2)/(a^2 - 2*a*b + b^2)) + a + b + 2*c)/(a - b)) - 2*a + 2*
b)/(a - b) + 1) + 4*(a - b)*sqrt((a*b + (a + b)*c + c^2)/(a^2 - 2*a*b + b^2
))*dilog(-1/2*((2*(a + b + 2*c)*cos(d*x + c) - (2*I*a + 2*I*b + 4*I*c)*sin(
d*x + c) - 4*((a - b)*cos(d*x + c) + (-I*a + I*b)*sin(d*x + c))*sqrt((a*b +
 (a + b)*c + c^2)/(a^2 - 2*a*b + b^2)))*sqrt(-(2*(a - b)*sqrt((a*b + (a + b
```

```
)*c + c^2)/(a^2 - 2*a*b + b^2)) + a + b + 2*c)/(a - b)) + 2*a - 2*b)/(a - b
) + 1) + 4*(a - b)*sqrt((a*b + (a + b)*c + c^2)/(a^2 - 2*a*b + b^2))*dilog(
1/2*((2*(a + b + 2*c)*cos(d*x + c) + (-2*I*a - 2*I*b - 4*I*c)*sin(d*x + c)
 - 4*((a - b)*cos(d*x + c) - (I*a - I*b)*sin(d*x + c))*sqrt((a*b + (a + b)*c
 + c^2)/(a^2 - 2*a*b + b^2)))*sqrt(-(2*(a - b)*sqrt((a*b + (a + b)*c + c^2)
/(a^2 - 2*a*b + b^2)) + a + b + 2*c)/(a - b)) - 2*a + 2*b)/(a - b) + 1) + 4
*(a - b)*sqrt((a*b + (a + b)*c + c^2)/(a^2 - 2*a*b + b^2))*dilog(-1/2*((2*(
a + b + 2*c)*cos(d*x + c) - (-2*I*a - 2*I*b - 4*I*c)*sin(d*x + c) - 4*((a -
 b)*cos(d*x + c) + (I*a - I*b)*sin(d*x + c))*sqrt((a*b + (a + b)*c + c^2)/(
a^2 - 2*a*b + b^2)))*sqrt(-(2*(a - b)*sqrt((a*b + (a + b)*c + c^2)/(a^2 - 2
*a*b + b^2)) + a + b + 2*c)/(a - b)) + 2*a - 2*b)/(a - b) + 1) - 4*(a - b)*
sqrt((a*b + (a + b)*c + c^2)/(a^2 - 2*a*b + b^2))*dilog(1/2*((2*(a + b + 2*
c)*cos(d*x + c) + (2*I*a + 2*I*b + 4*I*c)*sin(d*x + c) + 4*((a - b)*cos(d*x
 + c) + (I*a - I*b)*sin(d*x + c))*sqrt((a*b + (a + b)*c + c^2)/(a^2 - 2*a*b
 + b^2)))*sqrt((2*(a - b)*sqrt((a*b + (a + b)*c + c^2)/(a^2 - 2*a*b + b^2))
 - a - b - 2*c)/(a - b)) - 2*a + 2*b)/(a - b) + 1) - 4*(a - b)*sqrt((a*b +
(a + b)*c + c^2)/(a^2 - 2*a*b + b^2))*dilog(-1/2*((2*(a + b + 2*c)*cos(d*x
+ c) - (2*I*a + 2*I*b + 4*I*c)*sin(d*x + c) + 4*((a - b)*cos(d*x + c) - (I*
a - I*b)*sin(d*x + c))*sqrt((a*b + (a + b)*c + c^2)/(a^2 - 2*a*b + b^2)))*s
qrt((2*(a - b)*sqrt((a*b + (a + b)*c + c^2)/(a^2 - 2*a*b + b^2)) - a - b -
2*c)/(a - b)) + 2*a - 2*b)/(a - b) + 1) - 4*(a - b)*sqrt((a*b + (a + b)*c +
 c^2)/(a^2 - 2*a*b + b^2))*dilog(1/2*((2*(a + b + 2*c)*cos(d*x + c) + (-2*I
*a - 2*I*b - 4*I*c)*sin(d*x + c) + 4*((a - b)*cos(d*x + c) + (-I*a + I*b)*s
in(d*x + c))*sqrt((a*b + (a + b)*c + c^2)/(a^2 - 2*a*b + b^2)))*sqrt((2*(a
- b)*sqrt((a*b + (a + b)*c + c^2)/(a^2 - 2*a*b + b^2)) - a - b - 2*c)/(a -
b)) - 2*a + 2*b)/(a - b) + 1) - 4*(a - b)*sqrt((a*b + (a + b)*c + c^2)/(a^2
 - 2*a*b + b^2))*dilog(-1/2*((2*(a + b + 2*c)*cos(d*x + c) - (-2*I*a - 2*I*
b - 4*I*c)*sin(d*x + c) + 4*((a - b)*cos(d*x + c) - (-I*a + I*b)*sin(d*x +
c))*sqrt((a*b + (a + b)*c + c^2)/(a^2 - 2*a*b + b^2)))*sqrt((2*(a - b)*sqrt
((a*b + (a + b)*c + c^2)/(a^2 - 2*a*b + b^2)) - a - b - 2*c)/(a - b)) + 2*a
 - 2*b)/(a - b) + 1) + 4*(I*(a - b)*d*x + I*(a - b)*c)*sqrt((a*b + (a + b)*
c + c^2)/(a^2 - 2*a*b + b^2))*log(-1/2*((2*(a + b + 2*c)*cos(d*x + c) + (2*
I*a + 2*I*b + 4*I*c)*sin(d*x + c) - 4*((a - b)*cos(d*x + c) - (-I*a + I*b)*
sin(d*x + c))*sqrt((a*b + (a + b)*c + c^2)/(a^2 - 2*a*b + b^2)))*sqrt(-(2*(
a - b)*sqrt((a*b + (a + b)*c + c^2)/(a^2 - 2*a*b + b^2)) + a + b + 2*c)/(a
- b)) - 2*a + 2*b)/(a - b)) + 4*(-I*(a - b)*d*x - I*(a - b)*c)*sqrt((a*b +
(a + b)*c + c^2)/(a^2 - 2*a*b + b^2))*log(1/2*((2*(a + b + 2*c)*cos(d*x + c
) - (2*I*a + 2*I*b + 4*I*c)*sin(d*x + c) - 4*((a - b)*cos(d*x + c) + (-I*a
+ I*b)*sin(d*x + c))*sqrt((a*b + (a + b)*c + c^2)/(a^2 - 2*a*b + b^2)))*sqr
t(-(2*(a - b)*sqrt((a*b + (a + b)*c + c^2)/(a^2 - 2*a*b + b^2)) + a + b + 2
*c)/(a - b)) + 2*a - 2*b)/(a - b)) + 4*(-I*(a - b)*d*x - I*(a - b)*c)*sqrt(
(a*b + (a + b)*c + c^2)/(a^2 - 2*a*b + b^2))*log(-1/2*((2*(a + b + 2*c)*cos
(d*x + c) + (-2*I*a - 2*I*b - 4*I*c)*sin(d*x + c) - 4*((a - b)*cos(d*x + c)
 - (I*a - I*b)*sin(d*x + c))*sqrt((a*b + (a + b)*c + c^2)/(a^2 - 2*a*b + b^
2)))*sqrt(-(2*(a - b)*sqrt((a*b + (a + b)*c + c^2)/(a^2 - 2*a*b + b^2)) + a
```

```
 + b + 2*c)/(a - b)) - 2*a + 2*b)/(a - b)) + 4*(I*(a - b)*d*x + I*(a - b)*c
)*sqrt((a*b + (a + b)*c + c^2)/(a^2 - 2*a*b + b^2))*log(1/2*((2*(a + b + 2*
c)*cos(d*x + c) - (-2*I*a - 2*I*b - 4*I*c)*sin(d*x + c) - 4*((a - b)*cos(d*
x + c) + (I*a - I*b)*sin(d*x + c))*sqrt((a*b + (a + b)*c + c^2)/(a^2 - 2*a*
b + b^2)))*sqrt(-(2*(a - b)*sqrt((a*b + (a + b)*c + c^2)/(a^2 - 2*a*b + b^2
)) + a + b + 2*c)/(a - b)) + 2*a - 2*b)/(a - b)) + 4*(-I*(a - b)*d*x - I*(a
 - b)*c)*sqrt((a*b + (a + b)*c + c^2)/(a^2 - 2*a*b + b^2))*log(-1/2*((2*(a
+ b + 2*c)*cos(d*x + c) + (2*I*a + 2*I*b + 4*I*c)*sin(d*x + c) + 4*((a - b)
*cos(d*x + c) + (I*a - I*b)*sin(d*x + c))*sqrt((a*b + (a + b)*c + c^2)/(a^2
 - 2*a*b + b^2)))*sqrt((2*(a - b)*sqrt((a*b + (a + b)*c + c^2)/(a^2 - 2*a*b
 + b^2)) - a - b - 2*c)/(a - b)) - 2*a + 2*b)/(a - b)) + 4*(I*(a - b)*d*x +
 I*(a - b)*c)*sqrt((a*b + (a + b)*c + c^2)/(a^2 - 2*a*b + b^2))*log(1/2*((2
*(a + b + 2*c)*cos(d*x + c) - (2*I*a + 2*I*b + 4*I*c)*sin(d*x + c) + 4*((a
- b)*cos(d*x + c) - (I*a - I*b)*sin(d*x + c))*sqrt((a*b + (a + b)*c + c^2)/
(a^2 - 2*a*b + b^2)))*sqrt((2*(a - b)*sqrt((a*b + (a + b)*c + c^2)/(a^2 - 2
*a*b + b^2)) - a - b - 2*c)/(a - b)) + 2*a - 2*b)/(a - b)) + 4*(I*(a - b)*d
*x + I*(a - b)*c)*sqrt((a*b + (a + b)*c + c^2)/(a^2 - 2*a*b + b^2))*log(-1/
2*((2*(a + b + 2*c)*cos(d*x + c) + (-2*I*a - 2*I*b - 4*I*c)*sin(d*x + c) +
4*((a - b)*cos(d*x + c) + (-I*a + I*b)*sin(d*x + c))*sqrt((a*b + (a + b)*c
+ c^2)/(a^2 - 2*a*b + b^2)))*sqrt((2*(a - b)*sqrt((a*b + (a + b)*c + c^2)/(
a^2 - 2*a*b + b^2)) - a - b - 2*c)/(a - b)) - 2*a + 2*b)/(a - b)) + 4*(-I*(
a - b)*d*x - I*(a - b)*c)*sqrt((a*b + (a + b)*c + c^2)/(a^2 - 2*a*b + b^2))
*log(1/2*((2*(a + b + 2*c)*cos(d*x + c) - (-2*I*a - 2*I*b - 4*I*c)*sin(d*x
+ c) + 4*((a - b)*cos(d*x + c) - (-I*a + I*b)*sin(d*x + c))*sqrt((a*b + (a
+ b)*c + c^2)/(a^2 - 2*a*b + b^2)))*sqrt((2*(a - b)*sqrt((a*b + (a + b)*c +
 c^2)/(a^2 - 2*a*b + b^2)) - a - b - 2*c)/(a - b)) + 2*a - 2*b)/(a - b)))/(
(a*b + (a + b)*c + c^2)*d^2)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x \sec (dx + c)^2}{c \sec (dx + c)^2 + b \tan (dx + c)^2 + a}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sec(d*x+c)^2/(a+c*sec(d*x+c)^2+b*tan(d*x+c)^2),x, algorithm="gi
ac")
```

```
[Out] integrate(x*sec(d*x + c)^2/(c*sec(d*x + c)^2 + b*tan(d*x + c)^2 + a), x)
```

maple [B] time = 0.60, size = 1670, normalized size = 6.25

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*sec(d*x+c)^2/(a+c*sec(d*x+c)^2+b*tan(d*x+c)^2),x)
```

```
[Out] -1/d/((a+c)*(b+c))^(1/2)/(-2*((a+c)*(b+c))^(1/2)-a-b-2*c)*a*c*x-1/d/((a+c)*
(b+c))^(1/2)/(-2*((a+c)*(b+c))^(1/2)-a-b-2*c)*b*c*x-I/d^2/((a+c)*(b+c))^(1/
2)/(-2*((a+c)*(b+c))^(1/2)-a-b-2*c)*ln(1-(a-b)*exp(2*I*(d*x+c))/(-2*((a+c)*
(b+c))^(1/2)-a-b-2*c))*c^2-1/2*I/d^2/((a+c)*(b+c))^(1/2)/(-2*((a+c)*(b+c))^
(1/2)-a-b-2*c)*ln(1-(a-b)*exp(2*I*(d*x+c))/(-2*((a+c)*(b+c))^(1/2)-a-b-2*c)
)*b*c-1/2*I/d/((a+c)*(b+c))^(1/2)/(-2*((a+c)*(b+c))^(1/2)-a-b-2*c)*ln(1-(a-
b)*exp(2*I*(d*x+c))/(-2*((a+c)*(b+c))^(1/2)-a-b-2*c))*a*x-1/2*I/d/((a+c)*(b
+c))^(1/2)/(-2*((a+c)*(b+c))^(1/2)-a-b-2*c)*ln(1-(a-b)*exp(2*I*(d*x+c))/(-2
*((a+c)*(b+c))^(1/2)-a-b-2*c))*b*x-1/2*I/d^2/((a+c)*(b+c))^(1/2)*ln(1-(a-b)
*exp(2*I*(d*x+c))/(2*((a+c)*(b+c))^(1/2)-a-b-2*c))*c-I/d^2/(-2*((a+c)*(b+c)
)^(1/2)-a-b-2*c)*ln(1-(a-b)*exp(2*I*(d*x+c))/(-2*((a+c)*(b+c))^(1/2)-a-b-2*
c))*c-1/2*I/d/((a+c)*(b+c))^(1/2)*ln(1-(a-b)*exp(2*I*(d*x+c))/(2*((a+c)*(b+
c))^(1/2)-a-b-2*c))*x-I/d/(-2*((a+c)*(b+c))^(1/2)-a-b-2*c)*ln(1-(a-b)*exp(2
*I*(d*x+c))/(-2*((a+c)*(b+c))^(1/2)-a-b-2*c))*x-I/d^2*c/(a*b+a*c+b*c+c^2)^(
1/2)*arctanh(1/4*(2*(a-b)*exp(2*I*(d*x+c))+2*a+2*b+4*c)/(a*b+a*c+b*c+c^2)^(
1/2))-1/d/((a+c)*(b+c))^(1/2)*c*x-2/d/(-2*((a+c)*(b+c))^(1/2)-a-b-2*c)*c*x-
1/d^2/((a+c)*(b+c))^(1/2)/(-2*((a+c)*(b+c))^(1/2)-a-b-2*c)*c^3-1/2/((a+c)*(
b+c))^(1/2)/(-2*((a+c)*(b+c))^(1/2)-a-b-2*c)*b*x^2-1/((a+c)*(b+c))^(1/2)/(-
2*((a+c)*(b+c))^(1/2)-a-b-2*c)*c*x^2-1/2/((a+c)*(b+c))^(1/2)/(-2*((a+c)*(b+
c))^(1/2)-a-b-2*c)*a*x^2-I/d/((a+c)*(b+c))^(1/2)/(-2*((a+c)*(b+c))^(1/2)-a-
b-2*c)*ln(1-(a-b)*exp(2*I*(d*x+c))/(-2*((a+c)*(b+c))^(1/2)-a-b-2*c))*c*x-1/
2*I/d^2/((a+c)*(b+c))^(1/2)/(-2*((a+c)*(b+c))^(1/2)-a-b-2*c)*ln(1-(a-b)*exp
(2*I*(d*x+c))/(-2*((a+c)*(b+c))^(1/2)-a-b-2*c))*a*c-2/d/((a+c)*(b+c))^(1/2)
/(-2*((a+c)*(b+c))^(1/2)-a-b-2*c)*c^2*x-1/2/d^2/((a+c)*(b+c))^(1/2)/(-2*((a
+c)*(b+c))^(1/2)-a-b-2*c)*a*c^2-1/2/d^2/((a+c)*(b+c))^(1/2)/(-2*((a+c)*(b+c
))^(1/2)-a-b-2*c)*b*c^2-1/4/d^2/((a+c)*(b+c))^(1/2)/(-2*((a+c)*(b+c))^(1/2)
-a-b-2*c)*polylog(2,(a-b)*exp(2*I*(d*x+c))/(-2*((a+c)*(b+c))^(1/2)-a-b-2*c)
)*a-1/4/d^2/((a+c)*(b+c))^(1/2)/(-2*((a+c)*(b+c))^(1/2)-a-b-2*c)*polylog(2,
(a-b)*exp(2*I*(d*x+c))/(-2*((a+c)*(b+c))^(1/2)-a-b-2*c))*b-1/2/d^2/((a+c)*(
b+c))^(1/2)/(-2*((a+c)*(b+c))^(1/2)-a-b-2*c)*polylog(2,(a-b)*exp(2*I*(d*x+c
))/(-2*((a+c)*(b+c))^(1/2)-a-b-2*c))*c-1/4/d^2/((a+c)*(b+c))^(1/2)*polylog(
2,(a-b)*exp(2*I*(d*x+c))/(2*((a+c)*(b+c))^(1/2)-a-b-2*c))-1/2/d^2/(-2*((a+c
)*(b+c))^(1/2)-a-b-2*c)*polylog(2,(a-b)*exp(2*I*(d*x+c))/(-2*((a+c)*(b+c))^
(1/2)-a-b-2*c))-1/2/d^2/((a+c)*(b+c))^(1/2)*c^2-1/d^2/(-2*((a+c)*(b+c))^(1/
2)-a-b-2*c)*c^2-1/2/((a+c)*(b+c))^(1/2)*x^2-1/(-2*((a+c)*(b+c))^(1/2)-a-b-2
*c)*x^2
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x \sec(dx+c)^2}{c \sec(dx+c)^2 + b \tan(dx+c)^2 + a}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sec(d*x+c)^2/(a+c*sec(d*x+c)^2+b*tan(d*x+c)^2),x, algorithm="ma
xima")
```

```
[Out] integrate(x*sec(d*x + c)^2/(c*sec(d*x + c)^2 + b*tan(d*x + c)^2 + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x}{\cos\left(c+d\,x\right)^2\,\left(a+\frac{c}{\cos(c+d\,x)^2}+b\,\tan\left(c+d\,x\right)^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(cos(c + d*x)^2*(a + c/cos(c + d*x)^2 + b*tan(c + d*x)^2)),x)
```

```
[Out] int(x/(cos(c + d*x)^2*(a + c/cos(c + d*x)^2 + b*tan(c + d*x)^2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x\sec^2(c+dx)}{a+b\tan^2(c+dx)+c\sec^2(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sec(d*x+c)**2/(a+c*sec(d*x+c)**2+b*tan(d*x+c)**2),x)
```

```
[Out] Integral(x*sec(c + d*x)**2/(a + b*tan(c + d*x)**2 + c*sec(c + d*x)**2), x)
```

3.164 $\int \frac{x^2 \sec^2(c+dx)}{a+c\sec^2(c+dx)+b\tan^2(c+dx)}\,dx$

Optimal. Leaf size=407

$$-\frac{i\text{Li}_3\left(-\frac{(a-b)e^{2i(c+dx)}}{a+b+2c-2\sqrt{a+c}\sqrt{b+c}}\right)}{4d^3\sqrt{a+c}\sqrt{b+c}}+\frac{i\text{Li}_3\left(-\frac{(a-b)e^{2i(c+dx)}}{a+b+2\left(c+\sqrt{a+c}\sqrt{b+c}\right)}\right)}{4d^3\sqrt{a+c}\sqrt{b+c}}-\frac{x\text{Li}_2\left(-\frac{(a-b)e^{2i(c+dx)}}{a+b+2c-2\sqrt{a+c}\sqrt{b+c}}\right)}{2d^2\sqrt{a+c}\sqrt{b+c}}+\frac{x\text{Li}_2\left(-\frac{(a-b)e^{2i(c+dx)}}{a+b+2\left(c+\sqrt{a+c}\sqrt{b+c}\right)}\right)}{2d^2\sqrt{a+c}\sqrt{b+c}}$$

```
[Out] -1/2*I*x^2*ln(1+(a-b)*exp(2*I*(d*x+c))/(a+b+2*c-2*(a+c)^(1/2)*(b+c)^(1/2)))
/d/(a+c)^(1/2)/(b+c)^(1/2)+1/2*I*x^2*ln(1+(a-b)*exp(2*I*(d*x+c))/(a+b+2*c+2
*(a+c)^(1/2)*(b+c)^(1/2)))/d/(a+c)^(1/2)/(b+c)^(1/2)-1/2*x*polylog(2,-(a-b)
*exp(2*I*(d*x+c))/(a+b+2*c-2*(a+c)^(1/2)*(b+c)^(1/2)))/d^2/(a+c)^(1/2)/(b+c
)^(1/2)+1/2*x*polylog(2,-(a-b)*exp(2*I*(d*x+c))/(a+b+2*c+2*(a+c)^(1/2)*(b+c
)^(1/2)))/d^2/(a+c)^(1/2)/(b+c)^(1/2)-1/4*I*polylog(3,-(a-b)*exp(2*I*(d*x+c
))/(a+b+2*c-2*(a+c)^(1/2)*(b+c)^(1/2)))/d^3/(a+c)^(1/2)/(b+c)^(1/2)+1/4*I*p
olylog(3,-(a-b)*exp(2*I*(d*x+c))/(a+b+2*c+2*(a+c)^(1/2)*(b+c)^(1/2)))/d^3/(
a+c)^(1/2)/(b+c)^(1/2)
```

Rubi [A] time = 1.08, antiderivative size = 407, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 7, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.194, Rules used = {4589, 3321, 2264, 2190, 2531, 2282, 6589}

$$-\frac{x\text{PolyLog}\left(2,-\frac{(a-b)e^{2i(c+dx)}}{-2\sqrt{a+c}\sqrt{b+c}+a+b+2c}\right)}{2d^2\sqrt{a+c}\sqrt{b+c}}+\frac{x\text{PolyLog}\left(2,-\frac{(a-b)e^{2i(c+dx)}}{2\left(\sqrt{a+c}\sqrt{b+c}+c\right)+a+b}\right)}{2d^2\sqrt{a+c}\sqrt{b+c}}-\frac{i\text{PolyLog}\left(3,-\frac{(a-b)e^{2i(c+dx)}}{-2\sqrt{a+c}\sqrt{b+c}+a+b+2c}\right)}{4d^3\sqrt{a+c}\sqrt{b+c}}+$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*Sec[c + d*x]^2)/(a + c*Sec[c + d*x]^2 + b*Tan[c + d*x]^2),x]
```

```
[Out] ((-I/2)*x^2*Log[1 + ((a - b)*E^((2*I)*(c + d*x)))/(a + b + 2*c - 2*Sqrt[a +
 c]*Sqrt[b + c])])/(Sqrt[a + c]*Sqrt[b + c]*d) + ((I/2)*x^2*Log[1 + ((a - b
)*E^((2*I)*(c + d*x)))/(a + b + 2*(c + Sqrt[a + c]*Sqrt[b + c]))])/(Sqrt[a
+ c]*Sqrt[b + c]*d) - (x*PolyLog[2, -(((a - b)*E^((2*I)*(c + d*x)))/(a + b
+ 2*c - 2*Sqrt[a + c]*Sqrt[b + c]))])/(2*Sqrt[a + c]*Sqrt[b + c]*d^2) + (x*
PolyLog[2, -(((a - b)*E^((2*I)*(c + d*x)))/(a + b + 2*(c + Sqrt[a + c]*Sqrt
[b + c])))])/(2*Sqrt[a + c]*Sqrt[b + c]*d^2) - ((I/4)*PolyLog[3, -(((a - b)
*E^((2*I)*(c + d*x)))/(a + b + 2*c - 2*Sqrt[a + c]*Sqrt[b + c]))])/(Sqrt[a
+ c]*Sqrt[b + c]*d^3) + ((I/4)*PolyLog[3, -(((a - b)*E^((2*I)*(c + d*x)))/(
a + b + 2*(c + Sqrt[a + c]*Sqrt[b + c])))])/(Sqrt[a + c]*Sqrt[b + c]*d^3)
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
```

```
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2264

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c)/q, Int[
((f + g*x)^m*F^u)/(b - q + 2*c*F^u), x], x] - Dist[(2*c)/q, Int[((f + g*x)^
m*F^u)/(b + q + 2*c*F^u), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 3321

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(
x_)]), x_Symbol] :> Dist[2, Int[((c + d*x)^m*E^(I*Pi*(k - 1/2))*E^(I*(e + f
*x)))/(b + 2*a*E^(I*Pi*(k - 1/2))*E^(I*(e + f*x)) - b*E^(2*I*k*Pi)*E^(2*I*(
e + f*x))), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[2*k] && NeQ[
a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 4589

```
Int[(((f_.) + (g_.)*(x_))^(m_.)*Sec[(d_.) + (e_.)*(x_)]^2)/((b_.) + (a_.)*S
ec[(d_.) + (e_.)*(x_)]^2 + (c_.)*Tan[(d_.) + (e_.)*(x_)]^2), x_Symbol] :> D
ist[2, Int[(f + g*x)^m/(2*a + b + c + (b - c)*Cos[2*d + 2*e*x]), x], x] /;
FreeQ[{a, b, c, d, e, f, g}, x] && IGtQ[m, 0] && NeQ[a + b, 0] && NeQ[a + c
, 0]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{x^2\sec^2(c+dx)}{a+c\sec^2(c+dx)+b\tan^2(c+dx)}\,dx = 2\int \frac{x^2}{a+b+2c+(a-b)\cos(2c+2dx)}\,dx$$

$$= 4\int \frac{e^{i(2c+2dx)}x^2}{a-b+2(a+b+2c)e^{i(2c+2dx)}+(a-b)e^{2i(2c+2dx)}}\,dx$$

$$= \frac{(2(a-b))\int \frac{e^{i(2c+2dx)}x^2}{-4\sqrt{a+c}\sqrt{b+c}+2(a+b+2c)+2(a-b)e^{i(2c+2dx)}}\,dx}{\sqrt{a+c}\sqrt{b+c}} - \frac{(2(a-b))\int \frac{}{4\sqrt{a+}}$$

$$= -\frac{ix^2\log\left(1+\frac{(a-b)e^{2i(c+dx)}}{a+b+2c-2\sqrt{a+c}\sqrt{b+c}}\right)}{2\sqrt{a+c}\sqrt{b+c}\,d} + \frac{ix^2\log\left(1+\frac{(a-b)e^{2i(c+dx)}}{a+b+2\left(c+\sqrt{a+c}\sqrt{b+c}\right)}\right)}{2\sqrt{a+c}\sqrt{b+c}\,d}$$

$$= -\frac{ix^2\log\left(1+\frac{(a-b)e^{2i(c+dx)}}{a+b+2c-2\sqrt{a+c}\sqrt{b+c}}\right)}{2\sqrt{a+c}\sqrt{b+c}\,d} + \frac{ix^2\log\left(1+\frac{(a-b)e^{2i(c+dx)}}{a+b+2\left(c+\sqrt{a+c}\sqrt{b+c}\right)}\right)}{2\sqrt{a+c}\sqrt{b+c}\,d}$$

$$= -\frac{ix^2\log\left(1+\frac{(a-b)e^{2i(c+dx)}}{a+b+2c-2\sqrt{a+c}\sqrt{b+c}}\right)}{2\sqrt{a+c}\sqrt{b+c}\,d} + \frac{ix^2\log\left(1+\frac{(a-b)e^{2i(c+dx)}}{a+b+2\left(c+\sqrt{a+c}\sqrt{b+c}\right)}\right)}{2\sqrt{a+c}\sqrt{b+c}\,d}$$

$$= -\frac{ix^2\log\left(1+\frac{(a-b)e^{2i(c+dx)}}{a+b+2c-2\sqrt{a+c}\sqrt{b+c}}\right)}{2\sqrt{a+c}\sqrt{b+c}\,d} + \frac{ix^2\log\left(1+\frac{(a-b)e^{2i(c+dx)}}{a+b+2\left(c+\sqrt{a+c}\sqrt{b+c}\right)}\right)}{2\sqrt{a+c}\sqrt{b+c}\,d}$$

Mathematica [A] time = 2.21, size = 499, normalized size = 1.23

$$-\frac{ie^{2ic}\left(2d^2x^2\log\left(1+\frac{(a-b)e^{2i(2c+dx)}}{-2\sqrt{e^{4ic}(a+c)(b+c)}+ae^{2ic}+be^{2ic}+2ce^{2ic}}\right)-2d^2x^2\log\left(1+\frac{(a-b)e^{2i(2c+dx)}}{2\sqrt{e^{4ic}(a+c)(b+c)}+ae^{2ic}+be^{2ic}+2ce^{2ic}}\right)-2idx\text{Li}_2\left(\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*Sec[c + d*x]^2)/(a + c*Sec[c + d*x]^2 + b*Tan[c + d*x]^2),x]
```

```
[Out] ((-1/4*I)*E^((2*I)*c)*(2*d^2*x^2*Log[1 + ((a - b)*E^((2*I)*(2*c + d*x)))/(a
*E^((2*I)*c) + b*E^((2*I)*c) + 2*c*E^((2*I)*c) - 2*Sqrt[(a + c)*(b + c)*E^(
(4*I)*c)])] - 2*d^2*x^2*Log[1 + ((a - b)*E^((2*I)*(2*c + d*x)))/(a*E^((2*I)
*c) + b*E^((2*I)*c) + 2*c*E^((2*I)*c) + 2*Sqrt[(a + c)*(b + c)*E^((4*I)*c)]
)] - (2*I)*d*x*PolyLog[2, ((-a + b)*E^((2*I)*(2*c + d*x)))/(a*E^((2*I)*c) +
 b*E^((2*I)*c) + 2*c*E^((2*I)*c) - 2*Sqrt[(a + c)*(b + c)*E^((4*I)*c)])] +
(2*I)*d*x*PolyLog[2, ((-a + b)*E^((2*I)*(2*c + d*x)))/(a*E^((2*I)*c) + b*E^
((2*I)*c) + 2*c*E^((2*I)*c) + 2*Sqrt[(a + c)*(b + c)*E^((4*I)*c)])] + PolyL
og[3, ((-a + b)*E^((2*I)*(2*c + d*x)))/(a*E^((2*I)*c) + b*E^((2*I)*c) + 2*c
*E^((2*I)*c) - 2*Sqrt[(a + c)*(b + c)*E^((4*I)*c)])] - PolyLog[3, ((-a + b)
*E^((2*I)*(2*c + d*x)))/(a*E^((2*I)*c) + b*E^((2*I)*c) + 2*c*E^((2*I)*c) +
2*Sqrt[(a + c)*(b + c)*E^((4*I)*c)])]))/(d^3*Sqrt[(a + c)*(b + c)*E^((4*I)*
c)])
```

fricas [C] time = 1.28, size = 5772, normalized size = 14.18

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*sec(d*x+c)^2/(a+c*sec(d*x+c)^2+b*tan(d*x+c)^2),x, algorithm="
fricas")
```

```
[Out] 1/16*(8*(a - b)*d*x*sqrt((a*b + (a + b)*c + c^2)/(a^2 - 2*a*b + b^2))*dilog
(1/2*((2*(a + b + 2*c)*cos(d*x + c) + (2*I*a + 2*I*b + 4*I*c)*sin(d*x + c)
- 4*((a - b)*cos(d*x + c) - (-I*a + I*b)*sin(d*x + c))*sqrt((a*b + (a + b)*
c + c^2)/(a^2 - 2*a*b + b^2)))*sqrt(-(2*(a - b)*sqrt((a*b + (a + b)*c + c^2
)/(a^2 - 2*a*b + b^2)) + a + b + 2*c)/(a - b)) - 2*a + 2*b)/(a - b) + 1) +
8*(a - b)*d*x*sqrt((a*b + (a + b)*c + c^2)/(a^2 - 2*a*b + b^2))*dilog(-1/2*
((2*(a + b + 2*c)*cos(d*x + c) - (2*I*a + 2*I*b + 4*I*c)*sin(d*x + c) - 4*(
(a - b)*cos(d*x + c) + (-I*a + I*b)*sin(d*x + c))*sqrt((a*b + (a + b)*c + c
^2)/(a^2 - 2*a*b + b^2)))*sqrt(-(2*(a - b)*sqrt((a*b + (a + b)*c + c^2)/(a^
2 - 2*a*b + b^2)) + a + b + 2*c)/(a - b)) + 2*a - 2*b)/(a - b) + 1) + 8*(a
- b)*d*x*sqrt((a*b + (a + b)*c + c^2)/(a^2 - 2*a*b + b^2))*dilog(1/2*((2*(a
 + b + 2*c)*cos(d*x + c) + (-2*I*a - 2*I*b - 4*I*c)*sin(d*x + c) - 4*((a -
b)*cos(d*x + c) - (I*a - I*b)*sin(d*x + c))*sqrt((a*b + (a + b)*c + c^2)/(a
^2 - 2*a*b + b^2)))*sqrt(-(2*(a - b)*sqrt((a*b + (a + b)*c + c^2)/(a^2 - 2*
a*b + b^2)) + a + b + 2*c)/(a - b)) - 2*a + 2*b)/(a - b) + 1) + 8*(a - b)*d
*x*sqrt((a*b + (a + b)*c + c^2)/(a^2 - 2*a*b + b^2))*dilog(-1/2*((2*(a + b
+ 2*c)*cos(d*x + c) - (-2*I*a - 2*I*b - 4*I*c)*sin(d*x + c) - 4*((a - b)*co
s(d*x + c) + (I*a - I*b)*sin(d*x + c))*sqrt((a*b + (a + b)*c + c^2)/(a^2 -
2*a*b + b^2)))*sqrt(-(2*(a - b)*sqrt((a*b + (a + b)*c + c^2)/(a^2 - 2*a*b +
 b^2)) + a + b + 2*c)/(a - b)) + 2*a - 2*b)/(a - b) + 1) - 8*(a - b)*d*x*sq
rt((a*b + (a + b)*c + c^2)/(a^2 - 2*a*b + b^2))*dilog(1/2*((2*(a + b + 2*c)
*cos(d*x + c) + (2*I*a + 2*I*b + 4*I*c)*sin(d*x + c) + 4*((a - b)*cos(d*x +
 c) + (I*a - I*b)*sin(d*x + c))*sqrt((a*b + (a + b)*c + c^2)/(a^2 - 2*a*b +
```

```
b^2)))*sqrt((2*(a - b)*sqrt((a*b + (a + b)*c + c^2)/(a^2 - 2*a*b + b^2)) -
 a - b - 2*c)/(a - b)) - 2*a + 2*b)/(a - b) + 1) - 8*(a - b)*d*x*sqrt((a*b
+ (a + b)*c + c^2)/(a^2 - 2*a*b + b^2))*dilog(-1/2*((2*(a + b + 2*c)*cos(d*
x + c) - (2*I*a + 2*I*b + 4*I*c)*sin(d*x + c) + 4*((a - b)*cos(d*x + c) - (
I*a - I*b)*sin(d*x + c))*sqrt((a*b + (a + b)*c + c^2)/(a^2 - 2*a*b + b^2)))
*sqrt((2*(a - b)*sqrt((a*b + (a + b)*c + c^2)/(a^2 - 2*a*b + b^2)) - a - b
- 2*c)/(a - b)) + 2*a - 2*b)/(a - b) + 1) - 8*(a - b)*d*x*sqrt((a*b + (a +
b)*c + c^2)/(a^2 - 2*a*b + b^2))*dilog(1/2*((2*(a + b + 2*c)*cos(d*x + c) +
 (-2*I*a - 2*I*b - 4*I*c)*sin(d*x + c) + 4*((a - b)*cos(d*x + c) + (-I*a +
I*b)*sin(d*x + c))*sqrt((a*b + (a + b)*c + c^2)/(a^2 - 2*a*b + b^2)))*sqrt(
(2*(a - b)*sqrt((a*b + (a + b)*c + c^2)/(a^2 - 2*a*b + b^2)) - a - b - 2*c)
/(a - b)) - 2*a + 2*b)/(a - b) + 1) - 8*(a - b)*d*x*sqrt((a*b + (a + b)*c +
 c^2)/(a^2 - 2*a*b + b^2))*dilog(-1/2*((2*(a + b + 2*c)*cos(d*x + c) - (-2*
I*a - 2*I*b - 4*I*c)*sin(d*x + c) + 4*((a - b)*cos(d*x + c) - (-I*a + I*b)*
sin(d*x + c))*sqrt((a*b + (a + b)*c + c^2)/(a^2 - 2*a*b + b^2)))*sqrt((2*(a
 - b)*sqrt((a*b + (a + b)*c + c^2)/(a^2 - 2*a*b + b^2)) - a - b - 2*c)/(a -
 b)) + 2*a - 2*b)/(a - b) + 1) + 4*I*(a - b)*c^2*sqrt((a*b + (a + b)*c + c^
2)/(a^2 - 2*a*b + b^2))*log(2*sqrt(-(2*(a - b)*sqrt((a*b + (a + b)*c + c^2)
/(a^2 - 2*a*b + b^2)) + a + b + 2*c)/(a - b)) + 2*cos(d*x + c) + 2*I*sin(d*
x + c)) - 4*I*(a - b)*c^2*sqrt((a*b + (a + b)*c + c^2)/(a^2 - 2*a*b + b^2))
*log(2*sqrt(-(2*(a - b)*sqrt((a*b + (a + b)*c + c^2)/(a^2 - 2*a*b + b^2)) +
 a + b + 2*c)/(a - b)) + 2*cos(d*x + c) - 2*I*sin(d*x + c)) - 4*I*(a - b)*c
^2*sqrt((a*b + (a + b)*c + c^2)/(a^2 - 2*a*b + b^2))*log(2*sqrt(-(2*(a - b)
*sqrt((a*b + (a + b)*c + c^2)/(a^2 - 2*a*b + b^2)) + a + b + 2*c)/(a - b))
- 2*cos(d*x + c) + 2*I*sin(d*x + c)) + 4*I*(a - b)*c^2*sqrt((a*b + (a + b)*
c + c^2)/(a^2 - 2*a*b + b^2))*log(2*sqrt(-(2*(a - b)*sqrt((a*b + (a + b)*c
+ c^2)/(a^2 - 2*a*b + b^2)) + a + b + 2*c)/(a - b)) - 2*cos(d*x + c) - 2*I*
sin(d*x + c)) - 4*I*(a - b)*c^2*sqrt((a*b + (a + b)*c + c^2)/(a^2 - 2*a*b +
 b^2))*log(2*sqrt((2*(a - b)*sqrt((a*b + (a + b)*c + c^2)/(a^2 - 2*a*b + b^
2)) - a - b - 2*c)/(a - b)) + 2*cos(d*x + c) + 2*I*sin(d*x + c)) + 4*I*(a -
 b)*c^2*sqrt((a*b + (a + b)*c + c^2)/(a^2 - 2*a*b + b^2))*log(2*sqrt((2*(a
- b)*sqrt((a*b + (a + b)*c + c^2)/(a^2 - 2*a*b + b^2)) - a - b - 2*c)/(a -
b)) + 2*cos(d*x + c) - 2*I*sin(d*x + c)) + 4*I*(a - b)*c^2*sqrt((a*b + (a +
 b)*c + c^2)/(a^2 - 2*a*b + b^2))*log(2*sqrt((2*(a - b)*sqrt((a*b + (a + b)
*c + c^2)/(a^2 - 2*a*b + b^2)) - a - b - 2*c)/(a - b)) - 2*cos(d*x + c) + 2
*I*sin(d*x + c)) - 4*I*(a - b)*c^2*sqrt((a*b + (a + b)*c + c^2)/(a^2 - 2*a*
b + b^2))*log(2*sqrt((2*(a - b)*sqrt((a*b + (a + b)*c + c^2)/(a^2 - 2*a*b +
 b^2)) - a - b - 2*c)/(a - b)) - 2*cos(d*x + c) - 2*I*sin(d*x + c)) + 4*(I*
(a - b)*d^2*x^2 - I*(a - b)*c^2)*sqrt((a*b + (a + b)*c + c^2)/(a^2 - 2*a*b
+ b^2))*log(-1/2*((2*(a + b + 2*c)*cos(d*x + c) + (2*I*a + 2*I*b + 4*I*c)*s
in(d*x + c) - 4*((a - b)*cos(d*x + c) - (-I*a + I*b)*sin(d*x + c))*sqrt((a*
b + (a + b)*c + c^2)/(a^2 - 2*a*b + b^2)))*sqrt(-(2*(a - b)*sqrt((a*b + (a
+ b)*c + c^2)/(a^2 - 2*a*b + b^2)) + a + b + 2*c)/(a - b)) - 2*a + 2*b)/(a
- b)) + 4*(-I*(a - b)*d^2*x^2 + I*(a - b)*c^2)*sqrt((a*b + (a + b)*c + c^2)
/(a^2 - 2*a*b + b^2))*log(1/2*((2*(a + b + 2*c)*cos(d*x + c) - (2*I*a + 2*I
```

```
*b + 4*I*c)*sin(d*x + c) - 4*((a - b)*cos(d*x + c) + (-I*a + I*b)*sin(d*x +
 c))*sqrt((a*b + (a + b)*c + c^2)/(a^2 - 2*a*b + b^2)))*sqrt(-(2*(a - b)*sq
rt((a*b + (a + b)*c + c^2)/(a^2 - 2*a*b + b^2)) + a + b + 2*c)/(a - b)) + 2
*a - 2*b)/(a - b)) + 4*(-I*(a - b)*d^2*x^2 + I*(a - b)*c^2)*sqrt((a*b + (a
+ b)*c + c^2)/(a^2 - 2*a*b + b^2))*log(-1/2*((2*(a + b + 2*c)*cos(d*x + c)
+ (-2*I*a - 2*I*b - 4*I*c)*sin(d*x + c) - 4*((a - b)*cos(d*x + c) - (I*a -
I*b)*sin(d*x + c))*sqrt((a*b + (a + b)*c + c^2)/(a^2 - 2*a*b + b^2)))*sqrt(
-(2*(a - b)*sqrt((a*b + (a + b)*c + c^2)/(a^2 - 2*a*b + b^2)) + a + b + 2*c
)/(a - b)) - 2*a + 2*b)/(a - b)) + 4*(I*(a - b)*d^2*x^2 - I*(a - b)*c^2)*sq
rt((a*b + (a + b)*c + c^2)/(a^2 - 2*a*b + b^2))*log(1/2*((2*(a + b + 2*c)*c
os(d*x + c) - (-2*I*a - 2*I*b - 4*I*c)*sin(d*x + c) - 4*((a - b)*cos(d*x +
c) + (I*a - I*b)*sin(d*x + c))*sqrt((a*b + (a + b)*c + c^2)/(a^2 - 2*a*b +
b^2)))*sqrt(-(2*(a - b)*sqrt((a*b + (a + b)*c + c^2)/(a^2 - 2*a*b + b^2)) +
 a + b + 2*c)/(a - b)) + 2*a - 2*b)/(a - b)) + 4*(-I*(a - b)*d^2*x^2 + I*(a
 - b)*c^2)*sqrt((a*b + (a + b)*c + c^2)/(a^2 - 2*a*b + b^2))*log(-1/2*((2*(
a + b + 2*c)*cos(d*x + c) + (2*I*a + 2*I*b + 4*I*c)*sin(d*x + c) + 4*((a -
b)*cos(d*x + c) + (I*a - I*b)*sin(d*x + c))*sqrt((a*b + (a + b)*c + c^2)/(a
^2 - 2*a*b + b^2)))*sqrt((2*(a - b)*sqrt((a*b + (a + b)*c + c^2)/(a^2 - 2*a
*b + b^2)) - a - b - 2*c)/(a - b)) - 2*a + 2*b)/(a - b)) + 4*(I*(a - b)*d^2
*x^2 - I*(a - b)*c^2)*sqrt((a*b + (a + b)*c + c^2)/(a^2 - 2*a*b + b^2))*log
(1/2*((2*(a + b + 2*c)*cos(d*x + c) - (2*I*a + 2*I*b + 4*I*c)*sin(d*x + c)
+ 4*((a - b)*cos(d*x + c) - (I*a - I*b)*sin(d*x + c))*sqrt((a*b + (a + b)*c
 + c^2)/(a^2 - 2*a*b + b^2)))*sqrt((2*(a - b)*sqrt((a*b + (a + b)*c + c^2)/
(a^2 - 2*a*b + b^2)) - a - b - 2*c)/(a - b)) + 2*a - 2*b)/(a - b)) + 4*(I*(
a - b)*d^2*x^2 - I*(a - b)*c^2)*sqrt((a*b + (a + b)*c + c^2)/(a^2 - 2*a*b +
 b^2))*log(-1/2*((2*(a + b + 2*c)*cos(d*x + c) + (-2*I*a - 2*I*b - 4*I*c)*s
in(d*x + c) + 4*((a - b)*cos(d*x + c) + (-I*a + I*b)*sin(d*x + c))*sqrt((a*
b + (a + b)*c + c^2)/(a^2 - 2*a*b + b^2)))*sqrt((2*(a - b)*sqrt((a*b + (a +
 b)*c + c^2)/(a^2 - 2*a*b + b^2)) - a - b - 2*c)/(a - b)) - 2*a + 2*b)/(a -
 b)) + 4*(-I*(a - b)*d^2*x^2 + I*(a - b)*c^2)*sqrt((a*b + (a + b)*c + c^2)/
(a^2 - 2*a*b + b^2))*log(1/2*((2*(a + b + 2*c)*cos(d*x + c) - (-2*I*a - 2*I
*b - 4*I*c)*sin(d*x + c) + 4*((a - b)*cos(d*x + c) - (-I*a + I*b)*sin(d*x +
 c))*sqrt((a*b + (a + b)*c + c^2)/(a^2 - 2*a*b + b^2)))*sqrt((2*(a - b)*sqr
t((a*b + (a + b)*c + c^2)/(a^2 - 2*a*b + b^2)) - a - b - 2*c)/(a - b)) + 2*
a - 2*b)/(a - b)) + 4*(2*I*a - 2*I*b)*sqrt((a*b + (a + b)*c + c^2)/(a^2 - 2
*a*b + b^2))*polylog(3, -1/2*(2*(a + b + 2*c)*cos(d*x + c) + (2*I*a + 2*I*b
 + 4*I*c)*sin(d*x + c) - 4*((a - b)*cos(d*x + c) - (-I*a + I*b)*sin(d*x + c
))*sqrt((a*b + (a + b)*c + c^2)/(a^2 - 2*a*b + b^2)))*sqrt(-(2*(a - b)*sqrt
((a*b + (a + b)*c + c^2)/(a^2 - 2*a*b + b^2)) + a + b + 2*c)/(a - b))/(a -
b)) + 4*(-2*I*a + 2*I*b)*sqrt((a*b + (a + b)*c + c^2)/(a^2 - 2*a*b + b^2))*
polylog(3, 1/2*(2*(a + b + 2*c)*cos(d*x + c) - (2*I*a + 2*I*b + 4*I*c)*sin(
d*x + c) - 4*((a - b)*cos(d*x + c) + (-I*a + I*b)*sin(d*x + c))*sqrt((a*b +
 (a + b)*c + c^2)/(a^2 - 2*a*b + b^2)))*sqrt(-(2*(a - b)*sqrt((a*b + (a + b
)*c + c^2)/(a^2 - 2*a*b + b^2)) + a + b + 2*c)/(a - b))/(a - b)) + 4*(-2*I*
a + 2*I*b)*sqrt((a*b + (a + b)*c + c^2)/(a^2 - 2*a*b + b^2))*polylog(3, -1/
```

```
2*(2*(a + b + 2*c)*cos(d*x + c) + (-2*I*a - 2*I*b - 4*I*c)*sin(d*x + c) - 4
*((a - b)*cos(d*x + c) - (I*a - I*b)*sin(d*x + c))*sqrt((a*b + (a + b)*c +
c^2)/(a^2 - 2*a*b + b^2)))*sqrt(-(2*(a - b)*sqrt((a*b + (a + b)*c + c^2)/(a
^2 - 2*a*b + b^2)) + a + b + 2*c)/(a - b))/(a - b)) + 4*(2*I*a - 2*I*b)*sqr
t((a*b + (a + b)*c + c^2)/(a^2 - 2*a*b + b^2))*polylog(3, 1/2*(2*(a + b + 2
*c)*cos(d*x + c) - (-2*I*a - 2*I*b - 4*I*c)*sin(d*x + c) - 4*((a - b)*cos(d
*x + c) + (I*a - I*b)*sin(d*x + c))*sqrt((a*b + (a + b)*c + c^2)/(a^2 - 2*a
*b + b^2)))*sqrt(-(2*(a - b)*sqrt((a*b + (a + b)*c + c^2)/(a^2 - 2*a*b + b^
2)) + a + b + 2*c)/(a - b))/(a - b)) + 4*(-2*I*a + 2*I*b)*sqrt((a*b + (a +
b)*c + c^2)/(a^2 - 2*a*b + b^2))*polylog(3, -1/2*(2*(a + b + 2*c)*cos(d*x +
 c) + (2*I*a + 2*I*b + 4*I*c)*sin(d*x + c) + 4*((a - b)*cos(d*x + c) + (I*a
 - I*b)*sin(d*x + c))*sqrt((a*b + (a + b)*c + c^2)/(a^2 - 2*a*b + b^2)))*sq
rt((2*(a - b)*sqrt((a*b + (a + b)*c + c^2)/(a^2 - 2*a*b + b^2)) - a - b - 2
*c)/(a - b))/(a - b)) + 4*(2*I*a - 2*I*b)*sqrt((a*b + (a + b)*c + c^2)/(a^2
 - 2*a*b + b^2))*polylog(3, 1/2*(2*(a + b + 2*c)*cos(d*x + c) - (2*I*a + 2*
I*b + 4*I*c)*sin(d*x + c) + 4*((a - b)*cos(d*x + c) - (I*a - I*b)*sin(d*x +
 c))*sqrt((a*b + (a + b)*c + c^2)/(a^2 - 2*a*b + b^2)))*sqrt((2*(a - b)*sqr
t((a*b + (a + b)*c + c^2)/(a^2 - 2*a*b + b^2)) - a - b - 2*c)/(a - b))/(a -
 b)) + 4*(2*I*a - 2*I*b)*sqrt((a*b + (a + b)*c + c^2)/(a^2 - 2*a*b + b^2))*
polylog(3, -1/2*(2*(a + b + 2*c)*cos(d*x + c) + (-2*I*a - 2*I*b - 4*I*c)*si
n(d*x + c) + 4*((a - b)*cos(d*x + c) + (-I*a + I*b)*sin(d*x + c))*sqrt((a*b
 + (a + b)*c + c^2)/(a^2 - 2*a*b + b^2)))*sqrt((2*(a - b)*sqrt((a*b + (a +
b)*c + c^2)/(a^2 - 2*a*b + b^2)) - a - b - 2*c)/(a - b))/(a - b)) + 4*(-2*I
*a + 2*I*b)*sqrt((a*b + (a + b)*c + c^2)/(a^2 - 2*a*b + b^2))*polylog(3, 1/
2*(2*(a + b + 2*c)*cos(d*x + c) - (-2*I*a - 2*I*b - 4*I*c)*sin(d*x + c) + 4
*((a - b)*cos(d*x + c) - (-I*a + I*b)*sin(d*x + c))*sqrt((a*b + (a + b)*c +
 c^2)/(a^2 - 2*a*b + b^2)))*sqrt((2*(a - b)*sqrt((a*b + (a + b)*c + c^2)/(a
^2 - 2*a*b + b^2)) - a - b - 2*c)/(a - b))/(a - b)))/((a*b + (a + b)*c + c^
2)*d^3)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2 \sec(dx+c)^2}{c\sec(dx+c)^2 + b\tan(dx+c)^2 + a}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*sec(d*x+c)^2/(a+c*sec(d*x+c)^2+b*tan(d*x+c)^2),x, algorithm="
giac")
```

```
[Out] integrate(x^2*sec(d*x + c)^2/(c*sec(d*x + c)^2 + b*tan(d*x + c)^2 + a), x)
```

maple [B] time = 0.55, size = 2061, normalized size = 5.06

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*sec(d*x+c)^2/(a+c*sec(d*x+c)^2+b*tan(d*x+c)^2),x)
```

```
[Out] -I/d/((a+c)*(b+c))^(1/2)/(-2*((a+c)*(b+c))^(1/2)-a-b-2*c)*c*ln(1-(a-b)*exp(
2*I*(d*x+c))/(-2*((a+c)*(b+c))^(1/2)-a-b-2*c))*x^2-1/2*I/d/((a+c)*(b+c))^(1
/2)/(-2*((a+c)*(b+c))^(1/2)-a-b-2*c)*b*ln(1-(a-b)*exp(2*I*(d*x+c))/(-2*((a+
c)*(b+c))^(1/2)-a-b-2*c))*x^2-1/2*I/d/((a+c)*(b+c))^(1/2)/(-2*((a+c)*(b+c))
^(1/2)-a-b-2*c)*a*ln(1-(a-b)*exp(2*I*(d*x+c))/(-2*((a+c)*(b+c))^(1/2)-a-b-2
*c))*x^2+1/2*I/d^3/((a+c)*(b+c))^(1/2)/(-2*((a+c)*(b+c))^(1/2)-a-b-2*c)*b*l
n(1-(a-b)*exp(2*I*(d*x+c))/(-2*((a+c)*(b+c))^(1/2)-a-b-2*c))*c^2+1/2*I/d^3/
((a+c)*(b+c))^(1/2)/(-2*((a+c)*(b+c))^(1/2)-a-b-2*c)*a*ln(1-(a-b)*exp(2*I*(
d*x+c))/(-2*((a+c)*(b+c))^(1/2)-a-b-2*c))*c^2-1/2/d^2/((a+c)*(b+c))^(1/2)/(
-2*((a+c)*(b+c))^(1/2)-a-b-2*c)*b*polylog(2,(a-b)*exp(2*I*(d*x+c))/(-2*((a+
c)*(b+c))^(1/2)-a-b-2*c))*x-1/d^2/((a+c)*(b+c))^(1/2)/(-2*((a+c)*(b+c))^(1/
2)-a-b-2*c)*c*polylog(2,(a-b)*exp(2*I*(d*x+c))/(-2*((a+c)*(b+c))^(1/2)-a-b-
2*c))*x-1/2/d^2/((a+c)*(b+c))^(1/2)/(-2*((a+c)*(b+c))^(1/2)-a-b-2*c)*a*poly
log(2,(a-b)*exp(2*I*(d*x+c))/(-2*((a+c)*(b+c))^(1/2)-a-b-2*c))*x+1/d^2/((a+
c)*(b+c))^(1/2)/(-2*((a+c)*(b+c))^(1/2)-a-b-2*c)*a*c^2*x+1/d^2/((a+c)*(b+c)
)^(1/2)/(-2*((a+c)*(b+c))^(1/2)-a-b-2*c)*b*c^2*x-1/4*I/d^3/((a+c)*(b+c))^(1
/2)/(-2*((a+c)*(b+c))^(1/2)-a-b-2*c)*a*polylog(3,(a-b)*exp(2*I*(d*x+c))/(-2
*((a+c)*(b+c))^(1/2)-a-b-2*c))-1/4*I/d^3/((a+c)*(b+c))^(1/2)/(-2*((a+c)*(b+
c))^(1/2)-a-b-2*c)*b*polylog(3,(a-b)*exp(2*I*(d*x+c))/(-2*((a+c)*(b+c))^(1/
2)-a-b-2*c))+2/d^2*c^3/((a+c)*(b+c))^(1/2)/(-2*((a+c)*(b+c))^(1/2)-a-b-2*c)
*x+2/3/d^3/((a+c)*(b+c))^(1/2)/(-2*((a+c)*(b+c))^(1/2)-a-b-2*c)*a*c^3+2/3/d
^3/((a+c)*(b+c))^(1/2)/(-2*((a+c)*(b+c))^(1/2)-a-b-2*c)*b*c^3+I/d^3*c^2/(a*
b+a*c+b*c+c^2)^(1/2)*arctanh(1/4*(2*(a-b)*exp(2*I*(d*x+c))+2*a+2*b+4*c)/(a*
b+a*c+b*c+c^2)^(1/2))-I/d/(-2*((a+c)*(b+c))^(1/2)-a-b-2*c)*ln(1-(a-b)*exp(2
*I*(d*x+c))/(-2*((a+c)*(b+c))^(1/2)-a-b-2*c))*x^2-1/2*I/d/((a+c)*(b+c))^(1/
2)*ln(1-(a-b)*exp(2*I*(d*x+c))/(2*((a+c)*(b+c))^(1/2)-a-b-2*c))*x^2+1/2*I/d
^3*c^2/((a+c)*(b+c))^(1/2)*ln(1-(a-b)*exp(2*I*(d*x+c))/(2*((a+c)*(b+c))^(1/
2)-a-b-2*c))+I/d^3*c^2/(-2*((a+c)*(b+c))^(1/2)-a-b-2*c)*ln(1-(a-b)*exp(2*I*
(d*x+c))/(-2*((a+c)*(b+c))^(1/2)-a-b-2*c))-1/3/((a+c)*(b+c))^(1/2)/(-2*((a+
c)*(b+c))^(1/2)-a-b-2*c)*a*x^3-1/3/((a+c)*(b+c))^(1/2)/(-2*((a+c)*(b+c))^(1
/2)-a-b-2*c)*b*x^3-2/3/((a+c)*(b+c))^(1/2)/(-2*((a+c)*(b+c))^(1/2)-a-b-2*c)
*c*x^3-1/2/d^2/((a+c)*(b+c))^(1/2)*polylog(2,(a-b)*exp(2*I*(d*x+c))/(2*((a+
c)*(b+c))^(1/2)-a-b-2*c))*x-1/d^2/(-2*((a+c)*(b+c))^(1/2)-a-b-2*c)*polylog(
2,(a-b)*exp(2*I*(d*x+c))/(-2*((a+c)*(b+c))^(1/2)-a-b-2*c))*x+4/3/d^3*c^4/((
a+c)*(b+c))^(1/2)/(-2*((a+c)*(b+c))^(1/2)-a-b-2*c)+1/d^2*c^2/((a+c)*(b+c))^
(1/2)*x+2/d^2*c^2/(-2*((a+c)*(b+c))^(1/2)-a-b-2*c)*x-1/4*I/d^3/((a+c)*(b+c)
)^(1/2)*polylog(3,(a-b)*exp(2*I*(d*x+c))/(2*((a+c)*(b+c))^(1/2)-a-b-2*c))-1
/2*I/d^3/(-2*((a+c)*(b+c))^(1/2)-a-b-2*c)*polylog(3,(a-b)*exp(2*I*(d*x+c))/
(-2*((a+c)*(b+c))^(1/2)-a-b-2*c))+2/3/d^3*c^3/((a+c)*(b+c))^(1/2)+4/3/d^3*c
^3/(-2*((a+c)*(b+c))^(1/2)-a-b-2*c)-1/2*I/d^3/((a+c)*(b+c))^(1/2)/(-2*((a+c
)*(b+c))^(1/2)-a-b-2*c)*c*polylog(3,(a-b)*exp(2*I*(d*x+c))/(-2*((a+c)*(b+c)
```

```
)^(1/2)-a-b-2*c))+I/d^3/((a+c)*(b+c))^(1/2)/(-2*((a+c)*(b+c))^(1/2)-a-b-2*c
)*c^3*ln(1-(a-b)*exp(2*I*(d*x+c))/(-2*((a+c)*(b+c))^(1/2)-a-b-2*c))-1/3/((a
+c)*(b+c))^(1/2)*x^3-2/3/(-2*((a+c)*(b+c))^(1/2)-a-b-2*c)*x^3
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2 \sec(dx+c)^2}{c \sec(dx+c)^2 + b \tan(dx+c)^2 + a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*sec(d*x+c)^2/(a+c*sec(d*x+c)^2+b*tan(d*x+c)^2),x, algorithm="
maxima")
```

```
[Out] integrate(x^2*sec(d*x + c)^2/(c*sec(d*x + c)^2 + b*tan(d*x + c)^2 + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^2}{\cos(c+d\,x)^2 \left(a + \frac{c}{\cos(c+d\,x)^2} + b \tan(c+d\,x)^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(cos(c + d*x)^2*(a + c/cos(c + d*x)^2 + b*tan(c + d*x)^2)),x)
```

```
[Out] int(x^2/(cos(c + d*x)^2*(a + c/cos(c + d*x)^2 + b*tan(c + d*x)^2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2 \sec^2(c+dx)}{a + b \tan^2(c+dx) + c \sec^2(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*sec(d*x+c)**2/(a+c*sec(d*x+c)**2+b*tan(d*x+c)**2),x)
```

```
[Out] Integral(x**2*sec(c + d*x)**2/(a + b*tan(c + d*x)**2 + c*sec(c + d*x)**2),
x)
```

3.165 $\int x^3\sqrt{a-a\sin(e+fx)}\sqrt{c+c\sin(e+fx)}\,dx$

Optimal. Leaf size=155

$$-\frac{6\sqrt{a-a\sin(e+fx)}\sqrt{c\sin(e+fx)+c}}{f^4}-\frac{6x\tan(e+fx)\sqrt{a-a\sin(e+fx)}\sqrt{c\sin(e+fx)+c}}{f^3}+\frac{3x^2\sqrt{a-a\sin}}{}$$

```
[Out] -6*(a-a*sin(f*x+e))^(1/2)*(c+c*sin(f*x+e))^(1/2)/f^4+3*x^2*(a-a*sin(f*x+e))
^(1/2)*(c+c*sin(f*x+e))^(1/2)/f^2-6*x*(a-a*sin(f*x+e))^(1/2)*(c+c*sin(f*x+e
))^(1/2)*tan(f*x+e)/f^3+x^3*(a-a*sin(f*x+e))^(1/2)*(c+c*sin(f*x+e))^(1/2)*t
an(f*x+e)/f
```

Rubi [A] time = 0.20, antiderivative size = 155, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {4604, 3296, 2638}

$$\frac{3x^2\sqrt{a-a\sin(e+fx)}\sqrt{c\sin(e+fx)+c}}{f^2}-\frac{6\sqrt{a-a\sin(e+fx)}\sqrt{c\sin(e+fx)+c}}{f^4}-\frac{6x\tan(e+fx)\sqrt{a-a\sin(e}}{f}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*Sqrt[a - a*Sin[e + f*x]]*Sqrt[c + c*Sin[e + f*x]],x]
```

```
[Out] (-6*Sqrt[a - a*Sin[e + f*x]]*Sqrt[c + c*Sin[e + f*x]])/f^4 + (3*x^2*Sqrt[a
 - a*Sin[e + f*x]]*Sqrt[c + c*Sin[e + f*x]])/f^2 - (6*x*Sqrt[a - a*Sin[e + f
*x]]*Sqrt[c + c*Sin[e + f*x]]*Tan[e + f*x])/f^3 + (x^3*Sqrt[a - a*Sin[e + f
*x]]*Sqrt[c + c*Sin[e + f*x]]*Tan[e + f*x])/f
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 4604

```
Int[((g_.) + (h_.)*(x_))^(p_.)*((a_) + (b_.)*Sin[(e_.) + (f_.)*(x_)])^(m_)*
((c_) + (d_.)*Sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Dist[(a^IntPart[m
]*c^IntPart[m]*(a + b*Sin[e + f*x])^FracPart[m]*(c + d*Sin[e + f*x])^FracPa
rt[m])/Cos[e + f*x]^(2*FracPart[m]), Int[(g + h*x)^p*Cos[e + f*x]^(2*m)*(c
```

```
+ d*Sin[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] &&
EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[p] && IntegerQ[2*m] && I
GeQ[n - m, 0]
```

Rubi steps

$$\int x^3\sqrt{a-a\sin(e+fx)}\,\sqrt{c+c\sin(e+fx)}\,dx = \left(\sec(e+fx)\sqrt{a-a\sin(e+fx)}\,\sqrt{c+c\sin(e+fx)}\right)\int x^3\cos(e$$

$$= \frac{x^3\sqrt{a-a\sin(e+fx)}\,\sqrt{c+c\sin(e+fx)}\,\tan(e+fx)}{f} - \frac{(3\sec(e}{}$$

$$= \frac{3x^2\sqrt{a-a\sin(e+fx)}\,\sqrt{c+c\sin(e+fx)}}{f^2} + \frac{x^3\sqrt{a-a\sin(e+f}}{}$$

$$= \frac{3x^2\sqrt{a-a\sin(e+fx)}\,\sqrt{c+c\sin(e+fx)}}{f^2} - \frac{6x\sqrt{a-a\sin(e+f}}{}$$

$$= -\frac{6\sqrt{a-a\sin(e+fx)}\,\sqrt{c+c\sin(e+fx)}}{f^4} + \frac{3x^2\sqrt{a-a\sin(e+f}}{}$$

Mathematica [A] time = 0.51, size = 61, normalized size = 0.39

$$\frac{\left(fx\left(f^2x^2-6\right)\tan(e+fx)+3f^2x^2-6\right)\sqrt{a-a\sin(e+fx)}\,\sqrt{c(\sin(e+fx)+1)}}{f^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*Sqrt[a - a*Sin[e + f*x]]*Sqrt[c + c*Sin[e + f*x]],x]
```

```
[Out] (Sqrt[c*(1 + Sin[e + f*x])]*Sqrt[a - a*Sin[e + f*x]]*(-6 + 3*f^2*x^2 + f*x*
(-6 + f^2*x^2)*Tan[e + f*x]))/f^4
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a-a*sin(f*x+e))^(1/2)*(c+c*sin(f*x+e))^(1/2),x, algorithm="f
ricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a-a*sin(f*x+e))^(1/2)*(c+c*sin(f*x+e))^(1/2),x, algorithm="g
iac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4
*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (
4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to
check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x
/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)sqrt(2*a)*sqrt(2*c)*(1/2*(6*
sign(sin(1/2*(f*x+exp(1))-1/4*pi))*sign(cos(1/2*(f*x+exp(1))-1/4*pi))-3*f^2
*x^2*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*sign(cos(1/2*(f*x+exp(1))-1/4*pi)))
*cos(f*x+exp(1))/f^4-1/2*(-6*f*x*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*sign(co
s(1/2*(f*x+exp(1))-1/4*pi))+f^3*x^3*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*sign
(cos(1/2*(f*x+exp(1))-1/4*pi)))*sin(f*x+exp(1))/f^4)
```

maple [F] time = 0.40, size = 0, normalized size = 0.00

$$\int x^{3} \sqrt{a - a \sin\left(f x + e\right)}\, \sqrt{c + c \sin\left(f x + e\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a-a*sin(f*x+e))^(1/2)*(c+c*sin(f*x+e))^(1/2),x)
```

```
[Out] int(x^3*(a-a*sin(f*x+e))^(1/2)*(c+c*sin(f*x+e))^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{-a \sin\left(f x + e\right) + a}\, \sqrt{c \sin\left(f x + e\right) + c}\, x^{3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a-a*sin(f*x+e))^(1/2)*(c+c*sin(f*x+e))^(1/2),x, algorithm="m
axima")
```

```
[Out] integrate(sqrt(-a*sin(f*x + e) + a)*sqrt(c*sin(f*x + e) + c)*x^3, x)
```

mupad [B] time = 3.05, size = 111, normalized size = 0.72

$$-\frac{\sqrt{-a\,\left(\sin\left(e+f\,x\right)-1\right)}\,\sqrt{c\,\left(\sin\left(e+f\,x\right)+1\right)}\,\left(6\,\cos\left(2\,e+2\,f\,x\right)-3\,f^2\,x^2+6\,f\,x\,\sin\left(2\,e+2\,f\,x\right)-3\,f^2\,x^2\right.}{f^4\,\left(\cos\left(2\,e+2\,f\,x\right)+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a - a*sin(e + f*x))^(1/2)*(c + c*sin(e + f*x))^(1/2),x)
```

```
[Out] -((-a*(sin(e + f*x) - 1))^(1/2)*(c*(sin(e + f*x) + 1))^(1/2)*(6*cos(2*e + 2
*f*x) - 3*f^2*x^2 + 6*f*x*sin(2*e + 2*f*x) - 3*f^2*x^2*cos(2*e + 2*f*x) - f
^3*x^3*sin(2*e + 2*f*x) + 6))/(f^4*(cos(2*e + 2*f*x) + 1))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^{3}\sqrt{c\left(\sin\left(e+fx\right)+1\right)}\,\sqrt{-a\left(\sin\left(e+fx\right)-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(a-a*sin(f*x+e))**(1/2)*(c+c*sin(f*x+e))**(1/2),x)
```

```
[Out] Integral(x**3*sqrt(c*(sin(e + f*x) + 1))*sqrt(-a*(sin(e + f*x) - 1)), x)
```

3.166 $\int x^2\sqrt{a-a\sin(e+fx)}\sqrt{c+c\sin(e+fx)}\,dx$

Optimal. Leaf size=118

$$-\frac{2\tan(e+fx)\sqrt{a-a\sin(e+fx)}\sqrt{c\sin(e+fx)+c}}{f^3}+\frac{2x\sqrt{a-a\sin(e+fx)}\sqrt{c\sin(e+fx)+c}}{f^2}+\frac{x^2\tan(e+fx)}{}$$

[Out] 2*x*(a-a*sin(f*x+e))^(1/2)*(c+c*sin(f*x+e))^(1/2)/f^2-2*(a-a*sin(f*x+e))^(1/2)*(c+c*sin(f*x+e))^(1/2)*tan(f*x+e)/f^3+x^2*(a-a*sin(f*x+e))^(1/2)*(c+c*sin(f*x+e))^(1/2)*tan(f*x+e)/f

Rubi [A] time = 0.18, antiderivative size = 118, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {4604, 3296, 2637}

$$\frac{2x\sqrt{a-a\sin(e+fx)}\sqrt{c\sin(e+fx)+c}}{f^2}-\frac{2\tan(e+fx)\sqrt{a-a\sin(e+fx)}\sqrt{c\sin(e+fx)+c}}{f^3}+\frac{x^2\tan(e+fx)}{}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*Sqrt[a - a*Sin[e + f*x]]*Sqrt[c + c*Sin[e + f*x]],x]
```

```
[Out] (2*x*Sqrt[a - a*Sin[e + f*x]]*Sqrt[c + c*Sin[e + f*x]])/f^2 - (2*Sqrt[a - a
*Sin[e + f*x]]*Sqrt[c + c*Sin[e + f*x]]*Tan[e + f*x])/f^3 + (x^2*Sqrt[a - a
*Sin[e + f*x]]*Sqrt[c + c*Sin[e + f*x]]*Tan[e + f*x])/f
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 4604

```
Int[((g_.) + (h_.)*(x_))^(p_.)*((a_) + (b_.)*Sin[(e_.) + (f_.)*(x_)])^(m_)*
((c_) + (d_.)*Sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Dist[(a^IntPart[m
]*c^IntPart[m]*(a + b*Sin[e + f*x])^FracPart[m]*(c + d*Sin[e + f*x])^FracPa
rt[m])/Cos[e + f*x]^(2*FracPart[m]), Int[(g + h*x)^p*Cos[e + f*x]^(2*m)*(c
+ d*Sin[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] &&
EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[p] && IntegerQ[2*m] && I
```

```
GeQ[n - m, 0]
```

Rubi steps

$$\int x^2\sqrt{a-a\sin(e+fx)}\,\sqrt{c+c\sin(e+fx)}\,dx=\left(\sec(e+fx)\sqrt{a-a\sin(e+fx)}\,\sqrt{c+c\sin(e+fx)}\right)\int x^2\cos(e$$
$$=\frac{x^2\sqrt{a-a\sin(e+fx)}\,\sqrt{c+c\sin(e+fx)}\,\tan(e+fx)}{f}-\frac{\left(2\sec(e\right.}{}$$
$$=\frac{2x\sqrt{a-a\sin(e+fx)}\,\sqrt{c+c\sin(e+fx)}}{f^2}+\frac{x^2\sqrt{a-a\sin(e+fx}}{}$$
$$=\frac{2x\sqrt{a-a\sin(e+fx)}\,\sqrt{c+c\sin(e+fx)}}{f^2}-\frac{2\sqrt{a-a\sin(e+fx)}}{}$$

Mathematica [A] time = 0.33, size = 54, normalized size = 0.46

$$\frac{\left(\left(f^2x^2-2\right)\tan(e+fx)+2fx\right)\sqrt{a-a\sin(e+fx)}\,\sqrt{c(\sin(e+fx)+1)}}{f^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*Sqrt[a - a*Sin[e + f*x]]*Sqrt[c + c*Sin[e + f*x]],x]
```

```
[Out] (Sqrt[c*(1 + Sin[e + f*x])]*Sqrt[a - a*Sin[e + f*x]]*(2*f*x + (-2 + f^2*x^2
)*Tan[e + f*x]))/f^3
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a-a*sin(f*x+e))^(1/2)*(c+c*sin(f*x+e))^(1/2),x, algorithm="f
ricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a-a*sin(f*x+e))^(1/2)*(c+c*sin(f*x+e))^(1/2),x, algorithm="g
iac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4
*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (
4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to
check sign: (4*pi/x/2)>(-4*pi/x/2)sqrt(2*a)*sqrt(2*c)*(-2*f^3*(-2*sign(sin(
1/2*(f*x+exp(1))-1/4*pi))*sign(cos(1/2*(f*x+exp(1))-1/4*pi))+f^2*x^2*sign(s
in(1/2*(f*x+exp(1))-1/4*pi))*sign(cos(1/2*(f*x+exp(1))-1/4*pi)))*sin(f*x+ex
p(1))/(-2*f^3)^2-4*f^4*x*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*sign(cos(1/2*(f
*x+exp(1))-1/4*pi))*cos(f*x+exp(1))/(-2*f^3)^2)
```

maple [F] time = 0.18, size = 0, normalized size = 0.00

$$\int x^{2}\sqrt{a-a\sin\left(fx+e\right)}\,\sqrt{c+c\sin\left(fx+e\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a-a*sin(f*x+e))^(1/2)*(c+c*sin(f*x+e))^(1/2),x)
```

```
[Out] int(x^2*(a-a*sin(f*x+e))^(1/2)*(c+c*sin(f*x+e))^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{-a\sin\left(fx+e\right)+a}\,\sqrt{c\sin\left(fx+e\right)+c}\,x^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a-a*sin(f*x+e))^(1/2)*(c+c*sin(f*x+e))^(1/2),x, algorithm="m
axima")
```

```
[Out] integrate(sqrt(-a*sin(f*x + e) + a)*sqrt(c*sin(f*x + e) + c)*x^2, x)
```

mupad [B] time = 2.81, size = 86, normalized size = 0.73

$$\frac{\sqrt{-a\,\left(\sin\left(e+f\,x\right)-1\right)}\,\sqrt{c\,\left(\sin\left(e+f\,x\right)+1\right)}\,\left(2\,f\,x-2\,\sin\left(2\,e+2\,f\,x\right)+2\,f\,x\,\left(2\cos\left(e+f\,x\right)^{2}-1\right)+f^{2}\,x^{2}\right.}{2\,f^{3}\cos\left(e+f\,x\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a - a*sin(e + f*x))^(1/2)*(c + c*sin(e + f*x))^(1/2),x)
```

```
[Out] ((-a*(sin(e + f*x) - 1))^(1/2)*(c*(sin(e + f*x) + 1))^(1/2)*(2*f*x - 2*sin(
2*e + 2*f*x) + 2*f*x*(2*cos(e + f*x)^2 - 1) + f^2*x^2*sin(2*e + 2*f*x)))/(2
*f^3*cos(e + f*x)^2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2\sqrt{c\left(\sin\left(e+fx\right)+1\right)}\,\sqrt{-a\left(\sin\left(e+fx\right)-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a-a*sin(f*x+e))**(1/2)*(c+c*sin(f*x+e))**(1/2),x)
[Out] Integral(x**2*sqrt(c*(sin(e + f*x) + 1))*sqrt(-a*(sin(e + f*x) - 1)), x)
```

3.167 $\int x\sqrt{a-a\sin(e+fx)}\sqrt{c+c\sin(e+fx)}\,dx$

Optimal. Leaf size=74

$$\frac{\sqrt{a-a\sin(e+fx)}\sqrt{c\sin(e+fx)+c}}{f^2}+\frac{x\tan(e+fx)\sqrt{a-a\sin(e+fx)}\sqrt{c\sin(e+fx)+c}}{f}$$

```
[Out] (a-a*sin(f*x+e))^(1/2)*(c+c*sin(f*x+e))^(1/2)/f^2+x*(a-a*sin(f*x+e))^(1/2)*
(c+c*sin(f*x+e))^(1/2)*tan(f*x+e)/f
```

Rubi [A] time = 0.11, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.097, Rules used = {4604, 3296, 2638}

$$\frac{\sqrt{a-a\sin(e+fx)}\sqrt{c\sin(e+fx)+c}}{f^2}+\frac{x\tan(e+fx)\sqrt{a-a\sin(e+fx)}\sqrt{c\sin(e+fx)+c}}{f}$$

Antiderivative was successfully verified.

```
[In] Int[x*Sqrt[a - a*Sin[e + f*x]]*Sqrt[c + c*Sin[e + f*x]],x]
```

```
[Out] (Sqrt[a - a*Sin[e + f*x]]*Sqrt[c + c*Sin[e + f*x]])/f^2 + (x*Sqrt[a - a*Sin
[e + f*x]]*Sqrt[c + c*Sin[e + f*x]]*Tan[e + f*x])/f
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 4604

```
Int[((g_.) + (h_.)*(x_))^(p_.)*((a_) + (b_.)*Sin[(e_.) + (f_.)*(x_)])^(m_)*
((c_) + (d_.)*Sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Dist[(a^IntPart[m
]*c^IntPart[m]*(a + b*Sin[e + f*x])^FracPart[m]*(c + d*Sin[e + f*x])^FracPa
rt[m])/Cos[e + f*x]^(2*FracPart[m]), Int[(g + h*x)^p*Cos[e + f*x]^(2*m)*(c
+ d*Sin[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] &&
EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[p] && IntegerQ[2*m] && I
GeQ[n - m, 0]
```

Rubi steps

$$\int x\sqrt{a-a\sin(e+fx)}\,\sqrt{c+c\sin(e+fx)}\,dx = \left(\sec(e+fx)\sqrt{a-a\sin(e+fx)}\,\sqrt{c+c\sin(e+fx)}\right)\int x\cos(e+$$

$$= \frac{x\sqrt{a-a\sin(e+fx)}\,\sqrt{c+c\sin(e+fx)}\,\tan(e+fx)}{f} - \frac{(\sec(e+f}{}$$

$$= \frac{\sqrt{a-a\sin(e+fx)}\,\sqrt{c+c\sin(e+fx)}}{f^2} + \frac{x\sqrt{a-a\sin(e+fx)}\,\sqrt{c}}{}$$

Mathematica [A] time = 0.20, size = 44, normalized size = 0.59

$$\frac{(fx\tan(e+fx)+1)\sqrt{a-a\sin(e+fx)}\,\sqrt{c(\sin(e+fx)+1)}}{f^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Sqrt[a - a*Sin[e + f*x]]*Sqrt[c + c*Sin[e + f*x]],x]
```

```
[Out] (Sqrt[c*(1 + Sin[e + f*x])]*Sqrt[a - a*Sin[e + f*x]]*(1 + f*x*Tan[e + f*x])
)/f^2
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a-a*sin(f*x+e))^(1/2)*(c+c*sin(f*x+e))^(1/2),x, algorithm="fri
cas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a-a*sin(f*x+e))^(1/2)*(c+c*sin(f*x+e))^(1/2),x, algorithm="gia
c")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4
```

```
*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (
4*pi/x/2)>(-4*pi/x/2)sqrt(2*a)*sqrt(2*c)*(-1/2*sign(sin(1/2*(f*x+exp(1))-1/
4*pi))*sign(cos(1/2*(f*x+exp(1))-1/4*pi))*cos(f*x+exp(1))/f^2-1/2*x*sign(si
n(1/2*(f*x+exp(1))-1/4*pi))*sign(cos(1/2*(f*x+exp(1))-1/4*pi))*sin(f*x+exp(
1))/f)
```

maple [F] time = 0.17, size = 0, normalized size = 0.00

$$\int x\sqrt{a - a\sin\left(fx + e\right)}\,\sqrt{c + c\sin\left(fx + e\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a-a*sin(f*x+e))^(1/2)*(c+c*sin(f*x+e))^(1/2),x)
```

```
[Out] int(x*(a-a*sin(f*x+e))^(1/2)*(c+c*sin(f*x+e))^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{-a\sin\left(fx + e\right) + a}\,\sqrt{c\sin\left(fx + e\right) + c}\,x\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a-a*sin(f*x+e))^(1/2)*(c+c*sin(f*x+e))^(1/2),x, algorithm="max
ima")
```

```
[Out] integrate(sqrt(-a*sin(f*x + e) + a)*sqrt(c*sin(f*x + e) + c)*x, x)
```

mupad [B] time = 2.69, size = 61, normalized size = 0.82

$$\frac{\sqrt{-a\,\left(\sin\left(e+f\,x\right)-1\right)}\,\left(2\cos\left(e+f\,x\right)^2+f\,x\,\sin\left(2\,e+2\,f\,x\right)\right)\,\sqrt{c\,\left(\sin\left(e+f\,x\right)+1\right)}}{2\,f^2\cos\left(e+f\,x\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a - a*sin(e + f*x))^(1/2)*(c + c*sin(e + f*x))^(1/2),x)
```

```
[Out] ((-a*(sin(e + f*x) - 1))^(1/2)*(2*cos(e + f*x)^2 + f*x*sin(2*e + 2*f*x))*(c
*(sin(e + f*x) + 1))^(1/2))/(2*f^2*cos(e + f*x)^2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x\sqrt{c\left(\sin\left(e + fx\right) + 1\right)}\,\sqrt{-a\left(\sin\left(e + fx\right) - 1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a-a*sin(f*x+e))**(1/2)*(c+c*sin(f*x+e))**(1/2),x)
[Out] Integral(x*sqrt(c*(sin(e + f*x) + 1))*sqrt(-a*(sin(e + f*x) - 1)), x)
```

3.168 $\int \frac{\sqrt{a-a\sin(e+fx)}\,\sqrt{c+c\sin(e+fx)}}{x}\,dx$

Optimal. Leaf size=86

$\cos(e)\text{Ci}(fx)\sec(e+fx)\sqrt{a-a\sin(e+fx)}\,\sqrt{c\sin(e+fx)+c}-\sin(e)\text{Si}(fx)\sec(e+fx)\sqrt{a-a\sin(e+fx)}\,\sqrt{c\sin}$

```
[Out] Ci(f*x)*cos(e)*sec(f*x+e)*(a-a*sin(f*x+e))^(1/2)*(c+c*sin(f*x+e))^(1/2)-sec
(f*x+e)*Si(f*x)*sin(e)*(a-a*sin(f*x+e))^(1/2)*(c+c*sin(f*x+e))^(1/2)
```

Rubi [A] time = 0.18, antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.121, Rules used = {4604, 3303, 3299, 3302}

$\cos(e)\text{CosIntegral}(fx)\sec(e+fx)\sqrt{a-a\sin(e+fx)}\,\sqrt{c\sin(e+fx)+c}-\sin(e)\text{Si}(fx)\sec(e+fx)\sqrt{a-a\sin(e+}$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[a - a*Sin[e + f*x]]*Sqrt[c + c*Sin[e + f*x]])/x,x]
```

```
[Out] Cos[e]*CosIntegral[f*x]*Sec[e + f*x]*Sqrt[a - a*Sin[e + f*x]]*Sqrt[c + c*Si
n[e + f*x]] - Sec[e + f*x]*Sin[e]*Sqrt[a - a*Sin[e + f*x]]*Sqrt[c + c*Sin[e
 + f*x]]*SinIntegral[f*x]
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3302

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 4604

```
Int[((g_.) + (h_.)*(x_))^(p_.)*((a_) + (b_.)*Sin[(e_.) + (f_.)*(x_)])^(m_)*
((c_) + (d_.)*Sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Dist[(a^IntPart[m
```

```
]*c^IntPart[m]*(a + b*Sin[e + f*x])^FracPart[m]*(c + d*Sin[e + f*x])^FracPa
rt[m])/Cos[e + f*x]^(2*FracPart[m]), Int[(g + h*x)^p*Cos[e + f*x]^(2*m)*(c
+ d*Sin[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] &&
EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[p] && IntegerQ[2*m] && I
GeQ[n - m, 0]
```

Rubi steps

$$\int \frac{\sqrt{a-a\sin(e+fx)}\,\sqrt{c+c\sin(e+fx)}}{x}\,dx = \left(\sec(e+fx)\sqrt{a-a\sin(e+fx)}\,\sqrt{c+c\sin(e+fx)}\right)\int \frac{\cos(e+fx}{x}$$

$$= \left(\cos(e)\sec(e+fx)\sqrt{a-a\sin(e+fx)}\,\sqrt{c+c\sin(e+fx)}\right)\int \frac{\cos(}{x}$$

$$= \cos(e)\mathrm{Ci}(fx)\sec(e+fx)\sqrt{a-a\sin(e+fx)}\,\sqrt{c+c\sin(e+fx)} -$$

Mathematica [A] time = 0.21, size = 52, normalized size = 0.60

$$\sec(e+fx)\sqrt{a-a\sin(e+fx)}\,\sqrt{c(\sin(e+fx)+1)}\,(\cos(e)\mathrm{Ci}(fx)-\sin(e)\mathrm{Si}(fx))$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[a - a*Sin[e + f*x]]*Sqrt[c + c*Sin[e + f*x]])/x,x]
```

```
[Out] Sec[e + f*x]*Sqrt[c*(1 + Sin[e + f*x])]*Sqrt[a - a*Sin[e + f*x]]*(Cos[e]*Co
sIntegral[f*x] - Sin[e]*SinIntegral[f*x])
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-a*sin(f*x+e))^(1/2)*(c+c*sin(f*x+e))^(1/2)/x,x, algorithm="fri
cas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-a*sin(f*x+e))^(1/2)*(c+c*sin(f*x+e))^(1/2)/x,x, algorithm="gia
c")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4
*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (
4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to
check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x
/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/
x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check
 sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Un
able to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>
(-4*pi/x/2)sqrt(2*a)*sqrt(2*c)*(-sign(sin(1/2*(f*x+exp(1))-1/4*pi))*sign(co
s(1/2*(f*x+exp(1))-1/4*pi))*re(Ci(f*x))-sign(sin(1/2*(f*x+exp(1))-1/4*pi))*
sign(cos(1/2*(f*x+exp(1))-1/4*pi))*re(Ci(-f*x))+4*Si(f*x)*sign(sin(1/2*(f*x
+exp(1))-1/4*pi))*sign(cos(1/2*(f*x+exp(1))-1/4*pi))*tan(1/2*exp(1))+2*sign
(sin(1/2*(f*x+exp(1))-1/4*pi))*sign(cos(1/2*(f*x+exp(1))-1/4*pi))*im(Ci(f*x
))*tan(1/2*exp(1))-2*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*sign(cos(1/2*(f*x+e
xp(1))-1/4*pi))*im(Ci(-f*x))*tan(1/2*exp(1))+sign(sin(1/2*(f*x+exp(1))-1/4*
pi))*sign(cos(1/2*(f*x+exp(1))-1/4*pi))*re(Ci(f*x))*tan(1/2*exp(1))^2+sign(
sin(1/2*(f*x+exp(1))-1/4*pi))*sign(cos(1/2*(f*x+exp(1))-1/4*pi))*re(Ci(-f*x
))*tan(1/2*exp(1))^2)/(4*tan(1/2*exp(1))^2+4)
```

maple [F] time = 0.17, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a-a\sin\left(fx+e\right)}\,\sqrt{c+c\sin\left(fx+e\right)}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a-a*sin(f*x+e))^(1/2)*(c+c*sin(f*x+e))^(1/2)/x,x)
```

```
[Out] int((a-a*sin(f*x+e))^(1/2)*(c+c*sin(f*x+e))^(1/2)/x,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-a\sin\left(fx+e\right)+a}\,\sqrt{c\sin\left(fx+e\right)+c}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-a*sin(f*x+e))^(1/2)*(c+c*sin(f*x+e))^(1/2)/x,x, algorithm="max
ima")
```

```
[Out] integrate(sqrt(-a*sin(f*x + e) + a)*sqrt(c*sin(f*x + e) + c)/x, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{a-a\,\sin\left(e+f\,x\right)}\,\sqrt{c+c\,\sin\left(e+f\,x\right)}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a - a*sin(e + f*x))^(1/2)*(c + c*sin(e + f*x))^(1/2))/x,x)
[Out] int(((a - a*sin(e + f*x))^(1/2)*(c + c*sin(e + f*x))^(1/2))/x, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{c\left(\sin\left(e+fx\right)+1\right)}\,\sqrt{-a\left(\sin\left(e+fx\right)-1\right)}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-a*sin(f*x+e))**(1/2)*(c+c*sin(f*x+e))**(1/2)/x,x)
[Out] Integral(sqrt(c*(sin(e + f*x) + 1))*sqrt(-a*(sin(e + f*x) - 1))/x, x)
```

3.169 $\int \frac{\sqrt{a-a\sin(e+fx)}\sqrt{c+c\sin(e+fx)}}{x^2}\,dx$

Optimal. Leaf size=123

$$-f\sin(e)\text{Ci}(fx)\sec(e+fx)\sqrt{a-a\sin(e+fx)}\sqrt{c\sin(e+fx)+c}-f\cos(e)\text{Si}(fx)\sec(e+fx)\sqrt{a-a\sin(e+fx)}$$

```
[Out] -(a-a*sin(f*x+e))^(1/2)*(c+c*sin(f*x+e))^(1/2)/x-f*cos(e)*sec(f*x+e)*Si(f*x
)*(a-a*sin(f*x+e))^(1/2)*(c+c*sin(f*x+e))^(1/2)-f*Ci(f*x)*sec(f*x+e)*sin(e)
*(a-a*sin(f*x+e))^(1/2)*(c+c*sin(f*x+e))^(1/2)
```

Rubi [A] time = 0.20, antiderivative size = 123, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.152, Rules used = {4604, 3297, 3303, 3299, 3302}

$$-f\sin(e)\text{CosIntegral}(fx)\sec(e+fx)\sqrt{a-a\sin(e+fx)}\sqrt{c\sin(e+fx)+c}-f\cos(e)\text{Si}(fx)\sec(e+fx)\sqrt{a-a\sin}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[a - a*Sin[e + f*x]]*Sqrt[c + c*Sin[e + f*x]])/x^2,x]
```

```
[Out] -((Sqrt[a - a*Sin[e + f*x]]*Sqrt[c + c*Sin[e + f*x]])/x) - f*CosIntegral[f*
x]*Sec[e + f*x]*Sin[e]*Sqrt[a - a*Sin[e + f*x]]*Sqrt[c + c*Sin[e + f*x]] -
f*Cos[e]*Sec[e + f*x]*Sqrt[a - a*Sin[e + f*x]]*Sqrt[c + c*Sin[e + f*x]]*Sin
Integral[f*x]
```

Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[((
c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3302

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 4604

```
Int[((g_.) + (h_.)*(x_))^(p_.)*((a_) + (b_.)*Sin[(e_.) + (f_.)*(x_)])^(m_)*
((c_) + (d_.)*Sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Dist[(a^IntPart[m
]*c^IntPart[m]*(a + b*Sin[e + f*x])^FracPart[m]*(c + d*Sin[e + f*x])^FracPa
rt[m])/Cos[e + f*x]^(2*FracPart[m]), Int[(g + h*x)^p*Cos[e + f*x]^(2*m)*(c
+ d*Sin[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] &&
EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[p] && IntegerQ[2*m] && I
GeQ[n - m, 0]
```

Rubi steps

$$\int \frac{\sqrt{a-a\sin(e+fx)}\,\sqrt{c+c\sin(e+fx)}}{x^2}\,dx = \left(\sec(e+fx)\sqrt{a-a\sin(e+fx)}\,\sqrt{c+c\sin(e+fx)}\right)\int \frac{\cos(e+fx}{x^2}$$

$$= -\frac{\sqrt{a-a\sin(e+fx)}\,\sqrt{c+c\sin(e+fx)}}{x} - \left(f\sec(e+fx)\sqrt{a-a\,\mathrm{si}\right.$$

$$= -\frac{\sqrt{a-a\sin(e+fx)}\,\sqrt{c+c\sin(e+fx)}}{x} - \left(f\cos(e)\sec(e+fx)\sqrt{\right.$$

$$= -\frac{\sqrt{a-a\sin(e+fx)}\,\sqrt{c+c\sin(e+fx)}}{x} - f\mathrm{Ci}(fx)\sec(e+fx)\,\mathrm{si}$$

Mathematica [A] time = 0.24, size = 65, normalized size = 0.53

$$-\frac{\sec(e+fx)\sqrt{a-a\sin(e+fx)}\,\sqrt{c(\sin(e+fx)+1)}\,(fx\sin(e)\mathrm{Ci}(fx)+fx\cos(e)\mathrm{Si}(fx)+\cos(e+fx))}{x}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[a - a*Sin[e + f*x]]*Sqrt[c + c*Sin[e + f*x]])/x^2,x]
```

```
[Out] -((Sec[e + f*x]*Sqrt[c*(1 + Sin[e + f*x])]*Sqrt[a - a*Sin[e + f*x]]*(Cos[e
+ f*x] + f*x*CosIntegral[f*x]*Sin[e] + f*x*Cos[e]*SinIntegral[f*x]))/x)
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-a*sin(f*x+e))^(1/2)*(c+c*sin(f*x+e))^(1/2)/x^2,x, algorithm="f
ricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-a*sin(f*x+e))^(1/2)*(c+c*sin(f*x+e))^(1/2)/x^2,x, algorithm="g
iac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4
*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (
4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to
check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x
/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/
x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check
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able to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>
(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign
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to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*p
i/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*
pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to ch
eck sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2
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2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check s
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le to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-
4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign:
(4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to
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x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi
/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to chec
k sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)U
nable to check sign: (4*pi/x/2)>(-4*pi/x/2)sqrt(2*a)*sqrt(2*c)*(2*sign(sin(
1/2*(f*x+exp(1))-1/4*pi))*sign(cos(1/2*(f*x+exp(1))-1/4*pi))-2*sign(sin(1/2
*(f*x+exp(1))-1/4*pi))*sign(cos(1/2*(f*x+exp(1))-1/4*pi))*tan(1/2*exp(1))^2
-2*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*sign(cos(1/2*(f*x+exp(1))-1/4*pi))*ta
```

```
n(1/2*f*x)^2+2*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*sign(cos(1/2*(f*x+exp(1))
-1/4*pi))*tan(1/2*exp(1))^2*tan(1/2*f*x)^2-8*sign(sin(1/2*(f*x+exp(1))-1/4*
pi))*sign(cos(1/2*(f*x+exp(1))-1/4*pi))*tan(1/2*exp(1))*tan(1/2*f*x)+2*f*x*
Si(f*x)*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*sign(cos(1/2*(f*x+exp(1))-1/4*pi
))+f*x*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*sign(cos(1/2*(f*x+exp(1))-1/4*pi)
)*im(Ci(f*x))-f*x*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*sign(cos(1/2*(f*x+exp(
1))-1/4*pi))*im(Ci(-f*x))-2*f*x*Si(f*x)*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*
sign(cos(1/2*(f*x+exp(1))-1/4*pi))*tan(1/2*exp(1))^2+2*f*x*Si(f*x)*sign(sin
(1/2*(f*x+exp(1))-1/4*pi))*sign(cos(1/2*(f*x+exp(1))-1/4*pi))*tan(1/2*f*x)^
2-f*x*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*sign(cos(1/2*(f*x+exp(1))-1/4*pi))
*im(Ci(f*x))*tan(1/2*exp(1))^2+f*x*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*sign(
cos(1/2*(f*x+exp(1))-1/4*pi))*im(Ci(f*x))*tan(1/2*f*x)^2+f*x*sign(sin(1/2*(
f*x+exp(1))-1/4*pi))*sign(cos(1/2*(f*x+exp(1))-1/4*pi))*im(Ci(-f*x))*tan(1/
2*exp(1))^2-f*x*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*sign(cos(1/2*(f*x+exp(1)
)-1/4*pi))*im(Ci(-f*x))*tan(1/2*f*x)^2+2*f*x*sign(sin(1/2*(f*x+exp(1))-1/4*
pi))*sign(cos(1/2*(f*x+exp(1))-1/4*pi))*re(Ci(f*x))*tan(1/2*exp(1))+2*f*x*s
ign(sin(1/2*(f*x+exp(1))-1/4*pi))*sign(cos(1/2*(f*x+exp(1))-1/4*pi))*re(Ci(
-f*x))*tan(1/2*exp(1))-2*f*x*Si(f*x)*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*sig
n(cos(1/2*(f*x+exp(1))-1/4*pi))*tan(1/2*exp(1))^2*tan(1/2*f*x)^2-f*x*sign(s
in(1/2*(f*x+exp(1))-1/4*pi))*sign(cos(1/2*(f*x+exp(1))-1/4*pi))*im(Ci(f*x))
*tan(1/2*exp(1))^2*tan(1/2*f*x)^2+f*x*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*si
gn(cos(1/2*(f*x+exp(1))-1/4*pi))*im(Ci(-f*x))*tan(1/2*exp(1))^2*tan(1/2*f*x
)^2+2*f*x*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*sign(cos(1/2*(f*x+exp(1))-1/4*
pi))*re(Ci(f*x))*tan(1/2*exp(1))*tan(1/2*f*x)^2+2*f*x*sign(sin(1/2*(f*x+exp
(1))-1/4*pi))*sign(cos(1/2*(f*x+exp(1))-1/4*pi))*re(Ci(-f*x))*tan(1/2*exp(1
))*tan(1/2*f*x)^2)/(4*x*tan(1/2*exp(1))^2*tan(1/2*f*x)^2+4*x*tan(1/2*exp(1)
)^2+4*x*tan(1/2*f*x)^2+4*x)
```

maple [F] time = 0.18, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a-a\sin\left(fx+e\right)}\,\sqrt{c+c\sin\left(fx+e\right)}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a-a*sin(f*x+e))^(1/2)*(c+c*sin(f*x+e))^(1/2)/x^2,x)
```

```
[Out] int((a-a*sin(f*x+e))^(1/2)*(c+c*sin(f*x+e))^(1/2)/x^2,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-a\sin\left(fx+e\right)+a}\,\sqrt{c\sin\left(fx+e\right)+c}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-a*sin(f*x+e))^(1/2)*(c+c*sin(f*x+e))^(1/2)/x^2,x, algorithm="m
axima")
```

```
[Out] integrate(sqrt(-a*sin(f*x + e) + a)*sqrt(c*sin(f*x + e) + c)/x^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{a-a\,\sin\left(e+f\,x\right)}\,\sqrt{c+c\,\sin\left(e+f\,x\right)}}{x^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a - a*sin(e + f*x))^(1/2)*(c + c*sin(e + f*x))^(1/2))/x^2,x)
```

```
[Out] int(((a - a*sin(e + f*x))^(1/2)*(c + c*sin(e + f*x))^(1/2))/x^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{c\left(\sin\left(e+fx\right)+1\right)}\,\sqrt{-a\left(\sin\left(e+fx\right)-1\right)}}{x^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-a*sin(f*x+e))**(1/2)*(c+c*sin(f*x+e))**(1/2)/x**2,x)
```

```
[Out] Integral(sqrt(c*(sin(e + f*x) + 1))*sqrt(-a*(sin(e + f*x) - 1))/x**2, x)
```

3.170 $\int \frac{\sqrt{a-a\sin(e+fx)}\sqrt{c+c\sin(e+fx)}}{x^3}\,dx$

Optimal. Leaf size=176

$$-\frac{1}{2}f^2\cos(e)\text{Ci}(fx)\sec(e+fx)\sqrt{a-a\sin(e+fx)}\sqrt{c\sin(e+fx)+c}+\frac{1}{2}f^2\sin(e)\text{Si}(fx)\sec(e+fx)\sqrt{a-a\sin(e+$$

```
[Out] -1/2*(a-a*sin(f*x+e))^(1/2)*(c+c*sin(f*x+e))^(1/2)/x^2-1/2*f^2*Ci(f*x)*cos(
e)*sec(f*x+e)*(a-a*sin(f*x+e))^(1/2)*(c+c*sin(f*x+e))^(1/2)+1/2*f^2*sec(f*x
+e)*Si(f*x)*sin(e)*(a-a*sin(f*x+e))^(1/2)*(c+c*sin(f*x+e))^(1/2)+1/2*f*(a-a
*sin(f*x+e))^(1/2)*(c+c*sin(f*x+e))^(1/2)*tan(f*x+e)/x
```

Rubi [A] time = 0.23, antiderivative size = 176, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.152, Rules used = {4604, 3297, 3303, 3299, 3302}

$$-\frac{1}{2}f^2\cos(e)\text{CosIntegral}(fx)\sec(e+fx)\sqrt{a-a\sin(e+fx)}\sqrt{c\sin(e+fx)+c}+\frac{1}{2}f^2\sin(e)\text{Si}(fx)\sec(e+fx)\sqrt{a-$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[a - a*Sin[e + f*x]]*Sqrt[c + c*Sin[e + f*x]])/x^3,x]
```

```
[Out] -(Sqrt[a - a*Sin[e + f*x]]*Sqrt[c + c*Sin[e + f*x]])/(2*x^2) - (f^2*Cos[e]*
CosIntegral[f*x]*Sec[e + f*x]*Sqrt[a - a*Sin[e + f*x]]*Sqrt[c + c*Sin[e + f
*x]])/2 + (f^2*Sec[e + f*x]*Sin[e]*Sqrt[a - a*Sin[e + f*x]]*Sqrt[c + c*Sin[
e + f*x]]*SinIntegral[f*x])/2 + (f*Sqrt[a - a*Sin[e + f*x]]*Sqrt[c + c*Sin[
e + f*x]]*Tan[e + f*x])/(2*x)
```

Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[((
c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3302

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
```

```
c*f, 0]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 4604

```
Int[((g_.) + (h_.)*(x_))^(p_.)*((a_) + (b_.)*Sin[(e_.) + (f_.)*(x_)])^(m_)*
((c_) + (d_.)*Sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Dist[(a^IntPart[m
]*c^IntPart[m]*(a + b*Sin[e + f*x])^FracPart[m]*(c + d*Sin[e + f*x])^FracPa
rt[m])/Cos[e + f*x]^(2*FracPart[m]), Int[(g + h*x)^p*Cos[e + f*x]^(2*m)*(c
+ d*Sin[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] &&
EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[p] && IntegerQ[2*m] && I
GeQ[n - m, 0]
```

Rubi steps

$$\int \frac{\sqrt{a-a\sin(e+fx)}\,\sqrt{c+c\sin(e+fx)}}{x^3}\,dx = \left(\sec(e+fx)\sqrt{a-a\sin(e+fx)}\,\sqrt{c+c\sin(e+fx)}\right)\int \frac{\cos(e+f}{x^3}$$

$$= -\frac{\sqrt{a-a\sin(e+fx)}\,\sqrt{c+c\sin(e+fx)}}{2x^2} - \frac{1}{2}\left(f\sec(e+fx)\sqrt{a-}\right.$$

$$= -\frac{\sqrt{a-a\sin(e+fx)}\,\sqrt{c+c\sin(e+fx)}}{2x^2} + \frac{f\sqrt{a-a\sin(e+fx)}}{}$$

$$= -\frac{\sqrt{a-a\sin(e+fx)}\,\sqrt{c+c\sin(e+fx)}}{2x^2} + \frac{f\sqrt{a-a\sin(e+fx)}}{}$$

$$= -\frac{\sqrt{a-a\sin(e+fx)}\,\sqrt{c+c\sin(e+fx)}}{2x^2} - \frac{1}{2}f^2\cos(e)\mathrm{Ci}(fx)\sec$$

Mathematica [A] time = 0.28, size = 87, normalized size = 0.49

$$\frac{\sec(e+fx)\sqrt{a-a\sin(e+fx)}\,\sqrt{c(\sin(e+fx)+1)}\,\left(-f^2x^2\cos(e)\mathrm{Ci}(fx)+f^2x^2\sin(e)\mathrm{Si}(fx)+fx\sin(e+fx)-\right.}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[a - a*Sin[e + f*x]]*Sqrt[c + c*Sin[e + f*x]])/x^3,x]
```

```
[Out] (Sec[e + f*x]*Sqrt[c*(1 + Sin[e + f*x])]*Sqrt[a - a*Sin[e + f*x]]*(-Cos[e +
 f*x] - f^2*x^2*Cos[e]*CosIntegral[f*x] + f*x*Sin[e + f*x] + f^2*x^2*Sin[e]
*SinIntegral[f*x]))/(2*x^2)
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-a*sin(f*x+e))^(1/2)*(c+c*sin(f*x+e))^(1/2)/x^3,x, algorithm="f
ricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-a*sin(f*x+e))^(1/2)*(c+c*sin(f*x+e))^(1/2)/x^3,x, algorithm="g
iac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4
*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (
4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to
check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x
/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/
x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check
 sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Un
able to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>
(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign
: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable
to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*p
i/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*
pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to ch
eck sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2
)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/
2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check s
ign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unab
le to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-
4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign:
(4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to
 check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/
```

```
x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi
/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to chec
k sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)U
nable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)
>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sig
n: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)sqrt(2
*a)*sqrt(2*c)*(2*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*sign(cos(1/2*(f*x+exp(1
))-1/4*pi))-2*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*sign(cos(1/2*(f*x+exp(1))-
1/4*pi))*tan(1/2*exp(1))^2-2*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*sign(cos(1/
2*(f*x+exp(1))-1/4*pi))*tan(1/2*f*x)^2+2*sign(sin(1/2*(f*x+exp(1))-1/4*pi))
*sign(cos(1/2*(f*x+exp(1))-1/4*pi))*tan(1/2*exp(1))^2*tan(1/2*f*x)^2-8*sign
(sin(1/2*(f*x+exp(1))-1/4*pi))*sign(cos(1/2*(f*x+exp(1))-1/4*pi))*tan(1/2*e
xp(1))*tan(1/2*f*x)-4*f*x*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*sign(cos(1/2*(
f*x+exp(1))-1/4*pi))*tan(1/2*exp(1))-4*f*x*sign(sin(1/2*(f*x+exp(1))-1/4*pi
))*sign(cos(1/2*(f*x+exp(1))-1/4*pi))*tan(1/2*f*x)+f^2*x^2*sign(sin(1/2*(f*
x+exp(1))-1/4*pi))*sign(cos(1/2*(f*x+exp(1))-1/4*pi))*re(Ci(f*x))+f^2*x^2*s
ign(sin(1/2*(f*x+exp(1))-1/4*pi))*sign(cos(1/2*(f*x+exp(1))-1/4*pi))*re(Ci(
-f*x))+4*f*x*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*sign(cos(1/2*(f*x+exp(1))-1
/4*pi))*tan(1/2*exp(1))^2*tan(1/2*f*x)+4*f*x*sign(sin(1/2*(f*x+exp(1))-1/4*
pi))*sign(cos(1/2*(f*x+exp(1))-1/4*pi))*tan(1/2*exp(1))*tan(1/2*f*x)^2-4*f^
2*x^2*Si(f*x)*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*sign(cos(1/2*(f*x+exp(1))-
1/4*pi))*tan(1/2*exp(1))-2*f^2*x^2*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*sign(
cos(1/2*(f*x+exp(1))-1/4*pi))*im(Ci(f*x))*tan(1/2*exp(1))+2*f^2*x^2*sign(si
n(1/2*(f*x+exp(1))-1/4*pi))*sign(cos(1/2*(f*x+exp(1))-1/4*pi))*im(Ci(-f*x))
*tan(1/2*exp(1))-f^2*x^2*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*sign(cos(1/2*(f
*x+exp(1))-1/4*pi))*re(Ci(f*x))*tan(1/2*exp(1))^2+f^2*x^2*sign(sin(1/2*(f*x
+exp(1))-1/4*pi))*sign(cos(1/2*(f*x+exp(1))-1/4*pi))*re(Ci(f*x))*tan(1/2*f*
x)^2-f^2*x^2*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*sign(cos(1/2*(f*x+exp(1))-1
/4*pi))*re(Ci(-f*x))*tan(1/2*exp(1))^2+f^2*x^2*sign(sin(1/2*(f*x+exp(1))-1/
4*pi))*sign(cos(1/2*(f*x+exp(1))-1/4*pi))*re(Ci(-f*x))*tan(1/2*f*x)^2-4*f^2
*x^2*Si(f*x)*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*sign(cos(1/2*(f*x+exp(1))-1
/4*pi))*tan(1/2*exp(1))*tan(1/2*f*x)^2-2*f^2*x^2*sign(sin(1/2*(f*x+exp(1))-
1/4*pi))*sign(cos(1/2*(f*x+exp(1))-1/4*pi))*im(Ci(f*x))*tan(1/2*exp(1))*tan
(1/2*f*x)^2+2*f^2*x^2*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*sign(cos(1/2*(f*x+
exp(1))-1/4*pi))*im(Ci(-f*x))*tan(1/2*exp(1))*tan(1/2*f*x)^2-f^2*x^2*sign(s
in(1/2*(f*x+exp(1))-1/4*pi))*sign(cos(1/2*(f*x+exp(1))-1/4*pi))*re(Ci(f*x))
*tan(1/2*exp(1))^2*tan(1/2*f*x)^2-f^2*x^2*sign(sin(1/2*(f*x+exp(1))-1/4*pi)
)*sign(cos(1/2*(f*x+exp(1))-1/4*pi))*re(Ci(-f*x))*tan(1/2*exp(1))^2*tan(1/2
*f*x)^2)/(8*x^2*tan(1/2*exp(1))^2*tan(1/2*f*x)^2+8*x^2*tan(1/2*exp(1))^2+8*
x^2*tan(1/2*f*x)^2+8*x^2)
```

maple [F] time = 0.18, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a-a\sin\left(fx+e\right)}\,\sqrt{c+c\sin\left(fx+e\right)}}{x^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a-a*sin(f*x+e))^(1/2)*(c+c*sin(f*x+e))^(1/2)/x^3,x)
```

```
[Out] int((a-a*sin(f*x+e))^(1/2)*(c+c*sin(f*x+e))^(1/2)/x^3,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-a\sin\left(fx+e\right)+a}\,\sqrt{c\sin\left(fx+e\right)+c}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-a*sin(f*x+e))^(1/2)*(c+c*sin(f*x+e))^(1/2)/x^3,x, algorithm="m
axima")
```

```
[Out] integrate(sqrt(-a*sin(f*x + e) + a)*sqrt(c*sin(f*x + e) + c)/x^3, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{a-a\,\sin\left(e+f\,x\right)}\,\sqrt{c+c\,\sin\left(e+f\,x\right)}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a - a*sin(e + f*x))^(1/2)*(c + c*sin(e + f*x))^(1/2))/x^3,x)
```

```
[Out] int(((a - a*sin(e + f*x))^(1/2)*(c + c*sin(e + f*x))^(1/2))/x^3, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{c\left(\sin\left(e+fx\right)+1\right)}\,\sqrt{-a\left(\sin\left(e+fx\right)-1\right)}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-a*sin(f*x+e))**(1/2)*(c+c*sin(f*x+e))**(1/2)/x**3,x)
```

```
[Out] Integral(sqrt(c*(sin(e + f*x) + 1))*sqrt(-a*(sin(e + f*x) - 1))/x**3, x)
```

3.171 $\int x^3\sqrt{a-a\sin(e+fx)}\,(c+c\sin(e+fx))^{3/2}\,dx$

Optimal. Leaf size=393

$$-\frac{3c\sin(e+fx)\sqrt{a-a\sin(e+fx)}\,\sqrt{c\sin(e+fx)+c}}{8f^4}-\frac{6c\sqrt{a-a\sin(e+fx)}\,\sqrt{c\sin(e+fx)+c}}{f^4}-\frac{3cx\sin(e+f$$

```
[Out] 1/2*x^3*sec(f*x+e)*(c+c*sin(f*x+e))^(5/2)*(a-a*sin(f*x+e))^(1/2)/c/f-6*c*(a
-a*sin(f*x+e))^(1/2)*(c+c*sin(f*x+e))^(1/2)/f^4+3*c*x^2*(a-a*sin(f*x+e))^(1
/2)*(c+c*sin(f*x+e))^(1/2)/f^2+3/8*c*x*sec(f*x+e)*(a-a*sin(f*x+e))^(1/2)*(c
+c*sin(f*x+e))^(1/2)/f^3-3/4*c*x^3*sec(f*x+e)*(a-a*sin(f*x+e))^(1/2)*(c+c*s
in(f*x+e))^(1/2)/f-3/8*c*sin(f*x+e)*(a-a*sin(f*x+e))^(1/2)*(c+c*sin(f*x+e))
^(1/2)/f^4+3/4*c*x^2*sin(f*x+e)*(a-a*sin(f*x+e))^(1/2)*(c+c*sin(f*x+e))^(1/
2)/f^2-6*c*x*(a-a*sin(f*x+e))^(1/2)*(c+c*sin(f*x+e))^(1/2)*tan(f*x+e)/f^3-3
/4*c*x*sin(f*x+e)*(a-a*sin(f*x+e))^(1/2)*(c+c*sin(f*x+e))^(1/2)*tan(f*x+e)/
f^3
```

Rubi [A] time = 0.38, antiderivative size = 393, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 9, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {4604, 4422, 3317, 3296, 2638, 3311, 30, 2635, 8}

$$\frac{3cx^2\sin(e+fx)\sqrt{a-a\sin(e+fx)}\,\sqrt{c\sin(e+fx)+c}}{4f^2}+\frac{3cx^2\sqrt{a-a\sin(e+fx)}\,\sqrt{c\sin(e+fx)+c}}{f^2}-\frac{3c\sin(e+$$

Antiderivative was successfully verified.

```
[In] Int[x^3*Sqrt[a - a*Sin[e + f*x]]*(c + c*Sin[e + f*x])^(3/2),x]
```

```
[Out] (-6*c*Sqrt[a - a*Sin[e + f*x]]*Sqrt[c + c*Sin[e + f*x]])/f^4 + (3*c*x^2*Sqr
t[a - a*Sin[e + f*x]]*Sqrt[c + c*Sin[e + f*x]])/f^2 + (3*c*x*Sec[e + f*x]*S
qrt[a - a*Sin[e + f*x]]*Sqrt[c + c*Sin[e + f*x]])/(8*f^3) - (3*c*x^3*Sec[e
+ f*x]*Sqrt[a - a*Sin[e + f*x]]*Sqrt[c + c*Sin[e + f*x]])/(4*f) - (3*c*Sin[
e + f*x]*Sqrt[a - a*Sin[e + f*x]]*Sqrt[c + c*Sin[e + f*x]])/(8*f^4) + (3*c*
x^2*Sin[e + f*x]*Sqrt[a - a*Sin[e + f*x]]*Sqrt[c + c*Sin[e + f*x]])/(4*f^2)
 + (x^3*Sec[e + f*x]*Sqrt[a - a*Sin[e + f*x]]*(c + c*Sin[e + f*x])^(5/2))/(
2*c*f) - (6*c*x*Sqrt[a - a*Sin[e + f*x]]*Sqrt[c + c*Sin[e + f*x]]*Tan[e + f
*x])/f^3 - (3*c*x*Sin[e + f*x]*Sqrt[a - a*Sin[e + f*x]]*Sqrt[c + c*Sin[e +
f*x]]*Tan[e + f*x])/(4*f^3)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3311

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[(d*m*(c + d*x)^(m - 1)*(b*Sin[e + f*x])^n)/(f^2*n^2), x] + (Dist
[(b^2*(n - 1))/n, Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Dist[(
d^2*m*(m - 1))/(f^2*n^2), Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x]
- Simp[(b*(c + d*x)^m*Cos[e + f*x]*(b*Sin[e + f*x])^(n - 1))/(f*n), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

Rule 3317

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*Sin[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[n, 0] && (EqQ[n, 1] || IGtQ[
m, 0] || NeQ[a^2 - b^2, 0])
```

Rule 4422

```
Int[Cos[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.)*((a_) + (b_.)*Sin[(c
_.) + (d_.)*(x_)])^(n_.), x_Symbol] :> Simp[((e + f*x)^m*(a + b*Sin[c + d*x
])^(n + 1))/(b*d*(n + 1)), x] - Dist[(f*m)/(b*d*(n + 1)), Int[(e + f*x)^(m
- 1)*(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x
] && IGtQ[m, 0] && NeQ[n, -1]
```

Rule 4604

```
Int[((g_.) + (h_.)*(x_))^(p_.)*((a_) + (b_.)*Sin[(e_.) + (f_.)*(x_)])^(m_)*
((c_) + (d_.)*Sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Dist[(a^IntPart[m
]*c^IntPart[m]*(a + b*Sin[e + f*x])^FracPart[m]*(c + d*Sin[e + f*x])^FracPa
rt[m])/Cos[e + f*x]^(2*FracPart[m]), Int[(g + h*x)^p*Cos[e + f*x]^(2*m)*(c
+ d*Sin[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] &&
EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[p] && IntegerQ[2*m] && I
GeQ[n - m, 0]
```

Rubi steps

$$\int x^3\sqrt{a-a\sin(e+fx)}\,(c+c\sin(e+fx))^{3/2}\,dx = \left(\sec(e+fx)\sqrt{a-a\sin(e+fx)}\,\sqrt{c+c\sin(e+fx)}\right)\int x^3\cos$$

$$= \frac{x^3\sec(e+fx)\sqrt{a-a\sin(e+fx)}\,(c+c\sin(e+fx))^{5/2}}{2cf} - \frac{(3\,\text{s}}{}$$

$$= \frac{x^3\sec(e+fx)\sqrt{a-a\sin(e+fx)}\,(c+c\sin(e+fx))^{5/2}}{2cf} - \frac{(3\,\text{s}}{}$$

$$= -\frac{cx^3\sec(e+fx)\sqrt{a-a\sin(e+fx)}\,\sqrt{c+c\sin(e+fx)}}{2f} + \frac{x^3}{}$$

$$= \frac{3cx^2\sqrt{a-a\sin(e+fx)}\,\sqrt{c+c\sin(e+fx)}}{f^2} - \frac{cx^3\sec(e+fx)}{}$$

$$= \frac{3cx^2\sqrt{a-a\sin(e+fx)}\,\sqrt{c+c\sin(e+fx)}}{f^2} - \frac{3cx^3\sec(e+f}{}$$

$$= -\frac{6c\sqrt{a-a\sin(e+fx)}\,\sqrt{c+c\sin(e+fx)}}{f^4} + \frac{3cx^2\sqrt{a-a\sin}}{}$$

Mathematica [A] time = 1.16, size = 113, normalized size = 0.29

$$\frac{c\sqrt{a-a\sin(e+fx)}\,\sqrt{c(\sin(e+fx)+1)}\,\left(\left(6f^2x^2-3\right)\sin(e+fx)+8\left(fx\left(f^2x^2-6\right)\tan(e+fx)+3f^2x^2-6\right)\right.}{8f^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*Sqrt[a - a*Sin[e + f*x]]*(c + c*Sin[e + f*x])^(3/2),x]
```

```
[Out] (c*Sqrt[c*(1 + Sin[e + f*x])]*Sqrt[a - a*Sin[e + f*x]]*(-(f*x*(-3 + 2*f^2*x
^2)*Cos[2*(e + f*x)]*Sec[e + f*x]) + (-3 + 6*f^2*x^2)*Sin[e + f*x] + 8*(-6
+ 3*f^2*x^2 + f*x*(-6 + f^2*x^2)*Tan[e + f*x])))/(8*f^4)
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c+c*sin(f*x+e))^(3/2)*(a-a*sin(f*x+e))^(1/2),x, algorithm="f
ricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c+c*sin(f*x+e))^(3/2)*(a-a*sin(f*x+e))^(1/2),x, algorithm="g
iac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4
*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (
4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to
check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x
/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/
x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check
 sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Un
able to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>
(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign
: (4*pi/x/2)>(-4*pi/x/2)sqrt(2*a)*sqrt(2*c)*(1/2*(6*c*sign(sin(1/2*(f*x+exp
(1))-1/4*pi))*sign(cos(1/2*(f*x+exp(1))-1/4*pi))-3*c*f^2*x^2*sign(sin(1/2*(
f*x+exp(1))-1/4*pi))*sign(cos(1/2*(f*x+exp(1))-1/4*pi)))*cos(f*x+exp(1))/f^
4+1/64*(6*c*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*sign(cos(1/2*(f*x+exp(1))-1/
4*pi))-12*c*f^2*x^2*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*sign(cos(1/2*(f*x+ex
p(1))-1/4*pi)))*sin(2*f*x+2*exp(1))/f^4-1/2*(-6*c*f*x*sign(sin(1/2*(f*x+exp
(1))-1/4*pi))*sign(cos(1/2*(f*x+exp(1))-1/4*pi))+c*f^3*x^3*sign(sin(1/2*(f*
x+exp(1))-1/4*pi))*sign(cos(1/2*(f*x+exp(1))-1/4*pi)))*sin(f*x+exp(1))/f^4+
1/64*(-12*c*f*x*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*sign(cos(1/2*(f*x+exp(1)
)-1/4*pi))+8*c*f^3*x^3*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*sign(cos(1/2*(f*x
+exp(1))-1/4*pi)))*cos(2*f*x+2*exp(1))/f^4)
```

maple [F] time = 0.18, size = 0, normalized size = 0.00

$$\int x^{3} \left(c +c \sin \left(f x +e \right)\right)^{\frac{3}{2}} \sqrt{a -a \sin \left(f x +e \right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(c+c*sin(f*x+e))^(3/2)*(a-a*sin(f*x+e))^(1/2),x)
```

```
[Out] int(x^3*(c+c*sin(f*x+e))^(3/2)*(a-a*sin(f*x+e))^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{-a\sin\left(fx+e\right)+a}\left(c\sin\left(fx+e\right)+c\right)^{\frac{3}{2}}x^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c+c*sin(f*x+e))^(3/2)*(a-a*sin(f*x+e))^(1/2),x, algorithm="m
axima")
```

```
[Out] integrate(sqrt(-a*sin(f*x + e) + a)*(c*sin(f*x + e) + c)^(3/2)*x^3, x)
```

mupad [B] time = 4.15, size = 216, normalized size = 0.55

$$-\frac{c\sqrt{-a\,\left(\sin\left(e+f\,x\right)-1\right)}\,\sqrt{c\,\left(\sin\left(e+f\,x\right)+1\right)}\,\left(3\,\sin\left(e+f\,x\right)+96\,\cos\left(2\,e+2\,f\,x\right)+3\,\sin\left(3\,e+3\,f\,x\right)-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a - a*sin(e + f*x))^(1/2)*(c + c*sin(e + f*x))^(3/2),x)
```

```
[Out] -(c*(-a*(sin(e + f*x) - 1))^(1/2)*(c*(sin(e + f*x) + 1))^(1/2)*(3*sin(e + f
*x) + 96*cos(2*e + 2*f*x) + 3*sin(3*e + 3*f*x) - 48*f^2*x^2 - 6*f*x*cos(3*e
 + 3*f*x) + 96*f*x*sin(2*e + 2*f*x) + 4*f^3*x^3*cos(e + f*x) - 6*f^2*x^2*si
n(e + f*x) - 6*f*x*cos(e + f*x) - 48*f^2*x^2*cos(2*e + 2*f*x) + 4*f^3*x^3*c
os(3*e + 3*f*x) - 6*f^2*x^2*sin(3*e + 3*f*x) - 16*f^3*x^3*sin(2*e + 2*f*x)
+ 96))/(16*f^4*(cos(2*e + 2*f*x) + 1))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(c+c*sin(f*x+e))**(3/2)*(a-a*sin(f*x+e))**(1/2),x)
```

```
[Out] Timed out
```

3.172 $\int x^2\sqrt{a-a\sin(e+fx)}\,(c+c\sin(e+fx))^{3/2}\,dx$

Optimal. Leaf size=265

$$-\frac{c\sin(e+fx)\tan(e+fx)\sqrt{a-a\sin(e+fx)}\,\sqrt{c\sin(e+fx)+c}}{4f^3}-\frac{2c\tan(e+fx)\sqrt{a-a\sin(e+fx)}\,\sqrt{c\sin(e+f}}{f^3}$$

```
[Out] 1/2*x^2*sec(f*x+e)*(c+c*sin(f*x+e))^(5/2)*(a-a*sin(f*x+e))^(1/2)/c/f+2*c*x*
(a-a*sin(f*x+e))^(1/2)*(c+c*sin(f*x+e))^(1/2)/f^2-3/4*c*x^2*sec(f*x+e)*(a-a
*sin(f*x+e))^(1/2)*(c+c*sin(f*x+e))^(1/2)/f+1/2*c*x*sin(f*x+e)*(a-a*sin(f*x
+e))^(1/2)*(c+c*sin(f*x+e))^(1/2)/f^2-2*c*(a-a*sin(f*x+e))^(1/2)*(c+c*sin(f
*x+e))^(1/2)*tan(f*x+e)/f^3-1/4*c*sin(f*x+e)*(a-a*sin(f*x+e))^(1/2)*(c+c*si
n(f*x+e))^(1/2)*tan(f*x+e)/f^3
```

Rubi [A] time = 0.27, antiderivative size = 265, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.212, Rules used = {4604, 4422, 3317, 3296, 2637, 3310, 30}

$$\frac{2cx\sqrt{a-a\sin(e+fx)}\,\sqrt{c\sin(e+fx)+c}}{f^2}+\frac{cx\sin(e+fx)\sqrt{a-a\sin(e+fx)}\,\sqrt{c\sin(e+fx)+c}}{2f^2}-\frac{c\sin(e+fx)\,\text{ta}}{}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*Sqrt[a - a*Sin[e + f*x]]*(c + c*Sin[e + f*x])^(3/2),x]
```

```
[Out] (2*c*x*Sqrt[a - a*Sin[e + f*x]]*Sqrt[c + c*Sin[e + f*x]])/f^2 - (3*c*x^2*Se
c[e + f*x]*Sqrt[a - a*Sin[e + f*x]]*Sqrt[c + c*Sin[e + f*x]])/(4*f) + (c*x*
Sin[e + f*x]*Sqrt[a - a*Sin[e + f*x]]*Sqrt[c + c*Sin[e + f*x]])/(2*f^2) + (
x^2*Sec[e + f*x]*Sqrt[a - a*Sin[e + f*x]]*(c + c*Sin[e + f*x])^(5/2))/(2*c*
f) - (2*c*Sqrt[a - a*Sin[e + f*x]]*Sqrt[c + c*Sin[e + f*x]]*Tan[e + f*x])/f
^3 - (c*Sin[e + f*x]*Sqrt[a - a*Sin[e + f*x]]*Sqrt[c + c*Sin[e + f*x]]*Tan[
e + f*x])/(4*f^3)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3310

```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[(d*(b*Sin[e + f*x])^n)/(f^2*n^2), x] + (Dist[(b^2*(n - 1))/n, Int[(c
+ d*x)*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[(b*(c + d*x)*Cos[e + f*x]*(b
*Sin[e + f*x])^(n - 1))/(f*n), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1
]
```

Rule 3317

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*Sin[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[n, 0] && (EqQ[n, 1] || IGtQ[
m, 0] || NeQ[a^2 - b^2, 0])
```

Rule 4422

```
Int[Cos[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.)*((a_) + (b_.)*Sin[(c
_.) + (d_.)*(x_)])^(n_.), x_Symbol] :> Simp[((e + f*x)^m*(a + b*Sin[c + d*x
])^(n + 1))/(b*d*(n + 1)), x] - Dist[(f*m)/(b*d*(n + 1)), Int[(e + f*x)^(m
- 1)*(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x
] && IGtQ[m, 0] && NeQ[n, -1]
```

Rule 4604

```
Int[((g_.) + (h_.)*(x_))^(p_.)*((a_) + (b_.)*Sin[(e_.) + (f_.)*(x_)])^(m_)*
((c_) + (d_.)*Sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Dist[(a^IntPart[m
]*c^IntPart[m]*(a + b*Sin[e + f*x])^FracPart[m]*(c + d*Sin[e + f*x])^FracPa
rt[m])/Cos[e + f*x]^(2*FracPart[m]), Int[(g + h*x)^p*Cos[e + f*x]^(2*m)*(c
+ d*Sin[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] &&
EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[p] && IntegerQ[2*m] && I
GeQ[n - m, 0]
```

Rubi steps

$$\int x^2\sqrt{a-a\sin(e+fx)}\,(c+c\sin(e+fx))^{3/2}\,dx = \left(\sec(e+fx)\sqrt{a-a\sin(e+fx)}\,\sqrt{c+c\sin(e+fx)}\right)\int x^2\cos($$

$$= \frac{x^2\sec(e+fx)\sqrt{a-a\sin(e+fx)}\,(c+c\sin(e+fx))^{5/2}}{2cf} - \frac{(\sec(}{}$$

$$= \frac{x^2\sec(e+fx)\sqrt{a-a\sin(e+fx)}\,(c+c\sin(e+fx))^{5/2}}{2cf} - \frac{(\sec(}{}$$

$$= -\frac{cx^2\sec(e+fx)\sqrt{a-a\sin(e+fx)}\,\sqrt{c+c\sin(e+fx)}}{2f} + \frac{x^2\,\mathrm{s}}{}$$

$$= \frac{2cx\sqrt{a-a\sin(e+fx)}\,\sqrt{c+c\sin(e+fx)}}{f^2} - \frac{cx^2\sec(e+fx)\sqrt{}}{}$$

$$= \frac{2cx\sqrt{a-a\sin(e+fx)}\,\sqrt{c+c\sin(e+fx)}}{f^2} - \frac{3cx^2\sec(e+fx)}{}$$

Mathematica [A] time = 0.81, size = 95, normalized size = 0.36

$$\frac{c\sqrt{a-a\sin(e+fx)}\,\sqrt{c(\sin(e+fx)+1)}\,\left(8\left(f^2x^2-2\right)\tan(e+fx)-\left(2f^2x^2-1\right)\cos(2(e+fx))\sec(e+fx)+4f\right.}{8f^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*Sqrt[a - a*Sin[e + f*x]]*(c + c*Sin[e + f*x])^(3/2),x]
```

```
[Out] (c*Sqrt[c*(1 + Sin[e + f*x])]*Sqrt[a - a*Sin[e + f*x]]*(16*f*x - (-1 + 2*f^
2*x^2)*Cos[2*(e + f*x)]*Sec[e + f*x] + 4*f*x*Sin[e + f*x] + 8*(-2 + f^2*x^2
)*Tan[e + f*x]))/(8*f^3)
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c+c*sin(f*x+e))^(3/2)*(a-a*sin(f*x+e))^(1/2),x, algorithm="f
ricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c+c*sin(f*x+e))^(3/2)*(a-a*sin(f*x+e))^(1/2),x, algorithm="g
iac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4
*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (
4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to
check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x
/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/
x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check
 sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)sq
rt(2*a)*sqrt(2*c)*(-2*f^3*(-2*c*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*sign(cos
(1/2*(f*x+exp(1))-1/4*pi))+c*f^2*x^2*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*sig
n(cos(1/2*(f*x+exp(1))-1/4*pi)))*sin(f*x+exp(1))/(-2*f^3)^2+32*f^3*(-2*c*si
gn(sin(1/2*(f*x+exp(1))-1/4*pi))*sign(cos(1/2*(f*x+exp(1))-1/4*pi))+4*c*f^2
*x^2*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*sign(cos(1/2*(f*x+exp(1))-1/4*pi)))
*cos(2*f*x+2*exp(1))/(-32*f^3)^2-128*c*f^4*x*sign(sin(1/2*(f*x+exp(1))-1/4*
pi))*sign(cos(1/2*(f*x+exp(1))-1/4*pi))*sin(2*f*x+2*exp(1))/(-32*f^3)^2-4*c
*f^4*x*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*sign(cos(1/2*(f*x+exp(1))-1/4*pi)
)*cos(f*x+exp(1))/(-2*f^3)^2)
```

maple [F] time = 0.17, size = 0, normalized size = 0.00

$$\int x^{2} \left(c+c \sin \left(f x+e\right)\right)^{\frac{3}{2}} \sqrt{a-a \sin \left(f x+e\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(c+c*sin(f*x+e))^(3/2)*(a-a*sin(f*x+e))^(1/2),x)
```

```
[Out] int(x^2*(c+c*sin(f*x+e))^(3/2)*(a-a*sin(f*x+e))^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{-a \sin \left(f x+e\right)+a} \left(c \sin \left(f x+e\right)+c\right)^{\frac{3}{2}} x^{2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c+c*sin(f*x+e))^(3/2)*(a-a*sin(f*x+e))^(1/2),x, algorithm="m
axima")
```

```
[Out] integrate(sqrt(-a*sin(f*x + e) + a)*(c*sin(f*x + e) + c)^(3/2)*x^2, x)
```

mupad [B] time = 3.77, size = 159, normalized size = 0.60

$$\frac{c\,\sqrt{-a\,\left(\sin\left(e+f\,x\right)-1\right)}\,\sqrt{c\,\left(\sin\left(e+f\,x\right)+1\right)}\,\left(\cos\left(e+f\,x\right)+\cos\left(3\,e+3\,f\,x\right)-16\,\sin\left(2\,e+2\,f\,x\right)+16\,f\,x\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a - a*sin(e + f*x))^(1/2)*(c + c*sin(e + f*x))^(3/2),x)
```

```
[Out] (c*(-a*(sin(e + f*x) - 1))^(1/2)*(c*(sin(e + f*x) + 1))^(1/2)*(cos(e + f*x)
 + cos(3*e + 3*f*x) - 16*sin(2*e + 2*f*x) + 16*f*x + 16*f*x*cos(2*e + 2*f*x
) + 2*f*x*sin(3*e + 3*f*x) - 2*f^2*x^2*cos(e + f*x) + 2*f*x*sin(e + f*x) -
2*f^2*x^2*cos(3*e + 3*f*x) + 8*f^2*x^2*sin(2*e + 2*f*x)))/(8*f^3*(cos(2*e +
 2*f*x) + 1))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^{2} \left(c \left(\sin{\left(e + f x \right)} + 1\right)\right)^{\frac{3}{2}} \sqrt{- a \left(\sin{\left(e + f x \right)} - 1\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(c+c*sin(f*x+e))**(3/2)*(a-a*sin(f*x+e))**(1/2),x)
```

```
[Out] Integral(x**2*(c*(sin(e + f*x) + 1))**(3/2)*sqrt(-a*(sin(e + f*x) - 1)), x)
```

3.173 $\int x\sqrt{a-a\sin(e+fx)}\,(c+c\sin(e+fx))^{3/2}\,dx$

Optimal. Leaf size=168

$$\frac{c\sin(e+fx)\sqrt{a-a\sin(e+fx)}\,\sqrt{c\sin(e+fx)+c}}{4f^2}+\frac{c\sqrt{a-a\sin(e+fx)}\,\sqrt{c\sin(e+fx)+c}}{f^2}+\frac{x\sec(e+fx)\sqrt{a-}$$

```
[Out] 1/2*x*sec(f*x+e)*(c+c*sin(f*x+e))^(5/2)*(a-a*sin(f*x+e))^(1/2)/c/f+c*(a-a*s
in(f*x+e))^(1/2)*(c+c*sin(f*x+e))^(1/2)/f^2-3/4*c*x*sec(f*x+e)*(a-a*sin(f*x
+e))^(1/2)*(c+c*sin(f*x+e))^(1/2)/f+1/4*c*sin(f*x+e)*(a-a*sin(f*x+e))^(1/2)
*(c+c*sin(f*x+e))^(1/2)/f^2
```

Rubi [A] time = 0.14, antiderivative size = 168, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.097, Rules used = {4604, 4422, 2644}

$$\frac{c\sin(e+fx)\sqrt{a-a\sin(e+fx)}\,\sqrt{c\sin(e+fx)+c}}{4f^2}+\frac{c\sqrt{a-a\sin(e+fx)}\,\sqrt{c\sin(e+fx)+c}}{f^2}+\frac{x\sec(e+fx)\sqrt{a-}$$

Antiderivative was successfully verified.

```
[In] Int[x*Sqrt[a - a*Sin[e + f*x]]*(c + c*Sin[e + f*x])^(3/2),x]
```

```
[Out] (c*Sqrt[a - a*Sin[e + f*x]]*Sqrt[c + c*Sin[e + f*x]])/f^2 - (3*c*x*Sec[e +
f*x]*Sqrt[a - a*Sin[e + f*x]]*Sqrt[c + c*Sin[e + f*x]])/(4*f) + (c*Sin[e +
f*x]*Sqrt[a - a*Sin[e + f*x]]*Sqrt[c + c*Sin[e + f*x]])/(4*f^2) + (x*Sec[e
+ f*x]*Sqrt[a - a*Sin[e + f*x]]*(c + c*Sin[e + f*x])^(5/2))/(2*c*f)
```

Rule 2644

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^2, x_Symbol] :> Simp[((2*a^2 + b
^2)*x)/2, x] + (-Simp[(2*a*b*Cos[c + d*x])/d, x] - Simp[(b^2*Cos[c + d*x]*S
in[c + d*x])/(2*d), x]) /; FreeQ[{a, b, c, d}, x]
```

Rule 4422

```
Int[Cos[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.)*((a_) + (b_.)*Sin[(c
_.) + (d_.)*(x_)])^(n_.), x_Symbol] :> Simp[((e + f*x)^m*(a + b*Sin[c + d*x
])^(n + 1))/(b*d*(n + 1)), x] - Dist[(f*m)/(b*d*(n + 1)), Int[(e + f*x)^(m
- 1)*(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x
] && IGtQ[m, 0] && NeQ[n, -1]
```

Rule 4604

```
Int[((g_.) + (h_.)*(x_))^(p_.)*((a_) + (b_.)*Sin[(e_.) + (f_.)*(x_)])^(m_)*
((c_) + (d_.)*Sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Dist[(a^IntPart[m
```

```
]*c^IntPart[m]*(a + b*Sin[e + f*x])^FracPart[m]*(c + d*Sin[e + f*x])^FracPa
rt[m])/Cos[e + f*x]^(2*FracPart[m]), Int[(g + h*x)^p*Cos[e + f*x]^(2*m)*(c
+ d*Sin[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] &&
EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[p] && IntegerQ[2*m] && I
GeQ[n - m, 0]
```

Rubi steps

$$\int x\sqrt{a-a\sin(e+fx)}\,(c+c\sin(e+fx))^{3/2}\,dx = \left(\sec(e+fx)\sqrt{a-a\sin(e+fx)}\,\sqrt{c+c\sin(e+fx)}\right)\int x\cos(e$$

$$= \frac{x\sec(e+fx)\sqrt{a-a\sin(e+fx)}\,(c+c\sin(e+fx))^{5/2}}{2cf} - \frac{(\sec(e}{}$$

$$= \frac{c\sqrt{a-a\sin(e+fx)}\,\sqrt{c+c\sin(e+fx)}}{f^2} - \frac{3cx\sec(e+fx)\sqrt{a-}}{}$$

Mathematica [A] time = 0.64, size = 73, normalized size = 0.43

$$\frac{c\sqrt{a-a\sin(e+fx)}\,\sqrt{c(\sin(e+fx)+1)}\,(\sin(e+fx)+4fx\tan(e+fx)-fx\cos(2(e+fx))\sec(e+fx)+4)}{4f^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Sqrt[a - a*Sin[e + f*x]]*(c + c*Sin[e + f*x])^(3/2),x]
```

```
[Out] (c*Sqrt[c*(1 + Sin[e + f*x])]*Sqrt[a - a*Sin[e + f*x]]*(4 - f*x*Cos[2*(e +
f*x)]*Sec[e + f*x] + Sin[e + f*x] + 4*f*x*Tan[e + f*x]))/(4*f^2)
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c+c*sin(f*x+e))^(3/2)*(a-a*sin(f*x+e))^(1/2),x, algorithm="fri
cas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c+c*sin(f*x+e))^(3/2)*(a-a*sin(f*x+e))^(1/2),x, algorithm="gia
c")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4
*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (
4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to
check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x
/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)sqrt(2*a)*sqrt(2*c)*(-1/16*c
*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*sign(cos(1/2*(f*x+exp(1))-1/4*pi))*sin(
2*f*x+2*exp(1))/f^2-1/2*c*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*sign(cos(1/2*(
f*x+exp(1))-1/4*pi))*cos(f*x+exp(1))/f^2-1/2*c*x*sign(sin(1/2*(f*x+exp(1))-
1/4*pi))*sign(cos(1/2*(f*x+exp(1))-1/4*pi))*sin(f*x+exp(1))/f+1/8*c*x*sign(
sin(1/2*(f*x+exp(1))-1/4*pi))*sign(cos(1/2*(f*x+exp(1))-1/4*pi))*cos(2*f*x+
2*exp(1))/f)
```

maple [F] time = 0.17, size = 0, normalized size = 0.00

$$\int x\left(c+c\sin\left(fx+e\right)\right)^{\frac{3}{2}}\sqrt{a-a\sin\left(fx+e\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(c+c*sin(f*x+e))^(3/2)*(a-a*sin(f*x+e))^(1/2),x)
```

```
[Out] int(x*(c+c*sin(f*x+e))^(3/2)*(a-a*sin(f*x+e))^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{-a\sin\left(fx+e\right)+a}\left(c\sin\left(fx+e\right)+c\right)^{\frac{3}{2}}x\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c+c*sin(f*x+e))^(3/2)*(a-a*sin(f*x+e))^(1/2),x, algorithm="max
ima")
```

```
[Out] integrate(sqrt(-a*sin(f*x + e) + a)*(c*sin(f*x + e) + c)^(3/2)*x, x)
```

mupad [B] time = 1.21, size = 123, normalized size = 0.73

$$-\frac{c\sqrt{-a\left(\sin\left(e+f\,x\right)-1\right)}\,\sqrt{c\left(\sin\left(e+f\,x\right)+1\right)}\,\left(-16\sin\left(e+f\,x\right)^2+\sin\left(e+f\,x\right)+\sin\left(3\,e+3\,f\,x\right)+8\,f\,x}{8\,f^2\,\left(2\sin\left(e+f\,x\right)^2-2\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a - a*sin(e + f*x))^(1/2)*(c + c*sin(e + f*x))^(3/2),x)
```

```
[Out] -(c*(-a*(sin(e + f*x) - 1))^(1/2)*(c*(sin(e + f*x) + 1))^(1/2)*(sin(e + f*x
) + sin(3*e + 3*f*x) - 16*sin(e + f*x)^2 + 8*f*x*sin(2*e + 2*f*x) + 2*f*x*(
2*sin(e/2 + (f*x)/2)^2 - 1) + 2*f*x*(2*sin((3*e)/2 + (3*f*x)/2)^2 - 1) + 16
))/(8*f^2*(2*sin(e + f*x)^2 - 2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x\left(c\left(\sin\left(e+fx\right)+1\right)\right)^{\frac{3}{2}}\sqrt{-a\left(\sin\left(e+fx\right)-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c+c*sin(f*x+e))**(3/2)*(a-a*sin(f*x+e))**(1/2),x)
```

```
[Out] Integral(x*(c*(sin(e + f*x) + 1))**(3/2)*sqrt(-a*(sin(e + f*x) - 1)), x)
```

3.174 $\int \frac{\sqrt{a-a\sin(e+fx)}\,(c+c\sin(e+fx))^{3/2}}{x}\,dx$

Optimal. Leaf size=186

$$\frac{1}{2}c\sin(2e)\text{Ci}(2fx)\sec(e+fx)\sqrt{a-a\sin(e+fx)}\,\sqrt{c\sin(e+fx)+c}+c\cos(e)\text{Ci}(fx)\sec(e+fx)\sqrt{a-a\sin(e+fx}$$

```
[Out] c*Ci(f*x)*cos(e)*sec(f*x+e)*(a-a*sin(f*x+e))^(1/2)*(c+c*sin(f*x+e))^(1/2)+1
/2*c*cos(2*e)*sec(f*x+e)*Si(2*f*x)*(a-a*sin(f*x+e))^(1/2)*(c+c*sin(f*x+e))^
(1/2)-c*sec(f*x+e)*Si(f*x)*sin(e)*(a-a*sin(f*x+e))^(1/2)*(c+c*sin(f*x+e))^(
1/2)+1/2*c*Ci(2*f*x)*sec(f*x+e)*sin(2*e)*(a-a*sin(f*x+e))^(1/2)*(c+c*sin(f*
x+e))^(1/2)
```

Rubi [A] time = 0.66, antiderivative size = 186, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 7, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.212, Rules used = {4604, 6741, 12, 6742, 3303, 3299, 3302}

$$\frac{1}{2}c\sin(2e)\text{CosIntegral}(2fx)\sec(e+fx)\sqrt{a-a\sin(e+fx)}\,\sqrt{c\sin(e+fx)+c}+c\cos(e)\text{CosIntegral}(fx)\sec(e+$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[a - a*Sin[e + f*x]]*(c + c*Sin[e + f*x])^(3/2))/x,x]
```

```
[Out] c*Cos[e]*CosIntegral[f*x]*Sec[e + f*x]*Sqrt[a - a*Sin[e + f*x]]*Sqrt[c + c*
Sin[e + f*x]] + (c*CosIntegral[2*f*x]*Sec[e + f*x]*Sin[2*e]*Sqrt[a - a*Sin[
e + f*x]]*Sqrt[c + c*Sin[e + f*x]])/2 - c*Sec[e + f*x]*Sin[e]*Sqrt[a - a*Si
n[e + f*x]]*Sqrt[c + c*Sin[e + f*x]]*SinIntegral[f*x] + (c*Cos[2*e]*Sec[e +
 f*x]*Sqrt[a - a*Sin[e + f*x]]*Sqrt[c + c*Sin[e + f*x]]*SinIntegral[2*f*x])
/2
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3302

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
```

```
c*f, 0]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 4604

```
Int[((g_.) + (h_.)*(x_))^(p_.)*((a_) + (b_.)*Sin[(e_.) + (f_.)*(x_)])^(m_)*
((c_) + (d_.)*Sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Dist[(a^IntPart[m
]*c^IntPart[m]*(a + b*Sin[e + f*x])^FracPart[m]*(c + d*Sin[e + f*x])^FracPa
rt[m])/Cos[e + f*x]^(2*FracPart[m]), Int[(g + h*x)^p*Cos[e + f*x]^(2*m)*(c
+ d*Sin[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] &&
EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[p] && IntegerQ[2*m] && I
GeQ[n - m, 0]
```

Rule 6741

```
Int[u_, x_Symbol] :> With[{v = NormalizeIntegrand[u, x]}, Int[v, x] /; v =!
= u]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int \frac{\sqrt{a-a\sin(e+fx)}\,(c+c\sin(e+fx))^{3/2}}{x}\,dx = \left(\sec(e+fx)\sqrt{a-a\sin(e+fx)}\,\sqrt{c+c\sin(e+fx)}\right)\int \frac{\cos(e+$$

$$= \left(\sec(e+fx)\sqrt{a-a\sin(e+fx)}\,\sqrt{c+c\sin(e+fx)}\right)\int \frac{c\cos(e$$

$$= \left(c\sec(e+fx)\sqrt{a-a\sin(e+fx)}\,\sqrt{c+c\sin(e+fx)}\right)\int \frac{\cos(e$$

$$= \left(c\sec(e+fx)\sqrt{a-a\sin(e+fx)}\,\sqrt{c+c\sin(e+fx)}\right)\int \Big(\frac{\cos($$

$$= \frac{1}{2}\left(c\sec(e+fx)\sqrt{a-a\sin(e+fx)}\,\sqrt{c+c\sin(e+fx)}\right)\int \frac{\sin($$

$$= \left(c\cos(e)\sec(e+fx)\sqrt{a-a\sin(e+fx)}\,\sqrt{c+c\sin(e+fx)}\right)\int$$

$$= c\cos(e)\text{Ci}(fx)\sec(e+fx)\sqrt{a-a\sin(e+fx)}\,\sqrt{c+c\sin(e+fx$$

Mathematica [**C**] time = 1.22, size = 150, normalized size = 0.81

$$\frac{ce^{-i(e-fx)}\sqrt{-ice^{-i(e+fx)}\left(e^{i(e+fx)}+i\right)^2}\,\left(2e^{ie}\text{Ei}(-ifx)+2e^{3ie}\text{Ei}(ifx)+i\left(\text{Ei}(-2ifx)-e^{4ie}\text{Ei}(2ifx)\right)\right)\sqrt{a-a\sin(e+}}{2\sqrt{2}\left(1+e^{2i(e+fx)}\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[a - a*Sin[e + f*x]]*(c + c*Sin[e + f*x])^(3/2))/x,x]
```

```
[Out] (c*Sqrt[((-I)*c*(I + E^(I*(e + f*x)))^2)/E^(I*(e + f*x))]*(2*E^(I*e)*ExpInt
egralEi[(-I)*f*x] + 2*E^((3*I)*e)*ExpIntegralEi[I*f*x] + I*(ExpIntegralEi[(
-2*I)*f*x] - E^((4*I)*e)*ExpIntegralEi[(2*I)*f*x]))*Sqrt[a - a*Sin[e + f*x]
])/(2*Sqrt[2]*E^(I*(e - f*x))*(1 + E^((2*I)*(e + f*x))))
```

fricas [**F(-2)**] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+c*sin(f*x+e))^(3/2)*(a-a*sin(f*x+e))^(1/2)/x,x, algorithm="fri
cas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+c*sin(f*x+e))^(3/2)*(a-a*sin(f*x+e))^(1/2)/x,x, algorithm="gia
c")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4
*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (
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eck sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2
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nable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)
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heck sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/
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ble to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(
```

```
-4*pi/x/2)sqrt(2*a)*sqrt(2*c)*(-2*c*Si(2*f*x)*sign(sin(1/2*(f*x+exp(1))-1/4
*pi))*sign(cos(1/2*(f*x+exp(1))-1/4*pi))-c*sign(sin(1/2*(f*x+exp(1))-1/4*pi
))*sign(cos(1/2*(f*x+exp(1))-1/4*pi))*im(Ci(2*f*x))+c*sign(sin(1/2*(f*x+exp
(1))-1/4*pi))*sign(cos(1/2*(f*x+exp(1))-1/4*pi))*im(Ci(-2*f*x))-2*c*sign(si
n(1/2*(f*x+exp(1))-1/4*pi))*sign(cos(1/2*(f*x+exp(1))-1/4*pi))*re(Ci(f*x))-
2*c*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*sign(cos(1/2*(f*x+exp(1))-1/4*pi))*r
e(Ci(-f*x))+8*c*Si(f*x)*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*sign(cos(1/2*(f*
x+exp(1))-1/4*pi))*tan(1/2*exp(1))-2*c*Si(2*f*x)*sign(sin(1/2*(f*x+exp(1))-
1/4*pi))*sign(cos(1/2*(f*x+exp(1))-1/4*pi))*tan(1/2*exp(1))^2+2*c*Si(2*f*x)
*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*sign(cos(1/2*(f*x+exp(1))-1/4*pi))*tan(
exp(1))^2+4*c*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*sign(cos(1/2*(f*x+exp(1))-
1/4*pi))*im(Ci(f*x))*tan(1/2*exp(1))-c*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*s
ign(cos(1/2*(f*x+exp(1))-1/4*pi))*im(Ci(2*f*x))*tan(1/2*exp(1))^2+c*sign(si
n(1/2*(f*x+exp(1))-1/4*pi))*sign(cos(1/2*(f*x+exp(1))-1/4*pi))*im(Ci(2*f*x)
)*tan(exp(1))^2-4*c*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*sign(cos(1/2*(f*x+ex
p(1))-1/4*pi))*im(Ci(-f*x))*tan(1/2*exp(1))+c*sign(sin(1/2*(f*x+exp(1))-1/4
*pi))*sign(cos(1/2*(f*x+exp(1))-1/4*pi))*im(Ci(-2*f*x))*tan(1/2*exp(1))^2-c
*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*sign(cos(1/2*(f*x+exp(1))-1/4*pi))*im(C
i(-2*f*x))*tan(exp(1))^2+2*c*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*sign(cos(1/
2*(f*x+exp(1))-1/4*pi))*re(Ci(f*x))*tan(1/2*exp(1))^2-2*c*sign(sin(1/2*(f*x
+exp(1))-1/4*pi))*sign(cos(1/2*(f*x+exp(1))-1/4*pi))*re(Ci(f*x))*tan(exp(1)
)^2-2*c*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*sign(cos(1/2*(f*x+exp(1))-1/4*pi
))*re(Ci(2*f*x))*tan(exp(1))+2*c*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*sign(co
s(1/2*(f*x+exp(1))-1/4*pi))*re(Ci(-f*x))*tan(1/2*exp(1))^2-2*c*sign(sin(1/2
*(f*x+exp(1))-1/4*pi))*sign(cos(1/2*(f*x+exp(1))-1/4*pi))*re(Ci(-f*x))*tan(
exp(1))^2-2*c*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*sign(cos(1/2*(f*x+exp(1))-
1/4*pi))*re(Ci(-2*f*x))*tan(exp(1))+8*c*Si(f*x)*sign(sin(1/2*(f*x+exp(1))-1
/4*pi))*sign(cos(1/2*(f*x+exp(1))-1/4*pi))*tan(1/2*exp(1))*tan(exp(1))^2+2*
c*Si(2*f*x)*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*sign(cos(1/2*(f*x+exp(1))-1/
4*pi))*tan(1/2*exp(1))^2*tan(exp(1))^2+4*c*sign(sin(1/2*(f*x+exp(1))-1/4*pi
))*sign(cos(1/2*(f*x+exp(1))-1/4*pi))*im(Ci(f*x))*tan(1/2*exp(1))*tan(exp(1
))^2+c*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*sign(cos(1/2*(f*x+exp(1))-1/4*pi)
)*im(Ci(2*f*x))*tan(1/2*exp(1))^2*tan(exp(1))^2-4*c*sign(sin(1/2*(f*x+exp(1
))-1/4*pi))*sign(cos(1/2*(f*x+exp(1))-1/4*pi))*im(Ci(-f*x))*tan(1/2*exp(1))
*tan(exp(1))^2-c*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*sign(cos(1/2*(f*x+exp(1
))-1/4*pi))*im(Ci(-2*f*x))*tan(1/2*exp(1))^2*tan(exp(1))^2+2*c*sign(sin(1/2
*(f*x+exp(1))-1/4*pi))*sign(cos(1/2*(f*x+exp(1))-1/4*pi))*re(Ci(f*x))*tan(1
/2*exp(1))^2*tan(exp(1))^2-2*c*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*sign(cos(
1/2*(f*x+exp(1))-1/4*pi))*re(Ci(2*f*x))*tan(1/2*exp(1))^2*tan(exp(1))+2*c*s
ign(sin(1/2*(f*x+exp(1))-1/4*pi))*sign(cos(1/2*(f*x+exp(1))-1/4*pi))*re(Ci(
-f*x))*tan(1/2*exp(1))^2*tan(exp(1))^2-2*c*sign(sin(1/2*(f*x+exp(1))-1/4*pi
))*sign(cos(1/2*(f*x+exp(1))-1/4*pi))*re(Ci(-2*f*x))*tan(1/2*exp(1))^2*tan(
exp(1)))/(8*tan(1/2*exp(1))^2*tan(exp(1))^2+8*tan(1/2*exp(1))^2+8*tan(exp(1
))^2+8)
```

maple [F] time = 0.17, size = 0, normalized size = 0.00

$$\int \frac{\left(c+c\sin\left(fx+e\right)\right)^{\frac{3}{2}}\sqrt{a-a\sin\left(fx+e\right)}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+c*sin(f*x+e))^(3/2)*(a-a*sin(f*x+e))^(1/2)/x,x)
[Out] int((c+c*sin(f*x+e))^(3/2)*(a-a*sin(f*x+e))^(1/2)/x,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-a\sin\left(fx+e\right)+a}\left(c\sin\left(fx+e\right)+c\right)^{\frac{3}{2}}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+c*sin(f*x+e))^(3/2)*(a-a*sin(f*x+e))^(1/2)/x,x, algorithm="max
ima")
[Out] integrate(sqrt(-a*sin(f*x + e) + a)*(c*sin(f*x + e) + c)^(3/2)/x, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{a-a\,\sin\left(e+f\,x\right)}\,\left(c+c\,\sin\left(e+f\,x\right)\right)^{3/2}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a - a*sin(e + f*x))^(1/2)*(c + c*sin(e + f*x))^(3/2))/x,x)
[Out] int(((a - a*sin(e + f*x))^(1/2)*(c + c*sin(e + f*x))^(3/2))/x, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(c\left(\sin\left(e+fx\right)+1\right)\right)^{\frac{3}{2}}\sqrt{-a\left(\sin\left(e+fx\right)-1\right)}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+c*sin(f*x+e))**(3/2)*(a-a*sin(f*x+e))**(1/2)/x,x)
[Out] Integral((c*(sin(e + f*x) + 1))**(3/2)*sqrt(-a*(sin(e + f*x) - 1))/x, x)
```

3.175 $\int \frac{\sqrt{a-a\sin(e+fx)}\,(c+c\sin(e+fx))^{3/2}}{x^2}\,dx$

Optimal. Leaf size=273

$$-cf\sin(e)\text{Ci}(fx)\sec(e+fx)\sqrt{a-a\sin(e+fx)}\,\sqrt{c\sin(e+fx)+c}+cf\cos(2e)\text{Ci}(2fx)\sec(e+fx)\sqrt{a-a\sin(e+$$

```
[Out] -c*(a-a*sin(f*x+e))^(1/2)*(c+c*sin(f*x+e))^(1/2)/x+c*f*Ci(2*f*x)*cos(2*e)*s
ec(f*x+e)*(a-a*sin(f*x+e))^(1/2)*(c+c*sin(f*x+e))^(1/2)-c*f*cos(e)*sec(f*x+
e)*Si(f*x)*(a-a*sin(f*x+e))^(1/2)*(c+c*sin(f*x+e))^(1/2)-c*f*Ci(f*x)*sec(f*
x+e)*sin(e)*(a-a*sin(f*x+e))^(1/2)*(c+c*sin(f*x+e))^(1/2)-c*f*sec(f*x+e)*Si
(2*f*x)*sin(2*e)*(a-a*sin(f*x+e))^(1/2)*(c+c*sin(f*x+e))^(1/2)-1/2*c*sec(f*
x+e)*sin(2*f*x+2*e)*(a-a*sin(f*x+e))^(1/2)*(c+c*sin(f*x+e))^(1/2)/x
```

Rubi [A] time = 0.66, antiderivative size = 273, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 8, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.242, Rules used = {4604, 6741, 12, 6742, 3297, 3303, 3299, 3302}

$$-cf\sin(e)\text{CosIntegral}(fx)\sec(e+fx)\sqrt{a-a\sin(e+fx)}\,\sqrt{c\sin(e+fx)+c}+cf\cos(2e)\text{CosIntegral}(2fx)\sec($$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[a - a*Sin[e + f*x]]*(c + c*Sin[e + f*x])^(3/2))/x^2,x]
```

```
[Out] -((c*Sqrt[a - a*Sin[e + f*x]]*Sqrt[c + c*Sin[e + f*x]])/x) + c*f*Cos[2*e]*C
osIntegral[2*f*x]*Sec[e + f*x]*Sqrt[a - a*Sin[e + f*x]]*Sqrt[c + c*Sin[e +
f*x]] - c*f*CosIntegral[f*x]*Sec[e + f*x]*Sin[e]*Sqrt[a - a*Sin[e + f*x]]*S
qrt[c + c*Sin[e + f*x]] - (c*Sec[e + f*x]*Sqrt[a - a*Sin[e + f*x]]*Sqrt[c +
 c*Sin[e + f*x]]*Sin[2*e + 2*f*x])/(2*x) - c*f*Cos[e]*Sec[e + f*x]*Sqrt[a -
 a*Sin[e + f*x]]*Sqrt[c + c*Sin[e + f*x]]*SinIntegral[f*x] - c*f*Sec[e + f*
x]*Sin[2*e]*Sqrt[a - a*Sin[e + f*x]]*Sqrt[c + c*Sin[e + f*x]]*SinIntegral[2
*f*x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[((
c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
```

```
]
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3302

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 4604

```
Int[((g_.) + (h_.)*(x_))^(p_.)*((a_) + (b_.)*Sin[(e_.) + (f_.)*(x_)])^(m_)*
((c_) + (d_.)*Sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Dist[(a^IntPart[m
]*c^IntPart[m]*(a + b*Sin[e + f*x])^FracPart[m]*(c + d*Sin[e + f*x])^FracPa
rt[m])/Cos[e + f*x]^(2*FracPart[m]), Int[(g + h*x)^p*Cos[e + f*x]^(2*m)*(c
+ d*Sin[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] &&
EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[p] && IntegerQ[2*m] && I
GeQ[n - m, 0]
```

Rule 6741

```
Int[u_, x_Symbol] :> With[{v = NormalizeIntegrand[u, x]}, Int[v, x] /; v =!
= u]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int \frac{\sqrt{a-a\sin(e+fx)}\,(c+c\sin(e+fx))^{3/2}}{x^2}\,dx = \left(\sec(e+fx)\sqrt{a-a\sin(e+fx)}\,\sqrt{c+c\sin(e+fx)}\right)\int \frac{\cos(e+$$
$$= \left(\sec(e+fx)\sqrt{a-a\sin(e+fx)}\,\sqrt{c+c\sin(e+fx)}\right)\int \frac{c\cos(e$$
$$= \left(c\sec(e+fx)\sqrt{a-a\sin(e+fx)}\,\sqrt{c+c\sin(e+fx)}\right)\int \frac{\cos(e$$
$$= \left(c\sec(e+fx)\sqrt{a-a\sin(e+fx)}\,\sqrt{c+c\sin(e+fx)}\right)\int \left(\frac{\cos(e$$
$$= \frac{1}{2}\left(c\sec(e+fx)\sqrt{a-a\sin(e+fx)}\,\sqrt{c+c\sin(e+fx)}\right)\int \frac{\sin($$
$$= -\frac{c\sqrt{a-a\sin(e+fx)}\,\sqrt{c+c\sin(e+fx)}}{x} - \frac{c\sec(e+fx)\sqrt{a-$$
$$= -\frac{c\sqrt{a-a\sin(e+fx)}\,\sqrt{c+c\sin(e+fx)}}{x} - \frac{c\sec(e+fx)\sqrt{a-$$
$$= -\frac{c\sqrt{a-a\sin(e+fx)}\,\sqrt{c+c\sin(e+fx)}}{x} + cf\cos(2e)\text{Ci}(2fx)$$

Mathematica [C] time = 1.44, size = 231, normalized size = 0.85

$$\frac{ce^{-i(e+fx)}\sqrt{-ice^{-i(e+fx)}\left(e^{i(e+fx)}+i\right)^2}\left(-2ifxe^{i(e+2fx)}\text{Ei}(-ifx)+2ifxe^{3ie+2ifx}\text{Ei}(ifx)+2fxe^{2i(2e+fx)}\text{Ei}(2ifx)-2e^{}\right.}{2\sqrt{2}\,x\left(1+e^{2i(e+fx)}\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[a - a*Sin[e + f*x]]*(c + c*Sin[e + f*x])^(3/2))/x^2,x]
```

```
[Out] (c*Sqrt[((-I)*c*(I + E^(I*(e + f*x)))^2)/E^(I*(e + f*x))]*(-I - 2*E^(I*(e +
 f*x)) - 2*E^((3*I)*(e + f*x)) + I*E^((4*I)*(e + f*x)) - (2*I)*E^(I*(e + 2*
f*x))*f*x*ExpIntegralEi[(-I)*f*x] + (2*I)*E^((3*I)*e + (2*I)*f*x)*f*x*ExpIn
tegralEi[I*f*x] + 2*E^((2*I)*f*x)*f*x*ExpIntegralEi[(-2*I)*f*x] + 2*E^((2*I
)*(2*e + f*x))*f*x*ExpIntegralEi[(2*I)*f*x])*Sqrt[a - a*Sin[e + f*x]])/(2*S
qrt[2]*E^(I*(e + f*x))*(1 + E^((2*I)*(e + f*x)))*x)
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+c*sin(f*x+e))^(3/2)*(a-a*sin(f*x+e))^(1/2)/x^2,x, algorithm="f
ricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+c*sin(f*x+e))^(3/2)*(a-a*sin(f*x+e))^(1/2)/x^2,x, algorithm="g
iac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4
*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (
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sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Una
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1/4*pi))*tan(f*x)*tan(exp(1))^2-8*c*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*sign
(cos(1/2*(f*x+exp(1))-1/4*pi))*tan(1/2*exp(1))*tan(1/2*f*x)+2*c*f*x*Si(f*x)
*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*sign(cos(1/2*(f*x+exp(1))-1/4*pi))+c*f*
x*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*sign(cos(1/2*(f*x+exp(1))-1/4*pi))*im(
Ci(f*x))-c*f*x*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*sign(cos(1/2*(f*x+exp(1))
-1/4*pi))*im(Ci(-f*x))-c*f*x*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*sign(cos(1/
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2*(f*x+exp(1))-1/4*pi))*re(Ci(2*f*x))-c*f*x*sign(sin(1/2*(f*x+exp(1))-1/4*p
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+exp(1))-1/4*pi))*sign(cos(1/2*(f*x+exp(1))-1/4*pi))*tan(f*x)^2*tan(1/2*exp
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f*x+exp(1))-1/4*pi))*tan(f*x)^2*tan(1/2*exp(1))^2*tan(exp(1))^2-2*c*sign(si
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s(1/2*(f*x+exp(1))-1/4*pi))*tan(f*x)^2*tan(1/2*f*x)^2*tan(exp(1))^2-2*c*sig
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^2*tan(1/2*f*x)^2*tan(exp(1))-8*c*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*sign(c
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gn(sin(1/2*(f*x+exp(1))-1/4*pi))*sign(cos(1/2*(f*x+exp(1))-1/4*pi))*tan(1/2
*exp(1))^2*tan(1/2*f*x)^2*tan(exp(1))^2+2*c*sign(sin(1/2*(f*x+exp(1))-1/4*p
i))*sign(cos(1/2*(f*x+exp(1))-1/4*pi))*tan(1/2*exp(1))^2*tan(1/2*f*x)^2*tan
(exp(1))+2*c*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*sign(cos(1/2*(f*x+exp(1))-1
/4*pi))*tan(f*x)*tan(1/2*exp(1))^2*tan(1/2*f*x)^2-2*c*sign(sin(1/2*(f*x+exp
(1))-1/4*pi))*sign(cos(1/2*(f*x+exp(1))-1/4*pi))*tan(f*x)*tan(1/2*exp(1))^2
*tan(exp(1))^2-2*c*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*sign(cos(1/2*(f*x+exp
(1))-1/4*pi))*tan(f*x)*tan(1/2*f*x)^2*tan(exp(1))^2-8*c*sign(sin(1/2*(f*x+e
xp(1))-1/4*pi))*sign(cos(1/2*(f*x+exp(1))-1/4*pi))*tan(1/2*exp(1))*tan(1/2*
f*x)*tan(exp(1))^2+2*c*f*x*Si(f*x)*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*sign(
cos(1/2*(f*x+exp(1))-1/4*pi))*tan(f*x)^2-2*c*f*x*Si(f*x)*sign(sin(1/2*(f*x+
exp(1))-1/4*pi))*sign(cos(1/2*(f*x+exp(1))-1/4*pi))*tan(1/2*exp(1))^2+2*c*f
*x*Si(f*x)*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*sign(cos(1/2*(f*x+exp(1))-1/4
*pi))*tan(1/2*f*x)^2+2*c*f*x*Si(f*x)*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*sig
n(cos(1/2*(f*x+exp(1))-1/4*pi))*tan(exp(1))^2+4*c*f*x*Si(2*f*x)*sign(sin(1/
2*(f*x+exp(1))-1/4*pi))*sign(cos(1/2*(f*x+exp(1))-1/4*pi))*tan(exp(1))+c*f*
x*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*sign(cos(1/2*(f*x+exp(1))-1/4*pi))*im(
Ci(f*x))*tan(f*x)^2-c*f*x*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*sign(cos(1/2*(
f*x+exp(1))-1/4*pi))*im(Ci(f*x))*tan(1/2*exp(1))^2+c*f*x*sign(sin(1/2*(f*x+
exp(1))-1/4*pi))*sign(cos(1/2*(f*x+exp(1))-1/4*pi))*im(Ci(f*x))*tan(1/2*f*x
)^2+c*f*x*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*sign(cos(1/2*(f*x+exp(1))-1/4*
pi))*im(Ci(f*x))*tan(exp(1))^2+2*c*f*x*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*s
ign(cos(1/2*(f*x+exp(1))-1/4*pi))*im(Ci(2*f*x))*tan(exp(1))-c*f*x*sign(sin(
1/2*(f*x+exp(1))-1/4*pi))*sign(cos(1/2*(f*x+exp(1))-1/4*pi))*im(Ci(-f*x))*t
an(f*x)^2+c*f*x*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*sign(cos(1/2*(f*x+exp(1)
)-1/4*pi))*im(Ci(-f*x))*tan(1/2*exp(1))^2-c*f*x*sign(sin(1/2*(f*x+exp(1))-1
/4*pi))*sign(cos(1/2*(f*x+exp(1))-1/4*pi))*im(Ci(-f*x))*tan(1/2*f*x)^2-c*f*
x*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*sign(cos(1/2*(f*x+exp(1))-1/4*pi))*im(
Ci(-f*x))*tan(exp(1))^2-2*c*f*x*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*sign(cos
(1/2*(f*x+exp(1))-1/4*pi))*im(Ci(-2*f*x))*tan(exp(1))+2*c*f*x*sign(sin(1/2*
(f*x+exp(1))-1/4*pi))*sign(cos(1/2*(f*x+exp(1))-1/4*pi))*re(Ci(f*x))*tan(1/
2*exp(1))-c*f*x*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*sign(cos(1/2*(f*x+exp(1)
)-1/4*pi))*re(Ci(2*f*x))*tan(f*x)^2-c*f*x*sign(sin(1/2*(f*x+exp(1))-1/4*pi)
)*sign(cos(1/2*(f*x+exp(1))-1/4*pi))*re(Ci(2*f*x))*tan(1/2*exp(1))^2-c*f*x*
```

```
sign(sin(1/2*(f*x+exp(1))-1/4*pi))*sign(cos(1/2*(f*x+exp(1))-1/4*pi))*re(Ci
(2*f*x))*tan(1/2*f*x)^2+c*f*x*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*sign(cos(1
/2*(f*x+exp(1))-1/4*pi))*re(Ci(2*f*x))*tan(exp(1))^2+2*c*f*x*sign(sin(1/2*(
f*x+exp(1))-1/4*pi))*sign(cos(1/2*(f*x+exp(1))-1/4*pi))*re(Ci(-f*x))*tan(1/
2*exp(1))-c*f*x*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*sign(cos(1/2*(f*x+exp(1)
)-1/4*pi))*re(Ci(-2*f*x))*tan(f*x)^2-c*f*x*sign(sin(1/2*(f*x+exp(1))-1/4*pi
))*sign(cos(1/2*(f*x+exp(1))-1/4*pi))*re(Ci(-2*f*x))*tan(1/2*exp(1))^2-c*f*
x*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*sign(cos(1/2*(f*x+exp(1))-1/4*pi))*re(
Ci(-2*f*x))*tan(1/2*f*x)^2+c*f*x*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*sign(co
s(1/2*(f*x+exp(1))-1/4*pi))*re(Ci(-2*f*x))*tan(exp(1))^2+2*c*sign(sin(1/2*(
f*x+exp(1))-1/4*pi))*sign(cos(1/2*(f*x+exp(1))-1/4*pi))*tan(f*x)^2*tan(1/2*
exp(1))^2*tan(1/2*f*x)^2*tan(exp(1))^2-2*c*sign(sin(1/2*(f*x+exp(1))-1/4*pi
))*sign(cos(1/2*(f*x+exp(1))-1/4*pi))*tan(f*x)^2*tan(1/2*exp(1))^2*tan(1/2*
f*x)^2*tan(exp(1))-8*c*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*sign(cos(1/2*(f*x
+exp(1))-1/4*pi))*tan(f*x)^2*tan(1/2*exp(1))*tan(1/2*f*x)*tan(exp(1))^2-2*c
*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*sign(cos(1/2*(f*x+exp(1))-1/4*pi))*tan(
f*x)*tan(1/2*exp(1))^2*tan(1/2*f*x)^2*tan(exp(1))^2-2*c*f*x*Si(f*x)*sign(si
n(1/2*(f*x+exp(1))-1/4*pi))*sign(cos(1/2*(f*x+exp(1))-1/4*pi))*tan(f*x)^2*t
an(1/2*exp(1))^2+2*c*f*x*Si(f*x)*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*sign(co
s(1/2*(f*x+exp(1))-1/4*pi))*tan(f*x)^2*tan(1/2*f*x)^2+2*c*f*x*Si(f*x)*sign(
sin(1/2*(f*x+exp(1))-1/4*pi))*sign(cos(1/2*(f*x+exp(1))-1/4*pi))*tan(f*x)^2
*tan(exp(1))^2-2*c*f*x*Si(f*x)*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*sign(cos(
1/2*(f*x+exp(1))-1/4*pi))*tan(1/2*exp(1))^2*tan(1/2*f*x)^2-2*c*f*x*Si(f*x)*
sign(sin(1/2*(f*x+exp(1))-1/4*pi))*sign(cos(1/2*(f*x+exp(1))-1/4*pi))*tan(1
/2*exp(1))^2*tan(exp(1))^2+2*c*f*x*Si(f*x)*sign(sin(1/2*(f*x+exp(1))-1/4*pi
))*sign(cos(1/2*(f*x+exp(1))-1/4*pi))*tan(1/2*f*x)^2*tan(exp(1))^2+4*c*f*x*
Si(2*f*x)*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*sign(cos(1/2*(f*x+exp(1))-1/4*
pi))*tan(f*x)^2*tan(exp(1))+4*c*f*x*Si(2*f*x)*sign(sin(1/2*(f*x+exp(1))-1/4
*pi))*sign(cos(1/2*(f*x+exp(1))-1/4*pi))*tan(1/2*exp(1))^2*tan(exp(1))+4*c*
f*x*Si(2*f*x)*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*sign(cos(1/2*(f*x+exp(1))-
1/4*pi))*tan(1/2*f*x)^2*tan(exp(1))-c*f*x*sign(sin(1/2*(f*x+exp(1))-1/4*pi)
)*sign(cos(1/2*(f*x+exp(1))-1/4*pi))*im(Ci(f*x))*tan(f*x)^2*tan(1/2*exp(1))
^2+c*f*x*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*sign(cos(1/2*(f*x+exp(1))-1/4*p
i))*im(Ci(f*x))*tan(f*x)^2*tan(1/2*f*x)^2+c*f*x*sign(sin(1/2*(f*x+exp(1))-1
/4*pi))*sign(cos(1/2*(f*x+exp(1))-1/4*pi))*im(Ci(f*x))*tan(f*x)^2*tan(exp(1
))^2-c*f*x*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*sign(cos(1/2*(f*x+exp(1))-1/4
*pi))*im(Ci(f*x))*tan(1/2*exp(1))^2*tan(1/2*f*x)^2-c*f*x*sign(sin(1/2*(f*x+
exp(1))-1/4*pi))*sign(cos(1/2*(f*x+exp(1))-1/4*pi))*im(Ci(f*x))*tan(1/2*exp
(1))^2*tan(exp(1))^2+c*f*x*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*sign(cos(1/2*
(f*x+exp(1))-1/4*pi))*im(Ci(f*x))*tan(1/2*f*x)^2*tan(exp(1))^2+2*c*f*x*sign
(sin(1/2*(f*x+exp(1))-1/4*pi))*sign(cos(1/2*(f*x+exp(1))-1/4*pi))*im(Ci(2*f
*x))*tan(f*x)^2*tan(exp(1))+2*c*f*x*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*sign
(cos(1/2*(f*x+exp(1))-1/4*pi))*im(Ci(2*f*x))*tan(1/2*exp(1))^2*tan(exp(1))+
2*c*f*x*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*sign(cos(1/2*(f*x+exp(1))-1/4*pi
))*im(Ci(2*f*x))*tan(1/2*f*x)^2*tan(exp(1))+c*f*x*sign(sin(1/2*(f*x+exp(1))
```

-1/4*pi))*sign(cos(1/2*(f*x+exp(1))-1/4*pi))*im(Ci(-f*x))*tan(f*x)^2*tan(1/2*exp(1))^2-c*f*x*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*sign(cos(1/2*(f*x+exp(1))-1/4*pi))*im(Ci(-f*x))*tan(f*x)^2*tan(1/2*f*x)^2-c*f*x*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*sign(cos(1/2*(f*x+exp(1))-1/4*pi))*im(Ci(-f*x))*tan(f*x)^2*tan(exp(1))^2+c*f*x*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*sign(cos(1/2*(f*x+exp(1))-1/4*pi))*im(Ci(-f*x))*tan(1/2*exp(1))^2*tan(1/2*f*x)^2+c*f*x*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*sign(cos(1/2*(f*x+exp(1))-1/4*pi))*im(Ci(-f*x))*tan(1/2*exp(1))^2*tan(exp(1))^2-c*f*x*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*sign(cos(1/2*(f*x+exp(1))-1/4*pi))*im(Ci(-f*x))*tan(1/2*f*x)^2*tan(exp(1))^2-2*c*f*x*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*sign(cos(1/2*(f*x+exp(1))-1/4*pi))*im(Ci(-2*f*x))*tan(f*x)^2*tan(exp(1))-2*c*f*x*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*sign(cos(1/2*(f*x+exp(1))-1/4*pi))*im(Ci(-2*f*x))*tan(1/2*exp(1))^2*tan(exp(1))-2*c*f*x*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*sign(cos(1/2*(f*x+exp(1))-1/4*pi))*im(Ci(-2*f*x))*tan(1/2*f*x)^2*tan(exp(1))+2*c*f*x*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*sign(cos(1/2*(f*x+exp(1))-1/4*pi))*re(Ci(f*x))*tan(f*x)^2*tan(1/2*exp(1))+2*c*f*x*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*sign(cos(1/2*(f*x+exp(1))-1/4*pi))*re(Ci(f*x))*tan(1/2*exp(1))*tan(1/2*f*x)^2+2*c*f*x*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*sign(cos(1/2*(f*x+exp(1))-1/4*pi))*re(Ci(f*x))*tan(1/2*exp(1))*tan(exp(1))^2-c*f*x*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*sign(cos(1/2*(f*x+exp(1))-1/4*pi))*re(Ci(2*f*x))*tan(f*x)^2*tan(1/2*exp(1))^2-c*f*x*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*sign(cos(1/2*(f*x+exp(1))-1/4*pi))*re(Ci(2*f*x))*tan(f*x)^2*tan(1/2*f*x)^2+c*f*x*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*sign(cos(1/2*(f*x+exp(1))-1/4*pi))*re(Ci(2*f*x))*tan(f*x)^2*tan(exp(1))^2-c*f*x*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*sign(cos(1/2*(f*x+exp(1))-1/4*pi))*re(Ci(2*f*x))*tan(1/2*exp(1))^2*tan(1/2*f*x)^2+c*f*x*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*sign(cos(1/2*(f*x+exp(1))-1/4*pi))*re(Ci(2*f*x))*tan(1/2*exp(1))^2*tan(exp(1))^2+c*f*x*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*sign(cos(1/2*(f*x+exp(1))-1/4*pi))*re(Ci(2*f*x))*tan(1/2*f*x)^2*tan(exp(1))^2+2*c*f*x*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*sign(cos(1/2*(f*x+exp(1))-1/4*pi))*re(Ci(-f*x))*tan(f*x)^2*tan(1/2*exp(1))+2*c*f*x*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*sign(cos(1/2*(f*x+exp(1))-1/4*pi))*re(Ci(-f*x))*tan(1/2*exp(1))*tan(1/2*f*x)^2+2*c*f*x*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*sign(cos(1/2*(f*x+exp(1))-1/4*pi))*re(Ci(-f*x))*tan(1/2*exp(1))*tan(exp(1))^2-c*f*x*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*sign(cos(1/2*(f*x+exp(1))-1/4*pi))*re(Ci(-2*f*x))*tan(f*x)^2*tan(1/2*exp(1))^2-c*f*x*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*sign(cos(1/2*(f*x+exp(1))-1/4*pi))*re(Ci(-2*f*x))*tan(f*x)^2*tan(1/2*f*x)^2+c*f*x*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*sign(cos(1/2*(f*x+exp(1))-1/4*pi))*re(Ci(-2*f*x))*tan(f*x)^2*tan(exp(1))^2-c*f*x*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*sign(cos(1/2*(f*x+exp(1))-1/4*pi))*re(Ci(-2*f*x))*tan(1/2*exp(1))^2*tan(1/2*f*x)^2+c*f*x*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*sign(cos(1/2*(f*x+exp(1))-1/4*pi))*re(Ci(-2*f*x))*tan(1/2*exp(1))^2*tan(exp(1))^2+c*f*x*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*sign(cos(1/2*(f*x+exp(1))-1/4*pi))*re(Ci(-2*f*x))*tan(1/2*f*x)^2*tan(exp(1))^2-2*c*f*x*Si(f*x)*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*sign(cos(1/2*(f*x+exp(1))-1/4*pi))*tan(f*x)^2*tan(1/2*exp(1))^2*tan(1/2*f*x)^2-2*c*f*x*Si(f*x)*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*sign(

```
cos(1/2*(f*x+exp(1))-1/4*pi))*tan(f*x)^2*tan(1/2*exp(1))^2*tan(exp(1))^2+2*
c*f*x*Si(f*x)*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*sign(cos(1/2*(f*x+exp(1))-
1/4*pi))*tan(f*x)^2*tan(1/2*f*x)^2*tan(exp(1))^2-2*c*f*x*Si(f*x)*sign(sin(1
/2*(f*x+exp(1))-1/4*pi))*sign(cos(1/2*(f*x+exp(1))-1/4*pi))*tan(1/2*exp(1))
^2*tan(1/2*f*x)^2*tan(exp(1))^2+4*c*f*x*Si(2*f*x)*sign(sin(1/2*(f*x+exp(1))
-1/4*pi))*sign(cos(1/2*(f*x+exp(1))-1/4*pi))*tan(f*x)^2*tan(1/2*exp(1))^2*t
an(exp(1))+4*c*f*x*Si(2*f*x)*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*sign(cos(1/
2*(f*x+exp(1))-1/4*pi))*tan(f*x)^2*tan(1/2*f*x)^2*tan(exp(1))+4*c*f*x*Si(2*
f*x)*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*sign(cos(1/2*(f*x+exp(1))-1/4*pi))*
tan(1/2*exp(1))^2*tan(1/2*f*x)^2*tan(exp(1))-c*f*x*sign(sin(1/2*(f*x+exp(1)
)-1/4*pi))*sign(cos(1/2*(f*x+exp(1))-1/4*pi))*im(Ci(f*x))*tan(f*x)^2*tan(1/
2*exp(1))^2*tan(1/2*f*x)^2-c*f*x*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*sign(co
s(1/2*(f*x+exp(1))-1/4*pi))*im(Ci(f*x))*tan(f*x)^2*tan(1/2*exp(1))^2*tan(ex
p(1))^2+c*f*x*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*sign(cos(1/2*(f*x+exp(1))-
1/4*pi))*im(Ci(f*x))*tan(f*x)^2*tan(1/2*f*x)^2*tan(exp(1))^2-c*f*x*sign(sin
(1/2*(f*x+exp(1))-1/4*pi))*sign(cos(1/2*(f*x+exp(1))-1/4*pi))*im(Ci(f*x))*t
an(1/2*exp(1))^2*tan(1/2*f*x)^2*tan(exp(1))^2+2*c*f*x*sign(sin(1/2*(f*x+exp
(1))-1/4*pi))*sign(cos(1/2*(f*x+exp(1))-1/4*pi))*im(Ci(2*f*x))*tan(f*x)^2*t
an(1/2*exp(1))^2*tan(exp(1))+2*c*f*x*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*sig
n(cos(1/2*(f*x+exp(1))-1/4*pi))*im(Ci(2*f*x))*tan(f*x)^2*tan(1/2*f*x)^2*tan
(exp(1))+2*c*f*x*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*sign(cos(1/2*(f*x+exp(1
))-1/4*pi))*im(Ci(2*f*x))*tan(1/2*exp(1))^2*tan(1/2*f*x)^2*tan(exp(1))+c*f*
x*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*sign(cos(1/2*(f*x+exp(1))-1/4*pi))*im(
Ci(-f*x))*tan(f*x)^2*tan(1/2*exp(1))^2*tan(1/2*f*x)^2+c*f*x*sign(sin(1/2*(f
*x+exp(1))-1/4*pi))*sign(cos(1/2*(f*x+exp(1))-1/4*pi))*im(Ci(-f*x))*tan(f*x
)^2*tan(1/2*exp(1))^2*tan(exp(1))^2-c*f*x*sign(sin(1/2*(f*x+exp(1))-1/4*pi)
)*sign(cos(1/2*(f*x+exp(1))-1/4*pi))*im(Ci(-f*x))*tan(f*x)^2*tan(1/2*f*x)^2
*tan(exp(1))^2+c*f*x*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*sign(cos(1/2*(f*x+e
xp(1))-1/4*pi))*im(Ci(-f*x))*tan(1/2*exp(1))^2*tan(1/2*f*x)^2*tan(exp(1))^2
-2*c*f*x*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*sign(cos(1/2*(f*x+exp(1))-1/4*p
i))*im(Ci(-2*f*x))*tan(f*x)^2*tan(1/2*exp(1))^2*tan(exp(1))-2*c*f*x*sign(si
n(1/2*(f*x+exp(1))-1/4*pi))*sign(cos(1/2*(f*x+exp(1))-1/4*pi))*im(Ci(-2*f*x
))*tan(f*x)^2*tan(1/2*f*x)^2*tan(exp(1))-2*c*f*x*sign(sin(1/2*(f*x+exp(1))-
1/4*pi))*sign(cos(1/2*(f*x+exp(1))-1/4*pi))*im(Ci(-2*f*x))*tan(1/2*exp(1))^
2*tan(1/2*f*x)^2*tan(exp(1))+2*c*f*x*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*sig
n(cos(1/2*(f*x+exp(1))-1/4*pi))*re(Ci(f*x))*tan(f*x)^2*tan(1/2*exp(1))*tan(
1/2*f*x)^2+2*c*f*x*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*sign(cos(1/2*(f*x+exp
(1))-1/4*pi))*re(Ci(f*x))*tan(f*x)^2*tan(1/2*exp(1))*tan(exp(1))^2+2*c*f*x*
sign(sin(1/2*(f*x+exp(1))-1/4*pi))*sign(cos(1/2*(f*x+exp(1))-1/4*pi))*re(Ci
(f*x))*tan(1/2*exp(1))*tan(1/2*f*x)^2*tan(exp(1))^2-c*f*x*sign(sin(1/2*(f*x
+exp(1))-1/4*pi))*sign(cos(1/2*(f*x+exp(1))-1/4*pi))*re(Ci(2*f*x))*tan(f*x)
^2*tan(1/2*exp(1))^2*tan(1/2*f*x)^2+c*f*x*sign(sin(1/2*(f*x+exp(1))-1/4*pi)
)*sign(cos(1/2*(f*x+exp(1))-1/4*pi))*re(Ci(2*f*x))*tan(f*x)^2*tan(1/2*exp(1
))^2*tan(exp(1))^2+c*f*x*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*sign(cos(1/2*(f
*x+exp(1))-1/4*pi))*re(Ci(2*f*x))*tan(f*x)^2*tan(1/2*f*x)^2*tan(exp(1))^2+c
```

```
*f*x*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*sign(cos(1/2*(f*x+exp(1))-1/4*pi))*
re(Ci(2*f*x))*tan(1/2*exp(1))^2*tan(1/2*f*x)^2*tan(exp(1))^2+2*c*f*x*sign(s
in(1/2*(f*x+exp(1))-1/4*pi))*sign(cos(1/2*(f*x+exp(1))-1/4*pi))*re(Ci(-f*x)
)*tan(f*x)^2*tan(1/2*exp(1))*tan(1/2*f*x)^2+2*c*f*x*sign(sin(1/2*(f*x+exp(1
))-1/4*pi))*sign(cos(1/2*(f*x+exp(1))-1/4*pi))*re(Ci(-f*x))*tan(f*x)^2*tan(
1/2*exp(1))*tan(exp(1))^2+2*c*f*x*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*sign(c
os(1/2*(f*x+exp(1))-1/4*pi))*re(Ci(-f*x))*tan(1/2*exp(1))*tan(1/2*f*x)^2*ta
n(exp(1))^2-c*f*x*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*sign(cos(1/2*(f*x+exp(
1))-1/4*pi))*re(Ci(-2*f*x))*tan(f*x)^2*tan(1/2*exp(1))^2*tan(1/2*f*x)^2+c*f
*x*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*sign(cos(1/2*(f*x+exp(1))-1/4*pi))*re
(Ci(-2*f*x))*tan(f*x)^2*tan(1/2*exp(1))^2*tan(exp(1))^2+c*f*x*sign(sin(1/2*
(f*x+exp(1))-1/4*pi))*sign(cos(1/2*(f*x+exp(1))-1/4*pi))*re(Ci(-2*f*x))*tan
(f*x)^2*tan(1/2*f*x)^2*tan(exp(1))^2+c*f*x*sign(sin(1/2*(f*x+exp(1))-1/4*pi
))*sign(cos(1/2*(f*x+exp(1))-1/4*pi))*re(Ci(-2*f*x))*tan(1/2*exp(1))^2*tan(
1/2*f*x)^2*tan(exp(1))^2-2*c*f*x*Si(f*x)*sign(sin(1/2*(f*x+exp(1))-1/4*pi))
*sign(cos(1/2*(f*x+exp(1))-1/4*pi))*tan(f*x)^2*tan(1/2*exp(1))^2*tan(1/2*f*
x)^2*tan(exp(1))^2+4*c*f*x*Si(2*f*x)*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*sig
n(cos(1/2*(f*x+exp(1))-1/4*pi))*tan(f*x)^2*tan(1/2*exp(1))^2*tan(1/2*f*x)^2
*tan(exp(1))-c*f*x*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*sign(cos(1/2*(f*x+exp
(1))-1/4*pi))*im(Ci(f*x))*tan(f*x)^2*tan(1/2*exp(1))^2*tan(1/2*f*x)^2*tan(e
xp(1))^2+2*c*f*x*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*sign(cos(1/2*(f*x+exp(1
))-1/4*pi))*im(Ci(2*f*x))*tan(f*x)^2*tan(1/2*exp(1))^2*tan(1/2*f*x)^2*tan(e
xp(1))+c*f*x*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*sign(cos(1/2*(f*x+exp(1))-1
/4*pi))*im(Ci(-f*x))*tan(f*x)^2*tan(1/2*exp(1))^2*tan(1/2*f*x)^2*tan(exp(1)
)^2-2*c*f*x*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*sign(cos(1/2*(f*x+exp(1))-1/
4*pi))*im(Ci(-2*f*x))*tan(f*x)^2*tan(1/2*exp(1))^2*tan(1/2*f*x)^2*tan(exp(1
))+2*c*f*x*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*sign(cos(1/2*(f*x+exp(1))-1/4
*pi))*re(Ci(f*x))*tan(f*x)^2*tan(1/2*exp(1))*tan(1/2*f*x)^2*tan(exp(1))^2+c
*f*x*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*sign(cos(1/2*(f*x+exp(1))-1/4*pi))*
re(Ci(2*f*x))*tan(f*x)^2*tan(1/2*exp(1))^2*tan(1/2*f*x)^2*tan(exp(1))^2+2*c
*f*x*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*sign(cos(1/2*(f*x+exp(1))-1/4*pi))*
re(Ci(-f*x))*tan(f*x)^2*tan(1/2*exp(1))*tan(1/2*f*x)^2*tan(exp(1))^2+c*f*x*
sign(sin(1/2*(f*x+exp(1))-1/4*pi))*sign(cos(1/2*(f*x+exp(1))-1/4*pi))*re(Ci
(-2*f*x))*tan(f*x)^2*tan(1/2*exp(1))^2*tan(1/2*f*x)^2*tan(exp(1))^2)/(4*x*t
an(f*x)^2*tan(1/2*exp(1))^2*tan(1/2*f*x)^2*tan(exp(1))^2+4*x*tan(f*x)^2*tan
(1/2*exp(1))^2*tan(1/2*f*x)^2+4*x*tan(f*x)^2*tan(1/2*exp(1))^2*tan(exp(1))^
2+4*x*tan(f*x)^2*tan(1/2*f*x)^2*tan(exp(1))^2+4*x*tan(1/2*exp(1))^2*tan(1/2
*f*x)^2*tan(exp(1))^2+4*x*tan(f*x)^2*tan(1/2*exp(1))^2+4*x*tan(f*x)^2*tan(1
/2*f*x)^2+4*x*tan(f*x)^2*tan(exp(1))^2+4*x*tan(1/2*exp(1))^2*tan(1/2*f*x)^2
+4*x*tan(1/2*exp(1))^2*tan(exp(1))^2+4*x*tan(1/2*f*x)^2*tan(exp(1))^2+4*x*t
an(f*x)^2+4*x*tan(1/2*exp(1))^2+4*x*tan(1/2*f*x)^2+4*x*tan(exp(1))^2+4*x)
```

maple [F] time = 0.17, size = 0, normalized size = 0.00

$$\int \frac{\left(c+c\sin\left(fx+e\right)\right)^{\frac{3}{2}}\sqrt{a-a\sin\left(fx+e\right)}}{x^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+c*sin(f*x+e))^(3/2)*(a-a*sin(f*x+e))^(1/2)/x^2,x)
```

```
[Out] int((c+c*sin(f*x+e))^(3/2)*(a-a*sin(f*x+e))^(1/2)/x^2,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-a\sin\left(fx+e\right)+a}\left(c\sin\left(fx+e\right)+c\right)^{\frac{3}{2}}}{x^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+c*sin(f*x+e))^(3/2)*(a-a*sin(f*x+e))^(1/2)/x^2,x, algorithm="m
axima")
```

```
[Out] integrate(sqrt(-a*sin(f*x + e) + a)*(c*sin(f*x + e) + c)^(3/2)/x^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{a-a\,\sin\left(e+f\,x\right)}\,{\left(c+c\,\sin\left(e+f\,x\right)\right)}^{3/2}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a - a*sin(e + f*x))^(1/2)*(c + c*sin(e + f*x))^(3/2))/x^2,x)
```

```
[Out] int(((a - a*sin(e + f*x))^(1/2)*(c + c*sin(e + f*x))^(3/2))/x^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(c\left(\sin\left(e+fx\right)+1\right)\right)^{\frac{3}{2}}\sqrt{-a\left(\sin\left(e+fx\right)-1\right)}}{x^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+c*sin(f*x+e))**(3/2)*(a-a*sin(f*x+e))**(1/2)/x**2,x)
```

```
[Out] Integral((c*(sin(e + f*x) + 1))**(3/2)*sqrt(-a*(sin(e + f*x) - 1))/x**2, x)
```

3.176 $\int \frac{\sqrt{a-a\sin(e+fx)}\,(c+c\sin(e+fx))^{3/2}}{x^3}\,dx$

Optimal. Leaf size=385

$$-cf^2\sin(2e)\text{Ci}(2fx)\sec(e+fx)\sqrt{a-a\sin(e+fx)}\sqrt{c\sin(e+fx)+c}-\frac{1}{2}cf^2\cos(e)\text{Ci}(fx)\sec(e+fx)\sqrt{a-a\sin($$

```
[Out] -1/2*c*(a-a*sin(f*x+e))^(1/2)*(c+c*sin(f*x+e))^(1/2)/x^2-1/2*c*f^2*Ci(f*x)*
cos(e)*sec(f*x+e)*(a-a*sin(f*x+e))^(1/2)*(c+c*sin(f*x+e))^(1/2)-1/2*c*f*cos
(2*f*x+2*e)*sec(f*x+e)*(a-a*sin(f*x+e))^(1/2)*(c+c*sin(f*x+e))^(1/2)/x-c*f^
2*cos(2*e)*sec(f*x+e)*Si(2*f*x)*(a-a*sin(f*x+e))^(1/2)*(c+c*sin(f*x+e))^(1/
2)+1/2*c*f^2*sec(f*x+e)*Si(f*x)*sin(e)*(a-a*sin(f*x+e))^(1/2)*(c+c*sin(f*x+
e))^(1/2)-c*f^2*Ci(2*f*x)*sec(f*x+e)*sin(2*e)*(a-a*sin(f*x+e))^(1/2)*(c+c*s
in(f*x+e))^(1/2)-1/4*c*sec(f*x+e)*sin(2*f*x+2*e)*(a-a*sin(f*x+e))^(1/2)*(c+
c*sin(f*x+e))^(1/2)/x^2+1/2*c*f*(a-a*sin(f*x+e))^(1/2)*(c+c*sin(f*x+e))^(1/
2)*tan(f*x+e)/x
```

Rubi [A] time = 0.74, antiderivative size = 385, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 8, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.242, Rules used = {4604, 6741, 12, 6742, 3297, 3303, 3299, 3302}

$$-cf^2\sin(2e)\text{CosIntegral}(2fx)\sec(e+fx)\sqrt{a-a\sin(e+fx)}\sqrt{c\sin(e+fx)+c}-\frac{1}{2}cf^2\cos(e)\text{CosIntegral}(fx)\,\text{s}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[a - a*Sin[e + f*x]]*(c + c*Sin[e + f*x])^(3/2))/x^3,x]
```

```
[Out] -(c*Sqrt[a - a*Sin[e + f*x]]*Sqrt[c + c*Sin[e + f*x]])/(2*x^2) - (c*f*Cos[2
*e + 2*f*x]*Sec[e + f*x]*Sqrt[a - a*Sin[e + f*x]]*Sqrt[c + c*Sin[e + f*x]])
/(2*x) - (c*f^2*Cos[e]*CosIntegral[f*x]*Sec[e + f*x]*Sqrt[a - a*Sin[e + f*x
]]*Sqrt[c + c*Sin[e + f*x]])/2 - c*f^2*CosIntegral[2*f*x]*Sec[e + f*x]*Sin[
2*e]*Sqrt[a - a*Sin[e + f*x]]*Sqrt[c + c*Sin[e + f*x]] - (c*Sec[e + f*x]*Sq
rt[a - a*Sin[e + f*x]]*Sqrt[c + c*Sin[e + f*x]]*Sin[2*e + 2*f*x])/(4*x^2) +
 (c*f^2*Sec[e + f*x]*Sin[e]*Sqrt[a - a*Sin[e + f*x]]*Sqrt[c + c*Sin[e + f*x
]]*SinIntegral[f*x])/2 - c*f^2*Cos[2*e]*Sec[e + f*x]*Sqrt[a - a*Sin[e + f*x
]]*Sqrt[c + c*Sin[e + f*x]]*SinIntegral[2*f*x] + (c*f*Sqrt[a - a*Sin[e + f*
x]]*Sqrt[c + c*Sin[e + f*x]]*Tan[e + f*x])/(2*x)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[((
c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3302

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 4604

```
Int[((g_.) + (h_.)*(x_))^(p_.)*((a_) + (b_.)*Sin[(e_.) + (f_.)*(x_)])^(m_)*
((c_) + (d_.)*Sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Dist[(a^IntPart[m
]*c^IntPart[m]*(a + b*Sin[e + f*x])^FracPart[m]*(c + d*Sin[e + f*x])^FracPa
rt[m])/Cos[e + f*x]^(2*FracPart[m]), Int[(g + h*x)^p*Cos[e + f*x]^(2*m)*(c
+ d*Sin[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] &&
EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[p] && IntegerQ[2*m] && I
GeQ[n - m, 0]
```

Rule 6741

```
Int[u_, x_Symbol] :> With[{v = NormalizeIntegrand[u, x]}, Int[v, x] /; v =!
= u]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int \frac{\sqrt{a-a\sin(e+fx)}\,(c+c\sin(e+fx))^{3/2}}{x^3}\,dx = \left(\sec(e+fx)\sqrt{a-a\sin(e+fx)}\,\sqrt{c+c\sin(e+fx)}\right)\int \frac{\cos(e+$$

$$= \left(\sec(e+fx)\sqrt{a-a\sin(e+fx)}\,\sqrt{c+c\sin(e+fx)}\right)\int \frac{c\cos(e$$

$$= \left(c\sec(e+fx)\sqrt{a-a\sin(e+fx)}\,\sqrt{c+c\sin(e+fx)}\right)\int \frac{\cos(e$$

$$= \left(c\sec(e+fx)\sqrt{a-a\sin(e+fx)}\,\sqrt{c+c\sin(e+fx)}\right)\int \left(\frac{\cos(e$$

$$= \frac{1}{2}\left(c\sec(e+fx)\sqrt{a-a\sin(e+fx)}\,\sqrt{c+c\sin(e+fx)}\right)\int \frac{\sin($$

$$= -\frac{c\sqrt{a-a\sin(e+fx)}\,\sqrt{c+c\sin(e+fx)}}{2x^2} - \frac{c\sec(e+fx)\sqrt{a-}$$

$$= -\frac{c\sqrt{a-a\sin(e+fx)}\,\sqrt{c+c\sin(e+fx)}}{2x^2} - \frac{cf\cos(2e+2fx)\,\mathrm{s}$$

$$= -\frac{c\sqrt{a-a\sin(e+fx)}\,\sqrt{c+c\sin(e+fx)}}{2x^2} - \frac{cf\cos(2e+2fx)\,\mathrm{s}$$

$$= -\frac{c\sqrt{a-a\sin(e+fx)}\,\sqrt{c+c\sin(e+fx)}}{2x^2} - \frac{cf\cos(2e+2fx)\,\mathrm{s}$$

Mathematica [C] time = 1.84, size = 317, normalized size = 0.82

$$\frac{c^2e^{-2i(e+fx)}\left(e^{i(e+fx)}+i\right)\left(2if^2x^2e^{i(e+2fx)}\mathrm{Ei}(-ifx)+2if^2x^2e^{3ie+2ifx}\mathrm{Ei}(ifx)+4f^2x^2e^{2i(2e+fx)}\mathrm{Ei}(2ifx)+2fxe^{i(e+fx)}}{4\sqrt{2}\,x^2\left(e^{i(e+fx)}-i\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[a - a*Sin[e + f*x]]*(c + c*Sin[e + f*x])^(3/2))/x^3,x]
```

```
[Out] (c^2*(I + E^(I*(e + f*x)))*(-1 + (2*I)*E^(I*(e + f*x)) + (2*I)*E^((3*I)*(e
+ f*x)) + E^((4*I)*(e + f*x)) + (2*I)*f*x + 2*E^(I*(e + f*x))*f*x - 2*E^((3
*I)*(e + f*x))*f*x + (2*I)*E^((4*I)*(e + f*x))*f*x + (2*I)*E^(I*(e + 2*f*x)
)*f^2*x^2*ExpIntegralEi[(-I)*f*x] + (2*I)*E^((3*I)*e + (2*I)*f*x)*f^2*x^2*E
xpIntegralEi[I*f*x] - 4*E^((2*I)*f*x)*f^2*x^2*ExpIntegralEi[(-2*I)*f*x] + 4
*E^((2*I)*(2*e + f*x))*f^2*x^2*ExpIntegralEi[(2*I)*f*x])*Sqrt[a - a*Sin[e +
 f*x]])/(4*Sqrt[2]*E^((2*I)*(e + f*x))*(-I + E^(I*(e + f*x)))*Sqrt[((-I)*c*
(I + E^(I*(e + f*x)))^2)/E^(I*(e + f*x))]*x^2)
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+c*sin(f*x+e))^(3/2)*(a-a*sin(f*x+e))^(1/2)/x^3,x, algorithm="f
ricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+c*sin(f*x+e))^(3/2)*(a-a*sin(f*x+e))^(1/2)/x^3,x, algorithm="g
iac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4
*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (
4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to
check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x
/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/
x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check
 sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Un
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))*sign(cos(1/2*(f*x+exp(1))-1/4*pi))*tan(f*x)^2*tan(exp(1))+2*c*sign(sin(1
/2*(f*x+exp(1))-1/4*pi))*sign(cos(1/2*(f*x+exp(1))-1/4*pi))*tan(1/2*exp(1))
^2*tan(1/2*f*x)^2-2*c*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*sign(cos(1/2*(f*x+
exp(1))-1/4*pi))*tan(1/2*exp(1))^2*tan(exp(1))^2+2*c*sign(sin(1/2*(f*x+exp(
1))-1/4*pi))*sign(cos(1/2*(f*x+exp(1))-1/4*pi))*tan(1/2*exp(1))^2*tan(exp(1
))-2*c*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*sign(cos(1/2*(f*x+exp(1))-1/4*pi)
)*tan(1/2*f*x)^2*tan(exp(1))^2+2*c*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*sign(
cos(1/2*(f*x+exp(1))-1/4*pi))*tan(1/2*f*x)^2*tan(exp(1))+2*c*sign(sin(1/2*(
f*x+exp(1))-1/4*pi))*sign(cos(1/2*(f*x+exp(1))-1/4*pi))*tan(f*x)*tan(1/2*ex
p(1))^2+2*c*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*sign(cos(1/2*(f*x+exp(1))-1/
4*pi))*tan(f*x)*tan(1/2*f*x)^2-2*c*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*sign(
cos(1/2*(f*x+exp(1))-1/4*pi))*tan(f*x)*tan(exp(1))^2-8*c*sign(sin(1/2*(f*x+
exp(1))-1/4*pi))*sign(cos(1/2*(f*x+exp(1))-1/4*pi))*tan(1/2*exp(1))*tan(1/2
*f*x)-2*c*f*x*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*sign(cos(1/2*(f*x+exp(1))-
```

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1/4*pi))*tan(f*x)^2+2*c*f*x*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*sign(cos(1/2
*(f*x+exp(1))-1/4*pi))*tan(1/2*exp(1))^2+2*c*f*x*sign(sin(1/2*(f*x+exp(1))-
1/4*pi))*sign(cos(1/2*(f*x+exp(1))-1/4*pi))*tan(1/2*f*x)^2-2*c*f*x*sign(sin
(1/2*(f*x+exp(1))-1/4*pi))*sign(cos(1/2*(f*x+exp(1))-1/4*pi))*tan(exp(1))^2
-4*c*f*x*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*sign(cos(1/2*(f*x+exp(1))-1/4*p
i))*tan(1/2*exp(1))-4*c*f*x*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*sign(cos(1/2
*(f*x+exp(1))-1/4*pi))*tan(1/2*f*x)+4*c*f^2*x^2*Si(2*f*x)*sign(sin(1/2*(f*x
+exp(1))-1/4*pi))*sign(cos(1/2*(f*x+exp(1))-1/4*pi))+2*c*f^2*x^2*sign(sin(1
/2*(f*x+exp(1))-1/4*pi))*sign(cos(1/2*(f*x+exp(1))-1/4*pi))*im(Ci(2*f*x))-2
*c*f^2*x^2*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*sign(cos(1/2*(f*x+exp(1))-1/4
*pi))*im(Ci(-2*f*x))+c*f^2*x^2*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*sign(cos(
1/2*(f*x+exp(1))-1/4*pi))*re(Ci(f*x))+c*f^2*x^2*sign(sin(1/2*(f*x+exp(1))-1
/4*pi))*sign(cos(1/2*(f*x+exp(1))-1/4*pi))*re(Ci(-f*x))+2*c*sign(sin(1/2*(f
*x+exp(1))-1/4*pi))*sign(cos(1/2*(f*x+exp(1))-1/4*pi))*tan(f*x)^2*tan(1/2*e
xp(1))^2*tan(1/2*f*x)^2-2*c*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*sign(cos(1/2
*(f*x+exp(1))-1/4*pi))*tan(f*x)^2*tan(1/2*exp(1))^2*tan(exp(1))^2-2*c*sign(
sin(1/2*(f*x+exp(1))-1/4*pi))*sign(cos(1/2*(f*x+exp(1))-1/4*pi))*tan(f*x)^2
*tan(1/2*exp(1))^2*tan(exp(1))-2*c*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*sign(
cos(1/2*(f*x+exp(1))-1/4*pi))*tan(f*x)^2*tan(1/2*f*x)^2*tan(exp(1))^2-2*c*s
ign(sin(1/2*(f*x+exp(1))-1/4*pi))*sign(cos(1/2*(f*x+exp(1))-1/4*pi))*tan(f*
x)^2*tan(1/2*f*x)^2*tan(exp(1))-8*c*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*sign
(cos(1/2*(f*x+exp(1))-1/4*pi))*tan(f*x)^2*tan(1/2*exp(1))*tan(1/2*f*x)+2*c*
sign(sin(1/2*(f*x+exp(1))-1/4*pi))*sign(cos(1/2*(f*x+exp(1))-1/4*pi))*tan(1
/2*exp(1))^2*tan(1/2*f*x)^2*tan(exp(1))^2+2*c*sign(sin(1/2*(f*x+exp(1))-1/4
*pi))*sign(cos(1/2*(f*x+exp(1))-1/4*pi))*tan(1/2*exp(1))^2*tan(1/2*f*x)^2*t
an(exp(1))+2*c*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*sign(cos(1/2*(f*x+exp(1))
-1/4*pi))*tan(f*x)*tan(1/2*exp(1))^2*tan(1/2*f*x)^2-2*c*sign(sin(1/2*(f*x+e
xp(1))-1/4*pi))*sign(cos(1/2*(f*x+exp(1))-1/4*pi))*tan(f*x)*tan(1/2*exp(1))
^2*tan(exp(1))^2-2*c*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*sign(cos(1/2*(f*x+e
xp(1))-1/4*pi))*tan(f*x)*tan(1/2*f*x)^2*tan(exp(1))^2-8*c*sign(sin(1/2*(f*x
+exp(1))-1/4*pi))*sign(cos(1/2*(f*x+exp(1))-1/4*pi))*tan(1/2*exp(1))*tan(1/
2*f*x)*tan(exp(1))^2-2*c*f*x*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*sign(cos(1/
2*(f*x+exp(1))-1/4*pi))*tan(f*x)^2*tan(1/2*exp(1))^2-2*c*f*x*sign(sin(1/2*(
f*x+exp(1))-1/4*pi))*sign(cos(1/2*(f*x+exp(1))-1/4*pi))*tan(f*x)^2*tan(1/2*
f*x)^2+2*c*f*x*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*sign(cos(1/2*(f*x+exp(1))
-1/4*pi))*tan(f*x)^2*tan(exp(1))^2-4*c*f*x*sign(sin(1/2*(f*x+exp(1))-1/4*pi
))*sign(cos(1/2*(f*x+exp(1))-1/4*pi))*tan(f*x)^2*tan(1/2*exp(1))-4*c*f*x*si
gn(sin(1/2*(f*x+exp(1))-1/4*pi))*sign(cos(1/2*(f*x+exp(1))-1/4*pi))*tan(f*x
)^2*tan(1/2*f*x)+2*c*f*x*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*sign(cos(1/2*(f
*x+exp(1))-1/4*pi))*tan(1/2*exp(1))^2*tan(1/2*f*x)^2-2*c*f*x*sign(sin(1/2*(
f*x+exp(1))-1/4*pi))*sign(cos(1/2*(f*x+exp(1))-1/4*pi))*tan(1/2*exp(1))^2*t
an(exp(1))^2+4*c*f*x*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*sign(cos(1/2*(f*x+e
xp(1))-1/4*pi))*tan(1/2*exp(1))^2*tan(1/2*f*x)-2*c*f*x*sign(sin(1/2*(f*x+ex
p(1))-1/4*pi))*sign(cos(1/2*(f*x+exp(1))-1/4*pi))*tan(1/2*f*x)^2*tan(exp(1)
)^2-8*c*f*x*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*sign(cos(1/2*(f*x+exp(1))-1/
```

```
4*pi))*tan(f*x)*tan(exp(1))+4*c*f*x*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*sign
(cos(1/2*(f*x+exp(1))-1/4*pi))*tan(1/2*exp(1))*tan(1/2*f*x)^2-4*c*f*x*sign(
sin(1/2*(f*x+exp(1))-1/4*pi))*sign(cos(1/2*(f*x+exp(1))-1/4*pi))*tan(1/2*ex
p(1))*tan(exp(1))^2-4*c*f*x*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*sign(cos(1/2
*(f*x+exp(1))-1/4*pi))*tan(1/2*f*x)*tan(exp(1))^2-4*c*f^2*x^2*Si(f*x)*sign(
sin(1/2*(f*x+exp(1))-1/4*pi))*sign(cos(1/2*(f*x+exp(1))-1/4*pi))*tan(1/2*ex
p(1))+4*c*f^2*x^2*Si(2*f*x)*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*sign(cos(1/2
*(f*x+exp(1))-1/4*pi))*tan(f*x)^2+4*c*f^2*x^2*Si(2*f*x)*sign(sin(1/2*(f*x+e
xp(1))-1/4*pi))*sign(cos(1/2*(f*x+exp(1))-1/4*pi))*tan(1/2*exp(1))^2+4*c*f^
2*x^2*Si(2*f*x)*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*sign(cos(1/2*(f*x+exp(1)
)-1/4*pi))*tan(1/2*f*x)^2-4*c*f^2*x^2*Si(2*f*x)*sign(sin(1/2*(f*x+exp(1))-1
/4*pi))*sign(cos(1/2*(f*x+exp(1))-1/4*pi))*tan(exp(1))^2-2*c*f^2*x^2*sign(s
in(1/2*(f*x+exp(1))-1/4*pi))*sign(cos(1/2*(f*x+exp(1))-1/4*pi))*im(Ci(f*x))
*tan(1/2*exp(1))+2*c*f^2*x^2*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*sign(cos(1/
2*(f*x+exp(1))-1/4*pi))*im(Ci(2*f*x))*tan(f*x)^2+2*c*f^2*x^2*sign(sin(1/2*(
f*x+exp(1))-1/4*pi))*sign(cos(1/2*(f*x+exp(1))-1/4*pi))*im(Ci(2*f*x))*tan(1
/2*exp(1))^2+2*c*f^2*x^2*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*sign(cos(1/2*(f
*x+exp(1))-1/4*pi))*im(Ci(2*f*x))*tan(1/2*f*x)^2-2*c*f^2*x^2*sign(sin(1/2*(
f*x+exp(1))-1/4*pi))*sign(cos(1/2*(f*x+exp(1))-1/4*pi))*im(Ci(2*f*x))*tan(e
xp(1))^2+2*c*f^2*x^2*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*sign(cos(1/2*(f*x+e
xp(1))-1/4*pi))*im(Ci(-f*x))*tan(1/2*exp(1))-2*c*f^2*x^2*sign(sin(1/2*(f*x+
exp(1))-1/4*pi))*sign(cos(1/2*(f*x+exp(1))-1/4*pi))*im(Ci(-2*f*x))*tan(f*x)
^2-2*c*f^2*x^2*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*sign(cos(1/2*(f*x+exp(1))
-1/4*pi))*im(Ci(-2*f*x))*tan(1/2*exp(1))^2-2*c*f^2*x^2*sign(sin(1/2*(f*x+ex
p(1))-1/4*pi))*sign(cos(1/2*(f*x+exp(1))-1/4*pi))*im(Ci(-2*f*x))*tan(1/2*f*
x)^2+2*c*f^2*x^2*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*sign(cos(1/2*(f*x+exp(1
))-1/4*pi))*im(Ci(-2*f*x))*tan(exp(1))^2+c*f^2*x^2*sign(sin(1/2*(f*x+exp(1)
)-1/4*pi))*sign(cos(1/2*(f*x+exp(1))-1/4*pi))*re(Ci(f*x))*tan(f*x)^2-c*f^2*
x^2*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*sign(cos(1/2*(f*x+exp(1))-1/4*pi))*r
e(Ci(f*x))*tan(1/2*exp(1))^2+c*f^2*x^2*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*s
ign(cos(1/2*(f*x+exp(1))-1/4*pi))*re(Ci(f*x))*tan(1/2*f*x)^2+c*f^2*x^2*sign
(sin(1/2*(f*x+exp(1))-1/4*pi))*sign(cos(1/2*(f*x+exp(1))-1/4*pi))*re(Ci(f*x
))*tan(exp(1))^2+4*c*f^2*x^2*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*sign(cos(1/
2*(f*x+exp(1))-1/4*pi))*re(Ci(2*f*x))*tan(exp(1))+c*f^2*x^2*sign(sin(1/2*(f
*x+exp(1))-1/4*pi))*sign(cos(1/2*(f*x+exp(1))-1/4*pi))*re(Ci(-f*x))*tan(f*x
)^2-c*f^2*x^2*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*sign(cos(1/2*(f*x+exp(1))-
1/4*pi))*re(Ci(-f*x))*tan(1/2*exp(1))^2+c*f^2*x^2*sign(sin(1/2*(f*x+exp(1))
-1/4*pi))*sign(cos(1/2*(f*x+exp(1))-1/4*pi))*re(Ci(-f*x))*tan(1/2*f*x)^2+c*
f^2*x^2*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*sign(cos(1/2*(f*x+exp(1))-1/4*pi
))*re(Ci(-f*x))*tan(exp(1))^2+4*c*f^2*x^2*sign(sin(1/2*(f*x+exp(1))-1/4*pi)
)*sign(cos(1/2*(f*x+exp(1))-1/4*pi))*re(Ci(-2*f*x))*tan(exp(1))+2*c*sign(si
n(1/2*(f*x+exp(1))-1/4*pi))*sign(cos(1/2*(f*x+exp(1))-1/4*pi))*tan(f*x)^2*t
an(1/2*exp(1))^2*tan(1/2*f*x)^2*tan(exp(1))^2-2*c*sign(sin(1/2*(f*x+exp(1))
-1/4*pi))*sign(cos(1/2*(f*x+exp(1))-1/4*pi))*tan(f*x)^2*tan(1/2*exp(1))^2*t
an(1/2*f*x)^2*tan(exp(1))-8*c*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*sign(cos(1
```

```
/2*(f*x+exp(1))-1/4*pi))*tan(f*x)^2*tan(1/2*exp(1))*tan(1/2*f*x)*tan(exp(1)
)^2-2*c*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*sign(cos(1/2*(f*x+exp(1))-1/4*pi
))*tan(f*x)*tan(1/2*exp(1))^2*tan(1/2*f*x)^2*tan(exp(1))^2-2*c*f*x*sign(sin
(1/2*(f*x+exp(1))-1/4*pi))*sign(cos(1/2*(f*x+exp(1))-1/4*pi))*tan(f*x)^2*ta
n(1/2*exp(1))^2*tan(1/2*f*x)^2+2*c*f*x*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*s
ign(cos(1/2*(f*x+exp(1))-1/4*pi))*tan(f*x)^2*tan(1/2*exp(1))^2*tan(exp(1))^
2+4*c*f*x*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*sign(cos(1/2*(f*x+exp(1))-1/4*
pi))*tan(f*x)^2*tan(1/2*exp(1))^2*tan(1/2*f*x)+2*c*f*x*sign(sin(1/2*(f*x+ex
p(1))-1/4*pi))*sign(cos(1/2*(f*x+exp(1))-1/4*pi))*tan(f*x)^2*tan(1/2*f*x)^2
*tan(exp(1))^2+4*c*f*x*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*sign(cos(1/2*(f*x
+exp(1))-1/4*pi))*tan(f*x)^2*tan(1/2*exp(1))*tan(1/2*f*x)^2-4*c*f*x*sign(si
n(1/2*(f*x+exp(1))-1/4*pi))*sign(cos(1/2*(f*x+exp(1))-1/4*pi))*tan(f*x)^2*t
an(1/2*exp(1))*tan(exp(1))^2-4*c*f*x*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*sig
n(cos(1/2*(f*x+exp(1))-1/4*pi))*tan(f*x)^2*tan(1/2*f*x)*tan(exp(1))^2-2*c*f
*x*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*sign(cos(1/2*(f*x+exp(1))-1/4*pi))*ta
n(1/2*exp(1))^2*tan(1/2*f*x)^2*tan(exp(1))^2+4*c*f*x*sign(sin(1/2*(f*x+exp(
1))-1/4*pi))*sign(cos(1/2*(f*x+exp(1))-1/4*pi))*tan(1/2*exp(1))^2*tan(1/2*f
*x)*tan(exp(1))^2-8*c*f*x*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*sign(cos(1/2*(
f*x+exp(1))-1/4*pi))*tan(f*x)*tan(1/2*exp(1))^2*tan(exp(1))-8*c*f*x*sign(si
n(1/2*(f*x+exp(1))-1/4*pi))*sign(cos(1/2*(f*x+exp(1))-1/4*pi))*tan(f*x)*tan
(1/2*f*x)^2*tan(exp(1))+4*c*f*x*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*sign(cos
(1/2*(f*x+exp(1))-1/4*pi))*tan(1/2*exp(1))*tan(1/2*f*x)^2*tan(exp(1))^2-4*c
*f^2*x^2*Si(f*x)*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*sign(cos(1/2*(f*x+exp(1
))-1/4*pi))*tan(f*x)^2*tan(1/2*exp(1))-4*c*f^2*x^2*Si(f*x)*sign(sin(1/2*(f*
x+exp(1))-1/4*pi))*sign(cos(1/2*(f*x+exp(1))-1/4*pi))*tan(1/2*exp(1))*tan(1
/2*f*x)^2-4*c*f^2*x^2*Si(f*x)*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*sign(cos(1
/2*(f*x+exp(1))-1/4*pi))*tan(1/2*exp(1))*tan(exp(1))^2+4*c*f^2*x^2*Si(2*f*x
)*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*sign(cos(1/2*(f*x+exp(1))-1/4*pi))*tan
(f*x)^2*tan(1/2*exp(1))^2+4*c*f^2*x^2*Si(2*f*x)*sign(sin(1/2*(f*x+exp(1))-1
/4*pi))*sign(cos(1/2*(f*x+exp(1))-1/4*pi))*tan(f*x)^2*tan(1/2*f*x)^2-4*c*f^
2*x^2*Si(2*f*x)*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*sign(cos(1/2*(f*x+exp(1)
)-1/4*pi))*tan(f*x)^2*tan(exp(1))^2+4*c*f^2*x^2*Si(2*f*x)*sign(sin(1/2*(f*x
+exp(1))-1/4*pi))*sign(cos(1/2*(f*x+exp(1))-1/4*pi))*tan(1/2*exp(1))^2*tan(
1/2*f*x)^2-4*c*f^2*x^2*Si(2*f*x)*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*sign(co
s(1/2*(f*x+exp(1))-1/4*pi))*tan(1/2*exp(1))^2*tan(exp(1))^2-4*c*f^2*x^2*Si(
2*f*x)*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*sign(cos(1/2*(f*x+exp(1))-1/4*pi)
)*tan(1/2*f*x)^2*tan(exp(1))^2-2*c*f^2*x^2*sign(sin(1/2*(f*x+exp(1))-1/4*pi
))*sign(cos(1/2*(f*x+exp(1))-1/4*pi))*im(Ci(f*x))*tan(f*x)^2*tan(1/2*exp(1)
)-2*c*f^2*x^2*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*sign(cos(1/2*(f*x+exp(1))-
1/4*pi))*im(Ci(f*x))*tan(1/2*exp(1))*tan(1/2*f*x)^2-2*c*f^2*x^2*sign(sin(1/
2*(f*x+exp(1))-1/4*pi))*sign(cos(1/2*(f*x+exp(1))-1/4*pi))*im(Ci(f*x))*tan(
1/2*exp(1))*tan(exp(1))^2+2*c*f^2*x^2*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*si
gn(cos(1/2*(f*x+exp(1))-1/4*pi))*im(Ci(2*f*x))*tan(f*x)^2*tan(1/2*exp(1))^2
+2*c*f^2*x^2*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*sign(cos(1/2*(f*x+exp(1))-1
/4*pi))*im(Ci(2*f*x))*tan(f*x)^2*tan(1/2*f*x)^2-2*c*f^2*x^2*sign(sin(1/2*(f
```

```
*x+exp(1))-1/4*pi))*sign(cos(1/2*(f*x+exp(1))-1/4*pi))*im(Ci(2*f*x))*tan(f*
x)^2*tan(exp(1))^2+2*c*f^2*x^2*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*sign(cos(
1/2*(f*x+exp(1))-1/4*pi))*im(Ci(2*f*x))*tan(1/2*exp(1))^2*tan(1/2*f*x)^2-2*
c*f^2*x^2*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*sign(cos(1/2*(f*x+exp(1))-1/4*
pi))*im(Ci(2*f*x))*tan(1/2*exp(1))^2*tan(exp(1))^2-2*c*f^2*x^2*sign(sin(1/2
*(f*x+exp(1))-1/4*pi))*sign(cos(1/2*(f*x+exp(1))-1/4*pi))*im(Ci(2*f*x))*tan
(1/2*f*x)^2*tan(exp(1))^2+2*c*f^2*x^2*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*si
gn(cos(1/2*(f*x+exp(1))-1/4*pi))*im(Ci(-f*x))*tan(f*x)^2*tan(1/2*exp(1))+2*
c*f^2*x^2*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*sign(cos(1/2*(f*x+exp(1))-1/4*
pi))*im(Ci(-f*x))*tan(1/2*exp(1))*tan(1/2*f*x)^2+2*c*f^2*x^2*sign(sin(1/2*(
f*x+exp(1))-1/4*pi))*sign(cos(1/2*(f*x+exp(1))-1/4*pi))*im(Ci(-f*x))*tan(1/
2*exp(1))*tan(exp(1))^2-2*c*f^2*x^2*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*sign
(cos(1/2*(f*x+exp(1))-1/4*pi))*im(Ci(-2*f*x))*tan(f*x)^2*tan(1/2*exp(1))^2-
2*c*f^2*x^2*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*sign(cos(1/2*(f*x+exp(1))-1/
4*pi))*im(Ci(-2*f*x))*tan(f*x)^2*tan(1/2*f*x)^2+2*c*f^2*x^2*sign(sin(1/2*(f
*x+exp(1))-1/4*pi))*sign(cos(1/2*(f*x+exp(1))-1/4*pi))*im(Ci(-2*f*x))*tan(f
*x)^2*tan(exp(1))^2-2*c*f^2*x^2*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*sign(cos
(1/2*(f*x+exp(1))-1/4*pi))*im(Ci(-2*f*x))*tan(1/2*exp(1))^2*tan(1/2*f*x)^2+
2*c*f^2*x^2*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*sign(cos(1/2*(f*x+exp(1))-1/
4*pi))*im(Ci(-2*f*x))*tan(1/2*exp(1))^2*tan(exp(1))^2+2*c*f^2*x^2*sign(sin(
1/2*(f*x+exp(1))-1/4*pi))*sign(cos(1/2*(f*x+exp(1))-1/4*pi))*im(Ci(-2*f*x))
*tan(1/2*f*x)^2*tan(exp(1))^2-c*f^2*x^2*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*
sign(cos(1/2*(f*x+exp(1))-1/4*pi))*re(Ci(f*x))*tan(f*x)^2*tan(1/2*exp(1))^2
+c*f^2*x^2*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*sign(cos(1/2*(f*x+exp(1))-1/4
*pi))*re(Ci(f*x))*tan(f*x)^2*tan(1/2*f*x)^2+c*f^2*x^2*sign(sin(1/2*(f*x+exp
(1))-1/4*pi))*sign(cos(1/2*(f*x+exp(1))-1/4*pi))*re(Ci(f*x))*tan(f*x)^2*tan
(exp(1))^2-c*f^2*x^2*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*sign(cos(1/2*(f*x+e
xp(1))-1/4*pi))*re(Ci(f*x))*tan(1/2*exp(1))^2*tan(1/2*f*x)^2-c*f^2*x^2*sign
(sin(1/2*(f*x+exp(1))-1/4*pi))*sign(cos(1/2*(f*x+exp(1))-1/4*pi))*re(Ci(f*x
))*tan(1/2*exp(1))^2*tan(exp(1))^2+c*f^2*x^2*sign(sin(1/2*(f*x+exp(1))-1/4*
pi))*sign(cos(1/2*(f*x+exp(1))-1/4*pi))*re(Ci(f*x))*tan(1/2*f*x)^2*tan(exp(
1))^2+4*c*f^2*x^2*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*sign(cos(1/2*(f*x+exp(
1))-1/4*pi))*re(Ci(2*f*x))*tan(f*x)^2*tan(exp(1))+4*c*f^2*x^2*sign(sin(1/2*
(f*x+exp(1))-1/4*pi))*sign(cos(1/2*(f*x+exp(1))-1/4*pi))*re(Ci(2*f*x))*tan(
1/2*exp(1))^2*tan(exp(1))+4*c*f^2*x^2*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*si
gn(cos(1/2*(f*x+exp(1))-1/4*pi))*re(Ci(2*f*x))*tan(1/2*f*x)^2*tan(exp(1))-c
*f^2*x^2*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*sign(cos(1/2*(f*x+exp(1))-1/4*p
i))*re(Ci(-f*x))*tan(f*x)^2*tan(1/2*exp(1))^2+c*f^2*x^2*sign(sin(1/2*(f*x+e
xp(1))-1/4*pi))*sign(cos(1/2*(f*x+exp(1))-1/4*pi))*re(Ci(-f*x))*tan(f*x)^2*
tan(1/2*f*x)^2+c*f^2*x^2*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*sign(cos(1/2*(f
*x+exp(1))-1/4*pi))*re(Ci(-f*x))*tan(f*x)^2*tan(exp(1))^2-c*f^2*x^2*sign(si
n(1/2*(f*x+exp(1))-1/4*pi))*sign(cos(1/2*(f*x+exp(1))-1/4*pi))*re(Ci(-f*x))
*tan(1/2*exp(1))^2*tan(1/2*f*x)^2-c*f^2*x^2*sign(sin(1/2*(f*x+exp(1))-1/4*p
i))*sign(cos(1/2*(f*x+exp(1))-1/4*pi))*re(Ci(-f*x))*tan(1/2*exp(1))^2*tan(e
xp(1))^2+c*f^2*x^2*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*sign(cos(1/2*(f*x+exp
```

```
(1))-1/4*pi))*re(Ci(-f*x))*tan(1/2*f*x)^2*tan(exp(1))^2+4*c*f^2*x^2*sign(si
n(1/2*(f*x+exp(1))-1/4*pi))*sign(cos(1/2*(f*x+exp(1))-1/4*pi))*re(Ci(-2*f*x
))*tan(f*x)^2*tan(exp(1))+4*c*f^2*x^2*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*si
gn(cos(1/2*(f*x+exp(1))-1/4*pi))*re(Ci(-2*f*x))*tan(1/2*exp(1))^2*tan(exp(1
))+4*c*f^2*x^2*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*sign(cos(1/2*(f*x+exp(1))
-1/4*pi))*re(Ci(-2*f*x))*tan(1/2*f*x)^2*tan(exp(1))+2*c*f*x*sign(sin(1/2*(f
*x+exp(1))-1/4*pi))*sign(cos(1/2*(f*x+exp(1))-1/4*pi))*tan(f*x)^2*tan(1/2*e
xp(1))^2*tan(1/2*f*x)^2*tan(exp(1))^2+4*c*f*x*sign(sin(1/2*(f*x+exp(1))-1/4
*pi))*sign(cos(1/2*(f*x+exp(1))-1/4*pi))*tan(f*x)^2*tan(1/2*exp(1))^2*tan(1
/2*f*x)*tan(exp(1))^2+4*c*f*x*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*sign(cos(1
/2*(f*x+exp(1))-1/4*pi))*tan(f*x)^2*tan(1/2*exp(1))*tan(1/2*f*x)^2*tan(exp(
1))^2-8*c*f*x*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*sign(cos(1/2*(f*x+exp(1))-
1/4*pi))*tan(f*x)*tan(1/2*exp(1))^2*tan(1/2*f*x)^2*tan(exp(1))-4*c*f^2*x^2*
Si(f*x)*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*sign(cos(1/2*(f*x+exp(1))-1/4*pi
))*tan(f*x)^2*tan(1/2*exp(1))*tan(1/2*f*x)^2-4*c*f^2*x^2*Si(f*x)*sign(sin(1
/2*(f*x+exp(1))-1/4*pi))*sign(cos(1/2*(f*x+exp(1))-1/4*pi))*tan(f*x)^2*tan(
1/2*exp(1))*tan(exp(1))^2-4*c*f^2*x^2*Si(f*x)*sign(sin(1/2*(f*x+exp(1))-1/4
*pi))*sign(cos(1/2*(f*x+exp(1))-1/4*pi))*tan(1/2*exp(1))*tan(1/2*f*x)^2*tan
(exp(1))^2+4*c*f^2*x^2*Si(2*f*x)*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*sign(co
s(1/2*(f*x+exp(1))-1/4*pi))*tan(f*x)^2*tan(1/2*exp(1))^2*tan(1/2*f*x)^2-4*c
*f^2*x^2*Si(2*f*x)*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*sign(cos(1/2*(f*x+exp
(1))-1/4*pi))*tan(f*x)^2*tan(1/2*exp(1))^2*tan(exp(1))^2-4*c*f^2*x^2*Si(2*f
*x)*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*sign(cos(1/2*(f*x+exp(1))-1/4*pi))*t
an(f*x)^2*tan(1/2*f*x)^2*tan(exp(1))^2-4*c*f^2*x^2*Si(2*f*x)*sign(sin(1/2*(
f*x+exp(1))-1/4*pi))*sign(cos(1/2*(f*x+exp(1))-1/4*pi))*tan(1/2*exp(1))^2*t
an(1/2*f*x)^2*tan(exp(1))^2-2*c*f^2*x^2*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*
sign(cos(1/2*(f*x+exp(1))-1/4*pi))*im(Ci(f*x))*tan(f*x)^2*tan(1/2*exp(1))*t
an(1/2*f*x)^2-2*c*f^2*x^2*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*sign(cos(1/2*(
f*x+exp(1))-1/4*pi))*im(Ci(f*x))*tan(f*x)^2*tan(1/2*exp(1))*tan(exp(1))^2-2
*c*f^2*x^2*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*sign(cos(1/2*(f*x+exp(1))-1/4
*pi))*im(Ci(f*x))*tan(1/2*exp(1))*tan(1/2*f*x)^2*tan(exp(1))^2+2*c*f^2*x^2*
sign(sin(1/2*(f*x+exp(1))-1/4*pi))*sign(cos(1/2*(f*x+exp(1))-1/4*pi))*im(Ci
(2*f*x))*tan(f*x)^2*tan(1/2*exp(1))^2*tan(1/2*f*x)^2-2*c*f^2*x^2*sign(sin(1
/2*(f*x+exp(1))-1/4*pi))*sign(cos(1/2*(f*x+exp(1))-1/4*pi))*im(Ci(2*f*x))*t
an(f*x)^2*tan(1/2*exp(1))^2*tan(exp(1))^2-2*c*f^2*x^2*sign(sin(1/2*(f*x+exp
(1))-1/4*pi))*sign(cos(1/2*(f*x+exp(1))-1/4*pi))*im(Ci(2*f*x))*tan(f*x)^2*t
an(1/2*f*x)^2*tan(exp(1))^2-2*c*f^2*x^2*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*
sign(cos(1/2*(f*x+exp(1))-1/4*pi))*im(Ci(2*f*x))*tan(1/2*exp(1))^2*tan(1/2*
f*x)^2*tan(exp(1))^2+2*c*f^2*x^2*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*sign(co
s(1/2*(f*x+exp(1))-1/4*pi))*im(Ci(-f*x))*tan(f*x)^2*tan(1/2*exp(1))*tan(1/2
*f*x)^2+2*c*f^2*x^2*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*sign(cos(1/2*(f*x+ex
p(1))-1/4*pi))*im(Ci(-f*x))*tan(f*x)^2*tan(1/2*exp(1))*tan(exp(1))^2+2*c*f^
2*x^2*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*sign(cos(1/2*(f*x+exp(1))-1/4*pi))
*im(Ci(-f*x))*tan(1/2*exp(1))*tan(1/2*f*x)^2*tan(exp(1))^2-2*c*f^2*x^2*sign
(sin(1/2*(f*x+exp(1))-1/4*pi))*sign(cos(1/2*(f*x+exp(1))-1/4*pi))*im(Ci(-2*
```

```
f*x))*tan(f*x)^2*tan(1/2*exp(1))^2*tan(1/2*f*x)^2+2*c*f^2*x^2*sign(sin(1/2*
(f*x+exp(1))-1/4*pi))*sign(cos(1/2*(f*x+exp(1))-1/4*pi))*im(Ci(-2*f*x))*tan
(f*x)^2*tan(1/2*exp(1))^2*tan(exp(1))^2+2*c*f^2*x^2*sign(sin(1/2*(f*x+exp(1
))-1/4*pi))*sign(cos(1/2*(f*x+exp(1))-1/4*pi))*im(Ci(-2*f*x))*tan(f*x)^2*ta
n(1/2*f*x)^2*tan(exp(1))^2+2*c*f^2*x^2*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*s
ign(cos(1/2*(f*x+exp(1))-1/4*pi))*im(Ci(-2*f*x))*tan(1/2*exp(1))^2*tan(1/2*
f*x)^2*tan(exp(1))^2-c*f^2*x^2*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*sign(cos(
1/2*(f*x+exp(1))-1/4*pi))*re(Ci(f*x))*tan(f*x)^2*tan(1/2*exp(1))^2*tan(1/2*
f*x)^2-c*f^2*x^2*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*sign(cos(1/2*(f*x+exp(1
))-1/4*pi))*re(Ci(f*x))*tan(f*x)^2*tan(1/2*exp(1))^2*tan(exp(1))^2+c*f^2*x^
2*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*sign(cos(1/2*(f*x+exp(1))-1/4*pi))*re(
Ci(f*x))*tan(f*x)^2*tan(1/2*f*x)^2*tan(exp(1))^2-c*f^2*x^2*sign(sin(1/2*(f*
x+exp(1))-1/4*pi))*sign(cos(1/2*(f*x+exp(1))-1/4*pi))*re(Ci(f*x))*tan(1/2*e
xp(1))^2*tan(1/2*f*x)^2*tan(exp(1))^2+4*c*f^2*x^2*sign(sin(1/2*(f*x+exp(1))
-1/4*pi))*sign(cos(1/2*(f*x+exp(1))-1/4*pi))*re(Ci(2*f*x))*tan(f*x)^2*tan(1
/2*exp(1))^2*tan(exp(1))+4*c*f^2*x^2*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*sig
n(cos(1/2*(f*x+exp(1))-1/4*pi))*re(Ci(2*f*x))*tan(f*x)^2*tan(1/2*f*x)^2*tan
(exp(1))+4*c*f^2*x^2*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*sign(cos(1/2*(f*x+e
xp(1))-1/4*pi))*re(Ci(2*f*x))*tan(1/2*exp(1))^2*tan(1/2*f*x)^2*tan(exp(1))-
c*f^2*x^2*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*sign(cos(1/2*(f*x+exp(1))-1/4*
pi))*re(Ci(-f*x))*tan(f*x)^2*tan(1/2*exp(1))^2*tan(1/2*f*x)^2-c*f^2*x^2*sig
n(sin(1/2*(f*x+exp(1))-1/4*pi))*sign(cos(1/2*(f*x+exp(1))-1/4*pi))*re(Ci(-f
*x))*tan(f*x)^2*tan(1/2*exp(1))^2*tan(exp(1))^2+c*f^2*x^2*sign(sin(1/2*(f*x
+exp(1))-1/4*pi))*sign(cos(1/2*(f*x+exp(1))-1/4*pi))*re(Ci(-f*x))*tan(f*x)^
2*tan(1/2*f*x)^2*tan(exp(1))^2-c*f^2*x^2*sign(sin(1/2*(f*x+exp(1))-1/4*pi))
*sign(cos(1/2*(f*x+exp(1))-1/4*pi))*re(Ci(-f*x))*tan(1/2*exp(1))^2*tan(1/2*
f*x)^2*tan(exp(1))^2+4*c*f^2*x^2*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*sign(co
s(1/2*(f*x+exp(1))-1/4*pi))*re(Ci(-2*f*x))*tan(f*x)^2*tan(1/2*exp(1))^2*tan
(exp(1))+4*c*f^2*x^2*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*sign(cos(1/2*(f*x+e
xp(1))-1/4*pi))*re(Ci(-2*f*x))*tan(f*x)^2*tan(1/2*f*x)^2*tan(exp(1))+4*c*f^
2*x^2*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*sign(cos(1/2*(f*x+exp(1))-1/4*pi))
*re(Ci(-2*f*x))*tan(1/2*exp(1))^2*tan(1/2*f*x)^2*tan(exp(1))-4*c*f^2*x^2*Si
(f*x)*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*sign(cos(1/2*(f*x+exp(1))-1/4*pi))
*tan(f*x)^2*tan(1/2*exp(1))*tan(1/2*f*x)^2*tan(exp(1))^2-4*c*f^2*x^2*Si(2*f
*x)*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*sign(cos(1/2*(f*x+exp(1))-1/4*pi))*t
an(f*x)^2*tan(1/2*exp(1))^2*tan(1/2*f*x)^2*tan(exp(1))^2-2*c*f^2*x^2*sign(s
in(1/2*(f*x+exp(1))-1/4*pi))*sign(cos(1/2*(f*x+exp(1))-1/4*pi))*im(Ci(f*x))
*tan(f*x)^2*tan(1/2*exp(1))*tan(1/2*f*x)^2*tan(exp(1))^2-2*c*f^2*x^2*sign(s
in(1/2*(f*x+exp(1))-1/4*pi))*sign(cos(1/2*(f*x+exp(1))-1/4*pi))*im(Ci(2*f*x
))*tan(f*x)^2*tan(1/2*exp(1))^2*tan(1/2*f*x)^2*tan(exp(1))^2+2*c*f^2*x^2*si
gn(sin(1/2*(f*x+exp(1))-1/4*pi))*sign(cos(1/2*(f*x+exp(1))-1/4*pi))*im(Ci(-
f*x))*tan(f*x)^2*tan(1/2*exp(1))*tan(1/2*f*x)^2*tan(exp(1))^2+2*c*f^2*x^2*s
ign(sin(1/2*(f*x+exp(1))-1/4*pi))*sign(cos(1/2*(f*x+exp(1))-1/4*pi))*im(Ci(
-2*f*x))*tan(f*x)^2*tan(1/2*exp(1))^2*tan(1/2*f*x)^2*tan(exp(1))^2-c*f^2*x^
2*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*sign(cos(1/2*(f*x+exp(1))-1/4*pi))*re(
```

```
Ci(f*x))*tan(f*x)^2*tan(1/2*exp(1))^2*tan(1/2*f*x)^2*tan(exp(1))^2+4*c*f^2*
x^2*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*sign(cos(1/2*(f*x+exp(1))-1/4*pi))*r
e(Ci(2*f*x))*tan(f*x)^2*tan(1/2*exp(1))^2*tan(1/2*f*x)^2*tan(exp(1))-c*f^2*
x^2*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*sign(cos(1/2*(f*x+exp(1))-1/4*pi))*r
e(Ci(-f*x))*tan(f*x)^2*tan(1/2*exp(1))^2*tan(1/2*f*x)^2*tan(exp(1))^2+4*c*f
^2*x^2*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*sign(cos(1/2*(f*x+exp(1))-1/4*pi)
)*re(Ci(-2*f*x))*tan(f*x)^2*tan(1/2*exp(1))^2*tan(1/2*f*x)^2*tan(exp(1)))/(
8*x^2*tan(f*x)^2*tan(1/2*exp(1))^2*tan(1/2*f*x)^2*tan(exp(1))^2+8*x^2*tan(f
*x)^2*tan(1/2*exp(1))^2*tan(1/2*f*x)^2+8*x^2*tan(f*x)^2*tan(1/2*exp(1))^2*t
an(exp(1))^2+8*x^2*tan(f*x)^2*tan(1/2*f*x)^2*tan(exp(1))^2+8*x^2*tan(1/2*ex
p(1))^2*tan(1/2*f*x)^2*tan(exp(1))^2+8*x^2*tan(f*x)^2*tan(1/2*exp(1))^2+8*x
^2*tan(f*x)^2*tan(1/2*f*x)^2+8*x^2*tan(f*x)^2*tan(exp(1))^2+8*x^2*tan(1/2*e
xp(1))^2*tan(1/2*f*x)^2+8*x^2*tan(1/2*exp(1))^2*tan(exp(1))^2+8*x^2*tan(1/2
*f*x)^2*tan(exp(1))^2+8*x^2*tan(f*x)^2+8*x^2*tan(1/2*exp(1))^2+8*x^2*tan(1/
2*f*x)^2+8*x^2*tan(exp(1))^2+8*x^2)
```

maple [F] time = 0.17, size = 0, normalized size = 0.00

$$\int \frac{\left(c+c\sin\left(fx+e\right)\right)^{\frac{3}{2}}\sqrt{a-a\sin\left(fx+e\right)}}{x^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+c*sin(f*x+e))^(3/2)*(a-a*sin(f*x+e))^(1/2)/x^3,x)
```

```
[Out] int((c+c*sin(f*x+e))^(3/2)*(a-a*sin(f*x+e))^(1/2)/x^3,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-a\sin\left(fx+e\right)+a}\left(c\sin\left(fx+e\right)+c\right)^{\frac{3}{2}}}{x^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+c*sin(f*x+e))^(3/2)*(a-a*sin(f*x+e))^(1/2)/x^3,x, algorithm="m
axima")
```

```
[Out] integrate(sqrt(-a*sin(f*x + e) + a)*(c*sin(f*x + e) + c)^(3/2)/x^3, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{a-a\,\sin\left(e+f\,x\right)}\,{\left(c+c\,\sin\left(e+f\,x\right)\right)}^{3/2}}{x^3} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a - a*sin(e + f*x))^(1/2)*(c + c*sin(e + f*x))^(3/2))/x^3,x)
```

```
[Out] int(((a - a*sin(e + f*x))^(1/2)*(c + c*sin(e + f*x))^(3/2))/x^3, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(c\left(\sin\left(e+fx\right)+1\right)\right)^{\frac{3}{2}}\sqrt{-a\left(\sin\left(e+fx\right)-1\right)}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+c*sin(f*x+e))**(3/2)*(a-a*sin(f*x+e))**(1/2)/x**3,x)
```

```
[Out] Integral((c*(sin(e + f*x) + 1))**(3/2)*sqrt(-a*(sin(e + f*x) - 1))/x**3, x)
```

3.177 $\int \frac{(g+hx)^3\sqrt{a-a\sin(e+fx)}}{\sqrt{c+c\sin(e+fx)}}\,dx$

Optimal. Leaf size=767

$$-\frac{6iah^3\text{Li}_4\left(-ie^{i(e+fx)}\right)\cos(e+fx)}{f^4\sqrt{a-a\sin(e+fx)}\sqrt{c\sin(e+fx)+c}}+\frac{6iah^3\text{Li}_4\left(ie^{i(e+fx)}\right)\cos(e+fx)}{f^4\sqrt{a-a\sin(e+fx)}\sqrt{c\sin(e+fx)+c}}+\frac{3iah^3\text{Li}_4\left(-e^{2i(e+fx)}\right)\text{c}}{4f^4\sqrt{a-a\sin(e+fx)}\sqrt{c}}$$

```
[Out] -1/4*I*a*(h*x+g)^4*cos(f*x+e)/h/(a-a*sin(f*x+e))^(1/2)/(c+c*sin(f*x+e))^(1/
2)-2*I*a*(h*x+g)^3*arctan(exp(I*(f*x+e)))*cos(f*x+e)/f/(a-a*sin(f*x+e))^(1/
2)/(c+c*sin(f*x+e))^(1/2)+a*(h*x+g)^3*cos(f*x+e)*ln(1+exp(2*I*(f*x+e)))/f/(
a-a*sin(f*x+e))^(1/2)/(c+c*sin(f*x+e))^(1/2)+3*I*a*h*(h*x+g)^2*cos(f*x+e)*p
olylog(2,-I*exp(I*(f*x+e)))/f^2/(a-a*sin(f*x+e))^(1/2)/(c+c*sin(f*x+e))^(1/
2)-3*I*a*h*(h*x+g)^2*cos(f*x+e)*polylog(2,I*exp(I*(f*x+e)))/f^2/(a-a*sin(f*
x+e))^(1/2)/(c+c*sin(f*x+e))^(1/2)-3/2*I*a*h*(h*x+g)^2*cos(f*x+e)*polylog(2
,-exp(2*I*(f*x+e)))/f^2/(a-a*sin(f*x+e))^(1/2)/(c+c*sin(f*x+e))^(1/2)-6*a*h
^2*(h*x+g)*cos(f*x+e)*polylog(3,-I*exp(I*(f*x+e)))/f^3/(a-a*sin(f*x+e))^(1/
2)/(c+c*sin(f*x+e))^(1/2)+6*a*h^2*(h*x+g)*cos(f*x+e)*polylog(3,I*exp(I*(f*x
+e)))/f^3/(a-a*sin(f*x+e))^(1/2)/(c+c*sin(f*x+e))^(1/2)+3/2*a*h^2*(h*x+g)*c
os(f*x+e)*polylog(3,-exp(2*I*(f*x+e)))/f^3/(a-a*sin(f*x+e))^(1/2)/(c+c*sin(
f*x+e))^(1/2)-6*I*a*h^3*cos(f*x+e)*polylog(4,-I*exp(I*(f*x+e)))/f^4/(a-a*si
n(f*x+e))^(1/2)/(c+c*sin(f*x+e))^(1/2)+6*I*a*h^3*cos(f*x+e)*polylog(4,I*exp
(I*(f*x+e)))/f^4/(a-a*sin(f*x+e))^(1/2)/(c+c*sin(f*x+e))^(1/2)+3/4*I*a*h^3*
cos(f*x+e)*polylog(4,-exp(2*I*(f*x+e)))/f^4/(a-a*sin(f*x+e))^(1/2)/(c+c*sin
(f*x+e))^(1/2)
```

Rubi [A] time = 1.35, antiderivative size = 767, normalized size of antiderivative = 1.00, number of steps used = 20, number of rules used = 11, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.297, Rules used = {4604, 6741, 12, 6742, 4181, 2531, 6609, 2282, 6589, 3719, 2190}

$$-\frac{6ah^2(g+hx)\cos(e+fx)\text{PolyLog}\left(3,-ie^{i(e+fx)}\right)}{f^3\sqrt{a-a\sin(e+fx)}\sqrt{c\sin(e+fx)+c}}+\frac{6ah^2(g+hx)\cos(e+fx)\text{PolyLog}\left(3,ie^{i(e+fx)}\right)}{f^3\sqrt{a-a\sin(e+fx)}\sqrt{c\sin(e+fx)+c}}+\frac{3ah^2(g+hx)}{2f^3\sqrt{a-a}}$$

Antiderivative was successfully verified.

```
[In] Int[((g + h*x)^3*Sqrt[a - a*Sin[e + f*x]])/Sqrt[c + c*Sin[e + f*x]],x]
```

```
[Out] ((-I/4)*a*(g + h*x)^4*Cos[e + f*x])/(h*Sqrt[a - a*Sin[e + f*x]]*Sqrt[c + c*
Sin[e + f*x]]) - ((2*I)*a*(g + h*x)^3*ArcTan[E^(I*(e + f*x))]*Cos[e + f*x])
/(f*Sqrt[a - a*Sin[e + f*x]]*Sqrt[c + c*Sin[e + f*x]]) + (a*(g + h*x)^3*Cos
[e + f*x]*Log[1 + E^((2*I)*(e + f*x))])/(f*Sqrt[a - a*Sin[e + f*x]]*Sqrt[c
+ c*Sin[e + f*x]]) + ((3*I)*a*h*(g + h*x)^2*Cos[e + f*x]*PolyLog[2, (-I)*E^
(I*(e + f*x))])/(f^2*Sqrt[a - a*Sin[e + f*x]]*Sqrt[c + c*Sin[e + f*x]]) - (
```

```
(3*I)*a*h*(g + h*x)^2*Cos[e + f*x]*PolyLog[2, I*E^(I*(e + f*x))])/(f^2*Sqrt
[a - a*Sin[e + f*x]]*Sqrt[c + c*Sin[e + f*x]]) - (((3*I)/2)*a*h*(g + h*x)^2
*Cos[e + f*x]*PolyLog[2, -E^((2*I)*(e + f*x))])/(f^2*Sqrt[a - a*Sin[e + f*x
]]*Sqrt[c + c*Sin[e + f*x]]) - (6*a*h^2*(g + h*x)*Cos[e + f*x]*PolyLog[3, (
-I)*E^(I*(e + f*x))])/(f^3*Sqrt[a - a*Sin[e + f*x]]*Sqrt[c + c*Sin[e + f*x]
]) + (6*a*h^2*(g + h*x)*Cos[e + f*x]*PolyLog[3, I*E^(I*(e + f*x))])/(f^3*Sq
rt[a - a*Sin[e + f*x]]*Sqrt[c + c*Sin[e + f*x]]) + (3*a*h^2*(g + h*x)*Cos[e
 + f*x]*PolyLog[3, -E^((2*I)*(e + f*x))])/(2*f^3*Sqrt[a - a*Sin[e + f*x]]*S
qrt[c + c*Sin[e + f*x]]) - ((6*I)*a*h^3*Cos[e + f*x]*PolyLog[4, (-I)*E^(I*(
e + f*x))])/(f^4*Sqrt[a - a*Sin[e + f*x]]*Sqrt[c + c*Sin[e + f*x]]) + ((6*I
)*a*h^3*Cos[e + f*x]*PolyLog[4, I*E^(I*(e + f*x))])/(f^4*Sqrt[a - a*Sin[e +
 f*x]]*Sqrt[c + c*Sin[e + f*x]]) + (((3*I)/4)*a*h^3*Cos[e + f*x]*PolyLog[4,
 -E^((2*I)*(e + f*x))])/(f^4*Sqrt[a - a*Sin[e + f*x]]*Sqrt[c + c*Sin[e + f*
x]])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 3719

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
I*(c + d*x)^(m + 1))/(d*(m + 1)), x] - Dist[2*I, Int[((c + d*x)^m*E^(2*I*(e
 + f*x)))/(1 + E^(2*I*(e + f*x))), x], x] /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 4181

```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))])/f, x] + (-Di
st[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x))
], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 4604

```
Int[((g_.) + (h_.)*(x_))^(p_.)*((a_) + (b_.)*Sin[(e_.) + (f_.)*(x_)])^(m_)*
((c_) + (d_.)*Sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Dist[(a^IntPart[m
]*c^IntPart[m]*(a + b*Sin[e + f*x])^FracPart[m]*(c + d*Sin[e + f*x])^FracPa
rt[m])/Cos[e + f*x]^(2*FracPart[m]), Int[(g + h*x)^p*Cos[e + f*x]^(2*m)*(c
+ d*Sin[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] &&
EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[p] && IntegerQ[2*m] && I
GeQ[n - m, 0]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rule 6741

```
Int[u_, x_Symbol] :> With[{v = NormalizeIntegrand[u, x]}, Int[v, x] /; v =!
= u]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int \frac{(g+hx)^3\sqrt{a-a\sin(e+fx)}}{\sqrt{c+c\sin(e+fx)}}\,dx = \frac{\cos(e+fx)\int (g+hx)^3\sec(e+fx)(a-a\sin(e+fx))\,dx}{\sqrt{a-a\sin(e+fx)}\sqrt{c+c\sin(e+fx)}}$$

$$= \frac{\cos(e+fx)\int a(g+hx)^3\sec(e+fx)(1-\sin(e+fx))\,dx}{\sqrt{a-a\sin(e+fx)}\sqrt{c+c\sin(e+fx)}}$$

$$= \frac{(a\cos(e+fx))\int (g+hx)^3\sec(e+fx)(1-\sin(e+fx))\,dx}{\sqrt{a-a\sin(e+fx)}\sqrt{c+c\sin(e+fx)}}$$

$$= \frac{(a\cos(e+fx))\int \left((g+hx)^3\sec(e+fx)-(g+hx)^3\tan(e+fx)\right)\,dx}{\sqrt{a-a\sin(e+fx)}\sqrt{c+c\sin(e+fx)}}$$

$$= \frac{(a\cos(e+fx))\int (g+hx)^3\sec(e+fx)\,dx}{\sqrt{a-a\sin(e+fx)}\sqrt{c+c\sin(e+fx)}} - \frac{(a\cos(e+fx))\int (g+hx)^3\tan}{\sqrt{a-a\sin(e+fx)}\sqrt{c+c\sin}}$$

$$= -\frac{ia(g+hx)^4\cos(e+fx)}{4h\sqrt{a-a\sin(e+fx)}\sqrt{c+c\sin(e+fx)}} - \frac{2ia(g+hx)^3\tan^{-1}\left(e^{i(e+fx)}\right)c}{f\sqrt{a-a\sin(e+fx)}\sqrt{c+c\,s}}$$

$$= -\frac{ia(g+hx)^4\cos(e+fx)}{4h\sqrt{a-a\sin(e+fx)}\sqrt{c+c\sin(e+fx)}} - \frac{2ia(g+hx)^3\tan^{-1}\left(e^{i(e+fx)}\right)c}{f\sqrt{a-a\sin(e+fx)}\sqrt{c+c\,s}}$$

$$= -\frac{ia(g+hx)^4\cos(e+fx)}{4h\sqrt{a-a\sin(e+fx)}\sqrt{c+c\sin(e+fx)}} - \frac{2ia(g+hx)^3\tan^{-1}\left(e^{i(e+fx)}\right)c}{f\sqrt{a-a\sin(e+fx)}\sqrt{c+c\,s}}$$

$$= -\frac{ia(g+hx)^4\cos(e+fx)}{4h\sqrt{a-a\sin(e+fx)}\sqrt{c+c\sin(e+fx)}} - \frac{2ia(g+hx)^3\tan^{-1}\left(e^{i(e+fx)}\right)c}{f\sqrt{a-a\sin(e+fx)}\sqrt{c+c\,s}}$$

$$= -\frac{ia(g+hx)^4\cos(e+fx)}{4h\sqrt{a-a\sin(e+fx)}\sqrt{c+c\sin(e+fx)}} - \frac{2ia(g+hx)^3\tan^{-1}\left(e^{i(e+fx)}\right)c}{f\sqrt{a-a\sin(e+fx)}\sqrt{c+c\,s}}$$

$$= -\frac{ia(g+hx)^4\cos(e+fx)}{4h\sqrt{a-a\sin(e+fx)}\sqrt{c+c\sin(e+fx)}} - \frac{2ia(g+hx)^3\tan^{-1}\left(e^{i(e+fx)}\right)c}{f\sqrt{a-a\sin(e+fx)}\sqrt{c+c\,s}}$$

Mathematica [A] time = 2.66, size = 247, normalized size = 0.32

$$\frac{\left(\frac{1}{4}+\frac{i}{4}\right)e^{-\frac{1}{2}i(e+fx)}\left(e^{i(e+fx)}+i\right)\sqrt{a-a\sin(e+fx)}\left(\frac{24h\left(f^2(g+hx)^2\text{Li}_2\left(-ie^{-i(e+fx)}\right)-2h\left(if(g+hx)\text{Li}_3\left(-ie^{-i(e+fx)}\right)+h\text{Li}_4\left(-ie^{-i(e+fx)}\right)\right)\right)}{f^4}\right.}{\sqrt{2}\sqrt{-ice^{-i(e+fx)}\left(e^{i(e+fx)}+i\right)^2}\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[((g + h*x)^3*Sqrt[a - a*Sin[e + f*x]])/Sqrt[c + c*Sin[e + f*x]],x
]
```

```
[Out] ((1/4 + I/4)*(I + E^(I*(e + f*x)))*((g + h*x)^4/h - ((8*I)*(g + h*x)^3*Log[
1 + I/E^(I*(e + f*x))])/f + (24*h*(f^2*(g + h*x)^2*PolyLog[2, (-I)/E^(I*(e
+ f*x))] - 2*h*(I*f*(g + h*x)*PolyLog[3, (-I)/E^(I*(e + f*x))] + h*PolyLog[
4, (-I)/E^(I*(e + f*x))])))/f^4)*Sqrt[a - a*Sin[e + f*x]])/(Sqrt[2]*E^((I/2
)*(e + f*x))*Sqrt[((-I)*c*(I + E^(I*(e + f*x)))^2)/E^(I*(e + f*x))]*(Cos[(e
 + f*x)/2] - Sin[(e + f*x)/2]))
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)^3*(a-a*sin(f*x+e))^(1/2)/(c+c*sin(f*x+e))^(1/2),x, algori
thm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(hx+g\right)^3\sqrt{-a\sin\left(fx+e\right)+a}}{\sqrt{c\sin\left(fx+e\right)+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)^3*(a-a*sin(f*x+e))^(1/2)/(c+c*sin(f*x+e))^(1/2),x, algori
thm="giac")
```

```
[Out] integrate((h*x + g)^3*sqrt(-a*sin(f*x + e) + a)/sqrt(c*sin(f*x + e) + c), x
)
```

maple [F] time = 0.21, size = 0, normalized size = 0.00

$$\int \frac{\left(hx+g\right)^{3} \sqrt{a-a\sin\left(fx+e\right)}}{\sqrt{c+c\sin\left(fx+e\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((h*x+g)^3*(a-a*sin(f*x+e))^(1/2)/(c+c*sin(f*x+e))^(1/2),x)
```

```
[Out] int((h*x+g)^3*(a-a*sin(f*x+e))^(1/2)/(c+c*sin(f*x+e))^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(hx+g\right)^{3} \sqrt{-a\sin\left(fx+e\right)+a}}{\sqrt{c\sin\left(fx+e\right)+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)^3*(a-a*sin(f*x+e))^(1/2)/(c+c*sin(f*x+e))^(1/2),x, algori
thm="maxima")
```

```
[Out] integrate((h*x + g)^3*sqrt(-a*sin(f*x + e) + a)/sqrt(c*sin(f*x + e) + c), x
)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(g+h\,x\right)}^3\,\sqrt{a-a\,\sin\left(e+f\,x\right)}}{\sqrt{c+c\,\sin\left(e+f\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((g + h*x)^3*(a - a*sin(e + f*x))^(1/2))/(c + c*sin(e + f*x))^(1/2),x)
```

```
[Out] int(((g + h*x)^3*(a - a*sin(e + f*x))^(1/2))/(c + c*sin(e + f*x))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-a\left(\sin\left(e+fx\right)-1\right)} \left(g+hx\right)^{3}}{\sqrt{c\left(\sin\left(e+fx\right)+1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)**3*(a-a*sin(f*x+e))**(1/2)/(c+c*sin(f*x+e))**(1/2),x)
```

```
[Out] Integral(sqrt(-a*(sin(e + f*x) - 1))*(g + h*x)**3/sqrt(c*(sin(e + f*x) + 1)
), x)
```

3.178 $\int \frac{(g+hx)^2 \sqrt{a-a\sin(e+fx)}}{\sqrt{c+c\sin(e+fx)}}\,dx$

Optimal. Leaf size=555

$$-\frac{2ah^2\text{Li}_3\left(-ie^{i(e+fx)}\right)\cos(e+fx)}{f^3\sqrt{a-a\sin(e+fx)}\sqrt{c\sin(e+fx)+c}}+\frac{2ah^2\text{Li}_3\left(ie^{i(e+fx)}\right)\cos(e+fx)}{f^3\sqrt{a-a\sin(e+fx)}\sqrt{c\sin(e+fx)+c}}+\frac{ah^2\text{Li}_3\left(-e^{2i(e+fx)}\right)}{2f^3\sqrt{a-a\sin(e+fx)}\sqrt{}}$$

```
[Out] -1/3*I*a*(h*x+g)^3*cos(f*x+e)/h/(a-a*sin(f*x+e))^(1/2)/(c+c*sin(f*x+e))^(1/
2)-2*I*a*(h*x+g)^2*arctan(exp(I*(f*x+e)))*cos(f*x+e)/f/(a-a*sin(f*x+e))^(1/
2)/(c+c*sin(f*x+e))^(1/2)+a*(h*x+g)^2*cos(f*x+e)*ln(1+exp(2*I*(f*x+e)))/f/(
a-a*sin(f*x+e))^(1/2)/(c+c*sin(f*x+e))^(1/2)+2*I*a*h*(h*x+g)*cos(f*x+e)*pol
ylog(2,-I*exp(I*(f*x+e)))/f^2/(a-a*sin(f*x+e))^(1/2)/(c+c*sin(f*x+e))^(1/2)
-2*I*a*h*(h*x+g)*cos(f*x+e)*polylog(2,I*exp(I*(f*x+e)))/f^2/(a-a*sin(f*x+e)
)^(1/2)/(c+c*sin(f*x+e))^(1/2)-I*a*h*(h*x+g)*cos(f*x+e)*polylog(2,-exp(2*I*
(f*x+e)))/f^2/(a-a*sin(f*x+e))^(1/2)/(c+c*sin(f*x+e))^(1/2)-2*a*h^2*cos(f*x
+e)*polylog(3,-I*exp(I*(f*x+e)))/f^3/(a-a*sin(f*x+e))^(1/2)/(c+c*sin(f*x+e)
)^(1/2)+2*a*h^2*cos(f*x+e)*polylog(3,I*exp(I*(f*x+e)))/f^3/(a-a*sin(f*x+e))
^(1/2)/(c+c*sin(f*x+e))^(1/2)+1/2*a*h^2*cos(f*x+e)*polylog(3,-exp(2*I*(f*x+
e)))/f^3/(a-a*sin(f*x+e))^(1/2)/(c+c*sin(f*x+e))^(1/2)
```

Rubi [A] time = 0.88, antiderivative size = 555, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 10, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.270, Rules used = {4604, 6741, 12, 6742, 4181, 2531, 2282, 6589, 3719, 2190}

$$\frac{2iah(g+hx)\cos(e+fx)\text{PolyLog}\left(2,-ie^{i(e+fx)}\right)}{f^2\sqrt{a-a\sin(e+fx)}\sqrt{c\sin(e+fx)+c}}-\frac{2iah(g+hx)\cos(e+fx)\text{PolyLog}\left(2,ie^{i(e+fx)}\right)}{f^2\sqrt{a-a\sin(e+fx)}\sqrt{c\sin(e+fx)+c}}-\frac{iah(g+hx)\text{co}}{f^2\sqrt{a-a\,\text{si}}}$$

Antiderivative was successfully verified.

```
[In] Int[((g + h*x)^2*Sqrt[a - a*Sin[e + f*x]])/Sqrt[c + c*Sin[e + f*x]],x]
```

```
[Out] ((-I/3)*a*(g + h*x)^3*Cos[e + f*x])/(h*Sqrt[a - a*Sin[e + f*x]]*Sqrt[c + c*
Sin[e + f*x]]) - ((2*I)*a*(g + h*x)^2*ArcTan[E^(I*(e + f*x))]*Cos[e + f*x])
/(f*Sqrt[a - a*Sin[e + f*x]]*Sqrt[c + c*Sin[e + f*x]]) + (a*(g + h*x)^2*Cos
[e + f*x]*Log[1 + E^((2*I)*(e + f*x))])/(f*Sqrt[a - a*Sin[e + f*x]]*Sqrt[c
+ c*Sin[e + f*x]]) + ((2*I)*a*h*(g + h*x)*Cos[e + f*x]*PolyLog[2, (-I)*E^(I
*(e + f*x))])/(f^2*Sqrt[a - a*Sin[e + f*x]]*Sqrt[c + c*Sin[e + f*x]]) - ((2
*I)*a*h*(g + h*x)*Cos[e + f*x]*PolyLog[2, I*E^(I*(e + f*x))])/(f^2*Sqrt[a -
 a*Sin[e + f*x]]*Sqrt[c + c*Sin[e + f*x]]) - (I*a*h*(g + h*x)*Cos[e + f*x]*
PolyLog[2, -E^((2*I)*(e + f*x))])/(f^2*Sqrt[a - a*Sin[e + f*x]]*Sqrt[c + c*
Sin[e + f*x]]) - (2*a*h^2*Cos[e + f*x]*PolyLog[3, (-I)*E^(I*(e + f*x))])/(f
^3*Sqrt[a - a*Sin[e + f*x]]*Sqrt[c + c*Sin[e + f*x]]) + (2*a*h^2*Cos[e + f*
```

```
x]*PolyLog[3, I*E^(I*(e + f*x))])/(f^3*Sqrt[a - a*Sin[e + f*x]]*Sqrt[c + c*
Sin[e + f*x]]) + (a*h^2*Cos[e + f*x]*PolyLog[3, -E^((2*I)*(e + f*x))])/(2*f
^3*Sqrt[a - a*Sin[e + f*x]]*Sqrt[c + c*Sin[e + f*x]])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 3719

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
I*(c + d*x)^(m + 1))/(d*(m + 1)), x] - Dist[2*I, Int[((c + d*x)^m*E^(2*I*(e
 + f*x)))/(1 + E^(2*I*(e + f*x))), x], x] /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 4181

```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))])/f, x] + (-Di
st[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x))
```

```
], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 4604

```
Int[((g_.) + (h_.)*(x_))^(p_.)*((a_) + (b_.)*Sin[(e_.) + (f_.)*(x_)])^(m_)*
((c_) + (d_.)*Sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Dist[(a^IntPart[m
]*c^IntPart[m]*(a + b*Sin[e + f*x])^FracPart[m]*(c + d*Sin[e + f*x])^FracPa
rt[m])/Cos[e + f*x]^(2*FracPart[m]), Int[(g + h*x)^p*Cos[e + f*x]^(2*m)*(c
+ d*Sin[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] &&
EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[p] && IntegerQ[2*m] && I
GeQ[n - m, 0]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6741

```
Int[u_, x_Symbol] :> With[{v = NormalizeIntegrand[u, x]}, Int[v, x] /; v =!
= u]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int \frac{(g+hx)^2\sqrt{a-a\sin(e+fx)}}{\sqrt{c+c\sin(e+fx)}}\,dx = \frac{\cos(e+fx)\int (g+hx)^2\sec(e+fx)(a-a\sin(e+fx))\,dx}{\sqrt{a-a\sin(e+fx)}\sqrt{c+c\sin(e+fx)}}$$
$$= \frac{\cos(e+fx)\int a(g+hx)^2\sec(e+fx)(1-\sin(e+fx))\,dx}{\sqrt{a-a\sin(e+fx)}\sqrt{c+c\sin(e+fx)}}$$
$$= \frac{(a\cos(e+fx))\int (g+hx)^2\sec(e+fx)(1-\sin(e+fx))\,dx}{\sqrt{a-a\sin(e+fx)}\sqrt{c+c\sin(e+fx)}}$$
$$= \frac{(a\cos(e+fx))\int \left((g+hx)^2\sec(e+fx)-(g+hx)^2\tan(e+fx)\right)\,dx}{\sqrt{a-a\sin(e+fx)}\sqrt{c+c\sin(e+fx)}}$$
$$= \frac{(a\cos(e+fx))\int (g+hx)^2\sec(e+fx)\,dx}{\sqrt{a-a\sin(e+fx)}\sqrt{c+c\sin(e+fx)}} - \frac{(a\cos(e+fx))\int (g+hx)^2\tan(}{\sqrt{a-a\sin(e+fx)}\sqrt{c+c\sin}}$$
$$= -\frac{ia(g+hx)^3\cos(e+fx)}{3h\sqrt{a-a\sin(e+fx)}\sqrt{c+c\sin(e+fx)}} - \frac{2ia(g+hx)^2\tan^{-1}\left(e^{i(e+fx)}\right)\text{co}}{f\sqrt{a-a\sin(e+fx)}\sqrt{c+c\sin}}$$
$$= -\frac{ia(g+hx)^3\cos(e+fx)}{3h\sqrt{a-a\sin(e+fx)}\sqrt{c+c\sin(e+fx)}} - \frac{2ia(g+hx)^2\tan^{-1}\left(e^{i(e+fx)}\right)\text{co}}{f\sqrt{a-a\sin(e+fx)}\sqrt{c+c\sin}}$$
$$= -\frac{ia(g+hx)^3\cos(e+fx)}{3h\sqrt{a-a\sin(e+fx)}\sqrt{c+c\sin(e+fx)}} - \frac{2ia(g+hx)^2\tan^{-1}\left(e^{i(e+fx)}\right)\text{co}}{f\sqrt{a-a\sin(e+fx)}\sqrt{c+c\sin}}$$
$$= -\frac{ia(g+hx)^3\cos(e+fx)}{3h\sqrt{a-a\sin(e+fx)}\sqrt{c+c\sin(e+fx)}} - \frac{2ia(g+hx)^2\tan^{-1}\left(e^{i(e+fx)}\right)\text{co}}{f\sqrt{a-a\sin(e+fx)}\sqrt{c+c\sin}}$$
$$= -\frac{ia(g+hx)^3\cos(e+fx)}{3h\sqrt{a-a\sin(e+fx)}\sqrt{c+c\sin(e+fx)}} - \frac{2ia(g+hx)^2\tan^{-1}\left(e^{i(e+fx)}\right)\text{co}}{f\sqrt{a-a\sin(e+fx)}\sqrt{c+c\sin}}$$

Mathematica [A] time = 1.89, size = 194, normalized size = 0.35

$$\frac{\sqrt{2}\left(e^{i(e+fx)}+i\right)\sqrt{a-a\sin(e+fx)}\left(f^2(g+hx)^2\left(f(g+hx)-6ih\log\left(1+ie^{-i(e+fx)}\right)\right)+12fh^2(g+hx)\text{Li}_2\left(-ie^{-i(e}\right.\right.}{3f^3h\left(e^{i(e+fx)}-i\right)\sqrt{-ice^{-i(e+fx)}\left(e^{i(e+fx)}+i\right)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((g + h*x)^2*Sqrt[a - a*Sin[e + f*x]])/Sqrt[c + c*Sin[e + f*x]],x
]
```

```
[Out] (Sqrt[2]*(I + E^(I*(e + f*x)))*(f^2*(g + h*x)^2*(f*(g + h*x) - (6*I)*h*Log[
1 + I/E^(I*(e + f*x))]) + 12*f*h^2*(g + h*x)*PolyLog[2, (-I)/E^(I*(e + f*x)
```

```
)] - (12*I)*h^3*PolyLog[3, (-I)/E^(I*(e + f*x))])*Sqrt[a - a*Sin[e + f*x]])
/(3*(-I + E^(I*(e + f*x)))*Sqrt[((-I)*c*(I + E^(I*(e + f*x)))^2)/E^(I*(e +
f*x))]*f^3*h)
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)^2*(a-a*sin(f*x+e))^(1/2)/(c+c*sin(f*x+e))^(1/2),x, algori
thm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(hx+g)^2\sqrt{-a\sin(fx+e)+a}}{\sqrt{c\sin(fx+e)+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)^2*(a-a*sin(f*x+e))^(1/2)/(c+c*sin(f*x+e))^(1/2),x, algori
thm="giac")
```

```
[Out] integrate((h*x + g)^2*sqrt(-a*sin(f*x + e) + a)/sqrt(c*sin(f*x + e) + c), x
)
```

maple [F] time = 0.19, size = 0, normalized size = 0.00

$$\int \frac{(hx+g)^2\sqrt{a-a\sin(fx+e)}}{\sqrt{c+c\sin(fx+e)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((h*x+g)^2*(a-a*sin(f*x+e))^(1/2)/(c+c*sin(f*x+e))^(1/2),x)
```

```
[Out] int((h*x+g)^2*(a-a*sin(f*x+e))^(1/2)/(c+c*sin(f*x+e))^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(hx+g)^2\sqrt{-a\sin(fx+e)+a}}{\sqrt{c\sin(fx+e)+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)^2*(a-a*sin(f*x+e))^(1/2)/(c+c*sin(f*x+e))^(1/2),x, algori
thm="maxima")
```

```
[Out] integrate((h*x + g)^2*sqrt(-a*sin(f*x + e) + a)/sqrt(c*sin(f*x + e) + c), x
)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(g+h\,x\right)^2\,\sqrt{a-a\,\sin\left(e+f\,x\right)}}{\sqrt{c+c\,\sin\left(e+f\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((g + h*x)^2*(a - a*sin(e + f*x))^(1/2))/(c + c*sin(e + f*x))^(1/2),x)
```

```
[Out] int(((g + h*x)^2*(a - a*sin(e + f*x))^(1/2))/(c + c*sin(e + f*x))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-a\left(\sin\left(e+fx\right)-1\right)}\left(g+hx\right)^2}{\sqrt{c\left(\sin\left(e+fx\right)+1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)**2*(a-a*sin(f*x+e))**(1/2)/(c+c*sin(f*x+e))**(1/2),x)
```

```
[Out] Integral(sqrt(-a*(sin(e + f*x) - 1))*(g + h*x)**2/sqrt(c*(sin(e + f*x) + 1)
), x)
```

3.179 $\int \frac{(g+hx)\sqrt{a-a\sin(e+fx)}}{\sqrt{c+c\sin(e+fx)}}\,dx$

Optimal. Leaf size=355

$$\frac{iah\text{Li}_2\left(-ie^{i(e+fx)}\right)\cos(e+fx)}{f^2\sqrt{a-a\sin(e+fx)}\sqrt{c\sin(e+fx)+c}}-\frac{iah\text{Li}_2\left(ie^{i(e+fx)}\right)\cos(e+fx)}{f^2\sqrt{a-a\sin(e+fx)}\sqrt{c\sin(e+fx)+c}}-\frac{iah\text{Li}_2\left(-e^{2i(e+fx)}\right)\text{co}}{2f^2\sqrt{a-a\sin(e+fx)}\sqrt{c}}$$

```
[Out] -1/2*I*a*(h*x+g)^2*cos(f*x+e)/h/(a-a*sin(f*x+e))^(1/2)/(c+c*sin(f*x+e))^(1/
2)-2*I*a*(h*x+g)*arctan(exp(I*(f*x+e)))*cos(f*x+e)/f/(a-a*sin(f*x+e))^(1/2)
/(c+c*sin(f*x+e))^(1/2)+a*(h*x+g)*cos(f*x+e)*ln(1+exp(2*I*(f*x+e)))/f/(a-a*
sin(f*x+e))^(1/2)/(c+c*sin(f*x+e))^(1/2)+I*a*h*cos(f*x+e)*polylog(2,-I*exp(
I*(f*x+e)))/f^2/(a-a*sin(f*x+e))^(1/2)/(c+c*sin(f*x+e))^(1/2)-I*a*h*cos(f*x
+e)*polylog(2,I*exp(I*(f*x+e)))/f^2/(a-a*sin(f*x+e))^(1/2)/(c+c*sin(f*x+e))
^(1/2)-1/2*I*a*h*cos(f*x+e)*polylog(2,-exp(2*I*(f*x+e)))/f^2/(a-a*sin(f*x+e
))^(1/2)/(c+c*sin(f*x+e))^(1/2)
```

Rubi [A] time = 0.51, antiderivative size = 355, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 9, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.257, Rules used = {4604, 6741, 12, 6742, 4181, 2279, 2391, 3719, 2190}

$$\frac{iah\cos(e+fx)\text{PolyLog}\left(2,-ie^{i(e+fx)}\right)}{f^2\sqrt{a-a\sin(e+fx)}\sqrt{c\sin(e+fx)+c}}-\frac{iah\cos(e+fx)\text{PolyLog}\left(2,ie^{i(e+fx)}\right)}{f^2\sqrt{a-a\sin(e+fx)}\sqrt{c\sin(e+fx)+c}}-\frac{iah\cos(e+fx)\text{PolyLog}}{2f^2\sqrt{a-a\sin(e+fx)}\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In] Int[((g + h*x)*Sqrt[a - a*Sin[e + f*x]])/Sqrt[c + c*Sin[e + f*x]],x]
```

```
[Out] ((-I/2)*a*(g + h*x)^2*Cos[e + f*x])/(h*Sqrt[a - a*Sin[e + f*x]]*Sqrt[c + c*
Sin[e + f*x]]) - ((2*I)*a*(g + h*x)*ArcTan[E^(I*(e + f*x))]*Cos[e + f*x])/(
f*Sqrt[a - a*Sin[e + f*x]]*Sqrt[c + c*Sin[e + f*x]]) + (a*(g + h*x)*Cos[e +
 f*x]*Log[1 + E^((2*I)*(e + f*x))])/(f*Sqrt[a - a*Sin[e + f*x]]*Sqrt[c + c*
Sin[e + f*x]]) + (I*a*h*Cos[e + f*x]*PolyLog[2, (-I)*E^(I*(e + f*x))])/(f^2
*Sqrt[a - a*Sin[e + f*x]]*Sqrt[c + c*Sin[e + f*x]]) - (I*a*h*Cos[e + f*x]*P
olyLog[2, I*E^(I*(e + f*x))])/(f^2*Sqrt[a - a*Sin[e + f*x]]*Sqrt[c + c*Sin[
e + f*x]]) - ((I/2)*a*h*Cos[e + f*x]*PolyLog[2, -E^((2*I)*(e + f*x))])/(f^2
*Sqrt[a - a*Sin[e + f*x]]*Sqrt[c + c*Sin[e + f*x]])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3719

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
I*(c + d*x)^(m + 1))/(d*(m + 1)), x] - Dist[2*I, Int[((c + d*x)^m*E^(2*I*(e
 + f*x)))/(1 + E^(2*I*(e + f*x))), x], x] /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 4181

```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))])/f, x] + (-Di
st[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x))
], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 4604

```
Int[((g_.) + (h_.)*(x_))^(p_.)*((a_) + (b_.)*Sin[(e_.) + (f_.)*(x_)])^(m_)*
((c_) + (d_.)*Sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Dist[(a^IntPart[m
]*c^IntPart[m]*(a + b*Sin[e + f*x])^FracPart[m]*(c + d*Sin[e + f*x])^FracPa
rt[m])/Cos[e + f*x]^(2*FracPart[m]), Int[(g + h*x)^p*Cos[e + f*x]^(2*m)*(c
+ d*Sin[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] &&
EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[p] && IntegerQ[2*m] && I
GeQ[n - m, 0]
```

Rule 6741

```
Int[u_, x_Symbol] :> With[{v = NormalizeIntegrand[u, x]}, Int[v, x] /; v =!
= u]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int \frac{(g+hx)\sqrt{a-a\sin(e+fx)}}{\sqrt{c+c\sin(e+fx)}}\,dx = \frac{\cos(e+fx)\int (g+hx)\sec(e+fx)(a-a\sin(e+fx))\,dx}{\sqrt{a-a\sin(e+fx)}\sqrt{c+c\sin(e+fx)}}$$

$$= \frac{\cos(e+fx)\int a(g+hx)\sec(e+fx)(1-\sin(e+fx))\,dx}{\sqrt{a-a\sin(e+fx)}\sqrt{c+c\sin(e+fx)}}$$

$$= \frac{(a\cos(e+fx))\int (g+hx)\sec(e+fx)(1-\sin(e+fx))\,dx}{\sqrt{a-a\sin(e+fx)}\sqrt{c+c\sin(e+fx)}}$$

$$= \frac{(a\cos(e+fx))\int ((g+hx)\sec(e+fx)-(g+hx)\tan(e+fx))\,dx}{\sqrt{a-a\sin(e+fx)}\sqrt{c+c\sin(e+fx)}}$$

$$= \frac{(a\cos(e+fx))\int (g+hx)\sec(e+fx)\,dx}{\sqrt{a-a\sin(e+fx)}\sqrt{c+c\sin(e+fx)}} - \frac{(a\cos(e+fx))\int (g+hx)\tan(e}{\sqrt{a-a\sin(e+fx)}\sqrt{c+c\sin(}$$

$$= -\frac{ia(g+hx)^2\cos(e+fx)}{2h\sqrt{a-a\sin(e+fx)}\sqrt{c+c\sin(e+fx)}} - \frac{2ia(g+hx)\tan^{-1}\left(e^{i(e+fx)}\right)\mathrm{co}}{f\sqrt{a-a\sin(e+fx)}\sqrt{c+c\,\mathrm{si}}}$$

$$= -\frac{ia(g+hx)^2\cos(e+fx)}{2h\sqrt{a-a\sin(e+fx)}\sqrt{c+c\sin(e+fx)}} - \frac{2ia(g+hx)\tan^{-1}\left(e^{i(e+fx)}\right)\mathrm{co}}{f\sqrt{a-a\sin(e+fx)}\sqrt{c+c\,\mathrm{si}}}$$

$$= -\frac{ia(g+hx)^2\cos(e+fx)}{2h\sqrt{a-a\sin(e+fx)}\sqrt{c+c\sin(e+fx)}} - \frac{2ia(g+hx)\tan^{-1}\left(e^{i(e+fx)}\right)\mathrm{co}}{f\sqrt{a-a\sin(e+fx)}\sqrt{c+c\,\mathrm{si}}}$$

$$= -\frac{ia(g+hx)^2\cos(e+fx)}{2h\sqrt{a-a\sin(e+fx)}\sqrt{c+c\sin(e+fx)}} - \frac{2ia(g+hx)\tan^{-1}\left(e^{i(e+fx)}\right)\mathrm{co}}{f\sqrt{a-a\sin(e+fx)}\sqrt{c+c\,\mathrm{si}}}$$

Mathematica [A] time = 1.22, size = 154, normalized size = 0.43

$$\frac{\left(e^{i(e+fx)}+i\right)\sqrt{a-a\sin(e+fx)}\left(f\left(fx(2g+hx)-4i(g+hx)\log\left(1+ie^{-i(e+fx)}\right)\right)+4h\mathrm{Li}_2\left(-ie^{-i(e+fx)}\right)\right)}{\sqrt{2}f^2\left(e^{i(e+fx)}-i\right)\sqrt{-ice^{-i(e+fx)}\left(e^{i(e+fx)}+i\right)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((g + h*x)*Sqrt[a - a*Sin[e + f*x]])/Sqrt[c + c*Sin[e + f*x]],x]
```

```
[Out] ((I + E^(I*(e + f*x)))*(f*(f*x*(2*g + h*x) - (4*I)*(g + h*x)*Log[1 + I/E^(I
*(e + f*x))]) + 4*h*PolyLog[2, (-I)/E^(I*(e + f*x))])*Sqrt[a - a*Sin[e + f*
x]])/(Sqrt[2]*(-I + E^(I*(e + f*x)))*Sqrt[((-I)*c*(I + E^(I*(e + f*x)))^2)/
E^(I*(e + f*x))]*f^2)
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)*(a-a*sin(f*x+e))^(1/2)/(c+c*sin(f*x+e))^(1/2),x, algorith
m="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(hx+g)\sqrt{-a\sin(fx+e)+a}}{\sqrt{c\sin(fx+e)+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)*(a-a*sin(f*x+e))^(1/2)/(c+c*sin(f*x+e))^(1/2),x, algorith
m="giac")
```

```
[Out] integrate((h*x + g)*sqrt(-a*sin(f*x + e) + a)/sqrt(c*sin(f*x + e) + c), x)
```

maple [F] time = 0.35, size = 0, normalized size = 0.00

$$\int \frac{(hx+g)\sqrt{a-a\sin(fx+e)}}{\sqrt{c+c\sin(fx+e)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((h*x+g)*(a-a*sin(f*x+e))^(1/2)/(c+c*sin(f*x+e))^(1/2),x)
```

```
[Out] int((h*x+g)*(a-a*sin(f*x+e))^(1/2)/(c+c*sin(f*x+e))^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(hx+g)\sqrt{-a\sin(fx+e)+a}}{\sqrt{c\sin(fx+e)+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)*(a-a*sin(f*x+e))^(1/2)/(c+c*sin(f*x+e))^(1/2),x, algorith
m="maxima")
```

```
[Out] integrate((h*x + g)*sqrt(-a*sin(f*x + e) + a)/sqrt(c*sin(f*x + e) + c), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(g+h\,x)\,\sqrt{a-a\,\sin(e+f\,x)}}{\sqrt{c+c\,\sin(e+f\,x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((g + h*x)*(a - a*sin(e + f*x))^(1/2))/(c + c*sin(e + f*x))^(1/2),x)
```

```
[Out] int(((g + h*x)*(a - a*sin(e + f*x))^(1/2))/(c + c*sin(e + f*x))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-a(\sin(e+fx)-1)}\,(g+hx)}{\sqrt{c(\sin(e+fx)+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)*(a-a*sin(f*x+e))**(1/2)/(c+c*sin(f*x+e))**(1/2),x)
```

```
[Out] Integral(sqrt(-a*(sin(e + f*x) - 1))*(g + h*x)/sqrt(c*(sin(e + f*x) + 1)),
x)
```

3.180 $\int \frac{\sqrt{a-a\sin(e+fx)}}{(g+hx)\sqrt{c+c\sin(e+fx)}}\,dx$

Optimal. Leaf size=110

$$\frac{a\cos(e+fx)\text{Int}\left(\frac{\sec(e+fx)}{g+hx},x\right)}{\sqrt{a-a\sin(e+fx)}\sqrt{c\sin(e+fx)+c}}-\frac{a\cos(e+fx)\text{Int}\left(\frac{\tan(e+fx)}{g+hx},x\right)}{\sqrt{a-a\sin(e+fx)}\sqrt{c\sin(e+fx)+c}}$$

```
[Out] a*cos(f*x+e)*Unintegrable(sec(f*x+e)/(h*x+g),x)/(a-a*sin(f*x+e))^(1/2)/(c+c
*sin(f*x+e))^(1/2)-a*cos(f*x+e)*Unintegrable(tan(f*x+e)/(h*x+g),x)/(a-a*sin
(f*x+e))^(1/2)/(c+c*sin(f*x+e))^(1/2)
```

Rubi [A] time = 0.65, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\sqrt{a-a\sin(e+fx)}}{(g+hx)\sqrt{c+c\sin(e+fx)}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Sqrt[a - a*Sin[e + f*x]]/((g + h*x)*Sqrt[c + c*Sin[e + f*x]]),x]
```

```
[Out] (a*Cos[e + f*x]*Defer[Int][Sec[e + f*x]/(g + h*x), x])/(Sqrt[a - a*Sin[e +
f*x]]*Sqrt[c + c*Sin[e + f*x]]) - (a*Cos[e + f*x]*Defer[Int][Tan[e + f*x]/(
g + h*x), x])/(Sqrt[a - a*Sin[e + f*x]]*Sqrt[c + c*Sin[e + f*x]])
```

Rubi steps

$$\int \frac{\sqrt{a-a\sin(e+fx)}}{(g+hx)\sqrt{c+c\sin(e+fx)}}\,dx = \frac{\cos(e+fx)\int \frac{\sec(e+fx)(a-a\sin(e+fx))}{g+hx}\,dx}{\sqrt{a-a\sin(e+fx)}\,\sqrt{c+c\sin(e+fx)}}$$
$$= \frac{\cos(e+fx)\int \frac{a\sec(e+fx)(1-\sin(e+fx))}{g+hx}\,dx}{\sqrt{a-a\sin(e+fx)}\,\sqrt{c+c\sin(e+fx)}}$$
$$= \frac{(a\cos(e+fx))\int \frac{\sec(e+fx)(1-\sin(e+fx))}{g+hx}\,dx}{\sqrt{a-a\sin(e+fx)}\,\sqrt{c+c\sin(e+fx)}}$$
$$= \frac{(a\cos(e+fx))\int \left(\frac{\sec(e+fx)}{g+hx}-\frac{\tan(e+fx)}{g+hx}\right)dx}{\sqrt{a-a\sin(e+fx)}\,\sqrt{c+c\sin(e+fx)}}$$
$$= \frac{(a\cos(e+fx))\int \frac{\sec(e+fx)}{g+hx}\,dx}{\sqrt{a-a\sin(e+fx)}\,\sqrt{c+c\sin(e+fx)}} - \frac{(a\cos(e+fx))\int \frac{\tan(e+fx)}{g+hx}\,dx}{\sqrt{a-a\sin(e+fx)}\,\sqrt{c+c\sin(e+}$$

Mathematica [A] time = 3.54, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a-a\sin(e+fx)}}{(g+hx)\sqrt{c+c\sin(e+fx)}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Sqrt[a - a*Sin[e + f*x]]/((g + h*x)*Sqrt[c + c*Sin[e + f*x]]),x]
[Out] Integrate[Sqrt[a - a*Sin[e + f*x]]/((g + h*x)*Sqrt[c + c*Sin[e + f*x]]), x]
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-a*sin(f*x+e))^(1/2)/(h*x+g)/(c+c*sin(f*x+e))^(1/2),x, algorith
m="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-a\sin\left(fx+e\right)+a}}{\left(hx+g\right)\sqrt{c\sin\left(fx+e\right)+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-a*sin(f*x+e))^(1/2)/(h*x+g)/(c+c*sin(f*x+e))^(1/2),x, algorith
m="giac")
```

```
[Out] integrate(sqrt(-a*sin(f*x + e) + a)/((h*x + g)*sqrt(c*sin(f*x + e) + c)), x
)
```

maple [A] time = 0.19, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a-a\sin\left(fx+e\right)}}{\left(hx+g\right)\sqrt{c+c\sin\left(fx+e\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a-a*sin(f*x+e))^(1/2)/(h*x+g)/(c+c*sin(f*x+e))^(1/2),x)
```

```
[Out] int((a-a*sin(f*x+e))^(1/2)/(h*x+g)/(c+c*sin(f*x+e))^(1/2),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-a\sin\left(fx+e\right)+a}}{\left(hx+g\right)\sqrt{c\sin\left(fx+e\right)+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-a*sin(f*x+e))^(1/2)/(h*x+g)/(c+c*sin(f*x+e))^(1/2),x, algorith
m="maxima")
```

```
[Out] integrate(sqrt(-a*sin(f*x + e) + a)/((h*x + g)*sqrt(c*sin(f*x + e) + c)), x
)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{a-a\,\sin\left(e+f\,x\right)}}{\left(g+h\,x\right)\,\sqrt{c+c\,\sin\left(e+f\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a - a*sin(e + f*x))^(1/2)/((g + h*x)*(c + c*sin(e + f*x))^(1/2)),x)
```

```
[Out] int((a - a*sin(e + f*x))^(1/2)/((g + h*x)*(c + c*sin(e + f*x))^(1/2)), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-a\left(\sin\left(e+fx\right)-1\right)}}{\sqrt{c\left(\sin\left(e+fx\right)+1\right)}\left(g+hx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-a*sin(f*x+e))**(1/2)/(h*x+g)/(c+c*sin(f*x+e))**(1/2),x)
```

```
[Out] Integral(sqrt(-a*(sin(e + f*x) - 1))/(sqrt(c*(sin(e + f*x) + 1))*(g + h*x))
, x)
```

3.181 $\int \frac{x^3 \sqrt{a-a \sin(e+fx)}}{(c+c \sin(e+fx))^{3/2}} \, dx$

Optimal. Leaf size=536

$$\frac{6ia\text{Li}_2\left(-ie^{i(e+fx)}\right)\cos(e+fx)}{cf^4\sqrt{a-a\sin(e+fx)}\sqrt{c\sin(e+fx)+c}}-\frac{6ia\text{Li}_2\left(ie^{i(e+fx)}\right)\cos(e+fx)}{cf^4\sqrt{a-a\sin(e+fx)}\sqrt{c\sin(e+fx)+c}}-\frac{3ia\text{Li}_2\left(-e^{2i(e+fx)}\right)\text{co}}{cf^4\sqrt{a-a\sin(e+fx)}\sqrt{c}}$$

```
[Out] -3*a*x^2/c/f^2/(a-a*sin(f*x+e))^(1/2)/(c+c*sin(f*x+e))^(1/2)-3*I*a*x^2*cos(
f*x+e)/c/f^2/(a-a*sin(f*x+e))^(1/2)/(c+c*sin(f*x+e))^(1/2)-12*I*a*x*arctan(
exp(I*(f*x+e)))*cos(f*x+e)/c/f^3/(a-a*sin(f*x+e))^(1/2)/(c+c*sin(f*x+e))^(1
/2)+6*a*x*cos(f*x+e)*ln(1+exp(2*I*(f*x+e)))/c/f^3/(a-a*sin(f*x+e))^(1/2)/(c
+c*sin(f*x+e))^(1/2)+6*I*a*cos(f*x+e)*polylog(2,-I*exp(I*(f*x+e)))/c/f^4/(a
-a*sin(f*x+e))^(1/2)/(c+c*sin(f*x+e))^(1/2)-6*I*a*cos(f*x+e)*polylog(2,I*ex
p(I*(f*x+e)))/c/f^4/(a-a*sin(f*x+e))^(1/2)/(c+c*sin(f*x+e))^(1/2)-3*I*a*cos
(f*x+e)*polylog(2,-exp(2*I*(f*x+e)))/c/f^4/(a-a*sin(f*x+e))^(1/2)/(c+c*sin(
f*x+e))^(1/2)-a*x^3*sec(f*x+e)/c/f/(a-a*sin(f*x+e))^(1/2)/(c+c*sin(f*x+e))^
(1/2)+3*a*x^2*sin(f*x+e)/c/f^2/(a-a*sin(f*x+e))^(1/2)/(c+c*sin(f*x+e))^(1/2
)+a*x^3*tan(f*x+e)/c/f/(a-a*sin(f*x+e))^(1/2)/(c+c*sin(f*x+e))^(1/2)
```

Rubi [A] time = 3.55, antiderivative size = 536, normalized size of antiderivative = 1.00, number of steps used = 51, number of rules used = 17, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.515, Rules used = {4604, 6741, 12, 6742, 4186, 4181, 2279, 2391, 2531, 6609, 2282, 6589, 3757, 4184, 3719, 2190, 4413}

$$\frac{6ia\cos(e+fx)\text{PolyLog}\left(2,-ie^{i(e+fx)}\right)}{cf^4\sqrt{a-a\sin(e+fx)}\sqrt{c\sin(e+fx)+c}}-\frac{6ia\cos(e+fx)\text{PolyLog}\left(2,ie^{i(e+fx)}\right)}{cf^4\sqrt{a-a\sin(e+fx)}\sqrt{c\sin(e+fx)+c}}-\frac{3ia\cos(e+fx)\text{PolyLog}}{cf^4\sqrt{a-a\sin(e+fx)}\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*Sqrt[a - a*Sin[e + f*x]])/(c + c*Sin[e + f*x])^(3/2),x]
```

```
[Out] (-3*a*x^2)/(c*f^2*Sqrt[a - a*Sin[e + f*x]]*Sqrt[c + c*Sin[e + f*x]]) - ((3*
I)*a*x^2*Cos[e + f*x])/(c*f^2*Sqrt[a - a*Sin[e + f*x]]*Sqrt[c + c*Sin[e + f
*x]]) - ((12*I)*a*x*ArcTan[E^(I*(e + f*x))]*Cos[e + f*x])/(c*f^3*Sqrt[a - a
*Sin[e + f*x]]*Sqrt[c + c*Sin[e + f*x]]) + (6*a*x*Cos[e + f*x]*Log[1 + E^((
2*I)*(e + f*x))])/(c*f^3*Sqrt[a - a*Sin[e + f*x]]*Sqrt[c + c*Sin[e + f*x]])
 + ((6*I)*a*Cos[e + f*x]*PolyLog[2, (-I)*E^(I*(e + f*x))])/(c*f^4*Sqrt[a -
a*Sin[e + f*x]]*Sqrt[c + c*Sin[e + f*x]]) - ((6*I)*a*Cos[e + f*x]*PolyLog[2
, I*E^(I*(e + f*x))])/(c*f^4*Sqrt[a - a*Sin[e + f*x]]*Sqrt[c + c*Sin[e + f*
x]]) - ((3*I)*a*Cos[e + f*x]*PolyLog[2, -E^((2*I)*(e + f*x))])/(c*f^4*Sqrt[
a - a*Sin[e + f*x]]*Sqrt[c + c*Sin[e + f*x]]) - (a*x^3*Sec[e + f*x])/(c*f*S
qrt[a - a*Sin[e + f*x]]*Sqrt[c + c*Sin[e + f*x]]) + (3*a*x^2*Sin[e + f*x])/
```

```
(c*f^2*Sqrt[a - a*Sin[e + f*x]]*Sqrt[c + c*Sin[e + f*x]]) + (a*x^3*Tan[e +
f*x])/(c*f*Sqrt[a - a*Sin[e + f*x]]*Sqrt[c + c*Sin[e + f*x]])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 3719

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
I*(c + d*x)^(m + 1))/(d*(m + 1)), x] - Dist[2*I, Int[((c + d*x)^m*E^(2*I*(e
 + f*x)))/(1 + E^(2*I*(e + f*x))), x], x] /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 3757

```
Int[(x_)^(m_.)*Sec[(a_.) + (b_.)*(x_)^(n_.)]^(p_.)*Tan[(a_.) + (b_.)*(x_)^(
n_.)]^(q_.), x_Symbol] :> Simp[(x^(m - n + 1)*Sec[a + b*x^n]^p)/(b*n*p), x]
 - Dist[(m - n + 1)/(b*n*p), Int[x^(m - n)*Sec[a + b*x^n]^p, x], x] /; Free
Q[{a, b, p}, x] && IntegerQ[n] && GeQ[m, n] && EqQ[q, 1]
```

Rule 4181

```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))])/f, x] + (-Di
st[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x))
], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 4184

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> -Sim
p[((c + d*x)^m*Cot[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Co
t[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 4186

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_))^(m_), x_Symbo
l] :> -Simp[(b^2*(c + d*x)^m*Cot[e + f*x]*(b*Csc[e + f*x])^(n - 2))/(f*(n -
 1)), x] + (Dist[(b^2*d^2*m*(m - 1))/(f^2*(n - 1)*(n - 2)), Int[(c + d*x)^(
m - 2)*(b*Csc[e + f*x])^(n - 2), x], x] + Dist[(b^2*(n - 2))/(n - 1), Int[(
c + d*x)^m*(b*Csc[e + f*x])^(n - 2), x], x] - Simp[(b^2*d*m*(c + d*x)^(m -
1)*(b*Csc[e + f*x])^(n - 2))/(f^2*(n - 1)*(n - 2)), x]) /; FreeQ[{b, c, d,
e, f}, x] && GtQ[n, 1] && NeQ[n, 2] && GtQ[m, 1]
```

Rule 4413

```
Int[((c_.) + (d_.)*(x_))^(m_.)*Sec[(a_.) + (b_.)*(x_)]*Tan[(a_.) + (b_.)*(x
_)]^(p_), x_Symbol] :> -Int[(c + d*x)^m*Sec[a + b*x]*Tan[a + b*x]^(p - 2),
x] + Int[(c + d*x)^m*Sec[a + b*x]^3*Tan[a + b*x]^(p - 2), x] /; FreeQ[{a, b
, c, d, m}, x] && IGtQ[p/2, 0]
```

Rule 4604

```
Int[((g_.) + (h_.)*(x_))^(p_.)*((a_) + (b_.)*Sin[(e_.) + (f_.)*(x_)])^(m_)*
((c_) + (d_.)*Sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Dist[(a^IntPart[m
]*c^IntPart[m]*(a + b*Sin[e + f*x])^FracPart[m]*(c + d*Sin[e + f*x])^FracPa
rt[m])/Cos[e + f*x]^(2*FracPart[m]), Int[(g + h*x)^p*Cos[e + f*x]^(2*m)*(c
+ d*Sin[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] &&
EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[p] && IntegerQ[2*m] && I
GeQ[n - m, 0]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rule 6741

```
Int[u_, x_Symbol] :> With[{v = NormalizeIntegrand[u, x]}, Int[v, x] /; v =!
= u]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int \frac{x^3\sqrt{a-a\sin(e+fx)}}{(c+c\sin(e+fx))^{3/2}}\,dx = \frac{\cos(e+fx)\int x^3\sec^3(e+fx)(a-a\sin(e+fx))^2\,dx}{ac\sqrt{a-a\sin(e+fx)}\,\sqrt{c+c\sin(e+fx)}}$$
$$= \frac{\cos(e+fx)\int a^2x^3\sec^3(e+fx)(1-\sin(e+fx))^2\,dx}{ac\sqrt{a-a\sin(e+fx)}\,\sqrt{c+c\sin(e+fx)}}$$
$$= \frac{(a\cos(e+fx))\int x^3\sec^3(e+fx)(1-\sin(e+fx))^2\,dx}{c\sqrt{a-a\sin(e+fx)}\,\sqrt{c+c\sin(e+fx)}}$$
$$= \frac{(a\cos(e+fx))\int \left(x^3\sec^3(e+fx)-2x^3\sec^2(e+fx)\tan(e+fx)+x^3\sec(e+fx)\right.}{c\sqrt{a-a\sin(e+fx)}\,\sqrt{c+c\sin(e+fx)}}$$
$$= \frac{(a\cos(e+fx))\int x^3\sec^3(e+fx)\,dx}{c\sqrt{a-a\sin(e+fx)}\,\sqrt{c+c\sin(e+fx)}} + \frac{(a\cos(e+fx))\int x^3\sec(e+fx)\tan^2(e}{c\sqrt{a-a\sin(e+fx)}\,\sqrt{c+c\sin(e+}$$
$$= -\frac{3ax^2}{2cf^2\sqrt{a-a\sin(e+fx)}\,\sqrt{c+c\sin(e+fx)}} - \frac{ax^3\sec(e+fx)}{cf\sqrt{a-a\sin(e+fx)}\,\sqrt{c+c\sin(e}$$
$$= -\frac{3ax^2}{cf^2\sqrt{a-a\sin(e+fx)}\,\sqrt{c+c\sin(e+fx)}} - \frac{6iax\tan^{-1}\left(e^{i(e+fx)}\right)\cos(e+fx}{cf^3\sqrt{a-a\sin(e+fx)}\,\sqrt{c+c\sin(e}$$
$$= -\frac{3ax^2}{cf^2\sqrt{a-a\sin(e+fx)}\,\sqrt{c+c\sin(e+fx)}} - \frac{3iax^2\cos(e+fx)}{cf^2\sqrt{a-a\sin(e+fx)}\,\sqrt{c+c\sin(e}$$
$$= -\frac{3ax^2}{cf^2\sqrt{a-a\sin(e+fx)}\,\sqrt{c+c\sin(e+fx)}} - \frac{3iax^2\cos(e+fx)}{cf^2\sqrt{a-a\sin(e+fx)}\,\sqrt{c+c\sin(e}$$
$$= -\frac{3ax^2}{cf^2\sqrt{a-a\sin(e+fx)}\,\sqrt{c+c\sin(e+fx)}} - \frac{3iax^2\cos(e+fx)}{cf^2\sqrt{a-a\sin(e+fx)}\,\sqrt{c+c\sin(e}$$
$$= -\frac{3ax^2}{cf^2\sqrt{a-a\sin(e+fx)}\,\sqrt{c+c\sin(e+fx)}} - \frac{3iax^2\cos(e+fx)}{cf^2\sqrt{a-a\sin(e+fx)}\,\sqrt{c+c\sin(e}$$
$$= -\frac{3ax^2}{cf^2\sqrt{a-a\sin(e+fx)}\,\sqrt{c+c\sin(e+fx)}} - \frac{3iax^2\cos(e+fx)}{cf^2\sqrt{a-a\sin(e+fx)}\,\sqrt{c+c\sin(e}$$

<u>Mathematica</u> [A] time = 2.08, size = 193, normalized size = 0.36

$$-\frac{\sqrt{a-a\sin(e+fx)}\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)\left(fx\left(-12\log\left(1-ie^{i(e+fx)}\right)+3fx\cos(e+fx)+3i\left(fx+\right.\right.\right.}{f^4(c(\sin(e+fx)+1))^{3/2}\left(\cos\left(\frac{1}{2}(e+fx)\right)\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*Sqrt[a - a*Sin[e + f*x]])/(c + c*Sin[e + f*x])^(3/2),x]
```

```
[Out] -(((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*Sqrt[a - a*Sin[e + f*x]]*((12*I)*P
olyLog[2, I*E^(I*(e + f*x))]*(1 + Sin[e + f*x]) + f*x*((3*I)*f*x + f^2*x^2
+ 3*f*x*Cos[e + f*x] - 12*Log[1 - I*E^(I*(e + f*x))] + (3*I)*(f*x + (4*I)*L
og[1 - I*E^(I*(e + f*x))])*Sin[e + f*x])))/(f^4*(Cos[(e + f*x)/2] - Sin[(e
+ f*x)/2])*(c*(1 + Sin[e + f*x]))^(3/2)))
```

fricas [F] time = 0.73, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\sqrt{-a\sin\left(fx+e\right)+a}\,\sqrt{c\sin\left(fx+e\right)+c}\,x^{3}}{c^{2}\cos\left(fx+e\right)^{2}-2\,c^{2}\sin\left(fx+e\right)-2\,c^{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a-a*sin(f*x+e))^(1/2)/(c+c*sin(f*x+e))^(3/2),x, algorithm="f
ricas")
```

```
[Out] integral(-sqrt(-a*sin(f*x + e) + a)*sqrt(c*sin(f*x + e) + c)*x^3/(c^2*cos(f
*x + e)^2 - 2*c^2*sin(f*x + e) - 2*c^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-a\sin\left(fx+e\right)+a}\,x^{3}}{\left(c\sin\left(fx+e\right)+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a-a*sin(f*x+e))^(1/2)/(c+c*sin(f*x+e))^(3/2),x, algorithm="g
iac")
```

```
[Out] integrate(sqrt(-a*sin(f*x + e) + a)*x^3/(c*sin(f*x + e) + c)^(3/2), x)
```

maple [F] time = 0.14, size = 0, normalized size = 0.00

$$\int \frac{x^{3}\sqrt{a-a\sin\left(fx+e\right)}}{\left(c+c\sin\left(fx+e\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a-a*sin(f*x+e))^(1/2)/(c+c*sin(f*x+e))^(3/2),x)
```

```
[Out] int(x^3*(a-a*sin(f*x+e))^(1/2)/(c+c*sin(f*x+e))^(3/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-a\sin\left(fx+e\right)+a}\,x^3}{\left(c\sin\left(fx+e\right)+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a-a*sin(f*x+e))^(1/2)/(c+c*sin(f*x+e))^(3/2),x, algorithm="m
axima")
```

```
[Out] integrate(sqrt(-a*sin(f*x + e) + a)*x^3/(c*sin(f*x + e) + c)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^3\,\sqrt{a-a\,\sin\left(e+f\,x\right)}}{{\left(c+c\,\sin\left(e+f\,x\right)\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(a - a*sin(e + f*x))^(1/2))/(c + c*sin(e + f*x))^(3/2),x)
```

```
[Out] int((x^3*(a - a*sin(e + f*x))^(1/2))/(c + c*sin(e + f*x))^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3\sqrt{-a\left(\sin\left(e+fx\right)-1\right)}}{\left(c\left(\sin\left(e+fx\right)+1\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(a-a*sin(f*x+e))**(1/2)/(c+c*sin(f*x+e))**(3/2),x)
```

```
[Out] Integral(x**3*sqrt(-a*(sin(e + f*x) - 1))/(c*(sin(e + f*x) + 1))**(3/2), x)
```

3.182 $\int \frac{x^2\sqrt{a-a\sin(e+fx)}}{(c+c\sin(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=280

$$\frac{2a\cos(e+fx)\log(\cos(e+fx))}{cf^3\sqrt{a-a\sin(e+fx)}\sqrt{c\sin(e+fx)+c}}+\frac{2a\cos(e+fx)\tanh^{-1}(\sin(e+fx))}{cf^3\sqrt{a-a\sin(e+fx)}\sqrt{c\sin(e+fx)+c}}+\frac{2ax\sin(e+}{cf^2\sqrt{a-a\sin(e+fx)}\sqrt{}}$$

```
[Out] -2*a*x/c/f^2/(a-a*sin(f*x+e))^(1/2)/(c+c*sin(f*x+e))^(1/2)+2*a*arctanh(sin(
f*x+e))*cos(f*x+e)/c/f^3/(a-a*sin(f*x+e))^(1/2)/(c+c*sin(f*x+e))^(1/2)+2*a*
cos(f*x+e)*ln(cos(f*x+e))/c/f^3/(a-a*sin(f*x+e))^(1/2)/(c+c*sin(f*x+e))^(1/
2)-a*x^2*sec(f*x+e)/c/f/(a-a*sin(f*x+e))^(1/2)/(c+c*sin(f*x+e))^(1/2)+2*a*x
*sin(f*x+e)/c/f^2/(a-a*sin(f*x+e))^(1/2)/(c+c*sin(f*x+e))^(1/2)+a*x^2*tan(f
*x+e)/c/f/(a-a*sin(f*x+e))^(1/2)/(c+c*sin(f*x+e))^(1/2)
```

Rubi [A] time = 2.16, antiderivative size = 280, normalized size of antiderivative = 1.00, number of steps used = 34, number of rules used = 14, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.424, Rules used = {4604, 6741, 12, 6742, 4186, 3770, 4181, 2531, 2282, 6589, 3757, 4184, 3475, 4413}

$$\frac{2ax\sin(e+fx)}{cf^2\sqrt{a-a\sin(e+fx)}\sqrt{c\sin(e+fx)+c}}-\frac{2ax}{cf^2\sqrt{a-a\sin(e+fx)}\sqrt{c\sin(e+fx)+c}}+\frac{2a\cos(e+fx)\log(}{cf^3\sqrt{a-a\sin(e+fx)}\sqrt{}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*Sqrt[a - a*Sin[e + f*x]])/(c + c*Sin[e + f*x])^(3/2),x]
```

```
[Out] (-2*a*x)/(c*f^2*Sqrt[a - a*Sin[e + f*x]]*Sqrt[c + c*Sin[e + f*x]]) + (2*a*A
rcTanh[Sin[e + f*x]]*Cos[e + f*x])/(c*f^3*Sqrt[a - a*Sin[e + f*x]]*Sqrt[c +
 c*Sin[e + f*x]]) + (2*a*Cos[e + f*x]*Log[Cos[e + f*x]])/(c*f^3*Sqrt[a - a*
Sin[e + f*x]]*Sqrt[c + c*Sin[e + f*x]]) - (a*x^2*Sec[e + f*x])/(c*f*Sqrt[a
- a*Sin[e + f*x]]*Sqrt[c + c*Sin[e + f*x]]) + (2*a*x*Sin[e + f*x])/(c*f^2*S
qrt[a - a*Sin[e + f*x]]*Sqrt[c + c*Sin[e + f*x]]) + (a*x^2*Tan[e + f*x])/(c
*f*Sqrt[a - a*Sin[e + f*x]]*Sqrt[c + c*Sin[e + f*x]])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
```

```
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3757

```
Int[(x_)^(m_.)*Sec[(a_.) + (b_.)*(x_)^(n_.)]^(p_.)*Tan[(a_.) + (b_.)*(x_)^(
n_.)]^(q_.), x_Symbol] :> Simp[(x^(m - n + 1)*Sec[a + b*x^n]^p)/(b*n*p), x]
 - Dist[(m - n + 1)/(b*n*p), Int[x^(m - n)*Sec[a + b*x^n]^p, x], x] /; Free
Q[{a, b, p}, x] && IntegerQ[n] && GeQ[m, n] && EqQ[q, 1]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 4181

```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))])/f, x] + (-Di
st[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x))
], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 4184

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> -Sim
p[((c + d*x)^m*Cot[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Co
t[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 4186

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_))^(m_), x_Symbo
l] :> -Simp[(b^2*(c + d*x)^m*Cot[e + f*x]*(b*Csc[e + f*x])^(n - 2))/(f*(n -
 1)), x] + (Dist[(b^2*d^2*m*(m - 1))/(f^2*(n - 1)*(n - 2)), Int[(c + d*x)^(
m - 2)*(b*Csc[e + f*x])^(n - 2), x], x] + Dist[(b^2*(n - 2))/(n - 1), Int[(
c + d*x)^m*(b*Csc[e + f*x])^(n - 2), x], x] - Simp[(b^2*d*m*(c + d*x)^(m -
1)*(b*Csc[e + f*x])^(n - 2))/(f^2*(n - 1)*(n - 2)), x]) /; FreeQ[{b, c, d,
e, f}, x] && GtQ[n, 1] && NeQ[n, 2] && GtQ[m, 1]
```

Rule 4413

```
Int[((c_.) + (d_.)*(x_))^(m_.)*Sec[(a_.) + (b_.)*(x_)]*Tan[(a_.) + (b_.)*(x
_)]^(p_), x_Symbol] :> -Int[(c + d*x)^m*Sec[a + b*x]*Tan[a + b*x]^(p - 2),
x] + Int[(c + d*x)^m*Sec[a + b*x]^3*Tan[a + b*x]^(p - 2), x] /; FreeQ[{a, b
, c, d, m}, x] && IGtQ[p/2, 0]
```

Rule 4604

```
Int[((g_.) + (h_.)*(x_))^(p_.)*((a_) + (b_.)*Sin[(e_.) + (f_.)*(x_)])^(m_)*
((c_) + (d_.)*Sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Dist[(a^IntPart[m
]*c^IntPart[m]*(a + b*Sin[e + f*x])^FracPart[m]*(c + d*Sin[e + f*x])^FracPa
rt[m])/Cos[e + f*x]^(2*FracPart[m]), Int[(g + h*x)^p*Cos[e + f*x]^(2*m)*(c
+ d*Sin[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] &&
EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[p] && IntegerQ[2*m] && I
GeQ[n - m, 0]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6741

```
Int[u_, x_Symbol] :> With[{v = NormalizeIntegrand[u, x]}, Int[v, x] /; v =!
= u]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int \frac{x^2\sqrt{a-a\sin(e+fx)}}{(c+c\sin(e+fx))^{3/2}}\,dx = \frac{\cos(e+fx)\int x^2\sec^3(e+fx)(a-a\sin(e+fx))^2\,dx}{ac\sqrt{a-a\sin(e+fx)}\sqrt{c+c\sin(e+fx)}}$$

$$= \frac{\cos(e+fx)\int a^2x^2\sec^3(e+fx)(1-\sin(e+fx))^2\,dx}{ac\sqrt{a-a\sin(e+fx)}\sqrt{c+c\sin(e+fx)}}$$

$$= \frac{(a\cos(e+fx))\int x^2\sec^3(e+fx)(1-\sin(e+fx))^2\,dx}{c\sqrt{a-a\sin(e+fx)}\sqrt{c+c\sin(e+fx)}}$$

$$= \frac{(a\cos(e+fx))\int \left(x^2\sec^3(e+fx)-2x^2\sec^2(e+fx)\tan(e+fx)+x^2\sec(e+fx)\right.}{c\sqrt{a-a\sin(e+fx)}\sqrt{c+c\sin(e+fx)}}$$

$$= \frac{(a\cos(e+fx))\int x^2\sec^3(e+fx)\,dx}{c\sqrt{a-a\sin(e+fx)}\sqrt{c+c\sin(e+fx)}} + \frac{(a\cos(e+fx))\int x^2\sec(e+fx)\tan^2(e}{c\sqrt{a-a\sin(e+fx)}\sqrt{c+c\sin(e+}}$$

$$= -\frac{ax}{cf^2\sqrt{a-a\sin(e+fx)}\sqrt{c+c\sin(e+fx)}} - \frac{ax^2\sec(e+fx)}{cf\sqrt{a-a\sin(e+fx)}\sqrt{c+c\sin(e+}}$$

$$= -\frac{2ax}{cf^2\sqrt{a-a\sin(e+fx)}\sqrt{c+c\sin(e+fx)}} + \frac{iax^2\tan^{-1}\left(e^{i(e+fx)}\right)\cos(e+fx)}{cf\sqrt{a-a\sin(e+fx)}\sqrt{c+c\sin(e+}}$$

$$= -\frac{2ax}{cf^2\sqrt{a-a\sin(e+fx)}\sqrt{c+c\sin(e+fx)}} + \frac{2a\tanh^{-1}(\sin(e+fx))\cos(e+f}{cf^3\sqrt{a-a\sin(e+fx)}\sqrt{c+c\sin(e}}$$

$$= -\frac{2ax}{cf^2\sqrt{a-a\sin(e+fx)}\sqrt{c+c\sin(e+fx)}} + \frac{2a\tanh^{-1}(\sin(e+fx))\cos(e+f}{cf^3\sqrt{a-a\sin(e+fx)}\sqrt{c+c\sin(e}}$$

$$= -\frac{2ax}{cf^2\sqrt{a-a\sin(e+fx)}\sqrt{c+c\sin(e+fx)}} + \frac{2a\tanh^{-1}(\sin(e+fx))\cos(e+f}{cf^3\sqrt{a-a\sin(e+fx)}\sqrt{c+c\sin(e}}$$

$$= -\frac{2ax}{cf^2\sqrt{a-a\sin(e+fx)}\sqrt{c+c\sin(e+fx)}} + \frac{2a\tanh^{-1}(\sin(e+fx))\cos(e+f}{cf^3\sqrt{a-a\sin(e+fx)}\sqrt{c+c\sin(e}}$$

Mathematica [C] time = 1.56, size = 154, normalized size = 0.55

$$-\frac{\sqrt{a-a\sin(e+fx)}\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)\left(-4\log\left(e^{i(e+fx)}+i\right)+2fx\cos(e+fx)+\left(2ifx-4\log\right.\right.}{f^3(c(\sin(e+fx)+1))^{3/2}\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*Sqrt[a - a*Sin[e + f*x]])/(c + c*Sin[e + f*x])^(3/2),x]
```

```
[Out] -(((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*Sqrt[a - a*Sin[e + f*x]]*((2*I)*f*
x + f^2*x^2 + 2*f*x*Cos[e + f*x] - 4*Log[I + E^(I*(e + f*x))] + ((2*I)*f*x
- 4*Log[I + E^(I*(e + f*x))])*Sin[e + f*x]))/(f^3*(Cos[(e + f*x)/2] - Sin[(
e + f*x)/2])*(c*(1 + Sin[e + f*x]))^(3/2)))
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a-a*sin(f*x+e))^(1/2)/(c+c*sin(f*x+e))^(3/2),x, algorithm="f
ricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-a\sin\left(fx+e\right)+a}\,x^2}{\left(c\sin\left(fx+e\right)+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a-a*sin(f*x+e))^(1/2)/(c+c*sin(f*x+e))^(3/2),x, algorithm="g
iac")
```

```
[Out] integrate(sqrt(-a*sin(f*x + e) + a)*x^2/(c*sin(f*x + e) + c)^(3/2), x)
```

maple [F] time = 0.15, size = 0, normalized size = 0.00

$$\int \frac{x^2\sqrt{a-a\sin\left(fx+e\right)}}{\left(c+c\sin\left(fx+e\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a-a*sin(f*x+e))^(1/2)/(c+c*sin(f*x+e))^(3/2),x)
```

```
[Out] int(x^2*(a-a*sin(f*x+e))^(1/2)/(c+c*sin(f*x+e))^(3/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-a\sin\left(fx+e\right)+a}\,x^2}{\left(c\sin\left(fx+e\right)+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a-a*sin(f*x+e))^(1/2)/(c+c*sin(f*x+e))^(3/2),x, algorithm="m
axima")
```

```
[Out] integrate(sqrt(-a*sin(f*x + e) + a)*x^2/(c*sin(f*x + e) + c)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^2\,\sqrt{a-a\,\sin\left(e+f\,x\right)}}{{\left(c+c\,\sin\left(e+f\,x\right)\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(a - a*sin(e + f*x))^(1/2))/(c + c*sin(e + f*x))^(3/2),x)
```

```
[Out] int((x^2*(a - a*sin(e + f*x))^(1/2))/(c + c*sin(e + f*x))^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{2}\sqrt{-a\left(\sin\left(e+fx\right)-1\right)}}{\left(c\left(\sin\left(e+fx\right)+1\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a-a*sin(f*x+e))**(1/2)/(c+c*sin(f*x+e))**(3/2),x)
```

```
[Out] Integral(x**2*sqrt(-a*(sin(e + f*x) - 1))/(c*(sin(e + f*x) + 1))**(3/2), x)
```

3.183 $\int \frac{x\sqrt{a-a\sin(e+fx)}}{(c+c\sin(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=171

$$\frac{a\sin(e+fx)}{cf^2\sqrt{a-a\sin(e+fx)}\,\sqrt{c\sin(e+fx)+c}}-\frac{a}{cf^2\sqrt{a-a\sin(e+fx)}\,\sqrt{c\sin(e+fx)+c}}+\frac{ax\tan(e+}{cf\sqrt{a-a\sin(e+fx)}\,\sqrt{c}}$$

```
[Out] -a/c/f^2/(a-a*sin(f*x+e))^(1/2)/(c+c*sin(f*x+e))^(1/2)-a*x*sec(f*x+e)/c/f/(
a-a*sin(f*x+e))^(1/2)/(c+c*sin(f*x+e))^(1/2)+a*sin(f*x+e)/c/f^2/(a-a*sin(f*
x+e))^(1/2)/(c+c*sin(f*x+e))^(1/2)+a*x*tan(f*x+e)/c/f/(a-a*sin(f*x+e))^(1/2
)/(c+c*sin(f*x+e))^(1/2)
```

Rubi [A] time = 0.97, antiderivative size = 171, normalized size of antiderivative = 1.00, number of steps used = 26, number of rules used = 12, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.387, Rules used = {4604, 6741, 12, 6742, 4185, 4181, 2279, 2391, 3757, 3767, 8, 4413}

$$\frac{a\sin(e+fx)}{cf^2\sqrt{a-a\sin(e+fx)}\,\sqrt{c\sin(e+fx)+c}}-\frac{a}{cf^2\sqrt{a-a\sin(e+fx)}\,\sqrt{c\sin(e+fx)+c}}+\frac{ax\tan(e+}{cf\sqrt{a-a\sin(e+fx)}\,\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In] Int[(x*Sqrt[a - a*Sin[e + f*x]])/(c + c*Sin[e + f*x])^(3/2),x]
```

```
[Out] -(a/(c*f^2*Sqrt[a - a*Sin[e + f*x]]*Sqrt[c + c*Sin[e + f*x]])) - (a*x*Sec[e
 + f*x])/(c*f*Sqrt[a - a*Sin[e + f*x]]*Sqrt[c + c*Sin[e + f*x]]) + (a*Sin[e
 + f*x])/(c*f^2*Sqrt[a - a*Sin[e + f*x]]*Sqrt[c + c*Sin[e + f*x]]) + (a*x*T
an[e + f*x])/(c*f*Sqrt[a - a*Sin[e + f*x]]*Sqrt[c + c*Sin[e + f*x]])
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3757

```
Int[(x_)^(m_.)*Sec[(a_.) + (b_.)*(x_)^(n_.)]^(p_.)*Tan[(a_.) + (b_.)*(x_)^(
n_.)]^(q_.), x_Symbol] :> Simp[(x^(m - n + 1)*Sec[a + b*x^n]^p)/(b*n*p), x]
 - Dist[(m - n + 1)/(b*n*p), Int[x^(m - n)*Sec[a + b*x^n]^p, x], x] /; Free
Q[{a, b, p}, x] && IntegerQ[n] && GeQ[m, n] && EqQ[q, 1]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 4181

```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))])/f, x] + (-Di
st[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x))
], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 4185

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_)), x_Symbol] :>
 -Simp[(b^2*(c + d*x)*Cot[e + f*x]*(b*Csc[e + f*x])^(n - 2))/(f*(n - 1)), x
] + (Dist[(b^2*(n - 2))/(n - 1), Int[(c + d*x)*(b*Csc[e + f*x])^(n - 2), x]
, x] - Simp[(b^2*d*(b*Csc[e + f*x])^(n - 2))/(f^2*(n - 1)*(n - 2)), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && NeQ[n, 2]
```

Rule 4413

```
Int[((c_.) + (d_.)*(x_))^(m_.)*Sec[(a_.) + (b_.)*(x_)]*Tan[(a_.) + (b_.)*(x
_)]^(p_), x_Symbol] :> -Int[(c + d*x)^m*Sec[a + b*x]*Tan[a + b*x]^(p - 2),
x] + Int[(c + d*x)^m*Sec[a + b*x]^3*Tan[a + b*x]^(p - 2), x] /; FreeQ[{a, b
, c, d, m}, x] && IGtQ[p/2, 0]
```

Rule 4604

```
Int[((g_.) + (h_.)*(x_))^(p_.)*((a_) + (b_.)*Sin[(e_.) + (f_.)*(x_)])^(m_)*
((c_) + (d_.)*Sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Dist[(a^IntPart[m
```

```
]*c^IntPart[m]*(a + b*Sin[e + f*x])^FracPart[m]*(c + d*Sin[e + f*x])^FracPa
rt[m])/Cos[e + f*x]^(2*FracPart[m]), Int[(g + h*x)^p*Cos[e + f*x]^(2*m)*(c
+ d*Sin[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] &&
EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[p] && IntegerQ[2*m] && I
GeQ[n - m, 0]
```

Rule 6741

```
Int[u_, x_Symbol] :> With[{v = NormalizeIntegrand[u, x]}, Int[v, x] /; v =!
= u]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int \frac{x\sqrt{a-a\sin(e+fx)}}{(c+c\sin(e+fx))^{3/2}}\,dx = \frac{\cos(e+fx)\int x\sec^3(e+fx)(a-a\sin(e+fx))^2\,dx}{ac\sqrt{a-a\sin(e+fx)}\sqrt{c+c\sin(e+fx)}}$$

$$= \frac{\cos(e+fx)\int a^2x\sec^3(e+fx)(1-\sin(e+fx))^2\,dx}{ac\sqrt{a-a\sin(e+fx)}\sqrt{c+c\sin(e+fx)}}$$

$$= \frac{(a\cos(e+fx))\int x\sec^3(e+fx)(1-\sin(e+fx))^2\,dx}{c\sqrt{a-a\sin(e+fx)}\sqrt{c+c\sin(e+fx)}}$$

$$= \frac{(a\cos(e+fx))\int \left(x\sec^3(e+fx)-2x\sec^2(e+fx)\tan(e+fx)+x\sec(e+fx)\tan\right.}{c\sqrt{a-a\sin(e+fx)}\sqrt{c+c\sin(e+fx)}}$$

$$= \frac{(a\cos(e+fx))\int x\sec^3(e+fx)\,dx}{c\sqrt{a-a\sin(e+fx)}\sqrt{c+c\sin(e+fx)}} + \frac{(a\cos(e+fx))\int x\sec(e+fx)\tan^2(e+}{c\sqrt{a-a\sin(e+fx)}\sqrt{c+c\sin(e+f}$$

$$= -\frac{a}{2cf^2\sqrt{a-a\sin(e+fx)}\sqrt{c+c\sin(e+fx)}} - \frac{ax\sec(e+fx)}{cf\sqrt{a-a\sin(e+fx)}\sqrt{c+c\sin(e+}$$

$$= -\frac{a}{cf^2\sqrt{a-a\sin(e+fx)}\sqrt{c+c\sin(e+fx)}} + \frac{iax\tan^{-1}\left(e^{i(e+fx)}\right)\cos(e+fx)}{cf\sqrt{a-a\sin(e+fx)}\sqrt{c+c\sin(e+}$$

$$= -\frac{a}{cf^2\sqrt{a-a\sin(e+fx)}\sqrt{c+c\sin(e+fx)}} - \frac{ax\sec(e+fx)}{cf\sqrt{a-a\sin(e+fx)}\sqrt{c+c\sin(e+}$$

$$= -\frac{a}{cf^2\sqrt{a-a\sin(e+fx)}\sqrt{c+c\sin(e+fx)}} - \frac{ia\cos(e+fx)\mathrm{Li}_2\left(-ie^{i(e+fx)}\right)}{2cf^2\sqrt{a-a\sin(e+fx)}\sqrt{c+c\sin(e}$$

$$= -\frac{a}{cf^2\sqrt{a-a\sin(e+fx)}\sqrt{c+c\sin(e+fx)}} - \frac{ax\sec(e+fx)}{cf\sqrt{a-a\sin(e+fx)}\sqrt{c+c\sin(e+}$$

Mathematica [A] time = 0.60, size = 150, normalized size = 0.88

$$-\frac{\sqrt{a-a\sin(e+fx)}\sqrt{c(\sin(e+fx)+1)}\left(fx\sin\left(\frac{e}{2}\right)-\sin\left(\frac{e}{2}+fx\right)+\cos\left(\frac{e}{2}\right)(fx-1)+\cos\left(\frac{e}{2}+fx\right)+\sin\left(\frac{e}{2}\right)\right)}{c^2f^2\left(\sin\left(\frac{e}{2}\right)+\cos\left(\frac{e}{2}\right)\right)\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*Sqrt[a - a*Sin[e + f*x]])/(c + c*Sin[e + f*x])^(3/2),x]
```

```
[Out] -((((-1 + f*x)*Cos[e/2] + Cos[e/2 + f*x] + Sin[e/2] + f*x*Sin[e/2] - Sin[e/
2 + f*x])*Sqrt[c*(1 + Sin[e + f*x])]*Sqrt[a - a*Sin[e + f*x]])/(c^2*f^2*(Co
```

```
s[e/2] + Sin[e/2])*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(Cos[(e + f*x)/2]
+ Sin[(e + f*x)/2])^3))
```

fricas [A] time = 0.56, size = 72, normalized size = 0.42

$$-\frac{\left(fx+\cos\left(fx+e\right)\right)\sqrt{-a\sin\left(fx+e\right)+a}\,\sqrt{c\sin\left(fx+e\right)+c}}{c^2f^2\cos\left(fx+e\right)\sin\left(fx+e\right)+c^2f^2\cos\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a-a*sin(f*x+e))^(1/2)/(c+c*sin(f*x+e))^(3/2),x, algorithm="fri
cas")
```

```
[Out] -(f*x + cos(f*x + e))*sqrt(-a*sin(f*x + e) + a)*sqrt(c*sin(f*x + e) + c)/(c
^2*f^2*cos(f*x + e)*sin(f*x + e) + c^2*f^2*cos(f*x + e))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sqrt{-a\sin\left(fx+e\right)+a}\,x}{\left(c\sin\left(fx+e\right)+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a-a*sin(f*x+e))^(1/2)/(c+c*sin(f*x+e))^(3/2),x, algorithm="gia
c")
```

```
[Out] integrate(sqrt(-a*sin(f*x + e) + a)*x/(c*sin(f*x + e) + c)^(3/2), x)
```

maple [F] time = 0.15, size = 0, normalized size = 0.00

$$\int\frac{x\sqrt{a-a\sin\left(fx+e\right)}}{\left(c+c\sin\left(fx+e\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a-a*sin(f*x+e))^(1/2)/(c+c*sin(f*x+e))^(3/2),x)
```

```
[Out] int(x*(a-a*sin(f*x+e))^(1/2)/(c+c*sin(f*x+e))^(3/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sqrt{-a\sin\left(fx+e\right)+a}\,x}{\left(c\sin\left(fx+e\right)+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a-a*sin(f*x+e))^(1/2)/(c+c*sin(f*x+e))^(3/2),x, algorithm="max
ima")
```

```
[Out] integrate(sqrt(-a*sin(f*x + e) + a)*x/(c*sin(f*x + e) + c)^(3/2), x)
```

mupad [B] time = 3.32, size = 88, normalized size = 0.51

$$-\frac{\sqrt{-a\left(\sin\left(e+f\,x\right)-1\right)}\,\left(\cos\left(2\,e+2\,f\,x\right)+2\,f\,x\,\cos\left(e+f\,x\right)+1-\cos\left(e+f\,x\right)\,2\mathrm{i}-\sin\left(2\,e+2\,f\,x\right)\,1\mathrm{i}\right)}{c\,f^2\,\left(\cos\left(2\,e+2\,f\,x\right)+1\right)\,\sqrt{c\,\left(\sin\left(e+f\,x\right)+1\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(a - a*sin(e + f*x))^(1/2))/(c + c*sin(e + f*x))^(3/2),x)
```

```
[Out] -((-a*(sin(e + f*x) - 1))^(1/2)*(cos(2*e + 2*f*x) - cos(e + f*x)*2i - sin(2
*e + 2*f*x)*1i + 2*f*x*cos(e + f*x) + 1))/(c*f^2*(cos(2*e + 2*f*x) + 1)*(c*
(sin(e + f*x) + 1))^(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x\sqrt{-a\left(\sin\left(e+fx\right)-1\right)}}{\left(c\left(\sin\left(e+fx\right)+1\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a-a*sin(f*x+e))**(1/2)/(c+c*sin(f*x+e))**(3/2),x)
```

```
[Out] Integral(x*sqrt(-a*(sin(e + f*x) - 1))/(c*(sin(e + f*x) + 1))**(3/2), x)
```

3.184 $\int \frac{z^2 \sqrt{1+\cos(z)}}{\sqrt{1-\cos(z)}} \, dz$

Optimal. Leaf size=300

$$\frac{2iz\text{Li}_2\left(-e^{iz}\right)\sin(z)}{\sqrt{1-\cos(z)}\sqrt{\cos(z)+1}}-\frac{2iz\text{Li}_2\left(e^{iz}\right)\sin(z)}{\sqrt{1-\cos(z)}\sqrt{\cos(z)+1}}-\frac{iz\text{Li}_2\left(e^{2iz}\right)\sin(z)}{\sqrt{1-\cos(z)}\sqrt{\cos(z)+1}}-\frac{2\text{Li}_3\left(-e^{iz}\right)\sin(z)}{\sqrt{1-\cos(z)}\sqrt{\cos(z)+1}}+\frac{}{\sqrt{1}}$$

```
[Out] -1/3*I*z^3*sin(z)/(1-cos(z))^(1/2)/(1+cos(z))^(1/2)-2*z^2*arctanh(exp(I*z))
*sin(z)/(1-cos(z))^(1/2)/(1+cos(z))^(1/2)+z^2*ln(1-exp(2*I*z))*sin(z)/(1-co
s(z))^(1/2)/(1+cos(z))^(1/2)+2*I*z*polylog(2,-exp(I*z))*sin(z)/(1-cos(z))^(
1/2)/(1+cos(z))^(1/2)-2*I*z*polylog(2,exp(I*z))*sin(z)/(1-cos(z))^(1/2)/(1+
cos(z))^(1/2)-I*z*polylog(2,exp(2*I*z))*sin(z)/(1-cos(z))^(1/2)/(1+cos(z))^
(1/2)-2*polylog(3,-exp(I*z))*sin(z)/(1-cos(z))^(1/2)/(1+cos(z))^(1/2)+2*pol
ylog(3,exp(I*z))*sin(z)/(1-cos(z))^(1/2)/(1+cos(z))^(1/2)+1/2*polylog(3,exp
(2*I*z))*sin(z)/(1-cos(z))^(1/2)/(1+cos(z))^(1/2)
```

Rubi [A] time = 0.44, antiderivative size = 300, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 8, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {4605, 6742, 3717, 2190, 2531, 2282, 6589, 4183}

$$\frac{2iz\sin(z)\text{PolyLog}\left(2,-e^{iz}\right)}{\sqrt{1-\cos(z)}\sqrt{\cos(z)+1}}-\frac{2iz\sin(z)\text{PolyLog}\left(2,e^{iz}\right)}{\sqrt{1-\cos(z)}\sqrt{\cos(z)+1}}-\frac{iz\sin(z)\text{PolyLog}\left(2,e^{2iz}\right)}{\sqrt{1-\cos(z)}\sqrt{\cos(z)+1}}-\frac{2\sin(z)\text{PolyLog}\left(3,-e^{iz}\right)}{\sqrt{1-\cos(z)}\sqrt{\cos(z)+1}}+$$

Antiderivative was successfully verified.

```
[In] Int[(z^2*Sqrt[1 + Cos[z]])/Sqrt[1 - Cos[z]],z]
```

```
[Out] ((-I/3)*z^3*Sin[z])/(Sqrt[1 - Cos[z]]*Sqrt[1 + Cos[z]]) - (2*z^2*ArcTanh[E^
(I*z)]*Sin[z])/(Sqrt[1 - Cos[z]]*Sqrt[1 + Cos[z]]) + (z^2*Log[1 - E^((2*I)*
z)]*Sin[z])/(Sqrt[1 - Cos[z]]*Sqrt[1 + Cos[z]]) + ((2*I)*z*PolyLog[2, -E^(I
*z)]*Sin[z])/(Sqrt[1 - Cos[z]]*Sqrt[1 + Cos[z]]) - ((2*I)*z*PolyLog[2, E^(I
*z)]*Sin[z])/(Sqrt[1 - Cos[z]]*Sqrt[1 + Cos[z]]) - (I*z*PolyLog[2, E^((2*I)
*z)]*Sin[z])/(Sqrt[1 - Cos[z]]*Sqrt[1 + Cos[z]]) - (2*PolyLog[3, -E^(I*z)]*
Sin[z])/(Sqrt[1 - Cos[z]]*Sqrt[1 + Cos[z]]) + (2*PolyLog[3, E^(I*z)]*Sin[z]
)/(Sqrt[1 - Cos[z]]*Sqrt[1 + Cos[z]]) + (PolyLog[3, E^((2*I)*z)]*Sin[z])/(2
*Sqrt[1 - Cos[z]]*Sqrt[1 + Cos[z]])
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 3717

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] - Dist[2*I, Int[((c + d*x)^
m*E^(2*I*k*Pi)*E^(2*I*(e + f*x)))/(1 + E^(2*I*k*Pi)*E^(2*I*(e + f*x))), x],
 x] /; FreeQ[{c, d, e, f}, x] && IntegerQ[4*k] && IGtQ[m, 0]
```

Rule 4183

```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[(
-2*(c + d*x)^m*ArcTanh[E^(I*(e + f*x))])/f, x] + (-Dist[(d*m)/f, Int[(c + d
*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Dist[(d*m)/f, Int[(c + d*x)^
(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 4605

```
Int[(Cos[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(Cos[(e_.) + (f_.)*(x_)]*(d
_.) + (c_))^(n_)*((g_.) + (h_.)*(x_))^(p_.), x_Symbol] :> Dist[(a^IntPart[m
]*c^IntPart[m]*(a + b*Cos[e + f*x])^FracPart[m]*(c + d*Cos[e + f*x])^FracPa
rt[m])/Sin[e + f*x]^(2*FracPart[m]), Int[(g + h*x)^p*Sin[e + f*x]^(2*m)*(c
+ d*Cos[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] &&
EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[p] && IntegerQ[2*m] && I
GeQ[n - m, 0]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
```

```
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int \frac{z^2\sqrt{1+\cos(z)}}{\sqrt{1-\cos(z)}}\,dz = \frac{\sin(z)\int z^2(1+\cos(z))\csc(z)\,dz}{\sqrt{1-\cos(z)}\,\sqrt{1+\cos(z)}}$$
$$= \frac{\sin(z)\int \left(z^2\cot(z)+z^2\csc(z)\right)\,dz}{\sqrt{1-\cos(z)}\,\sqrt{1+\cos(z)}}$$
$$= \frac{\sin(z)\int z^2\cot(z)\,dz}{\sqrt{1-\cos(z)}\,\sqrt{1+\cos(z)}} + \frac{\sin(z)\int z^2\csc(z)\,dz}{\sqrt{1-\cos(z)}\,\sqrt{1+\cos(z)}}$$
$$= -\frac{iz^3\sin(z)}{3\sqrt{1-\cos(z)}\,\sqrt{1+\cos(z)}} - \frac{2z^2\tanh^{-1}\left(e^{iz}\right)\sin(z)}{\sqrt{1-\cos(z)}\,\sqrt{1+\cos(z)}} - \frac{(2i\sin(z))\int \frac{e^{2iz}z^2}{1-e^{2iz}}\,dz}{\sqrt{1-\cos(z)}\,\sqrt{1+\cos(z)}} - \cdots$$
$$= -\frac{iz^3\sin(z)}{3\sqrt{1-\cos(z)}\,\sqrt{1+\cos(z)}} - \frac{2z^2\tanh^{-1}\left(e^{iz}\right)\sin(z)}{\sqrt{1-\cos(z)}\,\sqrt{1+\cos(z)}} + \frac{z^2\log\left(1-e^{2iz}\right)\sin(z)}{\sqrt{1-\cos(z)}\,\sqrt{1+\cos(z)}} + \cdots$$
$$= -\frac{iz^3\sin(z)}{3\sqrt{1-\cos(z)}\,\sqrt{1+\cos(z)}} - \frac{2z^2\tanh^{-1}\left(e^{iz}\right)\sin(z)}{\sqrt{1-\cos(z)}\,\sqrt{1+\cos(z)}} + \frac{z^2\log\left(1-e^{2iz}\right)\sin(z)}{\sqrt{1-\cos(z)}\,\sqrt{1+\cos(z)}} + \cdots$$
$$= -\frac{iz^3\sin(z)}{3\sqrt{1-\cos(z)}\,\sqrt{1+\cos(z)}} - \frac{2z^2\tanh^{-1}\left(e^{iz}\right)\sin(z)}{\sqrt{1-\cos(z)}\,\sqrt{1+\cos(z)}} + \frac{z^2\log\left(1-e^{2iz}\right)\sin(z)}{\sqrt{1-\cos(z)}\,\sqrt{1+\cos(z)}} + \cdots$$
$$= -\frac{iz^3\sin(z)}{3\sqrt{1-\cos(z)}\,\sqrt{1+\cos(z)}} - \frac{2z^2\tanh^{-1}\left(e^{iz}\right)\sin(z)}{\sqrt{1-\cos(z)}\,\sqrt{1+\cos(z)}} + \frac{z^2\log\left(1-e^{2iz}\right)\sin(z)}{\sqrt{1-\cos(z)}\,\sqrt{1+\cos(z)}} + \cdots$$

Mathematica [A] time = 0.09, size = 85, normalized size = 0.28

$$\frac{\sqrt{\cos(z)+1}\,\tan\left(\frac{z}{2}\right)\left(12iz\text{Li}_2\left(e^{-iz}\right)+12\text{Li}_3\left(e^{-iz}\right)+iz^3+6z^2\log\left(1-e^{-iz}\right)-i\pi^3\right)}{3\sqrt{1-\cos(z)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(z^2*Sqrt[1 + Cos[z]])/Sqrt[1 - Cos[z]],z]
```

```
[Out] (Sqrt[1 + Cos[z]]*((-I)*Pi^3 + I*z^3 + 6*z^2*Log[1 - E^((-I)*z)] + (12*I)*z
*PolyLog[2, E^((-I)*z)] + 12*PolyLog[3, E^((-I)*z)])*Tan[z/2])/(3*Sqrt[1 -
Cos[z]])
```

fricas [C] time = 0.77, size = 75, normalized size = 0.25

$$z^2 \log(-\cos(z) + i \sin(z) + 1) + z^2 \log(-\cos(z) - i \sin(z) + 1) - 2iz\mathrm{Li}_2(\cos(z) + i \sin(z)) + 2iz\mathrm{Li}_2(\cos(z) - i \sin$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(z^2*(1+cos(z))^(1/2)/(1-cos(z))^(1/2),z, algorithm="fricas")
```

```
[Out] z^2*log(-cos(z) + I*sin(z) + 1) + z^2*log(-cos(z) - I*sin(z) + 1) - 2*I*z*d
ilog(cos(z) + I*sin(z)) + 2*I*z*dilog(cos(z) - I*sin(z)) + 2*polylog(3, cos
(z) + I*sin(z)) + 2*polylog(3, cos(z) - I*sin(z))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{z^2\sqrt{\cos(z)+1}}{\sqrt{-\cos(z)+1}}\,dz$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(z^2*(1+cos(z))^(1/2)/(1-cos(z))^(1/2),z, algorithm="giac")
```

```
[Out] integrate(z^2*sqrt(cos(z) + 1)/sqrt(-cos(z) + 1), z)
```

maple [A] time = 0.14, size = 154, normalized size = 0.51

$$\frac{(e^{iz}-1)\sqrt{(e^{iz}+1)^2 e^{-iz}}\, z^3}{3\sqrt{-(e^{iz}-1)^2 e^{-iz}}\,(e^{iz}+1)} + \frac{2i(e^{iz}-1)\sqrt{(e^{iz}+1)^2 e^{-iz}}\left(\frac{iz^3}{3} - z^2\ln(1-e^{iz}) + 2iz\,\mathrm{polylog}(2, e^{iz}) - 2\,\mathrm{polylog}\right.}{\sqrt{-(e^{iz}-1)^2 e^{-iz}}\,(e^{iz}+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(z^2*(1+cos(z))^(1/2)/(1-cos(z))^(1/2),z)
```

```
[Out] 1/3/(-(exp(I*z)-1)^2*exp(-I*z))^(1/2)*(exp(I*z)-1)*((exp(I*z)+1)^2*exp(-I*z
))^(1/2)/(exp(I*z)+1)*z^3+2*I/(-(exp(I*z)-1)^2*exp(-I*z))^(1/2)*(exp(I*z)-1
)*((exp(I*z)+1)^2*exp(-I*z))^(1/2)/(exp(I*z)+1)*(1/3*I*z^3-z^2*ln(1-exp(I*z
))+2*I*z*polylog(2,exp(I*z))-2*polylog(3,exp(I*z)))
```

maxima [A] time = 0.49, size = 56, normalized size = 0.19

$$\frac{1}{3}iz^3 + 2iz^2\arctan(\sin(z), -\cos(z)+1) - z^2\log\left(\cos(z)^2 + \sin(z)^2 - 2\cos(z) + 1\right) + 4iz\mathrm{Li}_2\left(e^{(iz)}\right) - 4\,\mathrm{Li}_3(e^{(iz)})$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(z^2*(1+cos(z))^(1/2)/(1-cos(z))^(1/2),z, algorithm="maxima")
```

```
[Out] 1/3*I*z^3 + 2*I*z^2*arctan2(sin(z), -cos(z) + 1) - z^2*log(cos(z)^2 + sin(z
)^2 - 2*cos(z) + 1) + 4*I*z*dilog(e^(I*z)) - 4*polylog(3, e^(I*z))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{z^2\,\sqrt{\cos(z)+1}}{\sqrt{1-\cos(z)}}\,dz$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((z^2*(cos(z) + 1)^(1/2))/(1 - cos(z))^(1/2),z)
```

```
[Out] int((z^2*(cos(z) + 1)^(1/2))/(1 - cos(z))^(1/2), z)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{z^2\sqrt{\cos(z)+1}}{\sqrt{1-\cos(z)}}\,dz$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(z**2*(1+cos(z))**(1/2)/(1-cos(z))**(1/2),z)
```

```
[Out] Integral(z**2*sqrt(cos(z) + 1)/sqrt(1 - cos(z)), z)
```

3.185 $\int (a + a\cos(x))(A + B\sec(x))\,dx$

Optimal. Leaf size=18

$$ax(A+B) + aA\sin(x) + aB\tanh^{-1}(\sin(x))$$

[Out] a*(A+B)*x+a*B*arctanh(sin(x))+a*A*sin(x)

Rubi [A] time = 0.10, antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {2828, 2968, 3023, 2735, 3770}

$$ax(A+B) + aA\sin(x) + aB\tanh^{-1}(\sin(x))$$

Antiderivative was successfully verified.

[In] Int[(a + a*Cos[x])*(A + B*Sec[x]),x]

[Out] a*(A + B)*x + a*B*ArcTanh[Sin[x]] + a*A*Sin[x]

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2828

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.)*((a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)])^(m_.), x_Symbol] :> Int[((a + b*Sin[e + f*x])^m*(d + c*Sin[e +
 f*x])^n)/Sin[e + f*x]^n, x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IntegerQ
[n]
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
```

```
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int (a + a\cos(x))(A + B\sec(x))\,dx &= \int (a + a\cos(x))(B + A\cos(x))\sec(x)\,dx\\ &= \int \left(aB + (aA + aB)\cos(x) + aA\cos^2(x)\right)\sec(x)\,dx\\ &= aA\sin(x) + \int (aB + a(A + B)\cos(x))\sec(x)\,dx\\ &= a(A + B)x + aA\sin(x) + (aB)\int \sec(x)\,dx\\ &= a(A + B)x + aB\tanh^{-1}(\sin(x)) + aA\sin(x)\end{aligned}$$

Mathematica [B] time = 0.01, size = 51, normalized size = 2.83

$$aAx + aA\sin(x) + aBx - aB\log\left(\cos\left(\frac{x}{2}\right) - \sin\left(\frac{x}{2}\right)\right) + aB\log\left(\sin\left(\frac{x}{2}\right) + \cos\left(\frac{x}{2}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[x])*(A + B*Sec[x]),x]
```

```
[Out] a*A*x + a*B*x - a*B*Log[Cos[x/2] - Sin[x/2]] + a*B*Log[Cos[x/2] + Sin[x/2]]
 + a*A*Sin[x]
```

fricas [A] time = 0.67, size = 32, normalized size = 1.78

$$(A + B)ax + \frac{1}{2}Ba\log(\sin(x) + 1) - \frac{1}{2}Ba\log(-\sin(x) + 1) + Aa\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(x))*(A+B*sec(x)),x, algorithm="fricas")
```

```
[Out] (A + B)*a*x + 1/2*B*a*log(sin(x) + 1) - 1/2*B*a*log(-sin(x) + 1) + A*a*sin(
x)
```

giac [B] time = 0.18, size = 51, normalized size = 2.83

$$Ba\log\left(\left|\tan\left(\frac{1}{2}x\right)+1\right|\right)-Ba\log\left(\left|\tan\left(\frac{1}{2}x\right)-1\right|\right)+(Aa+Ba)x+\frac{2\,Aa\tan\left(\frac{1}{2}x\right)}{\tan\left(\frac{1}{2}x\right)^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(x))*(A+B*sec(x)),x, algorithm="giac")
```

```
[Out] B*a*log(abs(tan(1/2*x) + 1)) - B*a*log(abs(tan(1/2*x) - 1)) + (A*a + B*a)*x
 + 2*A*a*tan(1/2*x)/(tan(1/2*x)^2 + 1)
```

maple [A] time = 0.10, size = 24, normalized size = 1.33

$$aA\sin(x)+Bax+aAx+Ba\ln\left(\sec(x)+\tan(x)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(x))*(A+B*sec(x)),x)
```

```
[Out] a*A*sin(x)+B*a*x+a*A*x+B*a*ln(sec(x)+tan(x))
```

maxima [A] time = 0.31, size = 23, normalized size = 1.28

$$Aax+Bax+Ba\log\left(\sec(x)+\tan(x)\right)+Aa\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(x))*(A+B*sec(x)),x, algorithm="maxima")
```

```
[Out] A*a*x + B*a*x + B*a*log(sec(x) + tan(x)) + A*a*sin(x)
```

mupad [B] time = 2.48, size = 54, normalized size = 3.00

$$2\,A\,a\,\mathrm{atan}\left(\frac{\sin\left(\frac{x}{2}\right)}{\cos\left(\frac{x}{2}\right)}\right)+2\,B\,a\,\mathrm{atan}\left(\frac{\sin\left(\frac{x}{2}\right)}{\cos\left(\frac{x}{2}\right)}\right)+2\,B\,a\,\mathrm{atanh}\left(\frac{\sin\left(\frac{x}{2}\right)}{\cos\left(\frac{x}{2}\right)}\right)+A\,a\,\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*cos(x))*(A + B/cos(x)),x)
```

```
[Out] 2*A*a*atan(sin(x/2)/cos(x/2)) + 2*B*a*atan(sin(x/2)/cos(x/2)) + 2*B*a*atanh
(sin(x/2)/cos(x/2)) + A*a*sin(x)
```

sympy [A] time = 2.17, size = 27, normalized size = 1.50

$$Aax+Aa\sin\left(x\right)+Bax+Ba\log\left(\tan\left(x\right)+\sec\left(x\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(x))*(A+B*sec(x)),x)
```

```
[Out] A*a*x + A*a*sin(x) + B*a*x + B*a*log(tan(x) + sec(x))
```

3.186 $\int (a+a\cos(x))^2(A+B\sec(x))\,dx$

Optimal. Leaf size=57

$$\frac{1}{2}a^2x(3A+4B)+\frac{1}{2}a^2(3A+2B)\sin(x)+\frac{1}{2}A\sin(x)\left(a^2\cos(x)+a^2\right)+a^2B\tanh^{-1}(\sin(x))$$

```
[Out] 1/2*a^2*(3*A+4*B)*x+a^2*B*arctanh(sin(x))+1/2*a^2*(3*A+2*B)*sin(x)+1/2*A*(a
^2+a^2*cos(x))*sin(x)
```

Rubi [A] time = 0.20, antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {2828, 2976, 2968, 3023, 2735, 3770}

$$\frac{1}{2}a^2x(3A+4B)+\frac{1}{2}a^2(3A+2B)\sin(x)+\frac{1}{2}A\sin(x)\left(a^2\cos(x)+a^2\right)+a^2B\tanh^{-1}(\sin(x))$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[x])^2*(A + B*Sec[x]),x]
```

```
[Out] (a^2*(3*A + 4*B)*x)/2 + a^2*B*ArcTanh[Sin[x]] + (a^2*(3*A + 2*B)*Sin[x])/2
+ (A*(a^2 + a^2*Cos[x])*Sin[x])/2
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2828

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.)*((a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)])^(m_.), x_Symbol] :> Int[((a + b*Sin[e + f*x])^m*(d + c*Sin[e +
 f*x])^n)/Sin[e + f*x]^n, x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IntegerQ
[n]
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 2976

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n +
 1))/(d*f*(m + n + 1)), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Sin[e + f*x
])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 1) + B*(a*c*(m - 1) +
 b*d*(n + 1)) + (A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e + f*x
], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0]
 && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1] &
& IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int (a+a\cos(x))^2(A+B\sec(x))\,dx &= \int (a+a\cos(x))^2(B+A\cos(x))\sec(x)\,dx\\
&= \frac{1}{2}A\left(a^2+a^2\cos(x)\right)\sin(x)+\frac{1}{2}\int (a+a\cos(x))(2aB+a(3A+2B)\cos(x)\\
&= \frac{1}{2}A\left(a^2+a^2\cos(x)\right)\sin(x)+\frac{1}{2}\int \left(2a^2B+\left(2a^2B+a^2(3A+2B)\right)\cos(x)\right.\\
&= \frac{1}{2}a^2(3A+2B)\sin(x)+\frac{1}{2}A\left(a^2+a^2\cos(x)\right)\sin(x)+\frac{1}{2}\int \left(2a^2B+a^2(3A\right.\\
&= \frac{1}{2}a^2(3A+4B)x+\frac{1}{2}a^2(3A+2B)\sin(x)+\frac{1}{2}A\left(a^2+a^2\cos(x)\right)\sin(x)+\left(a\right.\\
&= \frac{1}{2}a^2(3A+4B)x+a^2B\tanh^{-1}(\sin(x))+\frac{1}{2}a^2(3A+2B)\sin(x)+\frac{1}{2}A\left(a^2+\right.\end{aligned}$$

Mathematica [A] time = 0.08, size = 67, normalized size = 1.18

$$\frac{1}{4}a^2\left(4(2A+B)\sin(x)+6Ax+A\sin(2x)+8Bx-4B\log\left(\cos\left(\frac{x}{2}\right)-\sin\left(\frac{x}{2}\right)\right)+4B\log\left(\sin\left(\frac{x}{2}\right)+\cos\left(\frac{x}{2}\right)\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[x])^2*(A + B*Sec[x]),x]
```

```
[Out] (a^2*(6*A*x + 8*B*x - 4*B*Log[Cos[x/2] - Sin[x/2]] + 4*B*Log[Cos[x/2] + Sin
[x/2]] + 4*(2*A + B)*Sin[x] + A*Sin[2*x]))/4
```

fricas [A] time = 0.77, size = 60, normalized size = 1.05

$$\frac{1}{2}(3A+4B)a^2x+\frac{1}{2}Ba^2\log(\sin(x)+1)-\frac{1}{2}Ba^2\log(-\sin(x)+1)+\frac{1}{2}\left(Aa^2\cos(x)+2(2A+B)a^2\right)\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(x))^2*(A+B*sec(x)),x, algorithm="fricas")
```

```
[Out] 1/2*(3*A + 4*B)*a^2*x + 1/2*B*a^2*log(sin(x) + 1) - 1/2*B*a^2*log(-sin(x) +
 1) + 1/2*(A*a^2*cos(x) + 2*(2*A + B)*a^2)*sin(x)
```

giac [A] time = 0.16, size = 100, normalized size = 1.75

$$Ba^2\log\left(\left|\tan\left(\frac{1}{2}x\right)+1\right|\right)-Ba^2\log\left(\left|\tan\left(\frac{1}{2}x\right)-1\right|\right)+\frac{1}{2}\left(3Aa^2+4Ba^2\right)x+\frac{3Aa^2\tan\left(\frac{1}{2}x\right)^3+2Ba^2\tan\left(\frac{1}{2}x\right)^3+5}{\left(\tan\left(\frac{1}{2}x\right)^2\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(x))^2*(A+B*sec(x)),x, algorithm="giac")
```

```
[Out] B*a^2*log(abs(tan(1/2*x) + 1)) - B*a^2*log(abs(tan(1/2*x) - 1)) + 1/2*(3*A*
a^2 + 4*B*a^2)*x + (3*A*a^2*tan(1/2*x)^3 + 2*B*a^2*tan(1/2*x)^3 + 5*A*a^2*t
an(1/2*x) + 2*B*a^2*tan(1/2*x))/(tan(1/2*x)^2 + 1)^2
```

maple [A] time = 0.10, size = 52, normalized size = 0.91

$$\frac{a^2A\sin(x)\cos(x)}{2}+\frac{3a^2Ax}{2}+a^2B\sin(x)+2a^2A\sin(x)+2a^2Bx+a^2B\ln(\sec(x)+\tan(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(x))^2*(A+B*sec(x)),x)
```

```
[Out] 1/2*a^2*A*sin(x)*cos(x)+3/2*a^2*A*x+a^2*B*sin(x)+2*a^2*A*sin(x)+2*a^2*B*x+a
^2*B*ln(sec(x)+tan(x))
```

maxima [A] time = 0.32, size = 54, normalized size = 0.95

$$\frac{1}{4}Aa^2(2x+\sin(2x))+Aa^2x+2Ba^2x+Ba^2\log(\sec(x)+\tan(x))+2Aa^2\sin(x)+Ba^2\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*cos(x))^2*(A+B*sec(x)),x, algorithm="maxima")

[Out] 1/4*A*a^2*(2*x + sin(2*x)) + A*a^2*x + 2*B*a^2*x + B*a^2*log(sec(x) + tan(x
)) + 2*A*a^2*sin(x) + B*a^2*sin(x)

mupad [B] time = 2.46, size = 403, normalized size = 7.07

$$\frac{\left(3\,A\,a^2+2\,B\,a^2\right)\,\tan\left(\frac{x}{2}\right)^3+\left(5\,A\,a^2+2\,B\,a^2\right)\,\tan\left(\frac{x}{2}\right)}{\tan\left(\frac{x}{2}\right)^4+2\tan\left(\frac{x}{2}\right)^2+1}+a^2\,\mathrm{atan}\left(\frac{216\,A^3\,a^6\,\tan\left(\frac{x}{2}\right)}{216\,A^3\,a^6+864\,A^2\,B\,a^6+1248\,A\,B^2\,a^6+640\,$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + a*cos(x))^2*(A + B/cos(x)),x)

[Out] (tan(x/2)^3*(3*A*a^2 + 2*B*a^2) + tan(x/2)*(5*A*a^2 + 2*B*a^2))/(2*tan(x/2)
^2 + tan(x/2)^4 + 1) + a^2*atan((216*A^3*a^6*tan(x/2))/(216*A^3*a^6 + 640*B
^3*a^6 + 1248*A*B^2*a^6 + 864*A^2*B*a^6) + (640*B^3*a^6*tan(x/2))/(216*A^3*
a^6 + 640*B^3*a^6 + 1248*A*B^2*a^6 + 864*A^2*B*a^6) + (1248*A*B^2*a^6*tan(x
/2))/(216*A^3*a^6 + 640*B^3*a^6 + 1248*A*B^2*a^6 + 864*A^2*B*a^6) + (864*A^
2*B*a^6*tan(x/2))/(216*A^3*a^6 + 640*B^3*a^6 + 1248*A*B^2*a^6 + 864*A^2*B*a
^6))*(3*A + 4*B) + 2*B*a^2*atanh((320*B^3*a^6*tan(x/2))/(320*B^3*a^6 + 384*
A*B^2*a^6 + 144*A^2*B*a^6) + (384*A*B^2*a^6*tan(x/2))/(320*B^3*a^6 + 384*A*
B^2*a^6 + 144*A^2*B*a^6) + (144*A^2*B*a^6*tan(x/2))/(320*B^3*a^6 + 384*A*B^
2*a^6 + 144*A^2*B*a^6))

sympy [A] time = 3.54, size = 61, normalized size = 1.07

$$\frac{3Aa^2x}{2}+2Aa^2\sin(x)+\frac{Aa^2\sin(2x)}{4}+2Ba^2x+Ba^2\log(\tan(x)+\sec(x))+Ba^2\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*cos(x))**2*(A+B*sec(x)),x)

[Out] 3*A*a**2*x/2 + 2*A*a**2*sin(x) + A*a**2*sin(2*x)/4 + 2*B*a**2*x + B*a**2*lo
g(tan(x) + sec(x)) + B*a**2*sin(x)

3.187 $\int (a+a\cos(x))^3(A+B\sec(x))\,dx$

Optimal. Leaf size=75

$$\frac{1}{2}a^3x(5A+7B)+\frac{5}{2}a^3(A+B)\sin(x)+\frac{1}{6}(5A+3B)\sin(x)\left(a^3\cos(x)+a^3\right)+a^3B\tanh^{-1}(\sin(x))+\frac{1}{3}aA\sin(x)(a\cos(x)+$$

```
[Out] 1/2*a^3*(5*A+7*B)*x+a^3*B*arctanh(sin(x))+5/2*a^3*(A+B)*sin(x)+1/3*a*A*(a+a
*cos(x))^2*sin(x)+1/6*(5*A+3*B)*(a^3+a^3*cos(x))*sin(x)
```

Rubi [A] time = 0.30, antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {2828, 2976, 2968, 3023, 2735, 3770}

$$\frac{1}{2}a^3x(5A+7B)+\frac{5}{2}a^3(A+B)\sin(x)+\frac{1}{6}(5A+3B)\sin(x)\left(a^3\cos(x)+a^3\right)+a^3B\tanh^{-1}(\sin(x))+\frac{1}{3}aA\sin(x)(a\cos(x)+$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[x])^3*(A + B*Sec[x]),x]
```

```
[Out] (a^3*(5*A + 7*B)*x)/2 + a^3*B*ArcTanh[Sin[x]] + (5*a^3*(A + B)*Sin[x])/2 +
(a*A*(a + a*Cos[x])^2*Sin[x])/3 + ((5*A + 3*B)*(a^3 + a^3*Cos[x])*Sin[x])/6
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2828

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.)*((a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)])^(m_.), x_Symbol] :> Int[((a + b*Sin[e + f*x])^m*(d + c*Sin[e +
 f*x])^n)/Sin[e + f*x]^n, x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IntegerQ
[n]
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 2976

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n +
 1))/(d*f*(m + n + 1)), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Sin[e + f*x
])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 1) + B*(a*c*(m - 1) +
 b*d*(n + 1)) + (A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e + f*x
], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0]
 && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1] &
& IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (a+a\cos(x))^3(A+B\sec(x))\,dx = \int (a+a\cos(x))^3(B+A\cos(x))\sec(x)\,dx$$
$$= \frac{1}{3}aA(a+a\cos(x))^2\sin(x)+\frac{1}{3}\int (a+a\cos(x))^2(3aB+a(5A+3B)\cos(x$$
$$= \frac{1}{3}aA(a+a\cos(x))^2\sin(x)+\frac{1}{6}(5A+3B)\left(a^3+a^3\cos(x)\right)\sin(x)+\frac{1}{6}\int (a$$
$$= \frac{1}{3}aA(a+a\cos(x))^2\sin(x)+\frac{1}{6}(5A+3B)\left(a^3+a^3\cos(x)\right)\sin(x)+\frac{1}{6}\int \big(6$$
$$= \frac{5}{2}a^3(A+B)\sin(x)+\frac{1}{3}aA(a+a\cos(x))^2\sin(x)+\frac{1}{6}(5A+3B)\big(a^3+a^3\cos$$
$$= \frac{1}{2}a^3(5A+7B)x+\frac{5}{2}a^3(A+B)\sin(x)+\frac{1}{3}aA(a+a\cos(x))^2\sin(x)+\frac{1}{6}(5A$$
$$= \frac{1}{2}a^3(5A+7B)x+a^3B\tanh^{-1}(\sin(x))+\frac{5}{2}a^3(A+B)\sin(x)+\frac{1}{3}aA(a+a\cos$$

Mathematica [A] time = 0.10, size = 80, normalized size = 1.07

$$\frac{1}{12}a^3\left(9(5A+4B)\sin(x)+3(3A+B)\sin(2x)+30Ax+A\sin(3x)+42Bx-12B\log\left(\cos\left(\frac{x}{2}\right)-\sin\left(\frac{x}{2}\right)\right)+12B\ \mathrm{l}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[x])^3*(A + B*Sec[x]),x]
```

```
[Out] (a^3*(30*A*x + 42*B*x - 12*B*Log[Cos[x/2] - Sin[x/2]] + 12*B*Log[Cos[x/2] +
 Sin[x/2]] + 9*(5*A + 4*B)*Sin[x] + 3*(3*A + B)*Sin[2*x] + A*Sin[3*x]))/12
```

fricas [A] time = 1.19, size = 77, normalized size = 1.03

$$\frac{1}{2}(5A+7B)a^3x+\frac{1}{2}Ba^3\log(\sin(x)+1)-\frac{1}{2}Ba^3\log(-\sin(x)+1)+\frac{1}{6}\left(2Aa^3\cos(x)^2+3(3A+B)a^3\cos(x)+2(1\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(x))^3*(A+B*sec(x)),x, algorithm="fricas")
```

```
[Out] 1/2*(5*A + 7*B)*a^3*x + 1/2*B*a^3*log(sin(x) + 1) - 1/2*B*a^3*log(-sin(x) +
 1) + 1/6*(2*A*a^3*cos(x)^2 + 3*(3*A + B)*a^3*cos(x) + 2*(11*A + 9*B)*a^3)*
sin(x)
```

giac [A] time = 0.18, size = 125, normalized size = 1.67

$$Ba^3\log\left(\left|\tan\left(\frac{1}{2}x\right)+1\right|\right)-Ba^3\log\left(\left|\tan\left(\frac{1}{2}x\right)-1\right|\right)+\frac{1}{2}\left(5Aa^3+7Ba^3\right)x+\frac{15Aa^3\tan\left(\frac{1}{2}x\right)^5+15Ba^3\tan\left(\frac{1}{2}x\right)^5-\cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(x))^3*(A+B*sec(x)),x, algorithm="giac")
```

```
[Out] B*a^3*log(abs(tan(1/2*x) + 1)) - B*a^3*log(abs(tan(1/2*x) - 1)) + 1/2*(5*A*
a^3 + 7*B*a^3)*x + 1/3*(15*A*a^3*tan(1/2*x)^5 + 15*B*a^3*tan(1/2*x)^5 + 40*
A*a^3*tan(1/2*x)^3 + 36*B*a^3*tan(1/2*x)^3 + 33*A*a^3*tan(1/2*x) + 21*B*a^3
*tan(1/2*x))/(tan(1/2*x)^2 + 1)^3
```

maple [A] time = 0.11, size = 77, normalized size = 1.03

$$\frac{Aa^3\left(2+\cos^2(x)\right)\sin(x)}{3}+\frac{Ba^3\sin(x)\cos(x)}{2}+\frac{7Ba^3x}{2}+\frac{3Aa^3\sin(x)\cos(x)}{2}+\frac{5Aa^3x}{2}+3Ba^3\sin(x)+3Aa^3\sin(x)+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(x))^3*(A+B*sec(x)),x)
```

```
[Out] 1/3*A*a^3*(2+cos(x)^2)*sin(x)+1/2*B*a^3*sin(x)*cos(x)+7/2*B*a^3*x+3/2*A*a^3
*sin(x)*cos(x)+5/2*A*a^3*x+3*B*a^3*sin(x)+3*A*a^3*sin(x)+B*a^3*ln(sec(x)+ta
n(x))
```

maxima [A] time = 0.31, size = 84, normalized size = 1.12

$$-\frac{1}{3}\left(\sin(x)^3 - 3\sin(x)\right)Aa^3+\frac{3}{4}Aa^3(2x+\sin(2x))+\frac{1}{4}Ba^3(2x+\sin(2x))+Aa^3x+3Ba^3x+Ba^3\log(\sec(x)+\tan$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(x))^3*(A+B*sec(x)),x, algorithm="maxima")
```

```
[Out] -1/3*(sin(x)^3 - 3*sin(x))*A*a^3 + 3/4*A*a^3*(2*x + sin(2*x)) + 1/4*B*a^3*(
2*x + sin(2*x)) + A*a^3*x + 3*B*a^3*x + B*a^3*log(sec(x) + tan(x)) + 3*A*a^
3*sin(x) + 3*B*a^3*sin(x)
```

mupad [B] time = 2.48, size = 431, normalized size = 5.75

$$\frac{\left(5\,A\,a^3+5\,B\,a^3\right)\,\tan\left(\frac{x}{2}\right)^5+\left(\frac{40\,A\,a^3}{3}+12\,B\,a^3\right)\,\tan\left(\frac{x}{2}\right)^3+\left(11\,A\,a^3+7\,B\,a^3\right)\,\tan\left(\frac{x}{2}\right)}{\tan\left(\frac{x}{2}\right)^6+3\,\tan\left(\frac{x}{2}\right)^4+3\,\tan\left(\frac{x}{2}\right)^2+1}+a^3\,\mathrm{atan}\left(\frac{}{1000\,A^3\,a^9+42}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*cos(x))^3*(A + B/cos(x)),x)
```

```
[Out] (tan(x/2)^5*(5*A*a^3 + 5*B*a^3) + tan(x/2)^3*((40*A*a^3)/3 + 12*B*a^3) + ta
n(x/2)*(11*A*a^3 + 7*B*a^3))/(3*tan(x/2)^2 + 3*tan(x/2)^4 + tan(x/2)^6 + 1)
 + a^3*atan((1000*A^3*a^9*tan(x/2))/(1000*A^3*a^9 + 2968*B^3*a^9 + 6040*A*B
^2*a^9 + 4200*A^2*B*a^9) + (2968*B^3*a^9*tan(x/2))/(1000*A^3*a^9 + 2968*B^3
*a^9 + 6040*A*B^2*a^9 + 4200*A^2*B*a^9) + (6040*A*B^2*a^9*tan(x/2))/(1000*A
^3*a^9 + 2968*B^3*a^9 + 6040*A*B^2*a^9 + 4200*A^2*B*a^9) + (4200*A^2*B*a^9*
tan(x/2))/(1000*A^3*a^9 + 2968*B^3*a^9 + 6040*A*B^2*a^9 + 4200*A^2*B*a^9))*
(5*A + 7*B) + 2*B*a^3*atanh((848*B^3*a^9*tan(x/2))/(848*B^3*a^9 + 1120*A*B^
2*a^9 + 400*A^2*B*a^9) + (1120*A*B^2*a^9*tan(x/2))/(848*B^3*a^9 + 1120*A*B^
2*a^9 + 400*A^2*B*a^9) + (400*A^2*B*a^9*tan(x/2))/(848*B^3*a^9 + 1120*A*B^2
*a^9 + 400*A^2*B*a^9))
```

sympy [A] time = 7.63, size = 92, normalized size = 1.23

$$\frac{5Aa^3x}{2}-\frac{Aa^3\sin^3(x)}{3}+4Aa^3\sin(x)+\frac{3Aa^3\sin(2x)}{4}+\frac{7Ba^3x}{2}+Ba^3\log(\tan(x)+\sec(x))+\frac{Ba^3\sin(x)\cos(x)}{2}+3B$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(x))**3*(A+B*sec(x)),x)
```

```
[Out] 5*A*a**3*x/2 - A*a**3*sin(x)**3/3 + 4*A*a**3*sin(x) + 3*A*a**3*sin(2*x)/4 +
 7*B*a**3*x/2 + B*a**3*log(tan(x) + sec(x)) + B*a**3*sin(x)*cos(x)/2 + 3*B*
a**3*sin(x)
```

3.188 $\int (a+a\cos(x))^4(A+B\sec(x))\,dx$

Optimal. Leaf size=104

$$\frac{1}{8}a^4x(35A+48B)+\frac{5}{8}a^4(7A+8B)\sin(x)+\frac{1}{24}(35A+32B)\sin(x)\left(a^4\cos(x)+a^4\right)+a^4B\tanh^{-1}(\sin(x))+\frac{1}{12}(7A+4B$$

```
[Out] 1/8*a^4*(35*A+48*B)*x+a^4*B*arctanh(sin(x))+5/8*a^4*(7*A+8*B)*sin(x)+1/4*a*
A*(a+a*cos(x))^3*sin(x)+1/12*(7*A+4*B)*(a^2+a^2*cos(x))^2*sin(x)+1/24*(35*A
+32*B)*(a^4+a^4*cos(x))*sin(x)
```

Rubi [A] time = 0.40, antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {2828, 2976, 2968, 3023, 2735, 3770}

$$\frac{1}{8}a^4x(35A+48B)+\frac{5}{8}a^4(7A+8B)\sin(x)+\frac{1}{12}(7A+4B)\sin(x)\left(a^2\cos(x)+a^2\right)^2+\frac{1}{24}(35A+32B)\sin(x)\left(a^4\cos(x)+\right.$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[x])^4*(A + B*Sec[x]),x]
```

```
[Out] (a^4*(35*A + 48*B)*x)/8 + a^4*B*ArcTanh[Sin[x]] + (5*a^4*(7*A + 8*B)*Sin[x]
)/8 + (a*A*(a + a*Cos[x])^3*Sin[x])/4 + ((7*A + 4*B)*(a^2 + a^2*Cos[x])^2*S
in[x])/12 + ((35*A + 32*B)*(a^4 + a^4*Cos[x])*Sin[x])/24
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2828

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.)*((a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)])^(m_.), x_Symbol] :> Int[((a + b*Sin[e + f*x])^m*(d + c*Sin[e +
 f*x])^n)/Sin[e + f*x]^n, x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IntegerQ
[n]
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 2976

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n +
 1))/(d*f*(m + n + 1)), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Sin[e + f*x
])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 1) + B*(a*c*(m - 1) +
 b*d*(n + 1)) + (A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e + f*x
], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0]
 && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1] &
& IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (a+a\cos(x))^4(A+B\sec(x))\,dx = \int (a+a\cos(x))^4(B+A\cos(x))\sec(x)\,dx$$
$$= \frac{1}{4}aA(a+a\cos(x))^3\sin(x)+\frac{1}{4}\int (a+a\cos(x))^3(4aB+a(7A+4B)\cos(x$$
$$= \frac{1}{4}aA(a+a\cos(x))^3\sin(x)+\frac{1}{12}(7A+4B)\left(a^2+a^2\cos(x)\right)^2\sin(x)+\frac{1}{12}\int$$
$$= \frac{1}{4}aA(a+a\cos(x))^3\sin(x)+\frac{1}{12}(7A+4B)\left(a^2+a^2\cos(x)\right)^2\sin(x)+\frac{1}{24}(3$$
$$= \frac{1}{4}aA(a+a\cos(x))^3\sin(x)+\frac{1}{12}(7A+4B)\left(a^2+a^2\cos(x)\right)^2\sin(x)+\frac{1}{24}(3$$
$$= \frac{5}{8}a^4(7A+8B)\sin(x)+\frac{1}{4}aA(a+a\cos(x))^3\sin(x)+\frac{1}{12}(7A+4B)\left(a^2+a^2\right.$$
$$= \frac{1}{8}a^4(35A+48B)x+\frac{5}{8}a^4(7A+8B)\sin(x)+\frac{1}{4}aA(a+a\cos(x))^3\sin(x)+$$
$$= \frac{1}{8}a^4(35A+48B)x+a^4B\tanh^{-1}(\sin(x))+\frac{5}{8}a^4(7A+8B)\sin(x)+\frac{1}{4}aA(a$$

Mathematica [A] time = 0.12, size = 97, normalized size = 0.93

$$\frac{1}{96}a^4\left(24(28A+27B)\sin(x)+24(7A+4B)\sin(2x)+420Ax+32A\sin(3x)+3A\sin(4x)+576Bx+8B\sin(3x\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[x])^4*(A + B*Sec[x]),x]
```

```
[Out] (a^4*(420*A*x + 576*B*x - 96*B*Log[Cos[x/2] - Sin[x/2]] + 96*B*Log[Cos[x/2]
 + Sin[x/2]] + 24*(28*A + 27*B)*Sin[x] + 24*(7*A + 4*B)*Sin[2*x] + 32*A*Sin
[3*x] + 8*B*Sin[3*x] + 3*A*Sin[4*x]))/96
```

fricas [A] time = 0.70, size = 89, normalized size = 0.86

$$\frac{1}{8}(35A+48B)a^4x+\frac{1}{2}Ba^4\log(\sin(x)+1)-\frac{1}{2}Ba^4\log(-\sin(x)+1)+\frac{1}{24}\left(6Aa^4\cos(x)^3+8(4A+B)a^4\cos(x)^2\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(x))^4*(A+B*sec(x)),x, algorithm="fricas")
```

```
[Out] 1/8*(35*A + 48*B)*a^4*x + 1/2*B*a^4*log(sin(x) + 1) - 1/2*B*a^4*log(-sin(x)
 + 1) + 1/24*(6*A*a^4*cos(x)^3 + 8*(4*A + B)*a^4*cos(x)^2 + 3*(27*A + 16*B)
*a^4*cos(x) + 160*(A + B)*a^4)*sin(x)
```

giac [A] time = 0.16, size = 149, normalized size = 1.43

$$Ba^4\log\left(\left|\tan\left(\frac{1}{2}x\right)+1\right|\right)-Ba^4\log\left(\left|\tan\left(\frac{1}{2}x\right)-1\right|\right)+\frac{1}{8}\left(35\,Aa^4+48\,Ba^4\right)x+\frac{105\,Aa^4\tan\left(\frac{1}{2}x\right)^7+120\,Ba^4\tan\left(\frac{1}{2}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(x))^4*(A+B*sec(x)),x, algorithm="giac")
```

```
[Out] B*a^4*log(abs(tan(1/2*x) + 1)) - B*a^4*log(abs(tan(1/2*x) - 1)) + 1/8*(35*A
*a^4 + 48*B*a^4)*x + 1/12*(105*A*a^4*tan(1/2*x)^7 + 120*B*a^4*tan(1/2*x)^7
+ 385*A*a^4*tan(1/2*x)^5 + 424*B*a^4*tan(1/2*x)^5 + 511*A*a^4*tan(1/2*x)^3
+ 520*B*a^4*tan(1/2*x)^3 + 279*A*a^4*tan(1/2*x) + 216*B*a^4*tan(1/2*x))/(ta
n(1/2*x)^2 + 1)^4
```

maple [A] time = 0.12, size = 103, normalized size = 0.99

$$\frac{A\,a^4\sin(x)\left(\cos^3(x)\right)}{4}+\frac{27A\,a^4\sin(x)\cos(x)}{8}+\frac{35A\,a^4x}{8}+\frac{B\,a^4\left(2+\cos^2(x)\right)\sin(x)}{3}+\frac{4A\,a^4\left(2+\cos^2(x)\right)\sin(x)}{3}+2I$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(x))^4*(A+B*sec(x)),x)
```

```
[Out] 1/4*A*a^4*sin(x)*cos(x)^3+27/8*A*a^4*sin(x)*cos(x)+35/8*A*a^4*x+1/3*B*a^4*(
2+cos(x)^2)*sin(x)+4/3*A*a^4*(2+cos(x)^2)*sin(x)+2*B*a^4*sin(x)*cos(x)+6*B*
a^4*x+6*B*a^4*sin(x)+4*A*a^4*sin(x)+B*a^4*ln(sec(x)+tan(x))
```

maxima [A] time = 0.31, size = 118, normalized size = 1.13

$$-\frac{4}{3}\left(\sin(x)^3-3\,\sin(x)\right)Aa^4-\frac{1}{3}\left(\sin(x)^3-3\,\sin(x)\right)Ba^4+\frac{1}{32}\,Aa^4(12\,x+\sin(4\,x)+8\,\sin(2\,x))+\frac{3}{2}\,Aa^4(2\,x+\sin($$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(x))^4*(A+B*sec(x)),x, algorithm="maxima")
```

```
[Out] -4/3*(sin(x)^3 - 3*sin(x))*A*a^4 - 1/3*(sin(x)^3 - 3*sin(x))*B*a^4 + 1/32*A
*a^4*(12*x + sin(4*x) + 8*sin(2*x)) + 3/2*A*a^4*(2*x + sin(2*x)) + B*a^4*(2
*x + sin(2*x)) + A*a^4*x + 4*B*a^4*x + B*a^4*log(sec(x) + tan(x)) + 4*A*a^4
*sin(x) + 6*B*a^4*sin(x)
```

mupad [B] time = 2.52, size = 460, normalized size = 4.42

$$\frac{\left(\frac{35\,A\,a^4}{4}+10\,B\,a^4\right)\,\tan\left(\frac{x}{2}\right)^7+\left(\frac{385\,A\,a^4}{12}+\frac{106\,B\,a^4}{3}\right)\,\tan\left(\frac{x}{2}\right)^5+\left(\frac{511\,A\,a^4}{12}+\frac{130\,B\,a^4}{3}\right)\,\tan\left(\frac{x}{2}\right)^3+\left(\frac{93\,A\,a^4}{4}+18\,B\,a^4\right)\,\mathrm{ta}}{\tan\left(\frac{x}{2}\right)^8+4\,\tan\left(\frac{x}{2}\right)^6+6\,\tan\left(\frac{x}{2}\right)^4+4\,\tan\left(\frac{x}{2}\right)^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*cos(x))^4*(A + B/cos(x)),x)
```

```
[Out] (tan(x/2)^7*((35*A*a^4)/4 + 10*B*a^4) + tan(x/2)^5*((385*A*a^4)/12 + (106*B
*a^4)/3) + tan(x/2)^3*((511*A*a^4)/12 + (130*B*a^4)/3) + tan(x/2)*((93*A*a^
4)/4 + 18*B*a^4))/(4*tan(x/2)^2 + 6*tan(x/2)^4 + 4*tan(x/2)^6 + tan(x/2)^8
+ 1) + (a^4*atan((42875*A^3*a^12*tan(x/2))/(8*((42875*A^3*a^12)/8 + 14208*B
^3*a^12 + 30520*A*B^2*a^12 + 22050*A^2*B*a^12)) + (14208*B^3*a^12*tan(x/2))
/((42875*A^3*a^12)/8 + 14208*B^3*a^12 + 30520*A*B^2*a^12 + 22050*A^2*B*a^12
) + (30520*A*B^2*a^12*tan(x/2))/((42875*A^3*a^12)/8 + 14208*B^3*a^12 + 3052
0*A*B^2*a^12 + 22050*A^2*B*a^12) + (22050*A^2*B*a^12*tan(x/2))/((42875*A^3*
a^12)/8 + 14208*B^3*a^12 + 30520*A*B^2*a^12 + 22050*A^2*B*a^12))*(35*A + 48
*B))/4 + 2*B*a^4*atanh((2368*B^3*a^12*tan(x/2))/(2368*B^3*a^12 + 3360*A*B^2
*a^12 + 1225*A^2*B*a^12) + (3360*A*B^2*a^12*tan(x/2))/(2368*B^3*a^12 + 3360
*A*B^2*a^12 + 1225*A^2*B*a^12) + (1225*A^2*B*a^12*tan(x/2))/(2368*B^3*a^12
+ 3360*A*B^2*a^12 + 1225*A^2*B*a^12))
```

sympy [A] time = 15.26, size = 116, normalized size = 1.12

$$\frac{35Aa^4x}{8}-\frac{4Aa^4\sin^3(x)}{3}+8Aa^4\sin(x)+\frac{7Aa^4\sin(2x)}{4}+\frac{Aa^4\sin(4x)}{32}+6Ba^4x+Ba^4\log(\tan(x)+\sec(x))-\frac{Ba^4\sin}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(x))**4*(A+B*sec(x)),x)
```

```
[Out] 35*A*a**4*x/8 - 4*A*a**4*sin(x)**3/3 + 8*A*a**4*sin(x) + 7*A*a**4*sin(2*x)/
4 + A*a**4*sin(4*x)/32 + 6*B*a**4*x + B*a**4*log(tan(x) + sec(x)) - B*a**4*
sin(x)**3/3 + 2*B*a**4*sin(x)*cos(x) + 7*B*a**4*sin(x)
```

3.189 $\int \frac{A+B\sec(x)}{a+a\cos(x)}\,dx$

Optimal. Leaf size=25

$$\frac{(A-B)\sin(x)}{a\cos(x)+a}+\frac{B\tanh^{-1}(\sin(x))}{a}$$

```
[Out] B*arctanh(sin(x))/a+(A-B)*sin(x)/(a+a*cos(x))
```

Rubi [A] time = 0.09, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {2828, 2978, 12, 3770}

$$\frac{(A-B)\sin(x)}{a\cos(x)+a}+\frac{B\tanh^{-1}(\sin(x))}{a}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sec[x])/(a + a*Cos[x]),x]
```

```
[Out] (B*ArcTanh[Sin[x]])/a + ((A - B)*Sin[x])/(a + a*Cos[x])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2828

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.)*((a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)])^(m_.), x_Symbol] :> Int[((a + b*Sin[e + f*x])^m*(d + c*Sin[e +
 f*x])^n)/Sin[e + f*x]^n, x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IntegerQ
[n]
```

Rule 2978

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(
n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{A+B\sec(x)}{a+a\cos(x)}\,dx &= \int \frac{(B+A\cos(x))\sec(x)}{a+a\cos(x)}\,dx\\ &= \frac{(A-B)\sin(x)}{a+a\cos(x)}+\frac{\int aB\sec(x)\,dx}{a^2}\\ &= \frac{(A-B)\sin(x)}{a+a\cos(x)}+\frac{B\int \sec(x)\,dx}{a}\\ &= \frac{B\tanh^{-1}(\sin(x))}{a}+\frac{(A-B)\sin(x)}{a+a\cos(x)}\end{aligned}$$

Mathematica [B] time = 0.08, size = 71, normalized size = 2.84

$$-\frac{2\cos\left(\frac{x}{2}\right)\left((B-A)\sin\left(\frac{x}{2}\right)+B\cos\left(\frac{x}{2}\right)\left(\log\left(\cos\left(\frac{x}{2}\right)-\sin\left(\frac{x}{2}\right)\right)-\log\left(\sin\left(\frac{x}{2}\right)+\cos\left(\frac{x}{2}\right)\right)\right)\right)}{a(\cos(x)+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Sec[x])/(a + a*Cos[x]),x]
```

```
[Out] (-2*Cos[x/2]*(B*Cos[x/2]*(Log[Cos[x/2] - Sin[x/2]] - Log[Cos[x/2] + Sin[x/2
]]) + (-A + B)*Sin[x/2]))/(a*(1 + Cos[x]))
```

fricas [A] time = 1.14, size = 47, normalized size = 1.88

$$\frac{(B\cos(x)+B)\log(\sin(x)+1)-(B\cos(x)+B)\log(-\sin(x)+1)+2\,(A-B)\sin(x)}{2\,(a\cos(x)+a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(x))/(a+a*cos(x)),x, algorithm="fricas")
```

```
[Out] 1/2*((B*cos(x) + B)*log(sin(x) + 1) - (B*cos(x) + B)*log(-sin(x) + 1) + 2*(
A - B)*sin(x))/(a*cos(x) + a)
```

giac [A] time = 0.15, size = 46, normalized size = 1.84

$$\frac{B\log\left(\left|\tan\left(\frac{1}{2}\,x\right)+1\right|\right)}{a}-\frac{B\log\left(\left|\tan\left(\frac{1}{2}\,x\right)-1\right|\right)}{a}+\frac{A\tan\left(\frac{1}{2}\,x\right)-B\tan\left(\frac{1}{2}\,x\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(x))/(a+a*cos(x)),x, algorithm="giac")
```

```
[Out] B*log(abs(tan(1/2*x) + 1))/a - B*log(abs(tan(1/2*x) - 1))/a + (A*tan(1/2*x)
 - B*tan(1/2*x))/a
```

maple [A] time = 0.09, size = 46, normalized size = 1.84

$$\frac{A\tan\left(\frac{x}{2}\right)}{a}-\frac{B\tan\left(\frac{x}{2}\right)}{a}-\frac{B\ln\left(\tan\left(\frac{x}{2}\right)-1\right)}{a}+\frac{B\ln\left(1+\tan\left(\frac{x}{2}\right)\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sec(x))/(a+a*cos(x)),x)
```

```
[Out] 1/a*A*tan(1/2*x)-1/a*B*tan(1/2*x)-1/a*B*ln(tan(1/2*x)-1)+1/a*B*ln(1+tan(1/2
*x))
```

maxima [B] time = 0.42, size = 63, normalized size = 2.52

$$B\left(\frac{\log\left(\frac{\sin(x)}{\cos(x)+1}+1\right)}{a}-\frac{\log\left(\frac{\sin(x)}{\cos(x)+1}-1\right)}{a}-\frac{\sin(x)}{a(\cos(x)+1)}\right)+\frac{A\sin(x)}{a(\cos(x)+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(x))/(a+a*cos(x)),x, algorithm="maxima")
```

```
[Out] B*(log(sin(x)/(cos(x) + 1) + 1)/a - log(sin(x)/(cos(x) + 1) - 1)/a - sin(x)
/(a*(cos(x) + 1))) + A*sin(x)/(a*(cos(x) + 1))
```

mupad [B] time = 2.36, size = 25, normalized size = 1.00

$$\frac{2\,B\,\mathrm{atanh}\left(\tan\left(\frac{x}{2}\right)\right)}{a}+\frac{\tan\left(\frac{x}{2}\right)\,\left(A-B\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B/cos(x))/(a + a*cos(x)),x)
```

```
[Out] (2*B*atanh(tan(x/2)))/a + (tan(x/2)*(A - B))/a
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{A}{\cos(x)+1}\,dx+\int \frac{B\sec(x)}{\cos(x)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(x))/(a+a*cos(x)),x)
```

```
[Out] (Integral(A/(cos(x) + 1), x) + Integral(B*sec(x)/(cos(x) + 1), x))/a
```

3.190 $\int \frac{A+B\sec(x)}{(a+a\cos(x))^2}\,dx$

Optimal. Leaf size=48

$$\frac{(A-4B)\sin(x)}{3a^2(\cos(x)+1)}+\frac{B\tanh^{-1}(\sin(x))}{a^2}+\frac{(A-B)\sin(x)}{3(a\cos(x)+a)^2}$$

```
[Out] B*arctanh(sin(x))/a^2+1/3*(A-4*B)*sin(x)/a^2/(1+cos(x))+1/3*(A-B)*sin(x)/(a
+a*cos(x))^2
```

Rubi [A] time = 0.18, antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {2828, 2978, 12, 3770}

$$\frac{(A-4B)\sin(x)}{3a^2(\cos(x)+1)}+\frac{B\tanh^{-1}(\sin(x))}{a^2}+\frac{(A-B)\sin(x)}{3(a\cos(x)+a)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sec[x])/(a + a*Cos[x])^2,x]
```

```
[Out] (B*ArcTanh[Sin[x]])/a^2 + ((A - 4*B)*Sin[x])/(3*a^2*(1 + Cos[x])) + ((A - B
)*Sin[x])/(3*(a + a*Cos[x])^2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2828

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.)*((a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)])^(m_.), x_Symbol] :> Int[((a + b*Sin[e + f*x])^m*(d + c*Sin[e +
 f*x])^n)/Sin[e + f*x]^n, x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IntegerQ
[n]
```

Rule 2978

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(
n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
```

```
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{A+B\sec(x)}{(a+a\cos(x))^2}\,dx &= \int \frac{(B+A\cos(x))\sec(x)}{(a+a\cos(x))^2}\,dx \\
&= \frac{(A-B)\sin(x)}{3(a+a\cos(x))^2} + \frac{\int \frac{(3aB+a(A-B)\cos(x))\sec(x)}{a+a\cos(x)}\,dx}{3a^2} \\
&= \frac{(A-4B)\sin(x)}{3a^2(1+\cos(x))} + \frac{(A-B)\sin(x)}{3(a+a\cos(x))^2} + \frac{\int 3a^2B\sec(x)\,dx}{3a^4} \\
&= \frac{(A-4B)\sin(x)}{3a^2(1+\cos(x))} + \frac{(A-B)\sin(x)}{3(a+a\cos(x))^2} + \frac{B\int \sec(x)\,dx}{a^2} \\
&= \frac{B\tanh^{-1}(\sin(x))}{a^2} + \frac{(A-4B)\sin(x)}{3a^2(1+\cos(x))} + \frac{(A-B)\sin(x)}{3(a+a\cos(x))^2}
\end{aligned}
$$

Mathematica [A] time = 0.21, size = 76, normalized size = 1.58

$$
\frac{\sin(x)((A-4B)\cos(x)+2A-5B)-12B\cos^4\left(\frac{x}{2}\right)\left(\log\left(\cos\left(\frac{x}{2}\right)-\sin\left(\frac{x}{2}\right)\right)-\log\left(\sin\left(\frac{x}{2}\right)+\cos\left(\frac{x}{2}\right)\right)\right)}{3a^2(\cos(x)+1)^2}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Sec[x])/(a + a*Cos[x])^2,x]
```

```
[Out] (-12*B*Cos[x/2]^4*(Log[Cos[x/2] - Sin[x/2]] - Log[Cos[x/2] + Sin[x/2]]) + (
2*A - 5*B + (A - 4*B)*Cos[x])*Sin[x])/(3*a^2*(1 + Cos[x])^2)
```

fricas [A] time = 0.74, size = 85, normalized size = 1.77

$$
\frac{3\left(B\cos(x)^2+2B\cos(x)+B\right)\log\left(\sin(x)+1\right)-3\left(B\cos(x)^2+2B\cos(x)+B\right)\log\left(-\sin(x)+1\right)+2\left((A-4B\right.}{6\left(a^2\cos(x)^2+2a^2\cos(x)+a^2\right)}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(x))/(a+a*cos(x))^2,x, algorithm="fricas")
```

```
[Out] 1/6*(3*(B*cos(x)^2 + 2*B*cos(x) + B)*log(sin(x) + 1) - 3*(B*cos(x)^2 + 2*B*
cos(x) + B)*log(-sin(x) + 1) + 2*((A - 4*B)*cos(x) + 2*A - 5*B)*sin(x))/(a^
2*cos(x)^2 + 2*a^2*cos(x) + a^2)
```

giac [A] time = 0.16, size = 77, normalized size = 1.60

$$\frac{B\log\left(\left|\tan\left(\frac{1}{2}\,x\right)+1\right|\right)}{a^2}-\frac{B\log\left(\left|\tan\left(\frac{1}{2}\,x\right)-1\right|\right)}{a^2}+\frac{Aa^4\tan\left(\frac{1}{2}\,x\right)^3-Ba^4\tan\left(\frac{1}{2}\,x\right)^3+3\,Aa^4\tan\left(\frac{1}{2}\,x\right)-9\,Ba^4\tan\left(\frac{1}{2}\,x\right)}{6\,a^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(x))/(a+a*cos(x))^2,x, algorithm="giac")
```

```
[Out] B*log(abs(tan(1/2*x) + 1))/a^2 - B*log(abs(tan(1/2*x) - 1))/a^2 + 1/6*(A*a^
4*tan(1/2*x)^3 - B*a^4*tan(1/2*x)^3 + 3*A*a^4*tan(1/2*x) - 9*B*a^4*tan(1/2*
x))/a^6
```

maple [A] time = 0.09, size = 71, normalized size = 1.48

$$\frac{\left(\tan^3\left(\frac{x}{2}\right)\right)A}{6a^2}-\frac{\left(\tan^3\left(\frac{x}{2}\right)\right)B}{6a^2}+\frac{A\tan\left(\frac{x}{2}\right)}{2a^2}-\frac{3B\tan\left(\frac{x}{2}\right)}{2a^2}-\frac{B\ln\left(\tan\left(\frac{x}{2}\right)-1\right)}{a^2}+\frac{B\ln\left(1+\tan\left(\frac{x}{2}\right)\right)}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sec(x))/(a+a*cos(x))^2,x)
```

```
[Out] 1/6/a^2*tan(1/2*x)^3*A-1/6/a^2*tan(1/2*x)^3*B+1/2/a^2*A*tan(1/2*x)-3/2/a^2*
B*tan(1/2*x)-1/a^2*B*ln(tan(1/2*x)-1)+1/a^2*B*ln(1+tan(1/2*x))
```

maxima [B] time = 0.46, size = 93, normalized size = 1.94

$$-\frac{1}{6}\,B\left(\frac{\frac{9\,\sin(x)}{\cos(x)+1}+\frac{\sin(x)^3}{(\cos(x)+1)^3}}{a^2}-\frac{6\,\log\left(\frac{\sin(x)}{\cos(x)+1}+1\right)}{a^2}+\frac{6\,\log\left(\frac{\sin(x)}{\cos(x)+1}-1\right)}{a^2}\right)+\frac{A\left(\frac{3\,\sin(x)}{\cos(x)+1}+\frac{\sin(x)^3}{(\cos(x)+1)^3}\right)}{6\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(x))/(a+a*cos(x))^2,x, algorithm="maxima")
```

```
[Out] -1/6*B*((9*sin(x)/(cos(x) + 1) + sin(x)^3/(cos(x) + 1)^3)/a^2 - 6*log(sin(x
)/(cos(x) + 1) + 1)/a^2 + 6*log(sin(x)/(cos(x) + 1) - 1)/a^2) + 1/6*A*(3*si
n(x)/(cos(x) + 1) + sin(x)^3/(cos(x) + 1)^3)/a^2
```

mupad [B] time = 2.35, size = 50, normalized size = 1.04

$$\tan\left(\frac{x}{2}\right)\left(\frac{A-B}{2\,a^2}-\frac{B}{a^2}\right)+\frac{\tan\left(\frac{x}{2}\right)^3\,(A-B)}{6\,a^2}+\frac{2\,B\,\mathrm{atanh}\left(\tan\left(\frac{x}{2}\right)\right)}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B/cos(x))/(a + a*cos(x))^2,x)
```

```
[Out] tan(x/2)*((A - B)/(2*a^2) - B/a^2) + (tan(x/2)^3*(A - B))/(6*a^2) + (2*B*at
anh(tan(x/2)))/a^2
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{A}{\cos^2(x)+2\cos(x)+1}\,dx+\int \frac{B\sec(x)}{\cos^2(x)+2\cos(x)+1}\,dx}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(x))/(a+a*cos(x))**2,x)
```

```
[Out] (Integral(A/(cos(x)**2 + 2*cos(x) + 1), x) + Integral(B*sec(x)/(cos(x)**2 +
 2*cos(x) + 1), x))/a**2
```

3.191 $\int \frac{A+B\sec(x)}{(a+a\cos(x))^3}\,dx$

Optimal. Leaf size=75

$$\frac{2(A-11B)\sin(x)}{15\left(a^3\cos(x)+a^3\right)}+\frac{B\tanh^{-1}(\sin(x))}{a^3}+\frac{(2A-7B)\sin(x)}{15a(a\cos(x)+a)^2}+\frac{(A-B)\sin(x)}{5(a\cos(x)+a)^3}$$

```
[Out] B*arctanh(sin(x))/a^3+1/5*(A-B)*sin(x)/(a+a*cos(x))^3+1/15*(2*A-7*B)*sin(x)
/a/(a+a*cos(x))^2+2/15*(A-11*B)*sin(x)/(a^3+a^3*cos(x))
```

Rubi [A] time = 0.31, antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {2828, 2978, 12, 3770}

$$\frac{2(A-11B)\sin(x)}{15\left(a^3\cos(x)+a^3\right)}+\frac{B\tanh^{-1}(\sin(x))}{a^3}+\frac{(2A-7B)\sin(x)}{15a(a\cos(x)+a)^2}+\frac{(A-B)\sin(x)}{5(a\cos(x)+a)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sec[x])/(a + a*Cos[x])^3,x]
```

```
[Out] (B*ArcTanh[Sin[x]])/a^3 + ((A - B)*Sin[x])/(5*(a + a*Cos[x])^3) + ((2*A - 7
*B)*Sin[x])/(15*a*(a + a*Cos[x])^2) + (2*(A - 11*B)*Sin[x])/(15*(a^3 + a^3*
Cos[x]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2828

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.)*((a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)])^(m_.), x_Symbol] :> Int[((a + b*Sin[e + f*x])^m*(d + c*Sin[e +
 f*x])^n)/Sin[e + f*x]^n, x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IntegerQ
[n]
```

Rule 2978

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(
n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
```

```
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{A+B\sec(x)}{(a+a\cos(x))^3}\,dx &= \int \frac{(B+A\cos(x))\sec(x)}{(a+a\cos(x))^3}\,dx\\
&= \frac{(A-B)\sin(x)}{5(a+a\cos(x))^3} + \frac{\int \frac{(5aB+2a(A-B)\cos(x))\sec(x)}{(a+a\cos(x))^2}\,dx}{5a^2}\\
&= \frac{(A-B)\sin(x)}{5(a+a\cos(x))^3} + \frac{(2A-7B)\sin(x)}{15a(a+a\cos(x))^2} + \frac{\int \frac{\left(15a^2B+a^2(2A-7B)\cos(x)\right)\sec(x)}{a+a\cos(x)}\,dx}{15a^4}\\
&= \frac{(A-B)\sin(x)}{5(a+a\cos(x))^3} + \frac{(2A-7B)\sin(x)}{15a(a+a\cos(x))^2} + \frac{2(A-11B)\sin(x)}{15\left(a^3+a^3\cos(x)\right)} + \frac{\int 15a^3B\sec(x)\,dx}{15a^6}\\
&= \frac{(A-B)\sin(x)}{5(a+a\cos(x))^3} + \frac{(2A-7B)\sin(x)}{15a(a+a\cos(x))^2} + \frac{2(A-11B)\sin(x)}{15\left(a^3+a^3\cos(x)\right)} + \frac{B\int \sec(x)\,dx}{a^3}\\
&= \frac{B\tanh^{-1}(\sin(x))}{a^3} + \frac{(A-B)\sin(x)}{5(a+a\cos(x))^3} + \frac{(2A-7B)\sin(x)}{15a(a+a\cos(x))^2} + \frac{2(A-11B)\sin(x)}{15\left(a^3+a^3\cos(x)\right)}
\end{aligned}$$

Mathematica [A] time = 0.35, size = 88, normalized size = 1.17

$$\frac{\sin(x)((6A-51B)\cos(x)+(A-11B)\cos(2x)+8A-43B)-120B\cos^6\left(\frac{x}{2}\right)\left(\log\left(\cos\left(\frac{x}{2}\right)-\sin\left(\frac{x}{2}\right)\right)-\log\left(\sin\right.\right.}{15a^3(\cos(x)+1)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Sec[x])/(a + a*Cos[x])^3,x]
```

```
[Out] (-120*B*Cos[x/2]^6*(Log[Cos[x/2] - Sin[x/2]] - Log[Cos[x/2] + Sin[x/2]]) +
(8*A - 43*B + (6*A - 51*B)*Cos[x] + (A - 11*B)*Cos[2*x])*Sin[x])/(15*a^3*(1
 + Cos[x])^3)
```

fricas [A] time = 0.74, size = 122, normalized size = 1.63

$$\frac{15\left(B\cos(x)^3+3\,B\cos(x)^2+3\,B\cos(x)+B\right)\log\left(\sin(x)+1\right)-15\left(B\cos(x)^3+3\,B\cos(x)^2+3\,B\cos(x)+B\right)\log}{30\left(a^3\cos(x)^3+3\,a^3\cos(x)^2+3\,a^3\cos\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(x))/(a+a*cos(x))^3,x, algorithm="fricas")
```

```
[Out] 1/30*(15*(B*cos(x)^3 + 3*B*cos(x)^2 + 3*B*cos(x) + B)*log(sin(x) + 1) - 15*
(B*cos(x)^3 + 3*B*cos(x)^2 + 3*B*cos(x) + B)*log(-sin(x) + 1) + 2*(2*(A - 1
1*B)*cos(x)^2 + 3*(2*A - 17*B)*cos(x) + 7*A - 32*B)*sin(x))/(a^3*cos(x)^3 +
 3*a^3*cos(x)^2 + 3*a^3*cos(x) + a^3)
```

giac [A] time = 0.16, size = 102, normalized size = 1.36

$$\frac{B\log\left(\left|\tan\left(\frac{1}{2}\,x\right)+1\right|\right)}{a^3}-\frac{B\log\left(\left|\tan\left(\frac{1}{2}\,x\right)-1\right|\right)}{a^3}+\frac{3\,Aa^{12}\tan\left(\frac{1}{2}\,x\right)^5-3\,Ba^{12}\tan\left(\frac{1}{2}\,x\right)^5+10\,Aa^{12}\tan\left(\frac{1}{2}\,x\right)^3-20\,}{60\,a^{15}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(x))/(a+a*cos(x))^3,x, algorithm="giac")
```

```
[Out] B*log(abs(tan(1/2*x) + 1))/a^3 - B*log(abs(tan(1/2*x) - 1))/a^3 + 1/60*(3*A
*a^12*tan(1/2*x)^5 - 3*B*a^12*tan(1/2*x)^5 + 10*A*a^12*tan(1/2*x)^3 - 20*B*
a^12*tan(1/2*x)^3 + 15*A*a^12*tan(1/2*x) - 105*B*a^12*tan(1/2*x))/a^15
```

maple [A] time = 0.10, size = 95, normalized size = 1.27

$$\frac{\left(\tan^3\left(\frac{x}{2}\right)\right)A}{6a^3}-\frac{\left(\tan^3\left(\frac{x}{2}\right)\right)B}{3a^3}-\frac{B\ln\left(\tan\left(\frac{x}{2}\right)-1\right)}{a^3}+\frac{A\tan\left(\frac{x}{2}\right)}{4a^3}-\frac{7B\tan\left(\frac{x}{2}\right)}{4a^3}+\frac{B\ln\left(1+\tan\left(\frac{x}{2}\right)\right)}{a^3}+\frac{\left(\tan^5\left(\frac{x}{2}\right)\right)A}{20a^3}-\frac{\left(\tan^5\right.}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sec(x))/(a+a*cos(x))^3,x)
```

```
[Out] 1/6/a^3*tan(1/2*x)^3*A-1/3/a^3*tan(1/2*x)^3*B-1/a^3*B*ln(tan(1/2*x)-1)+1/4/
a^3*A*tan(1/2*x)-7/4/a^3*B*tan(1/2*x)+1/a^3*B*ln(1+tan(1/2*x))+1/20/a^3*tan
(1/2*x)^5*A-1/20/a^3*tan(1/2*x)^5*B
```

maxima [A] time = 0.64, size = 119, normalized size = 1.59

$$-\frac{1}{60}\,B\left(\frac{\frac{105\,\sin(x)}{\cos(x)+1}+\frac{20\,\sin(x)^3}{(\cos(x)+1)^3}+\frac{3\,\sin(x)^5}{(\cos(x)+1)^5}}{a^3}-\frac{60\,\log\left(\frac{\sin(x)}{\cos(x)+1}+1\right)}{a^3}+\frac{60\,\log\left(\frac{\sin(x)}{\cos(x)+1}-1\right)}{a^3}\right)+\frac{A\left(\frac{15\,\sin(x)}{\cos(x)+1}+\frac{10\,\sin(x)^3}{(\cos(x)+1)^3}\right.}{60\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(x))/(a+a*cos(x))^3,x, algorithm="maxima")
```

```
[Out] -1/60*B*((105*sin(x)/(cos(x) + 1) + 20*sin(x)^3/(cos(x) + 1)^3 + 3*sin(x)^5
/(cos(x) + 1)^5)/a^3 - 60*log(sin(x)/(cos(x) + 1) + 1)/a^3 + 60*log(sin(x)/
(cos(x) + 1) - 1)/a^3) + 1/60*A*(15*sin(x)/(cos(x) + 1) + 10*sin(x)^3/(cos(
x) + 1)^3 + 3*sin(x)^5/(cos(x) + 1)^5)/a^3
```

mupad [B] time = 2.37, size = 92, normalized size = 1.23

$$\tan\left(\frac{x}{2}\right)^3\left(\frac{A-B}{12\,a^3}+\frac{A-3\,B}{12\,a^3}\right)+\tan\left(\frac{x}{2}\right)\left(\frac{A-B}{4\,a^3}+\frac{A-3\,B}{4\,a^3}-\frac{A+3\,B}{4\,a^3}\right)+\frac{\tan\left(\frac{x}{2}\right)^5\,\left(A-B\right)}{20\,a^3}+\frac{2\,B\,\mathrm{atanh}\left(\tan\left(\frac{x}{2}\right)\right)}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B/cos(x))/(a + a*cos(x))^3,x)
```

```
[Out] tan(x/2)^3*((A - B)/(12*a^3) + (A - 3*B)/(12*a^3)) + tan(x/2)*((A - B)/(4*a
^3) + (A - 3*B)/(4*a^3) - (A + 3*B)/(4*a^3)) + (tan(x/2)^5*(A - B))/(20*a^3
) + (2*B*atanh(tan(x/2)))/a^3
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{A}{\cos^3(x)+3\cos^2(x)+3\cos(x)+1}\,dx+\int \frac{B\sec(x)}{\cos^3(x)+3\cos^2(x)+3\cos(x)+1}\,dx}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(x))/(a+a*cos(x))**3,x)
```

```
[Out] (Integral(A/(cos(x)**3 + 3*cos(x)**2 + 3*cos(x) + 1), x) + Integral(B*sec(x
)/(cos(x)**3 + 3*cos(x)**2 + 3*cos(x) + 1), x))/a**3
```

3.192 $\int \frac{A+B\sec(x)}{(a+a\cos(x))^4}\,dx$

Optimal. Leaf size=96

$$\frac{2(3A-80B)\sin(x)}{105a^4(\cos(x)+1)}+\frac{(6A-55B)\sin(x)}{105a^4(\cos(x)+1)^2}+\frac{B\tanh^{-1}(\sin(x))}{a^4}+\frac{(3A-10B)\sin(x)}{35a(a\cos(x)+a)^3}+\frac{(A-B)\sin(x)}{7(a\cos(x)+a)^4}$$

```
[Out] B*arctanh(sin(x))/a^4+1/105*(6*A-55*B)*sin(x)/a^4/(1+cos(x))^2+2/105*(3*A-8
0*B)*sin(x)/a^4/(1+cos(x))+1/7*(A-B)*sin(x)/(a+a*cos(x))^4+1/35*(3*A-10*B)*
sin(x)/a/(a+a*cos(x))^3
```

Rubi [A] time = 0.41, antiderivative size = 96, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {2828, 2978, 12, 3770}

$$\frac{2(3A-80B)\sin(x)}{105a^4(\cos(x)+1)}+\frac{(6A-55B)\sin(x)}{105a^4(\cos(x)+1)^2}+\frac{B\tanh^{-1}(\sin(x))}{a^4}+\frac{(3A-10B)\sin(x)}{35a(a\cos(x)+a)^3}+\frac{(A-B)\sin(x)}{7(a\cos(x)+a)^4}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sec[x])/(a + a*Cos[x])^4,x]
```

```
[Out] (B*ArcTanh[Sin[x]])/a^4 + ((6*A - 55*B)*Sin[x])/(105*a^4*(1 + Cos[x])^2) +
(2*(3*A - 80*B)*Sin[x])/(105*a^4*(1 + Cos[x])) + ((A - B)*Sin[x])/(7*(a + a
*Cos[x])^4) + ((3*A - 10*B)*Sin[x])/(35*a*(a + a*Cos[x])^3)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2828

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.)*((a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)])^(m_.), x_Symbol] :> Int[((a + b*Sin[e + f*x])^m*(d + c*Sin[e +
 f*x])^n)/Sin[e + f*x]^n, x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IntegerQ
[n]
```

Rule 2978

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(
n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
```

```
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{A+B\sec(x)}{(a+a\cos(x))^4}\,dx = \int \frac{(B+A\cos(x))\sec(x)}{(a+a\cos(x))^4}\,dx$$

$$= \frac{(A-B)\sin(x)}{7(a+a\cos(x))^4} + \frac{\int \frac{(7aB+3a(A-B)\cos(x))\sec(x)}{(a+a\cos(x))^3}\,dx}{7a^2}$$

$$= \frac{(A-B)\sin(x)}{7(a+a\cos(x))^4} + \frac{(3A-10B)\sin(x)}{35a(a+a\cos(x))^3} + \frac{\int \frac{\left(35a^2B+2a^2(3A-10B)\cos(x)\right)\sec(x)}{(a+a\cos(x))^2}\,dx}{35a^4}$$

$$= \frac{(6A-55B)\sin(x)}{105a^4(1+\cos(x))^2} + \frac{(A-B)\sin(x)}{7(a+a\cos(x))^4} + \frac{(3A-10B)\sin(x)}{35a(a+a\cos(x))^3} + \frac{\int \frac{\left(105a^3B+a^3(6A-55B)\cos(x)\right.}{a+a\cos(x)}}{105a^6}$$

$$= \frac{(6A-55B)\sin(x)}{105a^4(1+\cos(x))^2} + \frac{(A-B)\sin(x)}{7(a+a\cos(x))^4} + \frac{(3A-10B)\sin(x)}{35a(a+a\cos(x))^3} + \frac{2(3A-80B)\sin(x)}{105\left(a^4+a^4\cos(x)\right)} +$$

$$= \frac{(6A-55B)\sin(x)}{105a^4(1+\cos(x))^2} + \frac{(A-B)\sin(x)}{7(a+a\cos(x))^4} + \frac{(3A-10B)\sin(x)}{35a(a+a\cos(x))^3} + \frac{2(3A-80B)\sin(x)}{105\left(a^4+a^4\cos(x)\right)} +$$

$$= \frac{B\tanh^{-1}(\sin(x))}{a^4} + \frac{(6A-55B)\sin(x)}{105a^4(1+\cos(x))^2} + \frac{(A-B)\sin(x)}{7(a+a\cos(x))^4} + \frac{(3A-10B)\sin(x)}{35a(a+a\cos(x))^3} + \frac{2(3}{105}$$

Mathematica [A] time = 0.75, size = 104, normalized size = 1.08

$$\frac{\sin(x)((87A-1480B)\cos(x)+(24A-535B)\cos(2x)+3A\cos(3x)+96A-80B\cos(3x)-1055B)-3360B\,\mathrm{co}}{210a^4(\cos(x)+1)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Sec[x])/(a + a*Cos[x])^4,x]
```

```
[Out] (-3360*B*Cos[x/2]^8*(Log[Cos[x/2] - Sin[x/2]] - Log[Cos[x/2] + Sin[x/2]]) +
 (96*A - 1055*B + (87*A - 1480*B)*Cos[x] + (24*A - 535*B)*Cos[2*x] + 3*A*Co
s[3*x] - 80*B*Cos[3*x])*Sin[x])/(210*a^4*(1 + Cos[x])^4)
```

fricas [A] time = 0.57, size = 158, normalized size = 1.65

$$\frac{105\,(B\cos(x)^4+4\,B\cos(x)^3+6\,B\cos(x)^2+4\,B\cos(x)+B)\log(\sin(x)+1)-105\,(B\cos(x)^4+4\,B\cos(x)^3+6\,B}{210\,(a^4\cos(x)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(x))/(a+a*cos(x))^4,x, algorithm="fricas")
```

```
[Out] 1/210*(105*(B*cos(x)^4 + 4*B*cos(x)^3 + 6*B*cos(x)^2 + 4*B*cos(x) + B)*log(
sin(x) + 1) - 105*(B*cos(x)^4 + 4*B*cos(x)^3 + 6*B*cos(x)^2 + 4*B*cos(x) +
B)*log(-sin(x) + 1) + 2*(2*(3*A - 80*B)*cos(x)^3 + (24*A - 535*B)*cos(x)^2
+ (39*A - 620*B)*cos(x) + 36*A - 260*B)*sin(x))/(a^4*cos(x)^4 + 4*a^4*cos(x
)^3 + 6*a^4*cos(x)^2 + 4*a^4*cos(x) + a^4)
```

giac [A] time = 0.15, size = 126, normalized size = 1.31

$$\frac{B\log\left(\left|\tan\left(\frac{1}{2}\,x\right)+1\right|\right)}{a^4}-\frac{B\log\left(\left|\tan\left(\frac{1}{2}\,x\right)-1\right|\right)}{a^4}+\frac{15\,Aa^{24}\tan\left(\frac{1}{2}\,x\right)^7-15\,Ba^{24}\tan\left(\frac{1}{2}\,x\right)^7+63\,Aa^{24}\tan\left(\frac{1}{2}\,x\right)^5-1}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(x))/(a+a*cos(x))^4,x, algorithm="giac")
```

```
[Out] B*log(abs(tan(1/2*x) + 1))/a^4 - B*log(abs(tan(1/2*x) - 1))/a^4 + 1/840*(15
*A*a^24*tan(1/2*x)^7 - 15*B*a^24*tan(1/2*x)^7 + 63*A*a^24*tan(1/2*x)^5 - 10
5*B*a^24*tan(1/2*x)^5 + 105*A*a^24*tan(1/2*x)^3 - 385*B*a^24*tan(1/2*x)^3 +
 105*A*a^24*tan(1/2*x) - 1575*B*a^24*tan(1/2*x))/a^28
```

maple [A] time = 0.10, size = 119, normalized size = 1.24

$$\frac{\left(\tan^7\left(\frac{x}{2}\right)\right)A}{56a^4}-\frac{\left(\tan^7\left(\frac{x}{2}\right)\right)B}{56a^4}+\frac{\left(\tan^3\left(\frac{x}{2}\right)\right)A}{8a^4}-\frac{11\left(\tan^3\left(\frac{x}{2}\right)\right)B}{24a^4}-\frac{B\ln\left(\tan\left(\frac{x}{2}\right)-1\right)}{a^4}+\frac{A\tan\left(\frac{x}{2}\right)}{8a^4}-\frac{15B\tan\left(\frac{x}{2}\right)}{8a^4}+\frac{B\ln(1}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sec(x))/(a+a*cos(x))^4,x)
```

```
[Out] 1/56/a^4*tan(1/2*x)^7*A-1/56/a^4*tan(1/2*x)^7*B+1/8/a^4*tan(1/2*x)^3*A-11/2
4/a^4*tan(1/2*x)^3*B-1/a^4*B*ln(tan(1/2*x)-1)+1/8/a^4*A*tan(1/2*x)-15/8/a^4
*B*tan(1/2*x)+1/a^4*B*ln(1+tan(1/2*x))+3/40/a^4*tan(1/2*x)^5*A-1/8/a^4*tan(
1/2*x)^5*B
```

maxima [A]   time = 0.77, size = 143, normalized size = 1.49

$$-\frac{1}{168}B\left(\frac{\frac{315\sin(x)}{\cos(x)+1}+\frac{77\sin(x)^3}{(\cos(x)+1)^3}+\frac{21\sin(x)^5}{(\cos(x)+1)^5}+\frac{3\sin(x)^7}{(\cos(x)+1)^7}}{a^4}-\frac{168\log\left(\frac{\sin(x)}{\cos(x)+1}+1\right)}{a^4}+\frac{168\log\left(\frac{\sin(x)}{\cos(x)+1}-1\right)}{a^4}\right)+\frac{A\left(\frac{35\ldots}{\mathrm{co}\ldots}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(x))/(a+a*cos(x))^4,x, algorithm="maxima")
```

```
[Out] -1/168*B*((315*sin(x)/(cos(x) + 1) + 77*sin(x)^3/(cos(x) + 1)^3 + 21*sin(x)
^5/(cos(x) + 1)^5 + 3*sin(x)^7/(cos(x) + 1)^7)/a^4 - 168*log(sin(x)/(cos(x)
 + 1) + 1)/a^4 + 168*log(sin(x)/(cos(x) + 1) - 1)/a^4) + 1/280*A*(35*sin(x)
/(cos(x) + 1) + 35*sin(x)^3/(cos(x) + 1)^3 + 21*sin(x)^5/(cos(x) + 1)^5 + 5
*sin(x)^7/(cos(x) + 1)^7)/a^4
```

mupad [B]   time = 2.34, size = 140, normalized size = 1.46

$$\tan\left(\frac{x}{2}\right)\left(\frac{A-B}{8a^4}-\frac{3B}{4a^4}+\frac{2A-4B}{8a^4}-\frac{2A+4B}{8a^4}\right)+\tan\left(\frac{x}{2}\right)^5\left(\frac{A-B}{40a^4}+\frac{2A-4B}{40a^4}\right)+\tan\left(\frac{x}{2}\right)^3\left(\frac{A-B}{24a^4}-\frac{B}{4a^4}+\ldots\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B/cos(x))/(a + a*cos(x))^4,x)
```

```
[Out] tan(x/2)*((A - B)/(8*a^4) - (3*B)/(4*a^4) + (2*A - 4*B)/(8*a^4) - (2*A + 4*
B)/(8*a^4)) + tan(x/2)^5*((A - B)/(40*a^4) + (2*A - 4*B)/(40*a^4)) + tan(x/
2)^3*((A - B)/(24*a^4) - B/(4*a^4) + (2*A - 4*B)/(24*a^4)) + (tan(x/2)^7*(A
 - B))/(56*a^4) + (2*B*atanh(tan(x/2)))/a^4
```

sympy [F]   time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int\frac{A}{\cos^4(x)+4\cos^3(x)+6\cos^2(x)+4\cos(x)+1}\,dx+\int\frac{B\sec(x)}{\cos^4(x)+4\cos^3(x)+6\cos^2(x)+4\cos(x)+1}\,dx}{a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(x))/(a+a*cos(x))**4,x)
```

```
[Out] (Integral(A/(cos(x)**4 + 4*cos(x)**3 + 6*cos(x)**2 + 4*cos(x) + 1), x) + In
tegral(B*sec(x)/(cos(x)**4 + 4*cos(x)**3 + 6*cos(x)**2 + 4*cos(x) + 1), x))
/a**4
```

3.193 $\int (a+a\cos(x))^{5/2}(A+B\sec(x))\,dx$

Optimal. Leaf size=98

$$2a^{5/2}B\tanh^{-1}\left(\frac{\sqrt{a}\,\sin(x)}{\sqrt{a\cos(x)+a}}\right)+\frac{2a^3(32A+35B)\sin(x)}{15\sqrt{a\cos(x)+a}}+\frac{2}{15}a^2(8A+5B)\sin(x)\sqrt{a\cos(x)+a}+\frac{2}{5}aA\sin(x)(a\cos($$

```
[Out] 2*a^(5/2)*B*arctanh(sin(x)*a^(1/2)/(a+a*cos(x))^(1/2))+2/5*a*A*(a+a*cos(x))
^(3/2)*sin(x)+2/15*a^3*(32*A+35*B)*sin(x)/(a+a*cos(x))^(1/2)+2/15*a^2*(8*A+
5*B)*sin(x)*(a+a*cos(x))^(1/2)
```

Rubi [A] time = 0.43, antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.294, Rules used = {2828, 2976, 2981, 2773, 206}

$$\frac{2a^3(32A+35B)\sin(x)}{15\sqrt{a\cos(x)+a}}+\frac{2}{15}a^2(8A+5B)\sin(x)\sqrt{a\cos(x)+a}+2a^{5/2}B\tanh^{-1}\left(\frac{\sqrt{a}\,\sin(x)}{\sqrt{a\cos(x)+a}}\right)+\frac{2}{5}aA\sin(x)(a\cos($$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[x])^(5/2)*(A + B*Sec[x]),x]
```

```
[Out] 2*a^(5/2)*B*ArcTanh[(Sqrt[a]*Sin[x])/Sqrt[a + a*Cos[x]]] + (2*a^3*(32*A + 3
5*B)*Sin[x])/(15*Sqrt[a + a*Cos[x]]) + (2*a^2*(8*A + 5*B)*Sqrt[a + a*Cos[x]
]*Sin[x])/15 + (2*a*A*(a + a*Cos[x])^(3/2)*Sin[x])/5
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2773

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[(-2*b)/f, Subst[Int[1/(b*c + a*d - d*x^2), x
], x, (b*Cos[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2828

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.)*((a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)])^(m_.), x_Symbol] :> Int[((a + b*Sin[e + f*x])^m*(d + c*Sin[e +
 f*x])^n)/Sin[e + f*x]^n, x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IntegerQ
[n]
```

Rule 2976

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n +
 1))/(d*f*(m + n + 1)), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Sin[e + f*x
])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 1) + B*(a*c*(m - 1) +
 b*d*(n + 1)) + (A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e + f*x
], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0]
 && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1] &
& IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2981

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[(-2*b*B*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(2*n + 3)*Sqrt[a +
 b*Sin[e + f*x]]), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c - 2*a*d*(n + 1)))/(b
*d*(2*n + 3)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x], x]
/; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[n, -1]
```

Rubi steps

$$\begin{aligned}
\int (a+a\cos(x))^{5/2}(A+B\sec(x))\,dx &= \int (a+a\cos(x))^{5/2}(B+A\cos(x))\sec(x)\,dx\\
&= \frac{2}{5}aA(a+a\cos(x))^{3/2}\sin(x)+\frac{2}{5}\int (a+a\cos(x))^{3/2}\left(\frac{5aB}{2}+\frac{1}{2}a(8A+5B\right.\\
&= \frac{2}{15}a^2(8A+5B)\sqrt{a+a\cos(x)}\,\sin(x)+\frac{2}{5}aA(a+a\cos(x))^{3/2}\sin(x)+\frac{4}{15}\\
&= \frac{2a^3(32A+35B)\sin(x)}{15\sqrt{a+a\cos(x)}}+\frac{2}{15}a^2(8A+5B)\sqrt{a+a\cos(x)}\,\sin(x)+\frac{2}{5}aA(a\\
&= \frac{2a^3(32A+35B)\sin(x)}{15\sqrt{a+a\cos(x)}}+\frac{2}{15}a^2(8A+5B)\sqrt{a+a\cos(x)}\,\sin(x)+\frac{2}{5}aA(a\\
&= 2a^{5/2}B\tanh^{-1}\left(\frac{\sqrt{a}\,\sin(x)}{\sqrt{a+a\cos(x)}}\right)+\frac{2a^3(32A+35B)\sin(x)}{15\sqrt{a+a\cos(x)}}+\frac{2}{15}a^2(8A+
\end{aligned}$$

Mathematica [A] time = 0.16, size = 78, normalized size = 0.80

$$\frac{1}{30}a^2\sec\left(\frac{x}{2}\right)\sqrt{a(\cos(x)+1)}\left(2\sin\left(\frac{x}{2}\right)(2(14A+5B)\cos(x)+3A\cos(2x)+89A+80B)+30\sqrt{2}\,B\tanh^{-1}\left(\sqrt{2}\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[x])^(5/2)*(A + B*Sec[x]),x]
```

```
[Out] (a^2*Sqrt[a*(1 + Cos[x])]*Sec[x/2]*(30*Sqrt[2]*B*ArcTanh[Sqrt[2]*Sin[x/2]]
+ 2*(89*A + 80*B + 2*(14*A + 5*B)*Cos[x] + 3*A*Cos[2*x])*Sin[x/2]))/30
```

fricas [A] time = 0.73, size = 123, normalized size = 1.26

$$\frac{15\left(Ba^2\cos(x)+Ba^2\right)\sqrt{a}\,\log\left(\frac{a\cos(x)^3-7\,a\cos(x)^2-4\,\sqrt{a\cos(x)+a}\,\sqrt{a}\,(\cos(x)-2)\sin(x)+8\,a}{\cos(x)^3+\cos(x)^2}\right)+4\left(3\,Aa^2\cos(x)^2+(14\,A+5\,B)\right.}{30\,(\cos(x)+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(x))^(5/2)*(A+B*sec(x)),x, algorithm="fricas")
```

```
[Out] 1/30*(15*(B*a^2*cos(x) + B*a^2)*sqrt(a)*log((a*cos(x)^3 - 7*a*cos(x)^2 - 4*
sqrt(a*cos(x) + a)*sqrt(a)*(cos(x) - 2)*sin(x) + 8*a)/(cos(x)^3 + cos(x)^2)
) + 4*(3*A*a^2*cos(x)^2 + (14*A + 5*B)*a^2*cos(x) + (43*A + 40*B)*a^2)*sqrt
(a*cos(x) + a)*sin(x))/(cos(x) + 1)
```

giac [A] time = 0.21, size = 134, normalized size = 1.37

$$\frac{1}{30}\sqrt{2}\left(48\,Aa^2\mathrm{sgn}\left(\cos\left(\frac{1}{2}\,x\right)\right)\sin\left(\frac{1}{2}\,x\right)^5-160\,Aa^2\mathrm{sgn}\left(\cos\left(\frac{1}{2}\,x\right)\right)\sin\left(\frac{1}{2}\,x\right)^3-40\,Ba^2\mathrm{sgn}\left(\cos\left(\frac{1}{2}\,x\right)\right)\sin\left(\frac{1}{2}\,x\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(x))^(5/2)*(A+B*sec(x)),x, algorithm="giac")
```

```
[Out] 1/30*sqrt(2)*(48*A*a^2*sgn(cos(1/2*x))*sin(1/2*x)^5 - 160*A*a^2*sgn(cos(1/2
*x))*sin(1/2*x)^3 - 40*B*a^2*sgn(cos(1/2*x))*sin(1/2*x)^3 - 15*sqrt(2)*B*a^
2*log(abs(-2*sqrt(2) + 4*sin(1/2*x))/abs(2*sqrt(2) + 4*sin(1/2*x)))*sgn(cos
(1/2*x)) + 240*A*a^2*sgn(cos(1/2*x))*sin(1/2*x) + 180*B*a^2*sgn(cos(1/2*x))
*sin(1/2*x))*sqrt(a)
```

maple [B] time = 0.38, size = 228, normalized size = 2.33

$$\frac{a^{\frac{3}{2}}\cos\left(\frac{x}{2}\right)\sqrt{a\left(\sin^2\left(\frac{x}{2}\right)\right)}\left(24A\sqrt{2}\,\sqrt{a\left(\sin^2\left(\frac{x}{2}\right)\right)}\,\sqrt{a}\,\left(\sin^4\left(\frac{x}{2}\right)\right)-20\sqrt{2}\,\sqrt{a\left(\sin^2\left(\frac{x}{2}\right)\right)}\,\sqrt{a}\,(4A+B)\left(\sin^2\left(\frac{x}{2}\right)\right)\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(x))^(5/2)*(A+B*sec(x)),x)
```

```
[Out] 1/15*a^(3/2)*cos(1/2*x)*(a*sin(1/2*x)^2)^(1/2)*(24*A*2^(1/2)*(a*sin(1/2*x)^
2)^(1/2)*a^(1/2)*sin(1/2*x)^4-20*2^(1/2)*(a*sin(1/2*x)^2)^(1/2)*a^(1/2)*(4*
A+B)*sin(1/2*x)^2+120*A*2^(1/2)*(a*sin(1/2*x)^2)^(1/2)*a^(1/2)+90*B*2^(1/2)
*(a*sin(1/2*x)^2)^(1/2)*a^(1/2)+15*B*ln(4/(2*cos(1/2*x)+2^(1/2))*(a*2^(1/2)
*cos(1/2*x)+a^(1/2)*2^(1/2)*(a*sin(1/2*x)^2)^(1/2)+2*a))*a+15*B*ln(-4/(-2*c
os(1/2*x)+2^(1/2))*(a^(1/2)*2^(1/2)*(a*sin(1/2*x)^2)^(1/2)-a*2^(1/2)*cos(1/
2*x)+2*a))*a)/sin(1/2*x)/(cos(1/2*x)^2*a)^(1/2)
```

maxima [A] time = 1.12, size = 43, normalized size = 0.44

$$\frac{1}{30}\left(3\sqrt{2}\,a^2\sin\left(\frac{5}{2}\,x\right)+25\sqrt{2}\,a^2\sin\left(\frac{3}{2}\,x\right)+150\sqrt{2}\,a^2\sin\left(\frac{1}{2}\,x\right)\right)A\sqrt{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(x))^(5/2)*(A+B*sec(x)),x, algorithm="maxima")
```

```
[Out] 1/30*(3*sqrt(2)*a^2*sin(5/2*x) + 25*sqrt(2)*a^2*sin(3/2*x) + 150*sqrt(2)*a^
2*sin(1/2*x))*A*sqrt(a)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int (a+a\,\cos(x))^{5/2}\left(A+\frac{B}{\cos(x)}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*cos(x))^(5/2)*(A + B/cos(x)),x)
```

```
[Out] int((a + a*cos(x))^(5/2)*(A + B/cos(x)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(x))**(5/2)*(A+B*sec(x)),x)
```

```
[Out] Timed out
```

3.194 $\int (a+a\cos(x))^{3/2}(A+B\sec(x))\,dx$

Optimal. Leaf size=72

$$2a^{3/2}B\tanh^{-1}\left(\frac{\sqrt{a}\,\sin(x)}{\sqrt{a\cos(x)+a}}\right)+\frac{2a^2(4A+3B)\sin(x)}{3\sqrt{a\cos(x)+a}}+\frac{2}{3}aA\sin(x)\sqrt{a\cos(x)+a}$$

```
[Out] 2*a^(3/2)*B*arctanh(sin(x)*a^(1/2)/(a+a*cos(x))^(1/2))+2/3*a^2*(4*A+3*B)*si
n(x)/(a+a*cos(x))^(1/2)+2/3*a*A*sin(x)*(a+a*cos(x))^(1/2)
```

Rubi [A] time = 0.29, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.294, Rules used = {2828, 2976, 2981, 2773, 206}

$$\frac{2a^2(4A+3B)\sin(x)}{3\sqrt{a\cos(x)+a}}+2a^{3/2}B\tanh^{-1}\left(\frac{\sqrt{a}\,\sin(x)}{\sqrt{a\cos(x)+a}}\right)+\frac{2}{3}aA\sin(x)\sqrt{a\cos(x)+a}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[x])^(3/2)*(A + B*Sec[x]),x]
```

```
[Out] 2*a^(3/2)*B*ArcTanh[(Sqrt[a]*Sin[x])/Sqrt[a + a*Cos[x]]] + (2*a^2*(4*A + 3*
B)*Sin[x])/(3*Sqrt[a + a*Cos[x]]) + (2*a*A*Sqrt[a + a*Cos[x]]*Sin[x])/3
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2773

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[(-2*b)/f, Subst[Int[1/(b*c + a*d - d*x^2), x
], x, (b*Cos[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2828

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.)*((a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)])^(m_.), x_Symbol] :> Int[((a + b*Sin[e + f*x])^m*(d + c*Sin[e +
 f*x])^n)/Sin[e + f*x]^n, x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IntegerQ
[n]
```

Rule 2976

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n +
 1))/(d*f*(m + n + 1)), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Sin[e + f*x
])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 1) + B*(a*c*(m - 1) +
 b*d*(n + 1)) + (A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e + f*x
], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0]
 && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1] &
& IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2981

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[(-2*b*B*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(2*n + 3)*Sqrt[a +
 b*Sin[e + f*x]]), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c - 2*a*d*(n + 1)))/(b
*d*(2*n + 3)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x], x]
/; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[n, -1]
```

Rubi steps

$$\int (a+a\cos(x))^{3/2}(A+B\sec(x))\,dx = \int (a+a\cos(x))^{3/2}(B+A\cos(x))\sec(x)\,dx$$

$$= \frac{2}{3}aA\sqrt{a+a\cos(x)}\,\sin(x) + \frac{2}{3}\int \sqrt{a+a\cos(x)}\left(\frac{3aB}{2} + \frac{1}{2}a(4A+3B)\,\mathrm{co}\right.$$

$$= \frac{2a^2(4A+3B)\sin(x)}{3\sqrt{a+a\cos(x)}} + \frac{2}{3}aA\sqrt{a+a\cos(x)}\,\sin(x) + (aB)\int \sqrt{a+a\cos(}$$

$$= \frac{2a^2(4A+3B)\sin(x)}{3\sqrt{a+a\cos(x)}} + \frac{2}{3}aA\sqrt{a+a\cos(x)}\,\sin(x) - \left(2a^2B\right)\mathrm{Subst}\left(\int \frac{}{a\,\cdot}\right.$$

$$= 2a^{3/2}B\tanh^{-1}\left(\frac{\sqrt{a}\,\sin(x)}{\sqrt{a+a\cos(x)}}\right) + \frac{2a^2(4A+3B)\sin(x)}{3\sqrt{a+a\cos(x)}} + \frac{2}{3}aA\sqrt{a+a\,\mathrm{co}}$$

Mathematica [A] time = 0.10, size = 62, normalized size = 0.86

$$\frac{1}{3}a\sec\left(\frac{x}{2}\right)\sqrt{a(\cos(x)+1)}\left(2\sin\left(\frac{x}{2}\right)(A\cos(x)+5A+3B)+3\sqrt{2}B\tanh^{-1}\left(\sqrt{2}\,\sin\left(\frac{x}{2}\right)\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[x])^(3/2)*(A + B*Sec[x]),x]
```

[Out] (a*Sqrt[a*(1 + Cos[x])]*Sec[x/2]*(3*Sqrt[2]*B*ArcTanh[Sqrt[2]*Sin[x/2]] + 2
*(5*A + 3*B + A*Cos[x])*Sin[x/2]))/3

fricas [A] time = 0.66, size = 99, normalized size = 1.38

$$\frac{3\,(Ba\cos(x) + Ba)\sqrt{a}\,\log\left(\frac{a\cos(x)^3 - 7\,a\cos(x)^2 - 4\,\sqrt{a\cos(x) + a}\,\sqrt{a}\,(\cos(x) - 2)\sin(x) + 8\,a}{\cos(x)^3 + \cos(x)^2}\right) + 4\,(Aa\cos(x) + (5\,A + 3\,B)a)\sqrt{a\cos}}{6\,(\cos(x) + 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*cos(x))^(3/2)*(A+B*sec(x)),x, algorithm="fricas")

[Out] 1/6*(3*(B*a*cos(x) + B*a)*sqrt(a)*log((a*cos(x)^3 - 7*a*cos(x)^2 - 4*sqrt(a
*cos(x) + a)*sqrt(a)*(cos(x) - 2)*sin(x) + 8*a)/(cos(x)^3 + cos(x)^2)) + 4*
(A*a*cos(x) + (5*A + 3*B)*a)*sqrt(a*cos(x) + a)*sin(x))/(cos(x) + 1)

giac [A] time = 0.19, size = 92, normalized size = 1.28

$$-\frac{1}{6}\,\sqrt{2}\left(8\,Aa\mathrm{sgn}\left(\cos\left(\frac{1}{2}\,x\right)\right)\sin\left(\frac{1}{2}\,x\right)^3 + 3\,\sqrt{2}\,Ba\log\left(\frac{\left|-2\,\sqrt{2} + 4\,\sin\left(\frac{1}{2}\,x\right)\right|}{\left|2\,\sqrt{2} + 4\,\sin\left(\frac{1}{2}\,x\right)\right|}\right)\mathrm{sgn}\left(\cos\left(\frac{1}{2}\,x\right)\right) - 24\,Aa\mathrm{sgn}\left(\cos\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*cos(x))^(3/2)*(A+B*sec(x)),x, algorithm="giac")

[Out] -1/6*sqrt(2)*(8*A*a*sgn(cos(1/2*x))*sin(1/2*x)^3 + 3*sqrt(2)*B*a*log(abs(-2
*sqrt(2) + 4*sin(1/2*x))/abs(2*sqrt(2) + 4*sin(1/2*x)))*sgn(cos(1/2*x)) - 2
4*A*a*sgn(cos(1/2*x))*sin(1/2*x) - 12*B*a*sgn(cos(1/2*x))*sin(1/2*x))*sqrt(
a)

maple [B] time = 0.31, size = 199, normalized size = 2.76

$$\frac{\sqrt{a}\,\cos\left(\frac{x}{2}\right)\sqrt{a\left(\sin^2\left(\frac{x}{2}\right)\right)}\left(-4A\sqrt{a}\,\sqrt{2}\,\sqrt{a\left(\sin^2\left(\frac{x}{2}\right)\right)}\,\left(\sin^2\left(\frac{x}{2}\right)\right) + 12A\sqrt{2}\,\sqrt{a\left(\sin^2\left(\frac{x}{2}\right)\right)}\,\sqrt{a} + 6B\sqrt{2}\,\sqrt{a\left(\sin\right.}\right.}{3\sin\left(\frac{x}{2}\right)\sqrt{\left(\cos^2\right.}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+a*cos(x))^(3/2)*(A+B*sec(x)),x)

[Out] 1/3*a^(1/2)*cos(1/2*x)*(a*sin(1/2*x)^2)^(1/2)*(-4*A*a^(1/2)*2^(1/2)*(a*sin(
1/2*x)^2)^(1/2)*sin(1/2*x)^2+12*A*2^(1/2)*(a*sin(1/2*x)^2)^(1/2)*a^(1/2)+6*
B*2^(1/2)*(a*sin(1/2*x)^2)^(1/2)*a^(1/2)+3*B*ln(4/(2*cos(1/2*x)+2^(1/2))*(a

```
*2^(1/2)*cos(1/2*x)+a^(1/2)*2^(1/2)*(a*sin(1/2*x)^2)^(1/2)+2*a))*a+3*B*ln(-
4/(-2*cos(1/2*x)+2^(1/2))*(a^(1/2)*2^(1/2)*(a*sin(1/2*x)^2)^(1/2)-a*2^(1/2)
*cos(1/2*x)+2*a))*a)/sin(1/2*x)/(cos(1/2*x)^2*a)^(1/2)
```

maxima [A] time = 1.11, size = 26, normalized size = 0.36

$$\frac{1}{3}\left(\sqrt{2}\,a\sin\left(\frac{3}{2}\,x\right)+9\,\sqrt{2}\,a\sin\left(\frac{1}{2}\,x\right)\right)A\sqrt{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(x))^(3/2)*(A+B*sec(x)),x, algorithm="maxima")
```

```
[Out] 1/3*(sqrt(2)*a*sin(3/2*x) + 9*sqrt(2)*a*sin(1/2*x))*A*sqrt(a)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int (a+a\,\cos(x))^{3/2}\left(A+\frac{B}{\cos(x)}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*cos(x))^(3/2)*(A + B/cos(x)),x)
```

```
[Out] int((a + a*cos(x))^(3/2)*(A + B/cos(x)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a(\cos(x)+1))^{\frac{3}{2}}(A+B\sec(x))\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(x))**(3/2)*(A+B*sec(x)),x)
```

```
[Out] Integral((a*(cos(x) + 1))**(3/2)*(A + B*sec(x)), x)
```

3.195 $\int \sqrt{a+a\cos(x)}\,(A+B\sec(x))\,dx$

Optimal. Leaf size=44

$$\frac{2aA\sin(x)}{\sqrt{a\cos(x)+a}}+2\sqrt{a}\,B\tanh^{-1}\left(\frac{\sqrt{a}\,\sin(x)}{\sqrt{a\cos(x)+a}}\right)$$

```
[Out] 2*B*arctanh(sin(x)*a^(1/2)/(a+a*cos(x))^(1/2))*a^(1/2)+2*a*A*sin(x)/(a+a*co
s(x))^(1/2)
```

Rubi [A] time = 0.16, antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {2828, 2981, 2773, 206}

$$\frac{2aA\sin(x)}{\sqrt{a\cos(x)+a}}+2\sqrt{a}\,B\tanh^{-1}\left(\frac{\sqrt{a}\,\sin(x)}{\sqrt{a\cos(x)+a}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + a*Cos[x]]*(A + B*Sec[x]),x]
```

```
[Out] 2*Sqrt[a]*B*ArcTanh[(Sqrt[a]*Sin[x])/Sqrt[a + a*Cos[x]]] + (2*a*A*Sin[x])/S
qrt[a + a*Cos[x]]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2773

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[(-2*b)/f, Subst[Int[1/(b*c + a*d - d*x^2), x
], x, (b*Cos[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2828

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.)*((a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)])^(m_.), x_Symbol] :> Int[((a + b*Sin[e + f*x])^m*(d + c*Sin[e +
 f*x])^n)/Sin[e + f*x]^n, x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IntegerQ
[n]
```

Rule 2981

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[(-2*b*B*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(2*n + 3)*Sqrt[a +
 b*Sin[e + f*x]]), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c - 2*a*d*(n + 1)))/(b
*d*(2*n + 3)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x], x]
/; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
b^2, 0] && NeQ[c^2 - d^2, 0] && !LtQ[n, -1]
```

Rubi steps

$$\begin{aligned}\int \sqrt{a+a\cos(x)}\,(A+B\sec(x))\,dx &= \int \sqrt{a+a\cos(x)}\,(B+A\cos(x))\sec(x)\,dx\\ &= \frac{2aA\sin(x)}{\sqrt{a+a\cos(x)}}+B\int \sqrt{a+a\cos(x)}\,\sec(x)\,dx\\ &= \frac{2aA\sin(x)}{\sqrt{a+a\cos(x)}}-(2aB)\,\mathrm{Subst}\left(\int \frac{1}{a-x^2}\,dx,x,-\frac{a\sin(x)}{\sqrt{a+a\cos(x)}}\right)\\ &= 2\sqrt{a}\,B\tanh^{-1}\left(\frac{\sqrt{a}\,\sin(x)}{\sqrt{a+a\cos(x)}}\right)+\frac{2aA\sin(x)}{\sqrt{a+a\cos(x)}}\end{aligned}$$

Mathematica [A] time = 0.04, size = 47, normalized size = 1.07

$$\sec\left(\frac{x}{2}\right)\sqrt{a(\cos(x)+1)}\,\left(2A\sin\left(\frac{x}{2}\right)+\sqrt{2}\,B\tanh^{-1}\left(\sqrt{2}\,\sin\left(\frac{x}{2}\right)\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + a*Cos[x]]*(A + B*Sec[x]),x]
```

```
[Out] Sqrt[a*(1 + Cos[x])]*Sec[x/2]*(Sqrt[2]*B*ArcTanh[Sqrt[2]*Sin[x/2]] + 2*A*Si
n[x/2])
```

fricas [B] time = 0.78, size = 81, normalized size = 1.84

$$\frac{(B\cos(x)+B)\sqrt{a}\,\log\left(\frac{a\cos(x)^3-7\,a\cos(x)^2-4\,\sqrt{a\cos(x)+a}\,\sqrt{a}\,(\cos(x)-2)\sin(x)+8\,a}{\cos(x)^3+\cos(x)^2}\right)+4\,\sqrt{a\cos(x)+a}\,A\sin(x)}{2\,(\cos(x)+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(x))^(1/2)*(A+B*sec(x)),x, algorithm="fricas")
```

```
[Out] 1/2*((B*cos(x) + B)*sqrt(a)*log((a*cos(x)^3 - 7*a*cos(x)^2 - 4*sqrt(a*cos(x
) + a)*sqrt(a)*(cos(x) - 2)*sin(x) + 8*a)/(cos(x)^3 + cos(x)^2)) + 4*sqrt(a
*cos(x) + a)*A*sin(x))/(cos(x) + 1)
```

giac [A] time = 0.18, size = 61, normalized size = 1.39

$$-\frac{1}{2}\sqrt{2}\left(\sqrt{2}B\log\left(\frac{\left|-2\sqrt{2}+4\sin\left(\frac{1}{2}x\right)\right|}{\left|2\sqrt{2}+4\sin\left(\frac{1}{2}x\right)\right|}\right)\mathrm{sgn}\left(\cos\left(\frac{1}{2}x\right)\right)-4A\mathrm{sgn}\left(\cos\left(\frac{1}{2}x\right)\right)\sin\left(\frac{1}{2}x\right)\right)\sqrt{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(x))^(1/2)*(A+B*sec(x)),x, algorithm="giac")
```

```
[Out] -1/2*sqrt(2)*(sqrt(2)*B*log(abs(-2*sqrt(2) + 4*sin(1/2*x))/abs(2*sqrt(2) +
4*sin(1/2*x)))*sgn(cos(1/2*x)) - 4*A*sgn(cos(1/2*x))*sin(1/2*x))*sqrt(a)
```

maple [B] time = 0.30, size = 152, normalized size = 3.45

$$\frac{\cos\left(\frac{x}{2}\right)\sqrt{a\left(\sin^2\left(\frac{x}{2}\right)\right)}\left(2A\sqrt{2}\sqrt{a\left(\sin^2\left(\frac{x}{2}\right)\right)}\sqrt{a}+B\ln\left(\frac{4a\sqrt{2}\cos\left(\frac{x}{2}\right)+4\sqrt{a}\sqrt{2}\sqrt{a\left(\sin^2\left(\frac{x}{2}\right)\right)}+8a}{2\cos\left(\frac{x}{2}\right)+\sqrt{2}}\right)a+B\ln\left(-\frac{4\left(\sqrt{a}\sqrt{2}\sqrt{a}\right.}{}\right.}{\sqrt{a}\sin\left(\frac{x}{2}\right)\sqrt{\left(\cos^2\left(\frac{x}{2}\right)\right)a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(x))^(1/2)*(A+B*sec(x)),x)
```

```
[Out] 1/a^(1/2)*cos(1/2*x)*(a*sin(1/2*x)^2)^(1/2)*(2*A*2^(1/2)*(a*sin(1/2*x)^2)^(
1/2)*a^(1/2)+B*ln(4/(2*cos(1/2*x)+2^(1/2))*(a*2^(1/2)*cos(1/2*x)+a^(1/2)*2^
(1/2)*(a*sin(1/2*x)^2)^(1/2)+2*a))*a+B*ln(-4/(-2*cos(1/2*x)+2^(1/2))*(a^(1/
2)*2^(1/2)*(a*sin(1/2*x)^2)^(1/2)-a*2^(1/2)*cos(1/2*x)+2*a))*a)/sin(1/2*x)/
(cos(1/2*x)^2*a)^(1/2)
```

maxima [A] time = 0.47, size = 13, normalized size = 0.30

$$2\sqrt{2}A\sqrt{a}\sin\left(\frac{1}{2}x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(x))^(1/2)*(A+B*sec(x)),x, algorithm="maxima")
```

```
[Out] 2*sqrt(2)*A*sqrt(a)*sin(1/2*x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \sqrt{a+a\cos(x)}\left(A+\frac{B}{\cos(x)}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*cos(x))^(1/2)*(A + B/cos(x)),x)
```

```
[Out] int((a + a*cos(x))^(1/2)*(A + B/cos(x)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a\left(\cos\left(x\right)+1\right)}\,\left(A+B\sec\left(x\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(x))**(1/2)*(A+B*sec(x)),x)
```

```
[Out] Integral(sqrt(a*(cos(x) + 1))*(A + B*sec(x)), x)
```

3.196 $\int \frac{A+B\sec(x)}{\sqrt{a+a\cos(x)}}\,dx$

Optimal. Leaf size=68

$$\frac{\sqrt{2}\,(A-B)\tanh^{-1}\left(\frac{\sqrt{a}\,\sin(x)}{\sqrt{2}\,\sqrt{a\cos(x)+a}}\right)}{\sqrt{a}}+\frac{2B\tanh^{-1}\left(\frac{\sqrt{a}\,\sin(x)}{\sqrt{a\cos(x)+a}}\right)}{\sqrt{a}}$$

```
[Out] 2*B*arctanh(sin(x)*a^(1/2)/(a+a*cos(x))^(1/2))/a^(1/2)+(A-B)*arctanh(1/2*si
n(x)*a^(1/2)*2^(1/2)/(a+a*cos(x))^(1/2))*2^(1/2)/a^(1/2)
```

Rubi [A] time = 0.20, antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.294, Rules used = {2828, 2985, 2649, 206, 2773}

$$\frac{\sqrt{2}\,(A-B)\tanh^{-1}\left(\frac{\sqrt{a}\,\sin(x)}{\sqrt{2}\,\sqrt{a\cos(x)+a}}\right)}{\sqrt{a}}+\frac{2B\tanh^{-1}\left(\frac{\sqrt{a}\,\sin(x)}{\sqrt{a\cos(x)+a}}\right)}{\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sec[x])/Sqrt[a + a*Cos[x]],x]
```

```
[Out] (2*B*ArcTanh[(Sqrt[a]*Sin[x])/Sqrt[a + a*Cos[x]]])/Sqrt[a] + (Sqrt[2]*(A -
B)*ArcTanh[(Sqrt[a]*Sin[x])/(Sqrt[2]*Sqrt[a + a*Cos[x]])])/Sqrt[a]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2773

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[(-2*b)/f, Subst[Int[1/(b*c + a*d - d*x^2), x
], x, (b*Cos[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2828

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.)*((a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)])^(m_.), x_Symbol] :> Int[((a + b*Sin[e + f*x])^m*(d + c*Sin[e +
 f*x])^n)/Sin[e + f*x]^n, x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IntegerQ
[n]
```

Rule 2985

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A
*b - a*B)/(b*c - a*d), Int[1/Sqrt[a + b*Sin[e + f*x]], x], x] + Dist[(B*c -
 A*d)/(b*c - a*d), Int[Sqrt[a + b*Sin[e + f*x]]/(c + d*Sin[e + f*x]), x], x
] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b
^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{A+B\sec(x)}{\sqrt{a+a\cos(x)}}\,dx &= \int \frac{(B+A\cos(x))\sec(x)}{\sqrt{a+a\cos(x)}}\,dx \\
&= \frac{B\int \sqrt{a+a\cos(x)}\,\sec(x)\,dx}{a} - (-A+B)\int \frac{1}{\sqrt{a+a\cos(x)}}\,dx \\
&= -\left((2(A-B))\,\text{Subst}\left(\int \frac{1}{2a-x^2}\,dx, x, -\frac{a\sin(x)}{\sqrt{a+a\cos(x)}}\right)\right) - (2B)\,\text{Subst}\left(\int \frac{1}{a-x^2}\,dx, x,\right. \\
&= \frac{2B\tanh^{-1}\left(\frac{\sqrt{a}\,\sin(x)}{\sqrt{a+a\cos(x)}}\right)}{\sqrt{a}} + \frac{\sqrt{2}\,(A-B)\tanh^{-1}\left(\frac{\sqrt{a}\,\sin(x)}{\sqrt{2}\,\sqrt{a+a\cos(x)}}\right)}{\sqrt{a}}
\end{aligned}$$

Mathematica [A] time = 0.05, size = 52, normalized size = 0.76

$$\frac{2\cos\left(\frac{x}{2}\right)\left((A-B)\tanh^{-1}\left(\sin\left(\frac{x}{2}\right)\right)+\sqrt{2}\,B\tanh^{-1}\left(\sqrt{2}\,\sin\left(\frac{x}{2}\right)\right)\right)}{\sqrt{a(\cos(x)+1)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Sec[x])/Sqrt[a + a*Cos[x]],x]
```

```
[Out] (2*((A - B)*ArcTanh[Sin[x/2]] + Sqrt[2]*B*ArcTanh[Sqrt[2]*Sin[x/2]])*Cos[x/
2])/Sqrt[a*(1 + Cos[x])]
```

fricas [B] time = 0.61, size = 116, normalized size = 1.71

$$-\frac{\sqrt{2}(A-B)\sqrt{a}\log\left(-\frac{\cos(x)^2+\frac{2\sqrt{2}\sqrt{a\cos(x)+a}\sin(x)}{\sqrt{a}}-2\cos(x)-3}{\cos(x)^2+2\cos(x)+1}\right)-B\sqrt{a}\log\left(\frac{a\cos(x)^3-7a\cos(x)^2-4\sqrt{a\cos(x)+a}\sqrt{a}(\cos(x)-2)\sin(x}{\cos(x)^3+\cos(x)^2}\right.}{2a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(x))/(a+a*cos(x))^(1/2),x, algorithm="fricas")
```

```
[Out] -1/2*(sqrt(2)*(A - B)*sqrt(a)*log(-(cos(x)^2 + 2*sqrt(2)*sqrt(a*cos(x) + a)
*sin(x)/sqrt(a) - 2*cos(x) - 3)/(cos(x)^2 + 2*cos(x) + 1)) - B*sqrt(a)*log(
(a*cos(x)^3 - 7*a*cos(x)^2 - 4*sqrt(a*cos(x) + a)*sqrt(a)*(cos(x) - 2)*sin(
x) + 8*a)/(cos(x)^3 + cos(x)^2)))/a
```

giac [B] time = 0.70, size = 133, normalized size = 1.96

$$-\frac{\sqrt{2}\left(A\sqrt{a}-B\sqrt{a}\right)\log\left(\left(\sqrt{a}\tan\left(\frac{1}{2}x\right)-\sqrt{a\tan\left(\frac{1}{2}x\right)^2+a}\right)^2\right)}{2a}+\frac{B\log\left(\left|\left(\sqrt{a}\tan\left(\frac{1}{2}x\right)-\sqrt{a\tan\left(\frac{1}{2}x\right)^2+a}\right)^2-\right.\right.}{\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(x))/(a+a*cos(x))^(1/2),x, algorithm="giac")
```

```
[Out] -1/2*sqrt(2)*(A*sqrt(a) - B*sqrt(a))*log((sqrt(a)*tan(1/2*x) - sqrt(a*tan(1
/2*x)^2 + a))^2)/a + B*log(abs((sqrt(a)*tan(1/2*x) - sqrt(a*tan(1/2*x)^2 +
a))^2 - a*(2*sqrt(2) + 3)))/sqrt(a) - B*log(abs((sqrt(a)*tan(1/2*x) - sqrt(
a*tan(1/2*x)^2 + a))^2 + a*(2*sqrt(2) - 3)))/sqrt(a)
```

maple [B] time = 0.35, size = 192, normalized size = 2.82

$$\frac{\cos\left(\frac{x}{2}\right)\sqrt{a\left(\sin^2\left(\frac{x}{2}\right)\right)}\left(\sqrt{2}\ln\left(\frac{4\sqrt{a}\sqrt{a\left(\sin^2\left(\frac{x}{2}\right)\right)}+4a}{\cos\left(\frac{x}{2}\right)}\right)A-\sqrt{2}\ln\left(\frac{4\sqrt{a}\sqrt{a\left(\sin^2\left(\frac{x}{2}\right)\right)}+4a}{\cos\left(\frac{x}{2}\right)}\right)B+B\ln\left(\frac{4a\sqrt{2}\cos\left(\frac{x}{2}\right)+4\sqrt{a}\sqrt{2}\sqrt{}}{2\cos\left(\frac{x}{2}\right)+\sqrt{}}\right.\right.}{\sqrt{a}\sin\left(\frac{x}{2}\right)\sqrt{\left(\cos^2\left(\frac{x}{2}\right)\right)a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sec(x))/(a+a*cos(x))^(1/2),x)
```

```
[Out] cos(1/2*x)*(a*sin(1/2*x)^2)^(1/2)*(2^(1/2)*ln(4/cos(1/2*x)*(a^(1/2)*(a*sin(
1/2*x)^2)^(1/2)+a))*A-2^(1/2)*ln(4/cos(1/2*x)*(a^(1/2)*(a*sin(1/2*x)^2)^(1/
2)+a))*B+B*ln(4/(2*cos(1/2*x)+2^(1/2))*(a*2^(1/2)*cos(1/2*x)+a^(1/2)*2^(1/2
```

```
)*(a*sin(1/2*x)^2)^(1/2)+2*a))+B*ln(-4/(-2*cos(1/2*x)+2^(1/2))*(a^(1/2)*2^(
1/2)*(a*sin(1/2*x)^2)^(1/2)-a*2^(1/2)*cos(1/2*x)+2*a)))/a^(1/2)/sin(1/2*x)/
(cos(1/2*x)^2*a)^(1/2)
```

maxima [A] time = 1.05, size = 58, normalized size = 0.85

$$\frac{\left(\sqrt{2}\log\left(\cos\left(\frac{1}{2}x\right)^2+\sin\left(\frac{1}{2}x\right)^2+2\sin\left(\frac{1}{2}x\right)+1\right)-\sqrt{2}\log\left(\cos\left(\frac{1}{2}x\right)^2+\sin\left(\frac{1}{2}x\right)^2-2\sin\left(\frac{1}{2}x\right)+1\right)\right)A}{2\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(x))/(a+a*cos(x))^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*(sqrt(2)*log(cos(1/2*x)^2 + sin(1/2*x)^2 + 2*sin(1/2*x) + 1) - sqrt(2)*
log(cos(1/2*x)^2 + sin(1/2*x)^2 - 2*sin(1/2*x) + 1))*A/sqrt(a)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{A+\frac{B}{\cos(x)}}{\sqrt{a+a\cos(x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B/cos(x))/(a + a*cos(x))^(1/2),x)
```

```
[Out] int((A + B/cos(x))/(a + a*cos(x))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A+B\sec(x)}{\sqrt{a(\cos(x)+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(x))/(a+a*cos(x))**(1/2),x)
```

```
[Out] Integral((A + B*sec(x))/sqrt(a*(cos(x) + 1)), x)
```

3.197 $\int \frac{A+B\sec(x)}{(a+a\cos(x))^{3/2}}\,dx$

Optimal. Leaf size=92

$$\frac{(A-5B)\tanh^{-1}\left(\frac{\sqrt{a}\sin(x)}{\sqrt{2}\sqrt{a\cos(x)+a}}\right)}{2\sqrt{2}\,a^{3/2}}+\frac{2B\tanh^{-1}\left(\frac{\sqrt{a}\sin(x)}{\sqrt{a\cos(x)+a}}\right)}{a^{3/2}}+\frac{(A-B)\sin(x)}{2(a\cos(x)+a)^{3/2}}$$

```
[Out] 2*B*arctanh(sin(x)*a^(1/2)/(a+a*cos(x))^(1/2))/a^(3/2)+1/2*(A-B)*sin(x)/(a+
a*cos(x))^(3/2)+1/4*(A-5*B)*arctanh(1/2*sin(x)*a^(1/2)*2^(1/2)/(a+a*cos(x))
^(1/2))/a^(3/2)*2^(1/2)
```

Rubi [A] time = 0.34, antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.353, Rules used = {2828, 2978, 2985, 2649, 206, 2773}

$$\frac{(A-5B)\tanh^{-1}\left(\frac{\sqrt{a}\sin(x)}{\sqrt{2}\sqrt{a\cos(x)+a}}\right)}{2\sqrt{2}\,a^{3/2}}+\frac{2B\tanh^{-1}\left(\frac{\sqrt{a}\sin(x)}{\sqrt{a\cos(x)+a}}\right)}{a^{3/2}}+\frac{(A-B)\sin(x)}{2(a\cos(x)+a)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sec[x])/(a + a*Cos[x])^(3/2),x]
```

```
[Out] (2*B*ArcTanh[(Sqrt[a]*Sin[x])/Sqrt[a + a*Cos[x]]])/a^(3/2) + ((A - 5*B)*Arc
Tanh[(Sqrt[a]*Sin[x])/(Sqrt[2]*Sqrt[a + a*Cos[x]])])/(2*Sqrt[2]*a^(3/2)) +
((A - B)*Sin[x])/(2*(a + a*Cos[x])^(3/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2773

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[(-2*b)/f, Subst[Int[1/(b*c + a*d - d*x^2), x
], x, (b*Cos[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, c, d,
```

```
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2828

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.)*((a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)])^(m_.), x_Symbol] :> Int[((a + b*Sin[e + f*x])^m*(d + c*Sin[e +
 f*x])^n)/Sin[e + f*x]^n, x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IntegerQ
[n]
```

Rule 2978

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(
n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2985

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A
*b - a*B)/(b*c - a*d), Int[1/Sqrt[a + b*Sin[e + f*x]], x], x] + Dist[(B*c -
 A*d)/(b*c - a*d), Int[Sqrt[a + b*Sin[e + f*x]]/(c + d*Sin[e + f*x]), x], x
] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b
^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{A+B\sec(x)}{(a+a\cos(x))^{3/2}}\,dx = \int \frac{(B+A\cos(x))\sec(x)}{(a+a\cos(x))^{3/2}}\,dx$$
$$= \frac{(A-B)\sin(x)}{2(a+a\cos(x))^{3/2}} + \frac{\int \frac{\left(2aB+\frac{1}{2}a(A-B)\cos(x)\right)\sec(x)}{\sqrt{a+a\cos(x)}}\,dx}{2a^2}$$
$$= \frac{(A-B)\sin(x)}{2(a+a\cos(x))^{3/2}} + \frac{(A-5B)\int \frac{1}{\sqrt{a+a\cos(x)}}\,dx}{4a} + \frac{B\int \sqrt{a+a\cos(x)}\,\sec(x)\,dx}{a^2}$$
$$= \frac{(A-B)\sin(x)}{2(a+a\cos(x))^{3/2}} - \frac{(A-5B)\,\text{Subst}\left(\int \frac{1}{2a-x^2}\,dx, x, -\frac{a\sin(x)}{\sqrt{a+a\cos(x)}}\right)}{2a} - \frac{(2B)\,\text{Subst}\left(\int \frac{1}{a-x^2}\,d\right.}{a}$$
$$= \frac{2B\tanh^{-1}\left(\frac{\sqrt{a}\sin(x)}{\sqrt{a+a\cos(x)}}\right)}{a^{3/2}} + \frac{(A-5B)\tanh^{-1}\left(\frac{\sqrt{a}\sin(x)}{\sqrt{2}\sqrt{a+a\cos(x)}}\right)}{2\sqrt{2}\,a^{3/2}} + \frac{(A-B)\sin(x)}{2(a+a\cos(x))^{3/2}}$$

Mathematica [A] time = 0.37, size = 73, normalized size = 0.79

$$\frac{\frac{1}{2}(A-B)\sin(x)+(A-5B)\cos^3\left(\frac{x}{2}\right)\tanh^{-1}\left(\sin\left(\frac{x}{2}\right)\right)+4\sqrt{2}\,B\cos^3\left(\frac{x}{2}\right)\tanh^{-1}\left(\sqrt{2}\,\sin\left(\frac{x}{2}\right)\right)}{(a(\cos(x)+1))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Sec[x])/(a + a*Cos[x])^(3/2),x]
```

```
[Out] ((A - 5*B)*ArcTanh[Sin[x/2]]*Cos[x/2]^3 + 4*Sqrt[2]*B*ArcTanh[Sqrt[2]*Sin[x
/2]]*Cos[x/2]^3 + ((A - B)*Sin[x])/2)/(a*(1 + Cos[x]))^(3/2)
```

fricas [B] time = 0.70, size = 187, normalized size = 2.03

$$-\frac{\sqrt{2}\left((A-5\,B)\cos(x)^2+2\,(A-5\,B)\cos(x)+A-5\,B\right)\sqrt{a}\,\log\left(-\frac{a\cos(x)^2+2\,\sqrt{2}\,\sqrt{a\cos(x)+a}\,\sqrt{a}\,\sin(x)-2\,a\cos(x)-3\,a}{\cos(x)^2+2\,\cos(x)+1}\right)-4\,(}{8\left(a^2\cos(x)^2\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(x))/(a+a*cos(x))^(3/2),x, algorithm="fricas")
```

```
[Out] -1/8*(sqrt(2)*((A - 5*B)*cos(x)^2 + 2*(A - 5*B)*cos(x) + A - 5*B)*sqrt(a)*l
og(-(a*cos(x)^2 + 2*sqrt(2)*sqrt(a*cos(x) + a)*sqrt(a)*sin(x) - 2*a*cos(x)
- 3*a)/(cos(x)^2 + 2*cos(x) + 1)) - 4*(B*cos(x)^2 + 2*B*cos(x) + B)*sqrt(a)
*log((a*cos(x)^3 - 7*a*cos(x)^2 - 4*sqrt(a*cos(x) + a)*sqrt(a)*(cos(x) - 2)
```

*sin(x) + 8*a)/(cos(x)^3 + cos(x)^2)) - 4*sqrt(a*cos(x) + a)*(A - B)*sin(x)
)/(a^2*cos(x)^2 + 2*a^2*cos(x) + a^2)

giac [B] time = 0.76, size = 168, normalized size = 1.83

$$-\frac{\sqrt{2}\left(A\sqrt{a}-5\,B\sqrt{a}\right)\log\left(\left(\sqrt{a}\,\tan\left(\frac{1}{2}\,x\right)-\sqrt{a\tan\left(\frac{1}{2}\,x\right)^{2}+a}\right)^{2}\right)}{8\,a^{2}}+\frac{B\log\left(\left|\left(\sqrt{a}\,\tan\left(\frac{1}{2}\,x\right)-\sqrt{a\tan\left(\frac{1}{2}\,x\right)^{2}+a}\right.\right.\right.}{a^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(x))/(a+a*cos(x))^(3/2),x, algorithm="giac")
```

```
[Out] -1/8*sqrt(2)*(A*sqrt(a) - 5*B*sqrt(a))*log((sqrt(a)*tan(1/2*x) - sqrt(a*tan
(1/2*x)^2 + a))^2)/a^2 + B*log(abs((sqrt(a)*tan(1/2*x) - sqrt(a*tan(1/2*x)^
2 + a))^2 - a*(2*sqrt(2) + 3)))/a^(3/2) - B*log(abs((sqrt(a)*tan(1/2*x) - s
qrt(a*tan(1/2*x)^2 + a))^2 + a*(2*sqrt(2) - 3)))/a^(3/2) + 1/4*sqrt(a*tan(1
/2*x)^2 + a)*(sqrt(2)*A*a - sqrt(2)*B*a)*tan(1/2*x)/a^3
```

maple [B] time = 0.34, size = 270, normalized size = 2.93

$$\frac{\sqrt{a\left(\sin^{2}\left(\frac{x}{2}\right)\right)}\,\left(A\sqrt{2}\,\ln\left(\frac{4\sqrt{a}\,\sqrt{a\left(\sin^{2}\left(\frac{x}{2}\right)\right)}+4a}{\cos\left(\frac{x}{2}\right)}\right)\left(\cos^{2}\left(\frac{x}{2}\right)\right)a-5B\sqrt{2}\,\ln\left(\frac{4\sqrt{a}\,\sqrt{a\left(\sin^{2}\left(\frac{x}{2}\right)\right)}+4a}{\cos\left(\frac{x}{2}\right)}\right)\left(\cos^{2}\left(\frac{x}{2}\right)\right)a+4B\ln\left(\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sec(x))/(a+a*cos(x))^(3/2),x)
```

```
[Out] 1/4/a^(5/2)/cos(1/2*x)*(a*sin(1/2*x)^2)^(1/2)*(A*2^(1/2)*ln(2*(2*a^(1/2)*(a
*sin(1/2*x)^2)^(1/2)+2*a)/cos(1/2*x))*cos(1/2*x)^2*a-5*B*2^(1/2)*ln(2*(2*a^
(1/2)*(a*sin(1/2*x)^2)^(1/2)+2*a)/cos(1/2*x))*cos(1/2*x)^2*a+4*B*ln(4/(2*co
s(1/2*x)+2^(1/2))*(a*2^(1/2)*cos(1/2*x)+a^(1/2)*2^(1/2)*(a*sin(1/2*x)^2)^(1
/2)+2*a))*cos(1/2*x)^2*a+4*B*ln(-4*(a*2^(1/2)*cos(1/2*x)-a^(1/2)*2^(1/2)*(a
*sin(1/2*x)^2)^(1/2)-2*a)/(2*cos(1/2*x)-2^(1/2)))*cos(1/2*x)^2*a+A*2^(1/2)*
(a*sin(1/2*x)^2)^(1/2)*a^(1/2)-B*2^(1/2)*(a*sin(1/2*x)^2)^(1/2)*a^(1/2))/si
n(1/2*x)/(cos(1/2*x)^2*a)^(1/2)
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(x))/(a+a*cos(x))^(3/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{A + \frac{B}{\cos(x)}}{(a + a\,\cos(x))^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B/cos(x))/(a + a*cos(x))^(3/2),x)
```

```
[Out] int((A + B/cos(x))/(a + a*cos(x))^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A + B\sec{(x)}}{(a(\cos{(x)} + 1))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(x))/(a+a*cos(x))**(3/2),x)
```

```
[Out] Integral((A + B*sec(x))/(a*(cos(x) + 1))**(3/2), x)
```

3.198 $\int \frac{A+B\sec(x)}{(a+a\cos(x))^{5/2}}\,dx$

Optimal. Leaf size=120

$$\frac{(3A-43B)\tanh^{-1}\left(\frac{\sqrt{a}\sin(x)}{\sqrt{2}\sqrt{a\cos(x)+a}}\right)}{16\sqrt{2}\,a^{5/2}}+\frac{2B\tanh^{-1}\left(\frac{\sqrt{a}\sin(x)}{\sqrt{a\cos(x)+a}}\right)}{a^{5/2}}+\frac{(3A-11B)\sin(x)}{16a(a\cos(x)+a)^{3/2}}+\frac{(A-B)\sin(x)}{4(a\cos(x)+a)^{5/2}}$$

```
[Out] 2*B*arctanh(sin(x)*a^(1/2)/(a+a*cos(x))^(1/2))/a^(5/2)+1/4*(A-B)*sin(x)/(a+
a*cos(x))^(5/2)+1/16*(3*A-11*B)*sin(x)/a/(a+a*cos(x))^(3/2)+1/32*(3*A-43*B)
*arctanh(1/2*sin(x)*a^(1/2)*2^(1/2)/(a+a*cos(x))^(1/2))/a^(5/2)*2^(1/2)
```

Rubi [A] time = 0.48, antiderivative size = 120, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.353, Rules used = {2828, 2978, 2985, 2649, 206, 2773}

$$\frac{(3A-43B)\tanh^{-1}\left(\frac{\sqrt{a}\sin(x)}{\sqrt{2}\sqrt{a\cos(x)+a}}\right)}{16\sqrt{2}\,a^{5/2}}+\frac{2B\tanh^{-1}\left(\frac{\sqrt{a}\sin(x)}{\sqrt{a\cos(x)+a}}\right)}{a^{5/2}}+\frac{(3A-11B)\sin(x)}{16a(a\cos(x)+a)^{3/2}}+\frac{(A-B)\sin(x)}{4(a\cos(x)+a)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sec[x])/(a + a*Cos[x])^(5/2),x]
```

```
[Out] (2*B*ArcTanh[(Sqrt[a]*Sin[x])/Sqrt[a + a*Cos[x]]])/a^(5/2) + ((3*A - 43*B)*
ArcTanh[(Sqrt[a]*Sin[x])/(Sqrt[2]*Sqrt[a + a*Cos[x]])])/(16*Sqrt[2]*a^(5/2)
) + ((A - B)*Sin[x])/(4*(a + a*Cos[x])^(5/2)) + ((3*A - 11*B)*Sin[x])/(16*a
*(a + a*Cos[x])^(3/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2773

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[(-2*b)/f, Subst[Int[1/(b*c + a*d - d*x^2), x
```

```
], x, (b*Cos[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2828

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.)*((a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)])^(m_.), x_Symbol] :> Int[((a + b*Sin[e + f*x])^m*(d + c*Sin[e +
 f*x])^n)/Sin[e + f*x]^n, x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IntegerQ
[n]
```

Rule 2978

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(
n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2985

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A
*b - a*B)/(b*c - a*d), Int[1/Sqrt[a + b*Sin[e + f*x]], x], x] + Dist[(B*c -
 A*d)/(b*c - a*d), Int[Sqrt[a + b*Sin[e + f*x]]/(c + d*Sin[e + f*x]), x], x
] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b
^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{A+B\sec(x)}{(a+a\cos(x))^{5/2}}\,dx=\int \frac{(B+A\cos(x))\sec(x)}{(a+a\cos(x))^{5/2}}\,dx$$

$$=\frac{(A-B)\sin(x)}{4(a+a\cos(x))^{5/2}}+\frac{\int \frac{\left(4aB+\frac{3}{2}a(A-B)\cos(x)\right)\sec(x)}{(a+a\cos(x))^{3/2}}\,dx}{4a^2}$$

$$=\frac{(A-B)\sin(x)}{4(a+a\cos(x))^{5/2}}+\frac{(3A-11B)\sin(x)}{16a(a+a\cos(x))^{3/2}}+\frac{\int \frac{\left(8a^2B+\frac{1}{4}a^2(3A-11B)\cos(x)\right)\sec(x)}{\sqrt{a+a\cos(x)}}\,dx}{8a^4}$$

$$=\frac{(A-B)\sin(x)}{4(a+a\cos(x))^{5/2}}+\frac{(3A-11B)\sin(x)}{16a(a+a\cos(x))^{3/2}}+\frac{(3A-43B)\int \frac{1}{\sqrt{a+a\cos(x)}}\,dx}{32a^2}+\frac{B\int \sqrt{a+a}$$

$$=\frac{(A-B)\sin(x)}{4(a+a\cos(x))^{5/2}}+\frac{(3A-11B)\sin(x)}{16a(a+a\cos(x))^{3/2}}-\frac{(3A-43B)\,\text{Subst}\left(\int \frac{1}{2a-x^2}\,dx,x,-\frac{a\sin(x)}{\sqrt{a+a\cos}}\right.}{16a^2}$$

$$=\frac{2B\tanh^{-1}\left(\frac{\sqrt{a}\,\sin(x)}{\sqrt{a+a\cos(x)}}\right)}{a^{5/2}}+\frac{(3A-43B)\tanh^{-1}\left(\frac{\sqrt{a}\,\sin(x)}{\sqrt{2}\,\sqrt{a+a\cos(x)}}\right)}{16\sqrt{2}\,a^{5/2}}+\frac{(A-B)\sin(x)}{4(a+a\cos(x))^{5/2}}+\frac{}{1}$$

Mathematica [A] time = 0.50, size = 95, normalized size = 0.79

$$\frac{\tan\left(\frac{x}{2}\right)(3A\cos(x)+7A-11B\cos(x)-15B)+2(3A-43B)\cos^3\left(\frac{x}{2}\right)\tanh^{-1}\left(\sin\left(\frac{x}{2}\right)\right)+64\sqrt{2}B\cos^3\left(\frac{x}{2}\right)\tanh}{16a(a(\cos(x)+1))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Sec[x])/(a + a*Cos[x])^(5/2),x]
```

```
[Out] (2*(3*A - 43*B)*ArcTanh[Sin[x/2]]*Cos[x/2]^3 + 64*Sqrt[2]*B*ArcTanh[Sqrt[2]
*Sin[x/2]]*Cos[x/2]^3 + (7*A - 15*B + 3*A*Cos[x] - 11*B*Cos[x])*Tan[x/2])/(
16*a*(a*(1 + Cos[x]))^(3/2))
```

fricas [B] time = 0.73, size = 234, normalized size = 1.95

$$-\frac{\sqrt{2}\left((3A-43B)\cos(x)^3+3(3A-43B)\cos(x)^2+3(3A-43B)\cos(x)+3A-43B\right)\sqrt{a}\,\log\left(-\frac{a\cos(x)^2+2\sqrt{2}}{}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(x))/(a+a*cos(x))^(5/2),x, algorithm="fricas")
```

```
[Out] -1/64*(sqrt(2)*((3*A - 43*B)*cos(x)^3 + 3*(3*A - 43*B)*cos(x)^2 + 3*(3*A -
43*B)*cos(x) + 3*A - 43*B)*sqrt(a)*log(-(a*cos(x)^2 + 2*sqrt(2)*sqrt(a*cos(
x) + a)*sqrt(a)*sin(x) - 2*a*cos(x) - 3*a)/(cos(x)^2 + 2*cos(x) + 1)) - 32*
(B*cos(x)^3 + 3*B*cos(x)^2 + 3*B*cos(x) + B)*sqrt(a)*log((a*cos(x)^3 - 7*a*
cos(x)^2 - 4*sqrt(a*cos(x) + a)*sqrt(a)*(cos(x) - 2)*sin(x) + 8*a)/(cos(x)^
3 + cos(x)^2)) - 4*((3*A - 11*B)*cos(x) + 7*A - 15*B)*sqrt(a*cos(x) + a)*si
n(x))/(a^3*cos(x)^3 + 3*a^3*cos(x)^2 + 3*a^3*cos(x) + a^3)
```

giac [B] time = 0.95, size = 199, normalized size = 1.66

$$\frac{1}{32}\sqrt{a\tan\left(\frac{1}{2}x\right)^2+a}\left(\frac{2\sqrt{2}\left(Aa^5-Ba^5\right)\tan\left(\frac{1}{2}x\right)^2}{a^8}+\frac{\sqrt{2}\left(5\,Aa^5-13\,Ba^5\right)}{a^8}\right)\tan\left(\frac{1}{2}x\right)-\frac{\sqrt{2}\left(3\,A\sqrt{a}\,-43\,B\sqrt{a}\,\right)\mathrm{l}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(x))/(a+a*cos(x))^(5/2),x, algorithm="giac")
```

```
[Out] 1/32*sqrt(a*tan(1/2*x)^2 + a)*(2*sqrt(2)*(A*a^5 - B*a^5)*tan(1/2*x)^2/a^8 +
 sqrt(2)*(5*A*a^5 - 13*B*a^5)/a^8)*tan(1/2*x) - 1/64*sqrt(2)*(3*A*sqrt(a) -
 43*B*sqrt(a))*log((sqrt(a)*tan(1/2*x) - sqrt(a*tan(1/2*x)^2 + a))^2)/a^3 +
 B*log(abs((sqrt(a)*tan(1/2*x) - sqrt(a*tan(1/2*x)^2 + a))^2 - a*(2*sqrt(2)
 + 3)))/a^(5/2) - B*log(abs((sqrt(a)*tan(1/2*x) - sqrt(a*tan(1/2*x)^2 + a))
^2 + a*(2*sqrt(2) - 3)))/a^(5/2)
```

maple [B] time = 0.36, size = 322, normalized size = 2.68

$$\frac{\sqrt{a\left(\sin^2\left(\frac{x}{2}\right)\right)}\left(3A\sqrt{2}\;\ln\left(\frac{4\sqrt{a}\;\sqrt{a\left(\sin^2\left(\frac{x}{2}\right)\right)}+4a}{\cos\left(\frac{x}{2}\right)}\right)\left(\cos^4\left(\frac{x}{2}\right)\right)a-43B\sqrt{2}\;\ln\left(\frac{4\sqrt{a}\;\sqrt{a\left(\sin^2\left(\frac{x}{2}\right)\right)}+4a}{\cos\left(\frac{x}{2}\right)}\right)a\left(\cos^4\left(\frac{x}{2}\right)\right)+32B\ln}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sec(x))/(a+a*cos(x))^(5/2),x)
```

```
[Out] 1/32/a^(7/2)/cos(1/2*x)^3*(a*sin(1/2*x)^2)^(1/2)*(3*A*2^(1/2)*ln(2*(2*a^(1/
2)*(a*sin(1/2*x)^2)^(1/2)+2*a)/cos(1/2*x))*cos(1/2*x)^4*a-43*B*2^(1/2)*ln(2
*(2*a^(1/2)*(a*sin(1/2*x)^2)^(1/2)+2*a)/cos(1/2*x))*a*cos(1/2*x)^4+32*B*ln(
4/(2*cos(1/2*x)+2^(1/2))*(a*2^(1/2)*cos(1/2*x)+a^(1/2)*2^(1/2)*(a*sin(1/2*x
)^2)^(1/2)+2*a))*a*cos(1/2*x)^4+32*B*ln(-4*(a*2^(1/2)*cos(1/2*x)-a^(1/2)*2^
(1/2)*(a*sin(1/2*x)^2)^(1/2)-2*a)/(2*cos(1/2*x)-2^(1/2)))*a*cos(1/2*x)^4+3*
A*a^(1/2)*2^(1/2)*(a*sin(1/2*x)^2)^(1/2)*cos(1/2*x)^2-11*B*a^(1/2)*2^(1/2)*
```

```
(a*sin(1/2*x)^2)^(1/2)*cos(1/2*x)^2+2*A*2^(1/2)*(a*sin(1/2*x)^2)^(1/2)*a^(1
/2)-2*B*2^(1/2)*(a*sin(1/2*x)^2)^(1/2)*a^(1/2))/sin(1/2*x)/(cos(1/2*x)^2*a)
^(1/2)
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(x))/(a+a*cos(x))^(5/2),x, algorithm="maxima")
[Out] Timed out
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{A+\frac{B}{\cos(x)}}{(a+a\cos(x))^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B/cos(x))/(a + a*cos(x))^(5/2),x)
[Out] int((A + B/cos(x))/(a + a*cos(x))^(5/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A+B\sec(x)}{(a(\cos(x)+1))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(x))/(a+a*cos(x))**(5/2),x)
[Out] Integral((A + B*sec(x))/(a*(cos(x) + 1))**(5/2), x)
```

3.199 $\int \frac{x(b+a\sin(x))}{(a+b\sin(x))^2}\,dx$

Optimal. Leaf size=25

$$\frac{\log(a+b\sin(x))}{b}-\frac{x\cos(x)}{a+b\sin(x)}$$

```
[Out] ln(a+b*sin(x))/b-x*cos(x)/(a+b*sin(x))
```

Rubi [A] time = 0.05, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {4592, 2668, 31}

$$\frac{\log(a+b\sin(x))}{b}-\frac{x\cos(x)}{a+b\sin(x)}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(b + a*Sin[x]))/(a + b*Sin[x])^2,x]
```

```
[Out] Log[a + b*Sin[x]]/b - (x*Cos[x])/(a + b*Sin[x])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2668

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rule 4592

```
Int[(((e_.) + (f_.)*(x_))*((A_) + (B_.)*Sin[(c_.) + (d_.)*(x_)]))/((a_) + (
b_.)*Sin[(c_.) + (d_.)*(x_)])^2, x_Symbol] :> -Simp[(B*(e + f*x)*Cos[c + d*
x])/(a*d*(a + b*Sin[c + d*x])), x] + Dist[(B*f)/(a*d), Int[Cos[c + d*x]/(a
+ b*Sin[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && EqQ[a*A
- b*B, 0]
```

Rubi steps

$$\int \frac{x(b+a\sin(x))}{(a+b\sin(x))^2}\,dx = -\frac{x\cos(x)}{a+b\sin(x)} + \int \frac{\cos(x)}{a+b\sin(x)}\,dx$$
$$= -\frac{x\cos(x)}{a+b\sin(x)} + \frac{\text{Subst}\left(\int \frac{1}{a+x}\,dx, x, b\sin(x)\right)}{b}$$
$$= \frac{\log(a+b\sin(x))}{b} - \frac{x\cos(x)}{a+b\sin(x)}$$

Mathematica [A] time = 0.20, size = 25, normalized size = 1.00

$$\frac{\log(a+b\sin(x))}{b} - \frac{x\cos(x)}{a+b\sin(x)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(b + a*Sin[x]))/(a + b*Sin[x])^2,x]
[Out] Log[a + b*Sin[x]]/b - (x*Cos[x])/(a + b*Sin[x])
```

fricas [A] time = 0.71, size = 35, normalized size = 1.40

$$-\frac{bx\cos(x) - (b\sin(x)+a)\log(b\sin(x)+a)}{b^2\sin(x)+ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b+a*sin(x))/(a+b*sin(x))^2,x, algorithm="fricas")
[Out] -(b*x*cos(x) - (b*sin(x) + a)*log(b*sin(x) + a))/(b^2*sin(x) + a*b)
```

giac [B] time = 0.32, size = 283, normalized size = 11.32

$$4\,bx\tan\left(\frac{1}{2}\,x\right)^2 + a\log\left(\frac{4\left(a^2\tan\left(\frac{1}{2}\,x\right)^4 + 4\,ab\tan\left(\frac{1}{2}\,x\right)^3 + 2\,a^2\tan\left(\frac{1}{2}\,x\right)^2 + 4\,b^2\tan\left(\frac{1}{2}\,x\right)^2 + 4\,ab\tan\left(\frac{1}{2}\,x\right) + a^2\right)}{\tan\left(\frac{1}{2}\,x\right)^4 + 2\,\tan\left(\frac{1}{2}\,x\right)^2 + 1}\right)\tan\left(\frac{1}{2}\,x\right)^2 + 2\,b\log\left(\frac{4(}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b+a*sin(x))/(a+b*sin(x))^2,x, algorithm="giac")
[Out] 1/2*(4*b*x*tan(1/2*x)^2 + a*log(4*(a^2*tan(1/2*x)^4 + 4*a*b*tan(1/2*x)^3 +
2*a^2*tan(1/2*x)^2 + 4*b^2*tan(1/2*x)^2 + 4*a*b*tan(1/2*x) + a^2)/(tan(1/2*
```

```
x)^4 + 2*tan(1/2*x)^2 + 1))*tan(1/2*x)^2 + 2*b*log(4*(a^2*tan(1/2*x)^4 + 4*
a*b*tan(1/2*x)^3 + 2*a^2*tan(1/2*x)^2 + 4*b^2*tan(1/2*x)^2 + 4*a*b*tan(1/2*
x) + a^2)/(tan(1/2*x)^4 + 2*tan(1/2*x)^2 + 1))*tan(1/2*x) - 4*b*x + a*log(4
*(a^2*tan(1/2*x)^4 + 4*a*b*tan(1/2*x)^3 + 2*a^2*tan(1/2*x)^2 + 4*b^2*tan(1/
2*x)^2 + 4*a*b*tan(1/2*x) + a^2)/(tan(1/2*x)^4 + 2*tan(1/2*x)^2 + 1)))/(a*b
*tan(1/2*x)^2 + 2*b^2*tan(1/2*x) + a*b)
```

maple [B] time = 0.70, size = 80, normalized size = 3.20

$$\frac{x\left(\tan^4\left(\frac{x}{2}\right)\right)-x}{\left(1+\tan^2\left(\frac{x}{2}\right)\right)\left(a\left(\tan^2\left(\frac{x}{2}\right)\right)+2b\tan\left(\frac{x}{2}\right)+a\right)}+\frac{\ln\left(a\left(\tan^2\left(\frac{x}{2}\right)\right)+2b\tan\left(\frac{x}{2}\right)+a\right)}{b}-\frac{\ln\left(1+\tan^2\left(\frac{x}{2}\right)\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b+a*sin(x))/(a+b*sin(x))^2,x)
```

```
[Out] (x*tan(1/2*x)^4-x)/(1+tan(1/2*x)^2)/(a*tan(1/2*x)^2+2*b*tan(1/2*x)+a)+1/b*l
n(a*tan(1/2*x)^2+2*b*tan(1/2*x)+a)-1/b*ln(1+tan(1/2*x)^2)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b+a*sin(x))/(a+b*sin(x))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more details)Is 4*b^2-4*a^2 positive or negative?
```

mupad [F] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{x\ (b+a\ \sin(x))}{(a+b\ \sin(x))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(b + a*sin(x)))/(a + b*sin(x))^2,x)
```

```
[Out] int((x*(b + a*sin(x)))/(a + b*sin(x))^2, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b+a*sin(x))/(a+b*sin(x))**2,x)
```

```
[Out] Timed out
```

3.200 $\int \frac{x(b+a\cos(x))}{(a+b\cos(x))^2}\,dx$

Optimal. Leaf size=24

$$\frac{\log(a+b\cos(x))}{b}+\frac{x\sin(x)}{a+b\cos(x)}$$

```
[Out] ln(a+b*cos(x))/b+x*sin(x)/(a+b*cos(x))
```

Rubi [A] time = 0.06, antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {4593, 2668, 31}

$$\frac{\log(a+b\cos(x))}{b}+\frac{x\sin(x)}{a+b\cos(x)}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(b + a*Cos[x]))/(a + b*Cos[x])^2,x]
```

```
[Out] Log[a + b*Cos[x]]/b + (x*Sin[x])/(a + b*Cos[x])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2668

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p - 1)/2] && NeQ[a^2 - b^2, 0]
```

Rule 4593

```
Int[((Cos[(c_.) + (d_.)*(x_)]*(B_.) + (A_))*((e_.) + (f_.)*(x_)))/(Cos[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^2, x_Symbol] :> Simp[(B*(e + f*x)*Sin[c + d*x])/(a*d*(a + b*Cos[c + d*x])), x] - Dist[(B*f)/(a*d), Int[Sin[c + d*x]/(a + b*Cos[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && EqQ[a*A - b*B, 0]
```

Rubi steps

$$\int \frac{x(b+a\cos(x))}{(a+b\cos(x))^2}\,dx = \frac{x\sin(x)}{a+b\cos(x)} - \int \frac{\sin(x)}{a+b\cos(x)}\,dx$$
$$= \frac{x\sin(x)}{a+b\cos(x)} + \frac{\text{Subst}\left(\int \frac{1}{a+x}\,dx, x, b\cos(x)\right)}{b}$$
$$= \frac{\log(a+b\cos(x))}{b} + \frac{x\sin(x)}{a+b\cos(x)}$$

Mathematica [A] time = 0.13, size = 24, normalized size = 1.00

$$\frac{\log(a+b\cos(x))}{b} + \frac{x\sin(x)}{a+b\cos(x)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(b + a*Cos[x]))/(a + b*Cos[x])^2,x]
```

```
[Out] Log[a + b*Cos[x]]/b + (x*Sin[x])/(a + b*Cos[x])
```

fricas [A] time = 0.74, size = 36, normalized size = 1.50

$$\frac{bx\sin(x) + (b\cos(x)+a)\log\left(-b\cos(x)-a\right)}{b^2\cos(x)+ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b+a*cos(x))/(a+b*cos(x))^2,x, algorithm="fricas")
```

```
[Out] (b*x*sin(x) + (b*cos(x) + a)*log(-b*cos(x) - a))/(b^2*cos(x) + a*b)
```

giac [B] time = 0.32, size = 397, normalized size = 16.54

$$a\log\left(\frac{4\left(a^2\tan\left(\frac{1}{2}x\right)^4 - 2ab\tan\left(\frac{1}{2}x\right)^4 + b^2\tan\left(\frac{1}{2}x\right)^4 + 2a^2\tan\left(\frac{1}{2}x\right)^2 - 2b^2\tan\left(\frac{1}{2}x\right)^2 + a^2 + 2ab + b^2\right)}{\tan\left(\frac{1}{2}x\right)^4 + 2\tan\left(\frac{1}{2}x\right)^2 + 1}\right)\tan\left(\frac{1}{2}x\right)^2 - b\log\left(\frac{4\left(a^2\tan\left(\frac{1}{2}x\right)^4 - 2a\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b+a*cos(x))/(a+b*cos(x))^2,x, algorithm="giac")
```

```
[Out] 1/2*(a*log(4*(a^2*tan(1/2*x)^4 - 2*a*b*tan(1/2*x)^4 + b^2*tan(1/2*x)^4 + 2*
a^2*tan(1/2*x)^2 - 2*b^2*tan(1/2*x)^2 + a^2 + 2*a*b + b^2)/(tan(1/2*x)^4 +
```

```
2*tan(1/2*x)^2 + 1))*tan(1/2*x)^2 - b*log(4*(a^2*tan(1/2*x)^4 - 2*a*b*tan(1
/2*x)^4 + b^2*tan(1/2*x)^4 + 2*a^2*tan(1/2*x)^2 - 2*b^2*tan(1/2*x)^2 + a^2
+ 2*a*b + b^2)/(tan(1/2*x)^4 + 2*tan(1/2*x)^2 + 1))*tan(1/2*x)^2 + 8*b*x*ta
n(1/2*x) + a*log(4*(a^2*tan(1/2*x)^4 - 2*a*b*tan(1/2*x)^4 + b^2*tan(1/2*x)^
4 + 2*a^2*tan(1/2*x)^2 - 2*b^2*tan(1/2*x)^2 + a^2 + 2*a*b + b^2)/(tan(1/2*x
)^4 + 2*tan(1/2*x)^2 + 1)) + b*log(4*(a^2*tan(1/2*x)^4 - 2*a*b*tan(1/2*x)^4
 + b^2*tan(1/2*x)^4 + 2*a^2*tan(1/2*x)^2 - 2*b^2*tan(1/2*x)^2 + a^2 + 2*a*b
 + b^2)/(tan(1/2*x)^4 + 2*tan(1/2*x)^2 + 1)))/(a*b*tan(1/2*x)^2 - b^2*tan(1
/2*x)^2 + a*b + b^2)
```

maple [B] time = 0.41, size = 91, normalized size = 3.79

$$\frac{2x\tan\left(\frac{x}{2}\right)+2x\left(\tan^{3}\left(\frac{x}{2}\right)\right)}{\left(1+\tan^{2}\left(\frac{x}{2}\right)\right)\left(a\left(\tan^{2}\left(\frac{x}{2}\right)\right)-b\left(\tan^{2}\left(\frac{x}{2}\right)\right)+a+b\right)}+\frac{\ln\left(a\left(\tan^{2}\left(\frac{x}{2}\right)\right)-b\left(\tan^{2}\left(\frac{x}{2}\right)\right)+a+b\right)}{b}-\frac{\ln\left(1+\tan^{2}\left(\frac{x}{2}\right)\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b+a*cos(x))/(a+b*cos(x))^2,x)
```

```
[Out] (2*x*tan(1/2*x)+2*x*tan(1/2*x)^3)/(1+tan(1/2*x)^2)/(a*tan(1/2*x)^2-b*tan(1/
2*x)^2+a+b)+1/b*ln(a*tan(1/2*x)^2-b*tan(1/2*x)^2+a+b)-1/b*ln(1+tan(1/2*x)^2
)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b+a*cos(x))/(a+b*cos(x))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` for
more details)Is 4*a^2-4*b^2 positive or negative?
```

mupad [B] time = 2.74, size = 68, normalized size = 2.83

$$\frac{\ln\left(b+2\,a\,\mathrm{e}^{x\,1\mathrm{i}}+b\,\mathrm{e}^{x\,2\mathrm{i}}\right)}{b}-\frac{x\,2\mathrm{i}}{b}+\frac{x\,2\mathrm{i}+\frac{a\,x\,\mathrm{e}^{x\,1\mathrm{i}}\,2\mathrm{i}}{b}}{b+2\,a\,\mathrm{e}^{x\,1\mathrm{i}}+b\,\mathrm{e}^{x\,2\mathrm{i}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(b + a*cos(x)))/(a + b*cos(x))^2,x)
```

```
[Out] log(b + 2*a*exp(x*1i) + b*exp(x*2i))/b - (x*2i)/b + (x*2i + (a*x*exp(x*1i)*
2i)/b)/(b + 2*a*exp(x*1i) + b*exp(x*2i))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b+a*cos(x))/(a+b*cos(x))**2,x)
```

```
[Out] Timed out
```

3.201 $\int \frac{1+\sin^2(x)}{1-\sin^2(x)} \, dx$

Optimal. Leaf size=8

$$2\tan(x) - x$$

```
[Out] -x+2*tan(x)
```

Rubi [A] time = 0.04, antiderivative size = 8, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {3171, 3175, 3767, 8}

$$2\tan(x) - x$$

Antiderivative was successfully verified.

```
[In] Int[(1 + Sin[x]^2)/(1 - Sin[x]^2),x]
```

```
[Out] -x + 2*Tan[x]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3171

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)]^2)/((a_) + (b_.)*sin[(e_.) + (f_
.)*(x_)]^2), x_Symbol] :> Simp[(B*x)/b, x] + Dist[(A*b - a*B)/b, Int[1/(a +
 b*Sin[e + f*x]^2), x], x] /; FreeQ[{a, b, e, f, A, B}, x]
```

Rule 3175

```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Dist[
a^p, Int[ActivateTrig[u*cos[e + f*x]^(2*p)], x], x] /; FreeQ[{a, b, e, f, p
}, x] && EqQ[a + b, 0] && IntegerQ[p]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1+\sin^2(x)}{1-\sin^2(x)}\,dx &= -x+2\int \frac{1}{1-\sin^2(x)}\,dx \\
&= -x+2\int \sec^2(x)\,dx \\
&= -x-2\,\mathrm{Subst}\left(\int 1\,dx, x, -\tan(x)\right) \\
&= -x+2\tan(x)
\end{aligned}
$$

Mathematica [A] time = 0.01, size = 8, normalized size = 1.00

$$2\tan(x)-x$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + Sin[x]^2)/(1 - Sin[x]^2),x]
[Out] -x + 2*Tan[x]
```

fricas [A] time = 0.65, size = 15, normalized size = 1.88

$$-\frac{x\cos(x)-2\,\sin(x)}{\cos(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+sin(x)^2)/(1-sin(x)^2),x, algorithm="fricas")
[Out] -(x*cos(x) - 2*sin(x))/cos(x)
```

giac [A] time = 0.13, size = 8, normalized size = 1.00

$$-x+2\,\tan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+sin(x)^2)/(1-sin(x)^2),x, algorithm="giac")
[Out] -x + 2*tan(x)
```

maple [A] time = 0.08, size = 9, normalized size = 1.12

$$-x+2\tan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+sin(x)^2)/(1-sin(x)^2),x)
```

```
[Out] -x+2*tan(x)
```

maxima [A] time = 0.44, size = 8, normalized size = 1.00

$$-x + 2\,\tan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+sin(x)^2)/(1-sin(x)^2),x, algorithm="maxima")
```

```
[Out] -x + 2*tan(x)
```

mupad [B] time = 2.31, size = 8, normalized size = 1.00

$$2\tan(x) - x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(sin(x)^2 + 1)/(sin(x)^2 - 1),x)
```

```
[Out] 2*tan(x) - x
```

sympy [B] time = 0.97, size = 41, normalized size = 5.12

$$-\frac{x\tan^2\left(\frac{x}{2}\right)}{\tan^2\left(\frac{x}{2}\right)-1} + \frac{x}{\tan^2\left(\frac{x}{2}\right)-1} - \frac{4\tan\left(\frac{x}{2}\right)}{\tan^2\left(\frac{x}{2}\right)-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+sin(x)**2)/(1-sin(x)**2),x)
```

```
[Out] -x*tan(x/2)**2/(tan(x/2)**2 - 1) + x/(tan(x/2)**2 - 1) - 4*tan(x/2)/(tan(x/
2)**2 - 1)
```

3.202 $\int \frac{1-\sin^2(x)}{1+\sin^2(x)}\,dx$

Optimal. Leaf size=36

$$\sqrt{2}\,x - x + \sqrt{2}\,\tan^{-1}\left(\frac{\sin(x)\cos(x)}{\sin^2(x) + \sqrt{2} + 1}\right)$$

```
[Out] -x+x*2^(1/2)+arctan(cos(x)*sin(x)/(1+sin(x)^2+2^(1/2)))*2^(1/2)
```

Rubi [A] time = 0.04, antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {3171, 3181, 203}

$$\sqrt{2}\,x - x + \sqrt{2}\,\tan^{-1}\left(\frac{\sin(x)\cos(x)}{\sin^2(x) + \sqrt{2} + 1}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(1 - Sin[x]^2)/(1 + Sin[x]^2),x]
```

```
[Out] -x + Sqrt[2]*x + Sqrt[2]*ArcTan[(Cos[x]*Sin[x])/(1 + Sqrt[2] + Sin[x]^2)]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 3171

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)]^2)/((a_) + (b_.)*sin[(e_.) + (f_
.)*(x_)]^2), x_Symbol] :> Simp[(B*x)/b, x] + Dist[(A*b - a*B)/b, Int[1/(a +
 b*Sin[e + f*x]^2), x], x] /; FreeQ[{a, b, e, f, A, B}, x]
```

Rule 3181

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(-1), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[1/(a + (a + b)*ff^2*x^2
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x]
```

Rubi steps

$$\int \frac{1-\sin^2(x)}{1+\sin^2(x)}\,dx = -x+2\int \frac{1}{1+\sin^2(x)}\,dx$$
$$= -x+2\,\text{Subst}\left(\int \frac{1}{1+2x^2}\,dx,x,\tan(x)\right)$$
$$= -x+\sqrt{2}\,x+\sqrt{2}\,\tan^{-1}\left(\frac{\cos(x)\sin(x)}{1+\sqrt{2}+\sin^2(x)}\right)$$

Mathematica [A] time = 0.03, size = 24, normalized size = 0.67

$$-2\left(\frac{x}{2}-\frac{\tan^{-1}\left(\sqrt{2}\,\tan(x)\right)}{\sqrt{2}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - Sin[x]^2)/(1 + Sin[x]^2),x]
```

```
[Out] -2*(x/2 - ArcTan[Sqrt[2]*Tan[x]]/Sqrt[2])
```

fricas [A] time = 0.65, size = 35, normalized size = 0.97

$$-\frac{1}{2}\,\sqrt{2}\,\arctan\left(\frac{3\,\sqrt{2}\,\cos(x)^2-2\,\sqrt{2}}{4\,\cos(x)\sin(x)}\right)-x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-sin(x)^2)/(1+sin(x)^2),x, algorithm="fricas")
```

```
[Out] -1/2*sqrt(2)*arctan(1/4*(3*sqrt(2)*cos(x)^2 - 2*sqrt(2))/(cos(x)*sin(x))) -
 x
```

giac [A] time = 0.13, size = 49, normalized size = 1.36

$$\sqrt{2}\left(x+\arctan\left(-\frac{\sqrt{2}\,\sin\left(2\,x\right)-2\,\sin\left(2\,x\right)}{\sqrt{2}\,\cos\left(2\,x\right)+\sqrt{2}\,-2\,\cos\left(2\,x\right)+2}\right)\right)-x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-sin(x)^2)/(1+sin(x)^2),x, algorithm="giac")
```

```
[Out] sqrt(2)*(x + arctan(-(sqrt(2)*sin(2*x) - 2*sin(2*x))/(sqrt(2)*cos(2*x) + sq
rt(2) - 2*cos(2*x) + 2))) - x
```

maple [A] time = 0.08, size = 16, normalized size = 0.44

$$\sqrt{2}\ \arctan\left(\sqrt{2}\ \tan(x)\right) - x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1-sin(x)^2)/(1+sin(x)^2),x)
```

```
[Out] 2^(1/2)*arctan(2^(1/2)*tan(x))-x
```

maxima [A] time = 0.55, size = 15, normalized size = 0.42

$$\sqrt{2}\arctan\left(\sqrt{2}\tan(x)\right) - x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-sin(x)^2)/(1+sin(x)^2),x, algorithm="maxima")
```

```
[Out] sqrt(2)*arctan(sqrt(2)*tan(x)) - x
```

mupad [B] time = 2.32, size = 26, normalized size = 0.72

$$\sqrt{2}\ \left(x - \operatorname{atan}\left(\tan(x)\right)\right) - x + \sqrt{2}\ \operatorname{atan}\left(\sqrt{2}\ \tan(x)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(sin(x)^2 - 1)/(sin(x)^2 + 1),x)
```

```
[Out] 2^(1/2)*(x - atan(tan(x))) - x + 2^(1/2)*atan(2^(1/2)*tan(x))
```

sympy [B] time = 48.46, size = 248, normalized size = 6.89

$$-\frac{22619537x}{15994428\sqrt{2}+22619537}-\frac{15994428\sqrt{2}\,x}{15994428\sqrt{2}+22619537}+\frac{54608393\sqrt{2}\sqrt{3-2\sqrt{2}}\left(\operatorname{atan}\left(\frac{\tan\left(\frac{x}{2}\right)}{\sqrt{3-2\sqrt{2}}}\right)+\pi\left\lfloor\frac{\frac{x}{2}-\frac{\pi}{2}}{\pi}\right\rfloor\right)}{15994428\sqrt{2}+22619537}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-sin(x)**2)/(1+sin(x)**2),x)
```

```
[Out] -22619537*x/(15994428*sqrt(2) + 22619537) - 15994428*sqrt(2)*x/(15994428*sq
rt(2) + 22619537) + 54608393*sqrt(2)*sqrt(3 - 2*sqrt(2))*(atan(tan(x/2)/sqr
t(3 - 2*sqrt(2))) + pi*floor((x/2 - pi/2)/pi))/(15994428*sqrt(2) + 22619537
) + 77227930*sqrt(3 - 2*sqrt(2))*(atan(tan(x/2)/sqrt(3 - 2*sqrt(2))) + pi*f
loor((x/2 - pi/2)/pi))/(15994428*sqrt(2) + 22619537) + 9369319*sqrt(2)*sqrt
```

```
(2*sqrt(2) + 3)*(atan(tan(x/2)/sqrt(2*sqrt(2) + 3)) + pi*floor((x/2 - pi/2)
/pi))/(15994428*sqrt(2) + 22619537) + 13250218*sqrt(2*sqrt(2) + 3)*(atan(ta
n(x/2)/sqrt(2*sqrt(2) + 3)) + pi*floor((x/2 - pi/2)/pi))/(15994428*sqrt(2)
+ 22619537)
```

3.203 $\int \frac{1+\cos^2(x)}{1-\cos^2(x)} dx$

Optimal. Leaf size=8

$$-x - 2\cot(x)$$

```
[Out] -x-2*cot(x)
```

Rubi [A] time = 0.04, antiderivative size = 8, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {3171, 3175, 3767, 8}

$$-x - 2\cot(x)$$

Antiderivative was successfully verified.

```
[In] Int[(1 + Cos[x]^2)/(1 - Cos[x]^2),x]
```

```
[Out] -x - 2*Cot[x]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3171

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)]^2)/((a_) + (b_.)*sin[(e_.) + (f_
.)*(x_)]^2), x_Symbol] :> Simp[(B*x)/b, x] + Dist[(A*b - a*B)/b, Int[1/(a +
 b*Sin[e + f*x]^2), x], x] /; FreeQ[{a, b, e, f, A, B}, x]
```

Rule 3175

```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Dist[
a^p, Int[ActivateTrig[u*cos[e + f*x]^(2*p)], x], x] /; FreeQ[{a, b, e, f, p
}, x] && EqQ[a + b, 0] && IntegerQ[p]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\int \frac{1+\cos^2(x)}{1-\cos^2(x)}\,dx = -x + 2\int \frac{1}{1-\cos^2(x)}\,dx$$
$$= -x + 2\int \csc^2(x)\,dx$$
$$= -x - 2\,\mathrm{Subst}(\int 1\,dx, x, \cot(x))$$
$$= -x - 2\cot(x)$$

Mathematica [A] time = 0.01, size = 8, normalized size = 1.00

$$-x - 2\cot(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + Cos[x]^2)/(1 - Cos[x]^2),x]
```

```
[Out] -x - 2*Cot[x]
```

fricas [A] time = 0.65, size = 15, normalized size = 1.88

$$-\frac{x\sin(x) + 2\cos(x)}{\sin(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+cos(x)^2)/(1-cos(x)^2),x, algorithm="fricas")
```

```
[Out] -(x*sin(x) + 2*cos(x))/sin(x)
```

giac [A] time = 0.15, size = 16, normalized size = 2.00

$$-x - \frac{1}{\tan\left(\frac{1}{2}x\right)} + \tan\left(\frac{1}{2}x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+cos(x)^2)/(1-cos(x)^2),x, algorithm="giac")
```

```
[Out] -x - 1/tan(1/2*x) + tan(1/2*x)
```

maple [A] time = 0.08, size = 11, normalized size = 1.38

$$-\frac{2}{\tan(x)} - x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+cos(x)^2)/(1-cos(x)^2),x)
```

```
[Out] -2/tan(x)-x
```

maxima [A] time = 0.77, size = 10, normalized size = 1.25

$$-x - \frac{2}{\tan(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+cos(x)^2)/(1-cos(x)^2),x, algorithm="maxima")
```

```
[Out] -x - 2/tan(x)
```

mupad [B] time = 2.29, size = 8, normalized size = 1.00

$$-x - 2\cot(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(cos(x)^2 + 1)/(cos(x)^2 - 1),x)
```

```
[Out] - x - 2*cot(x)
```

sympy [A] time = 0.86, size = 12, normalized size = 1.50

$$-x + \tan\left(\frac{x}{2}\right) - \frac{1}{\tan\left(\frac{x}{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+cos(x)**2)/(1-cos(x)**2),x)
```

```
[Out] -x + tan(x/2) - 1/tan(x/2)
```

3.204 $\int \frac{1-\cos^2(x)}{1+\cos^2(x)}\,dx$

Optimal. Leaf size=37

$$\sqrt{2}\,x - x - \sqrt{2}\,\tan^{-1}\left(\frac{\sin(x)\cos(x)}{\cos^2(x)+\sqrt{2}+1}\right)$$

```
[Out] -x+x*2^(1/2)-arctan(cos(x)*sin(x)/(1+cos(x)^2+2^(1/2)))*2^(1/2)
```

Rubi [A] time = 0.04, antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {3171, 3181, 203}

$$\sqrt{2}\,x - x - \sqrt{2}\,\tan^{-1}\left(\frac{\sin(x)\cos(x)}{\cos^2(x)+\sqrt{2}+1}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(1 - Cos[x]^2)/(1 + Cos[x]^2),x]
```

```
[Out] -x + Sqrt[2]*x - Sqrt[2]*ArcTan[(Cos[x]*Sin[x])/(1 + Sqrt[2] + Cos[x]^2)]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 3171

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)]^2)/((a_) + (b_.)*sin[(e_.) + (f_
.)*(x_)]^2), x_Symbol] :> Simp[(B*x)/b, x] + Dist[(A*b - a*B)/b, Int[1/(a +
 b*Sin[e + f*x]^2), x], x] /; FreeQ[{a, b, e, f, A, B}, x]
```

Rule 3181

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(-1), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[1/(a + (a + b)*ff^2*x^2
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{1-\cos^2(x)}{1+\cos^2(x)}\,dx &= -x+2\int \frac{1}{1+\cos^2(x)}\,dx\\ &= -x-2\,\text{Subst}\left(\int \frac{1}{1+2x^2}\,dx, x, \cot(x)\right)\\ &= -x+\sqrt{2}\,x-\sqrt{2}\,\tan^{-1}\left(\frac{\cos(x)\sin(x)}{1+\sqrt{2}+\cos^2(x)}\right)\end{aligned}$$

Mathematica [A] time = 0.03, size = 23, normalized size = 0.62

$$2\left(\frac{\tan^{-1}\left(\frac{\tan(x)}{\sqrt{2}}\right)}{\sqrt{2}}-\frac{x}{2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - Cos[x]^2)/(1 + Cos[x]^2),x]
[Out] 2*(-1/2*x + ArcTan[Tan[x]/Sqrt[2]]/Sqrt[2])
```

fricas [A] time = 1.14, size = 35, normalized size = 0.95

$$-\frac{1}{2}\sqrt{2}\,\arctan\left(\frac{3\sqrt{2}\,\cos(x)^2-\sqrt{2}}{4\,\cos(x)\sin(x)}\right)-x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-cos(x)^2)/(1+cos(x)^2),x, algorithm="fricas")
[Out] -1/2*sqrt(2)*arctan(1/4*(3*sqrt(2)*cos(x)^2 - sqrt(2))/(cos(x)*sin(x))) - x
```

giac [A] time = 0.13, size = 49, normalized size = 1.32

$$\sqrt{2}\left(x+\arctan\left(-\frac{\sqrt{2}\,\sin(2x)-\sin(2x)}{\sqrt{2}\,\cos(2x)+\sqrt{2}-\cos(2x)+1}\right)\right)-x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-cos(x)^2)/(1+cos(x)^2),x, algorithm="giac")
[Out] sqrt(2)*(x + arctan(-(sqrt(2)*sin(2*x) - sin(2*x))/(sqrt(2)*cos(2*x) + sqrt
(2) - cos(2*x) + 1))) - x
```

maple [A] time = 0.08, size = 17, normalized size = 0.46

$$\sqrt{2}\ \arctan\left(\frac{\sqrt{2}\ \tan(x)}{2}\right)-x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1-cos(x)^2)/(1+cos(x)^2),x)
[Out] 2^(1/2)*arctan(1/2*2^(1/2)*tan(x))-x
```

maxima [A] time = 0.42, size = 16, normalized size = 0.43

$$\sqrt{2}\ \arctan\left(\frac{1}{2}\sqrt{2}\ \tan(x)\right)-x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-cos(x)^2)/(1+cos(x)^2),x, algorithm="maxima")
[Out] sqrt(2)*arctan(1/2*sqrt(2)*tan(x)) - x
```

mupad [B] time = 2.29, size = 27, normalized size = 0.73

$$\sqrt{2}\ \left(x-\operatorname{atan}\left(\tan(x)\right)\right)-x+\sqrt{2}\ \operatorname{atan}\left(\frac{\sqrt{2}\ \tan(x)}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(cos(x)^2 - 1)/(cos(x)^2 + 1),x)
[Out] 2^(1/2)*(x - atan(tan(x))) - x + 2^(1/2)*atan((2^(1/2)*tan(x))/2)
```

sympy [A] time = 2.66, size = 61, normalized size = 1.65

$$-x+\sqrt{2}\left(\operatorname{atan}\left(\sqrt{2}\ \tan\left(\frac{x}{2}\right)-1\right)+\pi\left\lfloor\frac{\frac{x}{2}-\frac{\pi}{2}}{\pi}\right\rfloor\right)+\sqrt{2}\left(\operatorname{atan}\left(\sqrt{2}\ \tan\left(\frac{x}{2}\right)+1\right)+\pi\left\lfloor\frac{\frac{x}{2}-\frac{\pi}{2}}{\pi}\right\rfloor\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-cos(x)**2)/(1+cos(x)**2),x)
[Out] -x + sqrt(2)*(atan(sqrt(2)*tan(x/2) - 1) + pi*floor((x/2 - pi/2)/pi)) + sqr
t(2)*(atan(sqrt(2)*tan(x/2) + 1) + pi*floor((x/2 - pi/2)/pi))
```

3.205 $\int \frac{-1+\frac{c^2}{d^2}+\sin^2(x)}{c+d\cos(x)}\,dx$

Optimal. Leaf size=14

$$\frac{cx}{d^2}-\frac{\sin(x)}{d}$$

[Out] c*x/d^2-sin(x)/d

Rubi [A] time = 0.13, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {4397, 3016, 2637}

$$\frac{cx}{d^2}-\frac{\sin(x)}{d}$$

Antiderivative was successfully verified.

[In] Int[(-1 + c^2/d^2 + Sin[x]^2)/(c + d*Cos[x]),x]

[Out] (c*x)/d^2 - Sin[x]/d

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3016

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (C_.)*sin[(e_.) +
 (f_.)*(x_)]^2), x_Symbol] :> Dist[C/b^2, Int[(a + b*Sin[e + f*x])^(m + 1)*
Simp[-a + b*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, C, m}, x] &&
EqQ[A*b^2 + a^2*C, 0]
```

Rule 4397

```
Int[u_, x_Symbol] :> Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]
```

Rubi steps

$$\int \frac{-1+\frac{c^2}{d^2}+\sin^2(x)}{c+d\cos(x)}\,dx = \int \frac{\frac{c^2}{d^2}-\cos^2(x)}{c+d\cos(x)}\,dx$$
$$= -\frac{\int(-c+d\cos(x))\,dx}{d^2}$$
$$= \frac{cx}{d^2}-\frac{\int\cos(x)\,dx}{d}$$
$$= \frac{cx}{d^2}-\frac{\sin(x)}{d}$$

Mathematica [A] time = 0.01, size = 14, normalized size = 1.00

$$\frac{cx}{d^2}-\frac{\sin(x)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-1 + c^2/d^2 + Sin[x]^2)/(c + d*Cos[x]),x]
[Out] (c*x)/d^2 - Sin[x]/d
```

fricas [A] time = 0.63, size = 13, normalized size = 0.93

$$\frac{cx-d\sin(x)}{d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+c^2/d^2+sin(x)^2)/(c+d*cos(x)),x, algorithm="fricas")
[Out] (c*x - d*sin(x))/d^2
```

giac [A] time = 0.15, size = 26, normalized size = 1.86

$$\frac{cx}{d^2}-\frac{2\tan\left(\frac{1}{2}x\right)}{\left(\tan\left(\frac{1}{2}x\right)^2+1\right)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+c^2/d^2+sin(x)^2)/(c+d*cos(x)),x, algorithm="giac")
[Out] c*x/d^2 - 2*tan(1/2*x)/((tan(1/2*x)^2 + 1)*d)
```

maple [B] time = 0.10, size = 32, normalized size = 2.29

$$-\frac{2\tan\left(\frac{x}{2}\right)}{d\left(1+\tan^2\left(\frac{x}{2}\right)\right)}+\frac{2c\arctan\left(\tan\left(\frac{x}{2}\right)\right)}{d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-1+c^2/d^2+sin(x)^2)/(c+d*cos(x)),x)
```

```
[Out] -2/d*tan(1/2*x)/(1+tan(1/2*x)^2)+2/d^2*c*arctan(tan(1/2*x))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+c^2/d^2+sin(x)^2)/(c+d*cos(x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*d^2-4*c^2>0)', see `assume?` for
more details)Is 4*d^2-4*c^2 positive or negative?
```

mupad [B] time = 2.47, size = 13, normalized size = 0.93

$$\frac{c\,x-d\,\sin(x)}{d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((sin(x)^2 + c^2/d^2 - 1)/(c + d*cos(x)),x)
```

```
[Out] (c*x - d*sin(x))/d^2
```

sympy [B] time = 56.76, size = 61, normalized size = 4.36

$$\frac{cx\tan^2\left(\frac{x}{2}\right)}{d^2\tan^2\left(\frac{x}{2}\right)+d^2}+\frac{cx}{d^2\tan^2\left(\frac{x}{2}\right)+d^2}-\frac{2d\tan\left(\frac{x}{2}\right)}{d^2\tan^2\left(\frac{x}{2}\right)+d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+c**2/d**2+sin(x)**2)/(c+d*cos(x)),x)
```

```
[Out] c*x*tan(x/2)**2/(d**2*tan(x/2)**2 + d**2) + c*x/(d**2*tan(x/2)**2 + d**2) -
 2*d*tan(x/2)/(d**2*tan(x/2)**2 + d**2)
```

3.206 $\int \frac{a+b\sin^2(x)}{c+d\cos(x)}\,dx$

Optimal. Leaf size=105

$$\frac{2a\tan^{-1}\left(\frac{\sqrt{c-d}\,\tan\left(\frac{x}{2}\right)}{\sqrt{c+d}}\right)}{\sqrt{c-d}\,\sqrt{c+d}}+\frac{bcx}{d^2}-\frac{2b\sqrt{c-d}\,\sqrt{c+d}\,\tan^{-1}\left(\frac{\sqrt{c-d}\,\tan\left(\frac{x}{2}\right)}{\sqrt{c+d}}\right)}{d^2}-\frac{b\sin(x)}{d}$$

```
[Out] b*c*x/d^2-b*sin(x)/d+2*a*arctan((c-d)^(1/2)*tan(1/2*x)/(c+d)^(1/2))/(c-d)^(
1/2)/(c+d)^(1/2)-2*b*arctan((c-d)^(1/2)*tan(1/2*x)/(c+d)^(1/2))*(c-d)^(1/2)
*(c+d)^(1/2)/d^2
```

Rubi [A] time = 0.26, antiderivative size = 105, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.294, Rules used = {4401, 2659, 205, 2695, 2735}

$$\frac{2a\tan^{-1}\left(\frac{\sqrt{c-d}\,\tan\left(\frac{x}{2}\right)}{\sqrt{c+d}}\right)}{\sqrt{c-d}\,\sqrt{c+d}}+\frac{bcx}{d^2}-\frac{2b\sqrt{c-d}\,\sqrt{c+d}\,\tan^{-1}\left(\frac{\sqrt{c-d}\,\tan\left(\frac{x}{2}\right)}{\sqrt{c+d}}\right)}{d^2}-\frac{b\sin(x)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[x]^2)/(c + d*Cos[x]),x]
```

```
[Out] (b*c*x)/d^2 + (2*a*ArcTan[(Sqrt[c - d]*Tan[x/2])/Sqrt[c + d]])/(Sqrt[c - d]
*Sqrt[c + d]) - (2*b*Sqrt[c - d]*Sqrt[c + d]*ArcTan[(Sqrt[c - d]*Tan[x/2])/
Sqrt[c + d]])/d^2 - (b*Sin[x])/d
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 2695

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(g*(g*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x
```

```
])^(m + 1))/(b*f*(m + p)), x] + Dist[(g^2*(p - 1))/(b*(m + p)), Int[(g*Cos[
e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^m*(b + a*Sin[e + f*x]), x], x] /; Fr
eeQ[{a, b, e, f, g, m}, x] && NeQ[a^2 - b^2, 0] && GtQ[p, 1] && NeQ[m + p,
0] && IntegersQ[2*m, 2*p]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 4401

```
Int[u_, x_Symbol] :> With[{v = ExpandTrig[u, x]}, Int[v, x] /; SumQ[v]] /;
 !InertTrigFreeQ[u]
```

Rubi steps

$$
\begin{aligned}
\int \frac{a+b\sin^2(x)}{c+d\cos(x)}\,dx &= \int \left(\frac{a}{c+d\cos(x)}+\frac{b\sin^2(x)}{c+d\cos(x)}\right)dx \\
&= a\int \frac{1}{c+d\cos(x)}\,dx + b\int \frac{\sin^2(x)}{c+d\cos(x)}\,dx \\
&= -\frac{b\sin(x)}{d} + (2a)\,\text{Subst}\left(\int \frac{1}{c+d+(c-d)x^2}\,dx, x, \tan\left(\frac{x}{2}\right)\right) - \frac{b\int \frac{-d-c\cos(x)}{c+d\cos(x)}\,dx}{d} \\
&= \frac{bcx}{d^2} + \frac{2a\tan^{-1}\left(\frac{\sqrt{c-d}\,\tan\left(\frac{x}{2}\right)}{\sqrt{c+d}}\right)}{\sqrt{c-d}\,\sqrt{c+d}} - \frac{b\sin(x)}{d} + \frac{\left(b\left(-c^2+d^2\right)\right)\int \frac{1}{c+d\cos(x)}\,dx}{d^2} \\
&= \frac{bcx}{d^2} + \frac{2a\tan^{-1}\left(\frac{\sqrt{c-d}\,\tan\left(\frac{x}{2}\right)}{\sqrt{c+d}}\right)}{\sqrt{c-d}\,\sqrt{c+d}} - \frac{b\sin(x)}{d} + \frac{\left(2b\left(-c^2+d^2\right)\right)\text{Subst}\left(\int \frac{1}{c+d+(c-d)x^2}\,dx, x, \tan\left(\right.\right.}{d^2} \\
&= \frac{bcx}{d^2} + \frac{2a\tan^{-1}\left(\frac{\sqrt{c-d}\,\tan\left(\frac{x}{2}\right)}{\sqrt{c+d}}\right)}{\sqrt{c-d}\,\sqrt{c+d}} - \frac{2b\sqrt{c-d}\,\sqrt{c+d}\,\tan^{-1}\left(\frac{\sqrt{c-d}\,\tan\left(\frac{x}{2}\right)}{\sqrt{c+d}}\right)}{d^2} - \frac{b\sin(x)}{d}
\end{aligned}
$$

Mathematica [A] time = 0.15, size = 73, normalized size = 0.70

$$
\frac{-\frac{2\left(ad^2+b\left(d^2-c^2\right)\right)\tanh^{-1}\left(\frac{(c-d)\tan\left(\frac{x}{2}\right)}{\sqrt{d^2-c^2}}\right)}{\sqrt{d^2-c^2}}+bcx-bd\sin(x)}{d^2}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sin[x]^2)/(c + d*Cos[x]),x]
```

```
[Out] (b*c*x - (2*(a*d^2 + b*(-c^2 + d^2))*ArcTanh[((c - d)*Tan[x/2])/Sqrt[-c^2 +
 d^2]])/Sqrt[-c^2 + d^2] - b*d*Sin[x])/d^2
```

fricas [A] time = 1.02, size = 254, normalized size = 2.42

$$\left[\frac{\left(bc^2-(a+b)d^2\right)\sqrt{-c^2+d^2}\,\log\left(\frac{2\,cd\cos(x)+\left(2\,c^2-d^2\right)\cos(x)^2+2\,\sqrt{-c^2+d^2}\,(c\cos(x)+d)\sin(x)-c^2+2\,d^2}{d^2\cos(x)^2+2\,cd\cos(x)+c^2}\right)+2\left(bc^3-bcd^2\right)x-2\left(b\right.}{2\left(c^2d^2-d^4\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(x)^2)/(c+d*cos(x)),x, algorithm="fricas")
```

```
[Out] [1/2*((b*c^2 - (a + b)*d^2)*sqrt(-c^2 + d^2)*log((2*c*d*cos(x) + (2*c^2 - d
^2)*cos(x)^2 + 2*sqrt(-c^2 + d^2)*(c*cos(x) + d)*sin(x) - c^2 + 2*d^2)/(d^2
*cos(x)^2 + 2*c*d*cos(x) + c^2)) + 2*(b*c^3 - b*c*d^2)*x - 2*(b*c^2*d - b*d
^3)*sin(x))/(c^2*d^2 - d^4), -((b*c^2 - (a + b)*d^2)*sqrt(c^2 - d^2)*arctan
(-(c*cos(x) + d)/(sqrt(c^2 - d^2)*sin(x))) - (b*c^3 - b*c*d^2)*x + (b*c^2*d
 - b*d^3)*sin(x))/(c^2*d^2 - d^4)]
```

giac [A] time = 0.14, size = 110, normalized size = 1.05

$$\frac{bcx}{d^2}-\frac{2\,b\tan\left(\frac{1}{2}\,x\right)}{\left(\tan\left(\frac{1}{2}\,x\right)^2+1\right)d}+\frac{2\left(bc^2-ad^2-bd^2\right)\left(\pi\left\lfloor\frac{x}{2\,\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}\left(-2\,c+2\,d\right)+\arctan\left(-\frac{c\tan\left(\frac{1}{2}\,x\right)-d\tan\left(\frac{1}{2}\,x\right)}{\sqrt{c^2-d^2}}\right)\right)}{\sqrt{c^2-d^2}\,d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(x)^2)/(c+d*cos(x)),x, algorithm="giac")
```

```
[Out] b*c*x/d^2 - 2*b*tan(1/2*x)/((tan(1/2*x)^2 + 1)*d) + 2*(b*c^2 - a*d^2 - b*d^
2)*(pi*floor(1/2*x/pi + 1/2)*sgn(-2*c + 2*d) + arctan(-(c*tan(1/2*x) - d*ta
n(1/2*x))/sqrt(c^2 - d^2)))/(sqrt(c^2 - d^2)*d^2)
```

maple [A] time = 0.11, size = 148, normalized size = 1.41

$$\frac{2\arctan\left(\frac{(c-d)\tan\left(\frac{x}{2}\right)}{\sqrt{(c+d)(c-d)}}\right)a}{\sqrt{(c+d)\,(c-d)}}-\frac{2\arctan\left(\frac{(c-d)\tan\left(\frac{x}{2}\right)}{\sqrt{(c+d)(c-d)}}\right)c^2b}{d^2\sqrt{(c+d)\,(c-d)}}+\frac{2\arctan\left(\frac{(c-d)\tan\left(\frac{x}{2}\right)}{\sqrt{(c+d)(c-d)}}\right)b}{\sqrt{(c+d)\,(c-d)}}-\frac{2b\tan\left(\frac{x}{2}\right)}{d\left(1+\tan^2\left(\frac{x}{2}\right)\right)}+\frac{2bc\arctan\left(\tan\left(\right.\right.}{d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(x)^2)/(c+d*cos(x)),x)
```

```
[Out] 2/((c+d)*(c-d))^(1/2)*arctan((c-d)*tan(1/2*x)/((c+d)*(c-d))^(1/2))*a-2/d^2/
((c+d)*(c-d))^(1/2)*arctan((c-d)*tan(1/2*x)/((c+d)*(c-d))^(1/2))*c^2*b+2/((
c+d)*(c-d))^(1/2)*arctan((c-d)*tan(1/2*x)/((c+d)*(c-d))^(1/2))*b-2*b/d*tan(
1/2*x)/(1+tan(1/2*x)^2)+2*b/d^2*c*arctan(tan(1/2*x))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(x)^2)/(c+d*cos(x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*d^2-4*c^2>0)', see `assume?` for
more details)Is 4*d^2-4*c^2 positive or negative?
```

mupad [B] time = 4.04, size = 2429, normalized size = 23.13

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(x)^2)/(c + d*cos(x)),x)
```

```
[Out] (b*c^2*d*sin(x))/(d^4 - c^2*d^2) - (b*d^3*sin(x))/(d^4 - c^2*d^2) - (2*b*c^
3*atan(sin(x/2)/cos(x/2)))/(d^4 - c^2*d^2) - (a*d^2*atan((a^2*d^7*sin(x/2)*
(d^2 - c^2)^(1/2)*1i - b^2*c^5*sin(x/2)*(d^2 - c^2)^(3/2)*2i - b^2*c^7*sin(
x/2)*(d^2 - c^2)^(1/2)*2i + b^2*d^7*sin(x/2)*(d^2 - c^2)^(1/2)*1i - a^2*c*d
^4*sin(x/2)*(d^2 - c^2)^(3/2)*2i + a^2*c*d^6*sin(x/2)*(d^2 - c^2)^(1/2)*1i
- b^2*c*d^4*sin(x/2)*(d^2 - c^2)^(3/2)*2i + b^2*c*d^6*sin(x/2)*(d^2 - c^2)^
(1/2)*1i - a^2*c^2*d^5*sin(x/2)*(d^2 - c^2)^(1/2)*1i - a^2*c^3*d^4*sin(x/2)
*(d^2 - c^2)^(1/2)*1i - b^2*c^2*d^5*sin(x/2)*(d^2 - c^2)^(1/2)*2i + b^2*c^3
*d^2*sin(x/2)*(d^2 - c^2)^(3/2)*4i - b^2*c^3*d^4*sin(x/2)*(d^2 - c^2)^(1/2)
*4i + b^2*c^4*d^3*sin(x/2)*(d^2 - c^2)^(1/2)*1i + b^2*c^5*d^2*sin(x/2)*(d^2
 - c^2)^(1/2)*5i + a*b*d^7*sin(x/2)*(d^2 - c^2)^(1/2)*2i - a*b*c*d^4*sin(x/
2)*(d^2 - c^2)^(3/2)*4i + a*b*c*d^6*sin(x/2)*(d^2 - c^2)^(1/2)*2i - a*b*c^2
*d^5*sin(x/2)*(d^2 - c^2)^(1/2)*4i + a*b*c^3*d^2*sin(x/2)*(d^2 - c^2)^(3/2)
*4i - a*b*c^3*d^4*sin(x/2)*(d^2 - c^2)^(1/2)*4i + a*b*c^4*d^3*sin(x/2)*(d^2
 - c^2)^(1/2)*2i + a*b*c^5*d^2*sin(x/2)*(d^2 - c^2)^(1/2)*2i)/(a^2*d^8*cos(
x/2) + b^2*d^8*cos(x/2) + 2*a*b*d^8*cos(x/2) - 2*a^2*c^2*d^6*cos(x/2) + a^2
*c^4*d^4*cos(x/2) - 3*b^2*c^2*d^6*cos(x/2) + 3*b^2*c^4*d^4*cos(x/2) - b^2*c
```

```
^6*d^2*cos(x/2) - 6*a*b*c^2*d^6*cos(x/2) + 6*a*b*c^4*d^4*cos(x/2) - 2*a*b*c
^6*d^2*cos(x/2)))*(d^2 - c^2)^(1/2)*2i)/(d^4 - c^2*d^2) + (b*c^2*atan((a^2*
d^7*sin(x/2)*(d^2 - c^2)^(1/2)*1i - b^2*c^5*sin(x/2)*(d^2 - c^2)^(3/2)*2i -
 b^2*c^7*sin(x/2)*(d^2 - c^2)^(1/2)*2i + b^2*d^7*sin(x/2)*(d^2 - c^2)^(1/2)
*1i - a^2*c*d^4*sin(x/2)*(d^2 - c^2)^(3/2)*2i + a^2*c*d^6*sin(x/2)*(d^2 - c
^2)^(1/2)*1i - b^2*c*d^4*sin(x/2)*(d^2 - c^2)^(3/2)*2i + b^2*c*d^6*sin(x/2)
*(d^2 - c^2)^(1/2)*1i - a^2*c^2*d^5*sin(x/2)*(d^2 - c^2)^(1/2)*1i - a^2*c^3
*d^4*sin(x/2)*(d^2 - c^2)^(1/2)*1i - b^2*c^2*d^5*sin(x/2)*(d^2 - c^2)^(1/2)
*2i + b^2*c^3*d^2*sin(x/2)*(d^2 - c^2)^(3/2)*4i - b^2*c^3*d^4*sin(x/2)*(d^2
 - c^2)^(1/2)*4i + b^2*c^4*d^3*sin(x/2)*(d^2 - c^2)^(1/2)*1i + b^2*c^5*d^2*
sin(x/2)*(d^2 - c^2)^(1/2)*5i + a*b*d^7*sin(x/2)*(d^2 - c^2)^(1/2)*2i - a*b
*c*d^4*sin(x/2)*(d^2 - c^2)^(3/2)*4i + a*b*c*d^6*sin(x/2)*(d^2 - c^2)^(1/2)
*2i - a*b*c^2*d^5*sin(x/2)*(d^2 - c^2)^(1/2)*4i + a*b*c^3*d^2*sin(x/2)*(d^2
 - c^2)^(3/2)*4i - a*b*c^3*d^4*sin(x/2)*(d^2 - c^2)^(1/2)*4i + a*b*c^4*d^3*
sin(x/2)*(d^2 - c^2)^(1/2)*2i + a*b*c^5*d^2*sin(x/2)*(d^2 - c^2)^(1/2)*2i)/
(a^2*d^8*cos(x/2) + b^2*d^8*cos(x/2) + 2*a*b*d^8*cos(x/2) - 2*a^2*c^2*d^6*c
os(x/2) + a^2*c^4*d^4*cos(x/2) - 3*b^2*c^2*d^6*cos(x/2) + 3*b^2*c^4*d^4*cos
(x/2) - b^2*c^6*d^2*cos(x/2) - 6*a*b*c^2*d^6*cos(x/2) + 6*a*b*c^4*d^4*cos(x
/2) - 2*a*b*c^6*d^2*cos(x/2)))*(d^2 - c^2)^(1/2)*2i)/(d^4 - c^2*d^2) - (b*d
^2*atan((a^2*d^7*sin(x/2)*(d^2 - c^2)^(1/2)*1i - b^2*c^5*sin(x/2)*(d^2 - c^
2)^(3/2)*2i - b^2*c^7*sin(x/2)*(d^2 - c^2)^(1/2)*2i + b^2*d^7*sin(x/2)*(d^2
 - c^2)^(1/2)*1i - a^2*c*d^4*sin(x/2)*(d^2 - c^2)^(3/2)*2i + a^2*c*d^6*sin(
x/2)*(d^2 - c^2)^(1/2)*1i - b^2*c*d^4*sin(x/2)*(d^2 - c^2)^(3/2)*2i + b^2*c
*d^6*sin(x/2)*(d^2 - c^2)^(1/2)*1i - a^2*c^2*d^5*sin(x/2)*(d^2 - c^2)^(1/2)
*1i - a^2*c^3*d^4*sin(x/2)*(d^2 - c^2)^(1/2)*1i - b^2*c^2*d^5*sin(x/2)*(d^2
 - c^2)^(1/2)*2i + b^2*c^3*d^2*sin(x/2)*(d^2 - c^2)^(3/2)*4i - b^2*c^3*d^4*
sin(x/2)*(d^2 - c^2)^(1/2)*4i + b^2*c^4*d^3*sin(x/2)*(d^2 - c^2)^(1/2)*1i +
 b^2*c^5*d^2*sin(x/2)*(d^2 - c^2)^(1/2)*5i + a*b*d^7*sin(x/2)*(d^2 - c^2)^(
1/2)*2i - a*b*c*d^4*sin(x/2)*(d^2 - c^2)^(3/2)*4i + a*b*c*d^6*sin(x/2)*(d^2
 - c^2)^(1/2)*2i - a*b*c^2*d^5*sin(x/2)*(d^2 - c^2)^(1/2)*4i + a*b*c^3*d^2*
sin(x/2)*(d^2 - c^2)^(3/2)*4i - a*b*c^3*d^4*sin(x/2)*(d^2 - c^2)^(1/2)*4i +
 a*b*c^4*d^3*sin(x/2)*(d^2 - c^2)^(1/2)*2i + a*b*c^5*d^2*sin(x/2)*(d^2 - c^
2)^(1/2)*2i)/(a^2*d^8*cos(x/2) + b^2*d^8*cos(x/2) + 2*a*b*d^8*cos(x/2) - 2*
a^2*c^2*d^6*cos(x/2) + a^2*c^4*d^4*cos(x/2) - 3*b^2*c^2*d^6*cos(x/2) + 3*b^
2*c^4*d^4*cos(x/2) - b^2*c^6*d^2*cos(x/2) - 6*a*b*c^2*d^6*cos(x/2) + 6*a*b*
c^4*d^4*cos(x/2) - 2*a*b*c^6*d^2*cos(x/2)))*(d^2 - c^2)^(1/2)*2i)/(d^4 - c^
2*d^2) + (2*b*c*d^2*atan(sin(x/2)/cos(x/2)))/(d^4 - c^2*d^2)
```

sympy [A] time = 111.37, size = 2608, normalized size = 24.84

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(x)**2)/(c+d*cos(x)),x)
```

```
[Out] Piecewise((zoo*(-a*log(tan(x/2) - 1)*tan(x/2)**2/(tan(x/2)**2 + 1) - a*log(
tan(x/2) - 1)/(tan(x/2)**2 + 1) + a*log(tan(x/2) + 1)*tan(x/2)**2/(tan(x/2)
**2 + 1) + a*log(tan(x/2) + 1)/(tan(x/2)**2 + 1) - b*log(tan(x/2) - 1)*tan(
x/2)**2/(tan(x/2)**2 + 1) - b*log(tan(x/2) - 1)/(tan(x/2)**2 + 1) + b*log(t
an(x/2) + 1)*tan(x/2)**2/(tan(x/2)**2 + 1) + b*log(tan(x/2) + 1)/(tan(x/2)*
*2 + 1) - 2*b*tan(x/2)/(tan(x/2)**2 + 1)), Eq(c, 0) & Eq(d, 0)), (a*tan(x/2
)**3/(d*tan(x/2)**2 + d) + a*tan(x/2)/(d*tan(x/2)**2 + d) + b*x*tan(x/2)**2
/(d*tan(x/2)**2 + d) + b*x/(d*tan(x/2)**2 + d) - 2*b*tan(x/2)/(d*tan(x/2)**
2 + d), Eq(c, d)), (a*tan(x/2)**2/(d*tan(x/2)**3 + d*tan(x/2)) + a/(d*tan(x
/2)**3 + d*tan(x/2)) - b*x*tan(x/2)**3/(d*tan(x/2)**3 + d*tan(x/2)) - b*x*t
an(x/2)/(d*tan(x/2)**3 + d*tan(x/2)) - 2*b*tan(x/2)**2/(d*tan(x/2)**3 + d*t
an(x/2)), Eq(c, -d)), ((a*x + b*x*sin(x)**2/2 + b*x*cos(x)**2/2 - b*sin(x)*
cos(x)/2)/c, Eq(d, 0)), (a*d**2*log(-sqrt(-c/(c - d) - d/(c - d)) + tan(x/2
))*tan(x/2)**2/(c*d**2*sqrt(-c/(c - d) - d/(c - d))*tan(x/2)**2 + c*d**2*sq
rt(-c/(c - d) - d/(c - d)) - d**3*sqrt(-c/(c - d) - d/(c - d))*tan(x/2)**2
- d**3*sqrt(-c/(c - d) - d/(c - d))) + a*d**2*log(-sqrt(-c/(c - d) - d/(c -
 d)) + tan(x/2))/(c*d**2*sqrt(-c/(c - d) - d/(c - d))*tan(x/2)**2 + c*d**2*
sqrt(-c/(c - d) - d/(c - d)) - d**3*sqrt(-c/(c - d) - d/(c - d))*tan(x/2)**
2 - d**3*sqrt(-c/(c - d) - d/(c - d))) - a*d**2*log(sqrt(-c/(c - d) - d/(c
- d)) + tan(x/2))*tan(x/2)**2/(c*d**2*sqrt(-c/(c - d) - d/(c - d))*tan(x/2)
**2 + c*d**2*sqrt(-c/(c - d) - d/(c - d)) - d**3*sqrt(-c/(c - d) - d/(c - d
))*tan(x/2)**2 - d**3*sqrt(-c/(c - d) - d/(c - d))) - a*d**2*log(sqrt(-c/(c
 - d) - d/(c - d)) + tan(x/2))/(c*d**2*sqrt(-c/(c - d) - d/(c - d))*tan(x/2
)**2 + c*d**2*sqrt(-c/(c - d) - d/(c - d)) - d**3*sqrt(-c/(c - d) - d/(c -
d))*tan(x/2)**2 - d**3*sqrt(-c/(c - d) - d/(c - d))) + b*c**2*x*sqrt(-c/(c
- d) - d/(c - d))*tan(x/2)**2/(c*d**2*sqrt(-c/(c - d) - d/(c - d))*tan(x/2)
**2 + c*d**2*sqrt(-c/(c - d) - d/(c - d)) - d**3*sqrt(-c/(c - d) - d/(c - d
))*tan(x/2)**2 - d**3*sqrt(-c/(c - d) - d/(c - d))) + b*c**2*x*sqrt(-c/(c -
 d) - d/(c - d))/(c*d**2*sqrt(-c/(c - d) - d/(c - d))*tan(x/2)**2 + c*d**2*
sqrt(-c/(c - d) - d/(c - d)) - d**3*sqrt(-c/(c - d) - d/(c - d))*tan(x/2)**
2 - d**3*sqrt(-c/(c - d) - d/(c - d))) - b*c**2*log(-sqrt(-c/(c - d) - d/(c
 - d)) + tan(x/2))*tan(x/2)**2/(c*d**2*sqrt(-c/(c - d) - d/(c - d))*tan(x/2
)**2 + c*d**2*sqrt(-c/(c - d) - d/(c - d)) - d**3*sqrt(-c/(c - d) - d/(c -
d))*tan(x/2)**2 - d**3*sqrt(-c/(c - d) - d/(c - d))) - b*c**2*log(-sqrt(-c/
(c - d) - d/(c - d)) + tan(x/2))/(c*d**2*sqrt(-c/(c - d) - d/(c - d))*tan(x
/2)**2 + c*d**2*sqrt(-c/(c - d) - d/(c - d)) - d**3*sqrt(-c/(c - d) - d/(c
- d))*tan(x/2)**2 - d**3*sqrt(-c/(c - d) - d/(c - d))) + b*c**2*log(sqrt(-c
/(c - d) - d/(c - d)) + tan(x/2))*tan(x/2)**2/(c*d**2*sqrt(-c/(c - d) - d/(
c - d))*tan(x/2)**2 + c*d**2*sqrt(-c/(c - d) - d/(c - d)) - d**3*sqrt(-c/(c
 - d) - d/(c - d))*tan(x/2)**2 - d**3*sqrt(-c/(c - d) - d/(c - d))) + b*c**
2*log(sqrt(-c/(c - d) - d/(c - d)) + tan(x/2))/(c*d**2*sqrt(-c/(c - d) - d/
(c - d))*tan(x/2)**2 + c*d**2*sqrt(-c/(c - d) - d/(c - d)) - d**3*sqrt(-c/(
c - d) - d/(c - d))*tan(x/2)**2 - d**3*sqrt(-c/(c - d) - d/(c - d))) - b*c*
d*x*sqrt(-c/(c - d) - d/(c - d))*tan(x/2)**2/(c*d**2*sqrt(-c/(c - d) - d/(c
 - d))*tan(x/2)**2 + c*d**2*sqrt(-c/(c - d) - d/(c - d)) - d**3*sqrt(-c/(c
```

```
- d) - d/(c - d))*tan(x/2)**2 - d**3*sqrt(-c/(c - d) - d/(c - d))) - b*c*d*
x*sqrt(-c/(c - d) - d/(c - d))/(c*d**2*sqrt(-c/(c - d) - d/(c - d))*tan(x/2
)**2 + c*d**2*sqrt(-c/(c - d) - d/(c - d)) - d**3*sqrt(-c/(c - d) - d/(c -
d))*tan(x/2)**2 - d**3*sqrt(-c/(c - d) - d/(c - d))) - 2*b*c*d*sqrt(-c/(c -
 d) - d/(c - d))*tan(x/2)/(c*d**2*sqrt(-c/(c - d) - d/(c - d))*tan(x/2)**2
+ c*d**2*sqrt(-c/(c - d) - d/(c - d)) - d**3*sqrt(-c/(c - d) - d/(c - d))*t
an(x/2)**2 - d**3*sqrt(-c/(c - d) - d/(c - d))) + 2*b*d**2*sqrt(-c/(c - d)
- d/(c - d))*tan(x/2)/(c*d**2*sqrt(-c/(c - d) - d/(c - d))*tan(x/2)**2 + c*
d**2*sqrt(-c/(c - d) - d/(c - d)) - d**3*sqrt(-c/(c - d) - d/(c - d))*tan(x
/2)**2 - d**3*sqrt(-c/(c - d) - d/(c - d))) + b*d**2*log(-sqrt(-c/(c - d) -
 d/(c - d)) + tan(x/2))*tan(x/2)**2/(c*d**2*sqrt(-c/(c - d) - d/(c - d))*ta
n(x/2)**2 + c*d**2*sqrt(-c/(c - d) - d/(c - d)) - d**3*sqrt(-c/(c - d) - d/
(c - d))*tan(x/2)**2 - d**3*sqrt(-c/(c - d) - d/(c - d))) + b*d**2*log(-sqr
t(-c/(c - d) - d/(c - d)) + tan(x/2))/(c*d**2*sqrt(-c/(c - d) - d/(c - d))*
tan(x/2)**2 + c*d**2*sqrt(-c/(c - d) - d/(c - d)) - d**3*sqrt(-c/(c - d) -
d/(c - d))*tan(x/2)**2 - d**3*sqrt(-c/(c - d) - d/(c - d))) - b*d**2*log(sq
rt(-c/(c - d) - d/(c - d)) + tan(x/2))*tan(x/2)**2/(c*d**2*sqrt(-c/(c - d)
- d/(c - d))*tan(x/2)**2 + c*d**2*sqrt(-c/(c - d) - d/(c - d)) - d**3*sqrt(
-c/(c - d) - d/(c - d))*tan(x/2)**2 - d**3*sqrt(-c/(c - d) - d/(c - d))) -
b*d**2*log(sqrt(-c/(c - d) - d/(c - d)) + tan(x/2))/(c*d**2*sqrt(-c/(c - d)
 - d/(c - d))*tan(x/2)**2 + c*d**2*sqrt(-c/(c - d) - d/(c - d)) - d**3*sqrt
(-c/(c - d) - d/(c - d))*tan(x/2)**2 - d**3*sqrt(-c/(c - d) - d/(c - d))),
True))
```

3.207 $\int \frac{a+b\sin^2(x)}{c+c\cos^2(x)}\,dx$

Optimal. Leaf size=57

$$\frac{x(a+2b)}{\sqrt{2}\,c}-\frac{(a+2b)\tan^{-1}\left(\frac{\sin(x)\cos(x)}{\cos^2(x)+\sqrt{2}+1}\right)}{\sqrt{2}\,c}-\frac{bx}{c}$$

```
[Out] -b*x/c+1/2*(a+2*b)*x/c*2^(1/2)-1/2*(a+2*b)*arctan(cos(x)*sin(x)/(1+cos(x)^2
+2^(1/2)))/c*2^(1/2)
```

Rubi [A] time = 0.13, antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {12, 1166, 203}

$$\frac{x(a+2b)}{\sqrt{2}\,c}-\frac{(a+2b)\tan^{-1}\left(\frac{\sin(x)\cos(x)}{\cos^2(x)+\sqrt{2}+1}\right)}{\sqrt{2}\,c}-\frac{bx}{c}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[x]^2)/(c + c*Cos[x]^2),x]
```

```
[Out] -((b*x)/c) + ((a + 2*b)*x)/(Sqrt[2]*c) - ((a + 2*b)*ArcTan[(Cos[x]*Sin[x])/
(1 + Sqrt[2] + Cos[x]^2)])/(Sqrt[2]*c)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rubi steps

$$\begin{aligned}\int \frac{a+b\sin^2(x)}{c+c\cos^2(x)}\,dx &= \text{Subst}\left(\int \frac{a+(a+b)x^2}{c\left(2+3x^2+x^4\right)}\,dx,x,\tan(x)\right)\\ &= \frac{\text{Subst}\left(\int \frac{a+(a+b)x^2}{2+3x^2+x^4}\,dx,x,\tan(x)\right)}{c}\\ &= -\frac{b\,\text{Subst}\left(\int \frac{1}{1+x^2}\,dx,x,\tan(x)\right)}{c}+\frac{(a+2b)\,\text{Subst}\left(\int \frac{1}{2+x^2}\,dx,x,\tan(x)\right)}{c}\\ &= -\frac{bx}{c}+\frac{(a+2b)x}{\sqrt{2}\,c}-\frac{(a+2b)\tan^{-1}\left(\frac{\cos(x)\sin(x)}{1+\sqrt{2}+\cos^2(x)}\right)}{\sqrt{2}\,c}\end{aligned}$$

Mathematica [A] time = 0.08, size = 34, normalized size = 0.60

$$-\frac{(-a-2b)\tan^{-1}\left(\frac{\tan(x)}{\sqrt{2}}\right)}{\sqrt{2}\,c}-\frac{bx}{c}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sin[x]^2)/(c + c*Cos[x]^2),x]
```

```
[Out] -((b*x)/c) - ((-a - 2*b)*ArcTan[Tan[x]/Sqrt[2]])/(Sqrt[2]*c)
```

fricas [A] time = 1.33, size = 45, normalized size = 0.79

$$-\frac{\sqrt{2}\,(a+2\,b)\arctan\left(\frac{3\,\sqrt{2}\,\cos(x)^2-\sqrt{2}}{4\,\cos(x)\sin(x)}\right)+4\,bx}{4\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(x)^2)/(c+c*cos(x)^2),x, algorithm="fricas")
```

```
[Out] -1/4*(sqrt(2)*(a + 2*b)*arctan(1/4*(3*sqrt(2)*cos(x)^2 - sqrt(2))/(cos(x)*s
in(x))) + 4*b*x)/c
```

giac [A] time = 0.14, size = 62, normalized size = 1.09

$$\frac{\sqrt{2}\,(a+2\,b)\left(x+\arctan\left(-\frac{\sqrt{2}\,\sin(2\,x)-\sin(2\,x)}{\sqrt{2}\,\cos(2\,x)+\sqrt{2}-\cos(2\,x)+1}\right)\right)}{2\,c}-\frac{bx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(x)^2)/(c+c*cos(x)^2),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(2)*(a + 2*b)*(x + arctan(-(sqrt(2)*sin(2*x) - sin(2*x))/(sqrt(2)*c
os(2*x) + sqrt(2) - cos(2*x) + 1)))/c - b*x/c
```

maple [A] time = 0.13, size = 44, normalized size = 0.77

$$\frac{\sqrt{2}\ \arctan\left(\frac{\sqrt{2}\ \tan(x)}{2}\right) a}{2c}+\frac{\sqrt{2}\ \arctan\left(\frac{\sqrt{2}\ \tan(x)}{2}\right) b}{c}-\frac{b\arctan\left(\tan(x)\right)}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(x)^2)/(c+c*cos(x)^2),x)
```

```
[Out] 1/2/c*2^(1/2)*arctan(1/2*2^(1/2)*tan(x))*a+1/c*2^(1/2)*arctan(1/2*2^(1/2)*t
an(x))*b-1/c*b*arctan(tan(x))
```

maxima [A] time = 0.42, size = 29, normalized size = 0.51

$$\frac{\sqrt{2}\,(a+2\,b)\arctan\left(\frac{1}{2}\,\sqrt{2}\ \tan(x)\right)}{2\,c}-\frac{bx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(x)^2)/(c+c*cos(x)^2),x, algorithm="maxima")
```

```
[Out] 1/2*sqrt(2)*(a + 2*b)*arctan(1/2*sqrt(2)*tan(x))/c - b*x/c
```

mupad [B] time = 2.43, size = 242, normalized size = 4.25

$$\frac{\sqrt{2}\ \mathrm{atan}\left(\frac{\sqrt{2}\ a^3\tan(x)}{2\left(a^3+6\,a^2\,b+10\,a\,b^2+4\,b^3\right)}+\frac{2\ \sqrt{2}\ b^3\tan(x)}{a^3+6\,a^2\,b+10\,a\,b^2+4\,b^3}+\frac{5\ \sqrt{2}\ a\,b^2\tan(x)}{a^3+6\,a^2\,b+10\,a\,b^2+4\,b^3}+\frac{3\ \sqrt{2}\ a^2\,b\tan(x)}{a^3+6\,a^2\,b+10\,a\,b^2+4\,b^3}\right)\,(a+2\,b)}{2\,c}-\frac{b\,\mathrm{atan}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(x)^2)/(c + c*cos(x)^2),x)
```

```
[Out] (2^(1/2)*atan((2^(1/2)*a^3*tan(x))/(2*(10*a*b^2 + 6*a^2*b + a^3 + 4*b^3)) +
 (2*2^(1/2)*b^3*tan(x))/(10*a*b^2 + 6*a^2*b + a^3 + 4*b^3) + (5*2^(1/2)*a*b
^2*tan(x))/(10*a*b^2 + 6*a^2*b + a^3 + 4*b^3) + (3*2^(1/2)*a^2*b*tan(x))/(1
0*a*b^2 + 6*a^2*b + a^3 + 4*b^3))*(a + 2*b))/(2*c) - (b*atan((4*b^3*tan(x))
/(8*a*b^2 + 2*a^2*b + 4*b^3) + (8*a*b^2*tan(x))/(8*a*b^2 + 2*a^2*b + 4*b^3)
 + (2*a^2*b*tan(x))/(8*a*b^2 + 2*a^2*b + 4*b^3)))/c
```

sympy [B] time = 10.67, size = 143, normalized size = 2.51

$$\frac{\sqrt{2}\,a\left(\operatorname{atan}\left(\sqrt{2}\,\tan\left(\frac{x}{2}\right)-1\right)+\pi\left\lfloor\frac{\frac{x}{2}-\frac{\pi}{2}}{\pi}\right\rfloor\right)}{2c}+\frac{\sqrt{2}\,a\left(\operatorname{atan}\left(\sqrt{2}\,\tan\left(\frac{x}{2}\right)+1\right)+\pi\left\lfloor\frac{\frac{x}{2}-\frac{\pi}{2}}{\pi}\right\rfloor\right)}{2c}-\frac{bx}{c}+\frac{\sqrt{2}\,b\left(\operatorname{atan}\left(\sqrt{2}\,\tan\left(\frac{x}{2}\right.\right.\right.}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(x)**2)/(c+c*cos(x)**2),x)
```

```
[Out] sqrt(2)*a*(atan(sqrt(2)*tan(x/2) - 1) + pi*floor((x/2 - pi/2)/pi))/(2*c) +
sqrt(2)*a*(atan(sqrt(2)*tan(x/2) + 1) + pi*floor((x/2 - pi/2)/pi))/(2*c) -
b*x/c + sqrt(2)*b*(atan(sqrt(2)*tan(x/2) - 1) + pi*floor((x/2 - pi/2)/pi))/
c + sqrt(2)*b*(atan(sqrt(2)*tan(x/2) + 1) + pi*floor((x/2 - pi/2)/pi))/c
```

3.208 $\int \frac{a+b\sin^2(x)}{c-c\cos^2(x)}\,dx$

Optimal. Leaf size=15

$$\frac{bx}{c}-\frac{a\cot(x)}{c}$$

```
[Out] b*x/c-a*cot(x)/c
```

Rubi [A] time = 0.09, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {453, 205}

$$\frac{bx}{c}-\frac{a\cot(x)}{c}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[x]^2)/(c - c*Cos[x]^2),x]
```

```
[Out] (b*x)/c - (a*Cot[x])/c
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{a+b\sin^2(x)}{c-c\cos^2(x)}\,dx &= \text{Subst}\left(\int \frac{a+(a+b)x^2}{x^2\left(c+cx^2\right)}\,dx,x,\tan(x)\right)\\
&= -\frac{a\cot(x)}{c}+b\,\text{Subst}\left(\int \frac{1}{c+cx^2}\,dx,x,\tan(x)\right)\\
&= \frac{bx}{c}-\frac{a\cot(x)}{c}
\end{aligned}$$

Mathematica [A] time = 0.01, size = 15, normalized size = 1.00

$$\frac{bx}{c} - \frac{a\cot(x)}{c}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sin[x]^2)/(c - c*Cos[x]^2),x]
```

```
[Out] (b*x)/c - (a*Cot[x])/c
```

fricas [A] time = 0.61, size = 19, normalized size = 1.27

$$\frac{bx\sin(x) - a\cos(x)}{c\sin(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(x)^2)/(c-c*cos(x)^2),x, algorithm="fricas")
```

```
[Out] (b*x*sin(x) - a*cos(x))/(c*sin(x))
```

giac [A] time = 0.15, size = 29, normalized size = 1.93

$$\frac{bx}{c} + \frac{a\tan\left(\frac{1}{2}x\right)}{2c} - \frac{a}{2c\tan\left(\frac{1}{2}x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(x)^2)/(c-c*cos(x)^2),x, algorithm="giac")
```

```
[Out] b*x/c + 1/2*a*tan(1/2*x)/c - 1/2*a/(c*tan(1/2*x))
```

maple [A] time = 0.11, size = 20, normalized size = 1.33

$$-\frac{a}{c\tan(x)} + \frac{b\arctan(\tan(x))}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(x)^2)/(c-c*cos(x)^2),x)
```

```
[Out] -1/c*a/tan(x)+1/c*b*arctan(tan(x))
```

maxima [A] time = 0.42, size = 17, normalized size = 1.13

$$\frac{bx}{c} - \frac{a}{c\tan(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(x)^2)/(c-c*cos(x)^2),x, algorithm="maxima")
```

```
[Out] b*x/c - a/(c*tan(x))
```

mupad [B] time = 2.31, size = 13, normalized size = 0.87

$$\frac{b\,x-a\cot(x)}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(x)^2)/(c - c*cos(x)^2),x)
```

```
[Out] (b*x - a*cot(x))/c
```

sympy [B] time = 1.18, size = 24, normalized size = 1.60

$$\frac{a\tan\left(\frac{x}{2}\right)}{2c}-\frac{a}{2c\tan\left(\frac{x}{2}\right)}+\frac{bx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(x)**2)/(c-c*cos(x)**2),x)
```

```
[Out] a*tan(x/2)/(2*c) - a/(2*c*tan(x/2)) + b*x/c
```

3.209 $\int \frac{a+b\sin^2(x)}{c+d\cos^2(x)}\,dx$

Optimal. Leaf size=49

$$\frac{(ad+b(c+d))\tan^{-1}\left(\frac{\sqrt{c}\tan(x)}{\sqrt{c+d}}\right)}{\sqrt{c}\,d\sqrt{c+d}}-\frac{bx}{d}$$

```
[Out] -b*x/d+(a*d+b*(c+d))*arctan(c^(1/2)*tan(x)/(c+d)^(1/2))/d/c^(1/2)/(c+d)^(1/
2)
```

Rubi [A] time = 0.15, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {522, 203, 205}

$$\frac{(ad+b(c+d))\tan^{-1}\left(\frac{\sqrt{c}\tan(x)}{\sqrt{c+d}}\right)}{\sqrt{c}\,d\sqrt{c+d}}-\frac{bx}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[x]^2)/(c + d*Cos[x]^2),x]
```

```
[Out] -((b*x)/d) + ((a*d + b*(c + d))*ArcTan[(Sqrt[c]*Tan[x])/Sqrt[c + d]])/(Sqrt
[c]*d*Sqrt[c + d])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rubi steps

$$\int \frac{a+b\sin^2(x)}{c+d\cos^2(x)}\,dx = \text{Subst}\left(\int \frac{a+(a+b)x^2}{(1+x^2)\,(c+d+cx^2)}\,dx, x, \tan(x)\right)$$
$$= -\frac{b\,\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \tan(x)\right)}{d} + \frac{(ad+b(c+d))\,\text{Subst}\left(\int \frac{1}{c+d+cx^2}\,dx, x, \tan(x)\right)}{d}$$
$$= -\frac{bx}{d} + \frac{(ad+b(c+d))\tan^{-1}\left(\frac{\sqrt{c}\,\tan(x)}{\sqrt{c+d}}\right)}{\sqrt{c}\,d\sqrt{c+d}}$$

Mathematica [A] time = 0.16, size = 47, normalized size = 0.96

$$\frac{\frac{(ad+b(c+d))\tan^{-1}\left(\frac{\sqrt{c}\,\tan(x)}{\sqrt{c+d}}\right)}{\sqrt{c}\,\sqrt{c+d}} - bx}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sin[x]^2)/(c + d*Cos[x]^2),x]
```

```
[Out] (-(b*x) + ((a*d + b*(c + d))*ArcTan[(Sqrt[c]*Tan[x])/Sqrt[c + d]])/(Sqrt[c]
*Sqrt[c + d]))/d
```

fricas [A] time = 0.79, size = 228, normalized size = 4.65

$$\left[-\frac{(bc+(a+b)d)\sqrt{-c^2-cd}\,\log\left(\frac{(8\,c^2+8\,cd+d^2)\cos(x)^4-2\,(4\,c^2+3\,cd)\cos(x)^2+4\,((2\,c+d)\cos(x)^3-c\cos(x))\sqrt{-c^2-cd}\,\sin(x)+c^2}{d^2\cos(x)^4+2\,cd\cos(x)^2+c^2}\right)+4\,(b}{4\,(c^2d+cd^2)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(x)^2)/(c+d*cos(x)^2),x, algorithm="fricas")
```

```
[Out] [-1/4*((b*c + (a + b)*d)*sqrt(-c^2 - c*d)*log(((8*c^2 + 8*c*d + d^2)*cos(x)
^4 - 2*(4*c^2 + 3*c*d)*cos(x)^2 + 4*((2*c + d)*cos(x)^3 - c*cos(x))*sqrt(-c
^2 - c*d)*sin(x) + c^2)/(d^2*cos(x)^4 + 2*c*d*cos(x)^2 + c^2)) + 4*(b*c^2 +
 b*c*d)*x)/(c^2*d + c*d^2), -1/2*((b*c + (a + b)*d)*sqrt(c^2 + c*d)*arctan(
1/2*((2*c + d)*cos(x)^2 - c)/(sqrt(c^2 + c*d)*cos(x)*sin(x))) + 2*(b*c^2 +
b*c*d)*x)/(c^2*d + c*d^2)]
```

giac [A] time = 0.15, size = 58, normalized size = 1.18

$$-\frac{bx}{d} + \frac{\left(\pi\left\lfloor\frac{x}{\pi}+\frac{1}{2}\right\rfloor\text{sgn}(c)+\arctan\left(\frac{c\tan(x)}{\sqrt{c^2+cd}}\right)\right)(bc+ad+bd)}{\sqrt{c^2+cd}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(x)^2)/(c+d*cos(x)^2),x, algorithm="giac")
```

```
[Out] -b*x/d + (pi*floor(x/pi + 1/2)*sgn(c) + arctan(c*tan(x)/sqrt(c^2 + c*d)))*(
b*c + a*d + b*d)/(sqrt(c^2 + c*d)*d)
```

maple [A] time = 0.12, size = 78, normalized size = 1.59

$$\frac{\arctan\left(\frac{c\tan(x)}{\sqrt{(c+d)c}}\right)a}{\sqrt{(c+d)\,c}}+\frac{\arctan\left(\frac{c\tan(x)}{\sqrt{(c+d)c}}\right)cb}{d\sqrt{(c+d)\,c}}+\frac{\arctan\left(\frac{c\tan(x)}{\sqrt{(c+d)c}}\right)b}{\sqrt{(c+d)\,c}}-\frac{b\arctan(\tan(x))}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(x)^2)/(c+d*cos(x)^2),x)
```

```
[Out] 1/((c+d)*c)^(1/2)*arctan(c*tan(x)/((c+d)*c)^(1/2))*a+1/d/((c+d)*c)^(1/2)*ar
ctan(c*tan(x)/((c+d)*c)^(1/2))*c*b+1/((c+d)*c)^(1/2)*arctan(c*tan(x)/((c+d)
*c)^(1/2))*b-b/d*arctan(tan(x))
```

maxima [A] time = 1.34, size = 40, normalized size = 0.82

$$-\frac{bx}{d}+\frac{(bc+(a+b)d)\arctan\left(\frac{c\tan(x)}{\sqrt{(c+d)c}}\right)}{\sqrt{(c+d)c}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(x)^2)/(c+d*cos(x)^2),x, algorithm="maxima")
```

```
[Out] -b*x/d + (b*c + (a + b)*d)*arctan(c*tan(x)/sqrt((c + d)*c))/(sqrt((c + d)*c
)*d)
```

mupad [B] time = 2.85, size = 1987, normalized size = 40.55

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(x)^2)/(c + d*cos(x)^2),x)
```

```
[Out] - (b*c^2*x)/(c*d^2 + c^2*d) - (a*d*atan((a^2*d^3*tan(x)*(- c*d - c^2)^(3/2)
*1i + b^2*c^3*tan(x)*(- c*d - c^2)^(3/2)*2i + b^2*c^5*tan(x)*(- c*d - c^2)^
(1/2)*2i + b^2*d^3*tan(x)*(- c*d - c^2)^(3/2)*1i + a^2*c*d^2*tan(x)*(- c*d
- c^2)^(3/2)*2i + a^2*c*d^4*tan(x)*(- c*d - c^2)^(1/2)*1i + b^2*c*d^2*tan(x
)*(- c*d - c^2)^(3/2)*4i + b^2*c*d^4*tan(x)*(- c*d - c^2)^(1/2)*1i + b^2*c^
2*d*tan(x)*(- c*d - c^2)^(3/2)*5i + b^2*c^4*d*tan(x)*(- c*d - c^2)^(1/2)*6i
```

```
 + a^2*c^2*d^3*tan(x)*(- c*d - c^2)^(1/2)*2i + a^2*c^3*d^2*tan(x)*(- c*d -
c^2)^(1/2)*1i + b^2*c^2*d^3*tan(x)*(- c*d - c^2)^(1/2)*4i + b^2*c^3*d^2*tan
(x)*(- c*d - c^2)^(1/2)*7i + a*b*d^3*tan(x)*(- c*d - c^2)^(3/2)*2i + a*b*c*
d^2*tan(x)*(- c*d - c^2)^(3/2)*6i + a*b*c*d^4*tan(x)*(- c*d - c^2)^(1/2)*2i
 + a*b*c^2*d*tan(x)*(- c*d - c^2)^(3/2)*4i + a*b*c^4*d*tan(x)*(- c*d - c^2)
^(1/2)*2i + a*b*c^2*d^3*tan(x)*(- c*d - c^2)^(1/2)*6i + a*b*c^3*d^2*tan(x)*
(- c*d - c^2)^(1/2)*6i)/(b^2*c^5*d + a^2*c^2*d^4 + 2*a^2*c^3*d^3 + a^2*c^4*
d^2 + b^2*c^2*d^4 + 3*b^2*c^3*d^3 + 3*b^2*c^4*d^2 + 2*a*b*c^5*d + 2*a*b*c^2
*d^4 + 6*a*b*c^3*d^3 + 6*a*b*c^4*d^2))*(- c*d - c^2)^(1/2)*1i)/(c*d^2 + c^2
*d) - (b*c*atan((a^2*d^3*tan(x)*(- c*d - c^2)^(3/2)*1i + b^2*c^3*tan(x)*(-
c*d - c^2)^(3/2)*2i + b^2*c^5*tan(x)*(- c*d - c^2)^(1/2)*2i + b^2*d^3*tan(x
)*(- c*d - c^2)^(3/2)*1i + a^2*c*d^2*tan(x)*(- c*d - c^2)^(3/2)*2i + a^2*c*
d^4*tan(x)*(- c*d - c^2)^(1/2)*1i + b^2*c*d^2*tan(x)*(- c*d - c^2)^(3/2)*4i
 + b^2*c*d^4*tan(x)*(- c*d - c^2)^(1/2)*1i + b^2*c^2*d*tan(x)*(- c*d - c^2)
^(3/2)*5i + b^2*c^4*d*tan(x)*(- c*d - c^2)^(1/2)*6i + a^2*c^2*d^3*tan(x)*(-
 c*d - c^2)^(1/2)*2i + a^2*c^3*d^2*tan(x)*(- c*d - c^2)^(1/2)*1i + b^2*c^2*
d^3*tan(x)*(- c*d - c^2)^(1/2)*4i + b^2*c^3*d^2*tan(x)*(- c*d - c^2)^(1/2)*
7i + a*b*d^3*tan(x)*(- c*d - c^2)^(3/2)*2i + a*b*c*d^2*tan(x)*(- c*d - c^2)
^(3/2)*6i + a*b*c*d^4*tan(x)*(- c*d - c^2)^(1/2)*2i + a*b*c^2*d*tan(x)*(- c
*d - c^2)^(3/2)*4i + a*b*c^4*d*tan(x)*(- c*d - c^2)^(1/2)*2i + a*b*c^2*d^3*
tan(x)*(- c*d - c^2)^(1/2)*6i + a*b*c^3*d^2*tan(x)*(- c*d - c^2)^(1/2)*6i)/
(b^2*c^5*d + a^2*c^2*d^4 + 2*a^2*c^3*d^3 + a^2*c^4*d^2 + b^2*c^2*d^4 + 3*b^
2*c^3*d^3 + 3*b^2*c^4*d^2 + 2*a*b*c^5*d + 2*a*b*c^2*d^4 + 6*a*b*c^3*d^3 + 6
*a*b*c^4*d^2))*(- c*d - c^2)^(1/2)*1i)/(c*d^2 + c^2*d) - (b*d*atan((a^2*d^3
*tan(x)*(- c*d - c^2)^(3/2)*1i + b^2*c^3*tan(x)*(- c*d - c^2)^(3/2)*2i + b^
2*c^5*tan(x)*(- c*d - c^2)^(1/2)*2i + b^2*d^3*tan(x)*(- c*d - c^2)^(3/2)*1i
 + a^2*c*d^2*tan(x)*(- c*d - c^2)^(3/2)*2i + a^2*c*d^4*tan(x)*(- c*d - c^2)
^(1/2)*1i + b^2*c*d^2*tan(x)*(- c*d - c^2)^(3/2)*4i + b^2*c*d^4*tan(x)*(- c
*d - c^2)^(1/2)*1i + b^2*c^2*d*tan(x)*(- c*d - c^2)^(3/2)*5i + b^2*c^4*d*ta
n(x)*(- c*d - c^2)^(1/2)*6i + a^2*c^2*d^3*tan(x)*(- c*d - c^2)^(1/2)*2i + a
^2*c^3*d^2*tan(x)*(- c*d - c^2)^(1/2)*1i + b^2*c^2*d^3*tan(x)*(- c*d - c^2)
^(1/2)*4i + b^2*c^3*d^2*tan(x)*(- c*d - c^2)^(1/2)*7i + a*b*d^3*tan(x)*(- c
*d - c^2)^(3/2)*2i + a*b*c*d^2*tan(x)*(- c*d - c^2)^(3/2)*6i + a*b*c*d^4*ta
n(x)*(- c*d - c^2)^(1/2)*2i + a*b*c^2*d*tan(x)*(- c*d - c^2)^(3/2)*4i + a*b
*c^4*d*tan(x)*(- c*d - c^2)^(1/2)*2i + a*b*c^2*d^3*tan(x)*(- c*d - c^2)^(1/
2)*6i + a*b*c^3*d^2*tan(x)*(- c*d - c^2)^(1/2)*6i)/(b^2*c^5*d + a^2*c^2*d^4
 + 2*a^2*c^3*d^3 + a^2*c^4*d^2 + b^2*c^2*d^4 + 3*b^2*c^3*d^3 + 3*b^2*c^4*d^
2 + 2*a*b*c^5*d + 2*a*b*c^2*d^4 + 6*a*b*c^3*d^3 + 6*a*b*c^4*d^2))*(- c*d -
c^2)^(1/2)*1i)/(c*d^2 + c^2*d) - (b*c*d*x)/(c*d^2 + c^2*d)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(x)**2)/(c+d*cos(x)**2),x)
```

```
[Out] Timed out
```

3.210 $\int \frac{-1+\frac{c^2}{d^2}+\cos^2(x)}{c+d\sin(x)}\,dx$

Optimal. Leaf size=13

$$\frac{cx}{d^2}+\frac{\cos(x)}{d}$$

[Out] c*x/d^2+cos(x)/d

Rubi [A] time = 0.12, antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {4397, 3016, 2638}

$$\frac{cx}{d^2}+\frac{\cos(x)}{d}$$

Antiderivative was successfully verified.

[In] Int[(-1 + c^2/d^2 + Cos[x]^2)/(c + d*Sin[x]),x]

[Out] (c*x)/d^2 + Cos[x]/d

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3016

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (C_.)*sin[(e_.) +
 (f_.)*(x_)]^2), x_Symbol] :> Dist[C/b^2, Int[(a + b*Sin[e + f*x])^(m + 1)*
Simp[-a + b*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, C, m}, x] &&
EqQ[A*b^2 + a^2*C, 0]
```

Rule 4397

```
Int[u_, x_Symbol] :> Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]
```

Rubi steps

$$\int \frac{-1+\frac{c^2}{d^2}+\cos^2(x)}{c+d\sin(x)}\,dx = \int \frac{\frac{c^2}{d^2}-\sin^2(x)}{c+d\sin(x)}\,dx$$
$$= -\frac{\int (-c+d\sin(x))\,dx}{d^2}$$
$$= \frac{cx}{d^2} - \frac{\int \sin(x)\,dx}{d}$$
$$= \frac{cx}{d^2} + \frac{\cos(x)}{d}$$

Mathematica [A] time = 0.01, size = 13, normalized size = 1.00

$$\frac{cx}{d^2} + \frac{\cos(x)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-1 + c^2/d^2 + Cos[x]^2)/(c + d*Sin[x]),x]
[Out] (c*x)/d^2 + Cos[x]/d
```

fricas [A] time = 0.68, size = 12, normalized size = 0.92

$$\frac{cx + d\cos(x)}{d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+c^2/d^2+cos(x)^2)/(c+d*sin(x)),x, algorithm="fricas")
[Out] (c*x + d*cos(x))/d^2
```

giac [A] time = 0.15, size = 22, normalized size = 1.69

$$\frac{cx}{d^2} + \frac{2}{\left(\tan\left(\frac{1}{2}x\right)^2 + 1\right)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+c^2/d^2+cos(x)^2)/(c+d*sin(x)),x, algorithm="giac")
[Out] c*x/d^2 + 2/((tan(1/2*x)^2 + 1)*d)
```

maple [B] time = 0.12, size = 28, normalized size = 2.15

$$\frac{2}{d\left(1+\tan^2\left(\frac{x}{2}\right)\right)}+\frac{2c\arctan\left(\tan\left(\frac{x}{2}\right)\right)}{d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-1+c^2/d^2+cos(x)^2)/(c+d*sin(x)),x)
```

```
[Out] 2/d/(1+tan(1/2*x)^2)+2/d^2*c*arctan(tan(1/2*x))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+c^2/d^2+cos(x)^2)/(c+d*sin(x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*d^2-4*c^2>0)', see `assume?` for
more details)Is 4*d^2-4*c^2 positive or negative?
```

mupad [B] time = 2.46, size = 13, normalized size = 1.00

$$\frac{\cos(x)}{d}+\frac{c\,x}{d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(x)^2 + c^2/d^2 - 1)/(c + d*sin(x)),x)
```

```
[Out] cos(x)/d + (c*x)/d^2
```

sympy [B] time = 103.32, size = 56, normalized size = 4.31

$$\frac{cx\tan^2\left(\frac{x}{2}\right)}{d^2\tan^2\left(\frac{x}{2}\right)+d^2}+\frac{cx}{d^2\tan^2\left(\frac{x}{2}\right)+d^2}+\frac{2d}{d^2\tan^2\left(\frac{x}{2}\right)+d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+c**2/d**2+cos(x)**2)/(c+d*sin(x)),x)
```

```
[Out] c*x*tan(x/2)**2/(d**2*tan(x/2)**2 + d**2) + c*x/(d**2*tan(x/2)**2 + d**2) +
 2*d/(d**2*tan(x/2)**2 + d**2)
```

3.211 $\int \frac{a+b\cos^2(x)}{c+d\sin(x)}\,dx$

Optimal. Leaf size=100

$$\frac{2a\tan^{-1}\left(\frac{c\tan\left(\frac{x}{2}\right)+d}{\sqrt{c^2-d^2}}\right)}{\sqrt{c^2-d^2}}-\frac{2b\sqrt{c^2-d^2}\ \tan^{-1}\left(\frac{c\tan\left(\frac{x}{2}\right)+d}{\sqrt{c^2-d^2}}\right)}{d^2}+\frac{bcx}{d^2}+\frac{b\cos(x)}{d}$$

```
[Out] b*c*x/d^2+b*cos(x)/d+2*a*arctan((d+c*tan(1/2*x))/(c^2-d^2)^(1/2))/(c^2-d^2)
^(1/2)-2*b*arctan((d+c*tan(1/2*x))/(c^2-d^2)^(1/2))*(c^2-d^2)^(1/2)/d^2
```

Rubi [A] time = 0.24, antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 6, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.353, Rules used = {4401, 2660, 618, 204, 2695, 2735}

$$\frac{2a\tan^{-1}\left(\frac{c\tan\left(\frac{x}{2}\right)+d}{\sqrt{c^2-d^2}}\right)}{\sqrt{c^2-d^2}}-\frac{2b\sqrt{c^2-d^2}\ \tan^{-1}\left(\frac{c\tan\left(\frac{x}{2}\right)+d}{\sqrt{c^2-d^2}}\right)}{d^2}+\frac{bcx}{d^2}+\frac{b\cos(x)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[x]^2)/(c + d*Sin[x]),x]
```

```
[Out] (b*c*x)/d^2 + (2*a*ArcTan[(d + c*Tan[x/2])/Sqrt[c^2 - d^2]])/Sqrt[c^2 - d^2
] - (2*b*Sqrt[c^2 - d^2]*ArcTan[(d + c*Tan[x/2])/Sqrt[c^2 - d^2]])/d^2 + (b
*Cos[x])/d
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
```

```
a^2 - b^2, 0]
```

Rule 2695

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(g*(g*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x
])^(m + 1))/(b*f*(m + p)), x] + Dist[(g^2*(p - 1))/(b*(m + p)), Int[(g*Cos[
e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^m*(b + a*Sin[e + f*x]), x], x] /; Fr
eeQ[{a, b, e, f, g, m}, x] && NeQ[a^2 - b^2, 0] && GtQ[p, 1] && NeQ[m + p,
0] && IntegersQ[2*m, 2*p]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 4401

```
Int[u_, x_Symbol] :> With[{v = ExpandTrig[u, x]}, Int[v, x] /; SumQ[v]] /;
 !InertTrigFreeQ[u]
```

Rubi steps

$$\int \frac{a+b\cos^2(x)}{c+d\sin(x)}\,dx = \int \left(\frac{a}{c+d\sin(x)}+\frac{b\cos^2(x)}{c+d\sin(x)}\right)dx$$
$$= a\int \frac{1}{c+d\sin(x)}\,dx + b\int \frac{\cos^2(x)}{c+d\sin(x)}\,dx$$
$$= \frac{b\cos(x)}{d} + (2a)\,\text{Subst}\left(\int \frac{1}{c+2dx+cx^2}\,dx, x, \tan\left(\frac{x}{2}\right)\right) + \frac{b\int \frac{d+c\sin(x)}{c+d\sin(x)}\,dx}{d}$$
$$= \frac{bcx}{d^2} + \frac{b\cos(x)}{d} - (4a)\,\text{Subst}\left(\int \frac{1}{-4\left(c^2-d^2\right)-x^2}\,dx, x, 2d+2c\tan\left(\frac{x}{2}\right)\right) - \frac{\left(b\left(c^2-d^2\right)\right)\int}{d^2}$$
$$= \frac{bcx}{d^2} + \frac{2a\tan^{-1}\left(\frac{d+c\tan\left(\frac{x}{2}\right)}{\sqrt{c^2-d^2}}\right)}{\sqrt{c^2-d^2}} + \frac{b\cos(x)}{d} - \frac{\left(2b\left(c^2-d^2\right)\right)\text{Subst}\left(\int \frac{1}{c+2dx+cx^2}\,dx, x, \tan\left(\frac{x}{2}\right)\right)}{d^2}$$
$$= \frac{bcx}{d^2} + \frac{2a\tan^{-1}\left(\frac{d+c\tan\left(\frac{x}{2}\right)}{\sqrt{c^2-d^2}}\right)}{\sqrt{c^2-d^2}} + \frac{b\cos(x)}{d} + \frac{\left(4b\left(c^2-d^2\right)\right)\text{Subst}\left(\int \frac{1}{-4\left(c^2-d^2\right)-x^2}\,dx, x, 2d+2c\right.}{d^2}$$
$$= \frac{bcx}{d^2} + \frac{2a\tan^{-1}\left(\frac{d+c\tan\left(\frac{x}{2}\right)}{\sqrt{c^2-d^2}}\right)}{\sqrt{c^2-d^2}} - \frac{2b\sqrt{c^2-d^2}\,\tan^{-1}\left(\frac{d+c\tan\left(\frac{x}{2}\right)}{\sqrt{c^2-d^2}}\right)}{d^2} + \frac{b\cos(x)}{d}$$

Mathematica [A] time = 0.18, size = 72, normalized size = 0.72

$$\frac{\frac{2\left(ad^2+b\left(d^2-c^2\right)\right)\tan^{-1}\left(\frac{c\tan\left(\frac{x}{2}\right)+d}{\sqrt{c^2-d^2}}\right)}{\sqrt{c^2-d^2}} + b(cx+d\cos(x))}{d^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[x]^2)/(c + d*Sin[x]),x]
```

```
[Out] ((2*(a*d^2 + b*(-c^2 + d^2))*ArcTan[(d + c*Tan[x/2])/Sqrt[c^2 - d^2]])/Sqrt
[c^2 - d^2] + b*(c*x + d*Cos[x]))/d^2
```

fricas [A] time = 0.78, size = 262, normalized size = 2.62

$$\left[\frac{\left(bc^2-(a+b)d^2\right)\sqrt{-c^2+d^2}\,\log\left(\frac{\left(2c^2-d^2\right)\cos(x)^2-2cd\sin(x)-c^2-d^2+2\left(c\cos(x)\sin(x)+d\cos(x)\right)\sqrt{-c^2+d^2}}{d^2\cos(x)^2-2cd\sin(x)-c^2-d^2}\right)+2\left(bc^3-bcd^2\right)x+}{2\left(c^2d^2-d^4\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*cos(x)^2)/(c+d*sin(x)),x, algorithm="fricas")

[Out] [1/2*((b*c^2 - (a + b)*d^2)*sqrt(-c^2 + d^2)*log(((2*c^2 - d^2)*cos(x)^2 - 2*c*d*sin(x) - c^2 - d^2 + 2*(c*cos(x)*sin(x) + d*cos(x))*sqrt(-c^2 + d^2))/(d^2*cos(x)^2 - 2*c*d*sin(x) - c^2 - d^2)) + 2*(b*c^3 - b*c*d^2)*x + 2*(b*c^2*d - b*d^3)*cos(x))/(c^2*d^2 - d^4), ((b*c^2 - (a + b)*d^2)*sqrt(c^2 - d^2)*arctan(-(c*sin(x) + d)/(sqrt(c^2 - d^2)*cos(x))) + (b*c^3 - b*c*d^2)*x + (b*c^2*d - b*d^3)*cos(x))/(c^2*d^2 - d^4)]

giac [A] time = 0.15, size = 93, normalized size = 0.93

$$\frac{bcx}{d^2} - \frac{2\left(bc^2 - ad^2 - bd^2\right)\left(\pi\left\lfloor\frac{x}{2\pi} + \frac{1}{2}\right\rfloor\operatorname{sgn}(c) + \arctan\left(\frac{c\tan\left(\frac{1}{2}x\right)+d}{\sqrt{c^2-d^2}}\right)\right)}{\sqrt{c^2-d^2}\,d^2} + \frac{2\,b}{\left(\tan\left(\frac{1}{2}x\right)^2 + 1\right)d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*cos(x)^2)/(c+d*sin(x)),x, algorithm="giac")

[Out] b*c*x/d^2 - 2*(b*c^2 - a*d^2 - b*d^2)*(pi*floor(1/2*x/pi + 1/2)*sgn(c) + arctan((c*tan(1/2*x) + d)/sqrt(c^2 - d^2)))/(sqrt(c^2 - d^2)*d^2) + 2*b/((tan(1/2*x)^2 + 1)*d)

maple [A] time = 0.08, size = 153, normalized size = 1.53

$$\frac{2\arctan\left(\frac{2c\tan\left(\frac{x}{2}\right)+2d}{2\sqrt{c^2-d^2}}\right)a}{\sqrt{c^2-d^2}} - \frac{2\arctan\left(\frac{2c\tan\left(\frac{x}{2}\right)+2d}{2\sqrt{c^2-d^2}}\right)c^2b}{d^2\sqrt{c^2-d^2}} + \frac{2\arctan\left(\frac{2c\tan\left(\frac{x}{2}\right)+2d}{2\sqrt{c^2-d^2}}\right)b}{\sqrt{c^2-d^2}} + \frac{2b}{d\left(1+\tan^2\left(\frac{x}{2}\right)\right)} + \frac{2bc\arctan\left(\text{ta}\right.}{d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b*cos(x)^2)/(c+d*sin(x)),x)

[Out] 2/(c^2-d^2)^(1/2)*arctan(1/2*(2*c*tan(1/2*x)+2*d)/(c^2-d^2)^(1/2))*a-2/d^2/(c^2-d^2)^(1/2)*arctan(1/2*(2*c*tan(1/2*x)+2*d)/(c^2-d^2)^(1/2))*c^2*b+2/(c^2-d^2)^(1/2)*arctan(1/2*(2*c*tan(1/2*x)+2*d)/(c^2-d^2)^(1/2))*b+2*b/d/(1+tan(1/2*x)^2)+2*b/d^2*c*arctan(tan(1/2*x))

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(x)^2)/(c+d*sin(x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*d^2-4*c^2>0)', see `assume?` for
more details)Is 4*d^2-4*c^2 positive or negative?
```

mupad [B] time = 5.15, size = 1646, normalized size = 16.46

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*cos(x)^2)/(c + d*sin(x)),x)
```

```
[Out] (2*b)/(d*(tan(x/2)^2 + 1)) - (atan((((-(c + d)*(c - d))^(1/2)*((32*b^2*c^4)
/d - (32*tan(x/2)*(a^2*c*d^5 + b^2*c*d^5 + 2*b^2*c^5*d - 4*b^2*c^3*d^3 + 2*
a*b*c*d^5 - 2*a*b*c^3*d^3))/d^3 + ((-(c + d)*(c - d))^(1/2)*(32*a*c^2*d^2 +
 (32*tan(x/2)*(2*a*c*d^6 - 2*b*c^3*d^4 + 2*b*c*d^6))/d^3 + ((-(c + d)*(c -
d))^(1/2)*(32*c^2*d^3 + (32*tan(x/2)*(3*c*d^7 - 2*c^3*d^5))/d^3)*(a*d^2 - b
*c^2 + b*d^2))/(d^4 - c^2*d^2))*(a*d^2 - b*c^2 + b*d^2))/(d^4 - c^2*d^2))*(
a*d^2 - b*c^2 + b*d^2)*1i)/(d^4 - c^2*d^2) - ((-(c + d)*(c - d))^(1/2)*((32
*tan(x/2)*(a^2*c*d^5 + b^2*c*d^5 + 2*b^2*c^5*d - 4*b^2*c^3*d^3 + 2*a*b*c*d^
5 - 2*a*b*c^3*d^3))/d^3 - (32*b^2*c^4)/d + ((-(c + d)*(c - d))^(1/2)*(32*a*
c^2*d^2 + (32*tan(x/2)*(2*a*c*d^6 - 2*b*c^3*d^4 + 2*b*c*d^6))/d^3 - ((-(c +
 d)*(c - d))^(1/2)*(32*c^2*d^3 + (32*tan(x/2)*(3*c*d^7 - 2*c^3*d^5))/d^3)*(
a*d^2 - b*c^2 + b*d^2))/(d^4 - c^2*d^2))*(a*d^2 - b*c^2 + b*d^2))/(d^4 - c^
2*d^2))*(a*d^2 - b*c^2 + b*d^2)*1i)/(d^4 - c^2*d^2))/((64*(b^3*c^2*d^2 - a*
b^2*c^4 - b^3*c^4 + 2*a*b^2*c^2*d^2 + a^2*b*c^2*d^2))/d^2 + (64*tan(x/2)*(2
*b^3*c^3*d^2 - 2*b^3*c^5 + 2*a*b^2*c^3*d^2))/d^3 + ((-(c + d)*(c - d))^(1/2
)*((32*b^2*c^4)/d - (32*tan(x/2)*(a^2*c*d^5 + b^2*c*d^5 + 2*b^2*c^5*d - 4*b
^2*c^3*d^3 + 2*a*b*c*d^5 - 2*a*b*c^3*d^3))/d^3 + ((-(c + d)*(c - d))^(1/2)*
(32*a*c^2*d^2 + (32*tan(x/2)*(2*a*c*d^6 - 2*b*c^3*d^4 + 2*b*c*d^6))/d^3 + (
(-(c + d)*(c - d))^(1/2)*(32*c^2*d^3 + (32*tan(x/2)*(3*c*d^7 - 2*c^3*d^5))/
d^3)*(a*d^2 - b*c^2 + b*d^2))/(d^4 - c^2*d^2))*(a*d^2 - b*c^2 + b*d^2))/(d^
4 - c^2*d^2))*(a*d^2 - b*c^2 + b*d^2))/(d^4 - c^2*d^2) + ((-(c + d)*(c - d)
)^(1/2)*((32*tan(x/2)*(a^2*c*d^5 + b^2*c*d^5 + 2*b^2*c^5*d - 4*b^2*c^3*d^3
+ 2*a*b*c*d^5 - 2*a*b*c^3*d^3))/d^3 - (32*b^2*c^4)/d + ((-(c + d)*(c - d))^
(1/2)*(32*a*c^2*d^2 + (32*tan(x/2)*(2*a*c*d^6 - 2*b*c^3*d^4 + 2*b*c*d^6))/d
^3 - ((-(c + d)*(c - d))^(1/2)*(32*c^2*d^3 + (32*tan(x/2)*(3*c*d^7 - 2*c^3*
d^5))/d^3)*(a*d^2 - b*c^2 + b*d^2))/(d^4 - c^2*d^2))*(a*d^2 - b*c^2 + b*d^2
))/(d^4 - c^2*d^2))*(a*d^2 - b*c^2 + b*d^2))/(d^4 - c^2*d^2)))*(-(c + d)*(c
 - d))^(1/2)*(a*d^2 - b*c^2 + b*d^2)*2i)/(d^4 - c^2*d^2) + (2*b*c*atan((64*
b^3*c^2*tan(x/2))/(64*b^3*c^2 + 128*a*b^2*c^2 + 64*a^2*b*c^2 - (64*b^3*c^4)
/d^2 - (128*a*b^2*c^4)/d^2) - (64*b^3*c^4*tan(x/2))/(64*b^3*c^2*d^2 - 128*a
*b^2*c^4 - 64*b^3*c^4 + 128*a*b^2*c^2*d^2 + 64*a^2*b*c^2*d^2) - (128*a*b^2*
```

```
c^4*tan(x/2))/(64*b^3*c^2*d^2 - 128*a*b^2*c^4 - 64*b^3*c^4 + 128*a*b^2*c^2*
d^2 + 64*a^2*b*c^2*d^2) + (128*a*b^2*c^2*tan(x/2))/(64*b^3*c^2 + 128*a*b^2*
c^2 + 64*a^2*b*c^2 - (64*b^3*c^4)/d^2 - (128*a*b^2*c^4)/d^2) + (64*a^2*b*c^
2*tan(x/2))/(64*b^3*c^2 + 128*a*b^2*c^2 + 64*a^2*b*c^2 - (64*b^3*c^4)/d^2 -
 (128*a*b^2*c^4)/d^2)))/d^2
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(x)**2)/(c+d*sin(x)),x)
```

```
[Out] Timed out
```

3.212 $\int \frac{a+b\cos^2(x)}{c+c\sin^2(x)}\,dx$

Optimal. Leaf size=56

$$\frac{x(a+2b)}{\sqrt{2}\,c}+\frac{(a+2b)\tan^{-1}\left(\frac{\sin(x)\cos(x)}{\sin^2(x)+\sqrt{2}+1}\right)}{\sqrt{2}\,c}-\frac{bx}{c}$$

```
[Out] -b*x/c+1/2*(a+2*b)*x/c*2^(1/2)+1/2*(a+2*b)*arctan(cos(x)*sin(x)/(1+sin(x)^2
+2^(1/2)))/c*2^(1/2)
```

Rubi [A] time = 0.20, antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {1166, 205}

$$\frac{x(a+2b)}{\sqrt{2}\,c}+\frac{(a+2b)\tan^{-1}\left(\frac{\sin(x)\cos(x)}{\sin^2(x)+\sqrt{2}+1}\right)}{\sqrt{2}\,c}-\frac{bx}{c}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[x]^2)/(c + c*Sin[x]^2),x]
```

```
[Out] -((b*x)/c) + ((a + 2*b)*x)/(Sqrt[2]*c) + ((a + 2*b)*ArcTan[(Cos[x]*Sin[x])/
(1 + Sqrt[2] + Sin[x]^2)])/(Sqrt[2]*c)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{a+b\cos^2(x)}{c+c\sin^2(x)}\,dx = \text{Subst}\left(\int \frac{a+b+ax^2}{c+3cx^2+2cx^4}\,dx, x, \tan(x)\right)$$
$$= -\left((2b)\,\text{Subst}\left(\int \frac{1}{2c+2cx^2}\,dx, x, \tan(x)\right)\right) + (a+2b)\,\text{Subst}\left(\int \frac{1}{c+2cx^2}\,dx, x, \tan(x)\right)$$
$$= -\frac{bx}{c} + \frac{(a+2b)x}{\sqrt{2}\,c} + \frac{(a+2b)\tan^{-1}\left(\frac{\cos(x)\sin(x)}{1+\sqrt{2}+\sin^2(x)}\right)}{\sqrt{2}\,c}$$

Mathematica [A] time = 0.08, size = 31, normalized size = 0.55

$$\frac{(a+2b)\tan^{-1}\left(\sqrt{2}\,\tan(x)\right)}{\sqrt{2}\,c} - \frac{bx}{c}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[x]^2)/(c + c*Sin[x]^2),x]
```

```
[Out] -((b*x)/c) + ((a + 2*b)*ArcTan[Sqrt[2]*Tan[x]])/(Sqrt[2]*c)
```

fricas [A] time = 1.01, size = 45, normalized size = 0.80

$$-\frac{\sqrt{2}\,(a+2\,b)\arctan\left(\frac{3\,\sqrt{2}\,\cos(x)^2-2\,\sqrt{2}}{4\,\cos(x)\sin(x)}\right)+4\,bx}{4\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(x)^2)/(c+c*sin(x)^2),x, algorithm="fricas")
```

```
[Out] -1/4*(sqrt(2)*(a + 2*b)*arctan(1/4*(3*sqrt(2)*cos(x)^2 - 2*sqrt(2))/(cos(x)
*sin(x))) + 4*b*x)/c
```

giac [A] time = 0.15, size = 62, normalized size = 1.11

$$\frac{\sqrt{2}\,(a+2\,b)\left(x+\arctan\left(-\frac{\sqrt{2}\,\sin(2\,x)-2\,\sin(2\,x)}{\sqrt{2}\,\cos(2\,x)+\sqrt{2}-2\,\cos(2\,x)+2}\right)\right)}{2\,c} - \frac{bx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(x)^2)/(c+c*sin(x)^2),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(2)*(a + 2*b)*(x + arctan(-(sqrt(2)*sin(2*x) - 2*sin(2*x))/(sqrt(2)
*cos(2*x) + sqrt(2) - 2*cos(2*x) + 2)))/c - b*x/c
```

maple [A] time = 0.12, size = 42, normalized size = 0.75

$$\frac{\sqrt{2}\ \arctan\left(\sqrt{2}\ \tan(x)\right) a}{2c}+\frac{\sqrt{2}\ \arctan\left(\sqrt{2}\ \tan(x)\right) b}{c}-\frac{b \arctan\left(\tan(x)\right)}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(x)^2)/(c+c*sin(x)^2),x)
```

```
[Out] 1/2/c*2^(1/2)*arctan(2^(1/2)*tan(x))*a+1/c*2^(1/2)*arctan(2^(1/2)*tan(x))*b
-1/c*b*arctan(tan(x))
```

maxima [A] time = 0.43, size = 28, normalized size = 0.50

$$\frac{\sqrt{2}\,(a+2\,b) \arctan\left(\sqrt{2}\ \tan(x)\right)}{2\,c}-\frac{bx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(x)^2)/(c+c*sin(x)^2),x, algorithm="maxima")
```

```
[Out] 1/2*sqrt(2)*(a + 2*b)*arctan(sqrt(2)*tan(x))/c - b*x/c
```

mupad [B] time = 2.39, size = 249, normalized size = 4.45

$$\frac{\sqrt{2}\ \mathrm{atan}\left(\frac{4\,\sqrt{2}\,a^3 \tan(x)}{4\,a^3+24\,a^2\,b+40\,a\,b^2+16\,b^3}+\frac{16\,\sqrt{2}\,b^3 \tan(x)}{4\,a^3+24\,a^2\,b+40\,a\,b^2+16\,b^3}+\frac{40\,\sqrt{2}\,a\,b^2 \tan(x)}{4\,a^3+24\,a^2\,b+40\,a\,b^2+16\,b^3}+\frac{24\,\sqrt{2}\,a^2\,b \tan(x)}{4\,a^3+24\,a^2\,b+40\,a\,b^2+16\,b^3}\right)(a+2\,b)}{2\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*cos(x)^2)/(c + c*sin(x)^2),x)
```

```
[Out] (2^(1/2)*atan((4*2^(1/2)*a^3*tan(x))/(40*a*b^2 + 24*a^2*b + 4*a^3 + 16*b^3)
 + (16*2^(1/2)*b^3*tan(x))/(40*a*b^2 + 24*a^2*b + 4*a^3 + 16*b^3) + (40*2^(
1/2)*a*b^2*tan(x))/(40*a*b^2 + 24*a^2*b + 4*a^3 + 16*b^3) + (24*2^(1/2)*a^2
*b*tan(x))/(40*a*b^2 + 24*a^2*b + 4*a^3 + 16*b^3))*(a + 2*b))/(2*c) - (b*at
an((8*b^3*tan(x))/(16*a*b^2 + 4*a^2*b + 8*b^3) + (16*a*b^2*tan(x))/(16*a*b^
2 + 4*a^2*b + 8*b^3) + (4*a^2*b*tan(x))/(16*a*b^2 + 4*a^2*b + 8*b^3)))/c
```

sympy [B] time = 55.65, size = 520, normalized size = 9.29

$$\frac{54608393\sqrt{2}\,a\sqrt{3-2\sqrt{2}}\left(\mathrm{atan}\left(\frac{\tan\left(\frac{x}{2}\right)}{\sqrt{3-2\sqrt{2}}}\right)+\pi\left\lfloor\frac{\frac{x}{2}-\frac{\pi}{2}}{\pi}\right\rfloor\right)}{31988856\sqrt{2}\,c+45239074c}+\frac{77227930a\sqrt{3-2\sqrt{2}}\left(\mathrm{atan}\left(\frac{\tan\left(\frac{x}{2}\right)}{\sqrt{3-2\sqrt{2}}}\right)+\pi\left\lfloor\frac{\frac{x}{2}-\frac{\pi}{2}}{\pi}\right\rfloor\right)}{31988856\sqrt{2}\,c+45239074c}+ \cdots 93$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(x)**2)/(c+c*sin(x)**2),x)

[Out] 54608393*sqrt(2)*a*sqrt(3 - 2*sqrt(2))*(atan(tan(x/2)/sqrt(3 - 2*sqrt(2)))
+ pi*floor((x/2 - pi/2)/pi))/(31988856*sqrt(2)*c + 45239074*c) + 77227930*a
*sqrt(3 - 2*sqrt(2))*(atan(tan(x/2)/sqrt(3 - 2*sqrt(2))) + pi*floor((x/2 -
pi/2)/pi))/(31988856*sqrt(2)*c + 45239074*c) + 9369319*sqrt(2)*a*sqrt(2*sqr
t(2) + 3)*(atan(tan(x/2)/sqrt(2*sqrt(2) + 3)) + pi*floor((x/2 - pi/2)/pi))/
(31988856*sqrt(2)*c + 45239074*c) + 13250218*a*sqrt(2*sqrt(2) + 3)*(atan(ta
n(x/2)/sqrt(2*sqrt(2) + 3)) + pi*floor((x/2 - pi/2)/pi))/(31988856*sqrt(2)*
c + 45239074*c) - 45239074*b*x/(31988856*sqrt(2)*c + 45239074*c) - 31988856
*sqrt(2)*b*x/(31988856*sqrt(2)*c + 45239074*c) + 109216786*sqrt(2)*b*sqrt(3
 - 2*sqrt(2))*(atan(tan(x/2)/sqrt(3 - 2*sqrt(2))) + pi*floor((x/2 - pi/2)/p
i))/(31988856*sqrt(2)*c + 45239074*c) + 154455860*b*sqrt(3 - 2*sqrt(2))*(at
an(tan(x/2)/sqrt(3 - 2*sqrt(2))) + pi*floor((x/2 - pi/2)/pi))/(31988856*sqr
t(2)*c + 45239074*c) + 18738638*sqrt(2)*b*sqrt(2*sqrt(2) + 3)*(atan(tan(x/2
)/sqrt(2*sqrt(2) + 3)) + pi*floor((x/2 - pi/2)/pi))/(31988856*sqrt(2)*c + 4
5239074*c) + 26500436*b*sqrt(2*sqrt(2) + 3)*(atan(tan(x/2)/sqrt(2*sqrt(2) +
 3)) + pi*floor((x/2 - pi/2)/pi))/(31988856*sqrt(2)*c + 45239074*c)
```

3.213 $\int \frac{a+b\cos^2(x)}{c-c\sin^2(x)}\,dx$

Optimal. Leaf size=14

$$\frac{a\tan(x)}{c}+\frac{bx}{c}$$

[Out] b*x/c+a*tan(x)/c

Rubi [A] time = 0.06, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {3175, 3012, 8}

$$\frac{a\tan(x)}{c}+\frac{bx}{c}$$

Antiderivative was successfully verified.

[In] Int[(a + b*Cos[x]^2)/(c - c*Sin[x]^2),x]

[Out] (b*x)/c + (a*Tan[x])/c

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3012

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_) + (C_.)*sin[(e_.) + (f_.)*(x
_)]^2), x_Symbol] :> Simp[(A*Cos[e + f*x]*(b*Sin[e + f*x])^(m + 1))/(b*f*(m
 + 1)), x] + Dist[(A*(m + 2) + C*(m + 1))/(b^2*(m + 1)), Int[(b*Sin[e + f*x
])^(m + 2), x], x] /; FreeQ[{b, e, f, A, C}, x] && LtQ[m, -1]
```

Rule 3175

```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Dist[
a^p, Int[ActivateTrig[u*cos[e + f*x]^(2*p)], x], x] /; FreeQ[{a, b, e, f, p
}, x] && EqQ[a + b, 0] && IntegerQ[p]
```

Rubi steps

$$\int \frac{a+b\cos^2(x)}{c-c\sin^2(x)}\,dx = \frac{\int \left(a+b\cos^2(x)\right)\sec^2(x)\,dx}{c}$$
$$= \frac{a\tan(x)}{c} + \frac{b\int 1\,dx}{c}$$
$$= \frac{bx}{c} + \frac{a\tan(x)}{c}$$

Mathematica [A] time = 0.01, size = 14, normalized size = 1.00

$$\frac{a\tan(x)}{c} + \frac{bx}{c}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[x]^2)/(c - c*Sin[x]^2),x]
[Out] (b*x)/c + (a*Tan[x])/c
```

fricas [A] time = 1.08, size = 18, normalized size = 1.29

$$\frac{bx\cos(x) + a\sin(x)}{c\cos(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(x)^2)/(c-c*sin(x)^2),x, algorithm="fricas")
[Out] (b*x*cos(x) + a*sin(x))/(c*cos(x))
```

giac [A] time = 0.15, size = 23, normalized size = 1.64

$$\frac{b\arctan\left(\frac{|c|\tan(x)}{c}\right)}{|c|} + \frac{a\tan(x)}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(x)^2)/(c-c*sin(x)^2),x, algorithm="giac")
[Out] b*arctan(abs(c)*tan(x)/c)/abs(c) + a*tan(x)/c
```

maple [A] time = 0.11, size = 17, normalized size = 1.21

$$\frac{a\tan(x)}{c} + \frac{b\arctan(\tan(x))}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(x)^2)/(c-c*sin(x)^2),x)
```

```
[Out] a*tan(x)/c+1/c*b*arctan(tan(x))
```

maxima [A] time = 0.43, size = 14, normalized size = 1.00

$$\frac{bx}{c} + \frac{a\tan(x)}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(x)^2)/(c-c*sin(x)^2),x, algorithm="maxima")
```

```
[Out] b*x/c + a*tan(x)/c
```

mupad [B] time = 2.31, size = 12, normalized size = 0.86

$$\frac{b\,x + a\tan(x)}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*cos(x)^2)/(c - c*sin(x)^2),x)
```

```
[Out] (b*x + a*tan(x))/c
```

sympy [B] time = 1.33, size = 51, normalized size = 3.64

$$-\frac{2a\tan\left(\frac{x}{2}\right)}{c\tan^2\left(\frac{x}{2}\right) - c} + \frac{bx\tan^2\left(\frac{x}{2}\right)}{c\tan^2\left(\frac{x}{2}\right) - c} - \frac{bx}{c\tan^2\left(\frac{x}{2}\right) - c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(x)**2)/(c-c*sin(x)**2),x)
```

```
[Out] -2*a*tan(x/2)/(c*tan(x/2)**2 - c) + b*x*tan(x/2)**2/(c*tan(x/2)**2 - c) - b
*x/(c*tan(x/2)**2 - c)
```

3.214 $\int \frac{a+b\cos^2(x)}{c+d\sin^2(x)}\,dx$

Optimal. Leaf size=49

$$\frac{(ad+b(c+d))\tan^{-1}\left(\frac{\sqrt{c+d}\,\tan(x)}{\sqrt{c}}\right)}{\sqrt{c}\,d\sqrt{c+d}}-\frac{bx}{d}$$

```
[Out] -b*x/d+(a*d+b*(c+d))*arctan((c+d)^(1/2)*tan(x)/c^(1/2))/d/c^(1/2)/(c+d)^(1/
2)
```

Rubi [A] time = 0.16, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {522, 203, 205}

$$\frac{(ad+b(c+d))\tan^{-1}\left(\frac{\sqrt{c+d}\,\tan(x)}{\sqrt{c}}\right)}{\sqrt{c}\,d\sqrt{c+d}}-\frac{bx}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[x]^2)/(c + d*Sin[x]^2),x]
```

```
[Out] -((b*x)/d) + ((a*d + b*(c + d))*ArcTan[(Sqrt[c + d]*Tan[x])/Sqrt[c]])/(Sqrt
[c]*d*Sqrt[c + d])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{a+b\cos^2(x)}{c+d\sin^2(x)}\,dx &= \text{Subst}\left(\int \frac{a+b+ax^2}{\left(1+x^2\right)\left(c+(c+d)x^2\right)}\,dx, x, \tan(x)\right)\\
&= -\frac{b\,\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \tan(x)\right)}{d} + \frac{(-ac+(a+b)(c+d))\,\text{Subst}\left(\int \frac{1}{c+(c+d)x^2}\,dx, x, \tan(x)\right)}{d}\\
&= -\frac{bx}{d} + \frac{(ad+b(c+d))\tan^{-1}\left(\frac{\sqrt{c+d}\,\tan(x)}{\sqrt{c}}\right)}{\sqrt{c}\,d\sqrt{c+d}}
\end{aligned}
$$

Mathematica [**A**] time = 0.15, size = 47, normalized size = 0.96

$$
\frac{\frac{(ad+b(c+d))\tan^{-1}\left(\frac{\sqrt{c+d}\,\tan(x)}{\sqrt{c}}\right)}{\sqrt{c}\,\sqrt{c+d}} - bx}{d}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[x]^2)/(c + d*Sin[x]^2),x]
```

```
[Out] (-(b*x) + ((a*d + b*(c + d))*ArcTan[(Sqrt[c + d]*Tan[x])/Sqrt[c]])/(Sqrt[c]
*Sqrt[c + d]))/d
```

fricas [**B**] time = 2.03, size = 255, normalized size = 5.20

$$
\left[-\frac{(bc+(a+b)d)\sqrt{-c^2-cd}\,\log\left(\frac{\left(8c^2+8cd+d^2\right)\cos(x)^4-2\left(4c^2+5cd+d^2\right)\cos(x)^2+4\left((2c+d)\cos(x)^3-(c+d)\cos(x)\right)\sqrt{-c^2-cd}\,\sin(x)+c^2+2}{d^2\cos(x)^4-2\left(cd+d^2\right)\cos(x)^2+c^2+2cd+d^2}\right.}{4\left(c^2d+cd^2\right)}\right.
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(x)^2)/(c+d*sin(x)^2),x, algorithm="fricas")
```

```
[Out] [-1/4*((b*c + (a + b)*d)*sqrt(-c^2 - c*d)*log(((8*c^2 + 8*c*d + d^2)*cos(x)
^4 - 2*(4*c^2 + 5*c*d + d^2)*cos(x)^2 + 4*((2*c + d)*cos(x)^3 - (c + d)*cos
(x))*sqrt(-c^2 - c*d)*sin(x) + c^2 + 2*c*d + d^2)/(d^2*cos(x)^4 - 2*(c*d +
d^2)*cos(x)^2 + c^2 + 2*c*d + d^2)) + 4*(b*c^2 + b*c*d)*x)/(c^2*d + c*d^2),
 -1/2*((b*c + (a + b)*d)*sqrt(c^2 + c*d)*arctan(1/2*((2*c + d)*cos(x)^2 - c
 - d)/(sqrt(c^2 + c*d)*cos(x)*sin(x))) + 2*(b*c^2 + b*c*d)*x)/(c^2*d + c*d^
2)]
```

giac [A] time = 0.14, size = 70, normalized size = 1.43

$$-\frac{bx}{d}+\frac{\left(\pi\left\lfloor\frac{x}{\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}\left(2\,c+2\,d\right)+\arctan\left(\frac{c\tan(x)+d\tan(x)}{\sqrt{c^2+cd}}\right)\right)(bc+ad+bd)}{\sqrt{c^2+cd}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(x)^2)/(c+d*sin(x)^2),x, algorithm="giac")
```

```
[Out] -b*x/d + (pi*floor(x/pi + 1/2)*sgn(2*c + 2*d) + arctan((c*tan(x) + d*tan(x)
)/sqrt(c^2 + c*d)))*(b*c + a*d + b*d)/(sqrt(c^2 + c*d)*d)
```

maple [B] time = 0.12, size = 84, normalized size = 1.71

$$\frac{\arctan\left(\frac{(c+d)\tan(x)}{\sqrt{(c+d)c}}\right)a}{\sqrt{(c+d)\,c}}+\frac{\arctan\left(\frac{(c+d)\tan(x)}{\sqrt{(c+d)c}}\right)cb}{d\sqrt{(c+d)\,c}}+\frac{\arctan\left(\frac{(c+d)\tan(x)}{\sqrt{(c+d)c}}\right)b}{\sqrt{(c+d)\,c}}-\frac{b\arctan\left(\tan(x)\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(x)^2)/(c+d*sin(x)^2),x)
```

```
[Out] 1/((c+d)*c)^(1/2)*arctan((c+d)*tan(x)/((c+d)*c)^(1/2))*a+1/d/((c+d)*c)^(1/2
)*arctan((c+d)*tan(x)/((c+d)*c)^(1/2))*c*b+1/((c+d)*c)^(1/2)*arctan((c+d)*t
an(x)/((c+d)*c)^(1/2))*b-b/d*arctan(tan(x))
```

maxima [A] time = 0.43, size = 42, normalized size = 0.86

$$-\frac{bx}{d}+\frac{(bc+(a+b)d)\arctan\left(\frac{(c+d)\tan(x)}{\sqrt{(c+d)c}}\right)}{\sqrt{(c+d)c}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(x)^2)/(c+d*sin(x)^2),x, algorithm="maxima")
```

```
[Out] -b*x/d + (b*c + (a + b)*d)*arctan((c + d)*tan(x)/sqrt((c + d)*c))/(sqrt((c
+ d)*c)*d)
```

mupad [B] time = 2.95, size = 1774, normalized size = 36.20

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*cos(x)^2)/(c + d*sin(x)^2),x)
```

```
[Out] - (b*c^2*x)/(c*d^2 + c^2*d) - (a*d*atan((a^2*d^3*tan(x)*(- c*d - c^2)^(3/2)
*1i + b^2*c^3*tan(x)*(- c*d - c^2)^(3/2)*2i + b^2*c^5*tan(x)*(- c*d - c^2)^
(1/2)*2i + b^2*d^3*tan(x)*(- c*d - c^2)^(3/2)*1i + a^2*c*d^2*tan(x)*(- c*d
- c^2)^(3/2)*2i + b^2*c*d^2*tan(x)*(- c*d - c^2)^(3/2)*4i + b^2*c^2*d*tan(x
)*(- c*d - c^2)^(3/2)*5i + b^2*c^4*d*tan(x)*(- c*d - c^2)^(1/2)*6i + a^2*c^
2*d^3*tan(x)*(- c*d - c^2)^(1/2)*1i + a^2*c^3*d^2*tan(x)*(- c*d - c^2)^(1/2
)*1i + b^2*c^2*d^3*tan(x)*(- c*d - c^2)^(1/2)*2i + b^2*c^3*d^2*tan(x)*(- c*
d - c^2)^(1/2)*6i + a*b*d^3*tan(x)*(- c*d - c^2)^(3/2)*2i + a*b*c*d^2*tan(x
)*(- c*d - c^2)^(3/2)*6i + a*b*c^2*d*tan(x)*(- c*d - c^2)^(3/2)*4i + a*b*c^
4*d*tan(x)*(- c*d - c^2)^(1/2)*2i + a*b*c^2*d^3*tan(x)*(- c*d - c^2)^(1/2)*
2i + a*b*c^3*d^2*tan(x)*(- c*d - c^2)^(1/2)*4i)/(b^2*c^5*d + a^2*c^2*d^4 +
2*a^2*c^3*d^3 + a^2*c^4*d^2 + b^2*c^2*d^4 + 3*b^2*c^3*d^3 + 3*b^2*c^4*d^2 +
 2*a*b*c^5*d + 2*a*b*c^2*d^4 + 6*a*b*c^3*d^3 + 6*a*b*c^4*d^2))*(- c*d - c^2
)^(1/2)*1i)/(c*d^2 + c^2*d) - (b*c*atan((a^2*d^3*tan(x)*(- c*d - c^2)^(3/2)
*1i + b^2*c^3*tan(x)*(- c*d - c^2)^(3/2)*2i + b^2*c^5*tan(x)*(- c*d - c^2)^
(1/2)*2i + b^2*d^3*tan(x)*(- c*d - c^2)^(3/2)*1i + a^2*c*d^2*tan(x)*(- c*d
- c^2)^(3/2)*2i + b^2*c*d^2*tan(x)*(- c*d - c^2)^(3/2)*4i + b^2*c^2*d*tan(x
)*(- c*d - c^2)^(3/2)*5i + b^2*c^4*d*tan(x)*(- c*d - c^2)^(1/2)*6i + a^2*c^
2*d^3*tan(x)*(- c*d - c^2)^(1/2)*1i + a^2*c^3*d^2*tan(x)*(- c*d - c^2)^(1/2
)*1i + b^2*c^2*d^3*tan(x)*(- c*d - c^2)^(1/2)*2i + b^2*c^3*d^2*tan(x)*(- c*
d - c^2)^(1/2)*6i + a*b*d^3*tan(x)*(- c*d - c^2)^(3/2)*2i + a*b*c*d^2*tan(x
)*(- c*d - c^2)^(3/2)*6i + a*b*c^2*d*tan(x)*(- c*d - c^2)^(3/2)*4i + a*b*c^
4*d*tan(x)*(- c*d - c^2)^(1/2)*2i + a*b*c^2*d^3*tan(x)*(- c*d - c^2)^(1/2)*
2i + a*b*c^3*d^2*tan(x)*(- c*d - c^2)^(1/2)*4i)/(b^2*c^5*d + a^2*c^2*d^4 +
2*a^2*c^3*d^3 + a^2*c^4*d^2 + b^2*c^2*d^4 + 3*b^2*c^3*d^3 + 3*b^2*c^4*d^2 +
 2*a*b*c^5*d + 2*a*b*c^2*d^4 + 6*a*b*c^3*d^3 + 6*a*b*c^4*d^2))*(- c*d - c^2
)^(1/2)*1i)/(c*d^2 + c^2*d) - (b*d*atan((a^2*d^3*tan(x)*(- c*d - c^2)^(3/2)
*1i + b^2*c^3*tan(x)*(- c*d - c^2)^(3/2)*2i + b^2*c^5*tan(x)*(- c*d - c^2)^
(1/2)*2i + b^2*d^3*tan(x)*(- c*d - c^2)^(3/2)*1i + a^2*c*d^2*tan(x)*(- c*d
- c^2)^(3/2)*2i + b^2*c*d^2*tan(x)*(- c*d - c^2)^(3/2)*4i + b^2*c^2*d*tan(x
)*(- c*d - c^2)^(3/2)*5i + b^2*c^4*d*tan(x)*(- c*d - c^2)^(1/2)*6i + a^2*c^
2*d^3*tan(x)*(- c*d - c^2)^(1/2)*1i + a^2*c^3*d^2*tan(x)*(- c*d - c^2)^(1/2
)*1i + b^2*c^2*d^3*tan(x)*(- c*d - c^2)^(1/2)*2i + b^2*c^3*d^2*tan(x)*(- c*
d - c^2)^(1/2)*6i + a*b*d^3*tan(x)*(- c*d - c^2)^(3/2)*2i + a*b*c*d^2*tan(x
)*(- c*d - c^2)^(3/2)*6i + a*b*c^2*d*tan(x)*(- c*d - c^2)^(3/2)*4i + a*b*c^
4*d*tan(x)*(- c*d - c^2)^(1/2)*2i + a*b*c^2*d^3*tan(x)*(- c*d - c^2)^(1/2)*
2i + a*b*c^3*d^2*tan(x)*(- c*d - c^2)^(1/2)*4i)/(b^2*c^5*d + a^2*c^2*d^4 +
2*a^2*c^3*d^3 + a^2*c^4*d^2 + b^2*c^2*d^4 + 3*b^2*c^3*d^3 + 3*b^2*c^4*d^2 +
 2*a*b*c^5*d + 2*a*b*c^2*d^4 + 6*a*b*c^3*d^3 + 6*a*b*c^4*d^2))*(- c*d - c^2
)^(1/2)*1i)/(c*d^2 + c^2*d) - (b*c*d*x)/(c*d^2 + c^2*d)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(x)**2)/(c+d*sin(x)**2),x)
[Out] Timed out
```

3.215 $\int \frac{a+b\sec^2(x)}{c+d\cos(x)}\,dx$

Optimal. Leaf size=74

$$\frac{2\left(ac^2+bd^2\right)\tan^{-1}\left(\frac{\sqrt{c-d}\,\tan\left(\frac{x}{2}\right)}{\sqrt{c+d}}\right)}{c^2\sqrt{c-d}\,\sqrt{c+d}}-\frac{bd\tanh^{-1}(\sin(x))}{c^2}+\frac{b\tan(x)}{c}$$

```
[Out] -b*d*arctanh(sin(x))/c^2+2*(a*c^2+b*d^2)*arctan((c-d)^(1/2)*tan(1/2*x)/(c+d
)^(1/2))/c^2/(c-d)^(1/2)/(c+d)^(1/2)+b*tan(x)/c
```

Rubi [A] time = 0.25, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.353, Rules used = {4234, 3056, 3001, 3770, 2659, 205}

$$\frac{2\left(ac^2+bd^2\right)\tan^{-1}\left(\frac{\sqrt{c-d}\,\tan\left(\frac{x}{2}\right)}{\sqrt{c+d}}\right)}{c^2\sqrt{c-d}\,\sqrt{c+d}}-\frac{bd\tanh^{-1}(\sin(x))}{c^2}+\frac{b\tan(x)}{c}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[x]^2)/(c + d*Cos[x]),x]
```

```
[Out] (2*(a*c^2 + b*d^2)*ArcTan[(Sqrt[c - d]*Tan[x/2])/Sqrt[c + d]])/(c^2*Sqrt[c
- d]*Sqrt[c + d]) - (b*d*ArcTanh[Sin[x]])/c^2 + (b*Tan[x])/c
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 3001

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A*b
- a*B)/(b*c - a*d), Int[1/(a + b*Sin[e + f*x]), x], x] + Dist[(B*c - A*d)/(
b*c - a*d), Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f,
```

```
A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3056

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
-Simp[((A*b^2 + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin
[e + f*x])^(n + 1))/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2)), x] + Dist[1/((m +
1)*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e
+ f*x])^n*Simp[a*(m + 1)*(b*c - a*d)*(A + C) + d*(A*b^2 + a^2*C)*(m + n + 2
) - (c*(A*b^2 + a^2*C) + b*(m + 1)*(b*c - a*d)*(A + C))*Sin[e + f*x] - d*(A
*b^2 + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d,
e, f, A, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d
^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(
IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0]
)))
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 4234

```
Int[(u_)*((A_) + (C_.)*sec[(a_.) + (b_.)*(x_)]^2), x_Symbol] :> Int[(Activa
teTrig[u]*(C + A*Cos[a + b*x]^2))/Cos[a + b*x]^2, x] /; FreeQ[{a, b, A, C},
 x] && KnownSineIntegrandQ[u, x]
```

Rubi steps

$$\int \frac{a+b\sec^2(x)}{c+d\cos(x)}\,dx=\int \frac{\left(b+a\cos^2(x)\right)\sec^2(x)}{c+d\cos(x)}\,dx$$
$$=\frac{b\tan(x)}{c}+\frac{\int \frac{(-bd+ac\cos(x))\sec(x)}{c+d\cos(x)}\,dx}{c}$$
$$=\frac{b\tan(x)}{c}-\frac{(bd)\int \sec(x)\,dx}{c^2}+\left(a+\frac{bd^2}{c^2}\right)\int \frac{1}{c+d\cos(x)}\,dx$$
$$=-\frac{bd\tanh^{-1}(\sin(x))}{c^2}+\frac{b\tan(x)}{c}+\left(2\left(a+\frac{bd^2}{c^2}\right)\right)\text{Subst}\left(\int \frac{1}{c+d+(c-d)x^2}\,dx,x,\tan\left(\frac{x}{2}\right)\right)$$
$$=\frac{2\left(a+\frac{bd^2}{c^2}\right)\tan^{-1}\left(\frac{\sqrt{c-d}\,\tan\left(\frac{x}{2}\right)}{\sqrt{c+d}}\right)}{\sqrt{c-d}\,\sqrt{c+d}}-\frac{bd\tanh^{-1}(\sin(x))}{c^2}+\frac{b\tan(x)}{c}$$

Mathematica [A] time = 0.45, size = 98, normalized size = 1.32

$$\frac{-\frac{2\left(ac^2+bd^2\right)\tanh^{-1}\left(\frac{(c-d)\tan\left(\frac{x}{2}\right)}{\sqrt{d^2-c^2}}\right)}{\sqrt{d^2-c^2}}+bc\tan(x)+bd\left(\log\left(\cos\left(\frac{x}{2}\right)-\sin\left(\frac{x}{2}\right)\right)-\log\left(\sin\left(\frac{x}{2}\right)+\cos\left(\frac{x}{2}\right)\right)\right)}{c^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sec[x]^2)/(c + d*Cos[x]),x]
```

```
[Out] ((-2*(a*c^2 + b*d^2)*ArcTanh[((c - d)*Tan[x/2])/Sqrt[-c^2 + d^2]])/Sqrt[-c^
2 + d^2] + b*d*(Log[Cos[x/2] - Sin[x/2]] - Log[Cos[x/2] + Sin[x/2]]) + b*c*
Tan[x])/c^2
```

fricas [B] time = 3.46, size = 318, normalized size = 4.30

$$\left[-\frac{\left(ac^2+bd^2\right)\sqrt{-c^2+d^2}\,\cos(x)\log\left(\frac{2\,cd\cos(x)+\left(2\,c^2-d^2\right)\cos(x)^2+2\,\sqrt{-c^2+d^2}\,(c\cos(x)+d)\sin(x)-c^2+2\,d^2}{d^2\cos(x)^2+2\,cd\cos(x)+c^2}\right)+\left(bc^2d-bd^3\right)\cos(x)}{2\left(c^4-c^2d^2\right)\cos(x)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(x)^2)/(c+d*cos(x)),x, algorithm="fricas")
```

```
[Out] [-1/2*((a*c^2 + b*d^2)*sqrt(-c^2 + d^2)*cos(x)*log((2*c*d*cos(x) + (2*c^2 -
 d^2)*cos(x)^2 + 2*sqrt(-c^2 + d^2)*(c*cos(x) + d)*sin(x) - c^2 + 2*d^2)/(d
```

```
^2*cos(x)^2 + 2*c*d*cos(x) + c^2)) + (b*c^2*d - b*d^3)*cos(x)*log(sin(x) +
1) - (b*c^2*d - b*d^3)*cos(x)*log(-sin(x) + 1) - 2*(b*c^3 - b*c*d^2)*sin(x)
)/((c^4 - c^2*d^2)*cos(x)), 1/2*(2*(a*c^2 + b*d^2)*sqrt(c^2 - d^2)*arctan(-
(c*cos(x) + d)/(sqrt(c^2 - d^2)*sin(x)))*cos(x) - (b*c^2*d - b*d^3)*cos(x)*
log(sin(x) + 1) + (b*c^2*d - b*d^3)*cos(x)*log(-sin(x) + 1) + 2*(b*c^3 - b*
c*d^2)*sin(x))/((c^4 - c^2*d^2)*cos(x))]
```

giac [A] time = 0.17, size = 125, normalized size = 1.69

$$-\frac{bd\log\left(\left|\tan\left(\frac{1}{2}x\right)+1\right|\right)}{c^2}+\frac{bd\log\left(\left|\tan\left(\frac{1}{2}x\right)-1\right|\right)}{c^2}-\frac{2b\tan\left(\frac{1}{2}x\right)}{\left(\tan\left(\frac{1}{2}x\right)^2-1\right)c}-\frac{2\left(ac^2+bd^2\right)\left(\pi\left\lfloor\frac{x}{2\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(-2c+2d\right.}{\sqrt{c^2-d^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(x)^2)/(c+d*cos(x)),x, algorithm="giac")
```

```
[Out] -b*d*log(abs(tan(1/2*x) + 1))/c^2 + b*d*log(abs(tan(1/2*x) - 1))/c^2 - 2*b*
tan(1/2*x)/((tan(1/2*x)^2 - 1)*c) - 2*(a*c^2 + b*d^2)*(pi*floor(1/2*x/pi +
1/2)*sgn(-2*c + 2*d) + arctan(-(c*tan(1/2*x) - d*tan(1/2*x))/sqrt(c^2 - d^2
)))/(sqrt(c^2 - d^2)*c^2)
```

maple [B] time = 0.10, size = 135, normalized size = 1.82

$$\frac{2\arctan\left(\frac{(c-d)\tan\left(\frac{x}{2}\right)}{\sqrt{(c+d)(c-d)}}\right)a}{\sqrt{(c+d)(c-d)}}+\frac{2\arctan\left(\frac{(c-d)\tan\left(\frac{x}{2}\right)}{\sqrt{(c+d)(c-d)}}\right)b\,d^2}{c^2\sqrt{(c+d)(c-d)}}-\frac{b}{c\left(\tan\left(\frac{x}{2}\right)-1\right)}+\frac{db\ln\left(\tan\left(\frac{x}{2}\right)-1\right)}{c^2}-\frac{b}{c\left(1+\tan\left(\frac{x}{2}\right)\right)}-\frac{db\ln}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(x)^2)/(c+d*cos(x)),x)
```

```
[Out] 2/((c+d)*(c-d))^(1/2)*arctan((c-d)*tan(1/2*x)/((c+d)*(c-d))^(1/2))*a+2/c^2/
((c+d)*(c-d))^(1/2)*arctan((c-d)*tan(1/2*x)/((c+d)*(c-d))^(1/2))*b*d^2-b/c/
(tan(1/2*x)-1)+d*b/c^2*ln(tan(1/2*x)-1)-b/c/(1+tan(1/2*x))-d*b/c^2*ln(1+tan
(1/2*x))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(x)^2)/(c+d*cos(x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*d^2-4*c^2>0)', see `assume?` for
more details)Is 4*d^2-4*c^2 positive or negative?
```

mupad [B] time = 3.51, size = 1302, normalized size = 17.59

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(x)^2)/(c + d*cos(x)),x)
```

```
[Out] (b*c^3*sin(x))/(c^4*cos(x) - c^2*d^2*cos(x)) - (b*c*d^2*sin(x))/(c^4*cos(x)
 - c^2*d^2*cos(x)) + (2*b*d^3*atanh(sin(x/2)/cos(x/2))*cos(x))/(c^4*cos(x)
- c^2*d^2*cos(x)) + (a*c^2*atan((a^2*c^7*sin(x/2)*(d^2 - c^2)^(1/2)*1i + b^
2*d^5*sin(x/2)*(d^2 - c^2)^(3/2)*2i - b^2*d^7*sin(x/2)*(d^2 - c^2)^(1/2)*2i
 + a^2*c^4*d*sin(x/2)*(d^2 - c^2)^(3/2)*2i + a^2*c^6*d*sin(x/2)*(d^2 - c^2)
^(1/2)*1i - a^2*c^4*d^3*sin(x/2)*(d^2 - c^2)^(1/2)*1i - a^2*c^5*d^2*sin(x/2
)*(d^2 - c^2)^(1/2)*1i + b^2*c^2*d^5*sin(x/2)*(d^2 - c^2)^(1/2)*3i - b^2*c^
3*d^4*sin(x/2)*(d^2 - c^2)^(1/2)*1i - b^2*c^4*d^3*sin(x/2)*(d^2 - c^2)^(1/2
)*1i + b^2*c^5*d^2*sin(x/2)*(d^2 - c^2)^(1/2)*1i + a*b*c^2*d^3*sin(x/2)*(d^
2 - c^2)^(3/2)*4i - a*b*c^2*d^5*sin(x/2)*(d^2 - c^2)^(1/2)*2i - a*b*c^3*d^4
*sin(x/2)*(d^2 - c^2)^(1/2)*2i + a*b*c^4*d^3*sin(x/2)*(d^2 - c^2)^(1/2)*2i
+ a*b*c^5*d^2*sin(x/2)*(d^2 - c^2)^(1/2)*2i)/(a^2*c^8*cos(x/2) + a^2*c^4*d^
4*cos(x/2) - 2*a^2*c^6*d^2*cos(x/2) + b^2*c^2*d^6*cos(x/2) - 2*b^2*c^4*d^4*
cos(x/2) + b^2*c^6*d^2*cos(x/2) + 2*a*b*c^2*d^6*cos(x/2) - 4*a*b*c^4*d^4*co
s(x/2) + 2*a*b*c^6*d^2*cos(x/2)))*cos(x)*(d^2 - c^2)^(1/2)*2i)/(c^4*cos(x)
- c^2*d^2*cos(x)) + (b*d^2*atan((a^2*c^7*sin(x/2)*(d^2 - c^2)^(1/2)*1i + b^
2*d^5*sin(x/2)*(d^2 - c^2)^(3/2)*2i - b^2*d^7*sin(x/2)*(d^2 - c^2)^(1/2)*2i
 + a^2*c^4*d*sin(x/2)*(d^2 - c^2)^(3/2)*2i + a^2*c^6*d*sin(x/2)*(d^2 - c^2)
^(1/2)*1i - a^2*c^4*d^3*sin(x/2)*(d^2 - c^2)^(1/2)*1i - a^2*c^5*d^2*sin(x/2
)*(d^2 - c^2)^(1/2)*1i + b^2*c^2*d^5*sin(x/2)*(d^2 - c^2)^(1/2)*3i - b^2*c^
3*d^4*sin(x/2)*(d^2 - c^2)^(1/2)*1i - b^2*c^4*d^3*sin(x/2)*(d^2 - c^2)^(1/2
)*1i + b^2*c^5*d^2*sin(x/2)*(d^2 - c^2)^(1/2)*1i + a*b*c^2*d^3*sin(x/2)*(d^
2 - c^2)^(3/2)*4i - a*b*c^2*d^5*sin(x/2)*(d^2 - c^2)^(1/2)*2i - a*b*c^3*d^4
*sin(x/2)*(d^2 - c^2)^(1/2)*2i + a*b*c^4*d^3*sin(x/2)*(d^2 - c^2)^(1/2)*2i
+ a*b*c^5*d^2*sin(x/2)*(d^2 - c^2)^(1/2)*2i)/(a^2*c^8*cos(x/2) + a^2*c^4*d^
4*cos(x/2) - 2*a^2*c^6*d^2*cos(x/2) + b^2*c^2*d^6*cos(x/2) - 2*b^2*c^4*d^4*
cos(x/2) + b^2*c^6*d^2*cos(x/2) + 2*a*b*c^2*d^6*cos(x/2) - 4*a*b*c^4*d^4*co
s(x/2) + 2*a*b*c^6*d^2*cos(x/2)))*cos(x)*(d^2 - c^2)^(1/2)*2i)/(c^4*cos(x)
- c^2*d^2*cos(x)) - (2*b*c^2*d*atanh(sin(x/2)/cos(x/2))*cos(x))/(c^4*cos(x)
 - c^2*d^2*cos(x))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a + b\sec^2(x)}{c + d\cos(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(x)**2)/(c+d*cos(x)),x)
```

```
[Out] Integral((a + b*sec(x)**2)/(c + d*cos(x)), x)
```

3.216 $\int \frac{a+b\csc^2(x)}{c+d\sin(x)}\,dx$

Optimal. Leaf size=72

$$\frac{2\left(ac^2+bd^2\right)\tan^{-1}\left(\frac{c\tan\left(\frac{x}{2}\right)+d}{\sqrt{c^2-d^2}}\right)}{c^2\sqrt{c^2-d^2}}+\frac{bd\tanh^{-1}(\cos(x))}{c^2}-\frac{b\cot(x)}{c}$$

```
[Out] b*d*arctanh(cos(x))/c^2-b*cot(x)/c+2*(a*c^2+b*d^2)*arctan((d+c*tan(1/2*x))/
(c^2-d^2)^(1/2))/c^2/(c^2-d^2)^(1/2)
```

Rubi [A] time = 0.24, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.412, Rules used = {4233, 3056, 3001, 3770, 2660, 618, 204}

$$\frac{2\left(ac^2+bd^2\right)\tan^{-1}\left(\frac{c\tan\left(\frac{x}{2}\right)+d}{\sqrt{c^2-d^2}}\right)}{c^2\sqrt{c^2-d^2}}+\frac{bd\tanh^{-1}(\cos(x))}{c^2}-\frac{b\cot(x)}{c}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Csc[x]^2)/(c + d*Sin[x]),x]
```

```
[Out] (2*(a*c^2 + b*d^2)*ArcTan[(d + c*Tan[x/2])/Sqrt[c^2 - d^2]])/(c^2*Sqrt[c^2
- d^2]) + (b*d*ArcTanh[Cos[x]])/c^2 - (b*Cot[x])/c
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 3001

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A*b
- a*B)/(b*c - a*d), Int[1/(a + b*Sin[e + f*x]), x], x] + Dist[(B*c - A*d)/(
b*c - a*d), Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f,
A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3056

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
-Simp[((A*b^2 + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin
[e + f*x])^(n + 1))/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2)), x] + Dist[1/((m +
1)*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e
+ f*x])^n*Simp[a*(m + 1)*(b*c - a*d)*(A + C) + d*(A*b^2 + a^2*C)*(m + n + 2
) - (c*(A*b^2 + a^2*C) + b*(m + 1)*(b*c - a*d)*(A + C))*Sin[e + f*x] - d*(A
*b^2 + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d,
e, f, A, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d
^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(
IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0]
)))
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 4233

```
Int[(csc[(a_.) + (b_.)*(x_)]^2*(C_.) + (A_))*(u_), x_Symbol] :> Int[(Activa
teTrig[u]*(C + A*Sin[a + b*x]^2))/Sin[a + b*x]^2, x] /; FreeQ[{a, b, A, C},
 x] && KnownSineIntegrandQ[u, x]
```

Rubi steps

$$\int \frac{a+b\csc^2(x)}{c+d\sin(x)}\,dx = \int \frac{\csc^2(x)\left(b+a\sin^2(x)\right)}{c+d\sin(x)}\,dx$$
$$= -\frac{b\cot(x)}{c} + \frac{\int \frac{\csc(x)(-bd+ac\sin(x))}{c+d\sin(x)}\,dx}{c}$$
$$= -\frac{b\cot(x)}{c} - \frac{(bd)\int \csc(x)\,dx}{c^2} + \left(a+\frac{bd^2}{c^2}\right)\int \frac{1}{c+d\sin(x)}\,dx$$
$$= \frac{bd\tanh^{-1}(\cos(x))}{c^2} - \frac{b\cot(x)}{c} + \left(2\left(a+\frac{bd^2}{c^2}\right)\right)\text{Subst}\left(\int \frac{1}{c+2dx+cx^2}\,dx, x, \tan\left(\frac{x}{2}\right)\right)$$
$$= \frac{bd\tanh^{-1}(\cos(x))}{c^2} - \frac{b\cot(x)}{c} - \left(4\left(a+\frac{bd^2}{c^2}\right)\right)\text{Subst}\left(\int \frac{1}{-4\left(c^2-d^2\right)-x^2}\,dx, x, 2d+2c\tan$$
$$= \frac{2\left(a+\frac{bd^2}{c^2}\right)\tan^{-1}\left(\frac{d+c\tan\left(\frac{x}{2}\right)}{\sqrt{c^2-d^2}}\right)}{\sqrt{c^2-d^2}} + \frac{bd\tanh^{-1}(\cos(x))}{c^2} - \frac{b\cot(x)}{c}$$

Mathematica [A] time = 0.53, size = 102, normalized size = 1.42

$$\frac{\csc\left(\frac{x}{2}\right)\sec\left(\frac{x}{2}\right)\left(\frac{2\sin(x)\left(ac^2+bd^2\right)\tan^{-1}\left(\frac{c\tan\left(\frac{x}{2}\right)+d}{\sqrt{c^2-d^2}}\right)}{\sqrt{c^2-d^2}} - b\left(c\cos(x)+d\sin(x)\left(\log\left(\sin\left(\frac{x}{2}\right)\right)-\log\left(\cos\left(\frac{x}{2}\right)\right)\right)\right)\right)}{2c^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Csc[x]^2)/(c + d*Sin[x]),x]
```

```
[Out] (Csc[x/2]*Sec[x/2]*((2*(a*c^2 + b*d^2)*ArcTan[(d + c*Tan[x/2])/Sqrt[c^2 - d
^2]]*Sin[x])/Sqrt[c^2 - d^2] - b*(c*Cos[x] + d*(-Log[Cos[x/2]] + Log[Sin[x/
2]])*Sin[x])))/(2*c^2)
```

fricas [B] time = 3.95, size = 332, normalized size = 4.61

$$\left[-\frac{\left(ac^2+bd^2\right)\sqrt{-c^2+d^2}\log\left(\frac{\left(2c^2-d^2\right)\cos(x)^2-2cd\sin(x)-c^2-d^2+2\left(c\cos(x)\sin(x)+d\cos(x)\right)\sqrt{-c^2+d^2}}{d^2\cos(x)^2-2cd\sin(x)-c^2-d^2}\right)\sin(x)-\left(bc^2d-bd^3\right)\log}{2\left(c^4-c^2d^2\right)\sin(x)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*csc(x)^2)/(c+d*sin(x)),x, algorithm="fricas")
```

```
[Out] [-1/2*((a*c^2 + b*d^2)*sqrt(-c^2 + d^2)*log(((2*c^2 - d^2)*cos(x)^2 - 2*c*d
*sin(x) - c^2 - d^2 + 2*(c*cos(x)*sin(x) + d*cos(x))*sqrt(-c^2 + d^2))/(d^2
*cos(x)^2 - 2*c*d*sin(x) - c^2 - d^2))*sin(x) - (b*c^2*d - b*d^3)*log(1/2*c
os(x) + 1/2)*sin(x) + (b*c^2*d - b*d^3)*log(-1/2*cos(x) + 1/2)*sin(x) + 2*(
b*c^3 - b*c*d^2)*cos(x))/((c^4 - c^2*d^2)*sin(x)), -1/2*(2*(a*c^2 + b*d^2)*
sqrt(c^2 - d^2)*arctan(-(c*sin(x) + d)/(sqrt(c^2 - d^2)*cos(x)))*sin(x) - (
b*c^2*d - b*d^3)*log(1/2*cos(x) + 1/2)*sin(x) + (b*c^2*d - b*d^3)*log(-1/2*
cos(x) + 1/2)*sin(x) + 2*(b*c^3 - b*c*d^2)*cos(x))/((c^4 - c^2*d^2)*sin(x))
]
```

giac [A] time = 0.17, size = 110, normalized size = 1.53

$$-\frac{bd\log\left(\left|\tan\left(\frac{1}{2}x\right)\right|\right)}{c^2}+\frac{b\tan\left(\frac{1}{2}x\right)}{2c}+\frac{2\left(ac^2+bd^2\right)\left(\pi\left\lfloor\frac{x}{2\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(c)+\arctan\left(\frac{c\tan\left(\frac{1}{2}x\right)+d}{\sqrt{c^2-d^2}}\right)\right)}{\sqrt{c^2-d^2}\,c^2}+\frac{2\,bd\tan\left(\frac{1}{2}x\right)-b}{2\,c^2\tan\left(\frac{1}{2}x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*csc(x)^2)/(c+d*sin(x)),x, algorithm="giac")
```

```
[Out] -b*d*log(abs(tan(1/2*x)))/c^2 + 1/2*b*tan(1/2*x)/c + 2*(a*c^2 + b*d^2)*(pi*
floor(1/2*x/pi + 1/2)*sgn(c) + arctan((c*tan(1/2*x) + d)/sqrt(c^2 - d^2)))/
(sqrt(c^2 - d^2)*c^2) + 1/2*(2*b*d*tan(1/2*x) - b*c)/(c^2*tan(1/2*x))
```

maple [A] time = 0.11, size = 120, normalized size = 1.67

$$\frac{b\tan\left(\frac{x}{2}\right)}{2c}-\frac{b}{2c\tan\left(\frac{x}{2}\right)}-\frac{db\ln\left(\tan\left(\frac{x}{2}\right)\right)}{c^2}+\frac{2\arctan\left(\frac{2c\tan\left(\frac{x}{2}\right)+2d}{2\sqrt{c^2-d^2}}\right)a}{\sqrt{c^2-d^2}}+\frac{2\arctan\left(\frac{2c\tan\left(\frac{x}{2}\right)+2d}{2\sqrt{c^2-d^2}}\right)b\,d^2}{c^2\sqrt{c^2-d^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*csc(x)^2)/(c+d*sin(x)),x)
```

```
[Out] 1/2*b/c*tan(1/2*x)-1/2*b/c/tan(1/2*x)-1/c^2*d*b*ln(tan(1/2*x))+2/(c^2-d^2)^
(1/2)*arctan(1/2*(2*c*tan(1/2*x)+2*d)/(c^2-d^2)^(1/2))*a+2/c^2/(c^2-d^2)^(1
/2)*arctan(1/2*(2*c*tan(1/2*x)+2*d)/(c^2-d^2)^(1/2))*b*d^2
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*csc(x)^2)/(c+d*sin(x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*d^2-4*c^2>0)', see `assume?` for
more details)Is 4*d^2-4*c^2 positive or negative?
```

mupad [B] time = 2.83, size = 463, normalized size = 6.43

$$b\,d^3\,\ln\left(\tan\left(\frac{x}{2}\right)\right)-b\,c^2\,d\,\ln\left(\tan\left(\frac{x}{2}\right)\right)+a\,c^2\,\mathrm{atan}\left(\frac{a\,c^3\,\sqrt{d^2-c^2}\,1\mathrm{i}+b\,d^3\,\tan\left(\frac{x}{2}\right)\,\sqrt{d^2-c^2}\,4\mathrm{i}+b\,c\,d^2\,\sqrt{d^2-c^2}\,2\mathrm{i}+a\,c^2\,d\,\tan\left(\frac{x}{2}\right)\,\sqrt{d^2-c^2}\,2\mathrm{i}-}{4\,b\,d^4\,\tan\left(\frac{x}{2}\right)-a\,c^4\,\tan\left(\frac{x}{2}\right)+a\,c^3\,d+2\,b\,c\,d^3-b\,c^3\,d+2\,a\,c^2\,d^2\,\tan\left(\frac{x}{2}\right)-3\,b\,c}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/sin(x)^2)/(c + d*sin(x)),x)
```

```
[Out] (b*d^3*log(tan(x/2)) + a*c^2*atan((a*c^3*(d^2 - c^2)^(1/2)*1i + b*d^3*tan(x
/2)*(d^2 - c^2)^(1/2)*4i + b*c*d^2*(d^2 - c^2)^(1/2)*2i + a*c^2*d*tan(x/2)*
(d^2 - c^2)^(1/2)*2i - b*c^2*d*tan(x/2)*(d^2 - c^2)^(1/2)*1i)/(4*b*d^4*tan(
x/2) - a*c^4*tan(x/2) + a*c^3*d + 2*b*c*d^3 - b*c^3*d + 2*a*c^2*d^2*tan(x/2
) - 3*b*c^2*d^2*tan(x/2)))*(d^2 - c^2)^(1/2)*2i + b*d^2*atan((a*c^3*(d^2 -
c^2)^(1/2)*1i + b*d^3*tan(x/2)*(d^2 - c^2)^(1/2)*4i + b*c*d^2*(d^2 - c^2)^(
1/2)*2i + a*c^2*d*tan(x/2)*(d^2 - c^2)^(1/2)*2i - b*c^2*d*tan(x/2)*(d^2 - c
^2)^(1/2)*1i)/(4*b*d^4*tan(x/2) - a*c^4*tan(x/2) + a*c^3*d + 2*b*c*d^3 - b*
c^3*d + 2*a*c^2*d^2*tan(x/2) - 3*b*c^2*d^2*tan(x/2)))*(d^2 - c^2)^(1/2)*2i
- b*c^2*d*log(tan(x/2))))/(c^4 - c^2*d^2) - (b*c^3 - b*c*d^2)/(c^4*tan(x) -
c^2*d^2*tan(x))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a+b\csc^2(x)}{c+d\sin(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*csc(x)**2)/(c+d*sin(x)),x)
```

```
[Out] Integral((a + b*csc(x)**2)/(c + d*sin(x)), x)
```

3.217 $\int (a\cos(c+dx)+b\sin(c+dx))^n\,dx$

Optimal. Leaf size=136

$$-\frac{\sin\left(-\tan^{-1}(a,b)+c+dx\right)(a\cos(c+dx)+b\sin(c+dx))^n\left(\frac{a\cos(c+dx)+b\sin(c+dx)}{\sqrt{a^2+b^2}}\right)^{-n}\cos^{n+1}\left(-\tan^{-1}(a,b)+c+\right.}{d(n+1)\sqrt{\sin^2\left(-\tan^{-1}(a,b)+c+dx\right)}}$$

```
[Out] -cos(c+d*x-arctan(a,b))^(1+n)*hypergeom([1/2, 1/2+1/2*n],[3/2+1/2*n],cos(c+
d*x-arctan(a,b))^2)*(a*cos(d*x+c)+b*sin(d*x+c))^n*sin(c+d*x-arctan(a,b))/d/
(1+n)/(((a*cos(d*x+c)+b*sin(d*x+c))/(a^2+b^2)^(1/2))^n)/(sin(c+d*x-arctan(a
,b))^2)^(1/2)
```

Rubi [A] time = 0.06, antiderivative size = 136, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {3078, 2643}

$$-\frac{\sin\left(-\tan^{-1}(a,b)+c+dx\right)(a\cos(c+dx)+b\sin(c+dx))^n\left(\frac{a\cos(c+dx)+b\sin(c+dx)}{\sqrt{a^2+b^2}}\right)^{-n}\cos^{n+1}\left(-\tan^{-1}(a,b)+c+\right.}{d(n+1)\sqrt{\sin^2\left(-\tan^{-1}(a,b)+c+dx\right)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Cos[c + d*x] + b*Sin[c + d*x])^n,x]
```

```
[Out] -((Cos[c + d*x - ArcTan[a, b]]^(1 + n)*Hypergeometric2F1[1/2, (1 + n)/2, (3
 + n)/2, Cos[c + d*x - ArcTan[a, b]]^2]*(a*Cos[c + d*x] + b*Sin[c + d*x])^n
*Sin[c + d*x - ArcTan[a, b]])/(d*(1 + n)*((a*Cos[c + d*x] + b*Sin[c + d*x])
/Sqrt[a^2 + b^2])^n*Sqrt[Sin[c + d*x - ArcTan[a, b]]^2]))
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rule 3078

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x
_Symbol] :> Dist[(a*Cos[c + d*x] + b*Sin[c + d*x])^n/((a*Cos[c + d*x] + b*S
in[c + d*x])/Sqrt[a^2 + b^2])^n, Int[Cos[c + d*x - ArcTan[a, b]]^n, x], x]
/; FreeQ[{a, b, c, d, n}, x] && !(GeQ[n, 1] || LeQ[n, -1]) && !(GtQ[a^2 +
 b^2, 0] || EqQ[a^2 + b^2, 0])
```

Rubi steps

$$\int (a\cos(c+dx)+b\sin(c+dx))^n\,dx = \left((a\cos(c+dx)+b\sin(c+dx))^n\left(\frac{a\cos(c+dx)+b\sin(c+dx)}{\sqrt{a^2+b^2}}\right)^{-n}\right)\int \text{co}$$

$$= -\frac{\cos^{1+n}\left(c+dx-\tan^{-1}(a,b)\right)\,{}_2F_1\left(\frac{1}{2},\frac{1+n}{2};\frac{3+n}{2};\cos^2\left(c+dx-\tan^{-1}(a,b\right.\right.}{d(1+n)\sqrt{\text{si}}}$$

Mathematica [A] time = 0.24, size = 94, normalized size = 0.69

$$-\frac{\sin\left(2\left(\tan^{-1}\left(\frac{a}{b}\right)+c+dx\right)\right)\sin^2\left(\tan^{-1}\left(\frac{a}{b}\right)+c+dx\right)^{-\frac{n}{2}-\frac{1}{2}}(a\cos(c+dx)+b\sin(c+dx))^n\,{}_2F_1\left(\frac{1}{2},\frac{1-n}{2};\frac{3}{2};\cos^2\left(\right.\right.}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Cos[c + d*x] + b*Sin[c + d*x])^n,x]
```

```
[Out] -1/2*(Hypergeometric2F1[1/2, (1 - n)/2, 3/2, Cos[c + d*x + ArcTan[a/b]]^2]*
(a*Cos[c + d*x] + b*Sin[c + d*x])^n*(Sin[c + d*x + ArcTan[a/b]]^2)^(-1/2 -
n/2)*Sin[2*(c + d*x + ArcTan[a/b])])/d
```

fricas [F] time = 1.86, size = 0, normalized size = 0.00

$$\text{integral}\left((a\cos(dx+c)+b\sin(dx+c))^n, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cos(d*x+c)+b*sin(d*x+c))^n,x, algorithm="fricas")
```

```
[Out] integral((a*cos(d*x + c) + b*sin(d*x + c))^n, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a\cos(dx+c)+b\sin(dx+c))^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cos(d*x+c)+b*sin(d*x+c))^n,x, algorithm="giac")
```

```
[Out] integrate((a*cos(d*x + c) + b*sin(d*x + c))^n, x)
```

maple [F] time = 1.00, size = 0, normalized size = 0.00

$$\int (a\cos(dx+c)+b\sin(dx+c))^n\ dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*cos(d*x+c)+b*sin(d*x+c))^n,x)
[Out] int((a*cos(d*x+c)+b*sin(d*x+c))^n,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a\cos(dx+c)+b\sin(dx+c))^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cos(d*x+c)+b*sin(d*x+c))^n,x, algorithm="maxima")
[Out] integrate((a*cos(d*x + c) + b*sin(d*x + c))^n, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int (a\ \cos(c+d\,x)+b\ \sin(c+d\,x))^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*cos(c + d*x) + b*sin(c + d*x))^n,x)
[Out] int((a*cos(c + d*x) + b*sin(c + d*x))^n, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cos(d*x+c)+b*sin(d*x+c))**n,x)
[Out] Timed out
```

3.218 $\int (2\cos(c+dx)+3\sin(c+dx))^n\,dx$

Optimal. Leaf size=95

$$-\frac{13^{n/2}\sin\left(c+dx-\tan^{-1}\left(\frac{3}{2}\right)\right)\cos^{n+1}\left(c+dx-\tan^{-1}\left(\frac{3}{2}\right)\right)\,{}_2F_1\left(\frac{1}{2},\frac{n+1}{2};\frac{n+3}{2};\cos^2\left(c+dx-\tan^{-1}\left(\frac{3}{2}\right)\right)\right)}{d(n+1)\sqrt{\sin^2\left(c+dx-\tan^{-1}\left(\frac{3}{2}\right)\right)}}$$

```
[Out] -13^(1/2*n)*cos(c+d*x-arctan(3/2))^(1+n)*hypergeom([1/2, 1/2+1/2*n],[3/2+1/
2*n],cos(c+d*x-arctan(3/2))^2)*sin(c+d*x-arctan(3/2))/d/(1+n)/(sin(c+d*x-ar
ctan(3/2))^2)^(1/2)
```

Rubi [A] time = 0.05, antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {3077, 2643}

$$-\frac{13^{n/2}\sin\left(c+dx-\tan^{-1}\left(\frac{3}{2}\right)\right)\cos^{n+1}\left(c+dx-\tan^{-1}\left(\frac{3}{2}\right)\right)\,{}_2F_1\left(\frac{1}{2},\frac{n+1}{2};\frac{n+3}{2};\cos^2\left(c+dx-\tan^{-1}\left(\frac{3}{2}\right)\right)\right)}{d(n+1)\sqrt{\sin^2\left(c+dx-\tan^{-1}\left(\frac{3}{2}\right)\right)}}$$

Antiderivative was successfully verified.

```
[In] Int[(2*Cos[c + d*x] + 3*Sin[c + d*x])^n,x]
```

```
[Out] -((13^(n/2)*Cos[c + d*x - ArcTan[3/2]]^(1 + n)*Hypergeometric2F1[1/2, (1 +
n)/2, (3 + n)/2, Cos[c + d*x - ArcTan[3/2]]^2]*Sin[c + d*x - ArcTan[3/2]])/
(d*(1 + n)*Sqrt[Sin[c + d*x - ArcTan[3/2]]^2]))
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rule 3077

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x
_Symbol] :> Dist[(a^2 + b^2)^(n/2), Int[Cos[c + d*x - ArcTan[a, b]]^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] &&  !(GeQ[n, 1] || LeQ[n, -1]) && GtQ[a^2 +
 b^2, 0]
```

Rubi steps

$$\int (2\cos(c+dx)+3\sin(c+dx))^n\,dx = 13^{n/2}\int \cos^n\left(c+dx-\tan^{-1}\left(\frac{3}{2}\right)\right)dx$$

$$= -\frac{13^{n/2}\cos^{1+n}\left(c+dx-\tan^{-1}\left(\frac{3}{2}\right)\right)\,{}_2F_1\left(\frac{1}{2},\frac{1+n}{2};\frac{3+n}{2};\cos^2\left(c+dx-\tan\right.\right.}{d(1+n)\sqrt{\sin^2\left(c+dx-\tan^{-1}\left(\frac{3}{2}\right)\right)}}$$

Mathematica [A] time = 0.17, size = 88, normalized size = 0.93

$$-\frac{\sin\left(2\left(c+dx+\tan^{-1}\left(\frac{2}{3}\right)\right)\right)\sin^2\left(c+dx+\tan^{-1}\left(\frac{2}{3}\right)\right)^{-\frac{n}{2}-\frac{1}{2}}(3\sin(c+dx)+2\cos(c+dx))^n\,{}_2F_1\left(\frac{1}{2},\frac{1-n}{2};\frac{3}{2};\cos\right.}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2*Cos[c + d*x] + 3*Sin[c + d*x])^n,x]
[Out] -1/2*(Hypergeometric2F1[1/2, (1 - n)/2, 3/2, Cos[c + d*x + ArcTan[2/3]]^2]*
(2*Cos[c + d*x] + 3*Sin[c + d*x])^n*(Sin[c + d*x + ArcTan[2/3]]^2)^(-1/2 -
n/2)*Sin[2*(c + d*x + ArcTan[2/3])])/d
```

fricas [F] time = 1.18, size = 0, normalized size = 0.00

$$\text{integral}\left((2\cos(dx+c)+3\sin(dx+c))^n, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*cos(d*x+c)+3*sin(d*x+c))^n,x, algorithm="fricas")
[Out] integral((2*cos(d*x + c) + 3*sin(d*x + c))^n, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (2\cos(dx+c)+3\sin(dx+c))^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*cos(d*x+c)+3*sin(d*x+c))^n,x, algorithm="giac")
[Out] integrate((2*cos(d*x + c) + 3*sin(d*x + c))^n, x)
```

maple [F] time = 1.06, size = 0, normalized size = 0.00

$$\int (2\cos(dx+c)+3\sin(dx+c))^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*cos(d*x+c)+3*sin(d*x+c))^n,x)
```

```
[Out] int((2*cos(d*x+c)+3*sin(d*x+c))^n,x)
```

maxima **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int (2\cos(dx+c)+3\sin(dx+c))^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*cos(d*x+c)+3*sin(d*x+c))^n,x, algorithm="maxima")
```

```
[Out] integrate((2*cos(d*x + c) + 3*sin(d*x + c))^n, x)
```

mupad **[F]** time = 0.00, size = -1, normalized size = -0.01

$$\int (2\cos(c+d\,x)+3\sin(c+d\,x))^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*cos(c + d*x) + 3*sin(c + d*x))^n,x)
```

```
[Out] int((2*cos(c + d*x) + 3*sin(c + d*x))^n, x)
```

sympy **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int (3\sin(c+dx)+2\cos(c+dx))^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*cos(d*x+c)+3*sin(d*x+c))**n,x)
```

```
[Out] Integral((3*sin(c + d*x) + 2*cos(c + d*x))**n, x)
```

3.219 $\int (a\cos(c+dx)+b\sin(c+dx))^7\,dx$

Optimal. Leaf size=127

$$-\frac{3\left(a^2+b^2\right)(b\cos(c+dx)-a\sin(c+dx))^5}{5d}+\frac{\left(a^2+b^2\right)^2(b\cos(c+dx)-a\sin(c+dx))^3}{d}-\frac{\left(a^2+b^2\right)^3(b\cos(c+\ldots}{d}$$

```
[Out] -(a^2+b^2)^3*(b*cos(d*x+c)-a*sin(d*x+c))/d+(a^2+b^2)^2*(b*cos(d*x+c)-a*sin(
d*x+c))^3/d-3/5*(a^2+b^2)*(b*cos(d*x+c)-a*sin(d*x+c))^5/d+1/7*(b*cos(d*x+c)
-a*sin(d*x+c))^7/d
```

Rubi [A] time = 0.08, antiderivative size = 127, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {3072, 194}

$$-\frac{3\left(a^2+b^2\right)(b\cos(c+dx)-a\sin(c+dx))^5}{5d}+\frac{\left(a^2+b^2\right)^2(b\cos(c+dx)-a\sin(c+dx))^3}{d}-\frac{\left(a^2+b^2\right)^3(b\cos(c+\ldots}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Cos[c + d*x] + b*Sin[c + d*x])^7,x]
```

```
[Out] -(((a^2 + b^2)^3*(b*Cos[c + d*x] - a*Sin[c + d*x]))/d) + ((a^2 + b^2)^2*(b*
Cos[c + d*x] - a*Sin[c + d*x])^3)/d - (3*(a^2 + b^2)*(b*Cos[c + d*x] - a*Si
n[c + d*x])^5)/(5*d) + (b*Cos[c + d*x] - a*Sin[c + d*x])^7/(7*d)
```

Rule 194

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 3072

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x
_Symbol] :> -Dist[d^(-1), Subst[Int[(a^2 + b^2 - x^2)^((n - 1)/2), x], x, b
*Cos[c + d*x] - a*Sin[c + d*x]], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 +
b^2, 0] && IGtQ[(n - 1)/2, 0]
```

Rubi steps

$$\int (a\cos(c+dx)+b\sin(c+dx))^7\,dx = -\frac{\text{Subst}\left(\int \left(a^2+b^2-x^2\right)^3\,dx, x, b\cos(c+dx)-a\sin(c+dx)\right)}{d}$$
$$= -\frac{\text{Subst}\left(\int \left(a^6\left(1+\frac{3a^4b^2+3a^2b^4+b^6}{a^6}\right)-3a^4\left(1+\frac{2a^2b^2+b^4}{a^4}\right)x^2+3a^2\left(1+\frac{b^2}{a^2}\right)x\right.\right.}{d}$$
$$= -\frac{\left(a^2+b^2\right)^3(b\cos(c+dx)-a\sin(c+dx))}{d}+\frac{\left(a^2+b^2\right)^2(b\cos(c+dx)-}{d}$$

Mathematica [A] time = 1.02, size = 246, normalized size = 1.94

$$\frac{1225a\left(a^2+b^2\right)^3\sin(c+dx)+245a\left(a^2-3b^2\right)\left(a^2+b^2\right)^2\sin(3(c+dx))-1225b\left(a^2+b^2\right)^3\cos(c+dx)+245b\left(\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Cos[c + d*x] + b*Sin[c + d*x])^7,x]
```

```
[Out] (-1225*b*(a^2 + b^2)^3*Cos[c + d*x] + 245*b*(-3*a^2 + b^2)*(a^2 + b^2)^2*Co
s[3*(c + d*x)] - 49*b*(5*a^6 - 5*a^4*b^2 - 9*a^2*b^4 + b^6)*Cos[5*(c + d*x)
] + 5*b*(-7*a^6 + 35*a^4*b^2 - 21*a^2*b^4 + b^6)*Cos[7*(c + d*x)] + 1225*a*
(a^2 + b^2)^3*Sin[c + d*x] + 245*a*(a^2 - 3*b^2)*(a^2 + b^2)^2*Sin[3*(c + d
*x)] + 49*a*(a^6 - 9*a^4*b^2 - 5*a^2*b^4 + 5*b^6)*Sin[5*(c + d*x)] + 5*a*(a
^6 - 21*a^4*b^2 + 35*a^2*b^4 - 7*b^6)*Sin[7*(c + d*x)])/(2240*d)
```

fricas [B] time = 1.01, size = 257, normalized size = 2.02

$$-\frac{35\,b^7\cos(dx+c)+5\left(7\,a^6b-35\,a^4b^3+21\,a^2b^5-b^7\right)\cos(dx+c)^7+7\left(35\,a^4b^3-42\,a^2b^5+3\,b^7\right)\cos(dx+c)^5}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cos(d*x+c)+b*sin(d*x+c))^7,x, algorithm="fricas")
```

```
[Out] -1/35*(35*b^7*cos(d*x + c) + 5*(7*a^6*b - 35*a^4*b^3 + 21*a^2*b^5 - b^7)*co
s(d*x + c)^7 + 7*(35*a^4*b^3 - 42*a^2*b^5 + 3*b^7)*cos(d*x + c)^5 + 35*(7*a
^2*b^5 - b^7)*cos(d*x + c)^3 - (16*a^7 + 56*a^5*b^2 + 70*a^3*b^4 + 35*a*b^6
 + 5*(a^7 - 21*a^5*b^2 + 35*a^3*b^4 - 7*a*b^6)*cos(d*x + c)^6 + (6*a^7 + 21
*a^5*b^2 - 280*a^3*b^4 + 105*a*b^6)*cos(d*x + c)^4 + (8*a^7 + 28*a^5*b^2 +
35*a^3*b^4 - 105*a*b^6)*cos(d*x + c)^2)*sin(d*x + c))/d
```

giac [B] time = 0.66, size = 316, normalized size = 2.49

$$-\frac{\left(7\,a^6b-35\,a^4b^3+21\,a^2b^5-b^7\right)\cos(7\,dx+7\,c)}{448\,d}-\frac{7\left(5\,a^6b-5\,a^4b^3-9\,a^2b^5+b^7\right)\cos(5\,dx+5\,c)}{320\,d}-\frac{7\left(3\,a^6b+5\,a\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cos(d*x+c)+b*sin(d*x+c))^7,x, algorithm="giac")
```

```
[Out] -1/448*(7*a^6*b - 35*a^4*b^3 + 21*a^2*b^5 - b^7)*cos(7*d*x + 7*c)/d - 7/320
*(5*a^6*b - 5*a^4*b^3 - 9*a^2*b^5 + b^7)*cos(5*d*x + 5*c)/d - 7/64*(3*a^6*b
 + 5*a^4*b^3 + a^2*b^5 - b^7)*cos(3*d*x + 3*c)/d - 35/64*(a^6*b + 3*a^4*b^3
 + 3*a^2*b^5 + b^7)*cos(d*x + c)/d + 1/448*(a^7 - 21*a^5*b^2 + 35*a^3*b^4 -
 7*a*b^6)*sin(7*d*x + 7*c)/d + 7/320*(a^7 - 9*a^5*b^2 - 5*a^3*b^4 + 5*a*b^6
)*sin(5*d*x + 5*c)/d + 7/64*(a^7 - a^5*b^2 - 5*a^3*b^4 - 3*a*b^6)*sin(3*d*x
 + 3*c)/d + 35/64*(a^7 + 3*a^5*b^2 + 3*a^3*b^4 + a*b^6)*sin(d*x + c)/d
```

maple [B] time = 0.41, size = 321, normalized size = 2.53

$$-\frac{b^{7}\left(\frac{16}{5}+\sin^{6}(dx+c)+\frac{6\left(\sin^{4}(dx+c)\right)}{5}+\frac{8\left(\sin^{2}(dx+c)\right)}{5}\right)\cos(dx+c)}{7}+a\,b^{6}\left(\sin^{7}\left(dx+c\right)\right)+21a^{2}b^{5}\left(-\frac{\left(\sin^{4}(dx+c)\right)\left(\cos^{3}(dx+c)\right)}{7}-\frac{4\left(\sin^{2}(dx+\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*cos(d*x+c)+b*sin(d*x+c))^7,x)
```

```
[Out] 1/d*(-1/7*b^7*(16/5+sin(d*x+c)^6+6/5*sin(d*x+c)^4+8/5*sin(d*x+c)^2)*cos(d*x
+c)+a*b^6*sin(d*x+c)^7+21*a^2*b^5*(-1/7*sin(d*x+c)^4*cos(d*x+c)^3-4/35*sin(
d*x+c)^2*cos(d*x+c)^3-8/105*cos(d*x+c)^3)+35*a^3*b^4*(-1/7*sin(d*x+c)^3*cos
(d*x+c)^4-3/35*sin(d*x+c)*cos(d*x+c)^4+1/35*(2+cos(d*x+c)^2)*sin(d*x+c))+35
*a^4*b^3*(-1/7*sin(d*x+c)^2*cos(d*x+c)^5-2/35*cos(d*x+c)^5)+21*a^5*b^2*(-1/
7*sin(d*x+c)*cos(d*x+c)^6+1/35*(8/3+cos(d*x+c)^4+4/3*cos(d*x+c)^2)*sin(d*x+
c))-a^6*b*cos(d*x+c)^7+1/7*a^7*(16/5+cos(d*x+c)^6+6/5*cos(d*x+c)^4+8/5*cos(
d*x+c)^2)*sin(d*x+c))
```

maxima [B] time = 0.33, size = 257, normalized size = 2.02

$$-\frac{35\,a^{6}b\cos\left(dx+c\right)^{7}-35\,ab^{6}\sin\left(dx+c\right)^{7}+\left(5\,\sin\left(dx+c\right)^{7}-21\,\sin\left(dx+c\right)^{5}+35\,\sin\left(dx+c\right)^{3}-35\,\sin\left(dx\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cos(d*x+c)+b*sin(d*x+c))^7,x, algorithm="maxima")
```

```
[Out] -1/35*(35*a^6*b*cos(d*x + c)^7 - 35*a*b^6*sin(d*x + c)^7 + (5*sin(d*x + c)^
7 - 21*sin(d*x + c)^5 + 35*sin(d*x + c)^3 - 35*sin(d*x + c))*a^7 - 7*(15*si
n(d*x + c)^7 - 42*sin(d*x + c)^5 + 35*sin(d*x + c)^3)*a^5*b^2 - 35*(5*cos(d
*x + c)^7 - 7*cos(d*x + c)^5)*a^4*b^3 + 35*(5*sin(d*x + c)^7 - 7*sin(d*x +
c)^5)*a^3*b^4 + 7*(15*cos(d*x + c)^7 - 42*cos(d*x + c)^5 + 35*cos(d*x + c)^
```

```
3)*a^2*b^5 - (5*cos(d*x + c)^7 - 21*cos(d*x + c)^5 + 35*cos(d*x + c)^3 - 35
*cos(d*x + c))*b^7)/d
```

mupad [B] time = 6.16, size = 422, normalized size = 3.32

$$-\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8\,\left(70\,a^6\,b-140\,a^4\,b^3+224\,a^2\,b^5\right)-2\,a^7\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{13}-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7\,\left(\frac{424\,a^7}{35}+\frac{912\,a^5\,b^2}{5}-192\,a^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*cos(c + d*x) + b*sin(c + d*x))^7,x)
```

```
[Out] -(tan(c/2 + (d*x)/2)^8*(70*a^6*b + 224*a^2*b^5 - 140*a^4*b^3) - 2*a^7*tan(c
/2 + (d*x)/2)^13 - tan(c/2 + (d*x)/2)^7*(128*a*b^6 + (424*a^7)/35 - 192*a^3
*b^4 + (912*a^5*b^2)/5) + tan(c/2 + (d*x)/2)^4*(42*a^6*b + (96*b^7)/5 + (33
6*a^2*b^5)/5 - 56*a^4*b^3) + 2*a^6*b + (32*b^7)/35 - tan(c/2 + (d*x)/2)^5*(
(86*a^7)/5 + 224*a^3*b^4 - (224*a^5*b^2)/5) - tan(c/2 + (d*x)/2)^9*((86*a^7
)/5 + 224*a^3*b^4 - (224*a^5*b^2)/5) + tan(c/2 + (d*x)/2)^2*((32*b^7)/5 + (
112*a^2*b^5)/5 + 28*a^4*b^3) + tan(c/2 + (d*x)/2)^6*(32*b^7 - 112*a^2*b^5 +
 280*a^4*b^3) + (16*a^2*b^5)/5 + 4*a^4*b^3 - tan(c/2 + (d*x)/2)^3*(4*a^7 +
56*a^5*b^2) - tan(c/2 + (d*x)/2)^11*(4*a^7 + 56*a^5*b^2) - 2*a^7*tan(c/2 +
(d*x)/2) + 140*a^4*b^3*tan(c/2 + (d*x)/2)^10 + 14*a^6*b*tan(c/2 + (d*x)/2)^
12)/(d*(tan(c/2 + (d*x)/2)^2 + 1)^7)
```

sympy [A] time = 6.56, size = 461, normalized size = 3.63

$$\begin{cases} \frac{16a^7\sin^7(c+dx)}{35d}+\frac{8a^7\sin^5(c+dx)\cos^2(c+dx)}{5d}+\frac{2a^7\sin^3(c+dx)\cos^4(c+dx)}{d}+\frac{a^7\sin(c+dx)\cos^6(c+dx)}{d}-\frac{a^6b\cos^7(c+dx)}{d}+\frac{8a^5b^2\sin^7(c+d} {5d} \\ x\,(a\cos(c)+b\sin(c))^7 \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cos(d*x+c)+b*sin(d*x+c))**7,x)
```

```
[Out] Piecewise((16*a**7*sin(c + d*x)**7/(35*d) + 8*a**7*sin(c + d*x)**5*cos(c +
d*x)**2/(5*d) + 2*a**7*sin(c + d*x)**3*cos(c + d*x)**4/d + a**7*sin(c + d*x
)*cos(c + d*x)**6/d - a**6*b*cos(c + d*x)**7/d + 8*a**5*b**2*sin(c + d*x)**
7/(5*d) + 28*a**5*b**2*sin(c + d*x)**5*cos(c + d*x)**2/(5*d) + 7*a**5*b**2*
sin(c + d*x)**3*cos(c + d*x)**4/d - 7*a**4*b**3*sin(c + d*x)**2*cos(c + d*x
)**5/d - 2*a**4*b**3*cos(c + d*x)**7/d + 2*a**3*b**4*sin(c + d*x)**7/d + 7*
a**3*b**4*sin(c + d*x)**5*cos(c + d*x)**2/d - 7*a**2*b**5*sin(c + d*x)**4*c
os(c + d*x)**3/d - 28*a**2*b**5*sin(c + d*x)**2*cos(c + d*x)**5/(5*d) - 8*a
**2*b**5*cos(c + d*x)**7/(5*d) + a*b**6*sin(c + d*x)**7/d - b**7*sin(c + d*
```

```
x)**6*cos(c + d*x)/d - 2*b**7*sin(c + d*x)**4*cos(c + d*x)**3/d - 8*b**7*si
n(c + d*x)**2*cos(c + d*x)**5/(5*d) - 16*b**7*cos(c + d*x)**7/(35*d), Ne(d,
 0)), (x*(a*cos(c) + b*sin(c))**7, True))
```

3.220 $\int (a\cos(c+dx)+b\sin(c+dx))^6\,dx$

Optimal. Leaf size=161

$$-\frac{5\left(a^2+b^2\right)(b\cos(c+dx)-a\sin(c+dx))(a\cos(c+dx)+b\sin(c+dx))^3}{24d}-\frac{5\left(a^2+b^2\right)^2(b\cos(c+dx)-a\sin(c+$$

```
[Out] 5/16*(a^2+b^2)^3*x-5/16*(a^2+b^2)^2*(b*cos(d*x+c)-a*sin(d*x+c))*(a*cos(d*x+
c)+b*sin(d*x+c))/d-5/24*(a^2+b^2)*(b*cos(d*x+c)-a*sin(d*x+c))*(a*cos(d*x+c)
+b*sin(d*x+c))^3/d-1/6*(b*cos(d*x+c)-a*sin(d*x+c))*(a*cos(d*x+c)+b*sin(d*x+
c))^5/d
```

Rubi [A] time = 0.08, antiderivative size = 161, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {3073, 8}

$$-\frac{5\left(a^2+b^2\right)(b\cos(c+dx)-a\sin(c+dx))(a\cos(c+dx)+b\sin(c+dx))^3}{24d}-\frac{5\left(a^2+b^2\right)^2(b\cos(c+dx)-a\sin(c+$$

Antiderivative was successfully verified.

```
[In] Int[(a*Cos[c + d*x] + b*Sin[c + d*x])^6,x]
```

```
[Out] (5*(a^2 + b^2)^3*x)/16 - (5*(a^2 + b^2)^2*(b*Cos[c + d*x] - a*Sin[c + d*x])
*(a*Cos[c + d*x] + b*Sin[c + d*x]))/(16*d) - (5*(a^2 + b^2)*(b*Cos[c + d*x]
 - a*Sin[c + d*x])*(a*Cos[c + d*x] + b*Sin[c + d*x])^3)/(24*d) - ((b*Cos[c
+ d*x] - a*Sin[c + d*x])*(a*Cos[c + d*x] + b*Sin[c + d*x])^5)/(6*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3073

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x
_Symbol] :> -Simp[((b*Cos[c + d*x] - a*Sin[c + d*x])*(a*Cos[c + d*x] + b*Si
n[c + d*x])^(n - 1))/(d*n), x] + Dist[((n - 1)*(a^2 + b^2))/n, Int[(a*Cos[c
 + d*x] + b*Sin[c + d*x])^(n - 2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 + b^2, 0] &&  !IntegerQ[(n - 1)/2] && GtQ[n, 1]
```

Rubi steps

$$\int (a\cos(c+dx)+b\sin(c+dx))^6\,dx = -\frac{(b\cos(c+dx)-a\sin(c+dx))(a\cos(c+dx)+b\sin(c+dx))^5}{6d} + \frac{1}{6}\big(5\big($$

$$= -\frac{5\left(a^2+b^2\right)(b\cos(c+dx)-a\sin(c+dx))(a\cos(c+dx)+b\sin(c+dx}{24d}$$

$$= -\frac{5\left(a^2+b^2\right)^2(b\cos(c+dx)-a\sin(c+dx))(a\cos(c+dx)+b\sin(c+d}{16d}$$

$$= \frac{5}{16}\left(a^2+b^2\right)^3 x - \frac{5\left(a^2+b^2\right)^2(b\cos(c+dx)-a\sin(c+dx))(a\cos(c+}{16d}$$

Mathematica [A] time = 0.69, size = 192, normalized size = 1.19

$$\frac{-36ab\left(a^4-b^4\right)\cos(4(c+dx))+60\left(a^2+b^2\right)^3(c+dx)+45\left(a^2-b^2\right)\left(a^2+b^2\right)^2\sin(2(c+dx))-90ab\left(a^2+b^2\right)}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Cos[c + d*x] + b*Sin[c + d*x])^6,x]
```

```
[Out] (60*(a^2 + b^2)^3*(c + d*x) - 90*a*b*(a^2 + b^2)^2*Cos[2*(c + d*x)] - 36*a*
b*(a^4 - b^4)*Cos[4*(c + d*x)] - 2*a*b*(3*a^4 - 10*a^2*b^2 + 3*b^4)*Cos[6*(
c + d*x)] + 45*(a^2 - b^2)*(a^2 + b^2)^2*Sin[2*(c + d*x)] + 9*(a^6 - 5*a^4*
b^2 - 5*a^2*b^4 + b^6)*Sin[4*(c + d*x)] + (a^6 - 15*a^4*b^2 + 15*a^2*b^4 -
b^6)*Sin[6*(c + d*x)])/(192*d)
```

fricas [A] time = 1.18, size = 219, normalized size = 1.36

$$-\frac{144\,ab^5\cos(dx+c)^2+16\left(3\,a^5b-10\,a^3b^3+3\,ab^5\right)\cos(dx+c)^6+48\left(5\,a^3b^3-3\,ab^5\right)\cos(dx+c)^4-15\left(a^6+\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cos(d*x+c)+b*sin(d*x+c))^6,x, algorithm="fricas")
```

```
[Out] -1/48*(144*a*b^5*cos(d*x + c)^2 + 16*(3*a^5*b - 10*a^3*b^3 + 3*a*b^5)*cos(d
*x + c)^6 + 48*(5*a^3*b^3 - 3*a*b^5)*cos(d*x + c)^4 - 15*(a^6 + 3*a^4*b^2 +
 3*a^2*b^4 + b^6)*d*x - (8*(a^6 - 15*a^4*b^2 + 15*a^2*b^4 - b^6)*cos(d*x +
c)^5 + 2*(5*a^6 + 15*a^4*b^2 - 105*a^2*b^4 + 13*b^6)*cos(d*x + c)^3 + 3*(5*
a^6 + 15*a^4*b^2 + 15*a^2*b^4 - 11*b^6)*cos(d*x + c))*sin(d*x + c))/d
```

giac [A] time = 0.32, size = 235, normalized size = 1.46

$$\frac{5}{16}\left(a^6+3\,a^4b^2+3\,a^2b^4+b^6\right)x-\frac{\left(3\,a^5b-10\,a^3b^3+3\,ab^5\right)\cos(6\,dx+6\,c)}{96\,d}-\frac{3\left(a^5b-ab^5\right)\cos(4\,dx+4\,c)}{16\,d}-\frac{15\left(a^5\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cos(d*x+c)+b*sin(d*x+c))^6,x, algorithm="giac")
```

```
[Out] 5/16*(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*x - 1/96*(3*a^5*b - 10*a^3*b^3 + 3
*a*b^5)*cos(6*d*x + 6*c)/d - 3/16*(a^5*b - a*b^5)*cos(4*d*x + 4*c)/d - 15/3
2*(a^5*b + 2*a^3*b^3 + a*b^5)*cos(2*d*x + 2*c)/d + 1/192*(a^6 - 15*a^4*b^2
+ 15*a^2*b^4 - b^6)*sin(6*d*x + 6*c)/d + 3/64*(a^6 - 5*a^4*b^2 - 5*a^2*b^4
+ b^6)*sin(4*d*x + 4*c)/d + 15/64*(a^6 + a^4*b^2 - a^2*b^4 - b^6)*sin(2*d*x
 + 2*c)/d
```

maple [A] time = 0.34, size = 285, normalized size = 1.77

$$b^6\left(-\frac{\left(\sin^5(dx+c)+\frac{5\left(\sin^3(dx+c)\right)}{4}+\frac{15\sin(dx+c)}{8}\right)\cos(dx+c)}{6}+\frac{5dx}{16}+\frac{5c}{16}\right)+a\,b^5\left(\sin^6(dx+c)\right)+15a^2b^4\left(-\frac{\left(\sin^3(dx+c)\right)\left(\cos^3(dx+c)\right)}{6}-\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*cos(d*x+c)+b*sin(d*x+c))^6,x)
```

```
[Out] 1/d*(b^6*(-1/6*(sin(d*x+c)^5+5/4*sin(d*x+c)^3+15/8*sin(d*x+c))*cos(d*x+c)+5
/16*d*x+5/16*c)+a*b^5*sin(d*x+c)^6+15*a^2*b^4*(-1/6*sin(d*x+c)^3*cos(d*x+c)
^3-1/8*sin(d*x+c)*cos(d*x+c)^3+1/16*sin(d*x+c)*cos(d*x+c)+1/16*d*x+1/16*c)+
20*a^3*b^3*(-1/6*sin(d*x+c)^2*cos(d*x+c)^4-1/12*cos(d*x+c)^4)+15*a^4*b^2*(-
1/6*sin(d*x+c)*cos(d*x+c)^5+1/24*(cos(d*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+1
/16*d*x+1/16*c)-a^5*b*cos(d*x+c)^6+a^6*(1/6*(cos(d*x+c)^5+5/4*cos(d*x+c)^3+
15/8*cos(d*x+c))*sin(d*x+c)+5/16*d*x+5/16*c))
```

maxima [A] time = 0.33, size = 238, normalized size = 1.48

$$-\frac{192\,a^5b\cos(dx+c)^6-192\,ab^5\sin(dx+c)^6+\left(4\,\sin(2\,dx+2\,c)^3-60\,dx-60\,c-9\,\sin(4\,dx+4\,c)-48\,\sin(2\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cos(d*x+c)+b*sin(d*x+c))^6,x, algorithm="maxima")
```

```
[Out] -1/192*(192*a^5*b*cos(d*x + c)^6 - 192*a*b^5*sin(d*x + c)^6 + (4*sin(2*d*x
+ 2*c)^3 - 60*d*x - 60*c - 9*sin(4*d*x + 4*c) - 48*sin(2*d*x + 2*c))*a^6 -
15*(4*sin(2*d*x + 2*c)^3 + 12*d*x + 12*c - 3*sin(4*d*x + 4*c))*a^4*b^2 + 32
0*(2*sin(d*x + c)^6 - 3*sin(d*x + c)^4)*a^3*b^3 + 15*(4*sin(2*d*x + 2*c)^3
- 12*d*x - 12*c + 3*sin(4*d*x + 4*c))*a^2*b^4 - (4*sin(2*d*x + 2*c)^3 + 60*
d*x + 60*c + 9*sin(4*d*x + 4*c) - 48*sin(2*d*x + 2*c))*b^6)/d
```

mupad [B] time = 4.12, size = 519, normalized size = 3.22

$$\frac{5\,\mathrm{atan}\left(\frac{5\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\left(a^2+b^2\right)^3}{8\left(\frac{5\,a^6}{8}+\frac{15\,a^4\,b^2}{8}+\frac{15\,a^2\,b^4}{8}+\frac{5\,b^6}{8}\right)}\right)\left(a^2+b^2\right)^3}{8\,d}-\frac{5\left(\mathrm{atan}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)-\frac{d\,x}{2}\right)\left(a^2+b^2\right)^3}{8\,d}+\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6\left(40\,a^5\,b\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*cos(c + d*x) + b*sin(c + d*x))^6,x)
```

```
[Out] (5*atan((5*tan(c/2 + (d*x)/2)*(a^2 + b^2)^3)/(8*((5*a^6)/8 + (5*b^6)/8 + (1
5*a^2*b^4)/8 + (15*a^4*b^2)/8)))*(a^2 + b^2)^3)/(8*d) - (5*(atan(tan(c/2 +
(d*x)/2)) - (d*x)/2)*(a^2 + b^2)^3)/(8*d) + (tan(c/2 + (d*x)/2)^6*(64*a*b^5
 + 40*a^5*b - (160*a^3*b^3)/3) - tan(c/2 + (d*x)/2)*((5*b^6)/8 - (11*a^6)/8
 + (15*a^2*b^4)/8 + (15*a^4*b^2)/8) + tan(c/2 + (d*x)/2)^11*((5*b^6)/8 - (1
1*a^6)/8 + (15*a^2*b^4)/8 + (15*a^4*b^2)/8) - tan(c/2 + (d*x)/2)^3*((5*a^6)
/24 + (85*b^6)/24 + (85*a^2*b^4)/8 - (235*a^4*b^2)/8) + tan(c/2 + (d*x)/2)^
9*((5*a^6)/24 + (85*b^6)/24 + (85*a^2*b^4)/8 - (235*a^4*b^2)/8) + tan(c/2 +
 (d*x)/2)^5*((15*a^6)/4 - (33*b^6)/4 + (285*a^2*b^4)/4 - (195*a^4*b^2)/4) -
 tan(c/2 + (d*x)/2)^7*((15*a^6)/4 - (33*b^6)/4 + (285*a^2*b^4)/4 - (195*a^4
*b^2)/4) + 80*a^3*b^3*tan(c/2 + (d*x)/2)^4 + 80*a^3*b^3*tan(c/2 + (d*x)/2)^
8 + 12*a^5*b*tan(c/2 + (d*x)/2)^2 + 12*a^5*b*tan(c/2 + (d*x)/2)^10)/(d*(6*t
an(c/2 + (d*x)/2)^2 + 15*tan(c/2 + (d*x)/2)^4 + 20*tan(c/2 + (d*x)/2)^6 + 1
5*tan(c/2 + (d*x)/2)^8 + 6*tan(c/2 + (d*x)/2)^10 + tan(c/2 + (d*x)/2)^12 +
1))
```

sympy [A] time = 4.62, size = 770, normalized size = 4.78

$$\begin{cases} \frac{5a^6x\sin^6(c+dx)}{16}+\frac{15a^6x\sin^4(c+dx)\cos^2(c+dx)}{16}+\frac{15a^6x\sin^2(c+dx)\cos^4(c+dx)}{16}+\frac{5a^6x\cos^6(c+dx)}{16}+\frac{5a^6\sin^5(c+dx)\cos(c+dx)}{16d}+\frac{5a^6\sin}{} \\ x\left(a\cos(c)+b\sin(c)\right)^6 \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cos(d*x+c)+b*sin(d*x+c))**6,x)
```

```
[Out] Piecewise((5*a**6*x*sin(c + d*x)**6/16 + 15*a**6*x*sin(c + d*x)**4*cos(c +
d*x)**2/16 + 15*a**6*x*sin(c + d*x)**2*cos(c + d*x)**4/16 + 5*a**6*x*cos(c
+ d*x)**6/16 + 5*a**6*sin(c + d*x)**5*cos(c + d*x)/(16*d) + 5*a**6*sin(c +
d*x)**3*cos(c + d*x)**3/(6*d) + 11*a**6*sin(c + d*x)*cos(c + d*x)**5/(16*d)
 - a**5*b*cos(c + d*x)**6/d + 15*a**4*b**2*x*sin(c + d*x)**6/16 + 45*a**4*b
**2*x*sin(c + d*x)**4*cos(c + d*x)**2/16 + 45*a**4*b**2*x*sin(c + d*x)**2*c
```

```
os(c + d*x)**4/16 + 15*a**4*b**2*x*cos(c + d*x)**6/16 + 15*a**4*b**2*sin(c
+ d*x)**5*cos(c + d*x)/(16*d) + 5*a**4*b**2*sin(c + d*x)**3*cos(c + d*x)**3
/(2*d) - 15*a**4*b**2*sin(c + d*x)*cos(c + d*x)**5/(16*d) + 5*a**3*b**3*sin
(c + d*x)**6/(3*d) + 5*a**3*b**3*sin(c + d*x)**4*cos(c + d*x)**2/d + 15*a**
2*b**4*x*sin(c + d*x)**6/16 + 45*a**2*b**4*x*sin(c + d*x)**4*cos(c + d*x)**
2/16 + 45*a**2*b**4*x*sin(c + d*x)**2*cos(c + d*x)**4/16 + 15*a**2*b**4*x*c
os(c + d*x)**6/16 + 15*a**2*b**4*sin(c + d*x)**5*cos(c + d*x)/(16*d) - 5*a*
*2*b**4*sin(c + d*x)**3*cos(c + d*x)**3/(2*d) - 15*a**2*b**4*sin(c + d*x)*c
os(c + d*x)**5/(16*d) + a*b**5*sin(c + d*x)**6/d + 5*b**6*x*sin(c + d*x)**6
/16 + 15*b**6*x*sin(c + d*x)**4*cos(c + d*x)**2/16 + 15*b**6*x*sin(c + d*x)
**2*cos(c + d*x)**4/16 + 5*b**6*x*cos(c + d*x)**6/16 - 11*b**6*sin(c + d*x)
**5*cos(c + d*x)/(16*d) - 5*b**6*sin(c + d*x)**3*cos(c + d*x)**3/(6*d) - 5*
b**6*sin(c + d*x)*cos(c + d*x)**5/(16*d), Ne(d, 0)), (x*(a*cos(c) + b*sin(c
))**6, True))
```

3.221 $\int (a\cos(c+dx)+b\sin(c+dx))^5\,dx$

Optimal. Leaf size=94

$$\frac{2\left(a^2+b^2\right)(b\cos(c+dx)-a\sin(c+dx))^3}{3d}-\frac{\left(a^2+b^2\right)^2(b\cos(c+dx)-a\sin(c+dx))}{d}-\frac{(b\cos(c+dx)-a\sin(c+dx))^5}{5d}$$

```
[Out] -(a^2+b^2)^2*(b*cos(d*x+c)-a*sin(d*x+c))/d+2/3*(a^2+b^2)*(b*cos(d*x+c)-a*si
n(d*x+c))^3/d-1/5*(b*cos(d*x+c)-a*sin(d*x+c))^5/d
```

Rubi [A] time = 0.05, antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {3072, 194}

$$\frac{2\left(a^2+b^2\right)(b\cos(c+dx)-a\sin(c+dx))^3}{3d}-\frac{\left(a^2+b^2\right)^2(b\cos(c+dx)-a\sin(c+dx))}{d}-\frac{(b\cos(c+dx)-a\sin(c+dx))^5}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Cos[c + d*x] + b*Sin[c + d*x])^5,x]
```

```
[Out] -(((a^2 + b^2)^2*(b*Cos[c + d*x] - a*Sin[c + d*x]))/d) + (2*(a^2 + b^2)*(b*
Cos[c + d*x] - a*Sin[c + d*x])^3)/(3*d) - (b*Cos[c + d*x] - a*Sin[c + d*x])
^5/(5*d)
```

Rule 194

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 3072

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x
_Symbol] :> -Dist[d^(-1), Subst[Int[(a^2 + b^2 - x^2)^((n - 1)/2), x], x, b
*Cos[c + d*x] - a*Sin[c + d*x]], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 +
b^2, 0] && IGtQ[(n - 1)/2, 0]
```

Rubi steps

$$\int (a\cos(c+dx)+b\sin(c+dx))^5\,dx = -\frac{\text{Subst}\left(\int \left(a^2+b^2-x^2\right)^2\,dx, x, b\cos(c+dx)-a\sin(c+dx)\right)}{d}$$

$$= -\frac{\text{Subst}\left(\int \left(a^4\left(1+\frac{2a^2b^2+b^4}{a^4}\right)-2a^2\left(1+\frac{b^2}{a^2}\right)x^2+x^4\right)\,dx, x, b\cos(c+dx) -$$

$$= -\frac{\left(a^2+b^2\right)^2 (b\cos(c+dx)-a\sin(c+dx))}{d} + \frac{2\left(a^2+b^2\right)(b\cos(c+dx) -}{3d}$$

Mathematica [A] time = 0.46, size = 156, normalized size = 1.66

$$\frac{150a\left(a^2+b^2\right)^2\sin(c+dx)-150b\left(a^2+b^2\right)^2\cos(c+dx)+25a\left(a^4-2a^2b^2-3b^4\right)\sin(3(c+dx))+3a\left(a^4-10a^2\right.}{240d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Cos[c + d*x] + b*Sin[c + d*x])^5,x]
```

```
[Out] (-150*b*(a^2 + b^2)^2*Cos[c + d*x] + 25*b*(-3*a^4 - 2*a^2*b^2 + b^4)*Cos[3*
(c + d*x)] - 3*b*(5*a^4 - 10*a^2*b^2 + b^4)*Cos[5*(c + d*x)] + 150*a*(a^2 +
 b^2)^2*Sin[c + d*x] + 25*a*(a^4 - 2*a^2*b^2 - 3*b^4)*Sin[3*(c + d*x)] + 3*
a*(a^4 - 10*a^2*b^2 + 5*b^4)*Sin[5*(c + d*x)])/(240*d)
```

fricas [A] time = 0.86, size = 155, normalized size = 1.65

$$-\frac{15\,b^5\cos(dx+c)+3\left(5\,a^4b-10\,a^2b^3+b^5\right)\cos(dx+c)^5+10\left(5\,a^2b^3-b^5\right)\cos(dx+c)^3-\left(8\,a^5+20\,a^3b^2+15\right.}{15\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cos(d*x+c)+b*sin(d*x+c))^5,x, algorithm="fricas")
```

```
[Out] -1/15*(15*b^5*cos(d*x + c) + 3*(5*a^4*b - 10*a^2*b^3 + b^5)*cos(d*x + c)^5
+ 10*(5*a^2*b^3 - b^5)*cos(d*x + c)^3 - (8*a^5 + 20*a^3*b^2 + 15*a*b^4 + 3*
(a^5 - 10*a^3*b^2 + 5*a*b^4)*cos(d*x + c)^4 + 2*(2*a^5 + 5*a^3*b^2 - 15*a*b
^4)*cos(d*x + c)^2)*sin(d*x + c))/d
```

giac [B] time = 0.29, size = 187, normalized size = 1.99

$$-\frac{\left(5\,a^4b-10\,a^2b^3+b^5\right)\cos(5\,dx+5\,c)}{80\,d}-\frac{5\left(3\,a^4b+2\,a^2b^3-b^5\right)\cos(3\,dx+3\,c)}{48\,d}-\frac{5\left(a^4b+2\,a^2b^3+b^5\right)\cos(dx+c}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cos(d*x+c)+b*sin(d*x+c))^5,x, algorithm="giac")
```

```
[Out] -1/80*(5*a^4*b - 10*a^2*b^3 + b^5)*cos(5*d*x + 5*c)/d - 5/48*(3*a^4*b + 2*a
^2*b^3 - b^5)*cos(3*d*x + 3*c)/d - 5/8*(a^4*b + 2*a^2*b^3 + b^5)*cos(d*x +
c)/d + 1/80*(a^5 - 10*a^3*b^2 + 5*a*b^4)*sin(5*d*x + 5*c)/d + 5/48*(a^5 - 2
*a^3*b^2 - 3*a*b^4)*sin(3*d*x + 3*c)/d + 5/8*(a^5 + 2*a^3*b^2 + a*b^4)*sin(
d*x + c)/d
```

maple [A] time = 0.27, size = 175, normalized size = 1.86

$$\frac{-\frac{b^5\left(\frac{8}{3}+\sin^4(dx+c)+\frac{4\left(\sin^2(dx+c)\right)}{3}\right)\cos(dx+c)}{5}+a\,b^4\left(\sin^5(dx+c)\right)+10a^2b^3\left(-\frac{\left(\sin^2(dx+c)\right)\left(\cos^3(dx+c)\right)}{5}-\frac{2\left(\cos^3(dx+c)\right)}{15}\right)+10a^3b}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*cos(d*x+c)+b*sin(d*x+c))^5,x)
```

```
[Out] 1/d*(-1/5*b^5*(8/3+sin(d*x+c)^4+4/3*sin(d*x+c)^2)*cos(d*x+c)+a*b^4*sin(d*x+
c)^5+10*a^2*b^3*(-1/5*sin(d*x+c)^2*cos(d*x+c)^3-2/15*cos(d*x+c)^3)+10*a^3*b
^2*(-1/5*sin(d*x+c)*cos(d*x+c)^4+1/15*(2+cos(d*x+c)^2)*sin(d*x+c))-a^4*b*co
s(d*x+c)^5+1/5*a^5*(8/3+cos(d*x+c)^4+4/3*cos(d*x+c)^2)*sin(d*x+c))
```

maxima [A] time = 0.75, size = 172, normalized size = 1.83

$$-\frac{a^4b\cos(dx+c)^5}{d}+\frac{ab^4\sin(dx+c)^5}{d}+\frac{\left(3\sin(dx+c)^5-10\sin(dx+c)^3+15\sin(dx+c)\right)a^5}{15\,d}-\frac{2\left(3\sin(dx+c\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cos(d*x+c)+b*sin(d*x+c))^5,x, algorithm="maxima")
```

```
[Out] -a^4*b*cos(d*x + c)^5/d + a*b^4*sin(d*x + c)^5/d + 1/15*(3*sin(d*x + c)^5 -
 10*sin(d*x + c)^3 + 15*sin(d*x + c))*a^5/d - 2/3*(3*sin(d*x + c)^5 - 5*sin
(d*x + c)^3)*a^3*b^2/d + 2/3*(3*cos(d*x + c)^5 - 5*cos(d*x + c)^3)*a^2*b^3/
d - 1/15*(3*cos(d*x + c)^5 - 10*cos(d*x + c)^3 + 15*cos(d*x + c))*b^5/d
```

mupad [B] time = 2.72, size = 248, normalized size = 2.64

$$\frac{2\left(\frac{3\sin(c+d\,x)\,a^5\cos(c+d\,x)^4}{2}+2\sin(c+d\,x)\,a^5\cos(c+d\,x)^2+4\sin(c+d\,x)\,a^5-\frac{15\,a^4\,b\cos(c+d\,x)^5}{2}-15\sin(c+d\,x\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*cos(c + d*x) + b*sin(c + d*x))^5,x)
```

```
[Out] (2*(4*a^5*sin(c + d*x) - (15*b^5*cos(c + d*x))/2 + 5*b^5*cos(c + d*x)^3 - (
3*b^5*cos(c + d*x)^5)/2 - (15*a^4*b*cos(c + d*x)^5)/2 + 2*a^5*cos(c + d*x)^
2*sin(c + d*x) + (3*a^5*cos(c + d*x)^4*sin(c + d*x))/2 + 10*a^3*b^2*sin(c +
 d*x) - 25*a^2*b^3*cos(c + d*x)^3 + 15*a^2*b^3*cos(c + d*x)^5 + (15*a*b^4*s
in(c + d*x))/2 + 5*a^3*b^2*cos(c + d*x)^2*sin(c + d*x) - 15*a^3*b^2*cos(c +
 d*x)^4*sin(c + d*x) - 15*a*b^4*cos(c + d*x)^2*sin(c + d*x) + (15*a*b^4*cos
(c + d*x)^4*sin(c + d*x))/2))/(15*d)
```

sympy [A] time = 2.22, size = 267, normalized size = 2.84

$$\begin{cases} \frac{8a^5 \sin^5{(c+dx)}}{15d} + \frac{4a^5 \sin^3{(c+dx)} \cos^2{(c+dx)}}{3d} + \frac{a^5 \sin{(c+dx)} \cos^4{(c+dx)}}{d} - \frac{a^4 b \cos^5{(c+dx)}}{d} + \frac{4a^3b^2 \sin^5{(c+dx)}}{3d} + \frac{10a^3b^2 \sin^3{(c+dx)} \cos^2{(c+} \\ x\left(a\cos{(c)} + b\sin{(c)}\right)^5 \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cos(d*x+c)+b*sin(d*x+c))**5,x)
```

```
[Out] Piecewise((8*a**5*sin(c + d*x)**5/(15*d) + 4*a**5*sin(c + d*x)**3*cos(c + d
*x)**2/(3*d) + a**5*sin(c + d*x)*cos(c + d*x)**4/d - a**4*b*cos(c + d*x)**5
/d + 4*a**3*b**2*sin(c + d*x)**5/(3*d) + 10*a**3*b**2*sin(c + d*x)**3*cos(c
 + d*x)**2/(3*d) - 10*a**2*b**3*sin(c + d*x)**2*cos(c + d*x)**3/(3*d) - 4*a
**2*b**3*cos(c + d*x)**5/(3*d) + a*b**4*sin(c + d*x)**5/d - b**5*sin(c + d*
x)**4*cos(c + d*x)/d - 4*b**5*sin(c + d*x)**2*cos(c + d*x)**3/(3*d) - 8*b**
5*cos(c + d*x)**5/(15*d), Ne(d, 0)), (x*(a*cos(c) + b*sin(c))**5, True))
```

3.222 $\int (a\cos(c+dx)+b\sin(c+dx))^4\,dx$

Optimal. Leaf size=108

$$-\frac{3\left(a^2+b^2\right)(b\cos(c+dx)-a\sin(c+dx))(a\cos(c+dx)+b\sin(c+dx))}{8d}+\frac{3}{8}x\left(a^2+b^2\right)^2-\frac{(b\cos(c+dx)-a\sin}{}$$

```
[Out] 3/8*(a^2+b^2)^2*x-3/8*(a^2+b^2)*(b*cos(d*x+c)-a*sin(d*x+c))*(a*cos(d*x+c)+b
*sin(d*x+c))/d-1/4*(b*cos(d*x+c)-a*sin(d*x+c))*(a*cos(d*x+c)+b*sin(d*x+c))^
3/d
```

Rubi [A] time = 0.04, antiderivative size = 108, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {3073, 8}

$$-\frac{3\left(a^2+b^2\right)(b\cos(c+dx)-a\sin(c+dx))(a\cos(c+dx)+b\sin(c+dx))}{8d}+\frac{3}{8}x\left(a^2+b^2\right)^2-\frac{(b\cos(c+dx)-a\sin}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Cos[c + d*x] + b*Sin[c + d*x])^4,x]
```

```
[Out] (3*(a^2 + b^2)^2*x)/8 - (3*(a^2 + b^2)*(b*Cos[c + d*x] - a*Sin[c + d*x])*(a
*Cos[c + d*x] + b*Sin[c + d*x]))/(8*d) - ((b*Cos[c + d*x] - a*Sin[c + d*x])
*(a*Cos[c + d*x] + b*Sin[c + d*x])^3)/(4*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3073

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x
_Symbol] :> -Simp[((b*Cos[c + d*x] - a*Sin[c + d*x])*(a*Cos[c + d*x] + b*Si
n[c + d*x])^(n - 1))/(d*n), x] + Dist[((n - 1)*(a^2 + b^2))/n, Int[(a*Cos[c
 + d*x] + b*Sin[c + d*x])^(n - 2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 + b^2, 0] &&  !IntegerQ[(n - 1)/2] && GtQ[n, 1]
```

Rubi steps

$$\int (a\cos(c+dx)+b\sin(c+dx))^4\,dx = -\frac{(b\cos(c+dx)-a\sin(c+dx))(a\cos(c+dx)+b\sin(c+dx))^3}{4d}+\frac{1}{4}\Big(3\Big(a$$

$$= -\frac{3\left(a^2+b^2\right)(b\cos(c+dx)-a\sin(c+dx))(a\cos(c+dx)+b\sin(c+dx))}{8d}$$

$$= \frac{3}{8}\left(a^2+b^2\right)^2 x - \frac{3\left(a^2+b^2\right)(b\cos(c+dx)-a\sin(c+dx))(a\cos(c+dx)}{8d}$$

Mathematica [**A**] time = 0.30, size = 107, normalized size = 0.99

$$\frac{8\left(a^4-b^4\right)\sin(2(c+dx))+12\left(a^2+b^2\right)^2(c+dx)-16ab\left(a^2+b^2\right)\cos(2(c+dx))-4ab\left(a^2-b^2\right)\cos(4(c+dx))}{32d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Cos[c + d*x] + b*Sin[c + d*x])^4,x]
```

```
[Out] (12*(a^2 + b^2)^2*(c + d*x) - 16*a*b*(a^2 + b^2)*Cos[2*(c + d*x)] - 4*a*b*(
a^2 - b^2)*Cos[4*(c + d*x)] + 8*(a^4 - b^4)*Sin[2*(c + d*x)] + (a^4 - 6*a^2
*b^2 + b^4)*Sin[4*(c + d*x)])/(32*d)
```

fricas [**A**] time = 1.11, size = 121, normalized size = 1.12

$$-\frac{16\,ab^3\cos(dx+c)^2+8\left(a^3b-ab^3\right)\cos(dx+c)^4-3\left(a^4+2\,a^2b^2+b^4\right)dx-\left(2\left(a^4-6\,a^2b^2+b^4\right)\cos(dx+c)^3+\right.}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cos(d*x+c)+b*sin(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] -1/8*(16*a*b^3*cos(d*x + c)^2 + 8*(a^3*b - a*b^3)*cos(d*x + c)^4 - 3*(a^4 +
 2*a^2*b^2 + b^4)*d*x - (2*(a^4 - 6*a^2*b^2 + b^4)*cos(d*x + c)^3 + (3*a^4
+ 6*a^2*b^2 - 5*b^4)*cos(d*x + c))*sin(d*x + c))/d
```

giac [**A**] time = 0.19, size = 122, normalized size = 1.13

$$\frac{3}{8}\left(a^4+2\,a^2b^2+b^4\right)x-\frac{\left(a^3b-ab^3\right)\cos(4\,dx+4\,c)}{8\,d}-\frac{\left(a^3b+ab^3\right)\cos(2\,dx+2\,c)}{2\,d}+\frac{\left(a^4-6\,a^2b^2+b^4\right)\sin(4\,dx+4}{32\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cos(d*x+c)+b*sin(d*x+c))^4,x, algorithm="giac")
```

```
[Out] 3/8*(a^4 + 2*a^2*b^2 + b^4)*x - 1/8*(a^3*b - a*b^3)*cos(4*d*x + 4*c)/d - 1/
2*(a^3*b + a*b^3)*cos(2*d*x + 2*c)/d + 1/32*(a^4 - 6*a^2*b^2 + b^4)*sin(4*d
*x + 4*c)/d + 1/4*(a^4 - b^4)*sin(2*d*x + 2*c)/d
```

maple [A] time = 0.25, size = 153, normalized size = 1.42

$$\frac{b^4\left(-\frac{\left(\sin^3(dx+c)+\frac{3\sin(dx+c)}{2}\right)\cos(dx+c)}{4}+\frac{3dx}{8}+\frac{3c}{8}\right)+a\,b^3\left(\sin^4\left(dx+c\right)\right)+6a^2b^2\left(-\frac{\sin(dx+c)\left(\cos^3(dx+c)\right)}{4}+\frac{\sin(dx+c)\cos(dx+}{8}\right.}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*cos(d*x+c)+b*sin(d*x+c))^4,x)
```

```
[Out] 1/d*(b^4*(-1/4*(sin(d*x+c)^3+3/2*sin(d*x+c))*cos(d*x+c)+3/8*d*x+3/8*c)+a*b^
3*sin(d*x+c)^4+6*a^2*b^2*(-1/4*sin(d*x+c)*cos(d*x+c)^3+1/8*sin(d*x+c)*cos(d
*x+c)+1/8*d*x+1/8*c)-cos(d*x+c)^4*a^3*b+a^4*(1/4*(cos(d*x+c)^3+3/2*cos(d*x+
c))*sin(d*x+c)+3/8*d*x+3/8*c))
```

maxima [A] time = 0.32, size = 136, normalized size = 1.26

$$-\frac{a^3b\cos\left(dx+c\right)^4}{d}+\frac{ab^3\sin\left(dx+c\right)^4}{d}+\frac{\left(12\,dx+12\,c+\sin\left(4\,dx+4\,c\right)+8\,\sin\left(2\,dx+2\,c\right)\right)a^4}{32\,d}+\frac{3\left(4\,dx+4\,c-\mathrm{si}\right.}{1\mathrm{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cos(d*x+c)+b*sin(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] -a^3*b*cos(d*x + c)^4/d + a*b^3*sin(d*x + c)^4/d + 1/32*(12*d*x + 12*c + si
n(4*d*x + 4*c) + 8*sin(2*d*x + 2*c))*a^4/d + 3/16*(4*d*x + 4*c - sin(4*d*x
+ 4*c))*a^2*b^2/d + 1/32*(12*d*x + 12*c + sin(4*d*x + 4*c) - 8*sin(2*d*x +
2*c))*b^4/d
```

mupad [B] time = 3.46, size = 320, normalized size = 2.96

$$\frac{3\,\mathrm{atan}\left(\frac{3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\left(a^2+b^2\right)^2}{4\left(\frac{3\,a^4}{4}+\frac{3\,a^2\,b^2}{2}+\frac{3\,b^4}{4}\right)}\right)\left(a^2+b^2\right)^2}{4\,d}+\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7\left(-\frac{5\,a^4}{4}+\frac{3\,a^2\,b^2}{2}+\frac{3\,b^4}{4}\right)-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3\left(\frac{3\,a^4}{4}-\frac{21\,a^2\,b^2}{2}+\frac{11}{}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*cos(c + d*x) + b*sin(c + d*x))^4,x)
```

```
[Out] (3*atan((3*tan(c/2 + (d*x)/2)*(a^2 + b^2)^2)/(4*((3*a^4)/4 + (3*b^4)/4 + (3
*a^2*b^2)/2)))*(a^2 + b^2)^2)/(4*d) + (tan(c/2 + (d*x)/2)^7*((3*b^4)/4 - (5
```

```
*a^4)/4 + (3*a^2*b^2)/2) - tan(c/2 + (d*x)/2)^3*((3*a^4)/4 + (11*b^4)/4 - (
21*a^2*b^2)/2) + tan(c/2 + (d*x)/2)^5*((3*a^4)/4 + (11*b^4)/4 - (21*a^2*b^2
)/2) - tan(c/2 + (d*x)/2)*((3*b^4)/4 - (5*a^4)/4 + (3*a^2*b^2)/2) + 8*a^3*b
*tan(c/2 + (d*x)/2)^2 + 16*a*b^3*tan(c/2 + (d*x)/2)^4 + 8*a^3*b*tan(c/2 + (
d*x)/2)^6)/(d*(4*tan(c/2 + (d*x)/2)^2 + 6*tan(c/2 + (d*x)/2)^4 + 4*tan(c/2
+ (d*x)/2)^6 + tan(c/2 + (d*x)/2)^8 + 1)) - (3*(atan(tan(c/2 + (d*x)/2)) -
(d*x)/2)*(a^2 + b^2)^2)/(4*d)
```

sympy [A] time = 1.27, size = 381, normalized size = 3.53

$$\begin{cases} \frac{3a^4x\sin^4(c+dx)}{8} + \frac{3a^4x\sin^2(c+dx)\cos^2(c+dx)}{4} + \frac{3a^4x\cos^4(c+dx)}{8} + \frac{3a^4\sin^3(c+dx)\cos(c+dx)}{8d} + \frac{5a^4\sin(c+dx)\cos^3(c+dx)}{8d} - \frac{a^3b\cos^4(c+a}{d} \\ x\left(a\cos(c) + b\sin(c)\right)^4 \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cos(d*x+c)+b*sin(d*x+c))**4,x)
```

```
[Out] Piecewise((3*a**4*x*sin(c + d*x)**4/8 + 3*a**4*x*sin(c + d*x)**2*cos(c + d*
x)**2/4 + 3*a**4*x*cos(c + d*x)**4/8 + 3*a**4*sin(c + d*x)**3*cos(c + d*x)/
(8*d) + 5*a**4*sin(c + d*x)*cos(c + d*x)**3/(8*d) - a**3*b*cos(c + d*x)**4/
d + 3*a**2*b**2*x*sin(c + d*x)**4/4 + 3*a**2*b**2*x*sin(c + d*x)**2*cos(c +
 d*x)**2/2 + 3*a**2*b**2*x*cos(c + d*x)**4/4 + 3*a**2*b**2*sin(c + d*x)**3*
cos(c + d*x)/(4*d) - 3*a**2*b**2*sin(c + d*x)*cos(c + d*x)**3/(4*d) + a*b**
3*sin(c + d*x)**4/d + 3*b**4*x*sin(c + d*x)**4/8 + 3*b**4*x*sin(c + d*x)**2
*cos(c + d*x)**2/4 + 3*b**4*x*cos(c + d*x)**4/8 - 5*b**4*sin(c + d*x)**3*co
s(c + d*x)/(8*d) - 3*b**4*sin(c + d*x)*cos(c + d*x)**3/(8*d), Ne(d, 0)), (x
*(a*cos(c) + b*sin(c))**4, True))
```

3.223 $\int (a\cos(c+dx)+b\sin(c+dx))^3\,dx$

Optimal. Leaf size=58

$$\frac{(b\cos(c+dx)-a\sin(c+dx))^3}{3d}-\frac{\left(a^2+b^2\right)(b\cos(c+dx)-a\sin(c+dx))}{d}$$

```
[Out] -(a^2+b^2)*(b*cos(d*x+c)-a*sin(d*x+c))/d+1/3*(b*cos(d*x+c)-a*sin(d*x+c))^3/
d
```

Rubi [A] time = 0.02, antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {3072}

$$\frac{(b\cos(c+dx)-a\sin(c+dx))^3}{3d}-\frac{\left(a^2+b^2\right)(b\cos(c+dx)-a\sin(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Cos[c + d*x] + b*Sin[c + d*x])^3,x]
```

```
[Out] -(((a^2 + b^2)*(b*Cos[c + d*x] - a*Sin[c + d*x]))/d) + (b*Cos[c + d*x] - a*
Sin[c + d*x])^3/(3*d)
```

Rule 3072

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x
_Symbol] :> -Dist[d^(-1), Subst[Int[(a^2 + b^2 - x^2)^((n - 1)/2), x], x, b
*Cos[c + d*x] - a*Sin[c + d*x]], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 +
b^2, 0] && IGtQ[(n - 1)/2, 0]
```

Rubi steps

$$\begin{aligned}\int (a\cos(c+dx)+b\sin(c+dx))^3\,dx &= -\frac{\text{Subst}\left(\int \left(a^2+b^2-x^2\right)\,dx, x, b\cos(c+dx)-a\sin(c+dx)\right)}{d}\\ &= -\frac{\left(a^2+b^2\right)(b\cos(c+dx)-a\sin(c+dx))}{d}+\frac{(b\cos(c+dx)-a\sin(c+d\end{aligned}$$

Mathematica [A] time = 0.32, size = 81, normalized size = 1.40

$$\frac{\left(b^3-3a^2b\right)\cos(3(c+dx))-9b\left(a^2+b^2\right)\cos(c+dx)+2a\sin(c+dx)\left(\left(a^2-3b^2\right)\cos(2(c+dx))+5a^2+3b^2\right)}{12d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Cos[c + d*x] + b*Sin[c + d*x])^3,x]
```

```
[Out] (-9*b*(a^2 + b^2)*Cos[c + d*x] + (-3*a^2*b + b^3)*Cos[3*(c + d*x)] + 2*a*(5
*a^2 + 3*b^2 + (a^2 - 3*b^2)*Cos[2*(c + d*x)])*Sin[c + d*x])/(12*d)
```

fricas [A] time = 1.32, size = 77, normalized size = 1.33

$$-\frac{3\,b^3\cos\left(dx+c\right)+\left(3\,a^2b-b^3\right)\cos\left(dx+c\right)^3-\left(2\,a^3+3\,ab^2+\left(a^3-3\,ab^2\right)\cos\left(dx+c\right)^2\right)\sin\left(dx+c\right)}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cos(d*x+c)+b*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] -1/3*(3*b^3*cos(d*x + c) + (3*a^2*b - b^3)*cos(d*x + c)^3 - (2*a^3 + 3*a*b^
2 + (a^3 - 3*a*b^2)*cos(d*x + c)^2)*sin(d*x + c))/d
```

giac [A] time = 0.15, size = 91, normalized size = 1.57

$$-\frac{\left(3\,a^2b-b^3\right)\cos\left(3\,dx+3\,c\right)}{12\,d}-\frac{3\left(a^2b+b^3\right)\cos\left(dx+c\right)}{4\,d}+\frac{\left(a^3-3\,ab^2\right)\sin\left(3\,dx+3\,c\right)}{12\,d}+\frac{3\left(a^3+ab^2\right)\sin\left(dx+c\right)}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cos(d*x+c)+b*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -1/12*(3*a^2*b - b^3)*cos(3*d*x + 3*c)/d - 3/4*(a^2*b + b^3)*cos(d*x + c)/d
 + 1/12*(a^3 - 3*a*b^2)*sin(3*d*x + 3*c)/d + 3/4*(a^3 + a*b^2)*sin(d*x + c)
/d
```

maple [A] time = 0.24, size = 75, normalized size = 1.29

$$\frac{-\frac{b^3\left(2+\sin^2(dx+c)\right)\cos(dx+c)}{3}+a\,b^2\left(\sin^3\left(dx+c\right)\right)-a^2b\left(\cos^3\left(dx+c\right)\right)+\frac{a^3\left(2+\cos^2(dx+c)\right)\sin(dx+c)}{3}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*cos(d*x+c)+b*sin(d*x+c))^3,x)
```

```
[Out] 1/d*(-1/3*b^3*(2+sin(d*x+c)^2)*cos(d*x+c)+a*b^2*sin(d*x+c)^3-a^2*b*cos(d*x+
c)^3+1/3*a^3*(2+cos(d*x+c)^2)*sin(d*x+c))
```

maxima [A] time = 0.31, size = 84, normalized size = 1.45

$$-\frac{a^2b\cos\left(dx+c\right)^3}{d}+\frac{ab^2\sin\left(dx+c\right)^3}{d}-\frac{\left(\sin\left(dx+c\right)^3-3\,\sin\left(dx+c\right)\right)a^3}{3\,d}+\frac{\left(\cos\left(dx+c\right)^3-3\,\cos\left(dx+c\right)\right)b^3}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cos(d*x+c)+b*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -a^2*b*cos(d*x + c)^3/d + a*b^2*sin(d*x + c)^3/d - 1/3*(sin(d*x + c)^3 - 3*
sin(d*x + c))*a^3/d + 1/3*(cos(d*x + c)^3 - 3*cos(d*x + c))*b^3/d
```

mupad [B] time = 2.48, size = 104, normalized size = 1.79

$$\frac{\frac{\sin(c+d\,x)\,a^3\,{\cos(c+d\,x)}^2}{3}+\frac{2\,\sin(c+d\,x)\,a^3}{3}-a^2\,b\,{\cos\left(c+d\,x\right)}^3-\sin\left(c+d\,x\right)\,a\,b^2\,{\cos\left(c+d\,x\right)}^2+\sin\left(c+d\,x\right)\,a\,b^2+\frac{b^3}{}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*cos(c + d*x) + b*sin(c + d*x))^3,x)
```

```
[Out] ((2*a^3*sin(c + d*x))/3 - b^3*cos(c + d*x) + (b^3*cos(c + d*x)^3)/3 - a^2*b
*cos(c + d*x)^3 + (a^3*cos(c + d*x)^2*sin(c + d*x))/3 + a*b^2*sin(c + d*x)
- a*b^2*cos(c + d*x)^2*sin(c + d*x))/d
```

sympy [A] time = 0.52, size = 117, normalized size = 2.02

$$\begin{cases} \frac{2a^3\sin^3(c+dx)}{3d}+\frac{a^3\sin(c+dx)\cos^2(c+dx)}{d}-\frac{a^2b\cos^3(c+dx)}{d}+\frac{ab^2\sin^3(c+dx)}{d}-\frac{b^3\sin^2(c+dx)\cos(c+dx)}{d}-\frac{2b^3\cos^3(c+dx)}{3d} & \text{for } d \neq \\ x\left(a\cos(c)+b\sin(c)\right)^3 & \text{otherw} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cos(d*x+c)+b*sin(d*x+c))**3,x)
```

```
[Out] Piecewise((2*a**3*sin(c + d*x)**3/(3*d) + a**3*sin(c + d*x)*cos(c + d*x)**2
/d - a**2*b*cos(c + d*x)**3/d + a*b**2*sin(c + d*x)**3/d - b**3*sin(c + d*x
)**2*cos(c + d*x)/d - 2*b**3*cos(c + d*x)**3/(3*d), Ne(d, 0)), (x*(a*cos(c)
 + b*sin(c))**3, True))
```

3.224 $\int (a\cos(c+dx)+b\sin(c+dx))^2\,dx$

Optimal. Leaf size=55

$$\frac{1}{2}x\left(a^2+b^2\right)-\frac{(b\cos(c+dx)-a\sin(c+dx))(a\cos(c+dx)+b\sin(c+dx))}{2d}$$

```
[Out] 1/2*(a^2+b^2)*x-1/2*(b*cos(d*x+c)-a*sin(d*x+c))*(a*cos(d*x+c)+b*sin(d*x+c))
/d
```

Rubi [A] time = 0.02, antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {3073, 8}

$$\frac{1}{2}x\left(a^2+b^2\right)-\frac{(b\cos(c+dx)-a\sin(c+dx))(a\cos(c+dx)+b\sin(c+dx))}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Cos[c + d*x] + b*Sin[c + d*x])^2,x]
```

```
[Out] ((a^2 + b^2)*x)/2 - ((b*Cos[c + d*x] - a*Sin[c + d*x])*(a*Cos[c + d*x] + b*
Sin[c + d*x]))/(2*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3073

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x
_Symbol] :> -Simp[((b*Cos[c + d*x] - a*Sin[c + d*x])*(a*Cos[c + d*x] + b*Si
n[c + d*x])^(n - 1))/(d*n), x] + Dist[((n - 1)*(a^2 + b^2))/n, Int[(a*Cos[c
 + d*x] + b*Sin[c + d*x])^(n - 2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 + b^2, 0] &&  !IntegerQ[(n - 1)/2] && GtQ[n, 1]
```

Rubi steps

$$\begin{aligned}\int (a\cos(c+dx)+b\sin(c+dx))^2\,dx &= -\frac{(b\cos(c+dx)-a\sin(c+dx))(a\cos(c+dx)+b\sin(c+dx))}{2d}+\frac{1}{2}\left(a^2+\right.\\ &= \frac{1}{2}\left(a^2+b^2\right)x-\frac{(b\cos(c+dx)-a\sin(c+dx))(a\cos(c+dx)+b\sin(c+d}{2d}\end{aligned}$$

Mathematica [A] time = 0.10, size = 52, normalized size = 0.95

$$\frac{2\left(a^2+b^2\right)(c+dx)+\left(a^2-b^2\right)\sin(2(c+dx))-2ab\cos(2(c+dx))}{4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Cos[c + d*x] + b*Sin[c + d*x])^2,x]
```

```
[Out] (2*(a^2 + b^2)*(c + d*x) - 2*a*b*Cos[2*(c + d*x)] + (a^2 - b^2)*Sin[2*(c +
d*x)])/(4*d)
```

fricas [A] time = 1.10, size = 52, normalized size = 0.95

$$-\frac{2\,ab\cos\left(dx+c\right)^2-\left(a^2+b^2\right)dx-\left(a^2-b^2\right)\cos\left(dx+c\right)\sin\left(dx+c\right)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cos(d*x+c)+b*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -1/2*(2*a*b*cos(d*x + c)^2 - (a^2 + b^2)*d*x - (a^2 - b^2)*cos(d*x + c)*sin
(d*x + c))/d
```

giac [A] time = 0.15, size = 50, normalized size = 0.91

$$\frac{1}{2}\left(a^2+b^2\right)x-\frac{ab\cos\left(2\,dx+2\,c\right)}{2\,d}+\frac{\left(a^2-b^2\right)\sin\left(2\,dx+2\,c\right)}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cos(d*x+c)+b*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/2*(a^2 + b^2)*x - 1/2*a*b*cos(2*d*x + 2*c)/d + 1/4*(a^2 - b^2)*sin(2*d*x
+ 2*c)/d
```

maple [A] time = 0.24, size = 70, normalized size = 1.27

$$\frac{b^2\left(-\frac{\sin(dx+c)\cos(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)-\left(\cos^2\left(dx+c\right)\right)ab+a^2\left(\frac{\sin(dx+c)\cos(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*cos(d*x+c)+b*sin(d*x+c))^2,x)
```

```
[Out] 1/d*(b^2*(-1/2*sin(d*x+c)*cos(d*x+c)+1/2*d*x+1/2*c)-cos(d*x+c)^2*a*b+a^2*(1
/2*sin(d*x+c)*cos(d*x+c)+1/2*d*x+1/2*c))
```

maxima [A] time = 0.47, size = 68, normalized size = 1.24

$$-\frac{ab\cos(dx+c)^2}{d}+\frac{(2\,dx+2\,c+\sin(2\,dx+2\,c))a^2}{4\,d}+\frac{(2\,dx+2\,c-\sin(2\,dx+2\,c))b^2}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cos(d*x+c)+b*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -a*b*cos(d*x + c)^2/d + 1/4*(2*d*x + 2*c + sin(2*d*x + 2*c))*a^2/d + 1/4*(2
*d*x + 2*c - sin(2*d*x + 2*c))*b^2/d
```

mupad [B] time = 2.42, size = 63, normalized size = 1.15

$$\frac{a^2\,x}{2}+\frac{b^2\,x}{2}+\frac{a^2\,\sin(2\,c+2\,d\,x)}{4\,d}-\frac{b^2\,\sin(2\,c+2\,d\,x)}{4\,d}-\frac{a\,b\,\cos(2\,c+2\,d\,x)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*cos(c + d*x) + b*sin(c + d*x))^2,x)
```

```
[Out] (a^2*x)/2 + (b^2*x)/2 + (a^2*sin(2*c + 2*d*x))/(4*d) - (b^2*sin(2*c + 2*d*x
))/(4*d) - (a*b*cos(2*c + 2*d*x))/(2*d)
```

sympy [A] time = 0.28, size = 128, normalized size = 2.33

$$\begin{cases}\frac{a^2x\sin^2(c+dx)}{2}+\frac{a^2x\cos^2(c+dx)}{2}+\frac{a^2\sin(c+dx)\cos(c+dx)}{2d}-\frac{ab\cos^2(c+dx)}{d}+\frac{b^2x\sin^2(c+dx)}{2}+\frac{b^2x\cos^2(c+dx)}{2}-\frac{b^2\sin(c+dx)\cos(c+dx)}{2d} \\ x\,(a\cos(c)+b\sin(c))^2\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cos(d*x+c)+b*sin(d*x+c))**2,x)
```

```
[Out] Piecewise((a**2*x*sin(c + d*x)**2/2 + a**2*x*cos(c + d*x)**2/2 + a**2*sin(c
 + d*x)*cos(c + d*x)/(2*d) - a*b*cos(c + d*x)**2/d + b**2*x*sin(c + d*x)**2
/2 + b**2*x*cos(c + d*x)**2/2 - b**2*sin(c + d*x)*cos(c + d*x)/(2*d), Ne(d,
 0)), (x*(a*cos(c) + b*sin(c))**2, True))
```

3.225 $\int (a\cos(c+dx)+b\sin(c+dx))\,dx$

Optimal. Leaf size=24

$$\frac{a\sin(c+dx)}{d}-\frac{b\cos(c+dx)}{d}$$

```
[Out] -b*cos(d*x+c)/d+a*sin(d*x+c)/d
```

Rubi [A] time = 0.01, antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {2637, 2638}

$$\frac{a\sin(c+dx)}{d}-\frac{b\cos(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[a*Cos[c + d*x] + b*Sin[c + d*x],x]
```

```
[Out] -((b*Cos[c + d*x])/d) + (a*Sin[c + d*x])/d
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int (a\cos(c+dx)+b\sin(c+dx))\,dx &= a\int \cos(c+dx)\,dx+b\int \sin(c+dx)\,dx\\ &= -\frac{b\cos(c+dx)}{d}+\frac{a\sin(c+dx)}{d}\end{aligned}$$

Mathematica [A] time = 0.01, size = 46, normalized size = 1.92

$$\frac{a\sin(c)\cos(dx)}{d}+\frac{a\cos(c)\sin(dx)}{d}+\frac{b\sin(c)\sin(dx)}{d}-\frac{b\cos(c)\cos(dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[a*Cos[c + d*x] + b*Sin[c + d*x],x]
```

```
[Out] -((b*Cos[c]*Cos[d*x])/d) + (a*Cos[d*x]*Sin[c])/d + (a*Cos[c]*Sin[d*x])/d +
(b*Sin[c]*Sin[d*x])/d
```

fricas [A] time = 1.27, size = 23, normalized size = 0.96

$$-\frac{b\cos(dx+c)-a\sin(dx+c)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a*cos(d*x+c)+b*sin(d*x+c),x, algorithm="fricas")
```

```
[Out] -(b*cos(d*x + c) - a*sin(d*x + c))/d
```

giac [A] time = 0.14, size = 24, normalized size = 1.00

$$-\frac{b\cos(dx+c)}{d}+\frac{a\sin(dx+c)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a*cos(d*x+c)+b*sin(d*x+c),x, algorithm="giac")
```

```
[Out] -b*cos(d*x + c)/d + a*sin(d*x + c)/d
```

maple [A] time = 0.04, size = 25, normalized size = 1.04

$$-\frac{b\cos(dx+c)}{d}+\frac{a\sin(dx+c)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(a*cos(d*x+c)+b*sin(d*x+c),x)
```

```
[Out] -b*cos(d*x+c)/d+a*sin(d*x+c)/d
```

maxima [A] time = 0.40, size = 24, normalized size = 1.00

$$-\frac{b\cos(dx+c)}{d}+\frac{a\sin(dx+c)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a*cos(d*x+c)+b*sin(d*x+c),x, algorithm="maxima")
```

```
[Out] -b*cos(d*x + c)/d + a*sin(d*x + c)/d
```

mupad [B] time = 2.32, size = 38, normalized size = 1.58

$$-\frac{2\,\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(b\,\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)-a\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(a*cos(c + d*x) + b*sin(c + d*x),x)
```

```
[Out] -(2*cos(c/2 + (d*x)/2)*(b*cos(c/2 + (d*x)/2) - a*sin(c/2 + (d*x)/2)))/d
```

sympy [A] time = 0.15, size = 31, normalized size = 1.29

$$a\left(\begin{cases}\frac{\sin(c+dx)}{d} & \text{for}\ d\neq 0\\ x\cos(c) & \text{otherwise}\end{cases}\right)+b\left(\begin{cases}-\frac{\cos(c+dx)}{d} & \text{for}\ d\neq 0\\ x\sin(c) & \text{otherwise}\end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a*cos(d*x+c)+b*sin(d*x+c),x)
```

```
[Out] a*Piecewise((sin(c + d*x)/d, Ne(d, 0)), (x*cos(c), True)) + b*Piecewise((-c
os(c + d*x)/d, Ne(d, 0)), (x*sin(c), True))
```

3.226 $\int \frac{1}{a\cos(c+dx)+b\sin(c+dx)}\,dx$

Optimal. Leaf size=47

$$-\frac{\tanh^{-1}\left(\frac{b\cos(c+dx)-a\sin(c+dx)}{\sqrt{a^2+b^2}}\right)}{d\sqrt{a^2+b^2}}$$

```
[Out] -arctanh((b*cos(d*x+c)-a*sin(d*x+c))/(a^2+b^2)^(1/2))/d/(a^2+b^2)^(1/2)
```

Rubi [A] time = 0.03, antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {3074, 206}

$$-\frac{\tanh^{-1}\left(\frac{b\cos(c+dx)-a\sin(c+dx)}{\sqrt{a^2+b^2}}\right)}{d\sqrt{a^2+b^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Cos[c + d*x] + b*Sin[c + d*x])^(-1),x]
```

```
[Out] -(ArcTanh[(b*Cos[c + d*x] - a*Sin[c + d*x])/Sqrt[a^2 + b^2]]/(Sqrt[a^2 + b^
2]*d))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3074

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x
_Symbol] :> -Dist[d^(-1), Subst[Int[1/(a^2 + b^2 - x^2), x], x, b*Cos[c + d
*x] - a*Sin[c + d*x]], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{a\cos(c+dx)+b\sin(c+dx)}\,dx &= -\frac{\text{Subst}\left(\int \frac{1}{a^2+b^2-x^2}\,dx, x, b\cos(c+dx)-a\sin(c+dx)\right)}{d}\\ &= -\frac{\tanh^{-1}\left(\frac{b\cos(c+dx)-a\sin(c+dx)}{\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}\,d}\end{aligned}$$

Mathematica [A] time = 0.06, size = 45, normalized size = 0.96

$$\frac{2\tanh^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)-b}{\sqrt{a^2+b^2}}\right)}{d\sqrt{a^2+b^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Cos[c + d*x] + b*Sin[c + d*x])^(-1),x]
```

```
[Out] (2*ArcTanh[(-b + a*Tan[(c + d*x)/2])/Sqrt[a^2 + b^2]])/(Sqrt[a^2 + b^2]*d)
```

fricas [B] time = 1.74, size = 131, normalized size = 2.79

$$\frac{\log\left(-\frac{2\,ab\cos(dx+c)\sin(dx+c)+\left(a^2-b^2\right)\cos(dx+c)^2-2\,a^2-b^2+2\,\sqrt{a^2+b^2}\,(b\cos(dx+c)-a\sin(dx+c))}{2\,ab\cos(dx+c)\sin(dx+c)+\left(a^2-b^2\right)\cos(dx+c)^2+b^2}\right)}{2\,\sqrt{a^2+b^2}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cos(d*x+c)+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/2*log(-(2*a*b*cos(d*x + c)*sin(d*x + c) + (a^2 - b^2)*cos(d*x + c)^2 - 2*
a^2 - b^2 + 2*sqrt(a^2 + b^2)*(b*cos(d*x + c) - a*sin(d*x + c)))/(2*a*b*cos
(d*x + c)*sin(d*x + c) + (a^2 - b^2)*cos(d*x + c)^2 + b^2))/(sqrt(a^2 + b^2
)*d)
```

giac [A] time = 0.22, size = 74, normalized size = 1.57

$$-\frac{\log\left(\frac{\left|2\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-2\,b-2\,\sqrt{a^2+b^2}\right|}{\left|2\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-2\,b+2\,\sqrt{a^2+b^2}\right|}\right)}{\sqrt{a^2+b^2}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cos(d*x+c)+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -log(abs(2*a*tan(1/2*d*x + 1/2*c) - 2*b - 2*sqrt(a^2 + b^2))/abs(2*a*tan(1/
2*d*x + 1/2*c) - 2*b + 2*sqrt(a^2 + b^2)))/(sqrt(a^2 + b^2)*d)
```

maple [A] time = 0.40, size = 43, normalized size = 0.91

$$\frac{2\,\mathrm{arctanh}\left(\frac{2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)a-2b}{2\sqrt{a^2+b^2}}\right)}{d\sqrt{a^2+b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*cos(d*x+c)+b*sin(d*x+c)),x)
```

```
[Out] 2/d/(a^2+b^2)^(1/2)*arctanh(1/2*(2*tan(1/2*d*x+1/2*c)*a-2*b)/(a^2+b^2)^(1/2
))
```

maxima [A] time = 0.68, size = 80, normalized size = 1.70

$$-\frac{\log\left(\frac{b-\frac{a\sin(dx+c)}{\cos(dx+c)+1}+\sqrt{a^2+b^2}}{b-\frac{a\sin(dx+c)}{\cos(dx+c)+1}-\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cos(d*x+c)+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -log((b - a*sin(d*x + c)/(cos(d*x + c) + 1) + sqrt(a^2 + b^2))/(b - a*sin(d
*x + c)/(cos(d*x + c) + 1) - sqrt(a^2 + b^2)))/(sqrt(a^2 + b^2)*d)
```

mupad [B] time = 2.80, size = 39, normalized size = 0.83

$$-\frac{2\,\mathrm{atanh}\left(\frac{b-a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{d\,\sqrt{a^2+b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*cos(c + d*x) + b*sin(c + d*x)),x)
```

```
[Out] -(2*atanh((b - a*tan(c/2 + (d*x)/2))/(a^2 + b^2)^(1/2)))/(d*(a^2 + b^2)^(1/
2))
```

sympy [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: AttributeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cos(d*x+c)+b*sin(d*x+c)),x)
```

```
[Out] Exception raised: AttributeError
```

3.227 $\int \frac{1}{(a\cos(c+dx)+b\sin(c+dx))^2}\,dx$

Optimal. Leaf size=32

$$\frac{\sin(c+dx)}{ad(a\cos(c+dx)+b\sin(c+dx))}$$

```
[Out] sin(d*x+c)/a/d/(a*cos(d*x+c)+b*sin(d*x+c))
```

Rubi [A] time = 0.02, antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {3075}

$$\frac{\sin(c+dx)}{ad(a\cos(c+dx)+b\sin(c+dx))}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Cos[c + d*x] + b*Sin[c + d*x])^(-2),x]
```

```
[Out] Sin[c + d*x]/(a*d*(a*Cos[c + d*x] + b*Sin[c + d*x]))
```

Rule 3075

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-2), x
_Symbol] :> Simp[Sin[c + d*x]/(a*d*(a*Cos[c + d*x] + b*Sin[c + d*x])), x] /
; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

Rubi steps

$$\int \frac{1}{(a\cos(c+dx)+b\sin(c+dx))^2}\,dx = \frac{\sin(c+dx)}{ad(a\cos(c+dx)+b\sin(c+dx))}$$

Mathematica [A] time = 0.04, size = 32, normalized size = 1.00

$$\frac{\sin(c+dx)}{ad(a\cos(c+dx)+b\sin(c+dx))}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Cos[c + d*x] + b*Sin[c + d*x])^(-2),x]
```

```
[Out] Sin[c + d*x]/(a*d*(a*Cos[c + d*x] + b*Sin[c + d*x]))
```

fricas [A] time = 0.59, size = 57, normalized size = 1.78

$$-\frac{b\cos(dx+c)-a\sin(dx+c)}{\left(a^3+ab^2\right)d\cos(dx+c)+\left(a^2b+b^3\right)d\sin(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cos(d*x+c)+b*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -(b*cos(d*x + c) - a*sin(d*x + c))/((a^3 + a*b^2)*d*cos(d*x + c) + (a^2*b +
 b^3)*d*sin(d*x + c))
```

giac [A] time = 0.15, size = 20, normalized size = 0.62

$$-\frac{1}{(b\tan(dx+c)+a)bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cos(d*x+c)+b*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/((b*tan(d*x + c) + a)*b*d)
```

maple [A] time = 0.49, size = 21, normalized size = 0.66

$$-\frac{1}{db(a+b\tan(dx+c))}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*cos(d*x+c)+b*sin(d*x+c))^2,x)
```

```
[Out] -1/d/b/(a+b*tan(d*x+c))
```

maxima [A] time = 0.52, size = 21, normalized size = 0.66

$$-\frac{1}{\left(b^2\tan(dx+c)+ab\right)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cos(d*x+c)+b*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -1/((b^2*tan(d*x + c) + a*b)*d)
```

mupad [B] time = 2.34, size = 47, normalized size = 1.47

$$\frac{2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{a\,d\,\left(-a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+2\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*cos(c + d*x) + b*sin(c + d*x))^2,x)
```

```
[Out] (2*tan(c/2 + (d*x)/2))/(a*d*(a + 2*b*tan(c/2 + (d*x)/2) - a*tan(c/2 + (d*x)
/2)^2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a\cos\left(c+dx\right)+b\sin\left(c+dx\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cos(d*x+c)+b*sin(d*x+c))**2,x)
```

```
[Out] Integral((a*cos(c + d*x) + b*sin(c + d*x))**(-2), x)
```

3.228 $\int \frac{1}{(a\cos(c+dx)+b\sin(c+dx))^3}\,dx$

Optimal. Leaf size=103

$$-\frac{b\cos(c+dx)-a\sin(c+dx)}{2d\left(a^2+b^2\right)(a\cos(c+dx)+b\sin(c+dx))^2}-\frac{\tanh^{-1}\left(\frac{b\cos(c+dx)-a\sin(c+dx)}{\sqrt{a^2+b^2}}\right)}{2d\left(a^2+b^2\right)^{3/2}}$$

```
[Out] -1/2*arctanh((b*cos(d*x+c)-a*sin(d*x+c))/(a^2+b^2)^(1/2))/(a^2+b^2)^(3/2)/d
+1/2*(-b*cos(d*x+c)+a*sin(d*x+c))/(a^2+b^2)/d/(a*cos(d*x+c)+b*sin(d*x+c))^2
```

Rubi [A] time = 0.06, antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {3076, 3074, 206}

$$-\frac{b\cos(c+dx)-a\sin(c+dx)}{2d\left(a^2+b^2\right)(a\cos(c+dx)+b\sin(c+dx))^2}-\frac{\tanh^{-1}\left(\frac{b\cos(c+dx)-a\sin(c+dx)}{\sqrt{a^2+b^2}}\right)}{2d\left(a^2+b^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Cos[c + d*x] + b*Sin[c + d*x])^(-3),x]
```

```
[Out] -ArcTanh[(b*Cos[c + d*x] - a*Sin[c + d*x])/Sqrt[a^2 + b^2]]/(2*(a^2 + b^2)^
(3/2)*d) - (b*Cos[c + d*x] - a*Sin[c + d*x])/(2*(a^2 + b^2)*d*(a*Cos[c + d*
x] + b*Sin[c + d*x])^2)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3074

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x
_Symbol] :> -Dist[d^(-1), Subst[Int[1/(a^2 + b^2 - x^2), x], x, b*Cos[c + d
*x] - a*Sin[c + d*x]], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

Rule 3076

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x
_Symbol] :> Simp[((b*Cos[c + d*x] - a*Sin[c + d*x])*(a*Cos[c + d*x] + b*Sin
[c + d*x])^(n + 1))/(d*(n + 1)*(a^2 + b^2)), x] + Dist[(n + 2)/((n + 1)*(a^
2 + b^2)), Int[(a*Cos[c + d*x] + b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{
```

```
a, b, c, d}, x] && NeQ[a^2 + b^2, 0] && LtQ[n, -1] && NeQ[n, -2]
```

Rubi steps

$$\int \frac{1}{(a\cos(c+dx)+b\sin(c+dx))^3}\,dx = -\frac{b\cos(c+dx)-a\sin(c+dx)}{2\left(a^2+b^2\right)d(a\cos(c+dx)+b\sin(c+dx))^2}+\frac{\int \frac{1}{a\cos(c+dx)+b\sin(c+dx)}\,d}{2\left(a^2+b^2\right)}$$

$$= -\frac{b\cos(c+dx)-a\sin(c+dx)}{2\left(a^2+b^2\right)d(a\cos(c+dx)+b\sin(c+dx))^2}-\frac{\text{Subst}\left(\int \frac{1}{a^2+b^2-x^2}\,dx,x\right.}{2}$$

$$= -\frac{\tanh^{-1}\left(\frac{b\cos(c+dx)-a\sin(c+dx)}{\sqrt{a^2+b^2}}\right)}{2\left(a^2+b^2\right)^{3/2}d}-\frac{b\cos(c+dx)-a\sin(c+dx)}{2\left(a^2+b^2\right)d(a\cos(c+dx)+b\sin(c+$$

Mathematica [C] time = 0.29, size = 132, normalized size = 1.28

$$\frac{\left(a^2+b^2\right)(a\sin(c+dx)-b\cos(c+dx))+2\sqrt{a^2+b^2}(a\cos(c+dx)+b\sin(c+dx))^2\tanh^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)-b}{\sqrt{a^2+b^2}}\right)}{2d(a-ib)^2(a+ib)^2(a\cos(c+dx)+b\sin(c+dx))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Cos[c + d*x] + b*Sin[c + d*x])^(-3),x]
```

```
[Out] ((a^2 + b^2)*(-(b*Cos[c + d*x]) + a*Sin[c + d*x]) + 2*Sqrt[a^2 + b^2]*ArcTa
nh[(-b + a*Tan[(c + d*x)/2])/Sqrt[a^2 + b^2]]*(a*Cos[c + d*x] + b*Sin[c + d
*x])^2)/(2*(a - I*b)^2*(a + I*b)^2*d*(a*Cos[c + d*x] + b*Sin[c + d*x])^2)
```

fricas [B] time = 1.38, size = 294, normalized size = 2.85

$$\frac{\left(2\,ab\cos(dx+c)\sin(dx+c)+\left(a^2-b^2\right)\cos(dx+c)^2+b^2\right)\sqrt{a^2+b^2}\,\log\left(-\frac{2\,ab\cos(dx+c)\sin(dx+c)+\left(a^2-b^2\right)\cos(dx+c)^2}{2\,ab\cos(dx+c)\sin(dx+c}\right.}{4\left(\left(a^6+a^4b^2-a^2b^4-b^6\right)d\cos(dx+c)^2+2\left(a^5b+2\,a^3b^3+ab^5\right)d\,c\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cos(d*x+c)+b*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/4*((2*a*b*cos(d*x + c)*sin(d*x + c) + (a^2 - b^2)*cos(d*x + c)^2 + b^2)*s
qrt(a^2 + b^2)*log(-(2*a*b*cos(d*x + c)*sin(d*x + c) + (a^2 - b^2)*cos(d*x
+ c)^2 - 2*a^2 - b^2 + 2*sqrt(a^2 + b^2)*(b*cos(d*x + c) - a*sin(d*x + c)))
```

```
/(2*a*b*cos(d*x + c)*sin(d*x + c) + (a^2 - b^2)*cos(d*x + c)^2 + b^2)) - 2*
(a^2*b + b^3)*cos(d*x + c) + 2*(a^3 + a*b^2)*sin(d*x + c))/((a^6 + a^4*b^2
- a^2*b^4 - b^6)*d*cos(d*x + c)^2 + 2*(a^5*b + 2*a^3*b^3 + a*b^5)*d*cos(d*x
 + c)*sin(d*x + c) + (a^4*b^2 + 2*a^2*b^4 + b^6)*d)
```

giac [B] time = 0.23, size = 221, normalized size = 2.15

$$-\frac{\frac{\log\left(\frac{\left|2\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-2\,b-2\,\sqrt{a^2+b^2}\right|}{\left|2\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-2\,b+2\,\sqrt{a^2+b^2}\right|}\right)}{\left(a^2+b^2\right)^{\frac{3}{2}}}-\frac{2\left(a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+2\,ab^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+a^2b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-2\,b^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-2}{\left(a^4+a^2b^2\right)\left(a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-2\,b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-a\right)^2}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cos(d*x+c)+b*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -1/2*(log(abs(2*a*tan(1/2*d*x + 1/2*c) - 2*b - 2*sqrt(a^2 + b^2))/abs(2*a*t
an(1/2*d*x + 1/2*c) - 2*b + 2*sqrt(a^2 + b^2)))/(a^2 + b^2)^(3/2) - 2*(a^3*
tan(1/2*d*x + 1/2*c)^3 + 2*a*b^2*tan(1/2*d*x + 1/2*c)^3 + a^2*b*tan(1/2*d*x
 + 1/2*c)^2 - 2*b^3*tan(1/2*d*x + 1/2*c)^2 + a^3*tan(1/2*d*x + 1/2*c) - 2*a
*b^2*tan(1/2*d*x + 1/2*c) - a^2*b)/((a^4 + a^2*b^2)*(a*tan(1/2*d*x + 1/2*c)
^2 - 2*b*tan(1/2*d*x + 1/2*c) - a)^2))/d
```

maple [A] time = 0.53, size = 191, normalized size = 1.85

$$-\frac{\frac{2\left(-\frac{\left(a^2+2b^2\right)\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2\left(a^2+b^2\right)a}-\frac{b\left(a^2-2b^2\right)\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2\left(a^2+b^2\right)a^2}-\frac{\left(a^2-2b^2\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2a\left(a^2+b^2\right)}+\frac{b}{2a^2+2b^2}\right)}{\left(a\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-2b\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-a\right)^2}+\frac{\operatorname{arctanh}\left(\frac{2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)a-2b}{2\sqrt{a^2+b^2}}\right)}{\left(a^2+b^2\right)^{\frac{3}{2}}}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*cos(d*x+c)+b*sin(d*x+c))^3,x)
```

```
[Out] 1/d*(-2*(-1/2*(a^2+2*b^2)/(a^2+b^2)/a*tan(1/2*d*x+1/2*c)^3-1/2*b*(a^2-2*b^2
)/(a^2+b^2)/a^2*tan(1/2*d*x+1/2*c)^2-1/2*(a^2-2*b^2)/a/(a^2+b^2)*tan(1/2*d*
x+1/2*c)+1/2*b/(a^2+b^2))/(a*tan(1/2*d*x+1/2*c)^2-2*b*tan(1/2*d*x+1/2*c)-a)
^2+1/(a^2+b^2)^(3/2)*arctanh(1/2*(2*tan(1/2*d*x+1/2*c)*a-2*b)/(a^2+b^2)^(1/
2)))
```

maxima [B] time = 0.46, size = 326, normalized size = 3.17

$$-\frac{\frac{2\left(a^2b-\frac{\left(a^3-2\,ab^2\right)\sin\left(dx+c\right)}{\cos\left(dx+c\right)+1}-\frac{\left(a^2b-2\,b^3\right)\sin\left(dx+c\right)^2}{\left(\cos\left(dx+c\right)+1\right)^2}-\frac{\left(a^3+2\,ab^2\right)\sin\left(dx+c\right)^3}{\left(\cos\left(dx+c\right)+1\right)^3}\right)}{a^6+a^4b^2+\frac{4\left(a^5b+a^3b^3\right)\sin\left(dx+c\right)}{\cos\left(dx+c\right)+1}-\frac{2\left(a^6-a^4b^2-2\,a^2b^4\right)\sin\left(dx+c\right)^2}{\left(\cos\left(dx+c\right)+1\right)^2}-\frac{4\left(a^5b+a^3b^3\right)\sin\left(dx+c\right)^3}{\left(\cos\left(dx+c\right)+1\right)^3}+\frac{\left(a^6+a^4b^2\right)\sin\left(dx+c\right)^4}{\left(\cos\left(dx+c\right)+1\right)^4}}+\frac{\log\left(\frac{b-\frac{a\sin\left(dx+c\right)}{\cos\left(dx+c\right)+1}+\sqrt{a^2+b^2}}{b-\frac{a\sin\left(dx+c\right)}{\cos\left(dx+c\right)+1}-\sqrt{a^2+b^2}}\right)}{\left(a^2+b^2\right)^{\frac{3}{2}}}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cos(d*x+c)+b*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -1/2*(2*(a^2*b - (a^3 - 2*a*b^2)*sin(d*x + c)/(cos(d*x + c) + 1) - (a^2*b -
 2*b^3)*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 - (a^3 + 2*a*b^2)*sin(d*x + c)^
3/(cos(d*x + c) + 1)^3)/(a^6 + a^4*b^2 + 4*(a^5*b + a^3*b^3)*sin(d*x + c)/(
cos(d*x + c) + 1) - 2*(a^6 - a^4*b^2 - 2*a^2*b^4)*sin(d*x + c)^2/(cos(d*x +
 c) + 1)^2 - 4*(a^5*b + a^3*b^3)*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + (a^6
 + a^4*b^2)*sin(d*x + c)^4/(cos(d*x + c) + 1)^4) + log((b - a*sin(d*x + c)/
(cos(d*x + c) + 1) + sqrt(a^2 + b^2))/(b - a*sin(d*x + c)/(cos(d*x + c) + 1
) - sqrt(a^2 + b^2)))/(a^2 + b^2)^(3/2))/d
```

mupad [B] time = 4.55, size = 260, normalized size = 2.52

$$\frac{\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\left(a^2-2\,b^2\right)}{a\,\left(a^2+b^2\right)}-\frac{b}{a^2+b^2}+\frac{{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^3\,\left(a^2+2\,b^2\right)}{a\,\left(a^2+b^2\right)}+\frac{b\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2\,\left(a^2-2\,b^2\right)}{a^2\,\left(a^2+b^2\right)}}{d\,\left(a^2\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^4-{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2\,\left(2\,a^2-4\,b^2\right)+a^2-4\,a\,b\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^3+4\,a\,b\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}+\mathrm{atanh}\left(\left(\left(2\,a\,t\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*cos(c + d*x) + b*sin(c + d*x))^3,x)
```

```
[Out] ((tan(c/2 + (d*x)/2)*(a^2 - 2*b^2))/(a*(a^2 + b^2)) - b/(a^2 + b^2) + (tan(
c/2 + (d*x)/2)^3*(a^2 + 2*b^2))/(a*(a^2 + b^2)) + (b*tan(c/2 + (d*x)/2)^2*(
a^2 - 2*b^2))/(a^2*(a^2 + b^2)))/(d*(a^2*tan(c/2 + (d*x)/2)^4 - tan(c/2 + (
d*x)/2)^2*(2*a^2 - 4*b^2) + a^2 - 4*a*b*tan(c/2 + (d*x)/2)^3 + 4*a*b*tan(c/
2 + (d*x)/2))) + atanh(((2*a*tan(c/2 + (d*x)/2) - (2*a^2*b + 2*b^3)/(a^2 +
b^2))*(a^2/2 + b^2/2))/(a^2 + b^2)^(3/2))/(d*(a^2 + b^2)^(3/2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cos(d*x+c)+b*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

3.229 $\int \frac{1}{(a\cos(c+dx)+b\sin(c+dx))^4}\,dx$

Optimal. Leaf size=98

$$\frac{2\sin(c+dx)}{3ad\left(a^2+b^2\right)(a\cos(c+dx)+b\sin(c+dx))}-\frac{b\cos(c+dx)-a\sin(c+dx)}{3d\left(a^2+b^2\right)(a\cos(c+dx)+b\sin(c+dx))^3}$$

```
[Out] 1/3*(-b*cos(d*x+c)+a*sin(d*x+c))/(a^2+b^2)/d/(a*cos(d*x+c)+b*sin(d*x+c))^3+
2/3*sin(d*x+c)/a/(a^2+b^2)/d/(a*cos(d*x+c)+b*sin(d*x+c))
```

Rubi [A] time = 0.04, antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {3076, 3075}

$$\frac{2\sin(c+dx)}{3ad\left(a^2+b^2\right)(a\cos(c+dx)+b\sin(c+dx))}-\frac{b\cos(c+dx)-a\sin(c+dx)}{3d\left(a^2+b^2\right)(a\cos(c+dx)+b\sin(c+dx))^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Cos[c + d*x] + b*Sin[c + d*x])^(-4),x]
```

```
[Out] -(b*Cos[c + d*x] - a*Sin[c + d*x])/(3*(a^2 + b^2)*d*(a*Cos[c + d*x] + b*Sin
[c + d*x])^3) + (2*Sin[c + d*x])/(3*a*(a^2 + b^2)*d*(a*Cos[c + d*x] + b*Sin
[c + d*x]))
```

Rule 3075

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-2), x
_Symbol] :> Simp[Sin[c + d*x]/(a*d*(a*Cos[c + d*x] + b*Sin[c + d*x])), x] /
; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

Rule 3076

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x
_Symbol] :> Simp[((b*Cos[c + d*x] - a*Sin[c + d*x])*(a*Cos[c + d*x] + b*Sin
[c + d*x])^(n + 1))/(d*(n + 1)*(a^2 + b^2)), x] + Dist[(n + 2)/((n + 1)*(a^
2 + b^2)), Int[(a*Cos[c + d*x] + b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{
a, b, c, d}, x] && NeQ[a^2 + b^2, 0] && LtQ[n, -1] && NeQ[n, -2]
```

Rubi steps

$$\int \frac{1}{(a\cos(c+dx)+b\sin(c+dx))^4}\,dx = -\frac{b\cos(c+dx)-a\sin(c+dx)}{3\left(a^2+b^2\right)d(a\cos(c+dx)+b\sin(c+dx))^3} + \frac{2\int \frac{1}{(a\cos(c+dx)+b\sin(c+d}}{3\left(a^2+b^2\right)}$$

$$= -\frac{b\cos(c+dx)-a\sin(c+dx)}{3\left(a^2+b^2\right)d(a\cos(c+dx)+b\sin(c+dx))^3} + \frac{2\sin(}{3a\left(a^2+b^2\right)d(a\cos(c}$$

Mathematica [A] time = 0.29, size = 85, normalized size = 0.87

$$\frac{\sin(c+dx)\left(\left(a^2-b^2\right)\cos(2(c+dx))+2a^2+b^2\right)-ab\cos(3(c+dx))}{3ad\left(a^2+b^2\right)(a\cos(c+dx)+b\sin(c+dx))^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Cos[c + d*x] + b*Sin[c + d*x])^(-4),x]
```

```
[Out] (-(a*b*Cos[3*(c + d*x)]) + (2*a^2 + b^2 + (a^2 - b^2)*Cos[2*(c + d*x)])*Sin
[c + d*x])/(3*a*(a^2 + b^2)*d*(a*Cos[c + d*x] + b*Sin[c + d*x])^3)
```

fricas [B] time = 0.67, size = 217, normalized size = 2.21

$$-\frac{2\left(3a^2b-b^3\right)\cos(dx+c)^3-3\left(a^2b-b^3\right)\cos(dx+c)-\left(a^3+3ab^2+2\left(a^3-3a\right.\right.}{3\left(\left(a^7-a^5b^2-5a^3b^4-3ab^6\right)d\cos(dx+c)^3+3\left(a^5b^2+2a^3b^4+ab^6\right)d\cos(dx+c)+\left(\left(3a^6b+5a^4b^3+a^2b^5\right.\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cos(d*x+c)+b*sin(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] -1/3*(2*(3*a^2*b - b^3)*cos(d*x + c)^3 - 3*(a^2*b - b^3)*cos(d*x + c) - (a^
3 + 3*a*b^2 + 2*(a^3 - 3*a*b^2)*cos(d*x + c)^2)*sin(d*x + c))/((a^7 - a^5*b
^2 - 5*a^3*b^4 - 3*a*b^6)*d*cos(d*x + c)^3 + 3*(a^5*b^2 + 2*a^3*b^4 + a*b^6
)*d*cos(d*x + c) + ((3*a^6*b + 5*a^4*b^3 + a^2*b^5 - b^7)*d*cos(d*x + c)^2
+ (a^4*b^3 + 2*a^2*b^5 + b^7)*d)*sin(d*x + c))
```

giac [A] time = 0.15, size = 50, normalized size = 0.51

$$-\frac{3b^2\tan(dx+c)^2+3ab\tan(dx+c)+a^2+b^2}{3(b\tan(dx+c)+a)^3b^3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cos(d*x+c)+b*sin(d*x+c))^4,x, algorithm="giac")
```

```
[Out] -1/3*(3*b^2*tan(d*x + c)^2 + 3*a*b*tan(d*x + c) + a^2 + b^2)/((b*tan(d*x +
c) + a)^3*b^3*d)
```

maple [A] time = 0.56, size = 64, normalized size = 0.65

$$\frac{\frac{a}{b^3(a+b\tan(dx+c))^2}-\frac{a^2+b^2}{3b^3(a+b\tan(dx+c))^3}-\frac{1}{b^3(a+b\tan(dx+c))}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*cos(d*x+c)+b*sin(d*x+c))^4,x)
```

```
[Out] 1/d*(a/b^3/(a+b*tan(d*x+c))^2-1/3*(a^2+b^2)/b^3/(a+b*tan(d*x+c))^3-1/b^3/(a
+b*tan(d*x+c)))
```

maxima [A] time = 0.34, size = 85, normalized size = 0.87

$$-\frac{3\,b^2\tan(dx+c)^2+3\,ab\tan(dx+c)+a^2+b^2}{3\left(b^6\tan(dx+c)^3+3\,ab^5\tan(dx+c)^2+3\,a^2b^4\tan(dx+c)+a^3b^3\right)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cos(d*x+c)+b*sin(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] -1/3*(3*b^2*tan(d*x + c)^2 + 3*a*b*tan(d*x + c) + a^2 + b^2)/((b^6*tan(d*x
+ c)^3 + 3*a*b^5*tan(d*x + c)^2 + 3*a^2*b^4*tan(d*x + c) + a^3*b^3)*d)
```

mupad [B] time = 3.12, size = 222, normalized size = 2.27

$$\frac{\frac{2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5}{a}+\frac{2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{a}-\frac{4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3\left(a^2-2\,b^2\right)}{3\,a^3}+\frac{4\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{a^2}-\frac{4\,b\,\mathrm{t}}{}}{d\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\left(12\,a\,b^2-3\,a^3\right)-a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4\left(12\,a\,b^2-3\,a^3\right)-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3\left(12\,a^2\,b-8\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*cos(c + d*x) + b*sin(c + d*x))^4,x)
```

```
[Out] ((2*tan(c/2 + (d*x)/2)^5)/a + (2*tan(c/2 + (d*x)/2))/a - (4*tan(c/2 + (d*x)
/2)^3*(a^2 - 2*b^2))/(3*a^3) + (4*b*tan(c/2 + (d*x)/2)^2)/a^2 - (4*b*tan(c/
2 + (d*x)/2)^4)/a^2)/(d*(tan(c/2 + (d*x)/2)^2*(12*a*b^2 - 3*a^3) - a^3*tan(
c/2 + (d*x)/2)^6 - tan(c/2 + (d*x)/2)^4*(12*a*b^2 - 3*a^3) - tan(c/2 + (d*x
)/2)^3*(12*a^2*b - 8*b^3) + a^3 + 6*a^2*b*tan(c/2 + (d*x)/2) + 6*a^2*b*tan(
c/2 + (d*x)/2)^5))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cos(d*x+c)+b*sin(d*x+c))**4,x)
```

```
[Out] Timed out
```

3.230 $\int \frac{1}{(a\cos(c+dx)+b\sin(c+dx))^5}\,dx$

Optimal. Leaf size=156

$$-\frac{3(b\cos(c+dx)-a\sin(c+dx))}{8d\left(a^2+b^2\right)^2(a\cos(c+dx)+b\sin(c+dx))^2}-\frac{b\cos(c+dx)-a\sin(c+dx)}{4d\left(a^2+b^2\right)(a\cos(c+dx)+b\sin(c+dx))^4}-\frac{3\tanh^{-1}\left(\frac{b\cos(c+dx}{\sqrt{a}}\right.}{8d\left(a^2+b^2\right.}$$

```
[Out] -3/8*arctanh((b*cos(d*x+c)-a*sin(d*x+c))/(a^2+b^2)^(1/2))/(a^2+b^2)^(5/2)/d
+1/4*(-b*cos(d*x+c)+a*sin(d*x+c))/(a^2+b^2)/d/(a*cos(d*x+c)+b*sin(d*x+c))^4
-3/8*(b*cos(d*x+c)-a*sin(d*x+c))/(a^2+b^2)^2/d/(a*cos(d*x+c)+b*sin(d*x+c))^
2
```

Rubi [A] time = 0.09, antiderivative size = 156, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {3076, 3074, 206}

$$-\frac{3(b\cos(c+dx)-a\sin(c+dx))}{8d\left(a^2+b^2\right)^2(a\cos(c+dx)+b\sin(c+dx))^2}-\frac{b\cos(c+dx)-a\sin(c+dx)}{4d\left(a^2+b^2\right)(a\cos(c+dx)+b\sin(c+dx))^4}-\frac{3\tanh^{-1}\left(\frac{b\cos(c+dx}{\sqrt{a}}\right.}{8d\left(a^2+b^2\right.}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Cos[c + d*x] + b*Sin[c + d*x])^(-5),x]
```

```
[Out] (-3*ArcTanh[(b*Cos[c + d*x] - a*Sin[c + d*x])/Sqrt[a^2 + b^2]])/(8*(a^2 + b
^2)^(5/2)*d) - (b*Cos[c + d*x] - a*Sin[c + d*x])/(4*(a^2 + b^2)*d*(a*Cos[c
+ d*x] + b*Sin[c + d*x])^4) - (3*(b*Cos[c + d*x] - a*Sin[c + d*x]))/(8*(a^2
 + b^2)^2*d*(a*Cos[c + d*x] + b*Sin[c + d*x])^2)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3074

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x
_Symbol] :> -Dist[d^(-1), Subst[Int[1/(a^2 + b^2 - x^2), x], x, b*Cos[c + d
*x] - a*Sin[c + d*x]], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

Rule 3076

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x
_Symbol] :> Simp[((b*Cos[c + d*x] - a*Sin[c + d*x])*(a*Cos[c + d*x] + b*Sin
[c + d*x])^(n + 1))/(d*(n + 1)*(a^2 + b^2)), x] + Dist[(n + 2)/((n + 1)*(a^
2 + b^2)), Int[(a*Cos[c + d*x] + b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{
a, b, c, d}, x] && NeQ[a^2 + b^2, 0] && LtQ[n, -1] && NeQ[n, -2]
```

Rubi steps

$$\int \frac{1}{(a\cos(c+dx)+b\sin(c+dx))^5}\,dx = -\frac{b\cos(c+dx)-a\sin(c+dx)}{4\left(a^2+b^2\right)d(a\cos(c+dx)+b\sin(c+dx))^4}+\frac{3\int \frac{1}{(a\cos(c+dx)+b\sin(c+d}}{4\left(a^2+b^2\right)}$$

$$= -\frac{b\cos(c+dx)-a\sin(c+dx)}{4\left(a^2+b^2\right)d(a\cos(c+dx)+b\sin(c+dx))^4}-\frac{3(b\cos(c+dx)}{8\left(a^2+b^2\right)^2 d(a\cos(c}$$

$$= -\frac{b\cos(c+dx)-a\sin(c+dx)}{4\left(a^2+b^2\right)d(a\cos(c+dx)+b\sin(c+dx))^4}-\frac{3(b\cos(c+dx)}{8\left(a^2+b^2\right)^2 d(a\cos(c}$$

$$= -\frac{3\tanh^{-1}\left(\frac{b\cos(c+dx)-a\sin(c+dx)}{\sqrt{a^2+b^2}}\right)}{8\left(a^2+b^2\right)^{5/2}d}-\frac{b\cos(c+dx)-a\sin(c+dx}{4\left(a^2+b^2\right)d(a\cos(c+dx)+b\sin(c}$$

Mathematica [A] time = 1.15, size = 157, normalized size = 1.01

$$\frac{\frac{6\tanh^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)-b}{\sqrt{a^2+b^2}}\right)}{\left(a^2+b^2\right)^{5/2}}+\frac{\left(3b^3-9a^2b\right)\cos(3(c+dx))-11b\left(a^2+b^2\right)\cos(c+dx)+2a\sin(c+dx)\left(3\left(a^2-3b^2\right)\cos(2(c+dx))+7a^2+b^2\right)}{4\left(a^2+b^2\right)^2(a\cos(c+dx)+b\sin(c+dx))^4}}{8d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Cos[c + d*x] + b*Sin[c + d*x])^(-5),x]
```

```
[Out] ((6*ArcTanh[(-b + a*Tan[(c + d*x)/2])/Sqrt[a^2 + b^2]])/(a^2 + b^2)^(5/2) +
 (-11*b*(a^2 + b^2)*Cos[c + d*x] + (-9*a^2*b + 3*b^3)*Cos[3*(c + d*x)] + 2*
a*(7*a^2 + b^2 + 3*(a^2 - 3*b^2)*Cos[2*(c + d*x)])*Sin[c + d*x])/(4*(a^2 +
b^2)^2*(a*Cos[c + d*x] + b*Sin[c + d*x])^4))/(8*d)
```

fricas [B] time = 1.13, size = 544, normalized size = 3.49

$$-\frac{6\left(3a^4b+2a^2b^3-b^5\right)\cos(dx+c)^3-3\left(\left(a^4-6a^2b^2+b^4\right)\cos(dx+c)^4+b^4+2\left(3a^2b^2-b^4\right)\cos(dx+c)^2+\right.}{16\left(\left(a^{10}-3a^8b^2-14a^6b^4-14a^4b^6-3a^2b^8+b^{10}\right)d\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cos(d*x+c)+b*sin(d*x+c))^5,x, algorithm="fricas")
```

```
[Out] -1/16*(6*(3*a^4*b + 2*a^2*b^3 - b^5)*cos(d*x + c)^3 - 3*((a^4 - 6*a^2*b^2 +
 b^4)*cos(d*x + c)^4 + b^4 + 2*(3*a^2*b^2 - b^4)*cos(d*x + c)^2 + 4*(a*b^3*
cos(d*x + c) + (a^3*b - a*b^3)*cos(d*x + c)^3)*sin(d*x + c))*sqrt(a^2 + b^2
)*log(-(2*a*b*cos(d*x + c)*sin(d*x + c) + (a^2 - b^2)*cos(d*x + c)^2 - 2*a^
2 - b^2 + 2*sqrt(a^2 + b^2)*(b*cos(d*x + c) - a*sin(d*x + c)))/(2*a*b*cos(d
*x + c)*sin(d*x + c) + (a^2 - b^2)*cos(d*x + c)^2 + b^2)) - 2*(4*a^4*b - a^
2*b^3 - 5*b^5)*cos(d*x + c) - 2*(2*a^5 + 7*a^3*b^2 + 5*a*b^4 + 3*(a^5 - 2*a
^3*b^2 - 3*a*b^4)*cos(d*x + c)^2)*sin(d*x + c))/((a^10 - 3*a^8*b^2 - 14*a^6
*b^4 - 14*a^4*b^6 - 3*a^2*b^8 + b^10)*d*cos(d*x + c)^4 + 2*(3*a^8*b^2 + 8*a
^6*b^4 + 6*a^4*b^6 - b^10)*d*cos(d*x + c)^2 + (a^6*b^4 + 3*a^4*b^6 + 3*a^2*
b^8 + b^10)*d + 4*((a^9*b + 2*a^7*b^3 - 2*a^3*b^7 - a*b^9)*d*cos(d*x + c)^3
 + (a^7*b^3 + 3*a^5*b^5 + 3*a^3*b^7 + a*b^9)*d*cos(d*x + c))*sin(d*x + c))
```

giac [B] time = 0.31, size = 588, normalized size = 3.77

$$-\frac{\frac{3\log\left(\frac{\left|2a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-2b-2\sqrt{a^2+b^2}\right|}{\left|2a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-2b+2\sqrt{a^2+b^2}\right|}\right)}{\left(a^4+2a^2b^2+b^4\right)\sqrt{a^2+b^2}}-\frac{2\left(5a^7\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^7+16a^5b^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^7+8a^3b^4\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^7-3a^6b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^6-48a^4b^3\tan\cdots\right.}{\cdots}}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cos(d*x+c)+b*sin(d*x+c))^5,x, algorithm="giac")
```

```
[Out] -1/8*(3*log(abs(2*a*tan(1/2*d*x + 1/2*c) - 2*b - 2*sqrt(a^2 + b^2))/abs(2*a
*tan(1/2*d*x + 1/2*c) - 2*b + 2*sqrt(a^2 + b^2)))/((a^4 + 2*a^2*b^2 + b^4)*
sqrt(a^2 + b^2)) - 2*(5*a^7*tan(1/2*d*x + 1/2*c)^7 + 16*a^5*b^2*tan(1/2*d*x
 + 1/2*c)^7 + 8*a^3*b^4*tan(1/2*d*x + 1/2*c)^7 - 3*a^6*b*tan(1/2*d*x + 1/2*
c)^6 - 48*a^4*b^3*tan(1/2*d*x + 1/2*c)^6 - 24*a^2*b^5*tan(1/2*d*x + 1/2*c)^
6 + 3*a^7*tan(1/2*d*x + 1/2*c)^5 - 36*a^5*b^2*tan(1/2*d*x + 1/2*c)^5 + 56*a
^3*b^4*tan(1/2*d*x + 1/2*c)^5 + 32*a*b^6*tan(1/2*d*x + 1/2*c)^5 - 15*a^6*b*
tan(1/2*d*x + 1/2*c)^4 + 114*a^4*b^3*tan(1/2*d*x + 1/2*c)^4 + 8*a^2*b^5*tan
(1/2*d*x + 1/2*c)^4 - 16*b^7*tan(1/2*d*x + 1/2*c)^4 + 3*a^7*tan(1/2*d*x + 1
/2*c)^3 + 84*a^5*b^2*tan(1/2*d*x + 1/2*c)^3 - 56*a^3*b^4*tan(1/2*d*x + 1/2*
c)^3 - 32*a*b^6*tan(1/2*d*x + 1/2*c)^3 + 23*a^6*b*tan(1/2*d*x + 1/2*c)^2 -
64*a^4*b^3*tan(1/2*d*x + 1/2*c)^2 - 24*a^2*b^5*tan(1/2*d*x + 1/2*c)^2 + 5*a
^7*tan(1/2*d*x + 1/2*c) - 24*a^5*b^2*tan(1/2*d*x + 1/2*c) - 8*a^3*b^4*tan(1
/2*d*x + 1/2*c) - 5*a^6*b - 2*a^4*b^3)/((a^8 + 2*a^6*b^2 + a^4*b^4)*(a*tan(
1/2*d*x + 1/2*c)^2 - 2*b*tan(1/2*d*x + 1/2*c) - a)^4))/d
```

maple [B] time = 0.60, size = 514, normalized size = 3.29

$$-\frac{2\left(-\frac{(5a^4+16a^2b^2+8b^4)\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{8a(a^4+2a^2b^2+b^4)}+\frac{3b(a^4+16a^2b^2+8b^4)\left(\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{8a^2(a^4+2a^2b^2+b^4)}-\frac{(3a^6-36a^4b^2+56a^2b^4+32b^6)\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{8a^3(a^4+2a^2b^2+b^4)}+\frac{b(15a^6-114a^4b^2-8a^2b^4+16b^6)\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right.\right.}{8a^4(a^4+2a^2b^2+b^4)}\right.}{\left(a\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-2b\tan\left(\frac{dx}{2}+\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*cos(d*x+c)+b*sin(d*x+c))^5,x)
```

```
[Out] 1/d*(-2*(-1/8*(5*a^4+16*a^2*b^2+8*b^4)/a/(a^4+2*a^2*b^2+b^4)*tan(1/2*d*x+1/
2*c)^7+3/8*b*(a^4+16*a^2*b^2+8*b^4)/a^2/(a^4+2*a^2*b^2+b^4)*tan(1/2*d*x+1/2
*c)^6-1/8/a^3*(3*a^6-36*a^4*b^2+56*a^2*b^4+32*b^6)/(a^4+2*a^2*b^2+b^4)*tan(
1/2*d*x+1/2*c)^5+1/8/a^4*b*(15*a^6-114*a^4*b^2-8*a^2*b^4+16*b^6)/(a^4+2*a^2
*b^2+b^4)*tan(1/2*d*x+1/2*c)^4-1/8/a^3*(3*a^6+84*a^4*b^2-56*a^2*b^4-32*b^6)
/(a^4+2*a^2*b^2+b^4)*tan(1/2*d*x+1/2*c)^3-1/8*b*(23*a^4-64*a^2*b^2-24*b^4)/
a^2/(a^4+2*a^2*b^2+b^4)*tan(1/2*d*x+1/2*c)^2-1/8*(5*a^4-24*a^2*b^2-8*b^4)/a
/(a^4+2*a^2*b^2+b^4)*tan(1/2*d*x+1/2*c)+1/8*b*(5*a^2+2*b^2)/(a^4+2*a^2*b^2+
b^4))/(a*tan(1/2*d*x+1/2*c)^2-2*b*tan(1/2*d*x+1/2*c)-a)^4+3/4/(a^4+2*a^2*b^
2+b^4)/(a^2+b^2)^(1/2)*arctanh(1/2*(2*tan(1/2*d*x+1/2*c)*a-2*b)/(a^2+b^2)^(
1/2)))
```

maxima [B] time = 1.04, size = 822, normalized size = 5.27

$$-\frac{2\left(5a^6b+2a^4b^3-\frac{(5a^7-24a^5b^2-8a^3b^4)\sin(dx+c)}{\cos(dx+c)+1}-\frac{(23a^6b-64a^4b^3-24a^2b^5)\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-\frac{(3a^7+84a^5b^2-56a^3b^4-32ab^6)\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\right.}{a^{12}+2a^{10}b^2+a^8b^4+\frac{8(a^{11}b+2a^9b^3+a^7b^5)\sin(dx+c)}{\cos(dx+c)+1}-\frac{4(a^{12}-4a^{10}b^2-11a^8b^4-6a^6b^6)\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-\frac{8(3a^{11}b+2a^9b^3-5a^7b^5-4a^5b^7)\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{2(3a^{12}-18a^{10}b^2-3}{(}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cos(d*x+c)+b*sin(d*x+c))^5,x, algorithm="maxima")
```

```
[Out] -1/8*(2*(5*a^6*b + 2*a^4*b^3 - (5*a^7 - 24*a^5*b^2 - 8*a^3*b^4)*sin(d*x + c
)/(cos(d*x + c) + 1) - (23*a^6*b - 64*a^4*b^3 - 24*a^2*b^5)*sin(d*x + c)^2/
(cos(d*x + c) + 1)^2 - (3*a^7 + 84*a^5*b^2 - 56*a^3*b^4 - 32*a*b^6)*sin(d*x
 + c)^3/(cos(d*x + c) + 1)^3 + (15*a^6*b - 114*a^4*b^3 - 8*a^2*b^5 + 16*b^7
)*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 - (3*a^7 - 36*a^5*b^2 + 56*a^3*b^4 +
32*a*b^6)*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + 3*(a^6*b + 16*a^4*b^3 + 8*a
^2*b^5)*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 - (5*a^7 + 16*a^5*b^2 + 8*a^3*b
^4)*sin(d*x + c)^7/(cos(d*x + c) + 1)^7)/(a^12 + 2*a^10*b^2 + a^8*b^4 + 8*(
a^11*b + 2*a^9*b^3 + a^7*b^5)*sin(d*x + c)/(cos(d*x + c) + 1) - 4*(a^12 - 4
*a^10*b^2 - 11*a^8*b^4 - 6*a^6*b^6)*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 - 8
```

```
*(3*a^11*b + 2*a^9*b^3 - 5*a^7*b^5 - 4*a^5*b^7)*sin(d*x + c)^3/(cos(d*x + c
) + 1)^3 + 2*(3*a^12 - 18*a^10*b^2 - 37*a^8*b^4 - 8*a^6*b^6 + 8*a^4*b^8)*si
n(d*x + c)^4/(cos(d*x + c) + 1)^4 + 8*(3*a^11*b + 2*a^9*b^3 - 5*a^7*b^5 - 4
*a^5*b^7)*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 4*(a^12 - 4*a^10*b^2 - 11*a
^8*b^4 - 6*a^6*b^6)*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 - 8*(a^11*b + 2*a^9
*b^3 + a^7*b^5)*sin(d*x + c)^7/(cos(d*x + c) + 1)^7 + (a^12 + 2*a^10*b^2 +
a^8*b^4)*sin(d*x + c)^8/(cos(d*x + c) + 1)^8) + 3*log((b - a*sin(d*x + c)/(
cos(d*x + c) + 1) + sqrt(a^2 + b^2))/(b - a*sin(d*x + c)/(cos(d*x + c) + 1)
 - sqrt(a^2 + b^2)))/((a^4 + 2*a^2*b^2 + b^4)*sqrt(a^2 + b^2)))/d
```

mupad [B] time = 6.09, size = 719, normalized size = 4.61

$$-\frac{\frac{5a^2b+2b^3}{4(a^4+2a^2b^2+b^4)}+\frac{3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^6(a^4b+16a^2b^3+8b^5)}{4a^2(a^4+2a^2b^2+b^4)}+\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2(-23a^4b+64a^2b^3+24b^5)}{4a^2(a^4+2a^2b^2+b^4)}+\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)(-5a^4+24a^2b^2+8b^4)}{4a(a^4+2a^2b^2+b^4)}-\cdots}{d\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4(6a^4-48a^2b^2+16b^4)+a^4\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^8+a^4-\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2(4a^4-24a^2b^2)-\tan\left(\frac{c}{2}+\cdots\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*cos(c + d*x) + b*sin(c + d*x))^5,x)
```

```
[Out] (atan((a^4*b*1i + b^5*1i + a^2*b^3*2i - a^5*tan(c/2 + (d*x)/2)*1i - a*b^4*t
an(c/2 + (d*x)/2)*1i - a^3*b^2*tan(c/2 + (d*x)/2)*2i)/(a^2 + b^2)^(5/2))*3i
)/(4*d*(a^2 + b^2)^(5/2)) - ((5*a^2*b + 2*b^3)/(4*(a^4 + b^4 + 2*a^2*b^2))
+ (3*tan(c/2 + (d*x)/2)^6*(a^4*b + 8*b^5 + 16*a^2*b^3))/(4*a^2*(a^4 + b^4 +
 2*a^2*b^2)) + (tan(c/2 + (d*x)/2)^2*(24*b^5 - 23*a^4*b + 64*a^2*b^3))/(4*a
^2*(a^4 + b^4 + 2*a^2*b^2)) + (tan(c/2 + (d*x)/2)*(8*b^4 - 5*a^4 + 24*a^2*b
^2))/(4*a*(a^4 + b^4 + 2*a^2*b^2)) - (tan(c/2 + (d*x)/2)^5*(3*a^6 + 32*b^6
+ 56*a^2*b^4 - 36*a^4*b^2))/(4*a^3*(a^4 + b^4 + 2*a^2*b^2)) - (tan(c/2 + (d
*x)/2)^3*(3*a^6 - 32*b^6 - 56*a^2*b^4 + 84*a^4*b^2))/(4*a^3*(a^4 + b^4 + 2*
a^2*b^2)) - (tan(c/2 + (d*x)/2)^7*(5*a^4 + 8*b^4 + 16*a^2*b^2))/(4*a*(a^4 +
 b^4 + 2*a^2*b^2)) + (tan(c/2 + (d*x)/2)^4*(5*a^2*b + 2*b^3)*(3*a^4 + 8*b^4
 - 24*a^2*b^2))/(4*a^4*(a^4 + b^4 + 2*a^2*b^2)))/(d*(tan(c/2 + (d*x)/2)^4*(
6*a^4 + 16*b^4 - 48*a^2*b^2) + a^4*tan(c/2 + (d*x)/2)^8 + a^4 - tan(c/2 + (
d*x)/2)^2*(4*a^4 - 24*a^2*b^2) - tan(c/2 + (d*x)/2)^6*(4*a^4 - 24*a^2*b^2)
+ tan(c/2 + (d*x)/2)^3*(32*a*b^3 - 24*a^3*b) - tan(c/2 + (d*x)/2)^5*(32*a*b
^3 - 24*a^3*b) + 8*a^3*b*tan(c/2 + (d*x)/2) - 8*a^3*b*tan(c/2 + (d*x)/2)^7)
)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cos(d*x+c)+b*sin(d*x+c))**5,x)
```

```
[Out] Timed out
```

3.231 $\int \frac{1}{(a\cos(c+dx)+b\sin(c+dx))^6}\,dx$

Optimal. Leaf size=151

$$\frac{8\sin(c+dx)}{15ad\left(a^2+b^2\right)^2(a\cos(c+dx)+b\sin(c+dx))}-\frac{4(b\cos(c+dx)-a\sin(c+dx))}{15d\left(a^2+b^2\right)^2(a\cos(c+dx)+b\sin(c+dx))^3}-\frac{b\cos(c+}{5d\left(a^2+b^2\right)(a\,\mathrm{co}}$$

```
[Out] 1/5*(-b*cos(d*x+c)+a*sin(d*x+c))/(a^2+b^2)/d/(a*cos(d*x+c)+b*sin(d*x+c))^5-
4/15*(b*cos(d*x+c)-a*sin(d*x+c))/(a^2+b^2)^2/d/(a*cos(d*x+c)+b*sin(d*x+c))^
3+8/15*sin(d*x+c)/a/(a^2+b^2)^2/d/(a*cos(d*x+c)+b*sin(d*x+c))
```

Rubi [A] time = 0.07, antiderivative size = 151, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {3076, 3075}

$$\frac{8\sin(c+dx)}{15ad\left(a^2+b^2\right)^2(a\cos(c+dx)+b\sin(c+dx))}-\frac{4(b\cos(c+dx)-a\sin(c+dx))}{15d\left(a^2+b^2\right)^2(a\cos(c+dx)+b\sin(c+dx))^3}-\frac{b\cos(c+}{5d\left(a^2+b^2\right)(a\,\mathrm{co}}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Cos[c + d*x] + b*Sin[c + d*x])^(-6),x]
```

```
[Out] -(b*Cos[c + d*x] - a*Sin[c + d*x])/(5*(a^2 + b^2)*d*(a*Cos[c + d*x] + b*Sin
[c + d*x])^5) - (4*(b*Cos[c + d*x] - a*Sin[c + d*x]))/(15*(a^2 + b^2)^2*d*(
a*Cos[c + d*x] + b*Sin[c + d*x])^3) + (8*Sin[c + d*x])/(15*a*(a^2 + b^2)^2*
d*(a*Cos[c + d*x] + b*Sin[c + d*x]))
```

Rule 3075

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-2), x
_Symbol] :> Simp[Sin[c + d*x]/(a*d*(a*Cos[c + d*x] + b*Sin[c + d*x])), x] /
; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

Rule 3076

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x
_Symbol] :> Simp[((b*Cos[c + d*x] - a*Sin[c + d*x])*(a*Cos[c + d*x] + b*Sin
[c + d*x])^(n + 1))/(d*(n + 1)*(a^2 + b^2)), x] + Dist[(n + 2)/((n + 1)*(a^
2 + b^2)), Int[(a*Cos[c + d*x] + b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{
a, b, c, d}, x] && NeQ[a^2 + b^2, 0] && LtQ[n, -1] && NeQ[n, -2]
```

Rubi steps

$$\int \frac{1}{(a\cos(c+dx)+b\sin(c+dx))^6}\,dx = -\frac{b\cos(c+dx)-a\sin(c+dx)}{5\left(a^2+b^2\right)d(a\cos(c+dx)+b\sin(c+dx))^5} + \frac{4\int \frac{1}{(a\cos(c+dx)+b\sin(c+d}}{5\left(a^2+b^2\right)}$$

$$= -\frac{b\cos(c+dx)-a\sin(c+dx)}{5\left(a^2+b^2\right)d(a\cos(c+dx)+b\sin(c+dx))^5} - \frac{4(b\cos(c+dx)}{15\left(a^2+b^2\right)^2 d(a\cos(c}$$

$$= -\frac{b\cos(c+dx)-a\sin(c+dx)}{5\left(a^2+b^2\right)d(a\cos(c+dx)+b\sin(c+dx))^5} - \frac{4(b\cos(c+dx)}{15\left(a^2+b^2\right)^2 d(a\cos(c}$$

Mathematica [A] time = 0.52, size = 182, normalized size = 1.21

$$\frac{10a^4\sin(c+dx)+5a^4\sin(3(c+dx))+a^4\sin(5(c+dx))+\left(4ab^3-4a^3b\right)\cos(5(c+dx))+20a^2b^2\sin(c+dx)-}{30ad\left(a^2+b^2\right)^2(a\cos(c}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Cos[c + d*x] + b*Sin[c + d*x])^(-6),x]
```

```
[Out] (-10*a*b*(a^2 + b^2)*Cos[3*(c + d*x)] + (-4*a^3*b + 4*a*b^3)*Cos[5*(c + d*x
)] + 10*a^4*Sin[c + d*x] + 20*a^2*b^2*Sin[c + d*x] + 10*b^4*Sin[c + d*x] +
5*a^4*Sin[3*(c + d*x)] - 5*b^4*Sin[3*(c + d*x)] + a^4*Sin[5*(c + d*x)] - 6*
a^2*b^2*Sin[5*(c + d*x)] + b^4*Sin[5*(c + d*x)])/(30*a*(a^2 + b^2)^2*d*(a*C
os[c + d*x] + b*Sin[c + d*x])^5)
```

fricas [B] time = 2.05, size = 441, normalized size = 2.92

$$-\frac{8\left(5\,a^4b-10\,a^2b^3+b^5\right)\cos\left(dx+c\right)^5-20\left(a^4b-6\,a\right.}{15\left(\left(a^{11}-7\,a^9b^2-22\,a^7b^4-14\,a^5b^6+5\,a^3b^8+5\,ab^{10}\right)d\cos\left(dx+c\right)^5+10\left(a^9b^2+2\,a^7b^4-2\,a^3b^8-ab^{10}\right)d\cos\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cos(d*x+c)+b*sin(d*x+c))^6,x, algorithm="fricas")
```

```
[Out] -1/15*(8*(5*a^4*b - 10*a^2*b^3 + b^5)*cos(d*x + c)^5 - 20*(a^4*b - 6*a^2*b^
3 + b^5)*cos(d*x + c)^3 - 5*(a^4*b + 6*a^2*b^3 - 3*b^5)*cos(d*x + c) - (3*a
^5 + 10*a^3*b^2 + 15*a*b^4 + 8*(a^5 - 10*a^3*b^2 + 5*a*b^4)*cos(d*x + c)^4
+ 4*(a^5 + 10*a^3*b^2 - 15*a*b^4)*cos(d*x + c)^2)*sin(d*x + c))/((a^11 - 7*
a^9*b^2 - 22*a^7*b^4 - 14*a^5*b^6 + 5*a^3*b^8 + 5*a*b^10)*d*cos(d*x + c)^5
+ 10*(a^9*b^2 + 2*a^7*b^4 - 2*a^3*b^8 - a*b^10)*d*cos(d*x + c)^3 + 5*(a^7*b
^4 + 3*a^5*b^6 + 3*a^3*b^8 + a*b^10)*d*cos(d*x + c) + ((5*a^10*b + 5*a^8*b^
```

```
3 - 14*a^6*b^5 - 22*a^4*b^7 - 7*a^2*b^9 + b^11)*d*cos(d*x + c)^4 + 2*(5*a^8
*b^3 + 14*a^6*b^5 + 12*a^4*b^7 + 2*a^2*b^9 - b^11)*d*cos(d*x + c)^2 + (a^6*
b^5 + 3*a^4*b^7 + 3*a^2*b^9 + b^11)*d)*sin(d*x + c))
```

giac [A] time = 0.22, size = 118, normalized size = 0.78

$$-\frac{15\,b^4\tan\left(dx+c\right)^4+30\,ab^3\tan\left(dx+c\right)^3+30\,a^2b^2\tan\left(dx+c\right)^2+10\,b^4\tan\left(dx+c\right)^2+15\,a^3b\tan\left(dx+c\right)+5\,a}{15\left(b\tan\left(dx+c\right)+a\right)^5b^5d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cos(d*x+c)+b*sin(d*x+c))^6,x, algorithm="giac")
```

```
[Out] -1/15*(15*b^4*tan(d*x + c)^4 + 30*a*b^3*tan(d*x + c)^3 + 30*a^2*b^2*tan(d*x
 + c)^2 + 10*b^4*tan(d*x + c)^2 + 15*a^3*b*tan(d*x + c) + 5*a*b^3*tan(d*x +
 c) + 3*a^4 + a^2*b^2 + 3*b^4)/((b*tan(d*x + c) + a)^5*b^5*d)
```

maple [A] time = 0.64, size = 125, normalized size = 0.83

$$\frac{-\frac{a^4+2a^2b^2+b^4}{5b^5(a+b\tan(dx+c))^5}+\frac{a\left(a^2+b^2\right)}{b^5(a+b\tan(dx+c))^4}-\frac{6a^2+2b^2}{3b^5(a+b\tan(dx+c))^3}+\frac{2a}{b^5(a+b\tan(dx+c))^2}-\frac{1}{b^5(a+b\tan(dx+c))}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*cos(d*x+c)+b*sin(d*x+c))^6,x)
```

```
[Out] 1/d*(-1/5*(a^4+2*a^2*b^2+b^4)/b^5/(a+b*tan(d*x+c))^5+a*(a^2+b^2)/b^5/(a+b*t
an(d*x+c))^4-1/3*(6*a^2+2*b^2)/b^5/(a+b*tan(d*x+c))^3+2*a/b^5/(a+b*tan(d*x+
c))^2-1/b^5/(a+b*tan(d*x+c)))
```

maxima [A] time = 0.79, size = 174, normalized size = 1.15

$$-\frac{15\,b^4\tan\left(dx+c\right)^4+30\,ab^3\tan\left(dx+c\right)^3+3\,a^4+a^2b^2+3\,b^4+10\left(3\,a^2b^2+b^4\right)\tan\left(dx+c\right)^2+5\left(3\,a^3b+ab^3\right)}{15\left(b^{10}\tan\left(dx+c\right)^5+5\,ab^9\tan\left(dx+c\right)^4+10\,a^2b^8\tan\left(dx+c\right)^3+10\,a^3b^7\tan\left(dx+c\right)^2+5\,a^4b^6\tan\left(dx+c\right)\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cos(d*x+c)+b*sin(d*x+c))^6,x, algorithm="maxima")
```

```
[Out] -1/15*(15*b^4*tan(d*x + c)^4 + 30*a*b^3*tan(d*x + c)^3 + 3*a^4 + a^2*b^2 +
3*b^4 + 10*(3*a^2*b^2 + b^4)*tan(d*x + c)^2 + 5*(3*a^3*b + a*b^3)*tan(d*x +
 c))/((b^10*tan(d*x + c)^5 + 5*a*b^9*tan(d*x + c)^4 + 10*a^2*b^8*tan(d*x +
c)^3 + 10*a^3*b^7*tan(d*x + c)^2 + 5*a^4*b^6*tan(d*x + c) + a^5*b^5)*d)
```

mupad [B] time = 5.15, size = 470, normalized size = 3.11

$$\frac{\frac{2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^9}{a}+\frac{2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{a}-}{d\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4\left(10a^5-120a^3b^2+80ab^4\right)-\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3\left(40a^4b-80a^2b^3\right)-\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^7\left(40a^4b-8\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*cos(c + d*x) + b*sin(c + d*x))^6,x)
```

```
[Out] ((2*tan(c/2 + (d*x)/2)^9)/a + (2*tan(c/2 + (d*x)/2))/a - (8*tan(c/2 + (d*x)
/2)^4*(7*a^2*b - 6*b^3))/(3*a^4) + (8*tan(c/2 + (d*x)/2)^6*(7*a^2*b - 6*b^3
))/(3*a^4) - (8*tan(c/2 + (d*x)/2)^3*(a^2 - 6*b^2))/(3*a^3) - (8*tan(c/2 +
(d*x)/2)^7*(a^2 - 6*b^2))/(3*a^3) + (8*b*tan(c/2 + (d*x)/2)^2)/a^2 - (8*b*t
an(c/2 + (d*x)/2)^8)/a^2 + (4*tan(c/2 + (d*x)/2)^5*(29*a^4 + 24*b^4 - 112*a
^2*b^2))/(15*a^5))/(d*(tan(c/2 + (d*x)/2)^4*(80*a*b^4 + 10*a^5 - 120*a^3*b^
2) - tan(c/2 + (d*x)/2)^3*(40*a^4*b - 80*a^2*b^3) - tan(c/2 + (d*x)/2)^7*(4
0*a^4*b - 80*a^2*b^3) - a^5*tan(c/2 + (d*x)/2)^10 - tan(c/2 + (d*x)/2)^6*(8
0*a*b^4 + 10*a^5 - 120*a^3*b^2) + tan(c/2 + (d*x)/2)^5*(60*a^4*b + 32*b^5 -
 160*a^2*b^3) + a^5 - tan(c/2 + (d*x)/2)^2*(5*a^5 - 40*a^3*b^2) + tan(c/2 +
 (d*x)/2)^8*(5*a^5 - 40*a^3*b^2) + 10*a^4*b*tan(c/2 + (d*x)/2) + 10*a^4*b*t
an(c/2 + (d*x)/2)^9))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cos(d*x+c)+b*sin(d*x+c))**6,x)
```

```
[Out] Timed out
```

3.232 $\int (a\cos(c+dx)+b\sin(c+dx))^{7/2}\,dx$

Optimal. Leaf size=186

$$\frac{10\left(a^2+b^2\right)^2\sqrt{\frac{a\cos(c+dx)+b\sin(c+dx)}{\sqrt{a^2+b^2}}}\,F\left(\frac{1}{2}\left(c+dx-\tan^{-1}(a,b)\right)\middle|2\right)}{21d\sqrt{a\cos(c+dx)+b\sin(c+dx)}}-\frac{10\left(a^2+b^2\right)(b\cos(c+dx)-a\sin(c+dx))\sqrt{a}}{21d}$$

```
[Out] -2/7*(b*cos(d*x+c)-a*sin(d*x+c))*(a*cos(d*x+c)+b*sin(d*x+c))^(5/2)/d-10/21*
(a^2+b^2)*(b*cos(d*x+c)-a*sin(d*x+c))*(a*cos(d*x+c)+b*sin(d*x+c))^(1/2)/d+1
0/21*(a^2+b^2)^2*(cos(1/2*c+1/2*d*x-1/2*arctan(a,b))^2)^(1/2)/cos(1/2*c+1/2
*d*x-1/2*arctan(a,b))*EllipticF(sin(1/2*c+1/2*d*x-1/2*arctan(a,b)),2^(1/2))
*((a*cos(d*x+c)+b*sin(d*x+c))/(a^2+b^2)^(1/2))^(1/2)/d/(a*cos(d*x+c)+b*sin(
d*x+c))^(1/2)
```

Rubi [A] time = 0.10, antiderivative size = 186, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3073, 3078, 2641}

$$\frac{10\left(a^2+b^2\right)^2\sqrt{\frac{a\cos(c+dx)+b\sin(c+dx)}{\sqrt{a^2+b^2}}}\,F\left(\frac{1}{2}\left(c+dx-\tan^{-1}(a,b)\right)\middle|2\right)}{21d\sqrt{a\cos(c+dx)+b\sin(c+dx)}}-\frac{10\left(a^2+b^2\right)(b\cos(c+dx)-a\sin(c+dx))\sqrt{a}}{21d}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Cos[c + d*x] + b*Sin[c + d*x])^(7/2),x]
```

```
[Out] (-10*(a^2 + b^2)*(b*Cos[c + d*x] - a*Sin[c + d*x])*Sqrt[a*Cos[c + d*x] + b*
Sin[c + d*x]])/(21*d) - (2*(b*Cos[c + d*x] - a*Sin[c + d*x])*(a*Cos[c + d*x
] + b*Sin[c + d*x])^(5/2))/(7*d) + (10*(a^2 + b^2)^2*EllipticF[(c + d*x - A
rcTan[a, b])/2, 2]*Sqrt[(a*Cos[c + d*x] + b*Sin[c + d*x])/Sqrt[a^2 + b^2]])
/(21*d*Sqrt[a*Cos[c + d*x] + b*Sin[c + d*x]])
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3073

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x
_Symbol] :> -Simp[((b*Cos[c + d*x] - a*Sin[c + d*x])*(a*Cos[c + d*x] + b*Si
n[c + d*x])^(n - 1))/(d*n), x] + Dist[((n - 1)*(a^2 + b^2))/n, Int[(a*Cos[c
 + d*x] + b*Sin[c + d*x])^(n - 2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 + b^2, 0] &&  !IntegerQ[(n - 1)/2] && GtQ[n, 1]
```

Rule 3078

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x
_Symbol] :> Dist[(a*Cos[c + d*x] + b*Sin[c + d*x])^n/((a*Cos[c + d*x] + b*S
in[c + d*x])/Sqrt[a^2 + b^2])^n, Int[Cos[c + d*x - ArcTan[a, b]]^n, x], x]
/; FreeQ[{a, b, c, d, n}, x] && !(GeQ[n, 1] || LeQ[n, -1]) && !(GtQ[a^2 +
 b^2, 0] || EqQ[a^2 + b^2, 0])
```

Rubi steps

$$\int (a\cos(c+dx)+b\sin(c+dx))^{7/2}\,dx = -\frac{2(b\cos(c+dx)-a\sin(c+dx))(a\cos(c+dx)+b\sin(c+dx))^{5/2}}{7d}+\frac{1}{7}$$

$$= -\frac{10\left(a^2+b^2\right)(b\cos(c+dx)-a\sin(c+dx))\sqrt{a\cos(c+dx)+b\sin(c+}}{21d}$$

$$= -\frac{10\left(a^2+b^2\right)(b\cos(c+dx)-a\sin(c+dx))\sqrt{a\cos(c+dx)+b\sin(c+}}{21d}$$

$$= -\frac{10\left(a^2+b^2\right)(b\cos(c+dx)-a\sin(c+dx))\sqrt{a\cos(c+dx)+b\sin(c+}}{21d}$$

Mathematica [C] time = 1.82, size = 205, normalized size = 1.10

$$\frac{\frac{20\left(a^2+b^2\right)^2\tan\left(\tan^{-1}\left(\frac{a}{b}\right)+c+dx\right)\sqrt{\cos^2\left(\tan^{-1}\left(\frac{a}{b}\right)+c+dx\right)}\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};\sin^2\left(c+dx+\tan^{-1}\left(\frac{a}{b}\right)\right)\right)}{\sqrt{b\sqrt{\frac{a^2}{b^2}+1}\sin\left(\tan^{-1}\left(\frac{a}{b}\right)+c+dx\right)}}+\sqrt{a\cos(c+dx)+b\sin(c+dx)}\,\left(\left(3b^3\right.\right.}{42}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Cos[c + d*x] + b*Sin[c + d*x])^(7/2),x]
```

```
[Out] (Sqrt[a*Cos[c + d*x] + b*Sin[c + d*x]]*(-23*b*(a^2 + b^2)*Cos[c + d*x] + (-
9*a^2*b + 3*b^3)*Cos[3*(c + d*x)] + 2*a*(13*a^2 + 7*b^2 + 3*(a^2 - 3*b^2)*C
os[2*(c + d*x)])*Sin[c + d*x]) + (20*(a^2 + b^2)^2*Sqrt[Cos[c + d*x + ArcTa
n[a/b]]^2]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[c + d*x + ArcTan[a/b]]^
2]*Tan[c + d*x + ArcTan[a/b]])/Sqrt[Sqrt[1 + a^2/b^2]*b*Sin[c + d*x + ArcTa
n[a/b]]])/(42*d)
```

fricas [F] time = 1.10, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(3\,ab^2\cos(dx+c)+\left(a^3-3\,ab^2\right)\cos(dx+c)^3+\left(b^3+\left(3\,a^2b-b^3\right)\cos(dx+c)^2\right)\sin(dx+c)\right)\sqrt{a\cos}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cos(d*x+c)+b*sin(d*x+c))^(7/2),x, algorithm="fricas")
```

```
[Out] integral((3*a*b^2*cos(d*x + c) + (a^3 - 3*a*b^2)*cos(d*x + c)^3 + (b^3 + (3
*a^2*b - b^3)*cos(d*x + c)^2)*sin(d*x + c))*sqrt(a*cos(d*x + c) + b*sin(d*x
 + c)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a\cos(dx+c)+b\sin(dx+c))^{\frac{7}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cos(d*x+c)+b*sin(d*x+c))^(7/2),x, algorithm="giac")
```

```
[Out] integrate((a*cos(d*x + c) + b*sin(d*x + c))^(7/2), x)
```

maple [A] time = 0.44, size = 183, normalized size = 0.98

$$\frac{\left(a^2+b^2\right)^2\left(6\left(\sin^5\left(dx+c-\arctan(-a,b)\right)\right)+5\sqrt{1+\sin\left(dx+c-\arctan(-a,b)\right)}\,\sqrt{-2\sin\left(dx+c-\arctan(-a,\right.}\right.}{21\cos(dx+}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*cos(d*x+c)+b*sin(d*x+c))^(7/2),x)
```

```
[Out] 1/21*(a^2+b^2)^2*(6*sin(d*x+c-arctan(-a,b))^5+5*(1+sin(d*x+c-arctan(-a,b)))
^(1/2)*(-2*sin(d*x+c-arctan(-a,b))+2)^(1/2)*(-sin(d*x+c-arctan(-a,b)))^(1/2
)*EllipticF((1+sin(d*x+c-arctan(-a,b)))^(1/2),1/2*2^(1/2))+4*sin(d*x+c-arct
an(-a,b))^3-10*sin(d*x+c-arctan(-a,b)))/cos(d*x+c-arctan(-a,b))/(sin(d*x+c-
arctan(-a,b))*(a^2+b^2)^(1/2))^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a\cos(dx+c)+b\sin(dx+c))^{\frac{7}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cos(d*x+c)+b*sin(d*x+c))^(7/2),x, algorithm="maxima")
```

```
[Out] integrate((a*cos(d*x + c) + b*sin(d*x + c))^(7/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int {\left(a\,\cos\left(c+d\,x\right)+b\,\sin\left(c+d\,x\right)\right)}^{7/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*cos(c + d*x) + b*sin(c + d*x))^(7/2),x)
```

```
[Out] int((a*cos(c + d*x) + b*sin(c + d*x))^(7/2), x)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cos(d*x+c)+b*sin(d*x+c))**(7/2),x)
```

```
[Out] Timed out
```

3.233 $\int (a\cos(c+dx)+b\sin(c+dx))^{5/2}\,dx$

Optimal. Leaf size=131

$$\frac{6\left(a^2+b^2\right)\sqrt{a\cos(c+dx)+b\sin(c+dx)}\,E\left(\frac{1}{2}\left(c+dx-\tan^{-1}(a,b)\right)\middle|2\right)}{5d\sqrt{\frac{a\cos(c+dx)+b\sin(c+dx)}{\sqrt{a^2+b^2}}}}-\frac{2(b\cos(c+dx)-a\sin(c+dx))(a\cos(c+dx)+b\sin(c+dx))^{3/2}}{5d}$$

```
[Out] -2/5*(b*cos(d*x+c)-a*sin(d*x+c))*(a*cos(d*x+c)+b*sin(d*x+c))^(3/2)/d+6/5*(a
^2+b^2)*(cos(1/2*c+1/2*d*x-1/2*arctan(a,b))^2)^(1/2)/cos(1/2*c+1/2*d*x-1/2*
arctan(a,b))*EllipticE(sin(1/2*c+1/2*d*x-1/2*arctan(a,b)),2^(1/2))*(a*cos(d
*x+c)+b*sin(d*x+c))^(1/2)/d/((a*cos(d*x+c)+b*sin(d*x+c))/(a^2+b^2)^(1/2))^(
1/2)
```

Rubi [A] time = 0.06, antiderivative size = 131, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3073, 3078, 2639}

$$\frac{6\left(a^2+b^2\right)\sqrt{a\cos(c+dx)+b\sin(c+dx)}\,E\left(\frac{1}{2}\left(c+dx-\tan^{-1}(a,b)\right)\middle|2\right)}{5d\sqrt{\frac{a\cos(c+dx)+b\sin(c+dx)}{\sqrt{a^2+b^2}}}}-\frac{2(b\cos(c+dx)-a\sin(c+dx))(a\cos(c+dx)+b\sin(c+dx))^{3/2}}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Cos[c + d*x] + b*Sin[c + d*x])^(5/2),x]
```

```
[Out] (-2*(b*Cos[c + d*x] - a*Sin[c + d*x])*(a*Cos[c + d*x] + b*Sin[c + d*x])^(3/
2))/(5*d) + (6*(a^2 + b^2)*EllipticE[(c + d*x - ArcTan[a, b])/2, 2]*Sqrt[a*
Cos[c + d*x] + b*Sin[c + d*x]])/(5*d*Sqrt[(a*Cos[c + d*x] + b*Sin[c + d*x])
/Sqrt[a^2 + b^2]])
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3073

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x
_Symbol] :> -Simp[((b*Cos[c + d*x] - a*Sin[c + d*x])*(a*Cos[c + d*x] + b*Si
n[c + d*x])^(n - 1))/(d*n), x] + Dist[((n - 1)*(a^2 + b^2))/n, Int[(a*Cos[c
 + d*x] + b*Sin[c + d*x])^(n - 2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 + b^2, 0] &&  !IntegerQ[(n - 1)/2] && GtQ[n, 1]
```

Rule 3078

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x
_Symbol] :> Dist[(a*Cos[c + d*x] + b*Sin[c + d*x])^n/((a*Cos[c + d*x] + b*S
in[c + d*x])/Sqrt[a^2 + b^2])^n, Int[Cos[c + d*x - ArcTan[a, b]]^n, x], x]
/; FreeQ[{a, b, c, d, n}, x] &&  !(GeQ[n, 1] || LeQ[n, -1]) &&  !(GtQ[a^2 +
 b^2, 0] || EqQ[a^2 + b^2, 0])
```

Rubi steps

$$\int (a\cos(c+dx)+b\sin(c+dx))^{5/2}\,dx = -\frac{2(b\cos(c+dx)-a\sin(c+dx))(a\cos(c+dx)+b\sin(c+dx))^{3/2}}{5d} + \frac{1}{5}$$

$$= -\frac{2(b\cos(c+dx)-a\sin(c+dx))(a\cos(c+dx)+b\sin(c+dx))^{3/2}}{5d} + \frac{(}{}$$

$$= -\frac{2(b\cos(c+dx)-a\sin(c+dx))(a\cos(c+dx)+b\sin(c+dx))^{3/2}}{5d} + \frac{6}{}$$

Mathematica [C] time = 1.60, size = 256, normalized size = 1.95

$$\sqrt{a\cos(c+dx)+b\sin(c+dx)}\,\left(b\left(a^2-b^2\right)\sin(2(c+dx))+6a\left(a^2+b^2\right)-2ab^2\cos(2(c+dx))\right)-\frac{3\left(a^2+b^2\right)^2\cos\left(-\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Cos[c + d*x] + b*Sin[c + d*x])^(5/2),x]
```

```
[Out] (Sqrt[a*Cos[c + d*x] + b*Sin[c + d*x]]*(6*a*(a^2 + b^2) - 2*a*b^2*Cos[2*(c
+ d*x)] + b*(a^2 - b^2)*Sin[2*(c + d*x)]) - (3*(a^2 + b^2)^2*Cos[c + d*x -
ArcTan[b/a]]*(b*HypergeometricPFQ[{-1/2, -1/4}, {3/4}, Cos[c + d*x - ArcTan
[b/a]]^2]*Sin[c + d*x - ArcTan[b/a]] + Sqrt[Sin[c + d*x - ArcTan[b/a]]^2]*(
2*a*Cos[c + d*x - ArcTan[b/a]] - b*Sin[c + d*x - ArcTan[b/a]])))/((a*Sqrt[1
 + b^2/a^2]*Cos[c + d*x - ArcTan[b/a]])^(3/2)*Sqrt[Sin[c + d*x - ArcTan[b/a
]]^2]))/(5*b*d)
```

fricas [F] time = 0.93, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(2\,ab\cos(dx+c)\sin(dx+c)+\left(a^2-b^2\right)\cos(dx+c)^2+b^2\right)\sqrt{a\cos(dx+c)+b\sin(dx+c)}\,,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cos(d*x+c)+b*sin(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] integral((2*a*b*cos(d*x + c)*sin(d*x + c) + (a^2 - b^2)*cos(d*x + c)^2 + b^
2)*sqrt(a*cos(d*x + c) + b*sin(d*x + c)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a\cos(dx+c)+b\sin(dx+c))^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cos(d*x+c)+b*sin(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((a*cos(d*x + c) + b*sin(d*x + c))^(5/2), x)
```

maple [A] time = 0.37, size = 246, normalized size = 1.88

$$-\frac{\left(a^{2}+b^{2}\right)^{\frac{3}{2}}\left(6\sqrt{1+\sin(dx+c-\arctan(-a,b))}\,\sqrt{-2\sin(dx+c-\arctan(-a,b))+2}\,\sqrt{-\sin(dx+c-\arctan(}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*cos(d*x+c)+b*sin(d*x+c))^(5/2),x)
```

```
[Out] -1/5*(a^2+b^2)^(3/2)*(6*(1+sin(d*x+c-arctan(-a,b)))^(1/2)*(-2*sin(d*x+c-arc
tan(-a,b))+2)^(1/2)*(-sin(d*x+c-arctan(-a,b)))^(1/2)*EllipticE((1+sin(d*x+c
-arctan(-a,b)))^(1/2),1/2*2^(1/2))-3*(1+sin(d*x+c-arctan(-a,b)))^(1/2)*(-2*
sin(d*x+c-arctan(-a,b))+2)^(1/2)*(-sin(d*x+c-arctan(-a,b)))^(1/2)*EllipticF
((1+sin(d*x+c-arctan(-a,b)))^(1/2),1/2*2^(1/2))-2*sin(d*x+c-arctan(-a,b))^4
+2*sin(d*x+c-arctan(-a,b))^2)/cos(d*x+c-arctan(-a,b))/(sin(d*x+c-arctan(-a,
b))*(a^2+b^2)^(1/2))^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a\cos(dx+c)+b\sin(dx+c))^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cos(d*x+c)+b*sin(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((a*cos(d*x + c) + b*sin(d*x + c))^(5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int (a\cos(c+dx)+b\sin(c+dx))^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*cos(c + d*x) + b*sin(c + d*x))^(5/2),x)
[Out] int((a*cos(c + d*x) + b*sin(c + d*x))^(5/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cos(d*x+c)+b*sin(d*x+c))**(5/2),x)
[Out] Timed out
```

3.234 $\int (a\cos(c+dx)+b\sin(c+dx))^{3/2}\,dx$

Optimal. Leaf size=131

$$\frac{2\left(a^2+b^2\right)\sqrt{\frac{a\cos(c+dx)+b\sin(c+dx)}{\sqrt{a^2+b^2}}}\,F\left(\frac{1}{2}\left(c+dx-\tan^{-1}(a,b)\right)\middle|2\right)}{3d\sqrt{a\cos(c+dx)+b\sin(c+dx)}}-\frac{2(b\cos(c+dx)-a\sin(c+dx))\sqrt{a\cos(c+dx)+}}{3d}$$

```
[Out] -2/3*(b*cos(d*x+c)-a*sin(d*x+c))*(a*cos(d*x+c)+b*sin(d*x+c))^(1/2)/d+2/3*(a
^2+b^2)*(cos(1/2*c+1/2*d*x-1/2*arctan(a,b))^2)^(1/2)/cos(1/2*c+1/2*d*x-1/2*
arctan(a,b))*EllipticF(sin(1/2*c+1/2*d*x-1/2*arctan(a,b)),2^(1/2))*((a*cos(
d*x+c)+b*sin(d*x+c))/(a^2+b^2)^(1/2))^(1/2)/d/(a*cos(d*x+c)+b*sin(d*x+c))^(
1/2)
```

Rubi [A] time = 0.06, antiderivative size = 131, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3073, 3078, 2641}

$$\frac{2\left(a^2+b^2\right)\sqrt{\frac{a\cos(c+dx)+b\sin(c+dx)}{\sqrt{a^2+b^2}}}\,F\left(\frac{1}{2}\left(c+dx-\tan^{-1}(a,b)\right)\middle|2\right)}{3d\sqrt{a\cos(c+dx)+b\sin(c+dx)}}-\frac{2(b\cos(c+dx)-a\sin(c+dx))\sqrt{a\cos(c+dx)+}}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Cos[c + d*x] + b*Sin[c + d*x])^(3/2),x]
```

```
[Out] (-2*(b*Cos[c + d*x] - a*Sin[c + d*x])*Sqrt[a*Cos[c + d*x] + b*Sin[c + d*x]]
)/(3*d) + (2*(a^2 + b^2)*EllipticF[(c + d*x - ArcTan[a, b])/2, 2]*Sqrt[(a*C
os[c + d*x] + b*Sin[c + d*x])/Sqrt[a^2 + b^2]])/(3*d*Sqrt[a*Cos[c + d*x] +
b*Sin[c + d*x]])
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3073

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x
_Symbol] :> -Simp[((b*Cos[c + d*x] - a*Sin[c + d*x])*(a*Cos[c + d*x] + b*Si
n[c + d*x])^(n - 1))/(d*n), x] + Dist[((n - 1)*(a^2 + b^2))/n, Int[(a*Cos[c
 + d*x] + b*Sin[c + d*x])^(n - 2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 + b^2, 0] &&  !IntegerQ[(n - 1)/2] && GtQ[n, 1]
```

Rule 3078

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x
_Symbol] :> Dist[(a*Cos[c + d*x] + b*Sin[c + d*x])^n/((a*Cos[c + d*x] + b*S
in[c + d*x])/Sqrt[a^2 + b^2])^n, Int[Cos[c + d*x - ArcTan[a, b]]^n, x], x]
/; FreeQ[{a, b, c, d, n}, x] && !(GeQ[n, 1] || LeQ[n, -1]) && !(GtQ[a^2 +
 b^2, 0] || EqQ[a^2 + b^2, 0])
```

Rubi steps

$$\int (a\cos(c+dx)+b\sin(c+dx))^{3/2}\,dx = -\frac{2(b\cos(c+dx)-a\sin(c+dx))\sqrt{a\cos(c+dx)+b\sin(c+dx)}}{3d} + \frac{1}{3}\Big($$

$$= -\frac{2(b\cos(c+dx)-a\sin(c+dx))\sqrt{a\cos(c+dx)+b\sin(c+dx)}}{3d} + \frac{\Big(\Big(a}{}$$

$$= -\frac{2(b\cos(c+dx)-a\sin(c+dx))\sqrt{a\cos(c+dx)+b\sin(c+dx)}}{3d} + \frac{2\Big(}{}$$

Mathematica [C] time = 1.29, size = 143, normalized size = 1.09

$$\frac{2\left(\frac{(a^2+b^2)\tan\left(\tan^{-1}\left(\frac{a}{b}\right)+c+dx\right)\sqrt{\cos^2\left(\tan^{-1}\left(\frac{a}{b}\right)+c+dx\right)}\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};\sin^2\left(c+dx+\tan^{-1}\left(\frac{a}{b}\right)\right)\right)}{\sqrt{b\sqrt{\frac{a^2}{b^2}+1}\,\sin\left(\tan^{-1}\left(\frac{a}{b}\right)+c+dx\right)}} + \sqrt{a\cos(c+dx)+b\sin(c+dx)}\,(a\sin(\right.}{3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Cos[c + d*x] + b*Sin[c + d*x])^(3/2),x]
```

```
[Out] (2*((-(b*Cos[c + d*x]) + a*Sin[c + d*x])*Sqrt[a*Cos[c + d*x] + b*Sin[c + d*
x]] + ((a^2 + b^2)*Sqrt[Cos[c + d*x + ArcTan[a/b]]^2]*HypergeometricPFQ[{1/
4, 1/2}, {5/4}, Sin[c + d*x + ArcTan[a/b]]^2]*Tan[c + d*x + ArcTan[a/b]])/S
qrt[Sqrt[1 + a^2/b^2]*b*Sin[c + d*x + ArcTan[a/b]]]))/(3*d)
```

fricas [F] time = 0.81, size = 0, normalized size = 0.00

$$\text{integral}\left((a\cos(dx+c)+b\sin(dx+c))^{\frac{3}{2}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cos(d*x+c)+b*sin(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] integral((a*cos(d*x + c) + b*sin(d*x + c))^(3/2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a\cos(dx+c)+b\sin(dx+c))^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cos(d*x+c)+b*sin(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((a*cos(d*x + c) + b*sin(d*x + c))^(3/2), x)
```

maple [A] time = 0.37, size = 163, normalized size = 1.24

$$\frac{\left(a^{2}+b^{2}\right)\left(\sqrt{1+\sin(dx+c-\arctan(-a,b))}\,\sqrt{-2\sin(dx+c-\arctan(-a,b))+2}\,\sqrt{-\sin(dx+c-\arctan(-a,}\right.}{3\cos(dx+c-\arctan(-a,b))}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*cos(d*x+c)+b*sin(d*x+c))^(3/2),x)
```

```
[Out] 1/3*(a^2+b^2)*((1+sin(d*x+c-arctan(-a,b)))^(1/2)*(-2*sin(d*x+c-arctan(-a,b)
)+2)^(1/2)*(-sin(d*x+c-arctan(-a,b)))^(1/2)*EllipticF((1+sin(d*x+c-arctan(-
a,b)))^(1/2),1/2*2^(1/2))+2*sin(d*x+c-arctan(-a,b))^3-2*sin(d*x+c-arctan(-a
,b)))/cos(d*x+c-arctan(-a,b))/(sin(d*x+c-arctan(-a,b))*(a^2+b^2)^(1/2))^(1/
2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a\cos(dx+c)+b\sin(dx+c))^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cos(d*x+c)+b*sin(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((a*cos(d*x + c) + b*sin(d*x + c))^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int (a\,\cos(c+d\,x)+b\,\sin(c+d\,x))^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*cos(c + d*x) + b*sin(c + d*x))^(3/2),x)
[Out] int((a*cos(c + d*x) + b*sin(c + d*x))^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a\cos(c+dx) + b\sin(c+dx))^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cos(d*x+c)+b*sin(d*x+c))**(3/2),x)
[Out] Integral((a*cos(c + d*x) + b*sin(c + d*x))**(3/2), x)
```

3.235 $\int \sqrt{a\cos(c+dx)+b\sin(c+dx)}\,dx$

Optimal. Leaf size=75

$$\frac{2\sqrt{a\cos(c+dx)+b\sin(c+dx)}\,E\left(\frac{1}{2}\left(c+dx-\tan^{-1}(a,b)\right)\middle|2\right)}{d\sqrt{\frac{a\cos(c+dx)+b\sin(c+dx)}{\sqrt{a^2+b^2}}}}$$

```
[Out] 2*(cos(1/2*c+1/2*d*x-1/2*arctan(a,b))^2)^(1/2)/cos(1/2*c+1/2*d*x-1/2*arctan
(a,b))*EllipticE(sin(1/2*c+1/2*d*x-1/2*arctan(a,b)),2^(1/2))*(a*cos(d*x+c)+
b*sin(d*x+c))^(1/2)/d/((a*cos(d*x+c)+b*sin(d*x+c))/(a^2+b^2)^(1/2))^(1/2)
```

Rubi [A] time = 0.03, antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {3078, 2639}

$$\frac{2\sqrt{a\cos(c+dx)+b\sin(c+dx)}\,E\left(\frac{1}{2}\left(c+dx-\tan^{-1}(a,b)\right)\middle|2\right)}{d\sqrt{\frac{a\cos(c+dx)+b\sin(c+dx)}{\sqrt{a^2+b^2}}}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a*Cos[c + d*x] + b*Sin[c + d*x]],x]
```

```
[Out] (2*EllipticE[(c + d*x - ArcTan[a, b])/2, 2]*Sqrt[a*Cos[c + d*x] + b*Sin[c +
 d*x]])/(d*Sqrt[(a*Cos[c + d*x] + b*Sin[c + d*x])/Sqrt[a^2 + b^2]])
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3078

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x
_Symbol] :> Dist[(a*Cos[c + d*x] + b*Sin[c + d*x])^n/((a*Cos[c + d*x] + b*S
in[c + d*x])/Sqrt[a^2 + b^2])^n, Int[Cos[c + d*x - ArcTan[a, b]]^n, x], x]
/; FreeQ[{a, b, c, d, n}, x] &&  !(GeQ[n, 1] || LeQ[n, -1]) &&  !(GtQ[a^2 +
 b^2, 0] || EqQ[a^2 + b^2, 0])
```

Rubi steps

$$\int \sqrt{a\cos(c+dx)+b\sin(c+dx)}\,dx=\frac{\sqrt{a\cos(c+dx)+b\sin(c+dx)}\int\sqrt{\cos\left(c+dx-\tan^{-1}(a,b)\right)}\,dx}{\sqrt{\frac{a\cos(c+dx)+b\sin(c+dx)}{\sqrt{a^2+b^2}}}}$$
$$=\frac{2E\left(\frac{1}{2}\left(c+dx-\tan^{-1}(a,b)\right)\middle|2\right)\sqrt{a\cos(c+dx)+b\sin(c+dx)}}{d\sqrt{\frac{a\cos(c+dx)+b\sin(c+dx)}{\sqrt{a^2+b^2}}}}$$

Mathematica [C] time = 1.10, size = 268, normalized size = 3.57

$$\frac{\cos\left(-\tan^{-1}\left(\frac{b}{a}\right)+c+dx\right)\left(\sqrt{\sin^2\left(-\tan^{-1}\left(\frac{b}{a}\right)+c+dx\right)}\left(b\left(a^2+b^2\right)\sin\left(-\tan^{-1}\left(\frac{b}{a}\right)+c+dx\right)-2a\left(a^2+b^2\right)\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a*Cos[c + d*x] + b*Sin[c + d*x]],x]
```

```
[Out] (Cos[c + d*x - ArcTan[b/a]]*(-(b*(a^2 + b^2)*HypergeometricPFQ[{-1/2, -1/4}
, {3/4}, Cos[c + d*x - ArcTan[b/a]]^2]*Sin[c + d*x - ArcTan[b/a]]) + Sqrt[S
in[c + d*x - ArcTan[b/a]]^2]*(-2*a*(a^2 + b^2)*Cos[c + d*x - ArcTan[b/a]] +
 2*a^2*Sqrt[1 + b^2/a^2]*Sqrt[a*Sqrt[1 + b^2/a^2]*Cos[c + d*x - ArcTan[b/a]
]]*Sqrt[a*Cos[c + d*x]] + b*Sin[c + d*x]] + b*(a^2 + b^2)*Sin[c + d*x - ArcT
an[b/a]])))/(b*d*(a*Sqrt[1 + b^2/a^2]*Cos[c + d*x - ArcTan[b/a]])^(3/2)*Sqr
t[Sin[c + d*x - ArcTan[b/a]]^2])
```

fricas [F] time = 0.80, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\sqrt{a\cos\left(dx+c\right)+b\sin\left(dx+c\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cos(d*x+c)+b*sin(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(a*cos(d*x + c) + b*sin(d*x + c)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{a\cos\left(dx+c\right)+b\sin\left(dx+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cos(d*x+c)+b*sin(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(a*cos(d*x + c) + b*sin(d*x + c)), x)
```

maple [A] time = 0.36, size = 159, normalized size = 2.12

$$-\frac{\sqrt{a^2+b^2}\,\sqrt{1+\sin\left(dx+c-\arctan\left(-a,b\right)\right)}\,\sqrt{-2\sin\left(dx+c-\arctan\left(-a,b\right)\right)+2}\,\sqrt{-\sin\left(dx+c-\arctan\left(-a,\right.\right.}}{\cos\left(dx+c-\arctan\left(-a,b\right)\right)\sqrt{\sin\left(\right.}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*cos(d*x+c)+b*sin(d*x+c))^(1/2),x)
```

```
[Out] -(a^2+b^2)^(1/2)*(1+sin(d*x+c-arctan(-a,b)))^(1/2)*(-2*sin(d*x+c-arctan(-a,
b))+2)^(1/2)*(-sin(d*x+c-arctan(-a,b)))^(1/2)*(2*EllipticE((1+sin(d*x+c-arc
tan(-a,b)))^(1/2),1/2*2^(1/2))-EllipticF((1+sin(d*x+c-arctan(-a,b)))^(1/2),
1/2*2^(1/2)))/cos(d*x+c-arctan(-a,b))/(sin(d*x+c-arctan(-a,b))*(a^2+b^2)^(1
/2))^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{a\cos\left(dx+c\right)+b\sin\left(dx+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cos(d*x+c)+b*sin(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(a*cos(d*x + c) + b*sin(d*x + c)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int\sqrt{a\,\cos\left(c+d\,x\right)+b\,\sin\left(c+d\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*cos(c + d*x) + b*sin(c + d*x))^(1/2),x)
```

```
[Out] int((a*cos(c + d*x) + b*sin(c + d*x))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{a\cos\left(c+dx\right)+b\sin\left(c+dx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cos(d*x+c)+b*sin(d*x+c))**(1/2),x)
```

```
[Out] Integral(sqrt(a*cos(c + d*x) + b*sin(c + d*x)), x)
```

3.236 $\int \frac{1}{\sqrt{a\cos(c+dx)+b\sin(c+dx)}}\,dx$

Optimal. Leaf size=75

$$\frac{2\sqrt{\frac{a\cos(c+dx)+b\sin(c+dx)}{\sqrt{a^2+b^2}}}\,F\left(\frac{1}{2}\left(c+dx-\tan^{-1}(a,b)\right)\middle|2\right)}{d\sqrt{a\cos(c+dx)+b\sin(c+dx)}}$$

```
[Out] 2*(cos(1/2*c+1/2*d*x-1/2*arctan(a,b))^2)^(1/2)/cos(1/2*c+1/2*d*x-1/2*arctan
(a,b))*EllipticF(sin(1/2*c+1/2*d*x-1/2*arctan(a,b)),2^(1/2))*((a*cos(d*x+c)
+b*sin(d*x+c))/(a^2+b^2)^(1/2))^(1/2)/d/(a*cos(d*x+c)+b*sin(d*x+c))^(1/2)
```

Rubi [A] time = 0.03, antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {3078, 2641}

$$\frac{2\sqrt{\frac{a\cos(c+dx)+b\sin(c+dx)}{\sqrt{a^2+b^2}}}\,F\left(\frac{1}{2}\left(c+dx-\tan^{-1}(a,b)\right)\middle|2\right)}{d\sqrt{a\cos(c+dx)+b\sin(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[a*Cos[c + d*x] + b*Sin[c + d*x]],x]
```

```
[Out] (2*EllipticF[(c + d*x - ArcTan[a, b])/2, 2]*Sqrt[(a*Cos[c + d*x] + b*Sin[c
+ d*x])/Sqrt[a^2 + b^2]])/(d*Sqrt[a*Cos[c + d*x] + b*Sin[c + d*x]])
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3078

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x
_Symbol] :> Dist[(a*Cos[c + d*x] + b*Sin[c + d*x])^n/((a*Cos[c + d*x] + b*S
in[c + d*x])/Sqrt[a^2 + b^2])^n, Int[Cos[c + d*x - ArcTan[a, b]]^n, x], x]
/; FreeQ[{a, b, c, d, n}, x] &&  !(GeQ[n, 1] || LeQ[n, -1]) &&  !(GtQ[a^2 +
 b^2, 0] || EqQ[a^2 + b^2, 0])
```

Rubi steps

$$\int \frac{1}{\sqrt{a\cos(c+dx)+b\sin(c+dx)}}\,dx = \frac{\sqrt{\frac{a\cos(c+dx)+b\sin(c+dx)}{\sqrt{a^2+b^2}}}\int \frac{1}{\sqrt{\cos\left(c+dx-\tan^{-1}(a,b)\right)}}\,dx}{\sqrt{a\cos(c+dx)+b\sin(c+dx)}}$$
$$= \frac{2F\left(\frac{1}{2}\left(c+dx-\tan^{-1}(a,b)\right)\middle|2\right)\sqrt{\frac{a\cos(c+dx)+b\sin(c+dx)}{\sqrt{a^2+b^2}}}}{d\sqrt{a\cos(c+dx)+b\sin(c+dx)}}$$

Mathematica [C] time = 0.19, size = 92, normalized size = 1.23

$$\frac{2\tan\left(\tan^{-1}\left(\frac{a}{b}\right)+c+dx\right)\sqrt{\cos^2\left(\tan^{-1}\left(\frac{a}{b}\right)+c+dx\right)}\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};\sin^2\left(c+dx+\tan^{-1}\left(\frac{a}{b}\right)\right)\right)}{d\sqrt{b\sqrt{\frac{a^2}{b^2}+1}\sin\left(\tan^{-1}\left(\frac{a}{b}\right)+c+dx\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[a*Cos[c + d*x] + b*Sin[c + d*x]],x]
```

```
[Out] (2*Sqrt[Cos[c + d*x + ArcTan[a/b]]^2]*HypergeometricPFQ[{1/4, 1/2}, {5/4},
Sin[c + d*x + ArcTan[a/b]]^2]*Tan[c + d*x + ArcTan[a/b]])/(d*Sqrt[Sqrt[1 +
a^2/b^2]*b*Sin[c + d*x + ArcTan[a/b]]])
```

fricas [F] time = 0.87, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{1}{\sqrt{a\cos(dx+c)+b\sin(dx+c)}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cos(d*x+c)+b*sin(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(1/sqrt(a*cos(d*x + c) + b*sin(d*x + c)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a\cos(dx+c)+b\sin(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cos(d*x+c)+b*sin(d*x+c))^(1/2),x, algorithm="giac")
```

[Out] integrate(1/sqrt(a*cos(d*x + c) + b*sin(d*x + c)), x)

maple [A] time = 0.25, size = 121, normalized size = 1.61

$$\frac{\sqrt{1+\sin\left(dx+c-\arctan\left(-a,b\right)\right)}\,\sqrt{-2\sin\left(dx+c-\arctan\left(-a,b\right)\right)+2}\,\sqrt{-\sin\left(dx+c-\arctan\left(-a,b\right)\right)}\ \mathrm{Ellipti}}{\cos\left(dx+c-\arctan\left(-a,b\right)\right)\sqrt{\sin\left(dx+c-\arctan\left(-a,b\right)\right)\sqrt{a^2+b^2}}\ }$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(a*cos(d*x+c)+b*sin(d*x+c))^(1/2),x)

[Out] (1+sin(d*x+c-arctan(-a,b)))^(1/2)*(-2*sin(d*x+c-arctan(-a,b))+2)^(1/2)*(-si
n(d*x+c-arctan(-a,b)))^(1/2)*EllipticF((1+sin(d*x+c-arctan(-a,b)))^(1/2),1/
2*2^(1/2))/cos(d*x+c-arctan(-a,b))/(sin(d*x+c-arctan(-a,b))*(a^2+b^2)^(1/2)
)^(1/2)/d

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a\cos\left(dx+c\right)+b\sin\left(dx+c\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a*cos(d*x+c)+b*sin(d*x+c))^(1/2),x, algorithm="maxima")

[Out] integrate(1/sqrt(a*cos(d*x + c) + b*sin(d*x + c)), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sqrt{a\,\cos\left(c+d\,x\right)+b\,\sin\left(c+d\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(a*cos(c + d*x) + b*sin(c + d*x))^(1/2),x)

[Out] int(1/(a*cos(c + d*x) + b*sin(c + d*x))^(1/2), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a\cos\left(c+dx\right)+b\sin\left(c+dx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a*cos(d*x+c)+b*sin(d*x+c))**(1/2),x)

[Out] Integral(1/sqrt(a*cos(c + d*x) + b*sin(c + d*x)), x)

3.237 $\int \frac{1}{(a\cos(c+dx)+b\sin(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=138

$$-\frac{2\sqrt{a\cos(c+dx)+b\sin(c+dx)}\,E\left(\frac{1}{2}\left(c+dx-\tan^{-1}(a,b)\right)\middle|2\right)}{d\left(a^2+b^2\right)\sqrt{\frac{a\cos(c+dx)+b\sin(c+dx)}{\sqrt{a^2+b^2}}}}-\frac{2(b\cos(c+dx)-a\sin(c+dx))}{d\left(a^2+b^2\right)\sqrt{a\cos(c+dx)+b\sin(c+dx)}}$$

```
[Out] -2*(b*cos(d*x+c)-a*sin(d*x+c))/(a^2+b^2)/d/(a*cos(d*x+c)+b*sin(d*x+c))^(1/2
)-2*(cos(1/2*c+1/2*d*x-1/2*arctan(a,b))^2)^(1/2)/cos(1/2*c+1/2*d*x-1/2*arct
an(a,b))*EllipticE(sin(1/2*c+1/2*d*x-1/2*arctan(a,b)),2^(1/2))*(a*cos(d*x+c
)+b*sin(d*x+c))^(1/2)/(a^2+b^2)/d/((a*cos(d*x+c)+b*sin(d*x+c))/(a^2+b^2)^(1
/2))^(1/2)
```

Rubi [A] time = 0.06, antiderivative size = 138, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3076, 3078, 2639}

$$-\frac{2\sqrt{a\cos(c+dx)+b\sin(c+dx)}\,E\left(\frac{1}{2}\left(c+dx-\tan^{-1}(a,b)\right)\middle|2\right)}{d\left(a^2+b^2\right)\sqrt{\frac{a\cos(c+dx)+b\sin(c+dx)}{\sqrt{a^2+b^2}}}}-\frac{2(b\cos(c+dx)-a\sin(c+dx))}{d\left(a^2+b^2\right)\sqrt{a\cos(c+dx)+b\sin(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Cos[c + d*x] + b*Sin[c + d*x])^(-3/2),x]
```

```
[Out] (-2*(b*Cos[c + d*x] - a*Sin[c + d*x]))/((a^2 + b^2)*d*Sqrt[a*Cos[c + d*x] +
 b*Sin[c + d*x]]) - (2*EllipticE[(c + d*x - ArcTan[a, b])/2, 2]*Sqrt[a*Cos[
c + d*x] + b*Sin[c + d*x]])/((a^2 + b^2)*d*Sqrt[(a*Cos[c + d*x] + b*Sin[c +
 d*x])/Sqrt[a^2 + b^2]])
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3076

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x
_Symbol] :> Simp[((b*Cos[c + d*x] - a*Sin[c + d*x])*(a*Cos[c + d*x] + b*Sin
[c + d*x])^(n + 1))/(d*(n + 1)*(a^2 + b^2)), x] + Dist[(n + 2)/((n + 1)*(a^
2 + b^2)), Int[(a*Cos[c + d*x] + b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{
a, b, c, d}, x] && NeQ[a^2 + b^2, 0] && LtQ[n, -1] && NeQ[n, -2]
```

Rule 3078

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x
_Symbol] :> Dist[(a*Cos[c + d*x] + b*Sin[c + d*x])^n/((a*Cos[c + d*x] + b*S
in[c + d*x])/Sqrt[a^2 + b^2])^n, Int[Cos[c + d*x - ArcTan[a, b]]^n, x], x]
/; FreeQ[{a, b, c, d, n}, x] &&  !(GeQ[n, 1] || LeQ[n, -1]) &&  !(GtQ[a^2 +
 b^2, 0] || EqQ[a^2 + b^2, 0])
```

Rubi steps

$$\int \frac{1}{(a\cos(c+dx)+b\sin(c+dx))^{3/2}}\,dx = -\frac{2(b\cos(c+dx)-a\sin(c+dx))}{\left(a^2+b^2\right)d\sqrt{a\cos(c+dx)+b\sin(c+dx)}} - \frac{\int \sqrt{a\cos(c+dx)+b\sin}}{a^2+b^2}$$

$$= -\frac{2(b\cos(c+dx)-a\sin(c+dx))}{\left(a^2+b^2\right)d\sqrt{a\cos(c+dx)+b\sin(c+dx)}} - \frac{\sqrt{a\cos(c+dx)+b\sin(c}}{\left(a^2+b^2\right.}$$

$$= -\frac{2(b\cos(c+dx)-a\sin(c+dx))}{\left(a^2+b^2\right)d\sqrt{a\cos(c+dx)+b\sin(c+dx)}} - \frac{2E\left(\frac{1}{2}\left(c+dx-\tan^{-1}(a\right.\right.}{\left(a^2+b^2\right)}$$

Mathematica [C] time = 3.06, size = 219, normalized size = 1.59

$$\frac{\frac{\tan\left(-\tan^{-1}\left(\frac{b}{a}\right)+c+dx\right)\sqrt{a\sqrt{\frac{b^2}{a^2}+1}\cos\left(-\tan^{-1}\left(\frac{b}{a}\right)+c+dx\right)}\,{}_2F_1\left(-\frac{1}{2},-\frac{1}{4};\frac{3}{4};\cos^2\left(c+dx-\tan^{-1}\left(\frac{b}{a}\right)\right)\right)}{\sqrt{\sin^2\left(-\tan^{-1}\left(\frac{b}{a}\right)+c+dx\right)}} - \tan\left(-\tan^{-1}\left(\frac{b}{a}\right)+c+dx\right)\sqrt{a\sqrt{\frac{b^2}{a^2}}}}{d\left(a^2+b^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Cos[c + d*x] + b*Sin[c + d*x])^(-3/2),x]
```

```
[Out] ((-2*b*Cos[c + d*x])/Sqrt[a*Cos[c + d*x] + b*Sin[c + d*x]] + (2*a*Sin[c + d
*x])/Sqrt[a*Cos[c + d*x] + b*Sin[c + d*x]] - Sqrt[a*Sqrt[1 + b^2/a^2]*Cos[c
 + d*x - ArcTan[b/a]]]*Tan[c + d*x - ArcTan[b/a]] + (Sqrt[a*Sqrt[1 + b^2/a^
2]*Cos[c + d*x - ArcTan[b/a]]]*HypergeometricPFQ[{-1/2, -1/4}, {3/4}, Cos[c
 + d*x - ArcTan[b/a]]^2]*Tan[c + d*x - ArcTan[b/a]])/Sqrt[Sin[c + d*x - Arc
Tan[b/a]]^2])/((a^2 + b^2)*d)
```

fricas [F] time = 1.93, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{a\cos(dx+c)+b\sin(dx+c)}}{2\,ab\cos(dx+c)\sin(dx+c)+\left(a^2-b^2\right)\cos(dx+c)^2+b^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cos(d*x+c)+b*sin(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(a*cos(d*x + c) + b*sin(d*x + c))/(2*a*b*cos(d*x + c)*sin(d*x
+ c) + (a^2 - b^2)*cos(d*x + c)^2 + b^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{(a\cos(dx+c)+b\sin(dx+c))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cos(d*x+c)+b*sin(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((a*cos(d*x + c) + b*sin(d*x + c))^(-3/2), x)
```

maple [A] time = 0.35, size = 228, normalized size = 1.65

$$2\sqrt{1+\sin(dx+c-\arctan(-a,b))}\,\sqrt{-2\sin(dx+c-\arctan(-a,b))+2}\,\sqrt{-\sin(dx+c-\arctan(-a,b))}\ \text{Elli}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*cos(d*x+c)+b*sin(d*x+c))^(3/2),x)
```

```
[Out] (2*(1+sin(d*x+c-arctan(-a,b)))^(1/2)*(-2*sin(d*x+c-arctan(-a,b))+2)^(1/2)*(
-sin(d*x+c-arctan(-a,b)))^(1/2)*EllipticE((1+sin(d*x+c-arctan(-a,b)))^(1/2)
,1/2*2^(1/2))-(1+sin(d*x+c-arctan(-a,b)))^(1/2)*(-2*sin(d*x+c-arctan(-a,b))
+2)^(1/2)*(-sin(d*x+c-arctan(-a,b)))^(1/2)*EllipticF((1+sin(d*x+c-arctan(-a
,b)))^(1/2),1/2*2^(1/2))-2*cos(d*x+c-arctan(-a,b))^2)/(a^2+b^2)^(1/2)/cos(d
*x+c-arctan(-a,b))/(sin(d*x+c-arctan(-a,b))*(a^2+b^2)^(1/2))^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{(a\cos(dx+c)+b\sin(dx+c))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cos(d*x+c)+b*sin(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((a*cos(d*x + c) + b*sin(d*x + c))^(-3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\left(a\,\cos\left(c+d\,x\right)+b\,\sin\left(c+d\,x\right)\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*cos(c + d*x) + b*sin(c + d*x))^(3/2),x)
```

```
[Out] int(1/(a*cos(c + d*x) + b*sin(c + d*x))^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a\cos\left(c+dx\right)+b\sin\left(c+dx\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cos(d*x+c)+b*sin(d*x+c))**(3/2),x)
```

```
[Out] Integral((a*cos(c + d*x) + b*sin(c + d*x))**(-3/2), x)
```

3.238 $\int \frac{1}{(a\cos(c+dx)+b\sin(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=142

$$\frac{2\sqrt{\frac{a\cos(c+dx)+b\sin(c+dx)}{\sqrt{a^2+b^2}}}\,F\left(\frac{1}{2}\left(c+dx-\tan^{-1}(a,b)\right)\middle|2\right)}{3d\left(a^2+b^2\right)\sqrt{a\cos(c+dx)+b\sin(c+dx)}}-\frac{2(b\cos(c+dx)-a\sin(c+dx))}{3d\left(a^2+b^2\right)(a\cos(c+dx)+b\sin(c+dx))^{3/2}}$$

```
[Out] -2/3*(b*cos(d*x+c)-a*sin(d*x+c))/(a^2+b^2)/d/(a*cos(d*x+c)+b*sin(d*x+c))^(3
/2)+2/3*(cos(1/2*c+1/2*d*x-1/2*arctan(a,b))^2)^(1/2)/cos(1/2*c+1/2*d*x-1/2*
arctan(a,b))*EllipticF(sin(1/2*c+1/2*d*x-1/2*arctan(a,b)),2^(1/2))*((a*cos(
d*x+c)+b*sin(d*x+c))/(a^2+b^2)^(1/2))^(1/2)/(a^2+b^2)/d/(a*cos(d*x+c)+b*sin
(d*x+c))^(1/2)
```

Rubi [A] time = 0.06, antiderivative size = 142, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3076, 3078, 2641}

$$\frac{2\sqrt{\frac{a\cos(c+dx)+b\sin(c+dx)}{\sqrt{a^2+b^2}}}\,F\left(\frac{1}{2}\left(c+dx-\tan^{-1}(a,b)\right)\middle|2\right)}{3d\left(a^2+b^2\right)\sqrt{a\cos(c+dx)+b\sin(c+dx)}}-\frac{2(b\cos(c+dx)-a\sin(c+dx))}{3d\left(a^2+b^2\right)(a\cos(c+dx)+b\sin(c+dx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Cos[c + d*x] + b*Sin[c + d*x])^(-5/2),x]
```

```
[Out] (-2*(b*Cos[c + d*x] - a*Sin[c + d*x]))/(3*(a^2 + b^2)*d*(a*Cos[c + d*x] + b
*Sin[c + d*x])^(3/2)) + (2*EllipticF[(c + d*x - ArcTan[a, b])/2, 2]*Sqrt[(a
*Cos[c + d*x] + b*Sin[c + d*x])/Sqrt[a^2 + b^2]])/(3*(a^2 + b^2)*d*Sqrt[a*C
os[c + d*x] + b*Sin[c + d*x]])
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3076

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x
_Symbol] :> Simp[((b*Cos[c + d*x] - a*Sin[c + d*x])*(a*Cos[c + d*x] + b*Sin
[c + d*x])^(n + 1))/(d*(n + 1)*(a^2 + b^2)), x] + Dist[(n + 2)/((n + 1)*(a^
2 + b^2)), Int[(a*Cos[c + d*x] + b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{
a, b, c, d}, x] && NeQ[a^2 + b^2, 0] && LtQ[n, -1] && NeQ[n, -2]
```

Rule 3078

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x
_Symbol] :> Dist[(a*Cos[c + d*x] + b*Sin[c + d*x])^n/((a*Cos[c + d*x] + b*S
in[c + d*x])/Sqrt[a^2 + b^2])^n, Int[Cos[c + d*x - ArcTan[a, b]]^n, x], x]
/; FreeQ[{a, b, c, d, n}, x] && !(GeQ[n, 1] || LeQ[n, -1]) && !(GtQ[a^2 +
 b^2, 0] || EqQ[a^2 + b^2, 0])
```

Rubi steps

$$\int \frac{1}{(a\cos(c+dx)+b\sin(c+dx))^{5/2}}\,dx = -\frac{2(b\cos(c+dx)-a\sin(c+dx))}{3\left(a^2+b^2\right)d(a\cos(c+dx)+b\sin(c+dx))^{3/2}}+\frac{\int \frac{1}{\sqrt{a\cos(c+dx)+b\sin(c+a}}}{3\left(a^2+b^2\right)}$$

$$= -\frac{2(b\cos(c+dx)-a\sin(c+dx))}{3\left(a^2+b^2\right)d(a\cos(c+dx)+b\sin(c+dx))^{3/2}}+\frac{\sqrt{\frac{a\cos(c+dx)+b\sin(c+dx}{\sqrt{a^2+b^2}}}}{3\left(a^2+b^2\right)\sqrt{a\cos}}$$

$$= -\frac{2(b\cos(c+dx)-a\sin(c+dx))}{3\left(a^2+b^2\right)d(a\cos(c+dx)+b\sin(c+dx))^{3/2}}+\frac{2F\left(\frac{1}{2}\left(c+dx-\tan^{-}\right.\right.}{3\left(a^2+b^2\right)d\sqrt{a}}$$

Mathematica [C] time = 1.68, size = 145, normalized size = 1.02

$$\frac{2\left(\frac{\tan\left(\tan^{-1}\left(\frac{a}{b}\right)+c+dx\right)\sqrt{\cos^2\left(\tan^{-1}\left(\frac{a}{b}\right)+c+dx\right)}\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};\sin^2\left(c+dx+\tan^{-1}\left(\frac{a}{b}\right)\right)\right)}{\sqrt{b\sqrt{\frac{a^2}{b^2}+1}\,\sin\left(\tan^{-1}\left(\frac{a}{b}\right)+c+dx\right)}}+\frac{a\sin(c+dx)-b\cos(c+dx)}{(a\cos(c+dx)+b\sin(c+dx))^{3/2}}\right)}{3d\left(a^2+b^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Cos[c + d*x] + b*Sin[c + d*x])^(-5/2),x]
```

```
[Out] (2*((-(b*Cos[c + d*x]) + a*Sin[c + d*x])/(a*Cos[c + d*x] + b*Sin[c + d*x])^
(3/2) + (Sqrt[Cos[c + d*x + ArcTan[a/b]]^2]*HypergeometricPFQ[{1/4, 1/2}, {
5/4}, Sin[c + d*x + ArcTan[a/b]]^2]*Tan[c + d*x + ArcTan[a/b]])/Sqrt[Sqrt[1
 + a^2/b^2]*b*Sin[c + d*x + ArcTan[a/b]]]))/(3*(a^2 + b^2)*d)
```

fricas [F] time = 1.88, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{a\cos(dx+c)+b\sin(dx+c)}}{3\,ab^2\cos(dx+c)+\left(a^3-3\,ab^2\right)\cos(dx+c)^3+\left(b^3+\left(3\,a^2b-b^3\right)\cos(dx+c)^2\right)\sin(dx+c)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cos(d*x+c)+b*sin(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(a*cos(d*x + c) + b*sin(d*x + c))/(3*a*b^2*cos(d*x + c) + (a^3
 - 3*a*b^2)*cos(d*x + c)^3 + (b^3 + (3*a^2*b - b^3)*cos(d*x + c)^2)*sin(d*x
 + c)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a\cos\left(dx+c\right)+b\sin\left(dx+c\right)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cos(d*x+c)+b*sin(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((a*cos(d*x + c) + b*sin(d*x + c))^(-5/2), x)
```

maple [A] time = 0.34, size = 178, normalized size = 1.25

$$\frac{\sqrt{1+\sin\left(dx+c-\arctan\left(-a,b\right)\right)}\,\sqrt{-2\sin\left(dx+c-\arctan\left(-a,b\right)\right)+2}\,\sqrt{-\sin\left(dx+c-\arctan\left(-a,b\right)\right)}\,\operatorname{Ellip}}{3\sin\left(dx+c-\arctan\left(-a,b\right)\right)\left(a^{2}+b^{2}\right)\cos\left(dx+c-\operatorname{arc}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*cos(d*x+c)+b*sin(d*x+c))^(5/2),x)
```

```
[Out] 1/3/sin(d*x+c-arctan(-a,b))/(a^2+b^2)*((1+sin(d*x+c-arctan(-a,b)))^(1/2)*(-
2*sin(d*x+c-arctan(-a,b))+2)^(1/2)*(-sin(d*x+c-arctan(-a,b)))^(1/2)*Ellipti
cF((1+sin(d*x+c-arctan(-a,b)))^(1/2),1/2*2^(1/2))*sin(d*x+c-arctan(-a,b))-2
*cos(d*x+c-arctan(-a,b))^2)/cos(d*x+c-arctan(-a,b))/(sin(d*x+c-arctan(-a,b)
)*(a^2+b^2)^(1/2))^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a\cos\left(dx+c\right)+b\sin\left(dx+c\right)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cos(d*x+c)+b*sin(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((a*cos(d*x + c) + b*sin(d*x + c))^(-5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\left(a\cos\left(c+d\,x\right)+b\sin\left(c+d\,x\right)\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*cos(c + d*x) + b*sin(c + d*x))^(5/2),x)
[Out] int(1/(a*cos(c + d*x) + b*sin(c + d*x))^(5/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cos(d*x+c)+b*sin(d*x+c))**(5/2),x)
[Out] Timed out
```

3.239 $\int \frac{1}{(a\cos(c+dx)+b\sin(c+dx))^{7/2}}\,dx$

Optimal. Leaf size=197

$$-\frac{6\sqrt{a\cos(c+dx)+b\sin(c+dx)}\,E\left(\frac{1}{2}\left(c+dx-\tan^{-1}(a,b)\right)\middle|2\right)}{5d\left(a^2+b^2\right)^2\sqrt{\frac{a\cos(c+dx)+b\sin(c+dx)}{\sqrt{a^2+b^2}}}}-\frac{6(b\cos(c+dx)-a\sin(c+dx))}{5d\left(a^2+b^2\right)^2\sqrt{a\cos(c+dx)+b\sin(c+dx)}}-$$

```
[Out] -2/5*(b*cos(d*x+c)-a*sin(d*x+c))/(a^2+b^2)/d/(a*cos(d*x+c)+b*sin(d*x+c))^(5
/2)-6/5*(b*cos(d*x+c)-a*sin(d*x+c))/(a^2+b^2)^2/d/(a*cos(d*x+c)+b*sin(d*x+c
))^(1/2)-6/5*(cos(1/2*c+1/2*d*x-1/2*arctan(a,b))^2)^(1/2)/cos(1/2*c+1/2*d*x
-1/2*arctan(a,b))*EllipticE(sin(1/2*c+1/2*d*x-1/2*arctan(a,b)),2^(1/2))*(a*
cos(d*x+c)+b*sin(d*x+c))^(1/2)/(a^2+b^2)^2/d/((a*cos(d*x+c)+b*sin(d*x+c))/(
a^2+b^2)^(1/2))^(1/2)
```

Rubi [A] time = 0.09, antiderivative size = 197, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3076, 3078, 2639}

$$-\frac{6\sqrt{a\cos(c+dx)+b\sin(c+dx)}\,E\left(\frac{1}{2}\left(c+dx-\tan^{-1}(a,b)\right)\middle|2\right)}{5d\left(a^2+b^2\right)^2\sqrt{\frac{a\cos(c+dx)+b\sin(c+dx)}{\sqrt{a^2+b^2}}}}-\frac{6(b\cos(c+dx)-a\sin(c+dx))}{5d\left(a^2+b^2\right)^2\sqrt{a\cos(c+dx)+b\sin(c+dx)}}-$$

Antiderivative was successfully verified.

```
[In] Int[(a*Cos[c + d*x] + b*Sin[c + d*x])^(-7/2),x]
```

```
[Out] (-2*(b*Cos[c + d*x] - a*Sin[c + d*x]))/(5*(a^2 + b^2)*d*(a*Cos[c + d*x] + b
*Sin[c + d*x])^(5/2)) - (6*(b*Cos[c + d*x] - a*Sin[c + d*x]))/(5*(a^2 + b^2
)^2*d*Sqrt[a*Cos[c + d*x] + b*Sin[c + d*x]]) - (6*EllipticE[(c + d*x - ArcT
an[a, b])/2, 2]*Sqrt[a*Cos[c + d*x] + b*Sin[c + d*x]])/(5*(a^2 + b^2)^2*d*S
qrt[(a*Cos[c + d*x] + b*Sin[c + d*x])/Sqrt[a^2 + b^2]])
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3076

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x
_Symbol] :> Simp[((b*Cos[c + d*x] - a*Sin[c + d*x])*(a*Cos[c + d*x] + b*Sin
[c + d*x])^(n + 1))/(d*(n + 1)*(a^2 + b^2)), x] + Dist[(n + 2)/((n + 1)*(a^
```

```
2 + b^2)), Int[(a*Cos[c + d*x] + b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{
a, b, c, d}, x] && NeQ[a^2 + b^2, 0] && LtQ[n, -1] && NeQ[n, -2]
```

Rule 3078

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x
_Symbol] :> Dist[(a*Cos[c + d*x] + b*Sin[c + d*x])^n/((a*Cos[c + d*x] + b*S
in[c + d*x])/Sqrt[a^2 + b^2])^n, Int[Cos[c + d*x - ArcTan[a, b]]^n, x], x]
/; FreeQ[{a, b, c, d, n}, x] &&  !(GeQ[n, 1] || LeQ[n, -1]) &&  !(GtQ[a^2 +
 b^2, 0] || EqQ[a^2 + b^2, 0])
```

Rubi steps

$$\int \frac{1}{(a\cos(c+dx)+b\sin(c+dx))^{7/2}}\,dx = -\frac{2(b\cos(c+dx)-a\sin(c+dx))}{5\left(a^2+b^2\right)d(a\cos(c+dx)+b\sin(c+dx))^{5/2}} + \frac{3\int \frac{1}{(a\cos(c+dx)+b\sin(c+}}{5\left(a^2+b^2\right)}$$

$$= -\frac{2(b\cos(c+dx)-a\sin(c+dx))}{5\left(a^2+b^2\right)d(a\cos(c+dx)+b\sin(c+dx))^{5/2}} - \frac{6(b\cos(c+dx}{5\left(a^2+b^2\right)^2 d\sqrt{a\cos}}$$

$$= -\frac{2(b\cos(c+dx)-a\sin(c+dx))}{5\left(a^2+b^2\right)d(a\cos(c+dx)+b\sin(c+dx))^{5/2}} - \frac{6(b\cos(c+dx}{5\left(a^2+b^2\right)^2 d\sqrt{a\cos}}$$

$$= -\frac{2(b\cos(c+dx)-a\sin(c+dx))}{5\left(a^2+b^2\right)d(a\cos(c+dx)+b\sin(c+dx))^{5/2}} - \frac{6(b\cos(c+dx}{5\left(a^2+b^2\right)^2 d\sqrt{a\cos}}$$

Mathematica [C] time = 2.44, size = 277, normalized size = 1.41

$$\frac{\frac{\cos\left(-\tan^{-1}\left(\frac{b}{a}\right)+c+dx\right)\left(3b\sin\left(-\tan^{-1}\left(\frac{b}{a}\right)+c+dx\right){}_2F_1\left(-\frac{1}{2},-\frac{1}{4};\frac{3}{4};\cos^2\left(c+dx-\tan^{-1}\left(\frac{b}{a}\right)\right)\right)-3\sqrt{\sin^2\left(-\tan^{-1}\left(\frac{b}{a}\right)+c+dx\right)}\left(b\sin\left(-\tan^{-1}\left(\frac{b}{a}\right)+c+dx\right)-2a\,c\right.\right.}{\sqrt{\sin^2\left(-\tan^{-1}\left(\frac{b}{a}\right)+c+dx\right)}\left(a\sqrt{\frac{b^2}{a^2}+1}\cos\left(-\tan^{-1}\left(\frac{b}{a}\right)+c+dx\right)\right)^{3/2}}}{5bd\left(a^2+b^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Cos[c + d*x] + b*Sin[c + d*x])^(-7/2),x]
```

```
[Out] ((-2*(3*a^2*Cos[c + d*x]^3 - a*b*Sin[c + d*x] + 6*a*b*Cos[c + d*x]^2*Sin[c
+ d*x] + b^2*Cos[c + d*x]*(1 + 3*Sin[c + d*x]^2)))/(a*Cos[c + d*x] + b*Sin[
```

```
c + d*x])^(5/2) + (Cos[c + d*x - ArcTan[b/a]]*(3*b*HypergeometricPFQ[{-1/2,
 -1/4}, {3/4}, Cos[c + d*x - ArcTan[b/a]]^2]*Sin[c + d*x - ArcTan[b/a]] - 3
*Sqrt[Sin[c + d*x - ArcTan[b/a]]^2]*(-2*a*Cos[c + d*x - ArcTan[b/a]] + b*Si
n[c + d*x - ArcTan[b/a]])))/((a*Sqrt[1 + b^2/a^2]*Cos[c + d*x - ArcTan[b/a]
])^(3/2)*Sqrt[Sin[c + d*x - ArcTan[b/a]]^2]))/(5*b*(a^2 + b^2)*d)
```

fricas [F] time = 1.39, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{a\cos(dx+c)+b\sin(dx+c)}}{\left(a^4-6\,a^2b^2+b^4\right)\cos(dx+c)^4+b^4+2\left(3\,a^2b^2-b^4\right)\cos(dx+c)^2+4\left(ab^3\cos(dx+c)+\left(a^3b-ab^3\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cos(d*x+c)+b*sin(d*x+c))^(7/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(a*cos(d*x + c) + b*sin(d*x + c))/((a^4 - 6*a^2*b^2 + b^4)*cos
(d*x + c)^4 + b^4 + 2*(3*a^2*b^2 - b^4)*cos(d*x + c)^2 + 4*(a*b^3*cos(d*x +
 c) + (a^3*b - a*b^3)*cos(d*x + c)^3)*sin(d*x + c)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a\cos(dx+c)+b\sin(dx+c))^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cos(d*x+c)+b*sin(d*x+c))^(7/2),x, algorithm="giac")
```

```
[Out] integrate((a*cos(d*x + c) + b*sin(d*x + c))^(-7/2), x)
```

maple [A] time = 0.38, size = 309, normalized size = 1.57

$$\frac{\sqrt{a^2+b^2}\,\left(6\sqrt{1+\sin(dx+c-\arctan(-a,b))}\,\sqrt{-2\sin(dx+c-\arctan(-a,b))+2}\,\sqrt{-\sin(dx+c-\arctan(\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*cos(d*x+c)+b*sin(d*x+c))^(7/2),x)
```

```
[Out] 1/5/sin(d*x+c-arctan(-a,b))^2*(a^2+b^2)^(1/2)*(6*(1+sin(d*x+c-arctan(-a,b))
)^(1/2)*(-2*sin(d*x+c-arctan(-a,b))+2)^(1/2)*(-sin(d*x+c-arctan(-a,b)))^(1/
2)*sin(d*x+c-arctan(-a,b))^2*EllipticE((1+sin(d*x+c-arctan(-a,b)))^(1/2),1/
2*2^(1/2))-3*(1+sin(d*x+c-arctan(-a,b)))^(1/2)*(-2*sin(d*x+c-arctan(-a,b))+
2)^(1/2)*(-sin(d*x+c-arctan(-a,b)))^(1/2)*sin(d*x+c-arctan(-a,b))^2*Ellipti
cF((1+sin(d*x+c-arctan(-a,b)))^(1/2),1/2*2^(1/2))+6*sin(d*x+c-arctan(-a,b))
```

```
^4-4*sin(d*x+c-arctan(-a,b))^2-2)/(a^4+2*a^2*b^2+b^4)/cos(d*x+c-arctan(-a,b
))/(sin(d*x+c-arctan(-a,b))*(a^2+b^2)^(1/2))^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a\cos(dx+c)+b\sin(dx+c))^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cos(d*x+c)+b*sin(d*x+c))^(7/2),x, algorithm="maxima")
[Out] integrate((a*cos(d*x + c) + b*sin(d*x + c))^(-7/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{(a\,\cos(c+d\,x)+b\,\sin(c+d\,x))^{7/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*cos(c + d*x) + b*sin(c + d*x))^(7/2),x)
[Out] int(1/(a*cos(c + d*x) + b*sin(c + d*x))^(7/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cos(d*x+c)+b*sin(d*x+c))**(7/2),x)
[Out] Timed out
```

3.240 $\int (2\cos(c+dx)+3\sin(c+dx))^{7/2}\,dx$

Optimal. Leaf size=120

$$-\frac{2(3\cos(c+dx)-2\sin(c+dx))(3\sin(c+dx)+2\cos(c+dx))^{5/2}}{7d}-\frac{130(3\cos(c+dx)-2\sin(c+dx))\sqrt{3\sin(c}}{21d}$$

```
[Out] 130/21*13^(3/4)*(cos(1/2*c+1/2*d*x-1/2*arctan(3/2))^2)^(1/2)/cos(1/2*c+1/2*
d*x-1/2*arctan(3/2))*EllipticF(sin(1/2*c+1/2*d*x-1/2*arctan(3/2)),2^(1/2))/
d-2/7*(3*cos(d*x+c)-2*sin(d*x+c))*(2*cos(d*x+c)+3*sin(d*x+c))^(5/2)/d-130/2
1*(3*cos(d*x+c)-2*sin(d*x+c))*(2*cos(d*x+c)+3*sin(d*x+c))^(1/2)/d
```

Rubi [A] time = 0.07, antiderivative size = 120, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3073, 3077, 2641}

$$-\frac{2(3\cos(c+dx)-2\sin(c+dx))(3\sin(c+dx)+2\cos(c+dx))^{5/2}}{7d}-\frac{130(3\cos(c+dx)-2\sin(c+dx))\sqrt{3\sin(c}}{21d}$$

Antiderivative was successfully verified.

```
[In] Int[(2*Cos[c + d*x] + 3*Sin[c + d*x])^(7/2),x]
```

```
[Out] (130*13^(3/4)*EllipticF[(c + d*x - ArcTan[3/2])/2, 2])/(21*d) - (130*(3*Cos
[c + d*x] - 2*Sin[c + d*x])*Sqrt[2*Cos[c + d*x] + 3*Sin[c + d*x]])/(21*d) -
 (2*(3*Cos[c + d*x] - 2*Sin[c + d*x])*(2*Cos[c + d*x] + 3*Sin[c + d*x])^(5/
2))/(7*d)
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3073

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x
_Symbol] :> -Simp[((b*Cos[c + d*x] - a*Sin[c + d*x])*(a*Cos[c + d*x] + b*Si
n[c + d*x])^(n - 1))/(d*n), x] + Dist[((n - 1)*(a^2 + b^2))/n, Int[(a*Cos[c
 + d*x] + b*Sin[c + d*x])^(n - 2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 + b^2, 0] &&  !IntegerQ[(n - 1)/2] && GtQ[n, 1]
```

Rule 3077

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x
_Symbol] :> Dist[(a^2 + b^2)^(n/2), Int[Cos[c + d*x - ArcTan[a, b]]^n, x],
```

```
x] /; FreeQ[{a, b, c, d, n}, x] &&  !(GeQ[n, 1] || LeQ[n, -1]) && GtQ[a^2 +
 b^2, 0]
```

Rubi steps

$$\int (2\cos(c+dx)+3\sin(c+dx))^{7/2}\,dx = -\frac{2(3\cos(c+dx)-2\sin(c+dx))(2\cos(c+dx)+3\sin(c+dx))^{5/2}}{7d}+\frac{65}{7}$$

$$= -\frac{130(3\cos(c+dx)-2\sin(c+dx))\sqrt{2\cos(c+dx)+3\sin(c+dx)}}{21d}-\frac{2(}{}$$

$$= -\frac{130(3\cos(c+dx)-2\sin(c+dx))\sqrt{2\cos(c+dx)+3\sin(c+dx)}}{21d}-\frac{2(}{}$$

$$= \frac{130\,13^{3/4}F\left(\frac{1}{2}\left(c+dx-\tan^{-1}\left(\frac{3}{2}\right)\right)\middle|2\right)}{21d}-\frac{130(3\cos(c+dx)-2\sin(c+}{}$$

Mathematica [C] time = 0.48, size = 153, normalized size = 1.28

$$\frac{260\,13^{3/4}\sqrt{-\left(\left(\sin\left(c+dx+\tan^{-1}\left(\frac{2}{3}\right)\right)-1\right)\sin\left(c+dx+\tan^{-1}\left(\frac{2}{3}\right)\right)\right)}\sqrt{\sin\left(c+dx+\tan^{-1}\left(\frac{2}{3}\right)\right)+1}\sec\left(c+d\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(2*Cos[c + d*x] + 3*Sin[c + d*x])^(7/2),x]
```

```
[Out] (-(Sqrt[2*Cos[c + d*x] + 3*Sin[c + d*x]]*(897*Cos[c + d*x] + 27*Cos[3*(c +
d*x)] - 598*Sin[c + d*x] + 138*Sin[3*(c + d*x)])) + 260*13^(3/4)*Hypergeome
tricPFQ[{1/4, 1/2}, {5/4}, Sin[c + d*x + ArcTan[2/3]]^2]*Sec[c + d*x + ArcT
an[2/3]]*Sqrt[-((-1 + Sin[c + d*x + ArcTan[2/3]])*Sin[c + d*x + ArcTan[2/3]
])]*Sqrt[1 + Sin[c + d*x + ArcTan[2/3]]])/(42*d)
```

fricas [F] time = 0.60, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(-\left(46\cos(dx+c)^3-9\left(\cos(dx+c)^2+3\right)\sin(dx+c)-54\cos(dx+c)\right)\sqrt{2\cos(dx+c)+3\sin(dx+}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*cos(d*x+c)+3*sin(d*x+c))^(7/2),x, algorithm="fricas")
```

```
[Out] integral(-(46*cos(d*x + c)^3 - 9*(cos(d*x + c)^2 + 3)*sin(d*x + c) - 54*cos
(d*x + c))*sqrt(2*cos(d*x + c) + 3*sin(d*x + c)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (2 \cos(dx+c) + 3 \sin(dx+c))^{\frac{7}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*cos(d*x+c)+3*sin(d*x+c))^(7/2),x, algorithm="giac")
```

```
[Out] integrate((2*cos(d*x + c) + 3*sin(d*x + c))^(7/2), x)
```

maple [A] time = 0.35, size = 128, normalized size = 1.07

$$\frac{\frac{338 \sin\left(dx+c+\arctan\left(\frac{2}{3}\right)\right)\left(\cos^4\left(dx+c+\arctan\left(\frac{2}{3}\right)\right)\right)}{7} + \frac{845\sqrt{1+\sin\left(dx+c+\arctan\left(\frac{2}{3}\right)\right)}\,\sqrt{-2\sin\left(dx+c+\arctan\left(\frac{2}{3}\right)\right)+2}\,\sqrt{-\sin\left(dx+c+\arctan\left(\frac{2}{3}\right)\right)}}{21}}{\cos\left(dx+c+\arctan\left(\frac{2}{3}\right)\right)\sqrt{\sqrt{13}\,\sin\left(dx+c+a\right.}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*cos(d*x+c)+3*sin(d*x+c))^(7/2),x)
```

```
[Out] (338/7*sin(d*x+c+arctan(2/3))*cos(d*x+c+arctan(2/3))^4+845/21*(1+sin(d*x+c+
arctan(2/3)))^(1/2)*(-2*sin(d*x+c+arctan(2/3))+2)^(1/2)*(-sin(d*x+c+arctan(
2/3)))^(1/2)*EllipticF((1+sin(d*x+c+arctan(2/3)))^(1/2),1/2*2^(1/2))-2704/2
1*cos(d*x+c+arctan(2/3))^2*sin(d*x+c+arctan(2/3)))/cos(d*x+c+arctan(2/3))/(
13^(1/2)*sin(d*x+c+arctan(2/3)))^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (2 \cos(dx+c) + 3 \sin(dx+c))^{\frac{7}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*cos(d*x+c)+3*sin(d*x+c))^(7/2),x, algorithm="maxima")
```

```
[Out] integrate((2*cos(d*x + c) + 3*sin(d*x + c))^(7/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int (2 \cos(c + d\,x) + 3 \sin(c + d\,x))^{7/2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*cos(c + d*x) + 3*sin(c + d*x))^(7/2),x)
```

```
[Out] int((2*cos(c + d*x) + 3*sin(c + d*x))^(7/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*cos(d*x+c)+3*sin(d*x+c))**(7/2),x)
```

```
[Out] Timed out
```

3.241 $\int (2\cos(c+dx)+3\sin(c+dx))^{5/2}\,dx$

Optimal. Leaf size=75

$$\frac{78\sqrt[4]{13}\,E\left(\frac{1}{2}\left(c+dx-\tan^{-1}\left(\frac{3}{2}\right)\right)\middle|2\right)}{5d}-\frac{2(3\cos(c+dx)-2\sin(c+dx))(3\sin(c+dx)+2\cos(c+dx))^{3/2}}{5d}$$

```
[Out] 78/5*13^(1/4)*(cos(1/2*c+1/2*d*x-1/2*arctan(3/2))^2)^(1/2)/cos(1/2*c+1/2*d*
x-1/2*arctan(3/2))*EllipticE(sin(1/2*c+1/2*d*x-1/2*arctan(3/2)),2^(1/2))/d-
2/5*(3*cos(d*x+c)-2*sin(d*x+c))*(2*cos(d*x+c)+3*sin(d*x+c))^(3/2)/d
```

Rubi [A] time = 0.04, antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3073, 3077, 2639}

$$\frac{78\sqrt[4]{13}\,E\left(\frac{1}{2}\left(c+dx-\tan^{-1}\left(\frac{3}{2}\right)\right)\middle|2\right)}{5d}-\frac{2(3\cos(c+dx)-2\sin(c+dx))(3\sin(c+dx)+2\cos(c+dx))^{3/2}}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[(2*Cos[c + d*x] + 3*Sin[c + d*x])^(5/2),x]
```

```
[Out] (78*13^(1/4)*EllipticE[(c + d*x - ArcTan[3/2])/2, 2])/(5*d) - (2*(3*Cos[c +
 d*x] - 2*Sin[c + d*x])*(2*Cos[c + d*x] + 3*Sin[c + d*x])^(3/2))/(5*d)
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3073

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x
_Symbol] :> -Simp[((b*Cos[c + d*x] - a*Sin[c + d*x])*(a*Cos[c + d*x] + b*Si
n[c + d*x])^(n - 1))/(d*n), x] + Dist[((n - 1)*(a^2 + b^2))/n, Int[(a*Cos[c
 + d*x] + b*Sin[c + d*x])^(n - 2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 + b^2, 0] &&  !IntegerQ[(n - 1)/2] && GtQ[n, 1]
```

Rule 3077

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x
_Symbol] :> Dist[(a^2 + b^2)^(n/2), Int[Cos[c + d*x - ArcTan[a, b]]^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] &&  !(GeQ[n, 1] || LeQ[n, -1]) && GtQ[a^2 +
 b^2, 0]
```

Rubi steps

$$\int (2\cos(c+dx)+3\sin(c+dx))^{5/2}\,dx = -\frac{2(3\cos(c+dx)-2\sin(c+dx))(2\cos(c+dx)+3\sin(c+dx))^{3/2}}{5d} + \frac{39}{5}$$

$$= -\frac{2(3\cos(c+dx)-2\sin(c+dx))(2\cos(c+dx)+3\sin(c+dx))^{3/2}}{5d} + \frac{1}{5}$$

$$= \frac{78\sqrt[4]{13}\,E\left(\frac{1}{2}\left(c+dx-\tan^{-1}\left(\frac{3}{2}\right)\right)\middle|2\right)}{5d} - \frac{2(3\cos(c+dx)-2\sin(c+dx))(}{5}$$

Mathematica [C] time = 0.82, size = 199, normalized size = 2.65

$$\frac{-\frac{39\sqrt[4]{13}\sin\left(c+dx-\tan^{-1}\left(\frac{3}{2}\right)\right){}_2F_1\left(-\frac{1}{2},-\frac{1}{4};\frac{3}{4};\cos^2\left(c+dx-\tan^{-1}\left(\frac{3}{2}\right)\right)\right)}{\sqrt{-\left(\left(\cos\left(c+dx-\tan^{-1}\left(\frac{3}{2}\right)\right)-1\right)\cos\left(c+dx-\tan^{-1}\left(\frac{3}{2}\right)\right)\right)}\sqrt{\cos\left(c+dx-\tan^{-1}\left(\frac{3}{2}\right)\right)+1}} + \sqrt{3\sin(c+dx)+2\cos(c+dx)}\,(-5\sin(2(c+d}{5d}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(2*Cos[c + d*x] + 3*Sin[c + d*x])^(5/2),x]
```

```
[Out] (Sqrt[2*Cos[c + d*x] + 3*Sin[c + d*x]]*(52 - 12*Cos[2*(c + d*x)] - 5*Sin[2*
(c + d*x)]) - (13*13^(1/4)*(4*Cos[c + d*x - ArcTan[3/2]] - 3*Sin[c + d*x -
ArcTan[3/2]]))/Sqrt[Cos[c + d*x - ArcTan[3/2]]] - (39*13^(1/4)*Hypergeometr
icPFQ[{-1/2, -1/4}, {3/4}, Cos[c + d*x - ArcTan[3/2]]^2]*Sin[c + d*x - ArcT
an[3/2]])/(Sqrt[-((-1 + Cos[c + d*x - ArcTan[3/2]])*Cos[c + d*x - ArcTan[3/
2]])]*Sqrt[1 + Cos[c + d*x - ArcTan[3/2]]]))/(5*d)
```

fricas [F] time = 0.97, size = 0, normalized size = 0.00

$$\text{integral}\left(-\left(5\cos(dx+c)^2-12\cos(dx+c)\sin(dx+c)-9\right)\sqrt{2\cos(dx+c)+3\sin(dx+c)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*cos(d*x+c)+3*sin(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] integral(-(5*cos(d*x + c)^2 - 12*cos(d*x + c)*sin(d*x + c) - 9)*sqrt(2*cos(
d*x + c) + 3*sin(d*x + c)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (2\cos(dx+c)+3\sin(dx+c))^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*cos(d*x+c)+3*sin(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((2*cos(d*x + c) + 3*sin(d*x + c))^(5/2), x)
```

maple [A] time = 0.37, size = 174, normalized size = 2.32

$$-\frac{13\sqrt{13}\left(6\sqrt{1+\sin\left(dx+c+\arctan\left(\frac{2}{3}\right)\right)}\,\sqrt{-2\sin\left(dx+c+\arctan\left(\frac{2}{3}\right)\right)+2}\,\sqrt{-\sin\left(dx+c+\arctan\left(\frac{2}{3}\right)\right)}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*cos(d*x+c)+3*sin(d*x+c))^(5/2),x)
```

```
[Out] -13/5*13^(1/2)*(6*(1+sin(d*x+c+arctan(2/3)))^(1/2)*(-2*sin(d*x+c+arctan(2/3
))+2)^(1/2)*(-sin(d*x+c+arctan(2/3)))^(1/2)*EllipticE((1+sin(d*x+c+arctan(2
/3)))^(1/2),1/2*2^(1/2))-3*(1+sin(d*x+c+arctan(2/3)))^(1/2)*(-2*sin(d*x+c+a
rctan(2/3))+2)^(1/2)*(-sin(d*x+c+arctan(2/3)))^(1/2)*EllipticF((1+sin(d*x+c
+arctan(2/3)))^(1/2),1/2*2^(1/2))-2*sin(d*x+c+arctan(2/3))^4+2*sin(d*x+c+ar
ctan(2/3))^2)/cos(d*x+c+arctan(2/3))/(13^(1/2)*sin(d*x+c+arctan(2/3)))^(1/2
)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (2\cos(dx+c) + 3\sin(dx+c))^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*cos(d*x+c)+3*sin(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((2*cos(d*x + c) + 3*sin(d*x + c))^(5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int (2\cos(c+d\,x) + 3\sin(c+d\,x))^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*cos(c + d*x) + 3*sin(c + d*x))^(5/2),x)
```

```
[Out] int((2*cos(c + d*x) + 3*sin(c + d*x))^(5/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*cos(d*x+c)+3*sin(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

3.242 $\int (2\cos(c+dx)+3\sin(c+dx))^{3/2}\,dx$

Optimal. Leaf size=75

$$\frac{2\,13^{3/4}F\left(\frac{1}{2}\left(c+dx-\tan^{-1}\left(\frac{3}{2}\right)\right)\middle|2\right)}{3d}-\frac{2(3\cos(c+dx)-2\sin(c+dx))\sqrt{3\sin(c+dx)+2\cos(c+dx)}}{3d}$$

```
[Out] 2/3*13^(3/4)*(cos(1/2*c+1/2*d*x-1/2*arctan(3/2))^2)^(1/2)/cos(1/2*c+1/2*d*x
-1/2*arctan(3/2))*EllipticF(sin(1/2*c+1/2*d*x-1/2*arctan(3/2)),2^(1/2))/d-2
/3*(3*cos(d*x+c)-2*sin(d*x+c))*(2*cos(d*x+c)+3*sin(d*x+c))^(1/2)/d
```

Rubi [A] time = 0.04, antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3073, 3077, 2641}

$$\frac{2\,13^{3/4}F\left(\frac{1}{2}\left(c+dx-\tan^{-1}\left(\frac{3}{2}\right)\right)\middle|2\right)}{3d}-\frac{2(3\cos(c+dx)-2\sin(c+dx))\sqrt{3\sin(c+dx)+2\cos(c+dx)}}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[(2*Cos[c + d*x] + 3*Sin[c + d*x])^(3/2),x]
```

```
[Out] (2*13^(3/4)*EllipticF[(c + d*x - ArcTan[3/2])/2, 2])/(3*d) - (2*(3*Cos[c +
d*x] - 2*Sin[c + d*x])*Sqrt[2*Cos[c + d*x] + 3*Sin[c + d*x]])/(3*d)
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3073

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x
_Symbol] :> -Simp[((b*Cos[c + d*x] - a*Sin[c + d*x])*(a*Cos[c + d*x] + b*Si
n[c + d*x])^(n - 1))/(d*n), x] + Dist[((n - 1)*(a^2 + b^2))/n, Int[(a*Cos[c
 + d*x] + b*Sin[c + d*x])^(n - 2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 + b^2, 0] &&  !IntegerQ[(n - 1)/2] && GtQ[n, 1]
```

Rule 3077

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x
_Symbol] :> Dist[(a^2 + b^2)^(n/2), Int[Cos[c + d*x - ArcTan[a, b]]^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] &&  !(GeQ[n, 1] || LeQ[n, -1]) && GtQ[a^2 +
 b^2, 0]
```

Rubi steps

$$\int (2\cos(c+dx)+3\sin(c+dx))^{3/2}\,dx = -\frac{2(3\cos(c+dx)-2\sin(c+dx))\sqrt{2\cos(c+dx)+3\sin(c+dx)}}{3d}+\frac{13}{3}\ldots$$

$$= -\frac{2(3\cos(c+dx)-2\sin(c+dx))\sqrt{2\cos(c+dx)+3\sin(c+dx)}}{3d}+\frac{1}{3}13\ldots$$

$$= \frac{2\,13^{3/4}F\left(\frac{1}{2}\left(c+dx-\tan^{-1}\left(\frac{3}{2}\right)\right)\middle|2\right)}{3d}-\frac{2(3\cos(c+dx)-2\sin(c+dx))\ldots}{3d\ldots}$$

Mathematica [C] time = 0.30, size = 133, normalized size = 1.77

$$\frac{2\,13^{3/4}\sqrt{-\left(\left(\sin\left(c+dx+\tan^{-1}\left(\frac{2}{3}\right)\right)-1\right)\sin\left(c+dx+\tan^{-1}\left(\frac{2}{3}\right)\right)\right)}\sqrt{\sin\left(c+dx+\tan^{-1}\left(\frac{2}{3}\right)\right)+1}\,\sec\left(c+dx\ldots\right.}{\ldots}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(2*Cos[c + d*x] + 3*Sin[c + d*x])^(3/2),x]
```

```
[Out] (2*(-3*Cos[c + d*x] + 2*Sin[c + d*x])*Sqrt[2*Cos[c + d*x] + 3*Sin[c + d*x]]
 + 2*13^(3/4)*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[c + d*x + ArcTan[2/3
]]^2]*Sec[c + d*x + ArcTan[2/3]]*Sqrt[-((-1 + Sin[c + d*x + ArcTan[2/3]])*S
in[c + d*x + ArcTan[2/3]])]*Sqrt[1 + Sin[c + d*x + ArcTan[2/3]]])/(3*d)
```

fricas [F] time = 1.25, size = 0, normalized size = 0.00

$$\text{integral}\left((2\cos(dx+c)+3\sin(dx+c))^{\frac{3}{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*cos(d*x+c)+3*sin(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] integral((2*cos(d*x + c) + 3*sin(d*x + c))^(3/2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (2\cos(dx+c)+3\sin(dx+c))^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*cos(d*x+c)+3*sin(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((2*cos(d*x + c) + 3*sin(d*x + c))^(3/2), x)
```

maple [A] time = 0.34, size = 108, normalized size = 1.44

$$\frac{\frac{13\sqrt{1+\sin\left(dx+c+\arctan\left(\frac{2}{3}\right)\right)}\,\sqrt{-2\sin\left(dx+c+\arctan\left(\frac{2}{3}\right)\right)+2}\,\sqrt{-\sin\left(dx+c+\arctan\left(\frac{2}{3}\right)\right)}\,\mathrm{EllipticF}\left(\sqrt{1+\sin\left(dx+c+\arctan\left(\frac{2}{3}\right)\right)},\frac{\sqrt{2}}{2}\right)}{3}-\frac{26\left(\cos^2\left(\ldots\right)\right)}{\ldots}}{\cos\left(dx+c+\arctan\left(\frac{2}{3}\right)\right)\sqrt{\sqrt{13}\,\sin\left(dx+c+\arctan\left(\frac{2}{3}\right)\right)}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*cos(d*x+c)+3*sin(d*x+c))^(3/2),x)
```

```
[Out] (13/3*(1+sin(d*x+c+arctan(2/3)))^(1/2)*(-2*sin(d*x+c+arctan(2/3))+2)^(1/2)*
(-sin(d*x+c+arctan(2/3)))^(1/2)*EllipticF((1+sin(d*x+c+arctan(2/3)))^(1/2),
1/2*2^(1/2))-26/3*cos(d*x+c+arctan(2/3))^2*sin(d*x+c+arctan(2/3)))/cos(d*x+
c+arctan(2/3))/(13^(1/2)*sin(d*x+c+arctan(2/3)))^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (2\cos(dx+c)+3\sin(dx+c))^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*cos(d*x+c)+3*sin(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((2*cos(d*x + c) + 3*sin(d*x + c))^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int (2\cos(c+d\,x)+3\sin(c+d\,x))^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*cos(c + d*x) + 3*sin(c + d*x))^(3/2),x)
```

```
[Out] int((2*cos(c + d*x) + 3*sin(c + d*x))^(3/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*cos(d*x+c)+3*sin(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

3.243 $\int \sqrt{2 \cos(c+dx)+3 \sin(c+dx)}\,dx$

Optimal. Leaf size=27

$$\frac{2\sqrt[4]{13}\,E\left(\frac{1}{2}\left(c+dx-\tan^{-1}\left(\frac{3}{2}\right)\right)\middle|2\right)}{d}$$

```
[Out] 2*13^(1/4)*(cos(1/2*c+1/2*d*x-1/2*arctan(3/2))^2)^(1/2)/cos(1/2*c+1/2*d*x-1
/2*arctan(3/2))*EllipticE(sin(1/2*c+1/2*d*x-1/2*arctan(3/2)),2^(1/2))/d
```

Rubi [A] time = 0.02, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {3077, 2639}

$$\frac{2\sqrt[4]{13}\,E\left(\frac{1}{2}\left(c+dx-\tan^{-1}\left(\frac{3}{2}\right)\right)\middle|2\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[2*Cos[c + d*x] + 3*Sin[c + d*x]],x]
```

```
[Out] (2*13^(1/4)*EllipticE[(c + d*x - ArcTan[3/2])/2, 2])/d
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3077

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x
_Symbol] :> Dist[(a^2 + b^2)^(n/2), Int[Cos[c + d*x - ArcTan[a, b]]^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && !(GeQ[n, 1] || LeQ[n, -1]) && GtQ[a^2 +
 b^2, 0]
```

Rubi steps

$$\begin{aligned}\int \sqrt{2\cos(c+dx)+3\sin(c+dx)}\,dx &= \sqrt[4]{13}\int \sqrt{\cos\left(c+dx-\tan^{-1}\left(\frac{3}{2}\right)\right)}\,dx\\ &= \frac{2\sqrt[4]{13}\,E\left(\frac{1}{2}\left(c+dx-\tan^{-1}\left(\frac{3}{2}\right)\right)\middle|2\right)}{d}\end{aligned}$$

Mathematica [C] time = 0.81, size = 184, normalized size = 6.81

$$-\frac{-\frac{3\sqrt[4]{13}\sin\left(c+dx-\tan^{-1}\left(\frac{3}{2}\right)\right){}_2F_1\left(-\frac{1}{2},-\frac{1}{4};\frac{3}{4};\cos^2\left(c+dx-\tan^{-1}\left(\frac{3}{2}\right)\right)\right)}{\sqrt{-\left(\left(\cos\left(c+dx-\tan^{-1}\left(\frac{3}{2}\right)\right)-1\right)\cos\left(c+dx-\tan^{-1}\left(\frac{3}{2}\right)\right)\right)}\sqrt{\cos\left(c+dx-\tan^{-1}\left(\frac{3}{2}\right)\right)+1}}+4\sqrt{3\sin(c+dx)+2\cos(c+dx)}-4\sqrt[4]{13}\sqrt{\mathrm{co}}}{3d}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[2*Cos[c + d*x] + 3*Sin[c + d*x]],x]
```

```
[Out] (-4*13^(1/4)*Sqrt[Cos[c + d*x - ArcTan[3/2]]] + 4*Sqrt[2*Cos[c + d*x] + 3*S
in[c + d*x]] + (3*13^(1/4)*Sin[c + d*x - ArcTan[3/2]])/Sqrt[Cos[c + d*x - A
rcTan[3/2]]] - (3*13^(1/4)*HypergeometricPFQ[{-1/2, -1/4}, {3/4}, Cos[c + d
*x - ArcTan[3/2]]^2]*Sin[c + d*x - ArcTan[3/2]])/(Sqrt[-((-1 + Cos[c + d*x
- ArcTan[3/2]])*Cos[c + d*x - ArcTan[3/2]])]*Sqrt[1 + Cos[c + d*x - ArcTan[
3/2]]]))/(3*d)
```

fricas [F] time = 1.19, size = 0, normalized size = 0.00

$$\text{integral}\left(\sqrt{2\cos(dx+c)+3\sin(dx+c)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*cos(d*x+c)+3*sin(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(2*cos(d*x + c) + 3*sin(d*x + c)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{2\cos(dx+c)+3\sin(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*cos(d*x+c)+3*sin(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(2*cos(d*x + c) + 3*sin(d*x + c)), x)
```

maple [A] time = 0.33, size = 112, normalized size = 4.15

$$-\frac{\sqrt{13}\sqrt{1+\sin\left(dx+c+\arctan\left(\frac{2}{3}\right)\right)}\sqrt{-2\sin\left(dx+c+\arctan\left(\frac{2}{3}\right)\right)+2}\sqrt{-\sin\left(dx+c+\arctan\left(\frac{2}{3}\right)\right)}\left(2\,\mathrm{E}\right.}{\cos\left(dx+c+\arctan\left(\frac{2}{3}\right)\right)\sqrt{\sqrt{13}\,\mathrm{si}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*cos(d*x+c)+3*sin(d*x+c))^(1/2),x)
```

```
[Out] -13^(1/2)*(1+sin(d*x+c+arctan(2/3)))^(1/2)*(-2*sin(d*x+c+arctan(2/3))+2)^(1
/2)*(-sin(d*x+c+arctan(2/3)))^(1/2)*(2*EllipticE((1+sin(d*x+c+arctan(2/3)))
^(1/2),1/2*2^(1/2))-EllipticF((1+sin(d*x+c+arctan(2/3)))^(1/2),1/2*2^(1/2))
)/cos(d*x+c+arctan(2/3))/(13^(1/2)*sin(d*x+c+arctan(2/3)))^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{2 \cos(dx+c) + 3 \sin(dx+c)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*cos(d*x+c)+3*sin(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(2*cos(d*x + c) + 3*sin(d*x + c)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.04

$$\int \sqrt{2 \cos(c+d\,x) + 3 \sin(c+d\,x)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*cos(c + d*x) + 3*sin(c + d*x))^(1/2),x)
```

```
[Out] int((2*cos(c + d*x) + 3*sin(c + d*x))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{3 \sin(c+dx) + 2 \cos(c+dx)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*cos(d*x+c)+3*sin(d*x+c))**(1/2),x)
```

```
[Out] Integral(sqrt(3*sin(c + d*x) + 2*cos(c + d*x)), x)
```

3.244 $\int \frac{1}{\sqrt{2\cos(c+dx)+3\sin(c+dx)}}\,dx$

Optimal. Leaf size=27

$$\frac{2F\left(\frac{1}{2}\left(c+dx-\tan^{-1}\left(\frac{3}{2}\right)\right)\middle|2\right)}{\sqrt[4]{13}\,d}$$

```
[Out] 2/13*13^(3/4)*(cos(1/2*c+1/2*d*x-1/2*arctan(3/2))^2)^(1/2)/cos(1/2*c+1/2*d*
x-1/2*arctan(3/2))*EllipticF(sin(1/2*c+1/2*d*x-1/2*arctan(3/2)),2^(1/2))/d
```

Rubi [A] time = 0.02, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {3077, 2641}

$$\frac{2F\left(\frac{1}{2}\left(c+dx-\tan^{-1}\left(\frac{3}{2}\right)\right)\middle|2\right)}{\sqrt[4]{13}\,d}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[2*Cos[c + d*x] + 3*Sin[c + d*x]],x]
```

```
[Out] (2*EllipticF[(c + d*x - ArcTan[3/2])/2, 2])/(13^(1/4)*d)
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3077

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x
_Symbol] :> Dist[(a^2 + b^2)^(n/2), Int[Cos[c + d*x - ArcTan[a, b]]^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] &&  !(GeQ[n, 1] || LeQ[n, -1]) && GtQ[a^2 +
 b^2, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{2\cos(c+dx)+3\sin(c+dx)}}\,dx = \frac{\int \frac{1}{\sqrt{\cos\left(c+dx-\tan^{-1}\left(\frac{3}{2}\right)\right)}}\,dx}{\sqrt[4]{13}}$$
$$= \frac{2F\left(\frac{1}{2}\left(c+dx-\tan^{-1}\left(\frac{3}{2}\right)\right)\middle|2\right)}{\sqrt[4]{13}\,d}$$

Mathematica [C] time = 0.09, size = 88, normalized size = 3.26

$$\frac{2\sqrt{-\left(\left(\sin\left(c+dx+\tan^{-1}\left(\frac{2}{3}\right)\right)-1\right)\sin\left(c+dx+\tan^{-1}\left(\frac{2}{3}\right)\right)\right)}\sqrt{\sin\left(c+dx+\tan^{-1}\left(\frac{2}{3}\right)\right)+1}\sec\left(c+dx+\tan\right.}{\sqrt[4]{13}\,d}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/Sqrt[2*Cos[c + d*x] + 3*Sin[c + d*x]],x]
[Out] (2*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[c + d*x + ArcTan[2/3]]^2]*Sec[c
 + d*x + ArcTan[2/3]]*Sqrt[-((-1 + Sin[c + d*x + ArcTan[2/3]])*Sin[c + d*x
+ ArcTan[2/3]])]*Sqrt[1 + Sin[c + d*x + ArcTan[2/3]]])/(13^(1/4)*d)
```

fricas [F] time = 1.77, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{1}{\sqrt{2\,\cos\left(dx+c\right)+3\,\sin\left(dx+c\right)}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*cos(d*x+c)+3*sin(d*x+c))^(1/2),x, algorithm="fricas")
[Out] integral(1/sqrt(2*cos(d*x + c) + 3*sin(d*x + c)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{2\,\cos\left(dx+c\right)+3\,\sin\left(dx+c\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*cos(d*x+c)+3*sin(d*x+c))^(1/2),x, algorithm="giac")
[Out] integrate(1/sqrt(2*cos(d*x + c) + 3*sin(d*x + c)), x)
```

maple [A] time = 0.24, size = 85, normalized size = 3.15

$$\frac{\sqrt{1+\sin\left(dx+c+\arctan\left(\frac{2}{3}\right)\right)}\,\sqrt{-2\sin\left(dx+c+\arctan\left(\frac{2}{3}\right)\right)+2}\,\sqrt{-\sin\left(dx+c+\arctan\left(\frac{2}{3}\right)\right)}\,\mathrm{EllipticF}\left(\right.}{\cos\left(dx+c+\arctan\left(\frac{2}{3}\right)\right)\sqrt{\sqrt{13}\,\sin\left(dx+c+\arctan\left(\frac{2}{3}\right)\right)}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2*cos(d*x+c)+3*sin(d*x+c))^(1/2),x)
```

```
[Out] (1+sin(d*x+c+arctan(2/3)))^(1/2)*(-2*sin(d*x+c+arctan(2/3))+2)^(1/2)*(-sin(
d*x+c+arctan(2/3)))^(1/2)*EllipticF((1+sin(d*x+c+arctan(2/3)))^(1/2),1/2*2^
(1/2))/cos(d*x+c+arctan(2/3))/(13^(1/2)*sin(d*x+c+arctan(2/3)))^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{2\,\cos\left(dx+c\right)+3\,\sin\left(dx+c\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*cos(d*x+c)+3*sin(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(2*cos(d*x + c) + 3*sin(d*x + c)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{1}{\sqrt{2\,\cos\left(c+d\,x\right)+3\,\sin\left(c+d\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2*cos(c + d*x) + 3*sin(c + d*x))^(1/2),x)
```

```
[Out] int(1/(2*cos(c + d*x) + 3*sin(c + d*x))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{3\sin\left(c+dx\right)+2\cos\left(c+dx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*cos(d*x+c)+3*sin(d*x+c))**(1/2),x)
```

```
[Out] Integral(1/sqrt(3*sin(c + d*x) + 2*cos(c + d*x)), x)
```

3.245 $\int \frac{1}{(2\cos(c+dx)+3\sin(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=73

$$-\frac{2(3\cos(c+dx)-2\sin(c+dx))}{13d\sqrt{3\sin(c+dx)+2\cos(c+dx)}}-\frac{2E\left(\frac{1}{2}\left(c+dx-\tan^{-1}\left(\frac{3}{2}\right)\right)\middle|2\right)}{13^{3/4}d}$$

```
[Out] -2/13*13^(1/4)*(cos(1/2*c+1/2*d*x-1/2*arctan(3/2))^2)^(1/2)/cos(1/2*c+1/2*d
*x-1/2*arctan(3/2))*EllipticE(sin(1/2*c+1/2*d*x-1/2*arctan(3/2)),2^(1/2))/d
-2/13*(3*cos(d*x+c)-2*sin(d*x+c))/d/(2*cos(d*x+c)+3*sin(d*x+c))^(1/2)
```

Rubi [A] time = 0.04, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3076, 3077, 2639}

$$-\frac{2(3\cos(c+dx)-2\sin(c+dx))}{13d\sqrt{3\sin(c+dx)+2\cos(c+dx)}}-\frac{2E\left(\frac{1}{2}\left(c+dx-\tan^{-1}\left(\frac{3}{2}\right)\right)\middle|2\right)}{13^{3/4}d}$$

Antiderivative was successfully verified.

```
[In] Int[(2*Cos[c + d*x] + 3*Sin[c + d*x])^(-3/2),x]
```

```
[Out] (-2*EllipticE[(c + d*x - ArcTan[3/2])/2, 2])/(13^(3/4)*d) - (2*(3*Cos[c + d
*x] - 2*Sin[c + d*x]))/(13*d*Sqrt[2*Cos[c + d*x] + 3*Sin[c + d*x]])
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3076

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x
_Symbol] :> Simp[((b*Cos[c + d*x] - a*Sin[c + d*x])*(a*Cos[c + d*x] + b*Sin
[c + d*x])^(n + 1))/(d*(n + 1)*(a^2 + b^2)), x] + Dist[(n + 2)/((n + 1)*(a^
2 + b^2)), Int[(a*Cos[c + d*x] + b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{
a, b, c, d}, x] && NeQ[a^2 + b^2, 0] && LtQ[n, -1] && NeQ[n, -2]
```

Rule 3077

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x
_Symbol] :> Dist[(a^2 + b^2)^(n/2), Int[Cos[c + d*x - ArcTan[a, b]]^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] &&  !(GeQ[n, 1] || LeQ[n, -1]) && GtQ[a^2 +
```

```
b^2, 0]
```

Rubi steps

$$\int \frac{1}{(2\cos(c+dx)+3\sin(c+dx))^{3/2}}\,dx = -\frac{2(3\cos(c+dx)-2\sin(c+dx))}{13d\sqrt{2\cos(c+dx)+3\sin(c+dx)}} - \frac{1}{13}\int \sqrt{2\cos(c+dx)+3\sin(}$$

$$= -\frac{2(3\cos(c+dx)-2\sin(c+dx))}{13d\sqrt{2\cos(c+dx)+3\sin(c+dx)}} - \frac{\int \sqrt{\cos\left(c+dx-\tan^{-1}\left(\frac{3}{2}\right)\right)}}{13^{3/4}}$$

$$= -\frac{2E\left(\frac{1}{2}\left(c+dx-\tan^{-1}\left(\frac{3}{2}\right)\right)\middle|2\right)}{13^{3/4}d} - \frac{2(3\cos(c+dx)-2\sin(c+dx))}{13d\sqrt{2\cos(c+dx)+3\sin(c+dx)}}$$

Mathematica [C] time = 1.03, size = 190, normalized size = 2.60

$$\frac{\frac{3\sin\left(c+dx-\tan^{-1}\left(\frac{3}{2}\right)\right){}_2F_1\left(-\frac{1}{2},-\frac{1}{4};\frac{3}{4};\cos^2\left(c+dx-\tan^{-1}\left(\frac{3}{2}\right)\right)\right)}{13^{3/4}\sqrt{-\left(\left(\cos\left(c+dx-\tan^{-1}\left(\frac{3}{2}\right)\right)-1\right)\cos\left(c+dx-\tan^{-1}\left(\frac{3}{2}\right)\right)\right)}\sqrt{\cos\left(c+dx-\tan^{-1}\left(\frac{3}{2}\right)\right)+1}} - \frac{2\cos(c+dx)}{\sqrt{3\sin(c+dx)+2\cos(c+dx)}} + \frac{4\sqrt{\cos\left(c+dx-\tan^{-1}\left(\frac{3}{2}\right)\right)}}{13^{3/4}}}{3d}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(2*Cos[c + d*x] + 3*Sin[c + d*x])^(-3/2),x]
```

```
[Out] ((4*Sqrt[Cos[c + d*x - ArcTan[3/2]]])/13^(3/4) - (2*Cos[c + d*x])/Sqrt[2*Co
s[c + d*x] + 3*Sin[c + d*x]] - (3*Sin[c + d*x - ArcTan[3/2]])/(13^(3/4)*Sqr
t[Cos[c + d*x - ArcTan[3/2]]]) + (3*HypergeometricPFQ[{-1/2, -1/4}, {3/4},
Cos[c + d*x - ArcTan[3/2]]^2]*Sin[c + d*x - ArcTan[3/2]])/(13^(3/4)*Sqrt[-(
(-1 + Cos[c + d*x - ArcTan[3/2]])*Cos[c + d*x - ArcTan[3/2]])]*Sqrt[1 + Cos
[c + d*x - ArcTan[3/2]]]))/(3*d)
```

fricas [F] time = 1.09, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\sqrt{2\cos(dx+c)+3\sin(dx+c)}}{5\cos(dx+c)^2-12\cos(dx+c)\sin(dx+c)-9},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*cos(d*x+c)+3*sin(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(2*cos(d*x + c) + 3*sin(d*x + c))/(5*cos(d*x + c)^2 - 12*cos(
d*x + c)*sin(d*x + c) - 9), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(2 \cos\left(dx+c\right)+3 \sin\left(dx+c\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*cos(d*x+c)+3*sin(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((2*cos(d*x + c) + 3*sin(d*x + c))^(-3/2), x)
```

maple [A] time = 0.34, size = 162, normalized size = 2.22

$$\frac{\sqrt{13}\,\left(2\sqrt{1+\sin\left(dx+c+\arctan\left(\frac{2}{3}\right)\right)}\,\sqrt{-2\sin\left(dx+c+\arctan\left(\frac{2}{3}\right)\right)+2}\,\sqrt{-\sin\left(dx+c+\arctan\left(\frac{2}{3}\right)\right)}\ \mathrm{Ellip}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2*cos(d*x+c)+3*sin(d*x+c))^(3/2),x)
```

```
[Out] 1/13*13^(1/2)*(2*(1+sin(d*x+c+arctan(2/3)))^(1/2)*(-2*sin(d*x+c+arctan(2/3)
)+2)^(1/2)*(-sin(d*x+c+arctan(2/3)))^(1/2)*EllipticE((1+sin(d*x+c+arctan(2/
3)))^(1/2),1/2*2^(1/2))-(1+sin(d*x+c+arctan(2/3)))^(1/2)*(-2*sin(d*x+c+arct
an(2/3))+2)^(1/2)*(-sin(d*x+c+arctan(2/3)))^(1/2)*EllipticF((1+sin(d*x+c+ar
ctan(2/3)))^(1/2),1/2*2^(1/2))-2*cos(d*x+c+arctan(2/3))^2)/cos(d*x+c+arctan
(2/3))/(13^(1/2)*sin(d*x+c+arctan(2/3)))^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(2 \cos\left(dx+c\right)+3 \sin\left(dx+c\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*cos(d*x+c)+3*sin(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((2*cos(d*x + c) + 3*sin(d*x + c))^(-3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{{\left(2\,\cos\left(c+d\,x\right)+3\,\sin\left(c+d\,x\right)\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2*cos(c + d*x) + 3*sin(c + d*x))^(3/2),x)
```

```
[Out] int(1/(2*cos(c + d*x) + 3*sin(c + d*x))^(3/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*cos(d*x+c)+3*sin(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

3.246 $\int \frac{1}{(2\cos(c+dx)+3\sin(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=75

$$\frac{2F\left(\frac{1}{2}\left(c+dx-\tan^{-1}\left(\frac{3}{2}\right)\right)\middle|2\right)}{39\sqrt[4]{13}\,d}-\frac{2(3\cos(c+dx)-2\sin(c+dx))}{39d(3\sin(c+dx)+2\cos(c+dx))^{3/2}}$$

```
[Out] 2/507*13^(3/4)*(cos(1/2*c+1/2*d*x-1/2*arctan(3/2))^2)^(1/2)/cos(1/2*c+1/2*d
*x-1/2*arctan(3/2))*EllipticF(sin(1/2*c+1/2*d*x-1/2*arctan(3/2)),2^(1/2))/d
-2/39*(3*cos(d*x+c)-2*sin(d*x+c))/d/(2*cos(d*x+c)+3*sin(d*x+c))^(3/2)
```

Rubi [A] time = 0.04, antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3076, 3077, 2641}

$$\frac{2F\left(\frac{1}{2}\left(c+dx-\tan^{-1}\left(\frac{3}{2}\right)\right)\middle|2\right)}{39\sqrt[4]{13}\,d}-\frac{2(3\cos(c+dx)-2\sin(c+dx))}{39d(3\sin(c+dx)+2\cos(c+dx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(2*Cos[c + d*x] + 3*Sin[c + d*x])^(-5/2),x]
```

```
[Out] (2*EllipticF[(c + d*x - ArcTan[3/2])/2, 2])/(39*13^(1/4)*d) - (2*(3*Cos[c +
 d*x] - 2*Sin[c + d*x]))/(39*d*(2*Cos[c + d*x] + 3*Sin[c + d*x])^(3/2))
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3076

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x
_Symbol] :> Simp[((b*Cos[c + d*x] - a*Sin[c + d*x])*(a*Cos[c + d*x] + b*Sin
[c + d*x])^(n + 1))/(d*(n + 1)*(a^2 + b^2)), x] + Dist[(n + 2)/((n + 1)*(a^
2 + b^2)), Int[(a*Cos[c + d*x] + b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{
a, b, c, d}, x] && NeQ[a^2 + b^2, 0] && LtQ[n, -1] && NeQ[n, -2]
```

Rule 3077

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x
_Symbol] :> Dist[(a^2 + b^2)^(n/2), Int[Cos[c + d*x - ArcTan[a, b]]^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] &&  !(GeQ[n, 1] || LeQ[n, -1]) && GtQ[a^2 +
```

```
b^2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(2\cos(c+dx)+3\sin(c+dx))^{5/2}}\,dx &= -\frac{2(3\cos(c+dx)-2\sin(c+dx))}{39d(2\cos(c+dx)+3\sin(c+dx))^{3/2}}+\frac{1}{39}\int \frac{1}{\sqrt{2\cos(c+dx)+3\sin(c+dx)}}\,dx\\ &= -\frac{2(3\cos(c+dx)-2\sin(c+dx))}{39d(2\cos(c+dx)+3\sin(c+dx))^{3/2}}+\frac{\int \frac{1}{\sqrt{\cos\left(c+dx-\tan^{-1}\left(\frac{3}{2}\right)\right)}}\,dx}{39\sqrt[4]{13}}\\ &= \frac{2F\left(\frac{1}{2}\left(c+dx-\tan^{-1}\left(\frac{3}{2}\right)\right)\middle|2\right)}{39\sqrt[4]{13}\,d}-\frac{2(3\cos(c+dx)-2\sin(c+dx))}{39d(2\cos(c+dx)+3\sin(c+dx))^{3/2}}\end{aligned}$$

Mathematica [C] time = 0.71, size = 157, normalized size = 2.09

$$\frac{\sqrt{2}\,13^{3/4}\sqrt{\sin\left(c+dx+\tan^{-1}\left(\frac{2}{3}\right)\right)+1}\,(3\sin(c+dx)+2\cos(c+dx))^{3/2}\sec\left(c+dx+\tan^{-1}\left(\frac{2}{3}\right)\right)\sqrt{2\sin\left(c+d \cdots\right.}}{507d(3\sin(c \cdots}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(2*Cos[c + d*x] + 3*Sin[c + d*x])^(-5/2),x]
```

```
[Out] (-78*Cos[c + d*x] + 52*Sin[c + d*x] + Sqrt[2]*13^(3/4)*HypergeometricPFQ[{1
/4, 1/2}, {5/4}, Sin[c + d*x + ArcTan[2/3]]^2]*Sec[c + d*x + ArcTan[2/3]]*(
2*Cos[c + d*x] + 3*Sin[c + d*x])^(3/2)*Sqrt[1 + Sin[c + d*x + ArcTan[2/3]]]
*Sqrt[-1 + Cos[2*(c + d*x + ArcTan[2/3])] + 2*Sin[c + d*x + ArcTan[2/3]]])/
(507*d*(2*Cos[c + d*x] + 3*Sin[c + d*x])^(3/2))
```

fricas [F] time = 0.50, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\sqrt{2\cos(dx+c)+3\sin(dx+c)}}{46\cos(dx+c)^3-9\left(\cos(dx+c)^2+3\right)\sin(dx+c)-54\cos(dx+c)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*cos(d*x+c)+3*sin(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(2*cos(d*x + c) + 3*sin(d*x + c))/(46*cos(d*x + c)^3 - 9*(cos
(d*x + c)^2 + 3)*sin(d*x + c) - 54*cos(d*x + c)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(2\cos\left(dx+c\right)+3\sin\left(dx+c\right)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*cos(d*x+c)+3*sin(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((2*cos(d*x + c) + 3*sin(d*x + c))^(-5/2), x)
```

maple [A] time = 0.33, size = 118, normalized size = 1.57

$$\frac{\sqrt{1+\sin\left(dx+c+\arctan\left(\frac{2}{3}\right)\right)}\,\sqrt{-2\sin\left(dx+c+\arctan\left(\frac{2}{3}\right)\right)+2}\,\sqrt{-\sin\left(dx+c+\arctan\left(\frac{2}{3}\right)\right)}\,\mathrm{EllipticF}\left(\sqrt{\right.}}{39\sin\left(dx+c+\arctan\left(\frac{2}{3}\right)\right)\cos\left(dx+c+\arctan\left(\frac{2}{3}\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2*cos(d*x+c)+3*sin(d*x+c))^(5/2),x)
```

```
[Out] 1/39/sin(d*x+c+arctan(2/3))*((1+sin(d*x+c+arctan(2/3)))^(1/2)*(-2*sin(d*x+c
+arctan(2/3))+2)^(1/2)*(-sin(d*x+c+arctan(2/3)))^(1/2)*EllipticF((1+sin(d*x
+c+arctan(2/3)))^(1/2),1/2*2^(1/2))*sin(d*x+c+arctan(2/3))-2*cos(d*x+c+arct
an(2/3))^2)/cos(d*x+c+arctan(2/3))/(13^(1/2)*sin(d*x+c+arctan(2/3)))^(1/2)/
d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(2\cos\left(dx+c\right)+3\sin\left(dx+c\right)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*cos(d*x+c)+3*sin(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((2*cos(d*x + c) + 3*sin(d*x + c))^(-5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{{\left(2\,\cos\left(c+d\,x\right)+3\,\sin\left(c+d\,x\right)\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2*cos(c + d*x) + 3*sin(c + d*x))^(5/2),x)
```

```
[Out] int(1/(2*cos(c + d*x) + 3*sin(c + d*x))^(5/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*cos(d*x+c)+3*sin(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

3.247 $\int \frac{1}{(2\cos(c+dx)+3\sin(c+dx))^{7/2}}\,dx$

Optimal. Leaf size=120

$$-\frac{6(3\cos(c+dx)-2\sin(c+dx))}{845d\sqrt{3\sin(c+dx)+2\cos(c+dx)}}-\frac{2(3\cos(c+dx)-2\sin(c+dx))}{65d(3\sin(c+dx)+2\cos(c+dx))^{5/2}}-\frac{6E\left(\frac{1}{2}\left(c+dx-\tan^{-1}\left(\frac{3}{2}\right)\right)\middle|2\right)}{65\,13^{3/4}d}$$

```
[Out] -6/845*13^(1/4)*(cos(1/2*c+1/2*d*x-1/2*arctan(3/2))^2)^(1/2)/cos(1/2*c+1/2*
d*x-1/2*arctan(3/2))*EllipticE(sin(1/2*c+1/2*d*x-1/2*arctan(3/2)),2^(1/2))/
d-2/65*(3*cos(d*x+c)-2*sin(d*x+c))/d/(2*cos(d*x+c)+3*sin(d*x+c))^(5/2)-6/84
5*(3*cos(d*x+c)-2*sin(d*x+c))/d/(2*cos(d*x+c)+3*sin(d*x+c))^(1/2)
```

Rubi [A] time = 0.06, antiderivative size = 120, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3076, 3077, 2639}

$$-\frac{6(3\cos(c+dx)-2\sin(c+dx))}{845d\sqrt{3\sin(c+dx)+2\cos(c+dx)}}-\frac{2(3\cos(c+dx)-2\sin(c+dx))}{65d(3\sin(c+dx)+2\cos(c+dx))^{5/2}}-\frac{6E\left(\frac{1}{2}\left(c+dx-\tan^{-1}\left(\frac{3}{2}\right)\right)\middle|2\right)}{65\,13^{3/4}d}$$

Antiderivative was successfully verified.

```
[In] Int[(2*Cos[c + d*x] + 3*Sin[c + d*x])^(-7/2),x]
```

```
[Out] (-6*EllipticE[(c + d*x - ArcTan[3/2])/2, 2])/(65*13^(3/4)*d) - (2*(3*Cos[c
+ d*x] - 2*Sin[c + d*x]))/(65*d*(2*Cos[c + d*x] + 3*Sin[c + d*x])^(5/2)) -
(6*(3*Cos[c + d*x] - 2*Sin[c + d*x]))/(845*d*Sqrt[2*Cos[c + d*x] + 3*Sin[c
+ d*x]])
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3076

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x
_Symbol] :> Simp[((b*Cos[c + d*x] - a*Sin[c + d*x])*(a*Cos[c + d*x] + b*Sin
[c + d*x])^(n + 1))/(d*(n + 1)*(a^2 + b^2)), x] + Dist[(n + 2)/((n + 1)*(a^
2 + b^2)), Int[(a*Cos[c + d*x] + b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{
a, b, c, d}, x] && NeQ[a^2 + b^2, 0] && LtQ[n, -1] && NeQ[n, -2]
```

Rule 3077

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x
_Symbol] :> Dist[(a^2 + b^2)^(n/2), Int[Cos[c + d*x - ArcTan[a, b]]^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] &&  !(GeQ[n, 1] || LeQ[n, -1]) && GtQ[a^2 +
 b^2, 0]
```

Rubi steps

$$\int \frac{1}{(2\cos(c+dx)+3\sin(c+dx))^{7/2}}\,dx = -\frac{2(3\cos(c+dx)-2\sin(c+dx))}{65d(2\cos(c+dx)+3\sin(c+dx))^{5/2}} + \frac{3}{65}\int \frac{1}{(2\cos(c+dx)+3\sin$$

$$= -\frac{2(3\cos(c+dx)-2\sin(c+dx))}{65d(2\cos(c+dx)+3\sin(c+dx))^{5/2}} - \frac{6(3\cos(c+dx)-2\sin(c}{845d\sqrt{2\cos(c+dx)+3\sin}}$$

$$= -\frac{2(3\cos(c+dx)-2\sin(c+dx))}{65d(2\cos(c+dx)+3\sin(c+dx))^{5/2}} - \frac{6(3\cos(c+dx)-2\sin(c}{845d\sqrt{2\cos(c+dx)+3\sin}}$$

$$= -\frac{6E\left(\frac{1}{2}\left(c+dx-\tan^{-1}\left(\frac{3}{2}\right)\right)\middle|2\right)}{65\,13^{3/4}d} - \frac{2(3\cos(c+dx)-2\sin(c+dx))}{65d(2\cos(c+dx)+3\sin(c+dx))}$$

Mathematica [C] time = 1.95, size = 224, normalized size = 1.87

$$\frac{\frac{3\sin\left(c+dx-\tan^{-1}\left(\frac{3}{2}\right)\right){}_2F_1\left(-\frac{1}{2},-\frac{1}{4};\frac{3}{4};\cos^2\left(c+dx-\tan^{-1}\left(\frac{3}{2}\right)\right)\right)}{13^{3/4}\sqrt{-\left(\left(\cos\left(c+dx-\tan^{-1}\left(\frac{3}{2}\right)\right)-1\right)\cos\left(c+dx-\tan^{-1}\left(\frac{3}{2}\right)\right)\right)}\sqrt{\cos\left(c+dx-\tan^{-1}\left(\frac{3}{2}\right)\right)+1}} + \frac{-4(\sin(c+dx)+3\sin(3(c+dx)))-33\cos(c+dx)+5\cos(3(c+}{2(3\sin(c+dx)+2\cos(c+dx))^{5/2}}}{65d}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(2*Cos[c + d*x] + 3*Sin[c + d*x])^(-7/2),x]
```

```
[Out] ((4*Sqrt[Cos[c + d*x - ArcTan[3/2]]])/13^(3/4) + (-33*Cos[c + d*x] + 5*Cos[
3*(c + d*x)] - 4*(Sin[c + d*x] + 3*Sin[3*(c + d*x)]))/(2*(2*Cos[c + d*x] +
3*Sin[c + d*x])^(5/2)) - (3*Sin[c + d*x - ArcTan[3/2]])/(13^(3/4)*Sqrt[Cos[
c + d*x - ArcTan[3/2]]]) + (3*HypergeometricPFQ[{-1/2, -1/4}, {3/4}, Cos[c
+ d*x - ArcTan[3/2]]^2]*Sin[c + d*x - ArcTan[3/2]])/(13^(3/4)*Sqrt[-((-1 +
Cos[c + d*x - ArcTan[3/2]])*Cos[c + d*x - ArcTan[3/2]])]*Sqrt[1 + Cos[c + d
*x - ArcTan[3/2]]]))/(65*d)
```

fricas [F] time = 1.54, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(-\frac{\sqrt{2\cos(dx+c)+3\sin(dx+c)}}{119\cos(dx+c)^4-54\cos(dx+c)^2+24\left(5\cos(dx+c)^3-9\cos(dx+c)\right)\sin(dx+c)-81}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*cos(d*x+c)+3*sin(d*x+c))^(7/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(2*cos(d*x + c) + 3*sin(d*x + c))/(119*cos(d*x + c)^4 - 54*co
s(d*x + c)^2 + 24*(5*cos(d*x + c)^3 - 9*cos(d*x + c))*sin(d*x + c) - 81), x
)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(2 \cos(dx+c) + 3 \sin(dx+c))^{\frac{7}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*cos(d*x+c)+3*sin(d*x+c))^(7/2),x, algorithm="giac")
```

```
[Out] integrate((2*cos(d*x + c) + 3*sin(d*x + c))^(-7/2), x)
```

maple [A] time = 0.35, size = 205, normalized size = 1.71

$$\frac{\sqrt{13}\left(6\sqrt{1+\sin\left(dx+c+\arctan\left(\frac{2}{3}\right)\right)}\,\sqrt{-2\sin\left(dx+c+\arctan\left(\frac{2}{3}\right)\right)+2}\,\sqrt{-\sin\left(dx+c+\arctan\left(\frac{2}{3}\right)\right)}\,\left(\sin\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2*cos(d*x+c)+3*sin(d*x+c))^(7/2),x)
```

```
[Out] 1/845*13^(1/2)/sin(d*x+c+arctan(2/3))^2*(6*(1+sin(d*x+c+arctan(2/3)))^(1/2)
*(-2*sin(d*x+c+arctan(2/3))+2)^(1/2)*(-sin(d*x+c+arctan(2/3)))^(1/2)*sin(d*
x+c+arctan(2/3))^2*EllipticE((1+sin(d*x+c+arctan(2/3)))^(1/2),1/2*2^(1/2))-
3*(1+sin(d*x+c+arctan(2/3)))^(1/2)*(-2*sin(d*x+c+arctan(2/3))+2)^(1/2)*(-si
n(d*x+c+arctan(2/3)))^(1/2)*sin(d*x+c+arctan(2/3))^2*EllipticF((1+sin(d*x+c
+arctan(2/3)))^(1/2),1/2*2^(1/2))+6*sin(d*x+c+arctan(2/3))^4-4*sin(d*x+c+ar
ctan(2/3))^2-2)/cos(d*x+c+arctan(2/3))/(13^(1/2)*sin(d*x+c+arctan(2/3)))^(1
/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(2 \cos(dx+c) + 3 \sin(dx+c))^{\frac{7}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*cos(d*x+c)+3*sin(d*x+c))^(7/2),x, algorithm="maxima")
[Out] integrate((2*cos(d*x + c) + 3*sin(d*x + c))^(-7/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\left(2\cos\left(c+d\,x\right)+3\sin\left(c+d\,x\right)\right)^{7/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2*cos(c + d*x) + 3*sin(c + d*x))^(7/2),x)
[Out] int(1/(2*cos(c + d*x) + 3*sin(c + d*x))^(7/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*cos(d*x+c)+3*sin(d*x+c))**(7/2),x)
[Out] Timed out
```

3.248 $\int (a\cos(c+dx)+ia\sin(c+dx))^n\,dx$

Optimal. Leaf size=32

$$-\frac{i(a\cos(c+dx)+ia\sin(c+dx))^n}{dn}$$

```
[Out] -I*(a*cos(d*x+c)+I*a*sin(d*x+c))^n/d/n
```

Rubi [A] time = 0.02, antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {3071}

$$-\frac{i(a\cos(c+dx)+ia\sin(c+dx))^n}{dn}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Cos[c + d*x] + I*a*Sin[c + d*x])^n,x]
```

```
[Out] ((-I)*(a*Cos[c + d*x] + I*a*Sin[c + d*x])^n)/(d*n)
```

Rule 3071

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x
_Symbol] :> Simp[(a*(a*Cos[c + d*x] + b*Sin[c + d*x])^n)/(b*d*n), x] /; Fre
eQ[{a, b, c, d, n}, x] && EqQ[a^2 + b^2, 0]
```

Rubi steps

$$\int (a\cos(c+dx)+ia\sin(c+dx))^n\,dx=-\frac{i(a\cos(c+dx)+ia\sin(c+dx))^n}{dn}$$

Mathematica [A] time = 0.09, size = 31, normalized size = 0.97

$$-\frac{i(a(\cos(c+dx)+i\sin(c+dx)))^n}{dn}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Cos[c + d*x] + I*a*Sin[c + d*x])^n,x]
```

```
[Out] ((-I)*(a*(Cos[c + d*x] + I*Sin[c + d*x]))^n)/(d*n)
```

fricas [A] time = 1.83, size = 23, normalized size = 0.72

$$-\frac{i\,e^{\left(i\,dnx+i\,cn+n\log\left(a\right)\right)}}{dn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cos(d*x+c)+I*a*sin(d*x+c))^n,x, algorithm="fricas")
[Out] -I*e^(I*d*n*x + I*c*n + n*log(a))/(d*n)
```

giac [A] time = 0.45, size = 23, normalized size = 0.72

$$-\frac{i\,e^{\left(i\,dnx+i\,cn+n\log\left(a\right)\right)}}{dn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cos(d*x+c)+I*a*sin(d*x+c))^n,x, algorithm="giac")
[Out] -I*e^(I*d*n*x + I*c*n + n*log(a))/(d*n)
```

maple [A] time = 0.26, size = 31, normalized size = 0.97

$$-\frac{i\left(a\cos\left(dx+c\right)+ia\sin\left(dx+c\right)\right)^{n}}{dn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*cos(d*x+c)+I*a*sin(d*x+c))^n,x)
[Out] -I*(a*cos(d*x+c)+I*a*sin(d*x+c))^n/d/n
```

maxima [B] time = 0.42, size = 59, normalized size = 1.84

$$-\frac{i\,a^{n}e^{\left(-n\log\left(\frac{\sin\left(dx+c\right)}{\cos\left(dx+c\right)+1}+i\right)+n\log\left(-\frac{\sin\left(dx+c\right)}{\cos\left(dx+c\right)+1}+i\right)\right)}}{dn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cos(d*x+c)+I*a*sin(d*x+c))^n,x, algorithm="maxima")
[Out] -I*a^n*e^(-n*log(sin(d*x + c)/(cos(d*x + c) + 1) + I) + n*log(-sin(d*x + c)
/(cos(d*x + c) + 1) + I))/(d*n)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int \left(a\,\cos\left(c+d\,x\right)+a\,\sin\left(c+d\,x\right)\,1\mathrm{i}\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*cos(c + d*x) + a*sin(c + d*x)*1i)^n,x)
```

```
[Out] int((a*cos(c + d*x) + a*sin(c + d*x)*1i)^n, x)
```

sympy [A] time = 0.25, size = 44, normalized size = 1.38

$$\begin{cases} x & \text{for } n = 0 \wedge (d = 0 \vee n = 0) \\ x\left(ia\sin\left(c\right) + a\cos\left(c\right)\right)^{n} & \text{for } d = 0 \\ -\frac{i\left(ia\sin\left(c+dx\right)+a\cos\left(c+dx\right)\right)^{n}}{dn} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cos(d*x+c)+I*a*sin(d*x+c))**n,x)
```

```
[Out] Piecewise((x, Eq(n, 0) & (Eq(d, 0) | Eq(n, 0))), (x*(I*a*sin(c) + a*cos(c))
**n, Eq(d, 0)), (-I*(I*a*sin(c + d*x) + a*cos(c + d*x))**n/(d*n), True))
```

3.249 $\int (a\cos(c+dx)+ia\sin(c+dx))^4\,dx$

Optimal. Leaf size=31

$$-\frac{i(a\cos(c+dx)+ia\sin(c+dx))^4}{4d}$$

```
[Out] -1/4*I*(a*cos(d*x+c)+I*a*sin(d*x+c))^4/d
```

Rubi [A] time = 0.02, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {3071}

$$-\frac{i(a\cos(c+dx)+ia\sin(c+dx))^4}{4d}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Cos[c + d*x] + I*a*Sin[c + d*x])^4,x]
```

```
[Out] ((-I/4)*(a*Cos[c + d*x] + I*a*Sin[c + d*x])^4)/d
```

Rule 3071

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x
_Symbol] :> Simp[(a*(a*Cos[c + d*x] + b*Sin[c + d*x])^n)/(b*d*n), x] /; Fre
eQ[{a, b, c, d, n}, x] && EqQ[a^2 + b^2, 0]
```

Rubi steps

$$\int (a\cos(c+dx)+ia\sin(c+dx))^4\,dx = -\frac{i(a\cos(c+dx)+ia\sin(c+dx))^4}{4d}$$

Mathematica [A] time = 0.11, size = 31, normalized size = 1.00

$$-\frac{i(a\cos(c+dx)+ia\sin(c+dx))^4}{4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Cos[c + d*x] + I*a*Sin[c + d*x])^4,x]
```

```
[Out] ((-1/4*I)*(a*Cos[c + d*x] + I*a*Sin[c + d*x])^4)/d
```

fricas [A] time = 1.83, size = 17, normalized size = 0.55

$$-\frac{i\,a^4 e^{(4i\,dx+4i\,c)}}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cos(d*x+c)+I*a*sin(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] -1/4*I*a^4*e^(4*I*d*x + 4*I*c)/d
```

giac [B] time = 0.19, size = 52, normalized size = 1.68

$$-\frac{i\,a^4 e^{(4i\,dx+4i\,c)}}{8\,d}-\frac{i\,a^4 e^{(-4i\,dx-4i\,c)}}{8\,d}+\frac{a^4\sin\left(4\,dx+4\,c\right)}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cos(d*x+c)+I*a*sin(d*x+c))^4,x, algorithm="giac")
```

```
[Out] -1/8*I*a^4*e^(4*I*d*x + 4*I*c)/d - 1/8*I*a^4*e^(-4*I*d*x - 4*I*c)/d + 1/4*a
^4*sin(4*d*x + 4*c)/d
```

maple [B] time = 0.29, size = 151, normalized size = 4.87

$$\frac{a^4\left(-\frac{\left(\sin^3(dx+c)+\frac{3\sin(dx+c)}{2}\right)\cos(dx+c)}{4}+\frac{3dx}{8}+\frac{3c}{8}\right)-ia^4\left(\sin^4\left(dx+c\right)\right)-6a^4\left(-\frac{\sin(dx+c)\left(\cos^3(dx+c)\right)}{4}+\frac{\sin(dx+c)\cos(dx+c)}{8}+\right.}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*cos(d*x+c)+I*a*sin(d*x+c))^4,x)
```

```
[Out] 1/d*(a^4*(-1/4*(sin(d*x+c)^3+3/2*sin(d*x+c))*cos(d*x+c)+3/8*d*x+3/8*c)-I*a^
4*sin(d*x+c)^4-6*a^4*(-1/4*sin(d*x+c)*cos(d*x+c)^3+1/8*sin(d*x+c)*cos(d*x+c
)+1/8*d*x+1/8*c)-I*a^4*cos(d*x+c)^4+a^4*(1/4*(cos(d*x+c)^3+3/2*cos(d*x+c))*
sin(d*x+c)+3/8*d*x+3/8*c))
```

maxima [B] time = 0.46, size = 132, normalized size = 4.26

$$-\frac{i\,a^4\cos\left(dx+c\right)^4}{d}-\frac{i\,a^4\sin\left(dx+c\right)^4}{d}+\frac{\left(12\,dx+12\,c+\sin\left(4\,dx+4\,c\right)+8\,\sin\left(2\,dx+2\,c\right)\right)a^4}{32\,d}+\frac{(12\,dx+12\,c+\sin}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cos(d*x+c)+I*a*sin(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] -I*a^4*cos(d*x + c)^4/d - I*a^4*sin(d*x + c)^4/d + 1/32*(12*d*x + 12*c + si
n(4*d*x + 4*c) + 8*sin(2*d*x + 2*c))*a^4/d + 1/32*(12*d*x + 12*c + sin(4*d*
x + 4*c) - 8*sin(2*d*x + 2*c))*a^4/d - 3/16*(4*d*x + 4*c - sin(4*d*x + 4*c)
)*a^4/d
```

mupad [B] time = 2.55, size = 84, normalized size = 2.71

$$-\frac{2\,a^4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left({\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2-1\right)}{d\,\left({\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^4+{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^3\,4{}\mathrm{i}-6\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,4{}\mathrm{i}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*cos(c + d*x) + a*sin(c + d*x)*1i)^4,x)
```

```
[Out] -(2*a^4*tan(c/2 + (d*x)/2)*(tan(c/2 + (d*x)/2)^2 - 1))/(d*(tan(c/2 + (d*x)/
2)^3*4i - 6*tan(c/2 + (d*x)/2)^2 - tan(c/2 + (d*x)/2)*4i + tan(c/2 + (d*x)/
2)^4 + 1))
```

sympy [A] time = 0.14, size = 37, normalized size = 1.19

$$\begin{cases} - \frac{i a^{4} e^{4 i c} e^{4 i d x}}{4 d} & \text{for}\: 4 d \neq 0 \\a^{4} x e^{4 i c} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cos(d*x+c)+I*a*sin(d*x+c))**4,x)
```

```
[Out] Piecewise((-I*a**4*exp(4*I*c)*exp(4*I*d*x)/(4*d), Ne(4*d, 0)), (a**4*x*exp(
4*I*c), True))
```

3.250 $\int (a\cos(c+dx)+ia\sin(c+dx))^3\,dx$

Optimal. Leaf size=31

$$-\frac{i(a\cos(c+dx)+ia\sin(c+dx))^3}{3d}$$

```
[Out] -1/3*I*(a*cos(d*x+c)+I*a*sin(d*x+c))^3/d
```

Rubi [A] time = 0.02, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {3071}

$$-\frac{i(a\cos(c+dx)+ia\sin(c+dx))^3}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Cos[c + d*x] + I*a*Sin[c + d*x])^3,x]
```

```
[Out] ((-I/3)*(a*Cos[c + d*x] + I*a*Sin[c + d*x])^3)/d
```

Rule 3071

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x
_Symbol] :> Simp[(a*(a*Cos[c + d*x] + b*Sin[c + d*x])^n)/(b*d*n), x] /; Fre
eQ[{a, b, c, d, n}, x] && EqQ[a^2 + b^2, 0]
```

Rubi steps

$$\int (a\cos(c+dx)+ia\sin(c+dx))^3\,dx=-\frac{i(a\cos(c+dx)+ia\sin(c+dx))^3}{3d}$$

Mathematica [A] time = 0.08, size = 31, normalized size = 1.00

$$-\frac{i(a\cos(c+dx)+ia\sin(c+dx))^3}{3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Cos[c + d*x] + I*a*Sin[c + d*x])^3,x]
```

```
[Out] ((-1/3*I)*(a*Cos[c + d*x] + I*a*Sin[c + d*x])^3)/d
```

fricas [A] time = 0.85, size = 17, normalized size = 0.55

$$-\frac{i a^{3} e^{(3 i d x+3 i c)}}{3 d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cos(d*x+c)+I*a*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] -1/3*I*a^3*e^(3*I*d*x + 3*I*c)/d
```

giac [B] time = 0.18, size = 52, normalized size = 1.68

$$-\frac{i a^{3} e^{(3 i d x+3 i c)}}{6 d}-\frac{i a^{3} e^{(-3 i d x-3 i c)}}{6 d}+\frac{a^{3} \sin (3 d x+3 c)}{3 d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cos(d*x+c)+I*a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -1/6*I*a^3*e^(3*I*d*x + 3*I*c)/d - 1/6*I*a^3*e^(-3*I*d*x - 3*I*c)/d + 1/3*a
^3*sin(3*d*x + 3*c)/d
```

maple [B] time = 0.28, size = 76, normalized size = 2.45

$$\frac{\frac{i a^{3}\left(2+\sin^{2}(d x+c)\right) \cos(d x+c)}{3}-a^{3}\left(\sin^{3}(d x+c)\right)-i a^{3}\left(\cos^{3}(d x+c)\right)+\frac{a^{3}\left(2+\cos^{2}(d x+c)\right) \sin(d x+c)}{3}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*cos(d*x+c)+I*a*sin(d*x+c))^3,x)
```

```
[Out] 1/d*(1/3*I*a^3*(2+sin(d*x+c)^2)*cos(d*x+c)-a^3*sin(d*x+c)^3-I*a^3*cos(d*x+c
)^3+1/3*a^3*(2+cos(d*x+c)^2)*sin(d*x+c))
```

maxima [B] time = 0.31, size = 83, normalized size = 2.68

$$-\frac{i a^{3} \cos (d x+c)^{3}}{d}-\frac{a^{3} \sin (d x+c)^{3}}{d}-\frac{i\left(\cos (d x+c)^{3}-3 \cos (d x+c)\right) a^{3}}{3 d}-\frac{\left(\sin (d x+c)^{3}-3 \sin (d x+c)\right) a^{3}}{3 d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cos(d*x+c)+I*a*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -I*a^3*cos(d*x + c)^3/d - a^3*sin(d*x + c)^3/d - 1/3*I*(cos(d*x + c)^3 - 3*
cos(d*x + c))*a^3/d - 1/3*(sin(d*x + c)^3 - 3*sin(d*x + c))*a^3/d
```

mupad [B] time = 2.47, size = 66, normalized size = 2.13

$$-\frac{2\,a^3\,\left(3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2-1\right)}{3\,d\,\left(-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\,3{}\mathrm{i}+3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+1{}\mathrm{i}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*cos(c + d*x) + a*sin(c + d*x)*1i)^3,x)
```

```
[Out] -(2*a^3*(3*tan(c/2 + (d*x)/2)^2 - 1))/(3*d*(3*tan(c/2 + (d*x)/2) - tan(c/2
+ (d*x)/2)^2*3i - tan(c/2 + (d*x)/2)^3 + 1i))
```

sympy [A] time = 0.14, size = 37, normalized size = 1.19

$$\begin{cases} -\frac{i a^3 e^{3ic} e^{3idx}}{3d} & \text{for}\ 3d \neq 0 \\ a^3 x e^{3ic} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cos(d*x+c)+I*a*sin(d*x+c))**3,x)
```

```
[Out] Piecewise((-I*a**3*exp(3*I*c)*exp(3*I*d*x)/(3*d), Ne(3*d, 0)), (a**3*x*exp(
3*I*c), True))
```

3.251 $\int (a\cos(c+dx)+ia\sin(c+dx))^2\,dx$

Optimal. Leaf size=31

$$-\frac{i(a\cos(c+dx)+ia\sin(c+dx))^2}{2d}$$

```
[Out] -1/2*I*(a*cos(d*x+c)+I*a*sin(d*x+c))^2/d
```

Rubi [A] time = 0.01, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {3071}

$$-\frac{i(a\cos(c+dx)+ia\sin(c+dx))^2}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Cos[c + d*x] + I*a*Sin[c + d*x])^2,x]
```

```
[Out] ((-I/2)*(a*Cos[c + d*x] + I*a*Sin[c + d*x])^2)/d
```

Rule 3071

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x
_Symbol] :> Simp[(a*(a*Cos[c + d*x] + b*Sin[c + d*x])^n)/(b*d*n), x] /; Fre
eQ[{a, b, c, d, n}, x] && EqQ[a^2 + b^2, 0]
```

Rubi steps

$$\int (a\cos(c+dx)+ia\sin(c+dx))^2\,dx=-\frac{i(a\cos(c+dx)+ia\sin(c+dx))^2}{2d}$$

Mathematica [A] time = 0.05, size = 31, normalized size = 1.00

$$-\frac{i(a\cos(c+dx)+ia\sin(c+dx))^2}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Cos[c + d*x] + I*a*Sin[c + d*x])^2,x]
```

```
[Out] ((-1/2*I)*(a*Cos[c + d*x] + I*a*Sin[c + d*x])^2)/d
```

fricas [A] time = 1.69, size = 17, normalized size = 0.55

$$-\frac{i\,a^2 e^{(2i\,dx+2i\,c)}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cos(d*x+c)+I*a*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -1/2*I*a^2*e^(2*I*d*x + 2*I*c)/d
```

giac [B] time = 0.15, size = 52, normalized size = 1.68

$$-\frac{i\,a^2 e^{(2i\,dx+2i\,c)}}{4\,d}-\frac{i\,a^2 e^{(-2i\,dx-2i\,c)}}{4\,d}+\frac{a^2\sin\left(2\,dx+2\,c\right)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cos(d*x+c)+I*a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/4*I*a^2*e^(2*I*d*x + 2*I*c)/d - 1/4*I*a^2*e^(-2*I*d*x - 2*I*c)/d + 1/2*a
^2*sin(2*d*x + 2*c)/d
```

maple [B] time = 0.26, size = 73, normalized size = 2.35

$$\frac{-a^2\left(-\frac{\sin(dx+c)\cos(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)-ia^2\left(\cos^2\left(dx+c\right)\right)+a^2\left(\frac{\sin(dx+c)\cos(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*cos(d*x+c)+I*a*sin(d*x+c))^2,x)
```

```
[Out] 1/d*(-a^2*(-1/2*sin(d*x+c)*cos(d*x+c)+1/2*d*x+1/2*c)-I*a^2*cos(d*x+c)^2+a^2
*(1/2*sin(d*x+c)*cos(d*x+c)+1/2*d*x+1/2*c))
```

maxima [B] time = 0.54, size = 69, normalized size = 2.23

$$-\frac{i\,a^2\cos\left(dx+c\right)^2}{d}+\frac{\left(2\,dx+2\,c+\sin\left(2\,dx+2\,c\right)\right)a^2}{4\,d}-\frac{\left(2\,dx+2\,c-\sin\left(2\,dx+2\,c\right)\right)a^2}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cos(d*x+c)+I*a*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -I*a^2*cos(d*x + c)^2/d + 1/4*(2*d*x + 2*c + sin(2*d*x + 2*c))*a^2/d - 1/4*
(2*d*x + 2*c - sin(2*d*x + 2*c))*a^2/d
```

mupad [B] time = 2.42, size = 44, normalized size = 1.42

$$-\frac{2\,a^2\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{d\,\left({\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,2{}\mathrm{i}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*cos(c + d*x) + a*sin(c + d*x)*1i)^2,x)
```

```
[Out] -(2*a^2*tan(c/2 + (d*x)/2))/(d*(tan(c/2 + (d*x)/2)*2i + tan(c/2 + (d*x)/2)^
2 - 1))
```

sympy [A] time = 0.13, size = 37, normalized size = 1.19

$$\begin{cases} -\frac{ia^{2}e^{2ic}e^{2idx}}{2d} & \text{for}\: 2d \neq 0 \\ a^{2}xe^{2ic} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cos(d*x+c)+I*a*sin(d*x+c))**2,x)
```

```
[Out] Piecewise((-I*a**2*exp(2*I*c)*exp(2*I*d*x)/(2*d), Ne(2*d, 0)), (a**2*x*exp(
2*I*c), True))
```

3.252 $\int (a\cos(c+dx)+ia\sin(c+dx))\,dx$

Optimal. Leaf size=26

$$\frac{a\sin(c+dx)}{d}-\frac{ia\cos(c+dx)}{d}$$

```
[Out] -I*a*cos(d*x+c)/d+a*sin(d*x+c)/d
```

Rubi [A] time = 0.01, antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {2637, 2638}

$$\frac{a\sin(c+dx)}{d}-\frac{ia\cos(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[a*Cos[c + d*x] + I*a*Sin[c + d*x],x]
```

```
[Out] ((-I)*a*Cos[c + d*x])/d + (a*Sin[c + d*x])/d
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int (a\cos(c+dx)+ia\sin(c+dx))\,dx &= (ia)\int \sin(c+dx)\,dx+a\int \cos(c+dx)\,dx\\ &= -\frac{ia\cos(c+dx)}{d}+\frac{a\sin(c+dx)}{d}\end{aligned}$$

Mathematica [A] time = 0.01, size = 51, normalized size = 1.96

$$\frac{ia\sin(c)\sin(dx)}{d}-\frac{ia\cos(c)\cos(dx)}{d}+\frac{a\sin(c)\cos(dx)}{d}+\frac{a\cos(c)\sin(dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[a*Cos[c + d*x] + I*a*Sin[c + d*x],x]
```

```
[Out] ((-I)*a*Cos[c]*Cos[d*x])/d + (a*Cos[d*x]*Sin[c])/d + (a*Cos[c]*Sin[d*x])/d
+ (I*a*Sin[c]*Sin[d*x])/d
```

fricas [A] time = 1.93, size = 15, normalized size = 0.58

$$-\frac{i\,ae^{(i\,dx+i\,c)}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a*cos(d*x+c)+I*a*sin(d*x+c),x, algorithm="fricas")
```

```
[Out] -I*a*e^(I*d*x + I*c)/d
```

giac [A] time = 0.15, size = 24, normalized size = 0.92

$$-\frac{i\,a\cos\left(dx+c\right)}{d}+\frac{a\sin\left(dx+c\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a*cos(d*x+c)+I*a*sin(d*x+c),x, algorithm="giac")
```

```
[Out] -I*a*cos(d*x + c)/d + a*sin(d*x + c)/d
```

maple [A] time = 0.00, size = 26, normalized size = 1.00

$$-\frac{ia\cos\left(dx+c\right)}{d}+\frac{a\sin\left(dx+c\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(a*cos(d*x+c)+I*a*sin(d*x+c),x)
```

```
[Out] -I*a*cos(d*x+c)/d+a*sin(d*x+c)/d
```

maxima [A] time = 1.12, size = 24, normalized size = 0.92

$$-\frac{i\,a\cos\left(dx+c\right)}{d}+\frac{a\sin\left(dx+c\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a*cos(d*x+c)+I*a*sin(d*x+c),x, algorithm="maxima")
```

```
[Out] -I*a*cos(d*x + c)/d + a*sin(d*x + c)/d
```

mupad [B] time = 2.39, size = 20, normalized size = 0.77

$$\frac{2a}{d\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)+1\mathrm{i}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(a*cos(c + d*x) + a*sin(c + d*x)*1i,x)
```

```
[Out] (2*a)/(d*(tan(c/2 + (d*x)/2) + 1i))
```

sympy [A] time = 0.12, size = 26, normalized size = 1.00

$$\begin{cases} -\frac{iae^{ic}e^{idx}}{d} & \text{for}\ d \neq 0 \\ axe^{ic} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a*cos(d*x+c)+I*a*sin(d*x+c),x)
```

```
[Out] Piecewise((-I*a*exp(I*c)*exp(I*d*x)/d, Ne(d, 0)), (a*x*exp(I*c), True))
```

3.253 $\int \frac{1}{a\cos(c+dx)+ia\sin(c+dx)}\,dx$

Optimal. Leaf size=29

$$\frac{i}{d(a\cos(c+dx)+ia\sin(c+dx))}$$

```
[Out] I/d/(a*cos(d*x+c)+I*a*sin(d*x+c))
```

Rubi [A] time = 0.01, antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {3071}

$$\frac{i}{d(a\cos(c+dx)+ia\sin(c+dx))}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Cos[c + d*x] + I*a*Sin[c + d*x])^(-1),x]
```

```
[Out] I/(d*(a*Cos[c + d*x] + I*a*Sin[c + d*x]))
```

Rule 3071

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x
_Symbol] :> Simp[(a*(a*Cos[c + d*x] + b*Sin[c + d*x])^n)/(b*d*n), x] /; Fre
eQ[{a, b, c, d, n}, x] && EqQ[a^2 + b^2, 0]
```

Rubi steps

$$\int \frac{1}{a\cos(c+dx)+ia\sin(c+dx)}\,dx = \frac{i}{d(a\cos(c+dx)+ia\sin(c+dx))}$$

Mathematica [A] time = 0.03, size = 29, normalized size = 1.00

$$\frac{i}{d(a\cos(c+dx)+ia\sin(c+dx))}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Cos[c + d*x] + I*a*Sin[c + d*x])^(-1),x]
```

```
[Out] I/(d*(a*Cos[c + d*x] + I*a*Sin[c + d*x]))
```

fricas [A] time = 0.92, size = 17, normalized size = 0.59

$$\frac{ie^{(-idx-ic)}}{ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cos(d*x+c)+I*a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] I*e^(-I*d*x - I*c)/(a*d)
```

giac [A] time = 0.13, size = 21, normalized size = 0.72

$$\frac{2}{ad\left(\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right) - i\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cos(d*x+c)+I*a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 2/(a*d*(tan(1/2*d*x + 1/2*c) - I))
```

maple [A] time = 0.40, size = 23, normalized size = 0.79

$$\frac{2}{da\left(\tan\left(\frac{dx}{2} + \frac{c}{2}\right) - i\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*cos(d*x+c)+I*a*sin(d*x+c)),x)
```

```
[Out] 2/d/a/(tan(1/2*d*x+1/2*c)-I)
```

maxima [A] time = 0.32, size = 29, normalized size = 1.00

$$\frac{2}{\left(-i\,a + \frac{a\sin(dx+c)}{\cos(dx+c)+1}\right)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cos(d*x+c)+I*a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 2/((-I*a + a*sin(d*x + c)/(cos(d*x + c) + 1))*d)
```

mupad [B] time = 2.39, size = 25, normalized size = 0.86

$$\frac{2\mathrm{i}}{a\,d\,\left(1+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,1\mathrm{i}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*cos(c + d*x) + a*sin(c + d*x)*1i),x)
```

```
[Out] 2i/(a*d*(tan(c/2 + (d*x)/2)*1i + 1))
```

sympy [A] time = 0.14, size = 31, normalized size = 1.07

$$\begin{cases} \frac{ie^{-ic}e^{-idx}}{ad} & \text{for } ade^{ic} \neq 0 \\ \frac{xe^{-ic}}{a} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cos(d*x+c)+I*a*sin(d*x+c)),x)
```

```
[Out] Piecewise((I*exp(-I*c)*exp(-I*d*x)/(a*d), Ne(a*d*exp(I*c), 0)), (x*exp(-I*c
)/a, True))
```

3.254 $\int \frac{1}{(a\cos(c+dx)+ia\sin(c+dx))^2}\,dx$

Optimal. Leaf size=31

$$\frac{i}{2d(a\cos(c+dx)+ia\sin(c+dx))^2}$$

```
[Out] 1/2*I/d/(a*cos(d*x+c)+I*a*sin(d*x+c))^2
```

Rubi [A] time = 0.02, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {3071}

$$\frac{i}{2d(a\cos(c+dx)+ia\sin(c+dx))^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Cos[c + d*x] + I*a*Sin[c + d*x])^(-2),x]
```

```
[Out] (I/2)/(d*(a*Cos[c + d*x] + I*a*Sin[c + d*x])^2)
```

Rule 3071

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x
_Symbol] :> Simp[(a*(a*Cos[c + d*x] + b*Sin[c + d*x])^n)/(b*d*n), x] /; Fre
eQ[{a, b, c, d, n}, x] && EqQ[a^2 + b^2, 0]
```

Rubi steps

$$\int \frac{1}{(a\cos(c+dx)+ia\sin(c+dx))^2}\,dx = \frac{i}{2d(a\cos(c+dx)+ia\sin(c+dx))^2}$$

Mathematica [A] time = 0.04, size = 31, normalized size = 1.00

$$\frac{i}{2d(a\cos(c+dx)+ia\sin(c+dx))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Cos[c + d*x] + I*a*Sin[c + d*x])^(-2),x]
```

```
[Out] (I/2)/(d*(a*Cos[c + d*x] + I*a*Sin[c + d*x])^2)
```

fricas [A]   time = 0.93, size = 17, normalized size = 0.55

$$\frac{i\,e^{(-2i\,dx-2i\,c)}}{2\,a^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cos(d*x+c)+I*a*sin(d*x+c))^2,x, algorithm="fricas")
[Out] 1/2*I*e^(-2*I*d*x - 2*I*c)/(a^2*d)
```

giac [A]   time = 0.15, size = 30, normalized size = 0.97

$$-\frac{2\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{a^2d\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-i\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cos(d*x+c)+I*a*sin(d*x+c))^2,x, algorithm="giac")
[Out] -2*tan(1/2*d*x + 1/2*c)/(a^2*d*(tan(1/2*d*x + 1/2*c) - I)^2)
```

maple [A]   time = 0.40, size = 23, normalized size = 0.74

$$\frac{i}{d\,a^2\,(1+i\tan\,(dx+c))}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*cos(d*x+c)+I*a*sin(d*x+c))^2,x)
[Out] I/d/a^2/(1+I*tan(d*x+c))
```

maxima [A]   time = 0.33, size = 22, normalized size = 0.71

$$\frac{1}{\left(a^2\tan\,(dx+c)-i\,a^2\right)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cos(d*x+c)+I*a*sin(d*x+c))^2,x, algorithm="maxima")
[Out] 1/((a^2*tan(d*x + c) - I*a^2)*d)
```

mupad [B] time = 2.41, size = 31, normalized size = 1.00

$$-\frac{2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{a^2\,d\,\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)-\mathrm{i}\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*cos(c + d*x) + a*sin(c + d*x)*1i)^2,x)
```

```
[Out] -(2*tan(c/2 + (d*x)/2))/(a^2*d*(tan(c/2 + (d*x)/2) - 1i)^2)
```

sympy [A] time = 0.14, size = 46, normalized size = 1.48

$$\begin{cases} \frac{ie^{-2ic}e^{-2idx}}{2a^2d} & \text{for } 2a^2de^{2ic} \neq 0 \\ \frac{xe^{-2ic}}{a^2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cos(d*x+c)+I*a*sin(d*x+c))**2,x)
```

```
[Out] Piecewise((I*exp(-2*I*c)*exp(-2*I*d*x)/(2*a**2*d), Ne(2*a**2*d*exp(2*I*c),
0)), (x*exp(-2*I*c)/a**2, True))
```

3.255 $\int \frac{1}{(a\cos(c+dx)+ia\sin(c+dx))^3}\,dx$

Optimal. Leaf size=31

$$\frac{i}{3d(a\cos(c+dx)+ia\sin(c+dx))^3}$$

```
[Out] 1/3*I/d/(a*cos(d*x+c)+I*a*sin(d*x+c))^3
```

Rubi [A] time = 0.02, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {3071}

$$\frac{i}{3d(a\cos(c+dx)+ia\sin(c+dx))^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Cos[c + d*x] + I*a*Sin[c + d*x])^(-3),x]
```

```
[Out] (I/3)/(d*(a*Cos[c + d*x] + I*a*Sin[c + d*x])^3)
```

Rule 3071

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x
_Symbol] :> Simp[(a*(a*Cos[c + d*x] + b*Sin[c + d*x])^n)/(b*d*n), x] /; Fre
eQ[{a, b, c, d, n}, x] && EqQ[a^2 + b^2, 0]
```

Rubi steps

$$\int \frac{1}{(a\cos(c+dx)+ia\sin(c+dx))^3}\,dx = \frac{i}{3d(a\cos(c+dx)+ia\sin(c+dx))^3}$$

Mathematica [A] time = 0.04, size = 31, normalized size = 1.00

$$\frac{i}{3d(a\cos(c+dx)+ia\sin(c+dx))^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Cos[c + d*x] + I*a*Sin[c + d*x])^(-3),x]
```

```
[Out] (I/3)/(d*(a*Cos[c + d*x] + I*a*Sin[c + d*x])^3)
```

fricas [A] time = 1.62, size = 17, normalized size = 0.55

$$\frac{i\,e^{(-3i\,dx-3i\,c)}}{3\,a^3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cos(d*x+c)+I*a*sin(d*x+c))^3,x, algorithm="fricas")
[Out] 1/3*I*e^(-3*I*d*x - 3*I*c)/(a^3*d)
```

giac [A] time = 0.15, size = 36, normalized size = 1.16

$$\frac{2\left(3\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1\right)}{3\,a^3d\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-i\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cos(d*x+c)+I*a*sin(d*x+c))^3,x, algorithm="giac")
[Out] 2/3*(3*tan(1/2*d*x + 1/2*c)^2 - 1)/(a^3*d*(tan(1/2*d*x + 1/2*c) - I)^3)
```

maple [B] time = 0.43, size = 57, normalized size = 1.84

$$\frac{\frac{2}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-i}-\frac{8}{3\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-i\right)^3}+\frac{4i}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-i\right)^2}}{d\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*cos(d*x+c)+I*a*sin(d*x+c))^3,x)
[Out] 2/d/a^3*(1/(tan(1/2*d*x+1/2*c)-I)-4/3/(tan(1/2*d*x+1/2*c)-I)^3+2*I/(tan(1/2
*d*x+1/2*c)-I)^2)
```

maxima [A] time = 0.63, size = 29, normalized size = 0.94

$$\frac{i\,\cos\left(3\,dx+3\,c\right)+\sin\left(3\,dx+3\,c\right)}{3\,a^3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cos(d*x+c)+I*a*sin(d*x+c))^3,x, algorithm="maxima")
[Out] 1/3*(I*cos(3*d*x + 3*c) + sin(3*d*x + 3*c))/(a^3*d)
```

mupad [B] time = 2.46, size = 68, normalized size = 2.19

$$-\frac{2\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\,3\mathrm{i}-\mathrm{i}\right)}{3\,a^3\,d\,\left(-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3\,1\mathrm{i}-3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,3\mathrm{i}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*cos(c + d*x) + a*sin(c + d*x)*1i)^3,x)
```

```
[Out] -(2*(tan(c/2 + (d*x)/2)^2*3i - 1i))/(3*a^3*d*(tan(c/2 + (d*x)/2)*3i - 3*tan
(c/2 + (d*x)/2)^2 - tan(c/2 + (d*x)/2)^3*1i + 1))
```

sympy [A] time = 0.14, size = 46, normalized size = 1.48

$$\begin{cases} \frac{ie^{-3ic}e^{-3idx}}{3a^3d} & \text{for } 3a^3de^{3ic} \neq 0 \\ \frac{xe^{-3ic}}{a^3} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cos(d*x+c)+I*a*sin(d*x+c))**3,x)
```

```
[Out] Piecewise((I*exp(-3*I*c)*exp(-3*I*d*x)/(3*a**3*d), Ne(3*a**3*d*exp(3*I*c),
0)), (x*exp(-3*I*c)/a**3, True))
```

3.256 $\int \frac{1}{(a\cos(c+dx)+ia\sin(c+dx))^4}\,dx$

Optimal. Leaf size=31

$$\frac{i}{4d(a\cos(c+dx)+ia\sin(c+dx))^4}$$

```
[Out] 1/4*I/d/(a*cos(d*x+c)+I*a*sin(d*x+c))^4
```

Rubi [A] time = 0.01, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {3071}

$$\frac{i}{4d(a\cos(c+dx)+ia\sin(c+dx))^4}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Cos[c + d*x] + I*a*Sin[c + d*x])^(-4),x]
[Out] (I/4)/(d*(a*Cos[c + d*x] + I*a*Sin[c + d*x])^4)
```

Rule 3071

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x
_Symbol] :> Simp[(a*(a*Cos[c + d*x] + b*Sin[c + d*x])^n)/(b*d*n), x] /; Fre
eQ[{a, b, c, d, n}, x] && EqQ[a^2 + b^2, 0]
```

Rubi steps

$$\int \frac{1}{(a\cos(c+dx)+ia\sin(c+dx))^4}\,dx=\frac{i}{4d(a\cos(c+dx)+ia\sin(c+dx))^4}$$

Mathematica [A] time = 0.05, size = 31, normalized size = 1.00

$$\frac{i}{4d(a\cos(c+dx)+ia\sin(c+dx))^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Cos[c + d*x] + I*a*Sin[c + d*x])^(-4),x]
[Out] (I/4)/(d*(a*Cos[c + d*x] + I*a*Sin[c + d*x])^4)
```

fricas [A] time = 1.33, size = 17, normalized size = 0.55

$$\frac{i\,e^{(-4i\,dx-4i\,c)}}{4\,a^4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cos(d*x+c)+I*a*sin(d*x+c))^4,x, algorithm="fricas")
[Out] 1/4*I*e^(-4*I*d*x - 4*I*c)/(a^4*d)
```

giac [A] time = 0.16, size = 44, normalized size = 1.42

$$-\frac{2\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{a^4d\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-i\right)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cos(d*x+c)+I*a*sin(d*x+c))^4,x, algorithm="giac")
[Out] -2*(tan(1/2*d*x + 1/2*c)^3 - tan(1/2*d*x + 1/2*c))/(a^4*d*(tan(1/2*d*x + 1/
2*c) - I)^4)
```

maple [A] time = 0.42, size = 36, normalized size = 1.16

$$\frac{-\frac{i}{(\tan(dx+c)-i)^2}-\frac{1}{\tan(dx+c)-i}}{d\,a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*cos(d*x+c)+I*a*sin(d*x+c))^4,x)
[Out] 1/d/a^4*(-I/(tan(d*x+c)-I)^2-1/(tan(d*x+c)-I))
```

maxima [A] time = 0.57, size = 29, normalized size = 0.94

$$\frac{i\,\cos\left(4\,dx+4\,c\right)+\sin\left(4\,dx+4\,c\right)}{4\,a^4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cos(d*x+c)+I*a*sin(d*x+c))^4,x, algorithm="maxima")
[Out] 1/4*(I*cos(4*d*x + 4*c) + sin(4*d*x + 4*c))/(a^4*d)
```

mupad [B] time = 2.56, size = 91, normalized size = 2.94

$$-\frac{2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left({\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2\,1{}\mathrm{i}-\mathrm{i}\right)}{a^4\,d\,\left({\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^4\,1{}\mathrm{i}+4\,{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^3-{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2\,6{}\mathrm{i}-4\,\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)+1{}\mathrm{i}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*cos(c + d*x) + a*sin(c + d*x)*1i)^4,x)
```

```
[Out] -(2*tan(c/2 + (d*x)/2)*(tan(c/2 + (d*x)/2)^2*1i - 1i))/(a^4*d*(4*tan(c/2 +
(d*x)/2)^3 - tan(c/2 + (d*x)/2)^2*6i - 4*tan(c/2 + (d*x)/2) + tan(c/2 + (d*
x)/2)^4*1i + 1i))
```

sympy [A] time = 0.15, size = 46, normalized size = 1.48

$$\begin{cases} \frac{ie^{-4ic}e^{-4idx}}{4a^4d} & \text{for } 4a^4de^{4ic} \neq 0 \\ \frac{xe^{-4ic}}{a^4} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cos(d*x+c)+I*a*sin(d*x+c))**4,x)
```

```
[Out] Piecewise(((I*exp(-4*I*c)*exp(-4*I*d*x)/(4*a**4*d), Ne(4*a**4*d*exp(4*I*c),
0)), (x*exp(-4*I*c)/a**4, True))
```

3.257 $\int (a\cos(c+dx)+ia\sin(c+dx))^{5/2}\,dx$

Optimal. Leaf size=33

$$-\frac{2i(a\cos(c+dx)+ia\sin(c+dx))^{5/2}}{5d}$$

```
[Out] -2/5*I*(a*cos(d*x+c)+I*a*sin(d*x+c))^(5/2)/d
```

Rubi [A] time = 0.02, antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.042, Rules used = {3071}

$$-\frac{2i(a\cos(c+dx)+ia\sin(c+dx))^{5/2}}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Cos[c + d*x] + I*a*Sin[c + d*x])^(5/2),x]
[Out] (((-2*I)/5)*(a*Cos[c + d*x] + I*a*Sin[c + d*x])^(5/2))/d
```

Rule 3071

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x
_Symbol] :> Simp[(a*(a*Cos[c + d*x] + b*Sin[c + d*x])^n)/(b*d*n), x] /; Fre
eQ[{a, b, c, d, n}, x] && EqQ[a^2 + b^2, 0]
```

Rubi steps

$$\int (a\cos(c+dx)+ia\sin(c+dx))^{5/2}\,dx=-\frac{2i(a\cos(c+dx)+ia\sin(c+dx))^{5/2}}{5d}$$

Mathematica [A] time = 0.03, size = 32, normalized size = 0.97

$$-\frac{2i(a(\cos(c+dx)+i\sin(c+dx)))^{5/2}}{5d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Cos[c + d*x] + I*a*Sin[c + d*x])^(5/2),x]
[Out] (((-2*I)/5)*(a*(Cos[c + d*x] + I*Sin[c + d*x]))^(5/2))/d
```

fricas [A] time = 1.47, size = 17, normalized size = 0.52

$$-\frac{2i\,a^{\frac{5}{2}}e^{\left(\frac{5}{2}i\,dx+\frac{5}{2}i\,c\right)}}{5\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cos(d*x+c)+I*a*sin(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] -2/5*I*a^(5/2)*e^(5/2*I*d*x + 5/2*I*c)/d
```

giac [A] time = 0.57, size = 17, normalized size = 0.52

$$-\frac{2i\,a^{\frac{5}{2}}e^{\left(\frac{5}{2}i\,dx+\frac{5}{2}i\,c\right)}}{5\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cos(d*x+c)+I*a*sin(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] -2/5*I*a^(5/2)*e^(5/2*I*d*x + 5/2*I*c)/d
```

maple [A] time = 0.36, size = 28, normalized size = 0.85

$$-\frac{2i\left(a\cos\left(dx+c\right)+ia\sin\left(dx+c\right)\right)^{\frac{5}{2}}}{5d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*cos(d*x+c)+I*a*sin(d*x+c))^(5/2),x)
```

```
[Out] -2/5*I*(a*cos(d*x+c)+I*a*sin(d*x+c))^(5/2)/d
```

maxima [B] time = 0.69, size = 51, normalized size = 1.55

$$-\frac{2i\,a^{\frac{5}{2}}\left(-\frac{\sin(dx+c)}{\cos(dx+c)+1}+i\right)^{\frac{5}{2}}}{5\,d\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+i\right)^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cos(d*x+c)+I*a*sin(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] -2/5*I*a^(5/2)*(-sin(d*x + c)/(cos(d*x + c) + 1) + I)^(5/2)/(d*(sin(d*x + c
)/(cos(d*x + c) + 1) + I)^(5/2))
```

mupad [B] time = 0.43, size = 35, normalized size = 1.06

$$-\frac{a^2\,{\mathrm{e}}^{c\,2{}\mathrm{i}}\,{\mathrm{e}}^{d\,x\,2{}\mathrm{i}}\,\sqrt{a\,{\mathrm{e}}^{c\,1{}\mathrm{i}}\,{\mathrm{e}}^{d\,x\,1{}\mathrm{i}}}\,2{}\mathrm{i}}{5\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*cos(c + d*x) + a*sin(c + d*x)*1i)^(5/2),x)
[Out] -(a^2*exp(c*2i)*exp(d*x*2i)*(a*exp(c*1i)*exp(d*x*1i))^(1/2)*2i)/(5*d)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cos(d*x+c)+I*a*sin(d*x+c))**(5/2),x)
[Out] Timed out
```

3.258 $\int (a\cos(c+dx)+ia\sin(c+dx))^{3/2}\,dx$

Optimal. Leaf size=33

$$-\frac{2i(a\cos(c+dx)+ia\sin(c+dx))^{3/2}}{3d}$$

```
[Out] -2/3*I*(a*cos(d*x+c)+I*a*sin(d*x+c))^(3/2)/d
```

Rubi [A] time = 0.02, antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.042, Rules used = {3071}

$$-\frac{2i(a\cos(c+dx)+ia\sin(c+dx))^{3/2}}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Cos[c + d*x] + I*a*Sin[c + d*x])^(3/2),x]
```

```
[Out] (((-2*I)/3)*(a*Cos[c + d*x] + I*a*Sin[c + d*x])^(3/2))/d
```

Rule 3071

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x
_Symbol] :> Simp[(a*(a*Cos[c + d*x] + b*Sin[c + d*x])^n)/(b*d*n), x] /; Fre
eQ[{a, b, c, d, n}, x] && EqQ[a^2 + b^2, 0]
```

Rubi steps

$$\int (a\cos(c+dx)+ia\sin(c+dx))^{3/2}\,dx=-\frac{2i(a\cos(c+dx)+ia\sin(c+dx))^{3/2}}{3d}$$

Mathematica [A] time = 0.03, size = 32, normalized size = 0.97

$$-\frac{2i(a(\cos(c+dx)+i\sin(c+dx)))^{3/2}}{3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Cos[c + d*x] + I*a*Sin[c + d*x])^(3/2),x]
```

```
[Out] (((-2*I)/3)*(a*(Cos[c + d*x] + I*Sin[c + d*x]))^(3/2))/d
```

fricas [A] time = 1.08, size = 17, normalized size = 0.52

$$-\frac{2i\,a^{\frac{3}{2}}e^{\left(\frac{3}{2}i\,dx+\frac{3}{2}i\,c\right)}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cos(d*x+c)+I*a*sin(d*x+c))^(3/2),x, algorithm="fricas")
[Out] -2/3*I*a^(3/2)*e^(3/2*I*d*x + 3/2*I*c)/d
```

giac [A] time = 0.46, size = 17, normalized size = 0.52

$$-\frac{2i\,a^{\frac{3}{2}}e^{\left(\frac{3}{2}i\,dx+\frac{3}{2}i\,c\right)}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cos(d*x+c)+I*a*sin(d*x+c))^(3/2),x, algorithm="giac")
[Out] -2/3*I*a^(3/2)*e^(3/2*I*d*x + 3/2*I*c)/d
```

maple [A] time = 0.22, size = 28, normalized size = 0.85

$$-\frac{2i\,(a\cos(dx+c)+ia\sin(dx+c))^{\frac{3}{2}}}{3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*cos(d*x+c)+I*a*sin(d*x+c))^(3/2),x)
[Out] -2/3*I*(a*cos(d*x+c)+I*a*sin(d*x+c))^(3/2)/d
```

maxima [B] time = 0.90, size = 51, normalized size = 1.55

$$-\frac{2i\,a^{\frac{3}{2}}\left(-\frac{\sin(dx+c)}{\cos(dx+c)+1}+i\right)^{\frac{3}{2}}}{3\,d\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+i\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cos(d*x+c)+I*a*sin(d*x+c))^(3/2),x, algorithm="maxima")
[Out] -2/3*I*a^(3/2)*(-sin(d*x + c)/(cos(d*x + c) + 1) + I)^(3/2)/(d*(sin(d*x + c
)/(cos(d*x + c) + 1) + I)^(3/2))
```

mupad [B] time = 2.38, size = 33, normalized size = 1.00

$$-\frac{a\,\mathrm{e}^{c\,1\mathrm{i}}\,\mathrm{e}^{d\,x\,1\mathrm{i}}\,\sqrt{a\,\mathrm{e}^{c\,1\mathrm{i}}\,\mathrm{e}^{d\,x\,1\mathrm{i}}}\,2\mathrm{i}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*cos(c + d*x) + a*sin(c + d*x)*1i)^(3/2),x)
[Out] -(a*exp(c*1i)*exp(d*x*1i)*(a*exp(c*1i)*exp(d*x*1i))^(1/2)*2i)/(3*d)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (ia\sin(c+dx)+a\cos(c+dx))^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cos(d*x+c)+I*a*sin(d*x+c))**(3/2),x)
[Out] Integral((I*a*sin(c + d*x) + a*cos(c + d*x))**(3/2), x)
```

3.259 $\int \sqrt{a\cos(c+dx)+ia\sin(c+dx)}\,dx$

Optimal. Leaf size=31

$$-\frac{2i\sqrt{a\cos(c+dx)+ia\sin(c+dx)}}{d}$$

```
[Out] -2*I*(a*cos(d*x+c)+I*a*sin(d*x+c))^(1/2)/d
```

Rubi [A] time = 0.02, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.042, Rules used = {3071}

$$-\frac{2i\sqrt{a\cos(c+dx)+ia\sin(c+dx)}}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a*Cos[c + d*x] + I*a*Sin[c + d*x]],x]
```

```
[Out] ((-2*I)*Sqrt[a*Cos[c + d*x] + I*a*Sin[c + d*x]])/d
```

Rule 3071

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x
_Symbol] :> Simp[(a*(a*Cos[c + d*x] + b*Sin[c + d*x])^n)/(b*d*n), x] /; Fre
eQ[{a, b, c, d, n}, x] && EqQ[a^2 + b^2, 0]
```

Rubi steps

$$\int \sqrt{a\cos(c+dx)+ia\sin(c+dx)}\,dx = -\frac{2i\sqrt{a\cos(c+dx)+ia\sin(c+dx)}}{d}$$

Mathematica [A] time = 0.02, size = 30, normalized size = 0.97

$$-\frac{2i\sqrt{a(\cos(c+dx)+i\sin(c+dx))}}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a*Cos[c + d*x] + I*a*Sin[c + d*x]],x]
```

```
[Out] ((-2*I)*Sqrt[a*(Cos[c + d*x] + I*Sin[c + d*x])])/d
```

fricas [A] time = 2.39, size = 17, normalized size = 0.55

$$-\frac{2i\sqrt{a}e^{\left(\frac{1}{2}idx+\frac{1}{2}ic\right)}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cos(d*x+c)+I*a*sin(d*x+c))^(1/2),x, algorithm="fricas")
[Out] -2*I*sqrt(a)*e^(1/2*I*d*x + 1/2*I*c)/d
```

giac [A] time = 0.14, size = 25, normalized size = 0.81

$$-\frac{2i\sqrt{a\cos(dx+c)+ia\sin(dx+c)}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cos(d*x+c)+I*a*sin(d*x+c))^(1/2),x, algorithm="giac")
[Out] -2*I*sqrt(a*cos(d*x + c) + I*a*sin(d*x + c))/d
```

maple [A] time = 0.22, size = 28, normalized size = 0.90

$$-\frac{2i\sqrt{a\cos(dx+c)+ia\sin(dx+c)}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*cos(d*x+c)+I*a*sin(d*x+c))^(1/2),x)
[Out] -2*I*(a*cos(d*x+c)+I*a*sin(d*x+c))^(1/2)/d
```

maxima [B] time = 0.84, size = 51, normalized size = 1.65

$$-\frac{2i\sqrt{a}\sqrt{-\frac{\sin(dx+c)}{\cos(dx+c)+1}+i}}{d\sqrt{\frac{\sin(dx+c)}{\cos(dx+c)+1}+i}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cos(d*x+c)+I*a*sin(d*x+c))^(1/2),x, algorithm="maxima")
[Out] -2*I*sqrt(a)*sqrt(-sin(d*x + c)/(cos(d*x + c) + 1) + I)/(d*sqrt(sin(d*x + c
)/(cos(d*x + c) + 1) + I))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int \sqrt{a\,\cos\left(c+d\,x\right)+a\,\sin\left(c+d\,x\right)\,1\mathrm{i}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*cos(c + d*x) + a*sin(c + d*x)*1i)^(1/2),x)
[Out] int((a*cos(c + d*x) + a*sin(c + d*x)*1i)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{ia\sin\left(c+dx\right)+a\cos\left(c+dx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cos(d*x+c)+I*a*sin(d*x+c))**(1/2),x)
[Out] Integral(sqrt(I*a*sin(c + d*x) + a*cos(c + d*x)), x)
```

3.260 $\int \frac{1}{\sqrt{a\cos(c+dx)+ia\sin(c+dx)}}\,dx$

Optimal. Leaf size=31

$$\frac{2i}{d\sqrt{a\cos(c+dx)+ia\sin(c+dx)}}$$

```
[Out] 2*I/d/(a*cos(d*x+c)+I*a*sin(d*x+c))^(1/2)
```

Rubi [A] time = 0.02, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.042, Rules used = {3071}

$$\frac{2i}{d\sqrt{a\cos(c+dx)+ia\sin(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[a*Cos[c + d*x] + I*a*Sin[c + d*x]],x]
```

```
[Out] (2*I)/(d*Sqrt[a*Cos[c + d*x] + I*a*Sin[c + d*x]])
```

Rule 3071

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x
_Symbol] :> Simp[(a*(a*Cos[c + d*x] + b*Sin[c + d*x])^n)/(b*d*n), x] /; Fre
eQ[{a, b, c, d, n}, x] && EqQ[a^2 + b^2, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{a\cos(c+dx)+ia\sin(c+dx)}}\,dx=\frac{2i}{d\sqrt{a\cos(c+dx)+ia\sin(c+dx)}}$$

Mathematica [A] time = 0.03, size = 30, normalized size = 0.97

$$\frac{2i}{d\sqrt{a(\cos(c+dx)+i\sin(c+dx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[a*Cos[c + d*x] + I*a*Sin[c + d*x]],x]
```

```
[Out] (2*I)/(d*Sqrt[a*(Cos[c + d*x] + I*Sin[c + d*x])])
```

fricas [A] time = 1.04, size = 17, normalized size = 0.55

$$\frac{2i\,e^{\left(-\frac{1}{2}i\,dx-\frac{1}{2}i\,c\right)}}{\sqrt{a}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cos(d*x+c)+I*a*sin(d*x+c))^(1/2),x, algorithm="fricas")
[Out] 2*I*e^(-1/2*I*d*x - 1/2*I*c)/(sqrt(a)*d)
```

giac [A] time = 0.33, size = 37, normalized size = 1.19

$$\frac{2i}{d\sqrt{-\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-i\,a}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+i}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cos(d*x+c)+I*a*sin(d*x+c))^(1/2),x, algorithm="giac")
[Out] 2*I/(d*sqrt(-(a*tan(1/2*d*x + 1/2*c) - I*a)/(tan(1/2*d*x + 1/2*c) + I)))
```

maple [A] time = 0.22, size = 28, normalized size = 0.90

$$\frac{2i}{d\sqrt{a\cos\left(dx+c\right)+ia\sin\left(dx+c\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*cos(d*x+c)+I*a*sin(d*x+c))^(1/2),x)
[Out] 2*I/d/(a*cos(d*x+c)+I*a*sin(d*x+c))^(1/2)
```

maxima [B] time = 0.43, size = 51, normalized size = 1.65

$$\frac{2i\sqrt{\frac{\sin(dx+c)}{\cos(dx+c)+1}+i}}{\sqrt{a}\,d\sqrt{-\frac{\sin(dx+c)}{\cos(dx+c)+1}+i}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cos(d*x+c)+I*a*sin(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] 2*I*sqrt(sin(d*x + c)/(cos(d*x + c) + 1) + I)/(sqrt(a)*d*sqrt(-sin(d*x + c)
/(cos(d*x + c) + 1) + I))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{1}{\sqrt{a\ \cos\left(c+d\,x\right)+a\ \sin\left(c+d\,x\right)\,1\mathrm{i}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*cos(c + d*x) + a*sin(c + d*x)*1i)^(1/2),x)
```

```
[Out] int(1/(a*cos(c + d*x) + a*sin(c + d*x)*1i)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{ia\sin\left(c+dx\right)+a\cos\left(c+dx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cos(d*x+c)+I*a*sin(d*x+c))**(1/2),x)
```

```
[Out] Integral(1/sqrt(I*a*sin(c + d*x) + a*cos(c + d*x)), x)
```

3.261 $\int \frac{1}{(a\cos(c+dx)+ia\sin(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=33

$$\frac{2i}{3d(a\cos(c+dx)+ia\sin(c+dx))^{3/2}}$$

```
[Out] 2/3*I/d/(a*cos(d*x+c)+I*a*sin(d*x+c))^(3/2)
```

Rubi [A] time = 0.02, antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.042, Rules used = {3071}

$$\frac{2i}{3d(a\cos(c+dx)+ia\sin(c+dx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Cos[c + d*x] + I*a*Sin[c + d*x])^(-3/2),x]
```

```
[Out] ((2*I)/3)/(d*(a*Cos[c + d*x] + I*a*Sin[c + d*x])^(3/2))
```

Rule 3071

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x
_Symbol] :> Simp[(a*(a*Cos[c + d*x] + b*Sin[c + d*x])^n)/(b*d*n), x] /; Fre
eQ[{a, b, c, d, n}, x] && EqQ[a^2 + b^2, 0]
```

Rubi steps

$$\int \frac{1}{(a\cos(c+dx)+ia\sin(c+dx))^{3/2}}\,dx = \frac{2i}{3d(a\cos(c+dx)+ia\sin(c+dx))^{3/2}}$$

Mathematica [A] time = 0.03, size = 32, normalized size = 0.97

$$\frac{2i}{3d(a(\cos(c+dx)+i\sin(c+dx)))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Cos[c + d*x] + I*a*Sin[c + d*x])^(-3/2),x]
```

```
[Out] ((2*I)/3)/(d*(a*(Cos[c + d*x] + I*Sin[c + d*x]))^(3/2))
```

fricas [A] time = 1.49, size = 17, normalized size = 0.52

$$\frac{2i\,e^{\left(-\frac{3}{2}i\,dx-\frac{3}{2}i\,c\right)}}{3\,a^{\frac{3}{2}}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cos(d*x+c)+I*a*sin(d*x+c))^(3/2),x, algorithm="fricas")
[Out] 2/3*I*e^(-3/2*I*d*x - 3/2*I*c)/(a^(3/2)*d)
```

giac [B] time = 1.23, size = 65, normalized size = 1.97

$$-\frac{2i\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+i\right)}{3\left(a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-i\,a\right)d\sqrt{-\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-i\,a}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+i}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cos(d*x+c)+I*a*sin(d*x+c))^(3/2),x, algorithm="giac")
[Out] -2/3*I*(tan(1/2*d*x + 1/2*c) + I)/((a*tan(1/2*d*x + 1/2*c) - I*a)*d*sqrt(-(
a*tan(1/2*d*x + 1/2*c) - I*a)/(tan(1/2*d*x + 1/2*c) + I)))
```

maple [A] time = 0.21, size = 28, normalized size = 0.85

$$\frac{2i}{3d\left(a\cos\left(dx+c\right)+ia\sin\left(dx+c\right)\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*cos(d*x+c)+I*a*sin(d*x+c))^(3/2),x)
[Out] 2/3*I/d/(a*cos(d*x+c)+I*a*sin(d*x+c))^(3/2)
```

maxima [B] time = 1.06, size = 51, normalized size = 1.55

$$\frac{2i\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+i\right)^{\frac{3}{2}}}{3\,a^{\frac{3}{2}}d\left(-\frac{\sin(dx+c)}{\cos(dx+c)+1}+i\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cos(d*x+c)+I*a*sin(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] 2/3*I*(sin(d*x + c)/(cos(d*x + c) + 1) + I)^(3/2)/(a^(3/2)*d*(-sin(d*x + c)
/(cos(d*x + c) + 1) + I)^(3/2))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{1}{(a\ \cos(c+d\,x)+a\ \sin(c+d\,x)\ 1\mathrm{i})^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*cos(c + d*x) + a*sin(c + d*x)*1i)^(3/2),x)
```

```
[Out] int(1/(a*cos(c + d*x) + a*sin(c + d*x)*1i)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(ia\sin(c+dx)+a\cos(c+dx))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cos(d*x+c)+I*a*sin(d*x+c))**(3/2),x)
```

```
[Out] Integral((I*a*sin(c + d*x) + a*cos(c + d*x))**(-3/2), x)
```

3.262 $\int \frac{1}{(a\cos(c+dx)+ia\sin(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=33

$$\frac{2i}{5d(a\cos(c+dx)+ia\sin(c+dx))^{5/2}}$$

```
[Out] 2/5*I/d/(a*cos(d*x+c)+I*a*sin(d*x+c))^(5/2)
```

Rubi [A] time = 0.02, antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.042, Rules used = {3071}

$$\frac{2i}{5d(a\cos(c+dx)+ia\sin(c+dx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Cos[c + d*x] + I*a*Sin[c + d*x])^(-5/2),x]
[Out] ((2*I)/5)/(d*(a*Cos[c + d*x] + I*a*Sin[c + d*x])^(5/2))
```

Rule 3071

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x
_Symbol] :> Simp[(a*(a*Cos[c + d*x] + b*Sin[c + d*x])^n)/(b*d*n), x] /; Fre
eQ[{a, b, c, d, n}, x] && EqQ[a^2 + b^2, 0]
```

Rubi steps

$$\int \frac{1}{(a\cos(c+dx)+ia\sin(c+dx))^{5/2}}\,dx = \frac{2i}{5d(a\cos(c+dx)+ia\sin(c+dx))^{5/2}}$$

Mathematica [A] time = 0.03, size = 32, normalized size = 0.97

$$\frac{2i}{5d(a(\cos(c+dx)+i\sin(c+dx)))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Cos[c + d*x] + I*a*Sin[c + d*x])^(-5/2),x]
[Out] ((2*I)/5)/(d*(a*(Cos[c + d*x] + I*Sin[c + d*x]))^(5/2))
```

fricas [A] time = 0.97, size = 17, normalized size = 0.52

$$\frac{2i\,e^{\left(-\frac{5}{2}i\,dx-\frac{5}{2}i\,c\right)}}{5\,a^{\frac{5}{2}}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cos(d*x+c)+I*a*sin(d*x+c))^(5/2),x, algorithm="fricas")
[Out] 2/5*I*e^(-5/2*I*d*x - 5/2*I*c)/(a^(5/2)*d)
```

giac [B] time = 3.30, size = 67, normalized size = 2.03

$$\frac{2i\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+i\right)^{2}}{5\left(a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-i\,a\right)^{2}d\sqrt{-\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-i\,a}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+i}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cos(d*x+c)+I*a*sin(d*x+c))^(5/2),x, algorithm="giac")
[Out] 2/5*I*(tan(1/2*d*x + 1/2*c) + I)^2/((a*tan(1/2*d*x + 1/2*c) - I*a)^2*d*sqrt
(-(a*tan(1/2*d*x + 1/2*c) - I*a)/(tan(1/2*d*x + 1/2*c) + I)))
```

maple [A] time = 0.21, size = 28, normalized size = 0.85

$$\frac{2i}{5d\left(a\cos\left(dx+c\right)+ia\sin\left(dx+c\right)\right)^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*cos(d*x+c)+I*a*sin(d*x+c))^(5/2),x)
[Out] 2/5*I/d/(a*cos(d*x+c)+I*a*sin(d*x+c))^(5/2)
```

maxima [B] time = 0.66, size = 51, normalized size = 1.55

$$\frac{2i\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+i\right)^{\frac{5}{2}}}{5\,a^{\frac{5}{2}}d\left(-\frac{\sin(dx+c)}{\cos(dx+c)+1}+i\right)^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cos(d*x+c)+I*a*sin(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] 2/5*I*(sin(d*x + c)/(cos(d*x + c) + 1) + I)^(5/2)/(a^(5/2)*d*(-sin(d*x + c)
/(cos(d*x + c) + 1) + I)^(5/2))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{1}{(a\,\cos\left(c+d\,x\right)+a\,\sin\left(c+d\,x\right)\,1\mathrm{i})^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*cos(c + d*x) + a*sin(c + d*x)*1i)^(5/2),x)
```

```
[Out] int(1/(a*cos(c + d*x) + a*sin(c + d*x)*1i)^(5/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(ia\sin\left(c+dx\right)+a\cos\left(c+dx\right))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cos(d*x+c)+I*a*sin(d*x+c))**(5/2),x)
```

```
[Out] Integral((I*a*sin(c + d*x) + a*cos(c + d*x))**(-5/2), x)
```

3.263 $\int (a\sec(x)+b\tan(x))^5\,dx$

Optimal. Leaf size=149

$$-\frac{1}{16}(a+b)^3\left(3a^2-9ab+8b^2\right)\log(1-\sin(x))+\frac{1}{16}(a-b)^3\left(3a^2+9ab+8b^2\right)\log(\sin(x)+1)+\frac{1}{8}\sec^2(x)(a+b\sin(x))^2$$

```
[Out] -1/16*(a+b)^3*(3*a^2-9*a*b+8*b^2)*ln(1-sin(x))+1/16*(a-b)^3*(3*a^2+9*a*b+8*
b^2)*ln(1+sin(x))-1/8*a*(7-3*a^2/b^2)*b^4*sin(x)+1/4*sec(x)^4*(b+a*sin(x))*
(a+b*sin(x))^4+1/8*sec(x)^2*(a+b*sin(x))^2*(2*b*(a^2-2*b^2)+a*(3*a^2-5*b^2)
*sin(x))
```

Rubi [A] time = 0.21, antiderivative size = 149, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.636, Rules used = {4391, 2668, 739, 819, 774, 633, 31}

$$-\frac{1}{8}ab^4\left(7-\frac{3a^2}{b^2}\right)\sin(x)-\frac{1}{16}(a+b)^3\left(3a^2-9ab+8b^2\right)\log(1-\sin(x))+\frac{1}{16}(a-b)^3\left(3a^2+9ab+8b^2\right)\log(\sin(x)+1)$$

Antiderivative was successfully verified.

```
[In] Int[(a*Sec[x] + b*Tan[x])^5,x]
```

```
[Out] -((a + b)^3*(3*a^2 - 9*a*b + 8*b^2)*Log[1 - Sin[x]])/16 + ((a - b)^3*(3*a^2
 + 9*a*b + 8*b^2)*Log[1 + Sin[x]])/16 - (a*(7 - (3*a^2)/b^2)*b^4*Sin[x])/8
+ (Sec[x]^4*(b + a*Sin[x])*(a + b*Sin[x])^4)/4 + (Sec[x]^2*(a + b*Sin[x])^2
*(2*b*(a^2 - 2*b^2) + a*(3*a^2 - 5*b^2)*Sin[x]))/8
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 633

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> With[{q = Rt[-(
a*c), 2]}, Dist[e/2 + (c*d)/(2*q), Int[1/(-q + c*x), x], x] + Dist[e/2 - (c
*d)/(2*q), Int[1/(q + c*x), x], x]] /; FreeQ[{a, c, d, e}, x] && NiceSqrtQ[
-(a*c)]
```

Rule 739

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
((d + e*x)^(m - 1)*(a*e - c*d*x)*(a + c*x^2)^(p + 1))/(2*a*c*(p + 1)), x] +
 Dist[1/((p + 1)*(-2*a*c)), Int[(d + e*x)^(m - 2)*Simp[a*e^2*(m - 1) - c*d^
2*(2*p + 3) - d*c*e*(m + 2*p + 2)*x, x]*(a + c*x^2)^(p + 1), x], x] /; Free
```

```
Q[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 1] && I
ntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 774

```
Int[(((d_.) + (e_.)*(x_))*((f_) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2), x_Sym
bol] :> Simp[(e*g*x)/c, x] + Dist[1/c, Int[(c*d*f - a*e*g + c*(e*f + d*g)*x
)/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e, f, g}, x]
```

Rule 819

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x)^(m - 1)*(a + c*x^2)^(p + 1)*(a*(e*f + d*g
) - (c*d*f - a*e*g)*x))/(2*a*c*(p + 1)), x] - Dist[1/(2*a*c*(p + 1)), Int[(
d + e*x)^(m - 2)*(a + c*x^2)^(p + 1)*Simp[a*e*(e*f*(m - 1) + d*g*m) - c*d^2
*f*(2*p + 3) + e*(a*e*g*m - c*d*f*(m + 2*p + 2))*x, x], x], x] /; FreeQ[{a,
 c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 1] &&
(EqQ[d, 0] || (EqQ[m, 2] && EqQ[p, -3] && RationalQ[a, c, d, e, f, g]) ||
!ILtQ[m + 2*p + 3, 0])
```

Rule 2668

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rule 4391

```
Int[(u_.)*((b_.)*sec[(c_.) + (d_.)*(x_)]^(n_.) + (a_.)*tan[(c_.) + (d_.)*(x
_)]^(n_.))^(p_), x_Symbol] :> Int[ActivateTrig[u]*Sec[c + d*x]^(n*p)*(b + a
*Sin[c + d*x]^n)^p, x] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p]
```

Rubi steps

$$\int (a\sec(x)+b\tan(x))^5\,dx = \int \sec^5(x)(a+b\sin(x))^5\,dx$$
$$= b^5\,\text{Subst}\left(\int \frac{(a+x)^5}{\left(b^2-x^2\right)^3}\,dx, x, b\sin(x)\right)$$
$$= \frac{1}{4}\sec^4(x)(b+a\sin(x))(a+b\sin(x))^4 - \frac{1}{4}b^3\,\text{Subst}\left(\int \frac{(a+x)^3\left(-3a^2+4b^2+ax\right)}{\left(b^2-x^2\right)^2}\right.$$
$$= \frac{1}{4}\sec^4(x)(b+a\sin(x))(a+b\sin(x))^4 + \frac{1}{8}\sec^2(x)(a+b\sin(x))^2\left(2b\left(a^2-2b^2\right)+\right.$$
$$= \frac{1}{8}ab^2\left(3a^2-7b^2\right)\sin(x) + \frac{1}{4}\sec^4(x)(b+a\sin(x))(a+b\sin(x))^4 + \frac{1}{8}\sec^2(x)(a+b$$
$$= \frac{1}{8}ab^2\left(3a^2-7b^2\right)\sin(x) + \frac{1}{4}\sec^4(x)(b+a\sin(x))(a+b\sin(x))^4 + \frac{1}{8}\sec^2(x)(a+b$$
$$= -\frac{1}{16}(a+b)^3\left(3a^2-9ab+8b^2\right)\log(1-\sin(x)) + \frac{1}{16}(a-b)^3\left(3a^2+9ab+8b^2\right)\log$$

Mathematica [B] time = 1.24, size = 303, normalized size = 2.03

$$-\frac{\left(a^2-b^2\right)^2\left((a+b)^3\left(3a^2-9ab+8b^2\right)\log(1-\sin(x))-(a-b)^3\left(3a^2+9ab+8b^2\right)\log(\sin(x)+1)\right)+4\left(b^2-a\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Sec[x] + b*Tan[x])^5,x]
```

```
[Out] -1/16*((a^2 - b^2)^2*((a + b)^3*(3*a^2 - 9*a*b + 8*b^2)*Log[1 - Sin[x]] - (
a - b)^3*(3*a^2 + 9*a*b + 8*b^2)*Log[1 + Sin[x]]) - 10*a*b^2*(9*a^6 - 6*a^4
*b^2 + 8*a^2*b^4 - 3*b^6)*Sin[x] + 8*b^3*(-15*a^6 - 4*a^4*b^2 - 2*a^2*b^4 +
 b^6)*Sin[x]^2 - 10*a*b^4*(9*a^4 + 8*a^2*b^2 - b^4)*Sin[x]^3 + 4*b^5*(-9*a^
4 - 12*a^2*b^2 + b^4)*Sin[x]^4 - 2*a*b^6*(3*a^2 + 5*b^2)*Sin[x]^5 + 4*(-a^2
 + b^2)*Sec[x]^4*(-b + a*Sin[x])*(a + b*Sin[x])^6 + 2*Sec[x]^2*(a + b*Sin[x
])^6*(6*a^2*b + 2*b^3 - 3*a^3*Sin[x] - 5*a*b^2*Sin[x]))/(a^2 - b^2)^2
```

fricas [A] time = 0.92, size = 166, normalized size = 1.11

$$\frac{\left(3a^5-10a^3b^2+15ab^4-8b^5\right)\cos(x)^4\log(\sin(x)+1)-\left(3a^5-10a^3b^2+15ab^4+8b^5\right)\cos(x)^4\log(-\sin(x)+}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sec(x)+b*tan(x))^5,x, algorithm="fricas")
```

```
[Out] 1/16*((3*a^5 - 10*a^3*b^2 + 15*a*b^4 - 8*b^5)*cos(x)^4*log(sin(x) + 1) - (3
*a^5 - 10*a^3*b^2 + 15*a*b^4 + 8*b^5)*cos(x)^4*log(-sin(x) + 1) + 20*a^4*b
+ 40*a^2*b^3 + 4*b^5 - 16*(5*a^2*b^3 + b^5)*cos(x)^2 + 2*(2*a^5 + 20*a^3*b^
2 + 10*a*b^4 + (3*a^5 - 10*a^3*b^2 - 25*a*b^4)*cos(x)^2)*sin(x))/cos(x)^4
```

giac [A] time = 0.15, size = 178, normalized size = 1.19

$$\frac{1}{16}\left(3\,a^5-10\,a^3b^2+15\,ab^4-8\,b^5\right)\log\left(\sin(x)+1\right)-\frac{1}{16}\left(3\,a^5-10\,a^3b^2+15\,ab^4+8\,b^5\right)\log\left(-\sin(x)+1\right)+\frac{6\,b^5\sin}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sec(x)+b*tan(x))^5,x, algorithm="giac")
```

```
[Out] 1/16*(3*a^5 - 10*a^3*b^2 + 15*a*b^4 - 8*b^5)*log(sin(x) + 1) - 1/16*(3*a^5
- 10*a^3*b^2 + 15*a*b^4 + 8*b^5)*log(-sin(x) + 1) + 1/8*(6*b^5*sin(x)^4 - 3
*a^5*sin(x)^3 + 10*a^3*b^2*sin(x)^3 + 25*a*b^4*sin(x)^3 + 40*a^2*b^3*sin(x)
^2 - 4*b^5*sin(x)^2 + 5*a^5*sin(x) + 10*a^3*b^2*sin(x) - 15*a*b^4*sin(x) +
10*a^4*b - 20*a^2*b^3)/(sin(x)^2 - 1)^2
```

maple [A] time = 0.18, size = 199, normalized size = 1.34

$$\frac{a^5\tan(x)\left(\sec^3(x)\right)}{4}+\frac{3a^5\sec(x)\tan(x)}{8}+\frac{3a^5\ln\left(\sec(x)+\tan(x)\right)}{8}+\frac{5a^4b}{4\cos(x)^4}+\frac{5a^3b^2\left(\sin^3(x)\right)}{2\cos(x)^4}+\frac{5a^3b^2\left(\sin^3(x)\right)}{4\cos(x)^2}+\frac{5}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*sec(x)+b*tan(x))^5,x)
```

```
[Out] 1/4*a^5*tan(x)*sec(x)^3+3/8*a^5*sec(x)*tan(x)+3/8*a^5*ln(sec(x)+tan(x))+5/4
*a^4*b/cos(x)^4+5/2*a^3*b^2*sin(x)^3/cos(x)^4+5/4*a^3*b^2*sin(x)^3/cos(x)^2
+5/4*a^3*b^2*sin(x)-5/4*a^3*b^2*ln(sec(x)+tan(x))+5/2*a^2*b^3*sin(x)^4/cos(
x)^4+5/4*a*b^4*sin(x)^5/cos(x)^4-5/8*a*b^4*sin(x)^5/cos(x)^2-5/8*a*b^4*sin(
x)^3-15/8*a*b^4*sin(x)+15/8*a*b^4*ln(sec(x)+tan(x))+1/4*b^5*tan(x)^4-1/2*b^
5*tan(x)^2-b^5*ln(cos(x))
```

maxima [A] time = 0.32, size = 204, normalized size = 1.37

$$\frac{5}{2}\,a^2b^3\,\tan(x)^4+\frac{5}{16}\,ab^4\left(\frac{2\left(5\,\sin(x)^3-3\,\sin(x)\right)}{\sin(x)^4-2\,\sin(x)^2+1}+3\,\log\left(\sin(x)+1\right)-3\,\log\left(\sin(x)-1\right)\right)-\frac{1}{16}\,a^5\left(\frac{2\left(3\,\sin(x)^3-5\right.}{\sin(x)^4-2\,\sin}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sec(x)+b*tan(x))^5,x, algorithm="maxima")
```

```
[Out] 5/2*a^2*b^3*tan(x)^4 + 5/16*a*b^4*(2*(5*sin(x)^3 - 3*sin(x))/(sin(x)^4 - 2*
sin(x)^2 + 1) + 3*log(sin(x) + 1) - 3*log(sin(x) - 1)) - 1/16*a^5*(2*(3*sin
(x)^3 - 5*sin(x))/(sin(x)^4 - 2*sin(x)^2 + 1) - 3*log(sin(x) + 1) + 3*log(s
in(x) - 1)) + 5/8*a^3*b^2*(2*(sin(x)^3 + sin(x))/(sin(x)^4 - 2*sin(x)^2 + 1
) - log(sin(x) + 1) + log(sin(x) - 1)) + 1/4*b^5*((4*sin(x)^2 - 3)/(sin(x)^
4 - 2*sin(x)^2 + 1) - 2*log(sin(x)^2 - 1)) + 5/4*a^4*b/(sin(x)^2 - 1)^2
```

mupad [B] time = 2.88, size = 272, normalized size = 1.83

$$\frac{\left(\frac{5\,a^5}{4}+\frac{5\,a^3\,b^2}{2}-\frac{15\,a\,b^4}{4}\right)\,\tan\left(\frac{x}{2}\right)^7+\left(10\,a^4\,b-2\,b^5\right)\,\tan\left(\frac{x}{2}\right)^6+\left(\frac{3\,a^5}{4}+\frac{35\,a^3\,b^2}{2}+\frac{55\,a\,b^4}{4}\right)\,\tan\left(\frac{x}{2}\right)^5+\left(40\,a^2\,b^3+8\,b^5\right.}{\tan\left(\frac{x}{2}\right)^8-4\,\tan\left(\frac{x}{2}\right)^6+6\,\tan}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*tan(x) + a/cos(x))^5,x)
```

```
[Out] (tan(x/2)^2*(10*a^4*b - 2*b^5) + tan(x/2)^6*(10*a^4*b - 2*b^5) + tan(x/2)^4
*(8*b^5 + 40*a^2*b^3) + tan(x/2)*((5*a^5)/4 - (15*a*b^4)/4 + (5*a^3*b^2)/2)
 + tan(x/2)^7*((5*a^5)/4 - (15*a*b^4)/4 + (5*a^3*b^2)/2) + tan(x/2)^3*((55*
a*b^4)/4 + (3*a^5)/4 + (35*a^3*b^2)/2) + tan(x/2)^5*((55*a*b^4)/4 + (3*a^5)
/4 + (35*a^3*b^2)/2))/(6*tan(x/2)^4 - 4*tan(x/2)^2 - 4*tan(x/2)^6 + tan(x/2
)^8 + 1) + b^5*log(tan(x/2)^2 + 1) - (log(tan(x/2) - 1)*(a + b)^3*(3*a^2 -
9*a*b + 8*b^2))/8 + log(tan(x/2) + 1)*(a - b)^3*((9*a*b)/8 + (3*a^2)/8 + b^
2)
```

sympy [B] time = 7.12, size = 308, normalized size = 2.07

$$-\frac{3a^5\log\left(\sin\left(x\right)-1\right)}{16}+\frac{3a^5\log\left(\sin\left(x\right)+1\right)}{16}-\frac{3a^5\sin^3\left(x\right)}{8\sin^4\left(x\right)-16\sin^2\left(x\right)+8}+\frac{5a^5\sin\left(x\right)}{8\sin^4\left(x\right)-16\sin^2\left(x\right)+8}+\frac{5a^4b\sec^4}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sec(x)+b*tan(x))**5,x)
```

```
[Out] -3*a**5*log(sin(x) - 1)/16 + 3*a**5*log(sin(x) + 1)/16 - 3*a**5*sin(x)**3/(
8*sin(x)**4 - 16*sin(x)**2 + 8) + 5*a**5*sin(x)/(8*sin(x)**4 - 16*sin(x)**2
 + 8) + 5*a**4*b*sec(x)**4/4 + 5*a**3*b**2*log(sin(x) - 1)/8 - 5*a**3*b**2*
log(sin(x) + 1)/8 + 10*a**3*b**2*sin(x)**3/(8*sin(x)**4 - 16*sin(x)**2 + 8)
 + 10*a**3*b**2*sin(x)/(8*sin(x)**4 - 16*sin(x)**2 + 8) + 5*a**2*b**3*tan(x
)**4/2 - 15*a*b**4*log(sin(x) - 1)/16 + 15*a*b**4*log(sin(x) + 1)/16 + 25*a
*b**4*sin(x)**3/(8*sin(x)**4 - 16*sin(x)**2 + 8) - 15*a*b**4*sin(x)/(8*sin(
x)**4 - 16*sin(x)**2 + 8) + b**5*log(sec(x)**2)/2 + b**5*sec(x)**4/4 - b**5
*sec(x)**2
```

3.264 $\int (a\sec(x) + b\tan(x))^4 \, dx$

Optimal. Leaf size=100

$$\frac{4}{3}ab\left(a^2-2b^2\right)\cos(x)+\frac{1}{3}b^2\left(2a^2-3b^2\right)\sin(x)\cos(x)-\frac{1}{3}\sec(x)(a+b\sin(x))^2\left(ab-\left(2a^2-3b^2\right)\sin(x)\right)+\frac{1}{3}\sec^3(x)($$

```
[Out] b^4*x+4/3*a*b*(a^2-2*b^2)*cos(x)+1/3*b^2*(2*a^2-3*b^2)*cos(x)*sin(x)+1/3*se
c(x)^3*(b+a*sin(x))*(a+b*sin(x))^3-1/3*sec(x)*(a+b*sin(x))^2*(a*b-(2*a^2-3*
b^2)*sin(x))
```

Rubi [A] time = 0.20, antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {4391, 2691, 2861, 2734}

$$\frac{4}{3}ab\left(a^2-2b^2\right)\cos(x)+\frac{1}{3}b^2\left(2a^2-3b^2\right)\sin(x)\cos(x)-\frac{1}{3}\sec(x)(a+b\sin(x))^2\left(ab-\left(2a^2-3b^2\right)\sin(x)\right)+\frac{1}{3}\sec^3(x)($$

Antiderivative was successfully verified.

```
[In] Int[(a*Sec[x] + b*Tan[x])^4,x]
```

```
[Out] b^4*x + (4*a*b*(a^2 - 2*b^2)*Cos[x])/3 + (b^2*(2*a^2 - 3*b^2)*Cos[x]*Sin[x]
)/3 + (Sec[x]^3*(b + a*Sin[x])*(a + b*Sin[x])^3)/3 - (Sec[x]*(a + b*Sin[x])
^2*(a*b - (2*a^2 - 3*b^2)*Sin[x]))/3
```

Rule 2691

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Simp[((g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x]
)^(m - 1)*(b + a*Sin[e + f*x]))/(f*g*(p + 1)), x] + Dist[1/(g^2*(p + 1)), I
nt[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^(m - 2)*(b^2*(m - 1) + a^2
*(p + 2) + a*b*(m + p + 1)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f, g},
x] && NeQ[a^2 - b^2, 0] && GtQ[m, 1] && LtQ[p, -1] && (IntegersQ[2*m, 2*p]
|| IntegerQ[m])
```

Rule 2734

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[((2*a*c + b*d)*x)/2, x] + (-Simp[((b*c + a*d)*Co
s[e + f*x])/f, x] - Simp[(b*d*Cos[e + f*x]*Sin[e + f*x])/(2*f), x]) /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2861

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> -Simp[((g*
```

```
Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m*(d + c*Sin[e + f*x]))/(f*g*(p
+ 1)), x] + Dist[1/(g^2*(p + 1)), Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e
 + f*x])^(m - 1)*Simp[a*c*(p + 2) + b*d*m + b*c*(m + p + 2)*Sin[e + f*x], x
], x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 - b^2, 0] && GtQ[m,
 0] && LtQ[p, -1] && IntegerQ[2*m] &&  !(EqQ[m, 1] && NeQ[c^2 - d^2, 0] &&
SimplerQ[c + d*x, a + b*x])
```

Rule 4391

```
Int[(u_.)*((b_.)*sec[(c_.) + (d_.)*(x_)]^(n_.) + (a_.)*tan[(c_.) + (d_.)*(x
_)]^(n_.))^(p_), x_Symbol] :> Int[ActivateTrig[u]*Sec[c + d*x]^(n*p)*(b + a
*Sin[c + d*x]^n)^p, x] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p]
```

Rubi steps

$$\int (a\sec(x)+b\tan(x))^4\,dx = \int \sec^4(x)(a+b\sin(x))^4\,dx$$
$$= \frac{1}{3}\sec^3(x)(b+a\sin(x))(a+b\sin(x))^3 - \frac{1}{3}\int \sec^2(x)(a+b\sin(x))^2\left(-2a^2+3b^2+\right.$$
$$= \frac{1}{3}\sec^3(x)(b+a\sin(x))(a+b\sin(x))^3 - \frac{1}{3}\sec(x)(a+b\sin(x))^2\left(ab-\left(2a^2-3b^2\right)\mathrm{s}\right.$$
$$= b^4x + \frac{4}{3}ab\left(a^2-2b^2\right)\cos(x) + \frac{1}{3}b^2\left(2a^2-3b^2\right)\cos(x)\sin(x) + \frac{1}{3}\sec^3(x)(b+a\sin$$

Mathematica [A] time = 0.20, size = 96, normalized size = 0.96

$$\frac{1}{12}\sec^3(x)\left(6a^4\sin(x)+2a^4\sin(3x)+16a^3b+18a^2b^2\sin(x)-6a^2b^2\sin(3x)-24ab^3\cos(2x)-8ab^3-4b^4\sin(3\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Sec[x] + b*Tan[x])^4,x]
```

```
[Out] (Sec[x]^3*(16*a^3*b - 8*a*b^3 + 9*b^4*x*Cos[x] - 24*a*b^3*Cos[2*x] + 3*b^4*
x*Cos[3*x] + 6*a^4*Sin[x] + 18*a^2*b^2*Sin[x] + 2*a^4*Sin[3*x] - 6*a^2*b^2*
Sin[3*x] - 4*b^4*Sin[3*x]))/12
```

fricas [A] time = 0.84, size = 80, normalized size = 0.80

$$\frac{3\,b^4x\cos(x)^3-12\,ab^3\cos(x)^2+4\,a^3b+4\,ab^3+\left(a^4+6\,a^2b^2+b^4+2\left(a^4-3\,a^2b^2-2\,b^4\right)\cos(x)^2\right)\sin(x)}{3\,\cos(x)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sec(x)+b*tan(x))^4,x, algorithm="fricas")
```

```
[Out] 1/3*(3*b^4*x*cos(x)^3 - 12*a*b^3*cos(x)^2 + 4*a^3*b + 4*a*b^3 + (a^4 + 6*a^
2*b^2 + b^4 + 2*(a^4 - 3*a^2*b^2 - 2*b^4)*cos(x)^2)*sin(x))/cos(x)^3
```

giac [A] time = 0.16, size = 131, normalized size = 1.31

$$b^4x - \frac{2\left(3\,a^4\tan\left(\frac{1}{2}\,x\right)^5 - 3\,b^4\tan\left(\frac{1}{2}\,x\right)^5 + 12\,a^3b\tan\left(\frac{1}{2}\,x\right)^4 - 2\,a^4\tan\left(\frac{1}{2}\,x\right)^3 + 24\,a^2b^2\tan\left(\frac{1}{2}\,x\right)^3 + 10\,b^4\tan\left(\frac{1}{2}\,x\right.\right.}{3\left(\tan\left(\frac{1}{2}\,x\right)^2 - 1\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sec(x)+b*tan(x))^4,x, algorithm="giac")
```

```
[Out] b^4*x - 2/3*(3*a^4*tan(1/2*x)^5 - 3*b^4*tan(1/2*x)^5 + 12*a^3*b*tan(1/2*x)^
4 - 2*a^4*tan(1/2*x)^3 + 24*a^2*b^2*tan(1/2*x)^3 + 10*b^4*tan(1/2*x)^3 + 24
*a*b^3*tan(1/2*x)^2 + 3*a^4*tan(1/2*x) - 3*b^4*tan(1/2*x) + 4*a^3*b - 8*a*b
^3)/(tan(1/2*x)^2 - 1)^3
```

maple [A] time = 0.09, size = 96, normalized size = 0.96

$$-a^4\left(-\frac{2}{3} - \frac{\left(\sec^2(x)\right)}{3}\right)\tan(x) + \frac{4a^3b}{3\cos(x)^3} + \frac{2a^2b^2\left(\sin^3(x)\right)}{\cos(x)^3} + 4a\,b^3\left(\frac{\sin^4(x)}{3\cos(x)^3} - \frac{\sin^4(x)}{3\cos(x)} - \frac{\left(2+\sin^2(x)\right)\cos(x)}{3}\right) + b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*sec(x)+b*tan(x))^4,x)
```

```
[Out] -a^4*(-2/3-1/3*sec(x)^2)*tan(x)+4/3*a^3*b/cos(x)^3+2*a^2*b^2*sin(x)^3/cos(x
)^3+4*a*b^3*(1/3*sin(x)^4/cos(x)^3-1/3*sin(x)^4/cos(x)-1/3*(2+sin(x)^2)*cos
(x))+b^4*(1/3*tan(x)^3-tan(x)+x)
```

maxima [A] time = 1.05, size = 72, normalized size = 0.72

$$2\,a^2b^2\tan(x)^3 + \frac{1}{3}\left(\tan(x)^3 + 3\,\tan(x)\right)a^4 + \frac{1}{3}\left(\tan(x)^3 + 3\,x - 3\,\tan(x)\right)b^4 - \frac{4\left(3\,\cos(x)^2 - 1\right)ab^3}{3\,\cos(x)^3} + \frac{4\,a^3b}{3\,\cos(x)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sec(x)+b*tan(x))^4,x, algorithm="maxima")
```

```
[Out] 2*a^2*b^2*tan(x)^3 + 1/3*(tan(x)^3 + 3*tan(x))*a^4 + 1/3*(tan(x)^3 + 3*x -
3*tan(x))*b^4 - 4/3*(3*cos(x)^2 - 1)*a*b^3/cos(x)^3 + 4/3*a^3*b/cos(x)^3
```

mupad [B] time = 2.53, size = 115, normalized size = 1.15

$$b^4\,x-\frac{\tan\left(\frac{x}{2}\right)\,\left(2\,a^4-2\,b^4\right)-\frac{16\,a\,b^3}{3}+\frac{8\,a^3\,b}{3}+\tan\left(\frac{x}{2}\right)^3\,\left(-\frac{4\,a^4}{3}+16\,a^2\,b^2+\frac{20\,b^4}{3}\right)+\tan\left(\frac{x}{2}\right)^5\,\left(2\,a^4-2\,b^4\right)+16\,a\,b^3}{\left(\tan\left(\frac{x}{2}\right)^2-1\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*tan(x) + a/cos(x))^4,x)
```

```
[Out] b^4*x - (tan(x/2)*(2*a^4 - 2*b^4) - (16*a*b^3)/3 + (8*a^3*b)/3 + tan(x/2)^3
*((20*b^4)/3 - (4*a^4)/3 + 16*a^2*b^2) + tan(x/2)^5*(2*a^4 - 2*b^4) + 16*a*
b^3*tan(x/2)^2 + 8*a^3*b*tan(x/2)^4)/(tan(x/2)^2 - 1)^3
```

sympy [A] time = 4.22, size = 97, normalized size = 0.97

$$\frac{a^4\tan^3(x)}{3}+a^4\tan(x)+\frac{4a^3b\sec^3(x)}{3}+2a^2b^2\tan^3(x)+\frac{4ab^3\sec^3(x)}{3}-4ab^3\sec(x)+b^4x+\frac{b^4\sin^3(x)}{3\cos^3(x)}-\frac{b^4\sin(x)}{\cos(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sec(x)+b*tan(x))**4,x)
```

```
[Out] a**4*tan(x)**3/3 + a**4*tan(x) + 4*a**3*b*sec(x)**3/3 + 2*a**2*b**2*tan(x)*
*3 + 4*a*b**3*sec(x)**3/3 - 4*a*b**3*sec(x) + b**4*x + b**4*sin(x)**3/(3*co
s(x)**3) - b**4*sin(x)/cos(x)
```

3.265 $\int (a\sec(x) + b\tan(x))^3\,dx$

Optimal. Leaf size=75

$$\frac{1}{2}ab^2\sin(x)+\frac{1}{4}(a+2b)(a-b)^2\log(\sin(x)+1)-\frac{1}{4}(a-2b)(a+b)^2\log(1-\sin(x))+\frac{1}{2}\sec^2(x)(a\sin(x)+b)(a+b\sin(x))^2$$

```
[Out] -1/4*(a-2*b)*(a+b)^2*ln(1-sin(x))+1/4*(a-b)^2*(a+2*b)*ln(1+sin(x))+1/2*a*b^
2*sin(x)+1/2*sec(x)^2*(b+a*sin(x))*(a+b*sin(x))^2
```

Rubi [A] time = 0.14, antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.546, Rules used = {4391, 2668, 739, 774, 633, 31}

$$\frac{1}{2}ab^2\sin(x)+\frac{1}{4}(a+2b)(a-b)^2\log(\sin(x)+1)-\frac{1}{4}(a-2b)(a+b)^2\log(1-\sin(x))+\frac{1}{2}\sec^2(x)(a\sin(x)+b)(a+b\sin(x))^2$$

Antiderivative was successfully verified.

```
[In] Int[(a*Sec[x] + b*Tan[x])^3,x]
```

```
[Out] -((a - 2*b)*(a + b)^2*Log[1 - Sin[x]])/4 + ((a - b)^2*(a + 2*b)*Log[1 + Sin
[x]])/4 + (a*b^2*Sin[x])/2 + (Sec[x]^2*(b + a*Sin[x])*(a + b*Sin[x])^2)/2
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 633

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> With[{q = Rt[-(
a*c), 2]}, Dist[e/2 + (c*d)/(2*q), Int[1/(-q + c*x), x], x] + Dist[e/2 - (c
*d)/(2*q), Int[1/(q + c*x), x], x]] /; FreeQ[{a, c, d, e}, x] && NiceSqrtQ[
-(a*c)]
```

Rule 739

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
((d + e*x)^(m - 1)*(a*e - c*d*x)*(a + c*x^2)^(p + 1))/(2*a*c*(p + 1)), x] +
 Dist[1/((p + 1)*(-2*a*c)), Int[(d + e*x)^(m - 2)*Simp[a*e^2*(m - 1) - c*d^
2*(2*p + 3) - d*c*e*(m + 2*p + 2)*x, x]*(a + c*x^2)^(p + 1), x], x] /; Free
Q[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 1] && I
ntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 774

```
Int[(((d_.) + (e_.)*(x_))*((f_) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2), x_Sym
bol] :> Simp[(e*g*x)/c, x] + Dist[1/c, Int[(c*d*f - a*e*g + c*(e*f + d*g)*x
)/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e, f, g}, x]
```

Rule 2668

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rule 4391

```
Int[(u_.)*((b_.)*sec[(c_.) + (d_.)*(x_)]^(n_.) + (a_.)*tan[(c_.) + (d_.)*(x
_)]^(n_.))^(p_), x_Symbol] :> Int[ActivateTrig[u]*Sec[c + d*x]^(n*p)*(b + a
*Sin[c + d*x]^n)^p, x] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p]
```

Rubi steps

$$\begin{aligned}
\int (a\sec(x)+b\tan(x))^3\,dx &= \int \sec^3(x)(a+b\sin(x))^3\,dx\\
&= b^3\,\mathrm{Subst}\left(\int \frac{(a+x)^3}{\left(b^2-x^2\right)^2}\,dx,x,b\sin(x)\right)\\
&= \frac{1}{2}\sec^2(x)(b+a\sin(x))(a+b\sin(x))^2-\frac{1}{2}b\,\mathrm{Subst}\left(\int \frac{(a+x)\left(-a^2+2b^2+ax\right)}{b^2-x^2}\,dx\right.\\
&= \frac{1}{2}ab^2\sin(x)+\frac{1}{2}\sec^2(x)(b+a\sin(x))(a+b\sin(x))^2+\frac{1}{2}b\,\mathrm{Subst}\left(\int \frac{-ab^2-a\left(-a^2\right.}{b^2}\right.\\
&= \frac{1}{2}ab^2\sin(x)+\frac{1}{2}\sec^2(x)(b+a\sin(x))(a+b\sin(x))^2+\frac{1}{4}\left((a-2b)(a+b)^2\right)\mathrm{Subst}\left(\right.\\
&= -\frac{1}{4}(a-2b)(a+b)^2\log(1-\sin(x))+\frac{1}{4}(a-b)^2(a+2b)\log(1+\sin(x))+\frac{1}{2}ab^2\sin
\end{aligned}$$

Mathematica [A] time = 0.57, size = 123, normalized size = 1.64

$$\frac{2a^4b\sec^2(x)+\left(a^2-b^2\right)\left((a-2b)(a+b)^2\log(1-\sin(x))-(a-b)^2(a+2b)\log(\sin(x)+1)\right)+\left(-8a^4b+4a^2b^3\right.}{4\left(b^2-a^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Sec[x] + b*Tan[x])^3,x]
```

```
[Out] ((a^2 - b^2)*((a - 2*b)*(a + b)^2*Log[1 - Sin[x]] - (a - b)^2*(a + 2*b)*Log
[1 + Sin[x]]) + 2*a^4*b*Sec[x]^2 - 2*a*(a^4 + 2*a^2*b^2 - 3*b^4)*Sec[x]*Tan
[x] + (-8*a^4*b + 4*a^2*b^3 + 2*b^5)*Tan[x]^2)/(4*(-a^2 + b^2))
```

fricas [A] time = 2.07, size = 85, normalized size = 1.13

$$\frac{\left(a^3-3\,ab^2+2\,b^3\right)\cos(x)^2\log\left(\sin(x)+1\right)-\left(a^3-3\,ab^2-2\,b^3\right)\cos(x)^2\log\left(-\sin(x)+1\right)+6\,a^2b+2\,b^3+2\left(a^3+\right.}{4\,\cos(x)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sec(x)+b*tan(x))^3,x, algorithm="fricas")
```

```
[Out] 1/4*((a^3 - 3*a*b^2 + 2*b^3)*cos(x)^2*log(sin(x) + 1) - (a^3 - 3*a*b^2 - 2*
b^3)*cos(x)^2*log(-sin(x) + 1) + 6*a^2*b + 2*b^3 + 2*(a^3 + 3*a*b^2)*sin(x)
)/cos(x)^2
```

giac [A] time = 0.14, size = 86, normalized size = 1.15

$$\frac{1}{4}\left(a^3-3\,ab^2+2\,b^3\right)\log\left(\sin(x)+1\right)-\frac{1}{4}\left(a^3-3\,ab^2-2\,b^3\right)\log\left(-\sin(x)+1\right)-\frac{b^3\sin(x)^2+a^3\sin(x)+3\,ab^2\sin(x}{2\left(\sin(x)^2-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sec(x)+b*tan(x))^3,x, algorithm="giac")
```

```
[Out] 1/4*(a^3 - 3*a*b^2 + 2*b^3)*log(sin(x) + 1) - 1/4*(a^3 - 3*a*b^2 - 2*b^3)*l
og(-sin(x) + 1) - 1/2*(b^3*sin(x)^2 + a^3*sin(x) + 3*a*b^2*sin(x) + 3*a^2*b
)/(sin(x)^2 - 1)
```

maple [A] time = 0.08, size = 82, normalized size = 1.09

$$\frac{a^3\sec(x)\tan(x)}{2}+\frac{a^3\ln\left(\sec(x)+\tan(x)\right)}{2}+\frac{3a^2b}{2\cos(x)^2}+\frac{3a\,b^2\left(\sin^3(x)\right)}{2\cos(x)^2}+\frac{3a\,b^2\sin(x)}{2}-\frac{3a\,b^2\ln\left(\sec(x)+\tan(x)\right)}{2}+\frac{b}{\ }$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*sec(x)+b*tan(x))^3,x)
```

```
[Out] 1/2*a^3*sec(x)*tan(x)+1/2*a^3*ln(sec(x)+tan(x))+3/2*a^2*b/cos(x)^2+3/2*a*b^
2*sin(x)^3/cos(x)^2+3/2*a*b^2*sin(x)-3/2*a*b^2*ln(sec(x)+tan(x))+1/2*b^3*ta
n(x)^2+b^3*ln(cos(x))
```

maxima [A] time = 0.35, size = 95, normalized size = 1.27

$$\frac{3}{2}\,a^2b\tan(x)^2-\frac{3}{4}\,ab^2\left(\frac{2\,\sin(x)}{\sin(x)^2-1}+\log\left(\sin(x)+1\right)-\log\left(\sin(x)-1\right)\right)-\frac{1}{4}\,a^3\left(\frac{2\,\sin(x)}{\sin(x)^2-1}-\log\left(\sin(x)+1\right)+\log\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sec(x)+b*tan(x))^3,x, algorithm="maxima")
```

```
[Out] 3/2*a^2*b*tan(x)^2 - 3/4*a*b^2*(2*sin(x)/(sin(x)^2 - 1) + log(sin(x) + 1) -
 log(sin(x) - 1)) - 1/4*a^3*(2*sin(x)/(sin(x)^2 - 1) - log(sin(x) + 1) + lo
g(sin(x) - 1)) - 1/2*b^3*(1/(sin(x)^2 - 1) - log(sin(x)^2 - 1))
```

mupad [B] time = 2.50, size = 126, normalized size = 1.68

$$\frac{\left(a^3+3\,a\,b^2\right)\,\tan\left(\frac{x}{2}\right)^3+\left(6\,a^2\,b+2\,b^3\right)\,\tan\left(\frac{x}{2}\right)^2+\left(a^3+3\,a\,b^2\right)\,\tan\left(\frac{x}{2}\right)}{\tan\left(\frac{x}{2}\right)^4-2\,\tan\left(\frac{x}{2}\right)^2+1}-b^3\,\ln\left(\tan\left(\frac{x}{2}\right)^2+1\right)-\frac{\ln\left(\tan\left(\frac{x}{2}\right)-1\right)}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*tan(x) + a/cos(x))^3,x)
```

```
[Out] (tan(x/2)^2*(6*a^2*b + 2*b^3) + tan(x/2)*(3*a*b^2 + a^3) + tan(x/2)^3*(3*a*
b^2 + a^3))/(tan(x/2)^4 - 2*tan(x/2)^2 + 1) - b^3*log(tan(x/2)^2 + 1) - (lo
g(tan(x/2) - 1)*(a + b)^2*(a - 2*b))/2 + (log(tan(x/2) + 1)*(a - b)^2*(a +
2*b))/2
```

sympy [A] time = 4.94, size = 122, normalized size = 1.63

$$-\frac{a^3\log\left(\sin\left(x\right)-1\right)}{4}+\frac{a^3\log\left(\sin\left(x\right)+1\right)}{4}-\frac{a^3\sin\left(x\right)}{2\sin^2\left(x\right)-2}+\frac{3a^2b\sec^2\left(x\right)}{2}+\frac{3ab^2\log\left(\sin\left(x\right)-1\right)}{4}-\frac{3ab^2\log\left(\sin\left(x\right.\right.}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sec(x)+b*tan(x))**3,x)
```

```
[Out] -a**3*log(sin(x) - 1)/4 + a**3*log(sin(x) + 1)/4 - a**3*sin(x)/(2*sin(x)**2
 - 2) + 3*a**2*b*sec(x)**2/2 + 3*a*b**2*log(sin(x) - 1)/4 - 3*a*b**2*log(si
n(x) + 1)/4 - 3*a*b**2*sin(x)/(2*sin(x)**2 - 2) - b**3*log(sec(x)**2)/2 + b
**3*sec(x)**2/2
```

3.266 $\int (a\sec(x)+b\tan(x))^2\,dx$

Optimal. Leaf size=27

$$ab\cos(x)+\sec(x)(a\sin(x)+b)(a+b\sin(x))+b^2(-x)$$

```
[Out] -b^2*x+a*b*cos(x)+sec(x)*(b+a*sin(x))*(a+b*sin(x))
```

Rubi [A] time = 0.05, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {4391, 2691, 2638}

$$ab\cos(x)+\sec(x)(a\sin(x)+b)(a+b\sin(x))+b^2(-x)$$

Antiderivative was successfully verified.

```
[In] Int[(a*Sec[x] + b*Tan[x])^2,x]
```

```
[Out] -(b^2*x) + a*b*Cos[x] + Sec[x]*(b + a*Sin[x])*(a + b*Sin[x])
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 2691

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Simp[((g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x]
)^(m - 1)*(b + a*Sin[e + f*x]))/(f*g*(p + 1)), x] + Dist[1/(g^2*(p + 1)), I
nt[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^(m - 2)*(b^2*(m - 1) + a^2
*(p + 2) + a*b*(m + p + 1)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f, g},
x] && NeQ[a^2 - b^2, 0] && GtQ[m, 1] && LtQ[p, -1] && (IntegersQ[2*m, 2*p]
|| IntegerQ[m])
```

Rule 4391

```
Int[(u_.)*((b_.)*sec[(c_.) + (d_.)*(x_)]^(n_.) + (a_.)*tan[(c_.) + (d_.)*(x
_)]^(n_.))^(p_), x_Symbol] :> Int[ActivateTrig[u]*Sec[c + d*x]^(n*p)*(b + a
*Sin[c + d*x]^n)^p, x] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p]
```

Rubi steps

$$\begin{aligned}\int (a\sec(x)+b\tan(x))^2\,dx &= \int \sec^2(x)(a+b\sin(x))^2\,dx\\ &= \sec(x)(b+a\sin(x))(a+b\sin(x)) - \int \left(b^2+ab\sin(x)\right)\,dx\\ &= -b^2x+\sec(x)(b+a\sin(x))(a+b\sin(x))-(ab)\int \sin(x)\,dx\\ &= -b^2x+ab\cos(x)+\sec(x)(b+a\sin(x))(a+b\sin(x))\end{aligned}$$

Mathematica [A] time = 0.05, size = 25, normalized size = 0.93

$$\left(a^2+b^2\right)\tan(x)+2ab\sec(x)+b^2\left(-\tan^{-1}(\tan(x))\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Sec[x] + b*Tan[x])^2,x]
[Out] -(b^2*ArcTan[Tan[x]]) + 2*a*b*Sec[x] + (a^2 + b^2)*Tan[x]
```

fricas [A] time = 1.62, size = 29, normalized size = 1.07

$$-\frac{b^2x\cos(x)-2\,ab-\left(a^2+b^2\right)\sin(x)}{\cos(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sec(x)+b*tan(x))^2,x, algorithm="fricas")
[Out] -(b^2*x*cos(x) - 2*a*b - (a^2 + b^2)*sin(x))/cos(x)
```

giac [A] time = 0.16, size = 40, normalized size = 1.48

$$-b^2x-\frac{2\left(a^2\tan\left(\frac{1}{2}\,x\right)+b^2\tan\left(\frac{1}{2}\,x\right)+2\,ab\right)}{\tan\left(\frac{1}{2}\,x\right)^2-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sec(x)+b*tan(x))^2,x, algorithm="giac")
[Out] -b^2*x - 2*(a^2*tan(1/2*x) + b^2*tan(1/2*x) + 2*a*b)/(tan(1/2*x)^2 - 1)
```

maple [A] time = 0.05, size = 26, normalized size = 0.96

$$a^2\tan(x)+\frac{2ab}{\cos(x)}+b^2\left(\tan(x)-x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*sec(x)+b*tan(x))^2,x)
```

```
[Out] a^2*tan(x)+2*a*b/cos(x)+b^2*(tan(x)-x)
```

maxima [A] time = 0.41, size = 26, normalized size = 0.96

$$-b^2(x - \tan(x)) + a^2 \tan(x) + \frac{2\,ab}{\cos(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sec(x)+b*tan(x))^2,x, algorithm="maxima")
```

```
[Out] -b^2*(x - tan(x)) + a^2*tan(x) + 2*a*b/cos(x)
```

mupad [B] time = 2.38, size = 40, normalized size = 1.48

$$-\frac{4\,a\,b + \tan\left(\frac{x}{2}\right)\,\left(2\,a^2 + 2\,b^2\right)}{\tan\left(\frac{x}{2}\right)^2 - 1} - b^2\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*tan(x) + a/cos(x))^2,x)
```

```
[Out] - (4*a*b + tan(x/2)*(2*a^2 + 2*b^2))/(tan(x/2)^2 - 1) - b^2*x
```

sympy [A] time = 1.25, size = 22, normalized size = 0.81

$$a^2 \tan{(x)} + 2ab \sec{(x)} + b^2 \left(-x + \tan{(x)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sec(x)+b*tan(x))**2,x)
```

```
[Out] a**2*tan(x) + 2*a*b*sec(x) + b**2*(-x + tan(x))
```

3.267 $\int (a\sec(x)+b\tan(x))\,dx$

Optimal. Leaf size=12

$$a\tanh^{-1}(\sin(x))-b\log(\cos(x))$$

```
[Out] a*arctanh(sin(x))-b*ln(cos(x))
```

Rubi [A] time = 0.01, antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {3770, 3475}

$$a\tanh^{-1}(\sin(x))-b\log(\cos(x))$$

Antiderivative was successfully verified.

```
[In] Int[a*Sec[x] + b*Tan[x],x]
```

```
[Out] a*ArcTanh[Sin[x]] - b*Log[Cos[x]]
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int (a\sec(x)+b\tan(x))\,dx &= a\int \sec(x)\,dx+b\int \tan(x)\,dx\\ &= a\tanh^{-1}(\sin(x))-b\log(\cos(x))\end{aligned}$$

Mathematica [B] time = 0.01, size = 42, normalized size = 3.50

$$-a\log\left(\cos\left(\frac{x}{2}\right)-\sin\left(\frac{x}{2}\right)\right)+a\log\left(\sin\left(\frac{x}{2}\right)+\cos\left(\frac{x}{2}\right)\right)-b\log(\cos(x))$$

Antiderivative was successfully verified.

```
[In] Integrate[a*Sec[x] + b*Tan[x],x]
```

[Out] -(b*Log[Cos[x]]) - a*Log[Cos[x/2] - Sin[x/2]] + a*Log[Cos[x/2] + Sin[x/2]]

fricas [B] time = 1.49, size = 25, normalized size = 2.08

$$\frac{1}{2}(a-b)\log(\sin(x)+1)-\frac{1}{2}(a+b)\log(-\sin(x)+1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(a*sec(x)+b*tan(x),x, algorithm="fricas")

[Out] 1/2*(a - b)*log(sin(x) + 1) - 1/2*(a + b)*log(-sin(x) + 1)

giac [B] time = 0.13, size = 34, normalized size = 2.83

$$\frac{1}{4}a\left(\log\left(\left|\frac{1}{\sin(x)}+\sin(x)+2\right|\right)-\log\left(\left|\frac{1}{\sin(x)}+\sin(x)-2\right|\right)\right)-b\log(|\cos(x)|)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(a*sec(x)+b*tan(x),x, algorithm="giac")

[Out] 1/4*a*(log(abs(1/sin(x) + sin(x) + 2)) - log(abs(1/sin(x) + sin(x) - 2))) - b*log(abs(cos(x)))

maple [A] time = 0.00, size = 16, normalized size = 1.33

$$a\ln(\sec(x)+\tan(x))-b\ln(\cos(x))$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(a*sec(x)+b*tan(x),x)

[Out] a*ln(sec(x)+tan(x))-b*ln(cos(x))

maxima [A] time = 0.30, size = 14, normalized size = 1.17

$$a\log(\sec(x)+\tan(x))+b\log(\sec(x))$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(a*sec(x)+b*tan(x),x, algorithm="maxima")

[Out] a*log(sec(x) + tan(x)) + b*log(sec(x))

mupad [B] time = 2.49, size = 37, normalized size = 3.08

$$b\ \ln\left(\tan\left(\frac{x}{2}\right)^2+1\right)-\ln\left(\tan\left(\frac{x}{2}\right)-1\right)\ (a+b)+\ln\left(\tan\left(\frac{x}{2}\right)+1\right)\ (a-b)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(b*tan(x) + a/cos(x),x)
```

```
[Out] b*log(tan(x/2)^2 + 1) - log(tan(x/2) - 1)*(a + b) + log(tan(x/2) + 1)*(a -
b)
```

sympy [A] time = 0.09, size = 24, normalized size = 2.00

$$a\left(-\frac{\log\left(\sin\left(x\right)-1\right)}{2}+\frac{\log\left(\sin\left(x\right)+1\right)}{2}\right)-b\log\left(\cos\left(x\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a*sec(x)+b*tan(x),x)
```

```
[Out] a*(-log(sin(x) - 1)/2 + log(sin(x) + 1)/2) - b*log(cos(x))
```

3.268 $\int \frac{1}{a \sec(x)+b \tan(x)} dx$

Optimal. Leaf size=11

$$\frac{\log(a + b\sin(x))}{b}$$

[Out] ln(a+b*sin(x))/b

Rubi [A] time = 0.03, antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {3159, 2668, 31}

$$\frac{\log(a + b\sin(x))}{b}$$

Antiderivative was successfully verified.

[In] Int[(a*Sec[x] + b*Tan[x])^(-1),x]

[Out] Log[a + b*Sin[x]]/b

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2668

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rule 3159

```
Int[((a_.) + (b_.)*sec[(d_.) + (e_.)*(x_)] + (c_.)*tan[(d_.) + (e_.)*(x_)])
^(-1), x_Symbol] :> Int[Cos[d + e*x]/(b + a*Cos[d + e*x] + c*Sin[d + e*x]),
 x] /; FreeQ[{a, b, c, d, e}, x]
```

Rubi steps

$$\int \frac{1}{a\sec(x)+b\tan(x)}\,dx = \int \frac{\cos(x)}{a+b\sin(x)}\,dx$$
$$= \frac{\text{Subst}\left(\int \frac{1}{a+x}\,dx, x, b\sin(x)\right)}{b}$$
$$= \frac{\log(a+b\sin(x))}{b}$$

Mathematica [A] time = 0.01, size = 11, normalized size = 1.00

$$\frac{\log(a+b\sin(x))}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Sec[x] + b*Tan[x])^(-1),x]
[Out] Log[a + b*Sin[x]]/b
```

fricas [A] time = 1.02, size = 11, normalized size = 1.00

$$\frac{\log(b\sin(x)+a)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sec(x)+b*tan(x)),x, algorithm="fricas")
[Out] log(b*sin(x) + a)/b
```

giac [A] time = 0.13, size = 12, normalized size = 1.09

$$\frac{\log(|b\sin(x)+a|)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sec(x)+b*tan(x)),x, algorithm="giac")
[Out] log(abs(b*sin(x) + a))/b
```

maple [A] time = 0.12, size = 12, normalized size = 1.09

$$\frac{\ln(a+b\sin(x))}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*sec(x)+b*tan(x)),x)
```

```
[Out] ln(a+b*sin(x))/b
```

maxima [B] time = 0.50, size = 50, normalized size = 4.55

$$\frac{\log\left(a + \frac{2\,b\sin(x)}{\cos(x)+1} + \frac{a\sin(x)^2}{(\cos(x)+1)^2}\right)}{b} - \frac{\log\left(\frac{\sin(x)^2}{(\cos(x)+1)^2} + 1\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sec(x)+b*tan(x)),x, algorithm="maxima")
```

```
[Out] log(a + 2*b*sin(x)/(cos(x) + 1) + a*sin(x)^2/(cos(x) + 1)^2)/b - log(sin(x)
^2/(cos(x) + 1)^2 + 1)/b
```

mupad [B] time = 3.79, size = 55, normalized size = 5.00

$$\frac{2\,\mathrm{atanh}\left(\frac{b\left(2\,a^3\,\sin(x)+\frac{5\,a^2\,b}{2}-b^3-\frac{a^2\,b\,\cos(2\,x)}{2}\right)}{\left(2\,a^2+\sin(x)\,a\,b-b^2\right)^2}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*tan(x) + a/cos(x)),x)
```

```
[Out] (2*atanh((b*(2*a^3*sin(x) + (5*a^2*b)/2 - b^3 - (a^2*b*cos(2*x))/2))/(2*a^2
 - b^2 + a*b*sin(x))^2))/b
```

sympy [A] time = 0.42, size = 32, normalized size = 2.91

$$\begin{cases} \frac{\log\left(\frac{a\sec(x)}{b}+\tan(x)\right)}{b} - \frac{\log\left(\tan^2(x)+1\right)}{2b} & \text{for}\ b \neq 0 \\ \frac{\tan(x)}{a\sec(x)} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sec(x)+b*tan(x)),x)
```

```
[Out] Piecewise((log(a*sec(x)/b + tan(x))/b - log(tan(x)**2 + 1)/(2*b), Ne(b, 0))
, (tan(x)/(a*sec(x)), True))
```

3.269 $\int \frac{1}{(a\sec(x)+b\tan(x))^2}\,dx$

Optimal. Leaf size=66

$$\frac{2a\tan^{-1}\left(\frac{a\tan\left(\frac{x}{2}\right)+b}{\sqrt{a^2-b^2}}\right)}{b^2\sqrt{a^2-b^2}}-\frac{\cos(x)}{b(a+b\sin(x))}-\frac{x}{b^2}$$

```
[Out] -x/b^2-cos(x)/b/(a+b*sin(x))+2*a*arctan((b+a*tan(1/2*x))/(a^2-b^2)^(1/2))/b
^2/(a^2-b^2)^(1/2)
```

Rubi [A] time = 0.13, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.546, Rules used = {4391, 2693, 2735, 2660, 618, 204}

$$\frac{2a\tan^{-1}\left(\frac{a\tan\left(\frac{x}{2}\right)+b}{\sqrt{a^2-b^2}}\right)}{b^2\sqrt{a^2-b^2}}-\frac{\cos(x)}{b(a+b\sin(x))}-\frac{x}{b^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Sec[x] + b*Tan[x])^(-2),x]
```

```
[Out] -(x/b^2) + (2*a*ArcTan[(b + a*Tan[x/2])/Sqrt[a^2 - b^2]])/(b^2*Sqrt[a^2 - b
^2]) - Cos[x]/(b*(a + b*Sin[x]))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2693

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(g*(g*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x
])^(m + 1))/(b*f*(m + 1)), x] + Dist[(g^2*(p - 1))/(b*(m + 1)), Int[(g*Cos[
e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 1)*Sin[e + f*x], x], x] /; Free
Q[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[p, 1] && In
tegersQ[2*m, 2*p]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 4391

```
Int[(u_.)*((b_.)*sec[(c_.) + (d_.)*(x_)]^(n_.) + (a_.)*tan[(c_.) + (d_.)*(x
_)]^(n_.))^(p_), x_Symbol] :> Int[ActivateTrig[u]*Sec[c + d*x]^(n*p)*(b + a
*Sin[c + d*x]^n)^p, x] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{(a\sec(x)+b\tan(x))^2}\,dx &= \int \frac{\cos^2(x)}{(a+b\sin(x))^2}\,dx\\
&= -\frac{\cos(x)}{b(a+b\sin(x))}-\frac{\int \frac{\sin(x)}{a+b\sin(x)}\,dx}{b}\\
&= -\frac{x}{b^2}-\frac{\cos(x)}{b(a+b\sin(x))}+\frac{a\int \frac{1}{a+b\sin(x)}\,dx}{b^2}\\
&= -\frac{x}{b^2}-\frac{\cos(x)}{b(a+b\sin(x))}+\frac{(2a)\,\mathrm{Subst}\left(\int \frac{1}{a+2bx+ax^2}\,dx,x,\tan\left(\frac{x}{2}\right)\right)}{b^2}\\
&= -\frac{x}{b^2}-\frac{\cos(x)}{b(a+b\sin(x))}-\frac{(4a)\,\mathrm{Subst}\left(\int \frac{1}{-4\left(a^2-b^2\right)-x^2}\,dx,x,2b+2a\tan\left(\frac{x}{2}\right)\right)}{b^2}\\
&= -\frac{x}{b^2}+\frac{2a\tan^{-1}\left(\frac{b+a\tan\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2}}\right)}{b^2\sqrt{a^2-b^2}}-\frac{\cos(x)}{b(a+b\sin(x))}
\end{aligned}
$$

Mathematica [B] time = 2.02, size = 344, normalized size = 5.21

$$\frac{(\sin(x)+1)\cos(x)\left(2a(a-b)\sqrt{1-\sin(x)}\,(a+b\sin(x))\tanh^{-1}\left(\frac{\sqrt{a-b}\sqrt{-\frac{b(\sin(x)+1)}{a-b}}}{\sqrt{a+b}\sqrt{-\frac{b(\sin(x)-1)}{a+b}}}\right)+\sqrt{a+b}\left(-(b-a)\sqrt{\frac{b-b\sin(x)}{a+b}}\right.\right.}{(a-b)^{5/2}(a+b)^{3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Sec[x] + b*Tan[x])^(-2),x]
```

```
[Out] (Cos[x]*(1 + Sin[x])*(2*a*(a - b)*ArcTanh[(Sqrt[a - b]*Sqrt[-((b*(1 + Sin[x
]))/(a - b))])/(Sqrt[a + b]*Sqrt[-((b*(-1 + Sin[x]))/(a + b))])]*Sqrt[1 - S
in[x]]*(a + b*Sin[x]) + Sqrt[a + b]*(-2*a*Sqrt[a - b]*ArcTanh[Sqrt[(b*(1 +
Sin[x]))/(-a + b)]/Sqrt[(b - b*Sin[x])/(a + b)]]*Sqrt[1 - Sin[x]]*(a + b*Si
n[x]) - (-a + b)*Sqrt[(b - b*Sin[x])/(a + b)]*(Sqrt[a - b]*(a + b)*Sqrt[1 -
 Sin[x]]*Sqrt[-((b*(1 + Sin[x]))/(a - b))] + 2*Sqrt[b]*ArcSinh[(Sqrt[a - b]
*Sqrt[-((b*(1 + Sin[x]))/(a - b))])/(Sqrt[2]*Sqrt[b])]*(a + b*Sin[x])))))/(
(a - b)^(5/2)*(a + b)^(3/2)*Sqrt[1 - Sin[x]]*(-((b*(1 + Sin[x]))/(a - b)))^
(3/2)*Sqrt[(b - b*Sin[x])/(a + b)]*(a + b*Sin[x]))
```

fricas [B] time = 1.06, size = 308, normalized size = 4.67

$$\left[-\frac{2\left(a^{2}b-b^{3}\right)x\sin(x)+\left(ab\sin(x)+a^{2}\right)\sqrt{-a^{2}+b^{2}}\,\log\left(\frac{\left(2a^{2}-b^{2}\right)\cos(x)^{2}-2ab\sin(x)-a^{2}-b^{2}+2\left(a\cos(x)\sin(x)+b\cos(x)\right)\sqrt{-a^{2}+\cdots}}{b^{2}\cos(x)^{2}-2ab\sin(x)-a^{2}-b^{2}}\right.}{2\left(a^{3}b^{2}-ab^{4}+\left(a^{2}b^{3}-b^{5}\right)\sin(x)\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sec(x)+b*tan(x))^2,x, algorithm="fricas")
```

```
[Out] [-1/2*(2*(a^2*b - b^3)*x*sin(x) + (a*b*sin(x) + a^2)*sqrt(-a^2 + b^2)*log((
(2*a^2 - b^2)*cos(x)^2 - 2*a*b*sin(x) - a^2 - b^2 + 2*(a*cos(x)*sin(x) + b*
cos(x))*sqrt(-a^2 + b^2))/(b^2*cos(x)^2 - 2*a*b*sin(x) - a^2 - b^2)) + 2*(a
^3 - a*b^2)*x + 2*(a^2*b - b^3)*cos(x))/(a^3*b^2 - a*b^4 + (a^2*b^3 - b^5)*
sin(x)), -((a^2*b - b^3)*x*sin(x) + (a*b*sin(x) + a^2)*sqrt(a^2 - b^2)*arct
an(-(a*sin(x) + b)/(sqrt(a^2 - b^2)*cos(x))) + (a^3 - a*b^2)*x + (a^2*b - b
^3)*cos(x))/(a^3*b^2 - a*b^4 + (a^2*b^3 - b^5)*sin(x))]
```

giac [A] time = 0.17, size = 94, normalized size = 1.42

$$\frac{2\left(\pi\left\lfloor\frac{x}{2\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(a)+\arctan\left(\frac{a\tan\left(\frac{1}{2}x\right)+b}{\sqrt{a^{2}-b^{2}}}\right)\right)a}{\sqrt{a^{2}-b^{2}}\,b^{2}}-\frac{x}{b^{2}}-\frac{2\left(b\tan\left(\frac{1}{2}x\right)+a\right)}{\left(a\tan\left(\frac{1}{2}x\right)^{2}+2b\tan\left(\frac{1}{2}x\right)+a\right)ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sec(x)+b*tan(x))^2,x, algorithm="giac")
```

```
[Out] 2*(pi*floor(1/2*x/pi + 1/2)*sgn(a) + arctan((a*tan(1/2*x) + b)/sqrt(a^2 - b
^2)))*a/(sqrt(a^2 - b^2)*b^2) - x/b^2 - 2*(b*tan(1/2*x) + a)/((a*tan(1/2*x)
^2 + 2*b*tan(1/2*x) + a)*a*b)
```

maple [A] time = 0.16, size = 106, normalized size = 1.61

$$-\frac{2\tan\left(\frac{x}{2}\right)}{\left(a\left(\tan^2\left(\frac{x}{2}\right)\right)+2b\tan\left(\frac{x}{2}\right)+a\right)a}-\frac{2}{b\left(a\left(\tan^2\left(\frac{x}{2}\right)\right)+2b\tan\left(\frac{x}{2}\right)+a\right)}+\frac{2a\arctan\left(\frac{2a\tan\left(\frac{x}{2}\right)+2b}{2\sqrt{a^2-b^2}}\right)}{b^2\sqrt{a^2-b^2}}-\frac{2\arctan\left(\tan\left(\frac{x}{2}\right.\right.}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*sec(x)+b*tan(x))^2,x)
```

```
[Out] -2/(a*tan(1/2*x)^2+2*b*tan(1/2*x)+a)/a*tan(1/2*x)-2/b/(a*tan(1/2*x)^2+2*b*t
an(1/2*x)+a)+2/b^2*a/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*x)+2*b)/(a^2-b
^2)^(1/2))-2/b^2*arctan(tan(1/2*x))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sec(x)+b*tan(x))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more details)Is 4*b^2-4*a^2 positive or negative?
```

mupad [B] time = 2.81, size = 604, normalized size = 9.15

$$-\frac{x}{b^2}-\frac{\frac{2\tan\left(\frac{x}{2}\right)}{a}+\frac{2}{b}}{a\tan\left(\frac{x}{2}\right)^2+2\,b\tan\left(\frac{x}{2}\right)+a}-\frac{a\,\mathrm{atan}\left(\frac{\frac{a\left(\frac{32\,a^2}{b}+\frac{32\tan\left(\frac{x}{2}\right)\left(2\,a\,b^3-2\,a^3\,b\right)}{b^3}+\frac{a\left(32\,a\,b^2+64\,a^2\,b\tan\left(\frac{x}{2}\right)+\frac{a\left(32\,a^2\,b^3+\frac{32\tan\left(\frac{x}{2}\right)\left(3\,a\,b^7-2\,a^3\,b \cdots\right.}{b^3}\cdots\right)}{b^2\,\sqrt{b^2-a^2}}\cdots\right)}{b^2\,\sqrt{b^2-a^2}}\cdots\right)}{b^2\,\sqrt{b^2-a^2}}\cdots}{\frac{128\,a^2\tan\left(\frac{x}{2}\right)}{b^3}+\frac{a\left(\frac{32\,a^2}{b}+\frac{32\tan\left(\frac{x}{2}\right)\left(2\,a\,b^3-2\,a^3\,b\right)}{b^3}+\frac{a\left(32\,a\,b^2+64\,a^2\,b\tan\left(\frac{x}{2}\right)+\frac{a\left(32\,a^2\,b^3+\frac{32\tan\left(\frac{x}{2}\right)\left(3 \cdots\right.}{b^3 \cdots}\cdots\right)}{b^2\,\sqrt{b^2-a^2}}\cdots\right)}{b^2\,\sqrt{b^2-a^2}}\cdots\right)}{b^2\,\sqrt{b^2-a^2}}\cdots}\right)\cdots}{b^2 \cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*tan(x) + a/cos(x))^2,x)
```

```
[Out] - x/b^2 - ((2*tan(x/2))/a + 2/b)/(a + 2*b*tan(x/2) + a*tan(x/2)^2) - (a*ata
n(((a*((32*a^2)/b + (32*tan(x/2)*(2*a*b^3 - 2*a^3*b))/b^3 + (a*(32*a*b^2 +
64*a^2*b*tan(x/2) + (a*(32*a^2*b^3 + (32*tan(x/2)*(3*a*b^7 - 2*a^3*b^5))/b^
3))/(b^2*(b^2 - a^2)^(1/2))))/(b^2*(b^2 - a^2)^(1/2)))*1i)/(b^2*(b^2 - a^2)
^(1/2)) + (a*((32*a^2)/b + (32*tan(x/2)*(2*a*b^3 - 2*a^3*b))/b^3 - (a*(32*a
*b^2 + 64*a^2*b*tan(x/2) - (a*(32*a^2*b^3 + (32*tan(x/2)*(3*a*b^7 - 2*a^3*b
^5))/b^3))/(b^2*(b^2 - a^2)^(1/2))))/(b^2*(b^2 - a^2)^(1/2)))*1i)/(b^2*(b^2
 - a^2)^(1/2)))/((128*a^2*tan(x/2))/b^3 + (a*((32*a^2)/b + (32*tan(x/2)*(2*
a*b^3 - 2*a^3*b))/b^3 + (a*(32*a*b^2 + 64*a^2*b*tan(x/2) + (a*(32*a^2*b^3 +
 (32*tan(x/2)*(3*a*b^7 - 2*a^3*b^5))/b^3))/(b^2*(b^2 - a^2)^(1/2))))/(b^2*(
b^2 - a^2)^(1/2))))/(b^2*(b^2 - a^2)^(1/2)) - (a*((32*a^2)/b + (32*tan(x/2)
*(2*a*b^3 - 2*a^3*b))/b^3 - (a*(32*a*b^2 + 64*a^2*b*tan(x/2) - (a*(32*a^2*b
^3 + (32*tan(x/2)*(3*a*b^7 - 2*a^3*b^5))/b^3))/(b^2*(b^2 - a^2)^(1/2))))/(b
^2*(b^2 - a^2)^(1/2))))/(b^2*(b^2 - a^2)^(1/2))))*2i)/(b^2*(b^2 - a^2)^(1/2
))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a\sec(x) + b\tan(x))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sec(x)+b*tan(x))**2,x)
```

```
[Out] Integral((a*sec(x) + b*tan(x))**(-2), x)
```

3.270 $\int \frac{1}{(a\sec(x)+b\tan(x))^3}\,dx$

Optimal. Leaf size=51

$$\frac{a^2-b^2}{2b^3(a+b\sin(x))^2}-\frac{2a}{b^3(a+b\sin(x))}-\frac{\log(a+b\sin(x))}{b^3}$$

```
[Out] -ln(a+b*sin(x))/b^3+1/2*(a^2-b^2)/b^3/(a+b*sin(x))^2-2*a/b^3/(a+b*sin(x))
```

Rubi [A] time = 0.08, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {4391, 2668, 697}

$$\frac{a^2-b^2}{2b^3(a+b\sin(x))^2}-\frac{2a}{b^3(a+b\sin(x))}-\frac{\log(a+b\sin(x))}{b^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Sec[x] + b*Tan[x])^(-3),x]
```

```
[Out] -(Log[a + b*Sin[x]]/b^3) + (a^2 - b^2)/(2*b^3*(a + b*Sin[x])^2) - (2*a)/(b^
3*(a + b*Sin[x]))
```

Rule 697

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m},
 x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0]
```

Rule 2668

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rule 4391

```
Int[(u_.)*((b_.)*sec[(c_.) + (d_.)*(x_)]^(n_.) + (a_.)*tan[(c_.) + (d_.)*(x
_)]^(n_.))^(p_), x_Symbol] :> Int[ActivateTrig[u]*Sec[c + d*x]^(n*p)*(b + a
*Sin[c + d*x]^n)^p, x] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p]
```

Rubi steps

$$\int \frac{1}{(a\sec(x)+b\tan(x))^3}\,dx = \int \frac{\cos^3(x)}{(a+b\sin(x))^3}\,dx$$
$$= \frac{\text{Subst}\left(\int \frac{b^2-x^2}{(a+x)^3}\,dx, x, b\sin(x)\right)}{b^3}$$
$$= \frac{\text{Subst}\left(\int \left(\frac{1}{-a-x}+\frac{-a^2+b^2}{(a+x)^3}+\frac{2a}{(a+x)^2}\right)dx, x, b\sin(x)\right)}{b^3}$$
$$= -\frac{\log(a+b\sin(x))}{b^3}+\frac{a^2-b^2}{2b^3(a+b\sin(x))^2}-\frac{2a}{b^3(a+b\sin(x))}$$

Mathematica [A] time = 0.17, size = 40, normalized size = 0.78

$$-\frac{\frac{3a^2+4ab\sin(x)+b^2}{2(a+b\sin(x))^2}+\log(a+b\sin(x))}{b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Sec[x] + b*Tan[x])^(-3),x]
```

```
[Out] -((Log[a + b*Sin[x]] + (3*a^2 + b^2 + 4*a*b*Sin[x])/(2*(a + b*Sin[x])^2))/b
^3)
```

fricas [A] time = 0.95, size = 83, normalized size = 1.63

$$\frac{4\,ab\sin(x)+3\,a^2+b^2-2\left(b^2\cos(x)^2-2\,ab\sin(x)-a^2-b^2\right)\log\left(b\sin(x)+a\right)}{2\left(b^5\cos(x)^2-2\,ab^4\sin(x)-a^2b^3-b^5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sec(x)+b*tan(x))^3,x, algorithm="fricas")
```

```
[Out] 1/2*(4*a*b*sin(x) + 3*a^2 + b^2 - 2*(b^2*cos(x)^2 - 2*a*b*sin(x) - a^2 - b^
2)*log(b*sin(x) + a))/(b^5*cos(x)^2 - 2*a*b^4*sin(x) - a^2*b^3 - b^5)
```

giac [A] time = 0.14, size = 43, normalized size = 0.84

$$-\frac{\log\left(|b\sin(x)+a|\right)}{b^3}+\frac{3\,b\sin(x)^2+2\,a\sin(x)-b}{2\left(b\sin(x)+a\right)^2b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sec(x)+b*tan(x))^3,x, algorithm="giac")
```

```
[Out] -log(abs(b*sin(x) + a))/b^3 + 1/2*(3*b*sin(x)^2 + 2*a*sin(x) - b)/((b*sin(x
) + a)^2*b^2)
```

maple [A] time = 0.17, size = 57, normalized size = 1.12

$$-\frac{2a}{b^3\left(a+b\sin(x)\right)}-\frac{\ln\left(a+b\sin(x)\right)}{b^3}+\frac{a^2}{2b^3\left(a+b\sin(x)\right)^2}-\frac{1}{2b\left(a+b\sin(x)\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*sec(x)+b*tan(x))^3,x)
```

```
[Out] -2*a/b^3/(a+b*sin(x))-ln(a+b*sin(x))/b^3+1/2/b^3/(a+b*sin(x))^2*a^2-1/2/b/(
a+b*sin(x))^2
```

maxima [B] time = 0.51, size = 201, normalized size = 3.94

$$\frac{2\left(\frac{\left(a^3+ab^2\right)\sin(x)}{\cos(x)+1}+\frac{\left(3\,a^2b+b^3\right)\sin(x)^2}{\left(\cos(x)+1\right)^2}+\frac{\left(a^3+ab^2\right)\sin(x)^3}{\left(\cos(x)+1\right)^3}\right)}{a^4b^2+\frac{4\,a^3b^3\sin(x)}{\cos(x)+1}+\frac{4\,a^3b^3\sin(x)^3}{\left(\cos(x)+1\right)^3}+\frac{a^4b^2\sin(x)^4}{\left(\cos(x)+1\right)^4}+\frac{2\left(a^4b^2+2\,a^2b^4\right)\sin(x)^2}{\left(\cos(x)+1\right)^2}}-\frac{\log\left(a+\frac{2\,b\sin(x)}{\cos(x)+1}+\frac{a\sin(x)^2}{\left(\cos(x)+1\right)^2}\right)}{b^3}+\frac{\log\left(\frac{\sin(x)^2}{\left(\cos(x)+1\right)^2}\right.}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sec(x)+b*tan(x))^3,x, algorithm="maxima")
```

```
[Out] 2*((a^3 + a*b^2)*sin(x)/(cos(x) + 1) + (3*a^2*b + b^3)*sin(x)^2/(cos(x) + 1
)^2 + (a^3 + a*b^2)*sin(x)^3/(cos(x) + 1)^3)/(a^4*b^2 + 4*a^3*b^3*sin(x)/(c
os(x) + 1) + 4*a^3*b^3*sin(x)^3/(cos(x) + 1)^3 + a^4*b^2*sin(x)^4/(cos(x) +
 1)^4 + 2*(a^4*b^2 + 2*a^2*b^4)*sin(x)^2/(cos(x) + 1)^2) - log(a + 2*b*sin(
x)/(cos(x) + 1) + a*sin(x)^2/(cos(x) + 1)^2)/b^3 + log(sin(x)^2/(cos(x) + 1
)^2 + 1)/b^3
```

mupad [B] time = 2.70, size = 106, normalized size = 2.08

$$\frac{2\,a^3\,b\,\sin(x)+3\,a^2\,b^2\,\sin(x)^2+2\,a\,b^3\,\sin(x)+b^4\,\sin(x)^2}{2\,a^4\,b^3+4\,a^3\,b^4\,\sin(x)+2\,a^2\,b^5\,\sin(x)^2}-\frac{2\,\mathrm{atanh}\left(\frac{b^2+a\,\sin(x)\,b}{2\,a^2+\sin(x)\,a\,b-b^2}\right)}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*tan(x) + a/cos(x))^3,x)
```

```
[Out] (b^4*sin(x)^2 + 3*a^2*b^2*sin(x)^2 + 2*a*b^3*sin(x) + 2*a^3*b*sin(x))/(2*a^
4*b^3 + 2*a^2*b^5*sin(x)^2 + 4*a^3*b^4*sin(x)) - (2*atanh((b^2 + a*b*sin(x)
)/(2*a^2 - b^2 + a*b*sin(x))))/b^3
```

sympy [A] time = 2.76, size = 503, normalized size = 9.86

$$\begin{cases} -\frac{2a^2\log\left(\frac{a\sec(x)}{b}+\tan(x)\right)\sec^2(x)}{2a^2b^3\sec^2(x)+4ab^4\tan(x)\sec(x)+2b^5\tan^2(x)} + \frac{a^2\log\left(\tan^2(x)+1\right)\sec^2(x)}{2a^2b^3\sec^2(x)+4ab^4\tan(x)\sec(x)+2b^5\tan^2(x)} - \frac{a^2\sec^2(x)}{2a^2b^3\sec^2(x)+4ab^4\tan(x)\sec(x)+2b^5\tan^2 \\ \frac{\frac{2\tan^3(x)}{3\sec^3(x)}+\frac{\tan(x)}{\sec^3(x)}}{a^3} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sec(x)+b*tan(x))**3,x)
```

```
[Out] Piecewise((-2*a**2*log(a*sec(x)/b + tan(x))*sec(x)**2/(2*a**2*b**3*sec(x)**
2 + 4*a*b**4*tan(x)*sec(x) + 2*b**5*tan(x)**2) + a**2*log(tan(x)**2 + 1)*se
c(x)**2/(2*a**2*b**3*sec(x)**2 + 4*a*b**4*tan(x)*sec(x) + 2*b**5*tan(x)**2)
 - a**2*sec(x)**2/(2*a**2*b**3*sec(x)**2 + 4*a*b**4*tan(x)*sec(x) + 2*b**5*
tan(x)**2) - 4*a*b*log(a*sec(x)/b + tan(x))*tan(x)*sec(x)/(2*a**2*b**3*sec(
x)**2 + 4*a*b**4*tan(x)*sec(x) + 2*b**5*tan(x)**2) + 2*a*b*log(tan(x)**2 +
1)*tan(x)*sec(x)/(2*a**2*b**3*sec(x)**2 + 4*a*b**4*tan(x)*sec(x) + 2*b**5*t
an(x)**2) - 2*b**2*log(a*sec(x)/b + tan(x))*tan(x)**2/(2*a**2*b**3*sec(x)**
2 + 4*a*b**4*tan(x)*sec(x) + 2*b**5*tan(x)**2) + b**2*log(tan(x)**2 + 1)*ta
n(x)**2/(2*a**2*b**3*sec(x)**2 + 4*a*b**4*tan(x)*sec(x) + 2*b**5*tan(x)**2)
 + b**2*tan(x)**2/(2*a**2*b**3*sec(x)**2 + 4*a*b**4*tan(x)*sec(x) + 2*b**5*
tan(x)**2) - b**2/(2*a**2*b**3*sec(x)**2 + 4*a*b**4*tan(x)*sec(x) + 2*b**5*
tan(x)**2), Ne(b, 0)), ((2*tan(x)**3/(3*sec(x)**3) + tan(x)/sec(x)**3)/a**3
, True))
```

3.271 $\int \frac{1}{(a\sec(x)+b\tan(x))^4}\,dx$

Optimal. Leaf size=156

$$\frac{a\cos^3(x)}{2b\left(a^2-b^2\right)(a+b\sin(x))^2}-\frac{a\left(2a^2-3b^2\right)\tan^{-1}\left(\frac{a\tan\left(\frac{x}{2}\right)+b}{\sqrt{a^2-b^2}}\right)}{b^4\left(a^2-b^2\right)^{3/2}}+\frac{\cos(x)\left(2\left(a^2-b^2\right)+ab\sin(x)\right)}{2b^3\left(a^2-b^2\right)(a+b\sin(x))}-\frac{\cos^3(x)}{3b(a+b\sin(x))^3}$$

```
[Out] x/b^4-a*(2*a^2-3*b^2)*arctan((b+a*tan(1/2*x))/(a^2-b^2)^(1/2))/b^4/(a^2-b^2
)^(3/2)-1/3*cos(x)^3/b/(a+b*sin(x))^3+1/2*a*cos(x)^3/b/(a^2-b^2)/(a+b*sin(x
))^2+1/2*cos(x)*(2*a^2-2*b^2+a*b*sin(x))/b^3/(a^2-b^2)/(a+b*sin(x))
```

Rubi [A] time = 0.34, antiderivative size = 156, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.727, Rules used = {4391, 2693, 2864, 2863, 2735, 2660, 618, 204}

$$-\frac{a\left(2a^2-3b^2\right)\tan^{-1}\left(\frac{a\tan\left(\frac{x}{2}\right)+b}{\sqrt{a^2-b^2}}\right)}{b^4\left(a^2-b^2\right)^{3/2}}+\frac{a\cos^3(x)}{2b\left(a^2-b^2\right)(a+b\sin(x))^2}+\frac{\cos(x)\left(2\left(a^2-b^2\right)+ab\sin(x)\right)}{2b^3\left(a^2-b^2\right)(a+b\sin(x))}-\frac{\cos^3(x)}{3b(a+b\sin(x))^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Sec[x] + b*Tan[x])^(-4),x]
```

```
[Out] x/b^4 - (a*(2*a^2 - 3*b^2)*ArcTan[(b + a*Tan[x/2])/Sqrt[a^2 - b^2]])/(b^4*(
a^2 - b^2)^(3/2)) - Cos[x]^3/(3*b*(a + b*Sin[x])^3) + (a*Cos[x]^3)/(2*b*(a^
2 - b^2)*(a + b*Sin[x])^2) + (Cos[x]*(2*(a^2 - b^2) + a*b*Sin[x]))/(2*b^3*(
a^2 - b^2)*(a + b*Sin[x]))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2693

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(g*(g*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x
])^(m + 1))/(b*f*(m + 1)), x] + Dist[(g^2*(p - 1))/(b*(m + 1)), Int[(g*Cos[
e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 1)*Sin[e + f*x], x], x] /; Free
Q[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[p, 1] && In
tegersQ[2*m, 2*p]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2863

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(g*(g*
Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x])^(m + 1)*(b*c*(m + p + 1) - a*d*p
 + b*d*(m + 1)*Sin[e + f*x]))/(b^2*f*(m + 1)*(m + p + 1)), x] + Dist[(g^2*(
p - 1))/(b^2*(m + 1)*(m + p + 1)), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[
e + f*x])^(m + 1)*Simp[b*d*(m + 1) + (b*c*(m + p + 1) - a*d*p)*Sin[e + f*x]
, x], x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 - b^2, 0] && LtQ
[m, -1] && GtQ[p, 1] && NeQ[m + p + 1, 0] && IntegerQ[2*m]
```

Rule 2864

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> -Simp[((b*c
 - a*d)*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^(m + 1))/(f*g*(a^2 -
b^2)*(m + 1)), x] + Dist[1/((a^2 - b^2)*(m + 1)), Int[(g*Cos[e + f*x])^p*(a
 + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m + p +
2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[a^
2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rule 4391

```
Int[(u_.)*((b_.)*sec[(c_.) + (d_.)*(x_)]^(n_.) + (a_.)*tan[(c_.) + (d_.)*(x
_)]^(n_.))^(p_), x_Symbol] :> Int[ActivateTrig[u]*Sec[c + d*x]^(n*p)*(b + a
```

```
*Sin[c + d*x]^n)^p, x] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p]
```

Rubi steps

$$\int \frac{1}{(a\sec(x)+b\tan(x))^4}\,dx = \int \frac{\cos^4(x)}{(a+b\sin(x))^4}\,dx$$

$$= -\frac{\cos^3(x)}{3b(a+b\sin(x))^3} - \frac{\int \frac{\cos^2(x)\sin(x)}{(a+b\sin(x))^3}\,dx}{b}$$

$$= -\frac{\cos^3(x)}{3b(a+b\sin(x))^3} + \frac{a\cos^3(x)}{2b\left(a^2-b^2\right)(a+b\sin(x))^2} + \frac{\int \frac{\cos^2(x)(2b+a\sin(x))}{(a+b\sin(x))^2}\,dx}{2b\left(a^2-b^2\right)}$$

$$= -\frac{\cos^3(x)}{3b(a+b\sin(x))^3} + \frac{a\cos^3(x)}{2b\left(a^2-b^2\right)(a+b\sin(x))^2} + \frac{\cos(x)\left(2\left(a^2-b^2\right)+ab\sin(x\right.}{2b^3\left(a^2-b^2\right)(a+b\sin(x))}$$

$$= \frac{x}{b^4} - \frac{\cos^3(x)}{3b(a+b\sin(x))^3} + \frac{a\cos^3(x)}{2b\left(a^2-b^2\right)(a+b\sin(x))^2} + \frac{\cos(x)\left(2\left(a^2-b^2\right)+ab\,\mathrm{s}\right.}{2b^3\left(a^2-b^2\right)(a+b\sin}$$

$$= \frac{x}{b^4} - \frac{\cos^3(x)}{3b(a+b\sin(x))^3} + \frac{a\cos^3(x)}{2b\left(a^2-b^2\right)(a+b\sin(x))^2} + \frac{\cos(x)\left(2\left(a^2-b^2\right)+ab\,\mathrm{s}\right.}{2b^3\left(a^2-b^2\right)(a+b\sin}$$

$$= \frac{x}{b^4} - \frac{\cos^3(x)}{3b(a+b\sin(x))^3} + \frac{a\cos^3(x)}{2b\left(a^2-b^2\right)(a+b\sin(x))^2} + \frac{\cos(x)\left(2\left(a^2-b^2\right)+ab\,\mathrm{s}\right.}{2b^3\left(a^2-b^2\right)(a+b\sin}$$

$$= \frac{x}{b^4} - \frac{a\left(2a^2-3b^2\right)\tan^{-1}\left(\frac{b+a\tan\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2}}\right)}{b^4\left(a^2-b^2\right)^{3/2}} - \frac{\cos^3(x)}{3b(a+b\sin(x))^3} + \frac{a\cos^3(x)}{2b\left(a^2-b^2\right)(a+b\sin}$$

Mathematica [B] time = 6.36, size = 2661, normalized size = 17.06

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a*Sec[x] + b*Tan[x])^(-4),x]
```

```
[Out] (Sec[x]*(a + b*Sin[x])^4*(-1/3*(b*(-(b/(a - b)) - (b*Sin[x])/(a - b))^(5/2)
*(b/(a + b) - (b*Sin[x])/(a + b))^(5/2))/(((a*b)/(a - b) - b^2/(a - b))*((a
*b)/(a + b) + b^2/(a + b))*(a + b*Sin[x])^3) - ((a*b^3*(-(b/(a - b)) - (b*S
```

```
in[x])/(a - b))^(5/2)*(b/(a + b) - (b*Sin[x])/(a + b))^(5/2))/(2*(a^2 - b^2
)*((a*b)/(a - b) - b^2/(a - b))*((a*b)/(a + b) + b^2/(a + b))*(a + b*Sin[x]
)^2) - (-((((-3*a^2*b^5)/((a - b)^2*(a + b)^2) + (2*b^5*(3*a^2 - 2*b^2))/((
a - b)^2*(a + b)^2))*(-(b/(a - b)) - (b*Sin[x])/(a - b))^(5/2)*(b/(a + b) -
 (b*Sin[x])/(a + b))^(5/2))/(((a*b)/(a - b) - b^2/(a - b))*((a*b)/(a + b) +
 b^2/(a + b))*(a + b*Sin[x]))) - ((16*Sqrt[2]*b^6*(3*a^2 - 4*b^2)*(-(b/(a -
 b)) - (b*Sin[x])/(a - b))^(5/2)*Sqrt[b/(a + b) - (b*Sin[x])/(a + b)]*(1 +
((a - b)*(-(b/(a - b)) - (b*Sin[x])/(a - b)))/(2*b))^(5/2)*((5*(1/(2*(1 + (
(a - b)*(-(b/(a - b)) - (b*Sin[x])/(a - b)))/(2*b))^2) + (1 + ((a - b)*(-(b
/(a - b)) - (b*Sin[x])/(a - b)))/(2*b))^(-1)))/8 - (15*b^3*(((a - b)*(-(b/(
a - b)) - (b*Sin[x])/(a - b)))/b - ((a - b)^2*(-(b/(a - b)) - (b*Sin[x])/(a
 - b))^2)/(3*b^2) - (Sqrt[2]*Sqrt[a - b]*ArcSinh[(Sqrt[a - b]*Sqrt[-(b/(a -
 b)) - (b*Sin[x])/(a - b)])/(Sqrt[2]*Sqrt[b])]*Sqrt[-(b/(a - b)) - (b*Sin[x
])/(a - b)])/(Sqrt[b]*Sqrt[1 + ((a - b)*(-(b/(a - b)) - (b*Sin[x])/(a - b))
)/(2*b)])))/(32*(a - b)^3*(-(b/(a - b)) - (b*Sin[x])/(a - b))^3*(1 + ((a -
b)*(-(b/(a - b)) - (b*Sin[x])/(a - b)))/(2*b))^2)))/(5*(a - b)^2*(a + b)^4*
Sqrt[((a + b)*(b/(a + b) - (b*Sin[x])/(a + b)))/b]) + ((-((a*b^7*(6*a^2 - 7
*b^2))/((a - b)^3*(a + b)^3)) + (4*a*b^7*(3*a^2 - 4*b^2))/((a - b)^3*(a + b
)^3))*((4*Sqrt[2]*(-(b/(a - b)) - (b*Sin[x])/(a - b))^(3/2)*Sqrt[b/(a + b)
- (b*Sin[x])/(a + b)]*(1 + ((a - b)*(-(b/(a - b)) - (b*Sin[x])/(a - b)))/(2
*b))^(5/2)*((3/(4*(1 + ((a - b)*(-(b/(a - b)) - (b*Sin[x])/(a - b)))/(2*b))
^2) + (1 + ((a - b)*(-(b/(a - b)) - (b*Sin[x])/(a - b)))/(2*b))^(-1))/2 + (
3*b^2*(((a - b)*(-(b/(a - b)) - (b*Sin[x])/(a - b)))/b - (Sqrt[2]*Sqrt[a -
b]*ArcSinh[(Sqrt[a - b]*Sqrt[-(b/(a - b)) - (b*Sin[x])/(a - b)])/(Sqrt[2]*S
qrt[b])]*Sqrt[-(b/(a - b)) - (b*Sin[x])/(a - b)])/(Sqrt[b]*Sqrt[1 + ((a - b
)*(-(b/(a - b)) - (b*Sin[x])/(a - b)))/(2*b)])))/(8*(a - b)^2*(-(b/(a - b))
 - (b*Sin[x])/(a - b))^2*(1 + ((a - b)*(-(b/(a - b)) - (b*Sin[x])/(a - b)))
/(2*b))^2)))/(3*(a + b)*Sqrt[((a + b)*(b/(a + b) - (b*Sin[x])/(a + b)))/b])
 - ((-((a*b)/(a - b)) + b^2/(a - b))*(-(((-((a*b)/(a - b)) + b^2/(a - b))*(
-(((-((a*b)/(a + b)) - b^2/(a + b))*((2*Sqrt[a - b]*ArcTanh[(Sqrt[a - b]*Sq
rt[-(b/(a - b)) - (b*Sin[x])/(a - b)])/(Sqrt[a + b]*Sqrt[b/(a + b) - (b*Sin
[x])/(a + b)])])/(b*Sqrt[a + b]) - (2*Sqrt[-((a*b)/(a + b)) - b^2/(a + b)]*
ArcTanh[(Sqrt[-((a*b)/(a + b)) - b^2/(a + b)]*Sqrt[-(b/(a - b)) - (b*Sin[x]
)/(a - b)])/(Sqrt[-((a*b)/(a - b)) + b^2/(a - b)]*Sqrt[b/(a + b) - (b*Sin[x
])/(a + b)])])/(b*Sqrt[-((a*b)/(a - b)) + b^2/(a - b)])))/b) + (2*Sqrt[2]*(
a - b)*Sqrt[-(b/(a - b)) - (b*Sin[x])/(a - b)]*Sqrt[b/(a + b) - (b*Sin[x])/
(a + b)]*(1 + ((a - b)*(-(b/(a - b)) - (b*Sin[x])/(a - b)))/(2*b))^(3/2)*((
Sqrt[b]*ArcSinh[(Sqrt[a - b]*Sqrt[-(b/(a - b)) - (b*Sin[x])/(a - b)])/(Sqrt
[2]*Sqrt[b])])/(Sqrt[2]*Sqrt[a - b]*Sqrt[-(b/(a - b)) - (b*Sin[x])/(a - b)]
*(1 + ((a - b)*(-(b/(a - b)) - (b*Sin[x])/(a - b)))/(2*b))^(3/2)) + 1/(2*(1
 + ((a - b)*(-(b/(a - b)) - (b*Sin[x])/(a - b)))/(2*b)))))/(b*(a + b)*Sqrt[
((a + b)*(b/(a + b) - (b*Sin[x])/(a + b)))/b])))/b) + (4*Sqrt[2]*Sqrt[-(b/(
a - b)) - (b*Sin[x])/(a - b)]*Sqrt[b/(a + b) - (b*Sin[x])/(a + b)]*(1 + ((a
 - b)*(-(b/(a - b)) - (b*Sin[x])/(a - b)))/(2*b))^(5/2)*((3*Sqrt[b]*ArcSinh
[(Sqrt[a - b]*Sqrt[-(b/(a - b)) - (b*Sin[x])/(a - b)])/(Sqrt[2]*Sqrt[b])])/
```

```
(4*Sqrt[2]*Sqrt[a - b]*Sqrt[-(b/(a - b)) - (b*Sin[x])/(a - b)]*(1 + ((a - b
)*(-(b/(a - b)) - (b*Sin[x])/(a - b)))/(2*b))^(5/2)) + (3/(2*(1 + ((a - b)*
(-(b/(a - b)) - (b*Sin[x])/(a - b)))/(2*b))^2) + (1 + ((a - b)*(-(b/(a - b)
) - (b*Sin[x])/(a - b)))/(2*b))^(-1))/4))/((a + b)*Sqrt[((a + b)*(b/(a + b)
 - (b*Sin[x])/(a + b)))/b]))))/b))/b)/(((a*b)/(a - b) - b^2/(a - b))*((a*b)/
(a + b) + b^2/(a + b))))/(2*((a*b)/(a - b) - b^2/(a - b))*((a*b)/(a + b) +
b^2/(a + b))))/(3*((a*b)/(a - b) - b^2/(a - b))*((a*b)/(a + b) + b^2/(a + b
)))))/((1 - (a + b*Sin[x])/(a - b))^(3/2)*(1 - (a + b*Sin[x])/(a + b))^(3/2
)*(a*Sec[x] + b*Tan[x])^4)
```

fricas [B] time = 1.40, size = 931, normalized size = 5.97

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sec(x)+b*tan(x))^4,x, algorithm="fricas")
```

```
[Out] [-1/12*(36*(a^5*b^2 - 2*a^3*b^4 + a*b^6)*x*cos(x)^2 + 2*(11*a^4*b^3 - 19*a^
2*b^5 + 8*b^7)*cos(x)^3 + 3*(2*a^6 + 3*a^4*b^2 - 9*a^2*b^4 - 3*(2*a^4*b^2 -
 3*a^2*b^4)*cos(x)^2 + (6*a^5*b - 7*a^3*b^3 - 3*a*b^5 - (2*a^3*b^3 - 3*a*b^
5)*cos(x)^2)*sin(x))*sqrt(-a^2 + b^2)*log(-((2*a^2 - b^2)*cos(x)^2 - 2*a*b*
sin(x) - a^2 - b^2 - 2*(a*cos(x)*sin(x) + b*cos(x))*sqrt(-a^2 + b^2))/(b^2*
cos(x)^2 - 2*a*b*sin(x) - a^2 - b^2)) - 12*(a^7 + a^5*b^2 - 5*a^3*b^4 + 3*a
*b^6)*x - 12*(a^6*b - 2*a^2*b^5 + b^7)*cos(x) + 6*(2*(a^4*b^3 - 2*a^2*b^5 +
 b^7)*x*cos(x)^2 - 2*(3*a^6*b - 5*a^4*b^3 + a^2*b^5 + b^7)*x - (5*a^5*b^2 -
 8*a^3*b^4 + 3*a*b^6)*cos(x))*sin(x))/(a^7*b^4 + a^5*b^6 - 5*a^3*b^8 + 3*a*
b^10 - 3*(a^5*b^6 - 2*a^3*b^8 + a*b^10)*cos(x)^2 + (3*a^6*b^5 - 5*a^4*b^7 +
 a^2*b^9 + b^11 - (a^4*b^7 - 2*a^2*b^9 + b^11)*cos(x)^2)*sin(x)), -1/6*(18*
(a^5*b^2 - 2*a^3*b^4 + a*b^6)*x*cos(x)^2 + (11*a^4*b^3 - 19*a^2*b^5 + 8*b^7
)*cos(x)^3 - 3*(2*a^6 + 3*a^4*b^2 - 9*a^2*b^4 - 3*(2*a^4*b^2 - 3*a^2*b^4)*c
os(x)^2 + (6*a^5*b - 7*a^3*b^3 - 3*a*b^5 - (2*a^3*b^3 - 3*a*b^5)*cos(x)^2)*
sin(x))*sqrt(a^2 - b^2)*arctan(-(a*sin(x) + b)/(sqrt(a^2 - b^2)*cos(x))) -
6*(a^7 + a^5*b^2 - 5*a^3*b^4 + 3*a*b^6)*x - 6*(a^6*b - 2*a^2*b^5 + b^7)*cos
(x) + 3*(2*(a^4*b^3 - 2*a^2*b^5 + b^7)*x*cos(x)^2 - 2*(3*a^6*b - 5*a^4*b^3
+ a^2*b^5 + b^7)*x - (5*a^5*b^2 - 8*a^3*b^4 + 3*a*b^6)*cos(x))*sin(x))/(a^7
*b^4 + a^5*b^6 - 5*a^3*b^8 + 3*a*b^10 - 3*(a^5*b^6 - 2*a^3*b^8 + a*b^10)*co
s(x)^2 + (3*a^6*b^5 - 5*a^4*b^7 + a^2*b^9 + b^11 - (a^4*b^7 - 2*a^2*b^9 + b
^11)*cos(x)^2)*sin(x))]
```

giac [B] time = 0.17, size = 369, normalized size = 2.37

$$-\frac{\left(2\,a^{3}-3\,ab^{2}\right)\left(\pi\left\lfloor\frac{x}{2\,\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(a)+\arctan\left(\frac{a\tan\left(\frac{1}{2}\,x\right)+b}{\sqrt{a^{2}-b^{2}}}\right)\right)}{\left(a^{2}b^{4}-b^{6}\right)\sqrt{a^{2}-b^{2}}}+\frac{3\,a^{6}b\tan\left(\frac{1}{2}\,x\right)^{5}-6\,a^{4}b^{3}\tan\left(\frac{1}{2}\,x\right)^{5}+6\,a^{2}b^{5}\tan\left(\frac{1}{2}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sec(x)+b*tan(x))^4,x, algorithm="giac")
```

```
[Out] -(2*a^3 - 3*a*b^2)*(pi*floor(1/2*x/pi + 1/2)*sgn(a) + arctan((a*tan(1/2*x)
+ b)/sqrt(a^2 - b^2)))/((a^2*b^4 - b^6)*sqrt(a^2 - b^2)) + 1/3*(3*a^6*b*tan
(1/2*x)^5 - 6*a^4*b^3*tan(1/2*x)^5 + 6*a^2*b^5*tan(1/2*x)^5 + 6*a^7*tan(1/2
*x)^4 + 9*a^5*b^2*tan(1/2*x)^4 - 12*a^3*b^4*tan(1/2*x)^4 + 12*a*b^6*tan(1/2
*x)^4 + 36*a^6*b*tan(1/2*x)^3 - 6*a^4*b^3*tan(1/2*x)^3 - 8*a^2*b^5*tan(1/2*
x)^3 + 8*b^7*tan(1/2*x)^3 + 12*a^7*tan(1/2*x)^2 + 48*a^5*b^2*tan(1/2*x)^2 -
 42*a^3*b^4*tan(1/2*x)^2 + 12*a*b^6*tan(1/2*x)^2 + 33*a^6*b*tan(1/2*x) - 24
*a^4*b^3*tan(1/2*x) + 6*a^2*b^5*tan(1/2*x) + 6*a^7 - 5*a^5*b^2 + 2*a^3*b^4)
/((a^5*b^3 - a^3*b^5)*(a*tan(1/2*x)^2 + 2*b*tan(1/2*x) + a)^3) + x/b^4
```

maple [B] time = 0.21, size = 967, normalized size = 6.20

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*sec(x)+b*tan(x))^4,x)
```

```
[Out] 1/b^2/(a*tan(1/2*x)^2+2*b*tan(1/2*x)+a)^3*a^3/(a^2-b^2)*tan(1/2*x)^5-2/(a*t
an(1/2*x)^2+2*b*tan(1/2*x)+a)^3*a/(a^2-b^2)*tan(1/2*x)^5+2*b^2/(a*tan(1/2*x
)^2+2*b*tan(1/2*x)+a)^3/a/(a^2-b^2)*tan(1/2*x)^5+2/b^3/(a*tan(1/2*x)^2+2*b*
tan(1/2*x)+a)^3/(a^2-b^2)*a^4*tan(1/2*x)^4+3/b/(a*tan(1/2*x)^2+2*b*tan(1/2*
x)+a)^3/(a^2-b^2)*a^2*tan(1/2*x)^4-4*b/(a*tan(1/2*x)^2+2*b*tan(1/2*x)+a)^3/
(a^2-b^2)*tan(1/2*x)^4+4*b^3/(a*tan(1/2*x)^2+2*b*tan(1/2*x)+a)^3/(a^2-b^2)/
a^2*tan(1/2*x)^4+12/b^2/(a*tan(1/2*x)^2+2*b*tan(1/2*x)+a)^3*a^3/(a^2-b^2)*t
an(1/2*x)^3-2/(a*tan(1/2*x)^2+2*b*tan(1/2*x)+a)^3*a/(a^2-b^2)*tan(1/2*x)^3-
8/3*b^2/(a*tan(1/2*x)^2+2*b*tan(1/2*x)+a)^3/a/(a^2-b^2)*tan(1/2*x)^3+8/3*b^
4/(a*tan(1/2*x)^2+2*b*tan(1/2*x)+a)^3/a^3/(a^2-b^2)*tan(1/2*x)^3+4/b^3/(a*t
an(1/2*x)^2+2*b*tan(1/2*x)+a)^3/(a^2-b^2)*a^4*tan(1/2*x)^2+16/b/(a*tan(1/2*
x)^2+2*b*tan(1/2*x)+a)^3/(a^2-b^2)*a^2*tan(1/2*x)^2-14*b/(a*tan(1/2*x)^2+2*
b*tan(1/2*x)+a)^3/(a^2-b^2)*tan(1/2*x)^2+4*b^3/(a*tan(1/2*x)^2+2*b*tan(1/2*
x)+a)^3/(a^2-b^2)/a^2*tan(1/2*x)^2+11/b^2/(a*tan(1/2*x)^2+2*b*tan(1/2*x)+a)
^3*a^3/(a^2-b^2)*tan(1/2*x)-8/(a*tan(1/2*x)^2+2*b*tan(1/2*x)+a)^3*a/(a^2-b^
2)*tan(1/2*x)+2*b^2/(a*tan(1/2*x)^2+2*b*tan(1/2*x)+a)^3/a/(a^2-b^2)*tan(1/2
*x)+2/b^3/(a*tan(1/2*x)^2+2*b*tan(1/2*x)+a)^3/(a^2-b^2)*a^4-5/3/b/(a*tan(1/
2*x)^2+2*b*tan(1/2*x)+a)^3/(a^2-b^2)*a^2+2/3*b/(a*tan(1/2*x)^2+2*b*tan(1/2*
x)+a)^3/(a^2-b^2)-2/b^4*a^3/(a^2-b^2)^(3/2)*arctan(1/2*(2*a*tan(1/2*x)+2*b)
/(a^2-b^2)^(1/2))+3/b^2*a/(a^2-b^2)^(3/2)*arctan(1/2*(2*a*tan(1/2*x)+2*b)/(
a^2-b^2)^(1/2))+2/b^4*arctan(tan(1/2*x))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sec(x)+b*tan(x))^4,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more details)Is 4*b^2-4*a^2 positive or negative?
```

mupad [B] time = 6.37, size = 2782, normalized size = 17.83

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*tan(x) + a/cos(x))^4,x)
```

```
[Out] ((6*a^4 + 2*b^4 - 5*a^2*b^2)/(3*b^3*(a^2 - b^2)) + (tan(x/2)*(11*a^4 + 2*b^
4 - 8*a^2*b^2))/(a*b^2*(a^2 - b^2)) + (tan(x/2)^5*(a^4 + 2*b^4 - 2*a^2*b^2)
)/(a*b^2*(a^2 - b^2)) + (tan(x/2)^4*(2*a^6 + 4*b^6 - 4*a^2*b^4 + 3*a^4*b^2)
)/(a^2*b^3*(a^2 - b^2)) + (2*tan(x/2)^2*(2*a^6 + 2*b^6 - 7*a^2*b^4 + 8*a^4*
b^2))/(a^2*b^3*(a^2 - b^2)) + (2*tan(x/2)^3*(3*a^2 + 2*b^2)*(6*a^4 + 2*b^4
- 5*a^2*b^2))/(3*a^3*b^2*(a^2 - b^2)))/(tan(x/2)^2*(12*a*b^2 + 3*a^3) + tan
(x/2)^4*(12*a*b^2 + 3*a^3) + tan(x/2)^3*(12*a^2*b + 8*b^3) + a^3 + a^3*tan(
x/2)^6 + 6*a^2*b*tan(x/2)^5 + 6*a^2*b*tan(x/2)) + (2*atan((48*a^3*b^3*tan(x
/2))/((176*a^3*b^15)/(b^12 - 2*a^2*b^10 + a^4*b^8) - (160*a^5*b^13)/(b^12 -
 2*a^2*b^10 + a^4*b^8) + (48*a^7*b^11)/(b^12 - 2*a^2*b^10 + a^4*b^8) - (64*
a*b^17)/(b^12 - 2*a^2*b^10 + a^4*b^8)) - (64*a*b^5*tan(x/2))/((176*a^3*b^15
)/(b^12 - 2*a^2*b^10 + a^4*b^8) - (160*a^5*b^13)/(b^12 - 2*a^2*b^10 + a^4*b
^8) + (48*a^7*b^11)/(b^12 - 2*a^2*b^10 + a^4*b^8) - (64*a*b^17)/(b^12 - 2*a
^2*b^10 + a^4*b^8))))/b^4 + (a*atan(((a*(2*a^2 - 3*b^2)*(-(a + b)^3*(a - b)
^3)^(1/2)*((8*(4*a^2*b^7 - 8*a^4*b^5 + 4*a^6*b^3))/(b^12 - 2*a^2*b^10 + a^4
*b^8) + (8*tan(x/2)*(8*a*b^9 - 29*a^3*b^7 + 28*a^5*b^5 - 8*a^7*b^3))/(b^13
- 2*a^2*b^11 + a^4*b^9) - (a*(2*a^2 - 3*b^2)*(-(a + b)^3*(a - b)^3)^(1/2)*(
(8*(4*a*b^12 - 6*a^3*b^10 + 2*a^5*b^8))/(b^12 - 2*a^2*b^10 + a^4*b^8) + (8*
tan(x/2)*(12*a^2*b^12 - 20*a^4*b^10 + 8*a^6*b^8))/(b^13 - 2*a^2*b^11 + a^4*
b^9) - (a*((8*(4*a^2*b^15 - 8*a^4*b^13 + 4*a^6*b^11))/(b^12 - 2*a^2*b^10 +
a^4*b^8) + (8*tan(x/2)*(12*a*b^17 - 32*a^3*b^15 + 28*a^5*b^13 - 8*a^7*b^11)
)/(b^13 - 2*a^2*b^11 + a^4*b^9))*(2*a^2 - 3*b^2)*(-(a + b)^3*(a - b)^3)^(1/
2))/(2*(b^10 - 3*a^2*b^8 + 3*a^4*b^6 - a^6*b^4))))/(2*(b^10 - 3*a^2*b^8 + 3
*a^4*b^6 - a^6*b^4)))*1i)/(2*(b^10 - 3*a^2*b^8 + 3*a^4*b^6 - a^6*b^4)) + (a
*(2*a^2 - 3*b^2)*(-(a + b)^3*(a - b)^3)^(1/2)*((8*(4*a^2*b^7 - 8*a^4*b^5 +
4*a^6*b^3))/(b^12 - 2*a^2*b^10 + a^4*b^8) + (8*tan(x/2)*(8*a*b^9 - 29*a^3*b
^7 + 28*a^5*b^5 - 8*a^7*b^3))/(b^13 - 2*a^2*b^11 + a^4*b^9) + (a*(2*a^2 - 3
*b^2)*(-(a + b)^3*(a - b)^3)^(1/2)*((8*(4*a*b^12 - 6*a^3*b^10 + 2*a^5*b^8))
/(b^12 - 2*a^2*b^10 + a^4*b^8) + (8*tan(x/2)*(12*a^2*b^12 - 20*a^4*b^10 + 8
```

```
*a^6*b^8))/(b^13 - 2*a^2*b^11 + a^4*b^9) + (a*((8*(4*a^2*b^15 - 8*a^4*b^13
+ 4*a^6*b^11))/(b^12 - 2*a^2*b^10 + a^4*b^8) + (8*tan(x/2)*(12*a*b^17 - 32*
a^3*b^15 + 28*a^5*b^13 - 8*a^7*b^11))/(b^13 - 2*a^2*b^11 + a^4*b^9))*(2*a^2
 - 3*b^2)*(-(a + b)^3*(a - b)^3)^(1/2))/(2*(b^10 - 3*a^2*b^8 + 3*a^4*b^6 -
a^6*b^4))))/(2*(b^10 - 3*a^2*b^8 + 3*a^4*b^6 - a^6*b^4)))*1i)/(2*(b^10 - 3*
a^2*b^8 + 3*a^4*b^6 - a^6*b^4)))/((16*(2*a^5 - 3*a^3*b^2))/(b^12 - 2*a^2*b^
10 + a^4*b^8) + (16*tan(x/2)*(8*a^6 + 12*a^2*b^4 - 20*a^4*b^2))/(b^13 - 2*a
^2*b^11 + a^4*b^9) - (a*(2*a^2 - 3*b^2)*(-(a + b)^3*(a - b)^3)^(1/2)*((8*(4
*a^2*b^7 - 8*a^4*b^5 + 4*a^6*b^3))/(b^12 - 2*a^2*b^10 + a^4*b^8) + (8*tan(x
/2)*(8*a*b^9 - 29*a^3*b^7 + 28*a^5*b^5 - 8*a^7*b^3))/(b^13 - 2*a^2*b^11 + a
^4*b^9) - (a*(2*a^2 - 3*b^2)*(-(a + b)^3*(a - b)^3)^(1/2)*((8*(4*a*b^12 - 6
*a^3*b^10 + 2*a^5*b^8))/(b^12 - 2*a^2*b^10 + a^4*b^8) + (8*tan(x/2)*(12*a^2
*b^12 - 20*a^4*b^10 + 8*a^6*b^8))/(b^13 - 2*a^2*b^11 + a^4*b^9) - (a*((8*(4
*a^2*b^15 - 8*a^4*b^13 + 4*a^6*b^11))/(b^12 - 2*a^2*b^10 + a^4*b^8) + (8*ta
n(x/2)*(12*a*b^17 - 32*a^3*b^15 + 28*a^5*b^13 - 8*a^7*b^11))/(b^13 - 2*a^2*
b^11 + a^4*b^9))*(2*a^2 - 3*b^2)*(-(a + b)^3*(a - b)^3)^(1/2))/(2*(b^10 - 3
*a^2*b^8 + 3*a^4*b^6 - a^6*b^4))))/(2*(b^10 - 3*a^2*b^8 + 3*a^4*b^6 - a^6*b
^4))))/(2*(b^10 - 3*a^2*b^8 + 3*a^4*b^6 - a^6*b^4)) + (a*(2*a^2 - 3*b^2)*(-
(a + b)^3*(a - b)^3)^(1/2)*((8*(4*a^2*b^7 - 8*a^4*b^5 + 4*a^6*b^3))/(b^12 -
 2*a^2*b^10 + a^4*b^8) + (8*tan(x/2)*(8*a*b^9 - 29*a^3*b^7 + 28*a^5*b^5 - 8
*a^7*b^3))/(b^13 - 2*a^2*b^11 + a^4*b^9) + (a*(2*a^2 - 3*b^2)*(-(a + b)^3*(
a - b)^3)^(1/2)*((8*(4*a*b^12 - 6*a^3*b^10 + 2*a^5*b^8))/(b^12 - 2*a^2*b^10
 + a^4*b^8) + (8*tan(x/2)*(12*a^2*b^12 - 20*a^4*b^10 + 8*a^6*b^8))/(b^13 -
2*a^2*b^11 + a^4*b^9) + (a*((8*(4*a^2*b^15 - 8*a^4*b^13 + 4*a^6*b^11))/(b^1
2 - 2*a^2*b^10 + a^4*b^8) + (8*tan(x/2)*(12*a*b^17 - 32*a^3*b^15 + 28*a^5*b
^13 - 8*a^7*b^11))/(b^13 - 2*a^2*b^11 + a^4*b^9))*(2*a^2 - 3*b^2)*(-(a + b)
^3*(a - b)^3)^(1/2))/(2*(b^10 - 3*a^2*b^8 + 3*a^4*b^6 - a^6*b^4))))/(2*(b^1
0 - 3*a^2*b^8 + 3*a^4*b^6 - a^6*b^4))))/(2*(b^10 - 3*a^2*b^8 + 3*a^4*b^6 -
a^6*b^4))))*(2*a^2 - 3*b^2)*(-(a + b)^3*(a - b)^3)^(1/2)*1i)/(b^10 - 3*a^2*
b^8 + 3*a^4*b^6 - a^6*b^4)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a\sec\left(x\right) + b\tan\left(x\right)\right)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sec(x)+b*tan(x))**4,x)
```

```
[Out] Integral((a*sec(x) + b*tan(x))**(-4), x)
```

3.272 $\int \frac{1}{(a\sec(x)+b\tan(x))^5}\,dx$

Optimal. Leaf size=101

$$-\frac{\left(a^2-b^2\right)^2}{4b^5(a+b\sin(x))^4}+\frac{4a\left(a^2-b^2\right)}{3b^5(a+b\sin(x))^3}-\frac{3a^2-b^2}{b^5(a+b\sin(x))^2}+\frac{4a}{b^5(a+b\sin(x))}+\frac{\log(a+b\sin(x))}{b^5}$$

```
[Out] ln(a+b*sin(x))/b^5-1/4*(a^2-b^2)^2/b^5/(a+b*sin(x))^4+4/3*a*(a^2-b^2)/b^5/(
a+b*sin(x))^3+(-3*a^2+b^2)/b^5/(a+b*sin(x))^2+4*a/b^5/(a+b*sin(x))
```

Rubi [A] time = 0.12, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {4391, 2668, 697}

$$-\frac{\left(a^2-b^2\right)^2}{4b^5(a+b\sin(x))^4}+\frac{4a\left(a^2-b^2\right)}{3b^5(a+b\sin(x))^3}-\frac{3a^2-b^2}{b^5(a+b\sin(x))^2}+\frac{4a}{b^5(a+b\sin(x))}+\frac{\log(a+b\sin(x))}{b^5}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Sec[x] + b*Tan[x])^(-5),x]
```

```
[Out] Log[a + b*Sin[x]]/b^5 - (a^2 - b^2)^2/(4*b^5*(a + b*Sin[x])^4) + (4*a*(a^2
- b^2))/(3*b^5*(a + b*Sin[x])^3) - (3*a^2 - b^2)/(b^5*(a + b*Sin[x])^2) + (
4*a)/(b^5*(a + b*Sin[x]))
```

Rule 697

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m},
 x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0]
```

Rule 2668

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rule 4391

```
Int[(u_.)*((b_.)*sec[(c_.) + (d_.)*(x_)]^(n_.) + (a_.)*tan[(c_.) + (d_.)*(x
_)]^(n_.))^(p_), x_Symbol] :> Int[ActivateTrig[u]*Sec[c + d*x]^(n*p)*(b + a
*Sin[c + d*x]^n)^p, x] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p]
```

Rubi steps

$$\int \frac{1}{(a\sec(x)+b\tan(x))^5}\,dx = \int \frac{\cos^5(x)}{(a+b\sin(x))^5}\,dx$$
$$= \frac{\text{Subst}\left(\int \frac{\left(b^2-x^2\right)^2}{(a+x)^5}\,dx, x, b\sin(x)\right)}{b^5}$$
$$= \frac{\text{Subst}\left(\int \left(\frac{\left(a^2-b^2\right)^2}{(a+x)^5}-\frac{4\left(a^3-ab^2\right)}{(a+x)^4}+\frac{2\left(3a^2-b^2\right)}{(a+x)^3}-\frac{4a}{(a+x)^2}+\frac{1}{a+x}\right)dx, x, b\sin(x)\right)}{b^5}$$
$$= \frac{\log(a+b\sin(x))}{b^5}-\frac{\left(a^2-b^2\right)^2}{4b^5(a+b\sin(x))^4}+\frac{4a\left(a^2-b^2\right)}{3b^5(a+b\sin(x))^3}-\frac{3a^2-b^2}{b^5(a+b\sin(x))^2}+$$

Mathematica [A] time = 0.34, size = 86, normalized size = 0.85

$$\frac{\frac{25a^4+12b^2\left(9a^2+b^2\right)\sin^2(x)+8ab\left(11a^2+b^2\right)\sin(x)+2a^2b^2+48ab^3\sin^3(x)-3b^4}{12(a+b\sin(x))^4}+\log(a+b\sin(x))}{b^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Sec[x] + b*Tan[x])^(-5),x]
```

```
[Out] (Log[a + b*Sin[x]] + (25*a^4 + 2*a^2*b^2 - 3*b^4 + 8*a*b*(11*a^2 + b^2)*Sin
[x] + 12*b^2*(9*a^2 + b^2)*Sin[x]^2 + 48*a*b^3*Sin[x]^3)/(12*(a + b*Sin[x])
^4))/b^5
```

fricas [B] time = 1.06, size = 217, normalized size = 2.15

$$\frac{25\,a^4+110\,a^2b^2+9\,b^4-12\left(9\,a^2b^2+b^4\right)\cos(x)^2+12\left(b^4\cos(x)^4+a^4+6\,a^2b^2+b^4-2\left(3\,a^2b^2+b^4\right)\cos(x)^2-4}{12\left(b^9\cos(x)^4+a^4b^5+6\,a^2b^7+b^9-2\left(3\,a^2b^7+b^9\right)\cos(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sec(x)+b*tan(x))^5,x, algorithm="fricas")
```

```
[Out] 1/12*(25*a^4 + 110*a^2*b^2 + 9*b^4 - 12*(9*a^2*b^2 + b^4)*cos(x)^2 + 12*(b^
4*cos(x)^4 + a^4 + 6*a^2*b^2 + b^4 - 2*(3*a^2*b^2 + b^4)*cos(x)^2 - 4*(a*b^
3*cos(x)^2 - a^3*b - a*b^3)*sin(x))*log(b*sin(x) + a) - 8*(6*a*b^3*cos(x)^2
 - 11*a^3*b - 7*a*b^3)*sin(x))/(b^9*cos(x)^4 + a^4*b^5 + 6*a^2*b^7 + b^9 -
2*(3*a^2*b^7 + b^9)*cos(x)^2 - 4*(a*b^8*cos(x)^2 - a^3*b^6 - a*b^8)*sin(x))
```

giac [A] time = 0.14, size = 91, normalized size = 0.90

$$\frac{\log\left(|b\sin(x)+a|\right)}{b^5}-\frac{25\,b^3\sin(x)^4+52\,ab^2\sin(x)^3+42\,a^2b\sin(x)^2-12\,b^3\sin(x)^2+12\,a^3\sin(x)-8\,ab^2\sin(x)-}{12\,(b\sin(x)+a)^4b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sec(x)+b*tan(x))^5,x, algorithm="giac")
```

```
[Out] log(abs(b*sin(x) + a))/b^5 - 1/12*(25*b^3*sin(x)^4 + 52*a*b^2*sin(x)^3 + 42
*a^2*b*sin(x)^2 - 12*b^3*sin(x)^2 + 12*a^3*sin(x) - 8*a*b^2*sin(x) - 2*a^2*
b + 3*b^3)/((b*sin(x) + a)^4*b^4)
```

maple [A] time = 0.21, size = 130, normalized size = 1.29

$$\frac{4a}{b^5\left(a+b\sin(x)\right)}-\frac{a^4}{4b^5\left(a+b\sin(x)\right)^4}+\frac{a^2}{2b^3\left(a+b\sin(x)\right)^4}-\frac{1}{4b\left(a+b\sin(x)\right)^4}+\frac{4a^3}{3b^5\left(a+b\sin(x)\right)^3}-\frac{4a}{3b^3\left(a+b\sin\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*sec(x)+b*tan(x))^5,x)
```

```
[Out] 4*a/b^5/(a+b*sin(x))-1/4/b^5/(a+b*sin(x))^4*a^4+1/2/b^3/(a+b*sin(x))^4*a^2-
1/4/b/(a+b*sin(x))^4+4/3*a^3/b^5/(a+b*sin(x))^3-4/3*a/b^3/(a+b*sin(x))^3+ln
(a+b*sin(x))/b^5-3/b^5/(a+b*sin(x))^2*a^2+1/b^3/(a+b*sin(x))^2
```

maxima [B] time = 0.51, size = 483, normalized size = 4.78

$$-\frac{2\left(\frac{3\left(a^7-a^3b^4\right)\sin(x)}{\cos(x)+1}+\frac{3\left(7\,a^6b-3\,a^2b^5\right)\sin(x)^2}{(\cos(x)+1)^2}+\frac{\left(9\,a^7+52\,a^5b^2-a^3b^4-12\,ab^6\right)\sin(x)^3}{(\cos(x)+1)^3}+\frac{2\left(21\,a^6b+25\,a^4b^3-7\,a^2b^5-3\,b^7\right)\sin(x)^4}{(\cos(x)+1)^4}+\frac{\left(9\,a^7+5\right.}{}}{3\left(a^8b^4+\frac{8\,a^7b^5\sin(x)}{\cos(x)+1}+\frac{8\,a^7b^5\sin(x)^7}{(\cos(x)+1)^7}+\frac{a^8b^4\sin(x)^8}{(\cos(x)+1)^8}+\frac{4\left(a^8b^4+6\,a^6b^6\right)\sin(x)^2}{(\cos(x)+1)^2}+\frac{8\left(3\,a^7b^5+4\,a^5b^7\right)\sin(x)^3}{(\cos(x)+1)^3}+\frac{2\left(3\,a^8b^4+24\,a^6b^6\right.}{(\cos(x)}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sec(x)+b*tan(x))^5,x, algorithm="maxima")
```

```
[Out] -2/3*(3*(a^7 - a^3*b^4)*sin(x)/(cos(x) + 1) + 3*(7*a^6*b - 3*a^2*b^5)*sin(x
)^2/(cos(x) + 1)^2 + (9*a^7 + 52*a^5*b^2 - a^3*b^4 - 12*a*b^6)*sin(x)^3/(co
s(x) + 1)^3 + 2*(21*a^6*b + 25*a^4*b^3 - 7*a^2*b^5 - 3*b^7)*sin(x)^4/(cos(x
) + 1)^4 + (9*a^7 + 52*a^5*b^2 - a^3*b^4 - 12*a*b^6)*sin(x)^5/(cos(x) + 1)^
5 + 3*(7*a^6*b - 3*a^2*b^5)*sin(x)^6/(cos(x) + 1)^6 + 3*(a^7 - a^3*b^4)*sin
(x)^7/(cos(x) + 1)^7)/(a^8*b^4 + 8*a^7*b^5*sin(x)/(cos(x) + 1) + 8*a^7*b^5*
sin(x)^7/(cos(x) + 1)^7 + a^8*b^4*sin(x)^8/(cos(x) + 1)^8 + 4*(a^8*b^4 + 6*
a^6*b^6)*sin(x)^2/(cos(x) + 1)^2 + 8*(3*a^7*b^5 + 4*a^5*b^7)*sin(x)^3/(cos(
x) + 1)^3 + 2*(3*a^8*b^4 + 24*a^6*b^6 + 8*a^4*b^8)*sin(x)^4/(cos(x) + 1)^4
```

```
+ 8*(3*a^7*b^5 + 4*a^5*b^7)*sin(x)^5/(cos(x) + 1)^5 + 4*(a^8*b^4 + 6*a^6*b^
6)*sin(x)^6/(cos(x) + 1)^6) + log(a + 2*b*sin(x)/(cos(x) + 1) + a*sin(x)^2/
(cos(x) + 1)^2)/b^5 - log(sin(x)^2/(cos(x) + 1)^2 + 1)/b^5
```

mupad [B] time = 3.84, size = 541, normalized size = 5.36

$$\frac{2\,\mathrm{atanh}\left(\frac{16\,a}{\frac{32\,a^3}{b^2}-16\,a\,\tan\left(\frac{x}{2}\right)^2-16\,a+\frac{32\,a^2\,\tan\left(\frac{x}{2}\right)}{b}+\frac{32\,a^3\,\tan\left(\frac{x}{2}\right)^2}{b^2}}+\frac{16\,a\,\tan\left(\frac{x}{2}\right)^2}{\frac{32\,a^3}{b^2}-16\,a\,\tan\left(\frac{x}{2}\right)^2-16\,a+\frac{32\,a^2\,\tan\left(\frac{x}{2}\right)}{b}+\frac{32\,a^3\,\tan\left(\frac{x}{2}\right)^2}{b^2}}+\frac{32\,\cdots}{32\,a^2\,\tan\left(\frac{x}{2}\right)-16\,a\,b+\frac{32\,a\cdots}{b}\cdots}\cdots\right.}{b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*tan(x) + a/cos(x))^5,x)
```

```
[Out] (2*atanh((16*a)/((32*a^3)/b^2 - 16*a*tan(x/2)^2 - 16*a + (32*a^2*tan(x/2))/
b + (32*a^3*tan(x/2)^2)/b^2) + (16*a*tan(x/2)^2)/((32*a^3)/b^2 - 16*a*tan(x
/2)^2 - 16*a + (32*a^2*tan(x/2))/b + (32*a^3*tan(x/2)^2)/b^2) + (32*a^2*tan
(x/2))/(32*a^2*tan(x/2) - 16*a*b + (32*a^3)/b + (32*a^3*tan(x/2)^2)/b - 16*
a*b*tan(x/2)^2)))/b^5 - ((2*tan(x/2)^2*(7*a^4 - 3*b^4))/(a^2*b^3) + (2*tan(
x/2)^6*(7*a^4 - 3*b^4))/(a^2*b^3) + (2*tan(x/2)*(a^4 - b^4))/(a*b^4) + (2*t
an(x/2)^7*(a^4 - b^4))/(a*b^4) + (4*tan(x/2)^4*(21*a^6 - 3*b^6 - 7*a^2*b^4
+ 25*a^4*b^2))/(3*a^4*b^3) + (2*tan(x/2)^3*(9*a^6 - 12*b^6 - a^2*b^4 + 52*a
^4*b^2))/(3*a^3*b^4) + (2*tan(x/2)^5*(9*a^6 - 12*b^6 - a^2*b^4 + 52*a^4*b^2
))/(3*a^3*b^4))/(tan(x/2)^2*(4*a^4 + 24*a^2*b^2) + tan(x/2)^6*(4*a^4 + 24*a
^2*b^2) + tan(x/2)^3*(32*a*b^3 + 24*a^3*b) + tan(x/2)^5*(32*a*b^3 + 24*a^3*
b) + tan(x/2)^4*(6*a^4 + 16*b^4 + 48*a^2*b^2) + a^4 + a^4*tan(x/2)^8 + 8*a^
3*b*tan(x/2)^7 + 8*a^3*b*tan(x/2))
```

sympy [A] time = 15.19, size = 1719, normalized size = 17.02

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sec(x)+b*tan(x))**5,x)
```

```
[Out] Piecewise((36*a**4*log(a*sec(x)/b + tan(x))*sec(x)**4/(36*a**4*b**5*sec(x)*
*4 + 144*a**3*b**6*tan(x)*sec(x)**3 + 216*a**2*b**7*tan(x)**2*sec(x)**2 + 1
44*a*b**8*tan(x)**3*sec(x) + 36*b**9*tan(x)**4) - 18*a**4*log(tan(x)**2 + 1
)*sec(x)**4/(36*a**4*b**5*sec(x)**4 + 144*a**3*b**6*tan(x)*sec(x)**3 + 216*
a**2*b**7*tan(x)**2*sec(x)**2 + 144*a*b**8*tan(x)**3*sec(x) + 36*b**9*tan(x
)**4) + 20*a**4*sec(x)**4/(36*a**4*b**5*sec(x)**4 + 144*a**3*b**6*tan(x)*se
c(x)**3 + 216*a**2*b**7*tan(x)**2*sec(x)**2 + 144*a*b**8*tan(x)**3*sec(x) +
 36*b**9*tan(x)**4) + 144*a**3*b*log(a*sec(x)/b + tan(x))*tan(x)*sec(x)**3/
```

```
(36*a**4*b**5*sec(x)**4 + 144*a**3*b**6*tan(x)*sec(x)**3 + 216*a**2*b**7*ta
n(x)**2*sec(x)**2 + 144*a*b**8*tan(x)**3*sec(x) + 36*b**9*tan(x)**4) - 72*a
**3*b*log(tan(x)**2 + 1)*tan(x)*sec(x)**3/(36*a**4*b**5*sec(x)**4 + 144*a**
3*b**6*tan(x)*sec(x)**3 + 216*a**2*b**7*tan(x)**2*sec(x)**2 + 144*a*b**8*ta
n(x)**3*sec(x) + 36*b**9*tan(x)**4) + 44*a**3*b*tan(x)*sec(x)**3/(36*a**4*b
**5*sec(x)**4 + 144*a**3*b**6*tan(x)*sec(x)**3 + 216*a**2*b**7*tan(x)**2*se
c(x)**2 + 144*a*b**8*tan(x)**3*sec(x) + 36*b**9*tan(x)**4) + 216*a**2*b**2*
log(a*sec(x)/b + tan(x))*tan(x)**2*sec(x)**2/(36*a**4*b**5*sec(x)**4 + 144*
a**3*b**6*tan(x)*sec(x)**3 + 216*a**2*b**7*tan(x)**2*sec(x)**2 + 144*a*b**8
*tan(x)**3*sec(x) + 36*b**9*tan(x)**4) - 108*a**2*b**2*log(tan(x)**2 + 1)*t
an(x)**2*sec(x)**2/(36*a**4*b**5*sec(x)**4 + 144*a**3*b**6*tan(x)*sec(x)**3
 + 216*a**2*b**7*tan(x)**2*sec(x)**2 + 144*a*b**8*tan(x)**3*sec(x) + 36*b**
9*tan(x)**4) + 6*a**2*b**2*sec(x)**2/(36*a**4*b**5*sec(x)**4 + 144*a**3*b**
6*tan(x)*sec(x)**3 + 216*a**2*b**7*tan(x)**2*sec(x)**2 + 144*a*b**8*tan(x)*
*3*sec(x) + 36*b**9*tan(x)**4) + 144*a*b**3*log(a*sec(x)/b + tan(x))*tan(x)
**3*sec(x)/(36*a**4*b**5*sec(x)**4 + 144*a**3*b**6*tan(x)*sec(x)**3 + 216*a
**2*b**7*tan(x)**2*sec(x)**2 + 144*a*b**8*tan(x)**3*sec(x) + 36*b**9*tan(x)
**4) - 72*a*b**3*log(tan(x)**2 + 1)*tan(x)**3*sec(x)/(36*a**4*b**5*sec(x)**
4 + 144*a**3*b**6*tan(x)*sec(x)**3 + 216*a**2*b**7*tan(x)**2*sec(x)**2 + 14
4*a*b**8*tan(x)**3*sec(x) + 36*b**9*tan(x)**4) - 52*a*b**3*tan(x)**3*sec(x)
/(36*a**4*b**5*sec(x)**4 + 144*a**3*b**6*tan(x)*sec(x)**3 + 216*a**2*b**7*t
an(x)**2*sec(x)**2 + 144*a*b**8*tan(x)**3*sec(x) + 36*b**9*tan(x)**4) + 24*
a*b**3*tan(x)*sec(x)/(36*a**4*b**5*sec(x)**4 + 144*a**3*b**6*tan(x)*sec(x)*
*3 + 216*a**2*b**7*tan(x)**2*sec(x)**2 + 144*a*b**8*tan(x)**3*sec(x) + 36*b
**9*tan(x)**4) + 36*b**4*log(a*sec(x)/b + tan(x))*tan(x)**4/(36*a**4*b**5*s
ec(x)**4 + 144*a**3*b**6*tan(x)*sec(x)**3 + 216*a**2*b**7*tan(x)**2*sec(x)*
*2 + 144*a*b**8*tan(x)**3*sec(x) + 36*b**9*tan(x)**4) - 18*b**4*log(tan(x)*
*2 + 1)*tan(x)**4/(36*a**4*b**5*sec(x)**4 + 144*a**3*b**6*tan(x)*sec(x)**3
+ 216*a**2*b**7*tan(x)**2*sec(x)**2 + 144*a*b**8*tan(x)**3*sec(x) + 36*b**9
*tan(x)**4) - 28*b**4*tan(x)**4/(36*a**4*b**5*sec(x)**4 + 144*a**3*b**6*tan
(x)*sec(x)**3 + 216*a**2*b**7*tan(x)**2*sec(x)**2 + 144*a*b**8*tan(x)**3*se
c(x) + 36*b**9*tan(x)**4) + 18*b**4*tan(x)**2/(36*a**4*b**5*sec(x)**4 + 144
*a**3*b**6*tan(x)*sec(x)**3 + 216*a**2*b**7*tan(x)**2*sec(x)**2 + 144*a*b**
8*tan(x)**3*sec(x) + 36*b**9*tan(x)**4) - 9*b**4/(36*a**4*b**5*sec(x)**4 +
144*a**3*b**6*tan(x)*sec(x)**3 + 216*a**2*b**7*tan(x)**2*sec(x)**2 + 144*a*
b**8*tan(x)**3*sec(x) + 36*b**9*tan(x)**4), Ne(b, 0)), ((8*tan(x)**5/(15*se
c(x)**5) + 4*tan(x)**3/(3*sec(x)**5) + tan(x)/sec(x)**5)/a**5, True))
```

3.273 $\int (\sec(x)+\tan(x))^5 \, dx$

Optimal. Leaf size=30

$$-\frac{4}{1-\sin(x)}+\frac{2}{(1-\sin(x))^2}-\log(1-\sin(x))$$

```
[Out] -ln(1-sin(x))+2/(1-sin(x))^2-4/(1-sin(x))
```

Rubi [A] time = 0.05, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {4391, 2667, 43}

$$-\frac{4}{1-\sin(x)}+\frac{2}{(1-\sin(x))^2}-\log(1-\sin(x))$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[x] + Tan[x])^5,x]
```

```
[Out] -Log[1 - Sin[x]] + 2/(1 - Sin[x])^2 - 4/(1 - Sin[x])
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2667

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rule 4391

```
Int[(u_.)*((b_.)*sec[(c_.) + (d_.)*(x_)]^(n_.) + (a_.)*tan[(c_.) + (d_.)*(x
_)]^(n_.))^(p_), x_Symbol] :> Int[ActivateTrig[u]*Sec[c + d*x]^(n*p)*(b + a
*Sin[c + d*x]^n)^p, x] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p]
```

Rubi steps

$$\begin{aligned}\int (\sec(x)+\tan(x))^5\,dx &= \int \sec^5(x)(1+\sin(x))^5\,dx\\ &= \text{Subst}\left(\int \frac{(1+x)^2}{(1-x)^3}\,dx, x, \sin(x)\right)\\ &= \text{Subst}\left(\int \left(\frac{1}{1-x}-\frac{4}{(-1+x)^3}-\frac{4}{(-1+x)^2}\right)dx, x, \sin(x)\right)\\ &= -\log(1-\sin(x))+\frac{2}{(1-\sin(x))^2}-\frac{4}{1-\sin(x)}\end{aligned}$$

Mathematica [A] time = 0.11, size = 54, normalized size = 1.80

$$\frac{11\tan^4(x)}{4}-\frac{\tan^2(x)}{2}+\frac{5\sec^4(x)}{4}+\tanh^{-1}(\sin(x))-\log(\cos(x))-\tan(x)\sec^3(x)+5\tan^3(x)\sec(x)+\tan(x)\sec(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[x] + Tan[x])^5,x]
```

```
[Out] ArcTanh[Sin[x]] - Log[Cos[x]] + (5*Sec[x]^4)/4 + Sec[x]*Tan[x] - Sec[x]^3*T
an[x] - Tan[x]^2/2 + 5*Sec[x]*Tan[x]^3 + (11*Tan[x]^4)/4
```

fricas [A] time = 0.96, size = 38, normalized size = 1.27

$$-\frac{\left(\cos(x)^2+2\,\sin(x)-2\right)\log\left(-\sin(x)+1\right)+4\,\sin(x)-2}{\cos(x)^2+2\,\sin(x)-2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((sec(x)+tan(x))^5,x, algorithm="fricas")
```

```
[Out] -((cos(x)^2 + 2*sin(x) - 2)*log(-sin(x) + 1) + 4*sin(x) - 2)/(cos(x)^2 + 2*
sin(x) - 2)
```

giac [B] time = 0.14, size = 62, normalized size = 2.07

$$\frac{25\,\tan\left(\frac{1}{2}\,x\right)^4-100\,\tan\left(\frac{1}{2}\,x\right)^3+198\,\tan\left(\frac{1}{2}\,x\right)^2-100\,\tan\left(\frac{1}{2}\,x\right)+25}{6\left(\tan\left(\frac{1}{2}\,x\right)-1\right)^4}+\log\left(\tan\left(\frac{1}{2}\,x\right)^2+1\right)-2\,\log\left(\left|\tan\left(\frac{1}{2}\,x\right)\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((sec(x)+tan(x))^5,x, algorithm="giac")
```

```
[Out] 1/6*(25*tan(1/2*x)^4 - 100*tan(1/2*x)^3 + 198*tan(1/2*x)^2 - 100*tan(1/2*x)
 + 25)/(tan(1/2*x) - 1)^4 + log(tan(1/2*x)^2 + 1) - 2*log(abs(tan(1/2*x) -
1))
```

maple [B] time = 0.12, size = 106, normalized size = 3.53

$$-\left(-\frac{\left(\sec^3(x)\right)}{4}-\frac{3\sec(x)}{8}\right)\tan(x)+\ln\left(\sec(x)+\tan(x)\right)+\frac{5}{4\cos(x)^4}+\frac{5\left(\sin^3(x)\right)}{2\cos(x)^4}+\frac{5\left(\sin^3(x)\right)}{4\cos(x)^2}-\frac{5\sin(x)}{8}+\frac{5\left(\sin^4(x\right.}{2\cos(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((sec(x)+tan(x))^5,x)
```

```
[Out] -(-1/4*sec(x)^3-3/8*sec(x))*tan(x)+ln(sec(x)+tan(x))+5/4/cos(x)^4+5/2*sin(x
)^3/cos(x)^4+5/4*sin(x)^3/cos(x)^2-5/8*sin(x)+5/2*sin(x)^4/cos(x)^4+5/4*sin
(x)^5/cos(x)^4-5/8*sin(x)^5/cos(x)^2-5/8*sin(x)^3+1/4*tan(x)^4-1/2*tan(x)^2
-ln(cos(x))
```

maxima [B] time = 0.34, size = 141, normalized size = 4.70

$$\frac{5}{2}\tan(x)^4+\frac{5\left(5\sin(x)^3-3\sin(x)\right)}{8\left(\sin(x)^4-2\sin(x)^2+1\right)}-\frac{3\sin(x)^3-5\sin(x)}{8\left(\sin(x)^4-2\sin(x)^2+1\right)}+\frac{5\left(\sin(x)^3+\sin(x)\right)}{4\left(\sin(x)^4-2\sin(x)^2+1\right)}+\frac{4\sin(x)}{4\left(\sin(x)^4-2\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((sec(x)+tan(x))^5,x, algorithm="maxima")
```

```
[Out] 5/2*tan(x)^4 + 5/8*(5*sin(x)^3 - 3*sin(x))/(sin(x)^4 - 2*sin(x)^2 + 1) - 1/
8*(3*sin(x)^3 - 5*sin(x))/(sin(x)^4 - 2*sin(x)^2 + 1) + 5/4*(sin(x)^3 + sin
(x))/(sin(x)^4 - 2*sin(x)^2 + 1) + 1/4*(4*sin(x)^2 - 3)/(sin(x)^4 - 2*sin(x
)^2 + 1) + 5/4/(sin(x)^2 - 1)^2 - 1/2*log(sin(x)^2 - 1) + 1/2*log(sin(x) +
1) - 1/2*log(sin(x) - 1)
```

mupad [B] time = 2.44, size = 59, normalized size = 1.97

$$\ln\left(\tan\left(\frac{x}{2}\right)^2+1\right)-2\,\ln\left(\tan\left(\frac{x}{2}\right)-1\right)+\frac{8\tan\left(\frac{x}{2}\right)^2}{\tan\left(\frac{x}{2}\right)^4-4\tan\left(\frac{x}{2}\right)^3+6\tan\left(\frac{x}{2}\right)^2-4\tan\left(\frac{x}{2}\right)+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((tan(x) + 1/cos(x))^5,x)
```

```
[Out] log(tan(x/2)^2 + 1) - 2*log(tan(x/2) - 1) + (8*tan(x/2)^2)/(6*tan(x/2)^2 -
4*tan(x/2) - 4*tan(x/2)^3 + tan(x/2)^4 + 1)
```

sympy [B] time = 7.13, size = 68, normalized size = 2.27

$$-\frac{\log\left(\sin\left(x\right)-1\right)}{2}+\frac{\log\left(\sin\left(x\right)+1\right)}{2}+\frac{\log\left(\sec^{2}\left(x\right)\right)}{2}+\frac{5\tan^{4}\left(x\right)}{2}+\frac{3\sec^{4}\left(x\right)}{2}-\sec^{2}\left(x\right)+\frac{32\sin^{3}\left(x\right)}{8\sin^{4}\left(x\right)-16\sin^{2}\left(x\right)+8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((sec(x)+tan(x))**5,x)
```

```
[Out] -log(sin(x) - 1)/2 + log(sin(x) + 1)/2 + log(sec(x)**2)/2 + 5*tan(x)**4/2 +
 3*sec(x)**4/2 - sec(x)**2 + 32*sin(x)**3/(8*sin(x)**4 - 16*sin(x)**2 + 8)
```

3.274 $\int (\sec(x) + \tan(x))^4 \, dx$

Optimal. Leaf size=30

$$x + \frac{2\cos^3(x)}{3(1-\sin(x))^3} - \frac{2\cos(x)}{1-\sin(x)}$$

```
[Out] x+2/3*cos(x)^3/(1-sin(x))^3-2*cos(x)/(1-sin(x))
```

Rubi [A] time = 0.10, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.571, Rules used = {4391, 2670, 2680, 8}

$$x + \frac{2\cos^3(x)}{3(1-\sin(x))^3} - \frac{2\cos(x)}{1-\sin(x)}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[x] + Tan[x])^4,x]
```

```
[Out] x + (2*Cos[x]^3)/(3*(1 - Sin[x])^3) - (2*Cos[x])/(1 - Sin[x])
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2670

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Dist[(a/g)^(2*m), Int[(g*Cos[e + f*x])^(2*m + p)/(a
 - b*Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2,
 0] && IntegerQ[m] && LtQ[p, -1] && GeQ[2*m + p, 0]
```

Rule 2680

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(2*g*(g*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f
*x])^(m + 1))/(b*f*(2*m + p + 1)), x] + Dist[(g^2*(p - 1))/(b^2*(2*m + p +
1)), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 2), x], x] /; F
reeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && LeQ[m, -2] && GtQ[p, 1] &&
 NeQ[2*m + p + 1, 0] &&  !ILtQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

Rule 4391

```
Int[(u_.)*((b_.)*sec[(c_.) + (d_.)*(x_)]^(n_.) + (a_.)*tan[(c_.) + (d_.)*(x
_)]^(n_.))^(p_), x_Symbol] :> Int[ActivateTrig[u]*Sec[c + d*x]^(n*p)*(b + a
```

```
*Sin[c + d*x]^n)^p, x] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p]
```

Rubi steps

$$\begin{aligned}\int (\sec(x)+\tan(x))^4\,dx &= \int \sec^4(x)(1+\sin(x))^4\,dx\\ &= \int \frac{\cos^4(x)}{(1-\sin(x))^4}\,dx\\ &= \frac{2\cos^3(x)}{3(1-\sin(x))^3}-\int \frac{\cos^2(x)}{(1-\sin(x))^2}\,dx\\ &= \frac{2\cos^3(x)}{3(1-\sin(x))^3}-\frac{2\cos(x)}{1-\sin(x)}+\int 1\,dx\\ &= x+\frac{2\cos^3(x)}{3(1-\sin(x))^3}-\frac{2\cos(x)}{1-\sin(x)}\end{aligned}$$

Mathematica [B] time = 0.13, size = 64, normalized size = 2.13

$$-\frac{-3(3x+8)\cos\left(\frac{x}{2}\right)+(3x+16)\cos\left(\frac{3x}{2}\right)+6\sin\left(\frac{x}{2}\right)(2x+x\cos(x)+4)}{6\left(\cos\left(\frac{x}{2}\right)-\sin\left(\frac{x}{2}\right)\right)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[x] + Tan[x])^4,x]
[Out] -1/6*(-3*(8 + 3*x)*Cos[x/2] + (16 + 3*x)*Cos[(3*x)/2] + 6*(4 + 2*x + x*Cos[
x])*Sin[x/2])/(Cos[x/2] - Sin[x/2])^3
```

fricas [B] time = 0.91, size = 61, normalized size = 2.03

$$\frac{(3x+8)\cos(x)^2-(3x-4)\cos(x)+((3x-8)\cos(x)+6x-4)\sin(x)-6x-4}{3\left(\cos(x)^2+(\cos(x)+2)\sin(x)-\cos(x)-2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((sec(x)+tan(x))^4,x, algorithm="fricas")
[Out] 1/3*((3*x + 8)*cos(x)^2 - (3*x - 4)*cos(x) + ((3*x - 8)*cos(x) + 6*x - 4)*s
in(x) - 6*x - 4)/(cos(x)^2 + (cos(x) + 2)*sin(x) - cos(x) - 2)
```

giac [A] time = 0.15, size = 20, normalized size = 0.67

$$x - \frac{8 \left(3 \tan\left(\frac{1}{2} x\right) - 1\right)}{3 \left(\tan\left(\frac{1}{2} x\right) - 1\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((sec(x)+tan(x))^4,x, algorithm="giac")
[Out] x - 8/3*(3*tan(1/2*x) - 1)/(tan(1/2*x) - 1)^3
```

maple [B] time = 0.10, size = 71, normalized size = 2.37

$$-\left(-\frac{2}{3} - \frac{\left(\sec^{2}(x)\right)}{3}\right)\tan(x) + \frac{4}{3\cos(x)^{3}} + \frac{2\left(\sin^{3}(x)\right)}{\cos(x)^{3}} + \frac{4\left(\sin^{4}(x)\right)}{3\cos(x)^{3}} - \frac{4\left(\sin^{4}(x)\right)}{3\cos(x)} - \frac{4\left(2+\sin^{2}(x)\right)\cos(x)}{3} + \frac{\left(\tan^{3}(x)\right)}{3} -$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((sec(x)+tan(x))^4,x)
[Out] -(-2/3-1/3*sec(x)^2)*tan(x)+4/3/cos(x)^3+2*sin(x)^3/cos(x)^3+4/3*sin(x)^4/c
os(x)^3-4/3*sin(x)^4/cos(x)-4/3*(2+sin(x)^2)*cos(x)+1/3*tan(x)^3-tan(x)+x
```

maxima [A] time = 0.44, size = 28, normalized size = 0.93

$$\frac{8}{3} \tan(x)^{3} + x - \frac{4 \left(3 \cos(x)^{2} - 1\right)}{3 \cos(x)^{3}} + \frac{4}{3 \cos(x)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((sec(x)+tan(x))^4,x, algorithm="maxima")
[Out] 8/3*tan(x)^3 + x - 4/3*(3*cos(x)^2 - 1)/cos(x)^3 + 4/3/cos(x)^3
```

mupad [B] time = 2.37, size = 20, normalized size = 0.67

$$x - \frac{8\tan\left(\frac{x}{2}\right) - \frac{8}{3}}{\left(\tan\left(\frac{x}{2}\right) - 1\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((tan(x) + 1/cos(x))^4,x)
[Out] x - (8*tan(x/2) - 8/3)/(tan(x/2) - 1)^3
```

sympy [A] time = 4.10, size = 44, normalized size = 1.47

$$x + \frac{\sin^3(x)}{3\cos^3(x)} - \frac{\sin(x)}{\cos(x)} + \frac{7\tan^3(x)}{3} + \tan(x) + \frac{8\sec^3(x)}{3} - 4\sec(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((sec(x)+tan(x))**4,x)
```

```
[Out] x + sin(x)**3/(3*cos(x)**3) - sin(x)/cos(x) + 7*tan(x)**3/3 + tan(x) + 8*se
c(x)**3/3 - 4*sec(x)
```

3.275 $\int (\sec(x)+\tan(x))^3\,dx$

Optimal. Leaf size=18

$$\frac{2}{1-\sin(x)}+\log(1-\sin(x))$$

[Out] ln(1-sin(x))+2/(1-sin(x))

Rubi [A] time = 0.04, antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {4391, 2667, 43}

$$\frac{2}{1-\sin(x)}+\log(1-\sin(x))$$

Antiderivative was successfully verified.

[In] Int[(Sec[x] + Tan[x])^3,x]

[Out] Log[1 - Sin[x]] + 2/(1 - Sin[x])

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2667

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rule 4391

```
Int[(u_.)*((b_.)*sec[(c_.) + (d_.)*(x_)]^(n_.) + (a_.)*tan[(c_.) + (d_.)*(x
_)]^(n_.))^(p_), x_Symbol] :> Int[ActivateTrig[u]*Sec[c + d*x]^(n*p)*(b + a
*Sin[c + d*x]^n)^p, x] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p]
```

Rubi steps

$$\begin{aligned}\int (\sec(x)+\tan(x))^3\,dx &= \int \sec^3(x)(1+\sin(x))^3\,dx\\ &= \text{Subst}\left(\int \frac{1+x}{(1-x)^2}\,dx, x, \sin(x)\right)\\ &= \text{Subst}\left(\int \left(\frac{2}{(-1+x)^2}+\frac{1}{-1+x}\right)dx, x, \sin(x)\right)\\ &= \log(1-\sin(x))+\frac{2}{1-\sin(x)}\end{aligned}$$

Mathematica [A] time = 0.02, size = 31, normalized size = 1.72

$$\frac{\tan^2(x)}{2}+\frac{3\sec^2(x)}{2}-\tanh^{-1}(\sin(x))+\log(\cos(x))+2\tan(x)\sec(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[x] + Tan[x])^3,x]
```

```
[Out] -ArcTanh[Sin[x]] + Log[Cos[x]] + (3*Sec[x]^2)/2 + 2*Sec[x]*Tan[x] + Tan[x]^
2/2
```

fricas [A] time = 0.91, size = 21, normalized size = 1.17

$$\frac{(\sin(x)-1)\log(-\sin(x)+1)-2}{\sin(x)-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((sec(x)+tan(x))^3,x, algorithm="fricas")
```

```
[Out] ((sin(x) - 1)*log(-sin(x) + 1) - 2)/(sin(x) - 1)
```

giac [B] time = 0.17, size = 48, normalized size = 2.67

$$-\frac{3\tan\left(\frac{1}{2}x\right)^2-10\tan\left(\frac{1}{2}x\right)+3}{\left(\tan\left(\frac{1}{2}x\right)-1\right)^2}-\log\left(\tan\left(\frac{1}{2}x\right)^2+1\right)+2\log\left(\left|\tan\left(\frac{1}{2}x\right)-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((sec(x)+tan(x))^3,x, algorithm="giac")
```

[Out] -(3*tan(1/2*x)^2 - 10*tan(1/2*x) + 3)/(tan(1/2*x) - 1)^2 - log(tan(1/2*x)^2 + 1) + 2*log(abs(tan(1/2*x) - 1))

maple [B] time = 0.10, size = 45, normalized size = 2.50

$$\frac{\sec(x)\tan(x)}{2}-\ln(\sec(x)+\tan(x))+\frac{3}{2\cos(x)^2}+\frac{3\left(\sin^3(x)\right)}{2\cos(x)^2}+\frac{3\sin(x)}{2}+\frac{\left(\tan^2(x)\right)}{2}+\ln(\cos(x))$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((sec(x)+tan(x))^3,x)

[Out] 1/2*sec(x)*tan(x)-ln(sec(x)+tan(x))+3/2/cos(x)^2+3/2*sin(x)^3/cos(x)^2+3/2*sin(x)+1/2*tan(x)^2+ln(cos(x))

maxima [B] time = 0.33, size = 52, normalized size = 2.89

$$\frac{3}{2}\tan(x)^2-\frac{2\sin(x)}{\sin(x)^2-1}-\frac{1}{2\left(\sin(x)^2-1\right)}+\frac{1}{2}\log\left(\sin(x)^2-1\right)-\frac{1}{2}\log(\sin(x)+1)+\frac{1}{2}\log(\sin(x)-1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((sec(x)+tan(x))^3,x, algorithm="maxima")

[Out] 3/2*tan(x)^2 - 2*sin(x)/(sin(x)^2 - 1) - 1/2/(sin(x)^2 - 1) + 1/2*log(sin(x)^2 - 1) - 1/2*log(sin(x) + 1) + 1/2*log(sin(x) - 1)

mupad [B] time = 2.40, size = 43, normalized size = 2.39

$$2\ln\left(\tan\left(\frac{x}{2}\right)-1\right)-\ln\left(\tan\left(\frac{x}{2}\right)^2+1\right)+\frac{4\tan\left(\frac{x}{2}\right)}{\tan\left(\frac{x}{2}\right)^2-2\tan\left(\frac{x}{2}\right)+1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((tan(x) + 1/cos(x))^3,x)

[Out] 2*log(tan(x/2) - 1) - log(tan(x/2)^2 + 1) + (4*tan(x/2))/(tan(x/2)^2 - 2*tan(x/2) + 1)

sympy [B] time = 4.81, size = 44, normalized size = 2.44

$$\frac{\log(\sin(x)-1)}{2}-\frac{\log(\sin(x)+1)}{2}-\frac{\log\left(\sec^2(x)\right)}{2}+2\sec^2(x)-\frac{4\sin(x)}{2\sin^2(x)-2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((sec(x)+tan(x))**3,x)

[Out] log(sin(x) - 1)/2 - log(sin(x) + 1)/2 - log(sec(x)**2)/2 + 2*sec(x)**2 - 4*
sin(x)/(2*sin(x)**2 - 2)
```

3.276 $\int (\sec(x)+\tan(x))^2\,dx$

Optimal. Leaf size=16

$$\frac{2\cos(x)}{1-\sin(x)}-x$$

[Out] -x+2*cos(x)/(1-sin(x))

Rubi [A] time = 0.07, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.571, Rules used = {4391, 2670, 2680, 8}

$$\frac{2\cos(x)}{1-\sin(x)}-x$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[x] + Tan[x])^2,x]
```

```
[Out] -x + (2*Cos[x])/(1 - Sin[x])
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2670

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Dist[(a/g)^(2*m), Int[(g*Cos[e + f*x])^(2*m + p)/(a
 - b*Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2,
 0] && IntegerQ[m] && LtQ[p, -1] && GeQ[2*m + p, 0]
```

Rule 2680

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(2*g*(g*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f
*x])^(m + 1))/(b*f*(2*m + p + 1)), x] + Dist[(g^2*(p - 1))/(b^2*(2*m + p +
1)), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 2), x], x] /; F
reeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && LeQ[m, -2] && GtQ[p, 1] &&
 NeQ[2*m + p + 1, 0] &&  !ILtQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

Rule 4391

```
Int[(u_.)*((b_.)*sec[(c_.) + (d_.)*(x_)]^(n_.) + (a_.)*tan[(c_.) + (d_.)*(x
_)]^(n_.))^(p_), x_Symbol] :> Int[ActivateTrig[u]*Sec[c + d*x]^(n*p)*(b + a
```

```
*Sin[c + d*x]^n)^p, x] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p]
```

Rubi steps

$$\begin{aligned}\int (\sec(x)+\tan(x))^2\,dx &= \int \sec^2(x)(1+\sin(x))^2\,dx\\ &= \int \frac{\cos^2(x)}{(1-\sin(x))^2}\,dx\\ &= \frac{2\cos(x)}{1-\sin(x)} - \int 1\,dx\\ &= -x+\frac{2\cos(x)}{1-\sin(x)}\end{aligned}$$

Mathematica [A] time = 0.01, size = 14, normalized size = 0.88

$$-\tan^{-1}(\tan(x))+2\tan(x)+2\sec(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[x] + Tan[x])^2,x]
[Out] -ArcTan[Tan[x]] + 2*Sec[x] + 2*Tan[x]
```

fricas [A] time = 0.89, size = 28, normalized size = 1.75

$$-\frac{(x-2)\cos(x)-(x+2)\sin(x)+x-2}{\cos(x)-\sin(x)+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((sec(x)+tan(x))^2,x, algorithm="fricas")
[Out] -((x - 2)*cos(x) - (x + 2)*sin(x) + x - 2)/(cos(x) - sin(x) + 1)
```

giac [A] time = 0.13, size = 14, normalized size = 0.88

$$-x-\frac{4}{\tan\left(\frac{1}{2}\,x\right)-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((sec(x)+tan(x))^2,x, algorithm="giac")
```

```
[Out] -x - 4/(tan(1/2*x) - 1)
```

maple [A] time = 0.05, size = 15, normalized size = 0.94

$$2\tan(x) + \frac{2}{\cos(x)} - x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((sec(x)+tan(x))^2,x)
[Out] 2*tan(x)+2/cos(x)-x
```

maxima [A] time = 0.45, size = 14, normalized size = 0.88

$$-x + \frac{2}{\cos(x)} + 2\tan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((sec(x)+tan(x))^2,x, algorithm="maxima")
[Out] -x + 2/cos(x) + 2*tan(x)
```

mupad [B] time = 2.36, size = 14, normalized size = 0.88

$$-x - \frac{4}{\tan\left(\frac{x}{2}\right) - 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((tan(x) + 1/cos(x))^2,x)
[Out] - x - 4/(tan(x/2) - 1)
```

sympy [A] time = 1.11, size = 10, normalized size = 0.62

$$-x + 2\tan(x) + 2\sec(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((sec(x)+tan(x))**2,x)
[Out] -x + 2*tan(x) + 2*sec(x)
```

3.277 $\int (\sec(x) + \tan(x))\,dx$

Optimal. Leaf size=13

$$-2\log\left(\cos\left(\frac{1}{4}(2x+\pi)\right)\right)$$

```
[Out] -2*ln(cos(1/4*Pi+1/2*x))
```

Rubi [A] time = 0.01, antiderivative size = 9, normalized size of antiderivative = 0.69, number of steps used = 3, number of rules used = 2, integrand size = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {3770, 3475}

$$\tanh^{-1}(\sin(x)) - \log(\cos(x))$$

Antiderivative was successfully verified.

```
[In] Int[Sec[x] + Tan[x],x]
```

```
[Out] ArcTanh[Sin[x]] - Log[Cos[x]]
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int (\sec(x)+\tan(x))\,dx &= \int \sec(x)\,dx + \int \tan(x)\,dx\\ &= \tanh^{-1}(\sin(x)) - \log(\cos(x))\end{aligned}$$

Mathematica [B] time = 0.00, size = 38, normalized size = 2.92

$$-\log(\cos(x)) - \log\left(\cos\left(\frac{x}{2}\right) - \sin\left(\frac{x}{2}\right)\right) + \log\left(\sin\left(\frac{x}{2}\right) + \cos\left(\frac{x}{2}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[x] + Tan[x],x]
[Out] -Log[Cos[x]] - Log[Cos[x/2] - Sin[x/2]] + Log[Cos[x/2] + Sin[x/2]]
```

fricas [A] time = 3.01, size = 9, normalized size = 0.69

$$-\log\left(-\sin(x)+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)+tan(x),x, algorithm="fricas")
[Out] -log(-sin(x) + 1)
```

giac [B] time = 0.15, size = 31, normalized size = 2.38

$$\frac{1}{4}\log\left(\left|\frac{1}{\sin(x)}+\sin(x)+2\right|\right)-\frac{1}{4}\log\left(\left|\frac{1}{\sin(x)}+\sin(x)-2\right|\right)-\log\left(|\cos(x)|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)+tan(x),x, algorithm="giac")
[Out] 1/4*log(abs(1/sin(x) + sin(x) + 2)) - 1/4*log(abs(1/sin(x) + sin(x) - 2)) -
 log(abs(cos(x)))
```

maple [A] time = 0.00, size = 13, normalized size = 1.00

$$\ln\left(\sec(x)+\tan(x)\right)-\ln\left(\cos(x)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(x)+tan(x),x)
[Out] ln(sec(x)+tan(x))-ln(cos(x))
```

maxima [A] time = 0.31, size = 10, normalized size = 0.77

$$\log\left(\sec(x)+\tan(x)\right)+\log\left(\sec(x)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)+tan(x),x, algorithm="maxima")
[Out] log(sec(x) + tan(x)) + log(sec(x))
```

mupad [B] time = 2.40, size = 19, normalized size = 1.46

$$\ln\left(\tan\left(\frac{x}{2}\right)^2+1\right)-2\,\ln\left(\tan\left(\frac{x}{2}\right)-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(x) + 1/cos(x),x)
```

```
[Out] log(tan(x/2)^2 + 1) - 2*log(tan(x/2) - 1)
```

sympy [A] time = 0.09, size = 20, normalized size = 1.54

$$-\frac{\log\left(\sin\left(x\right)-1\right)}{2}+\frac{\log\left(\sin\left(x\right)+1\right)}{2}-\log\left(\cos\left(x\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)+tan(x),x)
```

```
[Out] -log(sin(x) - 1)/2 + log(sin(x) + 1)/2 - log(cos(x))
```

3.278 $\int \frac{1}{\sec(x)+\tan(x)}\,dx$

Optimal. Leaf size=5

$$\log(\sin(x)+1)$$

```
[Out] ln(1+sin(x))
```

Rubi [A] time = 0.02, antiderivative size = 5, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {3159, 2667, 31}

$$\log(\sin(x)+1)$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[x] + Tan[x])^(-1),x]
```

```
[Out] Log[1 + Sin[x]]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2667

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rule 3159

```
Int[((a_.) + (b_.)*sec[(d_.) + (e_.)*(x_)] + (c_.)*tan[(d_.) + (e_.)*(x_)])
^(-1), x_Symbol] :> Int[Cos[d + e*x]/(b + a*Cos[d + e*x] + c*Sin[d + e*x]),
 x] /; FreeQ[{a, b, c, d, e}, x]
```

Rubi steps

$$\int \frac{1}{\sec(x)+\tan(x)}\,dx = \int \frac{\cos(x)}{1+\sin(x)}\,dx$$
$$= \text{Subst}\left(\int \frac{1}{1+x}\,dx, x, \sin(x)\right)$$
$$= \log(1+\sin(x))$$

Mathematica [B] time = 0.01, size = 16, normalized size = 3.20

$$2\log\left(\sin\left(\frac{x}{2}\right)+\cos\left(\frac{x}{2}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[x] + Tan[x])^(-1),x]
[Out] 2*Log[Cos[x/2] + Sin[x/2]]
```

fricas [A] time = 0.61, size = 5, normalized size = 1.00

$$\log\left(\sin(x)+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sec(x)+tan(x)),x, algorithm="fricas")
[Out] log(sin(x) + 1)
```

giac [B] time = 0.14, size = 22, normalized size = 4.40

$$-\log\left(\tan\left(\frac{1}{2}\,x\right)^2+1\right)+2\,\log\left(\left|\tan\left(\frac{1}{2}\,x\right)+1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sec(x)+tan(x)),x, algorithm="giac")
[Out] -log(tan(1/2*x)^2 + 1) + 2*log(abs(tan(1/2*x) + 1))
```

maple [A] time = 0.12, size = 6, normalized size = 1.20

$$\ln\left(1+\sin(x)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sec(x)+tan(x)),x)
```

```
[Out] ln(1+sin(x))
```

maxima [B] time = 0.32, size = 31, normalized size = 6.20

$$2 \log\left(\frac{\sin(x)}{\cos(x)+1}+1\right)-\log\left(\frac{\sin(x)^2}{(\cos(x)+1)^2}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sec(x)+tan(x)),x, algorithm="maxima")
```

```
[Out] 2*log(sin(x)/(cos(x) + 1) + 1) - log(sin(x)^2/(cos(x) + 1)^2 + 1)
```

mupad [B] time = 2.78, size = 21, normalized size = 4.20

$$2 \ln\left(\tan\left(\frac{x}{2}\right)+1\right)-\ln\left(\tan\left(\frac{x}{2}\right)^2+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(tan(x) + 1/cos(x)),x)
```

```
[Out] 2*log(tan(x/2) + 1) - log(tan(x/2)^2 + 1)
```

sympy [B] time = 0.14, size = 17, normalized size = 3.40

$$\log\left(\tan\left(x\right)+\sec\left(x\right)\right)-\frac{\log\left(\tan^2\left(x\right)+1\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sec(x)+tan(x)),x)
```

```
[Out] log(tan(x) + sec(x)) - log(tan(x)**2 + 1)/2
```

3.279 $\int \frac{1}{(\sec(x)+\tan(x))^2}\,dx$

Optimal. Leaf size=14

$$-x-\frac{2\cos(x)}{\sin(x)+1}$$

```
[Out] -x-2*cos(x)/(1+sin(x))
```

Rubi [A] time = 0.04, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {4391, 2680, 8}

$$-x-\frac{2\cos(x)}{\sin(x)+1}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[x] + Tan[x])^(-2),x]
```

```
[Out] -x - (2*Cos[x])/(1 + Sin[x])
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2680

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(2*g*(g*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f
*x])^(m + 1))/(b*f*(2*m + p + 1)), x] + Dist[(g^2*(p - 1))/(b^2*(2*m + p +
1)), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 2), x], x] /; F
reeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && LeQ[m, -2] && GtQ[p, 1] &&
 NeQ[2*m + p + 1, 0] &&  !ILtQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

Rule 4391

```
Int[(u_.)*((b_.)*sec[(c_.) + (d_.)*(x_)]^(n_.) + (a_.)*tan[(c_.) + (d_.)*(x
_)]^(n_.))^(p_), x_Symbol] :> Int[ActivateTrig[u]*Sec[c + d*x]^(n*p)*(b + a
*Sin[c + d*x]^n)^p, x] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p]
```

Rubi steps

$$\int \frac{1}{(\sec(x)+\tan(x))^2}\,dx = \int \frac{\cos^2(x)}{(1+\sin(x))^2}\,dx$$
$$= -\frac{2\cos(x)}{1+\sin(x)} - \int 1\,dx$$
$$= -x - \frac{2\cos(x)}{1+\sin(x)}$$

Mathematica [A] time = 0.02, size = 27, normalized size = 1.93

$$\frac{4\sin\left(\frac{x}{2}\right)}{\sin\left(\frac{x}{2}\right)+\cos\left(\frac{x}{2}\right)} - x$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[x] + Tan[x])^(-2),x]
[Out] -x + (4*Sin[x/2])/(Cos[x/2] + Sin[x/2])
```

fricas [A] time = 1.01, size = 25, normalized size = 1.79

$$-\frac{(x+2)\cos(x)+(x-2)\sin(x)+x+2}{\cos(x)+\sin(x)+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sec(x)+tan(x))^2,x, algorithm="fricas")
[Out] -((x + 2)*cos(x) + (x - 2)*sin(x) + x + 2)/(cos(x) + sin(x) + 1)
```

giac [A] time = 0.15, size = 14, normalized size = 1.00

$$-x - \frac{4}{\tan\left(\frac{1}{2}\,x\right)+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sec(x)+tan(x))^2,x, algorithm="giac")
[Out] -x - 4/(tan(1/2*x) + 1)
```

maple [A] time = 0.12, size = 15, normalized size = 1.07

$$-\frac{4}{1+\tan\left(\frac{x}{2}\right)} - x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sec(x)+tan(x))^2,x)
```

```
[Out] -4/(1+tan(1/2*x))-x
```

maxima [A] time = 0.41, size = 28, normalized size = 2.00

$$-\frac{4}{\frac{\sin(x)}{\cos(x)+1}+1}-2\arctan\left(\frac{\sin(x)}{\cos(x)+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sec(x)+tan(x))^2,x, algorithm="maxima")
```

```
[Out] -4/(sin(x)/(cos(x) + 1) + 1) - 2*arctan(sin(x)/(cos(x) + 1))
```

mupad [B] time = 2.34, size = 14, normalized size = 1.00

$$-x-\frac{4}{\tan\left(\frac{x}{2}\right)+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(tan(x) + 1/cos(x))^2,x)
```

```
[Out] - x - 4/(tan(x/2) + 1)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{(\tan(x)+\sec(x))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sec(x)+tan(x))**2,x)
```

```
[Out] Integral((tan(x) + sec(x))**(-2), x)
```

3.280 $\int \frac{1}{(\sec(x)+\tan(x))^3} dx$

Optimal. Leaf size=16

$$-\frac{2}{\sin(x)+1} - \log(\sin(x)+1)$$

```
[Out] -ln(1+sin(x))-2/(1+sin(x))
```

Rubi [A] time = 0.05, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {4391, 2667, 43}

$$-\frac{2}{\sin(x)+1} - \log(\sin(x)+1)$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[x] + Tan[x])^(-3),x]
```

```
[Out] -Log[1 + Sin[x]] - 2/(1 + Sin[x])
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2667

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rule 4391

```
Int[(u_.)*((b_.)*sec[(c_.) + (d_.)*(x_)]^(n_.) + (a_.)*tan[(c_.) + (d_.)*(x
_)]^(n_.))^(p_), x_Symbol] :> Int[ActivateTrig[u]*Sec[c + d*x]^(n*p)*(b + a
*Sin[c + d*x]^n)^p, x] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p]
```

Rubi steps

$$\int \frac{1}{(\sec(x)+\tan(x))^3}\,dx=\int \frac{\cos^3(x)}{(1+\sin(x))^3}\,dx$$
$$=\text{Subst}\left(\int \frac{1-x}{(1+x)^2}\,dx,x,\sin(x)\right)$$
$$=\text{Subst}\left(\int \left(\frac{1}{-1-x}+\frac{2}{(1+x)^2}\right)dx,x,\sin(x)\right)$$
$$=-\log(1+\sin(x))-\frac{2}{1+\sin(x)}$$

Mathematica [B] time = 0.02, size = 34, normalized size = 2.12

$$-\frac{2}{\left(\sin\left(\frac{x}{2}\right)+\cos\left(\frac{x}{2}\right)\right)^2}-2\log\left(\sin\left(\frac{x}{2}\right)+\cos\left(\frac{x}{2}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[x] + Tan[x])^(-3),x]
[Out] -2*Log[Cos[x/2] + Sin[x/2]] - 2/(Cos[x/2] + Sin[x/2])^2
```

fricas [A] time = 1.07, size = 20, normalized size = 1.25

$$-\frac{(\sin(x)+1)\log(\sin(x)+1)+2}{\sin(x)+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sec(x)+tan(x))^3,x, algorithm="fricas")
[Out] -((sin(x) + 1)*log(sin(x) + 1) + 2)/(sin(x) + 1)
```

giac [B] time = 0.16, size = 45, normalized size = 2.81

$$\frac{3\,\tan\left(\frac{1}{2}\,x\right)^2+10\,\tan\left(\frac{1}{2}\,x\right)+3}{\left(\tan\left(\frac{1}{2}\,x\right)+1\right)^2}+\log\left(\tan\left(\frac{1}{2}\,x\right)^2+1\right)-2\,\log\left(\left|\tan\left(\frac{1}{2}\,x\right)+1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sec(x)+tan(x))^3,x, algorithm="giac")
```

```
[Out] (3*tan(1/2*x)^2 + 10*tan(1/2*x) + 3)/(tan(1/2*x) + 1)^2 + log(tan(1/2*x)^2
+ 1) - 2*log(abs(tan(1/2*x) + 1))
```

maple [A] time = 0.16, size = 17, normalized size = 1.06

$$-\ln\left(1+\sin(x)\right)-\frac{2}{1+\sin(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sec(x)+tan(x))^3,x)
```

```
[Out] -ln(1+sin(x))-2/(1+sin(x))
```

maxima [B] time = 0.44, size = 64, normalized size = 4.00

$$\frac{4\,\sin(x)}{\left(\frac{2\,\sin(x)}{\cos(x)+1}+\frac{\sin(x)^2}{(\cos(x)+1)^2}+1\right)(\cos(x)+1)}-2\,\log\left(\frac{\sin(x)}{\cos(x)+1}+1\right)+\log\left(\frac{\sin(x)^2}{(\cos(x)+1)^2}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sec(x)+tan(x))^3,x, algorithm="maxima")
```

```
[Out] 4*sin(x)/((2*sin(x)/(cos(x) + 1) + sin(x)^2/(cos(x) + 1)^2 + 1)*(cos(x) + 1
)) - 2*log(sin(x)/(cos(x) + 1) + 1) + log(sin(x)^2/(cos(x) + 1)^2 + 1)
```

mupad [B] time = 2.36, size = 41, normalized size = 2.56

$$\ln\left(\tan\left(\frac{x}{2}\right)^2+1\right)-2\,\ln\left(\tan\left(\frac{x}{2}\right)+1\right)+\frac{4\tan\left(\frac{x}{2}\right)}{\tan\left(\frac{x}{2}\right)^2+2\tan\left(\frac{x}{2}\right)+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(tan(x) + 1/cos(x))^3,x)
```

```
[Out] log(tan(x/2)^2 + 1) - 2*log(tan(x/2) + 1) + (4*tan(x/2))/(2*tan(x/2) + tan(
x/2)^2 + 1)
```

sympy [B] time = 0.83, size = 301, normalized size = 18.81

$$-\frac{2\log\left(\tan\left(x\right)+\sec\left(x\right)\right)\tan^2\left(x\right)}{2\tan^2\left(x\right)+4\tan\left(x\right)\sec\left(x\right)+2\sec^2\left(x\right)}-\frac{4\log\left(\tan\left(x\right)+\sec\left(x\right)\right)\tan\left(x\right)\sec\left(x\right)}{2\tan^2\left(x\right)+4\tan\left(x\right)\sec\left(x\right)+2\sec^2\left(x\right)}-\frac{2\log\left(\tan\left(x\right)+\sec\left(x\right)\right.}{2\tan^2\left(x\right)+4\tan\left(x\right)\sec\left(x\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sec(x)+tan(x))**3,x)
```

```
[Out] -2*log(tan(x) + sec(x))*tan(x)**2/(2*tan(x)**2 + 4*tan(x)*sec(x) + 2*sec(x)
**2) - 4*log(tan(x) + sec(x))*tan(x)*sec(x)/(2*tan(x)**2 + 4*tan(x)*sec(x)
+ 2*sec(x)**2) - 2*log(tan(x) + sec(x))*sec(x)**2/(2*tan(x)**2 + 4*tan(x)*s
ec(x) + 2*sec(x)**2) + log(tan(x)**2 + 1)*tan(x)**2/(2*tan(x)**2 + 4*tan(x)
*sec(x) + 2*sec(x)**2) + 2*log(tan(x)**2 + 1)*tan(x)*sec(x)/(2*tan(x)**2 +
4*tan(x)*sec(x) + 2*sec(x)**2) + log(tan(x)**2 + 1)*sec(x)**2/(2*tan(x)**2
+ 4*tan(x)*sec(x) + 2*sec(x)**2) + tan(x)**2/(2*tan(x)**2 + 4*tan(x)*sec(x)
 + 2*sec(x)**2) - sec(x)**2/(2*tan(x)**2 + 4*tan(x)*sec(x) + 2*sec(x)**2) -
 1/(2*tan(x)**2 + 4*tan(x)*sec(x) + 2*sec(x)**2)
```

3.281 $\int \frac{1}{(\sec(x)+\tan(x))^4}\,dx$

Optimal. Leaf size=26

$$x-\frac{2\cos^3(x)}{3(\sin(x)+1)^3}+\frac{2\cos(x)}{\sin(x)+1}$$

```
[Out] x-2/3*cos(x)^3/(1+sin(x))^3+2*cos(x)/(1+sin(x))
```

Rubi [A] time = 0.07, antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {4391, 2680, 8}

$$x-\frac{2\cos^3(x)}{3(\sin(x)+1)^3}+\frac{2\cos(x)}{\sin(x)+1}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[x] + Tan[x])^(-4),x]
```

```
[Out] x - (2*Cos[x]^3)/(3*(1 + Sin[x])^3) + (2*Cos[x])/(1 + Sin[x])
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2680

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(2*g*(g*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f
*x])^(m + 1))/(b*f*(2*m + p + 1)), x] + Dist[(g^2*(p - 1))/(b^2*(2*m + p +
1)), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 2), x], x] /; F
reeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && LeQ[m, -2] && GtQ[p, 1] &&
 NeQ[2*m + p + 1, 0] &&  !ILtQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

Rule 4391

```
Int[(u_.)*((b_.)*sec[(c_.) + (d_.)*(x_)]^(n_.) + (a_.)*tan[(c_.) + (d_.)*(x
_)]^(n_.))^(p_), x_Symbol] :> Int[ActivateTrig[u]*Sec[c + d*x]^(n*p)*(b + a
*Sin[c + d*x]^n)^p, x] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{(\sec(x)+\tan(x))^4}\,dx &= \int \frac{\cos^4(x)}{(1+\sin(x))^4}\,dx \\
&= -\frac{2\cos^3(x)}{3(1+\sin(x))^3} - \int \frac{\cos^2(x)}{(1+\sin(x))^2}\,dx \\
&= -\frac{2\cos^3(x)}{3(1+\sin(x))^3} + \frac{2\cos(x)}{1+\sin(x)} + \int 1\,dx \\
&= x - \frac{2\cos^3(x)}{3(1+\sin(x))^3} + \frac{2\cos(x)}{1+\sin(x)}
\end{aligned}
$$

Mathematica [B] time = 0.07, size = 62, normalized size = 2.38

$$
\frac{3(3x-8)\cos\left(\frac{x}{2}\right)+(16-3x)\cos\left(\frac{3x}{2}\right)+6\sin\left(\frac{x}{2}\right)(2x+x\cos(x)-4)}{6\left(\sin\left(\frac{x}{2}\right)+\cos\left(\frac{x}{2}\right)\right)^3}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[x] + Tan[x])^(-4),x]
```

```
[Out] (3*(-8 + 3*x)*Cos[x/2] + (16 - 3*x)*Cos[(3*x)/2] + 6*(-4 + 2*x + x*Cos[x])*
Sin[x/2])/(6*(Cos[x/2] + Sin[x/2])^3)
```

fricas [B] time = 0.86, size = 63, normalized size = 2.42

$$
\frac{(3x-8)\cos(x)^2-(3x+4)\cos(x)-((3x+8)\cos(x)+6x+4)\sin(x)-6x+4}{3\left(\cos(x)^2-(\cos(x)+2)\sin(x)-\cos(x)-2\right)}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sec(x)+tan(x))^4,x, algorithm="fricas")
```

```
[Out] 1/3*((3*x - 8)*cos(x)^2 - (3*x + 4)*cos(x) - ((3*x + 8)*cos(x) + 6*x + 4)*s
in(x) - 6*x + 4)/(cos(x)^2 - (cos(x) + 2)*sin(x) - cos(x) - 2)
```

giac [A] time = 0.16, size = 20, normalized size = 0.77

$$
x + \frac{8\left(3\tan\left(\frac{1}{2}x\right)+1\right)}{3\left(\tan\left(\frac{1}{2}x\right)+1\right)^3}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sec(x)+tan(x))^4,x, algorithm="giac")
```

```
[Out] x + 8/3*(3*tan(1/2*x) + 1)/(tan(1/2*x) + 1)^3
```

maple [A] time = 0.15, size = 23, normalized size = 0.88

$$-\frac{16}{3\left(1+\tan\left(\frac{x}{2}\right)\right)^{3}}+\frac{8}{\left(1+\tan\left(\frac{x}{2}\right)\right)^{2}}+x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sec(x)+tan(x))^4,x)
```

```
[Out] -16/3/(1+tan(1/2*x))^3+8/(1+tan(1/2*x))^2+x
```

maxima [B] time = 0.44, size = 64, normalized size = 2.46

$$\frac{8\left(\frac{3\,\sin(x)}{\cos(x)+1}+1\right)}{3\left(\frac{3\,\sin(x)}{\cos(x)+1}+\frac{3\,\sin(x)^{2}}{(\cos(x)+1)^{2}}+\frac{\sin(x)^{3}}{(\cos(x)+1)^{3}}+1\right)}+2\,\arctan\left(\frac{\sin(x)}{\cos(x)+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sec(x)+tan(x))^4,x, algorithm="maxima")
```

```
[Out] 8/3*(3*sin(x)/(cos(x) + 1) + 1)/(3*sin(x)/(cos(x) + 1) + 3*sin(x)^2/(cos(x)
 + 1)^2 + sin(x)^3/(cos(x) + 1)^3 + 1) + 2*arctan(sin(x)/(cos(x) + 1))
```

mupad [B] time = 2.34, size = 19, normalized size = 0.73

$$x+\frac{8\tan\left(\frac{x}{2}\right)+\frac{8}{3}}{\left(\tan\left(\frac{x}{2}\right)+1\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(tan(x) + 1/cos(x))^4,x)
```

```
[Out] x + (8*tan(x/2) + 8/3)/(tan(x/2) + 1)^3
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{\left(\tan\left(x\right)+\sec\left(x\right)\right)^{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sec(x)+tan(x))**4,x)
```

```
[Out] Integral((tan(x) + sec(x))**(-4), x)
```

3.282 $\int \frac{1}{(\sec(x)+\tan(x))^5} \, dx$

Optimal. Leaf size=22

$$\frac{4}{\sin(x)+1} - \frac{2}{(\sin(x)+1)^2} + \log(\sin(x)+1)$$

```
[Out] ln(1+sin(x))-2/(1+sin(x))^2+4/(1+sin(x))
```

Rubi [A] time = 0.05, antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {4391, 2667, 43}

$$\frac{4}{\sin(x)+1} - \frac{2}{(\sin(x)+1)^2} + \log(\sin(x)+1)$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[x] + Tan[x])^(-5),x]
```

```
[Out] Log[1 + Sin[x]] - 2/(1 + Sin[x])^2 + 4/(1 + Sin[x])
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2667

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rule 4391

```
Int[(u_.)*((b_.)*sec[(c_.) + (d_.)*(x_)]^(n_.) + (a_.)*tan[(c_.) + (d_.)*(x
_)]^(n_.))^(p_), x_Symbol] :> Int[ActivateTrig[u]*Sec[c + d*x]^(n*p)*(b + a
*Sin[c + d*x]^n)^p, x] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p]
```

Rubi steps

$$\int \frac{1}{(\sec(x)+\tan(x))^5}\,dx = \int \frac{\cos^5(x)}{(1+\sin(x))^5}\,dx$$
$$= \text{Subst}\left(\int \frac{(1-x)^2}{(1+x)^3}\,dx, x, \sin(x)\right)$$
$$= \text{Subst}\left(\int \left(\frac{4}{(1+x)^3} - \frac{4}{(1+x)^2} + \frac{1}{1+x}\right)dx, x, \sin(x)\right)$$
$$= \log(1+\sin(x)) - \frac{2}{(1+\sin(x))^2} + \frac{4}{1+\sin(x)}$$

Mathematica [A] time = 0.05, size = 39, normalized size = 1.77

$$\frac{4\sin(x)+2}{\left(\sin\left(\frac{x}{2}\right)+\cos\left(\frac{x}{2}\right)\right)^4} + 2\log\left(\sin\left(\frac{x}{2}\right)+\cos\left(\frac{x}{2}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[x] + Tan[x])^(-5),x]
[Out] 2*Log[Cos[x/2] + Sin[x/2]] + (2 + 4*Sin[x])/(Cos[x/2] + Sin[x/2])^4
```

fricas [A] time = 1.06, size = 35, normalized size = 1.59

$$\frac{\left(\cos(x)^2 - 2\sin(x) - 2\right)\log\left(\sin(x)+1\right) - 4\sin(x) - 2}{\cos(x)^2 - 2\sin(x) - 2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sec(x)+tan(x))^5,x, algorithm="fricas")
[Out] ((cos(x)^2 - 2*sin(x) - 2)*log(sin(x) + 1) - 4*sin(x) - 2)/(cos(x)^2 - 2*si
n(x) - 2)
```

giac [B] time = 0.15, size = 64, normalized size = 2.91

$$-\frac{25\tan\left(\frac{1}{2}x\right)^4 + 100\tan\left(\frac{1}{2}x\right)^3 + 198\tan\left(\frac{1}{2}x\right)^2 + 100\tan\left(\frac{1}{2}x\right) + 25}{6\left(\tan\left(\frac{1}{2}x\right)+1\right)^4} - \log\left(\tan\left(\frac{1}{2}x\right)^2+1\right) + 2\log\left(\left|\tan\left(\frac{1}{2}x\right)\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sec(x)+tan(x))^5,x, algorithm="giac")
```

```
[Out] -1/6*(25*tan(1/2*x)^4 + 100*tan(1/2*x)^3 + 198*tan(1/2*x)^2 + 100*tan(1/2*x
) + 25)/(tan(1/2*x) + 1)^4 - log(tan(1/2*x)^2 + 1) + 2*log(abs(tan(1/2*x) +
 1))
```

maple [A] time = 0.18, size = 23, normalized size = 1.05

$$\ln(1+\sin(x)) - \frac{2}{(1+\sin(x))^2} + \frac{4}{1+\sin(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sec(x)+tan(x))^5,x)
```

```
[Out] ln(1+sin(x))-2/(1+sin(x))^2+4/(1+sin(x))
```

maxima [B] time = 0.66, size = 92, normalized size = 4.18

$$-\frac{8\sin(x)^2}{\left(\frac{4\sin(x)}{\cos(x)+1} + \frac{6\sin(x)^2}{(\cos(x)+1)^2} + \frac{4\sin(x)^3}{(\cos(x)+1)^3} + \frac{\sin(x)^4}{(\cos(x)+1)^4} + 1\right)(\cos(x)+1)^2} + 2\log\left(\frac{\sin(x)}{\cos(x)+1} + 1\right) - \log\left(\frac{\sin(x)^2}{(\cos(x)+1)^2} + \right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sec(x)+tan(x))^5,x, algorithm="maxima")
```

```
[Out] -8*sin(x)^2/((4*sin(x)/(cos(x) + 1) + 6*sin(x)^2/(cos(x) + 1)^2 + 4*sin(x)^
3/(cos(x) + 1)^3 + sin(x)^4/(cos(x) + 1)^4 + 1)*(cos(x) + 1)^2) + 2*log(sin
(x)/(cos(x) + 1) + 1) - log(sin(x)^2/(cos(x) + 1)^2 + 1)
```

mupad [B] time = 2.38, size = 61, normalized size = 2.77

$$2\ln\left(\tan\left(\frac{x}{2}\right)+1\right) - \ln\left(\tan\left(\frac{x}{2}\right)^2+1\right) - \frac{8\tan\left(\frac{x}{2}\right)^2}{\tan\left(\frac{x}{2}\right)^4 + 4\tan\left(\frac{x}{2}\right)^3 + 6\tan\left(\frac{x}{2}\right)^2 + 4\tan\left(\frac{x}{2}\right) + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(tan(x) + 1/cos(x))^5,x)
```

```
[Out] 2*log(tan(x/2) + 1) - log(tan(x/2)^2 + 1) - (8*tan(x/2)^2)/(4*tan(x/2) + 6*
tan(x/2)^2 + 4*tan(x/2)^3 + tan(x/2)^4 + 1)
```

sympy [B] time = 2.76, size = 1059, normalized size = 48.14

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sec(x)+tan(x))**5,x)
```

```
[Out] 36*log(tan(x) + sec(x))*tan(x)**4/(36*tan(x)**4 + 144*tan(x)**3*sec(x) + 21
6*tan(x)**2*sec(x)**2 + 144*tan(x)*sec(x)**3 + 36*sec(x)**4) + 144*log(tan(
x) + sec(x))*tan(x)**3*sec(x)/(36*tan(x)**4 + 144*tan(x)**3*sec(x) + 216*ta
n(x)**2*sec(x)**2 + 144*tan(x)*sec(x)**3 + 36*sec(x)**4) + 216*log(tan(x) +
 sec(x))*tan(x)**2*sec(x)**2/(36*tan(x)**4 + 144*tan(x)**3*sec(x) + 216*tan
(x)**2*sec(x)**2 + 144*tan(x)*sec(x)**3 + 36*sec(x)**4) + 144*log(tan(x) +
sec(x))*tan(x)*sec(x)**3/(36*tan(x)**4 + 144*tan(x)**3*sec(x) + 216*tan(x)*
*2*sec(x)**2 + 144*tan(x)*sec(x)**3 + 36*sec(x)**4) + 36*log(tan(x) + sec(x
))*sec(x)**4/(36*tan(x)**4 + 144*tan(x)**3*sec(x) + 216*tan(x)**2*sec(x)**2
 + 144*tan(x)*sec(x)**3 + 36*sec(x)**4) - 18*log(tan(x)**2 + 1)*tan(x)**4/(
36*tan(x)**4 + 144*tan(x)**3*sec(x) + 216*tan(x)**2*sec(x)**2 + 144*tan(x)*
sec(x)**3 + 36*sec(x)**4) - 72*log(tan(x)**2 + 1)*tan(x)**3*sec(x)/(36*tan(
x)**4 + 144*tan(x)**3*sec(x) + 216*tan(x)**2*sec(x)**2 + 144*tan(x)*sec(x)*
*3 + 36*sec(x)**4) - 108*log(tan(x)**2 + 1)*tan(x)**2*sec(x)**2/(36*tan(x)*
*4 + 144*tan(x)**3*sec(x) + 216*tan(x)**2*sec(x)**2 + 144*tan(x)*sec(x)**3
+ 36*sec(x)**4) - 72*log(tan(x)**2 + 1)*tan(x)*sec(x)**3/(36*tan(x)**4 + 14
4*tan(x)**3*sec(x) + 216*tan(x)**2*sec(x)**2 + 144*tan(x)*sec(x)**3 + 36*se
c(x)**4) - 18*log(tan(x)**2 + 1)*sec(x)**4/(36*tan(x)**4 + 144*tan(x)**3*se
c(x) + 216*tan(x)**2*sec(x)**2 + 144*tan(x)*sec(x)**3 + 36*sec(x)**4) - 28*
tan(x)**4/(36*tan(x)**4 + 144*tan(x)**3*sec(x) + 216*tan(x)**2*sec(x)**2 +
144*tan(x)*sec(x)**3 + 36*sec(x)**4) - 52*tan(x)**3*sec(x)/(36*tan(x)**4 +
144*tan(x)**3*sec(x) + 216*tan(x)**2*sec(x)**2 + 144*tan(x)*sec(x)**3 + 36*
sec(x)**4) + 18*tan(x)**2/(36*tan(x)**4 + 144*tan(x)**3*sec(x) + 216*tan(x)
**2*sec(x)**2 + 144*tan(x)*sec(x)**3 + 36*sec(x)**4) + 44*tan(x)*sec(x)**3/
(36*tan(x)**4 + 144*tan(x)**3*sec(x) + 216*tan(x)**2*sec(x)**2 + 144*tan(x)
*sec(x)**3 + 36*sec(x)**4) + 24*tan(x)*sec(x)/(36*tan(x)**4 + 144*tan(x)**3
*sec(x) + 216*tan(x)**2*sec(x)**2 + 144*tan(x)*sec(x)**3 + 36*sec(x)**4) +
20*sec(x)**4/(36*tan(x)**4 + 144*tan(x)**3*sec(x) + 216*tan(x)**2*sec(x)**2
 + 144*tan(x)*sec(x)**3 + 36*sec(x)**4) + 6*sec(x)**2/(36*tan(x)**4 + 144*t
an(x)**3*sec(x) + 216*tan(x)**2*sec(x)**2 + 144*tan(x)*sec(x)**3 + 36*sec(x
)**4) - 9/(36*tan(x)**4 + 144*tan(x)**3*sec(x) + 216*tan(x)**2*sec(x)**2 +
144*tan(x)*sec(x)**3 + 36*sec(x)**4)
```

3.283 $\int (a\cot(x)+b\csc(x))^5\,dx$

Optimal. Leaf size=152

$$\frac{1}{8}a^2b\left(7a^2-3b^2\right)\cos(x)+\frac{1}{16}(a+b)^3\left(8a^2-9ab+3b^2\right)\log(1-\cos(x))+\frac{1}{16}(a-b)^3\left(8a^2+9ab+3b^2\right)\log(\cos(x)+1$$

```
[Out] 1/8*a^2*b*(7*a^2-3*b^2)*cos(x)+1/8*(b+a*cos(x))^2*(2*a*(2*a^2-b^2)+b*(5*a^2
-3*b^2)*cos(x))*csc(x)^2-1/4*(b+a*cos(x))^4*(a+b*cos(x))*csc(x)^4+1/16*(a+b
)^3*(8*a^2-9*a*b+3*b^2)*ln(1-cos(x))+1/16*(a-b)^3*(8*a^2+9*a*b+3*b^2)*ln(1+
cos(x))
```

Rubi [A] time = 0.22, antiderivative size = 152, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.636, Rules used = {4392, 2668, 739, 819, 774, 633, 31}

$$\frac{1}{8}a^2b\left(7a^2-3b^2\right)\cos(x)+\frac{1}{16}(a+b)^3\left(8a^2-9ab+3b^2\right)\log(1-\cos(x))+\frac{1}{16}(a-b)^3\left(8a^2+9ab+3b^2\right)\log(\cos(x)+1$$

Antiderivative was successfully verified.

```
[In] Int[(a*Cot[x] + b*Csc[x])^5,x]
```

```
[Out] (a^2*b*(7*a^2 - 3*b^2)*Cos[x])/8 + ((b + a*Cos[x])^2*(2*a*(2*a^2 - b^2) + b
*(5*a^2 - 3*b^2)*Cos[x])*Csc[x]^2)/8 - ((b + a*Cos[x])^4*(a + b*Cos[x])*Csc
[x]^4)/4 + ((a + b)^3*(8*a^2 - 9*a*b + 3*b^2)*Log[1 - Cos[x]])/16 + ((a - b
)^3*(8*a^2 + 9*a*b + 3*b^2)*Log[1 + Cos[x]])/16
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 633

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> With[{q = Rt[-(
a*c), 2]}, Dist[e/2 + (c*d)/(2*q), Int[1/(-q + c*x), x], x] + Dist[e/2 - (c
*d)/(2*q), Int[1/(q + c*x), x], x]] /; FreeQ[{a, c, d, e}, x] && NiceSqrtQ[
-(a*c)]
```

Rule 739

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
((d + e*x)^(m - 1)*(a*e - c*d*x)*(a + c*x^2)^(p + 1))/(2*a*c*(p + 1)), x] +
 Dist[1/((p + 1)*(-2*a*c)), Int[(d + e*x)^(m - 2)*Simp[a*e^2*(m - 1) - c*d^
2*(2*p + 3) - d*c*e*(m + 2*p + 2)*x, x]*(a + c*x^2)^(p + 1), x], x] /; Free
```

```
Q[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 1] && I
ntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 774

```
Int[(((d_.) + (e_.)*(x_))*((f_) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2), x_Sym
bol] :> Simp[(e*g*x)/c, x] + Dist[1/c, Int[(c*d*f - a*e*g + c*(e*f + d*g)*x
)/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e, f, g}, x]
```

Rule 819

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x)^(m - 1)*(a + c*x^2)^(p + 1)*(a*(e*f + d*g
) - (c*d*f - a*e*g)*x))/(2*a*c*(p + 1)), x] - Dist[1/(2*a*c*(p + 1)), Int[(
d + e*x)^(m - 2)*(a + c*x^2)^(p + 1)*Simp[a*e*(e*f*(m - 1) + d*g*m) - c*d^2
*f*(2*p + 3) + e*(a*e*g*m - c*d*f*(m + 2*p + 2))*x, x], x], x] /; FreeQ[{a,
 c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 1] &&
(EqQ[d, 0] || (EqQ[m, 2] && EqQ[p, -3] && RationalQ[a, c, d, e, f, g]) ||
!ILtQ[m + 2*p + 3, 0])
```

Rule 2668

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rule 4392

```
Int[(cot[(c_.) + (d_.)*(x_)]^(n_.)*(a_.) + csc[(c_.) + (d_.)*(x_)]^(n_.)*(b
_.))^(p_)*(u_.), x_Symbol] :> Int[ActivateTrig[u]*Csc[c + d*x]^(n*p)*(b + a
*Cos[c + d*x]^n)^p, x] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p]
```

Rubi steps

$$\int (a\cot(x)+b\csc(x))^5\,dx = \int (b+a\cos(x))^5\csc^5(x)\,dx$$
$$= -\left(a^5\,\text{Subst}\left(\int \frac{(b+x)^5}{\left(a^2-x^2\right)^3}\,dx, x, a\cos(x)\right)\right)$$
$$= -\frac{1}{4}(b+a\cos(x))^4(a+b\cos(x))\csc^4(x)+\frac{1}{4}a^3\,\text{Subst}\left(\int \frac{(b+x)^3\left(4a^2-3b^2+bx\right)}{\left(a^2-x^2\right)^2}\right.$$
$$= \frac{1}{8}(b+a\cos(x))^2\left(2a\left(2a^2-b^2\right)+b\left(5a^2-3b^2\right)\cos(x)\right)\csc^2(x)-\frac{1}{4}(b+a\cos(x))^4($$
$$= \frac{1}{8}a^2b\left(7a^2-3b^2\right)\cos(x)+\frac{1}{8}(b+a\cos(x))^2\left(2a\left(2a^2-b^2\right)+b\left(5a^2-3b^2\right)\cos(x)\right)$$
$$= \frac{1}{8}a^2b\left(7a^2-3b^2\right)\cos(x)+\frac{1}{8}(b+a\cos(x))^2\left(2a\left(2a^2-b^2\right)+b\left(5a^2-3b^2\right)\cos(x)\right)$$
$$= \frac{1}{8}a^2b\left(7a^2-3b^2\right)\cos(x)+\frac{1}{8}(b+a\cos(x))^2\left(2a\left(2a^2-b^2\right)+b\left(5a^2-3b^2\right)\cos(x)\right)$$

Mathematica [A] time = 0.70, size = 143, normalized size = 0.94

$$\frac{1}{64}\left(8(a+b)^3\left(8a^2-9ab+3b^2\right)\log\left(\sin\left(\frac{x}{2}\right)\right)+8\left(8a^2+9ab+3b^2\right)(a-b)^3\log\left(\cos\left(\frac{x}{2}\right)\right)-(a+b)^5\csc^4\left(\frac{x}{2}\right)+\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Cot[x] + b*Csc[x])^5,x]
```

```
[Out] (2*(7*a - 3*b)*(a + b)^4*Csc[x/2]^2 - (a + b)^5*Csc[x/2]^4 + 8*(a - b)^3*(8
*a^2 + 9*a*b + 3*b^2)*Log[Cos[x/2]] + 8*(a + b)^3*(8*a^2 - 9*a*b + 3*b^2)*L
og[Sin[x/2]] + 2*(a - b)^4*(7*a + 3*b)*Sec[x/2]^2 - (a - b)^5*Sec[x/2]^4)/6
4
```

fricas [B] time = 0.87, size = 292, normalized size = 1.92

$$\frac{12\,a^5+40\,a^3b^2-20\,ab^4-2\left(25\,a^4b+10\,a^2b^3-3\,b^5\right)\cos(x)^3-16\left(a^5+5\,a^3b^2\right)\cos(x)^2+10\left(3\,a^4b-2\,a^2b^3-b\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cot(x)+b*csc(x))^5,x, algorithm="fricas")
```

```
[Out] 1/16*(12*a^5 + 40*a^3*b^2 - 20*a*b^4 - 2*(25*a^4*b + 10*a^2*b^3 - 3*b^5)*co
s(x)^3 - 16*(a^5 + 5*a^3*b^2)*cos(x)^2 + 10*(3*a^4*b - 2*a^2*b^3 - b^5)*cos
(x) + (8*a^5 - 15*a^4*b + 10*a^2*b^3 - 3*b^5 + (8*a^5 - 15*a^4*b + 10*a^2*b
^3 - 3*b^5)*cos(x)^4 - 2*(8*a^5 - 15*a^4*b + 10*a^2*b^3 - 3*b^5)*cos(x)^2)*
log(1/2*cos(x) + 1/2) + (8*a^5 + 15*a^4*b - 10*a^2*b^3 + 3*b^5 + (8*a^5 + 1
5*a^4*b - 10*a^2*b^3 + 3*b^5)*cos(x)^4 - 2*(8*a^5 + 15*a^4*b - 10*a^2*b^3 +
 3*b^5)*cos(x)^2)*log(-1/2*cos(x) + 1/2))/(cos(x)^4 - 2*cos(x)^2 + 1)
```

giac [A] time = 0.13, size = 169, normalized size = 1.11

$$\frac{1}{16}\left(8\,a^5-15\,a^4b+10\,a^2b^3-3\,b^5\right)\log\left(\cos(x)+1\right)+\frac{1}{16}\left(8\,a^5+15\,a^4b-10\,a^2b^3+3\,b^5\right)\log\left(-\cos(x)+1\right)+\frac{6\,a^5+}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cot(x)+b*csc(x))^5,x, algorithm="giac")
```

```
[Out] 1/16*(8*a^5 - 15*a^4*b + 10*a^2*b^3 - 3*b^5)*log(cos(x) + 1) + 1/16*(8*a^5
+ 15*a^4*b - 10*a^2*b^3 + 3*b^5)*log(-cos(x) + 1) + 1/8*(6*a^5 + 20*a^3*b^2
 - 10*a*b^4 - (25*a^4*b + 10*a^2*b^3 - 3*b^5)*cos(x)^3 - 8*(a^5 + 5*a^3*b^2
)*cos(x)^2 + 5*(3*a^4*b - 2*a^2*b^3 - b^5)*cos(x))/((cos(x) + 1)^2*(cos(x)
- 1)^2)
```

maple [A] time = 0.14, size = 204, normalized size = 1.34

$$-\frac{a^5\left(\cot^4(x)\right)}{4}+\frac{a^5\left(\cot^2(x)\right)}{2}+a^5\ln\left(\sin(x)\right)-\frac{5a^4b\left(\cos^5(x)\right)}{4\sin(x)^4}+\frac{5a^4b\left(\cos^5(x)\right)}{8\sin(x)^2}+\frac{5\left(\cos^3(x)\right)a^4b}{8}+\frac{15a^4b\cos(x)}{8}+\frac{15a}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*cot(x)+b*csc(x))^5,x)
```

```
[Out] -1/4*a^5*cot(x)^4+1/2*a^5*cot(x)^2+a^5*ln(sin(x))-5/4*a^4*b/sin(x)^4*cos(x)
^5+5/8*a^4*b/sin(x)^2*cos(x)^5+5/8*cos(x)^3*a^4*b+15/8*a^4*b*cos(x)+15/8*a^
4*b*ln(csc(x)-cot(x))-5/2*a^3*b^2/sin(x)^4*cos(x)^4-5/2*a^2*b^3/sin(x)^4*co
s(x)^3-5/4*a^2*b^3/sin(x)^2*cos(x)^3-5/4*cos(x)*a^2*b^3-5/4*a^2*b^3*ln(csc(
x)-cot(x))-5/4*a*b^4/sin(x)^4-1/4*b^5*cot(x)*csc(x)^3-3/8*b^5*csc(x)*cot(x)
+3/8*b^5*ln(csc(x)-cot(x))
```

maxima [A] time = 0.31, size = 188, normalized size = 1.24

$$-\frac{5}{2}\,a^3b^2\cot(x)^4-\frac{5}{16}\,a^4b\left(\frac{2\left(5\,\cos(x)^3-3\,\cos(x)\right)}{\cos(x)^4-2\,\cos(x)^2+1}+3\,\log\left(\cos(x)+1\right)-3\,\log\left(\cos(x)-1\right)\right)+\frac{1}{16}\,b^5\left(\frac{2\left(3\,\cos(x)^3\right.}{\cos(x)^4-2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cot(x)+b*csc(x))^5,x, algorithm="maxima")
```

```
[Out] -5/2*a^3*b^2*cot(x)^4 - 5/16*a^4*b*(2*(5*cos(x)^3 - 3*cos(x))/(cos(x)^4 - 2
*cos(x)^2 + 1) + 3*log(cos(x) + 1) - 3*log(cos(x) - 1)) + 1/16*b^5*(2*(3*co
s(x)^3 - 5*cos(x))/(cos(x)^4 - 2*cos(x)^2 + 1) - 3*log(cos(x) + 1) + 3*log(
cos(x) - 1)) - 5/8*a^2*b^3*(2*(cos(x)^3 + cos(x))/(cos(x)^4 - 2*cos(x)^2 +
1) - log(cos(x) + 1) + log(cos(x) - 1)) + 1/4*a^5*((4*sin(x)^2 - 1)/sin(x)^
4 + 2*log(sin(x)^2)) - 5/4*a*b^4/sin(x)^4
```

mupad [B] time = 2.58, size = 174, normalized size = 1.14

$$\tan\left(\frac{x}{2}\right)^2\left(\frac{5\,(a+b)\,(a-b)^4}{32}+\frac{(a-b)^5}{32}\right)-\frac{\frac{5\,a\,b^4}{4}+\frac{5\,a^4\,b}{4}-\tan\left(\frac{x}{2}\right)^2\,\left(3\,a^5+10\,a^4\,b+10\,a^3\,b^2-5\,a\,b^4-2\,b^5\right)+\frac{a^5}{4}}{16\tan\left(\frac{x}{2}\right)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b/sin(x) + a*cot(x))^5,x)
```

```
[Out] tan(x/2)^2*((5*(a + b)*(a - b)^4)/32 + (a - b)^5/32) - ((5*a*b^4)/4 + (5*a^
4*b)/4 - tan(x/2)^2*(10*a^4*b - 5*a*b^4 + 3*a^5 - 2*b^5 + 10*a^3*b^2) + a^5
/4 + b^5/4 + (5*a^2*b^3)/2 + (5*a^3*b^2)/2)/(16*tan(x/2)^4) - a^5*log(tan(x
/2)^2 + 1) + log(tan(x/2))*((15*a^4*b)/8 + a^5 + (3*b^5)/8 - (5*a^2*b^3)/4)
 - (tan(x/2)^4*(a - b)^5)/64
```

sympy [B] time = 102.28, size = 308, normalized size = 2.03

$$-\frac{a^5\log\left(\csc^2(x)\right)}{2}-\frac{a^5\csc^4(x)}{4}+a^5\csc^2(x)+\frac{15a^4b\log(\cos(x)-1)}{16}-\frac{15a^4b\log(\cos(x)+1)}{16}-\frac{25a^4b\cos^3(}{8\cos^4(x)-16\cos}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cot(x)+b*csc(x))**5,x)
```

```
[Out] -a**5*log(csc(x)**2)/2 - a**5*csc(x)**4/4 + a**5*csc(x)**2 + 15*a**4*b*log(
cos(x) - 1)/16 - 15*a**4*b*log(cos(x) + 1)/16 - 25*a**4*b*cos(x)**3/(8*cos(
x)**4 - 16*cos(x)**2 + 8) + 15*a**4*b*cos(x)/(8*cos(x)**4 - 16*cos(x)**2 +
8) - 5*a**3*b**2*cot(x)**4/2 - 5*a**2*b**3*log(cos(x) - 1)/8 + 5*a**2*b**3*
log(cos(x) + 1)/8 - 10*a**2*b**3*cos(x)**3/(8*cos(x)**4 - 16*cos(x)**2 + 8)
 - 10*a**2*b**3*cos(x)/(8*cos(x)**4 - 16*cos(x)**2 + 8) - 5*a*b**4*csc(x)**
4/4 + 3*b**5*log(cos(x) - 1)/16 - 3*b**5*log(cos(x) + 1)/16 + 3*b**5*cos(x)
**3/(8*cos(x)**4 - 16*cos(x)**2 + 8) - 5*b**5*cos(x)/(8*cos(x)**4 - 16*cos(
x)**2 + 8)
```

3.284 $\int (a\cot(x)+b\csc(x))^4\,dx$

Optimal. Leaf size=101

$$a^4x+\frac{4}{3}ab\left(2a^2-b^2\right)\sin(x)+\frac{1}{3}a^2\left(3a^2-2b^2\right)\sin(x)\cos(x)+\frac{1}{3}\csc(x)(a\cos(x)+b)^2\left(\left(3a^2-2b^2\right)\cos(x)+ab\right)-\frac{1}{3}\csc$$

```
[Out] a^4*x+1/3*(b+a*cos(x))^2*(a*b+(3*a^2-2*b^2)*cos(x))*csc(x)-1/3*(b+a*cos(x))
^3*(a+b*cos(x))*csc(x)^3+4/3*a*b*(2*a^2-b^2)*sin(x)+1/3*a^2*(3*a^2-2*b^2)*c
os(x)*sin(x)
```

Rubi [A] time = 0.22, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {4392, 2691, 2861, 2734}

$$\frac{4}{3}ab\left(2a^2-b^2\right)\sin(x)+\frac{1}{3}a^2\left(3a^2-2b^2\right)\sin(x)\cos(x)+\frac{1}{3}\csc(x)(a\cos(x)+b)^2\left(\left(3a^2-2b^2\right)\cos(x)+ab\right)+a^4x-\frac{1}{3}\csc$$

Antiderivative was successfully verified.

```
[In] Int[(a*Cot[x] + b*Csc[x])^4,x]
```

```
[Out] a^4*x + ((b + a*Cos[x])^2*(a*b + (3*a^2 - 2*b^2)*Cos[x])*Csc[x])/3 - ((b +
a*Cos[x])^3*(a + b*Cos[x])*Csc[x]^3)/3 + (4*a*b*(2*a^2 - b^2)*Sin[x])/3 + (
a^2*(3*a^2 - 2*b^2)*Cos[x]*Sin[x])/3
```

Rule 2691

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Simp[((g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x]
)^(m - 1)*(b + a*Sin[e + f*x]))/(f*g*(p + 1)), x] + Dist[1/(g^2*(p + 1)), I
nt[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^(m - 2)*(b^2*(m - 1) + a^2
*(p + 2) + a*b*(m + p + 1)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f, g},
x] && NeQ[a^2 - b^2, 0] && GtQ[m, 1] && LtQ[p, -1] && (IntegersQ[2*m, 2*p]
|| IntegerQ[m])
```

Rule 2734

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[((2*a*c + b*d)*x)/2, x] + (-Simp[((b*c + a*d)*Co
s[e + f*x])/f, x] - Simp[(b*d*Cos[e + f*x]*Sin[e + f*x])/(2*f), x]) /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2861

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> -Simp[((g*
```

```
Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m*(d + c*Sin[e + f*x]))/(f*g*(p
+ 1)), x] + Dist[1/(g^2*(p + 1)), Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e
 + f*x])^(m - 1)*Simp[a*c*(p + 2) + b*d*m + b*c*(m + p + 2)*Sin[e + f*x], x
], x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 - b^2, 0] && GtQ[m,
 0] && LtQ[p, -1] && IntegerQ[2*m] &&  !(EqQ[m, 1] && NeQ[c^2 - d^2, 0] &&
SimplerQ[c + d*x, a + b*x])
```

Rule 4392

```
Int[(cot[(c_.) + (d_.)*(x_)]^(n_.)*(a_.) + csc[(c_.) + (d_.)*(x_)]^(n_.)*(b
_.))^(p_)*(u_.), x_Symbol] :> Int[ActivateTrig[u]*Csc[c + d*x]^(n*p)*(b + a
*Cos[c + d*x]^n)^p, x] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p]
```

Rubi steps

$$\int (a\cot(x)+b\csc(x))^4\,dx = \int (b+a\cos(x))^4\csc^4(x)\,dx$$
$$= -\frac{1}{3}(b+a\cos(x))^3(a+b\cos(x))\csc^3(x) - \frac{1}{3}\int (b+a\cos(x))^2\left(3a^2-2b^2+ab\cos(\right.$$
$$= \frac{1}{3}(b+a\cos(x))^2\left(ab+\left(3a^2-2b^2\right)\cos(x)\right)\csc(x) - \frac{1}{3}(b+a\cos(x))^3(a+b\cos(x))$$
$$= a^4x + \frac{1}{3}(b+a\cos(x))^2\left(ab+\left(3a^2-2b^2\right)\cos(x)\right)\csc(x) - \frac{1}{3}(b+a\cos(x))^3(a+b\,\mathrm{c}$$

Mathematica [A] time = 0.26, size = 95, normalized size = 0.94

$$-\frac{1}{12}\csc^3(x)\left(-9a^4x\sin(x)+3a^4x\sin(3x)+4a^4\cos(3x)+24a^3b\cos(2x)-8a^3b+6a^2b^2\cos(3x)+6b^2\left(3a^2+b\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Cot[x] + b*Csc[x])^4,x]
```

```
[Out] -1/12*(Csc[x]^3*(-8*a^3*b + 16*a*b^3 + 6*b^2*(3*a^2 + b^2)*Cos[x] + 24*a^3*
b*Cos[2*x] + 4*a^4*Cos[3*x] + 6*a^2*b^2*Cos[3*x] - 2*b^4*Cos[3*x] - 9*a^4*x
*Sin[x] + 3*a^4*x*Sin[3*x]))
```

fricas [A] time = 0.95, size = 95, normalized size = 0.94

$$\frac{12\,a^3b\cos(x)^2-8\,a^3b+4\,ab^3+2\left(2\,a^4+3\,a^2b^2-b^4\right)\cos(x)^3-3\left(a^4-b^4\right)\cos(x)+3\left(a^4x\cos(x)^2-a^4x\right)\sin(x}{3\left(\cos(x)^2-1\right)\sin(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cot(x)+b*csc(x))^4,x, algorithm="fricas")
```

```
[Out] 1/3*(12*a^3*b*cos(x)^2 - 8*a^3*b + 4*a*b^3 + 2*(2*a^4 + 3*a^2*b^2 - b^4)*co
s(x)^3 - 3*(a^4 - b^4)*cos(x) + 3*(a^4*x*cos(x)^2 - a^4*x)*sin(x))/((cos(x)
^2 - 1)*sin(x))
```

giac [B] time = 0.16, size = 215, normalized size = 2.13

$$\frac{1}{24}\,a^4\tan\left(\frac{1}{2}\,x\right)^3-\frac{1}{6}\,a^3b\tan\left(\frac{1}{2}\,x\right)^3+\frac{1}{4}\,a^2b^2\tan\left(\frac{1}{2}\,x\right)^3-\frac{1}{6}\,ab^3\tan\left(\frac{1}{2}\,x\right)^3+\frac{1}{24}\,b^4\tan\left(\frac{1}{2}\,x\right)^3+a^4x-\frac{5}{8}\,a^4\tan\left(\frac{1}{2}\,x\right)+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cot(x)+b*csc(x))^4,x, algorithm="giac")
```

```
[Out] 1/24*a^4*tan(1/2*x)^3 - 1/6*a^3*b*tan(1/2*x)^3 + 1/4*a^2*b^2*tan(1/2*x)^3 -
 1/6*a*b^3*tan(1/2*x)^3 + 1/24*b^4*tan(1/2*x)^3 + a^4*x - 5/8*a^4*tan(1/2*x
) + 3/2*a^3*b*tan(1/2*x) - 3/4*a^2*b^2*tan(1/2*x) - 1/2*a*b^3*tan(1/2*x) +
3/8*b^4*tan(1/2*x) + 1/24*(15*a^4*tan(1/2*x)^2 + 36*a^3*b*tan(1/2*x)^2 + 18
*a^2*b^2*tan(1/2*x)^2 - 12*a*b^3*tan(1/2*x)^2 - 9*b^4*tan(1/2*x)^2 - a^4 -
4*a^3*b - 6*a^2*b^2 - 4*a*b^3 - b^4)/tan(1/2*x)^3
```

maple [A] time = 0.07, size = 93, normalized size = 0.92

$$a^4\left(-\frac{\left(\cot^3(x)\right)}{3}+\cot(x)+x\right)+4a^3b\left(-\frac{\cos^4(x)}{3\sin(x)^3}+\frac{\cos^4(x)}{3\sin(x)}+\frac{\left(2+\cos^2(x)\right)\sin(x)}{3}\right)-\frac{2a^2b^2\left(\cos^3(x)\right)}{\sin(x)^3}-\frac{4a\,b^3}{3\sin(x)^3}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*cot(x)+b*csc(x))^4,x)
```

```
[Out] a^4*(-1/3*cot(x)^3+cot(x)+x)+4*a^3*b*(-1/3/sin(x)^3*cos(x)^4+1/3/sin(x)*cos
(x)^4+1/3*(2+cos(x)^2)*sin(x))-2*a^2*b^2/sin(x)^3*cos(x)^3-4/3*a*b^3/sin(x)
^3+b^4*(-2/3-1/3*csc(x)^2)*cot(x)
```

maxima [A] time = 0.41, size = 80, normalized size = 0.79

$$-2\,a^2b^2\cot(x)^3+\frac{1}{3}\,a^4\left(3\,x+\frac{3\,\tan(x)^2-1}{\tan(x)^3}\right)+\frac{4\left(3\,\sin(x)^2-1\right)a^3b}{3\,\sin(x)^3}-\frac{\left(3\,\tan(x)^2+1\right)b^4}{3\,\tan(x)^3}-\frac{4\,ab^3}{3\,\sin(x)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cot(x)+b*csc(x))^4,x, algorithm="maxima")
```

```
[Out] -2*a^2*b^2*cot(x)^3 + 1/3*a^4*(3*x + (3*tan(x)^2 - 1)/tan(x)^3) + 4/3*(3*si
n(x)^2 - 1)*a^3*b/sin(x)^3 - 1/3*(3*tan(x)^2 + 1)*b^4/tan(x)^3 - 4/3*a*b^3/
sin(x)^3
```

mupad [B] time = 2.53, size = 127, normalized size = 1.26

$$a^4\,x - \frac{\frac{4\,a\,b^3}{3} + \frac{4\,a^3\,b}{3} - \tan\left(\frac{x}{2}\right)^2\,\left(5\,a^4 + 12\,a^3\,b + 6\,a^2\,b^2 - 4\,a\,b^3 - 3\,b^4\right) + \frac{a^4}{3} + \frac{b^4}{3} + 2\,a^2\,b^2}{8\,\tan\left(\frac{x}{2}\right)^3} - \tan\left(\frac{x}{2}\right)\,\left(\frac{(a+b)\,(a-}{2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b/sin(x) + a*cot(x))^4,x)
```

```
[Out] a^4*x - ((4*a*b^3)/3 + (4*a^3*b)/3 - tan(x/2)^2*(12*a^3*b - 4*a*b^3 + 5*a^4
 - 3*b^4 + 6*a^2*b^2) + a^4/3 + b^4/3 + 2*a^2*b^2)/(8*tan(x/2)^3) - tan(x/2
)*(((a + b)*(a - b)^3)/2 + (a - b)^4/8) + (tan(x/2)^3*(a - b)^4)/24
```

sympy [A] time = 33.02, size = 97, normalized size = 0.96

$$a^4x + \frac{a^4\cos(x)}{\sin(x)} - \frac{a^4\cos^3(x)}{3\sin^3(x)} - \frac{4a^3b\csc^3(x)}{3} + 4a^3b\csc(x) - 2a^2b^2\cot^3(x) - \frac{4ab^3\csc^3(x)}{3} - \frac{b^4\cot^3(x)}{3} - b^4\cot(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cot(x)+b*csc(x))**4,x)
```

```
[Out] a**4*x + a**4*cos(x)/sin(x) - a**4*cos(x)**3/(3*sin(x)**3) - 4*a**3*b*csc(x
)**3/3 + 4*a**3*b*csc(x) - 2*a**2*b**2*cot(x)**3 - 4*a*b**3*csc(x)**3/3 - b
**4*cot(x)**3/3 - b**4*cot(x)
```

3.285 $\int (a\cot(x)+b\csc(x))^3\,dx$

Optimal. Leaf size=77

$$-\frac{1}{2}a^2b\cos(x)-\frac{1}{4}(2a-b)(a+b)^2\log(1-\cos(x))-\frac{1}{4}(a-b)^2(2a+b)\log(\cos(x)+1)-\frac{1}{2}\csc^2(x)(a\cos(x)+b)^2(a+b\cos(x))$$

```
[Out] -1/2*a^2*b*cos(x)-1/2*(b+a*cos(x))^2*(a+b*cos(x))*csc(x)^2-1/4*(2*a-b)*(a+b
)^2*ln(1-cos(x))-1/4*(a-b)^2*(2*a+b)*ln(1+cos(x))
```

Rubi [A] time = 0.14, antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.546, Rules used = {4392, 2668, 739, 774, 633, 31}

$$-\frac{1}{2}a^2b\cos(x)-\frac{1}{4}(2a-b)(a+b)^2\log(1-\cos(x))-\frac{1}{4}(a-b)^2(2a+b)\log(\cos(x)+1)-\frac{1}{2}\csc^2(x)(a\cos(x)+b)^2(a+b\cos(x))$$

Antiderivative was successfully verified.

```
[In] Int[(a*Cot[x] + b*Csc[x])^3,x]
```

```
[Out] -(a^2*b*Cos[x])/2 - ((b + a*Cos[x])^2*(a + b*Cos[x])*Csc[x]^2)/2 - ((2*a -
b)*(a + b)^2*Log[1 - Cos[x]])/4 - ((a - b)^2*(2*a + b)*Log[1 + Cos[x]])/4
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 633

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> With[{q = Rt[-(
a*c), 2]}, Dist[e/2 + (c*d)/(2*q), Int[1/(-q + c*x), x], x] + Dist[e/2 - (c
*d)/(2*q), Int[1/(q + c*x), x], x]] /; FreeQ[{a, c, d, e}, x] && NiceSqrtQ[
-(a*c)]
```

Rule 739

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
((d + e*x)^(m - 1)*(a*e - c*d*x)*(a + c*x^2)^(p + 1))/(2*a*c*(p + 1)), x] +
 Dist[1/((p + 1)*(-2*a*c)), Int[(d + e*x)^(m - 2)*Simp[a*e^2*(m - 1) - c*d^
2*(2*p + 3) - d*c*e*(m + 2*p + 2)*x, x]*(a + c*x^2)^(p + 1), x], x] /; Free
Q[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 1] && I
ntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 774

```
Int[(((d_.) + (e_.)*(x_))*((f_) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2), x_Sym
bol] :> Simp[(e*g*x)/c, x] + Dist[1/c, Int[(c*d*f - a*e*g + c*(e*f + d*g)*x
)/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e, f, g}, x]
```

Rule 2668

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rule 4392

```
Int[(cot[(c_.) + (d_.)*(x_)]^(n_.)*(a_.) + csc[(c_.) + (d_.)*(x_)]^(n_.)*(b
_.))^(p_)*(u_.), x_Symbol] :> Int[ActivateTrig[u]*Csc[c + d*x]^(n*p)*(b + a
*Cos[c + d*x]^n)^p, x] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p]
```

Rubi steps

$$\int (a\cot(x)+b\csc(x))^3\,dx = \int (b+a\cos(x))^3\csc^3(x)\,dx$$
$$= -\left(a^3\,\text{Subst}\left(\int \frac{(b+x)^3}{\left(a^2-x^2\right)^2}\,dx, x, a\cos(x)\right)\right)$$
$$= -\frac{1}{2}(b+a\cos(x))^2(a+b\cos(x))\csc^2(x)+\frac{1}{2}a\,\text{Subst}\left(\int \frac{(b+x)\left(2a^2-b^2+bx\right)}{a^2-x^2}\,dx\right.$$
$$= -\frac{1}{2}a^2b\cos(x)-\frac{1}{2}(b+a\cos(x))^2(a+b\cos(x))\csc^2(x)-\frac{1}{2}a\,\text{Subst}\left(\int \frac{-a^2b-b\left(2\right.}{a^2}\right.$$
$$= -\frac{1}{2}a^2b\cos(x)-\frac{1}{2}(b+a\cos(x))^2(a+b\cos(x))\csc^2(x)+\frac{1}{4}\left((2a-b)(a+b)^2\right)\text{Subs}$$
$$= -\frac{1}{2}a^2b\cos(x)-\frac{1}{2}(b+a\cos(x))^2(a+b\cos(x))\csc^2(x)-\frac{1}{4}(2a-b)(a+b)^2\log(1-$$

Mathematica [A] time = 0.29, size = 79, normalized size = 1.03

$$\frac{1}{8}\left(-(a+b)^3\csc^2\left(\frac{x}{2}\right)+(a-b)^3\left(-\sec^2\left(\frac{x}{2}\right)\right)-4(2a-b)(a+b)^2\log\left(\sin\left(\frac{x}{2}\right)\right)-4(2a+b)(a-b)^2\log\left(\cos\left(\frac{x}{2}\right)\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Cot[x] + b*Csc[x])^3,x]
```

```
[Out] (-((a + b)^3*Csc[x/2]^2) - 4*(a - b)^2*(2*a + b)*Log[Cos[x/2]] - 4*(2*a - b
)*(a + b)^2*Log[Sin[x/2]] - (a - b)^3*Sec[x/2]^2)/8
```

fricas [A] time = 0.97, size = 128, normalized size = 1.66

$$\frac{2\,a^3+6\,ab^2+2\left(3\,a^2b+b^3\right)\cos(x)+\left(2\,a^3-3\,a^2b+b^3-\left(2\,a^3-3\,a^2b+b^3\right)\cos(x)^2\right)\log\left(\frac{1}{2}\,\cos(x)+\frac{1}{2}\right)+\left(2\,a^3\right.}{4\left(\cos(x)^2-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cot(x)+b*csc(x))^3,x, algorithm="fricas")
```

```
[Out] 1/4*(2*a^3 + 6*a*b^2 + 2*(3*a^2*b + b^3)*cos(x) + (2*a^3 - 3*a^2*b + b^3 -
(2*a^3 - 3*a^2*b + b^3)*cos(x)^2)*log(1/2*cos(x) + 1/2) + (2*a^3 + 3*a^2*b
- b^3 - (2*a^3 + 3*a^2*b - b^3)*cos(x)^2)*log(-1/2*cos(x) + 1/2))/(cos(x)^2
 - 1)
```

giac [A] time = 0.15, size = 86, normalized size = 1.12

$$-\frac{1}{4}\left(2\,a^3-3\,a^2b+b^3\right)\log\left(\cos(x)+1\right)-\frac{1}{4}\left(2\,a^3+3\,a^2b-b^3\right)\log\left(-\cos(x)+1\right)+\frac{a^3+3\,ab^2+\left(3\,a^2b+b^3\right)\cos(x)}{2\left(\cos(x)+1\right)\left(\cos(x)-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cot(x)+b*csc(x))^3,x, algorithm="giac")
```

```
[Out] -1/4*(2*a^3 - 3*a^2*b + b^3)*log(cos(x) + 1) - 1/4*(2*a^3 + 3*a^2*b - b^3)*
log(-cos(x) + 1) + 1/2*(a^3 + 3*a*b^2 + (3*a^2*b + b^3)*cos(x))/((cos(x) +
1)*(cos(x) - 1))
```

maple [A] time = 0.07, size = 87, normalized size = 1.13

$$-\frac{a^3\left(\cot^2(x)\right)}{2}-a^3\ln\left(\sin(x)\right)-\frac{3a^2b\left(\cos^3(x)\right)}{2\sin(x)^2}-\frac{3a^2b\cos(x)}{2}-\frac{3a^2b\ln\left(\csc(x)-\cot(x)\right)}{2}-\frac{3a\,b^2}{2\sin(x)^2}-\frac{b^3\csc(x)\cot(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*cot(x)+b*csc(x))^3,x)
```

```
[Out] -1/2*a^3*cot(x)^2-a^3*ln(sin(x))-3/2*a^2*b/sin(x)^2*cos(x)^3-3/2*a^2*b*cos(
x)-3/2*a^2*b*ln(csc(x)-cot(x))-3/2*a*b^2/sin(x)^2-1/2*b^3*csc(x)*cot(x)+1/2
*b^3*ln(csc(x)-cot(x))
```

maxima [A] time = 0.31, size = 87, normalized size = 1.13

$$-\frac{3}{2}\,ab^2\cot(x)^2+\frac{3}{4}\,a^2b\left(\frac{2\,\cos(x)}{\cos(x)^2-1}+\log\left(\cos(x)+1\right)-\log\left(\cos(x)-1\right)\right)+\frac{1}{4}\,b^3\left(\frac{2\,\cos(x)}{\cos(x)^2-1}-\log\left(\cos(x)+1\right)+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cot(x)+b*csc(x))^3,x, algorithm="maxima")
```

```
[Out] -3/2*a*b^2*cot(x)^2 + 3/4*a^2*b*(2*cos(x)/(cos(x)^2 - 1) + log(cos(x) + 1)
- log(cos(x) - 1)) + 1/4*b^3*(2*cos(x)/(cos(x)^2 - 1) - log(cos(x) + 1) + l
og(cos(x) - 1)) - 1/2*a^3*(1/sin(x)^2 + log(sin(x)^2))
```

mupad [B] time = 2.45, size = 82, normalized size = 1.06

$$a^3\,\ln\left(\tan\left(\frac{x}{2}\right)^2+1\right)-\frac{\frac{a^3}{8}+\frac{3\,a^2\,b}{8}+\frac{3\,a\,b^2}{8}+\frac{b^3}{8}}{\tan\left(\frac{x}{2}\right)^2}-\ln\left(\tan\left(\frac{x}{2}\right)\right)\,\left(a^3+\frac{3\,a^2\,b}{2}-\frac{b^3}{2}\right)-\frac{\tan\left(\frac{x}{2}\right)^2\,{\left(a-b\right)}^3}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b/sin(x) + a*cot(x))^3,x)
```

```
[Out] a^3*log(tan(x/2)^2 + 1) - ((3*a*b^2)/8 + (3*a^2*b)/8 + a^3/8 + b^3/8)/tan(x
/2)^2 - log(tan(x/2))*((3*a^2*b)/2 + a^3 - b^3/2) - (tan(x/2)^2*(a - b)^3)/
8
```

sympy [A] time = 14.11, size = 124, normalized size = 1.61

$$\frac{a^3\log\left(-\csc^2(x)\right)}{2}-\frac{a^3\csc^2(x)}{2}-\frac{3a^2b\log\left(\cos(x)-1\right)}{4}+\frac{3a^2b\log\left(\cos(x)+1\right)}{4}+\frac{3a^2b\cos(x)}{2\cos^2(x)-2}-\frac{3ab^2\csc^2(x)}{2}+\frac{b^3}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cot(x)+b*csc(x))**3,x)
```

```
[Out] a**3*log(-csc(x)**2)/2 - a**3*csc(x)**2/2 - 3*a**2*b*log(cos(x) - 1)/4 + 3*
a**2*b*log(cos(x) + 1)/4 + 3*a**2*b*cos(x)/(2*cos(x)**2 - 2) - 3*a*b**2*csc
(x)**2/2 + b**3*log(cos(x) - 1)/4 - b**3*log(cos(x) + 1)/4 + b**3*cos(x)/(2
*cos(x)**2 - 2)
```

3.286 $\int (a\cot(x)+b\csc(x))^2\,dx$

Optimal. Leaf size=29

$$a^2(-x)-ab\sin(x)-\csc(x)(a\cos(x)+b)(a+b\cos(x))$$

```
[Out] -a^2*x-(b+a*cos(x))*(a+b*cos(x))*csc(x)-a*b*sin(x)
```

Rubi [A] time = 0.06, antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {4392, 2691, 2637}

$$a^2(-x)-ab\sin(x)-\csc(x)(a\cos(x)+b)(a+b\cos(x))$$

Antiderivative was successfully verified.

```
[In] Int[(a*Cot[x] + b*Csc[x])^2,x]
```

```
[Out] -(a^2*x) - (b + a*Cos[x])*(a + b*Cos[x])*Csc[x] - a*b*Sin[x]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 2691

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Simp[((g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x]
)^(m - 1)*(b + a*Sin[e + f*x]))/(f*g*(p + 1)), x] + Dist[1/(g^2*(p + 1)), I
nt[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^(m - 2)*(b^2*(m - 1) + a^2
*(p + 2) + a*b*(m + p + 1)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f, g},
x] && NeQ[a^2 - b^2, 0] && GtQ[m, 1] && LtQ[p, -1] && (IntegersQ[2*m, 2*p]
|| IntegerQ[m])
```

Rule 4392

```
Int[(cot[(c_.) + (d_.)*(x_)]^(n_.)*(a_.) + csc[(c_.) + (d_.)*(x_)]^(n_.)*(b
_.))^(p_)*(u_.), x_Symbol] :> Int[ActivateTrig[u]*Csc[c + d*x]^(n*p)*(b + a
*Cos[c + d*x]^n)^p, x] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p]
```

Rubi steps

$$
\begin{aligned}
\int (a\cot(x)+b\csc(x))^2\,dx &= \int (b+a\cos(x))^2\csc^2(x)\,dx\\
&= -(b+a\cos(x))(a+b\cos(x))\csc(x)-\int \left(a^2+ab\cos(x)\right)\,dx\\
&= -a^2x-(b+a\cos(x))(a+b\cos(x))\csc(x)-(ab)\int \cos(x)\,dx\\
&= -a^2x-(b+a\cos(x))(a+b\cos(x))\csc(x)-ab\sin(x)
\end{aligned}
$$

Mathematica [A] time = 0.13, size = 24, normalized size = 0.83

$$-\left(\left(a^2+b^2\right)\cot(x)\right)-a(ax+2b\csc(x))$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Cot[x] + b*Csc[x])^2,x]
[Out] -((a^2 + b^2)*Cot[x]) - a*(a*x + 2*b*Csc[x])
```

fricas [A] time = 0.92, size = 28, normalized size = 0.97

$$-\frac{a^2x\sin(x)+2\,ab+\left(a^2+b^2\right)\cos(x)}{\sin(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cot(x)+b*csc(x))^2,x, algorithm="fricas")
[Out] -(a^2*x*sin(x) + 2*a*b + (a^2 + b^2)*cos(x))/sin(x)
```

giac [A] time = 0.15, size = 52, normalized size = 1.79

$$-a^2x+\frac{1}{2}\,a^2\tan\left(\frac{1}{2}\,x\right)-ab\tan\left(\frac{1}{2}\,x\right)+\frac{1}{2}\,b^2\tan\left(\frac{1}{2}\,x\right)-\frac{a^2+2\,ab+b^2}{2\,\tan\left(\frac{1}{2}\,x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cot(x)+b*csc(x))^2,x, algorithm="giac")
[Out] -a^2*x + 1/2*a^2*tan(1/2*x) - a*b*tan(1/2*x) + 1/2*b^2*tan(1/2*x) - 1/2*(a^
2 + 2*a*b + b^2)/tan(1/2*x)
```

maple [A] time = 0.04, size = 29, normalized size = 1.00

$$a^2\left(-\cot(x)-x\right)-\frac{2ab}{\sin(x)}-b^2\cot(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*cot(x)+b*csc(x))^2,x)
```

```
[Out] a^2*(-cot(x)-x)-2*a*b/sin(x)-b^2*cot(x)
```

maxima [A] time = 0.42, size = 29, normalized size = 1.00

$$-a^2\left(x+\frac{1}{\tan(x)}\right)-\frac{2\,ab}{\sin(x)}-\frac{b^2}{\tan(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cot(x)+b*csc(x))^2,x, algorithm="maxima")
```

```
[Out] -a^2*(x + 1/tan(x)) - 2*a*b/sin(x) - b^2/tan(x)
```

mupad [B] time = 2.42, size = 30, normalized size = 1.03

$$-\frac{\cos(x)\,a^2+2\,a\,b+\cos(x)\,b^2}{\sin(x)}-a^2\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b/sin(x) + a*cot(x))^2,x)
```

```
[Out] - (2*a*b + a^2*cos(x) + b^2*cos(x))/sin(x) - a^2*x
```

sympy [A] time = 2.79, size = 31, normalized size = 1.07

$$-a^2x-\frac{a^2\cos{(x)}}{\sin{(x)}}-2ab\csc{(x)}-b^2\cot{(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cot(x)+b*csc(x))**2,x)
```

```
[Out] -a**2*x - a**2*cos(x)/sin(x) - 2*a*b*csc(x) - b**2*cot(x)
```

3.287 $\int (a\cot(x)+b\csc(x))\,dx$

Optimal. Leaf size=12

$$a\log(\sin(x))-b\tanh^{-1}(\cos(x))$$

```
[Out] -b*arctanh(cos(x))+a*ln(sin(x))
```

Rubi [A] time = 0.01, antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {3475, 3770}

$$a\log(\sin(x))-b\tanh^{-1}(\cos(x))$$

Antiderivative was successfully verified.

```
[In] Int[a*Cot[x] + b*Csc[x],x]
```

```
[Out] -(b*ArcTanh[Cos[x]]) + a*Log[Sin[x]]
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int (a\cot(x)+b\csc(x))\,dx &= a\int \cot(x)\,dx+b\int \csc(x)\,dx\\ &= -b\tanh^{-1}(\cos(x))+a\log(\sin(x))\end{aligned}$$

Mathematica [B] time = 0.01, size = 25, normalized size = 2.08

$$a\log(\sin(x))+b\log\left(\sin\left(\frac{x}{2}\right)\right)-b\log\left(\cos\left(\frac{x}{2}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[a*Cot[x] + b*Csc[x],x]
```

[Out] -(b*Log[Cos[x/2]]) + b*Log[Sin[x/2]] + a*Log[Sin[x]]

fricas [B] time = 0.92, size = 27, normalized size = 2.25

$$\frac{1}{2}(a-b)\log\left(\frac{1}{2}\cos(x)+\frac{1}{2}\right)+\frac{1}{2}(a+b)\log\left(-\frac{1}{2}\cos(x)+\frac{1}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a*cot(x)+b*csc(x),x, algorithm="fricas")
```

```
[Out] 1/2*(a - b)*log(1/2*cos(x) + 1/2) + 1/2*(a + b)*log(-1/2*cos(x) + 1/2)
```

giac [A] time = 0.14, size = 15, normalized size = 1.25

$$a\log\left(|\sin(x)|\right)+b\log\left(\left|\tan\left(\frac{1}{2}x\right)\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a*cot(x)+b*csc(x),x, algorithm="giac")
```

```
[Out] a*log(abs(sin(x))) + b*log(abs(tan(1/2*x)))
```

maple [A] time = 0.00, size = 16, normalized size = 1.33

$$a\ln\left(\sin(x)\right)-b\ln\left(\cot(x)+\csc(x)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(a*cot(x)+b*csc(x),x)
```

```
[Out] a*ln(sin(x))-b*ln(cot(x)+csc(x))
```

maxima [A] time = 0.31, size = 15, normalized size = 1.25

$$-b\log\left(\cot(x)+\csc(x)\right)+a\log\left(\sin(x)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a*cot(x)+b*csc(x),x, algorithm="maxima")
```

```
[Out] -b*log(cot(x) + csc(x)) + a*log(sin(x))
```

mupad [B] time = 2.41, size = 27, normalized size = 2.25

$$a\,\ln\left(\tan\left(\frac{x}{2}\right)\right)-a\,\ln\left({\tan\left(\frac{x}{2}\right)}^2+1\right)+b\,\ln\left(\tan\left(\frac{x}{2}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(b/sin(x) + a*cot(x),x)
[Out] a*log(tan(x/2)) - a*log(tan(x/2)^2 + 1) + b*log(tan(x/2))
```

sympy [A] time = 0.10, size = 24, normalized size = 2.00

$$a\log\left(\sin\left(x\right)\right)+b\left(\frac{\log\left(\cos\left(x\right)-1\right)}{2}-\frac{\log\left(\cos\left(x\right)+1\right)}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a*cot(x)+b*csc(x),x)
[Out] a*log(sin(x)) + b*(log(cos(x) - 1)/2 - log(cos(x) + 1)/2)
```

3.288 $\int \frac{1}{a\cot(x)+b\csc(x)}\,dx$

Optimal. Leaf size=12

$$-\frac{\log(a\cos(x)+b)}{a}$$

[Out] -ln(b+a*cos(x))/a

Rubi [A] time = 0.04, antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {3160, 2668, 31}

$$-\frac{\log(a\cos(x)+b)}{a}$$

Antiderivative was successfully verified.

[In] Int[(a*Cot[x] + b*Csc[x])^(-1),x]

[Out] -(Log[b + a*Cos[x]]/a)

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2668

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rule 3160

```
Int[((a_.) + csc[(d_.) + (e_.)*(x_)]*(b_.) + cot[(d_.) + (e_.)*(x_)]*(c_.))
^(-1), x_Symbol] :> Int[Sin[d + e*x]/(b + a*Sin[d + e*x] + c*Cos[d + e*x]),
 x] /; FreeQ[{a, b, c, d, e}, x]
```

Rubi steps

$$\int \frac{1}{a\cot(x)+b\csc(x)}\,dx = \int \frac{\sin(x)}{b+a\cos(x)}\,dx$$
$$= -\frac{\text{Subst}\left(\int \frac{1}{b+x}\,dx, x, a\cos(x)\right)}{a}$$
$$= -\frac{\log(b+a\cos(x))}{a}$$

Mathematica [A] time = 0.02, size = 12, normalized size = 1.00

$$-\frac{\log(a\cos(x)+b)}{a}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Cot[x] + b*Csc[x])^(-1),x]
[Out] -(Log[b + a*Cos[x]]/a)
```

fricas [A] time = 1.20, size = 12, normalized size = 1.00

$$-\frac{\log\left(a\cos(x)+b\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cot(x)+b*csc(x)),x, algorithm="fricas")
[Out] -log(a*cos(x) + b)/a
```

giac [A] time = 0.12, size = 13, normalized size = 1.08

$$-\frac{\log\left(|a\cos(x)+b|\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cot(x)+b*csc(x)),x, algorithm="giac")
[Out] -log(abs(a*cos(x) + b))/a
```

maple [A] time = 0.11, size = 13, normalized size = 1.08

$$-\frac{\ln\left(b+a\cos(x)\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*cot(x)+b*csc(x)),x)
```

```
[Out] -ln(b+a*cos(x))/a
```

maxima [B] time = 0.40, size = 45, normalized size = 3.75

$$-\frac{\log\left(a+b-\frac{(a-b)\sin(x)^2}{(\cos(x)+1)^2}\right)}{a}+\frac{\log\left(\frac{\sin(x)^2}{(\cos(x)+1)^2}+1\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cot(x)+b*csc(x)),x, algorithm="maxima")
```

```
[Out] -log(a + b - (a - b)*sin(x)^2/(cos(x) + 1)^2)/a + log(sin(x)^2/(cos(x) + 1)
^2 + 1)/a
```

mupad [B] time = 3.28, size = 36, normalized size = 3.00

$$\frac{\mathrm{atan}\left(\frac{a\sin\left(\frac{x}{2}\right)^2}{-1\mathrm{i}\,a\sin\left(\frac{x}{2}\right)^2+a\,1\mathrm{i}+b\,1\mathrm{i}}\right)2\mathrm{i}}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b/sin(x) + a*cot(x)),x)
```

```
[Out] (atan((a*sin(x/2)^2)/(a*1i + b*1i - a*sin(x/2)^2*1i))*2i)/a
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{a\cot(x)+b\csc(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cot(x)+b*csc(x)),x)
```

```
[Out] Integral(1/(a*cot(x) + b*csc(x)), x)
```

3.289 $\int \frac{1}{(a\cot(x)+b\csc(x))^2}\,dx$

Optimal. Leaf size=67

$$\frac{2b\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tan\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{a^2\sqrt{a-b}\,\sqrt{a+b}}-\frac{x}{a^2}+\frac{\sin(x)}{a(a\cos(x)+b)}$$

```
[Out] -x/a^2+sin(x)/a/(b+a*cos(x))+2*b*arctanh((a-b)^(1/2)*tan(1/2*x)/(a+b)^(1/2)
)/a^2/(a-b)^(1/2)/(a+b)^(1/2)
```

Rubi [A] time = 0.12, antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.454, Rules used = {4392, 2693, 2735, 2659, 208}

$$\frac{2b\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tan\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{a^2\sqrt{a-b}\,\sqrt{a+b}}-\frac{x}{a^2}+\frac{\sin(x)}{a(a\cos(x)+b)}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Cot[x] + b*Csc[x])^(-2),x]
```

```
[Out] -(x/a^2) + (2*b*ArcTanh[(Sqrt[a - b]*Tan[x/2])/Sqrt[a + b]])/(a^2*Sqrt[a -
b]*Sqrt[a + b]) + Sin[x]/(a*(b + a*Cos[x]))
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 2693

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(g*(g*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x
])^(m + 1))/(b*f*(m + 1)), x] + Dist[(g^2*(p - 1))/(b*(m + 1)), Int[(g*Cos[
e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 1)*Sin[e + f*x], x], x] /; Free
Q[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[p, 1] && In
```

```
tegersQ[2*m, 2*p]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 4392

```
Int[(cot[(c_.) + (d_.)*(x_)]^(n_.)*(a_.) + csc[(c_.) + (d_.)*(x_)]^(n_.)*(b
_.))^(p_)*(u_.), x_Symbol] :> Int[ActivateTrig[u]*Csc[c + d*x]^(n*p)*(b + a
*Cos[c + d*x]^n)^p, x] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{(a\cot(x)+b\csc(x))^2}\,dx &= \int \frac{\sin^2(x)}{(b+a\cos(x))^2}\,dx\\
&= \frac{\sin(x)}{a(b+a\cos(x))}-\frac{\int \frac{\cos(x)}{b+a\cos(x)}\,dx}{a}\\
&= -\frac{x}{a^2}+\frac{\sin(x)}{a(b+a\cos(x))}+\frac{b\int \frac{1}{b+a\cos(x)}\,dx}{a^2}\\
&= -\frac{x}{a^2}+\frac{\sin(x)}{a(b+a\cos(x))}+\frac{(2b)\,\mathrm{Subst}\left(\int \frac{1}{a+b+(-a+b)x^2}\,dx,x,\tan\left(\frac{x}{2}\right)\right)}{a^2}\\
&= -\frac{x}{a^2}+\frac{2b\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tan\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{a^2\sqrt{a-b}\,\sqrt{a+b}}+\frac{\sin(x)}{a(b+a\cos(x))}
\end{aligned}$$

Mathematica [A] time = 0.27, size = 71, normalized size = 1.06

$$-\frac{\frac{2b\tanh^{-1}\left(\frac{(b-a)\tan\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}}+\frac{-a\sin(x)+ax\cos(x)+bx}{a\cos(x)+b}}{a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Cot[x] + b*Csc[x])^(-2),x]
```

```
[Out] -(((2*b*ArcTanh[((-a + b)*Tan[x/2])/Sqrt[a^2 - b^2]])/Sqrt[a^2 - b^2] + (b*
x + a*x*Cos[x] - a*Sin[x])/(b + a*Cos[x]))/a^2)
```

fricas [B] time = 1.05, size = 307, normalized size = 4.58

$$\left[-\frac{2\left(a^3-ab^2\right)x\cos(x)-\left(ab\cos(x)+b^2\right)\sqrt{a^2-b^2}\,\log\left(\frac{2\,ab\cos(x)-\left(a^2-2\,b^2\right)\cos(x)^2+2\,\sqrt{a^2-b^2}\,(b\cos(x)+a)\sin(x)+2\,a^2-b^2}{a^2\cos(x)^2+2\,ab\cos(x)+b^2}\right)+}{2\left(a^4b-a^2b^3+\left(a^5-a^3b^2\right)\cos(x)\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cot(x)+b*csc(x))^2,x, algorithm="fricas")
```

```
[Out] [-1/2*(2*(a^3 - a*b^2)*x*cos(x) - (a*b*cos(x) + b^2)*sqrt(a^2 - b^2)*log((2
*a*b*cos(x) - (a^2 - 2*b^2)*cos(x)^2 + 2*sqrt(a^2 - b^2)*(b*cos(x) + a)*sin
(x) + 2*a^2 - b^2)/(a^2*cos(x)^2 + 2*a*b*cos(x) + b^2)) + 2*(a^2*b - b^3)*x
 - 2*(a^3 - a*b^2)*sin(x))/(a^4*b - a^2*b^3 + (a^5 - a^3*b^2)*cos(x)), -((a
^3 - a*b^2)*x*cos(x) - (a*b*cos(x) + b^2)*sqrt(-a^2 + b^2)*arctan(-sqrt(-a^
2 + b^2)*(b*cos(x) + a)/((a^2 - b^2)*sin(x))) + (a^2*b - b^3)*x - (a^3 - a*
b^2)*sin(x))/(a^4*b - a^2*b^3 + (a^5 - a^3*b^2)*cos(x))]
```

giac [A] time = 0.14, size = 107, normalized size = 1.60

$$\frac{2\left(\pi\left\lfloor\frac{x}{2\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}\left(-2\,a+2\,b\right)+\arctan\left(-\frac{a\tan\left(\frac{1}{2}\,x\right)-b\tan\left(\frac{1}{2}\,x\right)}{\sqrt{-a^2+b^2}}\right)\right)b}{\sqrt{-a^2+b^2}\,a^2}-\frac{x}{a^2}-\frac{2\,\tan\left(\frac{1}{2}\,x\right)}{\left(a\tan\left(\frac{1}{2}\,x\right)^2-b\tan\left(\frac{1}{2}\,x\right)^2-a-b\right)a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cot(x)+b*csc(x))^2,x, algorithm="giac")
```

```
[Out] 2*(pi*floor(1/2*x/pi + 1/2)*sgn(-2*a + 2*b) + arctan(-(a*tan(1/2*x) - b*tan
(1/2*x))/sqrt(-a^2 + b^2)))*b/(sqrt(-a^2 + b^2)*a^2) - x/a^2 - 2*tan(1/2*x)
/((a*tan(1/2*x)^2 - b*tan(1/2*x)^2 - a - b)*a)
```

maple [A] time = 0.12, size = 86, normalized size = 1.28

$$-\frac{2\tan\left(\frac{x}{2}\right)}{a\left(a\left(\tan^2\left(\frac{x}{2}\right)\right)-b\left(\tan^2\left(\frac{x}{2}\right)\right)-a-b\right)}+\frac{2b\,\mathrm{arctanh}\left(\frac{\tan\left(\frac{x}{2}\right)(a-b)}{\sqrt{(a+b)(a-b)}}\right)}{a^2\sqrt{(a+b)\,(a-b)}}-\frac{2\arctan\left(\tan\left(\frac{x}{2}\right)\right)}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*cot(x)+b*csc(x))^2,x)
```

```
[Out] -2/a*tan(1/2*x)/(a*tan(1/2*x)^2-b*tan(1/2*x)^2-a-b)+2/a^2*b/((a+b)*(a-b))^(
1/2)*arctanh(tan(1/2*x)*(a-b)/((a+b)*(a-b))^(1/2))-2/a^2*arctan(tan(1/2*x))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cot(x)+b*csc(x))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` for
more details)Is 4*a^2-4*b^2 positive or negative?
```

mupad [B] time = 3.04, size = 440, normalized size = 6.57

$$\frac{a^3\,\sin(x)+b^2\left(-a\,\sin(x)+\mathrm{atan}\left(\frac{-a^5\,\sin\left(\frac{x}{2}\right)\,\sqrt{a^2-b^2}\,1\mathrm{i}+b^3\,\sin\left(\frac{x}{2}\right)\,\left(a^2-b^2\right)^{3/2}\,2\mathrm{i}+b^5\,\sin\left(\frac{x}{2}\right)\,\sqrt{a^2-b^2}\,2\mathrm{i}+a^4\,b\,\sin\left(\frac{x}{2}\right)\,\sqrt{a^2-b^2}\,1\mathrm{i}-a^2\,b^3\,\sin}{\cos\left(\frac{x}{2}\right)\,a^6-2\,\cos\left(\frac{x}{2}\right)\,a^4\,b^2+\cos\left(\frac{x}{2}\right)\,a^2\,b^4}\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b/sin(x) + a*cot(x))^2,x)
```

```
[Out] (a^3*sin(x) + b^2*(atan((b^3*sin(x/2)*(a^2 - b^2)^(3/2)*2i - a^5*sin(x/2)*(
a^2 - b^2)^(1/2)*1i + b^5*sin(x/2)*(a^2 - b^2)^(1/2)*2i + a^4*b*sin(x/2)*(a
^2 - b^2)^(1/2)*1i - a^2*b^3*sin(x/2)*(a^2 - b^2)^(1/2)*3i + a^3*b^2*sin(x/
2)*(a^2 - b^2)^(1/2)*1i)/(a^6*cos(x/2) + a^2*b^4*cos(x/2) - 2*a^4*b^2*cos(x
/2)))*(a^2 - b^2)^(1/2)*2i - a*sin(x)) + a*b*atan((b^3*sin(x/2)*(a^2 - b^2)
^(3/2)*2i - a^5*sin(x/2)*(a^2 - b^2)^(1/2)*1i + b^5*sin(x/2)*(a^2 - b^2)^(1
/2)*2i + a^4*b*sin(x/2)*(a^2 - b^2)^(1/2)*1i - a^2*b^3*sin(x/2)*(a^2 - b^2)
^(1/2)*3i + a^3*b^2*sin(x/2)*(a^2 - b^2)^(1/2)*1i)/(a^6*cos(x/2) + a^2*b^4*
cos(x/2) - 2*a^4*b^2*cos(x/2)))*cos(x)*(a^2 - b^2)^(1/2)*2i)/(a^4*b - a^2*b
^3 + a^5*cos(x) - a^3*b^2*cos(x)) - (2*atan(sin(x/2)/cos(x/2)))/a^2
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a\cot(x)+b\csc(x)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cot(x)+b*csc(x))**2,x)
```

```
[Out] Integral((a*cot(x) + b*csc(x))**(-2), x)
```

3.290 $\int \frac{1}{(a\cot(x)+b\csc(x))^3}\,dx$

Optimal. Leaf size=50

$$\frac{2b}{a^3(a\cos(x)+b)}+\frac{\log(a\cos(x)+b)}{a^3}+\frac{a^2-b^2}{2a^3(a\cos(x)+b)^2}$$

```
[Out] 1/2*(a^2-b^2)/a^3/(b+a*cos(x))^2+2*b/a^3/(b+a*cos(x))+ln(b+a*cos(x))/a^3
```

Rubi [A] time = 0.08, antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {4392, 2668, 697}

$$\frac{a^2-b^2}{2a^3(a\cos(x)+b)^2}+\frac{2b}{a^3(a\cos(x)+b)}+\frac{\log(a\cos(x)+b)}{a^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Cot[x] + b*Csc[x])^(-3),x]
```

```
[Out] (a^2 - b^2)/(2*a^3*(b + a*Cos[x])^2) + (2*b)/(a^3*(b + a*Cos[x])) + Log[b +
 a*Cos[x]]/a^3
```

Rule 697

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m},
 x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0]
```

Rule 2668

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rule 4392

```
Int[(cot[(c_.) + (d_.)*(x_)]^(n_.)*(a_.) + csc[(c_.) + (d_.)*(x_)]^(n_.)*(b
_.))^(p_)*(u_.), x_Symbol] :> Int[ActivateTrig[u]*Csc[c + d*x]^(n*p)*(b + a
*Cos[c + d*x]^n)^p, x] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p]
```

Rubi steps

$$\int \frac{1}{(a\cot(x)+b\csc(x))^3}\,dx = \int \frac{\sin^3(x)}{(b+a\cos(x))^3}\,dx$$
$$= -\frac{\text{Subst}\left(\int \frac{a^2-x^2}{(b+x)^3}\,dx, x, a\cos(x)\right)}{a^3}$$
$$= -\frac{\text{Subst}\left(\int \left(\frac{1}{-b-x}+\frac{a^2-b^2}{(b+x)^3}+\frac{2b}{(b+x)^2}\right)dx, x, a\cos(x)\right)}{a^3}$$
$$= \frac{a^2-b^2}{2a^3(b+a\cos(x))^2}+\frac{2b}{a^3(b+a\cos(x))}+\frac{\log(b+a\cos(x))}{a^3}$$

Mathematica [A] time = 0.11, size = 77, normalized size = 1.54

$$\frac{a^2\cos(2x)\log(a\cos(x)+b)+a^2\log(a\cos(x)+b)+a^2+2b^2\log(a\cos(x)+b)+4ab\cos(x)(\log(a\cos(x)+b)+}{2a^3(a\cos(x)+b)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Cot[x] + b*Csc[x])^(-3),x]
```

```
[Out] (a^2 + 3*b^2 + a^2*Log[b + a*Cos[x]] + 2*b^2*Log[b + a*Cos[x]] + a^2*Cos[2*
x]*Log[b + a*Cos[x]] + 4*a*b*Cos[x]*(1 + Log[b + a*Cos[x]]))/(2*a^3*(b + a*
Cos[x])^2)
```

fricas [A] time = 0.84, size = 70, normalized size = 1.40

$$\frac{4\,ab\cos(x)+a^2+3\,b^2+2\left(a^2\cos(x)^2+2\,ab\cos(x)+b^2\right)\log\left(a\cos(x)+b\right)}{2\left(a^5\cos(x)^2+2\,a^4b\cos(x)+a^3b^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cot(x)+b*csc(x))^3,x, algorithm="fricas")
```

```
[Out] 1/2*(4*a*b*cos(x) + a^2 + 3*b^2 + 2*(a^2*cos(x)^2 + 2*a*b*cos(x) + b^2)*log
(a*cos(x) + b))/(a^5*cos(x)^2 + 2*a^4*b*cos(x) + a^3*b^2)
```

giac [A] time = 0.13, size = 45, normalized size = 0.90

$$\frac{\log\left(|a\cos(x)+b|\right)}{a^3}+\frac{4\,b\cos(x)+\frac{a^2+3\,b^2}{a}}{2\left(a\cos(x)+b\right)^2a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cot(x)+b*csc(x))^3,x, algorithm="giac")
```

```
[Out] log(abs(a*cos(x) + b))/a^3 + 1/2*(4*b*cos(x) + (a^2 + 3*b^2)/a)/((a*cos(x)
+ b)^2*a^2)
```

maple [A] time = 0.12, size = 56, normalized size = 1.12

$$\frac{\ln\left(b+a\cos(x)\right)}{a^3}+\frac{1}{2a\left(b+a\cos(x)\right)^2}-\frac{b^2}{2a^3\left(b+a\cos(x)\right)^2}+\frac{2b}{a^3\left(b+a\cos(x)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*cot(x)+b*csc(x))^3,x)
```

```
[Out] ln(b+a*cos(x))/a^3+1/2/a/(b+a*cos(x))^2-1/2/a^3/(b+a*cos(x))^2*b^2+2*b/a^3/
(b+a*cos(x))
```

maxima [B] time = 0.43, size = 177, normalized size = 3.54

$$\frac{2\left(ab+b^2+\frac{\left(a^2-2\,ab+b^2\right)\sin(x)^2}{\left(\cos(x)+1\right)^2}\right)}{a^5+a^4b-a^3b^2-a^2b^3-\frac{2\left(a^5-a^4b-a^3b^2+a^2b^3\right)\sin(x)^2}{\left(\cos(x)+1\right)^2}+\frac{\left(a^5-3\,a^4b+3\,a^3b^2-a^2b^3\right)\sin(x)^4}{\left(\cos(x)+1\right)^4}}+\frac{\log\left(a+b-\frac{(a-b)\sin(x)^2}{\left(\cos(x)+1\right)^2}\right)}{a^3}-\frac{\log\left(\frac{s}{(co}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cot(x)+b*csc(x))^3,x, algorithm="maxima")
```

```
[Out] 2*(a*b + b^2 + (a^2 - 2*a*b + b^2)*sin(x)^2/(cos(x) + 1)^2)/(a^5 + a^4*b -
a^3*b^2 - a^2*b^3 - 2*(a^5 - a^4*b - a^3*b^2 + a^2*b^3)*sin(x)^2/(cos(x) +
1)^2 + (a^5 - 3*a^4*b + 3*a^3*b^2 - a^2*b^3)*sin(x)^4/(cos(x) + 1)^4) + log
(a + b - (a - b)*sin(x)^2/(cos(x) + 1)^2)/a^3 - log(sin(x)^2/(cos(x) + 1)^2
 + 1)/a^3
```

mupad [B] time = 2.77, size = 311, normalized size = 6.22

$$\frac{\frac{2\left(b^2+a\,b\right)}{a^2\,(a-b)}+\frac{2\tan\left(\frac{x}{2}\right)^2(a-b)}{a^2}}{\tan\left(\frac{x}{2}\right)^4\,\left(a^2-2\,a\,b+b^2\right)+2\,a\,b-\tan\left(\frac{x}{2}\right)^2\,\left(2\,a^2-2\,b^2\right)+a^2+b^2}-\frac{2\,\mathrm{atanh}\left(\frac{32\tan\left(\frac{x}{2}\right)^2}{\frac{32\,b^3}{a^3}-\frac{32\,b^2}{a^2}-\frac{32\,b}{a}+\frac{32\,b\tan\left(\frac{x}{2}\right)^2}{a}-\frac{64\,b^2\tan\left(\frac{x}{2}\right)^2}{a^2}\cdots}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b/sin(x) + a*cot(x))^3,x)
```

```
[Out] ((2*(a*b + b^2))/(a^2*(a - b)) + (2*tan(x/2)^2*(a - b))/a^2)/(tan(x/2)^4*(a
^2 - 2*a*b + b^2) + 2*a*b - tan(x/2)^2*(2*a^2 - 2*b^2) + a^2 + b^2) - (2*at
anh((32*tan(x/2)^2)/((32*b^3)/a^3 - (32*b^2)/a^2 - (32*b)/a + (32*b*tan(x/2
)^2)/a - (64*b^2*tan(x/2)^2)/a^2 + (32*b^3*tan(x/2)^2)/a^3 + 32) - (64*b*ta
n(x/2)^2)/(32*a - 32*b + 32*b*tan(x/2)^2 - (32*b^2)/a + (32*b^3)/a^2 - (64*
b^2*tan(x/2)^2)/a + (32*b^3*tan(x/2)^2)/a^2) + (32*b^2*tan(x/2)^2)/(32*a^2
- 32*a*b - 32*b^2 - 64*b^2*tan(x/2)^2 + (32*b^3)/a + (32*b^3*tan(x/2)^2)/a
+ 32*a*b*tan(x/2)^2)))/a^3
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a\cot(x) + b\csc(x))^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cot(x)+b*csc(x))**3,x)
```

```
[Out] Integral((a*cot(x) + b*csc(x))**(-3), x)
```

3.291 $\int \frac{1}{(a\cot(x)+b\csc(x))^4}\,dx$

Optimal. Leaf size=159

$$\frac{x}{a^4}+\frac{b\sin^3(x)}{2a\left(a^2-b^2\right)(a\cos(x)+b)^2}-\frac{b\left(3a^2-2b^2\right)\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tan\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{a^4(a-b)^{3/2}(a+b)^{3/2}}-\frac{\sin(x)\left(2\left(a^2-b^2\right)-ab\cos(x)\right)}{2a^3\left(a^2-b^2\right)(a\cos(x)+b)}+\frac{\sin^3\cdots}{3a(a\cos\cdots}$$

```
[Out] x/a^4-b*(3*a^2-2*b^2)*arctanh((a-b)^(1/2)*tan(1/2*x)/(a+b)^(1/2))/a^4/(a-b)
^(3/2)/(a+b)^(3/2)-1/2*(2*a^2-2*b^2-a*b*cos(x))*sin(x)/a^3/(a^2-b^2)/(b+a*c
os(x))+1/3*sin(x)^3/a/(b+a*cos(x))^3+1/2*b*sin(x)^3/a/(a^2-b^2)/(b+a*cos(x)
)^2
```

Rubi [A] time = 0.34, antiderivative size = 159, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.636, Rules used = {4392, 2693, 2864, 2863, 2735, 2659, 208}

$$\frac{b\sin^3(x)}{2a\left(a^2-b^2\right)(a\cos(x)+b)^2}-\frac{\sin(x)\left(2\left(a^2-b^2\right)-ab\cos(x)\right)}{2a^3\left(a^2-b^2\right)(a\cos(x)+b)}-\frac{b\left(3a^2-2b^2\right)\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tan\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{a^4(a-b)^{3/2}(a+b)^{3/2}}+\frac{x}{a^4}+\frac{\sin^3\cdots}{3a(a\cos\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Cot[x] + b*Csc[x])^(-4),x]
```

```
[Out] x/a^4 - (b*(3*a^2 - 2*b^2)*ArcTanh[(Sqrt[a - b]*Tan[x/2])/Sqrt[a + b]])/(a^
4*(a - b)^(3/2)*(a + b)^(3/2)) - ((2*(a^2 - b^2) - a*b*Cos[x])*Sin[x])/(2*a
^3*(a^2 - b^2)*(b + a*Cos[x])) + Sin[x]^3/(3*a*(b + a*Cos[x])^3) + (b*Sin[x
]^3)/(2*a*(a^2 - b^2)*(b + a*Cos[x])^2)
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 2693

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(g*(g*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x
])^(m + 1))/(b*f*(m + 1)), x] + Dist[(g^2*(p - 1))/(b*(m + 1)), Int[(g*Cos[
e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 1)*Sin[e + f*x], x], x] /; Free
Q[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[p, 1] && In
tegersQ[2*m, 2*p]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2863

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(g*(g*
Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x])^(m + 1)*(b*c*(m + p + 1) - a*d*p
 + b*d*(m + 1)*Sin[e + f*x]))/(b^2*f*(m + 1)*(m + p + 1)), x] + Dist[(g^2*(
p - 1))/(b^2*(m + 1)*(m + p + 1)), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[
e + f*x])^(m + 1)*Simp[b*d*(m + 1) + (b*c*(m + p + 1) - a*d*p)*Sin[e + f*x]
, x], x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 - b^2, 0] && LtQ
[m, -1] && GtQ[p, 1] && NeQ[m + p + 1, 0] && IntegerQ[2*m]
```

Rule 2864

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> -Simp[((b*c
 - a*d)*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^(m + 1))/(f*g*(a^2 -
b^2)*(m + 1)), x] + Dist[1/((a^2 - b^2)*(m + 1)), Int[(g*Cos[e + f*x])^p*(a
 + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m + p +
2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[a^
2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rule 4392

```
Int[(cot[(c_.) + (d_.)*(x_)]^(n_.)*(a_.) + csc[(c_.) + (d_.)*(x_)]^(n_.)*(b
_.))^(p_)*(u_.), x_Symbol] :> Int[ActivateTrig[u]*Csc[c + d*x]^(n*p)*(b + a
*Cos[c + d*x]^n)^p, x] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p]
```

Rubi steps

$$\int \frac{1}{(a\cot(x)+b\csc(x))^4}\,dx = \int \frac{\sin^4(x)}{(b+a\cos(x))^4}\,dx$$

$$= \frac{\sin^3(x)}{3a(b+a\cos(x))^3} - \frac{\int \frac{\cos(x)\sin^2(x)}{(b+a\cos(x))^3}\,dx}{a}$$

$$= \frac{\sin^3(x)}{3a(b+a\cos(x))^3} + \frac{b\sin^3(x)}{2a\left(a^2-b^2\right)(b+a\cos(x))^2} - \frac{\int \frac{(2a+b\cos(x))\sin^2(x)}{(b+a\cos(x))^2}\,dx}{2a\left(a^2-b^2\right)}$$

$$= -\frac{\left(2\left(a^2-b^2\right)-ab\cos(x)\right)\sin(x)}{2a^3\left(a^2-b^2\right)(b+a\cos(x))} + \frac{\sin^3(x)}{3a(b+a\cos(x))^3} + \frac{b\sin^3(x)}{2a\left(a^2-b^2\right)(b+a\cos(x)}$$

$$= \frac{x}{a^4} - \frac{\left(2\left(a^2-b^2\right)-ab\cos(x)\right)\sin(x)}{2a^3\left(a^2-b^2\right)(b+a\cos(x))} + \frac{\sin^3(x)}{3a(b+a\cos(x))^3} + \frac{b\sin^3(x)}{2a\left(a^2-b^2\right)(b+a\cos}$$

$$= \frac{x}{a^4} - \frac{\left(2\left(a^2-b^2\right)-ab\cos(x)\right)\sin(x)}{2a^3\left(a^2-b^2\right)(b+a\cos(x))} + \frac{\sin^3(x)}{3a(b+a\cos(x))^3} + \frac{b\sin^3(x)}{2a\left(a^2-b^2\right)(b+a\cos}$$

$$= \frac{x}{a^4} - \frac{b\left(3a^2-2b^2\right)\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tan\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{a^4(a-b)^{3/2}(a+b)^{3/2}} - \frac{\left(2\left(a^2-b^2\right)-ab\cos(x)\right)\sin(x)}{2a^3\left(a^2-b^2\right)(b+a\cos(x))} + \frac{}{3a(b}$$

Mathematica [A] time = 0.48, size = 150, normalized size = 0.94

$$\frac{\sin(x)\left(-\frac{a\left(8a^2-11b^2\right)(a\cos(x)+b)^2}{(a-b)(a+b)} - \frac{6b\left(2b^2-3a^2\right)\csc(x)(a\cos(x)+b)^3\tanh^{-1}\left(\frac{(b-a)\tan\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{3/2}} + 2a\left(a^2-b^2\right) + 7ab(a\cos(x)+b) + 6}{6a^4(a\cos(x)+b)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Cot[x] + b*Csc[x])^(-4),x]
```

```
[Out] ((2*a*(a^2 - b^2) + 7*a*b*(b + a*Cos[x]) - (a*(8*a^2 - 11*b^2)*(b + a*Cos[x
])^2)/((a - b)*(a + b)) + 6*x*(b + a*Cos[x])^3*Csc[x] - (6*b*(-3*a^2 + 2*b^
2)*ArcTanh[((-a + b)*Tan[x/2])/Sqrt[a^2 - b^2]]*(b + a*Cos[x])^3*Csc[x])/(a
^2 - b^2)^(3/2))*Sin[x])/(6*a^4*(b + a*Cos[x])^3)
```

fricas [B] time = 2.08, size = 878, normalized size = 5.52

$$\left[\frac{12\left(a^7-2\,a^5b^2+a^3b^4\right)x\cos(x)^3+36\left(a^6b-2\,a^4b^3+a^2b^5\right)x\cos(x)^2+36\left(a^5b^2-2\,a^3b^4+ab^6\right)x\cos(x)+3\left(3\,a^2\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cot(x)+b*csc(x))^4,x, algorithm="fricas")
```

```
[Out] [1/12*(12*(a^7 - 2*a^5*b^2 + a^3*b^4)*x*cos(x)^3 + 36*(a^6*b - 2*a^4*b^3 +
a^2*b^5)*x*cos(x)^2 + 36*(a^5*b^2 - 2*a^3*b^4 + a*b^6)*x*cos(x) + 3*(3*a^2*
b^4 - 2*b^6 + (3*a^5*b - 2*a^3*b^3)*cos(x)^3 + 3*(3*a^4*b^2 - 2*a^2*b^4)*co
s(x)^2 + 3*(3*a^3*b^3 - 2*a*b^5)*cos(x))*sqrt(a^2 - b^2)*log((2*a*b*cos(x)
- (a^2 - 2*b^2)*cos(x)^2 - 2*sqrt(a^2 - b^2)*(b*cos(x) + a)*sin(x) + 2*a^2
- b^2)/(a^2*cos(x)^2 + 2*a*b*cos(x) + b^2)) + 12*(a^4*b^3 - 2*a^2*b^5 + b^7
)*x + 2*(2*a^7 - 7*a^5*b^2 + 11*a^3*b^4 - 6*a*b^6 - (8*a^7 - 19*a^5*b^2 + 1
1*a^3*b^4)*cos(x)^2 - 3*(3*a^6*b - 8*a^4*b^3 + 5*a^2*b^5)*cos(x))*sin(x))/(
a^8*b^3 - 2*a^6*b^5 + a^4*b^7 + (a^11 - 2*a^9*b^2 + a^7*b^4)*cos(x)^3 + 3*(
a^10*b - 2*a^8*b^3 + a^6*b^5)*cos(x)^2 + 3*(a^9*b^2 - 2*a^7*b^4 + a^5*b^6)*
cos(x)), 1/6*(6*(a^7 - 2*a^5*b^2 + a^3*b^4)*x*cos(x)^3 + 18*(a^6*b - 2*a^4*
b^3 + a^2*b^5)*x*cos(x)^2 + 18*(a^5*b^2 - 2*a^3*b^4 + a*b^6)*x*cos(x) - 3*(
3*a^2*b^4 - 2*b^6 + (3*a^5*b - 2*a^3*b^3)*cos(x)^3 + 3*(3*a^4*b^2 - 2*a^2*b
^4)*cos(x)^2 + 3*(3*a^3*b^3 - 2*a*b^5)*cos(x))*sqrt(-a^2 + b^2)*arctan(-sqr
t(-a^2 + b^2)*(b*cos(x) + a)/((a^2 - b^2)*sin(x))) + 6*(a^4*b^3 - 2*a^2*b^5
 + b^7)*x + (2*a^7 - 7*a^5*b^2 + 11*a^3*b^4 - 6*a*b^6 - (8*a^7 - 19*a^5*b^2
 + 11*a^3*b^4)*cos(x)^2 - 3*(3*a^6*b - 8*a^4*b^3 + 5*a^2*b^5)*cos(x))*sin(x
))/(a^8*b^3 - 2*a^6*b^5 + a^4*b^7 + (a^11 - 2*a^9*b^2 + a^7*b^4)*cos(x)^3 +
 3*(a^10*b - 2*a^8*b^3 + a^6*b^5)*cos(x)^2 + 3*(a^9*b^2 - 2*a^7*b^4 + a^5*b
^6)*cos(x))]
```

giac [A] time = 0.20, size = 282, normalized size = 1.77

$$-\frac{\left(3\,a^2b-2\,b^3\right)\left(\pi\left\lfloor\frac{x}{2\,\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}\left(-2\,a+2\,b\right)+\arctan\left(-\frac{a\tan\left(\frac{1}{2}\,x\right)-b\tan\left(\frac{1}{2}\,x\right)}{\sqrt{-a^2+b^2}}\right)\right)}{\left(a^6-a^4b^2\right)\sqrt{-a^2+b^2}}+\frac{6\,a^4\tan\left(\frac{1}{2}\,x\right)^5-9\,a^3b\tan\left(\frac{1}{2}\,x\right)^5-}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cot(x)+b*csc(x))^4,x, algorithm="giac")
```

```
[Out] -(3*a^2*b - 2*b^3)*(pi*floor(1/2*x/pi + 1/2)*sgn(-2*a + 2*b) + arctan(-(a*t
an(1/2*x) - b*tan(1/2*x))/sqrt(-a^2 + b^2)))/((a^6 - a^4*b^2)*sqrt(-a^2 + b
```

```
^2)) + 1/3*(6*a^4*tan(1/2*x)^5 - 9*a^3*b*tan(1/2*x)^5 - 6*a^2*b^2*tan(1/2*x
)^5 + 15*a*b^3*tan(1/2*x)^5 - 6*b^4*tan(1/2*x)^5 - 20*a^4*tan(1/2*x)^3 + 32
*a^2*b^2*tan(1/2*x)^3 - 12*b^4*tan(1/2*x)^3 + 6*a^4*tan(1/2*x) + 9*a^3*b*ta
n(1/2*x) - 6*a^2*b^2*tan(1/2*x) - 15*a*b^3*tan(1/2*x) - 6*b^4*tan(1/2*x))/(
(a^5 - a^3*b^2)*(a*tan(1/2*x)^2 - b*tan(1/2*x)^2 - a - b)^3) + x/a^4
```

maple [**B**] time = 0.14, size = 534, normalized size = 3.36

$$\frac{2\left(\tan^5\left(\frac{x}{2}\right)\right)}{\left(a\left(\tan^2\left(\frac{x}{2}\right)\right)-b\left(\tan^2\left(\frac{x}{2}\right)\right)-a-b\right)^3(a+b)}-\frac{\left(\tan^5\left(\frac{x}{2}\right)\right)b}{a\left(a\left(\tan^2\left(\frac{x}{2}\right)\right)-b\left(\tan^2\left(\frac{x}{2}\right)\right)-a-b\right)^3(a+b)}-\frac{3}{a^2\left(a\left(\tan^2\left(\frac{x}{2}\right)\right)-\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*cot(x)+b*csc(x))^4,x)
```

```
[Out] 2/(a*tan(1/2*x)^2-b*tan(1/2*x)^2-a-b)^3/(a+b)*tan(1/2*x)^5-1/a/(a*tan(1/2*x
)^2-b*tan(1/2*x)^2-a-b)^3/(a+b)*tan(1/2*x)^5*b-3/a^2/(a*tan(1/2*x)^2-b*tan(
1/2*x)^2-a-b)^3/(a+b)*tan(1/2*x)^5*b^2+2/a^3/(a*tan(1/2*x)^2-b*tan(1/2*x)^2
-a-b)^3/(a+b)*tan(1/2*x)^5*b^3-20/3/a/(a*tan(1/2*x)^2-b*tan(1/2*x)^2-a-b)^3
*tan(1/2*x)^3+4/a^3/(a*tan(1/2*x)^2-b*tan(1/2*x)^2-a-b)^3*tan(1/2*x)^3*b^2+
2/(a*tan(1/2*x)^2-b*tan(1/2*x)^2-a-b)^3/(a-b)*tan(1/2*x)+1/a/(a*tan(1/2*x)^
2-b*tan(1/2*x)^2-a-b)^3/(a-b)*tan(1/2*x)*b-3/a^2/(a*tan(1/2*x)^2-b*tan(1/2*
x)^2-a-b)^3/(a-b)*tan(1/2*x)*b^2-2/a^3/(a*tan(1/2*x)^2-b*tan(1/2*x)^2-a-b)^
3/(a-b)*tan(1/2*x)*b^3-3/a^2*b/(a^2-b^2)/((a+b)*(a-b))^(1/2)*arctanh(tan(1/
2*x)*(a-b)/((a+b)*(a-b))^(1/2))+2/a^4*b^3/(a^2-b^2)/((a+b)*(a-b))^(1/2)*arc
tanh(tan(1/2*x)*(a-b)/((a+b)*(a-b))^(1/2))+2/a^4*arctan(tan(1/2*x))
```

maxima [**F(-2)**] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cot(x)+b*csc(x))^4,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` for
more details)Is 4*a^2-4*b^2 positive or negative?
```

mupad [**B**] time = 8.25, size = 3068, normalized size = 19.30

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b/sin(x) + a*cot(x))^4,x)

[Out] (2*atan((((((8*(6*a^12*b - 4*a^13 + 4*a^8*b^5 - 2*a^9*b^4 - 10*a^10*b^3 + 6
*a^11*b^2))/(a^11*b + a^12 - a^9*b^3 - a^10*b^2) - (tan(x/2)*(8*a^13*b - 8*
a^8*b^6 + 8*a^9*b^5 + 16*a^10*b^4 - 16*a^11*b^3 - 8*a^12*b^2)*8i)/(a^4*(a^8
*b + a^9 - a^6*b^3 - a^7*b^2)))*1i)/a^4 + (8*tan(x/2)*(4*a^6 - 8*a^5*b - 8*
a*b^5 + 8*b^6 - 16*a^2*b^4 + 16*a^3*b^3 + 5*a^4*b^2))/(a^8*b + a^9 - a^6*b^
3 - a^7*b^2))/a^4 - ((((8*(6*a^12*b - 4*a^13 + 4*a^8*b^5 - 2*a^9*b^4 - 10*a
^10*b^3 + 6*a^11*b^2))/(a^11*b + a^12 - a^9*b^3 - a^10*b^2) + (tan(x/2)*(8*
a^13*b - 8*a^8*b^6 + 8*a^9*b^5 + 16*a^10*b^4 - 16*a^11*b^3 - 8*a^12*b^2)*8i
)/(a^4*(a^8*b + a^9 - a^6*b^3 - a^7*b^2)))*1i)/a^4 - (8*tan(x/2)*(4*a^6 - 8
*a^5*b - 8*a*b^5 + 8*b^6 - 16*a^2*b^4 + 16*a^3*b^3 + 5*a^4*b^2))/(a^8*b + a
^9 - a^6*b^3 - a^7*b^2))/a^4)/((16*(6*a^4*b - 2*a*b^4 + 4*b^5 - 10*a^2*b^3
+ 3*a^3*b^2))/(a^11*b + a^12 - a^9*b^3 - a^10*b^2) + (((((8*(6*a^12*b - 4*a
^13 + 4*a^8*b^5 - 2*a^9*b^4 - 10*a^10*b^3 + 6*a^11*b^2))/(a^11*b + a^12 - a
^9*b^3 - a^10*b^2) - (tan(x/2)*(8*a^13*b - 8*a^8*b^6 + 8*a^9*b^5 + 16*a^10*
b^4 - 16*a^11*b^3 - 8*a^12*b^2)*8i)/(a^4*(a^8*b + a^9 - a^6*b^3 - a^7*b^2))
)*1i)/a^4 + (8*tan(x/2)*(4*a^6 - 8*a^5*b - 8*a*b^5 + 8*b^6 - 16*a^2*b^4 + 1
6*a^3*b^3 + 5*a^4*b^2))/(a^8*b + a^9 - a^6*b^3 - a^7*b^2))*1i)/a^4 + (((((8
*(6*a^12*b - 4*a^13 + 4*a^8*b^5 - 2*a^9*b^4 - 10*a^10*b^3 + 6*a^11*b^2))/(a
^11*b + a^12 - a^9*b^3 - a^10*b^2) + (tan(x/2)*(8*a^13*b - 8*a^8*b^6 + 8*a^
9*b^5 + 16*a^10*b^4 - 16*a^11*b^3 - 8*a^12*b^2)*8i)/(a^4*(a^8*b + a^9 - a^6
*b^3 - a^7*b^2)))*1i)/a^4 - (8*tan(x/2)*(4*a^6 - 8*a^5*b - 8*a*b^5 + 8*b^6
- 16*a^2*b^4 + 16*a^3*b^3 + 5*a^4*b^2))/(a^8*b + a^9 - a^6*b^3 - a^7*b^2))*
1i)/a^4)))/a^4 + ((4*tan(x/2)^3*(5*a^2 - 3*b^2))/(3*a^3) - (tan(x/2)*(a + b
)*(a*b - 2*a^2 + 2*b^2))/(a^3*b - a^4) + (tan(x/2)^5*(3*a*b^2 + a^2*b - 2*a
^3 - 2*b^3))/(a^3*(a + b)))/(3*a*b^2 - tan(x/2)^6*(3*a*b^2 - 3*a^2*b + a^3
- b^3) + 3*a^2*b + tan(x/2)^2*(3*a*b^2 - 3*a^2*b - 3*a^3 + 3*b^3) - tan(x/2
)^4*(3*a*b^2 + 3*a^2*b - 3*a^3 - 3*b^3) + a^3 + b^3) + (b*atan(((b*(3*a^2 -
 2*b^2)*((a + b)^3*(a - b)^3)^(1/2)*((8*tan(x/2)*(4*a^6 - 8*a^5*b - 8*a*b^5
 + 8*b^6 - 16*a^2*b^4 + 16*a^3*b^3 + 5*a^4*b^2))/(a^8*b + a^9 - a^6*b^3 - a
^7*b^2) + (b*((8*(6*a^12*b - 4*a^13 + 4*a^8*b^5 - 2*a^9*b^4 - 10*a^10*b^3 +
 6*a^11*b^2))/(a^11*b + a^12 - a^9*b^3 - a^10*b^2) - (4*b*tan(x/2)*(3*a^2 -
 2*b^2)*((a + b)^3*(a - b)^3)^(1/2)*(8*a^13*b - 8*a^8*b^6 + 8*a^9*b^5 + 16*
a^10*b^4 - 16*a^11*b^3 - 8*a^12*b^2))/((a^8*b + a^9 - a^6*b^3 - a^7*b^2)*(a
^10 - a^4*b^6 + 3*a^6*b^4 - 3*a^8*b^2)))*(3*a^2 - 2*b^2)*((a + b)^3*(a - b)
^3)^(1/2))/(2*(a^10 - a^4*b^6 + 3*a^6*b^4 - 3*a^8*b^2)))*1i)/(2*(a^10 - a^4
*b^6 + 3*a^6*b^4 - 3*a^8*b^2)) + (b*(3*a^2 - 2*b^2)*((a + b)^3*(a - b)^3)^(
1/2)*((8*tan(x/2)*(4*a^6 - 8*a^5*b - 8*a*b^5 + 8*b^6 - 16*a^2*b^4 + 16*a^3*
b^3 + 5*a^4*b^2))/(a^8*b + a^9 - a^6*b^3 - a^7*b^2) - (b*((8*(6*a^12*b - 4*
a^13 + 4*a^8*b^5 - 2*a^9*b^4 - 10*a^10*b^3 + 6*a^11*b^2))/(a^11*b + a^12 -
a^9*b^3 - a^10*b^2) + (4*b*tan(x/2)*(3*a^2 - 2*b^2)*((a + b)^3*(a - b)^3)^(
1/2)*(8*a^13*b - 8*a^8*b^6 + 8*a^9*b^5 + 16*a^10*b^4 - 16*a^11*b^3 - 8*a^12
*b^2))/((a^8*b + a^9 - a^6*b^3 - a^7*b^2)*(a^10 - a^4*b^6 + 3*a^6*b^4 - 3*a
^8*b^2)))*(3*a^2 - 2*b^2)*((a + b)^3*(a - b)^3)^(1/2))/(2*(a^10 - a^4*b^6 +
```

```
 3*a^6*b^4 - 3*a^8*b^2)))*1i)/(2*(a^10 - a^4*b^6 + 3*a^6*b^4 - 3*a^8*b^2)))
/((16*(6*a^4*b - 2*a*b^4 + 4*b^5 - 10*a^2*b^3 + 3*a^3*b^2))/(a^11*b + a^12
- a^9*b^3 - a^10*b^2) + (b*(3*a^2 - 2*b^2)*((a + b)^3*(a - b)^3)^(1/2)*((8*
tan(x/2)*(4*a^6 - 8*a^5*b - 8*a*b^5 + 8*b^6 - 16*a^2*b^4 + 16*a^3*b^3 + 5*a
^4*b^2))/(a^8*b + a^9 - a^6*b^3 - a^7*b^2) + (b*((8*(6*a^12*b - 4*a^13 + 4*
a^8*b^5 - 2*a^9*b^4 - 10*a^10*b^3 + 6*a^11*b^2))/(a^11*b + a^12 - a^9*b^3 -
 a^10*b^2) - (4*b*tan(x/2)*(3*a^2 - 2*b^2)*((a + b)^3*(a - b)^3)^(1/2)*(8*a
^13*b - 8*a^8*b^6 + 8*a^9*b^5 + 16*a^10*b^4 - 16*a^11*b^3 - 8*a^12*b^2))/((
a^8*b + a^9 - a^6*b^3 - a^7*b^2)*(a^10 - a^4*b^6 + 3*a^6*b^4 - 3*a^8*b^2)))
*(3*a^2 - 2*b^2)*((a + b)^3*(a - b)^3)^(1/2))/(2*(a^10 - a^4*b^6 + 3*a^6*b^
4 - 3*a^8*b^2))))/(2*(a^10 - a^4*b^6 + 3*a^6*b^4 - 3*a^8*b^2)) - (b*(3*a^2
- 2*b^2)*((a + b)^3*(a - b)^3)^(1/2)*((8*tan(x/2)*(4*a^6 - 8*a^5*b - 8*a*b^
5 + 8*b^6 - 16*a^2*b^4 + 16*a^3*b^3 + 5*a^4*b^2))/(a^8*b + a^9 - a^6*b^3 -
a^7*b^2) - (b*((8*(6*a^12*b - 4*a^13 + 4*a^8*b^5 - 2*a^9*b^4 - 10*a^10*b^3
+ 6*a^11*b^2))/(a^11*b + a^12 - a^9*b^3 - a^10*b^2) + (4*b*tan(x/2)*(3*a^2
- 2*b^2)*((a + b)^3*(a - b)^3)^(1/2)*(8*a^13*b - 8*a^8*b^6 + 8*a^9*b^5 + 16
*a^10*b^4 - 16*a^11*b^3 - 8*a^12*b^2))/((a^8*b + a^9 - a^6*b^3 - a^7*b^2)*(
a^10 - a^4*b^6 + 3*a^6*b^4 - 3*a^8*b^2)))*(3*a^2 - 2*b^2)*((a + b)^3*(a - b
)^3)^(1/2))/(2*(a^10 - a^4*b^6 + 3*a^6*b^4 - 3*a^8*b^2)))))/(2*(a^10 - a^4*b
^6 + 3*a^6*b^4 - 3*a^8*b^2))))*(3*a^2 - 2*b^2)*((a + b)^3*(a - b)^3)^(1/2)*
1i)/(a^10 - a^4*b^6 + 3*a^6*b^4 - 3*a^8*b^2)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cot(x)+b*csc(x))**4,x)
```

```
[Out] Timed out
```

3.292 $\int \frac{1}{(a\cot(x)+b\csc(x))^5}\,dx$

Optimal. Leaf size=100

$$-\frac{4b}{a^5(a\cos(x)+b)}-\frac{\log(a\cos(x)+b)}{a^5}+\frac{\left(a^2-b^2\right)^2}{4a^5(a\cos(x)+b)^4}+\frac{4b\left(a^2-b^2\right)}{3a^5(a\cos(x)+b)^3}-\frac{a^2-3b^2}{a^5(a\cos(x)+b)^2}$$

```
[Out] 1/4*(a^2-b^2)^2/a^5/(b+a*cos(x))^4+4/3*b*(a^2-b^2)/a^5/(b+a*cos(x))^3+(-a^2
+3*b^2)/a^5/(b+a*cos(x))^2-4*b/a^5/(b+a*cos(x))-ln(b+a*cos(x))/a^5
```

Rubi [A] time = 0.12, antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {4392, 2668, 697}

$$\frac{\left(a^2-b^2\right)^2}{4a^5(a\cos(x)+b)^4}+\frac{4b\left(a^2-b^2\right)}{3a^5(a\cos(x)+b)^3}-\frac{a^2-3b^2}{a^5(a\cos(x)+b)^2}-\frac{4b}{a^5(a\cos(x)+b)}-\frac{\log(a\cos(x)+b)}{a^5}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Cot[x] + b*Csc[x])^(-5),x]
```

```
[Out] (a^2 - b^2)^2/(4*a^5*(b + a*Cos[x])^4) + (4*b*(a^2 - b^2))/(3*a^5*(b + a*Co
s[x])^3) - (a^2 - 3*b^2)/(a^5*(b + a*Cos[x])^2) - (4*b)/(a^5*(b + a*Cos[x])
) - Log[b + a*Cos[x]]/a^5
```

Rule 697

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m},
 x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0]
```

Rule 2668

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rule 4392

```
Int[(cot[(c_.) + (d_.)*(x_)]^(n_.)*(a_.) + csc[(c_.) + (d_.)*(x_)]^(n_.)*(b
_.))^(p_)*(u_.), x_Symbol] :> Int[ActivateTrig[u]*Csc[c + d*x]^(n*p)*(b + a
*Cos[c + d*x]^n)^p, x] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p]
```

Rubi steps

$$\int \frac{1}{(a\cot(x)+b\csc(x))^5}\,dx = \int \frac{\sin^5(x)}{(b+a\cos(x))^5}\,dx$$
$$= -\frac{\text{Subst}\left(\int \frac{\left(a^2-x^2\right)^2}{(b+x)^5}\,dx, x, a\cos(x)\right)}{a^5}$$
$$= -\frac{\text{Subst}\left(\int \left(\frac{\left(a^2-b^2\right)^2}{(b+x)^5} - \frac{4b\left(-a^2+b^2\right)}{(b+x)^4} - \frac{2\left(a^2-3b^2\right)}{(b+x)^3} - \frac{4b}{(b+x)^2} + \frac{1}{b+x}\right)dx, x, a\cos(x)\right)}{a^5}$$
$$= \frac{\left(a^2-b^2\right)^2}{4a^5(b+a\cos(x))^4} + \frac{4b\left(a^2-b^2\right)}{3a^5(b+a\cos(x))^3} - \frac{a^2-3b^2}{a^5(b+a\cos(x))^2} - \frac{4b}{a^5(b+a\cos(x))} -$$

Mathematica [A] time = 0.34, size = 138, normalized size = 1.38

$$-\frac{12a^4\cos^4(x)\log(a\cos(x)+b) - 3a^4 + 48a^3b\cos^3(x)(\log(a\cos(x)+b)+1) + 12a^2\cos^2(x)\left(a^2+6b^2\log(a\cos\right.}{12a^5(a\cos(}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Cot[x] + b*Csc[x])^(-5),x]
```

```
[Out] -1/12*(-3*a^4 + 2*a^2*b^2 + 25*b^4 + 12*b^4*Log[b + a*Cos[x]] + 12*a^4*Cos[
x]^4*Log[b + a*Cos[x]] + 48*a^3*b*Cos[x]^3*(1 + Log[b + a*Cos[x]]) + 12*a^2
*Cos[x]^2*(a^2 + 9*b^2 + 6*b^2*Log[b + a*Cos[x]]) + 8*a*b*Cos[x]*(a^2 + 11*
b^2 + 6*b^2*Log[b + a*Cos[x]]))/(a^5*(b + a*Cos[x])^4)
```

fricas [A] time = 2.03, size = 166, normalized size = 1.66

$$-\frac{48\,a^3b\cos(x)^3 - 3\,a^4 + 2\,a^2b^2 + 25\,b^4 + 12\left(a^4+9\,a^2b^2\right)\cos(x)^2 + 8\left(a^3b+11\,ab^3\right)\cos(x) + 12\left(a^4\cos(x)^4 + 4\right.}{12\left(a^9\cos(x)^4 + 4\,a^8b\cos(x)^3 + 6\,a^7b^2\cos(x)^2 + 4\,a^6b^3\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cot(x)+b*csc(x))^5,x, algorithm="fricas")
```

```
[Out] -1/12*(48*a^3*b*cos(x)^3 - 3*a^4 + 2*a^2*b^2 + 25*b^4 + 12*(a^4 + 9*a^2*b^2
)*cos(x)^2 + 8*(a^3*b + 11*a*b^3)*cos(x) + 12*(a^4*cos(x)^4 + 4*a^3*b*cos(x
)^3 + 6*a^2*b^2*cos(x)^2 + 4*a*b^3*cos(x) + b^4)*log(a*cos(x) + b))/(a^9*co
s(x)^4 + 4*a^8*b*cos(x)^3 + 6*a^7*b^2*cos(x)^2 + 4*a^6*b^3*cos(x) + a^5*b^4
)
```

giac [A] time = 0.13, size = 93, normalized size = 0.93

$$-\frac{\log\left(|a\cos(x)+b|\right)}{a^5}-\frac{48\,a^2b\cos(x)^3+12\left(a^3+9\,ab^2\right)\cos(x)^2+8\left(a^2b+11\,b^3\right)\cos(x)-\frac{3\,a^4-2\,a^2b^2-25\,b^4}{a}}{12\left(a\cos(x)+b\right)^4a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cot(x)+b*csc(x))^5,x, algorithm="giac")
```

```
[Out] -log(abs(a*cos(x) + b))/a^5 - 1/12*(48*a^2*b*cos(x)^3 + 12*(a^3 + 9*a*b^2)*
cos(x)^2 + 8*(a^2*b + 11*b^3)*cos(x) - (3*a^4 - 2*a^2*b^2 - 25*b^4)/a)/((a*
cos(x) + b)^4*a^4)
```

maple [A] time = 0.13, size = 132, normalized size = 1.32

$$\frac{4b}{3a^3\left(b+a\cos(x)\right)^3}-\frac{4b^3}{3a^5\left(b+a\cos(x)\right)^3}-\frac{4b}{a^5\left(b+a\cos(x)\right)}-\frac{\ln\left(b+a\cos(x)\right)}{a^5}-\frac{1}{a^3\left(b+a\cos(x)\right)^2}+\frac{3b^2}{a^5\left(b+a\cos(x)\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*cot(x)+b*csc(x))^5,x)
```

```
[Out] 4/3*b/a^3/(b+a*cos(x))^3-4/3*b^3/a^5/(b+a*cos(x))^3-4*b/a^5/(b+a*cos(x))-ln
(b+a*cos(x))/a^5-1/a^3/(b+a*cos(x))^2+3/a^5/(b+a*cos(x))^2*b^2+1/4/a/(b+a*c
os(x))^4-1/2/a^3/(b+a*cos(x))^4*b^2+1/4/a^5/(b+a*cos(x))^4*b^4
```

maxima [B] time = 0.48, size = 497, normalized size = 4.97

$$-\frac{2\left(5\,a^4b+10\,a^3b^2+2\,a^2b^3-6\,ab^4-3\,b^5+\frac{\left(3\,a^5-17\,a^4b-6\,a^3b^2+26\,a^2b^3+3\,ab^4-9\,b^5\right)\sin(x)^2}{\left(\cos(x)+1\right)^2}-\ldots\right.}{3\left(a^{10}+2\,a^9b-a^8b^2-4\,a^7b^3-a^6b^4+2\,a^5b^5+a^4b^6-\frac{4\left(a^{10}-3\,a^8b^2+3\,a^6b^4-a^4b^6\right)\sin(x)^2}{\left(\cos(x)+1\right)^2}+\frac{6\left(a^{10}-2\,a^9b-a^8b^2+4\,a^7b^3-a^6b^4-2\ldots\right.}{\left(\cos(x)+1\right)^4}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cot(x)+b*csc(x))^5,x, algorithm="maxima")
```

```
[Out] -2/3*(5*a^4*b + 10*a^3*b^2 + 2*a^2*b^3 - 6*a*b^4 - 3*b^5 + (3*a^5 - 17*a^4*
b - 6*a^3*b^2 + 26*a^2*b^3 + 3*a*b^4 - 9*b^5)*sin(x)^2/(cos(x) + 1)^2 - 3*(
4*a^5 - 13*a^4*b + 12*a^3*b^2 + 2*a^2*b^3 - 8*a*b^4 + 3*b^5)*sin(x)^4/(cos(
x) + 1)^4 + 3*(a^5 - 5*a^4*b + 10*a^3*b^2 - 10*a^2*b^3 + 5*a*b^4 - b^5)*sin
(x)^6/(cos(x) + 1)^6)/(a^10 + 2*a^9*b - a^8*b^2 - 4*a^7*b^3 - a^6*b^4 + 2*a
^5*b^5 + a^4*b^6 - 4*(a^10 - 3*a^8*b^2 + 3*a^6*b^4 - a^4*b^6)*sin(x)^2/(cos
(x) + 1)^2 + 6*(a^10 - 2*a^9*b - a^8*b^2 + 4*a^7*b^3 - a^6*b^4 - 2*a^5*b^5
+ a^4*b^6)*sin(x)^4/(cos(x) + 1)^4 - 4*(a^10 - 4*a^9*b + 5*a^8*b^2 - 5*a^6*
```

```
b^4 + 4*a^5*b^5 - a^4*b^6)*sin(x)^6/(cos(x) + 1)^6 + (a^10 - 6*a^9*b + 15*a
^8*b^2 - 20*a^7*b^3 + 15*a^6*b^4 - 6*a^5*b^5 + a^4*b^6)*sin(x)^8/(cos(x) +
1)^8) - log(a + b - (a - b)*sin(x)^2/(cos(x) + 1)^2)/a^5 + log(sin(x)^2/(co
s(x) + 1)^2 + 1)/a^5
```

mupad [B] time = 3.68, size = 538, normalized size = 5.38

$$\frac{2\,\mathrm{atanh}\left(\frac{32\tan\left(\frac{x}{2}\right)^2}{\frac{32\,b^3}{a^3}-\frac{32\,b^2}{a^2}-\frac{32\,b}{a}+\frac{32\,b\tan\left(\frac{x}{2}\right)^2}{a}-\frac{64\,b^2\tan\left(\frac{x}{2}\right)^2}{a^2}+\frac{32\,b^3\tan\left(\frac{x}{2}\right)^2}{a^3}+32}-\frac{64\,b\tan\left(\frac{x}{2}\right)^2}{32\,a-32\,b+32\,b\tan\left(\frac{x}{2}\right)^2-\frac{32\,b^2}{a}+\frac{32\,b^3}{a^2}-\frac{64\,b^2\tan\left(\frac{x}{2}\right)^2}{a}+\frac{32\,b^3\tan\left(\frac{x}{2}\right)^2}{a^2}}+\right.}{a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b/sin(x) + a*cot(x))^5,x)
```

```
[Out] (2*atanh((32*tan(x/2)^2)/((32*b^3)/a^3 - (32*b^2)/a^2 - (32*b)/a + (32*b*ta
n(x/2)^2)/a - (64*b^2*tan(x/2)^2)/a^2 + (32*b^3*tan(x/2)^2)/a^3 + 32) - (64
*b*tan(x/2)^2)/(32*a - 32*b + 32*b*tan(x/2)^2 - (32*b^2)/a + (32*b^3)/a^2 -
 (64*b^2*tan(x/2)^2)/a + (32*b^3*tan(x/2)^2)/a^2) + (32*b^2*tan(x/2)^2)/(32
*a^2 - 32*a*b - 32*b^2 - 64*b^2*tan(x/2)^2 + (32*b^3)/a + (32*b^3*tan(x/2)^
2)/a + 32*a*b*tan(x/2)^2)))/a^5 - ((2*tan(x/2)^6*(3*a*b^2 - 3*a^2*b + a^3 -
 b^3))/a^4 + (2*(5*a^4*b - 6*a*b^4 - 3*b^5 + 2*a^2*b^3 + 10*a^3*b^2))/(3*a^
4*(a - b)^2) + (2*tan(x/2)^4*(2*a*b^2 + 5*a^2*b - 4*a^3 - 3*b^3))/a^4 + (2*
tan(x/2)^2*(6*a*b^3 - 14*a^3*b + 3*a^4 + 9*b^4 - 20*a^2*b^2))/(3*a^4*(a - b
)))/(4*a*b^3 + 4*a^3*b + tan(x/2)^4*(6*a^4 + 6*b^4 - 12*a^2*b^2) + tan(x/2)
^2*(8*a*b^3 - 8*a^3*b - 4*a^4 + 4*b^4) - tan(x/2)^6*(8*a*b^3 - 8*a^3*b + 4*
a^4 - 4*b^4) + a^4 + b^4 + tan(x/2)^8*(a^4 - 4*a^3*b - 4*a*b^3 + b^4 + 6*a^
2*b^2) + 6*a^2*b^2)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cot(x)+b*csc(x))**5,x)
```

```
[Out] Timed out
```

3.293 $\int (\cot(x)+\csc(x))^5 \, dx$

Optimal. Leaf size=28

$$\frac{4}{1-\cos(x)}-\frac{2}{(1-\cos(x))^2}+\log(1-\cos(x))$$

```
[Out] -2/(1-cos(x))^2+4/(1-cos(x))+ln(1-cos(x))
```

Rubi [A] time = 0.05, antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {4392, 2667, 43}

$$\frac{4}{1-\cos(x)}-\frac{2}{(1-\cos(x))^2}+\log(1-\cos(x))$$

Antiderivative was successfully verified.

```
[In] Int[(Cot[x] + Csc[x])^5,x]
```

```
[Out] -2/(1 - Cos[x])^2 + 4/(1 - Cos[x]) + Log[1 - Cos[x]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2667

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rule 4392

```
Int[(cot[(c_.) + (d_.)*(x_)]^(n_.)*(a_.) + csc[(c_.) + (d_.)*(x_)]^(n_.)*(b
_.))^(p_)*(u_.), x_Symbol] :> Int[ActivateTrig[u]*Csc[c + d*x]^(n*p)*(b + a
*Cos[c + d*x]^n)^p, x] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p]
```

Rubi steps

$$
\begin{aligned}
\int (\cot(x)+\csc(x))^5\,dx &= \int (1+\cos(x))^5\csc^5(x)\,dx\\
&= -\operatorname{Subst}\left(\int \frac{(1+x)^2}{(1-x)^3}\,dx, x, \cos(x)\right)\\
&= -\operatorname{Subst}\left(\int \left(\frac{1}{1-x}-\frac{4}{(-1+x)^3}-\frac{4}{(-1+x)^2}\right)dx, x, \cos(x)\right)\\
&= -\frac{2}{(1-\cos(x))^2}+\frac{4}{1-\cos(x)}+\log(1-\cos(x))
\end{aligned}
$$

Mathematica [A] time = 0.07, size = 32, normalized size = 1.14

$$
-\frac{1}{2}\csc^4\left(\frac{x}{2}\right)+2\csc^2\left(\frac{x}{2}\right)+2\log\left(\sin\left(\frac{x}{2}\right)\right)
$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cot[x] + Csc[x])^5,x]
```

```
[Out] 2*Csc[x/2]^2 - Csc[x/2]^4/2 + 2*Log[Sin[x/2]]
```

fricas [A] time = 1.58, size = 37, normalized size = 1.32

$$
\frac{\left(\cos(x)^2-2\cos(x)+1\right)\log\left(-\frac{1}{2}\cos(x)+\frac{1}{2}\right)-4\cos(x)+2}{\cos(x)^2-2\cos(x)+1}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((cot(x)+csc(x))^5,x, algorithm="fricas")
```

```
[Out] ((cos(x)^2 - 2*cos(x) + 1)*log(-1/2*cos(x) + 1/2) - 4*cos(x) + 2)/(cos(x)^2
 - 2*cos(x) + 1)
```

giac [A] time = 0.12, size = 22, normalized size = 0.79

$$
-\frac{2(2\cos(x)-1)}{(\cos(x)-1)^2}+\log(-\cos(x)+1)
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((cot(x)+csc(x))^5,x, algorithm="giac")
```

```
[Out] -2*(2*cos(x) - 1)/(cos(x) - 1)^2 + log(-cos(x) + 1)
```

maple [B] time = 0.11, size = 105, normalized size = 3.75

$$-\frac{\left(\cot^4(x)\right)}{4}+\frac{\left(\cot^2(x)\right)}{2}+\ln\left(\sin(x)\right)-\frac{5\left(\cos^5(x)\right)}{4\sin(x)^4}+\frac{5\left(\cos^5(x)\right)}{8\sin(x)^2}+\frac{5\left(\cos^3(x)\right)}{8}+\frac{5\cos(x)}{8}+\ln\left(\csc(x)-\cot(x)\right)-\frac{5\left(\cos\right.}{2\sin}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cot(x)+csc(x))^5,x)
```

```
[Out] -1/4*cot(x)^4+1/2*cot(x)^2+ln(sin(x))-5/4/sin(x)^4*cos(x)^5+5/8/sin(x)^2*co
s(x)^5+5/8*cos(x)^3+5/8*cos(x)+ln(csc(x)-cot(x))-5/2/sin(x)^4*cos(x)^4-5/2/
sin(x)^4*cos(x)^3-5/4/sin(x)^2*cos(x)^3-5/4/sin(x)^4+(-1/4*csc(x)^3-3/8*csc
(x))*cot(x)
```

maxima [B] time = 0.31, size = 125, normalized size = 4.46

$$-\frac{5}{2}\cot(x)^4-\frac{5\left(5\cos(x)^3-3\cos(x)\right)}{8\left(\cos(x)^4-2\cos(x)^2+1\right)}+\frac{3\cos(x)^3-5\cos(x)}{8\left(\cos(x)^4-2\cos(x)^2+1\right)}-\frac{5\left(\cos(x)^3+\cos(x)\right)}{4\left(\cos(x)^4-2\cos(x)^2+1\right)}+\frac{4\sin(x)^2-}{4\sin(x)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((cot(x)+csc(x))^5,x, algorithm="maxima")
```

```
[Out] -5/2*cot(x)^4 - 5/8*(5*cos(x)^3 - 3*cos(x))/(cos(x)^4 - 2*cos(x)^2 + 1) + 1
/8*(3*cos(x)^3 - 5*cos(x))/(cos(x)^4 - 2*cos(x)^2 + 1) - 5/4*(cos(x)^3 + co
s(x))/(cos(x)^4 - 2*cos(x)^2 + 1) + 1/4*(4*sin(x)^2 - 1)/sin(x)^4 - 5/4/sin
(x)^4 + 1/2*log(sin(x)^2) - 1/2*log(cos(x) + 1) + 1/2*log(cos(x) - 1)
```

mupad [B] time = 2.42, size = 34, normalized size = 1.21

$$2\ln\left(\tan\left(\frac{x}{2}\right)\right)-\ln\left(\tan\left(\frac{x}{2}\right)^2+1\right)+\frac{\tan\left(\frac{x}{2}\right)^2-\frac{1}{2}}{\tan\left(\frac{x}{2}\right)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cot(x) + 1/sin(x))^5,x)
```

```
[Out] 2*log(tan(x/2)) - log(tan(x/2)^2 + 1) + (tan(x/2)^2 - 1/2)/tan(x/2)^4
```

sympy [B] time = 107.21, size = 68, normalized size = 2.43

$$\frac{\log\left(\cos\left(x\right)-1\right)}{2}-\frac{\log\left(\cos\left(x\right)+1\right)}{2}-\frac{\log\left(\csc^2\left(x\right)\right)}{2}-\frac{5\cot^4\left(x\right)}{2}-\frac{3\csc^4\left(x\right)}{2}+\csc^2\left(x\right)-\frac{32\cos^3\left(x\right)}{8\cos^4\left(x\right)-16\cos^2\left(x\right)+8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((cot(x)+csc(x))**5,x)
```

```
[Out] log(cos(x) - 1)/2 - log(cos(x) + 1)/2 - log(csc(x)**2)/2 - 5*cot(x)**4/2 -
3*csc(x)**4/2 + csc(x)**2 - 32*cos(x)**3/(8*cos(x)**4 - 16*cos(x)**2 + 8)
```

3.294 $\int (\cot(x) + \csc(x))^4 \, dx$

Optimal. Leaf size=30

$$x - \frac{2\sin^3(x)}{3(1-\cos(x))^3} + \frac{2\sin(x)}{1-\cos(x)}$$

```
[Out] x+2*sin(x)/(1-cos(x))-2/3*sin(x)^3/(1-cos(x))^3
```

Rubi [A] time = 0.10, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.571, Rules used = {4392, 2670, 2680, 8}

$$x - \frac{2\sin^3(x)}{3(1-\cos(x))^3} + \frac{2\sin(x)}{1-\cos(x)}$$

Antiderivative was successfully verified.

```
[In] Int[(Cot[x] + Csc[x])^4,x]
```

```
[Out] x + (2*Sin[x])/(1 - Cos[x]) - (2*Sin[x]^3)/(3*(1 - Cos[x])^3)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2670

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Dist[(a/g)^(2*m), Int[(g*Cos[e + f*x])^(2*m + p)/(a
 - b*Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2,
 0] && IntegerQ[m] && LtQ[p, -1] && GeQ[2*m + p, 0]
```

Rule 2680

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(2*g*(g*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f
*x])^(m + 1))/(b*f*(2*m + p + 1)), x] + Dist[(g^2*(p - 1))/(b^2*(2*m + p +
1)), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 2), x], x] /; F
reeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && LeQ[m, -2] && GtQ[p, 1] &&
 NeQ[2*m + p + 1, 0] &&  !ILtQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

Rule 4392

```
Int[(cot[(c_.) + (d_.)*(x_)]^(n_.)*(a_.) + csc[(c_.) + (d_.)*(x_)]^(n_.)*(b
_.))^(p_)*(u_.), x_Symbol] :> Int[ActivateTrig[u]*Csc[c + d*x]^(n*p)*(b + a
```

```
*Cos[c + d*x]^n)^p, x] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p]
```

Rubi steps

$$\begin{aligned}\int (\cot(x)+\csc(x))^4\,dx &= \int (1+\cos(x))^4\csc^4(x)\,dx\\ &= \int \frac{\sin^4(x)}{(1-\cos(x))^4}\,dx\\ &= -\frac{2\sin^3(x)}{3(1-\cos(x))^3}-\int \frac{\sin^2(x)}{(1-\cos(x))^2}\,dx\\ &= \frac{2\sin(x)}{1-\cos(x)}-\frac{2\sin^3(x)}{3(1-\cos(x))^3}+\int 1\,dx\\ &= x+\frac{2\sin(x)}{1-\cos(x)}-\frac{2\sin^3(x)}{3(1-\cos(x))^3}\end{aligned}$$

Mathematica [A] time = 0.04, size = 30, normalized size = 1.00

$$x+\frac{8}{3}\cot\left(\frac{x}{2}\right)-\frac{2}{3}\cot\left(\frac{x}{2}\right)\csc^2\left(\frac{x}{2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cot[x] + Csc[x])^4,x]
[Out] x + (8*Cot[x/2])/3 - (2*Cot[x/2]*Csc[x/2]^2)/3
```

fricas [A] time = 1.70, size = 36, normalized size = 1.20

$$\frac{8\ \cos(x)^2+3\,(x\cos(x)-x)\sin(x)+4\ \cos(x)-4}{3\,(\cos(x)-1)\sin(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((cot(x)+csc(x))^4,x, algorithm="fricas")
[Out] 1/3*(8*cos(x)^2 + 3*(x*cos(x) - x)*sin(x) + 4*cos(x) - 4)/((cos(x) - 1)*sin
(x))
```

giac [A] time = 0.13, size = 20, normalized size = 0.67

$$x+\frac{2\left(3\ \tan\left(\frac{1}{2}\,x\right)^2-1\right)}{3\ \tan\left(\frac{1}{2}\,x\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((cot(x)+csc(x))^4,x, algorithm="giac")

[Out] x + 2/3*(3*tan(1/2*x)^2 - 1)/tan(1/2*x)^3

maple [B] time = 0.09, size = 68, normalized size = 2.27

$$-\frac{\left(\cot^3(x)\right)}{3}+\cot(x)+x-\frac{4\left(\cos^4(x)\right)}{3\sin(x)^3}+\frac{4\left(\cos^4(x)\right)}{3\sin(x)}+\frac{4\left(2+\cos^2(x)\right)\sin(x)}{3}-\frac{2\left(\cos^3(x)\right)}{\sin(x)^3}-\frac{4}{3\sin(x)^3}+\left(-\frac{2}{3}-\frac{\left(\csc^2(x\right.}{3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((cot(x)+csc(x))^4,x)

[Out] -1/3*cot(x)^3+cot(x)+x-4/3/sin(x)^3*cos(x)^4+4/3/sin(x)*cos(x)^4+4/3*(2+cos(x)^2)*sin(x)-2/sin(x)^3*cos(x)^3-4/3/sin(x)^3+(-2/3-1/3*csc(x)^2)*cot(x)

maxima [B] time = 0.42, size = 56, normalized size = 1.87

$$-2\cot(x)^3+x+\frac{4\left(3\sin(x)^2-1\right)}{3\sin(x)^3}-\frac{3\tan(x)^2+1}{3\tan(x)^3}+\frac{3\tan(x)^2-1}{3\tan(x)^3}-\frac{4}{3\sin(x)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((cot(x)+csc(x))^4,x, algorithm="maxima")

[Out] -2*cot(x)^3 + x + 4/3*(3*sin(x)^2 - 1)/sin(x)^3 - 1/3*(3*tan(x)^2 + 1)/tan(x)^3 + 1/3*(3*tan(x)^2 - 1)/tan(x)^3 - 4/3/sin(x)^3

mupad [B] time = 2.41, size = 16, normalized size = 0.53

$$-\frac{2\cot\left(\frac{x}{2}\right)^3}{3}+2\cot\left(\frac{x}{2}\right)+x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((cot(x) + 1/sin(x))^4,x)

[Out] x + 2*cot(x/2) - (2*cot(x/2)^3)/3

sympy [A] time = 35.04, size = 44, normalized size = 1.47

$$x-\frac{7\cot^3(x)}{3}-\cot(x)-\frac{8\csc^3(x)}{3}+4\csc(x)+\frac{\cos(x)}{\sin(x)}-\frac{\cos^3(x)}{3\sin^3(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((cot(x)+csc(x))**4,x)
```

```
[Out] x - 7*cot(x)**3/3 - cot(x) - 8*csc(x)**3/3 + 4*csc(x) + cos(x)/sin(x) - cos
(x)**3/(3*sin(x)**3)
```

3.295 $\int (\cot(x)+\csc(x))^3\,dx$

Optimal. Leaf size=20

$$-\frac{2}{1-\cos(x)}-\log(1-\cos(x))$$

```
[Out] -2/(1-cos(x))-ln(1-cos(x))
```

Rubi [A] time = 0.05, antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {4392, 2667, 43}

$$-\frac{2}{1-\cos(x)}-\log(1-\cos(x))$$

Antiderivative was successfully verified.

```
[In] Int[(Cot[x] + Csc[x])^3,x]
```

```
[Out] -2/(1 - Cos[x]) - Log[1 - Cos[x]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2667

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rule 4392

```
Int[(cot[(c_.) + (d_.)*(x_)]^(n_.)*(a_.) + csc[(c_.) + (d_.)*(x_)]^(n_.)*(b
_.))^(p_)*(u_.), x_Symbol] :> Int[ActivateTrig[u]*Csc[c + d*x]^(n*p)*(b + a
*Cos[c + d*x]^n)^p, x] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p]
```

Rubi steps

$$\int (\cot(x)+\csc(x))^3\,dx = \int (1+\cos(x))^3\csc^3(x)\,dx$$
$$= -\text{Subst}\left(\int \frac{1+x}{(1-x)^2}\,dx, x, \cos(x)\right)$$
$$= -\text{Subst}\left(\int \left(\frac{2}{(-1+x)^2}+\frac{1}{-1+x}\right)dx, x, \cos(x)\right)$$
$$= -\frac{2}{1-\cos(x)} - \log(1-\cos(x))$$

Mathematica [A] time = 0.04, size = 20, normalized size = 1.00

$$-\csc^2\left(\frac{x}{2}\right) - 2\log\left(\sin\left(\frac{x}{2}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cot[x] + Csc[x])^3,x]
[Out] -Csc[x/2]^2 - 2*Log[Sin[x/2]]
```

fricas [A] time = 1.68, size = 22, normalized size = 1.10

$$-\frac{(\cos(x)-1)\log\left(-\frac{1}{2}\cos(x)+\frac{1}{2}\right)-2}{\cos(x)-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((cot(x)+csc(x))^3,x, algorithm="fricas")
[Out] -((cos(x) - 1)*log(-1/2*cos(x) + 1/2) - 2)/(cos(x) - 1)
```

giac [A] time = 0.15, size = 18, normalized size = 0.90

$$\frac{2}{\cos(x)-1} - \log(-\cos(x)+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((cot(x)+csc(x))^3,x, algorithm="giac")
[Out] 2/(cos(x) - 1) - log(-cos(x) + 1)
```

maple [B] time = 0.08, size = 49, normalized size = 2.45

$$-\frac{\left(\cot^{2}(x)\right)}{2}-\ln\left(\sin(x)\right)-\frac{3\left(\cos^{3}(x)\right)}{2\sin(x)^{2}}-\frac{3\cos(x)}{2}-\ln\left(\csc(x)-\cot(x)\right)-\frac{3}{2\sin(x)^{2}}-\frac{\cot(x)\csc(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cot(x)+csc(x))^3,x)
```

```
[Out] -1/2*cot(x)^2-ln(sin(x))-3/2/sin(x)^2*cos(x)^3-3/2*cos(x)-ln(csc(x)-cot(x))
-3/2/sin(x)^2-1/2*cot(x)*csc(x)
```

maxima [B] time = 0.31, size = 46, normalized size = 2.30

$$-\frac{3}{2}\cot(x)^{2}+\frac{2\cos(x)}{\cos(x)^{2}-1}-\frac{1}{2\sin(x)^{2}}-\frac{1}{2}\log\left(\sin(x)^{2}\right)+\frac{1}{2}\log\left(\cos(x)+1\right)-\frac{1}{2}\log\left(\cos(x)-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((cot(x)+csc(x))^3,x, algorithm="maxima")
```

```
[Out] -3/2*cot(x)^2 + 2*cos(x)/(cos(x)^2 - 1) - 1/2/sin(x)^2 - 1/2*log(sin(x)^2)
+ 1/2*log(cos(x) + 1) - 1/2*log(cos(x) - 1)
```

mupad [B] time = 2.39, size = 25, normalized size = 1.25

$$\ln\left(\tan\left(\frac{x}{2}\right)^{2}+1\right)-2\ln\left(\tan\left(\frac{x}{2}\right)\right)-\frac{1}{\tan\left(\frac{x}{2}\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cot(x) + 1/sin(x))^3,x)
```

```
[Out] log(tan(x/2)^2 + 1) - 2*log(tan(x/2)) - 1/tan(x/2)^2
```

sympy [B] time = 14.64, size = 46, normalized size = 2.30

$$-\frac{\log\left(\cos\left(x\right)-1\right)}{2}+\frac{\log\left(\cos\left(x\right)+1\right)}{2}+\frac{\log\left(-\csc^{2}\left(x\right)\right)}{2}-2\csc^{2}\left(x\right)+\frac{4\cos\left(x\right)}{2\cos^{2}\left(x\right)-2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((cot(x)+csc(x))**3,x)
```

```
[Out] -log(cos(x) - 1)/2 + log(cos(x) + 1)/2 + log(-csc(x)**2)/2 - 2*csc(x)**2 +
4*cos(x)/(2*cos(x)**2 - 2)
```

3.296 $\int (\cot(x)+\csc(x))^2\,dx$

Optimal. Leaf size=16

$$-x-\frac{2\sin(x)}{1-\cos(x)}$$

```
[Out] -x-2*sin(x)/(1-cos(x))
```

Rubi [A] time = 0.07, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.571, Rules used = {4392, 2670, 2680, 8}

$$-x-\frac{2\sin(x)}{1-\cos(x)}$$

Antiderivative was successfully verified.

```
[In] Int[(Cot[x] + Csc[x])^2,x]
```

```
[Out] -x - (2*Sin[x])/(1 - Cos[x])
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2670

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Dist[(a/g)^(2*m), Int[(g*Cos[e + f*x])^(2*m + p)/(a
 - b*Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2,
 0] && IntegerQ[m] && LtQ[p, -1] && GeQ[2*m + p, 0]
```

Rule 2680

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(2*g*(g*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f
*x])^(m + 1))/(b*f*(2*m + p + 1)), x] + Dist[(g^2*(p - 1))/(b^2*(2*m + p +
1)), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 2), x], x] /; F
reeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && LeQ[m, -2] && GtQ[p, 1] &&
 NeQ[2*m + p + 1, 0] &&  !ILtQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

Rule 4392

```
Int[(cot[(c_.) + (d_.)*(x_)]^(n_.)*(a_.) + csc[(c_.) + (d_.)*(x_)]^(n_.)*(b
_.))^(p_)*(u_.), x_Symbol] :> Int[ActivateTrig[u]*Csc[c + d*x]^(n*p)*(b + a
```

```
*Cos[c + d*x]^n)^p, x] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p]
```

Rubi steps

$$\begin{aligned}\int (\cot(x)+\csc(x))^2\,dx &= \int (1+\cos(x))^2\csc^2(x)\,dx\\ &= \int \frac{\sin^2(x)}{(1-\cos(x))^2}\,dx\\ &= -\frac{2\sin(x)}{1-\cos(x)}-\int 1\,dx\\ &= -x-\frac{2\sin(x)}{1-\cos(x)}\end{aligned}$$

Mathematica [A] time = 0.02, size = 12, normalized size = 0.75

$$-x-2\cot\left(\frac{x}{2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cot[x] + Csc[x])^2,x]
[Out] -x - 2*Cot[x/2]
```

fricas [A] time = 1.16, size = 16, normalized size = 1.00

$$-\frac{x\sin(x)+2\,\cos(x)+2}{\sin(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((cot(x)+csc(x))^2,x, algorithm="fricas")
[Out] -(x*sin(x) + 2*cos(x) + 2)/sin(x)
```

giac [A] time = 0.16, size = 12, normalized size = 0.75

$$-x-\frac{2}{\tan\left(\frac{1}{2}\,x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((cot(x)+csc(x))^2,x, algorithm="giac")
```

```
[Out] -x - 2/tan(1/2*x)
```

maple [A]   time = 0.04, size = 15, normalized size = 0.94

$$-2\cot(x) - x - \frac{2}{\sin(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cot(x)+csc(x))^2,x)
```

```
[Out] -2*cot(x)-x-2/sin(x)
```

maxima [A]   time = 0.41, size = 16, normalized size = 1.00

$$-x - \frac{2}{\sin(x)} - \frac{2}{\tan(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((cot(x)+csc(x))^2,x, algorithm="maxima")
```

```
[Out] -x - 2/sin(x) - 2/tan(x)
```

mupad [B]   time = 2.40, size = 10, normalized size = 0.62

$$-x - 2\cot\left(\frac{x}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cot(x) + 1/sin(x))^2,x)
```

```
[Out] - x - 2*cot(x/2)
```

sympy [A]   time = 2.70, size = 17, normalized size = 1.06

$$-x - \cot(x) - 2\csc(x) - \frac{\cos(x)}{\sin(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((cot(x)+csc(x))**2,x)
```

```
[Out] -x - cot(x) - 2*csc(x) - cos(x)/sin(x)
```

3.297 $\int (\cot(x) + \csc(x))\,dx$

Optimal. Leaf size=9

$$\log(\sin(x)) - \tanh^{-1}(\cos(x))$$

```
[Out] -arctanh(cos(x))+ln(sin(x))
```

Rubi [A] time = 0.01, antiderivative size = 9, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {3475, 3770}

$$\log(\sin(x)) - \tanh^{-1}(\cos(x))$$

Antiderivative was successfully verified.

```
[In] Int[Cot[x] + Csc[x],x]
```

```
[Out] -ArcTanh[Cos[x]] + Log[Sin[x]]
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int (\cot(x)+\csc(x))\,dx &= \int \cot(x)\,dx + \int \csc(x)\,dx\\ &= -\tanh^{-1}(\cos(x)) + \log(\sin(x))\end{aligned}$$

Mathematica [B] time = 0.00, size = 20, normalized size = 2.22

$$\log\left(\sin\left(\frac{x}{2}\right)\right) + \log(\sin(x)) - \log\left(\cos\left(\frac{x}{2}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[x] + Csc[x],x]
```

```
[Out] -Log[Cos[x/2]] + Log[Sin[x/2]] + Log[Sin[x]]
```

fricas [A] time = 1.03, size = 7, normalized size = 0.78

$$\log\left(-\frac{1}{2}\,\cos(x) + \frac{1}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)+csc(x),x, algorithm="fricas")
```

```
[Out] log(-1/2*cos(x) + 1/2)
```

giac [A] time = 0.15, size = 11, normalized size = 1.22

$$\log\left(|\sin(x)|\right) + \log\left(\left|\tan\left(\frac{1}{2}\,x\right)\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)+csc(x),x, algorithm="giac")
```

```
[Out] log(abs(sin(x))) + log(abs(tan(1/2*x)))
```

maple [A] time = 0.00, size = 13, normalized size = 1.44

$$\ln\left(\sin(x)\right) - \ln\left(\cot(x) + \csc(x)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(x)+csc(x),x)
```

```
[Out] ln(sin(x))-ln(cot(x)+csc(x))
```

maxima [A] time = 0.30, size = 12, normalized size = 1.33

$$-\log\left(\cot(x) + \csc(x)\right) + \log\left(\sin(x)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)+csc(x),x, algorithm="maxima")
```

```
[Out] -log(cot(x) + csc(x)) + log(sin(x))
```

mupad [B] time = 2.41, size = 19, normalized size = 2.11

$$2\,\ln\left(\tan\left(\frac{x}{2}\right)\right) - \ln\left(\tan\left(\frac{x}{2}\right)^2 + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(x) + 1/sin(x),x)
```

```
[Out] 2*log(tan(x/2)) - log(tan(x/2)^2 + 1)
```

sympy [**B**] time = 0.10, size = 20, normalized size = 2.22

$$\frac{\log\left(\cos\left(x\right)-1\right)}{2}-\frac{\log\left(\cos\left(x\right)+1\right)}{2}+\log\left(\sin\left(x\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)+csc(x),x)
```

```
[Out] log(cos(x) - 1)/2 - log(cos(x) + 1)/2 + log(sin(x))
```

3.298 $\int \frac{1}{\cot(x)+\csc(x)} \, dx$

Optimal. Leaf size=7

$$-\log(\cos(x)+1)$$

```
[Out] -ln(1+cos(x))
```

Rubi [A] time = 0.03, antiderivative size = 7, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {3160, 2667, 31}

$$-\log(\cos(x)+1)$$

Antiderivative was successfully verified.

```
[In] Int[(Cot[x] + Csc[x])^(-1),x]
```

```
[Out] -Log[1 + Cos[x]]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2667

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rule 3160

```
Int[((a_.) + csc[(d_.) + (e_.)*(x_)]*(b_.) + cot[(d_.) + (e_.)*(x_)]*(c_.))
^(-1), x_Symbol] :> Int[Sin[d + e*x]/(b + a*Sin[d + e*x] + c*Cos[d + e*x]),
 x] /; FreeQ[{a, b, c, d, e}, x]
```

Rubi steps

$$\int \frac{1}{\cot(x)+\csc(x)}\,dx = \int \frac{\sin(x)}{1+\cos(x)}\,dx$$
$$= -\operatorname{Subst}\left(\int \frac{1}{1+x}\,dx, x, \cos(x)\right)$$
$$= -\log(1+\cos(x))$$

Mathematica [A] time = 0.01, size = 9, normalized size = 1.29

$$-2\log\left(\cos\left(\frac{x}{2}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cot[x] + Csc[x])^(-1),x]
[Out] -2*Log[Cos[x/2]]
```

fricas [A] time = 0.91, size = 9, normalized size = 1.29

$$-\log\left(\frac{1}{2}\cos(x)+\frac{1}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(cot(x)+csc(x)),x, algorithm="fricas")
[Out] -log(1/2*cos(x) + 1/2)
```

giac [A] time = 0.13, size = 7, normalized size = 1.00

$$-\log\left(\cos(x)+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(cot(x)+csc(x)),x, algorithm="giac")
[Out] -log(cos(x) + 1)
```

maple [A] time = 0.11, size = 8, normalized size = 1.14

$$-\ln\left(1+\cos(x)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cot(x)+csc(x)),x)
```

```
[Out] -ln(1+cos(x))
```

maxima [A] time = 0.41, size = 14, normalized size = 2.00

$$\log\left(\frac{\sin(x)^2}{(\cos(x)+1)^2}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(cot(x)+csc(x)),x, algorithm="maxima")
```

```
[Out] log(sin(x)^2/(cos(x) + 1)^2 + 1)
```

mupad [B] time = 2.86, size = 9, normalized size = 1.29

$$\ln\left(\tan\left(\frac{x}{2}\right)^2+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cot(x) + 1/sin(x)),x)
```

```
[Out] log(tan(x/2)^2 + 1)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\cot(x)+\csc(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(cot(x)+csc(x)),x)
```

```
[Out] Integral(1/(cot(x) + csc(x)), x)
```

3.299 $\int \frac{1}{(\cot(x)+\csc(x))^2}\, dx$

Optimal. Leaf size=14

$$\frac{2\sin(x)}{\cos(x)+1} - x$$

```
[Out] -x+2*sin(x)/(1+cos(x))
```

Rubi [A] time = 0.04, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {4392, 2680, 8}

$$\frac{2\sin(x)}{\cos(x)+1} - x$$

Antiderivative was successfully verified.

```
[In] Int[(Cot[x] + Csc[x])^(-2),x]
```

```
[Out] -x + (2*Sin[x])/(1 + Cos[x])
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2680

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(2*g*(g*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f
*x])^(m + 1))/(b*f*(2*m + p + 1)), x] + Dist[(g^2*(p - 1))/(b^2*(2*m + p +
1)), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 2), x], x] /; F
reeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && LeQ[m, -2] && GtQ[p, 1] &&
 NeQ[2*m + p + 1, 0] &&  !ILtQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

Rule 4392

```
Int[(cot[(c_.) + (d_.)*(x_)]^(n_.)*(a_.) + csc[(c_.) + (d_.)*(x_)]^(n_.)*(b
_.))^(p_)*(u_.), x_Symbol] :> Int[ActivateTrig[u]*Csc[c + d*x]^(n*p)*(b + a
*Cos[c + d*x]^n)^p, x] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p]
```

Rubi steps

$$\int \frac{1}{(\cot(x)+\csc(x))^2}\,dx = \int \frac{\sin^2(x)}{(1+\cos(x))^2}\,dx$$
$$= \frac{2\sin(x)}{1+\cos(x)} - \int 1\,dx$$
$$= -x + \frac{2\sin(x)}{1+\cos(x)}$$

Mathematica [**A**] time = 0.01, size = 12, normalized size = 0.86

$$2\tan\left(\frac{x}{2}\right) - x$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cot[x] + Csc[x])^(-2),x]
[Out] -x + 2*Tan[x/2]
```

fricas [**A**] time = 1.10, size = 18, normalized size = 1.29

$$-\frac{x\cos(x) + x - 2\sin(x)}{\cos(x) + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(cot(x)+csc(x))^2,x, algorithm="fricas")
[Out] -(x*cos(x) + x - 2*sin(x))/(cos(x) + 1)
```

giac [**A**] time = 0.14, size = 10, normalized size = 0.71

$$-x + 2\tan\left(\frac{1}{2}x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(cot(x)+csc(x))^2,x, algorithm="giac")
[Out] -x + 2*tan(1/2*x)
```

maple [**A**] time = 0.11, size = 11, normalized size = 0.79

$$2\tan\left(\frac{x}{2}\right) - x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cot(x)+csc(x))^2,x)
```

```
[Out] 2*tan(1/2*x)-x
```

maxima [A] time = 0.41, size = 23, normalized size = 1.64

$$\frac{2\,\sin(x)}{\cos(x)+1}-2\,\arctan\left(\frac{\sin(x)}{\cos(x)+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(cot(x)+csc(x))^2,x, algorithm="maxima")
```

```
[Out] 2*sin(x)/(cos(x) + 1) - 2*arctan(sin(x)/(cos(x) + 1))
```

mupad [B] time = 2.41, size = 10, normalized size = 0.71

$$2\tan\left(\frac{x}{2}\right)-x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cot(x) + 1/sin(x))^2,x)
```

```
[Out] 2*tan(x/2) - x
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{\left(\cot{(x)}+\csc{(x)}\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(cot(x)+csc(x))**2,x)
```

```
[Out] Integral((cot(x) + csc(x))**(-2), x)
```

3.300 $\int \frac{1}{(\cot(x)+\csc(x))^3} dx$

Optimal. Leaf size=14

$$\frac{2}{\cos(x)+1}+\log(\cos(x)+1)$$

```
[Out] 2/(1+cos(x))+ln(1+cos(x))
```

Rubi [A] time = 0.05, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {4392, 2667, 43}

$$\frac{2}{\cos(x)+1}+\log(\cos(x)+1)$$

Antiderivative was successfully verified.

```
[In] Int[(Cot[x] + Csc[x])^(-3),x]
```

```
[Out] 2/(1 + Cos[x]) + Log[1 + Cos[x]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2667

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rule 4392

```
Int[(cot[(c_.) + (d_.)*(x_)]^(n_.)*(a_.) + csc[(c_.) + (d_.)*(x_)]^(n_.)*(b
_.))^(p_)*(u_.), x_Symbol] :> Int[ActivateTrig[u]*Csc[c + d*x]^(n*p)*(b + a
*Cos[c + d*x]^n)^p, x] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p]
```

Rubi steps

$$\int \frac{1}{(\cot(x)+\csc(x))^3}\,dx = \int \frac{\sin^3(x)}{(1+\cos(x))^3}\,dx$$
$$= -\operatorname{Subst}\left(\int \frac{1-x}{(1+x)^2}\,dx, x, \cos(x)\right)$$
$$= -\operatorname{Subst}\left(\int \left(\frac{1}{-1-x}+\frac{2}{(1+x)^2}\right) dx, x, \cos(x)\right)$$
$$= \frac{2}{1+\cos(x)} + \log(1+\cos(x))$$

Mathematica [**A**] time = 0.01, size = 18, normalized size = 1.29

$$\sec^2\left(\frac{x}{2}\right) + 2\log\left(\cos\left(\frac{x}{2}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cot[x] + Csc[x])^(-3),x]
[Out] 2*Log[Cos[x/2]] + Sec[x/2]^2
```

fricas [**A**] time = 0.86, size = 21, normalized size = 1.50

$$\frac{(\cos(x)+1)\log\left(\frac{1}{2}\cos(x)+\frac{1}{2}\right)+2}{\cos(x)+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(cot(x)+csc(x))^3,x, algorithm="fricas")
[Out] ((cos(x) + 1)*log(1/2*cos(x) + 1/2) + 2)/(cos(x) + 1)
```

giac [**A**] time = 0.14, size = 14, normalized size = 1.00

$$\frac{2}{\cos(x)+1} + \log(\cos(x)+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(cot(x)+csc(x))^3,x, algorithm="giac")
[Out] 2/(cos(x) + 1) + log(cos(x) + 1)
```

maple [A] time = 0.15, size = 15, normalized size = 1.07

$$\frac{2}{1+\cos(x)}+\ln\left(1+\cos(x)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cot(x)+csc(x))^3,x)
```

```
[Out] 2/(1+cos(x))+ln(1+cos(x))
```

maxima [A] time = 0.41, size = 28, normalized size = 2.00

$$\frac{\sin(x)^2}{\left(\cos(x)+1\right)^2}-\log\left(\frac{\sin(x)^2}{\left(\cos(x)+1\right)^2}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(cot(x)+csc(x))^3,x, algorithm="maxima")
```

```
[Out] sin(x)^2/(cos(x) + 1)^2 - log(sin(x)^2/(cos(x) + 1)^2 + 1)
```

mupad [B] time = 2.35, size = 18, normalized size = 1.29

$$\tan\left(\frac{x}{2}\right)^2-\ln\left(\tan\left(\frac{x}{2}\right)^2+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cot(x) + 1/sin(x))^3,x)
```

```
[Out] tan(x/2)^2 - log(tan(x/2)^2 + 1)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{\left(\cot\left(x\right)+\csc\left(x\right)\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(cot(x)+csc(x))**3,x)
```

```
[Out] Integral((cot(x) + csc(x))**(-3), x)
```

3.301 $\int \frac{1}{(\cot(x)+\csc(x))^4} \, dx$

Optimal. Leaf size=26

$$x+\frac{2\sin^3(x)}{3(\cos(x)+1)^3}-\frac{2\sin(x)}{\cos(x)+1}$$

```
[Out] x-2*sin(x)/(1+cos(x))+2/3*sin(x)^3/(1+cos(x))^3
```

Rubi [A] time = 0.07, antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {4392, 2680, 8}

$$x+\frac{2\sin^3(x)}{3(\cos(x)+1)^3}-\frac{2\sin(x)}{\cos(x)+1}$$

Antiderivative was successfully verified.

```
[In] Int[(Cot[x] + Csc[x])^(-4),x]
```

```
[Out] x - (2*Sin[x])/(1 + Cos[x]) + (2*Sin[x]^3)/(3*(1 + Cos[x])^3)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2680

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(2*g*(g*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f
*x])^(m + 1))/(b*f*(2*m + p + 1)), x] + Dist[(g^2*(p - 1))/(b^2*(2*m + p +
1)), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 2), x], x] /; F
reeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && LeQ[m, -2] && GtQ[p, 1] &&
 NeQ[2*m + p + 1, 0] &&  !ILtQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

Rule 4392

```
Int[(cot[(c_.) + (d_.)*(x_)]^(n_.)*(a_.) + csc[(c_.) + (d_.)*(x_)]^(n_.)*(b
_.))^(p_)*(u_.), x_Symbol] :> Int[ActivateTrig[u]*Csc[c + d*x]^(n*p)*(b + a
*Cos[c + d*x]^n)^p, x] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(\cot(x)+\csc(x))^4}\,dx &= \int \frac{\sin^4(x)}{(1+\cos(x))^4}\,dx\\ &= \frac{2\sin^3(x)}{3(1+\cos(x))^3} - \int \frac{\sin^2(x)}{(1+\cos(x))^2}\,dx\\ &= -\frac{2\sin(x)}{1+\cos(x)} + \frac{2\sin^3(x)}{3(1+\cos(x))^3} + \int 1\,dx\\ &= x - \frac{2\sin(x)}{1+\cos(x)} + \frac{2\sin^3(x)}{3(1+\cos(x))^3}\end{aligned}$$

Mathematica [A] time = 0.01, size = 30, normalized size = 1.15

$$x - \frac{8}{3}\tan\left(\frac{x}{2}\right) + \frac{2}{3}\tan\left(\frac{x}{2}\right)\sec^2\left(\frac{x}{2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cot[x] + Csc[x])^(-4),x]
```

```
[Out] x - (8*Tan[x/2])/3 + (2*Sec[x/2]^2*Tan[x/2])/3
```

fricas [A] time = 0.95, size = 40, normalized size = 1.54

$$\frac{3\,x\cos(x)^2 + 6\,x\cos(x) - 4\,(2\,\cos(x)+1)\sin(x) + 3\,x}{3\left(\cos(x)^2 + 2\,\cos(x) + 1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(cot(x)+csc(x))^4,x, algorithm="fricas")
```

```
[Out] 1/3*(3*x*cos(x)^2 + 6*x*cos(x) - 4*(2*cos(x) + 1)*sin(x) + 3*x)/(cos(x)^2 +
 2*cos(x) + 1)
```

giac [A] time = 0.13, size = 16, normalized size = 0.62

$$\frac{2}{3}\,\tan\left(\frac{1}{2}\,x\right)^3 + x - 2\,\tan\left(\frac{1}{2}\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(cot(x)+csc(x))^4,x, algorithm="giac")
```

```
[Out] 2/3*tan(1/2*x)^3 + x - 2*tan(1/2*x)
```

maple [A] time = 0.14, size = 17, normalized size = 0.65

$$\frac{2\left(\tan^{3}\left(\frac{x}{2}\right)\right)}{3}-2\tan\left(\frac{x}{2}\right)+x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cot(x)+csc(x))^4,x)
```

```
[Out] 2/3*tan(1/2*x)^3-2*tan(1/2*x)+x
```

maxima [A] time = 0.41, size = 35, normalized size = 1.35

$$-\frac{2\,\sin(x)}{\cos(x)+1}+\frac{2\,\sin(x)^{3}}{3\,(\cos(x)+1)^{3}}+2\,\arctan\left(\frac{\sin(x)}{\cos(x)+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(cot(x)+csc(x))^4,x, algorithm="maxima")
```

```
[Out] -2*sin(x)/(cos(x) + 1) + 2/3*sin(x)^3/(cos(x) + 1)^3 + 2*arctan(sin(x)/(cos
(x) + 1))
```

mupad [B] time = 2.37, size = 16, normalized size = 0.62

$$\frac{2\tan\left(\frac{x}{2}\right)^{3}}{3}-2\tan\left(\frac{x}{2}\right)+x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cot(x) + 1/sin(x))^4,x)
```

```
[Out] x - 2*tan(x/2) + (2*tan(x/2)^3)/3
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(cot(x)+csc(x))**4,x)
```

```
[Out] Timed out
```

3.302 $\int \frac{1}{(\cot(x)+\csc(x))^5} dx$

Optimal. Leaf size=24

$$-\frac{4}{\cos(x)+1}+\frac{2}{(\cos(x)+1)^2}-\log(\cos(x)+1)$$

```
[Out] 2/(1+cos(x))^2-4/(1+cos(x))-ln(1+cos(x))
```

Rubi [A] time = 0.05, antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {4392, 2667, 43}

$$-\frac{4}{\cos(x)+1}+\frac{2}{(\cos(x)+1)^2}-\log(\cos(x)+1)$$

Antiderivative was successfully verified.

```
[In] Int[(Cot[x] + Csc[x])^(-5),x]
```

```
[Out] 2/(1 + Cos[x])^2 - 4/(1 + Cos[x]) - Log[1 + Cos[x]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2667

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rule 4392

```
Int[(cot[(c_.) + (d_.)*(x_)]^(n_.)*(a_.) + csc[(c_.) + (d_.)*(x_)]^(n_.)*(b
_.))^(p_)*(u_.), x_Symbol] :> Int[ActivateTrig[u]*Csc[c + d*x]^(n*p)*(b + a
*Cos[c + d*x]^n)^p, x] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p]
```

Rubi steps

$$\int \frac{1}{(\cot(x)+\csc(x))^5}\,dx = \int \frac{\sin^5(x)}{(1+\cos(x))^5}\,dx$$
$$= -\operatorname{Subst}\left(\int \frac{(1-x)^2}{(1+x)^3}\,dx, x, \cos(x)\right)$$
$$= -\operatorname{Subst}\left(\int \left(\frac{4}{(1+x)^3} - \frac{4}{(1+x)^2} + \frac{1}{1+x}\right)dx, x, \cos(x)\right)$$
$$= \frac{2}{(1+\cos(x))^2} - \frac{4}{1+\cos(x)} - \log(1+\cos(x))$$

Mathematica [**A**] time = 0.01, size = 32, normalized size = 1.33

$$\frac{1}{2}\sec^4\left(\frac{x}{2}\right) - 2\sec^2\left(\frac{x}{2}\right) - 2\log\left(\cos\left(\frac{x}{2}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cot[x] + Csc[x])^(-5),x]
```

```
[Out] -2*Log[Cos[x/2]] - 2*Sec[x/2]^2 + Sec[x/2]^4/2
```

fricas [**A**] time = 1.99, size = 38, normalized size = 1.58

$$-\frac{\left(\cos(x)^2 + 2\cos(x) + 1\right)\log\left(\frac{1}{2}\cos(x) + \frac{1}{2}\right) + 4\cos(x) + 2}{\cos(x)^2 + 2\cos(x) + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(cot(x)+csc(x))^5,x, algorithm="fricas")
```

```
[Out] -((cos(x)^2 + 2*cos(x) + 1)*log(1/2*cos(x) + 1/2) + 4*cos(x) + 2)/(cos(x)^2
 + 2*cos(x) + 1)
```

giac [**A**] time = 0.12, size = 22, normalized size = 0.92

$$-\frac{2(2\cos(x)+1)}{(\cos(x)+1)^2} - \log(\cos(x)+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(cot(x)+csc(x))^5,x, algorithm="giac")
```

```
[Out] -2*(2*cos(x) + 1)/(cos(x) + 1)^2 - log(cos(x) + 1)
```

maple [A] time = 0.16, size = 25, normalized size = 1.04

$$\frac{2}{(1+\cos(x))^2}-\frac{4}{1+\cos(x)}-\ln(1+\cos(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cot(x)+csc(x))^5,x)
```

```
[Out] 2/(1+cos(x))^2-4/(1+cos(x))-ln(1+cos(x))
```

maxima [A] time = 0.41, size = 39, normalized size = 1.62

$$-\frac{\sin(x)^2}{(\cos(x)+1)^2}+\frac{\sin(x)^4}{2(\cos(x)+1)^4}+\log\left(\frac{\sin(x)^2}{(\cos(x)+1)^2}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(cot(x)+csc(x))^5,x, algorithm="maxima")
```

```
[Out] -sin(x)^2/(cos(x) + 1)^2 + 1/2*sin(x)^4/(cos(x) + 1)^4 + log(sin(x)^2/(cos(
x) + 1)^2 + 1)
```

mupad [B] time = 2.41, size = 26, normalized size = 1.08

$$\ln\left(\tan\left(\frac{x}{2}\right)^2+1\right)-\tan\left(\frac{x}{2}\right)^2+\frac{\tan\left(\frac{x}{2}\right)^4}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cot(x) + 1/sin(x))^5,x)
```

```
[Out] log(tan(x/2)^2 + 1) - tan(x/2)^2 + tan(x/2)^4/2
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(cot(x)+csc(x))**5,x)
```

```
[Out] Timed out
```

3.303 $\int (\csc(x) - \sin(x))^4 \, dx$

Optimal. Leaf size=44

$$\frac{35x}{8} - \frac{35\cot^3(x)}{24} + \frac{35\cot(x)}{8} + \frac{1}{4}\cos^4(x)\cot^3(x) + \frac{7}{8}\cos^2(x)\cot^3(x)$$

```
[Out] 35/8*x+35/8*cot(x)-35/24*cot(x)^3+7/8*cos(x)^2*cot(x)^3+1/4*cos(x)^4*cot(x)
^3
```

Rubi [A] time = 0.03, antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {290, 325, 203}

$$\frac{35x}{8} - \frac{35\cot^3(x)}{24} + \frac{35\cot(x)}{8} + \frac{1}{4}\cos^4(x)\cot^3(x) + \frac{7}{8}\cos^2(x)\cot^3(x)$$

Antiderivative was successfully verified.

```
[In] Int[(Csc[x] - Sin[x])^4,x]
```

```
[Out] (35*x)/8 + (35*Cot[x])/8 - (35*Cot[x]^3)/24 + (7*Cos[x]^2*Cot[x]^3)/8 + (Co
s[x]^4*Cot[x]^3)/4
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rubi steps

$$\int (\csc(x)-\sin(x))^4\,dx = \text{Subst}\left(\int \frac{1}{x^4\left(1+x^2\right)^3}\,dx, x, \tan(x)\right)$$
$$= \frac{1}{4}\cos^4(x)\cot^3(x) + \frac{7}{4}\text{Subst}\left(\int \frac{1}{x^4\left(1+x^2\right)^2}\,dx, x, \tan(x)\right)$$
$$= \frac{7}{8}\cos^2(x)\cot^3(x) + \frac{1}{4}\cos^4(x)\cot^3(x) + \frac{35}{8}\text{Subst}\left(\int \frac{1}{x^4\left(1+x^2\right)}\,dx, x, \tan(x)\right)$$
$$= -\frac{35}{24}\cot^3(x) + \frac{7}{8}\cos^2(x)\cot^3(x) + \frac{1}{4}\cos^4(x)\cot^3(x) - \frac{35}{8}\text{Subst}\left(\int \frac{1}{x^2\left(1+x^2\right)}\,dx, x,\right.$$
$$= \frac{35\cot(x)}{8} - \frac{35\cot^3(x)}{24} + \frac{7}{8}\cos^2(x)\cot^3(x) + \frac{1}{4}\cos^4(x)\cot^3(x) + \frac{35}{8}\text{Subst}\left(\int \frac{1}{1+x^2}\right.$$
$$= \frac{35x}{8} + \frac{35\cot(x)}{8} - \frac{35\cot^3(x)}{24} + \frac{7}{8}\cos^2(x)\cot^3(x) + \frac{1}{4}\cos^4(x)\cot^3(x)$$

Mathematica [A] time = 0.03, size = 38, normalized size = 0.86

$$\frac{35x}{8} + \frac{3}{4}\sin(2x) + \frac{1}{32}\sin(4x) + \frac{10\cot(x)}{3} - \frac{1}{3}\cot(x)\csc^2(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(Csc[x] - Sin[x])^4,x]
```

```
[Out] (35*x)/8 + (10*Cot[x])/3 - (Cot[x]*Csc[x]^2)/3 + (3*Sin[2*x])/4 + Sin[4*x]/
32
```

fricas [A] time = 2.05, size = 51, normalized size = 1.16

$$-\frac{6\cos(x)^7 + 21\cos(x)^5 - 140\cos(x)^3 - 105\left(x\cos(x)^2 - x\right)\sin(x) + 105\cos(x)}{24\left(\cos(x)^2 - 1\right)\sin(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((csc(x)-sin(x))^4,x, algorithm="fricas")
```

```
[Out] -1/24*(6*cos(x)^7 + 21*cos(x)^5 - 140*cos(x)^3 - 105*(x*cos(x)^2 - x)*sin(x
) + 105*cos(x))/((cos(x)^2 - 1)*sin(x))
```

giac [A] time = 0.13, size = 39, normalized size = 0.89

$$\frac{35}{8}\,x+\frac{11\,\tan(x)^{3}+13\,\tan(x)}{8\,\left(\tan(x)^{2}+1\right)^{2}}+\frac{9\,\tan(x)^{2}-1}{3\,\tan(x)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((csc(x)-sin(x))^4,x, algorithm="giac")
```

```
[Out] 35/8*x + 1/8*(11*tan(x)^3 + 13*tan(x))/(tan(x)^2 + 1)^2 + 1/3*(9*tan(x)^2 -
 1)/tan(x)^3
```

maple [A] time = 0.06, size = 39, normalized size = 0.89

$$-\frac{\left(\sin^{3}(x)+\frac{3\sin(x)}{2}\right)\cos(x)}{4}+\frac{35x}{8}+2\cos(x)\sin(x)+4\cot(x)+\left(-\frac{2}{3}-\frac{\left(\csc^{2}(x)\right)}{3}\right)\cot(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((csc(x)-sin(x))^4,x)
```

```
[Out] -1/4*(sin(x)^3+3/2*sin(x))*cos(x)+35/8*x+2*cos(x)*sin(x)+4*cot(x)+(-2/3-1/3
*csc(x)^2)*cot(x)
```

maxima [A] time = 0.31, size = 36, normalized size = 0.82

$$\frac{35}{8}\,x+\frac{4}{\tan(x)}-\frac{3\,\tan(x)^{2}+1}{3\,\tan(x)^{3}}+\frac{1}{32}\,\sin\left(4\,x\right)+\frac{3}{4}\,\sin\left(2\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((csc(x)-sin(x))^4,x, algorithm="maxima")
```

```
[Out] 35/8*x + 4/tan(x) - 1/3*(3*tan(x)^2 + 1)/tan(x)^3 + 1/32*sin(4*x) + 3/4*sin
(2*x)
```

mupad [B] time = 2.49, size = 59, normalized size = 1.34

$$\frac{\frac{\cos(x)^{7}}{4}+\frac{7\cos(x)^{5}}{8}-\frac{35\cos(x)^{3}}{6}+\frac{35\,\cos(x)}{8}}{\sin(x)-\cos(x)^{2}\,\sin(x)}-\frac{\frac{35\,x}{8}-\frac{35\,x\cos(x)^{2}}{8}}{\cos(x)^{2}-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((sin(x) - 1/sin(x))^4,x)
```

```
[Out] ((35*cos(x))/8 - (35*cos(x)^3)/6 + (7*cos(x)^5)/8 + cos(x)^7/4)/(sin(x) - c
os(x)^2*sin(x)) - ((35*x)/8 - (35*x*cos(x)^2)/8)/(cos(x)^2 - 1)
```

sympy [**A**] time = 8.78, size = 44, normalized size = 1.00

$$\frac{35x}{8} + 2\sin(x)\cos(x) - \frac{\sin(2x)}{4} + \frac{\sin(4x)}{32} - \frac{\cot^3(x)}{3} - \cot(x) + \frac{4\cos(x)}{\sin(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((csc(x)-sin(x))**4,x)
```

```
[Out] 35*x/8 + 2*sin(x)*cos(x) - sin(2*x)/4 + sin(4*x)/32 - cot(x)**3/3 - cot(x)
+ 4*cos(x)/sin(x)
```

3.304 $\int (\csc(x)-\sin(x))^3\,dx$

Optimal. Leaf size=34

$$-\frac{5\cos^3(x)}{6}-\frac{5\cos(x)}{2}-\frac{1}{2}\cos^3(x)\cot^2(x)+\frac{5}{2}\tanh^{-1}(\cos(x))$$

```
[Out] 5/2*arctanh(cos(x))-5/2*cos(x)-5/6*cos(x)^3-1/2*cos(x)^3*cot(x)^2
```

Rubi [A] time = 0.05, antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.556, Rules used = {4397, 2592, 288, 302, 206}

$$-\frac{5\cos^3(x)}{6}-\frac{5\cos(x)}{2}-\frac{1}{2}\cos^3(x)\cot^2(x)+\frac{5}{2}\tanh^{-1}(\cos(x))$$

Antiderivative was successfully verified.

```
[In] Int[(Csc[x] - Sin[x])^3,x]
```

```
[Out] (5*ArcTanh[Cos[x]])/2 - (5*Cos[x])/2 - (5*Cos[x]^3)/6 - (Cos[x]^3*Cot[x]^2)
/2
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 2592

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(
```

```
ff*x)^(m + n)/(a^2 - ff^2*x^2)^((n + 1)/2), x], x, (a*Sin[e + f*x])/ff], x]
] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2]
```

Rule 4397

```
Int[u_, x_Symbol] :> Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]
```

Rubi steps

$$\begin{aligned}
\int (\csc(x)-\sin(x))^3\,dx &= \int \cos^3(x)\cot^3(x)\,dx\\
&= -\text{Subst}\left(\int \frac{x^6}{\left(1-x^2\right)^2}\,dx, x, \cos(x)\right)\\
&= -\frac{1}{2}\cos^3(x)\cot^2(x)+\frac{5}{2}\text{Subst}\left(\int \frac{x^4}{1-x^2}\,dx, x, \cos(x)\right)\\
&= -\frac{1}{2}\cos^3(x)\cot^2(x)+\frac{5}{2}\text{Subst}\left(\int \left(-1-x^2+\frac{1}{1-x^2}\right)dx, x, \cos(x)\right)\\
&= -\frac{5\cos(x)}{2}-\frac{5\cos^3(x)}{6}-\frac{1}{2}\cos^3(x)\cot^2(x)+\frac{5}{2}\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \cos(x)\right)\\
&= \frac{5}{2}\tanh^{-1}(\cos(x))-\frac{5\cos(x)}{2}-\frac{5\cos^3(x)}{6}-\frac{1}{2}\cos^3(x)\cot^2(x)
\end{aligned}$$

Mathematica [A] time = 0.02, size = 61, normalized size = 1.79

$$-\frac{9\cos(x)}{4}-\frac{1}{12}\cos(3x)-\frac{1}{8}\csc^2\left(\frac{x}{2}\right)+\frac{1}{8}\sec^2\left(\frac{x}{2}\right)-\frac{5}{2}\log\left(\sin\left(\frac{x}{2}\right)\right)+\frac{5}{2}\log\left(\cos\left(\frac{x}{2}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(Csc[x] - Sin[x])^3,x]
```

```
[Out] (-9*Cos[x])/4 - Cos[3*x]/12 - Csc[x/2]^2/8 + (5*Log[Cos[x/2]])/2 - (5*Log[S
in[x/2]])/2 + Sec[x/2]^2/8
```

fricas [B] time = 1.77, size = 57, normalized size = 1.68

$$-\frac{4\ \cos(x)^5+20\ \cos(x)^3-15\left(\cos(x)^2-1\right)\log\left(\frac{1}{2}\ \cos(x)+\frac{1}{2}\right)+15\left(\cos(x)^2-1\right)\log\left(-\frac{1}{2}\ \cos(x)+\frac{1}{2}\right)-30\ \cos}{12\left(\cos(x)^2-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((csc(x)-sin(x))^3,x, algorithm="fricas")
```

```
[Out] -1/12*(4*cos(x)^5 + 20*cos(x)^3 - 15*(cos(x)^2 - 1)*log(1/2*cos(x) + 1/2) +
 15*(cos(x)^2 - 1)*log(-1/2*cos(x) + 1/2) - 30*cos(x))/(cos(x)^2 - 1)
```

giac [B] time = 0.16, size = 99, normalized size = 2.91

$$\frac{\left(\frac{10\,(\cos(x)-1)}{\cos(x)+1}+1\right)(\cos(x)+1)}{8\,(\cos(x)-1)}-\frac{\cos(x)-1}{8\,(\cos(x)+1)}-\frac{2\left(\frac{12\,(\cos(x)-1)}{\cos(x)+1}-\frac{9\,(\cos(x)-1)^2}{(\cos(x)+1)^2}-7\right)}{3\left(\frac{\cos(x)-1}{\cos(x)+1}-1\right)^3}-\frac{5}{4}\log\left(-\frac{\cos(x)-1}{\cos(x)+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((csc(x)-sin(x))^3,x, algorithm="giac")
```

```
[Out] 1/8*(10*(cos(x) - 1)/(cos(x) + 1) + 1)*(cos(x) + 1)/(cos(x) - 1) - 1/8*(cos
(x) - 1)/(cos(x) + 1) - 2/3*(12*(cos(x) - 1)/(cos(x) + 1) - 9*(cos(x) - 1)^
2/(cos(x) + 1)^2 - 7)/((cos(x) - 1)/(cos(x) + 1) - 1)^3 - 5/4*log(-(cos(x)
- 1)/(cos(x) + 1))
```

maple [A] time = 0.06, size = 32, normalized size = 0.94

$$\frac{\left(2+\sin^2(x)\right)\cos(x)}{3}-3\cos(x)-\frac{5\ln\left(\csc(x)-\cot(x)\right)}{2}-\frac{\cot(x)\csc(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((csc(x)-sin(x))^3,x)
```

```
[Out] 1/3*(2+sin(x)^2)*cos(x)-3*cos(x)-5/2*ln(csc(x)-cot(x))-1/2*cot(x)*csc(x)
```

maxima [A] time = 0.31, size = 37, normalized size = 1.09

$$-\frac{1}{3}\cos(x)^3+\frac{\cos(x)}{2\left(\cos(x)^2-1\right)}-2\cos(x)+\frac{5}{4}\log\left(\cos(x)+1\right)-\frac{5}{4}\log\left(\cos(x)-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((csc(x)-sin(x))^3,x, algorithm="maxima")
```

```
[Out] -1/3*cos(x)^3 + 1/2*cos(x)/(cos(x)^2 - 1) - 2*cos(x) + 5/4*log(cos(x) + 1)
- 5/4*log(cos(x) - 1)
```

mupad [B] time = 2.49, size = 75, normalized size = 2.21

$$\frac{\tan\left(\frac{x}{2}\right)^2}{8}-\frac{\frac{49\tan\left(\frac{x}{2}\right)^6}{8}+\frac{67\tan\left(\frac{x}{2}\right)^4}{8}+\frac{121\tan\left(\frac{x}{2}\right)^2}{24}+\frac{1}{8}}{\tan\left(\frac{x}{2}\right)^8+3\tan\left(\frac{x}{2}\right)^6+3\tan\left(\frac{x}{2}\right)^4+\tan\left(\frac{x}{2}\right)^2}-\frac{5\ln\left(\tan\left(\frac{x}{2}\right)\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(sin(x) - 1/sin(x))^3,x)
```

```
[Out] tan(x/2)^2/8 - ((121*tan(x/2)^2)/24 + (67*tan(x/2)^4)/8 + (49*tan(x/2)^6)/8
 + 1/8)/(tan(x/2)^2 + 3*tan(x/2)^4 + 3*tan(x/2)^6 + tan(x/2)^8) - (5*log(ta
n(x/2)))/2
```

sympy [A] time = 3.47, size = 42, normalized size = 1.24

$$-\frac{5\log(\cos(x)-1)}{4}+\frac{5\log(\cos(x)+1)}{4}-\frac{\cos^3(x)}{3}-2\cos(x)+\frac{\cos(x)}{2\cos^2(x)-2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((csc(x)-sin(x))**3,x)
```

```
[Out] -5*log(cos(x) - 1)/4 + 5*log(cos(x) + 1)/4 - cos(x)**3/3 - 2*cos(x) + cos(x
)/(2*cos(x)**2 - 2)
```

3.305 $\int (\csc(x)-\sin(x))^2\,dx$

Optimal. Leaf size=22

$$-\frac{3x}{2}-\frac{3\cot(x)}{2}+\frac{1}{2}\cos^2(x)\cot(x)$$

```
[Out] -3/2*x-3/2*cot(x)+1/2*cos(x)^2*cot(x)
```

Rubi [A] time = 0.02, antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {290, 325, 203}

$$-\frac{3x}{2}-\frac{3\cot(x)}{2}+\frac{1}{2}\cos^2(x)\cot(x)$$

Antiderivative was successfully verified.

```
[In] Int[(Csc[x] - Sin[x])^2,x]
```

```
[Out] (-3*x)/2 - (3*Cot[x])/2 + (Cos[x]^2*Cot[x])/2
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rubi steps

$$
\begin{aligned}
\int (\csc(x)-\sin(x))^2\,dx &= \text{Subst}\left(\int \frac{1}{x^2\left(1+x^2\right)^2}\,dx, x, \tan(x)\right)\\
&= \frac{1}{2}\cos^2(x)\cot(x)+\frac{3}{2}\text{Subst}\left(\int \frac{1}{x^2\left(1+x^2\right)}\,dx, x, \tan(x)\right)\\
&= -\frac{3\cot(x)}{2}+\frac{1}{2}\cos^2(x)\cot(x)-\frac{3}{2}\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \tan(x)\right)\\
&= -\frac{3x}{2}-\frac{3\cot(x)}{2}+\frac{1}{2}\cos^2(x)\cot(x)
\end{aligned}
$$

Mathematica [A] time = 0.02, size = 18, normalized size = 0.82

$$
-\frac{3x}{2}-\frac{1}{4}\sin(2x)-\cot(x)
$$

Antiderivative was successfully verified.

```
[In] Integrate[(Csc[x] - Sin[x])^2,x]
[Out] (-3*x)/2 - Cot[x] - Sin[2*x]/4
```

fricas [A] time = 1.33, size = 20, normalized size = 0.91

$$
\frac{\cos(x)^3-3\,x\sin(x)-3\,\cos(x)}{2\,\sin(x)}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((csc(x)-sin(x))^2,x, algorithm="fricas")
[Out] 1/2*(cos(x)^3 - 3*x*sin(x) - 3*cos(x))/sin(x)
```

giac [A] time = 0.15, size = 23, normalized size = 1.05

$$
-\frac{3}{2}\,x-\frac{3\,\tan(x)^2+2}{2\left(\tan(x)^3+\tan(x)\right)}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((csc(x)-sin(x))^2,x, algorithm="giac")
[Out] -3/2*x - 1/2*(3*tan(x)^2 + 2)/(tan(x)^3 + tan(x))
```

maple [A] time = 0.05, size = 15, normalized size = 0.68

$$-\frac{\cos(x)\sin(x)}{2}-\frac{3x}{2}-\cot(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((csc(x)-sin(x))^2,x)
```

```
[Out] -1/2*cos(x)*sin(x)-3/2*x-cot(x)
```

maxima [A] time = 0.31, size = 16, normalized size = 0.73

$$-\frac{3}{2}x-\frac{1}{\tan(x)}-\frac{1}{4}\sin(2x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((csc(x)-sin(x))^2,x, algorithm="maxima")
```

```
[Out] -3/2*x - 1/tan(x) - 1/4*sin(2*x)
```

mupad [B] time = 2.40, size = 21, normalized size = 0.95

$$-\frac{3x}{2}-\frac{\frac{3\cos(x)}{2}-\frac{\cos(x)^3}{2}}{\sin(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((sin(x) - 1/sin(x))^2,x)
```

```
[Out] - (3*x)/2 - ((3*cos(x))/2 - cos(x)^3/2)/sin(x)
```

sympy [A] time = 1.57, size = 15, normalized size = 0.68

$$-\frac{3x}{2}-\frac{\sin(2x)}{4}-\cot(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((csc(x)-sin(x))**2,x)
```

```
[Out] -3*x/2 - sin(2*x)/4 - cot(x)
```

3.306 $\int (\csc(x)-\sin(x))\,dx$

Optimal. Leaf size=8

$$\cos(x)-\tanh^{-1}(\cos(x))$$

```
[Out] -arctanh(cos(x))+cos(x)
```

Rubi [A] time = 0.01, antiderivative size = 8, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3770, 2638}

$$\cos(x)-\tanh^{-1}(\cos(x))$$

Antiderivative was successfully verified.

```
[In] Int[Csc[x] - Sin[x],x]
```

```
[Out] -ArcTanh[Cos[x]] + Cos[x]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int (\csc(x)-\sin(x))\,dx &= \int \csc(x)\,dx-\int \sin(x)\,dx\\ &= -\tanh^{-1}(\cos(x))+\cos(x)\end{aligned}$$

Mathematica [B] time = 0.00, size = 19, normalized size = 2.38

$$\cos(x)+\log\left(\sin\left(\frac{x}{2}\right)\right)-\log\left(\cos\left(\frac{x}{2}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[x] - Sin[x],x]
```

```
[Out] Cos[x] - Log[Cos[x/2]] + Log[Sin[x/2]]
```

fricas [B] time = 0.94, size = 21, normalized size = 2.62

$$\cos(x) - \frac{1}{2}\log\left(\frac{1}{2}\cos(x) + \frac{1}{2}\right) + \frac{1}{2}\log\left(-\frac{1}{2}\cos(x) + \frac{1}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)-sin(x),x, algorithm="fricas")
[Out] cos(x) - 1/2*log(1/2*cos(x) + 1/2) + 1/2*log(-1/2*cos(x) + 1/2)
```

giac [A] time = 0.15, size = 9, normalized size = 1.12

$$\cos(x) + \log\left(\left|\tan\left(\frac{1}{2}x\right)\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)-sin(x),x, algorithm="giac")
[Out] cos(x) + log(abs(tan(1/2*x)))
```

maple [A] time = 0.00, size = 12, normalized size = 1.50

$$\cos(x) - \ln(\cot(x) + \csc(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(x)-sin(x),x)
[Out] cos(x)-ln(cot(x)+csc(x))
```

maxima [A] time = 0.32, size = 11, normalized size = 1.38

$$\cos(x) - \log(\cot(x) + \csc(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)-sin(x),x, algorithm="maxima")
[Out] cos(x) - log(cot(x) + csc(x))
```

mupad [B] time = 0.02, size = 8, normalized size = 1.00

$$\ln\left(\tan\left(\frac{x}{2}\right)\right) + \cos(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/sin(x) - sin(x),x)
```

```
[Out] log(tan(x/2)) + cos(x)
```

sympy [B] time = 0.09, size = 19, normalized size = 2.38

$$\frac{\log(\cos(x)-1)}{2}-\frac{\log(\cos(x)+1)}{2}+\cos(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)-sin(x),x)
```

```
[Out] log(cos(x) - 1)/2 - log(cos(x) + 1)/2 + cos(x)
```

3.307 $\int \frac{1}{\csc(x)-\sin(x)}\,dx$

Optimal. Leaf size=2

$$\sec(x)$$

```
[Out] sec(x)
```

Rubi [A] time = 0.02, antiderivative size = 2, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {4397, 2606, 8}

$$\sec(x)$$

Antiderivative was successfully verified.

```
[In] Int[(Csc[x] - Sin[x])^(-1),x]
```

```
[Out] Sec[x]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 4397

```
Int[u_, x_Symbol] :> Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\csc(x)-\sin(x)}\,dx &= \int \sec(x)\tan(x)\,dx\\ &= \text{Subst}\left(\int 1\,dx, x, \sec(x)\right)\\ &= \sec(x)\end{aligned}$$

Mathematica [A] time = 0.00, size = 2, normalized size = 1.00

$$\sec(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(Csc[x] - Sin[x])^(-1),x]
```

```
[Out] Sec[x]
```

fricas [A] time = 2.20, size = 4, normalized size = 2.00

$$\frac{1}{\cos(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(csc(x)-sin(x)),x, algorithm="fricas")
```

```
[Out] 1/cos(x)
```

giac [B] time = 0.14, size = 17, normalized size = 8.50

$$\frac{2}{\frac{\cos(x)-1}{\cos(x)+1}+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(csc(x)-sin(x)),x, algorithm="giac")
```

```
[Out] 2/((cos(x) - 1)/(cos(x) + 1) + 1)
```

maple [A] time = 0.10, size = 5, normalized size = 2.50

$$\frac{1}{\cos(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(csc(x)-sin(x)),x)
```

```
[Out] 1/cos(x)
```

maxima [B] time = 0.31, size = 17, normalized size = 8.50

$$-\frac{2}{\frac{\sin(x)^2}{(\cos(x)+1)^2}-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(csc(x)-sin(x)),x, algorithm="maxima")
```

```
[Out] -2/(sin(x)^2/(cos(x) + 1)^2 - 1)
```

mupad [B] time = 2.46, size = 12, normalized size = 6.00

$$-\frac{2}{\tan\left(\frac{x}{2}\right)^2-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-1/(sin(x) - 1/sin(x)),x)
[Out] -2/(tan(x/2)^2 - 1)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{-\sin(x)+\csc(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(csc(x)-sin(x)),x)
[Out] Integral(1/(-sin(x) + csc(x)), x)
```

3.308 $\int \frac{1}{(\csc(x)-\sin(x))^2} dx$

Optimal. Leaf size=8

$$\frac{\tan^3(x)}{3}$$

[Out] 1/3*tan(x)^3

Rubi [A] time = 0.01, antiderivative size = 8, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {30}

$$\frac{\tan^3(x)}{3}$$

Antiderivative was successfully verified.

[In] Int[(Csc[x] - Sin[x])^(-2),x]

[Out] Tan[x]^3/3

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(\csc(x)-\sin(x))^2} dx &= \text{Subst}\left(\int x^2 dx, x, \tan(x)\right)\\ &= \frac{\tan^3(x)}{3}\end{aligned}$$

Mathematica [A] time = 0.00, size = 8, normalized size = 1.00

$$\frac{\tan^3(x)}{3}$$

Antiderivative was successfully verified.

[In] Integrate[(Csc[x] - Sin[x])^(-2),x]

[Out] Tan[x]^3/3

fricas [B] time = 1.70, size = 14, normalized size = 1.75

$$-\frac{\left(\cos(x)^2-1\right)\sin(x)}{3\,\cos(x)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(csc(x)-sin(x))^2,x, algorithm="fricas")
[Out] -1/3*(cos(x)^2 - 1)*sin(x)/cos(x)^3
```

giac [A] time = 0.13, size = 6, normalized size = 0.75

$$\frac{1}{3}\,\tan(x)^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(csc(x)-sin(x))^2,x, algorithm="giac")
[Out] 1/3*tan(x)^3
```

maple [A] time = 0.11, size = 7, normalized size = 0.88

$$\frac{\left(\tan^3(x)\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(csc(x)-sin(x))^2,x)
[Out] 1/3*tan(x)^3
```

maxima [A] time = 0.33, size = 6, normalized size = 0.75

$$\frac{1}{3}\,\tan(x)^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(csc(x)-sin(x))^2,x, algorithm="maxima")
[Out] 1/3*tan(x)^3
```

mupad [B] time = 2.41, size = 6, normalized size = 0.75

$$\frac{\tan(x)^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(x) - 1/sin(x))^2,x)
```

```
[Out] tan(x)^3/3
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(-\sin(x) + \csc(x))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(csc(x)-sin(x))**2,x)
```

```
[Out] Integral((-sin(x) + csc(x))**(-2), x)
```

3.309 $\int \frac{1}{(\csc(x)-\sin(x))^3} \, dx$

Optimal. Leaf size=17

$$\frac{\sec^5(x)}{5} - \frac{\sec^3(x)}{3}$$

```
[Out] -1/3*sec(x)^3+1/5*sec(x)^5
```

Rubi [A] time = 0.04, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {4397, 2606, 14}

$$\frac{\sec^5(x)}{5} - \frac{\sec^3(x)}{3}$$

Antiderivative was successfully verified.

```
[In] Int[(Csc[x] - Sin[x])^(-3),x]
```

```
[Out] -Sec[x]^3/3 + Sec[x]^5/5
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 4397

```
Int[u_, x_Symbol] :> Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]
```

Rubi steps

$$\int \frac{1}{(\csc(x)-\sin(x))^3}\,dx = \int \sec^3(x)\tan^3(x)\,dx$$
$$= \text{Subst}\left(\int x^2\left(-1+x^2\right)dx, x, \sec(x)\right)$$
$$= \text{Subst}\left(\int \left(-x^2+x^4\right)dx, x, \sec(x)\right)$$
$$= -\frac{1}{3}\sec^3(x)+\frac{\sec^5(x)}{5}$$

Mathematica [A] time = 0.02, size = 17, normalized size = 1.00

$$\frac{\sec^5(x)}{5}-\frac{\sec^3(x)}{3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Csc[x] - Sin[x])^(-3),x]
[Out] -1/3*Sec[x]^3 + Sec[x]^5/5
```

fricas [A] time = 1.20, size = 14, normalized size = 0.82

$$-\frac{5\,\cos(x)^2-3}{15\,\cos(x)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(csc(x)-sin(x))^3,x, algorithm="fricas")
[Out] -1/15*(5*cos(x)^2 - 3)/cos(x)^5
```

giac [B] time = 0.13, size = 59, normalized size = 3.47

$$-\frac{4\left(\frac{5(\cos(x)-1)}{\cos(x)+1}-\frac{5(\cos(x)-1)^2}{(\cos(x)+1)^2}+\frac{15(\cos(x)-1)^3}{(\cos(x)+1)^3}+1\right)}{15\left(\frac{\cos(x)-1}{\cos(x)+1}+1\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(csc(x)-sin(x))^3,x, algorithm="giac")
[Out] -4/15*(5*(cos(x) - 1)/(cos(x) + 1) - 5*(cos(x) - 1)^2/(cos(x) + 1)^2 + 15*(
cos(x) - 1)^3/(cos(x) + 1)^3 + 1)/((cos(x) - 1)/(cos(x) + 1) + 1)^5
```

maple [A] time = 0.12, size = 14, normalized size = 0.82

$$-\frac{1}{3\cos(x)^3}+\frac{1}{5\cos(x)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(csc(x)-sin(x))^3,x)
```

```
[Out] -1/3/cos(x)^3+1/5/cos(x)^5
```

maxima [B] time = 0.31, size = 103, normalized size = 6.06

$$-\frac{4\left(\frac{5\sin(x)^2}{(\cos(x)+1)^2}+\frac{5\sin(x)^4}{(\cos(x)+1)^4}+\frac{15\sin(x)^6}{(\cos(x)+1)^6}-1\right)}{15\left(\frac{5\sin(x)^2}{(\cos(x)+1)^2}-\frac{10\sin(x)^4}{(\cos(x)+1)^4}+\frac{10\sin(x)^6}{(\cos(x)+1)^6}-\frac{5\sin(x)^8}{(\cos(x)+1)^8}+\frac{\sin(x)^{10}}{(\cos(x)+1)^{10}}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(csc(x)-sin(x))^3,x, algorithm="maxima")
```

```
[Out] -4/15*(5*sin(x)^2/(cos(x) + 1)^2 + 5*sin(x)^4/(cos(x) + 1)^4 + 15*sin(x)^6/
(cos(x) + 1)^6 - 1)/(5*sin(x)^2/(cos(x) + 1)^2 - 10*sin(x)^4/(cos(x) + 1)^4
 + 10*sin(x)^6/(cos(x) + 1)^6 - 5*sin(x)^8/(cos(x) + 1)^8 + sin(x)^10/(cos(
x) + 1)^10 - 1)
```

mupad [B] time = 2.55, size = 13, normalized size = 0.76

$$\frac{1}{5\cos(x)^5}-\frac{1}{3\cos(x)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-1/(sin(x) - 1/sin(x))^3,x)
```

```
[Out] 1/(5*cos(x)^5) - 1/(3*cos(x)^3)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(-\sin(x)+\csc(x))^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(csc(x)-sin(x))**3,x)
```

```
[Out] Integral((-sin(x) + csc(x))**(-3), x)
```

3.310 $\int \frac{1}{(\csc(x)-\sin(x))^4}\,dx$

Optimal. Leaf size=17

$$\frac{\tan^7(x)}{7}+\frac{\tan^5(x)}{5}$$

```
[Out] 1/5*tan(x)^5+1/7*tan(x)^7
```

Rubi [A] time = 0.02, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\frac{\tan^7(x)}{7}+\frac{\tan^5(x)}{5}$$

Antiderivative was successfully verified.

```
[In] Int[(Csc[x] - Sin[x])^(-4),x]
```

```
[Out] Tan[x]^5/5 + Tan[x]^7/7
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(\csc(x)-\sin(x))^4}\,dx &= \text{Subst}\left(\int \left(x^4+x^6\right)\,dx, x, \tan(x)\right)\\ &= \frac{\tan^5(x)}{5}+\frac{\tan^7(x)}{7}\end{aligned}$$

Mathematica [B] time = 0.02, size = 37, normalized size = 2.18

$$\frac{2\tan(x)}{35}+\frac{1}{7}\tan(x)\sec^6(x)-\frac{8}{35}\tan(x)\sec^4(x)+\frac{1}{35}\tan(x)\sec^2(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(Csc[x] - Sin[x])^(-4),x]
```

```
[Out] (2*Tan[x])/35 + (Sec[x]^2*Tan[x])/35 - (8*Sec[x]^4*Tan[x])/35 + (Sec[x]^6*T
an[x])/7
```

fricas [A] time = 1.54, size = 26, normalized size = 1.53

$$\frac{\left(2\,\cos(x)^6+\cos(x)^4-8\,\cos(x)^2+5\right)\sin(x)}{35\,\cos(x)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(csc(x)-sin(x))^4,x, algorithm="fricas")
[Out] 1/35*(2*cos(x)^6 + cos(x)^4 - 8*cos(x)^2 + 5)*sin(x)/cos(x)^7
```

giac [A] time = 0.12, size = 13, normalized size = 0.76

$$\frac{1}{7}\tan(x)^7 + \frac{1}{5}\tan(x)^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(csc(x)-sin(x))^4,x, algorithm="giac")
[Out] 1/7*tan(x)^7 + 1/5*tan(x)^5
```

maple [A] time = 0.11, size = 14, normalized size = 0.82

$$\frac{\left(\tan^5(x)\right)}{5} + \frac{\left(\tan^7(x)\right)}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(csc(x)-sin(x))^4,x)
[Out] 1/5*tan(x)^5+1/7*tan(x)^7
```

maxima [A] time = 0.31, size = 13, normalized size = 0.76

$$\frac{1}{7}\tan(x)^7 + \frac{1}{5}\tan(x)^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(csc(x)-sin(x))^4,x, algorithm="maxima")
[Out] 1/7*tan(x)^7 + 1/5*tan(x)^5
```

mupad [B] time = 2.57, size = 23, normalized size = 1.35

$$\frac{2\cos(x)^2\sin(x)^5 + 5\sin(x)^5}{35\cos(x)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(x) - 1/sin(x))^4,x)
[Out] (5*sin(x)^5 + 2*cos(x)^2*sin(x)^5)/(35*cos(x)^7)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-\sin\left(x\right)+\csc\left(x\right)\right)^{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(csc(x)-sin(x))**4,x)
```

```
[Out] Integral((-sin(x) + csc(x))**(-4), x)
```

3.311 $\int \frac{1}{(\csc(x)-\sin(x))^5} dx$

Optimal. Leaf size=25

$$\frac{\sec^9(x)}{9}-\frac{2\sec^7(x)}{7}+\frac{\sec^5(x)}{5}$$

```
[Out] 1/5*sec(x)^5-2/7*sec(x)^7+1/9*sec(x)^9
```

Rubi [A] time = 0.04, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {4397, 2606, 270}

$$\frac{\sec^9(x)}{9}-\frac{2\sec^7(x)}{7}+\frac{\sec^5(x)}{5}$$

Antiderivative was successfully verified.

```
[In] Int[(Csc[x] - Sin[x])^(-5),x]
```

```
[Out] Sec[x]^5/5 - (2*Sec[x]^7)/7 + Sec[x]^9/9
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 4397

```
Int[u_, x_Symbol] :> Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]
```

Rubi steps

$$\int \frac{1}{(\csc(x)-\sin(x))^5}\,dx = \int \sec^5(x)\tan^5(x)\,dx$$
$$= \text{Subst}\left(\int x^4\left(-1+x^2\right)^2\,dx, x, \sec(x)\right)$$
$$= \text{Subst}\left(\int \left(x^4-2x^6+x^8\right)dx, x, \sec(x)\right)$$
$$= \frac{\sec^5(x)}{5} - \frac{2\sec^7(x)}{7} + \frac{\sec^9(x)}{9}$$

Mathematica [A] time = 0.02, size = 25, normalized size = 1.00

$$\frac{\sec^9(x)}{9} - \frac{2\sec^7(x)}{7} + \frac{\sec^5(x)}{5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Csc[x] - Sin[x])^(-5),x]
[Out] Sec[x]^5/5 - (2*Sec[x]^7)/7 + Sec[x]^9/9
```

fricas [A] time = 0.97, size = 20, normalized size = 0.80

$$\frac{63\cos(x)^4 - 90\cos(x)^2 + 35}{315\cos(x)^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(csc(x)-sin(x))^5,x, algorithm="fricas")
[Out] 1/315*(63*cos(x)^4 - 90*cos(x)^2 + 35)/cos(x)^9
```

giac [B] time = 0.15, size = 101, normalized size = 4.04

$$\frac{16\left(\frac{9(\cos(x)-1)}{\cos(x)+1} + \frac{36(\cos(x)-1)^2}{(\cos(x)+1)^2} - \frac{126(\cos(x)-1)^3}{(\cos(x)+1)^3} + \frac{441(\cos(x)-1)^4}{(\cos(x)+1)^4} - \frac{315(\cos(x)-1)^5}{(\cos(x)+1)^5} + \frac{210(\cos(x)-1)^6}{(\cos(x)+1)^6} + 1\right)}{315\left(\frac{\cos(x)-1}{\cos(x)+1} + 1\right)^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(csc(x)-sin(x))^5,x, algorithm="giac")
[Out] 16/315*(9*(cos(x) - 1)/(cos(x) + 1) + 36*(cos(x) - 1)^2/(cos(x) + 1)^2 - 12
6*(cos(x) - 1)^3/(cos(x) + 1)^3 + 441*(cos(x) - 1)^4/(cos(x) + 1)^4 - 315*(
```

```
cos(x) - 1)^5/(cos(x) + 1)^5 + 210*(cos(x) - 1)^6/(cos(x) + 1)^6 + 1)/((cos
(x) - 1)/(cos(x) + 1) + 1)^9
```

maple [A] time = 0.12, size = 20, normalized size = 0.80

$$-\frac{2}{7\cos(x)^7}+\frac{1}{9\cos(x)^9}+\frac{1}{5\cos(x)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(csc(x)-sin(x))^5,x)
```

```
[Out] -2/7/cos(x)^7+1/9/cos(x)^9+1/5/cos(x)^5
```

maxima [B] time = 0.36, size = 187, normalized size = 7.48

$$\frac{16\left(\frac{9\sin(x)^2}{(\cos(x)+1)^2}-\frac{36\sin(x)^4}{(\cos(x)+1)^4}-\frac{126\sin(x)^6}{(\cos(x)+1)^6}-\frac{441\sin(x)^8}{(\cos(x)+1)^8}-\frac{315\sin(x)^{10}}{(\cos(x)+1)^{10}}-\frac{210\sin(x)^{12}}{(\cos(x)+1)^{12}}-1\right)}{315\left(\frac{9\sin(x)^2}{(\cos(x)+1)^2}-\frac{36\sin(x)^4}{(\cos(x)+1)^4}+\frac{84\sin(x)^6}{(\cos(x)+1)^6}-\frac{126\sin(x)^8}{(\cos(x)+1)^8}+\frac{126\sin(x)^{10}}{(\cos(x)+1)^{10}}-\frac{84\sin(x)^{12}}{(\cos(x)+1)^{12}}+\frac{36\sin(x)^{14}}{(\cos(x)+1)^{14}}-\frac{9\sin(x)^{16}}{(\cos(x)+1)^{16}}+\frac{\sin(x)^{18}}{(\cos(x)+1)}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(csc(x)-sin(x))^5,x, algorithm="maxima")
```

```
[Out] 16/315*(9*sin(x)^2/(cos(x) + 1)^2 - 36*sin(x)^4/(cos(x) + 1)^4 - 126*sin(x)
^6/(cos(x) + 1)^6 - 441*sin(x)^8/(cos(x) + 1)^8 - 315*sin(x)^10/(cos(x) + 1
)^10 - 210*sin(x)^12/(cos(x) + 1)^12 - 1)/(9*sin(x)^2/(cos(x) + 1)^2 - 36*s
in(x)^4/(cos(x) + 1)^4 + 84*sin(x)^6/(cos(x) + 1)^6 - 126*sin(x)^8/(cos(x)
+ 1)^8 + 126*sin(x)^10/(cos(x) + 1)^10 - 84*sin(x)^12/(cos(x) + 1)^12 + 36*
sin(x)^14/(cos(x) + 1)^14 - 9*sin(x)^16/(cos(x) + 1)^16 + sin(x)^18/(cos(x)
 + 1)^18 - 1)
```

mupad [B] time = 2.94, size = 19, normalized size = 0.76

$$\frac{1}{5\cos(x)^5}-\frac{2}{7\cos(x)^7}+\frac{1}{9\cos(x)^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-1/(sin(x) - 1/sin(x))^5,x)
```

```
[Out] 1/(5*cos(x)^5) - 2/(7*cos(x)^7) + 1/(9*cos(x)^9)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{(-\sin{(x)}+\csc{(x)})^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(csc(x)-sin(x))**5,x)
```

```
[Out] Integral((-sin(x) + csc(x))**(-5), x)
```

3.312 $\int \frac{1}{(\csc(x)-\sin(x))^6} \, dx$

Optimal. Leaf size=25

$$\frac{\tan^{11}(x)}{11}+\frac{2\tan^9(x)}{9}+\frac{\tan^7(x)}{7}$$

```
[Out] 1/7*tan(x)^7+2/9*tan(x)^9+1/11*tan(x)^11
```

Rubi [A] time = 0.02, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 1, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {270}

$$\frac{\tan^{11}(x)}{11}+\frac{2\tan^9(x)}{9}+\frac{\tan^7(x)}{7}$$

Antiderivative was successfully verified.

```
[In] Int[(Csc[x] - Sin[x])^(-6),x]
```

```
[Out] Tan[x]^7/7 + (2*Tan[x]^9)/9 + Tan[x]^11/11
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(\csc(x)-\sin(x))^6} \, dx &= \text{Subst}\left(\int x^6\left(1+x^2\right)^2 \, dx, x, \tan(x)\right)\\ &= \text{Subst}\left(\int \left(x^6+2x^8+x^{10}\right) dx, x, \tan(x)\right)\\ &= \frac{\tan^7(x)}{7}+\frac{2\tan^9(x)}{9}+\frac{\tan^{11}(x)}{11}\end{aligned}$$

Mathematica [B] time = 0.02, size = 57, normalized size = 2.28

$$-\frac{8\tan(x)}{693}+\frac{1}{11}\tan(x)\sec^{10}(x)-\frac{23}{99}\tan(x)\sec^8(x)+\frac{113}{693}\tan(x)\sec^6(x)-\frac{1}{231}\tan(x)\sec^4(x)-\frac{4}{693}\tan(x)\sec^2(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(Csc[x] - Sin[x])^(-6),x]
```

```
[Out] (-8*Tan[x])/693 - (4*Sec[x]^2*Tan[x])/693 - (Sec[x]^4*Tan[x])/231 + (113*Se
c[x]^6*Tan[x])/693 - (23*Sec[x]^8*Tan[x])/99 + (Sec[x]^10*Tan[x])/11
```

fricas [B] time = 0.86, size = 40, normalized size = 1.60

$$-\frac{\left(8\,\cos(x)^{10}+4\,\cos(x)^{8}+3\,\cos(x)^{6}-113\,\cos(x)^{4}+161\,\cos(x)^{2}-63\right)\sin(x)}{693\,\cos(x)^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(csc(x)-sin(x))^6,x, algorithm="fricas")
```

```
[Out] -1/693*(8*cos(x)^10 + 4*cos(x)^8 + 3*cos(x)^6 - 113*cos(x)^4 + 161*cos(x)^2
 - 63)*sin(x)/cos(x)^11
```

giac [A] time = 0.13, size = 19, normalized size = 0.76

$$\frac{1}{11}\,\tan(x)^{11}+\frac{2}{9}\,\tan(x)^{9}+\frac{1}{7}\,\tan(x)^{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(csc(x)-sin(x))^6,x, algorithm="giac")
```

```
[Out] 1/11*tan(x)^11 + 2/9*tan(x)^9 + 1/7*tan(x)^7
```

maple [A] time = 0.12, size = 20, normalized size = 0.80

$$\frac{\left(\tan^{7}(x)\right)}{7}+\frac{2\left(\tan^{9}(x)\right)}{9}+\frac{\left(\tan^{11}(x)\right)}{11}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(csc(x)-sin(x))^6,x)
```

```
[Out] 1/7*tan(x)^7+2/9*tan(x)^9+1/11*tan(x)^11
```

maxima [A] time = 0.35, size = 19, normalized size = 0.76

$$\frac{1}{11}\,\tan(x)^{11}+\frac{2}{9}\,\tan(x)^{9}+\frac{1}{7}\,\tan(x)^{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(csc(x)-sin(x))^6,x, algorithm="maxima")
```

```
[Out] 1/11*tan(x)^11 + 2/9*tan(x)^9 + 1/7*tan(x)^7
```

mupad [B] time = 2.91, size = 33, normalized size = 1.32

$$\frac{8\cos(x)^4\sin(x)^7+28\cos(x)^2\sin(x)^7+63\sin(x)^7}{693\cos(x)^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(x) - 1/sin(x))^6,x)
```

```
[Out] (63*sin(x)^7 + 28*cos(x)^2*sin(x)^7 + 8*cos(x)^4*sin(x)^7)/(693*cos(x)^11)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{\left(-\sin\left(x\right)+\csc\left(x\right)\right)^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(csc(x)-sin(x))**6,x)
```

```
[Out] Integral((-sin(x) + csc(x))**(-6), x)
```

3.313 $\int \frac{1}{(\csc(x)-\sin(x))^7} \, dx$

Optimal. Leaf size=33

$$\frac{\sec^{13}(x)}{13} - \frac{3\sec^{11}(x)}{11} + \frac{\sec^9(x)}{3} - \frac{\sec^7(x)}{7}$$

```
[Out] -1/7*sec(x)^7+1/3*sec(x)^9-3/11*sec(x)^11+1/13*sec(x)^13
```

Rubi [A] time = 0.04, antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {4397, 2606, 270}

$$\frac{\sec^{13}(x)}{13} - \frac{3\sec^{11}(x)}{11} + \frac{\sec^9(x)}{3} - \frac{\sec^7(x)}{7}$$

Antiderivative was successfully verified.

```
[In] Int[(Csc[x] - Sin[x])^(-7),x]
```

```
[Out] -Sec[x]^7/7 + Sec[x]^9/3 - (3*Sec[x]^11)/11 + Sec[x]^13/13
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 && !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 4397

```
Int[u_, x_Symbol] :> Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]
```

Rubi steps

$$\int \frac{1}{(\csc(x)-\sin(x))^7}\,dx = \int \sec^7(x)\tan^7(x)\,dx$$
$$= \text{Subst}\left(\int x^6\left(-1+x^2\right)^3\,dx, x, \sec(x)\right)$$
$$= \text{Subst}\left(\int \left(-x^6+3x^8-3x^{10}+x^{12}\right)dx, x, \sec(x)\right)$$
$$= -\frac{1}{7}\sec^7(x)+\frac{\sec^9(x)}{3}-\frac{3\sec^{11}(x)}{11}+\frac{\sec^{13}(x)}{13}$$

Mathematica [A] time = 0.02, size = 33, normalized size = 1.00

$$\frac{\sec^{13}(x)}{13}-\frac{3\sec^{11}(x)}{11}+\frac{\sec^9(x)}{3}-\frac{\sec^7(x)}{7}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Csc[x] - Sin[x])^(-7),x]
```

```
[Out] -1/7*Sec[x]^7 + Sec[x]^9/3 - (3*Sec[x]^11)/11 + Sec[x]^13/13
```

fricas [A] time = 0.65, size = 26, normalized size = 0.79

$$-\frac{429\cos(x)^6-1001\cos(x)^4+819\cos(x)^2-231}{3003\cos(x)^{13}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(csc(x)-sin(x))^7,x, algorithm="fricas")
```

```
[Out] -1/3003*(429*cos(x)^6 - 1001*cos(x)^4 + 819*cos(x)^2 - 231)/cos(x)^13
```

giac [B] time = 0.14, size = 143, normalized size = 4.33

$$-\frac{32\left(\frac{13(\cos(x)-1)}{\cos(x)+1}+\frac{78(\cos(x)-1)^2}{(\cos(x)+1)^2}+\frac{286(\cos(x)-1)^3}{(\cos(x)+1)^3}-\frac{2288(\cos(x)-1)^4}{(\cos(x)+1)^4}+\frac{10296(\cos(x)-1)^5}{(\cos(x)+1)^5}-\frac{16302(\cos(x)-1)^6}{(\cos(x)+1)^6}+\frac{18018(\cos(x)-1)^7}{(\cos(x)+1)^7}\cdots\right.}{3003\left(\frac{\cos(x)-1}{\cos(x)+1}+1\right)^{13}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(csc(x)-sin(x))^7,x, algorithm="giac")
```

```
[Out] -32/3003*(13*(cos(x) - 1)/(cos(x) + 1) + 78*(cos(x) - 1)^2/(cos(x) + 1)^2 +
 286*(cos(x) - 1)^3/(cos(x) + 1)^3 - 2288*(cos(x) - 1)^4/(cos(x) + 1)^4 + 1
```

```
0296*(cos(x) - 1)^5/(cos(x) + 1)^5 - 16302*(cos(x) - 1)^6/(cos(x) + 1)^6 +
18018*(cos(x) - 1)^7/(cos(x) + 1)^7 - 9009*(cos(x) - 1)^8/(cos(x) + 1)^8 +
3003*(cos(x) - 1)^9/(cos(x) + 1)^9 + 1)/((cos(x) - 1)/(cos(x) + 1) + 1)^13
```

maple [A] time = 0.14, size = 26, normalized size = 0.79

$$\frac{1}{13\cos(x)^{13}} - \frac{1}{7\cos(x)^{7}} + \frac{1}{3\cos(x)^{9}} - \frac{3}{11\cos(x)^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(csc(x)-sin(x))^7,x)
```

```
[Out] 1/13/cos(x)^13-1/7/cos(x)^7+1/3/cos(x)^9-3/11/cos(x)^11
```

maxima [B] time = 0.34, size = 271, normalized size = 8.21

$$-\frac{32\left(\frac{13\sin(x)^{2}}{(\cos(x)+1)^{2}} - \frac{78\sin(x)^{4}}{(\cos(x)+1)^{4}} + \frac{286\sin(x)^{6}}{(\cos(x)+1)^{6}} + \frac{2288\sin(x)^{8}}{(\cos(x)+1)^{8}} + \frac{10296\sin(x)^{10}}{(\cos(x)+1)^{10}} + \frac{16302\sin(x)^{12}}{(\cos(x)+1)^{12}} + \cdots\right.}{3003\left(\frac{13\sin(x)^{2}}{(\cos(x)+1)^{2}} - \frac{78\sin(x)^{4}}{(\cos(x)+1)^{4}} + \frac{286\sin(x)^{6}}{(\cos(x)+1)^{6}} - \frac{715\sin(x)^{8}}{(\cos(x)+1)^{8}} + \frac{1287\sin(x)^{10}}{(\cos(x)+1)^{10}} - \frac{1716\sin(x)^{12}}{(\cos(x)+1)^{12}} + \frac{1716\sin(x)^{14}}{(\cos(x)+1)^{14}} - \frac{1287\sin(x)^{16}}{(\cos(x)+1)^{16}} + \cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(csc(x)-sin(x))^7,x, algorithm="maxima")
```

```
[Out] -32/3003*(13*sin(x)^2/(cos(x) + 1)^2 - 78*sin(x)^4/(cos(x) + 1)^4 + 286*sin
(x)^6/(cos(x) + 1)^6 + 2288*sin(x)^8/(cos(x) + 1)^8 + 10296*sin(x)^10/(cos(
x) + 1)^10 + 16302*sin(x)^12/(cos(x) + 1)^12 + 18018*sin(x)^14/(cos(x) + 1)
^14 + 9009*sin(x)^16/(cos(x) + 1)^16 + 3003*sin(x)^18/(cos(x) + 1)^18 - 1)/
(13*sin(x)^2/(cos(x) + 1)^2 - 78*sin(x)^4/(cos(x) + 1)^4 + 286*sin(x)^6/(co
s(x) + 1)^6 - 715*sin(x)^8/(cos(x) + 1)^8 + 1287*sin(x)^10/(cos(x) + 1)^10
- 1716*sin(x)^12/(cos(x) + 1)^12 + 1716*sin(x)^14/(cos(x) + 1)^14 - 1287*si
n(x)^16/(cos(x) + 1)^16 + 715*sin(x)^18/(cos(x) + 1)^18 - 286*sin(x)^20/(co
s(x) + 1)^20 + 78*sin(x)^22/(cos(x) + 1)^22 - 13*sin(x)^24/(cos(x) + 1)^24
+ sin(x)^26/(cos(x) + 1)^26 - 1)
```

mupad [B] time = 3.54, size = 25, normalized size = 0.76

$$\frac{1}{3\cos(x)^{9}} - \frac{1}{7\cos(x)^{7}} - \frac{3}{11\cos(x)^{11}} + \frac{1}{13\cos(x)^{13}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-1/(sin(x) - 1/sin(x))^7,x)
```

```
[Out] 1/(3*cos(x)^9) - 1/(7*cos(x)^7) - 3/(11*cos(x)^11) + 1/(13*cos(x)^13)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(csc(x)-sin(x))**7,x)
```

```
[Out] Timed out
```

3.314 $\int (\csc(x)-\sin(x))^{7/2}\,dx$

Optimal. Leaf size=73

$$\frac{2}{7}\cos^3(x)\cot^2(x)\sqrt{\cos(x)\cot(x)}+\frac{8}{7}\cos(x)\cot^2(x)\sqrt{\cos(x)\cot(x)}-\frac{64}{35}\cot(x)\csc(x)\sqrt{\cos(x)\cot(x)}+\frac{256}{35}\sec(x)$$

```
[Out] 8/7*cos(x)*cot(x)^2*(cos(x)*cot(x))^(1/2)+2/7*cos(x)^3*cot(x)^2*(cos(x)*cot
(x))^(1/2)-64/35*cot(x)*csc(x)*(cos(x)*cot(x))^(1/2)+256/35*sec(x)*(cos(x)*
cot(x))^(1/2)
```

Rubi [A] time = 0.15, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.454, Rules used = {4397, 4400, 2598, 2594, 2589}

$$\frac{2}{7}\cos^3(x)\cot^2(x)\sqrt{\cos(x)\cot(x)}+\frac{8}{7}\cos(x)\cot^2(x)\sqrt{\cos(x)\cot(x)}-\frac{64}{35}\cot(x)\csc(x)\sqrt{\cos(x)\cot(x)}+\frac{256}{35}\sec(x)$$

Antiderivative was successfully verified.

```
[In] Int[(Csc[x] - Sin[x])^(7/2),x]
```

```
[Out] (8*Cos[x]*Cot[x]^2*Sqrt[Cos[x]*Cot[x]])/7 + (2*Cos[x]^3*Cot[x]^2*Sqrt[Cos[x
]*Cot[x]])/7 - (64*Cot[x]*Sqrt[Cos[x]*Cot[x]]*Csc[x])/35 + (256*Sqrt[Cos[x]
*Cot[x]]*Sec[x])/35
```

Rule 2589

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> -Simp[(b*(a*Sin[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*m
), x] /; FreeQ[{a, b, e, f, m, n}, x] && EqQ[m + n - 1, 0]
```

Rule 2594

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(b*(a*Sin[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*(
n - 1)), x] - Dist[(b^2*(m + n - 1))/(n - 1), Int[(a*Sin[e + f*x])^m*(b*Tan
[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] && Int
egersQ[2*m, 2*n] &&  !(GtQ[m, 1] &&  !IntegerQ[(m - 1)/2])
```

Rule 2598

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> -Simp[(b*(a*Sin[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f
*m), x] + Dist[(a^2*(m + n - 1))/m, Int[(a*Sin[e + f*x])^(m - 2)*(b*Tan[e +
 f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && (GtQ[m, 1] || (EqQ[m, 1] &
```

```
& EqQ[n, 1/2])) && IntegersQ[2*m, 2*n]
```

Rule 4397

```
Int[u_, x_Symbol] :> Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]
```

Rule 4400

```
Int[(u_.)*((v_)^(m_.)*(w_)^(n_.))^(p_), x_Symbol] :> With[{uu = ActivateTri
g[u], vv = ActivateTrig[v], ww = ActivateTrig[w]}, Dist[(vv^m*ww^n)^FracPar
t[p]/(vv^(m*FracPart[p])*ww^(n*FracPart[p])), Int[uu*vv^(m*p)*ww^(n*p), x],
 x]] /; FreeQ[{m, n, p}, x] &&  !IntegerQ[p] && ( !InertTrigFreeQ[v] ||  !I
nertTrigFreeQ[w])
```

Rubi steps

$$\int (\csc(x)-\sin(x))^{7/2}\,dx = \int (\cos(x)\cot(x))^{7/2}\,dx$$

$$= \frac{\sqrt{\cos(x)\cot(x)}\int \cos^{\frac{7}{2}}(x)\cot^{\frac{7}{2}}(x)\,dx}{\sqrt{\cos(x)}\sqrt{\cot(x)}}$$

$$= \frac{2}{7}\cos^3(x)\cot^2(x)\sqrt{\cos(x)\cot(x)} + \frac{\left(12\sqrt{\cos(x)\cot(x)}\right)\int \cos^{\frac{3}{2}}(x)\cot^{\frac{7}{2}}(x)\,dx}{7\sqrt{\cos(x)}\sqrt{\cot(x)}}$$

$$= \frac{8}{7}\cos(x)\cot^2(x)\sqrt{\cos(x)\cot(x)} + \frac{2}{7}\cos^3(x)\cot^2(x)\sqrt{\cos(x)\cot(x)} + \frac{\left(32\sqrt{\cos(x)\cot\right.}{7\sqrt{\cos(x}}$$

$$= \frac{8}{7}\cos(x)\cot^2(x)\sqrt{\cos(x)\cot(x)} + \frac{2}{7}\cos^3(x)\cot^2(x)\sqrt{\cos(x)\cot(x)} - \frac{64}{35}\cot(x)\sqrt{\cos(}$$

$$= \frac{8}{7}\cos(x)\cot^2(x)\sqrt{\cos(x)\cot(x)} + \frac{2}{7}\cos^3(x)\cot^2(x)\sqrt{\cos(x)\cot(x)} - \frac{64}{35}\cot(x)\sqrt{\cos(}$$

Mathematica [A] time = 0.08, size = 37, normalized size = 0.51

$$-\frac{1}{70}\sec(x)\sqrt{\cos(x)\cot(x)}\left(115\cos^2(x)+5\cos(3x)\cos(x)+28\cot^2(x)-512\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(Csc[x] - Sin[x])^(7/2),x]
```

[Out] -1/70*(Sqrt[Cos[x]*Cot[x]]*(-512 + 115*Cos[x]^2 + 5*Cos[x]*Cos[3*x] + 28*Co
t[x]^2)*Sec[x])

fricas [A] time = 1.00, size = 44, normalized size = 0.60

$$-\frac{2\left(5\cos(x)^{6}+20\cos(x)^{4}-160\cos(x)^{2}+128\right)\sqrt{\frac{\cos(x)^{2}}{\sin(x)}}}{35\left(\cos(x)^{3}-\cos(x)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((csc(x)-sin(x))^(7/2),x, algorithm="fricas")

[Out] -2/35*(5*cos(x)^6 + 20*cos(x)^4 - 160*cos(x)^2 + 128)*sqrt(cos(x)^2/sin(x))
/(cos(x)^3 - cos(x))

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int(\csc(x)-\sin(x))^{\frac{7}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((csc(x)-sin(x))^(7/2),x, algorithm="giac")

[Out] integrate((csc(x) - sin(x))^(7/2), x)

maple [A] time = 0.31, size = 40, normalized size = 0.55

$$\frac{2\left(5\left(\cos^{6}(x)\right)+20\left(\cos^{4}(x)\right)-160\left(\cos^{2}(x)\right)+128\right)\sin(x)\left(\frac{\cos^{2}(x)}{\sin(x)}\right)^{\frac{7}{2}}}{35\cos(x)^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((csc(x)-sin(x))^(7/2),x)

[Out] 2/35*(5*cos(x)^6+20*cos(x)^4-160*cos(x)^2+128)*sin(x)*(cos(x)^2/sin(x))^(7/
2)/cos(x)^7

maxima [B] time = 0.57, size = 578, normalized size = 7.92

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((csc(x)-sin(x))^(7/2),x, algorithm="maxima")

```
[Out] -1/280*(cos(x)^2 + sin(x)^2 + 2*cos(x) + 1)^(1/4)*(cos(x)^2 + sin(x)^2 - 2*
cos(x) + 1)^(1/4)*(((5*cos(21/2*x) + 105*cos(17/2*x) - 2275*cos(13/2*x) + 5
817*cos(9/2*x) - 5*cos(7/2*x) - 5817*cos(5/2*x) - 105*cos(3/2*x) + 2275*cos
(1/2*x) - 5*sin(21/2*x) - 105*sin(17/2*x) + 2275*sin(13/2*x) - 5817*sin(9/2
*x) - 5*sin(7/2*x) + 5817*sin(5/2*x) - 105*sin(3/2*x) - 2275*sin(1/2*x))*co
s(7/2*arctan2(sin(x), cos(x) - 1)) + (5*cos(21/2*x) + 105*cos(17/2*x) - 227
5*cos(13/2*x) + 5817*cos(9/2*x) - 5*cos(7/2*x) - 5817*cos(5/2*x) - 105*cos(
3/2*x) + 2275*cos(1/2*x) + 5*sin(21/2*x) + 105*sin(17/2*x) - 2275*sin(13/2*
x) + 5817*sin(9/2*x) + 5*sin(7/2*x) - 5817*sin(5/2*x) + 105*sin(3/2*x) + 22
75*sin(1/2*x))*sin(7/2*arctan2(sin(x), cos(x) - 1)))*cos(7/2*arctan2(sin(x)
, cos(x) + 1)) + ((5*cos(21/2*x) + 105*cos(17/2*x) - 2275*cos(13/2*x) + 581
7*cos(9/2*x) - 5*cos(7/2*x) - 5817*cos(5/2*x) - 105*cos(3/2*x) + 2275*cos(1
/2*x) + 5*sin(21/2*x) + 105*sin(17/2*x) - 2275*sin(13/2*x) + 5817*sin(9/2*x
) + 5*sin(7/2*x) - 5817*sin(5/2*x) + 105*sin(3/2*x) + 2275*sin(1/2*x))*cos(
7/2*arctan2(sin(x), cos(x) - 1)) - (5*cos(21/2*x) + 105*cos(17/2*x) - 2275*
cos(13/2*x) + 5817*cos(9/2*x) - 5*cos(7/2*x) - 5817*cos(5/2*x) - 105*cos(3/
2*x) + 2275*cos(1/2*x) - 5*sin(21/2*x) - 105*sin(17/2*x) + 2275*sin(13/2*x)
 - 5817*sin(9/2*x) - 5*sin(7/2*x) + 5817*sin(5/2*x) - 105*sin(3/2*x) - 2275
*sin(1/2*x))*sin(7/2*arctan2(sin(x), cos(x) - 1)))*sin(7/2*arctan2(sin(x),
cos(x) + 1)))/(cos(x)^8 + sin(x)^8 + 4*(cos(x)^2 + 1)*sin(x)^6 - 4*cos(x)^6
 + 2*(3*cos(x)^4 + 2*cos(x)^2 + 3)*sin(x)^4 + 6*cos(x)^4 + 4*(cos(x)^6 - co
s(x)^4 - cos(x)^2 + 1)*sin(x)^2 - 4*cos(x)^2 + 1)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \left(\frac{1}{\sin(x)} - \sin(x)\right)^{7/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/sin(x) - sin(x))^(7/2),x)
```

```
[Out] int((1/sin(x) - sin(x))^(7/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((csc(x)-sin(x))**(7/2),x)
```

```
[Out] Timed out
```

3.315 $\int (\csc(x)-\sin(x))^{5/2}\,dx$

Optimal. Leaf size=50

$$\frac{2}{5}\cos^2(x)\cot(x)\sqrt{\cos(x)\cot(x)}-\frac{16}{15}\cot(x)\sqrt{\cos(x)\cot(x)}-\frac{64}{15}\tan(x)\sqrt{\cos(x)\cot(x)}$$

```
[Out] -16/15*cot(x)*(cos(x)*cot(x))^(1/2)+2/5*cos(x)^2*cot(x)*(cos(x)*cot(x))^(1/
2)-64/15*(cos(x)*cot(x))^(1/2)*tan(x)
```

Rubi [A] time = 0.11, antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.454, Rules used = {4397, 4400, 2598, 2594, 2589}

$$\frac{2}{5}\cos^2(x)\cot(x)\sqrt{\cos(x)\cot(x)}-\frac{16}{15}\cot(x)\sqrt{\cos(x)\cot(x)}-\frac{64}{15}\tan(x)\sqrt{\cos(x)\cot(x)}$$

Antiderivative was successfully verified.

```
[In] Int[(Csc[x] - Sin[x])^(5/2),x]
```

```
[Out] (-16*Cot[x]*Sqrt[Cos[x]*Cot[x]])/15 + (2*Cos[x]^2*Cot[x]*Sqrt[Cos[x]*Cot[x]
])/5 - (64*Sqrt[Cos[x]*Cot[x]]*Tan[x])/15
```

Rule 2589

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> -Simp[(b*(a*Sin[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*m
), x] /; FreeQ[{a, b, e, f, m, n}, x] && EqQ[m + n - 1, 0]
```

Rule 2594

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(b*(a*Sin[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*(
n - 1)), x] - Dist[(b^2*(m + n - 1))/(n - 1), Int[(a*Sin[e + f*x])^m*(b*Tan
[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] && Int
egersQ[2*m, 2*n] &&  !(GtQ[m, 1] &&  !IntegerQ[(m - 1)/2])
```

Rule 2598

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> -Simp[(b*(a*Sin[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f
*m), x] + Dist[(a^2*(m + n - 1))/m, Int[(a*Sin[e + f*x])^(m - 2)*(b*Tan[e +
 f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && (GtQ[m, 1] || (EqQ[m, 1] &
& EqQ[n, 1/2])) && IntegersQ[2*m, 2*n]
```

Rule 4397

```
Int[u_, x_Symbol] :> Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]
```

Rule 4400

```
Int[(u_.)*((v_)^(m_.)*(w_)^(n_.))^(p_), x_Symbol] :> With[{uu = ActivateTri
g[u], vv = ActivateTrig[v], ww = ActivateTrig[w]}, Dist[(vv^m*ww^n)^FracPar
t[p]/(vv^(m*FracPart[p])*ww^(n*FracPart[p])), Int[uu*vv^(m*p)*ww^(n*p), x],
 x]] /; FreeQ[{m, n, p}, x] &&  !IntegerQ[p] && ( !InertTrigFreeQ[v] ||  !I
nertTrigFreeQ[w])
```

Rubi steps

$$\int (\csc(x)-\sin(x))^{5/2}\,dx = \int (\cos(x)\cot(x))^{5/2}\,dx$$
$$= \frac{\sqrt{\cos(x)\cot(x)}\int \cos^{\frac{5}{2}}(x)\cot^{\frac{5}{2}}(x)\,dx}{\sqrt{\cos(x)}\sqrt{\cot(x)}}$$
$$= \frac{2}{5}\cos^2(x)\cot(x)\sqrt{\cos(x)\cot(x)} + \frac{\left(8\sqrt{\cos(x)\cot(x)}\right)\int \sqrt{\cos(x)}\cot^{\frac{5}{2}}(x)\,dx}{5\sqrt{\cos(x)}\sqrt{\cot(x)}}$$
$$= -\frac{16}{15}\cot(x)\sqrt{\cos(x)\cot(x)} + \frac{2}{5}\cos^2(x)\cot(x)\sqrt{\cos(x)\cot(x)} - \frac{\left(32\sqrt{\cos(x)\cot(x)}\right)\int}{15\sqrt{\cos(x)}}$$
$$= -\frac{16}{15}\cot(x)\sqrt{\cos(x)\cot(x)} + \frac{2}{5}\cos^2(x)\cot(x)\sqrt{\cos(x)\cot(x)} - \frac{64}{15}\sqrt{\cos(x)\cot(x)}\tan$$

Mathematica [A] time = 0.08, size = 29, normalized size = 0.58

$$-\frac{2}{15}\tan(x)\sqrt{\cos(x)\cot(x)}\left(3\cos^2(x)+5\cot^2(x)+32\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(Csc[x] - Sin[x])^(5/2),x]
[Out] (-2*Sqrt[Cos[x]*Cot[x]]*(32 + 3*Cos[x]^2 + 5*Cot[x]^2)*Tan[x])/15
```

fricas [A] time = 1.72, size = 35, normalized size = 0.70

$$\frac{2\left(3\cos(x)^4+24\cos(x)^2-32\right)\sqrt{\frac{\cos(x)^2}{\sin(x)}}}{15\cos(x)\sin(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((csc(x)-sin(x))^(5/2),x, algorithm="fricas")
```

```
[Out] 2/15*(3*cos(x)^4 + 24*cos(x)^2 - 32)*sqrt(cos(x)^2/sin(x))/(cos(x)*sin(x))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (\csc(x) - \sin(x))^{\frac{5}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((csc(x)-sin(x))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((csc(x) - sin(x))^(5/2), x)
```

maple [A] time = 0.28, size = 34, normalized size = 0.68

$$\frac{2\left(3\left(\cos^{4}(x)\right)+24\left(\cos^{2}(x)\right)-32\right)\left(\frac{\cos^{2}(x)}{\sin(x)}\right)^{\frac{5}{2}}\sin(x)}{15\cos(x)^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((csc(x)-sin(x))^(5/2),x)
```

```
[Out] 2/15*(3*cos(x)^4+24*cos(x)^2-32)*(cos(x)^2/sin(x))^(5/2)*sin(x)/cos(x)^5
```

maxima [B] time = 0.56, size = 427, normalized size = 8.54

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((csc(x)-sin(x))^(5/2),x, algorithm="maxima")
```

```
[Out] -1/60*(((3*cos(15/2*x) + 105*cos(11/2*x) - 410*cos(7/2*x) - 3*cos(5/2*x) +
410*cos(3/2*x) - 105*cos(1/2*x) + 3*sin(15/2*x) + 105*sin(11/2*x) - 410*sin
(7/2*x) + 3*sin(5/2*x) + 410*sin(3/2*x) + 105*sin(1/2*x))*cos(5/2*arctan2(s
in(x), cos(x) - 1)) - (3*cos(15/2*x) + 105*cos(11/2*x) - 410*cos(7/2*x) - 3
*cos(5/2*x) + 410*cos(3/2*x) - 105*cos(1/2*x) - 3*sin(15/2*x) - 105*sin(11/
2*x) + 410*sin(7/2*x) - 3*sin(5/2*x) - 410*sin(3/2*x) - 105*sin(1/2*x))*sin
(5/2*arctan2(sin(x), cos(x) - 1)))*cos(5/2*arctan2(sin(x), cos(x) + 1)) - (
(3*cos(15/2*x) + 105*cos(11/2*x) - 410*cos(7/2*x) - 3*cos(5/2*x) + 410*cos(
3/2*x) - 105*cos(1/2*x) - 3*sin(15/2*x) - 105*sin(11/2*x) + 410*sin(7/2*x)
- 3*sin(5/2*x) - 410*sin(3/2*x) - 105*sin(1/2*x))*cos(5/2*arctan2(sin(x), c
os(x) - 1)) + (3*cos(15/2*x) + 105*cos(11/2*x) - 410*cos(7/2*x) - 3*cos(5/2
```

```
*x) + 410*cos(3/2*x) - 105*cos(1/2*x) + 3*sin(15/2*x) + 105*sin(11/2*x) - 4
10*sin(7/2*x) + 3*sin(5/2*x) + 410*sin(3/2*x) + 105*sin(1/2*x))*sin(5/2*arc
tan2(sin(x), cos(x) - 1)))*sin(5/2*arctan2(sin(x), cos(x) + 1)))/((cos(x)^4
 + sin(x)^4 + 2*(cos(x)^2 + 1)*sin(x)^2 - 2*cos(x)^2 + 1)*(cos(x)^2 + sin(x
)^2 + 2*cos(x) + 1)^(1/4)*(cos(x)^2 + sin(x)^2 - 2*cos(x) + 1)^(1/4))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \left(\frac{1}{\sin(x)} - \sin(x)\right)^{5/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/sin(x) - sin(x))^(5/2),x)
```

```
[Out] int((1/sin(x) - sin(x))^(5/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((csc(x)-sin(x))**(5/2),x)
```

```
[Out] Timed out
```

3.316 $\int (\csc(x)-\sin(x))^{3/2}\,dx$

Optimal. Leaf size=31

$$\frac{2}{3}\cos(x)\sqrt{\cos(x)\cot(x)}-\frac{8}{3}\sec(x)\sqrt{\cos(x)\cot(x)}$$

```
[Out] 2/3*cos(x)*(cos(x)*cot(x))^(1/2)-8/3*sec(x)*(cos(x)*cot(x))^(1/2)
```

Rubi [A] time = 0.08, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {4397, 4400, 2598, 2589}

$$\frac{2}{3}\cos(x)\sqrt{\cos(x)\cot(x)}-\frac{8}{3}\sec(x)\sqrt{\cos(x)\cot(x)}$$

Antiderivative was successfully verified.

```
[In] Int[(Csc[x] - Sin[x])^(3/2),x]
```

```
[Out] (2*Cos[x]*Sqrt[Cos[x]*Cot[x]])/3 - (8*Sqrt[Cos[x]*Cot[x]]*Sec[x])/3
```

Rule 2589

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> -Simp[(b*(a*Sin[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*m
), x] /; FreeQ[{a, b, e, f, m, n}, x] && EqQ[m + n - 1, 0]
```

Rule 2598

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> -Simp[(b*(a*Sin[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f
*m), x] + Dist[(a^2*(m + n - 1))/m, Int[(a*Sin[e + f*x])^(m - 2)*(b*Tan[e +
 f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && (GtQ[m, 1] || (EqQ[m, 1] &
& EqQ[n, 1/2])) && IntegersQ[2*m, 2*n]
```

Rule 4397

```
Int[u_, x_Symbol] :> Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]
```

Rule 4400

```
Int[(u_.)*((v_)^(m_.)*(w_)^(n_.))^(p_), x_Symbol] :> With[{uu = ActivateTri
g[u], vv = ActivateTrig[v], ww = ActivateTrig[w]}, Dist[(vv^m*ww^n)^FracPar
t[p]/(vv^(m*FracPart[p])*ww^(n*FracPart[p])), Int[uu*vv^(m*p)*ww^(n*p), x],
 x]] /; FreeQ[{m, n, p}, x] &&  !IntegerQ[p] && ( !InertTrigFreeQ[v] ||  !I
```

```
nertTrigFreeQ[w])
```

Rubi steps

$$\int (\csc(x)-\sin(x))^{3/2}\,dx = \int (\cos(x)\cot(x))^{3/2}\,dx$$
$$= \frac{\sqrt{\cos(x)\cot(x)}\int \cos^{\frac{3}{2}}(x)\cot^{\frac{3}{2}}(x)\,dx}{\sqrt{\cos(x)}\sqrt{\cot(x)}}$$
$$= \frac{2}{3}\cos(x)\sqrt{\cos(x)\cot(x)} + \frac{\left(4\sqrt{\cos(x)\cot(x)}\right)\int \frac{\cot^{\frac{3}{2}}(x)}{\sqrt{\cos(x)}}\,dx}{3\sqrt{\cos(x)}\sqrt{\cot(x)}}$$
$$= \frac{2}{3}\cos(x)\sqrt{\cos(x)\cot(x)} - \frac{8}{3}\sqrt{\cos(x)\cot(x)}\sec(x)$$

Mathematica [A] time = 0.04, size = 21, normalized size = 0.68

$$\frac{2}{3}\left(\cos^2(x)-4\right)\sec(x)\sqrt{\cos(x)\cot(x)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Csc[x] - Sin[x])^(3/2),x]
[Out] (2*(-4 + Cos[x]^2)*Sqrt[Cos[x]*Cot[x]]*Sec[x])/3
```

fricas [A] time = 0.91, size = 23, normalized size = 0.74

$$\frac{2\left(\cos(x)^2-4\right)\sqrt{\frac{\cos(x)^2}{\sin(x)}}}{3\cos(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((csc(x)-sin(x))^(3/2),x, algorithm="fricas")
[Out] 2/3*(cos(x)^2 - 4)*sqrt(cos(x)^2/sin(x))/cos(x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (\csc(x)-\sin(x))^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((csc(x)-sin(x))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((csc(x) - sin(x))^(3/2), x)
```

maple [A] time = 0.24, size = 26, normalized size = 0.84

$$\frac{2\left(\cos^{2}(x)-4\right)\left(\frac{\cos^{2}(x)}{\sin(x)}\right)^{\frac{3}{2}}\sin(x)}{3\cos(x)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((csc(x)-sin(x))^(3/2),x)
```

```
[Out] 2/3*(cos(x)^2-4)*(cos(x)^2/sin(x))^(3/2)*sin(x)/cos(x)^3
```

maxima [B] time = 0.52, size = 314, normalized size = 10.13

$$\left(\cos(x)^{2}+\sin(x)^{2}+2\cos(x)+1\right)^{\frac{1}{4}}\left(\cos(x)^{2}+\sin(x)^{2}-2\cos(x)+1\right)^{\frac{1}{4}}\left(\left(\left(\cos\left(\frac{9}{2}x\right)-15\cos\left(\frac{5}{2}x\right)-\cos\left(\frac{3}{2}x\right)\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((csc(x)-sin(x))^(3/2),x, algorithm="maxima")
```

```
[Out] 1/6*(cos(x)^2 + sin(x)^2 + 2*cos(x) + 1)^(1/4)*(cos(x)^2 + sin(x)^2 - 2*cos
(x) + 1)^(1/4)*(((cos(9/2*x) - 15*cos(5/2*x) - cos(3/2*x) + 15*cos(1/2*x) -
 sin(9/2*x) + 15*sin(5/2*x) - sin(3/2*x) - 15*sin(1/2*x))*cos(3/2*arctan2(s
in(x), cos(x) - 1)) + (cos(9/2*x) - 15*cos(5/2*x) - cos(3/2*x) + 15*cos(1/2
*x) + sin(9/2*x) - 15*sin(5/2*x) + sin(3/2*x) + 15*sin(1/2*x))*sin(3/2*arct
an2(sin(x), cos(x) - 1)))*cos(3/2*arctan2(sin(x), cos(x) + 1)) + ((cos(9/2*
x) - 15*cos(5/2*x) - cos(3/2*x) + 15*cos(1/2*x) + sin(9/2*x) - 15*sin(5/2*x
) + sin(3/2*x) + 15*sin(1/2*x))*cos(3/2*arctan2(sin(x), cos(x) - 1)) - (cos
(9/2*x) - 15*cos(5/2*x) - cos(3/2*x) + 15*cos(1/2*x) - sin(9/2*x) + 15*sin(
5/2*x) - sin(3/2*x) - 15*sin(1/2*x))*sin(3/2*arctan2(sin(x), cos(x) - 1)))*
sin(3/2*arctan2(sin(x), cos(x) + 1)))/(cos(x)^4 + sin(x)^4 + 2*(cos(x)^2 +
1)*sin(x)^2 - 2*cos(x)^2 + 1)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int\left(\frac{1}{\sin(x)}-\sin(x)\right)^{3/2}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/sin(x) - sin(x))^(3/2),x)
```

```
[Out] int((1/sin(x) - sin(x))^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (-\sin(x) + \csc(x))^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((csc(x)-sin(x))**(3/2),x)
```

```
[Out] Integral((-sin(x) + csc(x))**(3/2), x)
```

3.317 $\int \sqrt{\csc(x)-\sin(x)}\,dx$

Optimal. Leaf size=13

$$2\tan(x)\sqrt{\cos(x)\cot(x)}$$

[Out] 2*(cos(x)*cot(x))^(1/2)*tan(x)

Rubi [A] time = 0.05, antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {4397, 4400, 2589}

$$2\tan(x)\sqrt{\cos(x)\cot(x)}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[Csc[x] - Sin[x]],x]

[Out] 2*Sqrt[Cos[x]*Cot[x]]*Tan[x]

Rule 2589

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> -Simp[(b*(a*Sin[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*m
), x] /; FreeQ[{a, b, e, f, m, n}, x] && EqQ[m + n - 1, 0]
```

Rule 4397

```
Int[u_, x_Symbol] :> Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]
```

Rule 4400

```
Int[(u_.)*((v_)^(m_.)*(w_)^(n_.))^(p_), x_Symbol] :> With[{uu = ActivateTri
g[u], vv = ActivateTrig[v], ww = ActivateTrig[w]}, Dist[(vv^m*ww^n)^FracPar
t[p]/(vv^(m*FracPart[p])*ww^(n*FracPart[p])), Int[uu*vv^(m*p)*ww^(n*p), x],
 x]] /; FreeQ[{m, n, p}, x] && !IntegerQ[p] && ( !InertTrigFreeQ[v] ||  !I
nertTrigFreeQ[w])
```

Rubi steps

$$\int \sqrt{\csc(x)-\sin(x)}\,dx = \int \sqrt{\cos(x)\cot(x)}\,dx$$
$$= \frac{\sqrt{\cos(x)\cot(x)}\int \sqrt{\cos(x)}\,\sqrt{\cot(x)}\,dx}{\sqrt{\cos(x)}\,\sqrt{\cot(x)}}$$
$$= 2\sqrt{\cos(x)\cot(x)}\,\tan(x)$$

Mathematica [A] time = 0.02, size = 13, normalized size = 1.00

$$2\tan(x)\sqrt{\cos(x)\cot(x)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Csc[x] - Sin[x]],x]
[Out] 2*Sqrt[Cos[x]*Cot[x]]*Tan[x]
```

fricas [A] time = 0.77, size = 19, normalized size = 1.46

$$\frac{2\sqrt{\frac{\cos(x)^2}{\sin(x)}}\,\sin(x)}{\cos(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((csc(x)-sin(x))^(1/2),x, algorithm="fricas")
[Out] 2*sqrt(cos(x)^2/sin(x))*sin(x)/cos(x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{\csc(x)-\sin(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((csc(x)-sin(x))^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(csc(x) - sin(x)), x)
```

maple [A] time = 0.23, size = 20, normalized size = 1.54

$$\frac{2\sin(x)\sqrt{\frac{\cos^2(x)}{\sin(x)}}}{\cos(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((csc(x)-sin(x))^(1/2),x)
```

```
[Out] 2*sin(x)*(cos(x)^2/sin(x))^(1/2)/cos(x)
```

maxima [B] time = 0.51, size = 188, normalized size = 14.46

$$\frac{\left(\left(\cos\left(\frac{3}{2}\,x\right) - \cos\left(\frac{1}{2}\,x\right) + \sin\left(\frac{3}{2}\,x\right) + \sin\left(\frac{1}{2}\,x\right)\right)\cos\left(\frac{1}{2}\,\arctan\left(\sin(x), \cos(x) - 1\right)\right) - \left(\cos\left(\frac{3}{2}\,x\right) - \cos\left(\frac{1}{2}\,x\right) - \mathrm{si}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((csc(x)-sin(x))^(1/2),x, algorithm="maxima")
```

```
[Out] (((cos(3/2*x) - cos(1/2*x) + sin(3/2*x) + sin(1/2*x))*cos(1/2*arctan2(sin(x
), cos(x) - 1)) - (cos(3/2*x) - cos(1/2*x) - sin(3/2*x) - sin(1/2*x))*sin(1
/2*arctan2(sin(x), cos(x) - 1)))*cos(1/2*arctan2(sin(x), cos(x) + 1)) - ((c
os(3/2*x) - cos(1/2*x) - sin(3/2*x) - sin(1/2*x))*cos(1/2*arctan2(sin(x), c
os(x) - 1)) + (cos(3/2*x) - cos(1/2*x) + sin(3/2*x) + sin(1/2*x))*sin(1/2*a
rctan2(sin(x), cos(x) - 1)))*sin(1/2*arctan2(sin(x), cos(x) + 1)))/((cos(x)
^2 + sin(x)^2 + 2*cos(x) + 1)^(1/4)*(cos(x)^2 + sin(x)^2 - 2*cos(x) + 1)^(1
/4))
```

mupad [B] time = 2.48, size = 15, normalized size = 1.15

$$\frac{2\,\left|\cos(x)\right|}{\cos(x)\,\sqrt{\frac{1}{\sin(x)}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/sin(x) - sin(x))^(1/2),x)
```

```
[Out] (2*abs(cos(x)))/(cos(x)*(1/sin(x))^(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{-\sin\left(x\right) + \csc\left(x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((csc(x)-sin(x))**(1/2),x)
```

```
[Out] Integral(sqrt(-sin(x) + csc(x)), x)
```

3.318 $\int \frac{1}{\sqrt{\csc(x)-\sin(x)}}\,dx$

Optimal. Leaf size=60

$$\frac{\cos(x)\tan^{-1}\left(\sqrt{-\sin(x)}\right)}{\sqrt{-\sin(x)}\sqrt{\cos(x)\cot(x)}}-\frac{\cos(x)\tanh^{-1}\left(\sqrt{-\sin(x)}\right)}{\sqrt{-\sin(x)}\sqrt{\cos(x)\cot(x)}}$$

```
[Out] arctan((-sin(x))^(1/2))*cos(x)/(cos(x)*cot(x))^(1/2)/(-sin(x))^(1/2)-arctan
h((-sin(x))^(1/2))*cos(x)/(cos(x)*cot(x))^(1/2)/(-sin(x))^(1/2)
```

Rubi [A] time = 0.09, antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.727, Rules used = {4397, 4400, 2601, 2564, 329, 298, 203, 206}

$$\frac{\cos(x)\tan^{-1}\left(\sqrt{-\sin(x)}\right)}{\sqrt{-\sin(x)}\sqrt{\cos(x)\cot(x)}}-\frac{\cos(x)\tanh^{-1}\left(\sqrt{-\sin(x)}\right)}{\sqrt{-\sin(x)}\sqrt{\cos(x)\cot(x)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[Csc[x] - Sin[x]],x]
```

```
[Out] (ArcTan[Sqrt[-Sin[x]]]*Cos[x])/(Sqrt[Cos[x]*Cot[x]]*Sqrt[-Sin[x]]) - (ArcTa
nh[Sqrt[-Sin[x]]]*Cos[x])/(Sqrt[Cos[x]*Cot[x]]*Sqrt[-Sin[x]])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2564

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 2601

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Dist[(Cos[e + f*x]^n*(b*Tan[e + f*x])^n)/(a*Sin[e + f*x])
^n, Int[(a*Sin[e + f*x])^(m + n)/Cos[e + f*x]^n, x], x] /; FreeQ[{a, b, e,
f, m, n}, x] &&  !IntegerQ[n] && (ILtQ[m, 0] || (EqQ[m, 1] && EqQ[n, -2^(-1
)]) || IntegersQ[m - 1/2, n - 1/2])
```

Rule 4397

```
Int[u_, x_Symbol] :> Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]
```

Rule 4400

```
Int[(u_.)*((v_)^(m_.)*(w_)^(n_.))^(p_), x_Symbol] :> With[{uu = ActivateTri
g[u], vv = ActivateTrig[v], ww = ActivateTrig[w]}, Dist[(vv^m*ww^n)^FracPar
t[p]/(vv^(m*FracPart[p])*ww^(n*FracPart[p])), Int[uu*vv^(m*p)*ww^(n*p), x],
 x]] /; FreeQ[{m, n, p}, x] &&  !IntegerQ[p] && ( !InertTrigFreeQ[v] ||  !I
nertTrigFreeQ[w])
```

Rubi steps

$$\int \frac{1}{\sqrt{\csc(x)-\sin(x)}}\,dx = \int \frac{1}{\sqrt{\cos(x)\cot(x)}}\,dx$$

$$= \frac{\left(\sqrt{\cos(x)}\sqrt{\cot(x)}\right)\int \frac{1}{\sqrt{\cos(x)}\sqrt{\cot(x)}}\,dx}{\sqrt{\cos(x)\cot(x)}}$$

$$= \frac{\cos(x)\int \sec(x)\sqrt{-\sin(x)}\,dx}{\sqrt{\cos(x)\cot(x)}\sqrt{-\sin(x)}}$$

$$= -\frac{\cos(x)\,\mathrm{Subst}\left(\int \frac{\sqrt{x}}{1-x^2}\,dx, x, -\sin(x)\right)}{\sqrt{\cos(x)\cot(x)}\sqrt{-\sin(x)}}$$

$$= -\frac{(2\cos(x))\,\mathrm{Subst}\left(\int \frac{x^2}{1-x^4}\,dx, x, \sqrt{-\sin(x)}\right)}{\sqrt{\cos(x)\cot(x)}\sqrt{-\sin(x)}}$$

$$= -\frac{\cos(x)\,\mathrm{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \sqrt{-\sin(x)}\right)}{\sqrt{\cos(x)\cot(x)}\sqrt{-\sin(x)}} + \frac{\cos(x)\,\mathrm{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \sqrt{-\sin(x)}\right)}{\sqrt{\cos(x)\cot(x)}\sqrt{-\sin(x)}}$$

$$= \frac{\tan^{-1}\left(\sqrt{-\sin(x)}\right)\cos(x)}{\sqrt{\cos(x)\cot(x)}\sqrt{-\sin(x)}} - \frac{\tanh^{-1}\left(\sqrt{-\sin(x)}\right)\cos(x)}{\sqrt{\cos(x)\cot(x)}\sqrt{-\sin(x)}}$$

Mathematica [A] time = 0.27, size = 44, normalized size = 0.73

$$-\frac{\sin(x)\tan(x)\sqrt{\cos(x)\cot(x)}\left(\tan^{-1}\left(\sqrt[4]{\sin^2(x)}\right)-\tanh^{-1}\left(\sqrt[4]{\sin^2(x)}\right)\right)}{\sin^2(x)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[Csc[x] - Sin[x]],x]
```

```
[Out] -(((ArcTan[(Sin[x]^2)^(1/4)] - ArcTanh[(Sin[x]^2)^(1/4)])*Sqrt[Cos[x]*Cot[x
]]*Sin[x]*Tan[x])/(Sin[x]^2)^(3/4))
```

fricas [B] time = 1.06, size = 124, normalized size = 2.07

$$\frac{1}{2}\arctan\left(\frac{2\sqrt{\frac{\cos(x)^2}{\sin(x)}}\sin(x)}{\cos(x)\sin(x)-\cos(x)}\right)+\frac{1}{4}\log\left(\frac{\cos(x)^3-5\cos(x)^2-\left(\cos(x)^2+6\cos(x)+4\right)\sin(x)+4\left(\cos(x)^2-\right.}{\cos(x)^3+3\cos(x)^2-\left(\cos(x)^2-2\cos(x)-\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(csc(x)-sin(x))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*arctan(2*sqrt(cos(x)^2/sin(x))*sin(x)/(cos(x)*sin(x) - cos(x))) + 1/4*l
og((cos(x)^3 - 5*cos(x)^2 - (cos(x)^2 + 6*cos(x) + 4)*sin(x) + 4*(cos(x)^2
- (cos(x) + 1)*sin(x) - 1)*sqrt(cos(x)^2/sin(x)) - 2*cos(x) + 4)/(cos(x)^3
+ 3*cos(x)^2 - (cos(x)^2 - 2*cos(x) - 4)*sin(x) - 2*cos(x) - 4))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{\csc(x) - \sin(x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(csc(x)-sin(x))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(csc(x) - sin(x)), x)
```

maple [F] time = 0.26, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{\csc(x) - \sin(x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(csc(x)-sin(x))^(1/2),x)
```

```
[Out] int(1/(csc(x)-sin(x))^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{\csc(x) - \sin(x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(csc(x)-sin(x))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(csc(x) - sin(x)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{\sqrt{\frac{1}{\sin(x)} - \sin(x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1/sin(x) - sin(x))^(1/2),x)
```

```
[Out] int(1/(1/sin(x) - sin(x))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-\sin(x) + \csc(x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(csc(x)-sin(x))**(1/2),x)
```

```
[Out] Integral(1/sqrt(-sin(x) + csc(x)), x)
```

3.319 $\int \frac{1}{(\csc(x)-\sin(x))^{3/2}}\,dx$

Optimal. Leaf size=80

$$\frac{\sec(x)}{2\sqrt{\cos(x)\cot(x)}}+\frac{\sqrt{-\sin(x)}\,\cot(x)\tan^{-1}\left(\sqrt{-\sin(x)}\right)}{4\sqrt{\cos(x)\cot(x)}}+\frac{\sqrt{-\sin(x)}\,\cot(x)\tanh^{-1}\left(\sqrt{-\sin(x)}\right)}{4\sqrt{\cos(x)\cot(x)}}$$

```
[Out] 1/2*sec(x)/(cos(x)*cot(x))^(1/2)+1/4*arctan((-sin(x))^(1/2))*cot(x)*(-sin(x
))^(1/2)/(cos(x)*cot(x))^(1/2)+1/4*arctanh((-sin(x))^(1/2))*cot(x)*(-sin(x)
)^(1/2)/(cos(x)*cot(x))^(1/2)
```

Rubi [A] time = 0.12, antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.818, Rules used = {4397, 4400, 2597, 2601, 2564, 329, 212, 206, 203}

$$\frac{\sec(x)}{2\sqrt{\cos(x)\cot(x)}}+\frac{\sqrt{-\sin(x)}\,\cot(x)\tan^{-1}\left(\sqrt{-\sin(x)}\right)}{4\sqrt{\cos(x)\cot(x)}}+\frac{\sqrt{-\sin(x)}\,\cot(x)\tanh^{-1}\left(\sqrt{-\sin(x)}\right)}{4\sqrt{\cos(x)\cot(x)}}$$

Antiderivative was successfully verified.

```
[In] Int[(Csc[x] - Sin[x])^(-3/2),x]
```

```
[Out] Sec[x]/(2*Sqrt[Cos[x]*Cot[x]]) + (ArcTan[Sqrt[-Sin[x]]]*Cot[x]*Sqrt[-Sin[x]
])/(4*Sqrt[Cos[x]*Cot[x]]) + (ArcTanh[Sqrt[-Sin[x]]]*Cot[x]*Sqrt[-Sin[x]])/
(4*Sqrt[Cos[x]*Cot[x]])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2564

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 2597

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[((a*Sin[e + f*x])^m*(b*Tan[e + f*x])^(n + 1))/(b*f*(
m + n + 1)), x] - Dist[(n + 1)/(b^2*(m + n + 1)), Int[(a*Sin[e + f*x])^m*(b
*Tan[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && LtQ[n, -1] &
& NeQ[m + n + 1, 0] && IntegersQ[2*m, 2*n] &&  !(EqQ[n, -3/2] && EqQ[m, 1])
```

Rule 2601

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Dist[(Cos[e + f*x]^n*(b*Tan[e + f*x])^n)/(a*Sin[e + f*x])
^n, Int[(a*Sin[e + f*x])^(m + n)/Cos[e + f*x]^n, x], x] /; FreeQ[{a, b, e,
f, m, n}, x] &&  !IntegerQ[n] && (ILtQ[m, 0] || (EqQ[m, 1] && EqQ[n, -2^(-1
)]) || IntegersQ[m - 1/2, n - 1/2])
```

Rule 4397

```
Int[u_, x_Symbol] :> Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]
```

Rule 4400

```
Int[(u_.)*((v_)^(m_.)*(w_)^(n_.))^(p_), x_Symbol] :> With[{uu = ActivateTri
g[u], vv = ActivateTrig[v], ww = ActivateTrig[w]}, Dist[(vv^m*ww^n)^FracPar
t[p]/(vv^(m*FracPart[p])*ww^(n*FracPart[p])), Int[uu*vv^(m*p)*ww^(n*p), x],
 x]] /; FreeQ[{m, n, p}, x] &&  !IntegerQ[p] && ( !InertTrigFreeQ[v] ||  !I
nertTrigFreeQ[w])
```

Rubi steps

$$\int \frac{1}{(\csc(x)-\sin(x))^{3/2}}\,dx = \int \frac{1}{(\cos(x)\cot(x))^{3/2}}\,dx$$

$$= \frac{\left(\sqrt{\cos(x)}\sqrt{\cot(x)}\right)\int \frac{1}{\cos^{\frac{3}{2}}(x)\cot^{\frac{3}{2}}(x)}\,dx}{\sqrt{\cos(x)\cot(x)}}$$

$$= \frac{\sec(x)}{2\sqrt{\cos(x)\cot(x)}} - \frac{\left(\sqrt{\cos(x)}\sqrt{\cot(x)}\right)\int \frac{\sqrt{\cot(x)}}{\cos^{\frac{3}{2}}(x)}\,dx}{4\sqrt{\cos(x)\cot(x)}}$$

$$= \frac{\sec(x)}{2\sqrt{\cos(x)\cot(x)}} - \frac{\left(\cot(x)\sqrt{-\sin(x)}\right)\int \frac{\sec(x)}{\sqrt{-\sin(x)}}\,dx}{4\sqrt{\cos(x)\cot(x)}}$$

$$= \frac{\sec(x)}{2\sqrt{\cos(x)\cot(x)}} + \frac{\left(\cot(x)\sqrt{-\sin(x)}\right)\text{Subst}\left(\int \frac{1}{\sqrt{x}\,(1-x^2)}\,dx, x, -\sin(x)\right)}{4\sqrt{\cos(x)\cot(x)}}$$

$$= \frac{\sec(x)}{2\sqrt{\cos(x)\cot(x)}} + \frac{\left(\cot(x)\sqrt{-\sin(x)}\right)\text{Subst}\left(\int \frac{1}{1-x^4}\,dx, x, \sqrt{-\sin(x)}\right)}{2\sqrt{\cos(x)\cot(x)}}$$

$$= \frac{\sec(x)}{2\sqrt{\cos(x)\cot(x)}} + \frac{\left(\cot(x)\sqrt{-\sin(x)}\right)\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \sqrt{-\sin(x)}\right)}{4\sqrt{\cos(x)\cot(x)}} + \frac{(\cot(x)\sqrt{}}{}$$

$$= \frac{\sec(x)}{2\sqrt{\cos(x)\cot(x)}} + \frac{\tan^{-1}\left(\sqrt{-\sin(x)}\right)\cot(x)\sqrt{-\sin(x)}}{4\sqrt{\cos(x)\cot(x)}} + \frac{\tanh^{-1}\left(\sqrt{-\sin(x)}\right)\cot(}{4\sqrt{\cos(x)\cot(}}$$

<u>**Mathematica**</u> **[A]** time = 0.15, size = 60, normalized size = 0.75

$$\frac{2\sqrt[4]{\sin^2(x)}\,\sec(x)+\cos(x)\left(-\tan^{-1}\left(\sqrt[4]{\sin^2(x)}\right)\right)-\cos(x)\tanh^{-1}\left(\sqrt[4]{\sin^2(x)}\right)}{4\sqrt[4]{\sin^2(x)}\,\sqrt{\cos(x)\cot(x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Csc[x] - Sin[x])^(-3/2),x]
```

```
[Out] (-(ArcTan[(Sin[x]^2)^(1/4)]*Cos[x]) - ArcTanh[(Sin[x]^2)^(1/4)]*Cos[x] + 2*
Sec[x]*(Sin[x]^2)^(1/4))/(4*Sqrt[Cos[x]*Cot[x]]*(Sin[x]^2)^(1/4))
```

fricas [B] time = 1.80, size = 152, normalized size = 1.90

$$\frac{2\arctan\left(\frac{2\sqrt{\frac{\cos(x)^2}{\sin(x)}}\sin(x)}{\cos(x)\sin(x)-\cos(x)}\right)\cos(x)^3+\cos(x)^3\log\left(\frac{\cos(x)^3-5\cos(x)^2-\left(\cos(x)^2+6\cos(x)+4\right)\sin(x)-4\left(\cos(x)^2-(\cos(x)+1)\sin(x)-1\right)\cdots}{\cos(x)^3+3\cos(x)^2-\left(\cos(x)^2-2\cos(x)-4\right)\sin(x)-2\cos(x)-4\cdots}\right)\cdots}{16\cos(x)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(csc(x)-sin(x))^(3/2),x, algorithm="fricas")
```

```
[Out] 1/16*(2*arctan(2*sqrt(cos(x)^2/sin(x))*sin(x)/(cos(x)*sin(x) - cos(x)))*cos
(x)^3 + cos(x)^3*log((cos(x)^3 - 5*cos(x)^2 - (cos(x)^2 + 6*cos(x) + 4)*sin
(x) - 4*(cos(x)^2 - (cos(x) + 1)*sin(x) - 1)*sqrt(cos(x)^2/sin(x)) - 2*cos(
x) + 4)/(cos(x)^3 + 3*cos(x)^2 - (cos(x)^2 - 2*cos(x) - 4)*sin(x) - 2*cos(x
) - 4)) + 8*sqrt(cos(x)^2/sin(x))*sin(x))/cos(x)^3
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{(\csc(x)-\sin(x))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(csc(x)-sin(x))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((csc(x) - sin(x))^(-3/2), x)
```

maple [C] time = 0.55, size = 450, normalized size = 5.62

$$\frac{(-1+\cos(x))\left(i\sin(x)\left(\cos^2(x)\right)\mathrm{EllipticPi}\left(\sqrt{\frac{i\cos(x)+\sin(x)-i}{\sin(x)}},\frac{1}{2}-\frac{i}{2},\frac{\sqrt{2}}{2}\right)\sqrt{\frac{-i\cos(x)+\sin(x)+i}{\sin(x)}}\sqrt{\frac{i\cos(x)+\sin(x)-i}{\sin(x)}}\sqrt{-\frac{i(\cdots}{\cdots}}\cdots\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(csc(x)-sin(x))^(3/2),x)
```

```
[Out] 1/8*(-1+cos(x))*(I*sin(x)*cos(x)^2*EllipticPi(((I*cos(x)+sin(x)-I)/sin(x))^
(1/2),1/2-1/2*I,1/2*2^(1/2))*((-I*cos(x)+sin(x)+I)/sin(x))^(1/2)*((I*cos(x)
+sin(x)-I)/sin(x))^(1/2)*(-I*(-1+cos(x))/sin(x))^(1/2)+I*sin(x)*cos(x)^2*El
lipticPi(((I*cos(x)+sin(x)-I)/sin(x))^(1/2),1/2+1/2*I,1/2*2^(1/2))*((-I*cos
(x)+sin(x)+I)/sin(x))^(1/2)*((I*cos(x)+sin(x)-I)/sin(x))^(1/2)*(-I*(-1+cos(
x))/sin(x))^(1/2)-2*I*sin(x)*cos(x)^2*EllipticF(((I*cos(x)+sin(x)-I)/sin(x)
```

```
)^(1/2),1/2*2^(1/2))*((-I*cos(x)+sin(x)+I)/sin(x))^(1/2)*((I*cos(x)+sin(x)-
I)/sin(x))^(1/2)*(-I*(-1+cos(x))/sin(x))^(1/2)-sin(x)*cos(x)^2*EllipticPi((
(I*cos(x)+sin(x)-I)/sin(x))^(1/2),1/2-1/2*I,1/2*2^(1/2))*((-I*cos(x)+sin(x)
+I)/sin(x))^(1/2)*((I*cos(x)+sin(x)-I)/sin(x))^(1/2)*(-I*(-1+cos(x))/sin(x)
)^(1/2)+sin(x)*cos(x)^2*EllipticPi(((I*cos(x)+sin(x)-I)/sin(x))^(1/2),1/2+1
/2*I,1/2*2^(1/2))*((-I*cos(x)+sin(x)+I)/sin(x))^(1/2)*((I*cos(x)+sin(x)-I)/
sin(x))^(1/2)*(-I*(-1+cos(x))/sin(x))^(1/2)+2*cos(x)*2^(1/2)-2*2^(1/2))*cos
(x)*(1+cos(x))^2/(cos(x)^2/sin(x))^(3/2)/sin(x)^5*2^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(\csc(x) - \sin(x))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(csc(x)-sin(x))^(3/2),x, algorithm="maxima")
[Out] integrate((csc(x) - sin(x))^(-3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{{\left(\frac{1}{\sin(x)} - \sin(x)\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1/sin(x) - sin(x))^(3/2),x)
[Out] int(1/(1/sin(x) - sin(x))^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(-\sin{(x)} + \csc{(x)})^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(csc(x)-sin(x))**(3/2),x)
[Out] Integral((-sin(x) + csc(x))**(-3/2), x)
```

3.320 $\int \frac{1}{(\csc(x)-\sin(x))^{5/2}}\,dx$

Optimal. Leaf size=99

$$-\frac{3\tan(x)}{16\sqrt{\cos(x)\cot(x)}}-\frac{3\cos(x)\tan^{-1}\left(\sqrt{-\sin(x)}\right)}{32\sqrt{-\sin(x)}\,\sqrt{\cos(x)\cot(x)}}+\frac{3\cos(x)\tanh^{-1}\left(\sqrt{-\sin(x)}\right)}{32\sqrt{-\sin(x)}\,\sqrt{\cos(x)\cot(x)}}+\frac{\tan(x)\sec^2(x)}{4\sqrt{\cos(x)\cot(x)}}$$

```
[Out] -3/32*arctan((-sin(x))^(1/2))*cos(x)/(cos(x)*cot(x))^(1/2)/(-sin(x))^(1/2)+
3/32*arctanh((-sin(x))^(1/2))*cos(x)/(cos(x)*cot(x))^(1/2)/(-sin(x))^(1/2)-
3/16*tan(x)/(cos(x)*cot(x))^(1/2)+1/4*sec(x)^2*tan(x)/(cos(x)*cot(x))^(1/2)
```

Rubi [A] time = 0.15, antiderivative size = 99, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 10, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.909, Rules used = {4397, 4400, 2597, 2599, 2601, 2564, 329, 298, 203, 206}

$$-\frac{3\tan(x)}{16\sqrt{\cos(x)\cot(x)}}-\frac{3\cos(x)\tan^{-1}\left(\sqrt{-\sin(x)}\right)}{32\sqrt{-\sin(x)}\,\sqrt{\cos(x)\cot(x)}}+\frac{3\cos(x)\tanh^{-1}\left(\sqrt{-\sin(x)}\right)}{32\sqrt{-\sin(x)}\,\sqrt{\cos(x)\cot(x)}}+\frac{\tan(x)\sec^2(x)}{4\sqrt{\cos(x)\cot(x)}}$$

Antiderivative was successfully verified.

```
[In] Int[(Csc[x] - Sin[x])^(-5/2),x]
```

```
[Out] (-3*ArcTan[Sqrt[-Sin[x]]]*Cos[x])/(32*Sqrt[Cos[x]*Cot[x]]*Sqrt[-Sin[x]]) +
(3*ArcTanh[Sqrt[-Sin[x]]]*Cos[x])/(32*Sqrt[Cos[x]*Cot[x]]*Sqrt[-Sin[x]]) -
(3*Tan[x])/(16*Sqrt[Cos[x]*Cot[x]]) + (Sec[x]^2*Tan[x])/(4*Sqrt[Cos[x]*Cot[
x]])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
```

```
tQ[a/b, 0]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2564

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 2597

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[((a*Sin[e + f*x])^m*(b*Tan[e + f*x])^(n + 1))/(b*f*(
m + n + 1)), x] - Dist[(n + 1)/(b^2*(m + n + 1)), Int[(a*Sin[e + f*x])^m*(b
*Tan[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && LtQ[n, -1] &
& NeQ[m + n + 1, 0] && IntegersQ[2*m, 2*n] &&  !(EqQ[n, -3/2] && EqQ[m, 1])
```

Rule 2599

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_.), x_Symbol] :> Simp[(b*(a*Sin[e + f*x])^(m + 2)*(b*Tan[e + f*x])^(n - 1)
)/(a^2*f*(m + n + 1)), x] + Dist[(m + 2)/(a^2*(m + n + 1)), Int[(a*Sin[e +
f*x])^(m + 2)*(b*Tan[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && Lt
Q[m, -1] && NeQ[m + n + 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 2601

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Dist[(Cos[e + f*x]^n*(b*Tan[e + f*x])^n)/(a*Sin[e + f*x])
^n, Int[(a*Sin[e + f*x])^(m + n)/Cos[e + f*x]^n, x], x] /; FreeQ[{a, b, e,
f, m, n}, x] &&  !IntegerQ[n] && (ILtQ[m, 0] || (EqQ[m, 1] && EqQ[n, -2^(-1
)]) || IntegersQ[m - 1/2, n - 1/2])
```

Rule 4397

```
Int[u_, x_Symbol] :> Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]
```

Rule 4400

```
Int[(u_.)*((v_)^(m_.)*(w_)^(n_.))^(p_), x_Symbol] :> With[{uu = ActivateTri
g[u], vv = ActivateTrig[v], ww = ActivateTrig[w]}, Dist[(vv^m*ww^n)^FracPar
t[p]/(vv^(m*FracPart[p])*ww^(n*FracPart[p])), Int[uu*vv^(m*p)*ww^(n*p), x],
 x]] /; FreeQ[{m, n, p}, x] &&  !IntegerQ[p] && ( !InertTrigFreeQ[v] ||  !I
nertTrigFreeQ[w])
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{(\csc(x)-\sin(x))^{5/2}}\,dx &= \int \frac{1}{(\cos(x)\cot(x))^{5/2}}\,dx\\
&= \frac{\left(\sqrt{\cos(x)}\sqrt{\cot(x)}\right)\int \frac{1}{\cos^{\frac{5}{2}}(x)\cot^{\frac{5}{2}}(x)}\,dx}{\sqrt{\cos(x)\cot(x)}}\\
&= \frac{\sec^2(x)\tan(x)}{4\sqrt{\cos(x)\cot(x)}}-\frac{\left(3\sqrt{\cos(x)}\sqrt{\cot(x)}\right)\int \frac{1}{\cos^{\frac{5}{2}}(x)\sqrt{\cot(x)}}\,dx}{8\sqrt{\cos(x)\cot(x)}}\\
&= -\frac{3\tan(x)}{16\sqrt{\cos(x)\cot(x)}}+\frac{\sec^2(x)\tan(x)}{4\sqrt{\cos(x)\cot(x)}}-\frac{\left(3\sqrt{\cos(x)}\sqrt{\cot(x)}\right)\int \frac{1}{\sqrt{\cos(x)}\sqrt{\cot(x)}}\,dx}{32\sqrt{\cos(x)\cot(x)}}\\
&= -\frac{3\tan(x)}{16\sqrt{\cos(x)\cot(x)}}+\frac{\sec^2(x)\tan(x)}{4\sqrt{\cos(x)\cot(x)}}-\frac{(3\cos(x))\int \sec(x)\sqrt{-\sin(x)}\,dx}{32\sqrt{\cos(x)\cot(x)}\sqrt{-\sin(x)}}\\
&= -\frac{3\tan(x)}{16\sqrt{\cos(x)\cot(x)}}+\frac{\sec^2(x)\tan(x)}{4\sqrt{\cos(x)\cot(x)}}+\frac{(3\cos(x))\,\text{Subst}\left(\int \frac{\sqrt{x}}{1-x^2}\,dx,x,-\sin(x)\right)}{32\sqrt{\cos(x)\cot(x)}\sqrt{-\sin(x)}}\\
&= -\frac{3\tan(x)}{16\sqrt{\cos(x)\cot(x)}}+\frac{\sec^2(x)\tan(x)}{4\sqrt{\cos(x)\cot(x)}}+\frac{(3\cos(x))\,\text{Subst}\left(\int \frac{x^2}{1-x^4}\,dx,x,\sqrt{-\sin(x)}\right)}{16\sqrt{\cos(x)\cot(x)}\sqrt{-\sin(x)}}\\
&= -\frac{3\tan(x)}{16\sqrt{\cos(x)\cot(x)}}+\frac{\sec^2(x)\tan(x)}{4\sqrt{\cos(x)\cot(x)}}+\frac{(3\cos(x))\,\text{Subst}\left(\int \frac{1}{1-x^2}\,dx,x,\sqrt{-\sin(x)}\right)}{32\sqrt{\cos(x)\cot(x)}\sqrt{-\sin(x)}}\\
&= -\frac{3\tan^{-1}\left(\sqrt{-\sin(x)}\right)\cos(x)}{32\sqrt{\cos(x)\cot(x)}\sqrt{-\sin(x)}}+\frac{3\tanh^{-1}\left(\sqrt{-\sin(x)}\right)\cos(x)}{32\sqrt{\cos(x)\cot(x)}\sqrt{-\sin(x)}}-\frac{3\tan(x)}{16\sqrt{\cos(x)\cot(x)}}+
\end{aligned}$$

Mathematica [A] time = 0.51, size = 69, normalized size = 0.70

$$-\frac{\sin(x)\tan(x)\sqrt{\cos(x)\cot(x)}\left(-3\tan^{-1}\left(\sqrt[4]{\sin^2(x)}\right)+3\tanh^{-1}\left(\sqrt[4]{\sin^2(x)}\right)+\sin^2(x)^{3/4}(3\cos(2x)-5)\sec^4(x)\right)}{32\sin^2(x)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Csc[x] - Sin[x])^(-5/2),x]
```

```
[Out] -1/32*(Sqrt[Cos[x]*Cot[x]]*Sin[x]*(-3*ArcTan[(Sin[x]^2)^(1/4)] + 3*ArcTanh[
(Sin[x]^2)^(1/4)] + (-5 + 3*Cos[2*x])*Sec[x]^4*(Sin[x]^2)^(3/4))*Tan[x])/(S
in[x]^2)^(3/4)
```

fricas [B] time = 1.95, size = 165, normalized size = 1.67

$$-\frac{6\arctan\left(\frac{2\sqrt{\frac{\cos(x)^2}{\sin(x)}}\sin(x)}{\cos(x)\sin(x)-\cos(x)}\right)\cos(x)^5-3\cos(x)^5\log\left(\frac{\cos(x)^3-5\cos(x)^2-\left(\cos(x)^2+6\cos(x)+4\right)\sin(x)-4\left(\cos(x)^2-(\cos(x)+1)\sin\right.}{\cos(x)^3+3\cos(x)^2-\left(\cos(x)^2-2\cos(x)-4\right)\sin(x)-2\cos}\right.}{128\cos(x)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(csc(x)-sin(x))^(5/2),x, algorithm="fricas")
```

```
[Out] -1/128*(6*arctan(2*sqrt(cos(x)^2/sin(x))*sin(x)/(cos(x)*sin(x) - cos(x)))*c
os(x)^5 - 3*cos(x)^5*log((cos(x)^3 - 5*cos(x)^2 - (cos(x)^2 + 6*cos(x) + 4)
*sin(x) - 4*(cos(x)^2 - (cos(x) + 1)*sin(x) - 1)*sqrt(cos(x)^2/sin(x)) - 2*
cos(x) + 4)/(cos(x)^3 + 3*cos(x)^2 - (cos(x)^2 - 2*cos(x) - 4)*sin(x) - 2*c
os(x) - 4)) - 8*(3*cos(x)^4 - 7*cos(x)^2 + 4)*sqrt(cos(x)^2/sin(x)))/cos(x)
^5
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(\csc(x)-\sin(x))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(csc(x)-sin(x))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((csc(x) - sin(x))^(-5/2), x)
```

maple [C] time = 0.35, size = 382, normalized size = 3.86

$$-\frac{(-1+\cos(x))\left(3i\left(\cos^4(x)\right)\mathrm{EllipticPi}\left(\sqrt{\frac{i\cos(x)+\sin(x)-i}{\sin(x)}},\frac{1}{2}-\frac{i}{2},\frac{\sqrt{2}}{2}\right)\sqrt{\frac{-i\cos(x)+\sin(x)+i}{\sin(x)}}\sqrt{\frac{i\cos(x)+\sin(x)-i}{\sin(x)}}\sqrt{-\frac{i(-1-}{\mathrm{si}}}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(csc(x)-sin(x))^(5/2),x)
```

```
[Out] -1/64*(-1+cos(x))*(3*I*cos(x)^4*EllipticPi(((I*cos(x)+sin(x)-I)/sin(x))^(1/
2),1/2-1/2*I,1/2*2^(1/2))*((-I*cos(x)+sin(x)+I)/sin(x))^(1/2)*((I*cos(x)+si
n(x)-I)/sin(x))^(1/2)*(-I*(-1+cos(x))/sin(x))^(1/2)-3*I*cos(x)^4*EllipticPi
(((I*cos(x)+sin(x)-I)/sin(x))^(1/2),1/2+1/2*I,1/2*2^(1/2))*((-I*cos(x)+sin(
x)+I)/sin(x))^(1/2)*((I*cos(x)+sin(x)-I)/sin(x))^(1/2)*(-I*(-1+cos(x))/sin(
x))^(1/2)-3*cos(x)^4*EllipticPi(((I*cos(x)+sin(x)-I)/sin(x))^(1/2),1/2-1/2*
I,1/2*2^(1/2))*((-I*cos(x)+sin(x)+I)/sin(x))^(1/2)*((I*cos(x)+sin(x)-I)/sin
(x))^(1/2)*(-I*(-1+cos(x))/sin(x))^(1/2)-3*cos(x)^4*EllipticPi(((I*cos(x)+s
in(x)-I)/sin(x))^(1/2),1/2+1/2*I,1/2*2^(1/2))*((-I*cos(x)+sin(x)+I)/sin(x))
^(1/2)*((I*cos(x)+sin(x)-I)/sin(x))^(1/2)*(-I*(-1+cos(x))/sin(x))^(1/2)+6*c
os(x)^3*2^(1/2)-6*cos(x)^2*2^(1/2)-8*cos(x)*2^(1/2)+8*2^(1/2))*cos(x)*(1+co
s(x))^2/sin(x)^5/(cos(x)^2/sin(x))^(5/2)*2^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(\csc(x) - \sin(x))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(csc(x)-sin(x))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((csc(x) - sin(x))^(-5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\left(\frac{1}{\sin(x)} - \sin(x)\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1/sin(x) - sin(x))^(5/2),x)
```

```
[Out] int(1/(1/sin(x) - sin(x))^(5/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(-\sin(x) + \csc(x))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(csc(x)-sin(x))**(5/2),x)
```

```
[Out] Integral((-sin(x) + csc(x))**(-5/2), x)
```

3.321 $\int \frac{1}{(\csc(x)-\sin(x))^{7/2}} dx$

Optimal. Leaf size=118

$$-\frac{5\sec^3(x)}{48\sqrt{\cos(x)\cot(x)}}+\frac{5\sec(x)}{192\sqrt{\cos(x)\cot(x)}}-\frac{5\sqrt{-\sin(x)}\,\cot(x)\tan^{-1}\left(\sqrt{-\sin(x)}\right)}{128\sqrt{\cos(x)\cot(x)}}-\frac{5\sqrt{-\sin(x)}\,\cot(x)\tanh^{-1}\left(\sqrt{-\right.}}{128\sqrt{\cos(x)\cot(x)}}$$

```
[Out] 5/192*sec(x)/(cos(x)*cot(x))^(1/2)-5/48*sec(x)^3/(cos(x)*cot(x))^(1/2)-5/12
8*arctan((-sin(x))^(1/2))*cot(x)*(-sin(x))^(1/2)/(cos(x)*cot(x))^(1/2)-5/12
8*arctanh((-sin(x))^(1/2))*cot(x)*(-sin(x))^(1/2)/(cos(x)*cot(x))^(1/2)+1/6
*sec(x)^3*tan(x)^2/(cos(x)*cot(x))^(1/2)
```

Rubi [A] time = 0.18, antiderivative size = 118, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 10, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.909, Rules used = {4397, 4400, 2597, 2599, 2601, 2564, 329, 212, 206, 203}

$$-\frac{5\sec^3(x)}{48\sqrt{\cos(x)\cot(x)}}+\frac{5\sec(x)}{192\sqrt{\cos(x)\cot(x)}}-\frac{5\sqrt{-\sin(x)}\,\cot(x)\tan^{-1}\left(\sqrt{-\sin(x)}\right)}{128\sqrt{\cos(x)\cot(x)}}-\frac{5\sqrt{-\sin(x)}\,\cot(x)\tanh^{-1}\left(\sqrt{-\right.}}{128\sqrt{\cos(x)\cot(x)}}$$

Antiderivative was successfully verified.

```
[In] Int[(Csc[x] - Sin[x])^(-7/2),x]
```

```
[Out] (5*Sec[x])/(192*Sqrt[Cos[x]*Cot[x]]) - (5*Sec[x]^3)/(48*Sqrt[Cos[x]*Cot[x]]
) - (5*ArcTan[Sqrt[-Sin[x]]]*Cot[x]*Sqrt[-Sin[x]])/(128*Sqrt[Cos[x]*Cot[x]]
) - (5*ArcTanh[Sqrt[-Sin[x]]]*Cot[x]*Sqrt[-Sin[x]])/(128*Sqrt[Cos[x]*Cot[x]
]) + (Sec[x]^3*Tan[x]^2)/(6*Sqrt[Cos[x]*Cot[x]])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
```

```
x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] && !GtQ
[a/b, 0]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2564

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] && !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 2597

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[((a*Sin[e + f*x])^m*(b*Tan[e + f*x])^(n + 1))/(b*f*(
m + n + 1)), x] - Dist[(n + 1)/(b^2*(m + n + 1)), Int[(a*Sin[e + f*x])^m*(b
*Tan[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && LtQ[n, -1] &
& NeQ[m + n + 1, 0] && IntegersQ[2*m, 2*n] && !(EqQ[n, -3/2] && EqQ[m, 1])
```

Rule 2599

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_.), x_Symbol] :> Simp[(b*(a*Sin[e + f*x])^(m + 2)*(b*Tan[e + f*x])^(n - 1)
)/(a^2*f*(m + n + 1)), x] + Dist[(m + 2)/(a^2*(m + n + 1)), Int[(a*Sin[e +
f*x])^(m + 2)*(b*Tan[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && Lt
Q[m, -1] && NeQ[m + n + 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 2601

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Dist[(Cos[e + f*x]^n*(b*Tan[e + f*x])^n)/(a*Sin[e + f*x])
^n, Int[(a*Sin[e + f*x])^(m + n)/Cos[e + f*x]^n, x], x] /; FreeQ[{a, b, e,
f, m, n}, x] && !IntegerQ[n] && (ILtQ[m, 0] || (EqQ[m, 1] && EqQ[n, -2^(-1
)]) || IntegersQ[m - 1/2, n - 1/2])
```

Rule 4397

```
Int[u_, x_Symbol] :> Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]
```

Rule 4400

```
Int[(u_.)*((v_)^(m_.)*(w_)^(n_.))^(p_), x_Symbol] :> With[{uu = ActivateTri
g[u], vv = ActivateTrig[v], ww = ActivateTrig[w]}, Dist[(vv^m*ww^n)^FracPar
t[p]/(vv^(m*FracPart[p])*ww^(n*FracPart[p])), Int[uu*vv^(m*p)*ww^(n*p), x],
 x]] /; FreeQ[{m, n, p}, x] &&  !IntegerQ[p] && ( !InertTrigFreeQ[v] ||  !I
nertTrigFreeQ[w])
```

Rubi steps

$$\int \frac{1}{(\csc(x)-\sin(x))^{7/2}}\,dx = \int \frac{1}{(\cos(x)\cot(x))^{7/2}}\,dx$$

$$= \frac{\left(\sqrt{\cos(x)}\sqrt{\cot(x)}\right)\int \frac{1}{\cos^{\frac{7}{2}}(x)\cot^{\frac{7}{2}}(x)}\,dx}{\sqrt{\cos(x)\cot(x)}}$$

$$= \frac{\sec^3(x)\tan^2(x)}{6\sqrt{\cos(x)\cot(x)}} - \frac{\left(5\sqrt{\cos(x)}\sqrt{\cot(x)}\right)\int \frac{1}{\cos^{\frac{7}{2}}(x)\cot^{\frac{3}{2}}(x)}\,dx}{12\sqrt{\cos(x)\cot(x)}}$$

$$= -\frac{5\sec^3(x)}{48\sqrt{\cos(x)\cot(x)}} + \frac{\sec^3(x)\tan^2(x)}{6\sqrt{\cos(x)\cot(x)}} + \frac{\left(5\sqrt{\cos(x)}\sqrt{\cot(x)}\right)\int \frac{\sqrt{\cot(x)}}{\cos^{\frac{7}{2}}(x)}\,dx}{96\sqrt{\cos(x)\cot(x)}}$$

$$= \frac{5\sec(x)}{192\sqrt{\cos(x)\cot(x)}} - \frac{5\sec^3(x)}{48\sqrt{\cos(x)\cot(x)}} + \frac{\sec^3(x)\tan^2(x)}{6\sqrt{\cos(x)\cot(x)}} + \frac{\left(5\sqrt{\cos(x)}\sqrt{\cot(x)}\right.}{128\sqrt{\cos(x)}\,\mathrm{c}}$$

$$= \frac{5\sec(x)}{192\sqrt{\cos(x)\cot(x)}} - \frac{5\sec^3(x)}{48\sqrt{\cos(x)\cot(x)}} + \frac{\sec^3(x)\tan^2(x)}{6\sqrt{\cos(x)\cot(x)}} + \frac{\left(5\cot(x)\sqrt{-\sin(x)}\right.}{128\sqrt{\cos(x)}}$$

$$= \frac{5\sec(x)}{192\sqrt{\cos(x)\cot(x)}} - \frac{5\sec^3(x)}{48\sqrt{\cos(x)\cot(x)}} + \frac{\sec^3(x)\tan^2(x)}{6\sqrt{\cos(x)\cot(x)}} - \frac{\left(5\cot(x)\sqrt{-\sin(x)}\right)}{12}$$

$$= \frac{5\sec(x)}{192\sqrt{\cos(x)\cot(x)}} - \frac{5\sec^3(x)}{48\sqrt{\cos(x)\cot(x)}} + \frac{\sec^3(x)\tan^2(x)}{6\sqrt{\cos(x)\cot(x)}} - \frac{\left(5\cot(x)\sqrt{-\sin(x)}\right)}{64}$$

$$= \frac{5\sec(x)}{192\sqrt{\cos(x)\cot(x)}} - \frac{5\sec^3(x)}{48\sqrt{\cos(x)\cot(x)}} + \frac{\sec^3(x)\tan^2(x)}{6\sqrt{\cos(x)\cot(x)}} - \frac{\left(5\cot(x)\sqrt{-\sin(x)}\right)}{128}$$

$$= \frac{5\sec(x)}{192\sqrt{\cos(x)\cot(x)}} - \frac{5\sec^3(x)}{48\sqrt{\cos(x)\cot(x)}} - \frac{5\tan^{-1}\left(\sqrt{-\sin(x)}\right)\cot(x)\sqrt{-\sin(x)}}{128\sqrt{\cos(x)\cot(x)}} -$$

Mathematica [A] time = 0.26, size = 74, normalized size = 0.63

$$\frac{2\sqrt[4]{\sin^2(x)}\sec(x)\left(32\sec^4(x)-52\sec^2(x)+5\right)+15\cos(x)\tan^{-1}\left(\sqrt[4]{\sin^2(x)}\right)+15\cos(x)\tanh^{-1}\left(\sqrt[4]{\sin^2(x)}\right)}{384\sqrt[4]{\sin^2(x)}\sqrt{\cos(x)\cot(x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Csc[x] - Sin[x])^(-7/2),x]
```

```
[Out] (15*ArcTan[(Sin[x]^2)^(1/4)]*Cos[x] + 15*ArcTanh[(Sin[x]^2)^(1/4)]*Cos[x] +
 2*Sec[x]*(5 - 52*Sec[x]^2 + 32*Sec[x]^4)*(Sin[x]^2)^(1/4))/(384*Sqrt[Cos[x
]*Cot[x]]*(Sin[x]^2)^(1/4))
```

fricas [A] time = 1.12, size = 167, normalized size = 1.42

$$-\frac{30\arctan\left(\frac{2\sqrt{\frac{\cos(x)^2}{\sin(x)}}\sin(x)}{\cos(x)\sin(x)-\cos(x)}\right)\cos(x)^7-15\cos(x)^7\log\left(\frac{\cos(x)^3-5\cos(x)^2-\left(\cos(x)^2+6\cos(x)+4\right)\sin(x)+4\left(\cos(x)^2-(\cos(x)+1)\sin\right.}{\cos(x)^3+3\cos(x)^2-\left(\cos(x)^2-2\cos(x)-4\right)\sin(x)-2\cos}\right.}{1536\cos(x)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(csc(x)-sin(x))^(7/2),x, algorithm="fricas")
```

```
[Out] -1/1536*(30*arctan(2*sqrt(cos(x)^2/sin(x))*sin(x)/(cos(x)*sin(x) - cos(x)))
*cos(x)^7 - 15*cos(x)^7*log((cos(x)^3 - 5*cos(x)^2 - (cos(x)^2 + 6*cos(x) +
 4)*sin(x) + 4*(cos(x)^2 - (cos(x) + 1)*sin(x) - 1)*sqrt(cos(x)^2/sin(x)) -
 2*cos(x) + 4)/(cos(x)^3 + 3*cos(x)^2 - (cos(x)^2 - 2*cos(x) - 4)*sin(x) -
2*cos(x) - 4)) - 8*(5*cos(x)^4 - 52*cos(x)^2 + 32)*sqrt(cos(x)^2/sin(x))*si
n(x))/cos(x)^7
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{(\csc(x)-\sin(x))^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(csc(x)-sin(x))^(7/2),x, algorithm="giac")
```

```
[Out] integrate((csc(x) - sin(x))^(-7/2), x)
```

maple [C] time = 0.39, size = 487, normalized size = 4.13

$$\frac{(-1+\cos(x))\left(-15i\sin(x)\left(\cos^6(x)\right)\sqrt{\frac{i\cos(x)+\sin(x)-i}{\sin(x)}}\sqrt{\frac{-i\cos(x)+\sin(x)+i}{\sin(x)}}\sqrt{-\frac{i(-1+\cos(x))}{\sin(x)}}\operatorname{EllipticPi}\left(\sqrt{\frac{i\cos(x)+\sin(x)-}{\sin(x)}}\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(csc(x)-sin(x))^(7/2),x)
```

```
[Out] 1/768*(-1+cos(x))*(-15*I*sin(x)*cos(x)^6*((I*cos(x)+sin(x)-I)/sin(x))^(1/2)
*((-I*cos(x)+sin(x)+I)/sin(x))^(1/2)*(-I*(-1+cos(x))/sin(x))^(1/2)*Elliptic
Pi(((I*cos(x)+sin(x)-I)/sin(x))^(1/2),1/2-1/2*I,1/2*2^(1/2))-15*I*sin(x)*co
s(x)^6*((I*cos(x)+sin(x)-I)/sin(x))^(1/2)*((-I*cos(x)+sin(x)+I)/sin(x))^(1/
2)*(-I*(-1+cos(x))/sin(x))^(1/2)*EllipticPi(((I*cos(x)+sin(x)-I)/sin(x))^(1
/2),1/2+1/2*I,1/2*2^(1/2))+30*I*sin(x)*cos(x)^6*((I*cos(x)+sin(x)-I)/sin(x)
)^(1/2)*((-I*cos(x)+sin(x)+I)/sin(x))^(1/2)*(-I*(-1+cos(x))/sin(x))^(1/2)*E
llipticF(((I*cos(x)+sin(x)-I)/sin(x))^(1/2),1/2*2^(1/2))+15*sin(x)*cos(x)^6
*((I*cos(x)+sin(x)-I)/sin(x))^(1/2)*((-I*cos(x)+sin(x)+I)/sin(x))^(1/2)*(-I
*(-1+cos(x))/sin(x))^(1/2)*EllipticPi(((I*cos(x)+sin(x)-I)/sin(x))^(1/2),1/
2-1/2*I,1/2*2^(1/2))-15*sin(x)*cos(x)^6*((-I*cos(x)+sin(x)+I)/sin(x))^(1/2)
*((I*cos(x)+sin(x)-I)/sin(x))^(1/2)*(-I*(-1+cos(x))/sin(x))^(1/2)*EllipticP
i(((I*cos(x)+sin(x)-I)/sin(x))^(1/2),1/2+1/2*I,1/2*2^(1/2))+10*2^(1/2)*cos(
x)^5-10*2^(1/2)*cos(x)^4-104*cos(x)^3*2^(1/2)+104*cos(x)^2*2^(1/2)+64*cos(x
)*2^(1/2)-64*2^(1/2))*cos(x)*(1+cos(x))^2/(cos(x)^2/sin(x))^(7/2)/sin(x)^7*
2^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(\csc(x) - \sin(x))^{\frac{7}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(csc(x)-sin(x))^(7/2),x, algorithm="maxima")
```

```
[Out] integrate((csc(x) - sin(x))^(-7/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\left(\frac{1}{\sin(x)} - \sin(x)\right)^{7/2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1/sin(x) - sin(x))^(7/2),x)
```

```
[Out] int(1/(1/sin(x) - sin(x))^(7/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(csc(x)-sin(x))**(7/2),x)
```

```
[Out] Timed out
```

3.322 $\int (-\cos(x) + \sec(x))^4 \, dx$

Optimal. Leaf size=44

$$\frac{35x}{8} + \frac{35\tan^3(x)}{24} - \frac{35\tan(x)}{8} - \frac{1}{4}\sin^4(x)\tan^3(x) - \frac{7}{8}\sin^2(x)\tan^3(x)$$

```
[Out] 35/8*x-35/8*tan(x)+35/24*tan(x)^3-7/8*sin(x)^2*tan(x)^3-1/4*sin(x)^4*tan(x)
^3
```

Rubi [A] time = 0.03, antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {288, 302, 203}

$$\frac{35x}{8} + \frac{35\tan^3(x)}{24} - \frac{35\tan(x)}{8} - \frac{1}{4}\sin^4(x)\tan^3(x) - \frac{7}{8}\sin^2(x)\tan^3(x)$$

Antiderivative was successfully verified.

```
[In] Int[(-Cos[x] + Sec[x])^4,x]
```

```
[Out] (35*x)/8 - (35*Tan[x])/8 + (35*Tan[x]^3)/24 - (7*Sin[x]^2*Tan[x]^3)/8 - (Si
n[x]^4*Tan[x]^3)/4
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rubi steps

$$\int (-\cos(x)+\sec(x))^4\,dx = \text{Subst}\left(\int \frac{x^8}{(1+x^2)^3}\,dx, x, \tan(x)\right)$$
$$= -\frac{1}{4}\sin^4(x)\tan^3(x) + \frac{7}{4}\text{Subst}\left(\int \frac{x^6}{(1+x^2)^2}\,dx, x, \tan(x)\right)$$
$$= -\frac{7}{8}\sin^2(x)\tan^3(x) - \frac{1}{4}\sin^4(x)\tan^3(x) + \frac{35}{8}\text{Subst}\left(\int \frac{x^4}{1+x^2}\,dx, x, \tan(x)\right)$$
$$= -\frac{7}{8}\sin^2(x)\tan^3(x) - \frac{1}{4}\sin^4(x)\tan^3(x) + \frac{35}{8}\text{Subst}\left(\int \left(-1+x^2+\frac{1}{1+x^2}\right)dx, x, \tan$$
$$= -\frac{35\tan(x)}{8} + \frac{35\tan^3(x)}{24} - \frac{7}{8}\sin^2(x)\tan^3(x) - \frac{1}{4}\sin^4(x)\tan^3(x) + \frac{35}{8}\text{Subst}\left(\int \frac{1}{1+}\right.$$
$$= \frac{35x}{8} - \frac{35\tan(x)}{8} + \frac{35\tan^3(x)}{24} - \frac{7}{8}\sin^2(x)\tan^3(x) - \frac{1}{4}\sin^4(x)\tan^3(x)$$

Mathematica [A] time = 0.03, size = 38, normalized size = 0.86

$$\frac{35x}{8} - \frac{3}{4}\sin(2x) + \frac{1}{32}\sin(4x) - \frac{10\tan(x)}{3} + \frac{1}{3}\tan(x)\sec^2(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-Cos[x] + Sec[x])^4,x]
```

```
[Out] (35*x)/8 - (3*Sin[2*x])/4 + Sin[4*x]/32 - (10*Tan[x])/3 + (Sec[x]^2*Tan[x])
/3
```

fricas [A] time = 1.85, size = 37, normalized size = 0.84

$$\frac{105\,x\cos(x)^3 + \left(6\cos(x)^6 - 39\cos(x)^4 - 80\cos(x)^2 + 8\right)\sin(x)}{24\cos(x)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-cos(x)+sec(x))^4,x, algorithm="fricas")
```

```
[Out] 1/24*(105*x*cos(x)^3 + (6*cos(x)^6 - 39*cos(x)^4 - 80*cos(x)^2 + 8)*sin(x))
/cos(x)^3
```

giac [A] time = 0.14, size = 35, normalized size = 0.80

$$\frac{1}{3}\tan(x)^3 + \frac{35}{8}x - \frac{13\tan(x)^3 + 11\tan(x)}{8\left(\tan(x)^2+1\right)^2} - 3\tan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-cos(x)+sec(x))^4,x, algorithm="giac")
```

```
[Out] 1/3*tan(x)^3 + 35/8*x - 1/8*(13*tan(x)^3 + 11*tan(x))/(tan(x)^2 + 1)^2 - 3*
tan(x)
```

maple [A] time = 0.06, size = 40, normalized size = 0.91

$$-\left(-\frac{2}{3}-\frac{\left(\sec^2(x)\right)}{3}\right)\tan(x)-4\tan(x)+\frac{35x}{8}-2\cos(x)\sin(x)+\frac{\left(\cos^3(x)+\frac{3\cos(x)}{2}\right)\sin(x)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-cos(x)+sec(x))^4,x)
```

```
[Out] -(-2/3-1/3*sec(x)^2)*tan(x)-4*tan(x)+35/8*x-2*cos(x)*sin(x)+1/4*(cos(x)^3+3
/2*cos(x))*sin(x)
```

maxima [A] time = 0.32, size = 26, normalized size = 0.59

$$\frac{1}{3}\tan(x)^3+\frac{35}{8}x+\frac{1}{32}\sin(4x)-\frac{3}{4}\sin(2x)-3\tan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-cos(x)+sec(x))^4,x, algorithm="maxima")
```

```
[Out] 1/3*tan(x)^3 + 35/8*x + 1/32*sin(4*x) - 3/4*sin(2*x) - 3*tan(x)
```

mupad [B] time = 2.57, size = 80, normalized size = 1.82

$$\frac{35x}{8}+\frac{\frac{35\tan\left(\frac{x}{2}\right)^{13}}{4}+\frac{35\tan\left(\frac{x}{2}\right)^{11}}{6}-\frac{329\tan\left(\frac{x}{2}\right)^{9}}{12}-17\tan\left(\frac{x}{2}\right)^7-\frac{329\tan\left(\frac{x}{2}\right)^{5}}{12}+\frac{35\tan\left(\frac{x}{2}\right)^{3}}{6}+\frac{35\tan\left(\frac{x}{2}\right)}{4}}{\left(\tan\left(\frac{x}{2}\right)^2-1\right)^3\left(\tan\left(\frac{x}{2}\right)^2+1\right)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(x) - 1/cos(x))^4,x)
```

```
[Out] (35*x)/8 + ((35*tan(x/2))/4 + (35*tan(x/2)^3)/6 - (329*tan(x/2)^5)/12 - 17*
tan(x/2)^7 - (329*tan(x/2)^9)/12 + (35*tan(x/2)^11)/6 + (35*tan(x/2)^13)/4)
/((tan(x/2)^2 - 1)^3*(tan(x/2)^2 + 1)^4)
```

sympy [A] time = 8.97, size = 44, normalized size = 1.00

$$\frac{35x}{8}-2\sin(x)\cos(x)-\frac{4\sin(x)}{\cos(x)}+\frac{\sin(2x)}{4}+\frac{\sin(4x)}{32}+\frac{\tan^3(x)}{3}+\tan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-cos(x)+sec(x))**4,x)
```

```
[Out] 35*x/8 - 2*sin(x)*cos(x) - 4*sin(x)/cos(x) + sin(2*x)/4 + sin(4*x)/32 + tan
(x)**3/3 + tan(x)
```

3.323 $\int (-\cos(x)+\sec(x))^3\,dx$

Optimal. Leaf size=34

$$\frac{5\sin^3(x)}{6}+\frac{5\sin(x)}{2}+\frac{1}{2}\sin^3(x)\tan^2(x)-\frac{5}{2}\tanh^{-1}(\sin(x))$$

```
[Out] -5/2*arctanh(sin(x))+5/2*sin(x)+5/6*sin(x)^3+1/2*sin(x)^3*tan(x)^2
```

Rubi [A] time = 0.04, antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.556, Rules used = {4397, 2592, 288, 302, 206}

$$\frac{5\sin^3(x)}{6}+\frac{5\sin(x)}{2}+\frac{1}{2}\sin^3(x)\tan^2(x)-\frac{5}{2}\tanh^{-1}(\sin(x))$$

Antiderivative was successfully verified.

```
[In] Int[(-Cos[x] + Sec[x])^3,x]
```

```
[Out] (-5*ArcTanh[Sin[x]])/2 + (5*Sin[x])/2 + (5*Sin[x]^3)/6 + (Sin[x]^3*Tan[x]^2
)/2
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 2592

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(
```

```
ff*x)^(m + n)/(a^2 - ff^2*x^2)^((n + 1)/2), x], x, (a*Sin[e + f*x])/ff], x]
] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2]
```

Rule 4397

```
Int[u_, x_Symbol] :> Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]
```

Rubi steps

$$\begin{aligned}
\int (-\cos(x)+\sec(x))^3\,dx &= \int \sin^3(x)\tan^3(x)\,dx\\
&= \text{Subst}\left(\int \frac{x^6}{\left(1-x^2\right)^2}\,dx, x, \sin(x)\right)\\
&= \frac{1}{2}\sin^3(x)\tan^2(x)-\frac{5}{2}\text{Subst}\left(\int \frac{x^4}{1-x^2}\,dx, x, \sin(x)\right)\\
&= \frac{1}{2}\sin^3(x)\tan^2(x)-\frac{5}{2}\text{Subst}\left(\int \left(-1-x^2+\frac{1}{1-x^2}\right)dx, x, \sin(x)\right)\\
&= \frac{5\sin(x)}{2}+\frac{5\sin^3(x)}{6}+\frac{1}{2}\sin^3(x)\tan^2(x)-\frac{5}{2}\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \sin(x)\right)\\
&= -\frac{5}{2}\tanh^{-1}(\sin(x))+\frac{5\sin(x)}{2}+\frac{5\sin^3(x)}{6}+\frac{1}{2}\sin^3(x)\tan^2(x)
\end{aligned}$$

Mathematica [A] time = 0.01, size = 38, normalized size = 1.12

$$-\frac{1}{3}\sin^3(x)\tan^2(x)-\frac{5}{3}\sin(x)\tan^2(x)-\frac{5}{2}\tanh^{-1}(\sin(x))+\frac{5}{2}\tan(x)\sec(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-Cos[x] + Sec[x])^3,x]
```

```
[Out] (-5*ArcTanh[Sin[x]])/2 + (5*Sec[x]*Tan[x])/2 - (5*Sin[x]*Tan[x]^2)/3 - (Sin
[x]^3*Tan[x]^2)/3
```

fricas [A] time = 0.94, size = 49, normalized size = 1.44

$$-\frac{15\cos(x)^2\log(\sin(x)+1)-15\cos(x)^2\log(-\sin(x)+1)+2\left(2\cos(x)^4-14\cos(x)^2-3\right)\sin(x)}{12\cos(x)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-cos(x)+sec(x))^3,x, algorithm="fricas")
```

```
[Out] -1/12*(15*cos(x)^2*log(sin(x) + 1) - 15*cos(x)^2*log(-sin(x) + 1) + 2*(2*co
s(x)^4 - 14*cos(x)^2 - 3)*sin(x))/cos(x)^2
```

giac [A] time = 0.15, size = 39, normalized size = 1.15

$$\frac{1}{3}\,\sin(x)^3-\frac{\sin(x)}{2\left(\sin(x)^2-1\right)}-\frac{5}{4}\,\log\left(\sin(x)+1\right)+\frac{5}{4}\,\log\left(-\sin(x)+1\right)+2\,\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-cos(x)+sec(x))^3,x, algorithm="giac")
```

```
[Out] 1/3*sin(x)^3 - 1/2*sin(x)/(sin(x)^2 - 1) - 5/4*log(sin(x) + 1) + 5/4*log(-s
in(x) + 1) + 2*sin(x)
```

maple [A] time = 0.06, size = 30, normalized size = 0.88

$$\frac{\sec(x)\tan(x)}{2}-\frac{5\ln\left(\sec(x)+\tan(x)\right)}{2}+3\sin(x)-\frac{\left(2+\cos^2(x)\right)\sin(x)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-cos(x)+sec(x))^3,x)
```

```
[Out] 1/2*sec(x)*tan(x)-5/2*ln(sec(x)+tan(x))+3*sin(x)-1/3*(2+cos(x)^2)*sin(x)
```

maxima [A] time = 0.32, size = 37, normalized size = 1.09

$$\frac{1}{3}\,\sin(x)^3-\frac{\sin(x)}{2\left(\sin(x)^2-1\right)}-\frac{5}{4}\,\log\left(\sin(x)+1\right)+\frac{5}{4}\,\log\left(\sin(x)-1\right)+2\,\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-cos(x)+sec(x))^3,x, algorithm="maxima")
```

```
[Out] 1/3*sin(x)^3 - 1/2*sin(x)/(sin(x)^2 - 1) - 5/4*log(sin(x) + 1) + 5/4*log(si
n(x) - 1) + 2*sin(x)
```

mupad [B] time = 2.46, size = 68, normalized size = 2.00

$$\frac{5\tan\left(\frac{x}{2}\right)^9+\frac{20\tan\left(\frac{x}{2}\right)^7}{3}-\frac{22\tan\left(\frac{x}{2}\right)^5}{3}+\frac{20\tan\left(\frac{x}{2}\right)^3}{3}+5\tan\left(\frac{x}{2}\right)}{\left(\tan\left(\frac{x}{2}\right)^2-1\right)^2\left(\tan\left(\frac{x}{2}\right)^2+1\right)^3}-5\,\mathrm{atanh}\left(\tan\left(\frac{x}{2}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(cos(x) - 1/cos(x))^3,x)
```

```
[Out] (5*tan(x/2) + (20*tan(x/2)^3)/3 - (22*tan(x/2)^5)/3 + (20*tan(x/2)^7)/3 + 5
*tan(x/2)^9)/((tan(x/2)^2 - 1)^2*(tan(x/2)^2 + 1)^3) - 5*atanh(tan(x/2))
```

sympy [A] time = 3.74, size = 42, normalized size = 1.24

$$\frac{5\log\left(\sin\left(x\right)-1\right)}{4}-\frac{5\log\left(\sin\left(x\right)+1\right)}{4}+\frac{\sin^{3}\left(x\right)}{3}+2\sin\left(x\right)-\frac{\sin\left(x\right)}{2\sin^{2}\left(x\right)-2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-cos(x)+sec(x))**3,x)
```

```
[Out] 5*log(sin(x) - 1)/4 - 5*log(sin(x) + 1)/4 + sin(x)**3/3 + 2*sin(x) - sin(x)
/(2*sin(x)**2 - 2)
```

3.324 $\int (-\cos(x) + \sec(x))^2 \, dx$

Optimal. Leaf size=22

$$-\frac{3x}{2} + \frac{3\tan(x)}{2} - \frac{1}{2}\sin^2(x)\tan(x)$$

```
[Out] -3/2*x+3/2*tan(x)-1/2*sin(x)^2*tan(x)
```

Rubi [A] time = 0.02, antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {288, 321, 203}

$$-\frac{3x}{2} + \frac{3\tan(x)}{2} - \frac{1}{2}\sin^2(x)\tan(x)$$

Antiderivative was successfully verified.

```
[In] Int[(-Cos[x] + Sec[x])^2,x]
```

```
[Out] (-3*x)/2 + (3*Tan[x])/2 - (Sin[x]^2*Tan[x])/2
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] && !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\begin{aligned}\int (-\cos(x)+\sec(x))^2\,dx &= \text{Subst}\left(\int \frac{x^4}{\left(1+x^2\right)^2}\,dx,x,\tan(x)\right)\\ &= -\frac{1}{2}\sin^2(x)\tan(x)+\frac{3}{2}\text{Subst}\left(\int \frac{x^2}{1+x^2}\,dx,x,\tan(x)\right)\\ &= \frac{3\tan(x)}{2}-\frac{1}{2}\sin^2(x)\tan(x)-\frac{3}{2}\text{Subst}\left(\int \frac{1}{1+x^2}\,dx,x,\tan(x)\right)\\ &= -\frac{3x}{2}+\frac{3\tan(x)}{2}-\frac{1}{2}\sin^2(x)\tan(x)\end{aligned}$$

Mathematica [A] time = 0.02, size = 16, normalized size = 0.73

$$-\frac{3x}{2}+\frac{1}{4}\sin(2x)+\tan(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-Cos[x] + Sec[x])^2,x]
[Out] (-3*x)/2 + Sin[2*x]/4 + Tan[x]
```

fricas [A] time = 0.95, size = 22, normalized size = 1.00

$$-\frac{3\,x\cos(x)-\left(\cos(x)^2+2\right)\sin(x)}{2\,\cos(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-cos(x)+sec(x))^2,x, algorithm="fricas")
[Out] -1/2*(3*x*cos(x) - (cos(x)^2 + 2)*sin(x))/cos(x)
```

giac [A] time = 0.14, size = 18, normalized size = 0.82

$$-\frac{3}{2}\,x+\frac{\tan(x)}{2\left(\tan(x)^2+1\right)}+\tan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-cos(x)+sec(x))^2,x, algorithm="giac")
[Out] -3/2*x + 1/2*tan(x)/(tan(x)^2 + 1) + tan(x)
```

maple [A] time = 0.05, size = 13, normalized size = 0.59

$$\tan(x) - \frac{3x}{2} + \frac{\cos(x)\sin(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-cos(x)+sec(x))^2,x)
```

```
[Out] tan(x)-3/2*x+1/2*cos(x)*sin(x)
```

maxima [A] time = 0.31, size = 12, normalized size = 0.55

$$-\frac{3}{2}x + \frac{1}{4}\sin(2x) + \tan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-cos(x)+sec(x))^2,x, algorithm="maxima")
```

```
[Out] -3/2*x + 1/4*sin(2*x) + tan(x)
```

mupad [B] time = 2.41, size = 49, normalized size = 2.23

$$-\frac{3x}{2} - \frac{3\tan\left(\frac{x}{2}\right)^5 + 2\tan\left(\frac{x}{2}\right)^3 + 3\tan\left(\frac{x}{2}\right)}{\left(\tan\left(\frac{x}{2}\right)^2 - 1\right)\left(\tan\left(\frac{x}{2}\right)^2 + 1\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(x) - 1/cos(x))^2,x)
```

```
[Out] - (3*x)/2 - (3*tan(x/2) + 2*tan(x/2)^3 + 3*tan(x/2)^5)/((tan(x/2)^2 - 1)*(t
an(x/2)^2 + 1)^2)
```

sympy [A] time = 1.62, size = 14, normalized size = 0.64

$$-\frac{3x}{2} + \frac{\sin(2x)}{4} + \tan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-cos(x)+sec(x))**2,x)
```

```
[Out] -3*x/2 + sin(2*x)/4 + tan(x)
```

3.325 $\int (-\cos(x) + \sec(x))\,dx$

Optimal. Leaf size=8

$$\tanh^{-1}(\sin(x)) - \sin(x)$$

```
[Out] arctanh(sin(x))-sin(x)
```

Rubi [A] time = 0.00, antiderivative size = 8, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {2637, 3770}

$$\tanh^{-1}(\sin(x)) - \sin(x)$$

Antiderivative was successfully verified.

```
[In] Int[-Cos[x] + Sec[x],x]
```

```
[Out] ArcTanh[Sin[x]] - Sin[x]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned} \int (-\cos(x) + \sec(x))\,dx &= -\int \cos(x)\,dx + \int \sec(x)\,dx \\ &= \tanh^{-1}(\sin(x)) - \sin(x) \end{aligned}$$

Mathematica [B] time = 0.00, size = 37, normalized size = 4.62

$$-\sin(x) - \log\left(\cos\left(\frac{x}{2}\right) - \sin\left(\frac{x}{2}\right)\right) + \log\left(\sin\left(\frac{x}{2}\right) + \cos\left(\frac{x}{2}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[-Cos[x] + Sec[x],x]
```

```
[Out] -Log[Cos[x/2] - Sin[x/2]] + Log[Cos[x/2] + Sin[x/2]] - Sin[x]
```

fricas [B] time = 0.95, size = 21, normalized size = 2.62

$$\frac{1}{2}\log\left(\sin(x)+1\right)-\frac{1}{2}\log\left(-\sin(x)+1\right)-\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-cos(x)+sec(x),x, algorithm="fricas")
[Out] 1/2*log(sin(x) + 1) - 1/2*log(-sin(x) + 1) - sin(x)
```

giac [B] time = 0.15, size = 29, normalized size = 3.62

$$\frac{1}{4}\log\left(\left|\frac{1}{\sin(x)}+\sin(x)+2\right|\right)-\frac{1}{4}\log\left(\left|\frac{1}{\sin(x)}+\sin(x)-2\right|\right)-\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-cos(x)+sec(x),x, algorithm="giac")
[Out] 1/4*log(abs(1/sin(x) + sin(x) + 2)) - 1/4*log(abs(1/sin(x) + sin(x) - 2)) -
 sin(x)
```

maple [A] time = 0.00, size = 12, normalized size = 1.50

$$-\sin(x)+\ln\left(\sec(x)+\tan(x)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-cos(x)+sec(x),x)
[Out] -sin(x)+ln(sec(x)+tan(x))
```

maxima [A] time = 0.32, size = 11, normalized size = 1.38

$$\log\left(\sec(x)+\tan(x)\right)-\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-cos(x)+sec(x),x, algorithm="maxima")
[Out] log(sec(x) + tan(x)) - sin(x)
```

mupad [B] time = 2.30, size = 14, normalized size = 1.75

$$\ln\left(\tan\left(\frac{x}{2}+\frac{\pi}{4}\right)\right)-\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/cos(x) - cos(x),x)
```

```
[Out] log(tan(x/2 + pi/4)) - sin(x)
```

sympy [B] time = 0.09, size = 19, normalized size = 2.38

$$-\frac{\log\left(\sin\left(x\right)-1\right)}{2}+\frac{\log\left(\sin\left(x\right)+1\right)}{2}-\sin\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-cos(x)+sec(x),x)
```

```
[Out] -log(sin(x) - 1)/2 + log(sin(x) + 1)/2 - sin(x)
```

3.326 $\int \frac{1}{-\cos(x)+\sec(x)}\,dx$

Optimal. Leaf size=4

$$-\csc(x)$$

```
[Out] -csc(x)
```

Rubi [A] time = 0.02, antiderivative size = 4, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {4397, 2606, 8}

$$-\csc(x)$$

Antiderivative was successfully verified.

```
[In] Int[(-Cos[x] + Sec[x])^(-1),x]
```

```
[Out] -Csc[x]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 4397

```
Int[u_, x_Symbol] :> Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{-\cos(x)+\sec(x)}\,dx &= \int \cot(x)\csc(x)\,dx\\
&= -\text{Subst}\left(\int 1\,dx, x, \csc(x)\right)\\
&= -\csc(x)
\end{aligned}$$

Mathematica [A] time = 0.00, size = 4, normalized size = 1.00

$$-\csc(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-Cos[x] + Sec[x])^(-1),x]
```

```
[Out] -Csc[x]
```

fricas [A] time = 0.83, size = 6, normalized size = 1.50

$$-\frac{1}{\sin(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-cos(x)+sec(x)),x, algorithm="fricas")
```

```
[Out] -1/sin(x)
```

giac [A] time = 0.15, size = 6, normalized size = 1.50

$$-\frac{1}{\sin(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-cos(x)+sec(x)),x, algorithm="giac")
```

```
[Out] -1/sin(x)
```

maple [A] time = 0.10, size = 7, normalized size = 1.75

$$-\frac{1}{\sin(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-cos(x)+sec(x)),x)
```

```
[Out] -1/sin(x)
```

maxima [B] time = 0.32, size = 21, normalized size = 5.25

$$-\frac{\cos(x) + 1}{2\sin(x)} - \frac{\sin(x)}{2(\cos(x) + 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-cos(x)+sec(x)),x, algorithm="maxima")
```

```
[Out] -1/2*(cos(x) + 1)/sin(x) - 1/2*sin(x)/(cos(x) + 1)
```

mupad [B] time = 2.37, size = 6, normalized size = 1.50

$$-\frac{1}{\sin(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-1/(cos(x) - 1/cos(x)),x)
[Out] -1/sin(x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{1}{\cos(x) - \sec(x)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-cos(x)+sec(x)),x)
[Out] -Integral(1/(cos(x) - sec(x)), x)
```

3.327 $\int \frac{1}{(-\cos(x)+\sec(x))^2} dx$

Optimal. Leaf size=8

$$-\frac{1}{3}\cot^3(x)$$

```
[Out] -1/3*cot(x)^3
```

Rubi [A] time = 0.01, antiderivative size = 8, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {30}

$$-\frac{1}{3}\cot^3(x)$$

Antiderivative was successfully verified.

```
[In] Int[(-Cos[x] + Sec[x])^(-2),x]
```

```
[Out] -Cot[x]^3/3
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned} \int \frac{1}{(-\cos(x)+\sec(x))^2} dx &= \text{Subst}\left(\int \frac{1}{x^4} dx, x, \tan(x)\right) \\ &= -\frac{1}{3}\cot^3(x) \end{aligned}$$

Mathematica [A] time = 0.00, size = 8, normalized size = 1.00

$$-\frac{1}{3}\cot^3(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-Cos[x] + Sec[x])^(-2),x]
```

```
[Out] -1/3*Cot[x]^3
```

fricas [B] time = 0.90, size = 18, normalized size = 2.25

$$\frac{\cos(x)^3}{3\left(\cos(x)^2-1\right)\sin(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-cos(x)+sec(x))^2,x, algorithm="fricas")
[Out] 1/3*cos(x)^3/((cos(x)^2 - 1)*sin(x))
```

giac [A] time = 0.15, size = 6, normalized size = 0.75

$$-\frac{1}{3\tan(x)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-cos(x)+sec(x))^2,x, algorithm="giac")
[Out] -1/3/tan(x)^3
```

maple [A] time = 0.10, size = 7, normalized size = 0.88

$$-\frac{1}{3\tan(x)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-cos(x)+sec(x))^2,x)
[Out] -1/3/tan(x)^3
```

maxima [A] time = 0.32, size = 6, normalized size = 0.75

$$-\frac{1}{3\tan(x)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-cos(x)+sec(x))^2,x, algorithm="maxima")
[Out] -1/3/tan(x)^3
```

mupad [B] time = 2.45, size = 6, normalized size = 0.75

$$-\frac{\cot(x)^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(x) - 1/cos(x))^2,x)
```

```
[Out] -cot(x)^3/3
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(-\cos(x) + \sec(x))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-cos(x)+sec(x))**2,x)
```

```
[Out] Integral((-cos(x) + sec(x))**(-2), x)
```

3.328 $\int \frac{1}{(-\cos(x)+\sec(x))^3}\,dx$

Optimal. Leaf size=17

$$\frac{\csc^3(x)}{3}-\frac{\csc^5(x)}{5}$$

```
[Out] 1/3*csc(x)^3-1/5*csc(x)^5
```

Rubi [A] time = 0.04, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {4397, 2606, 14}

$$\frac{\csc^3(x)}{3}-\frac{\csc^5(x)}{5}$$

Antiderivative was successfully verified.

```
[In] Int[(-Cos[x] + Sec[x])^(-3),x]
```

```
[Out] Csc[x]^3/3 - Csc[x]^5/5
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 4397

```
Int[u_, x_Symbol] :> Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]
```

Rubi steps

$$\int \frac{1}{(-\cos(x)+\sec(x))^3}\,dx = \int \cot^3(x)\csc^3(x)\,dx$$
$$= -\text{Subst}\left(\int x^2\left(-1+x^2\right)\,dx, x, \csc(x)\right)$$
$$= -\text{Subst}\left(\int \left(-x^2+x^4\right)\,dx, x, \csc(x)\right)$$
$$= \frac{\csc^3(x)}{3} - \frac{\csc^5(x)}{5}$$

Mathematica [A] time = 0.01, size = 17, normalized size = 1.00

$$\frac{\csc^3(x)}{3} - \frac{\csc^5(x)}{5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-Cos[x] + Sec[x])^(-3),x]
[Out] Csc[x]^3/3 - Csc[x]^5/5
```

fricas [B] time = 1.02, size = 28, normalized size = 1.65

$$-\frac{5\,\cos(x)^2-2}{15\left(\cos(x)^4-2\,\cos(x)^2+1\right)\sin(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-cos(x)+sec(x))^3,x, algorithm="fricas")
[Out] -1/15*(5*cos(x)^2 - 2)/((cos(x)^4 - 2*cos(x)^2 + 1)*sin(x))
```

giac [A] time = 0.13, size = 14, normalized size = 0.82

$$\frac{5\,\sin(x)^2-3}{15\,\sin(x)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-cos(x)+sec(x))^3,x, algorithm="giac")
[Out] 1/15*(5*sin(x)^2 - 3)/sin(x)^5
```

maple [A] time = 0.10, size = 14, normalized size = 0.82

$$\frac{1}{3\sin(x)^3} - \frac{1}{5\sin(x)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-cos(x)+sec(x))^3,x)
```

```
[Out] 1/3/sin(x)^3-1/5/sin(x)^5
```

maxima [B] time = 0.32, size = 73, normalized size = 4.29

$$\frac{\left(\frac{5\,\sin(x)^2}{(\cos(x)+1)^2} + \frac{30\,\sin(x)^4}{(\cos(x)+1)^4} - 3\right)(\cos(x)+1)^5}{480\,\sin(x)^5} + \frac{\sin(x)}{16\,(\cos(x)+1)} + \frac{\sin(x)^3}{96\,(\cos(x)+1)^3} - \frac{\sin(x)^5}{160\,(\cos(x)+1)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-cos(x)+sec(x))^3,x, algorithm="maxima")
```

```
[Out] 1/480*(5*sin(x)^2/(cos(x) + 1)^2 + 30*sin(x)^4/(cos(x) + 1)^4 - 3)*(cos(x)
+ 1)^5/sin(x)^5 + 1/16*sin(x)/(cos(x) + 1) + 1/96*sin(x)^3/(cos(x) + 1)^3 -
 1/160*sin(x)^5/(cos(x) + 1)^5
```

mupad [B] time = 2.38, size = 14, normalized size = 0.82

$$\frac{5\sin(x)^2 - 3}{15\sin(x)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-1/(cos(x) - 1/cos(x))^3,x)
```

```
[Out] (5*sin(x)^2 - 3)/(15*sin(x)^5)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{1}{\cos^3(x) - 3\cos^2(x)\sec(x) + 3\cos(x)\sec^2(x) - \sec^3(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-cos(x)+sec(x))**3,x)
```

```
[Out] -Integral(1/(cos(x)**3 - 3*cos(x)**2*sec(x) + 3*cos(x)*sec(x)**2 - sec(x)**
3), x)
```

3.329 $\int \frac{1}{(-\cos(x)+\sec(x))^4} dx$

Optimal. Leaf size=17

$$-\frac{1}{7}\cot^7(x) - \frac{\cot^5(x)}{5}$$

```
[Out] -1/5*cot(x)^5-1/7*cot(x)^7
```

Rubi [A] time = 0.02, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$-\frac{1}{7}\cot^7(x) - \frac{\cot^5(x)}{5}$$

Antiderivative was successfully verified.

```
[In] Int[(-Cos[x] + Sec[x])^(-4),x]
```

```
[Out] -Cot[x]^5/5 - Cot[x]^7/7
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(-\cos(x)+\sec(x))^4} dx &= \text{Subst}\left(\int \left(\frac{1}{x^8}+\frac{1}{x^6}\right) dx, x, \tan(x)\right)\\ &= -\frac{1}{5}\cot^5(x) - \frac{\cot^7(x)}{7}\end{aligned}$$

Mathematica [B] time = 0.02, size = 37, normalized size = 2.18

$$-\frac{2\cot(x)}{35} - \frac{1}{7}\cot(x)\csc^6(x) + \frac{8}{35}\cot(x)\csc^4(x) - \frac{1}{35}\cot(x)\csc^2(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-Cos[x] + Sec[x])^(-4),x]
```

```
[Out] (-2*Cot[x])/35 - (Cot[x]*Csc[x]^2)/35 + (8*Cot[x]*Csc[x]^4)/35 - (Cot[x]*Cs
c[x]^6)/7
```

fricas [B] time = 0.65, size = 39, normalized size = 2.29

$$-\frac{2\cos(x)^7 - 7\cos(x)^5}{35\left(\cos(x)^6 - 3\cos(x)^4 + 3\cos(x)^2 - 1\right)\sin(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-cos(x)+sec(x))^4,x, algorithm="fricas")
```

```
[Out] -1/35*(2*cos(x)^7 - 7*cos(x)^5)/((cos(x)^6 - 3*cos(x)^4 + 3*cos(x)^2 - 1)*s
in(x))
```

giac [A] time = 0.14, size = 14, normalized size = 0.82

$$-\frac{7\,\tan(x)^2+5}{35\,\tan(x)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-cos(x)+sec(x))^4,x, algorithm="giac")
```

```
[Out] -1/35*(7*tan(x)^2 + 5)/tan(x)^7
```

maple [A] time = 0.11, size = 14, normalized size = 0.82

$$-\frac{1}{7\tan(x)^7}-\frac{1}{5\tan(x)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-cos(x)+sec(x))^4,x)
```

```
[Out] -1/7/tan(x)^7-1/5/tan(x)^5
```

maxima [A] time = 0.32, size = 14, normalized size = 0.82

$$-\frac{7\,\tan(x)^2+5}{35\,\tan(x)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-cos(x)+sec(x))^4,x, algorithm="maxima")
```

```
[Out] -1/35*(7*tan(x)^2 + 5)/tan(x)^7
```

mupad [B] time = 2.43, size = 16, normalized size = 0.94

$$\frac{\cos(x)^5\,\left(\cos\left(2\,x\right)-6\right)}{35\sin(x)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(x) - 1/cos(x))^4,x)
```

```
[Out] (cos(x)^5*(cos(2*x) - 6))/(35*sin(x)^7)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(-\cos(x) + \sec(x))^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-cos(x)+sec(x))**4,x)
```

```
[Out] Integral((-cos(x) + sec(x))**(-4), x)
```

3.330 $\int \frac{1}{(-\cos(x)+\sec(x))^5} dx$

Optimal. Leaf size=25

$$-\frac{1}{9}\csc^9(x)+\frac{2\csc^7(x)}{7}-\frac{\csc^5(x)}{5}$$

```
[Out] -1/5*csc(x)^5+2/7*csc(x)^7-1/9*csc(x)^9
```

Rubi [A] time = 0.04, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {4397, 2606, 270}

$$-\frac{1}{9}\csc^9(x)+\frac{2\csc^7(x)}{7}-\frac{\csc^5(x)}{5}$$

Antiderivative was successfully verified.

```
[In] Int[(-Cos[x] + Sec[x])^(-5),x]
```

```
[Out] -Csc[x]^5/5 + (2*Csc[x]^7)/7 - Csc[x]^9/9
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 4397

```
Int[u_, x_Symbol] :> Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]
```

Rubi steps

$$\int \frac{1}{(-\cos(x)+\sec(x))^5}\,dx = \int \cot^5(x)\csc^5(x)\,dx$$
$$= -\text{Subst}\left(\int x^4\left(-1+x^2\right)^2\,dx, x, \csc(x)\right)$$
$$= -\text{Subst}\left(\int \left(x^4-2x^6+x^8\right)\,dx, x, \csc(x)\right)$$
$$= -\frac{1}{5}\csc^5(x)+\frac{2\csc^7(x)}{7}-\frac{\csc^9(x)}{9}$$

Mathematica [A] time = 0.01, size = 25, normalized size = 1.00

$$-\frac{1}{9}\csc^9(x)+\frac{2\csc^7(x)}{7}-\frac{\csc^5(x)}{5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-Cos[x] + Sec[x])^(-5),x]
```

```
[Out] -1/5*Csc[x]^5 + (2*Csc[x]^7)/7 - Csc[x]^9/9
```

fricas [B] time = 1.38, size = 46, normalized size = 1.84

$$-\frac{63\,\cos(x)^4-36\,\cos(x)^2+8}{315\left(\cos(x)^8-4\,\cos(x)^6+6\,\cos(x)^4-4\,\cos(x)^2+1\right)\sin(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-cos(x)+sec(x))^5,x, algorithm="fricas")
```

```
[Out] -1/315*(63*cos(x)^4 - 36*cos(x)^2 + 8)/((cos(x)^8 - 4*cos(x)^6 + 6*cos(x)^4
 - 4*cos(x)^2 + 1)*sin(x))
```

giac [A] time = 0.14, size = 20, normalized size = 0.80

$$-\frac{63\,\sin(x)^4-90\,\sin(x)^2+35}{315\,\sin(x)^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-cos(x)+sec(x))^5,x, algorithm="giac")
```

```
[Out] -1/315*(63*sin(x)^4 - 90*sin(x)^2 + 35)/sin(x)^9
```

maple [A] time = 0.11, size = 20, normalized size = 0.80

$$-\frac{1}{9\sin(x)^9}+\frac{2}{7\sin(x)^7}-\frac{1}{5\sin(x)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-cos(x)+sec(x))^5,x)
```

```
[Out] -1/9/sin(x)^9+2/7/sin(x)^7-1/5/sin(x)^5
```

maxima [B] time = 0.33, size = 121, normalized size = 4.84

$$\frac{\left(\frac{45\,\sin(x)^2}{(\cos(x)+1)^2}+\frac{252\,\sin(x)^4}{(\cos(x)+1)^4}-\frac{420\,\sin(x)^6}{(\cos(x)+1)^6}-\frac{1890\,\sin(x)^8}{(\cos(x)+1)^8}-35\right)(\cos(x)+1)^9}{161280\,\sin(x)^9}-\frac{3\,\sin(x)}{256\,(\cos(x)+1)}-\frac{\sin(x)^3}{384\,(\cos(x)+1)^3}+\frac{\text{s}}{640\,(\text{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-cos(x)+sec(x))^5,x, algorithm="maxima")
```

```
[Out] 1/161280*(45*sin(x)^2/(cos(x) + 1)^2 + 252*sin(x)^4/(cos(x) + 1)^4 - 420*si
n(x)^6/(cos(x) + 1)^6 - 1890*sin(x)^8/(cos(x) + 1)^8 - 35)*(cos(x) + 1)^9/s
in(x)^9 - 3/256*sin(x)/(cos(x) + 1) - 1/384*sin(x)^3/(cos(x) + 1)^3 + 1/640
*sin(x)^5/(cos(x) + 1)^5 + 1/3584*sin(x)^7/(cos(x) + 1)^7 - 1/4608*sin(x)^9
/(cos(x) + 1)^9
```

mupad [B] time = 2.42, size = 20, normalized size = 0.80

$$-\frac{63\sin(x)^4-90\sin(x)^2+35}{315\sin(x)^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-1/(cos(x) - 1/cos(x))^5,x)
```

```
[Out] -(63*sin(x)^4 - 90*sin(x)^2 + 35)/(315*sin(x)^9)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-cos(x)+sec(x))**5,x)
```

```
[Out] Timed out
```

3.331 $\int \frac{1}{(-\cos(x)+\sec(x))^6} \, dx$

Optimal. Leaf size=25

$$-\frac{1}{11}\cot^{11}(x)-\frac{2\cot^9(x)}{9}-\frac{\cot^7(x)}{7}$$

```
[Out] -1/7*cot(x)^7-2/9*cot(x)^9-1/11*cot(x)^11
```

Rubi [A] time = 0.02, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 1, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {270}

$$-\frac{1}{11}\cot^{11}(x)-\frac{2\cot^9(x)}{9}-\frac{\cot^7(x)}{7}$$

Antiderivative was successfully verified.

```
[In] Int[(-Cos[x] + Sec[x])^(-6),x]
```

```
[Out] -Cot[x]^7/7 - (2*Cot[x]^9)/9 - Cot[x]^11/11
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(-\cos(x)+\sec(x))^6} \, dx &= \text{Subst}\left(\int \frac{\left(1+x^2\right)^2}{x^{12}} \, dx, x, \tan(x)\right)\\ &= \text{Subst}\left(\int \left(\frac{1}{x^{12}}+\frac{2}{x^{10}}+\frac{1}{x^8}\right) dx, x, \tan(x)\right)\\ &= -\frac{1}{7}\cot^7(x)-\frac{2\cot^9(x)}{9}-\frac{\cot^{11}(x)}{11}\end{aligned}$$

Mathematica [B] time = 0.02, size = 57, normalized size = 2.28

$$\frac{8\cot(x)}{693}-\frac{1}{11}\cot(x)\csc^{10}(x)+\frac{23}{99}\cot(x)\csc^8(x)-\frac{113}{693}\cot(x)\csc^6(x)+\frac{1}{231}\cot(x)\csc^4(x)+\frac{4}{693}\cot(x)\csc^2(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-Cos[x] + Sec[x])^(-6),x]
```

```
[Out] (8*Cot[x])/693 + (4*Cot[x]*Csc[x]^2)/693 + (Cot[x]*Csc[x]^4)/231 - (113*Cot
[x]*Csc[x]^6)/693 + (23*Cot[x]*Csc[x]^8)/99 - (Cot[x]*Csc[x]^10)/11
```

fricas [B] time = 1.47, size = 57, normalized size = 2.28

$$\frac{8 \cos(x)^{11} - 44 \cos(x)^{9} + 99 \cos(x)^{7}}{693 \left(\cos(x)^{10} - 5 \cos(x)^{8} + 10 \cos(x)^{6} - 10 \cos(x)^{4} + 5 \cos(x)^{2} - 1\right) \sin(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-cos(x)+sec(x))^6,x, algorithm="fricas")
```

```
[Out] 1/693*(8*cos(x)^11 - 44*cos(x)^9 + 99*cos(x)^7)/((cos(x)^10 - 5*cos(x)^8 +
10*cos(x)^6 - 10*cos(x)^4 + 5*cos(x)^2 - 1)*sin(x))
```

giac [A] time = 0.12, size = 20, normalized size = 0.80

$$-\frac{99 \tan(x)^{4} + 154 \tan(x)^{2} + 63}{693 \tan(x)^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-cos(x)+sec(x))^6,x, algorithm="giac")
```

```
[Out] -1/693*(99*tan(x)^4 + 154*tan(x)^2 + 63)/tan(x)^11
```

maple [A] time = 0.11, size = 20, normalized size = 0.80

$$-\frac{2}{9 \tan(x)^{9}} - \frac{1}{7 \tan(x)^{7}} - \frac{1}{11 \tan(x)^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-cos(x)+sec(x))^6,x)
```

```
[Out] -2/9/tan(x)^9-1/7/tan(x)^7-1/11/tan(x)^11
```

maxima [A] time = 0.32, size = 20, normalized size = 0.80

$$-\frac{99 \tan(x)^{4} + 154 \tan(x)^{2} + 63}{693 \tan(x)^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-cos(x)+sec(x))^6,x, algorithm="maxima")
```

```
[Out] -1/693*(99*tan(x)^4 + 154*tan(x)^2 + 63)/tan(x)^11
```

mupad [B] time = 2.45, size = 46, normalized size = 1.84

$$-\frac{80\cos(x)^7 - 18\cos(x)^7\left(2\cos(x)^2 - 1\right) + \cos(x)^7\left(2\left(2\cos(x)^2 - 1\right)^2 - 1\right)}{693\sin(x)^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(x) - 1/cos(x))^6,x)
```

```
[Out] -(80*cos(x)^7 - 18*cos(x)^7*(2*cos(x)^2 - 1) + cos(x)^7*(2*(2*cos(x)^2 - 1)
^2 - 1))/(693*sin(x)^11)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-cos(x)+sec(x))**6,x)
```

```
[Out] Timed out
```

3.332 $\int \frac{1}{(-\cos(x)+\sec(x))^7} dx$

Optimal. Leaf size=33

$$-\frac{1}{13}\csc^{13}(x)+\frac{3\csc^{11}(x)}{11}-\frac{\csc^9(x)}{3}+\frac{\csc^7(x)}{7}$$

```
[Out] 1/7*csc(x)^7-1/3*csc(x)^9+3/11*csc(x)^11-1/13*csc(x)^13
```

Rubi [A] time = 0.04, antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {4397, 2606, 270}

$$-\frac{1}{13}\csc^{13}(x)+\frac{3\csc^{11}(x)}{11}-\frac{\csc^9(x)}{3}+\frac{\csc^7(x)}{7}$$

Antiderivative was successfully verified.

```
[In] Int[(-Cos[x] + Sec[x])^(-7),x]
```

```
[Out] Csc[x]^7/7 - Csc[x]^9/3 + (3*Csc[x]^11)/11 - Csc[x]^13/13
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 4397

```
Int[u_, x_Symbol] :> Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]
```

Rubi steps

$$\int \frac{1}{(-\cos(x)+\sec(x))^7}\,dx = \int \cot^7(x)\csc^7(x)\,dx$$
$$= -\text{Subst}\left(\int x^6\left(-1+x^2\right)^3\,dx, x, \csc(x)\right)$$
$$= -\text{Subst}\left(\int \left(-x^6+3x^8-3x^{10}+x^{12}\right)\,dx, x, \csc(x)\right)$$
$$= \frac{\csc^7(x)}{7}-\frac{\csc^9(x)}{3}+\frac{3\csc^{11}(x)}{11}-\frac{\csc^{13}(x)}{13}$$

Mathematica [**A**] time = 0.01, size = 33, normalized size = 1.00

$$-\frac{1}{13}\csc^{13}(x)+\frac{3\csc^{11}(x)}{11}-\frac{\csc^9(x)}{3}+\frac{\csc^7(x)}{7}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-Cos[x] + Sec[x])^(-7),x]
[Out] Csc[x]^7/7 - Csc[x]^9/3 + (3*Csc[x]^11)/11 - Csc[x]^13/13
```

fricas [**B**] time = 2.27, size = 64, normalized size = 1.94

$$-\frac{429\,\cos(x)^6-286\,\cos(x)^4+104\,\cos(x)^2-16}{3003\left(\cos(x)^{12}-6\,\cos(x)^{10}+15\,\cos(x)^8-20\,\cos(x)^6+15\,\cos(x)^4-6\,\cos(x)^2+1\right)\sin(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-cos(x)+sec(x))^7,x, algorithm="fricas")
[Out] -1/3003*(429*cos(x)^6 - 286*cos(x)^4 + 104*cos(x)^2 - 16)/((cos(x)^12 - 6*c
os(x)^10 + 15*cos(x)^8 - 20*cos(x)^6 + 15*cos(x)^4 - 6*cos(x)^2 + 1)*sin(x)
)
```

giac [**A**] time = 0.15, size = 26, normalized size = 0.79

$$\frac{429\,\sin(x)^6-1001\,\sin(x)^4+819\,\sin(x)^2-231}{3003\,\sin(x)^{13}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-cos(x)+sec(x))^7,x, algorithm="giac")
[Out] 1/3003*(429*sin(x)^6 - 1001*sin(x)^4 + 819*sin(x)^2 - 231)/sin(x)^13
```

maple [A] time = 0.11, size = 26, normalized size = 0.79

$$-\frac{1}{13\sin(x)^{13}}-\frac{1}{3\sin(x)^{9}}+\frac{1}{7\sin(x)^{7}}+\frac{3}{11\sin(x)^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-cos(x)+sec(x))^7,x)
```

```
[Out] -1/13/sin(x)^13-1/3/sin(x)^9+1/7/sin(x)^7+3/11/sin(x)^11
```

maxima [B] time = 0.33, size = 169, normalized size = 5.12

$$\frac{\left(\frac{273\sin(x)^2}{(\cos(x)+1)^2}+\frac{2002\sin(x)^4}{(\cos(x)+1)^4}-\frac{2574\sin(x)^6}{(\cos(x)+1)^6}-\frac{9009\sin(x)^8}{(\cos(x)+1)^8}+\frac{15015\sin(x)^{10}}{(\cos(x)+1)^{10}}+\frac{60060\sin(x)^{12}}{(\cos(x)+1)^{12}}-231\right)(\cos(x)+1)^{13}}{24600576\sin(x)^{13}}+\frac{5\sin(}{2048(\cos(}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-cos(x)+sec(x))^7,x, algorithm="maxima")
```

```
[Out] 1/24600576*(273*sin(x)^2/(cos(x) + 1)^2 + 2002*sin(x)^4/(cos(x) + 1)^4 - 25
74*sin(x)^6/(cos(x) + 1)^6 - 9009*sin(x)^8/(cos(x) + 1)^8 + 15015*sin(x)^10
/(cos(x) + 1)^10 + 60060*sin(x)^12/(cos(x) + 1)^12 - 231)*(cos(x) + 1)^13/s
in(x)^13 + 5/2048*sin(x)/(cos(x) + 1) + 5/8192*sin(x)^3/(cos(x) + 1)^3 - 3/
8192*sin(x)^5/(cos(x) + 1)^5 - 3/28672*sin(x)^7/(cos(x) + 1)^7 + 1/12288*si
n(x)^9/(cos(x) + 1)^9 + 1/90112*sin(x)^11/(cos(x) + 1)^11 - 1/106496*sin(x)
^13/(cos(x) + 1)^13
```

mupad [B] time = 2.51, size = 109, normalized size = 3.30

$$-\frac{\cot\left(\frac{x}{2}\right)^{13}}{106496}+\frac{\cot\left(\frac{x}{2}\right)^{11}}{90112}+\frac{\cot\left(\frac{x}{2}\right)^{9}}{12288}-\frac{3\cot\left(\frac{x}{2}\right)^{7}}{28672}-\frac{3\cot\left(\frac{x}{2}\right)^{5}}{8192}+\frac{5\cot\left(\frac{x}{2}\right)^{3}}{8192}+\frac{5\cot\left(\frac{x}{2}\right)}{2048}-\frac{\tan\left(\frac{x}{2}\right)^{13}}{106496}+\frac{\tan\left(\frac{x}{2}\right)^{11}}{90112}+\frac{\tan\left(\frac{x}{2}\right)^{9}}{12288}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-1/(cos(x) - 1/cos(x))^7,x)
```

```
[Out] (5*cot(x/2))/2048 + (5*tan(x/2))/2048 + (5*cot(x/2)^3)/8192 - (3*cot(x/2)^5
)/8192 - (3*cot(x/2)^7)/28672 + cot(x/2)^9/12288 + cot(x/2)^11/90112 - cot(
x/2)^13/106496 + (5*tan(x/2)^3)/8192 - (3*tan(x/2)^5)/8192 - (3*tan(x/2)^7)
/28672 + tan(x/2)^9/12288 + tan(x/2)^11/90112 - tan(x/2)^13/106496
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-cos(x)+sec(x))**7,x)
```

```
[Out] Timed out
```

3.333 $\int (-\cos(x) + \sec(x))^{7/2}\,dx$

Optimal. Leaf size=73

$$-\frac{2}{7}\sin^3(x)\tan^2(x)\sqrt{\sin(x)\tan(x)}-\frac{8}{7}\sin(x)\tan^2(x)\sqrt{\sin(x)\tan(x)}-\frac{256}{35}\csc(x)\sqrt{\sin(x)\tan(x)}+\frac{64}{35}\tan(x)\sec(x$$

```
[Out] -256/35*csc(x)*(sin(x)*tan(x))^(1/2)+64/35*sec(x)*(sin(x)*tan(x))^(1/2)*tan
(x)-8/7*sin(x)*(sin(x)*tan(x))^(1/2)*tan(x)^2-2/7*sin(x)^3*(sin(x)*tan(x))^
(1/2)*tan(x)^2
```

Rubi [A] time = 0.11, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.454, Rules used = {4397, 4400, 2598, 2594, 2589}

$$-\frac{2}{7}\sin^3(x)\tan^2(x)\sqrt{\sin(x)\tan(x)}-\frac{8}{7}\sin(x)\tan^2(x)\sqrt{\sin(x)\tan(x)}-\frac{256}{35}\csc(x)\sqrt{\sin(x)\tan(x)}+\frac{64}{35}\tan(x)\sec(x$$

Antiderivative was successfully verified.

```
[In] Int[(-Cos[x] + Sec[x])^(7/2),x]
```

```
[Out] (-256*Csc[x]*Sqrt[Sin[x]*Tan[x]])/35 + (64*Sec[x]*Tan[x]*Sqrt[Sin[x]*Tan[x]
])/35 - (8*Sin[x]*Tan[x]^2*Sqrt[Sin[x]*Tan[x]])/7 - (2*Sin[x]^3*Tan[x]^2*Sq
rt[Sin[x]*Tan[x]])/7
```

Rule 2589

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> -Simp[(b*(a*Sin[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*m
), x] /; FreeQ[{a, b, e, f, m, n}, x] && EqQ[m + n - 1, 0]
```

Rule 2594

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(b*(a*Sin[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*(
n - 1)), x] - Dist[(b^2*(m + n - 1))/(n - 1), Int[(a*Sin[e + f*x])^m*(b*Tan
[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] && Int
egersQ[2*m, 2*n] &&  !(GtQ[m, 1] &&  !IntegerQ[(m - 1)/2])
```

Rule 2598

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> -Simp[(b*(a*Sin[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f
*m), x] + Dist[(a^2*(m + n - 1))/m, Int[(a*Sin[e + f*x])^(m - 2)*(b*Tan[e +
 f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && (GtQ[m, 1] || (EqQ[m, 1] &
```

```
& EqQ[n, 1/2])) && IntegersQ[2*m, 2*n]
```

Rule 4397

```
Int[u_, x_Symbol] :> Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]
```

Rule 4400

```
Int[(u_.)*((v_)^(m_.)*(w_)^(n_.))^(p_), x_Symbol] :> With[{uu = ActivateTri
g[u], vv = ActivateTrig[v], ww = ActivateTrig[w]}, Dist[(vv^m*ww^n)^FracPar
t[p]/(vv^(m*FracPart[p])*ww^(n*FracPart[p])), Int[uu*vv^(m*p)*ww^(n*p), x],
 x]] /; FreeQ[{m, n, p}, x] &&  !IntegerQ[p] && ( !InertTrigFreeQ[v] ||  !I
nertTrigFreeQ[w])
```

Rubi steps

$$\int(-\cos(x)+\sec(x))^{7/2}\,dx=\int(\sin(x)\tan(x))^{7/2}\,dx$$

$$=\frac{\sqrt{\sin(x)\tan(x)}\int\sin^{\frac{7}{2}}(x)\tan^{\frac{7}{2}}(x)\,dx}{\sqrt{\sin(x)}\sqrt{\tan(x)}}$$

$$=-\frac{2}{7}\sin^3(x)\tan^2(x)\sqrt{\sin(x)\tan(x)}+\frac{\left(12\sqrt{\sin(x)\tan(x)}\right)\int\sin^{\frac{3}{2}}(x)\tan^{\frac{7}{2}}(x)\,dx}{7\sqrt{\sin(x)}\sqrt{\tan(x)}}$$

$$=-\frac{8}{7}\sin(x)\tan^2(x)\sqrt{\sin(x)\tan(x)}-\frac{2}{7}\sin^3(x)\tan^2(x)\sqrt{\sin(x)\tan(x)}+\frac{\left(32\sqrt{\sin(x)\right.}{7\sqrt{\sin}}$$

$$=\frac{64}{35}\sec(x)\tan(x)\sqrt{\sin(x)\tan(x)}-\frac{8}{7}\sin(x)\tan^2(x)\sqrt{\sin(x)\tan(x)}-\frac{2}{7}\sin^3(x)\tan^2$$

$$=-\frac{256}{35}\csc(x)\sqrt{\sin(x)\tan(x)}+\frac{64}{35}\sec(x)\tan(x)\sqrt{\sin(x)\tan(x)}-\frac{8}{7}\sin(x)\tan^2(x)\sqrt{}$$

Mathematica [A] time = 0.21, size = 37, normalized size = 0.51

$$\frac{1}{70}\sec(x)\sqrt{\sin(x)\tan(x)}\,(28\tan(x)-512\cot(x)-5(\sin(3x)-23\sin(x))\cos(x))$$

Antiderivative was successfully verified.

```
[In] Integrate[(-Cos[x] + Sec[x])^(7/2),x]
```

```
[Out] (Sec[x]*Sqrt[Sin[x]*Tan[x]]*(-512*Cot[x] - 5*Cos[x]*(-23*Sin[x] + Sin[3*x])
 + 28*Tan[x]))/70
```

fricas [A] time = 0.99, size = 44, normalized size = 0.60

$$\frac{2\left(5\cos(x)^6-35\cos(x)^4-105\cos(x)^2+7\right)\sqrt{-\frac{\cos(x)^2-1}{\cos(x)}}}{35\cos(x)^2\sin(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-cos(x)+sec(x))^(7/2),x, algorithm="fricas")
```

```
[Out] 2/35*(5*cos(x)^6 - 35*cos(x)^4 - 105*cos(x)^2 + 7)*sqrt(-(cos(x)^2 - 1)/cos
(x))/(cos(x)^2*sin(x))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int(-\cos(x)+\sec(x))^{\frac{7}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-cos(x)+sec(x))^(7/2),x, algorithm="giac")
```

```
[Out] integrate((-cos(x) + sec(x))^(7/2), x)
```

maple [B] time = 0.41, size = 603, normalized size = 8.26

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-cos(x)+sec(x))^(7/2),x)
```

```
[Out] 1/70*(-1+cos(x))^2*(-105*cos(x)^4*(-cos(x)/(1+cos(x))^2)^(3/2)*ln(-2*(2*cos
(x)^2*(-cos(x)/(1+cos(x))^2)^(1/2)-cos(x)^2+2*cos(x)-2*(-cos(x)/(1+cos(x))^
2)^(1/2)-1)/sin(x)^2)+105*cos(x)^4*(-cos(x)/(1+cos(x))^2)^(3/2)*ln(-(2*cos(
x)^2*(-cos(x)/(1+cos(x))^2)^(1/2)-cos(x)^2+2*cos(x)-2*(-cos(x)/(1+cos(x))^2
)^(1/2)-1)/sin(x)^2)-315*cos(x)^3*ln(-2*(2*cos(x)^2*(-cos(x)/(1+cos(x))^2)^
(1/2)-cos(x)^2+2*cos(x)-2*(-cos(x)/(1+cos(x))^2)^(1/2)-1)/sin(x)^2)*(-cos(x
)/(1+cos(x))^2)^(3/2)+315*cos(x)^3*ln(-(2*cos(x)^2*(-cos(x)/(1+cos(x))^2)^(
1/2)-cos(x)^2+2*cos(x)-2*(-cos(x)/(1+cos(x))^2)^(1/2)-1)/sin(x)^2)*(-cos(x)
/(1+cos(x))^2)^(3/2)+20*cos(x)^6-315*ln(-2*(2*cos(x)^2*(-cos(x)/(1+cos(x))^
2)^(1/2)-cos(x)^2+2*cos(x)-2*(-cos(x)/(1+cos(x))^2)^(1/2)-1)/sin(x)^2)*cos(
x)^2*(-cos(x)/(1+cos(x))^2)^(3/2)+315*ln(-(2*cos(x)^2*(-cos(x)/(1+cos(x))^2
)^(1/2)-cos(x)^2+2*cos(x)-2*(-cos(x)/(1+cos(x))^2)^(1/2)-1)/sin(x)^2)*cos(x
```

```
)^2*(-cos(x)/(1+cos(x))^2)^(3/2)-105*cos(x)*ln(-2*(2*cos(x)^2*(-cos(x)/(1+c
os(x))^2)^(1/2)-cos(x)^2+2*cos(x)-2*(-cos(x)/(1+cos(x))^2)^(1/2)-1)/sin(x)^
2)*(-cos(x)/(1+cos(x))^2)^(3/2)+105*cos(x)*ln(-(2*cos(x)^2*(-cos(x)/(1+cos(
x))^2)^(1/2)-cos(x)^2+2*cos(x)-2*(-cos(x)/(1+cos(x))^2)^(1/2)-1)/sin(x)^2)*
(-cos(x)/(1+cos(x))^2)^(3/2)-140*cos(x)^4-420*cos(x)^2+28)*cos(x)*(1+cos(x)
)^2*(-(-1+cos(x)^2)/cos(x))^(7/2)/sin(x)^11
```

maxima [A] time = 0.44, size = 82, normalized size = 1.12

$$\frac{128\left(\frac{7\,\sin(x)^4}{(\cos(x)+1)^4} - \frac{7\,\sin(x)^{10}}{(\cos(x)+1)^{10}} + \frac{2\,\sin(x)^{14}}{(\cos(x)+1)^{14}} - 2\right)}{35\left(\frac{\sin(x)}{\cos(x)+1} + 1\right)^{\frac{7}{2}}\left(-\frac{\sin(x)}{\cos(x)+1} + 1\right)^{\frac{7}{2}}\left(\frac{\sin(x)^2}{(\cos(x)+1)^2} + 1\right)^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-cos(x)+sec(x))^(7/2),x, algorithm="maxima")
```

```
[Out] 128/35*(7*sin(x)^4/(cos(x) + 1)^4 - 7*sin(x)^10/(cos(x) + 1)^10 + 2*sin(x)^
14/(cos(x) + 1)^14 - 2)/((sin(x)/(cos(x) + 1) + 1)^(7/2)*(-sin(x)/(cos(x) +
 1) + 1)^(7/2)*(sin(x)^2/(cos(x) + 1)^2 + 1)^(7/2))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \left(\frac{1}{\cos(x)} - \cos(x)\right)^{7/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/cos(x) - cos(x))^(7/2),x)
```

```
[Out] int((1/cos(x) - cos(x))^(7/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-cos(x)+sec(x))**(7/2),x)
```

```
[Out] Timed out
```

3.334 $\int (-\cos(x) + \sec(x))^{5/2} \, dx$

Optimal. Leaf size=50

$$-\frac{2}{5}\sin^2(x)\tan(x)\sqrt{\sin(x)\tan(x)} + \frac{16}{15}\tan(x)\sqrt{\sin(x)\tan(x)} + \frac{64}{15}\cot(x)\sqrt{\sin(x)\tan(x)}$$

```
[Out] 64/15*cot(x)*(sin(x)*tan(x))^(1/2)+16/15*(sin(x)*tan(x))^(1/2)*tan(x)-2/5*s
in(x)^2*(sin(x)*tan(x))^(1/2)*tan(x)
```

Rubi [A] time = 0.09, antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.454, Rules used = {4397, 4400, 2598, 2594, 2589}

$$-\frac{2}{5}\sin^2(x)\tan(x)\sqrt{\sin(x)\tan(x)} + \frac{16}{15}\tan(x)\sqrt{\sin(x)\tan(x)} + \frac{64}{15}\cot(x)\sqrt{\sin(x)\tan(x)}$$

Antiderivative was successfully verified.

```
[In] Int[(-Cos[x] + Sec[x])^(5/2),x]
```

```
[Out] (64*Cot[x]*Sqrt[Sin[x]*Tan[x]])/15 + (16*Tan[x]*Sqrt[Sin[x]*Tan[x]])/15 - (
2*Sin[x]^2*Tan[x]*Sqrt[Sin[x]*Tan[x]])/5
```

Rule 2589

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> -Simp[(b*(a*Sin[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*m
), x] /; FreeQ[{a, b, e, f, m, n}, x] && EqQ[m + n - 1, 0]
```

Rule 2594

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(b*(a*Sin[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*(
n - 1)), x] - Dist[(b^2*(m + n - 1))/(n - 1), Int[(a*Sin[e + f*x])^m*(b*Tan
[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] && Int
egersQ[2*m, 2*n] &&  !(GtQ[m, 1] &&  !IntegerQ[(m - 1)/2])
```

Rule 2598

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> -Simp[(b*(a*Sin[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f
*m), x] + Dist[(a^2*(m + n - 1))/m, Int[(a*Sin[e + f*x])^(m - 2)*(b*Tan[e +
 f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && (GtQ[m, 1] || (EqQ[m, 1] &
& EqQ[n, 1/2])) && IntegersQ[2*m, 2*n]
```

Rule 4397

```
Int[u_, x_Symbol] :> Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]
```

Rule 4400

```
Int[(u_.)*((v_)^(m_.)*(w_)^(n_.))^(p_), x_Symbol] :> With[{uu = ActivateTri
g[u], vv = ActivateTrig[v], ww = ActivateTrig[w]}, Dist[(vv^m*ww^n)^FracPar
t[p]/(vv^(m*FracPart[p])*ww^(n*FracPart[p])), Int[uu*vv^(m*p)*ww^(n*p), x],
 x]] /; FreeQ[{m, n, p}, x] &&  !IntegerQ[p] && ( !InertTrigFreeQ[v] ||  !I
nertTrigFreeQ[w])
```

Rubi steps

$$\begin{aligned}
\int (-\cos(x)+\sec(x))^{5/2}\,dx &= \int (\sin(x)\tan(x))^{5/2}\,dx \\
&= \frac{\sqrt{\sin(x)\tan(x)}\int \sin^{\frac{5}{2}}(x)\tan^{\frac{5}{2}}(x)\,dx}{\sqrt{\sin(x)}\sqrt{\tan(x)}} \\
&= -\frac{2}{5}\sin^2(x)\tan(x)\sqrt{\sin(x)\tan(x)} + \frac{\left(8\sqrt{\sin(x)\tan(x)}\right)\int \sqrt{\sin(x)}\tan^{\frac{5}{2}}(x)\,dx}{5\sqrt{\sin(x)}\sqrt{\tan(x)}} \\
&= \frac{16}{15}\tan(x)\sqrt{\sin(x)\tan(x)} - \frac{2}{5}\sin^2(x)\tan(x)\sqrt{\sin(x)\tan(x)} - \frac{\left(32\sqrt{\sin(x)\tan(x)}\right)\ldots}{15\sqrt{\sin(x}\ldots} \\
&= \frac{64}{15}\cot(x)\sqrt{\sin(x)\tan(x)} + \frac{16}{15}\tan(x)\sqrt{\sin(x)\tan(x)} - \frac{2}{5}\sin^2(x)\tan(x)\sqrt{\sin(x)\tan\ldots}
\end{aligned}$$

Mathematica [A] time = 0.08, size = 29, normalized size = 0.58

$$\frac{2}{15}\tan(x)\sqrt{\sin(x)\tan(x)}\left(3\cos^2(x)+32\cot^2(x)+5\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-Cos[x] + Sec[x])^(5/2),x]
[Out] (2*(5 + 3*Cos[x]^2 + 32*Cot[x]^2)*Tan[x]*Sqrt[Sin[x]*Tan[x]])/15
```

fricas [A] time = 0.86, size = 38, normalized size = 0.76

$$-\frac{2\left(3\cos(x)^4-30\cos(x)^2-5\right)\sqrt{-\frac{\cos(x)^2-1}{\cos(x)}}}{15\cos(x)\sin(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-cos(x)+sec(x))^(5/2),x, algorithm="fricas")
```

```
[Out] -2/15*(3*cos(x)^4 - 30*cos(x)^2 - 5)*sqrt(-(cos(x)^2 - 1)/cos(x))/(cos(x)*s
in(x))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (-\cos(x) + \sec(x))^{\frac{5}{2}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-cos(x)+sec(x))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((-cos(x) + sec(x))^(5/2), x)
```

maple [B] time = 0.33, size = 321, normalized size = 6.42

$$-\frac{(-1+\cos(x))^2 \left(6\left(\cos^4(x)\right) - 15\left(\cos^2(x)\right)\sqrt{-\frac{\cos(x)}{(1+\cos(x))^2}}\ \ln\left(-\frac{2\left(\cos^2(x)\right)\sqrt{-\frac{\cos(x)}{(1+\cos(x))^2}} - \left(\cos^2(x)\right) + 2\cos(x) - 2\sqrt{-\frac{\cos(x)}{(1+\cos(x))^2}}}{\sin(x)^2}\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-cos(x)+sec(x))^(5/2),x)
```

```
[Out] -1/15*(-1+cos(x))^2*(6*cos(x)^4-15*cos(x)^2*(-cos(x)/(1+cos(x))^2)^(1/2)*ln
(-(2*cos(x)^2*(-cos(x)/(1+cos(x))^2)^(1/2)-cos(x)^2+2*cos(x)-2*(-cos(x)/(1+
cos(x))^2)^(1/2)-1)/sin(x)^2)+15*cos(x)^2*(-cos(x)/(1+cos(x))^2)^(1/2)*ln(-
2*(2*cos(x)^2*(-cos(x)/(1+cos(x))^2)^(1/2)-cos(x)^2+2*cos(x)-2*(-cos(x)/(1+
cos(x))^2)^(1/2)-1)/sin(x)^2)-15*cos(x)*(-cos(x)/(1+cos(x))^2)^(1/2)*ln(-(2
*cos(x)^2*(-cos(x)/(1+cos(x))^2)^(1/2)-cos(x)^2+2*cos(x)-2*(-cos(x)/(1+cos(
x))^2)^(1/2)-1)/sin(x)^2)+15*cos(x)*(-cos(x)/(1+cos(x))^2)^(1/2)*ln(-2*(2*c
os(x)^2*(-cos(x)/(1+cos(x))^2)^(1/2)-cos(x)^2+2*cos(x)-2*(-cos(x)/(1+cos(x)
)^2)^(1/2)-1)/sin(x)^2)-60*cos(x)^2-10)*cos(x)*(1+cos(x))^2*(-(-1+cos(x)^2)
/cos(x))^(5/2)/sin(x)^9
```

maxima [B] time = 0.43, size = 82, normalized size = 1.64

$$-\frac{32\left(\frac{5\sin(x)^4}{(\cos(x)+1)^4} - \frac{5\sin(x)^6}{(\cos(x)+1)^6} + \frac{2\sin(x)^{10}}{(\cos(x)+1)^{10}} - 2\right)}{15\left(\frac{\sin(x)}{\cos(x)+1} + 1\right)^{\frac{5}{2}}\left(-\frac{\sin(x)}{\cos(x)+1} + 1\right)^{\frac{5}{2}}\left(\frac{\sin(x)^2}{(\cos(x)+1)^2} + 1\right)^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-cos(x)+sec(x))^(5/2),x, algorithm="maxima")
```

```
[Out] -32/15*(5*sin(x)^4/(cos(x) + 1)^4 - 5*sin(x)^6/(cos(x) + 1)^6 + 2*sin(x)^10
/(cos(x) + 1)^10 - 2)/((sin(x)/(cos(x) + 1) + 1)^(5/2)*(-sin(x)/(cos(x) + 1
) + 1)^(5/2)*(sin(x)^2/(cos(x) + 1)^2 + 1)^(5/2))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \left(\frac{1}{\cos(x)} - \cos(x)\right)^{5/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/cos(x) - cos(x))^(5/2),x)
```

```
[Out] int((1/cos(x) - cos(x))^(5/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-cos(x)+sec(x))**(5/2),x)
```

```
[Out] Timed out
```

3.335 $\int (-\cos(x) + \sec(x))^{3/2}\, dx$

Optimal. Leaf size=31

$$\frac{8}{3}\csc(x)\sqrt{\sin(x)\tan(x)} - \frac{2}{3}\sin(x)\sqrt{\sin(x)\tan(x)}$$

```
[Out] 8/3*csc(x)*(sin(x)*tan(x))^(1/2)-2/3*sin(x)*(sin(x)*tan(x))^(1/2)
```

Rubi [A] time = 0.06, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {4397, 4400, 2598, 2589}

$$\frac{8}{3}\csc(x)\sqrt{\sin(x)\tan(x)} - \frac{2}{3}\sin(x)\sqrt{\sin(x)\tan(x)}$$

Antiderivative was successfully verified.

```
[In] Int[(-Cos[x] + Sec[x])^(3/2),x]
```

```
[Out] (8*Csc[x]*Sqrt[Sin[x]*Tan[x]])/3 - (2*Sin[x]*Sqrt[Sin[x]*Tan[x]])/3
```

Rule 2589

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> -Simp[(b*(a*Sin[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*m
), x] /; FreeQ[{a, b, e, f, m, n}, x] && EqQ[m + n - 1, 0]
```

Rule 2598

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> -Simp[(b*(a*Sin[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f
*m), x] + Dist[(a^2*(m + n - 1))/m, Int[(a*Sin[e + f*x])^(m - 2)*(b*Tan[e +
 f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && (GtQ[m, 1] || (EqQ[m, 1] &
& EqQ[n, 1/2])) && IntegersQ[2*m, 2*n]
```

Rule 4397

```
Int[u_, x_Symbol] :> Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]
```

Rule 4400

```
Int[(u_.)*((v_)^(m_.)*(w_)^(n_.))^(p_), x_Symbol] :> With[{uu = ActivateTri
g[u], vv = ActivateTrig[v], ww = ActivateTrig[w]}, Dist[(vv^m*ww^n)^FracPar
t[p]/(vv^(m*FracPart[p])*ww^(n*FracPart[p])), Int[uu*vv^(m*p)*ww^(n*p), x],
 x]] /; FreeQ[{m, n, p}, x] && !IntegerQ[p] && ( !InertTrigFreeQ[v] ||  !I
```

```
nertTrigFreeQ[w])
```

Rubi steps

$$\begin{aligned}\int (-\cos(x)+\sec(x))^{3/2}\,dx &= \int (\sin(x)\tan(x))^{3/2}\,dx\\ &= \frac{\sqrt{\sin(x)\tan(x)}\int \sin^{\frac{3}{2}}(x)\tan^{\frac{3}{2}}(x)\,dx}{\sqrt{\sin(x)}\sqrt{\tan(x)}}\\ &= -\frac{2}{3}\sin(x)\sqrt{\sin(x)\tan(x)}+\frac{\left(4\sqrt{\sin(x)\tan(x)}\right)\int \frac{\tan^{\frac{3}{2}}(x)}{\sqrt{\sin(x)}}\,dx}{3\sqrt{\sin(x)}\sqrt{\tan(x)}}\\ &= \frac{8}{3}\csc(x)\sqrt{\sin(x)\tan(x)}-\frac{2}{3}\sin(x)\sqrt{\sin(x)\tan(x)}\end{aligned}$$

Mathematica [A] time = 0.04, size = 23, normalized size = 0.74

$$\frac{2}{3}\sin(x)\left(4\csc^2(x)-1\right)\sqrt{\sin(x)\tan(x)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-Cos[x] + Sec[x])^(3/2),x]
[Out] (2*(-1 + 4*Csc[x]^2)*Sin[x]*Sqrt[Sin[x]*Tan[x]])/3
```

fricas [A] time = 1.25, size = 26, normalized size = 0.84

$$\frac{2\left(\cos(x)^2+3\right)\sqrt{-\frac{\cos(x)^2-1}{\cos(x)}}}{3\,\sin(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-cos(x)+sec(x))^(3/2),x, algorithm="fricas")
[Out] 2/3*(cos(x)^2 + 3)*sqrt(-(cos(x)^2 - 1)/cos(x))/sin(x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (-\cos(x)+\sec(x))^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-cos(x)+sec(x))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((-cos(x) + sec(x))^(3/2), x)
```

maple [B] time = 0.27, size = 584, normalized size = 18.84

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-cos(x)+sec(x))^(3/2),x)
```

```
[Out] 1/6*(-1+cos(x))^2*(3*cos(x)^3*ln(-2*(2*cos(x)^2*(-cos(x)/(1+cos(x))^2)^(1/2
)-cos(x)^2+2*cos(x)-2*(-cos(x)/(1+cos(x))^2)^(1/2)-1)/sin(x)^2)*(-cos(x)/(1
+cos(x))^2)^(3/2)-3*cos(x)^3*ln(-(2*cos(x)^2*(-cos(x)/(1+cos(x))^2)^(1/2)-c
os(x)^2+2*cos(x)-2*(-cos(x)/(1+cos(x))^2)^(1/2)-1)/sin(x)^2)*(-cos(x)/(1+co
s(x))^2)^(3/2)+9*ln(-2*(2*cos(x)^2*(-cos(x)/(1+cos(x))^2)^(1/2)-cos(x)^2+2*
cos(x)-2*(-cos(x)/(1+cos(x))^2)^(1/2)-1)/sin(x)^2)*cos(x)^2*(-cos(x)/(1+cos
(x))^2)^(3/2)-9*ln(-(2*cos(x)^2*(-cos(x)/(1+cos(x))^2)^(1/2)-cos(x)^2+2*cos
(x)-2*(-cos(x)/(1+cos(x))^2)^(1/2)-1)/sin(x)^2)*cos(x)^2*(-cos(x)/(1+cos(x)
)^2)^(3/2)+9*cos(x)*ln(-2*(2*cos(x)^2*(-cos(x)/(1+cos(x))^2)^(1/2)-cos(x)^2
+2*cos(x)-2*(-cos(x)/(1+cos(x))^2)^(1/2)-1)/sin(x)^2)*(-cos(x)/(1+cos(x))^2
)^(3/2)-9*cos(x)*ln(-(2*cos(x)^2*(-cos(x)/(1+cos(x))^2)^(1/2)-cos(x)^2+2*co
s(x)-2*(-cos(x)/(1+cos(x))^2)^(1/2)-1)/sin(x)^2)*(-cos(x)/(1+cos(x))^2)^(3/
2)+3*ln(-2*(2*cos(x)^2*(-cos(x)/(1+cos(x))^2)^(1/2)-cos(x)^2+2*cos(x)-2*(-c
os(x)/(1+cos(x))^2)^(1/2)-1)/sin(x)^2)*(-cos(x)/(1+cos(x))^2)^(3/2)-3*ln(-(
2*cos(x)^2*(-cos(x)/(1+cos(x))^2)^(1/2)-cos(x)^2+2*cos(x)-2*(-cos(x)/(1+cos
(x))^2)^(1/2)-1)/sin(x)^2)*(-cos(x)/(1+cos(x))^2)^(3/2)+4*cos(x)^3+12*cos(x
))*(1+cos(x))^2*(-(-1+cos(x)^2)/cos(x))^(3/2)/sin(x)^7
```

maxima [B] time = 0.43, size = 57, normalized size = 1.84

$$-\frac{8\left(\frac{\sin(x)^6}{(\cos(x)+1)^6}-1\right)}{3\left(\frac{\sin(x)}{\cos(x)+1}+1\right)^{\frac{3}{2}}\left(-\frac{\sin(x)}{\cos(x)+1}+1\right)^{\frac{3}{2}}\left(\frac{\sin(x)^2}{(\cos(x)+1)^2}+1\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-cos(x)+sec(x))^(3/2),x, algorithm="maxima")
```

```
[Out] -8/3*(sin(x)^6/(cos(x) + 1)^6 - 1)/((sin(x)/(cos(x) + 1) + 1)^(3/2)*(-sin(x
)/(cos(x) + 1) + 1)^(3/2)*(sin(x)^2/(cos(x) + 1)^2 + 1)^(3/2))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int \left(\frac{1}{\cos(x)}-\cos(x)\right)^{3/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/cos(x) - cos(x))^(3/2),x)
```

```
[Out] int((1/cos(x) - cos(x))^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (-\cos(x) + \sec(x))^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-cos(x)+sec(x))**(3/2),x)
```

```
[Out] Integral((-cos(x) + sec(x))**(3/2), x)
```

3.336 $\int \sqrt{-\cos(x)+\sec(x)}\,dx$

Optimal. Leaf size=13

$$-2\cot(x)\sqrt{\sin(x)\tan(x)}$$

```
[Out] -2*cot(x)*(sin(x)*tan(x))^(1/2)
```

Rubi [A] time = 0.04, antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {4397, 4400, 2589}

$$-2\cot(x)\sqrt{\sin(x)\tan(x)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[-Cos[x] + Sec[x]],x]
```

```
[Out] -2*Cot[x]*Sqrt[Sin[x]*Tan[x]]
```

Rule 2589

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> -Simp[(b*(a*Sin[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*m
), x] /; FreeQ[{a, b, e, f, m, n}, x] && EqQ[m + n - 1, 0]
```

Rule 4397

```
Int[u_, x_Symbol] :> Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]
```

Rule 4400

```
Int[(u_.)*((v_)^(m_.)*(w_)^(n_.))^(p_), x_Symbol] :> With[{uu = ActivateTri
g[u], vv = ActivateTrig[v], ww = ActivateTrig[w]}, Dist[(vv^m*ww^n)^FracPar
t[p]/(vv^(m*FracPart[p])*ww^(n*FracPart[p])), Int[uu*vv^(m*p)*ww^(n*p), x],
 x]] /; FreeQ[{m, n, p}, x] && !IntegerQ[p] && ( !InertTrigFreeQ[v] ||  !I
nertTrigFreeQ[w])
```

Rubi steps

$$\int \sqrt{-\cos(x)+\sec(x)}\,dx = \int \sqrt{\sin(x)\tan(x)}\,dx$$
$$= \frac{\sqrt{\sin(x)\tan(x)}\int \sqrt{\sin(x)}\sqrt{\tan(x)}\,dx}{\sqrt{\sin(x)}\sqrt{\tan(x)}}$$
$$= -2\cot(x)\sqrt{\sin(x)\tan(x)}$$

Mathematica [A] time = 0.03, size = 13, normalized size = 1.00

$$-2\cot(x)\sqrt{\sin(x)\tan(x)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[-Cos[x] + Sec[x]],x]
[Out] -2*Cot[x]*Sqrt[Sin[x]*Tan[x]]
```

fricas [A] time = 1.26, size = 22, normalized size = 1.69

$$-\frac{2\sqrt{-\frac{\cos(x)^2-1}{\cos(x)}}\cos(x)}{\sin(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-cos(x)+sec(x))^(1/2),x, algorithm="fricas")
[Out] -2*sqrt(-(cos(x)^2 - 1)/cos(x))*cos(x)/sin(x)
```

giac [B] time = 0.20, size = 46, normalized size = 3.54

$$-\frac{4\,\mathrm{sgn}\left(-\tan\left(\frac{1}{2}x\right)^3-\tan\left(\frac{1}{2}x\right)\right)\mathrm{sgn}\left(\cos(x)\right)}{\frac{\sqrt{-\tan\left(\frac{1}{2}x\right)^4+1}-1}{\tan\left(\frac{1}{2}x\right)^2}-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-cos(x)+sec(x))^(1/2),x, algorithm="giac")
[Out] -4*sgn(-tan(1/2*x)^3 - tan(1/2*x))*sgn(cos(x))/((sqrt(-tan(1/2*x)^4 + 1) -
1)/tan(1/2*x)^2 - 1)
```

maple [B] time = 0.31, size = 174, normalized size = 13.38

$$\frac{(-1+\cos(x))\left(4\cos(x)\sqrt{-\frac{\cos(x)}{(1+\cos(x))^2}}+4\sqrt{-\frac{\cos(x)}{(1+\cos(x))^2}}+\ln\left(-\frac{2\left(2\left(\cos^2(x)\right)\sqrt{-\frac{\cos(x)}{(1+\cos(x))^2}}-\left(\cos^2(x)\right)+2\cos(x)-2\sqrt{-\frac{\cos(x)}{(1+\cos(x))^2}}\right.}{\sin(x)^2}\right.\right.}{2\sqrt{-\frac{\cos(x)}{(1+\cos(x))^2}}\sin(x)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-cos(x)+sec(x))^(1/2),x)
```

```
[Out] 1/2*(-1+cos(x))*(4*cos(x)*(-cos(x)/(1+cos(x))^2)^(1/2)+4*(-cos(x)/(1+cos(x)
)^2)^(1/2)+ln(-2*(2*cos(x)^2*(-cos(x)/(1+cos(x))^2)^(1/2)-cos(x)^2+2*cos(x)
-2*(-cos(x)/(1+cos(x))^2)^(1/2)-1)/sin(x)^2)-ln(-(2*cos(x)^2*(-cos(x)/(1+co
s(x))^2)^(1/2)-cos(x)^2+2*cos(x)-2*(-cos(x)/(1+cos(x))^2)^(1/2)-1)/sin(x)^2
))*cos(x)*(-(-1+cos(x)^2)/cos(x))^(1/2)/(-cos(x)/(1+cos(x))^2)^(1/2)/sin(x)
^3
```

maxima [B] time = 0.43, size = 57, normalized size = 4.38

$$\frac{2\left(\frac{\sin(x)^2}{(\cos(x)+1)^2}-1\right)}{\sqrt{\frac{\sin(x)}{\cos(x)+1}+1}\sqrt{-\frac{\sin(x)}{\cos(x)+1}+1}\sqrt{\frac{\sin(x)^2}{(\cos(x)+1)^2}+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-cos(x)+sec(x))^(1/2),x, algorithm="maxima")
```

```
[Out] 2*(sin(x)^2/(cos(x) + 1)^2 - 1)/(sqrt(sin(x)/(cos(x) + 1) + 1)*sqrt(-sin(x)
/(cos(x) + 1) + 1)*sqrt(sin(x)^2/(cos(x) + 1)^2 + 1))
```

mupad [B] time = 2.42, size = 20, normalized size = 1.54

$$-\frac{2\sin(x)}{\sqrt{\frac{1}{\cos(x)}}\sqrt{1-\cos(x)^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/cos(x) - cos(x))^(1/2),x)
```

```
[Out] -(2*sin(x))/((1/cos(x))^(1/2)*(1 - cos(x)^2)^(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{-\cos(x) + \sec(x)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-cos(x)+sec(x))**(1/2),x)
```

```
[Out] Integral(sqrt(-cos(x) + sec(x)), x)
```

3.337 $\int \frac{1}{\sqrt{-\cos(x)+\sec(x)}}\,dx$

Optimal. Leaf size=52

$$\frac{\sin(x)\tan^{-1}\left(\sqrt{\cos(x)}\right)}{\sqrt{\cos(x)}\sqrt{\sin(x)\tan(x)}}-\frac{\sin(x)\tanh^{-1}\left(\sqrt{\cos(x)}\right)}{\sqrt{\cos(x)}\sqrt{\sin(x)\tan(x)}}$$

```
[Out] arctan(cos(x)^(1/2))*sin(x)/cos(x)^(1/2)/(sin(x)*tan(x))^(1/2)-arctanh(cos(
x)^(1/2))*sin(x)/cos(x)^(1/2)/(sin(x)*tan(x))^(1/2)
```

Rubi [A] time = 0.08, antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.727, Rules used = {4397, 4400, 2601, 2565, 329, 298, 203, 206}

$$\frac{\sin(x)\tan^{-1}\left(\sqrt{\cos(x)}\right)}{\sqrt{\cos(x)}\sqrt{\sin(x)\tan(x)}}-\frac{\sin(x)\tanh^{-1}\left(\sqrt{\cos(x)}\right)}{\sqrt{\cos(x)}\sqrt{\sin(x)\tan(x)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[-Cos[x] + Sec[x]],x]
```

```
[Out] (ArcTan[Sqrt[Cos[x]]]*Sin[x])/(Sqrt[Cos[x]]*Sqrt[Sin[x]*Tan[x]]) - (ArcTanh
[Sqrt[Cos[x]]]*Sin[x])/(Sqrt[Cos[x]]*Sqrt[Sin[x]*Tan[x]])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2565

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> -Dist[(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 2601

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Dist[(Cos[e + f*x]^n*(b*Tan[e + f*x])^n)/(a*Sin[e + f*x])
^n, Int[(a*Sin[e + f*x])^(m + n)/Cos[e + f*x]^n, x], x] /; FreeQ[{a, b, e,
f, m, n}, x] &&  !IntegerQ[n] && (ILtQ[m, 0] || (EqQ[m, 1] && EqQ[n, -2^(-1
)]) || IntegersQ[m - 1/2, n - 1/2])
```

Rule 4397

```
Int[u_, x_Symbol] :> Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]
```

Rule 4400

```
Int[(u_.)*((v_)^(m_.)*(w_)^(n_.))^(p_), x_Symbol] :> With[{uu = ActivateTri
g[u], vv = ActivateTrig[v], ww = ActivateTrig[w]}, Dist[(vv^m*ww^n)^FracPar
t[p]/(vv^(m*FracPart[p])*ww^(n*FracPart[p])), Int[uu*vv^(m*p)*ww^(n*p), x],
 x]] /; FreeQ[{m, n, p}, x] &&  !IntegerQ[p] && ( !InertTrigFreeQ[v] ||  !I
nertTrigFreeQ[w])
```

Rubi steps

$$\int \frac{1}{\sqrt{-\cos(x)+\sec(x)}}\,dx = \int \frac{1}{\sqrt{\sin(x)\tan(x)}}\,dx$$
$$= \frac{\left(\sqrt{\sin(x)}\sqrt{\tan(x)}\right)\int \frac{1}{\sqrt{\sin(x)}\sqrt{\tan(x)}}\,dx}{\sqrt{\sin(x)\tan(x)}}$$
$$= \frac{\sin(x)\int \sqrt{\cos(x)}\csc(x)\,dx}{\sqrt{\cos(x)}\sqrt{\sin(x)\tan(x)}}$$
$$= -\frac{\sin(x)\,\text{Subst}\left(\int \frac{\sqrt{x}}{1-x^2}\,dx, x, \cos(x)\right)}{\sqrt{\cos(x)}\sqrt{\sin(x)\tan(x)}}$$
$$= -\frac{(2\sin(x))\,\text{Subst}\left(\int \frac{x^2}{1-x^4}\,dx, x, \sqrt{\cos(x)}\right)}{\sqrt{\cos(x)}\sqrt{\sin(x)\tan(x)}}$$
$$= -\frac{\sin(x)\,\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \sqrt{\cos(x)}\right)}{\sqrt{\cos(x)}\sqrt{\sin(x)\tan(x)}} + \frac{\sin(x)\,\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \sqrt{\cos(x)}\right)}{\sqrt{\cos(x)}\sqrt{\sin(x)\tan(x)}}$$
$$= \frac{\tan^{-1}\left(\sqrt{\cos(x)}\right)\sin(x)}{\sqrt{\cos(x)}\sqrt{\sin(x)\tan(x)}} - \frac{\tanh^{-1}\left(\sqrt{\cos(x)}\right)\sin(x)}{\sqrt{\cos(x)}\sqrt{\sin(x)\tan(x)}}$$

Mathematica [A] time = 0.25, size = 43, normalized size = 0.83

$$\frac{\cos(x)\cot(x)\sqrt{\sin(x)\tan(x)}\left(\tan^{-1}\left(\sqrt[4]{\cos^2(x)}\right)-\tanh^{-1}\left(\sqrt[4]{\cos^2(x)}\right)\right)}{\cos^2(x)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[-Cos[x] + Sec[x]],x]
[Out] ((ArcTan[(Cos[x]^2)^(1/4)] - ArcTanh[(Cos[x]^2)^(1/4)])*Cos[x]*Cot[x]*Sqrt[
Sin[x]*Tan[x]])/(Cos[x]^2)^(3/4)
```

fricas [A] time = 1.00, size = 72, normalized size = 1.38

$$-\frac{1}{2}\arctan\left(\frac{2\sqrt{-\frac{\cos(x)^2-1}{\cos(x)}}\cos(x)}{(\cos(x)-1)\sin(x)}\right)+\frac{1}{2}\log\left(\frac{(\cos(x)+1)\sin(x)-2\sqrt{-\frac{\cos(x)^2-1}{\cos(x)}}\cos(x)}{(\cos(x)-1)\sin(x)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-cos(x)+sec(x))^(1/2),x, algorithm="fricas")
```

```
[Out] -1/2*arctan(2*sqrt(-(cos(x)^2 - 1)/cos(x))*cos(x)/((cos(x) - 1)*sin(x))) +
1/2*log(((cos(x) + 1)*sin(x) - 2*sqrt(-(cos(x)^2 - 1)/cos(x))*cos(x))/((cos
(x) - 1)*sin(x)))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-\cos(x)+\sec(x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-cos(x)+sec(x))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(-cos(x) + sec(x)), x)
```

maple [B] time = 0.28, size = 105, normalized size = 2.02

$$-\frac{\left(\arctan\left(\frac{1}{2\sqrt{-\frac{\cos(x)}{(1+\cos(x))^2}}}\right)+\ln\left(-\frac{2\left(\cos^2(x)\right)\sqrt{-\frac{\cos(x)}{(1+\cos(x))^2}}-\left(\cos^2(x)\right)+2\cos(x)-2\sqrt{-\frac{\cos(x)}{(1+\cos(x))^2}}-1}{\sin(x)^2}\right)\right)(1+\cos(x))\sqrt{-\frac{\cos(x)}{(1+\cos(x))^2}}}{2\sin(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-cos(x)+sec(x))^(1/2),x)
```

```
[Out] -1/2*(arctan(1/2/(-cos(x)/(1+cos(x))^2)^(1/2))+ln(-(2*cos(x)^2*(-cos(x)/(1+
cos(x))^2)^(1/2)-cos(x)^2+2*cos(x)-2*(-cos(x)/(1+cos(x))^2)^(1/2)-1)/sin(x)
^2))*(1+cos(x))*(-cos(x)/(1+cos(x))^2)^(1/2)*((1-cos(x)^2)/cos(x))^(1/2)/si
n(x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-\cos(x)+\sec(x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-cos(x)+sec(x))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(-cos(x) + sec(x)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{\sqrt{\frac{1}{\cos(x)}-\cos(x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1/cos(x) - cos(x))^(1/2),x)
```

```
[Out] int(1/(1/cos(x) - cos(x))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-\cos(x) + \sec(x)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-cos(x)+sec(x))**(1/2),x)
```

```
[Out] Integral(1/sqrt(-cos(x) + sec(x)), x)
```

3.338 $\int \frac{1}{(-\cos(x)+\sec(x))^{3/2}}\,dx$

Optimal. Leaf size=72

$$\frac{\sin(x)\tan^{-1}\left(\sqrt{\cos(x)}\right)}{4\sqrt{\cos(x)}\sqrt{\sin(x)\tan(x)}}-\frac{\csc(x)}{2\sqrt{\sin(x)\tan(x)}}+\frac{\sin(x)\tanh^{-1}\left(\sqrt{\cos(x)}\right)}{4\sqrt{\cos(x)}\sqrt{\sin(x)\tan(x)}}$$

```
[Out] -1/2*csc(x)/(sin(x)*tan(x))^(1/2)+1/4*arctan(cos(x)^(1/2))*sin(x)/cos(x)^(1
/2)/(sin(x)*tan(x))^(1/2)+1/4*arctanh(cos(x)^(1/2))*sin(x)/cos(x)^(1/2)/(si
n(x)*tan(x))^(1/2)
```

Rubi [A] time = 0.09, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.818, Rules used = {4397, 4400, 2597, 2601, 2565, 329, 212, 206, 203}

$$\frac{\sin(x)\tan^{-1}\left(\sqrt{\cos(x)}\right)}{4\sqrt{\cos(x)}\sqrt{\sin(x)\tan(x)}}-\frac{\csc(x)}{2\sqrt{\sin(x)\tan(x)}}+\frac{\sin(x)\tanh^{-1}\left(\sqrt{\cos(x)}\right)}{4\sqrt{\cos(x)}\sqrt{\sin(x)\tan(x)}}$$

Antiderivative was successfully verified.

```
[In] Int[(-Cos[x] + Sec[x])^(-3/2),x]
```

```
[Out] -Csc[x]/(2*Sqrt[Sin[x]*Tan[x]]) + (ArcTan[Sqrt[Cos[x]]]*Sin[x])/(4*Sqrt[Cos
[x]]*Sqrt[Sin[x]*Tan[x]]) + (ArcTanh[Sqrt[Cos[x]]]*Sin[x])/(4*Sqrt[Cos[x]]*
Sqrt[Sin[x]*Tan[x]])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2565

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> -Dist[(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 2597

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[((a*Sin[e + f*x])^m*(b*Tan[e + f*x])^(n + 1))/(b*f*(
m + n + 1)), x] - Dist[(n + 1)/(b^2*(m + n + 1)), Int[(a*Sin[e + f*x])^m*(b
*Tan[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && LtQ[n, -1] &
& NeQ[m + n + 1, 0] && IntegersQ[2*m, 2*n] &&  !(EqQ[n, -3/2] && EqQ[m, 1])
```

Rule 2601

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Dist[(Cos[e + f*x]^n*(b*Tan[e + f*x])^n)/(a*Sin[e + f*x])
^n, Int[(a*Sin[e + f*x])^(m + n)/Cos[e + f*x]^n, x], x] /; FreeQ[{a, b, e,
f, m, n}, x] &&  !IntegerQ[n] && (ILtQ[m, 0] || (EqQ[m, 1] && EqQ[n, -2^(-1
)]) || IntegersQ[m - 1/2, n - 1/2])
```

Rule 4397

```
Int[u_, x_Symbol] :> Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]
```

Rule 4400

```
Int[(u_.)*((v_)^(m_.)*(w_)^(n_.))^(p_), x_Symbol] :> With[{uu = ActivateTri
g[u], vv = ActivateTrig[v], ww = ActivateTrig[w]}, Dist[(vv^m*ww^n)^FracPar
t[p]/(vv^(m*FracPart[p])*ww^(n*FracPart[p])), Int[uu*vv^(m*p)*ww^(n*p), x],
 x]] /; FreeQ[{m, n, p}, x] &&  !IntegerQ[p] && ( !InertTrigFreeQ[v] ||  !I
nertTrigFreeQ[w])
```

Rubi steps

$$\int \frac{1}{(-\cos(x)+\sec(x))^{3/2}}\,dx = \int \frac{1}{(\sin(x)\tan(x))^{3/2}}\,dx$$
$$= \frac{\left(\sqrt{\sin(x)}\sqrt{\tan(x)}\right)\int \frac{1}{\sin^{\frac{3}{2}}(x)\tan^{\frac{3}{2}}(x)}\,dx}{\sqrt{\sin(x)\tan(x)}}$$
$$= -\frac{\csc(x)}{2\sqrt{\sin(x)\tan(x)}} - \frac{\left(\sqrt{\sin(x)}\sqrt{\tan(x)}\right)\int \frac{\sqrt{\tan(x)}}{\sin^{\frac{3}{2}}(x)}\,dx}{4\sqrt{\sin(x)\tan(x)}}$$
$$= -\frac{\csc(x)}{2\sqrt{\sin(x)\tan(x)}} - \frac{\sin(x)\int \frac{\csc(x)}{\sqrt{\cos(x)}}\,dx}{4\sqrt{\cos(x)}\sqrt{\sin(x)\tan(x)}}$$
$$= -\frac{\csc(x)}{2\sqrt{\sin(x)\tan(x)}} + \frac{\sin(x)\,\mathrm{Subst}\left(\int \frac{1}{\sqrt{x}(1-x^2)}\,dx, x, \cos(x)\right)}{4\sqrt{\cos(x)}\sqrt{\sin(x)\tan(x)}}$$
$$= -\frac{\csc(x)}{2\sqrt{\sin(x)\tan(x)}} + \frac{\sin(x)\,\mathrm{Subst}\left(\int \frac{1}{1-x^4}\,dx, x, \sqrt{\cos(x)}\right)}{2\sqrt{\cos(x)}\sqrt{\sin(x)\tan(x)}}$$
$$= -\frac{\csc(x)}{2\sqrt{\sin(x)\tan(x)}} + \frac{\sin(x)\,\mathrm{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \sqrt{\cos(x)}\right)}{4\sqrt{\cos(x)}\sqrt{\sin(x)\tan(x)}} + \frac{\sin(x)\,\mathrm{Subst}\left(\int \frac{1}{1+x^2}\right.}{4\sqrt{\cos(x)}\sqrt{\sin}}$$
$$= -\frac{\csc(x)}{2\sqrt{\sin(x)\tan(x)}} + \frac{\tan^{-1}\left(\sqrt{\cos(x)}\right)\sin(x)}{4\sqrt{\cos(x)}\sqrt{\sin(x)\tan(x)}} + \frac{\tanh^{-1}\left(\sqrt{\cos(x)}\right)\sin(x)}{4\sqrt{\cos(x)}\sqrt{\sin(x)\tan(x)}}$$

Mathematica [A] time = 0.18, size = 56, normalized size = 0.78

$$\frac{\cot(x)\sqrt{\sin(x)\tan(x)}\left(\tan^{-1}\left(\sqrt[4]{\cos^2(x)}\right) - 2\sqrt[4]{\cos^2(x)}\csc^2(x) + \tanh^{-1}\left(\sqrt[4]{\cos^2(x)}\right)\right)}{4\sqrt[4]{\cos^2(x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-Cos[x] + Sec[x])^(-3/2),x]
```

```
[Out] (Cot[x]*(ArcTan[(Cos[x]^2)^(1/4)] + ArcTanh[(Cos[x]^2)^(1/4)] - 2*(Cos[x]^2
)^(1/4)*Csc[x]^2)*Sqrt[Sin[x]*Tan[x]])/(4*(Cos[x]^2)^(1/4))
```

fricas [B] time = 0.97, size = 119, normalized size = 1.65

$$-\frac{\left(\cos(x)^2-1\right)\arctan\left(\frac{2\sqrt{-\frac{\cos(x)^2-1}{\cos(x)}}\cos(x)}{(\cos(x)-1)\sin(x)}\right)\sin(x)-\left(\cos(x)^2-1\right)\log\left(\frac{(\cos(x)+1)\sin(x)+2\sqrt{-\frac{\cos(x)^2-1}{\cos(x)}}\cos(x)}{(\cos(x)-1)\sin(x)}\right)\sin(x)-4}{8\left(\cos(x)^2-1\right)\sin(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-cos(x)+sec(x))^(3/2),x, algorithm="fricas")
```

```
[Out] -1/8*((cos(x)^2 - 1)*arctan(2*sqrt(-(cos(x)^2 - 1)/cos(x))*cos(x)/((cos(x)
- 1)*sin(x)))*sin(x) - (cos(x)^2 - 1)*log(((cos(x) + 1)*sin(x) + 2*sqrt(-(c
os(x)^2 - 1)/cos(x))*cos(x))/((cos(x) - 1)*sin(x)))*sin(x) - 4*sqrt(-(cos(x
)^2 - 1)/cos(x))*cos(x))/((cos(x)^2 - 1)*sin(x))
```

giac [B] time = 0.37, size = 114, normalized size = 1.58

$$-\frac{\frac{\tan\left(\frac{1}{2}x\right)^2}{\sqrt{-\tan\left(\frac{1}{2}x\right)^4+1}-1}-2\sqrt{-\tan\left(\frac{1}{2}x\right)^4+1}-\frac{\sqrt{-\tan\left(\frac{1}{2}x\right)^4+1}-1}{\tan\left(\frac{1}{2}x\right)^2}-2\arcsin\left(\tan\left(\frac{1}{2}x\right)^2\right)-2\log\left(-\frac{\sqrt{-\tan\left(\frac{1}{2}x\right)^4+1}-1}{\tan\left(\frac{1}{2}x\right)^2}\right)}{16\,\mathrm{sgn}\left(\tan\left(\frac{1}{2}x\right)^4-1\right)\mathrm{sgn}\left(\tan\left(\frac{1}{2}x\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-cos(x)+sec(x))^(3/2),x, algorithm="giac")
```

```
[Out] -1/16*(tan(1/2*x)^2/(sqrt(-tan(1/2*x)^4 + 1) - 1) - 2*sqrt(-tan(1/2*x)^4 +
1) - (sqrt(-tan(1/2*x)^4 + 1) - 1)/tan(1/2*x)^2 - 2*arcsin(tan(1/2*x)^2) -
2*log(-(sqrt(-tan(1/2*x)^4 + 1) - 1)/tan(1/2*x)^2))/(sgn(tan(1/2*x)^4 - 1)*
sgn(tan(1/2*x)))
```

maple [B] time = 0.31, size = 265, normalized size = 3.68

$$-\frac{(-1+\cos(x))\left(8\left(\cos^2(x)\right)\left(-\frac{\cos(x)}{(1+\cos(x))^2}\right)^{\frac{3}{2}}+16\cos(x)\left(-\frac{\cos(x)}{(1+\cos(x))^2}\right)^{\frac{3}{2}}-\left(\cos^2(x)\right)\ln\left(-\frac{2\left(\cos^2(x)\right)\sqrt{-\frac{\cos(x)}{(1+\cos(x))^2}}-\left(\cos^2\right.}{\sin}\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-cos(x)+sec(x))^(3/2),x)
```

```
[Out] -1/8*(-1+cos(x))*(8*cos(x)^2*(-cos(x)/(1+cos(x))^2)^(3/2)+16*cos(x)*(-cos(x
)/(1+cos(x))^2)^(3/2)-cos(x)^2*ln(-(2*cos(x)^2*(-cos(x)/(1+cos(x))^2)^(1/2)
-cos(x)^2+2*cos(x)-2*(-cos(x)/(1+cos(x))^2)^(1/2)-1)/sin(x)^2)+cos(x)^2*arc
tan(1/2/(-cos(x)/(1+cos(x))^2)^(1/2))+8*(-cos(x)/(1+cos(x))^2)^(3/2)+4*cos(
x)*(-cos(x)/(1+cos(x))^2)^(1/2)-4*(-cos(x)/(1+cos(x))^2)^(1/2)+ln(-(2*cos(x
)^2*(-cos(x)/(1+cos(x))^2)^(1/2)-cos(x)^2+2*cos(x)-2*(-cos(x)/(1+cos(x))^2)
^(1/2)-1)/sin(x)^2)-arctan(1/2/(-cos(x)/(1+cos(x))^2)^(1/2)))/(-(-1+cos(x)^
2)/cos(x))^(3/2)/cos(x)/sin(x)/(-cos(x)/(1+cos(x))^2)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(-\cos(x)+\sec(x))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-cos(x)+sec(x))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((-cos(x) + sec(x))^(-3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\left(\frac{1}{\cos(x)}-\cos(x)\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1/cos(x) - cos(x))^(3/2),x)
```

```
[Out] int(1/(1/cos(x) - cos(x))^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(-\cos(x)+\sec(x))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-cos(x)+sec(x))**(3/2),x)
```

```
[Out] Integral((-cos(x) + sec(x))**(-3/2), x)
```

3.339 $\int \frac{1}{(-\cos(x)+\sec(x))^{5/2}}\,dx$

Optimal. Leaf size=91

$$-\frac{3\sin(x)\tan^{-1}\left(\sqrt{\cos(x)}\right)}{32\sqrt{\cos(x)}\,\sqrt{\sin(x)\tan(x)}}+\frac{3\cot(x)}{16\sqrt{\sin(x)\tan(x)}}+\frac{3\sin(x)\tanh^{-1}\left(\sqrt{\cos(x)}\right)}{32\sqrt{\cos(x)}\,\sqrt{\sin(x)\tan(x)}}-\frac{\cot(x)\csc^2(x)}{4\sqrt{\sin(x)\tan(x)}}$$

```
[Out] 3/16*cot(x)/(sin(x)*tan(x))^(1/2)-1/4*cot(x)*csc(x)^2/(sin(x)*tan(x))^(1/2)
-3/32*arctan(cos(x)^(1/2))*sin(x)/cos(x)^(1/2)/(sin(x)*tan(x))^(1/2)+3/32*a
rctanh(cos(x)^(1/2))*sin(x)/cos(x)^(1/2)/(sin(x)*tan(x))^(1/2)
```

Rubi [A] time = 0.12, antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 10, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.909, Rules used = {4397, 4400, 2597, 2599, 2601, 2565, 329, 298, 203, 206}

$$-\frac{3\sin(x)\tan^{-1}\left(\sqrt{\cos(x)}\right)}{32\sqrt{\cos(x)}\,\sqrt{\sin(x)\tan(x)}}+\frac{3\cot(x)}{16\sqrt{\sin(x)\tan(x)}}+\frac{3\sin(x)\tanh^{-1}\left(\sqrt{\cos(x)}\right)}{32\sqrt{\cos(x)}\,\sqrt{\sin(x)\tan(x)}}-\frac{\cot(x)\csc^2(x)}{4\sqrt{\sin(x)\tan(x)}}$$

Antiderivative was successfully verified.

```
[In] Int[(-Cos[x] + Sec[x])^(-5/2),x]
```

```
[Out] (3*Cot[x])/(16*Sqrt[Sin[x]*Tan[x]]) - (Cot[x]*Csc[x]^2)/(4*Sqrt[Sin[x]*Tan[
x]]) - (3*ArcTan[Sqrt[Cos[x]]]*Sin[x])/(32*Sqrt[Cos[x]]*Sqrt[Sin[x]*Tan[x]]
) + (3*ArcTanh[Sqrt[Cos[x]]]*Sin[x])/(32*Sqrt[Cos[x]]*Sqrt[Sin[x]*Tan[x]])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2565

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> -Dist[(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 2597

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[((a*Sin[e + f*x])^m*(b*Tan[e + f*x])^(n + 1))/(b*f*(
m + n + 1)), x] - Dist[(n + 1)/(b^2*(m + n + 1)), Int[(a*Sin[e + f*x])^m*(b
*Tan[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && LtQ[n, -1] &
& NeQ[m + n + 1, 0] && IntegersQ[2*m, 2*n] &&  !(EqQ[n, -3/2] && EqQ[m, 1])
```

Rule 2599

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_.), x_Symbol] :> Simp[(b*(a*Sin[e + f*x])^(m + 2)*(b*Tan[e + f*x])^(n - 1)
)/(a^2*f*(m + n + 1)), x] + Dist[(m + 2)/(a^2*(m + n + 1)), Int[(a*Sin[e +
f*x])^(m + 2)*(b*Tan[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && Lt
Q[m, -1] && NeQ[m + n + 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 2601

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Dist[(Cos[e + f*x]^n*(b*Tan[e + f*x])^n)/(a*Sin[e + f*x])
^n, Int[(a*Sin[e + f*x])^(m + n)/Cos[e + f*x]^n, x], x] /; FreeQ[{a, b, e,
f, m, n}, x] &&  !IntegerQ[n] && (ILtQ[m, 0] || (EqQ[m, 1] && EqQ[n, -2^(-1
)]) || IntegersQ[m - 1/2, n - 1/2])
```

Rule 4397

```
Int[u_, x_Symbol] :> Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]
```

Rule 4400

```
Int[(u_.)*((v_)^(m_.)*(w_)^(n_.))^(p_), x_Symbol] :> With[{uu = ActivateTri
```

```
g[u], vv = ActivateTrig[v], ww = ActivateTrig[w]}, Dist[(vv^m*ww^n)^FracPar
t[p]/(vv^(m*FracPart[p])*ww^(n*FracPart[p])), Int[uu*vv^(m*p)*ww^(n*p), x],
 x]] /; FreeQ[{m, n, p}, x] && !IntegerQ[p] && ( !InertTrigFreeQ[v] || !I
nertTrigFreeQ[w])
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{(-\cos(x)+\sec(x))^{5/2}}\,dx &= \int \frac{1}{(\sin(x)\tan(x))^{5/2}}\,dx \\
&= \frac{\left(\sqrt{\sin(x)}\sqrt{\tan(x)}\right)\int \frac{1}{\sin^{\frac{5}{2}}(x)\tan^{\frac{5}{2}}(x)}\,dx}{\sqrt{\sin(x)\tan(x)}} \\
&= -\frac{\cot(x)\csc^2(x)}{4\sqrt{\sin(x)\tan(x)}} - \frac{\left(3\sqrt{\sin(x)}\sqrt{\tan(x)}\right)\int \frac{1}{\sin^{\frac{5}{2}}(x)\sqrt{\tan(x)}}\,dx}{8\sqrt{\sin(x)\tan(x)}} \\
&= \frac{3\cot(x)}{16\sqrt{\sin(x)\tan(x)}} - \frac{\cot(x)\csc^2(x)}{4\sqrt{\sin(x)\tan(x)}} - \frac{\left(3\sqrt{\sin(x)}\sqrt{\tan(x)}\right)\int \frac{1}{\sqrt{\sin(x)}\sqrt{\tan(x)}}\,dx}{32\sqrt{\sin(x)\tan(x)}} \\
&= \frac{3\cot(x)}{16\sqrt{\sin(x)\tan(x)}} - \frac{\cot(x)\csc^2(x)}{4\sqrt{\sin(x)\tan(x)}} - \frac{(3\sin(x))\int \sqrt{\cos(x)}\csc(x)\,dx}{32\sqrt{\cos(x)}\sqrt{\sin(x)\tan(x)}} \\
&= \frac{3\cot(x)}{16\sqrt{\sin(x)\tan(x)}} - \frac{\cot(x)\csc^2(x)}{4\sqrt{\sin(x)\tan(x)}} + \frac{(3\sin(x))\operatorname{Subst}\left(\int \frac{\sqrt{x}}{1-x^2}\,dx, x, \cos(x)\right)}{32\sqrt{\cos(x)}\sqrt{\sin(x)\tan(x)}} \\
&= \frac{3\cot(x)}{16\sqrt{\sin(x)\tan(x)}} - \frac{\cot(x)\csc^2(x)}{4\sqrt{\sin(x)\tan(x)}} + \frac{(3\sin(x))\operatorname{Subst}\left(\int \frac{x^2}{1-x^4}\,dx, x, \sqrt{\cos(x)}\right)}{16\sqrt{\cos(x)}\sqrt{\sin(x)\tan(x)}} \\
&= \frac{3\cot(x)}{16\sqrt{\sin(x)\tan(x)}} - \frac{\cot(x)\csc^2(x)}{4\sqrt{\sin(x)\tan(x)}} + \frac{(3\sin(x))\operatorname{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \sqrt{\cos(x)}\right)}{32\sqrt{\cos(x)}\sqrt{\sin(x)\tan(x)}} \\
&= \frac{3\cot(x)}{16\sqrt{\sin(x)\tan(x)}} - \frac{\cot(x)\csc^2(x)}{4\sqrt{\sin(x)\tan(x)}} - \frac{3\tan^{-1}\left(\sqrt{\cos(x)}\right)\sin(x)}{32\sqrt{\cos(x)}\sqrt{\sin(x)\tan(x)}} + \frac{3\tanh^{-1}\left(\right.}{32\sqrt{\cos(x}}
\end{aligned}
$$

Mathematica [A] time = 0.65, size = 73, normalized size = 0.80

$$
-\frac{\cot(x)\sqrt{\sin(x)\tan(x)}\left(3\cos(x)\tan^{-1}\left(\sqrt[4]{\cos^2(x)}\right)-3\cos(x)\tanh^{-1}\left(\sqrt[4]{\cos^2(x)}\right)+\cos^2(x)^{3/4}(3\cos(2x)+5)\,\mathrm{co}}{32\cos^2(x)^{3/4}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(-Cos[x] + Sec[x])^(-5/2),x]
```

```
[Out] -1/32*(Cot[x]*(3*ArcTan[(Cos[x]^2)^(1/4)]*Cos[x] - 3*ArcTanh[(Cos[x]^2)^(1/
4)]*Cos[x] + (Cos[x]^2)^(3/4)*(5 + 3*Cos[2*x])*Cot[x]*Csc[x]^3)*Sqrt[Sin[x]
*Tan[x]])/(Cos[x]^2)^(3/4)
```

fricas [B] time = 1.02, size = 147, normalized size = 1.62

$$\frac{3\left(\cos(x)^4-2\cos(x)^2+1\right)\arctan\left(\frac{2\sqrt{-\frac{\cos(x)^2-1}{\cos(x)}}\cos(x)}{(\cos(x)-1)\sin(x)}\right)\sin(x)+3\left(\cos(x)^4-2\cos(x)^2+1\right)\log\left(\frac{(\cos(x)+1)\sin(x)+2\sqrt{\cdots}}{(\cos(x)-1)\cdots}\right)\cdots}{64\left(\cos(x)^4-2\cos(x)^2+1\right)\sin(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-cos(x)+sec(x))^(5/2),x, algorithm="fricas")
```

```
[Out] 1/64*(3*(cos(x)^4 - 2*cos(x)^2 + 1)*arctan(2*sqrt(-(cos(x)^2 - 1)/cos(x))*c
os(x)/((cos(x) - 1)*sin(x)))*sin(x) + 3*(cos(x)^4 - 2*cos(x)^2 + 1)*log(((c
os(x) + 1)*sin(x) + 2*sqrt(-(cos(x)^2 - 1)/cos(x))*cos(x))/((cos(x) - 1)*si
n(x)))*sin(x) - 4*(3*cos(x)^4 + cos(x)^2)*sqrt(-(cos(x)^2 - 1)/cos(x)))/((c
os(x)^4 - 2*cos(x)^2 + 1)*sin(x))
```

giac [B] time = 0.40, size = 170, normalized size = 1.87

$$\frac{\frac{\left(\frac{4\left(\sqrt{-\tan\left(\frac{1}{2}x\right)^4+1}-1\right)}{\tan\left(\frac{1}{2}x\right)^2}+1\right)\tan\left(\frac{1}{2}x\right)^4}{\left(\sqrt{-\tan\left(\frac{1}{2}x\right)^4+1}-1\right)^2}-4\sqrt{-\tan\left(\frac{1}{2}x\right)^4+1}\left(\tan\left(\frac{1}{2}x\right)^2-2\right)-\frac{4\left(\sqrt{-\tan\left(\frac{1}{2}x\right)^4+1}-1\right)}{\tan\left(\frac{1}{2}x\right)^2}-\frac{\left(\sqrt{-\tan\left(\frac{1}{2}x\right)^4+1}-1\right)^2}{\tan\left(\frac{1}{2}x\right)^4}-1\cdots}{256\,\mathrm{sgn}\left(\tan\left(\frac{1}{2}x\right)^4-1\right)\mathrm{sgn}\left(\tan\left(\frac{1}{2}x\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-cos(x)+sec(x))^(5/2),x, algorithm="giac")
```

```
[Out] 1/256*((4*(sqrt(-tan(1/2*x)^4 + 1) - 1)/tan(1/2*x)^2 + 1)*tan(1/2*x)^4/(sqr
t(-tan(1/2*x)^4 + 1) - 1)^2 - 4*sqrt(-tan(1/2*x)^4 + 1)*(tan(1/2*x)^2 - 2)
- 4*(sqrt(-tan(1/2*x)^4 + 1) - 1)/tan(1/2*x)^2 - (sqrt(-tan(1/2*x)^4 + 1) -
 1)^2/tan(1/2*x)^4 - 12*arcsin(tan(1/2*x)^2) + 12*log(-(sqrt(-tan(1/2*x)^4
+ 1) - 1)/tan(1/2*x)^2))/(sgn(tan(1/2*x)^4 - 1)*sgn(tan(1/2*x)))
```

maple [B] time = 0.32, size = 454, normalized size = 4.99

$$\left(24\left(\cos^{3}(x)\right)\left(-\frac{\cos(x)}{(1+\cos(x))^{2}}\right)^{\frac{3}{2}}+40\left(\cos^{2}(x)\right)\left(-\frac{\cos(x)}{(1+\cos(x))^{2}}\right)^{\frac{3}{2}}-12\left(\cos^{3}(x)\right)\sqrt{-\frac{\cos(x)}{(1+\cos(x))^{2}}}-3\left(\cos^{3}(x)\right)\ln\left(-\frac{2\left(\cos\right.}{}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-cos(x)+sec(x))^(5/2),x)
```

```
[Out] 1/64*(24*cos(x)^3*(-cos(x)/(1+cos(x))^2)^(3/2)+40*cos(x)^2*(-cos(x)/(1+cos(
x))^2)^(3/2)-12*cos(x)^3*(-cos(x)/(1+cos(x))^2)^(1/2)-3*cos(x)^3*ln(-(2*cos
(x)^2*(-cos(x)/(1+cos(x))^2)^(1/2)-cos(x)^2+2*cos(x)-2*(-cos(x)/(1+cos(x))^
2)^(1/2)-1)/sin(x)^2)-3*cos(x)^3*arctan(1/2/(-cos(x)/(1+cos(x))^2)^(1/2))+8
*cos(x)*(-cos(x)/(1+cos(x))^2)^(3/2)+24*cos(x)^2*(-cos(x)/(1+cos(x))^2)^(1/
2)+3*cos(x)^2*ln(-(2*cos(x)^2*(-cos(x)/(1+cos(x))^2)^(1/2)-cos(x)^2+2*cos(x
)-2*(-cos(x)/(1+cos(x))^2)^(1/2)-1)/sin(x)^2)+3*cos(x)^2*arctan(1/2/(-cos(x
)/(1+cos(x))^2)^(1/2))-8*(-cos(x)/(1+cos(x))^2)^(3/2)-12*cos(x)*(-cos(x)/(1
+cos(x))^2)^(1/2)+3*cos(x)*ln(-(2*cos(x)^2*(-cos(x)/(1+cos(x))^2)^(1/2)-cos
(x)^2+2*cos(x)-2*(-cos(x)/(1+cos(x))^2)^(1/2)-1)/sin(x)^2)+3*cos(x)*arctan(
1/2/(-cos(x)/(1+cos(x))^2)^(1/2))-3*ln(-(2*cos(x)^2*(-cos(x)/(1+cos(x))^2)^
(1/2)-cos(x)^2+2*cos(x)-2*(-cos(x)/(1+cos(x))^2)^(1/2)-1)/sin(x)^2)-3*arcta
n(1/2/(-cos(x)/(1+cos(x))^2)^(1/2)))*sin(x)/(-(-1+cos(x)^2)/cos(x))^(5/2)/c
os(x)^2/(-cos(x)/(1+cos(x))^2)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{\left(-\cos(x)+\sec(x)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-cos(x)+sec(x))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((-cos(x) + sec(x))^(-5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int\frac{1}{\left(\frac{1}{\cos(x)}-\cos(x)\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1/cos(x) - cos(x))^(5/2),x)
```

```
[Out] int(1/(1/cos(x) - cos(x))^(5/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(-\cos(x) + \sec(x))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-cos(x)+sec(x))**(5/2),x)
```

```
[Out] Integral((-cos(x) + sec(x))**(-5/2), x)
```

3.340 $\int \frac{1}{(-\cos(x)+\sec(x))^{7/2}} \, dx$

Optimal. Leaf size=110

$$-\frac{5\sin(x)\tan^{-1}\left(\sqrt{\cos(x)}\right)}{128\sqrt{\cos(x)}\sqrt{\sin(x)\tan(x)}}+\frac{5\csc^3(x)}{48\sqrt{\sin(x)\tan(x)}}-\frac{5\csc(x)}{192\sqrt{\sin(x)\tan(x)}}-\frac{5\sin(x)\tanh^{-1}\left(\sqrt{\cos(x)}\right)}{128\sqrt{\cos(x)}\sqrt{\sin(x)\tan(x)}}-\frac{\cot^2(x)\,c}{6\sqrt{\sin(x)}}$$

```
[Out] -5/192*csc(x)/(sin(x)*tan(x))^(1/2)+5/48*csc(x)^3/(sin(x)*tan(x))^(1/2)-1/6
*cot(x)^2*csc(x)^3/(sin(x)*tan(x))^(1/2)-5/128*arctan(cos(x)^(1/2))*sin(x)/
cos(x)^(1/2)/(sin(x)*tan(x))^(1/2)-5/128*arctanh(cos(x)^(1/2))*sin(x)/cos(x
)^(1/2)/(sin(x)*tan(x))^(1/2)
```

Rubi [A] time = 0.14, antiderivative size = 110, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 10, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.909, Rules used = {4397, 4400, 2597, 2599, 2601, 2565, 329, 212, 206, 203}

$$-\frac{5\sin(x)\tan^{-1}\left(\sqrt{\cos(x)}\right)}{128\sqrt{\cos(x)}\sqrt{\sin(x)\tan(x)}}+\frac{5\csc^3(x)}{48\sqrt{\sin(x)\tan(x)}}-\frac{5\csc(x)}{192\sqrt{\sin(x)\tan(x)}}-\frac{5\sin(x)\tanh^{-1}\left(\sqrt{\cos(x)}\right)}{128\sqrt{\cos(x)}\sqrt{\sin(x)\tan(x)}}-\frac{\cot^2(x)\,c}{6\sqrt{\sin(x)}}$$

Antiderivative was successfully verified.

```
[In] Int[(-Cos[x] + Sec[x])^(-7/2),x]
```

```
[Out] (-5*Csc[x])/(192*Sqrt[Sin[x]*Tan[x]]) + (5*Csc[x]^3)/(48*Sqrt[Sin[x]*Tan[x]
]) - (Cot[x]^2*Csc[x]^3)/(6*Sqrt[Sin[x]*Tan[x]]) - (5*ArcTan[Sqrt[Cos[x]]]*
Sin[x])/(128*Sqrt[Cos[x]]*Sqrt[Sin[x]*Tan[x]]) - (5*ArcTanh[Sqrt[Cos[x]]]*S
in[x])/(128*Sqrt[Cos[x]]*Sqrt[Sin[x]*Tan[x]])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
```

```
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2565

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> -Dist[(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 2597

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[((a*Sin[e + f*x])^m*(b*Tan[e + f*x])^(n + 1))/(b*f*(
m + n + 1)), x] - Dist[(n + 1)/(b^2*(m + n + 1)), Int[(a*Sin[e + f*x])^m*(b
*Tan[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && LtQ[n, -1] &
& NeQ[m + n + 1, 0] && IntegersQ[2*m, 2*n] &&  !(EqQ[n, -3/2] && EqQ[m, 1])
```

Rule 2599

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_.), x_Symbol] :> Simp[(b*(a*Sin[e + f*x])^(m + 2)*(b*Tan[e + f*x])^(n - 1)
)/(a^2*f*(m + n + 1)), x] + Dist[(m + 2)/(a^2*(m + n + 1)), Int[(a*Sin[e +
f*x])^(m + 2)*(b*Tan[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && Lt
Q[m, -1] && NeQ[m + n + 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 2601

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Dist[(Cos[e + f*x]^n*(b*Tan[e + f*x])^n)/(a*Sin[e + f*x])
^n, Int[(a*Sin[e + f*x])^(m + n)/Cos[e + f*x]^n, x], x] /; FreeQ[{a, b, e,
f, m, n}, x] &&  !IntegerQ[n] && (ILtQ[m, 0] || (EqQ[m, 1] && EqQ[n, -2^(-1
)]) || IntegersQ[m - 1/2, n - 1/2])
```

Rule 4397

```
Int[u_, x_Symbol] :> Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]
```

Rule 4400

```
Int[(u_.)*((v_)^(m_.)*(w_)^(n_.))^(p_), x_Symbol] :> With[{uu = ActivateTri
g[u], vv = ActivateTrig[v], ww = ActivateTrig[w]}, Dist[(vv^m*ww^n)^FracPar
t[p]/(vv^(m*FracPart[p])*ww^(n*FracPart[p])), Int[uu*vv^(m*p)*ww^(n*p), x],
 x]] /; FreeQ[{m, n, p}, x] &&  !IntegerQ[p] && ( !InertTrigFreeQ[v] ||  !I
nertTrigFreeQ[w])
```

Rubi steps

$$\int \frac{1}{(-\cos(x)+\sec(x))^{7/2}}\,dx = \int \frac{1}{(\sin(x)\tan(x))^{7/2}}\,dx$$

$$= \frac{\left(\sqrt{\sin(x)}\sqrt{\tan(x)}\right)\int \frac{1}{\sin^{\frac{7}{2}}(x)\tan^{\frac{7}{2}}(x)}\,dx}{\sqrt{\sin(x)\tan(x)}}$$

$$= -\frac{\cot^2(x)\csc^3(x)}{6\sqrt{\sin(x)\tan(x)}} - \frac{\left(5\sqrt{\sin(x)}\sqrt{\tan(x)}\right)\int \frac{1}{\sin^{\frac{7}{2}}(x)\tan^{\frac{3}{2}}(x)}\,dx}{12\sqrt{\sin(x)\tan(x)}}$$

$$= \frac{5\csc^3(x)}{48\sqrt{\sin(x)\tan(x)}} - \frac{\cot^2(x)\csc^3(x)}{6\sqrt{\sin(x)\tan(x)}} + \frac{\left(5\sqrt{\sin(x)}\sqrt{\tan(x)}\right)\int \frac{\sqrt{\tan(x)}}{\sin^{\frac{7}{2}}(x)}\,dx}{96\sqrt{\sin(x)\tan(x)}}$$

$$= -\frac{5\csc(x)}{192\sqrt{\sin(x)\tan(x)}} + \frac{5\csc^3(x)}{48\sqrt{\sin(x)\tan(x)}} - \frac{\cot^2(x)\csc^3(x)}{6\sqrt{\sin(x)\tan(x)}} + \frac{\left(5\sqrt{\sin(x)}\sqrt{\tan\right.}{128\sqrt{\sin}}$$

$$= -\frac{5\csc(x)}{192\sqrt{\sin(x)\tan(x)}} + \frac{5\csc^3(x)}{48\sqrt{\sin(x)\tan(x)}} - \frac{\cot^2(x)\csc^3(x)}{6\sqrt{\sin(x)\tan(x)}} + \frac{(5\sin(x))\int}{128\sqrt{\cos(x)}\sqrt{\mathrm{si}}}$$

$$= -\frac{5\csc(x)}{192\sqrt{\sin(x)\tan(x)}} + \frac{5\csc^3(x)}{48\sqrt{\sin(x)\tan(x)}} - \frac{\cot^2(x)\csc^3(x)}{6\sqrt{\sin(x)\tan(x)}} - \frac{(5\sin(x))\,\mathrm{Subst}}{128\sqrt{\cos}}$$

$$= -\frac{5\csc(x)}{192\sqrt{\sin(x)\tan(x)}} + \frac{5\csc^3(x)}{48\sqrt{\sin(x)\tan(x)}} - \frac{\cot^2(x)\csc^3(x)}{6\sqrt{\sin(x)\tan(x)}} - \frac{(5\sin(x))\,\mathrm{Subst}}{64\sqrt{\cos(}}$$

$$= -\frac{5\csc(x)}{192\sqrt{\sin(x)\tan(x)}} + \frac{5\csc^3(x)}{48\sqrt{\sin(x)\tan(x)}} - \frac{\cot^2(x)\csc^3(x)}{6\sqrt{\sin(x)\tan(x)}} - \frac{(5\sin(x))\,\mathrm{Subst}}{128\sqrt{\cos(}}$$

$$= -\frac{5\csc(x)}{192\sqrt{\sin(x)\tan(x)}} + \frac{5\csc^3(x)}{48\sqrt{\sin(x)\tan(x)}} - \frac{\cot^2(x)\csc^3(x)}{6\sqrt{\sin(x)\tan(x)}} - \frac{5\tan^{-1}\left(\sqrt{\cos(\right.}{128\sqrt{\cos(x)}\sqrt{\mathrm{si}}}$$

Mathematica [A] time = 0.36, size = 74, normalized size = 0.67

$$-\frac{\cot(x)\sqrt{\sin(x)\tan(x)}\left(15\tan^{-1}\left(\sqrt[4]{\cos^2(x)}\right)+2\sqrt[4]{\cos^2(x)}\left(32\csc^4(x)-52\csc^2(x)+5\right)\csc^2(x)+15\tanh^{-1}\left(\sqrt[4]{c\right.\right.}{384\sqrt[4]{\cos^2(x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-Cos[x] + Sec[x])^(-7/2),x]
```

```
[Out] -1/384*(Cot[x]*(15*ArcTan[(Cos[x]^2)^(1/4)] + 15*ArcTanh[(Cos[x]^2)^(1/4)]
+ 2*(Cos[x]^2)^(1/4)*Csc[x]^2*(5 - 52*Csc[x]^2 + 32*Csc[x]^4))*Sqrt[Sin[x]*
Tan[x]])/(Cos[x]^2)^(1/4)
```

fricas [B] time = 1.63, size = 171, normalized size = 1.55

$$\frac{15\left(\cos(x)^6-3\cos(x)^4+3\cos(x)^2-1\right)\arctan\left(\frac{2\sqrt{-\frac{\cos(x)^2-1}{\cos(x)}}\cos(x)}{(\cos(x)-1)\sin(x)}\right)\sin(x)+15\left(\cos(x)^6-3\cos(x)^4+3\cos(x)^2\right.}{768\left(\cos(x)^6-3\cos(x)^4+3\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-cos(x)+sec(x))^(7/2),x, algorithm="fricas")
```

```
[Out] 1/768*(15*(cos(x)^6 - 3*cos(x)^4 + 3*cos(x)^2 - 1)*arctan(2*sqrt(-(cos(x)^2
 - 1)/cos(x))*cos(x)/((cos(x) - 1)*sin(x)))*sin(x) + 15*(cos(x)^6 - 3*cos(x
)^4 + 3*cos(x)^2 - 1)*log(((cos(x) + 1)*sin(x) - 2*sqrt(-(cos(x)^2 - 1)/cos
(x))*cos(x))/((cos(x) - 1)*sin(x)))*sin(x) + 4*(5*cos(x)^5 + 42*cos(x)^3 -
15*cos(x))*sqrt(-(cos(x)^2 - 1)/cos(x)))/((cos(x)^6 - 3*cos(x)^4 + 3*cos(x)
^2 - 1)*sin(x))
```

giac [B] time = 0.45, size = 229, normalized size = 2.08

$$-\frac{\frac{\left(\frac{3\left(\sqrt{-\tan\left(\frac{1}{2}x\right)^4+1}-1\right)}{\tan\left(\frac{1}{2}x\right)^2}-\frac{27\left(\sqrt{-\tan\left(\frac{1}{2}x\right)^4+1}-1\right)^2}{\tan\left(\frac{1}{2}x\right)^4}+1\right)\tan\left(\frac{1}{2}x\right)^6}{\left(\sqrt{-\tan\left(\frac{1}{2}x\right)^4+1}-1\right)^3}-4\sqrt{-\tan\left(\frac{1}{2}x\right)^4+1}\left(\left(2\tan\left(\frac{1}{2}x\right)^2-3\right)\tan\left(\frac{1}{2}x\right)^2-14\right)+\frac{27\left(\right.}{}}{3072\,\mathrm{sgn}\left(\tan\left(\frac{1}{2}x\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-cos(x)+sec(x))^(7/2),x, algorithm="giac")
```

```
[Out] -1/3072*((3*(sqrt(-tan(1/2*x)^4 + 1) - 1)/tan(1/2*x)^2 - 27*(sqrt(-tan(1/2*
x)^4 + 1) - 1)^2/tan(1/2*x)^4 + 1)*tan(1/2*x)^6/(sqrt(-tan(1/2*x)^4 + 1) -
1)^3 - 4*sqrt(-tan(1/2*x)^4 + 1)*((2*tan(1/2*x)^2 - 3)*tan(1/2*x)^2 - 14) +
 27*(sqrt(-tan(1/2*x)^4 + 1) - 1)/tan(1/2*x)^2 - 3*(sqrt(-tan(1/2*x)^4 + 1)
 - 1)^2/tan(1/2*x)^4 - (sqrt(-tan(1/2*x)^4 + 1) - 1)^3/tan(1/2*x)^6 + 60*ar
csin(tan(1/2*x)^2) + 60*log(-(sqrt(-tan(1/2*x)^4 + 1) - 1)/tan(1/2*x)^2))/(
sgn(tan(1/2*x)^4 - 1)*sgn(tan(1/2*x)))
```

maple [B] time = 0.36, size = 494, normalized size = 4.49

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-cos(x)+sec(x))^(7/2),x)
```

```
[Out] 1/768*(56*cos(x)^4*(-cos(x)/(1+cos(x))^2)^(3/2)-16*cos(x)^3*(-cos(x)/(1+cos
(x))^2)^(3/2)-15*cos(x)^4*ln(-(2*cos(x)^2*(-cos(x)/(1+cos(x))^2)^(1/2)-cos(
x)^2+2*cos(x)-2*(-cos(x)/(1+cos(x))^2)^(1/2)-1)/sin(x)^2)+15*cos(x)^4*arcta
n(1/2/(-cos(x)/(1+cos(x))^2)^(1/2))-192*cos(x)^2*(-cos(x)/(1+cos(x))^2)^(3/
2)+76*cos(x)^3*(-cos(x)/(1+cos(x))^2)^(1/2)+30*cos(x)^3*ln(-(2*cos(x)^2*(-c
os(x)/(1+cos(x))^2)^(1/2)-cos(x)^2+2*cos(x)-2*(-cos(x)/(1+cos(x))^2)^(1/2)-
1)/sin(x)^2)-30*cos(x)^3*arctan(1/2/(-cos(x)/(1+cos(x))^2)^(1/2))+16*cos(x)
*(-cos(x)/(1+cos(x))^2)^(3/2)-148*cos(x)^2*(-cos(x)/(1+cos(x))^2)^(1/2)+136
*(-cos(x)/(1+cos(x))^2)^(3/2)+196*cos(x)*(-cos(x)/(1+cos(x))^2)^(1/2)-30*co
s(x)*ln(-(2*cos(x)^2*(-cos(x)/(1+cos(x))^2)^(1/2)-cos(x)^2+2*cos(x)-2*(-cos
(x)/(1+cos(x))^2)^(1/2)-1)/sin(x)^2)+30*cos(x)*arctan(1/2/(-cos(x)/(1+cos(x
))^2)^(1/2))-60*(-cos(x)/(1+cos(x))^2)^(1/2)+15*ln(-(2*cos(x)^2*(-cos(x)/(1
+cos(x))^2)^(1/2)-cos(x)^2+2*cos(x)-2*(-cos(x)/(1+cos(x))^2)^(1/2)-1)/sin(x
)^2)-15*arctan(1/2/(-cos(x)/(1+cos(x))^2)^(1/2)))*sin(x)^3/(-1+cos(x))/(-(-
1+cos(x)^2)/cos(x))^(7/2)/cos(x)^3/(-cos(x)/(1+cos(x))^2)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(-\cos(x) + \sec(x))^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-cos(x)+sec(x))^(7/2),x, algorithm="maxima")
```

```
[Out] integrate((-cos(x) + sec(x))^(-7/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{{\left(\frac{1}{\cos(x)} - \cos(x)\right)}^{7/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1/cos(x) - cos(x))^(7/2),x)
```

```
[Out] int(1/(1/cos(x) - cos(x))^(7/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-cos(x)+sec(x))**(7/2),x)
```

```
[Out] Timed out
```

3.341 $\int (\sin(x)+\tan(x))^4\,dx$

Optimal. Leaf size=55

$$-\frac{61x}{8}-\frac{4\sin^3(x)}{3}+\frac{\tan^3(x)}{3}+5\tan(x)-2\tanh^{-1}(\sin(x))+\frac{1}{4}\sin(x)\cos^3(x)+\frac{19}{8}\sin(x)\cos(x)+2\tan(x)\sec(x)$$

```
[Out] -61/8*x-2*arctanh(sin(x))+19/8*cos(x)*sin(x)+1/4*cos(x)^3*sin(x)-4/3*sin(x)
^3+5*tan(x)+2*sec(x)*tan(x)+1/3*tan(x)^3
```

Rubi [A] time = 0.11, antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 18, number of rules used = 9, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.286, Rules used = {4397, 2709, 2637, 2635, 8, 2633, 3770, 3767, 3768}

$$-\frac{61x}{8}-\frac{4\sin^3(x)}{3}+\frac{\tan^3(x)}{3}+5\tan(x)-2\tanh^{-1}(\sin(x))+\frac{1}{4}\sin(x)\cos^3(x)+\frac{19}{8}\sin(x)\cos(x)+2\tan(x)\sec(x)$$

Antiderivative was successfully verified.

```
[In] Int[(Sin[x] + Tan[x])^4,x]
```

```
[Out] (-61*x)/8 - 2*ArcTanh[Sin[x]] + (19*Cos[x]*Sin[x])/8 + (Cos[x]^3*Sin[x])/4
- (4*Sin[x]^3)/3 + 5*Tan[x] + 2*Sec[x]*Tan[x] + Tan[x]^3/3
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 2709

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*tan[(e_.) + (f_.)*(x_)]^(p_
), x_Symbol] :> Dist[a^p, Int[ExpandIntegrand[(Sin[e + f*x]^p*(a + b*Sin[e
+ f*x])^(m - p/2))/(a - b*Sin[e + f*x])^(p/2), x], x], x] /; FreeQ[{a, b, e
, f}, x] && EqQ[a^2 - b^2, 0] && IntegersQ[m, p/2] && (LtQ[p, 0] || GtQ[m -
 p/2, 0])
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 4397

```
Int[u_, x_Symbol] :> Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]
```

Rubi steps

$$\int (\sin(x)+\tan(x))^4\,dx = \int (1+\cos(x))^4\tan^4(x)\,dx$$
$$= \int \left(-10-4\cos(x)+4\cos^2(x)+4\cos^3(x)+\cos^4(x)-4\sec(x)+4\sec^2(x)+4\sec^3(x\right.$$
$$= -10x-4\int \cos(x)\,dx+4\int \cos^2(x)\,dx+4\int \cos^3(x)\,dx-4\int \sec(x)\,dx+4\int \sec^2$$
$$= -10x-4\tanh^{-1}(\sin(x))-4\sin(x)+2\cos(x)\sin(x)+\frac{1}{4}\cos^3(x)\sin(x)+2\sec(x)\tan$$
$$= -8x-2\tanh^{-1}(\sin(x))+\frac{19}{8}\cos(x)\sin(x)+\frac{1}{4}\cos^3(x)\sin(x)-\frac{4\sin^3(x)}{3}+5\tan(x)$$
$$= -\frac{61x}{8}-2\tanh^{-1}(\sin(x))+\frac{19}{8}\cos(x)\sin(x)+\frac{1}{4}\cos^3(x)\sin(x)-\frac{4\sin^3(x)}{3}+5\tan(x$$

Mathematica [B] time = 0.20, size = 129, normalized size = 2.35

$$\frac{1}{768}\sec^3(x)\left(1395\sin(x)+672\sin(2x)+1265\sin(3x)+129\sin(5x)+32\sin(6x)+3\sin(7x)-72\cos(x)\left(61x\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sin[x] + Tan[x])^4,x]
```

```
[Out] (Sec[x]^3*(-72*Cos[x]*(61*x - 16*Log[Cos[x/2] - Sin[x/2]] + 16*Log[Cos[x/2]
 + Sin[x/2]]) - 24*Cos[3*x]*(61*x - 16*Log[Cos[x/2] - Sin[x/2]] + 16*Log[Co
s[x/2] + Sin[x/2]]) + 1395*Sin[x] + 672*Sin[2*x] + 1265*Sin[3*x] + 129*Sin[
5*x] + 32*Sin[6*x] + 3*Sin[7*x]))/768
```

fricas [A] time = 1.74, size = 78, normalized size = 1.42

$$-\frac{183\,x\cos(x)^3+24\,\cos(x)^3\log(\sin(x)+1)-24\,\cos(x)^3\log(-\sin(x)+1)-\left(6\,\cos(x)^6+32\,\cos(x)^5+57\,\cos\right.}{24\,\cos(x)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((sin(x)+tan(x))^4,x, algorithm="fricas")
```

```
[Out] -1/24*(183*x*cos(x)^3 + 24*cos(x)^3*log(sin(x) + 1) - 24*cos(x)^3*log(-sin(
x) + 1) - (6*cos(x)^6 + 32*cos(x)^5 + 57*cos(x)^4 - 32*cos(x)^3 + 112*cos(x
)^2 + 48*cos(x) + 8)*sin(x))/cos(x)^3
```

giac [B] time = 5.64, size = 1375, normalized size = 25.00

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((sin(x)+tan(x))^4,x, algorithm="giac")
```

```
[Out] 1/24*(8*tan(1/2*x)^10*tan(x)^5 - 183*x*tan(1/2*x)^10*tan(x)^2 - 24*log(2*(t
an(1/2*x)^2 + 2*tan(1/2*x) + 1)/(tan(1/2*x)^2 + 1))*tan(1/2*x)^10*tan(x)^2
+ 24*log(2*(tan(1/2*x)^2 - 2*tan(1/2*x) + 1)/(tan(1/2*x)^2 + 1))*tan(1/2*x)
^10*tan(x)^2 + 128*tan(1/2*x)^10*tan(x)^3 + 8*tan(1/2*x)^8*tan(x)^5 - 183*x
*tan(1/2*x)^10 - 24*log(2*(tan(1/2*x)^2 + 2*tan(1/2*x) + 1)/(tan(1/2*x)^2 +
 1))*tan(1/2*x)^10 + 24*log(2*(tan(1/2*x)^2 - 2*tan(1/2*x) + 1)/(tan(1/2*x)
^2 + 1))*tan(1/2*x)^10 + 180*tan(1/2*x)^10*tan(x) - 183*x*tan(1/2*x)^8*tan(
x)^2 - 24*log(2*(tan(1/2*x)^2 + 2*tan(1/2*x) + 1)/(tan(1/2*x)^2 + 1))*tan(1
/2*x)^8*tan(x)^2 + 24*log(2*(tan(1/2*x)^2 - 2*tan(1/2*x) + 1)/(tan(1/2*x)^2
 + 1))*tan(1/2*x)^8*tan(x)^2 + 96*tan(1/2*x)^9*tan(x)^2 + 128*tan(1/2*x)^8*
tan(x)^3 - 16*tan(1/2*x)^6*tan(x)^5 - 183*x*tan(1/2*x)^8 - 24*log(2*(tan(1/
2*x)^2 + 2*tan(1/2*x) + 1)/(tan(1/2*x)^2 + 1))*tan(1/2*x)^8 + 24*log(2*(tan
(1/2*x)^2 - 2*tan(1/2*x) + 1)/(tan(1/2*x)^2 + 1))*tan(1/2*x)^8 + 96*tan(1/2
*x)^9 + 180*tan(1/2*x)^8*tan(x) + 366*x*tan(1/2*x)^6*tan(x)^2 + 48*log(2*(t
an(1/2*x)^2 + 2*tan(1/2*x) + 1)/(tan(1/2*x)^2 + 1))*tan(1/2*x)^6*tan(x)^2 -
 48*log(2*(tan(1/2*x)^2 - 2*tan(1/2*x) + 1)/(tan(1/2*x)^2 + 1))*tan(1/2*x)^
6*tan(x)^2 + 128*tan(1/2*x)^7*tan(x)^2 - 256*tan(1/2*x)^6*tan(x)^3 - 16*tan
(1/2*x)^4*tan(x)^5 + 366*x*tan(1/2*x)^6 + 48*log(2*(tan(1/2*x)^2 + 2*tan(1/
2*x) + 1)/(tan(1/2*x)^2 + 1))*tan(1/2*x)^6 - 48*log(2*(tan(1/2*x)^2 - 2*tan
(1/2*x) + 1)/(tan(1/2*x)^2 + 1))*tan(1/2*x)^6 + 128*tan(1/2*x)^7 - 360*tan(
1/2*x)^6*tan(x) + 366*x*tan(1/2*x)^4*tan(x)^2 + 48*log(2*(tan(1/2*x)^2 + 2*
tan(1/2*x) + 1)/(tan(1/2*x)^2 + 1))*tan(1/2*x)^4*tan(x)^2 - 48*log(2*(tan(1
/2*x)^2 - 2*tan(1/2*x) + 1)/(tan(1/2*x)^2 + 1))*tan(1/2*x)^4*tan(x)^2 + 108
8*tan(1/2*x)^5*tan(x)^2 - 256*tan(1/2*x)^4*tan(x)^3 + 8*tan(1/2*x)^2*tan(x)
^5 + 366*x*tan(1/2*x)^4 + 48*log(2*(tan(1/2*x)^2 + 2*tan(1/2*x) + 1)/(tan(1
/2*x)^2 + 1))*tan(1/2*x)^4 - 48*log(2*(tan(1/2*x)^2 - 2*tan(1/2*x) + 1)/(ta
n(1/2*x)^2 + 1))*tan(1/2*x)^4 + 1088*tan(1/2*x)^5 - 360*tan(1/2*x)^4*tan(x)
 - 183*x*tan(1/2*x)^2*tan(x)^2 - 24*log(2*(tan(1/2*x)^2 + 2*tan(1/2*x) + 1)
/(tan(1/2*x)^2 + 1))*tan(1/2*x)^2*tan(x)^2 + 24*log(2*(tan(1/2*x)^2 - 2*tan
(1/2*x) + 1)/(tan(1/2*x)^2 + 1))*tan(1/2*x)^2*tan(x)^2 + 128*tan(1/2*x)^3*t
an(x)^2 + 128*tan(1/2*x)^2*tan(x)^3 + 8*tan(x)^5 - 183*x*tan(1/2*x)^2 - 24*
log(2*(tan(1/2*x)^2 + 2*tan(1/2*x) + 1)/(tan(1/2*x)^2 + 1))*tan(1/2*x)^2 +
24*log(2*(tan(1/2*x)^2 - 2*tan(1/2*x) + 1)/(tan(1/2*x)^2 + 1))*tan(1/2*x)^2
 + 128*tan(1/2*x)^3 + 180*tan(1/2*x)^2*tan(x) - 183*x*tan(x)^2 - 24*log(2*(
tan(1/2*x)^2 + 2*tan(1/2*x) + 1)/(tan(1/2*x)^2 + 1))*tan(x)^2 + 24*log(2*(t
an(1/2*x)^2 - 2*tan(1/2*x) + 1)/(tan(1/2*x)^2 + 1))*tan(x)^2 + 96*tan(1/2*x
)*tan(x)^2 + 128*tan(x)^3 - 183*x - 24*log(2*(tan(1/2*x)^2 + 2*tan(1/2*x) +
 1)/(tan(1/2*x)^2 + 1)) + 24*log(2*(tan(1/2*x)^2 - 2*tan(1/2*x) + 1)/(tan(1
/2*x)^2 + 1)) + 96*tan(1/2*x) + 180*tan(x))/(tan(1/2*x)^10*tan(x)^2 + tan(1
/2*x)^10 + tan(1/2*x)^8*tan(x)^2 + tan(1/2*x)^8 - 2*tan(1/2*x)^6*tan(x)^2 -
 2*tan(1/2*x)^6 - 2*tan(1/2*x)^4*tan(x)^2 - 2*tan(1/2*x)^4 + tan(1/2*x)^2*t
```

```
an(x)^2 + tan(1/2*x)^2 + tan(x)^2 + 1) + 1/32*sin(4*x)
```

maple [A] time = 0.07, size = 66, normalized size = 1.20

$$\frac{23\left(\sin^3(x)+\frac{3\sin(x)}{2}\right)\cos(x)}{4}-\frac{61x}{8}+\frac{2\left(\sin^3(x)\right)}{3}+2\sin(x)-2\ln\left(\sec(x)+\tan(x)\right)+\frac{6\left(\sin^5(x)\right)}{\cos(x)}+\frac{2\left(\sin^5(x)\right)}{\cos(x)^2}+\frac{(\tan$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((sin(x)+tan(x))^4,x)
```

```
[Out] 23/4*(sin(x)^3+3/2*sin(x))*cos(x)-61/8*x+2/3*sin(x)^3+2*sin(x)-2*ln(sec(x)+
tan(x))+6*sin(x)^5/cos(x)+2*sin(x)^5/cos(x)^2+1/3*tan(x)^3-tan(x)
```

maxima [A] time = 0.42, size = 68, normalized size = 1.24

$$-\frac{4}{3}\sin(x)^3+\frac{1}{3}\tan(x)^3-\frac{61}{8}x-\frac{2\sin(x)}{\sin(x)^2-1}+\frac{3\tan(x)}{\tan(x)^2+1}-\log\left(\sin(x)+1\right)+\log\left(\sin(x)-1\right)+\frac{1}{32}\sin\left(4x\right)-\frac{1}{4}\sin($$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((sin(x)+tan(x))^4,x, algorithm="maxima")
```

```
[Out] -4/3*sin(x)^3 + 1/3*tan(x)^3 - 61/8*x - 2*sin(x)/(sin(x)^2 - 1) + 3*tan(x)/
(tan(x)^2 + 1) - log(sin(x) + 1) + log(sin(x) - 1) + 1/32*sin(4*x) - 1/4*si
n(2*x) + 5*tan(x)
```

mupad [B] time = 2.54, size = 88, normalized size = 1.60

$$-\frac{61x}{8}-4\,\mathrm{atanh}\left(\tan\left(\frac{x}{2}\right)\right)-\frac{\frac{45\tan\left(\frac{x}{2}\right)^{13}}{4}+\frac{29\tan\left(\frac{x}{2}\right)^{11}}{6}-\frac{455\tan\left(\frac{x}{2}\right)^{9}}{12}-15\tan\left(\frac{x}{2}\right)^{7}+\frac{179\tan\left(\frac{x}{2}\right)^{5}}{4}+\frac{31\tan\left(\frac{x}{2}\right)^{3}}{2}+\frac{77\tan\left(\frac{x}{2}\right)}{4}}{\left(\tan\left(\frac{x}{2}\right)^2-1\right)^3\left(\tan\left(\frac{x}{2}\right)^2+1\right)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((sin(x) + tan(x))^4,x)
```

```
[Out] - (61*x)/8 - 4*atanh(tan(x/2)) - ((77*tan(x/2))/4 + (31*tan(x/2)^3)/2 + (17
9*tan(x/2)^5)/4 - 15*tan(x/2)^7 - (455*tan(x/2)^9)/12 + (29*tan(x/2)^11)/6
+ (45*tan(x/2)^13)/4)/((tan(x/2)^2 - 1)^3*(tan(x/2)^2 + 1)^4)
```

sympy [A] time = 4.63, size = 90, normalized size = 1.64

$$-\frac{61x}{8}+\log\left(\sin(x)-1\right)-\log\left(\sin(x)+1\right)-\frac{4\sin^3(x)}{3}+\frac{6\sin^3(x)}{\cos(x)}+\frac{\sin^3(x)}{3\cos^3(x)}+9\sin(x)\cos(x)-\frac{\sin(x)}{\cos(x)}-\frac{\sin(2x)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((sin(x)+tan(x))**4,x)
```

```
[Out] -61*x/8 + log(sin(x) - 1) - log(sin(x) + 1) - 4*sin(x)**3/3 + 6*sin(x)**3/c
os(x) + sin(x)**3/(3*cos(x)**3) + 9*sin(x)*cos(x) - sin(x)/cos(x) - sin(2*x
)/4 + sin(4*x)/32 - 4*sin(x)/(2*sin(x)**2 - 2)
```

3.342 $\int (\sin(x) + \tan(x))^3 \, dx$

Optimal. Leaf size=38

$$\frac{\cos^3(x)}{3} + \frac{3\cos^2(x)}{2} + 2\cos(x) + \frac{\sec^2(x)}{2} + 3\sec(x) - 2\log(\cos(x))$$

```
[Out] 2*cos(x)+3/2*cos(x)^2+1/3*cos(x)^3-2*ln(cos(x))+3*sec(x)+1/2*sec(x)^2
```

Rubi [A] time = 0.05, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {4397, 2707, 75}

$$\frac{\cos^3(x)}{3} + \frac{3\cos^2(x)}{2} + 2\cos(x) + \frac{\sec^2(x)}{2} + 3\sec(x) - 2\log(\cos(x))$$

Antiderivative was successfully verified.

```
[In] Int[(Sin[x] + Tan[x])^3,x]
```

```
[Out] 2*Cos[x] + (3*Cos[x]^2)/2 + Cos[x]^3/3 - 2*Log[Cos[x]] + 3*Sec[x] + Sec[x]^
2/2
```

Rule 75

```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_Sym
bol] :> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[
{a, b, d, e, f, n}, x] && IGtQ[p, 0] && EqQ[b*e + a*f, 0] && !(ILtQ[n + p
+ 2, 0] && GtQ[n + 2*p, 0])
```

Rule 2707

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Dist[1/f, Subst[Int[(x^p*(a + x)^(m - (p + 1)/2))/(a - x)
^((p + 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && Eq
Q[a^2 - b^2, 0] && IntegerQ[(p + 1)/2]
```

Rule 4397

```
Int[u_, x_Symbol] :> Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]
```

Rubi steps

$$\int (\sin(x)+\tan(x))^3\,dx = \int (1+\cos(x))^3\tan^3(x)\,dx$$
$$= -\text{Subst}\left(\int \frac{(1-x)(1+x)^4}{x^3}\,dx, x, \cos(x)\right)$$
$$= -\text{Subst}\left(\int \left(-2+\frac{1}{x^3}+\frac{3}{x^2}+\frac{2}{x}-3x-x^2\right)dx, x, \cos(x)\right)$$
$$= 2\cos(x)+\frac{3\cos^2(x)}{2}+\frac{\cos^3(x)}{3}-2\log(\cos(x))+3\sec(x)+\frac{\sec^2(x)}{2}$$

Mathematica [A] time = 0.04, size = 40, normalized size = 1.05

$$\frac{9\cos(x)}{4}+\frac{3}{4}\cos(2x)+\frac{1}{12}\cos(3x)+\frac{\sec^2(x)}{2}+3\sec(x)-2\log(\cos(x))$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sin[x] + Tan[x])^3,x]
```

```
[Out] (9*Cos[x])/4 + (3*Cos[2*x])/4 + Cos[3*x]/12 - 2*Log[Cos[x]] + 3*Sec[x] + Se
c[x]^2/2
```

fricas [A] time = 2.02, size = 47, normalized size = 1.24

$$\frac{4\cos(x)^5+18\cos(x)^4+24\cos(x)^3-24\cos(x)^2\log(-\cos(x))-9\cos(x)^2+36\cos(x)+6}{12\cos(x)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((sin(x)+tan(x))^3,x, algorithm="fricas")
```

```
[Out] 1/12*(4*cos(x)^5 + 18*cos(x)^4 + 24*cos(x)^3 - 24*cos(x)^2*log(-cos(x)) - 9
*cos(x)^2 + 36*cos(x) + 6)/cos(x)^2
```

giac [B] time = 0.45, size = 173, normalized size = 4.55

$$\frac{\tan\left(\frac{1}{2}x\right)^4\tan(x)^4-2\log\left(\frac{4}{\tan(x)^2+1}\right)\tan\left(\frac{1}{2}x\right)^4\tan(x)^2-10\tan\left(\frac{1}{2}x\right)^4\tan(x)^2-2\log\left(\frac{4}{\tan(x)^2+1}\right)\tan\left(\frac{1}{2}x\right)^4-8}{2\left(\tan\left(\frac{1}{2}x\right)^4\tan(x\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((sin(x)+tan(x))^3,x, algorithm="giac")
```

```
[Out] 1/2*(tan(1/2*x)^4*tan(x)^4 - 2*log(4/(tan(x)^2 + 1))*tan(1/2*x)^4*tan(x)^2
- 10*tan(1/2*x)^4*tan(x)^2 - 2*log(4/(tan(x)^2 + 1))*tan(1/2*x)^4 - 8*tan(1
/2*x)^4 - 3*tan(1/2*x)^2*tan(x)^2 - tan(x)^4 + 2*log(4/(tan(x)^2 + 1))*tan(
x)^2 - 3*tan(1/2*x)^2 - 11*tan(x)^2 + 2*log(4/(tan(x)^2 + 1)) - 13)/(tan(1/
2*x)^4*tan(x)^2 + tan(1/2*x)^4 - tan(x)^2 - 1) + 1/12*cos(3*x)
```

maple [A] time = 0.07, size = 39, normalized size = 1.03

$$\frac{8\left(2+\sin^{2}(x)\right)\cos(x)}{3}-\frac{3\left(\sin^{2}(x)\right)}{2}-2\ln\left(\cos(x)\right)+\frac{3\left(\sin^{4}(x)\right)}{\cos(x)}+\frac{\left(\tan^{2}(x)\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((sin(x)+tan(x))^3,x)
```

```
[Out] 8/3*(2+sin(x)^2)*cos(x)-3/2*sin(x)^2-2*ln(cos(x))+3*sin(x)^4/cos(x)+1/2*tan
(x)^2
```

maxima [A] time = 0.32, size = 42, normalized size = 1.11

$$\frac{1}{3}\cos(x)^{3}-\frac{3}{2}\sin(x)^{2}-\frac{1}{2\left(\sin(x)^{2}-1\right)}+\frac{3}{\cos(x)}+2\cos(x)-\log\left(\sin(x)^{2}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((sin(x)+tan(x))^3,x, algorithm="maxima")
```

```
[Out] 1/3*cos(x)^3 - 3/2*sin(x)^2 - 1/2/(sin(x)^2 - 1) + 3/cos(x) + 2*cos(x) - lo
g(sin(x)^2 - 1)
```

mupad [B] time = 2.45, size = 65, normalized size = 1.71

$$4\,\mathrm{atanh}\left(\tan\left(\frac{x}{2}\right)^{2}\right)+\frac{-4\tan\left(\frac{x}{2}\right)^{8}-4\tan\left(\frac{x}{2}\right)^{6}+\frac{20\tan\left(\frac{x}{2}\right)^{4}}{3}+\frac{20\tan\left(\frac{x}{2}\right)^{2}}{3}+\frac{32}{3}}{\left(\tan\left(\frac{x}{2}\right)^{2}-1\right)^{2}\left(\tan\left(\frac{x}{2}\right)^{2}+1\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((sin(x) + tan(x))^3,x)
```

```
[Out] 4*atanh(tan(x/2)^2) + ((20*tan(x/2)^2)/3 + (20*tan(x/2)^4)/3 - 4*tan(x/2)^6
 - 4*tan(x/2)^8 + 32/3)/((tan(x/2)^2 - 1)^2*(tan(x/2)^2 + 1)^3)
```

sympy [A] time = 5.99, size = 46, normalized size = 1.21

$$-3\log\left(\cos\left(x\right)\right)-\frac{\log\left(\sec^{2}\left(x\right)\right)}{2}+\frac{\cos^{3}\left(x\right)}{3}+\frac{3\cos^{2}\left(x\right)}{2}+2\cos\left(x\right)+\frac{\sec^{2}\left(x\right)}{2}+\frac{3}{\cos\left(x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((sin(x)+tan(x))**3,x)
```

```
[Out] -3*log(cos(x)) - log(sec(x)**2)/2 + cos(x)**3/3 + 3*cos(x)**2/2 + 2*cos(x)
+ sec(x)**2/2 + 3/cos(x)
```

3.343 $\int (\sin(x)+\tan(x))^2\,dx$

Optimal. Leaf size=25

$$-\frac{x}{2}-2\sin(x)+\tan(x)+2\tanh^{-1}(\sin(x))-\frac{1}{2}\sin(x)\cos(x)$$

```
[Out] -1/2*x+2*arctanh(sin(x))-2*sin(x)-1/2*cos(x)*sin(x)+tan(x)
```

Rubi [A] time = 0.06, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.000, Rules used = {4397, 2709, 2637, 2635, 8, 3770, 3767}

$$-\frac{x}{2}-2\sin(x)+\tan(x)+2\tanh^{-1}(\sin(x))-\frac{1}{2}\sin(x)\cos(x)$$

Antiderivative was successfully verified.

```
[In] Int[(Sin[x] + Tan[x])^2,x]
```

```
[Out] -x/2 + 2*ArcTanh[Sin[x]] - 2*Sin[x] - (Cos[x]*Sin[x])/2 + Tan[x]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 2709

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*tan[(e_.) + (f_.)*(x_)]^(p_
), x_Symbol] :> Dist[a^p, Int[ExpandIntegrand[(Sin[e + f*x]^p*(a + b*Sin[e
+ f*x])^(m - p/2))/(a - b*Sin[e + f*x])^(p/2), x], x], x] /; FreeQ[{a, b, e
, f}, x] && EqQ[a^2 - b^2, 0] && IntegersQ[m, p/2] && (LtQ[p, 0] || GtQ[m -
 p/2, 0])
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 4397

```
Int[u_, x_Symbol] :> Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]
```

Rubi steps

$$
\begin{aligned}
\int (\sin(x)+\tan(x))^2\,dx &= \int (1+\cos(x))^2\tan^2(x)\,dx \\
&= \int \left(-2\cos(x)-\cos^2(x)+2\sec(x)+\sec^2(x)\right)\,dx \\
&= -(2\int \cos(x)\,dx)+2\int \sec(x)\,dx-\int \cos^2(x)\,dx+\int \sec^2(x)\,dx \\
&= 2\tanh^{-1}(\sin(x))-2\sin(x)-\frac{1}{2}\cos(x)\sin(x)-\frac{\int 1\,dx}{2}-\text{Subst}(\int 1\,dx,x,-\tan(x)) \\
&= -\frac{x}{2}+2\tanh^{-1}(\sin(x))-2\sin(x)-\frac{1}{2}\cos(x)\sin(x)+\tan(x)
\end{aligned}
$$

Mathematica [B] time = 0.09, size = 60, normalized size = 2.40

$$
-\frac{x}{2}-2\sin(x)+\frac{7\tan(x)}{8}-\frac{1}{8}\sin(3x)\sec(x)-2\log\left(\cos\left(\frac{x}{2}\right)-\sin\left(\frac{x}{2}\right)\right)+2\log\left(\sin\left(\frac{x}{2}\right)+\cos\left(\frac{x}{2}\right)\right)
$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sin[x] + Tan[x])^2,x]
```

```
[Out] -1/2*x - 2*Log[Cos[x/2] - Sin[x/2]] + 2*Log[Cos[x/2] + Sin[x/2]] - 2*Sin[x]
 - (Sec[x]*Sin[3*x])/8 + (7*Tan[x])/8
```

fricas [B] time = 0.97, size = 44, normalized size = 1.76

$$
-\frac{x\cos(x)-2\cos(x)\log(\sin(x)+1)+2\cos(x)\log(-\sin(x)+1)+\left(\cos(x)^2+4\cos(x)-2\right)\sin(x)}{2\cos(x)}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((sin(x)+tan(x))^2,x, algorithm="fricas")
```

```
[Out] -1/2*(x*cos(x) - 2*cos(x)*log(sin(x) + 1) + 2*cos(x)*log(-sin(x) + 1) + (co
s(x)^2 + 4*cos(x) - 2)*sin(x))/cos(x)
```

giac [B] time = 0.20, size = 177, normalized size = 7.08

$$\frac{1}{2}\,x - \frac{x\tan\left(\frac{1}{2}\,x\right)^2 - \log\left(\frac{2\left(\tan\left(\frac{1}{2}\,x\right)^2+2\,\tan\left(\frac{1}{2}\,x\right)+1\right)}{\tan\left(\frac{1}{2}\,x\right)^2+1}\right)\tan\left(\frac{1}{2}\,x\right)^2 + \log\left(\frac{2\left(\tan\left(\frac{1}{2}\,x\right)^2-2\,\tan\left(\frac{1}{2}\,x\right)+1\right)}{\tan\left(\frac{1}{2}\,x\right)^2+1}\right)\tan\left(\frac{1}{2}\,x\right)^2 - \tan\left(\frac{1}{2}\,x\right)}{\tan\left(\frac{1}{2}\,x\right)^2 +}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((sin(x)+tan(x))^2,x, algorithm="giac")
```

```
[Out] 1/2*x - (x*tan(1/2*x)^2 - log(2*(tan(1/2*x)^2 + 2*tan(1/2*x) + 1)/(tan(1/2*
x)^2 + 1))*tan(1/2*x)^2 + log(2*(tan(1/2*x)^2 - 2*tan(1/2*x) + 1)/(tan(1/2*
x)^2 + 1))*tan(1/2*x)^2 - tan(1/2*x)^2*tan(x) + x - log(2*(tan(1/2*x)^2 + 2
*tan(1/2*x) + 1)/(tan(1/2*x)^2 + 1)) + log(2*(tan(1/2*x)^2 - 2*tan(1/2*x) +
 1)/(tan(1/2*x)^2 + 1)) + 4*tan(1/2*x) - tan(x))/(tan(1/2*x)^2 + 1) - 1/4*s
in(2*x)
```

maple [A] time = 0.05, size = 25, normalized size = 1.00

$$-\frac{\cos(x)\sin(x)}{2} - \frac{x}{2} - 2\sin(x) + 2\ln\left(\sec(x) + \tan(x)\right) + \tan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((sin(x)+tan(x))^2,x)
```

```
[Out] -1/2*cos(x)*sin(x)-1/2*x-2*sin(x)+2*ln(sec(x)+tan(x))+tan(x)
```

maxima [A] time = 0.41, size = 28, normalized size = 1.12

$$-\frac{1}{2}\,x + \log\left(\sin(x) + 1\right) - \log\left(\sin(x) - 1\right) - \frac{1}{4}\,\sin\left(2\,x\right) - 2\,\sin(x) + \tan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((sin(x)+tan(x))^2,x, algorithm="maxima")
```

```
[Out] -1/2*x + log(sin(x) + 1) - log(sin(x) - 1) - 1/4*sin(2*x) - 2*sin(x) + tan(
x)
```

mupad [B] time = 2.42, size = 61, normalized size = 2.44

$$4\,\mathrm{atanh}\left(\tan\left(\frac{x}{2}\right)\right)-\frac{x}{2}+\frac{5\tan\left(\frac{x}{2}\right)^5+6\tan\left(\frac{x}{2}\right)^3-3\tan\left(\frac{x}{2}\right)}{-\tan\left(\frac{x}{2}\right)^6-\tan\left(\frac{x}{2}\right)^4+\tan\left(\frac{x}{2}\right)^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((sin(x) + tan(x))^2,x)
```

```
[Out] 4*atanh(tan(x/2)) - x/2 + (6*tan(x/2)^3 - 3*tan(x/2) + 5*tan(x/2)^5)/(tan(x
/2)^2 - tan(x/2)^4 - tan(x/2)^6 + 1)
```

sympy [A] time = 1.66, size = 31, normalized size = 1.24

$$-\frac{x}{2}-\log\left(\sin\left(x\right)-1\right)+\log\left(\sin\left(x\right)+1\right)-2\sin\left(x\right)-\frac{\sin\left(2x\right)}{4}+\tan\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((sin(x)+tan(x))**2,x)
```

```
[Out] -x/2 - log(sin(x) - 1) + log(sin(x) + 1) - 2*sin(x) - sin(2*x)/4 + tan(x)
```

3.344 $\int (\sin(x) + \tan(x))\,dx$

Optimal. Leaf size=10

$$-\cos(x) - \log(\cos(x))$$

```
[Out] -cos(x)-ln(cos(x))
```

Rubi [A] time = 0.00, antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {2638, 3475}

$$-\cos(x) - \log(\cos(x))$$

Antiderivative was successfully verified.

```
[In] Int[Sin[x] + Tan[x],x]
```

```
[Out] -Cos[x] - Log[Cos[x]]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned} \int (\sin(x) + \tan(x))\,dx &= \int \sin(x)\,dx + \int \tan(x)\,dx \\ &= -\cos(x) - \log(\cos(x)) \end{aligned}$$

Mathematica [A] time = 0.00, size = 10, normalized size = 1.00

$$-\cos(x) - \log(\cos(x))$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[x] + Tan[x],x]
```

```
[Out] -Cos[x] - Log[Cos[x]]
```

fricas [A] time = 1.43, size = 12, normalized size = 1.20

$$-\cos(x) - \log(-\cos(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)+tan(x),x, algorithm="fricas")
[Out] -cos(x) - log(-cos(x))
```

giac [A] time = 0.14, size = 11, normalized size = 1.10

$$-\cos(x) - \log(|\cos(x)|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)+tan(x),x, algorithm="giac")
[Out] -cos(x) - log(abs(cos(x)))
```

maple [A] time = 0.00, size = 11, normalized size = 1.10

$$-\cos(x) - \ln(\cos(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)+tan(x),x)
[Out] -cos(x)-ln(cos(x))
```

maxima [A] time = 0.31, size = 8, normalized size = 0.80

$$-\cos(x) + \log(\sec(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)+tan(x),x, algorithm="maxima")
[Out] -cos(x) + log(sec(x))
```

mupad [B] time = 2.40, size = 22, normalized size = 2.20

$$2\,\mathrm{atanh}\left(\tan\left(\frac{x}{2}\right)^2\right) - \frac{2}{\tan\left(\frac{x}{2}\right)^2 + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x) + tan(x),x)
```

```
[Out] 2*atanh(tan(x/2)^2) - 2/(tan(x/2)^2 + 1)
```

sympy [A] time = 0.05, size = 8, normalized size = 0.80

$$-\log(\cos(x)) - \cos(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)+tan(x),x)
[Out] -log(cos(x)) - cos(x)
```

3.345 $\int \frac{1}{\sin(x)+\tan(x)}\,dx$

Optimal. Leaf size=24

$$-\frac{1}{2}\csc^2(x)-\frac{1}{2}\tanh^{-1}(\cos(x))+\frac{1}{2}\cot(x)\csc(x)$$

```
[Out] -1/2*arctanh(cos(x))+1/2*cot(x)*csc(x)-1/2*csc(x)^2
```

Rubi [A] time = 0.06, antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.857, Rules used = {4397, 2706, 2606, 30, 2611, 3770}

$$-\frac{1}{2}\csc^2(x)-\frac{1}{2}\tanh^{-1}(\cos(x))+\frac{1}{2}\cot(x)\csc(x)$$

Antiderivative was successfully verified.

```
[In] Int[(Sin[x] + Tan[x])^(-1),x]
```

```
[Out] -ArcTanh[Cos[x]]/2 + (Cot[x]*Csc[x])/2 - Csc[x]^2/2
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 2611

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(b*(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*(
m + n - 1)), x] - Dist[(b^2*(n - 1))/(m + n - 1), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 2706

```
Int[((g_.)*tan[(e_.) + (f_.)*(x_)])^(p_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Dist[1/a, Int[Sec[e + f*x]^2*(g*Tan[e + f*x])^p, x], x]
```

```
 - Dist[1/(b*g), Int[Sec[e + f*x]*(g*Tan[e + f*x])^(p + 1), x], x] /; FreeQ
[{a, b, e, f, g, p}, x] && EqQ[a^2 - b^2, 0] && NeQ[p, -1]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 4397

```
Int[u_, x_Symbol] :> Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sin(x)+\tan(x)}\,dx &= \int \frac{\cot(x)}{1+\cos(x)}\,dx\\ &= -\int \cot^2(x)\csc(x)\,dx+\int \cot(x)\csc^2(x)\,dx\\ &= \frac{1}{2}\cot(x)\csc(x)+\frac{1}{2}\int \csc(x)\,dx-\text{Subst}(\int x\,dx,x,\csc(x))\\ &= -\frac{1}{2}\tanh^{-1}(\cos(x))+\frac{1}{2}\cot(x)\csc(x)-\frac{\csc^2(x)}{2}\end{aligned}$$

Mathematica [A] time = 0.01, size = 35, normalized size = 1.46

$$-\frac{1}{4}\sec^2\left(\frac{x}{2}\right)+\frac{1}{2}\log\left(\sin\left(\frac{x}{2}\right)\right)-\frac{1}{2}\log\left(\cos\left(\frac{x}{2}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sin[x] + Tan[x])^(-1),x]
[Out] -1/2*Log[Cos[x/2]] + Log[Sin[x/2]]/2 - Sec[x/2]^2/4
```

fricas [A] time = 1.90, size = 35, normalized size = 1.46

$$-\frac{(\cos(x)+1)\log\left(\frac{1}{2}\cos(x)+\frac{1}{2}\right)-(\cos(x)+1)\log\left(-\frac{1}{2}\cos(x)+\frac{1}{2}\right)+2}{4(\cos(x)+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sin(x)+tan(x)),x, algorithm="fricas")
```

```
[Out] -1/4*((cos(x) + 1)*log(1/2*cos(x) + 1/2) - (cos(x) + 1)*log(-1/2*cos(x) + 1
/2) + 2)/(cos(x) + 1)
```

giac [A] time = 0.14, size = 28, normalized size = 1.17

$$\frac{\cos(x)-1}{4\,(\cos(x)+1)}+\frac{1}{4}\,\log\left(-\frac{\cos(x)-1}{\cos(x)+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sin(x)+tan(x)),x, algorithm="giac")
[Out] 1/4*(cos(x) - 1)/(cos(x) + 1) + 1/4*log(-(cos(x) - 1)/(cos(x) + 1))
```

maple [A] time = 0.11, size = 24, normalized size = 1.00

$$\frac{\ln\left(-1+\cos(x)\right)}{4}-\frac{1}{2\,(1+\cos(x))}-\frac{\ln\left(1+\cos(x)\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(x)+tan(x)),x)
[Out] 1/4*ln(-1+cos(x))-1/2/(1+cos(x))-1/4*ln(1+cos(x))
```

maxima [A] time = 0.31, size = 25, normalized size = 1.04

$$-\frac{\sin(x)^2}{4\,(\cos(x)+1)^2}+\frac{1}{2}\,\log\left(\frac{\sin(x)}{\cos(x)+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sin(x)+tan(x)),x, algorithm="maxima")
[Out] -1/4*sin(x)^2/(cos(x) + 1)^2 + 1/2*log(sin(x)/(cos(x) + 1))
```

mupad [B] time = 2.47, size = 16, normalized size = 0.67

$$\frac{\ln\left(\tan\left(\frac{x}{2}\right)\right)}{2}-\frac{\tan\left(\frac{x}{2}\right)^2}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(x) + tan(x)),x)
[Out] log(tan(x/2))/2 - tan(x/2)^2/4
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sin(x) + \tan(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sin(x)+tan(x)),x)
```

```
[Out] Integral(1/(sin(x) + tan(x)), x)
```

3.346 $\int \frac{1}{(\sin(x)+\tan(x))^2}\,dx$

Optimal. Leaf size=33

$$-\frac{2}{5}\cot^5(x)-\frac{\cot^3(x)}{3}+\frac{2\csc^5(x)}{5}-\frac{2\csc^3(x)}{3}$$

```
[Out] -1/3*cot(x)^3-2/5*cot(x)^5-2/3*csc(x)^3+2/5*csc(x)^5
```

Rubi [A] time = 0.13, antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 6, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.857, Rules used = {4397, 2711, 2607, 30, 2606, 14}

$$-\frac{2}{5}\cot^5(x)-\frac{\cot^3(x)}{3}+\frac{2\csc^5(x)}{5}-\frac{2\csc^3(x)}{3}$$

Antiderivative was successfully verified.

```
[In] Int[(Sin[x] + Tan[x])^(-2),x]
```

```
[Out] -Cot[x]^3/3 - (2*Cot[x]^5)/5 - (2*Csc[x]^3)/3 + (2*Csc[x]^5)/5
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
```

```
2] && LtQ[0, n, m - 1])
```

Rule 2711

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((g_.)*tan[(e_.) + (f_.)*(x
_)])^(p_.), x_Symbol] :> Dist[a^(2*m), Int[ExpandIntegrand[(g*Tan[e + f*x])
^p/Sec[e + f*x]^m, (a*Sec[e + f*x] - b*Tan[e + f*x])^(-m), x], x], x] /; Fr
eeQ[{a, b, e, f, g, p}, x] && EqQ[a^2 - b^2, 0] && ILtQ[m, 0]
```

Rule 4397

```
Int[u_, x_Symbol] :> Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{(\sin(x)+\tan(x))^2}\,dx &= \int \frac{\cot^2(x)}{(1+\cos(x))^2}\,dx\\
&= \int \left(\cot^4(x)\csc^2(x)-2\cot^3(x)\csc^3(x)+\cot^2(x)\csc^4(x)\right)dx\\
&= -\left(2\int \cot^3(x)\csc^3(x)\,dx\right)+\int \cot^4(x)\csc^2(x)\,dx+\int \cot^2(x)\csc^4(x)\,dx\\
&= 2\,\text{Subst}\left(\int x^2\left(-1+x^2\right)dx,x,\csc(x)\right)+\text{Subst}\left(\int x^4\,dx,x,-\cot(x)\right)+\text{Subst}\left(\int x^{2}\right.\\
&= -\frac{1}{5}\cot^5(x)+2\,\text{Subst}\left(\int\left(-x^2+x^4\right)dx,x,\csc(x)\right)+\text{Subst}\left(\int\left(x^2+x^4\right)dx,x,-\text{co}\right.\\
&= -\frac{1}{3}\cot^3(x)-\frac{2\cot^5(x)}{5}-\frac{2\csc^3(x)}{3}+\frac{2\csc^5(x)}{5}
\end{aligned}$$

Mathematica [A] time = 0.02, size = 57, normalized size = 1.73

$$-\frac{7}{120}\tan\left(\frac{x}{2}\right)-\frac{1}{8}\cot\left(\frac{x}{2}\right)+\frac{1}{40}\tan\left(\frac{x}{2}\right)\sec^4\left(\frac{x}{2}\right)-\frac{11}{120}\tan\left(\frac{x}{2}\right)\sec^2\left(\frac{x}{2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sin[x] + Tan[x])^(-2),x]
```

```
[Out] -1/8*Cot[x/2] - (7*Tan[x/2])/120 - (11*Sec[x/2]^2*Tan[x/2])/120 + (Sec[x/2]
^4*Tan[x/2])/40
```

fricas [A] time = 1.45, size = 34, normalized size = 1.03

$$-\frac{\cos(x)^3+2\cos(x)^2+8\cos(x)+4}{15\left(\cos(x)^2+2\cos(x)+1\right)\sin(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sin(x)+tan(x))^2,x, algorithm="fricas")
```

```
[Out] -1/15*(cos(x)^3 + 2*cos(x)^2 + 8*cos(x) + 4)/((cos(x)^2 + 2*cos(x) + 1)*sin
(x))
```

giac [A] time = 0.14, size = 31, normalized size = 0.94

$$\frac{1}{40}\tan\left(\frac{1}{2}x\right)^{5}-\frac{1}{24}\tan\left(\frac{1}{2}x\right)^{3}-\frac{1}{8\tan\left(\frac{1}{2}x\right)}-\frac{1}{8}\tan\left(\frac{1}{2}x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sin(x)+tan(x))^2,x, algorithm="giac")
```

```
[Out] 1/40*tan(1/2*x)^5 - 1/24*tan(1/2*x)^3 - 1/8/tan(1/2*x) - 1/8*tan(1/2*x)
```

maple [A] time = 0.12, size = 32, normalized size = 0.97

$$\frac{\left(\tan^{5}\left(\frac{x}{2}\right)\right)}{40}-\frac{\left(\tan^{3}\left(\frac{x}{2}\right)\right)}{24}-\frac{\tan\left(\frac{x}{2}\right)}{8}-\frac{1}{8\tan\left(\frac{x}{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(x)+tan(x))^2,x)
```

```
[Out] 1/40*tan(1/2*x)^5-1/24*tan(1/2*x)^3-1/8*tan(1/2*x)-1/8/tan(1/2*x)
```

maxima [A] time = 0.31, size = 45, normalized size = 1.36

$$-\frac{\cos(x)+1}{8\sin(x)}-\frac{\sin(x)}{8(\cos(x)+1)}-\frac{\sin(x)^{3}}{24(\cos(x)+1)^{3}}+\frac{\sin(x)^{5}}{40(\cos(x)+1)^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sin(x)+tan(x))^2,x, algorithm="maxima")
```

```
[Out] -1/8*(cos(x) + 1)/sin(x) - 1/8*sin(x)/(cos(x) + 1) - 1/24*sin(x)^3/(cos(x)
+ 1)^3 + 1/40*sin(x)^5/(cos(x) + 1)^5
```

mupad [B] time = 2.39, size = 40, normalized size = 1.21

$$-\frac{8\cos\left(\frac{x}{2}\right)^{6}-4\cos\left(\frac{x}{2}\right)^{4}+14\cos\left(\frac{x}{2}\right)^{2}-3}{120\cos\left(\frac{x}{2}\right)^{5}\sin\left(\frac{x}{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(x) + tan(x))^2,x)
```

```
[Out] -(14*cos(x/2)^2 - 4*cos(x/2)^4 + 8*cos(x/2)^6 - 3)/(120*cos(x/2)^5*sin(x/2)
)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(\sin{(x)} + \tan{(x)})^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sin(x)+tan(x))**2,x)
```

```
[Out] Integral((sin(x) + tan(x))**(-2), x)
```

3.347 $\int \frac{1}{(\sin(x)+\tan(x))^3} \, dx$

Optimal. Leaf size=60

$$-\frac{1}{32(1-\cos(x))}-\frac{1}{16(\cos(x)+1)}-\frac{3}{32(\cos(x)+1)^2}+\frac{1}{6(\cos(x)+1)^3}-\frac{1}{16(\cos(x)+1)^4}+\frac{1}{32}\tanh^{-1}(\cos(x))$$

```
[Out] 1/32*arctanh(cos(x))-1/32/(1-cos(x))-1/16/(1+cos(x))^4+1/6/(1+cos(x))^3-3/3
2/(1+cos(x))^2-1/16/(1+cos(x))
```

Rubi [A] time = 0.07, antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.571, Rules used = {4397, 2707, 88, 207}

$$-\frac{1}{32(1-\cos(x))}-\frac{1}{16(\cos(x)+1)}-\frac{3}{32(\cos(x)+1)^2}+\frac{1}{6(\cos(x)+1)^3}-\frac{1}{16(\cos(x)+1)^4}+\frac{1}{32}\tanh^{-1}(\cos(x))$$

Antiderivative was successfully verified.

```
[In] Int[(Sin[x] + Tan[x])^(-3),x]
```

```
[Out] ArcTanh[Cos[x]]/32 - 1/(32*(1 - Cos[x])) - 1/(16*(1 + Cos[x])^4) + 1/(6*(1
+ Cos[x])^3) - 3/(32*(1 + Cos[x])^2) - 1/(16*(1 + Cos[x]))
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 2707

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Dist[1/f, Subst[Int[(x^p*(a + x)^(m - (p + 1)/2))/(a - x)
^((p + 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && Eq
Q[a^2 - b^2, 0] && IntegerQ[(p + 1)/2]
```

Rule 4397

```
Int[u_, x_Symbol] :> Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]
```

Rubi steps

$$\int \frac{1}{(\sin(x)+\tan(x))^3}\,dx = \int \frac{\cot^3(x)}{(1+\cos(x))^3}\,dx$$
$$= -\text{Subst}\left(\int \frac{x^3}{(1-x)^2(1+x)^5}\,dx, x, \cos(x)\right)$$
$$= -\text{Subst}\left(\int \left(\frac{1}{32(-1+x)^2} - \frac{1}{4(1+x)^5} + \frac{1}{2(1+x)^4} - \frac{3}{16(1+x)^3} - \frac{1}{16(1+x)^2} + \frac{}{32(\cdots}\right.\right.$$
$$= -\frac{1}{32(1-\cos(x))} - \frac{1}{16(1+\cos(x))^4} + \frac{1}{6(1+\cos(x))^3} - \frac{3}{32(1+\cos(x))^2} - \frac{1}{16(1+\cos\cdots}$$
$$= \frac{1}{32}\tanh^{-1}(\cos(x)) - \frac{1}{32(1-\cos(x))} - \frac{1}{16(1+\cos(x))^4} + \frac{1}{6(1+\cos(x))^3} - \frac{3}{32(1+\text{co}\cdots}$$

Mathematica [A] time = 0.02, size = 83, normalized size = 1.38

$$-\frac{1}{64}\csc^2\left(\frac{x}{2}\right)-\frac{1}{256}\sec^8\left(\frac{x}{2}\right)+\frac{1}{48}\sec^6\left(\frac{x}{2}\right)-\frac{3}{128}\sec^4\left(\frac{x}{2}\right)-\frac{1}{32}\sec^2\left(\frac{x}{2}\right)-\frac{1}{32}\log\left(\sin\left(\frac{x}{2}\right)\right)+\frac{1}{32}\log\left(\cos\left(\frac{x}{2}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sin[x] + Tan[x])^(-3),x]
```

```
[Out] -1/64*Csc[x/2]^2 + Log[Cos[x/2]]/32 - Log[Sin[x/2]]/32 - Sec[x/2]^2/32 - (3
*Sec[x/2]^4)/128 + Sec[x/2]^6/48 - Sec[x/2]^8/256
```

fricas [B] time = 1.82, size = 130, normalized size = 2.17

$$-\frac{6\cos(x)^4+18\cos(x)^3-50\cos(x)^2-3\left(\cos(x)^5+3\cos(x)^4+2\cos(x)^3-2\cos(x)^2-3\cos(x)-1\right)\log\left(\frac{1}{2}\cdots\right.}{192\left(\cos(x)^5+3\cos(x)^4+2\cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sin(x)+tan(x))^3,x, algorithm="fricas")
```

```
[Out] -1/192*(6*cos(x)^4 + 18*cos(x)^3 - 50*cos(x)^2 - 3*(cos(x)^5 + 3*cos(x)^4 +
 2*cos(x)^3 - 2*cos(x)^2 - 3*cos(x) - 1)*log(1/2*cos(x) + 1/2) + 3*(cos(x)^
5 + 3*cos(x)^4 + 2*cos(x)^3 - 2*cos(x)^2 - 3*cos(x) - 1)*log(-1/2*cos(x) +
1/2) - 54*cos(x) - 16)/(cos(x)^5 + 3*cos(x)^4 + 2*cos(x)^3 - 2*cos(x)^2 - 3
*cos(x) - 1)
```

giac [B] time = 0.16, size = 95, normalized size = 1.58

$$\frac{\left(\frac{\cos(x)-1}{\cos(x)+1}+1\right)(\cos(x)+1)}{64\,(\cos(x)-1)}+\frac{\cos(x)-1}{32\,(\cos(x)+1)}+\frac{(\cos(x)-1)^2}{64\,(\cos(x)+1)^2}-\frac{(\cos(x)-1)^3}{192\,(\cos(x)+1)^3}-\frac{(\cos(x)-1)^4}{256\,(\cos(x)+1)^4}-\frac{1}{64}\log\left(-\frac{\cos(x)-1}{\cos(x)+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sin(x)+tan(x))^3,x, algorithm="giac")
```

```
[Out] 1/64*((cos(x) - 1)/(cos(x) + 1) + 1)*(cos(x) + 1)/(cos(x) - 1) + 1/32*(cos(
x) - 1)/(cos(x) + 1) + 1/64*(cos(x) - 1)^2/(cos(x) + 1)^2 - 1/192*(cos(x) -
 1)^3/(cos(x) + 1)^3 - 1/256*(cos(x) - 1)^4/(cos(x) + 1)^4 - 1/64*log(-(cos
(x) - 1)/(cos(x) + 1))
```

maple [A] time = 0.13, size = 56, normalized size = 0.93

$$\frac{1}{-32+32\cos(x)}-\frac{\ln(-1+\cos(x))}{64}-\frac{1}{16\,(1+\cos(x))^4}+\frac{1}{6\,(1+\cos(x))^3}-\frac{3}{32\,(1+\cos(x))^2}-\frac{1}{16\,(1+\cos(x))}+\frac{\ln(1+\cos(x))}{64}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(x)+tan(x))^3,x)
```

```
[Out] 1/32/(-1+cos(x))-1/64*ln(-1+cos(x))-1/16/(1+cos(x))^4+1/6/(1+cos(x))^3-3/32
/(1+cos(x))^2-1/16/(1+cos(x))+1/64*ln(1+cos(x))
```

maxima [A] time = 0.31, size = 73, normalized size = 1.22

$$-\frac{(\cos(x)+1)^2}{64\,\sin(x)^2}-\frac{\sin(x)^2}{32\,(\cos(x)+1)^2}+\frac{\sin(x)^4}{64\,(\cos(x)+1)^4}+\frac{\sin(x)^6}{192\,(\cos(x)+1)^6}-\frac{\sin(x)^8}{256\,(\cos(x)+1)^8}-\frac{1}{32}\log\left(\frac{\sin(x)}{\cos(x)+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sin(x)+tan(x))^3,x, algorithm="maxima")
```

```
[Out] -1/64*(cos(x) + 1)^2/sin(x)^2 - 1/32*sin(x)^2/(cos(x) + 1)^2 + 1/64*sin(x)^
4/(cos(x) + 1)^4 + 1/192*sin(x)^6/(cos(x) + 1)^6 - 1/256*sin(x)^8/(cos(x) +
 1)^8 - 1/32*log(sin(x)/(cos(x) + 1))
```

mupad [B] time = 2.39, size = 48, normalized size = 0.80

$$\frac{\tan\left(\frac{x}{2}\right)^4}{64}-\frac{1}{64\tan\left(\frac{x}{2}\right)^2}-\frac{\tan\left(\frac{x}{2}\right)^2}{32}-\frac{\ln\left(\tan\left(\frac{x}{2}\right)\right)}{32}+\frac{\tan\left(\frac{x}{2}\right)^6}{192}-\frac{\tan\left(\frac{x}{2}\right)^8}{256}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(x) + tan(x))^3,x)
```

```
[Out] tan(x/2)^4/64 - 1/(64*tan(x/2)^2) - tan(x/2)^2/32 - log(tan(x/2))/32 + tan(
x/2)^6/192 - tan(x/2)^8/256
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(\sin(x) + \tan(x))^3} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sin(x)+tan(x))**3,x)
```

```
[Out] Integral((sin(x) + tan(x))**(-3), x)
```

3.348 $\int \frac{1}{(\sin(x)+\tan(x))^4} \, dx$

Optimal. Leaf size=65

$$-\frac{8}{11}\cot^{11}(x)-\frac{16\cot^9(x)}{9}-\frac{9\cot^7(x)}{7}-\frac{\cot^5(x)}{5}+\frac{8\csc^{11}(x)}{11}-\frac{20\csc^9(x)}{9}+\frac{16\csc^7(x)}{7}-\frac{4\csc^5(x)}{5}$$

```
[Out] -1/5*cot(x)^5-9/7*cot(x)^7-16/9*cot(x)^9-8/11*cot(x)^11-4/5*csc(x)^5+16/7*c
sc(x)^7-20/9*csc(x)^9+8/11*csc(x)^11
```

Rubi [A] time = 0.21, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 18, number of rules used = 6, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.857, Rules used = {4397, 2711, 2607, 14, 2606, 270}

$$-\frac{8}{11}\cot^{11}(x)-\frac{16\cot^9(x)}{9}-\frac{9\cot^7(x)}{7}-\frac{\cot^5(x)}{5}+\frac{8\csc^{11}(x)}{11}-\frac{20\csc^9(x)}{9}+\frac{16\csc^7(x)}{7}-\frac{4\csc^5(x)}{5}$$

Antiderivative was successfully verified.

```
[In] Int[(Sin[x] + Tan[x])^(-4),x]
```

```
[Out] -Cot[x]^5/5 - (9*Cot[x]^7)/7 - (16*Cot[x]^9)/9 - (8*Cot[x]^11)/11 - (4*Csc[
x]^5)/5 + (16*Csc[x]^7)/7 - (20*Csc[x]^9)/9 + (8*Csc[x]^11)/11
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 2711

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((g_.)*tan[(e_.) + (f_.)*(x
_)])^(p_.), x_Symbol] :> Dist[a^(2*m), Int[ExpandIntegrand[(g*Tan[e + f*x])
^p/Sec[e + f*x]^m, (a*Sec[e + f*x] - b*Tan[e + f*x])^(-m), x], x], x] /; Fr
eeQ[{a, b, e, f, g, p}, x] && EqQ[a^2 - b^2, 0] && ILtQ[m, 0]
```

Rule 4397

```
Int[u_, x_Symbol] :> Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(\sin(x)+\tan(x))^4}\,dx &= \int \frac{\cot^4(x)}{(1+\cos(x))^4}\,dx\\ &= \int \left(\cot^8(x)\csc^4(x) - 4\cot^7(x)\csc^5(x) + 6\cot^6(x)\csc^6(x) - 4\cot^5(x)\csc^7(x) + \cot^4(\right.\\ &= -\left(4\int \cot^7(x)\csc^5(x)\,dx\right) - 4\int \cot^5(x)\csc^7(x)\,dx + 6\int \cot^6(x)\csc^6(x)\,dx + \int \mathrm{cc}\\ &= 4\,\mathrm{Subst}\left(\int x^6\left(-1+x^2\right)^2\,dx, x, \csc(x)\right) + 4\,\mathrm{Subst}\left(\int x^4\left(-1+x^2\right)^3\,dx, x, \csc(x)\right) +\\ &= 4\,\mathrm{Subst}\left(\int \left(-x^4+3x^6-3x^8+x^{10}\right)\,dx, x, \csc(x)\right) + 4\,\mathrm{Subst}\left(\int \left(x^6-2x^8+x^{10}\right)\,dx\right.\\ &= -\frac{1}{5}\cot^5(x) - \frac{9\cot^7(x)}{7} - \frac{16\cot^9(x)}{9} - \frac{8\cot^{11}(x)}{11} - \frac{4\csc^5(x)}{5} + \frac{16\csc^7(x)}{7} - \frac{20\csc^9}{9}\end{aligned}$$

Mathematica [A] time = 0.02, size = 129, normalized size = 1.98

$$-\frac{2749\tan\left(\frac{x}{2}\right)}{110880}+\frac{1}{96}\cot\left(\frac{x}{2}\right)-\frac{1}{384}\cot\left(\frac{x}{2}\right)\csc^2\left(\frac{x}{2}\right)+\frac{\tan\left(\frac{x}{2}\right)\sec^{10}\left(\frac{x}{2}\right)}{1408}-\frac{7\tan\left(\frac{x}{2}\right)\sec^8\left(\frac{x}{2}\right)}{1584}+\frac{641\tan\left(\frac{x}{2}\right)\sec^6\left(\frac{x}{2}\right)}{88704}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sin[x] + Tan[x])^(-4),x]
```

```
[Out] Cot[x/2]/96 - (Cot[x/2]*Csc[x/2]^2)/384 - (2749*Tan[x/2])/110880 - (2033*Se
c[x/2]^2*Tan[x/2])/443520 + (179*Sec[x/2]^4*Tan[x/2])/73920 + (641*Sec[x/2]
```

```
^6*Tan[x/2])/88704 - (7*Sec[x/2]^8*Tan[x/2])/1584 + (Sec[x/2]^10*Tan[x/2])/
1408
```

fricas [A] time = 1.06, size = 78, normalized size = 1.20

$$\frac{122\cos(x)^7 + 488\cos(x)^6 + 549\cos(x)^5 - 244\cos(x)^4 - 64\cos(x)^3 + 144\cos(x)^2 + 128\cos(x) + 32}{3465\left(\cos(x)^6 + 4\cos(x)^5 + 5\cos(x)^4 - 5\cos(x)^2 - 4\cos(x) - 1\right)\sin(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sin(x)+tan(x))^4,x, algorithm="fricas")
```

```
[Out] 1/3465*(122*cos(x)^7 + 488*cos(x)^6 + 549*cos(x)^5 - 244*cos(x)^4 - 64*cos(
x)^3 + 144*cos(x)^2 + 128*cos(x) + 32)/((cos(x)^6 + 4*cos(x)^5 + 5*cos(x)^4
 - 5*cos(x)^2 - 4*cos(x) - 1)*sin(x))
```

giac [A] time = 0.15, size = 65, normalized size = 1.00

$$\frac{1}{1408}\tan\left(\frac{1}{2}x\right)^{11} - \frac{1}{1152}\tan\left(\frac{1}{2}x\right)^{9} - \frac{3}{896}\tan\left(\frac{1}{2}x\right)^{7} + \frac{3}{640}\tan\left(\frac{1}{2}x\right)^{5} + \frac{1}{128}\tan\left(\frac{1}{2}x\right)^{3} + \frac{3\tan\left(\frac{1}{2}x\right)^2 - 1}{384\tan\left(\frac{1}{2}x\right)^3} - \frac{3}{128}\mathrm{t}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sin(x)+tan(x))^4,x, algorithm="giac")
```

```
[Out] 1/1408*tan(1/2*x)^11 - 1/1152*tan(1/2*x)^9 - 3/896*tan(1/2*x)^7 + 3/640*tan
(1/2*x)^5 + 1/128*tan(1/2*x)^3 + 1/384*(3*tan(1/2*x)^2 - 1)/tan(1/2*x)^3 -
3/128*tan(1/2*x)
```

maple [A] time = 0.13, size = 64, normalized size = 0.98

$$\frac{\left(\tan^{11}\left(\frac{x}{2}\right)\right)}{1408} - \frac{\left(\tan^{9}\left(\frac{x}{2}\right)\right)}{1152} - \frac{3\left(\tan^{7}\left(\frac{x}{2}\right)\right)}{896} + \frac{3\left(\tan^{5}\left(\frac{x}{2}\right)\right)}{640} + \frac{\left(\tan^{3}\left(\frac{x}{2}\right)\right)}{128} - \frac{3\tan\left(\frac{x}{2}\right)}{128} + \frac{1}{128\tan\left(\frac{x}{2}\right)} - \frac{1}{384\tan\left(\frac{x}{2}\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(x)+tan(x))^4,x)
```

```
[Out] 1/1408*tan(1/2*x)^11-1/1152*tan(1/2*x)^9-3/896*tan(1/2*x)^7+3/640*tan(1/2*x
)^5+1/128*tan(1/2*x)^3-3/128*tan(1/2*x)+1/128/tan(1/2*x)-1/384/tan(1/2*x)^3
```

maxima [A] time = 0.32, size = 97, normalized size = 1.49

$$\frac{\left(\frac{3\sin(x)^2}{(\cos(x)+1)^2} - 1\right)(\cos(x)+1)^3}{384\sin(x)^3} - \frac{3\sin(x)}{128(\cos(x)+1)} + \frac{\sin(x)^3}{128(\cos(x)+1)^3} + \frac{3\sin(x)^5}{640(\cos(x)+1)^5} - \frac{3\sin(x)^7}{896(\cos(x)+1)^7} - \frac{\mathrm{si}}{1152(\mathrm{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sin(x)+tan(x))^4,x, algorithm="maxima")
```

```
[Out] 1/384*(3*sin(x)^2/(cos(x) + 1)^2 - 1)*(cos(x) + 1)^3/sin(x)^3 - 3/128*sin(x
)/(cos(x) + 1) + 1/128*sin(x)^3/(cos(x) + 1)^3 + 3/640*sin(x)^5/(cos(x) + 1
)^5 - 3/896*sin(x)^7/(cos(x) + 1)^7 - 1/1152*sin(x)^9/(cos(x) + 1)^9 + 1/14
08*sin(x)^11/(cos(x) + 1)^11
```

mupad [B] time = 2.45, size = 87, normalized size = 1.34

$$-\frac{15616\cos\left(\frac{x}{2}\right)^{14}-23424\cos\left(\frac{x}{2}\right)^{12}+5856\cos\left(\frac{x}{2}\right)^{10}+976\cos\left(\frac{x}{2}\right)^{8}+7296\cos\left(\frac{x}{2}\right)^{6}-7440\cos\left(\frac{x}{2}\right)^{4}+2590\cos\left(\frac{x}{2}\right)^{2}-315}{443520\left(\cos\left(\frac{x}{2}\right)^{11}\sin\left(\frac{x}{2}\right)-\cos\left(\frac{x}{2}\right)^{13}\sin\left(\frac{x}{2}\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(x) + tan(x))^4,x)
```

```
[Out] -(2590*cos(x/2)^2 - 7440*cos(x/2)^4 + 7296*cos(x/2)^6 + 976*cos(x/2)^8 + 58
56*cos(x/2)^10 - 23424*cos(x/2)^12 + 15616*cos(x/2)^14 - 315)/(443520*(cos(
x/2)^11*sin(x/2) - cos(x/2)^13*sin(x/2)))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(\sin\left(x\right)+\tan\left(x\right)\right)^{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sin(x)+tan(x))**4,x)
```

```
[Out] Integral((sin(x) + tan(x))**(-4), x)
```

3.349 $\int \frac{A+C\sin(x)}{b\cos(x)+c\sin(x)}\,dx$

Optimal. Leaf size=74

$$-\frac{A\tanh^{-1}\left(\frac{c\cos(x)-b\sin(x)}{\sqrt{b^2+c^2}}\right)}{\sqrt{b^2+c^2}}+\frac{cCx}{b^2+c^2}-\frac{bC\log(b\cos(x)+c\sin(x))}{b^2+c^2}$$

```
[Out] c*C*x/(b^2+c^2)-b*C*ln(b*cos(x)+c*sin(x))/(b^2+c^2)-A*arctanh((c*cos(x)-b*s
in(x))/(b^2+c^2)^(1/2))/(b^2+c^2)^(1/2)
```

Rubi [A] time = 0.06, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3137, 3074, 206}

$$-\frac{A\tanh^{-1}\left(\frac{c\cos(x)-b\sin(x)}{\sqrt{b^2+c^2}}\right)}{\sqrt{b^2+c^2}}+\frac{cCx}{b^2+c^2}-\frac{bC\log(b\cos(x)+c\sin(x))}{b^2+c^2}$$

Antiderivative was successfully verified.

```
[In] Int[(A + C*Sin[x])/(b*Cos[x] + c*Sin[x]),x]
```

```
[Out] (c*C*x)/(b^2 + c^2) - (A*ArcTanh[(c*Cos[x] - b*Sin[x])/Sqrt[b^2 + c^2]])/Sq
rt[b^2 + c^2] - (b*C*Log[b*Cos[x] + c*Sin[x]])/(b^2 + c^2)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3074

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x
_Symbol] :> -Dist[d^(-1), Subst[Int[1/(a^2 + b^2 - x^2), x], x, b*Cos[c + d
*x] - a*Sin[c + d*x]], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

Rule 3137

```
Int[((A_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])/((a_.) + cos[(d_.) + (e_.)*(x_)
]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)]), x_Symbol] :> Simp[(c*C*(d + e*x))
/(e*(b^2 + c^2)), x] + (Dist[(A*(b^2 + c^2) - a*c*C)/(b^2 + c^2), Int[1/(a
+ b*Cos[d + e*x] + c*Sin[d + e*x]), x], x] - Simp[(b*C*Log[a + b*Cos[d + e*
x] + c*Sin[d + e*x]])/(e*(b^2 + c^2)), x]) /; FreeQ[{a, b, c, d, e, A, C},
```

```
x] && NeQ[b^2 + c^2, 0] && NeQ[A*(b^2 + c^2) - a*c*C, 0]
```

Rubi steps

$$\int \frac{A+C\sin(x)}{b\cos(x)+c\sin(x)}\,dx = \frac{cCx}{b^2+c^2}-\frac{bC\log(b\cos(x)+c\sin(x))}{b^2+c^2}+A\int \frac{1}{b\cos(x)+c\sin(x)}\,dx$$
$$= \frac{cCx}{b^2+c^2}-\frac{bC\log(b\cos(x)+c\sin(x))}{b^2+c^2}-A\,\text{Subst}\left(\int \frac{1}{b^2+c^2-x^2}\,dx,x,c\cos(x)-b\,s\right.$$
$$= \frac{cCx}{b^2+c^2}-\frac{A\tanh^{-1}\left(\frac{c\cos(x)-b\sin(x)}{\sqrt{b^2+c^2}}\right)}{\sqrt{b^2+c^2}}-\frac{bC\log(b\cos(x)+c\sin(x))}{b^2+c^2}$$

Mathematica [A] time = 0.20, size = 68, normalized size = 0.92

$$\frac{2A\tanh^{-1}\left(\frac{b\tan\left(\frac{x}{2}\right)-c}{\sqrt{b^2+c^2}}\right)}{\sqrt{b^2+c^2}}+\frac{C(cx-b\log(b\cos(x)+c\sin(x)))}{b^2+c^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + C*Sin[x])/(b*Cos[x] + c*Sin[x]),x]
```

```
[Out] (2*A*ArcTanh[(-c + b*Tan[x/2])/Sqrt[b^2 + c^2]])/Sqrt[b^2 + c^2] + (C*(c*x
- b*Log[b*Cos[x] + c*Sin[x]]))/(b^2 + c^2)
```

fricas [B] time = 0.93, size = 144, normalized size = 1.95

$$\frac{2\,Ccx-Cb\log\left(2\,bc\cos(x)\sin(x)+\left(b^2-c^2\right)\cos(x)^2+c^2\right)+\sqrt{b^2+c^2}\,A\log\left(-\frac{2\,bc\cos(x)\sin(x)+\left(b^2-c^2\right)\cos(x)^2-2\,b^2-c}{2\,bc\cos(x)\sin(x)+\left(b^2-c\right.}\right.}{2\left(b^2+c^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*sin(x))/(b*cos(x)+c*sin(x)),x, algorithm="fricas")
```

```
[Out] 1/2*(2*C*c*x - C*b*log(2*b*c*cos(x)*sin(x) + (b^2 - c^2)*cos(x)^2 + c^2) +
sqrt(b^2 + c^2)*A*log(-(2*b*c*cos(x)*sin(x) + (b^2 - c^2)*cos(x)^2 - 2*b^2
- c^2 + 2*sqrt(b^2 + c^2)*(c*cos(x) - b*sin(x)))/(2*b*c*cos(x)*sin(x) + (b^
2 - c^2)*cos(x)^2 + c^2)))/(b^2 + c^2)
```

giac [A] time = 0.21, size = 131, normalized size = 1.77

$$\frac{Ccx}{b^2+c^2}+\frac{Cb\log\left(\tan\left(\frac{1}{2}x\right)^2+1\right)}{b^2+c^2}-\frac{Cb\log\left(\left|b\tan\left(\frac{1}{2}x\right)^2-2c\tan\left(\frac{1}{2}x\right)-b\right|\right)}{b^2+c^2}-\frac{A\log\left(\frac{\left|2b\tan\left(\frac{1}{2}x\right)-2c-2\sqrt{b^2+c^2}\right|}{\left|2b\tan\left(\frac{1}{2}x\right)-2c+2\sqrt{b^2+c^2}\right|}\right)}{\sqrt{b^2+c^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*sin(x))/(b*cos(x)+c*sin(x)),x, algorithm="giac")
```

```
[Out] C*c*x/(b^2 + c^2) + C*b*log(tan(1/2*x)^2 + 1)/(b^2 + c^2) - C*b*log(abs(b*t
an(1/2*x)^2 - 2*c*tan(1/2*x) - b))/(b^2 + c^2) - A*log(abs(2*b*tan(1/2*x) -
 2*c - 2*sqrt(b^2 + c^2))/abs(2*b*tan(1/2*x) - 2*c + 2*sqrt(b^2 + c^2)))/sq
rt(b^2 + c^2)
```

maple [B] time = 0.14, size = 150, normalized size = 2.03

$$-\frac{bC\ln\left(b\left(\tan^2\left(\frac{x}{2}\right)\right)-2c\tan\left(\frac{x}{2}\right)-b\right)}{b^2+c^2}+\frac{2\,\mathrm{arctanh}\left(\frac{2b\tan\left(\frac{x}{2}\right)-2c}{2\sqrt{b^2+c^2}}\right)A\,b^2}{\left(b^2+c^2\right)^{\frac{3}{2}}}+\frac{2\,\mathrm{arctanh}\left(\frac{2b\tan\left(\frac{x}{2}\right)-2c}{2\sqrt{b^2+c^2}}\right)A\,c^2}{\left(b^2+c^2\right)^{\frac{3}{2}}}+\frac{Cb\ln\left(1+\mathrm{ta}\right.}{b^2+c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+C*sin(x))/(b*cos(x)+c*sin(x)),x)
```

```
[Out] -1/(b^2+c^2)*b*C*ln(b*tan(1/2*x)^2-2*c*tan(1/2*x)-b)+2/(b^2+c^2)^(3/2)*arct
anh(1/2*(2*b*tan(1/2*x)-2*c)/(b^2+c^2)^(1/2))*A*b^2+2/(b^2+c^2)^(3/2)*arcta
nh(1/2*(2*b*tan(1/2*x)-2*c)/(b^2+c^2)^(1/2))*A*c^2+C/(b^2+c^2)*b*ln(1+tan(1
/2*x)^2)+2*C/(b^2+c^2)*c*arctan(tan(1/2*x))
```

maxima [B] time = 0.42, size = 153, normalized size = 2.07

$$C\left(\frac{2c\arctan\left(\frac{\sin(x)}{\cos(x)+1}\right)}{b^2+c^2}-\frac{b\log\left(-b-\frac{2c\sin(x)}{\cos(x)+1}+\frac{b\sin(x)^2}{(\cos(x)+1)^2}\right)}{b^2+c^2}+\frac{b\log\left(\frac{\sin(x)^2}{(\cos(x)+1)^2}+1\right)}{b^2+c^2}\right)-\frac{A\log\left(\frac{c-\frac{b\sin(x)}{\cos(x)+1}+\sqrt{b^2+c^2}}{c-\frac{b\sin(x)}{\cos(x)+1}-\sqrt{b^2+c^2}}\right)}{\sqrt{b^2+c^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*sin(x))/(b*cos(x)+c*sin(x)),x, algorithm="maxima")
```

```
[Out] C*(2*c*arctan(sin(x)/(cos(x) + 1))/(b^2 + c^2) - b*log(-b - 2*c*sin(x)/(cos
(x) + 1) + b*sin(x)^2/(cos(x) + 1)^2)/(b^2 + c^2) + b*log(sin(x)^2/(cos(x)
+ 1)^2 + 1)/(b^2 + c^2)) - A*log((c - b*sin(x)/(cos(x) + 1) + sqrt(b^2 + c^
2))/(c - b*sin(x)/(cos(x) + 1) - sqrt(b^2 + c^2)))/sqrt(b^2 + c^2)
```

mupad [B] time = 7.02, size = 695, normalized size = 9.39

$$-\ln\left(-32\,A\,C^2\,b^2-\frac{\left(A\,\sqrt{{\left(b^2+c^2\right)}^3}+C\,b^3+C\,b\,c^2\right)\left(64\,A^2\,b^2\,c+32\,C^2\,b^2\,c-32\,b\,\tan\left(\frac{x}{2}\right)\,\left(A^2\,b^2-A^2\,c^2-4\right.\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + C*sin(x))/(b*cos(x) + c*sin(x)),x)
```

```
[Out] (C*log(tan(x/2) + 1i))/(b - c*1i) - log(- 32*A*C^2*b^2 - ((C*b^3 - A*((b^2
+ c^2)^3)^(1/2) + C*b*c^2)*(64*A^2*b^2*c + 32*C^2*b^2*c - 32*b*tan(x/2)*(A^
2*b^2 - A^2*c^2 + 2*C^2*c^2 - 4*A*C*b*c) + 64*A*C*b^3 + ((C*b^3 - A*((b^2 +
 c^2)^3)^(1/2) + C*b*c^2)*(32*A*b^4 + 32*A*b^2*c^2 + 32*b*tan(x/2)*(2*A*c^3
 - 2*C*b^3 + 2*A*b^2*c + C*b*c^2) - 32*C*b*c^3 + 64*C*b^3*c - (96*b*c*(b +
c*tan(x/2))*(C*b^3 - A*((b^2 + c^2)^3)^(1/2) + C*b*c^2))/(b^2 + c^2)))/(b^2
 + c^2)^2))/(b^2 + c^2)^2 - 32*A^2*C*b*c - 32*C*b*tan(x/2)*(2*C^2*b - A^2*b
 + 2*A*C*c))*((C*b)/(b^2 + c^2) - (A*((b^2 + c^2)^3)^(1/2))/(b^2 + c^2)^2)
- log(- 32*A*C^2*b^2 - ((A*((b^2 + c^2)^3)^(1/2) + C*b^3 + C*b*c^2)*(64*A^2
*b^2*c + 32*C^2*b^2*c - 32*b*tan(x/2)*(A^2*b^2 - A^2*c^2 + 2*C^2*c^2 - 4*A*
C*b*c) + 64*A*C*b^3 + ((A*((b^2 + c^2)^3)^(1/2) + C*b^3 + C*b*c^2)*(32*A*b^
4 + 32*A*b^2*c^2 + 32*b*tan(x/2)*(2*A*c^3 - 2*C*b^3 + 2*A*b^2*c + C*b*c^2)
- 32*C*b*c^3 + 64*C*b^3*c - (96*b*c*(b + c*tan(x/2))*(A*((b^2 + c^2)^3)^(1/
2) + C*b^3 + C*b*c^2))/(b^2 + c^2)))/(b^2 + c^2)^2))/(b^2 + c^2)^2 - 32*A^2
*C*b*c - 32*C*b*tan(x/2)*(2*C^2*b - A^2*b + 2*A*C*c))*((C*b)/(b^2 + c^2) +
(A*((b^2 + c^2)^3)^(1/2))/(b^2 + c^2)^2) + (C*log(tan(x/2) - 1i)*1i)/(b*1i
- c)
```

sympy [A] time = 36.72, size = 641, normalized size = 8.66

$$\begin{cases} \tilde{\infty}\left(A\log\left(\tan\left(\frac{x}{2}\right)\right)+Cx\right) \\ \frac{A\log\left(\tan\left(\frac{x}{2}\right)\right)+Cx}{c} \\ \frac{2A}{-2ic\sin(x)-2c\cos(x)}-\frac{iCx\sin(x)}{-2ic\sin(x)-2c\cos(x)}-\frac{Cx\cos(x)}{-2ic\sin(x)-2c\cos(x)}+\frac{C\sin(x)}{-2ic\sin(x)-2c\cos(x)} \\ \frac{2A}{2ic\sin(x)-2c\cos(x)}+\frac{iCx\sin(x)}{2ic\sin(x)-2c\cos(x)}-\frac{Cx\cos(x)}{2ic\sin(x)-2c\cos(x)}+\frac{C\sin(x)}{2ic\sin(x)-2c\cos(x)} \\ -\frac{Ab^2\log\left(\tan\left(\frac{x}{2}\right)-\frac{c}{b}-\frac{\sqrt{b^2+c^2}}{b}\right)}{b^2\sqrt{b^2+c^2}+c^2\sqrt{b^2+c^2}}+\frac{Ab^2\log\left(\tan\left(\frac{x}{2}\right)-\frac{c}{b}+\frac{\sqrt{b^2+c^2}}{b}\right)}{b^2\sqrt{b^2+c^2}+c^2\sqrt{b^2+c^2}}-\frac{Ac^2\log\left(\tan\left(\frac{x}{2}\right)-\frac{c}{b}-\frac{\sqrt{b^2+c^2}}{b}\right)}{b^2\sqrt{b^2+c^2}+c^2\sqrt{b^2+c^2}}+\frac{Ac^2\log\left(\tan\left(\frac{x}{2}\right)-\frac{c}{b}+\frac{\sqrt{b^2+c^2}}{b}\right)}{b^2\sqrt{b^2+c^2}+c^2\sqrt{b^2+c^2}}+\frac{Cb\sqrt{b^2}}{b^2\sqrt{}} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*sin(x))/(b*cos(x)+c*sin(x)),x)
```

```
[Out] Piecewise((zoo*(A*log(tan(x/2)) + C*x), Eq(b, 0) & Eq(c, 0)), ((A*log(tan(x
/2)) + C*x)/c, Eq(b, 0)), (2*A/(-2*I*c*sin(x) - 2*c*cos(x)) - I*C*x*sin(x)/
(-2*I*c*sin(x) - 2*c*cos(x)) - C*x*cos(x)/(-2*I*c*sin(x) - 2*c*cos(x)) + C*
sin(x)/(-2*I*c*sin(x) - 2*c*cos(x)), Eq(b, -I*c)), (2*A/(2*I*c*sin(x) - 2*c
*cos(x)) + I*C*x*sin(x)/(2*I*c*sin(x) - 2*c*cos(x)) - C*x*cos(x)/(2*I*c*sin
(x) - 2*c*cos(x)) + C*sin(x)/(2*I*c*sin(x) - 2*c*cos(x)), Eq(b, I*c)), (-A*
b**2*log(tan(x/2) - c/b - sqrt(b**2 + c**2)/b)/(b**2*sqrt(b**2 + c**2) + c*
*2*sqrt(b**2 + c**2)) + A*b**2*log(tan(x/2) - c/b + sqrt(b**2 + c**2)/b)/(b
**2*sqrt(b**2 + c**2) + c**2*sqrt(b**2 + c**2)) - A*c**2*log(tan(x/2) - c/b
 - sqrt(b**2 + c**2)/b)/(b**2*sqrt(b**2 + c**2) + c**2*sqrt(b**2 + c**2)) +
 A*c**2*log(tan(x/2) - c/b + sqrt(b**2 + c**2)/b)/(b**2*sqrt(b**2 + c**2) +
 c**2*sqrt(b**2 + c**2)) + C*b*sqrt(b**2 + c**2)*log(tan(x/2)**2 + 1)/(b**2
*sqrt(b**2 + c**2) + c**2*sqrt(b**2 + c**2)) - C*b*sqrt(b**2 + c**2)*log(ta
n(x/2) - c/b - sqrt(b**2 + c**2)/b)/(b**2*sqrt(b**2 + c**2) + c**2*sqrt(b**
2 + c**2)) - C*b*sqrt(b**2 + c**2)*log(tan(x/2) - c/b + sqrt(b**2 + c**2)/b
)/(b**2*sqrt(b**2 + c**2) + c**2*sqrt(b**2 + c**2)) + C*c*x*sqrt(b**2 + c**
2)/(b**2*sqrt(b**2 + c**2) + c**2*sqrt(b**2 + c**2)), True))
```

3.350 $\int \frac{A+C\sin(x)}{(b\cos(x)+c\sin(x))^2}\,dx$

Optimal. Leaf size=75

$$\frac{Ab\sin(x)-Ac\cos(x)+bC}{\left(b^2+c^2\right)(b\cos(x)+c\sin(x))}-\frac{cC\tanh^{-1}\left(\frac{c\cos(x)-b\sin(x)}{\sqrt{b^2+c^2}}\right)}{\left(b^2+c^2\right)^{3/2}}$$

```
[Out] -c*C*arctanh((c*cos(x)-b*sin(x))/(b^2+c^2)^(1/2))/(b^2+c^2)^(3/2)+(b*C-A*c*
cos(x)+A*b*sin(x))/(b^2+c^2)/(b*cos(x)+c*sin(x))
```

Rubi [A] time = 0.06, antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3154, 3074, 206}

$$\frac{Ab\sin(x)-Ac\cos(x)+bC}{\left(b^2+c^2\right)(b\cos(x)+c\sin(x))}-\frac{cC\tanh^{-1}\left(\frac{c\cos(x)-b\sin(x)}{\sqrt{b^2+c^2}}\right)}{\left(b^2+c^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + C*Sin[x])/(b*Cos[x] + c*Sin[x])^2,x]
```

```
[Out] -((c*C*ArcTanh[(c*Cos[x] - b*Sin[x])/Sqrt[b^2 + c^2]])/(b^2 + c^2)^(3/2)) +
 (b*C - A*c*Cos[x] + A*b*Sin[x])/((b^2 + c^2)*(b*Cos[x] + c*Sin[x]))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3074

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x
_Symbol] :> -Dist[d^(-1), Subst[Int[1/(a^2 + b^2 - x^2), x], x, b*Cos[c + d
*x] - a*Sin[c + d*x]], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

Rule 3154

```
Int[((A_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])/((a_.) + cos[(d_.) + (e_.)*(x_)
]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)])^2, x_Symbol] :> -Simp[(b*C + (a*C
- c*A)*Cos[d + e*x] + b*A*Sin[d + e*x])/(e*(a^2 - b^2 - c^2)*(a + b*Cos[d +
 e*x] + c*Sin[d + e*x])), x] + Dist[(a*A - c*C)/(a^2 - b^2 - c^2), Int[1/(a
 + b*Cos[d + e*x] + c*Sin[d + e*x]), x], x] /; FreeQ[{a, b, c, d, e, A, C},
```

```
x] && NeQ[a^2 - b^2 - c^2, 0] && NeQ[a*A - c*C, 0]
```

Rubi steps

$$\int \frac{A+C\sin(x)}{(b\cos(x)+c\sin(x))^2}\,dx = \frac{bC-Ac\cos(x)+Ab\sin(x)}{\left(b^2+c^2\right)(b\cos(x)+c\sin(x))}+\frac{(cC)\int\frac{1}{b\cos(x)+c\sin(x)}\,dx}{b^2+c^2}$$

$$= \frac{bC-Ac\cos(x)+Ab\sin(x)}{\left(b^2+c^2\right)(b\cos(x)+c\sin(x))}-\frac{(cC)\,\text{Subst}\left(\int\frac{1}{b^2+c^2-x^2}\,dx,x,c\cos(x)-b\sin(x)\right)}{b^2+c^2}$$

$$= -\frac{cC\tanh^{-1}\left(\frac{c\cos(x)-b\sin(x)}{\sqrt{b^2+c^2}}\right)}{\left(b^2+c^2\right)^{3/2}}+\frac{bC-Ac\cos(x)+Ab\sin(x)}{\left(b^2+c^2\right)(b\cos(x)+c\sin(x))}$$

Mathematica [A] time = 0.31, size = 82, normalized size = 1.09

$$\frac{A\left(b^2+c^2\right)\sin(x)+b^2C}{b\left(b^2+c^2\right)(b\cos(x)+c\sin(x))}+\frac{2cC\tanh^{-1}\left(\frac{b\tan\left(\frac{x}{2}\right)-c}{\sqrt{b^2+c^2}}\right)}{\left(b^2+c^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + C*Sin[x])/(b*Cos[x] + c*Sin[x])^2,x]
```

```
[Out] (2*c*C*ArcTanh[(-c + b*Tan[x/2])/Sqrt[b^2 + c^2]])/(b^2 + c^2)^(3/2) + (b^2
*C + A*(b^2 + c^2)*Sin[x])/(b*(b^2 + c^2)*(b*Cos[x] + c*Sin[x]))
```

fricas [B] time = 1.97, size = 200, normalized size = 2.67

$$\frac{2\,Cb^3+2\,Cbc^2+\left(Cbc\cos(x)+Cc^2\sin(x)\right)\sqrt{b^2+c^2}\,\log\left(-\frac{2\,bc\cos(x)\sin(x)+\left(b^2-c^2\right)\cos(x)^2-2\,b^2-c^2+2\,\sqrt{b^2+c^2}\,(c\cos(x)-b\sin(x}{2\,bc\cos(x)\sin(x)+\left(b^2-c^2\right)\cos(x)^2+c^2}\right.}{2\left(\left(b^5+2\,b^3c^2+bc^4\right)\cos(x)+\left(b^4c+2\,b^2c^3+c^5\right)\sin(x)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*sin(x))/(b*cos(x)+c*sin(x))^2,x, algorithm="fricas")
```

```
[Out] 1/2*(2*C*b^3 + 2*C*b*c^2 + (C*b*c*cos(x) + C*c^2*sin(x))*sqrt(b^2 + c^2)*lo
g(-(2*b*c*cos(x)*sin(x) + (b^2 - c^2)*cos(x)^2 - 2*b^2 - c^2 + 2*sqrt(b^2 +
 c^2)*(c*cos(x) - b*sin(x)))/(2*b*c*cos(x)*sin(x) + (b^2 - c^2)*cos(x)^2 +
```

```
c^2)) - 2*(A*b^2*c + A*c^3)*cos(x) + 2*(A*b^3 + A*b*c^2)*sin(x))/((b^5 + 2*
b^3*c^2 + b*c^4)*cos(x) + (b^4*c + 2*b^2*c^3 + c^5)*sin(x))
```

giac [A] time = 0.19, size = 130, normalized size = 1.73

$$-\frac{Cc\log\left(\frac{\left|2\,b\tan\left(\frac{1}{2}\,x\right)-2\,c-2\,\sqrt{b^2+c^2}\right|}{\left|2\,b\tan\left(\frac{1}{2}\,x\right)-2\,c+2\,\sqrt{b^2+c^2}\right|}\right)}{\left(b^2+c^2\right)^{\frac{3}{2}}}-\frac{2\left(Ab^2\tan\left(\frac{1}{2}\,x\right)+Cbc\tan\left(\frac{1}{2}\,x\right)+Ac^2\tan\left(\frac{1}{2}\,x\right)+Cb^2\right)}{\left(b^3+bc^2\right)\left(b\tan\left(\frac{1}{2}\,x\right)^2-2\,c\tan\left(\frac{1}{2}\,x\right)-b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*sin(x))/(b*cos(x)+c*sin(x))^2,x, algorithm="giac")
```

```
[Out] -C*c*log(abs(2*b*tan(1/2*x) - 2*c - 2*sqrt(b^2 + c^2))/abs(2*b*tan(1/2*x) -
 2*c + 2*sqrt(b^2 + c^2)))/(b^2 + c^2)^(3/2) - 2*(A*b^2*tan(1/2*x) + C*b*c*
tan(1/2*x) + A*c^2*tan(1/2*x) + C*b^2)/((b^3 + b*c^2)*(b*tan(1/2*x)^2 - 2*c
*tan(1/2*x) - b))
```

maple [A] time = 0.17, size = 108, normalized size = 1.44

$$\frac{-\frac{2\left(A\,b^2+A\,c^2+Cbc\right)\tan\left(\frac{x}{2}\right)}{b\left(b^2+c^2\right)}-\frac{2Cb}{b^2+c^2}}{b\left(\tan^2\left(\frac{x}{2}\right)\right)-2c\tan\left(\frac{x}{2}\right)-b}+\frac{2Cc\,\mathrm{arctanh}\left(\frac{2b\tan\left(\frac{x}{2}\right)-2c}{2\sqrt{b^2+c^2}}\right)}{\left(b^2+c^2\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+C*sin(x))/(b*cos(x)+c*sin(x))^2,x)
```

```
[Out] 2*(-(A*b^2+A*c^2+C*b*c)/b/(b^2+c^2)*tan(1/2*x)-C*b/(b^2+c^2))/(b*tan(1/2*x)
^2-2*c*tan(1/2*x)-b)+2*C*c/(b^2+c^2)^(3/2)*arctanh(1/2*(2*b*tan(1/2*x)-2*c)
/(b^2+c^2)^(1/2))
```

maxima [A] time = 0.42, size = 146, normalized size = 1.95

$$-C\left(\frac{c\log\left(\frac{c-\frac{b\sin(x)}{\cos(x)+1}+\sqrt{b^2+c^2}}{c-\frac{b\sin(x)}{\cos(x)+1}-\sqrt{b^2+c^2}}\right)}{\left(b^2+c^2\right)^{\frac{3}{2}}}-\frac{2\left(b+\frac{c\sin(x)}{\cos(x)+1}\right)}{b^3+bc^2+\frac{2\left(b^2c+c^3\right)\sin(x)}{\cos(x)+1}-\frac{\left(b^3+bc^2\right)\sin(x)^2}{\left(\cos(x)+1\right)^2}}\right)-\frac{A}{c^2\tan(x)+bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*sin(x))/(b*cos(x)+c*sin(x))^2,x, algorithm="maxima")
```

```
[Out] -C*(c*log((c - b*sin(x)/(cos(x) + 1) + sqrt(b^2 + c^2))/(c - b*sin(x)/(cos(
x) + 1) - sqrt(b^2 + c^2)))/(b^2 + c^2)^(3/2) - 2*(b + c*sin(x)/(cos(x) + 1
))/(b^3 + b*c^2 + 2*(b^2*c + c^3)*sin(x)/(cos(x) + 1) - (b^3 + b*c^2)*sin(x
)^2/(cos(x) + 1)^2)) - A/(c^2*tan(x) + b*c)
```

mupad [B] time = 2.58, size = 105, normalized size = 1.40

$$\frac{\frac{2\,C\,b}{b^2+c^2}+\frac{2\tan\left(\frac{x}{2}\right)\left(A\,b^2+C\,b\,c+A\,c^2\right)}{b\left(b^2+c^2\right)}}{-b\tan\left(\frac{x}{2}\right)^2+2\,c\tan\left(\frac{x}{2}\right)+b}-\frac{2\,C\,c\,\mathrm{atanh}\left(\frac{2\,c-2\,b\tan\left(\frac{x}{2}\right)}{2\,\sqrt{b^2+c^2}}\right)}{\left(b^2+c^2\right)^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + C*sin(x))/(b*cos(x) + c*sin(x))^2,x)
```

```
[Out] ((2*C*b)/(b^2 + c^2) + (2*tan(x/2)*(A*b^2 + A*c^2 + C*b*c))/(b*(b^2 + c^2))
)/(b + 2*c*tan(x/2) - b*tan(x/2)^2) - (2*C*c*atanh((2*c - 2*b*tan(x/2))/(2*
(b^2 + c^2)^(1/2))))/(b^2 + c^2)^(3/2)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*sin(x))/(b*cos(x)+c*sin(x))**2,x)
```

```
[Out] Timed out
```

3.351 $\int \frac{A+C\sin(x)}{(b\cos(x)+c\sin(x))^3}\,dx$

Optimal. Leaf size=116

$$\frac{Ab\sin(x)-Ac\cos(x)+bC}{2\left(b^2+c^2\right)(b\cos(x)+c\sin(x))^2}-\frac{A\tanh^{-1}\left(\frac{c\cos(x)-b\sin(x)}{\sqrt{b^2+c^2}}\right)}{2\left(b^2+c^2\right)^{3/2}}-\frac{c^2C\cos(x)-bcC\sin(x)}{\left(b^2+c^2\right)^2(b\cos(x)+c\sin(x))}$$

```
[Out] -1/2*A*arctanh((c*cos(x)-b*sin(x))/(b^2+c^2)^(1/2))/(b^2+c^2)^(3/2)+1/2*(b*
C-A*c*cos(x)+A*b*sin(x))/(b^2+c^2)/(b*cos(x)+c*sin(x))^2+(-c^2*C*cos(x)+b*c
*C*sin(x))/(b^2+c^2)^2/(b*cos(x)+c*sin(x))
```

Rubi [A] time = 0.11, antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {3157, 3153, 3074, 206}

$$\frac{Ab\sin(x)-Ac\cos(x)+bC}{2\left(b^2+c^2\right)(b\cos(x)+c\sin(x))^2}-\frac{A\tanh^{-1}\left(\frac{c\cos(x)-b\sin(x)}{\sqrt{b^2+c^2}}\right)}{2\left(b^2+c^2\right)^{3/2}}-\frac{c^2C\cos(x)-bcC\sin(x)}{\left(b^2+c^2\right)^2(b\cos(x)+c\sin(x))}$$

Antiderivative was successfully verified.

```
[In] Int[(A + C*Sin[x])/(b*Cos[x] + c*Sin[x])^3,x]
```

```
[Out] -(A*ArcTanh[(c*Cos[x] - b*Sin[x])/Sqrt[b^2 + c^2]])/(2*(b^2 + c^2)^(3/2)) +
 (b*C - A*c*Cos[x] + A*b*Sin[x])/(2*(b^2 + c^2)*(b*Cos[x] + c*Sin[x])^2) -
(c^2*C*Cos[x] - b*c*C*Sin[x])/((b^2 + c^2)^2*(b*Cos[x] + c*Sin[x]))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3074

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x
_Symbol] :> -Dist[d^(-1), Subst[Int[1/(a^2 + b^2 - x^2), x], x, b*Cos[c + d
*x] - a*Sin[c + d*x]], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

Rule 3153

```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])
/((a_.) + cos[(d_.) + (e_.)*(x_)]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)])^2,
 x_Symbol] :> Simp[(c*B - b*C - (a*C - c*A)*Cos[d + e*x] + (a*B - b*A)*Sin[
```

```
d + e*x])/(e*(a^2 - b^2 - c^2)*(a + b*Cos[d + e*x] + c*Sin[d + e*x])), x] +
 Dist[(a*A - b*B - c*C)/(a^2 - b^2 - c^2), Int[1/(a + b*Cos[d + e*x] + c*Si
n[d + e*x]), x], x] /; FreeQ[{a, b, c, d, e, A, B, C}, x] && NeQ[a^2 - b^2
- c^2, 0] && NeQ[a*A - b*B - c*C, 0]
```

Rule 3157

```
Int[((a_.) + cos[(d_.) + (e_.)*(x_)]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)])
^(n_)*((A_.) + (C_.)*sin[(d_.) + (e_.)*(x_)]), x_Symbol] :> Simp[((b*C + (a
*C - c*A)*Cos[d + e*x] + b*A*Sin[d + e*x])*(a + b*Cos[d + e*x] + c*Sin[d +
e*x])^(n + 1))/(e*(n + 1)*(a^2 - b^2 - c^2)), x] + Dist[1/((n + 1)*(a^2 - b
^2 - c^2)), Int[(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n + 1)*Simp[(n + 1)*
(a*A - c*C) - (n + 2)*b*A*Cos[d + e*x] + (n + 2)*(a*C - c*A)*Sin[d + e*x],
x], x], x] /; FreeQ[{a, b, c, d, e, A, C}, x] && LtQ[n, -1] && NeQ[a^2 - b^
2 - c^2, 0] && NeQ[n, -2]
```

Rubi steps

$$\int \frac{A+C\sin(x)}{(b\cos(x)+c\sin(x))^3}\,dx = \frac{bC-Ac\cos(x)+Ab\sin(x)}{2\left(b^2+c^2\right)(b\cos(x)+c\sin(x))^2} + \frac{\int \frac{2cC+Ab\cos(x)+Ac\sin(x)}{(b\cos(x)+c\sin(x))^2}\,dx}{2\left(b^2+c^2\right)}$$

$$= \frac{bC-Ac\cos(x)+Ab\sin(x)}{2\left(b^2+c^2\right)(b\cos(x)+c\sin(x))^2} - \frac{c^2C\cos(x)-bcC\sin(x)}{\left(b^2+c^2\right)^2(b\cos(x)+c\sin(x))} + \frac{A\int \frac{1}{b\cos(x)+c\,s}}{2\left(b^2+c\right.}$$

$$= \frac{bC-Ac\cos(x)+Ab\sin(x)}{2\left(b^2+c^2\right)(b\cos(x)+c\sin(x))^2} - \frac{c^2C\cos(x)-bcC\sin(x)}{\left(b^2+c^2\right)^2(b\cos(x)+c\sin(x))} - \frac{A\,\mathrm{Subst}\left(\int \frac{}{b^2}\right.}{}$$

$$= -\frac{A\tanh^{-1}\left(\frac{c\cos(x)-b\sin(x)}{\sqrt{b^2+c^2}}\right)}{2\left(b^2+c^2\right)^{3/2}} + \frac{bC-Ac\cos(x)+Ab\sin(x)}{2\left(b^2+c^2\right)(b\cos(x)+c\sin(x))^2} - \frac{c^2C\cos(x)-b}{\left(b^2+c^2\right)^2(b\cos(}$$

Mathematica [C] time = 0.39, size = 132, normalized size = 1.14

$$\frac{\left(b^2+c^2\right)\left(Ab^2\sin(x)-Abc\cos(x)+bC(b+c\sin(2x))+2c^2C\sin^2(x)\right)+2Ab\sqrt{b^2+c^2}\,(b\cos(x)+c\sin(x))^2\tanh}{2b(b-ic)^2(b+ic)^2(b\cos(x)+c\sin(x))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + C*Sin[x])/(b*Cos[x] + c*Sin[x])^3,x]
```

```
[Out] (2*A*b*Sqrt[b^2 + c^2]*ArcTanh[(-c + b*Tan[x/2])/Sqrt[b^2 + c^2]]*(b*Cos[x]
 + c*Sin[x])^2 + (b^2 + c^2)*(-(A*b*c*Cos[x]) + A*b^2*Sin[x] + 2*c^2*C*Sin[
x]^2 + b*C*(b + c*Sin[2*x])))/(2*b*(b - I*c)^2*(b + I*c)^2*(b*Cos[x] + c*Si
n[x])^2)
```

fricas [B] time = 0.92, size = 279, normalized size = 2.41

$$-\frac{8\,Cbc^2\cos(x)^2 - 2\,Cb^3 - 6\,Cbc^2 - \left(2\,Abc\cos(x)\sin(x) + Ac^2 + \left(Ab^2 - Ac^2\right)\cos(x)^2\right)\sqrt{b^2+c^2}\,\log\left(-\frac{2\,bc\cos(x)}{}\right.}{4\left(b^4c^2 + 2\,b^2c^4 + c^6 + \left(b^6 + b^4c^2 - b^2c^4\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*sin(x))/(b*cos(x)+c*sin(x))^3,x, algorithm="fricas")
```

```
[Out] -1/4*(8*C*b*c^2*cos(x)^2 - 2*C*b^3 - 6*C*b*c^2 - (2*A*b*c*cos(x)*sin(x) + A
*c^2 + (A*b^2 - A*c^2)*cos(x)^2)*sqrt(b^2 + c^2)*log(-(2*b*c*cos(x)*sin(x)
+ (b^2 - c^2)*cos(x)^2 - 2*b^2 - c^2 + 2*sqrt(b^2 + c^2)*(c*cos(x) - b*sin(
x)))/(2*b*c*cos(x)*sin(x) + (b^2 - c^2)*cos(x)^2 + c^2)) + 2*(A*b^2*c + A*c
^3)*cos(x) - 2*(A*b^3 + A*b*c^2 + 2*(C*b^2*c - C*c^3)*cos(x))*sin(x))/(b^4*
c^2 + 2*b^2*c^4 + c^6 + (b^6 + b^4*c^2 - b^2*c^4 - c^6)*cos(x)^2 + 2*(b^5*c
 + 2*b^3*c^3 + b*c^5)*cos(x)*sin(x))
```

giac [A] time = 0.24, size = 199, normalized size = 1.72

$$\frac{A\log\left(\frac{\left|-2\,b\tan\left(\frac{1}{2}\,x\right)+2\,c-2\,\sqrt{b^2+c^2}\right|}{\left|-2\,b\tan\left(\frac{1}{2}\,x\right)+2\,c+2\,\sqrt{b^2+c^2}\right|}\right)}{2\left(b^2+c^2\right)^{\frac{3}{2}}} + \frac{Ab^3\tan\left(\frac{1}{2}\,x\right)^3 + 2\,Abc^2\tan\left(\frac{1}{2}\,x\right)^3 + 2\,Cb^3\tan\left(\frac{1}{2}\,x\right)^2 + Ab^2c\tan\left(\frac{1}{2}\,x\right)^2 + 2\,C}{\left(b^4+b^2c^2\right)\left(b\tan\left(\frac{1}{2}\,x\right)\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*sin(x))/(b*cos(x)+c*sin(x))^3,x, algorithm="giac")
```

```
[Out] 1/2*A*log(abs(-2*b*tan(1/2*x) + 2*c - 2*sqrt(b^2 + c^2))/abs(-2*b*tan(1/2*x
) + 2*c + 2*sqrt(b^2 + c^2)))/(b^2 + c^2)^(3/2) + (A*b^3*tan(1/2*x)^3 + 2*A
*b*c^2*tan(1/2*x)^3 + 2*C*b^3*tan(1/2*x)^2 + A*b^2*c*tan(1/2*x)^2 + 2*C*b*c
^2*tan(1/2*x)^2 - 2*A*c^3*tan(1/2*x)^2 + A*b^3*tan(1/2*x) - 2*A*b*c^2*tan(1
/2*x) - A*b^2*c)/((b^4 + b^2*c^2)*(b*tan(1/2*x)^2 - 2*c*tan(1/2*x) - b)^2)
```

maple [A] time = 0.19, size = 177, normalized size = 1.53

$$-\frac{2\left(-\frac{A\left(b^2+2c^2\right)\left(\tan^3\left(\frac{x}{2}\right)\right)}{2\left(b^2+c^2\right)b} - \frac{\left(A\,b^2c-2A\,c^3+2C\,b^3+2Cb\,c^2\right)\left(\tan^2\left(\frac{x}{2}\right)\right)}{2\left(b^2+c^2\right)b^2} - \frac{A\left(b^2-2c^2\right)\tan\left(\frac{x}{2}\right)}{2\left(b^2+c^2\right)b} + \frac{Ac}{2b^2+2c^2}\right)}{\left(b\left(\tan^2\left(\frac{x}{2}\right)\right) - 2c\tan\left(\frac{x}{2}\right) - b\right)^2} + \frac{A\,\mathrm{arctanh}\left(\frac{2b\tan\left(\frac{x}{2}\right)-2c}{2\sqrt{b^2+c^2}}\right)}{\left(b^2+c^2\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+C*sin(x))/(b*cos(x)+c*sin(x))^3,x)
```

```
[Out] -2*(-1/2*A*(b^2+2*c^2)/(b^2+c^2)/b*tan(1/2*x)^3-1/2*(A*b^2*c-2*A*c^3+2*C*b^
3+2*C*b*c^2)/(b^2+c^2)/b^2*tan(1/2*x)^2-1/2*A*(b^2-2*c^2)/(b^2+c^2)/b*tan(1
/2*x)+1/2*A*c/(b^2+c^2))/(b*tan(1/2*x)^2-2*c*tan(1/2*x)-b)^2+A/(b^2+c^2)^(3
/2)*arctanh(1/2*(2*b*tan(1/2*x)-2*c)/(b^2+c^2)^(1/2))
```

maxima [B] time = 0.44, size = 338, normalized size = 2.91

$$-\frac{1}{2}A\left(\frac{2\left(b^2c-\frac{(b^3-2bc^2)\sin(x)}{\cos(x)+1}-\frac{(b^2c-2c^3)\sin(x)^2}{(\cos(x)+1)^2}-\frac{(b^3+2bc^2)\sin(x)^3}{(\cos(x)+1)^3}\right)}{b^6+b^4c^2+\frac{4(b^5c+b^3c^3)\sin(x)}{\cos(x)+1}-\frac{2(b^6-b^4c^2-2b^2c^4)\sin(x)^2}{(\cos(x)+1)^2}-\frac{4(b^5c+b^3c^3)\sin(x)^3}{(\cos(x)+1)^3}+\frac{(b^6+b^4c^2)\sin(x)^4}{(\cos(x)+1)^4}}+\frac{\log\left(\frac{c-\frac{b\sin(x)}{\cos(x)+1}+\sqrt{b^2+\cdots}}{c-\frac{b\sin(x)}{\cos(x)+1}-\sqrt{b^2+\cdots}}\right)}{(b^2+c^2)^{\frac{3}{2}}}\cdots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*sin(x))/(b*cos(x)+c*sin(x))^3,x, algorithm="maxima")
```

```
[Out] -1/2*A*(2*(b^2*c - (b^3 - 2*b*c^2)*sin(x)/(cos(x) + 1) - (b^2*c - 2*c^3)*si
n(x)^2/(cos(x) + 1)^2 - (b^3 + 2*b*c^2)*sin(x)^3/(cos(x) + 1)^3)/(b^6 + b^4
*c^2 + 4*(b^5*c + b^3*c^3)*sin(x)/(cos(x) + 1) - 2*(b^6 - b^4*c^2 - 2*b^2*c
^4)*sin(x)^2/(cos(x) + 1)^2 - 4*(b^5*c + b^3*c^3)*sin(x)^3/(cos(x) + 1)^3 +
 (b^6 + b^4*c^2)*sin(x)^4/(cos(x) + 1)^4) + log((c - b*sin(x)/(cos(x) + 1)
+ sqrt(b^2 + c^2))/(c - b*sin(x)/(cos(x) + 1) - sqrt(b^2 + c^2)))/(b^2 + c^
2)^(3/2)) + 2*C*sin(x)^2/((b^3 + 4*b^2*c*sin(x)/(cos(x) + 1) - 4*b^2*c*sin(
x)^3/(cos(x) + 1)^3 + b^3*sin(x)^4/(cos(x) + 1)^4 - 2*(b^3 - 2*b*c^2)*sin(x
)^2/(cos(x) + 1)^2)*(cos(x) + 1)^2)
```

mupad [B] time = 2.86, size = 227, normalized size = 1.96

$$\frac{\frac{\tan\left(\frac{x}{2}\right)^3(A\,b^2+2\,A\,c^2)}{b\,(b^2+c^2)}-\frac{A\,c}{b^2+c^2}+\frac{\tan\left(\frac{x}{2}\right)^2(2\,C\,b^3+A\,b^2\,c+2\,C\,b\,c^2-2\,A\,c^3)}{b^2\,(b^2+c^2)}+\frac{\tan\left(\frac{x}{2}\right)(A\,b^2-2\,A\,c^2)}{b\,(b^2+c^2)}}{b^2-\tan\left(\frac{x}{2}\right)^2(2\,b^2-4\,c^2)+b^2\tan\left(\frac{x}{2}\right)^4+4\,b\,c\tan\left(\frac{x}{2}\right)-4\,b\,c\tan\left(\frac{x}{2}\right)^3}+\frac{A\,\mathrm{atan}\left(\frac{b^2\,c\,1\mathrm{i}+c^3\,1\mathrm{i}-b\tan\left(\frac{x}{2}\right)(b^2+c^2)\,1\mathrm{i}\cdots}{(b^2+c^2)^{3/2}}\right)}{(b^2+c^2)^{3/2}}\cdots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + C*sin(x))/(b*cos(x) + c*sin(x))^3,x)
```

```
[Out] ((tan(x/2)^3*(A*b^2 + 2*A*c^2))/(b*(b^2 + c^2)) - (A*c)/(b^2 + c^2) + (tan(
x/2)^2*(2*C*b^3 - 2*A*c^3 + A*b^2*c + 2*C*b*c^2))/(b^2*(b^2 + c^2)) + (tan(
x/2)*(A*b^2 - 2*A*c^2))/(b*(b^2 + c^2)))/(b^2 - tan(x/2)^2*(2*b^2 - 4*c^2)
+ b^2*tan(x/2)^4 + 4*b*c*tan(x/2) - 4*b*c*tan(x/2)^3) + (A*atan((b^2*c*1i +
```

```
 c^3*1i - b*tan(x/2)*(b^2 + c^2)*1i)/(b^2 + c^2)^(3/2))*1i)/(b^2 + c^2)^(3/
2)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*sin(x))/(b*cos(x)+c*sin(x))**3,x)
[Out] Timed out
```

3.352 $\int \frac{A+B\cos(x)}{b\cos(x)+c\sin(x)}\,dx$

Optimal. Leaf size=73

$$-\frac{A\tanh^{-1}\left(\frac{c\cos(x)-b\sin(x)}{\sqrt{b^2+c^2}}\right)}{\sqrt{b^2+c^2}}+\frac{bBx}{b^2+c^2}+\frac{Bc\log(b\cos(x)+c\sin(x))}{b^2+c^2}$$

```
[Out] b*B*x/(b^2+c^2)+B*c*ln(b*cos(x)+c*sin(x))/(b^2+c^2)-A*arctanh((c*cos(x)-b*s
in(x))/(b^2+c^2)^(1/2))/(b^2+c^2)^(1/2)
```

Rubi [A] time = 0.05, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3138, 3074, 206}

$$-\frac{A\tanh^{-1}\left(\frac{c\cos(x)-b\sin(x)}{\sqrt{b^2+c^2}}\right)}{\sqrt{b^2+c^2}}+\frac{bBx}{b^2+c^2}+\frac{Bc\log(b\cos(x)+c\sin(x))}{b^2+c^2}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cos[x])/(b*Cos[x] + c*Sin[x]),x]
```

```
[Out] (b*B*x)/(b^2 + c^2) - (A*ArcTanh[(c*Cos[x] - b*Sin[x])/Sqrt[b^2 + c^2]])/Sq
rt[b^2 + c^2] + (B*c*Log[b*Cos[x] + c*Sin[x]])/(b^2 + c^2)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3074

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x
_Symbol] :> -Dist[d^(-1), Subst[Int[1/(a^2 + b^2 - x^2), x], x, b*Cos[c + d
*x] - a*Sin[c + d*x]], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

Rule 3138

```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.))/((a_.) + cos[(d_.) + (e_.)*(x_)
]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)]), x_Symbol] :> Simp[(b*B*(d + e*x))
/(e*(b^2 + c^2)), x] + (Dist[(A*(b^2 + c^2) - a*b*B)/(b^2 + c^2), Int[1/(a
+ b*Cos[d + e*x] + c*Sin[d + e*x]), x], x] + Simp[(c*B*Log[a + b*Cos[d + e*
x] + c*Sin[d + e*x]])/(e*(b^2 + c^2)), x]) /; FreeQ[{a, b, c, d, e, A, B},
```

```
x] && NeQ[b^2 + c^2, 0] && NeQ[A*(b^2 + c^2) - a*b*B, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{A+B\cos(x)}{b\cos(x)+c\sin(x)}\,dx &= \frac{bBx}{b^2+c^2}+\frac{Bc\log(b\cos(x)+c\sin(x))}{b^2+c^2}+A\int \frac{1}{b\cos(x)+c\sin(x)}\,dx\\ &= \frac{bBx}{b^2+c^2}+\frac{Bc\log(b\cos(x)+c\sin(x))}{b^2+c^2}-A\,\text{Subst}\left(\int \frac{1}{b^2+c^2-x^2}\,dx,x,c\cos(x)-b\,s\right.\\ &= \frac{bBx}{b^2+c^2}-\frac{A\tanh^{-1}\left(\frac{c\cos(x)-b\sin(x)}{\sqrt{b^2+c^2}}\right)}{\sqrt{b^2+c^2}}+\frac{Bc\log(b\cos(x)+c\sin(x))}{b^2+c^2}\end{aligned}$$

Mathematica [A] time = 0.14, size = 67, normalized size = 0.92

$$\frac{2A\tanh^{-1}\left(\frac{b\tan\left(\frac{x}{2}\right)-c}{\sqrt{b^2+c^2}}\right)}{\sqrt{b^2+c^2}}+\frac{B(c\log(b\cos(x)+c\sin(x))+bx)}{b^2+c^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Cos[x])/(b*Cos[x] + c*Sin[x]),x]
```

```
[Out] (2*A*ArcTanh[(-c + b*Tan[x/2])/Sqrt[b^2 + c^2]])/Sqrt[b^2 + c^2] + (B*(b*x
+ c*Log[b*Cos[x] + c*Sin[x]]))/(b^2 + c^2)
```

fricas [B] time = 1.66, size = 143, normalized size = 1.96

$$\frac{2\,Bbx+Bc\log\left(2\,bc\cos(x)\sin(x)+\left(b^2-c^2\right)\cos(x)^2+c^2\right)+\sqrt{b^2+c^2}\,A\log\left(-\frac{2\,bc\cos(x)\sin(x)+\left(b^2-c^2\right)\cos(x)^2-2\,b^2-c}{2\,bc\cos(x)\sin(x)+\left(b^2-c\right.}\right.}{2\left(b^2+c^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(x))/(b*cos(x)+c*sin(x)),x, algorithm="fricas")
```

```
[Out] 1/2*(2*B*b*x + B*c*log(2*b*c*cos(x)*sin(x) + (b^2 - c^2)*cos(x)^2 + c^2) +
sqrt(b^2 + c^2)*A*log(-(2*b*c*cos(x)*sin(x) + (b^2 - c^2)*cos(x)^2 - 2*b^2
- c^2 + 2*sqrt(b^2 + c^2)*(c*cos(x) - b*sin(x)))/(2*b*c*cos(x)*sin(x) + (b^
2 - c^2)*cos(x)^2 + c^2)))/(b^2 + c^2)
```

giac [A] time = 0.23, size = 131, normalized size = 1.79

$$\frac{Bbx}{b^2+c^2} - \frac{Bc\log\left(\tan\left(\frac{1}{2}x\right)^2+1\right)}{b^2+c^2} + \frac{Bc\log\left(\left|b\tan\left(\frac{1}{2}x\right)^2-2c\tan\left(\frac{1}{2}x\right)-b\right|\right)}{b^2+c^2} - \frac{A\log\left(\frac{\left|2b\tan\left(\frac{1}{2}x\right)-2c-2\sqrt{b^2+c^2}\right|}{\left|2b\tan\left(\frac{1}{2}x\right)-2c+2\sqrt{b^2+c^2}\right|}\right)}{\sqrt{b^2+c^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(x))/(b*cos(x)+c*sin(x)),x, algorithm="giac")
```

```
[Out] B*b*x/(b^2 + c^2) - B*c*log(tan(1/2*x)^2 + 1)/(b^2 + c^2) + B*c*log(abs(b*t
an(1/2*x)^2 - 2*c*tan(1/2*x) - b))/(b^2 + c^2) - A*log(abs(2*b*tan(1/2*x) -
 2*c - 2*sqrt(b^2 + c^2))/abs(2*b*tan(1/2*x) - 2*c + 2*sqrt(b^2 + c^2)))/sq
rt(b^2 + c^2)
```

maple [B] time = 0.13, size = 150, normalized size = 2.05

$$\frac{Bc\ln\left(b\left(\tan^2\left(\frac{x}{2}\right)\right)-2c\tan\left(\frac{x}{2}\right)-b\right)}{b^2+c^2} + \frac{2\,\mathrm{arctanh}\left(\frac{2b\tan\left(\frac{x}{2}\right)-2c}{2\sqrt{b^2+c^2}}\right)A\,b^2}{\left(b^2+c^2\right)^{\frac{3}{2}}} + \frac{2\,\mathrm{arctanh}\left(\frac{2b\tan\left(\frac{x}{2}\right)-2c}{2\sqrt{b^2+c^2}}\right)A\,c^2}{\left(b^2+c^2\right)^{\frac{3}{2}}} - \frac{Bc\ln\left(1+\tan^2\right.}{b^2+c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(x))/(b*cos(x)+c*sin(x)),x)
```

```
[Out] 1/(b^2+c^2)*B*c*ln(b*tan(1/2*x)^2-2*c*tan(1/2*x)-b)+2/(b^2+c^2)^(3/2)*arcta
nh(1/2*(2*b*tan(1/2*x)-2*c)/(b^2+c^2)^(1/2))*A*b^2+2/(b^2+c^2)^(3/2)*arctan
h(1/2*(2*b*tan(1/2*x)-2*c)/(b^2+c^2)^(1/2))*A*c^2-B/(b^2+c^2)*c*ln(1+tan(1/
2*x)^2)+2*B/(b^2+c^2)*b*arctan(tan(1/2*x))
```

maxima [B] time = 0.42, size = 153, normalized size = 2.10

$$B\left(\frac{2\,b\arctan\left(\frac{\sin(x)}{\cos(x)+1}\right)}{b^2+c^2} + \frac{c\log\left(-b-\frac{2\,c\sin(x)}{\cos(x)+1}+\frac{b\sin(x)^2}{(\cos(x)+1)^2}\right)}{b^2+c^2} - \frac{c\log\left(\frac{\sin(x)^2}{(\cos(x)+1)^2}+1\right)}{b^2+c^2}\right) - \frac{A\log\left(\frac{c-\frac{b\sin(x)}{\cos(x)+1}+\sqrt{b^2+c^2}}{c-\frac{b\sin(x)}{\cos(x)+1}-\sqrt{b^2+c^2}}\right)}{\sqrt{b^2+c^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(x))/(b*cos(x)+c*sin(x)),x, algorithm="maxima")
```

```
[Out] B*(2*b*arctan(sin(x)/(cos(x) + 1))/(b^2 + c^2) + c*log(-b - 2*c*sin(x)/(cos
(x) + 1) + b*sin(x)^2/(cos(x) + 1)^2)/(b^2 + c^2) - c*log(sin(x)^2/(cos(x)
+ 1)^2 + 1)/(b^2 + c^2)) - A*log((c - b*sin(x)/(cos(x) + 1) + sqrt(b^2 + c^
2))/(c - b*sin(x)/(cos(x) + 1) - sqrt(b^2 + c^2)))/sqrt(b^2 + c^2)
```

mupad [B] time = 6.53, size = 692, normalized size = 9.48

$$\ln\left(32\,A^2\,B\,b^2-32\,A\,B^2\,b^2-\frac{\left(A\,\sqrt{{\left(b^2+c^2\right)}^3}+B\,c^3+B\,b^2\,c\right)\left(32\,b\,\tan\left(\frac{x}{2}\right)\,\left(A^2\,b^2-A^2\,c^2+4\,A\,B\,c^2+B^2\,b^2-\right.\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cos(x))/(b*cos(x) + c*sin(x)),x)
```

```
[Out] log(32*A^2*B*b^2 - 32*A*B^2*b^2 - ((A*((b^2 + c^2)^3)^(1/2) + B*c^3 + B*b^2
*c)*(32*b*tan(x/2)*(A^2*b^2 - A^2*c^2 + B^2*b^2 - 3*B^2*c^2 + 4*A*B*c^2) -
64*A^2*b^2*c - 32*B^2*b^2*c + ((A*((b^2 + c^2)^3)^(1/2) + B*c^3 + B*b^2*c)*
(32*A*b^4 + 32*B*b^4 + 32*A*b^2*c^2 - 64*B*b^2*c^2 + 32*b*c*tan(x/2)*(2*A*b
^2 + 2*A*c^2 + 4*B*b^2 + B*c^2) + (96*b*c*(b + c*tan(x/2))*(A*((b^2 + c^2)^
3)^(1/2) + B*c^3 + B*b^2*c))/(b^2 + c^2)))/(b^2 + c^2)^2 + 64*A*B*b^2*c))/(
b^2 + c^2)^2 + 32*B*b*c*tan(x/2)*(A - B)^2)*((B*c)/(b^2 + c^2) + (A*((b^2 +
 c^2)^3)^(1/2))/(b^2 + c^2)^2) + log(32*A^2*B*b^2 - 32*A*B^2*b^2 - ((B*c^3
- A*((b^2 + c^2)^3)^(1/2) + B*b^2*c)*(32*b*tan(x/2)*(A^2*b^2 - A^2*c^2 + B^
2*b^2 - 3*B^2*c^2 + 4*A*B*c^2) - 64*A^2*b^2*c - 32*B^2*b^2*c + ((B*c^3 - A*
((b^2 + c^2)^3)^(1/2) + B*b^2*c)*(32*A*b^4 + 32*B*b^4 + 32*A*b^2*c^2 - 64*B
*b^2*c^2 + 32*b*c*tan(x/2)*(2*A*b^2 + 2*A*c^2 + 4*B*b^2 + B*c^2) + (96*b*c*
(b + c*tan(x/2))*(B*c^3 - A*((b^2 + c^2)^3)^(1/2) + B*b^2*c))/(b^2 + c^2)))
/(b^2 + c^2)^2 + 64*A*B*b^2*c))/(b^2 + c^2)^2 + 32*B*b*c*tan(x/2)*(A - B)^2
)*((B*c)/(b^2 + c^2) - (A*((b^2 + c^2)^3)^(1/2))/(b^2 + c^2)^2) - (B*log(ta
n(x/2) - 1i)*1i)/(b + c*1i) - (B*log(tan(x/2) + 1i))/(b*1i + c)
```

sympy [A] time = 37.03, size = 678, normalized size = 9.29

$$\begin{cases} \tilde{\infty}\left(A\log\left(\tan\left(\frac{x}{2}\right)\right)-B\log\left(\tan^2\left(\frac{x}{2}\right)+1\right)+B\log\left(\tan\left(\frac{x}{2}\right)\right)\right) \\ \frac{A\log\left(\tan\left(\frac{x}{2}\right)\right)-B\log\left(\tan^2\left(\frac{x}{2}\right)+1\right)+B\log\left(\tan\left(\frac{x}{2}\right)\right)}{c} \\ \frac{2A}{-2ic\sin(x)-2c\cos(x)}+\frac{Bx\sin(x)}{-2ic\sin(x)-2c\cos(x)}-\frac{iBx\cos(x)}{-2ic\sin(x)-2c\cos(x)}-\frac{iB\sin(x)}{-2ic\sin(x)-2c\cos(x)} \\ \frac{2A}{2ic\sin(x)-2c\cos(x)}+\frac{Bx\sin(x)}{2ic\sin(x)-2c\cos(x)}+\frac{iBx\cos(x)}{2ic\sin(x)-2c\cos(x)}+\frac{iB\sin(x)}{2ic\sin(x)-2c\cos(x)} \\ -\frac{Ab^2\log\left(\tan\left(\frac{x}{2}\right)-\frac{c}{b}-\frac{\sqrt{b^2+c^2}}{b}\right)}{b^2\sqrt{b^2+c^2}+c^2\sqrt{b^2+c^2}}+\frac{Ab^2\log\left(\tan\left(\frac{x}{2}\right)-\frac{c}{b}+\frac{\sqrt{b^2+c^2}}{b}\right)}{b^2\sqrt{b^2+c^2}+c^2\sqrt{b^2+c^2}}-\frac{Ac^2\log\left(\tan\left(\frac{x}{2}\right)-\frac{c}{b}-\frac{\sqrt{b^2+c^2}}{b}\right)}{b^2\sqrt{b^2+c^2}+c^2\sqrt{b^2+c^2}}+\frac{Ac^2\log\left(\tan\left(\frac{x}{2}\right)-\frac{c}{b}+\frac{\sqrt{b^2+c^2}}{b}\right)}{b^2\sqrt{b^2+c^2}+c^2\sqrt{b^2+c^2}}+\frac{Bb}{b^2\sqrt{b^2+\dots}} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(x))/(b*cos(x)+c*sin(x)),x)
```

```
[Out] Piecewise((zoo*(A*log(tan(x/2)) - B*log(tan(x/2)**2 + 1) + B*log(tan(x/2)))
, Eq(b, 0) & Eq(c, 0)), ((A*log(tan(x/2)) - B*log(tan(x/2)**2 + 1) + B*log(
tan(x/2)))/c, Eq(b, 0)), (2*A/(-2*I*c*sin(x) - 2*c*cos(x)) + B*x*sin(x)/(-2
*I*c*sin(x) - 2*c*cos(x)) - I*B*x*cos(x)/(-2*I*c*sin(x) - 2*c*cos(x)) - I*B
*sin(x)/(-2*I*c*sin(x) - 2*c*cos(x)), Eq(b, -I*c)), (2*A/(2*I*c*sin(x) - 2*
c*cos(x)) + B*x*sin(x)/(2*I*c*sin(x) - 2*c*cos(x)) + I*B*x*cos(x)/(2*I*c*si
n(x) - 2*c*cos(x)) + I*B*sin(x)/(2*I*c*sin(x) - 2*c*cos(x)), Eq(b, I*c)), (
-A*b**2*log(tan(x/2) - c/b - sqrt(b**2 + c**2)/b)/(b**2*sqrt(b**2 + c**2) +
 c**2*sqrt(b**2 + c**2)) + A*b**2*log(tan(x/2) - c/b + sqrt(b**2 + c**2)/b)
/(b**2*sqrt(b**2 + c**2) + c**2*sqrt(b**2 + c**2)) - A*c**2*log(tan(x/2) -
c/b - sqrt(b**2 + c**2)/b)/(b**2*sqrt(b**2 + c**2) + c**2*sqrt(b**2 + c**2)
) + A*c**2*log(tan(x/2) - c/b + sqrt(b**2 + c**2)/b)/(b**2*sqrt(b**2 + c**2
) + c**2*sqrt(b**2 + c**2)) + B*b*x*sqrt(b**2 + c**2)/(b**2*sqrt(b**2 + c**
2) + c**2*sqrt(b**2 + c**2)) - B*c*sqrt(b**2 + c**2)*log(tan(x/2)**2 + 1)/(
b**2*sqrt(b**2 + c**2) + c**2*sqrt(b**2 + c**2)) + B*c*sqrt(b**2 + c**2)*lo
g(tan(x/2) - c/b - sqrt(b**2 + c**2)/b)/(b**2*sqrt(b**2 + c**2) + c**2*sqrt
(b**2 + c**2)) + B*c*sqrt(b**2 + c**2)*log(tan(x/2) - c/b + sqrt(b**2 + c**
2)/b)/(b**2*sqrt(b**2 + c**2) + c**2*sqrt(b**2 + c**2)), True))
```

3.353 $\int \frac{A+B\cos(x)}{(b\cos(x)+c\sin(x))^2}\,dx$

Optimal. Leaf size=76

$$-\frac{-Ab\sin(x)+Ac\cos(x)+Bc}{\left(b^2+c^2\right)(b\cos(x)+c\sin(x))}-\frac{bB\tanh^{-1}\left(\frac{c\cos(x)-b\sin(x)}{\sqrt{b^2+c^2}}\right)}{\left(b^2+c^2\right)^{3/2}}$$

```
[Out] -b*B*arctanh((c*cos(x)-b*sin(x))/(b^2+c^2)^(1/2))/(b^2+c^2)^(3/2)+(-B*c-A*c
*cos(x)+A*b*sin(x))/(b^2+c^2)/(b*cos(x)+c*sin(x))
```

Rubi [A] time = 0.05, antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3155, 3074, 206}

$$-\frac{-Ab\sin(x)+Ac\cos(x)+Bc}{\left(b^2+c^2\right)(b\cos(x)+c\sin(x))}-\frac{bB\tanh^{-1}\left(\frac{c\cos(x)-b\sin(x)}{\sqrt{b^2+c^2}}\right)}{\left(b^2+c^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cos[x])/(b*Cos[x] + c*Sin[x])^2,x]
```

```
[Out] -((b*B*ArcTanh[(c*Cos[x] - b*Sin[x])/Sqrt[b^2 + c^2]])/(b^2 + c^2)^(3/2)) -
 (B*c + A*c*Cos[x] - A*b*Sin[x])/((b^2 + c^2)*(b*Cos[x] + c*Sin[x]))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3074

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x
_Symbol] :> -Dist[d^(-1), Subst[Int[1/(a^2 + b^2 - x^2), x], x, b*Cos[c + d
*x] - a*Sin[c + d*x]], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

Rule 3155

```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.))/((a_.) + cos[(d_.) + (e_.)*(x_)
]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)])^2, x_Symbol] :> Simp[(c*B + c*A*Co
s[d + e*x] + (a*B - b*A)*Sin[d + e*x])/(e*(a^2 - b^2 - c^2)*(a + b*Cos[d +
e*x] + c*Sin[d + e*x])), x] + Dist[(a*A - b*B)/(a^2 - b^2 - c^2), Int[1/(a
+ b*Cos[d + e*x] + c*Sin[d + e*x]), x], x] /; FreeQ[{a, b, c, d, e, A, B},
```

```
x] && NeQ[a^2 - b^2 - c^2, 0] && NeQ[a*A - b*B, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{A+B\cos(x)}{(b\cos(x)+c\sin(x))^2}\,dx &= -\frac{Bc+Ac\cos(x)-Ab\sin(x)}{\left(b^2+c^2\right)(b\cos(x)+c\sin(x))}+\frac{(bB)\int \frac{1}{b\cos(x)+c\sin(x)}\,dx}{b^2+c^2}\\ &= -\frac{Bc+Ac\cos(x)-Ab\sin(x)}{\left(b^2+c^2\right)(b\cos(x)+c\sin(x))}-\frac{(bB)\operatorname{Subst}\left(\int \frac{1}{b^2+c^2-x^2}\,dx,x,c\cos(x)-b\sin(x)\right)}{b^2+c^2}\\ &= -\frac{bB\tanh^{-1}\left(\frac{c\cos(x)-b\sin(x)}{\sqrt{b^2+c^2}}\right)}{\left(b^2+c^2\right)^{3/2}}-\frac{Bc+Ac\cos(x)-Ab\sin(x)}{\left(b^2+c^2\right)(b\cos(x)+c\sin(x))}\end{aligned}$$

Mathematica [A] time = 0.23, size = 82, normalized size = 1.08

$$\frac{A\left(b^2+c^2\right)\sin(x)-bBc}{b\left(b^2+c^2\right)(b\cos(x)+c\sin(x))}+\frac{2bB\tanh^{-1}\left(\frac{b\tan\left(\frac{x}{2}\right)-c}{\sqrt{b^2+c^2}}\right)}{\left(b^2+c^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Cos[x])/(b*Cos[x] + c*Sin[x])^2,x]
```

```
[Out] (2*b*B*ArcTanh[(-c + b*Tan[x/2])/Sqrt[b^2 + c^2]])/(b^2 + c^2)^(3/2) + (-(b
*B*c) + A*(b^2 + c^2)*Sin[x])/(b*(b^2 + c^2)*(b*Cos[x] + c*Sin[x]))
```

fricas [B] time = 0.68, size = 201, normalized size = 2.64

$$-\frac{2\,Bb^2c+2\,Bc^3-\left(Bb^2\cos(x)+Bbc\sin(x)\right)\sqrt{b^2+c^2}\,\log\left(-\frac{2\,bc\cos(x)\sin(x)+\left(b^2-c^2\right)\cos(x)^2-2\,b^2-c^2+2\,\sqrt{b^2+c^2}\,(c\cos(x)-b\sin(x))}{2\,bc\cos(x)\sin(x)+\left(b^2-c^2\right)\cos(x)^2+c^2}\right)}{2\left(\left(b^5+2\,b^3c^2+bc^4\right)\cos(x)+\left(b^4c+2\,b^2c^3+c^5\right)\sin(x)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(x))/(b*cos(x)+c*sin(x))^2,x, algorithm="fricas")
```

```
[Out] -1/2*(2*B*b^2*c + 2*B*c^3 - (B*b^2*cos(x) + B*b*c*sin(x))*sqrt(b^2 + c^2)*l
og(-(2*b*c*cos(x)*sin(x) + (b^2 - c^2)*cos(x)^2 - 2*b^2 - c^2 + 2*sqrt(b^2
+ c^2)*(c*cos(x) - b*sin(x)))/(2*b*c*cos(x)*sin(x) + (b^2 - c^2)*cos(x)^2 +
```

```
c^2)) + 2*(A*b^2*c + A*c^3)*cos(x) - 2*(A*b^3 + A*b*c^2)*sin(x))/((b^5 + 2
*b^3*c^2 + b*c^4)*cos(x) + (b^4*c + 2*b^2*c^3 + c^5)*sin(x))
```

giac [A] time = 0.19, size = 132, normalized size = 1.74

$$-\frac{Bb\log\left(\frac{\left|2\,b\tan\left(\frac{1}{2}\,x\right)-2\,c-2\,\sqrt{b^2+c^2}\right|}{\left|2\,b\tan\left(\frac{1}{2}\,x\right)-2\,c+2\,\sqrt{b^2+c^2}\right|}\right)}{\left(b^2+c^2\right)^{\frac{3}{2}}}-\frac{2\left(Ab^2\tan\left(\frac{1}{2}\,x\right)+Ac^2\tan\left(\frac{1}{2}\,x\right)-Bc^2\tan\left(\frac{1}{2}\,x\right)-Bbc\right)}{\left(b^3+bc^2\right)\left(b\tan\left(\frac{1}{2}\,x\right)^2-2\,c\tan\left(\frac{1}{2}\,x\right)-b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(x))/(b*cos(x)+c*sin(x))^2,x, algorithm="giac")
```

```
[Out] -B*b*log(abs(2*b*tan(1/2*x) - 2*c - 2*sqrt(b^2 + c^2))/abs(2*b*tan(1/2*x) -
 2*c + 2*sqrt(b^2 + c^2)))/(b^2 + c^2)^(3/2) - 2*(A*b^2*tan(1/2*x) + A*c^2*
tan(1/2*x) - B*c^2*tan(1/2*x) - B*b*c)/((b^3 + b*c^2)*(b*tan(1/2*x)^2 - 2*c
*tan(1/2*x) - b))
```

maple [A] time = 0.16, size = 109, normalized size = 1.43

$$\frac{-\frac{2\left(A\,b^2+A\,c^2-B\,c^2\right)\tan\left(\frac{x}{2}\right)}{b\left(b^2+c^2\right)}+\frac{2Bc}{b^2+c^2}}{b\left(\tan^2\left(\frac{x}{2}\right)\right)-2c\tan\left(\frac{x}{2}\right)-b}+\frac{2bB\,\mathrm{arctanh}\left(\frac{2b\tan\left(\frac{x}{2}\right)-2c}{2\sqrt{b^2+c^2}}\right)}{\left(b^2+c^2\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(x))/(b*cos(x)+c*sin(x))^2,x)
```

```
[Out] 2*(-(A*b^2+A*c^2-B*c^2)/b/(b^2+c^2)*tan(1/2*x)+B*c/(b^2+c^2))/(b*tan(1/2*x)
^2-2*c*tan(1/2*x)-b)+2*b*B/(b^2+c^2)^(3/2)*arctanh(1/2*(2*b*tan(1/2*x)-2*c)
/(b^2+c^2)^(1/2))
```

maxima [B] time = 0.42, size = 156, normalized size = 2.05

$$-B\left(\frac{b\log\left(\frac{c-\frac{b\sin(x)}{\cos(x)+1}+\sqrt{b^2+c^2}}{c-\frac{b\sin(x)}{\cos(x)+1}-\sqrt{b^2+c^2}}\right)}{\left(b^2+c^2\right)^{\frac{3}{2}}}+\frac{2\left(bc+\frac{c^2\sin(x)}{\cos(x)+1}\right)}{b^4+b^2c^2+\frac{2\left(b^3c+bc^3\right)\sin(x)}{\cos(x)+1}-\frac{\left(b^4+b^2c^2\right)\sin(x)^2}{\left(\cos(x)+1\right)^2}}\right)-\frac{A}{c^2\tan(x)+bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(x))/(b*cos(x)+c*sin(x))^2,x, algorithm="maxima")
```

```
[Out] -B*(b*log((c - b*sin(x)/(cos(x) + 1) + sqrt(b^2 + c^2))/(c - b*sin(x)/(cos(
x) + 1) - sqrt(b^2 + c^2)))/(b^2 + c^2)^(3/2) + 2*(b*c + c^2*sin(x)/(cos(x)
 + 1))/(b^4 + b^2*c^2 + 2*(b^3*c + b*c^3)*sin(x)/(cos(x) + 1) - (b^4 + b^2*
c^2)*sin(x)^2/(cos(x) + 1)^2)) - A/(c^2*tan(x) + b*c)
```

mupad [B] time = 2.62, size = 126, normalized size = 1.66

$$-\frac{\frac{2\,B\,c}{b^2+c^2}-\frac{2\,\tan\left(\frac{x}{2}\right)\,\left(A\,b^2+A\,c^2-B\,c^2\right)}{b\,\left(b^2+c^2\right)}}{-b\,\tan\left(\frac{x}{2}\right)^2+2\,c\,\tan\left(\frac{x}{2}\right)+b}+\frac{B\,b\,\mathrm{atan}\left(\frac{b^2\,c\,1{}\mathrm{i}+c^3\,1{}\mathrm{i}-b\,\tan\left(\frac{x}{2}\right)\,\left(b^2+c^2\right)\,1{}\mathrm{i}}{{\left(b^2+c^2\right)}^{3/2}}\right)\,2{}\mathrm{i}}{{\left(b^2+c^2\right)}^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cos(x))/(b*cos(x) + c*sin(x))^2,x)
```

```
[Out] (B*b*atan((b^2*c*1i + c^3*1i - b*tan(x/2)*(b^2 + c^2)*1i)/(b^2 + c^2)^(3/2)
)*2i)/(b^2 + c^2)^(3/2) - ((2*B*c)/(b^2 + c^2) - (2*tan(x/2)*(A*b^2 + A*c^2
 - B*c^2))/(b*(b^2 + c^2)))/(b + 2*c*tan(x/2) - b*tan(x/2)^2)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(x))/(b*cos(x)+c*sin(x))**2,x)
```

```
[Out] Timed out
```

3.354 $\int \frac{A+B\cos(x)}{(b\cos(x)+c\sin(x))^3}\,dx$

Optimal. Leaf size=116

$$-\frac{-Ab\sin(x)+Ac\cos(x)+Bc}{2\left(b^2+c^2\right)(b\cos(x)+c\sin(x))^2}-\frac{A\tanh^{-1}\left(\frac{c\cos(x)-b\sin(x)}{\sqrt{b^2+c^2}}\right)}{2\left(b^2+c^2\right)^{3/2}}-\frac{bBc\cos(x)-b^2B\sin(x)}{\left(b^2+c^2\right)^2(b\cos(x)+c\sin(x))}$$

```
[Out] -1/2*A*arctanh((c*cos(x)-b*sin(x))/(b^2+c^2)^(1/2))/(b^2+c^2)^(3/2)+1/2*(-B
*c-A*c*cos(x)+A*b*sin(x))/(b^2+c^2)/(b*cos(x)+c*sin(x))^2+(-b*B*c*cos(x)+b^
2*B*sin(x))/(b^2+c^2)^2/(b*cos(x)+c*sin(x))
```

Rubi [A] time = 0.11, antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {3158, 3153, 3074, 206}

$$-\frac{-Ab\sin(x)+Ac\cos(x)+Bc}{2\left(b^2+c^2\right)(b\cos(x)+c\sin(x))^2}-\frac{A\tanh^{-1}\left(\frac{c\cos(x)-b\sin(x)}{\sqrt{b^2+c^2}}\right)}{2\left(b^2+c^2\right)^{3/2}}-\frac{bBc\cos(x)-b^2B\sin(x)}{\left(b^2+c^2\right)^2(b\cos(x)+c\sin(x))}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cos[x])/(b*Cos[x] + c*Sin[x])^3,x]
```

```
[Out] -(A*ArcTanh[(c*Cos[x] - b*Sin[x])/Sqrt[b^2 + c^2]])/(2*(b^2 + c^2)^(3/2)) -
 (B*c + A*c*Cos[x] - A*b*Sin[x])/(2*(b^2 + c^2)*(b*Cos[x] + c*Sin[x])^2) -
(b*B*c*Cos[x] - b^2*B*Sin[x])/((b^2 + c^2)^2*(b*Cos[x] + c*Sin[x]))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3074

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x
_Symbol] :> -Dist[d^(-1), Subst[Int[1/(a^2 + b^2 - x^2), x], x, b*Cos[c + d
*x] - a*Sin[c + d*x]], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

Rule 3153

```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])
/((a_.) + cos[(d_.) + (e_.)*(x_)]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)])^2,
 x_Symbol] :> Simp[(c*B - b*C - (a*C - c*A)*Cos[d + e*x] + (a*B - b*A)*Sin[
```

```
d + e*x])/(e*(a^2 - b^2 - c^2)*(a + b*Cos[d + e*x] + c*Sin[d + e*x])), x] +
 Dist[(a*A - b*B - c*C)/(a^2 - b^2 - c^2), Int[1/(a + b*Cos[d + e*x] + c*Si
n[d + e*x]), x], x] /; FreeQ[{a, b, c, d, e, A, B, C}, x] && NeQ[a^2 - b^2
- c^2, 0] && NeQ[a*A - b*B - c*C, 0]
```

Rule 3158

```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.))*((a_.) + cos[(d_.) + (e_.)*(x_)
]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)])^(n_), x_Symbol] :> -Simp[((c*B + c
*A*Cos[d + e*x] + (a*B - b*A)*Sin[d + e*x])*(a + b*Cos[d + e*x] + c*Sin[d +
 e*x])^(n + 1))/(e*(n + 1)*(a^2 - b^2 - c^2)), x] + Dist[1/((n + 1)*(a^2 -
b^2 - c^2)), Int[(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n + 1)*Simp[(n + 1)
*(a*A - b*B) + (n + 2)*(a*B - b*A)*Cos[d + e*x] - (n + 2)*c*A*Sin[d + e*x],
 x], x], x] /; FreeQ[{a, b, c, d, e, A, B}, x] && LtQ[n, -1] && NeQ[a^2 - b
^2 - c^2, 0] && NeQ[n, -2]
```

Rubi steps

$$\int \frac{A+B\cos(x)}{(b\cos(x)+c\sin(x))^3}\,dx = -\frac{Bc+Ac\cos(x)-Ab\sin(x)}{2\left(b^2+c^2\right)(b\cos(x)+c\sin(x))^2}+\frac{\int \frac{2bB+Ab\cos(x)+Ac\sin(x)}{(b\cos(x)+c\sin(x))^2}\,dx}{2\left(b^2+c^2\right)}$$

$$= -\frac{Bc+Ac\cos(x)-Ab\sin(x)}{2\left(b^2+c^2\right)(b\cos(x)+c\sin(x))^2}-\frac{bBc\cos(x)-b^2B\sin(x)}{\left(b^2+c^2\right)^2(b\cos(x)+c\sin(x))}+\frac{A\int \frac{1}{b\cos(x)+\ldots}}{2\left(b^2+\ldots\right.}$$

$$= -\frac{Bc+Ac\cos(x)-Ab\sin(x)}{2\left(b^2+c^2\right)(b\cos(x)+c\sin(x))^2}-\frac{bBc\cos(x)-b^2B\sin(x)}{\left(b^2+c^2\right)^2(b\cos(x)+c\sin(x))}-\frac{A\,\mathrm{Subst}\left(\int \ldots\right.}{\ldots}$$

$$= -\frac{A\tanh^{-1}\left(\frac{c\cos(x)-b\sin(x)}{\sqrt{b^2+c^2}}\right)}{2\left(b^2+c^2\right)^{3/2}}-\frac{Bc+Ac\cos(x)-Ab\sin(x)}{2\left(b^2+c^2\right)(b\cos(x)+c\sin(x))^2}-\frac{bBc\cos(x)-b\ldots}{\left(b^2+c^2\right)^2(b\cos(\ldots}$$

Mathematica [C] time = 0.34, size = 118, normalized size = 1.02

$$\frac{\left(b^2+c^2\right)(b\sin(x)(A+2B\cos(x))-Ac\cos(x)-Bc\cos(2x))+2A\sqrt{b^2+c^2}\,(b\cos(x)+c\sin(x))^2\tanh^{-1}\left(\frac{b\tan\left(\frac{x}{2}\right)\ldots}{\sqrt{b^2+c^2}}\right.}{2(b-ic)^2(b+ic)^2(b\cos(x)+c\sin(x))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Cos[x])/(b*Cos[x] + c*Sin[x])^3,x]
```

```
[Out] (2*A*Sqrt[b^2 + c^2]*ArcTanh[(-c + b*Tan[x/2])/Sqrt[b^2 + c^2]]*(b*Cos[x] +
 c*Sin[x])^2 + (b^2 + c^2)*(-(A*c*Cos[x]) - B*c*Cos[2*x] + b*(A + 2*B*Cos[x
])*Sin[x]))/(2*(b - I*c)^2*(b + I*c)^2*(b*Cos[x] + c*Sin[x])^2)
```

fricas [B] time = 0.93, size = 279, normalized size = 2.41

$$-\frac{8\,Bb^2c\cos(x)^2 - 2\,Bb^2c + 2\,Bc^3 - \left(2\,Abc\cos(x)\sin(x) + Ac^2 + \left(Ab^2 - Ac^2\right)\cos(x)^2\right)\sqrt{b^2+c^2}\,\log\Bigg(-\frac{2\,bc\cos(x)\cdots}{\cdots}}{4\,\Big(b^4c^2 + 2\,b^2c^4 + c^6 + \Big(b^6 + b^4c^2 - b^2c^4 \cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(x))/(b*cos(x)+c*sin(x))^3,x, algorithm="fricas")
```

```
[Out] -1/4*(8*B*b^2*c*cos(x)^2 - 2*B*b^2*c + 2*B*c^3 - (2*A*b*c*cos(x)*sin(x) + A
*c^2 + (A*b^2 - A*c^2)*cos(x)^2)*sqrt(b^2 + c^2)*log(-(2*b*c*cos(x)*sin(x)
+ (b^2 - c^2)*cos(x)^2 - 2*b^2 - c^2 + 2*sqrt(b^2 + c^2)*(c*cos(x) - b*sin(
x)))/(2*b*c*cos(x)*sin(x) + (b^2 - c^2)*cos(x)^2 + c^2)) + 2*(A*b^2*c + A*c
^3)*cos(x) - 2*(A*b^3 + A*b*c^2 + 2*(B*b^3 - B*b*c^2)*cos(x))*sin(x))/(b^4*
c^2 + 2*b^2*c^4 + c^6 + (b^6 + b^4*c^2 - b^2*c^4 - c^6)*cos(x)^2 + 2*(b^5*c
 + 2*b^3*c^3 + b*c^5)*cos(x)*sin(x))
```

giac [B] time = 0.25, size = 245, normalized size = 2.11

$$\frac{A\log\left(\frac{\left|-2\,b\tan\left(\frac{1}{2}\,x\right)+2\,c-2\,\sqrt{b^2+c^2}\right|}{\left|-2\,b\tan\left(\frac{1}{2}\,x\right)+2\,c+2\,\sqrt{b^2+c^2}\right|}\right)}{2\left(b^2+c^2\right)^{\frac{3}{2}}} + \frac{Ab^3\tan\left(\frac{1}{2}\,x\right)^3 - 2\,Bb^3\tan\left(\frac{1}{2}\,x\right)^3 + 2\,Abc^2\tan\left(\frac{1}{2}\,x\right)^3 - 2\,Bbc^2\tan\left(\frac{1}{2}\,x\right)^3 + A\cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(x))/(b*cos(x)+c*sin(x))^3,x, algorithm="giac")
```

```
[Out] 1/2*A*log(abs(-2*b*tan(1/2*x) + 2*c - 2*sqrt(b^2 + c^2))/abs(-2*b*tan(1/2*x
) + 2*c + 2*sqrt(b^2 + c^2)))/(b^2 + c^2)^(3/2) + (A*b^3*tan(1/2*x)^3 - 2*B
*b^3*tan(1/2*x)^3 + 2*A*b*c^2*tan(1/2*x)^3 - 2*B*b*c^2*tan(1/2*x)^3 + A*b^2
*c*tan(1/2*x)^2 + 2*B*b^2*c*tan(1/2*x)^2 - 2*A*c^3*tan(1/2*x)^2 + 2*B*c^3*t
an(1/2*x)^2 + A*b^3*tan(1/2*x) + 2*B*b^3*tan(1/2*x) - 2*A*b*c^2*tan(1/2*x)
+ 2*B*b*c^2*tan(1/2*x) - A*b^2*c)/((b^4 + b^2*c^2)*(b*tan(1/2*x)^2 - 2*c*ta
n(1/2*x) - b)^2)
```

maple [A] time = 0.18, size = 204, normalized size = 1.76

$$-\frac{2\left(-\frac{\left(A\,b^2+2A\,c^2-2B\,b^2-2B\,c^2\right)\left(\tan^3\left(\frac{x}{2}\right)\right)}{2\left(b^2+c^2\right)b} - \frac{c\left(A\,b^2-2A\,c^2+2B\,b^2+2B\,c^2\right)\left(\tan^2\left(\frac{x}{2}\right)\right)}{2\left(b^2+c^2\right)b^2} - \frac{\left(A\,b^2-2A\,c^2+2B\,b^2+2B\,c^2\right)\tan\left(\frac{x}{2}\right)}{2\left(b^2+c^2\right)b} + \frac{Ac}{2b^2+2c^2}\right)}{\left(b\left(\tan^2\left(\frac{x}{2}\right)\right) - 2c\tan\left(\frac{x}{2}\right) - b\right)^2} + \frac{A\,\mathrm{a}\cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(x))/(b*cos(x)+c*sin(x))^3,x)
```

```
[Out] -2*(-1/2*(A*b^2+2*A*c^2-2*B*b^2-2*B*c^2)/(b^2+c^2)/b*tan(1/2*x)^3-1/2*c*(A*
b^2-2*A*c^2+2*B*b^2+2*B*c^2)/(b^2+c^2)/b^2*tan(1/2*x)^2-1/2*(A*b^2-2*A*c^2+
2*B*b^2+2*B*c^2)/(b^2+c^2)/b*tan(1/2*x)+1/2*A*c/(b^2+c^2))/(b*tan(1/2*x)^2-
2*c*tan(1/2*x)-b)^2+A/(b^2+c^2)^(3/2)*arctanh(1/2*(2*b*tan(1/2*x)-2*c)/(b^2
+c^2)^(1/2))
```

maxima [B] time = 0.44, size = 366, normalized size = 3.16

$$-\frac{1}{2}A\left(\frac{2\left(b^2c-\frac{(b^3-2bc^2)\sin(x)}{\cos(x)+1}-\frac{(b^2c-2c^3)\sin(x)^2}{(\cos(x)+1)^2}-\frac{(b^3+2bc^2)\sin(x)^3}{(\cos(x)+1)^3}\right)}{b^6+b^4c^2+\frac{4(b^5c+b^3c^3)\sin(x)}{\cos(x)+1}-\frac{2(b^6-b^4c^2-2b^2c^4)\sin(x)^2}{(\cos(x)+1)^2}-\frac{4(b^5c+b^3c^3)\sin(x)^3}{(\cos(x)+1)^3}+\frac{(b^6+b^4c^2)\sin(x)^4}{(\cos(x)+1)^4}}+\frac{\log\left(\frac{c-\frac{b\sin(x)}{\cos(x)+1}+\sqrt{b^2+}}{c-\frac{b\sin(x)}{\cos(x)+1}-\sqrt{b^2+}}\right)}{(b^2+c^2)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(x))/(b*cos(x)+c*sin(x))^3,x, algorithm="maxima")
```

```
[Out] -1/2*A*(2*(b^2*c - (b^3 - 2*b*c^2)*sin(x)/(cos(x) + 1) - (b^2*c - 2*c^3)*si
n(x)^2/(cos(x) + 1)^2 - (b^3 + 2*b*c^2)*sin(x)^3/(cos(x) + 1)^3)/(b^6 + b^4
*c^2 + 4*(b^5*c + b^3*c^3)*sin(x)/(cos(x) + 1) - 2*(b^6 - b^4*c^2 - 2*b^2*c
^4)*sin(x)^2/(cos(x) + 1)^2 - 4*(b^5*c + b^3*c^3)*sin(x)^3/(cos(x) + 1)^3 +
 (b^6 + b^4*c^2)*sin(x)^4/(cos(x) + 1)^4) + log((c - b*sin(x)/(cos(x) + 1)
+ sqrt(b^2 + c^2))/(c - b*sin(x)/(cos(x) + 1) - sqrt(b^2 + c^2)))/(b^2 + c^
2)^(3/2)) + 2*B*(b*sin(x)/(cos(x) + 1) + c*sin(x)^2/(cos(x) + 1)^2 - b*sin(
x)^3/(cos(x) + 1)^3)/(b^4 + 4*b^3*c*sin(x)/(cos(x) + 1) - 4*b^3*c*sin(x)^3/
(cos(x) + 1)^3 + b^4*sin(x)^4/(cos(x) + 1)^4 - 2*(b^4 - 2*b^2*c^2)*sin(x)^2
/(cos(x) + 1)^2)
```

mupad [B] time = 2.88, size = 251, normalized size = 2.16

$$\frac{\frac{\tan\left(\frac{x}{2}\right)\left(A\,b^2-2\,A\,c^2+2\,B\,b^2+2\,B\,c^2\right)}{b\,\left(b^2+c^2\right)}-\frac{A\,c}{b^2+c^2}+\frac{{\tan\left(\frac{x}{2}\right)}^2\,\left(2\,B\,c^3-2\,A\,c^3+A\,b^2\,c+2\,B\,b^2\,c\right)}{b^2\,\left(b^2+c^2\right)}+\frac{{\tan\left(\frac{x}{2}\right)}^3\,\left(A\,b^2+2\,A\,c^2-2\,B\,b^2-2\,B\,c^2\right)}{b\,\left(b^2+c^2\right)}}{b^2-{\tan\left(\frac{x}{2}\right)}^2\,\left(2\,b^2-4\,c^2\right)+b^2\,{\tan\left(\frac{x}{2}\right)}^4+4\,b\,c\,\tan\left(\frac{x}{2}\right)-4\,b\,c\,{\tan\left(\frac{x}{2}\right)}^3}+\frac{A\,\mathrm{atan}\left(\frac{b^2\,c}{}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cos(x))/(b*cos(x) + c*sin(x))^3,x)
```

```
[Out] ((tan(x/2)*(A*b^2 - 2*A*c^2 + 2*B*b^2 + 2*B*c^2))/(b*(b^2 + c^2)) - (A*c)/(
b^2 + c^2) + (tan(x/2)^2*(2*B*c^3 - 2*A*c^3 + A*b^2*c + 2*B*b^2*c))/(b^2*(b
^2 + c^2)) + (tan(x/2)^3*(A*b^2 + 2*A*c^2 - 2*B*b^2 - 2*B*c^2))/(b*(b^2 + c
```

```
^2)))/(b^2 - tan(x/2)^2*(2*b^2 - 4*c^2) + b^2*tan(x/2)^4 + 4*b*c*tan(x/2) -
 4*b*c*tan(x/2)^3) + (A*atan((b^2*c*1i + c^3*1i - b*tan(x/2)*(b^2 + c^2)*1i
)/(b^2 + c^2)^(3/2))*1i)/(b^2 + c^2)^(3/2)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(x))/(b*cos(x)+c*sin(x))**3,x)
```

```
[Out] Timed out
```

3.355 $\int \left(\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)\right)^4 \, dx$

Optimal. Leaf size=246

$$\frac{35b\left(b^2+c^2\right)^{3/2}\sin(d+ex)}{8e}-\frac{35c\left(b^2+c^2\right)^{3/2}\cos(d+ex)}{8e}-\frac{(c\cos(d+ex)-b\sin(d+ex))\left(\sqrt{b^2+c^2}+b\cos(d+e\right.}{4e}$$

```
[Out] 35/8*(b^2+c^2)^2*x-35/8*c*(b^2+c^2)^(3/2)*cos(e*x+d)/e+35/8*b*(b^2+c^2)^(3/
2)*sin(e*x+d)/e-35/24*(b^2+c^2)*(c*cos(e*x+d)-b*sin(e*x+d))*(b*cos(e*x+d)+c
*sin(e*x+d)+(b^2+c^2)^(1/2))/e-7/12*(c*cos(e*x+d)-b*sin(e*x+d))*(b^2+c^2)^(
1/2)*(b*cos(e*x+d)+c*sin(e*x+d)+(b^2+c^2)^(1/2))^2/e-1/4*(c*cos(e*x+d)-b*si
n(e*x+d))*(b*cos(e*x+d)+c*sin(e*x+d)+(b^2+c^2)^(1/2))^3/e
```

Rubi [A] time = 0.17, antiderivative size = 246, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {3113, 2637, 2638}

$$\frac{35b\left(b^2+c^2\right)^{3/2}\sin(d+ex)}{8e}-\frac{35c\left(b^2+c^2\right)^{3/2}\cos(d+ex)}{8e}-\frac{(c\cos(d+ex)-b\sin(d+ex))\left(\sqrt{b^2+c^2}+b\cos(d+e\right.}{4e}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[b^2 + c^2] + b*Cos[d + e*x] + c*Sin[d + e*x])^4,x]
```

```
[Out] (35*(b^2 + c^2)^2*x)/8 - (35*c*(b^2 + c^2)^(3/2)*Cos[d + e*x])/(8*e) + (35*
b*(b^2 + c^2)^(3/2)*Sin[d + e*x])/(8*e) - (35*(b^2 + c^2)*(c*Cos[d + e*x] -
 b*Sin[d + e*x])*(Sqrt[b^2 + c^2] + b*Cos[d + e*x] + c*Sin[d + e*x]))/(24*e
) - (7*Sqrt[b^2 + c^2]*(c*Cos[d + e*x] - b*Sin[d + e*x])*(Sqrt[b^2 + c^2] +
 b*Cos[d + e*x] + c*Sin[d + e*x])^2)/(12*e) - ((c*Cos[d + e*x] - b*Sin[d +
e*x])*(Sqrt[b^2 + c^2] + b*Cos[d + e*x] + c*Sin[d + e*x])^3)/(4*e)
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3113

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(n_), x_Symbol] :> -Simp[((c*Cos[d + e*x] - b*Sin[d + e*x])*(a + b*Cos[d +
e*x] + c*Sin[d + e*x])^(n - 1))/(e*n), x] + Dist[(a*(2*n - 1))/n, Int[(a +
b*Cos[d + e*x] + c*Sin[d + e*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e},
x] && EqQ[a^2 - b^2 - c^2, 0] && GtQ[n, 0]
```

Rubi steps

$$\int\left(\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)\right)^4\,dx=-\frac{(c\cos(d+ex)-b\sin(d+ex))\left(\sqrt{b^2+c^2}+b\cos(d+ex)\right.}{4e}$$

$$=-\frac{7\sqrt{b^2+c^2}\,(c\cos(d+ex)-b\sin(d+ex))\left(\sqrt{b^2+c^2}+b\,c\right.}{12e}$$

$$=-\frac{35\left(b^2+c^2\right)(c\cos(d+ex)-b\sin(d+ex))\left(\sqrt{b^2+c^2}+b\right.}{24e}$$

$$=\frac{35}{8}\left(b^2+c^2\right)^2x-\frac{35\left(b^2+c^2\right)(c\cos(d+ex)-b\sin(d+ex}{}$$

$$=\frac{35}{8}\left(b^2+c^2\right)^2x-\frac{35c\left(b^2+c^2\right)^{3/2}\cos(d+ex)}{8e}+\frac{35b\left(b^2+\right.}{}$$

Mathematica [C] time = 1.42, size = 238, normalized size = 0.97

$$\frac{168\left(b^4-c^4\right)\sin(2(d+ex))+420\left(b^2+c^2\right)^2(d+ex)+672b(b-ic)(b+ic)\sqrt{b^2+c^2}\sin(d+ex)+32b\left(b^2-3c^2\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[b^2 + c^2] + b*Cos[d + e*x] + c*Sin[d + e*x])^4,x]
```

```
[Out] (420*(b^2 + c^2)^2*(d + e*x) - 672*(b - I*c)*(b + I*c)*c*Sqrt[b^2 + c^2]*Co
s[d + e*x] - 336*b*c*(b^2 + c^2)*Cos[2*(d + e*x)] + 32*c*(-3*b^2 + c^2)*Sqr
t[b^2 + c^2]*Cos[3*(d + e*x)] - 12*b*c*(b^2 - c^2)*Cos[4*(d + e*x)] + 672*b
*(b - I*c)*(b + I*c)*Sqrt[b^2 + c^2]*Sin[d + e*x] + 168*(b^4 - c^4)*Sin[2*(
d + e*x)] + 32*b*(b^2 - 3*c^2)*Sqrt[b^2 + c^2]*Sin[3*(d + e*x)] + 3*(b^4 -
6*b^2*c^2 + c^4)*Sin[4*(d + e*x)])/(96*e)
```

fricas [A] time = 1.06, size = 221, normalized size = 0.90

$$-\frac{24\left(b^3c-bc^3\right)\cos(ex+d)^4-105\left(b^4+2b^2c^2+c^4\right)ex+48\left(3b^3c+4bc^3\right)\cos(ex+d)^2-3\left(2\left(b^4-6b^2c^2+c\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(e*x+d)+c*sin(e*x+d)+(b^2+c^2)^(1/2))^4,x, algorithm="frica
s")
```

```
[Out] -1/24*(24*(b^3*c - b*c^3)*cos(e*x + d)^4 - 105*(b^4 + 2*b^2*c^2 + c^4)*e*x
+ 48*(3*b^3*c + 4*b*c^3)*cos(e*x + d)^2 - 3*(2*(b^4 - 6*b^2*c^2 + c^4)*cos(
e*x + d)^3 + (27*b^4 + 6*b^2*c^2 - 29*c^4)*cos(e*x + d))*sin(e*x + d) + 32*
((3*b^2*c - c^3)*cos(e*x + d)^3 + 3*(b^2*c + 2*c^3)*cos(e*x + d) - (5*b^3 +
 6*b*c^2 + (b^3 - 3*b*c^2)*cos(e*x + d)^2)*sin(e*x + d))*sqrt(b^2 + c^2))/e
```

giac [A] time = 0.35, size = 287, normalized size = 1.17

$$-\frac{1}{8}\left(b^{3}c-bc^{3}\right)\cos\left(4\,xe+4\,d\right)e^{(-1)}-\frac{1}{3}\left(3\,\sqrt{b^{2}+c^{2}}\,b^{2}c-\sqrt{b^{2}+c^{2}}\,c^{3}\right)\cos\left(3\,xe+3\,d\right)e^{(-1)}-\frac{7}{2}\left(b^{3}c+bc^{3}\right)\cos\left(2\,xe\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(e*x+d)+c*sin(e*x+d)+(b^2+c^2)^(1/2))^4,x, algorithm="giac"
)
```

```
[Out] -1/8*(b^3*c - b*c^3)*cos(4*x*e + 4*d)*e^(-1) - 1/3*(3*sqrt(b^2 + c^2)*b^2*c
 - sqrt(b^2 + c^2)*c^3)*cos(3*x*e + 3*d)*e^(-1) - 7/2*(b^3*c + b*c^3)*cos(2
*x*e + 2*d)*e^(-1) - 7*(sqrt(b^2 + c^2)*b^2*c + sqrt(b^2 + c^2)*c^3)*cos(x*
e + d)*e^(-1) + 1/32*(b^4 - 6*b^2*c^2 + c^4)*e^(-1)*sin(4*x*e + 4*d) + 1/3*
(sqrt(b^2 + c^2)*b^3 - 3*sqrt(b^2 + c^2)*b*c^2)*e^(-1)*sin(3*x*e + 3*d) + 7
/4*(b^4 - c^4)*e^(-1)*sin(2*x*e + 2*d) + 7*(sqrt(b^2 + c^2)*b^3 + sqrt(b^2
+ c^2)*b*c^2)*e^(-1)*sin(x*e + d) + 35/8*(b^4 + 2*b^2*c^2 + c^4)*x
```

maple [B] time = 0.30, size = 514, normalized size = 2.09

$$b^{4}\left(ex+d\right)+2b^{2}c^{2}\left(ex+d\right)+c^{4}\left(ex+d\right)+b^{4}\left(\frac{\left(\cos^{3}(ex+d)+\frac{3\cos(ex+d)}{2}\right)\sin(ex+d)}{4}+\frac{3ex}{8}+\frac{3d}{8}\right)+6b^{4}\left(\frac{\sin(ex+d)\cos(ex+d)}{2}+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*cos(e*x+d)+c*sin(e*x+d)+(b^2+c^2)^(1/2))^4,x)
```

```
[Out] 1/e*(b^4*(e*x+d)+2*b^2*c^2*(e*x+d)+c^4*(e*x+d)+b^4*(1/4*(cos(e*x+d)^3+3/2*c
os(e*x+d))*sin(e*x+d)+3/8*e*x+3/8*d)+6*b^4*(1/2*sin(e*x+d)*cos(e*x+d)+1/2*e
*x+1/2*d)+c^4*(-1/4*(sin(e*x+d)^3+3/2*sin(e*x+d))*cos(e*x+d)+3/8*e*x+3/8*d)
+6*c^4*(-1/2*sin(e*x+d)*cos(e*x+d)+1/2*e*x+1/2*d)-4*(b^2+c^2)^(1/2)*b^2*c*c
os(e*x+d)^3+4*(b^2+c^2)^(1/2)*b*c^2*sin(e*x+d)^3+4/3*(b^2+c^2)^(1/2)*b^3*(2
+cos(e*x+d)^2)*sin(e*x+d)+6*b^2*c^2*(1/2*sin(e*x+d)*cos(e*x+d)+1/2*e*x+1/2*
d)+4*(b^2+c^2)^(1/2)*b^3*sin(e*x+d)-4/3*(b^2+c^2)^(1/2)*c^3*(2+sin(e*x+d)^2
```

```
)*cos(e*x+d)+6*b^2*c^2*(-1/2*sin(e*x+d)*cos(e*x+d)+1/2*e*x+1/2*d)-4*(b^2+c^
2)^(1/2)*c^3*cos(e*x+d)-cos(e*x+d)^4*b^3*c+6*b^2*c^2*(-1/4*sin(e*x+d)*cos(e
*x+d)^3+1/8*sin(e*x+d)*cos(e*x+d)+1/8*e*x+1/8*d)+c^3*b*sin(e*x+d)^4-6*cos(e
*x+d)^2*b^3*c-6*cos(e*x+d)^2*b*c^3+4*(b^2+c^2)^(1/2)*b*c^2*sin(e*x+d)-4*(b^
2+c^2)^(1/2)*b^2*c*cos(e*x+d))
```

maxima [A] time = 0.33, size = 354, normalized size = 1.44

$$-\frac{b^3c\cos(ex+d)^4}{e}+\frac{bc^3\sin(ex+d)^4}{e}+\frac{(12\,ex+12\,d+\sin(4\,ex+4\,d)+8\,\sin(2\,ex+2\,d))b^4}{32\,e}+\frac{3\,(4\,ex+4\,d-\sin}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(e*x+d)+c*sin(e*x+d)+(b^2+c^2)^(1/2))^4,x, algorithm="maxim
a")
```

```
[Out] -b^3*c*cos(e*x + d)^4/e + b*c^3*sin(e*x + d)^4/e + 1/32*(12*e*x + 12*d + si
n(4*e*x + 4*d) + 8*sin(2*e*x + 2*d))*b^4/e + 3/16*(4*e*x + 4*d - sin(4*e*x
+ 4*d))*b^2*c^2/e + 1/32*(12*e*x + 12*d + sin(4*e*x + 4*d) - 8*sin(2*e*x +
2*d))*c^4/e + (b^2 + c^2)^2*x - 4*(b^2 + c^2)^(3/2)*(c*cos(e*x + d)/e - b*s
in(e*x + d)/e) - 3/2*(4*b*c*cos(e*x + d)^2/e - (2*e*x + 2*d + sin(2*e*x + 2
*d))*b^2/e - (2*e*x + 2*d - sin(2*e*x + 2*d))*c^2/e)*(b^2 + c^2) - 4/3*(3*b
^2*c*cos(e*x + d)^3/e - 3*b*c^2*sin(e*x + d)^3/e + (sin(e*x + d)^3 - 3*sin(
e*x + d))*b^3/e - (cos(e*x + d)^3 - 3*cos(e*x + d))*c^3/e)*sqrt(b^2 + c^2)
```

mupad [B] time = 7.32, size = 522, normalized size = 2.12

$$\frac{35\,\mathrm{atan}\left(\frac{35\tan\left(\frac{d}{2}+\frac{e\,x}{2}\right)\left(b^2+c^2\right)^2}{4\left(\frac{35\,b^4}{4}+\frac{35\,b^2\,c^2}{2}+\frac{35\,c^4}{4}\right)}\right)\left(b^2+c^2\right)^2}{4\,e}-\frac{35\,\left(\mathrm{atan}\left(\tan\left(\frac{d}{2}+\frac{e\,x}{2}\right)\right)-\frac{e\,x}{2}\right)\,\left(b^2+c^2\right)^2}{4\,e}+\frac{\tan\left(\frac{d}{2}+\frac{e\,x}{2}\right)\,\left(\left(16\,b^3+8\,b\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*cos(d + e*x) + c*sin(d + e*x) + (b^2 + c^2)^(1/2))^4,x)
```

```
[Out] (35*atan((35*tan(d/2 + (e*x)/2)*(b^2 + c^2)^2)/(4*((35*b^4)/4 + (35*c^4)/4
+ (35*b^2*c^2)/2)))*(b^2 + c^2)^2)/(4*e) - (35*(atan(tan(d/2 + (e*x)/2)) -
(e*x)/2)*(b^2 + c^2)^2)/(4*e) + (tan(d/2 + (e*x)/2)*((8*b*c^2 + 16*b^3)*(b^
2 + c^2)^(1/2) + (29*b^4)/4 - (27*c^4)/4 - (3*b^2*c^2)/2) + tan(d/2 + (e*x)
/2)^6*(24*b*c^3 + 32*b^3*c - (32*b^2*c + 8*c^3)*(b^2 + c^2)^(1/2)) + tan(d/
2 + (e*x)/2)^4*(64*b*c^3 + 48*b^3*c - (48*b^2*c + 40*c^3)*(b^2 + c^2)^(1/2)
) + tan(d/2 + (e*x)/2)^2*(24*b*c^3 + 32*b^3*c - (32*b^2*c + (136*c^3)/3)*(b
^2 + c^2)^(1/2)) - (16*b^2*c + (40*c^3)/3)*(b^2 + c^2)^(1/2) + tan(d/2 + (e
```

```
*x)/2)^7*((8*b*c^2 + 16*b^3)*(b^2 + c^2)^(1/2) - (29*b^4)/4 + (27*c^4)/4 +
(3*b^2*c^2)/2) + tan(d/2 + (e*x)/2)^3*((56*b*c^2 + (112*b^3)/3)*(b^2 + c^2)
^(1/2) + (21*b^4)/4 - (35*c^4)/4 + (21*b^2*c^2)/2) + tan(d/2 + (e*x)/2)^5*(
(56*b*c^2 + (112*b^3)/3)*(b^2 + c^2)^(1/2) - (21*b^4)/4 + (35*c^4)/4 - (21*
b^2*c^2)/2))/(e*(4*tan(d/2 + (e*x)/2)^2 + 6*tan(d/2 + (e*x)/2)^4 + 4*tan(d/
2 + (e*x)/2)^6 + tan(d/2 + (e*x)/2)^8 + 1))
```

sympy [A] time = 3.04, size = 857, normalized size = 3.48

$$\begin{cases} \frac{3b^4x\sin^4(d+ex)}{8} + \frac{3b^4x\sin^2(d+ex)\cos^2(d+ex)}{4} + 3b^4x\sin^2(d+ex) + \frac{3b^4x\cos^4(d+ex)}{8} + 3b^4x\cos^2(d+ex) + b^4x + \frac{3b^4\sin^3(d\cdots}{} \\ x\left(b\cos(d) + c\sin(d) + \sqrt{b^2+c^2}\right)^4 \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(e*x+d)+c*sin(e*x+d)+(b**2+c**2)**(1/2))**4,x)
```

```
[Out] Piecewise((3*b**4*x*sin(d + e*x)**4/8 + 3*b**4*x*sin(d + e*x)**2*cos(d + e*
x)**2/4 + 3*b**4*x*sin(d + e*x)**2 + 3*b**4*x*cos(d + e*x)**4/8 + 3*b**4*x*
cos(d + e*x)**2 + b**4*x + 3*b**4*sin(d + e*x)**3*cos(d + e*x)/(8*e) + 5*b*
*4*sin(d + e*x)*cos(d + e*x)**3/(8*e) + 3*b**4*sin(d + e*x)*cos(d + e*x)/e
- b**3*c*cos(d + e*x)**4/e - 6*b**3*c*cos(d + e*x)**2/e + 8*b**3*sqrt(b**2
+ c**2)*sin(d + e*x)**3/(3*e) + 4*b**3*sqrt(b**2 + c**2)*sin(d + e*x)*cos(d
 + e*x)**2/e + 4*b**3*sqrt(b**2 + c**2)*sin(d + e*x)/e + 3*b**2*c**2*x*sin(
d + e*x)**4/4 + 3*b**2*c**2*x*sin(d + e*x)**2*cos(d + e*x)**2/2 + 6*b**2*c*
*2*x*sin(d + e*x)**2 + 3*b**2*c**2*x*cos(d + e*x)**4/4 + 6*b**2*c**2*x*cos(
d + e*x)**2 + 2*b**2*c**2*x + 3*b**2*c**2*sin(d + e*x)**3*cos(d + e*x)/(4*e
) - 3*b**2*c**2*sin(d + e*x)*cos(d + e*x)**3/(4*e) - 4*b**2*c*sqrt(b**2 + c
**2)*cos(d + e*x)**3/e - 4*b**2*c*sqrt(b**2 + c**2)*cos(d + e*x)/e + b*c**3
*sin(d + e*x)**4/e - 6*b*c**3*cos(d + e*x)**2/e + 4*b*c**2*sqrt(b**2 + c**2
)*sin(d + e*x)**3/e + 4*b*c**2*sqrt(b**2 + c**2)*sin(d + e*x)/e + 3*c**4*x*
sin(d + e*x)**4/8 + 3*c**4*x*sin(d + e*x)**2*cos(d + e*x)**2/4 + 3*c**4*x*s
in(d + e*x)**2 + 3*c**4*x*cos(d + e*x)**4/8 + 3*c**4*x*cos(d + e*x)**2 + c*
*4*x - 5*c**4*sin(d + e*x)**3*cos(d + e*x)/(8*e) - 3*c**4*sin(d + e*x)*cos(
d + e*x)**3/(8*e) - 3*c**4*sin(d + e*x)*cos(d + e*x)/e - 4*c**3*sqrt(b**2 +
 c**2)*sin(d + e*x)**2*cos(d + e*x)/e - 8*c**3*sqrt(b**2 + c**2)*cos(d + e*
x)**3/(3*e) - 4*c**3*sqrt(b**2 + c**2)*cos(d + e*x)/e, Ne(e, 0)), (x*(b*cos
(d) + c*sin(d) + sqrt(b**2 + c**2))**4, True))
```

3.356 $\int \left(\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)\right)^3\,dx$

Optimal. Leaf size=178

$$\frac{5b\left(b^2+c^2\right)\sin(d+ex)}{2e}-\frac{5c\left(b^2+c^2\right)\cos(d+ex)}{2e}-\frac{(c\cos(d+ex)-b\sin(d+ex))\left(\sqrt{b^2+c^2}+b\cos(d+ex)+c\right.}{3e}$$

```
[Out] 5/2*(b^2+c^2)^(3/2)*x-5/2*c*(b^2+c^2)*cos(e*x+d)/e+5/2*b*(b^2+c^2)*sin(e*x+
d)/e-5/6*(c*cos(e*x+d)-b*sin(e*x+d))*(b^2+c^2)^(1/2)*(b*cos(e*x+d)+c*sin(e*
x+d)+(b^2+c^2)^(1/2))/e-1/3*(c*cos(e*x+d)-b*sin(e*x+d))*(b*cos(e*x+d)+c*sin
(e*x+d)+(b^2+c^2)^(1/2))^2/e
```

Rubi [A] time = 0.10, antiderivative size = 178, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {3113, 2637, 2638}

$$\frac{5b\left(b^2+c^2\right)\sin(d+ex)}{2e}-\frac{5c\left(b^2+c^2\right)\cos(d+ex)}{2e}-\frac{(c\cos(d+ex)-b\sin(d+ex))\left(\sqrt{b^2+c^2}+b\cos(d+ex)+c\right.}{3e}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[b^2 + c^2] + b*Cos[d + e*x] + c*Sin[d + e*x])^3,x]
```

```
[Out] (5*(b^2 + c^2)^(3/2)*x)/2 - (5*c*(b^2 + c^2)*Cos[d + e*x])/(2*e) + (5*b*(b^
2 + c^2)*Sin[d + e*x])/(2*e) - (5*Sqrt[b^2 + c^2]*(c*Cos[d + e*x] - b*Sin[d
 + e*x])*(Sqrt[b^2 + c^2] + b*Cos[d + e*x] + c*Sin[d + e*x]))/(6*e) - ((c*C
os[d + e*x] - b*Sin[d + e*x])*(Sqrt[b^2 + c^2] + b*Cos[d + e*x] + c*Sin[d +
 e*x])^2)/(3*e)
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3113

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(n_), x_Symbol] :> -Simp[((c*Cos[d + e*x] - b*Sin[d + e*x])*(a + b*Cos[d +
e*x] + c*Sin[d + e*x])^(n - 1))/(e*n), x] + Dist[(a*(2*n - 1))/n, Int[(a +
```

```
b*Cos[d + e*x] + c*Sin[d + e*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e},
x] && EqQ[a^2 - b^2 - c^2, 0] && GtQ[n, 0]
```

Rubi steps

$$\int \left(\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)\right)^3 dx = -\frac{(c\cos(d+ex)-b\sin(d+ex))\left(\sqrt{b^2+c^2}+b\cos(d+ex)+\cdots\right.}{3e}$$

$$= -\frac{5\sqrt{b^2+c^2}\,(c\cos(d+ex)-b\sin(d+ex))\left(\sqrt{b^2+c^2}+b\cos\cdots\right.}{6e}$$

$$= \frac{5}{2}\left(b^2+c^2\right)^{3/2}x-\frac{5\sqrt{b^2+c^2}\,(c\cos(d+ex)-b\sin(d+ex))\left(\cdots\right.}{6\cdots}$$

$$= \frac{5}{2}\left(b^2+c^2\right)^{3/2}x-\frac{5c\left(b^2+c^2\right)\cos(d+ex)}{2e}+\frac{5b\left(b^2+c^2\right)\sin\cdots}{2e}$$

Mathematica [C] time = 0.60, size = 163, normalized size = 0.92

$$\frac{30(b-ic)(b+ic)\sqrt{b^2+c^2}\,(d+ex)+45b\left(b^2+c^2\right)\sin(d+ex)+9\left(b^2-c^2\right)\sqrt{b^2+c^2}\,\sin(2(d+ex))+b\left(b^2-3c^2\right)\cdots}{12e}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[b^2 + c^2] + b*Cos[d + e*x] + c*Sin[d + e*x])^3,x]
```

```
[Out] (30*(b - I*c)*(b + I*c)*Sqrt[b^2 + c^2]*(d + e*x) - 45*c*(b^2 + c^2)*Cos[d
+ e*x] - 18*b*c*Sqrt[b^2 + c^2]*Cos[2*(d + e*x)] + c*(-3*b^2 + c^2)*Cos[3*(
d + e*x)] + 45*b*(b^2 + c^2)*Sin[d + e*x] + 9*(b^2 - c^2)*Sqrt[b^2 + c^2]*S
in[2*(d + e*x)] + b*(b^2 - 3*c^2)*Sin[3*(d + e*x)])/(12*e)
```

fricas [A] time = 0.85, size = 145, normalized size = 0.81

$$-\frac{2\left(3\,b^2c-c^3\right)\cos\left(ex+d\right)^3+6\left(3\,b^2c+4\,c^3\right)\cos\left(ex+d\right)-2\left(11\,b^3+12\,bc^2+\left(b^3-3\,bc^2\right)\cos\left(ex+d\right)^2\right)\sin\left(ex\cdots\right.}{6\,e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(e*x+d)+c*sin(e*x+d)+(b^2+c^2)^(1/2))^3,x, algorithm="frica
s")
```

```
[Out] -1/6*(2*(3*b^2*c - c^3)*cos(e*x + d)^3 + 6*(3*b^2*c + 4*c^3)*cos(e*x + d) -
 2*(11*b^3 + 12*b*c^2 + (b^3 - 3*b*c^2)*cos(e*x + d)^2)*sin(e*x + d) + 3*(6
*b*c*cos(e*x + d)^2 - 5*(b^2 + c^2)*e*x - 3*(b^2 - c^2)*cos(e*x + d)*sin(e*
x + d))*sqrt(b^2 + c^2))/e
```

giac [A] time = 0.24, size = 199, normalized size = 1.12

$$-\frac{3}{2}\sqrt{b^2+c^2}\,bc\cos(2\,xe+2\,d)\,e^{(-1)}-\frac{1}{12}\left(3\,b^2c-c^3\right)\cos(3\,xe+3\,d)\,e^{(-1)}-\frac{15}{4}\left(b^2c+c^3\right)\cos(xe+d)\,e^{(-1)}+\frac{1}{12}\left(b^3\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(e*x+d)+c*sin(e*x+d)+(b^2+c^2)^(1/2))^3,x, algorithm="giac"
)
```

```
[Out] -3/2*sqrt(b^2 + c^2)*b*c*cos(2*x*e + 2*d)*e^(-1) - 1/12*(3*b^2*c - c^3)*cos
(3*x*e + 3*d)*e^(-1) - 15/4*(b^2*c + c^3)*cos(x*e + d)*e^(-1) + 1/12*(b^3 -
 3*b*c^2)*e^(-1)*sin(3*x*e + 3*d) + 3/4*(sqrt(b^2 + c^2)*b^2 - sqrt(b^2 + c
^2)*c^2)*e^(-1)*sin(2*x*e + 2*d) + 15/4*(b^3 + b*c^2)*e^(-1)*sin(x*e + d) +
 (b^2 + c^2)^(3/2)*x + 3/2*(sqrt(b^2 + c^2)*b^2 + sqrt(b^2 + c^2)*c^2)*x
```

maple [A] time = 0.25, size = 250, normalized size = 1.40

$$\frac{\frac{b^3\left(2+\cos^2(ex+d)\right)\sin(ex+d)}{3}-\left(\cos^3(ex+d)\right)b^2c+3\sqrt{b^2+c^2}\,b^2\left(\frac{\sin(ex+d)\cos(ex+d)}{2}+\frac{ex}{2}+\frac{d}{2}\right)+c^2b\left(\sin^3(ex+d)\right)-3\cdots}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*cos(e*x+d)+c*sin(e*x+d)+(b^2+c^2)^(1/2))^3,x)
```

```
[Out] 1/e*(1/3*b^3*(2+cos(e*x+d)^2)*sin(e*x+d)-cos(e*x+d)^3*b^2*c+3*(b^2+c^2)^(1/
2)*b^2*(1/2*sin(e*x+d)*cos(e*x+d)+1/2*e*x+1/2*d)+c^2*b*sin(e*x+d)^3-3*(b^2+
c^2)^(1/2)*b*c*cos(e*x+d)^2+3*sin(e*x+d)*b^3+3*c^2*b*sin(e*x+d)-1/3*c^3*(2+
sin(e*x+d)^2)*cos(e*x+d)+3*(b^2+c^2)^(1/2)*c^2*(-1/2*sin(e*x+d)*cos(e*x+d)+
1/2*e*x+1/2*d)-3*cos(e*x+d)*b^2*c-3*cos(e*x+d)*c^3+(b^2+c^2)^(1/2)*b^2*(e*x
+d)+(b^2+c^2)^(1/2)*c^2*(e*x+d))
```

maxima [A] time = 0.32, size = 207, normalized size = 1.16

$$-\frac{b^2c\cos(ex+d)^3}{e}+\frac{bc^2\sin(ex+d)^3}{e}-\frac{\left(\sin(ex+d)^3-3\,\sin(ex+d)\right)b^3}{3\,e}+\frac{\left(\cos(ex+d)^3-3\,\cos(ex+d)\right)c^3}{3\,e}+\left(b^3\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(e*x+d)+c*sin(e*x+d)+(b^2+c^2)^(1/2))^3,x, algorithm="maxim
a")
```

```
[Out] -b^2*c*cos(e*x + d)^3/e + b*c^2*sin(e*x + d)^3/e - 1/3*(sin(e*x + d)^3 - 3*
sin(e*x + d))*b^3/e + 1/3*(cos(e*x + d)^3 - 3*cos(e*x + d))*c^3/e + (b^2 +
c^2)^(3/2)*x - 3*(b^2 + c^2)*(c*cos(e*x + d)/e - b*sin(e*x + d)/e) - 3/4*(4
*b*c*cos(e*x + d)^2/e - (2*e*x + 2*d + sin(2*e*x + 2*d))*b^2/e - (2*e*x + 2
*d - sin(2*e*x + 2*d))*c^2/e)*sqrt(b^2 + c^2)
```

mupad [B] time = 7.32, size = 261, normalized size = 1.47

$$\frac{5\,x\,{\left(b^2+c^2\right)}^{3/2}}{2}-\frac{8\,b^2\,c-\tan\left(\frac{d}{2}+\frac{e\,x}{2}\right)\,\left(\left(3\,b^2-3\,c^2\right)\,\sqrt{b^2+c^2}+6\,b\,c^2+8\,b^3\right)-{\tan\left(\frac{d}{2}+\frac{e\,x}{2}\right)}^3\,\left(\frac{40\,b^3}{3}+20\,b\,c^2\right)+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*cos(d + e*x) + c*sin(d + e*x) + (b^2 + c^2)^(1/2))^3,x)
```

```
[Out] (5*x*(b^2 + c^2)^(3/2))/2 - (8*b^2*c - tan(d/2 + (e*x)/2)*((3*b^2 - 3*c^2)*
(b^2 + c^2)^(1/2) + 6*b*c^2 + 8*b^3) - tan(d/2 + (e*x)/2)^3*(20*b*c^2 + (40
*b^3)/3) + (22*c^3)/3 - tan(d/2 + (e*x)/2)^5*(6*b*c^2 - (3*b^2 - 3*c^2)*(b^
2 + c^2)^(1/2) + 8*b^3) + tan(d/2 + (e*x)/2)^4*(12*b^2*c + 6*c^3 - 12*b*c*(
b^2 + c^2)^(1/2)) + tan(d/2 + (e*x)/2)^2*(12*b^2*c + 16*c^3 - 12*b*c*(b^2 +
 c^2)^(1/2)))/(e*(3*tan(d/2 + (e*x)/2)^2 + 3*tan(d/2 + (e*x)/2)^4 + tan(d/2
 + (e*x)/2)^6 + 1))
```

sympy [A] time = 1.37, size = 415, normalized size = 2.33

$$\begin{cases} \frac{2b^3\sin^3(d+ex)}{3e}+\frac{b^3\sin(d+ex)\cos^2(d+ex)}{e}+\frac{3b^3\sin(d+ex)}{e}-\frac{b^2c\cos^3(d+ex)}{e}-\frac{3b^2c\cos(d+ex)}{e}+\frac{3b^2x\sqrt{b^2+c^2}\sin^2(d+ex)}{2}+\frac{3b^2x\sqrt{b^2+c^2}}{2} \\ x\left(b\cos(d)+c\sin(d)+\sqrt{b^2+c^2}\right)^3 \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(e*x+d)+c*sin(e*x+d)+(b**2+c**2)**(1/2))**3,x)
```

```
[Out] Piecewise((2*b**3*sin(d + e*x)**3/(3*e) + b**3*sin(d + e*x)*cos(d + e*x)**2
/e + 3*b**3*sin(d + e*x)/e - b**2*c*cos(d + e*x)**3/e - 3*b**2*c*cos(d + e*
x)/e + 3*b**2*x*sqrt(b**2 + c**2)*sin(d + e*x)**2/2 + 3*b**2*x*sqrt(b**2 +
c**2)*cos(d + e*x)**2/2 + b**2*x*sqrt(b**2 + c**2) + 3*b**2*sqrt(b**2 + c**
2)*sin(d + e*x)*cos(d + e*x)/(2*e) + b*c**2*sin(d + e*x)**3/e + 3*b*c**2*si
n(d + e*x)/e - 3*b*c*sqrt(b**2 + c**2)*cos(d + e*x)**2/e - c**3*sin(d + e*x
)**2*cos(d + e*x)/e - 2*c**3*cos(d + e*x)**3/(3*e) - 3*c**3*cos(d + e*x)/e
+ 3*c**2*x*sqrt(b**2 + c**2)*sin(d + e*x)**2/2 + 3*c**2*x*sqrt(b**2 + c**2)
*cos(d + e*x)**2/2 + c**2*x*sqrt(b**2 + c**2) - 3*c**2*sqrt(b**2 + c**2)*si
n(d + e*x)*cos(d + e*x)/(2*e), Ne(e, 0)), (x*(b*cos(d) + c*sin(d) + sqrt(b*
*2 + c**2))**3, True))
```

3.357 $\int \left(\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)\right)^2\,dx$

Optimal. Leaf size=116

$$\frac{3b\sqrt{b^2+c^2}\,\sin(d+ex)}{2e}-\frac{3c\sqrt{b^2+c^2}\,\cos(d+ex)}{2e}-\frac{(c\cos(d+ex)-b\sin(d+ex))\left(\sqrt{b^2+c^2}+b\cos(d+ex)+c\right.}{2e}$$

```
[Out] 3/2*(b^2+c^2)*x-3/2*c*cos(e*x+d)*(b^2+c^2)^(1/2)/e+3/2*b*sin(e*x+d)*(b^2+c^
2)^(1/2)/e-1/2*(c*cos(e*x+d)-b*sin(e*x+d))*(b*cos(e*x+d)+c*sin(e*x+d)+(b^2+
c^2)^(1/2))/e
```

Rubi [A] time = 0.06, antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {3113, 2637, 2638}

$$\frac{3b\sqrt{b^2+c^2}\,\sin(d+ex)}{2e}-\frac{3c\sqrt{b^2+c^2}\,\cos(d+ex)}{2e}-\frac{(c\cos(d+ex)-b\sin(d+ex))\left(\sqrt{b^2+c^2}+b\cos(d+ex)+c\right.}{2e}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[b^2 + c^2] + b*Cos[d + e*x] + c*Sin[d + e*x])^2,x]
```

```
[Out] (3*(b^2 + c^2)*x)/2 - (3*c*Sqrt[b^2 + c^2]*Cos[d + e*x])/(2*e) + (3*b*Sqrt[
b^2 + c^2]*Sin[d + e*x])/(2*e) - ((c*Cos[d + e*x] - b*Sin[d + e*x])*(Sqrt[b
^2 + c^2] + b*Cos[d + e*x] + c*Sin[d + e*x]))/(2*e)
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3113

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(n_), x_Symbol] :> -Simp[((c*Cos[d + e*x] - b*Sin[d + e*x])*(a + b*Cos[d +
e*x] + c*Sin[d + e*x])^(n - 1))/(e*n), x] + Dist[(a*(2*n - 1))/n, Int[(a +
b*Cos[d + e*x] + c*Sin[d + e*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e},
x] && EqQ[a^2 - b^2 - c^2, 0] && GtQ[n, 0]
```

Rubi steps

$$\int \left(\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)\right)^2 dx = -\frac{(c\cos(d+ex)-b\sin(d+ex))\left(\sqrt{b^2+c^2}+b\cos(d+ex)+\right.}{2e}$$

$$=\frac{3}{2}\left(b^2+c^2\right)x-\frac{(c\cos(d+ex)-b\sin(d+ex))\left(\sqrt{b^2+c^2}+b\right.}{2e}$$

$$=\frac{3}{2}\left(b^2+c^2\right)x-\frac{3c\sqrt{b^2+c^2}\cos(d+ex)}{2e}+\frac{3b\sqrt{b^2+c^2}\sin(d}{2e}$$

Mathematica [A] time = 0.21, size = 111, normalized size = 0.96

$$\frac{8b\sqrt{b^2+c^2}\sin(d+ex)-8c\sqrt{b^2+c^2}\cos(d+ex)+b^2\sin(2(d+ex))+6b^2d+6b^2ex-2bc\cos(2(d+ex))-c^2\sin}{4e}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[b^2 + c^2] + b*Cos[d + e*x] + c*Sin[d + e*x])^2,x]
```

```
[Out] (6*b^2*d + 6*c^2*d + 6*b^2*e*x + 6*c^2*e*x - 8*c*Sqrt[b^2 + c^2]*Cos[d + e*
x] - 2*b*c*Cos[2*(d + e*x)] + 8*b*Sqrt[b^2 + c^2]*Sin[d + e*x] + b^2*Sin[2*
(d + e*x)] - c^2*Sin[2*(d + e*x)])/(4*e)
```

fricas [A] time = 0.94, size = 81, normalized size = 0.70

$$-\frac{2\,bc\cos\left(ex+d\right)^2-3\left(b^2+c^2\right)ex-\left(b^2-c^2\right)\cos\left(ex+d\right)\sin\left(ex+d\right)+4\sqrt{b^2+c^2}\left(c\cos\left(ex+d\right)-b\sin\left(ex+d\right.\right.}{2\,e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(e*x+d)+c*sin(e*x+d)+(b^2+c^2)^(1/2))^2,x, algorithm="frica
s")
```

```
[Out] -1/2*(2*b*c*cos(e*x + d)^2 - 3*(b^2 + c^2)*e*x - (b^2 - c^2)*cos(e*x + d)*s
in(e*x + d) + 4*sqrt(b^2 + c^2)*(c*cos(e*x + d) - b*sin(e*x + d)))/e
```

giac [A] time = 0.17, size = 92, normalized size = 0.79

$$-\frac{1}{2}\,bc\cos\left(2\,xe+2\,d\right)e^{(-1)}-2\,\sqrt{b^2+c^2}\,c\cos\left(xe+d\right)e^{(-1)}+2\,\sqrt{b^2+c^2}\,be^{(-1)}\sin\left(xe+d\right)+\frac{1}{4}\left(b^2-c^2\right)e^{(-1)}\sin\left(2\,xe\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(e*x+d)+c*sin(e*x+d)+(b^2+c^2)^(1/2))^2,x, algorithm="giac"
)
```

```
[Out] -1/2*b*c*cos(2*x*e + 2*d)*e^(-1) - 2*sqrt(b^2 + c^2)*c*cos(x*e + d)*e^(-1)
+ 2*sqrt(b^2 + c^2)*b*e^(-1)*sin(x*e + d) + 1/4*(b^2 - c^2)*e^(-1)*sin(2*x*
e + 2*d) + 3/2*(b^2 + c^2)*x
```

maple [A] time = 0.22, size = 124, normalized size = 1.07

$$\frac{b^2\left(\frac{\sin(ex+d)\cos(ex+d)}{2}+\frac{ex}{2}+\frac{d}{2}\right)-\left(\cos^2(ex+d)\right)bc+c^2\left(-\frac{\sin(ex+d)\cos(ex+d)}{2}+\frac{ex}{2}+\frac{d}{2}\right)+2\sqrt{b^2+c^2}\,b\sin(ex+d)-}{e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*cos(e*x+d)+c*sin(e*x+d)+(b^2+c^2)^(1/2))^2,x)
```

```
[Out] 1/e*(b^2*(1/2*sin(e*x+d)*cos(e*x+d)+1/2*e*x+1/2*d)-cos(e*x+d)^2*b*c+c^2*(-1
/2*sin(e*x+d)*cos(e*x+d)+1/2*e*x+1/2*d)+2*(b^2+c^2)^(1/2)*b*sin(e*x+d)-2*(b
^2+c^2)^(1/2)*c*cos(e*x+d)+b^2*(e*x+d)+c^2*(e*x+d))
```

maxima [A] time = 0.31, size = 113, normalized size = 0.97

$$b^2x+c^2x-\frac{bc\cos(ex+d)^2}{e}+\frac{(2\,ex+2\,d+\sin(2\,ex+2\,d))b^2}{4\,e}+\frac{(2\,ex+2\,d-\sin(2\,ex+2\,d))c^2}{4\,e}-2\,\sqrt{b^2+c^2}\left(\frac{c\cos}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(e*x+d)+c*sin(e*x+d)+(b^2+c^2)^(1/2))^2,x, algorithm="maxim
a")
```

```
[Out] b^2*x + c^2*x - b*c*cos(e*x + d)^2/e + 1/4*(2*e*x + 2*d + sin(2*e*x + 2*d))
*b^2/e + 1/4*(2*e*x + 2*d - sin(2*e*x + 2*d))*c^2/e - 2*sqrt(b^2 + c^2)*(c*
cos(e*x + d)/e - b*sin(e*x + d)/e)
```

mupad [B] time = 3.08, size = 100, normalized size = 0.86

$$\frac{b^2\,\sin(2\,d+2\,e\,x)-c^2\,\sin(2\,d+2\,e\,x)+16\,c\sin\left(\frac{d}{2}+\frac{e\,x}{2}\right)^2\,\sqrt{b^2+c^2}\,+8\,b\,\sin(d+e\,x)\,\sqrt{b^2+c^2}\,+4\,b\,c\sin(d}{4\,e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*cos(d + e*x) + c*sin(d + e*x) + (b^2 + c^2)^(1/2))^2,x)
```

```
[Out] (b^2*sin(2*d + 2*e*x) - c^2*sin(2*d + 2*e*x) + 16*c*sin(d/2 + (e*x)/2)^2*(b
^2 + c^2)^(1/2) + 8*b*sin(d + e*x)*(b^2 + c^2)^(1/2) + 4*b*c*sin(d + e*x)^2
 + 6*b^2*e*x + 6*c^2*e*x)/(4*e)
```

sympy [A] time = 0.45, size = 192, normalized size = 1.66

$$\begin{cases} \frac{b^2 x \sin^2(d+ex)}{2} + \frac{b^2 x \cos^2(d+ex)}{2} + b^2 x + \frac{b^2 \sin(d+ex)\cos(d+ex)}{2e} - \frac{bc\cos^2(d+ex)}{e} + \frac{2b\sqrt{b^2+c^2}\sin(d+ex)}{e} + \frac{c^2 x \sin^2(d+ex)}{2} + \frac{c^2 x \cos^2(d+ex)}{2} \\ x\left(b\cos(d) + c\sin(d) + \sqrt{b^2+c^2}\right)^2 \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(e*x+d)+c*sin(e*x+d)+(b**2+c**2)**(1/2))**2,x)
```

```
[Out] Piecewise((b**2*x*sin(d + e*x)**2/2 + b**2*x*cos(d + e*x)**2/2 + b**2*x + b
**2*sin(d + e*x)*cos(d + e*x)/(2*e) - b*c*cos(d + e*x)**2/e + 2*b*sqrt(b**2
 + c**2)*sin(d + e*x)/e + c**2*x*sin(d + e*x)**2/2 + c**2*x*cos(d + e*x)**2
/2 + c**2*x - c**2*sin(d + e*x)*cos(d + e*x)/(2*e) - 2*c*sqrt(b**2 + c**2)*
cos(d + e*x)/e, Ne(e, 0)), (x*(b*cos(d) + c*sin(d) + sqrt(b**2 + c**2))**2,
 True))
```

3.358 $\int \left(\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)\right)dx$

Optimal. Leaf size=37

$$x\sqrt{b^2+c^2}+\frac{b\sin(d+ex)}{e}-\frac{c\cos(d+ex)}{e}$$

```
[Out] -c*cos(e*x+d)/e+b*sin(e*x+d)/e+x*(b^2+c^2)^(1/2)
```

Rubi [A] time = 0.02, antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {2637, 2638}

$$x\sqrt{b^2+c^2}+\frac{b\sin(d+ex)}{e}-\frac{c\cos(d+ex)}{e}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[b^2 + c^2] + b*Cos[d + e*x] + c*Sin[d + e*x],x]
[Out] Sqrt[b^2 + c^2]*x - (c*Cos[d + e*x])/e + (b*Sin[d + e*x])/e
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \left(\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)\right)dx &= \sqrt{b^2+c^2}\,x+b\int \cos(d+ex)\,dx+c\int \sin(d+ex)\,dx\\ &= \sqrt{b^2+c^2}\,x-\frac{c\cos(d+ex)}{e}+\frac{b\sin(d+ex)}{e}\end{aligned}$$

Mathematica [A] time = 0.03, size = 36, normalized size = 0.97

$$\frac{ex\sqrt{b^2+c^2}+b\sin(d+ex)-c\cos(d+ex)}{e}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[b^2 + c^2] + b*Cos[d + e*x] + c*Sin[d + e*x],x]
```

```
[Out] (Sqrt[b^2 + c^2]*e*x - c*Cos[d + e*x] + b*Sin[d + e*x])/e
```

fricas [A] time = 0.92, size = 34, normalized size = 0.92

$$\frac{\sqrt{b^2 + c^2}\, ex - c\cos(ex + d) + b\sin(ex + d)}{e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(b*cos(e*x+d)+c*sin(e*x+d)+(b^2+c^2)^(1/2),x, algorithm="fricas")
```

```
[Out] (sqrt(b^2 + c^2)*e*x - c*cos(e*x + d) + b*sin(e*x + d))/e
```

giac [A] time = 0.12, size = 35, normalized size = 0.95

$$-c\cos(xe + d)\, e^{(-1)} + be^{(-1)}\sin(xe + d) + \sqrt{b^2 + c^2}\, x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(b*cos(e*x+d)+c*sin(e*x+d)+(b^2+c^2)^(1/2),x, algorithm="giac")
```

```
[Out] -c*cos(x*e + d)*e^(-1) + b*e^(-1)*sin(x*e + d) + sqrt(b^2 + c^2)*x
```

maple [A] time = 0.04, size = 36, normalized size = 0.97

$$-\frac{c\cos(ex + d)}{e} + \frac{b\sin(ex + d)}{e} + x\sqrt{b^2 + c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(b*cos(e*x+d)+c*sin(e*x+d)+(b^2+c^2)^(1/2),x)
```

```
[Out] -c*cos(e*x+d)/e+b*sin(e*x+d)/e+x*(b^2+c^2)^(1/2)
```

maxima [A] time = 0.31, size = 35, normalized size = 0.95

$$\sqrt{b^2 + c^2}\, x - \frac{c\cos(ex + d)}{e} + \frac{b\sin(ex + d)}{e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(b*cos(e*x+d)+c*sin(e*x+d)+(b^2+c^2)^(1/2),x, algorithm="maxima")
```

```
[Out] sqrt(b^2 + c^2)*x - c*cos(e*x + d)/e + b*sin(e*x + d)/e
```

mupad [B] time = 2.67, size = 48, normalized size = 1.30

$$x\sqrt{b^2+c^2}-\frac{2\,c-2\,b\tan\left(\frac{d}{2}+\frac{e\,x}{2}\right)}{e\,\left({\tan\left(\frac{d}{2}+\frac{e\,x}{2}\right)}^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(b*cos(d + e*x) + c*sin(d + e*x) + (b^2 + c^2)^(1/2),x)
```

```
[Out] x*(b^2 + c^2)^(1/2) - (2*c - 2*b*tan(d/2 + (e*x)/2))/(e*(tan(d/2 + (e*x)/2)
^2 + 1))
```

sympy [A] time = 0.14, size = 42, normalized size = 1.14

$$b\left(\begin{cases}\frac{\sin(d+ex)}{e} & \text{for}\ e\neq 0\\ x\cos(d) & \text{otherwise}\end{cases}\right)+c\left(\begin{cases}-\frac{\cos(d+ex)}{e} & \text{for}\ e\neq 0\\ x\sin(d) & \text{otherwise}\end{cases}\right)+x\sqrt{b^2+c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(b*cos(e*x+d)+c*sin(e*x+d)+(b**2+c**2)**(1/2),x)
```

```
[Out] b*Piecewise((sin(d + e*x)/e, Ne(e, 0)), (x*cos(d), True)) + c*Piecewise((-c
os(d + e*x)/e, Ne(e, 0)), (x*sin(d), True)) + x*sqrt(b**2 + c**2)
```

3.359 $\int \frac{1}{\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)}\,dx$

Optimal. Leaf size=49

$$-\frac{c-\sqrt{b^2+c^2}\,\sin(d+ex)}{ce(c\cos(d+ex)-b\sin(d+ex))}$$

```
[Out] (-c+sin(e*x+d)*(b^2+c^2)^(1/2))/c/e/(c*cos(e*x+d)-b*sin(e*x+d))
```

Rubi [A] time = 0.04, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.033, Rules used = {3114}

$$-\frac{c-\sqrt{b^2+c^2}\,\sin(d+ex)}{ce(c\cos(d+ex)-b\sin(d+ex))}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[b^2 + c^2] + b*Cos[d + e*x] + c*Sin[d + e*x])^(-1),x]
```

```
[Out] -((c - Sqrt[b^2 + c^2]*Sin[d + e*x])/(c*e*(c*Cos[d + e*x] - b*Sin[d + e*x])
))
```

Rule 3114

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(-1), x_Symbol] :> -Simp[(c - a*Sin[d + e*x])/(c*e*(c*Cos[d + e*x] - b*Sin[
d + e*x])), x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[a^2 - b^2 - c^2, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)}\,dx=-\frac{c-\sqrt{b^2+c^2}\,\sin(d+ex)}{ce(c\cos(d+ex)-b\sin(d+ex))}$$

Mathematica [A] time = 0.10, size = 49, normalized size = 1.00

$$\frac{\sqrt{b^2+c^2}\,\sin(d+ex)-c}{ce(c\cos(d+ex)-b\sin(d+ex))}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[b^2 + c^2] + b*Cos[d + e*x] + c*Sin[d + e*x])^(-1),x]
```

```
[Out] (-c + Sqrt[b^2 + c^2]*Sin[d + e*x])/(c*e*(c*Cos[d + e*x] - b*Sin[d + e*x]))
```

fricas [A] time = 1.64, size = 75, normalized size = 1.53

$$-\frac{b^2+c^2-\sqrt{b^2+c^2}\,(b\cos{(ex+d)}+c\sin{(ex+d)})}{(b^2c+c^3)e\cos{(ex+d)}-(b^3+bc^2)e\sin{(ex+d)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*cos(e*x+d)+c*sin(e*x+d)+(b^2+c^2)^(1/2)),x, algorithm="frica
s")
```

```
[Out] -(b^2 + c^2 - sqrt(b^2 + c^2)*(b*cos(e*x + d) + c*sin(e*x + d)))/((b^2*c +
c^3)*e*cos(e*x + d) - (b^3 + b*c^2)*e*sin(e*x + d))
```

giac [A] time = 0.16, size = 43, normalized size = 0.88

$$-\frac{2\left(b+\sqrt{b^2+c^2}\right)e^{(-1)}}{\left(c\tan\left(\frac{1}{2}\,xe+\frac{1}{2}\,d\right)+b+\sqrt{b^2+c^2}\right)c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*cos(e*x+d)+c*sin(e*x+d)+(b^2+c^2)^(1/2)),x, algorithm="giac"
)
```

```
[Out] -2*(b + sqrt(b^2 + c^2))*e^(-1)/((c*tan(1/2*x*e + 1/2*d) + b + sqrt(b^2 + c
^2))*c)
```

maple [A] time = 0.29, size = 50, normalized size = 1.02

$$-\frac{2\left(\sqrt{b^2+c^2}+b\right)}{e\,c^2\left(\tan\left(\frac{d}{2}+\frac{ex}{2}\right)+\frac{\sqrt{b^2+c^2}}{c}+\frac{b}{c}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*cos(e*x+d)+c*sin(e*x+d)+(b^2+c^2)^(1/2)),x)
```

```
[Out] -2/e*((b^2+c^2)^(1/2)+b)/c^2/(tan(1/2*d+1/2*e*x)+1/c*(b^2+c^2)^(1/2)+b/c)
```

maxima [A] time = 0.31, size = 40, normalized size = 0.82

$$-\frac{2}{\left(c-\frac{\left(b-\sqrt{b^2+c^2}\right)\sin(ex+d)}{\cos(ex+d)+1}\right)e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*cos(e*x+d)+c*sin(e*x+d)+(b^2+c^2)^(1/2)),x, algorithm="maxim
a")
```

```
[Out] -2/((c - (b - sqrt(b^2 + c^2))*sin(e*x + d)/(cos(e*x + d) + 1))*e)
```

mupad [B] time = 2.82, size = 38, normalized size = 0.78

$$\frac{2\tan\left(\frac{d}{2}+\frac{e\,x}{2}\right)}{e\,\left(b+\sqrt{b^2+c^2}+c\,\tan\left(\frac{d}{2}+\frac{e\,x}{2}\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*cos(d + e*x) + c*sin(d + e*x) + (b^2 + c^2)^(1/2)),x)
```

```
[Out] (2*tan(d/2 + (e*x)/2))/(e*(b + (b^2 + c^2)^(1/2) + c*tan(d/2 + (e*x)/2)))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*cos(e*x+d)+c*sin(e*x+d)+(b**2+c**2)**(1/2)),x)
```

```
[Out] Timed out
```

3.360 $\int \frac{1}{\left(\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)\right)^2}\,dx$

Optimal. Leaf size=129

$$\frac{b\sin(d+ex)-c\cos(d+ex)}{3e\sqrt{b^2+c^2}\left(\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)\right)^2}-\frac{c-\sqrt{b^2+c^2}\,\sin(d+ex)}{3ce\sqrt{b^2+c^2}\,(c\cos(d+ex)-b\sin(d+ex))}$$

```
[Out] 1/3*(-c*cos(e*x+d)+b*sin(e*x+d))/e/(b^2+c^2)^(1/2)/(b*cos(e*x+d)+c*sin(e*x+
d)+(b^2+c^2)^(1/2))^2+1/3*(-c+sin(e*x+d)*(b^2+c^2)^(1/2))/c/e/(c*cos(e*x+d)
-b*sin(e*x+d))/(b^2+c^2)^(1/2)
```

Rubi [A] time = 0.09, antiderivative size = 129, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {3116, 3114}

$$-\frac{c\cos(d+ex)-b\sin(d+ex)}{3e\sqrt{b^2+c^2}\left(\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)\right)^2}-\frac{c-\sqrt{b^2+c^2}\,\sin(d+ex)}{3ce\sqrt{b^2+c^2}\,(c\cos(d+ex)-b\sin(d+ex))}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[b^2 + c^2] + b*Cos[d + e*x] + c*Sin[d + e*x])^(-2),x]
```

```
[Out] -(c*Cos[d + e*x] - b*Sin[d + e*x])/(3*Sqrt[b^2 + c^2]*e*(Sqrt[b^2 + c^2] +
b*Cos[d + e*x] + c*Sin[d + e*x])^2) - (c - Sqrt[b^2 + c^2]*Sin[d + e*x])/(3
*c*Sqrt[b^2 + c^2]*e*(c*Cos[d + e*x] - b*Sin[d + e*x]))
```

Rule 3114

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(-1), x_Symbol] :> -Simp[(c - a*Sin[d + e*x])/(c*e*(c*Cos[d + e*x] - b*Sin[
d + e*x])), x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[a^2 - b^2 - c^2, 0]
```

Rule 3116

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(n_), x_Symbol] :> Simp[((c*Cos[d + e*x] - b*Sin[d + e*x])*(a + b*Cos[d + e
*x] + c*Sin[d + e*x])^n)/(a*e*(2*n + 1)), x] + Dist[(n + 1)/(a*(2*n + 1)),
Int[(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n + 1), x], x] /; FreeQ[{a, b, c
, d, e}, x] && EqQ[a^2 - b^2 - c^2, 0] && LtQ[n, -1]
```

Rubi steps

$$\int \frac{1}{\left(\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)\right)^2}\,dx = -\frac{c\cos(d+ex)-b\sin(d+ex)}{3\sqrt{b^2+c^2}\,e\left(\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)\right)^2}+$$

$$= -\frac{c\cos(d+ex)-b\sin(d+ex)}{3\sqrt{b^2+c^2}\,e\left(\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)\right)^2}-$$

Mathematica [A] time = 0.25, size = 98, normalized size = 0.76

$$\frac{-2c\sqrt{b^2+c^2}+b^2\sin^3(d+ex)+2bc\cos^3(d+ex)+2c^2\sin(d+ex)+c^2\sin(d+ex)\cos^2(d+ex)}{3ce(c\cos(d+ex)-b\sin(d+ex))^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[b^2 + c^2] + b*Cos[d + e*x] + c*Sin[d + e*x])^(-2),x]
```

```
[Out] (-2*c*Sqrt[b^2 + c^2] + 2*b*c*Cos[d + e*x]^3 + 2*c^2*Sin[d + e*x] + c^2*Cos
[d + e*x]^2*Sin[d + e*x] + b^2*Sin[d + e*x]^3)/(3*c*e*(c*Cos[d + e*x] - b*S
in[d + e*x])^3)
```

fricas [A] time = 1.74, size = 192, normalized size = 1.49

$$-\frac{3\,b^3\cos\left(ex+d\right)-\left(b^3-3\,bc^2\right)\cos\left(ex+d\right)^3+\left(3\,b^2c+2\,c^3-\left(3\,b^2c-c^3\right)\cos\left(ex+d\right)^2\right)\sin\left(ex+d\right)-}{3\left(\left(3\,b^4c+2\,b^2c^3-c^5\right)e\cos\left(ex+d\right)^3-3\left(b^4c+b^2c^3\right)e\cos\left(ex+d\right)-\left(\left(b^5-2\,b^3c^2-3\,bc^4\right)e\cos\left(ex+d\right)^2-\left(b^5\right.\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*cos(e*x+d)+c*sin(e*x+d)+(b^2+c^2)^(1/2))^2,x, algorithm="fri
cas")
```

```
[Out] -1/3*(3*b^3*cos(e*x + d) - (b^3 - 3*b*c^2)*cos(e*x + d)^3 + (3*b^2*c + 2*c^
3 - (3*b^2*c - c^3)*cos(e*x + d)^2)*sin(e*x + d) - 2*(b^2 + c^2)^(3/2))/((3
*b^4*c + 2*b^2*c^3 - c^5)*e*cos(e*x + d)^3 - 3*(b^4*c + b^2*c^3)*e*cos(e*x
+ d) - ((b^5 - 2*b^3*c^2 - 3*b*c^4)*e*cos(e*x + d)^2 - (b^5 + b^3*c^2)*e)*s
in(e*x + d))
```

giac [A] time = 0.20, size = 160, normalized size = 1.24

$$-\frac{2\left(8\,b^4+10\,b^2c^2+2\,c^4+3\left(2\,b^2c^2+c^4+2\,\sqrt{b^2+c^2}\,bc^2\right)\tan\left(\frac{1}{2}\,xe+\frac{1}{2}\,d\right)^2+3\left(4\,b^3c+3\,bc^3+\left(4\,b^2c+c^3\right)\sqrt{b^2}\right.\right.}{3\left(c\tan\left(\frac{1}{2}\,xe+\frac{1}{2}\,d\right)+b+\sqrt{b^2+c^2}\right)^3c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*cos(e*x+d)+c*sin(e*x+d)+(b^2+c^2)^(1/2))^2,x, algorithm="gia
c")
```

```
[Out] -2/3*(8*b^4 + 10*b^2*c^2 + 2*c^4 + 3*(2*b^2*c^2 + c^4 + 2*sqrt(b^2 + c^2)*b
*c^2)*tan(1/2*x*e + 1/2*d)^2 + 3*(4*b^3*c + 3*b*c^3 + (4*b^2*c + c^3)*sqrt(
b^2 + c^2))*tan(1/2*x*e + 1/2*d) + 2*(4*b^3 + 3*b*c^2)*sqrt(b^2 + c^2))*e^(
-1)/((c*tan(1/2*x*e + 1/2*d) + b + sqrt(b^2 + c^2))^3*c^3)
```

maple [A] time = 0.40, size = 233, normalized size = 1.81

$$\frac{2\left(\sqrt{b^2+c^2}+b\right)\left(-\frac{\left(\sqrt{b^2+c^2}+b\right)\left(\tan^2\left(\frac{d}{2}+\frac{ex}{2}\right)\right)}{c^2}-\frac{\left(2b^2+c^2+2\sqrt{b^2+c^2}\,b\right)\tan\left(\frac{d}{2}+\frac{ex}{2}\right)}{c^3}-\frac{2\left(2\sqrt{b^2+c^2}\,b^2+\sqrt{b^2+c^2}\,c^2+2b^3+2c^2b\right)}{3c^4}\right)}{e\,c^2\left(\tan^2\left(\frac{d}{2}+\frac{ex}{2}\right)+\frac{2\sqrt{b^2+c^2}\,\tan\left(\frac{d}{2}+\frac{ex}{2}\right)}{c}+\frac{2b\tan\left(\frac{d}{2}+\frac{ex}{2}\right)}{c}+\frac{2\sqrt{b^2+c^2}\,b}{c^2}+\frac{2b^2}{c^2}+1\right)\left(\tan\left(\frac{d}{2}+\frac{ex}{2}\right)+\frac{\sqrt{b^2+c^2}}{c}+\frac{b}{c}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*cos(e*x+d)+c*sin(e*x+d)+(b^2+c^2)^(1/2))^2,x)
```

```
[Out] 2/e*((b^2+c^2)^(1/2)+b)/c^2*(-((b^2+c^2)^(1/2)+b)/c^2*tan(1/2*d+1/2*e*x)^2-
1/c^3*(2*b^2+c^2+2*(b^2+c^2)^(1/2)*b)*tan(1/2*d+1/2*e*x)-2/3*(2*(b^2+c^2)^(
1/2)*b^2+(b^2+c^2)^(1/2)*c^2+2*b^3+2*c^2*b)/c^4)/(tan(1/2*d+1/2*e*x)^2+2/c*
(b^2+c^2)^(1/2)*tan(1/2*d+1/2*e*x)+2*b/c*tan(1/2*d+1/2*e*x)+2/c^2*(b^2+c^2)
^(1/2)*b+2/c^2*b^2+1)/(tan(1/2*d+1/2*e*x)+1/c*(b^2+c^2)^(1/2)+b/c)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*cos(e*x+d)+c*sin(e*x+d)+(b^2+c^2)^(1/2))^2,x, algorithm="max
ima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is und
efined.
```

mupad [B] time = 3.67, size = 274, normalized size = 2.12

$$-\frac{\tan\left(\frac{d}{2}+\frac{e\,x}{2}\right)^2\left(\frac{4\,b^2+2\,c^2}{c^4}+\frac{4\,b\,\sqrt{b^2+c^2}}{c^4}\right)+\frac{\frac{16\,b^4}{3}+\frac{20\,b^2\,c^2}{3}+\frac{4\,c^4}{3}}{c^6}+\tan\left(\frac{d}{2}+\frac{e\,x}{2}\right)\left(\frac{8\,b^3+6\,b\,c^2}{c^5}+\frac{\left(8\,b^2+2\,c^2\right)\sqrt{b^2+c^2}}{c^5}\right)+\frac{\left(\frac{16\,b^3}{3}+4\,b}{}}{e\left(\tan\left(\frac{d}{2}+\frac{e\,x}{2}\right)\left(\frac{6\,b^2+3\,c^2}{c^2}+\frac{6\,b\,\sqrt{b^2+c^2}}{c^2}\right)+\frac{4\,b^3+3\,b\,c^2}{c^3}+\tan\left(\frac{d}{2}+\frac{e\,x}{2}\right)^3+\tan\left(\frac{d}{2}+\frac{e\,x}{2}\right)^2\left(\frac{3\,\sqrt{b^2+c^2}}{c}+\frac{3\,b}{c}\right)+\frac{\left(4\,b^2+c^2\right)}{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*cos(d + e*x) + c*sin(d + e*x) + (b^2 + c^2)^(1/2))^2,x)
```

```
[Out] -(tan(d/2 + (e*x)/2)^2*((4*b^2 + 2*c^2)/c^4 + (4*b*(b^2 + c^2)^(1/2))/c^4)
+ ((16*b^4)/3 + (4*c^4)/3 + (20*b^2*c^2)/3)/c^6 + tan(d/2 + (e*x)/2)*((6*b*
c^2 + 8*b^3)/c^5 + ((8*b^2 + 2*c^2)*(b^2 + c^2)^(1/2))/c^5) + ((4*b*c^2 + (
16*b^3)/3)*(b^2 + c^2)^(1/2))/c^6)/(e*(tan(d/2 + (e*x)/2)*((6*b^2 + 3*c^2)/
c^2 + (6*b*(b^2 + c^2)^(1/2))/c^2) + (3*b*c^2 + 4*b^3)/c^3 + tan(d/2 + (e*x
)/2)^3 + tan(d/2 + (e*x)/2)^2*((3*(b^2 + c^2)^(1/2))/c + (3*b)/c) + ((4*b^2
 + c^2)*(b^2 + c^2)^(1/2))/c^3))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*cos(e*x+d)+c*sin(e*x+d)+(b**2+c**2)**(1/2))**2,x)
```

```
[Out] Timed out
```

3.361 $\int \frac{1}{\left(\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)\right)^3}\,dx$

Optimal. Leaf size=191

$$-\frac{2(c\cos(d+ex)-b\sin(d+ex))}{15e\left(b^2+c^2\right)\left(\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)\right)^2}+\frac{b\sin(d+ex)-c\cos(d+ex)}{5e\sqrt{b^2+c^2}\left(\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+e\right.}$$

```
[Out] 1/5*(-c*cos(e*x+d)+b*sin(e*x+d))/e/(b^2+c^2)^(1/2)/(b*cos(e*x+d)+c*sin(e*x+
d)+(b^2+c^2)^(1/2))^3-2/15*(c*cos(e*x+d)-b*sin(e*x+d))/(b^2+c^2)/e/(b*cos(e
*x+d)+c*sin(e*x+d)+(b^2+c^2)^(1/2))^2-2/15*(c-sin(e*x+d)*(b^2+c^2)^(1/2))/c
/(b^2+c^2)/e/(c*cos(e*x+d)-b*sin(e*x+d))
```

Rubi [A] time = 0.13, antiderivative size = 191, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {3116, 3114}

$$-\frac{2(c\cos(d+ex)-b\sin(d+ex))}{15e\left(b^2+c^2\right)\left(\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)\right)^2}-\frac{c\cos(d+ex)-b\sin(d+ex)}{5e\sqrt{b^2+c^2}\left(\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+e\right.}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[b^2 + c^2] + b*Cos[d + e*x] + c*Sin[d + e*x])^(-3),x]
```

```
[Out] -(c*Cos[d + e*x] - b*Sin[d + e*x])/(5*Sqrt[b^2 + c^2]*e*(Sqrt[b^2 + c^2] +
b*Cos[d + e*x] + c*Sin[d + e*x])^3) - (2*(c*Cos[d + e*x] - b*Sin[d + e*x]))
/(15*(b^2 + c^2)*e*(Sqrt[b^2 + c^2] + b*Cos[d + e*x] + c*Sin[d + e*x])^2) -
 (2*(c - Sqrt[b^2 + c^2]*Sin[d + e*x]))/(15*c*(b^2 + c^2)*e*(c*Cos[d + e*x]
 - b*Sin[d + e*x]))
```

Rule 3114

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(-1), x_Symbol] :> -Simp[(c - a*Sin[d + e*x])/(c*e*(c*Cos[d + e*x] - b*Sin[
d + e*x])), x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[a^2 - b^2 - c^2, 0]
```

Rule 3116

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(n_), x_Symbol] :> Simp[((c*Cos[d + e*x] - b*Sin[d + e*x])*(a + b*Cos[d + e
*x] + c*Sin[d + e*x])^n)/(a*e*(2*n + 1)), x] + Dist[(n + 1)/(a*(2*n + 1)),
```

```
Int[(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n + 1), x], x] /; FreeQ[{a, b, c
, d, e}, x] && EqQ[a^2 - b^2 - c^2, 0] && LtQ[n, -1]
```

Rubi steps

$$\int \frac{1}{\left(\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)\right)^3}\,dx = -\frac{c\cos(d+ex)-b\sin(d+ex)}{5\sqrt{b^2+c^2}\,e\left(\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)\right)^3}+$$

$$= -\frac{c\cos(d+ex)-b\sin(d+ex)}{5\sqrt{b^2+c^2}\,e\left(\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)\right)^3}-$$

$$= -\frac{c\cos(d+ex)-b\sin(d+ex)}{5\sqrt{b^2+c^2}\,e\left(\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)\right)^3}-$$

Mathematica [B] time = 2.59, size = 420, normalized size = 2.20

$$\frac{20c\left(c^4-b^4\right)\cos(2(d+ex))-76b^4c-40b^3c^2\sin(2(d+ex))-152b^2c^3+110b^2c^2\sqrt{b^2+c^2}\sin(d+ex)-6b^2c^2\sqrt{b^2}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[b^2 + c^2] + b*Cos[d + e*x] + c*Sin[d + e*x])^(-3),x]
```

```
[Out] (-76*b^4*c - 152*b^2*c^3 - 76*c^5 + 90*b*c*(b^2 + c^2)^(3/2)*Cos[d + e*x] +
 20*c*(-b^4 + c^4)*Cos[2*(d + e*x)] + 10*b^3*c*Sqrt[b^2 + c^2]*Cos[3*(d + e
*x)] + 10*b*c^3*Sqrt[b^2 + c^2]*Cos[3*(d + e*x)] - 4*b^3*c*Sqrt[b^2 + c^2]*
Cos[5*(d + e*x)] + 4*b*c^3*Sqrt[b^2 + c^2]*Cos[5*(d + e*x)] + 10*b^4*Sqrt[b
^2 + c^2]*Sin[d + e*x] + 110*b^2*c^2*Sqrt[b^2 + c^2]*Sin[d + e*x] + 100*c^4
*Sqrt[b^2 + c^2]*Sin[d + e*x] - 40*b^3*c^2*Sin[2*(d + e*x)] - 40*b*c^4*Sin[
2*(d + e*x)] - 5*b^4*Sqrt[b^2 + c^2]*Sin[3*(d + e*x)] + 5*c^4*Sqrt[b^2 + c^
2]*Sin[3*(d + e*x)] + b^4*Sqrt[b^2 + c^2]*Sin[5*(d + e*x)] - 6*b^2*c^2*Sqrt
[b^2 + c^2]*Sin[5*(d + e*x)] + c^4*Sqrt[b^2 + c^2]*Sin[5*(d + e*x)])/(120*c
*(b^2 + c^2)*e*(c*Cos[d + e*x] - b*Sin[d + e*x])^5)
```

fricas [B] time = 0.96, size = 490, normalized size = 2.57

$$-\frac{7\,b^6+26\,b^4c^2+31\,b^2c^4+12\,c^6+5\left(b^6+b^4c^2-b^2c^4-c^6\right)\cos\left(ex+d\right)^2+10\left(b^5c+2\,b^3c^3+bc^5\right)\cos\left(ex+d\right)\mathrm{si}}{15\left(\left(5\,b^8c-14\,b^4c^5-8\,b^2c^7+c^9\right)e\cos\left(ex+d\right)^5-10\left(b^8c+b^6\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*cos(e*x+d)+c*sin(e*x+d)+(b^2+c^2)^(1/2))^3,x, algorithm="fri
cas")
```

```
[Out] -1/15*(7*b^6 + 26*b^4*c^2 + 31*b^2*c^4 + 12*c^6 + 5*(b^6 + b^4*c^2 - b^2*c^
4 - c^6)*cos(e*x + d)^2 + 10*(b^5*c + 2*b^3*c^3 + b*c^5)*cos(e*x + d)*sin(e
*x + d) - (2*(b^5 - 10*b^3*c^2 + 5*b*c^4)*cos(e*x + d)^5 - 5*(b^5 - 6*b^3*c
^2 + b*c^4)*cos(e*x + d)^3 + 5*(3*b^5 + 3*b^3*c^2 + 2*b*c^4)*cos(e*x + d) +
 (15*b^4*c + 25*b^2*c^3 + 12*c^5 + 2*(5*b^4*c - 10*b^2*c^3 + c^5)*cos(e*x +
 d)^4 - (15*b^4*c - 10*b^2*c^3 - c^5)*cos(e*x + d)^2)*sin(e*x + d))*sqrt(b^
2 + c^2))/((5*b^8*c - 14*b^4*c^5 - 8*b^2*c^7 + c^9)*e*cos(e*x + d)^5 - 10*(
b^8*c + b^6*c^3 - b^4*c^5 - b^2*c^7)*e*cos(e*x + d)^3 + 5*(b^8*c + 2*b^6*c^
3 + b^4*c^5)*e*cos(e*x + d) - ((b^9 - 8*b^7*c^2 - 14*b^5*c^4 + 5*b*c^8)*e*c
os(e*x + d)^4 - 2*(b^9 - 3*b^7*c^2 - 9*b^5*c^4 - 5*b^3*c^6)*e*cos(e*x + d)^
2 + (b^9 + 2*b^7*c^2 + b^5*c^4)*e)*sin(e*x + d))
```

giac [A] time = 0.61, size = 346, normalized size = 1.81

$$-\frac{2\left(192\,b^7+352\,b^5c^2+200\,b^3c^4+35\,bc^6+15\left(4\,b^3c^4+3\,bc^6+\left(4\,b^2c^4+c^6\right)\sqrt{b^2+c^2}\right)\tan\left(\frac{1}{2}\,xe+\frac{1}{2}\,d\right)^4+30\left(\cdots\right.\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*cos(e*x+d)+c*sin(e*x+d)+(b^2+c^2)^(1/2))^3,x, algorithm="gia
c")
```

```
[Out] -2/15*(192*b^7 + 352*b^5*c^2 + 200*b^3*c^4 + 35*b*c^6 + 15*(4*b^3*c^4 + 3*b
*c^6 + (4*b^2*c^4 + c^6)*sqrt(b^2 + c^2))*tan(1/2*x*e + 1/2*d)^4 + 30*(8*b^
4*c^3 + 8*b^2*c^5 + c^7 + 4*(2*b^3*c^3 + b*c^5)*sqrt(b^2 + c^2))*tan(1/2*x*
e + 1/2*d)^3 + 20*(24*b^5*c^2 + 32*b^3*c^4 + 9*b*c^6 + 2*(12*b^4*c^2 + 10*b
^2*c^4 + c^6)*sqrt(b^2 + c^2))*tan(1/2*x*e + 1/2*d)^2 + 10*(48*b^6*c + 76*b
^4*c^3 + 31*b^2*c^5 + 2*c^7 + (48*b^5*c + 52*b^3*c^3 + 11*b*c^5)*sqrt(b^2 +
 c^2))*tan(1/2*x*e + 1/2*d) + (192*b^6 + 256*b^4*c^2 + 96*b^2*c^4 + 7*c^6)*
sqrt(b^2 + c^2))*e^(-1)/((c*tan(1/2*x*e + 1/2*d) + b + sqrt(b^2 + c^2))^5*c
^5)
```

maple [B] time = 0.52, size = 496, normalized size = 2.60

$$\frac{-\frac{2\left(4\sqrt{b^2+c^2}\,b^2+\sqrt{b^2+c^2}\,c^2+4b^3+3c^2b\right)\left(\tan^4\left(\frac{d}{2}+\frac{ex}{2}\right)\right)}{c^2}-\frac{4\left(8b^4+8b^2c^2+c^4+8\sqrt{b^2+c^2}\,b^3+4\sqrt{b^2+c^2}\,b\,c^2\right)\left(\tan^3\left(\frac{d}{2}+\frac{ex}{2}\right)\right)}{c^3}-\frac{8\left(24\sqrt{b^2+c^2}\,b^4+20\sqrt{b^2+c^2}\cdots\right.}{\cdots}}{e\,c^4\left(\tan^2\left(\cdots\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*cos(e*x+d)+c*sin(e*x+d)+(b^2+c^2)^(1/2))^3,x)
```

```
[Out] 2/e/c^4*(-(4*(b^2+c^2)^(1/2)*b^2+(b^2+c^2)^(1/2)*c^2+4*b^3+3*c^2*b)/c^2*tan
(1/2*d+1/2*e*x)^4-2*(8*b^4+8*b^2*c^2+c^4+8*(b^2+c^2)^(1/2)*b^3+4*(b^2+c^2)^
(1/2)*b*c^2)/c^3*tan(1/2*d+1/2*e*x)^3-4/3*(24*(b^2+c^2)^(1/2)*b^4+20*(b^2+c
^2)^(1/2)*b^2*c^2+2*(b^2+c^2)^(1/2)*c^4+24*b^5+32*b^3*c^2+9*c^4*b)/c^4*tan(
1/2*d+1/2*e*x)^2-2/3*(48*b^6+76*b^4*c^2+31*b^2*c^4+2*c^6+48*(b^2+c^2)^(1/2)
*b^5+52*(b^2+c^2)^(1/2)*b^3*c^2+11*(b^2+c^2)^(1/2)*b*c^4)/c^5*tan(1/2*d+1/2
*e*x)-1/15/c^6*(192*(b^2+c^2)^(1/2)*b^6+256*(b^2+c^2)^(1/2)*b^4*c^2+96*(b^2
+c^2)^(1/2)*b^2*c^4+7*(b^2+c^2)^(1/2)*c^6+192*b^7+352*b^5*c^2+200*b^3*c^4+3
5*c^6*b))/(tan(1/2*d+1/2*e*x)^2+2/c*(b^2+c^2)^(1/2)*tan(1/2*d+1/2*e*x)+2*b/
c*tan(1/2*d+1/2*e*x)+2/c^2*(b^2+c^2)^(1/2)*b+2/c^2*b^2+1)^2/(tan(1/2*d+1/2*
e*x)+1/c*(b^2+c^2)^(1/2)+b/c)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*cos(e*x+d)+c*sin(e*x+d)+(b^2+c^2)^(1/2))^3,x, algorithm="max
ima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is und
efined.
```

mupad [B] time = 8.12, size = 592, normalized size = 3.10

$$-\frac{\tan\left(\frac{d}{2}+\frac{e\,x}{2}\right)^3\,\left(\frac{32\,b^4+32\,b^2\,c^2+4\,c^4}{c^7}+\frac{\left(32\,b^3+16\,b\,c^2\right)\,\sqrt{b^2+c^2}}{c^7}\right)+\tan\left(\frac{d}{2}+\frac{e\,x}{2}\right)^4\,\left(\frac{8\,b^3+6\,b\,c^2}{c^6}+\frac{\left(8\,b^2+2\,c^2\right)\,\sqrt{b^2+c^2}}{c^6}\right)+\tan\left(\frac{d}{2}+\right.}{e\,\left(\frac{16\,b^5+20\,b^3\,c^2+5\,b\,c^4}{c^5}+\tan\left(\frac{d}{2}+\frac{e\,x}{2}\right)^2\,\left(\frac{40\,b^3+30\,b\,c^2}{c^3}+\frac{\left(40\,b^2+10\,c^2\right)\,\sqrt{b^2+c^2}}{c^3}\right)+\tan\left(\frac{d}{2}+\frac{e\,x}{2}\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*cos(d + e*x) + c*sin(d + e*x) + (b^2 + c^2)^(1/2))^3,x)
```

```
[Out] -(tan(d/2 + (e*x)/2)^3*((32*b^4 + 4*c^4 + 32*b^2*c^2)/c^7 + ((16*b*c^2 + 32
*b^3)*(b^2 + c^2)^(1/2))/c^7) + tan(d/2 + (e*x)/2)^4*((6*b*c^2 + 8*b^3)/c^6
 + ((8*b^2 + 2*c^2)*(b^2 + c^2)^(1/2))/c^6) + tan(d/2 + (e*x)/2)*((64*b^6 +
 (8*c^6)/3 + (124*b^2*c^4)/3 + (304*b^4*c^2)/3)/c^9 + ((b^2 + c^2)^(1/2)*((
44*b*c^4)/3 + 64*b^5 + (208*b^3*c^2)/3))/c^9) + tan(d/2 + (e*x)/2)^2*((24*b
*c^4 + 64*b^5 + (256*b^3*c^2)/3)/c^8 + ((b^2 + c^2)^(1/2)*(64*b^4 + (16*c^4
```

```
)/3 + (160*b^2*c^2)/3))/c^8) + ((14*b*c^6)/3 + (128*b^7)/5 + (80*b^3*c^4)/3
 + (704*b^5*c^2)/15)/c^10 + ((b^2 + c^2)^(1/2)*((128*b^6)/5 + (14*c^6)/15 +
 (64*b^2*c^4)/5 + (512*b^4*c^2)/15))/c^10)/(e*((5*b*c^4 + 16*b^5 + 20*b^3*c
^2)/c^5 + tan(d/2 + (e*x)/2)^2*((30*b*c^2 + 40*b^3)/c^3 + ((40*b^2 + 10*c^2
)*(b^2 + c^2)^(1/2))/c^3) + tan(d/2 + (e*x)/2)^5 + tan(d/2 + (e*x)/2)^3*((2
0*b^2 + 10*c^2)/c^2 + (20*b*(b^2 + c^2)^(1/2))/c^2) + tan(d/2 + (e*x)/2)*((
40*b^4 + 5*c^4 + 40*b^2*c^2)/c^4 + ((20*b*c^2 + 40*b^3)*(b^2 + c^2)^(1/2))/
c^4) + tan(d/2 + (e*x)/2)^4*((5*(b^2 + c^2)^(1/2))/c + (5*b)/c) + ((b^2 + c
^2)^(1/2)*(16*b^4 + c^4 + 12*b^2*c^2))/c^5))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*cos(e*x+d)+c*sin(e*x+d)+(b**2+c**2)**(1/2))**3,x)
```

```
[Out] Timed out
```

3.362 $\int \frac{1}{\left(\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)\right)^4}\,dx$

Optimal. Leaf size=259

$$-\frac{2(c\cos(d+ex)-b\sin(d+ex))}{35e\left(b^2+c^2\right)^{3/2}\left(\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)\right)^2}-\frac{3(c\cos(d+ex)-b\sin(d+ex))}{35e\left(b^2+c^2\right)\left(\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d\right.}$$

```
[Out] 1/7*(-c*cos(e*x+d)+b*sin(e*x+d))/e/(b^2+c^2)^(1/2)/(b*cos(e*x+d)+c*sin(e*x+
d)+(b^2+c^2)^(1/2))^4-3/35*(c*cos(e*x+d)-b*sin(e*x+d))/(b^2+c^2)/e/(b*cos(e
*x+d)+c*sin(e*x+d)+(b^2+c^2)^(1/2))^3-2/35*(c*cos(e*x+d)-b*sin(e*x+d))/(b^2
+c^2)^(3/2)/e/(b*cos(e*x+d)+c*sin(e*x+d)+(b^2+c^2)^(1/2))^2-2/35*(c-sin(e*x
+d)*(b^2+c^2)^(1/2))/c/(b^2+c^2)^(3/2)/e/(c*cos(e*x+d)-b*sin(e*x+d))
```

Rubi [A] time = 0.19, antiderivative size = 259, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {3116, 3114}

$$-\frac{2(c\cos(d+ex)-b\sin(d+ex))}{35e\left(b^2+c^2\right)^{3/2}\left(\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)\right)^2}-\frac{3(c\cos(d+ex)-b\sin(d+ex))}{35e\left(b^2+c^2\right)\left(\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d\right.}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[b^2 + c^2] + b*Cos[d + e*x] + c*Sin[d + e*x])^(-4),x]
```

```
[Out] -(c*Cos[d + e*x] - b*Sin[d + e*x])/(7*Sqrt[b^2 + c^2]*e*(Sqrt[b^2 + c^2] +
b*Cos[d + e*x] + c*Sin[d + e*x])^4) - (3*(c*Cos[d + e*x] - b*Sin[d + e*x]))
/(35*(b^2 + c^2)*e*(Sqrt[b^2 + c^2] + b*Cos[d + e*x] + c*Sin[d + e*x])^3) -
 (2*(c*Cos[d + e*x] - b*Sin[d + e*x]))/(35*(b^2 + c^2)^(3/2)*e*(Sqrt[b^2 +
c^2] + b*Cos[d + e*x] + c*Sin[d + e*x])^2) - (2*(c - Sqrt[b^2 + c^2]*Sin[d
+ e*x]))/(35*c*(b^2 + c^2)^(3/2)*e*(c*Cos[d + e*x] - b*Sin[d + e*x]))
```

Rule 3114

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(-1), x_Symbol] :> -Simp[(c - a*Sin[d + e*x])/(c*e*(c*Cos[d + e*x] - b*Sin[
d + e*x])), x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[a^2 - b^2 - c^2, 0]
```

Rule 3116

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(n_), x_Symbol] :> Simp[((c*Cos[d + e*x] - b*Sin[d + e*x])*(a + b*Cos[d + e
```

```
*x] + c*Sin[d + e*x])^n)/(a*e*(2*n + 1)), x] + Dist[(n + 1)/(a*(2*n + 1)),
Int[(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n + 1), x], x] /; FreeQ[{a, b, c
, d, e}, x] && EqQ[a^2 - b^2 - c^2, 0] && LtQ[n, -1]
```

Rubi steps

$$\int \frac{1}{\left(\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)\right)^4}\,dx = -\frac{c\cos(d+ex)-b\sin(d+ex)}{7\sqrt{b^2+c^2}\,e\left(\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)\right)^4}$$

$$= -\frac{c\cos(d+ex)-b\sin(d+ex)}{7\sqrt{b^2+c^2}\,e\left(\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)\right)^4}$$

$$= -\frac{c\cos(d+ex)-b\sin(d+ex)}{7\sqrt{b^2+c^2}\,e\left(\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)\right)^4}$$

$$= -\frac{c\cos(d+ex)-b\sin(d+ex)}{7\sqrt{b^2+c^2}\,e\left(\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)\right)^4}$$

Mathematica [B] time = 2.03, size = 533, normalized size = 2.06

$$\frac{-35b^6\sin(d+ex)+21b^6\sin(3(d+ex))-7b^6\sin(5(d+ex))+b^6\sin(7(d+ex))-112b^5c\cos(3(d+ex))+28b^5}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[b^2 + c^2] + b*Cos[d + e*x] + c*Sin[d + e*x])^(-4),x]
```

```
[Out] (832*b^4*c*Sqrt[b^2 + c^2] + 1664*b^2*c^3*Sqrt[b^2 + c^2] + 832*c^5*Sqrt[b^
2 + c^2] - 1190*b*c*(b^2 + c^2)^2*Cos[d + e*x] + 448*c*Sqrt[b^2 + c^2]*(b^4
 - c^4)*Cos[2*(d + e*x)] - 112*b^5*c*Cos[3*(d + e*x)] + 56*b^3*c^3*Cos[3*(d
 + e*x)] + 168*b*c^5*Cos[3*(d + e*x)] + 28*b^5*c*Cos[5*(d + e*x)] - 28*b*c^
5*Cos[5*(d + e*x)] - 6*b^5*c*Cos[7*(d + e*x)] + 20*b^3*c^3*Cos[7*(d + e*x)]
 - 6*b*c^5*Cos[7*(d + e*x)] - 35*b^6*Sin[d + e*x] - 1295*b^4*c^2*Sin[d + e*
x] - 2485*b^2*c^4*Sin[d + e*x] - 1225*c^6*Sin[d + e*x] + 896*b^3*c^2*Sqrt[b
^2 + c^2]*Sin[2*(d + e*x)] + 896*b*c^4*Sqrt[b^2 + c^2]*Sin[2*(d + e*x)] + 2
```

```
1*b^6*Sin[3*(d + e*x)] - 189*b^4*c^2*Sin[3*(d + e*x)] - 161*b^2*c^4*Sin[3*(
d + e*x)] + 49*c^6*Sin[3*(d + e*x)] - 7*b^6*Sin[5*(d + e*x)] + 35*b^4*c^2*S
in[5*(d + e*x)] + 35*b^2*c^4*Sin[5*(d + e*x)] - 7*c^6*Sin[5*(d + e*x)] + b^
6*Sin[7*(d + e*x)] - 15*b^4*c^2*Sin[7*(d + e*x)] + 15*b^2*c^4*Sin[7*(d + e*
x)] - c^6*Sin[7*(d + e*x)])/(1120*c*(b^2 + c^2)*e*(-(c*Cos[d + e*x]) + b*Si
n[d + e*x])^7)
```

fricas [B] time = 1.85, size = 739, normalized size = 2.85

$$\frac{2\left(b^7-21\,b^5c^2+35\,b^3c^4-7\,bc^6\right)\cos\left(ex+d\right)^7-7\left(b^7-15\,b^5c^2+15\,b^3c^4-bc^6\right)\cos\left(ex+d\right)^5-14\left(5\,b^5c^2-5\,b^3c\right.}{35\left(\left(7\,b^{10}c-21\,b^8c^3-42\,b^6c^5+6\,b^4c^7+19\,b^2c^9-c^{11}\right)e\cos\left(ex+d\right)^7-7\left(3\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*cos(e*x+d)+c*sin(e*x+d)+(b^2+c^2)^(1/2))^4,x, algorithm="fri
cas")
```

```
[Out] 1/35*(2*(b^7 - 21*b^5*c^2 + 35*b^3*c^4 - 7*b*c^6)*cos(e*x + d)^7 - 7*(b^7 -
 15*b^5*c^2 + 15*b^3*c^4 - b*c^6)*cos(e*x + d)^5 - 14*(5*b^5*c^2 - 5*b^3*c^
4 - 2*b*c^6)*cos(e*x + d)^3 - 7*(5*b^7 + 15*b^5*c^2 + 20*b^3*c^4 + 8*b*c^6)
*cos(e*x + d) - (35*b^6*c + 105*b^4*c^3 + 112*b^2*c^5 + 40*c^7 - 2*(7*b^6*c
 - 35*b^4*c^3 + 21*b^2*c^5 - c^7)*cos(e*x + d)^6 + (35*b^6*c - 105*b^4*c^3
+ 21*b^2*c^5 + c^7)*cos(e*x + d)^4 + 2*(35*b^4*c^3 + 7*b^2*c^5 - 4*c^7)*cos
(e*x + d)^2)*sin(e*x + d) + 4*(3*b^6 + 16*b^4*c^2 + 23*b^2*c^4 + 10*c^6 + 7
*(b^6 + b^4*c^2 - b^2*c^4 - c^6)*cos(e*x + d)^2 + 14*(b^5*c + 2*b^3*c^3 + b
*c^5)*cos(e*x + d)*sin(e*x + d))*sqrt(b^2 + c^2))/((7*b^10*c - 21*b^8*c^3 -
 42*b^6*c^5 + 6*b^4*c^7 + 19*b^2*c^9 - c^11)*e*cos(e*x + d)^7 - 7*(3*b^10*c
 - 4*b^8*c^3 - 14*b^6*c^5 - 4*b^4*c^7 + 3*b^2*c^9)*e*cos(e*x + d)^5 + 7*(3*
b^10*c + b^8*c^3 - 7*b^6*c^5 - 5*b^4*c^7)*e*cos(e*x + d)^3 - 7*(b^10*c + 2*
b^8*c^3 + b^6*c^5)*e*cos(e*x + d) - ((b^11 - 19*b^9*c^2 - 6*b^7*c^4 + 42*b^
5*c^6 + 21*b^3*c^8 - 7*b*c^10)*e*cos(e*x + d)^6 - (3*b^11 - 36*b^9*c^2 - 46
*b^7*c^4 + 28*b^5*c^6 + 35*b^3*c^8)*e*cos(e*x + d)^4 + 3*(b^11 - 5*b^9*c^2
- 13*b^7*c^4 - 7*b^5*c^6)*e*cos(e*x + d)^2 - (b^11 + 2*b^9*c^2 + b^7*c^4)*e
)*sin(e*x + d))
```

giac [B] time = 2.52, size = 599, normalized size = 2.31

$$-\frac{2\left(2560\,b^{10}+6528\,b^8c^2+5888\,b^6c^4+2248\,b^4c^6+340\,b^2c^8+12\,c^{10}+35\left(8\,b^4c^6+8\,b^2c^8+c^{10}+4\left(2\,b^3c^6+bc\right.\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*cos(e*x+d)+c*sin(e*x+d)+(b^2+c^2)^(1/2))^4,x, algorithm="gia
c")
```

```
[Out] -2/35*(2560*b^10 + 6528*b^8*c^2 + 5888*b^6*c^4 + 2248*b^4*c^6 + 340*b^2*c^8
 + 12*c^10 + 35*(8*b^4*c^6 + 8*b^2*c^8 + c^10 + 4*(2*b^3*c^6 + b*c^8)*sqrt(
b^2 + c^2))*tan(1/2*x*e + 1/2*d)^6 + 105*(16*b^5*c^5 + 20*b^3*c^7 + 5*b*c^9
 + (16*b^4*c^5 + 12*b^2*c^7 + c^9)*sqrt(b^2 + c^2))*tan(1/2*x*e + 1/2*d)^5
+ 70*(80*b^6*c^4 + 124*b^4*c^6 + 49*b^2*c^8 + 3*c^10 + (80*b^5*c^4 + 84*b^3
*c^6 + 17*b*c^8)*sqrt(b^2 + c^2))*tan(1/2*x*e + 1/2*d)^4 + 70*(160*b^7*c^3
+ 288*b^5*c^5 + 150*b^3*c^7 + 20*b*c^9 + (160*b^6*c^3 + 208*b^4*c^5 + 66*b^
2*c^7 + 3*c^9)*sqrt(b^2 + c^2))*tan(1/2*x*e + 1/2*d)^3 + 21*(640*b^8*c^2 +
1312*b^6*c^4 + 856*b^4*c^6 + 186*b^2*c^8 + 7*c^10 + 2*(320*b^7*c^2 + 496*b^
5*c^4 + 220*b^3*c^6 + 25*b*c^8)*sqrt(b^2 + c^2))*tan(1/2*x*e + 1/2*d)^2 + 7
*(1280*b^9*c + 2944*b^7*c^3 + 2288*b^5*c^5 + 676*b^3*c^7 + 57*b*c^9 + (1280
*b^8*c + 2304*b^6*c^3 + 1296*b^4*c^5 + 236*b^2*c^7 + 7*c^9)*sqrt(b^2 + c^2)
)*tan(1/2*x*e + 1/2*d) + 4*(640*b^9 + 1312*b^7*c^2 + 896*b^5*c^4 + 238*b^3*
c^6 + 21*b*c^8)*sqrt(b^2 + c^2))*e^(-1)/((c*tan(1/2*x*e + 1/2*d) + b + sqrt
(b^2 + c^2))^7*c^7)
```

maple [B] time = 0.71, size = 823, normalized size = 3.18

$$-\frac{2\left(\frac{\left(8b^4+8b^2c^2+c^4+8\sqrt{b^2+c^2}\,b^3+4\sqrt{b^2+c^2}\,b\,c^2\right)\left(\tan^6\left(\frac{d}{2}+\frac{ex}{2}\right)\right)}{c^2}+\frac{3\left(16\sqrt{b^2+c^2}\,b^4+12\sqrt{b^2+c^2}\,b^2c^2+\sqrt{b^2+c^2}\,c^4+16b^5+20b^3c^2+5c^4b\right)\left(\tan^5\left(\frac{d}{2}+\frac{e\ldots}{2}\right.\right.}{c^3}\right.}{\ldots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*cos(e*x+d)+c*sin(e*x+d)+(b^2+c^2)^(1/2))^4,x)
```

```
[Out] -2/e/c^6*((8*b^4+8*b^2*c^2+c^4+8*(b^2+c^2)^(1/2)*b^3+4*(b^2+c^2)^(1/2)*b*c^
2)/c^2*tan(1/2*d+1/2*e*x)^6+3*(16*(b^2+c^2)^(1/2)*b^4+12*(b^2+c^2)^(1/2)*b^
2*c^2+(b^2+c^2)^(1/2)*c^4+16*b^5+20*b^3*c^2+5*c^4*b)/c^3*tan(1/2*d+1/2*e*x)
^5+2*(80*(b^2+c^2)^(1/2)*b^5+84*(b^2+c^2)^(1/2)*b^3*c^2+17*(b^2+c^2)^(1/2)*
b*c^4+80*b^6+124*b^4*c^2+49*b^2*c^4+3*c^6)/c^4*tan(1/2*d+1/2*e*x)^4+2*(160*
b^7+288*b^5*c^2+150*b^3*c^4+20*c^6*b+160*(b^2+c^2)^(1/2)*b^6+208*(b^2+c^2)^
(1/2)*b^4*c^2+66*(b^2+c^2)^(1/2)*b^2*c^4+3*(b^2+c^2)^(1/2)*c^6)/c^5*tan(1/2
*d+1/2*e*x)^3+3/5*(640*b^7*(b^2+c^2)^(1/2)+992*(b^2+c^2)^(1/2)*b^5*c^2+440*
(b^2+c^2)^(1/2)*b^3*c^4+50*(b^2+c^2)^(1/2)*b*c^6+640*b^8+1312*b^6*c^2+856*b
^4*c^4+186*b^2*c^6+7*c^8)/c^6*tan(1/2*d+1/2*e*x)^2+1/5*(1280*b^9+2944*b^7*c
^2+2288*b^5*c^4+676*b^3*c^6+57*b*c^8+1280*(b^2+c^2)^(1/2)*b^8+2304*(b^2+c^2
)^(1/2)*b^6*c^2+1296*(b^2+c^2)^(1/2)*b^4*c^4+236*(b^2+c^2)^(1/2)*b^2*c^6+7*
(b^2+c^2)^(1/2)*c^8)/c^7*tan(1/2*d+1/2*e*x)+4/35*(640*(b^2+c^2)^(1/2)*b^9+1
312*(b^2+c^2)^(1/2)*b^7*c^2+896*(b^2+c^2)^(1/2)*b^5*c^4+238*(b^2+c^2)^(1/2)
*b^3*c^6+21*(b^2+c^2)^(1/2)*b*c^8+640*b^10+1632*b^8*c^2+1472*b^6*c^4+562*b^
4*c^6+85*b^2*c^8+3*c^10)/c^8)/(tan(1/2*d+1/2*e*x)^2+2/c*(b^2+c^2)^(1/2)*tan
```

```
(1/2*d+1/2*e*x)+2*b/c*tan(1/2*d+1/2*e*x)+2/c^2*(b^2+c^2)^(1/2)*b+2/c^2*b^2+
1)^3/(tan(1/2*d+1/2*e*x)+1/c*(b^2+c^2)^(1/2)+b/c)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*cos(e*x+d)+c*sin(e*x+d)+(b^2+c^2)^(1/2))^4,x, algorithm="max
ima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is und
efined.
```

mupad [B] time = 12.31, size = 1004, normalized size = 3.88

$$-\frac{\tan\left(\frac{d}{2}+\frac{e\,x}{2}\right)^6\left(\frac{16\,b^4+16\,b^2\,c^2+2\,c^4}{c^8}+\frac{\left(16\,b^3+8\,b\,c^2\right)\sqrt{b^2+c^2}}{c^8}\right)+\tan\left(\frac{d}{2}+\frac{e\,x}{2}\right)\left(\frac{512\,b^9+\frac{5888\,b^7\,c^2}{5}+\frac{4576\,b^5\,c^4}{5}+\frac{1352\,b^3\,c^6}{5}+\frac{114\,b\,c^8}{5}}{c^{13}}+\frac{\sqrt{b\cdots}}{\cdots}\right.}{e\left(\tan\left(\frac{d}{2}+\frac{e\,x}{2}\right)^3\left(\frac{280\,b^4+280\,b^2\,c^2+3\cdots}{c^4}\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*cos(d + e*x) + c*sin(d + e*x) + (b^2 + c^2)^(1/2))^4,x)
```

```
[Out] -(tan(d/2 + (e*x)/2)^6*((16*b^4 + 2*c^4 + 16*b^2*c^2)/c^8 + ((8*b*c^2 + 16*
b^3)*(b^2 + c^2)^(1/2))/c^8) + tan(d/2 + (e*x)/2)*(((114*b*c^8)/5 + 512*b^9
 + (1352*b^3*c^6)/5 + (4576*b^5*c^4)/5 + (5888*b^7*c^2)/5)/c^13 + ((b^2 + c
^2)^(1/2)*(512*b^8 + (14*c^8)/5 + (472*b^2*c^6)/5 + (2592*b^4*c^4)/5 + (460
8*b^6*c^2)/5))/c^13) + ((1024*b^10)/7 + (24*c^10)/35 + (136*b^2*c^8)/7 + (4
496*b^4*c^6)/35 + (11776*b^6*c^4)/35 + (13056*b^8*c^2)/35)/c^14 + tan(d/2 +
 (e*x)/2)^2*((768*b^8 + (42*c^8)/5 + (1116*b^2*c^6)/5 + (5136*b^4*c^4)/5 +
(7872*b^6*c^2)/5)/c^12 + ((b^2 + c^2)^(1/2)*(60*b*c^6 + 768*b^7 + 528*b^3*c
^4 + (5952*b^5*c^2)/5))/c^12) + tan(d/2 + (e*x)/2)^3*((80*b*c^6 + 640*b^7 +
 600*b^3*c^4 + 1152*b^5*c^2)/c^11 + ((b^2 + c^2)^(1/2)*(640*b^6 + 12*c^6 +
264*b^2*c^4 + 832*b^4*c^2))/c^11) + tan(d/2 + (e*x)/2)^4*((320*b^6 + 12*c^6
 + 196*b^2*c^4 + 496*b^4*c^2)/c^10 + ((b^2 + c^2)^(1/2)*(68*b*c^4 + 320*b^5
 + 336*b^3*c^2))/c^10) + tan(d/2 + (e*x)/2)^5*((30*b*c^4 + 96*b^5 + 120*b^3
*c^2)/c^9 + ((b^2 + c^2)^(1/2)*(96*b^4 + 6*c^4 + 72*b^2*c^2))/c^9) + ((b^2
+ c^2)^(1/2)*((24*b*c^8)/5 + (1024*b^9)/7 + (272*b^3*c^6)/5 + (1024*b^5*c^4
)/5 + (10496*b^7*c^2)/35))/c^14)/(e*(tan(d/2 + (e*x)/2)^3*((280*b^4 + 35*c^
4 + 280*b^2*c^2)/c^4 + ((140*b*c^2 + 280*b^3)*(b^2 + c^2)^(1/2))/c^4) + tan
(d/2 + (e*x)/2)^4*((105*b*c^2 + 140*b^3)/c^3 + ((140*b^2 + 35*c^2)*(b^2 + c
^2)^(1/2))/c^3) + tan(d/2 + (e*x)/2)^7 + tan(d/2 + (e*x)/2)*((224*b^6 + 7*c
```

```
^6 + 126*b^2*c^4 + 336*b^4*c^2)/c^6 + ((b^2 + c^2)^(1/2)*(42*b*c^4 + 224*b^
5 + 224*b^3*c^2))/c^6) + tan(d/2 + (e*x)/2)^5*((42*b^2 + 21*c^2)/c^2 + (42*
b*(b^2 + c^2)^(1/2))/c^2) + tan(d/2 + (e*x)/2)^6*((7*(b^2 + c^2)^(1/2))/c +
 (7*b)/c) + tan(d/2 + (e*x)/2)^2*((105*b*c^4 + 336*b^5 + 420*b^3*c^2)/c^5 +
 ((b^2 + c^2)^(1/2)*(336*b^4 + 21*c^4 + 252*b^2*c^2))/c^5) + (7*b*c^6 + 64*
b^7 + 56*b^3*c^4 + 112*b^5*c^2)/c^7 + ((b^2 + c^2)^(1/2)*(64*b^6 + c^6 + 24
*b^2*c^4 + 80*b^4*c^2))/c^7))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*cos(e*x+d)+c*sin(e*x+d)+(b**2+c**2)**(1/2))**4,x)
```

```
[Out] Timed out
```

3.363 $\int (2a + 2a\cos(d+ex) + 2c\sin(d+ex))^3\,dx$

Optimal. Leaf size=157

$$\frac{4a\left(15a^2+4c^2\right)\sin(d+ex)}{3e}-\frac{4c\left(15a^2+4c^2\right)\cos(d+ex)}{3e}+4ax\left(5a^2+3c^2\right)-\frac{20\left(ac\cos(d+ex)-a^2\sin(d+ex)\right)(a}{3e}$$

```
[Out] 4*a*(5*a^2+3*c^2)*x-4/3*c*(15*a^2+4*c^2)*cos(e*x+d)/e+4/3*a*(15*a^2+4*c^2)*
sin(e*x+d)/e-20/3*(a*c*cos(e*x+d)-a^2*sin(e*x+d))*(a+a*cos(e*x+d)+c*sin(e*x
+d))/e-8/3*(c*cos(e*x+d)-a*sin(e*x+d))*(a+a*cos(e*x+d)+c*sin(e*x+d))^2/e
```

Rubi [A] time = 0.14, antiderivative size = 157, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3120, 3146, 2637, 2638}

$$\frac{4a\left(15a^2+4c^2\right)\sin(d+ex)}{3e}-\frac{4c\left(15a^2+4c^2\right)\cos(d+ex)}{3e}+4ax\left(5a^2+3c^2\right)-\frac{20\left(ac\cos(d+ex)-a^2\sin(d+ex)\right)(a}{3e}$$

Antiderivative was successfully verified.

```
[In] Int[(2*a + 2*a*Cos[d + e*x] + 2*c*Sin[d + e*x])^3,x]
```

```
[Out] 4*a*(5*a^2 + 3*c^2)*x - (4*c*(15*a^2 + 4*c^2)*Cos[d + e*x])/(3*e) + (4*a*(1
5*a^2 + 4*c^2)*Sin[d + e*x])/(3*e) - (20*(a*c*Cos[d + e*x] - a^2*Sin[d + e*
x])*(a + a*Cos[d + e*x] + c*Sin[d + e*x]))/(3*e) - (8*(c*Cos[d + e*x] - a*S
in[d + e*x])*(a + a*Cos[d + e*x] + c*Sin[d + e*x])^2)/(3*e)
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3120

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(n_), x_Symbol] :> -Simp[((c*Cos[d + e*x] - b*Sin[d + e*x])*(a + b*Cos[d +
e*x] + c*Sin[d + e*x])^(n - 1))/(e*n), x] + Dist[1/n, Int[Simp[n*a^2 + (n -
 1)*(b^2 + c^2) + a*b*(2*n - 1)*Cos[d + e*x] + a*c*(2*n - 1)*Sin[d + e*x],
x]*(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n - 2), x], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[a^2 - b^2 - c^2, 0] && GtQ[n, 1]
```

Rule 3146

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(n_.)*((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)
]), x_Symbol] :> Simp[((B*c - b*C - a*C*Cos[d + e*x] + a*B*Sin[d + e*x])*(a
 + b*Cos[d + e*x] + c*Sin[d + e*x])^n)/(a*e*(n + 1)), x] + Dist[1/(a*(n + 1
)), Int[(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n - 1)*Simp[a*(b*B + c*C)*n
+ a^2*A*(n + 1) + (n*(a^2*B - B*c^2 + b*c*C) + a*b*A*(n + 1))*Cos[d + e*x]
+ (n*(b*B*c + a^2*C - b^2*C) + a*c*A*(n + 1))*Sin[d + e*x], x], x], x] /; F
reeQ[{a, b, c, d, e, A, B, C}, x] && GtQ[n, 0] && NeQ[a^2 - b^2 - c^2, 0]
```

Rubi steps

$$\int (2a + 2a\cos(d+ex) + 2c\sin(d+ex))^3\,dx = -\frac{8(c\cos(d+ex) - a\sin(d+ex))(a + a\cos(d+ex) + c\sin(d+e}{3e}$$

$$= -\frac{20\left(ac\cos(d+ex) - a^2\sin(d+ex)\right)(a + a\cos(d+ex) + c\sin(}{3e}$$

$$= 4a\left(5a^2+3c^2\right)x - \frac{20\left(ac\cos(d+ex) - a^2\sin(d+ex)\right)(a + a\cos}{3e}$$

$$= 4a\left(5a^2+3c^2\right)x - \frac{4c\left(15a^2+4c^2\right)\cos(d+ex)}{3e} + \frac{4a\left(15a^2+4c^2\right)}{3e}$$

Mathematica [A] time = 0.41, size = 135, normalized size = 0.86

$$\frac{2\left(6a\left(5a^2+3c^2\right)(d+ex) + 9a\left(5a^2+c^2\right)\sin(d+ex) + 9a\left(a^2-c^2\right)\sin(2(d+ex)) + a\left(a^2-3c^2\right)\sin(3(d+ex))}{3e}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2*a + 2*a*Cos[d + e*x] + 2*c*Sin[d + e*x])^3,x]
```

```
[Out] (2*(6*a*(5*a^2 + 3*c^2)*(d + e*x) - 9*c*(5*a^2 + c^2)*Cos[d + e*x] - 18*a^2
*c*Cos[2*(d + e*x)] + c*(-3*a^2 + c^2)*Cos[3*(d + e*x)] + 9*a*(5*a^2 + c^2)
*Sin[d + e*x] + 9*a*(a^2 - c^2)*Sin[2*(d + e*x)] + a*(a^2 - 3*c^2)*Sin[3*(d
 + e*x)]))/(3*e)
```

fricas [A] time = 1.99, size = 134, normalized size = 0.85

$$-\frac{4\left(18\,a^2c\cos(ex+d)^2 + 2\left(3\,a^2c - c^3\right)\cos(ex+d)^3 - 3\left(5\,a^3 + 3\,ac^2\right)ex + 6\left(3\,a^2c + c^3\right)\cos(ex+d) - \left(22\,a^3\right.}{3\,e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*a+2*a*cos(e*x+d)+2*c*sin(e*x+d))^3,x, algorithm="fricas")
```

```
[Out] -4/3*(18*a^2*c*cos(e*x + d)^2 + 2*(3*a^2*c - c^3)*cos(e*x + d)^3 - 3*(5*a^3
 + 3*a*c^2)*e*x + 6*(3*a^2*c + c^3)*cos(e*x + d) - (22*a^3 + 6*a*c^2 + 2*(a
^3 - 3*a*c^2)*cos(e*x + d)^2 + 9*(a^3 - a*c^2)*cos(e*x + d))*sin(e*x + d))/
e
```

giac [A] time = 0.18, size = 151, normalized size = 0.96

$$-12\,a^2c\cos\left(2\,xe+2\,d\right)e^{(-1)}-\frac{2}{3}\left(3\,a^2c-c^3\right)\cos\left(3\,xe+3\,d\right)e^{(-1)}-6\left(5\,a^2c+c^3\right)\cos\left(xe+d\right)e^{(-1)}+\frac{2}{3}\left(a^3-3\,ac^2\right)e^{(-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*a+2*a*cos(e*x+d)+2*c*sin(e*x+d))^3,x, algorithm="giac")
```

```
[Out] -12*a^2*c*cos(2*x*e + 2*d)*e^(-1) - 2/3*(3*a^2*c - c^3)*cos(3*x*e + 3*d)*e^
(-1) - 6*(5*a^2*c + c^3)*cos(x*e + d)*e^(-1) + 2/3*(a^3 - 3*a*c^2)*e^(-1)*s
in(3*x*e + 3*d) + 6*(a^3 - a*c^2)*e^(-1)*sin(2*x*e + 2*d) + 6*(5*a^3 + a*c^
2)*e^(-1)*sin(x*e + d) + 4*(5*a^3 + 3*a*c^2)*x
```

maple [A] time = 0.26, size = 177, normalized size = 1.13

$$\frac{8a^3\left(ex+d\right)+24a^3\sin\left(ex+d\right)-24a^2c\cos\left(ex+d\right)+24a^3\left(\frac{\sin(ex+d)\cos(ex+d)}{2}+\frac{ex}{2}+\frac{d}{2}\right)-24a^2c\left(\cos^2\left(ex+d\right)\right)+}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*a+2*a*cos(e*x+d)+2*c*sin(e*x+d))^3,x)
```

```
[Out] 8/e*(a^3*(e*x+d)+3*a^3*sin(e*x+d)-3*a^2*c*cos(e*x+d)+3*a^3*(1/2*sin(e*x+d)*
cos(e*x+d)+1/2*e*x+1/2*d)-3*a^2*c*cos(e*x+d)^2+3*a*c^2*(-1/2*sin(e*x+d)*cos
(e*x+d)+1/2*e*x+1/2*d)+1/3*a^3*(2+cos(e*x+d)^2)*sin(e*x+d)-a^2*c*cos(e*x+d)
^3+a*c^2*sin(e*x+d)^3-1/3*c^3*(2+sin(e*x+d)^2)*cos(e*x+d))
```

maxima [A] time = 0.32, size = 191, normalized size = 1.22

$$-\frac{8\,a^2c\cos\left(ex+d\right)^3}{e}+\frac{8\,ac^2\sin\left(ex+d\right)^3}{e}+8\,a^3x-\frac{8\left(\sin\left(ex+d\right)^3-3\,\sin\left(ex+d\right)\right)a^3}{3\,e}+\frac{8\left(\cos\left(ex+d\right)^3-3\,\cos\left(ex\right.\right.}{3\,e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*a+2*a*cos(e*x+d)+2*c*sin(e*x+d))^3,x, algorithm="maxima")
```

```
[Out] -8*a^2*c*cos(e*x + d)^3/e + 8*a*c^2*sin(e*x + d)^3/e + 8*a^3*x - 8/3*(sin(e
*x + d)^3 - 3*sin(e*x + d))*a^3/e + 8/3*(cos(e*x + d)^3 - 3*cos(e*x + d))*c
^3/e - 24*a^2*(c*cos(e*x + d)/e - a*sin(e*x + d)/e) - 6*(4*a*c*cos(e*x + d)
```

```
^2/e - (2*e*x + 2*d + sin(2*e*x + 2*d))*a^2/e - (2*e*x + 2*d - sin(2*e*x +
2*d))*c^2/e)*a
```

mupad [B] time = 2.57, size = 239, normalized size = 1.52

$$20\,a^3\,x-\frac{32\,c^3\cos\left(\frac{d}{2}+\frac{e\,x}{2}\right)^4}{e}+\frac{64\,c^3\cos\left(\frac{d}{2}+\frac{e\,x}{2}\right)^6}{3\,e}+12\,a\,c^2\,x-\frac{64\,a^2\,c\cos\left(\frac{d}{2}+\frac{e\,x}{2}\right)^6}{e}+\frac{40\,a^3\,\cos\left(\frac{d}{2}+\frac{e\,x}{2}\right)\,\sin\left(\frac{d}{2}+\right.}{e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*a + 2*a*cos(d + e*x) + 2*c*sin(d + e*x))^3,x)
```

```
[Out] 20*a^3*x - (32*c^3*cos(d/2 + (e*x)/2)^4)/e + (64*c^3*cos(d/2 + (e*x)/2)^6)/
(3*e) + 12*a*c^2*x - (64*a^2*c*cos(d/2 + (e*x)/2)^6)/e + (40*a^3*cos(d/2 +
(e*x)/2)*sin(d/2 + (e*x)/2))/e + (80*a^3*cos(d/2 + (e*x)/2)^3*sin(d/2 + (e*
x)/2))/(3*e) + (64*a^3*cos(d/2 + (e*x)/2)^5*sin(d/2 + (e*x)/2))/(3*e) + (16
*a*c^2*cos(d/2 + (e*x)/2)^3*sin(d/2 + (e*x)/2))/e - (64*a*c^2*cos(d/2 + (e*
x)/2)^5*sin(d/2 + (e*x)/2))/e + (24*a*c^2*cos(d/2 + (e*x)/2)*sin(d/2 + (e*x
)/2))/e
```

sympy [A] time = 0.76, size = 291, normalized size = 1.85

$$\begin{cases} 12a^3x\sin^2(d+ex)+12a^3x\cos^2(d+ex)+8a^3x+\frac{16a^3\sin^3(d+ex)}{3e}+\frac{8a^3\sin(d+ex)\cos^2(d+ex)}{e}+\frac{12a^3\sin(d+ex)\cos(d+ex)}{e} \\ x\,(2a\cos(d)+2a+2c\sin(d))^3 \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*a+2*a*cos(e*x+d)+2*c*sin(e*x+d))**3,x)
```

```
[Out] Piecewise((12*a**3*x*sin(d + e*x)**2 + 12*a**3*x*cos(d + e*x)**2 + 8*a**3*x
 + 16*a**3*sin(d + e*x)**3/(3*e) + 8*a**3*sin(d + e*x)*cos(d + e*x)**2/e +
12*a**3*sin(d + e*x)*cos(d + e*x)/e + 24*a**3*sin(d + e*x)/e - 8*a**2*c*cos
(d + e*x)**3/e - 24*a**2*c*cos(d + e*x)**2/e - 24*a**2*c*cos(d + e*x)/e + 1
2*a*c**2*x*sin(d + e*x)**2 + 12*a*c**2*x*cos(d + e*x)**2 + 8*a*c**2*sin(d +
 e*x)**3/e - 12*a*c**2*sin(d + e*x)*cos(d + e*x)/e - 8*c**3*sin(d + e*x)**2
*cos(d + e*x)/e - 16*c**3*cos(d + e*x)**3/(3*e), Ne(e, 0)), (x*(2*a*cos(d)
+ 2*a + 2*c*sin(d))**3, True))
```

3.364 $\int (2a+2a\cos(d+ex)+2c\sin(d+ex))^2\,dx$

Optimal. Leaf size=81

$$2x\left(3a^2+c^2\right)+\frac{6a^2\sin(d+ex)}{e}-\frac{6ac\cos(d+ex)}{e}-\frac{2(c\cos(d+ex)-a\sin(d+ex))(a\cos(d+ex)+a+c\sin(d+ex}{e}$$

```
[Out] 2*(3*a^2+c^2)*x-6*a*c*cos(e*x+d)/e+6*a^2*sin(e*x+d)/e-2*(c*cos(e*x+d)-a*sin
(e*x+d))*(a+a*cos(e*x+d)+c*sin(e*x+d))/e
```

Rubi [A] time = 0.05, antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {3120, 2637, 2638}

$$2x\left(3a^2+c^2\right)+\frac{6a^2\sin(d+ex)}{e}-\frac{6ac\cos(d+ex)}{e}-\frac{2(c\cos(d+ex)-a\sin(d+ex))(a\cos(d+ex)+a+c\sin(d+ex}{e}$$

Antiderivative was successfully verified.

```
[In] Int[(2*a + 2*a*Cos[d + e*x] + 2*c*Sin[d + e*x])^2,x]
```

```
[Out] 2*(3*a^2 + c^2)*x - (6*a*c*Cos[d + e*x])/e + (6*a^2*Sin[d + e*x])/e - (2*(c
*Cos[d + e*x] - a*Sin[d + e*x])*(a + a*Cos[d + e*x] + c*Sin[d + e*x]))/e
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3120

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(n_), x_Symbol] :> -Simp[((c*Cos[d + e*x] - b*Sin[d + e*x])*(a + b*Cos[d +
e*x] + c*Sin[d + e*x])^(n - 1))/(e*n), x] + Dist[1/n, Int[Simp[n*a^2 + (n -
 1)*(b^2 + c^2) + a*b*(2*n - 1)*Cos[d + e*x] + a*c*(2*n - 1)*Sin[d + e*x],
x]*(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n - 2), x], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[a^2 - b^2 - c^2, 0] && GtQ[n, 1]
```

Rubi steps

$$\int (2a + 2a\cos(d+ex) + 2c\sin(d+ex))^2\,dx = -\frac{2(c\cos(d+ex) - a\sin(d+ex))(a + a\cos(d+ex) + c\sin(d+e}{e}$$
$$= 2\left(3a^2 + c^2\right)x - \frac{2(c\cos(d+ex) - a\sin(d+ex))(a + a\cos(d+ex}{e}$$
$$= 2\left(3a^2 + c^2\right)x - \frac{6ac\cos(d+ex)}{e} + \frac{6a^2\sin(d+ex)}{e} - \frac{2(c\cos(d+}{}$$

Mathematica [A] time = 0.14, size = 92, normalized size = 1.14

$$4\left(\frac{\left(3a^2 + c^2\right)(d+ex)}{2e} + \frac{\left(a^2 - c^2\right)\sin(2(d+ex))}{4e} + \frac{2a^2\sin(d+ex)}{e} - \frac{2ac\cos(d+ex)}{e} - \frac{ac\cos(2(d+ex))}{2e}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(2*a + 2*a*Cos[d + e*x] + 2*c*Sin[d + e*x])^2,x]
```

```
[Out] 4*((((3*a^2 + c^2)*(d + e*x))/(2*e) - (2*a*c*Cos[d + e*x])/e - (a*c*Cos[2*(d
 + e*x)])/(2*e) + (2*a^2*Sin[d + e*x])/e + ((a^2 - c^2)*Sin[2*(d + e*x)])/(
4*e))
```

fricas [A] time = 1.42, size = 71, normalized size = 0.88

$$-\frac{2\left(2\,ac\cos\left(ex+d\right)^2 - \left(3\,a^2 + c^2\right)ex + 4\,ac\cos\left(ex+d\right) - \left(4\,a^2 + \left(a^2 - c^2\right)\cos\left(ex+d\right)\right)\sin\left(ex+d\right)\right)}{e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*a+2*a*cos(e*x+d)+2*c*sin(e*x+d))^2,x, algorithm="fricas")
```

```
[Out] -2*(2*a*c*cos(e*x + d)^2 - (3*a^2 + c^2)*e*x + 4*a*c*cos(e*x + d) - (4*a^2
+ (a^2 - c^2)*cos(e*x + d))*sin(e*x + d))/e
```

giac [A] time = 0.16, size = 78, normalized size = 0.96

$$-2\,ac\cos\left(2\,xe + 2\,d\right)e^{(-1)} - 8\,ac\cos\left(xe + d\right)e^{(-1)} + 8\,a^2e^{(-1)}\sin\left(xe + d\right) + \left(a^2 - c^2\right)e^{(-1)}\sin\left(2\,xe + 2\,d\right) + 2\left(3\,a^2 + c\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*a+2*a*cos(e*x+d)+2*c*sin(e*x+d))^2,x, algorithm="giac")
```

```
[Out] -2*a*c*cos(2*x*e + 2*d)*e^(-1) - 8*a*c*cos(x*e + d)*e^(-1) + 8*a^2*e^(-1)*s
in(x*e + d) + (a^2 - c^2)*e^(-1)*sin(2*x*e + 2*d) + 2*(3*a^2 + c^2)*x
```

maple [A] time = 0.22, size = 101, normalized size = 1.25

$$\frac{4a^2(ex+d)+8a^2\sin(ex+d)-8ac\cos(ex+d)+4a^2\left(\frac{\sin(ex+d)\cos(ex+d)}{2}+\frac{ex}{2}+\frac{d}{2}\right)-4ac\left(\cos^2(ex+d)\right)+4c^2\left(-\right.}{e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*a+2*a*cos(e*x+d)+2*c*sin(e*x+d))^2,x)
```

```
[Out] 4/e*(a^2*(e*x+d)+2*a^2*sin(e*x+d)-2*a*c*cos(e*x+d)+a^2*(1/2*sin(e*x+d)*cos(
e*x+d)+1/2*e*x+1/2*d)-a*c*cos(e*x+d)^2+c^2*(-1/2*sin(e*x+d)*cos(e*x+d)+1/2*
e*x+1/2*d))
```

maxima [A] time = 0.30, size = 99, normalized size = 1.22

$$4a^2x-\frac{4ac\cos(ex+d)^2}{e}+\frac{(2ex+2d+\sin(2ex+2d))a^2}{e}+\frac{(2ex+2d-\sin(2ex+2d))c^2}{e}-8a\left(\frac{c\cos(ex+d)}{e}-\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*a+2*a*cos(e*x+d)+2*c*sin(e*x+d))^2,x, algorithm="maxima")
```

```
[Out] 4*a^2*x - 4*a*c*cos(e*x + d)^2/e + (2*e*x + 2*d + sin(2*e*x + 2*d))*a^2/e +
 (2*e*x + 2*d - sin(2*e*x + 2*d))*c^2/e - 8*a*(c*cos(e*x + d)/e - a*sin(e*x
 + d)/e)
```

mupad [B] time = 3.21, size = 96, normalized size = 1.19

$$\frac{x\left(12a^2+4c^2\right)}{2}+\frac{\left(12a^2+4c^2\right)\tan\left(\frac{d}{2}+\frac{ex}{2}\right)^3+\left(20a^2-4c^2\right)\tan\left(\frac{d}{2}+\frac{ex}{2}\right)-16ac}{e\left(\tan\left(\frac{d}{2}+\frac{ex}{2}\right)^4+2\tan\left(\frac{d}{2}+\frac{ex}{2}\right)^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*a + 2*a*cos(d + e*x) + 2*c*sin(d + e*x))^2,x)
```

```
[Out] (x*(12*a^2 + 4*c^2))/2 + (tan(d/2 + (e*x)/2)^3*(12*a^2 + 4*c^2) - 16*a*c +
tan(d/2 + (e*x)/2)*(20*a^2 - 4*c^2))/(e*(2*tan(d/2 + (e*x)/2)^2 + tan(d/2 +
 (e*x)/2)^4 + 1))
```

sympy [A] time = 0.31, size = 170, normalized size = 2.10

$$\begin{cases} 2a^2x\sin^2(d+ex)+2a^2x\cos^2(d+ex)+4a^2x+\frac{2a^2\sin(d+ex)\cos(d+ex)}{e}+\frac{8a^2\sin(d+ex)}{e}-\frac{4ac\cos^2(d+ex)}{e}-\frac{8ac\cos(d+ex)}{e}+ \\ x\left(2a\cos(d)+2a+2c\sin(d)\right)^2 \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*a+2*a*cos(e*x+d)+2*c*sin(e*x+d))**2,x)
```

```
[Out] Piecewise((2*a**2*x*sin(d + e*x)**2 + 2*a**2*x*cos(d + e*x)**2 + 4*a**2*x +
 2*a**2*sin(d + e*x)*cos(d + e*x)/e + 8*a**2*sin(d + e*x)/e - 4*a*c*cos(d +
 e*x)**2/e - 8*a*c*cos(d + e*x)/e + 2*c**2*x*sin(d + e*x)**2 + 2*c**2*x*cos
(d + e*x)**2 - 2*c**2*sin(d + e*x)*cos(d + e*x)/e, Ne(e, 0)), (x*(2*a*cos(d
) + 2*a + 2*c*sin(d))**2, True))
```

3.365 $\int (2a + 2a\cos(d + ex) + 2c\sin(d + ex))\,dx$

Optimal. Leaf size=29

$$\frac{2a\sin(d + ex)}{e} + 2ax - \frac{2c\cos(d + ex)}{e}$$

```
[Out] 2*a*x-2*c*cos(e*x+d)/e+2*a*sin(e*x+d)/e
```

Rubi [A] time = 0.02, antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {2637, 2638}

$$\frac{2a\sin(d + ex)}{e} + 2ax - \frac{2c\cos(d + ex)}{e}$$

Antiderivative was successfully verified.

```
[In] Int[2*a + 2*a*Cos[d + e*x] + 2*c*Sin[d + e*x],x]
```

```
[Out] 2*a*x - (2*c*Cos[d + e*x])/e + (2*a*Sin[d + e*x])/e
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int (2a + 2a\cos(d + ex) + 2c\sin(d + ex))\,dx &= 2ax + (2a)\int \cos(d + ex)\,dx + (2c)\int \sin(d + ex)\,dx\\ &= 2ax - \frac{2c\cos(d + ex)}{e} + \frac{2a\sin(d + ex)}{e}\end{aligned}$$

Mathematica [A] time = 0.02, size = 53, normalized size = 1.83

$$\frac{2a\sin(d)\cos(ex)}{e} + \frac{2a\cos(d)\sin(ex)}{e} + 2ax + \frac{2c\sin(d)\sin(ex)}{e} - \frac{2c\cos(d)\cos(ex)}{e}$$

Antiderivative was successfully verified.

```
[In] Integrate[2*a + 2*a*Cos[d + e*x] + 2*c*Sin[d + e*x],x]
```

```
[Out] 2*a*x - (2*c*Cos[d]*Cos[e*x])/e + (2*a*Cos[e*x]*Sin[d])/e + (2*a*Cos[d]*Sin
[e*x])/e + (2*c*Sin[d]*Sin[e*x])/e
```

fricas [A] time = 0.88, size = 27, normalized size = 0.93

$$\frac{2\left(aex - c\cos\left(ex+d\right) + a\sin\left(ex+d\right)\right)}{e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(2*a+2*a*cos(e*x+d)+2*c*sin(e*x+d),x, algorithm="fricas")
```

```
[Out] 2*(a*e*x - c*cos(e*x + d) + a*sin(e*x + d))/e
```

giac [A] time = 0.14, size = 29, normalized size = 1.00

$$-2\,c\cos\left(xe+d\right)e^{(-1)} + 2\,ae^{(-1)}\sin\left(xe+d\right) + 2\,ax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(2*a+2*a*cos(e*x+d)+2*c*sin(e*x+d),x, algorithm="giac")
```

```
[Out] -2*c*cos(x*e + d)*e^(-1) + 2*a*e^(-1)*sin(x*e + d) + 2*a*x
```

maple [A] time = 0.00, size = 30, normalized size = 1.03

$$2ax - \frac{2c\cos\left(ex+d\right)}{e} + \frac{2a\sin\left(ex+d\right)}{e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(2*a+2*a*cos(e*x+d)+2*c*sin(e*x+d),x)
```

```
[Out] 2*a*x-2*c*cos(e*x+d)/e+2*a*sin(e*x+d)/e
```

maxima [A] time = 0.30, size = 29, normalized size = 1.00

$$2\,ax - \frac{2\,c\cos\left(ex+d\right)}{e} + \frac{2\,a\sin\left(ex+d\right)}{e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(2*a+2*a*cos(e*x+d)+2*c*sin(e*x+d),x, algorithm="maxima")
```

```
[Out] 2*a*x - 2*c*cos(e*x + d)/e + 2*a*sin(e*x + d)/e
```

mupad [B] time = 2.43, size = 29, normalized size = 1.00

$$2\,a\,x-\frac{2\,c\,\cos\left(d+e\,x\right)}{e}+\frac{2\,a\,\sin\left(d+e\,x\right)}{e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(2*a + 2*a*cos(d + e*x) + 2*c*sin(d + e*x),x)
```

```
[Out] 2*a*x - (2*c*cos(d + e*x))/e + (2*a*sin(d + e*x))/e
```

sympy [A] time = 0.14, size = 39, normalized size = 1.34

$$2ax+2a\left(\begin{cases}\frac{\sin\left(d+ex\right)}{e} & \text{for}\ e\neq 0\\ x\cos\left(d\right) & \text{otherwise}\end{cases}\right)+2c\left(\begin{cases}-\frac{\cos\left(d+ex\right)}{e} & \text{for}\ e\neq 0\\ x\sin\left(d\right) & \text{otherwise}\end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(2*a+2*a*cos(e*x+d)+2*c*sin(e*x+d),x)
```

```
[Out] 2*a*x + 2*a*Piecewise((sin(d + e*x)/e, Ne(e, 0)), (x*cos(d), True)) + 2*c*P
iecewise((-cos(d + e*x)/e, Ne(e, 0)), (x*sin(d), True))
```

3.366 $\int \frac{1}{2a+2a\cos(d+ex)+2c\sin(d+ex)}\,dx$

Optimal. Leaf size=25

$$\frac{\log\left(a+c\tan\left(\frac{1}{2}(d+ex)\right)\right)}{2ce}$$

```
[Out] 1/2*ln(a+c*tan(1/2*e*x+1/2*d))/c/e
```

Rubi [A] time = 0.02, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {3124, 31}

$$\frac{\log\left(a+c\tan\left(\frac{1}{2}(d+ex)\right)\right)}{2ce}$$

Antiderivative was successfully verified.

```
[In] Int[(2*a + 2*a*Cos[d + e*x] + 2*c*Sin[d + e*x])^(-1),x]
```

```
[Out] Log[a + c*Tan[(d + e*x)/2]]/(2*c*e)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 3124

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(-1), x_Symbol] :> Module[{f = FreeFactors[Tan[(d + e*x)/2], x]}, Dist[(2*f
)/e, Subst[Int[1/(a + b + 2*c*f*x + (a - b)*f^2*x^2), x], x, Tan[(d + e*x)/
2]/f], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{2a+2a\cos(d+ex)+2c\sin(d+ex)}\,dx &= \frac{2\,\text{Subst}\left(\int \frac{1}{4a+4cx}\,dx,x,\tan\left(\frac{1}{2}(d+ex)\right)\right)}{e}\\ &= \frac{\log\left(a+c\tan\left(\frac{1}{2}(d+ex)\right)\right)}{2ce}\end{aligned}$$

Mathematica [B] time = 0.05, size = 57, normalized size = 2.28

$$\frac{1}{2}\left(\frac{\log\left(a\cos\left(\frac{1}{2}(d+ex)\right)+c\sin\left(\frac{1}{2}(d+ex)\right)\right)}{ce}-\frac{\log\left(\cos\left(\frac{1}{2}(d+ex)\right)\right)}{ce}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(2*a + 2*a*Cos[d + e*x] + 2*c*Sin[d + e*x])^(-1),x]
```

```
[Out] (-(Log[Cos[(d + e*x)/2]]/(c*e)) + Log[a*Cos[(d + e*x)/2] + c*Sin[(d + e*x)/
2]]/(c*e))/2
```

fricas [B] time = 1.34, size = 60, normalized size = 2.40

$$\frac{\log\left(ac\sin\left(ex+d\right)+\frac{1}{2}a^2+\frac{1}{2}c^2+\frac{1}{2}\left(a^2-c^2\right)\cos\left(ex+d\right)\right)-\log\left(\frac{1}{2}\cos\left(ex+d\right)+\frac{1}{2}\right)}{4ce}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*a+2*a*cos(e*x+d)+2*c*sin(e*x+d)),x, algorithm="fricas")
```

```
[Out] 1/4*(log(a*c*sin(e*x + d) + 1/2*a^2 + 1/2*c^2 + 1/2*(a^2 - c^2)*cos(e*x + d
)) - log(1/2*cos(e*x + d) + 1/2))/(c*e)
```

giac [A] time = 0.13, size = 23, normalized size = 0.92

$$\frac{e^{(-1)}\log\left(\left|c\tan\left(\frac{1}{2}xe+\frac{1}{2}d\right)+a\right|\right)}{2c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*a+2*a*cos(e*x+d)+2*c*sin(e*x+d)),x, algorithm="giac")
```

```
[Out] 1/2*e^(-1)*log(abs(c*tan(1/2*x*e + 1/2*d) + a))/c
```

maple [A] time = 0.38, size = 23, normalized size = 0.92

$$\frac{\ln\left(a+c\tan\left(\frac{d}{2}+\frac{ex}{2}\right)\right)}{2ce}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2*a+2*a*cos(e*x+d)+2*c*sin(e*x+d)),x)
```

```
[Out] 1/2*ln(a+c*tan(1/2*d+1/2*e*x))/c/e
```

maxima [A] time = 0.31, size = 29, normalized size = 1.16

$$\frac{\log\left(a + \frac{c\sin(ex+d)}{\cos(ex+d)+1}\right)}{2\,ce}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*a+2*a*cos(e*x+d)+2*c*sin(e*x+d)),x, algorithm="maxima")
[Out] 1/2*log(a + c*sin(e*x + d)/(cos(e*x + d) + 1))/(c*e)
```

mupad [B] time = 2.82, size = 22, normalized size = 0.88

$$\frac{\ln\left(a + c\tan\left(\frac{d}{2} + \frac{e\,x}{2}\right)\right)}{2\,c\,e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2*a + 2*a*cos(d + e*x) + 2*c*sin(d + e*x)),x)
[Out] log(a + c*tan(d/2 + (e*x)/2))/(2*c*e)
```

sympy [A] time = 1.10, size = 63, normalized size = 2.52

$$\begin{cases} \frac{x}{2a\cos(d)+2a} & \text{for}\ c = 0 \wedge e = 0 \\ \frac{\tan\left(\frac{d}{2}+\frac{ex}{2}\right)}{2ae} & \text{for}\ c = 0 \\ \frac{x}{2a\cos(d)+2a+2c\sin(d)} & \text{for}\ e = 0 \\ \frac{\log\left(\frac{a}{c}+\tan\left(\frac{d}{2}+\frac{ex}{2}\right)\right)}{2ce} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*a+2*a*cos(e*x+d)+2*c*sin(e*x+d)),x)
[Out] Piecewise((x/(2*a*cos(d) + 2*a), Eq(c, 0) & Eq(e, 0)), (tan(d/2 + e*x/2)/(2
*a*e), Eq(c, 0)), (x/(2*a*cos(d) + 2*a + 2*c*sin(d)), Eq(e, 0)), (log(a/c +
 tan(d/2 + e*x/2))/(2*c*e), True))
```

3.367 $\int \frac{1}{(2a+2a\cos(d+ex)+2c\sin(d+ex))^2}\,dx$

Optimal. Leaf size=75

$$-\frac{a\log\left(a+c\tan\left(\frac{1}{2}(d+ex)\right)\right)}{4c^3e}-\frac{c\cos(d+ex)-a\sin(d+ex)}{4c^2e(a\cos(d+ex)+a+c\sin(d+ex))}$$

```
[Out] -1/4*a*ln(a+c*tan(1/2*e*x+1/2*d))/c^3/e+1/4*(-c*cos(e*x+d)+a*sin(e*x+d))/c^
2/e/(a+a*cos(e*x+d)+c*sin(e*x+d))
```

Rubi [A] time = 0.05, antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3129, 12, 3124, 31}

$$-\frac{a\log\left(a+c\tan\left(\frac{1}{2}(d+ex)\right)\right)}{4c^3e}-\frac{c\cos(d+ex)-a\sin(d+ex)}{4c^2e(a\cos(d+ex)+a+c\sin(d+ex))}$$

Antiderivative was successfully verified.

```
[In] Int[(2*a + 2*a*Cos[d + e*x] + 2*c*Sin[d + e*x])^(-2),x]
```

```
[Out] -(a*Log[a + c*Tan[(d + e*x)/2]])/(4*c^3*e) - (c*Cos[d + e*x] - a*Sin[d + e*
x])/(4*c^2*e*(a + a*Cos[d + e*x] + c*Sin[d + e*x]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 3124

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(-1), x_Symbol] :> Module[{f = FreeFactors[Tan[(d + e*x)/2], x]}, Dist[(2*f
)/e, Subst[Int[1/(a + b + 2*c*f*x + (a - b)*f^2*x^2), x], x, Tan[(d + e*x)/
2]/f], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0]
```

Rule 3129

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(n_), x_Symbol] :> Simp[((-(c*Cos[d + e*x]) + b*Sin[d + e*x])*(a + b*Cos[d
+ e*x] + c*Sin[d + e*x])^(n + 1))/(e*(n + 1)*(a^2 - b^2 - c^2)), x] + Dist[
1/((n + 1)*(a^2 - b^2 - c^2)), Int[(a*(n + 1) - b*(n + 2)*Cos[d + e*x] - c*
(n + 2)*Sin[d + e*x])*(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n + 1), x], x]
 /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0] && LtQ[n, -1] && N
eQ[n, -3/2]
```

Rubi steps

$$\int \frac{1}{(2a+2a\cos(d+ex)+2c\sin(d+ex))^2}\,dx = -\frac{c\cos(d+ex)-a\sin(d+ex)}{4c^2e(a+a\cos(d+ex)+c\sin(d+ex))} + \frac{\int -\frac{2a}{2a+2a\cos(d+ex)+2c\,s}}{4c^2}$$

$$= -\frac{c\cos(d+ex)-a\sin(d+ex)}{4c^2e(a+a\cos(d+ex)+c\sin(d+ex))} - \frac{a\int \frac{1}{2a+2a\cos(d+ex)+2c\,s}}{2c^2}$$

$$= -\frac{c\cos(d+ex)-a\sin(d+ex)}{4c^2e(a+a\cos(d+ex)+c\sin(d+ex))} - \frac{a\,\mathrm{Subst}\left(\int \frac{1}{4a+4cx}\,dx\right.}{c^2}$$

$$= -\frac{a\log\left(a+c\tan\left(\frac{1}{2}(d+ex)\right)\right)}{4c^3e} - \frac{c\cos(d+ex)-a\sin(d+}{4c^2e(a+a\cos(d+ex)+c\sin(}$$

Mathematica [A] time = 0.55, size = 115, normalized size = 1.53

$$\frac{\frac{c(a^2+c^2)\sin\left(\frac{1}{2}(d+ex)\right)}{a\left(a\cos\left(\frac{1}{2}(d+ex)\right)+c\sin\left(\frac{1}{2}(d+ex)\right)\right)} + 2a\left(\log\left(\cos\left(\frac{1}{2}(d+ex)\right)\right) - \log\left(a\cos\left(\frac{1}{2}(d+ex)\right)+c\sin\left(\frac{1}{2}(d+ex)\right)\right)\right) + c\tan\left(\frac{1}{2}\right.}{8c^3e}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2*a + 2*a*Cos[d + e*x] + 2*c*Sin[d + e*x])^(-2),x]
```

```
[Out] (2*a*(Log[Cos[(d + e*x)/2]] - Log[a*Cos[(d + e*x)/2] + c*Sin[(d + e*x)/2]])
 + (c*(a^2 + c^2)*Sin[(d + e*x)/2])/(a*(a*Cos[(d + e*x)/2] + c*Sin[(d + e*x
)/2])) + c*Tan[(d + e*x)/2])/(8*c^3*e)
```

fricas [B] time = 0.99, size = 154, normalized size = 2.05

$$-\frac{2c^2\cos(ex+d) - 2ac\sin(ex+d) + \left(a^2\cos(ex+d) + ac\sin(ex+d) + a^2\right)\log\left(ac\sin(ex+d) + \frac{1}{2}a^2 + \frac{1}{2}c^2\right.}{8\left(ac^3e\cos(ex+d) + c^4e\sin(ex\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*a+2*a*cos(e*x+d)+2*c*sin(e*x+d))^2,x, algorithm="fricas")
```

```
[Out] -1/8*(2*c^2*cos(e*x + d) - 2*a*c*sin(e*x + d) + (a^2*cos(e*x + d) + a*c*sin
(e*x + d) + a^2)*log(a*c*sin(e*x + d) + 1/2*a^2 + 1/2*c^2 + 1/2*(a^2 - c^2)
*cos(e*x + d)) - (a^2*cos(e*x + d) + a*c*sin(e*x + d) + a^2)*log(1/2*cos(e*
x + d) + 1/2))/(a*c^3*e*cos(e*x + d) + c^4*e*sin(e*x + d) + a*c^3*e)
```

giac [A] time = 0.14, size = 86, normalized size = 1.15

$$-\frac{1}{8}\left(\frac{2\,a\log\left(\left|c\tan\left(\frac{1}{2}\,xe+\frac{1}{2}\,d\right)+a\right|\right)}{c^3}-\frac{\tan\left(\frac{1}{2}\,xe+\frac{1}{2}\,d\right)}{c^2}-\frac{2\,ac\tan\left(\frac{1}{2}\,xe+\frac{1}{2}\,d\right)+a^2-c^2}{\left(c\tan\left(\frac{1}{2}\,xe+\frac{1}{2}\,d\right)+a\right)c^3}\right)e^{(-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*a+2*a*cos(e*x+d)+2*c*sin(e*x+d))^2,x, algorithm="giac")
```

```
[Out] -1/8*(2*a*log(abs(c*tan(1/2*x*e + 1/2*d) + a))/c^3 - tan(1/2*x*e + 1/2*d)/c
^2 - (2*a*c*tan(1/2*x*e + 1/2*d) + a^2 - c^2)/((c*tan(1/2*x*e + 1/2*d) + a)
*c^3))*e^(-1)
```

maple [A] time = 0.49, size = 91, normalized size = 1.21

$$\frac{\tan\left(\frac{d}{2}+\frac{ex}{2}\right)}{8e\,c^2}-\frac{a^2}{8e\,c^3\left(a+c\tan\left(\frac{d}{2}+\frac{ex}{2}\right)\right)}-\frac{1}{8ec\left(a+c\tan\left(\frac{d}{2}+\frac{ex}{2}\right)\right)}-\frac{a\ln\left(a+c\tan\left(\frac{d}{2}+\frac{ex}{2}\right)\right)}{4c^3e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2*a+2*a*cos(e*x+d)+2*c*sin(e*x+d))^2,x)
```

```
[Out] 1/8/e/c^2*tan(1/2*d+1/2*e*x)-1/8/e/c^3/(a+c*tan(1/2*d+1/2*e*x))*a^2-1/8/e/c
/(a+c*tan(1/2*d+1/2*e*x))-1/4*a*ln(a+c*tan(1/2*d+1/2*e*x))/c^3/e
```

maxima [A] time = 0.32, size = 90, normalized size = 1.20

$$-\frac{\frac{a^2+c^2}{ac^3+\frac{c^4\sin(ex+d)}{\cos(ex+d)+1}}+\frac{2\,a\log\left(a+\frac{c\sin(ex+d)}{\cos(ex+d)+1}\right)}{c^3}-\frac{\sin(ex+d)}{c^2(\cos(ex+d)+1)}}{8\,e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*a+2*a*cos(e*x+d)+2*c*sin(e*x+d))^2,x, algorithm="maxima")
```

```
[Out] -1/8*((a^2 + c^2)/(a*c^3 + c^4*sin(e*x + d)/(cos(e*x + d) + 1)) + 2*a*log(a
 + c*sin(e*x + d)/(cos(e*x + d) + 1))/c^3 - sin(e*x + d)/(c^2*(cos(e*x + d)
 + 1)))/e
```

mupad [B] time = 2.48, size = 79, normalized size = 1.05

$$\frac{\tan\left(\frac{d}{2}+\frac{e\,x}{2}\right)}{8\,c^2\,e}-\frac{a\,\ln\left(a+c\tan\left(\frac{d}{2}+\frac{e\,x}{2}\right)\right)}{4\,c^3\,e}-\frac{a^2+c^2}{c\,e\,\left(8\tan\left(\frac{d}{2}+\frac{e\,x}{2}\right)\,c^3+8\,a\,c^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2*a + 2*a*cos(d + e*x) + 2*c*sin(d + e*x))^2,x)
```

```
[Out] tan(d/2 + (e*x)/2)/(8*c^2*e) - (a*log(a + c*tan(d/2 + (e*x)/2)))/(4*c^3*e)
- (a^2 + c^2)/(c*e*(8*a*c^2 + 8*c^3*tan(d/2 + (e*x)/2)))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*a+2*a*cos(e*x+d)+2*c*sin(e*x+d))**2,x)
```

```
[Out] Timed out
```

3.368 $\int \frac{1}{(2a+2a\cos(d+ex)+2c\sin(d+ex))^3}\,dx$

Optimal. Leaf size=134

$$\frac{3\left(ac\cos(d+ex)-a^2\sin(d+ex)\right)}{16c^4e(a\cos(d+ex)+a+c\sin(d+ex))}+\frac{\left(3a^2+c^2\right)\log\left(a+c\tan\left(\frac{1}{2}(d+ex)\right)\right)}{16c^5e}-\frac{c\cos(d+ex)-a\sin(d+e}{16c^2e(a\cos(d+ex)+a+c\sin(}$$

```
[Out] 1/16*(3*a^2+c^2)*ln(a+c*tan(1/2*e*x+1/2*d))/c^5/e+1/16*(-c*cos(e*x+d)+a*sin
(e*x+d))/c^2/e/(a+a*cos(e*x+d)+c*sin(e*x+d))^2+3/16*(a*c*cos(e*x+d)-a^2*sin
(e*x+d))/c^4/e/(a+a*cos(e*x+d)+c*sin(e*x+d))
```

Rubi [A] time = 0.11, antiderivative size = 134, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3129, 3153, 3124, 31}

$$\frac{\left(3a^2+c^2\right)\log\left(a+c\tan\left(\frac{1}{2}(d+ex)\right)\right)}{16c^5e}+\frac{3\left(ac\cos(d+ex)-a^2\sin(d+ex)\right)}{16c^4e(a\cos(d+ex)+a+c\sin(d+ex))}-\frac{c\cos(d+ex)-a\sin(d+e}{16c^2e(a\cos(d+ex)+a+c\sin(}$$

Antiderivative was successfully verified.

```
[In] Int[(2*a + 2*a*Cos[d + e*x] + 2*c*Sin[d + e*x])^(-3),x]
```

```
[Out] ((3*a^2 + c^2)*Log[a + c*Tan[(d + e*x)/2]])/(16*c^5*e) - (c*Cos[d + e*x] -
a*Sin[d + e*x])/(16*c^2*e*(a + a*Cos[d + e*x] + c*Sin[d + e*x])^2) + (3*(a*
c*Cos[d + e*x] - a^2*Sin[d + e*x]))/(16*c^4*e*(a + a*Cos[d + e*x] + c*Sin[d
 + e*x]))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 3124

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(-1), x_Symbol] :> Module[{f = FreeFactors[Tan[(d + e*x)/2], x]}, Dist[(2*f
)/e, Subst[Int[1/(a + b + 2*c*f*x + (a - b)*f^2*x^2), x], x, Tan[(d + e*x)/
2]/f], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0]
```

Rule 3129

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(n_), x_Symbol] :> Simp[((-(c*Cos[d + e*x]) + b*Sin[d + e*x])*(a + b*Cos[d
+ e*x] + c*Sin[d + e*x])^(n + 1))/(e*(n + 1)*(a^2 - b^2 - c^2)), x] + Dist[
```

```
1/((n + 1)*(a^2 - b^2 - c^2)), Int[(a*(n + 1) - b*(n + 2)*Cos[d + e*x] - c*
(n + 2)*Sin[d + e*x])*(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n + 1), x], x]
 /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0] && LtQ[n, -1] && N
eQ[n, -3/2]
```

Rule 3153

```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])
/((a_.) + cos[(d_.) + (e_.)*(x_)]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)])^2,
 x_Symbol] :> Simp[(c*B - b*C - (a*C - c*A)*Cos[d + e*x] + (a*B - b*A)*Sin[
d + e*x])/(e*(a^2 - b^2 - c^2)*(a + b*Cos[d + e*x] + c*Sin[d + e*x])), x] +
 Dist[(a*A - b*B - c*C)/(a^2 - b^2 - c^2), Int[1/(a + b*Cos[d + e*x] + c*Si
n[d + e*x]), x], x] /; FreeQ[{a, b, c, d, e, A, B, C}, x] && NeQ[a^2 - b^2
- c^2, 0] && NeQ[a*A - b*B - c*C, 0]
```

Rubi steps

$$\int \frac{1}{(2a+2a\cos(d+ex)+2c\sin(d+ex))^3}\,dx = -\frac{c\cos(d+ex)-a\sin(d+ex)}{16c^2e(a+a\cos(d+ex)+c\sin(d+ex))^2}+\frac{\int \frac{-4a+2a\cos(d+ex)+}{(2a+2a\cos(d+ex)+2}}{8c^2}$$

$$= -\frac{c\cos(d+ex)-a\sin(d+ex)}{16c^2e(a+a\cos(d+ex)+c\sin(d+ex))^2}+\frac{3\left(ac\cos(d+e\right.}{16c^4e(a+a\cos(d}$$

$$= -\frac{c\cos(d+ex)-a\sin(d+ex)}{16c^2e(a+a\cos(d+ex)+c\sin(d+ex))^2}+\frac{3\left(ac\cos(d+e\right.}{16c^4e(a+a\cos(d}$$

$$= \frac{\left(3a^2+c^2\right)\log\left(a+c\tan\left(\frac{1}{2}(d+ex)\right)\right)}{16c^5e}-\frac{c\cos(d+ex)-a}{16c^2e(a+a\cos(d+ex}$$

Mathematica [A] time = 3.00, size = 186, normalized size = 1.39

$$-\frac{4\left(3a^2+c^2\right)\log\left(\cos\left(\frac{1}{2}(d+ex)\right)\right)+\frac{c^2\left(a^2+c^2\right)}{\left(a\cos\left(\frac{1}{2}(d+ex)\right)+c\sin\left(\frac{1}{2}(d+ex)\right)\right)^2}+\frac{6c\left(a^2+c^2\right)\sin\left(\frac{1}{2}(d+ex)\right)}{a\cos\left(\frac{1}{2}(d+ex)\right)+c\sin\left(\frac{1}{2}(d+ex)\right)}-4\left(3a^2+c^2\right)\log}{64c^5e}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2*a + 2*a*Cos[d + e*x] + 2*c*Sin[d + e*x])^(-3),x]
```

```
[Out] -1/64*(4*(3*a^2 + c^2)*Log[Cos[(d + e*x)/2]] - 4*(3*a^2 + c^2)*Log[a*Cos[(d
 + e*x)/2] + c*Sin[(d + e*x)/2]] - c^2*Sec[(d + e*x)/2]^2 + (c^2*(a^2 + c^2
```

```
))/(a*Cos[(d + e*x)/2] + c*Sin[(d + e*x)/2])^2 + (6*c*(a^2 + c^2)*Sin[(d +
e*x)/2])/(a*Cos[(d + e*x)/2] + c*Sin[(d + e*x)/2]) + 6*a*c*Tan[(d + e*x)/2]
)/(c^5*e)
```

fricas [B] time = 0.95, size = 433, normalized size = 3.23

$$12\,a^2c^2\cos\left(ex+d\right)^2-6\,a^2c^2+2\left(3\,a^2c^2-c^4\right)\cos\left(ex+d\right)+\left(3\,a^4+4\,a^2c^2+c^4+\left(3\,a^4-2\,a^2c^2-c^4\right)\cos\left(ex+d\right)\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*a+2*a*cos(e*x+d)+2*c*sin(e*x+d))^3,x, algorithm="fricas")
```

```
[Out] 1/32*(12*a^2*c^2*cos(e*x + d)^2 - 6*a^2*c^2 + 2*(3*a^2*c^2 - c^4)*cos(e*x +
 d) + (3*a^4 + 4*a^2*c^2 + c^4 + (3*a^4 - 2*a^2*c^2 - c^4)*cos(e*x + d)^2 +
 2*(3*a^4 + a^2*c^2)*cos(e*x + d) + 2*(3*a^3*c + a*c^3 + (3*a^3*c + a*c^3)*
cos(e*x + d))*sin(e*x + d))*log(a*c*sin(e*x + d) + 1/2*a^2 + 1/2*c^2 + 1/2*
(a^2 - c^2)*cos(e*x + d)) - (3*a^4 + 4*a^2*c^2 + c^4 + (3*a^4 - 2*a^2*c^2 -
 c^4)*cos(e*x + d)^2 + 2*(3*a^4 + a^2*c^2)*cos(e*x + d) + 2*(3*a^3*c + a*c^
3 + (3*a^3*c + a*c^3)*cos(e*x + d))*sin(e*x + d))*log(1/2*cos(e*x + d) + 1/
2) - 2*(3*a^3*c - a*c^3 + 3*(a^3*c - a*c^3)*cos(e*x + d))*sin(e*x + d))/(2*
a^2*c^5*e*cos(e*x + d) + (a^2*c^5 - c^7)*e*cos(e*x + d)^2 + (a^2*c^5 + c^7)
*e + 2*(a*c^6*e*cos(e*x + d) + a*c^6*e)*sin(e*x + d))
```

giac [A] time = 0.17, size = 171, normalized size = 1.28

$$\frac{1}{64}\left(\frac{4\left(3\,a^2+c^2\right)\log\left(\left|c\tan\left(\frac{1}{2}\,xe+\frac{1}{2}\,d\right)+a\right|\right)}{c^5}+\frac{c^3\tan\left(\frac{1}{2}\,xe+\frac{1}{2}\,d\right)^2-6\,ac^2\tan\left(\frac{1}{2}\,xe+\frac{1}{2}\,d\right)}{c^6}-\frac{18\,a^2c^2\tan\left(\frac{1}{2}\,xe\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*a+2*a*cos(e*x+d)+2*c*sin(e*x+d))^3,x, algorithm="giac")
```

```
[Out] 1/64*(4*(3*a^2 + c^2)*log(abs(c*tan(1/2*x*e + 1/2*d) + a))/c^5 + (c^3*tan(1
/2*x*e + 1/2*d)^2 - 6*a*c^2*tan(1/2*x*e + 1/2*d))/c^6 - (18*a^2*c^2*tan(1/2
*x*e + 1/2*d)^2 + 6*c^4*tan(1/2*x*e + 1/2*d)^2 + 28*a^3*c*tan(1/2*x*e + 1/2
*d) + 4*a*c^3*tan(1/2*x*e + 1/2*d) + 11*a^4 + c^4)/((c*tan(1/2*x*e + 1/2*d)
 + a)^2*c^5))*e^(-1)
```

maple [A] time = 0.56, size = 211, normalized size = 1.57

$$\frac{\tan^2\left(\frac{d}{2}+\frac{ex}{2}\right)}{64e\,c^3}-\frac{3\tan\left(\frac{d}{2}+\frac{ex}{2}\right)a}{32e\,c^4}-\frac{a^4}{64e\,c^5\left(a+c\tan\left(\frac{d}{2}+\frac{ex}{2}\right)\right)^2}-\frac{a^2}{32e\,c^3\left(a+c\tan\left(\frac{d}{2}+\frac{ex}{2}\right)\right)^2}-\frac{1}{64ec\left(a+c\tan\left(\frac{d}{2}+\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2*a+2*a*cos(e*x+d)+2*c*sin(e*x+d))^3,x)
```

```
[Out] 1/64/e/c^3*tan(1/2*d+1/2*e*x)^2-3/32/e/c^4*tan(1/2*d+1/2*e*x)*a-1/64/e/c^5/
(a+c*tan(1/2*d+1/2*e*x))^2*a^4-1/32/e/c^3/(a+c*tan(1/2*d+1/2*e*x))^2*a^2-1/
64/e/c/(a+c*tan(1/2*d+1/2*e*x))^2+1/8/e*a^3/c^5/(a+c*tan(1/2*d+1/2*e*x))+1/
8/e*a/c^3/(a+c*tan(1/2*d+1/2*e*x))+3/16/e/c^5*ln(a+c*tan(1/2*d+1/2*e*x))*a^
2+1/16/e/c^3*ln(a+c*tan(1/2*d+1/2*e*x))
```

maxima [A] time = 0.34, size = 190, normalized size = 1.42

$$\frac{\frac{7a^4+6a^2c^2-c^4+\frac{8(a^3c+ac^3)\sin(ex+d)}{\cos(ex+d)+1}}{a^2c^5+\frac{2ac^6\sin(ex+d)}{\cos(ex+d)+1}+\frac{c^7\sin(ex+d)^2}{(\cos(ex+d)+1)^2}}-\frac{\frac{6a\sin(ex+d)}{\cos(ex+d)+1}-\frac{c\sin(ex+d)^2}{(\cos(ex+d)+1)^2}}{c^4}+\frac{4(3a^2+c^2)\log\left(a+\frac{c\sin(ex+d)}{\cos(ex+d)+1}\right)}{c^5}}{64e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*a+2*a*cos(e*x+d)+2*c*sin(e*x+d))^3,x, algorithm="maxima")
```

```
[Out] 1/64*((7*a^4 + 6*a^2*c^2 - c^4 + 8*(a^3*c + a*c^3)*sin(e*x + d)/(cos(e*x +
d) + 1))/(a^2*c^5 + 2*a*c^6*sin(e*x + d)/(cos(e*x + d) + 1) + c^7*sin(e*x +
 d)^2/(cos(e*x + d) + 1)^2) - (6*a*sin(e*x + d)/(cos(e*x + d) + 1) - c*sin(
e*x + d)^2/(cos(e*x + d) + 1)^2)/c^4 + 4*(3*a^2 + c^2)*log(a + c*sin(e*x +
d)/(cos(e*x + d) + 1))/c^5)/e
```

mupad [B] time = 2.48, size = 162, normalized size = 1.21

$$\frac{\tan\left(\frac{d}{2}+\frac{e\,x}{2}\right)^2}{64\,c^3\,e}+\frac{\tan\left(\frac{d}{2}+\frac{e\,x}{2}\right)\,\left(4\,a^3+4\,a\,c^2\right)+\frac{7\,a^4+6\,a^2\,c^2-c^4}{2\,c}}{e\,\left(32\,a^2\,c^4+64\,a\,c^5\,\tan\left(\frac{d}{2}+\frac{e\,x}{2}\right)+32\,c^6\,\tan\left(\frac{d}{2}+\frac{e\,x}{2}\right)^2\right)}-\frac{3\,a\,\tan\left(\frac{d}{2}+\frac{e\,x}{2}\right)}{32\,c^4\,e}+\frac{\ln\left(a+c\,\tan\left(\frac{d}{2}+\frac{e}{2}\right.\right.}{16\,c^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2*a + 2*a*cos(d + e*x) + 2*c*sin(d + e*x))^3,x)
```

```
[Out] tan(d/2 + (e*x)/2)^2/(64*c^3*e) + (tan(d/2 + (e*x)/2)*(4*a*c^2 + 4*a^3) + (
7*a^4 - c^4 + 6*a^2*c^2)/(2*c))/(e*(32*c^6*tan(d/2 + (e*x)/2)^2 + 32*a^2*c^
4 + 64*a*c^5*tan(d/2 + (e*x)/2))) - (3*a*tan(d/2 + (e*x)/2))/(32*c^4*e) + (
log(a + c*tan(d/2 + (e*x)/2))*(3*a^2 + c^2))/(16*c^5*e)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*a+2*a*cos(e*x+d)+2*c*sin(e*x+d))**3,x)
```

```
[Out] Timed out
```

3.369 $\int \frac{1}{(2a+2a\cos(d+ex)+2c\sin(d+ex))^4}\,dx$

Optimal. Leaf size=207

$$\frac{5\left(ac\cos(d+ex)-a^2\sin(d+ex)\right)}{96c^4e(a\cos(d+ex)+a+c\sin(d+ex))^2}-\frac{a\left(5a^2+3c^2\right)\log\left(a+c\tan\left(\frac{1}{2}(d+ex)\right)\right)}{32c^7e}-\frac{c\left(15a^2+4c^2\right)\cos(d+ex)}{96c^6e(a\cos(d+ex)}$$

```
[Out] -1/32*a*(5*a^2+3*c^2)*ln(a+c*tan(1/2*e*x+1/2*d))/c^7/e+1/48*(-c*cos(e*x+d)+
a*sin(e*x+d))/c^2/e/(a+a*cos(e*x+d)+c*sin(e*x+d))^3+5/96*(a*c*cos(e*x+d)-a^
2*sin(e*x+d))/c^4/e/(a+a*cos(e*x+d)+c*sin(e*x+d))^2+1/96*(-c*(15*a^2+4*c^2)
*cos(e*x+d)+a*(15*a^2+4*c^2)*sin(e*x+d))/c^6/e/(a+a*cos(e*x+d)+c*sin(e*x+d)
)
```

Rubi [A] time = 0.25, antiderivative size = 207, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {3129, 3156, 3153, 3124, 31}

$$-\frac{a\left(5a^2+3c^2\right)\log\left(a+c\tan\left(\frac{1}{2}(d+ex)\right)\right)}{32c^7e}-\frac{c\left(15a^2+4c^2\right)\cos(d+ex)-a\left(15a^2+4c^2\right)\sin(d+ex)}{96c^6e(a\cos(d+ex)+a+c\sin(d+ex))}+\frac{5\left(ac\cos\right.}{96c^4e(a\cos}$$

Antiderivative was successfully verified.

```
[In] Int[(2*a + 2*a*Cos[d + e*x] + 2*c*Sin[d + e*x])^(-4),x]
```

```
[Out] -(a*(5*a^2 + 3*c^2)*Log[a + c*Tan[(d + e*x)/2]])/(32*c^7*e) - (c*Cos[d + e*
x] - a*Sin[d + e*x])/(48*c^2*e*(a + a*Cos[d + e*x] + c*Sin[d + e*x])^3) + (
5*(a*c*Cos[d + e*x] - a^2*Sin[d + e*x]))/(96*c^4*e*(a + a*Cos[d + e*x] + c*
Sin[d + e*x])^2) - (c*(15*a^2 + 4*c^2)*Cos[d + e*x] - a*(15*a^2 + 4*c^2)*Si
n[d + e*x])/(96*c^6*e*(a + a*Cos[d + e*x] + c*Sin[d + e*x]))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 3124

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(-1), x_Symbol] :> Module[{f = FreeFactors[Tan[(d + e*x)/2], x]}, Dist[(2*f
)/e, Subst[Int[1/(a + b + 2*c*f*x + (a - b)*f^2*x^2), x], x, Tan[(d + e*x)/
2]/f], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0]
```

Rule 3129

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(n_), x_Symbol] :> Simp[((-(c*Cos[d + e*x]) + b*Sin[d + e*x])*(a + b*Cos[d
+ e*x] + c*Sin[d + e*x])^(n + 1))/(e*(n + 1)*(a^2 - b^2 - c^2)), x] + Dist[
1/((n + 1)*(a^2 - b^2 - c^2)), Int[(a*(n + 1) - b*(n + 2)*Cos[d + e*x] - c*
(n + 2)*Sin[d + e*x])*(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n + 1), x], x]
 /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0] && LtQ[n, -1] && N
eQ[n, -3/2]
```

Rule 3153

```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])
/((a_.) + cos[(d_.) + (e_.)*(x_)]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)])^2,
 x_Symbol] :> Simp[(c*B - b*C - (a*C - c*A)*Cos[d + e*x] + (a*B - b*A)*Sin[
d + e*x])/(e*(a^2 - b^2 - c^2)*(a + b*Cos[d + e*x] + c*Sin[d + e*x])), x] +
 Dist[(a*A - b*B - c*C)/(a^2 - b^2 - c^2), Int[1/(a + b*Cos[d + e*x] + c*Si
n[d + e*x]), x], x] /; FreeQ[{a, b, c, d, e, A, B, C}, x] && NeQ[a^2 - b^2
- c^2, 0] && NeQ[a*A - b*B - c*C, 0]
```

Rule 3156

```
Int[((a_.) + cos[(d_.) + (e_.)*(x_)]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)])
^(n_)*((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)
]), x_Symbol] :> -Simp[((c*B - b*C - (a*C - c*A)*Cos[d + e*x] + (a*B - b*A)
*Sin[d + e*x])*(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n + 1))/(e*(n + 1)*(a
^2 - b^2 - c^2)), x] + Dist[1/((n + 1)*(a^2 - b^2 - c^2)), Int[(a + b*Cos[d
 + e*x] + c*Sin[d + e*x])^(n + 1)*Simp[(n + 1)*(a*A - b*B - c*C) + (n + 2)*
(a*B - b*A)*Cos[d + e*x] + (n + 2)*(a*C - c*A)*Sin[d + e*x], x], x], x] /;
FreeQ[{a, b, c, d, e, A, B, C}, x] && LtQ[n, -1] && NeQ[a^2 - b^2 - c^2, 0]
 && NeQ[n, -2]
```

Rubi steps

$$\int \frac{1}{(2a+2a\cos(d+ex)+2c\sin(d+ex))^4}\,dx = -\frac{c\cos(d+ex)-a\sin(d+ex)}{48c^2e(a+a\cos(d+ex)+c\sin(d+ex))^3} + \frac{\int \frac{-6a+4a\cos(d+ex)+}{(2a+2a\cos(d+ex)+2}}{12c^2}$$

$$= -\frac{c\cos(d+ex)-a\sin(d+ex)}{48c^2e(a+a\cos(d+ex)+c\sin(d+ex))^3} + \frac{5\left(ac\cos(d+e\right.}{96c^4e(a+a\cos(d}$$

$$= -\frac{c\cos(d+ex)-a\sin(d+ex)}{48c^2e(a+a\cos(d+ex)+c\sin(d+ex))^3} + \frac{5\left(ac\cos(d+e\right.}{96c^4e(a+a\cos(d}$$

$$= -\frac{c\cos(d+ex)-a\sin(d+ex)}{48c^2e(a+a\cos(d+ex)+c\sin(d+ex))^3} + \frac{5\left(ac\cos(d+e\right.}{96c^4e(a+a\cos(d}$$

$$= -\frac{a(5a^2+3c^2)\log\left(a+c\tan\left(\frac{1}{2}(d+ex)\right)\right)}{32c^7e} - \frac{c\cos(d+ex}{48c^2e(a+a\cos(d}$$

Mathematica [B] time = 1.73, size = 492, normalized size = 2.38

$$\frac{\cos\left(\frac{1}{2}(d+ex)\right)\left(a\cos\left(\frac{1}{2}(d+ex)\right)+c\sin\left(\frac{1}{2}(d+ex)\right)\right)\left(192\left(5a^3+3ac^2\right)\cos^3\left(\frac{1}{2}(d+ex)\right)\log\left(\cos\left(\frac{1}{2}(d+ex)\right)\right)\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2*a + 2*a*Cos[d + e*x] + 2*c*Sin[d + e*x])^(-4),x]
```

```
[Out] (Cos[(d + e*x)/2]*(a*Cos[(d + e*x)/2] + c*Sin[(d + e*x)/2])*(192*(5*a^3 + 3
*a*c^2)*Cos[(d + e*x)/2]^3*Log[Cos[(d + e*x)/2]]*(a*Cos[(d + e*x)/2] + c*Si
n[(d + e*x)/2])^3 - 192*(5*a^3 + 3*a*c^2)*Cos[(d + e*x)/2]^3*Log[a*Cos[(d +
 e*x)/2] + c*Sin[(d + e*x)/2]]*(a*Cos[(d + e*x)/2] + c*Sin[(d + e*x)/2])^3
+ (c*(150*a^5*c + 130*a^3*c^3 + 24*a*c^5 + 3*a*c*(25*a^4 + 25*a^2*c^2 - 4*c
^4)*Cos[d + e*x] - 6*(25*a^5*c + 15*a^3*c^3 + 4*a*c^5)*Cos[2*(d + e*x)] - 7
5*a^5*c*Cos[3*(d + e*x)] - 35*a^3*c^3*Cos[3*(d + e*x)] - 4*a*c^5*Cos[3*(d +
 e*x)] + 150*a^6*Sin[d + e*x] + 255*a^4*c^2*Sin[d + e*x] + 129*a^2*c^4*Sin[
d + e*x] + 12*c^6*Sin[d + e*x] + 120*a^6*Sin[2*(d + e*x)] + 72*a^4*c^2*Sin[
2*(d + e*x)] + 36*a^2*c^4*Sin[2*(d + e*x)] + 30*a^6*Sin[3*(d + e*x)] - 37*a
^4*c^2*Sin[3*(d + e*x)] - 27*a^2*c^4*Sin[3*(d + e*x)] - 4*c^6*Sin[3*(d + e*
x)]))/a))/(384*c^7*e*(a + a*Cos[d + e*x] + c*Sin[d + e*x])^4)
```

fricas [B] time = 1.05, size = 791, normalized size = 3.82

$$\frac{60a^4c^2+6a^2c^4-2\left(45a^4c^2-3a^2c^4-4c^6\right)\cos(ex+d)^3-12\left(10a^4c^2+a^2c^4\right)\cos(ex+d)^2+6\left(5a^4c^2-2a^2c\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*a+2*a*cos(e*x+d)+2*c*sin(e*x+d))^4,x, algorithm="fricas")
```

```
[Out] 1/192*(60*a^4*c^2 + 6*a^2*c^4 - 2*(45*a^4*c^2 - 3*a^2*c^4 - 4*c^6)*cos(e*x
+ d)^3 - 12*(10*a^4*c^2 + a^2*c^4)*cos(e*x + d)^2 + 6*(5*a^4*c^2 - 2*a^2*c^
4 - 2*c^6)*cos(e*x + d) - 3*(5*a^6 + 18*a^4*c^2 + 9*a^2*c^4 + (5*a^6 - 12*a
^4*c^2 - 9*a^2*c^4)*cos(e*x + d)^3 + 3*(5*a^6 - 2*a^4*c^2 - 3*a^2*c^4)*cos(
e*x + d)^2 + 3*(5*a^6 + 8*a^4*c^2 + 3*a^2*c^4)*cos(e*x + d) + (15*a^5*c + 1
4*a^3*c^3 + 3*a*c^5 + (15*a^5*c + 4*a^3*c^3 - 3*a*c^5)*cos(e*x + d)^2 + 6*(
5*a^5*c + 3*a^3*c^3)*cos(e*x + d))*sin(e*x + d))*log(a*c*sin(e*x + d) + 1/2
*a^2 + 1/2*c^2 + 1/2*(a^2 - c^2)*cos(e*x + d)) + 3*(5*a^6 + 18*a^4*c^2 + 9*
a^2*c^4 + (5*a^6 - 12*a^4*c^2 - 9*a^2*c^4)*cos(e*x + d)^3 + 3*(5*a^6 - 2*a^
4*c^2 - 3*a^2*c^4)*cos(e*x + d)^2 + 3*(5*a^6 + 8*a^4*c^2 + 3*a^2*c^4)*cos(e
*x + d) + (15*a^5*c + 14*a^3*c^3 + 3*a*c^5 + (15*a^5*c + 4*a^3*c^3 - 3*a*c^
5)*cos(e*x + d)^2 + 6*(5*a^5*c + 3*a^3*c^3)*cos(e*x + d))*sin(e*x + d))*log
(1/2*cos(e*x + d) + 1/2) + 2*(15*a^5*c + 14*a^3*c^3 + 6*a*c^5 + (15*a^5*c -
 41*a^3*c^3 - 12*a*c^5)*cos(e*x + d)^2 + 3*(10*a^5*c - 9*a^3*c^3 - a*c^5)*c
os(e*x + d))*sin(e*x + d))/((a^3*c^7 - 3*a*c^9)*e*cos(e*x + d)^3 + 3*(a^3*c
^7 - a*c^9)*e*cos(e*x + d)^2 + 3*(a^3*c^7 + a*c^9)*e*cos(e*x + d) + (a^3*c^
7 + 3*a*c^9)*e + (6*a^2*c^8*e*cos(e*x + d) + (3*a^2*c^8 - c^10)*e*cos(e*x +
 d)^2 + (3*a^2*c^8 + c^10)*e)*sin(e*x + d))
```

giac [A] time = 0.21, size = 304, normalized size = 1.47

$$-\frac{1}{384}\left(\frac{12\left(5a^3+3ac^2\right)\log\left(\left|c\tan\left(\frac{1}{2}xe+\frac{1}{2}d\right)+a\right|\right)}{c^7}-\frac{110a^3c^3\tan\left(\frac{1}{2}xe+\frac{1}{2}d\right)^3+66ac^5\tan\left(\frac{1}{2}xe+\frac{1}{2}d\right)^3+28\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*a+2*a*cos(e*x+d)+2*c*sin(e*x+d))^4,x, algorithm="giac")
```

```
[Out] -1/384*(12*(5*a^3 + 3*a*c^2)*log(abs(c*tan(1/2*x*e + 1/2*d) + a))/c^7 - (11
0*a^3*c^3*tan(1/2*x*e + 1/2*d)^3 + 66*a*c^5*tan(1/2*x*e + 1/2*d)^3 + 285*a^
4*c^2*tan(1/2*x*e + 1/2*d)^2 + 144*a^2*c^4*tan(1/2*x*e + 1/2*d)^2 - 9*c^6*t
an(1/2*x*e + 1/2*d)^2 + 249*a^5*c*tan(1/2*x*e + 1/2*d) + 108*a^3*c^3*tan(1/
2*x*e + 1/2*d) - 9*a*c^5*tan(1/2*x*e + 1/2*d) + 73*a^6 + 27*a^4*c^2 - 3*a^2
*c^4 - c^6)/((c*tan(1/2*x*e + 1/2*d) + a)^3*c^7) - (c^8*tan(1/2*x*e + 1/2*d
)^3 - 6*a*c^7*tan(1/2*x*e + 1/2*d)^2 + 30*a^2*c^6*tan(1/2*x*e + 1/2*d) + 9*
c^8*tan(1/2*x*e + 1/2*d))/c^12)*e^(-1)
```

maple [A] time = 0.60, size = 378, normalized size = 1.83

$$\frac{\tan^3\left(\frac{d}{2}+\frac{ex}{2}\right)}{384e\,c^4}-\frac{\left(\tan^2\left(\frac{d}{2}+\frac{ex}{2}\right)\right)a}{64e\,c^5}+\frac{5a^2\tan\left(\frac{d}{2}+\frac{ex}{2}\right)}{64e\,c^6}+\frac{3\tan\left(\frac{d}{2}+\frac{ex}{2}\right)}{128e\,c^4}+\frac{3a^5}{128e\,c^7\left(a+c\tan\left(\frac{d}{2}+\frac{ex}{2}\right)\right)^2}+\frac{}{64e\,c^5\left(a+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2*a+2*a*cos(e*x+d)+2*c*sin(e*x+d))^4,x)
```

```
[Out] 1/384/e/c^4*tan(1/2*d+1/2*e*x)^3-1/64/e/c^5*tan(1/2*d+1/2*e*x)^2*a+5/64/e/c
^6*a^2*tan(1/2*d+1/2*e*x)+3/128/e/c^4*tan(1/2*d+1/2*e*x)+3/128/e*a^5/c^7/(a
+c*tan(1/2*d+1/2*e*x))^2+3/64/e*a^3/c^5/(a+c*tan(1/2*d+1/2*e*x))^2+3/128/e*
a/c^3/(a+c*tan(1/2*d+1/2*e*x))^2-1/384/e/c^7/(a+c*tan(1/2*d+1/2*e*x))^3*a^6
-1/128/e/c^5/(a+c*tan(1/2*d+1/2*e*x))^3*a^4-1/128/e/c^3/(a+c*tan(1/2*d+1/2*
e*x))^3*a^2-1/384/e/c/(a+c*tan(1/2*d+1/2*e*x))^3-15/128/e/c^7/(a+c*tan(1/2*
d+1/2*e*x))*a^4-9/64/e/c^5/(a+c*tan(1/2*d+1/2*e*x))*a^2-3/128/e/c^3/(a+c*ta
n(1/2*d+1/2*e*x))-5/32/e*a^3/c^7*ln(a+c*tan(1/2*d+1/2*e*x))-3/32/e*a/c^5*ln
(a+c*tan(1/2*d+1/2*e*x))
```

maxima [A] time = 0.36, size = 307, normalized size = 1.48

$$-\frac{\frac{37\,a^6+39\,a^4c^2+3\,a^2c^4+c^6+\frac{9\left(9\,a^5c+10\,a^3c^3+ac^5\right)\sin(ex+d)}{\cos(ex+d)+1}+\frac{9\left(5\,a^4c^2+6\,a^2c^4+c^6\right)\sin(ex+d)^2}{(\cos(ex+d)+1)^2}}{a^3c^7+\frac{3\,a^2c^8\sin(ex+d)}{\cos(ex+d)+1}+\frac{3\,ac^9\sin(ex+d)^2}{(\cos(ex+d)+1)^2}+\frac{c^{10}\sin(ex+d)^3}{(\cos(ex+d)+1)^3}}+\frac{\frac{6\,ac\sin(ex+d)^2}{(\cos(ex+d)+1)^2}-\frac{c^2\sin(ex+d)^3}{(\cos(ex+d)+1)^3}-\frac{3\left(10\,a^2+3\,c^2\right)\sin(ex+d)}{\cos(ex+d)+1}}{c^6}+\frac{12}{}}{384\,e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*a+2*a*cos(e*x+d)+2*c*sin(e*x+d))^4,x, algorithm="maxima")
```

```
[Out] -1/384*((37*a^6 + 39*a^4*c^2 + 3*a^2*c^4 + c^6 + 9*(9*a^5*c + 10*a^3*c^3 +
a*c^5)*sin(e*x + d)/(cos(e*x + d) + 1) + 9*(5*a^4*c^2 + 6*a^2*c^4 + c^6)*si
n(e*x + d)^2/(cos(e*x + d) + 1)^2)/(a^3*c^7 + 3*a^2*c^8*sin(e*x + d)/(cos(e
*x + d) + 1) + 3*a*c^9*sin(e*x + d)^2/(cos(e*x + d) + 1)^2 + c^10*sin(e*x +
 d)^3/(cos(e*x + d) + 1)^3) + (6*a*c*sin(e*x + d)^2/(cos(e*x + d) + 1)^2 -
c^2*sin(e*x + d)^3/(cos(e*x + d) + 1)^3 - 3*(10*a^2 + 3*c^2)*sin(e*x + d)/(
cos(e*x + d) + 1))/c^6 + 12*(5*a^3 + 3*a*c^2)*log(a + c*sin(e*x + d)/(cos(e
*x + d) + 1))/c^7)/e
```

mupad [B] time = 2.53, size = 260, normalized size = 1.26

$$\frac{\tan\left(\frac{d}{2}+\frac{e\,x}{2}\right)^3}{384\,c^4\,e}+\frac{\tan\left(\frac{d}{2}+\frac{e\,x}{2}\right)\left(\frac{3}{128\,c^4}+\frac{5\,a^2}{64\,c^6}\right)}{e}-\frac{\tan\left(\frac{d}{2}+\frac{e\,x}{2}\right)\left(27\,a^5+30\,a^3\,c^2+3\,a\,c^4\right)+\frac{37\,a^6+39\,a^4\,c^2+3\,a^2\,c^4+c^6}{3\,c}+\mathrm{t}}{e\left(128\,a^3\,c^6+384\,a^2\,c^7\tan\left(\frac{d}{2}+\frac{e\,x}{2}\right)+384\,a\,c^8\tan\left(\frac{d}{2}+\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2*a + 2*a*cos(d + e*x) + 2*c*sin(d + e*x))^4,x)
```

```
[Out] tan(d/2 + (e*x)/2)^3/(384*c^4*e) + (tan(d/2 + (e*x)/2)*(3/(128*c^4) + (5*a^
2)/(64*c^6)))/e - (tan(d/2 + (e*x)/2)*(3*a*c^4 + 27*a^5 + 30*a^3*c^2) + (37
*a^6 + c^6 + 3*a^2*c^4 + 39*a^4*c^2)/(3*c) + tan(d/2 + (e*x)/2)^2*(15*a^4*c
 + 3*c^5 + 18*a^2*c^3))/(e*(128*c^9*tan(d/2 + (e*x)/2)^3 + 128*a^3*c^6 + 38
4*a^2*c^7*tan(d/2 + (e*x)/2) + 384*a*c^8*tan(d/2 + (e*x)/2)^2)) - (a*tan(d/
2 + (e*x)/2)^2)/(64*c^5*e) - (log(a + c*tan(d/2 + (e*x)/2))*(3*a*c^2 + 5*a^
3))/(32*c^7*e)
```

sympy **[F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*a+2*a*cos(e*x+d)+2*c*sin(e*x+d))**4,x)
```

```
[Out] Timed out
```

3.370 $\int \frac{1}{2a+2a\cos(d+ex)+2a\sin(d+ex)}\,dx$

Optimal. Leaf size=23

$$\frac{\log\left(\tan\left(\frac{1}{2}(d+ex)\right)+1\right)}{2ae}$$

```
[Out] 1/2*ln(1+tan(1/2*e*x+1/2*d))/a/e
```

Rubi [A] time = 0.02, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {3124, 31}

$$\frac{\log\left(\tan\left(\frac{1}{2}(d+ex)\right)+1\right)}{2ae}$$

Antiderivative was successfully verified.

```
[In] Int[(2*a + 2*a*Cos[d + e*x] + 2*a*Sin[d + e*x])^(-1),x]
```

```
[Out] Log[1 + Tan[(d + e*x)/2]]/(2*a*e)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 3124

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(-1), x_Symbol] :> Module[{f = FreeFactors[Tan[(d + e*x)/2], x]}, Dist[(2*f
)/e, Subst[Int[1/(a + b + 2*c*f*x + (a - b)*f^2*x^2), x], x, Tan[(d + e*x)/
2]/f], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{2a+2a\cos(d+ex)+2a\sin(d+ex)}\,dx &= \frac{2\,\text{Subst}\left(\int \frac{1}{4a+4ax}\,dx, x, \tan\left(\frac{1}{2}(d+ex)\right)\right)}{e}\\
&= \frac{\log\left(1+\tan\left(\frac{1}{2}(d+ex)\right)\right)}{2ae}
\end{aligned}$$

Mathematica [B] time = 0.03, size = 50, normalized size = 2.17

$$\frac{\frac{\log\left(\sin\left(\frac{1}{2}(d+ex)\right)+\cos\left(\frac{1}{2}(d+ex)\right)\right)}{e}-\frac{\log\left(\cos\left(\frac{1}{2}(d+ex)\right)\right)}{e}}{2a}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2*a + 2*a*Cos[d + e*x] + 2*a*Sin[d + e*x])^(-1),x]
[Out] (-(Log[Cos[(d + e*x)/2]]/e) + Log[Cos[(d + e*x)/2] + Sin[(d + e*x)/2]]/e)/(
2*a)
```

fricas [A] time = 1.17, size = 31, normalized size = 1.35

$$-\frac{\log\left(\frac{1}{2}\cos\left(ex+d\right)+\frac{1}{2}\right)-\log\left(\sin\left(ex+d\right)+1\right)}{4\,ae}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*a+2*a*cos(e*x+d)+2*a*sin(e*x+d)),x, algorithm="fricas")
[Out] -1/4*(log(1/2*cos(e*x + d) + 1/2) - log(sin(e*x + d) + 1))/(a*e)
```

giac [A] time = 0.16, size = 21, normalized size = 0.91

$$\frac{e^{(-1)}\log\left(\left|\tan\left(\frac{1}{2}\,xe+\frac{1}{2}\,d\right)+1\right|\right)}{2\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*a+2*a*cos(e*x+d)+2*a*sin(e*x+d)),x, algorithm="giac")
[Out] 1/2*e^(-1)*log(abs(tan(1/2*x*e + 1/2*d) + 1))/a
```

maple [A] time = 0.40, size = 21, normalized size = 0.91

$$\frac{\ln\left(1+\tan\left(\frac{d}{2}+\frac{ex}{2}\right)\right)}{2ae}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2*a+2*a*cos(e*x+d)+2*a*sin(e*x+d)),x)
[Out] 1/2*ln(1+tan(1/2*d+1/2*e*x))/a/e
```

maxima [A] time = 0.31, size = 28, normalized size = 1.22

$$\frac{\log\left(\frac{\sin(ex+d)}{\cos(ex+d)+1}+1\right)}{2\,ae}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*a+2*a*cos(e*x+d)+2*a*sin(e*x+d)),x, algorithm="maxima")
[Out] 1/2*log(sin(e*x + d)/(cos(e*x + d) + 1) + 1)/(a*e)
```

mupad [B] time = 2.49, size = 20, normalized size = 0.87

$$\frac{\ln\left(\tan\left(\frac{d}{2}+\frac{e\,x}{2}\right)+1\right)}{2\,a\,e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2*a + 2*a*cos(d + e*x) + 2*a*sin(d + e*x)),x)
[Out] log(tan(d/2 + (e*x)/2) + 1)/(2*a*e)
```

sympy [A] time = 0.61, size = 36, normalized size = 1.57

$$\begin{cases} \frac{\log\left(\tan\left(\frac{d}{2}+\frac{ex}{2}\right)+1\right)}{2ae} & \text{for}\ e \neq 0 \\ \frac{x}{2a\sin(d)+2a\cos(d)+2a} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*a+2*a*cos(e*x+d)+2*a*sin(e*x+d)),x)
[Out] Piecewise((log(tan(d/2 + e*x/2) + 1)/(2*a*e), Ne(e, 0)), (x/(2*a*sin(d) + 2
*a*cos(d) + 2*a), True))
```

3.371 $\int \frac{1}{(2a+2a\cos(d+ex)+2a\sin(d+ex))^2}\,dx$

Optimal. Leaf size=75

$$-\frac{a\cos(d+ex)-a\sin(d+ex)}{4e\left(a^3\sin(d+ex)+a^3\cos(d+ex)+a^3\right)}-\frac{\log\left(\tan\left(\frac{1}{2}(d+ex)\right)+1\right)}{4a^2e}$$

```
[Out] -1/4*ln(1+tan(1/2*e*x+1/2*d))/a^2/e+1/4*(-a*cos(e*x+d)+a*sin(e*x+d))/e/(a^3
+a^3*cos(e*x+d)+a^3*sin(e*x+d))
```

Rubi [A] time = 0.05, antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3129, 12, 3124, 31}

$$-\frac{\log\left(\tan\left(\frac{1}{2}(d+ex)\right)+1\right)}{4a^2e}-\frac{a\cos(d+ex)-a\sin(d+ex)}{4e\left(a^3\sin(d+ex)+a^3\cos(d+ex)+a^3\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(2*a + 2*a*Cos[d + e*x] + 2*a*Sin[d + e*x])^(-2),x]
```

```
[Out] -Log[1 + Tan[(d + e*x)/2]]/(4*a^2*e) - (a*Cos[d + e*x] - a*Sin[d + e*x])/(4
*e*(a^3 + a^3*Cos[d + e*x] + a^3*Sin[d + e*x]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 3124

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(-1), x_Symbol] :> Module[{f = FreeFactors[Tan[(d + e*x)/2], x]}, Dist[(2*f
)/e, Subst[Int[1/(a + b + 2*c*f*x + (a - b)*f^2*x^2), x], x, Tan[(d + e*x)/
2]/f], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0]
```

Rule 3129

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(n_), x_Symbol] :> Simp[((-(c*Cos[d + e*x]) + b*Sin[d + e*x])*(a + b*Cos[d
+ e*x] + c*Sin[d + e*x])^(n + 1))/(e*(n + 1)*(a^2 - b^2 - c^2)), x] + Dist[
1/((n + 1)*(a^2 - b^2 - c^2)), Int[(a*(n + 1) - b*(n + 2)*Cos[d + e*x] - c*
(n + 2)*Sin[d + e*x])*(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n + 1), x], x]
 /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0] && LtQ[n, -1] && N
eQ[n, -3/2]
```

Rubi steps

$$\int \frac{1}{(2a+2a\cos(d+ex)+2a\sin(d+ex))^2}\,dx = -\frac{a\cos(d+ex)-a\sin(d+ex)}{4e\left(a^3+a^3\cos(d+ex)+a^3\sin(d+ex)\right)} + \frac{\int -\frac{2a}{2a+2a\cos(d+ex)+\cdots}}{4a^2}$$

$$= -\frac{a\cos(d+ex)-a\sin(d+ex)}{4e\left(a^3+a^3\cos(d+ex)+a^3\sin(d+ex)\right)} - \frac{\int \frac{1}{2a+2a\cos(d+ex)+2a\cdots}}{2a}$$

$$= -\frac{a\cos(d+ex)-a\sin(d+ex)}{4e\left(a^3+a^3\cos(d+ex)+a^3\sin(d+ex)\right)} - \frac{\text{Subst}\left(\int \frac{1}{4a+4ax}\,d\cdots\right.}{\cdots}$$

$$= -\frac{\log\left(1+\tan\left(\frac{1}{2}(d+ex)\right)\right)}{4a^2e} - \frac{a\cos(d+ex)-a\sin(d+e\cdots}{4e\left(a^3+a^3\cos(d+ex)+a^3\sin(\cdots\right.}$$

Mathematica [A] time = 0.19, size = 93, normalized size = 1.24

$$\frac{\tan\left(\frac{1}{2}(d+ex)\right)+2\log\left(\cos\left(\frac{1}{2}(d+ex)\right)\right)+\frac{2\sin\left(\frac{1}{2}(d+ex)\right)}{\sin\left(\frac{1}{2}(d+ex)\right)+\cos\left(\frac{1}{2}(d+ex)\right)}-2\log\left(\sin\left(\frac{1}{2}(d+ex)\right)+\cos\left(\frac{1}{2}(d+ex)\right)\right)}{8a^2e}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2*a + 2*a*Cos[d + e*x] + 2*a*Sin[d + e*x])^(-2),x]
```

```
[Out] (2*Log[Cos[(d + e*x)/2]] - 2*Log[Cos[(d + e*x)/2] + Sin[(d + e*x)/2]] + (2*
Sin[(d + e*x)/2])/(Cos[(d + e*x)/2] + Sin[(d + e*x)/2]) + Tan[(d + e*x)/2])
/(8*a^2*e)
```

fricas [A] time = 1.21, size = 100, normalized size = 1.33

$$\frac{(\cos(ex+d)+\sin(ex+d)+1)\log\left(\frac{1}{2}\cos(ex+d)+\frac{1}{2}\right)-(\cos(ex+d)+\sin(ex+d)+1)\log(\sin(ex+d)+1\cdots}{8\left(a^2e\cos(ex+d)+a^2e\sin(ex+d)+a^2e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*a+2*a*cos(e*x+d)+2*a*sin(e*x+d))^2,x, algorithm="fricas")
```

```
[Out] 1/8*((cos(e*x + d) + sin(e*x + d) + 1)*log(1/2*cos(e*x + d) + 1/2) - (cos(e
*x + d) + sin(e*x + d) + 1)*log(sin(e*x + d) + 1) - 2*cos(e*x + d) + 2*sin(
e*x + d))/(a^2*e*cos(e*x + d) + a^2*e*sin(e*x + d) + a^2*e)
```

giac [A] time = 0.17, size = 68, normalized size = 0.91

$$-\frac{1}{8}\left(\frac{2\log\left(\left|\tan\left(\frac{1}{2}xe+\frac{1}{2}d\right)+1\right|\right)}{a^2}-\frac{\tan\left(\frac{1}{2}xe+\frac{1}{2}d\right)}{a^2}-\frac{2\tan\left(\frac{1}{2}xe+\frac{1}{2}d\right)}{a^2\left(\tan\left(\frac{1}{2}xe+\frac{1}{2}d\right)+1\right)}\right)e^{(-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*a+2*a*cos(e*x+d)+2*a*sin(e*x+d))^2,x, algorithm="giac")
```

```
[Out] -1/8*(2*log(abs(tan(1/2*x*e + 1/2*d) + 1))/a^2 - tan(1/2*x*e + 1/2*d)/a^2 -
 2*tan(1/2*x*e + 1/2*d)/(a^2*(tan(1/2*x*e + 1/2*d) + 1)))*e^(-1)
```

maple [A] time = 0.41, size = 60, normalized size = 0.80

$$\frac{\tan\left(\frac{d}{2}+\frac{ex}{2}\right)}{8e\,a^2}-\frac{1}{4e\,a^2\left(1+\tan\left(\frac{d}{2}+\frac{ex}{2}\right)\right)}-\frac{\ln\left(1+\tan\left(\frac{d}{2}+\frac{ex}{2}\right)\right)}{4a^2e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2*a+2*a*cos(e*x+d)+2*a*sin(e*x+d))^2,x)
```

```
[Out] 1/8/e/a^2*tan(1/2*d+1/2*e*x)-1/4/e/a^2/(1+tan(1/2*d+1/2*e*x))-1/4*ln(1+tan(
1/2*d+1/2*e*x))/a^2/e
```

maxima [A] time = 0.32, size = 80, normalized size = 1.07

$$-\frac{\frac{2}{a^2+\frac{a^2\sin(ex+d)}{\cos(ex+d)+1}}+\frac{2\log\left(\frac{\sin(ex+d)}{\cos(ex+d)+1}+1\right)}{a^2}-\frac{\sin(ex+d)}{a^2(\cos(ex+d)+1)}}{8\,e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*a+2*a*cos(e*x+d)+2*a*sin(e*x+d))^2,x, algorithm="maxima")
```

```
[Out] -1/8*(2/(a^2 + a^2*sin(e*x + d)/(cos(e*x + d) + 1)) + 2*log(sin(e*x + d)/(c
os(e*x + d) + 1) + 1)/a^2 - sin(e*x + d)/(a^2*(cos(e*x + d) + 1)))/e
```

mupad [B] time = 2.45, size = 59, normalized size = 0.79

$$\frac{\tan\left(\frac{d}{2}+\frac{e\,x}{2}\right)}{8\,a^2\,e}-\frac{\ln\left(\tan\left(\frac{d}{2}+\frac{e\,x}{2}\right)+1\right)}{4\,a^2\,e}-\frac{1}{4\,a^2\,e\,\left(\tan\left(\frac{d}{2}+\frac{e\,x}{2}\right)+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2*a + 2*a*cos(d + e*x) + 2*a*sin(d + e*x))^2,x)
```

```
[Out] tan(d/2 + (e*x)/2)/(8*a^2*e) - log(tan(d/2 + (e*x)/2) + 1)/(4*a^2*e) - 1/(4
*a^2*e*(tan(d/2 + (e*x)/2) + 1))
```

sympy [A] time = 1.83, size = 168, normalized size = 2.24

$$\begin{cases} -\frac{2\log\left(\tan\left(\frac{d}{2}+\frac{ex}{2}\right)+1\right)\tan\left(\frac{d}{2}+\frac{ex}{2}\right)}{8a^2e\tan\left(\frac{d}{2}+\frac{ex}{2}\right)+8a^2e}-\frac{2\log\left(\tan\left(\frac{d}{2}+\frac{ex}{2}\right)+1\right)}{8a^2e\tan\left(\frac{d}{2}+\frac{ex}{2}\right)+8a^2e}+\frac{\tan^2\left(\frac{d}{2}+\frac{ex}{2}\right)}{8a^2e\tan\left(\frac{d}{2}+\frac{ex}{2}\right)+8a^2e}-\frac{3}{8a^2e\tan\left(\frac{d}{2}+\frac{ex}{2}\right)+8a^2e} & \text{for } e\neq 0 \\ \frac{x}{(2a\sin(d)+2a\cos(d)+2a)^2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*a+2*a*cos(e*x+d)+2*a*sin(e*x+d))**2,x)
```

```
[Out] Piecewise((-2*log(tan(d/2 + e*x/2) + 1)*tan(d/2 + e*x/2)/(8*a**2*e*tan(d/2
+ e*x/2) + 8*a**2*e) - 2*log(tan(d/2 + e*x/2) + 1)/(8*a**2*e*tan(d/2 + e*x/
2) + 8*a**2*e) + tan(d/2 + e*x/2)**2/(8*a**2*e*tan(d/2 + e*x/2) + 8*a**2*e)
 - 3/(8*a**2*e*tan(d/2 + e*x/2) + 8*a**2*e), Ne(e, 0)), (x/(2*a*sin(d) + 2*
a*cos(d) + 2*a)**2, True))
```

3.372 $\int \frac{1}{(2a+2a\cos(d+ex)+2a\sin(d+ex))^3}\,dx$

Optimal. Leaf size=123

$$\frac{\log\left(\tan\left(\frac{1}{2}(d+ex)\right)+1\right)}{4a^3e}+\frac{3(\cos(d+ex)-\sin(d+ex))}{16e\left(a^3\sin(d+ex)+a^3\cos(d+ex)+a^3\right)}-\frac{a\cos(d+ex)-a\sin(d+ex)}{16e\left(a^2\sin(d+ex)+a^2\cos(d+ex)+a^2\right)^2}$$

```
[Out] 1/4*ln(1+tan(1/2*e*x+1/2*d))/a^3/e+1/16*(-a*cos(e*x+d)+a*sin(e*x+d))/e/(a^2
+a^2*cos(e*x+d)+a^2*sin(e*x+d))^2+3/16*(cos(e*x+d)-sin(e*x+d))/e/(a^3+a^3*c
os(e*x+d)+a^3*sin(e*x+d))
```

Rubi [A] time = 0.11, antiderivative size = 123, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3129, 3153, 3124, 31}

$$\frac{\log\left(\tan\left(\frac{1}{2}(d+ex)\right)+1\right)}{4a^3e}+\frac{3(\cos(d+ex)-\sin(d+ex))}{16e\left(a^3\sin(d+ex)+a^3\cos(d+ex)+a^3\right)}-\frac{a\cos(d+ex)-a\sin(d+ex)}{16e\left(a^2\sin(d+ex)+a^2\cos(d+ex)+a^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(2*a + 2*a*Cos[d + e*x] + 2*a*Sin[d + e*x])^(-3),x]
```

```
[Out] Log[1 + Tan[(d + e*x)/2]]/(4*a^3*e) - (a*Cos[d + e*x] - a*Sin[d + e*x])/(16
*e*(a^2 + a^2*Cos[d + e*x] + a^2*Sin[d + e*x])^2) + (3*(Cos[d + e*x] - Sin[
d + e*x]))/(16*e*(a^3 + a^3*Cos[d + e*x] + a^3*Sin[d + e*x]))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 3124

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(-1), x_Symbol] :> Module[{f = FreeFactors[Tan[(d + e*x)/2], x]}, Dist[(2*f
)/e, Subst[Int[1/(a + b + 2*c*f*x + (a - b)*f^2*x^2), x], x, Tan[(d + e*x)/
2]/f], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0]
```

Rule 3129

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(n_), x_Symbol] :> Simp[((-(c*Cos[d + e*x]) + b*Sin[d + e*x])*(a + b*Cos[d
+ e*x] + c*Sin[d + e*x])^(n + 1))/(e*(n + 1)*(a^2 - b^2 - c^2)), x] + Dist[
```

```
1/((n + 1)*(a^2 - b^2 - c^2)), Int[(a*(n + 1) - b*(n + 2)*Cos[d + e*x] - c*
(n + 2)*Sin[d + e*x])*(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n + 1), x], x]
 /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0] && LtQ[n, -1] && N
eQ[n, -3/2]
```

Rule 3153

```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])
/((a_.) + cos[(d_.) + (e_.)*(x_)]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)])^2,
 x_Symbol] :> Simp[(c*B - b*C - (a*C - c*A)*Cos[d + e*x] + (a*B - b*A)*Sin[
d + e*x])/(e*(a^2 - b^2 - c^2)*(a + b*Cos[d + e*x] + c*Sin[d + e*x])), x] +
 Dist[(a*A - b*B - c*C)/(a^2 - b^2 - c^2), Int[1/(a + b*Cos[d + e*x] + c*Si
n[d + e*x]), x], x] /; FreeQ[{a, b, c, d, e, A, B, C}, x] && NeQ[a^2 - b^2
- c^2, 0] && NeQ[a*A - b*B - c*C, 0]
```

Rubi steps

$$\int \frac{1}{(2a+2a\cos(d+ex)+2a\sin(d+ex))^3}\,dx = -\frac{a\cos(d+ex)-a\sin(d+ex)}{16e\left(a^2+a^2\cos(d+ex)+a^2\sin(d+ex)\right)^2}+\frac{\int\frac{-4a+2a\cos(d+ex)}{(2a+2a\cos(d+ex)}}{8a}$$

$$= -\frac{a\cos(d+ex)-a\sin(d+ex)}{16e\left(a^2+a^2\cos(d+ex)+a^2\sin(d+ex)\right)^2}+\frac{3(\cos(d}{16e\left(a^3+a^3\cos\right.}$$

$$= -\frac{a\cos(d+ex)-a\sin(d+ex)}{16e\left(a^2+a^2\cos(d+ex)+a^2\sin(d+ex)\right)^2}+\frac{3(\cos(d}{16e\left(a^3+a^3\cos\right.}$$

$$= \frac{\log\left(1+\tan\left(\frac{1}{2}(d+ex)\right)\right)}{4a^3e}-\frac{a\cos(d+ex)-a\sin(d+ex}{16e\left(a^2+a^2\cos(d+ex)+a^2\sin(d\right.}$$

Mathematica [A] time = 0.57, size = 135, normalized size = 1.10

$$\frac{\sec^2\left(\frac{1}{2}(d+ex)\right)+2\left(-3\tan\left(\frac{1}{2}(d+ex)\right)-8\log\left(\cos\left(\frac{1}{2}(d+ex)\right)\right)-\frac{6\sin\left(\frac{1}{2}(d+ex)\right)}{\sin\left(\frac{1}{2}(d+ex)\right)+\cos\left(\frac{1}{2}(d+ex)\right)}-\frac{1}{\left(\sin\left(\frac{1}{2}(d+ex)\right)+\cos\left(\frac{1}{2}(\right.\right.}\right.}{64a^3e}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2*a + 2*a*Cos[d + e*x] + 2*a*Sin[d + e*x])^(-3),x]
```

```
[Out] (Sec[(d + e*x)/2]^2 + 2*(-8*Log[Cos[(d + e*x)/2]] + 8*Log[Cos[(d + e*x)/2]
+ Sin[(d + e*x)/2]] - (Cos[(d + e*x)/2] + Sin[(d + e*x)/2])^(-2) - (6*Sin[(
d + e*x)/2])/(Cos[(d + e*x)/2] + Sin[(d + e*x)/2]) - 3*Tan[(d + e*x)/2]))/(
64*a^3*e)
```

fricas [A] time = 1.76, size = 143, normalized size = 1.16

$$\frac{6 \cos(ex+d)^2 - 4((\cos(ex+d)+1)\sin(ex+d)+\cos(ex+d)+1)\log\left(\frac{1}{2}\cos(ex+d)+\frac{1}{2}\right) + 4((\cos(ex+d)}{32\left(a^3e\cos(ex+d)+a^3e+\left(a^3e\cos(ex+\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*a+2*a*cos(e*x+d)+2*a*sin(e*x+d))^3,x, algorithm="fricas")
```

```
[Out] 1/32*(6*cos(e*x + d)^2 - 4*((cos(e*x + d) + 1)*sin(e*x + d) + cos(e*x + d)
+ 1)*log(1/2*cos(e*x + d) + 1/2) + 4*((cos(e*x + d) + 1)*sin(e*x + d) + cos
(e*x + d) + 1)*log(sin(e*x + d) + 1) + 2*cos(e*x + d) - 2*sin(e*x + d) - 3)
/(a^3*e*cos(e*x + d) + a^3*e + (a^3*e*cos(e*x + d) + a^3*e)*sin(e*x + d))
```

giac [A] time = 0.16, size = 107, normalized size = 0.87

$$\frac{1}{64}\left(\frac{16\log\left(\left|\tan\left(\frac{1}{2}xe+\frac{1}{2}d\right)+1\right|\right)}{a^3} - \frac{4\left(6\tan\left(\frac{1}{2}xe+\frac{1}{2}d\right)^2+8\tan\left(\frac{1}{2}xe+\frac{1}{2}d\right)+3\right)}{a^3\left(\tan\left(\frac{1}{2}xe+\frac{1}{2}d\right)+1\right)^2} + \frac{a^3\tan\left(\frac{1}{2}xe+\frac{1}{2}d\right)^2-6}{a^6}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*a+2*a*cos(e*x+d)+2*a*sin(e*x+d))^3,x, algorithm="giac")
```

```
[Out] 1/64*(16*log(abs(tan(1/2*x*e + 1/2*d) + 1))/a^3 - 4*(6*tan(1/2*x*e + 1/2*d)
^2 + 8*tan(1/2*x*e + 1/2*d) + 3)/(a^3*(tan(1/2*x*e + 1/2*d) + 1)^2) + (a^3*
tan(1/2*x*e + 1/2*d)^2 - 6*a^3*tan(1/2*x*e + 1/2*d))/a^6)*e^(-1)
```

maple [A] time = 0.45, size = 100, normalized size = 0.81

$$\frac{\tan^2\left(\frac{d}{2}+\frac{ex}{2}\right)}{64a^3e} - \frac{3\tan\left(\frac{d}{2}+\frac{ex}{2}\right)}{32a^3e} - \frac{1}{16a^3e\left(1+\tan\left(\frac{d}{2}+\frac{ex}{2}\right)\right)^2} + \frac{1}{4a^3e\left(1+\tan\left(\frac{d}{2}+\frac{ex}{2}\right)\right)} + \frac{\ln\left(1+\tan\left(\frac{d}{2}+\frac{ex}{2}\right)\right)}{4a^3e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2*a+2*a*cos(e*x+d)+2*a*sin(e*x+d))^3,x)
```

```
[Out] 1/64/a^3/e*tan(1/2*d+1/2*e*x)^2-3/32/a^3/e*tan(1/2*d+1/2*e*x)-1/16/a^3/e/(1
+tan(1/2*d+1/2*e*x))^2+1/4/a^3/e/(1+tan(1/2*d+1/2*e*x))+1/4*ln(1+tan(1/2*d+
1/2*e*x))/a^3/e
```

maxima [A] time = 0.34, size = 146, normalized size = 1.19

$$\frac{\frac{4\left(\frac{4\sin(ex+d)}{\cos(ex+d)+1}+3\right)}{a^3+\frac{2a^3\sin(ex+d)}{\cos(ex+d)+1}+\frac{a^3\sin(ex+d)^2}{(\cos(ex+d)+1)^2}}-\frac{\frac{6\sin(ex+d)}{\cos(ex+d)+1}-\frac{\sin(ex+d)^2}{(\cos(ex+d)+1)^2}}{a^3}+\frac{16\log\left(\frac{\sin(ex+d)}{\cos(ex+d)+1}+1\right)}{a^3}}{64e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*a+2*a*cos(e*x+d)+2*a*sin(e*x+d))^3,x, algorithm="maxima")
```

```
[Out] 1/64*(4*(4*sin(e*x + d)/(cos(e*x + d) + 1) + 3)/(a^3 + 2*a^3*sin(e*x + d)/(
cos(e*x + d) + 1) + a^3*sin(e*x + d)^2/(cos(e*x + d) + 1)^2) - (6*sin(e*x +
 d)/(cos(e*x + d) + 1) - sin(e*x + d)^2/(cos(e*x + d) + 1)^2)/a^3 + 16*log(
sin(e*x + d)/(cos(e*x + d) + 1) + 1)/a^3)/e
```

mupad [B] time = 2.45, size = 90, normalized size = 0.73

$$\frac{\tan\left(\frac{d}{2}+\frac{e\,x}{2}\right)^2}{64\,a^3\,e}+\frac{\ln\left(\tan\left(\frac{d}{2}+\frac{e\,x}{2}\right)+1\right)}{4\,a^3\,e}-\frac{3\tan\left(\frac{d}{2}+\frac{e\,x}{2}\right)}{32\,a^3\,e}+\frac{\frac{\tan\left(\frac{d}{2}+\frac{e\,x}{2}\right)}{4}+\frac{3}{16}}{a^3\,e\left(\tan\left(\frac{d}{2}+\frac{e\,x}{2}\right)+1\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2*a + 2*a*cos(d + e*x) + 2*a*sin(d + e*x))^3,x)
```

```
[Out] tan(d/2 + (e*x)/2)^2/(64*a^3*e) + log(tan(d/2 + (e*x)/2) + 1)/(4*a^3*e) - (
3*tan(d/2 + (e*x)/2))/(32*a^3*e) + (tan(d/2 + (e*x)/2)/4 + 3/16)/(a^3*e*(ta
n(d/2 + (e*x)/2) + 1)^2)
```

sympy [A] time = 6.49, size = 423, normalized size = 3.44

$$\begin{cases}\frac{16\log\left(\tan\left(\frac{d}{2}+\frac{ex}{2}\right)+1\right)\tan^2\left(\frac{d}{2}+\frac{ex}{2}\right)}{64a^3e\tan^2\left(\frac{d}{2}+\frac{ex}{2}\right)+128a^3e\tan\left(\frac{d}{2}+\frac{ex}{2}\right)+64a^3e}+\frac{32\log\left(\tan\left(\frac{d}{2}+\frac{ex}{2}\right)+1\right)\tan\left(\frac{d}{2}+\frac{ex}{2}\right)}{64a^3e\tan^2\left(\frac{d}{2}+\frac{ex}{2}\right)+128a^3e\tan\left(\frac{d}{2}+\frac{ex}{2}\right)+64a^3e}+\frac{16\log\left(\tan\left(\frac{d}{2}+\frac{ex}{2}\right)+1\right)}{64a^3e\tan^2\left(\frac{d}{2}+\frac{ex}{2}\right)+128a^3e\tan\left(\frac{d}{2}+\frac{ex}{2}\right)+64a} \\ \frac{x}{(2a\sin(d)+2a\cos(d)+2a)^3}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*a+2*a*cos(e*x+d)+2*a*sin(e*x+d))**3,x)
```

```
[Out] Piecewise((16*log(tan(d/2 + e*x/2) + 1)*tan(d/2 + e*x/2)**2/(64*a**3*e*tan(
d/2 + e*x/2)**2 + 128*a**3*e*tan(d/2 + e*x/2) + 64*a**3*e) + 32*log(tan(d/2
 + e*x/2) + 1)*tan(d/2 + e*x/2)/(64*a**3*e*tan(d/2 + e*x/2)**2 + 128*a**3*e
*tan(d/2 + e*x/2) + 64*a**3*e) + 16*log(tan(d/2 + e*x/2) + 1)/(64*a**3*e*ta
n(d/2 + e*x/2)**2 + 128*a**3*e*tan(d/2 + e*x/2) + 64*a**3*e) + tan(d/2 + e*
x/2)**4/(64*a**3*e*tan(d/2 + e*x/2)**2 + 128*a**3*e*tan(d/2 + e*x/2) + 64*a
**3*e) - 4*tan(d/2 + e*x/2)**3/(64*a**3*e*tan(d/2 + e*x/2)**2 + 128*a**3*e*
tan(d/2 + e*x/2) + 64*a**3*e) + 32*tan(d/2 + e*x/2)/(64*a**3*e*tan(d/2 + e*
x/2)**2 + 128*a**3*e*tan(d/2 + e*x/2) + 64*a**3*e) + 23/(64*a**3*e*tan(d/2
+ e*x/2)**2 + 128*a**3*e*tan(d/2 + e*x/2) + 64*a**3*e), Ne(e, 0)), (x/(2*a*
sin(d) + 2*a*cos(d) + 2*a)**3, True))
```

3.373 $\int \frac{1}{(2a+2a\cos(d+ex)+2a\sin(d+ex))^4}\,dx$

Optimal. Leaf size=168

$$-\frac{19(a\cos(d+ex)-a\sin(d+ex))}{96e\left(a^5\sin(d+ex)+a^5\cos(d+ex)+a^5\right)}-\frac{\log\left(\tan\left(\frac{1}{2}(d+ex)\right)+1\right)}{4a^4e}+\frac{5(\cos(d+ex)-\sin(d+ex))}{96e\left(a^2\sin(d+ex)+a^2\cos(d+ex)+a\right.}$$

```
[Out] -1/4*ln(1+tan(1/2*e*x+1/2*d))/a^4/e+1/48*(-cos(e*x+d)+sin(e*x+d))/a/e/(a+a*
cos(e*x+d)+a*sin(e*x+d))^3+5/96*(cos(e*x+d)-sin(e*x+d))/e/(a^2+a^2*cos(e*x+
d)+a^2*sin(e*x+d))^2-19/96*(a*cos(e*x+d)-a*sin(e*x+d))/e/(a^5+a^5*cos(e*x+d
)+a^5*sin(e*x+d))
```

Rubi [A] time = 0.19, antiderivative size = 168, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {3129, 3156, 3153, 3124, 31}

$$-\frac{\log\left(\tan\left(\frac{1}{2}(d+ex)\right)+1\right)}{4a^4e}-\frac{19(a\cos(d+ex)-a\sin(d+ex))}{96e\left(a^5\sin(d+ex)+a^5\cos(d+ex)+a^5\right)}+\frac{5(\cos(d+ex)-\sin(d+ex))}{96e\left(a^2\sin(d+ex)+a^2\cos(d+ex)+a\right.}$$

Antiderivative was successfully verified.

```
[In] Int[(2*a + 2*a*Cos[d + e*x] + 2*a*Sin[d + e*x])^(-4),x]
```

```
[Out] -Log[1 + Tan[(d + e*x)/2]]/(4*a^4*e) - (Cos[d + e*x] - Sin[d + e*x])/(48*a*
e*(a + a*Cos[d + e*x] + a*Sin[d + e*x])^3) + (5*(Cos[d + e*x] - Sin[d + e*x
]))/(96*e*(a^2 + a^2*Cos[d + e*x] + a^2*Sin[d + e*x])^2) - (19*(a*Cos[d + e
*x] - a*Sin[d + e*x]))/(96*e*(a^5 + a^5*Cos[d + e*x] + a^5*Sin[d + e*x]))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 3124

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(-1), x_Symbol] :> Module[{f = FreeFactors[Tan[(d + e*x)/2], x]}, Dist[(2*f
)/e, Subst[Int[1/(a + b + 2*c*f*x + (a - b)*f^2*x^2), x], x, Tan[(d + e*x)/
2]/f], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0]
```

Rule 3129

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(n_), x_Symbol] :> Simp[((-(c*Cos[d + e*x]) + b*Sin[d + e*x])*(a + b*Cos[d
+ e*x] + c*Sin[d + e*x])^(n + 1))/(e*(n + 1)*(a^2 - b^2 - c^2)), x] + Dist[
1/((n + 1)*(a^2 - b^2 - c^2)), Int[(a*(n + 1) - b*(n + 2)*Cos[d + e*x] - c*
(n + 2)*Sin[d + e*x])*(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n + 1), x], x]
 /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0] && LtQ[n, -1] && N
eQ[n, -3/2]
```

Rule 3153

```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])
/((a_.) + cos[(d_.) + (e_.)*(x_)]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)])^2,
 x_Symbol] :> Simp[(c*B - b*C - (a*C - c*A)*Cos[d + e*x] + (a*B - b*A)*Sin[
d + e*x])/(e*(a^2 - b^2 - c^2)*(a + b*Cos[d + e*x] + c*Sin[d + e*x])), x] +
 Dist[(a*A - b*B - c*C)/(a^2 - b^2 - c^2), Int[1/(a + b*Cos[d + e*x] + c*Si
n[d + e*x]), x], x] /; FreeQ[{a, b, c, d, e, A, B, C}, x] && NeQ[a^2 - b^2
- c^2, 0] && NeQ[a*A - b*B - c*C, 0]
```

Rule 3156

```
Int[((a_.) + cos[(d_.) + (e_.)*(x_)]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)])
^(n_)*((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)
]), x_Symbol] :> -Simp[((c*B - b*C - (a*C - c*A)*Cos[d + e*x] + (a*B - b*A)
*Sin[d + e*x])*(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n + 1))/(e*(n + 1)*(a
^2 - b^2 - c^2)), x] + Dist[1/((n + 1)*(a^2 - b^2 - c^2)), Int[(a + b*Cos[d
 + e*x] + c*Sin[d + e*x])^(n + 1)*Simp[(n + 1)*(a*A - b*B - c*C) + (n + 2)*
(a*B - b*A)*Cos[d + e*x] + (n + 2)*(a*C - c*A)*Sin[d + e*x], x], x], x] /;
FreeQ[{a, b, c, d, e, A, B, C}, x] && LtQ[n, -1] && NeQ[a^2 - b^2 - c^2, 0]
 && NeQ[n, -2]
```

Rubi steps

$$\int \frac{1}{(2a+2a\cos(d+ex)+2a\sin(d+ex))^4}\,dx = -\frac{\cos(d+ex)-\sin(d+ex)}{48ae(a+a\cos(d+ex)+a\sin(d+ex))^3} + \frac{\int \frac{-6a+4a\cos(d+ex)+\cdots}{(2a+2a\cos(d+ex)+2a\cdots}}{12a^2\cdots}$$

$$= -\frac{\cos(d+ex)-\sin(d+ex)}{48ae(a+a\cos(d+ex)+a\sin(d+ex))^3} + \frac{5(\cos(d+\cdots}{96e\left(a^2+a^2\cos(d\cdots\right.}$$

$$= -\frac{\cos(d+ex)-\sin(d+ex)}{48ae(a+a\cos(d+ex)+a\sin(d+ex))^3} + \frac{5(\cos(d+\cdots}{96e\left(a^2+a^2\cos(d\cdots\right.}$$

$$= -\frac{\cos(d+ex)-\sin(d+ex)}{48ae(a+a\cos(d+ex)+a\sin(d+ex))^3} + \frac{5(\cos(d+\cdots}{96e\left(a^2+a^2\cos(d\cdots\right.}$$

$$= -\frac{\log\left(1+\tan\left(\frac{1}{2}(d+ex)\right)\right)}{4a^4e} - \frac{\cos(d+ex)-\sin(d+ex)}{48ae(a+a\cos(d+ex)+a\sin(d\cdots}$$

Mathematica [A] time = 0.98, size = 247, normalized size = 1.47

$$\frac{19\tan\left(\frac{1}{2}(d+ex)\right)}{192a^4e} - \frac{\sec^2\left(\frac{1}{2}(d+ex)\right)}{64a^4e} + \frac{\log\left(\cos\left(\frac{1}{2}(d+ex)\right)\right)}{4a^4e} + \frac{19\sin\left(\frac{1}{2}(d+ex)\right)}{96a^4e\left(\sin\left(\frac{1}{2}(d+ex)\right)+\cos\left(\frac{1}{2}(d+ex)\right)\right)} + \frac{\cdots}{192a^{4}\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2*a + 2*a*Cos[d + e*x] + 2*a*Sin[d + e*x])^(-4),x]
```

```
[Out] Log[Cos[(d + e*x)/2]]/(4*a^4*e) - Log[Cos[(d + e*x)/2] + Sin[(d + e*x)/2]]/
(4*a^4*e) - Sec[(d + e*x)/2]^2/(64*a^4*e) + Sin[(d + e*x)/2]/(96*a^4*e*(Cos
[(d + e*x)/2] + Sin[(d + e*x)/2])^3) + 5/(192*a^4*e*(Cos[(d + e*x)/2] + Sin
[(d + e*x)/2])^2) + (19*Sin[(d + e*x)/2])/(96*a^4*e*(Cos[(d + e*x)/2] + Sin
[(d + e*x)/2])) + (19*Tan[(d + e*x)/2])/(192*a^4*e) + (Sec[(d + e*x)/2]^2*T
an[(d + e*x)/2])/(384*a^4*e)
```

fricas [A] time = 0.95, size = 237, normalized size = 1.41

$$\frac{38\cos(ex+d)^3+66\cos(ex+d)^2+24\left(\cos(ex+d)^3-\left(\cos(ex+d)^2+3\cos(ex+d)+2\right)\sin(ex+d)-3\,\mathrm{c}\cdots\right.}{192\left(\cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*a+2*a*cos(e*x+d)+2*a*sin(e*x+d))^4,x, algorithm="fricas")
```

```
[Out] 1/192*(38*cos(e*x + d)^3 + 66*cos(e*x + d)^2 + 24*(cos(e*x + d)^3 - (cos(e*
x + d)^2 + 3*cos(e*x + d) + 2)*sin(e*x + d) - 3*cos(e*x + d) - 2)*log(1/2*c
os(e*x + d) + 1/2) - 24*(cos(e*x + d)^3 - (cos(e*x + d)^2 + 3*cos(e*x + d)
+ 2)*sin(e*x + d) - 3*cos(e*x + d) - 2)*log(sin(e*x + d) + 1) + (38*cos(e*x
 + d)^2 - 35)*sin(e*x + d) - 3*cos(e*x + d) - 33)/(a^4*e*cos(e*x + d)^3 - 3
*a^4*e*cos(e*x + d) - 2*a^4*e - (a^4*e*cos(e*x + d)^2 + 3*a^4*e*cos(e*x + d
) + 2*a^4*e)*sin(e*x + d))
```

giac [A] time = 0.20, size = 139, normalized size = 0.83

$$-\frac{1}{384}\left(\frac{96\log\left(\left|\tan\left(\frac{1}{2}xe+\frac{1}{2}d\right)+1\right|\right)}{a^4}-\frac{4\left(44\tan\left(\frac{1}{2}xe+\frac{1}{2}d\right)^3+105\tan\left(\frac{1}{2}xe+\frac{1}{2}d\right)^2+87\tan\left(\frac{1}{2}xe+\frac{1}{2}d\right)+\right.}{a^4\left(\tan\left(\frac{1}{2}xe+\frac{1}{2}d\right)+1\right)^3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*a+2*a*cos(e*x+d)+2*a*sin(e*x+d))^4,x, algorithm="giac")
```

```
[Out] -1/384*(96*log(abs(tan(1/2*x*e + 1/2*d) + 1))/a^4 - 4*(44*tan(1/2*x*e + 1/2
*d)^3 + 105*tan(1/2*x*e + 1/2*d)^2 + 87*tan(1/2*x*e + 1/2*d) + 24)/(a^4*(ta
n(1/2*x*e + 1/2*d) + 1)^3) - (a^8*tan(1/2*x*e + 1/2*d)^3 - 6*a^8*tan(1/2*x*
e + 1/2*d)^2 + 39*a^8*tan(1/2*x*e + 1/2*d))/a^12)*e^(-1)
```

maple [A] time = 0.46, size = 140, normalized size = 0.83

$$\frac{\tan^3\left(\frac{d}{2}+\frac{ex}{2}\right)}{384a^4e}-\frac{\tan^2\left(\frac{d}{2}+\frac{ex}{2}\right)}{64a^4e}+\frac{13\tan\left(\frac{d}{2}+\frac{ex}{2}\right)}{128a^4e}-\frac{1}{48a^4e\left(1+\tan\left(\frac{d}{2}+\frac{ex}{2}\right)\right)^3}+\frac{3}{32a^4e\left(1+\tan\left(\frac{d}{2}+\frac{ex}{2}\right)\right)^2}-\frac{}{32a^4e\left(1+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2*a+2*a*cos(e*x+d)+2*a*sin(e*x+d))^4,x)
```

```
[Out] 1/384/a^4/e*tan(1/2*d+1/2*e*x)^3-1/64/a^4/e*tan(1/2*d+1/2*e*x)^2+13/128/a^4
/e*tan(1/2*d+1/2*e*x)-1/48/a^4/e/(1+tan(1/2*d+1/2*e*x))^3+3/32/a^4/e/(1+tan
(1/2*d+1/2*e*x))^2-9/32/a^4/e/(1+tan(1/2*d+1/2*e*x))-1/4*ln(1+tan(1/2*d+1/2
*e*x))/a^4/e
```

maxima [A] time = 0.35, size = 208, normalized size = 1.24

$$-\frac{\frac{4\left(\frac{45\sin(ex+d)}{\cos(ex+d)+1}+\frac{27\sin(ex+d)^2}{(\cos(ex+d)+1)^2}+20\right)}{a^4+\frac{3a^4\sin(ex+d)}{\cos(ex+d)+1}+\frac{3a^4\sin(ex+d)^2}{(\cos(ex+d)+1)^2}+\frac{a^4\sin(ex+d)^3}{(\cos(ex+d)+1)^3}}-\frac{\frac{39\sin(ex+d)}{\cos(ex+d)+1}-\frac{6\sin(ex+d)^2}{(\cos(ex+d)+1)^2}+\frac{\sin(ex+d)^3}{(\cos(ex+d)+1)^3}}{a^4}+\frac{96\log\left(\frac{\sin(ex+d)}{\cos(ex+d)+1}+1\right)}{a^4}}{384e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*a+2*a*cos(e*x+d)+2*a*sin(e*x+d))^4,x, algorithm="maxima")
```

```
[Out] -1/384*(4*(45*sin(e*x + d)/(cos(e*x + d) + 1) + 27*sin(e*x + d)^2/(cos(e*x
+ d) + 1)^2 + 20)/(a^4 + 3*a^4*sin(e*x + d)/(cos(e*x + d) + 1) + 3*a^4*sin(
e*x + d)^2/(cos(e*x + d) + 1)^2 + a^4*sin(e*x + d)^3/(cos(e*x + d) + 1)^3)
- (39*sin(e*x + d)/(cos(e*x + d) + 1) - 6*sin(e*x + d)^2/(cos(e*x + d) + 1)
^2 + sin(e*x + d)^3/(cos(e*x + d) + 1)^3)/a^4 + 96*log(sin(e*x + d)/(cos(e*
x + d) + 1) + 1)/a^4)/e
```

mupad [**B**] time = 2.44, size = 161, normalized size = 0.96

$$\frac{\tan\left(\frac{d}{2}+\frac{e\,x}{2}\right)^3}{384\,a^4\,e}-\frac{\tan\left(\frac{d}{2}+\frac{e\,x}{2}\right)^2}{64\,a^4\,e}-\frac{\ln\left(\tan\left(\frac{d}{2}+\frac{e\,x}{2}\right)+1\right)}{4\,a^4\,e}+\frac{13\tan\left(\frac{d}{2}+\frac{e\,x}{2}\right)}{128\,a^4\,e}-\frac{9\tan\left(\frac{d}{2}+\frac{e\,x}{2}\right)^2+15}{e\left(32\,a^4\tan\left(\frac{d}{2}+\frac{e\,x}{2}\right)^3+96\,a^4\tan\left(\frac{d}{2}+\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2*a + 2*a*cos(d + e*x) + 2*a*sin(d + e*x))^4,x)
```

```
[Out] tan(d/2 + (e*x)/2)^3/(384*a^4*e) - tan(d/2 + (e*x)/2)^2/(64*a^4*e) - log(ta
n(d/2 + (e*x)/2) + 1)/(4*a^4*e) + (13*tan(d/2 + (e*x)/2))/(128*a^4*e) - (15
*tan(d/2 + (e*x)/2) + 9*tan(d/2 + (e*x)/2)^2 + 20/3)/(e*(96*a^4*tan(d/2 + (
e*x)/2)^2 + 32*a^4*tan(d/2 + (e*x)/2)^3 + 32*a^4 + 96*a^4*tan(d/2 + (e*x)/2
)))
```

sympy [**A**] time = 23.78, size = 792, normalized size = 4.71

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*a+2*a*cos(e*x+d)+2*a*sin(e*x+d))**4,x)
```

```
[Out] Piecewise((-96*log(tan(d/2 + e*x/2) + 1)*tan(d/2 + e*x/2)**3/(384*a**4*e*ta
n(d/2 + e*x/2)**3 + 1152*a**4*e*tan(d/2 + e*x/2)**2 + 1152*a**4*e*tan(d/2 +
 e*x/2) + 384*a**4*e) - 288*log(tan(d/2 + e*x/2) + 1)*tan(d/2 + e*x/2)**2/(
384*a**4*e*tan(d/2 + e*x/2)**3 + 1152*a**4*e*tan(d/2 + e*x/2)**2 + 1152*a**
4*e*tan(d/2 + e*x/2) + 384*a**4*e) - 288*log(tan(d/2 + e*x/2) + 1)*tan(d/2
+ e*x/2)/(384*a**4*e*tan(d/2 + e*x/2)**3 + 1152*a**4*e*tan(d/2 + e*x/2)**2
+ 1152*a**4*e*tan(d/2 + e*x/2) + 384*a**4*e) - 96*log(tan(d/2 + e*x/2) + 1)
/(384*a**4*e*tan(d/2 + e*x/2)**3 + 1152*a**4*e*tan(d/2 + e*x/2)**2 + 1152*a
**4*e*tan(d/2 + e*x/2) + 384*a**4*e) + tan(d/2 + e*x/2)**6/(384*a**4*e*tan(
d/2 + e*x/2)**3 + 1152*a**4*e*tan(d/2 + e*x/2)**2 + 1152*a**4*e*tan(d/2 + e
*x/2) + 384*a**4*e) - 3*tan(d/2 + e*x/2)**5/(384*a**4*e*tan(d/2 + e*x/2)**3
```

```
 + 1152*a**4*e*tan(d/2 + e*x/2)**2 + 1152*a**4*e*tan(d/2 + e*x/2) + 384*a**
4*e) + 24*tan(d/2 + e*x/2)**4/(384*a**4*e*tan(d/2 + e*x/2)**3 + 1152*a**4*e
*tan(d/2 + e*x/2)**2 + 1152*a**4*e*tan(d/2 + e*x/2) + 384*a**4*e) - 297*tan
(d/2 + e*x/2)**2/(384*a**4*e*tan(d/2 + e*x/2)**3 + 1152*a**4*e*tan(d/2 + e*
x/2)**2 + 1152*a**4*e*tan(d/2 + e*x/2) + 384*a**4*e) - 441*tan(d/2 + e*x/2)
/(384*a**4*e*tan(d/2 + e*x/2)**3 + 1152*a**4*e*tan(d/2 + e*x/2)**2 + 1152*a
**4*e*tan(d/2 + e*x/2) + 384*a**4*e) - 180/(384*a**4*e*tan(d/2 + e*x/2)**3
+ 1152*a**4*e*tan(d/2 + e*x/2)**2 + 1152*a**4*e*tan(d/2 + e*x/2) + 384*a**4
*e), Ne(e, 0)), (x/(2*a*sin(d) + 2*a*cos(d) + 2*a)**4, True))
```

3.374 $\int (2a-2a\cos(d+ex)+2c\sin(d+ex))^3\,dx$

Optimal. Leaf size=157

$$-\frac{4a\left(15a^2+4c^2\right)\sin(d+ex)}{3e}-\frac{4c\left(15a^2+4c^2\right)\cos(d+ex)}{3e}+4ax\left(5a^2+3c^2\right)-\frac{20\left(a^2\sin(d+ex)+ac\cos(d+ex)\right.}{}$$

```
[Out] 4*a*(5*a^2+3*c^2)*x-4/3*c*(15*a^2+4*c^2)*cos(e*x+d)/e-4/3*a*(15*a^2+4*c^2)*
sin(e*x+d)/e-20/3*(a*c*cos(e*x+d)+a^2*sin(e*x+d))*(a-a*cos(e*x+d)+c*sin(e*x
+d))/e-8/3*(c*cos(e*x+d)+a*sin(e*x+d))*(a-a*cos(e*x+d)+c*sin(e*x+d))^2/e
```

Rubi [A] time = 0.13, antiderivative size = 157, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3120, 3146, 2637, 2638}

$$-\frac{4a\left(15a^2+4c^2\right)\sin(d+ex)}{3e}-\frac{4c\left(15a^2+4c^2\right)\cos(d+ex)}{3e}+4ax\left(5a^2+3c^2\right)-\frac{20\left(a^2\sin(d+ex)+ac\cos(d+ex)\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(2*a - 2*a*Cos[d + e*x] + 2*c*Sin[d + e*x])^3,x]
```

```
[Out] 4*a*(5*a^2 + 3*c^2)*x - (4*c*(15*a^2 + 4*c^2)*Cos[d + e*x])/(3*e) - (4*a*(1
5*a^2 + 4*c^2)*Sin[d + e*x])/(3*e) - (20*(a*c*Cos[d + e*x] + a^2*Sin[d + e*
x])*(a - a*Cos[d + e*x] + c*Sin[d + e*x]))/(3*e) - (8*(c*Cos[d + e*x] + a*S
in[d + e*x])*(a - a*Cos[d + e*x] + c*Sin[d + e*x])^2)/(3*e)
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3120

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(n_), x_Symbol] :> -Simp[((c*Cos[d + e*x] - b*Sin[d + e*x])*(a + b*Cos[d +
e*x] + c*Sin[d + e*x])^(n - 1))/(e*n), x] + Dist[1/n, Int[Simp[n*a^2 + (n -
 1)*(b^2 + c^2) + a*b*(2*n - 1)*Cos[d + e*x] + a*c*(2*n - 1)*Sin[d + e*x],
x]*(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n - 2), x], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[a^2 - b^2 - c^2, 0] && GtQ[n, 1]
```

Rule 3146

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(n_.)*((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)
]), x_Symbol] :> Simp[((B*c - b*C - a*C*Cos[d + e*x] + a*B*Sin[d + e*x])*(a
 + b*Cos[d + e*x] + c*Sin[d + e*x])^n)/(a*e*(n + 1)), x] + Dist[1/(a*(n + 1
)), Int[(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n - 1)*Simp[a*(b*B + c*C)*n
+ a^2*A*(n + 1) + (n*(a^2*B - B*c^2 + b*c*C) + a*b*A*(n + 1))*Cos[d + e*x]
+ (n*(b*B*c + a^2*C - b^2*C) + a*c*A*(n + 1))*Sin[d + e*x], x], x], x] /; F
reeQ[{a, b, c, d, e, A, B, C}, x] && GtQ[n, 0] && NeQ[a^2 - b^2 - c^2, 0]
```

Rubi steps

$$\int (2a - 2a\cos(d+ex) + 2c\sin(d+ex))^3\,dx = -\frac{8(c\cos(d+ex) + a\sin(d+ex))(a - a\cos(d+ex) + c\sin(d+ex)}{3e}$$

$$= -\frac{20\left(ac\cos(d+ex) + a^2\sin(d+ex)\right)(a - a\cos(d+ex) + c\sin(d}{3e}$$

$$= 4a\left(5a^2+3c^2\right)x - \frac{20\left(ac\cos(d+ex) + a^2\sin(d+ex)\right)(a - a\cos(}{3e}$$

$$= 4a\left(5a^2+3c^2\right)x - \frac{4c\left(15a^2+4c^2\right)\cos(d+ex)}{3e} - \frac{4a\left(15a^2+4c^2\right)s}{3e}$$

Mathematica [A] time = 0.43, size = 136, normalized size = 0.87

$$\frac{2\left(6a\left(5a^2+3c^2\right)(d+ex) - 9a\left(5a^2+c^2\right)\sin(d+ex) + 9a\left(a^2-c^2\right)\sin(2(d+ex)) - a\left(a^2-3c^2\right)\sin(3(d+ex)) -}{3e}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2*a - 2*a*Cos[d + e*x] + 2*c*Sin[d + e*x])^3,x]
```

```
[Out] (2*(6*a*(5*a^2 + 3*c^2)*(d + e*x) - 9*c*(5*a^2 + c^2)*Cos[d + e*x] + 18*a^2
*c*Cos[2*(d + e*x)] + c*(-3*a^2 + c^2)*Cos[3*(d + e*x)] - 9*a*(5*a^2 + c^2)
*Sin[d + e*x] + 9*a*(a^2 - c^2)*Sin[2*(d + e*x)] - a*(a^2 - 3*c^2)*Sin[3*(d
 + e*x)]))/(3*e)
```

fricas [A] time = 1.95, size = 134, normalized size = 0.85

$$\frac{4\left(18\,a^2c\cos(ex+d)^2 - 2\left(3\,a^2c - c^3\right)\cos(ex+d)^3 + 3\left(5\,a^3 + 3\,ac^2\right)ex - 6\left(3\,a^2c + c^3\right)\cos(ex+d) - \left(22\,a^3 + 6\right.}{3\,e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*a-2*a*cos(e*x+d)+2*c*sin(e*x+d))^3,x, algorithm="fricas")
```

```
[Out] 4/3*(18*a^2*c*cos(e*x + d)^2 - 2*(3*a^2*c - c^3)*cos(e*x + d)^3 + 3*(5*a^3
+ 3*a*c^2)*e*x - 6*(3*a^2*c + c^3)*cos(e*x + d) - (22*a^3 + 6*a*c^2 + 2*(a^
3 - 3*a*c^2)*cos(e*x + d)^2 - 9*(a^3 - a*c^2)*cos(e*x + d))*sin(e*x + d))/e
```

giac [A] time = 0.19, size = 151, normalized size = 0.96

$$12\,a^2c\cos\left(2\,xe+2\,d\right)e^{(-1)}-\frac{2}{3}\left(3\,a^2c-c^3\right)\cos\left(3\,xe+3\,d\right)e^{(-1)}-6\left(5\,a^2c+c^3\right)\cos\left(xe+d\right)e^{(-1)}-\frac{2}{3}\left(a^3-3\,ac^2\right)e^{(-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*a-2*a*cos(e*x+d)+2*c*sin(e*x+d))^3,x, algorithm="giac")
```

```
[Out] 12*a^2*c*cos(2*x*e + 2*d)*e^(-1) - 2/3*(3*a^2*c - c^3)*cos(3*x*e + 3*d)*e^(
-1) - 6*(5*a^2*c + c^3)*cos(x*e + d)*e^(-1) - 2/3*(a^3 - 3*a*c^2)*e^(-1)*si
n(3*x*e + 3*d) + 6*(a^3 - a*c^2)*e^(-1)*sin(2*x*e + 2*d) - 6*(5*a^3 + a*c^2
)*e^(-1)*sin(x*e + d) + 4*(5*a^3 + 3*a*c^2)*x
```

maple [A] time = 0.25, size = 178, normalized size = 1.13

$$\frac{-\frac{8a^3\left(2+\cos^2(ex+d)\right)\sin(ex+d)}{3}-8a^2c\left(\cos^3\left(ex+d\right)\right)+24a^3\left(\frac{\sin(ex+d)\cos(ex+d)}{2}+\frac{ex}{2}+\frac{d}{2}\right)-8a\,c^2\left(\sin^3\left(ex+d\right)\right)+24a^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*a-2*a*cos(e*x+d)+2*c*sin(e*x+d))^3,x)
```

```
[Out] 8/e*(-1/3*a^3*(2+cos(e*x+d)^2)*sin(e*x+d)-a^2*c*cos(e*x+d)^3+3*a^3*(1/2*sin
(e*x+d)*cos(e*x+d)+1/2*e*x+1/2*d)-a*c^2*sin(e*x+d)^3+3*a^2*c*cos(e*x+d)^2-3
*a^3*sin(e*x+d)-1/3*c^3*(2+sin(e*x+d)^2)*cos(e*x+d)+3*a*c^2*(-1/2*sin(e*x+d
)*cos(e*x+d)+1/2*e*x+1/2*d)-3*a^2*c*cos(e*x+d)+a^3*(e*x+d))
```

maxima [A] time = 0.33, size = 188, normalized size = 1.20

$$-\frac{8\,a^2c\cos\left(ex+d\right)^3}{e}-\frac{8\,ac^2\sin\left(ex+d\right)^3}{e}+8\,a^3x+\frac{8\left(\sin\left(ex+d\right)^3-3\,\sin\left(ex+d\right)\right)a^3}{3\,e}+\frac{8\left(\cos\left(ex+d\right)^3-3\,\cos\left(e\right.\right.}{3\,e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*a-2*a*cos(e*x+d)+2*c*sin(e*x+d))^3,x, algorithm="maxima")
```

```
[Out] -8*a^2*c*cos(e*x + d)^3/e - 8*a*c^2*sin(e*x + d)^3/e + 8*a^3*x + 8/3*(sin(e
*x + d)^3 - 3*sin(e*x + d))*a^3/e + 8/3*(cos(e*x + d)^3 - 3*cos(e*x + d))*c
^3/e - 24*a^2*(c*cos(e*x + d)/e + a*sin(e*x + d)/e) + 6*(4*a*c*cos(e*x + d)
```

```
^2/e + (2*e*x + 2*d + sin(2*e*x + 2*d))*a^2/e + (2*e*x + 2*d - sin(2*e*x +
2*d))*c^2/e)*a
```

mupad [B] time = 3.21, size = 258, normalized size = 1.64

$$\frac{8\,a\,\mathrm{atan}\left(\frac{8\,a\tan\left(\frac{d}{2}+\frac{e\,x}{2}\right)\left(5\,a^2+3\,c^2\right)}{40\,a^3+24\,a\,c^2}\right)\left(5\,a^2+3\,c^2\right)}{e}-\frac{\tan\left(\frac{d}{2}+\frac{e\,x}{2}\right)\left(40\,a^3+24\,a\,c^2\right)+64\,a^2\,c-\tan\left(\frac{d}{2}+\frac{e\,x}{2}\right)^5\,\left(24\,a\,c^2-\right.}{e\left(\tan\left(\frac{d}{2}\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*a - 2*a*cos(d + e*x) + 2*c*sin(d + e*x))^3,x)
```

```
[Out] (8*a*atan((8*a*tan(d/2 + (e*x)/2)*(5*a^2 + 3*c^2))/(24*a*c^2 + 40*a^3))*(5*
a^2 + 3*c^2))/e - (tan(d/2 + (e*x)/2)*(24*a*c^2 + 40*a^3) + 64*a^2*c - tan(
d/2 + (e*x)/2)^5*(24*a*c^2 - 88*a^3) + tan(d/2 + (e*x)/2)^3*(64*a*c^2 + (32
0*a^3)/3) + tan(d/2 + (e*x)/2)^2*(192*a^2*c + 32*c^3) + (32*c^3)/3 + 192*a^
2*c*tan(d/2 + (e*x)/2)^4)/(e*(3*tan(d/2 + (e*x)/2)^2 + 3*tan(d/2 + (e*x)/2)
^4 + tan(d/2 + (e*x)/2)^6 + 1)) - (8*a*(5*a^2 + 3*c^2)*(atan(tan(d/2 + (e*x
)/2)) - (e*x)/2))/e
```

sympy [A] time = 0.76, size = 291, normalized size = 1.85

$$\begin{cases} 12a^3x\sin^2(d+ex)+12a^3x\cos^2(d+ex)+8a^3x-\frac{16a^3\sin^3(d+ex)}{3e}-\frac{8a^3\sin(d+ex)\cos^2(d+ex)}{e}+\frac{12a^3\sin(d+ex)\cos(d+ex)}{e}- \\ x\left(-2a\cos(d)+2a+2c\sin(d)\right)^3 \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*a-2*a*cos(e*x+d)+2*c*sin(e*x+d))**3,x)
```

```
[Out] Piecewise((12*a**3*x*sin(d + e*x)**2 + 12*a**3*x*cos(d + e*x)**2 + 8*a**3*x
 - 16*a**3*sin(d + e*x)**3/(3*e) - 8*a**3*sin(d + e*x)*cos(d + e*x)**2/e +
12*a**3*sin(d + e*x)*cos(d + e*x)/e - 24*a**3*sin(d + e*x)/e - 8*a**2*c*cos
(d + e*x)**3/e + 24*a**2*c*cos(d + e*x)**2/e - 24*a**2*c*cos(d + e*x)/e + 1
2*a*c**2*x*sin(d + e*x)**2 + 12*a*c**2*x*cos(d + e*x)**2 - 8*a*c**2*sin(d +
 e*x)**3/e - 12*a*c**2*sin(d + e*x)*cos(d + e*x)/e - 8*c**3*sin(d + e*x)**2
*cos(d + e*x)/e - 16*c**3*cos(d + e*x)**3/(3*e), Ne(e, 0)), (x*(-2*a*cos(d)
 + 2*a + 2*c*sin(d))**3, True))
```

3.375 $\int (2a - 2a\cos(d+ex) + 2c\sin(d+ex))^2\,dx$

Optimal. Leaf size=81

$$2x\left(3a^2+c^2\right)-\frac{6a^2\sin(d+ex)}{e}-\frac{6ac\cos(d+ex)}{e}-\frac{2(a\sin(d+ex)+c\cos(d+ex))(a(-\cos(d+ex))+a+c\sin(d$$

```
[Out] 2*(3*a^2+c^2)*x-6*a*c*cos(e*x+d)/e-6*a^2*sin(e*x+d)/e-2*(c*cos(e*x+d)+a*sin
(e*x+d))*(a-a*cos(e*x+d)+c*sin(e*x+d))/e
```

Rubi [A] time = 0.05, antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {3120, 2637, 2638}

$$2x\left(3a^2+c^2\right)-\frac{6a^2\sin(d+ex)}{e}-\frac{6ac\cos(d+ex)}{e}-\frac{2(a\sin(d+ex)+c\cos(d+ex))(a(-\cos(d+ex))+a+c\sin(d$$

Antiderivative was successfully verified.

```
[In] Int[(2*a - 2*a*Cos[d + e*x] + 2*c*Sin[d + e*x])^2,x]
```

```
[Out] 2*(3*a^2 + c^2)*x - (6*a*c*Cos[d + e*x])/e - (6*a^2*Sin[d + e*x])/e - (2*(c
*Cos[d + e*x] + a*Sin[d + e*x])*(a - a*Cos[d + e*x] + c*Sin[d + e*x]))/e
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3120

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(n_), x_Symbol] :> -Simp[((c*Cos[d + e*x] - b*Sin[d + e*x])*(a + b*Cos[d +
e*x] + c*Sin[d + e*x])^(n - 1))/(e*n), x] + Dist[1/n, Int[Simp[n*a^2 + (n -
 1)*(b^2 + c^2) + a*b*(2*n - 1)*Cos[d + e*x] + a*c*(2*n - 1)*Sin[d + e*x],
x]*(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n - 2), x], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[a^2 - b^2 - c^2, 0] && GtQ[n, 1]
```

Rubi steps

$$\int (2a-2a\cos(d+ex)+2c\sin(d+ex))^2\,dx = -\frac{2(c\cos(d+ex)+a\sin(d+ex))(a-a\cos(d+ex)+c\sin(d+ex)}{e}$$
$$= 2\left(3a^2+c^2\right)x-\frac{2(c\cos(d+ex)+a\sin(d+ex))(a-a\cos(d+ex)}{e}$$
$$= 2\left(3a^2+c^2\right)x-\frac{6ac\cos(d+ex)}{e}-\frac{6a^2\sin(d+ex)}{e}-\frac{2(c\cos(d+e}{}$$

Mathematica [A] time = 0.15, size = 92, normalized size = 1.14

$$4\left(\frac{\left(3a^2+c^2\right)(d+ex)}{2e}+\frac{\left(a^2-c^2\right)\sin(2(d+ex))}{4e}-\frac{2a^2\sin(d+ex)}{e}-\frac{2ac\cos(d+ex)}{e}+\frac{ac\cos(2(d+ex))}{2e}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(2*a - 2*a*Cos[d + e*x] + 2*c*Sin[d + e*x])^2,x]
```

```
[Out] 4*(((3*a^2 + c^2)*(d + e*x))/(2*e) - (2*a*c*Cos[d + e*x])/e + (a*c*Cos[2*(d
 + e*x)])/(2*e) - (2*a^2*Sin[d + e*x])/e + ((a^2 - c^2)*Sin[2*(d + e*x)])/(
4*e))
```

fricas [A] time = 0.53, size = 71, normalized size = 0.88

$$\frac{2\left(2\,ac\cos\left(ex+d\right)^2+\left(3\,a^2+c^2\right)ex-4\,ac\cos\left(ex+d\right)-\left(4\,a^2-\left(a^2-c^2\right)\cos\left(ex+d\right)\right)\sin\left(ex+d\right)\right)}{e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*a-2*a*cos(e*x+d)+2*c*sin(e*x+d))^2,x, algorithm="fricas")
```

```
[Out] 2*(2*a*c*cos(e*x + d)^2 + (3*a^2 + c^2)*e*x - 4*a*c*cos(e*x + d) - (4*a^2 -
 (a^2 - c^2)*cos(e*x + d))*sin(e*x + d))/e
```

giac [A] time = 0.16, size = 78, normalized size = 0.96

$$2\,ac\cos\left(2\,xe+2\,d\right)e^{(-1)}-8\,ac\cos\left(xe+d\right)e^{(-1)}-8\,a^2e^{(-1)}\sin\left(xe+d\right)+\left(a^2-c^2\right)e^{(-1)}\sin\left(2\,xe+2\,d\right)+2\left(3\,a^2+c^2\right)x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*a-2*a*cos(e*x+d)+2*c*sin(e*x+d))^2,x, algorithm="giac")
```

```
[Out] 2*a*c*cos(2*x*e + 2*d)*e^(-1) - 8*a*c*cos(x*e + d)*e^(-1) - 8*a^2*e^(-1)*si
n(x*e + d) + (a^2 - c^2)*e^(-1)*sin(2*x*e + 2*d) + 2*(3*a^2 + c^2)*x
```

maple [A] time = 0.22, size = 100, normalized size = 1.23

$$\frac{4a^2(ex+d) - 8a^2\sin(ex+d) - 8ac\cos(ex+d) + 4a^2\left(\frac{\sin(ex+d)\cos(ex+d)}{2} + \frac{ex}{2} + \frac{d}{2}\right) + 4ac\left(\cos^2(ex+d)\right) + 4c^2\left(\cdots\right.}{e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*a-2*a*cos(e*x+d)+2*c*sin(e*x+d))^2,x)
```

```
[Out] 4/e*(a^2*(e*x+d)-2*a^2*sin(e*x+d)-2*a*c*cos(e*x+d)+a^2*(1/2*sin(e*x+d)*cos(
e*x+d)+1/2*e*x+1/2*d)+a*c*cos(e*x+d)^2+c^2*(-1/2*sin(e*x+d)*cos(e*x+d)+1/2*
e*x+1/2*d))
```

maxima [A] time = 0.32, size = 98, normalized size = 1.21

$$4\,a^2x + \frac{4\,ac\cos(ex+d)^2}{e} + \frac{(2\,ex+2\,d+\sin(2\,ex+2\,d))a^2}{e} + \frac{(2\,ex+2\,d-\sin(2\,ex+2\,d))c^2}{e} - 8\,a\left(\frac{c\cos(ex+d)}{e} + \cdots\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*a-2*a*cos(e*x+d)+2*c*sin(e*x+d))^2,x, algorithm="maxima")
```

```
[Out] 4*a^2*x + 4*a*c*cos(e*x + d)^2/e + (2*e*x + 2*d + sin(2*e*x + 2*d))*a^2/e +
 (2*e*x + 2*d - sin(2*e*x + 2*d))*c^2/e - 8*a*(c*cos(e*x + d)/e + a*sin(e*x
 + d)/e)
```

mupad [B] time = 2.50, size = 84, normalized size = 1.04

$$\frac{a^2\,\sin(2\,d+2\,e\,x) - 8\,a^2\,\sin(d+e\,x) - c^2\,\sin(2\,d+2\,e\,x) + 16\,a\,c\sin\left(\frac{d}{2}+\frac{e\,x}{2}\right)^2 - 4\,a\,c\sin(d+e\,x)^2 + 6\,a^2\,e\,x\cdots}{e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*a - 2*a*cos(d + e*x) + 2*c*sin(d + e*x))^2,x)
```

```
[Out] (a^2*sin(2*d + 2*e*x) - 8*a^2*sin(d + e*x) - c^2*sin(2*d + 2*e*x) + 16*a*c*
sin(d/2 + (e*x)/2)^2 - 4*a*c*sin(d + e*x)^2 + 6*a^2*e*x + 2*c^2*e*x)/e
```

sympy [A] time = 0.32, size = 170, normalized size = 2.10

$$\begin{cases} 2a^2x\sin^2(d+ex) + 2a^2x\cos^2(d+ex) + 4a^2x + \frac{2a^2\sin(d+ex)\cos(d+ex)}{e} - \frac{8a^2\sin(d+ex)}{e} + \frac{4ac\cos^2(d+ex)}{e} - \frac{8ac\cos(d+ex)}{e} \cdots \\ x(-2a\cos(d) + 2a + 2c\sin(d))^2 \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*a-2*a*cos(e*x+d)+2*c*sin(e*x+d))**2,x)
```

```
[Out] Piecewise((2*a**2*x*sin(d + e*x)**2 + 2*a**2*x*cos(d + e*x)**2 + 4*a**2*x +
 2*a**2*sin(d + e*x)*cos(d + e*x)/e - 8*a**2*sin(d + e*x)/e + 4*a*c*cos(d +
 e*x)**2/e - 8*a*c*cos(d + e*x)/e + 2*c**2*x*sin(d + e*x)**2 + 2*c**2*x*cos
(d + e*x)**2 - 2*c**2*sin(d + e*x)*cos(d + e*x)/e, Ne(e, 0)), (x*(-2*a*cos(
d) + 2*a + 2*c*sin(d))**2, True))
```

3.376 $\int (2a - 2a\cos(d+ex) + 2c\sin(d+ex))\,dx$

Optimal. Leaf size=29

$$-\frac{2a\sin(d+ex)}{e} + 2ax - \frac{2c\cos(d+ex)}{e}$$

```
[Out] 2*a*x-2*c*cos(e*x+d)/e-2*a*sin(e*x+d)/e
```

Rubi [A] time = 0.01, antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {2637, 2638}

$$-\frac{2a\sin(d+ex)}{e} + 2ax - \frac{2c\cos(d+ex)}{e}$$

Antiderivative was successfully verified.

```
[In] Int[2*a - 2*a*Cos[d + e*x] + 2*c*Sin[d + e*x],x]
```

```
[Out] 2*a*x - (2*c*Cos[d + e*x])/e - (2*a*Sin[d + e*x])/e
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int (2a - 2a\cos(d+ex) + 2c\sin(d+ex))\,dx &= 2ax - (2a)\int \cos(d+ex)\,dx + (2c)\int \sin(d+ex)\,dx\\ &= 2ax - \frac{2c\cos(d+ex)}{e} - \frac{2a\sin(d+ex)}{e}\end{aligned}$$

Mathematica [A] time = 0.01, size = 53, normalized size = 1.83

$$-\frac{2a\sin(d)\cos(ex)}{e} - \frac{2a\cos(d)\sin(ex)}{e} + 2ax + \frac{2c\sin(d)\sin(ex)}{e} - \frac{2c\cos(d)\cos(ex)}{e}$$

Antiderivative was successfully verified.

```
[In] Integrate[2*a - 2*a*Cos[d + e*x] + 2*c*Sin[d + e*x],x]
```

```
[Out] 2*a*x - (2*c*Cos[d]*Cos[e*x])/e - (2*a*Cos[e*x]*Sin[d])/e - (2*a*Cos[d]*Sin
[e*x])/e + (2*c*Sin[d]*Sin[e*x])/e
```

fricas [A] time = 0.84, size = 28, normalized size = 0.97

$$\frac{2\,(aex - c\cos{(ex + d)} - a\sin{(ex + d)})}{e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(2*a-2*a*cos(e*x+d)+2*c*sin(e*x+d),x, algorithm="fricas")
```

```
[Out] 2*(a*e*x - c*cos(e*x + d) - a*sin(e*x + d))/e
```

giac [A] time = 0.15, size = 29, normalized size = 1.00

$$-2\,c\cos{(xe + d)}\,e^{(-1)} - 2\,ae^{(-1)}\sin{(xe + d)} + 2\,ax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(2*a-2*a*cos(e*x+d)+2*c*sin(e*x+d),x, algorithm="giac")
```

```
[Out] -2*c*cos(x*e + d)*e^(-1) - 2*a*e^(-1)*sin(x*e + d) + 2*a*x
```

maple [A] time = 0.00, size = 30, normalized size = 1.03

$$2ax - \frac{2c\cos{(ex + d)}}{e} - \frac{2a\sin{(ex + d)}}{e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(2*a-2*a*cos(e*x+d)+2*c*sin(e*x+d),x)
```

```
[Out] 2*a*x-2*c*cos(e*x+d)/e-2*a*sin(e*x+d)/e
```

maxima [A] time = 0.31, size = 29, normalized size = 1.00

$$2\,ax - \frac{2\,c\cos{(ex + d)}}{e} - \frac{2\,a\sin{(ex + d)}}{e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(2*a-2*a*cos(e*x+d)+2*c*sin(e*x+d),x, algorithm="maxima")
```

```
[Out] 2*a*x - 2*c*cos(e*x + d)/e - 2*a*sin(e*x + d)/e
```

mupad [B] time = 2.43, size = 29, normalized size = 1.00

$$2\,a\,x-\frac{2\,c\,\cos\left(d+e\,x\right)}{e}-\frac{2\,a\,\sin\left(d+e\,x\right)}{e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(2*a - 2*a*cos(d + e*x) + 2*c*sin(d + e*x),x)
```

```
[Out] 2*a*x - (2*c*cos(d + e*x))/e - (2*a*sin(d + e*x))/e
```

sympy [A] time = 0.15, size = 39, normalized size = 1.34

$$2ax-2a\left(\begin{cases}\frac{\sin\left(d+ex\right)}{e} & \text{for}\: e \neq 0\\x\cos\left(d\right) & \text{otherwise}\end{cases}\right)+2c\left(\begin{cases}-\frac{\cos\left(d+ex\right)}{e} & \text{for}\: e \neq 0\\x\sin\left(d\right) & \text{otherwise}\end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(2*a-2*a*cos(e*x+d)+2*c*sin(e*x+d),x)
```

```
[Out] 2*a*x - 2*a*Piecewise((sin(d + e*x)/e, Ne(e, 0)), (x*cos(d), True)) + 2*c*P
iecewise((-cos(d + e*x)/e, Ne(e, 0)), (x*sin(d), True))
```

3.377 $\int \frac{1}{2a-2a\cos(d+ex)+2c\sin(d+ex)}\,dx$

Optimal. Leaf size=25

$$-\frac{\log\left(a+c\cot\left(\frac{1}{2}(d+ex)\right)\right)}{2ce}$$

```
[Out] -1/2*ln(a+c*cot(1/2*e*x+1/2*d))/c/e
```

Rubi [A] time = 0.02, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {3121, 31}

$$-\frac{\log\left(a+c\cot\left(\frac{1}{2}(d+ex)\right)\right)}{2ce}$$

Antiderivative was successfully verified.

```
[In] Int[(2*a - 2*a*Cos[d + e*x] + 2*c*Sin[d + e*x])^(-1),x]
```

```
[Out] -Log[a + c*Cot[(d + e*x)/2]]/(2*c*e)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 3121

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(-1), x_Symbol] :> Module[{f = FreeFactors[Cot[(d + e*x)/2], x]}, -Dist[f/e
, Subst[Int[1/(a + c*f*x), x], x, Cot[(d + e*x)/2]/f], x]] /; FreeQ[{a, b,
c, d, e}, x] && EqQ[a + b, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{2a-2a\cos(d+ex)+2c\sin(d+ex)}\,dx &= -\frac{\text{Subst}\left(\int \frac{1}{2a+2cx}\,dx, x, \cot\left(\frac{1}{2}(d+ex)\right)\right)}{e}\\ &= -\frac{\log\left(a+c\cot\left(\frac{1}{2}(d+ex)\right)\right)}{2ce}\end{aligned}$$

Mathematica [A] time = 0.15, size = 50, normalized size = 2.00

$$\frac{\log\left(\sin\left(\frac{1}{2}(d+ex)\right)\right)-\log\left(a\sin\left(\frac{1}{2}(d+ex)\right)+c\cos\left(\frac{1}{2}(d+ex)\right)\right)}{2ce}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2*a - 2*a*Cos[d + e*x] + 2*c*Sin[d + e*x])^(-1),x]
```

```
[Out] (Log[Sin[(d + e*x)/2]] - Log[c*Cos[(d + e*x)/2] + a*Sin[(d + e*x)/2]])/(2*c
*e)
```

fricas [B] time = 0.90, size = 60, normalized size = 2.40

$$-\frac{\log\left(ac\sin\left(ex+d\right)+\frac{1}{2}a^2+\frac{1}{2}c^2-\frac{1}{2}\left(a^2-c^2\right)\cos\left(ex+d\right)\right)-\log\left(-\frac{1}{2}\cos\left(ex+d\right)+\frac{1}{2}\right)}{4ce}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*a-2*a*cos(e*x+d)+2*c*sin(e*x+d)),x, algorithm="fricas")
```

```
[Out] -1/4*(log(a*c*sin(e*x + d) + 1/2*a^2 + 1/2*c^2 - 1/2*(a^2 - c^2)*cos(e*x +
d)) - log(-1/2*cos(e*x + d) + 1/2))/(c*e)
```

giac [A] time = 0.16, size = 42, normalized size = 1.68

$$-\frac{1}{2}\left(\frac{\log\left(\left|a\tan\left(\frac{1}{2}xe+\frac{1}{2}d\right)+c\right|\right)}{c}-\frac{\log\left(\left|\tan\left(\frac{1}{2}xe+\frac{1}{2}d\right)\right|\right)}{c}\right)e^{(-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*a-2*a*cos(e*x+d)+2*c*sin(e*x+d)),x, algorithm="giac")
```

```
[Out] -1/2*(log(abs(a*tan(1/2*x*e + 1/2*d) + c))/c - log(abs(tan(1/2*x*e + 1/2*d)
))/c)*e^(-1)
```

maple [A] time = 0.42, size = 42, normalized size = 1.68

$$-\frac{\ln\left(c+a\tan\left(\frac{d}{2}+\frac{ex}{2}\right)\right)}{2ec}+\frac{\ln\left(\tan\left(\frac{d}{2}+\frac{ex}{2}\right)\right)}{2ec}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2*a-2*a*cos(e*x+d)+2*c*sin(e*x+d)),x)
```

```
[Out] -1/2/e/c*ln(c+a*tan(1/2*d+1/2*e*x))+1/2/e/c*ln(tan(1/2*d+1/2*e*x))
```

maxima [B] time = 0.32, size = 54, normalized size = 2.16

$$-\frac{\frac{\log\left(c+\frac{a\sin(ex+d)}{\cos(ex+d)+1}\right)}{c}-\frac{\log\left(\frac{\sin(ex+d)}{\cos(ex+d)+1}\right)}{c}}{2e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*a-2*a*cos(e*x+d)+2*c*sin(e*x+d)),x, algorithm="maxima")
```

```
[Out] -1/2*(log(c + a*sin(e*x + d)/(cos(e*x + d) + 1))/c - log(sin(e*x + d)/(cos(
e*x + d) + 1))/c)/e
```

mupad [B] time = 2.62, size = 26, normalized size = 1.04

$$-\frac{\operatorname{atanh}\left(\frac{2\,a\tan\left(\frac{d}{2}+\frac{e\,x}{2}\right)}{c}+1\right)}{c\,e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2*a - 2*a*cos(d + e*x) + 2*c*sin(d + e*x)),x)
```

```
[Out] -atanh((2*a*tan(d/2 + (e*x)/2))/c + 1)/(c*e)
```

sympy [A] time = 1.30, size = 95, normalized size = 3.80

$$\begin{cases} \frac{\tilde{\infty}x}{\sin(d)} & \text{for } a=0 \wedge c=0 \wedge e=0 \\ -\frac{1}{2ae\tan\left(\frac{d}{2}+\frac{ex}{2}\right)} & \text{for } c=0 \\ \frac{\log\left(\tan\left(\frac{d}{2}+\frac{ex}{2}\right)\right)}{2ce} & \text{for } a=0 \\ \frac{x}{-2a\cos(d)+2a+2c\sin(d)} & \text{for } e=0 \\ -\frac{\log\left(\tan\left(\frac{d}{2}+\frac{ex}{2}\right)+\frac{c}{a}\right)}{2ce}+\frac{\log\left(\tan\left(\frac{d}{2}+\frac{ex}{2}\right)\right)}{2ce} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*a-2*a*cos(e*x+d)+2*c*sin(e*x+d)),x)
```

```
[Out] Piecewise((zoo*x/sin(d), Eq(a, 0) & Eq(c, 0) & Eq(e, 0)), (-1/(2*a*e*tan(d/
2 + e*x/2)), Eq(c, 0)), (log(tan(d/2 + e*x/2))/(2*c*e), Eq(a, 0)), (x/(-2*a
*cos(d) + 2*a + 2*c*sin(d)), Eq(e, 0)), (-log(tan(d/2 + e*x/2) + c/a)/(2*c*
e) + log(tan(d/2 + e*x/2))/(2*c*e), True))
```

3.378 $\int \frac{1}{(2a-2a\cos(d+ex)+2c\sin(d+ex))^2}\,dx$

Optimal. Leaf size=75

$$\frac{a\log\left(a+c\cot\left(\frac{1}{2}(d+ex)\right)\right)}{4c^3e}-\frac{a\sin(d+ex)+c\cos(d+ex)}{4c^2e(a(-\cos(d+ex))+a+c\sin(d+ex))}$$

```
[Out] 1/4*a*ln(a+c*cot(1/2*e*x+1/2*d))/c^3/e+1/4*(-c*cos(e*x+d)-a*sin(e*x+d))/c^2
/e/(a-a*cos(e*x+d)+c*sin(e*x+d))
```

Rubi [A] time = 0.05, antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3129, 12, 3121, 31}

$$\frac{a\log\left(a+c\cot\left(\frac{1}{2}(d+ex)\right)\right)}{4c^3e}-\frac{a\sin(d+ex)+c\cos(d+ex)}{4c^2e(a(-\cos(d+ex))+a+c\sin(d+ex))}$$

Antiderivative was successfully verified.

```
[In] Int[(2*a - 2*a*Cos[d + e*x] + 2*c*Sin[d + e*x])^(-2),x]
```

```
[Out] (a*Log[a + c*Cot[(d + e*x)/2]])/(4*c^3*e) - (c*Cos[d + e*x] + a*Sin[d + e*x
])/(4*c^2*e*(a - a*Cos[d + e*x] + c*Sin[d + e*x]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 3121

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(-1), x_Symbol] :> Module[{f = FreeFactors[Cot[(d + e*x)/2], x]}, -Dist[f/e
, Subst[Int[1/(a + c*f*x), x], x, Cot[(d + e*x)/2]/f], x]] /; FreeQ[{a, b,
c, d, e}, x] && EqQ[a + b, 0]
```

Rule 3129

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(n_), x_Symbol] :> Simp[((-(c*Cos[d + e*x]) + b*Sin[d + e*x])*(a + b*Cos[d
+ e*x] + c*Sin[d + e*x])^(n + 1))/(e*(n + 1)*(a^2 - b^2 - c^2)), x] + Dist[
1/((n + 1)*(a^2 - b^2 - c^2)), Int[(a*(n + 1) - b*(n + 2)*Cos[d + e*x] - c*
(n + 2)*Sin[d + e*x])*(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n + 1), x], x]
 /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0] && LtQ[n, -1] && N
eQ[n, -3/2]
```

Rubi steps

$$\int \frac{1}{(2a-2a\cos(d+ex)+2c\sin(d+ex))^2}\,dx = -\frac{c\cos(d+ex)+a\sin(d+ex)}{4c^2e(a-a\cos(d+ex)+c\sin(d+ex))} + \frac{\int -\frac{2a}{2a-2a\cos(d+ex)+2c\,\mathrm{s}}}{4c^2}$$

$$= -\frac{c\cos(d+ex)+a\sin(d+ex)}{4c^2e(a-a\cos(d+ex)+c\sin(d+ex))} - \frac{a\int \frac{1}{2a-2a\cos(d+ex)+2c\,\mathrm{s}}}{2c^2}$$

$$= -\frac{c\cos(d+ex)+a\sin(d+ex)}{4c^2e(a-a\cos(d+ex)+c\sin(d+ex))} + \frac{a\,\mathrm{Subst}\left(\int \frac{1}{2a+2cx}\,dx\right.}{2c^{}}$$

$$= \frac{a\log\left(a+c\cot\left(\frac{1}{2}(d+ex)\right)\right)}{4c^3e} - \frac{c\cos(d+ex)+a\sin(d+e}{4c^2e(a-a\cos(d+ex)+c\sin(d}$$

Mathematica [B] time = 0.42, size = 229, normalized size = 3.05

$$-\frac{\sin\left(\frac{1}{2}(d+ex)\right)\left(a\sin\left(\frac{1}{2}(d+ex)\right)+c\cos\left(\frac{1}{2}(d+ex)\right)\right)\left(\cos(d+ex)\left(2a^2\log\left(a\sin\left(\frac{1}{2}(d+ex)\right)+c\cos\left(\frac{1}{2}(d+\right.\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2*a - 2*a*Cos[d + e*x] + 2*c*Sin[d + e*x])^(-2),x]
```

```
[Out] -1/4*(Sin[(d + e*x)/2]*(c*Cos[(d + e*x)/2] + a*Sin[(d + e*x)/2])*(Cos[d + e
*x]*(a^2 + 2*c^2 - 2*a^2*Log[Sin[(d + e*x)/2]] + 2*a^2*Log[c*Cos[(d + e*x)/
2] + a*Sin[(d + e*x)/2]]) + a*(a*(-1 + 2*Log[Sin[(d + e*x)/2]] - 2*Log[c*Co
s[(d + e*x)/2] + a*Sin[(d + e*x)/2]]) + c*(1 + 2*Log[Sin[(d + e*x)/2]] - 2*
Log[c*Cos[(d + e*x)/2] + a*Sin[(d + e*x)/2]])*Sin[d + e*x])))/(c^3*e*(a - a
*Cos[d + e*x] + c*Sin[d + e*x])^2)
```

fricas [B] time = 0.96, size = 162, normalized size = 2.16

$$\frac{2\,c^2\cos\left(ex+d\right)+2\,ac\sin\left(ex+d\right)+\left(a^2\cos\left(ex+d\right)-ac\sin\left(ex+d\right)-a^2\right)\log\left(ac\sin\left(ex+d\right)+\frac{1}{2}\,a^2+\frac{1}{2}\,c^2-\right.}{8\left(ac^3e\cos\left(ex+d\right)-c^4e\sin\left(ex\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*a-2*a*cos(e*x+d)+2*c*sin(e*x+d))^2,x, algorithm="fricas")
```

```
[Out] 1/8*(2*c^2*cos(e*x + d) + 2*a*c*sin(e*x + d) + (a^2*cos(e*x + d) - a*c*sin(
e*x + d) - a^2)*log(a*c*sin(e*x + d) + 1/2*a^2 + 1/2*c^2 - 1/2*(a^2 - c^2)*
cos(e*x + d)) - (a^2*cos(e*x + d) - a*c*sin(e*x + d) - a^2)*log(-1/2*cos(e*
x + d) + 1/2))/(a*c^3*e*cos(e*x + d) - c^4*e*sin(e*x + d) - a*c^3*e)
```

giac [A] time = 0.17, size = 115, normalized size = 1.53

$$\frac{1}{8}\left(\frac{2\,a\log\left(\left|a\tan\left(\frac{1}{2}\,xe+\frac{1}{2}\,d\right)+c\right|\right)}{c^3}-\frac{2\,a\log\left(\left|\tan\left(\frac{1}{2}\,xe+\frac{1}{2}\,d\right)\right|\right)}{c^3}-\frac{2\,a^2\tan\left(\frac{1}{2}\,xe+\frac{1}{2}\,d\right)+c^2\tan\left(\frac{1}{2}\,xe+\frac{1}{2}\,d\right)+}{\left(a\tan\left(\frac{1}{2}\,xe+\frac{1}{2}\,d\right)^2+c\tan\left(\frac{1}{2}\,xe+\frac{1}{2}\,d\right)\right)ac}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*a-2*a*cos(e*x+d)+2*c*sin(e*x+d))^2,x, algorithm="giac")
```

```
[Out] 1/8*(2*a*log(abs(a*tan(1/2*x*e + 1/2*d) + c))/c^3 - 2*a*log(abs(tan(1/2*x*e
 + 1/2*d)))/c^3 - (2*a^2*tan(1/2*x*e + 1/2*d) + c^2*tan(1/2*x*e + 1/2*d) +
a*c)/((a*tan(1/2*x*e + 1/2*d)^2 + c*tan(1/2*x*e + 1/2*d))*a*c^2))*e^(-1)
```

maple [A] time = 0.47, size = 110, normalized size = 1.47

$$-\frac{a}{8e\,c^2\left(c+a\tan\left(\frac{d}{2}+\frac{ex}{2}\right)\right)}-\frac{1}{8ea\left(c+a\tan\left(\frac{d}{2}+\frac{ex}{2}\right)\right)}+\frac{a\ln\left(c+a\tan\left(\frac{d}{2}+\frac{ex}{2}\right)\right)}{4e\,c^3}-\frac{1}{8e\,c^2\tan\left(\frac{d}{2}+\frac{ex}{2}\right)}-\frac{a\ln\left(\tan\left(\frac{d}{2}\right.\right.}{4e\,c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2*a-2*a*cos(e*x+d)+2*c*sin(e*x+d))^2,x)
```

```
[Out] -1/8/e/c^2*a/(c+a*tan(1/2*d+1/2*e*x))-1/8/e/a/(c+a*tan(1/2*d+1/2*e*x))+1/4/
e/c^3*a*ln(c+a*tan(1/2*d+1/2*e*x))-1/8/e/c^2/tan(1/2*d+1/2*e*x)-1/4/e/c^3*a
*ln(tan(1/2*d+1/2*e*x))
```

maxima [A] time = 0.33, size = 137, normalized size = 1.83

$$-\frac{\frac{ac+\frac{\left(2\,a^2+c^2\right)\sin(ex+d)}{\cos(ex+d)+1}}{\frac{ac^3\sin(ex+d)}{\cos(ex+d)+1}+\frac{a^2c^2\sin(ex+d)^2}{(\cos(ex+d)+1)^2}}-\frac{2\,a\log\left(c+\frac{a\sin(ex+d)}{\cos(ex+d)+1}\right)}{c^3}+\frac{2\,a\log\left(\frac{\sin(ex+d)}{\cos(ex+d)+1}\right)}{c^3}}{8\,e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*a-2*a*cos(e*x+d)+2*c*sin(e*x+d))^2,x, algorithm="maxima")
```

```
[Out] -1/8*((a*c + (2*a^2 + c^2)*sin(e*x + d)/(cos(e*x + d) + 1))/(a*c^3*sin(e*x
+ d)/(cos(e*x + d) + 1) + a^2*c^2*sin(e*x + d)^2/(cos(e*x + d) + 1)^2) - 2*
a*log(c + a*sin(e*x + d)/(cos(e*x + d) + 1))/c^3 + 2*a*log(sin(e*x + d)/(co
s(e*x + d) + 1))/c^3)/e
```

mupad [B] time = 2.55, size = 91, normalized size = 1.21

$$\frac{a\,\mathrm{atanh}\left(\frac{2\,a\,\tan\left(\frac{d}{2}+\frac{e\,x}{2}\right)}{c}+1\right)}{2\,c^3\,e}-\frac{\frac{1}{c}+\frac{\tan\left(\frac{d}{2}+\frac{e\,x}{2}\right)\,\left(2\,a^2+c^2\right)}{a\,c^2}}{e\,\left(8\,a\,{\tan\left(\frac{d}{2}+\frac{e\,x}{2}\right)}^2+8\,c\,\tan\left(\frac{d}{2}+\frac{e\,x}{2}\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2*a - 2*a*cos(d + e*x) + 2*c*sin(d + e*x))^2,x)
```

```
[Out] (a*atanh((2*a*tan(d/2 + (e*x)/2))/c + 1))/(2*c^3*e) - (1/c + (tan(d/2 + (e*
x)/2)*(2*a^2 + c^2))/(a*c^2))/(e*(8*c*tan(d/2 + (e*x)/2) + 8*a*tan(d/2 + (e
*x)/2)^2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*a-2*a*cos(e*x+d)+2*c*sin(e*x+d))**2,x)
```

```
[Out] Timed out
```

3.379 $\int \frac{1}{(2a-2a\cos(d+ex)+2c\sin(d+ex))^3}\,dx$

Optimal. Leaf size=134

$$\frac{3\left(a^2\sin(d+ex)+ac\cos(d+ex)\right)}{16c^4e(a(-\cos(d+ex))+a+c\sin(d+ex))}-\frac{\left(3a^2+c^2\right)\log\left(a+c\cot\left(\frac{1}{2}(d+ex)\right)\right)}{16c^5e}-\frac{a\sin(d+ex)+c\cos(}{16c^2e(a(-\cos(d+ex))+a+}$$

```
[Out] -1/16*(3*a^2+c^2)*ln(a+c*cot(1/2*e*x+1/2*d))/c^5/e+1/16*(-c*cos(e*x+d)-a*si
n(e*x+d))/c^2/e/(a-a*cos(e*x+d)+c*sin(e*x+d))^2+3/16*(a*c*cos(e*x+d)+a^2*si
n(e*x+d))/c^4/e/(a-a*cos(e*x+d)+c*sin(e*x+d))
```

Rubi [A] time = 0.11, antiderivative size = 134, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3129, 3153, 3121, 31}

$$\frac{3\left(a^2\sin(d+ex)+ac\cos(d+ex)\right)}{16c^4e(a(-\cos(d+ex))+a+c\sin(d+ex))}-\frac{\left(3a^2+c^2\right)\log\left(a+c\cot\left(\frac{1}{2}(d+ex)\right)\right)}{16c^5e}-\frac{a\sin(d+ex)+c\cos(}{16c^2e(a(-\cos(d+ex))+a+}$$

Antiderivative was successfully verified.

```
[In] Int[(2*a - 2*a*Cos[d + e*x] + 2*c*Sin[d + e*x])^(-3),x]
```

```
[Out] -((3*a^2 + c^2)*Log[a + c*Cot[(d + e*x)/2]])/(16*c^5*e) - (c*Cos[d + e*x] +
 a*Sin[d + e*x])/(16*c^2*e*(a - a*Cos[d + e*x] + c*Sin[d + e*x])^2) + (3*(a
*c*Cos[d + e*x] + a^2*Sin[d + e*x]))/(16*c^4*e*(a - a*Cos[d + e*x] + c*Sin[
d + e*x]))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 3121

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(-1), x_Symbol] :> Module[{f = FreeFactors[Cot[(d + e*x)/2], x]}, -Dist[f/e
, Subst[Int[1/(a + c*f*x), x], x, Cot[(d + e*x)/2]/f], x]] /; FreeQ[{a, b,
c, d, e}, x] && EqQ[a + b, 0]
```

Rule 3129

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(n_), x_Symbol] :> Simp[((-(c*Cos[d + e*x]) + b*Sin[d + e*x])*(a + b*Cos[d
+ e*x] + c*Sin[d + e*x])^(n + 1))/(e*(n + 1)*(a^2 - b^2 - c^2)), x] + Dist[
```

```
1/((n + 1)*(a^2 - b^2 - c^2)), Int[(a*(n + 1) - b*(n + 2)*Cos[d + e*x] - c*
(n + 2)*Sin[d + e*x])*(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n + 1), x], x]
 /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0] && LtQ[n, -1] && N
eQ[n, -3/2]
```

Rule 3153

```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])
/((a_.) + cos[(d_.) + (e_.)*(x_)]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)])^2,
 x_Symbol] :> Simp[(c*B - b*C - (a*C - c*A)*Cos[d + e*x] + (a*B - b*A)*Sin[
d + e*x])/(e*(a^2 - b^2 - c^2)*(a + b*Cos[d + e*x] + c*Sin[d + e*x])), x] +
 Dist[(a*A - b*B - c*C)/(a^2 - b^2 - c^2), Int[1/(a + b*Cos[d + e*x] + c*Si
n[d + e*x]), x], x] /; FreeQ[{a, b, c, d, e, A, B, C}, x] && NeQ[a^2 - b^2
- c^2, 0] && NeQ[a*A - b*B - c*C, 0]
```

Rubi steps

$$\int \frac{1}{(2a-2a\cos(d+ex)+2c\sin(d+ex))^3}\,dx = -\frac{c\cos(d+ex)+a\sin(d+ex)}{16c^2e(a-a\cos(d+ex)+c\sin(d+ex))^2} + \frac{\int \frac{-4a-2a\cos(d+ex)+2}{(2a-2a\cos(d+ex)+2c}}{8c^2}$$

$$= -\frac{c\cos(d+ex)+a\sin(d+ex)}{16c^2e(a-a\cos(d+ex)+c\sin(d+ex))^2} + \frac{3\left(ac\cos(d+ex\right.}{16c^4e(a-a\cos(d}$$

$$= -\frac{c\cos(d+ex)+a\sin(d+ex)}{16c^2e(a-a\cos(d+ex)+c\sin(d+ex))^2} + \frac{3\left(ac\cos(d+ex\right.}{16c^4e(a-a\cos(d}$$

$$= -\frac{\left(3a^2+c^2\right)\log\left(a+c\cot\left(\frac{1}{2}(d+ex)\right)\right)}{16c^5e} - \frac{c\cos(d+ex)+}{16c^2e(a-a\cos(d+e}$$

Mathematica [C] time = 0.61, size = 350, normalized size = 2.61

$$\frac{\sin\left(\frac{1}{2}(d+ex)\right)\left(a\sin\left(\frac{1}{2}(d+ex)\right)+c\cos\left(\frac{1}{2}(d+ex)\right)\right)\left(-6a\left(a^2+c^2\right)\sin^3\left(\frac{1}{2}(d+ex)\right)\left(a\sin\left(\frac{1}{2}(d+ex)\right)+c\cos\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2*a - 2*a*Cos[d + e*x] + 2*c*Sin[d + e*x])^(-3),x]
```

```
[Out] (Sin[(d + e*x)/2]*(c*Cos[(d + e*x)/2] + a*Sin[(d + e*x)/2])*(c^2*((-I)*a +
c)*(I*a + c)*Sin[(d + e*x)/2]^2 - 6*a*(a^2 + c^2)*Sin[(d + e*x)/2]^3*(c*Cos
```

```
[(d + e*x)/2] + a*Sin[(d + e*x)/2]) - c^2*(c*Cos[(d + e*x)/2] + a*Sin[(d +
e*x)/2])^2 + 4*(3*a^2 + c^2)*Log[Sin[(d + e*x)/2]]*Sin[(d + e*x)/2]^2*(c*Co
s[(d + e*x)/2] + a*Sin[(d + e*x)/2])^2 - 4*(3*a^2 + c^2)*Log[c*Cos[(d + e*x
)/2] + a*Sin[(d + e*x)/2]]*Sin[(d + e*x)/2]^2*(c*Cos[(d + e*x)/2] + a*Sin[(
d + e*x)/2])^2 + 3*a*c*(c*Cos[(d + e*x)/2] + a*Sin[(d + e*x)/2])^2*Sin[d +
e*x]))/(8*c^5*e*(a - a*Cos[d + e*x] + c*Sin[d + e*x])^3)
```

fricas [B] time = 1.08, size = 438, normalized size = 3.27

$$\frac{12\,a^2c^2\cos\left(ex+d\right)^2-6\,a^2c^2-2\left(3\,a^2c^2-c^4\right)\cos\left(ex+d\right)+\left(3\,a^4+4\,a^2c^2+c^4+\left(3\,a^4-2\,a^2c^2-c^4\right)\cos\left(ex+d\right)\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*a-2*a*cos(e*x+d)+2*c*sin(e*x+d))^3,x, algorithm="fricas")
```

```
[Out] 1/32*(12*a^2*c^2*cos(e*x + d)^2 - 6*a^2*c^2 - 2*(3*a^2*c^2 - c^4)*cos(e*x +
 d) + (3*a^4 + 4*a^2*c^2 + c^4 + (3*a^4 - 2*a^2*c^2 - c^4)*cos(e*x + d)^2 -
 2*(3*a^4 + a^2*c^2)*cos(e*x + d) + 2*(3*a^3*c + a*c^3 - (3*a^3*c + a*c^3)*
cos(e*x + d))*sin(e*x + d))*log(a*c*sin(e*x + d) + 1/2*a^2 + 1/2*c^2 - 1/2*
(a^2 - c^2)*cos(e*x + d)) - (3*a^4 + 4*a^2*c^2 + c^4 + (3*a^4 - 2*a^2*c^2 -
 c^4)*cos(e*x + d)^2 - 2*(3*a^4 + a^2*c^2)*cos(e*x + d) + 2*(3*a^3*c + a*c^
3 - (3*a^3*c + a*c^3)*cos(e*x + d))*sin(e*x + d))*log(-1/2*cos(e*x + d) + 1
/2) - 2*(3*a^3*c - a*c^3 - 3*(a^3*c - a*c^3)*cos(e*x + d))*sin(e*x + d))/(2
*a^2*c^5*e*cos(e*x + d) - (a^2*c^5 - c^7)*e*cos(e*x + d)^2 - (a^2*c^5 + c^7
)*e + 2*(a*c^6*e*cos(e*x + d) - a*c^6*e)*sin(e*x + d))
```

giac [A] time = 0.21, size = 239, normalized size = 1.78

$$\frac{1}{64}\left(\frac{4\left(3\,a^2+c^2\right)\log\left(\left|\tan\left(\frac{1}{2}\,xe+\frac{1}{2}\,d\right)\right|\right)}{c^5}-\frac{4\left(3\,a^3+ac^2\right)\log\left(\left|a\tan\left(\frac{1}{2}\,xe+\frac{1}{2}\,d\right)+c\right|\right)}{ac^5}+\frac{12\,a^5\tan\left(\frac{1}{2}\,xe+\frac{1}{2}\,d\right)^3}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*a-2*a*cos(e*x+d)+2*c*sin(e*x+d))^3,x, algorithm="giac")
```

```
[Out] 1/64*(4*(3*a^2 + c^2)*log(abs(tan(1/2*x*e + 1/2*d)))/c^5 - 4*(3*a^3 + a*c^2
)*log(abs(a*tan(1/2*x*e + 1/2*d) + c))/(a*c^5) + (12*a^5*tan(1/2*x*e + 1/2*
d)^3 + 4*a^3*c^2*tan(1/2*x*e + 1/2*d)^3 - 2*a*c^4*tan(1/2*x*e + 1/2*d)^3 +
18*a^4*c*tan(1/2*x*e + 1/2*d)^2 + 6*a^2*c^3*tan(1/2*x*e + 1/2*d)^2 - c^5*ta
n(1/2*x*e + 1/2*d)^2 + 4*a^3*c^2*tan(1/2*x*e + 1/2*d) - a^2*c^3)/((a*tan(1/
2*x*e + 1/2*d)^2 + c*tan(1/2*x*e + 1/2*d))^2*a^2*c^4))*e^(-1)
```

maple [B] time = 0.57, size = 272, normalized size = 2.03

$$\frac{3a^2}{32e\,c^4\left(c+a\tan\left(\frac{d}{2}+\frac{ex}{2}\right)\right)}+\frac{1}{16e\,c^2\left(c+a\tan\left(\frac{d}{2}+\frac{ex}{2}\right)\right)}-\frac{1}{32e\,a^2\left(c+a\tan\left(\frac{d}{2}+\frac{ex}{2}\right)\right)}+\frac{a^2}{64e\,c^3\left(c+a\tan\left(\frac{d}{2}+\frac{ex}{2}\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2*a-2*a*cos(e*x+d)+2*c*sin(e*x+d))^3,x)
```

```
[Out] 3/32/e*a^2/c^4/(c+a*tan(1/2*d+1/2*e*x))+1/16/e/c^2/(c+a*tan(1/2*d+1/2*e*x))
-1/32/e/a^2/(c+a*tan(1/2*d+1/2*e*x))+1/64/e*a^2/c^3/(c+a*tan(1/2*d+1/2*e*x)
)^2+1/32/e/c/(c+a*tan(1/2*d+1/2*e*x))^2+1/64/e/a^2*c/(c+a*tan(1/2*d+1/2*e*x
))^2-3/16/e/c^5*ln(c+a*tan(1/2*d+1/2*e*x))*a^2-1/16/e/c^3*ln(c+a*tan(1/2*d+
1/2*e*x))-1/64/e/c^3/tan(1/2*d+1/2*e*x)^2+3/16/e/c^5*ln(tan(1/2*d+1/2*e*x))
*a^2+1/16/e/c^3*ln(tan(1/2*d+1/2*e*x))+3/32/e/c^4*a/tan(1/2*d+1/2*e*x)
```

maxima [B] time = 0.35, size = 265, normalized size = 1.98

$$-\frac{\frac{a^2c^3-\frac{4\,a^3c^2\sin(ex+d)}{\cos(ex+d)+1}-\frac{\left(18\,a^4c+6\,a^2c^3-c^5\right)\sin(ex+d)^2}{(\cos(ex+d)+1)^2}-\frac{2\left(6\,a^5+2\,a^3c^2-ac^4\right)\sin(ex+d)^3}{(\cos(ex+d)+1)^3}}{\frac{a^2c^6\sin(ex+d)^2}{(\cos(ex+d)+1)^2}+\frac{2\,a^3c^5\sin(ex+d)^3}{(\cos(ex+d)+1)^3}+\frac{a^4c^4\sin(ex+d)^4}{(\cos(ex+d)+1)^4}}+\frac{4\left(3\,a^2+c^2\right)\log\left(c+\frac{a\sin(ex+d)}{\cos(ex+d)+1}\right)}{c^5}-\frac{4\left(3\,a^2+c^2\right)\log\left(\frac{\sin(ex+d)}{\cos(ex+d)+1}\right)}{c^5}}{64\,e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*a-2*a*cos(e*x+d)+2*c*sin(e*x+d))^3,x, algorithm="maxima")
```

```
[Out] -1/64*((a^2*c^3 - 4*a^3*c^2*sin(e*x + d)/(cos(e*x + d) + 1) - (18*a^4*c + 6
*a^2*c^3 - c^5)*sin(e*x + d)^2/(cos(e*x + d) + 1)^2 - 2*(6*a^5 + 2*a^3*c^2
- a*c^4)*sin(e*x + d)^3/(cos(e*x + d) + 1)^3)/(a^2*c^6*sin(e*x + d)^2/(cos(
e*x + d) + 1)^2 + 2*a^3*c^5*sin(e*x + d)^3/(cos(e*x + d) + 1)^3 + a^4*c^4*s
in(e*x + d)^4/(cos(e*x + d) + 1)^4) + 4*(3*a^2 + c^2)*log(c + a*sin(e*x + d
)/(cos(e*x + d) + 1))/c^5 - 4*(3*a^2 + c^2)*log(sin(e*x + d)/(cos(e*x + d)
+ 1))/c^5)/e
```

mupad [B] time = 4.47, size = 186, normalized size = 1.39

$$\frac{\frac{2\,a\tan\left(\frac{d}{2}+\frac{e\,x}{2}\right)}{c^2}-\frac{1}{2\,c}+\frac{\tan\left(\frac{d}{2}+\frac{e\,x}{2}\right)^3\left(6\,a^4+2\,a^2\,c^2-c^4\right)}{a\,c^4}+\frac{\tan\left(\frac{d}{2}+\frac{e\,x}{2}\right)^2\left(18\,a^4+6\,a^2\,c^2-c^4\right)}{2\,a^2\,c^3}}{e\left(32\,a^2\tan\left(\frac{d}{2}+\frac{e\,x}{2}\right)^4+64\,a\,c\tan\left(\frac{d}{2}+\frac{e\,x}{2}\right)^3+32\,c^2\tan\left(\frac{d}{2}+\frac{e\,x}{2}\right)^2\right)}-\frac{\mathrm{atanh}\left(\frac{2\,a\tan\left(\frac{d}{2}+\frac{e\,x}{2}\right)}{c}+1\right)\left(3\,a^2+c^2\right)}{8\,c^5\,e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2*a - 2*a*cos(d + e*x) + 2*c*sin(d + e*x))^3,x)
```

```
[Out] ((2*a*tan(d/2 + (e*x)/2))/c^2 - 1/(2*c) + (tan(d/2 + (e*x)/2)^3*(6*a^4 - c^
4 + 2*a^2*c^2))/(a*c^4) + (tan(d/2 + (e*x)/2)^2*(18*a^4 - c^4 + 6*a^2*c^2))
/(2*a^2*c^3))/(e*(32*a^2*tan(d/2 + (e*x)/2)^4 + 32*c^2*tan(d/2 + (e*x)/2)^2
 + 64*a*c*tan(d/2 + (e*x)/2)^3)) - (atanh((2*a*tan(d/2 + (e*x)/2))/c + 1)*(
3*a^2 + c^2))/(8*c^5*e)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*a-2*a*cos(e*x+d)+2*c*sin(e*x+d))**3,x)
```

```
[Out] Timed out
```

3.380 $\int \frac{1}{(2a-2a\cos(d+ex)+2c\sin(d+ex))^4}\,dx$

Optimal. Leaf size=207

$$\frac{5\left(a^2\sin(d+ex)+ac\cos(d+ex)\right)}{96c^4e(a(-\cos(d+ex))+a+c\sin(d+ex))^2}+\frac{a\left(5a^2+3c^2\right)\log\left(a+c\cot\left(\frac{1}{2}(d+ex)\right)\right)}{32c^7e}-\frac{a\left(15a^2+4c^2\right)\sin(d+e}{96c^6e(a(-\cos(d+}$$

```
[Out] 1/32*a*(5*a^2+3*c^2)*ln(a+c*cot(1/2*e*x+1/2*d))/c^7/e+1/48*(-c*cos(e*x+d)-a
*sin(e*x+d))/c^2/e/(a-a*cos(e*x+d)+c*sin(e*x+d))^3+5/96*(a*c*cos(e*x+d)+a^2
*sin(e*x+d))/c^4/e/(a-a*cos(e*x+d)+c*sin(e*x+d))^2+1/96*(-c*(15*a^2+4*c^2)*
cos(e*x+d)-a*(15*a^2+4*c^2)*sin(e*x+d))/c^6/e/(a-a*cos(e*x+d)+c*sin(e*x+d))
```

Rubi [A] time = 0.24, antiderivative size = 207, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {3129, 3156, 3153, 3121, 31}

$$-\frac{a\left(15a^2+4c^2\right)\sin(d+ex)+c\left(15a^2+4c^2\right)\cos(d+ex)}{96c^6e(a(-\cos(d+ex))+a+c\sin(d+ex))}+\frac{5\left(a^2\sin(d+ex)+ac\cos(d+ex)\right)}{96c^4e(a(-\cos(d+ex))+a+c\sin(d+ex))^2}+\frac{a\left(5a^2\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(2*a - 2*a*Cos[d + e*x] + 2*c*Sin[d + e*x])^(-4),x]
```

```
[Out] (a*(5*a^2 + 3*c^2)*Log[a + c*Cot[(d + e*x)/2]])/(32*c^7*e) - (c*Cos[d + e*x
] + a*Sin[d + e*x])/(48*c^2*e*(a - a*Cos[d + e*x] + c*Sin[d + e*x])^3) + (5
*(a*c*Cos[d + e*x] + a^2*Sin[d + e*x]))/(96*c^4*e*(a - a*Cos[d + e*x] + c*S
in[d + e*x])^2) - (c*(15*a^2 + 4*c^2)*Cos[d + e*x] + a*(15*a^2 + 4*c^2)*Sin
[d + e*x])/(96*c^6*e*(a - a*Cos[d + e*x] + c*Sin[d + e*x]))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 3121

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(-1), x_Symbol] :> Module[{f = FreeFactors[Cot[(d + e*x)/2], x]}, -Dist[f/e
, Subst[Int[1/(a + c*f*x), x], x, Cot[(d + e*x)/2]/f], x]] /; FreeQ[{a, b,
c, d, e}, x] && EqQ[a + b, 0]
```

Rule 3129

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(n_), x_Symbol] :> Simp[((-(c*Cos[d + e*x]) + b*Sin[d + e*x])*(a + b*Cos[d
+ e*x] + c*Sin[d + e*x])^(n + 1))/(e*(n + 1)*(a^2 - b^2 - c^2)), x] + Dist[
1/((n + 1)*(a^2 - b^2 - c^2)), Int[(a*(n + 1) - b*(n + 2)*Cos[d + e*x] - c*
(n + 2)*Sin[d + e*x])*(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n + 1), x], x]
 /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0] && LtQ[n, -1] && N
eQ[n, -3/2]
```

Rule 3153

```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])
/((a_.) + cos[(d_.) + (e_.)*(x_)]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)])^2,
 x_Symbol] :> Simp[(c*B - b*C - (a*C - c*A)*Cos[d + e*x] + (a*B - b*A)*Sin[
d + e*x])/(e*(a^2 - b^2 - c^2)*(a + b*Cos[d + e*x] + c*Sin[d + e*x])), x] +
 Dist[(a*A - b*B - c*C)/(a^2 - b^2 - c^2), Int[1/(a + b*Cos[d + e*x] + c*Si
n[d + e*x]), x], x] /; FreeQ[{a, b, c, d, e, A, B, C}, x] && NeQ[a^2 - b^2
- c^2, 0] && NeQ[a*A - b*B - c*C, 0]
```

Rule 3156

```
Int[((a_.) + cos[(d_.) + (e_.)*(x_)]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)])
^(n_)*((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)
]), x_Symbol] :> -Simp[((c*B - b*C - (a*C - c*A)*Cos[d + e*x] + (a*B - b*A)
*Sin[d + e*x])*(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n + 1))/(e*(n + 1)*(a
^2 - b^2 - c^2)), x] + Dist[1/((n + 1)*(a^2 - b^2 - c^2)), Int[(a + b*Cos[d
 + e*x] + c*Sin[d + e*x])^(n + 1)*Simp[(n + 1)*(a*A - b*B - c*C) + (n + 2)*
(a*B - b*A)*Cos[d + e*x] + (n + 2)*(a*C - c*A)*Sin[d + e*x], x], x], x] /;
FreeQ[{a, b, c, d, e, A, B, C}, x] && LtQ[n, -1] && NeQ[a^2 - b^2 - c^2, 0]
 && NeQ[n, -2]
```

Rubi steps

$$\int \frac{1}{(2a-2a\cos(d+ex)+2c\sin(d+ex))^4}\,dx = -\frac{c\cos(d+ex)+a\sin(d+ex)}{48c^2e(a-a\cos(d+ex)+c\sin(d+ex))^3} + \frac{\int \frac{-6a-4a\cos(d+ex)+}{(2a-2a\cos(d+ex)+2c}}{12c^2}$$

$$= -\frac{c\cos(d+ex)+a\sin(d+ex)}{48c^2e(a-a\cos(d+ex)+c\sin(d+ex))^3} + \frac{5\left(ac\cos(d+e\right.}{96c^4e(a-a\cos(d}$$

$$= -\frac{c\cos(d+ex)+a\sin(d+ex)}{48c^2e(a-a\cos(d+ex)+c\sin(d+ex))^3} + \frac{5\left(ac\cos(d+e\right.}{96c^4e(a-a\cos(d}$$

$$= -\frac{c\cos(d+ex)+a\sin(d+ex)}{48c^2e(a-a\cos(d+ex)+c\sin(d+ex))^3} + \frac{5\left(ac\cos(d+e\right.}{96c^4e(a-a\cos(d}$$

$$= \frac{a\left(5a^2+3c^2\right)\log\left(a+c\cot\left(\frac{1}{2}(d+ex)\right)\right)}{32c^7e} - \frac{c\cos(d+ex)}{48c^2e(a-a\cos(d+}$$

Mathematica [B] time = 1.17, size = 494, normalized size = 2.39

$$\frac{\sin\left(\frac{1}{2}(d+ex)\right)\left(a\sin\left(\frac{1}{2}(d+ex)\right)+c\cos\left(\frac{1}{2}(d+ex)\right)\right)\left(-225a^6\cos(d+ex)+90a^6\cos(2(d+ex))-15a^6\cos(3(\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2*a - 2*a*Cos[d + e*x] + 2*c*Sin[d + e*x])^(-4),x]
```

```
[Out] (Sin[(d + e*x)/2]*(c*Cos[(d + e*x)/2] + a*Sin[(d + e*x)/2])*(150*a^6 + 130*
a^4*c^2 + 24*a^2*c^4 - 225*a^6*Cos[d + e*x] - 255*a^4*c^2*Cos[d + e*x] - 42
*a^2*c^4*Cos[d + e*x] - 24*c^6*Cos[d + e*x] + 90*a^6*Cos[2*(d + e*x)] + 174
*a^4*c^2*Cos[2*(d + e*x)] - 15*a^6*Cos[3*(d + e*x)] - 49*a^4*c^2*Cos[3*(d +
 e*x)] + 18*a^2*c^4*Cos[3*(d + e*x)] + 8*c^6*Cos[3*(d + e*x)] - 192*(5*a^3
+ 3*a*c^2)*Log[Sin[(d + e*x)/2]]*Sin[(d + e*x)/2]^3*(c*Cos[(d + e*x)/2] + a
*Sin[(d + e*x)/2])^3 + 192*(5*a^3 + 3*a*c^2)*Log[c*Cos[(d + e*x)/2] + a*Sin
[(d + e*x)/2]]*Sin[(d + e*x)/2]^3*(c*Cos[(d + e*x)/2] + a*Sin[(d + e*x)/2])
^3 + 75*a^5*c*Sin[d + e*x] + 75*a^3*c^3*Sin[d + e*x] - 12*a*c^5*Sin[d + e*x
] - 60*a^5*c*Sin[2*(d + e*x)] - 156*a^3*c^3*Sin[2*(d + e*x)] - 12*a*c^5*Sin
[2*(d + e*x)] + 15*a^5*c*Sin[3*(d + e*x)] + 79*a^3*c^3*Sin[3*(d + e*x)] + 2
0*a*c^5*Sin[3*(d + e*x)]))/(384*c^7*e*(a - a*Cos[d + e*x] + c*Sin[d + e*x])
^4)
```

fricas [B] time = 2.47, size = 796, normalized size = 3.85

$$\frac{60\,a^4c^2+6\,a^2c^4+2\left(45\,a^4c^2-3\,a^2c^4-4\,c^6\right)\cos\left(ex+d\right)^3-12\left(10\,a^4c^2+a^2c^4\right)\cos\left(ex+d\right)^2-6\left(5\,a^4c^2-2\,a^2c\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*a-2*a*cos(e*x+d)+2*c*sin(e*x+d))^4,x, algorithm="fricas")
```

```
[Out] 1/192*(60*a^4*c^2 + 6*a^2*c^4 + 2*(45*a^4*c^2 - 3*a^2*c^4 - 4*c^6)*cos(e*x
+ d)^3 - 12*(10*a^4*c^2 + a^2*c^4)*cos(e*x + d)^2 - 6*(5*a^4*c^2 - 2*a^2*c^
4 - 2*c^6)*cos(e*x + d) - 3*(5*a^6 + 18*a^4*c^2 + 9*a^2*c^4 - (5*a^6 - 12*a
^4*c^2 - 9*a^2*c^4)*cos(e*x + d)^3 + 3*(5*a^6 - 2*a^4*c^2 - 3*a^2*c^4)*cos(
e*x + d)^2 - 3*(5*a^6 + 8*a^4*c^2 + 3*a^2*c^4)*cos(e*x + d) + (15*a^5*c + 1
4*a^3*c^3 + 3*a*c^5 + (15*a^5*c + 4*a^3*c^3 - 3*a*c^5)*cos(e*x + d)^2 - 6*(
5*a^5*c + 3*a^3*c^3)*cos(e*x + d))*sin(e*x + d))*log(a*c*sin(e*x + d) + 1/2
*a^2 + 1/2*c^2 - 1/2*(a^2 - c^2)*cos(e*x + d)) + 3*(5*a^6 + 18*a^4*c^2 + 9*
a^2*c^4 - (5*a^6 - 12*a^4*c^2 - 9*a^2*c^4)*cos(e*x + d)^3 + 3*(5*a^6 - 2*a^
4*c^2 - 3*a^2*c^4)*cos(e*x + d)^2 - 3*(5*a^6 + 8*a^4*c^2 + 3*a^2*c^4)*cos(e
*x + d) + (15*a^5*c + 14*a^3*c^3 + 3*a*c^5 + (15*a^5*c + 4*a^3*c^3 - 3*a*c^
5)*cos(e*x + d)^2 - 6*(5*a^5*c + 3*a^3*c^3)*cos(e*x + d))*sin(e*x + d))*log
(-1/2*cos(e*x + d) + 1/2) + 2*(15*a^5*c + 14*a^3*c^3 + 6*a*c^5 + (15*a^5*c
- 41*a^3*c^3 - 12*a*c^5)*cos(e*x + d)^2 - 3*(10*a^5*c - 9*a^3*c^3 - a*c^5)*
cos(e*x + d))*sin(e*x + d))/((a^3*c^7 - 3*a*c^9)*e*cos(e*x + d)^3 - 3*(a^3*
c^7 - a*c^9)*e*cos(e*x + d)^2 + 3*(a^3*c^7 + a*c^9)*e*cos(e*x + d) - (a^3*c
^7 + 3*a*c^9)*e + (6*a^2*c^8*e*cos(e*x + d) - (3*a^2*c^8 - c^10)*e*cos(e*x
+ d)^2 - (3*a^2*c^8 + c^10)*e)*sin(e*x + d))
```

giac [A] time = 0.25, size = 363, normalized size = 1.75

$$-\frac{1}{384}\left(\frac{12\left(5\,a^{3}+3\,ac^{2}\right)\log\left(\left|\tan\left(\frac{1}{2}\,xe+\frac{1}{2}\,d\right)\right|\right)}{c^{7}}-\frac{12\left(5\,a^{4}+3\,a^{2}c^{2}\right)\log\left(\left|a\tan\left(\frac{1}{2}\,xe+\frac{1}{2}\,d\right)+c\right|\right)}{ac^{7}}+\frac{60\,a^{8}\tan\left(\frac{1}{2}\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*a-2*a*cos(e*x+d)+2*c*sin(e*x+d))^4,x, algorithm="giac")
```

```
[Out] -1/384*(12*(5*a^3 + 3*a*c^2)*log(abs(tan(1/2*x*e + 1/2*d)))/c^7 - 12*(5*a^4
 + 3*a^2*c^2)*log(abs(a*tan(1/2*x*e + 1/2*d) + c))/(a*c^7) + (60*a^8*tan(1/
2*x*e + 1/2*d)^5 + 36*a^6*c^2*tan(1/2*x*e + 1/2*d)^5 + 3*a^2*c^6*tan(1/2*x*
e + 1/2*d)^5 + 150*a^7*c*tan(1/2*x*e + 1/2*d)^4 + 90*a^5*c^3*tan(1/2*x*e +
1/2*d)^4 + 3*a*c^7*tan(1/2*x*e + 1/2*d)^4 + 110*a^6*c^2*tan(1/2*x*e + 1/2*d
)^3 + 66*a^4*c^4*tan(1/2*x*e + 1/2*d)^3 + 3*a^2*c^6*tan(1/2*x*e + 1/2*d)^3
+ c^8*tan(1/2*x*e + 1/2*d)^3 + 15*a^5*c^3*tan(1/2*x*e + 1/2*d)^2 + 9*a^3*c^
5*tan(1/2*x*e + 1/2*d)^2 - 3*a^4*c^4*tan(1/2*x*e + 1/2*d) + a^3*c^5)/((a*ta
n(1/2*x*e + 1/2*d)^2 + c*tan(1/2*x*e + 1/2*d))^3*a^3*c^6))*e^(-1)
```

maple [B] time = 0.59, size = 416, normalized size = 2.01

$$-\frac{a^3}{64e\,c^5\left(c+a\tan\left(\frac{d}{2}+\frac{ex}{2}\right)\right)^2}-\frac{3a}{128e\,c^3\left(c+a\tan\left(\frac{d}{2}+\frac{ex}{2}\right)\right)^2}+\frac{c}{128e\,a^3\left(c+a\tan\left(\frac{d}{2}+\frac{ex}{2}\right)\right)^2}-\frac{5a^3}{64e\,c^6\left(c+a\tan\left(\frac{d}{2}\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2*a-2*a*cos(e*x+d)+2*c*sin(e*x+d))^4,x)
```

```
[Out] -1/64/e*a^3/c^5/(c+a*tan(1/2*d+1/2*e*x))^2-3/128/e*a/c^3/(c+a*tan(1/2*d+1/2
*e*x))^2+1/128/e/a^3*c/(c+a*tan(1/2*d+1/2*e*x))^2-5/64/e/c^6*a^3/(c+a*tan(1
/2*d+1/2*e*x))-9/128/e/c^4*a/(c+a*tan(1/2*d+1/2*e*x))-1/128/e/a^3/(c+a*tan(
1/2*d+1/2*e*x))-1/384/e*a^3/c^4/(c+a*tan(1/2*d+1/2*e*x))^3-1/128/e*a/c^2/(c
+a*tan(1/2*d+1/2*e*x))^3-1/128/e/a/(c+a*tan(1/2*d+1/2*e*x))^3-1/384/e/a^3*c
^2/(c+a*tan(1/2*d+1/2*e*x))^3+5/32/e*a^3/c^7*ln(c+a*tan(1/2*d+1/2*e*x))+3/3
2/e*a/c^5*ln(c+a*tan(1/2*d+1/2*e*x))-1/384/e/c^4/tan(1/2*d+1/2*e*x)^3-5/64/
e/c^6/tan(1/2*d+1/2*e*x)*a^2-3/128/e/c^4/tan(1/2*d+1/2*e*x)+1/64/e/c^5*a/ta
n(1/2*d+1/2*e*x)^2-5/32/e*a^3/c^7*ln(tan(1/2*d+1/2*e*x))-3/32/e*a/c^5*ln(ta
n(1/2*d+1/2*e*x))
```

maxima [A] time = 0.38, size = 382, normalized size = 1.85

$$-\frac{\frac{a^3c^5-\frac{3a^4c^4\sin(ex+d)}{\cos(ex+d)+1}+\frac{3\left(5a^5c^3+3a^3c^5\right)\sin(ex+d)^2}{(\cos(ex+d)+1)^2}+\frac{\left(110a^6c^2+66a^4c^4+3a^2c^6+c^8\right)\sin(ex+d)^3}{(\cos(ex+d)+1)^3}+\frac{3\left(50a^7c+30a^5c^3+ac^7\right)\sin(ex+d)^4}{(\cos(ex+d)+1)^4}+\frac{3\left(20a^8+12a^6c^2+a^2c^6\right)\sin(ex+d}{(\cos(ex+d)+1)^5}}{\frac{a^3c^9\sin(ex+d)^3}{(\cos(ex+d)+1)^3}+\frac{3a^4c^8\sin(ex+d)^4}{(\cos(ex+d)+1)^4}+\frac{3a^5c^7\sin(ex+d)^5}{(\cos(ex+d)+1)^5}+\frac{a^6c^6\sin(ex+d)^6}{(\cos(ex+d)+1)^6}}}{384\,e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*a-2*a*cos(e*x+d)+2*c*sin(e*x+d))^4,x, algorithm="maxima")
```

```
[Out] -1/384*((a^3*c^5 - 3*a^4*c^4*sin(e*x + d)/(cos(e*x + d) + 1) + 3*(5*a^5*c^3
 + 3*a^3*c^5)*sin(e*x + d)^2/(cos(e*x + d) + 1)^2 + (110*a^6*c^2 + 66*a^4*c
^4 + 3*a^2*c^6 + c^8)*sin(e*x + d)^3/(cos(e*x + d) + 1)^3 + 3*(50*a^7*c + 3
0*a^5*c^3 + a*c^7)*sin(e*x + d)^4/(cos(e*x + d) + 1)^4 + 3*(20*a^8 + 12*a^6
*c^2 + a^2*c^6)*sin(e*x + d)^5/(cos(e*x + d) + 1)^5)/(a^3*c^9*sin(e*x + d)^
3/(cos(e*x + d) + 1)^3 + 3*a^4*c^8*sin(e*x + d)^4/(cos(e*x + d) + 1)^4 + 3*
a^5*c^7*sin(e*x + d)^5/(cos(e*x + d) + 1)^5 + a^6*c^6*sin(e*x + d)^6/(cos(e
*x + d) + 1)^6) - 12*(5*a^3 + 3*a*c^2)*log(c + a*sin(e*x + d)/(cos(e*x + d)
 + 1))/c^7 + 12*(5*a^3 + 3*a*c^2)*log(sin(e*x + d)/(cos(e*x + d) + 1))/c^7)
/e
```

mupad [B] time = 6.05, size = 301, normalized size = 1.45

$$\frac{a\,\mathrm{atanh}\left(\frac{a\left(c+2\,a\tan\left(\frac{d}{2}+\frac{e\,x}{2}\right)\right)\left(5\,a^2+3\,c^2\right)}{c\left(5\,a^3+3\,a\,c^2\right)}\right)\left(5\,a^2+3\,c^2\right)}{16\,c^7\,e}-\frac{\frac{1}{3\,c}-\frac{a\tan\left(\frac{d}{2}+\frac{e\,x}{2}\right)}{c^2}+\frac{\tan\left(\frac{d}{2}+\frac{e\,x}{2}\right)^2\left(5\,a^2+3\,c^2\right)}{c^3}+\frac{\tan\left(\frac{d}{2}+\frac{e\,x}{2}\right)^3\left(110\,a^6+66\,a^4\,c^2\cdots\right)}{3\,a^3\,c^4}}{e\left(128\,a^3\tan\left(\frac{d}{2}+\frac{e\,x}{2}\right)^6+384\,a^2\,c\tan\left(\frac{d}{2}+\frac{e\,x}{2}\right)\cdots\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2*a - 2*a*cos(d + e*x) + 2*c*sin(d + e*x))^4,x)
```

```
[Out] (a*atanh((a*(c + 2*a*tan(d/2 + (e*x)/2))*(5*a^2 + 3*c^2))/(c*(3*a*c^2 + 5*a
^3)))*(5*a^2 + 3*c^2))/(16*c^7*e) - (1/(3*c) - (a*tan(d/2 + (e*x)/2))/c^2 +
 (tan(d/2 + (e*x)/2)^2*(5*a^2 + 3*c^2))/c^3 + (tan(d/2 + (e*x)/2)^3*(110*a^
6 + c^6 + 3*a^2*c^4 + 66*a^4*c^2))/(3*a^3*c^4) + (tan(d/2 + (e*x)/2)^5*(20*
a^6 + c^6 + 12*a^4*c^2))/(a*c^6) + (tan(d/2 + (e*x)/2)^4*(50*a^6 + c^6 + 30
*a^4*c^2))/(a^2*c^5))/(e*(128*a^3*tan(d/2 + (e*x)/2)^6 + 128*c^3*tan(d/2 +
(e*x)/2)^3 + 384*a*c^2*tan(d/2 + (e*x)/2)^4 + 384*a^2*c*tan(d/2 + (e*x)/2)^
5))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*a-2*a*cos(e*x+d)+2*c*sin(e*x+d))**4,x)
```

```
[Out] Timed out
```

3.381 $\int (2a + 2b\cos(d + ex) + 2a\sin(d + ex))^3\,dx$

Optimal. Leaf size=157

$$\frac{4b(15a^2 + 4b^2)\sin(d + ex)}{3e} - \frac{4a(15a^2 + 4b^2)\cos(d + ex)}{3e} + 4ax(5a^2 + 3b^2) - \frac{20(a^2\cos(d + ex) - ab\sin(d + ex))}{3}$$

```
[Out] 4*a*(5*a^2+3*b^2)*x-4/3*a*(15*a^2+4*b^2)*cos(e*x+d)/e+4/3*b*(15*a^2+4*b^2)*
sin(e*x+d)/e-8/3*(a+b*cos(e*x+d)+a*sin(e*x+d))^2*(a*cos(e*x+d)-b*sin(e*x+d)
)/e-20/3*(a+b*cos(e*x+d)+a*sin(e*x+d))*(a^2*cos(e*x+d)-a*b*sin(e*x+d))/e
```

Rubi [A] time = 0.14, antiderivative size = 157, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3120, 3146, 2637, 2638}

$$\frac{4b(15a^2 + 4b^2)\sin(d + ex)}{3e} - \frac{4a(15a^2 + 4b^2)\cos(d + ex)}{3e} + 4ax(5a^2 + 3b^2) - \frac{20(a^2\cos(d + ex) - ab\sin(d + ex))}{3}$$

Antiderivative was successfully verified.

```
[In] Int[(2*a + 2*b*Cos[d + e*x] + 2*a*Sin[d + e*x])^3,x]
```

```
[Out] 4*a*(5*a^2 + 3*b^2)*x - (4*a*(15*a^2 + 4*b^2)*Cos[d + e*x])/(3*e) + (4*b*(1
5*a^2 + 4*b^2)*Sin[d + e*x])/(3*e) - (8*(a + b*Cos[d + e*x] + a*Sin[d + e*x
])^2*(a*Cos[d + e*x] - b*Sin[d + e*x]))/(3*e) - (20*(a + b*Cos[d + e*x] + a
*Sin[d + e*x])*(a^2*Cos[d + e*x] - a*b*Sin[d + e*x]))/(3*e)
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3120

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(n_), x_Symbol] :> -Simp[((c*Cos[d + e*x] - b*Sin[d + e*x])*(a + b*Cos[d +
e*x] + c*Sin[d + e*x])^(n - 1))/(e*n), x] + Dist[1/n, Int[Simp[n*a^2 + (n -
 1)*(b^2 + c^2) + a*b*(2*n - 1)*Cos[d + e*x] + a*c*(2*n - 1)*Sin[d + e*x],
x]*(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n - 2), x], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[a^2 - b^2 - c^2, 0] && GtQ[n, 1]
```

Rule 3146

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(n_.)*((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)
]), x_Symbol] :> Simp[((B*c - b*C - a*C*Cos[d + e*x] + a*B*Sin[d + e*x])*(a
 + b*Cos[d + e*x] + c*Sin[d + e*x])^n)/(a*e*(n + 1)), x] + Dist[1/(a*(n + 1
)), Int[(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n - 1)*Simp[a*(b*B + c*C)*n
+ a^2*A*(n + 1) + (n*(a^2*B - B*c^2 + b*c*C) + a*b*A*(n + 1))*Cos[d + e*x]
+ (n*(b*B*c + a^2*C - b^2*C) + a*c*A*(n + 1))*Sin[d + e*x], x], x], x] /; F
reeQ[{a, b, c, d, e, A, B, C}, x] && GtQ[n, 0] && NeQ[a^2 - b^2 - c^2, 0]
```

Rubi steps

$$\int (2a+2b\cos(d+ex)+2a\sin(d+ex))^3\,dx = -\frac{8(a+b\cos(d+ex)+a\sin(d+ex))^2(a\cos(d+ex)-b\sin(d+e$$

$$= -\frac{8(a+b\cos(d+ex)+a\sin(d+ex))^2(a\cos(d+ex)-b\sin(d+e$$

$$= 4a\left(5a^2+3b^2\right)x - \frac{8(a+b\cos(d+ex)+a\sin(d+ex))^2(a\cos(d+$$

$$= 4a\left(5a^2+3b^2\right)x - \frac{4a\left(15a^2+4b^2\right)\cos(d+ex)}{3e} + \frac{4b\left(15a^2+4b^2\right)}{3e}$$

Mathematica [A] time = 0.44, size = 135, normalized size = 0.86

$$\frac{2\left(6a\left(5a^2+3b^2\right)(d+ex)-9a\left(a^2-b^2\right)\sin(2(d+ex))+9b\left(5a^2+b^2\right)\sin(d+ex)+b\left(b^2-3a^2\right)\sin(3(d+ex))-\right.}{3e}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2*a + 2*b*Cos[d + e*x] + 2*a*Sin[d + e*x])^3,x]
```

```
[Out] (2*(6*a*(5*a^2 + 3*b^2)*(d + e*x) - 9*a*(5*a^2 + b^2)*Cos[d + e*x] - 18*a^2
*b*Cos[2*(d + e*x)] + a*(a^2 - 3*b^2)*Cos[3*(d + e*x)] + 9*b*(5*a^2 + b^2)*
Sin[d + e*x] - 9*a*(a^2 - b^2)*Sin[2*(d + e*x)] + b*(-3*a^2 + b^2)*Sin[3*(d
 + e*x)]))/(3*e)
```

fricas [A] time = 0.77, size = 127, normalized size = 0.81

$$-\frac{4\left(18\,a^2b\cos\left(ex+d\right)^2+24\,a^3\cos\left(ex+d\right)-2\left(a^3-3\,ab^2\right)\cos\left(ex+d\right)^3-3\left(5\,a^3+3\,ab^2\right)ex-\left(24\,a^2b+4\,b^3-\right.\right.}{3\,e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*a+2*b*cos(e*x+d)+2*a*sin(e*x+d))^3,x, algorithm="fricas")
```

```
[Out] -4/3*(18*a^2*b*cos(e*x + d)^2 + 24*a^3*cos(e*x + d) - 2*(a^3 - 3*a*b^2)*cos
(e*x + d)^3 - 3*(5*a^3 + 3*a*b^2)*e*x - (24*a^2*b + 4*b^3 - 2*(3*a^2*b - b^
3)*cos(e*x + d)^2 - 9*(a^3 - a*b^2)*cos(e*x + d))*sin(e*x + d))/e
```

giac [A] time = 0.20, size = 151, normalized size = 0.96

$$-12\,a^2b\cos\left(2\,xe+2\,d\right)e^{(-1)}+\frac{2}{3}\left(a^3-3\,ab^2\right)\cos\left(3\,xe+3\,d\right)e^{(-1)}-6\left(5\,a^3+ab^2\right)\cos\left(xe+d\right)e^{(-1)}-\frac{2}{3}\left(3\,a^2b-b^3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*a+2*b*cos(e*x+d)+2*a*sin(e*x+d))^3,x, algorithm="giac")
```

```
[Out] -12*a^2*b*cos(2*x*e + 2*d)*e^(-1) + 2/3*(a^3 - 3*a*b^2)*cos(3*x*e + 3*d)*e^
(-1) - 6*(5*a^3 + a*b^2)*cos(x*e + d)*e^(-1) - 2/3*(3*a^2*b - b^3)*e^(-1)*s
in(3*x*e + 3*d) - 6*(a^3 - a*b^2)*e^(-1)*sin(2*x*e + 2*d) + 6*(5*a^2*b + b^
3)*e^(-1)*sin(x*e + d) + 4*(5*a^3 + 3*a*b^2)*x
```

maple [A] time = 0.25, size = 177, normalized size = 1.13

$$\frac{8a^3\left(ex+d\right)+24\sin\left(ex+d\right)a^2b-24a^3\cos\left(ex+d\right)+24a\,b^2\left(\frac{\sin(ex+d)\cos(ex+d)}{2}+\frac{ex}{2}+\frac{d}{2}\right)-24\left(\cos^2\left(ex+d\right)\right)a^2}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*a+2*b*cos(e*x+d)+2*a*sin(e*x+d))^3,x)
```

```
[Out] 8/e*(a^3*(e*x+d)+3*sin(e*x+d)*a^2*b-3*a^3*cos(e*x+d)+3*a*b^2*(1/2*sin(e*x+d
)*cos(e*x+d)+1/2*e*x+1/2*d)-3*cos(e*x+d)^2*a^2*b+3*a^3*(-1/2*sin(e*x+d)*cos
(e*x+d)+1/2*e*x+1/2*d)+1/3*b^3*(2+cos(e*x+d)^2)*sin(e*x+d)-a*b^2*cos(e*x+d)
^3+a^2*b*sin(e*x+d)^3-1/3*a^3*(2+sin(e*x+d)^2)*cos(e*x+d))
```

maxima [A] time = 0.33, size = 191, normalized size = 1.22

$$-\frac{8\,ab^2\cos\left(ex+d\right)^3}{e}+\frac{8\,a^2b\sin\left(ex+d\right)^3}{e}+8\,a^3x+\frac{8\left(\cos\left(ex+d\right)^3-3\,\cos\left(ex+d\right)\right)a^3}{3\,e}-\frac{8\left(\sin\left(ex+d\right)^3-3\,\sin\left(\right.\right.}{3\,e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*a+2*b*cos(e*x+d)+2*a*sin(e*x+d))^3,x, algorithm="maxima")
```

```
[Out] -8*a*b^2*cos(e*x + d)^3/e + 8*a^2*b*sin(e*x + d)^3/e + 8*a^3*x + 8/3*(cos(e
*x + d)^3 - 3*cos(e*x + d))*a^3/e - 8/3*(sin(e*x + d)^3 - 3*sin(e*x + d))*b
^3/e - 24*a^2*(a*cos(e*x + d)/e - b*sin(e*x + d)/e) - 6*(4*a*b*cos(e*x + d)
```

```
^2/e - (2*e*x + 2*d - sin(2*e*x + 2*d))*a^2/e - (2*e*x + 2*d + sin(2*e*x +
2*d))*b^2/e)*a
```

mupad [B] time = 3.60, size = 292, normalized size = 1.86

$$\frac{\tan\left(\frac{d}{2}+\frac{e\,x}{2}\right)^5\,\left(24\,a^3+48\,a^2\,b-24\,a\,b^2+16\,b^3\right)-\tan\left(\frac{d}{2}+\frac{e\,x}{2}\right)^4\,\left(48\,a^3-96\,a^2\,b+48\,a\,b^2\right)-16\,a\,b^2+\tan\left(\frac{d}{2}+\right.}{e\,\left(\tan\left(\frac{d}{2}+\frac{e\,x}{2}\right)^6+3\tan\left(\frac{d}{2}\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*a + 2*b*cos(d + e*x) + 2*a*sin(d + e*x))^3,x)
```

```
[Out] (tan(d/2 + (e*x)/2)^5*(48*a^2*b - 24*a*b^2 + 24*a^3 + 16*b^3) - tan(d/2 + (
e*x)/2)^4*(48*a*b^2 - 96*a^2*b + 48*a^3) - 16*a*b^2 + tan(d/2 + (e*x)/2)^2*
(96*a^2*b - 128*a^3) + tan(d/2 + (e*x)/2)^3*(160*a^2*b + (32*b^3)/3) - (176
*a^3)/3 + tan(d/2 + (e*x)/2)*(24*a*b^2 + 48*a^2*b - 24*a^3 + 16*b^3))/(e*(3
*tan(d/2 + (e*x)/2)^2 + 3*tan(d/2 + (e*x)/2)^4 + tan(d/2 + (e*x)/2)^6 + 1))
 + (8*a*atan((8*a*tan(d/2 + (e*x)/2)*(5*a^2 + 3*b^2))/(24*a*b^2 + 40*a^3))*
(5*a^2 + 3*b^2))/e - (8*a*(5*a^2 + 3*b^2)*(atan(tan(d/2 + (e*x)/2)) - (e*x)
/2))/e
```

sympy [A] time = 0.75, size = 291, normalized size = 1.85

$$\begin{cases} 12a^3x\sin^2(d+ex)+12a^3x\cos^2(d+ex)+8a^3x-\frac{8a^3\sin^2(d+ex)\cos(d+ex)}{e}-\frac{12a^3\sin(d+ex)\cos(d+ex)}{e}-\frac{16a^3\cos^3(d+ex)}{3e}- \\ x\left(2a\sin(d)+2a+2b\cos(d)\right)^3 \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*a+2*b*cos(e*x+d)+2*a*sin(e*x+d))**3,x)
```

```
[Out] Piecewise((12*a**3*x*sin(d + e*x)**2 + 12*a**3*x*cos(d + e*x)**2 + 8*a**3*x
 - 8*a**3*sin(d + e*x)**2*cos(d + e*x)/e - 12*a**3*sin(d + e*x)*cos(d + e*x
)/e - 16*a**3*cos(d + e*x)**3/(3*e) - 24*a**3*cos(d + e*x)/e + 8*a**2*b*sin
(d + e*x)**3/e + 24*a**2*b*sin(d + e*x)/e - 24*a**2*b*cos(d + e*x)**2/e + 1
2*a*b**2*x*sin(d + e*x)**2 + 12*a*b**2*x*cos(d + e*x)**2 + 12*a*b**2*sin(d
+ e*x)*cos(d + e*x)/e - 8*a*b**2*cos(d + e*x)**3/e + 16*b**3*sin(d + e*x)**
3/(3*e) + 8*b**3*sin(d + e*x)*cos(d + e*x)**2/e, Ne(e, 0)), (x*(2*a*sin(d)
+ 2*a + 2*b*cos(d))**3, True))
```

3.382 $\int (2a+2b\cos(d+ex)+2a\sin(d+ex))^2\,dx$

Optimal. Leaf size=81

$$2x\left(3a^2+b^2\right)-\frac{6a^2\cos(d+ex)}{e}+\frac{6ab\sin(d+ex)}{e}-\frac{2(a\sin(d+ex)+a+b\cos(d+ex))(a\cos(d+ex)-b\sin(d+\dots}{e}$$

```
[Out] 2*(3*a^2+b^2)*x-6*a^2*cos(e*x+d)/e+6*a*b*sin(e*x+d)/e-2*(a+b*cos(e*x+d)+a*s
in(e*x+d))*(a*cos(e*x+d)-b*sin(e*x+d))/e
```

Rubi [A] time = 0.05, antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {3120, 2637, 2638}

$$2x\left(3a^2+b^2\right)-\frac{6a^2\cos(d+ex)}{e}+\frac{6ab\sin(d+ex)}{e}-\frac{2(a\sin(d+ex)+a+b\cos(d+ex))(a\cos(d+ex)-b\sin(d+\dots}{e}$$

Antiderivative was successfully verified.

```
[In] Int[(2*a + 2*b*Cos[d + e*x] + 2*a*Sin[d + e*x])^2,x]
```

```
[Out] 2*(3*a^2 + b^2)*x - (6*a^2*Cos[d + e*x])/e + (6*a*b*Sin[d + e*x])/e - (2*(a
 + b*Cos[d + e*x] + a*Sin[d + e*x])*(a*Cos[d + e*x] - b*Sin[d + e*x]))/e
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3120

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(n_), x_Symbol] :> -Simp[((c*Cos[d + e*x] - b*Sin[d + e*x])*(a + b*Cos[d +
e*x] + c*Sin[d + e*x])^(n - 1))/(e*n), x] + Dist[1/n, Int[Simp[n*a^2 + (n -
 1)*(b^2 + c^2) + a*b*(2*n - 1)*Cos[d + e*x] + a*c*(2*n - 1)*Sin[d + e*x],
x]*(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n - 2), x], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[a^2 - b^2 - c^2, 0] && GtQ[n, 1]
```

Rubi steps

$$\int (2a+2b\cos(d+ex)+2a\sin(d+ex))^2\,dx = -\frac{2(a+b\cos(d+ex)+a\sin(d+ex))(a\cos(d+ex)-b\sin(d+ex}{e}$$
$$= 2\left(3a^2+b^2\right)x - \frac{2(a+b\cos(d+ex)+a\sin(d+ex))(a\cos(d+ex)}{e}$$
$$= 2\left(3a^2+b^2\right)x - \frac{6a^2\cos(d+ex)}{e} + \frac{6ab\sin(d+ex)}{e} - \frac{2(a+b\cos(}{}$$

Mathematica [A] time = 0.14, size = 92, normalized size = 1.14

$$4\left(\frac{\left(3a^2+b^2\right)(d+ex)}{2e} - \frac{\left(a^2-b^2\right)\sin(2(d+ex))}{4e} - \frac{2a^2\cos(d+ex)}{e} + \frac{2ab\sin(d+ex)}{e} - \frac{ab\cos(2(d+ex))}{2e}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(2*a + 2*b*Cos[d + e*x] + 2*a*Sin[d + e*x])^2,x]
```

```
[Out] 4*(((3*a^2 + b^2)*(d + e*x))/(2*e) - (2*a^2*Cos[d + e*x])/e - (a*b*Cos[2*(d
 + e*x)])/(2*e) + (2*a*b*Sin[d + e*x])/e - ((a^2 - b^2)*Sin[2*(d + e*x)])/(
4*e))
```

fricas [A] time = 0.76, size = 72, normalized size = 0.89

$$-\frac{2\left(2\,ab\cos\left(ex+d\right)^2 - \left(3\,a^2+b^2\right)ex + 4\,a^2\cos\left(ex+d\right) - \left(4\,ab - \left(a^2-b^2\right)\cos\left(ex+d\right)\right)\sin\left(ex+d\right)\right)}{e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*a+2*b*cos(e*x+d)+2*a*sin(e*x+d))^2,x, algorithm="fricas")
```

```
[Out] -2*(2*a*b*cos(e*x + d)^2 - (3*a^2 + b^2)*e*x + 4*a^2*cos(e*x + d) - (4*a*b
- (a^2 - b^2)*cos(e*x + d))*sin(e*x + d))/e
```

giac [A] time = 0.16, size = 79, normalized size = 0.98

$$-2\,ab\cos\left(2\,xe+2\,d\right)e^{(-1)} - 8\,a^2\cos\left(xe+d\right)e^{(-1)} + 8\,abe^{(-1)}\sin\left(xe+d\right) - \left(a^2-b^2\right)e^{(-1)}\sin\left(2\,xe+2\,d\right) + 2\left(3\,a^2+b^2\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*a+2*b*cos(e*x+d)+2*a*sin(e*x+d))^2,x, algorithm="giac")
```

```
[Out] -2*a*b*cos(2*x*e + 2*d)*e^(-1) - 8*a^2*cos(x*e + d)*e^(-1) + 8*a*b*e^(-1)*s
in(x*e + d) - (a^2 - b^2)*e^(-1)*sin(2*x*e + 2*d) + 2*(3*a^2 + b^2)*x
```

maple [A] time = 0.23, size = 101, normalized size = 1.25

$$\frac{4a^2(ex+d)+8ab\sin(ex+d)-8a^2\cos(ex+d)+4b^2\left(\frac{\sin(ex+d)\cos(ex+d)}{2}+\frac{ex}{2}+\frac{d}{2}\right)-4\left(\cos^2(ex+d)\right)ab+4a^2\left(\right.}{e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*a+2*b*cos(e*x+d)+2*a*sin(e*x+d))^2,x)
```

```
[Out] 4/e*(a^2*(e*x+d)+2*a*b*sin(e*x+d)-2*a^2*cos(e*x+d)+b^2*(1/2*sin(e*x+d)*cos(
e*x+d)+1/2*e*x+1/2*d)-cos(e*x+d)^2*a*b+a^2*(-1/2*sin(e*x+d)*cos(e*x+d)+1/2*
e*x+1/2*d))
```

maxima [A] time = 0.32, size = 99, normalized size = 1.22

$$4a^2x-\frac{4ab\cos(ex+d)^2}{e}+\frac{(2ex+2d-\sin(2ex+2d))a^2}{e}+\frac{(2ex+2d+\sin(2ex+2d))b^2}{e}-8a\left(\frac{a\cos(ex+d)}{e}-\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*a+2*b*cos(e*x+d)+2*a*sin(e*x+d))^2,x, algorithm="maxima")
```

```
[Out] 4*a^2*x - 4*a*b*cos(e*x + d)^2/e + (2*e*x + 2*d - sin(2*e*x + 2*d))*a^2/e +
 (2*e*x + 2*d + sin(2*e*x + 2*d))*b^2/e - 8*a*(a*cos(e*x + d)/e - b*sin(e*x
 + d)/e)
```

mupad [B] time = 3.72, size = 127, normalized size = 1.57

$$\frac{x\left(12a^2+4b^2\right)}{2}+\frac{\tan\left(\frac{d}{2}+\frac{ex}{2}\right)^2\left(16ab-16a^2\right)+\tan\left(\frac{d}{2}+\frac{ex}{2}\right)^3\left(4a^2+16ab-4b^2\right)-16a^2+\tan\left(\frac{d}{2}+\frac{ex}{2}\right)\left(-\right.}{e\left(\tan\left(\frac{d}{2}+\frac{ex}{2}\right)^4+2\tan\left(\frac{d}{2}+\frac{ex}{2}\right)^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*a + 2*b*cos(d + e*x) + 2*a*sin(d + e*x))^2,x)
```

```
[Out] (x*(12*a^2 + 4*b^2))/2 + (tan(d/2 + (e*x)/2)^2*(16*a*b - 16*a^2) + tan(d/2
+ (e*x)/2)^3*(16*a*b + 4*a^2 - 4*b^2) - 16*a^2 + tan(d/2 + (e*x)/2)*(16*a*b
 - 4*a^2 + 4*b^2))/(e*(2*tan(d/2 + (e*x)/2)^2 + tan(d/2 + (e*x)/2)^4 + 1))
```

sympy [A] time = 0.31, size = 170, normalized size = 2.10

$$\begin{cases} 2a^2x\sin^2(d+ex)+2a^2x\cos^2(d+ex)+4a^2x-\frac{2a^2\sin(d+ex)\cos(d+ex)}{e}-\frac{8a^2\cos(d+ex)}{e}+\frac{8ab\sin(d+ex)}{e}-\frac{4ab\cos^2(d+ex)}{e} \\ x(2a\sin(d)+2a+2b\cos(d))^2 \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*a+2*b*cos(e*x+d)+2*a*sin(e*x+d))**2,x)
```

```
[Out] Piecewise((2*a**2*x*sin(d + e*x)**2 + 2*a**2*x*cos(d + e*x)**2 + 4*a**2*x -
 2*a**2*sin(d + e*x)*cos(d + e*x)/e - 8*a**2*cos(d + e*x)/e + 8*a*b*sin(d +
 e*x)/e - 4*a*b*cos(d + e*x)**2/e + 2*b**2*x*sin(d + e*x)**2 + 2*b**2*x*cos
(d + e*x)**2 + 2*b**2*sin(d + e*x)*cos(d + e*x)/e, Ne(e, 0)), (x*(2*a*sin(d
) + 2*a + 2*b*cos(d))**2, True))
```

3.383 $\int (2a + 2b\cos(d+ex) + 2a\sin(d+ex))\,dx$

Optimal. Leaf size=29

$$-\frac{2a\cos(d+ex)}{e} + 2ax + \frac{2b\sin(d+ex)}{e}$$

```
[Out] 2*a*x-2*a*cos(e*x+d)/e+2*b*sin(e*x+d)/e
```

Rubi [A] time = 0.02, antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {2637, 2638}

$$-\frac{2a\cos(d+ex)}{e} + 2ax + \frac{2b\sin(d+ex)}{e}$$

Antiderivative was successfully verified.

```
[In] Int[2*a + 2*b*Cos[d + e*x] + 2*a*Sin[d + e*x],x]
```

```
[Out] 2*a*x - (2*a*Cos[d + e*x])/e + (2*b*Sin[d + e*x])/e
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\int (2a + 2b\cos(d+ex) + 2a\sin(d+ex))\,dx &= 2ax + (2a)\int \sin(d+ex)\,dx + (2b)\int \cos(d+ex)\,dx \\
&= 2ax - \frac{2a\cos(d+ex)}{e} + \frac{2b\sin(d+ex)}{e}
\end{aligned}$$

Mathematica [A] time = 0.01, size = 53, normalized size = 1.83

$$\frac{2a\sin(d)\sin(ex)}{e} - \frac{2a\cos(d)\cos(ex)}{e} + 2ax + \frac{2b\sin(d)\cos(ex)}{e} + \frac{2b\cos(d)\sin(ex)}{e}$$

Antiderivative was successfully verified.

```
[In] Integrate[2*a + 2*b*Cos[d + e*x] + 2*a*Sin[d + e*x],x]
```

```
[Out] 2*a*x - (2*a*Cos[d]*Cos[e*x])/e + (2*b*Cos[e*x]*Sin[d])/e + (2*b*Cos[d]*Sin
[e*x])/e + (2*a*Sin[d]*Sin[e*x])/e
```

fricas [A] time = 2.09, size = 27, normalized size = 0.93

$$\frac{2\left(aex - a\cos\left(ex + d\right) + b\sin\left(ex + d\right)\right)}{e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(2*a+2*b*cos(e*x+d)+2*a*sin(e*x+d),x, algorithm="fricas")
```

```
[Out] 2*(a*e*x - a*cos(e*x + d) + b*sin(e*x + d))/e
```

giac [A] time = 0.14, size = 29, normalized size = 1.00

$$-2\,a\cos\left(xe + d\right)e^{(-1)} + 2\,be^{(-1)}\sin\left(xe + d\right) + 2\,ax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(2*a+2*b*cos(e*x+d)+2*a*sin(e*x+d),x, algorithm="giac")
```

```
[Out] -2*a*cos(x*e + d)*e^(-1) + 2*b*e^(-1)*sin(x*e + d) + 2*a*x
```

maple [A] time = 0.00, size = 30, normalized size = 1.03

$$2ax - \frac{2a\cos\left(ex + d\right)}{e} + \frac{2b\sin\left(ex + d\right)}{e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(2*a+2*b*cos(e*x+d)+2*a*sin(e*x+d),x)
```

```
[Out] 2*a*x-2*a*cos(e*x+d)/e+2*b*sin(e*x+d)/e
```

maxima [A] time = 0.31, size = 29, normalized size = 1.00

$$2\,ax - \frac{2\,a\cos\left(ex + d\right)}{e} + \frac{2\,b\sin\left(ex + d\right)}{e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(2*a+2*b*cos(e*x+d)+2*a*sin(e*x+d),x, algorithm="maxima")
```

```
[Out] 2*a*x - 2*a*cos(e*x + d)/e + 2*b*sin(e*x + d)/e
```

mupad [B] time = 2.44, size = 29, normalized size = 1.00

$$2\,a\,x-\frac{2\,a\,\cos\left(d+e\,x\right)}{e}+\frac{2\,b\,\sin\left(d+e\,x\right)}{e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(2*a + 2*b*cos(d + e*x) + 2*a*sin(d + e*x),x)
[Out] 2*a*x - (2*a*cos(d + e*x))/e + (2*b*sin(d + e*x))/e
```

sympy [A] time = 0.14, size = 39, normalized size = 1.34

$$2ax+2a\left(\begin{cases}-\frac{\cos\left(d+ex\right)}{e} & \text{for } e\neq 0\\ x\sin\left(d\right) & \text{otherwise}\end{cases}\right)+2b\left(\begin{cases}\frac{\sin\left(d+ex\right)}{e} & \text{for } e\neq 0\\ x\cos\left(d\right) & \text{otherwise}\end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(2*a+2*b*cos(e*x+d)+2*a*sin(e*x+d),x)
[Out] 2*a*x + 2*a*Piecewise((-cos(d + e*x)/e, Ne(e, 0)), (x*sin(d), True)) + 2*b*
Piecewise((sin(d + e*x)/e, Ne(e, 0)), (x*cos(d), True))
```

3.384 $\int \frac{1}{2a+2b\cos(d+ex)+2a\sin(d+ex)}\,dx$

Optimal. Leaf size=33

$$-\frac{\log\left(a+b\cot\left(\frac{d}{2}+\frac{ex}{2}+\frac{\pi}{4}\right)\right)}{2be}$$

```
[Out] -1/2*ln(a+b*cot(1/2*d+1/4*Pi+1/2*e*x))/b/e
```

Rubi [A] time = 0.02, antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {3123, 31}

$$-\frac{\log\left(a+b\cot\left(\frac{d}{2}+\frac{ex}{2}+\frac{\pi}{4}\right)\right)}{2be}$$

Antiderivative was successfully verified.

```
[In] Int[(2*a + 2*b*Cos[d + e*x] + 2*a*Sin[d + e*x])^(-1),x]
```

```
[Out] -Log[a + b*Cot[d/2 + Pi/4 + (e*x)/2]]/(2*b*e)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 3123

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(-1), x_Symbol] :> Module[{f = FreeFactors[Cot[(d + e*x)/2 + Pi/4], x]}, -D
ist[f/e, Subst[Int[1/(a + b*f*x), x], x, Cot[(d + e*x)/2 + Pi/4]/f], x]] /;
 FreeQ[{a, b, c, d, e}, x] && EqQ[a - c, 0] && NeQ[a - b, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{2a+2b\cos(d+ex)+2a\sin(d+ex)}\,dx &= -\frac{\text{Subst}\left(\int \frac{1}{2a+2bx}\,dx,x,\cot\left(\frac{\pi}{4}+\frac{1}{2}(d+ex)\right)\right)}{e}\\ &= -\frac{\log\left(a+b\cot\left(\frac{d}{2}+\frac{\pi}{4}+\frac{ex}{2}\right)\right)}{2be}\end{aligned}$$

Mathematica [B] time = 0.07, size = 93, normalized size = 2.82

$$\frac{1}{2}\left(\frac{\log\left(\sin\left(\frac{1}{2}(d+ex)\right)+\cos\left(\frac{1}{2}(d+ex)\right)\right)}{be}-\frac{\log\left(a\sin\left(\frac{1}{2}(d+ex)\right)+a\cos\left(\frac{1}{2}(d+ex)\right)-b\sin\left(\frac{1}{2}(d+ex)\right)+b\right.}{be}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(2*a + 2*b*Cos[d + e*x] + 2*a*Sin[d + e*x])^(-1),x]
```

```
[Out] (Log[Cos[(d + e*x)/2] + Sin[(d + e*x)/2]]/(b*e) - Log[a*Cos[(d + e*x)/2] +
b*Cos[(d + e*x)/2] + a*Sin[(d + e*x)/2] - b*Sin[(d + e*x)/2]]/(b*e))/2
```

fricas [B] time = 1.75, size = 54, normalized size = 1.64

$$-\frac{\log\left(2\,ab\cos\left(ex+d\right)+a^{2}+b^{2}+\left(a^{2}-b^{2}\right)\sin\left(ex+d\right)\right)-\log\left(\sin\left(ex+d\right)+1\right)}{4\,be}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*a+2*b*cos(e*x+d)+2*a*sin(e*x+d)),x, algorithm="fricas")
```

```
[Out] -1/4*(log(2*a*b*cos(e*x + d) + a^2 + b^2 + (a^2 - b^2)*sin(e*x + d)) - log(
sin(e*x + d) + 1))/(b*e)
```

giac [B] time = 0.20, size = 82, normalized size = 2.48

$$\frac{e^{(-1)}\log\left(\frac{\left|2\,a\tan\left(\frac{1}{2}\,xe+\frac{1}{2}\,d\right)-2\,b\tan\left(\frac{1}{2}\,xe+\frac{1}{2}\,d\right)+2\,a-2\,|b|\right|}{\left|2\,a\tan\left(\frac{1}{2}\,xe+\frac{1}{2}\,d\right)-2\,b\tan\left(\frac{1}{2}\,xe+\frac{1}{2}\,d\right)+2\,a+2\,|b|\right|}\right)}{2\,|b|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*a+2*b*cos(e*x+d)+2*a*sin(e*x+d)),x, algorithm="giac")
```

```
[Out] 1/2*e^(-1)*log(abs(2*a*tan(1/2*x*e + 1/2*d) - 2*b*tan(1/2*x*e + 1/2*d) + 2*
a - 2*abs(b))/abs(2*a*tan(1/2*x*e + 1/2*d) - 2*b*tan(1/2*x*e + 1/2*d) + 2*a
 + 2*abs(b)))/abs(b)
```

maple [B] time = 0.41, size = 104, normalized size = 3.15

$$-\frac{\ln\left(a\tan\left(\frac{d}{2}+\frac{ex}{2}\right)-b\tan\left(\frac{d}{2}+\frac{ex}{2}\right)+a+b\right)a}{2eb\left(a-b\right)}+\frac{\ln\left(a\tan\left(\frac{d}{2}+\frac{ex}{2}\right)-b\tan\left(\frac{d}{2}+\frac{ex}{2}\right)+a+b\right)}{2e\left(a-b\right)}+\frac{\ln\left(1+\tan\left(\frac{d}{2}+\right.\right.}{2eb}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2*a+2*b*cos(e*x+d)+2*a*sin(e*x+d)),x)
```

```
[Out] -1/2/e/b/(a-b)*ln(a*tan(1/2*d+1/2*e*x)-b*tan(1/2*d+1/2*e*x)+a+b)*a+1/2/e/(a
-b)*ln(a*tan(1/2*d+1/2*e*x)-b*tan(1/2*d+1/2*e*x)+a+b)+1/2/e/b*ln(1+tan(1/2*
d+1/2*e*x))
```

maxima [B] time = 0.32, size = 66, normalized size = 2.00

$$-\frac{\frac{\log\left(-a-b-\frac{(a-b)\sin(ex+d)}{\cos(ex+d)+1}\right)}{b}-\frac{\log\left(\frac{\sin(ex+d)}{\cos(ex+d)+1}+1\right)}{b}}{2\,e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*a+2*b*cos(e*x+d)+2*a*sin(e*x+d)),x, algorithm="maxima")
```

```
[Out] -1/2*(log(-a - b - (a - b)*sin(e*x + d)/(cos(e*x + d) + 1))/b - log(sin(e*x
 + d)/(cos(e*x + d) + 1) + 1)/b)/e
```

mupad [B] time = 2.83, size = 33, normalized size = 1.00

$$-\frac{\operatorname{atanh}\left(\frac{a+\frac{\tan\left(\frac{d}{2}+\frac{e\,x}{2}\right)\,(2\,a-2\,b)}{2}}{b}\right)}{b\,e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2*a + 2*b*cos(d + e*x) + 2*a*sin(d + e*x)),x)
```

```
[Out] -atanh((a + (tan(d/2 + (e*x)/2)*(2*a - 2*b))/2)/b)/(b*e)
```

sympy [A] time = 1.69, size = 107, normalized size = 3.24

$$\begin{cases} \frac{\tilde{\infty}x}{\cos(d)} & \text{for}\: a = 0 \wedge b = 0 \wedge e = 0 \\ -\frac{1}{ae\tan\left(\frac{d}{2}+\frac{ex}{2}\right)+ae} & \text{for}\: b = 0 \\ \frac{\log\left(\tan\left(\frac{d}{2}+\frac{ex}{2}\right)+1\right)}{2be} & \text{for}\: a = b \\ \frac{x}{2a\sin(d)+2a+2b\cos(d)} & \text{for}\: e = 0 \\ \frac{\log\left(\tan\left(\frac{d}{2}+\frac{ex}{2}\right)+1\right)}{2be}-\frac{\log\left(\frac{a}{a-b}+\frac{b}{a-b}+\tan\left(\frac{d}{2}+\frac{ex}{2}\right)\right)}{2be} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*a+2*b*cos(e*x+d)+2*a*sin(e*x+d)),x)
```

```
[Out] Piecewise((zoo*x/cos(d), Eq(a, 0) & Eq(b, 0) & Eq(e, 0)), (-1/(a*e*tan(d/2
+ e*x/2) + a*e), Eq(b, 0)), (log(tan(d/2 + e*x/2) + 1)/(2*b*e), Eq(a, b)),
(x/(2*a*sin(d) + 2*a + 2*b*cos(d)), Eq(e, 0)), (log(tan(d/2 + e*x/2) + 1)/(
2*b*e) - log(a/(a - b) + b/(a - b) + tan(d/2 + e*x/2))/(2*b*e), True))
```

3.385 $\int \frac{1}{(2a+2b\cos(d+ex)+2a\sin(d+ex))^2}\,dx$

Optimal. Leaf size=83

$$\frac{a\log\left(a+b\cot\left(\frac{d}{2}+\frac{ex}{2}+\frac{\pi}{4}\right)\right)}{4b^3e}-\frac{a\cos(d+ex)-b\sin(d+ex)}{4b^2e(a\sin(d+ex)+a+b\cos(d+ex))}$$

```
[Out] 1/4*a*ln(a+b*cot(1/2*d+1/4*Pi+1/2*e*x))/b^3/e+1/4*(-a*cos(e*x+d)+b*sin(e*x+
d))/b^2/e/(a+b*cos(e*x+d)+a*sin(e*x+d))
```

Rubi [A] time = 0.05, antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3129, 12, 3123, 31}

$$\frac{a\log\left(a+b\cot\left(\frac{d}{2}+\frac{ex}{2}+\frac{\pi}{4}\right)\right)}{4b^3e}-\frac{a\cos(d+ex)-b\sin(d+ex)}{4b^2e(a\sin(d+ex)+a+b\cos(d+ex))}$$

Antiderivative was successfully verified.

```
[In] Int[(2*a + 2*b*Cos[d + e*x] + 2*a*Sin[d + e*x])^(-2),x]
```

```
[Out] (a*Log[a + b*Cot[d/2 + Pi/4 + (e*x)/2]])/(4*b^3*e) - (a*Cos[d + e*x] - b*Si
n[d + e*x])/(4*b^2*e*(a + b*Cos[d + e*x] + a*Sin[d + e*x]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 3123

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(-1), x_Symbol] :> Module[{f = FreeFactors[Cot[(d + e*x)/2 + Pi/4], x]}, -D
ist[f/e, Subst[Int[1/(a + b*f*x), x], x, Cot[(d + e*x)/2 + Pi/4]/f], x]] /;
 FreeQ[{a, b, c, d, e}, x] && EqQ[a - c, 0] && NeQ[a - b, 0]
```

Rule 3129

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(n_), x_Symbol] :> Simp[((-(c*Cos[d + e*x]) + b*Sin[d + e*x])*(a + b*Cos[d
+ e*x] + c*Sin[d + e*x])^(n + 1))/(e*(n + 1)*(a^2 - b^2 - c^2)), x] + Dist[
1/((n + 1)*(a^2 - b^2 - c^2)), Int[(a*(n + 1) - b*(n + 2)*Cos[d + e*x] - c*
(n + 2)*Sin[d + e*x])*(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n + 1), x], x]
 /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0] && LtQ[n, -1] && N
eQ[n, -3/2]
```

Rubi steps

$$\int \frac{1}{(2a+2b\cos(d+ex)+2a\sin(d+ex))^2}\,dx = -\frac{a\cos(d+ex)-b\sin(d+ex)}{4b^2e(a+b\cos(d+ex)+a\sin(d+ex))} + \frac{\int -\frac{2a}{2a+2b\cos(d+ex)+2a}}{4b^2}$$

$$= -\frac{a\cos(d+ex)-b\sin(d+ex)}{4b^2e(a+b\cos(d+ex)+a\sin(d+ex))} - \frac{a\int \frac{1}{2a+2b\cos(d+ex)+2a}}{2b^2}$$

$$= -\frac{a\cos(d+ex)-b\sin(d+ex)}{4b^2e(a+b\cos(d+ex)+a\sin(d+ex))} + \frac{a\,\mathrm{Subst}\left(\int \frac{1}{2a+2bx}\,dx\right.}{}$$

$$= \frac{a\log\left(a+b\cot\left(\frac{d}{2}+\frac{\pi}{4}+\frac{ex}{2}\right)\right)}{4b^3e} - \frac{a\cos(d+ex)-b\sin(d+}{4b^2e(a+b\cos(d+ex)+a\sin}$$

Mathematica [A] time = 0.55, size = 162, normalized size = 1.95

$$\frac{\frac{b(a^2+b^2)\sin\left(\frac{1}{2}(d+ex)\right)}{(a+b)\left((a-b)\sin\left(\frac{1}{2}(d+ex)\right)+(a+b)\cos\left(\frac{1}{2}(d+ex)\right)\right)} + a\log\left((a-b)\sin\left(\frac{1}{2}(d+ex)\right)+(a+b)\cos\left(\frac{1}{2}(d+ex)\right)\right) - a\log\left(\sin\left(\frac{1}{2}(\right.\right.}{4b^3e}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2*a + 2*b*Cos[d + e*x] + 2*a*Sin[d + e*x])^(-2),x]
```

```
[Out] (-(a*Log[Cos[(d + e*x)/2] + Sin[(d + e*x)/2]]) + a*Log[(a + b)*Cos[(d + e*x
)/2] + (a - b)*Sin[(d + e*x)/2]] + (b*Sin[(d + e*x)/2])/(Cos[(d + e*x)/2] +
 Sin[(d + e*x)/2]) + (b*(a^2 + b^2)*Sin[(d + e*x)/2])/((a + b)*((a + b)*Cos
[(d + e*x)/2] + (a - b)*Sin[(d + e*x)/2])))/(4*b^3*e)
```

fricas [B] time = 1.25, size = 148, normalized size = 1.78

$$-\frac{2\,ab\cos(ex+d) - 2\,b^2\sin(ex+d) - \left(ab\cos(ex+d) + a^2\sin(ex+d) + a^2\right)\log\left(2\,ab\cos(ex+d) + a^2 + b^2 +\right.}{8\left(b^4e\cos(ex+d) + ab^3e\sin(ex+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*a+2*b*cos(e*x+d)+2*a*sin(e*x+d))^2,x, algorithm="fricas")
```

```
[Out] -1/8*(2*a*b*cos(e*x + d) - 2*b^2*sin(e*x + d) - (a*b*cos(e*x + d) + a^2*sin
(e*x + d) + a^2)*log(2*a*b*cos(e*x + d) + a^2 + b^2 + (a^2 - b^2)*sin(e*x +
 d)) + (a*b*cos(e*x + d) + a^2*sin(e*x + d) + a^2)*log(sin(e*x + d) + 1))/(
b^4*e*cos(e*x + d) + a*b^3*e*sin(e*x + d) + a*b^3*e)
```

giac [B] time = 0.22, size = 196, normalized size = 2.36

$$-\frac{1}{4}\left(\frac{2\left(a^2\tan\left(\frac{1}{2}\,xe+\frac{1}{2}\,d\right)-ab\tan\left(\frac{1}{2}\,xe+\frac{1}{2}\,d\right)+b^2\tan\left(\frac{1}{2}\,xe+\frac{1}{2}\,d\right)+a^2\right)}{\left(ab^2-b^3\right)\left(a\tan\left(\frac{1}{2}\,xe+\frac{1}{2}\,d\right)^2-b\tan\left(\frac{1}{2}\,xe+\frac{1}{2}\,d\right)^2+2\,a\tan\left(\frac{1}{2}\,xe+\frac{1}{2}\,d\right)+a+b\right)}+\frac{a\log\left(\frac{\left|2\,a\tan\left(\frac{1}{2}\,xe+\frac{1}{2}\,d\right)-2\right.}{\left|2\,a\tan\left(\frac{1}{2}\,xe+\frac{1}{2}\,d\right)-2\right.}\right.}{b^2|}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*a+2*b*cos(e*x+d)+2*a*sin(e*x+d))^2,x, algorithm="giac")
```

```
[Out] -1/4*(2*(a^2*tan(1/2*x*e + 1/2*d) - a*b*tan(1/2*x*e + 1/2*d) + b^2*tan(1/2*
x*e + 1/2*d) + a^2)/((a*b^2 - b^3)*(a*tan(1/2*x*e + 1/2*d)^2 - b*tan(1/2*x*
e + 1/2*d)^2 + 2*a*tan(1/2*x*e + 1/2*d) + a + b)) + a*log(abs(2*a*tan(1/2*x
*e + 1/2*d) - 2*b*tan(1/2*x*e + 1/2*d) + 2*a - 2*abs(b))/abs(2*a*tan(1/2*x*
e + 1/2*d) - 2*b*tan(1/2*x*e + 1/2*d) + 2*a + 2*abs(b)))/(b^2*abs(b)))*e^(-
1)
```

maple [B] time = 0.53, size = 166, normalized size = 2.00

$$-\frac{a^2}{4e\,b^2\,(a-b)\left(a\tan\left(\frac{d}{2}+\frac{ex}{2}\right)-b\tan\left(\frac{d}{2}+\frac{ex}{2}\right)+a+b\right)}-\frac{1}{4e\,(a-b)\left(a\tan\left(\frac{d}{2}+\frac{ex}{2}\right)-b\tan\left(\frac{d}{2}+\frac{ex}{2}\right)+a+b\right)}+\frac{a\ln}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2*a+2*b*cos(e*x+d)+2*a*sin(e*x+d))^2,x)
```

```
[Out] -1/4/e/b^2/(a-b)/(a*tan(1/2*d+1/2*e*x)-b*tan(1/2*d+1/2*e*x)+a+b)*a^2-1/4/e/
(a-b)/(a*tan(1/2*d+1/2*e*x)-b*tan(1/2*d+1/2*e*x)+a+b)+1/4/e*a/b^3*ln(a*tan(
1/2*d+1/2*e*x)-b*tan(1/2*d+1/2*e*x)+a+b)-1/4/e/b^2/(1+tan(1/2*d+1/2*e*x))-1
/4/e*a/b^3*ln(1+tan(1/2*d+1/2*e*x))
```

maxima [B] time = 0.34, size = 185, normalized size = 2.23

$$-\frac{\frac{2\left(a^2+\frac{(a^2-ab+b^2)\sin(ex+d)}{\cos(ex+d)+1}\right)}{a^2b^2-b^4+\frac{2(a^2b^2-ab^3)\sin(ex+d)}{\cos(ex+d)+1}+\frac{(a^2b^2-2ab^3+b^4)\sin(ex+d)^2}{(\cos(ex+d)+1)^2}}-\frac{a\log\left(-a-b-\frac{(a-b)\sin(ex+d)}{\cos(ex+d)+1}\right)}{b^3}+\frac{a\log\left(\frac{\sin(ex+d)}{\cos(ex+d)+1}+1\right)}{b^3}}{4e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*a+2*b*cos(e*x+d)+2*a*sin(e*x+d))^2,x, algorithm="maxima")
```

```
[Out] -1/4*(2*(a^2 + (a^2 - a*b + b^2)*sin(e*x + d)/(cos(e*x + d) + 1))/(a^2*b^2
- b^4 + 2*(a^2*b^2 - a*b^3)*sin(e*x + d)/(cos(e*x + d) + 1) + (a^2*b^2 - 2*
a*b^3 + b^4)*sin(e*x + d)^2/(cos(e*x + d) + 1)^2) - a*log(-a - b - (a - b)*
sin(e*x + d)/(cos(e*x + d) + 1))/b^3 + a*log(sin(e*x + d)/(cos(e*x + d) + 1
) + 1)/b^3)/e
```

mupad [B] time = 2.72, size = 126, normalized size = 1.52

$$\frac{a\,\mathrm{atanh}\left(\frac{a+\frac{\tan\left(\frac{d}{2}+\frac{e\,x}{2}\right)(2\,a-2\,b)}{2}}{b}\right)}{2\,b^3\,e}-\frac{\frac{a^2}{b^2\,(a-b)}+\frac{\tan\left(\frac{d}{2}+\frac{e\,x}{2}\right)\left(a^2-a\,b+b^2\right)}{b^2\,(a-b)}}{e\left((2\,a-2\,b)\,\tan\left(\frac{d}{2}+\frac{e\,x}{2}\right)^2+4\,a\tan\left(\frac{d}{2}+\frac{e\,x}{2}\right)+2\,a+2\,b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2*a + 2*b*cos(d + e*x) + 2*a*sin(d + e*x))^2,x)
```

```
[Out] (a*atanh((a + (tan(d/2 + (e*x)/2)*(2*a - 2*b))/2)/b))/(2*b^3*e) - (a^2/(b^2
*(a - b)) + (tan(d/2 + (e*x)/2)*(a^2 - a*b + b^2))/(b^2*(a - b)))/(e*(2*a +
 2*b + tan(d/2 + (e*x)/2)^2*(2*a - 2*b) + 4*a*tan(d/2 + (e*x)/2)))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*a+2*b*cos(e*x+d)+2*a*sin(e*x+d))**2,x)
```

```
[Out] Timed out
```

3.386 $\int \frac{1}{(2a+2b\cos(d+ex)+2a\sin(d+ex))^3}\,dx$

Optimal. Leaf size=142

$$\frac{3\left(a^2\cos(d+ex)-ab\sin(d+ex)\right)}{16b^4e(a\sin(d+ex)+a+b\cos(d+ex))}-\frac{\left(3a^2+b^2\right)\log\left(a+b\cot\left(\frac{d}{2}+\frac{ex}{2}+\frac{\pi}{4}\right)\right)}{16b^5e}-\frac{a\cos(d+ex)-b\sin(d+ex)}{16b^2e(a\sin(d+ex)+a+b\cos(d+ex))^2}$$

```
[Out] -1/16*(3*a^2+b^2)*ln(a+b*cot(1/2*d+1/4*Pi+1/2*e*x))/b^5/e+1/16*(-a*cos(e*x+
d)+b*sin(e*x+d))/b^2/e/(a+b*cos(e*x+d)+a*sin(e*x+d))^2+3/16*(a^2*cos(e*x+d)
-a*b*sin(e*x+d))/b^4/e/(a+b*cos(e*x+d)+a*sin(e*x+d))
```

Rubi [A] time = 0.11, antiderivative size = 142, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3129, 3153, 3123, 31}

$$\frac{3\left(a^2\cos(d+ex)-ab\sin(d+ex)\right)}{16b^4e(a\sin(d+ex)+a+b\cos(d+ex))}-\frac{\left(3a^2+b^2\right)\log\left(a+b\cot\left(\frac{d}{2}+\frac{ex}{2}+\frac{\pi}{4}\right)\right)}{16b^5e}-\frac{a\cos(d+ex)-b\sin(d+ex)}{16b^2e(a\sin(d+ex)+a+b\cos(d+ex))^2}$$

Antiderivative was successfully verified.

```
[In] Int[(2*a + 2*b*Cos[d + e*x] + 2*a*Sin[d + e*x])^(-3),x]
```

```
[Out] -((3*a^2 + b^2)*Log[a + b*Cot[d/2 + Pi/4 + (e*x)/2]])/(16*b^5*e) - (a*Cos[d
 + e*x] - b*Sin[d + e*x])/(16*b^2*e*(a + b*Cos[d + e*x] + a*Sin[d + e*x])^2
) + (3*(a^2*Cos[d + e*x] - a*b*Sin[d + e*x]))/(16*b^4*e*(a + b*Cos[d + e*x]
 + a*Sin[d + e*x]))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 3123

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(-1), x_Symbol] :> Module[{f = FreeFactors[Cot[(d + e*x)/2 + Pi/4], x]}, -D
ist[f/e, Subst[Int[1/(a + b*f*x), x], x, Cot[(d + e*x)/2 + Pi/4]/f], x]] /;
 FreeQ[{a, b, c, d, e}, x] && EqQ[a - c, 0] && NeQ[a - b, 0]
```

Rule 3129

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(n_), x_Symbol] :> Simp[((-(c*Cos[d + e*x]) + b*Sin[d + e*x])*(a + b*Cos[d
+ e*x] + c*Sin[d + e*x])^(n + 1))/(e*(n + 1)*(a^2 - b^2 - c^2)), x] + Dist[
```

```
1/((n + 1)*(a^2 - b^2 - c^2)), Int[(a*(n + 1) - b*(n + 2)*Cos[d + e*x] - c*
(n + 2)*Sin[d + e*x])*(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n + 1), x], x]
 /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0] && LtQ[n, -1] && N
eQ[n, -3/2]
```

Rule 3153

```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])
/((a_.) + cos[(d_.) + (e_.)*(x_)]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)])^2,
 x_Symbol] :> Simp[(c*B - b*C - (a*C - c*A)*Cos[d + e*x] + (a*B - b*A)*Sin[
d + e*x])/(e*(a^2 - b^2 - c^2)*(a + b*Cos[d + e*x] + c*Sin[d + e*x])), x] +
 Dist[(a*A - b*B - c*C)/(a^2 - b^2 - c^2), Int[1/(a + b*Cos[d + e*x] + c*Si
n[d + e*x]), x], x] /; FreeQ[{a, b, c, d, e, A, B, C}, x] && NeQ[a^2 - b^2
- c^2, 0] && NeQ[a*A - b*B - c*C, 0]
```

Rubi steps

$$\int \frac{1}{(2a+2b\cos(d+ex)+2a\sin(d+ex))^3}\,dx = -\frac{a\cos(d+ex)-b\sin(d+ex)}{16b^2e(a+b\cos(d+ex)+a\sin(d+ex))^2} + \frac{\int \frac{-4a+2b\cos(d+ex)+}{(2a+2b\cos(d+ex)+2}}{8b^2}$$

$$= -\frac{a\cos(d+ex)-b\sin(d+ex)}{16b^2e(a+b\cos(d+ex)+a\sin(d+ex))^2} + \frac{3\left(a^2\cos(d+e\right.}{16b^4e(a+b\cos(d}$$

$$= -\frac{a\cos(d+ex)-b\sin(d+ex)}{16b^2e(a+b\cos(d+ex)+a\sin(d+ex))^2} + \frac{3\left(a^2\cos(d+e\right.}{16b^4e(a+b\cos(d}$$

$$= -\frac{\left(3a^2+b^2\right)\log\left(a+b\cot\left(\frac{d}{2}+\frac{\pi}{4}+\frac{ex}{2}\right)\right)}{16b^5e} - \frac{a\cos(d+ex)}{16b^2e(a+b\cos(d+}$$

Mathematica [A] time = 2.43, size = 255, normalized size = 1.80

$$-\frac{-\frac{b^2\left(a^2+b^2\right)}{\left((a-b)\sin\left(\frac{1}{2}(d+ex)\right)+(a+b)\cos\left(\frac{1}{2}(d+ex)\right)\right)^2} + \frac{6ab\left(a^2+b^2\right)\sin\left(\frac{1}{2}(d+ex)\right)}{(a+b)\left((a-b)\sin\left(\frac{1}{2}(d+ex)\right)+(a+b)\cos\left(\frac{1}{2}(d+ex)\right)\right)} - 2\left(3a^2+b^2\right)\log\left(\sin\left(\frac{1}{2}(d+ex)\right)\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2*a + 2*b*Cos[d + e*x] + 2*a*Sin[d + e*x])^(-3),x]
```

```
[Out] -1/32*(-2*(3*a^2 + b^2)*Log[Cos[(d + e*x)/2] + Sin[(d + e*x)/2]] + 2*(3*a^2
 + b^2)*Log[(a + b)*Cos[(d + e*x)/2] + (a - b)*Sin[(d + e*x)/2]] + b^2/(Cos
```

```
[(d + e*x)/2] + Sin[(d + e*x)/2])^2 + (6*a*b*Sin[(d + e*x)/2])/(Cos[(d + e*
x)/2] + Sin[(d + e*x)/2]) - (b^2*(a^2 + b^2))/((a + b)*Cos[(d + e*x)/2] + (
a - b)*Sin[(d + e*x)/2])^2 + (6*a*b*(a^2 + b^2)*Sin[(d + e*x)/2])/((a + b)*
((a + b)*Cos[(d + e*x)/2] + (a - b)*Sin[(d + e*x)/2])))/(b^5*e)
```

fricas [B] time = 0.86, size = 420, normalized size = 2.96

$$\frac{12\,a^2b^2\cos\left(ex+d\right)^2-6\,a^2b^2+2\left(3\,a^3b-ab^3\right)\cos\left(ex+d\right)-\left(6\,a^4+2\,a^2b^2-\left(3\,a^4-2\,a^2b^2-b^4\right)\cos\left(ex+d\right)^2+\cdots\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*a+2*b*cos(e*x+d)+2*a*sin(e*x+d))^3,x, algorithm="fricas")
```

```
[Out] 1/32*(12*a^2*b^2*cos(e*x + d)^2 - 6*a^2*b^2 + 2*(3*a^3*b - a*b^3)*cos(e*x +
 d) - (6*a^4 + 2*a^2*b^2 - (3*a^4 - 2*a^2*b^2 - b^4)*cos(e*x + d)^2 + 2*(3*
a^3*b + a*b^3)*cos(e*x + d) + 2*(3*a^4 + a^2*b^2 + (3*a^3*b + a*b^3)*cos(e*
x + d))*sin(e*x + d))*log(2*a*b*cos(e*x + d) + a^2 + b^2 + (a^2 - b^2)*sin(
e*x + d)) + (6*a^4 + 2*a^2*b^2 - (3*a^4 - 2*a^2*b^2 - b^4)*cos(e*x + d)^2 +
 2*(3*a^3*b + a*b^3)*cos(e*x + d) + 2*(3*a^4 + a^2*b^2 + (3*a^3*b + a*b^3)*
cos(e*x + d))*sin(e*x + d))*log(sin(e*x + d) + 1) - 2*(3*a^2*b^2 - b^4 - 3*
(a^3*b - a*b^3)*cos(e*x + d))*sin(e*x + d))/(2*a*b^6*e*cos(e*x + d) + 2*a^2
*b^5*e - (a^2*b^5 - b^7)*e*cos(e*x + d)^2 + 2*(a*b^6*e*cos(e*x + d) + a^2*b
^5*e)*sin(e*x + d))
```

giac [B] time = 0.24, size = 481, normalized size = 3.39

$$\frac{1}{16}\left(\frac{2\left(3\,a^5\tan\left(\frac{1}{2}\,xe+\frac{1}{2}\,d\right)^3-9\,a^4b\tan\left(\frac{1}{2}\,xe+\frac{1}{2}\,d\right)^3+10\,a^3b^2\tan\left(\frac{1}{2}\,xe+\frac{1}{2}\,d\right)^3-6\,a^2b^3\tan\left(\frac{1}{2}\,xe+\frac{1}{2}\,d\right)^3+ab^4\cdots\right.}{\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*a+2*b*cos(e*x+d)+2*a*sin(e*x+d))^3,x, algorithm="giac")
```

```
[Out] 1/16*(2*(3*a^5*tan(1/2*x*e + 1/2*d)^3 - 9*a^4*b*tan(1/2*x*e + 1/2*d)^3 + 10
*a^3*b^2*tan(1/2*x*e + 1/2*d)^3 - 6*a^2*b^3*tan(1/2*x*e + 1/2*d)^3 + a*b^4*
tan(1/2*x*e + 1/2*d)^3 + b^5*tan(1/2*x*e + 1/2*d)^3 + 9*a^5*tan(1/2*x*e + 1
/2*d)^2 - 18*a^4*b*tan(1/2*x*e + 1/2*d)^2 + 12*a^3*b^2*tan(1/2*x*e + 1/2*d)
^2 - 6*a^2*b^3*tan(1/2*x*e + 1/2*d)^2 + a*b^4*tan(1/2*x*e + 1/2*d)^2 + 9*a^
5*tan(1/2*x*e + 1/2*d) - 9*a^4*b*tan(1/2*x*e + 1/2*d) - 2*a^3*b^2*tan(1/2*x
*e + 1/2*d) + 2*a^2*b^3*tan(1/2*x*e + 1/2*d) - 5*a*b^4*tan(1/2*x*e + 1/2*d)
 + b^5*tan(1/2*x*e + 1/2*d) + 3*a^5 - 4*a^3*b^2 - a*b^4)/((a^2*b^4 - 2*a*b^
```

```
5 + b^6)*(a*tan(1/2*x*e + 1/2*d)^2 - b*tan(1/2*x*e + 1/2*d)^2 + 2*a*tan(1/2
*x*e + 1/2*d) + a + b)^2) + (3*a^2 + b^2)*log(abs(2*a*tan(1/2*x*e + 1/2*d)
- 2*b*tan(1/2*x*e + 1/2*d) + 2*a - 2*abs(b))/abs(2*a*tan(1/2*x*e + 1/2*d) -
 2*b*tan(1/2*x*e + 1/2*d) + 2*a + 2*abs(b)))/(b^4*abs(b)))*e^(-1)
```

maple [B] time = 0.57, size = 639, normalized size = 4.50

$$-\frac{3\ln\left(a\tan\left(\frac{d}{2}+\frac{ex}{2}\right)-b\tan\left(\frac{d}{2}+\frac{ex}{2}\right)+a+b\right)a^{3}}{16e\,b^{5}\left(a-b\right)}+\frac{3\ln\left(a\tan\left(\frac{d}{2}+\frac{ex}{2}\right)-b\tan\left(\frac{d}{2}+\frac{ex}{2}\right)+a+b\right)a^{2}}{16e\,b^{4}\left(a-b\right)}-\frac{\ln\left(a\tan\left(\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2*a+2*b*cos(e*x+d)+2*a*sin(e*x+d))^3,x)
```

```
[Out] -3/16/e/b^5/(a-b)*ln(a*tan(1/2*d+1/2*e*x)-b*tan(1/2*d+1/2*e*x)+a+b)*a^3+3/1
6/e/b^4/(a-b)*ln(a*tan(1/2*d+1/2*e*x)-b*tan(1/2*d+1/2*e*x)+a+b)*a^2-1/16/e/
b^3/(a-b)*ln(a*tan(1/2*d+1/2*e*x)-b*tan(1/2*d+1/2*e*x)+a+b)*a+1/16/e/b^2/(a
-b)*ln(a*tan(1/2*d+1/2*e*x)-b*tan(1/2*d+1/2*e*x)+a+b)+1/16/e/b^3/(a-b)^2/(a
*tan(1/2*d+1/2*e*x)-b*tan(1/2*d+1/2*e*x)+a+b)^2*a^4+1/8/e/b/(a-b)^2/(a*tan(
1/2*d+1/2*e*x)-b*tan(1/2*d+1/2*e*x)+a+b)^2*a^2+1/16/e*b/(a-b)^2/(a*tan(1/2*
d+1/2*e*x)-b*tan(1/2*d+1/2*e*x)+a+b)^2+3/16/e/b^4/(a-b)^2/(a*tan(1/2*d+1/2*
e*x)-b*tan(1/2*d+1/2*e*x)+a+b)*a^4-1/4/e/b^3/(a-b)^2/(a*tan(1/2*d+1/2*e*x)-
b*tan(1/2*d+1/2*e*x)+a+b)*a^3+1/8/e/b^2/(a-b)^2/(a*tan(1/2*d+1/2*e*x)-b*tan
(1/2*d+1/2*e*x)+a+b)*a^2-1/4/e/b/(a-b)^2/(a*tan(1/2*d+1/2*e*x)-b*tan(1/2*d+
1/2*e*x)+a+b)*a-1/16/e/(a-b)^2/(a*tan(1/2*d+1/2*e*x)-b*tan(1/2*d+1/2*e*x)+a
+b)-1/16/e/b^3/(1+tan(1/2*d+1/2*e*x))^2+3/16/e/b^4/(1+tan(1/2*d+1/2*e*x))*a
+1/16/e/b^3/(1+tan(1/2*d+1/2*e*x))+3/16/e/b^5*ln(1+tan(1/2*d+1/2*e*x))*a^2+
1/16/e/b^3*ln(1+tan(1/2*d+1/2*e*x))
```

maxima [B] time = 0.35, size = 493, normalized size = 3.47

$$\frac{\frac{2\left(3a^{5}-4a^{3}b^{2}-ab^{4}+\frac{\left(9a^{5}-9a^{4}b-2a^{3}b^{2}+2a^{2}b^{3}-5ab^{4}+b^{5}\right)\sin(ex+d)}{\cos(ex+d)+1}+\frac{\left(9a^{5}-18a^{4}b+12a^{3}b^{2}-6a^{2}b^{3}+ab^{4}\right)\sin(ex+d)^{2}}{(\cos(ex+d)+1)^{2}}+\frac{\left(3a^{5}-9a^{4}b+10a^{3}b^{2}-6a^{2}b^{3}+ab^{4}+b^{5}\right)\mathrm{s}}{(\cos(ex+d)+1)^{3}}}{a^{4}b^{4}-2a^{2}b^{6}+b^{8}+\frac{4\left(a^{4}b^{4}-a^{3}b^{5}-a^{2}b^{6}+ab^{7}\right)\sin(ex+d)}{\cos(ex+d)+1}+\frac{2\left(3a^{4}b^{4}-6a^{3}b^{5}+2a^{2}b^{6}+2ab^{7}-b^{8}\right)\sin(ex+d)^{2}}{(\cos(ex+d)+1)^{2}}+\frac{4\left(a^{4}b^{4}-3a^{3}b^{5}+3a^{2}b^{6}-ab^{7}\right)\sin(ex+d)^{3}}{(\cos(ex+d)+1)^{3}}+\frac{\left(a^{4}b^{4}-4a^{3}b^{5}+6a^{2}b^{6}-\right.}{(\cos(ex+}}}{16e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*a+2*b*cos(e*x+d)+2*a*sin(e*x+d))^3,x, algorithm="maxima")
```

```
[Out] 1/16*(2*(3*a^5 - 4*a^3*b^2 - a*b^4 + (9*a^5 - 9*a^4*b - 2*a^3*b^2 + 2*a^2*b
^3 - 5*a*b^4 + b^5)*sin(e*x + d)/(cos(e*x + d) + 1) + (9*a^5 - 18*a^4*b + 1
2*a^3*b^2 - 6*a^2*b^3 + a*b^4)*sin(e*x + d)^2/(cos(e*x + d) + 1)^2 + (3*a^5
 - 9*a^4*b + 10*a^3*b^2 - 6*a^2*b^3 + a*b^4 + b^5)*sin(e*x + d)^3/(cos(e*x
```

```
+ d) + 1)^3)/(a^4*b^4 - 2*a^2*b^6 + b^8 + 4*(a^4*b^4 - a^3*b^5 - a^2*b^6 +
a*b^7)*sin(e*x + d)/(cos(e*x + d) + 1) + 2*(3*a^4*b^4 - 6*a^3*b^5 + 2*a^2*b
^6 + 2*a*b^7 - b^8)*sin(e*x + d)^2/(cos(e*x + d) + 1)^2 + 4*(a^4*b^4 - 3*a^
3*b^5 + 3*a^2*b^6 - a*b^7)*sin(e*x + d)^3/(cos(e*x + d) + 1)^3 + (a^4*b^4 -
 4*a^3*b^5 + 6*a^2*b^6 - 4*a*b^7 + b^8)*sin(e*x + d)^4/(cos(e*x + d) + 1)^4
) - (3*a^2 + b^2)*log(-a - b - (a - b)*sin(e*x + d)/(cos(e*x + d) + 1))/b^5
 + (3*a^2 + b^2)*log(sin(e*x + d)/(cos(e*x + d) + 1) + 1)/b^5)/e
```

mupad [B] time = 6.45, size = 360, normalized size = 2.54

$$-\frac{\frac{-3\,a^5+4\,a^3\,b^2+a\,b^4}{2\,b^4\,(a-b)^2}+\frac{\tan\left(\frac{d}{2}+\frac{e\,x}{2}\right)\left(-9\,a^5+9\,a^4\,b+2\,a^3\,b^2-2\,a^2\,b^3+5\,a\,b^4-b^5\right)}{2\,b^4\,(a-b)^2}+\frac{\tan\left(\frac{d}{2}+\frac{e\,x}{2}\right)^3\left(-3\,a^4+6\,a^3\,b-4\,a^2\,b^2+2\,a\,b^3+b^4\right)}{2\,b^4\,(a-b)}-\frac{\tan\left(\right.}{}}{e\left(8\,a\,b+\tan\left(\frac{d}{2}+\frac{e\,x}{2}\right)^2\left(24\,a^2-8\,b^2\right)-\tan\left(\frac{d}{2}+\frac{e\,x}{2}\right)^3\left(16\,a\,b-16\,a^2\right)+\tan\left(\frac{d}{2}+\frac{e\,x}{2}\right)^4\left(4\,a^2-8\,a\,b+4\,b^2\right)+4\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2*a + 2*b*cos(d + e*x) + 2*a*sin(d + e*x))^3,x)
```

```
[Out] - ((a*b^4 - 3*a^5 + 4*a^3*b^2)/(2*b^4*(a - b)^2) + (tan(d/2 + (e*x)/2)*(5*a
*b^4 + 9*a^4*b - 9*a^5 - b^5 - 2*a^2*b^3 + 2*a^3*b^2))/(2*b^4*(a - b)^2) +
(tan(d/2 + (e*x)/2)^3*(2*a*b^3 + 6*a^3*b - 3*a^4 + b^4 - 4*a^2*b^2))/(2*b^4
*(a - b)) - (tan(d/2 + (e*x)/2)^2*(a*b^4 - 18*a^4*b + 9*a^5 - 6*a^2*b^3 + 1
2*a^3*b^2))/(2*b^4*(a - b)^2))/(e*(8*a*b + tan(d/2 + (e*x)/2)^2*(24*a^2 - 8
*b^2) - tan(d/2 + (e*x)/2)^3*(16*a*b - 16*a^2) + tan(d/2 + (e*x)/2)^4*(4*a^
2 - 8*a*b + 4*b^2) + 4*a^2 + 4*b^2 + tan(d/2 + (e*x)/2)*(16*a*b + 16*a^2)))
 - (atanh((2*a + tan(d/2 + (e*x)/2)*(2*a - 2*b))/(2*b))*(3*a^2 + b^2))/(8*b
^5*e)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*a+2*b*cos(e*x+d)+2*a*sin(e*x+d))**3,x)
```

```
[Out] Timed out
```

3.387 $\int \frac{1}{(2a+2b\cos(d+ex)+2a\sin(d+ex))^4}\,dx$

Optimal. Leaf size=215

$$\frac{5\left(a^2\cos(d+ex)-ab\sin(d+ex)\right)}{96b^4e(a\sin(d+ex)+a+b\cos(d+ex))^2}+\frac{a\left(5a^2+3b^2\right)\log\left(a+b\cot\left(\frac{d}{2}+\frac{ex}{2}+\frac{\pi}{4}\right)\right)}{32b^7e}-\frac{a\left(15a^2+4b^2\right)\cos(d+e\ldots}{96b^6e(a\sin(d+e\ldots}$$

```
[Out] 1/32*a*(5*a^2+3*b^2)*ln(a+b*cot(1/2*d+1/4*Pi+1/2*e*x))/b^7/e+1/48*(-a*cos(e
*x+d)+b*sin(e*x+d))/b^2/e/(a+b*cos(e*x+d)+a*sin(e*x+d))^3+5/96*(a^2*cos(e*x
+d)-a*b*sin(e*x+d))/b^4/e/(a+b*cos(e*x+d)+a*sin(e*x+d))^2+1/96*(-a*(15*a^2+
4*b^2)*cos(e*x+d)+b*(15*a^2+4*b^2)*sin(e*x+d))/b^6/e/(a+b*cos(e*x+d)+a*sin(
e*x+d))
```

Rubi [A] time = 0.24, antiderivative size = 215, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {3129, 3156, 3153, 3123, 31}

$$\frac{5\left(a^2\cos(d+ex)-ab\sin(d+ex)\right)}{96b^4e(a\sin(d+ex)+a+b\cos(d+ex))^2}-\frac{a\left(15a^2+4b^2\right)\cos(d+ex)-b\left(15a^2+4b^2\right)\sin(d+ex)}{96b^6e(a\sin(d+ex)+a+b\cos(d+ex))}+\frac{a\left(5a^2+3b\ldots\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(2*a + 2*b*Cos[d + e*x] + 2*a*Sin[d + e*x])^(-4),x]
```

```
[Out] (a*(5*a^2 + 3*b^2)*Log[a + b*Cot[d/2 + Pi/4 + (e*x)/2]])/(32*b^7*e) - (a*Co
s[d + e*x] - b*Sin[d + e*x])/(48*b^2*e*(a + b*Cos[d + e*x] + a*Sin[d + e*x]
)^3) + (5*(a^2*Cos[d + e*x] - a*b*Sin[d + e*x]))/(96*b^4*e*(a + b*Cos[d + e
*x] + a*Sin[d + e*x])^2) - (a*(15*a^2 + 4*b^2)*Cos[d + e*x] - b*(15*a^2 + 4
*b^2)*Sin[d + e*x])/(96*b^6*e*(a + b*Cos[d + e*x] + a*Sin[d + e*x]))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 3123

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(-1), x_Symbol] :> Module[{f = FreeFactors[Cot[(d + e*x)/2 + Pi/4], x]}, -D
ist[f/e, Subst[Int[1/(a + b*f*x), x], x, Cot[(d + e*x)/2 + Pi/4]/f], x]] /;
 FreeQ[{a, b, c, d, e}, x] && EqQ[a - c, 0] && NeQ[a - b, 0]
```

Rule 3129

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(n_), x_Symbol] :> Simp[((-(c*Cos[d + e*x]) + b*Sin[d + e*x])*(a + b*Cos[d
+ e*x] + c*Sin[d + e*x])^(n + 1))/(e*(n + 1)*(a^2 - b^2 - c^2)), x] + Dist[
1/((n + 1)*(a^2 - b^2 - c^2)), Int[(a*(n + 1) - b*(n + 2)*Cos[d + e*x] - c*
(n + 2)*Sin[d + e*x])*(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n + 1), x], x]
 /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0] && LtQ[n, -1] && N
eQ[n, -3/2]
```

Rule 3153

```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])
/((a_.) + cos[(d_.) + (e_.)*(x_)]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)])^2,
 x_Symbol] :> Simp[(c*B - b*C - (a*C - c*A)*Cos[d + e*x] + (a*B - b*A)*Sin[
d + e*x])/(e*(a^2 - b^2 - c^2)*(a + b*Cos[d + e*x] + c*Sin[d + e*x])), x] +
 Dist[(a*A - b*B - c*C)/(a^2 - b^2 - c^2), Int[1/(a + b*Cos[d + e*x] + c*Si
n[d + e*x]), x], x] /; FreeQ[{a, b, c, d, e, A, B, C}, x] && NeQ[a^2 - b^2
- c^2, 0] && NeQ[a*A - b*B - c*C, 0]
```

Rule 3156

```
Int[((a_.) + cos[(d_.) + (e_.)*(x_)]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)])
^(n_)*((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)
]), x_Symbol] :> -Simp[((c*B - b*C - (a*C - c*A)*Cos[d + e*x] + (a*B - b*A)
*Sin[d + e*x])*(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n + 1))/(e*(n + 1)*(a
^2 - b^2 - c^2)), x] + Dist[1/((n + 1)*(a^2 - b^2 - c^2)), Int[(a + b*Cos[d
 + e*x] + c*Sin[d + e*x])^(n + 1)*Simp[(n + 1)*(a*A - b*B - c*C) + (n + 2)*
(a*B - b*A)*Cos[d + e*x] + (n + 2)*(a*C - c*A)*Sin[d + e*x], x], x], x] /;
FreeQ[{a, b, c, d, e, A, B, C}, x] && LtQ[n, -1] && NeQ[a^2 - b^2 - c^2, 0]
 && NeQ[n, -2]
```

Rubi steps

$$\int \frac{1}{(2a+2b\cos(d+ex)+2a\sin(d+ex))^4}\,dx = -\frac{a\cos(d+ex)-b\sin(d+ex)}{48b^2e(a+b\cos(d+ex)+a\sin(d+ex))^3} + \frac{\int \frac{-6a+4b\cos(d+ex)+}{(2a+2b\cos(d+ex)+2}}{12b^2}$$

$$= -\frac{a\cos(d+ex)-b\sin(d+ex)}{48b^2e(a+b\cos(d+ex)+a\sin(d+ex))^3} + \frac{5\left(a^2\cos(d+\right.}{96b^4e(a+b\cos(a}$$

$$= -\frac{a\cos(d+ex)-b\sin(d+ex)}{48b^2e(a+b\cos(d+ex)+a\sin(d+ex))^3} + \frac{5\left(a^2\cos(d+\right.}{96b^4e(a+b\cos(a}$$

$$= -\frac{a\cos(d+ex)-b\sin(d+ex)}{48b^2e(a+b\cos(d+ex)+a\sin(d+ex))^3} + \frac{5\left(a^2\cos(d+\right.}{96b^4e(a+b\cos(a}$$

$$= \frac{a\left(5a^2+3b^2\right)\log\left(a+b\cot\left(\frac{d}{2}+\frac{\pi}{4}+\frac{ex}{2}\right)\right)}{32b^7e} - \frac{a\cos(d+ex}{48b^2e(a+b\cos(d}$$

Mathematica [B] time = 3.01, size = 632, normalized size = 2.94

$$-12a\left(5a^2+3b^2\right)\log\left(\sin\left(\frac{1}{2}(d+ex)\right)+\cos\left(\frac{1}{2}(d+ex)\right)\right)+12a\left(5a^2+3b^2\right)\log\left((a-b)\sin\left(\frac{1}{2}(d+ex)\right)+(a+b\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(2*a + 2*b*Cos[d + e*x] + 2*a*Sin[d + e*x])^(-4),x]
```

```
[Out] (-12*a*(5*a^2 + 3*b^2)*Log[Cos[(d + e*x)/2] + Sin[(d + e*x)/2]] + 12*a*(5*a
^2 + 3*b^2)*Log[(a + b)*Cos[(d + e*x)/2] + (a - b)*Sin[(d + e*x)/2]] + (b*(
150*a^6 + 130*a^4*b^2 + 24*a^2*b^4 - 3*a^2*(25*a^4 - 50*a^3*b + 5*a^2*b^2 -
 30*a*b^3 + 4*b^4)*Cos[d + e*x] - 6*a^2*(15*a^4 + 20*a^3*b + 9*a^2*b^2 + 2*
a*b^3 - 2*b^4)*Cos[2*(d + e*x)] + 15*a^6*Cos[3*(d + e*x)] - 30*a^5*b*Cos[3*
(d + e*x)] - 41*a^4*b^2*Cos[3*(d + e*x)] - 38*a^3*b^3*Cos[3*(d + e*x)] - 12
*a^2*b^4*Cos[3*(d + e*x)] - 8*a*b^5*Cos[3*(d + e*x)] + 225*a^6*Sin[d + e*x]
 + 75*a^5*b*Sin[d + e*x] + 180*a^4*b^2*Sin[d + e*x] + 15*a^3*b^3*Sin[d + e*
x] + 27*a^2*b^4*Sin[d + e*x] + 12*a*b^5*Sin[d + e*x] + 12*b^6*Sin[d + e*x]
- 60*a^6*Sin[2*(d + e*x)] + 120*a^5*b*Sin[2*(d + e*x)] + 54*a^4*b^2*Sin[2*(
d + e*x)] + 102*a^3*b^3*Sin[2*(d + e*x)] + 6*a^2*b^4*Sin[2*(d + e*x)] + 6*a
*b^5*Sin[2*(d + e*x)] - 15*a^6*Sin[3*(d + e*x)] - 45*a^5*b*Sin[3*(d + e*x)]
 - 4*a^4*b^2*Sin[3*(d + e*x)] + 3*a^3*b^3*Sin[3*(d + e*x)] + 15*a^2*b^4*Sin
[3*(d + e*x)] + 4*a*b^5*Sin[3*(d + e*x)] + 4*b^6*Sin[3*(d + e*x)]]))/((a + b
)*(Cos[(d + e*x)/2] + Sin[(d + e*x)/2])^3*((a + b)*Cos[(d + e*x)/2] + (a -
b)*Sin[(d + e*x)/2])^3))/(384*b^7*e)
```

fricas [B] time = 1.09, size = 729, normalized size = 3.39

$$\frac{60\,a^4b^2 + 6\,a^2b^4 + 2\left(15\,a^5b - 41\,a^3b^3 - 12\,ab^5\right)\cos\left(ex+d\right)^3 - 12\left(10\,a^4b^2 + a^2b^4\right)\cos\left(ex+d\right)^2 - 6\left(10\,a^5b - 9\,a^3\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*a+2*b*cos(e*x+d)+2*a*sin(e*x+d))^4,x, algorithm="fricas")
```

```
[Out] 1/192*(60*a^4*b^2 + 6*a^2*b^4 + 2*(15*a^5*b - 41*a^3*b^3 - 12*a*b^5)*cos(e*
x + d)^3 - 12*(10*a^4*b^2 + a^2*b^4)*cos(e*x + d)^2 - 6*(10*a^5*b - 9*a^3*b
^3 - 2*a*b^5)*cos(e*x + d) + 3*(20*a^6 + 12*a^4*b^2 - (15*a^5*b + 4*a^3*b^3
 - 3*a*b^5)*cos(e*x + d)^3 - 3*(5*a^6 - 2*a^4*b^2 - 3*a^2*b^4)*cos(e*x + d)
^2 + 6*(5*a^5*b + 3*a^3*b^3)*cos(e*x + d) + (20*a^6 + 12*a^4*b^2 - (5*a^6 -
 12*a^4*b^2 - 9*a^2*b^4)*cos(e*x + d)^2 + 6*(5*a^5*b + 3*a^3*b^3)*cos(e*x +
 d))*sin(e*x + d))*log(2*a*b*cos(e*x + d) + a^2 + b^2 + (a^2 - b^2)*sin(e*x
 + d)) - 3*(20*a^6 + 12*a^4*b^2 - (15*a^5*b + 4*a^3*b^3 - 3*a*b^5)*cos(e*x
+ d)^3 - 3*(5*a^6 - 2*a^4*b^2 - 3*a^2*b^4)*cos(e*x + d)^2 + 6*(5*a^5*b + 3*
a^3*b^3)*cos(e*x + d) + (20*a^6 + 12*a^4*b^2 - (5*a^6 - 12*a^4*b^2 - 9*a^2*
b^4)*cos(e*x + d)^2 + 6*(5*a^5*b + 3*a^3*b^3)*cos(e*x + d))*sin(e*x + d))*l
og(sin(e*x + d) + 1) + 2*(30*a^4*b^2 + 3*a^2*b^4 + 2*b^6 - (45*a^4*b^2 - 3*
a^2*b^4 - 4*b^6)*cos(e*x + d)^2 - 3*(10*a^5*b - 9*a^3*b^3 - a*b^5)*cos(e*x
+ d))*sin(e*x + d))/(6*a^2*b^8*e*cos(e*x + d) + 4*a^3*b^7*e - (3*a^2*b^8 -
b^10)*e*cos(e*x + d)^3 - 3*(a^3*b^7 - a*b^9)*e*cos(e*x + d)^2 + (6*a^2*b^8*
e*cos(e*x + d) + 4*a^3*b^7*e - (a^3*b^7 - 3*a*b^9)*e*cos(e*x + d)^2)*sin(e*
x + d))
```

giac [B] time = 0.31, size = 1006, normalized size = 4.68

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*a+2*b*cos(e*x+d)+2*a*sin(e*x+d))^4,x, algorithm="giac")
```

```
[Out] -1/96*(2*(15*a^8*tan(1/2*x*e + 1/2*d)^5 - 75*a^7*b*tan(1/2*x*e + 1/2*d)^5 +
 159*a^6*b^2*tan(1/2*x*e + 1/2*d)^5 - 195*a^5*b^3*tan(1/2*x*e + 1/2*d)^5 +
165*a^4*b^4*tan(1/2*x*e + 1/2*d)^5 - 105*a^3*b^5*tan(1/2*x*e + 1/2*d)^5 + 5
1*a^2*b^6*tan(1/2*x*e + 1/2*d)^5 - 21*a*b^7*tan(1/2*x*e + 1/2*d)^5 + 6*b^8*
tan(1/2*x*e + 1/2*d)^5 + 75*a^8*tan(1/2*x*e + 1/2*d)^4 - 300*a^7*b*tan(1/2*
x*e + 1/2*d)^4 + 495*a^6*b^2*tan(1/2*x*e + 1/2*d)^4 - 480*a^5*b^3*tan(1/2*x
*e + 1/2*d)^4 + 345*a^4*b^4*tan(1/2*x*e + 1/2*d)^4 - 180*a^3*b^5*tan(1/2*x*
e + 1/2*d)^4 + 57*a^2*b^6*tan(1/2*x*e + 1/2*d)^4 - 12*a*b^7*tan(1/2*x*e + 1
/2*d)^4 + 150*a^8*tan(1/2*x*e + 1/2*d)^3 - 450*a^7*b*tan(1/2*x*e + 1/2*d)^3
 + 500*a^6*b^2*tan(1/2*x*e + 1/2*d)^3 - 300*a^5*b^3*tan(1/2*x*e + 1/2*d)^3
```

```
+ 126*a^4*b^4*tan(1/2*x*e + 1/2*d)^3 + 22*a^3*b^5*tan(1/2*x*e + 1/2*d)^3 -
48*a^2*b^6*tan(1/2*x*e + 1/2*d)^3 + 12*a*b^7*tan(1/2*x*e + 1/2*d)^3 - 4*b^8
*tan(1/2*x*e + 1/2*d)^3 + 150*a^8*tan(1/2*x*e + 1/2*d)^2 - 300*a^7*b*tan(1/
2*x*e + 1/2*d)^2 + 120*a^6*b^2*tan(1/2*x*e + 1/2*d)^2 + 60*a^5*b^3*tan(1/2*
x*e + 1/2*d)^2 - 102*a^4*b^4*tan(1/2*x*e + 1/2*d)^2 + 144*a^3*b^5*tan(1/2*x
*e + 1/2*d)^2 - 60*a^2*b^6*tan(1/2*x*e + 1/2*d)^2 + 12*a*b^7*tan(1/2*x*e +
1/2*d)^2 + 75*a^8*tan(1/2*x*e + 1/2*d) - 75*a^7*b*tan(1/2*x*e + 1/2*d) - 75
*a^6*b^2*tan(1/2*x*e + 1/2*d) + 75*a^5*b^3*tan(1/2*x*e + 1/2*d) - 39*a^4*b^
4*tan(1/2*x*e + 1/2*d) + 39*a^3*b^5*tan(1/2*x*e + 1/2*d) + 33*a^2*b^6*tan(1
/2*x*e + 1/2*d) - 15*a*b^7*tan(1/2*x*e + 1/2*d) + 6*b^8*tan(1/2*x*e + 1/2*d
) + 15*a^8 - 31*a^6*b^2 + 9*a^4*b^4 + 15*a^2*b^6)/((a^3*b^6 - 3*a^2*b^7 + 3
*a*b^8 - b^9)*(a*tan(1/2*x*e + 1/2*d)^2 - b*tan(1/2*x*e + 1/2*d)^2 + 2*a*ta
n(1/2*x*e + 1/2*d) + a + b)^3) + 3*(5*a^3 + 3*a*b^2)*log(abs(2*a*tan(1/2*x*
e + 1/2*d) - 2*b*tan(1/2*x*e + 1/2*d) + 2*a - 2*abs(b))/abs(2*a*tan(1/2*x*e
 + 1/2*d) - 2*b*tan(1/2*x*e + 1/2*d) + 2*a + 2*abs(b)))/(b^6*abs(b)))*e^(-1
)
```

maple [B] time = 0.59, size = 1069, normalized size = 4.97

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2*a+2*b*cos(e*x+d)+2*a*sin(e*x+d))^4,x)
```

```
[Out] -5/32/e*a^3/b^7*ln(1+tan(1/2*d+1/2*e*x))-3/32/e*a/b^5*ln(1+tan(1/2*d+1/2*e*
x))-1/16/e/(a-b)^3/(a*tan(1/2*d+1/2*e*x)-b*tan(1/2*d+1/2*e*x)+a+b)^3*a^2+3/
32/e/(a-b)^3/(a*tan(1/2*d+1/2*e*x)-b*tan(1/2*d+1/2*e*x)+a+b)^2*a-1/48/e*b^2
/(a-b)^3/(a*tan(1/2*d+1/2*e*x)-b*tan(1/2*d+1/2*e*x)+a+b)^3+1/32/e*b/(a-b)^3
/(a*tan(1/2*d+1/2*e*x)-b*tan(1/2*d+1/2*e*x)+a+b)^2+1/16/e/b^5/(1+tan(1/2*d+
1/2*e*x))^2*a-5/32/e/b^6/(1+tan(1/2*d+1/2*e*x))*a^2-1/16/e/b^5/(1+tan(1/2*d
+1/2*e*x))*a+5/32/e*a^4/b^7/(a-b)*ln(a*tan(1/2*d+1/2*e*x)-b*tan(1/2*d+1/2*e
*x)+a+b)-1/48/e/b^4/(1+tan(1/2*d+1/2*e*x))^3+1/32/e/b^4/(1+tan(1/2*d+1/2*e*
x))^2-1/16/e/b^4/(1+tan(1/2*d+1/2*e*x))-1/16/e/(a-b)^3/(a*tan(1/2*d+1/2*e*x
)-b*tan(1/2*d+1/2*e*x)+a+b)-5/32/e*a^3/b^6/(a-b)*ln(a*tan(1/2*d+1/2*e*x)-b*
tan(1/2*d+1/2*e*x)+a+b)+3/32/e*a^2/b^5/(a-b)*ln(a*tan(1/2*d+1/2*e*x)-b*tan(
1/2*d+1/2*e*x)+a+b)-3/32/e*a/b^4/(a-b)*ln(a*tan(1/2*d+1/2*e*x)-b*tan(1/2*d+
1/2*e*x)+a+b)-1/48/e/b^4/(a-b)^3/(a*tan(1/2*d+1/2*e*x)-b*tan(1/2*d+1/2*e*x)
+a+b)^3*a^6-1/16/e/b^2/(a-b)^3/(a*tan(1/2*d+1/2*e*x)-b*tan(1/2*d+1/2*e*x)+a
+b)^3*a^4-1/16/e/b^5/(a-b)^3/(a*tan(1/2*d+1/2*e*x)-b*tan(1/2*d+1/2*e*x)+a+b
)^2*a^6+3/32/e/b^4/(a-b)^3/(a*tan(1/2*d+1/2*e*x)-b*tan(1/2*d+1/2*e*x)+a+b)^
2*a^5-3/32/e/b^3/(a-b)^3/(a*tan(1/2*d+1/2*e*x)-b*tan(1/2*d+1/2*e*x)+a+b)^2*
a^4+3/16/e/b^2/(a-b)^3/(a*tan(1/2*d+1/2*e*x)-b*tan(1/2*d+1/2*e*x)+a+b)^2*a^
3-5/32/e/b^6/(a-b)^3/(a*tan(1/2*d+1/2*e*x)-b*tan(1/2*d+1/2*e*x)+a+b)*a^6+3/
8/e/b^5/(a-b)^3/(a*tan(1/2*d+1/2*e*x)-b*tan(1/2*d+1/2*e*x)+a+b)*a^5-3/8/e/b
^4/(a-b)^3/(a*tan(1/2*d+1/2*e*x)-b*tan(1/2*d+1/2*e*x)+a+b)*a^4+3/8/e/b^3/(a
```

```
-b)^3/(a*tan(1/2*d+1/2*e*x)-b*tan(1/2*d+1/2*e*x)+a+b)*a^3-9/32/e/b^2/(a-b)^
3/(a*tan(1/2*d+1/2*e*x)-b*tan(1/2*d+1/2*e*x)+a+b)*a^2
```

maxima [B] time = 0.39, size = 963, normalized size = 4.48

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*a+2*b*cos(e*x+d)+2*a*sin(e*x+d))^4,x, algorithm="maxima")
```

```
[Out] -1/96*(2*(15*a^8 - 31*a^6*b^2 + 9*a^4*b^4 + 15*a^2*b^6 + 3*(25*a^8 - 25*a^7
*b - 25*a^6*b^2 + 25*a^5*b^3 - 13*a^4*b^4 + 13*a^3*b^5 + 11*a^2*b^6 - 5*a*b
^7 + 2*b^8)*sin(e*x + d)/(cos(e*x + d) + 1) + 6*(25*a^8 - 50*a^7*b + 20*a^6
*b^2 + 10*a^5*b^3 - 17*a^4*b^4 + 24*a^3*b^5 - 10*a^2*b^6 + 2*a*b^7)*sin(e*x
 + d)^2/(cos(e*x + d) + 1)^2 + 2*(75*a^8 - 225*a^7*b + 250*a^6*b^2 - 150*a^
5*b^3 + 63*a^4*b^4 + 11*a^3*b^5 - 24*a^2*b^6 + 6*a*b^7 - 2*b^8)*sin(e*x + d
)^3/(cos(e*x + d) + 1)^3 + 3*(25*a^8 - 100*a^7*b + 165*a^6*b^2 - 160*a^5*b^
3 + 115*a^4*b^4 - 60*a^3*b^5 + 19*a^2*b^6 - 4*a*b^7)*sin(e*x + d)^4/(cos(e*
x + d) + 1)^4 + 3*(5*a^8 - 25*a^7*b + 53*a^6*b^2 - 65*a^5*b^3 + 55*a^4*b^4
- 35*a^3*b^5 + 17*a^2*b^6 - 7*a*b^7 + 2*b^8)*sin(e*x + d)^5/(cos(e*x + d) +
 1)^5)/(a^6*b^6 - 3*a^4*b^8 + 3*a^2*b^10 - b^12 + 6*(a^6*b^6 - a^5*b^7 - 2*
a^4*b^8 + 2*a^3*b^9 + a^2*b^10 - a*b^11)*sin(e*x + d)/(cos(e*x + d) + 1) +
3*(5*a^6*b^6 - 10*a^5*b^7 - a^4*b^8 + 12*a^3*b^9 - 5*a^2*b^10 - 2*a*b^11 +
b^12)*sin(e*x + d)^2/(cos(e*x + d) + 1)^2 + 4*(5*a^6*b^6 - 15*a^5*b^7 + 12*
a^4*b^8 + 4*a^3*b^9 - 9*a^2*b^10 + 3*a*b^11)*sin(e*x + d)^3/(cos(e*x + d) +
 1)^3 + 3*(5*a^6*b^6 - 20*a^5*b^7 + 29*a^4*b^8 - 16*a^3*b^9 - a^2*b^10 + 4*
a*b^11 - b^12)*sin(e*x + d)^4/(cos(e*x + d) + 1)^4 + 6*(a^6*b^6 - 5*a^5*b^7
 + 10*a^4*b^8 - 10*a^3*b^9 + 5*a^2*b^10 - a*b^11)*sin(e*x + d)^5/(cos(e*x +
 d) + 1)^5 + (a^6*b^6 - 6*a^5*b^7 + 15*a^4*b^8 - 20*a^3*b^9 + 15*a^2*b^10 -
 6*a*b^11 + b^12)*sin(e*x + d)^6/(cos(e*x + d) + 1)^6) - 3*(5*a^3 + 3*a*b^2
)*log(-a - b - (a - b)*sin(e*x + d)/(cos(e*x + d) + 1))/b^7 + 3*(5*a^3 + 3*
a*b^2)*log(sin(e*x + d)/(cos(e*x + d) + 1) + 1)/b^7)/e
```

mupad [B] time = 7.22, size = 730, normalized size = 3.40

$$\frac{a\,\mathrm{atanh}\left(\frac{a\left(2\,a+\tan\left(\frac{d}{2}+\frac{e\,x}{2}\right)\,\left(2\,a-2\,b\right)\right)\,\left(5\,a^2+3\,b^2\right)}{2\,b\,\left(5\,a^3+3\,a\,b^2\right)}\right)\,\left(5\,a^2+3\,b^2\right)}{16\,b^7\,e}-\frac{\frac{15\,a^8-31\,a^6\,b^2+9\,a^4\,b^4+15\,a^2\,b^6}{6\,b^6\,\left(a-b\right)^3}+\frac{{\tan\left(\frac{d}{2}+\frac{e\,x}{2}\right)}^2\,\left(25\,a^8-50\,a^7\,b+20\,a^6\,b^2+\dots\right)}{b^6\dots}}{e\,\left({\tan\left(\frac{d}{2}+\frac{e\,x}{2}\right)}^5\,\left(48\,a^3-96\,a^2\,b+48\,a\,b^2\right)\dots\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2*a + 2*b*cos(d + e*x) + 2*a*sin(d + e*x))^4,x)
```

```
[Out] (a*atanh((a*(2*a + tan(d/2 + (e*x)/2)*(2*a - 2*b))*(5*a^2 + 3*b^2))/(2*b*(3
*a*b^2 + 5*a^3)))*(5*a^2 + 3*b^2))/(16*b^7*e) - ((15*a^8 + 15*a^2*b^6 + 9*a
^4*b^4 - 31*a^6*b^2)/(6*b^6*(a - b)^3) + (tan(d/2 + (e*x)/2)^2*(2*a*b^7 - 5
0*a^7*b + 25*a^8 - 10*a^2*b^6 + 24*a^3*b^5 - 17*a^4*b^4 + 10*a^5*b^3 + 20*a
^6*b^2))/(b^6*(a - b)^3) + (tan(d/2 + (e*x)/2)^4*(4*a*b^6 - 75*a^6*b + 25*a
^7 - 15*a^2*b^5 + 45*a^3*b^4 - 70*a^4*b^3 + 90*a^5*b^2))/(2*b^6*(a - b)^2)
+ (tan(d/2 + (e*x)/2)^3*(6*a*b^7 - 225*a^7*b + 75*a^8 - 2*b^8 - 24*a^2*b^6
+ 11*a^3*b^5 + 63*a^4*b^4 - 150*a^5*b^3 + 250*a^6*b^2))/(3*b^6*(a - b)^3) +
 (tan(d/2 + (e*x)/2)^5*(5*a^6 - 15*a^5*b - 3*a*b^5 + 2*b^6 + 9*a^2*b^4 - 14
*a^3*b^3 + 18*a^4*b^2))/(2*b^6*(a - b)) + (tan(d/2 + (e*x)/2)*(25*a^8 - 25*
a^7*b - 5*a*b^7 + 2*b^8 + 11*a^2*b^6 + 13*a^3*b^5 - 13*a^4*b^4 + 25*a^5*b^3
 - 25*a^6*b^2))/(2*b^6*(a - b)^3))/(e*(tan(d/2 + (e*x)/2)^5*(48*a*b^2 - 96*
a^2*b + 48*a^3) + tan(d/2 + (e*x)/2)^6*(24*a*b^2 - 24*a^2*b + 8*a^3 - 8*b^3
) - tan(d/2 + (e*x)/2)^2*(24*a*b^2 - 120*a^2*b - 120*a^3 + 24*b^3) - tan(d/
2 + (e*x)/2)^4*(24*a*b^2 + 120*a^2*b - 120*a^3 - 24*b^3) + 24*a*b^2 + 24*a^
2*b - tan(d/2 + (e*x)/2)^3*(96*a*b^2 - 160*a^3) + tan(d/2 + (e*x)/2)*(48*a*
b^2 + 96*a^2*b + 48*a^3) + 8*a^3 + 8*b^3))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*a+2*b*cos(e*x+d)+2*a*sin(e*x+d))**4,x)
```

```
[Out] Timed out
```

3.388 $\int (2a + 2b\cos(d + ex) - 2a\sin(d + ex))^3 \, dx$

Optimal. Leaf size=157

$$\frac{4b(15a^2 + 4b^2)\sin(d + ex)}{3e} + \frac{4a(15a^2 + 4b^2)\cos(d + ex)}{3e} + 4ax(5a^2 + 3b^2) + \frac{20(a^2\cos(d + ex) + ab\sin(d + ex))}{3}$$

```
[Out] 4*a*(5*a^2+3*b^2)*x+4/3*a*(15*a^2+4*b^2)*cos(e*x+d)/e+4/3*b*(15*a^2+4*b^2)*
sin(e*x+d)/e+8/3*(a+b*cos(e*x+d)-a*sin(e*x+d))^2*(a*cos(e*x+d)+b*sin(e*x+d)
)/e+20/3*(a+b*cos(e*x+d)-a*sin(e*x+d))*(a^2*cos(e*x+d)+a*b*sin(e*x+d))/e
```

Rubi [A] time = 0.14, antiderivative size = 157, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3120, 3146, 2637, 2638}

$$\frac{4b(15a^2 + 4b^2)\sin(d + ex)}{3e} + \frac{4a(15a^2 + 4b^2)\cos(d + ex)}{3e} + 4ax(5a^2 + 3b^2) + \frac{20(a^2\cos(d + ex) + ab\sin(d + ex))}{3}$$

Antiderivative was successfully verified.

```
[In] Int[(2*a + 2*b*Cos[d + e*x] - 2*a*Sin[d + e*x])^3,x]
```

```
[Out] 4*a*(5*a^2 + 3*b^2)*x + (4*a*(15*a^2 + 4*b^2)*Cos[d + e*x])/(3*e) + (4*b*(1
5*a^2 + 4*b^2)*Sin[d + e*x])/(3*e) + (8*(a + b*Cos[d + e*x] - a*Sin[d + e*x
])^2*(a*Cos[d + e*x] + b*Sin[d + e*x]))/(3*e) + (20*(a + b*Cos[d + e*x] - a
*Sin[d + e*x])*(a^2*Cos[d + e*x] + a*b*Sin[d + e*x]))/(3*e)
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3120

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(n_), x_Symbol] :> -Simp[((c*Cos[d + e*x] - b*Sin[d + e*x])*(a + b*Cos[d +
e*x] + c*Sin[d + e*x])^(n - 1))/(e*n), x] + Dist[1/n, Int[Simp[n*a^2 + (n -
 1)*(b^2 + c^2) + a*b*(2*n - 1)*Cos[d + e*x] + a*c*(2*n - 1)*Sin[d + e*x],
x]*(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n - 2), x], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[a^2 - b^2 - c^2, 0] && GtQ[n, 1]
```

Rule 3146

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(n_.)*((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)
]), x_Symbol] :> Simp[((B*c - b*C - a*C*Cos[d + e*x] + a*B*Sin[d + e*x])*(a
 + b*Cos[d + e*x] + c*Sin[d + e*x])^n)/(a*e*(n + 1)), x] + Dist[1/(a*(n + 1
)), Int[(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n - 1)*Simp[a*(b*B + c*C)*n
+ a^2*A*(n + 1) + (n*(a^2*B - B*c^2 + b*c*C) + a*b*A*(n + 1))*Cos[d + e*x]
+ (n*(b*B*c + a^2*C - b^2*C) + a*c*A*(n + 1))*Sin[d + e*x], x], x], x] /; F
reeQ[{a, b, c, d, e, A, B, C}, x] && GtQ[n, 0] && NeQ[a^2 - b^2 - c^2, 0]
```

Rubi steps

$$\int (2a + 2b\cos(d+ex) - 2a\sin(d+ex))^3\,dx = \frac{8(a + b\cos(d+ex) - a\sin(d+ex))^2(a\cos(d+ex) + b\sin(d+e}{3e}$$

$$= \frac{8(a + b\cos(d+ex) - a\sin(d+ex))^2(a\cos(d+ex) + b\sin(d+e}{3e}$$

$$= 4a\left(5a^2+3b^2\right)x + \frac{8(a + b\cos(d+ex) - a\sin(d+ex))^2(a\cos(d}{3e}$$

$$= 4a\left(5a^2+3b^2\right)x + \frac{4a\left(15a^2+4b^2\right)\cos(d+ex)}{3e} + \frac{4b\left(15a^2+4b^2\right.}{3e}$$

Mathematica [A] time = 0.44, size = 136, normalized size = 0.87

$$\frac{2\left(6a\left(5a^2+3b^2\right)(d+ex) - 9a\left(a^2-b^2\right)\sin(2(d+ex)) + 9b\left(5a^2+b^2\right)\sin(d+ex) + b\left(b^2-3a^2\right)\sin(3(d+ex))\right.}{3e}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2*a + 2*b*Cos[d + e*x] - 2*a*Sin[d + e*x])^3,x]
```

```
[Out] (2*(6*a*(5*a^2 + 3*b^2)*(d + e*x) + 9*a*(5*a^2 + b^2)*Cos[d + e*x] + 18*a^2
*b*Cos[2*(d + e*x)] - a*(a^2 - 3*b^2)*Cos[3*(d + e*x)] + 9*b*(5*a^2 + b^2)*
Sin[d + e*x] - 9*a*(a^2 - b^2)*Sin[2*(d + e*x)] + b*(-3*a^2 + b^2)*Sin[3*(d
 + e*x)]))/(3*e)
```

fricas [A] time = 0.93, size = 126, normalized size = 0.80

$$\frac{4\left(18\,a^2b\cos\left(ex+d\right)^2 + 24\,a^3\cos\left(ex+d\right) - 2\left(a^3-3\,ab^2\right)\cos\left(ex+d\right)^3 + 3\left(5\,a^3+3\,ab^2\right)ex + \left(24\,a^2b + 4\,b^3 - \right.\right.}{3\,e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*a+2*b*cos(e*x+d)-2*a*sin(e*x+d))^3,x, algorithm="fricas")
```

```
[Out] 4/3*(18*a^2*b*cos(e*x + d)^2 + 24*a^3*cos(e*x + d) - 2*(a^3 - 3*a*b^2)*cos(
e*x + d)^3 + 3*(5*a^3 + 3*a*b^2)*e*x + (24*a^2*b + 4*b^3 - 2*(3*a^2*b - b^3
)*cos(e*x + d)^2 - 9*(a^3 - a*b^2)*cos(e*x + d))*sin(e*x + d))/e
```

giac [A] time = 0.18, size = 151, normalized size = 0.96

$$12\,a^2b\cos\left(2\,xe+2\,d\right)e^{(-1)}-\frac{2}{3}\left(a^3-3\,ab^2\right)\cos\left(3\,xe+3\,d\right)e^{(-1)}+6\left(5\,a^3+ab^2\right)\cos\left(xe+d\right)e^{(-1)}-\frac{2}{3}\left(3\,a^2b-b^3\right)e^{(-}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*a+2*b*cos(e*x+d)-2*a*sin(e*x+d))^3,x, algorithm="giac")
```

```
[Out] 12*a^2*b*cos(2*x*e + 2*d)*e^(-1) - 2/3*(a^3 - 3*a*b^2)*cos(3*x*e + 3*d)*e^(
-1) + 6*(5*a^3 + a*b^2)*cos(x*e + d)*e^(-1) - 2/3*(3*a^2*b - b^3)*e^(-1)*si
n(3*x*e + 3*d) - 6*(a^3 - a*b^2)*e^(-1)*sin(2*x*e + 2*d) + 6*(5*a^2*b + b^3
)*e^(-1)*sin(x*e + d) + 4*(5*a^3 + 3*a*b^2)*x
```

maple [A] time = 0.25, size = 176, normalized size = 1.12

$$\frac{8a^3\left(ex+d\right)+24\sin\left(ex+d\right)a^2b+24a^3\cos\left(ex+d\right)+24a\,b^2\left(\frac{\sin(ex+d)\cos(ex+d)}{2}+\frac{ex}{2}+\frac{d}{2}\right)+24\left(\cos^2\left(ex+d\right)\right)a^2b}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*a+2*b*cos(e*x+d)-2*a*sin(e*x+d))^3,x)
```

```
[Out] 8/e*(a^3*(e*x+d)+3*sin(e*x+d)*a^2*b+3*a^3*cos(e*x+d)+3*a*b^2*(1/2*sin(e*x+d
)*cos(e*x+d)+1/2*e*x+1/2*d)+3*cos(e*x+d)^2*a^2*b+3*a^3*(-1/2*sin(e*x+d)*cos
(e*x+d)+1/2*e*x+1/2*d)+1/3*b^3*(2+cos(e*x+d)^2)*sin(e*x+d)+a*b^2*cos(e*x+d)
^3+a^2*b*sin(e*x+d)^3+1/3*a^3*(2+sin(e*x+d)^2)*cos(e*x+d))
```

maxima [A] time = 0.32, size = 188, normalized size = 1.20

$$\frac{8\,ab^2\cos\left(ex+d\right)^3}{e}+\frac{8\,a^2b\sin\left(ex+d\right)^3}{e}+8\,a^3x-\frac{8\left(\cos\left(ex+d\right)^3-3\,\cos\left(ex+d\right)\right)a^3}{3\,e}-\frac{8\left(\sin\left(ex+d\right)^3-3\,\sin\left(ex+\right.\right.}{3\,e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*a+2*b*cos(e*x+d)-2*a*sin(e*x+d))^3,x, algorithm="maxima")
```

```
[Out] 8*a*b^2*cos(e*x + d)^3/e + 8*a^2*b*sin(e*x + d)^3/e + 8*a^3*x - 8/3*(cos(e*
x + d)^3 - 3*cos(e*x + d))*a^3/e - 8/3*(sin(e*x + d)^3 - 3*sin(e*x + d))*b^
3/e + 24*a^2*(a*cos(e*x + d)/e + b*sin(e*x + d)/e) + 6*(4*a*b*cos(e*x + d)^
```

```
2/e + (2*e*x + 2*d - sin(2*e*x + 2*d))*a^2/e + (2*e*x + 2*d + sin(2*e*x + 2
*d))*b^2/e)*a
```

mupad [B] time = 3.44, size = 292, normalized size = 1.86

$$\frac{\tan\left(\frac{d}{2}+\frac{e\,x}{2}\right)^4\,\left(48\,a^3-96\,a^2\,b+48\,a\,b^2\right)+\tan\left(\frac{d}{2}+\frac{e\,x}{2}\right)^5\,\left(24\,a^3+48\,a^2\,b-24\,a\,b^2+16\,b^3\right)+16\,a\,b^2-\tan\left(\frac{d}{2}+\right.}{e\,\left(\tan\left(\frac{d}{2}+\frac{e\,x}{2}\right)^6+3\tan\left(\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*a + 2*b*cos(d + e*x) - 2*a*sin(d + e*x))^3,x)
```

```
[Out] (tan(d/2 + (e*x)/2)^4*(48*a*b^2 - 96*a^2*b + 48*a^3) + tan(d/2 + (e*x)/2)^5
*(48*a^2*b - 24*a*b^2 + 24*a^3 + 16*b^3) + 16*a*b^2 - tan(d/2 + (e*x)/2)^2*
(96*a^2*b - 128*a^3) + tan(d/2 + (e*x)/2)^3*(160*a^2*b + (32*b^3)/3) + (176
*a^3)/3 + tan(d/2 + (e*x)/2)*(24*a*b^2 + 48*a^2*b - 24*a^3 + 16*b^3))/(e*(3
*tan(d/2 + (e*x)/2)^2 + 3*tan(d/2 + (e*x)/2)^4 + tan(d/2 + (e*x)/2)^6 + 1))
 + (8*a*atan((8*a*tan(d/2 + (e*x)/2)*(5*a^2 + 3*b^2))/(24*a*b^2 + 40*a^3))*
(5*a^2 + 3*b^2))/e - (8*a*(5*a^2 + 3*b^2)*(atan(tan(d/2 + (e*x)/2)) - (e*x)
/2))/e
```

sympy [A] time = 0.76, size = 291, normalized size = 1.85

$$\begin{cases} 12a^3x\sin^2(d+ex)+12a^3x\cos^2(d+ex)+8a^3x+\frac{8a^3\sin^2(d+ex)\cos(d+ex)}{e}-\frac{12a^3\sin(d+ex)\cos(d+ex)}{e}+\frac{16a^3\cos^3(d+ex)}{3e} \\ x\left(-2a\sin(d)+2a+2b\cos(d)\right)^3 \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*a+2*b*cos(e*x+d)-2*a*sin(e*x+d))**3,x)
```

```
[Out] Piecewise((12*a**3*x*sin(d + e*x)**2 + 12*a**3*x*cos(d + e*x)**2 + 8*a**3*x
 + 8*a**3*sin(d + e*x)**2*cos(d + e*x)/e - 12*a**3*sin(d + e*x)*cos(d + e*x
)/e + 16*a**3*cos(d + e*x)**3/(3*e) + 24*a**3*cos(d + e*x)/e + 8*a**2*b*sin
(d + e*x)**3/e + 24*a**2*b*sin(d + e*x)/e + 24*a**2*b*cos(d + e*x)**2/e + 1
2*a*b**2*x*sin(d + e*x)**2 + 12*a*b**2*x*cos(d + e*x)**2 + 12*a*b**2*sin(d
+ e*x)*cos(d + e*x)/e + 8*a*b**2*cos(d + e*x)**3/e + 16*b**3*sin(d + e*x)**
3/(3*e) + 8*b**3*sin(d + e*x)*cos(d + e*x)**2/e, Ne(e, 0)), (x*(-2*a*sin(d)
 + 2*a + 2*b*cos(d))**3, True))
```

3.389 $\int (2a + 2b\cos(d + ex) - 2a\sin(d + ex))^2\, dx$

Optimal. Leaf size=81

$$2x\left(3a^2 + b^2\right) + \frac{6a^2\cos(d + ex)}{e} + \frac{6ab\sin(d + ex)}{e} + \frac{2(a(-\sin(d + ex)) + a + b\cos(d + ex))(a\cos(d + ex) + b\sin(d$$

```
[Out] 2*(3*a^2+b^2)*x+6*a^2*cos(e*x+d)/e+6*a*b*sin(e*x+d)/e+2*(a+b*cos(e*x+d)-a*s
in(e*x+d))*(a*cos(e*x+d)+b*sin(e*x+d))/e
```

Rubi [A] time = 0.05, antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {3120, 2637, 2638}

$$2x\left(3a^2 + b^2\right) + \frac{6a^2\cos(d + ex)}{e} + \frac{6ab\sin(d + ex)}{e} + \frac{2(a(-\sin(d + ex)) + a + b\cos(d + ex))(a\cos(d + ex) + b\sin(d$$

Antiderivative was successfully verified.

```
[In] Int[(2*a + 2*b*Cos[d + e*x] - 2*a*Sin[d + e*x])^2,x]
```

```
[Out] 2*(3*a^2 + b^2)*x + (6*a^2*Cos[d + e*x])/e + (6*a*b*Sin[d + e*x])/e + (2*(a
 + b*Cos[d + e*x] - a*Sin[d + e*x])*(a*Cos[d + e*x] + b*Sin[d + e*x]))/e
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3120

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(n_), x_Symbol] :> -Simp[((c*Cos[d + e*x] - b*Sin[d + e*x])*(a + b*Cos[d +
e*x] + c*Sin[d + e*x])^(n - 1))/(e*n), x] + Dist[1/n, Int[Simp[n*a^2 + (n -
 1)*(b^2 + c^2) + a*b*(2*n - 1)*Cos[d + e*x] + a*c*(2*n - 1)*Sin[d + e*x],
x]*(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n - 2), x], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[a^2 - b^2 - c^2, 0] && GtQ[n, 1]
```

Rubi steps

$$\int (2a + 2b\cos(d+ex) - 2a\sin(d+ex))^2\,dx = \frac{2(a+b\cos(d+ex)-a\sin(d+ex))(a\cos(d+ex)+b\sin(d+ex}{e}$$
$$= 2\left(3a^2+b^2\right)x + \frac{2(a+b\cos(d+ex)-a\sin(d+ex))(a\cos(d+ex}{e}$$
$$= 2\left(3a^2+b^2\right)x + \frac{6a^2\cos(d+ex)}{e} + \frac{6ab\sin(d+ex)}{e} + \frac{2(a+b\cos}{}$$

Mathematica [A] time = 0.15, size = 92, normalized size = 1.14

$$4\left(\frac{\left(3a^2+b^2\right)(d+ex)}{2e} - \frac{\left(a^2-b^2\right)\sin(2(d+ex))}{4e} + \frac{2a^2\cos(d+ex)}{e} + \frac{2ab\sin(d+ex)}{e} + \frac{ab\cos(2(d+ex))}{2e}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(2*a + 2*b*Cos[d + e*x] - 2*a*Sin[d + e*x])^2,x]
```

```
[Out] 4*(((3*a^2 + b^2)*(d + e*x))/(2*e) + (2*a^2*Cos[d + e*x])/e + (a*b*Cos[2*(d
 + e*x)])/(2*e) + (2*a*b*Sin[d + e*x])/e - ((a^2 - b^2)*Sin[2*(d + e*x)])/(
4*e))
```

fricas [A] time = 0.90, size = 70, normalized size = 0.86

$$\frac{2\left(2\,ab\cos\left(ex+d\right)^2 + \left(3\,a^2+b^2\right)ex + 4\,a^2\cos\left(ex+d\right) + \left(4\,ab - \left(a^2-b^2\right)\cos\left(ex+d\right)\right)\sin\left(ex+d\right)\right)}{e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*a+2*b*cos(e*x+d)-2*a*sin(e*x+d))^2,x, algorithm="fricas")
```

```
[Out] 2*(2*a*b*cos(e*x + d)^2 + (3*a^2 + b^2)*e*x + 4*a^2*cos(e*x + d) + (4*a*b -
 (a^2 - b^2)*cos(e*x + d))*sin(e*x + d))/e
```

giac [A] time = 0.14, size = 79, normalized size = 0.98

$$2\,ab\cos\left(2\,xe+2\,d\right)e^{(-1)} + 8\,a^2\cos\left(xe+d\right)e^{(-1)} + 8\,abe^{(-1)}\sin\left(xe+d\right) - \left(a^2-b^2\right)e^{(-1)}\sin\left(2\,xe+2\,d\right) + 2\left(3\,a^2+b^2\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*a+2*b*cos(e*x+d)-2*a*sin(e*x+d))^2,x, algorithm="giac")
```

```
[Out] 2*a*b*cos(2*x*e + 2*d)*e^(-1) + 8*a^2*cos(x*e + d)*e^(-1) + 8*a*b*e^(-1)*si
n(x*e + d) - (a^2 - b^2)*e^(-1)*sin(2*x*e + 2*d) + 2*(3*a^2 + b^2)*x
```

maple [A] time = 0.23, size = 100, normalized size = 1.23

$$\frac{4a^2(ex+d)+8ab\sin(ex+d)+8a^2\cos(ex+d)+4b^2\left(\frac{\sin(ex+d)\cos(ex+d)}{2}+\frac{ex}{2}+\frac{d}{2}\right)+4\left(\cos^2(ex+d)\right)ab+4a^2\left(-\right.}{e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*a+2*b*cos(e*x+d)-2*a*sin(e*x+d))^2,x)
```

```
[Out] 4/e*(a^2*(e*x+d)+2*a*b*sin(e*x+d)+2*a^2*cos(e*x+d)+b^2*(1/2*sin(e*x+d)*cos(
e*x+d)+1/2*e*x+1/2*d)+cos(e*x+d)^2*a*b+a^2*(-1/2*sin(e*x+d)*cos(e*x+d)+1/2*
e*x+1/2*d))
```

maxima [A] time = 0.32, size = 98, normalized size = 1.21

$$4a^2x+\frac{4ab\cos(ex+d)^2}{e}+\frac{(2ex+2d-\sin(2ex+2d))a^2}{e}+\frac{(2ex+2d+\sin(2ex+2d))b^2}{e}+8a\left(\frac{a\cos(ex+d)}{e}+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*a+2*b*cos(e*x+d)-2*a*sin(e*x+d))^2,x, algorithm="maxima")
```

```
[Out] 4*a^2*x + 4*a*b*cos(e*x + d)^2/e + (2*e*x + 2*d - sin(2*e*x + 2*d))*a^2/e +
 (2*e*x + 2*d + sin(2*e*x + 2*d))*b^2/e + 8*a*(a*cos(e*x + d)/e + b*sin(e*x
 + d)/e)
```

mupad [B] time = 3.74, size = 128, normalized size = 1.58

$$\frac{x\left(12a^2+4b^2\right)}{2}+\frac{\tan\left(\frac{d}{2}+\frac{ex}{2}\right)^3\left(4a^2+16ab-4b^2\right)-\tan\left(\frac{d}{2}+\frac{ex}{2}\right)^2\left(16ab-16a^2\right)+16a^2+\tan\left(\frac{d}{2}+\frac{ex}{2}\right)\left(-4\right.}{e\left(\tan\left(\frac{d}{2}+\frac{ex}{2}\right)^4+2\tan\left(\frac{d}{2}+\frac{ex}{2}\right)^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*a + 2*b*cos(d + e*x) - 2*a*sin(d + e*x))^2,x)
```

```
[Out] (x*(12*a^2 + 4*b^2))/2 + (tan(d/2 + (e*x)/2)^3*(16*a*b + 4*a^2 - 4*b^2) - t
an(d/2 + (e*x)/2)^2*(16*a*b - 16*a^2) + 16*a^2 + tan(d/2 + (e*x)/2)*(16*a*b
 - 4*a^2 + 4*b^2))/(e*(2*tan(d/2 + (e*x)/2)^2 + tan(d/2 + (e*x)/2)^4 + 1))
```

sympy [A] time = 0.32, size = 170, normalized size = 2.10

$$\begin{cases}2a^2x\sin^2(d+ex)+2a^2x\cos^2(d+ex)+4a^2x-\frac{2a^2\sin(d+ex)\cos(d+ex)}{e}+\frac{8a^2\cos(d+ex)}{e}+\frac{8ab\sin(d+ex)}{e}+\frac{4ab\cos^2(d+ex)}{e} \\ x\left(-2a\sin(d)+2a+2b\cos(d)\right)^2\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*a+2*b*cos(e*x+d)-2*a*sin(e*x+d))**2,x)

[Out] Piecewise((2*a**2*x*sin(d + e*x)**2 + 2*a**2*x*cos(d + e*x)**2 + 4*a**2*x - 2*a**2*sin(d + e*x)*cos(d + e*x)/e + 8*a**2*cos(d + e*x)/e + 8*a*b*sin(d + e*x)/e + 4*a*b*cos(d + e*x)**2/e + 2*b**2*x*sin(d + e*x)**2 + 2*b**2*x*cos(d + e*x)**2 + 2*b**2*sin(d + e*x)*cos(d + e*x)/e, Ne(e, 0)), (x*(-2*a*sin(d) + 2*a + 2*b*cos(d))**2, True))

3.390 $\int (2a + 2b\cos(d + ex) - 2a\sin(d + ex))\,dx$

Optimal. Leaf size=29

$$\frac{2a\cos(d + ex)}{e} + 2ax + \frac{2b\sin(d + ex)}{e}$$

```
[Out] 2*a*x+2*a*cos(e*x+d)/e+2*b*sin(e*x+d)/e
```

Rubi [A] time = 0.01, antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {2637, 2638}

$$\frac{2a\cos(d + ex)}{e} + 2ax + \frac{2b\sin(d + ex)}{e}$$

Antiderivative was successfully verified.

```
[In] Int[2*a + 2*b*Cos[d + e*x] - 2*a*Sin[d + e*x],x]
```

```
[Out] 2*a*x + (2*a*Cos[d + e*x])/e + (2*b*Sin[d + e*x])/e
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int (2a + 2b\cos(d + ex) - 2a\sin(d + ex))\,dx &= 2ax - (2a)\int \sin(d + ex)\,dx + (2b)\int \cos(d + ex)\,dx\\ &= 2ax + \frac{2a\cos(d + ex)}{e} + \frac{2b\sin(d + ex)}{e}\end{aligned}$$

Mathematica [A] time = 0.01, size = 53, normalized size = 1.83

$$-\frac{2a\sin(d)\sin(ex)}{e} + \frac{2a\cos(d)\cos(ex)}{e} + 2ax + \frac{2b\sin(d)\cos(ex)}{e} + \frac{2b\cos(d)\sin(ex)}{e}$$

Antiderivative was successfully verified.

```
[In] Integrate[2*a + 2*b*Cos[d + e*x] - 2*a*Sin[d + e*x],x]
```

```
[Out] 2*a*x + (2*a*Cos[d]*Cos[e*x])/e + (2*b*Cos[e*x]*Sin[d])/e + (2*b*Cos[d]*Sin
[e*x])/e - (2*a*Sin[d]*Sin[e*x])/e
```

fricas [A] time = 2.90, size = 26, normalized size = 0.90

$$\frac{2\left(aex + a\cos\left(ex + d\right) + b\sin\left(ex + d\right)\right)}{e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(2*a+2*b*cos(e*x+d)-2*a*sin(e*x+d),x, algorithm="fricas")
```

```
[Out] 2*(a*e*x + a*cos(e*x + d) + b*sin(e*x + d))/e
```

giac [A] time = 0.14, size = 29, normalized size = 1.00

$$2\,a\cos\left(xe + d\right)e^{(-1)} + 2\,be^{(-1)}\sin\left(xe + d\right) + 2\,ax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(2*a+2*b*cos(e*x+d)-2*a*sin(e*x+d),x, algorithm="giac")
```

```
[Out] 2*a*cos(x*e + d)*e^(-1) + 2*b*e^(-1)*sin(x*e + d) + 2*a*x
```

maple [A] time = 0.00, size = 30, normalized size = 1.03

$$2ax + \frac{2a\cos\left(ex + d\right)}{e} + \frac{2b\sin\left(ex + d\right)}{e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(2*a+2*b*cos(e*x+d)-2*a*sin(e*x+d),x)
```

```
[Out] 2*a*x+2*a*cos(e*x+d)/e+2*b*sin(e*x+d)/e
```

maxima [A] time = 0.33, size = 29, normalized size = 1.00

$$2\,ax + \frac{2\,a\cos\left(ex + d\right)}{e} + \frac{2\,b\sin\left(ex + d\right)}{e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(2*a+2*b*cos(e*x+d)-2*a*sin(e*x+d),x, algorithm="maxima")
```

```
[Out] 2*a*x + 2*a*cos(e*x + d)/e + 2*b*sin(e*x + d)/e
```

mupad [B] time = 2.44, size = 29, normalized size = 1.00

$$2\,a\,x+\frac{2\,a\,\cos\left(d+e\,x\right)}{e}+\frac{2\,b\,\sin\left(d+e\,x\right)}{e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(2*a + 2*b*cos(d + e*x) - 2*a*sin(d + e*x),x)
```

```
[Out] 2*a*x + (2*a*cos(d + e*x))/e + (2*b*sin(d + e*x))/e
```

sympy [A] time = 0.14, size = 39, normalized size = 1.34

$$2ax-2a\left(\begin{cases}-\frac{\cos\left(d+ex\right)}{e} & \text{for}\ e\neq 0\\ x\sin\left(d\right) & \text{otherwise}\end{cases}\right)+2b\left(\begin{cases}\frac{\sin\left(d+ex\right)}{e} & \text{for}\ e\neq 0\\ x\cos\left(d\right) & \text{otherwise}\end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(2*a+2*b*cos(e*x+d)-2*a*sin(e*x+d),x)
```

```
[Out] 2*a*x - 2*a*Piecewise((-cos(d + e*x)/e, Ne(e, 0)), (x*sin(d), True)) + 2*b*
Piecewise((sin(d + e*x)/e, Ne(e, 0)), (x*cos(d), True))
```

3.391 $\int \frac{1}{2a+2b\cos(d+ex)-2a\sin(d+ex)}\,dx$

Optimal. Leaf size=33

$$\frac{\log\left(a+b\tan\left(\frac{d}{2}+\frac{ex}{2}+\frac{\pi}{4}\right)\right)}{2be}$$

```
[Out] 1/2*ln(a+b*tan(1/2*d+1/4*Pi+1/2*e*x))/b/e
```

Rubi [A] time = 0.02, antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {3122, 31}

$$\frac{\log\left(a+b\tan\left(\frac{d}{2}+\frac{ex}{2}+\frac{\pi}{4}\right)\right)}{2be}$$

Antiderivative was successfully verified.

```
[In] Int[(2*a + 2*b*Cos[d + e*x] - 2*a*Sin[d + e*x])^(-1),x]
```

```
[Out] Log[a + b*Tan[d/2 + Pi/4 + (e*x)/2]]/(2*b*e)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 3122

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^(-1), x_Symbol] :> Module[{f = FreeFactors[Tan[(d + e*x)/2 + Pi/4], x]}, Dist[f/e, Subst[Int[1/(a + b*f*x), x], x, Tan[(d + e*x)/2 + Pi/4]/f], x]] /; FreeQ[{a, b, c, d, e}, x] && EqQ[a + c, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{2a+2b\cos(d+ex)-2a\sin(d+ex)}\,dx &= \frac{\text{Subst}\left(\int \frac{1}{2a+2bx}\,dx, x, \tan\left(\frac{\pi}{4}+\frac{1}{2}(d+ex)\right)\right)}{e}\\ &= \frac{\log\left(a+b\tan\left(\frac{d}{2}+\frac{\pi}{4}+\frac{ex}{2}\right)\right)}{2be}\end{aligned}$$

Mathematica [B] time = 0.10, size = 96, normalized size = 2.91

$$\frac{\log\left(-a\sin\left(\frac{1}{2}(d+ex)\right)+a\cos\left(\frac{1}{2}(d+ex)\right)+b\sin\left(\frac{1}{2}(d+ex)\right)+b\cos\left(\frac{1}{2}(d+ex)\right)\right)}{2be}-\frac{\log\left(\cos\left(\frac{1}{2}(d+ex)\right)-\sin\left(\right.\right.}{2be}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2*a + 2*b*Cos[d + e*x] - 2*a*Sin[d + e*x])^(-1),x]
```

```
[Out] -1/2*Log[Cos[(d + e*x)/2] - Sin[(d + e*x)/2]]/(b*e) + Log[a*Cos[(d + e*x)/2
] + b*Cos[(d + e*x)/2] - a*Sin[(d + e*x)/2] + b*Sin[(d + e*x)/2]]/(2*b*e)
```

fricas [B] time = 1.98, size = 57, normalized size = 1.73

$$\frac{\log\left(2\,ab\cos\left(ex+d\right)+a^2+b^2-\left(a^2-b^2\right)\sin\left(ex+d\right)\right)-\log\left(-\sin\left(ex+d\right)+1\right)}{4\,be}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*a+2*b*cos(e*x+d)-2*a*sin(e*x+d)),x, algorithm="fricas")
```

```
[Out] 1/4*(log(2*a*b*cos(e*x + d) + a^2 + b^2 - (a^2 - b^2)*sin(e*x + d)) - log(-
sin(e*x + d) + 1))/(b*e)
```

giac [B] time = 0.21, size = 82, normalized size = 2.48

$$\frac{e^{(-1)}\log\left(\frac{\left|2\,a\tan\left(\frac{1}{2}\,xe+\frac{1}{2}\,d\right)-2\,b\tan\left(\frac{1}{2}\,xe+\frac{1}{2}\,d\right)-2\,a-2\,|b|\right|}{\left|2\,a\tan\left(\frac{1}{2}\,xe+\frac{1}{2}\,d\right)-2\,b\tan\left(\frac{1}{2}\,xe+\frac{1}{2}\,d\right)-2\,a+2\,|b|\right|}\right)}{2\,|b|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*a+2*b*cos(e*x+d)-2*a*sin(e*x+d)),x, algorithm="giac")
```

```
[Out] 1/2*e^(-1)*log(abs(2*a*tan(1/2*x*e + 1/2*d) - 2*b*tan(1/2*x*e + 1/2*d) - 2*
a - 2*abs(b))/abs(2*a*tan(1/2*x*e + 1/2*d) - 2*b*tan(1/2*x*e + 1/2*d) - 2*a
 + 2*abs(b)))/abs(b)
```

maple [B] time = 0.42, size = 61, normalized size = 1.85

$$\frac{\ln\left(a\tan\left(\frac{d}{2}+\frac{ex}{2}\right)-b\tan\left(\frac{d}{2}+\frac{ex}{2}\right)-a-b\right)}{2eb}-\frac{\ln\left(\tan\left(\frac{d}{2}+\frac{ex}{2}\right)-1\right)}{2eb}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2*a+2*b*cos(e*x+d)-2*a*sin(e*x+d)),x)
```

```
[Out] 1/2/e/b*ln(a*tan(1/2*d+1/2*e*x)-b*tan(1/2*d+1/2*e*x)-a-b)-1/2/e/b*ln(tan(1/
2*d+1/2*e*x)-1)
```

maxima [B] time = 0.33, size = 62, normalized size = 1.88

$$\frac{\frac{\log\left(a+b-\frac{(a-b)\sin(ex+d)}{\cos(ex+d)+1}\right)}{b}-\frac{\log\left(\frac{\sin(ex+d)}{\cos(ex+d)+1}-1\right)}{b}}{2\,e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*a+2*b*cos(e*x+d)-2*a*sin(e*x+d)),x, algorithm="maxima")
```

```
[Out] 1/2*(log(a + b - (a - b)*sin(e*x + d)/(cos(e*x + d) + 1))/b - log(sin(e*x +
 d)/(cos(e*x + d) + 1) - 1)/b)/e
```

mupad [B] time = 2.74, size = 32, normalized size = 0.97

$$\frac{\operatorname{atanh}\left(\frac{a-\frac{\tan\left(\frac{d}{2}+\frac{e\,x}{2}\right)\,(2\,a-2\,b)}{2}}{b}\right)}{b\,e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2*a + 2*b*cos(d + e*x) - 2*a*sin(d + e*x)),x)
```

```
[Out] atanh((a - (tan(d/2 + (e*x)/2)*(2*a - 2*b))/2)/b)/(b*e)
```

sympy [A] time = 1.72, size = 109, normalized size = 3.30

$$\begin{cases} \frac{\tilde{\infty}x}{\cos(d)} & \text{for}\ a=0 \wedge b=0 \wedge e=0 \\ -\frac{1}{ae\tan\left(\frac{d}{2}+\frac{ex}{2}\right)-ae} & \text{for}\ b=0 \\ -\frac{\log\left(\tan\left(\frac{d}{2}+\frac{ex}{2}\right)-1\right)}{2be} & \text{for}\ a=b \\ \frac{x}{-2a\sin(d)+2a+2b\cos(d)} & \text{for}\ e=0 \\ -\frac{\log\left(\tan\left(\frac{d}{2}+\frac{ex}{2}\right)-1\right)}{2be}+\frac{\log\left(-\frac{a}{a-b}-\frac{b}{a-b}+\tan\left(\frac{d}{2}+\frac{ex}{2}\right)\right)}{2be} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*a+2*b*cos(e*x+d)-2*a*sin(e*x+d)),x)
```

```
[Out] Piecewise((zoo*x/cos(d), Eq(a, 0) & Eq(b, 0) & Eq(e, 0)), (-1/(a*e*tan(d/2
+ e*x/2) - a*e), Eq(b, 0)), (-log(tan(d/2 + e*x/2) - 1)/(2*b*e), Eq(a, b)),
 (x/(-2*a*sin(d) + 2*a + 2*b*cos(d)), Eq(e, 0)), (-log(tan(d/2 + e*x/2) - 1
)/(2*b*e) + log(-a/(a - b) - b/(a - b) + tan(d/2 + e*x/2))/(2*b*e), True))
```

3.392 $\int \frac{1}{(2a+2b\cos(d+ex)-2a\sin(d+ex))^2}\,dx$

Optimal. Leaf size=83

$$\frac{a\cos(d+ex)+b\sin(d+ex)}{4b^2e(a(-\sin(d+ex))+a+b\cos(d+ex))}-\frac{a\log\left(a+b\tan\left(\frac{d}{2}+\frac{ex}{2}+\frac{\pi}{4}\right)\right)}{4b^3e}$$

```
[Out] -1/4*a*ln(a+b*tan(1/2*d+1/4*Pi+1/2*e*x))/b^3/e+1/4*(a*cos(e*x+d)+b*sin(e*x+
d))/b^2/e/(a+b*cos(e*x+d)-a*sin(e*x+d))
```

Rubi [A] time = 0.05, antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3129, 12, 3122, 31}

$$\frac{a\cos(d+ex)+b\sin(d+ex)}{4b^2e(a(-\sin(d+ex))+a+b\cos(d+ex))}-\frac{a\log\left(a+b\tan\left(\frac{d}{2}+\frac{ex}{2}+\frac{\pi}{4}\right)\right)}{4b^3e}$$

Antiderivative was successfully verified.

```
[In] Int[(2*a + 2*b*Cos[d + e*x] - 2*a*Sin[d + e*x])^(-2),x]
```

```
[Out] -(a*Log[a + b*Tan[d/2 + Pi/4 + (e*x)/2]])/(4*b^3*e) + (a*Cos[d + e*x] + b*S
in[d + e*x])/(4*b^2*e*(a + b*Cos[d + e*x] - a*Sin[d + e*x]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 3122

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(-1), x_Symbol] :> Module[{f = FreeFactors[Tan[(d + e*x)/2 + Pi/4], x]}, Di
st[f/e, Subst[Int[1/(a + b*f*x), x], x, Tan[(d + e*x)/2 + Pi/4]/f], x]] /;
FreeQ[{a, b, c, d, e}, x] && EqQ[a + c, 0]
```

Rule 3129

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(n_), x_Symbol] :> Simp[((-(c*Cos[d + e*x]) + b*Sin[d + e*x])*(a + b*Cos[d
+ e*x] + c*Sin[d + e*x])^(n + 1))/(e*(n + 1)*(a^2 - b^2 - c^2)), x] + Dist[
1/((n + 1)*(a^2 - b^2 - c^2)), Int[(a*(n + 1) - b*(n + 2)*Cos[d + e*x] - c*
(n + 2)*Sin[d + e*x])*(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n + 1), x], x]
 /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0] && LtQ[n, -1] && N
eQ[n, -3/2]
```

Rubi steps

$$\int \frac{1}{(2a+2b\cos(d+ex)-2a\sin(d+ex))^2}\,dx = \frac{a\cos(d+ex)+b\sin(d+ex)}{4b^2e(a+b\cos(d+ex)-a\sin(d+ex))} + \frac{\int -\frac{2a}{2a+2b\cos(d+ex)-2a\sin(\ldots}}{4b^2}$$

$$= \frac{a\cos(d+ex)+b\sin(d+ex)}{4b^2e(a+b\cos(d+ex)-a\sin(d+ex))} - \frac{a\int \frac{1}{2a+2b\cos(d+ex)-2a\sin(\ldots}}{2b^2}$$

$$= \frac{a\cos(d+ex)+b\sin(d+ex)}{4b^2e(a+b\cos(d+ex)-a\sin(d+ex))} - \frac{a\,\text{Subst}\left(\int \frac{1}{2a+2bx}\,dx, x, \ldots\right.}{2b^2 \ldots}$$

$$= -\frac{a\log\left(a+b\tan\left(\frac{d}{2}+\frac{\pi}{4}+\frac{ex}{2}\right)\right)}{4b^3e} + \frac{a\cos(d+ex)+b\sin(d+\ldots}{4b^2e(a+b\cos(d+ex)-a\sin\ldots}$$

Mathematica [A] time = 0.62, size = 166, normalized size = 2.00

$$\frac{\frac{b(a^2+b^2)\sin\left(\frac{1}{2}(d+ex)\right)}{(a+b)\left((b-a)\sin\left(\frac{1}{2}(d+ex)\right)+(a+b)\cos\left(\frac{1}{2}(d+ex)\right)\right)} - a\log\left((b-a)\sin\left(\frac{1}{2}(d+ex)\right)+(a+b)\cos\left(\frac{1}{2}(d+ex)\right)\right) + a\log\left(\cos\left(\frac{1}{2}(d \ldots\right.\right.}{4b^3e}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2*a + 2*b*Cos[d + e*x] - 2*a*Sin[d + e*x])^(-2),x]
```

```
[Out] (a*Log[Cos[(d + e*x)/2] - Sin[(d + e*x)/2]] - a*Log[(a + b)*Cos[(d + e*x)/2
] + (-a + b)*Sin[(d + e*x)/2]] + (b*Sin[(d + e*x)/2])/(Cos[(d + e*x)/2] - S
in[(d + e*x)/2]) + (b*(a^2 + b^2)*Sin[(d + e*x)/2])/((a + b)*((a + b)*Cos[(
d + e*x)/2] + (-a + b)*Sin[(d + e*x)/2])))/(4*b^3*e)
```

fricas [B] time = 0.84, size = 154, normalized size = 1.86

$$\frac{2\,ab\cos(ex+d)+2\,b^2\sin(ex+d)-\left(ab\cos(ex+d)-a^2\sin(ex+d)+a^2\right)\log\left(2\,ab\cos(ex+d)+a^2+b^2-\left(a^{\ldots}\right.\right.}{8\left(b^4e\cos(ex+d)-ab^3e\sin(ex+d)\ldots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(2*a+2*b*cos(e*x+d)-2*a*sin(e*x+d))^2,x, algorithm="fricas")

[Out] 1/8*(2*a*b*cos(e*x + d) + 2*b^2*sin(e*x + d) - (a*b*cos(e*x + d) - a^2*sin(e*x + d) + a^2)*log(2*a*b*cos(e*x + d) + a^2 + b^2 - (a^2 - b^2)*sin(e*x + d)) + (a*b*cos(e*x + d) - a^2*sin(e*x + d) + a^2)*log(-sin(e*x + d) + 1))/(b^4*e*cos(e*x + d) - a*b^3*e*sin(e*x + d) + a*b^3*e)

giac [B] time = 0.22, size = 198, normalized size = 2.39

$$-\frac{1}{4}\left(\frac{2\left(a^2\tan\left(\frac{1}{2}xe+\frac{1}{2}d\right)-ab\tan\left(\frac{1}{2}xe+\frac{1}{2}d\right)+b^2\tan\left(\frac{1}{2}xe+\frac{1}{2}d\right)-a^2\right)}{\left(ab^2-b^3\right)\left(a\tan\left(\frac{1}{2}xe+\frac{1}{2}d\right)^2-b\tan\left(\frac{1}{2}xe+\frac{1}{2}d\right)^2-2\,a\tan\left(\frac{1}{2}xe+\frac{1}{2}d\right)+a+b\right)}+\frac{a\log\left(\frac{\left|2\,a\tan\left(\frac{1}{2}xe+\frac{1}{2}d\right)-\right.}{\left|2\,a\tan\left(\frac{1}{2}xe+\frac{1}{2}d\right)-\right.}\right.}{b^{}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(2*a+2*b*cos(e*x+d)-2*a*sin(e*x+d))^2,x, algorithm="giac")

[Out] -1/4*(2*(a^2*tan(1/2*x*e + 1/2*d) - a*b*tan(1/2*x*e + 1/2*d) + b^2*tan(1/2*x*e + 1/2*d) - a^2)/((a*b^2 - b^3)*(a*tan(1/2*x*e + 1/2*d)^2 - b*tan(1/2*x*e + 1/2*d)^2 - 2*a*tan(1/2*x*e + 1/2*d) + a + b)) + a*log(abs(2*a*tan(1/2*x*e + 1/2*d) - 2*b*tan(1/2*x*e + 1/2*d) - 2*a - 2*abs(b))/abs(2*a*tan(1/2*x*e + 1/2*d) - 2*b*tan(1/2*x*e + 1/2*d) - 2*a + 2*abs(b)))/(b^2*abs(b)))*e^(-1)

maple [B] time = 0.51, size = 178, normalized size = 2.14

$$-\frac{a^2}{4e\,b^2\,(a-b)\left(a\tan\left(\frac{d}{2}+\frac{ex}{2}\right)-b\tan\left(\frac{d}{2}+\frac{ex}{2}\right)-a-b\right)}-\frac{1}{4e\,(a-b)\left(a\tan\left(\frac{d}{2}+\frac{ex}{2}\right)-b\tan\left(\frac{d}{2}+\frac{ex}{2}\right)-a-b\right)}-\frac{a\ln}{}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(2*a+2*b*cos(e*x+d)-2*a*sin(e*x+d))^2,x)

[Out] -1/4/e/b^2/(a-b)/(a*tan(1/2*d+1/2*e*x)-b*tan(1/2*d+1/2*e*x)-a-b)*a^2-1/4/e/(a-b)/(a*tan(1/2*d+1/2*e*x)-b*tan(1/2*d+1/2*e*x)-a-b)-1/4/e*a/b^3*ln(a*tan(1/2*d+1/2*e*x)-b*tan(1/2*d+1/2*e*x)-a-b)-1/4/e/b^2/(tan(1/2*d+1/2*e*x)-1)+1/4/e*a/b^3*ln(tan(1/2*d+1/2*e*x)-1)

maxima [B] time = 0.34, size = 182, normalized size = 2.19

$$\frac{\frac{2\left(a^2-\frac{\left(a^2-ab+b^2\right)\sin(ex+d)}{\cos(ex+d)+1}\right)}{a^2b^2-b^4-\frac{2\left(a^2b^2-ab^3\right)\sin(ex+d)}{\cos(ex+d)+1}+\frac{\left(a^2b^2-2\,ab^3+b^4\right)\sin(ex+d)^2}{(\cos(ex+d)+1)^2}}-\frac{a\log\left(a+b-\frac{(a-b)\sin(ex+d)}{\cos(ex+d)+1}\right)}{b^3}+\frac{a\log\left(\frac{\sin(ex+d)}{\cos(ex+d)+1}-1\right)}{b^3}}{4\,e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*a+2*b*cos(e*x+d)-2*a*sin(e*x+d))^2,x, algorithm="maxima")
```

```
[Out] 1/4*(2*(a^2 - (a^2 - a*b + b^2)*sin(e*x + d)/(cos(e*x + d) + 1))/(a^2*b^2 -
 b^4 - 2*(a^2*b^2 - a*b^3)*sin(e*x + d)/(cos(e*x + d) + 1) + (a^2*b^2 - 2*a
*b^3 + b^4)*sin(e*x + d)^2/(cos(e*x + d) + 1)^2) - a*log(a + b - (a - b)*si
n(e*x + d)/(cos(e*x + d) + 1))/b^3 + a*log(sin(e*x + d)/(cos(e*x + d) + 1)
- 1)/b^3)/e
```

mupad [B] time = 2.74, size = 126, normalized size = 1.52

$$\frac{\frac{a^2}{b^2\,(a-b)}-\frac{\tan\left(\frac{d}{2}+\frac{e\,x}{2}\right)\left(a^2-a\,b+b^2\right)}{b^2\,(a-b)}}{e\left((2\,a-2\,b)\,\tan\left(\frac{d}{2}+\frac{e\,x}{2}\right)^2-4\,a\tan\left(\frac{d}{2}+\frac{e\,x}{2}\right)+2\,a+2\,b\right)}-\frac{a\,\mathrm{atanh}\left(\frac{a-\frac{\tan\left(\frac{d}{2}+\frac{e\,x}{2}\right)(2\,a-2\,b)}{2}}{b}\right)}{2\,b^3\,e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2*a + 2*b*cos(d + e*x) - 2*a*sin(d + e*x))^2,x)
```

```
[Out] (a^2/(b^2*(a - b)) - (tan(d/2 + (e*x)/2)*(a^2 - a*b + b^2))/(b^2*(a - b)))/
(e*(2*a + 2*b + tan(d/2 + (e*x)/2)^2*(2*a - 2*b) - 4*a*tan(d/2 + (e*x)/2)))
 - (a*atanh((a - (tan(d/2 + (e*x)/2)*(2*a - 2*b))/2)/b))/(2*b^3*e)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*a+2*b*cos(e*x+d)-2*a*sin(e*x+d))**2,x)
```

```
[Out] Timed out
```

3.393 $\int \frac{1}{(2a+2b\cos(d+ex)-2a\sin(d+ex))^3}\,dx$

Optimal. Leaf size=142

$$-\frac{3\left(a^2\cos(d+ex)+ab\sin(d+ex)\right)}{16b^4e(a(-\sin(d+ex))+a+b\cos(d+ex))}+\frac{\left(3a^2+b^2\right)\log\left(a+b\tan\left(\frac{d}{2}+\frac{ex}{2}+\frac{\pi}{4}\right)\right)}{16b^5e}+\frac{a\cos(d+ex)+}{16b^2e(a(-\sin(d+ex))}$$

```
[Out] 1/16*(3*a^2+b^2)*ln(a+b*tan(1/2*d+1/4*Pi+1/2*e*x))/b^5/e+1/16*(a*cos(e*x+d)
+b*sin(e*x+d))/b^2/e/(a+b*cos(e*x+d)-a*sin(e*x+d))^2-3/16*(a^2*cos(e*x+d)+a
*b*sin(e*x+d))/b^4/e/(a+b*cos(e*x+d)-a*sin(e*x+d))
```

Rubi [A] time = 0.11, antiderivative size = 142, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3129, 3153, 3122, 31}

$$\frac{\left(3a^2+b^2\right)\log\left(a+b\tan\left(\frac{d}{2}+\frac{ex}{2}+\frac{\pi}{4}\right)\right)}{16b^5e}-\frac{3\left(a^2\cos(d+ex)+ab\sin(d+ex)\right)}{16b^4e(a(-\sin(d+ex))+a+b\cos(d+ex))}+\frac{a\cos(d+ex)+b}{16b^2e(a(-\sin(d+ex))+}$$

Antiderivative was successfully verified.

```
[In] Int[(2*a + 2*b*Cos[d + e*x] - 2*a*Sin[d + e*x])^(-3),x]
```

```
[Out] ((3*a^2 + b^2)*Log[a + b*Tan[d/2 + Pi/4 + (e*x)/2]])/(16*b^5*e) + (a*Cos[d
+ e*x] + b*Sin[d + e*x])/(16*b^2*e*(a + b*Cos[d + e*x] - a*Sin[d + e*x])^2)
 - (3*(a^2*Cos[d + e*x] + a*b*Sin[d + e*x]))/(16*b^4*e*(a + b*Cos[d + e*x]
- a*Sin[d + e*x]))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 3122

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(-1), x_Symbol] :> Module[{f = FreeFactors[Tan[(d + e*x)/2 + Pi/4], x]}, Di
st[f/e, Subst[Int[1/(a + b*f*x), x], x, Tan[(d + e*x)/2 + Pi/4]/f], x]] /;
FreeQ[{a, b, c, d, e}, x] && EqQ[a + c, 0]
```

Rule 3129

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(n_), x_Symbol] :> Simp[((-(c*Cos[d + e*x]) + b*Sin[d + e*x])*(a + b*Cos[d
+ e*x] + c*Sin[d + e*x])^(n + 1))/(e*(n + 1)*(a^2 - b^2 - c^2)), x] + Dist[
```

```
1/((n + 1)*(a^2 - b^2 - c^2)), Int[(a*(n + 1) - b*(n + 2)*Cos[d + e*x] - c*
(n + 2)*Sin[d + e*x])*(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n + 1), x], x]
 /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0] && LtQ[n, -1] && N
eQ[n, -3/2]
```

Rule 3153

```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])
/((a_.) + cos[(d_.) + (e_.)*(x_)]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)])^2,
 x_Symbol] :> Simp[(c*B - b*C - (a*C - c*A)*Cos[d + e*x] + (a*B - b*A)*Sin[
d + e*x])/(e*(a^2 - b^2 - c^2)*(a + b*Cos[d + e*x] + c*Sin[d + e*x])), x] +
 Dist[(a*A - b*B - c*C)/(a^2 - b^2 - c^2), Int[1/(a + b*Cos[d + e*x] + c*Si
n[d + e*x]), x], x] /; FreeQ[{a, b, c, d, e, A, B, C}, x] && NeQ[a^2 - b^2
- c^2, 0] && NeQ[a*A - b*B - c*C, 0]
```

Rubi steps

$$\int \frac{1}{(2a+2b\cos(d+ex)-2a\sin(d+ex))^3}\,dx = \frac{a\cos(d+ex)+b\sin(d+ex)}{16b^2e(a+b\cos(d+ex)-a\sin(d+ex))^2} + \frac{\int \frac{-4a+2b\cos(d+ex)-2a\sin}{(2a+2b\cos(d+ex)-2a\sin}}{8b^2}$$

$$= \frac{a\cos(d+ex)+b\sin(d+ex)}{16b^2e(a+b\cos(d+ex)-a\sin(d+ex))^2} - \frac{3\left(a^2\cos(d+ex)+\right.}{16b^4e(a+b\cos(d+e}$$

$$= \frac{a\cos(d+ex)+b\sin(d+ex)}{16b^2e(a+b\cos(d+ex)-a\sin(d+ex))^2} - \frac{3\left(a^2\cos(d+ex)+\right.}{16b^4e(a+b\cos(d+e}$$

$$= \frac{\left(3a^2+b^2\right)\log\left(a+b\tan\left(\frac{d}{2}+\frac{\pi}{4}+\frac{ex}{2}\right)\right)}{16b^5e} + \frac{a\cos(d+ex)+}{16b^2e(a+b\cos(d+ex}$$

Mathematica [A] time = 2.77, size = 261, normalized size = 1.84

$$-\frac{\frac{b^2\left(a^2+b^2\right)}{\left((b-a)\sin\left(\frac{1}{2}(d+ex)\right)+(a+b)\cos\left(\frac{1}{2}(d+ex)\right)\right)^2} + \frac{6ab\left(a^2+b^2\right)\sin\left(\frac{1}{2}(d+ex)\right)}{(a+b)\left((b-a)\sin\left(\frac{1}{2}(d+ex)\right)+(a+b)\cos\left(\frac{1}{2}(d+ex)\right)\right)} + 2\left(3a^2+b^2\right)\log\left(\cos\left(\frac{1}{2}(d+ex)\right) - \right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(2*a + 2*b*Cos[d + e*x] - 2*a*Sin[d + e*x])^(-3),x]
```

```
[Out] -1/32*(2*(3*a^2 + b^2)*Log[Cos[(d + e*x)/2] - Sin[(d + e*x)/2]] - 2*(3*a^2
+ b^2)*Log[(a + b)*Cos[(d + e*x)/2] + (-a + b)*Sin[(d + e*x)/2]] - b^2/(Cos
```

```
[(d + e*x)/2] - Sin[(d + e*x)/2])^2 + (6*a*b*Sin[(d + e*x)/2])/(Cos[(d + e*
x)/2] - Sin[(d + e*x)/2]) + (b^2*(a^2 + b^2))/((a + b)*Cos[(d + e*x)/2] + (
-a + b)*Sin[(d + e*x)/2])^2 + (6*a*b*(a^2 + b^2)*Sin[(d + e*x)/2])/((a + b)
*((a + b)*Cos[(d + e*x)/2] + (-a + b)*Sin[(d + e*x)/2])))/(b^5*e)
```

fricas [B] time = 1.51, size = 423, normalized size = 2.98

$$-\frac{12\,a^2b^2\cos(ex+d)^2 - 6\,a^2b^2 + 2\left(3\,a^3b - ab^3\right)\cos(ex+d) - \left(6\,a^4 + 2\,a^2b^2 - \left(3\,a^4 - 2\,a^2b^2 - b^4\right)\cos(ex+d)^2\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*a+2*b*cos(e*x+d)-2*a*sin(e*x+d))^3,x, algorithm="fricas")
```

```
[Out] -1/32*(12*a^2*b^2*cos(e*x + d)^2 - 6*a^2*b^2 + 2*(3*a^3*b - a*b^3)*cos(e*x
+ d) - (6*a^4 + 2*a^2*b^2 - (3*a^4 - 2*a^2*b^2 - b^4)*cos(e*x + d)^2 + 2*(3
*a^3*b + a*b^3)*cos(e*x + d) - 2*(3*a^4 + a^2*b^2 + (3*a^3*b + a*b^3)*cos(e
*x + d))*sin(e*x + d))*log(2*a*b*cos(e*x + d) + a^2 + b^2 - (a^2 - b^2)*sin
(e*x + d)) + (6*a^4 + 2*a^2*b^2 - (3*a^4 - 2*a^2*b^2 - b^4)*cos(e*x + d)^2
+ 2*(3*a^3*b + a*b^3)*cos(e*x + d) - 2*(3*a^4 + a^2*b^2 + (3*a^3*b + a*b^3)
*cos(e*x + d))*sin(e*x + d))*log(-sin(e*x + d) + 1) + 2*(3*a^2*b^2 - b^4 -
3*(a^3*b - a*b^3)*cos(e*x + d))*sin(e*x + d))/(2*a*b^6*e*cos(e*x + d) + 2*a
^2*b^5*e - (a^2*b^5 - b^7)*e*cos(e*x + d)^2 - 2*(a*b^6*e*cos(e*x + d) + a^2
*b^5*e)*sin(e*x + d))
```

giac [B] time = 0.24, size = 481, normalized size = 3.39

$$\frac{1}{16}\left(\frac{2\left(3\,a^5\tan\left(\frac{1}{2}\,xe + \frac{1}{2}\,d\right)^3 - 9\,a^4b\tan\left(\frac{1}{2}\,xe + \frac{1}{2}\,d\right)^3 + 10\,a^3b^2\tan\left(\frac{1}{2}\,xe + \frac{1}{2}\,d\right)^3 - 6\,a^2b^3\tan\left(\frac{1}{2}\,xe + \frac{1}{2}\,d\right)^3 + ab\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*a+2*b*cos(e*x+d)-2*a*sin(e*x+d))^3,x, algorithm="giac")
```

```
[Out] 1/16*(2*(3*a^5*tan(1/2*x*e + 1/2*d)^3 - 9*a^4*b*tan(1/2*x*e + 1/2*d)^3 + 10
*a^3*b^2*tan(1/2*x*e + 1/2*d)^3 - 6*a^2*b^3*tan(1/2*x*e + 1/2*d)^3 + a*b^4*
tan(1/2*x*e + 1/2*d)^3 + b^5*tan(1/2*x*e + 1/2*d)^3 - 9*a^5*tan(1/2*x*e + 1
/2*d)^2 + 18*a^4*b*tan(1/2*x*e + 1/2*d)^2 - 12*a^3*b^2*tan(1/2*x*e + 1/2*d)
^2 + 6*a^2*b^3*tan(1/2*x*e + 1/2*d)^2 - a*b^4*tan(1/2*x*e + 1/2*d)^2 + 9*a^
5*tan(1/2*x*e + 1/2*d) - 9*a^4*b*tan(1/2*x*e + 1/2*d) - 2*a^3*b^2*tan(1/2*x
*e + 1/2*d) + 2*a^2*b^3*tan(1/2*x*e + 1/2*d) - 5*a*b^4*tan(1/2*x*e + 1/2*d)
 + b^5*tan(1/2*x*e + 1/2*d) - 3*a^5 + 4*a^3*b^2 + a*b^4)/((a^2*b^4 - 2*a*b^
```

```
5 + b^6)*(a*tan(1/2*x*e + 1/2*d)^2 - b*tan(1/2*x*e + 1/2*d)^2 - 2*a*tan(1/2
*x*e + 1/2*d) + a + b)^2) + (3*a^2 + b^2)*log(abs(2*a*tan(1/2*x*e + 1/2*d)
- 2*b*tan(1/2*x*e + 1/2*d) - 2*a - 2*abs(b))/abs(2*a*tan(1/2*x*e + 1/2*d) -
 2*b*tan(1/2*x*e + 1/2*d) - 2*a + 2*abs(b)))/(b^4*abs(b)))*e^(-1)
```

maple [B] time = 0.53, size = 687, normalized size = 4.84

$$\frac{3\ln\left(a\tan\left(\frac{d}{2}+\frac{ex}{2}\right)-b\tan\left(\frac{d}{2}+\frac{ex}{2}\right)-a-b\right)a^{3}}{16e\,b^{5}\left(a-b\right)}-\frac{3\ln\left(a\tan\left(\frac{d}{2}+\frac{ex}{2}\right)-b\tan\left(\frac{d}{2}+\frac{ex}{2}\right)-a-b\right)a^{2}}{16e\,b^{4}\left(a-b\right)}+\frac{\ln\left(a\tan\left(\frac{d}{2}+\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2*a+2*b*cos(e*x+d)-2*a*sin(e*x+d))^3,x)
```

```
[Out] 3/16/e/b^5/(a-b)*ln(a*tan(1/2*d+1/2*e*x)-b*tan(1/2*d+1/2*e*x)-a-b)*a^3-3/16
/e/b^4/(a-b)*ln(a*tan(1/2*d+1/2*e*x)-b*tan(1/2*d+1/2*e*x)-a-b)*a^2+1/16/e/b
^3/(a-b)*ln(a*tan(1/2*d+1/2*e*x)-b*tan(1/2*d+1/2*e*x)-a-b)*a-1/16/e/b^2/(a-
b)*ln(a*tan(1/2*d+1/2*e*x)-b*tan(1/2*d+1/2*e*x)-a-b)-1/16/e/b^3/(a-b)^2/(a*
tan(1/2*d+1/2*e*x)-b*tan(1/2*d+1/2*e*x)-a-b)^2*a^4-1/8/e/b/(a-b)^2/(a*tan(1
/2*d+1/2*e*x)-b*tan(1/2*d+1/2*e*x)-a-b)^2*a^2-1/16/e*b/(a-b)^2/(a*tan(1/2*d
+1/2*e*x)-b*tan(1/2*d+1/2*e*x)-a-b)^2+3/16/e/b^4/(a-b)^2/(a*tan(1/2*d+1/2*e
*x)-b*tan(1/2*d+1/2*e*x)-a-b)*a^4-1/4/e/b^3/(a-b)^2/(a*tan(1/2*d+1/2*e*x)-b
*tan(1/2*d+1/2*e*x)-a-b)*a^3+1/8/e/b^2/(a-b)^2/(a*tan(1/2*d+1/2*e*x)-b*tan(
1/2*d+1/2*e*x)-a-b)*a^2-1/4/e/b/(a-b)^2/(a*tan(1/2*d+1/2*e*x)-b*tan(1/2*d+1
/2*e*x)-a-b)*a-1/16/e/(a-b)^2/(a*tan(1/2*d+1/2*e*x)-b*tan(1/2*d+1/2*e*x)-a-
b)+1/16/e/b^3/(tan(1/2*d+1/2*e*x)-1)^2+3/16/e/b^4/(tan(1/2*d+1/2*e*x)-1)*a+
1/16/e/b^3/(tan(1/2*d+1/2*e*x)-1)-3/16/e/b^5*ln(tan(1/2*d+1/2*e*x)-1)*a^2-1
/16/e/b^3*ln(tan(1/2*d+1/2*e*x)-1)
```

maxima [B] time = 0.36, size = 491, normalized size = 3.46

$$-\frac{\frac{2\left(3a^{5}-4a^{3}b^{2}-ab^{4}-\frac{\left(9a^{5}-9a^{4}b-2a^{3}b^{2}+2a^{2}b^{3}-5ab^{4}+b^{5}\right)\sin(ex+d)}{\cos(ex+d)+1}+\frac{\left(9a^{5}-18a^{4}b+12a^{3}b^{2}-6a^{2}b^{3}+ab^{4}\right)\sin(ex+d)^{2}}{(\cos(ex+d)+1)^{2}}-\frac{\left(3a^{5}-9a^{4}b+10a^{3}b^{2}-6a^{2}b^{3}+ab^{4}+b^{5}\right)\sin}{(\cos(ex+d)+1)^{3}}\right.}{a^{4}b^{4}-2a^{2}b^{6}+b^{8}-\frac{4\left(a^{4}b^{4}-a^{3}b^{5}-a^{2}b^{6}+ab^{7}\right)\sin(ex+d)}{\cos(ex+d)+1}+\frac{2\left(3a^{4}b^{4}-6a^{3}b^{5}+2a^{2}b^{6}+2ab^{7}-b^{8}\right)\sin(ex+d)^{2}}{(\cos(ex+d)+1)^{2}}-\frac{4\left(a^{4}b^{4}-3a^{3}b^{5}+3a^{2}b^{6}-ab^{7}\right)\sin(ex+d)^{3}}{(\cos(ex+d)+1)^{3}}+\frac{\left(a^{4}b^{4}-4a^{3}b^{5}+6a^{2}b^{6}-\right.}{(\cos(ex+}}}{16\,e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*a+2*b*cos(e*x+d)-2*a*sin(e*x+d))^3,x, algorithm="maxima")
```

```
[Out] -1/16*(2*(3*a^5 - 4*a^3*b^2 - a*b^4 - (9*a^5 - 9*a^4*b - 2*a^3*b^2 + 2*a^2*
b^3 - 5*a*b^4 + b^5)*sin(e*x + d)/(cos(e*x + d) + 1) + (9*a^5 - 18*a^4*b +
12*a^3*b^2 - 6*a^2*b^3 + a*b^4)*sin(e*x + d)^2/(cos(e*x + d) + 1)^2 - (3*a^
5 - 9*a^4*b + 10*a^3*b^2 - 6*a^2*b^3 + a*b^4 + b^5)*sin(e*x + d)^3/(cos(e*x
```

```
 + d) + 1)^3)/(a^4*b^4 - 2*a^2*b^6 + b^8 - 4*(a^4*b^4 - a^3*b^5 - a^2*b^6 +
 a*b^7)*sin(e*x + d)/(cos(e*x + d) + 1) + 2*(3*a^4*b^4 - 6*a^3*b^5 + 2*a^2*
b^6 + 2*a*b^7 - b^8)*sin(e*x + d)^2/(cos(e*x + d) + 1)^2 - 4*(a^4*b^4 - 3*a
^3*b^5 + 3*a^2*b^6 - a*b^7)*sin(e*x + d)^3/(cos(e*x + d) + 1)^3 + (a^4*b^4
- 4*a^3*b^5 + 6*a^2*b^6 - 4*a*b^7 + b^8)*sin(e*x + d)^4/(cos(e*x + d) + 1)^
4) - (3*a^2 + b^2)*log(a + b - (a - b)*sin(e*x + d)/(cos(e*x + d) + 1))/b^5
 + (3*a^2 + b^2)*log(sin(e*x + d)/(cos(e*x + d) + 1) - 1)/b^5)/e
```

mupad [B] time = 6.48, size = 361, normalized size = 2.54

$$\frac{\operatorname{atanh}\left(\frac{2\,a-\tan\left(\frac{d}{2}+\frac{e\,x}{2}\right)\,(2\,a-2\,b)}{2\,b}\right)\,\left(3\,a^2+b^2\right)}{8\,b^5\,e}-\frac{\frac{\tan\left(\frac{d}{2}+\frac{e\,x}{2}\right)\,\left(-9\,a^5+9\,a^4\,b+2\,a^3\,b^2-2\,a^2\,b^3+5\,a\,b^4-b^5\right)}{2\,b^4\,(a-b)^2}-\frac{-3\,a^5+4\,a^3\,b^2+a\,b^4}{2\,b^4\,(a-b)^2}+\cdots}{e\,\left(8\,a\,b+\tan\left(\frac{d}{2}+\frac{e\,x}{2}\right)^2\,\left(24\,a^2-8\,b^2\right)+\tan\left(\frac{d}{2}+\frac{e\,x}{2}\right)^3\,\left(16\,a\,b-16\,a^2\cdots\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2*a + 2*b*cos(d + e*x) - 2*a*sin(d + e*x))^3,x)
```

```
[Out] (atanh((2*a - tan(d/2 + (e*x)/2)*(2*a - 2*b))/(2*b))*(3*a^2 + b^2))/(8*b^5*
e) - ((tan(d/2 + (e*x)/2)*(5*a*b^4 + 9*a^4*b - 9*a^5 - b^5 - 2*a^2*b^3 + 2*
a^3*b^2))/(2*b^4*(a - b)^2) - (a*b^4 - 3*a^5 + 4*a^3*b^2)/(2*b^4*(a - b)^2)
 + (tan(d/2 + (e*x)/2)^3*(2*a*b^3 + 6*a^3*b - 3*a^4 + b^4 - 4*a^2*b^2))/(2*
b^4*(a - b)) + (tan(d/2 + (e*x)/2)^2*(a*b^4 - 18*a^4*b + 9*a^5 - 6*a^2*b^3
+ 12*a^3*b^2))/(2*b^4*(a - b)^2))/(e*(8*a*b + tan(d/2 + (e*x)/2)^2*(24*a^2
- 8*b^2) + tan(d/2 + (e*x)/2)^3*(16*a*b - 16*a^2) + tan(d/2 + (e*x)/2)^4*(4
*a^2 - 8*a*b + 4*b^2) + 4*a^2 + 4*b^2 - tan(d/2 + (e*x)/2)*(16*a*b + 16*a^2
)))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*a+2*b*cos(e*x+d)-2*a*sin(e*x+d))**3,x)
```

```
[Out] Timed out
```

3.394 $\int \frac{1}{(2a+2b\cos(d+ex)-2a\sin(d+ex))^4}\,dx$

Optimal. Leaf size=215

$$-\frac{5\left(a^2\cos(d+ex)+ab\sin(d+ex)\right)}{96b^4e(a(-\sin(d+ex))+a+b\cos(d+ex))^2}-\frac{a\left(5a^2+3b^2\right)\log\left(a+b\tan\left(\frac{d}{2}+\frac{ex}{2}+\frac{\pi}{4}\right)\right)}{32b^7e}+\frac{b\left(15a^2+4b^2\right)\sin(d}{96b^6e(a(-\sin(}$$

```
[Out] -1/32*a*(5*a^2+3*b^2)*ln(a+b*tan(1/2*d+1/4*Pi+1/2*e*x))/b^7/e+1/48*(a*cos(e
*x+d)+b*sin(e*x+d))/b^2/e/(a+b*cos(e*x+d)-a*sin(e*x+d))^3-5/96*(a^2*cos(e*x
+d)+a*b*sin(e*x+d))/b^4/e/(a+b*cos(e*x+d)-a*sin(e*x+d))^2+1/96*(a*(15*a^2+4
*b^2)*cos(e*x+d)+b*(15*a^2+4*b^2)*sin(e*x+d))/b^6/e/(a+b*cos(e*x+d)-a*sin(e
*x+d))
```

Rubi [A] time = 0.24, antiderivative size = 215, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {3129, 3156, 3153, 3122, 31}

$$-\frac{a\left(5a^2+3b^2\right)\log\left(a+b\tan\left(\frac{d}{2}+\frac{ex}{2}+\frac{\pi}{4}\right)\right)}{32b^7e}-\frac{5\left(a^2\cos(d+ex)+ab\sin(d+ex)\right)}{96b^4e(a(-\sin(d+ex))+a+b\cos(d+ex))^2}+\frac{b\left(15a^2+4b^2\right)\sin(d}{96b^6e(a(-\sin(}$$

Antiderivative was successfully verified.

```
[In] Int[(2*a + 2*b*Cos[d + e*x] - 2*a*Sin[d + e*x])^(-4),x]
```

```
[Out] -(a*(5*a^2 + 3*b^2)*Log[a + b*Tan[d/2 + Pi/4 + (e*x)/2]])/(32*b^7*e) + (a*C
os[d + e*x] + b*Sin[d + e*x])/(48*b^2*e*(a + b*Cos[d + e*x] - a*Sin[d + e*x
])^3) - (5*(a^2*Cos[d + e*x] + a*b*Sin[d + e*x]))/(96*b^4*e*(a + b*Cos[d +
e*x] - a*Sin[d + e*x])^2) + (a*(15*a^2 + 4*b^2)*Cos[d + e*x] + b*(15*a^2 +
4*b^2)*Sin[d + e*x])/(96*b^6*e*(a + b*Cos[d + e*x] - a*Sin[d + e*x]))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 3122

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(-1), x_Symbol] :> Module[{f = FreeFactors[Tan[(d + e*x)/2 + Pi/4], x]}, Di
st[f/e, Subst[Int[1/(a + b*f*x), x], x, Tan[(d + e*x)/2 + Pi/4]/f], x]] /;
FreeQ[{a, b, c, d, e}, x] && EqQ[a + c, 0]
```

Rule 3129

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(n_), x_Symbol] :> Simp[((-(c*Cos[d + e*x]) + b*Sin[d + e*x])*(a + b*Cos[d
+ e*x] + c*Sin[d + e*x])^(n + 1))/(e*(n + 1)*(a^2 - b^2 - c^2)), x] + Dist[
1/((n + 1)*(a^2 - b^2 - c^2)), Int[(a*(n + 1) - b*(n + 2)*Cos[d + e*x] - c*
(n + 2)*Sin[d + e*x])*(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n + 1), x], x]
 /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0] && LtQ[n, -1] && N
eQ[n, -3/2]
```

Rule 3153

```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])
/((a_.) + cos[(d_.) + (e_.)*(x_)]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)])^2,
 x_Symbol] :> Simp[(c*B - b*C - (a*C - c*A)*Cos[d + e*x] + (a*B - b*A)*Sin[
d + e*x])/(e*(a^2 - b^2 - c^2)*(a + b*Cos[d + e*x] + c*Sin[d + e*x])), x] +
 Dist[(a*A - b*B - c*C)/(a^2 - b^2 - c^2), Int[1/(a + b*Cos[d + e*x] + c*Si
n[d + e*x]), x], x] /; FreeQ[{a, b, c, d, e, A, B, C}, x] && NeQ[a^2 - b^2
- c^2, 0] && NeQ[a*A - b*B - c*C, 0]
```

Rule 3156

```
Int[((a_.) + cos[(d_.) + (e_.)*(x_)]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)])
^(n_)*((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)
]), x_Symbol] :> -Simp[((c*B - b*C - (a*C - c*A)*Cos[d + e*x] + (a*B - b*A)
*Sin[d + e*x])*(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n + 1))/(e*(n + 1)*(a
^2 - b^2 - c^2)), x] + Dist[1/((n + 1)*(a^2 - b^2 - c^2)), Int[(a + b*Cos[d
 + e*x] + c*Sin[d + e*x])^(n + 1)*Simp[(n + 1)*(a*A - b*B - c*C) + (n + 2)*
(a*B - b*A)*Cos[d + e*x] + (n + 2)*(a*C - c*A)*Sin[d + e*x], x], x], x] /;
FreeQ[{a, b, c, d, e, A, B, C}, x] && LtQ[n, -1] && NeQ[a^2 - b^2 - c^2, 0]
 && NeQ[n, -2]
```

Rubi steps

$$\int \frac{1}{(2a+2b\cos(d+ex)-2a\sin(d+ex))^4}\,dx = \frac{a\cos(d+ex)+b\sin(d+ex)}{48b^2e(a+b\cos(d+ex)-a\sin(d+ex))^3} + \frac{\int \frac{-6a+4b\cos(d+ex)-4a\sin}{(2a+2b\cos(d+ex)-2a\sin}}{12b^2}$$

$$= \frac{a\cos(d+ex)+b\sin(d+ex)}{48b^2e(a+b\cos(d+ex)-a\sin(d+ex))^3} - \frac{5\left(a^2\cos(d+ex)\right.}{96b^4e(a+b\cos(d+}$$

$$= \frac{a\cos(d+ex)+b\sin(d+ex)}{48b^2e(a+b\cos(d+ex)-a\sin(d+ex))^3} - \frac{5\left(a^2\cos(d+ex)\right.}{96b^4e(a+b\cos(d+}$$

$$= \frac{a\cos(d+ex)+b\sin(d+ex)}{48b^2e(a+b\cos(d+ex)-a\sin(d+ex))^3} - \frac{5\left(a^2\cos(d+ex)\right.}{96b^4e(a+b\cos(d+}$$

$$= -\frac{a\left(5a^2+3b^2\right)\log\left(a+b\tan\left(\frac{d}{2}+\frac{\pi}{4}+\frac{ex}{2}\right)\right)}{32b^7e} + \frac{a\cos(d+e}{48b^2e(a+b\cos(d}$$

Mathematica [B] time = 1.92, size = 636, normalized size = 2.96

$$12a\left(5a^2+3b^2\right)\log\left(\cos\left(\tfrac{1}{2}(d+ex)\right)-\sin\left(\tfrac{1}{2}(d+ex)\right)\right)-12a\left(5a^2+3b^2\right)\log\left((b-a)\sin\left(\tfrac{1}{2}(d+ex)\right)+(a+b)\mathrm{co}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(2*a + 2*b*Cos[d + e*x] - 2*a*Sin[d + e*x])^(-4),x]
```

```
[Out] (12*a*(5*a^2 + 3*b^2)*Log[Cos[(d + e*x)/2] - Sin[(d + e*x)/2]] - 12*a*(5*a^
2 + 3*b^2)*Log[(a + b)*Cos[(d + e*x)/2] + (-a + b)*Sin[(d + e*x)/2]] + (b*(
-150*a^6 - 130*a^4*b^2 - 24*a^2*b^4 + 3*a^2*(25*a^4 - 50*a^3*b + 5*a^2*b^2
- 30*a*b^3 + 4*b^4)*Cos[d + e*x] + 6*a^2*(15*a^4 + 20*a^3*b + 9*a^2*b^2 + 2
*a*b^3 - 2*b^4)*Cos[2*(d + e*x)] - 15*a^6*Cos[3*(d + e*x)] + 30*a^5*b*Cos[3
*(d + e*x)] + 41*a^4*b^2*Cos[3*(d + e*x)] + 38*a^3*b^3*Cos[3*(d + e*x)] + 1
2*a^2*b^4*Cos[3*(d + e*x)] + 8*a*b^5*Cos[3*(d + e*x)] + 225*a^6*Sin[d + e*x
] + 75*a^5*b*Sin[d + e*x] + 180*a^4*b^2*Sin[d + e*x] + 15*a^3*b^3*Sin[d + e
*x] + 27*a^2*b^4*Sin[d + e*x] + 12*a*b^5*Sin[d + e*x] + 12*b^6*Sin[d + e*x]
 - 60*a^6*Sin[2*(d + e*x)] + 120*a^5*b*Sin[2*(d + e*x)] + 54*a^4*b^2*Sin[2*
(d + e*x)] + 102*a^3*b^3*Sin[2*(d + e*x)] + 6*a^2*b^4*Sin[2*(d + e*x)] + 6*
a*b^5*Sin[2*(d + e*x)] - 15*a^6*Sin[3*(d + e*x)] - 45*a^5*b*Sin[3*(d + e*x)
] - 4*a^4*b^2*Sin[3*(d + e*x)] + 3*a^3*b^3*Sin[3*(d + e*x)] + 15*a^2*b^4*Si
n[3*(d + e*x)] + 4*a*b^5*Sin[3*(d + e*x)] + 4*b^6*Sin[3*(d + e*x)]))/((a +
b)*(Cos[(d + e*x)/2] - Sin[(d + e*x)/2])^3*((a + b)*Cos[(d + e*x)/2] + (-a
+ b)*Sin[(d + e*x)/2])^3))/(384*b^7*e)
```

fricas [B] time = 2.00, size = 735, normalized size = 3.42

$$-\frac{60\,a^4b^2+6\,a^2b^4+2\left(15\,a^5b-41\,a^3b^3-12\,ab^5\right)\cos\left(ex+d\right)^3-12\left(10\,a^4b^2+a^2b^4\right)\cos\left(ex+d\right)^2-6\left(10\,a^5b-\cdots\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*a+2*b*cos(e*x+d)-2*a*sin(e*x+d))^4,x, algorithm="fricas")
```

```
[Out] -1/192*(60*a^4*b^2 + 6*a^2*b^4 + 2*(15*a^5*b - 41*a^3*b^3 - 12*a*b^5)*cos(e
*x + d)^3 - 12*(10*a^4*b^2 + a^2*b^4)*cos(e*x + d)^2 - 6*(10*a^5*b - 9*a^3*
b^3 - 2*a*b^5)*cos(e*x + d) + 3*(20*a^6 + 12*a^4*b^2 - (15*a^5*b + 4*a^3*b^
3 - 3*a*b^5)*cos(e*x + d)^3 - 3*(5*a^6 - 2*a^4*b^2 - 3*a^2*b^4)*cos(e*x + d
)^2 + 6*(5*a^5*b + 3*a^3*b^3)*cos(e*x + d) - (20*a^6 + 12*a^4*b^2 - (5*a^6
- 12*a^4*b^2 - 9*a^2*b^4)*cos(e*x + d)^2 + 6*(5*a^5*b + 3*a^3*b^3)*cos(e*x
+ d))*sin(e*x + d))*log(2*a*b*cos(e*x + d) + a^2 + b^2 - (a^2 - b^2)*sin(e*
x + d)) - 3*(20*a^6 + 12*a^4*b^2 - (15*a^5*b + 4*a^3*b^3 - 3*a*b^5)*cos(e*x
 + d)^3 - 3*(5*a^6 - 2*a^4*b^2 - 3*a^2*b^4)*cos(e*x + d)^2 + 6*(5*a^5*b + 3
*a^3*b^3)*cos(e*x + d) - (20*a^6 + 12*a^4*b^2 - (5*a^6 - 12*a^4*b^2 - 9*a^2
*b^4)*cos(e*x + d)^2 + 6*(5*a^5*b + 3*a^3*b^3)*cos(e*x + d))*sin(e*x + d))*
log(-sin(e*x + d) + 1) - 2*(30*a^4*b^2 + 3*a^2*b^4 + 2*b^6 - (45*a^4*b^2 -
3*a^2*b^4 - 4*b^6)*cos(e*x + d)^2 - 3*(10*a^5*b - 9*a^3*b^3 - a*b^5)*cos(e*
x + d))*sin(e*x + d))/(6*a^2*b^8*e*cos(e*x + d) + 4*a^3*b^7*e - (3*a^2*b^8
- b^10)*e*cos(e*x + d)^3 - 3*(a^3*b^7 - a*b^9)*e*cos(e*x + d)^2 - (6*a^2*b^
8*e*cos(e*x + d) + 4*a^3*b^7*e - (a^3*b^7 - 3*a*b^9)*e*cos(e*x + d)^2)*sin(
e*x + d))
```

giac [B] time = 0.28, size = 1006, normalized size = 4.68

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*a+2*b*cos(e*x+d)-2*a*sin(e*x+d))^4,x, algorithm="giac")
```

```
[Out] -1/96*(2*(15*a^8*tan(1/2*x*e + 1/2*d)^5 - 75*a^7*b*tan(1/2*x*e + 1/2*d)^5 +
 159*a^6*b^2*tan(1/2*x*e + 1/2*d)^5 - 195*a^5*b^3*tan(1/2*x*e + 1/2*d)^5 +
165*a^4*b^4*tan(1/2*x*e + 1/2*d)^5 - 105*a^3*b^5*tan(1/2*x*e + 1/2*d)^5 + 5
1*a^2*b^6*tan(1/2*x*e + 1/2*d)^5 - 21*a*b^7*tan(1/2*x*e + 1/2*d)^5 + 6*b^8*
tan(1/2*x*e + 1/2*d)^5 - 75*a^8*tan(1/2*x*e + 1/2*d)^4 + 300*a^7*b*tan(1/2*
x*e + 1/2*d)^4 - 495*a^6*b^2*tan(1/2*x*e + 1/2*d)^4 + 480*a^5*b^3*tan(1/2*x
*e + 1/2*d)^4 - 345*a^4*b^4*tan(1/2*x*e + 1/2*d)^4 + 180*a^3*b^5*tan(1/2*x*
e + 1/2*d)^4 - 57*a^2*b^6*tan(1/2*x*e + 1/2*d)^4 + 12*a*b^7*tan(1/2*x*e + 1
/2*d)^4 + 150*a^8*tan(1/2*x*e + 1/2*d)^3 - 450*a^7*b*tan(1/2*x*e + 1/2*d)^3
 + 500*a^6*b^2*tan(1/2*x*e + 1/2*d)^3 - 300*a^5*b^3*tan(1/2*x*e + 1/2*d)^3
```

```
+ 126*a^4*b^4*tan(1/2*x*e + 1/2*d)^3 + 22*a^3*b^5*tan(1/2*x*e + 1/2*d)^3 -
48*a^2*b^6*tan(1/2*x*e + 1/2*d)^3 + 12*a*b^7*tan(1/2*x*e + 1/2*d)^3 - 4*b^8
*tan(1/2*x*e + 1/2*d)^3 - 150*a^8*tan(1/2*x*e + 1/2*d)^2 + 300*a^7*b*tan(1/
2*x*e + 1/2*d)^2 - 120*a^6*b^2*tan(1/2*x*e + 1/2*d)^2 - 60*a^5*b^3*tan(1/2*
x*e + 1/2*d)^2 + 102*a^4*b^4*tan(1/2*x*e + 1/2*d)^2 - 144*a^3*b^5*tan(1/2*x
*e + 1/2*d)^2 + 60*a^2*b^6*tan(1/2*x*e + 1/2*d)^2 - 12*a*b^7*tan(1/2*x*e +
1/2*d)^2 + 75*a^8*tan(1/2*x*e + 1/2*d) - 75*a^7*b*tan(1/2*x*e + 1/2*d) - 75
*a^6*b^2*tan(1/2*x*e + 1/2*d) + 75*a^5*b^3*tan(1/2*x*e + 1/2*d) - 39*a^4*b^
4*tan(1/2*x*e + 1/2*d) + 39*a^3*b^5*tan(1/2*x*e + 1/2*d) + 33*a^2*b^6*tan(1
/2*x*e + 1/2*d) - 15*a*b^7*tan(1/2*x*e + 1/2*d) + 6*b^8*tan(1/2*x*e + 1/2*d
) - 15*a^8 + 31*a^6*b^2 - 9*a^4*b^4 - 15*a^2*b^6)/((a^3*b^6 - 3*a^2*b^7 + 3
*a*b^8 - b^9)*(a*tan(1/2*x*e + 1/2*d)^2 - b*tan(1/2*x*e + 1/2*d)^2 - 2*a*ta
n(1/2*x*e + 1/2*d) + a + b)^3) + 3*(5*a^3 + 3*a*b^2)*log(abs(2*a*tan(1/2*x*
e + 1/2*d) - 2*b*tan(1/2*x*e + 1/2*d) - 2*a - 2*abs(b))/abs(2*a*tan(1/2*x*e
 + 1/2*d) - 2*b*tan(1/2*x*e + 1/2*d) - 2*a + 2*abs(b)))/(b^6*abs(b)))*e^(-1
)
```

maple [B] time = 0.61, size = 1149, normalized size = 5.34

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2*a+2*b*cos(e*x+d)-2*a*sin(e*x+d))^4,x)
```

```
[Out] 1/16/e/b^5/(a-b)^3/(a*tan(1/2*d+1/2*e*x)-b*tan(1/2*d+1/2*e*x)-a-b)^2*a^6-3/
32/e/b^4/(a-b)^3/(a*tan(1/2*d+1/2*e*x)-b*tan(1/2*d+1/2*e*x)-a-b)^2*a^5+3/32
/e/b^3/(a-b)^3/(a*tan(1/2*d+1/2*e*x)-b*tan(1/2*d+1/2*e*x)-a-b)^2*a^4-3/16/e
/b^2/(a-b)^3/(a*tan(1/2*d+1/2*e*x)-b*tan(1/2*d+1/2*e*x)-a-b)^2*a^3-5/32/e/b
^6/(a-b)^3/(a*tan(1/2*d+1/2*e*x)-b*tan(1/2*d+1/2*e*x)-a-b)*a^6+3/8/e/b^5/(a
-b)^3/(a*tan(1/2*d+1/2*e*x)-b*tan(1/2*d+1/2*e*x)-a-b)*a^5-3/8/e/b^4/(a-b)^3
/(a*tan(1/2*d+1/2*e*x)-b*tan(1/2*d+1/2*e*x)-a-b)*a^4+3/8/e/b^3/(a-b)^3/(a*t
an(1/2*d+1/2*e*x)-b*tan(1/2*d+1/2*e*x)-a-b)*a^3-9/32/e/b^2/(a-b)^3/(a*tan(1
/2*d+1/2*e*x)-b*tan(1/2*d+1/2*e*x)-a-b)*a^2-5/32/e*a^4/b^7/(a-b)*ln(a*tan(1
/2*d+1/2*e*x)-b*tan(1/2*d+1/2*e*x)-a-b)+5/32/e*a^3/b^6/(a-b)*ln(a*tan(1/2*d
+1/2*e*x)-b*tan(1/2*d+1/2*e*x)-a-b)-3/32/e*a^2/b^5/(a-b)*ln(a*tan(1/2*d+1/2
*e*x)-b*tan(1/2*d+1/2*e*x)-a-b)+3/32/e*a/b^4/(a-b)*ln(a*tan(1/2*d+1/2*e*x)-
b*tan(1/2*d+1/2*e*x)-a-b)-1/16/e/(a-b)^3/(a*tan(1/2*d+1/2*e*x)-b*tan(1/2*d+
1/2*e*x)-a-b)-1/48/e/b^4/(tan(1/2*d+1/2*e*x)-1)^3-1/32/e/b^4/(tan(1/2*d+1/2
*e*x)-1)^2-1/16/e/b^4/(tan(1/2*d+1/2*e*x)-1)-3/32/e/(a-b)^3/(a*tan(1/2*d+1/
2*e*x)-b*tan(1/2*d+1/2*e*x)-a-b)^2*a-1/48/e*b^2/(a-b)^3/(a*tan(1/2*d+1/2*e*
x)-b*tan(1/2*d+1/2*e*x)-a-b)^3-1/32/e*b/(a-b)^3/(a*tan(1/2*d+1/2*e*x)-b*tan
(1/2*d+1/2*e*x)-a-b)^2-1/16/e/b^5/(tan(1/2*d+1/2*e*x)-1)^2*a-5/32/e/b^6/(ta
n(1/2*d+1/2*e*x)-1)*a^2-1/16/e/b^5/(tan(1/2*d+1/2*e*x)-1)*a+5/32/e*a^3/b^7*
ln(tan(1/2*d+1/2*e*x)-1)+3/32/e*a/b^5*ln(tan(1/2*d+1/2*e*x)-1)-1/16/e/(a-b)
^3/(a*tan(1/2*d+1/2*e*x)-b*tan(1/2*d+1/2*e*x)-a-b)^3*a^2-1/48/e/b^4/(a-b)^3
```

```
/(a*tan(1/2*d+1/2*e*x)-b*tan(1/2*d+1/2*e*x)-a-b)^3*a^6-1/16/e/b^2/(a-b)^3/(
a*tan(1/2*d+1/2*e*x)-b*tan(1/2*d+1/2*e*x)-a-b)^3*a^4
```

maxima [B] time = 0.39, size = 959, normalized size = 4.46

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*a+2*b*cos(e*x+d)-2*a*sin(e*x+d))^4,x, algorithm="maxima")
```

```
[Out] 1/96*(2*(15*a^8 - 31*a^6*b^2 + 9*a^4*b^4 + 15*a^2*b^6 - 3*(25*a^8 - 25*a^7*
b - 25*a^6*b^2 + 25*a^5*b^3 - 13*a^4*b^4 + 13*a^3*b^5 + 11*a^2*b^6 - 5*a*b^
7 + 2*b^8)*sin(e*x + d)/(cos(e*x + d) + 1) + 6*(25*a^8 - 50*a^7*b + 20*a^6*
b^2 + 10*a^5*b^3 - 17*a^4*b^4 + 24*a^3*b^5 - 10*a^2*b^6 + 2*a*b^7)*sin(e*x
+ d)^2/(cos(e*x + d) + 1)^2 - 2*(75*a^8 - 225*a^7*b + 250*a^6*b^2 - 150*a^5
*b^3 + 63*a^4*b^4 + 11*a^3*b^5 - 24*a^2*b^6 + 6*a*b^7 - 2*b^8)*sin(e*x + d)
^3/(cos(e*x + d) + 1)^3 + 3*(25*a^8 - 100*a^7*b + 165*a^6*b^2 - 160*a^5*b^3
 + 115*a^4*b^4 - 60*a^3*b^5 + 19*a^2*b^6 - 4*a*b^7)*sin(e*x + d)^4/(cos(e*x
 + d) + 1)^4 - 3*(5*a^8 - 25*a^7*b + 53*a^6*b^2 - 65*a^5*b^3 + 55*a^4*b^4 -
 35*a^3*b^5 + 17*a^2*b^6 - 7*a*b^7 + 2*b^8)*sin(e*x + d)^5/(cos(e*x + d) +
1)^5)/(a^6*b^6 - 3*a^4*b^8 + 3*a^2*b^10 - b^12 - 6*(a^6*b^6 - a^5*b^7 - 2*a
^4*b^8 + 2*a^3*b^9 + a^2*b^10 - a*b^11)*sin(e*x + d)/(cos(e*x + d) + 1) + 3
*(5*a^6*b^6 - 10*a^5*b^7 - a^4*b^8 + 12*a^3*b^9 - 5*a^2*b^10 - 2*a*b^11 + b
^12)*sin(e*x + d)^2/(cos(e*x + d) + 1)^2 - 4*(5*a^6*b^6 - 15*a^5*b^7 + 12*a
^4*b^8 + 4*a^3*b^9 - 9*a^2*b^10 + 3*a*b^11)*sin(e*x + d)^3/(cos(e*x + d) +
1)^3 + 3*(5*a^6*b^6 - 20*a^5*b^7 + 29*a^4*b^8 - 16*a^3*b^9 - a^2*b^10 + 4*a
*b^11 - b^12)*sin(e*x + d)^4/(cos(e*x + d) + 1)^4 - 6*(a^6*b^6 - 5*a^5*b^7
+ 10*a^4*b^8 - 10*a^3*b^9 + 5*a^2*b^10 - a*b^11)*sin(e*x + d)^5/(cos(e*x +
d) + 1)^5 + (a^6*b^6 - 6*a^5*b^7 + 15*a^4*b^8 - 20*a^3*b^9 + 15*a^2*b^10 -
6*a*b^11 + b^12)*sin(e*x + d)^6/(cos(e*x + d) + 1)^6) - 3*(5*a^3 + 3*a*b^2)
*log(a + b - (a - b)*sin(e*x + d)/(cos(e*x + d) + 1))/b^7 + 3*(5*a^3 + 3*a*
b^2)*log(sin(e*x + d)/(cos(e*x + d) + 1) - 1)/b^7)/e
```

mupad [B] time = 7.09, size = 731, normalized size = 3.40

$$\frac{\frac{15\,a^8-31\,a^6\,b^2+9\,a^4\,b^4+15\,a^2\,b^6}{6\,b^6\,{\left(a-b\right)}^3}+\frac{{\mathrm{tan}\left(\frac{d}{2}+\frac{e\,x}{2}\right)}^2\,\left(25\,a^8-50\,a^7\,b+20\,a^6\,b^2+10\,a^5\,b^3-17\,a^4\,b^4+24\,a^3\,b^5-10\,a^2\,b^6+2\,a\,b^7\right)}{b^6\,{\left(a-b\right)}^3}+\frac{{\mathrm{tan}\left(\frac{d}{2}+\frac{e\,x}{2}\right)}^4\,\left(25\,a^7-75\,a^6}{}}{e\,\left({\mathrm{tan}\left(\frac{d}{2}+\frac{e\,x}{2}\right)}^6\,\left(8\,a^3-24\,a^2\,b+24\,a\,b^2-8\,b^3\right)-{\mathrm{tan}\left(\frac{d}{2}+\frac{e\,x}{2}\right)}^5\,\left(48\,a^3-96\,a^2\,b+48\,a\,b^2\right)-\mathrm{tan}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2*a + 2*b*cos(d + e*x) - 2*a*sin(d + e*x))^4,x)
```

```
[Out] ((15*a^8 + 15*a^2*b^6 + 9*a^4*b^4 - 31*a^6*b^2)/(6*b^6*(a - b)^3) + (tan(d/
2 + (e*x)/2)^2*(2*a*b^7 - 50*a^7*b + 25*a^8 - 10*a^2*b^6 + 24*a^3*b^5 - 17*
a^4*b^4 + 10*a^5*b^3 + 20*a^6*b^2))/(b^6*(a - b)^3) + (tan(d/2 + (e*x)/2)^4
*(4*a*b^6 - 75*a^6*b + 25*a^7 - 15*a^2*b^5 + 45*a^3*b^4 - 70*a^4*b^3 + 90*a
^5*b^2))/(2*b^6*(a - b)^2) - (tan(d/2 + (e*x)/2)^3*(6*a*b^7 - 225*a^7*b + 7
5*a^8 - 2*b^8 - 24*a^2*b^6 + 11*a^3*b^5 + 63*a^4*b^4 - 150*a^5*b^3 + 250*a^
6*b^2))/(3*b^6*(a - b)^3) - (tan(d/2 + (e*x)/2)^5*(5*a^6 - 15*a^5*b - 3*a*b
^5 + 2*b^6 + 9*a^2*b^4 - 14*a^3*b^3 + 18*a^4*b^2))/(2*b^6*(a - b)) - (tan(d
/2 + (e*x)/2)*(25*a^8 - 25*a^7*b - 5*a*b^7 + 2*b^8 + 11*a^2*b^6 + 13*a^3*b^
5 - 13*a^4*b^4 + 25*a^5*b^3 - 25*a^6*b^2))/(2*b^6*(a - b)^3))/(e*(tan(d/2 +
 (e*x)/2)^6*(24*a*b^2 - 24*a^2*b + 8*a^3 - 8*b^3) - tan(d/2 + (e*x)/2)^5*(4
8*a*b^2 - 96*a^2*b + 48*a^3) - tan(d/2 + (e*x)/2)^2*(24*a*b^2 - 120*a^2*b -
 120*a^3 + 24*b^3) - tan(d/2 + (e*x)/2)^4*(24*a*b^2 + 120*a^2*b - 120*a^3 -
 24*b^3) + 24*a*b^2 + 24*a^2*b + tan(d/2 + (e*x)/2)^3*(96*a*b^2 - 160*a^3)
- tan(d/2 + (e*x)/2)*(48*a*b^2 + 96*a^2*b + 48*a^3) + 8*a^3 + 8*b^3)) - (a*
atanh((a*(2*a - tan(d/2 + (e*x)/2)*(2*a - 2*b))*(5*a^2 + 3*b^2))/(2*b*(3*a*
b^2 + 5*a^3)))*(5*a^2 + 3*b^2))/(16*b^7*e)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*a+2*b*cos(e*x+d)-2*a*sin(e*x+d))**4,x)
```

```
[Out] Timed out
```

3.395 $\int (a+b\cos(d+ex)+c\sin(d+ex))^4\,dx$

Optimal. Leaf size=260

$$\frac{5ab\left(10a^2+11\left(b^2+c^2\right)\right)\sin(d+ex)}{24e}-\frac{5ac\left(10a^2+11\left(b^2+c^2\right)\right)\cos(d+ex)}{24e}-\frac{\left(c\left(26a^2+9\left(b^2+c^2\right)\right)\cos(d+ex)\right.}{}$$

```
[Out] 1/8*(8*a^4+24*a^2*(b^2+c^2)+3*(b^2+c^2)^2)*x-5/24*a*c*(10*a^2+11*b^2+11*c^2
)*cos(e*x+d)/e+5/24*a*b*(10*a^2+11*b^2+11*c^2)*sin(e*x+d)/e-7/12*(a*c*cos(e
*x+d)-a*b*sin(e*x+d))*(a+b*cos(e*x+d)+c*sin(e*x+d))^2/e-1/4*(c*cos(e*x+d)-b
*sin(e*x+d))*(a+b*cos(e*x+d)+c*sin(e*x+d))^3/e-1/24*(a+b*cos(e*x+d)+c*sin(e
*x+d))*(c*(26*a^2+9*b^2+9*c^2)*cos(e*x+d)-b*(26*a^2+9*b^2+9*c^2)*sin(e*x+d)
)/e
```

Rubi [A] time = 0.40, antiderivative size = 260, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {3120, 3146, 2637, 2638}

$$\frac{5ab\left(10a^2+11\left(b^2+c^2\right)\right)\sin(d+ex)}{24e}-\frac{5ac\left(10a^2+11\left(b^2+c^2\right)\right)\cos(d+ex)}{24e}-\frac{\left(c\left(26a^2+9\left(b^2+c^2\right)\right)\cos(d+ex)\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[d + e*x] + c*Sin[d + e*x])^4,x]
```

```
[Out] ((8*a^4 + 24*a^2*(b^2 + c^2) + 3*(b^2 + c^2)^2)*x)/8 - (5*a*c*(10*a^2 + 11*
(b^2 + c^2))*Cos[d + e*x])/(24*e) + (5*a*b*(10*a^2 + 11*(b^2 + c^2))*Sin[d
+ e*x])/(24*e) - (7*(a*c*Cos[d + e*x] - a*b*Sin[d + e*x])*(a + b*Cos[d + e*
x] + c*Sin[d + e*x])^2)/(12*e) - ((c*Cos[d + e*x] - b*Sin[d + e*x])*(a + b*
Cos[d + e*x] + c*Sin[d + e*x])^3)/(4*e) - ((a + b*Cos[d + e*x] + c*Sin[d +
e*x])*(c*(26*a^2 + 9*(b^2 + c^2))*Cos[d + e*x] - b*(26*a^2 + 9*(b^2 + c^2))
*Sin[d + e*x]))/(24*e)
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3120

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(n_), x_Symbol] :> -Simp[((c*Cos[d + e*x] - b*Sin[d + e*x])*(a + b*Cos[d +
e*x] + c*Sin[d + e*x])^(n - 1))/(e*n), x] + Dist[1/n, Int[Simp[n*a^2 + (n -
 1)*(b^2 + c^2) + a*b*(2*n - 1)*Cos[d + e*x] + a*c*(2*n - 1)*Sin[d + e*x],
x]*(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n - 2), x], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[a^2 - b^2 - c^2, 0] && GtQ[n, 1]
```

Rule 3146

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(n_.)*((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)
]), x_Symbol] :> Simp[((B*c - b*C - a*C*Cos[d + e*x] + a*B*Sin[d + e*x])*(a
 + b*Cos[d + e*x] + c*Sin[d + e*x])^n)/(a*e*(n + 1)), x] + Dist[1/(a*(n + 1
)), Int[(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n - 1)*Simp[a*(b*B + c*C)*n
+ a^2*A*(n + 1) + (n*(a^2*B - B*c^2 + b*c*C) + a*b*A*(n + 1))*Cos[d + e*x]
+ (n*(b*B*c + a^2*C - b^2*C) + a*c*A*(n + 1))*Sin[d + e*x], x], x], x] /; F
reeQ[{a, b, c, d, e, A, B, C}, x] && GtQ[n, 0] && NeQ[a^2 - b^2 - c^2, 0]
```

Rubi steps

$$\int (a+b\cos(d+ex)+c\sin(d+ex))^4\,dx = -\frac{(c\cos(d+ex)-b\sin(d+ex))(a+b\cos(d+ex)+c\sin(d+ex))^3}{4e} +$$

$$= -\frac{7(ac\cos(d+ex)-ab\sin(d+ex))(a+b\cos(d+ex)+c\sin(d+ex))^{2}}{12e}$$

$$= -\frac{7(ac\cos(d+ex)-ab\sin(d+ex))(a+b\cos(d+ex)+c\sin(d+ex))^{2}}{12e}$$

$$= \frac{1}{8}\left(8a^4+24a^2\left(b^2+c^2\right)+3\left(b^2+c^2\right)^2\right)x-\frac{7(ac\cos(d+ex)-ab\sin(}{}$$

$$= \frac{1}{8}\left(8a^4+24a^2\left(b^2+c^2\right)+3\left(b^2+c^2\right)^2\right)x-\frac{5ac\left(10a^2+11\left(b^2+c^2\right)\right)c}{24e}$$

Mathematica [A] time = 1.05, size = 237, normalized size = 0.91

$$\frac{96ab\left(4a^2+3\left(b^2+c^2\right)\right)\sin(d+ex)+24\left(b^2-c^2\right)\left(6a^2+b^2+c^2\right)\sin(2(d+ex))-96ac\left(4a^2+3\left(b^2+c^2\right)\right)\cos(d}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[d + e*x] + c*Sin[d + e*x])^4,x]
```

```
[Out] (12*(8*a^4 + 24*a^2*(b^2 + c^2) + 3*(b^2 + c^2)^2)*(d + e*x) - 96*a*c*(4*a^
2 + 3*(b^2 + c^2))*Cos[d + e*x] - 48*b*c*(6*a^2 + b^2 + c^2)*Cos[2*(d + e*x
)] + 32*a*c*(-3*b^2 + c^2)*Cos[3*(d + e*x)] - 12*b*c*(b^2 - c^2)*Cos[4*(d +
 e*x)] + 96*a*b*(4*a^2 + 3*(b^2 + c^2))*Sin[d + e*x] + 24*(b^2 - c^2)*(6*a^
2 + b^2 + c^2)*Sin[2*(d + e*x)] + 32*a*b*(b^2 - 3*c^2)*Sin[3*(d + e*x)] + 3
*(b^4 - 6*b^2*c^2 + c^4)*Sin[4*(d + e*x)])/(96*e)
```

fricas [A] time = 1.07, size = 255, normalized size = 0.98

$$-\frac{24\left(b^3c-bc^3\right)\cos\left(ex+d\right)^4+32\left(3\,ab^2c-ac^3\right)\cos\left(ex+d\right)^3-3\left(8\,a^4+24\,a^2b^2+3\,b^4+3\,c^4+6\left(4\,a^2+b^2\right)c^2\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(e*x+d)+c*sin(e*x+d))^4,x, algorithm="fricas")
```

```
[Out] -1/24*(24*(b^3*c - b*c^3)*cos(e*x + d)^4 + 32*(3*a*b^2*c - a*c^3)*cos(e*x +
 d)^3 - 3*(8*a^4 + 24*a^2*b^2 + 3*b^4 + 3*c^4 + 6*(4*a^2 + b^2)*c^2)*e*x +
48*(3*a^2*b*c + b*c^3)*cos(e*x + d)^2 + 96*(a^3*c + a*c^3)*cos(e*x + d) - (
96*a^3*b + 64*a*b^3 + 96*a*b*c^2 + 6*(b^4 - 6*b^2*c^2 + c^4)*cos(e*x + d)^3
 + 32*(a*b^3 - 3*a*b*c^2)*cos(e*x + d)^2 + 3*(24*a^2*b^2 + 3*b^4 - 5*c^4 -
6*(4*a^2 - b^2)*c^2)*cos(e*x + d))*sin(e*x + d))/e
```

giac [A] time = 0.30, size = 286, normalized size = 1.10

$$-\frac{1}{8}\left(b^3c-bc^3\right)\cos\left(4\,xe+4\,d\right)e^{(-1)}-\frac{1}{3}\left(3\,ab^2c-ac^3\right)\cos\left(3\,xe+3\,d\right)e^{(-1)}-\frac{1}{2}\left(6\,a^2bc+b^3c+bc^3\right)\cos\left(2\,xe+2\,d\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(e*x+d)+c*sin(e*x+d))^4,x, algorithm="giac")
```

```
[Out] -1/8*(b^3*c - b*c^3)*cos(4*x*e + 4*d)*e^(-1) - 1/3*(3*a*b^2*c - a*c^3)*cos(
3*x*e + 3*d)*e^(-1) - 1/2*(6*a^2*b*c + b^3*c + b*c^3)*cos(2*x*e + 2*d)*e^(-
1) - (4*a^3*c + 3*a*b^2*c + 3*a*c^3)*cos(x*e + d)*e^(-1) + 1/32*(b^4 - 6*b^
2*c^2 + c^4)*e^(-1)*sin(4*x*e + 4*d) + 1/3*(a*b^3 - 3*a*b*c^2)*e^(-1)*sin(3
*x*e + 3*d) + 1/4*(6*a^2*b^2 + b^4 - 6*a^2*c^2 - c^4)*e^(-1)*sin(2*x*e + 2*
d) + (4*a^3*b + 3*a*b^3 + 3*a*b*c^2)*e^(-1)*sin(x*e + d) + 1/8*(8*a^4 + 24*
a^2*b^2 + 3*b^4 + 24*a^2*c^2 + 6*b^2*c^2 + 3*c^4)*x
```

maple [A] time = 0.27, size = 335, normalized size = 1.29

$$\frac{b^4\left(\frac{\left(\cos^3(ex+d)+\frac{3\cos(ex+d)}{2}\right)\sin(ex+d)}{4}+\frac{3ex}{8}+\frac{3d}{8}\right)+c^4\left(-\frac{\left(\sin^3(ex+d)+\frac{3\sin(ex+d)}{2}\right)\cos(ex+d)}{4}+\frac{3ex}{8}+\frac{3d}{8}\right)-6a^2bc\left(\cos^2\left(ex+d\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(e*x+d)+c*sin(e*x+d))^4,x)
```

```
[Out] 1/e*(b^4*(1/4*(cos(e*x+d)^3+3/2*cos(e*x+d))*sin(e*x+d)+3/8*e*x+3/8*d)+c^4*(
-1/4*(sin(e*x+d)^3+3/2*sin(e*x+d))*cos(e*x+d)+3/8*e*x+3/8*d)-6*a^2*b*c*cos(
e*x+d)^2-4*a*b^2*c*cos(e*x+d)^3+4*a*b*c^2*sin(e*x+d)^3+4*a^3*b*sin(e*x+d)-4
*cos(e*x+d)*a^3*c+6*a^2*b^2*(1/2*sin(e*x+d)*cos(e*x+d)+1/2*e*x+1/2*d)+6*a^2
*c^2*(-1/2*sin(e*x+d)*cos(e*x+d)+1/2*e*x+1/2*d)+4/3*a*b^3*(2+cos(e*x+d)^2)*
sin(e*x+d)-4/3*a*c^3*(2+sin(e*x+d)^2)*cos(e*x+d)+a^4*(e*x+d)-cos(e*x+d)^4*b
^3*c+6*b^2*c^2*(-1/4*sin(e*x+d)*cos(e*x+d)^3+1/8*sin(e*x+d)*cos(e*x+d)+1/8*
e*x+1/8*d)+c^3*b*sin(e*x+d)^4)
```

maxima [A] time = 0.33, size = 330, normalized size = 1.27

$$-\frac{b^3c\cos(ex+d)^4}{e}+\frac{bc^3\sin(ex+d)^4}{e}+a^4x+\frac{(12\,ex+12\,d+\sin(4\,ex+4\,d)+8\,\sin(2\,ex+2\,d))b^4}{32\,e}+\frac{3\,(4\,ex+4\,d-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(e*x+d)+c*sin(e*x+d))^4,x, algorithm="maxima")
```

```
[Out] -b^3*c*cos(e*x + d)^4/e + b*c^3*sin(e*x + d)^4/e + a^4*x + 1/32*(12*e*x + 1
2*d + sin(4*e*x + 4*d) + 8*sin(2*e*x + 2*d))*b^4/e + 3/16*(4*e*x + 4*d - si
n(4*e*x + 4*d))*b^2*c^2/e + 1/32*(12*e*x + 12*d + sin(4*e*x + 4*d) - 8*sin(
2*e*x + 2*d))*c^4/e - 4*a^3*(c*cos(e*x + d)/e - b*sin(e*x + d)/e) - 3/2*(4*
b*c*cos(e*x + d)^2/e - (2*e*x + 2*d + sin(2*e*x + 2*d))*b^2/e - (2*e*x + 2*
d - sin(2*e*x + 2*d))*c^2/e)*a^2 - 4/3*(3*b^2*c*cos(e*x + d)^3/e - 3*b*c^2*
sin(e*x + d)^3/e + (sin(e*x + d)^3 - 3*sin(e*x + d))*b^3/e - (cos(e*x + d)^
3 - 3*cos(e*x + d))*c^3/e)*a
```

mupad [B] time = 3.37, size = 376, normalized size = 1.45

$$\frac{6\,b^4\,\sin(2\,d+2\,e\,x)+\frac{3\,b^4\,\sin(4\,d+4\,e\,x)}{4}-6\,c^4\,\sin(2\,d+2\,e\,x)+\frac{3\,c^4\,\sin(4\,d+4\,e\,x)}{4}+8\,a\,c^3\,\cos(3\,d+3\,e\,x)-12\,b\,c^3\,\mathrm{co}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*cos(d + e*x) + c*sin(d + e*x))^4,x)
```

```
[Out] (6*b^4*sin(2*d + 2*e*x) + (3*b^4*sin(4*d + 4*e*x))/4 - 6*c^4*sin(2*d + 2*e*
x) + (3*c^4*sin(4*d + 4*e*x))/4 + 8*a*c^3*cos(3*d + 3*e*x) - 12*b*c^3*cos(2
*d + 2*e*x) - 12*b^3*c*cos(2*d + 2*e*x) + 3*b*c^3*cos(4*d + 4*e*x) - 3*b^3*
c*cos(4*d + 4*e*x) + 8*a*b^3*sin(3*d + 3*e*x) + 36*a^2*b^2*sin(2*d + 2*e*x)
 - 36*a^2*c^2*sin(2*d + 2*e*x) - (9*b^2*c^2*sin(4*d + 4*e*x))/2 - 72*a*c^3*
cos(d + e*x) - 96*a^3*c*cos(d + e*x) + 72*a*b^3*sin(d + e*x) + 96*a^3*b*sin
(d + e*x) + 24*a^4*e*x + 9*b^4*e*x + 9*c^4*e*x - 72*a*b^2*c*cos(d + e*x) +
72*a*b*c^2*sin(d + e*x) - 72*a^2*b*c*cos(2*d + 2*e*x) - 24*a*b^2*c*cos(3*d
```

```
+ 3*e*x) - 24*a*b*c^2*sin(3*d + 3*e*x) + 72*a^2*b^2*e*x + 72*a^2*c^2*e*x +
18*b^2*c^2*e*x)/(24*e)
```

sympy [A] time = 1.86, size = 682, normalized size = 2.62

$$\begin{cases} a^4x + \frac{4a^3b\sin(d+ex)}{e} - \frac{4a^3c\cos(d+ex)}{e} + 3a^2b^2x\sin^2(d+ex) + 3a^2b^2x\cos^2(d+ex) + \frac{3a^2b^2\sin(d+ex)\cos(d+ex)}{e} - \frac{6a^2bc\,c \\ x(a+b\cos(d)+c\sin(d))^4 \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(e*x+d)+c*sin(e*x+d))**4,x)
```

```
[Out] Piecewise((a**4*x + 4*a**3*b*sin(d + e*x)/e - 4*a**3*c*cos(d + e*x)/e + 3*a
**2*b**2*x*sin(d + e*x)**2 + 3*a**2*b**2*x*cos(d + e*x)**2 + 3*a**2*b**2*si
n(d + e*x)*cos(d + e*x)/e - 6*a**2*b*c*cos(d + e*x)**2/e + 3*a**2*c**2*x*si
n(d + e*x)**2 + 3*a**2*c**2*x*cos(d + e*x)**2 - 3*a**2*c**2*sin(d + e*x)*co
s(d + e*x)/e + 8*a*b**3*sin(d + e*x)**3/(3*e) + 4*a*b**3*sin(d + e*x)*cos(d
 + e*x)**2/e - 4*a*b**2*c*cos(d + e*x)**3/e + 4*a*b*c**2*sin(d + e*x)**3/e
- 4*a*c**3*sin(d + e*x)**2*cos(d + e*x)/e - 8*a*c**3*cos(d + e*x)**3/(3*e)
+ 3*b**4*x*sin(d + e*x)**4/8 + 3*b**4*x*sin(d + e*x)**2*cos(d + e*x)**2/4 +
 3*b**4*x*cos(d + e*x)**4/8 + 3*b**4*sin(d + e*x)**3*cos(d + e*x)/(8*e) + 5
*b**4*sin(d + e*x)*cos(d + e*x)**3/(8*e) - b**3*c*cos(d + e*x)**4/e + 3*b**
2*c**2*x*sin(d + e*x)**4/4 + 3*b**2*c**2*x*sin(d + e*x)**2*cos(d + e*x)**2/
2 + 3*b**2*c**2*x*cos(d + e*x)**4/4 + 3*b**2*c**2*sin(d + e*x)**3*cos(d + e
*x)/(4*e) - 3*b**2*c**2*sin(d + e*x)*cos(d + e*x)**3/(4*e) + b*c**3*sin(d +
 e*x)**4/e + 3*c**4*x*sin(d + e*x)**4/8 + 3*c**4*x*sin(d + e*x)**2*cos(d +
e*x)**2/4 + 3*c**4*x*cos(d + e*x)**4/8 - 5*c**4*sin(d + e*x)**3*cos(d + e*x
)/(8*e) - 3*c**4*sin(d + e*x)*cos(d + e*x)**3/(8*e), Ne(e, 0)), (x*(a + b*c
os(d) + c*sin(d))**4, True))
```

3.396 $\int (a+b\cos(d+ex)+c\sin(d+ex))^3\,dx$

Optimal. Leaf size=170

$$\frac{b\left(11a^2+4\left(b^2+c^2\right)\right)\sin(d+ex)}{6e}-\frac{c\left(11a^2+4\left(b^2+c^2\right)\right)\cos(d+ex)}{6e}+\frac{1}{2}ax\left(2a^2+3\left(b^2+c^2\right)\right)-\frac{(c\cos(d+ex)-b$$

```
[Out] 1/2*a*(2*a^2+3*b^2+3*c^2)*x-1/6*c*(11*a^2+4*b^2+4*c^2)*cos(e*x+d)/e+1/6*b*(
11*a^2+4*b^2+4*c^2)*sin(e*x+d)/e-5/6*(a*c*cos(e*x+d)-a*b*sin(e*x+d))*(a+b*c
os(e*x+d)+c*sin(e*x+d))/e-1/3*(c*cos(e*x+d)-b*sin(e*x+d))*(a+b*cos(e*x+d)+c
*sin(e*x+d))^2/e
```

Rubi [A] time = 0.19, antiderivative size = 170, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {3120, 3146, 2637, 2638}

$$\frac{b\left(11a^2+4\left(b^2+c^2\right)\right)\sin(d+ex)}{6e}-\frac{c\left(11a^2+4\left(b^2+c^2\right)\right)\cos(d+ex)}{6e}+\frac{1}{2}ax\left(2a^2+3\left(b^2+c^2\right)\right)-\frac{(c\cos(d+ex)-b$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[d + e*x] + c*Sin[d + e*x])^3,x]
```

```
[Out] (a*(2*a^2 + 3*(b^2 + c^2))*x)/2 - (c*(11*a^2 + 4*(b^2 + c^2))*Cos[d + e*x])
/(6*e) + (b*(11*a^2 + 4*(b^2 + c^2))*Sin[d + e*x])/(6*e) - (5*(a*c*Cos[d +
e*x] - a*b*Sin[d + e*x])*(a + b*Cos[d + e*x] + c*Sin[d + e*x]))/(6*e) - ((c
*Cos[d + e*x] - b*Sin[d + e*x])*(a + b*Cos[d + e*x] + c*Sin[d + e*x])^2)/(3
*e)
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3120

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(n_), x_Symbol] :> -Simp[((c*Cos[d + e*x] - b*Sin[d + e*x])*(a + b*Cos[d +
e*x] + c*Sin[d + e*x])^(n - 1))/(e*n), x] + Dist[1/n, Int[Simp[n*a^2 + (n -
 1)*(b^2 + c^2) + a*b*(2*n - 1)*Cos[d + e*x] + a*c*(2*n - 1)*Sin[d + e*x],
x]*(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n - 2), x], x] /; FreeQ[{a, b, c,
```

```
 d, e}, x] && NeQ[a^2 - b^2 - c^2, 0] && GtQ[n, 1]
```

Rule 3146

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(n_.)*((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)
]), x_Symbol] :> Simp[((B*c - b*C - a*C*Cos[d + e*x] + a*B*Sin[d + e*x])*(a
 + b*Cos[d + e*x] + c*Sin[d + e*x])^n)/(a*e*(n + 1)), x] + Dist[1/(a*(n + 1
)), Int[(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n - 1)*Simp[a*(b*B + c*C)*n
+ a^2*A*(n + 1) + (n*(a^2*B - B*c^2 + b*c*C) + a*b*A*(n + 1))*Cos[d + e*x]
+ (n*(b*B*c + a^2*C - b^2*C) + a*c*A*(n + 1))*Sin[d + e*x], x], x], x] /; F
reeQ[{a, b, c, d, e, A, B, C}, x] && GtQ[n, 0] && NeQ[a^2 - b^2 - c^2, 0]
```

Rubi steps

$$\int (a+b\cos(d+ex)+c\sin(d+ex))^3\,dx = -\frac{(c\cos(d+ex)-b\sin(d+ex))(a+b\cos(d+ex)+c\sin(d+ex))^2}{3e}$$

$$= -\frac{5(ac\cos(d+ex)-ab\sin(d+ex))(a+b\cos(d+ex)+c\sin(d+ex)}{6e}$$

$$= \frac{1}{2}a\left(2a^2+3\left(b^2+c^2\right)\right)x - \frac{5(ac\cos(d+ex)-ab\sin(d+ex))(a+b\,c}{6e}$$

$$= \frac{1}{2}a\left(2a^2+3\left(b^2+c^2\right)\right)x - \frac{c\left(11a^2+4\left(b^2+c^2\right)\right)\cos(d+ex)}{6e} + \frac{b\,(11}{}$$

Mathematica [A] time = 0.43, size = 144, normalized size = 0.85

$$\frac{6a\left(2a^2+3\left(b^2+c^2\right)\right)(d+ex)+9b\left(4a^2+b^2+c^2\right)\sin(d+ex)-9c\left(4a^2+b^2+c^2\right)\cos(d+ex)+9a\left(b^2-c^2\right)\mathrm{si}}{12e}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[d + e*x] + c*Sin[d + e*x])^3,x]
```

```
[Out] (6*a*(2*a^2 + 3*(b^2 + c^2))*(d + e*x) - 9*c*(4*a^2 + b^2 + c^2)*Cos[d + e*
x] - 18*a*b*c*Cos[2*(d + e*x)] + c*(-3*b^2 + c^2)*Cos[3*(d + e*x)] + 9*b*(4
*a^2 + b^2 + c^2)*Sin[d + e*x] + 9*a*(b^2 - c^2)*Sin[2*(d + e*x)] + b*(b^2
- 3*c^2)*Sin[3*(d + e*x)])/(12*e)
```

fricas [A] time = 2.94, size = 147, normalized size = 0.86

$$-\frac{18\,abc\cos(ex+d)^2+2\left(3\,b^2c-c^3\right)\cos(ex+d)^3-3\left(2\,a^3+3\,ab^2+3\,ac^2\right)ex+6\left(3\,a^2c+c^3\right)\cos(ex+d)-(1}{6\,e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(e*x+d)+c*sin(e*x+d))^3,x, algorithm="fricas")
```

```
[Out] -1/6*(18*a*b*c*cos(e*x + d)^2 + 2*(3*b^2*c - c^3)*cos(e*x + d)^3 - 3*(2*a^3
 + 3*a*b^2 + 3*a*c^2)*e*x + 6*(3*a^2*c + c^3)*cos(e*x + d) - (18*a^2*b + 4*
b^3 + 6*b*c^2 + 2*(b^3 - 3*b*c^2)*cos(e*x + d)^2 + 9*(a*b^2 - a*c^2)*cos(e*
x + d))*sin(e*x + d))/e
```

giac [A] time = 0.20, size = 167, normalized size = 0.98

$$-\frac{3}{2}\,abc\cos\left(2\,xe+2\,d\right)e^{(-1)}-\frac{1}{12}\left(3\,b^2c-c^3\right)\cos\left(3\,xe+3\,d\right)e^{(-1)}-\frac{3}{4}\left(4\,a^2c+b^2c+c^3\right)\cos\left(xe+d\right)e^{(-1)}+\frac{1}{12}\left(b^3-\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(e*x+d)+c*sin(e*x+d))^3,x, algorithm="giac")
```

```
[Out] -3/2*a*b*c*cos(2*x*e + 2*d)*e^(-1) - 1/12*(3*b^2*c - c^3)*cos(3*x*e + 3*d)*
e^(-1) - 3/4*(4*a^2*c + b^2*c + c^3)*cos(x*e + d)*e^(-1) + 1/12*(b^3 - 3*b*
c^2)*e^(-1)*sin(3*x*e + 3*d) + 3/4*(a*b^2 - a*c^2)*e^(-1)*sin(2*x*e + 2*d)
+ 3/4*(4*a^2*b + b^3 + b*c^2)*e^(-1)*sin(x*e + d) + 1/2*(2*a^3 + 3*a*b^2 +
3*a*c^2)*x
```

maple [A] time = 0.24, size = 177, normalized size = 1.04

$$\frac{a^3\left(ex+d\right)+3\sin\left(ex+d\right)a^2b-3a^2c\cos\left(ex+d\right)+3a\,b^2\left(\frac{\sin(ex+d)\cos(ex+d)}{2}+\frac{ex}{2}+\frac{d}{2}\right)-3abc\left(\cos^2\left(ex+d\right)\right)+3a}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(e*x+d)+c*sin(e*x+d))^3,x)
```

```
[Out] 1/e*(a^3*(e*x+d)+3*sin(e*x+d)*a^2*b-3*a^2*c*cos(e*x+d)+3*a*b^2*(1/2*sin(e*x
+d)*cos(e*x+d)+1/2*e*x+1/2*d)-3*a*b*c*cos(e*x+d)^2+3*a*c^2*(-1/2*sin(e*x+d)
*cos(e*x+d)+1/2*e*x+1/2*d)+1/3*b^3*(2+cos(e*x+d)^2)*sin(e*x+d)-cos(e*x+d)^3
*b^2*c+c^2*b*sin(e*x+d)^3-1/3*c^3*(2+sin(e*x+d)^2)*cos(e*x+d))
```

maxima [A] time = 0.33, size = 189, normalized size = 1.11

$$-\frac{b^2c\cos\left(ex+d\right)^3}{e}+\frac{bc^2\sin\left(ex+d\right)^3}{e}+a^3x-\frac{\left(\sin\left(ex+d\right)^3-3\,\sin\left(ex+d\right)\right)b^3}{3\,e}+\frac{\left(\cos\left(ex+d\right)^3-3\,\cos\left(ex+d\right)\right)c^3}{3\,e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(e*x+d)+c*sin(e*x+d))^3,x, algorithm="maxima")
```

```
[Out] -b^2*c*cos(e*x + d)^3/e + b*c^2*sin(e*x + d)^3/e + a^3*x - 1/3*(sin(e*x + d
)^3 - 3*sin(e*x + d))*b^3/e + 1/3*(cos(e*x + d)^3 - 3*cos(e*x + d))*c^3/e -
 3*a^2*(c*cos(e*x + d)/e - b*sin(e*x + d)/e) - 3/4*(4*b*c*cos(e*x + d)^2/e
- (2*e*x + 2*d + sin(2*e*x + 2*d))*b^2/e - (2*e*x + 2*d - sin(2*e*x + 2*d))
*c^2/e)*a
```

mupad [B] time = 3.70, size = 333, normalized size = 1.96

$$\frac{a\,\mathrm{atan}\left(\frac{a\tan\left(\frac{d}{2}+\frac{e\,x}{2}\right)\left(2\,a^2+3\,b^2+3\,c^2\right)}{2\,a^3+3\,a\,b^2+3\,a\,c^2}\right)\left(2\,a^2+3\,b^2+3\,c^2\right)}{e}-\frac{a\,\left(\mathrm{atan}\left(\tan\left(\frac{d}{2}+\frac{e\,x}{2}\right)\right)-\frac{e\,x}{2}\right)\,\left(2\,a^2+3\,b^2+3\,c^2\right)}{e}-\frac{\tan\left(\frac{d}{2}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*cos(d + e*x) + c*sin(d + e*x))^3,x)
```

```
[Out] (a*atan((a*tan(d/2 + (e*x)/2)*(2*a^2 + 3*b^2 + 3*c^2))/(3*a*b^2 + 3*a*c^2 +
 2*a^3))*(2*a^2 + 3*b^2 + 3*c^2))/e - (a*(atan(tan(d/2 + (e*x)/2)) - (e*x)/
2)*(2*a^2 + 3*b^2 + 3*c^2))/e - (tan(d/2 + (e*x)/2)^2*(12*a^2*c + 4*c^3 - 1
2*a*b*c) - tan(d/2 + (e*x)/2)^3*(12*a^2*b + 8*b*c^2 + (4*b^3)/3) - tan(d/2
+ (e*x)/2)*(3*a*b^2 + 6*a^2*b - 3*a*c^2 + 2*b^3) + tan(d/2 + (e*x)/2)^4*(6*
a^2*c + 6*b^2*c - 12*a*b*c) + 6*a^2*c + 2*b^2*c - tan(d/2 + (e*x)/2)^5*(6*a
^2*b - 3*a*b^2 + 3*a*c^2 + 2*b^3) + (4*c^3)/3)/(e*(3*tan(d/2 + (e*x)/2)^2 +
 3*tan(d/2 + (e*x)/2)^4 + tan(d/2 + (e*x)/2)^6 + 1))
```

sympy [A] time = 0.76, size = 294, normalized size = 1.73

$$\begin{cases} a^3x+\frac{3a^2b\sin(d+ex)}{e}-\frac{3a^2c\cos(d+ex)}{e}+\frac{3ab^2x\sin^2(d+ex)}{2}+\frac{3ab^2x\cos^2(d+ex)}{2}+\frac{3ab^2\sin(d+ex)\cos(d+ex)}{2e}-\frac{3abc\cos^2(d+ex)}{e}+\frac{3ac^2}{} \\ x\left(a+b\cos(d)+c\sin(d)\right)^3 \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(e*x+d)+c*sin(e*x+d))**3,x)
```

```
[Out] Piecewise((a**3*x + 3*a**2*b*sin(d + e*x)/e - 3*a**2*c*cos(d + e*x)/e + 3*a
*b**2*x*sin(d + e*x)**2/2 + 3*a*b**2*x*cos(d + e*x)**2/2 + 3*a*b**2*sin(d +
 e*x)*cos(d + e*x)/(2*e) - 3*a*b*c*cos(d + e*x)**2/e + 3*a*c**2*x*sin(d + e
*x)**2/2 + 3*a*c**2*x*cos(d + e*x)**2/2 - 3*a*c**2*sin(d + e*x)*cos(d + e*x
)/(2*e) + 2*b**3*sin(d + e*x)**3/(3*e) + b**3*sin(d + e*x)*cos(d + e*x)**2/
e - b**2*c*cos(d + e*x)**3/e + b*c**2*sin(d + e*x)**3/e - c**3*sin(d + e*x)
**2*cos(d + e*x)/e - 2*c**3*cos(d + e*x)**3/(3*e), Ne(e, 0)), (x*(a + b*cos
(d) + c*sin(d))**3, True))
```

3.397 $\int (a+b\cos(d+ex)+c\sin(d+ex))^2\,dx$

Optimal. Leaf size=91

$$\frac{1}{2}x\left(2a^2+b^2+c^2\right)-\frac{(c\cos(d+ex)-b\sin(d+ex))(a+b\cos(d+ex)+c\sin(d+ex))}{2e}+\frac{3ab\sin(d+ex)}{2e}-\frac{3ac\cos(d}{2e}$$

```
[Out] 1/2*(2*a^2+b^2+c^2)*x-3/2*a*c*cos(e*x+d)/e+3/2*a*b*sin(e*x+d)/e-1/2*(c*cos(
e*x+d)-b*sin(e*x+d))*(a+b*cos(e*x+d)+c*sin(e*x+d))/e
```

Rubi [A] time = 0.05, antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {3120, 2637, 2638}

$$\frac{1}{2}x\left(2a^2+b^2+c^2\right)-\frac{(c\cos(d+ex)-b\sin(d+ex))(a+b\cos(d+ex)+c\sin(d+ex))}{2e}+\frac{3ab\sin(d+ex)}{2e}-\frac{3ac\cos(d}{2e}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[d + e*x] + c*Sin[d + e*x])^2,x]
```

```
[Out] ((2*a^2 + b^2 + c^2)*x)/2 - (3*a*c*Cos[d + e*x])/(2*e) + (3*a*b*Sin[d + e*x
])/(2*e) - ((c*Cos[d + e*x] - b*Sin[d + e*x])*(a + b*Cos[d + e*x] + c*Sin[d
 + e*x]))/(2*e)
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3120

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(n_), x_Symbol] :> -Simp[((c*Cos[d + e*x] - b*Sin[d + e*x])*(a + b*Cos[d +
e*x] + c*Sin[d + e*x])^(n - 1))/(e*n), x] + Dist[1/n, Int[Simp[n*a^2 + (n -
 1)*(b^2 + c^2) + a*b*(2*n - 1)*Cos[d + e*x] + a*c*(2*n - 1)*Sin[d + e*x],
x]*(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n - 2), x], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[a^2 - b^2 - c^2, 0] && GtQ[n, 1]
```

Rubi steps

$$\int (a + b\cos(d + ex) + c\sin(d + ex))^2\,dx = -\frac{(c\cos(d + ex) - b\sin(d + ex))(a + b\cos(d + ex) + c\sin(d + ex))}{2e} +$$
$$= \frac{1}{2}\left(2a^2 + b^2 + c^2\right)x - \frac{(c\cos(d + ex) - b\sin(d + ex))(a + b\cos(d + e}{2e}$$
$$= \frac{1}{2}\left(2a^2 + b^2 + c^2\right)x - \frac{3ac\cos(d + ex)}{2e} + \frac{3ab\sin(d + ex)}{2e} - \frac{(c\cos(d -}{}$$

Mathematica [A] time = 0.17, size = 77, normalized size = 0.85

$$\frac{2\left(2a^2 + b^2 + c^2\right)(d + ex) + 8ab\sin(d + ex) - 8ac\cos(d + ex) + \left(b^2 - c^2\right)\sin(2(d + ex)) - 2bc\cos(2(d + ex))}{4e}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[d + e*x] + c*Sin[d + e*x])^2,x]
```

```
[Out] (2*(2*a^2 + b^2 + c^2)*(d + e*x) - 8*a*c*Cos[d + e*x] - 2*b*c*Cos[2*(d + e*
x)] + 8*a*b*Sin[d + e*x] + (b^2 - c^2)*Sin[2*(d + e*x)])/(4*e)
```

fricas [A] time = 1.52, size = 73, normalized size = 0.80

$$-\frac{2\,bc\cos\left(ex + d\right)^2 - \left(2\,a^2 + b^2 + c^2\right)ex + 4\,ac\cos\left(ex + d\right) - \left(4\,ab + \left(b^2 - c^2\right)\cos\left(ex + d\right)\right)\sin\left(ex + d\right)}{2\,e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(e*x+d)+c*sin(e*x+d))^2,x, algorithm="fricas")
```

```
[Out] -1/2*(2*b*c*cos(e*x + d)^2 - (2*a^2 + b^2 + c^2)*e*x + 4*a*c*cos(e*x + d) -
 (4*a*b + (b^2 - c^2)*cos(e*x + d))*sin(e*x + d))/e
```

giac [A] time = 0.15, size = 81, normalized size = 0.89

$$-\frac{1}{2}\,bc\cos\left(2\,xe + 2\,d\right)e^{(-1)} - 2\,ac\cos\left(xe + d\right)e^{(-1)} + 2\,abe^{(-1)}\sin\left(xe + d\right) + \frac{1}{4}\left(b^2 - c^2\right)e^{(-1)}\sin\left(2\,xe + 2\,d\right) + \frac{1}{2}\left(2\,a^2\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(e*x+d)+c*sin(e*x+d))^2,x, algorithm="giac")
```

```
[Out] -1/2*b*c*cos(2*x*e + 2*d)*e^(-1) - 2*a*c*cos(x*e + d)*e^(-1) + 2*a*b*e^(-1)
*sin(x*e + d) + 1/4*(b^2 - c^2)*e^(-1)*sin(2*x*e + 2*d) + 1/2*(2*a^2 + b^2
+ c^2)*x
```

maple [A] time = 0.22, size = 99, normalized size = 1.09

$$\frac{a^2(ex+d)+2ab\sin(ex+d)-2ac\cos(ex+d)+b^2\left(\frac{\sin(ex+d)\cos(ex+d)}{2}+\frac{ex}{2}+\frac{d}{2}\right)-\left(\cos^2(ex+d)\right)bc+c^2\left(-\frac{\sin(ex-}{}\right.}{e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(e*x+d)+c*sin(e*x+d))^2,x)
```

```
[Out] 1/e*(a^2*(e*x+d)+2*a*b*sin(e*x+d)-2*a*c*cos(e*x+d)+b^2*(1/2*sin(e*x+d)*cos(
e*x+d)+1/2*e*x+1/2*d)-cos(e*x+d)^2*b*c+c^2*(-1/2*sin(e*x+d)*cos(e*x+d)+1/2*
e*x+1/2*d))
```

maxima [A] time = 0.31, size = 100, normalized size = 1.10

$$a^2x-\frac{bc\cos(ex+d)^2}{e}+\frac{(2\,ex+2\,d+\sin(2\,ex+2\,d))b^2}{4\,e}+\frac{(2\,ex+2\,d-\sin(2\,ex+2\,d))c^2}{4\,e}-2\,a\left(\frac{c\cos(ex+d)}{e}-\frac{b\sin}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(e*x+d)+c*sin(e*x+d))^2,x, algorithm="maxima")
```

```
[Out] a^2*x - b*c*cos(e*x + d)^2/e + 1/4*(2*e*x + 2*d + sin(2*e*x + 2*d))*b^2/e +
 1/4*(2*e*x + 2*d - sin(2*e*x + 2*d))*c^2/e - 2*a*(c*cos(e*x + d)/e - b*sin
(e*x + d)/e)
```

mupad [B] time = 3.78, size = 125, normalized size = 1.37

$$\frac{x\left(2\,a^2+b^2+c^2\right)}{2}-\frac{\left(b^2-4\,a\,b-c^2\right)\tan\left(\frac{d}{2}+\frac{e\,x}{2}\right)^3+(4\,a\,c-4\,b\,c)\tan\left(\frac{d}{2}+\frac{e\,x}{2}\right)^2+\left(-b^2-4\,a\,b+c^2\right)\tan\left(\frac{d}{2}+\frac{e\,x}{2}\right.}{e\left(\tan\left(\frac{d}{2}+\frac{e\,x}{2}\right)^4+2\tan\left(\frac{d}{2}+\frac{e\,x}{2}\right)^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*cos(d + e*x) + c*sin(d + e*x))^2,x)
```

```
[Out] (x*(2*a^2 + b^2 + c^2))/2 - (4*a*c + tan(d/2 + (e*x)/2)^2*(4*a*c - 4*b*c) -
 tan(d/2 + (e*x)/2)*(4*a*b + b^2 - c^2) - tan(d/2 + (e*x)/2)^3*(4*a*b - b^2
 + c^2))/(e*(2*tan(d/2 + (e*x)/2)^2 + tan(d/2 + (e*x)/2)^4 + 1))
```

sympy [A] time = 0.31, size = 162, normalized size = 1.78

$$\begin{cases} a^2x+\frac{2ab\sin(d+ex)}{e}-\frac{2ac\cos(d+ex)}{e}+\frac{b^2x\sin^2(d+ex)}{2}+\frac{b^2x\cos^2(d+ex)}{2}+\frac{b^2\sin(d+ex)\cos(d+ex)}{2e}-\frac{bc\cos^2(d+ex)}{e}+\frac{c^2x\sin^2(d+ex)}{2}+ \\ x\left(a+b\cos(d)+c\sin(d)\right)^2 \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(e*x+d)+c*sin(e*x+d))**2,x)
```

```
[Out] Piecewise((a**2*x + 2*a*b*sin(d + e*x)/e - 2*a*c*cos(d + e*x)/e + b**2*x*si
n(d + e*x)**2/2 + b**2*x*cos(d + e*x)**2/2 + b**2*sin(d + e*x)*cos(d + e*x)
/(2*e) - b*c*cos(d + e*x)**2/e + c**2*x*sin(d + e*x)**2/2 + c**2*x*cos(d +
e*x)**2/2 - c**2*sin(d + e*x)*cos(d + e*x)/(2*e), Ne(e, 0)), (x*(a + b*cos(
d) + c*sin(d))**2, True))
```

3.398 $\int (a + b\cos(d + ex) + c\sin(d + ex))\,dx$

Optimal. Leaf size=27

$$ax + \frac{b\sin(d + ex)}{e} - \frac{c\cos(d + ex)}{e}$$

```
[Out] a*x-c*cos(e*x+d)/e+b*sin(e*x+d)/e
```

Rubi [A] time = 0.02, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2637, 2638}

$$ax + \frac{b\sin(d + ex)}{e} - \frac{c\cos(d + ex)}{e}$$

Antiderivative was successfully verified.

```
[In] Int[a + b*Cos[d + e*x] + c*Sin[d + e*x],x]
```

```
[Out] a*x - (c*Cos[d + e*x])/e + (b*Sin[d + e*x])/e
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int (a + b\cos(d + ex) + c\sin(d + ex))\,dx &= ax + b\int \cos(d + ex)\,dx + c\int \sin(d + ex)\,dx\\ &= ax - \frac{c\cos(d + ex)}{e} + \frac{b\sin(d + ex)}{e}\end{aligned}$$

Mathematica [A] time = 0.01, size = 49, normalized size = 1.81

$$ax + \frac{b\sin(d)\cos(ex)}{e} + \frac{b\cos(d)\sin(ex)}{e} + \frac{c\sin(d)\sin(ex)}{e} - \frac{c\cos(d)\cos(ex)}{e}$$

Antiderivative was successfully verified.

```
[In] Integrate[a + b*Cos[d + e*x] + c*Sin[d + e*x],x]
```

```
[Out] a*x - (c*Cos[d]*Cos[e*x])/e + (b*Cos[e*x]*Sin[d])/e + (b*Cos[d]*Sin[e*x])/e
 + (c*Sin[d]*Sin[e*x])/e
```

fricas [A] time = 0.89, size = 26, normalized size = 0.96

$$\frac{aex - c\cos(ex + d) + b\sin(ex + d)}{e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*cos(e*x+d)+c*sin(e*x+d),x, algorithm="fricas")
```

```
[Out] (a*e*x - c*cos(e*x + d) + b*sin(e*x + d))/e
```

giac [A] time = 0.15, size = 27, normalized size = 1.00

$$-c\cos(xe + d)e^{(-1)} + be^{(-1)}\sin(xe + d) + ax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*cos(e*x+d)+c*sin(e*x+d),x, algorithm="giac")
```

```
[Out] -c*cos(x*e + d)*e^(-1) + b*e^(-1)*sin(x*e + d) + a*x
```

maple [A] time = 0.00, size = 28, normalized size = 1.04

$$ax - \frac{c\cos(ex + d)}{e} + \frac{b\sin(ex + d)}{e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(a+b*cos(e*x+d)+c*sin(e*x+d),x)
```

```
[Out] a*x-c*cos(e*x+d)/e+b*sin(e*x+d)/e
```

maxima [A] time = 0.31, size = 27, normalized size = 1.00

$$ax - \frac{c\cos(ex + d)}{e} + \frac{b\sin(ex + d)}{e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*cos(e*x+d)+c*sin(e*x+d),x, algorithm="maxima")
```

```
[Out] a*x - c*cos(e*x + d)/e + b*sin(e*x + d)/e
```

mupad [B] time = 2.51, size = 40, normalized size = 1.48

$$a\,x-\frac{2\,c-2\,b\,\tan\left(\frac{d}{2}+\frac{e\,x}{2}\right)}{e\,\left({\tan\left(\frac{d}{2}+\frac{e\,x}{2}\right)}^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(a + b*cos(d + e*x) + c*sin(d + e*x),x)
```

```
[Out] a*x - (2*c - 2*b*tan(d/2 + (e*x)/2))/(e*(tan(d/2 + (e*x)/2)^2 + 1))
```

sympy [A] time = 0.14, size = 34, normalized size = 1.26

$$ax+b\left(\begin{cases}\frac{\sin(d+ex)}{e} & \text{for } e\neq 0\\ x\cos(d) & \text{otherwise}\end{cases}\right)+c\left(\begin{cases}-\frac{\cos(d+ex)}{e} & \text{for } e\neq 0\\ x\sin(d) & \text{otherwise}\end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*cos(e*x+d)+c*sin(e*x+d),x)
```

```
[Out] a*x + b*Piecewise((sin(d + e*x)/e, Ne(e, 0)), (x*cos(d), True)) + c*Piecewi
se((-cos(d + e*x)/e, Ne(e, 0)), (x*sin(d), True))
```

3.399 $\int \frac{1}{a+b\cos(d+ex)+c\sin(d+ex)}\,dx$

Optimal. Leaf size=61

$$\frac{2\tan^{-1}\left(\frac{(a-b)\tan\left(\frac{1}{2}(d+ex)\right)+c}{\sqrt{a^2-b^2-c^2}}\right)}{e\sqrt{a^2-b^2-c^2}}$$

```
[Out] 2*arctan((c+(a-b)*tan(1/2*e*x+1/2*d))/(a^2-b^2-c^2)^(1/2))/e/(a^2-b^2-c^2)^
(1/2)
```

Rubi [A] time = 0.08, antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {3124, 618, 204}

$$\frac{2\tan^{-1}\left(\frac{(a-b)\tan\left(\frac{1}{2}(d+ex)\right)+c}{\sqrt{a^2-b^2-c^2}}\right)}{e\sqrt{a^2-b^2-c^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(-1),x]
```

```
[Out] (2*ArcTan[(c + (a - b)*Tan[(d + e*x)/2])/Sqrt[a^2 - b^2 - c^2]])/(Sqrt[a^2
- b^2 - c^2]*e)
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 3124

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(-1), x_Symbol] :> Module[{f = FreeFactors[Tan[(d + e*x)/2], x]}, Dist[(2*f
)/e, Subst[Int[1/(a + b + 2*c*f*x + (a - b)*f^2*x^2), x], x, Tan[(d + e*x)/
2]/f], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0]
```

Rubi steps

$$\int \frac{1}{a+b\cos(d+ex)+c\sin(d+ex)}\,dx = \frac{2\,\text{Subst}\left(\int \frac{1}{a+b+2cx+(a-b)x^2}\,dx, x, \tan\left(\frac{1}{2}(d+ex)\right)\right)}{e}$$

$$= -\frac{4\,\text{Subst}\left(\int \frac{1}{-4(a^2-b^2-c^2)-x^2}\,dx, x, 2c+2(a-b)\tan\left(\frac{1}{2}(d+ex)\right)\right)}{e}$$

$$= \frac{2\tan^{-1}\left(\frac{c+(a-b)\tan\left(\frac{1}{2}(d+ex)\right)}{\sqrt{a^2-b^2-c^2}}\right)}{\sqrt{a^2-b^2-c^2}\,e}$$

Mathematica [A] time = 0.12, size = 57, normalized size = 0.93

$$-\frac{2\tanh^{-1}\left(\frac{(a-b)\tan\left(\frac{1}{2}(d+ex)\right)+c}{\sqrt{-a^2+b^2+c^2}}\right)}{e\sqrt{-a^2+b^2+c^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(-1),x]
```

```
[Out] (-2*ArcTanh[(c + (a - b)*Tan[(d + e*x)/2])/Sqrt[-a^2 + b^2 + c^2]])/(Sqrt[-
a^2 + b^2 + c^2]*e)
```

fricas [B] time = 1.71, size = 434, normalized size = 7.11

$$\left[-\frac{\sqrt{-a^2+b^2+c^2}\,\log\left(-\frac{a^2b^2-2b^4-c^4-(a^2+3b^2)c^2-(2a^2b^2-b^4-2a^2c^2+c^4)\cos(ex+d)^2-2(ab^3+abc^2)\cos(ex+d)-2(ab^2c+ac^3-(bc^3-(2a^2b-b\ldots}{2ab\cos(ex+d)+(b^2-c^2)\cos(ex\ldots}\right.}{2(a^2-b^2-c^2\ldots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cos(e*x+d)+c*sin(e*x+d)),x, algorithm="fricas")
```

```
[Out] [-1/2*sqrt(-a^2 + b^2 + c^2)*log(-(a^2*b^2 - 2*b^4 - c^4 - (a^2 + 3*b^2)*c^
2 - (2*a^2*b^2 - b^4 - 2*a^2*c^2 + c^4)*cos(e*x + d)^2 - 2*(a*b^3 + a*b*c^2
)*cos(e*x + d) - 2*(a*b^2*c + a*c^3 - (b*c^3 - (2*a^2*b - b^3)*c)*cos(e*x +
 d))*sin(e*x + d) + 2*(2*a*b*c*cos(e*x + d)^2 - a*b*c + (b^2*c + c^3)*cos(e
*x + d) - (b^3 + b*c^2 + (a*b^2 - a*c^2)*cos(e*x + d))*sin(e*x + d))*sqrt(-
```

```
a^2 + b^2 + c^2))/(2*a*b*cos(e*x + d) + (b^2 - c^2)*cos(e*x + d)^2 + a^2 +
c^2 + 2*(b*c*cos(e*x + d) + a*c)*sin(e*x + d)))/((a^2 - b^2 - c^2)*e), arct
an(-(a*b*cos(e*x + d) + a*c*sin(e*x + d) + b^2 + c^2)*sqrt(a^2 - b^2 - c^2)
/((c^3 - (a^2 - b^2)*c)*cos(e*x + d) + (a^2*b - b^3 - b*c^2)*sin(e*x + d)))
/(sqrt(a^2 - b^2 - c^2)*e)]
```

giac [A] time = 0.16, size = 91, normalized size = 1.49

$$-\frac{2\left(\pi\left\lfloor\frac{xe+d}{2\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}\left(-2\,a+2\,b\right)+\arctan\left(-\frac{a\tan\left(\frac{1}{2}\,xe+\frac{1}{2}\,d\right)-b\tan\left(\frac{1}{2}\,xe+\frac{1}{2}\,d\right)+c}{\sqrt{a^2-b^2-c^2}}\right)\right)e^{(-1)}}{\sqrt{a^2-b^2-c^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cos(e*x+d)+c*sin(e*x+d)),x, algorithm="giac")
```

```
[Out] -2*(pi*floor(1/2*(x*e + d)/pi + 1/2)*sgn(-2*a + 2*b) + arctan(-(a*tan(1/2*x
*e + 1/2*d) - b*tan(1/2*x*e + 1/2*d) + c)/sqrt(a^2 - b^2 - c^2)))*e^(-1)/sq
rt(a^2 - b^2 - c^2)
```

maple [A] time = 0.38, size = 61, normalized size = 1.00

$$\frac{2\arctan\left(\frac{2(a-b)\tan\left(\frac{d}{2}+\frac{ex}{2}\right)+2c}{2\sqrt{a^2-b^2-c^2}}\right)}{e\sqrt{a^2-b^2-c^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*cos(e*x+d)+c*sin(e*x+d)),x)
```

```
[Out] 2/e/(a^2-b^2-c^2)^(1/2)*arctan(1/2*(2*(a-b)*tan(1/2*d+1/2*e*x)+2*c)/(a^2-b^
2-c^2)^(1/2))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cos(e*x+d)+c*sin(e*x+d)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(c^2+b^2-a^2>0)', see `assume?` for
more details)Is c^2+b^2-a^2 positive or negative?
```

mupad [B] time = 4.01, size = 75, normalized size = 1.23

$$\frac{2\,\mathrm{atan}\left(\frac{c}{\sqrt{a^2-b^2-c^2}}+\frac{\tan\left(\frac{d}{2}+\frac{e\,x}{2}\right)\,\left(2\,a-2\,b\right)}{2\,\sqrt{a^2-b^2-c^2}}\right)}{e\,\sqrt{a^2-b^2-c^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*cos(d + e*x) + c*sin(d + e*x)),x)
```

```
[Out] (2*atan(c/(a^2 - b^2 - c^2)^(1/2) + (tan(d/2 + (e*x)/2)*(2*a - 2*b))/(2*(a^
2 - b^2 - c^2)^(1/2))))/(e*(a^2 - b^2 - c^2)^(1/2))
```

sympy [A] time = 152.53, size = 3179, normalized size = 52.11

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cos(e*x+d)+c*sin(e*x+d)),x)
```

```
[Out] Piecewise((32*b**5/(32*b**6*e*tan(d/2 + e*x/2) - 16*b**5*c*e + 32*b**5*e*sq
rt(b**2 + c**2)*tan(d/2 + e*x/2) + 48*b**4*c**2*e*tan(d/2 + e*x/2) - 16*b**
4*c*e*sqrt(b**2 + c**2) - 20*b**3*c**3*e + 32*b**3*c**2*e*sqrt(b**2 + c**2)
*tan(d/2 + e*x/2) + 18*b**2*c**4*e*tan(d/2 + e*x/2) - 12*b**2*c**3*e*sqrt(b
**2 + c**2) - 5*b*c**5*e + 6*b*c**4*e*sqrt(b**2 + c**2)*tan(d/2 + e*x/2) +
c**6*e*tan(d/2 + e*x/2) - c**5*e*sqrt(b**2 + c**2)) + 32*b**4*sqrt(b**2 + c
**2)/(32*b**6*e*tan(d/2 + e*x/2) - 16*b**5*c*e + 32*b**5*e*sqrt(b**2 + c**2
)*tan(d/2 + e*x/2) + 48*b**4*c**2*e*tan(d/2 + e*x/2) - 16*b**4*c*e*sqrt(b**
2 + c**2) - 20*b**3*c**3*e + 32*b**3*c**2*e*sqrt(b**2 + c**2)*tan(d/2 + e*x
/2) + 18*b**2*c**4*e*tan(d/2 + e*x/2) - 12*b**2*c**3*e*sqrt(b**2 + c**2) -
5*b*c**5*e + 6*b*c**4*e*sqrt(b**2 + c**2)*tan(d/2 + e*x/2) + c**6*e*tan(d/2
 + e*x/2) - c**5*e*sqrt(b**2 + c**2)) + 40*b**3*c**2/(32*b**6*e*tan(d/2 + e
*x/2) - 16*b**5*c*e + 32*b**5*e*sqrt(b**2 + c**2)*tan(d/2 + e*x/2) + 48*b**
4*c**2*e*tan(d/2 + e*x/2) - 16*b**4*c*e*sqrt(b**2 + c**2) - 20*b**3*c**3*e
+ 32*b**3*c**2*e*sqrt(b**2 + c**2)*tan(d/2 + e*x/2) + 18*b**2*c**4*e*tan(d/
2 + e*x/2) - 12*b**2*c**3*e*sqrt(b**2 + c**2) - 5*b*c**5*e + 6*b*c**4*e*sqr
t(b**2 + c**2)*tan(d/2 + e*x/2) + c**6*e*tan(d/2 + e*x/2) - c**5*e*sqrt(b**
2 + c**2)) + 24*b**2*c**2*sqrt(b**2 + c**2)/(32*b**6*e*tan(d/2 + e*x/2) - 1
6*b**5*c*e + 32*b**5*e*sqrt(b**2 + c**2)*tan(d/2 + e*x/2) + 48*b**4*c**2*e*
tan(d/2 + e*x/2) - 16*b**4*c*e*sqrt(b**2 + c**2) - 20*b**3*c**3*e + 32*b**3
*c**2*e*sqrt(b**2 + c**2)*tan(d/2 + e*x/2) + 18*b**2*c**4*e*tan(d/2 + e*x/2
) - 12*b**2*c**3*e*sqrt(b**2 + c**2) - 5*b*c**5*e + 6*b*c**4*e*sqrt(b**2 +
c**2)*tan(d/2 + e*x/2) + c**6*e*tan(d/2 + e*x/2) - c**5*e*sqrt(b**2 + c**2)
) + 10*b*c**4/(32*b**6*e*tan(d/2 + e*x/2) - 16*b**5*c*e + 32*b**5*e*sqrt(b*
```

```
*2 + c**2)*tan(d/2 + e*x/2) + 48*b**4*c**2*e*tan(d/2 + e*x/2) - 16*b**4*c*e
*sqrt(b**2 + c**2) - 20*b**3*c**3*e + 32*b**3*c**2*e*sqrt(b**2 + c**2)*tan(
d/2 + e*x/2) + 18*b**2*c**4*e*tan(d/2 + e*x/2) - 12*b**2*c**3*e*sqrt(b**2 +
 c**2) - 5*b*c**5*e + 6*b*c**4*e*sqrt(b**2 + c**2)*tan(d/2 + e*x/2) + c**6*
e*tan(d/2 + e*x/2) - c**5*e*sqrt(b**2 + c**2)) + 2*c**4*sqrt(b**2 + c**2)/(
32*b**6*e*tan(d/2 + e*x/2) - 16*b**5*c*e + 32*b**5*e*sqrt(b**2 + c**2)*tan(
d/2 + e*x/2) + 48*b**4*c**2*e*tan(d/2 + e*x/2) - 16*b**4*c*e*sqrt(b**2 + c*
*2) - 20*b**3*c**3*e + 32*b**3*c**2*e*sqrt(b**2 + c**2)*tan(d/2 + e*x/2) +
18*b**2*c**4*e*tan(d/2 + e*x/2) - 12*b**2*c**3*e*sqrt(b**2 + c**2) - 5*b*c*
*5*e + 6*b*c**4*e*sqrt(b**2 + c**2)*tan(d/2 + e*x/2) + c**6*e*tan(d/2 + e*x
/2) - c**5*e*sqrt(b**2 + c**2)), Eq(a, -sqrt(b**2 + c**2))), (32*b**5/(32*b
**6*e*tan(d/2 + e*x/2) - 16*b**5*c*e - 32*b**5*e*sqrt(b**2 + c**2)*tan(d/2
+ e*x/2) + 48*b**4*c**2*e*tan(d/2 + e*x/2) + 16*b**4*c*e*sqrt(b**2 + c**2)
- 20*b**3*c**3*e - 32*b**3*c**2*e*sqrt(b**2 + c**2)*tan(d/2 + e*x/2) + 18*b
**2*c**4*e*tan(d/2 + e*x/2) + 12*b**2*c**3*e*sqrt(b**2 + c**2) - 5*b*c**5*e
 - 6*b*c**4*e*sqrt(b**2 + c**2)*tan(d/2 + e*x/2) + c**6*e*tan(d/2 + e*x/2)
+ c**5*e*sqrt(b**2 + c**2)) - 32*b**4*sqrt(b**2 + c**2)/(32*b**6*e*tan(d/2
+ e*x/2) - 16*b**5*c*e - 32*b**5*e*sqrt(b**2 + c**2)*tan(d/2 + e*x/2) + 48*
b**4*c**2*e*tan(d/2 + e*x/2) + 16*b**4*c*e*sqrt(b**2 + c**2) - 20*b**3*c**3
*e - 32*b**3*c**2*e*sqrt(b**2 + c**2)*tan(d/2 + e*x/2) + 18*b**2*c**4*e*tan
(d/2 + e*x/2) + 12*b**2*c**3*e*sqrt(b**2 + c**2) - 5*b*c**5*e - 6*b*c**4*e*
sqrt(b**2 + c**2)*tan(d/2 + e*x/2) + c**6*e*tan(d/2 + e*x/2) + c**5*e*sqrt(
b**2 + c**2)) + 40*b**3*c**2/(32*b**6*e*tan(d/2 + e*x/2) - 16*b**5*c*e - 32
*b**5*e*sqrt(b**2 + c**2)*tan(d/2 + e*x/2) + 48*b**4*c**2*e*tan(d/2 + e*x/2
) + 16*b**4*c*e*sqrt(b**2 + c**2) - 20*b**3*c**3*e - 32*b**3*c**2*e*sqrt(b*
*2 + c**2)*tan(d/2 + e*x/2) + 18*b**2*c**4*e*tan(d/2 + e*x/2) + 12*b**2*c**
3*e*sqrt(b**2 + c**2) - 5*b*c**5*e - 6*b*c**4*e*sqrt(b**2 + c**2)*tan(d/2 +
 e*x/2) + c**6*e*tan(d/2 + e*x/2) + c**5*e*sqrt(b**2 + c**2)) - 24*b**2*c**
2*sqrt(b**2 + c**2)/(32*b**6*e*tan(d/2 + e*x/2) - 16*b**5*c*e - 32*b**5*e*s
qrt(b**2 + c**2)*tan(d/2 + e*x/2) + 48*b**4*c**2*e*tan(d/2 + e*x/2) + 16*b*
*4*c*e*sqrt(b**2 + c**2) - 20*b**3*c**3*e - 32*b**3*c**2*e*sqrt(b**2 + c**2
)*tan(d/2 + e*x/2) + 18*b**2*c**4*e*tan(d/2 + e*x/2) + 12*b**2*c**3*e*sqrt(
b**2 + c**2) - 5*b*c**5*e - 6*b*c**4*e*sqrt(b**2 + c**2)*tan(d/2 + e*x/2) +
 c**6*e*tan(d/2 + e*x/2) + c**5*e*sqrt(b**2 + c**2)) + 10*b*c**4/(32*b**6*e
*tan(d/2 + e*x/2) - 16*b**5*c*e - 32*b**5*e*sqrt(b**2 + c**2)*tan(d/2 + e*x
/2) + 48*b**4*c**2*e*tan(d/2 + e*x/2) + 16*b**4*c*e*sqrt(b**2 + c**2) - 20*
b**3*c**3*e - 32*b**3*c**2*e*sqrt(b**2 + c**2)*tan(d/2 + e*x/2) + 18*b**2*c
**4*e*tan(d/2 + e*x/2) + 12*b**2*c**3*e*sqrt(b**2 + c**2) - 5*b*c**5*e - 6*
b*c**4*e*sqrt(b**2 + c**2)*tan(d/2 + e*x/2) + c**6*e*tan(d/2 + e*x/2) + c**
5*e*sqrt(b**2 + c**2)) - 2*c**4*sqrt(b**2 + c**2)/(32*b**6*e*tan(d/2 + e*x/
2) - 16*b**5*c*e - 32*b**5*e*sqrt(b**2 + c**2)*tan(d/2 + e*x/2) + 48*b**4*c
**2*e*tan(d/2 + e*x/2) + 16*b**4*c*e*sqrt(b**2 + c**2) - 20*b**3*c**3*e - 3
2*b**3*c**2*e*sqrt(b**2 + c**2)*tan(d/2 + e*x/2) + 18*b**2*c**4*e*tan(d/2 +
 e*x/2) + 12*b**2*c**3*e*sqrt(b**2 + c**2) - 5*b*c**5*e - 6*b*c**4*e*sqrt(b
**2 + c**2)*tan(d/2 + e*x/2) + c**6*e*tan(d/2 + e*x/2) + c**5*e*sqrt(b**2 +
```

```
c**2)), Eq(a, sqrt(b**2 + c**2))), (log(b/c + tan(d/2 + e*x/2))/(c*e), Eq(
a, b)), (x/(a + b*cos(d) + c*sin(d)), Eq(e, 0)), (log(c/(a - b) + tan(d/2 +
 e*x/2) - sqrt(-a**2 + b**2 + c**2)/(a - b))/(e*sqrt(-a**2 + b**2 + c**2))
- log(c/(a - b) + tan(d/2 + e*x/2) + sqrt(-a**2 + b**2 + c**2)/(a - b))/(e*
sqrt(-a**2 + b**2 + c**2)), True))
```

3.400 $\int \frac{1}{(a+b\cos(d+ex)+c\sin(d+ex))^2}\,dx$

Optimal. Leaf size=121

$$\frac{2a\tan^{-1}\left(\frac{(a-b)\tan\left(\frac{1}{2}(d+ex)\right)+c}{\sqrt{a^2-b^2-c^2}}\right)}{e\left(a^2-b^2-c^2\right)^{3/2}}+\frac{c\cos(d+ex)-b\sin(d+ex)}{e\left(a^2-b^2-c^2\right)(a+b\cos(d+ex)+c\sin(d+ex))}$$

```
[Out] 2*a*arctan((c+(a-b)*tan(1/2*e*x+1/2*d))/(a^2-b^2-c^2)^(1/2))/(a^2-b^2-c^2)^
(3/2)/e+(c*cos(e*x+d)-b*sin(e*x+d))/(a^2-b^2-c^2)/e/(a+b*cos(e*x+d)+c*sin(e
*x+d))
```

Rubi [A] time = 0.11, antiderivative size = 121, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {3129, 12, 3124, 618, 204}

$$\frac{2a\tan^{-1}\left(\frac{(a-b)\tan\left(\frac{1}{2}(d+ex)\right)+c}{\sqrt{a^2-b^2-c^2}}\right)}{e\left(a^2-b^2-c^2\right)^{3/2}}+\frac{c\cos(d+ex)-b\sin(d+ex)}{e\left(a^2-b^2-c^2\right)(a+b\cos(d+ex)+c\sin(d+ex))}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(-2),x]
```

```
[Out] (2*a*ArcTan[(c + (a - b)*Tan[(d + e*x)/2])/Sqrt[a^2 - b^2 - c^2]])/((a^2 -
b^2 - c^2)^(3/2)*e) + (c*Cos[d + e*x] - b*Sin[d + e*x])/((a^2 - b^2 - c^2)*
e*(a + b*Cos[d + e*x] + c*Sin[d + e*x]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
```

```
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 3124

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(-1), x_Symbol] :> Module[{f = FreeFactors[Tan[(d + e*x)/2], x]}, Dist[(2*f
)/e, Subst[Int[1/(a + b + 2*c*f*x + (a - b)*f^2*x^2), x], x, Tan[(d + e*x)/
2]/f], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0]
```

Rule 3129

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(n_), x_Symbol] :> Simp[((-(c*Cos[d + e*x]) + b*Sin[d + e*x])*(a + b*Cos[d
+ e*x] + c*Sin[d + e*x])^(n + 1))/(e*(n + 1)*(a^2 - b^2 - c^2)), x] + Dist[
1/((n + 1)*(a^2 - b^2 - c^2)), Int[(a*(n + 1) - b*(n + 2)*Cos[d + e*x] - c*
(n + 2)*Sin[d + e*x])*(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n + 1), x], x]
 /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0] && LtQ[n, -1] && N
eQ[n, -3/2]
```

Rubi steps

$$\int \frac{1}{(a+b\cos(d+ex)+c\sin(d+ex))^2}\,dx = \frac{c\cos(d+ex)-b\sin(d+ex)}{\left(a^2-b^2-c^2\right)e(a+b\cos(d+ex)+c\sin(d+ex))} - \frac{\int \frac{a}{a+b\cos(d+ex)+c\,s}}{-a^2+b^2\,-}$$

$$= \frac{c\cos(d+ex)-b\sin(d+ex)}{\left(a^2-b^2-c^2\right)e(a+b\cos(d+ex)+c\sin(d+ex))} + \frac{a\int \frac{1}{a+b\cos(d+ex)+}}{a^2-b^2\,-}$$

$$= \frac{c\cos(d+ex)-b\sin(d+ex)}{\left(a^2-b^2-c^2\right)e(a+b\cos(d+ex)+c\sin(d+ex))} + \frac{(2a)\,\mathrm{Subst}\left(\int \frac{}{a+}\right.}{}$$

$$= \frac{c\cos(d+ex)-b\sin(d+ex)}{\left(a^2-b^2-c^2\right)e(a+b\cos(d+ex)+c\sin(d+ex))} - \frac{(4a)\,\mathrm{Subst}\left(\int \frac{}{-4}\right.}{}$$

$$= \frac{2a\tan^{-1}\left(\frac{c+(a-b)\tan\left(\frac{1}{2}(d+ex)\right)}{\sqrt{a^2-b^2-c^2}}\right)}{\left(a^2-b^2-c^2\right)^{3/2}e} + \frac{c\cos(d+ex)-b\sin(d+e}{\left(a^2-b^2-c^2\right)e(a+b\cos(d+ex)+c}$$

Mathematica [A] time = 0.36, size = 116, normalized size = 0.96

$$\frac{\frac{ac+\left(b^2+c^2\right)\sin(d+ex)}{b\left(-a^2+b^2+c^2\right)(a+b\cos(d+ex)+c\sin(d+ex))}+\frac{2a\tanh^{-1}\left(\frac{(a-b)\tan\left(\frac{1}{2}(d+ex)\right)+c}{\sqrt{-a^2+b^2+c^2}}\right)}{\left(-a^2+b^2+c^2\right)^{3/2}}}{e}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(-2),x]
```

```
[Out] ((2*a*ArcTanh[(c + (a - b)*Tan[(d + e*x)/2])/Sqrt[-a^2 + b^2 + c^2]])/(-a^2
 + b^2 + c^2)^(3/2) + (a*c + (b^2 + c^2)*Sin[d + e*x])/(b*(-a^2 + b^2 + c^2
)*(a + b*Cos[d + e*x] + c*Sin[d + e*x])))/e
```

fricas [B] time = 1.04, size = 819, normalized size = 6.77

$$\left[\frac{\left(ab\cos(ex+d)+ac\sin(ex+d)+a^2\right)\sqrt{-a^2+b^2+c^2}\,\log\left(\frac{a^2b^2-2b^4-c^4-\left(a^2+3b^2\right)c^2-\left(2a^2b^2-b^4-2a^2c^2+c^4\right)\cos(ex+d)^2-2\,(\cdots}{\cdots}\right.}{2\left(\left(a^4b-2a^2b^3+b^5+bc^4-2\left(a^2b\cdots\right.\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cos(e*x+d)+c*sin(e*x+d))^2,x, algorithm="fricas")
```

```
[Out] [1/2*((a*b*cos(e*x + d) + a*c*sin(e*x + d) + a^2)*sqrt(-a^2 + b^2 + c^2)*lo
g((a^2*b^2 - 2*b^4 - c^4 - (a^2 + 3*b^2)*c^2 - (2*a^2*b^2 - b^4 - 2*a^2*c^2
 + c^4)*cos(e*x + d)^2 - 2*(a*b^3 + a*b*c^2)*cos(e*x + d) - 2*(a*b^2*c + a*
c^3 - (b*c^3 - (2*a^2*b - b^3)*c)*cos(e*x + d))*sin(e*x + d) - 2*(2*a*b*c*c
os(e*x + d)^2 - a*b*c + (b^2*c + c^3)*cos(e*x + d) - (b^3 + b*c^2 + (a*b^2
- a*c^2)*cos(e*x + d))*sin(e*x + d))*sqrt(-a^2 + b^2 + c^2))/(2*a*b*cos(e*x
 + d) + (b^2 - c^2)*cos(e*x + d)^2 + a^2 + c^2 + 2*(b*c*cos(e*x + d) + a*c)
*sin(e*x + d))) - 2*(c^3 - (a^2 - b^2)*c)*cos(e*x + d) - 2*(a^2*b - b^3 - b
*c^2)*sin(e*x + d))/((a^4*b - 2*a^2*b^3 + b^5 + b*c^4 - 2*(a^2*b - b^3)*c^2
)*e*cos(e*x + d) + (c^5 - 2*(a^2 - b^2)*c^3 + (a^4 - 2*a^2*b^2 + b^4)*c)*e*
sin(e*x + d) + (a^5 - 2*a^3*b^2 + a*b^4 + a*c^4 - 2*(a^3 - a*b^2)*c^2)*e),
((a*b*cos(e*x + d) + a*c*sin(e*x + d) + a^2)*sqrt(a^2 - b^2 - c^2)*arctan(-
(a*b*cos(e*x + d) + a*c*sin(e*x + d) + b^2 + c^2)*sqrt(a^2 - b^2 - c^2)/((c
^3 - (a^2 - b^2)*c)*cos(e*x + d) + (a^2*b - b^3 - b*c^2)*sin(e*x + d))) - (
c^3 - (a^2 - b^2)*c)*cos(e*x + d) - (a^2*b - b^3 - b*c^2)*sin(e*x + d))/((a
^4*b - 2*a^2*b^3 + b^5 + b*c^4 - 2*(a^2*b - b^3)*c^2)*e*cos(e*x + d) + (c^5
 - 2*(a^2 - b^2)*c^3 + (a^4 - 2*a^2*b^2 + b^4)*c)*e*sin(e*x + d) + (a^5 - 2
*a^3*b^2 + a*b^4 + a*c^4 - 2*(a^3 - a*b^2)*c^2)*e)]
```

giac [A] time = 0.16, size = 222, normalized size = 1.83

$$-2\left(\frac{\left(\pi\left\lfloor\frac{xe+d}{2\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(-2a+2b)+\arctan\left(-\frac{a\tan\left(\frac{1}{2}xe+\frac{1}{2}d\right)-b\tan\left(\frac{1}{2}xe+\frac{1}{2}d\right)+c}{\sqrt{a^2-b^2-c^2}}\right)\right)a}{\left(a^2-b^2-c^2\right)^{\frac{3}{2}}}+\frac{ab\tan\left(\frac{1}{2}xe\right.}{\left(a^3-a^2b-ab^2+b^3-ac^2+bc^2\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cos(e*x+d)+c*sin(e*x+d))^2,x, algorithm="giac")
```

```
[Out] -2*((pi*floor(1/2*(x*e + d)/pi + 1/2)*sgn(-2*a + 2*b) + arctan(-(a*tan(1/2*
x*e + 1/2*d) - b*tan(1/2*x*e + 1/2*d) + c)/sqrt(a^2 - b^2 - c^2)))*a/(a^2 -
 b^2 - c^2)^(3/2) + (a*b*tan(1/2*x*e + 1/2*d) - b^2*tan(1/2*x*e + 1/2*d) -
c^2*tan(1/2*x*e + 1/2*d) - a*c)/((a^3 - a^2*b - a*b^2 + b^3 - a*c^2 + b*c^2
)*(a*tan(1/2*x*e + 1/2*d)^2 - b*tan(1/2*x*e + 1/2*d)^2 + 2*c*tan(1/2*x*e +
1/2*d) + a + b)))*e^(-1)
```

maple [B] time = 0.50, size = 424, normalized size = 3.50

$$-\frac{2\tan\left(\frac{d}{2}+\frac{ex}{2}\right)ab}{e\left(a\left(\tan^2\left(\frac{d}{2}+\frac{ex}{2}\right)\right)-b\left(\tan^2\left(\frac{d}{2}+\frac{ex}{2}\right)\right)+2c\tan\left(\frac{d}{2}+\frac{ex}{2}\right)+a+b\right)\left(a^3-a^2b-a\,b^2-a\,c^2+b^3+c^2b\right)}+\frac{}{e\left(a\left(\tan\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*cos(e*x+d)+c*sin(e*x+d))^2,x)
```

```
[Out] -2/e/(a*tan(1/2*d+1/2*e*x)^2-b*tan(1/2*d+1/2*e*x)^2+2*c*tan(1/2*d+1/2*e*x)+
a+b)/(a^3-a^2*b-a*b^2-a*c^2+b^3+b*c^2)*tan(1/2*d+1/2*e*x)*a*b+2/e/(a*tan(1/
2*d+1/2*e*x)^2-b*tan(1/2*d+1/2*e*x)^2+2*c*tan(1/2*d+1/2*e*x)+a+b)/(a^3-a^2*
b-a*b^2-a*c^2+b^3+b*c^2)*tan(1/2*d+1/2*e*x)*b^2+2/e/(a*tan(1/2*d+1/2*e*x)^2
-b*tan(1/2*d+1/2*e*x)^2+2*c*tan(1/2*d+1/2*e*x)+a+b)/(a^3-a^2*b-a*b^2-a*c^2+
b^3+b*c^2)*tan(1/2*d+1/2*e*x)*c^2+2/e/(a*tan(1/2*d+1/2*e*x)^2-b*tan(1/2*d+1
/2*e*x)^2+2*c*tan(1/2*d+1/2*e*x)+a+b)*a*c/(a^3-a^2*b-a*b^2-a*c^2+b^3+b*c^2)
+2/e*a/(a^2-b^2-c^2)^(3/2)*arctan(1/2*(2*(a-b)*tan(1/2*d+1/2*e*x)+2*c)/(a^2
-b^2-c^2)^(1/2))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cos(e*x+d)+c*sin(e*x+d))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(c^2+b^2-a^2>0)', see `assume?` for
more details)Is c^2+b^2-a^2 positive or negative?
```

mupad [B] time = 3.06, size = 195, normalized size = 1.61

$$\frac{2\,a\,\mathrm{atanh}\left(\frac{\tan\left(\frac{d}{2}+\frac{e\,x}{2}\right)\,(2\,a-2\,b)+\frac{2\left(-a^2\,c+b^2\,c+c^3\right)}{-a^2+b^2+c^2}}{2\,\sqrt{-a^2+b^2+c^2}}\right)}{e\left(-a^2+b^2+c^2\right)^{3/2}}-\frac{\frac{2\,a\,c}{(a-b)\left(-a^2+b^2+c^2\right)}+\frac{2\tan\left(\frac{d}{2}+\frac{e\,x}{2}\right)\left(b^2-a\,b+c^2\right)}{(a-b)\left(-a^2+b^2+c^2\right)}}{e\left((a-b)\,\tan\left(\frac{d}{2}+\frac{e\,x}{2}\right)^2+2\,c\tan\left(\frac{d}{2}+\frac{e\,x}{2}\right)+a+b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*cos(d + e*x) + c*sin(d + e*x))^2,x)
```

```
[Out] (2*a*atanh((tan(d/2 + (e*x)/2)*(2*a - 2*b) + (2*(b^2*c - a^2*c + c^3))/(b^2
 - a^2 + c^2))/(2*(b^2 - a^2 + c^2)^(1/2))))/(e*(b^2 - a^2 + c^2)^(3/2)) -
((2*a*c)/((a - b)*(b^2 - a^2 + c^2)) + (2*tan(d/2 + (e*x)/2)*(b^2 - a*b + c
^2))/((a - b)*(b^2 - a^2 + c^2)))/(e*(a + b + tan(d/2 + (e*x)/2)^2*(a - b)
+ 2*c*tan(d/2 + (e*x)/2)))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cos(e*x+d)+c*sin(e*x+d))**2,x)
```

```
[Out] Timed out
```

3.401 $\int \frac{1}{(a+b\cos(d+ex)+c\sin(d+ex))^3} \, dx$

Optimal. Leaf size=197

$$\frac{(2a^2+b^2+c^2)\tan^{-1}\left(\frac{(a-b)\tan\left(\frac{1}{2}(d+ex)\right)+c}{\sqrt{a^2-b^2-c^2}}\right)}{e\,(a^2-b^2-c^2)^{5/2}}+\frac{3(ac\cos(d+ex)-ab\sin(d+ex))}{2e\,(a^2-b^2-c^2)^2\,(a+b\cos(d+ex)+c\sin(d+ex))}+\frac{c\,\text{co}}{2e\,(a^2-b^2-c^2}$$

```
[Out] (2*a^2+b^2+c^2)*arctan((c+(a-b)*tan(1/2*e*x+1/2*d))/(a^2-b^2-c^2)^(1/2))/(a
^2-b^2-c^2)^(5/2)/e+1/2*(c*cos(e*x+d)-b*sin(e*x+d))/(a^2-b^2-c^2)/e/(a+b*co
s(e*x+d)+c*sin(e*x+d))^2+3/2*(a*c*cos(e*x+d)-a*b*sin(e*x+d))/(a^2-b^2-c^2)^
2/e/(a+b*cos(e*x+d)+c*sin(e*x+d))
```

Rubi [A] time = 0.20, antiderivative size = 197, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {3129, 3153, 3124, 618, 204}

$$\frac{(2a^2+b^2+c^2)\tan^{-1}\left(\frac{(a-b)\tan\left(\frac{1}{2}(d+ex)\right)+c}{\sqrt{a^2-b^2-c^2}}\right)}{e\,(a^2-b^2-c^2)^{5/2}}+\frac{3(ac\cos(d+ex)-ab\sin(d+ex))}{2e\,(a^2-b^2-c^2)^2\,(a+b\cos(d+ex)+c\sin(d+ex))}+\frac{c\,\text{co}}{2e\,(a^2-b^2-c^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(-3),x]
```

```
[Out] ((2*a^2 + b^2 + c^2)*ArcTan[(c + (a - b)*Tan[(d + e*x)/2])/Sqrt[a^2 - b^2 -
 c^2]])/((a^2 - b^2 - c^2)^(5/2)*e) + (c*Cos[d + e*x] - b*Sin[d + e*x])/(2*
(a^2 - b^2 - c^2)*e*(a + b*Cos[d + e*x] + c*Sin[d + e*x])^2) + (3*(a*c*Cos[
d + e*x] - a*b*Sin[d + e*x]))/(2*(a^2 - b^2 - c^2)^2*e*(a + b*Cos[d + e*x]
+ c*Sin[d + e*x]))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 3124

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(-1), x_Symbol] :> Module[{f = FreeFactors[Tan[(d + e*x)/2], x]}, Dist[(2*f
)/e, Subst[Int[1/(a + b + 2*c*f*x + (a - b)*f^2*x^2), x], x, Tan[(d + e*x)/
2]/f], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0]
```

Rule 3129

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(n_), x_Symbol] :> Simp[((-(c*Cos[d + e*x]) + b*Sin[d + e*x])*(a + b*Cos[d
+ e*x] + c*Sin[d + e*x])^(n + 1))/(e*(n + 1)*(a^2 - b^2 - c^2)), x] + Dist[
1/((n + 1)*(a^2 - b^2 - c^2)), Int[(a*(n + 1) - b*(n + 2)*Cos[d + e*x] - c*
(n + 2)*Sin[d + e*x])*(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n + 1), x], x]
 /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0] && LtQ[n, -1] && N
eQ[n, -3/2]
```

Rule 3153

```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])
/((a_.) + cos[(d_.) + (e_.)*(x_)]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)])^2,
 x_Symbol] :> Simp[(c*B - b*C - (a*C - c*A)*Cos[d + e*x] + (a*B - b*A)*Sin[
d + e*x])/(e*(a^2 - b^2 - c^2)*(a + b*Cos[d + e*x] + c*Sin[d + e*x])), x] +
 Dist[(a*A - b*B - c*C)/(a^2 - b^2 - c^2), Int[1/(a + b*Cos[d + e*x] + c*Si
n[d + e*x]), x], x] /; FreeQ[{a, b, c, d, e, A, B, C}, x] && NeQ[a^2 - b^2
- c^2, 0] && NeQ[a*A - b*B - c*C, 0]
```

Rubi steps

$$\int \frac{1}{(a+b\cos(d+ex)+c\sin(d+ex))^3}\,dx = \frac{c\cos(d+ex)-b\sin(d+ex)}{2\left(a^2-b^2-c^2\right)e(a+b\cos(d+ex)+c\sin(d+ex))^2} - \frac{\int \frac{-2a+b\cos(d+e}{(a+b\cos(d+ex)}}{2\left(a^2-\right.}$$

$$= \frac{c\cos(d+ex)-b\sin(d+ex)}{2\left(a^2-b^2-c^2\right)e(a+b\cos(d+ex)+c\sin(d+ex))^2} + \frac{3(ac}{2\left(a^2-b^2-c\right.}$$

$$= \frac{c\cos(d+ex)-b\sin(d+ex)}{2\left(a^2-b^2-c^2\right)e(a+b\cos(d+ex)+c\sin(d+ex))^2} + \frac{3(ac}{2\left(a^2-b^2-c\right.}$$

$$= \frac{c\cos(d+ex)-b\sin(d+ex)}{2\left(a^2-b^2-c^2\right)e(a+b\cos(d+ex)+c\sin(d+ex))^2} + \frac{3(ac}{2\left(a^2-b^2-c\right.}$$

$$= \frac{\left(2a^2+b^2+c^2\right)\tan^{-1}\left(\frac{c+(a-b)\tan\left(\frac{1}{2}(d+ex)\right)}{\sqrt{a^2-b^2-c^2}}\right)}{\left(a^2-b^2-c^2\right)^{5/2}e} + \frac{c\cos(d+ex}{2\left(a^2-b^2-c^2\right)e(a+b\,c}$$

Mathematica [**A**] time = 0.95, size = 200, normalized size = 1.02

$$\frac{\frac{ac+\left(b^2+c^2\right)\sin(d+ex)}{b\left(-a^2+b^2+c^2\right)(a+b\cos(d+ex)+c\sin(d+ex))^2} - \frac{c\left(2a^2+b^2+c^2\right)+3a\left(b^2+c^2\right)\sin(d+ex)}{b\left(-a^2+b^2+c^2\right)^2(a+b\cos(d+ex)+c\sin(d+ex))} - \frac{2\left(2a^2+b^2+c^2\right)\tanh^{-1}\left(\frac{(a-b)\tan\left(\frac{1}{2}(d+ex)\right)+c}{\sqrt{-a^2+b^2+c^2}}\right)}{\left(-a^2+b^2+c^2\right)^{5/2}}}{2e}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(-3),x]
```

```
[Out] ((-2*(2*a^2 + b^2 + c^2)*ArcTanh[(c + (a - b)*Tan[(d + e*x)/2])/Sqrt[-a^2 +
 b^2 + c^2]])/(-a^2 + b^2 + c^2)^(5/2) + (a*c + (b^2 + c^2)*Sin[d + e*x])/(
b*(-a^2 + b^2 + c^2)*(a + b*Cos[d + e*x] + c*Sin[d + e*x])^2) - (c*(2*a^2 +
 b^2 + c^2) + 3*a*(b^2 + c^2)*Sin[d + e*x])/(b*(-a^2 + b^2 + c^2)^2*(a + b*
Cos[d + e*x] + c*Sin[d + e*x])))/(2*e)
```

fricas [**B**] time = 2.01, size = 1947, normalized size = 9.88

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cos(e*x+d)+c*sin(e*x+d))^3,x, algorithm="fricas")

[Out] [1/4*(6*a*b*c^3 - 12*(a*b*c^3 - (a^3*b - a*b^3)*c)*cos(e*x + d)^2 - (2*a^4
+ a^2*b^2 + c^4 + (3*a^2 + b^2)*c^2 + (2*a^2*b^2 + b^4 - 2*a^2*c^2 - c^4)*c
os(e*x + d)^2 + 2*(2*a^3*b + a*b^3 + a*b*c^2)*cos(e*x + d) + 2*(a*c^3 + (2*
a^3 + a*b^2)*c + (b*c^3 + (2*a^2*b + b^3)*c)*cos(e*x + d))*sin(e*x + d))*sq
rt(-a^2 + b^2 + c^2)*log(-(a^2*b^2 - 2*b^4 - c^4 - (a^2 + 3*b^2)*c^2 - (2*a
^2*b^2 - b^4 - 2*a^2*c^2 + c^4)*cos(e*x + d)^2 - 2*(a*b^3 + a*b*c^2)*cos(e*
x + d) - 2*(a*b^2*c + a*c^3 - (b*c^3 - (2*a^2*b - b^3)*c)*cos(e*x + d))*sin
(e*x + d) + 2*(2*a*b*c*cos(e*x + d)^2 - a*b*c + (b^2*c + c^3)*cos(e*x + d)
- (b^3 + b*c^2 + (a*b^2 - a*c^2)*cos(e*x + d))*sin(e*x + d))*sqrt(-a^2 + b^
2 + c^2))/(2*a*b*cos(e*x + d) + (b^2 - c^2)*cos(e*x + d)^2 + a^2 + c^2 + 2*
(b*c*cos(e*x + d) + a*c)*sin(e*x + d))) - 6*(a^3*b - a*b^3)*c + 2*(c^5 - (5
*a^2 - 2*b^2)*c^3 + (4*a^4 - 5*a^2*b^2 + b^4)*c)*cos(e*x + d) - 2*(4*a^4*b
- 5*a^2*b^3 + b^5 + b*c^4 - (5*a^2*b - 2*b^3)*c^2 + 3*(a^3*b^2 - a*b^4 - a^
3*c^2 + a*c^4)*cos(e*x + d))*sin(e*x + d))/((a^6*b^2 - 3*a^4*b^4 + 3*a^2*b^
6 - b^8 + c^8 - (3*a^2 - 2*b^2)*c^6 + 3*(a^4 - a^2*b^2)*c^4 - (a^6 - 3*a^2*
b^4 + 2*b^6)*c^2)*e*cos(e*x + d)^2 + 2*(a^7*b - 3*a^5*b^3 + 3*a^3*b^5 - a*b
^7 - a*b*c^6 + 3*(a^3*b - a*b^3)*c^4 - 3*(a^5*b - 2*a^3*b^3 + a*b^5)*c^2)*e
*cos(e*x + d) + (a^8 - 3*a^6*b^2 + 3*a^4*b^4 - a^2*b^6 - c^8 + (2*a^2 - 3*b
^2)*c^6 + 3*(a^2*b^2 - b^4)*c^4 - (2*a^6 - 3*a^4*b^2 + b^6)*c^2)*e - 2*((b*
c^7 - 3*(a^2*b - b^3)*c^5 + 3*(a^4*b - 2*a^2*b^3 + b^5)*c^3 - (a^6*b - 3*a^
4*b^3 + 3*a^2*b^5 - b^7)*c)*e*cos(e*x + d) + (a*c^7 - 3*(a^3 - a*b^2)*c^5 +
 3*(a^5 - 2*a^3*b^2 + a*b^4)*c^3 - (a^7 - 3*a^5*b^2 + 3*a^3*b^4 - a*b^6)*c)
*e)*sin(e*x + d)), 1/2*(3*a*b*c^3 - 6*(a*b*c^3 - (a^3*b - a*b^3)*c)*cos(e*x
 + d)^2 + (2*a^4 + a^2*b^2 + c^4 + (3*a^2 + b^2)*c^2 + (2*a^2*b^2 + b^4 - 2
*a^2*c^2 - c^4)*cos(e*x + d)^2 + 2*(2*a^3*b + a*b^3 + a*b*c^2)*cos(e*x + d)
 + 2*(a*c^3 + (2*a^3 + a*b^2)*c + (b*c^3 + (2*a^2*b + b^3)*c)*cos(e*x + d))
*sin(e*x + d))*sqrt(a^2 - b^2 - c^2)*arctan(-(a*b*cos(e*x + d) + a*c*sin(e*
x + d) + b^2 + c^2)*sqrt(a^2 - b^2 - c^2)/((c^3 - (a^2 - b^2)*c)*cos(e*x +
d) + (a^2*b - b^3 - b*c^2)*sin(e*x + d))) - 3*(a^3*b - a*b^3)*c + (c^5 - (5
*a^2 - 2*b^2)*c^3 + (4*a^4 - 5*a^2*b^2 + b^4)*c)*cos(e*x + d) - (4*a^4*b -
5*a^2*b^3 + b^5 + b*c^4 - (5*a^2*b - 2*b^3)*c^2 + 3*(a^3*b^2 - a*b^4 - a^3*
c^2 + a*c^4)*cos(e*x + d))*sin(e*x + d))/((a^6*b^2 - 3*a^4*b^4 + 3*a^2*b^6
- b^8 + c^8 - (3*a^2 - 2*b^2)*c^6 + 3*(a^4 - a^2*b^2)*c^4 - (a^6 - 3*a^2*b^
4 + 2*b^6)*c^2)*e*cos(e*x + d)^2 + 2*(a^7*b - 3*a^5*b^3 + 3*a^3*b^5 - a*b^7
 - a*b*c^6 + 3*(a^3*b - a*b^3)*c^4 - 3*(a^5*b - 2*a^3*b^3 + a*b^5)*c^2)*e*c
os(e*x + d) + (a^8 - 3*a^6*b^2 + 3*a^4*b^4 - a^2*b^6 - c^8 + (2*a^2 - 3*b^2
)*c^6 + 3*(a^2*b^2 - b^4)*c^4 - (2*a^6 - 3*a^4*b^2 + b^6)*c^2)*e - 2*((b*c^
7 - 3*(a^2*b - b^3)*c^5 + 3*(a^4*b - 2*a^2*b^3 + b^5)*c^3 - (a^6*b - 3*a^4*
b^3 + 3*a^2*b^5 - b^7)*c)*e*cos(e*x + d) + (a*c^7 - 3*(a^3 - a*b^2)*c^5 + 3
*(a^5 - 2*a^3*b^2 + a*b^4)*c^3 - (a^7 - 3*a^5*b^2 + 3*a^3*b^4 - a*b^6)*c)*e
)*sin(e*x + d))]
```

giac [B] time = 0.32, size = 892, normalized size = 4.53

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cos(e*x+d)+c*sin(e*x+d))^3,x, algorithm="giac")
```

```
[Out] -((pi*floor(1/2*(x*e + d)/pi + 1/2)*sgn(-2*a + 2*b) + arctan(-(a*tan(1/2*x*
e + 1/2*d) - b*tan(1/2*x*e + 1/2*d) + c)/sqrt(a^2 - b^2 - c^2)))*(2*a^2 + b
^2 + c^2)/((a^4 - 2*a^2*b^2 + b^4 - 2*a^2*c^2 + 2*b^2*c^2 + c^4)*sqrt(a^2 -
 b^2 - c^2)) + (4*a^4*b*tan(1/2*x*e + 1/2*d)^3 - 11*a^3*b^2*tan(1/2*x*e + 1
/2*d)^3 + 9*a^2*b^3*tan(1/2*x*e + 1/2*d)^3 - a*b^4*tan(1/2*x*e + 1/2*d)^3 -
 b^5*tan(1/2*x*e + 1/2*d)^3 - 5*a^3*c^2*tan(1/2*x*e + 1/2*d)^3 + 7*a^2*b*c^
2*tan(1/2*x*e + 1/2*d)^3 + a*b^2*c^2*tan(1/2*x*e + 1/2*d)^3 - 3*b^3*c^2*tan
(1/2*x*e + 1/2*d)^3 + 2*a*c^4*tan(1/2*x*e + 1/2*d)^3 - 2*b*c^4*tan(1/2*x*e
+ 1/2*d)^3 - 4*a^4*c*tan(1/2*x*e + 1/2*d)^2 + 12*a^3*b*c*tan(1/2*x*e + 1/2*
d)^2 - 13*a^2*b^2*c*tan(1/2*x*e + 1/2*d)^2 + 6*a*b^3*c*tan(1/2*x*e + 1/2*d)
^2 - b^4*c*tan(1/2*x*e + 1/2*d)^2 - 7*a^2*c^3*tan(1/2*x*e + 1/2*d)^2 + 6*a*
b*c^3*tan(1/2*x*e + 1/2*d)^2 + b^2*c^3*tan(1/2*x*e + 1/2*d)^2 + 2*c^5*tan(1
/2*x*e + 1/2*d)^2 + 4*a^4*b*tan(1/2*x*e + 1/2*d) - 5*a^3*b^2*tan(1/2*x*e +
1/2*d) - 3*a^2*b^3*tan(1/2*x*e + 1/2*d) + 5*a*b^4*tan(1/2*x*e + 1/2*d) - b^
5*tan(1/2*x*e + 1/2*d) - 11*a^3*c^2*tan(1/2*x*e + 1/2*d) + 3*a^2*b*c^2*tan(
1/2*x*e + 1/2*d) + 7*a*b^2*c^2*tan(1/2*x*e + 1/2*d) + b^3*c^2*tan(1/2*x*e +
 1/2*d) + 2*a*c^4*tan(1/2*x*e + 1/2*d) + 2*b*c^4*tan(1/2*x*e + 1/2*d) - 4*a
^4*c + 3*a^2*b^2*c + b^4*c + a^2*c^3 + b^2*c^3)/((a^6 - 2*a^5*b - a^4*b^2 +
 4*a^3*b^3 - a^2*b^4 - 2*a*b^5 + b^6 - 2*a^4*c^2 + 4*a^3*b*c^2 - 4*a*b^3*c^
2 + 2*b^4*c^2 + a^2*c^4 - 2*a*b*c^4 + b^2*c^4)*(a*tan(1/2*x*e + 1/2*d)^2 -
b*tan(1/2*x*e + 1/2*d)^2 + 2*c*tan(1/2*x*e + 1/2*d) + a + b)^2))*e^(-1)
```

maple [B] time = 0.56, size = 3933, normalized size = 19.96

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*cos(e*x+d)+c*sin(e*x+d))^3,x)
```

```
[Out] 3/e/(a*tan(1/2*d+1/2*e*x)^2-b*tan(1/2*d+1/2*e*x)^2+2*c*tan(1/2*d+1/2*e*x)+a
+b)^2/(a^4-2*a^2*b^2-2*a^2*c^2+b^4+2*b^2*c^2+c^4)/(a^2-2*a*b+b^2)*tan(1/2*d
+1/2*e*x)*a^2*b^3-5/e/(a*tan(1/2*d+1/2*e*x)^2-b*tan(1/2*d+1/2*e*x)^2+2*c*ta
n(1/2*d+1/2*e*x)+a+b)^2/(a^4-2*a^2*b^2-2*a^2*c^2+b^4+2*b^2*c^2+c^4)/(a^2-2*
a*b+b^2)*tan(1/2*d+1/2*e*x)*a*b^4-2/e/(a*tan(1/2*d+1/2*e*x)^2-b*tan(1/2*d+1
/2*e*x)^2+2*c*tan(1/2*d+1/2*e*x)+a+b)^2/(a^4-2*a^2*b^2-2*a^2*c^2+b^4+2*b^2*
c^2+c^4)/(a^2-2*a*b+b^2)*tan(1/2*d+1/2*e*x)*a*c^4-1/e/(a*tan(1/2*d+1/2*e*x)
^2-b*tan(1/2*d+1/2*e*x)^2+2*c*tan(1/2*d+1/2*e*x)+a+b)^2/(a^4-2*a^2*b^2-2*a^
```

2*c^2+b^4+2*b^2*c^2+c^4)/(a^2-2*a*b+b^2)*tan(1/2*d+1/2*e*x)*b^3*c^2-2/e/(a*
tan(1/2*d+1/2*e*x)^2-b*tan(1/2*d+1/2*e*x)^2+2*c*tan(1/2*d+1/2*e*x)+a+b)^2/(
a^4-2*a^2*b^2-2*a^2*c^2+b^4+2*b^2*c^2+c^4)/(a^2-2*a*b+b^2)*tan(1/2*d+1/2*e*
x)*c^4*b-3/e/(a*tan(1/2*d+1/2*e*x)^2-b*tan(1/2*d+1/2*e*x)^2+2*c*tan(1/2*d+1
/2*e*x)+a+b)^2*c/(a^4-2*a^2*b^2-2*a^2*c^2+b^4+2*b^2*c^2+c^4)/(a^2-2*a*b+b^2
)*a^2*b^2-4/e/(a*tan(1/2*d+1/2*e*x)^2-b*tan(1/2*d+1/2*e*x)^2+2*c*tan(1/2*d+
1/2*e*x)+a+b)^2/(a-b)/(a^4-2*a^2*b^2-2*a^2*c^2+b^4+2*b^2*c^2+c^4)*tan(1/2*d
+1/2*e*x)^3*a^3*b+7/e/(a*tan(1/2*d+1/2*e*x)^2-b*tan(1/2*d+1/2*e*x)^2+2*c*ta
n(1/2*d+1/2*e*x)+a+b)^2/(a-b)/(a^4-2*a^2*b^2-2*a^2*c^2+b^4+2*b^2*c^2+c^4)*t
an(1/2*d+1/2*e*x)^3*a^2*b^2+5/e/(a*tan(1/2*d+1/2*e*x)^2-b*tan(1/2*d+1/2*e*x
)^2+2*c*tan(1/2*d+1/2*e*x)+a+b)^2/(a-b)/(a^4-2*a^2*b^2-2*a^2*c^2+b^4+2*b^2*
c^2+c^4)*tan(1/2*d+1/2*e*x)^3*a^2*c^2-2/e/(a*tan(1/2*d+1/2*e*x)^2-b*tan(1/2
*d+1/2*e*x)^2+2*c*tan(1/2*d+1/2*e*x)+a+b)^2/(a-b)/(a^4-2*a^2*b^2-2*a^2*c^2+
b^4+2*b^2*c^2+c^4)*tan(1/2*d+1/2*e*x)^3*a*b^3-3/e/(a*tan(1/2*d+1/2*e*x)^2-b
*tan(1/2*d+1/2*e*x)^2+2*c*tan(1/2*d+1/2*e*x)+a+b)^2/(a-b)/(a^4-2*a^2*b^2-2*
a^2*c^2+b^4+2*b^2*c^2+c^4)*tan(1/2*d+1/2*e*x)^3*b^2*c^2+4/e/(a*tan(1/2*d+1/
2*e*x)^2-b*tan(1/2*d+1/2*e*x)^2+2*c*tan(1/2*d+1/2*e*x)+a+b)^2*c/(a^4-2*a^2*
b^2-2*a^2*c^2+b^4+2*b^2*c^2+c^4)/(a^2-2*a*b+b^2)*tan(1/2*d+1/2*e*x)^2*a^4+7
/e/(a*tan(1/2*d+1/2*e*x)^2-b*tan(1/2*d+1/2*e*x)^2+2*c*tan(1/2*d+1/2*e*x)+a+
b)^2*c^3/(a^4-2*a^2*b^2-2*a^2*c^2+b^4+2*b^2*c^2+c^4)/(a^2-2*a*b+b^2)*tan(1/
2*d+1/2*e*x)^2*a^2+1/e/(a*tan(1/2*d+1/2*e*x)^2-b*tan(1/2*d+1/2*e*x)^2+2*c*t
an(1/2*d+1/2*e*x)+a+b)^2*c/(a^4-2*a^2*b^2-2*a^2*c^2+b^4+2*b^2*c^2+c^4)/(a^2
-2*a*b+b^2)*tan(1/2*d+1/2*e*x)^2*b^4-1/e/(a*tan(1/2*d+1/2*e*x)^2-b*tan(1/2*
d+1/2*e*x)^2+2*c*tan(1/2*d+1/2*e*x)+a+b)^2*c^3/(a^4-2*a^2*b^2-2*a^2*c^2+b^4
+2*b^2*c^2+c^4)/(a^2-2*a*b+b^2)*tan(1/2*d+1/2*e*x)^2*b^2-4/e/(a*tan(1/2*d+1
/2*e*x)^2-b*tan(1/2*d+1/2*e*x)^2+2*c*tan(1/2*d+1/2*e*x)+a+b)^2/(a^4-2*a^2*b
^2-2*a^2*c^2+b^4+2*b^2*c^2+c^4)/(a^2-2*a*b+b^2)*tan(1/2*d+1/2*e*x)*a^4*b+5/
e/(a*tan(1/2*d+1/2*e*x)^2-b*tan(1/2*d+1/2*e*x)^2+2*c*tan(1/2*d+1/2*e*x)+a+b
)^2/(a^4-2*a^2*b^2-2*a^2*c^2+b^4+2*b^2*c^2+c^4)/(a^2-2*a*b+b^2)*tan(1/2*d+1
/2*e*x)*a^3*b^2+11/e/(a*tan(1/2*d+1/2*e*x)^2-b*tan(1/2*d+1/2*e*x)^2+2*c*tan
(1/2*d+1/2*e*x)+a+b)^2/(a^4-2*a^2*b^2-2*a^2*c^2+b^4+2*b^2*c^2+c^4)/(a^2-2*a
*b+b^2)*tan(1/2*d+1/2*e*x)*a^3*c^2+2/e/(a^4-2*a^2*b^2-2*a^2*c^2+b^4+2*b^2*c
^2+c^4)/(a^2-b^2-c^2)^(1/2)*arctan(1/2*(2*(a-b)*tan(1/2*d+1/2*e*x)+2*c)/(a^
2-b^2-c^2)^(1/2))*a^2+1/e/(a^4-2*a^2*b^2-2*a^2*c^2+b^4+2*b^2*c^2+c^4)/(a^2-
b^2-c^2)^(1/2)*arctan(1/2*(2*(a-b)*tan(1/2*d+1/2*e*x)+2*c)/(a^2-b^2-c^2)^(1
/2))*b^2+1/e/(a^4-2*a^2*b^2-2*a^2*c^2+b^4+2*b^2*c^2+c^4)/(a^2-b^2-c^2)^(1/2
)*arctan(1/2*(2*(a-b)*tan(1/2*d+1/2*e*x)+2*c)/(a^2-b^2-c^2)^(1/2))*c^2-1/e/
(a*tan(1/2*d+1/2*e*x)^2-b*tan(1/2*d+1/2*e*x)^2+2*c*tan(1/2*d+1/2*e*x)+a+b)^
2*c^3/(a^4-2*a^2*b^2-2*a^2*c^2+b^4+2*b^2*c^2+c^4)/(a^2-2*a*b+b^2)*a^2-1/e/(
a*tan(1/2*d+1/2*e*x)^2-b*tan(1/2*d+1/2*e*x)^2+2*c*tan(1/2*d+1/2*e*x)+a+b)^2
*c/(a^4-2*a^2*b^2-2*a^2*c^2+b^4+2*b^2*c^2+c^4)/(a^2-2*a*b+b^2)*b^4-1/e/(a*t
an(1/2*d+1/2*e*x)^2-b*tan(1/2*d+1/2*e*x)^2+2*c*tan(1/2*d+1/2*e*x)+a+b)^2*c^
3/(a^4-2*a^2*b^2-2*a^2*c^2+b^4+2*b^2*c^2+c^4)/(a^2-2*a*b+b^2)*b^2-1/e/(a*ta
n(1/2*d+1/2*e*x)^2-b*tan(1/2*d+1/2*e*x)^2+2*c*tan(1/2*d+1/2*e*x)+a+b)^2/(a-
b)/(a^4-2*a^2*b^2-2*a^2*c^2+b^4+2*b^2*c^2+c^4)*tan(1/2*d+1/2*e*x)^3*b^4-2/e

```
/(a*tan(1/2*d+1/2*e*x)^2-b*tan(1/2*d+1/2*e*x)^2+2*c*tan(1/2*d+1/2*e*x)+a+b)
^2/(a-b)/(a^4-2*a^2*b^2-2*a^2*c^2+b^4+2*b^2*c^2+c^4)*tan(1/2*d+1/2*e*x)^3*c
^4-2/e/(a*tan(1/2*d+1/2*e*x)^2-b*tan(1/2*d+1/2*e*x)^2+2*c*tan(1/2*d+1/2*e*x
)+a+b)^2*c^5/(a^4-2*a^2*b^2-2*a^2*c^2+b^4+2*b^2*c^2+c^4)/(a^2-2*a*b+b^2)*ta
n(1/2*d+1/2*e*x)^2+1/e/(a*tan(1/2*d+1/2*e*x)^2-b*tan(1/2*d+1/2*e*x)^2+2*c*t
an(1/2*d+1/2*e*x)+a+b)^2/(a^4-2*a^2*b^2-2*a^2*c^2+b^4+2*b^2*c^2+c^4)/(a^2-2
*a*b+b^2)*tan(1/2*d+1/2*e*x)*b^5+4/e/(a*tan(1/2*d+1/2*e*x)^2-b*tan(1/2*d+1/
2*e*x)^2+2*c*tan(1/2*d+1/2*e*x)+a+b)^2*c/(a^4-2*a^2*b^2-2*a^2*c^2+b^4+2*b^2
*c^2+c^4)/(a^2-2*a*b+b^2)*a^4+13/e/(a*tan(1/2*d+1/2*e*x)^2-b*tan(1/2*d+1/2*
e*x)^2+2*c*tan(1/2*d+1/2*e*x)+a+b)^2*c/(a^4-2*a^2*b^2-2*a^2*c^2+b^4+2*b^2*c
^2+c^4)/(a^2-2*a*b+b^2)*tan(1/2*d+1/2*e*x)^2*a^2*b^2-12/e/(a*tan(1/2*d+1/2*
e*x)^2-b*tan(1/2*d+1/2*e*x)^2+2*c*tan(1/2*d+1/2*e*x)+a+b)^2*c/(a^4-2*a^2*b^
2-2*a^2*c^2+b^4+2*b^2*c^2+c^4)/(a^2-2*a*b+b^2)*tan(1/2*d+1/2*e*x)^2*a^3*b-2
/e/(a*tan(1/2*d+1/2*e*x)^2-b*tan(1/2*d+1/2*e*x)^2+2*c*tan(1/2*d+1/2*e*x)+a+
b)^2/(a-b)/(a^4-2*a^2*b^2-2*a^2*c^2+b^4+2*b^2*c^2+c^4)*tan(1/2*d+1/2*e*x)^3
*a*b*c^2-6/e/(a*tan(1/2*d+1/2*e*x)^2-b*tan(1/2*d+1/2*e*x)^2+2*c*tan(1/2*d+1
/2*e*x)+a+b)^2*c/(a^4-2*a^2*b^2-2*a^2*c^2+b^4+2*b^2*c^2+c^4)/(a^2-2*a*b+b^2
)*tan(1/2*d+1/2*e*x)^2*a*b^3-6/e/(a*tan(1/2*d+1/2*e*x)^2-b*tan(1/2*d+1/2*e*
x)^2+2*c*tan(1/2*d+1/2*e*x)+a+b)^2*c^3/(a^4-2*a^2*b^2-2*a^2*c^2+b^4+2*b^2*c
^2+c^4)/(a^2-2*a*b+b^2)*tan(1/2*d+1/2*e*x)^2*a*b-3/e/(a*tan(1/2*d+1/2*e*x)^
2-b*tan(1/2*d+1/2*e*x)^2+2*c*tan(1/2*d+1/2*e*x)+a+b)^2/(a^4-2*a^2*b^2-2*a^2
*c^2+b^4+2*b^2*c^2+c^4)/(a^2-2*a*b+b^2)*tan(1/2*d+1/2*e*x)*a^2*b*c^2-7/e/(a
*tan(1/2*d+1/2*e*x)^2-b*tan(1/2*d+1/2*e*x)^2+2*c*tan(1/2*d+1/2*e*x)+a+b)^2/
(a^4-2*a^2*b^2-2*a^2*c^2+b^4+2*b^2*c^2+c^4)/(a^2-2*a*b+b^2)*tan(1/2*d+1/2*e
*x)*a*b^2*c^2
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cos(e*x+d)+c*sin(e*x+d))^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(c^2+b^2-a^2>0)', see `assume?` for
more details)Is c^2+b^2-a^2 positive or negative?
```

mupad [B] time = 6.06, size = 700, normalized size = 3.55

$$-\frac{\frac{-4\,a^4\,c+3\,a^2\,b^2\,c+a^2\,c^3+b^4\,c+b^2\,c^3}{{\left(a-b\right)}^2\,\left(a^4-2\,a^2\,b^2-2\,a^2\,c^2+b^4+2\,b^2\,c^2+c^4\right)}+\frac{\tan\left(\frac{d}{2}+\frac{e\,x}{2}\right)\,\left(4\,a^4\,b-5\,a^3\,b^2-11\,a^3\,c^2-3\,a^2\,b^3+3\,a^2\,b\,c^2+5\,a\,b^4+7\,a\,b^2\,c^2+2\,a\,c^4-b^5+b^3\,c^2+2\,b\,c^4\right)}{{\left(a-b\right)}^2\,\left(a^4-2\,a^2\,b^2-2\,a^2\,c^2+b^4+2\,b^2\,c^2+c^4\right)}+\cdots}{e\,\left(2\,a\,b+{\tan\left(\frac{d}{2}+\frac{e\,x}{2}\right)}^3\,\left(4\,a\,c-4\,b\,c\right)+{\tan\left(\frac{d}{2}+\frac{e\,x}{2}\right)}^2\,\left(2\,a^2-2\,b^2+4\,c\cdots\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*cos(d + e*x) + c*sin(d + e*x))^3,x)
```

```
[Out] - ((b^4*c - 4*a^4*c + a^2*c^3 + b^2*c^3 + 3*a^2*b^2*c)/((a - b)^2*(a^4 + b^
4 + c^4 - 2*a^2*b^2 - 2*a^2*c^2 + 2*b^2*c^2)) + (tan(d/2 + (e*x)/2)*(5*a*b^
4 + 4*a^4*b + 2*a*c^4 + 2*b*c^4 - b^5 - 3*a^2*b^3 - 5*a^3*b^2 - 11*a^3*c^2
+ b^3*c^2 + 7*a*b^2*c^2 + 3*a^2*b*c^2))/((a - b)^2*(a^4 + b^4 + c^4 - 2*a^2
*b^2 - 2*a^2*c^2 + 2*b^2*c^2)) + (tan(d/2 + (e*x)/2)^2*(2*c^5 - b^4*c - 4*a
^4*c - 7*a^2*c^3 + b^2*c^3 - 13*a^2*b^2*c + 6*a*b*c^3 + 6*a*b^3*c + 12*a^3*
b*c))/((a - b)^2*(a^4 + b^4 + c^4 - 2*a^2*b^2 - 2*a^2*c^2 + 2*b^2*c^2)) + (
tan(d/2 + (e*x)/2)^3*(2*a*b^3 + 4*a^3*b + b^4 + 2*c^4 - 7*a^2*b^2 - 5*a^2*c
^2 + 3*b^2*c^2 + 2*a*b*c^2))/((a - b)*(a^4 + b^4 + c^4 - 2*a^2*b^2 - 2*a^2*
c^2 + 2*b^2*c^2)))/(e*(2*a*b + tan(d/2 + (e*x)/2)^3*(4*a*c - 4*b*c) + tan(d
/2 + (e*x)/2)^2*(2*a^2 - 2*b^2 + 4*c^2) + tan(d/2 + (e*x)/2)^4*(a^2 - 2*a*b
 + b^2) + a^2 + b^2 + tan(d/2 + (e*x)/2)*(4*a*c + 4*b*c))) - (atanh((a^4*c
+ b^4*c + c^5 - 2*a^2*c^3 + 2*b^2*c^3 - 2*a^2*b^2*c)/(b^2 - a^2 + c^2)^(5/2
) + (tan(d/2 + (e*x)/2)*(2*a - 2*b)*(a^4 + b^4 + c^4 - 2*a^2*b^2 - 2*a^2*c^
2 + 2*b^2*c^2))/(2*(b^2 - a^2 + c^2)^(5/2)))*(2*a^2 + b^2 + c^2))/(e*(b^2 -
 a^2 + c^2)^(5/2))
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cos(e*x+d)+c*sin(e*x+d))**3,x)
```

```
[Out] Timed out
```

3.402 $\int \frac{1}{(a+b\cos(d+ex)+c\sin(d+ex))^4}\,dx$

Optimal. Leaf size=292

$$\frac{a\left(2a^2+3\left(b^2+c^2\right)\right)\tan^{-1}\left(\frac{(a-b)\tan\left(\frac{1}{2}(d+ex)\right)+c}{\sqrt{a^2-b^2-c^2}}\right)}{e\left(a^2-b^2-c^2\right)^{7/2}}+\frac{c\left(11a^2+4\left(b^2+c^2\right)\right)\cos(d+ex)-b\left(11a^2+4\left(b^2+c^2\right)\right)\sin(d+}{6e\left(a^2-b^2-c^2\right)^3(a+b\cos(d+ex)+c\sin(d+ex))}$$

```
[Out] a*(2*a^2+3*b^2+3*c^2)*arctan((c+(a-b)*tan(1/2*e*x+1/2*d))/(a^2-b^2-c^2)^(1/
2))/(a^2-b^2-c^2)^(7/2)/e+1/3*(c*cos(e*x+d)-b*sin(e*x+d))/(a^2-b^2-c^2)/e/(
a+b*cos(e*x+d)+c*sin(e*x+d))^3+5/6*(a*c*cos(e*x+d)-a*b*sin(e*x+d))/(a^2-b^2
-c^2)^2/e/(a+b*cos(e*x+d)+c*sin(e*x+d))^2+1/6*(c*(11*a^2+4*b^2+4*c^2)*cos(e
*x+d)-b*(11*a^2+4*b^2+4*c^2)*sin(e*x+d))/(a^2-b^2-c^2)^3/e/(a+b*cos(e*x+d)+
c*sin(e*x+d))
```

Rubi [A] time = 0.38, antiderivative size = 292, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {3129, 3156, 3153, 3124, 618, 204}

$$\frac{a\left(2a^2+3\left(b^2+c^2\right)\right)\tan^{-1}\left(\frac{(a-b)\tan\left(\frac{1}{2}(d+ex)\right)+c}{\sqrt{a^2-b^2-c^2}}\right)}{e\left(a^2-b^2-c^2\right)^{7/2}}+\frac{c\left(11a^2+4\left(b^2+c^2\right)\right)\cos(d+ex)-b\left(11a^2+4\left(b^2+c^2\right)\right)\sin(d+}{6e\left(a^2-b^2-c^2\right)^3(a+b\cos(d+ex)+c\sin(d+ex))}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(-4),x]
```

```
[Out] (a*(2*a^2 + 3*(b^2 + c^2))*ArcTan[(c + (a - b)*Tan[(d + e*x)/2])/Sqrt[a^2 -
 b^2 - c^2]])/((a^2 - b^2 - c^2)^(7/2)*e) + (c*Cos[d + e*x] - b*Sin[d + e*x
])/(3*(a^2 - b^2 - c^2)*e*(a + b*Cos[d + e*x] + c*Sin[d + e*x])^3) + (5*(a*
c*Cos[d + e*x] - a*b*Sin[d + e*x]))/(6*(a^2 - b^2 - c^2)^2*e*(a + b*Cos[d +
 e*x] + c*Sin[d + e*x])^2) + (c*(11*a^2 + 4*(b^2 + c^2))*Cos[d + e*x] - b*(
11*a^2 + 4*(b^2 + c^2))*Sin[d + e*x])/(6*(a^2 - b^2 - c^2)^3*e*(a + b*Cos[d
 + e*x] + c*Sin[d + e*x]))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 3124

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(-1), x_Symbol] :> Module[{f = FreeFactors[Tan[(d + e*x)/2], x]}, Dist[(2*f
)/e, Subst[Int[1/(a + b + 2*c*f*x + (a - b)*f^2*x^2), x], x, Tan[(d + e*x)/
2]/f], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0]
```

Rule 3129

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(n_), x_Symbol] :> Simp[((-(c*Cos[d + e*x]) + b*Sin[d + e*x])*(a + b*Cos[d
+ e*x] + c*Sin[d + e*x])^(n + 1))/(e*(n + 1)*(a^2 - b^2 - c^2)), x] + Dist[
1/((n + 1)*(a^2 - b^2 - c^2)), Int[(a*(n + 1) - b*(n + 2)*Cos[d + e*x] - c*
(n + 2)*Sin[d + e*x])*(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n + 1), x], x]
 /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0] && LtQ[n, -1] && N
eQ[n, -3/2]
```

Rule 3153

```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])
/((a_.) + cos[(d_.) + (e_.)*(x_)]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)])^2,
 x_Symbol] :> Simp[(c*B - b*C - (a*C - c*A)*Cos[d + e*x] + (a*B - b*A)*Sin[
d + e*x])/(e*(a^2 - b^2 - c^2)*(a + b*Cos[d + e*x] + c*Sin[d + e*x])), x] +
 Dist[(a*A - b*B - c*C)/(a^2 - b^2 - c^2), Int[1/(a + b*Cos[d + e*x] + c*Si
n[d + e*x]), x], x] /; FreeQ[{a, b, c, d, e, A, B, C}, x] && NeQ[a^2 - b^2
- c^2, 0] && NeQ[a*A - b*B - c*C, 0]
```

Rule 3156

```
Int[((a_.) + cos[(d_.) + (e_.)*(x_)]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)])
^(n_)*((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)
]), x_Symbol] :> -Simp[((c*B - b*C - (a*C - c*A)*Cos[d + e*x] + (a*B - b*A)
*Sin[d + e*x])*(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n + 1))/(e*(n + 1)*(a
^2 - b^2 - c^2)), x] + Dist[1/((n + 1)*(a^2 - b^2 - c^2)), Int[(a + b*Cos[d
 + e*x] + c*Sin[d + e*x])^(n + 1)*Simp[(n + 1)*(a*A - b*B - c*C) + (n + 2)*
(a*B - b*A)*Cos[d + e*x] + (n + 2)*(a*C - c*A)*Sin[d + e*x], x], x], x] /;
FreeQ[{a, b, c, d, e, A, B, C}, x] && LtQ[n, -1] && NeQ[a^2 - b^2 - c^2, 0]
 && NeQ[n, -2]
```

Rubi steps

$$\int \frac{1}{(a+b\cos(d+ex)+c\sin(d+ex))^4}\,dx = \frac{c\cos(d+ex)-b\sin(d+ex)}{3\left(a^2-b^2-c^2\right)e(a+b\cos(d+ex)+c\sin(d+ex))^3} - \frac{\int \frac{-3a+2b\cos(d+\dots}{(a+b\cos(d+e\dots}}{3\left(a^2-\dots\right.}$$

$$= \frac{c\cos(d+ex)-b\sin(d+ex)}{3\left(a^2-b^2-c^2\right)e(a+b\cos(d+ex)+c\sin(d+ex))^3} + \frac{5(a\dots}{6\left(a^2-b^2-c\dots\right.}$$

$$= \frac{c\cos(d+ex)-b\sin(d+ex)}{3\left(a^2-b^2-c^2\right)e(a+b\cos(d+ex)+c\sin(d+ex))^3} + \frac{5(a\dots}{6\left(a^2-b^2-c\dots\right.}$$

$$= \frac{c\cos(d+ex)-b\sin(d+ex)}{3\left(a^2-b^2-c^2\right)e(a+b\cos(d+ex)+c\sin(d+ex))^3} + \frac{5(a\dots}{6\left(a^2-b^2-c\dots\right.}$$

$$= \frac{c\cos(d+ex)-b\sin(d+ex)}{3\left(a^2-b^2-c^2\right)e(a+b\cos(d+ex)+c\sin(d+ex))^3} + \frac{5(a\dots}{6\left(a^2-b^2-c\dots\right.}$$

$$= \frac{a\left(2a^2+3\left(b^2+c^2\right)\right)\tan^{-1}\left(\frac{c+(a-b)\tan\left(\frac{1}{2}(d+ex)\right)}{\sqrt{a^2-b^2-c^2}}\right)}{\left(a^2-b^2-c^2\right)^{7/2}e} + \frac{c\cos(d\dots}{3\left(a^2-b^2-c^2\right)e(a\dots}$$

Mathematica [B] time = 2.11, size = 606, normalized size = 2.08

$$\frac{\frac{24a\left(2a^2+3\left(b^2+c^2\right)\right)\tanh^{-1}\left(\frac{(a-b)\tan\left(\frac{1}{2}(d+ex)\right)+c}{\sqrt{-a^2+b^2+c^2}}\right)}{\left(-a^2+b^2+c^2\right)^{7/2}} + \frac{44a^5c+72a^4b^2\sin(d+ex)+132a^4c^2\sin(d+ex)+54a^3b^3\sin(2(d+ex))+82a^3b^2c+78a^3bc^2\sin(2(d+ex)\dots}{\dots}}{\dots}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(-4),x]
```

```
[Out] ((24*a*(2*a^2 + 3*(b^2 + c^2))*ArcTanh[(c + (a - b)*Tan[(d + e*x)/2])/Sqrt[
-a^2 + b^2 + c^2]])/(-a^2 + b^2 + c^2)^(7/2) + (44*a^5*c + 82*a^3*b^2*c + 2
4*a*b^4*c + 82*a^3*c^3 + 48*a*b^2*c^3 + 24*a*c^5 + 30*a^2*b*c*(2*a^2 + 3*(b
^2 + c^2))*Cos[d + e*x] - 6*a*c*(-2*b^4 + 2*b^2*c^2 + 4*c^4 + a^2*(7*b^2 +
11*c^2))*Cos[2*(d + e*x)] - 22*a^2*b^3*c*Cos[3*(d + e*x)] - 8*b^5*c*Cos[3*(
d + e*x)] - 22*a^2*b*c^3*Cos[3*(d + e*x)] - 16*b^3*c^3*Cos[3*(d + e*x)] - 8
*b*c^5*Cos[3*(d + e*x)] + 72*a^4*b^2*Sin[d + e*x] - 9*a^2*b^4*Sin[d + e*x]
```

```
+ 12*b^6*Sin[d + e*x] + 132*a^4*c^2*Sin[d + e*x] + 72*a^2*b^2*c^2*Sin[d + e
*x] + 36*b^4*c^2*Sin[d + e*x] + 81*a^2*c^4*Sin[d + e*x] + 36*b^2*c^4*Sin[d
+ e*x] + 12*c^6*Sin[d + e*x] + 54*a^3*b^3*Sin[2*(d + e*x)] + 6*a*b^5*Sin[2*
(d + e*x)] + 78*a^3*b*c^2*Sin[2*(d + e*x)] + 48*a*b^3*c^2*Sin[2*(d + e*x)]
+ 42*a*b*c^4*Sin[2*(d + e*x)] + 11*a^2*b^4*Sin[3*(d + e*x)] + 4*b^6*Sin[3*(
d + e*x)] + 4*b^4*c^2*Sin[3*(d + e*x)] - 11*a^2*c^4*Sin[3*(d + e*x)] - 4*b^
2*c^4*Sin[3*(d + e*x)] - 4*c^6*Sin[3*(d + e*x)])/(b*(-a^2 + b^2 + c^2)^3*(a
 + b*Cos[d + e*x] + c*Sin[d + e*x])^3))/(24*e)
```

fricas [B] time = 1.63, size = 4069, normalized size = 13.93

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cos(e*x+d)+c*sin(e*x+d))^4,x, algorithm="fricas")

[Out] [1/12*(6*a*b*c^5 + 12*(4*a^3*b + a*b^3)*c^3 + 2*(4*c^7 + (7*a^2 - 4*b^2)*c^
5 - (11*a^4 + 14*a^2*b^2 + 20*b^4)*c^3 + 3*(11*a^4*b^2 - 7*a^2*b^4 - 4*b^6)
*c)*cos(e*x + d)^3 - 12*(a*b*c^5 + 2*(4*a^3*b + a*b^3)*c^3 - (9*a^5*b - 8*a
^3*b^3 - a*b^5)*c)*cos(e*x + d)^2 + 3*(2*a^6 + 3*a^4*b^2 + 9*a^2*c^4 + (2*a
^3*b^3 + 3*a*b^5 - 9*a*b*c^4 - 6*(a^3*b + a*b^3)*c^2)*cos(e*x + d)^3 + 9*(a
^4 + a^2*b^2)*c^2 + 3*(2*a^4*b^2 + 3*a^2*b^4 - 2*a^4*c^2 - 3*a^2*c^4)*cos(e
*x + d)^2 + 3*(2*a^5*b + 3*a^3*b^3 + 3*a*b*c^4 + (5*a^3*b + 3*a*b^3)*c^2)*c
os(e*x + d) + (3*a*c^5 + (11*a^3 + 3*a*b^2)*c^3 - (3*a*c^5 + 2*(a^3 - 3*a*b
^2)*c^3 - 3*(2*a^3*b^2 + 3*a*b^4)*c)*cos(e*x + d)^2 + 3*(2*a^5 + 3*a^3*b^2)
*c + 6*(3*a^2*b*c^3 + (2*a^4*b + 3*a^2*b^3)*c)*cos(e*x + d))*sin(e*x + d))*
sqrt(-a^2 + b^2 + c^2)*log((a^2*b^2 - 2*b^4 - c^4 - (a^2 + 3*b^2)*c^2 - (2*
a^2*b^2 - b^4 - 2*a^2*c^2 + c^4)*cos(e*x + d)^2 - 2*(a*b^3 + a*b*c^2)*cos(e
*x + d) - 2*(a*b^2*c + a*c^3 - (b*c^3 - (2*a^2*b - b^3)*c)*cos(e*x + d))*si
n(e*x + d) - 2*(2*a*b*c*cos(e*x + d)^2 - a*b*c + (b^2*c + c^3)*cos(e*x + d)
 - (b^3 + b*c^2 + (a*b^2 - a*c^2)*cos(e*x + d))*sin(e*x + d))*sqrt(-a^2 + b
^2 + c^2))/(2*a*b*cos(e*x + d) + (b^2 - c^2)*cos(e*x + d)^2 + a^2 + c^2 + 2
*(b*c*cos(e*x + d) + a*c)*sin(e*x + d))) - 6*(9*a^5*b - 8*a^3*b^3 - a*b^5)*
c - 6*(2*b^2*c^5 + 2*c^7 + (4*a^4 - 7*a^2*b^2 - 2*b^4)*c^3 - (6*a^6 - 15*a^
4*b^2 + 7*a^2*b^4 + 2*b^6)*c)*cos(e*x + d) - 2*(18*a^6*b - 23*a^4*b^3 + 7*a
^2*b^5 - 2*b^7 - 14*b^3*c^4 - 6*b*c^6 - (12*a^4*b - 7*a^2*b^3 + 10*b^5)*c^2
 + (11*a^4*b^3 - 7*a^2*b^5 - 4*b^7 + 12*b*c^6 + (21*a^2*b + 20*b^3)*c^4 - (
33*a^4*b - 14*a^2*b^3 - 4*b^5)*c^2)*cos(e*x + d)^2 + 3*(9*a^5*b^2 - 8*a^3*b
^4 - a*b^6 + a*c^6 + (8*a^3 + a*b^2)*c^4 - (9*a^5 + a*b^4)*c^2)*cos(e*x + d
))*sin(e*x + d))/((a^8*b^3 - 4*a^6*b^5 + 6*a^4*b^7 - 4*a^2*b^9 + b^11 - 3*b
*c^10 + (12*a^2*b - 11*b^3)*c^8 - 2*(9*a^4*b - 16*a^2*b^3 + 7*b^5)*c^6 + 6*
(2*a^6*b - 5*a^4*b^3 + 4*a^2*b^5 - b^7)*c^4 - (3*a^8*b - 8*a^6*b^3 + 6*a^4*
b^5 - b^9)*c^2)*e*cos(e*x + d)^3 + 3*(a^9*b^2 - 4*a^7*b^4 + 6*a^5*b^6 - 4*a
^3*b^8 + a*b^10 - a*c^10 + (4*a^3 - 3*a*b^2)*c^8 - 2*(3*a^5 - 4*a^3*b^2 + a
*b^4)*c^6 + 2*(2*a^7 - 3*a^5*b^2 + a*b^6)*c^4 - (a^9 - 6*a^5*b^4 + 8*a^3*b^
```

6 - 3*a*b^8)*c^2)*e*cos(e*x + d)^2 + 3*(a^10*b - 4*a^8*b^3 + 6*a^6*b^5 - 4*a^4*b^7 + a^2*b^9 + b*c^10 - (3*a^2*b - 4*b^3)*c^8 + 2*(a^4*b - 4*a^2*b^3 + 3*b^5)*c^6 + 2*(a^6*b - 3*a^2*b^5 + 2*b^7)*c^4 - (3*a^8*b - 8*a^6*b^3 + 6*a^4*b^5 - b^9)*c^2)*e*cos(e*x + d) + (a^11 - 4*a^9*b^2 + 6*a^7*b^4 - 4*a^5*b^6 + a^3*b^8 + 3*a*c^10 - (11*a^3 - 12*a*b^2)*c^8 + 2*(7*a^5 - 16*a^3*b^2 + 9*a*b^4)*c^6 - 6*(a^7 - 4*a^5*b^2 + 5*a^3*b^4 - 2*a*b^6)*c^4 - (a^9 - 6*a^5*b^4 + 8*a^3*b^6 - 3*a*b^8)*c^2)*e - ((c^11 - (4*a^2 - b^2)*c^9 + 6*(a^4 - b^4)*c^7 - 2*(2*a^6 + 3*a^4*b^2 - 12*a^2*b^4 + 7*b^6)*c^5 + (a^8 + 8*a^6*b^2 - 30*a^4*b^4 + 32*a^2*b^6 - 11*b^8)*c^3 - 3*(a^8*b^2 - 4*a^6*b^4 + 6*a^4*b^6 - 4*a^2*b^8 + b^10)*c)*e*cos(e*x + d)^2 - 6*(a*b*c^9 - 4*(a^3*b - a*b^3)*c^7 + 6*(a^5*b - 2*a^3*b^3 + a*b^5)*c^5 - 4*(a^7*b - 3*a^5*b^3 + 3*a^3*b^5 - a*b^7)*c^3 + (a^9*b - 4*a^7*b^3 + 6*a^5*b^5 - 4*a^3*b^7 + a*b^9)*c)*e*cos(e*x + d) - (c^11 - (a^2 - 4*b^2)*c^9 - 6*(a^4 - b^4)*c^7 + 2*(7*a^6 - 12*a^4*b^2 + 3*a^2*b^4 + 2*b^6)*c^5 - (11*a^8 - 32*a^6*b^2 + 30*a^4*b^4 - 8*a^2*b^6 - b^8)*c^3 + 3*(a^10 - 4*a^8*b^2 + 6*a^6*b^4 - 4*a^4*b^6 + a^2*b^8)*c)*e)*sin(e*x + d)), 1/6*(3*a*b*c^5 + 6*(4*a^3*b + a*b^3)*c^3 + (4*c^7 + (7*a^2 - 4*b^2)*c^5 - (11*a^4 + 14*a^2*b^2 + 20*b^4)*c^3 + 3*(11*a^4*b^2 - 7*a^2*b^4 - 4*b^6)*c)*cos(e*x + d)^3 - 6*(a*b*c^5 + 2*(4*a^3*b + a*b^3)*c^3 - (9*a^5*b - 8*a^3*b^3 - a*b^5)*c)*cos(e*x + d)^2 + 3*(2*a^6 + 3*a^4*b^2 + 9*a^2*c^4 + (2*a^3*b^3 + 3*a*b^5 - 9*a*b*c^4 - 6*(a^3*b + a*b^3)*c^2)*cos(e*x + d)^3 + 9*(a^4 + a^2*b^2)*c^2 + 3*(2*a^4*b^2 + 3*a^2*b^4 - 2*a^4*c^2 - 3*a^2*c^4)*cos(e*x + d)^2 + 3*(2*a^5*b + 3*a^3*b^3 + 3*a*b*c^4 + (5*a^3*b + 3*a*b^3)*c^2)*cos(e*x + d) + (3*a*c^5 + (11*a^3 + 3*a*b^2)*c^3 - (3*a*c^5 + 2*(a^3 - 3*a*b^2)*c^3 - 3*(2*a^3*b^2 + 3*a*b^4)*c)*cos(e*x + d)^2 + 3*(2*a^5 + 3*a^3*b^2)*c + 6*(3*a^2*b*c^3 + (2*a^4*b + 3*a^2*b^3)*c)*cos(e*x + d))*sin(e*x + d))*sqrt(a^2 - b^2 - c^2)*arctan(-(a*b*cos(e*x + d) + a*c*sin(e*x + d) + b^2 + c^2)*sqrt(a^2 - b^2 - c^2)/((c^3 - (a^2 - b^2)*c)*cos(e*x + d) + (a^2*b - b^3 - b*c^2)*sin(e*x + d))) - 3*(9*a^5*b - 8*a^3*b^3 - a*b^5)*c - 3*(2*b^2*c^5 + 2*c^7 + (4*a^4 - 7*a^2*b^2 - 2*b^4)*c^3 - (6*a^6 - 15*a^4*b^2 + 7*a^2*b^4 + 2*b^6)*c)*cos(e*x + d) - (18*a^6*b - 23*a^4*b^3 + 7*a^2*b^5 - 2*b^7 - 14*b^3*c^4 - 6*b*c^6 - (12*a^4*b - 7*a^2*b^3 + 10*b^5)*c^2 + (11*a^4*b^3 - 7*a^2*b^5 - 4*b^7 + 12*b*c^6 + (21*a^2*b + 20*b^3)*c^4 - (33*a^4*b - 14*a^2*b^3 - 4*b^5)*c^2)*cos(e*x + d)^2 + 3*(9*a^5*b^2 - 8*a^3*b^4 - a*b^6 + a*c^6 + (8*a^3 + a*b^2)*c^4 - (9*a^5 + a*b^4)*c^2)*cos(e*x + d))*sin(e*x + d))/((a^8*b^3 - 4*a^6*b^5 + 6*a^4*b^7 - 4*a^2*b^9 + b^11 - 3*b*c^10 + (12*a^2*b - 11*b^3)*c^8 - 2*(9*a^4*b - 16*a^2*b^3 + 7*b^5)*c^6 + 6*(2*a^6*b - 5*a^4*b^3 + 4*a^2*b^5 - b^7)*c^4 - (3*a^8*b - 8*a^6*b^3 + 6*a^4*b^5 - b^9)*c^2)*e*cos(e*x + d)^3 + 3*(a^9*b^2 - 4*a^7*b^4 + 6*a^5*b^6 - 4*a^3*b^8 + a*b^10 - a*c^10 + (4*a^3 - 3*a*b^2)*c^8 - 2*(3*a^5 - 4*a^3*b^2 + a*b^4)*c^6 + 2*(2*a^7 - 3*a^5*b^2 + a*b^6)*c^4 - (a^9 - 6*a^5*b^4 + 8*a^3*b^6 - 3*a*b^8)*c^2)*e*cos(e*x + d)^2 + 3*(a^10*b - 4*a^8*b^3 + 6*a^6*b^5 - 4*a^4*b^7 + a^2*b^9 + b*c^10 - (3*a^2*b - 4*b^3)*c^8 + 2*(a^4*b - 4*a^2*b^3 + 3*b^5)*c^6 + 2*(a^6*b - 3*a^2*b^5 + 2*b^7)*c^4 - (3*a^8*b - 8*a^6*b^3 + 6*a^4*b^5 - b^9)*c^2)*e*cos(e*x + d) + (a^11 - 4*a^9*b^2 + 6*a^7*b^4 - 4*a^5*b^6 + a^3*b^8 + 3*a*c^10 - (11*a^3 - 12*a*b^2)*c^8 + 2*(7*a^5 - 16*a^3*b^2

```
+ 9*a*b^4)*c^6 - 6*(a^7 - 4*a^5*b^2 + 5*a^3*b^4 - 2*a*b^6)*c^4 - (a^9 - 6*
a^5*b^4 + 8*a^3*b^6 - 3*a*b^8)*c^2)*e - ((c^11 - (4*a^2 - b^2)*c^9 + 6*(a^4
 - b^4)*c^7 - 2*(2*a^6 + 3*a^4*b^2 - 12*a^2*b^4 + 7*b^6)*c^5 + (a^8 + 8*a^6
*b^2 - 30*a^4*b^4 + 32*a^2*b^6 - 11*b^8)*c^3 - 3*(a^8*b^2 - 4*a^6*b^4 + 6*a
^4*b^6 - 4*a^2*b^8 + b^10)*c)*e*cos(e*x + d)^2 - 6*(a*b*c^9 - 4*(a^3*b - a*
b^3)*c^7 + 6*(a^5*b - 2*a^3*b^3 + a*b^5)*c^5 - 4*(a^7*b - 3*a^5*b^3 + 3*a^3
*b^5 - a*b^7)*c^3 + (a^9*b - 4*a^7*b^3 + 6*a^5*b^5 - 4*a^3*b^7 + a*b^9)*c)*
e*cos(e*x + d) - (c^11 - (a^2 - 4*b^2)*c^9 - 6*(a^4 - b^4)*c^7 + 2*(7*a^6 -
 12*a^4*b^2 + 3*a^2*b^4 + 2*b^6)*c^5 - (11*a^8 - 32*a^6*b^2 + 30*a^4*b^4 -
8*a^2*b^6 - b^8)*c^3 + 3*(a^10 - 4*a^8*b^2 + 6*a^6*b^4 - 4*a^4*b^6 + a^2*b^
8)*c)*e)*sin(e*x + d))]
```

giac [B] time = 0.58, size = 2671, normalized size = 9.15

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cos(e*x+d)+c*sin(e*x+d))^4,x, algorithm="giac")
```

```
[Out] -1/3*(3*(2*a^3 + 3*a*b^2 + 3*a*c^2)*(pi*floor(1/2*(x*e + d)/pi + 1/2)*sgn(-
2*a + 2*b) + arctan(-(a*tan(1/2*x*e + 1/2*d) - b*tan(1/2*x*e + 1/2*d) + c)/
sqrt(a^2 - b^2 - c^2)))/((a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6 - 3*a^4*c^2 + 6
*a^2*b^2*c^2 - 3*b^4*c^2 + 3*a^2*c^4 - 3*b^2*c^4 - c^6)*sqrt(a^2 - b^2 - c^
2)) + (18*a^7*b*tan(1/2*x*e + 1/2*d)^5 - 81*a^6*b^2*tan(1/2*x*e + 1/2*d)^5
+ 141*a^5*b^3*tan(1/2*x*e + 1/2*d)^5 - 120*a^4*b^4*tan(1/2*x*e + 1/2*d)^5 +
 60*a^3*b^5*tan(1/2*x*e + 1/2*d)^5 - 33*a^2*b^6*tan(1/2*x*e + 1/2*d)^5 + 21
*a*b^7*tan(1/2*x*e + 1/2*d)^5 - 6*b^8*tan(1/2*x*e + 1/2*d)^5 - 27*a^6*c^2*t
an(1/2*x*e + 1/2*d)^5 + 81*a^5*b*c^2*tan(1/2*x*e + 1/2*d)^5 - 72*a^4*b^2*c^
2*tan(1/2*x*e + 1/2*d)^5 + 18*a^3*b^3*c^2*tan(1/2*x*e + 1/2*d)^5 - 27*a^2*b
^4*c^2*tan(1/2*x*e + 1/2*d)^5 + 45*a*b^5*c^2*tan(1/2*x*e + 1/2*d)^5 - 18*b^
6*c^2*tan(1/2*x*e + 1/2*d)^5 + 18*a^4*c^4*tan(1/2*x*e + 1/2*d)^5 - 36*a^3*b
*c^4*tan(1/2*x*e + 1/2*d)^5 + 36*a*b^3*c^4*tan(1/2*x*e + 1/2*d)^5 - 18*b^4*
c^4*tan(1/2*x*e + 1/2*d)^5 - 6*a^2*c^6*tan(1/2*x*e + 1/2*d)^5 + 12*a*b*c^6*
tan(1/2*x*e + 1/2*d)^5 - 6*b^2*c^6*tan(1/2*x*e + 1/2*d)^5 - 18*a^7*c*tan(1/
2*x*e + 1/2*d)^4 + 108*a^6*b*c*tan(1/2*x*e + 1/2*d)^4 - 261*a^5*b^2*c*tan(1
/2*x*e + 1/2*d)^4 + 336*a^4*b^3*c*tan(1/2*x*e + 1/2*d)^4 - 264*a^3*b^4*c*ta
n(1/2*x*e + 1/2*d)^4 + 144*a^2*b^5*c*tan(1/2*x*e + 1/2*d)^4 - 57*a*b^6*c*ta
n(1/2*x*e + 1/2*d)^4 + 12*b^7*c*tan(1/2*x*e + 1/2*d)^4 - 81*a^5*c^3*tan(1/2
*x*e + 1/2*d)^4 + 216*a^4*b*c^3*tan(1/2*x*e + 1/2*d)^4 - 198*a^3*b^2*c^3*ta
n(1/2*x*e + 1/2*d)^4 + 108*a^2*b^3*c^3*tan(1/2*x*e + 1/2*d)^4 - 81*a*b^4*c^
3*tan(1/2*x*e + 1/2*d)^4 + 36*b^5*c^3*tan(1/2*x*e + 1/2*d)^4 + 36*a^3*c^5*t
an(1/2*x*e + 1/2*d)^4 - 36*a^2*b*c^5*tan(1/2*x*e + 1/2*d)^4 - 36*a*b^2*c^5*
tan(1/2*x*e + 1/2*d)^4 + 36*b^3*c^5*tan(1/2*x*e + 1/2*d)^4 - 12*a*c^7*tan(1
/2*x*e + 1/2*d)^4 + 12*b*c^7*tan(1/2*x*e + 1/2*d)^4 + 36*a^7*b*tan(1/2*x*e
+ 1/2*d)^3 - 108*a^6*b^2*tan(1/2*x*e + 1/2*d)^3 + 76*a^5*b^3*tan(1/2*x*e +
```

```
1/2*d)^3 + 60*a^4*b^4*tan(1/2*x*e + 1/2*d)^3 - 100*a^3*b^5*tan(1/2*x*e + 1/
2*d)^3 + 44*a^2*b^6*tan(1/2*x*e + 1/2*d)^3 - 12*a*b^7*tan(1/2*x*e + 1/2*d)^
3 + 4*b^8*tan(1/2*x*e + 1/2*d)^3 - 108*a^6*c^2*tan(1/2*x*e + 1/2*d)^3 + 240
*a^5*b*c^2*tan(1/2*x*e + 1/2*d)^3 - 162*a^4*b^2*c^2*tan(1/2*x*e + 1/2*d)^3
+ 122*a^3*b^3*c^2*tan(1/2*x*e + 1/2*d)^3 - 174*a^2*b^4*c^2*tan(1/2*x*e + 1/
2*d)^3 + 78*a*b^5*c^2*tan(1/2*x*e + 1/2*d)^3 + 4*b^6*c^2*tan(1/2*x*e + 1/2*
d)^3 - 42*a^4*c^4*tan(1/2*x*e + 1/2*d)^3 + 162*a^3*b*c^4*tan(1/2*x*e + 1/2*
d)^3 - 210*a^2*b^2*c^4*tan(1/2*x*e + 1/2*d)^3 + 102*a*b^3*c^4*tan(1/2*x*e +
 1/2*d)^3 - 12*b^4*c^4*tan(1/2*x*e + 1/2*d)^3 + 8*a^2*c^6*tan(1/2*x*e + 1/2
*d)^3 + 12*a*b*c^6*tan(1/2*x*e + 1/2*d)^3 - 20*b^2*c^6*tan(1/2*x*e + 1/2*d)
^3 - 8*c^8*tan(1/2*x*e + 1/2*d)^3 - 36*a^7*c*tan(1/2*x*e + 1/2*d)^2 + 108*a
^6*b*c*tan(1/2*x*e + 1/2*d)^2 - 108*a^5*b^2*c*tan(1/2*x*e + 1/2*d)^2 + 12*a
^4*b^3*c*tan(1/2*x*e + 1/2*d)^2 + 84*a^3*b^4*c*tan(1/2*x*e + 1/2*d)^2 - 108
*a^2*b^5*c*tan(1/2*x*e + 1/2*d)^2 + 60*a*b^6*c*tan(1/2*x*e + 1/2*d)^2 - 12*
b^7*c*tan(1/2*x*e + 1/2*d)^2 - 120*a^5*c^3*tan(1/2*x*e + 1/2*d)^2 + 132*a^4
*b*c^3*tan(1/2*x*e + 1/2*d)^2 + 42*a^3*b^2*c^3*tan(1/2*x*e + 1/2*d)^2 - 36*
a^2*b^3*c^3*tan(1/2*x*e + 1/2*d)^2 + 18*a*b^4*c^3*tan(1/2*x*e + 1/2*d)^2 -
36*b^5*c^3*tan(1/2*x*e + 1/2*d)^2 + 18*a^3*c^5*tan(1/2*x*e + 1/2*d)^2 + 72*
a^2*b*c^5*tan(1/2*x*e + 1/2*d)^2 - 54*a*b^2*c^5*tan(1/2*x*e + 1/2*d)^2 - 36
*b^3*c^5*tan(1/2*x*e + 1/2*d)^2 - 12*a*c^7*tan(1/2*x*e + 1/2*d)^2 - 12*b*c^
7*tan(1/2*x*e + 1/2*d)^2 + 18*a^7*b*tan(1/2*x*e + 1/2*d) - 27*a^6*b^2*tan(1
/2*x*e + 1/2*d) - 21*a^5*b^3*tan(1/2*x*e + 1/2*d) + 48*a^4*b^4*tan(1/2*x*e
+ 1/2*d) - 12*a^3*b^5*tan(1/2*x*e + 1/2*d) - 15*a^2*b^6*tan(1/2*x*e + 1/2*d
) + 15*a*b^7*tan(1/2*x*e + 1/2*d) - 6*b^8*tan(1/2*x*e + 1/2*d) - 81*a^6*c^2
*tan(1/2*x*e + 1/2*d) + 27*a^5*b*c^2*tan(1/2*x*e + 1/2*d) + 90*a^4*b^2*c^2*
tan(1/2*x*e + 1/2*d) + 9*a^2*b^4*c^2*tan(1/2*x*e + 1/2*d) - 27*a*b^5*c^2*ta
n(1/2*x*e + 1/2*d) - 18*b^6*c^2*tan(1/2*x*e + 1/2*d) + 12*a^4*c^4*tan(1/2*x
*e + 1/2*d) + 42*a^3*b*c^4*tan(1/2*x*e + 1/2*d) + 18*a^2*b^2*c^4*tan(1/2*x*
e + 1/2*d) - 54*a*b^3*c^4*tan(1/2*x*e + 1/2*d) - 18*b^4*c^4*tan(1/2*x*e + 1
/2*d) - 6*a^2*c^6*tan(1/2*x*e + 1/2*d) - 12*a*b*c^6*tan(1/2*x*e + 1/2*d) -
6*b^2*c^6*tan(1/2*x*e + 1/2*d) - 18*a^7*c + 21*a^5*b^2*c + 12*a^3*b^4*c - 1
5*a*b^6*c + 5*a^5*c^3 + 16*a^3*b^2*c^3 - 21*a*b^4*c^3 - 2*a^3*c^5 - 6*a*b^2
*c^5)/((a^9 - 3*a^8*b + 8*a^6*b^3 - 6*a^5*b^4 - 6*a^4*b^5 + 8*a^3*b^6 - 3*a
*b^8 + b^9 - 3*a^7*c^2 + 9*a^6*b*c^2 - 3*a^5*b^2*c^2 - 15*a^4*b^3*c^2 + 15*
a^3*b^4*c^2 + 3*a^2*b^5*c^2 - 9*a*b^6*c^2 + 3*b^7*c^2 + 3*a^5*c^4 - 9*a^4*b
*c^4 + 6*a^3*b^2*c^4 + 6*a^2*b^3*c^4 - 9*a*b^4*c^4 + 3*b^5*c^4 - a^3*c^6 +
3*a^2*b*c^6 - 3*a*b^2*c^6 + b^3*c^6)*(a*tan(1/2*x*e + 1/2*d)^2 - b*tan(1/2*
x*e + 1/2*d)^2 + 2*c*tan(1/2*x*e + 1/2*d) + a + b)^3))*e^(-1)
```

maple [B] time = 0.66, size = 16909, normalized size = 57.91

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*cos(e*x+d)+c*sin(e*x+d))^4,x)
```

[Out] result too large to display

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cos(e*x+d)+c*sin(e*x+d))^4,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(c^2+b^2-a^2>0)', see `assume?` for
more details)Is c^2+b^2-a^2 positive or negative?
```

mupad [B] time = 4.81, size = 1946, normalized size = 6.66

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*cos(d + e*x) + c*sin(d + e*x))^4,x)
```

```
[Out] (a*atanh((a*(2*a^2 + 3*b^2 + 3*c^2)*(2*b^6*c - 2*a^6*c + 2*c^7 - 6*a^2*c^5
+ 6*a^4*c^3 + 6*b^2*c^5 + 6*b^4*c^3 - 6*a^2*b^4*c + 6*a^4*b^2*c - 12*a^2*b^
2*c^3))/(2*(b^2 - a^2 + c^2)^(7/2)*(3*a*b^2 + 3*a*c^2 + 2*a^3)) + (a*tan(d/
2 + (e*x)/2)*(2*a - 2*b)*(2*a^2 + 3*b^2 + 3*c^2)*(b^6 - a^6 + c^6 - 3*a^2*b
^4 + 3*a^4*b^2 - 3*a^2*c^4 + 3*a^4*c^2 + 3*b^2*c^4 + 3*b^4*c^2 - 6*a^2*b^2*
c^2))/(2*(b^2 - a^2 + c^2)^(7/2)*(3*a*b^2 + 3*a*c^2 + 2*a^3)))*(2*a^2 + 3*b
^2 + 3*c^2))/(e*(b^2 - a^2 + c^2)^(7/2)) - ((18*a^7*c + 2*a^3*c^5 - 5*a^5*c
^3 + 6*a*b^2*c^5 + 21*a*b^4*c^3 - 12*a^3*b^4*c - 21*a^5*b^2*c - 16*a^3*b^2*
c^3 + 15*a*b^6*c)/(3*(a - b)^3*(b^6 - a^6 + c^6 - 3*a^2*b^4 + 3*a^4*b^2 - 3
*a^2*c^4 + 3*a^4*c^2 + 3*b^2*c^4 + 3*b^4*c^2 - 6*a^2*b^2*c^2)) + (tan(d/2 +
 (e*x)/2)*(2*b^8 - 6*a^7*b - 5*a*b^7 + 5*a^2*b^6 + 4*a^3*b^5 - 16*a^4*b^4 +
 7*a^5*b^3 + 9*a^6*b^2 + 2*a^2*c^6 - 4*a^4*c^4 + 27*a^6*c^2 + 2*b^2*c^6 + 6
*b^4*c^4 + 6*b^6*c^2 + 18*a*b^3*c^4 + 9*a*b^5*c^2 - 14*a^3*b*c^4 - 9*a^5*b*
c^2 - 6*a^2*b^2*c^4 - 3*a^2*b^4*c^2 - 30*a^4*b^2*c^2 + 4*a*b*c^6))/((a - b)
^3*(b^6 - a^6 + c^6 - 3*a^2*b^4 + 3*a^4*b^2 - 3*a^2*c^4 + 3*a^4*c^2 + 3*b^2
*c^4 + 3*b^4*c^2 - 6*a^2*b^2*c^2)) + (tan(d/2 + (e*x)/2)^4*(6*a^6*c + 4*b^6
*c + 4*c^7 - 12*a^2*c^5 + 27*a^4*c^3 + 12*b^2*c^5 + 12*b^4*c^3 - 15*a*b^3*c
^3 + 33*a^2*b^4*c - 45*a^3*b*c^3 - 55*a^3*b^3*c + 57*a^4*b^2*c + 21*a^2*b^2
*c^3 - 15*a*b^5*c - 30*a^5*b*c))/((a - b)^2*(b^6 - a^6 + c^6 - 3*a^2*b^4 +
3*a^4*b^2 - 3*a^2*c^4 + 3*a^4*c^2 + 3*b^2*c^4 + 3*b^4*c^2 - 6*a^2*b^2*c^2))
 - (2*tan(d/2 + (e*x)/2)^3*(18*a^7*b - 6*a*b^7 + 2*b^8 - 4*c^8 + 22*a^2*b^6
 - 50*a^3*b^5 + 30*a^4*b^4 + 38*a^5*b^3 - 54*a^6*b^2 + 4*a^2*c^6 - 21*a^4*c
^4 - 54*a^6*c^2 - 10*b^2*c^6 - 6*b^4*c^4 + 2*b^6*c^2 + 51*a*b^3*c^4 + 39*a*
b^5*c^2 + 81*a^3*b*c^4 + 120*a^5*b*c^2 - 105*a^2*b^2*c^4 - 87*a^2*b^4*c^2 +
```

```
 61*a^3*b^3*c^2 - 81*a^4*b^2*c^2 + 6*a*b*c^6))/(3*(a - b)^3*(b^6 - a^6 + c^
6 - 3*a^2*b^4 + 3*a^4*b^2 - 3*a^2*c^4 + 3*a^4*c^2 + 3*b^2*c^4 + 3*b^4*c^2 -
 6*a^2*b^2*c^2)) - (tan(d/2 + (e*x)/2)^5*(3*a*b^5 + 6*a^5*b - 2*b^6 - 2*c^6
 - 3*a^2*b^4 + 11*a^3*b^3 - 15*a^4*b^2 + 6*a^2*c^4 - 9*a^4*c^2 - 6*b^2*c^4
- 6*b^4*c^2 + 3*a*b^3*c^2 + 9*a^3*b*c^2 + 3*a^2*b^2*c^2))/((a - b)*(b^6 - a
^6 + c^6 - 3*a^2*b^4 + 3*a^4*b^2 - 3*a^2*c^4 + 3*a^4*c^2 + 3*b^2*c^4 + 3*b^
4*c^2 - 6*a^2*b^2*c^2)) + (2*tan(d/2 + (e*x)/2)^2*(2*a*c^7 + 6*a^7*c + 2*b*
c^7 + 2*b^7*c - 3*a^3*c^5 + 20*a^5*c^3 + 6*b^3*c^5 + 6*b^5*c^3 + 9*a*b^2*c^
5 - 3*a*b^4*c^3 - 12*a^2*b*c^5 + 18*a^2*b^5*c - 14*a^3*b^4*c - 22*a^4*b*c^3
 - 2*a^4*b^3*c + 18*a^5*b^2*c + 6*a^2*b^3*c^3 - 7*a^3*b^2*c^3 - 10*a*b^6*c
- 18*a^6*b*c))/((a - b)^3*(b^6 - a^6 + c^6 - 3*a^2*b^4 + 3*a^4*b^2 - 3*a^2*
c^4 + 3*a^4*c^2 + 3*b^2*c^4 + 3*b^4*c^2 - 6*a^2*b^2*c^2)))/(e*(tan(d/2 + (e
*x)/2)^3*(12*a^2*c - 12*b^2*c + 8*c^3) + tan(d/2 + (e*x)/2)*(6*a^2*c + 6*b^
2*c + 12*a*b*c) + tan(d/2 + (e*x)/2)^2*(3*a^2*b - 3*a*b^2 + 12*a*c^2 + 12*b
*c^2 + 3*a^3 - 3*b^3) - tan(d/2 + (e*x)/2)^4*(3*a*b^2 + 3*a^2*b - 12*a*c^2
+ 12*b*c^2 - 3*a^3 - 3*b^3) + tan(d/2 + (e*x)/2)^5*(6*a^2*c + 6*b^2*c - 12*
a*b*c) + 3*a*b^2 + 3*a^2*b + a^3 + b^3 + tan(d/2 + (e*x)/2)^6*(3*a*b^2 - 3*
a^2*b + a^3 - b^3)))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cos(e*x+d)+c*sin(e*x+d))**4,x)
```

```
[Out] Timed out
```

3.403 $\int (2+3\cos(d+ex)+5\sin(d+ex))^{5/2}\,dx$

Optimal. Leaf size=185

$$-\frac{2(5\cos(d+ex)-3\sin(d+ex))(5\sin(d+ex)+3\cos(d+ex)+2)^{3/2}}{5e}-\frac{32(5\cos(d+ex)-3\sin(d+ex))\sqrt{5\sin}}{15e}$$

```
[Out] -2/5*(5*cos(e*x+d)-3*sin(e*x+d))*(2+3*cos(e*x+d)+5*sin(e*x+d))^(3/2)/e-32/1
5*(5*cos(e*x+d)-3*sin(e*x+d))*(2+3*cos(e*x+d)+5*sin(e*x+d))^(1/2)/e+64*(cos
(1/2*d+1/2*e*x-1/2*arctan(5/3))^2)^(1/2)/cos(1/2*d+1/2*e*x-1/2*arctan(5/3))
*EllipticF(sin(1/2*d+1/2*e*x-1/2*arctan(5/3)),1/15*(510-30*34^(1/2))^(1/2))
/e/(2+34^(1/2))^(1/2)+796/15*(cos(1/2*d+1/2*e*x-1/2*arctan(5/3))^2)^(1/2)/c
os(1/2*d+1/2*e*x-1/2*arctan(5/3))*EllipticE(sin(1/2*d+1/2*e*x-1/2*arctan(5/
3)),1/15*(510-30*34^(1/2))^(1/2))*(2+34^(1/2))^(1/2)/e
```

Rubi [A] time = 0.27, antiderivative size = 185, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.318, Rules used = {3120, 3146, 3149, 3118, 2653, 3126, 2661}

$$-\frac{2(5\cos(d+ex)-3\sin(d+ex))(5\sin(d+ex)+3\cos(d+ex)+2)^{3/2}}{5e}-\frac{32(5\cos(d+ex)-3\sin(d+ex))\sqrt{5\sin}}{15e}$$

Antiderivative was successfully verified.

```
[In] Int[(2 + 3*Cos[d + e*x] + 5*Sin[d + e*x])^(5/2),x]
```

```
[Out] (796*Sqrt[2 + Sqrt[34]]*EllipticE[(d + e*x - ArcTan[5/3])/2, (2*(17 - Sqrt[
34]))/15])/(15*e) + (64*EllipticF[(d + e*x - ArcTan[5/3])/2, (2*(17 - Sqrt[
34]))/15])/(Sqrt[2 + Sqrt[34]]*e) - (32*(5*Cos[d + e*x] - 3*Sin[d + e*x])*S
qrt[2 + 3*Cos[d + e*x] + 5*Sin[d + e*x]])/(15*e) - (2*(5*Cos[d + e*x] - 3*S
in[d + e*x])*(2 + 3*Cos[d + e*x] + 5*Sin[d + e*x])^(3/2))/(5*e)
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
```

```
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 3118

```
Int[Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_
)]], x_Symbol] :> Int[Sqrt[a + Sqrt[b^2 + c^2]*Cos[d + e*x - ArcTan[b, c]]]
, x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 + c^2, 0] && GtQ[a + Sqrt[b^2
+ c^2], 0]
```

Rule 3120

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(n_), x_Symbol] :> -Simp[((c*Cos[d + e*x] - b*Sin[d + e*x])*(a + b*Cos[d +
e*x] + c*Sin[d + e*x])^(n - 1))/(e*n), x] + Dist[1/n, Int[Simp[n*a^2 + (n -
 1)*(b^2 + c^2) + a*b*(2*n - 1)*Cos[d + e*x] + a*c*(2*n - 1)*Sin[d + e*x],
x]*(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n - 2), x], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[a^2 - b^2 - c^2, 0] && GtQ[n, 1]
```

Rule 3126

```
Int[1/Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(
x_)]], x_Symbol] :> Int[1/Sqrt[a + Sqrt[b^2 + c^2]*Cos[d + e*x - ArcTan[b,
c]]], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 + c^2, 0] && GtQ[a + Sqrt[
b^2 + c^2], 0]
```

Rule 3146

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(n_.)*((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)
]), x_Symbol] :> Simp[((B*c - b*C - a*C*Cos[d + e*x] + a*B*Sin[d + e*x])*(a
 + b*Cos[d + e*x] + c*Sin[d + e*x])^n)/(a*e*(n + 1)), x] + Dist[1/(a*(n + 1
)), Int[(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n - 1)*Simp[a*(b*B + c*C)*n
+ a^2*A*(n + 1) + (n*(a^2*B - B*c^2 + b*c*C) + a*b*A*(n + 1))*Cos[d + e*x]
+ (n*(b*B*c + a^2*C - b^2*C) + a*c*A*(n + 1))*Sin[d + e*x], x], x], x] /; F
reeQ[{a, b, c, d, e, A, B, C}, x] && GtQ[n, 0] && NeQ[a^2 - b^2 - c^2, 0]
```

Rule 3149

```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])
/Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)]]
, x_Symbol] :> Dist[B/b, Int[Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]], x],
 x] + Dist[(A*b - a*B)/b, Int[1/Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]],
x], x] /; FreeQ[{a, b, c, d, e, A, B, C}, x] && EqQ[B*c - b*C, 0] && NeQ[A*
b - a*B, 0]
```

Rubi steps

$$\int (2+3\cos(d+ex)+5\sin(d+ex))^{5/2}\,dx = -\frac{2(5\cos(d+ex)-3\sin(d+ex))(2+3\cos(d+ex)+5\sin(d+ex)}{5e}$$

$$= -\frac{32(5\cos(d+ex)-3\sin(d+ex))\sqrt{2+3\cos(d+ex)+5\sin(d+}}{15e}$$

$$= -\frac{32(5\cos(d+ex)-3\sin(d+ex))\sqrt{2+3\cos(d+ex)+5\sin(d+}}{15e}$$

$$= -\frac{32(5\cos(d+ex)-3\sin(d+ex))\sqrt{2+3\cos(d+ex)+5\sin(d+}}{15e}$$

$$= \frac{796\sqrt{2+\sqrt{34}}\,E\left(\frac{1}{2}\left(d+ex-\tan^{-1}\left(\frac{5}{3}\right)\right)|\frac{2}{15}\left(17-\sqrt{34}\right)\right)}{15e} + \frac{64F\left(\right.}{}$$

Mathematica [C] time = 5.91, size = 399, normalized size = 2.16

$$\frac{1276\sqrt{\frac{10}{3}}\sqrt{\sqrt{34}\sin\left(d+ex+\tan^{-1}\left(\frac{3}{5}\right)\right)+2}\sqrt{\cos^2\left(d+ex+\tan^{-1}\left(\frac{3}{5}\right)\right)}\sec\left(d+ex+\tan^{-1}\left(\frac{3}{5}\right)\right)F_1\left(\frac{1}{2};\frac{1}{2},\frac{1}{2};\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(2 + 3*Cos[d + e*x] + 5*Sin[d + e*x])^(5/2),x]
```

```
[Out] (-2388*Sqrt[2 + Sqrt[34]*Cos[d + e*x - ArcTan[5/3]]] - 2*Sqrt[2 + 3*Cos[d +
 e*x] + 5*Sin[d + e*x]]*(550*Cos[d + e*x] + 3*(-398 + 75*Cos[2*(d + e*x)] -
 110*Sin[d + e*x] + 40*Sin[2*(d + e*x)])) + 1276*Sqrt[10/3]*AppellF1[1/2, 1
/2, 1/2, 3/2, (Sqrt[34] + 17*Sin[d + e*x + ArcTan[3/5]])/(-17 + Sqrt[34]),
(Sqrt[34] + 17*Sin[d + e*x + ArcTan[3/5]])/(17 + Sqrt[34])]*Sqrt[Cos[d + e*
x + ArcTan[3/5]]^2]*Sec[d + e*x + ArcTan[3/5]]*Sqrt[2 + Sqrt[34]*Sin[d + e*
x + ArcTan[3/5]]] + (1990*Sin[d + e*x - ArcTan[5/3]])/Sqrt[1/17 + Cos[d + e
*x - ArcTan[5/3]]/Sqrt[34]] - (1990*Sqrt[30]*AppellF1[-1/2, -1/2, -1/2, 1/2
, (Sqrt[34] + 17*Cos[d + e*x - ArcTan[5/3]])/(-17 + Sqrt[34]), (Sqrt[34] +
17*Cos[d + e*x - ArcTan[5/3]])/(17 + Sqrt[34])]*Csc[d + e*x - ArcTan[5/3]]*
Sqrt[Sin[d + e*x - ArcTan[5/3]]^2])/Sqrt[2 + Sqrt[34]*Cos[d + e*x - ArcTan[
5/3]]])/(75*e)
```

fricas [F] time = 1.18, size = 0, normalized size = 0.00

$$\text{integral}\left(-\left(16\,\cos\left(ex+d\right)^{2}-10\,\left(3\,\cos\left(ex+d\right)+2\right)\sin\left(ex+d\right)-12\,\cos\left(ex+d\right)-29\right)\sqrt{3\,\cos\left(ex+d\right)+5\,\sin$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+3*cos(e*x+d)+5*sin(e*x+d))^(5/2),x, algorithm="fricas")
```

```
[Out] integral(-(16*cos(e*x + d)^2 - 10*(3*cos(e*x + d) + 2)*sin(e*x + d) - 12*co
s(e*x + d) - 29)*sqrt(3*cos(e*x + d) + 5*sin(e*x + d) + 2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(3\,\cos\left(ex+d\right)+5\,\sin\left(ex+d\right)+2\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+3*cos(e*x+d)+5*sin(e*x+d))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((3*cos(e*x + d) + 5*sin(e*x + d) + 2)^(5/2), x)
```

maple [C] time = 0.66, size = 701, normalized size = 3.79

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2+3*cos(e*x+d)+5*sin(e*x+d))^(5/2),x)
```

```
[Out] (16/17*((17*sin(e*x+d+arctan(3/5))+34^(1/2))/(34^(1/2)+17))^(1/2)*17^(1/2)*
((1+sin(e*x+d+arctan(3/5)))/(-34^(1/2)+17))^(1/2)*(-17*(sin(e*x+d+arctan(3/
5))-1)/(34^(1/2)+17))^(1/2)*EllipticF(((17*sin(e*x+d+arctan(3/5))+34^(1/2))
/(34^(1/2)+17))^(1/2),I*(1/(-34^(1/2)+17)*(34^(1/2)+17))^(1/2))*34^(1/2)+16
*((17*sin(e*x+d+arctan(3/5))+34^(1/2))/(34^(1/2)+17))^(1/2)*17^(1/2)*((1+si
n(e*x+d+arctan(3/5)))/(-34^(1/2)+17))^(1/2)*(-17*(sin(e*x+d+arctan(3/5))-1)
/(34^(1/2)+17))^(1/2)*EllipticF(((17*sin(e*x+d+arctan(3/5))+34^(1/2))/(34^(
1/2)+17))^(1/2),I*(1/(-34^(1/2)+17)*(34^(1/2)+17))^(1/2))-44*17^(1/2)*((1+s
in(e*x+d+arctan(3/5)))/(-34^(1/2)+17))^(1/2)*(-17*(sin(e*x+d+arctan(3/5))-1
)/(34^(1/2)+17))^(1/2)*(-(17*sin(e*x+d+arctan(3/5))+34^(1/2))/(-34^(1/2)+17
))^(1/2)*EllipticF((-(17*sin(e*x+d+arctan(3/5))+34^(1/2))/(-34^(1/2)+17))^(
1/2),I*((-34^(1/2)+17)/(34^(1/2)+17))^(1/2))*34^(1/2)+796/17*17^(1/2)*((1+s
in(e*x+d+arctan(3/5)))/(-34^(1/2)+17))^(1/2)*(-17*(sin(e*x+d+arctan(3/5))-1
)/(34^(1/2)+17))^(1/2)*(-(17*sin(e*x+d+arctan(3/5))+34^(1/2))/(-34^(1/2)+17
))^(1/2)*EllipticE((-(17*sin(e*x+d+arctan(3/5))+34^(1/2))/(-34^(1/2)+17))^(
1/2),I*((-34^(1/2)+17)/(34^(1/2)+17))^(1/2))*34^(1/2)-48*(-(17*sin(e*x+d+ar
ctan(3/5))+34^(1/2))/(-34^(1/2)+17))^(1/2)*(-17*(sin(e*x+d+arctan(3/5))-1)/
```

```
(34^(1/2)+17))^(1/2)*17^(1/2)*((1+sin(e*x+d+arctan(3/5)))/(-34^(1/2)+17))^(
1/2)*EllipticF((-(17*sin(e*x+d+arctan(3/5))+34^(1/2))/(-34^(1/2)+17))^(1/2)
,I*((-34^(1/2)+17)/(34^(1/2)+17))^(1/2))+68/5*34^(1/2)*sin(e*x+d+arctan(3/5
))^4-116/15*34^(1/2)*sin(e*x+d+arctan(3/5))^2+1904/15*sin(e*x+d+arctan(3/5)
)^3-1904/15*sin(e*x+d+arctan(3/5))-88/15*34^(1/2))/cos(e*x+d+arctan(3/5))/(
34^(1/2)*sin(e*x+d+arctan(3/5))+2)^(1/2)/e
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (3 \cos(ex+d) + 5 \sin(ex+d) + 2)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+3*cos(e*x+d)+5*sin(e*x+d))^(5/2),x, algorithm="maxima")
[Out] integrate((3*cos(e*x + d) + 5*sin(e*x + d) + 2)^(5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int (3 \cos(d+e\,x) + 5 \sin(d+e\,x) + 2)^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*cos(d + e*x) + 5*sin(d + e*x) + 2)^(5/2),x)
[Out] int((3*cos(d + e*x) + 5*sin(d + e*x) + 2)^(5/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+3*cos(e*x+d)+5*sin(e*x+d))**(5/2),x)
[Out] Timed out
```

3.404 $\int (2+3\cos(d+ex)+5\sin(d+ex))^{3/2}\,dx$

Optimal. Leaf size=139

$$-\frac{2(5\cos(d+ex)-3\sin(d+ex))\sqrt{5\sin(d+ex)+3\cos(d+ex)+2}}{3e}+\frac{20F\left(\frac{1}{2}\left(d+ex-\tan^{-1}\left(\frac{5}{3}\right)\right)|\frac{2}{15}\left(17-\sqrt{34}\right.\right.}{\sqrt{2+\sqrt{34}}\,e}$$

```
[Out] -2/3*(5*cos(e*x+d)-3*sin(e*x+d))*(2+3*cos(e*x+d)+5*sin(e*x+d))^(1/2)/e+20*(
cos(1/2*d+1/2*e*x-1/2*arctan(5/3))^2)^(1/2)/cos(1/2*d+1/2*e*x-1/2*arctan(5/
3))*EllipticF(sin(1/2*d+1/2*e*x-1/2*arctan(5/3)),1/15*(510-30*34^(1/2))^(1/
2))/e/(2+34^(1/2))^(1/2)+16/3*(cos(1/2*d+1/2*e*x-1/2*arctan(5/3))^2)^(1/2)/
cos(1/2*d+1/2*e*x-1/2*arctan(5/3))*EllipticE(sin(1/2*d+1/2*e*x-1/2*arctan(5
/3)),1/15*(510-30*34^(1/2))^(1/2))*(2+34^(1/2))^(1/2)/e
```

Rubi [A] time = 0.14, antiderivative size = 139, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {3120, 3149, 3118, 2653, 3126, 2661}

$$-\frac{2(5\cos(d+ex)-3\sin(d+ex))\sqrt{5\sin(d+ex)+3\cos(d+ex)+2}}{3e}+\frac{20F\left(\frac{1}{2}\left(d+ex-\tan^{-1}\left(\frac{5}{3}\right)\right)|\frac{2}{15}\left(17-\sqrt{34}\right.\right.}{\sqrt{2+\sqrt{34}}\,e}$$

Antiderivative was successfully verified.

```
[In] Int[(2 + 3*Cos[d + e*x] + 5*Sin[d + e*x])^(3/2),x]
```

```
[Out] (16*Sqrt[2 + Sqrt[34]]*EllipticE[(d + e*x - ArcTan[5/3])/2, (2*(17 - Sqrt[3
4]))/15])/(3*e) + (20*EllipticF[(d + e*x - ArcTan[5/3])/2, (2*(17 - Sqrt[34
]))/15])/(Sqrt[2 + Sqrt[34]]*e) - (2*(5*Cos[d + e*x] - 3*Sin[d + e*x])*Sqrt
[2 + 3*Cos[d + e*x] + 5*Sin[d + e*x]])/(3*e)
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 3118

```
Int[Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_
)]], x_Symbol] :> Int[Sqrt[a + Sqrt[b^2 + c^2]*Cos[d + e*x - ArcTan[b, c]]]
, x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 + c^2, 0] && GtQ[a + Sqrt[b^2
+ c^2], 0]
```

Rule 3120

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(n_), x_Symbol] :> -Simp[((c*Cos[d + e*x] - b*Sin[d + e*x])*(a + b*Cos[d +
e*x] + c*Sin[d + e*x])^(n - 1))/(e*n), x] + Dist[1/n, Int[Simp[n*a^2 + (n -
 1)*(b^2 + c^2) + a*b*(2*n - 1)*Cos[d + e*x] + a*c*(2*n - 1)*Sin[d + e*x],
x]*(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n - 2), x], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[a^2 - b^2 - c^2, 0] && GtQ[n, 1]
```

Rule 3126

```
Int[1/Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(
x_)]], x_Symbol] :> Int[1/Sqrt[a + Sqrt[b^2 + c^2]*Cos[d + e*x - ArcTan[b,
c]]], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 + c^2, 0] && GtQ[a + Sqrt[
b^2 + c^2], 0]
```

Rule 3149

```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])
/Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)]]
, x_Symbol] :> Dist[B/b, Int[Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]], x],
 x] + Dist[(A*b - a*B)/b, Int[1/Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]],
x], x] /; FreeQ[{a, b, c, d, e, A, B, C}, x] && EqQ[B*c - b*C, 0] && NeQ[A*
b - a*B, 0]
```

Rubi steps

$$\int (2+3\cos(d+ex)+5\sin(d+ex))^{3/2}\,dx = -\frac{2(5\cos(d+ex)-3\sin(d+ex))\sqrt{2+3\cos(d+ex)+5\sin(d+ex)}}{3e}$$

$$= -\frac{2(5\cos(d+ex)-3\sin(d+ex))\sqrt{2+3\cos(d+ex)+5\sin(d+ex)}}{3e}$$

$$= -\frac{2(5\cos(d+ex)-3\sin(d+ex))\sqrt{2+3\cos(d+ex)+5\sin(d+ex)}}{3e}$$

$$= \frac{16\sqrt{2+\sqrt{34}}\,E\left(\frac{1}{2}\left(d+ex-\tan^{-1}\left(\frac{5}{3}\right)\right)|\frac{2}{15}\left(17-\sqrt{34}\right)\right)}{3e} + \frac{20F\left(\frac{1}{2}\left(\right.\right.}{}$$

Mathematica [C] time = 3.42, size = 349, normalized size = 2.51

$$\frac{2\left(\sqrt{\sin^2\left(d+ex-\tan^{-1}\left(\frac{5}{3}\right)\right)}\left(23\sqrt{30}\sqrt{\sqrt{34}\sin\left(d+ex+\tan^{-1}\left(\frac{3}{5}\right)\right)+2}\sqrt{\cos^2\left(d+ex+\tan^{-1}\left(\frac{3}{5}\right)\right)}\sqrt{\sqrt{34}}\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(2 + 3*Cos[d + e*x] + 5*Sin[d + e*x])^(3/2),x]
```

```
[Out] (2*(-60*Sqrt[30]*AppellF1[-1/2, -1/2, -1/2, 1/2, (Sqrt[34] + 17*Cos[d + e*x
 - ArcTan[5/3]])/(-17 + Sqrt[34]), (Sqrt[34] + 17*Cos[d + e*x - ArcTan[5/3]
])/(17 + Sqrt[34])]*Sin[d + e*x - ArcTan[5/3]] + (-15*(30*Cos[d + e*x] + 15
*Cos[2*(d + e*x)] - 18*Sin[d + e*x] + 8*Sin[2*(d + e*x)]) + 23*Sqrt[30]*App
ellF1[1/2, 1/2, 1/2, 3/2, (Sqrt[34] + 17*Sin[d + e*x + ArcTan[3/5]])/(-17 +
 Sqrt[34]), (Sqrt[34] + 17*Sin[d + e*x + ArcTan[3/5]])/(17 + Sqrt[34])]*Sqr
t[Cos[d + e*x + ArcTan[3/5]]^2]*Sqrt[2 + Sqrt[34]*Cos[d + e*x - ArcTan[5/3]
]]*Sec[d + e*x + ArcTan[3/5]]*Sqrt[2 + Sqrt[34]*Sin[d + e*x + ArcTan[3/5]]]
)*Sqrt[Sin[d + e*x - ArcTan[5/3]]^2]))/(45*e*Sqrt[2 + Sqrt[34]*Cos[d + e*x
- ArcTan[5/3]]]*Sqrt[Sin[d + e*x - ArcTan[5/3]]^2])
```

fricas [F] time = 0.87, size = 0, normalized size = 0.00

$$\mathrm{integral}\left((3\,\cos\left(ex+d\right)+5\,\sin\left(ex+d\right)+2)^{\frac{3}{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+3*cos(e*x+d)+5*sin(e*x+d))^(3/2),x, algorithm="fricas")
[Out] integral((3*cos(e*x + d) + 5*sin(e*x + d) + 2)^(3/2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (3\cos(ex+d)+5\sin(ex+d)+2)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+3*cos(e*x+d)+5*sin(e*x+d))^(3/2),x, algorithm="giac")
[Out] integrate((3*cos(e*x + d) + 5*sin(e*x + d) + 2)^(3/2), x)
```

maple [C] time = 0.46, size = 686, normalized size = 4.94

$$\frac{8\sqrt{\frac{17\sin\left(ex+d+\arctan\left(\frac{3}{5}\right)\right)+\sqrt{34}}{\sqrt{34}+17}}\,\sqrt{17}\,\sqrt{\frac{1+\sin\left(ex+d+\arctan\left(\frac{3}{5}\right)\right)}{-\sqrt{34}+17}}\,\sqrt{-\frac{17\left(\sin\left(ex+d+\arctan\left(\frac{3}{5}\right)\right)-1\right)}{\sqrt{34}+17}}\,\mathrm{EllipticF}\left(\sqrt{\frac{17\sin\left(ex+d+\arctan\left(\frac{3}{5}\right)\right)+\sqrt{34}}{\sqrt{34}+17}},i\sqrt{\frac{\sqrt{34}+17}{-\sqrt{34}+17}}\right)}{17}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2+3*cos(e*x+d)+5*sin(e*x+d))^(3/2),x)
[Out] (8/17*((17*sin(e*x+d+arctan(3/5))+34^(1/2))/(34^(1/2)+17))^(1/2)*17^(1/2)*(
(1+sin(e*x+d+arctan(3/5)))/(-34^(1/2)+17))^(1/2)*(-17*(sin(e*x+d+arctan(3/5
))-1)/(34^(1/2)+17))^(1/2)*EllipticF(((17*sin(e*x+d+arctan(3/5))+34^(1/2))/
(34^(1/2)+17))^(1/2),I*(1/(-34^(1/2)+17)*(34^(1/2)+17))^(1/2))*34^(1/2)+8*(
(17*sin(e*x+d+arctan(3/5))+34^(1/2))/(34^(1/2)+17))^(1/2)*17^(1/2)*((1+sin(
e*x+d+arctan(3/5)))/(-34^(1/2)+17))^(1/2)*(-17*(sin(e*x+d+arctan(3/5))-1)/(
34^(1/2)+17))^(1/2)*EllipticF(((17*sin(e*x+d+arctan(3/5))+34^(1/2))/(34^(1/
2)+17))^(1/2),I*(1/(-34^(1/2)+17)*(34^(1/2)+17))^(1/2))+68/3*sin(e*x+d+arct
an(3/5))^3-68/3*sin(e*x+d+arctan(3/5))+4/3*34^(1/2)*sin(e*x+d+arctan(3/5))^
2-4/3*34^(1/2)-4*17^(1/2)*((1+sin(e*x+d+arctan(3/5)))/(-34^(1/2)+17))^(1/2)
*(-17*(sin(e*x+d+arctan(3/5))-1)/(34^(1/2)+17))^(1/2)*(-(17*sin(e*x+d+arcta
n(3/5))+34^(1/2))/(-34^(1/2)+17))^(1/2)*EllipticF((-(17*sin(e*x+d+arctan(3/
5))+34^(1/2))/(-34^(1/2)+17))^(1/2),I*((-34^(1/2)+17)/(34^(1/2)+17))^(1/2))
*34^(1/2)-12*(-(17*sin(e*x+d+arctan(3/5))+34^(1/2))/(-34^(1/2)+17))^(1/2)*(
-17*(sin(e*x+d+arctan(3/5))-1)/(34^(1/2)+17))^(1/2)*17^(1/2)*((1+sin(e*x+d+
arctan(3/5)))/(-34^(1/2)+17))^(1/2)*EllipticF((-(17*sin(e*x+d+arctan(3/5))+
34^(1/2))/(-34^(1/2)+17))^(1/2),I*((-34^(1/2)+17)/(34^(1/2)+17))^(1/2))+80/
17*17^(1/2)*((1+sin(e*x+d+arctan(3/5)))/(-34^(1/2)+17))^(1/2)*(-17*(sin(e*x
```

```
+d+arctan(3/5))-1)/(34^(1/2)+17))^(1/2)*(-(17*sin(e*x+d+arctan(3/5))+34^(1/
2))/(-34^(1/2)+17))^(1/2)*EllipticE((-(17*sin(e*x+d+arctan(3/5))+34^(1/2))/
(-34^(1/2)+17))^(1/2),I*((-34^(1/2)+17)/(34^(1/2)+17))^(1/2))*34^(1/2))/cos
(e*x+d+arctan(3/5))/(34^(1/2)*sin(e*x+d+arctan(3/5))+2)^(1/2)/e
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (3\cos(ex+d)+5\sin(ex+d)+2)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+3*cos(e*x+d)+5*sin(e*x+d))^(3/2),x, algorithm="maxima")
[Out] integrate((3*cos(e*x + d) + 5*sin(e*x + d) + 2)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int (3\cos(d+e\,x)+5\sin(d+e\,x)+2)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*cos(d + e*x) + 5*sin(d + e*x) + 2)^(3/2),x)
[Out] int((3*cos(d + e*x) + 5*sin(d + e*x) + 2)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (5\sin(d+ex)+3\cos(d+ex)+2)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+3*cos(e*x+d)+5*sin(e*x+d))**(3/2),x)
[Out] Integral((5*sin(d + e*x) + 3*cos(d + e*x) + 2)**(3/2), x)
```

3.405 $\int \sqrt{2+3\cos(d+ex)+5\sin(d+ex)}\,dx$

Optimal. Leaf size=45

$$\frac{2\sqrt{2+\sqrt{34}}\,E\left(\frac{1}{2}\left(d+ex-\tan^{-1}\left(\frac{5}{3}\right)\right)|\frac{2}{15}\left(17-\sqrt{34}\right)\right)}{e}$$

```
[Out] 2*(cos(1/2*d+1/2*e*x-1/2*arctan(5/3))^2)^(1/2)/cos(1/2*d+1/2*e*x-1/2*arctan
(5/3))*EllipticE(sin(1/2*d+1/2*e*x-1/2*arctan(5/3)),1/15*(510-30*34^(1/2))^
(1/2))*(2+34^(1/2))^(1/2)/e
```

Rubi [A] time = 0.03, antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {3118, 2653}

$$\frac{2\sqrt{2+\sqrt{34}}\,E\left(\frac{1}{2}\left(d+ex-\tan^{-1}\left(\frac{5}{3}\right)\right)|\frac{2}{15}\left(17-\sqrt{34}\right)\right)}{e}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[2 + 3*Cos[d + e*x] + 5*Sin[d + e*x]],x]
```

```
[Out] (2*Sqrt[2 + Sqrt[34]]*EllipticE[(d + e*x - ArcTan[5/3])/2, (2*(17 - Sqrt[34
]))/15])/e
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 3118

```
Int[Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_
)]], x_Symbol] :> Int[Sqrt[a + Sqrt[b^2 + c^2]*Cos[d + e*x - ArcTan[b, c]]]
, x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 + c^2, 0] && GtQ[a + Sqrt[b^2
+ c^2], 0]
```

Rubi steps

$$\int \sqrt{2+3\cos(d+ex)+5\sin(d+ex)}\,dx = \int \sqrt{2+\sqrt{34}\cos\left(d+ex-\tan^{-1}\left(\frac{5}{3}\right)\right)}\,dx$$
$$= \frac{2\sqrt{2+\sqrt{34}}\,E\left(\frac{1}{2}\left(d+ex-\tan^{-1}\left(\frac{5}{3}\right)\right)|\frac{2}{15}\left(17-\sqrt{34}\right)\right)}{e}$$

Mathematica [C] time = 2.27, size = 326, normalized size = 7.24

$$\frac{\sqrt{\sin^2\left(d+ex-\tan^{-1}\left(\frac{5}{3}\right)\right)}\left(2\sqrt{30}\sqrt{\sqrt{34}\sin\left(d+ex+\tan^{-1}\left(\frac{3}{5}\right)\right)+2}\sqrt{\cos^2\left(d+ex+\tan^{-1}\left(\frac{3}{5}\right)\right)}\sqrt{\sqrt{34}\cos\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[2 + 3*Cos[d + e*x] + 5*Sin[d + e*x]],x]
```

```
[Out] (-15*Sqrt[30]*AppellF1[-1/2, -1/2, -1/2, 1/2, (Sqrt[34] + 17*Cos[d + e*x -
ArcTan[5/3]])/(-17 + Sqrt[34]), (Sqrt[34] + 17*Cos[d + e*x - ArcTan[5/3]])/
(17 + Sqrt[34])]*Sin[d + e*x - ArcTan[5/3]] + (-75*Cos[d + e*x] + 45*Sin[d
+ e*x] + 2*Sqrt[30]*AppellF1[1/2, 1/2, 1/2, 3/2, (Sqrt[34] + 17*Sin[d + e*x
 + ArcTan[3/5]])/(-17 + Sqrt[34]), (Sqrt[34] + 17*Sin[d + e*x + ArcTan[3/5]
])/(17 + Sqrt[34])]*Sqrt[Cos[d + e*x + ArcTan[3/5]]^2]*Sqrt[2 + Sqrt[34]*Co
s[d + e*x - ArcTan[5/3]]]*Sec[d + e*x + ArcTan[3/5]]*Sqrt[2 + Sqrt[34]*Sin[
d + e*x + ArcTan[3/5]]])*Sqrt[Sin[d + e*x - ArcTan[5/3]]^2])/(15*e*Sqrt[2 +
 Sqrt[34]*Cos[d + e*x - ArcTan[5/3]]]*Sqrt[Sin[d + e*x - ArcTan[5/3]]^2])
```

fricas [F] time = 0.52, size = 0, normalized size = 0.00

$$\text{integral}\left(\sqrt{3\cos\left(ex+d\right)+5\sin\left(ex+d\right)+2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+3*cos(e*x+d)+5*sin(e*x+d))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(3*cos(e*x + d) + 5*sin(e*x + d) + 2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{3\cos\left(ex+d\right)+5\sin\left(ex+d\right)+2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+3*cos(e*x+d)+5*sin(e*x+d))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(3*cos(e*x + d) + 5*sin(e*x + d) + 2), x)
```

maple [C] time = 0.40, size = 461, normalized size = 10.24

$$2\sqrt{-\frac{17\left(\sin\left(ex+d+\arctan\left(\frac{3}{5}\right)\right)-1\right)}{\sqrt{34}+17}}\,\sqrt{17}\,\sqrt{\frac{1+\sin\left(ex+d+\arctan\left(\frac{3}{5}\right)\right)}{-\sqrt{34}+17}}\,\left(15\sqrt{34}\,\sqrt{-\frac{17\sin\left(ex+d+\arctan\left(\frac{3}{5}\right)\right)+\sqrt{34}}{-\sqrt{34}+17}}\,\operatorname{EllipticE}\left(\sqrt{-\frac{17}{}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2+3*cos(e*x+d)+5*sin(e*x+d))^(1/2),x)
```

```
[Out] 2/17*(-17*(sin(e*x+d+arctan(3/5))-1)/(34^(1/2)+17))^(1/2)*17^(1/2)*((1+sin(
e*x+d+arctan(3/5)))/(-34^(1/2)+17))^(1/2)*(15*34^(1/2)*(-(17*sin(e*x+d+arct
an(3/5))+34^(1/2))/(-34^(1/2)+17))^(1/2)*EllipticE((-(17*sin(e*x+d+arctan(3
/5))+34^(1/2))/(-34^(1/2)+17))^(1/2),I*((-34^(1/2)+17)/(34^(1/2)+17))^(1/2)
)-17*34^(1/2)*(-(17*sin(e*x+d+arctan(3/5))+34^(1/2))/(-34^(1/2)+17))^(1/2)*
EllipticF((-(17*sin(e*x+d+arctan(3/5))+34^(1/2))/(-34^(1/2)+17))^(1/2),I*((
-34^(1/2)+17)/(34^(1/2)+17))^(1/2))+2*((17*sin(e*x+d+arctan(3/5))+34^(1/2))
/(34^(1/2)+17))^(1/2)*EllipticF(((17*sin(e*x+d+arctan(3/5))+34^(1/2))/(34^(
1/2)+17))^(1/2),I*(1/(-34^(1/2)+17)*(34^(1/2)+17))^(1/2))*34^(1/2)+34*(-(17
*sin(e*x+d+arctan(3/5))+34^(1/2))/(-34^(1/2)+17))^(1/2)*EllipticF((-(17*sin
(e*x+d+arctan(3/5))+34^(1/2))/(-34^(1/2)+17))^(1/2),I*((-34^(1/2)+17)/(34^(
1/2)+17))^(1/2))+34*((17*sin(e*x+d+arctan(3/5))+34^(1/2))/(34^(1/2)+17))^(1
/2)*EllipticF(((17*sin(e*x+d+arctan(3/5))+34^(1/2))/(34^(1/2)+17))^(1/2),I*
(1/(-34^(1/2)+17)*(34^(1/2)+17))^(1/2)))/cos(e*x+d+arctan(3/5))/(34^(1/2)*s
in(e*x+d+arctan(3/5))+2)^(1/2)/e
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{3\cos\left(ex+d\right)+5\sin\left(ex+d\right)+2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+3*cos(e*x+d)+5*sin(e*x+d))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(3*cos(e*x + d) + 5*sin(e*x + d) + 2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \sqrt{3\cos\left(d+e\,x\right)+5\sin\left(d+e\,x\right)+2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*cos(d + e*x) + 5*sin(d + e*x) + 2)^(1/2),x)
[Out] int((3*cos(d + e*x) + 5*sin(d + e*x) + 2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{5 \sin \left(d + ex\right) + 3 \cos \left(d + ex\right) + 2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+3*cos(e*x+d)+5*sin(e*x+d))**(1/2),x)
[Out] Integral(sqrt(5*sin(d + e*x) + 3*cos(d + e*x) + 2), x)
```

3.406 $\int \frac{1}{\sqrt{2+3\cos(d+ex)+5\sin(d+ex)}}\,dx$

Optimal. Leaf size=45

$$\frac{2F\left(\frac{1}{2}\left(d+ex-\tan^{-1}\left(\frac{5}{3}\right)\right)|\frac{2}{15}\left(17-\sqrt{34}\right)\right)}{\sqrt{2+\sqrt{34}}\,e}$$

```
[Out] 2*(cos(1/2*d+1/2*e*x-1/2*arctan(5/3))^2)^(1/2)/cos(1/2*d+1/2*e*x-1/2*arctan
(5/3))*EllipticF(sin(1/2*d+1/2*e*x-1/2*arctan(5/3)),1/15*(510-30*34^(1/2))^
(1/2))/e/(2+34^(1/2))^(1/2)
```

Rubi [A] time = 0.04, antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {3126, 2661}

$$\frac{2F\left(\frac{1}{2}\left(d+ex-\tan^{-1}\left(\frac{5}{3}\right)\right)|\frac{2}{15}\left(17-\sqrt{34}\right)\right)}{\sqrt{2+\sqrt{34}}\,e}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[2 + 3*Cos[d + e*x] + 5*Sin[d + e*x]],x]
```

```
[Out] (2*EllipticF[(d + e*x - ArcTan[5/3])/2, (2*(17 - Sqrt[34]))/15])/(Sqrt[2 +
Sqrt[34]]*e)
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 3126

```
Int[1/Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(
x_)]], x_Symbol] :> Int[1/Sqrt[a + Sqrt[b^2 + c^2]*Cos[d + e*x - ArcTan[b,
c]]], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 + c^2, 0] && GtQ[a + Sqrt[
b^2 + c^2], 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{2+3\cos(d+ex)+5\sin(d+ex)}}\,dx = \int \frac{1}{\sqrt{2+\sqrt{34}\cos\left(d+ex-\tan^{-1}\left(\frac{5}{3}\right)\right)}}\,dx$$
$$= \frac{2F\left(\frac{1}{2}\left(d+ex-\tan^{-1}\left(\frac{5}{3}\right)\right)|\frac{2}{15}\left(17-\sqrt{34}\right)\right)}{\sqrt{2+\sqrt{34}}\,e}$$

Mathematica [C] time = 0.26, size = 128, normalized size = 2.84

$$\frac{\sqrt{\frac{2}{15}}\sqrt{\sqrt{34}\sin\left(d+ex+\tan^{-1}\left(\frac{3}{5}\right)\right)+2}\sqrt{\cos^2\left(d+ex+\tan^{-1}\left(\frac{3}{5}\right)\right)}\sec\left(d+ex+\tan^{-1}\left(\frac{3}{5}\right)\right)F_1\left(\frac{1}{2};\frac{1}{2},\frac{1}{2};\frac{3}{2};\frac{17\mathrm{si}}{}\right.}{e}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/Sqrt[2 + 3*Cos[d + e*x] + 5*Sin[d + e*x]],x]
```

```
[Out] (Sqrt[2/15]*AppellF1[1/2, 1/2, 1/2, 3/2, (Sqrt[34] + 17*Sin[d + e*x + ArcTa
n[3/5]])/(-17 + Sqrt[34]), (Sqrt[34] + 17*Sin[d + e*x + ArcTan[3/5]])/(17 +
 Sqrt[34])]*Sqrt[Cos[d + e*x + ArcTan[3/5]]^2]*Sec[d + e*x + ArcTan[3/5]]*S
qrt[2 + Sqrt[34]*Sin[d + e*x + ArcTan[3/5]]])/e
```

fricas [F] time = 2.26, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{1}{\sqrt{3\cos\left(ex+d\right)+5\sin\left(ex+d\right)+2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2+3*cos(e*x+d)+5*sin(e*x+d))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(1/sqrt(3*cos(e*x + d) + 5*sin(e*x + d) + 2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{3\cos\left(ex+d\right)+5\sin\left(ex+d\right)+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2+3*cos(e*x+d)+5*sin(e*x+d))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(3*cos(e*x + d) + 5*sin(e*x + d) + 2), x)
```

maple [C] time = 0.31, size = 152, normalized size = 3.38

$$\frac{2\left(\sqrt{34}+17\right)\sqrt{\frac{17\sin\left(ex+d+\arctan\left(\frac{3}{5}\right)\right)+\sqrt{34}}{\sqrt{34}+17}}\,\sqrt{17}\,\sqrt{\frac{1+\sin\left(ex+d+\arctan\left(\frac{3}{5}\right)\right)}{-\sqrt{34}+17}}\,\sqrt{-\frac{17\left(\sin\left(ex+d+\arctan\left(\frac{3}{5}\right)\right)-1\right)}{\sqrt{34}+17}}\ \mathrm{EllipticF}\left(\sqrt{\cdots}\right.}{17\cos\left(ex+d+\arctan\left(\frac{3}{5}\right)\right)\sqrt{\sqrt{34}\ \sin\left(ex+d+\arctan\left(\frac{3}{5}\right)\right)+2}\ e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2+3*cos(e*x+d)+5*sin(e*x+d))^(1/2),x)
```

```
[Out] 2/17*(34^(1/2)+17)*((17*sin(e*x+d+arctan(3/5))+34^(1/2))/(34^(1/2)+17))^(1/
2)*17^(1/2)*((1+sin(e*x+d+arctan(3/5)))/(-34^(1/2)+17))^(1/2)*(-17*(sin(e*x
+d+arctan(3/5))-1)/(34^(1/2)+17))^(1/2)*EllipticF(((17*sin(e*x+d+arctan(3/5
))+34^(1/2))/(34^(1/2)+17))^(1/2),I*(1/(-34^(1/2)+17)*(34^(1/2)+17))^(1/2))
/cos(e*x+d+arctan(3/5))/(34^(1/2)*sin(e*x+d+arctan(3/5))+2)^(1/2)/e
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{3\cos\left(ex+d\right)+5\sin\left(ex+d\right)+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2+3*cos(e*x+d)+5*sin(e*x+d))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(3*cos(e*x + d) + 5*sin(e*x + d) + 2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{\sqrt{3\cos\left(d+e\,x\right)+5\sin\left(d+e\,x\right)+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(3*cos(d + e*x) + 5*sin(d + e*x) + 2)^(1/2),x)
```

```
[Out] int(1/(3*cos(d + e*x) + 5*sin(d + e*x) + 2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{5\sin\left(d+ex\right)+3\cos\left(d+ex\right)+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2+3*cos(e*x+d)+5*sin(e*x+d))**(1/2),x)
[Out] Integral(1/sqrt(5*sin(d + e*x) + 3*cos(d + e*x) + 2), x)
```

3.407 $\int \frac{1}{(2+3\cos(d+ex)+5\sin(d+ex))^{3/2}}\,dx$

Optimal. Leaf size=94

$$-\frac{5\cos(d+ex)-3\sin(d+ex)}{15e\sqrt{5\sin(d+ex)+3\cos(d+ex)+2}}-\frac{\sqrt{2+\sqrt{34}}\,E\left(\frac{1}{2}\left(d+ex-\tan^{-1}\left(\frac{5}{3}\right)\right)|\frac{2}{15}\left(17-\sqrt{34}\right)\right)}{15e}$$

```
[Out] 1/15*(-5*cos(e*x+d)+3*sin(e*x+d))/e/(2+3*cos(e*x+d)+5*sin(e*x+d))^(1/2)-1/1
5*(cos(1/2*d+1/2*e*x-1/2*arctan(5/3))^2)^(1/2)/cos(1/2*d+1/2*e*x-1/2*arctan
(5/3))*EllipticE(sin(1/2*d+1/2*e*x-1/2*arctan(5/3)),1/15*(510-30*34^(1/2))^
(1/2))*(2+34^(1/2))^(1/2)/e
```

Rubi [A] time = 0.05, antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {3128, 3118, 2653}

$$-\frac{5\cos(d+ex)-3\sin(d+ex)}{15e\sqrt{5\sin(d+ex)+3\cos(d+ex)+2}}-\frac{\sqrt{2+\sqrt{34}}\,E\left(\frac{1}{2}\left(d+ex-\tan^{-1}\left(\frac{5}{3}\right)\right)|\frac{2}{15}\left(17-\sqrt{34}\right)\right)}{15e}$$

Antiderivative was successfully verified.

```
[In] Int[(2 + 3*Cos[d + e*x] + 5*Sin[d + e*x])^(-3/2),x]
```

```
[Out] -(Sqrt[2 + Sqrt[34]]*EllipticE[(d + e*x - ArcTan[5/3])/2, (2*(17 - Sqrt[34]
))/15]])/(15*e) - (5*Cos[d + e*x] - 3*Sin[d + e*x])/(15*e*Sqrt[2 + 3*Cos[d +
 e*x] + 5*Sin[d + e*x]])
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 3118

```
Int[Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_
)]], x_Symbol] :> Int[Sqrt[a + Sqrt[b^2 + c^2]*Cos[d + e*x - ArcTan[b, c]]]
, x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 + c^2, 0] && GtQ[a + Sqrt[b^2
+ c^2], 0]
```

Rule 3128

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(-3/2), x_Symbol] :> Simp[(2*(c*Cos[d + e*x] - b*Sin[d + e*x]))/(e*(a^2 - b
```

```
^2 - c^2)*Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]]), x] + Dist[1/(a^2 - b^
2 - c^2), Int[Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]], x], x] /; FreeQ[{a
, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0]
```

Rubi steps

$$\int \frac{1}{(2+3\cos(d+ex)+5\sin(d+ex))^{3/2}}\,dx = -\frac{5\cos(d+ex)-3\sin(d+ex)}{15e\sqrt{2+3\cos(d+ex)+5\sin(d+ex)}} - \frac{1}{30}\int \sqrt{2+3\cos(d+e$$

$$= -\frac{5\cos(d+ex)-3\sin(d+ex)}{15e\sqrt{2+3\cos(d+ex)+5\sin(d+ex)}} - \frac{1}{30}\int \sqrt{2+\sqrt{34}\cos\Big(}$$

$$= -\frac{\sqrt{2+\sqrt{34}}\,E\left(\frac{1}{2}\left(d+ex-\tan^{-1}\left(\frac{5}{3}\right)\right)|\frac{2}{15}\left(17-\sqrt{34}\right)\right)}{15e} - \frac{5\,\mathrm{co}}{15e\sqrt{2+}}$$

Mathematica [C] time = 6.03, size = 390, normalized size = 4.15

$$-2\sqrt{30}\sqrt{\sqrt{34}\sin\left(d+ex+\tan^{-1}\left(\frac{3}{5}\right)\right)+2}\sqrt{\cos^2\left(d+ex+\tan^{-1}\left(\frac{3}{5}\right)\right)}\sec\left(d+ex+\tan^{-1}\left(\frac{3}{5}\right)\right)F_1\Big(\frac{1}{2};\frac{1}{2},\frac{1}{2};\frac{3}{2};$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(2 + 3*Cos[d + e*x] + 5*Sin[d + e*x])^(-3/2),x]
```

```
[Out] (18*Sqrt[2 + Sqrt[34]*Cos[d + e*x - ArcTan[5/3]]] - 68*Sqrt[2 + 3*Cos[d + e
*x] + 5*Sin[d + e*x]] + (20*(5 + 17*Sin[d + e*x]))/Sqrt[2 + 3*Cos[d + e*x]
+ 5*Sin[d + e*x]] - 2*Sqrt[30]*AppellF1[1/2, 1/2, 1/2, 3/2, (Sqrt[34] + 17*
Sin[d + e*x + ArcTan[3/5]])/(-17 + Sqrt[34]), (Sqrt[34] + 17*Sin[d + e*x +
ArcTan[3/5]])/(17 + Sqrt[34])]*Sqrt[Cos[d + e*x + ArcTan[3/5]]^2]*Sec[d + e
*x + ArcTan[3/5]]*Sqrt[2 + Sqrt[34]*Sin[d + e*x + ArcTan[3/5]]] - (15*Sin[d
 + e*x - ArcTan[5/3]])/Sqrt[1/17 + Cos[d + e*x - ArcTan[5/3]]/Sqrt[34]] + (
15*Sqrt[30]*AppellF1[-1/2, -1/2, -1/2, 1/2, (Sqrt[34] + 17*Cos[d + e*x - Ar
cTan[5/3]])/(-17 + Sqrt[34]), (Sqrt[34] + 17*Cos[d + e*x - ArcTan[5/3]])/(1
7 + Sqrt[34])]*Csc[d + e*x - ArcTan[5/3]]*Sqrt[Sin[d + e*x - ArcTan[5/3]]^2
])/Sqrt[2 + Sqrt[34]*Cos[d + e*x - ArcTan[5/3]]])/(450*e)
```

fricas [F] time = 1.60, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(-\frac{\sqrt{3\cos(ex+d)+5\sin(ex+d)+2}}{16\cos(ex+d)^2-10\,(3\cos(ex+d)+2)\sin(ex+d)-12\cos(ex+d)-29},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2+3*cos(e*x+d)+5*sin(e*x+d))^(3/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(3*cos(e*x + d) + 5*sin(e*x + d) + 2)/(16*cos(e*x + d)^2 - 10
*(3*cos(e*x + d) + 2)*sin(e*x + d) - 12*cos(e*x + d) - 29), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(3\cos\left(ex+d\right)+5\sin\left(ex+d\right)+2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2+3*cos(e*x+d)+5*sin(e*x+d))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((3*cos(e*x + d) + 5*sin(e*x + d) + 2)^(-3/2), x)
```

maple [C] time = 0.49, size = 437, normalized size = 4.65

$$\frac{\sqrt{34}\,\left(255\sqrt{\left(17\sin\left(ex+d+\arctan\left(\frac{3}{5}\right)\right)+\sqrt{34}\right)\sqrt{34}\,\left(\cos^{2}\left(ex+d+\arctan\left(\frac{3}{5}\right)\right)\right)}\,\sqrt{-\frac{17\sin\left(ex+d+\arctan\left(\frac{3}{5}\right)\right)+}{-\sqrt{34}+17}}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2+3*cos(e*x+d)+5*sin(e*x+d))^(3/2),x)
```

```
[Out] 1/4335*34^(1/2)*(255*((17*sin(e*x+d+arctan(3/5))+34^(1/2))*34^(1/2)*cos(e*x
+d+arctan(3/5))^2)^(1/2)*(-(17*sin(e*x+d+arctan(3/5))+34^(1/2))/(-34^(1/2)+
17))^(1/2)*(-17*(sin(e*x+d+arctan(3/5))-1)/(34^(1/2)+17))^(1/2)*((1+sin(e*x
+d+arctan(3/5)))/(-34^(1/2)+17))^(1/2)*EllipticF((-(17*sin(e*x+d+arctan(3/5
))+34^(1/2))/(-34^(1/2)+17))^(1/2),I*((-34^(1/2)+17)/(34^(1/2)+17))^(1/2))-
255*((17*sin(e*x+d+arctan(3/5))+34^(1/2))*34^(1/2)*cos(e*x+d+arctan(3/5))^2
)^(1/2)*(-(17*sin(e*x+d+arctan(3/5))+34^(1/2))/(-34^(1/2)+17))^(1/2)*(-17*(
sin(e*x+d+arctan(3/5))-1)/(34^(1/2)+17))^(1/2)*((1+sin(e*x+d+arctan(3/5)))/
(-34^(1/2)+17))^(1/2)*EllipticE((-(17*sin(e*x+d+arctan(3/5))+34^(1/2))/(-34
^(1/2)+17))^(1/2),I*((-34^(1/2)+17)/(34^(1/2)+17))^(1/2))+289*((34^(1/2)*si
n(e*x+d+arctan(3/5))+2)*cos(e*x+d+arctan(3/5))^2)^(1/2)*sin(e*x+d+arctan(3/
5))^2-289*((34^(1/2)*sin(e*x+d+arctan(3/5))+2)*cos(e*x+d+arctan(3/5))^2)^(1
/2))*17^(1/2)/((17*sin(e*x+d+arctan(3/5))+34^(1/2))*34^(1/2)*cos(e*x+d+arct
an(3/5))^2)^(1/2)/cos(e*x+d+arctan(3/5))/(34^(1/2)*sin(e*x+d+arctan(3/5))+2
)^(1/2)/e
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(3\cos(ex+d)+5\sin(ex+d)+2)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2+3*cos(e*x+d)+5*sin(e*x+d))^(3/2),x, algorithm="maxima")
[Out] integrate((3*cos(e*x + d) + 5*sin(e*x + d) + 2)^(-3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{(3\cos(d+e\,x)+5\sin(d+e\,x)+2)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(3*cos(d + e*x) + 5*sin(d + e*x) + 2)^(3/2),x)
[Out] int(1/(3*cos(d + e*x) + 5*sin(d + e*x) + 2)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(5\sin(d+ex)+3\cos(d+ex)+2)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2+3*cos(e*x+d)+5*sin(e*x+d))**(3/2),x)
[Out] Integral((5*sin(d + e*x) + 3*cos(d + e*x) + 2)**(-3/2), x)
```

3.408 $\int \frac{1}{(2+3\cos(d+ex)+5\sin(d+ex))^{5/2}}\,dx$

Optimal. Leaf size=187

$$\frac{4(5\cos(d+ex)-3\sin(d+ex))}{675e\sqrt{5\sin(d+ex)+3\cos(d+ex)+2}}-\frac{5\cos(d+ex)-3\sin(d+ex)}{45e(5\sin(d+ex)+3\cos(d+ex)+2)^{3/2}}+\frac{F\left(\frac{1}{2}\left(d+ex-\tan^{-1}\left(\frac{5}{3}\right)\right)|\right.}{45\sqrt{2+\sqrt{34}}}$$

```
[Out] 1/45*(-5*cos(e*x+d)+3*sin(e*x+d))/e/(2+3*cos(e*x+d)+5*sin(e*x+d))^(3/2)+4/6
75*(5*cos(e*x+d)-3*sin(e*x+d))/e/(2+3*cos(e*x+d)+5*sin(e*x+d))^(1/2)+1/45*(
cos(1/2*d+1/2*e*x-1/2*arctan(5/3))^2)^(1/2)/cos(1/2*d+1/2*e*x-1/2*arctan(5/
3))*EllipticF(sin(1/2*d+1/2*e*x-1/2*arctan(5/3)),1/15*(510-30*34^(1/2))^(1/
2))/e/(2+34^(1/2))^(1/2)+4/675*(cos(1/2*d+1/2*e*x-1/2*arctan(5/3))^2)^(1/2)
/cos(1/2*d+1/2*e*x-1/2*arctan(5/3))*EllipticE(sin(1/2*d+1/2*e*x-1/2*arctan(
5/3)),1/15*(510-30*34^(1/2))^(1/2))*(2+34^(1/2))^(1/2)/e
```

Rubi [A] time = 0.20, antiderivative size = 187, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.318, Rules used = {3129, 3156, 3149, 3118, 2653, 3126, 2661}

$$\frac{4(5\cos(d+ex)-3\sin(d+ex))}{675e\sqrt{5\sin(d+ex)+3\cos(d+ex)+2}}-\frac{5\cos(d+ex)-3\sin(d+ex)}{45e(5\sin(d+ex)+3\cos(d+ex)+2)^{3/2}}+\frac{F\left(\frac{1}{2}\left(d+ex-\tan^{-1}\left(\frac{5}{3}\right)\right)|\right.}{45\sqrt{2+\sqrt{34}}}$$

Antiderivative was successfully verified.

```
[In] Int[(2 + 3*Cos[d + e*x] + 5*Sin[d + e*x])^(-5/2),x]
```

```
[Out] (4*Sqrt[2 + Sqrt[34]]*EllipticE[(d + e*x - ArcTan[5/3])/2, (2*(17 - Sqrt[34
]))/15])/(675*e) + EllipticF[(d + e*x - ArcTan[5/3])/2, (2*(17 - Sqrt[34]))
/15]/(45*Sqrt[2 + Sqrt[34]]*e) - (5*Cos[d + e*x] - 3*Sin[d + e*x])/(45*e*(2
 + 3*Cos[d + e*x] + 5*Sin[d + e*x])^(3/2)) + (4*(5*Cos[d + e*x] - 3*Sin[d +
 e*x]))/(675*e*Sqrt[2 + 3*Cos[d + e*x] + 5*Sin[d + e*x]])
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
```

```
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 3118

```
Int[Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_
)]], x_Symbol] :> Int[Sqrt[a + Sqrt[b^2 + c^2]*Cos[d + e*x - ArcTan[b, c]]]
, x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 + c^2, 0] && GtQ[a + Sqrt[b^2
+ c^2], 0]
```

Rule 3126

```
Int[1/Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(
x_)]], x_Symbol] :> Int[1/Sqrt[a + Sqrt[b^2 + c^2]*Cos[d + e*x - ArcTan[b,
c]]], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 + c^2, 0] && GtQ[a + Sqrt[
b^2 + c^2], 0]
```

Rule 3129

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(n_), x_Symbol] :> Simp[((-(c*Cos[d + e*x]) + b*Sin[d + e*x])*(a + b*Cos[d
+ e*x] + c*Sin[d + e*x])^(n + 1))/(e*(n + 1)*(a^2 - b^2 - c^2)), x] + Dist[
1/((n + 1)*(a^2 - b^2 - c^2)), Int[(a*(n + 1) - b*(n + 2)*Cos[d + e*x] - c*
(n + 2)*Sin[d + e*x])*(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n + 1), x], x]
 /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0] && LtQ[n, -1] && N
eQ[n, -3/2]
```

Rule 3149

```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])
/Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)]]
, x_Symbol] :> Dist[B/b, Int[Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]], x],
 x] + Dist[(A*b - a*B)/b, Int[1/Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]],
x], x] /; FreeQ[{a, b, c, d, e, A, B, C}, x] && EqQ[B*c - b*C, 0] && NeQ[A*
b - a*B, 0]
```

Rule 3156

```
Int[((a_.) + cos[(d_.) + (e_.)*(x_)]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)])
^(n_)*((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)
]), x_Symbol] :> -Simp[((c*B - b*C - (a*C - c*A)*Cos[d + e*x] + (a*B - b*A)
*Sin[d + e*x])*(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n + 1))/(e*(n + 1)*(a
^2 - b^2 - c^2)), x] + Dist[1/((n + 1)*(a^2 - b^2 - c^2)), Int[(a + b*Cos[d
 + e*x] + c*Sin[d + e*x])^(n + 1)*Simp[(n + 1)*(a*A - b*B - c*C) + (n + 2)*
(a*B - b*A)*Cos[d + e*x] + (n + 2)*(a*C - c*A)*Sin[d + e*x], x], x], x] /;
FreeQ[{a, b, c, d, e, A, B, C}, x] && LtQ[n, -1] && NeQ[a^2 - b^2 - c^2, 0]
```

```
&& NeQ[n, -2]
```

Rubi steps

$$\int \frac{1}{(2+3\cos(d+ex)+5\sin(d+ex))^{5/2}}\,dx = -\frac{5\cos(d+ex)-3\sin(d+ex)}{45e(2+3\cos(d+ex)+5\sin(d+ex))^{3/2}} + \frac{1}{45}\int \frac{-3+\frac{3}{2}\cos(d}{(2+3\cos(d+}$$

$$= -\frac{5\cos(d+ex)-3\sin(d+ex)}{45e(2+3\cos(d+ex)+5\sin(d+ex))^{3/2}} + \frac{4(5\cos(d+ex)}{675e\sqrt{2+3\cos(d+}}$$

$$= -\frac{5\cos(d+ex)-3\sin(d+ex)}{45e(2+3\cos(d+ex)+5\sin(d+ex))^{3/2}} + \frac{4(5\cos(d+ex)}{675e\sqrt{2+3\cos(d+}}$$

$$= -\frac{5\cos(d+ex)-3\sin(d+ex)}{45e(2+3\cos(d+ex)+5\sin(d+ex))^{3/2}} + \frac{4(5\cos(d+ex)}{675e\sqrt{2+3\cos(d+}}$$

$$= \frac{4\sqrt{2+\sqrt{34}}\,E\left(\frac{1}{2}\left(d+ex-\tan^{-1}\left(\frac{5}{3}\right)\right)|\frac{2}{15}\left(17-\sqrt{34}\right)\right)}{675e} + \frac{F\left(\frac{1}{2}\left(d\right.\right.}{}$$

Mathematica [C] time = 3.18, size = 430, normalized size = 2.30

$$\frac{23\sqrt{\frac{10}{3}}\sqrt{\sqrt{34}\sin\left(d+ex+\tan^{-1}\left(\frac{3}{5}\right)\right)+2}\sqrt{\cos^2\left(d+ex+\tan^{-1}\left(\frac{3}{5}\right)\right)}\sec\left(d+ex+\tan^{-1}\left(\frac{3}{5}\right)\right)F_1\left(\frac{1}{2};\frac{1}{2},\frac{1}{2};\frac{3}{2};\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(2 + 3*Cos[d + e*x] + 5*Sin[d + e*x])^(-5/2),x]
```

```
[Out] (-24*Sqrt[2 + Sqrt[34]*Cos[d + e*x - ArcTan[5/3]]] + (272*Sqrt[2 + 3*Cos[d
+ e*x] + 5*Sin[d + e*x]])/3 + (100*(5 + 17*Sin[d + e*x]))/(2 + 3*Cos[d + e*
x] + 5*Sin[d + e*x])^(3/2) - (10*(115 + 136*Sin[d + e*x]))/(3*Sqrt[2 + 3*Co
s[d + e*x] + 5*Sin[d + e*x]]) + 23*Sqrt[10/3]*AppellF1[1/2, 1/2, 1/2, 3/2,
(Sqrt[34] + 17*Sin[d + e*x + ArcTan[3/5]])/(-17 + Sqrt[34]), (Sqrt[34] + 17
*Sin[d + e*x + ArcTan[3/5]])/(17 + Sqrt[34])]*Sqrt[Cos[d + e*x + ArcTan[3/5
]]^2]*Sec[d + e*x + ArcTan[3/5]]*Sqrt[2 + Sqrt[34]*Sin[d + e*x + ArcTan[3/5
]]] + (20*Sin[d + e*x - ArcTan[5/3]])/Sqrt[1/17 + Cos[d + e*x - ArcTan[5/3]
]/Sqrt[34]] - (20*Sqrt[30]*AppellF1[-1/2, -1/2, -1/2, 1/2, (Sqrt[34] + 17*C
```

```
os[d + e*x - ArcTan[5/3]])/(-17 + Sqrt[34]), (Sqrt[34] + 17*Cos[d + e*x - A
rcTan[5/3]])/(17 + Sqrt[34])]*Csc[d + e*x - ArcTan[5/3]]*Sqrt[Sin[d + e*x -
 ArcTan[5/3]]^2])/Sqrt[2 + Sqrt[34]*Cos[d + e*x - ArcTan[5/3]]])/(6750*e)
```

fricas [F] time = 2.88, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\sqrt{3\cos(ex+d)+5\sin(ex+d)+2}}{198\cos(ex+d)^3+96\cos(ex+d)^2-5\left(2\cos(ex+d)^2+36\cos(ex+d)+37\right)\sin(ex+d)-261\cos\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2+3*cos(e*x+d)+5*sin(e*x+d))^(5/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(3*cos(e*x + d) + 5*sin(e*x + d) + 2)/(198*cos(e*x + d)^3 + 9
6*cos(e*x + d)^2 - 5*(2*cos(e*x + d)^2 + 36*cos(e*x + d) + 37)*sin(e*x + d)
 - 261*cos(e*x + d) - 158), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(3\cos(ex+d)+5\sin(ex+d)+2)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2+3*cos(e*x+d)+5*sin(e*x+d))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((3*cos(e*x + d) + 5*sin(e*x + d) + 2)^(-5/2), x)
```

maple [C] time = 0.61, size = 542, normalized size = 2.90

$$\frac{\sqrt{-\left(-\sqrt{34}\sin\left(ex+d+\arctan\left(\frac{3}{5}\right)\right)-2\right)\left(\cos^2\left(ex+d+\arctan\left(\frac{3}{5}\right)\right)\right)}\left(-\frac{\sqrt{34}\sqrt{-\left(-\sqrt{34}\sin\left(ex+d+\arctan\left(\frac{3}{5}\right)\right)-2\right)\left(\cos\right.}}{1530\left(\sin\left(ex+d+\arctan\left(\frac{3}{5}\right)\right)+\right.}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2+3*cos(e*x+d)+5*sin(e*x+d))^(5/2),x)
```

```
[Out] (-(-34^(1/2)*sin(e*x+d+arctan(3/5))-2)*cos(e*x+d+arctan(3/5))^2)^(1/2)*(-1/
1530*34^(1/2)*(-(-34^(1/2)*sin(e*x+d+arctan(3/5))-2)*cos(e*x+d+arctan(3/5))
^2)^(1/2)/(sin(e*x+d+arctan(3/5))+1/17*34^(1/2))^2+68/675*34^(1/2)*cos(e*x+
d+arctan(3/5))^2/(-(-289*sin(e*x+d+arctan(3/5))-17*34^(1/2))*34^(1/2)*cos(e
```

```
*x+d+arctan(3/5))^2)^(1/2)+23/675*(-1+1/17*34^(1/2))*((-17*sin(e*x+d+arctan
(3/5))-34^(1/2))/(-34^(1/2)+17))^(1/2)*((-17*sin(e*x+d+arctan(3/5))+17)/(34
^(1/2)+17))^(1/2)*((17*sin(e*x+d+arctan(3/5))+17)/(-34^(1/2)+17))^(1/2)/(-(
-34^(1/2)*sin(e*x+d+arctan(3/5))-2)*cos(e*x+d+arctan(3/5))^2)^(1/2)*Ellipti
cF(((-17*sin(e*x+d+arctan(3/5))-34^(1/2))/(-34^(1/2)+17))^(1/2),I*((-34^(1/
2)+17)/(34^(1/2)+17))^(1/2))+4/675*34^(1/2)*(-1+1/17*34^(1/2))*((-17*sin(e*
x+d+arctan(3/5))-34^(1/2))/(-34^(1/2)+17))^(1/2)*((-17*sin(e*x+d+arctan(3/5
))+17)/(34^(1/2)+17))^(1/2)*((17*sin(e*x+d+arctan(3/5))+17)/(-34^(1/2)+17))
^(1/2)/(-(-34^(1/2)*sin(e*x+d+arctan(3/5))-2)*cos(e*x+d+arctan(3/5))^2)^(1/
2)*((-1/17*34^(1/2)-1)*EllipticE(((-17*sin(e*x+d+arctan(3/5))-34^(1/2))/(-3
4^(1/2)+17))^(1/2),I*((-34^(1/2)+17)/(34^(1/2)+17))^(1/2))+EllipticF(((-17*
sin(e*x+d+arctan(3/5))-34^(1/2))/(-34^(1/2)+17))^(1/2),I*((-34^(1/2)+17)/(3
4^(1/2)+17))^(1/2))))/cos(e*x+d+arctan(3/5))/(34^(1/2)*sin(e*x+d+arctan(3/5
))+2)^(1/2)/e
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(3 \cos(ex+d) + 5 \sin(ex+d) + 2)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2+3*cos(e*x+d)+5*sin(e*x+d))^(5/2),x, algorithm="maxima")
[Out] integrate((3*cos(e*x + d) + 5*sin(e*x + d) + 2)^(-5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{(3 \cos(d+e\,x) + 5 \sin(d+e\,x) + 2)^{5/2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(3*cos(d + e*x) + 5*sin(d + e*x) + 2)^(5/2),x)
[Out] int(1/(3*cos(d + e*x) + 5*sin(d + e*x) + 2)^(5/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(5\sin(d+ex) + 3\cos(d+ex) + 2)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2+3*cos(e*x+d)+5*sin(e*x+d))**(5/2),x)
[Out] Integral((5*sin(d + e*x) + 3*cos(d + e*x) + 2)**(-5/2), x)
```

3.409 $\int \frac{1}{(2+3\cos(d+ex)+5\sin(d+ex))^{7/2}}\,dx$

Optimal. Leaf size=233

$$-\frac{199(5\cos(d+ex)-3\sin(d+ex))}{101250e\sqrt{5\sin(d+ex)+3\cos(d+ex)+2}}+\frac{8(5\cos(d+ex)-3\sin(d+ex))}{3375e(5\sin(d+ex)+3\cos(d+ex)+2)^{3/2}}-\frac{5\cos(d+ex)-}{75e(5\sin(d+ex)+3}$$

```
[Out] 1/75*(-5*cos(e*x+d)+3*sin(e*x+d))/e/(2+3*cos(e*x+d)+5*sin(e*x+d))^(5/2)+8/3
375*(5*cos(e*x+d)-3*sin(e*x+d))/e/(2+3*cos(e*x+d)+5*sin(e*x+d))^(3/2)-199/1
01250*(5*cos(e*x+d)-3*sin(e*x+d))/e/(2+3*cos(e*x+d)+5*sin(e*x+d))^(1/2)-8/3
375*(cos(1/2*d+1/2*e*x-1/2*arctan(5/3))^2)^(1/2)/cos(1/2*d+1/2*e*x-1/2*arct
an(5/3))*EllipticF(sin(1/2*d+1/2*e*x-1/2*arctan(5/3)),1/15*(510-30*34^(1/2)
)^(1/2))/e/(2+34^(1/2))^(1/2)-199/101250*(cos(1/2*d+1/2*e*x-1/2*arctan(5/3)
)^2)^(1/2)/cos(1/2*d+1/2*e*x-1/2*arctan(5/3))*EllipticE(sin(1/2*d+1/2*e*x-1
/2*arctan(5/3)),1/15*(510-30*34^(1/2))^(1/2))*(2+34^(1/2))^(1/2)/e
```

Rubi [A] time = 0.26, antiderivative size = 233, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.318, Rules used = {3129, 3156, 3149, 3118, 2653, 3126, 2661}

$$-\frac{199(5\cos(d+ex)-3\sin(d+ex))}{101250e\sqrt{5\sin(d+ex)+3\cos(d+ex)+2}}+\frac{8(5\cos(d+ex)-3\sin(d+ex))}{3375e(5\sin(d+ex)+3\cos(d+ex)+2)^{3/2}}-\frac{5\cos(d+ex)-}{75e(5\sin(d+ex)+3}$$

Antiderivative was successfully verified.

```
[In] Int[(2 + 3*Cos[d + e*x] + 5*Sin[d + e*x])^(-7/2),x]
```

```
[Out] (-199*Sqrt[2 + Sqrt[34]]*EllipticE[(d + e*x - ArcTan[5/3])/2, (2*(17 - Sqrt
[34]))/15])/(101250*e) - (8*EllipticF[(d + e*x - ArcTan[5/3])/2, (2*(17 - S
qrt[34]))/15])/(3375*Sqrt[2 + Sqrt[34]]*e) - (5*Cos[d + e*x] - 3*Sin[d + e*
x])/(75*e*(2 + 3*Cos[d + e*x] + 5*Sin[d + e*x])^(5/2)) + (8*(5*Cos[d + e*x]
 - 3*Sin[d + e*x]))/(3375*e*(2 + 3*Cos[d + e*x] + 5*Sin[d + e*x])^(3/2)) -
(199*(5*Cos[d + e*x] - 3*Sin[d + e*x]))/(101250*e*Sqrt[2 + 3*Cos[d + e*x] +
 5*Sin[d + e*x]])
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 3118

```
Int[Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_
)]], x_Symbol] :> Int[Sqrt[a + Sqrt[b^2 + c^2]*Cos[d + e*x - ArcTan[b, c]]]
, x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 + c^2, 0] && GtQ[a + Sqrt[b^2
+ c^2], 0]
```

Rule 3126

```
Int[1/Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(
x_)]], x_Symbol] :> Int[1/Sqrt[a + Sqrt[b^2 + c^2]*Cos[d + e*x - ArcTan[b,
c]]], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 + c^2, 0] && GtQ[a + Sqrt[
b^2 + c^2], 0]
```

Rule 3129

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(n_), x_Symbol] :> Simp[((-(c*Cos[d + e*x]) + b*Sin[d + e*x])*(a + b*Cos[d
+ e*x] + c*Sin[d + e*x])^(n + 1))/(e*(n + 1)*(a^2 - b^2 - c^2)), x] + Dist[
1/((n + 1)*(a^2 - b^2 - c^2)), Int[(a*(n + 1) - b*(n + 2)*Cos[d + e*x] - c*
(n + 2)*Sin[d + e*x])*(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n + 1), x], x]
 /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0] && LtQ[n, -1] && N
eQ[n, -3/2]
```

Rule 3149

```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])
/Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)]]
, x_Symbol] :> Dist[B/b, Int[Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]], x],
 x] + Dist[(A*b - a*B)/b, Int[1/Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]],
x], x] /; FreeQ[{a, b, c, d, e, A, B, C}, x] && EqQ[B*c - b*C, 0] && NeQ[A*
b - a*B, 0]
```

Rule 3156

```
Int[((a_.) + cos[(d_.) + (e_.)*(x_)]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)])
^(n_)*((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)
]), x_Symbol] :> -Simp[((c*B - b*C - (a*C - c*A)*Cos[d + e*x] + (a*B - b*A)
*Sin[d + e*x])*(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n + 1))/(e*(n + 1)*(a
^2 - b^2 - c^2)), x] + Dist[1/((n + 1)*(a^2 - b^2 - c^2)), Int[(a + b*Cos[d
 + e*x] + c*Sin[d + e*x])^(n + 1)*Simp[(n + 1)*(a*A - b*B - c*C) + (n + 2)*
```

```
(a*B - b*A)*Cos[d + e*x] + (n + 2)*(a*C - c*A)*Sin[d + e*x], x], x], x] /;
FreeQ[{a, b, c, d, e, A, B, C}, x] && LtQ[n, -1] && NeQ[a^2 - b^2 - c^2, 0]
 && NeQ[n, -2]
```

Rubi steps

$$\int \frac{1}{(2+3\cos(d+ex)+5\sin(d+ex))^{7/2}}\,dx = -\frac{5\cos(d+ex)-3\sin(d+ex)}{75e(2+3\cos(d+ex)+5\sin(d+ex))^{5/2}} + \frac{1}{75}\int \frac{-5+\frac{9}{2}\cos(d+}{(2+3\cos(d+}$$

$$= -\frac{5\cos(d+ex)-3\sin(d+ex)}{75e(2+3\cos(d+ex)+5\sin(d+ex))^{5/2}} + \frac{8(5\cos(d+ex)}{3375e(2+3\cos(d+e}$$

$$= -\frac{5\cos(d+ex)-3\sin(d+ex)}{75e(2+3\cos(d+ex)+5\sin(d+ex))^{5/2}} + \frac{8(5\cos(d+ex)}{3375e(2+3\cos(d+e}$$

$$= -\frac{5\cos(d+ex)-3\sin(d+ex)}{75e(2+3\cos(d+ex)+5\sin(d+ex))^{5/2}} + \frac{8(5\cos(d+ex)}{3375e(2+3\cos(d+e}$$

$$= -\frac{5\cos(d+ex)-3\sin(d+ex)}{75e(2+3\cos(d+ex)+5\sin(d+ex))^{5/2}} + \frac{8(5\cos(d+ex)}{3375e(2+3\cos(d+e}$$

$$= -\frac{199\sqrt{2+\sqrt{34}}\,E\left(\frac{1}{2}\left(d+ex-\tan^{-1}\left(\frac{5}{3}\right)\right)|\frac{2}{15}\left(17-\sqrt{34}\right)\right)}{101250e} - \frac{8F\left(\frac{1}{2}\right.}{}$$

Mathematica [C] time = 3.84, size = 436, normalized size = 1.87

$$\frac{-638\sqrt{30}\sqrt{\sqrt{34}\sin\left(d+ex+\tan^{-1}\left(\frac{3}{5}\right)\right)+2}\sqrt{\cos^2\left(d+ex+\tan^{-1}\left(\frac{3}{5}\right)\right)}\sec\left(d+ex+\tan^{-1}\left(\frac{3}{5}\right)\right)F_1\left(\frac{1}{2};\frac{1}{2},\frac{1}{2};\frac{3}{2}\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(2 + 3*Cos[d + e*x] + 5*Sin[d + e*x])^(-7/2),x]
```

```
[Out] (-13532*Sqrt[2 + 3*Cos[d + e*x] + 5*Sin[d + e*x]] + (597*(12 + 43*Cos[d + e
*x] + 15*Sin[d + e*x]))/Sqrt[2 + Sqrt[34]*Cos[d + e*x - ArcTan[5/3]]] + (27
000*(5 + 17*Sin[d + e*x]))/(2 + 3*Cos[d + e*x] + 5*Sin[d + e*x])^(5/2) - (3
```

```
00*(305 + 272*Sin[d + e*x]))/(2 + 3*Cos[d + e*x] + 5*Sin[d + e*x])^(3/2) +
(20*(1595 + 3383*Sin[d + e*x]))/Sqrt[2 + 3*Cos[d + e*x] + 5*Sin[d + e*x]] -
 638*Sqrt[30]*AppellF1[1/2, 1/2, 1/2, 3/2, (Sqrt[34] + 17*Sin[d + e*x + Arc
Tan[3/5]])/(-17 + Sqrt[34]), (Sqrt[34] + 17*Sin[d + e*x + ArcTan[3/5]])/(17
 + Sqrt[34])]*Sqrt[Cos[d + e*x + ArcTan[3/5]]^2]*Sec[d + e*x + ArcTan[3/5]]
*Sqrt[2 + Sqrt[34]*Sin[d + e*x + ArcTan[3/5]]] + (2985*Sqrt[30]*AppellF1[-1
/2, -1/2, -1/2, 1/2, (Sqrt[34] + 17*Cos[d + e*x - ArcTan[5/3]])/(-17 + Sqrt
[34]), (Sqrt[34] + 17*Cos[d + e*x - ArcTan[5/3]])/(17 + Sqrt[34])]*Csc[d +
e*x - ArcTan[5/3]]*Sqrt[Sin[d + e*x - ArcTan[5/3]]^2])/Sqrt[2 + Sqrt[34]*Co
s[d + e*x - ArcTan[5/3]]]])/(3037500*e)
```

fricas [F] time = 0.84, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(-\frac{\sqrt{3\cos\left(ex+d\right)+5\sin\left(ex+d\right)+2}}{644\cos\left(ex+d\right)^{4}+1584\cos\left(ex+d\right)^{3}+284\cos\left(ex+d\right)^{2}+20\left(48\cos\left(ex+d\right)^{3}-4\cos\left(ex+d\right)^{2}\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2+3*cos(e*x+d)+5*sin(e*x+d))^(7/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(3*cos(e*x + d) + 5*sin(e*x + d) + 2)/(644*cos(e*x + d)^4 + 1
584*cos(e*x + d)^3 + 284*cos(e*x + d)^2 + 20*(48*cos(e*x + d)^3 - 4*cos(e*x
 + d)^2 - 111*cos(e*x + d) - 58)*sin(e*x + d) - 1896*cos(e*x + d) - 1241),
x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(3\cos\left(ex+d\right)+5\sin\left(ex+d\right)+2\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2+3*cos(e*x+d)+5*sin(e*x+d))^(7/2),x, algorithm="giac")
```

```
[Out] integrate((3*cos(e*x + d) + 5*sin(e*x + d) + 2)^(-7/2), x)
```

maple [C] time = 0.63, size = 571, normalized size = 2.45

$$\frac{\sqrt{-\left(-\sqrt{34}\,\sin\left(ex+d+\arctan\left(\frac{3}{5}\right)\right)-2\right)\left(\cos^{2}\left(ex+d+\arctan\left(\frac{3}{5}\right)\right)\right)}\left(-\frac{\sqrt{-\left(-\sqrt{34}\,\sin\left(ex+d+\arctan\left(\frac{3}{5}\right)\right)-2\right)\left(\cos^{2}\left(e\right.\right.}}{2550\left(\sin\left(ex+d+\arctan\left(\frac{3}{5}\right)\right)+\frac{\sqrt{}}{1}\right.}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2+3*cos(e*x+d)+5*sin(e*x+d))^(7/2),x)
```

```
[Out] (-(-34^(1/2)*sin(e*x+d+arctan(3/5))-2)*cos(e*x+d+arctan(3/5))^2)^(1/2)*(-1/
2550*(-(-34^(1/2)*sin(e*x+d+arctan(3/5))-2)*cos(e*x+d+arctan(3/5))^2)^(1/2)
/(sin(e*x+d+arctan(3/5))+1/17*34^(1/2))^3+4/57375*34^(1/2)*(-(-34^(1/2)*sin
(e*x+d+arctan(3/5))-2)*cos(e*x+d+arctan(3/5))^2)^(1/2)/(sin(e*x+d+arctan(3/
5))+1/17*34^(1/2))^2-3383/101250*34^(1/2)*cos(e*x+d+arctan(3/5))^2/(-(-289*
sin(e*x+d+arctan(3/5))-17*34^(1/2))*34^(1/2)*cos(e*x+d+arctan(3/5))^2)^(1/2
)-319/50625*(1/17*34^(1/2)+1)*((17*sin(e*x+d+arctan(3/5))+34^(1/2))/(34^(1/
2)+17))^(1/2)*((17*sin(e*x+d+arctan(3/5))+17)/(-34^(1/2)+17))^(1/2)*((-17*s
in(e*x+d+arctan(3/5))+17)/(34^(1/2)+17))^(1/2)/(-(-34^(1/2)*sin(e*x+d+arcta
n(3/5))-2)*cos(e*x+d+arctan(3/5))^2)^(1/2)*EllipticF(((17*sin(e*x+d+arctan(
3/5))+34^(1/2))/(34^(1/2)+17))^(1/2),I*(1/(-34^(1/2)+17)*(34^(1/2)+17))^(1/
2))-199/101250*34^(1/2)*(1/17*34^(1/2)+1)*((17*sin(e*x+d+arctan(3/5))+34^(1
/2))/(34^(1/2)+17))^(1/2)*((17*sin(e*x+d+arctan(3/5))+17)/(-34^(1/2)+17))^(
1/2)*((-17*sin(e*x+d+arctan(3/5))+17)/(34^(1/2)+17))^(1/2)/(-(-34^(1/2)*sin
(e*x+d+arctan(3/5))-2)*cos(e*x+d+arctan(3/5))^2)^(1/2)*((-1/17*34^(1/2)+1)*
EllipticE(((17*sin(e*x+d+arctan(3/5))+34^(1/2))/(34^(1/2)+17))^(1/2),I*(1/(
-34^(1/2)+17)*(34^(1/2)+17))^(1/2))-EllipticF(((17*sin(e*x+d+arctan(3/5))+3
4^(1/2))/(34^(1/2)+17))^(1/2),I*(1/(-34^(1/2)+17)*(34^(1/2)+17))^(1/2))))/c
os(e*x+d+arctan(3/5))/(34^(1/2)*sin(e*x+d+arctan(3/5))+2)^(1/2)/e
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(3 \cos(ex+d) + 5 \sin(ex+d) + 2)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2+3*cos(e*x+d)+5*sin(e*x+d))^(7/2),x, algorithm="maxima")
```

```
[Out] integrate((3*cos(e*x + d) + 5*sin(e*x + d) + 2)^(-7/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{(3 \cos(d+e\,x) + 5 \sin(d+e\,x) + 2)^{7/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(3*cos(d + e*x) + 5*sin(d + e*x) + 2)^(7/2),x)
```

```
[Out] int(1/(3*cos(d + e*x) + 5*sin(d + e*x) + 2)^(7/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2+3*cos(e*x+d)+5*sin(e*x+d))**(7/2),x)
```

```
[Out] Timed out
```

3.410 $\int (a+b\cos(d+ex)+c\sin(d+ex))^{5/2}\,dx$

Optimal. Leaf size=347

$$-\frac{16a\left(a^2-b^2-c^2\right)\sqrt{\frac{a+b\cos(d+ex)+c\sin(d+ex)}{a+\sqrt{b^2+c^2}}}\,F\left(\frac{1}{2}\left(d+ex-\tan^{-1}(b,c)\right)|\frac{2\sqrt{b^2+c^2}}{a+\sqrt{b^2+c^2}}\right)}{15e\sqrt{a+b\cos(d+ex)+c\sin(d+ex)}}+\frac{2\left(23a^2+9\left(b^2+c^2\right)\right)\sqrt{a+b\cos}}{}$$

```
[Out] -2/5*(c*cos(e*x+d)-b*sin(e*x+d))*(a+b*cos(e*x+d)+c*sin(e*x+d))^(3/2)/e-16/1
5*(a*c*cos(e*x+d)-a*b*sin(e*x+d))*(a+b*cos(e*x+d)+c*sin(e*x+d))^(1/2)/e+2/1
5*(23*a^2+9*b^2+9*c^2)*(cos(1/2*d+1/2*e*x-1/2*arctan(b,c))^2)^(1/2)/cos(1/2
*d+1/2*e*x-1/2*arctan(b,c))*EllipticE(sin(1/2*d+1/2*e*x-1/2*arctan(b,c)),2^
(1/2)*((b^2+c^2)^(1/2)/(a+(b^2+c^2)^(1/2)))^(1/2))*(a+b*cos(e*x+d)+c*sin(e*
x+d))^(1/2)/e/((a+b*cos(e*x+d)+c*sin(e*x+d))/(a+(b^2+c^2)^(1/2)))^(1/2)-16/
15*a*(a^2-b^2-c^2)*(cos(1/2*d+1/2*e*x-1/2*arctan(b,c))^2)^(1/2)/cos(1/2*d+1
/2*e*x-1/2*arctan(b,c))*EllipticF(sin(1/2*d+1/2*e*x-1/2*arctan(b,c)),2^(1/2
)*((b^2+c^2)^(1/2)/(a+(b^2+c^2)^(1/2)))^(1/2))*((a+b*cos(e*x+d)+c*sin(e*x+d
))/(a+(b^2+c^2)^(1/2)))^(1/2)/e/(a+b*cos(e*x+d)+c*sin(e*x+d))^(1/2)
```

Rubi [A] time = 0.53, antiderivative size = 347, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.318, Rules used = {3120, 3146, 3149, 3119, 2653, 3127, 2661}

$$-\frac{16a\left(a^2-b^2-c^2\right)\sqrt{\frac{a+b\cos(d+ex)+c\sin(d+ex)}{a+\sqrt{b^2+c^2}}}\,F\left(\frac{1}{2}\left(d+ex-\tan^{-1}(b,c)\right)|\frac{2\sqrt{b^2+c^2}}{a+\sqrt{b^2+c^2}}\right)}{15e\sqrt{a+b\cos(d+ex)+c\sin(d+ex)}}+\frac{2\left(23a^2+9\left(b^2+c^2\right)\right)\sqrt{a+b\cos}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(5/2),x]
```

```
[Out] (-16*(a*c*Cos[d + e*x] - a*b*Sin[d + e*x])*Sqrt[a + b*Cos[d + e*x] + c*Sin[
d + e*x]])/(15*e) - (2*(c*Cos[d + e*x] - b*Sin[d + e*x])*(a + b*Cos[d + e*x
] + c*Sin[d + e*x])^(3/2))/(5*e) + (2*(23*a^2 + 9*(b^2 + c^2))*EllipticE[(d
 + e*x - ArcTan[b, c])/2, (2*Sqrt[b^2 + c^2])/(a + Sqrt[b^2 + c^2])]*Sqrt[a
 + b*Cos[d + e*x] + c*Sin[d + e*x]])/(15*e*Sqrt[(a + b*Cos[d + e*x] + c*Sin
[d + e*x])/(a + Sqrt[b^2 + c^2])]) - (16*a*(a^2 - b^2 - c^2)*EllipticF[(d +
 e*x - ArcTan[b, c])/2, (2*Sqrt[b^2 + c^2])/(a + Sqrt[b^2 + c^2])]*Sqrt[(a
+ b*Cos[d + e*x] + c*Sin[d + e*x])/(a + Sqrt[b^2 + c^2])])/(15*e*Sqrt[a + b
*Cos[d + e*x] + c*Sin[d + e*x]])
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 3119

```
Int[Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_
)]], x_Symbol] :> Dist[Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]]/Sqrt[(a +
b*Cos[d + e*x] + c*Sin[d + e*x])/(a + Sqrt[b^2 + c^2])], Int[Sqrt[a/(a + Sq
rt[b^2 + c^2]) + (Sqrt[b^2 + c^2]*Cos[d + e*x - ArcTan[b, c]])/(a + Sqrt[b^
2 + c^2])], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0]
&& NeQ[b^2 + c^2, 0] &&  !GtQ[a + Sqrt[b^2 + c^2], 0]
```

Rule 3120

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(n_), x_Symbol] :> -Simp[((c*Cos[d + e*x] - b*Sin[d + e*x])*(a + b*Cos[d +
e*x] + c*Sin[d + e*x])^(n - 1))/(e*n), x] + Dist[1/n, Int[Simp[n*a^2 + (n -
 1)*(b^2 + c^2) + a*b*(2*n - 1)*Cos[d + e*x] + a*c*(2*n - 1)*Sin[d + e*x],
x]*(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n - 2), x], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[a^2 - b^2 - c^2, 0] && GtQ[n, 1]
```

Rule 3127

```
Int[1/Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(
x_)]], x_Symbol] :> Dist[Sqrt[(a + b*Cos[d + e*x] + c*Sin[d + e*x])/(a + Sq
rt[b^2 + c^2])]/Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]], Int[1/Sqrt[a/(a
+ Sqrt[b^2 + c^2]) + (Sqrt[b^2 + c^2]*Cos[d + e*x - ArcTan[b, c]])/(a + Sqr
t[b^2 + c^2])], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2,
 0] && NeQ[b^2 + c^2, 0] &&  !GtQ[a + Sqrt[b^2 + c^2], 0]
```

Rule 3146

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(n_.)*((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)
]), x_Symbol] :> Simp[((B*c - b*C - a*C*Cos[d + e*x] + a*B*Sin[d + e*x])*(a
 + b*Cos[d + e*x] + c*Sin[d + e*x])^n)/(a*e*(n + 1)), x] + Dist[1/(a*(n + 1
)), Int[(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n - 1)*Simp[a*(b*B + c*C)*n
+ a^2*A*(n + 1) + (n*(a^2*B - B*c^2 + b*c*C) + a*b*A*(n + 1))*Cos[d + e*x]
```

```
+ (n*(b*B*c + a^2*C - b^2*C) + a*c*A*(n + 1))*Sin[d + e*x], x], x], x] /; F
reeQ[{a, b, c, d, e, A, B, C}, x] && GtQ[n, 0] && NeQ[a^2 - b^2 - c^2, 0]
```

Rule 3149

```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])
/Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)]]
, x_Symbol] :> Dist[B/b, Int[Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]], x],
 x] + Dist[(A*b - a*B)/b, Int[1/Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]],
x], x] /; FreeQ[{a, b, c, d, e, A, B, C}, x] && EqQ[B*c - b*C, 0] && NeQ[A*
b - a*B, 0]
```

Rubi steps

$$\int (a+b\cos(d+ex)+c\sin(d+ex))^{5/2}\,dx = -\frac{2(c\cos(d+ex)-b\sin(d+ex))(a+b\cos(d+ex)+c\sin(d+ex))^{3/}}{5e}$$

$$= -\frac{16(ac\cos(d+ex)-ab\sin(d+ex))\sqrt{a+b\cos(d+ex)+c\sin(d+e}}{15e}$$

$$= -\frac{16(ac\cos(d+ex)-ab\sin(d+ex))\sqrt{a+b\cos(d+ex)+c\sin(d+e}}{15e}$$

$$= -\frac{16(ac\cos(d+ex)-ab\sin(d+ex))\sqrt{a+b\cos(d+ex)+c\sin(d+e}}{15e}$$

$$= -\frac{16(ac\cos(d+ex)-ab\sin(d+ex))\sqrt{a+b\cos(d+ex)+c\sin(d+e}}{15e}$$

Mathematica [C] time = 6.62, size = 3767, normalized size = 10.86

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(5/2),x]
```

```
[Out] (Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]]*((2*b*(23*a^2 + 9*b^2 + 9*c^2))/
(15*c) - (22*a*c*Cos[d + e*x])/15 - (2*b*c*Cos[2*(d + e*x)])/5 + (22*a*b*Si
n[d + e*x])/15 + ((b^2 - c^2)*Sin[2*(d + e*x)])/5))/e + (2*a^3*AppellF1[1/2
, 1/2, 1/2, 3/2, -((a + Sqrt[1 + b^2/c^2]*c*Sin[d + e*x + ArcTan[b/c]])/(Sq
rt[1 + b^2/c^2]*(1 - a/(Sqrt[1 + b^2/c^2]*c))*c)), -((a + Sqrt[1 + b^2/c^2]
*c*Sin[d + e*x + ArcTan[b/c]])/(Sqrt[1 + b^2/c^2]*(-1 - a/(Sqrt[1 + b^2/c^2
]*c))*c))]*Sec[d + e*x + ArcTan[b/c]]*Sqrt[(c*Sqrt[(b^2 + c^2)/c^2] - c*Sqr
t[(b^2 + c^2)/c^2]*Sin[d + e*x + ArcTan[b/c]])/(a + c*Sqrt[(b^2 + c^2)/c^2]
)]*Sqrt[a + c*Sqrt[(b^2 + c^2)/c^2]*Sin[d + e*x + ArcTan[b/c]]]*Sqrt[(c*Sqr
t[(b^2 + c^2)/c^2] + c*Sqrt[(b^2 + c^2)/c^2]*Sin[d + e*x + ArcTan[b/c]])/(-
a + c*Sqrt[(b^2 + c^2)/c^2])])/(Sqrt[1 + b^2/c^2]*c*e) + (34*a*b^2*AppellF1
[1/2, 1/2, 1/2, 3/2, -((a + Sqrt[1 + b^2/c^2]*c*Sin[d + e*x + ArcTan[b/c]])
/(Sqrt[1 + b^2/c^2]*(1 - a/(Sqrt[1 + b^2/c^2]*c))*c)), -((a + Sqrt[1 + b^2/
c^2]*c*Sin[d + e*x + ArcTan[b/c]])/(Sqrt[1 + b^2/c^2]*(-1 - a/(Sqrt[1 + b^2
/c^2]*c))*c))]*Sec[d + e*x + ArcTan[b/c]]*Sqrt[(c*Sqrt[(b^2 + c^2)/c^2] - c
*Sqrt[(b^2 + c^2)/c^2]*Sin[d + e*x + ArcTan[b/c]])/(a + c*Sqrt[(b^2 + c^2)/
c^2])]*Sqrt[a + c*Sqrt[(b^2 + c^2)/c^2]*Sin[d + e*x + ArcTan[b/c]]]*Sqrt[(c
*Sqrt[(b^2 + c^2)/c^2] + c*Sqrt[(b^2 + c^2)/c^2]*Sin[d + e*x + ArcTan[b/c]]
)/(-a + c*Sqrt[(b^2 + c^2)/c^2])])/(15*Sqrt[1 + b^2/c^2]*c*e) + (34*a*c*App
ellF1[1/2, 1/2, 1/2, 3/2, -((a + Sqrt[1 + b^2/c^2]*c*Sin[d + e*x + ArcTan[b
/c]])/(Sqrt[1 + b^2/c^2]*(1 - a/(Sqrt[1 + b^2/c^2]*c))*c)), -((a + Sqrt[1 +
 b^2/c^2]*c*Sin[d + e*x + ArcTan[b/c]])/(Sqrt[1 + b^2/c^2]*(-1 - a/(Sqrt[1
+ b^2/c^2]*c))*c))]*Sec[d + e*x + ArcTan[b/c]]*Sqrt[(c*Sqrt[(b^2 + c^2)/c^2
] - c*Sqrt[(b^2 + c^2)/c^2]*Sin[d + e*x + ArcTan[b/c]])/(a + c*Sqrt[(b^2 +
c^2)/c^2])]*Sqrt[a + c*Sqrt[(b^2 + c^2)/c^2]*Sin[d + e*x + ArcTan[b/c]]]*Sq
rt[(c*Sqrt[(b^2 + c^2)/c^2] + c*Sqrt[(b^2 + c^2)/c^2]*Sin[d + e*x + ArcTan[
b/c]])/(-a + c*Sqrt[(b^2 + c^2)/c^2])])/(15*Sqrt[1 + b^2/c^2]*e) + (23*a^2*
b^2*(-((c*AppellF1[-1/2, -1/2, -1/2, 1/2, -((a + b*Sqrt[1 + c^2/b^2]*Cos[d
+ e*x - ArcTan[c/b]])/(b*Sqrt[1 + c^2/b^2]*(1 - a/(b*Sqrt[1 + c^2/b^2])))),
 -((a + b*Sqrt[1 + c^2/b^2]*Cos[d + e*x - ArcTan[c/b]])/(b*Sqrt[1 + c^2/b^2
]*(-1 - a/(b*Sqrt[1 + c^2/b^2]))))]*Sin[d + e*x - ArcTan[c/b]])/(b*Sqrt[1 +
 c^2/b^2]*Sqrt[(b*Sqrt[(b^2 + c^2)/b^2] - b*Sqrt[(b^2 + c^2)/b^2]*Cos[d + e
*x - ArcTan[c/b]])/(a + b*Sqrt[(b^2 + c^2)/b^2])]*Sqrt[a + b*Sqrt[(b^2 + c^
2)/b^2]*Cos[d + e*x - ArcTan[c/b]]]*Sqrt[(b*Sqrt[(b^2 + c^2)/b^2] + b*Sqrt[
(b^2 + c^2)/b^2]*Cos[d + e*x - ArcTan[c/b]])/(-a + b*Sqrt[(b^2 + c^2)/b^2])
])) - ((2*b*(a + b*Sqrt[1 + c^2/b^2]*Cos[d + e*x - ArcTan[c/b]]))/(b^2 + c^
2) - (c*Sin[d + e*x - ArcTan[c/b]])/(b*Sqrt[1 + c^2/b^2]))/Sqrt[a + b*Sqrt[
1 + c^2/b^2]*Cos[d + e*x - ArcTan[c/b]]]))/(15*c*e) + (3*b^4*(-((c*AppellF1
[-1/2, -1/2, -1/2, 1/2, -((a + b*Sqrt[1 + c^2/b^2]*Cos[d + e*x - ArcTan[c/b
]])/(b*Sqrt[1 + c^2/b^2]*(1 - a/(b*Sqrt[1 + c^2/b^2])))), -((a + b*Sqrt[1 +
 c^2/b^2]*Cos[d + e*x - ArcTan[c/b]])/(b*Sqrt[1 + c^2/b^2]*(-1 - a/(b*Sqrt[
1 + c^2/b^2]))))]*Sin[d + e*x - ArcTan[c/b]])/(b*Sqrt[1 + c^2/b^2]*Sqrt[(b*
Sqrt[(b^2 + c^2)/b^2] - b*Sqrt[(b^2 + c^2)/b^2]*Cos[d + e*x - ArcTan[c/b]])
/(a + b*Sqrt[(b^2 + c^2)/b^2])]*Sqrt[a + b*Sqrt[(b^2 + c^2)/b^2]*Cos[d + e*
x - ArcTan[c/b]]]*Sqrt[(b*Sqrt[(b^2 + c^2)/b^2] + b*Sqrt[(b^2 + c^2)/b^2]*C
```

```
os[d + e*x - ArcTan[c/b]])/(-a + b*Sqrt[(b^2 + c^2)/b^2])])) - ((2*b*(a + b
*Sqrt[1 + c^2/b^2]*Cos[d + e*x - ArcTan[c/b]]))/(b^2 + c^2) - (c*Sin[d + e*
x - ArcTan[c/b]])/(b*Sqrt[1 + c^2/b^2]))/Sqrt[a + b*Sqrt[1 + c^2/b^2]*Cos[d
 + e*x - ArcTan[c/b]]]))/(5*c*e) + (23*a^2*c*(-((c*AppellF1[-1/2, -1/2, -1/
2, 1/2, -((a + b*Sqrt[1 + c^2/b^2]*Cos[d + e*x - ArcTan[c/b]])/(b*Sqrt[1 +
c^2/b^2]*(1 - a/(b*Sqrt[1 + c^2/b^2])))), -((a + b*Sqrt[1 + c^2/b^2]*Cos[d
+ e*x - ArcTan[c/b]])/(b*Sqrt[1 + c^2/b^2]*(-1 - a/(b*Sqrt[1 + c^2/b^2]))))
]*Sin[d + e*x - ArcTan[c/b]])/(b*Sqrt[1 + c^2/b^2]*Sqrt[(b*Sqrt[(b^2 + c^2)
/b^2] - b*Sqrt[(b^2 + c^2)/b^2]*Cos[d + e*x - ArcTan[c/b]])/(a + b*Sqrt[(b^
2 + c^2)/b^2])]*Sqrt[a + b*Sqrt[(b^2 + c^2)/b^2]*Cos[d + e*x - ArcTan[c/b]]
]*Sqrt[(b*Sqrt[(b^2 + c^2)/b^2] + b*Sqrt[(b^2 + c^2)/b^2]*Cos[d + e*x - Arc
Tan[c/b]])/(-a + b*Sqrt[(b^2 + c^2)/b^2])])) - ((2*b*(a + b*Sqrt[1 + c^2/b^
2]*Cos[d + e*x - ArcTan[c/b]]))/(b^2 + c^2) - (c*Sin[d + e*x - ArcTan[c/b]]
)/(b*Sqrt[1 + c^2/b^2]))/Sqrt[a + b*Sqrt[1 + c^2/b^2]*Cos[d + e*x - ArcTan[
c/b]]]))/(15*e) + (6*b^2*c*(-((c*AppellF1[-1/2, -1/2, -1/2, 1/2, -((a + b*S
qrt[1 + c^2/b^2]*Cos[d + e*x - ArcTan[c/b]])/(b*Sqrt[1 + c^2/b^2]*(1 - a/(b
*Sqrt[1 + c^2/b^2])))), -((a + b*Sqrt[1 + c^2/b^2]*Cos[d + e*x - ArcTan[c/b
]])/(b*Sqrt[1 + c^2/b^2]*(-1 - a/(b*Sqrt[1 + c^2/b^2]))))]*Sin[d + e*x - Ar
cTan[c/b]])/(b*Sqrt[1 + c^2/b^2]*Sqrt[(b*Sqrt[(b^2 + c^2)/b^2] - b*Sqrt[(b^
2 + c^2)/b^2]*Cos[d + e*x - ArcTan[c/b]])/(a + b*Sqrt[(b^2 + c^2)/b^2])]*Sq
rt[a + b*Sqrt[(b^2 + c^2)/b^2]*Cos[d + e*x - ArcTan[c/b]]]*Sqrt[(b*Sqrt[(b^
2 + c^2)/b^2] + b*Sqrt[(b^2 + c^2)/b^2]*Cos[d + e*x - ArcTan[c/b]])/(-a + b
*Sqrt[(b^2 + c^2)/b^2])])) - ((2*b*(a + b*Sqrt[1 + c^2/b^2]*Cos[d + e*x - A
rcTan[c/b]]))/(b^2 + c^2) - (c*Sin[d + e*x - ArcTan[c/b]])/(b*Sqrt[1 + c^2/
b^2]))/Sqrt[a + b*Sqrt[1 + c^2/b^2]*Cos[d + e*x - ArcTan[c/b]]]))/(5*e) + (
3*c^3*(-((c*AppellF1[-1/2, -1/2, -1/2, 1/2, -((a + b*Sqrt[1 + c^2/b^2]*Cos[
d + e*x - ArcTan[c/b]])/(b*Sqrt[1 + c^2/b^2]*(1 - a/(b*Sqrt[1 + c^2/b^2])))
), -((a + b*Sqrt[1 + c^2/b^2]*Cos[d + e*x - ArcTan[c/b]])/(b*Sqrt[1 + c^2/b
^2]*(-1 - a/(b*Sqrt[1 + c^2/b^2]))))]*Sin[d + e*x - ArcTan[c/b]])/(b*Sqrt[1
 + c^2/b^2]*Sqrt[(b*Sqrt[(b^2 + c^2)/b^2] - b*Sqrt[(b^2 + c^2)/b^2]*Cos[d +
 e*x - ArcTan[c/b]])/(a + b*Sqrt[(b^2 + c^2)/b^2])]*Sqrt[a + b*Sqrt[(b^2 +
c^2)/b^2]*Cos[d + e*x - ArcTan[c/b]]]*Sqrt[(b*Sqrt[(b^2 + c^2)/b^2] + b*Sqr
t[(b^2 + c^2)/b^2]*Cos[d + e*x - ArcTan[c/b]])/(-a + b*Sqrt[(b^2 + c^2)/b^2
])])) - ((2*b*(a + b*Sqrt[1 + c^2/b^2]*Cos[d + e*x - ArcTan[c/b]]))/(b^2 +
c^2) - (c*Sin[d + e*x - ArcTan[c/b]])/(b*Sqrt[1 + c^2/b^2]))/Sqrt[a + b*Sqr
t[1 + c^2/b^2]*Cos[d + e*x - ArcTan[c/b]]]))/(5*e)
```

fricas [F] time = 1.88, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(2\,ab\cos\left(ex+d\right)+\left(b^2-c^2\right)\cos\left(ex+d\right)^2+a^2+c^2+2\left(bc\cos\left(ex+d\right)+ac\right)\sin\left(ex+d\right)\right)\sqrt{b\cos\left(ex+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(e*x+d)+c*sin(e*x+d))^(5/2),x, algorithm="fricas")

[Out] integral((2*a*b*cos(e*x + d) + (b^2 - c^2)*cos(e*x + d)^2 + a^2 + c^2 + 2*(
```

```
b*c*cos(e*x + d) + a*c)*sin(e*x + d))*sqrt(b*cos(e*x + d) + c*sin(e*x + d)
+ a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b\cos(ex+d)+c\sin(ex+d)+a)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(e*x+d)+c*sin(e*x+d))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*cos(e*x + d) + c*sin(e*x + d) + a)^(5/2), x)
```

maple [B] time = 1.00, size = 2303, normalized size = 6.64

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(e*x+d)+c*sin(e*x+d))^(5/2),x)
```

```
[Out] (-(-b^2*sin(e*x+d-arctan(-b,c))-c^2*sin(e*x+d-arctan(-b,c))-a*(b^2+c^2)^(1/
2))*cos(e*x+d-arctan(-b,c))^2/(b^2+c^2)^(1/2))^(1/2)*((b^2+c^2)^(3/2)*(-2/5
/(b^2+c^2)^(1/2)*sin(e*x+d-arctan(-b,c))*(cos(e*x+d-arctan(-b,c))^2*((b^2+c
^2)^(1/2)*sin(e*x+d-arctan(-b,c))+a))^(1/2)+8/15/(b^2+c^2)*a*(cos(e*x+d-arc
tan(-b,c))^2*((b^2+c^2)^(1/2)*sin(e*x+d-arctan(-b,c))+a))^(1/2)+4/15/(b^2+c
^2)^(1/2)*a*(1/(b^2+c^2)^(1/2)*a-1)*((-(b^2+c^2)^(1/2)*sin(e*x+d-arctan(-b,
c))-a)/(-a+(b^2+c^2)^(1/2)))^(1/2)*((-sin(e*x+d-arctan(-b,c))+1)*(b^2+c^2)^
(1/2)/(a+(b^2+c^2)^(1/2)))^(1/2)*((1+sin(e*x+d-arctan(-b,c)))*(b^2+c^2)^(1/
2)/(-a+(b^2+c^2)^(1/2)))^(1/2)/(cos(e*x+d-arctan(-b,c))^2*((b^2+c^2)^(1/2)*
sin(e*x+d-arctan(-b,c))+a))^(1/2)*EllipticF(((-(b^2+c^2)^(1/2)*sin(e*x+d-ar
ctan(-b,c))-a)/(-a+(b^2+c^2)^(1/2)))^(1/2),((a-(b^2+c^2)^(1/2))/(a+(b^2+c^2
)^(1/2)))^(1/2))+2*(3/5+8/15/(b^2+c^2)*a^2)*(1/(b^2+c^2)^(1/2)*a-1)*((-(b^2
+c^2)^(1/2)*sin(e*x+d-arctan(-b,c))-a)/(-a+(b^2+c^2)^(1/2)))^(1/2)*((-sin(e
*x+d-arctan(-b,c))+1)*(b^2+c^2)^(1/2)/(a+(b^2+c^2)^(1/2)))^(1/2)*((1+sin(e*
x+d-arctan(-b,c)))*(b^2+c^2)^(1/2)/(-a+(b^2+c^2)^(1/2)))^(1/2)/(cos(e*x+d-a
rctan(-b,c))^2*((b^2+c^2)^(1/2)*sin(e*x+d-arctan(-b,c))+a))^(1/2)*((-1/(b^2
+c^2)^(1/2)*a-1)*EllipticE(((-(b^2+c^2)^(1/2)*sin(e*x+d-arctan(-b,c))-a)/(-
a+(b^2+c^2)^(1/2)))^(1/2),((a-(b^2+c^2)^(1/2))/(a+(b^2+c^2)^(1/2)))^(1/2))+
EllipticF(((-(b^2+c^2)^(1/2)*sin(e*x+d-arctan(-b,c))-a)/(-a+(b^2+c^2)^(1/2)
))^(1/2),((a-(b^2+c^2)^(1/2))/(a+(b^2+c^2)^(1/2)))^(1/2))))+(3*a*b^2+3*a*c^
2)*(-2/3/(b^2+c^2)^(1/2)*(cos(e*x+d-arctan(-b,c))^2*((b^2+c^2)^(1/2)*sin(e*
x+d-arctan(-b,c))+a))^(1/2)+2/3*(1/(b^2+c^2)^(1/2)*a-1)*((-(b^2+c^2)^(1/2)*
sin(e*x+d-arctan(-b,c))-a)/(-a+(b^2+c^2)^(1/2)))^(1/2)*((-sin(e*x+d-arctan(
-b,c))+1)*(b^2+c^2)^(1/2)/(a+(b^2+c^2)^(1/2)))^(1/2)*((1+sin(e*x+d-arctan(-
b,c)))*(b^2+c^2)^(1/2)/(-a+(b^2+c^2)^(1/2)))^(1/2)/(cos(e*x+d-arctan(-b,c))
```

```
^2*((b^2+c^2)^(1/2)*sin(e*x+d-arctan(-b,c))+a))^(1/2)*EllipticF(((-(b^2+c^2
)^(1/2)*sin(e*x+d-arctan(-b,c))-a)/(-a+(b^2+c^2)^(1/2)))^(1/2),((a-(b^2+c^2
)^(1/2))/(a+(b^2+c^2)^(1/2)))^(1/2))-4/3/(b^2+c^2)^(1/2)*a*(1/(b^2+c^2)^(1/
2)*a-1)*((-(b^2+c^2)^(1/2)*sin(e*x+d-arctan(-b,c))-a)/(-a+(b^2+c^2)^(1/2)))
^(1/2)*((-sin(e*x+d-arctan(-b,c))+1)*(b^2+c^2)^(1/2)/(a+(b^2+c^2)^(1/2)))^(
1/2)*((1+sin(e*x+d-arctan(-b,c)))*(b^2+c^2)^(1/2)/(-a+(b^2+c^2)^(1/2)))^(1/
2)/(cos(e*x+d-arctan(-b,c))^2*((b^2+c^2)^(1/2)*sin(e*x+d-arctan(-b,c))+a))^
(1/2)*((-1/(b^2+c^2)^(1/2)*a-1)*EllipticE(((-(b^2+c^2)^(1/2)*sin(e*x+d-arct
an(-b,c))-a)/(-a+(b^2+c^2)^(1/2)))^(1/2),((a-(b^2+c^2)^(1/2))/(a+(b^2+c^2)^
(1/2)))^(1/2))+EllipticF(((-(b^2+c^2)^(1/2)*sin(e*x+d-arctan(-b,c))-a)/(-a+
(b^2+c^2)^(1/2)))^(1/2),((a-(b^2+c^2)^(1/2))/(a+(b^2+c^2)^(1/2)))^(1/2)))+
6*a^2*(b^2+c^2)^(1/2)*(1/(b^2+c^2)^(1/2)*a-1)*((-(b^2+c^2)^(1/2)*sin(e*x+d-
arctan(-b,c))-a)/(-a+(b^2+c^2)^(1/2)))^(1/2)*((-sin(e*x+d-arctan(-b,c))+1)*
(b^2+c^2)^(1/2)/(a+(b^2+c^2)^(1/2)))^(1/2)*((1+sin(e*x+d-arctan(-b,c)))*(b^
2+c^2)^(1/2)/(-a+(b^2+c^2)^(1/2)))^(1/2)/(cos(e*x+d-arctan(-b,c))^2*((b^2+c
^2)^(1/2)*sin(e*x+d-arctan(-b,c))+a))^(1/2)*((-1/(b^2+c^2)^(1/2)*a-1)*Ellip
ticE(((-(b^2+c^2)^(1/2)*sin(e*x+d-arctan(-b,c))-a)/(-a+(b^2+c^2)^(1/2)))^(1
/2),((a-(b^2+c^2)^(1/2))/(a+(b^2+c^2)^(1/2)))^(1/2))+EllipticF(((-(b^2+c^2)
^(1/2)*sin(e*x+d-arctan(-b,c))-a)/(-a+(b^2+c^2)^(1/2)))^(1/2),((a-(b^2+c^2)
^(1/2))/(a+(b^2+c^2)^(1/2)))^(1/2)))+2*a^3*(1/(b^2+c^2)^(1/2)*a-1)*((-(b^2+
c^2)^(1/2)*sin(e*x+d-arctan(-b,c))-a)/(-a+(b^2+c^2)^(1/2)))^(1/2)*((-sin(e*
x+d-arctan(-b,c))+1)*(b^2+c^2)^(1/2)/(a+(b^2+c^2)^(1/2)))^(1/2)*((1+sin(e*x
+d-arctan(-b,c)))*(b^2+c^2)^(1/2)/(-a+(b^2+c^2)^(1/2)))^(1/2)/(-(-b^2*sin(e
*x+d-arctan(-b,c))-c^2*sin(e*x+d-arctan(-b,c))-a*(b^2+c^2)^(1/2))*cos(e*x+d
-arctan(-b,c))^2/(b^2+c^2)^(1/2))^(1/2)*EllipticF(((-(b^2+c^2)^(1/2)*sin(e*
x+d-arctan(-b,c))-a)/(-a+(b^2+c^2)^(1/2)))^(1/2),((a-(b^2+c^2)^(1/2))/(a+(b
^2+c^2)^(1/2)))^(1/2)))/cos(e*x+d-arctan(-b,c))/((b^2*sin(e*x+d-arctan(-b,c
))+c^2*sin(e*x+d-arctan(-b,c))+a*(b^2+c^2)^(1/2))/(b^2+c^2)^(1/2))^(1/2)/e
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b\cos(ex+d) + c\sin(ex+d) + a)^{\frac{5}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(e*x+d)+c*sin(e*x+d))^(5/2),x, algorithm="maxima")
[Out] integrate((b*cos(e*x + d) + c*sin(e*x + d) + a)^(5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int (a + b\,\cos(d + e\,x) + c\,\sin(d + e\,x))^{5/2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*cos(d + e*x) + c*sin(d + e*x))^(5/2),x)
[Out] int((a + b*cos(d + e*x) + c*sin(d + e*x))^(5/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(e*x+d)+c*sin(e*x+d))**(5/2),x)
[Out] Timed out
```

3.411 $\int (a+b\cos(d+ex)+c\sin(d+ex))^{3/2}\,dx$

Optimal. Leaf size=283

$$-\frac{2\left(a^2-b^2-c^2\right)\sqrt{\frac{a+b\cos(d+ex)+c\sin(d+ex)}{a+\sqrt{b^2+c^2}}}\,F\left(\frac{1}{2}\left(d+ex-\tan^{-1}(b,c)\right)|\frac{2\sqrt{b^2+c^2}}{a+\sqrt{b^2+c^2}}\right)}{3e\sqrt{a+b\cos(d+ex)+c\sin(d+ex)}}+\frac{8a\sqrt{a+b\cos(d+ex)+c\sin(d+}}{3e\sqrt{\frac{a+b}{}}}$$

```
[Out] -2/3*(c*cos(e*x+d)-b*sin(e*x+d))*(a+b*cos(e*x+d)+c*sin(e*x+d))^(1/2)/e+8/3*
a*(cos(1/2*d+1/2*e*x-1/2*arctan(b,c))^2)^(1/2)/cos(1/2*d+1/2*e*x-1/2*arctan
(b,c))*EllipticE(sin(1/2*d+1/2*e*x-1/2*arctan(b,c)),2^(1/2)*((b^2+c^2)^(1/2
)/(a+(b^2+c^2)^(1/2)))^(1/2))*(a+b*cos(e*x+d)+c*sin(e*x+d))^(1/2)/e/((a+b*c
os(e*x+d)+c*sin(e*x+d))/(a+(b^2+c^2)^(1/2)))^(1/2)-2/3*(a^2-b^2-c^2)*(cos(1
/2*d+1/2*e*x-1/2*arctan(b,c))^2)^(1/2)/cos(1/2*d+1/2*e*x-1/2*arctan(b,c))*E
llipticF(sin(1/2*d+1/2*e*x-1/2*arctan(b,c)),2^(1/2)*((b^2+c^2)^(1/2)/(a+(b^
2+c^2)^(1/2)))^(1/2))*((a+b*cos(e*x+d)+c*sin(e*x+d))/(a+(b^2+c^2)^(1/2)))^(
1/2)/e/(a+b*cos(e*x+d)+c*sin(e*x+d))^(1/2)
```

Rubi [A] time = 0.28, antiderivative size = 283, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {3120, 3149, 3119, 2653, 3127, 2661}

$$-\frac{2\left(a^2-b^2-c^2\right)\sqrt{\frac{a+b\cos(d+ex)+c\sin(d+ex)}{a+\sqrt{b^2+c^2}}}\,F\left(\frac{1}{2}\left(d+ex-\tan^{-1}(b,c)\right)|\frac{2\sqrt{b^2+c^2}}{a+\sqrt{b^2+c^2}}\right)}{3e\sqrt{a+b\cos(d+ex)+c\sin(d+ex)}}+\frac{8a\sqrt{a+b\cos(d+ex)+c\sin(d+}}{3e\sqrt{\frac{a+b}{}}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(3/2),x]
```

```
[Out] (-2*(c*Cos[d + e*x] - b*Sin[d + e*x])*Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e
*x]])/(3*e) + (8*a*EllipticE[(d + e*x - ArcTan[b, c])/2, (2*Sqrt[b^2 + c^2]
)/(a + Sqrt[b^2 + c^2])]*Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]])/(3*e*Sq
rt[(a + b*Cos[d + e*x] + c*Sin[d + e*x])/(a + Sqrt[b^2 + c^2])]) - (2*(a^2
- b^2 - c^2)*EllipticF[(d + e*x - ArcTan[b, c])/2, (2*Sqrt[b^2 + c^2])/(a +
 Sqrt[b^2 + c^2])]*Sqrt[(a + b*Cos[d + e*x] + c*Sin[d + e*x])/(a + Sqrt[b^2
 + c^2])])/(3*e*Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]])
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
```

```
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 3119

```
Int[Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_
)]], x_Symbol] :> Dist[Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]]/Sqrt[(a +
b*Cos[d + e*x] + c*Sin[d + e*x])/(a + Sqrt[b^2 + c^2])], Int[Sqrt[a/(a + Sq
rt[b^2 + c^2]) + (Sqrt[b^2 + c^2]*Cos[d + e*x - ArcTan[b, c]])/(a + Sqrt[b^
2 + c^2])], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0]
&& NeQ[b^2 + c^2, 0] &&  !GtQ[a + Sqrt[b^2 + c^2], 0]
```

Rule 3120

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(n_), x_Symbol] :> -Simp[((c*Cos[d + e*x] - b*Sin[d + e*x])*(a + b*Cos[d +
e*x] + c*Sin[d + e*x])^(n - 1))/(e*n), x] + Dist[1/n, Int[Simp[n*a^2 + (n -
 1)*(b^2 + c^2) + a*b*(2*n - 1)*Cos[d + e*x] + a*c*(2*n - 1)*Sin[d + e*x],
x]*(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n - 2), x], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[a^2 - b^2 - c^2, 0] && GtQ[n, 1]
```

Rule 3127

```
Int[1/Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(
x_)]], x_Symbol] :> Dist[Sqrt[(a + b*Cos[d + e*x] + c*Sin[d + e*x])/(a + Sq
rt[b^2 + c^2])]/Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]], Int[1/Sqrt[a/(a
+ Sqrt[b^2 + c^2]) + (Sqrt[b^2 + c^2]*Cos[d + e*x - ArcTan[b, c]])/(a + Sqr
t[b^2 + c^2])], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2,
 0] && NeQ[b^2 + c^2, 0] &&  !GtQ[a + Sqrt[b^2 + c^2], 0]
```

Rule 3149

```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])
/Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)]]
, x_Symbol] :> Dist[B/b, Int[Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]], x],
 x] + Dist[(A*b - a*B)/b, Int[1/Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]],
x], x] /; FreeQ[{a, b, c, d, e, A, B, C}, x] && EqQ[B*c - b*C, 0] && NeQ[A*
b - a*B, 0]
```

Rubi steps

$$\int (a+b\cos(d+ex)+c\sin(d+ex))^{3/2}\,dx = -\frac{2(c\cos(d+ex)-b\sin(d+ex))\sqrt{a+b\cos(d+ex)+c\sin(d+ex)}}{3e}$$

$$= -\frac{2(c\cos(d+ex)-b\sin(d+ex))\sqrt{a+b\cos(d+ex)+c\sin(d+ex)}}{3e}$$

$$= -\frac{2(c\cos(d+ex)-b\sin(d+ex))\sqrt{a+b\cos(d+ex)+c\sin(d+ex)}}{3e}$$

$$= -\frac{2(c\cos(d+ex)-b\sin(d+ex))\sqrt{a+b\cos(d+ex)+c\sin(d+ex)}}{3e}$$

Mathematica [C] time = 6.26, size = 2190, normalized size = 7.74

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(3/2),x]
```

```
[Out] (((8*a*b)/(3*c) - (2*c*Cos[d + e*x])/3 + (2*b*Sin[d + e*x])/3)*Sqrt[a + b*C
os[d + e*x] + c*Sin[d + e*x]])/e + (2*a^2*AppellF1[1/2, 1/2, 1/2, 3/2, -((a
 + Sqrt[1 + b^2/c^2]*c*Sin[d + e*x + ArcTan[b/c]])/(Sqrt[1 + b^2/c^2]*(1 -
a/(Sqrt[1 + b^2/c^2]*c))*c)), -((a + Sqrt[1 + b^2/c^2]*c*Sin[d + e*x + ArcT
an[b/c]])/(Sqrt[1 + b^2/c^2]*(-1 - a/(Sqrt[1 + b^2/c^2]*c))*c))]*Sec[d + e*
x + ArcTan[b/c]]*Sqrt[(c*Sqrt[(b^2 + c^2)/c^2] - c*Sqrt[(b^2 + c^2)/c^2]*Si
n[d + e*x + ArcTan[b/c]])/(a + c*Sqrt[(b^2 + c^2)/c^2])]*Sqrt[a + c*Sqrt[(b
^2 + c^2)/c^2]*Sin[d + e*x + ArcTan[b/c]]]*Sqrt[(c*Sqrt[(b^2 + c^2)/c^2] +
c*Sqrt[(b^2 + c^2)/c^2]*Sin[d + e*x + ArcTan[b/c]])/(-a + c*Sqrt[(b^2 + c^2
)/c^2])])/(Sqrt[1 + b^2/c^2]*c*e) + (2*b^2*AppellF1[1/2, 1/2, 1/2, 3/2, -((
a + Sqrt[1 + b^2/c^2]*c*Sin[d + e*x + ArcTan[b/c]])/(Sqrt[1 + b^2/c^2]*(1 -
 a/(Sqrt[1 + b^2/c^2]*c))*c)), -((a + Sqrt[1 + b^2/c^2]*c*Sin[d + e*x + Arc
Tan[b/c]])/(Sqrt[1 + b^2/c^2]*(-1 - a/(Sqrt[1 + b^2/c^2]*c))*c))]*Sec[d + e
*x + ArcTan[b/c]]*Sqrt[(c*Sqrt[(b^2 + c^2)/c^2] - c*Sqrt[(b^2 + c^2)/c^2]*S
in[d + e*x + ArcTan[b/c]])/(a + c*Sqrt[(b^2 + c^2)/c^2])]*Sqrt[a + c*Sqrt[(
b^2 + c^2)/c^2]*Sin[d + e*x + ArcTan[b/c]]]*Sqrt[(c*Sqrt[(b^2 + c^2)/c^2] +
```

```
 c*Sqrt[(b^2 + c^2)/c^2]*Sin[d + e*x + ArcTan[b/c]])/(-a + c*Sqrt[(b^2 + c^
2)/c^2])])/(3*Sqrt[1 + b^2/c^2]*c*e) + (2*c*AppellF1[1/2, 1/2, 1/2, 3/2, -(
(a + Sqrt[1 + b^2/c^2]*c*Sin[d + e*x + ArcTan[b/c]])/(Sqrt[1 + b^2/c^2]*(1
- a/(Sqrt[1 + b^2/c^2]*c))*c)), -((a + Sqrt[1 + b^2/c^2]*c*Sin[d + e*x + Ar
cTan[b/c]])/(Sqrt[1 + b^2/c^2]*(-1 - a/(Sqrt[1 + b^2/c^2]*c))*c))]*Sec[d +
e*x + ArcTan[b/c]]*Sqrt[(c*Sqrt[(b^2 + c^2)/c^2] - c*Sqrt[(b^2 + c^2)/c^2]*
Sin[d + e*x + ArcTan[b/c]])/(a + c*Sqrt[(b^2 + c^2)/c^2])]*Sqrt[a + c*Sqrt[
(b^2 + c^2)/c^2]*Sin[d + e*x + ArcTan[b/c]]]*Sqrt[(c*Sqrt[(b^2 + c^2)/c^2]
+ c*Sqrt[(b^2 + c^2)/c^2]*Sin[d + e*x + ArcTan[b/c]])/(-a + c*Sqrt[(b^2 + c
^2)/c^2])])/(3*Sqrt[1 + b^2/c^2]*e) + (4*a*b^2*(-((c*AppellF1[-1/2, -1/2, -
1/2, 1/2, -((a + b*Sqrt[1 + c^2/b^2]*Cos[d + e*x - ArcTan[c/b]])/(b*Sqrt[1
+ c^2/b^2]*(1 - a/(b*Sqrt[1 + c^2/b^2])))), -((a + b*Sqrt[1 + c^2/b^2]*Cos[
d + e*x - ArcTan[c/b]])/(b*Sqrt[1 + c^2/b^2]*(-1 - a/(b*Sqrt[1 + c^2/b^2]))
))]*Sin[d + e*x - ArcTan[c/b]])/(b*Sqrt[1 + c^2/b^2]*Sqrt[(b*Sqrt[(b^2 + c^
2)/b^2] - b*Sqrt[(b^2 + c^2)/b^2]*Cos[d + e*x - ArcTan[c/b]])/(a + b*Sqrt[(
b^2 + c^2)/b^2])]*Sqrt[a + b*Sqrt[(b^2 + c^2)/b^2]*Cos[d + e*x - ArcTan[c/b
]]]*Sqrt[(b*Sqrt[(b^2 + c^2)/b^2] + b*Sqrt[(b^2 + c^2)/b^2]*Cos[d + e*x - A
rcTan[c/b]])/(-a + b*Sqrt[(b^2 + c^2)/b^2])])) - ((2*b*(a + b*Sqrt[1 + c^2/
b^2]*Cos[d + e*x - ArcTan[c/b]]))/(b^2 + c^2) - (c*Sin[d + e*x - ArcTan[c/b
]])/(b*Sqrt[1 + c^2/b^2]))/Sqrt[a + b*Sqrt[1 + c^2/b^2]*Cos[d + e*x - ArcTa
n[c/b]]]))/(3*c*e) + (4*a*c*(-((c*AppellF1[-1/2, -1/2, -1/2, 1/2, -((a + b*
Sqrt[1 + c^2/b^2]*Cos[d + e*x - ArcTan[c/b]])/(b*Sqrt[1 + c^2/b^2]*(1 - a/(
b*Sqrt[1 + c^2/b^2])))), -((a + b*Sqrt[1 + c^2/b^2]*Cos[d + e*x - ArcTan[c/
b]])/(b*Sqrt[1 + c^2/b^2]*(-1 - a/(b*Sqrt[1 + c^2/b^2]))))]*Sin[d + e*x - A
rcTan[c/b]])/(b*Sqrt[1 + c^2/b^2]*Sqrt[(b*Sqrt[(b^2 + c^2)/b^2] - b*Sqrt[(b
^2 + c^2)/b^2]*Cos[d + e*x - ArcTan[c/b]])/(a + b*Sqrt[(b^2 + c^2)/b^2])]*S
qrt[a + b*Sqrt[(b^2 + c^2)/b^2]*Cos[d + e*x - ArcTan[c/b]]]*Sqrt[(b*Sqrt[(b
^2 + c^2)/b^2] + b*Sqrt[(b^2 + c^2)/b^2]*Cos[d + e*x - ArcTan[c/b]])/(-a +
b*Sqrt[(b^2 + c^2)/b^2])])) - ((2*b*(a + b*Sqrt[1 + c^2/b^2]*Cos[d + e*x -
ArcTan[c/b]]))/(b^2 + c^2) - (c*Sin[d + e*x - ArcTan[c/b]])/(b*Sqrt[1 + c^2
/b^2]))/Sqrt[a + b*Sqrt[1 + c^2/b^2]*Cos[d + e*x - ArcTan[c/b]]]))/(3*e)
```

fricas [F] time = 0.86, size = 0, normalized size = 0.00

$$\text{integral}\left((b\cos(ex+d)+c\sin(ex+d)+a)^{\frac{3}{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(e*x+d)+c*sin(e*x+d))^(3/2),x, algorithm="fricas")
```

```
[Out] integral((b*cos(e*x + d) + c*sin(e*x + d) + a)^(3/2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int(b\cos(ex+d)+c\sin(ex+d)+a)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*cos(e*x+d)+c*sin(e*x+d))^(3/2),x, algorithm="giac")

[Out] integrate((b*cos(e*x + d) + c*sin(e*x + d) + a)^(3/2), x)

maple [B] time = 0.71, size = 1516, normalized size = 5.36

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b*cos(e*x+d)+c*sin(e*x+d))^(3/2),x)

[Out] (-(-b^2*sin(e*x+d-arctan(-b,c))-c^2*sin(e*x+d-arctan(-b,c))-a*(b^2+c^2)^(1/2))*cos(e*x+d-arctan(-b,c))^2/(b^2+c^2)^(1/2))^(1/2)*((b^2+c^2)*(-2/3/(b^2+c^2)^(1/2)*(cos(e*x+d-arctan(-b,c))^2*((b^2+c^2)^(1/2)*sin(e*x+d-arctan(-b,c))+a))^(1/2)+2/3*(1/(b^2+c^2)^(1/2)*a-1)*((-(b^2+c^2)^(1/2)*sin(e*x+d-arctan(-b,c))-a)/(-a+(b^2+c^2)^(1/2)))^(1/2)*((-sin(e*x+d-arctan(-b,c))+1)*(b^2+c^2)^(1/2)/(a+(b^2+c^2)^(1/2)))^(1/2)*((1+sin(e*x+d-arctan(-b,c)))*(b^2+c^2)^(1/2)/(-a+(b^2+c^2)^(1/2)))^(1/2)/(cos(e*x+d-arctan(-b,c))^2*((b^2+c^2)^(1/2)*sin(e*x+d-arctan(-b,c))+a))^(1/2)*EllipticF(((-(b^2+c^2)^(1/2)*sin(e*x+d-arctan(-b,c))-a)/(-a+(b^2+c^2)^(1/2)))^(1/2),((a-(b^2+c^2)^(1/2))/(a+(b^2+c^2)^(1/2)))^(1/2))-4/3/(b^2+c^2)^(1/2)*a*(1/(b^2+c^2)^(1/2)*a-1)*((-(b^2+c^2)^(1/2)*sin(e*x+d-arctan(-b,c))-a)/(-a+(b^2+c^2)^(1/2)))^(1/2)*((-sin(e*x+d-arctan(-b,c))+1)*(b^2+c^2)^(1/2)/(a+(b^2+c^2)^(1/2)))^(1/2)*((1+sin(e*x+d-arctan(-b,c)))*(b^2+c^2)^(1/2)/(-a+(b^2+c^2)^(1/2)))^(1/2)/(cos(e*x+d-arctan(-b,c))^2*((b^2+c^2)^(1/2)*sin(e*x+d-arctan(-b,c))+a))^(1/2)*((-1/(b^2+c^2)^(1/2)*a-1)*EllipticE(((-(b^2+c^2)^(1/2)*sin(e*x+d-arctan(-b,c))-a)/(-a+(b^2+c^2)^(1/2)))^(1/2),((a-(b^2+c^2)^(1/2))/(a+(b^2+c^2)^(1/2)))^(1/2))+EllipticF(((-(b^2+c^2)^(1/2)*sin(e*x+d-arctan(-b,c))-a)/(-a+(b^2+c^2)^(1/2)))^(1/2),((a-(b^2+c^2)^(1/2))/(a+(b^2+c^2)^(1/2)))^(1/2))))+4*a*(b^2+c^2)^(1/2)*(1/(b^2+c^2)^(1/2)*a-1)*((-(b^2+c^2)^(1/2)*sin(e*x+d-arctan(-b,c))-a)/(-a+(b^2+c^2)^(1/2)))^(1/2)*((-sin(e*x+d-arctan(-b,c))+1)*(b^2+c^2)^(1/2)/(a+(b^2+c^2)^(1/2)))^(1/2)*((1+sin(e*x+d-arctan(-b,c)))*(b^2+c^2)^(1/2)/(-a+(b^2+c^2)^(1/2)))^(1/2)/(cos(e*x+d-arctan(-b,c))^2*((b^2+c^2)^(1/2)*sin(e*x+d-arctan(-b,c))+a))^(1/2)*((-1/(b^2+c^2)^(1/2)*a-1)*EllipticE(((-(b^2+c^2)^(1/2)*sin(e*x+d-arctan(-b,c))-a)/(-a+(b^2+c^2)^(1/2)))^(1/2),((a-(b^2+c^2)^(1/2))/(a+(b^2+c^2)^(1/2)))^(1/2))+EllipticF(((-(b^2+c^2)^(1/2)*sin(e*x+d-arctan(-b,c))-a)/(-a+(b^2+c^2)^(1/2)))^(1/2),((a-(b^2+c^2)^(1/2))/(a+(b^2+c^2)^(1/2)))^(1/2)))+2*a^2*(1/(b^2+c^2)^(1/2)*a-1)*((-(b^2+c^2)^(1/2)*sin(e*x+d-arctan(-b,c))-a)/(-a+(b^2+c^2)^(1/2)))^(1/2)*((-sin(e*x+d-arctan(-b,c))+1)*(b^2+c^2)^(1/2)/(a+(b^2+c^2)^(1/2)))^(1/2)*((1+sin(e*x+d-arctan(-b,c)))*(b^2+c^2)^(1/2)/(-a+(b^2+c^2)^(1/2)))^(1/2)/(-(-b^2*sin(e*x+d-arctan(-b,c))-c^2*sin(e*x+d-arctan(-b,c))-a*(b^2+c^2)^(1/2))*cos(e*x+d-arctan(-b,c))^2/(b^2+c^2)^(1/2))^(1/2)*EllipticF(((-(b^2+c^2)^(1/2)*sin(e*x+d-arctan(-b,c)

```
)-a)/(-a+(b^2+c^2)^(1/2)))^(1/2),((a-(b^2+c^2)^(1/2))/(a+(b^2+c^2)^(1/2)))^
(1/2)))/cos(e*x+d-arctan(-b,c))/((b^2*sin(e*x+d-arctan(-b,c))+c^2*sin(e*x+d
-arctan(-b,c))+a*(b^2+c^2)^(1/2))/(b^2+c^2)^(1/2))^(1/2)/e
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b\cos(ex+d) + c\sin(ex+d) + a)^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(e*x+d)+c*sin(e*x+d))^(3/2),x, algorithm="maxima")
[Out] integrate((b*cos(e*x + d) + c*sin(e*x + d) + a)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int {\left(a+b\,\cos\left(d+e\,x\right)+c\,\sin\left(d+e\,x\right)\right)}^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*cos(d + e*x) + c*sin(d + e*x))^(3/2),x)
[Out] int((a + b*cos(d + e*x) + c*sin(d + e*x))^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b\cos(d+ex) + c\sin(d+ex))^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(e*x+d)+c*sin(e*x+d))**(3/2),x)
[Out] Integral((a + b*cos(d + e*x) + c*sin(d + e*x))**(3/2), x)
```

3.412 $\int \sqrt{a+b\cos(d+ex)+c\sin(d+ex)}\,dx$

Optimal. Leaf size=108

$$\frac{2\sqrt{a+b\cos(d+ex)+c\sin(d+ex)}\,E\left(\frac{1}{2}\left(d+ex-\tan^{-1}(b,c)\right)|\frac{2\sqrt{b^2+c^2}}{a+\sqrt{b^2+c^2}}\right)}{e\sqrt{\frac{a+b\cos(d+ex)+c\sin(d+ex)}{a+\sqrt{b^2+c^2}}}}$$

```
[Out] 2*(cos(1/2*d+1/2*e*x-1/2*arctan(b,c))^2)^(1/2)/cos(1/2*d+1/2*e*x-1/2*arctan
(b,c))*EllipticE(sin(1/2*d+1/2*e*x-1/2*arctan(b,c)),2^(1/2)*((b^2+c^2)^(1/2
)/(a+(b^2+c^2)^(1/2)))^(1/2))*(a+b*cos(e*x+d)+c*sin(e*x+d))^(1/2)/e/((a+b*c
os(e*x+d)+c*sin(e*x+d))/(a+(b^2+c^2)^(1/2)))^(1/2)
```

Rubi [A] time = 0.07, antiderivative size = 108, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {3119, 2653}

$$\frac{2\sqrt{a+b\cos(d+ex)+c\sin(d+ex)}\,E\left(\frac{1}{2}\left(d+ex-\tan^{-1}(b,c)\right)|\frac{2\sqrt{b^2+c^2}}{a+\sqrt{b^2+c^2}}\right)}{e\sqrt{\frac{a+b\cos(d+ex)+c\sin(d+ex)}{a+\sqrt{b^2+c^2}}}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]],x]
```

```
[Out] (2*EllipticE[(d + e*x - ArcTan[b, c])/2, (2*Sqrt[b^2 + c^2])/(a + Sqrt[b^2
+ c^2])]*Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]])/(e*Sqrt[(a + b*Cos[d +
e*x] + c*Sin[d + e*x])/(a + Sqrt[b^2 + c^2])])
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 3119

```
Int[Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_
)]], x_Symbol] :> Dist[Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]]/Sqrt[(a +
b*Cos[d + e*x] + c*Sin[d + e*x])/(a + Sqrt[b^2 + c^2])], Int[Sqrt[a/(a + Sq
rt[b^2 + c^2]) + (Sqrt[b^2 + c^2]*Cos[d + e*x - ArcTan[b, c]])/(a + Sqrt[b^
2 + c^2])], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0]
```

```
&& NeQ[b^2 + c^2, 0] && !GtQ[a + Sqrt[b^2 + c^2], 0]
```

Rubi steps

$$\int \sqrt{a+b\cos(d+ex)+c\sin(d+ex)}\,dx = \frac{\sqrt{a+b\cos(d+ex)+c\sin(d+ex)}\int \sqrt{\frac{a}{a+\sqrt{b^2+c^2}}+\frac{\sqrt{b^2+c^2}\cos\left(d+ex-\text{ta}\right.}{a+\sqrt{b^2+c^2}}}}{\sqrt{\frac{a+b\cos(d+ex)+c\sin(d+ex)}{a+\sqrt{b^2+c^2}}}}$$
$$= \frac{2E\left(\frac{1}{2}\left(d+ex-\tan^{-1}(b,c)\right)|\frac{2\sqrt{b^2+c^2}}{a+\sqrt{b^2+c^2}}\right)\sqrt{a+b\cos(d+ex)+c\sin(d}}{e\sqrt{\frac{a+b\cos(d+ex)+c\sin(d+ex)}{a+\sqrt{b^2+c^2}}}}$$

Mathematica [C] time = 6.23, size = 1408, normalized size = 13.04

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]],x]
```

```
[Out] (2*b*Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]])/(c*e) + (2*a*AppellF1[1/2,
1/2, 1/2, 3/2, -((a + Sqrt[1 + b^2/c^2]*c*Sin[d + e*x + ArcTan[b/c]])/(Sqrt
[1 + b^2/c^2]*(1 - a/(Sqrt[1 + b^2/c^2]*c))*c)), -((a + Sqrt[1 + b^2/c^2]*c
*Sin[d + e*x + ArcTan[b/c]])/(Sqrt[1 + b^2/c^2]*(-1 - a/(Sqrt[1 + b^2/c^2]*
c))*c))]*Sec[d + e*x + ArcTan[b/c]]*Sqrt[(c*Sqrt[(b^2 + c^2)/c^2] - c*Sqrt[
(b^2 + c^2)/c^2]*Sin[d + e*x + ArcTan[b/c]])/(a + c*Sqrt[(b^2 + c^2)/c^2])]
*Sqrt[a + c*Sqrt[(b^2 + c^2)/c^2]*Sin[d + e*x + ArcTan[b/c]]]*Sqrt[(c*Sqrt[
(b^2 + c^2)/c^2] + c*Sqrt[(b^2 + c^2)/c^2]*Sin[d + e*x + ArcTan[b/c]])/(-a
+ c*Sqrt[(b^2 + c^2)/c^2])])/(Sqrt[1 + b^2/c^2]*c*e) + (b^2*(-((c*AppellF1[
-1/2, -1/2, -1/2, 1/2, -((a + b*Sqrt[1 + c^2/b^2]*Cos[d + e*x - ArcTan[c/b]
])/(b*Sqrt[1 + c^2/b^2]*(1 - a/(b*Sqrt[1 + c^2/b^2])))), -((a + b*Sqrt[1 +
c^2/b^2]*Cos[d + e*x - ArcTan[c/b]])/(b*Sqrt[1 + c^2/b^2]*(-1 - a/(b*Sqrt[1
 + c^2/b^2]))))]*Sin[d + e*x - ArcTan[c/b]])/(b*Sqrt[1 + c^2/b^2]*Sqrt[(b*S
qrt[(b^2 + c^2)/b^2] - b*Sqrt[(b^2 + c^2)/b^2]*Cos[d + e*x - ArcTan[c/b]])/
(a + b*Sqrt[(b^2 + c^2)/b^2])]*Sqrt[a + b*Sqrt[(b^2 + c^2)/b^2]*Cos[d + e*x
 - ArcTan[c/b]]]*Sqrt[(b*Sqrt[(b^2 + c^2)/b^2] + b*Sqrt[(b^2 + c^2)/b^2]*Co
s[d + e*x - ArcTan[c/b]])/(-a + b*Sqrt[(b^2 + c^2)/b^2])])) - ((2*b*(a + b*
Sqrt[1 + c^2/b^2]*Cos[d + e*x - ArcTan[c/b]]))/(b^2 + c^2) - (c*Sin[d + e*x
 - ArcTan[c/b]])/(b*Sqrt[1 + c^2/b^2]))/Sqrt[a + b*Sqrt[1 + c^2/b^2]*Cos[d
+ e*x - ArcTan[c/b]]]))/(c*e) + (c*(-((c*AppellF1[-1/2, -1/2, -1/2, 1/2, -(
```

```
(a + b*Sqrt[1 + c^2/b^2]*Cos[d + e*x - ArcTan[c/b]])/(b*Sqrt[1 + c^2/b^2]*(
1 - a/(b*Sqrt[1 + c^2/b^2])))), -((a + b*Sqrt[1 + c^2/b^2]*Cos[d + e*x - Ar
cTan[c/b]])/(b*Sqrt[1 + c^2/b^2]*(-1 - a/(b*Sqrt[1 + c^2/b^2]))))]*Sin[d +
e*x - ArcTan[c/b]])/(b*Sqrt[1 + c^2/b^2]*Sqrt[(b*Sqrt[(b^2 + c^2)/b^2] - b*
Sqrt[(b^2 + c^2)/b^2]*Cos[d + e*x - ArcTan[c/b]])/(a + b*Sqrt[(b^2 + c^2)/b
^2])]*Sqrt[a + b*Sqrt[(b^2 + c^2)/b^2]*Cos[d + e*x - ArcTan[c/b]]]*Sqrt[(b*
Sqrt[(b^2 + c^2)/b^2] + b*Sqrt[(b^2 + c^2)/b^2]*Cos[d + e*x - ArcTan[c/b]])
/(-a + b*Sqrt[(b^2 + c^2)/b^2])])) - ((2*b*(a + b*Sqrt[1 + c^2/b^2]*Cos[d +
 e*x - ArcTan[c/b]]))/(b^2 + c^2) - (c*Sin[d + e*x - ArcTan[c/b]])/(b*Sqrt[
1 + c^2/b^2]))/Sqrt[a + b*Sqrt[1 + c^2/b^2]*Cos[d + e*x - ArcTan[c/b]]]))/e
```

fricas [F] time = 0.75, size = 0, normalized size = 0.00

$$\text{integral}\left(\sqrt{b\cos\left(ex+d\right)+c\sin\left(ex+d\right)+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(e*x+d)+c*sin(e*x+d))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*cos(e*x + d) + c*sin(e*x + d) + a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{b\cos\left(ex+d\right)+c\sin\left(ex+d\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(e*x+d)+c*sin(e*x+d))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*cos(e*x + d) + c*sin(e*x + d) + a), x)
```

maple [B] time = 0.46, size = 720, normalized size = 6.67

$$\frac{2\left(-a+\sqrt{b^2+c^2}\right)\sqrt{-\frac{\sqrt{b^2+c^2}\,\sin(ex+d-\arctan(-b,c))+a}{-a+\sqrt{b^2+c^2}}}\,\sqrt{-\frac{(\sin(ex+d-\arctan(-b,c))-1)\sqrt{b^2+c^2}}{a+\sqrt{b^2+c^2}}}\,\sqrt{\frac{(1+\sin(ex+d-\arctan(-b,c)))\sqrt{b^2+c}}{-a+\sqrt{b^2+c^2}}}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(e*x+d)+c*sin(e*x+d))^(1/2),x)
```

```
[Out] 2/(b^2+c^2)^(1/2)*(-a+(b^2+c^2)^(1/2))*(-((b^2+c^2)^(1/2)*sin(e*x+d-arctan(
-b,c))+a)/(-a+(b^2+c^2)^(1/2)))^(1/2)*(-(sin(e*x+d-arctan(-b,c))-1)*(b^2+c^
2)^(1/2)/(a+(b^2+c^2)^(1/2)))^(1/2)*((1+sin(e*x+d-arctan(-b,c)))*(b^2+c^2)^
(1/2)/(-a+(b^2+c^2)^(1/2)))^(1/2)*((b^2+c^2)^(1/2)*sin(e*x+d-arctan(-b,c))*
```

```
cos(e*x+d-arctan(-b,c))^2+cos(e*x+d-arctan(-b,c))^2*a)^(1/2)*((b^2+c^2)^(1/
2)*EllipticE((-(b^2+c^2)^(1/2)/(-a+(b^2+c^2)^(1/2))*sin(e*x+d-arctan(-b,c))
-a/(-a+(b^2+c^2)^(1/2)))^(1/2),(-(-a+(b^2+c^2)^(1/2))/(a+(b^2+c^2)^(1/2)))^
(1/2))-(b^2+c^2)^(1/2)*EllipticF((-(b^2+c^2)^(1/2)/(-a+(b^2+c^2)^(1/2))*sin
(e*x+d-arctan(-b,c))-a/(-a+(b^2+c^2)^(1/2)))^(1/2),(-(-a+(b^2+c^2)^(1/2))/(
a+(b^2+c^2)^(1/2)))^(1/2))+EllipticE((-(b^2+c^2)^(1/2)/(-a+(b^2+c^2)^(1/2))
*sin(e*x+d-arctan(-b,c))-a/(-a+(b^2+c^2)^(1/2)))^(1/2),(-(-a+(b^2+c^2)^(1/2
))/(a+(b^2+c^2)^(1/2)))^(1/2))*a-EllipticF((-(b^2+c^2)^(1/2)/(-a+(b^2+c^2)^
(1/2))*sin(e*x+d-arctan(-b,c))-a/(-a+(b^2+c^2)^(1/2)))^(1/2),(-(-a+(b^2+c^2
)^(1/2))/(a+(b^2+c^2)^(1/2)))^(1/2))*a)/(cos(e*x+d-arctan(-b,c))^2*((b^2+c^
2)^(1/2)*sin(e*x+d-arctan(-b,c))+a))^(1/2)/cos(e*x+d-arctan(-b,c))/((b^2*si
n(e*x+d-arctan(-b,c))+c^2*sin(e*x+d-arctan(-b,c))+a*(b^2+c^2)^(1/2))/(b^2+c
^2)^(1/2))^(1/2)/e
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{b\cos(ex+d) + c\sin(ex+d) + a}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(e*x+d)+c*sin(e*x+d))^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(b*cos(e*x + d) + c*sin(e*x + d) + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \sqrt{a + b\,\cos(d + e\,x) + c\,\sin(d + e\,x)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*cos(d + e*x) + c*sin(d + e*x))^(1/2),x)
[Out] int((a + b*cos(d + e*x) + c*sin(d + e*x))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a + b\cos(d + ex) + c\sin(d + ex)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(e*x+d)+c*sin(e*x+d))**(1/2),x)
[Out] Integral(sqrt(a + b*cos(d + e*x) + c*sin(d + e*x)), x)
```

3.413 $\int \frac{1}{\sqrt{a+b\cos(d+ex)+c\sin(d+ex)}}\,dx$

Optimal. Leaf size=108

$$\frac{2\sqrt{\frac{a+b\cos(d+ex)+c\sin(d+ex)}{a+\sqrt{b^2+c^2}}}\,F\left(\frac{1}{2}\left(d+ex-\tan^{-1}(b,c)\right)|\frac{2\sqrt{b^2+c^2}}{a+\sqrt{b^2+c^2}}\right)}{e\sqrt{a+b\cos(d+ex)+c\sin(d+ex)}}$$

```
[Out] 2*(cos(1/2*d+1/2*e*x-1/2*arctan(b,c))^2)^(1/2)/cos(1/2*d+1/2*e*x-1/2*arctan
(b,c))*EllipticF(sin(1/2*d+1/2*e*x-1/2*arctan(b,c)),2^(1/2)*((b^2+c^2)^(1/2
)/(a+(b^2+c^2)^(1/2)))^(1/2))*((a+b*cos(e*x+d)+c*sin(e*x+d))/(a+(b^2+c^2)^(
1/2)))^(1/2)/e/(a+b*cos(e*x+d)+c*sin(e*x+d))^(1/2)
```

Rubi [A] time = 0.07, antiderivative size = 108, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {3127, 2661}

$$\frac{2\sqrt{\frac{a+b\cos(d+ex)+c\sin(d+ex)}{a+\sqrt{b^2+c^2}}}\,F\left(\frac{1}{2}\left(d+ex-\tan^{-1}(b,c)\right)|\frac{2\sqrt{b^2+c^2}}{a+\sqrt{b^2+c^2}}\right)}{e\sqrt{a+b\cos(d+ex)+c\sin(d+ex)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]],x]
```

```
[Out] (2*EllipticF[(d + e*x - ArcTan[b, c])/2, (2*Sqrt[b^2 + c^2])/(a + Sqrt[b^2
+ c^2])]*Sqrt[(a + b*Cos[d + e*x] + c*Sin[d + e*x])/(a + Sqrt[b^2 + c^2])])
/(e*Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]])
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 3127

```
Int[1/Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(
x_)]], x_Symbol] :> Dist[Sqrt[(a + b*Cos[d + e*x] + c*Sin[d + e*x])/(a + Sq
rt[b^2 + c^2])]/Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]], Int[1/Sqrt[a/(a
+ Sqrt[b^2 + c^2]) + (Sqrt[b^2 + c^2]*Cos[d + e*x - ArcTan[b, c]])/(a + Sqr
t[b^2 + c^2])], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2,
 0] && NeQ[b^2 + c^2, 0] &&  !GtQ[a + Sqrt[b^2 + c^2], 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{a+b\cos(d+ex)+c\sin(d+ex)}}\,dx = \frac{\sqrt{\frac{a+b\cos(d+ex)+c\sin(d+ex)}{a+\sqrt{b^2+c^2}}}\int \frac{1}{\sqrt{\frac{a}{a+\sqrt{b^2+c^2}}+\frac{\sqrt{b^2+c^2}\cos\left(d+ex-\tan^{-1}(b,c)\right)}{a+\sqrt{b^2+c^2}}}}\,dx}{\sqrt{a+b\cos(d+ex)+c\sin(d+ex)}}$$

$$= \frac{2F\left(\frac{1}{2}\left(d+ex-\tan^{-1}(b,c)\right)\middle|\frac{2\sqrt{b^2+c^2}}{a+\sqrt{b^2+c^2}}\right)\sqrt{\frac{a+b\cos(d+ex)+c\sin(d+ex)}{a+\sqrt{b^2+c^2}}}}{e\sqrt{a+b\cos(d+ex)+c\sin(d+ex)}}$$

Mathematica [C] time = 0.56, size = 285, normalized size = 2.64

$$\frac{2\sec\left(\tan^{-1}\left(\frac{b}{c}\right)+d+ex\right)\sqrt{-\frac{c\sqrt{\frac{b^2}{c^2}+1}\left(\sin\left(\tan^{-1}\left(\frac{b}{c}\right)+d+ex\right)-1\right)}{a+c\sqrt{\frac{b^2}{c^2}+1}}}\sqrt{\frac{c\sqrt{\frac{b^2}{c^2}+1}\left(\sin\left(\tan^{-1}\left(\frac{b}{c}\right)+d+ex\right)+1\right)}{c\sqrt{\frac{b^2}{c^2}+1}-a}}\sqrt{a+c\sqrt{\frac{b^2}{c^2}+1}\sin\left(\text{ta}\right.}}{ce\sqrt{\frac{b^2}{c^2}+1}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]],x]
```

```
[Out] (2*AppellF1[1/2, 1/2, 1/2, 3/2, (a + Sqrt[1 + b^2/c^2]*c*Sin[d + e*x + ArcT
an[b/c]])/(a - Sqrt[1 + b^2/c^2]*c), (a + Sqrt[1 + b^2/c^2]*c*Sin[d + e*x +
 ArcTan[b/c]])/(a + Sqrt[1 + b^2/c^2]*c)]*Sec[d + e*x + ArcTan[b/c]]*Sqrt[-
((Sqrt[1 + b^2/c^2]*c*(-1 + Sin[d + e*x + ArcTan[b/c]]))/(a + Sqrt[1 + b^2/
c^2]*c))]*Sqrt[(Sqrt[1 + b^2/c^2]*c*(1 + Sin[d + e*x + ArcTan[b/c]]))/(-a +
 Sqrt[1 + b^2/c^2]*c)]*Sqrt[a + Sqrt[1 + b^2/c^2]*c*Sin[d + e*x + ArcTan[b/
c]]])/(Sqrt[1 + b^2/c^2]*c*e)
```

fricas [F] time = 3.15, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{1}{\sqrt{b\cos(ex+d)+c\sin(ex+d)+a}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cos(e*x+d)+c*sin(e*x+d))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(1/sqrt(b*cos(e*x + d) + c*sin(e*x + d) + a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{b\cos(ex+d)+c\sin(ex+d)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cos(e*x+d)+c*sin(e*x+d))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(b*cos(e*x + d) + c*sin(e*x + d) + a), x)
```

maple [B] time = 0.37, size = 303, normalized size = 2.81

$$-\frac{2\left(-a+\sqrt{b^2+c^2}\right)\sqrt{-\frac{\sqrt{b^2+c^2}\ \sin(ex+d-\arctan(-b,c))+a}{-a+\sqrt{b^2+c^2}}}\ \sqrt{-\frac{(\sin(ex+d-\arctan(-b,c))-1)\sqrt{b^2+c^2}}{a+\sqrt{b^2+c^2}}}\ \sqrt{\frac{(1+\sin(ex+d-\arctan(-b,c)))\sqrt{b^2}}{-a+\sqrt{b^2+c^2}}}}{\sqrt{b^2+c^2}\ \cos(ex+d-\arctan(-b,c))\sqrt{\frac{b^2\sin(ex+d-\arctan(-b,c))+c^2\sin(ex+}{\sqrt{b^2+c^2}}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*cos(e*x+d)+c*sin(e*x+d))^(1/2),x)
```

```
[Out] -2*(-a+(b^2+c^2)^(1/2))*(-((b^2+c^2)^(1/2)*sin(e*x+d-arctan(-b,c))+a)/(-a+(
b^2+c^2)^(1/2)))^(1/2)*(-(sin(e*x+d-arctan(-b,c))-1)*(b^2+c^2)^(1/2)/(a+(b^
2+c^2)^(1/2)))^(1/2)*((1+sin(e*x+d-arctan(-b,c)))*(b^2+c^2)^(1/2)/(-a+(b^2+
c^2)^(1/2)))^(1/2)*EllipticF((-((b^2+c^2)^(1/2)*sin(e*x+d-arctan(-b,c))+a)/
(-a+(b^2+c^2)^(1/2)))^(1/2),(-(-a+(b^2+c^2)^(1/2))/(a+(b^2+c^2)^(1/2)))^(1/
2))/(b^2+c^2)^(1/2)/cos(e*x+d-arctan(-b,c))/((b^2*sin(e*x+d-arctan(-b,c))+c
^2*sin(e*x+d-arctan(-b,c))+a*(b^2+c^2)^(1/2))/(b^2+c^2)^(1/2))^(1/2)/e
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{b\cos(ex+d)+c\sin(ex+d)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cos(e*x+d)+c*sin(e*x+d))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(b*cos(e*x + d) + c*sin(e*x + d) + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sqrt{a+b\ \cos(d+e\,x)+c\ \sin(d+e\,x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*cos(d + e*x) + c*sin(d + e*x))^(1/2),x)
```

```
[Out] int(1/(a + b*cos(d + e*x) + c*sin(d + e*x))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a+b\cos(d+ex)+c\sin(d+ex)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cos(e*x+d)+c*sin(e*x+d))**(1/2),x)
```

```
[Out] Integral(1/sqrt(a + b*cos(d + e*x) + c*sin(d + e*x)), x)
```

3.414 $\int \frac{1}{(a+b\cos(d+ex)+c\sin(d+ex))^{3/2}}\,dx$

Optimal. Leaf size=186

$$\frac{2\sqrt{a+b\cos(d+ex)+c\sin(d+ex)}\,E\left(\frac{1}{2}\left(d+ex-\tan^{-1}(b,c)\right)|\frac{2\sqrt{b^2+c^2}}{a+\sqrt{b^2+c^2}}\right)}{e\left(a^2-b^2-c^2\right)\sqrt{\frac{a+b\cos(d+ex)+c\sin(d+ex)}{a+\sqrt{b^2+c^2}}}}+\frac{2(c\cos(d+ex)-b\sin(d+e$$

$$e\left(a^2-b^2-c^2\right)\sqrt{a+b\cos(d+ex)+c$$

```
[Out] 2*(c*cos(e*x+d)-b*sin(e*x+d))/(a^2-b^2-c^2)/e/(a+b*cos(e*x+d)+c*sin(e*x+d))
^(1/2)+2*(cos(1/2*d+1/2*e*x-1/2*arctan(b,c))^2)^(1/2)/cos(1/2*d+1/2*e*x-1/2
*arctan(b,c))*EllipticE(sin(1/2*d+1/2*e*x-1/2*arctan(b,c)),2^(1/2)*((b^2+c^
2)^(1/2)/(a+(b^2+c^2)^(1/2)))^(1/2))*(a+b*cos(e*x+d)+c*sin(e*x+d))^(1/2)/(a
^2-b^2-c^2)/e/((a+b*cos(e*x+d)+c*sin(e*x+d))/(a+(b^2+c^2)^(1/2)))^(1/2)
```

Rubi [A] time = 0.10, antiderivative size = 186, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {3128, 3119, 2653}

$$\frac{2\sqrt{a+b\cos(d+ex)+c\sin(d+ex)}\,E\left(\frac{1}{2}\left(d+ex-\tan^{-1}(b,c)\right)|\frac{2\sqrt{b^2+c^2}}{a+\sqrt{b^2+c^2}}\right)}{e\left(a^2-b^2-c^2\right)\sqrt{\frac{a+b\cos(d+ex)+c\sin(d+ex)}{a+\sqrt{b^2+c^2}}}}+\frac{2(c\cos(d+ex)-b\sin(d+e$$

$$e\left(a^2-b^2-c^2\right)\sqrt{a+b\cos(d+ex)+c$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(-3/2),x]
```

```
[Out] (2*(c*Cos[d + e*x] - b*Sin[d + e*x]))/((a^2 - b^2 - c^2)*e*Sqrt[a + b*Cos[d
 + e*x] + c*Sin[d + e*x]]) + (2*EllipticE[(d + e*x - ArcTan[b, c])/2, (2*Sq
rt[b^2 + c^2])/(a + Sqrt[b^2 + c^2])]*Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e
*x]])/((a^2 - b^2 - c^2)*e*Sqrt[(a + b*Cos[d + e*x] + c*Sin[d + e*x])/(a +
Sqrt[b^2 + c^2]])
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 3119

```
Int[Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_
)]], x_Symbol] :> Dist[Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]]/Sqrt[(a +
```

```
b*Cos[d + e*x] + c*Sin[d + e*x])/(a + Sqrt[b^2 + c^2])], Int[Sqrt[a/(a + Sq
rt[b^2 + c^2]) + (Sqrt[b^2 + c^2]*Cos[d + e*x - ArcTan[b, c]])/(a + Sqrt[b^
2 + c^2])], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0]
&& NeQ[b^2 + c^2, 0] &&  !GtQ[a + Sqrt[b^2 + c^2], 0]
```

Rule 3128

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(-3/2), x_Symbol] :> Simp[(2*(c*Cos[d + e*x] - b*Sin[d + e*x]))/(e*(a^2 - b
^2 - c^2)*Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]]), x] + Dist[1/(a^2 - b^
2 - c^2), Int[Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]], x], x] /; FreeQ[{a
, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0]
```

Rubi steps

$$\int \frac{1}{(a+b\cos(d+ex)+c\sin(d+ex))^{3/2}}\,dx = \frac{2(c\cos(d+ex)-b\sin(d+ex))}{\left(a^2-b^2-c^2\right)e\sqrt{a+b\cos(d+ex)+c\sin(d+ex)}} + \frac{\int\sqrt{a+b\cos}}{}$$

$$= \frac{2(c\cos(d+ex)-b\sin(d+ex))}{\left(a^2-b^2-c^2\right)e\sqrt{a+b\cos(d+ex)+c\sin(d+ex)}} + \frac{\sqrt{a+b\cos(}}{}$$

$$= \frac{2(c\cos(d+ex)-b\sin(d+ex))}{\left(a^2-b^2-c^2\right)e\sqrt{a+b\cos(d+ex)+c\sin(d+ex)}} + \frac{2E\left(\frac{1}{2}\left(d+e\right.\right.}{}$$

Mathematica [C] time = 6.35, size = 1540, normalized size = 8.28

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(-3/2),x]
```

```
[Out] (Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]]*((-2*(b^2 + c^2))/(b*c*(-a^2 + b
^2 + c^2)) + (2*(a*c + b^2*Sin[d + e*x] + c^2*Sin[d + e*x]))/(b*(-a^2 + b^2
 + c^2)*(a + b*Cos[d + e*x] + c*Sin[d + e*x]))))/e - (2*a*AppellF1[1/2, 1/2
, 1/2, 3/2, -((a + Sqrt[1 + b^2/c^2]*c*Sin[d + e*x + ArcTan[b/c]])/(Sqrt[1
+ b^2/c^2]*(1 - a/(Sqrt[1 + b^2/c^2]*c))*c)), -((a + Sqrt[1 + b^2/c^2]*c*Si
n[d + e*x + ArcTan[b/c]])/(Sqrt[1 + b^2/c^2]*(-1 - a/(Sqrt[1 + b^2/c^2]*c))
```

```
*c))]*Sec[d + e*x + ArcTan[b/c]]*Sqrt[(c*Sqrt[(b^2 + c^2)/c^2] - c*Sqrt[(b^
2 + c^2)/c^2]*Sin[d + e*x + ArcTan[b/c]])/(a + c*Sqrt[(b^2 + c^2)/c^2])]*Sq
rt[a + c*Sqrt[(b^2 + c^2)/c^2]*Sin[d + e*x + ArcTan[b/c]]]*Sqrt[(c*Sqrt[(b^
2 + c^2)/c^2] + c*Sqrt[(b^2 + c^2)/c^2]*Sin[d + e*x + ArcTan[b/c]])/(-a + c
*Sqrt[(b^2 + c^2)/c^2])])/(Sqrt[1 + b^2/c^2]*c*(-a^2 + b^2 + c^2)*e) - (b^2
*(-((c*AppellF1[-1/2, -1/2, -1/2, 1/2, -((a + b*Sqrt[1 + c^2/b^2]*Cos[d + e
*x - ArcTan[c/b]])/(b*Sqrt[1 + c^2/b^2]*(1 - a/(b*Sqrt[1 + c^2/b^2])))), -(
(a + b*Sqrt[1 + c^2/b^2]*Cos[d + e*x - ArcTan[c/b]])/(b*Sqrt[1 + c^2/b^2]*(
-1 - a/(b*Sqrt[1 + c^2/b^2]))))]*Sin[d + e*x - ArcTan[c/b]])/(b*Sqrt[1 + c^
2/b^2]*Sqrt[(b*Sqrt[(b^2 + c^2)/b^2] - b*Sqrt[(b^2 + c^2)/b^2]*Cos[d + e*x
- ArcTan[c/b]])/(a + b*Sqrt[(b^2 + c^2)/b^2])]*Sqrt[a + b*Sqrt[(b^2 + c^2)/
b^2]*Cos[d + e*x - ArcTan[c/b]]]*Sqrt[(b*Sqrt[(b^2 + c^2)/b^2] + b*Sqrt[(b^
2 + c^2)/b^2]*Cos[d + e*x - ArcTan[c/b]])/(-a + b*Sqrt[(b^2 + c^2)/b^2])]))
 - ((2*b*(a + b*Sqrt[1 + c^2/b^2]*Cos[d + e*x - ArcTan[c/b]]))/(b^2 + c^2)
- (c*Sin[d + e*x - ArcTan[c/b]])/(b*Sqrt[1 + c^2/b^2]))/Sqrt[a + b*Sqrt[1 +
 c^2/b^2]*Cos[d + e*x - ArcTan[c/b]]]))/(c*(-a^2 + b^2 + c^2)*e) - (c*(-((c
*AppellF1[-1/2, -1/2, -1/2, 1/2, -((a + b*Sqrt[1 + c^2/b^2]*Cos[d + e*x - A
rcTan[c/b]])/(b*Sqrt[1 + c^2/b^2]*(1 - a/(b*Sqrt[1 + c^2/b^2])))), -((a + b
*Sqrt[1 + c^2/b^2]*Cos[d + e*x - ArcTan[c/b]])/(b*Sqrt[1 + c^2/b^2]*(-1 - a
/(b*Sqrt[1 + c^2/b^2]))))]*Sin[d + e*x - ArcTan[c/b]])/(b*Sqrt[1 + c^2/b^2]
*Sqrt[(b*Sqrt[(b^2 + c^2)/b^2] - b*Sqrt[(b^2 + c^2)/b^2]*Cos[d + e*x - ArcT
an[c/b]])/(a + b*Sqrt[(b^2 + c^2)/b^2])]*Sqrt[a + b*Sqrt[(b^2 + c^2)/b^2]*C
os[d + e*x - ArcTan[c/b]]]*Sqrt[(b*Sqrt[(b^2 + c^2)/b^2] + b*Sqrt[(b^2 + c^
2)/b^2]*Cos[d + e*x - ArcTan[c/b]])/(-a + b*Sqrt[(b^2 + c^2)/b^2])])) - ((2
*b*(a + b*Sqrt[1 + c^2/b^2]*Cos[d + e*x - ArcTan[c/b]]))/(b^2 + c^2) - (c*S
in[d + e*x - ArcTan[c/b]])/(b*Sqrt[1 + c^2/b^2]))/Sqrt[a + b*Sqrt[1 + c^2/b
^2]*Cos[d + e*x - ArcTan[c/b]]]))/((-a^2 + b^2 + c^2)*e)
```

fricas [F] time = 0.74, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{b\cos(ex+d)+c\sin(ex+d)+a}}{2\,ab\cos(ex+d)+\left(b^2-c^2\right)\cos(ex+d)^2+a^2+c^2+2\,(bc\cos(ex+d)+ac)\sin(ex+d)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cos(e*x+d)+c*sin(e*x+d))^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*cos(e*x + d) + c*sin(e*x + d) + a)/(2*a*b*cos(e*x + d) + (b
^2 - c^2)*cos(e*x + d)^2 + a^2 + c^2 + 2*(b*c*cos(e*x + d) + a*c)*sin(e*x +
 d)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(b\cos(ex+d)+c\sin(ex+d)+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cos(e*x+d)+c*sin(e*x+d))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*cos(e*x + d) + c*sin(e*x + d) + a)^(-3/2), x)
```

maple [B] time = 0.77, size = 2388, normalized size = 12.84

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*cos(e*x+d)+c*sin(e*x+d))^(3/2),x)
```

```
[Out] (-(-b^2*sin(e*x+d-arctan(-b,c))-c^2*sin(e*x+d-arctan(-b,c))-a*(b^2+c^2)^(1/
2))*cos(e*x+d-arctan(-b,c))^2/(b^2+c^2)^(1/2))^(1/2)/(b^2+c^2)^(1/2)*(-(b^2
+c^2)^(1/2)*(-b^2-c^2)*cos(e*x+d-arctan(-b,c))^2/(a^2-b^2-c^2)/(-(-(b^2+c^2
)^(1/2)*sin(e*x+d-arctan(-b,c))-a)*cos(e*x+d-arctan(-b,c))^2*(b^2+c^2))^(1/
2)+a*(b^2+c^2)/(a^2-b^2-c^2)*(1/(b^2+c^2)^(1/2)*a-1)*((-(b^2+c^2)^(1/2)*sin
(e*x+d-arctan(-b,c))-a)/(-a+(b^2+c^2)^(1/2)))^(1/2)*((-sin(e*x+d-arctan(-b,
c))+1)*(b^2+c^2)^(1/2)/(a+(b^2+c^2)^(1/2)))^(1/2)*((1+sin(e*x+d-arctan(-b,c
)))*(b^2+c^2)^(1/2)/(-a+(b^2+c^2)^(1/2)))^(1/2)/(-(-(b^2+c^2)^(1/2)*sin(e*x
+d-arctan(-b,c))-a)*cos(e*x+d-arctan(-b,c))^2*(b^2+c^2))^(1/2)*EllipticF(((
-(b^2+c^2)^(1/2)*sin(e*x+d-arctan(-b,c))-a)/(-a+(b^2+c^2)^(1/2)))^(1/2),((a
-(b^2+c^2)^(1/2))/(a+(b^2+c^2)^(1/2)))^(1/2))+2*(-(b^2+c^2)^(3/2)+2*(b^2+c^
2)^(1/2)*b^2+2*(b^2+c^2)^(1/2)*c^2)/(2*a^2-2*b^2-2*c^2)*(1/(b^2+c^2)^(1/2)*
a-1)*((-(b^2+c^2)^(1/2)*sin(e*x+d-arctan(-b,c))-a)/(-a+(b^2+c^2)^(1/2)))^(1
/2)*((-sin(e*x+d-arctan(-b,c))+1)*(b^2+c^2)^(1/2)/(a+(b^2+c^2)^(1/2)))^(1/2
)*((1+sin(e*x+d-arctan(-b,c)))*(b^2+c^2)^(1/2)/(-a+(b^2+c^2)^(1/2)))^(1/2)/
(-(-(b^2+c^2)^(1/2)*sin(e*x+d-arctan(-b,c))-a)*cos(e*x+d-arctan(-b,c))^2*(b
^2+c^2))^(1/2)*((-1/(b^2+c^2)^(1/2)*a-1)*EllipticE(((-(b^2+c^2)^(1/2)*sin(e
*x+d-arctan(-b,c))-a)/(-a+(b^2+c^2)^(1/2)))^(1/2),((a-(b^2+c^2)^(1/2))/(a+(
b^2+c^2)^(1/2)))^(1/2))+EllipticF(((-(b^2+c^2)^(1/2)*sin(e*x+d-arctan(-b,c)
)-a)/(-a+(b^2+c^2)^(1/2)))^(1/2),((a-(b^2+c^2)^(1/2))/(a+(b^2+c^2)^(1/2)))^
(1/2)))+1/2*(b^2+c^2)*(1/(b^2+c^2)^(1/2)*a-1)*((-(b^2+c^2)^(1/2)*sin(e*x+d-
arctan(-b,c))-a)/(-a+(b^2+c^2)^(1/2)))^(1/2)*((-sin(e*x+d-arctan(-b,c))+1)*
(b^2+c^2)^(1/2)/(a+(b^2+c^2)^(1/2)))^(1/2)*((1+sin(e*x+d-arctan(-b,c)))*(b^
2+c^2)^(1/2)/(-a+(b^2+c^2)^(1/2)))^(1/2)/(-(-(b^2+c^2)^(1/2)*sin(e*x+d-arct
an(-b,c))-a)*cos(e*x+d-arctan(-b,c))^2*(b^2+c^2))^(1/2)/a*EllipticPi(((-(b^
2+c^2)^(1/2)*sin(e*x+d-arctan(-b,c))-a)/(-a+(b^2+c^2)^(1/2)))^(1/2),-1/2*(-
1/(b^2+c^2)^(1/2)*a+1)*(b^2+c^2)^(1/2)/a,((a-(b^2+c^2)^(1/2))/(a+(b^2+c^2)^
(1/2)))^(1/2))+(b^2+c^2)*cos(e*x+d-arctan(-b,c))^2/(a^2-b^2-c^2)/(-(-(b^2+c
^2)^(1/2)*sin(e*x+d-arctan(-b,c))-a)*cos(e*x+d-arctan(-b,c))^2)^(1/2)+1/(a^
2-b^2-c^2)*(b^2+c^2)^(1/2)*a*(1/(b^2+c^2)^(1/2)*a-1)*((-(b^2+c^2)^(1/2)*sin
(e*x+d-arctan(-b,c))-a)/(-a+(b^2+c^2)^(1/2)))^(1/2)*((-sin(e*x+d-arctan(-b,
c))+1)*(b^2+c^2)^(1/2)/(a+(b^2+c^2)^(1/2)))^(1/2)*((1+sin(e*x+d-arctan(-b,c
```

```
)))*(b^2+c^2)^(1/2)/(-a+(b^2+c^2)^(1/2)))^(1/2)/(-(-(b^2+c^2)^(1/2)*sin(e*x
+d-arctan(-b,c))-a)*cos(e*x+d-arctan(-b,c))^2)^(1/2)*EllipticF(((-(b^2+c^2)
^(1/2)*sin(e*x+d-arctan(-b,c))-a)/(-a+(b^2+c^2)^(1/2)))^(1/2),((a-(b^2+c^2)
^(1/2))/(a+(b^2+c^2)^(1/2)))^(1/2))+1/(a^2-b^2-c^2)*(b^2+c^2)*(1/(b^2+c^2)^
(1/2)*a-1)*((-(b^2+c^2)^(1/2)*sin(e*x+d-arctan(-b,c))-a)/(-a+(b^2+c^2)^(1/2
)))^(1/2)*((-sin(e*x+d-arctan(-b,c))+1)*(b^2+c^2)^(1/2)/(a+(b^2+c^2)^(1/2))
)^(1/2)*((1+sin(e*x+d-arctan(-b,c)))*(b^2+c^2)^(1/2)/(-a+(b^2+c^2)^(1/2)))^
(1/2)/(-(-(b^2+c^2)^(1/2)*sin(e*x+d-arctan(-b,c))-a)*cos(e*x+d-arctan(-b,c)
)^2)^(1/2)*((-1/(b^2+c^2)^(1/2)*a-1)*EllipticE(((-(b^2+c^2)^(1/2)*sin(e*x+d
-arctan(-b,c))-a)/(-a+(b^2+c^2)^(1/2)))^(1/2),((a-(b^2+c^2)^(1/2))/(a+(b^2+
c^2)^(1/2)))^(1/2))+EllipticF(((-(b^2+c^2)^(1/2)*sin(e*x+d-arctan(-b,c))-a)
/(-a+(b^2+c^2)^(1/2)))^(1/2),((a-(b^2+c^2)^(1/2))/(a+(b^2+c^2)^(1/2)))^(1/2
)))-(1/2*b^2+1/2*c^2)/(b^2+c^2)^(1/2)*(1/(b^2+c^2)^(1/2)*a-1)*((-(b^2+c^2)^
(1/2)*sin(e*x+d-arctan(-b,c))-a)/(-a+(b^2+c^2)^(1/2)))^(1/2)*((-sin(e*x+d-a
rctan(-b,c))+1)*(b^2+c^2)^(1/2)/(a+(b^2+c^2)^(1/2)))^(1/2)*((1+sin(e*x+d-ar
ctan(-b,c)))*(b^2+c^2)^(1/2)/(-a+(b^2+c^2)^(1/2)))^(1/2)/(-(-(b^2+c^2)^(1/2
)*sin(e*x+d-arctan(-b,c))-a)*cos(e*x+d-arctan(-b,c))^2)^(1/2)/a*EllipticPi(
((-(b^2+c^2)^(1/2)*sin(e*x+d-arctan(-b,c))-a)/(-a+(b^2+c^2)^(1/2)))^(1/2),-
1/2*(-1/(b^2+c^2)^(1/2)*a+1)*(b^2+c^2)^(1/2)/a,((a-(b^2+c^2)^(1/2))/(a+(b^2
+c^2)^(1/2)))^(1/2)))/cos(e*x+d-arctan(-b,c))/((b^2*sin(e*x+d-arctan(-b,c))
+c^2*sin(e*x+d-arctan(-b,c))+a*(b^2+c^2)^(1/2))/(b^2+c^2)^(1/2))^(1/2)/e
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(b\cos(ex+d)+c\sin(ex+d)+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cos(e*x+d)+c*sin(e*x+d))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*cos(e*x + d) + c*sin(e*x + d) + a)^(-3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{(a+b\,\cos(d+e\,x)+c\,\sin(d+e\,x))^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*cos(d + e*x) + c*sin(d + e*x))^(3/2),x)
```

```
[Out] int(1/(a + b*cos(d + e*x) + c*sin(d + e*x))^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a+b\cos(d+ex)+c\sin(d+ex))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cos(e*x+d)+c*sin(e*x+d))**(3/2),x)
```

```
[Out] Integral((a + b*cos(d + e*x) + c*sin(d + e*x))**(-3/2), x)
```

3.415 $\int \frac{1}{(a+b\cos(d+ex)+c\sin(d+ex))^{5/2}}\,dx$

Optimal. Leaf size=382

$$-\frac{2\sqrt{\frac{a+b\cos(d+ex)+c\sin(d+ex)}{a+\sqrt{b^2+c^2}}}\,F\left(\frac{1}{2}\left(d+ex-\tan^{-1}(b,c)\right)|\frac{2\sqrt{b^2+c^2}}{a+\sqrt{b^2+c^2}}\right)}{3e\left(a^2-b^2-c^2\right)\sqrt{a+b\cos(d+ex)+c\sin(d+ex)}}+\frac{8a\sqrt{a+b\cos(d+ex)+c\sin(d+ex)}\,E\left(\frac{1}{2}\left(d+\right.\right.}{3e\left(a^2-b^2-c^2\right)^2\sqrt{\frac{a+b\cos(d+ex}{a+\sqrt{b}}}}$$

```
[Out] 2/3*(c*cos(e*x+d)-b*sin(e*x+d))/(a^2-b^2-c^2)/e/(a+b*cos(e*x+d)+c*sin(e*x+d
))^(3/2)+8/3*(a*c*cos(e*x+d)-a*b*sin(e*x+d))/(a^2-b^2-c^2)^2/e/(a+b*cos(e*x
+d)+c*sin(e*x+d))^(1/2)+8/3*a*(cos(1/2*d+1/2*e*x-1/2*arctan(b,c))^2)^(1/2)/
cos(1/2*d+1/2*e*x-1/2*arctan(b,c))*EllipticE(sin(1/2*d+1/2*e*x-1/2*arctan(b
,c)),2^(1/2)*((b^2+c^2)^(1/2)/(a+(b^2+c^2)^(1/2)))^(1/2))*(a+b*cos(e*x+d)+c
*sin(e*x+d))^(1/2)/(a^2-b^2-c^2)^2/e/((a+b*cos(e*x+d)+c*sin(e*x+d))/(a+(b^2
+c^2)^(1/2)))^(1/2)-2/3*(cos(1/2*d+1/2*e*x-1/2*arctan(b,c))^2)^(1/2)/cos(1/
2*d+1/2*e*x-1/2*arctan(b,c))*EllipticF(sin(1/2*d+1/2*e*x-1/2*arctan(b,c)),2
^(1/2)*((b^2+c^2)^(1/2)/(a+(b^2+c^2)^(1/2)))^(1/2))*((a+b*cos(e*x+d)+c*sin(
e*x+d))/(a+(b^2+c^2)^(1/2)))^(1/2)/(a^2-b^2-c^2)/e/(a+b*cos(e*x+d)+c*sin(e*
x+d))^(1/2)
```

Rubi [A] time = 0.36, antiderivative size = 382, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.318, Rules used = {3129, 3156, 3149, 3119, 2653, 3127, 2661}

$$-\frac{2\sqrt{\frac{a+b\cos(d+ex)+c\sin(d+ex)}{a+\sqrt{b^2+c^2}}}\,F\left(\frac{1}{2}\left(d+ex-\tan^{-1}(b,c)\right)|\frac{2\sqrt{b^2+c^2}}{a+\sqrt{b^2+c^2}}\right)}{3e\left(a^2-b^2-c^2\right)\sqrt{a+b\cos(d+ex)+c\sin(d+ex)}}+\frac{8a\sqrt{a+b\cos(d+ex)+c\sin(d+ex)}\,E\left(\frac{1}{2}\left(d+\right.\right.}{3e\left(a^2-b^2-c^2\right)^2\sqrt{\frac{a+b\cos(d+ex}{a+\sqrt{b}}}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(-5/2),x]
```

```
[Out] (2*(c*Cos[d + e*x] - b*Sin[d + e*x]))/(3*(a^2 - b^2 - c^2)*e*(a + b*Cos[d +
 e*x] + c*Sin[d + e*x])^(3/2)) + (8*(a*c*Cos[d + e*x] - a*b*Sin[d + e*x]))/
(3*(a^2 - b^2 - c^2)^2*e*Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]]) + (8*a*
EllipticE[(d + e*x - ArcTan[b, c])/2, (2*Sqrt[b^2 + c^2])/(a + Sqrt[b^2 + c
^2])]*Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]])/(3*(a^2 - b^2 - c^2)^2*e*S
qrt[(a + b*Cos[d + e*x] + c*Sin[d + e*x])/(a + Sqrt[b^2 + c^2])]) - (2*Elli
pticF[(d + e*x - ArcTan[b, c])/2, (2*Sqrt[b^2 + c^2])/(a + Sqrt[b^2 + c^2])
]*Sqrt[(a + b*Cos[d + e*x] + c*Sin[d + e*x])/(a + Sqrt[b^2 + c^2])])/(3*(a^
2 - b^2 - c^2)*e*Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]])
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 3119

```
Int[Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_
)]], x_Symbol] :> Dist[Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]]/Sqrt[(a +
b*Cos[d + e*x] + c*Sin[d + e*x])/(a + Sqrt[b^2 + c^2])], Int[Sqrt[a/(a + Sq
rt[b^2 + c^2]) + (Sqrt[b^2 + c^2]*Cos[d + e*x - ArcTan[b, c]])/(a + Sqrt[b^
2 + c^2])], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0]
&& NeQ[b^2 + c^2, 0] &&  !GtQ[a + Sqrt[b^2 + c^2], 0]
```

Rule 3127

```
Int[1/Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(
x_)]], x_Symbol] :> Dist[Sqrt[(a + b*Cos[d + e*x] + c*Sin[d + e*x])/(a + Sq
rt[b^2 + c^2])]/Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]], Int[1/Sqrt[a/(a
+ Sqrt[b^2 + c^2]) + (Sqrt[b^2 + c^2]*Cos[d + e*x - ArcTan[b, c]])/(a + Sqr
t[b^2 + c^2])], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2,
 0] && NeQ[b^2 + c^2, 0] &&  !GtQ[a + Sqrt[b^2 + c^2], 0]
```

Rule 3129

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(n_), x_Symbol] :> Simp[((-(c*Cos[d + e*x]) + b*Sin[d + e*x])*(a + b*Cos[d
+ e*x] + c*Sin[d + e*x])^(n + 1))/(e*(n + 1)*(a^2 - b^2 - c^2)), x] + Dist[
1/((n + 1)*(a^2 - b^2 - c^2)), Int[(a*(n + 1) - b*(n + 2)*Cos[d + e*x] - c*
(n + 2)*Sin[d + e*x])*(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n + 1), x], x]
 /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0] && LtQ[n, -1] && N
eQ[n, -3/2]
```

Rule 3149

```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])
/Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)]]
, x_Symbol] :> Dist[B/b, Int[Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]], x],
```

```
x] + Dist[(A*b - a*B)/b, Int[1/Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]],
x], x] /; FreeQ[{a, b, c, d, e, A, B, C}, x] && EqQ[B*c - b*C, 0] && NeQ[A*
b - a*B, 0]
```

Rule 3156

```
Int[((a_.) + cos[(d_.) + (e_.)*(x_)]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)])
^(n_)*((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)
]), x_Symbol] :> -Simp[((c*B - b*C - (a*C - c*A)*Cos[d + e*x] + (a*B - b*A)
*Sin[d + e*x])*(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n + 1))/(e*(n + 1)*(a
^2 - b^2 - c^2)), x] + Dist[1/((n + 1)*(a^2 - b^2 - c^2)), Int[(a + b*Cos[d
 + e*x] + c*Sin[d + e*x])^(n + 1)*Simp[(n + 1)*(a*A - b*B - c*C) + (n + 2)*
(a*B - b*A)*Cos[d + e*x] + (n + 2)*(a*C - c*A)*Sin[d + e*x], x], x], x] /;
FreeQ[{a, b, c, d, e, A, B, C}, x] && LtQ[n, -1] && NeQ[a^2 - b^2 - c^2, 0]
 && NeQ[n, -2]
```

Rubi steps

$$\int \frac{1}{(a+b\cos(d+ex)+c\sin(d+ex))^{5/2}}\,dx = \frac{2(c\cos(d+ex)-b\sin(d+ex))}{3\left(a^2-b^2-c^2\right)e(a+b\cos(d+ex)+c\sin(d+ex))^{3/2}} - \frac{2\int \frac{-\frac{3a}{2}+\frac{1}{2}b}{(a+b\cos}}{3(}$$

$$= \frac{2(c\cos(d+ex)-b\sin(d+ex))}{3\left(a^2-b^2-c^2\right)e(a+b\cos(d+ex)+c\sin(d+ex))^{3/2}} + \frac{8}{3\left(a^2-b^2\right.}$$

$$= \frac{2(c\cos(d+ex)-b\sin(d+ex))}{3\left(a^2-b^2-c^2\right)e(a+b\cos(d+ex)+c\sin(d+ex))^{3/2}} + \frac{8}{3\left(a^2-b^2\right.}$$

$$= \frac{2(c\cos(d+ex)-b\sin(d+ex))}{3\left(a^2-b^2-c^2\right)e(a+b\cos(d+ex)+c\sin(d+ex))^{3/2}} + \frac{8}{3\left(a^2-b^2\right.}$$

$$= \frac{2(c\cos(d+ex)-b\sin(d+ex))}{3\left(a^2-b^2-c^2\right)e(a+b\cos(d+ex)+c\sin(d+ex))^{3/2}} + \frac{8}{3\left(a^2-b^2\right.}$$

Mathematica [C] time = 6.41, size = 2408, normalized size = 6.30

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(-5/2),x]
```

```
[Out] (Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]]*((8*a*(b^2 + c^2))/(3*b*c*(a^2 -
 b^2 - c^2)^2) + (2*(a*c + b^2*Sin[d + e*x] + c^2*Sin[d + e*x]))/(3*b*(-a^2
 + b^2 + c^2)*(a + b*Cos[d + e*x] + c*Sin[d + e*x])^2) - (2*(3*a^2*c + b^2*
c + c^3 + 4*a*b^2*Sin[d + e*x] + 4*a*c^2*Sin[d + e*x]))/(3*b*(-a^2 + b^2 +
c^2)^2*(a + b*Cos[d + e*x] + c*Sin[d + e*x]))))/e + (2*a^2*AppellF1[1/2, 1/
2, 1/2, 3/2, -((a + Sqrt[1 + b^2/c^2]*c*Sin[d + e*x + ArcTan[b/c]])/(Sqrt[1
 + b^2/c^2]*(1 - a/(Sqrt[1 + b^2/c^2]*c))*c)), -((a + Sqrt[1 + b^2/c^2]*c*S
in[d + e*x + ArcTan[b/c]])/(Sqrt[1 + b^2/c^2]*(-1 - a/(Sqrt[1 + b^2/c^2]*c)
)*c))]*Sec[d + e*x + ArcTan[b/c]]*Sqrt[(c*Sqrt[(b^2 + c^2)/c^2] - c*Sqrt[(b
^2 + c^2)/c^2]*Sin[d + e*x + ArcTan[b/c]])/(a + c*Sqrt[(b^2 + c^2)/c^2])]*S
qrt[a + c*Sqrt[(b^2 + c^2)/c^2]*Sin[d + e*x + ArcTan[b/c]]]*Sqrt[(c*Sqrt[(b
^2 + c^2)/c^2] + c*Sqrt[(b^2 + c^2)/c^2]*Sin[d + e*x + ArcTan[b/c]])/(-a +
c*Sqrt[(b^2 + c^2)/c^2])])/(Sqrt[1 + b^2/c^2]*c*(-a^2 + b^2 + c^2)^2*e) + (
2*b^2*AppellF1[1/2, 1/2, 1/2, 3/2, -((a + Sqrt[1 + b^2/c^2]*c*Sin[d + e*x +
 ArcTan[b/c]])/(Sqrt[1 + b^2/c^2]*(1 - a/(Sqrt[1 + b^2/c^2]*c))*c)), -((a +
 Sqrt[1 + b^2/c^2]*c*Sin[d + e*x + ArcTan[b/c]])/(Sqrt[1 + b^2/c^2]*(-1 - a
/(Sqrt[1 + b^2/c^2]*c))*c))]*Sec[d + e*x + ArcTan[b/c]]*Sqrt[(c*Sqrt[(b^2 +
 c^2)/c^2] - c*Sqrt[(b^2 + c^2)/c^2]*Sin[d + e*x + ArcTan[b/c]])/(a + c*Sqr
t[(b^2 + c^2)/c^2])]*Sqrt[a + c*Sqrt[(b^2 + c^2)/c^2]*Sin[d + e*x + ArcTan[
b/c]]]*Sqrt[(c*Sqrt[(b^2 + c^2)/c^2] + c*Sqrt[(b^2 + c^2)/c^2]*Sin[d + e*x
+ ArcTan[b/c]])/(-a + c*Sqrt[(b^2 + c^2)/c^2])])/(3*Sqrt[1 + b^2/c^2]*c*(-a
^2 + b^2 + c^2)^2*e) + (2*c*AppellF1[1/2, 1/2, 1/2, 3/2, -((a + Sqrt[1 + b^
2/c^2]*c*Sin[d + e*x + ArcTan[b/c]])/(Sqrt[1 + b^2/c^2]*(1 - a/(Sqrt[1 + b^
2/c^2]*c))*c)), -((a + Sqrt[1 + b^2/c^2]*c*Sin[d + e*x + ArcTan[b/c]])/(Sqr
t[1 + b^2/c^2]*(-1 - a/(Sqrt[1 + b^2/c^2]*c))*c))]*Sec[d + e*x + ArcTan[b/c
]]*Sqrt[(c*Sqrt[(b^2 + c^2)/c^2] - c*Sqrt[(b^2 + c^2)/c^2]*Sin[d + e*x + Ar
cTan[b/c]])/(a + c*Sqrt[(b^2 + c^2)/c^2])]*Sqrt[a + c*Sqrt[(b^2 + c^2)/c^2]
*Sin[d + e*x + ArcTan[b/c]]]*Sqrt[(c*Sqrt[(b^2 + c^2)/c^2] + c*Sqrt[(b^2 +
c^2)/c^2]*Sin[d + e*x + ArcTan[b/c]])/(-a + c*Sqrt[(b^2 + c^2)/c^2])])/(3*S
qrt[1 + b^2/c^2]*(-a^2 + b^2 + c^2)^2*e) + (4*a*b^2*(-((c*AppellF1[-1/2, -1
/2, -1/2, 1/2, -((a + b*Sqrt[1 + c^2/b^2]*Cos[d + e*x - ArcTan[c/b]])/(b*Sq
rt[1 + c^2/b^2]*(1 - a/(b*Sqrt[1 + c^2/b^2])))), -((a + b*Sqrt[1 + c^2/b^2]
*Cos[d + e*x - ArcTan[c/b]])/(b*Sqrt[1 + c^2/b^2]*(-1 - a/(b*Sqrt[1 + c^2/b
^2]))))]*Sin[d + e*x - ArcTan[c/b]])/(b*Sqrt[1 + c^2/b^2]*Sqrt[(b*Sqrt[(b^2
 + c^2)/b^2] - b*Sqrt[(b^2 + c^2)/b^2]*Cos[d + e*x - ArcTan[c/b]])/(a + b*S
qrt[(b^2 + c^2)/b^2])]*Sqrt[a + b*Sqrt[(b^2 + c^2)/b^2]*Cos[d + e*x - ArcTa
n[c/b]]]*Sqrt[(b*Sqrt[(b^2 + c^2)/b^2] + b*Sqrt[(b^2 + c^2)/b^2]*Cos[d + e*
```

```
x - ArcTan[c/b]])/(-a + b*Sqrt[(b^2 + c^2)/b^2])])) - ((2*b*(a + b*Sqrt[1 +
 c^2/b^2]*Cos[d + e*x - ArcTan[c/b]]))/(b^2 + c^2) - (c*Sin[d + e*x - ArcTa
n[c/b]])/(b*Sqrt[1 + c^2/b^2]))/Sqrt[a + b*Sqrt[1 + c^2/b^2]*Cos[d + e*x -
ArcTan[c/b]]]))/(3*c*(-a^2 + b^2 + c^2)^2*e) + (4*a*c*(-((c*AppellF1[-1/2,
-1/2, -1/2, 1/2, -((a + b*Sqrt[1 + c^2/b^2]*Cos[d + e*x - ArcTan[c/b]])/(b*
Sqrt[1 + c^2/b^2]*(1 - a/(b*Sqrt[1 + c^2/b^2])))), -((a + b*Sqrt[1 + c^2/b^
2]*Cos[d + e*x - ArcTan[c/b]])/(b*Sqrt[1 + c^2/b^2]*(-1 - a/(b*Sqrt[1 + c^2
/b^2]))))]*Sin[d + e*x - ArcTan[c/b]])/(b*Sqrt[1 + c^2/b^2]*Sqrt[(b*Sqrt[(b
^2 + c^2)/b^2] - b*Sqrt[(b^2 + c^2)/b^2]*Cos[d + e*x - ArcTan[c/b]])/(a + b
*Sqrt[(b^2 + c^2)/b^2])]*Sqrt[a + b*Sqrt[(b^2 + c^2)/b^2]*Cos[d + e*x - Arc
Tan[c/b]]]*Sqrt[(b*Sqrt[(b^2 + c^2)/b^2] + b*Sqrt[(b^2 + c^2)/b^2]*Cos[d +
e*x - ArcTan[c/b]])/(-a + b*Sqrt[(b^2 + c^2)/b^2])])) - ((2*b*(a + b*Sqrt[1
 + c^2/b^2]*Cos[d + e*x - ArcTan[c/b]]))/(b^2 + c^2) - (c*Sin[d + e*x - Arc
Tan[c/b]])/(b*Sqrt[1 + c^2/b^2]))/Sqrt[a + b*Sqrt[1 + c^2/b^2]*Cos[d + e*x
- ArcTan[c/b]]]))/(3*(-a^2 + b^2 + c^2)^2*e)
```

fricas [F] time = 2.96, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{b\cos\left(ex+d\right)+c\sin\left(ex+d\right)+a}}{\left(b^3-3\,bc^2\right)\cos\left(ex+d\right)^3+a^3+3\,ac^2+3\left(ab^2-ac^2\right)\cos\left(ex+d\right)^2+3\left(a^2b+bc^2\right)\cos\left(ex+d\right)+\left(6\,abc\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cos(e*x+d)+c*sin(e*x+d))^(5/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*cos(e*x + d) + c*sin(e*x + d) + a)/((b^3 - 3*b*c^2)*cos(e*x
 + d)^3 + a^3 + 3*a*c^2 + 3*(a*b^2 - a*c^2)*cos(e*x + d)^2 + 3*(a^2*b + b*c
^2)*cos(e*x + d) + (6*a*b*c*cos(e*x + d) + 3*a^2*c + c^3 + (3*b^2*c - c^3)*
cos(e*x + d)^2)*sin(e*x + d)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b\cos\left(ex+d\right)+c\sin\left(ex+d\right)+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cos(e*x+d)+c*sin(e*x+d))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*cos(e*x + d) + c*sin(e*x + d) + a)^(-5/2), x)
```

maple [B] time = 2.14, size = 2967, normalized size = 7.77

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*cos(e*x+d)+c*sin(e*x+d))^(5/2),x)

[Out] (-(-b^2*sin(e*x+d-arctan(-b,c))-c^2*sin(e*x+d-arctan(-b,c))-a*(b^2+c^2)^(1/
2))*cos(e*x+d-arctan(-b,c))^2/(b^2+c^2)^(1/2))^(1/2)*(-1/4/a/(a^2-b^2-c^2)*
(b^2+c^2)^(1/2)*(cos(e*x+d-arctan(-b,c))^2*(b^2+c^2)*((b^2+c^2)^(1/2)*sin(e
*x+d-arctan(-b,c))+a))^(1/2)/(b^2*sin(e*x+d-arctan(-b,c))+c^2*sin(e*x+d-arc
tan(-b,c))-a*(b^2+c^2)^(1/2))+1/3/(a^2-b^2-c^2)/(b^2+c^2)*(cos(e*x+d-arctan
(-b,c))^2*(b^2+c^2)*((b^2+c^2)^(1/2)*sin(e*x+d-arctan(-b,c))+a))^(1/2)/(sin
(e*x+d-arctan(-b,c))+1/(b^2+c^2)^(1/2)*a)^2-4/3*(-b^2-c^2)*cos(e*x+d-arctan
(-b,c))^2/(a^2-b^2-c^2)^2*a/(cos(e*x+d-arctan(-b,c))^2*(b^2+c^2)*((b^2+c^2)
^(1/2)*sin(e*x+d-arctan(-b,c))+a))^(1/2)+2*(-1/24*(b^2+c^2)^(1/2)/(a^2-b^2-
c^2)+2/3*a^2*(b^2+c^2)^(1/2)/(a^2-b^2-c^2)^2)*(1/(b^2+c^2)^(1/2)*a-1)*((-(b
^2+c^2)^(1/2)*sin(e*x+d-arctan(-b,c))-a)/(-a+(b^2+c^2)^(1/2)))^(1/2)*((-sin
(e*x+d-arctan(-b,c))+1)*(b^2+c^2)^(1/2)/(a+(b^2+c^2)^(1/2)))^(1/2)*((1+sin(
e*x+d-arctan(-b,c)))*(b^2+c^2)^(1/2)/(-a+(b^2+c^2)^(1/2)))^(1/2)/(cos(e*x+d
-arctan(-b,c))^2*(b^2+c^2)*((b^2+c^2)^(1/2)*sin(e*x+d-arctan(-b,c))+a))^(1/
2)*EllipticF(((-(b^2+c^2)^(1/2)*sin(e*x+d-arctan(-b,c))-a)/(-a+(b^2+c^2)^(1
/2)))^(1/2),((a-(b^2+c^2)^(1/2))/(a+(b^2+c^2)^(1/2)))^(1/2))+2*(13*a^2*b^2+
13*a^2*c^2+3*b^4+6*b^2*c^2+3*c^4)/(24*a^5-48*a^3*b^2-48*a^3*c^2+24*a*b^4+48
*a*b^2*c^2+24*a*c^4)*(1/(b^2+c^2)^(1/2)*a-1)*((-(b^2+c^2)^(1/2)*sin(e*x+d-a
rctan(-b,c))-a)/(-a+(b^2+c^2)^(1/2)))^(1/2)*((-sin(e*x+d-arctan(-b,c))+1)*(
b^2+c^2)^(1/2)/(a+(b^2+c^2)^(1/2)))^(1/2)*((1+sin(e*x+d-arctan(-b,c)))*(b^2
+c^2)^(1/2)/(-a+(b^2+c^2)^(1/2)))^(1/2)/(cos(e*x+d-arctan(-b,c))^2*(b^2+c^2
)*((b^2+c^2)^(1/2)*sin(e*x+d-arctan(-b,c))+a))^(1/2)*((-1/(b^2+c^2)^(1/2)*a
-1)*EllipticE(((-(b^2+c^2)^(1/2)*sin(e*x+d-arctan(-b,c))-a)/(-a+(b^2+c^2)^(
1/2)))^(1/2),((a-(b^2+c^2)^(1/2))/(a+(b^2+c^2)^(1/2)))^(1/2))+EllipticF(((-
(b^2+c^2)^(1/2)*sin(e*x+d-arctan(-b,c))-a)/(-a+(b^2+c^2)^(1/2)))^(1/2),((a-
(b^2+c^2)^(1/2))/(a+(b^2+c^2)^(1/2)))^(1/2)))-1/8*(5*a^2*b^2+5*a^2*c^2-b^4-
2*b^2*c^2-c^4)/a^2/(a^2-b^2-c^2)/(b^2+c^2)^(1/2)*(1/(b^2+c^2)^(1/2)*a-1)*((
-(b^2+c^2)^(1/2)*sin(e*x+d-arctan(-b,c))-a)/(-a+(b^2+c^2)^(1/2)))^(1/2)*((-
sin(e*x+d-arctan(-b,c))+1)*(b^2+c^2)^(1/2)/(a+(b^2+c^2)^(1/2)))^(1/2)*((1+s
in(e*x+d-arctan(-b,c)))*(b^2+c^2)^(1/2)/(-a+(b^2+c^2)^(1/2)))^(1/2)/(cos(e*
x+d-arctan(-b,c))^2*(b^2+c^2)*((b^2+c^2)^(1/2)*sin(e*x+d-arctan(-b,c))+a))^
(1/2)*EllipticPi(((-(b^2+c^2)^(1/2)*sin(e*x+d-arctan(-b,c))-a)/(-a+(b^2+c^2
)^(1/2)))^(1/2),-1/2*(-1/(b^2+c^2)^(1/2)*a+1)*(b^2+c^2)^(1/2)/a,((a-(b^2+c^
2)^(1/2))/(a+(b^2+c^2)^(1/2)))^(1/2))+1/4*(b^2+c^2)/a/(a^2-b^2-c^2)*(cos(e*
x+d-arctan(-b,c))^2*((b^2+c^2)^(1/2)*sin(e*x+d-arctan(-b,c))+a))^(1/2)/(b^2
*sin(e*x+d-arctan(-b,c))+c^2*sin(e*x+d-arctan(-b,c))-a*(b^2+c^2)^(1/2))+1/3
/(a^2-b^2-c^2)/(b^2+c^2)^(1/2)*(cos(e*x+d-arctan(-b,c))^2*((b^2+c^2)^(1/2)*
sin(e*x+d-arctan(-b,c))+a))^(1/2)/(sin(e*x+d-arctan(-b,c))+1/(b^2+c^2)^(1/2
)*a)^2+4/3*(b^2+c^2)^(1/2)*cos(e*x+d-arctan(-b,c))^2/(a^2-b^2-c^2)^2*a/(cos
(e*x+d-arctan(-b,c))^2*((b^2+c^2)^(1/2)*sin(e*x+d-arctan(-b,c))+a))^(1/2)+2
*(-7/24/(a^2-b^2-c^2)+2/3*a^2/(a^2-b^2-c^2)^2)*(1/(b^2+c^2)^(1/2)*a-1)*((-(
b^2+c^2)^(1/2)*sin(e*x+d-arctan(-b,c))-a)/(-a+(b^2+c^2)^(1/2)))^(1/2)*((-si
n(e*x+d-arctan(-b,c))+1)*(b^2+c^2)^(1/2)/(a+(b^2+c^2)^(1/2)))^(1/2)*((1+sin
```

```
(e*x+d-arctan(-b,c)))*(b^2+c^2)^(1/2)/(-a+(b^2+c^2)^(1/2)))^(1/2)/(cos(e*x+
d-arctan(-b,c))^2*((b^2+c^2)^(1/2)*sin(e*x+d-arctan(-b,c))+a))^(1/2)*Ellipt
icF(((-(b^2+c^2)^(1/2)*sin(e*x+d-arctan(-b,c))-a)/(-a+(b^2+c^2)^(1/2)))^(1/
2),((a-(b^2+c^2)^(1/2))/(a+(b^2+c^2)^(1/2)))^(1/2))+2*(1/8/a/(a^2-b^2-c^2)*
(b^2+c^2)^(1/2)+2/3*a*(b^2+c^2)^(1/2)/(a^2-b^2-c^2)^2)*(1/(b^2+c^2)^(1/2)*a
-1)*((-(b^2+c^2)^(1/2)*sin(e*x+d-arctan(-b,c))-a)/(-a+(b^2+c^2)^(1/2)))^(1/
2)*((-sin(e*x+d-arctan(-b,c))+1)*(b^2+c^2)^(1/2)/(a+(b^2+c^2)^(1/2)))^(1/2)
*((1+sin(e*x+d-arctan(-b,c)))*(b^2+c^2)^(1/2)/(-a+(b^2+c^2)^(1/2)))^(1/2)/(
cos(e*x+d-arctan(-b,c))^2*((b^2+c^2)^(1/2)*sin(e*x+d-arctan(-b,c))+a))^(1/2
)*((-1/(b^2+c^2)^(1/2)*a-1)*EllipticE(((-(b^2+c^2)^(1/2)*sin(e*x+d-arctan(-
b,c))-a)/(-a+(b^2+c^2)^(1/2)))^(1/2),((a-(b^2+c^2)^(1/2))/(a+(b^2+c^2)^(1/2
)))^(1/2))+EllipticF(((-(b^2+c^2)^(1/2)*sin(e*x+d-arctan(-b,c))-a)/(-a+(b^2
+c^2)^(1/2)))^(1/2),((a-(b^2+c^2)^(1/2))/(a+(b^2+c^2)^(1/2)))^(1/2)))+1/8*(
5*a^2-b^2-c^2)/a^2/(a^2-b^2-c^2)*(1/(b^2+c^2)^(1/2)*a-1)*((-(b^2+c^2)^(1/2)
*sin(e*x+d-arctan(-b,c))-a)/(-a+(b^2+c^2)^(1/2)))^(1/2)*((-sin(e*x+d-arctan
(-b,c))+1)*(b^2+c^2)^(1/2)/(a+(b^2+c^2)^(1/2)))^(1/2)*((1+sin(e*x+d-arctan(
-b,c)))*(b^2+c^2)^(1/2)/(-a+(b^2+c^2)^(1/2)))^(1/2)/(cos(e*x+d-arctan(-b,c)
)^2*((b^2+c^2)^(1/2)*sin(e*x+d-arctan(-b,c))+a))^(1/2)*EllipticPi(((-(b^2+c
^2)^(1/2)*sin(e*x+d-arctan(-b,c))-a)/(-a+(b^2+c^2)^(1/2)))^(1/2),-1/2*(-1/(
b^2+c^2)^(1/2)*a+1)*(b^2+c^2)^(1/2)/a,((a-(b^2+c^2)^(1/2))/(a+(b^2+c^2)^(1/
2)))^(1/2)))/cos(e*x+d-arctan(-b,c))/((b^2*sin(e*x+d-arctan(-b,c))+c^2*sin(
e*x+d-arctan(-b,c))+a*(b^2+c^2)^(1/2))/(b^2+c^2)^(1/2))^(1/2)/e
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(b\cos(ex+d)+c\sin(ex+d)+a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cos(e*x+d)+c*sin(e*x+d))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((b*cos(e*x + d) + c*sin(e*x + d) + a)^(-5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{(a+b\cos(d+e\,x)+c\sin(d+e\,x))^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*cos(d + e*x) + c*sin(d + e*x))^(5/2),x)
```

```
[Out] int(1/(a + b*cos(d + e*x) + c*sin(d + e*x))^(5/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cos(e*x+d)+c*sin(e*x+d))**(5/2),x)
```

```
[Out] Timed out
```

3.416 $\int \frac{1}{(a+b\cos(d+ex)+c\sin(d+ex))^{7/2}}\,dx$

Optimal. Leaf size=490

$$-\frac{16a\sqrt{\frac{a+b\cos(d+ex)+c\sin(d+ex)}{a+\sqrt{b^2+c^2}}}\,F\left(\frac{1}{2}\left(d+ex-\tan^{-1}(b,c)\right)|\frac{2\sqrt{b^2+c^2}}{a+\sqrt{b^2+c^2}}\right)}{15e\left(a^2-b^2-c^2\right)^2\sqrt{a+b\cos(d+ex)+c\sin(d+ex)}}+\frac{2\left(23a^2+9\left(b^2+c^2\right)\right)\sqrt{a+b\cos(d+ex)+c}}{15e\left(a^2-b^2-c^2\right)^3}$$

```
[Out] 2/5*(c*cos(e*x+d)-b*sin(e*x+d))/(a^2-b^2-c^2)/e/(a+b*cos(e*x+d)+c*sin(e*x+d
))^(5/2)+16/15*(a*c*cos(e*x+d)-a*b*sin(e*x+d))/(a^2-b^2-c^2)^2/e/(a+b*cos(e
*x+d)+c*sin(e*x+d))^(3/2)+2/15*(c*(23*a^2+9*b^2+9*c^2)*cos(e*x+d)-b*(23*a^2
+9*b^2+9*c^2)*sin(e*x+d))/(a^2-b^2-c^2)^3/e/(a+b*cos(e*x+d)+c*sin(e*x+d))^(
1/2)+2/15*(23*a^2+9*b^2+9*c^2)*(cos(1/2*d+1/2*e*x-1/2*arctan(b,c))^2)^(1/2)
/cos(1/2*d+1/2*e*x-1/2*arctan(b,c))*EllipticE(sin(1/2*d+1/2*e*x-1/2*arctan(
b,c)),2^(1/2)*((b^2+c^2)^(1/2)/(a+(b^2+c^2)^(1/2)))^(1/2))*(a+b*cos(e*x+d)+
c*sin(e*x+d))^(1/2)/(a^2-b^2-c^2)^3/e/((a+b*cos(e*x+d)+c*sin(e*x+d))/(a+(b^
2+c^2)^(1/2)))^(1/2)-16/15*a*(cos(1/2*d+1/2*e*x-1/2*arctan(b,c))^2)^(1/2)/c
os(1/2*d+1/2*e*x-1/2*arctan(b,c))*EllipticF(sin(1/2*d+1/2*e*x-1/2*arctan(b,
c)),2^(1/2)*((b^2+c^2)^(1/2)/(a+(b^2+c^2)^(1/2)))^(1/2))*((a+b*cos(e*x+d)+c
*sin(e*x+d))/(a+(b^2+c^2)^(1/2)))^(1/2)/(a^2-b^2-c^2)^2/e/(a+b*cos(e*x+d)+c
*sin(e*x+d))^(1/2)
```

Rubi [A] time = 0.62, antiderivative size = 490, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.318, Rules used = {3129, 3156, 3149, 3119, 2653, 3127, 2661}

$$-\frac{16a\sqrt{\frac{a+b\cos(d+ex)+c\sin(d+ex)}{a+\sqrt{b^2+c^2}}}\,F\left(\frac{1}{2}\left(d+ex-\tan^{-1}(b,c)\right)|\frac{2\sqrt{b^2+c^2}}{a+\sqrt{b^2+c^2}}\right)}{15e\left(a^2-b^2-c^2\right)^2\sqrt{a+b\cos(d+ex)+c\sin(d+ex)}}+\frac{2\left(23a^2+9\left(b^2+c^2\right)\right)\sqrt{a+b\cos(d+ex)+c}}{15e\left(a^2-b^2-c^2\right)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(-7/2),x]
```

```
[Out] (2*(c*Cos[d + e*x] - b*Sin[d + e*x]))/(5*(a^2 - b^2 - c^2)*e*(a + b*Cos[d +
 e*x] + c*Sin[d + e*x])^(5/2)) + (16*(a*c*Cos[d + e*x] - a*b*Sin[d + e*x]))
/(15*(a^2 - b^2 - c^2)^2*e*(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(3/2)) + (
2*(23*a^2 + 9*(b^2 + c^2))*EllipticE[(d + e*x - ArcTan[b, c])/2, (2*Sqrt[b^
2 + c^2])/(a + Sqrt[b^2 + c^2])]*Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]])
/(15*(a^2 - b^2 - c^2)^3*e*Sqrt[(a + b*Cos[d + e*x] + c*Sin[d + e*x])/(a +
Sqrt[b^2 + c^2])]) - (16*a*EllipticF[(d + e*x - ArcTan[b, c])/2, (2*Sqrt[b^
```

```
2 + c^2])/(a + Sqrt[b^2 + c^2])]*Sqrt[(a + b*Cos[d + e*x] + c*Sin[d + e*x])
/(a + Sqrt[b^2 + c^2])])/(15*(a^2 - b^2 - c^2)^2*e*Sqrt[a + b*Cos[d + e*x]
+ c*Sin[d + e*x]]) + (2*(c*(23*a^2 + 9*(b^2 + c^2))*Cos[d + e*x] - b*(23*a^
2 + 9*(b^2 + c^2))*Sin[d + e*x]))/(15*(a^2 - b^2 - c^2)^3*e*Sqrt[a + b*Cos[
d + e*x] + c*Sin[d + e*x]])
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 3119

```
Int[Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_
)]], x_Symbol] :> Dist[Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]]/Sqrt[(a +
b*Cos[d + e*x] + c*Sin[d + e*x])/(a + Sqrt[b^2 + c^2])], Int[Sqrt[a/(a + Sq
rt[b^2 + c^2]) + (Sqrt[b^2 + c^2]*Cos[d + e*x - ArcTan[b, c]])/(a + Sqrt[b^
2 + c^2])], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0]
&& NeQ[b^2 + c^2, 0] &&  !GtQ[a + Sqrt[b^2 + c^2], 0]
```

Rule 3127

```
Int[1/Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(
x_)]], x_Symbol] :> Dist[Sqrt[(a + b*Cos[d + e*x] + c*Sin[d + e*x])/(a + Sq
rt[b^2 + c^2])]/Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]], Int[1/Sqrt[a/(a
+ Sqrt[b^2 + c^2]) + (Sqrt[b^2 + c^2]*Cos[d + e*x - ArcTan[b, c]])/(a + Sqr
t[b^2 + c^2])], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2,
 0] && NeQ[b^2 + c^2, 0] &&  !GtQ[a + Sqrt[b^2 + c^2], 0]
```

Rule 3129

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(n_), x_Symbol] :> Simp[((-(c*Cos[d + e*x]) + b*Sin[d + e*x])*(a + b*Cos[d
+ e*x] + c*Sin[d + e*x])^(n + 1))/(e*(n + 1)*(a^2 - b^2 - c^2)), x] + Dist[
1/((n + 1)*(a^2 - b^2 - c^2)), Int[(a*(n + 1) - b*(n + 2)*Cos[d + e*x] - c*
(n + 2)*Sin[d + e*x])*(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n + 1), x], x]
 /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0] && LtQ[n, -1] && N
eQ[n, -3/2]
```

Rule 3149

```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])
/Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)]]
, x_Symbol] :> Dist[B/b, Int[Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]], x],
 x] + Dist[(A*b - a*B)/b, Int[1/Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]],
x], x] /; FreeQ[{a, b, c, d, e, A, B, C}, x] && EqQ[B*c - b*C, 0] && NeQ[A*
b - a*B, 0]
```

Rule 3156

```
Int[((a_.) + cos[(d_.) + (e_.)*(x_)]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)])
^(n_)*((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)
]), x_Symbol] :> -Simp[((c*B - b*C - (a*C - c*A)*Cos[d + e*x] + (a*B - b*A)
*Sin[d + e*x])*(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n + 1))/(e*(n + 1)*(a
^2 - b^2 - c^2)), x] + Dist[1/((n + 1)*(a^2 - b^2 - c^2)), Int[(a + b*Cos[d
 + e*x] + c*Sin[d + e*x])^(n + 1)*Simp[(n + 1)*(a*A - b*B - c*C) + (n + 2)*
(a*B - b*A)*Cos[d + e*x] + (n + 2)*(a*C - c*A)*Sin[d + e*x], x], x], x] /;
FreeQ[{a, b, c, d, e, A, B, C}, x] && LtQ[n, -1] && NeQ[a^2 - b^2 - c^2, 0]
 && NeQ[n, -2]
```

Rubi steps

$$\int \frac{1}{(a+b\cos(d+ex)+c\sin(d+ex))^{7/2}}\,dx = \frac{2(c\cos(d+ex)-b\sin(d+ex))}{5\left(a^2-b^2-c^2\right)e(a+b\cos(d+ex)+c\sin(d+ex))^{5/2}} - \frac{2\int \frac{-\frac{5a}{2}+\frac{3}{2}}{(a+b c}}{5}$$

$$= \frac{2(c\cos(d+ex)-b\sin(d+ex))}{5\left(a^2-b^2-c^2\right)e(a+b\cos(d+ex)+c\sin(d+ex))^{5/2}} + \frac{}{15\left(a^2-\right.}$$

$$= \frac{2(c\cos(d+ex)-b\sin(d+ex))}{5\left(a^2-b^2-c^2\right)e(a+b\cos(d+ex)+c\sin(d+ex))^{5/2}} + \frac{}{15\left(a^2-\right.}$$

$$= \frac{2(c\cos(d+ex)-b\sin(d+ex))}{5\left(a^2-b^2-c^2\right)e(a+b\cos(d+ex)+c\sin(d+ex))^{5/2}} + \frac{}{15\left(a^2-\right.}$$

$$= \frac{2(c\cos(d+ex)-b\sin(d+ex))}{5\left(a^2-b^2-c^2\right)e(a+b\cos(d+ex)+c\sin(d+ex))^{5/2}} + \frac{}{15\left(a^2-\right.}$$

$$= \frac{2(c\cos(d+ex)-b\sin(d+ex))}{5\left(a^2-b^2-c^2\right)e(a+b\cos(d+ex)+c\sin(d+ex))^{5/2}} + \frac{}{15\left(a^2-\right.}$$

Mathematica [C] time = 6.66, size = 4116, normalized size = 8.40

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(-7/2),x]
```

```
[Out] (Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]]*((-2*(b^2 + c^2)*(23*a^2 + 9*b^2
 + 9*c^2))/(15*b*c*(-a^2 + b^2 + c^2)^3) + (2*(a*c + b^2*Sin[d + e*x] + c^2
*Sin[d + e*x]))/(5*b*(-a^2 + b^2 + c^2)*(a + b*Cos[d + e*x] + c*Sin[d + e*x
])^3) - (2*(5*a^2*c + 3*b^2*c + 3*c^3 + 8*a*b^2*Sin[d + e*x] + 8*a*c^2*Sin[
d + e*x]))/(15*b*(-a^2 + b^2 + c^2)^2*(a + b*Cos[d + e*x] + c*Sin[d + e*x])
^2) + (2*(15*a^3*c + 17*a*b^2*c + 17*a*c^3 + 23*a^2*b^2*Sin[d + e*x] + 9*b^
4*Sin[d + e*x] + 23*a^2*c^2*Sin[d + e*x] + 18*b^2*c^2*Sin[d + e*x] + 9*c^4*
Sin[d + e*x]))/(15*b*(-a^2 + b^2 + c^2)^3*(a + b*Cos[d + e*x] + c*Sin[d + e
```

*x])))))/e - (2*a^3*AppellF1[1/2, 1/2, 1/2, 3/2, -((a + Sqrt[1 + b^2/c^2]*c*
Sin[d + e*x + ArcTan[b/c]])/(Sqrt[1 + b^2/c^2]*(1 - a/(Sqrt[1 + b^2/c^2]*c)
)*c)), -((a + Sqrt[1 + b^2/c^2]*c*Sin[d + e*x + ArcTan[b/c]])/(Sqrt[1 + b^2
/c^2]*(-1 - a/(Sqrt[1 + b^2/c^2]*c))*c))]*Sec[d + e*x + ArcTan[b/c]]*Sqrt[(
c*Sqrt[(b^2 + c^2)/c^2] - c*Sqrt[(b^2 + c^2)/c^2]*Sin[d + e*x + ArcTan[b/c]
])/(a + c*Sqrt[(b^2 + c^2)/c^2])]*Sqrt[a + c*Sqrt[(b^2 + c^2)/c^2]*Sin[d +
e*x + ArcTan[b/c]]]*Sqrt[(c*Sqrt[(b^2 + c^2)/c^2] + c*Sqrt[(b^2 + c^2)/c^2]
*Sin[d + e*x + ArcTan[b/c]])/(-a + c*Sqrt[(b^2 + c^2)/c^2])]])/(Sqrt[1 + b^2
/c^2]*c*(-a^2 + b^2 + c^2)^3*e) - (34*a*b^2*AppellF1[1/2, 1/2, 1/2, 3/2, -(
(a + Sqrt[1 + b^2/c^2]*c*Sin[d + e*x + ArcTan[b/c]])/(Sqrt[1 + b^2/c^2]*(1
- a/(Sqrt[1 + b^2/c^2]*c))*c)), -((a + Sqrt[1 + b^2/c^2]*c*Sin[d + e*x + Ar
cTan[b/c]])/(Sqrt[1 + b^2/c^2]*(-1 - a/(Sqrt[1 + b^2/c^2]*c))*c))]*Sec[d +
e*x + ArcTan[b/c]]*Sqrt[(c*Sqrt[(b^2 + c^2)/c^2] - c*Sqrt[(b^2 + c^2)/c^2]*
Sin[d + e*x + ArcTan[b/c]])/(a + c*Sqrt[(b^2 + c^2)/c^2])]*Sqrt[a + c*Sqrt[
(b^2 + c^2)/c^2]*Sin[d + e*x + ArcTan[b/c]]]*Sqrt[(c*Sqrt[(b^2 + c^2)/c^2]
+ c*Sqrt[(b^2 + c^2)/c^2]*Sin[d + e*x + ArcTan[b/c]])/(-a + c*Sqrt[(b^2 + c
^2)/c^2])]])/(15*Sqrt[1 + b^2/c^2]*c*(-a^2 + b^2 + c^2)^3*e) - (34*a*c*Appel
lF1[1/2, 1/2, 1/2, 3/2, -((a + Sqrt[1 + b^2/c^2]*c*Sin[d + e*x + ArcTan[b/c
]])/(Sqrt[1 + b^2/c^2]*(1 - a/(Sqrt[1 + b^2/c^2]*c))*c)), -((a + Sqrt[1 + b
^2/c^2]*c*Sin[d + e*x + ArcTan[b/c]])/(Sqrt[1 + b^2/c^2]*(-1 - a/(Sqrt[1 +
b^2/c^2]*c))*c))]*Sec[d + e*x + ArcTan[b/c]]*Sqrt[(c*Sqrt[(b^2 + c^2)/c^2]
- c*Sqrt[(b^2 + c^2)/c^2]*Sin[d + e*x + ArcTan[b/c]])/(a + c*Sqrt[(b^2 + c^
2)/c^2])]*Sqrt[a + c*Sqrt[(b^2 + c^2)/c^2]*Sin[d + e*x + ArcTan[b/c]]]*Sqrt
[(c*Sqrt[(b^2 + c^2)/c^2] + c*Sqrt[(b^2 + c^2)/c^2]*Sin[d + e*x + ArcTan[b/
c]])/(-a + c*Sqrt[(b^2 + c^2)/c^2])]])/(15*Sqrt[1 + b^2/c^2]*(-a^2 + b^2 + c
^2)^3*e) - (23*a^2*b^2*(-(((c*AppellF1[-1/2, -1/2, -1/2, 1/2, -((a + b*Sqrt[
1 + c^2/b^2]*Cos[d + e*x - ArcTan[c/b]])/(b*Sqrt[1 + c^2/b^2]*(1 - a/(b*Sqr
t[1 + c^2/b^2])))), -((a + b*Sqrt[1 + c^2/b^2]*Cos[d + e*x - ArcTan[c/b]])/
(b*Sqrt[1 + c^2/b^2]*(-1 - a/(b*Sqrt[1 + c^2/b^2]))))]*Sin[d + e*x - ArcTan
[c/b]])/(b*Sqrt[1 + c^2/b^2]*Sqrt[(b*Sqrt[(b^2 + c^2)/b^2] - b*Sqrt[(b^2 +
c^2)/b^2]*Cos[d + e*x - ArcTan[c/b]])/(a + b*Sqrt[(b^2 + c^2)/b^2])]*Sqrt[a
 + b*Sqrt[(b^2 + c^2)/b^2]*Cos[d + e*x - ArcTan[c/b]]]*Sqrt[(b*Sqrt[(b^2 +
c^2)/b^2] + b*Sqrt[(b^2 + c^2)/b^2]*Cos[d + e*x - ArcTan[c/b]])/(-a + b*Sqr
t[(b^2 + c^2)/b^2])]])) - ((2*b*(a + b*Sqrt[1 + c^2/b^2]*Cos[d + e*x - ArcTa
n[c/b]]))/(b^2 + c^2) - (c*Sin[d + e*x - ArcTan[c/b]])/(b*Sqrt[1 + c^2/b^2]
))/Sqrt[a + b*Sqrt[1 + c^2/b^2]*Cos[d + e*x - ArcTan[c/b]]]))/(15*c*(-a^2 +
 b^2 + c^2)^3*e) - (3*b^4*(-(((c*AppellF1[-1/2, -1/2, -1/2, 1/2, -((a + b*Sq
rt[1 + c^2/b^2]*Cos[d + e*x - ArcTan[c/b]])/(b*Sqrt[1 + c^2/b^2]*(1 - a/(b*
Sqrt[1 + c^2/b^2])))), -((a + b*Sqrt[1 + c^2/b^2]*Cos[d + e*x - ArcTan[c/b]
])/(b*Sqrt[1 + c^2/b^2]*(-1 - a/(b*Sqrt[1 + c^2/b^2]))))]*Sin[d + e*x - Arc
Tan[c/b]])/(b*Sqrt[1 + c^2/b^2]*Sqrt[(b*Sqrt[(b^2 + c^2)/b^2] - b*Sqrt[(b^2
 + c^2)/b^2]*Cos[d + e*x - ArcTan[c/b]])/(a + b*Sqrt[(b^2 + c^2)/b^2])]*Sqr
t[a + b*Sqrt[(b^2 + c^2)/b^2]*Cos[d + e*x - ArcTan[c/b]]]*Sqrt[(b*Sqrt[(b^2
 + c^2)/b^2] + b*Sqrt[(b^2 + c^2)/b^2]*Cos[d + e*x - ArcTan[c/b]])/(-a + b*
Sqrt[(b^2 + c^2)/b^2])]])) - ((2*b*(a + b*Sqrt[1 + c^2/b^2]*Cos[d + e*x - Ar

```
cTan[c/b]]))/(b^2 + c^2) - (c*Sin[d + e*x - ArcTan[c/b]])/(b*Sqrt[1 + c^2/b
^2]))/Sqrt[a + b*Sqrt[1 + c^2/b^2]*Cos[d + e*x - ArcTan[c/b]]]))/(5*c*(-a^2
 + b^2 + c^2)^3*e) - (23*a^2*c*(-((c*AppellF1[-1/2, -1/2, -1/2, 1/2, -((a +
 b*Sqrt[1 + c^2/b^2]*Cos[d + e*x - ArcTan[c/b]])/(b*Sqrt[1 + c^2/b^2]*(1 -
a/(b*Sqrt[1 + c^2/b^2])))), -((a + b*Sqrt[1 + c^2/b^2]*Cos[d + e*x - ArcTan
[c/b]])/(b*Sqrt[1 + c^2/b^2]*(-1 - a/(b*Sqrt[1 + c^2/b^2]))))]*Sin[d + e*x
- ArcTan[c/b]])/(b*Sqrt[1 + c^2/b^2]*Sqrt[(b*Sqrt[(b^2 + c^2)/b^2] - b*Sqrt
[(b^2 + c^2)/b^2]*Cos[d + e*x - ArcTan[c/b]])/(a + b*Sqrt[(b^2 + c^2)/b^2])
]*Sqrt[a + b*Sqrt[(b^2 + c^2)/b^2]*Cos[d + e*x - ArcTan[c/b]]]*Sqrt[(b*Sqrt
[(b^2 + c^2)/b^2] + b*Sqrt[(b^2 + c^2)/b^2]*Cos[d + e*x - ArcTan[c/b]])/(-a
 + b*Sqrt[(b^2 + c^2)/b^2])])) - ((2*b*(a + b*Sqrt[1 + c^2/b^2]*Cos[d + e*x
 - ArcTan[c/b]]))/(b^2 + c^2) - (c*Sin[d + e*x - ArcTan[c/b]])/(b*Sqrt[1 +
c^2/b^2]))/Sqrt[a + b*Sqrt[1 + c^2/b^2]*Cos[d + e*x - ArcTan[c/b]]]))/(15*(
-a^2 + b^2 + c^2)^3*e) - (6*b^2*c*(-((c*AppellF1[-1/2, -1/2, -1/2, 1/2, -((
a + b*Sqrt[1 + c^2/b^2]*Cos[d + e*x - ArcTan[c/b]])/(b*Sqrt[1 + c^2/b^2]*(1
 - a/(b*Sqrt[1 + c^2/b^2])))), -((a + b*Sqrt[1 + c^2/b^2]*Cos[d + e*x - Arc
Tan[c/b]])/(b*Sqrt[1 + c^2/b^2]*(-1 - a/(b*Sqrt[1 + c^2/b^2]))))]*Sin[d + e
*x - ArcTan[c/b]])/(b*Sqrt[1 + c^2/b^2]*Sqrt[(b*Sqrt[(b^2 + c^2)/b^2] - b*S
qrt[(b^2 + c^2)/b^2]*Cos[d + e*x - ArcTan[c/b]])/(a + b*Sqrt[(b^2 + c^2)/b^
2])]*Sqrt[a + b*Sqrt[(b^2 + c^2)/b^2]*Cos[d + e*x - ArcTan[c/b]]]*Sqrt[(b*S
qrt[(b^2 + c^2)/b^2] + b*Sqrt[(b^2 + c^2)/b^2]*Cos[d + e*x - ArcTan[c/b]])/
(-a + b*Sqrt[(b^2 + c^2)/b^2])])) - ((2*b*(a + b*Sqrt[1 + c^2/b^2]*Cos[d +
e*x - ArcTan[c/b]]))/(b^2 + c^2) - (c*Sin[d + e*x - ArcTan[c/b]])/(b*Sqrt[1
 + c^2/b^2]))/Sqrt[a + b*Sqrt[1 + c^2/b^2]*Cos[d + e*x - ArcTan[c/b]]]))/(5
*(-a^2 + b^2 + c^2)^3*e) - (3*c^3*(-((c*AppellF1[-1/2, -1/2, -1/2, 1/2, -((
a + b*Sqrt[1 + c^2/b^2]*Cos[d + e*x - ArcTan[c/b]])/(b*Sqrt[1 + c^2/b^2]*(1
 - a/(b*Sqrt[1 + c^2/b^2])))), -((a + b*Sqrt[1 + c^2/b^2]*Cos[d + e*x - Arc
Tan[c/b]])/(b*Sqrt[1 + c^2/b^2]*(-1 - a/(b*Sqrt[1 + c^2/b^2]))))]*Sin[d + e
*x - ArcTan[c/b]])/(b*Sqrt[1 + c^2/b^2]*Sqrt[(b*Sqrt[(b^2 + c^2)/b^2] - b*S
qrt[(b^2 + c^2)/b^2]*Cos[d + e*x - ArcTan[c/b]])/(a + b*Sqrt[(b^2 + c^2)/b^
2])]*Sqrt[a + b*Sqrt[(b^2 + c^2)/b^2]*Cos[d + e*x - ArcTan[c/b]]]*Sqrt[(b*S
qrt[(b^2 + c^2)/b^2] + b*Sqrt[(b^2 + c^2)/b^2]*Cos[d + e*x - ArcTan[c/b]])/
(-a + b*Sqrt[(b^2 + c^2)/b^2])])) - ((2*b*(a + b*Sqrt[1 + c^2/b^2]*Cos[d +
e*x - ArcTan[c/b]]))/(b^2 + c^2) - (c*Sin[d + e*x - ArcTan[c/b]])/(b*Sqrt[1
 + c^2/b^2]))/Sqrt[a + b*Sqrt[1 + c^2/b^2]*Cos[d + e*x - ArcTan[c/b]]]))/(5
*(-a^2 + b^2 + c^2)^3*e)
```

fricas [F] time = 1.12, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{}{\left(b^4 - 6\,b^2c^2 + c^4\right)\cos\left(ex + d\right)^4 + a^4 + 6\,a^2c^2 + c^4 + 4\left(ab^3 - 3\,abc^2\right)\cos\left(ex + d\right)^3 + 2\left(3\,a^2b^2 - c^4 - 3\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cos(e*x+d)+c*sin(e*x+d))^(7/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*cos(e*x + d) + c*sin(e*x + d) + a)/((b^4 - 6*b^2*c^2 + c^4)
*cos(e*x + d)^4 + a^4 + 6*a^2*c^2 + c^4 + 4*(a*b^3 - 3*a*b*c^2)*cos(e*x + d
)^3 + 2*(3*a^2*b^2 - c^4 - 3*(a^2 - b^2)*c^2)*cos(e*x + d)^2 + 4*(a^3*b + 3
*a*b*c^2)*cos(e*x + d) + 4*(a^3*c + a*c^3 + (b^3*c - b*c^3)*cos(e*x + d)^3
+ (3*a*b^2*c - a*c^3)*cos(e*x + d)^2 + (3*a^2*b*c + b*c^3)*cos(e*x + d))*si
n(e*x + d)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(b\cos(ex+d)+c\sin(ex+d)+a)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cos(e*x+d)+c*sin(e*x+d))^(7/2),x, algorithm="giac")
```

```
[Out] integrate((b*cos(e*x + d) + c*sin(e*x + d) + a)^(-7/2), x)
```

maple [B] time = 6.69, size = 3876, normalized size = 7.91

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*cos(e*x+d)+c*sin(e*x+d))^(7/2),x)
```

```
[Out] (-(-b^2*sin(e*x+d-arctan(-b,c))-c^2*sin(e*x+d-arctan(-b,c))-a*(b^2+c^2)^(1/
2))*cos(e*x+d-arctan(-b,c))^2/(b^2+c^2)^(1/2))^(1/2)/(b^2+c^2)^(1/2)*(-1/8/
a/(a^2-b^2-c^2)*(b^2+c^2)^(3/2)*(cos(e*x+d-arctan(-b,c))^2*(b^2+c^2)*((b^2+
c^2)^(1/2)*sin(e*x+d-arctan(-b,c))+a))^(1/2)/(b^2*sin(e*x+d-arctan(-b,c))+c
^2*sin(e*x+d-arctan(-b,c))-a*(b^2+c^2)^(1/2))^2+1/5/(a^2-b^2-c^2)/(b^2+c^2)
*(cos(e*x+d-arctan(-b,c))^2*(b^2+c^2)*((b^2+c^2)^(1/2)*sin(e*x+d-arctan(-b,
c))+a))^(1/2)/(sin(e*x+d-arctan(-b,c))+1/(b^2+c^2)^(1/2)*a)^3+3/32*(5*a^2*b
^2+5*a^2*c^2-b^4-2*b^2*c^2-c^4)/(a^2-b^2-c^2)^2/a^2*(cos(e*x+d-arctan(-b,c)
)^2*(b^2+c^2)*((b^2+c^2)^(1/2)*sin(e*x+d-arctan(-b,c))+a))^(1/2)/(b^2*sin(e
*x+d-arctan(-b,c))+c^2*sin(e*x+d-arctan(-b,c))-a*(b^2+c^2)^(1/2))+8/15/(a^2
-b^2-c^2)^2*a/(b^2+c^2)^(1/2)*(cos(e*x+d-arctan(-b,c))^2*(b^2+c^2)*((b^2+c^
2)^(1/2)*sin(e*x+d-arctan(-b,c))+a))^(1/2)/(sin(e*x+d-arctan(-b,c))+1/(b^2+
c^2)^(1/2)*a)^2-1/15*(b^2+c^2)^(1/2)*(-b^2-c^2)*cos(e*x+d-arctan(-b,c))^2/(
a^2-b^2-c^2)^3*(23*a^2+9*b^2+9*c^2)/(cos(e*x+d-arctan(-b,c))^2*(b^2+c^2)*((
b^2+c^2)^(1/2)*sin(e*x+d-arctan(-b,c))+a))^(1/2)+2*(-1/64*(11*a^2*b^2+11*a^
2*c^2+b^4+2*b^2*c^2+c^4)/a/(a^2-b^2-c^2)^2-4/15*a*(b^2+c^2)/(a^2-b^2-c^2)^2
+1/30*a*(b^2+c^2)*(23*a^2+9*b^2+9*c^2)/(a^2-b^2-c^2)^3)*(1/(b^2+c^2)^(1/2)*
a-1)*((-(b^2+c^2)^(1/2)*sin(e*x+d-arctan(-b,c))-a)/(-a+(b^2+c^2)^(1/2)))^(1
/2)*((-sin(e*x+d-arctan(-b,c))+1)*(b^2+c^2)^(1/2)/(a+(b^2+c^2)^(1/2)))^(1/2
)*((1+sin(e*x+d-arctan(-b,c)))*(b^2+c^2)^(1/2)/(-a+(b^2+c^2)^(1/2)))^(1/2)/
```

```
(cos(e*x+d-arctan(-b,c))^2*(b^2+c^2)*((b^2+c^2)^(1/2)*sin(e*x+d-arctan(-b,c
))+a))^(1/2)*EllipticF(((-(b^2+c^2)^(1/2)*sin(e*x+d-arctan(-b,c))-a)/(-a+(b
^2+c^2)^(1/2)))^(1/2),((a-(b^2+c^2)^(1/2))/(a+(b^2+c^2)^(1/2)))^(1/2))+2*(3
/64*(5*a^2*b^2+5*a^2*c^2-b^4-2*b^2*c^2-c^4)*(b^2+c^2)^(1/2)/(a^2-b^2-c^2)^2
/a^2-1/30*(b^2+c^2)^(3/2)*(23*a^2+9*b^2+9*c^2)/(a^2-b^2-c^2)^3+1/30*(b^2+c^
2)^(1/2)*(2*b^2+2*c^2)/(a^2-b^2-c^2)^3*(23*a^2+9*b^2+9*c^2))*(1/(b^2+c^2)^(
1/2)*a-1)*((-(b^2+c^2)^(1/2)*sin(e*x+d-arctan(-b,c))-a)/(-a+(b^2+c^2)^(1/2)
))^(1/2)*((-sin(e*x+d-arctan(-b,c))+1)*(b^2+c^2)^(1/2)/(a+(b^2+c^2)^(1/2)))
^(1/2)*((1+sin(e*x+d-arctan(-b,c)))*(b^2+c^2)^(1/2)/(-a+(b^2+c^2)^(1/2)))^(
1/2)/(cos(e*x+d-arctan(-b,c))^2*(b^2+c^2)*((b^2+c^2)^(1/2)*sin(e*x+d-arctan
(-b,c))+a))^(1/2)*((-1/(b^2+c^2)^(1/2)*a-1)*EllipticE(((-(b^2+c^2)^(1/2)*si
n(e*x+d-arctan(-b,c))-a)/(-a+(b^2+c^2)^(1/2)))^(1/2),((a-(b^2+c^2)^(1/2))/(
a+(b^2+c^2)^(1/2)))^(1/2))+EllipticF(((-(b^2+c^2)^(1/2)*sin(e*x+d-arctan(-b
,c))-a)/(-a+(b^2+c^2)^(1/2)))^(1/2),((a-(b^2+c^2)^(1/2))/(a+(b^2+c^2)^(1/2)
))^(1/2)))+1/64*(43*a^4*b^2+43*a^4*c^2+2*a^2*b^4+4*a^2*b^2*c^2+2*a^2*c^4+3*
b^6+9*b^4*c^2+9*b^2*c^4+3*c^6)/(a^2-b^2-c^2)^2/a^3*(1/(b^2+c^2)^(1/2)*a-1)*
((-(b^2+c^2)^(1/2)*sin(e*x+d-arctan(-b,c))-a)/(-a+(b^2+c^2)^(1/2)))^(1/2)*(
(-sin(e*x+d-arctan(-b,c))+1)*(b^2+c^2)^(1/2)/(a+(b^2+c^2)^(1/2)))^(1/2)*((1
+sin(e*x+d-arctan(-b,c)))*(b^2+c^2)^(1/2)/(-a+(b^2+c^2)^(1/2)))^(1/2)/(cos(
e*x+d-arctan(-b,c))^2*(b^2+c^2)*((b^2+c^2)^(1/2)*sin(e*x+d-arctan(-b,c))+a)
)^(1/2)*EllipticPi(((-(b^2+c^2)^(1/2)*sin(e*x+d-arctan(-b,c))-a)/(-a+(b^2+c
^2)^(1/2)))^(1/2),-1/2*(-1/(b^2+c^2)^(1/2)*a+1)*(b^2+c^2)^(1/2)/a,((a-(b^2+
c^2)^(1/2))/(a+(b^2+c^2)^(1/2)))^(1/2))+1/8/a*(b^4+2*b^2*c^2+c^4)/(a^2-b^2-
c^2)*(cos(e*x+d-arctan(-b,c))^2*((b^2+c^2)^(1/2)*sin(e*x+d-arctan(-b,c))+a)
)^(1/2)/(b^2*sin(e*x+d-arctan(-b,c))+c^2*sin(e*x+d-arctan(-b,c))-a*(b^2+c^2
)^(1/2))^2+1/5/(a^2-b^2-c^2)/(b^2+c^2)^(1/2)*(cos(e*x+d-arctan(-b,c))^2*((b
^2+c^2)^(1/2)*sin(e*x+d-arctan(-b,c))+a))^(1/2)/(sin(e*x+d-arctan(-b,c))+1/
(b^2+c^2)^(1/2)*a)^3-3/32*(5*a^2*b^2+5*a^2*c^2-b^4-2*b^2*c^2-c^4)*(b^2+c^2)
^(1/2)/(a^2-b^2-c^2)^2/a^2*(cos(e*x+d-arctan(-b,c))^2*((b^2+c^2)^(1/2)*sin(
e*x+d-arctan(-b,c))+a))^(1/2)/(b^2*sin(e*x+d-arctan(-b,c))+c^2*sin(e*x+d-ar
ctan(-b,c))-a*(b^2+c^2)^(1/2))+8/15/(a^2-b^2-c^2)^2*a*(cos(e*x+d-arctan(-b,
c))^2*((b^2+c^2)^(1/2)*sin(e*x+d-arctan(-b,c))+a))^(1/2)/(sin(e*x+d-arctan(
-b,c))+1/(b^2+c^2)^(1/2)*a)^2+1/15*(b^2+c^2)*cos(e*x+d-arctan(-b,c))^2/(a^2
-b^2-c^2)^3*(23*a^2+9*b^2+9*c^2)/(cos(e*x+d-arctan(-b,c))^2*((b^2+c^2)^(1/2
)*sin(e*x+d-arctan(-b,c))+a))^(1/2)+2*(1/64*(11*a^2+b^2+c^2)*(b^2+c^2)^(1/2
)/a/(a^2-b^2-c^2)^2-4/15*a*(b^2+c^2)^(1/2)/(a^2-b^2-c^2)^2+1/30*a*(b^2+c^2)
^(1/2)*(23*a^2+9*b^2+9*c^2)/(a^2-b^2-c^2)^3)*(1/(b^2+c^2)^(1/2)*a-1)*((-(b^
2+c^2)^(1/2)*sin(e*x+d-arctan(-b,c))-a)/(-a+(b^2+c^2)^(1/2)))^(1/2)*((-sin(
e*x+d-arctan(-b,c))+1)*(b^2+c^2)^(1/2)/(a+(b^2+c^2)^(1/2)))^(1/2)*((1+sin(e
*x+d-arctan(-b,c)))*(b^2+c^2)^(1/2)/(-a+(b^2+c^2)^(1/2)))^(1/2)/(cos(e*x+d-
arctan(-b,c))^2*((b^2+c^2)^(1/2)*sin(e*x+d-arctan(-b,c))+a))^(1/2)*Elliptic
F(((-(b^2+c^2)^(1/2)*sin(e*x+d-arctan(-b,c))-a)/(-a+(b^2+c^2)^(1/2)))^(1/2)
,((a-(b^2+c^2)^(1/2))/(a+(b^2+c^2)^(1/2)))^(1/2))+2*(-3/64*(5*a^2*b^2+5*a^2
*c^2-b^4-2*b^2*c^2-c^4)/(a^2-b^2-c^2)^2/a^2+1/30*(b^2+c^2)*(23*a^2+9*b^2+9*
c^2)/(a^2-b^2-c^2)^3)*(1/(b^2+c^2)^(1/2)*a-1)*((-(b^2+c^2)^(1/2)*sin(e*x+d-
```

```
arctan(-b,c))-a)/(-a+(b^2+c^2)^(1/2)))^(1/2)*((-sin(e*x+d-arctan(-b,c))+1)*
(b^2+c^2)^(1/2)/(a+(b^2+c^2)^(1/2)))^(1/2)*((1+sin(e*x+d-arctan(-b,c)))*(b^
2+c^2)^(1/2)/(-a+(b^2+c^2)^(1/2)))^(1/2)/(cos(e*x+d-arctan(-b,c))^2*((b^2+c
^2)^(1/2)*sin(e*x+d-arctan(-b,c))+a))^(1/2)*((-1/(b^2+c^2)^(1/2)*a-1)*Ellip
ticE(((-(b^2+c^2)^(1/2)*sin(e*x+d-arctan(-b,c))-a)/(-a+(b^2+c^2)^(1/2)))^(1
/2),((a-(b^2+c^2)^(1/2))/(a+(b^2+c^2)^(1/2)))^(1/2))+EllipticF(((-(b^2+c^2)
^(1/2)*sin(e*x+d-arctan(-b,c))-a)/(-a+(b^2+c^2)^(1/2)))^(1/2),((a-(b^2+c^2)
^(1/2))/(a+(b^2+c^2)^(1/2)))^(1/2)))-1/64*(43*a^4*b^2+43*a^4*c^2+2*a^2*b^4+
4*a^2*b^2*c^2+2*a^2*c^4+3*b^6+9*b^4*c^2+9*b^2*c^4+3*c^6)/(a^2-b^2-c^2)^2/a^
3/(b^2+c^2)^(1/2)*(1/(b^2+c^2)^(1/2)*a-1)*((-(b^2+c^2)^(1/2)*sin(e*x+d-arct
an(-b,c))-a)/(-a+(b^2+c^2)^(1/2)))^(1/2)*((-sin(e*x+d-arctan(-b,c))+1)*(b^2
+c^2)^(1/2)/(a+(b^2+c^2)^(1/2)))^(1/2)*((1+sin(e*x+d-arctan(-b,c)))*(b^2+c^
2)^(1/2)/(-a+(b^2+c^2)^(1/2)))^(1/2)/(cos(e*x+d-arctan(-b,c))^2*((b^2+c^2)^
(1/2)*sin(e*x+d-arctan(-b,c))+a))^(1/2)*EllipticPi(((-(b^2+c^2)^(1/2)*sin(e
*x+d-arctan(-b,c))-a)/(-a+(b^2+c^2)^(1/2)))^(1/2),-1/2*(-1/(b^2+c^2)^(1/2)*
a+1)*(b^2+c^2)^(1/2)/a,((a-(b^2+c^2)^(1/2))/(a+(b^2+c^2)^(1/2)))^(1/2)))/co
s(e*x+d-arctan(-b,c))/((b^2*sin(e*x+d-arctan(-b,c))+c^2*sin(e*x+d-arctan(-b
,c))+a*(b^2+c^2)^(1/2))/(b^2+c^2)^(1/2))^(1/2)/e
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(b\cos(ex+d)+c\sin(ex+d)+a)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cos(e*x+d)+c*sin(e*x+d))^(7/2),x, algorithm="maxima")
[Out] integrate((b*cos(e*x + d) + c*sin(e*x + d) + a)^(-7/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{(a+b\,\cos(d+e\,x)+c\,\sin(d+e\,x))^{7/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*cos(d + e*x) + c*sin(d + e*x))^(7/2),x)
[Out] int(1/(a + b*cos(d + e*x) + c*sin(d + e*x))^(7/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cos(e*x+d)+c*sin(e*x+d))**(7/2),x)
```

```
[Out] Timed out
```

3.417 $\int (5+4\cos(d+ex)+3\sin(d+ex))^{5/2}\,dx$

Optimal. Leaf size=139

$$-\frac{2(3\cos(d+ex)-4\sin(d+ex))(3\sin(d+ex)+4\cos(d+ex)+5)^{3/2}}{5e}-\frac{16(3\cos(d+ex)-4\sin(d+ex))\sqrt{3\sin(d}}{3e}$$

```
[Out] -2/5*(3*cos(e*x+d)-4*sin(e*x+d))*(5+4*cos(e*x+d)+3*sin(e*x+d))^(3/2)/e-320/
3*(3*cos(e*x+d)-4*sin(e*x+d))/e/(5+4*cos(e*x+d)+3*sin(e*x+d))^(1/2)-16/3*(3
*cos(e*x+d)-4*sin(e*x+d))*(5+4*cos(e*x+d)+3*sin(e*x+d))^(1/2)/e
```

Rubi [A] time = 0.06, antiderivative size = 139, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {3113, 3112}

$$-\frac{2(3\cos(d+ex)-4\sin(d+ex))(3\sin(d+ex)+4\cos(d+ex)+5)^{3/2}}{5e}-\frac{16(3\cos(d+ex)-4\sin(d+ex))\sqrt{3\sin(d}}{3e}$$

Antiderivative was successfully verified.

```
[In] Int[(5 + 4*Cos[d + e*x] + 3*Sin[d + e*x])^(5/2),x]
```

```
[Out] (-320*(3*Cos[d + e*x] - 4*Sin[d + e*x]))/(3*e*Sqrt[5 + 4*Cos[d + e*x] + 3*S
in[d + e*x]]) - (16*(3*Cos[d + e*x] - 4*Sin[d + e*x])*Sqrt[5 + 4*Cos[d + e*
x] + 3*Sin[d + e*x]])/(3*e) - (2*(3*Cos[d + e*x] - 4*Sin[d + e*x])*(5 + 4*C
os[d + e*x] + 3*Sin[d + e*x])^(3/2))/(5*e)
```

Rule 3112

```
Int[Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_
)]], x_Symbol] :> Simp[(-2*(c*Cos[d + e*x] - b*Sin[d + e*x]))/(e*Sqrt[a + b
*Cos[d + e*x] + c*Sin[d + e*x]]), x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[a^
2 - b^2 - c^2, 0]
```

Rule 3113

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(n_), x_Symbol] :> -Simp[((c*Cos[d + e*x] - b*Sin[d + e*x])*(a + b*Cos[d +
e*x] + c*Sin[d + e*x])^(n - 1))/(e*n), x] + Dist[(a*(2*n - 1))/n, Int[(a +
b*Cos[d + e*x] + c*Sin[d + e*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e},
x] && EqQ[a^2 - b^2 - c^2, 0] && GtQ[n, 0]
```

Rubi steps

$$\int (5+4\cos(d+ex)+3\sin(d+ex))^{5/2}\,dx = -\frac{2(3\cos(d+ex)-4\sin(d+ex))(5+4\cos(d+ex)+3\sin(d+ex)}{5e}$$
$$= -\frac{16(3\cos(d+ex)-4\sin(d+ex))\sqrt{5+4\cos(d+ex)+3\sin(d+}}{3e}$$
$$= -\frac{320(3\cos(d+ex)-4\sin(d+ex))}{3e\sqrt{5+4\cos(d+ex)+3\sin(d+ex)}} - \frac{16(3\cos(d+ex)-4\sin}{}$$

Mathematica [A] time = 0.60, size = 130, normalized size = 0.94

$$-\frac{(3\sin(d+ex)+4\cos(d+ex)+5)^{5/2}\left(3750\cos\left(\frac{1}{2}(d+ex)\right)+1625\cos\left(\frac{3}{2}(d+ex)\right)+3\left(-3750\sin\left(\frac{1}{2}(d+ex)\right)\right.\right.}{30e\left(\sin\left(\frac{1}{2}(d+ex)\right)+3\cos\left(\frac{1}{2}(d+ex)\right)\right)^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(5 + 4*Cos[d + e*x] + 3*Sin[d + e*x])^(5/2),x]
```

```
[Out] -1/30*((5 + 4*Cos[d + e*x] + 3*Sin[d + e*x])^(5/2)*(3750*Cos[(d + e*x)/2] +
 1625*Cos[(3*(d + e*x))/2] + 3*(79*Cos[(5*(d + e*x))/2] - 3750*Sin[(d + e*x
)/2] - 375*Sin[(3*(d + e*x))/2] + 3*Sin[(5*(d + e*x))/2])))/(e*(3*Cos[(d +
e*x)/2] + Sin[(d + e*x)/2])^5)
```

fricas [A] time = 1.09, size = 101, normalized size = 0.73

$$-\frac{2\left(237\cos(ex+d)^3+931\cos(ex+d)^2+9\left(\cos(ex+d)^2-62\cos(ex+d)-344\right)\sin(ex+d)+1166\cos(e\right.}{15\,(3\,e\cos(ex+d)+e\sin(ex+d)+3\,e)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5+4*cos(e*x+d)+3*sin(e*x+d))^(5/2),x, algorithm="fricas")
```

```
[Out] -2/15*(237*cos(e*x + d)^3 + 931*cos(e*x + d)^2 + 9*(cos(e*x + d)^2 - 62*cos
(e*x + d) - 344)*sin(e*x + d) + 1166*cos(e*x + d) + 472)*sqrt(4*cos(e*x + d
) + 3*sin(e*x + d) + 5)/(3*e*cos(e*x + d) + e*sin(e*x + d) + 3*e)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (4\cos(ex+d)+3\sin(ex+d)+5)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5+4*cos(e*x+d)+3*sin(e*x+d))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((4*cos(e*x + d) + 3*sin(e*x + d) + 5)^(5/2), x)
```

maple [A] time = 0.37, size = 74, normalized size = 0.53

$$\frac{50\left(1+\sin\left(ex+d+\arctan\left(\frac{4}{3}\right)\right)\right)\left(\sin\left(ex+d+\arctan\left(\frac{4}{3}\right)\right)-1\right)\left(3\left(\sin^{2}\left(ex+d+\arctan\left(\frac{4}{3}\right)\right)\right)+14\sin\left(ex+\right.\right.}{3\cos\left(ex+d+\arctan\left(\frac{4}{3}\right)\right)\sqrt{5+5\sin\left(ex+d+\arctan\left(\frac{4}{3}\right)\right)}\, e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5+4*cos(e*x+d)+3*sin(e*x+d))^(5/2),x)
```

```
[Out] 50/3*(1+sin(e*x+d+arctan(4/3)))*(sin(e*x+d+arctan(4/3))-1)*(3*sin(e*x+d+arc
tan(4/3))^2+14*sin(e*x+d+arctan(4/3))+43)/cos(e*x+d+arctan(4/3))/(5+5*sin(e
*x+d+arctan(4/3)))^(1/2)/e
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (4\cos(ex+d)+3\sin(ex+d)+5)^{\frac{5}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5+4*cos(e*x+d)+3*sin(e*x+d))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((4*cos(e*x + d) + 3*sin(e*x + d) + 5)^(5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int (4\cos(d+e\,x)+3\sin(d+e\,x)+5)^{5/2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((4*cos(d + e*x) + 3*sin(d + e*x) + 5)^(5/2),x)
```

```
[Out] int((4*cos(d + e*x) + 3*sin(d + e*x) + 5)^(5/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5+4*cos(e*x+d)+3*sin(e*x+d))**(5/2),x)
```

```
[Out] Timed out
```

3.418 $\int (5+4\cos(d+ex)+3\sin(d+ex))^{3/2}\,dx$

Optimal. Leaf size=93

$$-\frac{2\sqrt{3\sin(d+ex)+4\cos(d+ex)+5}\,(3\cos(d+ex)-4\sin(d+ex))}{3e}-\frac{40(3\cos(d+ex)-4\sin(d+ex))}{3e\sqrt{3\sin(d+ex)+4\cos(d+ex)+5}}$$

```
[Out] -40/3*(3*cos(e*x+d)-4*sin(e*x+d))/e/(5+4*cos(e*x+d)+3*sin(e*x+d))^(1/2)-2/3
*(3*cos(e*x+d)-4*sin(e*x+d))*(5+4*cos(e*x+d)+3*sin(e*x+d))^(1/2)/e
```

Rubi [A] time = 0.04, antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {3113, 3112}

$$-\frac{2\sqrt{3\sin(d+ex)+4\cos(d+ex)+5}\,(3\cos(d+ex)-4\sin(d+ex))}{3e}-\frac{40(3\cos(d+ex)-4\sin(d+ex))}{3e\sqrt{3\sin(d+ex)+4\cos(d+ex)+5}}$$

Antiderivative was successfully verified.

```
[In] Int[(5 + 4*Cos[d + e*x] + 3*Sin[d + e*x])^(3/2),x]
```

```
[Out] (-40*(3*Cos[d + e*x] - 4*Sin[d + e*x]))/(3*e*Sqrt[5 + 4*Cos[d + e*x] + 3*Si
n[d + e*x]]) - (2*(3*Cos[d + e*x] - 4*Sin[d + e*x])*Sqrt[5 + 4*Cos[d + e*x]
 + 3*Sin[d + e*x]])/(3*e)
```

Rule 3112

```
Int[Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_
)]], x_Symbol] :> Simp[(-2*(c*Cos[d + e*x] - b*Sin[d + e*x]))/(e*Sqrt[a + b
*Cos[d + e*x] + c*Sin[d + e*x]]), x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[a^
2 - b^2 - c^2, 0]
```

Rule 3113

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(n_), x_Symbol] :> -Simp[((c*Cos[d + e*x] - b*Sin[d + e*x])*(a + b*Cos[d +
e*x] + c*Sin[d + e*x])^(n - 1))/(e*n), x] + Dist[(a*(2*n - 1))/n, Int[(a +
b*Cos[d + e*x] + c*Sin[d + e*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e},
x] && EqQ[a^2 - b^2 - c^2, 0] && GtQ[n, 0]
```

Rubi steps

$$\int (5+4\cos(d+ex)+3\sin(d+ex))^{3/2}\,dx = -\frac{2(3\cos(d+ex)-4\sin(d+ex))\sqrt{5+4\cos(d+ex)+3\sin(d+ex)}}{3e}$$
$$= -\frac{40(3\cos(d+ex)-4\sin(d+ex))}{3e\sqrt{5+4\cos(d+ex)+3\sin(d+ex)}} - \frac{2(3\cos(d+ex)-4\sin(d$$

Mathematica [A] time = 0.33, size = 104, normalized size = 1.12

$$\frac{(3\sin(d+ex)+4\cos(d+ex)+5)^{3/2}\left(9\left(15\sin\left(\frac{1}{2}(d+ex)\right)+\sin\left(\frac{3}{2}(d+ex)\right)\right)-45\cos\left(\frac{1}{2}(d+ex)\right)-13\cos\left(\frac{3}{2}(d\right.\right.}{3e\left(\sin\left(\frac{1}{2}(d+ex)\right)+3\cos\left(\frac{1}{2}(d+ex)\right)\right)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(5 + 4*Cos[d + e*x] + 3*Sin[d + e*x])^(3/2),x]
```

```
[Out] ((5 + 4*Cos[d + e*x] + 3*Sin[d + e*x])^(3/2)*(-45*Cos[(d + e*x)/2] - 13*Cos
[(3*(d + e*x))/2] + 9*(15*Sin[(d + e*x)/2] + Sin[(3*(d + e*x))/2])))/(3*e*(
3*Cos[(d + e*x)/2] + Sin[(d + e*x)/2])^3)
```

fricas [A] time = 0.92, size = 81, normalized size = 0.87

$$-\frac{2\left(13\cos\left(ex+d\right)^2-9\left(\cos\left(ex+d\right)+8\right)\sin\left(ex+d\right)+29\cos\left(ex+d\right)+16\right)\sqrt{4\cos\left(ex+d\right)+3\sin\left(ex+d\right)+}}{3\left(3e\cos\left(ex+d\right)+e\sin\left(ex+d\right)+3e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5+4*cos(e*x+d)+3*sin(e*x+d))^(3/2),x, algorithm="fricas")
```

```
[Out] -2/3*(13*cos(e*x + d)^2 - 9*(cos(e*x + d) + 8)*sin(e*x + d) + 29*cos(e*x +
d) + 16)*sqrt(4*cos(e*x + d) + 3*sin(e*x + d) + 5)/(3*e*cos(e*x + d) + e*si
n(e*x + d) + 3*e)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(4\cos\left(ex+d\right)+3\sin\left(ex+d\right)+5\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5+4*cos(e*x+d)+3*sin(e*x+d))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((4*cos(e*x + d) + 3*sin(e*x + d) + 5)^(3/2), x)
```

maple [A] time = 0.35, size = 60, normalized size = 0.65

$$\frac{50\left(1+\sin\left(ex+d+\arctan\left(\frac{4}{3}\right)\right)\right)\left(\sin\left(ex+d+\arctan\left(\frac{4}{3}\right)\right)-1\right)\left(\sin\left(ex+d+\arctan\left(\frac{4}{3}\right)\right)+5\right)}{3\cos\left(ex+d+\arctan\left(\frac{4}{3}\right)\right)\sqrt{5+5\sin\left(ex+d+\arctan\left(\frac{4}{3}\right)\right)}\,e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5+4*cos(e*x+d)+3*sin(e*x+d))^(3/2),x)
```

```
[Out] 50/3*(1+sin(e*x+d+arctan(4/3)))*(sin(e*x+d+arctan(4/3))-1)*(sin(e*x+d+arcta
n(4/3))+5)/cos(e*x+d+arctan(4/3))/(5+5*sin(e*x+d+arctan(4/3)))^(1/2)/e
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (4\cos(ex+d)+3\sin(ex+d)+5)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5+4*cos(e*x+d)+3*sin(e*x+d))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((4*cos(e*x + d) + 3*sin(e*x + d) + 5)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int (4\cos(d+e\,x)+3\sin(d+e\,x)+5)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((4*cos(d + e*x) + 3*sin(d + e*x) + 5)^(3/2),x)
```

```
[Out] int((4*cos(d + e*x) + 3*sin(d + e*x) + 5)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (3\sin(d+ex)+4\cos(d+ex)+5)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5+4*cos(e*x+d)+3*sin(e*x+d))**(3/2),x)
```

```
[Out] Integral((3*sin(d + e*x) + 4*cos(d + e*x) + 5)**(3/2), x)
```

3.419 $\int \sqrt{5+4\cos(d+ex)+3\sin(d+ex)}\,dx$

Optimal. Leaf size=44

$$-\frac{2(3\cos(d+ex)-4\sin(d+ex))}{e\sqrt{3\sin(d+ex)+4\cos(d+ex)+5}}$$

```
[Out] -2*(3*cos(e*x+d)-4*sin(e*x+d))/e/(5+4*cos(e*x+d)+3*sin(e*x+d))^(1/2)
```

Rubi [A] time = 0.02, antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {3112}

$$-\frac{2(3\cos(d+ex)-4\sin(d+ex))}{e\sqrt{3\sin(d+ex)+4\cos(d+ex)+5}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[5 + 4*Cos[d + e*x] + 3*Sin[d + e*x]],x]
```

```
[Out] (-2*(3*Cos[d + e*x] - 4*Sin[d + e*x]))/(e*Sqrt[5 + 4*Cos[d + e*x] + 3*Sin[d
 + e*x]])
```

Rule 3112

```
Int[Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_
)]], x_Symbol] :> Simp[(-2*(c*Cos[d + e*x] - b*Sin[d + e*x]))/(e*Sqrt[a + b
*Cos[d + e*x] + c*Sin[d + e*x]]), x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[a^
2 - b^2 - c^2, 0]
```

Rubi steps

$$\int \sqrt{5+4\cos(d+ex)+3\sin(d+ex)}\,dx=-\frac{2(3\cos(d+ex)-4\sin(d+ex))}{e\sqrt{5+4\cos(d+ex)+3\sin(d+ex)}}$$

Mathematica [A] time = 0.04, size = 75, normalized size = 1.70

$$-\frac{2\left(\cos\left(\frac{1}{2}(d+ex)\right)-3\sin\left(\frac{1}{2}(d+ex)\right)\right)\sqrt{3\sin(d+ex)+4\cos(d+ex)+5}}{e\left(\sin\left(\frac{1}{2}(d+ex)\right)+3\cos\left(\frac{1}{2}(d+ex)\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[5 + 4*Cos[d + e*x] + 3*Sin[d + e*x]],x]
```

```
[Out] (-2*(Cos[(d + e*x)/2] - 3*Sin[(d + e*x)/2])*Sqrt[5 + 4*Cos[d + e*x] + 3*Sin
[d + e*x]])/(e*(3*Cos[(d + e*x)/2] + Sin[(d + e*x)/2]))
```

fricas [A] time = 1.84, size = 61, normalized size = 1.39

$$-\frac{2\sqrt{4\cos(ex+d)+3\sin(ex+d)+5}\,(\cos(ex+d)-3\sin(ex+d)+1)}{3e\cos(ex+d)+e\sin(ex+d)+3e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5+4*cos(e*x+d)+3*sin(e*x+d))^(1/2),x, algorithm="fricas")
```

```
[Out] -2*sqrt(4*cos(e*x + d) + 3*sin(e*x + d) + 5)*(cos(e*x + d) - 3*sin(e*x + d)
 + 1)/(3*e*cos(e*x + d) + e*sin(e*x + d) + 3*e)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{4\cos(ex+d)+3\sin(ex+d)+5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5+4*cos(e*x+d)+3*sin(e*x+d))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(4*cos(e*x + d) + 3*sin(e*x + d) + 5), x)
```

maple [A] time = 0.30, size = 50, normalized size = 1.14

$$\frac{10\left(\sin\left(ex+d+\arctan\left(\frac{4}{3}\right)\right)-1\right)\left(1+\sin\left(ex+d+\arctan\left(\frac{4}{3}\right)\right)\right)}{\cos\left(ex+d+\arctan\left(\frac{4}{3}\right)\right)\sqrt{5+5\sin\left(ex+d+\arctan\left(\frac{4}{3}\right)\right)}\,e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5+4*cos(e*x+d)+3*sin(e*x+d))^(1/2),x)
```

```
[Out] 10*(sin(e*x+d+arctan(4/3))-1)*(1+sin(e*x+d+arctan(4/3)))/cos(e*x+d+arctan(4
/3))/(5+5*sin(e*x+d+arctan(4/3)))^(1/2)/e
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{4\cos(ex+d)+3\sin(ex+d)+5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5+4*cos(e*x+d)+3*sin(e*x+d))^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(4*cos(e*x + d) + 3*sin(e*x + d) + 5), x)
```

mupad [B] time = 0.31, size = 39, normalized size = 0.89

$$-\frac{2\sqrt{5}\ (3\ \cos(d+e\,x)-4\ \sin(d+e\,x))}{5\,e\,\sqrt{\cos\left(d-\operatorname{atan}\left(\frac{3}{4}\right)+e\,x\right)+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((4*cos(d + e*x) + 3*sin(d + e*x) + 5)^(1/2),x)
[Out] -(2*5^(1/2)*(3*cos(d + e*x) - 4*sin(d + e*x)))/(5*e*(cos(d - atan(3/4) + e*
x) + 1)^(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{3\sin(d+ex)+4\cos(d+ex)+5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5+4*cos(e*x+d)+3*sin(e*x+d))**(1/2),x)
[Out] Integral(sqrt(3*sin(d + e*x) + 4*cos(d + e*x) + 5), x)
```

3.420 $\int \frac{1}{\sqrt{5+4\cos(d+ex)+3\sin(d+ex)}}\,dx$

Optimal. Leaf size=48

$$\frac{\sqrt{\frac{2}{5}}\tanh^{-1}\left(\frac{\sin\left(d+ex-\tan^{-1}\left(\frac{3}{4}\right)\right)}{\sqrt{2}\sqrt{\cos\left(d+ex-\tan^{-1}\left(\frac{3}{4}\right)\right)+1}}\right)}{e}$$

```
[Out] 1/5*arctanh(1/2*sin(d+e*x-arctan(3/4))*2^(1/2)/(1+cos(d+e*x-arctan(3/4)))^(
1/2))*10^(1/2)/e
```

Rubi [A] time = 0.06, antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {3115, 2649, 206}

$$\frac{\sqrt{\frac{2}{5}}\tanh^{-1}\left(\frac{\sin\left(d+ex-\tan^{-1}\left(\frac{3}{4}\right)\right)}{\sqrt{2}\sqrt{\cos\left(d+ex-\tan^{-1}\left(\frac{3}{4}\right)\right)+1}}\right)}{e}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[5 + 4*Cos[d + e*x] + 3*Sin[d + e*x]],x]
```

```
[Out] (Sqrt[2/5]*ArcTanh[Sin[d + e*x - ArcTan[3/4]]/(Sqrt[2]*Sqrt[1 + Cos[d + e*x
 - ArcTan[3/4]]])])/e
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3115

```
Int[1/Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(
x_)]], x_Symbol] :> Int[1/Sqrt[a + Sqrt[b^2 + c^2]*Cos[d + e*x - ArcTan[b,
```

```
c]]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[a^2 - b^2 - c^2, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{5+4\cos(d+ex)+3\sin(d+ex)}}\,dx = \int \frac{1}{\sqrt{5+5\cos\left(d+ex-\tan^{-1}\left(\frac{3}{4}\right)\right)}}\,dx$$

$$= -\frac{2\,\mathrm{Subst}\left(\int \frac{1}{10-x^2}\,dx, x, -\frac{5\sin\left(d+ex-\tan^{-1}\left(\frac{3}{4}\right)\right)}{\sqrt{5+5\cos\left(d+ex-\tan^{-1}\left(\frac{3}{4}\right)\right)}}\right)}{e}$$

$$= \frac{\sqrt{\frac{2}{5}}\tanh^{-1}\left(\frac{\sin\left(d+ex-\tan^{-1}\left(\frac{3}{4}\right)\right)}{\sqrt{2}\sqrt{1+\cos\left(d+ex-\tan^{-1}\left(\frac{3}{4}\right)\right)}}\right)}{e}$$

Mathematica [C] time = 0.10, size = 101, normalized size = 2.10

$$-\frac{\left(\frac{2}{5}+\frac{6i}{5}\right)\sqrt{\frac{4}{5}+\frac{3i}{5}}\tan^{-1}\left(\left(\frac{1}{10}+\frac{3i}{10}\right)\sqrt{\frac{4}{5}+\frac{3i}{5}}\left(3\tan\left(\frac{1}{4}(d+ex)\right)-1\right)\right)\left(\sin\left(\frac{1}{2}(d+ex)\right)+3\cos\left(\frac{1}{2}(d+ex)\right)\right)}{e\sqrt{3\sin(d+ex)+4\cos(d+ex)+5}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[5 + 4*Cos[d + e*x] + 3*Sin[d + e*x]],x]
```

```
[Out] ((-2/5 - (6*I)/5)*Sqrt[4/5 + (3*I)/5]*ArcTan[(1/10 + (3*I)/10)*Sqrt[4/5 + (
3*I)/5]*(-1 + 3*Tan[(d + e*x)/4])]*(3*Cos[(d + e*x)/2] + Sin[(d + e*x)/2]))
/(e*Sqrt[5 + 4*Cos[d + e*x] + 3*Sin[d + e*x]])
```

fricas [B] time = 1.17, size = 147, normalized size = 3.06

$$\frac{\sqrt{5}\sqrt{2}\log\left(-\frac{9\cos(ex+d)^2+(13\cos(ex+d)-6)\sin(ex+d)+2\left(\sqrt{5}\sqrt{2}\cos(ex+d)-3\sqrt{5}\sqrt{2}\sin(ex+d)+\sqrt{5}\sqrt{2}\right)\sqrt{4\cos(ex+d)+3\sin(ex+d)+5}-33\,c}{9\cos(ex+d)^2+(13\cos(ex+d)+14)\sin(ex+d)+27\cos(ex+d)+18}\right.}{10e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(5+4*cos(e*x+d)+3*sin(e*x+d))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/10*sqrt(5)*sqrt(2)*log(-(9*cos(e*x + d)^2 + (13*cos(e*x + d) - 6)*sin(e*x
 + d) + 2*(sqrt(5)*sqrt(2)*cos(e*x + d) - 3*sqrt(5)*sqrt(2)*sin(e*x + d) +
```

```
sqrt(5)*sqrt(2))*sqrt(4*cos(e*x + d) + 3*sin(e*x + d) + 5) - 33*cos(e*x + d
) - 42)/(9*cos(e*x + d)^2 + (13*cos(e*x + d) + 14)*sin(e*x + d) + 27*cos(e*
x + d) + 18))/e
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{4\,\cos\left(ex+d\right)+3\,\sin\left(ex+d\right)+5}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(5+4*cos(e*x+d)+3*sin(e*x+d))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(4*cos(e*x + d) + 3*sin(e*x + d) + 5), x)
```

maple [A] time = 0.25, size = 77, normalized size = 1.60

$$-\frac{\left(1+\sin\left(ex+d+\arctan\left(\frac{4}{3}\right)\right)\right)\sqrt{-5\sin\left(ex+d+\arctan\left(\frac{4}{3}\right)\right)+5}\,\sqrt{10}\,\operatorname{arctanh}\left(\frac{\sqrt{-5\sin\left(ex+d+\arctan\left(\frac{4}{3}\right)\right)+5}\,\sqrt{1}}{10}\right.}{5\cos\left(ex+d+\arctan\left(\frac{4}{3}\right)\right)\sqrt{5+5\sin\left(ex+d+\arctan\left(\frac{4}{3}\right)\right)}\,e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(5+4*cos(e*x+d)+3*sin(e*x+d))^(1/2),x)
```

```
[Out] -1/5*(1+sin(e*x+d+arctan(4/3)))*(-5*sin(e*x+d+arctan(4/3))+5)^(1/2)*10^(1/2
)*arctanh(1/10*(-5*sin(e*x+d+arctan(4/3))+5)^(1/2)*10^(1/2))/cos(e*x+d+arct
an(4/3))/(5+5*sin(e*x+d+arctan(4/3)))^(1/2)/e
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{4\,\cos\left(ex+d\right)+3\,\sin\left(ex+d\right)+5}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(5+4*cos(e*x+d)+3*sin(e*x+d))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(4*cos(e*x + d) + 3*sin(e*x + d) + 5), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{\sqrt{4\,\cos\left(d+e\,x\right)+3\,\sin\left(d+e\,x\right)+5}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(4*cos(d + e*x) + 3*sin(d + e*x) + 5)^(1/2),x)
```

```
[Out] int(1/(4*cos(d + e*x) + 3*sin(d + e*x) + 5)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{3\sin(d+ex)+4\cos(d+ex)+5}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(5+4*cos(e*x+d)+3*sin(e*x+d))**(1/2),x)
```

```
[Out] Integral(1/sqrt(3*sin(d + e*x) + 4*cos(d + e*x) + 5), x)
```

3.421 $\int \frac{1}{(5+4\cos(d+ex)+3\sin(d+ex))^{3/2}}\,dx$

Optimal. Leaf size=96

$$\frac{\tanh^{-1}\left(\frac{\sin\left(d+ex-\tan^{-1}\left(\frac{3}{4}\right)\right)}{\sqrt{2}\sqrt{\cos\left(d+ex-\tan^{-1}\left(\frac{3}{4}\right)\right)+1}}\right)}{10\sqrt{10}\,e}-\frac{3\cos(d+ex)-4\sin(d+ex)}{10e(3\sin(d+ex)+4\cos(d+ex)+5)^{3/2}}$$

```
[Out] 1/10*(-3*cos(e*x+d)+4*sin(e*x+d))/e/(5+4*cos(e*x+d)+3*sin(e*x+d))^(3/2)+1/1
00*arctanh(1/2*sin(d+e*x-arctan(3/4))*2^(1/2)/(1+cos(d+e*x-arctan(3/4)))^(1
/2))*10^(1/2)/e
```

Rubi [A] time = 0.05, antiderivative size = 96, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {3116, 3115, 2649, 206}

$$\frac{\tanh^{-1}\left(\frac{\sin\left(d+ex-\tan^{-1}\left(\frac{3}{4}\right)\right)}{\sqrt{2}\sqrt{\cos\left(d+ex-\tan^{-1}\left(\frac{3}{4}\right)\right)+1}}\right)}{10\sqrt{10}\,e}-\frac{3\cos(d+ex)-4\sin(d+ex)}{10e(3\sin(d+ex)+4\cos(d+ex)+5)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(5 + 4*Cos[d + e*x] + 3*Sin[d + e*x])^(-3/2),x]
```

```
[Out] ArcTanh[Sin[d + e*x - ArcTan[3/4]]/(Sqrt[2]*Sqrt[1 + Cos[d + e*x - ArcTan[3
/4]]])]/(10*Sqrt[10]*e) - (3*Cos[d + e*x] - 4*Sin[d + e*x])/(10*e*(5 + 4*Co
s[d + e*x] + 3*Sin[d + e*x])^(3/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3115

```
Int[1/Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(
x_)]], x_Symbol] :> Int[1/Sqrt[a + Sqrt[b^2 + c^2]*Cos[d + e*x - ArcTan[b,
c]]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[a^2 - b^2 - c^2, 0]
```

Rule 3116

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(n_), x_Symbol] :> Simp[((c*Cos[d + e*x] - b*Sin[d + e*x])*(a + b*Cos[d + e
*x] + c*Sin[d + e*x])^n)/(a*e*(2*n + 1)), x] + Dist[(n + 1)/(a*(2*n + 1)),
Int[(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n + 1), x], x] /; FreeQ[{a, b, c
, d, e}, x] && EqQ[a^2 - b^2 - c^2, 0] && LtQ[n, -1]
```

Rubi steps

$$\int \frac{1}{(5+4\cos(d+ex)+3\sin(d+ex))^{3/2}}\,dx = -\frac{3\cos(d+ex)-4\sin(d+ex)}{10e(5+4\cos(d+ex)+3\sin(d+ex))^{3/2}} + \frac{1}{20}\int \frac{}{\sqrt{5+4\cos(d+}}$$

$$= -\frac{3\cos(d+ex)-4\sin(d+ex)}{10e(5+4\cos(d+ex)+3\sin(d+ex))^{3/2}} + \frac{1}{20}\int \frac{}{\sqrt{5+5\cos\left(d\right.}}$$

$$= -\frac{3\cos(d+ex)-4\sin(d+ex)}{10e(5+4\cos(d+ex)+3\sin(d+ex))^{3/2}} - \frac{\text{Subst}\left(\int \frac{1}{10-x^2}\,dx, x, -\right.}{1}$$

$$= \frac{\tanh^{-1}\left(\frac{\sin\left(d+ex-\tan^{-1}\left(\frac{3}{4}\right)\right)}{\sqrt{2}\sqrt{1+\cos\left(d+ex-\tan^{-1}\left(\frac{3}{4}\right)\right)}}\right)}{10\sqrt{10}\,e} - \frac{3\cos(d+ex)-4\sin(d+}{10e(5+4\cos(d+ex)+3\sin(}$$

Mathematica [C] time = 0.29, size = 154, normalized size = 1.60

$$-\frac{\left(\frac{1}{250}-\frac{i}{125}\right)\left(\sin\left(\frac{1}{2}(d+ex)\right)+3\cos\left(\frac{1}{2}(d+ex)\right)\right)\left((5+10i)\left(\cos\left(\frac{1}{2}(d+ex)\right)-3\sin\left(\frac{1}{2}(d+ex)\right)\right)-(1-i)\sqrt{20+}\right.}{e(3\sin(d+ex)+4\cos(d+}$$

Antiderivative was successfully verified.

```
[In] Integrate[(5 + 4*Cos[d + e*x] + 3*Sin[d + e*x])^(-3/2),x]
```

```
[Out] ((-1/250 + I/125)*(3*Cos[(d + e*x)/2] + Sin[(d + e*x)/2])*((5 + 10*I)*(Cos[
(d + e*x)/2] - 3*Sin[(d + e*x)/2]) - (1 - I)*Sqrt[20 + 15*I]*ArcTan[(1/10 +
```

```
 (3*I)/10)*Sqrt[4/5 + (3*I)/5]*(-1 + 3*Tan[(d + e*x)/4])]*(3*Cos[(d + e*x)/
2] + Sin[(d + e*x)/2])^2))/(e*(5 + 4*Cos[d + e*x] + 3*Sin[d + e*x])^(3/2))
```

fricas [B] time = 0.93, size = 268, normalized size = 2.79

$$\frac{\left(9\sqrt{10}\cos\left(ex+d\right)^{2}+\left(13\sqrt{10}\cos\left(ex+d\right)+14\sqrt{10}\right)\sin\left(ex+d\right)+27\sqrt{10}\cos\left(ex+d\right)+18\sqrt{10}\right)\log\left(-\frac{9\,\mathrm{co}}{}\right.}{200\left(9\,e\,\mathrm{co}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(5+4*cos(e*x+d)+3*sin(e*x+d))^(3/2),x, algorithm="fricas")
```

```
[Out] 1/200*((9*sqrt(10)*cos(e*x + d)^2 + (13*sqrt(10)*cos(e*x + d) + 14*sqrt(10)
)*sin(e*x + d) + 27*sqrt(10)*cos(e*x + d) + 18*sqrt(10))*log(-(9*cos(e*x +
d)^2 + (13*cos(e*x + d) - 6)*sin(e*x + d) + 2*(sqrt(10)*cos(e*x + d) - 3*sq
rt(10)*sin(e*x + d) + sqrt(10))*sqrt(4*cos(e*x + d) + 3*sin(e*x + d) + 5) -
 33*cos(e*x + d) - 42)/(9*cos(e*x + d)^2 + (13*cos(e*x + d) + 14)*sin(e*x +
 d) + 27*cos(e*x + d) + 18)) - 20*sqrt(4*cos(e*x + d) + 3*sin(e*x + d) + 5)
*(cos(e*x + d) - 3*sin(e*x + d) + 1))/(9*e*cos(e*x + d)^2 + 27*e*cos(e*x +
d) + (13*e*cos(e*x + d) + 14*e)*sin(e*x + d) + 18*e)
```

giac [B] time = 0.74, size = 284, normalized size = 2.96

$$\frac{1}{100}\left(\frac{\sqrt{10}\log\left(\frac{\left|-2\sqrt{10}+2\sqrt{\tan\left(\frac{1}{2}xe+\frac{1}{2}d\right)^{2}+1}-2\tan\left(\frac{1}{2}xe+\frac{1}{2}d\right)-6\right|}{\left|2\sqrt{10}+2\sqrt{\tan\left(\frac{1}{2}xe+\frac{1}{2}d\right)^{2}+1}-2\tan\left(\frac{1}{2}xe+\frac{1}{2}d\right)-6\right|}\right)}{\mathrm{sgn}\left(\tan\left(\frac{1}{2}xe+\frac{1}{2}d\right)+3\right)}-\frac{20\left(19\left(\sqrt{\tan\left(\frac{1}{2}xe+\frac{1}{2}d\right)^{2}+1}-\tan\left(\frac{1}{2}xe+\frac{1}{2}d\right.\right.\right.}{\left(\left(\sqrt{\tan\left(\frac{1}{2}xe+\frac{1}{2}d\right)^{2}+1}-\tan\left(\right.\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(5+4*cos(e*x+d)+3*sin(e*x+d))^(3/2),x, algorithm="giac")
```

```
[Out] 1/100*(sqrt(10)*log(abs(-2*sqrt(10) + 2*sqrt(tan(1/2*x*e + 1/2*d)^2 + 1) -
2*tan(1/2*x*e + 1/2*d) - 6)/abs(2*sqrt(10) + 2*sqrt(tan(1/2*x*e + 1/2*d)^2
+ 1) - 2*tan(1/2*x*e + 1/2*d) - 6))/sgn(tan(1/2*x*e + 1/2*d) + 3) - 20*(19*
(sqrt(tan(1/2*x*e + 1/2*d)^2 + 1) - tan(1/2*x*e + 1/2*d))^3 - 51*(sqrt(tan(
1/2*x*e + 1/2*d)^2 + 1) - tan(1/2*x*e + 1/2*d))^2 - 17*sqrt(tan(1/2*x*e + 1
/2*d)^2 + 1) + 17*tan(1/2*x*e + 1/2*d) - 3)/(((sqrt(tan(1/2*x*e + 1/2*d)^2
+ 1) - tan(1/2*x*e + 1/2*d))^2 - 6*sqrt(tan(1/2*x*e + 1/2*d)^2 + 1) + 6*tan
(1/2*x*e + 1/2*d) - 1)^2*sgn(tan(1/2*x*e + 1/2*d) + 3)))*e^(-1)
```

maple [A] time = 0.31, size = 117, normalized size = 1.22

$$-\frac{\left(\sqrt{10}\ \operatorname{arctanh}\left(\frac{\sqrt{-5\sin\left(ex+d+\arctan\left(\frac{4}{3}\right)\right)+5}\ \sqrt{10}}{10}\right)\sin\left(ex+d+\arctan\left(\frac{4}{3}\right)\right)+\sqrt{10}\ \operatorname{arctanh}\left(\frac{\sqrt{-5\sin\left(ex+d+\arctan\left(\frac{4}{3}\right)\right)+}}{10}\right.\right.}{100\cos\left(ex+d+\arctan\left(\frac{4}{3}\right)\right)\sqrt{5+5\sin\left(ex\right.}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(5+4*cos(e*x+d)+3*sin(e*x+d))^(3/2),x)
```

```
[Out] -1/100*(10^(1/2)*arctanh(1/10*(-5*sin(e*x+d+arctan(4/3))+5)^(1/2)*10^(1/2))
*sin(e*x+d+arctan(4/3))+10^(1/2)*arctanh(1/10*(-5*sin(e*x+d+arctan(4/3))+5)
^(1/2)*10^(1/2))+2*(-5*sin(e*x+d+arctan(4/3))+5)^(1/2))*(-5*sin(e*x+d+arcta
n(4/3))+5)^(1/2)/cos(e*x+d+arctan(4/3))/(5+5*sin(e*x+d+arctan(4/3)))^(1/2)/
e
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(4\cos\left(ex+d\right)+3\sin\left(ex+d\right)+5\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(5+4*cos(e*x+d)+3*sin(e*x+d))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((4*cos(e*x + d) + 3*sin(e*x + d) + 5)^(-3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\left(4\cos\left(d+e\,x\right)+3\sin\left(d+e\,x\right)+5\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(4*cos(d + e*x) + 3*sin(d + e*x) + 5)^(3/2),x)
```

```
[Out] int(1/(4*cos(d + e*x) + 3*sin(d + e*x) + 5)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(3\sin\left(d+ex\right)+4\cos\left(d+ex\right)+5\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(5+4*cos(e*x+d)+3*sin(e*x+d))**(3/2),x)
[Out] Integral((3*sin(d + e*x) + 4*cos(d + e*x) + 5)**(-3/2), x)
```

3.422 $\int \frac{1}{(5+4\cos(d+ex)+3\sin(d+ex))^{5/2}}\,dx$

Optimal. Leaf size=142

$$-\frac{3(3\cos(d+ex)-4\sin(d+ex))}{400e(3\sin(d+ex)+4\cos(d+ex)+5)^{3/2}}-\frac{3\cos(d+ex)-4\sin(d+ex)}{20e(3\sin(d+ex)+4\cos(d+ex)+5)^{5/2}}+\frac{3\tanh^{-1}\left(\frac{\sin\left(d+ex-\tan^{-1}\right.}{\sqrt{2}\sqrt{\cos\left(d+ex-\tan\right.}}\right.}{400\sqrt{10}\,e}$$

```
[Out] 1/20*(-3*cos(e*x+d)+4*sin(e*x+d))/e/(5+4*cos(e*x+d)+3*sin(e*x+d))^(5/2)-3/4
00*(3*cos(e*x+d)-4*sin(e*x+d))/e/(5+4*cos(e*x+d)+3*sin(e*x+d))^(3/2)+3/4000
*arctanh(1/2*sin(d+e*x-arctan(3/4))*2^(1/2)/(1+cos(d+e*x-arctan(3/4)))^(1/2
))*10^(1/2)/e
```

Rubi [A] time = 0.08, antiderivative size = 142, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {3116, 3115, 2649, 206}

$$-\frac{3(3\cos(d+ex)-4\sin(d+ex))}{400e(3\sin(d+ex)+4\cos(d+ex)+5)^{3/2}}-\frac{3\cos(d+ex)-4\sin(d+ex)}{20e(3\sin(d+ex)+4\cos(d+ex)+5)^{5/2}}+\frac{3\tanh^{-1}\left(\frac{\sin\left(d+ex-\tan^{-1}\right.}{\sqrt{2}\sqrt{\cos\left(d+ex-\tan\right.}}\right.}{400\sqrt{10}\,e}$$

Antiderivative was successfully verified.

```
[In] Int[(5 + 4*Cos[d + e*x] + 3*Sin[d + e*x])^(-5/2),x]
```

```
[Out] (3*ArcTanh[Sin[d + e*x - ArcTan[3/4]]/(Sqrt[2]*Sqrt[1 + Cos[d + e*x - ArcTa
n[3/4]]])])/(400*Sqrt[10]*e) - (3*Cos[d + e*x] - 4*Sin[d + e*x])/(20*e*(5 +
 4*Cos[d + e*x] + 3*Sin[d + e*x])^(5/2)) - (3*(3*Cos[d + e*x] - 4*Sin[d + e
*x]))/(400*e*(5 + 4*Cos[d + e*x] + 3*Sin[d + e*x])^(3/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3115

```
Int[1/Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(
x_)]], x_Symbol] :> Int[1/Sqrt[a + Sqrt[b^2 + c^2]*Cos[d + e*x - ArcTan[b,
c]]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[a^2 - b^2 - c^2, 0]
```

Rule 3116

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(n_), x_Symbol] :> Simp[((c*Cos[d + e*x] - b*Sin[d + e*x])*(a + b*Cos[d + e
*x] + c*Sin[d + e*x])^n)/(a*e*(2*n + 1)), x] + Dist[(n + 1)/(a*(2*n + 1)),
Int[(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n + 1), x], x] /; FreeQ[{a, b, c
, d, e}, x] && EqQ[a^2 - b^2 - c^2, 0] && LtQ[n, -1]
```

Rubi steps

$$\int \frac{1}{(5+4\cos(d+ex)+3\sin(d+ex))^{5/2}}\,dx = -\frac{3\cos(d+ex)-4\sin(d+ex)}{20e(5+4\cos(d+ex)+3\sin(d+ex))^{5/2}}+\frac{3}{40}\int \frac{}{(5+4\cos(d+}$$

$$= -\frac{3\cos(d+ex)-4\sin(d+ex)}{20e(5+4\cos(d+ex)+3\sin(d+ex))^{5/2}}-\frac{3(3\cos(d+ex)}{400e(5+4\cos(d+e}$$

$$= -\frac{3\cos(d+ex)-4\sin(d+ex)}{20e(5+4\cos(d+ex)+3\sin(d+ex))^{5/2}}-\frac{3(3\cos(d+ex)}{400e(5+4\cos(d+e}$$

$$= -\frac{3\cos(d+ex)-4\sin(d+ex)}{20e(5+4\cos(d+ex)+3\sin(d+ex))^{5/2}}-\frac{3(3\cos(d+ex)}{400e(5+4\cos(d+e}$$

$$= \frac{3\tanh^{-1}\left(\frac{\sin\left(d+ex-\tan^{-1}\left(\frac{3}{4}\right)\right)}{\sqrt{2}\sqrt{1+\cos\left(d+ex-\tan^{-1}\left(\frac{3}{4}\right)\right)}}\right)}{400\sqrt{10}\,e}-\frac{3\cos(d+ex)-4\sin(}{20e(5+4\cos(d+ex)+3\,\mathrm{s}}$$

Mathematica [C] time = 0.41, size = 180, normalized size = 1.27

$$-\frac{\left(\frac{1}{20000}-\frac{i}{10000}\right)\left(\sin\left(\frac{1}{2}(d+ex)\right)+3\cos\left(\frac{1}{2}(d+ex)\right)\right)\left((5+10i)\left(-165\sin\left(\frac{1}{2}(d+ex)\right)-27\sin\left(\frac{3}{2}(d+ex)\right)+5\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(5 + 4*Cos[d + e*x] + 3*Sin[d + e*x])^(-5/2),x]
```

```
[Out] ((-1/20000 + I/10000)*(3*Cos[(d + e*x)/2] + Sin[(d + e*x)/2])*((-6 + 6*I)*S
qrt[20 + 15*I]*ArcTan[(1/10 + (3*I)/10)*Sqrt[4/5 + (3*I)/5]*(-1 + 3*Tan[(d
+ e*x)/4])]*(3*Cos[(d + e*x)/2] + Sin[(d + e*x)/2])^4 + (5 + 10*I)*(55*Cos[
(d + e*x)/2] + 39*Cos[(3*(d + e*x))/2] - 165*Sin[(d + e*x)/2] - 27*Sin[(3*(
d + e*x))/2])))/(e*(5 + 4*Cos[d + e*x] + 3*Sin[d + e*x])^(5/2))
```

fricas [B] time = 2.93, size = 341, normalized size = 2.40

$$\frac{3\left(3\sqrt{10}\cos\left(ex+d\right)^3-111\sqrt{10}\cos\left(ex+d\right)^2-\left(79\sqrt{10}\cos\left(ex+d\right)^2+202\sqrt{10}\cos\left(ex+d\right)+124\sqrt{10}\right)\sin\left(\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(5+4*cos(e*x+d)+3*sin(e*x+d))^(5/2),x, algorithm="fricas")
```

```
[Out] 1/8000*(3*(3*sqrt(10)*cos(e*x + d)^3 - 111*sqrt(10)*cos(e*x + d)^2 - (79*sq
rt(10)*cos(e*x + d)^2 + 202*sqrt(10)*cos(e*x + d) + 124*sqrt(10))*sin(e*x +
 d) - 246*sqrt(10)*cos(e*x + d) - 132*sqrt(10))*log(-(9*cos(e*x + d)^2 + (1
3*cos(e*x + d) - 6)*sin(e*x + d) + 2*(sqrt(10)*cos(e*x + d) - 3*sqrt(10)*si
n(e*x + d) + sqrt(10))*sqrt(4*cos(e*x + d) + 3*sin(e*x + d) + 5) - 33*cos(e
*x + d) - 42)/(9*cos(e*x + d)^2 + (13*cos(e*x + d) + 14)*sin(e*x + d) + 27*
cos(e*x + d) + 18)) + 20*(39*cos(e*x + d)^2 - 3*(9*cos(e*x + d) + 32)*sin(e
*x + d) + 47*cos(e*x + d) + 8)*sqrt(4*cos(e*x + d) + 3*sin(e*x + d) + 5))/(
3*e*cos(e*x + d)^3 - 111*e*cos(e*x + d)^2 - 246*e*cos(e*x + d) - (79*e*cos(
e*x + d)^2 + 202*e*cos(e*x + d) + 124*e)*sin(e*x + d) - 132*e)
```

giac [B] time = 0.97, size = 417, normalized size = 2.94

$$\frac{1}{4000}\left(\frac{3\sqrt{10}\log\left(\frac{\left|-2\sqrt{10}+2\sqrt{\tan\left(\frac{1}{2}xe+\frac{1}{2}d\right)^2+1}-2\tan\left(\frac{1}{2}xe+\frac{1}{2}d\right)-6\right|}{\left|2\sqrt{10}+2\sqrt{\tan\left(\frac{1}{2}xe+\frac{1}{2}d\right)^2+1}-2\tan\left(\frac{1}{2}xe+\frac{1}{2}d\right)-6\right|}\right)}{\operatorname{sgn}\left(\tan\left(\frac{1}{2}xe+\frac{1}{2}d\right)+3\right)}-\frac{20\left(797\left(\sqrt{\tan\left(\frac{1}{2}xe+\frac{1}{2}d\right)^2+1}-\tan\left(\frac{1}{2}xe+\right.\right.\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(5+4*cos(e*x+d)+3*sin(e*x+d))^(5/2),x, algorithm="giac")
```

```
[Out] 1/4000*(3*sqrt(10)*log(abs(-2*sqrt(10) + 2*sqrt(tan(1/2*x*e + 1/2*d)^2 + 1)
 - 2*tan(1/2*x*e + 1/2*d) - 6)/abs(2*sqrt(10) + 2*sqrt(tan(1/2*x*e + 1/2*d)
```

```
^2 + 1) - 2*tan(1/2*x*e + 1/2*d) - 6))/sgn(tan(1/2*x*e + 1/2*d) + 3) - 20*(
797*(sqrt(tan(1/2*x*e + 1/2*d)^2 + 1) - tan(1/2*x*e + 1/2*d))^7 - 7137*(sqr
t(tan(1/2*x*e + 1/2*d)^2 + 1) - tan(1/2*x*e + 1/2*d))^6 + 27543*(sqrt(tan(1
/2*x*e + 1/2*d)^2 + 1) - tan(1/2*x*e + 1/2*d))^5 - 30015*(sqrt(tan(1/2*x*e
+ 1/2*d)^2 + 1) - tan(1/2*x*e + 1/2*d))^4 - 27105*(sqrt(tan(1/2*x*e + 1/2*d
)^2 + 1) - tan(1/2*x*e + 1/2*d))^3 - 7491*(sqrt(tan(1/2*x*e + 1/2*d)^2 + 1)
 - tan(1/2*x*e + 1/2*d))^2 - 859*sqrt(tan(1/2*x*e + 1/2*d)^2 + 1) + 859*tan
(1/2*x*e + 1/2*d) - 69)/(((sqrt(tan(1/2*x*e + 1/2*d)^2 + 1) - tan(1/2*x*e +
 1/2*d))^2 - 6*sqrt(tan(1/2*x*e + 1/2*d)^2 + 1) + 6*tan(1/2*x*e + 1/2*d) -
1)^4*sgn(tan(1/2*x*e + 1/2*d) + 3)))*e^(-1)
```

maple [A] time = 0.32, size = 190, normalized size = 1.34

$$-\frac{\left(3\sqrt{10}\ \operatorname{arctanh}\left(\frac{\sqrt{-5\sin\left(ex+d+\arctan\left(\frac{4}{3}\right)\right)+5}\ \sqrt{10}}{10}\right)\left(\sin^2\left(ex+d+\arctan\left(\frac{4}{3}\right)\right)\right)+6\sqrt{10}\ \operatorname{arctanh}\left(\frac{\sqrt{-5\sin\left(ex+d+\mathrm{arc}\right.}}{10}\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(5+4*cos(e*x+d)+3*sin(e*x+d))^(5/2),x)
```

```
[Out] -1/4000*(3*10^(1/2)*arctanh(1/10*(-5*sin(e*x+d+arctan(4/3))+5)^(1/2)*10^(1/
2))*sin(e*x+d+arctan(4/3))^2+6*10^(1/2)*arctanh(1/10*(-5*sin(e*x+d+arctan(4
/3))+5)^(1/2)*10^(1/2))*sin(e*x+d+arctan(4/3))+6*(-5*sin(e*x+d+arctan(4/3))
+5)^(1/2)*sin(e*x+d+arctan(4/3))+3*10^(1/2)*arctanh(1/10*(-5*sin(e*x+d+arct
an(4/3))+5)^(1/2)*10^(1/2))+14*(-5*sin(e*x+d+arctan(4/3))+5)^(1/2))*(-5*sin
(e*x+d+arctan(4/3))+5)^(1/2)/(1+sin(e*x+d+arctan(4/3)))/cos(e*x+d+arctan(4/
3))/(5+5*sin(e*x+d+arctan(4/3)))^(1/2)/e
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(4\cos(ex+d)+3\sin(ex+d)+5)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(5+4*cos(e*x+d)+3*sin(e*x+d))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((4*cos(e*x + d) + 3*sin(e*x + d) + 5)^(-5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{(4\cos(d+e\,x)+3\sin(d+e\,x)+5)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(4*cos(d + e*x) + 3*sin(d + e*x) + 5)^(5/2),x)
```

```
[Out] int(1/(4*cos(d + e*x) + 3*sin(d + e*x) + 5)^(5/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(3\sin(d+ex)+4\cos(d+ex)+5)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(5+4*cos(e*x+d)+3*sin(e*x+d))**(5/2),x)
```

```
[Out] Integral((3*sin(d + e*x) + 4*cos(d + e*x) + 5)**(-5/2), x)
```

3.423 $\int (-5+4\cos(d+ex)+3\sin(d+ex))^{7/2}\,dx$

Optimal. Leaf size=185

$$-\frac{2(3\cos(d+ex)-4\sin(d+ex))(3\sin(d+ex)+4\cos(d+ex)-5)^{5/2}}{7e}+\frac{24(3\cos(d+ex)-4\sin(d+ex))(3\sin(\cdots}{7e}$$

```
[Out] 24/7*(3*cos(e*x+d)-4*sin(e*x+d))*(-5+4*cos(e*x+d)+3*sin(e*x+d))^(3/2)/e-2/7
*(3*cos(e*x+d)-4*sin(e*x+d))*(-5+4*cos(e*x+d)+3*sin(e*x+d))^(5/2)/e+6400/7*
(3*cos(e*x+d)-4*sin(e*x+d))/e/(-5+4*cos(e*x+d)+3*sin(e*x+d))^(1/2)-320/7*(3
*cos(e*x+d)-4*sin(e*x+d))*(-5+4*cos(e*x+d)+3*sin(e*x+d))^(1/2)/e
```

Rubi [A] time = 0.09, antiderivative size = 185, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {3113, 3112}

$$-\frac{2(3\cos(d+ex)-4\sin(d+ex))(3\sin(d+ex)+4\cos(d+ex)-5)^{5/2}}{7e}+\frac{24(3\cos(d+ex)-4\sin(d+ex))(3\sin(\cdots}{7e}$$

Antiderivative was successfully verified.

```
[In] Int[(-5 + 4*Cos[d + e*x] + 3*Sin[d + e*x])^(7/2),x]
```

```
[Out] (6400*(3*Cos[d + e*x] - 4*Sin[d + e*x]))/(7*e*Sqrt[-5 + 4*Cos[d + e*x] + 3*
Sin[d + e*x]]) - (320*(3*Cos[d + e*x] - 4*Sin[d + e*x])*Sqrt[-5 + 4*Cos[d +
 e*x] + 3*Sin[d + e*x]])/(7*e) + (24*(3*Cos[d + e*x] - 4*Sin[d + e*x])*(-5
+ 4*Cos[d + e*x] + 3*Sin[d + e*x])^(3/2))/(7*e) - (2*(3*Cos[d + e*x] - 4*Si
n[d + e*x])*(-5 + 4*Cos[d + e*x] + 3*Sin[d + e*x])^(5/2))/(7*e)
```

Rule 3112

```
Int[Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_
)]], x_Symbol] :> Simp[(-2*(c*Cos[d + e*x] - b*Sin[d + e*x]))/(e*Sqrt[a + b
*Cos[d + e*x] + c*Sin[d + e*x]]), x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[a^
2 - b^2 - c^2, 0]
```

Rule 3113

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(n_), x_Symbol] :> -Simp[((c*Cos[d + e*x] - b*Sin[d + e*x])*(a + b*Cos[d +
e*x] + c*Sin[d + e*x])^(n - 1))/(e*n), x] + Dist[(a*(2*n - 1))/n, Int[(a +
b*Cos[d + e*x] + c*Sin[d + e*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e},
x] && EqQ[a^2 - b^2 - c^2, 0] && GtQ[n, 0]
```

Rubi steps

$$\int (-5+4\cos(d+ex)+3\sin(d+ex))^{7/2}\,dx = -\frac{2(3\cos(d+ex)-4\sin(d+ex))(-5+4\cos(d+ex)+3\sin(d+e}{7e}$$
$$= \frac{24(3\cos(d+ex)-4\sin(d+ex))(-5+4\cos(d+ex)+3\sin(d+e}{7e}$$
$$= -\frac{320(3\cos(d+ex)-4\sin(d+ex))\sqrt{-5+4\cos(d+ex)+3\sin(d}}{7e}$$
$$= \frac{6400(3\cos(d+ex)-4\sin(d+ex))}{7e\sqrt{-5+4\cos(d+ex)+3\sin(d+ex)}} - \frac{320(3\cos(d+ex)-4\sin}{}$$

Mathematica [A] time = 1.82, size = 151, normalized size = 0.82

$$\frac{(3\sin(d+ex)+4\cos(d+ex)-5)^{7/2}\left(30625\sin\left(\frac{1}{2}(d+ex)\right)-15925\sin\left(\frac{3}{2}(d+ex)\right)+3871\sin\left(\frac{5}{2}(d+ex)\right)-30}{28e\left(\cos\left(\frac{1}{2}(d+ex)\right)-\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-5 + 4*Cos[d + e*x] + 3*Sin[d + e*x])^(7/2),x]
```

```
[Out] ((-5 + 4*Cos[d + e*x] + 3*Sin[d + e*x])^(7/2)*(91875*Cos[(d + e*x)/2] - 110
25*Cos[(3*(d + e*x))/2] - 147*Cos[(5*(d + e*x))/2] + 249*Cos[(7*(d + e*x))/
2] + 30625*Sin[(d + e*x)/2] - 15925*Sin[(3*(d + e*x))/2] + 3871*Sin[(5*(d +
 e*x))/2] - 307*Sin[(7*(d + e*x))/2]))/(28*e*(Cos[(d + e*x)/2] - 3*Sin[(d +
 e*x)/2])^7)
```

fricas [A] time = 0.85, size = 121, normalized size = 0.65

$$-\frac{2\left(249\cos(ex+d)^{4}+51\cos(ex+d)^{3}-3042\cos(ex+d)^{2}-\left(307\cos(ex+d)^{3}-1782\cos(ex+d)^{2}+2860\right.\right.}{7\left(e\cos(ex+d)-3\,e\sin\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-5+4*cos(e*x+d)+3*sin(e*x+d))^(7/2),x, algorithm="fricas")
```

```
[Out] -2/7*(249*cos(e*x + d)^4 + 51*cos(e*x + d)^3 - 3042*cos(e*x + d)^2 - (307*c
os(e*x + d)^3 - 1782*cos(e*x + d)^2 + 2860*cos(e*x + d) - 1392)*sin(e*x + d
) + 10068*cos(e*x + d) + 12912)*sqrt(4*cos(e*x + d) + 3*sin(e*x + d) - 5)/(
e*cos(e*x + d) - 3*e*sin(e*x + d) + e)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (4\cos(ex+d)+3\sin(ex+d)-5)^{\frac{7}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-5+4*cos(e*x+d)+3*sin(e*x+d))^(7/2),x, algorithm="giac")
```

```
[Out] integrate((4*cos(e*x + d) + 3*sin(e*x + d) - 5)^(7/2), x)
```

maple [A] time = 0.28, size = 86, normalized size = 0.46

$$\frac{250\left(\sin\left(ex+d+\arctan\left(\frac{4}{3}\right)\right)-1\right)\left(1+\sin\left(ex+d+\arctan\left(\frac{4}{3}\right)\right)\right)\left(5\left(\sin^3\left(ex+d+\arctan\left(\frac{4}{3}\right)\right)\right)-27\left(\sin^2\right.\right.}{7\cos\left(ex+d+\arctan\left(\frac{4}{3}\right)\right)\sqrt{-5+5\sin\left(ex+d+\arctan\right.}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-5+4*cos(e*x+d)+3*sin(e*x+d))^(7/2),x)
```

```
[Out] 250/7*(sin(e*x+d+arctan(4/3))-1)*(1+sin(e*x+d+arctan(4/3)))*(5*sin(e*x+d+ar
ctan(4/3))^3-27*sin(e*x+d+arctan(4/3))^2+71*sin(e*x+d+arctan(4/3))-177)/cos
(e*x+d+arctan(4/3))/(-5+5*sin(e*x+d+arctan(4/3)))^(1/2)/e
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int(4\cos(ex+d)+3\sin(ex+d)-5)^{\frac{7}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-5+4*cos(e*x+d)+3*sin(e*x+d))^(7/2),x, algorithm="maxima")
```

```
[Out] integrate((4*cos(e*x + d) + 3*sin(e*x + d) - 5)^(7/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int(4\cos(d+e\,x)+3\sin(d+e\,x)-5)^{7/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((4*cos(d + e*x) + 3*sin(d + e*x) - 5)^(7/2),x)
```

```
[Out] int((4*cos(d + e*x) + 3*sin(d + e*x) - 5)^(7/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-5+4*cos(e*x+d)+3*sin(e*x+d))**(7/2),x)
```

```
[Out] Timed out
```

3.424 $\int (-5+4\cos(d+ex)+3\sin(d+ex))^{5/2}\,dx$

Optimal. Leaf size=139

$$-\frac{2(3\cos(d+ex)-4\sin(d+ex))(3\sin(d+ex)+4\cos(d+ex)-5)^{3/2}}{5e}+\frac{16(3\cos(d+ex)-4\sin(d+ex))\sqrt{3\sin(d+ex)+4\cos(d+ex)-5}}{3e}$$

```
[Out] -2/5*(3*cos(e*x+d)-4*sin(e*x+d))*(-5+4*cos(e*x+d)+3*sin(e*x+d))^(3/2)/e-320
/3*(3*cos(e*x+d)-4*sin(e*x+d))/e/(-5+4*cos(e*x+d)+3*sin(e*x+d))^(1/2)+16/3*
(3*cos(e*x+d)-4*sin(e*x+d))*(-5+4*cos(e*x+d)+3*sin(e*x+d))^(1/2)/e
```

Rubi [A] time = 0.07, antiderivative size = 139, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {3113, 3112}

$$-\frac{2(3\cos(d+ex)-4\sin(d+ex))(3\sin(d+ex)+4\cos(d+ex)-5)^{3/2}}{5e}+\frac{16(3\cos(d+ex)-4\sin(d+ex))\sqrt{3\sin(d+ex)+4\cos(d+ex)-5}}{3e}$$

Antiderivative was successfully verified.

```
[In] Int[(-5 + 4*Cos[d + e*x] + 3*Sin[d + e*x])^(5/2),x]
```

```
[Out] (-320*(3*Cos[d + e*x] - 4*Sin[d + e*x]))/(3*e*Sqrt[-5 + 4*Cos[d + e*x] + 3*
Sin[d + e*x]]) + (16*(3*Cos[d + e*x] - 4*Sin[d + e*x])*Sqrt[-5 + 4*Cos[d +
e*x] + 3*Sin[d + e*x]])/(3*e) - (2*(3*Cos[d + e*x] - 4*Sin[d + e*x])*(-5 +
4*Cos[d + e*x] + 3*Sin[d + e*x])^(3/2))/(5*e)
```

Rule 3112

```
Int[Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_
)]], x_Symbol] :> Simp[(-2*(c*Cos[d + e*x] - b*Sin[d + e*x]))/(e*Sqrt[a + b
*Cos[d + e*x] + c*Sin[d + e*x]]), x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[a^
2 - b^2 - c^2, 0]
```

Rule 3113

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(n_), x_Symbol] :> -Simp[((c*Cos[d + e*x] - b*Sin[d + e*x])*(a + b*Cos[d +
e*x] + c*Sin[d + e*x])^(n - 1))/(e*n), x] + Dist[(a*(2*n - 1))/n, Int[(a +
b*Cos[d + e*x] + c*Sin[d + e*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e},
x] && EqQ[a^2 - b^2 - c^2, 0] && GtQ[n, 0]
```

Rubi steps

$$\int (-5+4\cos(d+ex)+3\sin(d+ex))^{5/2}\,dx = -\frac{2(3\cos(d+ex)-4\sin(d+ex))(-5+4\cos(d+ex)+3\sin(d+ex)}{5e}$$
$$= \frac{16(3\cos(d+ex)-4\sin(d+ex))\sqrt{-5+4\cos(d+ex)+3\sin(d+}}{3e}$$
$$= -\frac{320(3\cos(d+ex)-4\sin(d+ex))}{3e\sqrt{-5+4\cos(d+ex)+3\sin(d+ex)}} + \frac{16(3\cos(d+ex)-4}{}$$

Mathematica [A] time = 0.49, size = 127, normalized size = 0.91

$$\frac{(3\sin(d+ex)+4\cos(d+ex)-5)^{5/2}\left(3750\sin\left(\frac{1}{2}(d+ex)\right)-1625\sin\left(\frac{3}{2}(d+ex)\right)+237\sin\left(\frac{5}{2}(d+ex)\right)+1125\right.}{30e\left(\cos\left(\frac{1}{2}(d+ex)\right)-3\sin\left(\frac{1}{2}(d+ex)\right)\right)^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-5 + 4*Cos[d + e*x] + 3*Sin[d + e*x])^(5/2),x]
```

```
[Out] ((-5 + 4*Cos[d + e*x] + 3*Sin[d + e*x])^(5/2)*(11250*Cos[(d + e*x)/2] - 112
5*Cos[(3*(d + e*x))/2] - 9*Cos[(5*(d + e*x))/2] + 3750*Sin[(d + e*x)/2] - 1
625*Sin[(3*(d + e*x))/2] + 237*Sin[(5*(d + e*x))/2]))/(30*e*(Cos[(d + e*x)/
2] - 3*Sin[(d + e*x)/2])^5)
```

fricas [A] time = 0.65, size = 101, normalized size = 0.73

$$-\frac{2\left(9\cos(ex+d)^3+567\cos(ex+d)^2-\left(237\cos(ex+d)^2-694\cos(ex+d)+472\right)\sin(ex+d)-2538\cos\right.}{15\,(e\cos(ex+d)-3\,e\sin(ex+d)+e)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-5+4*cos(e*x+d)+3*sin(e*x+d))^(5/2),x, algorithm="fricas")
```

```
[Out] -2/15*(9*cos(e*x + d)^3 + 567*cos(e*x + d)^2 - (237*cos(e*x + d)^2 - 694*co
s(e*x + d) + 472)*sin(e*x + d) - 2538*cos(e*x + d) - 3096)*sqrt(4*cos(e*x +
 d) + 3*sin(e*x + d) - 5)/(e*cos(e*x + d) - 3*e*sin(e*x + d) + e)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (4\cos(ex+d)+3\sin(ex+d)-5)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-5+4*cos(e*x+d)+3*sin(e*x+d))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((4*cos(e*x + d) + 3*sin(e*x + d) - 5)^(5/2), x)
```

maple [A] time = 0.31, size = 74, normalized size = 0.53

$$\frac{50\left(\sin\left(ex+d+\arctan\left(\frac{4}{3}\right)\right)-1\right)\left(1+\sin\left(ex+d+\arctan\left(\frac{4}{3}\right)\right)\right)\left(3\left(\sin^2\left(ex+d+\arctan\left(\frac{4}{3}\right)\right)\right)-14\sin\left(ex+\right.\right.}{3\cos\left(ex+d+\arctan\left(\frac{4}{3}\right)\right)\sqrt{-5+5\sin\left(ex+d+\arctan\left(\frac{4}{3}\right)\right)}\, e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-5+4*cos(e*x+d)+3*sin(e*x+d))^(5/2),x)
```

```
[Out] 50/3*(sin(e*x+d+arctan(4/3))-1)*(1+sin(e*x+d+arctan(4/3)))*(3*sin(e*x+d+arc
tan(4/3))^2-14*sin(e*x+d+arctan(4/3))+43)/cos(e*x+d+arctan(4/3))/(-5+5*sin(
e*x+d+arctan(4/3)))^(1/2)/e
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (4\cos(ex+d)+3\sin(ex+d)-5)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-5+4*cos(e*x+d)+3*sin(e*x+d))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((4*cos(e*x + d) + 3*sin(e*x + d) - 5)^(5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int (4\cos(d+e\,x)+3\sin(d+e\,x)-5)^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((4*cos(d + e*x) + 3*sin(d + e*x) - 5)^(5/2),x)
```

```
[Out] int((4*cos(d + e*x) + 3*sin(d + e*x) - 5)^(5/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-5+4*cos(e*x+d)+3*sin(e*x+d))**(5/2),x)
```

```
[Out] Timed out
```

3.425 $\int (-5+4\cos(d+ex)+3\sin(d+ex))^{3/2}\,dx$

Optimal. Leaf size=93

$$\frac{40(3\cos(d+ex)-4\sin(d+ex))}{3e\sqrt{3\sin(d+ex)+4\cos(d+ex)-5}}-\frac{2(3\cos(d+ex)-4\sin(d+ex))\sqrt{3\sin(d+ex)+4\cos(d+ex)-5}}{3e}$$

```
[Out] 40/3*(3*cos(e*x+d)-4*sin(e*x+d))/e/(-5+4*cos(e*x+d)+3*sin(e*x+d))^(1/2)-2/3
*(3*cos(e*x+d)-4*sin(e*x+d))*(-5+4*cos(e*x+d)+3*sin(e*x+d))^(1/2)/e
```

Rubi [A] time = 0.04, antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {3113, 3112}

$$\frac{40(3\cos(d+ex)-4\sin(d+ex))}{3e\sqrt{3\sin(d+ex)+4\cos(d+ex)-5}}-\frac{2(3\cos(d+ex)-4\sin(d+ex))\sqrt{3\sin(d+ex)+4\cos(d+ex)-5}}{3e}$$

Antiderivative was successfully verified.

```
[In] Int[(-5 + 4*Cos[d + e*x] + 3*Sin[d + e*x])^(3/2),x]
```

```
[Out] (40*(3*Cos[d + e*x] - 4*Sin[d + e*x]))/(3*e*Sqrt[-5 + 4*Cos[d + e*x] + 3*Si
n[d + e*x]]) - (2*(3*Cos[d + e*x] - 4*Sin[d + e*x])*Sqrt[-5 + 4*Cos[d + e*x
] + 3*Sin[d + e*x]])/(3*e)
```

Rule 3112

```
Int[Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_
)]], x_Symbol] :> Simp[(-2*(c*Cos[d + e*x] - b*Sin[d + e*x]))/(e*Sqrt[a + b
*Cos[d + e*x] + c*Sin[d + e*x]]), x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[a^
2 - b^2 - c^2, 0]
```

Rule 3113

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(n_), x_Symbol] :> -Simp[((c*Cos[d + e*x] - b*Sin[d + e*x])*(a + b*Cos[d +
e*x] + c*Sin[d + e*x])^(n - 1))/(e*n), x] + Dist[(a*(2*n - 1))/n, Int[(a +
b*Cos[d + e*x] + c*Sin[d + e*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e},
x] && EqQ[a^2 - b^2 - c^2, 0] && GtQ[n, 0]
```

Rubi steps

$$\int (-5+4\cos(d+ex)+3\sin(d+ex))^{3/2}\,dx = -\frac{2(3\cos(d+ex)-4\sin(d+ex))\sqrt{-5+4\cos(d+ex)+3\sin(d+}}{3e}$$
$$= \frac{40(3\cos(d+ex)-4\sin(d+ex))}{3e\sqrt{-5+4\cos(d+ex)+3\sin(d+ex)}} - \frac{2(3\cos(d+ex)-4\sin(}{}$$

Mathematica [A] time = 0.22, size = 103, normalized size = 1.11

$$\frac{(3\sin(d+ex)+4\cos(d+ex)-5)^{3/2}\left(45\sin\left(\frac{1}{2}(d+ex)\right)-13\sin\left(\frac{3}{2}(d+ex)\right)+135\cos\left(\frac{1}{2}(d+ex)\right)-9\cos\left(\frac{3}{2}(d\right.\right.}{3e\left(\cos\left(\frac{1}{2}(d+ex)\right)-3\sin\left(\frac{1}{2}(d+ex)\right)\right)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-5 + 4*Cos[d + e*x] + 3*Sin[d + e*x])^(3/2),x]
```

```
[Out] ((-5 + 4*Cos[d + e*x] + 3*Sin[d + e*x])^(3/2)*(135*Cos[(d + e*x)/2] - 9*Cos
[(3*(d + e*x))/2] + 45*Sin[(d + e*x)/2] - 13*Sin[(3*(d + e*x))/2]))/(3*e*(C
os[(d + e*x)/2] - 3*Sin[(d + e*x)/2])^3)
```

fricas [A] time = 0.73, size = 80, normalized size = 0.86

$$\frac{2\left(9\cos(ex+d)^2+(13\cos(ex+d)-16)\sin(ex+d)-63\cos(ex+d)-72\right)\sqrt{4\cos(ex+d)+3\sin(ex+d)-}}{3\left(e\cos(ex+d)-3e\sin(ex+d)+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-5+4*cos(e*x+d)+3*sin(e*x+d))^(3/2),x, algorithm="fricas")
```

```
[Out] 2/3*(9*cos(e*x + d)^2 + (13*cos(e*x + d) - 16)*sin(e*x + d) - 63*cos(e*x +
d) - 72)*sqrt(4*cos(e*x + d) + 3*sin(e*x + d) - 5)/(e*cos(e*x + d) - 3*e*si
n(e*x + d) + e)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (4\cos(ex+d)+3\sin(ex+d)-5)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-5+4*cos(e*x+d)+3*sin(e*x+d))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((4*cos(e*x + d) + 3*sin(e*x + d) - 5)^(3/2), x)
```

maple [A] time = 0.31, size = 60, normalized size = 0.65

$$\frac{50\left(\sin\left(ex+d+\arctan\left(\frac{4}{3}\right)\right)-1\right)\left(1+\sin\left(ex+d+\arctan\left(\frac{4}{3}\right)\right)\right)\left(\sin\left(ex+d+\arctan\left(\frac{4}{3}\right)\right)-5\right)}{3\cos\left(ex+d+\arctan\left(\frac{4}{3}\right)\right)\sqrt{-5+5\sin\left(ex+d+\arctan\left(\frac{4}{3}\right)\right)}\,e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-5+4*cos(e*x+d)+3*sin(e*x+d))^(3/2),x)
```

```
[Out] 50/3*(sin(e*x+d+arctan(4/3))-1)*(1+sin(e*x+d+arctan(4/3)))*(sin(e*x+d+arcta
n(4/3))-5)/cos(e*x+d+arctan(4/3))/(-5+5*sin(e*x+d+arctan(4/3)))^(1/2)/e
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (4\cos(ex+d)+3\sin(ex+d)-5)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-5+4*cos(e*x+d)+3*sin(e*x+d))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((4*cos(e*x + d) + 3*sin(e*x + d) - 5)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int (4\cos(d+e\,x)+3\sin(d+e\,x)-5)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((4*cos(d + e*x) + 3*sin(d + e*x) - 5)^(3/2),x)
```

```
[Out] int((4*cos(d + e*x) + 3*sin(d + e*x) - 5)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (3\sin(d+ex)+4\cos(d+ex)-5)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-5+4*cos(e*x+d)+3*sin(e*x+d))**(3/2),x)
```

```
[Out] Integral((3*sin(d + e*x) + 4*cos(d + e*x) - 5)**(3/2), x)
```

3.426 $\int \sqrt{-5+4\cos(d+ex)+3\sin(d+ex)}\,dx$

Optimal. Leaf size=44

$$-\frac{2(3\cos(d+ex)-4\sin(d+ex))}{e\sqrt{3\sin(d+ex)+4\cos(d+ex)-5}}$$

```
[Out] -2*(3*cos(e*x+d)-4*sin(e*x+d))/e/(-5+4*cos(e*x+d)+3*sin(e*x+d))^(1/2)
```

Rubi [A] time = 0.02, antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {3112}

$$-\frac{2(3\cos(d+ex)-4\sin(d+ex))}{e\sqrt{3\sin(d+ex)+4\cos(d+ex)-5}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[-5 + 4*Cos[d + e*x] + 3*Sin[d + e*x]],x]
```

```
[Out] (-2*(3*Cos[d + e*x] - 4*Sin[d + e*x]))/(e*Sqrt[-5 + 4*Cos[d + e*x] + 3*Sin[
d + e*x]])
```

Rule 3112

```
Int[Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_
)]], x_Symbol] :> Simp[(-2*(c*Cos[d + e*x] - b*Sin[d + e*x]))/(e*Sqrt[a + b
*Cos[d + e*x] + c*Sin[d + e*x]]), x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[a^
2 - b^2 - c^2, 0]
```

Rubi steps

$$\int \sqrt{-5+4\cos(d+ex)+3\sin(d+ex)}\,dx = -\frac{2(3\cos(d+ex)-4\sin(d+ex))}{e\sqrt{-5+4\cos(d+ex)+3\sin(d+ex)}}$$

Mathematica [A] time = 0.04, size = 75, normalized size = 1.70

$$\frac{2\left(\sin\left(\frac{1}{2}(d+ex)\right)+3\cos\left(\frac{1}{2}(d+ex)\right)\right)\sqrt{3\sin(d+ex)+4\cos(d+ex)-5}}{e\left(\cos\left(\frac{1}{2}(d+ex)\right)-3\sin\left(\frac{1}{2}(d+ex)\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[-5 + 4*Cos[d + e*x] + 3*Sin[d + e*x]],x]
```

```
[Out] (2*(3*Cos[(d + e*x)/2] + Sin[(d + e*x)/2])*Sqrt[-5 + 4*Cos[d + e*x] + 3*Sin
[d + e*x]])/(e*(Cos[(d + e*x)/2] - 3*Sin[(d + e*x)/2]))
```

fricas [A] time = 0.65, size = 59, normalized size = 1.34

$$\frac{2 \sqrt{4 \cos\left(ex+d\right) + 3 \sin\left(ex+d\right) - 5} \left(3 \cos\left(ex+d\right) + \sin\left(ex+d\right) + 3\right)}{e \cos\left(ex+d\right) - 3 e \sin\left(ex+d\right) + e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-5+4*cos(e*x+d)+3*sin(e*x+d))^(1/2),x, algorithm="fricas")
```

```
[Out] 2*sqrt(4*cos(e*x + d) + 3*sin(e*x + d) - 5)*(3*cos(e*x + d) + sin(e*x + d)
+ 3)/(e*cos(e*x + d) - 3*e*sin(e*x + d) + e)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{4 \cos\left(ex+d\right) + 3 \sin\left(ex+d\right) - 5}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-5+4*cos(e*x+d)+3*sin(e*x+d))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(4*cos(e*x + d) + 3*sin(e*x + d) - 5), x)
```

maple [A] time = 0.30, size = 50, normalized size = 1.14

$$\frac{10 \left(\sin\left(ex+d+\arctan\left(\frac{4}{3}\right)\right)-1\right) \left(1+\sin\left(ex+d+\arctan\left(\frac{4}{3}\right)\right)\right)}{\cos\left(ex+d+\arctan\left(\frac{4}{3}\right)\right) \sqrt{-5+5 \sin\left(ex+d+\arctan\left(\frac{4}{3}\right)\right)}\, e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-5+4*cos(e*x+d)+3*sin(e*x+d))^(1/2),x)
```

```
[Out] 10*(sin(e*x+d+arctan(4/3))-1)*(1+sin(e*x+d+arctan(4/3)))/cos(e*x+d+arctan(4
/3))/(-5+5*sin(e*x+d+arctan(4/3)))^(1/2)/e
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{4 \cos\left(ex+d\right) + 3 \sin\left(ex+d\right) - 5}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-5+4*cos(e*x+d)+3*sin(e*x+d))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(4*cos(e*x + d) + 3*sin(e*x + d) - 5), x)
```

mupad [B] time = 0.42, size = 39, normalized size = 0.89

$$-\frac{2\sqrt{5}\,\left(3\,\cos\left(d+e\,x\right)-4\,\sin\left(d+e\,x\right)\right)}{5\,e\,\sqrt{\cos\left(d-\mathrm{atan}\left(\frac{3}{4}\right)+e\,x\right)-1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((4*cos(d + e*x) + 3*sin(d + e*x) - 5)^(1/2),x)
```

```
[Out] -(2*5^(1/2)*(3*cos(d + e*x) - 4*sin(d + e*x)))/(5*e*(cos(d - atan(3/4) + e*
x) - 1)^(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{3\sin\left(d+ex\right)+4\cos\left(d+ex\right)-5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-5+4*cos(e*x+d)+3*sin(e*x+d))**(1/2),x)
```

```
[Out] Integral(sqrt(3*sin(d + e*x) + 4*cos(d + e*x) - 5), x)
```

3.427 $\int \frac{1}{\sqrt{-5+4\cos(d+ex)+3\sin(d+ex)}}\,dx$

Optimal. Leaf size=49

$$-\frac{\sqrt{\frac{2}{5}}\tan^{-1}\left(\frac{\sin\left(d+ex-\tan^{-1}\left(\frac{3}{4}\right)\right)}{\sqrt{2}\sqrt{\cos\left(d+ex-\tan^{-1}\left(\frac{3}{4}\right)\right)-1}}\right)}{e}$$

```
[Out] -1/5*arctan(1/2*sin(d+e*x-arctan(3/4))*2^(1/2)/(-1+cos(d+e*x-arctan(3/4)))^
(1/2))*10^(1/2)/e
```

Rubi [A] time = 0.06, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {3115, 2649, 204}

$$-\frac{\sqrt{\frac{2}{5}}\tan^{-1}\left(\frac{\sin\left(d+ex-\tan^{-1}\left(\frac{3}{4}\right)\right)}{\sqrt{2}\sqrt{\cos\left(d+ex-\tan^{-1}\left(\frac{3}{4}\right)\right)-1}}\right)}{e}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[-5 + 4*Cos[d + e*x] + 3*Sin[d + e*x]],x]
```

```
[Out] -((Sqrt[2/5]*ArcTan[Sin[d + e*x - ArcTan[3/4]]/(Sqrt[2]*Sqrt[-1 + Cos[d + e
*x - ArcTan[3/4]]])])/e)
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3115

```
Int[1/Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(
x_)]], x_Symbol] :> Int[1/Sqrt[a + Sqrt[b^2 + c^2]*Cos[d + e*x - ArcTan[b,
```

```
c]]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[a^2 - b^2 - c^2, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{-5+4\cos(d+ex)+3\sin(d+ex)}}\,dx = \int \frac{1}{\sqrt{-5+5\cos\left(d+ex-\tan^{-1}\left(\frac{3}{4}\right)\right)}}\,dx$$
$$= -\frac{2\,\text{Subst}\left(\int \frac{1}{-10-x^2}\,dx, x, -\frac{5\sin\left(d+ex-\tan^{-1}\left(\frac{3}{4}\right)\right)}{\sqrt{-5+5\cos\left(d+ex-\tan^{-1}\left(\frac{3}{4}\right)\right)}}\right)}{e}$$
$$= -\frac{\sqrt{\frac{2}{5}}\,\tan^{-1}\left(\frac{\sin\left(d+ex-\tan^{-1}\left(\frac{3}{4}\right)\right)}{\sqrt{2}\sqrt{-1+\cos\left(d+ex-\tan^{-1}\left(\frac{3}{4}\right)\right)}}\right)}{e}$$

Mathematica [C] time = 0.09, size = 99, normalized size = 2.02

$$\frac{\left(\frac{2}{5}+\frac{6i}{5}\right)\sqrt{-\frac{4}{5}-\frac{3i}{5}}\left(\cos\left(\frac{1}{2}(d+ex)\right)-3\sin\left(\frac{1}{2}(d+ex)\right)\right)\tanh^{-1}\left(\left(\frac{1}{10}+\frac{3i}{10}\right)\sqrt{-\frac{4}{5}-\frac{3i}{5}}\left(\tan\left(\frac{1}{4}(d+ex)\right)+3\right)\right)}{e\sqrt{3\sin(d+ex)+4\cos(d+ex)-5}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[-5 + 4*Cos[d + e*x] + 3*Sin[d + e*x]],x]
```

```
[Out] ((2/5 + (6*I)/5)*Sqrt[-4/5 - (3*I)/5]*ArcTanh[(1/10 + (3*I)/10)*Sqrt[-4/5 -
 (3*I)/5]*(3 + Tan[(d + e*x)/4])]*(Cos[(d + e*x)/2] - 3*Sin[(d + e*x)/2]))/
(e*Sqrt[-5 + 4*Cos[d + e*x] + 3*Sin[d + e*x]])
```

fricas [B] time = 2.11, size = 88, normalized size = 1.80

$$\frac{\sqrt{5}\,\sqrt{2}\,\arctan\left(-\frac{\left(3\sqrt{5}\sqrt{2}\cos(ex+d)+\sqrt{5}\sqrt{2}\sin(ex+d)+3\sqrt{5}\sqrt{2}\right)\sqrt{4\cos(ex+d)+3\sin(ex+d)-5}}{10\,(\cos(ex+d)-3\sin(ex+d)+1)}\right)}{5\,e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-5+4*cos(e*x+d)+3*sin(e*x+d))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/5*sqrt(5)*sqrt(2)*arctan(-1/10*(3*sqrt(5)*sqrt(2)*cos(e*x + d) + sqrt(5)*
sqrt(2)*sin(e*x + d) + 3*sqrt(5)*sqrt(2))*sqrt(4*cos(e*x + d) + 3*sin(e*x +
 d) - 5)/(cos(e*x + d) - 3*sin(e*x + d) + 1))/e
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{4 \cos(ex+d) + 3 \sin(ex+d) - 5}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-5+4*cos(e*x+d)+3*sin(e*x+d))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(4*cos(e*x + d) + 3*sin(e*x + d) - 5), x)
```

maple [A] time = 0.33, size = 77, normalized size = 1.57

$$\frac{\left(\sin\left(ex+d+\arctan\left(\frac{4}{3}\right)\right)-1\right)\sqrt{-5\sin\left(ex+d+\arctan\left(\frac{4}{3}\right)\right)-5}\,\sqrt{10}\,\arctan\left(\frac{\sqrt{-5\sin\left(ex+d+\arctan\left(\frac{4}{3}\right)\right)-5}\,\sqrt{10}}{10}\right)}{5\cos\left(ex+d+\arctan\left(\frac{4}{3}\right)\right)\sqrt{-5+5\sin\left(ex+d+\arctan\left(\frac{4}{3}\right)\right)}\,e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-5+4*cos(e*x+d)+3*sin(e*x+d))^(1/2),x)
```

```
[Out] 1/5*(sin(e*x+d+arctan(4/3))-1)*(-5*sin(e*x+d+arctan(4/3))-5)^(1/2)*10^(1/2)
*arctan(1/10*(-5*sin(e*x+d+arctan(4/3))-5)^(1/2)*10^(1/2))/cos(e*x+d+arctan
(4/3))/(-5+5*sin(e*x+d+arctan(4/3)))^(1/2)/e
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{4 \cos(ex+d) + 3 \sin(ex+d) - 5}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-5+4*cos(e*x+d)+3*sin(e*x+d))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(4*cos(e*x + d) + 3*sin(e*x + d) - 5), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{\sqrt{4 \cos(d+e\,x) + 3 \sin(d+e\,x) - 5}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(4*cos(d + e*x) + 3*sin(d + e*x) - 5)^(1/2),x)
```

```
[Out] int(1/(4*cos(d + e*x) + 3*sin(d + e*x) - 5)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{3\sin(d+ex)+4\cos(d+ex)-5}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-5+4*cos(e*x+d)+3*sin(e*x+d))**(1/2),x)
```

```
[Out] Integral(1/sqrt(3*sin(d + e*x) + 4*cos(d + e*x) - 5), x)
```

3.428 $\int \frac{1}{(-5+4\cos(d+ex)+3\sin(d+ex))^{3/2}}\,dx$

Optimal. Leaf size=96

$$\frac{3\cos(d+ex)-4\sin(d+ex)}{10e(3\sin(d+ex)+4\cos(d+ex)-5)^{3/2}}+\frac{\tan^{-1}\left(\frac{\sin\left(d+ex-\tan^{-1}\left(\frac{3}{4}\right)\right)}{\sqrt{2}\sqrt{\cos\left(d+ex-\tan^{-1}\left(\frac{3}{4}\right)\right)-1}}\right)}{10\sqrt{10}\,e}$$

```
[Out] 1/10*(3*cos(e*x+d)-4*sin(e*x+d))/e/(-5+4*cos(e*x+d)+3*sin(e*x+d))^(3/2)+1/1
00*arctan(1/2*sin(d+e*x-arctan(3/4))*2^(1/2)/(-1+cos(d+e*x-arctan(3/4)))^(1
/2))*10^(1/2)/e
```

Rubi [A] time = 0.05, antiderivative size = 96, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {3116, 3115, 2649, 204}

$$\frac{3\cos(d+ex)-4\sin(d+ex)}{10e(3\sin(d+ex)+4\cos(d+ex)-5)^{3/2}}+\frac{\tan^{-1}\left(\frac{\sin\left(d+ex-\tan^{-1}\left(\frac{3}{4}\right)\right)}{\sqrt{2}\sqrt{\cos\left(d+ex-\tan^{-1}\left(\frac{3}{4}\right)\right)-1}}\right)}{10\sqrt{10}\,e}$$

Antiderivative was successfully verified.

```
[In] Int[(-5 + 4*Cos[d + e*x] + 3*Sin[d + e*x])^(-3/2),x]
```

```
[Out] ArcTan[Sin[d + e*x - ArcTan[3/4]]/(Sqrt[2]*Sqrt[-1 + Cos[d + e*x - ArcTan[3
/4]]])]/(10*Sqrt[10]*e) + (3*Cos[d + e*x] - 4*Sin[d + e*x])/(10*e*(-5 + 4*C
os[d + e*x] + 3*Sin[d + e*x])^(3/2))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3115

```
Int[1/Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(
x_)]], x_Symbol] :> Int[1/Sqrt[a + Sqrt[b^2 + c^2]*Cos[d + e*x - ArcTan[b,
c]]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[a^2 - b^2 - c^2, 0]
```

Rule 3116

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(n_), x_Symbol] :> Simp[((c*Cos[d + e*x] - b*Sin[d + e*x])*(a + b*Cos[d + e
*x] + c*Sin[d + e*x])^n)/(a*e*(2*n + 1)), x] + Dist[(n + 1)/(a*(2*n + 1)),
Int[(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n + 1), x], x] /; FreeQ[{a, b, c
, d, e}, x] && EqQ[a^2 - b^2 - c^2, 0] && LtQ[n, -1]
```

Rubi steps

$$\int \frac{1}{(-5+4\cos(d+ex)+3\sin(d+ex))^{3/2}}\,dx = \frac{3\cos(d+ex)-4\sin(d+ex)}{10e(-5+4\cos(d+ex)+3\sin(d+ex))^{3/2}} - \frac{1}{20}\int \frac{}{\sqrt{-5+4\cos(}}$$

$$= \frac{3\cos(d+ex)-4\sin(d+ex)}{10e(-5+4\cos(d+ex)+3\sin(d+ex))^{3/2}} - \frac{1}{20}\int \frac{}{\sqrt{-5+5\cos}}$$

$$= \frac{3\cos(d+ex)-4\sin(d+ex)}{10e(-5+4\cos(d+ex)+3\sin(d+ex))^{3/2}} + \frac{\text{Subst}\left(\int \frac{1}{-10-x^2}\,dx,\right.}{}$$

$$= \frac{\tan^{-1}\left(\frac{\sin\left(d+ex-\tan^{-1}\left(\frac{3}{4}\right)\right)}{\sqrt{2}\sqrt{-1+\cos\left(d+ex-\tan^{-1}\left(\frac{3}{4}\right)\right)}}\right)}{10\sqrt{10}\,e} + \frac{3\cos(d+ex)-4\sin(}{10e(-5+4\cos(d+ex)+3\sin}$$

Mathematica [C] time = 0.31, size = 152, normalized size = 1.58

$$\frac{\left(\frac{1}{250}-\frac{i}{125}\right)\left(\cos\left(\frac{1}{2}(d+ex)\right)-3\sin\left(\frac{1}{2}(d+ex)\right)\right)\left((5+10i)\left(\sin\left(\frac{1}{2}(d+ex)\right)+3\cos\left(\frac{1}{2}(d+ex)\right)\right)-(1-i)\sqrt{-20-}\right.}{e(3\sin(d+ex)+4\cos(d+}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-5 + 4*Cos[d + e*x] + 3*Sin[d + e*x])^(-3/2),x]
```

```
[Out] ((1/250 - I/125)*(Cos[(d + e*x)/2] - 3*Sin[(d + e*x)/2])*((-1 + I)*Sqrt[-20
 - 15*I]*ArcTanh[(1/10 + (3*I)/10)*Sqrt[-4/5 - (3*I)/5]*(3 + Tan[(d + e*x)/
```

```
4])]*(Cos[(d + e*x)/2] - 3*Sin[(d + e*x)/2])^2 + (5 + 10*I)*(3*Cos[(d + e*x
)/2] + Sin[(d + e*x)/2])))/(e*(-5 + 4*Cos[d + e*x] + 3*Sin[d + e*x])^(3/2))
```

fricas [B] time = 1.03, size = 210, normalized size = 2.19

$$-\frac{\left(13\sqrt{10}\cos\left(ex+d\right)^{2}-9\left(\sqrt{10}\cos\left(ex+d\right)-2\sqrt{10}\right)\sin\left(ex+d\right)-\sqrt{10}\cos\left(ex+d\right)-14\sqrt{10}\right)\arctan\left(-\frac{(3}{}\right.}{100\left(13\,e\cos\left(ex+d\right)^{2}-e\cos\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-5+4*cos(e*x+d)+3*sin(e*x+d))^(3/2),x, algorithm="fricas")
```

```
[Out] -1/100*((13*sqrt(10)*cos(e*x + d)^2 - 9*(sqrt(10)*cos(e*x + d) - 2*sqrt(10)
)*sin(e*x + d) - sqrt(10)*cos(e*x + d) - 14*sqrt(10))*arctan(-1/10*(3*sqrt(
10)*cos(e*x + d) + sqrt(10)*sin(e*x + d) + 3*sqrt(10))*sqrt(4*cos(e*x + d)
+ 3*sin(e*x + d) - 5)/(cos(e*x + d) - 3*sin(e*x + d) + 1)) + 10*sqrt(4*cos(
e*x + d) + 3*sin(e*x + d) - 5)*(3*cos(e*x + d) + sin(e*x + d) + 3))/(13*e*c
os(e*x + d)^2 - e*cos(e*x + d) - 9*(e*cos(e*x + d) - 2*e)*sin(e*x + d) - 14
*e)
```

giac [C] time = 0.53, size = 249, normalized size = 2.59

$$-\frac{1}{450}\left(\frac{9\sqrt{10}\arctan\left(\frac{1}{10}\sqrt{10}\left(-3i\sqrt{\tan\left(\frac{1}{2}xe+\frac{1}{2}d\right)^{2}+1}+3i\tan\left(\frac{1}{2}xe+\frac{1}{2}d\right)-i\right)\right)}{\operatorname{sgn}\left(-3\tan\left(\frac{1}{2}xe+\frac{1}{2}d\right)+1\right)}+\frac{10\left(33i\left(\sqrt{\tan\left(\frac{1}{2}xe+\right.}\right.\right.}{\left(-3i\left(\sqrt{\tan}\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-5+4*cos(e*x+d)+3*sin(e*x+d))^(3/2),x, algorithm="giac")
```

```
[Out] -1/450*(9*sqrt(10)*arctan(1/10*sqrt(10)*(-3*I*sqrt(tan(1/2*x*e + 1/2*d)^2 +
 1) + 3*I*tan(1/2*x*e + 1/2*d) - I))/sgn(-3*tan(1/2*x*e + 1/2*d) + 1) + 10*
(33*I*(sqrt(tan(1/2*x*e + 1/2*d)^2 + 1) - tan(1/2*x*e + 1/2*d))^3 - 7*I*(sq
rt(tan(1/2*x*e + 1/2*d)^2 + 1) - tan(1/2*x*e + 1/2*d))^2 + 21*I*sqrt(tan(1/
2*x*e + 1/2*d)^2 + 1) - 21*I*tan(1/2*x*e + 1/2*d) + 9*I)/((-3*I*(sqrt(tan(1
/2*x*e + 1/2*d)^2 + 1) - tan(1/2*x*e + 1/2*d))^2 - 2*I*sqrt(tan(1/2*x*e + 1
/2*d)^2 + 1) + 2*I*tan(1/2*x*e + 1/2*d) + 3*I)^2*sgn(-3*tan(1/2*x*e + 1/2*d
) + 1)))*e^(-1)
```

maple [A] time = 0.37, size = 118, normalized size = 1.23

$$\frac{\left(-\sqrt{10}\ \arctan\left(\frac{\sqrt{-5\sin\left(ex+d+\arctan\left(\frac{4}{3}\right)\right)-5}\ \sqrt{10}}{10}\right)\sin\left(ex+d+\arctan\left(\frac{4}{3}\right)\right)+\sqrt{10}\ \arctan\left(\frac{\sqrt{-5\sin\left(ex+d+\arctan\left(\frac{4}{3}\right)\right)-5}\ \sqrt{}}{10}\right.\right.}{100\cos\left(ex+d+\arctan\left(\frac{4}{3}\right)\right)\sqrt{-5+5\sin\left(ex+\right.}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-5+4*cos(e*x+d)+3*sin(e*x+d))^(3/2),x)
```

```
[Out] 1/100*(-10^(1/2)*arctan(1/10*(-5*sin(e*x+d+arctan(4/3))-5)^(1/2)*10^(1/2))*
sin(e*x+d+arctan(4/3))+10^(1/2)*arctan(1/10*(-5*sin(e*x+d+arctan(4/3))-5)^(
1/2)*10^(1/2))+2*(-5*sin(e*x+d+arctan(4/3))-5)^(1/2))*(-5*sin(e*x+d+arctan(
4/3))-5)^(1/2)/cos(e*x+d+arctan(4/3))/(-5+5*sin(e*x+d+arctan(4/3)))^(1/2)/e
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(4\cos(ex+d)+3\sin(ex+d)-5)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-5+4*cos(e*x+d)+3*sin(e*x+d))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((4*cos(e*x + d) + 3*sin(e*x + d) - 5)^(-3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{(4\cos(d+e\,x)+3\sin(d+e\,x)-5)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(4*cos(d + e*x) + 3*sin(d + e*x) - 5)^(3/2),x)
```

```
[Out] int(1/(4*cos(d + e*x) + 3*sin(d + e*x) - 5)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(3\sin(d+ex)+4\cos(d+ex)-5)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-5+4*cos(e*x+d)+3*sin(e*x+d))**(3/2),x)
```

```
[Out] Integral((3*sin(d + e*x) + 4*cos(d + e*x) - 5)**(-3/2), x)
```

3.429 $\int \frac{1}{(-5+4\cos(d+ex)+3\sin(d+ex))^{5/2}}\,dx$

Optimal. Leaf size=142

$$-\frac{3(3\cos(d+ex)-4\sin(d+ex))}{400e(3\sin(d+ex)+4\cos(d+ex)-5)^{3/2}}+\frac{3\cos(d+ex)-4\sin(d+ex)}{20e(3\sin(d+ex)+4\cos(d+ex)-5)^{5/2}}-\frac{3\tan^{-1}\left(\frac{\sin\left(d+ex-\tan^{-}\right.}{\sqrt{2}\sqrt{\cos\left(d+ex-\tan\right.}}\right.}{400\sqrt{10}\,e}$$

```
[Out] 1/20*(3*cos(e*x+d)-4*sin(e*x+d))/e/(-5+4*cos(e*x+d)+3*sin(e*x+d))^(5/2)-3/4
00*(3*cos(e*x+d)-4*sin(e*x+d))/e/(-5+4*cos(e*x+d)+3*sin(e*x+d))^(3/2)-3/400
0*arctan(1/2*sin(d+e*x-arctan(3/4))*2^(1/2)/(-1+cos(d+e*x-arctan(3/4)))^(1/
2))*10^(1/2)/e
```

Rubi [A] time = 0.08, antiderivative size = 142, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {3116, 3115, 2649, 204}

$$-\frac{3(3\cos(d+ex)-4\sin(d+ex))}{400e(3\sin(d+ex)+4\cos(d+ex)-5)^{3/2}}+\frac{3\cos(d+ex)-4\sin(d+ex)}{20e(3\sin(d+ex)+4\cos(d+ex)-5)^{5/2}}-\frac{3\tan^{-1}\left(\frac{\sin\left(d+ex-\tan^{-}\right.}{\sqrt{2}\sqrt{\cos\left(d+ex-\tan\right.}}\right.}{400\sqrt{10}\,e}$$

Antiderivative was successfully verified.

```
[In] Int[(-5 + 4*Cos[d + e*x] + 3*Sin[d + e*x])^(-5/2),x]
```

```
[Out] (-3*ArcTan[Sin[d + e*x - ArcTan[3/4]]/(Sqrt[2]*Sqrt[-1 + Cos[d + e*x - ArcT
an[3/4]]])])/(400*Sqrt[10]*e) + (3*Cos[d + e*x] - 4*Sin[d + e*x])/(20*e*(-5
 + 4*Cos[d + e*x] + 3*Sin[d + e*x])^(5/2)) - (3*(3*Cos[d + e*x] - 4*Sin[d +
 e*x]))/(400*e*(-5 + 4*Cos[d + e*x] + 3*Sin[d + e*x])^(3/2))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3115

```
Int[1/Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(
x_)]], x_Symbol] :> Int[1/Sqrt[a + Sqrt[b^2 + c^2]*Cos[d + e*x - ArcTan[b,
c]]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[a^2 - b^2 - c^2, 0]
```

Rule 3116

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(n_), x_Symbol] :> Simp[((c*Cos[d + e*x] - b*Sin[d + e*x])*(a + b*Cos[d + e
*x] + c*Sin[d + e*x])^n)/(a*e*(2*n + 1)), x] + Dist[(n + 1)/(a*(2*n + 1)),
Int[(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n + 1), x], x] /; FreeQ[{a, b, c
, d, e}, x] && EqQ[a^2 - b^2 - c^2, 0] && LtQ[n, -1]
```

Rubi steps

$$\int \frac{1}{(-5+4\cos(d+ex)+3\sin(d+ex))^{5/2}}\,dx = \frac{3\cos(d+ex)-4\sin(d+ex)}{20e(-5+4\cos(d+ex)+3\sin(d+ex))^{5/2}} - \frac{3}{40}\int \frac{}{(-5+4\cos(d}$$

$$= \frac{3\cos(d+ex)-4\sin(d+ex)}{20e(-5+4\cos(d+ex)+3\sin(d+ex))^{5/2}} - \frac{3(3\cos(d+ex}{400e(-5+4\cos(d+}$$

$$= \frac{3\cos(d+ex)-4\sin(d+ex)}{20e(-5+4\cos(d+ex)+3\sin(d+ex))^{5/2}} - \frac{3(3\cos(d+ex}{400e(-5+4\cos(d+}$$

$$= \frac{3\cos(d+ex)-4\sin(d+ex)}{20e(-5+4\cos(d+ex)+3\sin(d+ex))^{5/2}} - \frac{3(3\cos(d+ex}{400e(-5+4\cos(d+}$$

$$= -\frac{3\tan^{-1}\left(\frac{\sin\left(d+ex-\tan^{-1}\left(\frac{3}{4}\right)\right)}{\sqrt{2}\sqrt{-1+\cos\left(d+ex-\tan^{-1}\left(\frac{3}{4}\right)\right)}}\right)}{400\sqrt{10}\,e} + \frac{3\cos(d+ex)-4\,\mathrm{si}}{20e(-5+4\cos(d+ex)+}$$

Mathematica [C] time = 0.38, size = 178, normalized size = 1.25

$$\frac{\left(\frac{1}{10000}+\frac{i}{20000}\right)\left(\cos\left(\frac{1}{2}(d+ex)\right)-3\sin\left(\frac{1}{2}(d+ex)\right)\right)\left((10-5i)\left(55\sin\left(\frac{1}{2}(d+ex)\right)-39\sin\left(\frac{3}{2}(d+ex)\right)+165\cos\right.\right.}{e}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-5 + 4*Cos[d + e*x] + 3*Sin[d + e*x])^(-5/2),x]
```

```
[Out] ((1/10000 + I/20000)*(Cos[(d + e*x)/2] - 3*Sin[(d + e*x)/2])*((6 + 6*I)*Sqr
t[-20 - 15*I]*ArcTanh[(1/10 + (3*I)/10)*Sqrt[-4/5 - (3*I)/5]*(3 + Tan[(d +
e*x)/4])]*(Cos[(d + e*x)/2] - 3*Sin[(d + e*x)/2])^4 + (10 - 5*I)*(165*Cos[(
d + e*x)/2] - 27*Cos[(3*(d + e*x))/2] + 55*Sin[(d + e*x)/2] - 39*Sin[(3*(d
+ e*x))/2])))/(e*(-5 + 4*Cos[d + e*x] + 3*Sin[d + e*x])^(5/2))
```

fricas [B] time = 0.89, size = 280, normalized size = 1.97

$$\frac{3\left(79\sqrt{10}\cos\left(ex+d\right)^{3}-123\sqrt{10}\cos\left(ex+d\right)^{2}+3\left(\sqrt{10}\cos\left(ex+d\right)^{2}+38\sqrt{10}\cos\left(ex+d\right)-44\sqrt{10}\right)\sin\left(e\right.}{4000\left(7\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-5+4*cos(e*x+d)+3*sin(e*x+d))^(5/2),x, algorithm="fricas")
```

```
[Out] 1/4000*(3*(79*sqrt(10)*cos(e*x + d)^3 - 123*sqrt(10)*cos(e*x + d)^2 + 3*(sq
rt(10)*cos(e*x + d)^2 + 38*sqrt(10)*cos(e*x + d) - 44*sqrt(10))*sin(e*x + d
) - 78*sqrt(10)*cos(e*x + d) + 124*sqrt(10))*arctan(-1/10*(3*sqrt(10)*cos(e
*x + d) + sqrt(10)*sin(e*x + d) + 3*sqrt(10))*sqrt(4*cos(e*x + d) + 3*sin(e
*x + d) - 5)/(cos(e*x + d) - 3*sin(e*x + d) + 1)) + 10*(27*cos(e*x + d)^2 +
 (39*cos(e*x + d) - 8)*sin(e*x + d) - 69*cos(e*x + d) - 96)*sqrt(4*cos(e*x
+ d) + 3*sin(e*x + d) - 5))/(79*e*cos(e*x + d)^3 - 123*e*cos(e*x + d)^2 - 7
8*e*cos(e*x + d) + 3*(e*cos(e*x + d)^2 + 38*e*cos(e*x + d) - 44*e)*sin(e*x
+ d) + 124*e)
```

giac [C] time = 0.80, size = 381, normalized size = 2.68

$$-\frac{1}{162000}\left(\frac{243\sqrt{10}\arctan\left(\frac{1}{10}\sqrt{10}\left(3i\sqrt{\tan\left(\frac{1}{2}xe+\frac{1}{2}d\right)^{2}+1}-3i\tan\left(\frac{1}{2}xe+\frac{1}{2}d\right)+i\right)\right)}{\operatorname{sgn}\left(-3\tan\left(\frac{1}{2}xe+\frac{1}{2}d\right)+1\right)}+\frac{10\left(15039i\left(\sqrt{\tan\right.\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-5+4*cos(e*x+d)+3*sin(e*x+d))^(5/2),x, algorithm="giac")
```

```
[Out] -1/162000*(243*sqrt(10)*arctan(1/10*sqrt(10)*(3*I*sqrt(tan(1/2*x*e + 1/2*d)
^2 + 1) - 3*I*tan(1/2*x*e + 1/2*d) + I))/sgn(-3*tan(1/2*x*e + 1/2*d) + 1) +
 10*(15039*I*(sqrt(tan(1/2*x*e + 1/2*d)^2 + 1) - tan(1/2*x*e + 1/2*d))^7 +
6291*I*(sqrt(tan(1/2*x*e + 1/2*d)^2 + 1) - tan(1/2*x*e + 1/2*d))^6 - 579*I*
(sqrt(tan(1/2*x*e + 1/2*d)^2 + 1) - tan(1/2*x*e + 1/2*d))^5 + 1645*I*(sqrt(
```

```
tan(1/2*x*e + 1/2*d)^2 + 1) - tan(1/2*x*e + 1/2*d))^4 + 25365*I*(sqrt(tan(1
/2*x*e + 1/2*d)^2 + 1) - tan(1/2*x*e + 1/2*d))^3 - 11367*I*(sqrt(tan(1/2*x*
e + 1/2*d)^2 + 1) - tan(1/2*x*e + 1/2*d))^2 + 4887*I*sqrt(tan(1/2*x*e + 1/2
*d)^2 + 1) - 4887*I*tan(1/2*x*e + 1/2*d) + 3807*I)/((3*I*(sqrt(tan(1/2*x*e
+ 1/2*d)^2 + 1) - tan(1/2*x*e + 1/2*d))^2 + 2*I*sqrt(tan(1/2*x*e + 1/2*d)^2
 + 1) - 2*I*tan(1/2*x*e + 1/2*d) - 3*I)^4*sgn(-3*tan(1/2*x*e + 1/2*d) + 1))
)*e^(-1)
```

maple [A] time = 0.34, size = 190, normalized size = 1.34

$$\frac{\left(3\sqrt{10}\ \arctan\left(\frac{\sqrt{-5\sin\left(ex+d+\arctan\left(\frac{4}{3}\right)\right)-5}\ \sqrt{10}}{10}\right)\left(\sin^2\left(ex+d+\arctan\left(\frac{4}{3}\right)\right)\right)-6\sqrt{10}\ \arctan\left(\frac{\sqrt{-5\sin\left(ex+d+\arctan\left(\frac{4}{3}\right)\right)}}{10}\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-5+4*cos(e*x+d)+3*sin(e*x+d))^(5/2),x)
```

```
[Out] 1/4000*(3*10^(1/2)*arctan(1/10*(-5*sin(e*x+d+arctan(4/3))-5)^(1/2)*10^(1/2)
)*sin(e*x+d+arctan(4/3))^2-6*10^(1/2)*arctan(1/10*(-5*sin(e*x+d+arctan(4/3)
)-5)^(1/2)*10^(1/2))*sin(e*x+d+arctan(4/3))+3*10^(1/2)*arctan(1/10*(-5*sin(
e*x+d+arctan(4/3))-5)^(1/2)*10^(1/2))-6*(-5*sin(e*x+d+arctan(4/3))-5)^(1/2)
*sin(e*x+d+arctan(4/3))+14*(-5*sin(e*x+d+arctan(4/3))-5)^(1/2))*(-5*sin(e*x
+d+arctan(4/3))-5)^(1/2)/(sin(e*x+d+arctan(4/3))-1)/cos(e*x+d+arctan(4/3))/
(-5+5*sin(e*x+d+arctan(4/3)))^(1/2)/e
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(4\cos(ex+d)+3\sin(ex+d)-5)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-5+4*cos(e*x+d)+3*sin(e*x+d))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((4*cos(e*x + d) + 3*sin(e*x + d) - 5)^(-5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{(4\cos(d+e\,x)+3\sin(d+e\,x)-5)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(4*cos(d + e*x) + 3*sin(d + e*x) - 5)^(5/2),x)
```

```
[Out] int(1/(4*cos(d + e*x) + 3*sin(d + e*x) - 5)^(5/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(3\sin(d+ex)+4\cos(d+ex)-5)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-5+4*cos(e*x+d)+3*sin(e*x+d))**(5/2),x)
```

```
[Out] Integral((3*sin(d + e*x) + 4*cos(d + e*x) - 5)**(-5/2), x)
```

3.430 $\int \left(\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)\right)^{7/2}\,dx$

Optimal. Leaf size=258

$$-\frac{2(c\cos(d+ex)-b\sin(d+ex))\left(\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)\right)^{5/2}}{7e}-\frac{24\sqrt{b^2+c^2}\,(c\cos(d+ex)-b\sin}{}$$

```
[Out] -24/35*(c*cos(e*x+d)-b*sin(e*x+d))*(b^2+c^2)^(1/2)*(b*cos(e*x+d)+c*sin(e*x+
d)+(b^2+c^2)^(1/2))^(3/2)/e-2/7*(c*cos(e*x+d)-b*sin(e*x+d))*(b*cos(e*x+d)+c
*sin(e*x+d)+(b^2+c^2)^(1/2))^(5/2)/e-256/35*(b^2+c^2)^(3/2)*(c*cos(e*x+d)-b
*sin(e*x+d))/e/(b*cos(e*x+d)+c*sin(e*x+d)+(b^2+c^2)^(1/2))^(1/2)-64/35*(b^2
+c^2)*(c*cos(e*x+d)-b*sin(e*x+d))*(b*cos(e*x+d)+c*sin(e*x+d)+(b^2+c^2)^(1/2
))^(1/2)/e
```

Rubi [A] time = 0.18, antiderivative size = 258, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {3113, 3112}

$$-\frac{2(c\cos(d+ex)-b\sin(d+ex))\left(\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)\right)^{5/2}}{7e}-\frac{24\sqrt{b^2+c^2}\,(c\cos(d+ex)-b\sin}{}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[b^2 + c^2] + b*Cos[d + e*x] + c*Sin[d + e*x])^(7/2),x]
```

```
[Out] (-256*(b^2 + c^2)^(3/2)*(c*Cos[d + e*x] - b*Sin[d + e*x]))/(35*e*Sqrt[Sqrt[
b^2 + c^2] + b*Cos[d + e*x] + c*Sin[d + e*x]]) - (64*(b^2 + c^2)*(c*Cos[d +
 e*x] - b*Sin[d + e*x])*Sqrt[Sqrt[b^2 + c^2] + b*Cos[d + e*x] + c*Sin[d + e
*x]])/(35*e) - (24*Sqrt[b^2 + c^2]*(c*Cos[d + e*x] - b*Sin[d + e*x])*(Sqrt[
b^2 + c^2] + b*Cos[d + e*x] + c*Sin[d + e*x])^(3/2))/(35*e) - (2*(c*Cos[d +
 e*x] - b*Sin[d + e*x])*(Sqrt[b^2 + c^2] + b*Cos[d + e*x] + c*Sin[d + e*x])
^(5/2))/(7*e)
```

Rule 3112

```
Int[Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_
)]], x_Symbol] :> Simp[(-2*(c*Cos[d + e*x] - b*Sin[d + e*x]))/(e*Sqrt[a + b
*Cos[d + e*x] + c*Sin[d + e*x]]), x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[a^
2 - b^2 - c^2, 0]
```

Rule 3113

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(n_), x_Symbol] :> -Simp[((c*Cos[d + e*x] - b*Sin[d + e*x])*(a + b*Cos[d +
e*x] + c*Sin[d + e*x])^(n - 1))/(e*n), x] + Dist[(a*(2*n - 1))/n, Int[(a +
b*Cos[d + e*x] + c*Sin[d + e*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e},
x] && EqQ[a^2 - b^2 - c^2, 0] && GtQ[n, 0]
```

Rubi steps

$$\int \left(\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)\right)^{7/2} dx = -\frac{2(c\cos(d+ex)-b\sin(d+ex))\left(\sqrt{b^2+c^2}+b\cos(d+e\right.}{7e}$$

$$= -\frac{24\sqrt{b^2+c^2}\,(c\cos(d+ex)-b\sin(d+ex))\left(\sqrt{b^2+c^2}+\right.}{35e}$$

$$= -\frac{64\left(b^2+c^2\right)(c\cos(d+ex)-b\sin(d+ex))\sqrt{\sqrt{b^2+c^2}+}}{35e}$$

$$= -\frac{256\left(b^2+c^2\right)^{3/2}(c\cos(d+ex)-b\sin(d+ex))}{35e\sqrt{\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)}} - \frac{64\left(b^2\right.}{}$$

Mathematica [C] time = 33.46, size = 11888, normalized size = 46.08

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Sqrt[b^2 + c^2] + b*Cos[d + e*x] + c*Sin[d + e*x])^(7/2),x]
[Out] Result too large to show
```

fricas [A] time = 1.52, size = 268, normalized size = 1.04

$$\frac{2\left(5\left(b^4-6b^2c^2+c^4\right)\cos(ex+d)^4-177b^4-310b^2c^2-128c^4+2\left(22b^4+15b^2c^2-27c^4\right)\cos(ex+d)^2+4\left(\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(e*x+d)+c*sin(e*x+d)+(b^2+c^2)^(1/2))^(7/2),x, algorithm="f
ricas")
```

```
[Out] 2/35*(5*(b^4 - 6*b^2*c^2 + c^4)*cos(e*x + d)^4 - 177*b^4 - 310*b^2*c^2 - 12
8*c^4 + 2*(22*b^4 + 15*b^2*c^2 - 27*c^4)*cos(e*x + d)^2 + 4*(5*(b^3*c - b*c
^3)*cos(e*x + d)^3 + (22*b^3*c + 27*b*c^3)*cos(e*x + d))*sin(e*x + d) + 2*(
11*(b^3 - 3*b*c^2)*cos(e*x + d)^3 + (53*b^3 + 86*b*c^2)*cos(e*x + d) + (53*
b^2*c + 64*c^3 + 11*(3*b^2*c - c^3)*cos(e*x + d)^2)*sin(e*x + d))*sqrt(b^2
+ c^2))*sqrt(b*cos(e*x + d) + c*sin(e*x + d) + sqrt(b^2 + c^2))/(c*e*cos(e*
x + d) - b*e*sin(e*x + d))
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(e*x+d)+c*sin(e*x+d)+(b^2+c^2)^(1/2))^(7/2),x, algorithm="g
iac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Simplification assuming b near 0Evaluation t
ime: 1.02sym2poly/r2sym(const gen & e,const index_m & i,const vecteur & l)
Error: Bad Argument Value
```

maple [A] time = 0.36, size = 306, normalized size = 1.19

$$\frac{2\left(1+\sin\left(ex+d-\arctan\left(-b,c\right)\right)\right)\left(\sin\left(ex+d-\arctan\left(-b,c\right)\right)-1\right)\left(5b^{4}\left(\sin^{3}\left(ex+d-\arctan\left(-b,c\right)\right)\right)+10b^{2}c\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*cos(e*x+d)+c*sin(e*x+d)+(b^2+c^2)^(1/2))^(7/2),x)
```

```
[Out] 2/35*(1+sin(e*x+d-arctan(-b,c)))*(sin(e*x+d-arctan(-b,c))-1)*(5*b^4*sin(e*x
+d-arctan(-b,c))^3+10*b^2*c^2*sin(e*x+d-arctan(-b,c))^3+5*c^4*sin(e*x+d-arc
tan(-b,c))^3+27*b^4*sin(e*x+d-arctan(-b,c))^2+54*b^2*c^2*sin(e*x+d-arctan(-
b,c))^2+27*c^4*sin(e*x+d-arctan(-b,c))^2+71*b^4*sin(e*x+d-arctan(-b,c))+142
*b^2*c^2*sin(e*x+d-arctan(-b,c))+71*c^4*sin(e*x+d-arctan(-b,c))+177*b^4+354
*b^2*c^2+177*c^4)/cos(e*x+d-arctan(-b,c))/((b^2*sin(e*x+d-arctan(-b,c))+c^2
*sin(e*x+d-arctan(-b,c))+b^2+c^2)/(b^2+c^2)^(1/2))^(1/2)/e
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(e*x+d)+c*sin(e*x+d)+(b^2+c^2)^(1/2))^(7/2),x, algorithm="m
axima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is und
efined.
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \left(b\,\cos\left(d+e\,x\right)+c\,\sin\left(d+e\,x\right)+\sqrt{b^2+c^2}\right)^{7/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*cos(d + e*x) + c*sin(d + e*x) + (b^2 + c^2)^(1/2))^(7/2),x)
```

```
[Out] int((b*cos(d + e*x) + c*sin(d + e*x) + (b^2 + c^2)^(1/2))^(7/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(e*x+d)+c*sin(e*x+d)+(b**2+c**2)**(1/2))**(7/2),x)
```

```
[Out] Timed out
```

3.431 $\int \left(\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)\right)^{5/2}\,dx$

Optimal. Leaf size=190

$$-\frac{2(c\cos(d+ex)-b\sin(d+ex))\left(\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)\right)^{3/2}}{5e}-\frac{16\sqrt{b^2+c^2}\,(c\cos(d+ex)-b\sin}{}$$

```
[Out] -2/5*(c*cos(e*x+d)-b*sin(e*x+d))*(b*cos(e*x+d)+c*sin(e*x+d)+(b^2+c^2)^(1/2)
)^(3/2)/e-64/15*(b^2+c^2)*(c*cos(e*x+d)-b*sin(e*x+d))/e/(b*cos(e*x+d)+c*sin
(e*x+d)+(b^2+c^2)^(1/2))^(1/2)-16/15*(c*cos(e*x+d)-b*sin(e*x+d))*(b^2+c^2)^
(1/2)*(b*cos(e*x+d)+c*sin(e*x+d)+(b^2+c^2)^(1/2))^(1/2)/e
```

Rubi [A] time = 0.12, antiderivative size = 190, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {3113, 3112}

$$-\frac{2(c\cos(d+ex)-b\sin(d+ex))\left(\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)\right)^{3/2}}{5e}-\frac{16\sqrt{b^2+c^2}\,(c\cos(d+ex)-b\sin}{}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[b^2 + c^2] + b*Cos[d + e*x] + c*Sin[d + e*x])^(5/2),x]
```

```
[Out] (-64*(b^2 + c^2)*(c*Cos[d + e*x] - b*Sin[d + e*x]))/(15*e*Sqrt[Sqrt[b^2 + c
^2] + b*Cos[d + e*x] + c*Sin[d + e*x]]) - (16*Sqrt[b^2 + c^2]*(c*Cos[d + e*
x] - b*Sin[d + e*x])*Sqrt[Sqrt[b^2 + c^2] + b*Cos[d + e*x] + c*Sin[d + e*x]
])/(15*e) - (2*(c*Cos[d + e*x] - b*Sin[d + e*x])*(Sqrt[b^2 + c^2] + b*Cos[d
 + e*x] + c*Sin[d + e*x])^(3/2))/(5*e)
```

Rule 3112

```
Int[Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_
)]], x_Symbol] :> Simp[(-2*(c*Cos[d + e*x] - b*Sin[d + e*x]))/(e*Sqrt[a + b
*Cos[d + e*x] + c*Sin[d + e*x]]), x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[a^
2 - b^2 - c^2, 0]
```

Rule 3113

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(n_), x_Symbol] :> -Simp[((c*Cos[d + e*x] - b*Sin[d + e*x])*(a + b*Cos[d +
e*x] + c*Sin[d + e*x])^(n - 1))/(e*n), x] + Dist[(a*(2*n - 1))/n, Int[(a +
b*Cos[d + e*x] + c*Sin[d + e*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e},
```

```
x] && EqQ[a^2 - b^2 - c^2, 0] && GtQ[n, 0]
```

Rubi steps

$$\int \left(\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)\right)^{5/2}dx = -\frac{2(c\cos(d+ex)-b\sin(d+ex))\left(\sqrt{b^2+c^2}+b\cos(d+e\right.}{5e}$$

$$= -\frac{16\sqrt{b^2+c^2}\,(c\cos(d+ex)-b\sin(d+ex))\sqrt{\sqrt{b^2+c^2}+}{15e}$$

$$= -\frac{64\left(b^2+c^2\right)(c\cos(d+ex)-b\sin(d+ex))}{15e\sqrt{\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)}} - \frac{16\sqrt{b^2}}{}$$

Mathematica [C] time = 34.26, size = 11771, normalized size = 61.95

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Sqrt[b^2 + c^2] + b*Cos[d + e*x] + c*Sin[d + e*x])^(5/2),x]
[Out] Result too large to show
```

fricas [A] time = 0.96, size = 189, normalized size = 0.99

$$\frac{2\left(3\left(b^3-3bc^2\right)\cos(ex+d)^3+\left(29b^3+38bc^2\right)\cos(ex+d)+\left(29b^2c+32c^3+3\left(3b^2c-c^3\right)\cos(ex+d)^2\right)\sin\right.}{15(}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(e*x+d)+c*sin(e*x+d)+(b^2+c^2)^(1/2))^(5/2),x, algorithm="f
ricas")
[Out] 2/15*(3*(b^3 - 3*b*c^2)*cos(e*x + d)^3 + (29*b^3 + 38*b*c^2)*cos(e*x + d) +
 (29*b^2*c + 32*c^3 + 3*(3*b^2*c - c^3)*cos(e*x + d)^2)*sin(e*x + d) + (22*
b*c*cos(e*x + d)*sin(e*x + d) + 11*(b^2 - c^2)*cos(e*x + d)^2 - 43*b^2 - 32
*c^2)*sqrt(b^2 + c^2))*sqrt(b*cos(e*x + d) + c*sin(e*x + d) + sqrt(b^2 + c^
2))/(c*e*cos(e*x + d) - b*e*sin(e*x + d))
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(e*x+d)+c*sin(e*x+d)+(b^2+c^2)^(1/2))^(5/2),x, algorithm="g
iac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Simplification assuming b near 0Evaluation t
ime: 0.43sym2poly/r2sym(const gen & e,const index_m & i,const vecteur & l)
Error: Bad Argument Value
```

maple [A] time = 0.37, size = 200, normalized size = 1.05

$$\frac{2\left(1+\sin\left(ex+d-\arctan\left(-b,c\right)\right)\right)\sqrt{b^2+c^2}\,\left(\sin\left(ex+d-\arctan\left(-b,c\right)\right)-1\right)\left(3b^2\left(\sin^2\left(ex+d-\arctan\left(-b,c\right)\right.\right.\right.}{15\cos\left(ex+d-\arctan\left(-b,c\right)\right)\sqrt{}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*cos(e*x+d)+c*sin(e*x+d)+(b^2+c^2)^(1/2))^(5/2),x)
```

```
[Out] 2/15*(1+sin(e*x+d-arctan(-b,c)))*(b^2+c^2)^(1/2)*(sin(e*x+d-arctan(-b,c))-1
)*(3*b^2*sin(e*x+d-arctan(-b,c))^2+3*c^2*sin(e*x+d-arctan(-b,c))^2+14*b^2*s
in(e*x+d-arctan(-b,c))+14*c^2*sin(e*x+d-arctan(-b,c))+43*b^2+43*c^2)/cos(e*
x+d-arctan(-b,c))/((b^2*sin(e*x+d-arctan(-b,c))+c^2*sin(e*x+d-arctan(-b,c))
+b^2+c^2)/(b^2+c^2)^(1/2))^(1/2)/e
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(e*x+d)+c*sin(e*x+d)+(b^2+c^2)^(1/2))^(5/2),x, algorithm="m
axima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is und
efined.
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \left(b\,\cos\left(d+e\,x\right)+c\,\sin\left(d+e\,x\right)+\sqrt{b^2+c^2}\right)^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*cos(d + e*x) + c*sin(d + e*x) + (b^2 + c^2)^(1/2))^(5/2),x)
```

```
[Out] int((b*cos(d + e*x) + c*sin(d + e*x) + (b^2 + c^2)^(1/2))^(5/2), x)
```

sympy **[F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(e*x+d)+c*sin(e*x+d)+(b**2+c**2)**(1/2))**(5/2),x)
[Out] Timed out
```

3.432 $\int \left(\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)\right)^{3/2}\,dx$

Optimal. Leaf size=126

$$-\frac{2\sqrt{\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)}\,(c\cos(d+ex)-b\sin(d+ex))}{3e}-\frac{8\sqrt{b^2+c^2}\,(c\cos(d+ex)-b\sin(}{3e\sqrt{\sqrt{b^2+c^2}+b\cos(d+ex)+c\,\mathrm{si}}}$$

```
[Out] -8/3*(c*cos(e*x+d)-b*sin(e*x+d))*(b^2+c^2)^(1/2)/e/(b*cos(e*x+d)+c*sin(e*x+
d)+(b^2+c^2)^(1/2))^(1/2)-2/3*(c*cos(e*x+d)-b*sin(e*x+d))*(b*cos(e*x+d)+c*s
in(e*x+d)+(b^2+c^2)^(1/2))^(1/2)/e
```

Rubi [A] time = 0.07, antiderivative size = 126, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {3113, 3112}

$$-\frac{2\sqrt{\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)}\,(c\cos(d+ex)-b\sin(d+ex))}{3e}-\frac{8\sqrt{b^2+c^2}\,(c\cos(d+ex)-b\sin(}{3e\sqrt{\sqrt{b^2+c^2}+b\cos(d+ex)+c\,\mathrm{si}}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[b^2 + c^2] + b*Cos[d + e*x] + c*Sin[d + e*x])^(3/2),x]
```

```
[Out] (-8*Sqrt[b^2 + c^2]*(c*Cos[d + e*x] - b*Sin[d + e*x]))/(3*e*Sqrt[Sqrt[b^2 +
 c^2] + b*Cos[d + e*x] + c*Sin[d + e*x]]) - (2*(c*Cos[d + e*x] - b*Sin[d +
e*x])*Sqrt[Sqrt[b^2 + c^2] + b*Cos[d + e*x] + c*Sin[d + e*x]])/(3*e)
```

Rule 3112

```
Int[Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_
)]], x_Symbol] :> Simp[(-2*(c*Cos[d + e*x] - b*Sin[d + e*x]))/(e*Sqrt[a + b
*Cos[d + e*x] + c*Sin[d + e*x]]), x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[a^
2 - b^2 - c^2, 0]
```

Rule 3113

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(n_), x_Symbol] :> -Simp[((c*Cos[d + e*x] - b*Sin[d + e*x])*(a + b*Cos[d +
e*x] + c*Sin[d + e*x])^(n - 1))/(e*n), x] + Dist[(a*(2*n - 1))/n, Int[(a +
b*Cos[d + e*x] + c*Sin[d + e*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e},
x] && EqQ[a^2 - b^2 - c^2, 0] && GtQ[n, 0]
```

Rubi steps

$$\int \left(\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)\right)^{3/2} dx = -\frac{2(c\cos(d+ex)-b\sin(d+ex))\sqrt{\sqrt{b^2+c^2}+b\cos(d+}{3e}$$

$$= -\frac{8\sqrt{b^2+c^2}\,(c\cos(d+ex)-b\sin(d+ex))}{3e\sqrt{\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)}} - \frac{2(c\cos}{}$$

Mathematica [C] time = 33.01, size = 11679, normalized size = 92.69

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Sqrt[b^2 + c^2] + b*Cos[d + e*x] + c*Sin[d + e*x])^(3/2),x]
```

```
[Out] Result too large to show
```

fricas [A] time = 0.68, size = 125, normalized size = 0.99

$$\frac{2\left(2\,bc\cos\left(ex+d\right)\sin\left(ex+d\right)+\left(b^2-c^2\right)\cos\left(ex+d\right)^2-5\,b^2-4\,c^2+4\,\sqrt{b^2+c^2}\,\left(b\cos\left(ex+d\right)+c\sin\left(ex+d\right.\right.\right.}{3\left(ce\cos\left(ex+d\right)-be\sin\left(ex+d\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(e*x+d)+c*sin(e*x+d)+(b^2+c^2)^(1/2))^(3/2),x, algorithm="f
ricas")
```

```
[Out] 2/3*(2*b*c*cos(e*x + d)*sin(e*x + d) + (b^2 - c^2)*cos(e*x + d)^2 - 5*b^2 -
 4*c^2 + 4*sqrt(b^2 + c^2)*(b*cos(e*x + d) + c*sin(e*x + d)))*sqrt(b*cos(e*
x + d) + c*sin(e*x + d) + sqrt(b^2 + c^2))/(c*e*cos(e*x + d) - b*e*sin(e*x
+ d))
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(e*x+d)+c*sin(e*x+d)+(b^2+c^2)^(1/2))^(3/2),x, algorithm="g
iac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Simplification assuming b near 0sym2poly/r2s
```

```
ym(const gen & e,const index_m & i,const vecteur & l) Error: Bad Argument V
alue
```

maple [A] time = 0.37, size = 126, normalized size = 1.00

$$\frac{2\left(1+\sin\left(ex+d-\arctan\left(-b,c\right)\right)\right)\left(b^{2}+c^{2}\right)\left(\sin\left(ex+d-\arctan\left(-b,c\right)\right)-1\right)\left(\sin\left(ex+d-\arctan\left(-b,c\right)\right)+5\right)}{3\cos\left(ex+d-\arctan\left(-b,c\right)\right)\sqrt{\frac{b^{2}\sin\left(ex+d-\arctan\left(-b,c\right)\right)+c^{2}\sin\left(ex+d-\arctan\left(-b,c\right)\right)+b^{2}+c^{2}}{\sqrt{b^{2}+c^{2}}}}\,e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*cos(e*x+d)+c*sin(e*x+d)+(b^2+c^2)^(1/2))^(3/2),x)
```

```
[Out] 2/3*(1+sin(e*x+d-arctan(-b,c)))*(b^2+c^2)*(sin(e*x+d-arctan(-b,c))-1)*(sin(
e*x+d-arctan(-b,c))+5)/cos(e*x+d-arctan(-b,c))/((b^2*sin(e*x+d-arctan(-b,c)
)+c^2*sin(e*x+d-arctan(-b,c))+b^2+c^2)/(b^2+c^2)^(1/2))^(1/2)/e
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(e*x+d)+c*sin(e*x+d)+(b^2+c^2)^(1/2))^(3/2),x, algorithm="m
axima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is und
efined.
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \left(b\,\cos\left(d+e\,x\right)+c\,\sin\left(d+e\,x\right)+\sqrt{b^2+c^2}\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*cos(d + e*x) + c*sin(d + e*x) + (b^2 + c^2)^(1/2))^(3/2),x)
```

```
[Out] int((b*cos(d + e*x) + c*sin(d + e*x) + (b^2 + c^2)^(1/2))^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\cos\left(d+ex\right)+c\sin\left(d+ex\right)+\sqrt{b^{2}+c^{2}}\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(e*x+d)+c*sin(e*x+d)+(b**2+c**2)**(1/2))**(3/2),x)
```

```
[Out] Integral((b*cos(d + e*x) + c*sin(d + e*x) + sqrt(b**2 + c**2))**(3/2), x)
```

3.433 $\int \sqrt{\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)}\,dx$

Optimal. Leaf size=55

$$-\frac{2(c\cos(d+ex)-b\sin(d+ex))}{e\sqrt{\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)}}$$

```
[Out] -2*(c*cos(e*x+d)-b*sin(e*x+d))/e/(b*cos(e*x+d)+c*sin(e*x+d)+(b^2+c^2)^(1/2)
)^(1/2)
```

Rubi [A] time = 0.03, antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.031, Rules used = {3112}

$$-\frac{2(c\cos(d+ex)-b\sin(d+ex))}{e\sqrt{\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Sqrt[b^2 + c^2] + b*Cos[d + e*x] + c*Sin[d + e*x]],x]
```

```
[Out] (-2*(c*Cos[d + e*x] - b*Sin[d + e*x]))/(e*Sqrt[Sqrt[b^2 + c^2] + b*Cos[d +
e*x] + c*Sin[d + e*x]])
```

Rule 3112

```
Int[Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_
)]], x_Symbol] :> Simp[(-2*(c*Cos[d + e*x] - b*Sin[d + e*x]))/(e*Sqrt[a + b
*Cos[d + e*x] + c*Sin[d + e*x]]), x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[a^
2 - b^2 - c^2, 0]
```

Rubi steps

$$\int \sqrt{\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)}\,dx=-\frac{2(c\cos(d+ex)-b\sin(d+ex))}{e\sqrt{\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)}}$$

Mathematica [C] time = 32.71, size = 11586, normalized size = 210.65

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[Sqrt[b^2 + c^2] + b*Cos[d + e*x] + c*Sin[d + e*x]],x]
```

```
[Out] Result too large to show
```

fricas [A] time = 0.91, size = 80, normalized size = 1.45

$$\frac{2\sqrt{b\cos(ex+d)+c\sin(ex+d)+\sqrt{b^2+c^2}}\left(b\cos(ex+d)+c\sin(ex+d)-\sqrt{b^2+c^2}\right)}{ce\cos(ex+d)-be\sin(ex+d)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(e*x+d)+c*sin(e*x+d)+(b^2+c^2)^(1/2))^(1/2),x, algorithm="f
ricas")
```

```
[Out] 2*sqrt(b*cos(e*x + d) + c*sin(e*x + d) + sqrt(b^2 + c^2))*(b*cos(e*x + d) +
 c*sin(e*x + d) - sqrt(b^2 + c^2))/(c*e*cos(e*x + d) - b*e*sin(e*x + d))
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(e*x+d)+c*sin(e*x+d)+(b^2+c^2)^(1/2))^(1/2),x, algorithm="g
iac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Simplification assuming b near 0sym2poly/r2s
ym(const gen & e,const index_m & i,const vecteur & l) Error: Bad Argument V
alue
```

maple [B] time = 0.36, size = 113, normalized size = 2.05

$$\frac{2\left(1+\sin\left(ex+d-\arctan\left(-b,c\right)\right)\right)\sqrt{b^2+c^2}\,\left(\sin\left(ex+d-\arctan\left(-b,c\right)\right)-1\right)}{\cos\left(ex+d-\arctan\left(-b,c\right)\right)\sqrt{\frac{b^2\sin(ex+d-\arctan(-b,c))+c^2\sin(ex+d-\arctan(-b,c))+b^2+c^2}{\sqrt{b^2+c^2}}}\;e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*cos(e*x+d)+c*sin(e*x+d)+(b^2+c^2)^(1/2))^(1/2),x)
```

```
[Out] 2*(1+sin(e*x+d-arctan(-b,c)))*(b^2+c^2)^(1/2)*(sin(e*x+d-arctan(-b,c))-1)/c
os(e*x+d-arctan(-b,c))/((b^2*sin(e*x+d-arctan(-b,c))+c^2*sin(e*x+d-arctan(-
b,c))+b^2+c^2)/(b^2+c^2)^(1/2))^(1/2)/e
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(e*x+d)+c*sin(e*x+d)+(b^2+c^2)^(1/2))^(1/2),x, algorithm="m
axima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is und
efined.
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \sqrt{b\,\cos\left(d+e\,x\right)+c\,\sin\left(d+e\,x\right)+\sqrt{b^2+c^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*cos(d + e*x) + c*sin(d + e*x) + (b^2 + c^2)^(1/2))^(1/2),x)
```

```
[Out] int((b*cos(d + e*x) + c*sin(d + e*x) + (b^2 + c^2)^(1/2))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{b\cos\left(d+ex\right)+c\sin\left(d+ex\right)+\sqrt{b^2+c^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(e*x+d)+c*sin(e*x+d)+(b**2+c**2)**(1/2))**(1/2),x)
```

```
[Out] Integral(sqrt(b*cos(d + e*x) + c*sin(d + e*x) + sqrt(b**2 + c**2)), x)
```

3.434 $\int \frac{1}{\sqrt{\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)}}\,dx$

Optimal. Leaf size=88

$$\frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt[4]{b^2+c^2}\sin\left(-\tan^{-1}(b,c)+d+ex\right)}{\sqrt{2}\sqrt{\sqrt{b^2+c^2}\cos\left(-\tan^{-1}(b,c)+d+ex\right)+\sqrt{b^2+c^2}}}\right)}{e\sqrt[4]{b^2+c^2}}$$

```
[Out] arctanh(1/2*(b^2+c^2)^(1/4)*sin(d+e*x-arctan(b,c))*2^(1/2)/((b^2+c^2)^(1/2)
+cos(d+e*x-arctan(b,c))*(b^2+c^2)^(1/2))^(1/2))*2^(1/2)/(b^2+c^2)^(1/4)/e
```

Rubi [A] time = 0.12, antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.094, Rules used = {3115, 2649, 206}

$$\frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt[4]{b^2+c^2}\sin\left(-\tan^{-1}(b,c)+d+ex\right)}{\sqrt{2}\sqrt{\sqrt{b^2+c^2}\cos\left(-\tan^{-1}(b,c)+d+ex\right)+\sqrt{b^2+c^2}}}\right)}{e\sqrt[4]{b^2+c^2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[Sqrt[b^2 + c^2] + b*Cos[d + e*x] + c*Sin[d + e*x]],x]
```

```
[Out] (Sqrt[2]*ArcTanh[((b^2 + c^2)^(1/4)*Sin[d + e*x - ArcTan[b, c]])/(Sqrt[2]*S
qrt[Sqrt[b^2 + c^2] + Sqrt[b^2 + c^2]*Cos[d + e*x - ArcTan[b, c]]])])/((b^2
 + c^2)^(1/4)*e)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3115

```
Int[1/Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(
x_)]], x_Symbol] :> Int[1/Sqrt[a + Sqrt[b^2 + c^2]*Cos[d + e*x - ArcTan[b,
```

```
c]]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[a^2 - b^2 - c^2, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)}}\,dx = \int \frac{1}{\sqrt{\sqrt{b^2+c^2}+\sqrt{b^2+c^2}\cos\left(d+ex-\tan^{-1}(b,c)\right)}}\,dx$$

$$= -\frac{2\,\text{Subst}\left(\int \frac{1}{2\sqrt{b^2+c^2}-x^2}\,dx, x, -\frac{\sqrt{b^2+c^2}\sin\left(d+ex-\tan^{-1}(b,c)\right)}{\sqrt{\sqrt{b^2+c^2}+\sqrt{b^2+c^2}\cos\left(d+ex-\tan^{-1}\right.}}\right.}{e}$$

$$= \frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt[4]{b^2+c^2}\sin\left(d+ex-\tan^{-1}(b,c)\right)}{\sqrt{2}\sqrt{\sqrt{b^2+c^2}+\sqrt{b^2+c^2}\cos\left(d+ex-\tan^{-1}(b,c)\right)}}\right)}{\sqrt[4]{b^2+c^2}\,e}$$

Mathematica [C] time = 33.83, size = 63264, normalized size = 718.91

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/Sqrt[Sqrt[b^2 + c^2] + b*Cos[d + e*x] + c*Sin[d + e*x]],x]
[Out] Result too large to show
```

fricas [B] time = 2.78, size = 349, normalized size = 3.97

$$\frac{\sqrt{2}\log\left(\frac{\left(3b^2c-c^3\right)\cos(ex+d)^3+\left(b^2c+4c^3\right)\cos(ex+d)-\left(3b^3+4bc^2+\left(b^3-3bc^2\right)\cos(ex+d)^2\right)\sin(ex+d)+\frac{2\sqrt{2}\left(2\left(b^3+bc^2\right)\cos(ex+d)+2\left(b^2c+c^3\right)\sin(ex+d)\right.}{}}{3b^2c\cos(ex+d)-\left(3b^2c-c^3\right)\cos(\right.}{2\left(b^2\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*cos(e*x+d)+c*sin(e*x+d)+(b^2+c^2)^(1/2))^(1/2),x, algorithm=
"fricas")
[Out] 1/2*sqrt(2)*log(((3*b^2*c - c^3)*cos(e*x + d)^3 + (b^2*c + 4*c^3)*cos(e*x +
 d) - (3*b^3 + 4*b*c^2 + (b^3 - 3*b*c^2)*cos(e*x + d)^2)*sin(e*x + d) + 2*s
```

```
qrt(2)*(2*(b^3 + b*c^2)*cos(e*x + d) + 2*(b^2*c + c^3)*sin(e*x + d) - (2*b*
c*cos(e*x + d)*sin(e*x + d) + (b^2 - c^2)*cos(e*x + d)^2 + b^2 + 2*c^2)*sqr
t(b^2 + c^2))*sqrt(b*cos(e*x + d) + c*sin(e*x + d) + sqrt(b^2 + c^2))/(b^2
+ c^2)^(1/4) - 4*(2*b*c*cos(e*x + d)^2 - (b^2 - c^2)*cos(e*x + d)*sin(e*x +
 d) - b*c)*sqrt(b^2 + c^2))/(3*b^2*c*cos(e*x + d) - (3*b^2*c - c^3)*cos(e*x
 + d)^3 - (b^3 - (b^3 - 3*b*c^2)*cos(e*x + d)^2)*sin(e*x + d)))/((b^2 + c^2
)^(1/4)*e)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*cos(e*x+d)+c*sin(e*x+d)+(b^2+c^2)^(1/2))^(1/2),x, algorithm=
"giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Simplification assuming b near 0sym2poly/r2s
ym(const gen & e,const index_m & i,const vecteur & l) Error: Bad Argument V
alue
```

maple [B] time = 0.38, size = 172, normalized size = 1.95

$$-\frac{(1+\sin(ex+d-\arctan(-b,c)))\sqrt{-\sqrt{b^2+c^2}\,(\sin(ex+d-\arctan(-b,c))-1)}\,\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{-\sqrt{b^2+c^2}\,(\sin(ex\cdots}}{2(b\cdots}\right)}{\left(b^2+c^2\right)^{\frac{1}{4}}\cos(ex+d-\arctan(-b,c))\sqrt{\frac{b^2\sin(ex+d-\arctan(-b,c))+c^2\sin(ex+d-\arctan(-b,c))+b^2+c^2}{\sqrt{b^2+c^2}}}\,e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*cos(e*x+d)+c*sin(e*x+d)+(b^2+c^2)^(1/2))^(1/2),x)
```

```
[Out] -(1+sin(e*x+d-arctan(-b,c)))*(-(b^2+c^2)^(1/2)*(sin(e*x+d-arctan(-b,c))-1))
^(1/2)*2^(1/2)/(b^2+c^2)^(1/4)*arctanh(1/2*(-(b^2+c^2)^(1/2)*(sin(e*x+d-arc
tan(-b,c))-1))^(1/2)*2^(1/2)/(b^2+c^2)^(1/4))/cos(e*x+d-arctan(-b,c))/((b^2
*sin(e*x+d-arctan(-b,c))+c^2*sin(e*x+d-arctan(-b,c))+b^2+c^2)/(b^2+c^2)^(1/
2))^(1/2)/e
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*cos(e*x+d)+c*sin(e*x+d)+(b^2+c^2)^(1/2))^(1/2),x, algorithm=
"maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is und
efined.
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sqrt{b\cos\left(d+e\,x\right)+c\sin\left(d+e\,x\right)+\sqrt{b^2+c^2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*cos(d + e*x) + c*sin(d + e*x) + (b^2 + c^2)^(1/2))^(1/2),x)
[Out] int(1/(b*cos(d + e*x) + c*sin(d + e*x) + (b^2 + c^2)^(1/2))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{b\cos\left(d+ex\right)+c\sin\left(d+ex\right)+\sqrt{b^2+c^2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*cos(e*x+d)+c*sin(e*x+d)+(b**2+c**2)**(1/2))**(1/2),x)
[Out] Integral(1/sqrt(b*cos(d + e*x) + c*sin(d + e*x) + sqrt(b**2 + c**2)), x)
```

3.435 $\int \frac{1}{\left(\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)\right)^{3/2}}\,dx$

Optimal. Leaf size=160

$$\frac{\tanh^{-1}\left(\frac{\sqrt[4]{b^2+c^2}\,\sin\left(-\tan^{-1}(b,c)+d+ex\right)}{\sqrt{2}\sqrt{\sqrt{b^2+c^2}\,\cos\left(-\tan^{-1}(b,c)+d+ex\right)+\sqrt{b^2+c^2}}}\right)}{2\sqrt{2}\,e\left(b^2+c^2\right)^{3/4}}-\frac{c\cos(d+ex)-b\sin(d+ex)}{2e\sqrt{b^2+c^2}\left(\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)\right)^{3/2}}$$

```
[Out] 1/4*arctanh(1/2*(b^2+c^2)^(1/4)*sin(d+e*x-arctan(b,c))*2^(1/2)/((b^2+c^2)^(
1/2)+cos(d+e*x-arctan(b,c))*(b^2+c^2)^(1/2))^(1/2))/(b^2+c^2)^(3/4)/e*2^(1/
2)+1/2*(-c*cos(e*x+d)+b*sin(e*x+d))/e/(b^2+c^2)^(1/2)/(b*cos(e*x+d)+c*sin(e
*x+d)+(b^2+c^2)^(1/2))^(3/2)
```

Rubi [A] time = 0.13, antiderivative size = 160, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {3116, 3115, 2649, 206}

$$\frac{\tanh^{-1}\left(\frac{\sqrt[4]{b^2+c^2}\,\sin\left(-\tan^{-1}(b,c)+d+ex\right)}{\sqrt{2}\sqrt{\sqrt{b^2+c^2}\,\cos\left(-\tan^{-1}(b,c)+d+ex\right)+\sqrt{b^2+c^2}}}\right)}{2\sqrt{2}\,e\left(b^2+c^2\right)^{3/4}}-\frac{c\cos(d+ex)-b\sin(d+ex)}{2e\sqrt{b^2+c^2}\left(\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[b^2 + c^2] + b*Cos[d + e*x] + c*Sin[d + e*x])^(-3/2),x]
```

```
[Out] ArcTanh[((b^2 + c^2)^(1/4)*Sin[d + e*x - ArcTan[b, c]])/(Sqrt[2]*Sqrt[Sqrt[
b^2 + c^2] + Sqrt[b^2 + c^2]*Cos[d + e*x - ArcTan[b, c]]])]/(2*Sqrt[2]*(b^2
 + c^2)^(3/4)*e) - (c*Cos[d + e*x] - b*Sin[d + e*x])/(2*Sqrt[b^2 + c^2]*e*(
Sqrt[b^2 + c^2] + b*Cos[d + e*x] + c*Sin[d + e*x])^(3/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
```

```
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3115

```
Int[1/Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(
x_)]], x_Symbol] :> Int[1/Sqrt[a + Sqrt[b^2 + c^2]*Cos[d + e*x - ArcTan[b,
c]]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[a^2 - b^2 - c^2, 0]
```

Rule 3116

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(n_), x_Symbol] :> Simp[((c*Cos[d + e*x] - b*Sin[d + e*x])*(a + b*Cos[d + e
*x] + c*Sin[d + e*x])^n)/(a*e*(2*n + 1)), x] + Dist[(n + 1)/(a*(2*n + 1)),
Int[(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n + 1), x], x] /; FreeQ[{a, b, c
, d, e}, x] && EqQ[a^2 - b^2 - c^2, 0] && LtQ[n, -1]
```

Rubi steps

$$\int \frac{1}{\left(\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)\right)^{3/2}}\,dx = -\frac{c\cos(d+ex)-b\sin(d+ex)}{2\sqrt{b^2+c^2}\,e\left(\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)\right)^{3}}$$

$$= -\frac{c\cos(d+ex)-b\sin(d+ex)}{2\sqrt{b^2+c^2}\,e\left(\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)\right)^{3}}$$

$$= -\frac{c\cos(d+ex)-b\sin(d+ex)}{2\sqrt{b^2+c^2}\,e\left(\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)\right)^{3}}$$

$$= \frac{\tanh^{-1}\left(\frac{\sqrt[4]{b^2+c^2}\,\sin\left(d+ex-\tan^{-1}(b,c)\right)}{\sqrt{2}\sqrt{\sqrt{b^2+c^2}+\sqrt{b^2+c^2}\,\cos\left(d+ex-\tan^{-1}(b,c)\right)}}\right)}{2\sqrt{2}\left(b^2+c^2\right)^{3/4}e} - \frac{}{2\sqrt{b^2+}}$$

Mathematica [F] time = 180.02, size = 0, normalized size = 0.00

$Aborted

Verification is Not applicable to the result.

```
[In] Integrate[(Sqrt[b^2 + c^2] + b*Cos[d + e*x] + c*Sin[d + e*x])^(-3/2),x]
```

```
[Out] $Aborted
```

fricas [B] time = 1.82, size = 646, normalized size = 4.04

$$-\frac{\left(3\sqrt{2}\,b^2c\cos\left(ex+d\right)-\sqrt{2}\left(3\,b^2c-c^3\right)\cos\left(ex+d\right)^3-\left(\sqrt{2}\,b^3-\sqrt{2}\left(b^3-3\,bc^2\right)\cos\left(ex+d\right)^2\right)\sin\left(ex+d\right)\right)\left(b^2\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*cos(e*x+d)+c*sin(e*x+d)+(b^2+c^2)^(1/2))^(3/2),x, algorithm=
"fricas")
```

```
[Out] -1/8*((3*sqrt(2)*b^2*c*cos(e*x + d) - sqrt(2)*(3*b^2*c - c^3)*cos(e*x + d)^
3 - (sqrt(2)*b^3 - sqrt(2)*(b^3 - 3*b*c^2)*cos(e*x + d)^2)*sin(e*x + d))*(b
^2 + c^2)^(1/4)*log(((3*b^2*c - c^3)*cos(e*x + d)^3 + (b^2*c + 4*c^3)*cos(e
*x + d) - (3*b^3 + 4*b*c^2 + (b^3 - 3*b*c^2)*cos(e*x + d)^2)*sin(e*x + d) -
 2*(2*sqrt(2)*b*c*cos(e*x + d)*sin(e*x + d) + sqrt(2)*(b^2 - c^2)*cos(e*x +
 d)^2 + sqrt(2)*(b^2 + 2*c^2) - 2*(sqrt(2)*b*cos(e*x + d) + sqrt(2)*c*sin(e
*x + d))*sqrt(b^2 + c^2))*(b^2 + c^2)^(1/4)*sqrt(b*cos(e*x + d) + c*sin(e*x
 + d) + sqrt(b^2 + c^2)) - 4*(2*b*c*cos(e*x + d)^2 - (b^2 - c^2)*cos(e*x +
d)*sin(e*x + d) - b*c)*sqrt(b^2 + c^2))/(3*b^2*c*cos(e*x + d) - (3*b^2*c -
c^3)*cos(e*x + d)^3 - (b^3 - (b^3 - 3*b*c^2)*cos(e*x + d)^2)*sin(e*x + d)))
 + 4*(2*(b^3 + b*c^2)*cos(e*x + d) + 2*(b^2*c + c^3)*sin(e*x + d) - (2*b*c*
cos(e*x + d)*sin(e*x + d) + (b^2 - c^2)*cos(e*x + d)^2 + b^2 + 2*c^2)*sqrt(
b^2 + c^2))*sqrt(b*cos(e*x + d) + c*sin(e*x + d) + sqrt(b^2 + c^2)))/((3*b^
4*c + 2*b^2*c^3 - c^5)*e*cos(e*x + d)^3 - 3*(b^4*c + b^2*c^3)*e*cos(e*x + d
) - ((b^5 - 2*b^3*c^2 - 3*b*c^4)*e*cos(e*x + d)^2 - (b^5 + b^3*c^2)*e)*sin(
e*x + d))
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*cos(e*x+d)+c*sin(e*x+d)+(b^2+c^2)^(1/2))^(3/2),x, algorithm=
"giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Simplification assuming b near 0sym2poly/r2s
ym(const gen & e,const index_m & i,const vecteur & l) Error: Bad Argument V
alue
```

maple [B] time = 0.41, size = 350, normalized size = 2.19

$$-\frac{\left(\sin\left(ex+d-\arctan\left(-b,c\right)\right)\sqrt{2}\ \operatorname{arctanh}\left(\frac{\sqrt{-\sqrt{b^2+c^2}\ \sin(ex+d-\arctan(-b,c))+\sqrt{b^2+c^2}}\ \sqrt{2}}{2\left(b^2+c^2\right)^{\frac{1}{4}}}\right)\left(b^2+c^2\right)+2\sqrt{-\sqrt{b^2+c^2}}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*cos(e*x+d)+c*sin(e*x+d)+(b^2+c^2)^(1/2))^(3/2),x)
```

```
[Out] -1/4/(b^2+c^2)^(7/4)*(sin(e*x+d-arctan(-b,c))*2^(1/2)*arctanh(1/2*(-(b^2+c^
2)^(1/2)*sin(e*x+d-arctan(-b,c))+(b^2+c^2)^(1/2))^(1/2)*2^(1/2)/(b^2+c^2)^(
1/4))*(b^2+c^2)+2*(-(b^2+c^2)^(1/2)*sin(e*x+d-arctan(-b,c))+(b^2+c^2)^(1/2)
)^(1/2)*(b^2+c^2)^(3/4)+2^(1/2)*arctanh(1/2*(-(b^2+c^2)^(1/2)*sin(e*x+d-arc
tan(-b,c))+(b^2+c^2)^(1/2))^(1/2)*2^(1/2)/(b^2+c^2)^(1/4))*b^2+2^(1/2)*arct
anh(1/2*(-(b^2+c^2)^(1/2)*sin(e*x+d-arctan(-b,c))+(b^2+c^2)^(1/2))^(1/2)*2^
(1/2)/(b^2+c^2)^(1/4))*c^2)*(-(b^2+c^2)^(1/2)*(sin(e*x+d-arctan(-b,c))-1))^
(1/2)/cos(e*x+d-arctan(-b,c))/((b^2*sin(e*x+d-arctan(-b,c))+c^2*sin(e*x+d-a
rctan(-b,c))+b^2+c^2)/(b^2+c^2)^(1/2))^(1/2)/e
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*cos(e*x+d)+c*sin(e*x+d)+(b^2+c^2)^(1/2))^(3/2),x, algorithm=
"maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is und
efined.
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{{\left(b\,\cos\left(d+e\,x\right)+c\,\sin\left(d+e\,x\right)+\sqrt{b^2+c^2}\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*cos(d + e*x) + c*sin(d + e*x) + (b^2 + c^2)^(1/2))^(3/2),x)
```

```
[Out] int(1/(b*cos(d + e*x) + c*sin(d + e*x) + (b^2 + c^2)^(1/2))^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b\cos\left(d+ex\right)+c\sin\left(d+ex\right)+\sqrt{b^2+c^2}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*cos(e*x+d)+c*sin(e*x+d)+(b**2+c**2)**(1/2))**(3/2),x)
[Out] Integral((b*cos(d + e*x) + c*sin(d + e*x) + sqrt(b**2 + c**2))**(-3/2), x)
```

3.436 $\int \frac{1}{\left(\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)\right)^{5/2}}\,dx$

Optimal. Leaf size=226

$$\frac{3\tanh^{-1}\left(\frac{\sqrt[4]{b^2+c^2}\sin\left(-\tan^{-1}(b,c)+d+ex\right)}{\sqrt{2}\sqrt{\sqrt{b^2+c^2}\cos\left(-\tan^{-1}(b,c)+d+ex\right)+\sqrt{b^2+c^2}}}\right)}{16\sqrt{2}\,e\left(b^2+c^2\right)^{5/4}}-\frac{3(c\cos(d+ex)-b\sin(d+ex))}{16e\left(b^2+c^2\right)\left(\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)\right)^{3/2}}-$$

```
[Out] 3/32*arctanh(1/2*(b^2+c^2)^(1/4)*sin(d+e*x-arctan(b,c))*2^(1/2)/((b^2+c^2)^
(1/2)+cos(d+e*x-arctan(b,c))*(b^2+c^2)^(1/2))^(1/2))/(b^2+c^2)^(5/4)/e*2^(1
/2)+1/4*(-c*cos(e*x+d)+b*sin(e*x+d))/e/(b^2+c^2)^(1/2)/(b*cos(e*x+d)+c*sin(
e*x+d)+(b^2+c^2)^(1/2))^(5/2)-3/16*(c*cos(e*x+d)-b*sin(e*x+d))/(b^2+c^2)/e/
(b*cos(e*x+d)+c*sin(e*x+d)+(b^2+c^2)^(1/2))^(3/2)
```

Rubi [A] time = 0.19, antiderivative size = 226, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {3116, 3115, 2649, 206}

$$\frac{3\tanh^{-1}\left(\frac{\sqrt[4]{b^2+c^2}\sin\left(-\tan^{-1}(b,c)+d+ex\right)}{\sqrt{2}\sqrt{\sqrt{b^2+c^2}\cos\left(-\tan^{-1}(b,c)+d+ex\right)+\sqrt{b^2+c^2}}}\right)}{16\sqrt{2}\,e\left(b^2+c^2\right)^{5/4}}-\frac{3(c\cos(d+ex)-b\sin(d+ex))}{16e\left(b^2+c^2\right)\left(\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)\right)^{3/2}}-$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[b^2 + c^2] + b*Cos[d + e*x] + c*Sin[d + e*x])^(-5/2),x]
```

```
[Out] (3*ArcTanh[((b^2 + c^2)^(1/4)*Sin[d + e*x - ArcTan[b, c]])/(Sqrt[2]*Sqrt[Sq
rt[b^2 + c^2] + Sqrt[b^2 + c^2]*Cos[d + e*x - ArcTan[b, c]]])])/(16*Sqrt[2]
*(b^2 + c^2)^(5/4)*e) - (c*Cos[d + e*x] - b*Sin[d + e*x])/(4*Sqrt[b^2 + c^2
]*e*(Sqrt[b^2 + c^2] + b*Cos[d + e*x] + c*Sin[d + e*x])^(5/2)) - (3*(c*Cos[
d + e*x] - b*Sin[d + e*x]))/(16*(b^2 + c^2)*e*(Sqrt[b^2 + c^2] + b*Cos[d +
e*x] + c*Sin[d + e*x])^(3/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3115

```
Int[1/Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(
x_)]], x_Symbol] :> Int[1/Sqrt[a + Sqrt[b^2 + c^2]*Cos[d + e*x - ArcTan[b,
c]]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[a^2 - b^2 - c^2, 0]
```

Rule 3116

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(n_), x_Symbol] :> Simp[((c*Cos[d + e*x] - b*Sin[d + e*x])*(a + b*Cos[d + e
*x] + c*Sin[d + e*x])^n)/(a*e*(2*n + 1)), x] + Dist[(n + 1)/(a*(2*n + 1)),
Int[(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n + 1), x], x] /; FreeQ[{a, b, c
, d, e}, x] && EqQ[a^2 - b^2 - c^2, 0] && LtQ[n, -1]
```

Rubi steps

$$\int \frac{1}{\left(\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)\right)^{5/2}}\,dx = -\frac{c\cos(d+ex)-b\sin(d+ex)}{4\sqrt{b^2+c^2}\,e\left(\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)\right)^{5}}$$

$$= -\frac{c\cos(d+ex)-b\sin(d+ex)}{4\sqrt{b^2+c^2}\,e\left(\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)\right)^{5}}$$

$$= -\frac{c\cos(d+ex)-b\sin(d+ex)}{4\sqrt{b^2+c^2}\,e\left(\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)\right)^{5}}$$

$$= -\frac{c\cos(d+ex)-b\sin(d+ex)}{4\sqrt{b^2+c^2}\,e\left(\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)\right)^{5}}$$

$$= \frac{3\tanh^{-1}\left(\frac{\sqrt[4]{b^2+c^2}\,\sin\left(d+ex-\tan^{-1}(b,c)\right)}{\sqrt{2}\sqrt{\sqrt{b^2+c^2}+\sqrt{b^2+c^2}\,\cos\left(d+ex-\tan^{-1}(b,c)\right)}}\right)}{16\sqrt{2}\left(b^2+c^2\right)^{5/4}e} - \frac{}{4\sqrt{b^2+}}$$

Mathematica [F] time = 180.03, size = 0, normalized size = 0.00

$Aborted

Verification is Not applicable to the result.

```
[In] Integrate[(Sqrt[b^2 + c^2] + b*Cos[d + e*x] + c*Sin[d + e*x])^(-5/2),x]
[Out] $Aborted
```

fricas [B] time = 2.53, size = 895, normalized size = 3.96

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*cos(e*x+d)+c*sin(e*x+d)+(b^2+c^2)^(1/2))^(5/2),x, algorithm=
"fricas")
```

```
[Out] 1/32*(3*sqrt(1/2)*(5*b^4*c*cos(e*x + d) + (5*b^4*c - 10*b^2*c^3 + c^5)*cos(
e*x + d)^5 - 10*(b^4*c - b^2*c^3)*cos(e*x + d)^3 - (b^5 + (b^5 - 10*b^3*c^2
 + 5*b*c^4)*cos(e*x + d)^4 - 2*(b^5 - 5*b^3*c^2)*cos(e*x + d)^2)*sin(e*x +
d))*log(((3*b^2*c - c^3)*cos(e*x + d)^3 + (b^2*c + 4*c^3)*cos(e*x + d) - (3
*b^3 + 4*b*c^2 + (b^3 - 3*b*c^2)*cos(e*x + d)^2)*sin(e*x + d) + 4*sqrt(1/2)
*(2*(b^3 + b*c^2)*cos(e*x + d) + 2*(b^2*c + c^3)*sin(e*x + d) - (2*b*c*cos(
e*x + d)*sin(e*x + d) + (b^2 - c^2)*cos(e*x + d)^2 + b^2 + 2*c^2)*sqrt(b^2
+ c^2))*sqrt(b*cos(e*x + d) + c*sin(e*x + d) + sqrt(b^2 + c^2))/(b^2 + c^2)
^(1/4) - 4*(2*b*c*cos(e*x + d)^2 - (b^2 - c^2)*cos(e*x + d)*sin(e*x + d) -
b*c)*sqrt(b^2 + c^2))/(3*b^2*c*cos(e*x + d) - (3*b^2*c - c^3)*cos(e*x + d)^
3 - (b^3 - (b^3 - 3*b*c^2)*cos(e*x + d)^2)*sin(e*x + d)))/(b^2 + c^2)^(1/4)
 + 2*(3*(b^4 - 6*b^2*c^2 + c^4)*cos(e*x + d)^4 - 7*b^4 - 26*b^2*c^2 - 16*c^
4 - 6*(2*b^4 - 3*b^2*c^2 - c^4)*cos(e*x + d)^2 + 12*((b^3*c - b*c^3)*cos(e*
x + d)^3 - (2*b^3*c + b*c^3)*cos(e*x + d))*sin(e*x + d) - 2*((b^3 - 3*b*c^2
)*cos(e*x + d)^3 - 3*(3*b^3 + 2*b*c^2)*cos(e*x + d) - (9*b^2*c + 8*c^3 - (3
*b^2*c - c^3)*cos(e*x + d)^2)*sin(e*x + d))*sqrt(b^2 + c^2))*sqrt(b*cos(e*x
 + d) + c*sin(e*x + d) + sqrt(b^2 + c^2)))/((5*b^6*c - 5*b^4*c^3 - 9*b^2*c^
5 + c^7)*e*cos(e*x + d)^5 - 10*(b^6*c - b^2*c^5)*e*cos(e*x + d)^3 + 5*(b^6*
c + b^4*c^3)*e*cos(e*x + d) - ((b^7 - 9*b^5*c^2 - 5*b^3*c^4 + 5*b*c^6)*e*co
s(e*x + d)^4 - 2*(b^7 - 4*b^5*c^2 - 5*b^3*c^4)*e*cos(e*x + d)^2 + (b^7 + b^
5*c^2)*e)*sin(e*x + d))
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*cos(e*x+d)+c*sin(e*x+d)+(b^2+c^2)^(1/2))^(5/2),x, algorithm=
"giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Simplification assuming b near 0Evaluation t
ime: 0.84sym2poly/r2sym(const gen & e,const index_m & i,const vecteur & l)
Error: Bad Argument Value
```

maple [A] time = 0.40, size = 350, normalized size = 1.55

$$\left(\sin\left(ex+d-\arctan\left(-b,c\right)\right)\sqrt{2}\ \operatorname{arctanh}\left(\frac{\sqrt{-\sqrt{b^2+c^2}\ \sin(ex+d-\arctan(-b,c))+\sqrt{b^2+c^2}}\ \sqrt{2}}{2\left(b^2+c^2\right)^{\frac{1}{4}}}\right)\left(b^2+c^2\right)+2\sqrt{-\sqrt{b^2+c^2}\ \sin\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*cos(e*x+d)+c*sin(e*x+d)+(b^2+c^2)^(1/2))^(5/2),x)
```

```
[Out] 1/4*(sin(e*x+d-arctan(-b,c))*2^(1/2)*arctanh(1/2*(-(b^2+c^2)^(1/2)*sin(e*x+
d-arctan(-b,c))+(b^2+c^2)^(1/2))^(1/2)*2^(1/2)/(b^2+c^2)^(1/4))*(b^2+c^2)+2
*(-(b^2+c^2)^(1/2)*sin(e*x+d-arctan(-b,c))+(b^2+c^2)^(1/2))^(1/2)*(b^2+c^2)
^(3/4)+2^(1/2)*arctanh(1/2*(-(b^2+c^2)^(1/2)*sin(e*x+d-arctan(-b,c))+(b^2+c
^2)^(1/2))^(1/2)*2^(1/2)/(b^2+c^2)^(1/4))*b^2+2^(1/2)*arctanh(1/2*(-(b^2+c^
2)^(1/2)*sin(e*x+d-arctan(-b,c))+(b^2+c^2)^(1/2))^(1/2)*2^(1/2)/(b^2+c^2)^(
1/4))*c^2)*(-(b^2+c^2)^(1/2)*(sin(e*x+d-arctan(-b,c))-1))^(1/2)/(b^2+c^2)^(
5/4)/cos(e*x+d-arctan(-b,c))/((b^2*sin(e*x+d-arctan(-b,c))+c^2*sin(e*x+d-ar
ctan(-b,c))+b^2+c^2)/(b^2+c^2)^(1/2))^(1/2)/e
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*cos(e*x+d)+c*sin(e*x+d)+(b^2+c^2)^(1/2))^(5/2),x, algorithm=
"maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is und
efined.
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\left(b\,\cos\left(d+e\,x\right)+c\,\sin\left(d+e\,x\right)+\sqrt{b^2+c^2}\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*cos(d + e*x) + c*sin(d + e*x) + (b^2 + c^2)^(1/2))^(5/2),x)
```

```
[Out] int(1/(b*cos(d + e*x) + c*sin(d + e*x) + (b^2 + c^2)^(1/2))^(5/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b\cos\left(d+ex\right)+c\sin\left(d+ex\right)+\sqrt{b^{2}+c^{2}}\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*cos(e*x+d)+c*sin(e*x+d)+(b**2+c**2)**(1/2))**(5/2),x)
```

```
[Out] Integral((b*cos(d + e*x) + c*sin(d + e*x) + sqrt(b**2 + c**2))**(-5/2), x)
```

3.437 $\int \left(-\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)\right)^{5/2}\,dx$

Optimal. Leaf size=196

$$-\frac{2(c\cos(d+ex)-b\sin(d+ex))\left(-\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)\right)^{3/2}}{5e}+\frac{16\sqrt{b^2+c^2}\,(c\cos(d+ex)-b\sin}{}$$

```
[Out] -2/5*(c*cos(e*x+d)-b*sin(e*x+d))*(b*cos(e*x+d)+c*sin(e*x+d)-(b^2+c^2)^(1/2)
)^(3/2)/e-64/15*(b^2+c^2)*(c*cos(e*x+d)-b*sin(e*x+d))/e/(b*cos(e*x+d)+c*sin
(e*x+d)-(b^2+c^2)^(1/2))^(1/2)+16/15*(c*cos(e*x+d)-b*sin(e*x+d))*(b^2+c^2)^
(1/2)*(b*cos(e*x+d)+c*sin(e*x+d)-(b^2+c^2)^(1/2))^(1/2)/e
```

Rubi [A] time = 0.13, antiderivative size = 196, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {3113, 3112}

$$-\frac{2(c\cos(d+ex)-b\sin(d+ex))\left(-\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)\right)^{3/2}}{5e}+\frac{16\sqrt{b^2+c^2}\,(c\cos(d+ex)-b\sin}{}$$

Antiderivative was successfully verified.

```
[In] Int[(-Sqrt[b^2 + c^2] + b*Cos[d + e*x] + c*Sin[d + e*x])^(5/2),x]
```

```
[Out] (-64*(b^2 + c^2)*(c*Cos[d + e*x] - b*Sin[d + e*x]))/(15*e*Sqrt[-Sqrt[b^2 +
c^2] + b*Cos[d + e*x] + c*Sin[d + e*x]]) + (16*Sqrt[b^2 + c^2]*(c*Cos[d + e
*x] - b*Sin[d + e*x])*Sqrt[-Sqrt[b^2 + c^2] + b*Cos[d + e*x] + c*Sin[d + e*
x]])/(15*e) - (2*(c*Cos[d + e*x] - b*Sin[d + e*x])*(-Sqrt[b^2 + c^2] + b*Co
s[d + e*x] + c*Sin[d + e*x])^(3/2))/(5*e)
```

Rule 3112

```
Int[Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_
)]], x_Symbol] :> Simp[(-2*(c*Cos[d + e*x] - b*Sin[d + e*x]))/(e*Sqrt[a + b
*Cos[d + e*x] + c*Sin[d + e*x]]), x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[a^
2 - b^2 - c^2, 0]
```

Rule 3113

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(n_), x_Symbol] :> -Simp[((c*Cos[d + e*x] - b*Sin[d + e*x])*(a + b*Cos[d +
e*x] + c*Sin[d + e*x])^(n - 1))/(e*n), x] + Dist[(a*(2*n - 1))/n, Int[(a +
b*Cos[d + e*x] + c*Sin[d + e*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e},
```

```
x] && EqQ[a^2 - b^2 - c^2, 0] && GtQ[n, 0]
```

Rubi steps

$$\int \left(-\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)\right)^{5/2} dx = -\frac{2(c\cos(d+ex)-b\sin(d+ex))\left(-\sqrt{b^2+c^2}+b\cos(d\right.}{5e}$$

$$= \frac{16\sqrt{b^2+c^2}\,(c\cos(d+ex)-b\sin(d+ex))\sqrt{-\sqrt{b^2+c^2}}}{15e}$$

$$= -\frac{64\left(b^2+c^2\right)(c\cos(d+ex)-b\sin(d+ex))}{15e\sqrt{-\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)}} + \frac{16}{}$$

Mathematica [C] time = 34.19, size = 11602, normalized size = 59.19

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(-Sqrt[b^2 + c^2] + b*Cos[d + e*x] + c*Sin[d + e*x])^(5/2),x]
[Out] Result too large to show
```

fricas [A] time = 1.12, size = 192, normalized size = 0.98

$$\frac{2\left(3\left(b^3-3bc^2\right)\cos(ex+d)^3+\left(29b^3+38bc^2\right)\cos(ex+d)+\left(29b^2c+32c^3+3\left(3b^2c-c^3\right)\cos(ex+d)^2\right)\sin}{15(}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(e*x+d)+c*sin(e*x+d)-(b^2+c^2)^(1/2))^(5/2),x, algorithm="f
ricas")
[Out] 2/15*(3*(b^3 - 3*b*c^2)*cos(e*x + d)^3 + (29*b^3 + 38*b*c^2)*cos(e*x + d) +
 (29*b^2*c + 32*c^3 + 3*(3*b^2*c - c^3)*cos(e*x + d)^2)*sin(e*x + d) - (22*
b*c*cos(e*x + d)*sin(e*x + d) + 11*(b^2 - c^2)*cos(e*x + d)^2 - 43*b^2 - 32
*c^2)*sqrt(b^2 + c^2))*sqrt(b*cos(e*x + d) + c*sin(e*x + d) - sqrt(b^2 + c^
2))/(c*e*cos(e*x + d) - b*e*sin(e*x + d))
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(e*x+d)+c*sin(e*x+d)-(b^2+c^2)^(1/2))^(5/2),x, algorithm="g
iac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Simplification assuming b near 0Evaluation t
ime: 0.44sym2poly/r2sym(const gen & e,const index_m & i,const vecteur & l)
Error: Bad Argument Value
```

maple [A] time = 0.38, size = 204, normalized size = 1.04

$$\frac{2\left(\sin\left(ex+d-\arctan\left(-b,c\right)\right)-1\right)\sqrt{b^{2}+c^{2}}\;\left(1+\sin\left(ex+d-\arctan\left(-b,c\right)\right)\right)\left(3b^{2}\left(\sin^{2}\left(ex+d-\arctan\left(-b,c\right)\right.\right.\right.}{15\cos\left(ex+d-\arctan\left(-b,c\right)\right)\sqrt{\cdots}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*cos(e*x+d)+c*sin(e*x+d)-(b^2+c^2)^(1/2))^(5/2),x)
```

```
[Out] 2/15*(sin(e*x+d-arctan(-b,c))-1)*(b^2+c^2)^(1/2)*(1+sin(e*x+d-arctan(-b,c))
)*(3*b^2*sin(e*x+d-arctan(-b,c))^2+3*c^2*sin(e*x+d-arctan(-b,c))^2-14*b^2*s
in(e*x+d-arctan(-b,c))-14*c^2*sin(e*x+d-arctan(-b,c))+43*b^2+43*c^2)/cos(e*
x+d-arctan(-b,c))/((b^2*sin(e*x+d-arctan(-b,c))+c^2*sin(e*x+d-arctan(-b,c))
-b^2-c^2)/(b^2+c^2)^(1/2))^(1/2)/e
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(e*x+d)+c*sin(e*x+d)-(b^2+c^2)^(1/2))^(5/2),x, algorithm="m
axima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is und
efined.
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \left(b\,\cos\left(d+e\,x\right)+c\,\sin\left(d+e\,x\right)-\sqrt{b^2+c^2}\right)^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*cos(d + e*x) + c*sin(d + e*x) - (b^2 + c^2)^(1/2))^(5/2),x)
```

```
[Out] int((b*cos(d + e*x) + c*sin(d + e*x) - (b^2 + c^2)^(1/2))^(5/2), x)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(e*x+d)+c*sin(e*x+d)-(b**2+c**2)**(1/2))**(5/2),x)
```

```
[Out] Timed out
```

3.438 $\int \left(-\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)\right)^{3/2}\,dx$

Optimal. Leaf size=130

$$\frac{8\sqrt{b^2+c^2}\,(c\cos(d+ex)-b\sin(d+ex))}{3e\sqrt{-\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)}}-\frac{2(c\cos(d+ex)-b\sin(d+ex))\sqrt{-\sqrt{b^2+c^2}+b\cos(d+ex)+c}}{3e}$$

```
[Out] 8/3*(c*cos(e*x+d)-b*sin(e*x+d))*(b^2+c^2)^(1/2)/e/(b*cos(e*x+d)+c*sin(e*x+d
)-(b^2+c^2)^(1/2))^(1/2)-2/3*(c*cos(e*x+d)-b*sin(e*x+d))*(b*cos(e*x+d)+c*si
n(e*x+d)-(b^2+c^2)^(1/2))^(1/2)/e
```

Rubi [A] time = 0.08, antiderivative size = 130, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {3113, 3112}

$$\frac{8\sqrt{b^2+c^2}\,(c\cos(d+ex)-b\sin(d+ex))}{3e\sqrt{-\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)}}-\frac{2(c\cos(d+ex)-b\sin(d+ex))\sqrt{-\sqrt{b^2+c^2}+b\cos(d+ex)+c}}{3e}$$

Antiderivative was successfully verified.

```
[In] Int[(-Sqrt[b^2 + c^2] + b*Cos[d + e*x] + c*Sin[d + e*x])^(3/2),x]
```

```
[Out] (8*Sqrt[b^2 + c^2]*(c*Cos[d + e*x] - b*Sin[d + e*x]))/(3*e*Sqrt[-Sqrt[b^2 +
 c^2] + b*Cos[d + e*x] + c*Sin[d + e*x]]) - (2*(c*Cos[d + e*x] - b*Sin[d +
e*x])*Sqrt[-Sqrt[b^2 + c^2] + b*Cos[d + e*x] + c*Sin[d + e*x]])/(3*e)
```

Rule 3112

```
Int[Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_
)]], x_Symbol] :> Simp[(-2*(c*Cos[d + e*x] - b*Sin[d + e*x]))/(e*Sqrt[a + b
*Cos[d + e*x] + c*Sin[d + e*x]]), x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[a^
2 - b^2 - c^2, 0]
```

Rule 3113

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(n_), x_Symbol] :> -Simp[((c*Cos[d + e*x] - b*Sin[d + e*x])*(a + b*Cos[d +
e*x] + c*Sin[d + e*x])^(n - 1))/(e*n), x] + Dist[(a*(2*n - 1))/n, Int[(a +
b*Cos[d + e*x] + c*Sin[d + e*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e},
x] && EqQ[a^2 - b^2 - c^2, 0] && GtQ[n, 0]
```

Rubi steps

$$\int \left(-\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)\right)^{3/2}\,dx = -\frac{2(c\cos(d+ex)-b\sin(d+ex))\sqrt{-\sqrt{b^2+c^2}+b\cos(d}}{3e}$$

$$= \frac{8\sqrt{b^2+c^2}\,(c\cos(d+ex)-b\sin(d+ex))}{3e\sqrt{-\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)}} - \frac{2(c\,\mathrm{cc}}{}$$

Mathematica [C] time = 21.61, size = 11512, normalized size = 88.55

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(-Sqrt[b^2 + c^2] + b*Cos[d + e*x] + c*Sin[d + e*x])^(3/2),x]
```

```
[Out] Result too large to show
```

fricas [A] time = 1.48, size = 127, normalized size = 0.98

$$\frac{2\left(2\,bc\cos\left(ex+d\right)\sin\left(ex+d\right)+\left(b^2-c^2\right)\cos\left(ex+d\right)^2-5\,b^2-4\,c^2-4\,\sqrt{b^2+c^2}\left(b\cos\left(ex+d\right)+c\sin\left(ex+d\right.\right.}{3\left(ce\cos\left(ex+d\right)-be\sin\left(ex+d\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(e*x+d)+c*sin(e*x+d)-(b^2+c^2)^(1/2))^(3/2),x, algorithm="f
ricas")
```

```
[Out] 2/3*(2*b*c*cos(e*x + d)*sin(e*x + d) + (b^2 - c^2)*cos(e*x + d)^2 - 5*b^2 -
 4*c^2 - 4*sqrt(b^2 + c^2)*(b*cos(e*x + d) + c*sin(e*x + d)))*sqrt(b*cos(e*
x + d) + c*sin(e*x + d) - sqrt(b^2 + c^2))/(c*e*cos(e*x + d) - b*e*sin(e*x
+ d))
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(e*x+d)+c*sin(e*x+d)-(b^2+c^2)^(1/2))^(3/2),x, algorithm="g
iac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Simplification assuming b near 0sym2poly/r2s
```

```
ym(const gen & e,const index_m & i,const vecteur & l) Error: Bad Argument V
alue
```

maple [A] time = 0.38, size = 130, normalized size = 1.00

$$\frac{2\left(\sin\left(ex+d-\arctan\left(-b,c\right)\right)-1\right)\left(b^{2}+c^{2}\right)\left(1+\sin\left(ex+d-\arctan\left(-b,c\right)\right)\right)\left(\sin\left(ex+d-\arctan\left(-b,c\right)\right)-5\right)}{3\cos\left(ex+d-\arctan\left(-b,c\right)\right)\sqrt{\frac{b^{2}\sin\left(ex+d-\arctan\left(-b,c\right)\right)+c^{2}\sin\left(ex+d-\arctan\left(-b,c\right)\right)-b^{2}-c^{2}}{\sqrt{b^{2}+c^{2}}}}\;e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*cos(e*x+d)+c*sin(e*x+d)-(b^2+c^2)^(1/2))^(3/2),x)
```

```
[Out] 2/3*(sin(e*x+d-arctan(-b,c))-1)*(b^2+c^2)*(1+sin(e*x+d-arctan(-b,c)))*(sin(
e*x+d-arctan(-b,c))-5)/cos(e*x+d-arctan(-b,c))/((b^2*sin(e*x+d-arctan(-b,c)
)+c^2*sin(e*x+d-arctan(-b,c))-b^2-c^2)/(b^2+c^2)^(1/2))^(1/2)/e
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(e*x+d)+c*sin(e*x+d)-(b^2+c^2)^(1/2))^(3/2),x, algorithm="m
axima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is und
efined.
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \left(b\,\cos\left(d+e\,x\right)+c\,\sin\left(d+e\,x\right)-\sqrt{b^2+c^2}\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*cos(d + e*x) + c*sin(d + e*x) - (b^2 + c^2)^(1/2))^(3/2),x)
```

```
[Out] int((b*cos(d + e*x) + c*sin(d + e*x) - (b^2 + c^2)^(1/2))^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\cos\left(d+ex\right)+c\sin\left(d+ex\right)-\sqrt{b^{2}+c^{2}}\right)^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(e*x+d)+c*sin(e*x+d)-(b**2+c**2)**(1/2))**(3/2),x)
```

```
[Out] Integral((b*cos(d + e*x) + c*sin(d + e*x) - sqrt(b**2 + c**2))**(3/2), x)
```

3.439 $\int \sqrt{-\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)}\,dx$

Optimal. Leaf size=57

$$-\frac{2(c\cos(d+ex)-b\sin(d+ex))}{e\sqrt{-\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)}}$$

```
[Out] -2*(c*cos(e*x+d)-b*sin(e*x+d))/e/(b*cos(e*x+d)+c*sin(e*x+d)-(b^2+c^2)^(1/2)
)^(1/2)
```

Rubi [A] time = 0.04, antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.029, Rules used = {3112}

$$-\frac{2(c\cos(d+ex)-b\sin(d+ex))}{e\sqrt{-\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[-Sqrt[b^2 + c^2] + b*Cos[d + e*x] + c*Sin[d + e*x]],x]
```

```
[Out] (-2*(c*Cos[d + e*x] - b*Sin[d + e*x]))/(e*Sqrt[-Sqrt[b^2 + c^2] + b*Cos[d +
 e*x] + c*Sin[d + e*x]])
```

Rule 3112

```
Int[Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_
)]], x_Symbol] :> Simp[(-2*(c*Cos[d + e*x] - b*Sin[d + e*x]))/(e*Sqrt[a + b
*Cos[d + e*x] + c*Sin[d + e*x]]), x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[a^
2 - b^2 - c^2, 0]
```

Rubi steps

$$\int \sqrt{-\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)}\,dx = -\frac{2(c\cos(d+ex)-b\sin(d+ex))}{e\sqrt{-\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)}}$$

Mathematica [C] time = 21.95, size = 11415, normalized size = 200.26

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[-Sqrt[b^2 + c^2] + b*Cos[d + e*x] + c*Sin[d + e*x]],x]
```

```
[Out] Result too large to show
```

fricas [A] time = 2.89, size = 80, normalized size = 1.40

$$\frac{2\left(b\cos\left(ex+d\right)+c\sin\left(ex+d\right)+\sqrt{b^2+c^2}\right)\sqrt{b\cos\left(ex+d\right)+c\sin\left(ex+d\right)-\sqrt{b^2+c^2}}}{ce\cos\left(ex+d\right)-be\sin\left(ex+d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(e*x+d)+c*sin(e*x+d)-(b^2+c^2)^(1/2))^(1/2),x, algorithm="f
ricas")
```

```
[Out] 2*(b*cos(e*x + d) + c*sin(e*x + d) + sqrt(b^2 + c^2))*sqrt(b*cos(e*x + d) +
 c*sin(e*x + d) - sqrt(b^2 + c^2))/(c*e*cos(e*x + d) - b*e*sin(e*x + d))
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(e*x+d)+c*sin(e*x+d)-(b^2+c^2)^(1/2))^(1/2),x, algorithm="g
iac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Simplification assuming b near 0sym2poly/r2s
ym(const gen & e,const index_m & i,const vecteur & l) Error: Bad Argument V
alue
```

maple [B] time = 0.36, size = 117, normalized size = 2.05

$$\frac{2\left(\sin\left(ex+d-\arctan\left(-b,c\right)\right)-1\right)\sqrt{b^2+c^2}\,\left(1+\sin\left(ex+d-\arctan\left(-b,c\right)\right)\right)}{\cos\left(ex+d-\arctan\left(-b,c\right)\right)\sqrt{\frac{b^2\sin\left(ex+d-\arctan\left(-b,c\right)\right)+c^2\sin\left(ex+d-\arctan\left(-b,c\right)\right)-b^2-c^2}{\sqrt{b^2+c^2}}}\;e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*cos(e*x+d)+c*sin(e*x+d)-(b^2+c^2)^(1/2))^(1/2),x)
```

```
[Out] 2*(sin(e*x+d-arctan(-b,c))-1)*(b^2+c^2)^(1/2)*(1+sin(e*x+d-arctan(-b,c)))/c
os(e*x+d-arctan(-b,c))/((b^2*sin(e*x+d-arctan(-b,c))+c^2*sin(e*x+d-arctan(-
b,c))-b^2-c^2)/(b^2+c^2)^(1/2))^(1/2)/e
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(e*x+d)+c*sin(e*x+d)-(b^2+c^2)^(1/2))^(1/2),x, algorithm="m
axima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is und
efined.
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \sqrt{b\,\cos\left(d+e\,x\right)+c\,\sin\left(d+e\,x\right)-\sqrt{b^2+c^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*cos(d + e*x) + c*sin(d + e*x) - (b^2 + c^2)^(1/2))^(1/2),x)
```

```
[Out] int((b*cos(d + e*x) + c*sin(d + e*x) - (b^2 + c^2)^(1/2))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{b\cos\left(d+ex\right)+c\sin\left(d+ex\right)-\sqrt{b^2+c^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(e*x+d)+c*sin(e*x+d)-(b**2+c**2)**(1/2))**(1/2),x)
```

```
[Out] Integral(sqrt(b*cos(d + e*x) + c*sin(d + e*x) - sqrt(b**2 + c**2)), x)
```

3.440 $\int \frac{1}{\sqrt{-\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)}}\,dx$

Optimal. Leaf size=91

$$-\frac{\sqrt{2}\tan^{-1}\left(\frac{\sqrt[4]{b^2+c^2}\sin\left(-\tan^{-1}(b,c)+d+ex\right)}{\sqrt{2}\sqrt{\sqrt{b^2+c^2}\cos\left(-\tan^{-1}(b,c)+d+ex\right)-\sqrt{b^2+c^2}}}\right)}{e\sqrt[4]{b^2+c^2}}$$

```
[Out] -arctan(1/2*(b^2+c^2)^(1/4)*sin(d+e*x-arctan(b,c))*2^(1/2)/(-(b^2+c^2)^(1/2
)+cos(d+e*x-arctan(b,c))*(b^2+c^2)^(1/2))^(1/2))*2^(1/2)/(b^2+c^2)^(1/4)/e
```

Rubi [A] time = 0.10, antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.088, Rules used = {3115, 2649, 204}

$$-\frac{\sqrt{2}\tan^{-1}\left(\frac{\sqrt[4]{b^2+c^2}\sin\left(-\tan^{-1}(b,c)+d+ex\right)}{\sqrt{2}\sqrt{\sqrt{b^2+c^2}\cos\left(-\tan^{-1}(b,c)+d+ex\right)-\sqrt{b^2+c^2}}}\right)}{e\sqrt[4]{b^2+c^2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[-Sqrt[b^2 + c^2] + b*Cos[d + e*x] + c*Sin[d + e*x]],x]
```

```
[Out] -((Sqrt[2]*ArcTan[((b^2 + c^2)^(1/4)*Sin[d + e*x - ArcTan[b, c]])/(Sqrt[2]*
Sqrt[-Sqrt[b^2 + c^2] + Sqrt[b^2 + c^2]*Cos[d + e*x - ArcTan[b, c]]])])/((b
^2 + c^2)^(1/4)*e))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3115

```
Int[1/Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(
x_)]], x_Symbol] :> Int[1/Sqrt[a + Sqrt[b^2 + c^2]*Cos[d + e*x - ArcTan[b,
```

```
c]]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[a^2 - b^2 - c^2, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{-\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)}}\,dx = \int \frac{1}{\sqrt{-\sqrt{b^2+c^2}+\sqrt{b^2+c^2}\cos\left(d+ex-\tan^{-1}(b,c)\right)}}\,dx$$

$$= -\frac{2\,\text{Subst}\left(\int \frac{1}{-2\sqrt{b^2+c^2}-x^2}\,dx, x, -\frac{\sqrt{b^2+c^2}\sin\left(d+ex-\tan^{-1}(\right.}{\sqrt{-\sqrt{b^2+c^2}+\sqrt{b^2+c^2}\cos\left(d+ex-\right.}}\right.}{e}$$

$$= -\frac{\sqrt{2}\tan^{-1}\left(\frac{\sqrt[4]{b^2+c^2}\sin\left(d+ex-\tan^{-1}(b,c)\right)}{\sqrt{2}\sqrt{-\sqrt{b^2+c^2}+\sqrt{b^2+c^2}\cos\left(d+ex-\tan^{-1}(b,c)\right)}}\right)}{\sqrt[4]{b^2+c^2}\,e}$$

Mathematica [C] time = 34.47, size = 61904, normalized size = 680.26

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/Sqrt[-Sqrt[b^2 + c^2] + b*Cos[d + e*x] + c*Sin[d + e*x]],x]
[Out] Result too large to show
```

fricas [B] time = 1.15, size = 107, normalized size = 1.18

$$\frac{\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(b\cos(ex+d)+c\sin(ex+d)+\sqrt{b^2+c^2}\right)\sqrt{b\cos(ex+d)+c\sin(ex+d)-\sqrt{b^2+c^2}}}{2\left(b^2+c^2\right)^{\frac{1}{4}}(c\cos(ex+d)-b\sin(ex+d))}\right)}{\left(b^2+c^2\right)^{\frac{1}{4}}e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*cos(e*x+d)+c*sin(e*x+d)-(b^2+c^2)^(1/2))^(1/2),x, algorithm=
"fricas")
[Out] sqrt(2)*arctan(-1/2*sqrt(2)*(b*cos(e*x + d) + c*sin(e*x + d) + sqrt(b^2 + c
^2))*sqrt(b*cos(e*x + d) + c*sin(e*x + d) - sqrt(b^2 + c^2))/((b^2 + c^2)^(
1/4)*(c*cos(e*x + d) - b*sin(e*x + d))))/((b^2 + c^2)^(1/4)*e)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*cos(e*x+d)+c*sin(e*x+d)-(b^2+c^2)^(1/2))^(1/2),x, algorithm=
"giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Simplification assuming b near 0sym2poly/r2s
ym(const gen & e,const index_m & i,const vecteur & l) Error: Bad Argument V
alue
```

maple [B] time = 0.30, size = 175, normalized size = 1.92

$$\frac{(\sin(ex+d-\arctan(-b,c))-1)\sqrt{-\sqrt{b^2+c^2}\,(1+\sin(ex+d-\arctan(-b,c)))}\,\sqrt{2}\,\arctan\left(\frac{\sqrt{-\sqrt{b^2+c^2}\,(1+\sin(ex+\cdots}}{2\left(b^2+c\cdots\right.}\right.}{\left(b^2+c^2\right)^{\frac{1}{4}}\cos(ex+d-\arctan(-b,c))\sqrt{\frac{b^2\sin(ex+d-\arctan(-b,c))+c^2\sin(ex+d-\arctan(-b,c))-b^2-c^2}{\sqrt{b^2+c^2}}}\,e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*cos(e*x+d)+c*sin(e*x+d)-(b^2+c^2)^(1/2))^(1/2),x)
```

```
[Out] (sin(e*x+d-arctan(-b,c))-1)*(-(b^2+c^2)^(1/2)*(1+sin(e*x+d-arctan(-b,c))))^
(1/2)*2^(1/2)/(b^2+c^2)^(1/4)*arctan(1/2*(-(b^2+c^2)^(1/2)*(1+sin(e*x+d-arc
tan(-b,c))))^(1/2)*2^(1/2)/(b^2+c^2)^(1/4))/cos(e*x+d-arctan(-b,c))/((b^2*s
in(e*x+d-arctan(-b,c))+c^2*sin(e*x+d-arctan(-b,c))-b^2-c^2)/(b^2+c^2)^(1/2)
)^(1/2)/e
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*cos(e*x+d)+c*sin(e*x+d)-(b^2+c^2)^(1/2))^(1/2),x, algorithm=
"maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is und
efined.
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sqrt{b\,\cos\left(d+e\,x\right)+c\,\sin\left(d+e\,x\right)-\sqrt{b^2+c^2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*cos(d + e*x) + c*sin(d + e*x) - (b^2 + c^2)^(1/2))^(1/2),x)
```

```
[Out] int(1/(b*cos(d + e*x) + c*sin(d + e*x) - (b^2 + c^2)^(1/2))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{b\cos(d+ex)+c\sin(d+ex)-\sqrt{b^2+c^2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*cos(e*x+d)+c*sin(e*x+d)-(b**2+c**2)**(1/2))**(1/2),x)
```

```
[Out] Integral(1/sqrt(b*cos(d + e*x) + c*sin(d + e*x) - sqrt(b**2 + c**2)), x)
```

3.441 $\int \frac{1}{\left(-\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)\right)^{3/2}}\,dx$

Optimal. Leaf size=164

$$\frac{\tan^{-1}\left(\frac{\sqrt[4]{b^2+c^2}\,\sin\left(-\tan^{-1}(b,c)+d+ex\right)}{\sqrt{2}\sqrt{\sqrt{b^2+c^2}\,\cos\left(-\tan^{-1}(b,c)+d+ex\right)-\sqrt{b^2+c^2}}}\right)}{2\sqrt{2}\,e\left(b^2+c^2\right)^{3/4}}+\frac{c\cos(d+ex)-b\sin(d+ex)}{2e\sqrt{b^2+c^2}\left(-\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)\right)^{3/2}}$$

```
[Out] 1/4*arctan(1/2*(b^2+c^2)^(1/4)*sin(d+e*x-arctan(b,c))*2^(1/2)/(-(b^2+c^2)^(
1/2)+cos(d+e*x-arctan(b,c))*(b^2+c^2)^(1/2))^(1/2))/(b^2+c^2)^(3/4)/e*2^(1/
2)+1/2*(c*cos(e*x+d)-b*sin(e*x+d))/e/(b*cos(e*x+d)+c*sin(e*x+d)-(b^2+c^2)^(
1/2))^(3/2)/(b^2+c^2)^(1/2)
```

Rubi [A] time = 0.13, antiderivative size = 164, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {3116, 3115, 2649, 204}

$$\frac{\tan^{-1}\left(\frac{\sqrt[4]{b^2+c^2}\,\sin\left(-\tan^{-1}(b,c)+d+ex\right)}{\sqrt{2}\sqrt{\sqrt{b^2+c^2}\,\cos\left(-\tan^{-1}(b,c)+d+ex\right)-\sqrt{b^2+c^2}}}\right)}{2\sqrt{2}\,e\left(b^2+c^2\right)^{3/4}}+\frac{c\cos(d+ex)-b\sin(d+ex)}{2e\sqrt{b^2+c^2}\left(-\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(-Sqrt[b^2 + c^2] + b*Cos[d + e*x] + c*Sin[d + e*x])^(-3/2),x]
```

```
[Out] ArcTan[((b^2 + c^2)^(1/4)*Sin[d + e*x - ArcTan[b, c]])/(Sqrt[2]*Sqrt[-Sqrt[
b^2 + c^2] + Sqrt[b^2 + c^2]*Cos[d + e*x - ArcTan[b, c]]])]/(2*Sqrt[2]*(b^2
 + c^2)^(3/4)*e) + (c*Cos[d + e*x] - b*Sin[d + e*x])/(2*Sqrt[b^2 + c^2]*e*(
-Sqrt[b^2 + c^2] + b*Cos[d + e*x] + c*Sin[d + e*x])^(3/2))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
```

```
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3115

```
Int[1/Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(
x_)]], x_Symbol] :> Int[1/Sqrt[a + Sqrt[b^2 + c^2]*Cos[d + e*x - ArcTan[b,
c]]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[a^2 - b^2 - c^2, 0]
```

Rule 3116

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(n_), x_Symbol] :> Simp[((c*Cos[d + e*x] - b*Sin[d + e*x])*(a + b*Cos[d + e
*x] + c*Sin[d + e*x])^n)/(a*e*(2*n + 1)), x] + Dist[(n + 1)/(a*(2*n + 1)),
Int[(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n + 1), x], x] /; FreeQ[{a, b, c
, d, e}, x] && EqQ[a^2 - b^2 - c^2, 0] && LtQ[n, -1]
```

Rubi steps

$$\int \frac{1}{\left(-\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)\right)^{3/2}}\,dx = \frac{c\cos(d+ex)-b\sin(d+ex)}{2\sqrt{b^2+c^2}\,e\left(-\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)\right.}$$

$$= \frac{c\cos(d+ex)-b\sin(d+ex)}{2\sqrt{b^2+c^2}\,e\left(-\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)\right.}$$

$$= \frac{c\cos(d+ex)-b\sin(d+ex)}{2\sqrt{b^2+c^2}\,e\left(-\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)\right.}$$

$$= \frac{\tan^{-1}\left(\frac{\sqrt[4]{b^2+c^2}\,\sin\left(d+ex-\tan^{-1}(b,c)\right)}{\sqrt{2}\sqrt{-\sqrt{b^2+c^2}+\sqrt{b^2+c^2}\,\cos\left(d+ex-\tan^{-1}(b,c)\right)}}\right)}{2\sqrt{2}\left(b^2+c^2\right)^{3/4}e} + \frac{}{2\sqrt{b^2+}}$$

Mathematica [F] time = 180.00, size = 0, normalized size = 0.00

$Aborted

Verification is Not applicable to the result.

```
[In] Integrate[(-Sqrt[b^2 + c^2] + b*Cos[d + e*x] + c*Sin[d + e*x])^(-3/2),x]
```

```
[Out] $Aborted
```

fricas [B] time = 0.86, size = 442, normalized size = 2.70

$$\frac{\left(3\sqrt{2}\,b^2c\cos\left(ex+d\right)-\sqrt{2}\left(3\,b^2c-c^3\right)\cos\left(ex+d\right)^3-\left(\sqrt{2}\,b^3-\sqrt{2}\left(b^3-3\,bc^2\right)\cos\left(ex+d\right)^2\right)\sin\left(ex+d\right)\right)\left(b^2+\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*cos(e*x+d)+c*sin(e*x+d)-(b^2+c^2)^(1/2))^(3/2),x, algorithm=
"fricas")
```

```
[Out] 1/4*((3*sqrt(2)*b^2*c*cos(e*x + d) - sqrt(2)*(3*b^2*c - c^3)*cos(e*x + d)^3
 - (sqrt(2)*b^3 - sqrt(2)*(b^3 - 3*b*c^2)*cos(e*x + d)^2)*sin(e*x + d))*(b^
2 + c^2)^(1/4)*arctan(-1/2*(b^2 + c^2)^(1/4)*sqrt(b*cos(e*x + d) + c*sin(e*
x + d) - sqrt(b^2 + c^2))*((sqrt(2)*b*cos(e*x + d) + sqrt(2)*c*sin(e*x + d)
)*sqrt(b^2 + c^2) + sqrt(2)*(b^2 + c^2))/((b^2*c + c^3)*cos(e*x + d) - (b^3
 + b*c^2)*sin(e*x + d))) - 2*(2*(b^3 + b*c^2)*cos(e*x + d) + 2*(b^2*c + c^3
)*sin(e*x + d) + (2*b*c*cos(e*x + d)*sin(e*x + d) + (b^2 - c^2)*cos(e*x + d
)^2 + b^2 + 2*c^2)*sqrt(b^2 + c^2))*sqrt(b*cos(e*x + d) + c*sin(e*x + d) -
sqrt(b^2 + c^2)))/((3*b^4*c + 2*b^2*c^3 - c^5)*e*cos(e*x + d)^3 - 3*(b^4*c
+ b^2*c^3)*e*cos(e*x + d) - ((b^5 - 2*b^3*c^2 - 3*b*c^4)*e*cos(e*x + d)^2 -
 (b^5 + b^3*c^2)*e)*sin(e*x + d))
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*cos(e*x+d)+c*sin(e*x+d)-(b^2+c^2)^(1/2))^(3/2),x, algorithm=
"giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Simplification assuming b near 0sym2poly/r2s
ym(const gen & e,const index_m & i,const vecteur & l) Error: Bad Argument V
alue
```

maple [B] time = 0.36, size = 363, normalized size = 2.21

$$\left(-\sin\left(ex+d-\arctan\left(-b,c\right)\right)\sqrt{2}\ \arctan\left(\frac{\sqrt{-\sqrt{b^2+c^2}\ \sin(ex+d-\arctan(-b,c))-\sqrt{b^2+c^2}}\ \sqrt{2}}{2\left(b^2+c^2\right)^{\frac{1}{4}}}\right)\left(b^2+c^2\right)+2\sqrt{-\sqrt{b^2+c^2}\ \ s}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*cos(e*x+d)+c*sin(e*x+d)-(b^2+c^2)^(1/2))^(3/2),x)
```

```
[Out] 1/4/(b^2+c^2)^(7/4)*(-sin(e*x+d-arctan(-b,c))*2^(1/2)*arctan(1/2*(-(b^2+c^2
)^(1/2)*sin(e*x+d-arctan(-b,c))-(b^2+c^2)^(1/2))^(1/2)*2^(1/2)/(b^2+c^2)^(1
/4))*(b^2+c^2)+2*(-(b^2+c^2)^(1/2)*sin(e*x+d-arctan(-b,c))-(b^2+c^2)^(1/2))
^(1/2)*(b^2+c^2)^(3/4)+2^(1/2)*arctan(1/2*(-(b^2+c^2)^(1/2)*sin(e*x+d-arcta
n(-b,c))-(b^2+c^2)^(1/2))^(1/2)*2^(1/2)/(b^2+c^2)^(1/4))*b^2+2^(1/2)*arctan
(1/2*(-(b^2+c^2)^(1/2)*sin(e*x+d-arctan(-b,c))-(b^2+c^2)^(1/2))^(1/2)*2^(1/
2)/(b^2+c^2)^(1/4))*c^2)*(-(b^2+c^2)^(1/2)*(1+sin(e*x+d-arctan(-b,c))))^(1/
2)/cos(e*x+d-arctan(-b,c))/((b^2*sin(e*x+d-arctan(-b,c))+c^2*sin(e*x+d-arct
an(-b,c))-b^2-c^2)/(b^2+c^2)^(1/2))^(1/2)/e
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*cos(e*x+d)+c*sin(e*x+d)-(b^2+c^2)^(1/2))^(3/2),x, algorithm=
"maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is und
efined.
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\left(b\ \cos\left(d+e\,x\right)+c\ \sin\left(d+e\,x\right)-\sqrt{b^2+c^2}\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*cos(d + e*x) + c*sin(d + e*x) - (b^2 + c^2)^(1/2))^(3/2),x)
```

```
[Out] int(1/(b*cos(d + e*x) + c*sin(d + e*x) - (b^2 + c^2)^(1/2))^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b\cos\left(d+ex\right)+c\sin\left(d+ex\right)-\sqrt{b^2+c^2}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*cos(e*x+d)+c*sin(e*x+d)-(b**2+c**2)**(1/2))**(3/2),x)
[Out] Integral((b*cos(d + e*x) + c*sin(d + e*x) - sqrt(b**2 + c**2))**(-3/2), x)
```

3.442 $\int \frac{1}{\left(-\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)\right)^{5/2}}\,dx$

Optimal. Leaf size=232

$$-\frac{3\tan^{-1}\left(\frac{\sqrt[4]{b^2+c^2}\,\sin\left(-\tan^{-1}(b,c)+d+ex\right)}{\sqrt{2}\sqrt{\sqrt{b^2+c^2}\,\cos\left(-\tan^{-1}(b,c)+d+ex\right)-\sqrt{b^2+c^2}}}\right)}{16\sqrt{2}\,e\left(b^2+c^2\right)^{5/4}}-\frac{3(c\cos(d+ex)-b\sin(d+ex))}{16e\left(b^2+c^2\right)\left(-\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)\right)^{3/2}}+$$

```
[Out] -3/32*arctan(1/2*(b^2+c^2)^(1/4)*sin(d+e*x-arctan(b,c))*2^(1/2)/(-(b^2+c^2)
^(1/2)+cos(d+e*x-arctan(b,c))*(b^2+c^2)^(1/2))^(1/2))/(b^2+c^2)^(5/4)/e*2^(
1/2)-3/16*(c*cos(e*x+d)-b*sin(e*x+d))/(b^2+c^2)/e/(b*cos(e*x+d)+c*sin(e*x+d
)-(b^2+c^2)^(1/2))^(3/2)+1/4*(c*cos(e*x+d)-b*sin(e*x+d))/e/(b*cos(e*x+d)+c*
sin(e*x+d)-(b^2+c^2)^(1/2))^(5/2)/(b^2+c^2)^(1/2)
```

Rubi [A] time = 0.17, antiderivative size = 232, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {3116, 3115, 2649, 204}

$$-\frac{3\tan^{-1}\left(\frac{\sqrt[4]{b^2+c^2}\,\sin\left(-\tan^{-1}(b,c)+d+ex\right)}{\sqrt{2}\sqrt{\sqrt{b^2+c^2}\,\cos\left(-\tan^{-1}(b,c)+d+ex\right)-\sqrt{b^2+c^2}}}\right)}{16\sqrt{2}\,e\left(b^2+c^2\right)^{5/4}}-\frac{3(c\cos(d+ex)-b\sin(d+ex))}{16e\left(b^2+c^2\right)\left(-\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)\right)^{3/2}}+$$

Antiderivative was successfully verified.

```
[In] Int[(-Sqrt[b^2 + c^2] + b*Cos[d + e*x] + c*Sin[d + e*x])^(-5/2),x]
```

```
[Out] (-3*ArcTan[((b^2 + c^2)^(1/4)*Sin[d + e*x - ArcTan[b, c]])/(Sqrt[2]*Sqrt[-S
qrt[b^2 + c^2] + Sqrt[b^2 + c^2]*Cos[d + e*x - ArcTan[b, c]]])])/(16*Sqrt[2
]*(b^2 + c^2)^(5/4)*e) + (c*Cos[d + e*x] - b*Sin[d + e*x])/(4*Sqrt[b^2 + c^
2]*e*(-Sqrt[b^2 + c^2] + b*Cos[d + e*x] + c*Sin[d + e*x])^(5/2)) - (3*(c*Co
s[d + e*x] - b*Sin[d + e*x]))/(16*(b^2 + c^2)*e*(-Sqrt[b^2 + c^2] + b*Cos[d
 + e*x] + c*Sin[d + e*x])^(3/2))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3115

```
Int[1/Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(
x_)]], x_Symbol] :> Int[1/Sqrt[a + Sqrt[b^2 + c^2]*Cos[d + e*x - ArcTan[b,
c]]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[a^2 - b^2 - c^2, 0]
```

Rule 3116

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(n_), x_Symbol] :> Simp[((c*Cos[d + e*x] - b*Sin[d + e*x])*(a + b*Cos[d + e
*x] + c*Sin[d + e*x])^n)/(a*e*(2*n + 1)), x] + Dist[(n + 1)/(a*(2*n + 1)),
Int[(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n + 1), x], x] /; FreeQ[{a, b, c
, d, e}, x] && EqQ[a^2 - b^2 - c^2, 0] && LtQ[n, -1]
```

Rubi steps

$$\int \frac{1}{\left(-\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)\right)^{5/2}}\,dx = \frac{c\cos(d+ex)-b\sin(d+ex)}{4\sqrt{b^2+c^2}\,e\left(-\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)\right)}$$

$$= \frac{c\cos(d+ex)-b\sin(d+ex)}{4\sqrt{b^2+c^2}\,e\left(-\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)\right)}$$

$$= \frac{c\cos(d+ex)-b\sin(d+ex)}{4\sqrt{b^2+c^2}\,e\left(-\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)\right)}$$

$$= \frac{c\cos(d+ex)-b\sin(d+ex)}{4\sqrt{b^2+c^2}\,e\left(-\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)\right)}$$

$$= -\frac{3\tan^{-1}\left(\frac{\sqrt[4]{b^2+c^2}\sin\left(d+ex-\tan^{-1}(b,c)\right)}{\sqrt{2}\sqrt{-\sqrt{b^2+c^2}+\sqrt{b^2+c^2}\cos\left(d+ex-\tan^{-1}(b,c)\right)}}\right)}{16\sqrt{2}\left(b^2+c^2\right)^{5/4}e} + \frac{}{4\sqrt{b}}$$

Mathematica [F] time = 180.06, size = 0, normalized size = 0.00

$Aborted

Verification is Not applicable to the result.

```
[In] Integrate[(-Sqrt[b^2 + c^2] + b*Cos[d + e*x] + c*Sin[d + e*x])^(-5/2),x]
[Out] $Aborted
```

fricas [B] time = 1.20, size = 655, normalized size = 2.82

$$\frac{3\sqrt{\frac{1}{2}}\left(5\,b^4c\cos(ex+d)+\left(5\,b^4c-10\,b^2c^3+c^5\right)\cos(ex+d)^5-10\left(b^4c-b^2c^3\right)\cos(ex+d)^3-\left(b^5+\left(b^5-10\,b^3c^2+5\,bc^4\right)\cos(ex+d)^4-2\left(b^5-5\,b^3c^2\right)\cos(ex+d)^2\right)\sin}{\frac{}{\left(b^2+c^2\right)^{\frac{1}{4}}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*cos(e*x+d)+c*sin(e*x+d)-(b^2+c^2)^(1/2))^(5/2),x, algorithm=
"fricas")
```

```
[Out] 1/16*(3*sqrt(1/2)*(5*b^4*c*cos(e*x + d) + (5*b^4*c - 10*b^2*c^3 + c^5)*cos(
e*x + d)^5 - 10*(b^4*c - b^2*c^3)*cos(e*x + d)^3 - (b^5 + (b^5 - 10*b^3*c^2
 + 5*b*c^4)*cos(e*x + d)^4 - 2*(b^5 - 5*b^3*c^2)*cos(e*x + d)^2)*sin(e*x +
d))*arctan(-sqrt(1/2)*(b*cos(e*x + d) + c*sin(e*x + d) + sqrt(b^2 + c^2))*s
qrt(b*cos(e*x + d) + c*sin(e*x + d) - sqrt(b^2 + c^2))/((b^2 + c^2)^(1/4)*(
c*cos(e*x + d) - b*sin(e*x + d))))/(b^2 + c^2)^(1/4) + (3*(b^4 - 6*b^2*c^2
+ c^4)*cos(e*x + d)^4 - 7*b^4 - 26*b^2*c^2 - 16*c^4 - 6*(2*b^4 - 3*b^2*c^2
- c^4)*cos(e*x + d)^2 + 12*((b^3*c - b*c^3)*cos(e*x + d)^3 - (2*b^3*c + b*c
^3)*cos(e*x + d))*sin(e*x + d) + 2*((b^3 - 3*b*c^2)*cos(e*x + d)^3 - 3*(3*b
^3 + 2*b*c^2)*cos(e*x + d) - (9*b^2*c + 8*c^3 - (3*b^2*c - c^3)*cos(e*x + d
)^2)*sin(e*x + d))*sqrt(b^2 + c^2))*sqrt(b*cos(e*x + d) + c*sin(e*x + d) -
sqrt(b^2 + c^2)))/((5*b^6*c - 5*b^4*c^3 - 9*b^2*c^5 + c^7)*e*cos(e*x + d)^5
 - 10*(b^6*c - b^2*c^5)*e*cos(e*x + d)^3 + 5*(b^6*c + b^4*c^3)*e*cos(e*x +
d) - ((b^7 - 9*b^5*c^2 - 5*b^3*c^4 + 5*b*c^6)*e*cos(e*x + d)^4 - 2*(b^7 - 4
*b^5*c^2 - 5*b^3*c^4)*e*cos(e*x + d)^2 + (b^7 + b^5*c^2)*e)*sin(e*x + d))
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*cos(e*x+d)+c*sin(e*x+d)-(b^2+c^2)^(1/2))^(5/2),x, algorithm=
"giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Simplification assuming b near 0Evaluation t
ime: 0.87sym2poly/r2sym(const gen & e,const index_m & i,const vecteur & l)
Error: Bad Argument Value
```

maple [A] time = 0.45, size = 363, normalized size = 1.56

$$-\frac{\left(-\sin\left(ex+d-\arctan\left(-b,c\right)\right)\sqrt{2}\ \arctan\left(\frac{\sqrt{-\sqrt{b^2+c^2}\ \sin(ex+d-\arctan(-b,c))-\sqrt{b^2+c^2}}\ \sqrt{2}}{2\left(b^2+c^2\right)^{\frac{1}{4}}}\right)\left(b^2+c^2\right)+2\sqrt{-\sqrt{b^2+c^2}\ \text{s}}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*cos(e*x+d)+c*sin(e*x+d)-(b^2+c^2)^(1/2))^(5/2),x)
```

```
[Out] -1/4*(-sin(e*x+d-arctan(-b,c))*2^(1/2)*arctan(1/2*(-(b^2+c^2)^(1/2)*sin(e*x
+d-arctan(-b,c))-(b^2+c^2)^(1/2))^(1/2)*2^(1/2)/(b^2+c^2)^(1/4))*(b^2+c^2)+
2*(-(b^2+c^2)^(1/2)*sin(e*x+d-arctan(-b,c))-(b^2+c^2)^(1/2))^(1/2)*(b^2+c^2
)^(3/4)+2^(1/2)*arctan(1/2*(-(b^2+c^2)^(1/2)*sin(e*x+d-arctan(-b,c))-(b^2+c
^2)^(1/2))^(1/2)*2^(1/2)/(b^2+c^2)^(1/4))*b^2+2^(1/2)*arctan(1/2*(-(b^2+c^2
)^(1/2)*sin(e*x+d-arctan(-b,c))-(b^2+c^2)^(1/2))^(1/2)*2^(1/2)/(b^2+c^2)^(1
/4))*c^2)*(-(b^2+c^2)^(1/2)*(1+sin(e*x+d-arctan(-b,c))))^(1/2)/(b^2+c^2)^(5
/4)/cos(e*x+d-arctan(-b,c))/((b^2*sin(e*x+d-arctan(-b,c))+c^2*sin(e*x+d-arc
tan(-b,c))-b^2-c^2)/(b^2+c^2)^(1/2))^(1/2)/e
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*cos(e*x+d)+c*sin(e*x+d)-(b^2+c^2)^(1/2))^(5/2),x, algorithm=
"maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is und
efined.
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{{\left(b\,\cos\left(d+e\,x\right)+c\,\sin\left(d+e\,x\right)-\sqrt{b^2+c^2}\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*cos(d + e*x) + c*sin(d + e*x) - (b^2 + c^2)^(1/2))^(5/2),x)
```

```
[Out] int(1/(b*cos(d + e*x) + c*sin(d + e*x) - (b^2 + c^2)^(1/2))^(5/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b \cos{\left(d + e x \right)} + c \sin{\left(d + e x \right)} - \sqrt{b^{2} + c^{2}}\right)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*cos(e*x+d)+c*sin(e*x+d)-(b**2+c**2)**(1/2))**(5/2),x)
```

```
[Out] Integral((b*cos(d + e*x) + c*sin(d + e*x) - sqrt(b**2 + c**2))**(-5/2), x)
```

3.443 $\int \frac{\sin(x)}{a+b\cos(x)+c\sin(x)}\,dx$

Optimal. Leaf size=101

$$-\frac{2ac\tan^{-1}\left(\frac{(a-b)\tan\left(\frac{x}{2}\right)+c}{\sqrt{a^2-b^2-c^2}}\right)}{\left(b^2+c^2\right)\sqrt{a^2-b^2-c^2}}-\frac{b\log(a+b\cos(x)+c\sin(x))}{b^2+c^2}+\frac{cx}{b^2+c^2}$$

```
[Out] c*x/(b^2+c^2)-b*ln(a+b*cos(x)+c*sin(x))/(b^2+c^2)-2*a*c*arctan((c+(a-b)*tan
(1/2*x))/(a^2-b^2-c^2)^(1/2))/(b^2+c^2)/(a^2-b^2-c^2)^(1/2)
```

Rubi [A] time = 0.10, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {3137, 3124, 618, 204}

$$-\frac{2ac\tan^{-1}\left(\frac{(a-b)\tan\left(\frac{x}{2}\right)+c}{\sqrt{a^2-b^2-c^2}}\right)}{\left(b^2+c^2\right)\sqrt{a^2-b^2-c^2}}-\frac{b\log(a+b\cos(x)+c\sin(x))}{b^2+c^2}+\frac{cx}{b^2+c^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[x]/(a + b*Cos[x] + c*Sin[x]),x]
```

```
[Out] (c*x)/(b^2 + c^2) - (2*a*c*ArcTan[(c + (a - b)*Tan[x/2])/Sqrt[a^2 - b^2 - c
^2]])/(Sqrt[a^2 - b^2 - c^2]*(b^2 + c^2)) - (b*Log[a + b*Cos[x] + c*Sin[x]]
)/(b^2 + c^2)
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 3124

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(-1), x_Symbol] :> Module[{f = FreeFactors[Tan[(d + e*x)/2], x]}, Dist[(2*f
)/e, Subst[Int[1/(a + b + 2*c*f*x + (a - b)*f^2*x^2), x], x, Tan[(d + e*x)/
```

```
2]/f], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0]
```

Rule 3137

```
Int[((A_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])/((a_.) + cos[(d_.) + (e_.)*(x_)
]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)]), x_Symbol] :> Simp[(c*C*(d + e*x))
/(e*(b^2 + c^2)), x] + (Dist[(A*(b^2 + c^2) - a*c*C)/(b^2 + c^2), Int[1/(a
+ b*Cos[d + e*x] + c*Sin[d + e*x]), x], x] - Simp[(b*C*Log[a + b*Cos[d + e*
x] + c*Sin[d + e*x]])/(e*(b^2 + c^2)), x]) /; FreeQ[{a, b, c, d, e, A, C},
x] && NeQ[b^2 + c^2, 0] && NeQ[A*(b^2 + c^2) - a*c*C, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\sin(x)}{a+b\cos(x)+c\sin(x)}\,dx &= \frac{cx}{b^2+c^2}-\frac{b\log(a+b\cos(x)+c\sin(x))}{b^2+c^2}-\frac{(ac)\int \frac{1}{a+b\cos(x)+c\sin(x)}\,dx}{b^2+c^2}\\
&= \frac{cx}{b^2+c^2}-\frac{b\log(a+b\cos(x)+c\sin(x))}{b^2+c^2}-\frac{(2ac)\,\mathrm{Subst}\left(\int \frac{1}{a+b+2cx+(a-b)x^2}\,dx,x,\mathrm{ta}\right.}{b^2+c^2}\\
&= \frac{cx}{b^2+c^2}-\frac{b\log(a+b\cos(x)+c\sin(x))}{b^2+c^2}+\frac{(4ac)\,\mathrm{Subst}\left(\int \frac{1}{-4\left(a^2-b^2-c^2\right)-x^2}\,dx,x,2c\right.}{b^2+c^2}\\
&= \frac{cx}{b^2+c^2}-\frac{2ac\tan^{-1}\left(\frac{c+(a-b)\tan\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2-c^2}}\right)}{\sqrt{a^2-b^2-c^2}\,\left(b^2+c^2\right)}-\frac{b\log(a+b\cos(x)+c\sin(x))}{b^2+c^2}\end{aligned}$$

Mathematica [A] time = 0.22, size = 80, normalized size = 0.79

$$\frac{\frac{2ac\tanh^{-1}\left(\frac{(a-b)\tan\left(\frac{x}{2}\right)+c}{\sqrt{-a^2+b^2+c^2}}\right)}{\sqrt{-a^2+b^2+c^2}}-b\log(a+b\cos(x)+c\sin(x))+cx}{b^2+c^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[x]/(a + b*Cos[x] + c*Sin[x]),x]
```

```
[Out] (c*x + (2*a*c*ArcTanh[(c + (a - b)*Tan[x/2])/Sqrt[-a^2 + b^2 + c^2]])/Sqrt[
-a^2 + b^2 + c^2] - b*Log[a + b*Cos[x] + c*Sin[x]])/(b^2 + c^2)
```

fricas [B] time = 1.21, size = 579, normalized size = 5.73

$$\left[-\frac{\sqrt{-a^2+b^2+c^2}\,ac\log\left(\frac{a^2b^2-2\,b^4-c^4-\left(a^2+3\,b^2\right)c^2-\left(2\,a^2b^2-b^4-2\,a^2c^2+c^4\right)\cos(x)^2-2\left(ab^3+abc^2\right)\cos(x)-2\left(ab^2c+ac^3-\left(bc^3-\left(2\,a^2b-b^3\right)c\right)\cos\right.}{2\,ab\cos(x)+\left(b^2-c^2\right)\cos(x)^2+a^2+c^2+2}\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)/(a+b*cos(x)+c*sin(x)),x, algorithm="fricas")
```

```
[Out] [-1/2*(sqrt(-a^2 + b^2 + c^2)*a*c*log((a^2*b^2 - 2*b^4 - c^4 - (a^2 + 3*b^2
)*c^2 - (2*a^2*b^2 - b^4 - 2*a^2*c^2 + c^4)*cos(x)^2 - 2*(a*b^3 + a*b*c^2)*
cos(x) - 2*(a*b^2*c + a*c^3 - (b*c^3 - (2*a^2*b - b^3)*c)*cos(x))*sin(x) -
2*(2*a*b*c*cos(x)^2 - a*b*c + (b^2*c + c^3)*cos(x) - (b^3 + b*c^2 + (a*b^2
- a*c^2)*cos(x))*sin(x))*sqrt(-a^2 + b^2 + c^2))/(2*a*b*cos(x) + (b^2 - c^2
)*cos(x)^2 + a^2 + c^2 + 2*(b*c*cos(x) + a*c)*sin(x))) + 2*(c^3 - (a^2 - b^
2)*c)*x + (a^2*b - b^3 - b*c^2)*log(2*a*b*cos(x) + (b^2 - c^2)*cos(x)^2 + a
^2 + c^2 + 2*(b*c*cos(x) + a*c)*sin(x)))/(a^2*b^2 - b^4 - c^4 + (a^2 - 2*b^
2)*c^2), -1/2*(2*sqrt(a^2 - b^2 - c^2)*a*c*arctan(-(a*b*cos(x) + a*c*sin(x)
 + b^2 + c^2)*sqrt(a^2 - b^2 - c^2)/((c^3 - (a^2 - b^2)*c)*cos(x) + (a^2*b
- b^3 - b*c^2)*sin(x))) + 2*(c^3 - (a^2 - b^2)*c)*x + (a^2*b - b^3 - b*c^2)
*log(2*a*b*cos(x) + (b^2 - c^2)*cos(x)^2 + a^2 + c^2 + 2*(b*c*cos(x) + a*c)
*sin(x)))/(a^2*b^2 - b^4 - c^4 + (a^2 - 2*b^2)*c^2)]
```

giac [A] time = 0.16, size = 160, normalized size = 1.58

$$\frac{2\left(\pi\left\lfloor\frac{x}{2\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}\left(-2\,a+2\,b\right)+\arctan\left(-\frac{a\tan\left(\frac{1}{2}\,x\right)-b\tan\left(\frac{1}{2}\,x\right)+c}{\sqrt{a^2-b^2-c^2}}\right)\right)ac}{\sqrt{a^2-b^2-c^2}\left(b^2+c^2\right)}+\frac{cx}{b^2+c^2}-\frac{b\log\left(-a\tan\left(\frac{1}{2}\,x\right)^2+b\tan\left(\frac{1}{2}\,x\right)^2\right.}{b^2+c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)/(a+b*cos(x)+c*sin(x)),x, algorithm="giac")
```

```
[Out] 2*(pi*floor(1/2*x/pi + 1/2)*sgn(-2*a + 2*b) + arctan(-(a*tan(1/2*x) - b*tan
(1/2*x) + c)/sqrt(a^2 - b^2 - c^2)))*a*c/(sqrt(a^2 - b^2 - c^2)*(b^2 + c^2)
) + c*x/(b^2 + c^2) - b*log(-a*tan(1/2*x)^2 + b*tan(1/2*x)^2 - 2*c*tan(1/2*
x) - a - b)/(b^2 + c^2) + b*log(tan(1/2*x)^2 + 1)/(b^2 + c^2)
```

maple [B] time = 0.12, size = 438, normalized size = 4.34

$$-\frac{2\ln\left(a\left(\tan^2\left(\frac{x}{2}\right)\right)-b\left(\tan^2\left(\frac{x}{2}\right)\right)+2c\tan\left(\frac{x}{2}\right)+a+b\right)ab}{\left(2b^2+2c^2\right)\left(a-b\right)}+\frac{2\ln\left(a\left(\tan^2\left(\frac{x}{2}\right)\right)-b\left(\tan^2\left(\frac{x}{2}\right)\right)+2c\tan\left(\frac{x}{2}\right)+a+b\right.}{\left(2b^2+2c^2\right)\left(a-b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)/(a+b*cos(x)+c*sin(x)),x)
```

```
[Out] -2/(2*b^2+2*c^2)/(a-b)*ln(a*tan(1/2*x)^2-b*tan(1/2*x)^2+2*c*tan(1/2*x)+a+b)
*a*b+2/(2*b^2+2*c^2)/(a-b)*ln(a*tan(1/2*x)^2-b*tan(1/2*x)^2+2*c*tan(1/2*x)+
a+b)*b^2-4/(2*b^2+2*c^2)/(a^2-b^2-c^2)^(1/2)*arctan(1/2*(2*(a-b)*tan(1/2*x)
+2*c)/(a^2-b^2-c^2)^(1/2))*a*c-4/(2*b^2+2*c^2)/(a^2-b^2-c^2)^(1/2)*arctan(1
/2*(2*(a-b)*tan(1/2*x)+2*c)/(a^2-b^2-c^2)^(1/2))*c*b+4/(2*b^2+2*c^2)/(a^2-b
^2-c^2)^(1/2)*arctan(1/2*(2*(a-b)*tan(1/2*x)+2*c)/(a^2-b^2-c^2)^(1/2))*c/(a
-b)*a*b-4/(2*b^2+2*c^2)/(a^2-b^2-c^2)^(1/2)*arctan(1/2*(2*(a-b)*tan(1/2*x)+
2*c)/(a^2-b^2-c^2)^(1/2))*c/(a-b)*b^2+2/(2*b^2+2*c^2)*b*ln(1+tan(1/2*x)^2)+
4/(2*b^2+2*c^2)*c*arctan(tan(1/2*x))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)/(a+b*cos(x)+c*sin(x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(c^2+b^2-a^2>0)', see `assume?` for
more details)Is c^2+b^2-a^2 positive or negative?
```

mupad [B] time = 11.44, size = 950, normalized size = 9.41

$$\frac{\ln\left(\tan\left(\frac{x}{2}\right)+1\mathrm{i}\right)}{b-c\,1\mathrm{i}}+\frac{\ln\left(64\tan\left(\frac{x}{2}\right)\,(a-b)^2-\frac{\left(a^2\,b-b\,c^2-b^3+a\,c\,\sqrt{-a^2+b^2+c^2}\right)\left(32\,a^2\,c+32\,b^2\,c-64\,a\,b\,c+64\tan\left(\frac{x}{2}\right)(a-b)\left(-a^2+b\,a+c^2\right)+\cdots\right.}{\cdots}\right)}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)/(a + b*cos(x) + c*sin(x)),x)
```

```
[Out] log(tan(x/2) + 1i)/(b - c*1i) + (log(tan(x/2) - 1i)*1i)/(b*1i - c) + (log(6
4*tan(x/2)*(a - b)^2 - ((a^2*b - b*c^2 - b^3 + a*c*(b^2 - a^2 + c^2)^(1/2))
*(32*a^2*c + 32*b^2*c - 64*a*b*c + 64*tan(x/2)*(a - b)*(a*b - a^2 + c^2) +
```

```
((a^2*b - b*c^2 - b^3 + a*c*(b^2 - a^2 + c^2)^(1/2))*(32*b*c^3 - 32*a*c^3 -
 64*b^3*c + 32*tan(x/2)*(a - b)*(2*a*b^2 - 2*a*c^2 + b*c^2 - 2*b^3) + 128*a
*b^2*c - 64*a^2*b*c + (32*(a - b)*(a^2*b - b*c^2 - b^3 + a*c*(b^2 - a^2 + c
^2)^(1/2))*(3*c^4*tan(x/2) + a*c^3 + 3*b*c^3 + 3*b^3*c + 2*a^2*b^2*tan(x/2)
 - 2*a^2*c^2*tan(x/2) + 3*b^2*c^2*tan(x/2) - 2*a*b^3*tan(x/2) + a*b^2*c - 4
*a^2*b*c - 2*a*b*c^2*tan(x/2)))/((b^2 + c^2)*(b^2 - a^2 + c^2))))/((b^2 + c
^2)*(b^2 - a^2 + c^2))))/((b^2 + c^2)*(b^2 - a^2 + c^2)))*(b*(a^2 - c^2) -
b^3 + a*c*(b^2 - a^2 + c^2)^(1/2)))/((b^2 + c^2)*(b^2 - a^2 + c^2)) - (log(
64*tan(x/2)*(a - b)^2 + ((b*c^2 - a^2*b + b^3 + a*c*(b^2 - a^2 + c^2)^(1/2)
)*(32*a^2*c + 32*b^2*c - 64*a*b*c + 64*tan(x/2)*(a - b)*(a*b - a^2 + c^2) +
 ((b*c^2 - a^2*b + b^3 + a*c*(b^2 - a^2 + c^2)^(1/2))*(32*a*c^3 - 32*b*c^3
+ 64*b^3*c - 32*tan(x/2)*(a - b)*(2*a*b^2 - 2*a*c^2 + b*c^2 - 2*b^3) - 128*
a*b^2*c + 64*a^2*b*c + (32*(a - b)*(b*c^2 - a^2*b + b^3 + a*c*(b^2 - a^2 +
c^2)^(1/2))*(3*c^4*tan(x/2) + a*c^3 + 3*b*c^3 + 3*b^3*c + 2*a^2*b^2*tan(x/2
) - 2*a^2*c^2*tan(x/2) + 3*b^2*c^2*tan(x/2) - 2*a*b^3*tan(x/2) + a*b^2*c -
4*a^2*b*c - 2*a*b*c^2*tan(x/2)))/((b^2 + c^2)*(b^2 - a^2 + c^2))))/((b^2 +
c^2)*(b^2 - a^2 + c^2))))/((b^2 + c^2)*(b^2 - a^2 + c^2)))*(b^3 - b*(a^2 -
c^2) + a*c*(b^2 - a^2 + c^2)^(1/2)))/((b^2 + c^2)*(b^2 - a^2 + c^2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)/(a+b*cos(x)+c*sin(x)),x)
```

```
[Out] Timed out
```

3.444 $\int \frac{\sin(x)}{1+\cos(x)+\sin(x)}\,dx$

Optimal. Leaf size=22

$$\frac{x}{2}-\log\left(\sin\left(\frac{x}{2}\right)+\cos\left(\frac{x}{2}\right)\right)$$

```
[Out] 1/2*x-ln(cos(1/2*x)+sin(1/2*x))
```

Rubi [A] time = 0.03, antiderivative size = 30, normalized size of antiderivative = 1.36, number of steps used = 3, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {3137, 3124, 31}

$$\frac{x}{2}-\frac{1}{2}\log\left(\tan\left(\frac{x}{2}\right)+1\right)-\frac{1}{2}\log(\sin(x)+\cos(x)+1)$$

Antiderivative was successfully verified.

```
[In] Int[Sin[x]/(1 + Cos[x] + Sin[x]),x]
```

```
[Out] x/2 - Log[1 + Cos[x] + Sin[x]]/2 - Log[1 + Tan[x/2]]/2
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 3124

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^(-1), x_Symbol] :> Module[{f = FreeFactors[Tan[(d + e*x)/2], x]}, Dist[(2*f)/e, Subst[Int[1/(a + b + 2*c*f*x + (a - b)*f^2*x^2), x], x, Tan[(d + e*x)/2]/f], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0]
```

Rule 3137

```
Int[((A_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])/((a_.) + cos[(d_.) + (e_.)*(x_)]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)]), x_Symbol] :> Simp[(c*C*(d + e*x))/(e*(b^2 + c^2)), x] + (Dist[(A*(b^2 + c^2) - a*c*C)/(b^2 + c^2), Int[1/(a + b*Cos[d + e*x] + c*Sin[d + e*x]), x], x] - Simp[(b*C*Log[a + b*Cos[d + e*x] + c*Sin[d + e*x]])/(e*(b^2 + c^2)), x]) /; FreeQ[{a, b, c, d, e, A, C}, x] && NeQ[b^2 + c^2, 0] && NeQ[A*(b^2 + c^2) - a*c*C, 0]
```

Rubi steps

$$\int \frac{\sin(x)}{1+\cos(x)+\sin(x)}\,dx = \frac{x}{2}-\frac{1}{2}\log(1+\cos(x)+\sin(x))-\frac{1}{2}\int \frac{1}{1+\cos(x)+\sin(x)}\,dx$$
$$= \frac{x}{2}-\frac{1}{2}\log(1+\cos(x)+\sin(x))-\text{Subst}\left(\int \frac{1}{2+2x}\,dx, x, \tan\left(\frac{x}{2}\right)\right)$$
$$= \frac{x}{2}-\frac{1}{2}\log(1+\cos(x)+\sin(x))-\frac{1}{2}\log\left(1+\tan\left(\frac{x}{2}\right)\right)$$

Mathematica [A] time = 0.04, size = 22, normalized size = 1.00

$$\frac{x}{2}-\log\left(\sin\left(\frac{x}{2}\right)+\cos\left(\frac{x}{2}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[x]/(1 + Cos[x] + Sin[x]),x]
[Out] x/2 - Log[Cos[x/2] + Sin[x/2]]
```

fricas [A] time = 2.12, size = 11, normalized size = 0.50

$$\frac{1}{2}\,x-\frac{1}{2}\,\log\left(\sin(x)+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)/(1+cos(x)+sin(x)),x, algorithm="fricas")
[Out] 1/2*x - 1/2*log(sin(x) + 1)
```

giac [A] time = 0.14, size = 25, normalized size = 1.14

$$\frac{1}{2}\,x+\frac{1}{2}\,\log\left(\tan\left(\frac{1}{2}\,x\right)^{2}+1\right)-\log\left(\left|\tan\left(\frac{1}{2}\,x\right)+1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)/(1+cos(x)+sin(x)),x, algorithm="giac")
[Out] 1/2*x + 1/2*log(tan(1/2*x)^2 + 1) - log(abs(tan(1/2*x) + 1))
```

maple [A] time = 0.11, size = 25, normalized size = 1.14

$$\frac{\ln\left(1+\tan^{2}\left(\frac{x}{2}\right)\right)}{2}-\ln\left(1+\tan\left(\frac{x}{2}\right)\right)+\frac{x}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)/(1+cos(x)+sin(x)),x)
```

```
[Out] 1/2*ln(1+tan(1/2*x)^2)-ln(1+tan(1/2*x))+1/2*x
```

maxima [B] time = 0.41, size = 41, normalized size = 1.86

$$\arctan\left(\frac{\sin(x)}{\cos(x)+1}\right)-\log\left(\frac{\sin(x)}{\cos(x)+1}+1\right)+\frac{1}{2}\log\left(\frac{\sin(x)^2}{(\cos(x)+1)^2}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)/(1+cos(x)+sin(x)),x, algorithm="maxima")
```

```
[Out] arctan(sin(x)/(cos(x) + 1)) - log(sin(x)/(cos(x) + 1) + 1) + 1/2*log(sin(x)
^2/(cos(x) + 1)^2 + 1)
```

mupad [B] time = 2.79, size = 34, normalized size = 1.55

$$-\ln\left(\tan\left(\frac{x}{2}\right)+1\right)+\ln\left(\tan\left(\frac{x}{2}\right)-\mathrm{i}\right)\left(\frac{1}{2}-\frac{1}{2}\mathrm{i}\right)+\ln\left(\tan\left(\frac{x}{2}\right)+1\mathrm{i}\right)\left(\frac{1}{2}+\frac{1}{2}\mathrm{i}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)/(cos(x) + sin(x) + 1),x)
```

```
[Out] log(tan(x/2) - 1i)*(1/2 - 1i/2) - log(tan(x/2) + 1) + log(tan(x/2) + 1i)*(1
/2 + 1i/2)
```

sympy [A] time = 0.26, size = 22, normalized size = 1.00

$$\frac{x}{2}-\log\left(\tan\left(\frac{x}{2}\right)+1\right)+\frac{\log\left(\tan^2\left(\frac{x}{2}\right)+1\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)/(1+cos(x)+sin(x)),x)
```

```
[Out] x/2 - log(tan(x/2) + 1) + log(tan(x/2)**2 + 1)/2
```

3.445 $\int \frac{1}{a+c\sec(x)+b\tan(x)}\,dx$

Optimal. Leaf size=97

$$\frac{2ac\tanh^{-1}\left(\frac{b-(a-c)\tan\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2-c^2}}\right)}{\left(a^2+b^2\right)\sqrt{a^2+b^2-c^2}}+\frac{b\log(a\cos(x)+b\sin(x)+c)}{a^2+b^2}+\frac{ax}{a^2+b^2}$$

```
[Out] a*x/(a^2+b^2)+b*ln(c+a*cos(x)+b*sin(x))/(a^2+b^2)+2*a*c*arctanh((b-(a-c)*ta
n(1/2*x))/(a^2+b^2-c^2)^(1/2))/(a^2+b^2)/(a^2+b^2-c^2)^(1/2)
```

Rubi [A] time = 0.13, antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {3159, 3138, 3124, 618, 206}

$$\frac{2ac\tanh^{-1}\left(\frac{b-(a-c)\tan\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2-c^2}}\right)}{\left(a^2+b^2\right)\sqrt{a^2+b^2-c^2}}+\frac{b\log(a\cos(x)+b\sin(x)+c)}{a^2+b^2}+\frac{ax}{a^2+b^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + c*Sec[x] + b*Tan[x])^(-1),x]
```

```
[Out] (a*x)/(a^2 + b^2) + (2*a*c*ArcTanh[(b - (a - c)*Tan[x/2])/Sqrt[a^2 + b^2 -
c^2]])/((a^2 + b^2)*Sqrt[a^2 + b^2 - c^2]) + (b*Log[c + a*Cos[x] + b*Sin[x]
])/(a^2 + b^2)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 3124

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(-1), x_Symbol] :> Module[{f = FreeFactors[Tan[(d + e*x)/2], x]}, Dist[(2*f
)/e, Subst[Int[1/(a + b + 2*c*f*x + (a - b)*f^2*x^2), x], x, Tan[(d + e*x)/
```

```
2]/f], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0]
```

Rule 3138

```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.))/((a_.) + cos[(d_.) + (e_.)*(x_)
]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)]), x_Symbol] :> Simp[(b*B*(d + e*x))
/(e*(b^2 + c^2)), x] + (Dist[(A*(b^2 + c^2) - a*b*B)/(b^2 + c^2), Int[1/(a
+ b*Cos[d + e*x] + c*Sin[d + e*x]), x], x] + Simp[(c*B*Log[a + b*Cos[d + e*
x] + c*Sin[d + e*x]])/(e*(b^2 + c^2)), x]) /; FreeQ[{a, b, c, d, e, A, B},
x] && NeQ[b^2 + c^2, 0] && NeQ[A*(b^2 + c^2) - a*b*B, 0]
```

Rule 3159

```
Int[((a_.) + (b_.)*sec[(d_.) + (e_.)*(x_)] + (c_.)*tan[(d_.) + (e_.)*(x_)])
^(-1), x_Symbol] :> Int[Cos[d + e*x]/(b + a*Cos[d + e*x] + c*Sin[d + e*x]),
 x] /; FreeQ[{a, b, c, d, e}, x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{a+c\sec(x)+b\tan(x)}\,dx &= \int \frac{\cos(x)}{c+a\cos(x)+b\sin(x)}\,dx\\
&= \frac{ax}{a^2+b^2}+\frac{b\log(c+a\cos(x)+b\sin(x))}{a^2+b^2}-\frac{(ac)\int \frac{1}{c+a\cos(x)+b\sin(x)}\,dx}{a^2+b^2}\\
&= \frac{ax}{a^2+b^2}+\frac{b\log(c+a\cos(x)+b\sin(x))}{a^2+b^2}-\frac{(2ac)\operatorname{Subst}\left(\int \frac{1}{a+c+2bx+(-a+c)x^2}\,dx,x,
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{ax}{a^2+b^2}+\frac{b\log(c+a\cos(x)+b\sin(x))}{a^2+b^2}+\frac{(4ac)\operatorname{Subst}\left(\int \frac{1}{4\left(a^2+b^2-c^2\right)-x^2}\,dx,x,2b
\end{aligned}
$$

$$
= \frac{ax}{a^2+b^2}+\frac{2ac\tanh^{-1}\left(\frac{b-(a-c)\tan\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2-c^2}}\right)}{\left(a^2+b^2\right)\sqrt{a^2+b^2-c^2}}+\frac{b\log(c+a\cos(x)+b\sin(x))}{a^2+b^2}
$$

Mathematica [A] time = 0.19, size = 79, normalized size = 0.81

$$
\frac{\frac{2ac\tanh^{-1}\left(\frac{(c-a)\tan\left(\frac{x}{2}\right)+b}{\sqrt{a^2+b^2-c^2}}\right)}{\sqrt{a^2+b^2-c^2}}+b\log(a\cos(x)+b\sin(x)+c)+ax}{a^2+b^2}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + c*Sec[x] + b*Tan[x])^(-1),x]
```

```
[Out] (a*x + (2*a*c*ArcTanh[(b + (-a + c)*Tan[x/2])/Sqrt[a^2 + b^2 - c^2]])/Sqrt[
a^2 + b^2 - c^2] + b*Log[c + a*Cos[x] + b*Sin[x]])/(a^2 + b^2)
```

fricas [B] time = 1.76, size = 553, normalized size = 5.70

$$\left[\frac{\sqrt{a^2+b^2-c^2}\,ac\log\left(\frac{2a^4+3a^2b^2+b^4-(a^2-b^2)c^2+2(a^3+ab^2)c\cos(x)-(a^4-b^4-2(a^2-b^2)c^2)\cos(x)^2+2((a^2b+b^3)c-(a^3b+ab^3-2abc^2)\cos(x))\,s}{2ac\cos(x)+(a^2-b^2)\cos(x)^2+b^2+c^2+2(ab\,c}\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+c*sec(x)+b*tan(x)),x, algorithm="fricas")
```

```
[Out] [1/2*(sqrt(a^2 + b^2 - c^2)*a*c*log((2*a^4 + 3*a^2*b^2 + b^4 - (a^2 - b^2)*
c^2 + 2*(a^3 + a*b^2)*c*cos(x) - (a^4 - b^4 - 2*(a^2 - b^2)*c^2)*cos(x)^2 +
 2*((a^2*b + b^3)*c - (a^3*b + a*b^3 - 2*a*b*c^2)*cos(x))*sin(x) + 2*(2*a*b
*c*cos(x)^2 - a*b*c + (a^2*b + b^3)*cos(x) - (a^3 + a*b^2 + (a^2 - b^2)*c*c
os(x))*sin(x))*sqrt(a^2 + b^2 - c^2))/(2*a*c*cos(x) + (a^2 - b^2)*cos(x)^2
+ b^2 + c^2 + 2*(a*b*cos(x) + b*c)*sin(x))) + 2*(a^3 + a*b^2 - a*c^2)*x + (
a^2*b + b^3 - b*c^2)*log(2*a*c*cos(x) + (a^2 - b^2)*cos(x)^2 + b^2 + c^2 +
2*(a*b*cos(x) + b*c)*sin(x)))/(a^4 + 2*a^2*b^2 + b^4 - (a^2 + b^2)*c^2), -1
/2*(2*sqrt(-a^2 - b^2 + c^2)*a*c*arctan((a*c*cos(x) + b*c*sin(x) + a^2 + b^
2)*sqrt(-a^2 - b^2 + c^2)/((a^2*b + b^3 - b*c^2)*cos(x) - (a^3 + a*b^2 - a*
c^2)*sin(x))) - 2*(a^3 + a*b^2 - a*c^2)*x - (a^2*b + b^3 - b*c^2)*log(2*a*c
*cos(x) + (a^2 - b^2)*cos(x)^2 + b^2 + c^2 + 2*(a*b*cos(x) + b*c)*sin(x)))/
(a^4 + 2*a^2*b^2 + b^4 - (a^2 + b^2)*c^2)]
```

giac [A] time = 0.15, size = 158, normalized size = 1.63

$$-\frac{2\left(\pi\left\lfloor\frac{x}{2\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(-2a+2c)+\arctan\left(-\frac{a\tan\left(\frac{1}{2}x\right)-c\tan\left(\frac{1}{2}x\right)-b}{\sqrt{-a^2-b^2+c^2}}\right)\right)ac}{(a^2+b^2)\sqrt{-a^2-b^2+c^2}}+\frac{ax}{a^2+b^2}+\frac{b\log\left(-a\tan\left(\frac{1}{2}x\right)^2+c\tan\left(\frac{1}{2}x\right)^2\right.}{a^2+b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+c*sec(x)+b*tan(x)),x, algorithm="giac")
```

```
[Out] -2*(pi*floor(1/2*x/pi + 1/2)*sgn(-2*a + 2*c) + arctan(-(a*tan(1/2*x) - c*ta
n(1/2*x) - b)/sqrt(-a^2 - b^2 + c^2)))*a*c/((a^2 + b^2)*sqrt(-a^2 - b^2 + c
^2)) + a*x/(a^2 + b^2) + b*log(-a*tan(1/2*x)^2 + c*tan(1/2*x)^2 + 2*b*tan(1
/2*x) + a + c)/(a^2 + b^2) - b*log(tan(1/2*x)^2 + 1)/(a^2 + b^2)
```

maple [B] time = 0.15, size = 414, normalized size = 4.27

$$\frac{\ln\left(a\left(\tan^2\left(\frac{x}{2}\right)\right)-c\left(\tan^2\left(\frac{x}{2}\right)\right)-2b\tan\left(\frac{x}{2}\right)-a-c\right)ab}{\left(a^2+b^2\right)(a-c)}-\frac{\ln\left(a\left(\tan^2\left(\frac{x}{2}\right)\right)-c\left(\tan^2\left(\frac{x}{2}\right)\right)-2b\tan\left(\frac{x}{2}\right)-a-c\right)cb}{\left(a^2+b^2\right)(a-c)}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+c*sec(x)+b*tan(x)),x)
```

```
[Out] 1/(a^2+b^2)/(a-c)*ln(a*tan(1/2*x)^2-c*tan(1/2*x)^2-2*b*tan(1/2*x)-a-c)*a*b-
1/(a^2+b^2)/(a-c)*ln(a*tan(1/2*x)^2-c*tan(1/2*x)^2-2*b*tan(1/2*x)-a-c)*c*b+
2/(a^2+b^2)/(-a^2-b^2+c^2)^(1/2)*arctan(1/2*(2*(a-c)*tan(1/2*x)-2*b)/(-a^2-
b^2+c^2)^(1/2))*a*c-2/(a^2+b^2)/(-a^2-b^2+c^2)^(1/2)*arctan(1/2*(2*(a-c)*ta
n(1/2*x)-2*b)/(-a^2-b^2+c^2)^(1/2))*b^2+2/(a^2+b^2)/(-a^2-b^2+c^2)^(1/2)*ar
ctan(1/2*(2*(a-c)*tan(1/2*x)-2*b)/(-a^2-b^2+c^2)^(1/2))*b^2/(a-c)*a-2/(a^2+
b^2)/(-a^2-b^2+c^2)^(1/2)*arctan(1/2*(2*(a-c)*tan(1/2*x)-2*b)/(-a^2-b^2+c^2
)^(1/2))*b^2/(a-c)*c-1/(a^2+b^2)*b*ln(1+tan(1/2*x)^2)+2/(a^2+b^2)*a*arctan(
tan(1/2*x))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+c*sec(x)+b*tan(x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(c^2-b^2-a^2>0)', see `assume?` for
more details)Is c^2-b^2-a^2 positive or negative?
```

mupad [B] time = 13.03, size = 988, normalized size = 10.19

$$\ln\left(32\,a\,c-32\,c^2+32\,b\tan\left(\frac{x}{2}\right)\,(a-c)+\frac{\left(32\,a^2\,b-32\,b\,c^2+32\tan\left(\frac{x}{2}\right)(a-c)\left(-a^2+2\,a\,c+3\,b^2-2\,c^2\right)-\frac{\left(a^2\,b-b\,c^2+b^3+a\,c\,\sqrt{a^2+b^2-c^2}\right)\left(32\,a^4-64\right.}{}\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*tan(x) + c/cos(x)),x)
```

```
[Out] (log(32*a*c - 32*c^2 + 32*b*tan(x/2)*(a - c) + ((32*a^2*b - 32*b*c^2 + 32*t
an(x/2)*(a - c)*(2*a*c - a^2 + 3*b^2 - 2*c^2) - ((a^2*b - b*c^2 + b^3 + a*c
*(a^2 + b^2 - c^2)^(1/2))*(32*a^4 - 64*a^3*c - 64*a^2*b^2 + 32*a^2*c^2 - 32
*b^2*c^2 + 96*a*b^2*c + 32*b*tan(x/2)*(a - c)*(4*a^2 - 4*a*c + b^2) + (32*(
a - c)*(a^2*b - b*c^2 + b^3 + a*c*(a^2 + b^2 - c^2)^(1/2))*(3*b^4*tan(x/2)
+ 3*a*b^3 + 3*a^3*b + b^3*c + 3*a^2*b^2*tan(x/2) + 2*a^2*c^2*tan(x/2) - 2*b
^2*c^2*tan(x/2) - 2*a^3*c*tan(x/2) - 4*a*b*c^2 + a^2*b*c - 2*a*b^2*c*tan(x/
2)))/((a^2 + b^2)*(a^2 + b^2 - c^2))))/((a^2 + b^2)*(a^2 + b^2 - c^2)))*(a^
2*b - b*c^2 + b^3 + a*c*(a^2 + b^2 - c^2)^(1/2)))/((a^2 + b^2)*(a^2 + b^2 -
 c^2)))*(b*(a^2 - c^2) + b^3 + a*c*(a^2 + b^2 - c^2)^(1/2)))/(c^2*(a^2 + b^
2 - c^2) + (a^2 + b^2 - c^2)^2) - log(tan(x/2) + 1i)/(a*1i + b) - (log(tan(
x/2) - 1i)*1i)/(a + b*1i) + (log(32*a*c - 32*c^2 + 32*b*tan(x/2)*(a - c) +
((32*a^2*b - 32*b*c^2 + 32*tan(x/2)*(a - c)*(2*a*c - a^2 + 3*b^2 - 2*c^2) -
 ((a^2*b - b*c^2 + b^3 - a*c*(a^2 + b^2 - c^2)^(1/2))*(32*a^4 - 64*a^3*c -
64*a^2*b^2 + 32*a^2*c^2 - 32*b^2*c^2 + 96*a*b^2*c + 32*b*tan(x/2)*(a - c)*(
4*a^2 - 4*a*c + b^2) + (32*(a - c)*(a^2*b - b*c^2 + b^3 - a*c*(a^2 + b^2 -
c^2)^(1/2))*(3*b^4*tan(x/2) + 3*a*b^3 + 3*a^3*b + b^3*c + 3*a^2*b^2*tan(x/2
) + 2*a^2*c^2*tan(x/2) - 2*b^2*c^2*tan(x/2) - 2*a^3*c*tan(x/2) - 4*a*b*c^2
+ a^2*b*c - 2*a*b^2*c*tan(x/2)))/((a^2 + b^2)*(a^2 + b^2 - c^2))))/((a^2 +
b^2)*(a^2 + b^2 - c^2)))*(a^2*b - b*c^2 + b^3 - a*c*(a^2 + b^2 - c^2)^(1/2)
))/((a^2 + b^2)*(a^2 + b^2 - c^2)))*(b*(a^2 - c^2) + b^3 - a*c*(a^2 + b^2 -
 c^2)^(1/2)))/(c^2*(a^2 + b^2 - c^2) + (a^2 + b^2 - c^2)^2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{a + b\tan(x) + c\sec(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+c*sec(x)+b*tan(x)),x)
```

```
[Out] Integral(1/(a + b*tan(x) + c*sec(x)), x)
```

3.446 $\int \frac{\sec(x)}{a+c\sec(x)+b\tan(x)}\,dx$

Optimal. Leaf size=51

$$-\frac{2\tanh^{-1}\left(\frac{b-(a-c)\tan\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2-c^2}}\right)}{\sqrt{a^2+b^2-c^2}}$$

```
[Out] -2*arctanh((b-(a-c)*tan(1/2*x))/(a^2+b^2-c^2)^(1/2))/(a^2+b^2-c^2)^(1/2)
```

Rubi [A] time = 0.08, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {3165, 3124, 618, 206}

$$-\frac{2\tanh^{-1}\left(\frac{b-(a-c)\tan\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2-c^2}}\right)}{\sqrt{a^2+b^2-c^2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[x]/(a + c*Sec[x] + b*Tan[x]),x]
```

```
[Out] (-2*ArcTanh[(b - (a - c)*Tan[x/2])/Sqrt[a^2 + b^2 - c^2]])/Sqrt[a^2 + b^2 -
 c^2]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 3124

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(-1), x_Symbol] :> Module[{f = FreeFactors[Tan[(d + e*x)/2], x]}, Dist[(2*f
)/e, Subst[Int[1/(a + b + 2*c*f*x + (a - b)*f^2*x^2), x], x, Tan[(d + e*x)/
2]/f], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0]
```

Rule 3165

```
Int[sec[(d_.) + (e_.)*(x_)]^(n_.)*((a_.) + (b_.)*sec[(d_.) + (e_.)*(x_)] +
(c_.)*tan[(d_.) + (e_.)*(x_)])^(m_), x_Symbol] :> Int[1/(b + a*Cos[d + e*x]
 + c*Sin[d + e*x])^n, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[m + n, 0] && I
ntegerQ[n]
```

Rubi steps

$$\begin{aligned}\int \frac{\sec(x)}{a+c\sec(x)+b\tan(x)}\,dx &= \int \frac{1}{c+a\cos(x)+b\sin(x)}\,dx\\ &= 2\,\text{Subst}\left(\int \frac{1}{a+c+2bx+(-a+c)x^2}\,dx, x, \tan\left(\frac{x}{2}\right)\right)\\ &= -\left(4\,\text{Subst}\left(\int \frac{1}{4\left(a^2+b^2-c^2\right)-x^2}\,dx, x, 2b+2(-a+c)\tan\left(\frac{x}{2}\right)\right)\right)\\ &= -\frac{2\tanh^{-1}\left(\frac{b-(a-c)\tan\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2-c^2}}\right)}{\sqrt{a^2+b^2-c^2}}\end{aligned}$$

Mathematica [A] time = 0.04, size = 50, normalized size = 0.98

$$-\frac{2\tanh^{-1}\left(\frac{(c-a)\tan\left(\frac{x}{2}\right)+b}{\sqrt{a^2+b^2-c^2}}\right)}{\sqrt{a^2+b^2-c^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[x]/(a + c*Sec[x] + b*Tan[x]),x]
```

```
[Out] (-2*ArcTanh[(b + (-a + c)*Tan[x/2])/Sqrt[a^2 + b^2 - c^2]])/Sqrt[a^2 + b^2
- c^2]
```

fricas [B] time = 0.51, size = 349, normalized size = 6.84

$$\left[\frac{\log\left(-\frac{2a^4+3a^2b^2+b^4-\left(a^2-b^2\right)c^2+2\left(a^3+ab^2\right)c\cos(x)-\left(a^4-b^4-2\left(a^2-b^2\right)c^2\right)\cos(x)^2+2\left(\left(a^2b+b^3\right)c-\left(a^3b+ab^3-2abc^2\right)\cos(x)\right)\sin(x)-2\left(2abc\cos(x\right.}{2ac\cos(x)+\left(a^2-b^2\right)\cos(x)^2+b^2+c^2+2(ab\cos(x)+bc)\sin(x)}\right.}{2\sqrt{a^2+b^2-c^2}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)/(a+c*sec(x)+b*tan(x)),x, algorithm="fricas")
```

```
[Out] [1/2*log(-(2*a^4 + 3*a^2*b^2 + b^4 - (a^2 - b^2)*c^2 + 2*(a^3 + a*b^2)*c*co
s(x) - (a^4 - b^4 - 2*(a^2 - b^2)*c^2)*cos(x)^2 + 2*((a^2*b + b^3)*c - (a^3
*b + a*b^3 - 2*a*b*c^2)*cos(x))*sin(x) - 2*(2*a*b*c*cos(x)^2 - a*b*c + (a^2
*b + b^3)*cos(x) - (a^3 + a*b^2 + (a^2 - b^2)*c*cos(x))*sin(x))*sqrt(a^2 +
b^2 - c^2))/(2*a*c*cos(x) + (a^2 - b^2)*cos(x)^2 + b^2 + c^2 + 2*(a*b*cos(x
) + b*c)*sin(x)))/sqrt(a^2 + b^2 - c^2), sqrt(-a^2 - b^2 + c^2)*arctan((a*c
*cos(x) + b*c*sin(x) + a^2 + b^2)*sqrt(-a^2 - b^2 + c^2)/((a^2*b + b^3 - b*
c^2)*cos(x) - (a^3 + a*b^2 - a*c^2)*sin(x)))/(a^2 + b^2 - c^2)]
```

giac [A] time = 0.17, size = 73, normalized size = 1.43

$$-\frac{2\left(\pi\left\lfloor\frac{x}{2\,\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}\left(2\,a-2\,c\right)+\arctan\left(\frac{a\tan\left(\frac{1}{2}\,x\right)-c\tan\left(\frac{1}{2}\,x\right)-b}{\sqrt{-a^{2}-b^{2}+c^{2}}}\right)\right)}{\sqrt{-a^{2}-b^{2}+c^{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)/(a+c*sec(x)+b*tan(x)),x, algorithm="giac")
```

```
[Out] -2*(pi*floor(1/2*x/pi + 1/2)*sgn(2*a - 2*c) + arctan((a*tan(1/2*x) - c*tan(
1/2*x) - b)/sqrt(-a^2 - b^2 + c^2)))/sqrt(-a^2 - b^2 + c^2)
```

maple [A] time = 0.13, size = 53, normalized size = 1.04

$$-\frac{2\arctan\left(\frac{2(a-c)\tan\left(\frac{x}{2}\right)-2b}{2\sqrt{-a^{2}-b^{2}+c^{2}}}\right)}{\sqrt{-a^{2}-b^{2}+c^{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(x)/(a+c*sec(x)+b*tan(x)),x)
```

```
[Out] -2/(-a^2-b^2+c^2)^(1/2)*arctan(1/2*(2*(a-c)*tan(1/2*x)-2*b)/(-a^2-b^2+c^2)^
(1/2))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)/(a+c*sec(x)+b*tan(x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
```

```
elp (example of legal syntax is 'assume(c^2-b^2-a^2>0)', see `assume?` for
more details)Is c^2-b^2-a^2 positive or negative?
```

mupad [B] time = 2.78, size = 47, normalized size = 0.92

$$-\frac{2\,\mathrm{atanh}\left(\frac{b-\frac{\tan\left(\frac{x}{2}\right)\,(2\,a-2\,c)}{2}}{\sqrt{a^2+b^2-c^2}}\right)}{\sqrt{a^2+b^2-c^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(x)*(a + b*tan(x) + c/cos(x))),x)
```

```
[Out] -(2*atanh((b - (tan(x/2)*(2*a - 2*c))/2)/(a^2 + b^2 - c^2)^(1/2)))/(a^2 + b
^2 - c^2)^(1/2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec(x)}{a+b\tan(x)+c\sec(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)/(a+c*sec(x)+b*tan(x)),x)
```

```
[Out] Integral(sec(x)/(a + b*tan(x) + c*sec(x)), x)
```

3.447 $\int \frac{\sec^2(x)}{a+c\sec(x)+b\tan(x)}\,dx$

Optimal. Leaf size=142

$$-\frac{2ac\tanh^{-1}\left(\frac{b-(a-c)\tan\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2-c^2}}\right)}{\left(b^2-c^2\right)\sqrt{a^2+b^2-c^2}}+\frac{b\log\left(-\left((a-c)\tan^2\left(\frac{x}{2}\right)\right)+a+2b\tan\left(\frac{x}{2}\right)+c\right)}{b^2-c^2}-\frac{\log\left(1-\tan\left(\frac{x}{2}\right)\right)}{b+c}-\frac{\log\left(\tan\left(\frac{x}{2}\right)+\right.}{b-c}$$

```
[Out] -ln(1-tan(1/2*x))/(b+c)-ln(1+tan(1/2*x))/(b-c)+b*ln(a+c+2*b*tan(1/2*x)-(a-c
)*tan(1/2*x)^2)/(b^2-c^2)-2*a*c*arctanh((b-(a-c)*tan(1/2*x))/(a^2+b^2-c^2)^
(1/2))/(b^2-c^2)/(a^2+b^2-c^2)^(1/2)
```

Rubi [A] time = 0.51, antiderivative size = 142, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 8, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.471, Rules used = {4397, 1075, 634, 618, 206, 628, 633, 31}

$$-\frac{2ac\tanh^{-1}\left(\frac{b-(a-c)\tan\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2-c^2}}\right)}{\left(b^2-c^2\right)\sqrt{a^2+b^2-c^2}}+\frac{b\log\left(-(a-c)\tan^2\left(\frac{x}{2}\right)+a+2b\tan\left(\frac{x}{2}\right)+c\right)}{b^2-c^2}-\frac{\log\left(1-\tan\left(\frac{x}{2}\right)\right)}{b+c}-\frac{\log\left(\tan\left(\frac{x}{2}\right)+1\right.}{b-c}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[x]^2/(a + c*Sec[x] + b*Tan[x]),x]
```

```
[Out] (-2*a*c*ArcTanh[(b - (a - c)*Tan[x/2])/Sqrt[a^2 + b^2 - c^2]])/((b^2 - c^2)
*Sqrt[a^2 + b^2 - c^2]) - Log[1 - Tan[x/2]]/(b + c) - Log[1 + Tan[x/2]]/(b
- c) + (b*Log[a + c + 2*b*Tan[x/2] - (a - c)*Tan[x/2]^2])/(b^2 - c^2)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
```

```
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 633

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> With[{q = Rt[-(
a*c), 2]}, Dist[e/2 + (c*d)/(2*q), Int[1/(-q + c*x), x], x] + Dist[e/2 - (c
*d)/(2*q), Int[1/(q + c*x), x], x]] /; FreeQ[{a, c, d, e}, x] && NiceSqrtQ[
-(a*c)]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1075

```
Int[((A_.) + (C_.)*(x_)^2)/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*((d_) + (f_.
)*(x_)^2)), x_Symbol] :> With[{q = c^2*d^2 + b^2*d*f - 2*a*c*d*f + a^2*f^2}
, Dist[1/q, Int[(A*c^2*d - a*c*C*d + A*b^2*f - a*A*c*f + a^2*C*f + c*(-(b*C
*d) + A*b*f)*x)/(a + b*x + c*x^2), x], x] + Dist[1/q, Int[(c*C*d^2 - A*c*d*
f - a*C*d*f + a*A*f^2 - f*(-(b*C*d) + A*b*f)*x)/(d + f*x^2), x], x] /; NeQ[
q, 0]] /; FreeQ[{a, b, c, d, f, A, C}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 4397

```
Int[u_, x_Symbol] :> Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]
```

Rubi steps

$$\int \frac{\sec^2(x)}{a+c\sec(x)+b\tan(x)}\,dx = \int \frac{\sec(x)}{c+a\cos(x)+b\sin(x)}\,dx$$
$$= 2\,\text{Subst}\left(\int \frac{1+x^2}{(1-x^2)\left(a+c+2bx-(a-c)x^2\right)}\,dx, x, \tan\left(\frac{x}{2}\right)\right)$$
$$= -\frac{\text{Subst}\left(\int \frac{4c-4bx}{1-x^2}\,dx, x, \tan\left(\frac{x}{2}\right)\right)}{2(b^2-c^2)} - \frac{\text{Subst}\left(\int \frac{-4b^2+(-a+c)^2-(a+c)^2-4b(-a+c)x}{a+c+2bx+(-a+c)x^2}\,dx, x, \tan\right.}{2(b^2-c^2)}$$
$$= \frac{\text{Subst}\left(\int \frac{1}{-1-x}\,dx, x, \tan\left(\frac{x}{2}\right)\right)}{b-c} + \frac{\text{Subst}\left(\int \frac{1}{1-x}\,dx, x, \tan\left(\frac{x}{2}\right)\right)}{b+c} + \frac{b\,\text{Subst}\left(\int \frac{2b-}{a+c+2}\right.}{}$$
$$= -\frac{\log\left(1-\tan\left(\frac{x}{2}\right)\right)}{b+c} - \frac{\log\left(1+\tan\left(\frac{x}{2}\right)\right)}{b-c} + \frac{b\log\left(a+c+2b\tan\left(\frac{x}{2}\right)-(a-c)\tan\right.}{b^2-c^2}$$
$$= -\frac{2ac\tanh^{-1}\left(\frac{b-(a-c)\tan\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2-c^2}}\right)}{(b^2-c^2)\sqrt{a^2+b^2-c^2}} - \frac{\log\left(1-\tan\left(\frac{x}{2}\right)\right)}{b+c} - \frac{\log\left(1+\tan\left(\frac{x}{2}\right)\right)}{b-c} + \frac{b\log\left(a\right.}{}$$

Mathematica [A] time = 0.28, size = 120, normalized size = 0.85

$$\frac{\frac{2ac\tanh^{-1}\left(\frac{(c-a)\tan\left(\frac{x}{2}\right)+b}{\sqrt{a^2+b^2-c^2}}\right)}{\sqrt{a^2+b^2-c^2}} - b\log(a\cos(x)+b\sin(x)+c)+(b-c)\log\left(\cos\left(\frac{x}{2}\right)-\sin\left(\frac{x}{2}\right)\right)+(b+c)\log\left(\sin\left(\frac{x}{2}\right)+\text{co}\right.}{(c-b)(b+c)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[x]^2/(a + c*Sec[x] + b*Tan[x]),x]
```

```
[Out] ((2*a*c*ArcTanh[(b + (-a + c)*Tan[x/2])/Sqrt[a^2 + b^2 - c^2]])/Sqrt[a^2 +
b^2 - c^2] + (b - c)*Log[Cos[x/2] - Sin[x/2]] + (b + c)*Log[Cos[x/2] + Sin[
x/2]] - b*Log[c + a*Cos[x] + b*Sin[x]])/((-b + c)*(b + c))
```

fricas [A] time = 5.00, size = 663, normalized size = 4.67

$$\left[-\frac{\sqrt{a^2+b^2-c^2}\,ac\log\left(\frac{2a^4+3a^2b^2+b^4-(a^2-b^2)c^2+2(a^3+ab^2)c\cos(x)-(a^4-b^4-2(a^2-b^2)c^2)\cos(x)^2+2\left((a^2b+b^3)c-(a^3b+ab^3-2abc^2)\cos(\right.}{2ac\cos(x)+(a^2-b^2)\cos(x)^2+b^2+c^2+2(}\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)^2/(a+c*sec(x)+b*tan(x)),x, algorithm="fricas")
```

```
[Out] [-1/2*(sqrt(a^2 + b^2 - c^2)*a*c*log((2*a^4 + 3*a^2*b^2 + b^4 - (a^2 - b^2)
*c^2 + 2*(a^3 + a*b^2)*c*cos(x) - (a^4 - b^4 - 2*(a^2 - b^2)*c^2)*cos(x)^2
+ 2*((a^2*b + b^3)*c - (a^3*b + a*b^3 - 2*a*b*c^2)*cos(x))*sin(x) + 2*(2*a*
b*c*cos(x)^2 - a*b*c + (a^2*b + b^3)*cos(x) - (a^3 + a*b^2 + (a^2 - b^2)*c*
cos(x))*sin(x))*sqrt(a^2 + b^2 - c^2))/(2*a*c*cos(x) + (a^2 - b^2)*cos(x)^2
 + b^2 + c^2 + 2*(a*b*cos(x) + b*c)*sin(x))) - (a^2*b + b^3 - b*c^2)*log(2*
a*c*cos(x) + (a^2 - b^2)*cos(x)^2 + b^2 + c^2 + 2*(a*b*cos(x) + b*c)*sin(x)
) + (a^2*b + b^3 - b*c^2 - c^3 + (a^2 + b^2)*c)*log(sin(x) + 1) + (a^2*b +
b^3 - b*c^2 + c^3 - (a^2 + b^2)*c)*log(-sin(x) + 1))/(a^2*b^2 + b^4 + c^4 -
 (a^2 + 2*b^2)*c^2), 1/2*(2*sqrt(-a^2 - b^2 + c^2)*a*c*arctan((a*c*cos(x) +
 b*c*sin(x) + a^2 + b^2)*sqrt(-a^2 - b^2 + c^2)/((a^2*b + b^3 - b*c^2)*cos(
x) - (a^3 + a*b^2 - a*c^2)*sin(x))) + (a^2*b + b^3 - b*c^2)*log(2*a*c*cos(x
) + (a^2 - b^2)*cos(x)^2 + b^2 + c^2 + 2*(a*b*cos(x) + b*c)*sin(x)) - (a^2*
b + b^3 - b*c^2 - c^3 + (a^2 + b^2)*c)*log(sin(x) + 1) - (a^2*b + b^3 - b*c
^2 + c^3 - (a^2 + b^2)*c)*log(-sin(x) + 1))/(a^2*b^2 + b^4 + c^4 - (a^2 + 2
*b^2)*c^2)]
```

giac [A] time = 0.17, size = 161, normalized size = 1.13

$$\frac{2\left(\pi\left\lfloor\frac{x}{2\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}\left(-2a+2c\right)+\arctan\left(-\frac{a\tan\left(\frac{1}{2}x\right)-c\tan\left(\frac{1}{2}x\right)-b}{\sqrt{-a^2-b^2+c^2}}\right)\right)ac}{\sqrt{-a^2-b^2+c^2}\left(b^2-c^2\right)}+\frac{b\log\left(-a\tan\left(\frac{1}{2}x\right)^2+c\tan\left(\frac{1}{2}x\right)^2+2b\tan\left(\right.\right.}{b^2-c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)^2/(a+c*sec(x)+b*tan(x)),x, algorithm="giac")
```

```
[Out] 2*(pi*floor(1/2*x/pi + 1/2)*sgn(-2*a + 2*c) + arctan(-(a*tan(1/2*x) - c*tan
(1/2*x) - b)/sqrt(-a^2 - b^2 + c^2)))*a*c/(sqrt(-a^2 - b^2 + c^2)*(b^2 - c^
2)) + b*log(-a*tan(1/2*x)^2 + c*tan(1/2*x)^2 + 2*b*tan(1/2*x) + a + c)/(b^2
 - c^2) - log(abs(tan(1/2*x) + 1))/(b - c) - log(abs(tan(1/2*x) - 1))/(b +
c)
```

maple [B] time = 0.14, size = 430, normalized size = 3.03

$$-\frac{2\ln\left(\tan\left(\frac{x}{2}\right)-1\right)}{2b+2c}+\frac{\ln\left(a\left(\tan^2\left(\frac{x}{2}\right)\right)-c\left(\tan^2\left(\frac{x}{2}\right)\right)-2b\tan\left(\frac{x}{2}\right)-a-c\right)ab}{(b-c)(b+c)(a-c)}-\frac{\ln\left(a\left(\tan^2\left(\frac{x}{2}\right)\right)-c\left(\tan^2\left(\frac{x}{2}\right)\right)-2b\right.}{(b-c)(b+c)(a-c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(x)^2/(a+c*sec(x)+b*tan(x)),x)
```

```
[Out] -2/(2*b+2*c)*ln(tan(1/2*x)-1)+1/(b-c)/(b+c)/(a-c)*ln(a*tan(1/2*x)^2-c*tan(1
/2*x)^2-2*b*tan(1/2*x)-a-c)*a*b-1/(b-c)/(b+c)/(a-c)*ln(a*tan(1/2*x)^2-c*tan
(1/2*x)^2-2*b*tan(1/2*x)-a-c)*c*b-2/(b-c)/(b+c)/(-a^2-b^2+c^2)^(1/2)*arctan
(1/2*(2*(a-c)*tan(1/2*x)-2*b)/(-a^2-b^2+c^2)^(1/2))*a*c-2/(b-c)/(b+c)/(-a^2
-b^2+c^2)^(1/2)*arctan(1/2*(2*(a-c)*tan(1/2*x)-2*b)/(-a^2-b^2+c^2)^(1/2))*b
^2+2/(b-c)/(b+c)/(-a^2-b^2+c^2)^(1/2)*arctan(1/2*(2*(a-c)*tan(1/2*x)-2*b)/(
-a^2-b^2+c^2)^(1/2))*b^2/(a-c)*a-2/(b-c)/(b+c)/(-a^2-b^2+c^2)^(1/2)*arctan(
1/2*(2*(a-c)*tan(1/2*x)-2*b)/(-a^2-b^2+c^2)^(1/2))*b^2/(a-c)*c-2/(-2*c+2*b)
*ln(1+tan(1/2*x))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)^2/(a+c*sec(x)+b*tan(x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(c^2-b^2-a^2>0)', see `assume?` for
more details)Is c^2-b^2-a^2 positive or negative?
```

mupad [B] time = 11.44, size = 977, normalized size = 6.88

$$\ln\left(32\,a\,c-32\,a^2-32\,b\,\tan\left(\frac{x}{2}\right)\,\left(a-c\right)-\frac{\left(a^2\,b-b\,c^2+b^3+a\,c\,\sqrt{a^2+b^2-c^2}\right)\,\left(32\,a^2\,b-32\,b\,c^2+32\,\tan\left(\frac{x}{2}\right)\,\left(a-c\right)\,\left(2\,a^2-2\,a\,c+3\,b^2+c^2\right)-\frac{\left(a^2\,b-\cdots\right.}{\cdots}\right.}{\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(x)^2*(a + b*tan(x) + c/cos(x))),x)
```

```
[Out] (log(32*a*c - 32*a^2 - 32*b*tan(x/2)*(a - c) - ((a^2*b - b*c^2 + b^3 + a*c*
(a^2 + b^2 - c^2)^(1/2))*(32*a^2*b - 32*b*c^2 + 32*tan(x/2)*(a - c)*(2*a^2
- 2*a*c + 3*b^2 + c^2) - ((a^2*b - b*c^2 + b^3 + a*c*(a^2 + b^2 - c^2)^(1/2
))*(32*c^4 - 64*a*c^3 + 32*a^2*b^2 + 32*a^2*c^2 + 64*b^2*c^2 - 96*a*b^2*c +
 32*b*tan(x/2)*(a - c)*(4*a*c + b^2 - 4*c^2) + (32*(a - c)*(a^2*b - b*c^2 +
 b^3 + a*c*(a^2 + b^2 - c^2)^(1/2))*(3*b^4*tan(x/2) + a*b^3 - 3*b*c^3 + 3*b
```

```
^3*c + 2*a^2*b^2*tan(x/2) + 2*a^2*c^2*tan(x/2) - 3*b^2*c^2*tan(x/2) - 2*a*c
^3*tan(x/2) - a*b*c^2 + 4*a^2*b*c + 2*a*b^2*c*tan(x/2)))/((b^2 - c^2)*(a^2
+ b^2 - c^2))))/((b^2 - c^2)*(a^2 + b^2 - c^2))))/((b^2 - c^2)*(a^2 + b^2 -
 c^2)))*(b*(a^2 - c^2) + b^3 + a*c*(a^2 + b^2 - c^2)^(1/2)))/((b^2 - c^2)*(
a^2 + b^2 - c^2)) - log(tan(x/2) - 1)/(b + c) - log(tan(x/2) + 1)/(b - c) +
 (log(32*a*c - 32*a^2 - 32*b*tan(x/2)*(a - c) - ((a^2*b - b*c^2 + b^3 - a*c
*(a^2 + b^2 - c^2)^(1/2))*(32*a^2*b - 32*b*c^2 + 32*tan(x/2)*(a - c)*(2*a^2
 - 2*a*c + 3*b^2 + c^2) - ((a^2*b - b*c^2 + b^3 - a*c*(a^2 + b^2 - c^2)^(1/
2))*(32*c^4 - 64*a*c^3 + 32*a^2*b^2 + 32*a^2*c^2 + 64*b^2*c^2 - 96*a*b^2*c
+ 32*b*tan(x/2)*(a - c)*(4*a*c + b^2 - 4*c^2) + (32*(a - c)*(a^2*b - b*c^2
+ b^3 - a*c*(a^2 + b^2 - c^2)^(1/2))*(3*b^4*tan(x/2) + a*b^3 - 3*b*c^3 + 3*
b^3*c + 2*a^2*b^2*tan(x/2) + 2*a^2*c^2*tan(x/2) - 3*b^2*c^2*tan(x/2) - 2*a*
c^3*tan(x/2) - a*b*c^2 + 4*a^2*b*c + 2*a*b^2*c*tan(x/2)))/((b^2 - c^2)*(a^2
 + b^2 - c^2))))/((b^2 - c^2)*(a^2 + b^2 - c^2))))/((b^2 - c^2)*(a^2 + b^2
- c^2)))*(b*(a^2 - c^2) + b^3 - a*c*(a^2 + b^2 - c^2)^(1/2)))/((b^2 - c^2)*
(a^2 + b^2 - c^2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec^2(x)}{a + b\tan(x) + c\sec(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)**2/(a+c*sec(x)+b*tan(x)),x)
```

```
[Out] Integral(sec(x)**2/(a + b*tan(x) + c*sec(x)), x)
```

3.448 $\int \frac{(a+b\sec(d+ex)+c\tan(d+ex))^{3/2}}{\sec^{\frac{3}{2}}(d+ex)}\,dx$

Optimal. Leaf size=371

$$\frac{2\left(a^2-b^2+c^2\right)\sqrt{\frac{a\cos(d+ex)+b+c\sin(d+ex)}{\sqrt{a^2+c^2}+b}}\,(a+b\sec(d+ex)+c\tan(d+ex))^{3/2}F\left(\frac{1}{2}\left(d+ex-\tan^{-1}(a,c)\right)|\frac{2\sqrt{a^2+c^2}}{b+\sqrt{a^2+c^2}}\right.}{3e\sec^{\frac{3}{2}}(d+ex)(a\cos(d+ex)+b+c\sin(d+ex))^2}$$

```
[Out] -2/3*(c*cos(e*x+d)-a*sin(e*x+d))*(a+b*sec(e*x+d)+c*tan(e*x+d))^(3/2)/e/sec(
e*x+d)^(3/2)/(b+a*cos(e*x+d)+c*sin(e*x+d))+8/3*b*(cos(1/2*d+1/2*e*x-1/2*arc
tan(a,c))^2)^(1/2)/cos(1/2*d+1/2*e*x-1/2*arctan(a,c))*EllipticE(sin(1/2*d+1
/2*e*x-1/2*arctan(a,c)),2^(1/2)*((a^2+c^2)^(1/2)/(b+(a^2+c^2)^(1/2)))^(1/2)
)*(a+b*sec(e*x+d)+c*tan(e*x+d))^(3/2)/e/sec(e*x+d)^(3/2)/(b+a*cos(e*x+d)+c*
sin(e*x+d))/((b+a*cos(e*x+d)+c*sin(e*x+d))/(b+(a^2+c^2)^(1/2)))^(1/2)+2/3*(
a^2-b^2+c^2)*(cos(1/2*d+1/2*e*x-1/2*arctan(a,c))^2)^(1/2)/cos(1/2*d+1/2*e*x
-1/2*arctan(a,c))*EllipticF(sin(1/2*d+1/2*e*x-1/2*arctan(a,c)),2^(1/2)*((a^
2+c^2)^(1/2)/(b+(a^2+c^2)^(1/2)))^(1/2))*((b+a*cos(e*x+d)+c*sin(e*x+d))/(b+
(a^2+c^2)^(1/2)))^(1/2)*(a+b*sec(e*x+d)+c*tan(e*x+d))^(3/2)/e/sec(e*x+d)^(3
/2)/(b+a*cos(e*x+d)+c*sin(e*x+d))^2
```

Rubi [A] time = 0.45, antiderivative size = 371, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.212, Rules used = {3167, 3120, 3149, 3119, 2653, 3127, 2661}

$$\frac{2\left(a^2-b^2+c^2\right)\sqrt{\frac{a\cos(d+ex)+b+c\sin(d+ex)}{\sqrt{a^2+c^2}+b}}\,(a+b\sec(d+ex)+c\tan(d+ex))^{3/2}F\left(\frac{1}{2}\left(d+ex-\tan^{-1}(a,c)\right)|\frac{2\sqrt{a^2+c^2}}{b+\sqrt{a^2+c^2}}\right.}{3e\sec^{\frac{3}{2}}(d+ex)(a\cos(d+ex)+b+c\sin(d+ex))^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[d + e*x] + c*Tan[d + e*x])^(3/2)/Sec[d + e*x]^(3/2),x]
```

```
[Out] (-2*(c*Cos[d + e*x] - a*Sin[d + e*x])*(a + b*Sec[d + e*x] + c*Tan[d + e*x])
^(3/2))/(3*e*Sec[d + e*x]^(3/2)*(b + a*Cos[d + e*x] + c*Sin[d + e*x])) + (8
*b*EllipticE[(d + e*x - ArcTan[a, c])/2, (2*Sqrt[a^2 + c^2])/(b + Sqrt[a^2
+ c^2])]*(a + b*Sec[d + e*x] + c*Tan[d + e*x])^(3/2))/(3*e*Sec[d + e*x]^(3/
2)*(b + a*Cos[d + e*x] + c*Sin[d + e*x])*Sqrt[(b + a*Cos[d + e*x] + c*Sin[d
 + e*x])/(b + Sqrt[a^2 + c^2])]) + (2*(a^2 - b^2 + c^2)*EllipticF[(d + e*x
- ArcTan[a, c])/2, (2*Sqrt[a^2 + c^2])/(b + Sqrt[a^2 + c^2])]*Sqrt[(b + a*C
os[d + e*x] + c*Sin[d + e*x])/(b + Sqrt[a^2 + c^2])]*(a + b*Sec[d + e*x] +
```

```
c*Tan[d + e*x])^(3/2))/(3*e*Sec[d + e*x]^(3/2)*(b + a*Cos[d + e*x] + c*Sin[
d + e*x])^2)
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 3119

```
Int[Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_
)]], x_Symbol] :> Dist[Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]]/Sqrt[(a +
b*Cos[d + e*x] + c*Sin[d + e*x])/(a + Sqrt[b^2 + c^2])], Int[Sqrt[a/(a + Sq
rt[b^2 + c^2]) + (Sqrt[b^2 + c^2]*Cos[d + e*x - ArcTan[b, c]])/(a + Sqrt[b^
2 + c^2])], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0]
&& NeQ[b^2 + c^2, 0] &&  !GtQ[a + Sqrt[b^2 + c^2], 0]
```

Rule 3120

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(n_), x_Symbol] :> -Simp[((c*Cos[d + e*x] - b*Sin[d + e*x])*(a + b*Cos[d +
e*x] + c*Sin[d + e*x])^(n - 1))/(e*n), x] + Dist[1/n, Int[Simp[n*a^2 + (n -
 1)*(b^2 + c^2) + a*b*(2*n - 1)*Cos[d + e*x] + a*c*(2*n - 1)*Sin[d + e*x],
x]*(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n - 2), x], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[a^2 - b^2 - c^2, 0] && GtQ[n, 1]
```

Rule 3127

```
Int[1/Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(
x_)]], x_Symbol] :> Dist[Sqrt[(a + b*Cos[d + e*x] + c*Sin[d + e*x])/(a + Sq
rt[b^2 + c^2])]/Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]], Int[1/Sqrt[a/(a
+ Sqrt[b^2 + c^2]) + (Sqrt[b^2 + c^2]*Cos[d + e*x - ArcTan[b, c]])/(a + Sqr
t[b^2 + c^2])], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2,
 0] && NeQ[b^2 + c^2, 0] &&  !GtQ[a + Sqrt[b^2 + c^2], 0]
```

Rule 3149

```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])
/Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)]]
```

```
, x_Symbol] :> Dist[B/b, Int[Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]], x],
 x] + Dist[(A*b - a*B)/b, Int[1/Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]],
x], x] /; FreeQ[{a, b, c, d, e, A, B, C}, x] && EqQ[B*c - b*C, 0] && NeQ[A*
b - a*B, 0]
```

Rule 3167

```
Int[sec[(d_.) + (e_.)*(x_)]^(n_.)*((a_.) + (b_.)*sec[(d_.) + (e_.)*(x_)] +
(c_.)*tan[(d_.) + (e_.)*(x_)])^(m_), x_Symbol] :> Dist[(Sec[d + e*x]^n*(b +
 a*Cos[d + e*x] + c*Sin[d + e*x])^n)/(a + b*Sec[d + e*x] + c*Tan[d + e*x])^
n, Int[1/(b + a*Cos[d + e*x] + c*Sin[d + e*x])^n, x], x] /; FreeQ[{a, b, c,
 d, e}, x] && EqQ[m + n, 0] &&  !IntegerQ[n]
```

Rubi steps

$$\int \frac{(a+b\sec(d+ex)+c\tan(d+ex))^{3/2}}{\sec^{\frac{3}{2}}(d+ex)}\,dx = \frac{(a+b\sec(d+ex)+c\tan(d+ex))^{3/2}\int (b+a\cos(d+ex)+c\sin(\ldots}{\sec^{\frac{3}{2}}(d+ex)(b+a\cos(d+ex)+c\sin(d+ex))^{3/2}}$$

$$= -\frac{2(c\cos(d+ex)-a\sin(d+ex))(a+b\sec(d+ex)+c\tan(d+ex\ldots}{3e\sec^{\frac{3}{2}}(d+ex)(b+a\cos(d+ex)+c\sin(d+ex))}$$

$$= -\frac{2(c\cos(d+ex)-a\sin(d+ex))(a+b\sec(d+ex)+c\tan(d+ex\ldots}{3e\sec^{\frac{3}{2}}(d+ex)(b+a\cos(d+ex)+c\sin(d+ex))}$$

$$= -\frac{2(c\cos(d+ex)-a\sin(d+ex))(a+b\sec(d+ex)+c\tan(d+ex\ldots}{3e\sec^{\frac{3}{2}}(d+ex)(b+a\cos(d+ex)+c\sin(d+ex))}$$

$$= -\frac{2(c\cos(d+ex)-a\sin(d+ex))(a+b\sec(d+ex)+c\tan(d+ex\ldots}{3e\sec^{\frac{3}{2}}(d+ex)(b+a\cos(d+ex)+c\sin(d+ex))}$$

Mathematica [C] time = 6.46, size = 2490, normalized size = 6.71

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*Sec[d + e*x] + c*Tan[d + e*x])^(3/2)/Sec[d + e*x]^(3/2),x]

[Out] (((8*a*b)/(3*c) - (2*c*Cos[d + e*x])/3 + (2*a*Sin[d + e*x])/3)*(a + b*Sec[d
 + e*x] + c*Tan[d + e*x])^(3/2))/(e*Sec[d + e*x]^(3/2)*(b + a*Cos[d + e*x]
+ c*Sin[d + e*x])) + (2*a^2*AppellF1[1/2, 1/2, 1/2, 3/2, -((b + Sqrt[1 + a^
2/c^2]*c*Sin[d + e*x + ArcTan[a/c]])/(Sqrt[1 + a^2/c^2]*(1 - b/(Sqrt[1 + a^
2/c^2]*c))*c)), -((b + Sqrt[1 + a^2/c^2]*c*Sin[d + e*x + ArcTan[a/c]])/(Sqr
t[1 + a^2/c^2]*(-1 - b/(Sqrt[1 + a^2/c^2]*c))*c))]*Sec[d + e*x + ArcTan[a/c
]]*Sqrt[(c*Sqrt[(a^2 + c^2)/c^2] - c*Sqrt[(a^2 + c^2)/c^2]*Sin[d + e*x + Ar
cTan[a/c]])/(b + c*Sqrt[(a^2 + c^2)/c^2])]*Sqrt[b + c*Sqrt[(a^2 + c^2)/c^2]
*Sin[d + e*x + ArcTan[a/c]]]*Sqrt[(c*Sqrt[(a^2 + c^2)/c^2] + c*Sqrt[(a^2 +
c^2)/c^2]*Sin[d + e*x + ArcTan[a/c]])/(-b + c*Sqrt[(a^2 + c^2)/c^2])]*(a +
b*Sec[d + e*x] + c*Tan[d + e*x])^(3/2))/(3*Sqrt[1 + a^2/c^2]*c*e*Sec[d + e*
x]^(3/2)*(b + a*Cos[d + e*x] + c*Sin[d + e*x])^(3/2)) + (2*b^2*AppellF1[1/2
, 1/2, 1/2, 3/2, -((b + Sqrt[1 + a^2/c^2]*c*Sin[d + e*x + ArcTan[a/c]])/(Sq
rt[1 + a^2/c^2]*(1 - b/(Sqrt[1 + a^2/c^2]*c))*c)), -((b + Sqrt[1 + a^2/c^2]
*c*Sin[d + e*x + ArcTan[a/c]])/(Sqrt[1 + a^2/c^2]*(-1 - b/(Sqrt[1 + a^2/c^2
]*c))*c))]*Sec[d + e*x + ArcTan[a/c]]*Sqrt[(c*Sqrt[(a^2 + c^2)/c^2] - c*Sqr
t[(a^2 + c^2)/c^2]*Sin[d + e*x + ArcTan[a/c]])/(b + c*Sqrt[(a^2 + c^2)/c^2]
)]*Sqrt[b + c*Sqrt[(a^2 + c^2)/c^2]*Sin[d + e*x + ArcTan[a/c]]]*Sqrt[(c*Sqr
t[(a^2 + c^2)/c^2] + c*Sqrt[(a^2 + c^2)/c^2]*Sin[d + e*x + ArcTan[a/c]])/(-
b + c*Sqrt[(a^2 + c^2)/c^2])]*(a + b*Sec[d + e*x] + c*Tan[d + e*x])^(3/2))/
(Sqrt[1 + a^2/c^2]*c*e*Sec[d + e*x]^(3/2)*(b + a*Cos[d + e*x] + c*Sin[d + e
*x])^(3/2)) + (2*c*AppellF1[1/2, 1/2, 1/2, 3/2, -((b + Sqrt[1 + a^2/c^2]*c*
Sin[d + e*x + ArcTan[a/c]])/(Sqrt[1 + a^2/c^2]*(1 - b/(Sqrt[1 + a^2/c^2]*c)
)*c)), -((b + Sqrt[1 + a^2/c^2]*c*Sin[d + e*x + ArcTan[a/c]])/(Sqrt[1 + a^2
/c^2]*(-1 - b/(Sqrt[1 + a^2/c^2]*c))*c))]*Sec[d + e*x + ArcTan[a/c]]*Sqrt[(
c*Sqrt[(a^2 + c^2)/c^2] - c*Sqrt[(a^2 + c^2)/c^2]*Sin[d + e*x + ArcTan[a/c]
])/(b + c*Sqrt[(a^2 + c^2)/c^2])]*Sqrt[b + c*Sqrt[(a^2 + c^2)/c^2]*Sin[d +
e*x + ArcTan[a/c]]]*Sqrt[(c*Sqrt[(a^2 + c^2)/c^2] + c*Sqrt[(a^2 + c^2)/c^2]
*Sin[d + e*x + ArcTan[a/c]])/(-b + c*Sqrt[(a^2 + c^2)/c^2])]*(a + b*Sec[d +
 e*x] + c*Tan[d + e*x])^(3/2))/(3*Sqrt[1 + a^2/c^2]*e*Sec[d + e*x]^(3/2)*(b
 + a*Cos[d + e*x] + c*Sin[d + e*x])^(3/2)) + (4*a^2*b*(-((c*AppellF1[-1/2,
-1/2, -1/2, 1/2, -((b + a*Sqrt[1 + c^2/a^2]*Cos[d + e*x - ArcTan[c/a]])/(a*
Sqrt[1 + c^2/a^2]*(1 - b/(a*Sqrt[1 + c^2/a^2])))), -((b + a*Sqrt[1 + c^2/a^
2]*Cos[d + e*x - ArcTan[c/a]])/(a*Sqrt[1 + c^2/a^2]*(-1 - b/(a*Sqrt[1 + c^2
/a^2]))))]*Sin[d + e*x - ArcTan[c/a]])/(a*Sqrt[1 + c^2/a^2]*Sqrt[(a*Sqrt[(a
^2 + c^2)/a^2] - a*Sqrt[(a^2 + c^2)/a^2]*Cos[d + e*x - ArcTan[c/a]])/(b + a
*Sqrt[(a^2 + c^2)/a^2])]*Sqrt[b + a*Sqrt[(a^2 + c^2)/a^2]*Cos[d + e*x - Arc
Tan[c/a]]]*Sqrt[(a*Sqrt[(a^2 + c^2)/a^2] + a*Sqrt[(a^2 + c^2)/a^2]*Cos[d +
e*x - ArcTan[c/a]])/(-b + a*Sqrt[(a^2 + c^2)/a^2])])) - ((2*a*(b + a*Sqrt[1
 + c^2/a^2]*Cos[d + e*x - ArcTan[c/a]]))/(a^2 + c^2) - (c*Sin[d + e*x - Arc
Tan[c/a]])/(a*Sqrt[1 + c^2/a^2]))/Sqrt[b + a*Sqrt[1 + c^2/a^2]*Cos[d + e*x
- ArcTan[c/a]]])*(a + b*Sec[d + e*x] + c*Tan[d + e*x])^(3/2))/(3*c*e*Sec[d
+ e*x]^(3/2)*(b + a*Cos[d + e*x] + c*Sin[d + e*x])^(3/2)) + (4*b*c*(-((c*Ap
```

```
pellF1[-1/2, -1/2, -1/2, 1/2, -((b + a*Sqrt[1 + c^2/a^2]*Cos[d + e*x - ArcT
an[c/a]])/(a*Sqrt[1 + c^2/a^2]*(1 - b/(a*Sqrt[1 + c^2/a^2])))), -((b + a*Sq
rt[1 + c^2/a^2]*Cos[d + e*x - ArcTan[c/a]])/(a*Sqrt[1 + c^2/a^2]*(-1 - b/(a
*Sqrt[1 + c^2/a^2]))))]*Sin[d + e*x - ArcTan[c/a]])/(a*Sqrt[1 + c^2/a^2]*Sq
rt[(a*Sqrt[(a^2 + c^2)/a^2] - a*Sqrt[(a^2 + c^2)/a^2]*Cos[d + e*x - ArcTan[
c/a]])/(b + a*Sqrt[(a^2 + c^2)/a^2])]*Sqrt[b + a*Sqrt[(a^2 + c^2)/a^2]*Cos[
d + e*x - ArcTan[c/a]]]*Sqrt[(a*Sqrt[(a^2 + c^2)/a^2] + a*Sqrt[(a^2 + c^2)/
a^2]*Cos[d + e*x - ArcTan[c/a]])/(-b + a*Sqrt[(a^2 + c^2)/a^2])])) - ((2*a*
(b + a*Sqrt[1 + c^2/a^2]*Cos[d + e*x - ArcTan[c/a]]))/(a^2 + c^2) - (c*Sin[
d + e*x - ArcTan[c/a]])/(a*Sqrt[1 + c^2/a^2]))/Sqrt[b + a*Sqrt[1 + c^2/a^2]
*Cos[d + e*x - ArcTan[c/a]]])*(a + b*Sec[d + e*x] + c*Tan[d + e*x])^(3/2))/
(3*e*Sec[d + e*x]^(3/2)*(b + a*Cos[d + e*x] + c*Sin[d + e*x])^(3/2))
```

fricas [F] time = 0.88, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{(b\sec(ex+d)+c\tan(ex+d)+a)^{\frac{3}{2}}}{\sec(ex+d)^{\frac{3}{2}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(e*x+d)+c*tan(e*x+d))^(3/2)/sec(e*x+d)^(3/2),x, algorithm
="fricas")
```

```
[Out] integral((b*sec(e*x + d) + c*tan(e*x + d) + a)^(3/2)/sec(e*x + d)^(3/2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b\sec(ex+d)+c\tan(ex+d)+a)^{\frac{3}{2}}}{\sec(ex+d)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(e*x+d)+c*tan(e*x+d))^(3/2)/sec(e*x+d)^(3/2),x, algorithm
="giac")
```

```
[Out] integrate((b*sec(e*x + d) + c*tan(e*x + d) + a)^(3/2)/sec(e*x + d)^(3/2), x
)
```

maple [C] time = 4.41, size = 21186, normalized size = 57.11

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(e*x+d)+c*tan(e*x+d))^(3/2)/sec(e*x+d)^(3/2),x)
```

```
[Out] result too large to display
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b\sec(ex+d) + c\tan(ex+d) + a)^{\frac{3}{2}}}{\sec(ex+d)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(e*x+d)+c*tan(e*x+d))^(3/2)/sec(e*x+d)^(3/2),x, algorithm
="maxima")
```

```
[Out] integrate((b*sec(e*x + d) + c*tan(e*x + d) + a)^(3/2)/sec(e*x + d)^(3/2), x
)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(a + c\tan(d+e\,x) + \frac{b}{\cos(d+e\,x)}\right)^{3/2}}{\left(\frac{1}{\cos(d+e\,x)}\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + c*tan(d + e*x) + b/cos(d + e*x))^(3/2)/(1/cos(d + e*x))^(3/2),x)
```

```
[Out] int((a + c*tan(d + e*x) + b/cos(d + e*x))^(3/2)/(1/cos(d + e*x))^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + b\sec(d+ex) + c\tan(d+ex))^{\frac{3}{2}}}{\sec^{\frac{3}{2}}(d+ex)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(e*x+d)+c*tan(e*x+d))**(3/2)/sec(e*x+d)**(3/2),x)
```

```
[Out] Integral((a + b*sec(d + e*x) + c*tan(d + e*x))**(3/2)/sec(d + e*x)**(3/2),
x)
```

3.449 $\int \frac{\sqrt{a+b\sec(d+ex)+c\tan(d+ex)}}{\sqrt{\sec(d+ex)}}\,dx$

Optimal. Leaf size=118

$$\frac{2\sqrt{a+b\sec(d+ex)+c\tan(d+ex)}\,E\left(\frac{1}{2}\left(d+ex-\tan^{-1}(a,c)\right)|\frac{2\sqrt{a^2+c^2}}{b+\sqrt{a^2+c^2}}\right)}{e\sqrt{\sec(d+ex)}\sqrt{\frac{a\cos(d+ex)+b+c\sin(d+ex)}{\sqrt{a^2+c^2}+b}}}$$

```
[Out] 2*(cos(1/2*d+1/2*e*x-1/2*arctan(a,c))^2)^(1/2)/cos(1/2*d+1/2*e*x-1/2*arctan
(a,c))*EllipticE(sin(1/2*d+1/2*e*x-1/2*arctan(a,c)),2^(1/2)*((a^2+c^2)^(1/2
)/(b+(a^2+c^2)^(1/2)))^(1/2))*(a+b*sec(e*x+d)+c*tan(e*x+d))^(1/2)/e/sec(e*x
+d)^(1/2)/((b+a*cos(e*x+d)+c*sin(e*x+d))/(b+(a^2+c^2)^(1/2)))^(1/2)
```

Rubi [A] time = 0.14, antiderivative size = 118, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {3167, 3119, 2653}

$$\frac{2\sqrt{a+b\sec(d+ex)+c\tan(d+ex)}\,E\left(\frac{1}{2}\left(d+ex-\tan^{-1}(a,c)\right)|\frac{2\sqrt{a^2+c^2}}{b+\sqrt{a^2+c^2}}\right)}{e\sqrt{\sec(d+ex)}\sqrt{\frac{a\cos(d+ex)+b+c\sin(d+ex)}{\sqrt{a^2+c^2}+b}}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*Sec[d + e*x] + c*Tan[d + e*x]]/Sqrt[Sec[d + e*x]],x]
```

```
[Out] (2*EllipticE[(d + e*x - ArcTan[a, c])/2, (2*Sqrt[a^2 + c^2])/(b + Sqrt[a^2
+ c^2])]*Sqrt[a + b*Sec[d + e*x] + c*Tan[d + e*x]])/(e*Sqrt[Sec[d + e*x]]*S
qrt[(b + a*Cos[d + e*x] + c*Sin[d + e*x])/(b + Sqrt[a^2 + c^2])])
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 3119

```
Int[Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_
)]], x_Symbol] :> Dist[Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]]/Sqrt[(a +
b*Cos[d + e*x] + c*Sin[d + e*x])/(a + Sqrt[b^2 + c^2])], Int[Sqrt[a/(a + Sq
rt[b^2 + c^2]) + (Sqrt[b^2 + c^2]*Cos[d + e*x - ArcTan[b, c]])/(a + Sqrt[b^
2 + c^2])], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0]
```

```
&& NeQ[b^2 + c^2, 0] &&  !GtQ[a + Sqrt[b^2 + c^2], 0]
```

Rule 3167

```
Int[sec[(d_.) + (e_.)*(x_)]^(n_.)*((a_.) + (b_.)*sec[(d_.) + (e_.)*(x_)] +
(c_.)*tan[(d_.) + (e_.)*(x_)])^(m_), x_Symbol] :> Dist[(Sec[d + e*x]^n*(b +
 a*Cos[d + e*x] + c*Sin[d + e*x])^n)/(a + b*Sec[d + e*x] + c*Tan[d + e*x])^
n, Int[1/(b + a*Cos[d + e*x] + c*Sin[d + e*x])^n, x], x] /; FreeQ[{a, b, c,
 d, e}, x] && EqQ[m + n, 0] &&  !IntegerQ[n]
```

Rubi steps

$$\int \frac{\sqrt{a+b\sec(d+ex)+c\tan(d+ex)}}{\sqrt{\sec(d+ex)}}\,dx = \frac{\sqrt{a+b\sec(d+ex)+c\tan(d+ex)}\int \sqrt{b+a\cos(d+ex)+c\sin(d+}}{\sqrt{\sec(d+ex)}\sqrt{b+a\cos(d+ex)+c\sin(d+ex)}}$$

$$= \frac{\sqrt{a+b\sec(d+ex)+c\tan(d+ex)}\int \sqrt{\frac{b}{b+\sqrt{a^2+c^2}}+\frac{\sqrt{a^2+c^2}\cos\left(d+ex-\tan\right.}{b+\sqrt{a^2+c^2}}}}{\sqrt{\sec(d+ex)}\sqrt{\frac{b+a\cos(d+ex)+c\sin(d+ex)}{b+\sqrt{a^2+c^2}}}}$$

$$= \frac{2E\left(\frac{1}{2}\left(d+ex-\tan^{-1}(a,c)\right)|\frac{2\sqrt{a^2+c^2}}{b+\sqrt{a^2+c^2}}\right)\sqrt{a+b\sec(d+ex)+c\tan(d-}}{e\sqrt{\sec(d+ex)}\sqrt{\frac{b+a\cos(d+ex)+c\sin(d+ex)}{b+\sqrt{a^2+c^2}}}}$$

Mathematica [C] time = 6.25, size = 1580, normalized size = 13.39

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[a + b*Sec[d + e*x] + c*Tan[d + e*x]]/Sqrt[Sec[d + e*x]],x]
```

```
[Out] (2*a*Sqrt[a + b*Sec[d + e*x] + c*Tan[d + e*x]])/(c*e*Sqrt[Sec[d + e*x]]) +
(2*b*AppellF1[1/2, 1/2, 1/2, 3/2, -((b + Sqrt[1 + a^2/c^2]*c*Sin[d + e*x +
ArcTan[a/c]])/(Sqrt[1 + a^2/c^2]*(1 - b/(Sqrt[1 + a^2/c^2]*c))*c)), -((b +
Sqrt[1 + a^2/c^2]*c*Sin[d + e*x + ArcTan[a/c]])/(Sqrt[1 + a^2/c^2]*(-1 - b/
(Sqrt[1 + a^2/c^2]*c))*c))]*Sec[d + e*x + ArcTan[a/c]]*Sqrt[(c*Sqrt[(a^2 +
c^2)/c^2] - c*Sqrt[(a^2 + c^2)/c^2]*Sin[d + e*x + ArcTan[a/c]])/(b + c*Sqrt
[(a^2 + c^2)/c^2])]*Sqrt[b + c*Sqrt[(a^2 + c^2)/c^2]*Sin[d + e*x + ArcTan[a
/c]]]*Sqrt[(c*Sqrt[(a^2 + c^2)/c^2] + c*Sqrt[(a^2 + c^2)/c^2]*Sin[d + e*x +
 ArcTan[a/c]])/(-b + c*Sqrt[(a^2 + c^2)/c^2])]*Sqrt[a + b*Sec[d + e*x] + c*
```

```
Tan[d + e*x]])/(Sqrt[1 + a^2/c^2]*c*e*Sqrt[Sec[d + e*x]]*Sqrt[b + a*Cos[d +
 e*x] + c*Sin[d + e*x]]) + (a^2*(-((c*AppellF1[-1/2, -1/2, -1/2, 1/2, -((b
+ a*Sqrt[1 + c^2/a^2]*Cos[d + e*x - ArcTan[c/a]])/(a*Sqrt[1 + c^2/a^2]*(1 -
 b/(a*Sqrt[1 + c^2/a^2])))), -((b + a*Sqrt[1 + c^2/a^2]*Cos[d + e*x - ArcTa
n[c/a]])/(a*Sqrt[1 + c^2/a^2]*(-1 - b/(a*Sqrt[1 + c^2/a^2]))))]*Sin[d + e*x
 - ArcTan[c/a]])/(a*Sqrt[1 + c^2/a^2]*Sqrt[(a*Sqrt[(a^2 + c^2)/a^2] - a*Sqr
t[(a^2 + c^2)/a^2]*Cos[d + e*x - ArcTan[c/a]])/(b + a*Sqrt[(a^2 + c^2)/a^2]
)]*Sqrt[b + a*Sqrt[(a^2 + c^2)/a^2]*Cos[d + e*x - ArcTan[c/a]]]*Sqrt[(a*Sqr
t[(a^2 + c^2)/a^2] + a*Sqrt[(a^2 + c^2)/a^2]*Cos[d + e*x - ArcTan[c/a]])/(-
b + a*Sqrt[(a^2 + c^2)/a^2])])) - ((2*a*(b + a*Sqrt[1 + c^2/a^2]*Cos[d + e*
x - ArcTan[c/a]]))/(a^2 + c^2) - (c*Sin[d + e*x - ArcTan[c/a]])/(a*Sqrt[1 +
 c^2/a^2]))/Sqrt[b + a*Sqrt[1 + c^2/a^2]*Cos[d + e*x - ArcTan[c/a]]])*Sqrt[
a + b*Sec[d + e*x] + c*Tan[d + e*x]])/(c*e*Sqrt[Sec[d + e*x]]*Sqrt[b + a*Co
s[d + e*x] + c*Sin[d + e*x]]) + (c*(-((c*AppellF1[-1/2, -1/2, -1/2, 1/2, -(
(b + a*Sqrt[1 + c^2/a^2]*Cos[d + e*x - ArcTan[c/a]])/(a*Sqrt[1 + c^2/a^2]*(
1 - b/(a*Sqrt[1 + c^2/a^2])))), -((b + a*Sqrt[1 + c^2/a^2]*Cos[d + e*x - Ar
cTan[c/a]])/(a*Sqrt[1 + c^2/a^2]*(-1 - b/(a*Sqrt[1 + c^2/a^2]))))]*Sin[d +
e*x - ArcTan[c/a]])/(a*Sqrt[1 + c^2/a^2]*Sqrt[(a*Sqrt[(a^2 + c^2)/a^2] - a*
Sqrt[(a^2 + c^2)/a^2]*Cos[d + e*x - ArcTan[c/a]])/(b + a*Sqrt[(a^2 + c^2)/a
^2])]*Sqrt[b + a*Sqrt[(a^2 + c^2)/a^2]*Cos[d + e*x - ArcTan[c/a]]]*Sqrt[(a*
Sqrt[(a^2 + c^2)/a^2] + a*Sqrt[(a^2 + c^2)/a^2]*Cos[d + e*x - ArcTan[c/a]])
/(-b + a*Sqrt[(a^2 + c^2)/a^2])])) - ((2*a*(b + a*Sqrt[1 + c^2/a^2]*Cos[d +
 e*x - ArcTan[c/a]]))/(a^2 + c^2) - (c*Sin[d + e*x - ArcTan[c/a]])/(a*Sqrt[
1 + c^2/a^2]))/Sqrt[b + a*Sqrt[1 + c^2/a^2]*Cos[d + e*x - ArcTan[c/a]]])*Sq
rt[a + b*Sec[d + e*x] + c*Tan[d + e*x]])/(e*Sqrt[Sec[d + e*x]]*Sqrt[b + a*C
os[d + e*x] + c*Sin[d + e*x]])
```

fricas [F] time = 0.65, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{b\sec\left(ex+d\right)+c\tan\left(ex+d\right)+a}}{\sqrt{\sec\left(ex+d\right)}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(e*x+d)+c*tan(e*x+d))^(1/2)/sec(e*x+d)^(1/2),x, algorithm
="fricas")
```

```
[Out] integral(sqrt(b*sec(e*x + d) + c*tan(e*x + d) + a)/sqrt(sec(e*x + d)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sqrt{b\sec\left(ex+d\right)+c\tan\left(ex+d\right)+a}}{\sqrt{\sec\left(ex+d\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(e*x+d)+c*tan(e*x+d))^(1/2)/sec(e*x+d)^(1/2),x, algorithm
="giac")
```

```
[Out] integrate(sqrt(b*sec(e*x + d) + c*tan(e*x + d) + a)/sqrt(sec(e*x + d)), x)
```

maple [C] time = 1.89, size = 12462, normalized size = 105.61

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(e*x+d)+c*tan(e*x+d))^(1/2)/sec(e*x+d)^(1/2),x)
```

```
[Out] result too large to display
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{b \sec\left(ex+d\right) + c \tan\left(ex+d\right) + a}}{\sqrt{\sec\left(ex+d\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(e*x+d)+c*tan(e*x+d))^(1/2)/sec(e*x+d)^(1/2),x, algorithm
="maxima")
```

```
[Out] integrate(sqrt(b*sec(e*x + d) + c*tan(e*x + d) + a)/sqrt(sec(e*x + d)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{a + c \tan\left(d + e\,x\right) + \frac{b}{\cos(d+e\,x)}}}{\sqrt{\frac{1}{\cos(d+e\,x)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + c*tan(d + e*x) + b/cos(d + e*x))^(1/2)/(1/cos(d + e*x))^(1/2),x)
```

```
[Out] int((a + c*tan(d + e*x) + b/cos(d + e*x))^(1/2)/(1/cos(d + e*x))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a + b \sec\left(d+ex\right) + c \tan\left(d+ex\right)}}{\sqrt{\sec\left(d+ex\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(e*x+d)+c*tan(e*x+d))**(1/2)/sec(e*x+d)**(1/2),x)
```

```
[Out] Integral(sqrt(a + b*sec(d + e*x) + c*tan(d + e*x))/sqrt(sec(d + e*x)), x)
```

3.450 $\int \frac{\sqrt{\sec(d+ex)}}{\sqrt{a+b\sec(d+ex)+c\tan(d+ex)}}\,dx$

Optimal. Leaf size=118

$$\frac{2\sqrt{\sec(d+ex)}\sqrt{\frac{a\cos(d+ex)+b+c\sin(d+ex)}{\sqrt{a^2+c^2}+b}}\,F\left(\frac{1}{2}\left(d+ex-\tan^{-1}(a,c)\right)|\frac{2\sqrt{a^2+c^2}}{b+\sqrt{a^2+c^2}}\right)}{e\sqrt{a+b\sec(d+ex)+c\tan(d+ex)}}$$

```
[Out] 2*(cos(1/2*d+1/2*e*x-1/2*arctan(a,c))^2)^(1/2)/cos(1/2*d+1/2*e*x-1/2*arctan
(a,c))*EllipticF(sin(1/2*d+1/2*e*x-1/2*arctan(a,c)),2^(1/2)*((a^2+c^2)^(1/2
)/(b+(a^2+c^2)^(1/2)))^(1/2))*sec(e*x+d)^(1/2)*((b+a*cos(e*x+d)+c*sin(e*x+d
))/(b+(a^2+c^2)^(1/2)))^(1/2)/e/(a+b*sec(e*x+d)+c*tan(e*x+d))^(1/2)
```

Rubi [A] time = 0.17, antiderivative size = 118, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {3167, 3127, 2661}

$$\frac{2\sqrt{\sec(d+ex)}\sqrt{\frac{a\cos(d+ex)+b+c\sin(d+ex)}{\sqrt{a^2+c^2}+b}}\,F\left(\frac{1}{2}\left(d+ex-\tan^{-1}(a,c)\right)|\frac{2\sqrt{a^2+c^2}}{b+\sqrt{a^2+c^2}}\right)}{e\sqrt{a+b\sec(d+ex)+c\tan(d+ex)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Sec[d + e*x]]/Sqrt[a + b*Sec[d + e*x] + c*Tan[d + e*x]],x]
```

```
[Out] (2*EllipticF[(d + e*x - ArcTan[a, c])/2, (2*Sqrt[a^2 + c^2])/(b + Sqrt[a^2
+ c^2])]*Sqrt[Sec[d + e*x]]*Sqrt[(b + a*Cos[d + e*x] + c*Sin[d + e*x])/(b +
 Sqrt[a^2 + c^2])])/(e*Sqrt[a + b*Sec[d + e*x] + c*Tan[d + e*x]])
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 3127

```
Int[1/Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(
x_)]], x_Symbol] :> Dist[Sqrt[(a + b*Cos[d + e*x] + c*Sin[d + e*x])/(a + Sq
rt[b^2 + c^2])]/Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]], Int[1/Sqrt[a/(a
+ Sqrt[b^2 + c^2]) + (Sqrt[b^2 + c^2]*Cos[d + e*x - ArcTan[b, c]])/(a + Sqr
t[b^2 + c^2])], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2,
 0] && NeQ[b^2 + c^2, 0] &&  !GtQ[a + Sqrt[b^2 + c^2], 0]
```

Rule 3167

```
Int[sec[(d_.) + (e_.)*(x_)]^(n_.)*((a_.) + (b_.)*sec[(d_.) + (e_.)*(x_)] +
(c_.)*tan[(d_.) + (e_.)*(x_)])^(m_), x_Symbol] :> Dist[(Sec[d + e*x]^n*(b +
 a*Cos[d + e*x] + c*Sin[d + e*x])^n)/(a + b*Sec[d + e*x] + c*Tan[d + e*x])^
n, Int[1/(b + a*Cos[d + e*x] + c*Sin[d + e*x])^n, x], x] /; FreeQ[{a, b, c,
 d, e}, x] && EqQ[m + n, 0] &&  !IntegerQ[n]
```

Rubi steps

$$\int \frac{\sqrt{\sec(d+ex)}}{\sqrt{a+b\sec(d+ex)+c\tan(d+ex)}}\,dx = \frac{\left(\sqrt{\sec(d+ex)}\sqrt{b+a\cos(d+ex)+c\sin(d+ex)}\right)\int \frac{1}{\sqrt{b+a\cos(d+ex)+c\,\mathrm{s}}}}{\sqrt{a+b\sec(d+ex)+c\tan(d+ex)}}$$

$$= \frac{\left(\sqrt{\sec(d+ex)}\sqrt{\frac{b+a\cos(d+ex)+c\sin(d+ex)}{b+\sqrt{a^2+c^2}}}\right)\int \frac{1}{\sqrt{\frac{b}{b+\sqrt{a^2+c^2}}+\frac{\sqrt{a^2+c^2}\cos\left(d+ex-\tan^{-}\right.}{b+\sqrt{a^2+c^2}}}}}{\sqrt{a+b\sec(d+ex)+c\tan(d+ex)}}$$

$$= \frac{2F\left(\frac{1}{2}\left(d+ex-\tan^{-1}(a,c)\right)|\frac{2\sqrt{a^2+c^2}}{b+\sqrt{a^2+c^2}}\right)\sqrt{\sec(d+ex)}\sqrt{\frac{b+a\cos(d+ex)+c\,\mathrm{s}}{b+\sqrt{a^2+c^2}}}}{e\sqrt{a+b\sec(d+ex)+c\tan(d+ex)}}$$

Mathematica [C] time = 0.87, size = 339, normalized size = 2.87

$$\frac{2\sqrt{\sec(d+ex)}\,\sec\left(\tan^{-1}\left(\frac{a}{c}\right)+d+ex\right)\sqrt{-\frac{c\sqrt{\frac{a^2}{c^2}+1}\left(\sin\left(\tan^{-1}\left(\frac{a}{c}\right)+d+ex\right)-1\right)}{c\sqrt{\frac{a^2}{c^2}+1}+b}}\sqrt{\frac{c\sqrt{\frac{a^2}{c^2}+1}\left(\sin\left(\tan^{-1}\left(\frac{a}{c}\right)+d+ex\right)+1\right)}{c\sqrt{\frac{a^2}{c^2}+1}-b}}\sqrt{c\sqrt{\frac{a^2}{c^2}+1}}}{ce\sqrt{\frac{a^2}{c^2}+1}\sqrt{a}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[Sec[d + e*x]]/Sqrt[a + b*Sec[d + e*x] + c*Tan[d + e*x]],x]
```

```
[Out] (2*AppellF1[1/2, 1/2, 1/2, 3/2, (b + Sqrt[1 + a^2/c^2]*c*Sin[d + e*x + ArcT
an[a/c]])/(b - Sqrt[1 + a^2/c^2]*c), (b + Sqrt[1 + a^2/c^2]*c*Sin[d + e*x +
 ArcTan[a/c]])/(b + Sqrt[1 + a^2/c^2]*c)]*Sqrt[Sec[d + e*x]]*Sec[d + e*x +
ArcTan[a/c]]*Sqrt[b + a*Cos[d + e*x] + c*Sin[d + e*x]]*Sqrt[-((Sqrt[1 + a^2
/c^2]*c*(-1 + Sin[d + e*x + ArcTan[a/c]]))/(b + Sqrt[1 + a^2/c^2]*c))]*Sqrt
[(Sqrt[1 + a^2/c^2]*c*(1 + Sin[d + e*x + ArcTan[a/c]]))/(-b + Sqrt[1 + a^2/
c^2]*c)]*Sqrt[b + Sqrt[1 + a^2/c^2]*c*Sin[d + e*x + ArcTan[a/c]]])/(Sqrt[1
+ a^2/c^2]*c*e*Sqrt[a + b*Sec[d + e*x] + c*Tan[d + e*x]])
```

fricas [F] time = 2.90, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{\sec\left(ex+d\right)}}{\sqrt{b\sec\left(ex+d\right)+c\tan\left(ex+d\right)+a}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(e*x+d)^(1/2)/(a+b*sec(e*x+d)+c*tan(e*x+d))^(1/2),x, algorithm
="fricas")
```

```
[Out] integral(sqrt(sec(e*x + d))/sqrt(b*sec(e*x + d) + c*tan(e*x + d) + a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\sec\left(ex+d\right)}}{\sqrt{b\sec\left(ex+d\right)+c\tan\left(ex+d\right)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(e*x+d)^(1/2)/(a+b*sec(e*x+d)+c*tan(e*x+d))^(1/2),x, algorithm
="giac")
```

```
[Out] integrate(sqrt(sec(e*x + d))/sqrt(b*sec(e*x + d) + c*tan(e*x + d) + a), x)
```

maple [C] time = 1.86, size = 722, normalized size = 6.12

$$-\frac{4i\,\mathrm{EllipticF}\left(\sqrt{\frac{\left(i\sin(ex+d)+\cos(ex+d)\right)\left(ia-ib-\sqrt{a^2-b^2+c^2}+c\right)}{ia-ib+\sqrt{a^2-b^2+c^2}-c}},\sqrt{\frac{\left(ia-ib+\sqrt{a^2-b^2+c^2}-c\right)\left(ia-ib+\sqrt{a^2-b^2+c^2}+c\right)}{\left(ia-ib-\sqrt{a^2-b^2+c^2}+c\right)\left(ia-ib-\sqrt{a^2-b^2+c^2}-c\right)}}\right)\sqrt{\frac{1}{\cos(ex+d)}}\,\sqrt{\frac{b+a}{}}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(e*x+d)^(1/2)/(a+b*sec(e*x+d)+c*tan(e*x+d))^(1/2),x)
```

```
[Out] -4*I/e*EllipticF(((I*sin(e*x+d)+cos(e*x+d))*(I*a-I*b-(a^2-b^2+c^2)^(1/2)+c)
/(I*a-I*b+(a^2-b^2+c^2)^(1/2)-c))^(1/2),((I*a-I*b+(a^2-b^2+c^2)^(1/2)-c)*(I
*a-I*b+(a^2-b^2+c^2)^(1/2)+c)/(I*a-I*b-(a^2-b^2+c^2)^(1/2)+c)/(I*a-I*b-(a^2
-b^2+c^2)^(1/2)-c))^(1/2))*(1/cos(e*x+d))^(1/2)*((b+a*cos(e*x+d)+c*sin(e*x+
d))/cos(e*x+d))^(1/2)*((I*sin(e*x+d)+cos(e*x+d))*(I*a-I*b-(a^2-b^2+c^2)^(1/
2)+c)/(I*a-I*b+(a^2-b^2+c^2)^(1/2)-c))^(1/2)*(-I*(cos(e*x+d)*(a^2-b^2+c^2)^
(1/2)-a*sin(e*x+d)+b*sin(e*x+d)+c*cos(e*x+d)+(a^2-b^2+c^2)^(1/2)+c)/(I*cos(
e*x+d)+sin(e*x+d)+I)/(I*a-I*b-(a^2-b^2+c^2)^(1/2)-c))^(1/2)*(I*(a*sin(e*x+d
)-b*sin(e*x+d)+cos(e*x+d)*(a^2-b^2+c^2)^(1/2)-c*cos(e*x+d)+(a^2-b^2+c^2)^(1
/2)-c)/(I*cos(e*x+d)+sin(e*x+d)+I)/(I*a-I*b+(a^2-b^2+c^2)^(1/2)-c))^(1/2)*(
```

```
cos(e*x+d)+1)^2*cos(e*x+d)*(cos(e*x+d)-1)^2*(I*a*cos(e*x+d)-I*cos(e*x+d)*b-
I*(a^2-b^2+c^2)^(1/2)*sin(e*x+d)+I*c*sin(e*x+d)+cos(e*x+d)*(a^2-b^2+c^2)^(1
/2)-c*cos(e*x+d)+a*sin(e*x+d)-b*sin(e*x+d))/sin(e*x+d)^4/(b+a*cos(e*x+d)+c*
sin(e*x+d))/(I*a-I*b-(a^2-b^2+c^2)^(1/2)+c)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\sec(ex+d)}}{\sqrt{b\sec(ex+d)+c\tan(ex+d)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(e*x+d)^(1/2)/(a+b*sec(e*x+d)+c*tan(e*x+d))^(1/2),x, algorithm
="maxima")
```

```
[Out] integrate(sqrt(sec(e*x + d))/sqrt(b*sec(e*x + d) + c*tan(e*x + d) + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{\frac{1}{\cos(d+e\,x)}}}{\sqrt{a+c\tan(d+e\,x)+\frac{b}{\cos(d+e\,x)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/cos(d + e*x))^(1/2)/(a + c*tan(d + e*x) + b/cos(d + e*x))^(1/2),x)
```

```
[Out] int((1/cos(d + e*x))^(1/2)/(a + c*tan(d + e*x) + b/cos(d + e*x))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\sec(d+ex)}}{\sqrt{a+b\sec(d+ex)+c\tan(d+ex)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(e*x+d)**(1/2)/(a+b*sec(e*x+d)+c*tan(e*x+d))**(1/2),x)
```

```
[Out] Integral(sqrt(sec(d + e*x))/sqrt(a + b*sec(d + e*x) + c*tan(d + e*x)), x)
```

3.451 $\int \frac{\sec^{\frac{3}{2}}(d+ex)}{(a+b\sec(d+ex)+c\tan(d+ex))^{3/2}}\,dx$

Optimal. Leaf size=240

$$-\frac{2\sec^{\frac{3}{2}}(d+ex)(a\cos(d+ex)+b+c\sin(d+ex))^2E\left(\frac{1}{2}\left(d+ex-\tan^{-1}(a,c)\right)|\frac{2\sqrt{a^2+c^2}}{b+\sqrt{a^2+c^2}}\right)}{e\left(a^2-b^2+c^2\right)\sqrt{\frac{a\cos(d+ex)+b+c\sin(d+ex)}{\sqrt{a^2+c^2}+b}}\,(a+b\sec(d+ex)+c\tan(d+ex))^{3/2}}-\frac{2\sec^{\frac{3}{2}}(d+ex)(c\cos(}{e\left(a^2-b\right.}$$

```
[Out] -2*sec(e*x+d)^(3/2)*(c*cos(e*x+d)-a*sin(e*x+d))*(b+a*cos(e*x+d)+c*sin(e*x+d
))/(a^2-b^2+c^2)/e/(a+b*sec(e*x+d)+c*tan(e*x+d))^(3/2)-2*(cos(1/2*d+1/2*e*x
-1/2*arctan(a,c))^2)^(1/2)/cos(1/2*d+1/2*e*x-1/2*arctan(a,c))*EllipticE(sin
(1/2*d+1/2*e*x-1/2*arctan(a,c)),2^(1/2)*((a^2+c^2)^(1/2)/(b+(a^2+c^2)^(1/2)
))^(1/2))*sec(e*x+d)^(3/2)*(b+a*cos(e*x+d)+c*sin(e*x+d))^2/(a^2-b^2+c^2)/e/
((b+a*cos(e*x+d)+c*sin(e*x+d))/(b+(a^2+c^2)^(1/2)))^(1/2)/(a+b*sec(e*x+d)+c
*tan(e*x+d))^(3/2)
```

Rubi [A] time = 0.22, antiderivative size = 240, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.121, Rules used = {3167, 3128, 3119, 2653}

$$-\frac{2\sec^{\frac{3}{2}}(d+ex)(a\cos(d+ex)+b+c\sin(d+ex))^2E\left(\frac{1}{2}\left(d+ex-\tan^{-1}(a,c)\right)|\frac{2\sqrt{a^2+c^2}}{b+\sqrt{a^2+c^2}}\right)}{e\left(a^2-b^2+c^2\right)\sqrt{\frac{a\cos(d+ex)+b+c\sin(d+ex)}{\sqrt{a^2+c^2}+b}}\,(a+b\sec(d+ex)+c\tan(d+ex))^{3/2}}-\frac{2\sec^{\frac{3}{2}}(d+ex)(c\cos(}{e\left(a^2-b\right.}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[d + e*x]^(3/2)/(a + b*Sec[d + e*x] + c*Tan[d + e*x])^(3/2),x]
```

```
[Out] (-2*Sec[d + e*x]^(3/2)*(c*Cos[d + e*x] - a*Sin[d + e*x])*(b + a*Cos[d + e*x
] + c*Sin[d + e*x]))/((a^2 - b^2 + c^2)*e*(a + b*Sec[d + e*x] + c*Tan[d + e
*x])^(3/2)) - (2*EllipticE[(d + e*x - ArcTan[a, c])/2, (2*Sqrt[a^2 + c^2])/
(b + Sqrt[a^2 + c^2])]*Sec[d + e*x]^(3/2)*(b + a*Cos[d + e*x] + c*Sin[d + e
*x])^2)/((a^2 - b^2 + c^2)*e*Sqrt[(b + a*Cos[d + e*x] + c*Sin[d + e*x])/(b
+ Sqrt[a^2 + c^2])]*(a + b*Sec[d + e*x] + c*Tan[d + e*x])^(3/2))
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 3119

```
Int[Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_
)]], x_Symbol] :> Dist[Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]]/Sqrt[(a +
b*Cos[d + e*x] + c*Sin[d + e*x])/(a + Sqrt[b^2 + c^2])], Int[Sqrt[a/(a + Sq
rt[b^2 + c^2]) + (Sqrt[b^2 + c^2]*Cos[d + e*x - ArcTan[b, c]])/(a + Sqrt[b^
2 + c^2])], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0]
&& NeQ[b^2 + c^2, 0] &&  !GtQ[a + Sqrt[b^2 + c^2], 0]
```

Rule 3128

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(-3/2), x_Symbol] :> Simp[(2*(c*Cos[d + e*x] - b*Sin[d + e*x]))/(e*(a^2 - b
^2 - c^2)*Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]]), x] + Dist[1/(a^2 - b^
2 - c^2), Int[Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]], x], x] /; FreeQ[{a
, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0]
```

Rule 3167

```
Int[sec[(d_.) + (e_.)*(x_)]^(n_.)*((a_.) + (b_.)*sec[(d_.) + (e_.)*(x_)] +
(c_.)*tan[(d_.) + (e_.)*(x_)])^(m_), x_Symbol] :> Dist[(Sec[d + e*x]^n*(b +
 a*Cos[d + e*x] + c*Sin[d + e*x])^n)/(a + b*Sec[d + e*x] + c*Tan[d + e*x])^
n, Int[1/(b + a*Cos[d + e*x] + c*Sin[d + e*x])^n, x], x] /; FreeQ[{a, b, c,
 d, e}, x] && EqQ[m + n, 0] &&  !IntegerQ[n]
```

Rubi steps

$$\int \frac{\sec^{\frac{3}{2}}(d+ex)}{(a+b\sec(d+ex)+c\tan(d+ex))^{3/2}}\,dx = \frac{\left(\sec^{\frac{3}{2}}(d+ex)(b+a\cos(d+ex)+c\sin(d+ex))^{3/2}\right)\int \frac{1}{(b+a\cos(d+ex)+}}{(a+b\sec(d+ex)+c\tan(d+ex))^{3/2}}$$

$$= -\frac{2\sec^{\frac{3}{2}}(d+ex)(c\cos(d+ex)-a\sin(d+ex))(b+a\cos(d+ex)+}{\left(a^2-b^2+c^2\right)e(a+b\sec(d+ex)+c\tan(d+ex))^{3/2}}$$

$$= -\frac{2\sec^{\frac{3}{2}}(d+ex)(c\cos(d+ex)-a\sin(d+ex))(b+a\cos(d+ex)+}{\left(a^2-b^2+c^2\right)e(a+b\sec(d+ex)+c\tan(d+ex))^{3/2}}$$

$$= -\frac{2\sec^{\frac{3}{2}}(d+ex)(c\cos(d+ex)-a\sin(d+ex))(b+a\cos(d+ex)+}{\left(a^2-b^2+c^2\right)e(a+b\sec(d+ex)+c\tan(d+ex))^{3/2}}$$

Mathematica [C] time = 6.40, size = 1732, normalized size = 7.22

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sec[d + e*x]^(3/2)/(a + b*Sec[d + e*x] + c*Tan[d + e*x])^(3/2),x]
```

```
[Out] (Sec[d + e*x]^(3/2)*(b + a*Cos[d + e*x] + c*Sin[d + e*x])^2*((-2*(a^2 + c^2
))/(a*c*(a^2 - b^2 + c^2)) + (2*(b*c + a^2*Sin[d + e*x] + c^2*Sin[d + e*x])
)/(a*(a^2 - b^2 + c^2)*(b + a*Cos[d + e*x] + c*Sin[d + e*x]))))/(e*(a + b*S
ec[d + e*x] + c*Tan[d + e*x])^(3/2)) - (2*b*AppellF1[1/2, 1/2, 1/2, 3/2, -(
(b + Sqrt[1 + a^2/c^2]*c*Sin[d + e*x + ArcTan[a/c]])/(Sqrt[1 + a^2/c^2]*(1
- b/(Sqrt[1 + a^2/c^2]*c))*c)), -((b + Sqrt[1 + a^2/c^2]*c*Sin[d + e*x + Ar
cTan[a/c]])/(Sqrt[1 + a^2/c^2]*(-1 - b/(Sqrt[1 + a^2/c^2]*c))*c))]*Sec[d +
e*x]^(3/2)*Sec[d + e*x + ArcTan[a/c]]*(b + a*Cos[d + e*x] + c*Sin[d + e*x])
^(3/2)*Sqrt[(c*Sqrt[(a^2 + c^2)/c^2] - c*Sqrt[(a^2 + c^2)/c^2]*Sin[d + e*x
+ ArcTan[a/c]])/(b + c*Sqrt[(a^2 + c^2)/c^2])]*Sqrt[b + c*Sqrt[(a^2 + c^2)/
c^2]*Sin[d + e*x + ArcTan[a/c]]]*Sqrt[(c*Sqrt[(a^2 + c^2)/c^2] + c*Sqrt[(a^
2 + c^2)/c^2]*Sin[d + e*x + ArcTan[a/c]])/(-b + c*Sqrt[(a^2 + c^2)/c^2])]])/
(Sqrt[1 + a^2/c^2]*c*(a^2 - b^2 + c^2)*e*(a + b*Sec[d + e*x] + c*Tan[d + e*
x])^(3/2)) - (a^2*Sec[d + e*x]^(3/2)*(b + a*Cos[d + e*x] + c*Sin[d + e*x])^
(3/2)*(-((c*AppellF1[-1/2, -1/2, -1/2, 1/2, -((b + a*Sqrt[1 + c^2/a^2]*Cos[
d + e*x - ArcTan[c/a]])/(a*Sqrt[1 + c^2/a^2]*(1 - b/(a*Sqrt[1 + c^2/a^2])))
), -((b + a*Sqrt[1 + c^2/a^2]*Cos[d + e*x - ArcTan[c/a]])/(a*Sqrt[1 + c^2/a
^2]*(-1 - b/(a*Sqrt[1 + c^2/a^2]))))]*Sin[d + e*x - ArcTan[c/a]])/(a*Sqrt[1
 + c^2/a^2]*Sqrt[(a*Sqrt[(a^2 + c^2)/a^2] - a*Sqrt[(a^2 + c^2)/a^2]*Cos[d +
 e*x - ArcTan[c/a]])/(b + a*Sqrt[(a^2 + c^2)/a^2])]*Sqrt[b + a*Sqrt[(a^2 +
c^2)/a^2]*Cos[d + e*x - ArcTan[c/a]]]*Sqrt[(a*Sqrt[(a^2 + c^2)/a^2] + a*Sqr
t[(a^2 + c^2)/a^2]*Cos[d + e*x - ArcTan[c/a]])/(-b + a*Sqrt[(a^2 + c^2)/a^2
])])) - ((2*a*(b + a*Sqrt[1 + c^2/a^2]*Cos[d + e*x - ArcTan[c/a]]))/(a^2 +
c^2) - (c*Sin[d + e*x - ArcTan[c/a]])/(a*Sqrt[1 + c^2/a^2]))/Sqrt[b + a*Sqr
t[1 + c^2/a^2]*Cos[d + e*x - ArcTan[c/a]]]))/(c*(a^2 - b^2 + c^2)*e*(a + b*
Sec[d + e*x] + c*Tan[d + e*x])^(3/2)) - (c*Sec[d + e*x]^(3/2)*(b + a*Cos[d
+ e*x] + c*Sin[d + e*x])^(3/2)*(-((c*AppellF1[-1/2, -1/2, -1/2, 1/2, -((b +
 a*Sqrt[1 + c^2/a^2]*Cos[d + e*x - ArcTan[c/a]])/(a*Sqrt[1 + c^2/a^2]*(1 -
b/(a*Sqrt[1 + c^2/a^2])))), -((b + a*Sqrt[1 + c^2/a^2]*Cos[d + e*x - ArcTan
[c/a]])/(a*Sqrt[1 + c^2/a^2]*(-1 - b/(a*Sqrt[1 + c^2/a^2]))))]*Sin[d + e*x
- ArcTan[c/a]])/(a*Sqrt[1 + c^2/a^2]*Sqrt[(a*Sqrt[(a^2 + c^2)/a^2] - a*Sqrt
[(a^2 + c^2)/a^2]*Cos[d + e*x - ArcTan[c/a]])/(b + a*Sqrt[(a^2 + c^2)/a^2])
]*Sqrt[b + a*Sqrt[(a^2 + c^2)/a^2]*Cos[d + e*x - ArcTan[c/a]]]*Sqrt[(a*Sqrt
[(a^2 + c^2)/a^2] + a*Sqrt[(a^2 + c^2)/a^2]*Cos[d + e*x - ArcTan[c/a]])/(-b
 + a*Sqrt[(a^2 + c^2)/a^2])])) - ((2*a*(b + a*Sqrt[1 + c^2/a^2]*Cos[d + e*x
 - ArcTan[c/a]]))/(a^2 + c^2) - (c*Sin[d + e*x - ArcTan[c/a]])/(a*Sqrt[1 +
c^2/a^2]))/Sqrt[b + a*Sqrt[1 + c^2/a^2]*Cos[d + e*x - ArcTan[c/a]]]))/((a^2
 - b^2 + c^2)*e*(a + b*Sec[d + e*x] + c*Tan[d + e*x])^(3/2))
```

fricas [F] time = 1.84, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{b\sec(ex+d)+c\tan(ex+d)+a}\,\sec(ex+d)^{\frac{3}{2}}}{b^2\sec(ex+d)^2+c^2\tan(ex+d)^2+2\,ab\sec(ex+d)+a^2+2\,(bc\sec(ex+d)+ac)\tan(ex+d)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(e*x+d)^(3/2)/(a+b*sec(e*x+d)+c*tan(e*x+d))^(3/2),x, algorithm
="fricas")
```

```
[Out] integral(sqrt(b*sec(e*x + d) + c*tan(e*x + d) + a)*sec(e*x + d)^(3/2)/(b^2*
sec(e*x + d)^2 + c^2*tan(e*x + d)^2 + 2*a*b*sec(e*x + d) + a^2 + 2*(b*c*sec
(e*x + d) + a*c)*tan(e*x + d)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sec(ex+d)^{\frac{3}{2}}}{(b\sec(ex+d)+c\tan(ex+d)+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(e*x+d)^(3/2)/(a+b*sec(e*x+d)+c*tan(e*x+d))^(3/2),x, algorithm
="giac")
```

```
[Out] integrate(sec(e*x + d)^(3/2)/(b*sec(e*x + d) + c*tan(e*x + d) + a)^(3/2), x
)
```

maple [C] time = 1.22, size = 12574, normalized size = 52.39

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(e*x+d)^(3/2)/(a+b*sec(e*x+d)+c*tan(e*x+d))^(3/2),x)
```

```
[Out] result too large to display
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sec(ex+d)^{\frac{3}{2}}}{(b\sec(ex+d)+c\tan(ex+d)+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(e*x+d)^(3/2)/(a+b*sec(e*x+d)+c*tan(e*x+d))^(3/2),x, algorithm
="maxima")
```

```
[Out] integrate(sec(e*x + d)^(3/2)/(b*sec(e*x + d) + c*tan(e*x + d) + a)^(3/2), x
)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(\frac{1}{\cos(d+e\,x)}\right)^{3/2}}{\left(a+c\tan\left(d+e\,x\right)+\frac{b}{\cos(d+e\,x)}\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/cos(d + e*x))^(3/2)/(a + c*tan(d + e*x) + b/cos(d + e*x))^(3/2),x)
```

```
[Out] int((1/cos(d + e*x))^(3/2)/(a + c*tan(d + e*x) + b/cos(d + e*x))^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec^{\frac{3}{2}}\left(d+ex\right)}{\left(a+b\sec\left(d+ex\right)+c\tan\left(d+ex\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(e*x+d)**(3/2)/(a+b*sec(e*x+d)+c*tan(e*x+d))**(3/2),x)
```

```
[Out] Integral(sec(d + e*x)**(3/2)/(a + b*sec(d + e*x) + c*tan(d + e*x))**(3/2),
x)
```

3.452 $\int \frac{\sec^{\frac{5}{2}}(d+ex)}{(a+b\sec(d+ex)+c\tan(d+ex))^{5/2}}\,dx$

Optimal. Leaf size=492

$$\frac{2\sec^{\frac{5}{2}}(d+ex)\sqrt{\frac{a\cos(d+ex)+b+c\sin(d+ex)}{\sqrt{a^2+c^2}+b}}\,(a\cos(d+ex)+b+c\sin(d+ex))^2F\left(\frac{1}{2}\left(d+ex-\tan^{-1}(a,c)\right)|\frac{2\sqrt{a^2+c^2}}{b+\sqrt{a^2+c^2}}\right)}{3e\left(a^2-b^2+c^2\right)(a+b\sec(d+ex)+c\tan(d+ex))^{5/2}}+\cdots$$

```
[Out] -2/3*sec(e*x+d)^(5/2)*(c*cos(e*x+d)-a*sin(e*x+d))*(b+a*cos(e*x+d)+c*sin(e*x
+d))/(a^2-b^2+c^2)/e/(a+b*sec(e*x+d)+c*tan(e*x+d))^(5/2)+8/3*sec(e*x+d)^(5/
2)*(b*c*cos(e*x+d)-a*b*sin(e*x+d))*(b+a*cos(e*x+d)+c*sin(e*x+d))^2/(a^2-b^2
+c^2)^2/e/(a+b*sec(e*x+d)+c*tan(e*x+d))^(5/2)+8/3*b*(cos(1/2*d+1/2*e*x-1/2*
arctan(a,c))^2)^(1/2)/cos(1/2*d+1/2*e*x-1/2*arctan(a,c))*EllipticE(sin(1/2*
d+1/2*e*x-1/2*arctan(a,c)),2^(1/2)*((a^2+c^2)^(1/2)/(b+(a^2+c^2)^(1/2)))^(1
/2))*sec(e*x+d)^(5/2)*(b+a*cos(e*x+d)+c*sin(e*x+d))^3/(a^2-b^2+c^2)^2/e/((b
+a*cos(e*x+d)+c*sin(e*x+d))/(b+(a^2+c^2)^(1/2)))^(1/2)/(a+b*sec(e*x+d)+c*ta
n(e*x+d))^(5/2)+2/3*(cos(1/2*d+1/2*e*x-1/2*arctan(a,c))^2)^(1/2)/cos(1/2*d+
1/2*e*x-1/2*arctan(a,c))*EllipticF(sin(1/2*d+1/2*e*x-1/2*arctan(a,c)),2^(1/
2)*((a^2+c^2)^(1/2)/(b+(a^2+c^2)^(1/2)))^(1/2))*sec(e*x+d)^(5/2)*(b+a*cos(e
*x+d)+c*sin(e*x+d))^2*((b+a*cos(e*x+d)+c*sin(e*x+d))/(b+(a^2+c^2)^(1/2)))^(
1/2)/(a^2-b^2+c^2)/e/(a+b*sec(e*x+d)+c*tan(e*x+d))^(5/2)
```

Rubi [A] time = 0.52, antiderivative size = 492, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.242, Rules used = {3167, 3129, 3156, 3149, 3119, 2653, 3127, 2661}

$$\frac{2\sec^{\frac{5}{2}}(d+ex)\sqrt{\frac{a\cos(d+ex)+b+c\sin(d+ex)}{\sqrt{a^2+c^2}+b}}\,(a\cos(d+ex)+b+c\sin(d+ex))^2F\left(\frac{1}{2}\left(d+ex-\tan^{-1}(a,c)\right)|\frac{2\sqrt{a^2+c^2}}{b+\sqrt{a^2+c^2}}\right)}{3e\left(a^2-b^2+c^2\right)(a+b\sec(d+ex)+c\tan(d+ex))^{5/2}}+\cdots$$

Antiderivative was successfully verified.

```
[In] Int[Sec[d + e*x]^(5/2)/(a + b*Sec[d + e*x] + c*Tan[d + e*x])^(5/2),x]
```

```
[Out] (-2*Sec[d + e*x]^(5/2)*(c*Cos[d + e*x] - a*Sin[d + e*x])*(b + a*Cos[d + e*x
] + c*Sin[d + e*x]))/(3*(a^2 - b^2 + c^2)*e*(a + b*Sec[d + e*x] + c*Tan[d +
 e*x])^(5/2)) + (8*Sec[d + e*x]^(5/2)*(b*c*Cos[d + e*x] - a*b*Sin[d + e*x])
*(b + a*Cos[d + e*x] + c*Sin[d + e*x])^2)/(3*(a^2 - b^2 + c^2)^2*e*(a + b*S
ec[d + e*x] + c*Tan[d + e*x])^(5/2)) + (8*b*EllipticE[(d + e*x - ArcTan[a,
c])/2, (2*Sqrt[a^2 + c^2])/(b + Sqrt[a^2 + c^2])]*Sec[d + e*x]^(5/2)*(b + a
```

```
*Cos[d + e*x] + c*Sin[d + e*x])^3)/(3*(a^2 - b^2 + c^2)^2*e*Sqrt[(b + a*Cos
[d + e*x] + c*Sin[d + e*x])/(b + Sqrt[a^2 + c^2])]*(a + b*Sec[d + e*x] + c*
Tan[d + e*x])^(5/2)) + (2*EllipticF[(d + e*x - ArcTan[a, c])/2, (2*Sqrt[a^2
 + c^2])/(b + Sqrt[a^2 + c^2])]*Sec[d + e*x]^(5/2)*(b + a*Cos[d + e*x] + c*
Sin[d + e*x])^2*Sqrt[(b + a*Cos[d + e*x] + c*Sin[d + e*x])/(b + Sqrt[a^2 +
c^2])])/(3*(a^2 - b^2 + c^2)*e*(a + b*Sec[d + e*x] + c*Tan[d + e*x])^(5/2))
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 3119

```
Int[Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_
)]], x_Symbol] :> Dist[Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]]/Sqrt[(a +
b*Cos[d + e*x] + c*Sin[d + e*x])/(a + Sqrt[b^2 + c^2])], Int[Sqrt[a/(a + Sq
rt[b^2 + c^2]) + (Sqrt[b^2 + c^2]*Cos[d + e*x - ArcTan[b, c]])/(a + Sqrt[b^
2 + c^2])], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0]
&& NeQ[b^2 + c^2, 0] &&  !GtQ[a + Sqrt[b^2 + c^2], 0]
```

Rule 3127

```
Int[1/Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(
x_)]], x_Symbol] :> Dist[Sqrt[(a + b*Cos[d + e*x] + c*Sin[d + e*x])/(a + Sq
rt[b^2 + c^2])]/Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]], Int[1/Sqrt[a/(a
+ Sqrt[b^2 + c^2]) + (Sqrt[b^2 + c^2]*Cos[d + e*x - ArcTan[b, c]])/(a + Sqr
t[b^2 + c^2])], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2,
 0] && NeQ[b^2 + c^2, 0] &&  !GtQ[a + Sqrt[b^2 + c^2], 0]
```

Rule 3129

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(n_), x_Symbol] :> Simp[((-(c*Cos[d + e*x]) + b*Sin[d + e*x])*(a + b*Cos[d
+ e*x] + c*Sin[d + e*x])^(n + 1))/(e*(n + 1)*(a^2 - b^2 - c^2)), x] + Dist[
1/((n + 1)*(a^2 - b^2 - c^2)), Int[(a*(n + 1) - b*(n + 2)*Cos[d + e*x] - c*
(n + 2)*Sin[d + e*x])*(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n + 1), x], x]
 /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0] && LtQ[n, -1] && N
```

```
eQ[n, -3/2]
```

Rule 3149

```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])
/Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)]]
, x_Symbol] :> Dist[B/b, Int[Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]], x],
 x] + Dist[(A*b - a*B)/b, Int[1/Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]],
x], x] /; FreeQ[{a, b, c, d, e, A, B, C}, x] && EqQ[B*c - b*C, 0] && NeQ[A*
b - a*B, 0]
```

Rule 3156

```
Int[((a_.) + cos[(d_.) + (e_.)*(x_)]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)])
^(n_)*((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)
]), x_Symbol] :> -Simp[((c*B - b*C - (a*C - c*A)*Cos[d + e*x] + (a*B - b*A)
*Sin[d + e*x])*(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n + 1))/(e*(n + 1)*(a
^2 - b^2 - c^2)), x] + Dist[1/((n + 1)*(a^2 - b^2 - c^2)), Int[(a + b*Cos[d
 + e*x] + c*Sin[d + e*x])^(n + 1)*Simp[(n + 1)*(a*A - b*B - c*C) + (n + 2)*
(a*B - b*A)*Cos[d + e*x] + (n + 2)*(a*C - c*A)*Sin[d + e*x], x], x], x] /;
FreeQ[{a, b, c, d, e, A, B, C}, x] && LtQ[n, -1] && NeQ[a^2 - b^2 - c^2, 0]
 && NeQ[n, -2]
```

Rule 3167

```
Int[sec[(d_.) + (e_.)*(x_)]^(n_.)*((a_.) + (b_.)*sec[(d_.) + (e_.)*(x_)] +
(c_.)*tan[(d_.) + (e_.)*(x_)])^(m_), x_Symbol] :> Dist[(Sec[d + e*x]^n*(b +
 a*Cos[d + e*x] + c*Sin[d + e*x])^n)/(a + b*Sec[d + e*x] + c*Tan[d + e*x])^
n, Int[1/(b + a*Cos[d + e*x] + c*Sin[d + e*x])^n, x], x] /; FreeQ[{a, b, c,
 d, e}, x] && EqQ[m + n, 0] &&  !IntegerQ[n]
```

Rubi steps

$$\int \frac{\sec^{\frac{5}{2}}(d+ex)}{(a+b\sec(d+ex)+c\tan(d+ex))^{5/2}}\,dx = \frac{\left(\sec^{\frac{5}{2}}(d+ex)(b+a\cos(d+ex)+c\sin(d+ex))^{5/2}\right)\int \frac{}{(b+a\cos(d+ex)}}{(a+b\sec(d+ex)+c\tan(d+ex))^{5/2}}$$

$$= -\frac{2\sec^{\frac{5}{2}}(d+ex)(c\cos(d+ex)-a\sin(d+ex))(b+a\cos(d+ex)+}{3\left(a^2-b^2+c^2\right)e(a+b\sec(d+ex)+c\tan(d+ex))^{5}}$$

$$= -\frac{2\sec^{\frac{5}{2}}(d+ex)(c\cos(d+ex)-a\sin(d+ex))(b+a\cos(d+ex)+}{3\left(a^2-b^2+c^2\right)e(a+b\sec(d+ex)+c\tan(d+ex))^{5}}$$

$$= -\frac{2\sec^{\frac{5}{2}}(d+ex)(c\cos(d+ex)-a\sin(d+ex))(b+a\cos(d+ex)+}{3\left(a^2-b^2+c^2\right)e(a+b\sec(d+ex)+c\tan(d+ex))^{5}}$$

$$= -\frac{2\sec^{\frac{5}{2}}(d+ex)(c\cos(d+ex)-a\sin(d+ex))(b+a\cos(d+ex)+}{3\left(a^2-b^2+c^2\right)e(a+b\sec(d+ex)+c\tan(d+ex))^{5}}$$

$$= -\frac{2\sec^{\frac{5}{2}}(d+ex)(c\cos(d+ex)-a\sin(d+ex))(b+a\cos(d+ex)+}{3\left(a^2-b^2+c^2\right)e(a+b\sec(d+ex)+c\tan(d+ex))^{5}}$$

Mathematica [C] time = 6.55, size = 2708, normalized size = 5.50

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sec[d + e*x]^(5/2)/(a + b*Sec[d + e*x] + c*Tan[d + e*x])^(5/2),x]
```

```
[Out] (Sec[d + e*x]^(5/2)*(b + a*Cos[d + e*x] + c*Sin[d + e*x])^3*((8*b*(a^2 + c^
2))/(3*a*c*(-a^2 + b^2 - c^2)^2) + (2*(b*c + a^2*Sin[d + e*x] + c^2*Sin[d +
 e*x]))/(3*a*(a^2 - b^2 + c^2)*(b + a*Cos[d + e*x] + c*Sin[d + e*x])^2) - (
2*(a^2*c + 3*b^2*c + c^3 + 4*a^2*b*Sin[d + e*x] + 4*b*c^2*Sin[d + e*x]))/(3
*a*(a^2 - b^2 + c^2)^2*(b + a*Cos[d + e*x] + c*Sin[d + e*x]))))/(e*(a + b*S
ec[d + e*x] + c*Tan[d + e*x])^(5/2)) + (2*a^2*AppellF1[1/2, 1/2, 1/2, 3/2,
-((b + Sqrt[1 + a^2/c^2]*c*Sin[d + e*x + ArcTan[a/c]])/(Sqrt[1 + a^2/c^2]*(
1 - b/(Sqrt[1 + a^2/c^2]*c))*c)), -((b + Sqrt[1 + a^2/c^2]*c*Sin[d + e*x +
```

```
ArcTan[a/c]])/(Sqrt[1 + a^2/c^2]*(-1 - b/(Sqrt[1 + a^2/c^2]*c))*c))]*Sec[d
+ e*x]^(5/2)*Sec[d + e*x + ArcTan[a/c]]*(b + a*Cos[d + e*x] + c*Sin[d + e*x
])^(5/2)*Sqrt[(c*Sqrt[(a^2 + c^2)/c^2] - c*Sqrt[(a^2 + c^2)/c^2]*Sin[d + e*
x + ArcTan[a/c]])/(b + c*Sqrt[(a^2 + c^2)/c^2])]*Sqrt[b + c*Sqrt[(a^2 + c^2
)/c^2]*Sin[d + e*x + ArcTan[a/c]]]*Sqrt[(c*Sqrt[(a^2 + c^2)/c^2] + c*Sqrt[(
a^2 + c^2)/c^2]*Sin[d + e*x + ArcTan[a/c]])/(-b + c*Sqrt[(a^2 + c^2)/c^2])]
)/(3*Sqrt[1 + a^2/c^2]*c*(a^2 - b^2 + c^2)^2*e*(a + b*Sec[d + e*x] + c*Tan[
d + e*x])^(5/2)) + (2*b^2*AppellF1[1/2, 1/2, 1/2, 3/2, -((b + Sqrt[1 + a^2/
c^2]*c*Sin[d + e*x + ArcTan[a/c]])/(Sqrt[1 + a^2/c^2]*(1 - b/(Sqrt[1 + a^2/
c^2]*c))*c)), -((b + Sqrt[1 + a^2/c^2]*c*Sin[d + e*x + ArcTan[a/c]])/(Sqrt[
1 + a^2/c^2]*(-1 - b/(Sqrt[1 + a^2/c^2]*c))*c))]*Sec[d + e*x]^(5/2)*Sec[d +
 e*x + ArcTan[a/c]]*(b + a*Cos[d + e*x] + c*Sin[d + e*x])^(5/2)*Sqrt[(c*Sqr
t[(a^2 + c^2)/c^2] - c*Sqrt[(a^2 + c^2)/c^2]*Sin[d + e*x + ArcTan[a/c]])/(b
 + c*Sqrt[(a^2 + c^2)/c^2])]*Sqrt[b + c*Sqrt[(a^2 + c^2)/c^2]*Sin[d + e*x +
 ArcTan[a/c]]]*Sqrt[(c*Sqrt[(a^2 + c^2)/c^2] + c*Sqrt[(a^2 + c^2)/c^2]*Sin[
d + e*x + ArcTan[a/c]])/(-b + c*Sqrt[(a^2 + c^2)/c^2])])/(Sqrt[1 + a^2/c^2]
*c*(a^2 - b^2 + c^2)^2*e*(a + b*Sec[d + e*x] + c*Tan[d + e*x])^(5/2)) + (2*
c*AppellF1[1/2, 1/2, 1/2, 3/2, -((b + Sqrt[1 + a^2/c^2]*c*Sin[d + e*x + Arc
Tan[a/c]])/(Sqrt[1 + a^2/c^2]*(1 - b/(Sqrt[1 + a^2/c^2]*c))*c)), -((b + Sqr
t[1 + a^2/c^2]*c*Sin[d + e*x + ArcTan[a/c]])/(Sqrt[1 + a^2/c^2]*(-1 - b/(Sq
rt[1 + a^2/c^2]*c))*c))]*Sec[d + e*x]^(5/2)*Sec[d + e*x + ArcTan[a/c]]*(b +
 a*Cos[d + e*x] + c*Sin[d + e*x])^(5/2)*Sqrt[(c*Sqrt[(a^2 + c^2)/c^2] - c*S
qrt[(a^2 + c^2)/c^2]*Sin[d + e*x + ArcTan[a/c]])/(b + c*Sqrt[(a^2 + c^2)/c^
2])]*Sqrt[b + c*Sqrt[(a^2 + c^2)/c^2]*Sin[d + e*x + ArcTan[a/c]]]*Sqrt[(c*S
qrt[(a^2 + c^2)/c^2] + c*Sqrt[(a^2 + c^2)/c^2]*Sin[d + e*x + ArcTan[a/c]])/
(-b + c*Sqrt[(a^2 + c^2)/c^2])])/(3*Sqrt[1 + a^2/c^2]*(a^2 - b^2 + c^2)^2*e
*(a + b*Sec[d + e*x] + c*Tan[d + e*x])^(5/2)) + (4*a^2*b*Sec[d + e*x]^(5/2)
*(b + a*Cos[d + e*x] + c*Sin[d + e*x])^(5/2)*(-((c*AppellF1[-1/2, -1/2, -1/
2, 1/2, -((b + a*Sqrt[1 + c^2/a^2]*Cos[d + e*x - ArcTan[c/a]])/(a*Sqrt[1 +
c^2/a^2]*(1 - b/(a*Sqrt[1 + c^2/a^2])))), -((b + a*Sqrt[1 + c^2/a^2]*Cos[d
+ e*x - ArcTan[c/a]])/(a*Sqrt[1 + c^2/a^2]*(-1 - b/(a*Sqrt[1 + c^2/a^2]))))
]*Sin[d + e*x - ArcTan[c/a]])/(a*Sqrt[1 + c^2/a^2]*Sqrt[(a*Sqrt[(a^2 + c^2)
/a^2] - a*Sqrt[(a^2 + c^2)/a^2]*Cos[d + e*x - ArcTan[c/a]])/(b + a*Sqrt[(a^
2 + c^2)/a^2])]*Sqrt[b + a*Sqrt[(a^2 + c^2)/a^2]*Cos[d + e*x - ArcTan[c/a]]
]*Sqrt[(a*Sqrt[(a^2 + c^2)/a^2] + a*Sqrt[(a^2 + c^2)/a^2]*Cos[d + e*x - Arc
Tan[c/a]])/(-b + a*Sqrt[(a^2 + c^2)/a^2])])) - ((2*a*(b + a*Sqrt[1 + c^2/a^
2]*Cos[d + e*x - ArcTan[c/a]]))/(a^2 + c^2) - (c*Sin[d + e*x - ArcTan[c/a]]
)/(a*Sqrt[1 + c^2/a^2]))/Sqrt[b + a*Sqrt[1 + c^2/a^2]*Cos[d + e*x - ArcTan[
c/a]]]))/(3*c*(a^2 - b^2 + c^2)^2*e*(a + b*Sec[d + e*x] + c*Tan[d + e*x])^(
5/2)) + (4*b*c*Sec[d + e*x]^(5/2)*(b + a*Cos[d + e*x] + c*Sin[d + e*x])^(5/
2)*(-((c*AppellF1[-1/2, -1/2, -1/2, 1/2, -((b + a*Sqrt[1 + c^2/a^2]*Cos[d +
 e*x - ArcTan[c/a]])/(a*Sqrt[1 + c^2/a^2]*(1 - b/(a*Sqrt[1 + c^2/a^2])))),
-((b + a*Sqrt[1 + c^2/a^2]*Cos[d + e*x - ArcTan[c/a]])/(a*Sqrt[1 + c^2/a^2]
*(-1 - b/(a*Sqrt[1 + c^2/a^2]))))]*Sin[d + e*x - ArcTan[c/a]])/(a*Sqrt[1 +
c^2/a^2]*Sqrt[(a*Sqrt[(a^2 + c^2)/a^2] - a*Sqrt[(a^2 + c^2)/a^2]*Cos[d + e*
```

```
x - ArcTan[c/a]])/(b + a*Sqrt[(a^2 + c^2)/a^2])]*Sqrt[b + a*Sqrt[(a^2 + c^2
)/a^2]*Cos[d + e*x - ArcTan[c/a]]]*Sqrt[(a*Sqrt[(a^2 + c^2)/a^2] + a*Sqrt[(
a^2 + c^2)/a^2]*Cos[d + e*x - ArcTan[c/a]])/(-b + a*Sqrt[(a^2 + c^2)/a^2])]
)) - ((2*a*(b + a*Sqrt[1 + c^2/a^2]*Cos[d + e*x - ArcTan[c/a]]))/(a^2 + c^2
) - (c*Sin[d + e*x - ArcTan[c/a]])/(a*Sqrt[1 + c^2/a^2]))/Sqrt[b + a*Sqrt[1
 + c^2/a^2]*Cos[d + e*x - ArcTan[c/a]]]))/(3*(a^2 - b^2 + c^2)^2*e*(a + b*S
ec[d + e*x] + c*Tan[d + e*x])^(5/2))
```

fricas [F] time = 0.96, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{b\sec\left(ex+d\right)+c\tan\left(ex+d\right)+a}}{b^{3}\sec\left(ex+d\right)^{3}+c^{3}\tan\left(ex+d\right)^{3}+3\,ab^{2}\sec\left(ex+d\right)^{2}+3\,a^{2}b\sec\left(ex+d\right)+a^{3}+3\left(bc^{2}\sec\left(ex+d\right)\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(e*x+d)^(5/2)/(a+b*sec(e*x+d)+c*tan(e*x+d))^(5/2),x, algorithm
="fricas")
```

```
[Out] integral(sqrt(b*sec(e*x + d) + c*tan(e*x + d) + a)*sec(e*x + d)^(5/2)/(b^3*
sec(e*x + d)^3 + c^3*tan(e*x + d)^3 + 3*a*b^2*sec(e*x + d)^2 + 3*a^2*b*sec(
e*x + d) + a^3 + 3*(b*c^2*sec(e*x + d) + a*c^2)*tan(e*x + d)^2 + 3*(b^2*c*s
ec(e*x + d)^2 + 2*a*b*c*sec(e*x + d) + a^2*c)*tan(e*x + d)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec\left(ex+d\right)^{\frac{5}{2}}}{\left(b\sec\left(ex+d\right)+c\tan\left(ex+d\right)+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(e*x+d)^(5/2)/(a+b*sec(e*x+d)+c*tan(e*x+d))^(5/2),x, algorithm
="giac")
```

```
[Out] integrate(sec(e*x + d)^(5/2)/(b*sec(e*x + d) + c*tan(e*x + d) + a)^(5/2), x
)
```

maple [C] time = 2.55, size = 64693, normalized size = 131.49

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(e*x+d)^(5/2)/(a+b*sec(e*x+d)+c*tan(e*x+d))^(5/2),x)
```

```
[Out] result too large to display
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec\left(ex+d\right)^{\frac{5}{2}}}{\left(b\sec\left(ex+d\right)+c\tan\left(ex+d\right)+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(e*x+d)^(5/2)/(a+b*sec(e*x+d)+c*tan(e*x+d))^(5/2),x, algorithm
="maxima")
```

```
[Out] integrate(sec(e*x + d)^(5/2)/(b*sec(e*x + d) + c*tan(e*x + d) + a)^(5/2), x
)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(\frac{1}{\cos\left(d+e\,x\right)}\right)}^{5/2}}{{\left(a+c\,\tan\left(d+e\,x\right)+\frac{b}{\cos\left(d+e\,x\right)}\right)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/cos(d + e*x))^(5/2)/(a + c*tan(d + e*x) + b/cos(d + e*x))^(5/2),x)
```

```
[Out] int((1/cos(d + e*x))^(5/2)/(a + c*tan(d + e*x) + b/cos(d + e*x))^(5/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(e*x+d)**(5/2)/(a+b*sec(e*x+d)+c*tan(e*x+d))**(5/2),x)
```

```
[Out] Timed out
```

3.453 $\int \cos^{\frac{3}{2}}(d+ex)(a+b\sec(d+ex)+c\tan(d+ex))^{3/2}\,dx$

Optimal. Leaf size=371

$$\frac{2\left(a^2-b^2+c^2\right)\cos^{\frac{3}{2}}(d+ex)\sqrt{\frac{a\cos(d+ex)+b+c\sin(d+ex)}{\sqrt{a^2+c^2}+b}}\,(a+b\sec(d+ex)+c\tan(d+ex))^{3/2}F\left(\frac{1}{2}\left(d+ex-\tan^{-1}(a\right.\right.}{3e(a\cos(d+ex)+b+c\sin(d+ex))^2}$$

```
[Out] -2/3*cos(e*x+d)^(3/2)*(c*cos(e*x+d)-a*sin(e*x+d))*(a+b*sec(e*x+d)+c*tan(e*x
+d))^(3/2)/e/(b+a*cos(e*x+d)+c*sin(e*x+d))+8/3*b*cos(e*x+d)^(3/2)*(cos(1/2*
d+1/2*e*x-1/2*arctan(a,c))^2)^(1/2)/cos(1/2*d+1/2*e*x-1/2*arctan(a,c))*Elli
pticE(sin(1/2*d+1/2*e*x-1/2*arctan(a,c)),2^(1/2)*((a^2+c^2)^(1/2)/(b+(a^2+c
^2)^(1/2)))^(1/2))*(a+b*sec(e*x+d)+c*tan(e*x+d))^(3/2)/e/(b+a*cos(e*x+d)+c*
sin(e*x+d))/((b+a*cos(e*x+d)+c*sin(e*x+d))/(b+(a^2+c^2)^(1/2)))^(1/2)+2/3*(
a^2-b^2+c^2)*cos(e*x+d)^(3/2)*(cos(1/2*d+1/2*e*x-1/2*arctan(a,c))^2)^(1/2)/
cos(1/2*d+1/2*e*x-1/2*arctan(a,c))*EllipticF(sin(1/2*d+1/2*e*x-1/2*arctan(a
,c)),2^(1/2)*((a^2+c^2)^(1/2)/(b+(a^2+c^2)^(1/2)))^(1/2))*((b+a*cos(e*x+d)+
c*sin(e*x+d))/(b+(a^2+c^2)^(1/2)))^(1/2)*(a+b*sec(e*x+d)+c*tan(e*x+d))^(3/2
)/e/(b+a*cos(e*x+d)+c*sin(e*x+d))^2
```

Rubi [A] time = 0.39, antiderivative size = 371, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.212, Rules used = {3163, 3120, 3149, 3119, 2653, 3127, 2661}

$$\frac{2\left(a^2-b^2+c^2\right)\cos^{\frac{3}{2}}(d+ex)\sqrt{\frac{a\cos(d+ex)+b+c\sin(d+ex)}{\sqrt{a^2+c^2}+b}}\,(a+b\sec(d+ex)+c\tan(d+ex))^{3/2}F\left(\frac{1}{2}\left(d+ex-\tan^{-1}(a\right.\right.}{3e(a\cos(d+ex)+b+c\sin(d+ex))^2}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[d + e*x]^(3/2)*(a + b*Sec[d + e*x] + c*Tan[d + e*x])^(3/2),x]
```

```
[Out] (-2*Cos[d + e*x]^(3/2)*(c*Cos[d + e*x] - a*Sin[d + e*x])*(a + b*Sec[d + e*x
] + c*Tan[d + e*x])^(3/2))/(3*e*(b + a*Cos[d + e*x] + c*Sin[d + e*x])) + (8
*b*Cos[d + e*x]^(3/2)*EllipticE[(d + e*x - ArcTan[a, c])/2, (2*Sqrt[a^2 + c
^2])/(b + Sqrt[a^2 + c^2])]*(a + b*Sec[d + e*x] + c*Tan[d + e*x])^(3/2))/(3
*e*(b + a*Cos[d + e*x] + c*Sin[d + e*x])*Sqrt[(b + a*Cos[d + e*x] + c*Sin[d
 + e*x])/(b + Sqrt[a^2 + c^2])]) + (2*(a^2 - b^2 + c^2)*Cos[d + e*x]^(3/2)*
EllipticF[(d + e*x - ArcTan[a, c])/2, (2*Sqrt[a^2 + c^2])/(b + Sqrt[a^2 + c
^2])]*Sqrt[(b + a*Cos[d + e*x] + c*Sin[d + e*x])/(b + Sqrt[a^2 + c^2])]*(a
```

```
+ b*Sec[d + e*x] + c*Tan[d + e*x])^(3/2))/(3*e*(b + a*Cos[d + e*x] + c*Sin[
d + e*x])^2)
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 3119

```
Int[Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_
)]], x_Symbol] :> Dist[Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]]/Sqrt[(a +
b*Cos[d + e*x] + c*Sin[d + e*x])/(a + Sqrt[b^2 + c^2])], Int[Sqrt[a/(a + Sq
rt[b^2 + c^2]) + (Sqrt[b^2 + c^2]*Cos[d + e*x - ArcTan[b, c]])/(a + Sqrt[b^
2 + c^2])], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0]
&& NeQ[b^2 + c^2, 0] &&  !GtQ[a + Sqrt[b^2 + c^2], 0]
```

Rule 3120

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(n_), x_Symbol] :> -Simp[((c*Cos[d + e*x] - b*Sin[d + e*x])*(a + b*Cos[d +
e*x] + c*Sin[d + e*x])^(n - 1))/(e*n), x] + Dist[1/n, Int[Simp[n*a^2 + (n -
 1)*(b^2 + c^2) + a*b*(2*n - 1)*Cos[d + e*x] + a*c*(2*n - 1)*Sin[d + e*x],
x]*(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n - 2), x], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[a^2 - b^2 - c^2, 0] && GtQ[n, 1]
```

Rule 3127

```
Int[1/Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(
x_)]], x_Symbol] :> Dist[Sqrt[(a + b*Cos[d + e*x] + c*Sin[d + e*x])/(a + Sq
rt[b^2 + c^2])]/Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]], Int[1/Sqrt[a/(a
+ Sqrt[b^2 + c^2]) + (Sqrt[b^2 + c^2]*Cos[d + e*x - ArcTan[b, c]])/(a + Sqr
t[b^2 + c^2])], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2,
 0] && NeQ[b^2 + c^2, 0] &&  !GtQ[a + Sqrt[b^2 + c^2], 0]
```

Rule 3149

```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])
/Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)]]
```

```
, x_Symbol] :> Dist[B/b, Int[Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]], x],
 x] + Dist[(A*b - a*B)/b, Int[1/Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]],
x], x] /; FreeQ[{a, b, c, d, e, A, B, C}, x] && EqQ[B*c - b*C, 0] && NeQ[A*
b - a*B, 0]
```

Rule 3163

```
Int[cos[(d_.) + (e_.)*(x_)]^(n_)*((a_.) + (b_.)*sec[(d_.) + (e_.)*(x_)] + (
c_.)*tan[(d_.) + (e_.)*(x_)])^(n_), x_Symbol] :> Dist[(Cos[d + e*x]^n*(a +
b*Sec[d + e*x] + c*Tan[d + e*x])^n)/(b + a*Cos[d + e*x] + c*Sin[d + e*x])^n
, Int[(b + a*Cos[d + e*x] + c*Sin[d + e*x])^n, x], x] /; FreeQ[{a, b, c, d,
 e}, x] && !IntegerQ[n]
```

Rubi steps

$$\int \cos^{\frac{3}{2}}(d+ex)(a+b\sec(d+ex)+c\tan(d+ex))^{3/2}\,dx = \frac{\left(\cos^{\frac{3}{2}}(d+ex)(a+b\sec(d+ex)+c\tan(d+ex))^{3/2}\right)\int}{(b+a\cos(d+ex)+c\sin}$$

$$= -\frac{2\cos^{\frac{3}{2}}(d+ex)(c\cos(d+ex)-a\sin(d+ex))(a+b\,\mathrm{s}}{3e(b+a\cos(d+ex)+c\sin(}$$

$$= -\frac{2\cos^{\frac{3}{2}}(d+ex)(c\cos(d+ex)-a\sin(d+ex))(a+b\,\mathrm{s}}{3e(b+a\cos(d+ex)+c\sin(}$$

$$= -\frac{2\cos^{\frac{3}{2}}(d+ex)(c\cos(d+ex)-a\sin(d+ex))(a+b\,\mathrm{s}}{3e(b+a\cos(d+ex)+c\sin(}$$

$$= -\frac{2\cos^{\frac{3}{2}}(d+ex)(c\cos(d+ex)-a\sin(d+ex))(a+b\,\mathrm{s}}{3e(b+a\cos(d+ex)+c\sin(}$$

Mathematica [F] time = 150.81, size = 0, normalized size = 0.00

$$\int \cos^{\frac{3}{2}}(d+ex)(a+b\sec(d+ex)+c\tan(d+ex))^{3/2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Cos[d + e*x]^(3/2)*(a + b*Sec[d + e*x] + c*Tan[d + e*x])^(3/2),x]
```

```
[Out] Integrate[Cos[d + e*x]^(3/2)*(a + b*Sec[d + e*x] + c*Tan[d + e*x])^(3/2), x
]
```

fricas [F] time = 0.93, size = 0, normalized size = 0.00

$$\text{integral}\Big((b\cos(ex+d)\sec(ex+d) + c\cos(ex+d)\tan(ex+d) + a\cos(ex+d))\sqrt{b\sec(ex+d) + c\tan(ex+d)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(e*x+d)^(3/2)*(a+b*sec(e*x+d)+c*tan(e*x+d))^(3/2),x, algorithm
="fricas")
```

```
[Out] integral((b*cos(e*x + d)*sec(e*x + d) + c*cos(e*x + d)*tan(e*x + d) + a*cos
(e*x + d))*sqrt(b*sec(e*x + d) + c*tan(e*x + d) + a)*sqrt(cos(e*x + d)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b\sec(ex+d) + c\tan(ex+d) + a)^{\frac{3}{2}}\cos(ex+d)^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(e*x+d)^(3/2)*(a+b*sec(e*x+d)+c*tan(e*x+d))^(3/2),x, algorithm
="giac")
```

```
[Out] integrate((b*sec(e*x + d) + c*tan(e*x + d) + a)^(3/2)*cos(e*x + d)^(3/2), x
)
```

maple [C] time = 1.29, size = 20776, normalized size = 56.00

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(e*x+d)^(3/2)*(a+b*sec(e*x+d)+c*tan(e*x+d))^(3/2),x)
```

```
[Out] result too large to display
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b\sec(ex+d) + c\tan(ex+d) + a)^{\frac{3}{2}}\cos(ex+d)^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(e*x+d)^(3/2)*(a+b*sec(e*x+d)+c*tan(e*x+d))^(3/2),x, algorithm
="maxima")
```

```
[Out] integrate((b*sec(e*x + d) + c*tan(e*x + d) + a)^(3/2)*cos(e*x + d)^(3/2), x
)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \cos(d+e\,x)^{3/2}\left(a+c\tan(d+e\,x)+\frac{b}{\cos(d+e\,x)}\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d + e*x)^(3/2)*(a + c*tan(d + e*x) + b/cos(d + e*x))^(3/2),x)
[Out] int(cos(d + e*x)^(3/2)*(a + c*tan(d + e*x) + b/cos(d + e*x))^(3/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(e*x+d)**(3/2)*(a+b*sec(e*x+d)+c*tan(e*x+d))**(3/2),x)
[Out] Timed out
```

3.454 $\int \sqrt{\cos(d+ex)}\sqrt{a+b\sec(d+ex)+c\tan(d+ex)}\,dx$

Optimal. Leaf size=118

$$\frac{2\sqrt{\cos(d+ex)}\sqrt{a+b\sec(d+ex)+c\tan(d+ex)}\,E\left(\frac{1}{2}\left(d+ex-\tan^{-1}(a,c)\right)|\frac{2\sqrt{a^2+c^2}}{b+\sqrt{a^2+c^2}}\right)}{e\sqrt{\frac{a\cos(d+ex)+b+c\sin(d+ex)}{\sqrt{a^2+c^2}+b}}}$$

```
[Out] 2*(cos(1/2*d+1/2*e*x-1/2*arctan(a,c))^2)^(1/2)/cos(1/2*d+1/2*e*x-1/2*arctan
(a,c))*EllipticE(sin(1/2*d+1/2*e*x-1/2*arctan(a,c)),2^(1/2)*((a^2+c^2)^(1/2
)/(b+(a^2+c^2)^(1/2)))^(1/2))*cos(e*x+d)^(1/2)*(a+b*sec(e*x+d)+c*tan(e*x+d)
)^(1/2)/e/((b+a*cos(e*x+d)+c*sin(e*x+d))/(b+(a^2+c^2)^(1/2)))^(1/2)
```

Rubi [A] time = 0.15, antiderivative size = 118, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {3163, 3119, 2653}

$$\frac{2\sqrt{\cos(d+ex)}\sqrt{a+b\sec(d+ex)+c\tan(d+ex)}\,E\left(\frac{1}{2}\left(d+ex-\tan^{-1}(a,c)\right)|\frac{2\sqrt{a^2+c^2}}{b+\sqrt{a^2+c^2}}\right)}{e\sqrt{\frac{a\cos(d+ex)+b+c\sin(d+ex)}{\sqrt{a^2+c^2}+b}}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Cos[d + e*x]]*Sqrt[a + b*Sec[d + e*x] + c*Tan[d + e*x]],x]
```

```
[Out] (2*Sqrt[Cos[d + e*x]]*EllipticE[(d + e*x - ArcTan[a, c])/2, (2*Sqrt[a^2 + c
^2])/(b + Sqrt[a^2 + c^2])]*Sqrt[a + b*Sec[d + e*x] + c*Tan[d + e*x]])/(e*S
qrt[(b + a*Cos[d + e*x] + c*Sin[d + e*x])/(b + Sqrt[a^2 + c^2])])
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 3119

```
Int[Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_
)]], x_Symbol] :> Dist[Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]]/Sqrt[(a +
b*Cos[d + e*x] + c*Sin[d + e*x])/(a + Sqrt[b^2 + c^2])], Int[Sqrt[a/(a + Sq
rt[b^2 + c^2]) + (Sqrt[b^2 + c^2]*Cos[d + e*x - ArcTan[b, c]])/(a + Sqrt[b^
2 + c^2])], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0]
```

```
&& NeQ[b^2 + c^2, 0] && !GtQ[a + Sqrt[b^2 + c^2], 0]
```

Rule 3163

```
Int[cos[(d_.) + (e_.)*(x_)]^(n_)*((a_.) + (b_.)*sec[(d_.) + (e_.)*(x_)] + (
c_.)*tan[(d_.) + (e_.)*(x_)])^(n_), x_Symbol] :> Dist[(Cos[d + e*x]^n*(a +
b*Sec[d + e*x] + c*Tan[d + e*x])^n)/(b + a*Cos[d + e*x] + c*Sin[d + e*x])^n
, Int[(b + a*Cos[d + e*x] + c*Sin[d + e*x])^n, x], x] /; FreeQ[{a, b, c, d,
 e}, x] && !IntegerQ[n]
```

Rubi steps

$$\int \sqrt{\cos(d+ex)}\sqrt{a+b\sec(d+ex)+c\tan(d+ex)}\,dx = \frac{\left(\sqrt{\cos(d+ex)}\sqrt{a+b\sec(d+ex)+c\tan(d+ex)}\right)\int \cdots}{\sqrt{b+a\cos(d+ex)+c\sin \cdots}}$$

$$= \frac{\left(\sqrt{\cos(d+ex)}\sqrt{a+b\sec(d+ex)+c\tan(d+ex)}\right)\int \cdots}{\sqrt{\frac{b+a\cos(d+ex)+c\sin(\cdots}{b+\sqrt{a^2+c^2}}}}$$

$$= \frac{2\sqrt{\cos(d+ex)}\,E\left(\frac{1}{2}\left(d+ex-\tan^{-1}(a,c)\right)|\frac{2\sqrt{a^2+c^2}}{b+\sqrt{a^2+c^2}}\right)\sqrt{\cdots}}{e\sqrt{\frac{b+a\cos(d+ex)+c\sin(d+e\cdots}{b+\sqrt{a^2+c^2}}}}$$

Mathematica [F] time = 21.22, size = 0, normalized size = 0.00

$$\int \sqrt{\cos(d+ex)}\sqrt{a+b\sec(d+ex)+c\tan(d+ex)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Sqrt[Cos[d + e*x]]*Sqrt[a + b*Sec[d + e*x] + c*Tan[d + e*x]],x]
[Out] Integrate[Sqrt[Cos[d + e*x]]*Sqrt[a + b*Sec[d + e*x] + c*Tan[d + e*x]], x]
```

fricas [F] time = 1.12, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\sqrt{b\sec(ex+d)+c\tan(ex+d)+a}\sqrt{\cos(ex+d)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(e*x+d)^(1/2)*(a+b*sec(e*x+d)+c*tan(e*x+d))^(1/2),x, algorithm
="fricas")
```

```
[Out] integral(sqrt(b*sec(e*x + d) + c*tan(e*x + d) + a)*sqrt(cos(e*x + d)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{b\sec\left(ex+d\right)+c\tan\left(ex+d\right)+a}\,\sqrt{\cos\left(ex+d\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(e*x+d)^(1/2)*(a+b*sec(e*x+d)+c*tan(e*x+d))^(1/2),x, algorithm
="giac")
```

```
[Out] integrate(sqrt(b*sec(e*x + d) + c*tan(e*x + d) + a)*sqrt(cos(e*x + d)), x)
```

maple [C] time = 1.02, size = 12459, normalized size = 105.58

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(e*x+d)^(1/2)*(a+b*sec(e*x+d)+c*tan(e*x+d))^(1/2),x)
```

```
[Out] result too large to display
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{b\sec\left(ex+d\right)+c\tan\left(ex+d\right)+a}\,\sqrt{\cos\left(ex+d\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(e*x+d)^(1/2)*(a+b*sec(e*x+d)+c*tan(e*x+d))^(1/2),x, algorithm
="maxima")
```

```
[Out] integrate(sqrt(b*sec(e*x + d) + c*tan(e*x + d) + a)*sqrt(cos(e*x + d)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \sqrt{\cos\left(d+e\,x\right)}\,\sqrt{a+c\,\tan\left(d+e\,x\right)+\frac{b}{\cos\left(d+e\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d + e*x)^(1/2)*(a + c*tan(d + e*x) + b/cos(d + e*x))^(1/2),x)
```

```
[Out] int(cos(d + e*x)^(1/2)*(a + c*tan(d + e*x) + b/cos(d + e*x))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a+b\sec\left(d+ex\right)+c\tan\left(d+ex\right)}\,\sqrt{\cos\left(d+ex\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(e*x+d)**(1/2)*(a+b*sec(e*x+d)+c*tan(e*x+d))**(1/2),x)
[Out] Integral(sqrt(a + b*sec(d + e*x) + c*tan(d + e*x))*sqrt(cos(d + e*x)), x)
```

3.455 $\int \frac{1}{\sqrt{\cos(d+ex)}\sqrt{a+b\sec(d+ex)+c\tan(d+ex)}}\,dx$

Optimal. Leaf size=118

$$\frac{2\sqrt{\frac{a\cos(d+ex)+b+c\sin(d+ex)}{\sqrt{a^2+c^2}+b}}\,F\left(\frac{1}{2}\left(d+ex-\tan^{-1}(a,c)\right)|\frac{2\sqrt{a^2+c^2}}{b+\sqrt{a^2+c^2}}\right)}{e\sqrt{\cos(d+ex)}\sqrt{a+b\sec(d+ex)+c\tan(d+ex)}}$$

```
[Out] 2*(cos(1/2*d+1/2*e*x-1/2*arctan(a,c))^2)^(1/2)/cos(1/2*d+1/2*e*x-1/2*arctan
(a,c))*EllipticF(sin(1/2*d+1/2*e*x-1/2*arctan(a,c)),2^(1/2)*((a^2+c^2)^(1/2
)/(b+(a^2+c^2)^(1/2)))^(1/2))*((b+a*cos(e*x+d)+c*sin(e*x+d))/(b+(a^2+c^2)^(
1/2)))^(1/2)/e/cos(e*x+d)^(1/2)/(a+b*sec(e*x+d)+c*tan(e*x+d))^(1/2)
```

Rubi [A] time = 0.15, antiderivative size = 118, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {3163, 3127, 2661}

$$\frac{2\sqrt{\frac{a\cos(d+ex)+b+c\sin(d+ex)}{\sqrt{a^2+c^2}+b}}\,F\left(\frac{1}{2}\left(d+ex-\tan^{-1}(a,c)\right)|\frac{2\sqrt{a^2+c^2}}{b+\sqrt{a^2+c^2}}\right)}{e\sqrt{\cos(d+ex)}\sqrt{a+b\sec(d+ex)+c\tan(d+ex)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[Cos[d + e*x]]*Sqrt[a + b*Sec[d + e*x] + c*Tan[d + e*x]]),x]
```

```
[Out] (2*EllipticF[(d + e*x - ArcTan[a, c])/2, (2*Sqrt[a^2 + c^2])/(b + Sqrt[a^2
+ c^2])]*Sqrt[(b + a*Cos[d + e*x] + c*Sin[d + e*x])/(b + Sqrt[a^2 + c^2])])
/(e*Sqrt[Cos[d + e*x]]*Sqrt[a + b*Sec[d + e*x] + c*Tan[d + e*x]])
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 3127

```
Int[1/Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(
x_)]], x_Symbol] :> Dist[Sqrt[(a + b*Cos[d + e*x] + c*Sin[d + e*x])/(a + Sq
rt[b^2 + c^2])]/Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]], Int[1/Sqrt[a/(a
+ Sqrt[b^2 + c^2]) + (Sqrt[b^2 + c^2]*Cos[d + e*x - ArcTan[b, c]])/(a + Sqr
t[b^2 + c^2])], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2,
 0] && NeQ[b^2 + c^2, 0] && !GtQ[a + Sqrt[b^2 + c^2], 0]
```

Rule 3163

```
Int[cos[(d_.) + (e_.)*(x_)]^(n_)*((a_.) + (b_.)*sec[(d_.) + (e_.)*(x_)] + (
c_.)*tan[(d_.) + (e_.)*(x_)])^(n_), x_Symbol] :> Dist[(Cos[d + e*x]^n*(a +
b*Sec[d + e*x] + c*Tan[d + e*x])^n)/(b + a*Cos[d + e*x] + c*Sin[d + e*x])^n
, Int[(b + a*Cos[d + e*x] + c*Sin[d + e*x])^n, x], x] /; FreeQ[{a, b, c, d,
 e}, x] && !IntegerQ[n]
```

Rubi steps

$$\int \frac{1}{\sqrt{\cos(d+ex)}\sqrt{a+b\sec(d+ex)+c\tan(d+ex)}}\,dx = \frac{\sqrt{b+a\cos(d+ex)+c\sin(d+ex)}\int \frac{1}{\sqrt{b+a\cos(d+ex)+c\,\mathrm{si}}}}{\sqrt{\cos(d+ex)}\sqrt{a+b\sec(d+ex)+c\tan(d+e}}$$

$$= \frac{\sqrt{\frac{b+a\cos(d+ex)+c\sin(d+ex)}{b+\sqrt{a^2+c^2}}}\int \frac{1}{\sqrt{\frac{b}{b+\sqrt{a^2+c^2}}+\frac{\sqrt{a^2+c^2}\cos\left(d+ex-\tan^{-}\right.}{b+\sqrt{a^2+c^2}}}}}{\sqrt{\cos(d+ex)}\sqrt{a+b\sec(d+ex)+c\tan(d+ex}}$$

$$= \frac{2F\left(\frac{1}{2}\left(d+ex-\tan^{-1}(a,c)\right)|\frac{2\sqrt{a^2+c^2}}{b+\sqrt{a^2+c^2}}\right)\sqrt{\frac{b+a\cos(d+ex)+}{b+\sqrt{a^2-}}}}{e\sqrt{\cos(d+ex)}\sqrt{a+b\sec(d+ex)+c\tan(d+e}}$$

Mathematica [C] time = 2.90, size = 506, normalized size = 4.29

$$\frac{4\left(\sqrt{a^2-b^2+c^2}+ia-ib+c\right)(\cos(d+ex)+i\sin(d+ex))\sqrt{-\frac{i\left(\sqrt{a^2-b^2+c^2}+(a-b)\tan\left(\frac{1}{2}(d+ex)\right)-c\right)}{\left(\sqrt{a^2-b^2+c^2}-ia+ib-c\right)\left(\tan\left(\frac{1}{2}(d+ex)\right)-i\right)}}\sqrt{-\frac{i\left(\sqrt{a^2-b^2+c^2}\right.}{\left(\sqrt{a^2-b^2+c^2}+i\right.}}}{e\left(a+i\left(\sqrt{a^2-b^2+c^2}+ib+c\right)\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(Sqrt[Cos[d + e*x]]*Sqrt[a + b*Sec[d + e*x] + c*Tan[d + e*x]]),
x]
```

```
[Out] (4*(I*a - I*b + c + Sqrt[a^2 - b^2 + c^2])*EllipticF[ArcSin[Sqrt[(((-I)*a +
 I*b + c + Sqrt[a^2 - b^2 + c^2])*(-Cos[d + e*x] + I*Sin[d + e*x]))/(I*a -
I*b + c + Sqrt[a^2 - b^2 + c^2])]], (b + I*Sqrt[a^2 - b^2 + c^2])/(b - I*Sq
rt[a^2 - b^2 + c^2])]*Sqrt[(((-I)*a + I*b + c + Sqrt[a^2 - b^2 + c^2])*(-Co
s[d + e*x] + I*Sin[d + e*x]))/(I*a - I*b + c + Sqrt[a^2 - b^2 + c^2])]*(Cos
[d + e*x] + I*Sin[d + e*x])*Sqrt[((-I)*(-c + Sqrt[a^2 - b^2 + c^2] + (a - b
)*Tan[(d + e*x)/2]))/(((-I)*a + I*b - c + Sqrt[a^2 - b^2 + c^2])*(-I + Tan[
```

```
(d + e*x)/2]))]*Sqrt[((-I)*(c + Sqrt[a^2 - b^2 + c^2] + (-a + b)*Tan[(d + e
*x)/2]))/((I*a - I*b + c + Sqrt[a^2 - b^2 + c^2])*(-I + Tan[(d + e*x)/2]))]
)/((a + I*(I*b + c + Sqrt[a^2 - b^2 + c^2]))*e*Sqrt[Cos[d + e*x]]*Sqrt[a +
b*Sec[d + e*x] + c*Tan[d + e*x]])
```

fricas [F] time = 0.89, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{b\sec(ex+d)+c\tan(ex+d)+a}\,\sqrt{\cos(ex+d)}}{b\cos(ex+d)\sec(ex+d)+c\cos(ex+d)\tan(ex+d)+a\cos(ex+d)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(e*x+d)^(1/2)/(a+b*sec(e*x+d)+c*tan(e*x+d))^(1/2),x, algorit
hm="fricas")
```

```
[Out] integral(sqrt(b*sec(e*x + d) + c*tan(e*x + d) + a)*sqrt(cos(e*x + d))/(b*co
s(e*x + d)*sec(e*x + d) + c*cos(e*x + d)*tan(e*x + d) + a*cos(e*x + d)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{b\sec(ex+d)+c\tan(ex+d)+a}\,\sqrt{\cos(ex+d)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(e*x+d)^(1/2)/(a+b*sec(e*x+d)+c*tan(e*x+d))^(1/2),x, algorit
hm="giac")
```

```
[Out] integrate(1/(sqrt(b*sec(e*x + d) + c*tan(e*x + d) + a)*sqrt(cos(e*x + d))),
 x)
```

maple [C] time = 1.53, size = 714, normalized size = 6.05

$$-\frac{4i\sqrt{\frac{b+a\cos(ex+d)+c\sin(ex+d)}{\cos(ex+d)}}\;\sqrt{\frac{(i\sin(ex+d)+\cos(ex+d))\left(ia-ib-\sqrt{a^2-b^2+c^2}+c\right)}{ia-ib+\sqrt{a^2-b^2+c^2}-c}}\;\sqrt{-\frac{i\left(\cos(ex+d)\sqrt{a^2-b^2+c^2}-a\sin(ex+d)+b\sin(ex+d)+c\cos\right.}{(i\cos(ex+d)+\sin(ex+d)+i)\left(ia-ib-\sqrt{a^2-\right.}}}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/cos(e*x+d)^(1/2)/(a+b*sec(e*x+d)+c*tan(e*x+d))^(1/2),x)
```

```
[Out] -4*I/e*((b+a*cos(e*x+d)+c*sin(e*x+d))/cos(e*x+d))^(1/2)*((I*sin(e*x+d)+cos(
e*x+d))*(I*a-I*b-(a^2-b^2+c^2)^(1/2)+c)/(I*a-I*b+(a^2-b^2+c^2)^(1/2)-c))^(1
/2)*(-I*(cos(e*x+d)*(a^2-b^2+c^2)^(1/2)-a*sin(e*x+d)+b*sin(e*x+d)+c*cos(e*x
+d)+(a^2-b^2+c^2)^(1/2)+c)/(I*cos(e*x+d)+sin(e*x+d)+I)/(I*a-I*b-(a^2-b^2+c^
```

```
2)^(1/2)-c))^(1/2)*(I*(a*sin(e*x+d)-b*sin(e*x+d)+cos(e*x+d)*(a^2-b^2+c^2)^(
1/2)-c*cos(e*x+d)+(a^2-b^2+c^2)^(1/2)-c)/(I*cos(e*x+d)+sin(e*x+d)+I)/(I*a-I
*b+(a^2-b^2+c^2)^(1/2)-c))^(1/2)*(cos(e*x+d)+1)^2*EllipticF(((I*sin(e*x+d)+
cos(e*x+d))*(I*a-I*b-(a^2-b^2+c^2)^(1/2)+c)/(I*a-I*b+(a^2-b^2+c^2)^(1/2)-c)
)^(1/2),((I*a-I*b+(a^2-b^2+c^2)^(1/2)-c)*(I*a-I*b+(a^2-b^2+c^2)^(1/2)+c)/(I
*a-I*b-(a^2-b^2+c^2)^(1/2)+c)/(I*a-I*b-(a^2-b^2+c^2)^(1/2)-c))^(1/2))*cos(e
*x+d)^(1/2)*(cos(e*x+d)-1)^2*(I*(a^2-b^2+c^2)^(1/2)*sin(e*x+d)-I*a*cos(e*x+
d)+I*cos(e*x+d)*b-I*c*sin(e*x+d)-cos(e*x+d)*(a^2-b^2+c^2)^(1/2)+c*cos(e*x+d
)-a*sin(e*x+d)+b*sin(e*x+d))/sin(e*x+d)^4/(b+a*cos(e*x+d)+c*sin(e*x+d))/(-I
*a+I*b+(a^2-b^2+c^2)^(1/2)-c)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{b\sec\left(ex+d\right)+c\tan\left(ex+d\right)+a}\,\sqrt{\cos\left(ex+d\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(e*x+d)^(1/2)/(a+b*sec(e*x+d)+c*tan(e*x+d))^(1/2),x, algorit
hm="maxima")
```

```
[Out] integrate(1/(sqrt(b*sec(e*x + d) + c*tan(e*x + d) + a)*sqrt(cos(e*x + d))),
 x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sqrt{\cos\left(d+e\,x\right)}\,\sqrt{a+c\,\tan\left(d+e\,x\right)+\frac{b}{\cos\left(d+e\,x\right)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(d + e*x)^(1/2)*(a + c*tan(d + e*x) + b/cos(d + e*x))^(1/2)),x)
```

```
[Out] int(1/(cos(d + e*x)^(1/2)*(a + c*tan(d + e*x) + b/cos(d + e*x))^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a+b\sec\left(d+ex\right)+c\tan\left(d+ex\right)}\,\sqrt{\cos\left(d+ex\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(e*x+d)**(1/2)/(a+b*sec(e*x+d)+c*tan(e*x+d))**(1/2),x)
```

```
[Out] Integral(1/(sqrt(a + b*sec(d + e*x) + c*tan(d + e*x))*sqrt(cos(d + e*x))),
x)
```

3.456 $\int \frac{1}{\cos^{\frac{3}{2}}(d+ex)(a+b\sec(d+ex)+c\tan(d+ex))^{3/2}}\,dx$

Optimal. Leaf size=240

$$-\frac{2(a\cos(d+ex)+b+c\sin(d+ex))^2 E\left(\frac{1}{2}\left(d+ex-\tan^{-1}(a,c)\right)|\frac{2\sqrt{a^2+c^2}}{b+\sqrt{a^2+c^2}}\right)}{e\left(a^2-b^2+c^2\right)\cos^{\frac{3}{2}}(d+ex)\sqrt{\frac{a\cos(d+ex)+b+c\sin(d+ex)}{\sqrt{a^2+c^2}+b}}\,(a+b\sec(d+ex)+c\tan(d+ex))^{3/2}}-\frac{2(c\cos(d+ex)-a\,\mathrm{s}}{e\left(a^2-b^2+c^2\right)\cos^{\frac{3}{2}}}$$

```
[Out] -2*(c*cos(e*x+d)-a*sin(e*x+d))*(b+a*cos(e*x+d)+c*sin(e*x+d))/(a^2-b^2+c^2)/
e/cos(e*x+d)^(3/2)/(a+b*sec(e*x+d)+c*tan(e*x+d))^(3/2)-2*(cos(1/2*d+1/2*e*x
-1/2*arctan(a,c))^2)^(1/2)/cos(1/2*d+1/2*e*x-1/2*arctan(a,c))*EllipticE(sin
(1/2*d+1/2*e*x-1/2*arctan(a,c)),2^(1/2)*((a^2+c^2)^(1/2)/(b+(a^2+c^2)^(1/2)
))^(1/2))*(b+a*cos(e*x+d)+c*sin(e*x+d))^2/(a^2-b^2+c^2)/e/cos(e*x+d)^(3/2)/
((b+a*cos(e*x+d)+c*sin(e*x+d))/(b+(a^2+c^2)^(1/2)))^(1/2)/(a+b*sec(e*x+d)+c
*tan(e*x+d))^(3/2)
```

Rubi [A] time = 0.21, antiderivative size = 240, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.121, Rules used = {3163, 3128, 3119, 2653}

$$-\frac{2(a\cos(d+ex)+b+c\sin(d+ex))^2 E\left(\frac{1}{2}\left(d+ex-\tan^{-1}(a,c)\right)|\frac{2\sqrt{a^2+c^2}}{b+\sqrt{a^2+c^2}}\right)}{e\left(a^2-b^2+c^2\right)\cos^{\frac{3}{2}}(d+ex)\sqrt{\frac{a\cos(d+ex)+b+c\sin(d+ex)}{\sqrt{a^2+c^2}+b}}\,(a+b\sec(d+ex)+c\tan(d+ex))^{3/2}}-\frac{2(c\cos(d+ex)-a\,\mathrm{s}}{e\left(a^2-b^2+c^2\right)\cos^{\frac{3}{2}}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Cos[d + e*x]^(3/2)*(a + b*Sec[d + e*x] + c*Tan[d + e*x])^(3/2)),x]
```

```
[Out] (-2*(c*Cos[d + e*x] - a*Sin[d + e*x])*(b + a*Cos[d + e*x] + c*Sin[d + e*x])
)/((a^2 - b^2 + c^2)*e*Cos[d + e*x]^(3/2)*(a + b*Sec[d + e*x] + c*Tan[d + e
*x])^(3/2)) - (2*EllipticE[(d + e*x - ArcTan[a, c])/2, (2*Sqrt[a^2 + c^2])/
(b + Sqrt[a^2 + c^2])]*(b + a*Cos[d + e*x] + c*Sin[d + e*x])^2)/((a^2 - b^2
 + c^2)*e*Cos[d + e*x]^(3/2)*Sqrt[(b + a*Cos[d + e*x] + c*Sin[d + e*x])/(b
+ Sqrt[a^2 + c^2])]*(a + b*Sec[d + e*x] + c*Tan[d + e*x])^(3/2))
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 3119

```
Int[Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_
)]], x_Symbol] :> Dist[Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]]/Sqrt[(a +
b*Cos[d + e*x] + c*Sin[d + e*x])/(a + Sqrt[b^2 + c^2])], Int[Sqrt[a/(a + Sq
rt[b^2 + c^2]) + (Sqrt[b^2 + c^2]*Cos[d + e*x - ArcTan[b, c]])/(a + Sqrt[b^
2 + c^2])], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0]
&& NeQ[b^2 + c^2, 0] &&  !GtQ[a + Sqrt[b^2 + c^2], 0]
```

Rule 3128

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(-3/2), x_Symbol] :> Simp[(2*(c*Cos[d + e*x] - b*Sin[d + e*x]))/(e*(a^2 - b
^2 - c^2)*Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]]), x] + Dist[1/(a^2 - b^
2 - c^2), Int[Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]], x], x] /; FreeQ[{a
, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0]
```

Rule 3163

```
Int[cos[(d_.) + (e_.)*(x_)]^(n_)*((a_.) + (b_.)*sec[(d_.) + (e_.)*(x_)] + (
c_.)*tan[(d_.) + (e_.)*(x_)])^(n_), x_Symbol] :> Dist[(Cos[d + e*x]^n*(a +
b*Sec[d + e*x] + c*Tan[d + e*x])^n)/(b + a*Cos[d + e*x] + c*Sin[d + e*x])^n
, Int[(b + a*Cos[d + e*x] + c*Sin[d + e*x])^n, x], x] /; FreeQ[{a, b, c, d,
 e}, x] &&  !IntegerQ[n]
```

Rubi steps

$$\int \frac{1}{\cos^{\frac{3}{2}}(d+ex)(a+b\sec(d+ex)+c\tan(d+ex))^{3/2}}\,dx = \frac{(b+a\cos(d+ex)+c\sin(d+ex))^{3/2}\int \frac{1}{(b+a\cos(d+ex)+c\ldots}}{\cos^{\frac{3}{2}}(d+ex)(a+b\sec(d+ex)+c\tan(d+e\ldots}$$

$$= -\frac{2(c\cos(d+ex)-a\sin(d+ex))(b+a\cos(d+ex)+\ldots}{\left(a^2-b^2+c^2\right)e\cos^{\frac{3}{2}}(d+ex)(a+b\sec(d+ex)+c\,\mathrm{t}\ldots}$$

$$= -\frac{2(c\cos(d+ex)-a\sin(d+ex))(b+a\cos(d+ex)+\ldots}{\left(a^2-b^2+c^2\right)e\cos^{\frac{3}{2}}(d+ex)(a+b\sec(d+ex)+c\,\mathrm{t}\ldots}$$

$$= -\frac{2(c\cos(d+ex)-a\sin(d+ex))(b+a\cos(d+ex)+\ldots}{\left(a^2-b^2+c^2\right)e\cos^{\frac{3}{2}}(d+ex)(a+b\sec(d+ex)+c\,\mathrm{t}\ldots}$$

Mathematica [F] time = 23.99, size = 0, normalized size = 0.00

$$\int \frac{1}{\cos^{\frac{3}{2}}(d+ex)(a+b\sec(d+ex)+c\tan(d+ex))^{3/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/(Cos[d + e*x]^(3/2)*(a + b*Sec[d + e*x] + c*Tan[d + e*x])^(3/2)
),x]
```

```
[Out] Integrate[1/(Cos[d + e*x]^(3/2)*(a + b*Sec[d + e*x] + c*Tan[d + e*x])^(3/2)
), x]
```

fricas [F] time = 0.90, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{b\sec(ex+d)+c\tan(ex+d)+a}\,\sqrt{\text{co}}}{b^2\cos(ex+d)^2\sec(ex+d)^2+c^2\cos(ex+d)^2\tan(ex+d)^2+2\,ab\cos(ex+d)^2\sec(ex+d)+a^2\cos(\text{e}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(e*x+d)^(3/2)/(a+b*sec(e*x+d)+c*tan(e*x+d))^(3/2),x, algorit
hm="fricas")
```

```
[Out] integral(sqrt(b*sec(e*x + d) + c*tan(e*x + d) + a)*sqrt(cos(e*x + d))/(b^2*
cos(e*x + d)^2*sec(e*x + d)^2 + c^2*cos(e*x + d)^2*tan(e*x + d)^2 + 2*a*b*c
os(e*x + d)^2*sec(e*x + d) + a^2*cos(e*x + d)^2 + 2*(b*c*cos(e*x + d)^2*sec
(e*x + d) + a*c*cos(e*x + d)^2)*tan(e*x + d)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(b\sec(ex+d)+c\tan(ex+d)+a)^{\frac{3}{2}}\cos(ex+d)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(e*x+d)^(3/2)/(a+b*sec(e*x+d)+c*tan(e*x+d))^(3/2),x, algorit
hm="giac")
```

```
[Out] integrate(1/((b*sec(e*x + d) + c*tan(e*x + d) + a)^(3/2)*cos(e*x + d)^(3/2)
), x)
```

maple [C] time = 0.95, size = 12564, normalized size = 52.35

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/cos(e*x+d)^(3/2)/(a+b*sec(e*x+d)+c*tan(e*x+d))^(3/2),x)
```

```
[Out] result too large to display
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(b\sec(ex+d)+c\tan(ex+d)+a)^{\frac{3}{2}}\cos(ex+d)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(e*x+d)^(3/2)/(a+b*sec(e*x+d)+c*tan(e*x+d))^(3/2),x, algorit
hm="maxima")
```

```
[Out] integrate(1/((b*sec(e*x + d) + c*tan(e*x + d) + a)^(3/2)*cos(e*x + d)^(3/2)
), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\cos(d+e\,x)^{3/2}\left(a+c\tan(d+e\,x)+\frac{b}{\cos(d+e\,x)}\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(d + e*x)^(3/2)*(a + c*tan(d + e*x) + b/cos(d + e*x))^(3/2)),x)
```

```
[Out] int(1/(cos(d + e*x)^(3/2)*(a + c*tan(d + e*x) + b/cos(d + e*x))^(3/2)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(e*x+d)**(3/2)/(a+b*sec(e*x+d)+c*tan(e*x+d))**(3/2),x)
```

```
[Out] Timed out
```

3.457 $\int \frac{1}{\cos^{\frac{5}{2}}(d+ex)(a+b\sec(d+ex)+c\tan(d+ex))^{5/2}}\,dx$

Optimal. Leaf size=492

$$\frac{2\sqrt{\frac{a\cos(d+ex)+b+c\sin(d+ex)}{\sqrt{a^2+c^2}+b}}\,(a\cos(d+ex)+b+c\sin(d+ex))^2F\left(\frac{1}{2}\left(d+ex-\tan^{-1}(a,c)\right)|\frac{2\sqrt{a^2+c^2}}{b+\sqrt{a^2+c^2}}\right)}{3e\left(a^2-b^2+c^2\right)\cos^{\frac{5}{2}}(d+ex)(a+b\sec(d+ex)+c\tan(d+ex))^{5/2}}+\frac{8b(a}{3e\left(a^2-b^2+\right.}$$

```
[Out] -2/3*(c*cos(e*x+d)-a*sin(e*x+d))*(b+a*cos(e*x+d)+c*sin(e*x+d))/(a^2-b^2+c^2
)/e/cos(e*x+d)^(5/2)/(a+b*sec(e*x+d)+c*tan(e*x+d))^(5/2)+8/3*(b*c*cos(e*x+d
)-a*b*sin(e*x+d))*(b+a*cos(e*x+d)+c*sin(e*x+d))^2/(a^2-b^2+c^2)^2/e/cos(e*x
+d)^(5/2)/(a+b*sec(e*x+d)+c*tan(e*x+d))^(5/2)+8/3*b*(cos(1/2*d+1/2*e*x-1/2*
arctan(a,c))^2)^(1/2)/cos(1/2*d+1/2*e*x-1/2*arctan(a,c))*EllipticE(sin(1/2*
d+1/2*e*x-1/2*arctan(a,c)),2^(1/2)*((a^2+c^2)^(1/2)/(b+(a^2+c^2)^(1/2)))^(1
/2))*(b+a*cos(e*x+d)+c*sin(e*x+d))^3/(a^2-b^2+c^2)^2/e/cos(e*x+d)^(5/2)/((b
+a*cos(e*x+d)+c*sin(e*x+d))/(b+(a^2+c^2)^(1/2)))^(1/2)/(a+b*sec(e*x+d)+c*ta
n(e*x+d))^(5/2)+2/3*(cos(1/2*d+1/2*e*x-1/2*arctan(a,c))^2)^(1/2)/cos(1/2*d+
1/2*e*x-1/2*arctan(a,c))*EllipticF(sin(1/2*d+1/2*e*x-1/2*arctan(a,c)),2^(1/
2)*((a^2+c^2)^(1/2)/(b+(a^2+c^2)^(1/2)))^(1/2))*(b+a*cos(e*x+d)+c*sin(e*x+d
))^2*((b+a*cos(e*x+d)+c*sin(e*x+d))/(b+(a^2+c^2)^(1/2)))^(1/2)/(a^2-b^2+c^2
)/e/cos(e*x+d)^(5/2)/(a+b*sec(e*x+d)+c*tan(e*x+d))^(5/2)
```

Rubi [A] time = 0.49, antiderivative size = 492, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.242, Rules used = {3163, 3129, 3156, 3149, 3119, 2653, 3127, 2661}

$$\frac{2\sqrt{\frac{a\cos(d+ex)+b+c\sin(d+ex)}{\sqrt{a^2+c^2}+b}}\,(a\cos(d+ex)+b+c\sin(d+ex))^2F\left(\frac{1}{2}\left(d+ex-\tan^{-1}(a,c)\right)|\frac{2\sqrt{a^2+c^2}}{b+\sqrt{a^2+c^2}}\right)}{3e\left(a^2-b^2+c^2\right)\cos^{\frac{5}{2}}(d+ex)(a+b\sec(d+ex)+c\tan(d+ex))^{5/2}}+\frac{8b(a}{3e\left(a^2-b^2+\right.}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Cos[d + e*x]^(5/2)*(a + b*Sec[d + e*x] + c*Tan[d + e*x])^(5/2)),x]
```

```
[Out] (-2*(c*Cos[d + e*x] - a*Sin[d + e*x])*(b + a*Cos[d + e*x] + c*Sin[d + e*x])
)/(3*(a^2 - b^2 + c^2)*e*Cos[d + e*x]^(5/2)*(a + b*Sec[d + e*x] + c*Tan[d +
 e*x])^(5/2)) + (8*(b*c*Cos[d + e*x] - a*b*Sin[d + e*x])*(b + a*Cos[d + e*x
] + c*Sin[d + e*x])^2)/(3*(a^2 - b^2 + c^2)^2*e*Cos[d + e*x]^(5/2)*(a + b*S
ec[d + e*x] + c*Tan[d + e*x])^(5/2)) + (8*b*EllipticE[(d + e*x - ArcTan[a,
c])/2, (2*Sqrt[a^2 + c^2])/(b + Sqrt[a^2 + c^2])]*(b + a*Cos[d + e*x] + c*S
in[d + e*x])^3)/(3*(a^2 - b^2 + c^2)^2*e*Cos[d + e*x]^(5/2)*Sqrt[(b + a*Cos
```

```
[d + e*x] + c*Sin[d + e*x])/(b + Sqrt[a^2 + c^2])]*(a + b*Sec[d + e*x] + c*
Tan[d + e*x])^(5/2)) + (2*EllipticF[(d + e*x - ArcTan[a, c])/2, (2*Sqrt[a^2
 + c^2])/(b + Sqrt[a^2 + c^2])]*(b + a*Cos[d + e*x] + c*Sin[d + e*x])^2*Sqr
t[(b + a*Cos[d + e*x] + c*Sin[d + e*x])/(b + Sqrt[a^2 + c^2])])/(3*(a^2 - b
^2 + c^2)*e*Cos[d + e*x]^(5/2)*(a + b*Sec[d + e*x] + c*Tan[d + e*x])^(5/2))
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 3119

```
Int[Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_
)]], x_Symbol] :> Dist[Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]]/Sqrt[(a +
b*Cos[d + e*x] + c*Sin[d + e*x])/(a + Sqrt[b^2 + c^2])], Int[Sqrt[a/(a + Sq
rt[b^2 + c^2]) + (Sqrt[b^2 + c^2]*Cos[d + e*x - ArcTan[b, c]])/(a + Sqrt[b^
2 + c^2])], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0]
&& NeQ[b^2 + c^2, 0] &&  !GtQ[a + Sqrt[b^2 + c^2], 0]
```

Rule 3127

```
Int[1/Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(
x_)]], x_Symbol] :> Dist[Sqrt[(a + b*Cos[d + e*x] + c*Sin[d + e*x])/(a + Sq
rt[b^2 + c^2])]/Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]], Int[1/Sqrt[a/(a
+ Sqrt[b^2 + c^2]) + (Sqrt[b^2 + c^2]*Cos[d + e*x - ArcTan[b, c]])/(a + Sqr
t[b^2 + c^2])], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2,
 0] && NeQ[b^2 + c^2, 0] &&  !GtQ[a + Sqrt[b^2 + c^2], 0]
```

Rule 3129

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(n_), x_Symbol] :> Simp[((-(c*Cos[d + e*x]) + b*Sin[d + e*x])*(a + b*Cos[d
+ e*x] + c*Sin[d + e*x])^(n + 1))/(e*(n + 1)*(a^2 - b^2 - c^2)), x] + Dist[
1/((n + 1)*(a^2 - b^2 - c^2)), Int[(a*(n + 1) - b*(n + 2)*Cos[d + e*x] - c*
(n + 2)*Sin[d + e*x])*(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n + 1), x], x]
 /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0] && LtQ[n, -1] && N
eQ[n, -3/2]
```

Rule 3149

```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])
/Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)]]
, x_Symbol] :> Dist[B/b, Int[Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]], x],
 x] + Dist[(A*b - a*B)/b, Int[1/Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]],
x], x] /; FreeQ[{a, b, c, d, e, A, B, C}, x] && EqQ[B*c - b*C, 0] && NeQ[A*
b - a*B, 0]
```

Rule 3156

```
Int[((a_.) + cos[(d_.) + (e_.)*(x_)]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)])
^(n_)*((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)
]), x_Symbol] :> -Simp[((c*B - b*C - (a*C - c*A)*Cos[d + e*x] + (a*B - b*A)
*Sin[d + e*x])*(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n + 1))/(e*(n + 1)*(a
^2 - b^2 - c^2)), x] + Dist[1/((n + 1)*(a^2 - b^2 - c^2)), Int[(a + b*Cos[d
 + e*x] + c*Sin[d + e*x])^(n + 1)*Simp[(n + 1)*(a*A - b*B - c*C) + (n + 2)*
(a*B - b*A)*Cos[d + e*x] + (n + 2)*(a*C - c*A)*Sin[d + e*x], x], x], x] /;
FreeQ[{a, b, c, d, e, A, B, C}, x] && LtQ[n, -1] && NeQ[a^2 - b^2 - c^2, 0]
 && NeQ[n, -2]
```

Rule 3163

```
Int[cos[(d_.) + (e_.)*(x_)]^(n_)*((a_.) + (b_.)*sec[(d_.) + (e_.)*(x_)] + (
c_.)*tan[(d_.) + (e_.)*(x_)])^(n_), x_Symbol] :> Dist[(Cos[d + e*x]^n*(a +
b*Sec[d + e*x] + c*Tan[d + e*x])^n)/(b + a*Cos[d + e*x] + c*Sin[d + e*x])^n
, Int[(b + a*Cos[d + e*x] + c*Sin[d + e*x])^n, x], x] /; FreeQ[{a, b, c, d,
 e}, x] &&  !IntegerQ[n]
```

Rubi steps

$$\int \frac{1}{\cos^{\frac{5}{2}}(d+ex)(a+b\sec(d+ex)+c\tan(d+ex))^{5/2}}\,dx = \frac{(b+a\cos(d+ex)+c\sin(d+ex))^{5/2}\int \frac{1}{(b+a\cos(d+ex)+c}}{\cos^{\frac{5}{2}}(d+ex)(a+b\sec(d+ex)+c\tan(d+e}$$

$$= -\frac{2(c\cos(d+ex)-a\sin(d+ex))(b+a\cos(d+ex)}{3\left(a^2-b^2+c^2\right)e\cos^{\frac{5}{2}}(d+ex)(a+b\sec(d+ex)+}$$

$$= -\frac{2(c\cos(d+ex)-a\sin(d+ex))(b+a\cos(d+ex)}{3\left(a^2-b^2+c^2\right)e\cos^{\frac{5}{2}}(d+ex)(a+b\sec(d+ex)+}$$

$$= -\frac{2(c\cos(d+ex)-a\sin(d+ex))(b+a\cos(d+ex)}{3\left(a^2-b^2+c^2\right)e\cos^{\frac{5}{2}}(d+ex)(a+b\sec(d+ex)+}$$

$$= -\frac{2(c\cos(d+ex)-a\sin(d+ex))(b+a\cos(d+ex)}{3\left(a^2-b^2+c^2\right)e\cos^{\frac{5}{2}}(d+ex)(a+b\sec(d+ex)+}$$

$$= -\frac{2(c\cos(d+ex)-a\sin(d+ex))(b+a\cos(d+ex)}{3\left(a^2-b^2+c^2\right)e\cos^{\frac{5}{2}}(d+ex)(a+b\sec(d+ex)+}$$

Mathematica [F] time = 27.74, size = 0, normalized size = 0.00

$$\int \frac{1}{\cos^{\frac{5}{2}}(d+ex)(a+b\sec(d+ex)+c\tan(d+ex))^{5/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/(Cos[d + e*x]^(5/2)*(a + b*Sec[d + e*x] + c*Tan[d + e*x])^(5/2)
),x]
```

```
[Out] Integrate[1/(Cos[d + e*x]^(5/2)*(a + b*Sec[d + e*x] + c*Tan[d + e*x])^(5/2)
), x]
```

fricas [F] time = 1.57, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{}{b^3\cos(ex+d)^3\sec(ex+d)^3+c^3\cos(ex+d)^3\tan(ex+d)^3+3ab^2\cos(ex+d)^3\sec(ex+d)^2+3a^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(e*x+d)^(5/2)/(a+b*sec(e*x+d)+c*tan(e*x+d))^(5/2),x, algorit
hm="fricas")
```

```
[Out] integral(sqrt(b*sec(e*x + d) + c*tan(e*x + d) + a)*sqrt(cos(e*x + d))/(b^3*
cos(e*x + d)^3*sec(e*x + d)^3 + c^3*cos(e*x + d)^3*tan(e*x + d)^3 + 3*a*b^2
*cos(e*x + d)^3*sec(e*x + d)^2 + 3*a^2*b*cos(e*x + d)^3*sec(e*x + d) + a^3*
cos(e*x + d)^3 + 3*(b*c^2*cos(e*x + d)^3*sec(e*x + d) + a*c^2*cos(e*x + d)^
3)*tan(e*x + d)^2 + 3*(b^2*c*cos(e*x + d)^3*sec(e*x + d)^2 + 2*a*b*c*cos(e*
x + d)^3*sec(e*x + d) + a^2*c*cos(e*x + d)^3)*tan(e*x + d)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(b\sec(ex+d)+c\tan(ex+d)+a)^{\frac{5}{2}}\cos(ex+d)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(e*x+d)^(5/2)/(a+b*sec(e*x+d)+c*tan(e*x+d))^(5/2),x, algorit
hm="giac")
```

```
[Out] integrate(1/((b*sec(e*x + d) + c*tan(e*x + d) + a)^(5/2)*cos(e*x + d)^(5/2)
), x)
```

maple [C] time = 1.74, size = 64683, normalized size = 131.47

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/cos(e*x+d)^(5/2)/(a+b*sec(e*x+d)+c*tan(e*x+d))^(5/2),x)
```

```
[Out] result too large to display
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(b\sec(ex+d)+c\tan(ex+d)+a)^{\frac{5}{2}}\cos(ex+d)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(e*x+d)^(5/2)/(a+b*sec(e*x+d)+c*tan(e*x+d))^(5/2),x, algorit
hm="maxima")
```

```
[Out] integrate(1/((b*sec(e*x + d) + c*tan(e*x + d) + a)^(5/2)*cos(e*x + d)^(5/2)
), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\cos (d+e\,x)^{5/2}\,\left(a+c\tan (d+e\,x)+\frac{b}{\cos(d+e\,x)}\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(d + e*x)^(5/2)*(a + c*tan(d + e*x) + b/cos(d + e*x))^(5/2)),x)
[Out] int(1/(cos(d + e*x)^(5/2)*(a + c*tan(d + e*x) + b/cos(d + e*x))^(5/2)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(e*x+d)**(5/2)/(a+b*sec(e*x+d)+c*tan(e*x+d))**(5/2),x)
[Out] Timed out
```

3.458 $\int \frac{1}{a+b\cot(x)+c\csc(x)}\,dx$

Optimal. Leaf size=98

$$\frac{2ac\tanh^{-1}\left(\frac{a-(b-c)\tan\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2-c^2}}\right)}{\left(a^2+b^2\right)\sqrt{a^2+b^2-c^2}}-\frac{b\log(a\sin(x)+b\cos(x)+c)}{a^2+b^2}+\frac{ax}{a^2+b^2}$$

```
[Out] a*x/(a^2+b^2)-b*ln(c+b*cos(x)+a*sin(x))/(a^2+b^2)+2*a*c*arctanh((a-(b-c)*ta
n(1/2*x))/(a^2+b^2-c^2)^(1/2))/(a^2+b^2)/(a^2+b^2-c^2)^(1/2)
```

Rubi [A] time = 0.10, antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {3160, 3137, 3124, 618, 206}

$$\frac{2ac\tanh^{-1}\left(\frac{a-(b-c)\tan\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2-c^2}}\right)}{\left(a^2+b^2\right)\sqrt{a^2+b^2-c^2}}-\frac{b\log(a\sin(x)+b\cos(x)+c)}{a^2+b^2}+\frac{ax}{a^2+b^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cot[x] + c*Csc[x])^(-1),x]
```

```
[Out] (a*x)/(a^2 + b^2) + (2*a*c*ArcTanh[(a - (b - c)*Tan[x/2])/Sqrt[a^2 + b^2 -
c^2]])/((a^2 + b^2)*Sqrt[a^2 + b^2 - c^2]) - (b*Log[c + b*Cos[x] + a*Sin[x]
])/(a^2 + b^2)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 3124

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(-1), x_Symbol] :> Module[{f = FreeFactors[Tan[(d + e*x)/2], x]}, Dist[(2*f
)/e, Subst[Int[1/(a + b + 2*c*f*x + (a - b)*f^2*x^2), x], x, Tan[(d + e*x)/
```

```
2]/f], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0]
```

Rule 3137

```
Int[((A_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])/((a_.) + cos[(d_.) + (e_.)*(x_)
]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)]), x_Symbol] :> Simp[(c*C*(d + e*x))
/(e*(b^2 + c^2)), x] + (Dist[(A*(b^2 + c^2) - a*c*C)/(b^2 + c^2), Int[1/(a
+ b*Cos[d + e*x] + c*Sin[d + e*x]), x], x] - Simp[(b*C*Log[a + b*Cos[d + e*
x] + c*Sin[d + e*x]])/(e*(b^2 + c^2)), x]) /; FreeQ[{a, b, c, d, e, A, C},
x] && NeQ[b^2 + c^2, 0] && NeQ[A*(b^2 + c^2) - a*c*C, 0]
```

Rule 3160

```
Int[((a_.) + csc[(d_.) + (e_.)*(x_)]*(b_.) + cot[(d_.) + (e_.)*(x_)]*(c_.))
^(-1), x_Symbol] :> Int[Sin[d + e*x]/(b + a*Sin[d + e*x] + c*Cos[d + e*x]),
 x] /; FreeQ[{a, b, c, d, e}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{a+b\cot(x)+c\csc(x)}\,dx &= \int \frac{\sin(x)}{c+b\cos(x)+a\sin(x)}\,dx\\ &= \frac{ax}{a^2+b^2}-\frac{b\log(c+b\cos(x)+a\sin(x))}{a^2+b^2}-\frac{(ac)\int \frac{1}{c+b\cos(x)+a\sin(x)}\,dx}{a^2+b^2}\\ &= \frac{ax}{a^2+b^2}-\frac{b\log(c+b\cos(x)+a\sin(x))}{a^2+b^2}-\frac{(2ac)\operatorname{Subst}\left(\int \frac{1}{b+c+2ax+(-b+c)x^2}\,dx,x,t\right.}{a^2+b^2}\\ &= \frac{ax}{a^2+b^2}-\frac{b\log(c+b\cos(x)+a\sin(x))}{a^2+b^2}+\frac{(4ac)\operatorname{Subst}\left(\int \frac{1}{4\left(a^2+b^2-c^2\right)-x^2}\,dx,x,2a\right.}{a^2+b^2}\\ &= \frac{ax}{a^2+b^2}+\frac{2ac\tanh^{-1}\left(\frac{a-(b-c)\tan\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2-c^2}}\right)}{\left(a^2+b^2\right)\sqrt{a^2+b^2-c^2}}-\frac{b\log(c+b\cos(x)+a\sin(x))}{a^2+b^2}\end{aligned}$$

Mathematica [A] time = 0.22, size = 80, normalized size = 0.82

$$\frac{\frac{2ac\tanh^{-1}\left(\frac{a+(c-b)\tan\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2-c^2}}\right)}{\sqrt{a^2+b^2-c^2}}-b\log(a\sin(x)+b\cos(x)+c)+ax}{a^2+b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cot[x] + c*Csc[x])^(-1),x]
```

```
[Out] (a*x + (2*a*c*ArcTanh[(a + (-b + c)*Tan[x/2])/Sqrt[a^2 + b^2 - c^2]])/Sqrt[
a^2 + b^2 - c^2] - b*Log[c + b*Cos[x] + a*Sin[x]])/(a^2 + b^2)
```

fricas [B] time = 2.05, size = 555, normalized size = 5.66

$$\left[\frac{\sqrt{a^2+b^2-c^2}\,ac\log\left(\frac{a^4+3a^2b^2+2b^4+(a^2-b^2)c^2+2(a^2b+b^3)c\cos(x)+(a^4-b^4-2(a^2-b^2)c^2)\cos(x)^2+2\left((a^3+ab^2)c-(a^3b+ab^3-2abc^2)\cos(x)\right)\sin \ldots}{2bc\cos(x)-(a^2-b^2)\cos(x)^2+a^2+c^2+2(ab\cos \ldots}\right.}{\ldots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cot(x)+c*csc(x)),x, algorithm="fricas")
```

```
[Out] [1/2*(sqrt(a^2 + b^2 - c^2)*a*c*log((a^4 + 3*a^2*b^2 + 2*b^4 + (a^2 - b^2)*
c^2 + 2*(a^2*b + b^3)*c*cos(x) + (a^4 - b^4 - 2*(a^2 - b^2)*c^2)*cos(x)^2 +
 2*((a^3 + a*b^2)*c - (a^3*b + a*b^3 - 2*a*b*c^2)*cos(x))*sin(x) + 2*(2*a*b
*c*cos(x)^2 - a*b*c + (a^3 + a*b^2)*cos(x) - (a^2*b + b^3 - (a^2 - b^2)*c*c
os(x))*sin(x))*sqrt(a^2 + b^2 - c^2))/(2*b*c*cos(x) - (a^2 - b^2)*cos(x)^2
+ a^2 + c^2 + 2*(a*b*cos(x) + a*c)*sin(x))) + 2*(a^3 + a*b^2 - a*c^2)*x - (
a^2*b + b^3 - b*c^2)*log(2*b*c*cos(x) - (a^2 - b^2)*cos(x)^2 + a^2 + c^2 +
2*(a*b*cos(x) + a*c)*sin(x)))/(a^4 + 2*a^2*b^2 + b^4 - (a^2 + b^2)*c^2), -1
/2*(2*sqrt(-a^2 - b^2 + c^2)*a*c*arctan((b*c*cos(x) + a*c*sin(x) + a^2 + b^
2)*sqrt(-a^2 - b^2 + c^2)/((a^3 + a*b^2 - a*c^2)*cos(x) - (a^2*b + b^3 - b*
c^2)*sin(x))) - 2*(a^3 + a*b^2 - a*c^2)*x + (a^2*b + b^3 - b*c^2)*log(2*b*c
*cos(x) - (a^2 - b^2)*cos(x)^2 + a^2 + c^2 + 2*(a*b*cos(x) + a*c)*sin(x)))/
(a^4 + 2*a^2*b^2 + b^4 - (a^2 + b^2)*c^2)]
```

giac [A] time = 0.14, size = 158, normalized size = 1.61

$$-\frac{2\left(\pi\left\lfloor\frac{x}{2\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(-2b+2c)+\arctan\left(-\frac{b\tan\left(\frac{1}{2}x\right)-c\tan\left(\frac{1}{2}x\right)-a}{\sqrt{-a^2-b^2+c^2}}\right)\right)ac}{(a^2+b^2)\sqrt{-a^2-b^2+c^2}}+\frac{ax}{a^2+b^2}-\frac{b\log\left(-b\tan\left(\frac{1}{2}x\right)^2+c\tan\left(\frac{1}{2}x\right)^2 \ldots\right.}{a^2+b^2} \ldots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cot(x)+c*csc(x)),x, algorithm="giac")
```

```
[Out] -2*(pi*floor(1/2*x/pi + 1/2)*sgn(-2*b + 2*c) + arctan(-(b*tan(1/2*x) - c*ta
n(1/2*x) - a)/sqrt(-a^2 - b^2 + c^2)))*a*c/((a^2 + b^2)*sqrt(-a^2 - b^2 + c
^2)) + a*x/(a^2 + b^2) - b*log(-b*tan(1/2*x)^2 + c*tan(1/2*x)^2 + 2*a*tan(1
/2*x) + b + c)/(a^2 + b^2) + b*log(tan(1/2*x)^2 + 1)/(a^2 + b^2)
```

maple [B] time = 0.13, size = 446, normalized size = 4.55

$$-\frac{2\ln\left(b\left(\tan^2\left(\frac{x}{2}\right)\right)-c\left(\tan^2\left(\frac{x}{2}\right)\right)-2a\tan\left(\frac{x}{2}\right)-b-c\right)b^2}{\left(2a^2+2b^2\right)(b-c)}+\frac{2\ln\left(b\left(\tan^2\left(\frac{x}{2}\right)\right)-c\left(\tan^2\left(\frac{x}{2}\right)\right)-2a\tan\left(\frac{x}{2}\right)-b-c\right.}{\left(2a^2+2b^2\right)(b-c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*cot(x)+c*csc(x)),x)
```

```
[Out] -2/(2*a^2+2*b^2)/(b-c)*ln(b*tan(1/2*x)^2-c*tan(1/2*x)^2-2*a*tan(1/2*x)-b-c)
*b^2+2/(2*a^2+2*b^2)/(b-c)*ln(b*tan(1/2*x)^2-c*tan(1/2*x)^2-2*a*tan(1/2*x)-
b-c)*c*b+4/(2*a^2+2*b^2)/(-a^2-b^2+c^2)^(1/2)*arctan(1/2*(2*(b-c)*tan(1/2*x
)-2*a)/(-a^2-b^2+c^2)^(1/2))*a*b+4/(2*a^2+2*b^2)/(-a^2-b^2+c^2)^(1/2)*arcta
n(1/2*(2*(b-c)*tan(1/2*x)-2*a)/(-a^2-b^2+c^2)^(1/2))*a*c-4/(2*a^2+2*b^2)/(-
a^2-b^2+c^2)^(1/2)*arctan(1/2*(2*(b-c)*tan(1/2*x)-2*a)/(-a^2-b^2+c^2)^(1/2)
)*a/(b-c)*b^2+4/(2*a^2+2*b^2)/(-a^2-b^2+c^2)^(1/2)*arctan(1/2*(2*(b-c)*tan(
1/2*x)-2*a)/(-a^2-b^2+c^2)^(1/2))*a/(b-c)*c*b+2/(2*a^2+2*b^2)*b*ln(1+tan(1/
2*x)^2)+4/(2*a^2+2*b^2)*a*arctan(tan(1/2*x))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cot(x)+c*csc(x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(c^2-b^2-a^2>0)', see `assume?` for
more details)Is c^2-b^2-a^2 positive or negative?
```

mupad [B] time = 13.76, size = 965, normalized size = 9.85

$$\frac{\ln\left(\tan\left(\frac{x}{2}\right)-\mathrm{i}\right)}{b+a\,1\mathrm{i}}-\frac{\ln\left(-64\tan\left(\frac{x}{2}\right)(b-c)^2-\frac{\left(a^2\,b-b\,c^2+b^3+a\,c\,\sqrt{a^2+b^2-c^2}\right)\left(32\,a\,b^2+32\,a\,c^2-64\,a\,b\,c-64\tan\left(\frac{x}{2}\right)(b-c)\left(a^2-c^2+b\,c\right)+\cdots\right.}{\cdots}\right)}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + c/sin(x) + b*cot(x)),x)
```

```
[Out] log(tan(x/2) - 1i)/(a*1i + b) + (log(tan(x/2) + 1i)*1i)/(a + b*1i) - (log(-
 64*tan(x/2)*(b - c)^2 - ((a^2*b - b*c^2 + b^3 + a*c*(a^2 + b^2 - c^2)^(1/2
))*(32*a*b^2 + 32*a*c^2 - 64*a*b*c - 64*tan(x/2)*(b - c)*(b*c + a^2 - c^2)
+ ((a^2*b - b*c^2 + b^3 + a*c*(a^2 + b^2 - c^2)^(1/2))*(64*a*b^3 - 32*a^3*b
 + 32*a^3*c + 32*tan(x/2)*(b - c)*(a^2*b - 2*a^2*c + 2*b^2*c - 2*b^3) + 64*
a*b*c^2 - 128*a*b^2*c - (32*(b - c)*(a^2*b - b*c^2 + b^3 + a*c*(a^2 + b^2 -
 c^2)^(1/2))*(3*a^4*tan(x/2) + 3*a*b^3 + 3*a^3*b + a^3*c + 3*a^2*b^2*tan(x/
2) - 2*a^2*c^2*tan(x/2) + 2*b^2*c^2*tan(x/2) - 2*b^3*c*tan(x/2) - 4*a*b*c^2
 + a*b^2*c - 2*a^2*b*c*tan(x/2)))/((a^2 + b^2)*(a^2 + b^2 - c^2))))/((a^2 +
 b^2)*(a^2 + b^2 - c^2))))/((a^2 + b^2)*(a^2 + b^2 - c^2)))*(b*(a^2 - c^2)
+ b^3 + a*c*(a^2 + b^2 - c^2)^(1/2)))/(c^2*(a^2 + b^2 - c^2) + (a^2 + b^2 -
 c^2)^2) - (log(- 64*tan(x/2)*(b - c)^2 - ((a^2*b - b*c^2 + b^3 - a*c*(a^2
+ b^2 - c^2)^(1/2))*(32*a*b^2 + 32*a*c^2 - 64*a*b*c - 64*tan(x/2)*(b - c)*(
b*c + a^2 - c^2) + ((a^2*b - b*c^2 + b^3 - a*c*(a^2 + b^2 - c^2)^(1/2))*(64
*a*b^3 - 32*a^3*b + 32*a^3*c + 32*tan(x/2)*(b - c)*(a^2*b - 2*a^2*c + 2*b^2
*c - 2*b^3) + 64*a*b*c^2 - 128*a*b^2*c - (32*(b - c)*(a^2*b - b*c^2 + b^3 -
 a*c*(a^2 + b^2 - c^2)^(1/2))*(3*a^4*tan(x/2) + 3*a*b^3 + 3*a^3*b + a^3*c +
 3*a^2*b^2*tan(x/2) - 2*a^2*c^2*tan(x/2) + 2*b^2*c^2*tan(x/2) - 2*b^3*c*tan
(x/2) - 4*a*b*c^2 + a*b^2*c - 2*a^2*b*c*tan(x/2)))/((a^2 + b^2)*(a^2 + b^2
- c^2))))/((a^2 + b^2)*(a^2 + b^2 - c^2))))/((a^2 + b^2)*(a^2 + b^2 - c^2))
)*(b*(a^2 - c^2) + b^3 - a*c*(a^2 + b^2 - c^2)^(1/2)))/(c^2*(a^2 + b^2 - c^
2) + (a^2 + b^2 - c^2)^2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{a + b\cot(x) + c\csc(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cot(x)+c*csc(x)),x)
```

```
[Out] Integral(1/(a + b*cot(x) + c*csc(x)), x)
```

3.459 $\int \frac{\csc(x)}{a+b\cot(x)+c\csc(x)}\,dx$

Optimal. Leaf size=51

$$-\frac{2\tanh^{-1}\left(\frac{a-(b-c)\tan\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2-c^2}}\right)}{\sqrt{a^2+b^2-c^2}}$$

```
[Out] -2*arctanh((a-(b-c)*tan(1/2*x))/(a^2+b^2-c^2)^(1/2))/(a^2+b^2-c^2)^(1/2)
```

Rubi [A] time = 0.07, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {3166, 3124, 618, 206}

$$-\frac{2\tanh^{-1}\left(\frac{a-(b-c)\tan\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2-c^2}}\right)}{\sqrt{a^2+b^2-c^2}}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[x]/(a + b*Cot[x] + c*Csc[x]),x]
```

```
[Out] (-2*ArcTanh[(a - (b - c)*Tan[x/2])/Sqrt[a^2 + b^2 - c^2]])/Sqrt[a^2 + b^2 -
 c^2]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 3124

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(-1), x_Symbol] :> Module[{f = FreeFactors[Tan[(d + e*x)/2], x]}, Dist[(2*f
)/e, Subst[Int[1/(a + b + 2*c*f*x + (a - b)*f^2*x^2), x], x, Tan[(d + e*x)/
2]/f], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0]
```

Rule 3166

```
Int[csc[(d_.) + (e_.)*(x_)]^(n_.)*((a_.) + csc[(d_.) + (e_.)*(x_)]*(b_.) +
cot[(d_.) + (e_.)*(x_)]*(c_.))^(m_), x_Symbol] :> Int[1/(b + a*Sin[d + e*x]
 + c*Cos[d + e*x])^n, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[m + n, 0] && I
ntegerQ[n]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\csc(x)}{a+b\cot(x)+c\csc(x)}\,dx &= \int \frac{1}{c+b\cos(x)+a\sin(x)}\,dx \\
&= 2\,\text{Subst}\left(\int \frac{1}{b+c+2ax+(-b+c)x^2}\,dx, x, \tan\left(\frac{x}{2}\right)\right) \\
&= -\left(4\,\text{Subst}\left(\int \frac{1}{4\left(a^2+b^2-c^2\right)-x^2}\,dx, x, 2a+2(-b+c)\tan\left(\frac{x}{2}\right)\right)\right) \\
&= -\frac{2\tanh^{-1}\left(\frac{a-(b-c)\tan\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2-c^2}}\right)}{\sqrt{a^2+b^2-c^2}}
\end{aligned}
$$

Mathematica [A] time = 0.04, size = 50, normalized size = 0.98

$$
-\frac{2\tanh^{-1}\left(\frac{a+(c-b)\tan\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2-c^2}}\right)}{\sqrt{a^2+b^2-c^2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[x]/(a + b*Cot[x] + c*Csc[x]),x]
```

```
[Out] (-2*ArcTanh[(a + (-b + c)*Tan[x/2])/Sqrt[a^2 + b^2 - c^2]])/Sqrt[a^2 + b^2
- c^2]
```

fricas [B] time = 0.81, size = 349, normalized size = 6.84

$$
\Bigg[\frac{\log\Bigg(-\frac{a^4+3\,a^2b^2+2\,b^4+\left(a^2-b^2\right)c^2+2\left(a^2b+b^3\right)c\cos(x)+\left(a^4-b^4-2\left(a^2-b^2\right)c^2\right)\cos(x)^2+2\left(\left(a^3+ab^2\right)c-\left(a^3b+ab^3-2\,abc^2\right)\cos(x)\right)\sin(x)-2\,\Big(2\,abc\cos(x}{2\,bc\cos(x)-\left(a^2-b^2\right)\cos(x)^2+a^2+c^2+2\left(ab\cos(x)+ac\right)\sin(x)}}{2\sqrt{a^2+b^2-c^2}}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)/(a+b*cot(x)+c*csc(x)),x, algorithm="fricas")
```

```
[Out] [1/2*log(-(a^4 + 3*a^2*b^2 + 2*b^4 + (a^2 - b^2)*c^2 + 2*(a^2*b + b^3)*c*co
s(x) + (a^4 - b^4 - 2*(a^2 - b^2)*c^2)*cos(x)^2 + 2*((a^3 + a*b^2)*c - (a^3
*b + a*b^3 - 2*a*b*c^2)*cos(x))*sin(x) - 2*(2*a*b*c*cos(x)^2 - a*b*c + (a^3
 + a*b^2)*cos(x) - (a^2*b + b^3 - (a^2 - b^2)*c*cos(x))*sin(x))*sqrt(a^2 +
b^2 - c^2))/(2*b*c*cos(x) - (a^2 - b^2)*cos(x)^2 + a^2 + c^2 + 2*(a*b*cos(x
) + a*c)*sin(x)))/sqrt(a^2 + b^2 - c^2), sqrt(-a^2 - b^2 + c^2)*arctan((b*c
*cos(x) + a*c*sin(x) + a^2 + b^2)*sqrt(-a^2 - b^2 + c^2)/((a^3 + a*b^2 - a*
c^2)*cos(x) - (a^2*b + b^3 - b*c^2)*sin(x)))/(a^2 + b^2 - c^2)]
```

giac [A] time = 0.15, size = 73, normalized size = 1.43

$$-\frac{2\left(\pi\left\lfloor\frac{x}{2\,\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}\left(2\,b-2\,c\right)+\arctan\left(\frac{b\tan\left(\frac{1}{2}\,x\right)-c\tan\left(\frac{1}{2}\,x\right)-a}{\sqrt{-a^2-b^2+c^2}}\right)\right)}{\sqrt{-a^2-b^2+c^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)/(a+b*cot(x)+c*csc(x)),x, algorithm="giac")
```

```
[Out] -2*(pi*floor(1/2*x/pi + 1/2)*sgn(2*b - 2*c) + arctan((b*tan(1/2*x) - c*tan(
1/2*x) - a)/sqrt(-a^2 - b^2 + c^2)))/sqrt(-a^2 - b^2 + c^2)
```

maple [A] time = 0.11, size = 53, normalized size = 1.04

$$-\frac{2\arctan\left(\frac{2(b-c)\tan\left(\frac{x}{2}\right)-2a}{2\sqrt{-a^2-b^2+c^2}}\right)}{\sqrt{-a^2-b^2+c^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(x)/(a+b*cot(x)+c*csc(x)),x)
```

```
[Out] -2/(-a^2-b^2+c^2)^(1/2)*arctan(1/2*(2*(b-c)*tan(1/2*x)-2*a)/(-a^2-b^2+c^2)^
(1/2))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)/(a+b*cot(x)+c*csc(x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
```

```
elp (example of legal syntax is 'assume(c^2-b^2-a^2>0)', see `assume?` for
more details)Is c^2-b^2-a^2 positive or negative?
```

mupad [B] time = 2.79, size = 47, normalized size = 0.92

$$-\frac{2\,\mathrm{atanh}\left(\frac{a-\frac{\tan\left(\frac{x}{2}\right)\,(2\,b-2\,c)}{2}}{\sqrt{a^2+b^2-c^2}}\right)}{\sqrt{a^2+b^2-c^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(x)*(a + c/sin(x) + b*cot(x))),x)
```

```
[Out] -(2*atanh((a - (tan(x/2)*(2*b - 2*c))/2)/(a^2 + b^2 - c^2)^(1/2)))/(a^2 + b
^2 - c^2)^(1/2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc(x)}{a + b\cot(x) + c\csc(x)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)/(a+b*cot(x)+c*csc(x)),x)
```

```
[Out] Integral(csc(x)/(a + b*cot(x) + c*csc(x)), x)
```

3.460 $\int \frac{\csc^2(x)}{a+b\cot(x)+c\csc(x)}\,dx$

Optimal. Leaf size=120

$$-\frac{2ac\tanh^{-1}\left(\frac{a-(b-c)\tan\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2-c^2}}\right)}{\left(b^2-c^2\right)\sqrt{a^2+b^2-c^2}}-\frac{b\log\left(2a\tan\left(\frac{x}{2}\right)-\left((b-c)\tan^2\left(\frac{x}{2}\right)\right)+b+c\right)}{b^2-c^2}+\frac{\log\left(\tan\left(\frac{x}{2}\right)\right)}{b+c}$$

```
[Out] ln(tan(1/2*x))/(b+c)-b*ln(b+c+2*a*tan(1/2*x)-(b-c)*tan(1/2*x)^2)/(b^2-c^2)-
2*a*c*arctanh((a-(b-c)*tan(1/2*x))/(a^2+b^2-c^2)^(1/2))/(b^2-c^2)/(a^2+b^2-
c^2)^(1/2)
```

Rubi [A] time = 0.53, antiderivative size = 120, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.412, Rules used = {4397, 12, 1628, 634, 618, 206, 628}

$$-\frac{2ac\tanh^{-1}\left(\frac{a-(b-c)\tan\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2-c^2}}\right)}{\left(b^2-c^2\right)\sqrt{a^2+b^2-c^2}}-\frac{b\log\left(2a\tan\left(\frac{x}{2}\right)-(b-c)\tan^2\left(\frac{x}{2}\right)+b+c\right)}{b^2-c^2}+\frac{\log\left(\tan\left(\frac{x}{2}\right)\right)}{b+c}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[x]^2/(a + b*Cot[x] + c*Csc[x]),x]
```

```
[Out] (-2*a*c*ArcTanh[(a - (b - c)*Tan[x/2])/Sqrt[a^2 + b^2 - c^2]])/((b^2 - c^2)
*Sqrt[a^2 + b^2 - c^2]) + Log[Tan[x/2]]/(b + c) - (b*Log[b + c + 2*a*Tan[x/
2] - (b - c)*Tan[x/2]^2])/(b^2 - c^2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
```

```
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1628

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 4397

```
Int[u_, x_Symbol] :> Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]
```

Rubi steps

$$\int \frac{\csc^2(x)}{a+b\cot(x)+c\csc(x)}\,dx = \int \frac{\csc(x)}{c+b\cos(x)+a\sin(x)}\,dx$$

$$= 2\,\text{Subst}\left(\int \frac{1+x^2}{2x\left(b+c+2ax+(-b+c)x^2\right)}\,dx, x, \tan\left(\frac{x}{2}\right)\right)$$

$$= \text{Subst}\left(\int \frac{1+x^2}{x\left(b+c+2ax+(-b+c)x^2\right)}\,dx, x, \tan\left(\frac{x}{2}\right)\right)$$

$$= \text{Subst}\left(\int \left(\frac{1}{(b+c)x}+\frac{2(-a+bx)}{(b+c)\left(b+c+2ax-(b-c)x^2\right)}\right)dx, x, \tan\left(\frac{x}{2}\right)\right)$$

$$= \frac{\log\left(\tan\left(\frac{x}{2}\right)\right)}{b+c}+\frac{2\,\text{Subst}\left(\int \frac{-a+bx}{b+c+2ax+(-b+c)x^2}\,dx, x, \tan\left(\frac{x}{2}\right)\right)}{b+c}$$

$$= \frac{\log\left(\tan\left(\frac{x}{2}\right)\right)}{b+c}-\frac{b\,\text{Subst}\left(\int \frac{2a+2(-b+c)x}{b+c+2ax+(-b+c)x^2}\,dx, x, \tan\left(\frac{x}{2}\right)\right)}{b^2-c^2}+\frac{(2ac)\,\text{Subst}\left(\int \frac{}{b+c+\cdots}\right.}{}$$

$$= \frac{\log\left(\tan\left(\frac{x}{2}\right)\right)}{b+c}-\frac{b\log\left(b+c+2a\tan\left(\frac{x}{2}\right)-(b-c)\tan^2\left(\frac{x}{2}\right)\right)}{b^2-c^2}-\frac{(4ac)\,\text{Subst}\left(\int \frac{}{4(\cdots}\right.}{}$$

$$= -\frac{2ac\tanh^{-1}\left(\frac{a-(b-c)\tan\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2-c^2}}\right)}{\left(b^2-c^2\right)\sqrt{a^2+b^2-c^2}}+\frac{\log\left(\tan\left(\frac{x}{2}\right)\right)}{b+c}-\frac{b\log\left(b+c+2a\tan\left(\frac{x}{2}\right)-(b-c\cdots\right.}{b^2-c^2}$$

Mathematica [A] time = 0.28, size = 104, normalized size = 0.87

$$\frac{\frac{2ac\tanh^{-1}\left(\frac{a+(c-b)\tan\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2-c^2}}\right)}{\sqrt{a^2+b^2-c^2}}+b\log(a\sin(x)+b\cos(x)+c)+(c-b)\log\left(\sin\left(\frac{x}{2}\right)\right)-(b+c)\log\left(\cos\left(\frac{x}{2}\right)\right)}{(c-b)(b+c)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[x]^2/(a + b*Cot[x] + c*Csc[x]),x]
```

```
[Out] ((2*a*c*ArcTanh[(a + (-b + c)*Tan[x/2])/Sqrt[a^2 + b^2 - c^2]])/Sqrt[a^2 +
b^2 - c^2] - (b + c)*Log[Cos[x/2]] + (-b + c)*Log[Sin[x/2]] + b*Log[c + b*C
os[x] + a*Sin[x]])/((-b + c)*(b + c))
```

fricas [B] time = 6.05, size = 669, normalized size = 5.58

$$\left[-\frac{\sqrt{a^2+b^2-c^2}\,ac\log\left(\frac{a^4+3\,a^2b^2+2\,b^4+\left(a^2-b^2\right)c^2+2\,\left(a^2b+b^3\right)c\cos(x)+\left(a^4-b^4-2\,\left(a^2-b^2\right)c^2\right)\cos(x)^2+2\,\left(\left(a^3+ab^2\right)c-\left(a^3b+ab^3-2\,abc^2\right)\cos(x)\right.}{2\,bc\cos(x)-\left(a^2-b^2\right)\cos(x)^2+a^2+c^2+2\,(ab}\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)^2/(a+b*cot(x)+c*csc(x)),x, algorithm="fricas")
```

```
[Out] [-1/2*(sqrt(a^2 + b^2 - c^2)*a*c*log((a^4 + 3*a^2*b^2 + 2*b^4 + (a^2 - b^2)
*c^2 + 2*(a^2*b + b^3)*c*cos(x) + (a^4 - b^4 - 2*(a^2 - b^2)*c^2)*cos(x)^2
+ 2*((a^3 + a*b^2)*c - (a^3*b + a*b^3 - 2*a*b*c^2)*cos(x))*sin(x) + 2*(2*a*
b*c*cos(x)^2 - a*b*c + (a^3 + a*b^2)*cos(x) - (a^2*b + b^3 - (a^2 - b^2)*c*
cos(x))*sin(x))*sqrt(a^2 + b^2 - c^2))/(2*b*c*cos(x) - (a^2 - b^2)*cos(x)^2
 + a^2 + c^2 + 2*(a*b*cos(x) + a*c)*sin(x))) + (a^2*b + b^3 - b*c^2)*log(2*
b*c*cos(x) - (a^2 - b^2)*cos(x)^2 + a^2 + c^2 + 2*(a*b*cos(x) + a*c)*sin(x)
) - (a^2*b + b^3 - b*c^2 - c^3 + (a^2 + b^2)*c)*log(1/2*cos(x) + 1/2) - (a^
2*b + b^3 - b*c^2 + c^3 - (a^2 + b^2)*c)*log(-1/2*cos(x) + 1/2))/(a^2*b^2 +
 b^4 + c^4 - (a^2 + 2*b^2)*c^2), 1/2*(2*sqrt(-a^2 - b^2 + c^2)*a*c*arctan((
b*c*cos(x) + a*c*sin(x) + a^2 + b^2)*sqrt(-a^2 - b^2 + c^2)/((a^3 + a*b^2 -
 a*c^2)*cos(x) - (a^2*b + b^3 - b*c^2)*sin(x))) - (a^2*b + b^3 - b*c^2)*log
(2*b*c*cos(x) - (a^2 - b^2)*cos(x)^2 + a^2 + c^2 + 2*(a*b*cos(x) + a*c)*sin
(x)) + (a^2*b + b^3 - b*c^2 - c^3 + (a^2 + b^2)*c)*log(1/2*cos(x) + 1/2) +
(a^2*b + b^3 - b*c^2 + c^3 - (a^2 + b^2)*c)*log(-1/2*cos(x) + 1/2))/(a^2*b^
2 + b^4 + c^4 - (a^2 + 2*b^2)*c^2)]
```

giac [A] time = 0.17, size = 142, normalized size = 1.18

$$\frac{2\left(\pi\left\lfloor\frac{x}{2\,\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}\left(-2\,b+2\,c\right)+\arctan\left(-\frac{b\tan\left(\frac{1}{2}\,x\right)-c\tan\left(\frac{1}{2}\,x\right)-a}{\sqrt{-a^2-b^2+c^2}}\right)\right)ac}{\sqrt{-a^2-b^2+c^2}\left(b^2-c^2\right)}-\frac{b\log\left(-b\tan\left(\frac{1}{2}\,x\right)^2+c\tan\left(\frac{1}{2}\,x\right)^2+2\,a\tan\left(\right.\right.}{b^2-c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)^2/(a+b*cot(x)+c*csc(x)),x, algorithm="giac")
```

```
[Out] 2*(pi*floor(1/2*x/pi + 1/2)*sgn(-2*b + 2*c) + arctan(-(b*tan(1/2*x) - c*tan
(1/2*x) - a)/sqrt(-a^2 - b^2 + c^2)))*a*c/(sqrt(-a^2 - b^2 + c^2)*(b^2 - c^
2)) - b*log(-b*tan(1/2*x)^2 + c*tan(1/2*x)^2 + 2*a*tan(1/2*x) + b + c)/(b^2
 - c^2) + log(abs(tan(1/2*x)))/(b + c)
```

maple [A] time = 0.11, size = 184, normalized size = 1.53

$$-\frac{b\ln\left(b\left(\tan^2\left(\frac{x}{2}\right)\right)-c\left(\tan^2\left(\frac{x}{2}\right)\right)-2a\tan\left(\frac{x}{2}\right)-b-c\right)}{(b+c)(b-c)}+\frac{2\arctan\left(\frac{2(b-c)\tan\left(\frac{x}{2}\right)-2a}{2\sqrt{-a^2-b^2+c^2}}\right)a}{(b+c)\sqrt{-a^2-b^2+c^2}}-\frac{2\arctan\left(\frac{2(b-c)\tan\left(\frac{x}{2}\right)-2a}{2\sqrt{-a^2-b^2+c^2}}\right)}{(b+c)\sqrt{-a^2-b^2+c^2}\ (b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(x)^2/(a+b*cot(x)+c*csc(x)),x)
```

```
[Out] -1/(b+c)*b/(b-c)*ln(b*tan(1/2*x)^2-c*tan(1/2*x)^2-2*a*tan(1/2*x)-b-c)+2/(b+
c)/(-a^2-b^2+c^2)^(1/2)*arctan(1/2*(2*(b-c)*tan(1/2*x)-2*a)/(-a^2-b^2+c^2)^
(1/2))*a-2/(b+c)/(-a^2-b^2+c^2)^(1/2)*arctan(1/2*(2*(b-c)*tan(1/2*x)-2*a)/(
-a^2-b^2+c^2)^(1/2))*b*a/(b-c)+ln(tan(1/2*x))/(b+c)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)^2/(a+b*cot(x)+c*csc(x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(c^2-b^2-a^2>0)', see `assume?` for
more details)Is c^2-b^2-a^2 positive or negative?
```

mupad [B] time = 8.70, size = 531, normalized size = 4.42

$$\frac{\ln\left(\tan\left(\frac{x}{2}\right)\right)}{b+c}-\frac{\ln\left(2\,a-2\,b\tan\left(\frac{x}{2}\right)-\frac{\left(\tan\left(\frac{x}{2}\right)\left(-8\,a^2-8\,b^2+6\,b\,c+2\,c^2\right)-4\,a\,c+\frac{2\,(b-c)\left(a^2\,b-b\,c^2+b^3+a\,c\,\sqrt{a^2+b^2-c^2}\right)\left(4\tan\left(\frac{x}{2}\right)a^2+a\,b+a\,c+3\tan\left(\frac{x}{2}\right)\right.}{\left(b^2-c^2\right)\left(a^2+b^2-c^2\right)}}{\left(b^2-c^2\right)\left(a^2+b^2-c^2\right)}\right.}{\left(b^2-c^2\right)\left(a^2+b^2-c^2\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(x)^2*(a + c/sin(x) + b*cot(x))),x)
```

```
[Out] log(tan(x/2))/(b + c) - (log(2*a - 2*b*tan(x/2) - ((tan(x/2)*(6*b*c - 8*a^2
 - 8*b^2 + 2*c^2) - 4*a*c + (2*(b - c)*(a^2*b - b*c^2 + b^3 + a*c*(a^2 + b^
2 - c^2)^(1/2))*(a*b + a*c + 4*a^2*tan(x/2) + 3*b^2*tan(x/2) - 3*c^2*tan(x/
2)))/((b^2 - c^2)*(a^2 + b^2 - c^2)))*(a^2*b - b*c^2 + b^3 + a*c*(a^2 + b^2
 - c^2)^(1/2)))/((b^2 - c^2)*(a^2 + b^2 - c^2)))*(b*(a^2 - c^2) + b^3 + a*c
```

```
*(a^2 + b^2 - c^2)^(1/2)))/((b^2 - c^2)*(a^2 + b^2 - c^2)) - (log(2*a - 2*b
*tan(x/2) - ((tan(x/2)*(6*b*c - 8*a^2 - 8*b^2 + 2*c^2) - 4*a*c + (2*(b - c)
*(a^2*b - b*c^2 + b^3 - a*c*(a^2 + b^2 - c^2)^(1/2))*(a*b + a*c + 4*a^2*tan
(x/2) + 3*b^2*tan(x/2) - 3*c^2*tan(x/2)))/((b^2 - c^2)*(a^2 + b^2 - c^2)))*
(a^2*b - b*c^2 + b^3 - a*c*(a^2 + b^2 - c^2)^(1/2)))/((b^2 - c^2)*(a^2 + b^
2 - c^2)))*(b*(a^2 - c^2) + b^3 - a*c*(a^2 + b^2 - c^2)^(1/2)))/((b^2 - c^2
)*(a^2 + b^2 - c^2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc^2(x)}{a + b\cot(x) + c\csc(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)**2/(a+b*cot(x)+c*csc(x)),x)
```

```
[Out] Integral(csc(x)**2/(a + b*cot(x) + c*csc(x)), x)
```

3.461 $\int \frac{\csc(x)}{2+2\cot(x)+3\csc(x)}\,dx$

Optimal. Leaf size=21

$$x+2\tan^{-1}\left(\frac{\cos(x)-\sin(x)}{\sin(x)+\cos(x)+2}\right)$$

```
[Out] x+2*arctan((cos(x)-sin(x))/(2+cos(x)+sin(x)))
```

Rubi [A] time = 0.05, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {3166, 3124, 618, 204}

$$x+2\tan^{-1}\left(\frac{\cos(x)-\sin(x)}{\sin(x)+\cos(x)+2}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Csc[x]/(2 + 2*Cot[x] + 3*Csc[x]),x]
```

```
[Out] x + 2*ArcTan[(Cos[x] - Sin[x])/(2 + Cos[x] + Sin[x])]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 3124

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(-1), x_Symbol] :> Module[{f = FreeFactors[Tan[(d + e*x)/2], x]}, Dist[(2*f
)/e, Subst[Int[1/(a + b + 2*c*f*x + (a - b)*f^2*x^2), x], x, Tan[(d + e*x)/
2]/f], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0]
```

Rule 3166

```
Int[csc[(d_.) + (e_.)*(x_)]^(n_.)*((a_.) + csc[(d_.) + (e_.)*(x_)]*(b_.) +
cot[(d_.) + (e_.)*(x_)]*(c_.))^(m_), x_Symbol] :> Int[1/(b + a*Sin[d + e*x]
 + c*Cos[d + e*x])^n, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[m + n, 0] && I
```

ntegerQ[n]

Rubi steps

$$
\begin{aligned}
\int \frac{\csc(x)}{2+2\cot(x)+3\csc(x)}\,dx &= \int \frac{1}{3+2\cos(x)+2\sin(x)}\,dx \\
&= 2\,\text{Subst}\left(\int \frac{1}{5+4x+x^2}\,dx, x, \tan\left(\frac{x}{2}\right)\right) \\
&= -\left(4\,\text{Subst}\left(\int \frac{1}{-4-x^2}\,dx, x, 4+2\tan\left(\frac{x}{2}\right)\right)\right) \\
&= x+2\tan^{-1}\left(\frac{\cos(x)-\sin(x)}{2+\cos(x)+\sin(x)}\right)
\end{aligned}
$$

Mathematica [B] time = 0.02, size = 51, normalized size = 2.43

$$
\tan^{-1}\left(\sec\left(\frac{x}{2}\right)\left(\sin\left(\frac{x}{2}\right)+2\cos\left(\frac{x}{2}\right)\right)\right)-\tan^{-1}\left(\frac{\cos\left(\frac{x}{2}\right)}{\sin\left(\frac{x}{2}\right)+2\cos\left(\frac{x}{2}\right)}\right)
$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[x]/(2 + 2*Cot[x] + 3*Csc[x]),x]
[Out] -ArcTan[Cos[x/2]/(2*Cos[x/2] + Sin[x/2])] + ArcTan[Sec[x/2]*(2*Cos[x/2] + S
in[x/2])]
```

fricas [A] time = 0.89, size = 24, normalized size = 1.14

$$
-\arctan\left(-\frac{3\cos(x)+3\sin(x)+4}{\cos(x)-\sin(x)}\right)
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)/(2+2*cot(x)+3*csc(x)),x, algorithm="fricas")
[Out] -arctan(-(3*cos(x) + 3*sin(x) + 4)/(cos(x) - sin(x)))
```

giac [A] time = 0.14, size = 22, normalized size = 1.05

$$
2\pi\left\lfloor\frac{x}{2\pi}+\frac{1}{2}\right\rfloor+2\arctan\left(\tan\left(\frac{1}{2}x\right)+2\right)
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)/(2+2*cot(x)+3*csc(x)),x, algorithm="giac")
[Out] 2*pi*floor(1/2*x/pi + 1/2) + 2*arctan(tan(1/2*x) + 2)
```

maple [A] time = 0.12, size = 10, normalized size = 0.48

$$2\arctan\left(2+\tan\left(\frac{x}{2}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(x)/(2+2*cot(x)+3*csc(x)),x)
[Out] 2*arctan(2+tan(1/2*x))
```

maxima [A] time = 0.41, size = 14, normalized size = 0.67

$$2\arctan\left(\frac{\sin(x)}{\cos(x)+1}+2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)/(2+2*cot(x)+3*csc(x)),x, algorithm="maxima")
[Out] 2*arctan(sin(x)/(cos(x) + 1) + 2)
```

mupad [B] time = 3.14, size = 9, normalized size = 0.43

$$2\,\mathrm{atan}\left(\tan\left(\frac{x}{2}\right)+2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(x)*(2*cot(x) + 3/sin(x) + 2)),x)
[Out] 2*atan(tan(x/2) + 2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\csc(x)}{2\cot(x)+3\csc(x)+2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)/(2+2*cot(x)+3*csc(x)),x)
[Out] Integral(csc(x)/(2*cot(x) + 3*csc(x) + 2), x)
```

3.462 $\int \frac{(a+c\cot(d+ex)+b\csc(d+ex))^{3/2}}{\csc^{\frac{3}{2}}(d+ex)}\,dx$

Optimal. Leaf size=371

$$\frac{2\left(a^2-b^2+c^2\right)\sqrt{\frac{a\sin(d+ex)+b+c\cos(d+ex)}{\sqrt{a^2+c^2}+b}}\,(a+b\csc(d+ex)+c\cot(d+ex))^{3/2}F\left(\frac{1}{2}\left(d+ex-\tan^{-1}(c,a)\right)|\frac{2\sqrt{a^2+c^2}}{b+\sqrt{a^2+c^2}}\right)}{3e\csc^{\frac{3}{2}}(d+ex)(a\sin(d+ex)+b+c\cos(d+ex))^2}$$

```
[Out] -2/3*(a+c*cot(e*x+d)+b*csc(e*x+d))^(3/2)*(a*cos(e*x+d)-c*sin(e*x+d))/e/csc(
e*x+d)^(3/2)/(b+c*cos(e*x+d)+a*sin(e*x+d))+8/3*b*(a+c*cot(e*x+d)+b*csc(e*x+
d))^(3/2)*(cos(1/2*d+1/2*e*x-1/2*arctan(c,a))^2)^(1/2)/cos(1/2*d+1/2*e*x-1/
2*arctan(c,a))*EllipticE(sin(1/2*d+1/2*e*x-1/2*arctan(c,a)),2^(1/2)*((a^2+c
^2)^(1/2)/(b+(a^2+c^2)^(1/2)))^(1/2))/e/csc(e*x+d)^(3/2)/(b+c*cos(e*x+d)+a*
sin(e*x+d))/((b+c*cos(e*x+d)+a*sin(e*x+d))/(b+(a^2+c^2)^(1/2)))^(1/2)+2/3*(
a^2-b^2+c^2)*(a+c*cot(e*x+d)+b*csc(e*x+d))^(3/2)*(cos(1/2*d+1/2*e*x-1/2*arc
tan(c,a))^2)^(1/2)/cos(1/2*d+1/2*e*x-1/2*arctan(c,a))*EllipticF(sin(1/2*d+1
/2*e*x-1/2*arctan(c,a)),2^(1/2)*((a^2+c^2)^(1/2)/(b+(a^2+c^2)^(1/2)))^(1/2)
)*((b+c*cos(e*x+d)+a*sin(e*x+d))/(b+(a^2+c^2)^(1/2)))^(1/2)/e/csc(e*x+d)^(3
/2)/(b+c*cos(e*x+d)+a*sin(e*x+d))^2
```

Rubi [A] time = 0.43, antiderivative size = 371, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.212, Rules used = {3168, 3120, 3149, 3119, 2653, 3127, 2661}

$$\frac{2\left(a^2-b^2+c^2\right)\sqrt{\frac{a\sin(d+ex)+b+c\cos(d+ex)}{\sqrt{a^2+c^2}+b}}\,(a+b\csc(d+ex)+c\cot(d+ex))^{3/2}F\left(\frac{1}{2}\left(d+ex-\tan^{-1}(c,a)\right)|\frac{2\sqrt{a^2+c^2}}{b+\sqrt{a^2+c^2}}\right)}{3e\csc^{\frac{3}{2}}(d+ex)(a\sin(d+ex)+b+c\cos(d+ex))^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + c*Cot[d + e*x] + b*Csc[d + e*x])^(3/2)/Csc[d + e*x]^(3/2),x]
```

```
[Out] (8*b*(a + c*Cot[d + e*x] + b*Csc[d + e*x])^(3/2)*EllipticE[(d + e*x - ArcTa
n[c, a])/2, (2*Sqrt[a^2 + c^2])/(b + Sqrt[a^2 + c^2])])/(3*e*Csc[d + e*x]^(
3/2)*(b + c*Cos[d + e*x] + a*Sin[d + e*x])*Sqrt[(b + c*Cos[d + e*x] + a*Sin
[d + e*x])/(b + Sqrt[a^2 + c^2])]) + (2*(a^2 - b^2 + c^2)*(a + c*Cot[d + e*
x] + b*Csc[d + e*x])^(3/2)*EllipticF[(d + e*x - ArcTan[c, a])/2, (2*Sqrt[a^
2 + c^2])/(b + Sqrt[a^2 + c^2])]*Sqrt[(b + c*Cos[d + e*x] + a*Sin[d + e*x])
/(b + Sqrt[a^2 + c^2])])/(3*e*Csc[d + e*x]^(3/2)*(b + c*Cos[d + e*x] + a*Si
n[d + e*x])^2) - (2*(a + c*Cot[d + e*x] + b*Csc[d + e*x])^(3/2)*(a*Cos[d +
```

```
e*x] - c*Sin[d + e*x]))/(3*e*Csc[d + e*x]^(3/2)*(b + c*Cos[d + e*x] + a*Sin
[d + e*x]))
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 3119

```
Int[Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_
)]], x_Symbol] :> Dist[Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]]/Sqrt[(a +
b*Cos[d + e*x] + c*Sin[d + e*x])/(a + Sqrt[b^2 + c^2])], Int[Sqrt[a/(a + Sq
rt[b^2 + c^2]) + (Sqrt[b^2 + c^2]*Cos[d + e*x - ArcTan[b, c]])/(a + Sqrt[b^
2 + c^2])], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0]
&& NeQ[b^2 + c^2, 0] &&  !GtQ[a + Sqrt[b^2 + c^2], 0]
```

Rule 3120

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(n_), x_Symbol] :> -Simp[((c*Cos[d + e*x] - b*Sin[d + e*x])*(a + b*Cos[d +
e*x] + c*Sin[d + e*x])^(n - 1))/(e*n), x] + Dist[1/n, Int[Simp[n*a^2 + (n -
 1)*(b^2 + c^2) + a*b*(2*n - 1)*Cos[d + e*x] + a*c*(2*n - 1)*Sin[d + e*x],
x]*(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n - 2), x], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[a^2 - b^2 - c^2, 0] && GtQ[n, 1]
```

Rule 3127

```
Int[1/Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(
x_)]], x_Symbol] :> Dist[Sqrt[(a + b*Cos[d + e*x] + c*Sin[d + e*x])/(a + Sq
rt[b^2 + c^2])]/Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]], Int[1/Sqrt[a/(a
+ Sqrt[b^2 + c^2]) + (Sqrt[b^2 + c^2]*Cos[d + e*x - ArcTan[b, c]])/(a + Sqr
t[b^2 + c^2])], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2,
 0] && NeQ[b^2 + c^2, 0] &&  !GtQ[a + Sqrt[b^2 + c^2], 0]
```

Rule 3149

```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])
/Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)]]
```

```
, x_Symbol] :> Dist[B/b, Int[Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]], x],
 x] + Dist[(A*b - a*B)/b, Int[1/Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]],
x], x] /; FreeQ[{a, b, c, d, e, A, B, C}, x] && EqQ[B*c - b*C, 0] && NeQ[A*
b - a*B, 0]
```

Rule 3168

```
Int[csc[(d_.) + (e_.)*(x_)]^(n_.)*((a_.) + csc[(d_.) + (e_.)*(x_)]*(b_.) +
cot[(d_.) + (e_.)*(x_)]*(c_.))^(m_), x_Symbol] :> Dist[(Csc[d + e*x]^n*(b +
 a*Sin[d + e*x] + c*Cos[d + e*x])^n)/(a + b*Csc[d + e*x] + c*Cot[d + e*x])^
n, Int[1/(b + a*Sin[d + e*x] + c*Cos[d + e*x])^n, x], x] /; FreeQ[{a, b, c,
 d, e}, x] && EqQ[m + n, 0] &&  !IntegerQ[n]
```

Rubi steps

$$\int \frac{(a+c\cot(d+ex)+b\csc(d+ex))^{3/2}}{\csc^{\frac{3}{2}}(d+ex)}\,dx = \frac{(a+c\cot(d+ex)+b\csc(d+ex))^{3/2}\int(b+c\cos(d+ex)+a\sin(d+}{\csc^{\frac{3}{2}}(d+ex)(b+c\cos(d+ex)+a\sin(d+ex))^{3/2}}$$

$$= -\frac{2(a+c\cot(d+ex)+b\csc(d+ex))^{3/2}(a\cos(d+ex)-c\sin(d+ex}{3e\csc^{\frac{3}{2}}(d+ex)(b+c\cos(d+ex)+a\sin(d+ex))}$$

$$= -\frac{2(a+c\cot(d+ex)+b\csc(d+ex))^{3/2}(a\cos(d+ex)-c\sin(d+ex}{3e\csc^{\frac{3}{2}}(d+ex)(b+c\cos(d+ex)+a\sin(d+ex))}$$

$$= -\frac{2(a+c\cot(d+ex)+b\csc(d+ex))^{3/2}(a\cos(d+ex)-c\sin(d+ex}{3e\csc^{\frac{3}{2}}(d+ex)(b+c\cos(d+ex)+a\sin(d+ex))}$$

$$= \frac{8b(a+c\cot(d+ex)+b\csc(d+ex))^{3/2}E\left(\frac{1}{2}\left(d+ex-\tan^{-1}(c,a)\right)|\frac{2}{b}\right.}{3e\csc^{\frac{3}{2}}(d+ex)(b+c\cos(d+ex)+a\sin(d+ex))\sqrt{\frac{b+c\cos(d+ex)+a\,s}{b+\sqrt{a^2+c^2}}}}$$

Mathematica [C] time = 6.44, size = 2490, normalized size = 6.71

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + c*Cot[d + e*x] + b*Csc[d + e*x])^(3/2)/Csc[d + e*x]^(3/2),x]

[Out] ((a + c*Cot[d + e*x] + b*Csc[d + e*x])^(3/2)*((8*b*c)/(3*a) - (2*a*Cos[d +
e*x])/3 + (2*c*Sin[d + e*x])/3))/(e*Csc[d + e*x]^(3/2)*(b + c*Cos[d + e*x]
+ a*Sin[d + e*x])) + (4*a*b*(a + c*Cot[d + e*x] + b*Csc[d + e*x])^(3/2)*(-(
(a*AppellF1[-1/2, -1/2, -1/2, 1/2, -((b + Sqrt[1 + a^2/c^2]*c*Cos[d + e*x -
 ArcTan[a/c]])/(Sqrt[1 + a^2/c^2]*(1 - b/(Sqrt[1 + a^2/c^2]*c))*c)), -((b +
 Sqrt[1 + a^2/c^2]*c*Cos[d + e*x - ArcTan[a/c]])/(Sqrt[1 + a^2/c^2]*(-1 - b
/(Sqrt[1 + a^2/c^2]*c))*c))]*Sin[d + e*x - ArcTan[a/c]])/(Sqrt[1 + a^2/c^2]
*c*Sqrt[(c*Sqrt[(a^2 + c^2)/c^2] - c*Sqrt[(a^2 + c^2)/c^2]*Cos[d + e*x - Ar
cTan[a/c]])/(b + c*Sqrt[(a^2 + c^2)/c^2])]*Sqrt[b + c*Sqrt[(a^2 + c^2)/c^2]
*Cos[d + e*x - ArcTan[a/c]]]*Sqrt[(c*Sqrt[(a^2 + c^2)/c^2] + c*Sqrt[(a^2 +
c^2)/c^2]*Cos[d + e*x - ArcTan[a/c]])/(-b + c*Sqrt[(a^2 + c^2)/c^2])])) - (
(2*c*(b + Sqrt[1 + a^2/c^2]*c*Cos[d + e*x - ArcTan[a/c]]))/(a^2 + c^2) - (a
*Sin[d + e*x - ArcTan[a/c]])/(Sqrt[1 + a^2/c^2]*c))/Sqrt[b + Sqrt[1 + a^2/c
^2]*c*Cos[d + e*x - ArcTan[a/c]]]))/(3*e*Csc[d + e*x]^(3/2)*(b + c*Cos[d +
e*x] + a*Sin[d + e*x])^(3/2)) + (4*b*c^2*(a + c*Cot[d + e*x] + b*Csc[d + e*
x])^(3/2)*(-((a*AppellF1[-1/2, -1/2, -1/2, 1/2, -((b + Sqrt[1 + a^2/c^2]*c*
Cos[d + e*x - ArcTan[a/c]])/(Sqrt[1 + a^2/c^2]*(1 - b/(Sqrt[1 + a^2/c^2]*c)
)*c)), -((b + Sqrt[1 + a^2/c^2]*c*Cos[d + e*x - ArcTan[a/c]])/(Sqrt[1 + a^2
/c^2]*(-1 - b/(Sqrt[1 + a^2/c^2]*c))*c))]*Sin[d + e*x - ArcTan[a/c]])/(Sqrt
[1 + a^2/c^2]*c*Sqrt[(c*Sqrt[(a^2 + c^2)/c^2] - c*Sqrt[(a^2 + c^2)/c^2]*Cos
[d + e*x - ArcTan[a/c]])/(b + c*Sqrt[(a^2 + c^2)/c^2])]*Sqrt[b + c*Sqrt[(a^
2 + c^2)/c^2]*Cos[d + e*x - ArcTan[a/c]]]*Sqrt[(c*Sqrt[(a^2 + c^2)/c^2] + c
*Sqrt[(a^2 + c^2)/c^2]*Cos[d + e*x - ArcTan[a/c]])/(-b + c*Sqrt[(a^2 + c^2)
/c^2])])) - ((2*c*(b + Sqrt[1 + a^2/c^2]*c*Cos[d + e*x - ArcTan[a/c]]))/(a^
2 + c^2) - (a*Sin[d + e*x - ArcTan[a/c]])/(Sqrt[1 + a^2/c^2]*c))/Sqrt[b + S
qrt[1 + a^2/c^2]*c*Cos[d + e*x - ArcTan[a/c]]]))/(3*a*e*Csc[d + e*x]^(3/2)*
(b + c*Cos[d + e*x] + a*Sin[d + e*x])^(3/2)) + (2*a*AppellF1[1/2, 1/2, 1/2,
 3/2, -((b + a*Sqrt[1 + c^2/a^2]*Sin[d + e*x + ArcTan[c/a]])/(a*Sqrt[1 + c^
2/a^2]*(1 - b/(a*Sqrt[1 + c^2/a^2])))), -((b + a*Sqrt[1 + c^2/a^2]*Sin[d +
e*x + ArcTan[c/a]])/(a*Sqrt[1 + c^2/a^2]*(-1 - b/(a*Sqrt[1 + c^2/a^2]))))]*
(a + c*Cot[d + e*x] + b*Csc[d + e*x])^(3/2)*Sec[d + e*x + ArcTan[c/a]]*Sqrt
[(a*Sqrt[(a^2 + c^2)/a^2] - a*Sqrt[(a^2 + c^2)/a^2]*Sin[d + e*x + ArcTan[c/
a]])/(b + a*Sqrt[(a^2 + c^2)/a^2])]*Sqrt[b + a*Sqrt[(a^2 + c^2)/a^2]*Sin[d
+ e*x + ArcTan[c/a]]]*Sqrt[(a*Sqrt[(a^2 + c^2)/a^2] + a*Sqrt[(a^2 + c^2)/a^
2]*Sin[d + e*x + ArcTan[c/a]])/(-b + a*Sqrt[(a^2 + c^2)/a^2])])/(3*Sqrt[1 +
 c^2/a^2]*e*Csc[d + e*x]^(3/2)*(b + c*Cos[d + e*x] + a*Sin[d + e*x])^(3/2))
 + (2*b^2*AppellF1[1/2, 1/2, 1/2, 3/2, -((b + a*Sqrt[1 + c^2/a^2]*Sin[d + e
*x + ArcTan[c/a]])/(a*Sqrt[1 + c^2/a^2]*(1 - b/(a*Sqrt[1 + c^2/a^2])))), -(
(b + a*Sqrt[1 + c^2/a^2]*Sin[d + e*x + ArcTan[c/a]])/(a*Sqrt[1 + c^2/a^2]*(
-1 - b/(a*Sqrt[1 + c^2/a^2]))))]*(a + c*Cot[d + e*x] + b*Csc[d + e*x])^(3/2
)*Sec[d + e*x + ArcTan[c/a]]*Sqrt[(a*Sqrt[(a^2 + c^2)/a^2] - a*Sqrt[(a^2 +
c^2)/a^2]*Sin[d + e*x + ArcTan[c/a]])/(b + a*Sqrt[(a^2 + c^2)/a^2])]*Sqrt[b
 + a*Sqrt[(a^2 + c^2)/a^2]*Sin[d + e*x + ArcTan[c/a]]]*Sqrt[(a*Sqrt[(a^2 +
```

```
c^2)/a^2] + a*Sqrt[(a^2 + c^2)/a^2]*Sin[d + e*x + ArcTan[c/a]])/(-b + a*Sqr
t[(a^2 + c^2)/a^2])])/(a*Sqrt[1 + c^2/a^2]*e*Csc[d + e*x]^(3/2)*(b + c*Cos[
d + e*x] + a*Sin[d + e*x])^(3/2)) + (2*c^2*AppellF1[1/2, 1/2, 1/2, 3/2, -((
b + a*Sqrt[1 + c^2/a^2]*Sin[d + e*x + ArcTan[c/a]])/(a*Sqrt[1 + c^2/a^2]*(1
 - b/(a*Sqrt[1 + c^2/a^2])))), -((b + a*Sqrt[1 + c^2/a^2]*Sin[d + e*x + Arc
Tan[c/a]])/(a*Sqrt[1 + c^2/a^2]*(-1 - b/(a*Sqrt[1 + c^2/a^2]))))]*(a + c*Co
t[d + e*x] + b*Csc[d + e*x])^(3/2)*Sec[d + e*x + ArcTan[c/a]]*Sqrt[(a*Sqrt[
(a^2 + c^2)/a^2] - a*Sqrt[(a^2 + c^2)/a^2]*Sin[d + e*x + ArcTan[c/a]])/(b +
 a*Sqrt[(a^2 + c^2)/a^2])]*Sqrt[b + a*Sqrt[(a^2 + c^2)/a^2]*Sin[d + e*x + A
rcTan[c/a]]]*Sqrt[(a*Sqrt[(a^2 + c^2)/a^2] + a*Sqrt[(a^2 + c^2)/a^2]*Sin[d
+ e*x + ArcTan[c/a]])/(-b + a*Sqrt[(a^2 + c^2)/a^2])])/(3*a*Sqrt[1 + c^2/a^
2]*e*Csc[d + e*x]^(3/2)*(b + c*Cos[d + e*x] + a*Sin[d + e*x])^(3/2))
```

fricas [F] time = 1.05, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{(c\cot(ex+d)+b\csc(ex+d)+a)^{\frac{3}{2}}}{\csc(ex+d)^{\frac{3}{2}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+c*cot(e*x+d)+b*csc(e*x+d))^(3/2)/csc(e*x+d)^(3/2),x, algorithm
="fricas")
```

```
[Out] integral((c*cot(e*x + d) + b*csc(e*x + d) + a)^(3/2)/csc(e*x + d)^(3/2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{(c\cot(ex+d)+b\csc(ex+d)+a)^{\frac{3}{2}}}{\csc(ex+d)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+c*cot(e*x+d)+b*csc(e*x+d))^(3/2)/csc(e*x+d)^(3/2),x, algorithm
="giac")
```

```
[Out] integrate((c*cot(e*x + d) + b*csc(e*x + d) + a)^(3/2)/csc(e*x + d)^(3/2), x
)
```

maple [C] time = 3.12, size = 20463, normalized size = 55.16

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+c*cot(e*x+d)+b*csc(e*x+d))^(3/2)/csc(e*x+d)^(3/2),x)
```

```
[Out] result too large to display
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(c\cot(ex+d)+b\csc(ex+d)+a)^{\frac{3}{2}}}{\csc(ex+d)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+c*cot(e*x+d)+b*csc(e*x+d))^(3/2)/csc(e*x+d)^(3/2),x, algorithm
="maxima")
```

```
[Out] integrate((c*cot(e*x + d) + b*csc(e*x + d) + a)^(3/2)/csc(e*x + d)^(3/2), x
)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(a+c\cot(d+e\,x)+\frac{b}{\sin(d+e\,x)}\right)^{3/2}}{\left(\frac{1}{\sin(d+e\,x)}\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + c*cot(d + e*x) + b/sin(d + e*x))^(3/2)/(1/sin(d + e*x))^(3/2),x)
```

```
[Out] int((a + c*cot(d + e*x) + b/sin(d + e*x))^(3/2)/(1/sin(d + e*x))^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+b\csc(d+ex)+c\cot(d+ex))^{\frac{3}{2}}}{\csc^{\frac{3}{2}}(d+ex)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+c*cot(e*x+d)+b*csc(e*x+d))**(3/2)/csc(e*x+d)**(3/2),x)
```

```
[Out] Integral((a + b*csc(d + e*x) + c*cot(d + e*x))**(3/2)/csc(d + e*x)**(3/2),
x)
```

3.463 $\int \frac{\sqrt{a+c\cot(d+ex)+b\csc(d+ex)}}{\sqrt{\csc(d+ex)}}\,dx$

Optimal. Leaf size=118

$$\frac{2\sqrt{a+b\csc(d+ex)+c\cot(d+ex)}\,E\left(\frac{1}{2}\left(d+ex-\tan^{-1}(c,a)\right)|\frac{2\sqrt{a^2+c^2}}{b+\sqrt{a^2+c^2}}\right)}{e\sqrt{\csc(d+ex)}\sqrt{\frac{a\sin(d+ex)+b+c\cos(d+ex)}{\sqrt{a^2+c^2}+b}}}$$

```
[Out] 2*(cos(1/2*d+1/2*e*x-1/2*arctan(c,a))^2)^(1/2)/cos(1/2*d+1/2*e*x-1/2*arctan
(c,a))*EllipticE(sin(1/2*d+1/2*e*x-1/2*arctan(c,a)),2^(1/2)*((a^2+c^2)^(1/2
)/(b+(a^2+c^2)^(1/2)))^(1/2))*(a+c*cot(e*x+d)+b*csc(e*x+d))^(1/2)/e/csc(e*x
+d)^(1/2)/((b+c*cos(e*x+d)+a*sin(e*x+d))/(b+(a^2+c^2)^(1/2)))^(1/2)
```

Rubi [A] time = 0.14, antiderivative size = 118, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {3168, 3119, 2653}

$$\frac{2\sqrt{a+b\csc(d+ex)+c\cot(d+ex)}\,E\left(\frac{1}{2}\left(d+ex-\tan^{-1}(c,a)\right)|\frac{2\sqrt{a^2+c^2}}{b+\sqrt{a^2+c^2}}\right)}{e\sqrt{\csc(d+ex)}\sqrt{\frac{a\sin(d+ex)+b+c\cos(d+ex)}{\sqrt{a^2+c^2}+b}}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + c*Cot[d + e*x] + b*Csc[d + e*x]]/Sqrt[Csc[d + e*x]],x]
```

```
[Out] (2*Sqrt[a + c*Cot[d + e*x] + b*Csc[d + e*x]]*EllipticE[(d + e*x - ArcTan[c,
 a])/2, (2*Sqrt[a^2 + c^2])/(b + Sqrt[a^2 + c^2])])/(e*Sqrt[Csc[d + e*x]]*S
qrt[(b + c*Cos[d + e*x] + a*Sin[d + e*x])/(b + Sqrt[a^2 + c^2])])
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 3119

```
Int[Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_
)]], x_Symbol] :> Dist[Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]]/Sqrt[(a +
b*Cos[d + e*x] + c*Sin[d + e*x])/(a + Sqrt[b^2 + c^2])], Int[Sqrt[a/(a + Sq
rt[b^2 + c^2]) + (Sqrt[b^2 + c^2]*Cos[d + e*x - ArcTan[b, c]])/(a + Sqrt[b^
2 + c^2])], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0]
```

```
&& NeQ[b^2 + c^2, 0] &&  !GtQ[a + Sqrt[b^2 + c^2], 0]
```

Rule 3168

```
Int[csc[(d_.) + (e_.)*(x_)]^(n_.)*((a_.) + csc[(d_.) + (e_.)*(x_)]*(b_.) +
cot[(d_.) + (e_.)*(x_)]*(c_.))^(m_), x_Symbol] :> Dist[(Csc[d + e*x]^n*(b +
 a*Sin[d + e*x] + c*Cos[d + e*x])^n)/(a + b*Csc[d + e*x] + c*Cot[d + e*x])^
n, Int[1/(b + a*Sin[d + e*x] + c*Cos[d + e*x])^n, x], x] /; FreeQ[{a, b, c,
 d, e}, x] && EqQ[m + n, 0] &&  !IntegerQ[n]
```

Rubi steps

$$\int \frac{\sqrt{a+c\cot(d+ex)+b\csc(d+ex)}}{\sqrt{\csc(d+ex)}}\,dx = \frac{\sqrt{a+c\cot(d+ex)+b\csc(d+ex)}\int\sqrt{b+c\cos(d+ex)+a\sin(d+}}{\sqrt{\csc(d+ex)}\sqrt{b+c\cos(d+ex)+a\sin(d+ex)}}$$

$$= \frac{\sqrt{a+c\cot(d+ex)+b\csc(d+ex)}\int\sqrt{\frac{b}{b+\sqrt{a^2+c^2}}+\frac{\sqrt{a^2+c^2}\cos\left(d+ex-\text{ta}\right.}{b+\sqrt{a^2+c^2}}}}{\sqrt{\csc(d+ex)}\sqrt{\frac{b+c\cos(d+ex)+a\sin(d+ex)}{b+\sqrt{a^2+c^2}}}}$$

$$= \frac{2\sqrt{a+c\cot(d+ex)+b\csc(d+ex)}\,E\left(\frac{1}{2}\left(d+ex-\tan^{-1}(c,a)\right)|\frac{2\sqrt{a^2}}{b+\sqrt{a}}\right.}{e\sqrt{\csc(d+ex)}\sqrt{\frac{b+c\cos(d+ex)+a\sin(d+ex)}{b+\sqrt{a^2+c^2}}}}$$

Mathematica [C] time = 6.24, size = 1580, normalized size = 13.39

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[a + c*Cot[d + e*x] + b*Csc[d + e*x]]/Sqrt[Csc[d + e*x]],x]
```

```
[Out] (2*c*Sqrt[a + c*Cot[d + e*x] + b*Csc[d + e*x]])/(a*e*Sqrt[Csc[d + e*x]]) +
(a*Sqrt[a + c*Cot[d + e*x] + b*Csc[d + e*x]]*(-((a*AppellF1[-1/2, -1/2, -1/
2, 1/2, -((b + Sqrt[1 + a^2/c^2]*c*Cos[d + e*x - ArcTan[a/c]])/(Sqrt[1 + a^
2/c^2]*(1 - b/(Sqrt[1 + a^2/c^2]*c))*c)), -((b + Sqrt[1 + a^2/c^2]*c*Cos[d
+ e*x - ArcTan[a/c]])/(Sqrt[1 + a^2/c^2]*(-1 - b/(Sqrt[1 + a^2/c^2]*c))*c))
]*Sin[d + e*x - ArcTan[a/c]])/(Sqrt[1 + a^2/c^2]*c*Sqrt[(c*Sqrt[(a^2 + c^2)
/c^2] - c*Sqrt[(a^2 + c^2)/c^2]*Cos[d + e*x - ArcTan[a/c]])/(b + c*Sqrt[(a^
2 + c^2)/c^2])]*Sqrt[b + c*Sqrt[(a^2 + c^2)/c^2]*Cos[d + e*x - ArcTan[a/c]]
]*Sqrt[(c*Sqrt[(a^2 + c^2)/c^2] + c*Sqrt[(a^2 + c^2)/c^2]*Cos[d + e*x - Arc
```

```
Tan[a/c]])/(-b + c*Sqrt[(a^2 + c^2)/c^2])])) - ((2*c*(b + Sqrt[1 + a^2/c^2]
*c*Cos[d + e*x - ArcTan[a/c]]))/(a^2 + c^2) - (a*Sin[d + e*x - ArcTan[a/c]]
)/(Sqrt[1 + a^2/c^2]*c))/Sqrt[b + Sqrt[1 + a^2/c^2]*c*Cos[d + e*x - ArcTan[
a/c]]]))/(e*Sqrt[Csc[d + e*x]]*Sqrt[b + c*Cos[d + e*x] + a*Sin[d + e*x]]) +
 (c^2*Sqrt[a + c*Cot[d + e*x] + b*Csc[d + e*x]]*(-((a*AppellF1[-1/2, -1/2,
-1/2, 1/2, -((b + Sqrt[1 + a^2/c^2]*c*Cos[d + e*x - ArcTan[a/c]])/(Sqrt[1 +
 a^2/c^2]*(1 - b/(Sqrt[1 + a^2/c^2]*c))*c)), -((b + Sqrt[1 + a^2/c^2]*c*Cos
[d + e*x - ArcTan[a/c]])/(Sqrt[1 + a^2/c^2]*(-1 - b/(Sqrt[1 + a^2/c^2]*c))*
c))]*Sin[d + e*x - ArcTan[a/c]])/(Sqrt[1 + a^2/c^2]*c*Sqrt[(c*Sqrt[(a^2 + c
^2)/c^2] - c*Sqrt[(a^2 + c^2)/c^2]*Cos[d + e*x - ArcTan[a/c]])/(b + c*Sqrt[
(a^2 + c^2)/c^2])]*Sqrt[b + c*Sqrt[(a^2 + c^2)/c^2]*Cos[d + e*x - ArcTan[a/
c]]]*Sqrt[(c*Sqrt[(a^2 + c^2)/c^2] + c*Sqrt[(a^2 + c^2)/c^2]*Cos[d + e*x -
ArcTan[a/c]])/(-b + c*Sqrt[(a^2 + c^2)/c^2])])) - ((2*c*(b + Sqrt[1 + a^2/c
^2]*c*Cos[d + e*x - ArcTan[a/c]]))/(a^2 + c^2) - (a*Sin[d + e*x - ArcTan[a/
c]])/(Sqrt[1 + a^2/c^2]*c))/Sqrt[b + Sqrt[1 + a^2/c^2]*c*Cos[d + e*x - ArcT
an[a/c]]]))/(a*e*Sqrt[Csc[d + e*x]]*Sqrt[b + c*Cos[d + e*x] + a*Sin[d + e*x
]]) + (2*b*AppellF1[1/2, 1/2, 1/2, 3/2, -((b + a*Sqrt[1 + c^2/a^2]*Sin[d +
e*x + ArcTan[c/a]])/(a*Sqrt[1 + c^2/a^2]*(1 - b/(a*Sqrt[1 + c^2/a^2])))), -
((b + a*Sqrt[1 + c^2/a^2]*Sin[d + e*x + ArcTan[c/a]])/(a*Sqrt[1 + c^2/a^2]*
(-1 - b/(a*Sqrt[1 + c^2/a^2]))))]*Sqrt[a + c*Cot[d + e*x] + b*Csc[d + e*x]]
*Sec[d + e*x + ArcTan[c/a]]*Sqrt[(a*Sqrt[(a^2 + c^2)/a^2] - a*Sqrt[(a^2 + c
^2)/a^2]*Sin[d + e*x + ArcTan[c/a]])/(b + a*Sqrt[(a^2 + c^2)/a^2])]*Sqrt[b
+ a*Sqrt[(a^2 + c^2)/a^2]*Sin[d + e*x + ArcTan[c/a]]]*Sqrt[(a*Sqrt[(a^2 + c
^2)/a^2] + a*Sqrt[(a^2 + c^2)/a^2]*Sin[d + e*x + ArcTan[c/a]])/(-b + a*Sqrt
[(a^2 + c^2)/a^2])])/(a*Sqrt[1 + c^2/a^2]*e*Sqrt[Csc[d + e*x]]*Sqrt[b + c*C
os[d + e*x] + a*Sin[d + e*x]])
```

fricas [F] time = 1.23, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{c\cot(ex+d)+b\csc(ex+d)+a}}{\sqrt{\csc(ex+d)}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+c*cot(e*x+d)+b*csc(e*x+d))^(1/2)/csc(e*x+d)^(1/2),x, algorithm
="fricas")
```

```
[Out] integral(sqrt(c*cot(e*x + d) + b*csc(e*x + d) + a)/sqrt(csc(e*x + d)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{c\cot(ex+d)+b\csc(ex+d)+a}}{\sqrt{\csc(ex+d)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+c*cot(e*x+d)+b*csc(e*x+d))^(1/2)/csc(e*x+d)^(1/2),x, algorithm
="giac")
```

```
[Out] integrate(sqrt(c*cot(e*x + d) + b*csc(e*x + d) + a)/sqrt(csc(e*x + d)), x)
```

maple [C] time = 1.76, size = 12367, normalized size = 104.81

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+c*cot(e*x+d)+b*csc(e*x+d))^(1/2)/csc(e*x+d)^(1/2),x)
```

```
[Out] result too large to display
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{c\cot\left(ex+d\right)+b\csc\left(ex+d\right)+a}}{\sqrt{\csc\left(ex+d\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+c*cot(e*x+d)+b*csc(e*x+d))^(1/2)/csc(e*x+d)^(1/2),x, algorithm
="maxima")
```

```
[Out] integrate(sqrt(c*cot(e*x + d) + b*csc(e*x + d) + a)/sqrt(csc(e*x + d)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{a+c\cot\left(d+e\,x\right)+\frac{b}{\sin\left(d+e\,x\right)}}}{\sqrt{\frac{1}{\sin\left(d+e\,x\right)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + c*cot(d + e*x) + b/sin(d + e*x))^(1/2)/(1/sin(d + e*x))^(1/2),x)
```

```
[Out] int((a + c*cot(d + e*x) + b/sin(d + e*x))^(1/2)/(1/sin(d + e*x))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a+b\csc\left(d+ex\right)+c\cot\left(d+ex\right)}}{\sqrt{\csc\left(d+ex\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+c*cot(e*x+d)+b*csc(e*x+d))**(1/2)/csc(e*x+d)**(1/2),x)
```

```
[Out] Integral(sqrt(a + b*csc(d + e*x) + c*cot(d + e*x))/sqrt(csc(d + e*x)), x)
```

3.464 $\int \frac{\sqrt{\csc(d+ex)}}{\sqrt{a+c\cot(d+ex)+b\csc(d+ex)}}\,dx$

Optimal. Leaf size=118

$$\frac{2\sqrt{\csc(d+ex)}\sqrt{\frac{a\sin(d+ex)+b+c\cos(d+ex)}{\sqrt{a^2+c^2}+b}}\,F\left(\frac{1}{2}\left(d+ex-\tan^{-1}(c,a)\right)|\frac{2\sqrt{a^2+c^2}}{b+\sqrt{a^2+c^2}}\right)}{e\sqrt{a+b\csc(d+ex)+c\cot(d+ex)}}$$

```
[Out] 2*(cos(1/2*d+1/2*e*x-1/2*arctan(c,a))^2)^(1/2)/cos(1/2*d+1/2*e*x-1/2*arctan
(c,a))*EllipticF(sin(1/2*d+1/2*e*x-1/2*arctan(c,a)),2^(1/2)*((a^2+c^2)^(1/2
)/(b+(a^2+c^2)^(1/2)))^(1/2))*csc(e*x+d)^(1/2)*((b+c*cos(e*x+d)+a*sin(e*x+d
))/(b+(a^2+c^2)^(1/2)))^(1/2)/e/(a+c*cot(e*x+d)+b*csc(e*x+d))^(1/2)
```

Rubi [A] time = 0.17, antiderivative size = 118, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {3168, 3127, 2661}

$$\frac{2\sqrt{\csc(d+ex)}\sqrt{\frac{a\sin(d+ex)+b+c\cos(d+ex)}{\sqrt{a^2+c^2}+b}}\,F\left(\frac{1}{2}\left(d+ex-\tan^{-1}(c,a)\right)|\frac{2\sqrt{a^2+c^2}}{b+\sqrt{a^2+c^2}}\right)}{e\sqrt{a+b\csc(d+ex)+c\cot(d+ex)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Csc[d + e*x]]/Sqrt[a + c*Cot[d + e*x] + b*Csc[d + e*x]],x]
```

```
[Out] (2*Sqrt[Csc[d + e*x]]*EllipticF[(d + e*x - ArcTan[c, a])/2, (2*Sqrt[a^2 + c
^2])/(b + Sqrt[a^2 + c^2])]*Sqrt[(b + c*Cos[d + e*x] + a*Sin[d + e*x])/(b +
 Sqrt[a^2 + c^2])])/(e*Sqrt[a + c*Cot[d + e*x] + b*Csc[d + e*x]])
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 3127

```
Int[1/Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(
x_)]], x_Symbol] :> Dist[Sqrt[(a + b*Cos[d + e*x] + c*Sin[d + e*x])/(a + Sq
rt[b^2 + c^2])]/Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]], Int[1/Sqrt[a/(a
+ Sqrt[b^2 + c^2]) + (Sqrt[b^2 + c^2]*Cos[d + e*x - ArcTan[b, c]])/(a + Sqr
t[b^2 + c^2])], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2,
 0] && NeQ[b^2 + c^2, 0] &&  !GtQ[a + Sqrt[b^2 + c^2], 0]
```

Rule 3168

```
Int[csc[(d_.) + (e_.)*(x_)]^(n_.)*((a_.) + csc[(d_.) + (e_.)*(x_)]*(b_.) +
cot[(d_.) + (e_.)*(x_)]*(c_.))^(m_), x_Symbol] :> Dist[(Csc[d + e*x]^n*(b +
 a*Sin[d + e*x] + c*Cos[d + e*x])^n)/(a + b*Csc[d + e*x] + c*Cot[d + e*x])^
n, Int[1/(b + a*Sin[d + e*x] + c*Cos[d + e*x])^n, x], x] /; FreeQ[{a, b, c,
 d, e}, x] && EqQ[m + n, 0] &&  !IntegerQ[n]
```

Rubi steps

$$\int \frac{\sqrt{\csc(d+ex)}}{\sqrt{a+c\cot(d+ex)+b\csc(d+ex)}}\,dx = \frac{\left(\sqrt{\csc(d+ex)}\sqrt{b+c\cos(d+ex)+a\sin(d+ex)}\right)\int \frac{1}{\sqrt{b+c\cos(d+ex)+a}}}{\sqrt{a+c\cot(d+ex)+b\csc(d+ex)}}$$

$$= \frac{\left(\sqrt{\csc(d+ex)}\sqrt{\frac{b+c\cos(d+ex)+a\sin(d+ex)}{b+\sqrt{a^2+c^2}}}\right)\int \frac{1}{\sqrt{\frac{b}{b+\sqrt{a^2+c^2}}+\frac{\sqrt{a^2+c^2}\cos(d+ex-\tan}{b+\sqrt{a^2+c^2}}}}}{\sqrt{a+c\cot(d+ex)+b\csc(d+ex)}}$$

$$= \frac{2\sqrt{\csc(d+ex)}\,F\left(\frac{1}{2}\left(d+ex-\tan^{-1}(c,a)\right)|\frac{2\sqrt{a^2+c^2}}{b+\sqrt{a^2+c^2}}\right)\sqrt{\frac{b+c\cos(d+ex)+a}{b+\sqrt{a^2+}}}}{e\sqrt{a+c\cot(d+ex)+b\csc(d+ex)}}$$

Mathematica [C] time = 0.91, size = 339, normalized size = 2.87

$$\frac{2\sqrt{\csc(d+ex)}\sec\left(\tan^{-1}\left(\frac{c}{a}\right)+d+ex\right)\sqrt{-\frac{a\sqrt{\frac{c^2}{a^2}+1}\left(\sin\left(\tan^{-1}\left(\frac{c}{a}\right)+d+ex\right)-1\right)}{a\sqrt{\frac{c^2}{a^2}+1}+b}}\sqrt{\frac{a\sqrt{\frac{c^2}{a^2}+1}\left(\sin\left(\tan^{-1}\left(\frac{c}{a}\right)+d+ex\right)+1\right)}{a\sqrt{\frac{c^2}{a^2}+1}-b}}\sqrt{a\sqrt{\frac{c^2}{a^2}+}}}{ae\sqrt{\frac{c^2}{a^2}+1}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[Csc[d + e*x]]/Sqrt[a + c*Cot[d + e*x] + b*Csc[d + e*x]],x]
```

```
[Out] (2*AppellF1[1/2, 1/2, 1/2, 3/2, (b + a*Sqrt[1 + c^2/a^2]*Sin[d + e*x + ArcT
an[c/a]])/(b - a*Sqrt[1 + c^2/a^2]), (b + a*Sqrt[1 + c^2/a^2]*Sin[d + e*x +
 ArcTan[c/a]])/(b + a*Sqrt[1 + c^2/a^2])]*Sqrt[Csc[d + e*x]]*Sec[d + e*x +
ArcTan[c/a]]*Sqrt[b + c*Cos[d + e*x] + a*Sin[d + e*x]]*Sqrt[-((a*Sqrt[1 + c
^2/a^2]*(-1 + Sin[d + e*x + ArcTan[c/a]]))/(b + a*Sqrt[1 + c^2/a^2]))]*Sqrt
[(a*Sqrt[1 + c^2/a^2]*(1 + Sin[d + e*x + ArcTan[c/a]]))/(-b + a*Sqrt[1 + c^
2/a^2])]*Sqrt[b + a*Sqrt[1 + c^2/a^2]*Sin[d + e*x + ArcTan[c/a]]])/(a*Sqrt[
1 + c^2/a^2]*e*Sqrt[a + c*Cot[d + e*x] + b*Csc[d + e*x]])
```

fricas [F] time = 0.67, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{\csc\left(ex+d\right)}}{\sqrt{c\cot\left(ex+d\right)+b\csc\left(ex+d\right)+a}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(e*x+d)^(1/2)/(a+c*cot(e*x+d)+b*csc(e*x+d))^(1/2),x, algorithm
="fricas")
```

```
[Out] integral(sqrt(csc(e*x + d))/sqrt(c*cot(e*x + d) + b*csc(e*x + d) + a), x)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(e*x+d)^(1/2)/(a+c*cot(e*x+d)+b*csc(e*x+d))^(1/2),x, algorithm
="giac")
```

```
[Out] Timed out
```

maple [C] time = 1.68, size = 713, normalized size = 6.04

$$-\frac{4i\sqrt{\frac{1}{\sin(ex+d)}}\sqrt{\frac{b+c\cos(ex+d)+a\sin(ex+d)}{\sin(ex+d)}}\sqrt{\frac{(i\sin(ex+d)+\cos(ex+d))\left(ib-ic-\sqrt{a^2-b^2+c^2}-a\right)}{ib-ic+\sqrt{a^2-b^2+c^2}+a}}\sqrt{-\frac{i\left(\cos(ex+d)\sqrt{a^2-b^2+c^2}-b\sin(ex+d)+c\sin\cdots\right.}{(i\cos(ex+d)+\sin(ex+d)+i)\cdots}}\cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(e*x+d)^(1/2)/(a+c*cot(e*x+d)+b*csc(e*x+d))^(1/2),x)
```

```
[Out] -4*I/e*(1/sin(e*x+d))^(1/2)*((b+c*cos(e*x+d)+a*sin(e*x+d))/sin(e*x+d))^(1/2
)*((I*sin(e*x+d)+cos(e*x+d))*(I*b-I*c-(a^2-b^2+c^2)^(1/2)-a)/(I*b-I*c+(a^2-
b^2+c^2)^(1/2)+a))^(1/2)*(-I*(cos(e*x+d)*(a^2-b^2+c^2)^(1/2)-b*sin(e*x+d)+c
*sin(e*x+d)-a*cos(e*x+d)+(a^2-b^2+c^2)^(1/2)-a)/(I*cos(e*x+d)+sin(e*x+d)+I)
/(I*b-I*c-(a^2-b^2+c^2)^(1/2)+a))^(1/2)*(I*(b*sin(e*x+d)-c*sin(e*x+d)+cos(e
*x+d)*(a^2-b^2+c^2)^(1/2)+a*cos(e*x+d)+(a^2-b^2+c^2)^(1/2)+a)/(I*cos(e*x+d)
+sin(e*x+d)+I)/(I*b-I*c+(a^2-b^2+c^2)^(1/2)+a))^(1/2)*(cos(e*x+d)+1)^2*Elli
pticF(((I*sin(e*x+d)+cos(e*x+d))*(I*b-I*c-(a^2-b^2+c^2)^(1/2)-a)/(I*b-I*c+(
a^2-b^2+c^2)^(1/2)+a))^(1/2),((I*b-I*c+(a^2-b^2+c^2)^(1/2)+a)*(I*b-I*c+(a^2
-b^2+c^2)^(1/2)-a)/(I*b-I*c-(a^2-b^2+c^2)^(1/2)-a)/(I*b-I*c-(a^2-b^2+c^2)^(
1/2)+a))^(1/2))*(cos(e*x+d)-1)^2*(I*cos(e*x+d)*b-I*cos(e*x+d)*c-I*(a^2-b^2+
```

```
c^2)^(1/2)*sin(e*x+d)-I*sin(e*x+d)*a+cos(e*x+d)*(a^2-b^2+c^2)^(1/2)+a*cos(e
*x+d)+b*sin(e*x+d)-c*sin(e*x+d))/sin(e*x+d)^3/(b+c*cos(e*x+d)+a*sin(e*x+d))
/(I*b-I*c-(a^2-b^2+c^2)^(1/2)-a)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\csc(ex+d)}}{\sqrt{c\cot(ex+d)+b\csc(ex+d)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(e*x+d)^(1/2)/(a+c*cot(e*x+d)+b*csc(e*x+d))^(1/2),x, algorithm
="maxima")
```

```
[Out] integrate(sqrt(csc(e*x + d))/sqrt(c*cot(e*x + d) + b*csc(e*x + d) + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{\frac{1}{\sin(d+e\,x)}}}{\sqrt{a+c\cot(d+e\,x)+\frac{b}{\sin(d+e\,x)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/sin(d + e*x))^(1/2)/(a + c*cot(d + e*x) + b/sin(d + e*x))^(1/2),x)
```

```
[Out] int((1/sin(d + e*x))^(1/2)/(a + c*cot(d + e*x) + b/sin(d + e*x))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\csc(d+ex)}}{\sqrt{a+b\csc(d+ex)+c\cot(d+ex)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(e*x+d)**(1/2)/(a+c*cot(e*x+d)+b*csc(e*x+d))**(1/2),x)
```

```
[Out] Integral(sqrt(csc(d + e*x))/sqrt(a + b*csc(d + e*x) + c*cot(d + e*x)), x)
```

3.465 $\int \frac{\csc^{\frac{3}{2}}(d+ex)}{(a+c\cot(d+ex)+b\csc(d+ex))^{3/2}}\,dx$

Optimal. Leaf size=240

$$-\frac{2\csc^{\frac{3}{2}}(d+ex)(a\sin(d+ex)+b+c\cos(d+ex))^2E\left(\frac{1}{2}\left(d+ex-\tan^{-1}(c,a)\right)|\frac{2\sqrt{a^2+c^2}}{b+\sqrt{a^2+c^2}}\right)}{e\left(a^2-b^2+c^2\right)\sqrt{\frac{a\sin(d+ex)+b+c\cos(d+ex)}{\sqrt{a^2+c^2}+b}}\,(a+b\csc(d+ex)+c\cot(d+ex))^{3/2}}-\frac{2\csc^{\frac{3}{2}}(d+ex)(a\cos(d}{e\left(a^2-b^2\right.}$$

```
[Out] -2*csc(e*x+d)^(3/2)*(b+c*cos(e*x+d)+a*sin(e*x+d))*(a*cos(e*x+d)-c*sin(e*x+d
))/(a^2-b^2+c^2)/e/(a+c*cot(e*x+d)+b*csc(e*x+d))^(3/2)-2*csc(e*x+d)^(3/2)*(
cos(1/2*d+1/2*e*x-1/2*arctan(c,a))^2)^(1/2)/cos(1/2*d+1/2*e*x-1/2*arctan(c,
a))*EllipticE(sin(1/2*d+1/2*e*x-1/2*arctan(c,a)),2^(1/2)*((a^2+c^2)^(1/2)/(
b+(a^2+c^2)^(1/2)))^(1/2))*(b+c*cos(e*x+d)+a*sin(e*x+d))^2/(a^2-b^2+c^2)/e/
(a+c*cot(e*x+d)+b*csc(e*x+d))^(3/2)/((b+c*cos(e*x+d)+a*sin(e*x+d))/(b+(a^2+
c^2)^(1/2)))^(1/2)
```

Rubi [A] time = 0.21, antiderivative size = 240, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.121, Rules used = {3168, 3128, 3119, 2653}

$$-\frac{2\csc^{\frac{3}{2}}(d+ex)(a\sin(d+ex)+b+c\cos(d+ex))^2E\left(\frac{1}{2}\left(d+ex-\tan^{-1}(c,a)\right)|\frac{2\sqrt{a^2+c^2}}{b+\sqrt{a^2+c^2}}\right)}{e\left(a^2-b^2+c^2\right)\sqrt{\frac{a\sin(d+ex)+b+c\cos(d+ex)}{\sqrt{a^2+c^2}+b}}\,(a+b\csc(d+ex)+c\cot(d+ex))^{3/2}}-\frac{2\csc^{\frac{3}{2}}(d+ex)(a\cos(d}{e\left(a^2-b^2\right.}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[d + e*x]^(3/2)/(a + c*Cot[d + e*x] + b*Csc[d + e*x])^(3/2),x]
```

```
[Out] (-2*Csc[d + e*x]^(3/2)*EllipticE[(d + e*x - ArcTan[c, a])/2, (2*Sqrt[a^2 +
c^2])/(b + Sqrt[a^2 + c^2])]*(b + c*Cos[d + e*x] + a*Sin[d + e*x])^2)/((a^2
 - b^2 + c^2)*e*(a + c*Cot[d + e*x] + b*Csc[d + e*x])^(3/2)*Sqrt[(b + c*Cos
[d + e*x] + a*Sin[d + e*x])/(b + Sqrt[a^2 + c^2])]) - (2*Csc[d + e*x]^(3/2)
*(b + c*Cos[d + e*x] + a*Sin[d + e*x])*(a*Cos[d + e*x] - c*Sin[d + e*x]))/(
(a^2 - b^2 + c^2)*e*(a + c*Cot[d + e*x] + b*Csc[d + e*x])^(3/2))
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 3119

```
Int[Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_
)]], x_Symbol] :> Dist[Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]]/Sqrt[(a +
b*Cos[d + e*x] + c*Sin[d + e*x])/(a + Sqrt[b^2 + c^2])], Int[Sqrt[a/(a + Sq
rt[b^2 + c^2]) + (Sqrt[b^2 + c^2]*Cos[d + e*x - ArcTan[b, c]])/(a + Sqrt[b^
2 + c^2])], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0]
&& NeQ[b^2 + c^2, 0] &&  !GtQ[a + Sqrt[b^2 + c^2], 0]
```

Rule 3128

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(-3/2), x_Symbol] :> Simp[(2*(c*Cos[d + e*x] - b*Sin[d + e*x]))/(e*(a^2 - b
^2 - c^2)*Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]]), x] + Dist[1/(a^2 - b^
2 - c^2), Int[Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]], x], x] /; FreeQ[{a
, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0]
```

Rule 3168

```
Int[csc[(d_.) + (e_.)*(x_)]^(n_.)*((a_.) + csc[(d_.) + (e_.)*(x_)]*(b_.) +
cot[(d_.) + (e_.)*(x_)]*(c_.))^(m_), x_Symbol] :> Dist[(Csc[d + e*x]^n*(b +
 a*Sin[d + e*x] + c*Cos[d + e*x])^n)/(a + b*Csc[d + e*x] + c*Cot[d + e*x])^
n, Int[1/(b + a*Sin[d + e*x] + c*Cos[d + e*x])^n, x], x] /; FreeQ[{a, b, c,
 d, e}, x] && EqQ[m + n, 0] &&  !IntegerQ[n]
```

Rubi steps

$$\int \frac{\csc^{\frac{3}{2}}(d+ex)}{(a+c\cot(d+ex)+b\csc(d+ex))^{3/2}}\,dx = \frac{\left(\csc^{\frac{3}{2}}(d+ex)(b+c\cos(d+ex)+a\sin(d+ex))^{3/2}\right)\int \frac{}{(b+c\cos(d+ex)}}{(a+c\cot(d+ex)+b\csc(d+ex))^{3/2}}$$

$$= -\frac{2\csc^{\frac{3}{2}}(d+ex)(b+c\cos(d+ex)+a\sin(d+ex))(a\cos(d+ex)-}{\left(a^2-b^2+c^2\right)e(a+c\cot(d+ex)+b\csc(d+ex))^{3/2}}$$

$$= -\frac{2\csc^{\frac{3}{2}}(d+ex)(b+c\cos(d+ex)+a\sin(d+ex))(a\cos(d+ex)-}{\left(a^2-b^2+c^2\right)e(a+c\cot(d+ex)+b\csc(d+ex))^{3/2}}$$

$$= -\frac{2\csc^{\frac{3}{2}}(d+ex)E\left(\frac{1}{2}\left(d+ex-\tan^{-1}(c,a)\right)|\frac{2\sqrt{a^2+c^2}}{b+\sqrt{a^2+c^2}}\right)(b+c\cos(d}{\left(a^2-b^2+c^2\right)e(a+c\cot(d+ex)+b\csc(d+ex))^{3/2}\sqrt{\frac{b+c\,c}{}}}$$

Mathematica [C] time = 6.40, size = 1732, normalized size = 7.22

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[Csc[d + e*x]^(3/2)/(a + c*Cot[d + e*x] + b*Csc[d + e*x])^(3/2),x]
```

```
[Out] (Csc[d + e*x]^(3/2)*(b + c*Cos[d + e*x] + a*Sin[d + e*x])^2*((-2*(a^2 + c^2
))/(a*c*(a^2 - b^2 + c^2)) + (2*(a*b + a^2*Sin[d + e*x] + c^2*Sin[d + e*x])
)/(c*(a^2 - b^2 + c^2)*(b + c*Cos[d + e*x] + a*Sin[d + e*x]))))/(e*(a + c*C
ot[d + e*x] + b*Csc[d + e*x])^(3/2)) - (a*Csc[d + e*x]^(3/2)*(b + c*Cos[d +
 e*x] + a*Sin[d + e*x])^(3/2)*(-((a*AppellF1[-1/2, -1/2, -1/2, 1/2, -((b +
Sqrt[1 + a^2/c^2]*c*Cos[d + e*x - ArcTan[a/c]])/(Sqrt[1 + a^2/c^2]*(1 - b/(
Sqrt[1 + a^2/c^2]*c))*c)), -((b + Sqrt[1 + a^2/c^2]*c*Cos[d + e*x - ArcTan[
a/c]])/(Sqrt[1 + a^2/c^2]*(-1 - b/(Sqrt[1 + a^2/c^2]*c))*c))]*Sin[d + e*x -
 ArcTan[a/c]])/(Sqrt[1 + a^2/c^2]*c*Sqrt[(c*Sqrt[(a^2 + c^2)/c^2] - c*Sqrt[
(a^2 + c^2)/c^2]*Cos[d + e*x - ArcTan[a/c]])/(b + c*Sqrt[(a^2 + c^2)/c^2])]
*Sqrt[b + c*Sqrt[(a^2 + c^2)/c^2]*Cos[d + e*x - ArcTan[a/c]]]*Sqrt[(c*Sqrt[
(a^2 + c^2)/c^2] + c*Sqrt[(a^2 + c^2)/c^2]*Cos[d + e*x - ArcTan[a/c]])/(-b
+ c*Sqrt[(a^2 + c^2)/c^2])])) - ((2*c*(b + Sqrt[1 + a^2/c^2]*c*Cos[d + e*x
- ArcTan[a/c]]))/(a^2 + c^2) - (a*Sin[d + e*x - ArcTan[a/c]])/(Sqrt[1 + a^2
/c^2]*c))/Sqrt[b + Sqrt[1 + a^2/c^2]*c*Cos[d + e*x - ArcTan[a/c]]]))/((a^2
- b^2 + c^2)*e*(a + c*Cot[d + e*x] + b*Csc[d + e*x])^(3/2)) - (c^2*Csc[d +
e*x]^(3/2)*(b + c*Cos[d + e*x] + a*Sin[d + e*x])^(3/2)*(-((a*AppellF1[-1/2,
 -1/2, -1/2, 1/2, -((b + Sqrt[1 + a^2/c^2]*c*Cos[d + e*x - ArcTan[a/c]])/(S
qrt[1 + a^2/c^2]*(1 - b/(Sqrt[1 + a^2/c^2]*c))*c)), -((b + Sqrt[1 + a^2/c^2
]*c*Cos[d + e*x - ArcTan[a/c]])/(Sqrt[1 + a^2/c^2]*(-1 - b/(Sqrt[1 + a^2/c^
2]*c))*c))]*Sin[d + e*x - ArcTan[a/c]])/(Sqrt[1 + a^2/c^2]*c*Sqrt[(c*Sqrt[(
a^2 + c^2)/c^2] - c*Sqrt[(a^2 + c^2)/c^2]*Cos[d + e*x - ArcTan[a/c]])/(b +
c*Sqrt[(a^2 + c^2)/c^2])]*Sqrt[b + c*Sqrt[(a^2 + c^2)/c^2]*Cos[d + e*x - Ar
cTan[a/c]]]*Sqrt[(c*Sqrt[(a^2 + c^2)/c^2] + c*Sqrt[(a^2 + c^2)/c^2]*Cos[d +
 e*x - ArcTan[a/c]])/(-b + c*Sqrt[(a^2 + c^2)/c^2])])) - ((2*c*(b + Sqrt[1
+ a^2/c^2]*c*Cos[d + e*x - ArcTan[a/c]]))/(a^2 + c^2) - (a*Sin[d + e*x - Ar
cTan[a/c]])/(Sqrt[1 + a^2/c^2]*c))/Sqrt[b + Sqrt[1 + a^2/c^2]*c*Cos[d + e*x
 - ArcTan[a/c]]]))/(a*(a^2 - b^2 + c^2)*e*(a + c*Cot[d + e*x] + b*Csc[d + e
*x])^(3/2)) - (2*b*AppellF1[1/2, 1/2, 1/2, 3/2, -((b + a*Sqrt[1 + c^2/a^2]*
Sin[d + e*x + ArcTan[c/a]])/(a*Sqrt[1 + c^2/a^2]*(1 - b/(a*Sqrt[1 + c^2/a^2
])))), -((b + a*Sqrt[1 + c^2/a^2]*Sin[d + e*x + ArcTan[c/a]])/(a*Sqrt[1 + c
^2/a^2]*(-1 - b/(a*Sqrt[1 + c^2/a^2]))))]*Csc[d + e*x]^(3/2)*Sec[d + e*x +
ArcTan[c/a]]*(b + c*Cos[d + e*x] + a*Sin[d + e*x])^(3/2)*Sqrt[(a*Sqrt[(a^2
+ c^2)/a^2] - a*Sqrt[(a^2 + c^2)/a^2]*Sin[d + e*x + ArcTan[c/a]])/(b + a*Sq
rt[(a^2 + c^2)/a^2])]*Sqrt[b + a*Sqrt[(a^2 + c^2)/a^2]*Sin[d + e*x + ArcTan
[c/a]]]*Sqrt[(a*Sqrt[(a^2 + c^2)/a^2] + a*Sqrt[(a^2 + c^2)/a^2]*Sin[d + e*x
 + ArcTan[c/a]])/(-b + a*Sqrt[(a^2 + c^2)/a^2])])/(a*(a^2 - b^2 + c^2)*Sqrt
[1 + c^2/a^2]*e*(a + c*Cot[d + e*x] + b*Csc[d + e*x])^(3/2))
```

fricas [F] time = 1.10, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{c\cot\left(ex+d\right)+b\csc\left(ex+d\right)+a}\,\csc\left(ex+d\right)^{\frac{3}{2}}}{c^{2}\cot\left(ex+d\right)^{2}+b^{2}\csc\left(ex+d\right)^{2}+2\,ac\cot\left(ex+d\right)+a^{2}+2\,\left(bc\cot\left(ex+d\right)+ab\right)\csc\left(ex+d\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(e*x+d)^(3/2)/(a+c*cot(e*x+d)+b*csc(e*x+d))^(3/2),x, algorithm
="fricas")
```

```
[Out] integral(sqrt(c*cot(e*x + d) + b*csc(e*x + d) + a)*csc(e*x + d)^(3/2)/(c^2*
cot(e*x + d)^2 + b^2*csc(e*x + d)^2 + 2*a*c*cot(e*x + d) + a^2 + 2*(b*c*cot
(e*x + d) + a*b)*csc(e*x + d)), x)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(e*x+d)^(3/2)/(a+c*cot(e*x+d)+b*csc(e*x+d))^(3/2),x, algorithm
="giac")
```

```
[Out] Timed out
```

maple [C] time = 1.12, size = 12236, normalized size = 50.98

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(e*x+d)^(3/2)/(a+c*cot(e*x+d)+b*csc(e*x+d))^(3/2),x)
```

```
[Out] result too large to display
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc\left(ex+d\right)^{\frac{3}{2}}}{\left(c\cot\left(ex+d\right)+b\csc\left(ex+d\right)+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(e*x+d)^(3/2)/(a+c*cot(e*x+d)+b*csc(e*x+d))^(3/2),x, algorithm
="maxima")
```

```
[Out] integrate(csc(e*x + d)^(3/2)/(c*cot(e*x + d) + b*csc(e*x + d) + a)^(3/2), x
)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(\frac{1}{\sin(d+e\,x)}\right)^{3/2}}{\left(a+c\cot\left(d+e\,x\right)+\frac{b}{\sin(d+e\,x)}\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/sin(d + e*x))^(3/2)/(a + c*cot(d + e*x) + b/sin(d + e*x))^(3/2),x)
```

```
[Out] int((1/sin(d + e*x))^(3/2)/(a + c*cot(d + e*x) + b/sin(d + e*x))^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc^{\frac{3}{2}}\left(d+ex\right)}{\left(a+b\csc\left(d+ex\right)+c\cot\left(d+ex\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(e*x+d)**(3/2)/(a+c*cot(e*x+d)+b*csc(e*x+d))**(3/2),x)
```

```
[Out] Integral(csc(d + e*x)**(3/2)/(a + b*csc(d + e*x) + c*cot(d + e*x))**(3/2),
x)
```

3.466 $\int \frac{\csc^{\frac{5}{2}}(d+ex)}{(a+c\cot(d+ex)+b\csc(d+ex))^{5/2}}\,dx$

Optimal. Leaf size=492

$$\frac{2\csc^{\frac{5}{2}}(d+ex)\sqrt{\frac{a\sin(d+ex)+b+c\cos(d+ex)}{\sqrt{a^2+c^2}+b}}\,(a\sin(d+ex)+b+c\cos(d+ex))^2F\left(\frac{1}{2}\left(d+ex-\tan^{-1}(c,a)\right)|\frac{2\sqrt{a^2+c^2}}{b+\sqrt{a^2+c^2}}\right)}{3e\left(a^2-b^2+c^2\right)(a+b\csc(d+ex)+c\cot(d+ex))^{5/2}}+$$

```
[Out] -2/3*csc(e*x+d)^(5/2)*(b+c*cos(e*x+d)+a*sin(e*x+d))*(a*cos(e*x+d)-c*sin(e*x
+d))/(a^2-b^2+c^2)/e/(a+c*cot(e*x+d)+b*csc(e*x+d))^(5/2)+8/3*csc(e*x+d)^(5/
2)*(b+c*cos(e*x+d)+a*sin(e*x+d))^2*(a*b*cos(e*x+d)-b*c*sin(e*x+d))/(a^2-b^2
+c^2)^2/e/(a+c*cot(e*x+d)+b*csc(e*x+d))^(5/2)+8/3*b*csc(e*x+d)^(5/2)*(cos(1
/2*d+1/2*e*x-1/2*arctan(c,a))^2)^(1/2)/cos(1/2*d+1/2*e*x-1/2*arctan(c,a))*E
llipticE(sin(1/2*d+1/2*e*x-1/2*arctan(c,a)),2^(1/2)*((a^2+c^2)^(1/2)/(b+(a^
2+c^2)^(1/2)))^(1/2))*(b+c*cos(e*x+d)+a*sin(e*x+d))^3/(a^2-b^2+c^2)^2/e/(a+
c*cot(e*x+d)+b*csc(e*x+d))^(5/2)/((b+c*cos(e*x+d)+a*sin(e*x+d))/(b+(a^2+c^2
)^(1/2)))^(1/2)+2/3*csc(e*x+d)^(5/2)*(cos(1/2*d+1/2*e*x-1/2*arctan(c,a))^2)
^(1/2)/cos(1/2*d+1/2*e*x-1/2*arctan(c,a))*EllipticF(sin(1/2*d+1/2*e*x-1/2*a
rctan(c,a)),2^(1/2)*((a^2+c^2)^(1/2)/(b+(a^2+c^2)^(1/2)))^(1/2))*(b+c*cos(e
*x+d)+a*sin(e*x+d))^2*((b+c*cos(e*x+d)+a*sin(e*x+d))/(b+(a^2+c^2)^(1/2)))^(
1/2)/(a^2-b^2+c^2)/e/(a+c*cot(e*x+d)+b*csc(e*x+d))^(5/2)
```

Rubi [A] time = 0.50, antiderivative size = 492, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.242, Rules used = {3168, 3129, 3156, 3149, 3119, 2653, 3127, 2661}

$$\frac{2\csc^{\frac{5}{2}}(d+ex)\sqrt{\frac{a\sin(d+ex)+b+c\cos(d+ex)}{\sqrt{a^2+c^2}+b}}\,(a\sin(d+ex)+b+c\cos(d+ex))^2F\left(\frac{1}{2}\left(d+ex-\tan^{-1}(c,a)\right)|\frac{2\sqrt{a^2+c^2}}{b+\sqrt{a^2+c^2}}\right)}{3e\left(a^2-b^2+c^2\right)(a+b\csc(d+ex)+c\cot(d+ex))^{5/2}}+$$

Antiderivative was successfully verified.

```
[In] Int[Csc[d + e*x]^(5/2)/(a + c*Cot[d + e*x] + b*Csc[d + e*x])^(5/2),x]
```

```
[Out] (8*b*Csc[d + e*x]^(5/2)*EllipticE[(d + e*x - ArcTan[c, a])/2, (2*Sqrt[a^2 +
 c^2])/(b + Sqrt[a^2 + c^2])]*(b + c*Cos[d + e*x] + a*Sin[d + e*x])^3)/(3*(
a^2 - b^2 + c^2)^2*e*(a + c*Cot[d + e*x] + b*Csc[d + e*x])^(5/2)*Sqrt[(b +
c*Cos[d + e*x] + a*Sin[d + e*x])/(b + Sqrt[a^2 + c^2])]) + (2*Csc[d + e*x]^
(5/2)*EllipticF[(d + e*x - ArcTan[c, a])/2, (2*Sqrt[a^2 + c^2])/(b + Sqrt[a
^2 + c^2])]*(b + c*Cos[d + e*x] + a*Sin[d + e*x])^2*Sqrt[(b + c*Cos[d + e*x
```

```
] + a*Sin[d + e*x])/(b + Sqrt[a^2 + c^2])]])/(3*(a^2 - b^2 + c^2)*e*(a + c*C
ot[d + e*x] + b*Csc[d + e*x])^(5/2)) - (2*Csc[d + e*x]^(5/2)*(b + c*Cos[d +
 e*x] + a*Sin[d + e*x])*(a*Cos[d + e*x] - c*Sin[d + e*x]))/(3*(a^2 - b^2 +
c^2)*e*(a + c*Cot[d + e*x] + b*Csc[d + e*x])^(5/2)) + (8*Csc[d + e*x]^(5/2)
*(b + c*Cos[d + e*x] + a*Sin[d + e*x])^2*(a*b*Cos[d + e*x] - b*c*Sin[d + e*
x]))/(3*(a^2 - b^2 + c^2)^2*e*(a + c*Cot[d + e*x] + b*Csc[d + e*x])^(5/2))
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 3119

```
Int[Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_
)]], x_Symbol] :> Dist[Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]]/Sqrt[(a +
b*Cos[d + e*x] + c*Sin[d + e*x])/(a + Sqrt[b^2 + c^2])], Int[Sqrt[a/(a + Sq
rt[b^2 + c^2]) + (Sqrt[b^2 + c^2]*Cos[d + e*x - ArcTan[b, c]])/(a + Sqrt[b^
2 + c^2])], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0]
&& NeQ[b^2 + c^2, 0] &&  !GtQ[a + Sqrt[b^2 + c^2], 0]
```

Rule 3127

```
Int[1/Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(
x_)]], x_Symbol] :> Dist[Sqrt[(a + b*Cos[d + e*x] + c*Sin[d + e*x])/(a + Sq
rt[b^2 + c^2])]/Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]], Int[1/Sqrt[a/(a
+ Sqrt[b^2 + c^2]) + (Sqrt[b^2 + c^2]*Cos[d + e*x - ArcTan[b, c]])/(a + Sqr
t[b^2 + c^2])], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2,
 0] && NeQ[b^2 + c^2, 0] &&  !GtQ[a + Sqrt[b^2 + c^2], 0]
```

Rule 3129

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(n_), x_Symbol] :> Simp[((-(c*Cos[d + e*x]) + b*Sin[d + e*x])*(a + b*Cos[d
+ e*x] + c*Sin[d + e*x])^(n + 1))/(e*(n + 1)*(a^2 - b^2 - c^2)), x] + Dist[
1/((n + 1)*(a^2 - b^2 - c^2)), Int[(a*(n + 1) - b*(n + 2)*Cos[d + e*x] - c*
(n + 2)*Sin[d + e*x])*(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n + 1), x], x]
 /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0] && LtQ[n, -1] && N
```

```
eQ[n, -3/2]
```

Rule 3149

```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])
/Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)]]
, x_Symbol] :> Dist[B/b, Int[Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]], x],
 x] + Dist[(A*b - a*B)/b, Int[1/Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]],
x], x] /; FreeQ[{a, b, c, d, e, A, B, C}, x] && EqQ[B*c - b*C, 0] && NeQ[A*
b - a*B, 0]
```

Rule 3156

```
Int[((a_.) + cos[(d_.) + (e_.)*(x_)]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)])
^(n_)*((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)
]), x_Symbol] :> -Simp[((c*B - b*C - (a*C - c*A)*Cos[d + e*x] + (a*B - b*A)
*Sin[d + e*x])*(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n + 1))/(e*(n + 1)*(a
^2 - b^2 - c^2)), x] + Dist[1/((n + 1)*(a^2 - b^2 - c^2)), Int[(a + b*Cos[d
 + e*x] + c*Sin[d + e*x])^(n + 1)*Simp[(n + 1)*(a*A - b*B - c*C) + (n + 2)*
(a*B - b*A)*Cos[d + e*x] + (n + 2)*(a*C - c*A)*Sin[d + e*x], x], x], x] /;
FreeQ[{a, b, c, d, e, A, B, C}, x] && LtQ[n, -1] && NeQ[a^2 - b^2 - c^2, 0]
 && NeQ[n, -2]
```

Rule 3168

```
Int[csc[(d_.) + (e_.)*(x_)]^(n_.)*((a_.) + csc[(d_.) + (e_.)*(x_)]*(b_.) +
cot[(d_.) + (e_.)*(x_)]*(c_.))^(m_), x_Symbol] :> Dist[(Csc[d + e*x]^n*(b +
 a*Sin[d + e*x] + c*Cos[d + e*x])^n)/(a + b*Csc[d + e*x] + c*Cot[d + e*x])^
n, Int[1/(b + a*Sin[d + e*x] + c*Cos[d + e*x])^n, x], x] /; FreeQ[{a, b, c,
 d, e}, x] && EqQ[m + n, 0] &&  !IntegerQ[n]
```

Rubi steps

$$\int \frac{\csc^{\frac{5}{2}}(d+ex)}{(a+c\cot(d+ex)+b\csc(d+ex))^{5/2}}\,dx = \frac{\left(\csc^{\frac{5}{2}}(d+ex)(b+c\cos(d+ex)+a\sin(d+ex))^{5/2}\right)\int \frac{1}{(b+c\cos(d+ex)+a}}{(a+c\cot(d+ex)+b\csc(d+ex))^{5/2}}$$

$$= -\frac{2\csc^{\frac{5}{2}}(d+ex)(b+c\cos(d+ex)+a\sin(d+ex))(a\cos(d+ex)-c}{3\left(a^2-b^2+c^2\right)e(a+c\cot(d+ex)+b\csc(d+ex))^{5/2}}$$

$$= -\frac{2\csc^{\frac{5}{2}}(d+ex)(b+c\cos(d+ex)+a\sin(d+ex))(a\cos(d+ex)-c}{3\left(a^2-b^2+c^2\right)e(a+c\cot(d+ex)+b\csc(d+ex))^{5/2}}$$

$$= -\frac{2\csc^{\frac{5}{2}}(d+ex)(b+c\cos(d+ex)+a\sin(d+ex))(a\cos(d+ex)-c}{3\left(a^2-b^2+c^2\right)e(a+c\cot(d+ex)+b\csc(d+ex))^{5/2}}$$

$$= -\frac{2\csc^{\frac{5}{2}}(d+ex)(b+c\cos(d+ex)+a\sin(d+ex))(a\cos(d+ex)-c}{3\left(a^2-b^2+c^2\right)e(a+c\cot(d+ex)+b\csc(d+ex))^{5/2}}$$

$$= \frac{8b\csc^{\frac{5}{2}}(d+ex)E\left(\frac{1}{2}\left(d+ex-\tan^{-1}(c,a)\right)|\frac{2\sqrt{a^2+c^2}}{b+\sqrt{a^2+c^2}}\right)(b+c\cos(d+}{3\left(a^2-b^2+c^2\right)^2e(a+c\cot(d+ex)+b\csc(d+ex))^{5/2}\sqrt{\frac{b+c\cos}{}}}$$

Mathematica [C] time = 6.49, size = 2708, normalized size = 5.50

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Csc[d + e*x]^(5/2)/(a + c*Cot[d + e*x] + b*Csc[d + e*x])^(5/2),x]
```

```
[Out] (Csc[d + e*x]^(5/2)*(b + c*Cos[d + e*x] + a*Sin[d + e*x])^3*((8*b*(a^2 + c^
2))/(3*a*c*(-a^2 + b^2 - c^2)^2) + (2*(a*b + a^2*Sin[d + e*x] + c^2*Sin[d +
 e*x]))/(3*c*(a^2 - b^2 + c^2)*(b + c*Cos[d + e*x] + a*Sin[d + e*x])^2) - (
2*(a^3 + 3*a*b^2 + a*c^2 + 4*a^2*b*Sin[d + e*x] + 4*b*c^2*Sin[d + e*x]))/(3
*c*(a^2 - b^2 + c^2)^2*(b + c*Cos[d + e*x] + a*Sin[d + e*x]))))/(e*(a + c*C
ot[d + e*x] + b*Csc[d + e*x])^(5/2)) + (4*a*b*Csc[d + e*x]^(5/2)*(b + c*Cos
[d + e*x] + a*Sin[d + e*x])^(5/2)*(-((a*AppellF1[-1/2, -1/2, -1/2, 1/2, -((
b + Sqrt[1 + a^2/c^2]*c*Cos[d + e*x - ArcTan[a/c]])/(Sqrt[1 + a^2/c^2]*(1 -
```

b/(Sqrt[1 + a^2/c^2]*c))*c)), -((b + Sqrt[1 + a^2/c^2]*c*Cos[d + e*x - Arc Tan[a/c]])/(Sqrt[1 + a^2/c^2]*(-1 - b/(Sqrt[1 + a^2/c^2]*c))*c))]*Sin[d + e *x - ArcTan[a/c]])/(Sqrt[1 + a^2/c^2]*c*Sqrt[(c*Sqrt[(a^2 + c^2)/c^2] - c*S qrt[(a^2 + c^2)/c^2]*Cos[d + e*x - ArcTan[a/c]])/(b + c*Sqrt[(a^2 + c^2)/c^ 2])]*Sqrt[b + c*Sqrt[(a^2 + c^2)/c^2]*Cos[d + e*x - ArcTan[a/c]]]*Sqrt[(c*S qrt[(a^2 + c^2)/c^2] + c*Sqrt[(a^2 + c^2)/c^2]*Cos[d + e*x - ArcTan[a/c]])/ (-b + c*Sqrt[(a^2 + c^2)/c^2])])) - ((2*c*(b + Sqrt[1 + a^2/c^2]*c*Cos[d + e*x - ArcTan[a/c]]))/(a^2 + c^2) - (a*Sin[d + e*x - ArcTan[a/c]])/(Sqrt[1 + a^2/c^2]*c))/Sqrt[b + Sqrt[1 + a^2/c^2]*c*Cos[d + e*x - ArcTan[a/c]]]))/(3 *(a^2 - b^2 + c^2)^2*e*(a + c*Cot[d + e*x] + b*Csc[d + e*x])^(5/2)) + (4*b* c^2*Csc[d + e*x]^(5/2)*(b + c*Cos[d + e*x] + a*Sin[d + e*x])^(5/2)*(-((a*Ap pellF1[-1/2, -1/2, -1/2, 1/2, -((b + Sqrt[1 + a^2/c^2]*c*Cos[d + e*x - ArcT an[a/c]])/(Sqrt[1 + a^2/c^2]*(1 - b/(Sqrt[1 + a^2/c^2]*c))*c)), -((b + Sqrt [1 + a^2/c^2]*c*Cos[d + e*x - ArcTan[a/c]])/(Sqrt[1 + a^2/c^2]*(-1 - b/(Sqr t[1 + a^2/c^2]*c))*c))]*Sin[d + e*x - ArcTan[a/c]])/(Sqrt[1 + a^2/c^2]*c*Sq rt[(c*Sqrt[(a^2 + c^2)/c^2] - c*Sqrt[(a^2 + c^2)/c^2]*Cos[d + e*x - ArcTan[a/c]])/(b + c*Sqrt[(a^2 + c^2)/c^2])]*Sqrt[b + c*Sqrt[(a^2 + c^2)/c^2]*Cos[d + e*x - ArcTan[a/c]]]*Sqrt[(c*Sqrt[(a^2 + c^2)/c^2] + c*Sqrt[(a^2 + c^2)/ c^2]*Cos[d + e*x - ArcTan[a/c]])/(-b + c*Sqrt[(a^2 + c^2)/c^2])])) - ((2*c* (b + Sqrt[1 + a^2/c^2]*c*Cos[d + e*x - ArcTan[a/c]]))/(a^2 + c^2) - (a*Sin[d + e*x - ArcTan[a/c]])/(Sqrt[1 + a^2/c^2]*c))/Sqrt[b + Sqrt[1 + a^2/c^2]*c *Cos[d + e*x - ArcTan[a/c]]]))/(3*a*(a^2 - b^2 + c^2)^2*e*(a + c*Cot[d + e* x] + b*Csc[d + e*x])^(5/2)) + (2*a*AppellF1[1/2, 1/2, 1/2, 3/2, -((b + a*Sq rt[1 + c^2/a^2]*Sin[d + e*x + ArcTan[c/a]])/(a*Sqrt[1 + c^2/a^2]*(1 - b/(a* Sqrt[1 + c^2/a^2])))), -((b + a*Sqrt[1 + c^2/a^2]*Sin[d + e*x + ArcTan[c/a]])/(a*Sqrt[1 + c^2/a^2]*(-1 - b/(a*Sqrt[1 + c^2/a^2]))))]*Csc[d + e*x]^(5/2)*Sec[d + e*x + ArcTan[c/a]]*(b + c*Cos[d + e*x] + a*Sin[d + e*x])^(5/2)*Sq rt[(a*Sqrt[(a^2 + c^2)/a^2] - a*Sqrt[(a^2 + c^2)/a^2]*Sin[d + e*x + ArcTan[c/a]])/(b + a*Sqrt[(a^2 + c^2)/a^2])]*Sqrt[b + a*Sqrt[(a^2 + c^2)/a^2]*Sin[d + e*x + ArcTan[c/a]]]*Sqrt[(a*Sqrt[(a^2 + c^2)/a^2] + a*Sqrt[(a^2 + c^2)/ a^2]*Sin[d + e*x + ArcTan[c/a]])/(-b + a*Sqrt[(a^2 + c^2)/a^2])])/(3*(a^2 - b^2 + c^2)^2*Sqrt[1 + c^2/a^2]*e*(a + c*Cot[d + e*x] + b*Csc[d + e*x])^(5/ 2)) + (2*b^2*AppellF1[1/2, 1/2, 1/2, 3/2, -((b + a*Sqrt[1 + c^2/a^2]*Sin[d + e*x + ArcTan[c/a]])/(a*Sqrt[1 + c^2/a^2]*(1 - b/(a*Sqrt[1 + c^2/a^2])))), -((b + a*Sqrt[1 + c^2/a^2]*Sin[d + e*x + ArcTan[c/a]])/(a*Sqrt[1 + c^2/a^2]*(-1 - b/(a*Sqrt[1 + c^2/a^2]))))]*Csc[d + e*x]^(5/2)*Sec[d + e*x + ArcTan [c/a]]*(b + c*Cos[d + e*x] + a*Sin[d + e*x])^(5/2)*Sqrt[(a*Sqrt[(a^2 + c^2) /a^2] - a*Sqrt[(a^2 + c^2)/a^2]*Sin[d + e*x + ArcTan[c/a]])/(b + a*Sqrt[(a^ 2 + c^2)/a^2])]*Sqrt[b + a*Sqrt[(a^2 + c^2)/a^2]*Sin[d + e*x + ArcTan[c/a]]]*Sqrt[(a*Sqrt[(a^2 + c^2)/a^2] + a*Sqrt[(a^2 + c^2)/a^2]*Sin[d + e*x + Arc Tan[c/a]])/(-b + a*Sqrt[(a^2 + c^2)/a^2])])/(a*(a^2 - b^2 + c^2)^2*Sqrt[1 + c^2/a^2]*e*(a + c*Cot[d + e*x] + b*Csc[d + e*x])^(5/2)) + (2*c^2*AppellF1[1/2, 1/2, 1/2, 3/2, -((b + a*Sqrt[1 + c^2/a^2]*Sin[d + e*x + ArcTan[c/a]])/ (a*Sqrt[1 + c^2/a^2]*(1 - b/(a*Sqrt[1 + c^2/a^2])))), -((b + a*Sqrt[1 + c^2 /a^2]*Sin[d + e*x + ArcTan[c/a]])/(a*Sqrt[1 + c^2/a^2]*(-1 - b/(a*Sqrt[1 +

```
c^2/a^2]))))]*Csc[d + e*x]^(5/2)*Sec[d + e*x + ArcTan[c/a]]*(b + c*Cos[d +
e*x] + a*Sin[d + e*x])^(5/2)*Sqrt[(a*Sqrt[(a^2 + c^2)/a^2] - a*Sqrt[(a^2 +
c^2)/a^2]*Sin[d + e*x + ArcTan[c/a]])/(b + a*Sqrt[(a^2 + c^2)/a^2])]*Sqrt[b
 + a*Sqrt[(a^2 + c^2)/a^2]*Sin[d + e*x + ArcTan[c/a]]]*Sqrt[(a*Sqrt[(a^2 +
c^2)/a^2] + a*Sqrt[(a^2 + c^2)/a^2]*Sin[d + e*x + ArcTan[c/a]])/(-b + a*Sqr
t[(a^2 + c^2)/a^2])])/(3*a*(a^2 - b^2 + c^2)^2*Sqrt[1 + c^2/a^2]*e*(a + c*C
ot[d + e*x] + b*Csc[d + e*x])^(5/2))
```

fricas [F] time = 0.68, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{c\cot(ex+d)+b\csc(ex+d)+a}\,\text{cs}}{c^3\cot(ex+d)^3+b^3\csc(ex+d)^3+3\,ac^2\cot(ex+d)^2+3\,a^2c\cot(ex+d)+a^3+3\left(b^2c\cot(ex+d)+a\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(e*x+d)^(5/2)/(a+c*cot(e*x+d)+b*csc(e*x+d))^(5/2),x, algorithm
="fricas")
```

```
[Out] integral(sqrt(c*cot(e*x + d) + b*csc(e*x + d) + a)*csc(e*x + d)^(5/2)/(c^3*
cot(e*x + d)^3 + b^3*csc(e*x + d)^3 + 3*a*c^2*cot(e*x + d)^2 + 3*a^2*c*cot(
e*x + d) + a^3 + 3*(b^2*c*cot(e*x + d) + a*b^2)*csc(e*x + d)^2 + 3*(b*c^2*c
ot(e*x + d)^2 + 2*a*b*c*cot(e*x + d) + a^2*b)*csc(e*x + d)), x)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(e*x+d)^(5/2)/(a+c*cot(e*x+d)+b*csc(e*x+d))^(5/2),x, algorithm
="giac")
```

```
[Out] Timed out
```

maple [C] time = 2.16, size = 62955, normalized size = 127.96

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(e*x+d)^(5/2)/(a+c*cot(e*x+d)+b*csc(e*x+d))^(5/2),x)
```

```
[Out] result too large to display
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc(ex+d)^{\frac{5}{2}}}{(c\cot(ex+d)+b\csc(ex+d)+a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(e*x+d)^(5/2)/(a+c*cot(e*x+d)+b*csc(e*x+d))^(5/2),x, algorithm
="maxima")
```

```
[Out] integrate(csc(e*x + d)^(5/2)/(c*cot(e*x + d) + b*csc(e*x + d) + a)^(5/2), x
)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(\frac{1}{\sin(d+e\,x)}\right)^{5/2}}{\left(a+c\cot\left(d+e\,x\right)+\frac{b}{\sin(d+e\,x)}\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/sin(d + e*x))^(5/2)/(a + c*cot(d + e*x) + b/sin(d + e*x))^(5/2),x)
```

```
[Out] int((1/sin(d + e*x))^(5/2)/(a + c*cot(d + e*x) + b/sin(d + e*x))^(5/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(e*x+d)**(5/2)/(a+c*cot(e*x+d)+b*csc(e*x+d))**(5/2),x)
```

```
[Out] Timed out
```

3.467 $\int (a+c\cot(d+ex)+b\csc(d+ex))^{3/2}\sin^{\frac{3}{2}}(d+ex)\,dx$

Optimal. Leaf size=371

$$\frac{2\left(a^2-b^2+c^2\right)\sin^{\frac{3}{2}}(d+ex)\sqrt{\frac{a\sin(d+ex)+b+c\cos(d+ex)}{\sqrt{a^2+c^2}+b}}\,(a+b\csc(d+ex)+c\cot(d+ex))^{3/2}F\left(\frac{1}{2}\left(d+ex-\tan^{-1}(c,a\right.\right.}{3e(a\sin(d+ex)+b+c\cos(d+ex))^2}$$

```
[Out] -2/3*(a+c*cot(e*x+d)+b*csc(e*x+d))^(3/2)*sin(e*x+d)^(3/2)*(a*cos(e*x+d)-c*s
in(e*x+d))/e/(b+c*cos(e*x+d)+a*sin(e*x+d))+8/3*b*(a+c*cot(e*x+d)+b*csc(e*x+
d))^(3/2)*(cos(1/2*d+1/2*e*x-1/2*arctan(c,a))^2)^(1/2)/cos(1/2*d+1/2*e*x-1/
2*arctan(c,a))*EllipticE(sin(1/2*d+1/2*e*x-1/2*arctan(c,a)),2^(1/2)*((a^2+c
^2)^(1/2)/(b+(a^2+c^2)^(1/2)))^(1/2))*sin(e*x+d)^(3/2)/e/(b+c*cos(e*x+d)+a*
sin(e*x+d))/((b+c*cos(e*x+d)+a*sin(e*x+d))/(b+(a^2+c^2)^(1/2)))^(1/2)+2/3*(
a^2-b^2+c^2)*(a+c*cot(e*x+d)+b*csc(e*x+d))^(3/2)*(cos(1/2*d+1/2*e*x-1/2*arc
tan(c,a))^2)^(1/2)/cos(1/2*d+1/2*e*x-1/2*arctan(c,a))*EllipticF(sin(1/2*d+1
/2*e*x-1/2*arctan(c,a)),2^(1/2)*((a^2+c^2)^(1/2)/(b+(a^2+c^2)^(1/2)))^(1/2)
)*sin(e*x+d)^(3/2)*((b+c*cos(e*x+d)+a*sin(e*x+d))/(b+(a^2+c^2)^(1/2)))^(1/2
)/e/(b+c*cos(e*x+d)+a*sin(e*x+d))^2
```

Rubi [A] time = 0.38, antiderivative size = 371, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.212, Rules used = {3164, 3120, 3149, 3119, 2653, 3127, 2661}

$$\frac{2\left(a^2-b^2+c^2\right)\sin^{\frac{3}{2}}(d+ex)\sqrt{\frac{a\sin(d+ex)+b+c\cos(d+ex)}{\sqrt{a^2+c^2}+b}}\,(a+b\csc(d+ex)+c\cot(d+ex))^{3/2}F\left(\frac{1}{2}\left(d+ex-\tan^{-1}(c,a\right.\right.}{3e(a\sin(d+ex)+b+c\cos(d+ex))^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + c*Cot[d + e*x] + b*Csc[d + e*x])^(3/2)*Sin[d + e*x]^(3/2),x]
```

```
[Out] (8*b*(a + c*Cot[d + e*x] + b*Csc[d + e*x])^(3/2)*EllipticE[(d + e*x - ArcTa
n[c, a])/2, (2*Sqrt[a^2 + c^2])/(b + Sqrt[a^2 + c^2])]*Sin[d + e*x]^(3/2))/
(3*e*(b + c*Cos[d + e*x] + a*Sin[d + e*x])*Sqrt[(b + c*Cos[d + e*x] + a*Sin
[d + e*x])/(b + Sqrt[a^2 + c^2])]) + (2*(a^2 - b^2 + c^2)*(a + c*Cot[d + e*
x] + b*Csc[d + e*x])^(3/2)*EllipticF[(d + e*x - ArcTan[c, a])/2, (2*Sqrt[a^
2 + c^2])/(b + Sqrt[a^2 + c^2])]*Sin[d + e*x]^(3/2)*Sqrt[(b + c*Cos[d + e*x
] + a*Sin[d + e*x])/(b + Sqrt[a^2 + c^2])])/(3*e*(b + c*Cos[d + e*x] + a*Si
n[d + e*x])^2) - (2*(a + c*Cot[d + e*x] + b*Csc[d + e*x])^(3/2)*Sin[d + e*x
```

```
]^(3/2)*(a*Cos[d + e*x] - c*Sin[d + e*x]))/(3*e*(b + c*Cos[d + e*x] + a*Sin
[d + e*x]))
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 3119

```
Int[Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_
)]], x_Symbol] :> Dist[Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]]/Sqrt[(a +
b*Cos[d + e*x] + c*Sin[d + e*x])/(a + Sqrt[b^2 + c^2])], Int[Sqrt[a/(a + Sq
rt[b^2 + c^2]) + (Sqrt[b^2 + c^2]*Cos[d + e*x - ArcTan[b, c]])/(a + Sqrt[b^
2 + c^2])], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0]
&& NeQ[b^2 + c^2, 0] &&  !GtQ[a + Sqrt[b^2 + c^2], 0]
```

Rule 3120

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(n_), x_Symbol] :> -Simp[((c*Cos[d + e*x] - b*Sin[d + e*x])*(a + b*Cos[d +
e*x] + c*Sin[d + e*x])^(n - 1))/(e*n), x] + Dist[1/n, Int[Simp[n*a^2 + (n -
 1)*(b^2 + c^2) + a*b*(2*n - 1)*Cos[d + e*x] + a*c*(2*n - 1)*Sin[d + e*x],
x]*(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n - 2), x], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[a^2 - b^2 - c^2, 0] && GtQ[n, 1]
```

Rule 3127

```
Int[1/Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(
x_)]], x_Symbol] :> Dist[Sqrt[(a + b*Cos[d + e*x] + c*Sin[d + e*x])/(a + Sq
rt[b^2 + c^2])]/Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]], Int[1/Sqrt[a/(a
+ Sqrt[b^2 + c^2]) + (Sqrt[b^2 + c^2]*Cos[d + e*x - ArcTan[b, c]])/(a + Sqr
t[b^2 + c^2])], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2,
 0] && NeQ[b^2 + c^2, 0] &&  !GtQ[a + Sqrt[b^2 + c^2], 0]
```

Rule 3149

```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])
/Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)]]
```

```
, x_Symbol] :> Dist[B/b, Int[Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]], x],
 x] + Dist[(A*b - a*B)/b, Int[1/Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]],
x], x] /; FreeQ[{a, b, c, d, e, A, B, C}, x] && EqQ[B*c - b*C, 0] && NeQ[A*
b - a*B, 0]
```

Rule 3164

```
Int[((a_.) + csc[(d_.) + (e_.)*(x_)]*(b_.) + cot[(d_.) + (e_.)*(x_)]*(c_.))
^(n_)*sin[(d_.) + (e_.)*(x_)]^(n_), x_Symbol] :> Dist[(Sin[d + e*x]^n*(a +
b*Csc[d + e*x] + c*Cot[d + e*x])^n)/(b + a*Sin[d + e*x] + c*Cos[d + e*x])^n
, Int[(b + a*Sin[d + e*x] + c*Cos[d + e*x])^n, x], x] /; FreeQ[{a, b, c, d,
 e}, x] &&  !IntegerQ[n]
```

Rubi steps

$$\int (a + c\cot(d+ex) + b\csc(d+ex))^{3/2}\sin^{\frac{3}{2}}(d+ex)\,dx = \frac{\left((a + c\cot(d+ex) + b\csc(d+ex))^{3/2}\sin^{\frac{3}{2}}(d+ex)\right)\int (b}{(b + c\cos(d+ex) + a\sin(d}$$

$$= -\frac{2(a + c\cot(d+ex) + b\csc(d+ex))^{3/2}\sin^{\frac{3}{2}}(d+ex)(a\,c}{3e(b + c\cos(d+ex) + a\sin(d+}$$

$$= -\frac{2(a + c\cot(d+ex) + b\csc(d+ex))^{3/2}\sin^{\frac{3}{2}}(d+ex)(a\,c}{3e(b + c\cos(d+ex) + a\sin(d+}$$

$$= -\frac{2(a + c\cot(d+ex) + b\csc(d+ex))^{3/2}\sin^{\frac{3}{2}}(d+ex)(a\,c}{3e(b + c\cos(d+ex) + a\sin(d+}$$

$$= \frac{8b(a + c\cot(d+ex) + b\csc(d+ex))^{3/2}E\left(\frac{1}{2}\left(d + ex - \mathrm{ta}\right.\right.}{3e(b + c\cos(d+ex) + a\sin(d+ex))\sqrt{\frac{b+}{}}}$$

Mathematica [F] time = 52.86, size = 0, normalized size = 0.00

$$\int (a + c\cot(d+ex) + b\csc(d+ex))^{3/2}\sin^{\frac{3}{2}}(d+ex)\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + c*Cot[d + e*x] + b*Csc[d + e*x])^(3/2)*Sin[d + e*x]^(3/2),x]
```

```
[Out] Integrate[(a + c*Cot[d + e*x] + b*Csc[d + e*x])^(3/2)*Sin[d + e*x]^(3/2), x
]
```

fricas [F] time = 0.63, size = 0, normalized size = 0.00

$$\text{integral}\left((c\cot(ex+d)+b\csc(ex+d)+a)^{\frac{3}{2}}\sin(ex+d)^{\frac{3}{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+c*cot(e*x+d)+b*csc(e*x+d))^(3/2)*sin(e*x+d)^(3/2),x, algorithm
="fricas")
```

```
[Out] integral((c*cot(e*x + d) + b*csc(e*x + d) + a)^(3/2)*sin(e*x + d)^(3/2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int(c\cot(ex+d)+b\csc(ex+d)+a)^{\frac{3}{2}}\sin(ex+d)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+c*cot(e*x+d)+b*csc(e*x+d))^(3/2)*sin(e*x+d)^(3/2),x, algorithm
="giac")
```

```
[Out] integrate((c*cot(e*x + d) + b*csc(e*x + d) + a)^(3/2)*sin(e*x + d)^(3/2), x
)
```

maple [C] time = 1.10, size = 20454, normalized size = 55.13

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+c*cot(e*x+d)+b*csc(e*x+d))^(3/2)*sin(e*x+d)^(3/2),x)
```

```
[Out] result too large to display
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int(c\cot(ex+d)+b\csc(ex+d)+a)^{\frac{3}{2}}\sin(ex+d)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+c*cot(e*x+d)+b*csc(e*x+d))^(3/2)*sin(e*x+d)^(3/2),x, algorithm
="maxima")
```

```
[Out] integrate((c*cot(e*x + d) + b*csc(e*x + d) + a)^(3/2)*sin(e*x + d)^(3/2), x
)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \sin(d+e\,x)^{3/2}\left(a+c\cot(d+e\,x)+\frac{b}{\sin(d+e\,x)}\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d + e*x)^(3/2)*(a + c*cot(d + e*x) + b/sin(d + e*x))^(3/2),x)
[Out] int(sin(d + e*x)^(3/2)*(a + c*cot(d + e*x) + b/sin(d + e*x))^(3/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+c*cot(e*x+d)+b*csc(e*x+d))**(3/2)*sin(e*x+d)**(3/2),x)
[Out] Timed out
```

3.468 $\int \sqrt{a+c\cot(d+ex)+b\csc(d+ex)}\sqrt{\sin(d+ex)}\,dx$

Optimal. Leaf size=118

$$\frac{2\sqrt{\sin(d+ex)}\sqrt{a+b\csc(d+ex)+c\cot(d+ex)}\,E\left(\frac{1}{2}\left(d+ex-\tan^{-1}(c,a)\right)|\frac{2\sqrt{a^2+c^2}}{b+\sqrt{a^2+c^2}}\right)}{e\sqrt{\frac{a\sin(d+ex)+b+c\cos(d+ex)}{\sqrt{a^2+c^2}+b}}}$$

```
[Out] 2*(cos(1/2*d+1/2*e*x-1/2*arctan(c,a))^2)^(1/2)/cos(1/2*d+1/2*e*x-1/2*arctan
(c,a))*EllipticE(sin(1/2*d+1/2*e*x-1/2*arctan(c,a)),2^(1/2)*((a^2+c^2)^(1/2
)/(b+(a^2+c^2)^(1/2)))^(1/2))*(a+c*cot(e*x+d)+b*csc(e*x+d))^(1/2)*sin(e*x+d
)^(1/2)/e/((b+c*cos(e*x+d)+a*sin(e*x+d))/(b+(a^2+c^2)^(1/2)))^(1/2)
```

Rubi [A] time = 0.14, antiderivative size = 118, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {3164, 3119, 2653}

$$\frac{2\sqrt{\sin(d+ex)}\sqrt{a+b\csc(d+ex)+c\cot(d+ex)}\,E\left(\frac{1}{2}\left(d+ex-\tan^{-1}(c,a)\right)|\frac{2\sqrt{a^2+c^2}}{b+\sqrt{a^2+c^2}}\right)}{e\sqrt{\frac{a\sin(d+ex)+b+c\cos(d+ex)}{\sqrt{a^2+c^2}+b}}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + c*Cot[d + e*x] + b*Csc[d + e*x]]*Sqrt[Sin[d + e*x]],x]
```

```
[Out] (2*Sqrt[a + c*Cot[d + e*x] + b*Csc[d + e*x]]*EllipticE[(d + e*x - ArcTan[c,
 a])/2, (2*Sqrt[a^2 + c^2])/(b + Sqrt[a^2 + c^2])]*Sqrt[Sin[d + e*x]])/(e*S
qrt[(b + c*Cos[d + e*x] + a*Sin[d + e*x])/(b + Sqrt[a^2 + c^2])])
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 3119

```
Int[Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_
)]], x_Symbol] :> Dist[Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]]/Sqrt[(a +
b*Cos[d + e*x] + c*Sin[d + e*x])/(a + Sqrt[b^2 + c^2])], Int[Sqrt[a/(a + Sq
rt[b^2 + c^2]) + (Sqrt[b^2 + c^2]*Cos[d + e*x - ArcTan[b, c]])/(a + Sqrt[b^
2 + c^2])], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0]
```

```
&& NeQ[b^2 + c^2, 0] &&  !GtQ[a + Sqrt[b^2 + c^2], 0]
```

Rule 3164

```
Int[((a_.) + csc[(d_.) + (e_.)*(x_)]*(b_.) + cot[(d_.) + (e_.)*(x_)]*(c_.))
^(n_)*sin[(d_.) + (e_.)*(x_)]^(n_), x_Symbol] :> Dist[(Sin[d + e*x]^n*(a +
b*Csc[d + e*x] + c*Cot[d + e*x])^n)/(b + a*Sin[d + e*x] + c*Cos[d + e*x])^n
, Int[(b + a*Sin[d + e*x] + c*Cos[d + e*x])^n, x], x] /; FreeQ[{a, b, c, d,
 e}, x] &&  !IntegerQ[n]
```

Rubi steps

$$\int \sqrt{a+c\cot(d+ex)+b\csc(d+ex)}\,\sqrt{\sin(d+ex)}\,dx = \frac{\left(\sqrt{a+c\cot(d+ex)+b\csc(d+ex)}\,\sqrt{\sin(d+ex)}\right)\int \sqrt{b}}{\sqrt{b+c\cos(d+ex)+a\sin(d}}$$

$$= \frac{\left(\sqrt{a+c\cot(d+ex)+b\csc(d+ex)}\,\sqrt{\sin(d+ex)}\right)\int \sqrt{\frac{}{b}}}{\sqrt{\frac{b+c\cos(d+ex)+a\sin(d+}{b+\sqrt{a^2+c^2}}}}$$

$$= \frac{2\sqrt{a+c\cot(d+ex)+b\csc(d+ex)}\,E\left(\frac{1}{2}\left(d+ex-\tan^{-1}\right.\right.}{e\sqrt{\frac{b+c\cos(d+ex)+a\sin(d+ex)}{b+\sqrt{a^2+c^2}}}}$$

Mathematica [F] time = 12.34, size = 0, normalized size = 0.00

$$\int \sqrt{a+c\cot(d+ex)+b\csc(d+ex)}\,\sqrt{\sin(d+ex)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Sqrt[a + c*Cot[d + e*x] + b*Csc[d + e*x]]*Sqrt[Sin[d + e*x]],x]
```

```
[Out] Integrate[Sqrt[a + c*Cot[d + e*x] + b*Csc[d + e*x]]*Sqrt[Sin[d + e*x]], x]
```

fricas [F] time = 1.30, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\sqrt{c\cot\left(ex+d\right)+b\csc\left(ex+d\right)+a}\,\sqrt{\sin\left(ex+d\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+c*cot(e*x+d)+b*csc(e*x+d))^(1/2)*sin(e*x+d)^(1/2),x, algorithm
="fricas")
```

```
[Out] integral(sqrt(c*cot(e*x + d) + b*csc(e*x + d) + a)*sqrt(sin(e*x + d)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{c\cot(ex+d)+b\csc(ex+d)+a}\sqrt{\sin(ex+d)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+c*cot(e*x+d)+b*csc(e*x+d))^(1/2)*sin(e*x+d)^(1/2),x, algorithm
="giac")
```

```
[Out] integrate(sqrt(c*cot(e*x + d) + b*csc(e*x + d) + a)*sqrt(sin(e*x + d)), x)
```

maple [C] time = 1.06, size = 12365, normalized size = 104.79

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+c*cot(e*x+d)+b*csc(e*x+d))^(1/2)*sin(e*x+d)^(1/2),x)
```

```
[Out] result too large to display
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{c\cot(ex+d)+b\csc(ex+d)+a}\sqrt{\sin(ex+d)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+c*cot(e*x+d)+b*csc(e*x+d))^(1/2)*sin(e*x+d)^(1/2),x, algorithm
="maxima")
```

```
[Out] integrate(sqrt(c*cot(e*x + d) + b*csc(e*x + d) + a)*sqrt(sin(e*x + d)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \sqrt{\sin(d+e\,x)}\,\sqrt{a+c\cot(d+e\,x)+\frac{b}{\sin(d+e\,x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d + e*x)^(1/2)*(a + c*cot(d + e*x) + b/sin(d + e*x))^(1/2),x)
```

```
[Out] int(sin(d + e*x)^(1/2)*(a + c*cot(d + e*x) + b/sin(d + e*x))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a+b\csc(d+ex)+c\cot(d+ex)}\sqrt{\sin(d+ex)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+c*cot(e*x+d)+b*csc(e*x+d))**(1/2)*sin(e*x+d)**(1/2),x)
```

```
[Out] Integral(sqrt(a + b*csc(d + e*x) + c*cot(d + e*x))*sqrt(sin(d + e*x)), x)
```

3.469 $\int \frac{1}{\sqrt{a+c\cot(d+ex)+b\csc(d+ex)}\sqrt{\sin(d+ex)}}\,dx$

Optimal. Leaf size=118

$$\frac{2\sqrt{\frac{a\sin(d+ex)+b+c\cos(d+ex)}{\sqrt{a^2+c^2}+b}}\,F\left(\frac{1}{2}\left(d+ex-\tan^{-1}(c,a)\right)|\frac{2\sqrt{a^2+c^2}}{b+\sqrt{a^2+c^2}}\right)}{e\sqrt{\sin(d+ex)}\sqrt{a+b\csc(d+ex)+c\cot(d+ex)}}$$

```
[Out] 2*(cos(1/2*d+1/2*e*x-1/2*arctan(c,a))^2)^(1/2)/cos(1/2*d+1/2*e*x-1/2*arctan
(c,a))*EllipticF(sin(1/2*d+1/2*e*x-1/2*arctan(c,a)),2^(1/2)*((a^2+c^2)^(1/2
)/(b+(a^2+c^2)^(1/2)))^(1/2))*((b+c*cos(e*x+d)+a*sin(e*x+d))/(b+(a^2+c^2)^(
1/2)))^(1/2)/e/(a+c*cot(e*x+d)+b*csc(e*x+d))^(1/2)/sin(e*x+d)^(1/2)
```

Rubi [A] time = 0.15, antiderivative size = 118, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {3164, 3127, 2661}

$$\frac{2\sqrt{\frac{a\sin(d+ex)+b+c\cos(d+ex)}{\sqrt{a^2+c^2}+b}}\,F\left(\frac{1}{2}\left(d+ex-\tan^{-1}(c,a)\right)|\frac{2\sqrt{a^2+c^2}}{b+\sqrt{a^2+c^2}}\right)}{e\sqrt{\sin(d+ex)}\sqrt{a+b\csc(d+ex)+c\cot(d+ex)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[a + c*Cot[d + e*x] + b*Csc[d + e*x]]*Sqrt[Sin[d + e*x]]),x]
```

```
[Out] (2*EllipticF[(d + e*x - ArcTan[c, a])/2, (2*Sqrt[a^2 + c^2])/(b + Sqrt[a^2
+ c^2])]*Sqrt[(b + c*Cos[d + e*x] + a*Sin[d + e*x])/(b + Sqrt[a^2 + c^2])])
/(e*Sqrt[a + c*Cot[d + e*x] + b*Csc[d + e*x]]*Sqrt[Sin[d + e*x]])
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 3127

```
Int[1/Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(
x_)]], x_Symbol] :> Dist[Sqrt[(a + b*Cos[d + e*x] + c*Sin[d + e*x])/(a + Sq
rt[b^2 + c^2])]/Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]], Int[1/Sqrt[a/(a
+ Sqrt[b^2 + c^2]) + (Sqrt[b^2 + c^2]*Cos[d + e*x - ArcTan[b, c]])/(a + Sqr
t[b^2 + c^2])], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2,
 0] && NeQ[b^2 + c^2, 0] &&  !GtQ[a + Sqrt[b^2 + c^2], 0]
```

Rule 3164

```
Int[((a_.) + csc[(d_.) + (e_.)*(x_)]*(b_.) + cot[(d_.) + (e_.)*(x_)]*(c_.))
^(n_)*sin[(d_.) + (e_.)*(x_)]^(n_), x_Symbol] :> Dist[(Sin[d + e*x]^n*(a +
b*Csc[d + e*x] + c*Cot[d + e*x])^n)/(b + a*Sin[d + e*x] + c*Cos[d + e*x])^n
, Int[(b + a*Sin[d + e*x] + c*Cos[d + e*x])^n, x], x] /; FreeQ[{a, b, c, d,
 e}, x] &&  !IntegerQ[n]
```

Rubi steps

$$\int \frac{1}{\sqrt{a+c\cot(d+ex)+b\csc(d+ex)}\sqrt{\sin(d+ex)}}\,dx = \frac{\sqrt{b+c\cos(d+ex)+a\sin(d+ex)}\int \frac{1}{\sqrt{b+c\cos(d+ex)+a\sin(d}}}{\sqrt{a+c\cot(d+ex)+b\csc(d+ex)}\sqrt{\sin(d+ex)}}$$

$$= \frac{\sqrt{\frac{b+c\cos(d+ex)+a\sin(d+ex)}{b+\sqrt{a^2+c^2}}}\int \frac{1}{\sqrt{\frac{b}{b+\sqrt{a^2+c^2}}+\frac{\sqrt{a^2+c^2}\cos\left(d+ex-\tan^{-1}(c,\right.}{b+\sqrt{a^2+c^2}}}}}{\sqrt{a+c\cot(d+ex)+b\csc(d+ex)}\sqrt{\sin(d+ex)}}$$

$$= \frac{2F\left(\frac{1}{2}\left(d+ex-\tan^{-1}(c,a)\right)|\frac{2\sqrt{a^2+c^2}}{b+\sqrt{a^2+c^2}}\right)\sqrt{\frac{b+c\cos(d+ex)+a\sin}{b+\sqrt{a^2+c^2}}}}{e\sqrt{a+c\cot(d+ex)+b\csc(d+ex)}\sqrt{\sin(d+ex)}}$$

Mathematica [C] time = 2.84, size = 519, normalized size = 4.40

$$\frac{4\left(i\sqrt{a^2-b^2+c^2}-ia-b+c\right)(\cos(d+ex)+i\sin(d+ex))\sqrt{-\frac{i\left(\sqrt{a^2-b^2+c^2}+a+(b-c)\tan\left(\frac{1}{2}(d+ex)\right)\right)}{\left(\sqrt{a^2-b^2+c^2}+a-ib+ic\right)\left(\tan\left(\frac{1}{2}(d+ex)\right)-i\right)}}\sqrt{-\frac{i\left(\sqrt{a^2-b^2+c^2}-a\right.}{\left(\sqrt{a^2-b^2+c^2}-a+\right.}}}{e\left(-\sqrt{a^2-b^2+c^2}+a+ib-ic\right)\sqrt{}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(Sqrt[a + c*Cot[d + e*x] + b*Csc[d + e*x]]*Sqrt[Sin[d + e*x]]),
x]
```

```
[Out] (4*((-I)*a - b + c + I*Sqrt[a^2 - b^2 + c^2])*EllipticF[ArcSin[Sqrt[((-a -
I*b + I*c + Sqrt[a^2 - b^2 + c^2])*(-Cos[d + e*x] + I*Sin[d + e*x]))/(-a +
I*b - I*c + Sqrt[a^2 - b^2 + c^2])]], (I*b + Sqrt[a^2 - b^2 + c^2])/(I*b -
Sqrt[a^2 - b^2 + c^2])]*Sqrt[((-a - I*b + I*c + Sqrt[a^2 - b^2 + c^2])*(-Co
s[d + e*x] + I*Sin[d + e*x]))/(-a + I*b - I*c + Sqrt[a^2 - b^2 + c^2])]*(Co
s[d + e*x] + I*Sin[d + e*x])*Sqrt[((-I)*(a + Sqrt[a^2 - b^2 + c^2] + (b - c
)*Tan[(d + e*x)/2]))/((a - I*b + I*c + Sqrt[a^2 - b^2 + c^2])*(-I + Tan[(d
```

```
+ e*x)/2]))]*Sqrt[((-I)*(-a + Sqrt[a^2 - b^2 + c^2] + (-b + c)*Tan[(d + e*x
)/2]))/((-a + I*b - I*c + Sqrt[a^2 - b^2 + c^2])*(-I + Tan[(d + e*x)/2]))])
/((a + I*b - I*c - Sqrt[a^2 - b^2 + c^2])*e*Sqrt[a + c*Cot[d + e*x] + b*Csc
[d + e*x]]*Sqrt[Sin[d + e*x]])
```

fricas [F] time = 0.86, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{1}{\sqrt{c\cot\left(ex+d\right)+b\csc\left(ex+d\right)+a}\,\sqrt{\sin\left(ex+d\right)}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+c*cot(e*x+d)+b*csc(e*x+d))^(1/2)/sin(e*x+d)^(1/2),x, algorit
hm="fricas")
```

```
[Out] integral(1/(sqrt(c*cot(e*x + d) + b*csc(e*x + d) + a)*sqrt(sin(e*x + d))),
x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{c\cot\left(ex+d\right)+b\csc\left(ex+d\right)+a}\,\sqrt{\sin\left(ex+d\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+c*cot(e*x+d)+b*csc(e*x+d))^(1/2)/sin(e*x+d)^(1/2),x, algorit
hm="giac")
```

```
[Out] integrate(1/(sqrt(c*cot(e*x + d) + b*csc(e*x + d) + a)*sqrt(sin(e*x + d))),
 x)
```

maple [C] time = 1.16, size = 705, normalized size = 5.97

$$4i\sqrt{\frac{b+c\cos(ex+d)+a\sin(ex+d)}{\sin(ex+d)}}\,\sqrt{\frac{\left(i\sin(ex+d)+\cos(ex+d)\right)\left(ib-ic-\sqrt{a^2-b^2+c^2}-a\right)}{ib-ic+\sqrt{a^2-b^2+c^2}+a}}\,\sqrt{-\frac{i\left(\cos(ex+d)\sqrt{a^2-b^2+c^2}-b\sin(ex+d)+c\sin(ex+d)-a\cos\right.}{\left(i\cos(ex+d)+\sin(ex+d)+i\right)\left(ib-ic-\sqrt{a^2-b\right.}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+c*cot(e*x+d)+b*csc(e*x+d))^(1/2)/sin(e*x+d)^(1/2),x)
```

```
[Out] 4*I/e*((b+c*cos(e*x+d)+a*sin(e*x+d))/sin(e*x+d))^(1/2)*((I*sin(e*x+d)+cos(e
*x+d))*(I*b-I*c-(a^2-b^2+c^2)^(1/2)-a)/(I*b-I*c+(a^2-b^2+c^2)^(1/2)+a))^(1/
2)*(-I*(cos(e*x+d)*(a^2-b^2+c^2)^(1/2)-b*sin(e*x+d)+c*sin(e*x+d)-a*cos(e*x+
d)+(a^2-b^2+c^2)^(1/2)-a)/(I*cos(e*x+d)+sin(e*x+d)+I)/(I*b-I*c-(a^2-b^2+c^2
```

```
)^(1/2)+a))^(1/2)*(I*(b*sin(e*x+d)-c*sin(e*x+d)+cos(e*x+d)*(a^2-b^2+c^2)^(1
/2)+a*cos(e*x+d)+(a^2-b^2+c^2)^(1/2)+a)/(I*cos(e*x+d)+sin(e*x+d)+I)/(I*b-I*
c+(a^2-b^2+c^2)^(1/2)+a))^(1/2)*(cos(e*x+d)+1)^2*EllipticF(((I*sin(e*x+d)+c
os(e*x+d))*(I*b-I*c-(a^2-b^2+c^2)^(1/2)-a)/(I*b-I*c+(a^2-b^2+c^2)^(1/2)+a))
^(1/2),((I*b-I*c+(a^2-b^2+c^2)^(1/2)+a)*(I*b-I*c+(a^2-b^2+c^2)^(1/2)-a)/(I*
b-I*c-(a^2-b^2+c^2)^(1/2)-a)/(I*b-I*c-(a^2-b^2+c^2)^(1/2)+a))^(1/2))*(cos(e
*x+d)-1)^2*(I*(a^2-b^2+c^2)^(1/2)*sin(e*x+d)+I*sin(e*x+d)*a-I*cos(e*x+d)*b+
I*cos(e*x+d)*c-b*sin(e*x+d)+c*sin(e*x+d)-cos(e*x+d)*(a^2-b^2+c^2)^(1/2)-a*c
os(e*x+d))/sin(e*x+d)^(7/2)/(b+c*cos(e*x+d)+a*sin(e*x+d))/(I*b-I*c-(a^2-b^2
+c^2)^(1/2)-a)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{c\cot(ex+d)+b\csc(ex+d)+a}\,\sqrt{\sin(ex+d)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+c*cot(e*x+d)+b*csc(e*x+d))^(1/2)/sin(e*x+d)^(1/2),x, algorit
hm="maxima")
```

```
[Out] integrate(1/(sqrt(c*cot(e*x + d) + b*csc(e*x + d) + a)*sqrt(sin(e*x + d))),
 x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sqrt{\sin(d+e\,x)}\,\sqrt{a+c\cot(d+e\,x)+\frac{b}{\sin(d+e\,x)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(d + e*x)^(1/2)*(a + c*cot(d + e*x) + b/sin(d + e*x))^(1/2)),x)
```

```
[Out] int(1/(sin(d + e*x)^(1/2)*(a + c*cot(d + e*x) + b/sin(d + e*x))^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a+b\csc(d+ex)+c\cot(d+ex)}\,\sqrt{\sin(d+ex)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+c*cot(e*x+d)+b*csc(e*x+d))**(1/2)/sin(e*x+d)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(a + b*csc(d + e*x) + c*cot(d + e*x))*sqrt(sin(d + e*x))),
x)
```

3.470 $\int \frac{1}{(a+c\cot(d+ex)+b\csc(d+ex))^{3/2}\sin^{\frac{3}{2}}(d+ex)}\,dx$

Optimal. Leaf size=240

$$-\frac{2(a\sin(d+ex)+b+c\cos(d+ex))^2E\left(\frac{1}{2}\left(d+ex-\tan^{-1}(c,a)\right)|\frac{2\sqrt{a^2+c^2}}{b+\sqrt{a^2+c^2}}\right)}{e\left(a^2-b^2+c^2\right)\sin^{\frac{3}{2}}(d+ex)\sqrt{\frac{a\sin(d+ex)+b+c\cos(d+ex)}{\sqrt{a^2+c^2}+b}}\,(a+b\csc(d+ex)+c\cot(d+ex))^{3/2}}-\frac{2(a\cos(d+ex)-c\,s}{e\left(a^2-b^2+c^2\right)\sin}$$

```
[Out] -2*(b+c*cos(e*x+d)+a*sin(e*x+d))*(a*cos(e*x+d)-c*sin(e*x+d))/(a^2-b^2+c^2)/
e/(a+c*cot(e*x+d)+b*csc(e*x+d))^(3/2)/sin(e*x+d)^(3/2)-2*(cos(1/2*d+1/2*e*x
-1/2*arctan(c,a))^2)^(1/2)/cos(1/2*d+1/2*e*x-1/2*arctan(c,a))*EllipticE(sin
(1/2*d+1/2*e*x-1/2*arctan(c,a)),2^(1/2)*((a^2+c^2)^(1/2)/(b+(a^2+c^2)^(1/2)
))^(1/2))*(b+c*cos(e*x+d)+a*sin(e*x+d))^2/(a^2-b^2+c^2)/e/(a+c*cot(e*x+d)+b
*csc(e*x+d))^(3/2)/sin(e*x+d)^(3/2)/((b+c*cos(e*x+d)+a*sin(e*x+d))/(b+(a^2+
c^2)^(1/2)))^(1/2)
```

Rubi [A] time = 0.21, antiderivative size = 240, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.121, Rules used = {3164, 3128, 3119, 2653}

$$-\frac{2(a\sin(d+ex)+b+c\cos(d+ex))^2E\left(\frac{1}{2}\left(d+ex-\tan^{-1}(c,a)\right)|\frac{2\sqrt{a^2+c^2}}{b+\sqrt{a^2+c^2}}\right)}{e\left(a^2-b^2+c^2\right)\sin^{\frac{3}{2}}(d+ex)\sqrt{\frac{a\sin(d+ex)+b+c\cos(d+ex)}{\sqrt{a^2+c^2}+b}}\,(a+b\csc(d+ex)+c\cot(d+ex))^{3/2}}-\frac{2(a\cos(d+ex)-c\,s}{e\left(a^2-b^2+c^2\right)\sin}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + c*Cot[d + e*x] + b*Csc[d + e*x])^(3/2)*Sin[d + e*x]^(3/2)),x]
```

```
[Out] (-2*EllipticE[(d + e*x - ArcTan[c, a])/2, (2*Sqrt[a^2 + c^2])/(b + Sqrt[a^2
 + c^2])]*(b + c*Cos[d + e*x] + a*Sin[d + e*x])^2)/((a^2 - b^2 + c^2)*e*(a
+ c*Cot[d + e*x] + b*Csc[d + e*x])^(3/2)*Sin[d + e*x]^(3/2)*Sqrt[(b + c*Cos
[d + e*x] + a*Sin[d + e*x])/(b + Sqrt[a^2 + c^2])]) - (2*(b + c*Cos[d + e*x
] + a*Sin[d + e*x])*(a*Cos[d + e*x] - c*Sin[d + e*x]))/((a^2 - b^2 + c^2)*e
*(a + c*Cot[d + e*x] + b*Csc[d + e*x])^(3/2)*Sin[d + e*x]^(3/2))
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 3119

```
Int[Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_
)]], x_Symbol] :> Dist[Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]]/Sqrt[(a +
b*Cos[d + e*x] + c*Sin[d + e*x])/(a + Sqrt[b^2 + c^2])], Int[Sqrt[a/(a + Sq
rt[b^2 + c^2]) + (Sqrt[b^2 + c^2]*Cos[d + e*x - ArcTan[b, c]])/(a + Sqrt[b^
2 + c^2])], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0]
&& NeQ[b^2 + c^2, 0] &&  !GtQ[a + Sqrt[b^2 + c^2], 0]
```

Rule 3128

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(-3/2), x_Symbol] :> Simp[(2*(c*Cos[d + e*x] - b*Sin[d + e*x]))/(e*(a^2 - b
^2 - c^2)*Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]]), x] + Dist[1/(a^2 - b^
2 - c^2), Int[Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]], x], x] /; FreeQ[{a
, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0]
```

Rule 3164

```
Int[((a_.) + csc[(d_.) + (e_.)*(x_)]*(b_.) + cot[(d_.) + (e_.)*(x_)]*(c_.))
^(n_)*sin[(d_.) + (e_.)*(x_)]^(n_), x_Symbol] :> Dist[(Sin[d + e*x]^n*(a +
b*Csc[d + e*x] + c*Cot[d + e*x])^n)/(b + a*Sin[d + e*x] + c*Cos[d + e*x])^n
, Int[(b + a*Sin[d + e*x] + c*Cos[d + e*x])^n, x], x] /; FreeQ[{a, b, c, d,
 e}, x] &&  !IntegerQ[n]
```

Rubi steps

$$\int \frac{1}{(a+c\cot(d+ex)+b\csc(d+ex))^{3/2}\sin^{\frac{3}{2}}(d+ex)}\,dx = \frac{(b+c\cos(d+ex)+a\sin(d+ex))^{3/2}\int \frac{1}{(b+c\cos(d+ex)+a\sin}}{(a+c\cot(d+ex)+b\csc(d+ex))^{3/2}\sin^{\frac{3}{2}}(d+}$$

$$= -\frac{2(b+c\cos(d+ex)+a\sin(d+ex))(a\cos(d+ex)-c}{\left(a^2-b^2+c^2\right)e(a+c\cot(d+ex)+b\csc(d+ex))^{3/2}\,\mathrm{s}}$$

$$= -\frac{2(b+c\cos(d+ex)+a\sin(d+ex))(a\cos(d+ex)-c}{\left(a^2-b^2+c^2\right)e(a+c\cot(d+ex)+b\csc(d+ex))^{3/2}\,\mathrm{s}}$$

$$= -\frac{2E\left(\frac{1}{2}\left(d+ex-\tan^{-1}(c,a)\right)|\frac{2\sqrt{a^2+c^2}}{b+\sqrt{a^2+c^2}}\right)(b+c\,\mathrm{c}}{\left(a^2-b^2+c^2\right)e(a+c\cot(d+ex)+b\csc(d+ex))^{3/2}\,\mathrm{s}}$$

Mathematica [F] time = 20.78, size = 0, normalized size = 0.00

$$\int \frac{1}{(a + c\cot(d + ex) + b\csc(d + ex))^{3/2}\sin^{\frac{3}{2}}(d + ex)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/((a + c*Cot[d + e*x] + b*Csc[d + e*x])^(3/2)*Sin[d + e*x]^(3/2)
),x]
```

```
[Out] Integrate[1/((a + c*Cot[d + e*x] + b*Csc[d + e*x])^(3/2)*Sin[d + e*x]^(3/2)
), x]
```

fricas [F] time = 1.20, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\sqrt{c\cot(ex + d) + b}}{a^2\cos(ex + d)^2 + \left(c^2\cos(ex + d)^2 - c^2\right)\cot(ex + d)^2 + \left(b^2\cos(ex + d)^2 - b^2\right)\csc(ex + d)^2 - a^2 + 2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+c*cot(e*x+d)+b*csc(e*x+d))^(3/2)/sin(e*x+d)^(3/2),x, algorit
hm="fricas")
```

```
[Out] integral(-sqrt(c*cot(e*x + d) + b*csc(e*x + d) + a)*sqrt(sin(e*x + d))/(a^2
*cos(e*x + d)^2 + (c^2*cos(e*x + d)^2 - c^2)*cot(e*x + d)^2 + (b^2*cos(e*x
+ d)^2 - b^2)*csc(e*x + d)^2 - a^2 + 2*(a*c*cos(e*x + d)^2 - a*c)*cot(e*x +
 d) + 2*(a*b*cos(e*x + d)^2 - a*b + (b*c*cos(e*x + d)^2 - b*c)*cot(e*x + d)
)*csc(e*x + d)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(c\cot(ex + d) + b\csc(ex + d) + a)^{\frac{3}{2}}\sin(ex + d)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+c*cot(e*x+d)+b*csc(e*x+d))^(3/2)/sin(e*x+d)^(3/2),x, algorit
hm="giac")
```

```
[Out] integrate(1/((c*cot(e*x + d) + b*csc(e*x + d) + a)^(3/2)*sin(e*x + d)^(3/2)
), x)
```

maple [C] time = 0.96, size = 12231, normalized size = 50.96

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+c*cot(e*x+d)+b*csc(e*x+d))^(3/2)/sin(e*x+d)^(3/2),x)
```

```
[Out] result too large to display
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(c \cot\left(ex+d\right)+b \csc\left(ex+d\right)+a\right)^{\frac{3}{2}} \sin\left(ex+d\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+c*cot(e*x+d)+b*csc(e*x+d))^(3/2)/sin(e*x+d)^(3/2),x, algorit
hm="maxima")
```

```
[Out] integrate(1/((c*cot(e*x + d) + b*csc(e*x + d) + a)^(3/2)*sin(e*x + d)^(3/2)
), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\sin\left(d+e\,x\right)^{3/2}\,{\left(a+c\,\mathrm{cot}\left(d+e\,x\right)+\frac{b}{\sin\left(d+e\,x\right)}\right)}^{3/2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(d + e*x)^(3/2)*(a + c*cot(d + e*x) + b/sin(d + e*x))^(3/2)),x)
```

```
[Out] int(1/(sin(d + e*x)^(3/2)*(a + c*cot(d + e*x) + b/sin(d + e*x))^(3/2)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+c*cot(e*x+d)+b*csc(e*x+d))**(3/2)/sin(e*x+d)**(3/2),x)
```

```
[Out] Timed out
```

3.471 $\int \frac{1}{(a+c\cot(d+ex)+b\csc(d+ex))^{5/2}\sin^{\frac{5}{2}}(d+ex)}\,dx$

Optimal. Leaf size=492

$$\frac{2\sqrt{\frac{a\sin(d+ex)+b+c\cos(d+ex)}{\sqrt{a^2+c^2}+b}}\,(a\sin(d+ex)+b+c\cos(d+ex))^2F\left(\frac{1}{2}\left(d+ex-\tan^{-1}(c,a)\right)|\frac{2\sqrt{a^2+c^2}}{b+\sqrt{a^2+c^2}}\right)}{3e\left(a^2-b^2+c^2\right)\sin^{\frac{5}{2}}(d+ex)(a+b\csc(d+ex)+c\cot(d+ex))^{5/2}}+\frac{8b(a}{3e\left(a^2-b^2\right.}$$

```
[Out] -2/3*(b+c*cos(e*x+d)+a*sin(e*x+d))*(a*cos(e*x+d)-c*sin(e*x+d))/(a^2-b^2+c^2
)/e/(a+c*cot(e*x+d)+b*csc(e*x+d))^(5/2)/sin(e*x+d)^(5/2)+8/3*(b+c*cos(e*x+d
)+a*sin(e*x+d))^2*(a*b*cos(e*x+d)-b*c*sin(e*x+d))/(a^2-b^2+c^2)^2/e/(a+c*co
t(e*x+d)+b*csc(e*x+d))^(5/2)/sin(e*x+d)^(5/2)+8/3*b*(cos(1/2*d+1/2*e*x-1/2*
arctan(c,a))^2)^(1/2)/cos(1/2*d+1/2*e*x-1/2*arctan(c,a))*EllipticE(sin(1/2*
d+1/2*e*x-1/2*arctan(c,a)),2^(1/2)*((a^2+c^2)^(1/2)/(b+(a^2+c^2)^(1/2)))^(1
/2))*(b+c*cos(e*x+d)+a*sin(e*x+d))^3/(a^2-b^2+c^2)^2/e/(a+c*cot(e*x+d)+b*cs
c(e*x+d))^(5/2)/sin(e*x+d)^(5/2)/((b+c*cos(e*x+d)+a*sin(e*x+d))/(b+(a^2+c^2
)^(1/2)))^(1/2)+2/3*(cos(1/2*d+1/2*e*x-1/2*arctan(c,a))^2)^(1/2)/cos(1/2*d+
1/2*e*x-1/2*arctan(c,a))*EllipticF(sin(1/2*d+1/2*e*x-1/2*arctan(c,a)),2^(1/
2)*((a^2+c^2)^(1/2)/(b+(a^2+c^2)^(1/2)))^(1/2))*(b+c*cos(e*x+d)+a*sin(e*x+d
))^2*((b+c*cos(e*x+d)+a*sin(e*x+d))/(b+(a^2+c^2)^(1/2)))^(1/2)/(a^2-b^2+c^2
)/e/(a+c*cot(e*x+d)+b*csc(e*x+d))^(5/2)/sin(e*x+d)^(5/2)
```

Rubi [A] time = 0.49, antiderivative size = 492, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.242, Rules used = {3164, 3129, 3156, 3149, 3119, 2653, 3127, 2661}

$$\frac{2\sqrt{\frac{a\sin(d+ex)+b+c\cos(d+ex)}{\sqrt{a^2+c^2}+b}}\,(a\sin(d+ex)+b+c\cos(d+ex))^2F\left(\frac{1}{2}\left(d+ex-\tan^{-1}(c,a)\right)|\frac{2\sqrt{a^2+c^2}}{b+\sqrt{a^2+c^2}}\right)}{3e\left(a^2-b^2+c^2\right)\sin^{\frac{5}{2}}(d+ex)(a+b\csc(d+ex)+c\cot(d+ex))^{5/2}}+\frac{8b(a}{3e\left(a^2-b^2\right.}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + c*Cot[d + e*x] + b*Csc[d + e*x])^(5/2)*Sin[d + e*x]^(5/2)),x]
```

```
[Out] (8*b*EllipticE[(d + e*x - ArcTan[c, a])/2, (2*Sqrt[a^2 + c^2])/(b + Sqrt[a^
2 + c^2])]*(b + c*Cos[d + e*x] + a*Sin[d + e*x])^3)/(3*(a^2 - b^2 + c^2)^2*
e*(a + c*Cot[d + e*x] + b*Csc[d + e*x])^(5/2)*Sin[d + e*x]^(5/2)*Sqrt[(b +
c*Cos[d + e*x] + a*Sin[d + e*x])/(b + Sqrt[a^2 + c^2])]) + (2*EllipticF[(d
+ e*x - ArcTan[c, a])/2, (2*Sqrt[a^2 + c^2])/(b + Sqrt[a^2 + c^2])]*(b + c*
Cos[d + e*x] + a*Sin[d + e*x])^2*Sqrt[(b + c*Cos[d + e*x] + a*Sin[d + e*x])
/(b + Sqrt[a^2 + c^2])])/(3*(a^2 - b^2 + c^2)*e*(a + c*Cot[d + e*x] + b*Csc
```

```
[d + e*x])^(5/2)*Sin[d + e*x]^(5/2)) - (2*(b + c*Cos[d + e*x] + a*Sin[d + e
*x])*(a*Cos[d + e*x] - c*Sin[d + e*x]))/(3*(a^2 - b^2 + c^2)*e*(a + c*Cot[d
 + e*x] + b*Csc[d + e*x])^(5/2)*Sin[d + e*x]^(5/2)) + (8*(b + c*Cos[d + e*x
] + a*Sin[d + e*x])^2*(a*b*Cos[d + e*x] - b*c*Sin[d + e*x]))/(3*(a^2 - b^2
+ c^2)^2*e*(a + c*Cot[d + e*x] + b*Csc[d + e*x])^(5/2)*Sin[d + e*x]^(5/2))
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 3119

```
Int[Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_
)]], x_Symbol] :> Dist[Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]]/Sqrt[(a +
b*Cos[d + e*x] + c*Sin[d + e*x])/(a + Sqrt[b^2 + c^2])], Int[Sqrt[a/(a + Sq
rt[b^2 + c^2]) + (Sqrt[b^2 + c^2]*Cos[d + e*x - ArcTan[b, c]])/(a + Sqrt[b^
2 + c^2])], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0]
&& NeQ[b^2 + c^2, 0] &&  !GtQ[a + Sqrt[b^2 + c^2], 0]
```

Rule 3127

```
Int[1/Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(
x_)]], x_Symbol] :> Dist[Sqrt[(a + b*Cos[d + e*x] + c*Sin[d + e*x])/(a + Sq
rt[b^2 + c^2])]/Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]], Int[1/Sqrt[a/(a
+ Sqrt[b^2 + c^2]) + (Sqrt[b^2 + c^2]*Cos[d + e*x - ArcTan[b, c]])/(a + Sqr
t[b^2 + c^2])], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2,
 0] && NeQ[b^2 + c^2, 0] &&  !GtQ[a + Sqrt[b^2 + c^2], 0]
```

Rule 3129

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(n_), x_Symbol] :> Simp[((-(c*Cos[d + e*x]) + b*Sin[d + e*x])*(a + b*Cos[d
+ e*x] + c*Sin[d + e*x])^(n + 1))/(e*(n + 1)*(a^2 - b^2 - c^2)), x] + Dist[
1/((n + 1)*(a^2 - b^2 - c^2)), Int[(a*(n + 1) - b*(n + 2)*Cos[d + e*x] - c*
(n + 2)*Sin[d + e*x])*(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n + 1), x], x]
 /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0] && LtQ[n, -1] && N
eQ[n, -3/2]
```

Rule 3149

```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])
/Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)]]
, x_Symbol] :> Dist[B/b, Int[Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]], x],
 x] + Dist[(A*b - a*B)/b, Int[1/Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]],
x], x] /; FreeQ[{a, b, c, d, e, A, B, C}, x] && EqQ[B*c - b*C, 0] && NeQ[A*
b - a*B, 0]
```

Rule 3156

```
Int[((a_.) + cos[(d_.) + (e_.)*(x_)]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)])
^(n_)*((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)
]), x_Symbol] :> -Simp[((c*B - b*C - (a*C - c*A)*Cos[d + e*x] + (a*B - b*A)
*Sin[d + e*x])*(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n + 1))/(e*(n + 1)*(a
^2 - b^2 - c^2)), x] + Dist[1/((n + 1)*(a^2 - b^2 - c^2)), Int[(a + b*Cos[d
 + e*x] + c*Sin[d + e*x])^(n + 1)*Simp[(n + 1)*(a*A - b*B - c*C) + (n + 2)*
(a*B - b*A)*Cos[d + e*x] + (n + 2)*(a*C - c*A)*Sin[d + e*x], x], x], x] /;
FreeQ[{a, b, c, d, e, A, B, C}, x] && LtQ[n, -1] && NeQ[a^2 - b^2 - c^2, 0]
 && NeQ[n, -2]
```

Rule 3164

```
Int[((a_.) + csc[(d_.) + (e_.)*(x_)]*(b_.) + cot[(d_.) + (e_.)*(x_)]*(c_.))
^(n_)*sin[(d_.) + (e_.)*(x_)]^(n_), x_Symbol] :> Dist[(Sin[d + e*x]^n*(a +
b*Csc[d + e*x] + c*Cot[d + e*x])^n)/(b + a*Sin[d + e*x] + c*Cos[d + e*x])^n
, Int[(b + a*Sin[d + e*x] + c*Cos[d + e*x])^n, x], x] /; FreeQ[{a, b, c, d,
 e}, x] &&  !IntegerQ[n]
```

Rubi steps

$$\int \frac{1}{(a+c\cot(d+ex)+b\csc(d+ex))^{5/2}\sin^{\frac{5}{2}}(d+ex)}\,dx = \frac{(b+c\cos(d+ex)+a\sin(d+ex))^{5/2}\int \frac{1}{(b+c\cos(d+ex)+a\sin}}{(a+c\cot(d+ex)+b\csc(d+ex))^{5/2}\sin^{\frac{5}{2}}(d+}$$

$$= -\frac{2(b+c\cos(d+ex)+a\sin(d+ex))(a\cos(d+ex)-}{3\left(a^2-b^2+c^2\right)e(a+c\cot(d+ex)+b\csc(d+ex))^{5/2}}$$

$$= -\frac{2(b+c\cos(d+ex)+a\sin(d+ex))(a\cos(d+ex)-}{3\left(a^2-b^2+c^2\right)e(a+c\cot(d+ex)+b\csc(d+ex))^{5/2}}$$

$$= -\frac{2(b+c\cos(d+ex)+a\sin(d+ex))(a\cos(d+ex)-}{3\left(a^2-b^2+c^2\right)e(a+c\cot(d+ex)+b\csc(d+ex))^{5/2}}$$

$$= -\frac{2(b+c\cos(d+ex)+a\sin(d+ex))(a\cos(d+ex)-}{3\left(a^2-b^2+c^2\right)e(a+c\cot(d+ex)+b\csc(d+ex))^{5/2}}$$

$$= \frac{8bE\left(\frac{1}{2}\left(d+ex-\tan^{-1}(c,a)\right)|\frac{2\sqrt{a^2+c^2}}{b+\sqrt{a^2+c^2}}\right)(b+c}{3\left(a^2-b^2+c^2\right)^2e(a+c\cot(d+ex)+b\csc(d+ex))^{5/2}}$$

Mathematica [F] time = 25.27, size = 0, normalized size = 0.00

$$\int \frac{1}{(a+c\cot(d+ex)+b\csc(d+ex))^{5/2}\sin^{\frac{5}{2}}(d+ex)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/((a + c*Cot[d + e*x] + b*Csc[d + e*x])^(5/2)*Sin[d + e*x]^(5/2)
),x]
```

```
[Out] Integrate[1/((a + c*Cot[d + e*x] + b*Csc[d + e*x])^(5/2)*Sin[d + e*x]^(5/2)
), x]
```

fricas [F] time = 1.94, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(-\frac{}{\left(a^3\cos(ex+d)^2+\left(c^3\cos(ex+d)^2-c^3\right)\cot(ex+d)^3+\left(b^3\cos(ex+d)^2-b^3\right)\csc(ex+d)^3-a^3+3\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+c*cot(e*x+d)+b*csc(e*x+d))^(5/2)/sin(e*x+d)^(5/2),x, algorit
hm="fricas")
```

```
[Out] integral(-sqrt(c*cot(e*x + d) + b*csc(e*x + d) + a)/((a^3*cos(e*x + d)^2 +
(c^3*cos(e*x + d)^2 - c^3)*cot(e*x + d)^3 + (b^3*cos(e*x + d)^2 - b^3)*csc(
e*x + d)^3 - a^3 + 3*(a*c^2*cos(e*x + d)^2 - a*c^2)*cot(e*x + d)^2 + 3*(a*b
^2*cos(e*x + d)^2 - a*b^2 + (b^2*c*cos(e*x + d)^2 - b^2*c)*cot(e*x + d))*cs
c(e*x + d)^2 + 3*(a^2*c*cos(e*x + d)^2 - a^2*c)*cot(e*x + d) + 3*(a^2*b*cos
(e*x + d)^2 - a^2*b + (b*c^2*cos(e*x + d)^2 - b*c^2)*cot(e*x + d)^2 + 2*(a*
b*c*cos(e*x + d)^2 - a*b*c)*cot(e*x + d))*csc(e*x + d))*sqrt(sin(e*x + d)))
, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(c\cot(ex+d)+b\csc(ex+d)+a)^{\frac{5}{2}}\sin(ex+d)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+c*cot(e*x+d)+b*csc(e*x+d))^(5/2)/sin(e*x+d)^(5/2),x, algorit
hm="giac")
```

```
[Out] integrate(1/((c*cot(e*x + d) + b*csc(e*x + d) + a)^(5/2)*sin(e*x + d)^(5/2)
), x)
```

maple [C] time = 1.65, size = 62945, normalized size = 127.94

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+c*cot(e*x+d)+b*csc(e*x+d))^(5/2)/sin(e*x+d)^(5/2),x)
```

```
[Out] result too large to display
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(c\cot(ex+d)+b\csc(ex+d)+a)^{\frac{5}{2}}\sin(ex+d)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+c*cot(e*x+d)+b*csc(e*x+d))^(5/2)/sin(e*x+d)^(5/2),x, algorit
hm="maxima")
```

```
[Out] integrate(1/((c*cot(e*x + d) + b*csc(e*x + d) + a)^(5/2)*sin(e*x + d)^(5/2)
), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\sin\left(d+e\,x\right)^{5/2}\,{\left(a+c\,\mathrm{cot}\left(d+e\,x\right)+\frac{b}{\sin\left(d+e\,x\right)}\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(d + e*x)^(5/2)*(a + c*cot(d + e*x) + b/sin(d + e*x))^(5/2)),x)
[Out] int(1/(sin(d + e*x)^(5/2)*(a + c*cot(d + e*x) + b/sin(d + e*x))^(5/2)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+c*cot(e*x+d)+b*csc(e*x+d))**(5/2)/sin(e*x+d)**(5/2),x)
[Out] Timed out
```

3.472 $\int \frac{1}{\cos^2(x)+\sin^2(x)}\,dx$

Optimal. Leaf size=1

$$x$$

```
[Out] x
```

Rubi [A] time = 0.01, antiderivative size = 1, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {4380, 8}

$$x$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[x]^2 + Sin[x]^2)^(-1),x]
[Out] x
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 4380

```
Int[(u_.)*((a_.) + cos[(d_.) + (e_.)*(x_)]^2*(b_.) + (c_.)*sin[(d_.) + (e_.
)*(x_)]^2)^(p_.), x_Symbol] :> Dist[(a + c)^p, Int[ActivateTrig[u], x], x]
/; FreeQ[{a, b, c, d, e, p}, x] && EqQ[b - c, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\cos^2(x)+\sin^2(x)}\,dx &= \int 1\,dx\\ &= x\end{aligned}$$

Mathematica [A] time = 0.00, size = 1, normalized size = 1.00

$$x$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[x]^2 + Sin[x]^2)^(-1),x]
[Out] x
```

fricas [A] time = 0.89, size = 1, normalized size = 1.00

$$x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(cos(x)^2+sin(x)^2),x, algorithm="fricas")
[Out] x
```

giac [A] time = 0.14, size = 1, normalized size = 1.00

$$x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(cos(x)^2+sin(x)^2),x, algorithm="giac")
[Out] x
```

maple [A] time = 0.06, size = 2, normalized size = 2.00

$$x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(x)^2+sin(x)^2),x)
[Out] x
```

maxima [A] time = 0.41, size = 1, normalized size = 1.00

$$x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(cos(x)^2+sin(x)^2),x, algorithm="maxima")
[Out] x
```

mupad [B] time = 2.64, size = 1, normalized size = 1.00

$$x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(x)^2 + sin(x)^2),x)
[Out] x
```

sympy [B] time = 0.36, size = 10, normalized size = 10.00

$$\frac{x}{\sin^2(x)+\cos^2(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(cos(x)**2+sin(x)**2),x)
```

```
[Out] x/(sin(x)**2 + cos(x)**2)
```

3.473 $\int \frac{1}{\left(\cos^2(x)+\sin^2(x)\right)^2} \, dx$

Optimal. Leaf size=1

$$x$$

[Out] x

Rubi [A] time = 0.01, antiderivative size = 1, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {4380, 8}

$$x$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[x]^2 + Sin[x]^2)^(-2),x]
```

```
[Out] x
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 4380

```
Int[(u_.)*((a_.) + cos[(d_.) + (e_.)*(x_)]^2*(b_.) + (c_.)*sin[(d_.) + (e_.
)*(x_)]^2)^(p_.), x_Symbol] :> Dist[(a + c)^p, Int[ActivateTrig[u], x], x]
/; FreeQ[{a, b, c, d, e, p}, x] && EqQ[b - c, 0]
```

Rubi steps

$$\begin{aligned} \int \frac{1}{\left(\cos^2(x)+\sin^2(x)\right)^2} \, dx &= \int 1 \, dx \\ &= x \end{aligned}$$

Mathematica [A] time = 0.00, size = 1, normalized size = 1.00

$$x$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[x]^2 + Sin[x]^2)^(-2),x]
```

[Out] x

fricas [A] time = 1.49, size = 1, normalized size = 1.00

$$x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(cos(x)^2+sin(x)^2)^2,x, algorithm="fricas")
[Out] x
```

giac [A] time = 0.12, size = 1, normalized size = 1.00

$$x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(cos(x)^2+sin(x)^2)^2,x, algorithm="giac")
[Out] x
```

maple [A] time = 0.05, size = 2, normalized size = 2.00

$$x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(x)^2+sin(x)^2)^2,x)
[Out] x
```

maxima [A] time = 0.41, size = 1, normalized size = 1.00

$$x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(cos(x)^2+sin(x)^2)^2,x, algorithm="maxima")
[Out] x
```

mupad [B] time = 2.63, size = 1, normalized size = 1.00

$$x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(x)^2 + sin(x)^2)^2,x)
```

[Out] x

sympy [B] time = 0.83, size = 22, normalized size = 22.00

$$\frac{x}{\sin^4(x) + 2\sin^2(x)\cos^2(x) + \cos^4(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(cos(x)**2+sin(x)**2)**2,x)
[Out] x/(sin(x)**4 + 2*sin(x)**2*cos(x)**2 + cos(x)**4)
```

3.474 $\int \frac{1}{\left(\cos^2(x)+\sin^2(x)\right)^3} \, dx$

Optimal. Leaf size=1

$$x$$

```
[Out] x
```

Rubi [A] time = 0.01, antiderivative size = 1, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {4380, 8}

$$x$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[x]^2 + Sin[x]^2)^(-3),x]
```

```
[Out] x
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 4380

```
Int[(u_.)*((a_.) + cos[(d_.) + (e_.)*(x_)]^2*(b_.) + (c_.)*sin[(d_.) + (e_.
)*(x_)]^2)^(p_.), x_Symbol] :> Dist[(a + c)^p, Int[ActivateTrig[u], x], x]
/; FreeQ[{a, b, c, d, e, p}, x] && EqQ[b - c, 0]
```

Rubi steps

$$\begin{aligned} \int \frac{1}{\left(\cos^2(x)+\sin^2(x)\right)^3} \, dx &= \int 1 \, dx \\ &= x \end{aligned}$$

Mathematica [A] time = 0.00, size = 1, normalized size = 1.00

$$x$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[x]^2 + Sin[x]^2)^(-3),x]
```

```
[Out] x
```

fricas [A] time = 0.99, size = 1, normalized size = 1.00

$$x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(cos(x)^2+sin(x)^2)^3,x, algorithm="fricas")
[Out] x
```

giac [A] time = 0.13, size = 1, normalized size = 1.00

$$x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(cos(x)^2+sin(x)^2)^3,x, algorithm="giac")
[Out] x
```

maple [A] time = 0.05, size = 2, normalized size = 2.00

$$x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(x)^2+sin(x)^2)^3,x)
[Out] x
```

maxima [A] time = 0.41, size = 1, normalized size = 1.00

$$x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(cos(x)^2+sin(x)^2)^3,x, algorithm="maxima")
[Out] x
```

mupad [B] time = 2.59, size = 1, normalized size = 1.00

$$x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(x)^2 + sin(x)^2)^3,x)
```

```
[Out] x
```

sympy [B] time = 2.10, size = 34, normalized size = 34.00

$$\frac{x}{\sin^{6}(x) + 3\sin^{4}(x)\cos^{2}(x) + 3\sin^{2}(x)\cos^{4}(x) + \cos^{6}(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(cos(x)**2+sin(x)**2)**3,x)
[Out] x/(sin(x)**6 + 3*sin(x)**4*cos(x)**2 + 3*sin(x)**2*cos(x)**4 + cos(x)**6)
```

3.475 $\int \frac{1}{\cos^2(x)-\sin^2(x)}\,dx$

Optimal. Leaf size=11

$$\frac{1}{2}\tanh^{-1}(2\sin(x)\cos(x))$$

```
[Out] 1/2*arctanh(2*cos(x)*sin(x))
```

Rubi [A] time = 0.02, antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {206}

$$\frac{1}{2}\tanh^{-1}(2\sin(x)\cos(x))$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[x]^2 - Sin[x]^2)^(-1),x]
```

```
[Out] ArcTanh[2*Cos[x]*Sin[x]]/2
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\cos^2(x)-\sin^2(x)}\,dx &= \text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \tan(x)\right)\\ &= \frac{1}{2}\tanh^{-1}(2\cos(x)\sin(x))\end{aligned}$$

Mathematica [B] time = 0.01, size = 23, normalized size = 2.09

$$\frac{1}{2}\log(\sin(x)+\cos(x))-\frac{1}{2}\log(\cos(x)-\sin(x))$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[x]^2 - Sin[x]^2)^(-1),x]
```

```
[Out] -1/2*Log[Cos[x] - Sin[x]] + Log[Cos[x] + Sin[x]]/2
```

fricas [B] time = 0.85, size = 23, normalized size = 2.09

$$\frac{1}{4}\,\log\left(2\,\cos(x)\sin(x)+1\right)-\frac{1}{4}\,\log\left(-2\,\cos(x)\sin(x)+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(cos(x)^2-sin(x)^2),x, algorithm="fricas")
```

```
[Out] 1/4*log(2*cos(x)*sin(x) + 1) - 1/4*log(-2*cos(x)*sin(x) + 1)
```

giac [B] time = 0.13, size = 33, normalized size = 3.00

$$\frac{1}{8}\,\log\left(\left|\frac{1}{\sin\left(2\,x\right)}+\sin\left(2\,x\right)+2\right|\right)-\frac{1}{8}\,\log\left(\left|\frac{1}{\sin\left(2\,x\right)}+\sin\left(2\,x\right)-2\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(cos(x)^2-sin(x)^2),x, algorithm="giac")
```

```
[Out] 1/8*log(abs(1/sin(2*x) + sin(2*x) + 2)) - 1/8*log(abs(1/sin(2*x) + sin(2*x)
 - 2))
```

maple [A] time = 0.10, size = 4, normalized size = 0.36

$$\operatorname{arctanh}\left(\tan(x)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(x)^2-sin(x)^2),x)
```

```
[Out] arctanh(tan(x))
```

maxima [A] time = 0.32, size = 15, normalized size = 1.36

$$\frac{1}{2}\,\log\left(\tan(x)+1\right)-\frac{1}{2}\,\log\left(\tan(x)-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(cos(x)^2-sin(x)^2),x, algorithm="maxima")
```

```
[Out] 1/2*log(tan(x) + 1) - 1/2*log(tan(x) - 1)
```

mupad [B] time = 2.90, size = 3, normalized size = 0.27

$$\operatorname{atanh}\left(\tan(x)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(x)^2 - sin(x)^2),x)
```

```
[Out] atanh(tan(x))
```

sympy [B] time = 0.35, size = 36, normalized size = 3.27

$$\frac{\log\left(\tan^2\left(\frac{x}{2}\right)-2\tan\left(\frac{x}{2}\right)-1\right)}{2}-\frac{\log\left(\tan^2\left(\frac{x}{2}\right)+2\tan\left(\frac{x}{2}\right)-1\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(cos(x)**2-sin(x)**2),x)
```

```
[Out] log(tan(x/2)**2 - 2*tan(x/2) - 1)/2 - log(tan(x/2)**2 + 2*tan(x/2) - 1)/2
```

3.476 $\int \frac{1}{\left(\cos^2(x)-\sin^2(x)\right)^2}\,dx$

Optimal. Leaf size=13

$$\frac{\tan(x)}{1-\tan^2(x)}$$

```
[Out] tan(x)/(1-tan(x)^2)
```

Rubi [A] time = 0.02, antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {383}

$$\frac{\tan(x)}{1-\tan^2(x)}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[x]^2 - Sin[x]^2)^(-2),x]
```

```
[Out] Tan[x]/(1 - Tan[x]^2)
```

Rule 383

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> S
imp[(c*x*(a + b*x^n)^(p + 1))/a, x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[
b*c - a*d, 0] && EqQ[a*d - b*c*(n*(p + 1) + 1), 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\left(\cos^2(x)-\sin^2(x)\right)^2}\,dx &= \text{Subst}\left(\int \frac{1+x^2}{\left(1-x^2\right)^2}\,dx, x, \tan(x)\right)\\ &= \frac{\tan(x)}{1-\tan^2(x)}\end{aligned}$$

Mathematica [A] time = 0.00, size = 8, normalized size = 0.62

$$\frac{1}{2}\tan(2x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[x]^2 - Sin[x]^2)^(-2),x]
```

```
[Out] Tan[2*x]/2
```

fricas [A] time = 1.31, size = 15, normalized size = 1.15

$$\frac{\cos(x)\sin(x)}{2\cos(x)^2 - 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(cos(x)^2-sin(x)^2)^2,x, algorithm="fricas")
[Out] cos(x)*sin(x)/(2*cos(x)^2 - 1)
```

giac [A] time = 0.12, size = 6, normalized size = 0.46

$$\frac{1}{2}\tan(2x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(cos(x)^2-sin(x)^2)^2,x, algorithm="giac")
[Out] 1/2*tan(2*x)
```

maple [A] time = 0.10, size = 18, normalized size = 1.38

$$-\frac{1}{2(1+\tan(x))} - \frac{1}{2(\tan(x)-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(x)^2-sin(x)^2)^2,x)
[Out] -1/2/(1+tan(x))-1/2/(tan(x)-1)
```

maxima [A] time = 0.30, size = 12, normalized size = 0.92

$$-\frac{\tan(x)}{\tan(x)^2 - 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(cos(x)^2-sin(x)^2)^2,x, algorithm="maxima")
[Out] -tan(x)/(tan(x)^2 - 1)
```

mupad [B] time = 2.63, size = 6, normalized size = 0.46

$$\frac{\tan(2x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(x)^2 - sin(x)^2)^2,x)
```

```
[Out] tan(2*x)/2
```

sympy [B] time = 1.34, size = 48, normalized size = 3.69

$$-\frac{2\tan^{3}\left(\frac{x}{2}\right)}{\tan^{4}\left(\frac{x}{2}\right)-6\tan^{2}\left(\frac{x}{2}\right)+1}+\frac{2\tan\left(\frac{x}{2}\right)}{\tan^{4}\left(\frac{x}{2}\right)-6\tan^{2}\left(\frac{x}{2}\right)+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(cos(x)**2-sin(x)**2)**2,x)
```

```
[Out] -2*tan(x/2)**3/(tan(x/2)**4 - 6*tan(x/2)**2 + 1) + 2*tan(x/2)/(tan(x/2)**4
- 6*tan(x/2)**2 + 1)
```

3.477 $\int \frac{1}{\left(\cos^2(x)-\sin^2(x)\right)^3} dx$

Optimal. Leaf size=32

$$\frac{\tan(x)\sec^2(x)}{2\left(1-\tan^2(x)\right)^2}+\frac{1}{4}\tanh^{-1}(2\sin(x)\cos(x))$$

```
[Out] 1/4*arctanh(2*cos(x)*sin(x))+1/2*sec(x)^2*tan(x)/(1-tan(x)^2)^2
```

Rubi [A] time = 0.03, antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {413, 21, 206}

$$\frac{\tan(x)\sec^2(x)}{2\left(1-\tan^2(x)\right)^2}+\frac{1}{4}\tanh^{-1}(2\sin(x)\cos(x))$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[x]^2 - Sin[x]^2)^(-3),x]
```

```
[Out] ArcTanh[2*Cos[x]*Sin[x]]/4 + (Sec[x]^2*Tan[x])/(2*(1 - Tan[x]^2)^2)
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 413

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[((a*d - c*b)*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1))/(a*b*n*(p +
 1)), x] - Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q -
 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(p
 + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

Rubi steps

$$\int \frac{1}{\left(\cos^2(x)-\sin^2(x)\right)^3}\,dx = \text{Subst}\left(\int \frac{\left(1+x^2\right)^2}{\left(1-x^2\right)^3}\,dx, x, \tan(x)\right)$$
$$= \frac{\sec^2(x)\tan(x)}{2\left(1-\tan^2(x)\right)^2} - \frac{1}{4}\text{Subst}\left(\int \frac{-2+2x^2}{\left(1-x^2\right)^2}\,dx, x, \tan(x)\right)$$
$$= \frac{\sec^2(x)\tan(x)}{2\left(1-\tan^2(x)\right)^2} + \frac{1}{2}\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \tan(x)\right)$$
$$= \frac{1}{4}\tanh^{-1}(2\cos(x)\sin(x)) + \frac{\sec^2(x)\tan(x)}{2\left(1-\tan^2(x)\right)^2}$$

Mathematica [A] time = 0.01, size = 22, normalized size = 0.69

$$\frac{1}{4}\tanh^{-1}(\sin(2x)) + \frac{1}{4}\tan(2x)\sec(2x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[x]^2 - Sin[x]^2)^(-3),x]
```

```
[Out] ArcTanh[Sin[2*x]]/4 + (Sec[2*x]*Tan[2*x])/4
```

fricas [B] time = 0.76, size = 74, normalized size = 2.31

$$\frac{\left(4\cos(x)^4-4\cos(x)^2+1\right)\log\left(2\cos(x)\sin(x)+1\right)-\left(4\cos(x)^4-4\cos(x)^2+1\right)\log\left(-2\cos(x)\sin(x)+1\right)+}{8\left(4\cos(x)^4-4\cos(x)^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(cos(x)^2-sin(x)^2)^3,x, algorithm="fricas")
```

```
[Out] 1/8*((4*cos(x)^4 - 4*cos(x)^2 + 1)*log(2*cos(x)*sin(x) + 1) - (4*cos(x)^4 -
 4*cos(x)^2 + 1)*log(-2*cos(x)*sin(x) + 1) + 4*cos(x)*sin(x))/(4*cos(x)^4 -
 4*cos(x)^2 + 1)
```

giac [A] time = 0.13, size = 37, normalized size = 1.16

$$-\frac{\sin(2x)}{4\left(\sin(2x)^2-1\right)} + \frac{1}{8}\log\left(\sin(2x)+1\right) - \frac{1}{8}\log\left(-\sin(2x)+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(cos(x)^2-sin(x)^2)^3,x, algorithm="giac")
```

```
[Out] -1/4*sin(2*x)/(sin(2*x)^2 - 1) + 1/8*log(sin(2*x) + 1) - 1/8*log(-sin(2*x)
+ 1)
```

maple [A] time = 0.12, size = 48, normalized size = 1.50

$$\frac{1}{4\left(\tan(x)-1\right)^{2}}+\frac{1}{4\tan(x)-4}-\frac{\ln\left(\tan(x)-1\right)}{4}-\frac{1}{4\left(1+\tan(x)\right)^{2}}+\frac{1}{4+4\tan(x)}+\frac{\ln\left(1+\tan(x)\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(x)^2-sin(x)^2)^3,x)
```

```
[Out] 1/4/(tan(x)-1)^2+1/4/(tan(x)-1)-1/4*ln(tan(x)-1)-1/4/(1+tan(x))^2+1/4/(1+ta
n(x))+1/4*ln(1+tan(x))
```

maxima [A] time = 0.31, size = 38, normalized size = 1.19

$$\frac{\tan(x)^{3}+\tan(x)}{2\left(\tan(x)^{4}-2\,\tan(x)^{2}+1\right)}+\frac{1}{4}\,\log\left(\tan(x)+1\right)-\frac{1}{4}\,\log\left(\tan(x)-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(cos(x)^2-sin(x)^2)^3,x, algorithm="maxima")
```

```
[Out] 1/2*(tan(x)^3 + tan(x))/(tan(x)^4 - 2*tan(x)^2 + 1) + 1/4*log(tan(x) + 1) -
 1/4*log(tan(x) - 1)
```

mupad [B] time = 2.65, size = 32, normalized size = 1.00

$$\frac{\mathrm{atanh}\left(\tan(x)\right)}{2}+\frac{\frac{\tan(x)^{3}}{2}+\frac{\tan(x)}{2}}{\tan(x)^{4}-2\tan(x)^{2}+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(x)^2 - sin(x)^2)^3,x)
```

```
[Out] atanh(tan(x))/2 + (tan(x)/2 + tan(x)^3/2)/(tan(x)^4 - 2*tan(x)^2 + 1)
```

sympy [B] time = 3.48, size = 765, normalized size = 23.91

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(cos(x)**2-sin(x)**2)**3,x)
```

```
[Out] log(tan(x/2)**2 - 2*tan(x/2) - 1)*tan(x/2)**8/(4*tan(x/2)**8 - 48*tan(x/2)*
*6 + 152*tan(x/2)**4 - 48*tan(x/2)**2 + 4) - 12*log(tan(x/2)**2 - 2*tan(x/2
) - 1)*tan(x/2)**6/(4*tan(x/2)**8 - 48*tan(x/2)**6 + 152*tan(x/2)**4 - 48*t
an(x/2)**2 + 4) + 38*log(tan(x/2)**2 - 2*tan(x/2) - 1)*tan(x/2)**4/(4*tan(x
/2)**8 - 48*tan(x/2)**6 + 152*tan(x/2)**4 - 48*tan(x/2)**2 + 4) - 12*log(ta
n(x/2)**2 - 2*tan(x/2) - 1)*tan(x/2)**2/(4*tan(x/2)**8 - 48*tan(x/2)**6 + 1
52*tan(x/2)**4 - 48*tan(x/2)**2 + 4) + log(tan(x/2)**2 - 2*tan(x/2) - 1)/(4
*tan(x/2)**8 - 48*tan(x/2)**6 + 152*tan(x/2)**4 - 48*tan(x/2)**2 + 4) - log
(tan(x/2)**2 + 2*tan(x/2) - 1)*tan(x/2)**8/(4*tan(x/2)**8 - 48*tan(x/2)**6
+ 152*tan(x/2)**4 - 48*tan(x/2)**2 + 4) + 12*log(tan(x/2)**2 + 2*tan(x/2) -
 1)*tan(x/2)**6/(4*tan(x/2)**8 - 48*tan(x/2)**6 + 152*tan(x/2)**4 - 48*tan(
x/2)**2 + 4) - 38*log(tan(x/2)**2 + 2*tan(x/2) - 1)*tan(x/2)**4/(4*tan(x/2)
**8 - 48*tan(x/2)**6 + 152*tan(x/2)**4 - 48*tan(x/2)**2 + 4) + 12*log(tan(x
/2)**2 + 2*tan(x/2) - 1)*tan(x/2)**2/(4*tan(x/2)**8 - 48*tan(x/2)**6 + 152*
tan(x/2)**4 - 48*tan(x/2)**2 + 4) - log(tan(x/2)**2 + 2*tan(x/2) - 1)/(4*ta
n(x/2)**8 - 48*tan(x/2)**6 + 152*tan(x/2)**4 - 48*tan(x/2)**2 + 4) - 4*tan(
x/2)**7/(4*tan(x/2)**8 - 48*tan(x/2)**6 + 152*tan(x/2)**4 - 48*tan(x/2)**2
+ 4) - 4*tan(x/2)**5/(4*tan(x/2)**8 - 48*tan(x/2)**6 + 152*tan(x/2)**4 - 48
*tan(x/2)**2 + 4) + 4*tan(x/2)**3/(4*tan(x/2)**8 - 48*tan(x/2)**6 + 152*tan
(x/2)**4 - 48*tan(x/2)**2 + 4) + 4*tan(x/2)/(4*tan(x/2)**8 - 48*tan(x/2)**6
 + 152*tan(x/2)**4 - 48*tan(x/2)**2 + 4)
```

3.478 $\int \frac{1}{\cos^2(x)+a^2\sin^2(x)}\,dx$

Optimal. Leaf size=9

$$\frac{\tan^{-1}(a\tan(x))}{a}$$

```
[Out] arctan(a*tan(x))/a
```

Rubi [A] time = 0.02, antiderivative size = 9, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {203}

$$\frac{\tan^{-1}(a\tan(x))}{a}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[x]^2 + a^2*Sin[x]^2)^(-1),x]
```

```
[Out] ArcTan[a*Tan[x]]/a
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\cos^2(x)+a^2\sin^2(x)}\,dx &= \text{Subst}\left(\int \frac{1}{1+a^2x^2}\,dx, x, \tan(x)\right)\\ &= \frac{\tan^{-1}(a\tan(x))}{a}\end{aligned}$$

Mathematica [A] time = 0.03, size = 9, normalized size = 1.00

$$\frac{\tan^{-1}(a\tan(x))}{a}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[x]^2 + a^2*Sin[x]^2)^(-1),x]
```

```
[Out] ArcTan[a*Tan[x]]/a
```

fricas [B] time = 0.69, size = 35, normalized size = 3.89

$$-\frac{\arctan\left(\frac{\left(a^{2}+1\right)\cos(x)^{2}-a^{2}}{2\,a\cos(x)\sin(x)}\right)}{2\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(cos(x)^2+a^2*sin(x)^2),x, algorithm="fricas")
[Out] -1/2*arctan(1/2*((a^2 + 1)*cos(x)^2 - a^2)/(a*cos(x)*sin(x)))/a
```

giac [B] time = 0.15, size = 20, normalized size = 2.22

$$\frac{\pi\left\lfloor\frac{x}{\pi}+\frac{1}{2}\right\rfloor+\arctan\left(a\tan(x)\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(cos(x)^2+a^2*sin(x)^2),x, algorithm="giac")
[Out] (pi*floor(x/pi + 1/2) + arctan(a*tan(x)))/a
```

maple [A] time = 0.12, size = 10, normalized size = 1.11

$$\frac{\arctan\left(a\tan(x)\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(x)^2+a^2*sin(x)^2),x)
[Out] arctan(a*tan(x))/a
```

maxima [A] time = 0.40, size = 9, normalized size = 1.00

$$\frac{\arctan\left(a\tan(x)\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(cos(x)^2+a^2*sin(x)^2),x, algorithm="maxima")
[Out] arctan(a*tan(x))/a
```

mupad [B] time = 2.83, size = 9, normalized size = 1.00

$$\frac{\mathrm{atan}\left(a\tan(x)\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(x)^2 + a^2*sin(x)^2),x)
```

```
[Out] atan(a*tan(x))/a
```

sympy [B] time = 22.40, size = 12007, normalized size = 1334.11

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(cos(x)**2+a**2*sin(x)**2),x)
```

```
[Out] Piecewise((64*a**7*sqrt(-2*a**2 - 2*a*sqrt(a**2 - 1) + 1)*log(-sqrt(-2*a**2
 + 2*a*sqrt(a**2 - 1) + 1) + tan(x/2))/(64*a**7*sqrt(-2*a**2 - 2*a*sqrt(a**
2 - 1) + 1)*sqrt(-2*a**2 + 2*a*sqrt(a**2 - 1) + 1) - 64*a**6*sqrt(a**2 - 1)
*sqrt(-2*a**2 - 2*a*sqrt(a**2 - 1) + 1)*sqrt(-2*a**2 + 2*a*sqrt(a**2 - 1) +
 1) - 96*a**5*sqrt(-2*a**2 - 2*a*sqrt(a**2 - 1) + 1)*sqrt(-2*a**2 + 2*a*sqr
t(a**2 - 1) + 1) + 64*a**4*sqrt(a**2 - 1)*sqrt(-2*a**2 - 2*a*sqrt(a**2 - 1)
 + 1)*sqrt(-2*a**2 + 2*a*sqrt(a**2 - 1) + 1) + 36*a**3*sqrt(-2*a**2 - 2*a*s
qrt(a**2 - 1) + 1)*sqrt(-2*a**2 + 2*a*sqrt(a**2 - 1) + 1) - 12*a**2*sqrt(a*
*2 - 1)*sqrt(-2*a**2 - 2*a*sqrt(a**2 - 1) + 1)*sqrt(-2*a**2 + 2*a*sqrt(a**2
 - 1) + 1) - 2*a*sqrt(-2*a**2 - 2*a*sqrt(a**2 - 1) + 1)*sqrt(-2*a**2 + 2*a*
sqrt(a**2 - 1) + 1)) - 64*a**7*sqrt(-2*a**2 - 2*a*sqrt(a**2 - 1) + 1)*log(s
qrt(-2*a**2 + 2*a*sqrt(a**2 - 1) + 1) + tan(x/2))/(64*a**7*sqrt(-2*a**2 - 2
*a*sqrt(a**2 - 1) + 1)*sqrt(-2*a**2 + 2*a*sqrt(a**2 - 1) + 1) - 64*a**6*sqr
t(a**2 - 1)*sqrt(-2*a**2 - 2*a*sqrt(a**2 - 1) + 1)*sqrt(-2*a**2 + 2*a*sqrt(
a**2 - 1) + 1) - 96*a**5*sqrt(-2*a**2 - 2*a*sqrt(a**2 - 1) + 1)*sqrt(-2*a**
2 + 2*a*sqrt(a**2 - 1) + 1) + 64*a**4*sqrt(a**2 - 1)*sqrt(-2*a**2 - 2*a*sqr
t(a**2 - 1) + 1)*sqrt(-2*a**2 + 2*a*sqrt(a**2 - 1) + 1) + 36*a**3*sqrt(-2*a
**2 - 2*a*sqrt(a**2 - 1) + 1)*sqrt(-2*a**2 + 2*a*sqrt(a**2 - 1) + 1) - 12*a
**2*sqrt(a**2 - 1)*sqrt(-2*a**2 - 2*a*sqrt(a**2 - 1) + 1)*sqrt(-2*a**2 + 2*
a*sqrt(a**2 - 1) + 1) - 2*a*sqrt(-2*a**2 - 2*a*sqrt(a**2 - 1) + 1)*sqrt(-2*
a**2 + 2*a*sqrt(a**2 - 1) + 1)) - 64*a**6*sqrt(a**2 - 1)*sqrt(-2*a**2 - 2*a
*sqrt(a**2 - 1) + 1)*log(-sqrt(-2*a**2 + 2*a*sqrt(a**2 - 1) + 1) + tan(x/2)
)/(64*a**7*sqrt(-2*a**2 - 2*a*sqrt(a**2 - 1) + 1)*sqrt(-2*a**2 + 2*a*sqrt(a
**2 - 1) + 1) - 64*a**6*sqrt(a**2 - 1)*sqrt(-2*a**2 - 2*a*sqrt(a**2 - 1) +
1)*sqrt(-2*a**2 + 2*a*sqrt(a**2 - 1) + 1) - 96*a**5*sqrt(-2*a**2 - 2*a*sqrt
(a**2 - 1) + 1)*sqrt(-2*a**2 + 2*a*sqrt(a**2 - 1) + 1) + 64*a**4*sqrt(a**2
- 1)*sqrt(-2*a**2 - 2*a*sqrt(a**2 - 1) + 1)*sqrt(-2*a**2 + 2*a*sqrt(a**2 -
1) + 1) + 36*a**3*sqrt(-2*a**2 - 2*a*sqrt(a**2 - 1) + 1)*sqrt(-2*a**2 + 2*a
*sqrt(a**2 - 1) + 1) - 12*a**2*sqrt(a**2 - 1)*sqrt(-2*a**2 - 2*a*sqrt(a**2
- 1) + 1)*sqrt(-2*a**2 + 2*a*sqrt(a**2 - 1) + 1) - 2*a*sqrt(-2*a**2 - 2*a*s
qrt(a**2 - 1) + 1)*sqrt(-2*a**2 + 2*a*sqrt(a**2 - 1) + 1)) + 64*a**6*sqrt(a
**2 - 1)*sqrt(-2*a**2 - 2*a*sqrt(a**2 - 1) + 1)*log(sqrt(-2*a**2 + 2*a*sqrt
```

(a**2 - 1) + 1) + tan(x/2))/(64*a**7*sqrt(-2*a**2 - 2*a*sqrt(a**2 - 1) + 1)
*sqrt(-2*a**2 + 2*a*sqrt(a**2 - 1) + 1) - 64*a**6*sqrt(a**2 - 1)*sqrt(-2*a*
*2 - 2*a*sqrt(a**2 - 1) + 1)*sqrt(-2*a**2 + 2*a*sqrt(a**2 - 1) + 1) - 96*a*
*5*sqrt(-2*a**2 - 2*a*sqrt(a**2 - 1) + 1)*sqrt(-2*a**2 + 2*a*sqrt(a**2 - 1)
 + 1) + 64*a**4*sqrt(a**2 - 1)*sqrt(-2*a**2 - 2*a*sqrt(a**2 - 1) + 1)*sqrt(
-2*a**2 + 2*a*sqrt(a**2 - 1) + 1) + 36*a**3*sqrt(-2*a**2 - 2*a*sqrt(a**2 -
1) + 1)*sqrt(-2*a**2 + 2*a*sqrt(a**2 - 1) + 1) - 12*a**2*sqrt(a**2 - 1)*sqr
t(-2*a**2 - 2*a*sqrt(a**2 - 1) + 1)*sqrt(-2*a**2 + 2*a*sqrt(a**2 - 1) + 1)
- 2*a*sqrt(-2*a**2 - 2*a*sqrt(a**2 - 1) + 1)*sqrt(-2*a**2 + 2*a*sqrt(a**2 -
 1) + 1)) - 112*a**5*sqrt(-2*a**2 - 2*a*sqrt(a**2 - 1) + 1)*log(-sqrt(-2*a*
*2 + 2*a*sqrt(a**2 - 1) + 1) + tan(x/2))/(64*a**7*sqrt(-2*a**2 - 2*a*sqrt(a
2 - 1) + 1)*sqrt(-2*a2 + 2*a*sqrt(a**2 - 1) + 1) - 64*a**6*sqrt(a**2 -
1)*sqrt(-2*a**2 - 2*a*sqrt(a**2 - 1) + 1)*sqrt(-2*a**2 + 2*a*sqrt(a**2 - 1)
 + 1) - 96*a**5*sqrt(-2*a**2 - 2*a*sqrt(a**2 - 1) + 1)*sqrt(-2*a**2 + 2*a*s
qrt(a**2 - 1) + 1) + 64*a**4*sqrt(a**2 - 1)*sqrt(-2*a**2 - 2*a*sqrt(a**2 -
1) + 1)*sqrt(-2*a**2 + 2*a*sqrt(a**2 - 1) + 1) + 36*a**3*sqrt(-2*a**2 - 2*a
*sqrt(a**2 - 1) + 1)*sqrt(-2*a**2 + 2*a*sqrt(a**2 - 1) + 1) - 12*a**2*sqrt(
a**2 - 1)*sqrt(-2*a**2 - 2*a*sqrt(a**2 - 1) + 1)*sqrt(-2*a**2 + 2*a*sqrt(a*
*2 - 1) + 1) - 2*a*sqrt(-2*a**2 - 2*a*sqrt(a**2 - 1) + 1)*sqrt(-2*a**2 + 2*
a*sqrt(a**2 - 1) + 1)) + 112*a**5*sqrt(-2*a**2 - 2*a*sqrt(a**2 - 1) + 1)*lo
g(sqrt(-2*a**2 + 2*a*sqrt(a**2 - 1) + 1) + tan(x/2))/(64*a**7*sqrt(-2*a**2
- 2*a*sqrt(a**2 - 1) + 1)*sqrt(-2*a**2 + 2*a*sqrt(a**2 - 1) + 1) - 64*a**6*
sqrt(a**2 - 1)*sqrt(-2*a**2 - 2*a*sqrt(a**2 - 1) + 1)*sqrt(-2*a**2 + 2*a*sq
rt(a**2 - 1) + 1) - 96*a**5*sqrt(-2*a**2 - 2*a*sqrt(a**2 - 1) + 1)*sqrt(-2*
a**2 + 2*a*sqrt(a**2 - 1) + 1) + 64*a**4*sqrt(a**2 - 1)*sqrt(-2*a**2 - 2*a*
sqrt(a**2 - 1) + 1)*sqrt(-2*a**2 + 2*a*sqrt(a**2 - 1) + 1) + 36*a**3*sqrt(-
2*a**2 - 2*a*sqrt(a**2 - 1) + 1)*sqrt(-2*a**2 + 2*a*sqrt(a**2 - 1) + 1) - 1
2*a**2*sqrt(a**2 - 1)*sqrt(-2*a**2 - 2*a*sqrt(a**2 - 1) + 1)*sqrt(-2*a**2 +
 2*a*sqrt(a**2 - 1) + 1) - 2*a*sqrt(-2*a**2 - 2*a*sqrt(a**2 - 1) + 1)*sqrt(
-2*a**2 + 2*a*sqrt(a**2 - 1) + 1)) + 16*a**5*sqrt(-2*a**2 + 2*a*sqrt(a**2 -
 1) + 1)*log(-sqrt(-2*a**2 - 2*a*sqrt(a**2 - 1) + 1) + tan(x/2))/(64*a**7*s
qrt(-2*a**2 - 2*a*sqrt(a**2 - 1) + 1)*sqrt(-2*a**2 + 2*a*sqrt(a**2 - 1) + 1
) - 64*a**6*sqrt(a**2 - 1)*sqrt(-2*a**2 - 2*a*sqrt(a**2 - 1) + 1)*sqrt(-2*a
2 + 2*a*sqrt(a2 - 1) + 1) - 96*a**5*sqrt(-2*a**2 - 2*a*sqrt(a**2 - 1) +
 1)*sqrt(-2*a**2 + 2*a*sqrt(a**2 - 1) + 1) + 64*a**4*sqrt(a**2 - 1)*sqrt(-2
*a**2 - 2*a*sqrt(a**2 - 1) + 1)*sqrt(-2*a**2 + 2*a*sqrt(a**2 - 1) + 1) + 36
*a**3*sqrt(-2*a**2 - 2*a*sqrt(a**2 - 1) + 1)*sqrt(-2*a**2 + 2*a*sqrt(a**2 -
 1) + 1) - 12*a**2*sqrt(a**2 - 1)*sqrt(-2*a**2 - 2*a*sqrt(a**2 - 1) + 1)*sq
rt(-2*a**2 + 2*a*sqrt(a**2 - 1) + 1) - 2*a*sqrt(-2*a**2 - 2*a*sqrt(a**2 - 1
) + 1)*sqrt(-2*a**2 + 2*a*sqrt(a**2 - 1) + 1)) - 16*a**5*sqrt(-2*a**2 + 2*a
*sqrt(a**2 - 1) + 1)*log(sqrt(-2*a**2 - 2*a*sqrt(a**2 - 1) + 1) + tan(x/2))
/(64*a**7*sqrt(-2*a**2 - 2*a*sqrt(a**2 - 1) + 1)*sqrt(-2*a**2 + 2*a*sqrt(a*
*2 - 1) + 1) - 64*a**6*sqrt(a**2 - 1)*sqrt(-2*a**2 - 2*a*sqrt(a**2 - 1) + 1
)*sqrt(-2*a**2 + 2*a*sqrt(a**2 - 1) + 1) - 96*a**5*sqrt(-2*a**2 - 2*a*sqrt(
a**2 - 1) + 1)*sqrt(-2*a**2 + 2*a*sqrt(a**2 - 1) + 1) + 64*a**4*sqrt(a**2 -

```
 1)*sqrt(-2*a**2 - 2*a*sqrt(a**2 - 1) + 1)*sqrt(-2*a**2 + 2*a*sqrt(a**2 - 1
) + 1) + 36*a**3*sqrt(-2*a**2 - 2*a*sqrt(a**2 - 1) + 1)*sqrt(-2*a**2 + 2*a*
sqrt(a**2 - 1) + 1) - 12*a**2*sqrt(a**2 - 1)*sqrt(-2*a**2 - 2*a*sqrt(a**2 -
 1) + 1)*sqrt(-2*a**2 + 2*a*sqrt(a**2 - 1) + 1) - 2*a*sqrt(-2*a**2 - 2*a*sq
rt(a**2 - 1) + 1)*sqrt(-2*a**2 + 2*a*sqrt(a**2 - 1) + 1)) + 80*a**4*sqrt(a*
*2 - 1)*sqrt(-2*a**2 - 2*a*sqrt(a**2 - 1) + 1)*log(-sqrt(-2*a**2 + 2*a*sqrt
(a**2 - 1) + 1) + tan(x/2))/(64*a**7*sqrt(-2*a**2 - 2*a*sqrt(a**2 - 1) + 1)
*sqrt(-2*a**2 + 2*a*sqrt(a**2 - 1) + 1) - 64*a**6*sqrt(a**2 - 1)*sqrt(-2*a*
*2 - 2*a*sqrt(a**2 - 1) + 1)*sqrt(-2*a**2 + 2*a*sqrt(a**2 - 1) + 1) - 96*a*
*5*sqrt(-2*a**2 - 2*a*sqrt(a**2 - 1) + 1)*sqrt(-2*a**2 + 2*a*sqrt(a**2 - 1)
 + 1) + 64*a**4*sqrt(a**2 - 1)*sqrt(-2*a**2 - 2*a*sqrt(a**2 - 1) + 1)*sqrt(
-2*a**2 + 2*a*sqrt(a**2 - 1) + 1) + 36*a**3*sqrt(-2*a**2 - 2*a*sqrt(a**2 -
1) + 1)*sqrt(-2*a**2 + 2*a*sqrt(a**2 - 1) + 1) - 12*a**2*sqrt(a**2 - 1)*sqr
t(-2*a**2 - 2*a*sqrt(a**2 - 1) + 1)*sqrt(-2*a**2 + 2*a*sqrt(a**2 - 1) + 1)
- 2*a*sqrt(-2*a**2 - 2*a*sqrt(a**2 - 1) + 1)*sqrt(-2*a**2 + 2*a*sqrt(a**2 -
 1) + 1)) - 80*a**4*sqrt(a**2 - 1)*sqrt(-2*a**2 - 2*a*sqrt(a**2 - 1) + 1)*l
og(sqrt(-2*a**2 + 2*a*sqrt(a**2 - 1) + 1) + tan(x/2))/(64*a**7*sqrt(-2*a**2
 - 2*a*sqrt(a**2 - 1) + 1)*sqrt(-2*a**2 + 2*a*sqrt(a**2 - 1) + 1) - 64*a**6
*sqrt(a**2 - 1)*sqrt(-2*a**2 - 2*a*sqrt(a**2 - 1) + 1)*sqrt(-2*a**2 + 2*a*s
qrt(a**2 - 1) + 1) - 96*a**5*sqrt(-2*a**2 - 2*a*sqrt(a**2 - 1) + 1)*sqrt(-2
*a**2 + 2*a*sqrt(a**2 - 1) + 1) + 64*a**4*sqrt(a**2 - 1)*sqrt(-2*a**2 - 2*a
*sqrt(a**2 - 1) + 1)*sqrt(-2*a**2 + 2*a*sqrt(a**2 - 1) + 1) + 36*a**3*sqrt(
-2*a**2 - 2*a*sqrt(a**2 - 1) + 1)*sqrt(-2*a**2 + 2*a*sqrt(a**2 - 1) + 1) -
12*a**2*sqrt(a**2 - 1)*sqrt(-2*a**2 - 2*a*sqrt(a**2 - 1) + 1)*sqrt(-2*a**2
+ 2*a*sqrt(a**2 - 1) + 1) - 2*a*sqrt(-2*a**2 - 2*a*sqrt(a**2 - 1) + 1)*sqrt
(-2*a**2 + 2*a*sqrt(a**2 - 1) + 1)) - 16*a**4*sqrt(a**2 - 1)*sqrt(-2*a**2 +
 2*a*sqrt(a**2 - 1) + 1)*log(-sqrt(-2*a**2 - 2*a*sqrt(a**2 - 1) + 1) + tan(
x/2))/(64*a**7*sqrt(-2*a**2 - 2*a*sqrt(a**2 - 1) + 1)*sqrt(-2*a**2 + 2*a*sq
rt(a**2 - 1) + 1) - 64*a**6*sqrt(a**2 - 1)*sqrt(-2*a**2 - 2*a*sqrt(a**2 - 1
) + 1)*sqrt(-2*a**2 + 2*a*sqrt(a**2 - 1) + 1) - 96*a**5*sqrt(-2*a**2 - 2*a*
sqrt(a**2 - 1) + 1)*sqrt(-2*a**2 + 2*a*sqrt(a**2 - 1) + 1) + 64*a**4*sqrt(a
**2 - 1)*sqrt(-2*a**2 - 2*a*sqrt(a**2 - 1) + 1)*sqrt(-2*a**2 + 2*a*sqrt(a**
2 - 1) + 1) + 36*a**3*sqrt(-2*a**2 - 2*a*sqrt(a**2 - 1) + 1)*sqrt(-2*a**2 +
 2*a*sqrt(a**2 - 1) + 1) - 12*a**2*sqrt(a**2 - 1)*sqrt(-2*a**2 - 2*a*sqrt(a
**2 - 1) + 1)*sqrt(-2*a**2 + 2*a*sqrt(a**2 - 1) + 1) - 2*a*sqrt(-2*a**2 - 2
*a*sqrt(a**2 - 1) + 1)*sqrt(-2*a**2 + 2*a*sqrt(a**2 - 1) + 1)) + 16*a**4*sq
rt(a**2 - 1)*sqrt(-2*a**2 + 2*a*sqrt(a**2 - 1) + 1)*log(sqrt(-2*a**2 - 2*a*
sqrt(a**2 - 1) + 1) + tan(x/2))/(64*a**7*sqrt(-2*a**2 - 2*a*sqrt(a**2 - 1)
+ 1)*sqrt(-2*a**2 + 2*a*sqrt(a**2 - 1) + 1) - 64*a**6*sqrt(a**2 - 1)*sqrt(-
2*a**2 - 2*a*sqrt(a**2 - 1) + 1)*sqrt(-2*a**2 + 2*a*sqrt(a**2 - 1) + 1) - 9
6*a**5*sqrt(-2*a**2 - 2*a*sqrt(a**2 - 1) + 1)*sqrt(-2*a**2 + 2*a*sqrt(a**2
- 1) + 1) + 64*a**4*sqrt(a**2 - 1)*sqrt(-2*a**2 - 2*a*sqrt(a**2 - 1) + 1)*s
qrt(-2*a**2 + 2*a*sqrt(a**2 - 1) + 1) + 36*a**3*sqrt(-2*a**2 - 2*a*sqrt(a**
2 - 1) + 1)*sqrt(-2*a**2 + 2*a*sqrt(a**2 - 1) + 1) - 12*a**2*sqrt(a**2 - 1)
*sqrt(-2*a**2 - 2*a*sqrt(a**2 - 1) + 1)*sqrt(-2*a**2 + 2*a*sqrt(a**2 - 1) +
```

```
 1) - 2*a*sqrt(-2*a**2 - 2*a*sqrt(a**2 - 1) + 1)*sqrt(-2*a**2 + 2*a*sqrt(a*
*2 - 1) + 1)) + 56*a**3*sqrt(-2*a**2 - 2*a*sqrt(a**2 - 1) + 1)*log(-sqrt(-2
*a**2 + 2*a*sqrt(a**2 - 1) + 1) + tan(x/2))/(64*a**7*sqrt(-2*a**2 - 2*a*sqr
t(a**2 - 1) + 1)*sqrt(-2*a**2 + 2*a*sqrt(a**2 - 1) + 1) - 64*a**6*sqrt(a**2
 - 1)*sqrt(-2*a**2 - 2*a*sqrt(a**2 - 1) + 1)*sqrt(-2*a**2 + 2*a*sqrt(a**2 -
 1) + 1) - 96*a**5*sqrt(-2*a**2 - 2*a*sqrt(a**2 - 1) + 1)*sqrt(-2*a**2 + 2*
a*sqrt(a**2 - 1) + 1) + 64*a**4*sqrt(a**2 - 1)*sqrt(-2*a**2 - 2*a*sqrt(a**2
 - 1) + 1)*sqrt(-2*a**2 + 2*a*sqrt(a**2 - 1) + 1) + 36*a**3*sqrt(-2*a**2 -
2*a*sqrt(a**2 - 1) + 1)*sqrt(-2*a**2 + 2*a*sqrt(a**2 - 1) + 1) - 12*a**2*sq
rt(a**2 - 1)*sqrt(-2*a**2 - 2*a*sqrt(a**2 - 1) + 1)*sqrt(-2*a**2 + 2*a*sqrt
(a**2 - 1) + 1) - 2*a*sqrt(-2*a**2 - 2*a*sqrt(a**2 - 1) + 1)*sqrt(-2*a**2 +
 2*a*sqrt(a**2 - 1) + 1)) - 56*a**3*sqrt(-2*a**2 - 2*a*sqrt(a**2 - 1) + 1)*
log(sqrt(-2*a**2 + 2*a*sqrt(a**2 - 1) + 1) + tan(x/2))/(64*a**7*sqrt(-2*a**
2 - 2*a*sqrt(a**2 - 1) + 1)*sqrt(-2*a**2 + 2*a*sqrt(a**2 - 1) + 1) - 64*a**
6*sqrt(a**2 - 1)*sqrt(-2*a**2 - 2*a*sqrt(a**2 - 1) + 1)*sqrt(-2*a**2 + 2*a*
sqrt(a**2 - 1) + 1) - 96*a**5*sqrt(-2*a**2 - 2*a*sqrt(a**2 - 1) + 1)*sqrt(-
2*a**2 + 2*a*sqrt(a**2 - 1) + 1) + 64*a**4*sqrt(a**2 - 1)*sqrt(-2*a**2 - 2*
a*sqrt(a**2 - 1) + 1)*sqrt(-2*a**2 + 2*a*sqrt(a**2 - 1) + 1) + 36*a**3*sqrt
(-2*a**2 - 2*a*sqrt(a**2 - 1) + 1)*sqrt(-2*a**2 + 2*a*sqrt(a**2 - 1) + 1) -
 12*a**2*sqrt(a**2 - 1)*sqrt(-2*a**2 - 2*a*sqrt(a**2 - 1) + 1)*sqrt(-2*a**2
 + 2*a*sqrt(a**2 - 1) + 1) - 2*a*sqrt(-2*a**2 - 2*a*sqrt(a**2 - 1) + 1)*sqr
t(-2*a**2 + 2*a*sqrt(a**2 - 1) + 1)) - 20*a**3*sqrt(-2*a**2 + 2*a*sqrt(a**2
 - 1) + 1)*log(-sqrt(-2*a**2 - 2*a*sqrt(a**2 - 1) + 1) + tan(x/2))/(64*a**7
*sqrt(-2*a**2 - 2*a*sqrt(a**2 - 1) + 1)*sqrt(-2*a**2 + 2*a*sqrt(a**2 - 1) +
 1) - 64*a**6*sqrt(a**2 - 1)*sqrt(-2*a**2 - 2*a*sqrt(a**2 - 1) + 1)*sqrt(-2
*a**2 + 2*a*sqrt(a**2 - 1) + 1) - 96*a**5*sqrt(-2*a**2 - 2*a*sqrt(a**2 - 1)
 + 1)*sqrt(-2*a**2 + 2*a*sqrt(a**2 - 1) + 1) + 64*a**4*sqrt(a**2 - 1)*sqrt(
-2*a**2 - 2*a*sqrt(a**2 - 1) + 1)*sqrt(-2*a**2 + 2*a*sqrt(a**2 - 1) + 1) +
36*a**3*sqrt(-2*a**2 - 2*a*sqrt(a**2 - 1) + 1)*sqrt(-2*a**2 + 2*a*sqrt(a**2
 - 1) + 1) - 12*a**2*sqrt(a**2 - 1)*sqrt(-2*a**2 - 2*a*sqrt(a**2 - 1) + 1)*
sqrt(-2*a**2 + 2*a*sqrt(a**2 - 1) + 1) - 2*a*sqrt(-2*a**2 - 2*a*sqrt(a**2 -
 1) + 1)*sqrt(-2*a**2 + 2*a*sqrt(a**2 - 1) + 1)) + 20*a**3*sqrt(-2*a**2 + 2
*a*sqrt(a**2 - 1) + 1)*log(sqrt(-2*a**2 - 2*a*sqrt(a**2 - 1) + 1) + tan(x/2
))/(64*a**7*sqrt(-2*a**2 - 2*a*sqrt(a**2 - 1) + 1)*sqrt(-2*a**2 + 2*a*sqrt(
a**2 - 1) + 1) - 64*a**6*sqrt(a**2 - 1)*sqrt(-2*a**2 - 2*a*sqrt(a**2 - 1) +
 1)*sqrt(-2*a**2 + 2*a*sqrt(a**2 - 1) + 1) - 96*a**5*sqrt(-2*a**2 - 2*a*sqr
t(a**2 - 1) + 1)*sqrt(-2*a**2 + 2*a*sqrt(a**2 - 1) + 1) + 64*a**4*sqrt(a**2
 - 1)*sqrt(-2*a**2 - 2*a*sqrt(a**2 - 1) + 1)*sqrt(-2*a**2 + 2*a*sqrt(a**2 -
 1) + 1) + 36*a**3*sqrt(-2*a**2 - 2*a*sqrt(a**2 - 1) + 1)*sqrt(-2*a**2 + 2*
a*sqrt(a**2 - 1) + 1) - 12*a**2*sqrt(a**2 - 1)*sqrt(-2*a**2 - 2*a*sqrt(a**2
 - 1) + 1)*sqrt(-2*a**2 + 2*a*sqrt(a**2 - 1) + 1) - 2*a*sqrt(-2*a**2 - 2*a*
sqrt(a**2 - 1) + 1)*sqrt(-2*a**2 + 2*a*sqrt(a**2 - 1) + 1)) - 24*a**2*sqrt(
a**2 - 1)*sqrt(-2*a**2 - 2*a*sqrt(a**2 - 1) + 1)*log(-sqrt(-2*a**2 + 2*a*sq
rt(a**2 - 1) + 1) + tan(x/2))/(64*a**7*sqrt(-2*a**2 - 2*a*sqrt(a**2 - 1) +
1)*sqrt(-2*a**2 + 2*a*sqrt(a**2 - 1) + 1) - 64*a**6*sqrt(a**2 - 1)*sqrt(-2*
```

a**2 - 2*a*sqrt(a**2 - 1) + 1)*sqrt(-2*a**2 + 2*a*sqrt(a**2 - 1) + 1) - 96*
a**5*sqrt(-2*a**2 - 2*a*sqrt(a**2 - 1) + 1)*sqrt(-2*a**2 + 2*a*sqrt(a**2 -
1) + 1) + 64*a**4*sqrt(a**2 - 1)*sqrt(-2*a**2 - 2*a*sqrt(a**2 - 1) + 1)*sqr
t(-2*a**2 + 2*a*sqrt(a**2 - 1) + 1) + 36*a**3*sqrt(-2*a**2 - 2*a*sqrt(a**2
- 1) + 1)*sqrt(-2*a**2 + 2*a*sqrt(a**2 - 1) + 1) - 12*a**2*sqrt(a**2 - 1)*s
qrt(-2*a**2 - 2*a*sqrt(a**2 - 1) + 1)*sqrt(-2*a**2 + 2*a*sqrt(a**2 - 1) + 1
) - 2*a*sqrt(-2*a**2 - 2*a*sqrt(a**2 - 1) + 1)*sqrt(-2*a**2 + 2*a*sqrt(a**2
 - 1) + 1)) + 24*a**2*sqrt(a**2 - 1)*sqrt(-2*a**2 - 2*a*sqrt(a**2 - 1) + 1)
*log(sqrt(-2*a**2 + 2*a*sqrt(a**2 - 1) + 1) + tan(x/2))/(64*a**7*sqrt(-2*a*
*2 - 2*a*sqrt(a**2 - 1) + 1)*sqrt(-2*a**2 + 2*a*sqrt(a**2 - 1) + 1) - 64*a*
*6*sqrt(a**2 - 1)*sqrt(-2*a**2 - 2*a*sqrt(a**2 - 1) + 1)*sqrt(-2*a**2 + 2*a
*sqrt(a**2 - 1) + 1) - 96*a**5*sqrt(-2*a**2 - 2*a*sqrt(a**2 - 1) + 1)*sqrt(
-2*a**2 + 2*a*sqrt(a**2 - 1) + 1) + 64*a**4*sqrt(a**2 - 1)*sqrt(-2*a**2 - 2
*a*sqrt(a**2 - 1) + 1)*sqrt(-2*a**2 + 2*a*sqrt(a**2 - 1) + 1) + 36*a**3*sqr
t(-2*a**2 - 2*a*sqrt(a**2 - 1) + 1)*sqrt(-2*a**2 + 2*a*sqrt(a**2 - 1) + 1)
- 12*a**2*sqrt(a**2 - 1)*sqrt(-2*a**2 - 2*a*sqrt(a**2 - 1) + 1)*sqrt(-2*a**
2 + 2*a*sqrt(a**2 - 1) + 1) - 2*a*sqrt(-2*a**2 - 2*a*sqrt(a**2 - 1) + 1)*sq
rt(-2*a**2 + 2*a*sqrt(a**2 - 1) + 1)) + 12*a**2*sqrt(a**2 - 1)*sqrt(-2*a**2
 + 2*a*sqrt(a**2 - 1) + 1)*log(-sqrt(-2*a**2 - 2*a*sqrt(a**2 - 1) + 1) + ta
n(x/2))/(64*a**7*sqrt(-2*a**2 - 2*a*sqrt(a**2 - 1) + 1)*sqrt(-2*a**2 + 2*a*
sqrt(a**2 - 1) + 1) - 64*a**6*sqrt(a**2 - 1)*sqrt(-2*a**2 - 2*a*sqrt(a**2 -
 1) + 1)*sqrt(-2*a**2 + 2*a*sqrt(a**2 - 1) + 1) - 96*a**5*sqrt(-2*a**2 - 2*
a*sqrt(a**2 - 1) + 1)*sqrt(-2*a**2 + 2*a*sqrt(a**2 - 1) + 1) + 64*a**4*sqrt
(a**2 - 1)*sqrt(-2*a**2 - 2*a*sqrt(a**2 - 1) + 1)*sqrt(-2*a**2 + 2*a*sqrt(a
2 - 1) + 1) + 36*a3*sqrt(-2*a**2 - 2*a*sqrt(a**2 - 1) + 1)*sqrt(-2*a**2
 + 2*a*sqrt(a**2 - 1) + 1) - 12*a**2*sqrt(a**2 - 1)*sqrt(-2*a**2 - 2*a*sqrt
(a**2 - 1) + 1)*sqrt(-2*a**2 + 2*a*sqrt(a**2 - 1) + 1) - 2*a*sqrt(-2*a**2 -
 2*a*sqrt(a**2 - 1) + 1)*sqrt(-2*a**2 + 2*a*sqrt(a**2 - 1) + 1)) - 12*a**2*
sqrt(a**2 - 1)*sqrt(-2*a**2 + 2*a*sqrt(a**2 - 1) + 1)*log(sqrt(-2*a**2 - 2*
a*sqrt(a**2 - 1) + 1) + tan(x/2))/(64*a**7*sqrt(-2*a**2 - 2*a*sqrt(a**2 - 1
) + 1)*sqrt(-2*a**2 + 2*a*sqrt(a**2 - 1) + 1) - 64*a**6*sqrt(a**2 - 1)*sqrt
(-2*a**2 - 2*a*sqrt(a**2 - 1) + 1)*sqrt(-2*a**2 + 2*a*sqrt(a**2 - 1) + 1) -
 96*a**5*sqrt(-2*a**2 - 2*a*sqrt(a**2 - 1) + 1)*sqrt(-2*a**2 + 2*a*sqrt(a**
2 - 1) + 1) + 64*a**4*sqrt(a**2 - 1)*sqrt(-2*a**2 - 2*a*sqrt(a**2 - 1) + 1)
*sqrt(-2*a**2 + 2*a*sqrt(a**2 - 1) + 1) + 36*a**3*sqrt(-2*a**2 - 2*a*sqrt(a
2 - 1) + 1)*sqrt(-2*a2 + 2*a*sqrt(a**2 - 1) + 1) - 12*a**2*sqrt(a**2 -
1)*sqrt(-2*a**2 - 2*a*sqrt(a**2 - 1) + 1)*sqrt(-2*a**2 + 2*a*sqrt(a**2 - 1)
 + 1) - 2*a*sqrt(-2*a**2 - 2*a*sqrt(a**2 - 1) + 1)*sqrt(-2*a**2 + 2*a*sqrt(
a**2 - 1) + 1)) - 7*a*sqrt(-2*a**2 - 2*a*sqrt(a**2 - 1) + 1)*log(-sqrt(-2*a
2 + 2*a*sqrt(a2 - 1) + 1) + tan(x/2))/(64*a**7*sqrt(-2*a**2 - 2*a*sqrt(
a**2 - 1) + 1)*sqrt(-2*a**2 + 2*a*sqrt(a**2 - 1) + 1) - 64*a**6*sqrt(a**2 -
 1)*sqrt(-2*a**2 - 2*a*sqrt(a**2 - 1) + 1)*sqrt(-2*a**2 + 2*a*sqrt(a**2 - 1
) + 1) - 96*a**5*sqrt(-2*a**2 - 2*a*sqrt(a**2 - 1) + 1)*sqrt(-2*a**2 + 2*a*
sqrt(a**2 - 1) + 1) + 64*a**4*sqrt(a**2 - 1)*sqrt(-2*a**2 - 2*a*sqrt(a**2 -
 1) + 1)*sqrt(-2*a**2 + 2*a*sqrt(a**2 - 1) + 1) + 36*a**3*sqrt(-2*a**2 - 2*

```
a*sqrt(a**2 - 1) + 1)*sqrt(-2*a**2 + 2*a*sqrt(a**2 - 1) + 1) - 12*a**2*sqrt
(a**2 - 1)*sqrt(-2*a**2 - 2*a*sqrt(a**2 - 1) + 1)*sqrt(-2*a**2 + 2*a*sqrt(a
**2 - 1) + 1) - 2*a*sqrt(-2*a**2 - 2*a*sqrt(a**2 - 1) + 1)*sqrt(-2*a**2 + 2
*a*sqrt(a**2 - 1) + 1)) + 7*a*sqrt(-2*a**2 - 2*a*sqrt(a**2 - 1) + 1)*log(sq
rt(-2*a**2 + 2*a*sqrt(a**2 - 1) + 1) + tan(x/2))/(64*a**7*sqrt(-2*a**2 - 2*
a*sqrt(a**2 - 1) + 1)*sqrt(-2*a**2 + 2*a*sqrt(a**2 - 1) + 1) - 64*a**6*sqrt
(a**2 - 1)*sqrt(-2*a**2 - 2*a*sqrt(a**2 - 1) + 1)*sqrt(-2*a**2 + 2*a*sqrt(a
**2 - 1) + 1) - 96*a**5*sqrt(-2*a**2 - 2*a*sqrt(a**2 - 1) + 1)*sqrt(-2*a**2
 + 2*a*sqrt(a**2 - 1) + 1) + 64*a**4*sqrt(a**2 - 1)*sqrt(-2*a**2 - 2*a*sqrt
(a**2 - 1) + 1)*sqrt(-2*a**2 + 2*a*sqrt(a**2 - 1) + 1) + 36*a**3*sqrt(-2*a*
*2 - 2*a*sqrt(a**2 - 1) + 1)*sqrt(-2*a**2 + 2*a*sqrt(a**2 - 1) + 1) - 12*a*
*2*sqrt(a**2 - 1)*sqrt(-2*a**2 - 2*a*sqrt(a**2 - 1) + 1)*sqrt(-2*a**2 + 2*a
*sqrt(a**2 - 1) + 1) - 2*a*sqrt(-2*a**2 - 2*a*sqrt(a**2 - 1) + 1)*sqrt(-2*a
**2 + 2*a*sqrt(a**2 - 1) + 1)) + 5*a*sqrt(-2*a**2 + 2*a*sqrt(a**2 - 1) + 1)
*log(-sqrt(-2*a**2 - 2*a*sqrt(a**2 - 1) + 1) + tan(x/2))/(64*a**7*sqrt(-2*a
**2 - 2*a*sqrt(a**2 - 1) + 1)*sqrt(-2*a**2 + 2*a*sqrt(a**2 - 1) + 1) - 64*a
**6*sqrt(a**2 - 1)*sqrt(-2*a**2 - 2*a*sqrt(a**2 - 1) + 1)*sqrt(-2*a**2 + 2*
a*sqrt(a**2 - 1) + 1) - 96*a**5*sqrt(-2*a**2 - 2*a*sqrt(a**2 - 1) + 1)*sqrt
(-2*a**2 + 2*a*sqrt(a**2 - 1) + 1) + 64*a**4*sqrt(a**2 - 1)*sqrt(-2*a**2 -
2*a*sqrt(a**2 - 1) + 1)*sqrt(-2*a**2 + 2*a*sqrt(a**2 - 1) + 1) + 36*a**3*sq
rt(-2*a**2 - 2*a*sqrt(a**2 - 1) + 1)*sqrt(-2*a**2 + 2*a*sqrt(a**2 - 1) + 1)
 - 12*a**2*sqrt(a**2 - 1)*sqrt(-2*a**2 - 2*a*sqrt(a**2 - 1) + 1)*sqrt(-2*a*
*2 + 2*a*sqrt(a**2 - 1) + 1) - 2*a*sqrt(-2*a**2 - 2*a*sqrt(a**2 - 1) + 1)*s
qrt(-2*a**2 + 2*a*sqrt(a**2 - 1) + 1)) - 5*a*sqrt(-2*a**2 + 2*a*sqrt(a**2 -
 1) + 1)*log(sqrt(-2*a**2 - 2*a*sqrt(a**2 - 1) + 1) + tan(x/2))/(64*a**7*sq
rt(-2*a**2 - 2*a*sqrt(a**2 - 1) + 1)*sqrt(-2*a**2 + 2*a*sqrt(a**2 - 1) + 1)
 - 64*a**6*sqrt(a**2 - 1)*sqrt(-2*a**2 - 2*a*sqrt(a**2 - 1) + 1)*sqrt(-2*a*
*2 + 2*a*sqrt(a**2 - 1) + 1) - 96*a**5*sqrt(-2*a**2 - 2*a*sqrt(a**2 - 1) +
1)*sqrt(-2*a**2 + 2*a*sqrt(a**2 - 1) + 1) + 64*a**4*sqrt(a**2 - 1)*sqrt(-2*
a**2 - 2*a*sqrt(a**2 - 1) + 1)*sqrt(-2*a**2 + 2*a*sqrt(a**2 - 1) + 1) + 36*
a**3*sqrt(-2*a**2 - 2*a*sqrt(a**2 - 1) + 1)*sqrt(-2*a**2 + 2*a*sqrt(a**2 -
1) + 1) - 12*a**2*sqrt(a**2 - 1)*sqrt(-2*a**2 - 2*a*sqrt(a**2 - 1) + 1)*sqr
t(-2*a**2 + 2*a*sqrt(a**2 - 1) + 1) - 2*a*sqrt(-2*a**2 - 2*a*sqrt(a**2 - 1)
 + 1)*sqrt(-2*a**2 + 2*a*sqrt(a**2 - 1) + 1)) + sqrt(a**2 - 1)*sqrt(-2*a**2
 - 2*a*sqrt(a**2 - 1) + 1)*log(-sqrt(-2*a**2 + 2*a*sqrt(a**2 - 1) + 1) + ta
n(x/2))/(64*a**7*sqrt(-2*a**2 - 2*a*sqrt(a**2 - 1) + 1)*sqrt(-2*a**2 + 2*a*
sqrt(a**2 - 1) + 1) - 64*a**6*sqrt(a**2 - 1)*sqrt(-2*a**2 - 2*a*sqrt(a**2 -
 1) + 1)*sqrt(-2*a**2 + 2*a*sqrt(a**2 - 1) + 1) - 96*a**5*sqrt(-2*a**2 - 2*
a*sqrt(a**2 - 1) + 1)*sqrt(-2*a**2 + 2*a*sqrt(a**2 - 1) + 1) + 64*a**4*sqrt
(a**2 - 1)*sqrt(-2*a**2 - 2*a*sqrt(a**2 - 1) + 1)*sqrt(-2*a**2 + 2*a*sqrt(a
**2 - 1) + 1) + 36*a**3*sqrt(-2*a**2 - 2*a*sqrt(a**2 - 1) + 1)*sqrt(-2*a**2
 + 2*a*sqrt(a**2 - 1) + 1) - 12*a**2*sqrt(a**2 - 1)*sqrt(-2*a**2 - 2*a*sqrt
(a**2 - 1) + 1)*sqrt(-2*a**2 + 2*a*sqrt(a**2 - 1) + 1) - 2*a*sqrt(-2*a**2 -
 2*a*sqrt(a**2 - 1) + 1)*sqrt(-2*a**2 + 2*a*sqrt(a**2 - 1) + 1)) - sqrt(a**
2 - 1)*sqrt(-2*a**2 - 2*a*sqrt(a**2 - 1) + 1)*log(sqrt(-2*a**2 + 2*a*sqrt(a
```

```
**2 - 1) + 1) + tan(x/2))/(64*a**7*sqrt(-2*a**2 - 2*a*sqrt(a**2 - 1) + 1)*s
qrt(-2*a**2 + 2*a*sqrt(a**2 - 1) + 1) - 64*a**6*sqrt(a**2 - 1)*sqrt(-2*a**2
 - 2*a*sqrt(a**2 - 1) + 1)*sqrt(-2*a**2 + 2*a*sqrt(a**2 - 1) + 1) - 96*a**5
*sqrt(-2*a**2 - 2*a*sqrt(a**2 - 1) + 1)*sqrt(-2*a**2 + 2*a*sqrt(a**2 - 1) +
 1) + 64*a**4*sqrt(a**2 - 1)*sqrt(-2*a**2 - 2*a*sqrt(a**2 - 1) + 1)*sqrt(-2
*a**2 + 2*a*sqrt(a**2 - 1) + 1) + 36*a**3*sqrt(-2*a**2 - 2*a*sqrt(a**2 - 1)
 + 1)*sqrt(-2*a**2 + 2*a*sqrt(a**2 - 1) + 1) - 12*a**2*sqrt(a**2 - 1)*sqrt(
-2*a**2 - 2*a*sqrt(a**2 - 1) + 1)*sqrt(-2*a**2 + 2*a*sqrt(a**2 - 1) + 1) -
2*a*sqrt(-2*a**2 - 2*a*sqrt(a**2 - 1) + 1)*sqrt(-2*a**2 + 2*a*sqrt(a**2 - 1
) + 1)) - sqrt(a**2 - 1)*sqrt(-2*a**2 + 2*a*sqrt(a**2 - 1) + 1)*log(-sqrt(-
2*a**2 - 2*a*sqrt(a**2 - 1) + 1) + tan(x/2))/(64*a**7*sqrt(-2*a**2 - 2*a*sq
rt(a**2 - 1) + 1)*sqrt(-2*a**2 + 2*a*sqrt(a**2 - 1) + 1) - 64*a**6*sqrt(a**
2 - 1)*sqrt(-2*a**2 - 2*a*sqrt(a**2 - 1) + 1)*sqrt(-2*a**2 + 2*a*sqrt(a**2
- 1) + 1) - 96*a**5*sqrt(-2*a**2 - 2*a*sqrt(a**2 - 1) + 1)*sqrt(-2*a**2 + 2
*a*sqrt(a**2 - 1) + 1) + 64*a**4*sqrt(a**2 - 1)*sqrt(-2*a**2 - 2*a*sqrt(a**
2 - 1) + 1)*sqrt(-2*a**2 + 2*a*sqrt(a**2 - 1) + 1) + 36*a**3*sqrt(-2*a**2 -
 2*a*sqrt(a**2 - 1) + 1)*sqrt(-2*a**2 + 2*a*sqrt(a**2 - 1) + 1) - 12*a**2*s
qrt(a**2 - 1)*sqrt(-2*a**2 - 2*a*sqrt(a**2 - 1) + 1)*sqrt(-2*a**2 + 2*a*sqr
t(a**2 - 1) + 1) - 2*a*sqrt(-2*a**2 - 2*a*sqrt(a**2 - 1) + 1)*sqrt(-2*a**2
+ 2*a*sqrt(a**2 - 1) + 1)) + sqrt(a**2 - 1)*sqrt(-2*a**2 + 2*a*sqrt(a**2 -
1) + 1)*log(sqrt(-2*a**2 - 2*a*sqrt(a**2 - 1) + 1) + tan(x/2))/(64*a**7*sqr
t(-2*a**2 - 2*a*sqrt(a**2 - 1) + 1)*sqrt(-2*a**2 + 2*a*sqrt(a**2 - 1) + 1)
- 64*a**6*sqrt(a**2 - 1)*sqrt(-2*a**2 - 2*a*sqrt(a**2 - 1) + 1)*sqrt(-2*a**
2 + 2*a*sqrt(a**2 - 1) + 1) - 96*a**5*sqrt(-2*a**2 - 2*a*sqrt(a**2 - 1) + 1
)*sqrt(-2*a**2 + 2*a*sqrt(a**2 - 1) + 1) + 64*a**4*sqrt(a**2 - 1)*sqrt(-2*a
**2 - 2*a*sqrt(a**2 - 1) + 1)*sqrt(-2*a**2 + 2*a*sqrt(a**2 - 1) + 1) + 36*a
**3*sqrt(-2*a**2 - 2*a*sqrt(a**2 - 1) + 1)*sqrt(-2*a**2 + 2*a*sqrt(a**2 - 1
) + 1) - 12*a**2*sqrt(a**2 - 1)*sqrt(-2*a**2 - 2*a*sqrt(a**2 - 1) + 1)*sqrt
(-2*a**2 + 2*a*sqrt(a**2 - 1) + 1) - 2*a*sqrt(-2*a**2 - 2*a*sqrt(a**2 - 1)
+ 1)*sqrt(-2*a**2 + 2*a*sqrt(a**2 - 1) + 1)), Ne(a, 0)), (-2*tan(x/2)/(tan(
x/2)**2 - 1), True))
```

3.479 $\int \frac{1}{b^2\cos^2(x)+\sin^2(x)}\,dx$

Optimal. Leaf size=11

$$\frac{\tan^{-1}\left(\frac{\tan(x)}{b}\right)}{b}$$

```
[Out] arctan(tan(x)/b)/b
```

Rubi [A] time = 0.02, antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {203}

$$\frac{\tan^{-1}\left(\frac{\tan(x)}{b}\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[(b^2*Cos[x]^2 + Sin[x]^2)^(-1),x]
```

```
[Out] ArcTan[Tan[x]/b]/b
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{b^2\cos^2(x)+\sin^2(x)}\,dx &= \text{Subst}\left(\int \frac{1}{b^2+x^2}\,dx, x, \tan(x)\right)\\ &= \frac{\tan^{-1}\left(\frac{\tan(x)}{b}\right)}{b}\end{aligned}$$

Mathematica [A] time = 0.03, size = 11, normalized size = 1.00

$$\frac{\tan^{-1}\left(\frac{\tan(x)}{b}\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b^2*Cos[x]^2 + Sin[x]^2)^(-1),x]
[Out] ArcTan[Tan[x]/b]/b
```

fricas [B] time = 2.04, size = 31, normalized size = 2.82

$$-\frac{\arctan\left(\frac{(b^2+1)\cos(x)^2-1}{2\,b\cos(x)\sin(x)}\right)}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b^2*cos(x)^2+sin(x)^2),x, algorithm="fricas")
[Out] -1/2*arctan(1/2*((b^2 + 1)*cos(x)^2 - 1)/(b*cos(x)*sin(x)))/b
```

giac [A] time = 0.13, size = 22, normalized size = 2.00

$$\frac{\pi\left\lfloor\frac{x}{\pi}+\frac{1}{2}\right\rfloor+\arctan\left(\frac{\tan(x)}{b}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b^2*cos(x)^2+sin(x)^2),x, algorithm="giac")
[Out] (pi*floor(x/pi + 1/2) + arctan(tan(x)/b))/b
```

maple [A] time = 0.11, size = 12, normalized size = 1.09

$$\frac{\arctan\left(\frac{\tan(x)}{b}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b^2*cos(x)^2+sin(x)^2),x)
[Out] arctan(tan(x)/b)/b
```

maxima [A] time = 0.41, size = 11, normalized size = 1.00

$$\frac{\arctan\left(\frac{\tan(x)}{b}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b^2*cos(x)^2+sin(x)^2),x, algorithm="maxima")
```

```
[Out] arctan(tan(x)/b)/b
```

mupad [B] time = 2.83, size = 11, normalized size = 1.00

$$\frac{\operatorname{atan}\left(\frac{\tan(x)}{b}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(x)^2 + b^2*cos(x)^2),x)
```

```
[Out] atan(tan(x)/b)/b
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b**2*cos(x)**2+sin(x)**2),x)
```

```
[Out] Timed out
```

3.480 $\int \frac{1}{b^2\cos^2(x)+a^2\sin^2(x)}\,dx$

Optimal. Leaf size=15

$$\frac{\tan^{-1}\left(\frac{a\tan(x)}{b}\right)}{ab}$$

```
[Out] arctan(a*tan(x)/b)/a/b
```

Rubi [A] time = 0.03, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {205}

$$\frac{\tan^{-1}\left(\frac{a\tan(x)}{b}\right)}{ab}$$

Antiderivative was successfully verified.

```
[In] Int[(b^2*Cos[x]^2 + a^2*Sin[x]^2)^(-1),x]
```

```
[Out] ArcTan[(a*Tan[x])/b]/(a*b)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{b^2\cos^2(x)+a^2\sin^2(x)}\,dx &= \text{Subst}\left(\int \frac{1}{b^2+a^2x^2}\,dx, x, \tan(x)\right)\\ &= \frac{\tan^{-1}\left(\frac{a\tan(x)}{b}\right)}{ab}\end{aligned}$$

Mathematica [A] time = 0.04, size = 15, normalized size = 1.00

$$\frac{\tan^{-1}\left(\frac{a\tan(x)}{b}\right)}{ab}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b^2*Cos[x]^2 + a^2*Sin[x]^2)^(-1),x]
```

[Out] ArcTan[(a*Tan[x])/b]/(a*b)

fricas [B] time = 2.39, size = 43, normalized size = 2.87

$$-\frac{\arctan\left(\frac{\left(a^2+b^2\right)\cos(x)^2-a^2}{2\,ab\cos(x)\sin(x)}\right)}{2\,ab}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b^2*cos(x)^2+a^2*sin(x)^2),x, algorithm="fricas")

[Out] -1/2*arctan(1/2*((a^2 + b^2)*cos(x)^2 - a^2)/(a*b*cos(x)*sin(x)))/(a*b)

giac [A] time = 0.13, size = 26, normalized size = 1.73

$$\frac{\pi\left\lfloor\frac{x}{\pi}+\frac{1}{2}\right\rfloor+\arctan\left(\frac{a\tan(x)}{b}\right)}{ab}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b^2*cos(x)^2+a^2*sin(x)^2),x, algorithm="giac")

[Out] (pi*floor(x/pi + 1/2) + arctan(a*tan(x)/b))/(a*b)

maple [A] time = 0.12, size = 16, normalized size = 1.07

$$\frac{\arctan\left(\frac{a\tan(x)}{b}\right)}{ab}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(b^2*cos(x)^2+a^2*sin(x)^2),x)

[Out] arctan(a*tan(x)/b)/a/b

maxima [A] time = 0.40, size = 15, normalized size = 1.00

$$\frac{\arctan\left(\frac{a\tan(x)}{b}\right)}{ab}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b^2*cos(x)^2+a^2*sin(x)^2),x, algorithm="maxima")

[Out] arctan(a*tan(x)/b)/(a*b)

mupad [B] time = 2.85, size = 15, normalized size = 1.00

$$\frac{\operatorname{atan}\left(\frac{a\tan(x)}{b}\right)}{a\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b^2*cos(x)^2 + a^2*sin(x)^2),x)
```

```
[Out] atan((a*tan(x))/b)/(a*b)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b**2*cos(x)**2+a**2*sin(x)**2),x)
```

```
[Out] Timed out
```

3.481 $\int \frac{1}{4\cos^2(1+2x)+3\sin^2(1+2x)}\,dx$

Optimal. Leaf size=53

$$\frac{x}{2\sqrt{3}}-\frac{\tan^{-1}\left(\frac{\sin(2x+1)\cos(2x+1)}{\cos^2(2x+1)+2\sqrt{3}+3}\right)}{4\sqrt{3}}$$

```
[Out] 1/6*x*3^(1/2)-1/12*arctan(cos(1+2*x)*sin(1+2*x)/(3+cos(1+2*x)^2+2*3^(1/2)))
*3^(1/2)
```

Rubi [A] time = 0.04, antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.043, Rules used = {203}

$$\frac{x}{2\sqrt{3}}-\frac{\tan^{-1}\left(\frac{\sin(2x+1)\cos(2x+1)}{\cos^2(2x+1)+2\sqrt{3}+3}\right)}{4\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[(4*Cos[1 + 2*x]^2 + 3*Sin[1 + 2*x]^2)^(-1),x]
```

```
[Out] x/(2*Sqrt[3]) - ArcTan[(Cos[1 + 2*x]*Sin[1 + 2*x])/(3 + 2*Sqrt[3] + Cos[1 +
 2*x]^2)]/(4*Sqrt[3])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{4\cos^2(1+2x)+3\sin^2(1+2x)}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{1}{4+3x^2}\,dx, x, \tan(1+2x)\right)\\ &= \frac{x}{2\sqrt{3}}-\frac{\tan^{-1}\left(\frac{\cos(1+2x)\sin(1+2x)}{3+2\sqrt{3}+\cos^2(1+2x)}\right)}{4\sqrt{3}}\end{aligned}$$

Mathematica [A] time = 0.04, size = 25, normalized size = 0.47

$$\frac{\tan^{-1}\left(\frac{1}{2}\sqrt{3}\,\tan(2x+1)\right)}{4\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(4*Cos[1 + 2*x]^2 + 3*Sin[1 + 2*x]^2)^(-1),x]
```

```
[Out] ArcTan[(Sqrt[3]*Tan[1 + 2*x])/2]/(4*Sqrt[3])
```

fricas [A] time = 0.99, size = 43, normalized size = 0.81

$$-\frac{1}{24}\sqrt{3}\arctan\left(\frac{7\sqrt{3}\cos\left(2x+1\right)^{2}-3\sqrt{3}}{12\cos\left(2x+1\right)\sin\left(2x+1\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(4*cos(1+2*x)^2+3*sin(1+2*x)^2),x, algorithm="fricas")
```

```
[Out] -1/24*sqrt(3)*arctan(1/12*(7*sqrt(3)*cos(2*x + 1)^2 - 3*sqrt(3))/(cos(2*x +
 1)*sin(2*x + 1)))
```

giac [A] time = 0.13, size = 61, normalized size = 1.15

$$\frac{1}{12}\sqrt{3}\left(2x+\arctan\left(-\frac{2\sqrt{3}\sin\left(4x+2\right)-3\sin\left(4x+2\right)}{2\sqrt{3}\cos\left(4x+2\right)+2\sqrt{3}-3\cos\left(4x+2\right)+3}\right)+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(4*cos(1+2*x)^2+3*sin(1+2*x)^2),x, algorithm="giac")
```

```
[Out] 1/12*sqrt(3)*(2*x + arctan(-(2*sqrt(3)*sin(4*x + 2) - 3*sin(4*x + 2))/(2*sq
rt(3)*cos(4*x + 2) + 2*sqrt(3) - 3*cos(4*x + 2) + 3)) + 1)
```

maple [A] time = 0.23, size = 18, normalized size = 0.34

$$\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\tan(1+2x)}{2}\right)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(4*cos(1+2*x)^2+3*sin(1+2*x)^2),x)
```

```
[Out] 1/12*3^(1/2)*arctan(1/2*3^(1/2)*tan(1+2*x))
```

maxima [A] time = 0.41, size = 17, normalized size = 0.32

$$\frac{1}{12}\sqrt{3}\arctan\left(\frac{1}{2}\sqrt{3}\tan\left(2x+1\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(4*cos(1+2*x)^2+3*sin(1+2*x)^2),x, algorithm="maxima")
```

```
[Out] 1/12*sqrt(3)*arctan(1/2*sqrt(3)*tan(2*x + 1))
```

mupad [B] time = 2.76, size = 36, normalized size = 0.68

$$\frac{\sqrt{3}\ (2\,x-\mathrm{atan}\,(\tan\,(2\,x+1)))}{12}+\frac{\sqrt{3}\ \mathrm{atan}\left(\frac{\sqrt{3}\ \tan(2\,x+1)}{2}\right)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(3*sin(2*x + 1)^2 + 4*cos(2*x + 1)^2),x)
```

```
[Out] (3^(1/2)*(2*x - atan(tan(2*x + 1))))/12 + (3^(1/2)*atan((3^(1/2)*tan(2*x +
1))/2))/12
```

sympy [A] time = 0.81, size = 87, normalized size = 1.64

$$\frac{\sqrt{3}\left(\mathrm{atan}\left(\frac{2\sqrt{3}\ \tan\left(x+\frac{1}{2}\right)}{3}-\frac{\sqrt{3}}{3}\right)+\pi\left\lfloor\frac{x-\frac{\pi}{2}+\frac{1}{2}}{\pi}\right\rfloor\right)}{12}+\frac{\sqrt{3}\left(\mathrm{atan}\left(\frac{2\sqrt{3}\ \tan\left(x+\frac{1}{2}\right)}{3}+\frac{\sqrt{3}}{3}\right)+\pi\left\lfloor\frac{x-\frac{\pi}{2}+\frac{1}{2}}{\pi}\right\rfloor\right)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(4*cos(1+2*x)**2+3*sin(1+2*x)**2),x)
```

```
[Out] sqrt(3)*(atan(2*sqrt(3)*tan(x + 1/2)/3 - sqrt(3)/3) + pi*floor((x - pi/2 +
1/2)/pi))/12 + sqrt(3)*(atan(2*sqrt(3)*tan(x + 1/2)/3 + sqrt(3)/3) + pi*flo
or((x - pi/2 + 1/2)/pi))/12
```

3.482 $\int \frac{\sin^2(x)}{a\cos^2(x)+b\sin^2(x)}\,dx$

Optimal. Leaf size=43

$$\frac{\sqrt{a}\,\tan^{-1}\left(\frac{\sqrt{b}\,\tan(x)}{\sqrt{a}}\right)}{\sqrt{b}\,(a-b)}-\frac{x}{a-b}$$

```
[Out] -x/(a-b)+arctan(b^(1/2)*tan(x)/a^(1/2))*a^(1/2)/(a-b)/b^(1/2)
```

Rubi [A] time = 0.15, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {481, 203, 205}

$$\frac{\sqrt{a}\,\tan^{-1}\left(\frac{\sqrt{b}\,\tan(x)}{\sqrt{a}}\right)}{\sqrt{b}\,(a-b)}-\frac{x}{a-b}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[x]^2/(a*Cos[x]^2 + b*Sin[x]^2),x]
```

```
[Out] -(x/(a - b)) + (Sqrt[a]*ArcTan[(Sqrt[b]*Tan[x])/Sqrt[a]])/((a - b)*Sqrt[b])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 481

```
Int[((e_.)*(x_))^(m_.)/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(n_))),
 x_Symbol] :> -Dist[(a*e^n)/(b*c - a*d), Int[(e*x)^(m - n)/(a + b*x^n), x],
 x] + Dist[(c*e^n)/(b*c - a*d), Int[(e*x)^(m - n)/(c + d*x^n), x], x] /; Fr
eeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LeQ[n, m,
2*n - 1]
```

Rubi steps

$$\int \frac{\sin^2(x)}{a\cos^2(x)+b\sin^2(x)}\,dx = \text{Subst}\left(\int \frac{x^2}{(1+x^2)(a+bx^2)}\,dx, x, \tan(x)\right)$$
$$= -\frac{\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \tan(x)\right)}{a-b} + \frac{a\,\text{Subst}\left(\int \frac{1}{a+bx^2}\,dx, x, \tan(x)\right)}{a-b}$$
$$= -\frac{x}{a-b} + \frac{\sqrt{a}\,\tan^{-1}\left(\frac{\sqrt{b}\tan(x)}{\sqrt{a}}\right)}{(a-b)\sqrt{b}}$$

Mathematica [A] time = 0.09, size = 36, normalized size = 0.84

$$\frac{x-\frac{\sqrt{a}\,\tan^{-1}\left(\frac{\sqrt{b}\tan(x)}{\sqrt{a}}\right)}{\sqrt{b}}}{b-a}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[x]^2/(a*Cos[x]^2 + b*Sin[x]^2),x]
[Out] (x - (Sqrt[a]*ArcTan[(Sqrt[b]*Tan[x])/Sqrt[a]])/Sqrt[b])/(-a + b)
```

fricas [A] time = 0.70, size = 182, normalized size = 4.23

$$\left[-\frac{\sqrt{-\frac{a}{b}}\log\left(\frac{(a^2+6ab+b^2)\cos(x)^4-2(3ab+b^2)\cos(x)^2+4\left((ab+b^2)\cos(x)^3-b^2\cos(x)\right)\sqrt{-\frac{a}{b}}\sin(x)+b^2}{(a^2-2ab+b^2)\cos(x)^4+2(ab-b^2)\cos(x)^2+b^2}\right)+4x}{4(a-b)}, -\frac{\sqrt{\frac{a}{b}}\arctan\left(\frac{((a+b)\cos}{2a\cos(}\right.}{2(a-}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^2/(a*cos(x)^2+b*sin(x)^2),x, algorithm="fricas")
[Out] [-1/4*(sqrt(-a/b)*log(((a^2 + 6*a*b + b^2)*cos(x)^4 - 2*(3*a*b + b^2)*cos(x
)^2 + 4*((a*b + b^2)*cos(x)^3 - b^2*cos(x))*sqrt(-a/b)*sin(x) + b^2)/((a^2
- 2*a*b + b^2)*cos(x)^4 + 2*(a*b - b^2)*cos(x)^2 + b^2)) + 4*x)/(a - b), -1
/2*(sqrt(a/b)*arctan(1/2*((a + b)*cos(x)^2 - b)*sqrt(a/b)/(a*cos(x)*sin(x))
) + 2*x)/(a - b)]
```

giac [A] time = 0.13, size = 48, normalized size = 1.12

$$\frac{\left(\pi\left\lfloor\frac{x}{\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(b)+\arctan\left(\frac{b\tan(x)}{\sqrt{ab}}\right)\right)a}{\sqrt{ab}\,(a-b)}-\frac{x}{a-b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^2/(a*cos(x)^2+b*sin(x)^2),x, algorithm="giac")
```

```
[Out] (pi*floor(x/pi + 1/2)*sgn(b) + arctan(b*tan(x)/sqrt(a*b)))*a/(sqrt(a*b)*(a
- b)) - x/(a - b)
```

maple [A] time = 0.11, size = 38, normalized size = 0.88

$$\frac{a\arctan\left(\frac{\tan(x)b}{\sqrt{ab}}\right)}{(a-b)\sqrt{ab}}-\frac{\arctan\left(\tan(x)\right)}{a-b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)^2/(a*cos(x)^2+b*sin(x)^2),x)
```

```
[Out] a/(a-b)/(a*b)^(1/2)*arctan(tan(x)*b/(a*b)^(1/2))-1/(a-b)*arctan(tan(x))
```

maxima [A] time = 0.41, size = 35, normalized size = 0.81

$$\frac{a\arctan\left(\frac{b\tan(x)}{\sqrt{ab}}\right)}{\sqrt{ab}\,(a-b)}-\frac{x}{a-b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^2/(a*cos(x)^2+b*sin(x)^2),x, algorithm="maxima")
```

```
[Out] a*arctan(b*tan(x)/sqrt(a*b))/(sqrt(a*b)*(a - b)) - x/(a - b)
```

mupad [B] time = 2.73, size = 51, normalized size = 1.19

$$\begin{cases} \frac{2x-\sin(2x)}{4b} & \text{if } a=b \\ -\frac{x-\frac{\sqrt{a}\,\mathrm{atan}\left(\frac{\sqrt{b}\,\tan(x)}{\sqrt{a}}\right)}{\sqrt{b}}}{a-b} & \text{if } a\neq b \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)^2/(b*sin(x)^2 + a*cos(x)^2),x)
```

```
[Out] piecewise(a == b, (2*x - sin(2*x))/(4*b), a ~= b, -(x - (a^(1/2)*atan((b^(1
/2)*tan(x))/a^(1/2)))/b^(1/2))/(a - b))
```

sympy [A] time = 1.56, size = 241, normalized size = 5.60

$$\begin{cases} \tilde{\infty}x & \text{for } a = 0 \wedge b = 0 \\ \frac{x\sin^2(x)}{2b\sin^2(x)+2b\cos^2(x)} + \frac{x\cos^2(x)}{2b\sin^2(x)+2b\cos^2(x)} - \frac{\sin(x)\cos(x)}{2b\sin^2(x)+2b\cos^2(x)} & \text{for } a = b \\ \frac{-x+\frac{\sin(x)}{\cos(x)}}{a} & \text{for } b = 0 \\ \frac{x}{b} & \text{for } a = 0 \\ -\frac{2i\sqrt{b}\,x\sqrt{\frac{1}{a}}}{2ia\sqrt{b}\sqrt{\frac{1}{a}}-2ib^{\frac{3}{2}}\sqrt{\frac{1}{a}}} - \frac{\log\left(-i\sqrt{b}\sqrt{\frac{1}{a}}\sin(x)+\cos(x)\right)}{2ia\sqrt{b}\sqrt{\frac{1}{a}}-2ib^{\frac{3}{2}}\sqrt{\frac{1}{a}}} + \frac{\log\left(i\sqrt{b}\sqrt{\frac{1}{a}}\sin(x)+\cos(x)\right)}{2ia\sqrt{b}\sqrt{\frac{1}{a}}-2ib^{\frac{3}{2}}\sqrt{\frac{1}{a}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)**2/(a*cos(x)**2+b*sin(x)**2),x)
```

```
[Out] Piecewise((zoo*x, Eq(a, 0) & Eq(b, 0)), (x*sin(x)**2/(2*b*sin(x)**2 + 2*b*c
os(x)**2) + x*cos(x)**2/(2*b*sin(x)**2 + 2*b*cos(x)**2) - sin(x)*cos(x)/(2*
b*sin(x)**2 + 2*b*cos(x)**2), Eq(a, b)), ((-x + sin(x)/cos(x))/a, Eq(b, 0))
, (x/b, Eq(a, 0)), (-2*I*sqrt(b)*x*sqrt(1/a)/(2*I*a*sqrt(b)*sqrt(1/a) - 2*I
*b**(3/2)*sqrt(1/a)) - log(-I*sqrt(b)*sqrt(1/a)*sin(x) + cos(x))/(2*I*a*sqr
t(b)*sqrt(1/a) - 2*I*b**(3/2)*sqrt(1/a)) + log(I*sqrt(b)*sqrt(1/a)*sin(x) +
 cos(x))/(2*I*a*sqrt(b)*sqrt(1/a) - 2*I*b**(3/2)*sqrt(1/a)), True))
```

3.483 $\int \frac{\cos^2(x)}{a\cos^2(x)+b\sin^2(x)}\,dx$

Optimal. Leaf size=43

$$\frac{x}{a-b}-\frac{\sqrt{b}\tan^{-1}\left(\frac{\sqrt{b}\tan(x)}{\sqrt{a}}\right)}{\sqrt{a}\,(a-b)}$$

```
[Out] x/(a-b)-arctan(b^(1/2)*tan(x)/a^(1/2))*b^(1/2)/(a-b)/a^(1/2)
```

Rubi [A] time = 0.11, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {391, 203, 205}

$$\frac{x}{a-b}-\frac{\sqrt{b}\tan^{-1}\left(\frac{\sqrt{b}\tan(x)}{\sqrt{a}}\right)}{\sqrt{a}\,(a-b)}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[x]^2/(a*Cos[x]^2 + b*Sin[x]^2),x]
```

```
[Out] x/(a - b) - (Sqrt[b]*ArcTan[(Sqrt[b]*Tan[x])/Sqrt[a]])/(Sqrt[a]*(a - b))
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 391

```
Int[1/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(n_))), x_Symbol] :> Dis
t[b/(b*c - a*d), Int[1/(a + b*x^n), x], x] - Dist[d/(b*c - a*d), Int[1/(c +
 d*x^n), x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0]
```

Rubi steps

$$\int \frac{\cos^2(x)}{a\cos^2(x)+b\sin^2(x)}\,dx = \text{Subst}\left(\int \frac{1}{(1+x^2)(a+bx^2)}\,dx, x, \tan(x)\right)$$
$$= \frac{\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \tan(x)\right)}{a-b} - \frac{b\,\text{Subst}\left(\int \frac{1}{a+bx^2}\,dx, x, \tan(x)\right)}{a-b}$$
$$= \frac{x}{a-b} - \frac{\sqrt{b}\tan^{-1}\left(\frac{\sqrt{b}\tan(x)}{\sqrt{a}}\right)}{\sqrt{a}(a-b)}$$

Mathematica [A] time = 0.05, size = 36, normalized size = 0.84

$$\frac{x - \frac{\sqrt{b}\tan^{-1}\left(\frac{\sqrt{b}\tan(x)}{\sqrt{a}}\right)}{\sqrt{a}}}{a-b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[x]^2/(a*Cos[x]^2 + b*Sin[x]^2),x]
```

```
[Out] (x - (Sqrt[b]*ArcTan[(Sqrt[b]*Tan[x])/Sqrt[a]])/Sqrt[a])/(a - b)
```

fricas [A] time = 1.49, size = 181, normalized size = 4.21

$$\left[-\frac{\sqrt{-\frac{b}{a}}\log\left(\frac{(a^2+6ab+b^2)\cos(x)^4-2(3ab+b^2)\cos(x)^2-4\left((a^2+ab)\cos(x)^3-ab\cos(x)\right)\sqrt{-\frac{b}{a}}\sin(x)+b^2}{(a^2-2ab+b^2)\cos(x)^4+2(ab-b^2)\cos(x)^2+b^2}\right)-4x}{4(a-b)}, \frac{\sqrt{\frac{b}{a}}\arctan\left(\frac{((a+b)\cos(x}{2b\cos(x)}\right.}{2(a-b)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^2/(a*cos(x)^2+b*sin(x)^2),x, algorithm="fricas")
```

```
[Out] [-1/4*(sqrt(-b/a)*log(((a^2 + 6*a*b + b^2)*cos(x)^4 - 2*(3*a*b + b^2)*cos(x
)^2 - 4*((a^2 + a*b)*cos(x)^3 - a*b*cos(x))*sqrt(-b/a)*sin(x) + b^2)/((a^2
- 2*a*b + b^2)*cos(x)^4 + 2*(a*b - b^2)*cos(x)^2 + b^2)) - 4*x)/(a - b), 1/
2*(sqrt(b/a)*arctan(1/2*((a + b)*cos(x)^2 - b)*sqrt(b/a)/(b*cos(x)*sin(x)))
 + 2*x)/(a - b)]
```

giac [A] time = 0.14, size = 48, normalized size = 1.12

$$-\frac{\left(\pi\left\lfloor\frac{x}{\pi}+\frac{1}{2}\right\rfloor\text{sgn}(b)+\arctan\left(\frac{b\tan(x)}{\sqrt{ab}}\right)\right)b}{\sqrt{ab}(a-b)}+\frac{x}{a-b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^2/(a*cos(x)^2+b*sin(x)^2),x, algorithm="giac")
```

```
[Out] -(pi*floor(x/pi + 1/2)*sgn(b) + arctan(b*tan(x)/sqrt(a*b)))*b/(sqrt(a*b)*(a
 - b)) + x/(a - b)
```

maple [A] time = 0.10, size = 36, normalized size = 0.84

$$-\frac{b \arctan\left(\frac{\tan(x) b}{\sqrt{ab}}\right)}{(a-b)\sqrt{ab}}+\frac{x}{a-b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)^2/(a*cos(x)^2+b*sin(x)^2),x)
```

```
[Out] -b/(a-b)/(a*b)^(1/2)*arctan(tan(x)*b/(a*b)^(1/2))+x/(a-b)
```

maxima [A] time = 0.41, size = 35, normalized size = 0.81

$$-\frac{b \arctan\left(\frac{b \tan(x)}{\sqrt{ab}}\right)}{\sqrt{ab}\,(a-b)}+\frac{x}{a-b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^2/(a*cos(x)^2+b*sin(x)^2),x, algorithm="maxima")
```

```
[Out] -b*arctan(b*tan(x)/sqrt(a*b))/(sqrt(a*b)*(a - b)) + x/(a - b)
```

mupad [B] time = 2.67, size = 48, normalized size = 1.12

$$\begin{cases} \frac{2x+\sin(2x)}{4b} & \text{if } a=b \\ \frac{x-\frac{\sqrt{b}\,\mathrm{atan}\left(\frac{\sqrt{b}\,\tan(x)}{\sqrt{a}}\right)}{\sqrt{a}}}{a-b} & \text{if } a\neq b \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)^2/(b*sin(x)^2 + a*cos(x)^2),x)
```

```
[Out] piecewise(a == b, (2*x + sin(2*x))/(4*b), a ~= b, (x - (b^(1/2)*atan((b^(1/
2)*tan(x))/a^(1/2)))/a^(1/2))/(a - b))
```

sympy [A] time = 1.62, size = 267, normalized size = 6.21

$$\begin{cases} \tilde{\infty}\left(-x - \frac{\cos(x)}{\sin(x)}\right) & \text{for } a = 0 \wedge b = 0 \\ \frac{x\sin^2(x)}{2b\sin^2(x)+2b\cos^2(x)} + \frac{x\cos^2(x)}{2b\sin^2(x)+2b\cos^2(x)} + \frac{\sin(x)\cos(x)}{2b\sin^2(x)+2b\cos^2(x)} & \text{for } a = b \\ \frac{x}{a} & \text{for } b = 0 \\ \frac{-x-\frac{\cos(x)}{\sin(x)}}{b} & \text{for } a = 0 \\ \frac{2ia\sqrt{b}\,x\sqrt{\frac{1}{a}}}{2ia^2\sqrt{b}\sqrt{\frac{1}{a}}-2iab^{\frac{3}{2}}\sqrt{\frac{1}{a}}} + \frac{b\log\left(-i\sqrt{b}\sqrt{\frac{1}{a}}\sin(x)+\cos(x)\right)}{2ia^2\sqrt{b}\sqrt{\frac{1}{a}}-2iab^{\frac{3}{2}}\sqrt{\frac{1}{a}}} - \frac{b\log\left(i\sqrt{b}\sqrt{\frac{1}{a}}\sin(x)+\cos(x)\right)}{2ia^2\sqrt{b}\sqrt{\frac{1}{a}}-2iab^{\frac{3}{2}}\sqrt{\frac{1}{a}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)**2/(a*cos(x)**2+b*sin(x)**2),x)
```

```
[Out] Piecewise((zoo*(-x - cos(x)/sin(x)), Eq(a, 0) & Eq(b, 0)), (x*sin(x)**2/(2*
b*sin(x)**2 + 2*b*cos(x)**2) + x*cos(x)**2/(2*b*sin(x)**2 + 2*b*cos(x)**2)
+ sin(x)*cos(x)/(2*b*sin(x)**2 + 2*b*cos(x)**2), Eq(a, b)), (x/a, Eq(b, 0))
, ((-x - cos(x)/sin(x))/b, Eq(a, 0)), (2*I*a*sqrt(b)*x*sqrt(1/a)/(2*I*a**2*
sqrt(b)*sqrt(1/a) - 2*I*a*b**(3/2)*sqrt(1/a)) + b*log(-I*sqrt(b)*sqrt(1/a)*
sin(x) + cos(x))/(2*I*a**2*sqrt(b)*sqrt(1/a) - 2*I*a*b**(3/2)*sqrt(1/a)) -
b*log(I*sqrt(b)*sqrt(1/a)*sin(x) + cos(x))/(2*I*a**2*sqrt(b)*sqrt(1/a) - 2*
I*a*b**(3/2)*sqrt(1/a)), True))
```

3.484 $\int \frac{1}{\sec^2(x)+\tan^2(x)} \, dx$

Optimal. Leaf size=36

$$\sqrt{2}\, x - x + \sqrt{2} \tan^{-1}\left(\frac{\sin(x)\cos(x)}{\sin^2(x) + \sqrt{2} + 1}\right)$$

```
[Out] -x+x*2^(1/2)+arctan(cos(x)*sin(x)/(1+sin(x)^2+2^(1/2)))*2^(1/2)
```

Rubi [A] time = 0.03, antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {1093, 203}

$$\sqrt{2}\, x - x + \sqrt{2} \tan^{-1}\left(\frac{\sin(x)\cos(x)}{\sin^2(x) + \sqrt{2} + 1}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[x]^2 + Tan[x]^2)^(-1),x]
```

```
[Out] -x + Sqrt[2]*x + Sqrt[2]*ArcTan[(Cos[x]*Sin[x])/(1 + Sqrt[2] + Sin[x]^2)]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 1093

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(-1), x_Symbol] :> With[{q = Rt[b^
2 - 4*a*c, 2]}, Dist[c/q, Int[1/(b/2 - q/2 + c*x^2), x], x] - Dist[c/q, Int
[1/(b/2 + q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c,
0] && PosQ[b^2 - 4*a*c]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{\sec^2(x) + \tan^2(x)} \, dx &= \text{Subst}\left(\int \frac{1}{1 + 3x^2 + 2x^4} \, dx, x, \tan(x)\right)\\
&= 2\,\text{Subst}\left(\int \frac{1}{1 + 2x^2} \, dx, x, \tan(x)\right) - 2\,\text{Subst}\left(\int \frac{1}{2 + 2x^2} \, dx, x, \tan(x)\right)\\
&= -x + \sqrt{2}\, x + \sqrt{2} \tan^{-1}\left(\frac{\cos(x)\sin(x)}{1 + \sqrt{2} + \sin^2(x)}\right)
\end{aligned}$$

Mathematica [A] time = 0.05, size = 19, normalized size = 0.53

$$\sqrt{2}\tan^{-1}\left(\sqrt{2}\tan(x)\right)-x$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[x]^2 + Tan[x]^2)^(-1),x]
```

```
[Out] -x + Sqrt[2]*ArcTan[Sqrt[2]*Tan[x]]
```

fricas [A] time = 1.08, size = 35, normalized size = 0.97

$$-\frac{1}{2}\sqrt{2}\arctan\left(\frac{3\sqrt{2}\cos(x)^2-2\sqrt{2}}{4\cos(x)\sin(x)}\right)-x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sec(x)^2+tan(x)^2),x, algorithm="fricas")
```

```
[Out] -1/2*sqrt(2)*arctan(1/4*(3*sqrt(2)*cos(x)^2 - 2*sqrt(2))/(cos(x)*sin(x))) -
 x
```

giac [A] time = 0.12, size = 15, normalized size = 0.42

$$\sqrt{2}\arctan\left(\sqrt{2}\tan(x)\right)-x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sec(x)^2+tan(x)^2),x, algorithm="giac")
```

```
[Out] sqrt(2)*arctan(sqrt(2)*tan(x)) - x
```

maple [A] time = 0.12, size = 16, normalized size = 0.44

$$\sqrt{2}\,\arctan\left(\sqrt{2}\,\tan(x)\right)-x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sec(x)^2+tan(x)^2),x)
```

```
[Out] 2^(1/2)*arctan(2^(1/2)*tan(x))-x
```

maxima [A] time = 0.41, size = 15, normalized size = 0.42

$$\sqrt{2}\arctan\left(\sqrt{2}\tan(x)\right)-x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sec(x)^2+tan(x)^2),x, algorithm="maxima")
```

```
[Out] sqrt(2)*arctan(sqrt(2)*tan(x)) - x
```

mupad [B] time = 2.65, size = 15, normalized size = 0.42

$$\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\,\tan(x)\right) - x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1/cos(x)^2 + tan(x)^2),x)
```

```
[Out] 2^(1/2)*atan(2^(1/2)*tan(x)) - x
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\tan^2(x) + \sec^2(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sec(x)**2+tan(x)**2),x)
```

```
[Out] Integral(1/(tan(x)**2 + sec(x)**2), x)
```

3.485 $\int \frac{1}{(\sec^2(x)+\tan^2(x))^2}\,dx$

Optimal. Leaf size=49

$$-\frac{x}{\sqrt{2}}+x+\frac{\tan(x)}{2\tan^2(x)+1}-\frac{\tan^{-1}\left(\frac{\sin(x)\cos(x)}{\sin^2(x)+\sqrt{2}+1}\right)}{\sqrt{2}}$$

```
[Out] x-1/2*x*2^(1/2)-1/2*arctan(cos(x)*sin(x)/(1+sin(x)^2+2^(1/2)))*2^(1/2)+tan(
x)/(1+2*tan(x)^2)
```

Rubi [A] time = 0.05, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {414, 12, 481, 203}

$$-\frac{x}{\sqrt{2}}+x+\frac{\tan(x)}{2\tan^2(x)+1}-\frac{\tan^{-1}\left(\frac{\sin(x)\cos(x)}{\sin^2(x)+\sqrt{2}+1}\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[x]^2 + Tan[x]^2)^(-2),x]
```

```
[Out] x - x/Sqrt[2] - ArcTan[(Cos[x]*Sin[x])/(1 + Sqrt[2] + Sin[x]^2)]/Sqrt[2] +
Tan[x]/(1 + 2*Tan[x]^2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 414

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c +
d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n, x]
, x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1]
 &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c,
```

```
d, n, p, q, x]
```

Rule 481

```
Int[((e_.)*(x_))^(m_.)/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(n_))),
 x_Symbol] :> -Dist[(a*e^n)/(b*c - a*d), Int[(e*x)^(m - n)/(a + b*x^n), x],
 x] + Dist[(c*e^n)/(b*c - a*d), Int[(e*x)^(m - n)/(c + d*x^n), x], x] /; Fr
eeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LeQ[n, m,
2*n - 1]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{\left(\sec^2(x)+\tan^2(x)\right)^2}\,dx &= \text{Subst}\left(\int \frac{1}{\left(1+x^2\right)\left(1+2x^2\right)^2}\,dx, x, \tan(x)\right)\\
&= \frac{\tan(x)}{1+2\tan^2(x)} - \frac{1}{2}\,\text{Subst}\left(\int -\frac{2x^2}{\left(1+x^2\right)\left(1+2x^2\right)}\,dx, x, \tan(x)\right)\\
&= \frac{\tan(x)}{1+2\tan^2(x)} + \text{Subst}\left(\int \frac{x^2}{\left(1+x^2\right)\left(1+2x^2\right)}\,dx, x, \tan(x)\right)\\
&= \frac{\tan(x)}{1+2\tan^2(x)} + \text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \tan(x)\right) - \text{Subst}\left(\int \frac{1}{1+2x^2}\,dx, x, \tan(x)\right)\\
&= x - \frac{x}{\sqrt{2}} - \frac{\tan^{-1}\left(\frac{\cos(x)\sin(x)}{1+\sqrt{2}+\sin^2(x)}\right)}{\sqrt{2}} + \frac{\tan(x)}{1+2\tan^2(x)}
\end{aligned}$$

Mathematica [A] time = 0.14, size = 42, normalized size = 0.86

$$\frac{-3x-\sin(2x)+x\cos(2x)}{\cos(2x)-3} - \frac{\tan^{-1}\left(\sqrt{2}\,\tan(x)\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[x]^2 + Tan[x]^2)^(-2),x]
```

```
[Out] -(ArcTan[Sqrt[2]*Tan[x]]/Sqrt[2]) + (-3*x + x*Cos[2*x] - Sin[2*x])/(-3 + Co
s[2*x])
```

fricas [A] time = 1.02, size = 68, normalized size = 1.39

$$\frac{4\,x\cos(x)^2 + \left(\sqrt{2}\,\cos(x)^2 - 2\,\sqrt{2}\right)\arctan\left(\frac{3\,\sqrt{2}\,\cos(x)^2-2\,\sqrt{2}}{4\,\cos(x)\sin(x)}\right) - 4\,\cos(x)\sin(x) - 8\,x}{4\left(\cos(x)^2-2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sec(x)^2+tan(x)^2)^2,x, algorithm="fricas")
```

```
[Out] 1/4*(4*x*cos(x)^2 + (sqrt(2)*cos(x)^2 - 2*sqrt(2))*arctan(1/4*(3*sqrt(2)*co
s(x)^2 - 2*sqrt(2))/(cos(x)*sin(x))) - 4*cos(x)*sin(x) - 8*x)/(cos(x)^2 - 2
)
```

giac [A] time = 0.15, size = 27, normalized size = 0.55

$$-\frac{1}{2}\sqrt{2}\arctan\left(\sqrt{2}\tan(x)\right)+x+\frac{\tan(x)}{2\tan(x)^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sec(x)^2+tan(x)^2)^2,x, algorithm="giac")
```

```
[Out] -1/2*sqrt(2)*arctan(sqrt(2)*tan(x)) + x + tan(x)/(2*tan(x)^2 + 1)
```

maple [A] time = 0.12, size = 27, normalized size = 0.55

$$\frac{\tan(x)}{2\left(\tan^2(x)\right)+1}-\frac{\sqrt{2}\ \arctan\left(\sqrt{2}\ \tan(x)\right)}{2}+x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sec(x)^2+tan(x)^2)^2,x)
```

```
[Out] 1/2*tan(x)/(tan(x)^2+1/2)-1/2*2^(1/2)*arctan(2^(1/2)*tan(x))+x
```

maxima [A] time = 0.40, size = 27, normalized size = 0.55

$$-\frac{1}{2}\sqrt{2}\arctan\left(\sqrt{2}\tan(x)\right)+x+\frac{\tan(x)}{2\tan(x)^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sec(x)^2+tan(x)^2)^2,x, algorithm="maxima")
```

```
[Out] -1/2*sqrt(2)*arctan(sqrt(2)*tan(x)) + x + tan(x)/(2*tan(x)^2 + 1)
```

mupad [B] time = 2.66, size = 27, normalized size = 0.55

$$x+\frac{\tan(x)}{2\left(\tan(x)^2+\frac{1}{2}\right)}-\frac{\sqrt{2}\ \mathrm{atan}\left(\sqrt{2}\ \tan(x)\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1/cos(x)^2 + tan(x)^2)^2,x)
```

```
[Out] x + tan(x)/(2*(tan(x)^2 + 1/2)) - (2^(1/2)*atan(2^(1/2)*tan(x)))/2
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(\tan^2(x) + \sec^2(x)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sec(x)**2+tan(x)**2)**2,x)
```

```
[Out] Integral((tan(x)**2 + sec(x)**2)**(-2), x)
```

3.486 $\int \frac{1}{\left(\sec^2(x)+\tan^2(x)\right)^3} \, dx$

Optimal. Leaf size=74

$$\frac{7x}{4\sqrt{2}}-x-\frac{\tan(x)}{4\left(2\tan^2(x)+1\right)}+\frac{\tan(x)}{2\left(2\tan^2(x)+1\right)^2}+\frac{7\tan^{-1}\left(\frac{\sin(x)\cos(x)}{\sin^2(x)+\sqrt{2}+1}\right)}{4\sqrt{2}}$$

```
[Out] -x+7/8*x*2^(1/2)+7/8*arctan(cos(x)*sin(x)/(1+sin(x)^2+2^(1/2)))*2^(1/2)+1/2
*tan(x)/(1+2*tan(x)^2)^2-1/4*tan(x)/(1+2*tan(x)^2)
```

Rubi [A] time = 0.05, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {414, 527, 522, 203}

$$\frac{7x}{4\sqrt{2}}-x-\frac{\tan(x)}{4\left(2\tan^2(x)+1\right)}+\frac{\tan(x)}{2\left(2\tan^2(x)+1\right)^2}+\frac{7\tan^{-1}\left(\frac{\sin(x)\cos(x)}{\sin^2(x)+\sqrt{2}+1}\right)}{4\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[x]^2 + Tan[x]^2)^(-3),x]
```

```
[Out] -x + (7*x)/(4*Sqrt[2]) + (7*ArcTan[(Cos[x]*Sin[x])/(1 + Sqrt[2] + Sin[x]^2)
])/(4*Sqrt[2]) + Tan[x]/(2*(1 + 2*Tan[x]^2)^2) - Tan[x]/(4*(1 + 2*Tan[x]^2)
)
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 414

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c +
d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n, x]
, x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1]
 &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c,
 d, n, p, q, x]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\left(\sec^2(x)+\tan^2(x)\right)^3}\,dx &= \text{Subst}\left(\int \frac{1}{\left(1+x^2\right)\left(1+2x^2\right)^3}\,dx,x,\tan(x)\right)\\
&= \frac{\tan(x)}{2\left(1+2\tan^2(x)\right)^2}-\frac{1}{4}\text{Subst}\left(\int \frac{-2-6x^2}{\left(1+x^2\right)\left(1+2x^2\right)^2}\,dx,x,\tan(x)\right)\\
&= \frac{\tan(x)}{2\left(1+2\tan^2(x)\right)^2}-\frac{\tan(x)}{4\left(1+2\tan^2(x)\right)}+\frac{1}{8}\text{Subst}\left(\int \frac{6-2x^2}{\left(1+x^2\right)\left(1+2x^2\right)}\,dx,x,\tan(x\right.\\
&= \frac{\tan(x)}{2\left(1+2\tan^2(x)\right)^2}-\frac{\tan(x)}{4\left(1+2\tan^2(x)\right)}+\frac{7}{4}\text{Subst}\left(\int \frac{1}{1+2x^2}\,dx,x,\tan(x)\right)-\text{Subs}\\
&= -x+\frac{7x}{4\sqrt{2}}+\frac{7\tan^{-1}\left(\frac{\cos(x)\sin(x)}{1+\sqrt{2}+\sin^2(x)}\right)}{4\sqrt{2}}+\frac{\tan(x)}{2\left(1+2\tan^2(x)\right)^2}-\frac{\tan(x)}{4\left(1+2\tan^2(x)\right)}\end{aligned}$$

Mathematica [A] time = 0.18, size = 79, normalized size = 1.07

$$-\frac{(\cos(2x)-3)\sec^6(x)\left(-76x-2\sin(2x)+3\sin(4x)+48x\cos(2x)-4x\cos(4x)+7\sqrt{2}(\cos(2x)-3)^2\tan^{-1}\left(\sqrt{2}\right.\right.}{64\left(\tan^2(x)+\sec^2(x)\right)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[x]^2 + Tan[x]^2)^(-3),x]
```

```
[Out] -1/64*((-3 + Cos[2*x])*Sec[x]^6*(-76*x + 7*Sqrt[2]*ArcTan[Sqrt[2]*Tan[x]]*(
-3 + Cos[2*x])^2 + 48*x*Cos[2*x] - 4*x*Cos[4*x] - 2*Sin[2*x] + 3*Sin[4*x]))
/(Sec[x]^2 + Tan[x]^2)^3
```

fricas [A] time = 2.12, size = 100, normalized size = 1.35

$$-\frac{16\,x\cos(x)^4-64\,x\cos(x)^2+7\left(\sqrt{2}\,\cos(x)^4-4\,\sqrt{2}\,\cos(x)^2+4\,\sqrt{2}\right)\arctan\left(\frac{3\,\sqrt{2}\,\cos(x)^2-2\,\sqrt{2}}{4\,\cos(x)\sin(x)}\right)-4\left(3\,\cos(x)^3-\right.}{16\left(\cos(x)^4-4\,\cos(x)^2+4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sec(x)^2+tan(x)^2)^3,x, algorithm="fricas")
```

```
[Out] -1/16*(16*x*cos(x)^4 - 64*x*cos(x)^2 + 7*(sqrt(2)*cos(x)^4 - 4*sqrt(2)*cos(
x)^2 + 4*sqrt(2))*arctan(1/4*(3*sqrt(2)*cos(x)^2 - 2*sqrt(2))/(cos(x)*sin(x
))) - 4*(3*cos(x)^3 - 2*cos(x))*sin(x) + 64*x)/(cos(x)^4 - 4*cos(x)^2 + 4)
```

giac [A] time = 0.15, size = 39, normalized size = 0.53

$$\frac{7}{8}\,\sqrt{2}\arctan\left(\sqrt{2}\,\tan(x)\right)-x-\frac{2\,\tan(x)^3-\tan(x)}{4\left(2\,\tan(x)^2+1\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sec(x)^2+tan(x)^2)^3,x, algorithm="giac")
```

```
[Out] 7/8*sqrt(2)*arctan(sqrt(2)*tan(x)) - x - 1/4*(2*tan(x)^3 - tan(x))/(2*tan(x
)^2 + 1)^2
```

maple [A] time = 0.13, size = 40, normalized size = 0.54

$$\frac{-\frac{\left(\tan^3(x)\right)}{2}+\frac{\tan(x)}{4}}{\left(2\left(\tan^2(x)\right)+1\right)^2}+\frac{7\sqrt{2}\,\arctan\left(\sqrt{2}\,\tan(x)\right)}{8}-x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sec(x)^2+tan(x)^2)^3,x)
```

```
[Out] 8*(-1/16*tan(x)^3+1/32*tan(x))/(1+2*tan(x)^2)^2+7/8*2^(1/2)*arctan(2^(1/2)*
tan(x))-x
```

maxima [A] time = 0.42, size = 45, normalized size = 0.61

$$\frac{7}{8}\,\sqrt{2}\arctan\left(\sqrt{2}\,\tan(x)\right)-x-\frac{2\,\tan(x)^3-\tan(x)}{4\left(4\,\tan(x)^4+4\,\tan(x)^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sec(x)^2+tan(x)^2)^3,x, algorithm="maxima")
```

```
[Out] 7/8*sqrt(2)*arctan(sqrt(2)*tan(x)) - x - 1/4*(2*tan(x)^3 - tan(x))/(4*tan(x
)^4 + 4*tan(x)^2 + 1)
```

mupad [B] time = 2.70, size = 40, normalized size = 0.54

$$\frac{\frac{\tan(x)}{16}-\frac{\tan(x)^3}{8}}{\tan(x)^4+\tan(x)^2+\frac{1}{4}}-x+\frac{7\sqrt{2}\,\operatorname{atan}\left(\sqrt{2}\,\tan(x)\right)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1/cos(x)^2 + tan(x)^2)^3,x)
```

```
[Out] (tan(x)/16 - tan(x)^3/8)/(tan(x)^2 + tan(x)^4 + 1/4) - x + (7*2^(1/2)*atan(
2^(1/2)*tan(x)))/8
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(\tan^2(x)+\sec^2(x)\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sec(x)**2+tan(x)**2)**3,x)
```

```
[Out] Integral((tan(x)**2 + sec(x)**2)**(-3), x)
```

3.487 $\int \frac{1}{\sec^2(x)-\tan^2(x)}\,dx$

Optimal. Leaf size=1

$$x$$

```
[Out] x
```

Rubi [A] time = 0.01, antiderivative size = 1, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {4381, 8}

$$x$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[x]^2 - Tan[x]^2)^(-1),x]
[Out] x
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 4381

```
Int[(u_.)*((a_.) + (c_.)*sec[(d_.) + (e_.)*(x_)]^2 + (b_.)*tan[(d_.) + (e_.
)*(x_)]^2)^(p_.), x_Symbol] :> Dist[(a + c)^p, Int[ActivateTrig[u], x], x]
/; FreeQ[{a, b, c, d, e, p}, x] && EqQ[b + c, 0]
```

Rubi steps

$$\begin{aligned} \int \frac{1}{\sec^2(x)-\tan^2(x)}\,dx &= \int 1\,dx\\ &= x \end{aligned}$$

Mathematica [A] time = 0.00, size = 1, normalized size = 1.00

$$x$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[x]^2 - Tan[x]^2)^(-1),x]
[Out] x
```

fricas [A] time = 0.76, size = 1, normalized size = 1.00

$$x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sec(x)^2-tan(x)^2),x, algorithm="fricas")
[Out] x
```

giac [A] time = 0.14, size = 1, normalized size = 1.00

$$x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sec(x)^2-tan(x)^2),x, algorithm="giac")
[Out] x
```

maple [C] time = 0.06, size = 4, normalized size = 4.00

$$\arctan(\tan(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sec(x)^2-tan(x)^2),x)
[Out] arctan(tan(x))
```

maxima [A] time = 0.43, size = 1, normalized size = 1.00

$$x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sec(x)^2-tan(x)^2),x, algorithm="maxima")
[Out] x
```

mupad [B] time = 2.75, size = 1, normalized size = 1.00

$$x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1/cos(x)^2 - tan(x)^2),x)
[Out] x
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(-\tan (x) + \sec (x)) (\tan (x) + \sec (x))} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sec(x)**2-tan(x)**2),x)
```

```
[Out] Integral(1/((-tan(x) + sec(x))*(tan(x) + sec(x))), x)
```

3.488 $\int \frac{1}{\left(\sec^2(x)-\tan^2(x)\right)^2} dx$

Optimal. Leaf size=1

$$x$$

```
[Out] x
```

Rubi [A] time = 0.01, antiderivative size = 1, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {4381, 8}

$$x$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[x]^2 - Tan[x]^2)^(-2),x]
```

```
[Out] x
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 4381

```
Int[(u_.)*((a_.) + (c_.)*sec[(d_.) + (e_.)*(x_)]^2 + (b_.)*tan[(d_.) + (e_.
)*(x_)]^2)^(p_.), x_Symbol] :> Dist[(a + c)^p, Int[ActivateTrig[u], x], x]
/; FreeQ[{a, b, c, d, e, p}, x] && EqQ[b + c, 0]
```

Rubi steps

$$\begin{aligned} \int \frac{1}{\left(\sec^2(x)-\tan^2(x)\right)^2} \, dx &= \int 1 \, dx \\ &= x \end{aligned}$$

Mathematica [A] time = 0.00, size = 1, normalized size = 1.00

$$x$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[x]^2 - Tan[x]^2)^(-2),x]
```

[Out] x

fricas [A] time = 0.84, size = 1, normalized size = 1.00

$$x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sec(x)^2-tan(x)^2)^2,x, algorithm="fricas")
```

[Out] x

giac [A] time = 0.15, size = 1, normalized size = 1.00

$$x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sec(x)^2-tan(x)^2)^2,x, algorithm="giac")
```

[Out] x

maple [C] time = 0.06, size = 4, normalized size = 4.00

$$\arctan(\tan(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sec(x)^2-tan(x)^2)^2,x)
```

[Out] arctan(tan(x))

maxima [A] time = 0.41, size = 1, normalized size = 1.00

$$x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sec(x)^2-tan(x)^2)^2,x, algorithm="maxima")
```

[Out] x

mupad [B] time = 2.58, size = 1, normalized size = 1.00

$$x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1/cos(x)^2 - tan(x)^2)^2,x)
```

```
[Out] x
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(-\tan (x) + \sec (x))^2 (\tan (x) + \sec (x))^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sec(x)**2-tan(x)**2)**2,x)
```

```
[Out] Integral(1/((-tan(x) + sec(x))**2*(tan(x) + sec(x))**2), x)
```

3.489 $\int \frac{1}{\left(\sec^2(x)-\tan^2(x)\right)^3} \, dx$

Optimal. Leaf size=1

$$x$$

```
[Out] x
```

Rubi [A] time = 0.01, antiderivative size = 1, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {4381, 8}

$$x$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[x]^2 - Tan[x]^2)^(-3),x]
```

```
[Out] x
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 4381

```
Int[(u_.)*((a_.) + (c_.)*sec[(d_.) + (e_.)*(x_)]^2 + (b_.)*tan[(d_.) + (e_.
)*(x_)]^2)^(p_.), x_Symbol] :> Dist[(a + c)^p, Int[ActivateTrig[u], x], x]
/; FreeQ[{a, b, c, d, e, p}, x] && EqQ[b + c, 0]
```

Rubi steps

$$\begin{aligned} \int \frac{1}{\left(\sec^2(x)-\tan^2(x)\right)^3} \, dx &= \int 1 \, dx\\ &= x \end{aligned}$$

Mathematica [A] time = 0.00, size = 1, normalized size = 1.00

$$x$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[x]^2 - Tan[x]^2)^(-3),x]
```

[Out] x

fricas [A] time = 1.53, size = 1, normalized size = 1.00

$$x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sec(x)^2-tan(x)^2)^3,x, algorithm="fricas")
[Out] x
```

giac [A] time = 0.14, size = 1, normalized size = 1.00

$$x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sec(x)^2-tan(x)^2)^3,x, algorithm="giac")
[Out] x
```

maple [C] time = 0.07, size = 4, normalized size = 4.00

$$\arctan(\tan(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sec(x)^2-tan(x)^2)^3,x)
[Out] arctan(tan(x))
```

maxima [A] time = 0.41, size = 1, normalized size = 1.00

$$x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sec(x)^2-tan(x)^2)^3,x, algorithm="maxima")
[Out] x
```

mupad [B] time = 2.57, size = 1, normalized size = 1.00

$$x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1/cos(x)^2 - tan(x)^2)^3,x)
```

```
[Out] x
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-\tan\left(x\right)+\sec\left(x\right)\right)^{3}\left(\tan\left(x\right)+\sec\left(x\right)\right)^{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sec(x)**2-tan(x)**2)**3,x)
```

```
[Out] Integral(1/((-tan(x) + sec(x))**3*(tan(x) + sec(x))**3), x)
```

3.490 $\int \frac{1}{\cot^2(x)+\csc^2(x)}\,dx$

Optimal. Leaf size=37

$$\sqrt{2}\,x - x - \sqrt{2}\,\tan^{-1}\left(\frac{\sin(x)\cos(x)}{\cos^2(x)+\sqrt{2}+1}\right)$$

```
[Out] -x+x*2^(1/2)-arctan(cos(x)*sin(x)/(1+cos(x)^2+2^(1/2)))*2^(1/2)
```

Rubi [A] time = 0.03, antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {1130, 203}

$$\sqrt{2}\,x - x - \sqrt{2}\,\tan^{-1}\left(\frac{\sin(x)\cos(x)}{\cos^2(x)+\sqrt{2}+1}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(Cot[x]^2 + Csc[x]^2)^(-1),x]
```

```
[Out] -x + Sqrt[2]*x - Sqrt[2]*ArcTan[(Cos[x]*Sin[x])/(1 + Sqrt[2] + Cos[x]^2)]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 1130

```
Int[((d_.)*(x_))^(m_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> Wi
th[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(d^2*(b/q + 1))/2, Int[(d*x)^(m - 2)/(b/2
 + q/2 + c*x^2), x], x] - Dist[(d^2*(b/q - 1))/2, Int[(d*x)^(m - 2)/(b/2 -
q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - 4*a*c, 0] && G
eQ[m, 2]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\cot^2(x)+\csc^2(x)}\,dx &= \text{Subst}\left(\int \frac{x^2}{2+3x^2+x^4}\,dx, x, \tan(x)\right)\\ &= 2\,\text{Subst}\left(\int \frac{1}{2+x^2}\,dx, x, \tan(x)\right) - \text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \tan(x)\right)\\ &= -x+\sqrt{2}\,x-\sqrt{2}\,\tan^{-1}\left(\frac{\cos(x)\sin(x)}{1+\sqrt{2}+\cos^2(x)}\right)\end{aligned}$$

Mathematica [A] time = 0.04, size = 19, normalized size = 0.51

$$\sqrt{2}\,\tan^{-1}\left(\frac{\tan(x)}{\sqrt{2}}\right)-x$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cot[x]^2 + Csc[x]^2)^(-1),x]
[Out] -x + Sqrt[2]*ArcTan[Tan[x]/Sqrt[2]]
```

fricas [A] time = 1.32, size = 35, normalized size = 0.95

$$-\frac{1}{2}\,\sqrt{2}\,\arctan\left(\frac{3\,\sqrt{2}\,\cos(x)^2-\sqrt{2}}{4\,\cos(x)\sin(x)}\right)-x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(cot(x)^2+csc(x)^2),x, algorithm="fricas")
[Out] -1/2*sqrt(2)*arctan(1/4*(3*sqrt(2)*cos(x)^2 - sqrt(2))/(cos(x)*sin(x))) - x
```

giac [A] time = 0.14, size = 49, normalized size = 1.32

$$\sqrt{2}\left(x+\arctan\left(-\frac{\sqrt{2}\,\sin(2\,x)-\sin(2\,x)}{\sqrt{2}\,\cos(2\,x)+\sqrt{2}-\cos(2\,x)+1}\right)\right)-x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(cot(x)^2+csc(x)^2),x, algorithm="giac")
[Out] sqrt(2)*(x + arctan(-(sqrt(2)*sin(2*x) - sin(2*x))/(sqrt(2)*cos(2*x) + sqrt
(2) - cos(2*x) + 1))) - x
```

maple [A] time = 0.16, size = 17, normalized size = 0.46

$$\sqrt{2}\, \arctan\left(\frac{\sqrt{2}\, \tan(x)}{2}\right) - x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cot(x)^2+csc(x)^2),x)
[Out] 2^(1/2)*arctan(1/2*2^(1/2)*tan(x))-x
```

maxima [A] time = 0.41, size = 16, normalized size = 0.43

$$\sqrt{2} \arctan\left(\frac{1}{2} \sqrt{2} \tan(x)\right) - x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(cot(x)^2+csc(x)^2),x, algorithm="maxima")
[Out] sqrt(2)*arctan(1/2*sqrt(2)*tan(x)) - x
```

mupad [B] time = 2.71, size = 16, normalized size = 0.43

$$\sqrt{2}\, \mathrm{atan}\left(\frac{\sqrt{2}\, \tan(x)}{2}\right) - x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cot(x)^2 + 1/sin(x)^2),x)
[Out] 2^(1/2)*atan((2^(1/2)*tan(x))/2) - x
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\cot^2(x) + \csc^2(x)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(cot(x)**2+csc(x)**2),x)
[Out] Integral(1/(cot(x)**2 + csc(x)**2), x)
```

3.491 $\int \frac{1}{\left(\cot^2(x)+\csc^2(x)\right)^2} dx$

Optimal. Leaf size=47

$$-\frac{x}{\sqrt{2}}+x-\frac{\tan(x)}{\tan^2(x)+2}+\frac{\tan^{-1}\left(\frac{\sin(x)\cos(x)}{\cos^2(x)+\sqrt{2}+1}\right)}{\sqrt{2}}$$

```
[Out] x-1/2*x*2^(1/2)+1/2*arctan(cos(x)*sin(x)/(1+cos(x)^2+2^(1/2)))*2^(1/2)-tan(
x)/(2+tan(x)^2)
```

Rubi [A] time = 0.04, antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {470, 12, 391, 203}

$$-\frac{x}{\sqrt{2}}+x-\frac{\tan(x)}{\tan^2(x)+2}+\frac{\tan^{-1}\left(\frac{\sin(x)\cos(x)}{\cos^2(x)+\sqrt{2}+1}\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cot[x]^2 + Csc[x]^2)^(-2),x]
```

```
[Out] x - x/Sqrt[2] + ArcTan[(Cos[x]*Sin[x])/(1 + Sqrt[2] + Cos[x]^2)]/Sqrt[2] -
Tan[x]/(2 + Tan[x]^2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 391

```
Int[1/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(n_))), x_Symbol] :> Dis
t[b/(b*c - a*d), Int[1/(a + b*x^n), x], x] - Dist[d/(b*c - a*d), Int[1/(c +
 d*x^n), x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[(a*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)^
(p + 1)*(c + d*x^n)^(q + 1))/(b*n*(b*c - a*d)*(p + 1)), x] + Dist[e^(2*n)/(
b*n*(b*c - a*d)*(p + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d*x^
n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^n,
x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n,
0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n,
p, q, x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{\left(\cot^2(x)+\csc^2(x)\right)^2}\,dx &= \text{Subst}\left(\int \frac{x^4}{\left(1+x^2\right)\left(2+x^2\right)^2}\,dx, x, \tan(x)\right)\\
&= -\frac{\tan(x)}{2+\tan^2(x)}+\frac{1}{2}\,\text{Subst}\left(\int \frac{2}{\left(1+x^2\right)\left(2+x^2\right)}\,dx, x, \tan(x)\right)\\
&= -\frac{\tan(x)}{2+\tan^2(x)}+\text{Subst}\left(\int \frac{1}{\left(1+x^2\right)\left(2+x^2\right)}\,dx, x, \tan(x)\right)\\
&= -\frac{\tan(x)}{2+\tan^2(x)}+\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \tan(x)\right)-\text{Subst}\left(\int \frac{1}{2+x^2}\,dx, x, \tan(x)\right)\\
&= x-\frac{x}{\sqrt{2}}+\frac{\tan^{-1}\left(\frac{\cos(x)\sin(x)}{1+\sqrt{2}+\cos^2(x)}\right)}{\sqrt{2}}-\frac{\tan(x)}{2+\tan^2(x)}
\end{aligned}
$$

Mathematica [A] time = 0.11, size = 64, normalized size = 1.36

$$
\frac{(\cos(2x)+3)\csc^4(x)\left(6x-2\sin(2x)+2x\cos(2x)-\sqrt{2}\,(\cos(2x)+3)\tan^{-1}\left(\frac{\tan(x)}{\sqrt{2}}\right)\right)}{8\left(\cot^2(x)+\csc^2(x)\right)^2}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cot[x]^2 + Csc[x]^2)^(-2),x]
```

```
[Out] ((3 + Cos[2*x])*Csc[x]^4*(6*x + 2*x*Cos[2*x] - Sqrt[2]*ArcTan[Tan[x]/Sqrt[2
]]*(3 + Cos[2*x]) - 2*Sin[2*x]))/(8*(Cot[x]^2 + Csc[x]^2)^2)
```

fricas [A] time = 1.07, size = 66, normalized size = 1.40

$$
\frac{4\,x\cos(x)^2+\left(\sqrt{2}\,\cos(x)^2+\sqrt{2}\right)\arctan\left(\frac{3\,\sqrt{2}\,\cos(x)^2-\sqrt{2}}{4\,\cos(x)\sin(x)}\right)-4\,\cos(x)\sin(x)+4\,x}{4\left(\cos(x)^2+1\right)}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(cot(x)^2+csc(x)^2)^2,x, algorithm="fricas")
```

```
[Out] 1/4*(4*x*cos(x)^2 + (sqrt(2)*cos(x)^2 + sqrt(2))*arctan(1/4*(3*sqrt(2)*cos(
x)^2 - sqrt(2))/(cos(x)*sin(x))) - 4*cos(x)*sin(x) + 4*x)/(cos(x)^2 + 1)
```

giac [A] time = 0.14, size = 60, normalized size = 1.28

$$-\frac{1}{2}\sqrt{2}\left(x+\arctan\left(-\frac{\sqrt{2}\sin\left(2x\right)-\sin\left(2x\right)}{\sqrt{2}\cos\left(2x\right)+\sqrt{2}-\cos\left(2x\right)+1}\right)\right)+x-\frac{\tan(x)}{\tan(x)^{2}+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(cot(x)^2+csc(x)^2)^2,x, algorithm="giac")
```

```
[Out] -1/2*sqrt(2)*(x + arctan(-(sqrt(2)*sin(2*x) - sin(2*x))/(sqrt(2)*cos(2*x) +
 sqrt(2) - cos(2*x) + 1))) + x - tan(x)/(tan(x)^2 + 2)
```

maple [A] time = 0.18, size = 28, normalized size = 0.60

$$-\frac{\tan(x)}{2+\tan^{2}(x)}-\frac{\sqrt{2}\ \arctan\left(\frac{\sqrt{2}\ \tan(x)}{2}\right)}{2}+x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cot(x)^2+csc(x)^2)^2,x)
```

```
[Out] -tan(x)/(2+tan(x)^2)-1/2*2^(1/2)*arctan(1/2*2^(1/2)*tan(x))+x
```

maxima [A] time = 0.40, size = 27, normalized size = 0.57

$$-\frac{1}{2}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\tan(x)\right)+x-\frac{\tan(x)}{\tan(x)^{2}+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(cot(x)^2+csc(x)^2)^2,x, algorithm="maxima")
```

```
[Out] -1/2*sqrt(2)*arctan(1/2*sqrt(2)*tan(x)) + x - tan(x)/(tan(x)^2 + 2)
```

mupad [B] time = 2.69, size = 27, normalized size = 0.57

$$x-\frac{\tan(x)}{{\tan(x)}^2+2}-\frac{\sqrt{2}\ \mathrm{atan}\left(\frac{\sqrt{2}\ \tan(x)}{2}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cot(x)^2 + 1/sin(x)^2)^2,x)
```

```
[Out] x - tan(x)/(tan(x)^2 + 2) - (2^(1/2)*atan((2^(1/2)*tan(x))/2))/2
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(\cot^2(x) + \csc^2(x)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(cot(x)**2+csc(x)**2)**2,x)
```

```
[Out] Integral((cot(x)**2 + csc(x)**2)**(-2), x)
```

3.492 $\int \frac{1}{\left(\cot^2(x)+\csc^2(x)\right)^3} dx$

Optimal. Leaf size=72

$$\frac{7x}{4\sqrt{2}} - x + \frac{\tan(x)}{4\left(\tan^2(x)+2\right)} - \frac{\tan^3(x)}{2\left(\tan^2(x)+2\right)^2} - \frac{7\tan^{-1}\left(\frac{\sin(x)\cos(x)}{\cos^2(x)+\sqrt{2}+1}\right)}{4\sqrt{2}}$$

```
[Out] -x+7/8*x*2^(1/2)-7/8*arctan(cos(x)*sin(x)/(1+cos(x)^2+2^(1/2)))*2^(1/2)-1/2
*tan(x)^3/(2+tan(x)^2)^2+1/4*tan(x)/(2+tan(x)^2)
```

Rubi [A] time = 0.08, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {470, 578, 522, 203}

$$\frac{7x}{4\sqrt{2}} - x - \frac{\tan^3(x)}{2\left(\tan^2(x)+2\right)^2} + \frac{\tan(x)}{4\left(\tan^2(x)+2\right)} - \frac{7\tan^{-1}\left(\frac{\sin(x)\cos(x)}{\cos^2(x)+\sqrt{2}+1}\right)}{4\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cot[x]^2 + Csc[x]^2)^(-3),x]
```

```
[Out] -x + (7*x)/(4*Sqrt[2]) - (7*ArcTan[(Cos[x]*Sin[x])/(1 + Sqrt[2] + Cos[x]^2)
])/(4*Sqrt[2]) - Tan[x]^3/(2*(2 + Tan[x]^2)^2) + Tan[x]/(4*(2 + Tan[x]^2))
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[(a*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)^
(p + 1)*(c + d*x^n)^(q + 1))/(b*n*(b*c - a*d)*(p + 1)), x] + Dist[e^(2*n)/(
b*n*(b*c - a*d)*(p + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d*x^
n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^n,
x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n,
0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n,
p, q, x]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 578

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(g^(n - 1)*(b*e - a*f)*
(g*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(b*n*(b*c - a*d)
*(p + 1)), x] - Dist[g^n/(b*n*(b*c - a*d)*(p + 1)), Int[(g*x)^(m - n)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f)*(m - n + 1) + (d*(b*e - a*f
)*(m + n*q + 1) - b*n*(c*f - d*e)*(p + 1))*x^n, x], x], x] /; FreeQ[{a, b,
c, d, e, f, g, q}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m - n + 1, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{\left(\cot^2(x)+\csc^2(x)\right)^3}\,dx &= \text{Subst}\left(\int \frac{x^6}{\left(1+x^2\right)\left(2+x^2\right)^3}\,dx,x,\tan(x)\right)\\
&= -\frac{\tan^3(x)}{2\left(2+\tan^2(x)\right)^2}+\frac{1}{4}\text{Subst}\left(\int \frac{x^2\left(6+2x^2\right)}{\left(1+x^2\right)\left(2+x^2\right)^2}\,dx,x,\tan(x)\right)\\
&= -\frac{\tan^3(x)}{2\left(2+\tan^2(x)\right)^2}+\frac{\tan(x)}{4\left(2+\tan^2(x)\right)}-\frac{1}{8}\text{Subst}\left(\int \frac{2-6x^2}{\left(1+x^2\right)\left(2+x^2\right)}\,dx,x,\tan(x)\right)\\
&= -\frac{\tan^3(x)}{2\left(2+\tan^2(x)\right)^2}+\frac{\tan(x)}{4\left(2+\tan^2(x)\right)}+\frac{7}{4}\text{Subst}\left(\int \frac{1}{2+x^2}\,dx,x,\tan(x)\right)-\text{Subst}\Bigg(\\
&= -x+\frac{7x}{4\sqrt{2}}-\frac{7\tan^{-1}\left(\frac{\cos(x)\sin(x)}{1+\sqrt{2}+\cos^2(x)}\right)}{4\sqrt{2}}-\frac{\tan^3(x)}{2\left(2+\tan^2(x)\right)^2}+\frac{\tan(x)}{4\left(2+\tan^2(x)\right)}
\end{aligned}
$$

Mathematica [A] time = 0.16, size = 66, normalized size = 0.92

$$
\frac{-76x+2\sin(2x)+3\sin(4x)-48x\cos(2x)-4x\cos(4x)+7\sqrt{2}\,(\cos(2x)+3)^2\tan^{-1}\left(\frac{\tan(x)}{\sqrt{2}}\right)}{8(\cos(2x)+3)^2}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cot[x]^2 + Csc[x]^2)^(-3),x]
```

```
[Out] (-76*x - 48*x*Cos[2*x] + 7*Sqrt[2]*ArcTan[Tan[x]/Sqrt[2]]*(3 + Cos[2*x])^2
- 4*x*Cos[4*x] + 2*Sin[2*x] + 3*Sin[4*x])/(8*(3 + Cos[2*x])^2)
```

fricas [A] time = 1.62, size = 98, normalized size = 1.36

$$-\frac{16\,x\cos(x)^4+32\,x\cos(x)^2+7\left(\sqrt{2}\,\cos(x)^4+2\,\sqrt{2}\,\cos(x)^2+\sqrt{2}\right)\arctan\left(\frac{3\,\sqrt{2}\,\cos(x)^2-\sqrt{2}}{4\,\cos(x)\sin(x)}\right)-4\left(3\,\cos(x)^3-c\right.}{16\left(\cos(x)^4+2\,\cos(x)^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(cot(x)^2+csc(x)^2)^3,x, algorithm="fricas")
```

```
[Out] -1/16*(16*x*cos(x)^4 + 32*x*cos(x)^2 + 7*(sqrt(2)*cos(x)^4 + 2*sqrt(2)*cos(
x)^2 + sqrt(2))*arctan(1/4*(3*sqrt(2)*cos(x)^2 - sqrt(2))/(cos(x)*sin(x)))
- 4*(3*cos(x)^3 - cos(x))*sin(x) + 16*x)/(cos(x)^4 + 2*cos(x)^2 + 1)
```

giac [A] time = 0.13, size = 69, normalized size = 0.96

$$\frac{7}{8}\,\sqrt{2}\left(x+\arctan\left(-\frac{\sqrt{2}\,\sin\left(2\,x\right)-\sin\left(2\,x\right)}{\sqrt{2}\,\cos\left(2\,x\right)+\sqrt{2}\,-\cos\left(2\,x\right)+1}\right)\right)-x-\frac{\tan(x)^3-2\,\tan(x)}{4\left(\tan(x)^2+2\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(cot(x)^2+csc(x)^2)^3,x, algorithm="giac")
```

```
[Out] 7/8*sqrt(2)*(x + arctan(-(sqrt(2)*sin(2*x) - sin(2*x))/(sqrt(2)*cos(2*x) +
sqrt(2) - cos(2*x) + 1))) - x - 1/4*(tan(x)^3 - 2*tan(x))/(tan(x)^2 + 2)^2
```

maple [A] time = 0.22, size = 39, normalized size = 0.54

$$\frac{-\frac{\left(\tan^3(x)\right)}{4}+\frac{\tan(x)}{2}}{\left(2+\tan^2(x)\right)^2}+\frac{7\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\,\tan(x)}{2}\right)}{8}-x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cot(x)^2+csc(x)^2)^3,x)
```

```
[Out] 2*(-1/8*tan(x)^3+1/4*tan(x))/(2+tan(x)^2)^2+7/8*2^(1/2)*arctan(1/2*2^(1/2)*
tan(x))-x
```

maxima [A] time = 0.42, size = 42, normalized size = 0.58

$$\frac{7}{8}\,\sqrt{2}\,\arctan\left(\frac{1}{2}\,\sqrt{2}\,\tan(x)\right)-x-\frac{\tan(x)^3-2\,\tan(x)}{4\left(\tan(x)^4+4\,\tan(x)^2+4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(cot(x)^2+csc(x)^2)^3,x, algorithm="maxima")
```

```
[Out] 7/8*sqrt(2)*arctan(1/2*sqrt(2)*tan(x)) - x - 1/4*(tan(x)^3 - 2*tan(x))/(tan
(x)^4 + 4*tan(x)^2 + 4)
```

mupad [B] time = 2.68, size = 43, normalized size = 0.60

$$\frac{\frac{\tan(x)}{2}-\frac{\tan(x)^3}{4}}{\tan(x)^4+4\tan(x)^2+4}-x+\frac{7\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}\,\tan(x)}{2}\right)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cot(x)^2 + 1/sin(x)^2)^3,x)
```

```
[Out] (tan(x)/2 - tan(x)^3/4)/(4*tan(x)^2 + tan(x)^4 + 4) - x + (7*2^(1/2)*atan((
2^(1/2)*tan(x))/2))/8
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(cot(x)**2+csc(x)**2)**3,x)
```

```
[Out] Timed out
```

3.493 $\int \frac{1}{\cot^2(x)-\csc^2(x)} \, dx$

Optimal. Leaf size=3

$$-x$$

```
[Out] -x
```

Rubi [A] time = 0.01, antiderivative size = 3, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {4382, 8}

$$-x$$

Antiderivative was successfully verified.

```
[In] Int[(Cot[x]^2 - Csc[x]^2)^(-1),x]
```

```
[Out] -x
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 4382

```
Int[((a_.) + cot[(d_.) + (e_.)*(x_)]^2*(b_.) + csc[(d_.) + (e_.)*(x_)]^2*(c
_.))^(p_.)*(u_.), x_Symbol] :> Dist[(a + c)^p, Int[ActivateTrig[u], x], x]
/; FreeQ[{a, b, c, d, e, p}, x] && EqQ[b + c, 0]
```

Rubi steps

$$\begin{aligned} \int \frac{1}{\cot^2(x) - \csc^2(x)} \, dx &= -\int 1 \, dx \\ &= -x \end{aligned}$$

Mathematica [A] time = 0.00, size = 3, normalized size = 1.00

$$-x$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cot[x]^2 - Csc[x]^2)^(-1),x]
```

```
[Out] -x
```

fricas [A] time = 1.12, size = 3, normalized size = 1.00

$$-x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(cot(x)^2-csc(x)^2),x, algorithm="fricas")
[Out] -x
```

giac [A] time = 0.15, size = 3, normalized size = 1.00

$$-x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(cot(x)^2-csc(x)^2),x, algorithm="giac")
[Out] -x
```

maple [C] time = 0.06, size = 6, normalized size = 2.00

$$-\arctan\left(\tan(x)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cot(x)^2-csc(x)^2),x)
[Out] -arctan(tan(x))
```

maxima [A] time = 0.42, size = 3, normalized size = 1.00

$$-x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(cot(x)^2-csc(x)^2),x, algorithm="maxima")
[Out] -x
```

mupad [B] time = 2.73, size = 3, normalized size = 1.00

$$-x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cot(x)^2 - 1/sin(x)^2),x)
[Out] -x
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(\cot(x) - \csc(x))(\cot(x) + \csc(x))}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(cot(x)**2-csc(x)**2),x)
```

```
[Out] Integral(1/((cot(x) - csc(x))*(cot(x) + csc(x))), x)
```

3.494 $\int \frac{1}{\left(\cot^2(x)-\csc^2(x)\right)^2} dx$

Optimal. Leaf size=1

$$x$$

[Out] x

Rubi [A] time = 0.01, antiderivative size = 1, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {4382, 8}

$$x$$

Antiderivative was successfully verified.

```
[In] Int[(Cot[x]^2 - Csc[x]^2)^(-2),x]
```

```
[Out] x
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 4382

```
Int[((a_.) + cot[(d_.) + (e_.)*(x_)]^2*(b_.) + csc[(d_.) + (e_.)*(x_)]^2*(c
_.))^(p_.)*(u_.), x_Symbol] :> Dist[(a + c)^p, Int[ActivateTrig[u], x], x]
/; FreeQ[{a, b, c, d, e, p}, x] && EqQ[b + c, 0]
```

Rubi steps

$$\begin{aligned} \int \frac{1}{\left(\cot^2(x)-\csc^2(x)\right)^2} dx &= \int 1 \, dx \\ &= x \end{aligned}$$

Mathematica [A] time = 0.00, size = 1, normalized size = 1.00

$$x$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cot[x]^2 - Csc[x]^2)^(-2),x]
```

```
[Out] x
```

fricas [A] time = 0.80, size = 1, normalized size = 1.00

$$x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(cot(x)^2-csc(x)^2)^2,x, algorithm="fricas")
[Out] x
```

giac [A] time = 0.15, size = 1, normalized size = 1.00

$$x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(cot(x)^2-csc(x)^2)^2,x, algorithm="giac")
[Out] x
```

maple [C] time = 0.06, size = 4, normalized size = 4.00

$$\arctan(\tan(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cot(x)^2-csc(x)^2)^2,x)
[Out] arctan(tan(x))
```

maxima [A] time = 0.42, size = 1, normalized size = 1.00

$$x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(cot(x)^2-csc(x)^2)^2,x, algorithm="maxima")
[Out] x
```

mupad [B] time = 2.65, size = 1, normalized size = 1.00

$$x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cot(x)^2 - 1/sin(x)^2)^2,x)
```

```
[Out] x
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(cot(x)**2-csc(x)**2)**2,x)
```

```
[Out] Timed out
```

3.495 $\int \frac{1}{\left(\cot^2(x)-\csc^2(x)\right)^3}\,dx$

Optimal. Leaf size=3

$$-x$$

```
[Out] -x
```

Rubi [A] time = 0.01, antiderivative size = 3, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {4382, 8}

$$-x$$

Antiderivative was successfully verified.

```
[In] Int[(Cot[x]^2 - Csc[x]^2)^(-3),x]
```

```
[Out] -x
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 4382

```
Int[((a_.) + cot[(d_.) + (e_.)*(x_)]^2*(b_.) + csc[(d_.) + (e_.)*(x_)]^2*(c
_.))^(p_.)*(u_.), x_Symbol] :> Dist[(a + c)^p, Int[ActivateTrig[u], x], x]
/; FreeQ[{a, b, c, d, e, p}, x] && EqQ[b + c, 0]
```

Rubi steps

$$\begin{aligned} \int \frac{1}{\left(\cot^2(x)-\csc^2(x)\right)^3}\,dx &= -\int 1\,dx\\ &= -x \end{aligned}$$

Mathematica [A] time = 0.00, size = 3, normalized size = 1.00

$$-x$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cot[x]^2 - Csc[x]^2)^(-3),x]
```

```
[Out] -x
```

fricas [A] time = 0.50, size = 3, normalized size = 1.00

$$-x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(cot(x)^2-csc(x)^2)^3,x, algorithm="fricas")
[Out] -x
```

giac [A] time = 0.14, size = 3, normalized size = 1.00

$$-x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(cot(x)^2-csc(x)^2)^3,x, algorithm="giac")
[Out] -x
```

maple [C] time = 0.07, size = 6, normalized size = 2.00

$$-\arctan\left(\tan(x)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cot(x)^2-csc(x)^2)^3,x)
[Out] -arctan(tan(x))
```

maxima [A] time = 0.41, size = 3, normalized size = 1.00

$$-x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(cot(x)^2-csc(x)^2)^3,x, algorithm="maxima")
[Out] -x
```

mupad [B] time = 2.62, size = 3, normalized size = 1.00

$$-x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cot(x)^2 - 1/sin(x)^2)^3,x)
```

```
[Out] -x
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(cot(x)**2-csc(x)**2)**3,x)
```

```
[Out] Timed out
```

3.496 $\int \frac{1}{a+b\cos^2(x)+c\sin^2(x)}\,dx$

Optimal. Leaf size=33

$$\frac{\tan^{-1}\left(\frac{\sqrt{a+c}\,\tan(x)}{\sqrt{a+b}}\right)}{\sqrt{a+b}\,\sqrt{a+c}}$$

```
[Out] arctan((a+c)^(1/2)*tan(x)/(a+b)^(1/2))/(a+b)^(1/2)/(a+c)^(1/2)
```

Rubi [A] time = 0.05, antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {205}

$$\frac{\tan^{-1}\left(\frac{\sqrt{a+c}\,\tan(x)}{\sqrt{a+b}}\right)}{\sqrt{a+b}\,\sqrt{a+c}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[x]^2 + c*Sin[x]^2)^(-1),x]
```

```
[Out] ArcTan[(Sqrt[a + c]*Tan[x])/Sqrt[a + b]]/(Sqrt[a + b]*Sqrt[a + c])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{a+b\cos^2(x)+c\sin^2(x)}\,dx &= \text{Subst}\left(\int \frac{1}{a+b+(a+c)x^2}\,dx, x, \tan(x)\right)\\ &= \frac{\tan^{-1}\left(\frac{\sqrt{a+c}\,\tan(x)}{\sqrt{a+b}}\right)}{\sqrt{a+b}\,\sqrt{a+c}}\end{aligned}$$

Mathematica [A] time = 0.06, size = 33, normalized size = 1.00

$$\frac{\tan^{-1}\left(\frac{\sqrt{a+c}\,\tan(x)}{\sqrt{a+b}}\right)}{\sqrt{a+b}\,\sqrt{a+c}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[x]^2 + c*Sin[x]^2)^(-1),x]
```

```
[Out] ArcTan[(Sqrt[a + c]*Tan[x])/Sqrt[a + b]]/(Sqrt[a + b]*Sqrt[a + c])
```

fricas [B] time = 0.96, size = 259, normalized size = 7.85

$$\left[-\frac{\sqrt{-a^2-ab-(a+b)c}\,\log\left(\frac{\left(8a^2+8ab+b^2+2(4a+3b)c+c^2\right)\cos(x)^4-2\left(4a^2+3ab+(5a+3b)c+c^2\right)\cos(x)^2+4\left((2a+b+c)\cos(x)^3-(a+c)\cos\right.}{\left(b^2-2bc+c^2\right)\cos(x)^4+2\left(ab-(a-b)c-c^2\right)\cos(x)^2+a^2+2ac+c^2}\right.}{4\left(a^2+ab+(a+b)c\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cos(x)^2+c*sin(x)^2),x, algorithm="fricas")
```

```
[Out] [-1/4*sqrt(-a^2 - a*b - (a + b)*c)*log(((8*a^2 + 8*a*b + b^2 + 2*(4*a + 3*b
)*c + c^2)*cos(x)^4 - 2*(4*a^2 + 3*a*b + (5*a + 3*b)*c + c^2)*cos(x)^2 + 4*
((2*a + b + c)*cos(x)^3 - (a + c)*cos(x))*sqrt(-a^2 - a*b - (a + b)*c)*sin(
x) + a^2 + 2*a*c + c^2)/((b^2 - 2*b*c + c^2)*cos(x)^4 + 2*(a*b - (a - b)*c
- c^2)*cos(x)^2 + a^2 + 2*a*c + c^2))/(a^2 + a*b + (a + b)*c), -1/2*arctan(
1/2*((2*a + b + c)*cos(x)^2 - a - c)/(sqrt(a^2 + a*b + (a + b)*c)*cos(x)*si
n(x)))/sqrt(a^2 + a*b + (a + b)*c)]
```

giac [B] time = 0.14, size = 61, normalized size = 1.85

$$\frac{\pi\left\lfloor\frac{x}{\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(2a+2c)+\arctan\left(\frac{a\tan(x)+c\tan(x)}{\sqrt{a^2+ab+ac+bc}}\right)}{\sqrt{a^2+ab+ac+bc}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cos(x)^2+c*sin(x)^2),x, algorithm="giac")
```

```
[Out] (pi*floor(x/pi + 1/2)*sgn(2*a + 2*c) + arctan((a*tan(x) + c*tan(x))/sqrt(a^
2 + a*b + a*c + b*c)))/sqrt(a^2 + a*b + a*c + b*c)
```

maple [A] time = 0.12, size = 27, normalized size = 0.82

$$\frac{\arctan\left(\frac{(a+c)\tan(x)}{\sqrt{(a+b)(a+c)}}\right)}{\sqrt{(a+b)(a+c)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*cos(x)^2+c*sin(x)^2),x)
```

```
[Out] 1/((a+b)*(a+c))^(1/2)*arctan((a+c)*tan(x)/((a+b)*(a+c))^(1/2))
```

maxima [A] time = 0.43, size = 26, normalized size = 0.79

$$\frac{\arctan\left(\frac{(a+c)\tan(x)}{\sqrt{(a+b)(a+c)}}\right)}{\sqrt{(a+b)(a+c)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cos(x)^2+c*sin(x)^2),x, algorithm="maxima")
```

```
[Out] arctan((a + c)*tan(x)/sqrt((a + b)*(a + c)))/sqrt((a + b)*(a + c))
```

mupad [B] time = 2.85, size = 43, normalized size = 1.30

$$\frac{\operatorname{atan}\left(\frac{\tan(x)\,(2\,a+2\,c)}{2\,\sqrt{a\,b+a\,c+b\,c+a^2}}\right)}{\sqrt{a\,b+a\,c+b\,c+a^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + c*sin(x)^2 + b*cos(x)^2),x)
```

```
[Out] atan((tan(x)*(2*a + 2*c))/(2*(a*b + a*c + b*c + a^2)^(1/2)))/(a*b + a*c + b
*c + a^2)^(1/2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{a+b\cos^2(x)+c\sin^2(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cos(x)**2+c*sin(x)**2),x)
```

```
[Out] Integral(1/(a + b*cos(x)**2 + c*sin(x)**2), x)
```

3.497 $\int \frac{x}{a+b\cos^2(x)+c\sin^2(x)}\,dx$

Optimal. Leaf size=239

$$-\frac{\text{Li}_2\left(-\frac{(b-c)e^{2ix}}{2a+b+c-2\sqrt{a+b}\sqrt{a+c}}\right)}{4\sqrt{a+b}\sqrt{a+c}}+\frac{\text{Li}_2\left(-\frac{(b-c)e^{2ix}}{2a+b+c+2\sqrt{a+b}\sqrt{a+c}}\right)}{4\sqrt{a+b}\sqrt{a+c}}-\frac{ix\log\left(1+\frac{e^{2ix}(b-c)}{-2\sqrt{a+b}\sqrt{a+c}+2a+b+c}\right)}{2\sqrt{a+b}\sqrt{a+c}}+\frac{ix\log\left(1+\frac{e^{2ix}(b-}{2\sqrt{a+b}\sqrt{a+c}}\right.}{2\sqrt{a+b}\sqrt{a+}}$$

```
[Out] -1/2*I*x*ln(1+(b-c)*exp(2*I*x)/(2*a+b+c-2*(a+b)^(1/2)*(a+c)^(1/2)))/(a+b)^(
1/2)/(a+c)^(1/2)+1/2*I*x*ln(1+(b-c)*exp(2*I*x)/(2*a+b+c+2*(a+b)^(1/2)*(a+c)
^(1/2)))/(a+b)^(1/2)/(a+c)^(1/2)-1/4*polylog(2,-(b-c)*exp(2*I*x)/(2*a+b+c-2
*(a+b)^(1/2)*(a+c)^(1/2)))/(a+b)^(1/2)/(a+c)^(1/2)+1/4*polylog(2,-(b-c)*exp
(2*I*x)/(2*a+b+c+2*(a+b)^(1/2)*(a+c)^(1/2)))/(a+b)^(1/2)/(a+c)^(1/2)
```

Rubi [A] time = 0.49, antiderivative size = 239, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {4587, 3321, 2264, 2190, 2279, 2391}

$$-\frac{\text{PolyLog}\left(2,-\frac{e^{2ix}(b-c)}{-2\sqrt{a+b}\sqrt{a+c}+2a+b+c}\right)}{4\sqrt{a+b}\sqrt{a+c}}+\frac{\text{PolyLog}\left(2,-\frac{e^{2ix}(b-c)}{2\sqrt{a+b}\sqrt{a+c}+2a+b+c}\right)}{4\sqrt{a+b}\sqrt{a+c}}-\frac{ix\log\left(1+\frac{e^{2ix}(b-c)}{-2\sqrt{a+b}\sqrt{a+c}+2a+b+c}\right)}{2\sqrt{a+b}\sqrt{a+c}}+ix\,\text{lo}$$

Antiderivative was successfully verified.

```
[In] Int[x/(a + b*Cos[x]^2 + c*Sin[x]^2),x]
```

```
[Out] ((-I/2)*x*Log[1 + ((b - c)*E^((2*I)*x))/(2*a + b + c - 2*Sqrt[a + b]*Sqrt[a
 + c])])/(Sqrt[a + b]*Sqrt[a + c]) + ((I/2)*x*Log[1 + ((b - c)*E^((2*I)*x))
/(2*a + b + c + 2*Sqrt[a + b]*Sqrt[a + c])])/(Sqrt[a + b]*Sqrt[a + c]) - Po
lyLog[2, -(((b - c)*E^((2*I)*x))/(2*a + b + c - 2*Sqrt[a + b]*Sqrt[a + c]))
]/(4*Sqrt[a + b]*Sqrt[a + c]) + PolyLog[2, -(((b - c)*E^((2*I)*x))/(2*a + b
 + c + 2*Sqrt[a + b]*Sqrt[a + c]))]/(4*Sqrt[a + b]*Sqrt[a + c])
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2264

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c)/q, Int[
```

```
((f + g*x)^m*F^u)/(b - q + 2*c*F^u), x], x] - Dist[(2*c)/q, Int[((f + g*x)^
m*F^u)/(b + q + 2*c*F^u), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3321

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(
x_)]), x_Symbol] :> Dist[2, Int[((c + d*x)^m*E^(I*Pi*(k - 1/2))*E^(I*(e + f
*x)))/(b + 2*a*E^(I*Pi*(k - 1/2))*E^(I*(e + f*x)) - b*E^(2*I*k*Pi)*E^(2*I*(
e + f*x))), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[2*k] && NeQ[
a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 4587

```
Int[((f_.) + (g_.)*(x_))^(m_.)/((a_.) + Cos[(d_.) + (e_.)*(x_)]^2*(b_.) + (
c_.)*Sin[(d_.) + (e_.)*(x_)]^2), x_Symbol] :> Dist[2, Int[(f + g*x)^m/(2*a
+ b + c + (b - c)*Cos[2*d + 2*e*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g},
 x] && IGtQ[m, 0] && NeQ[a + b, 0] && NeQ[a + c, 0]
```

Rubi steps

$$\int \frac{x}{a+b\cos^2(x)+c\sin^2(x)}\,dx = 2\int \frac{x}{2a+b+c+(b-c)\cos(2x)}\,dx$$
$$= 4\int \frac{e^{2ix}x}{b-c+2(2a+b+c)e^{2ix}+(b-c)e^{4ix}}\,dx$$
$$= \frac{(2(b-c))\int \frac{e^{2ix}x}{-4\sqrt{a+b}\sqrt{a+c}+2(2a+b+c)+2(b-c)e^{2ix}}\,dx}{\sqrt{a+b}\sqrt{a+c}} - \frac{(2(b-c))\int \frac{e^{2ix}x}{4\sqrt{a+b}\sqrt{a+c}+2(2a+b+}}{\sqrt{a+b}\sqrt{a+c}}$$
$$= -\frac{ix\log\left(1+\frac{(b-c)e^{2ix}}{2a+b+c-2\sqrt{a+b}\sqrt{a+c}}\right)}{2\sqrt{a+b}\sqrt{a+c}} + \frac{ix\log\left(1+\frac{(b-c)e^{2ix}}{2a+b+c+2\sqrt{a+b}\sqrt{a+c}}\right)}{2\sqrt{a+b}\sqrt{a+c}} + \frac{i\int\log(1}{}$$
$$= -\frac{ix\log\left(1+\frac{(b-c)e^{2ix}}{2a+b+c-2\sqrt{a+b}\sqrt{a+c}}\right)}{2\sqrt{a+b}\sqrt{a+c}} + \frac{ix\log\left(1+\frac{(b-c)e^{2ix}}{2a+b+c+2\sqrt{a+b}\sqrt{a+c}}\right)}{2\sqrt{a+b}\sqrt{a+c}} + \frac{\text{Subst}\left(\int\right.}{}$$
$$= -\frac{ix\log\left(1+\frac{(b-c)e^{2ix}}{2a+b+c-2\sqrt{a+b}\sqrt{a+c}}\right)}{2\sqrt{a+b}\sqrt{a+c}} + \frac{ix\log\left(1+\frac{(b-c)e^{2ix}}{2a+b+c+2\sqrt{a+b}\sqrt{a+c}}\right)}{2\sqrt{a+b}\sqrt{a+c}} - \frac{\text{Li}_2\left(-\frac{}{2a+}\right.}{4\sqrt{a}}$$

Mathematica [B] time = 3.02, size = 507, normalized size = 2.12

$$\frac{\tan^{-1}\left(\frac{\sqrt{a+c}\tan(x)}{\sqrt{a+b}}\right)\left(2x+\frac{i\left(-\text{Li}_2\left(\frac{\sqrt{a+b}-i\sqrt{a+c}\tan(x)}{\sqrt{a+b}-\sqrt{a+c}}\right)+\text{Li}_2\left(\frac{\sqrt{a+b}-i\sqrt{a+c}\tan(x)}{\sqrt{a+b}+\sqrt{a+c}}\right)-\text{Li}_2\left(\frac{i\sqrt{a+c}\tan(x)+\sqrt{a+b}}{\sqrt{a+b}-\sqrt{a+c}}\right)+\text{Li}_2\left(\frac{i\sqrt{a+c}\tan(x)+\sqrt{a+b}}{\sqrt{a+b}+\sqrt{a+c}}\right)+\log\left(\frac{\sqrt{a+}}{\sqrt{}}\right.\right.}{}\right.}{2}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x/(a + b*Cos[x]^2 + c*Sin[x]^2),x]
```

```
[Out] (ArcTan[(Sqrt[a + c]*Tan[x])/Sqrt[a + b]]*(2*x + (I*(Log[(Sqrt[a + c]*(1 +
I*Tan[x]))/(Sqrt[a + b] + Sqrt[a + c])]*Log[1 - (I*Sqrt[a + c]*Tan[x])/Sqrt
[a + b]] - Log[(I*Sqrt[a + c]*(I + Tan[x]))/(Sqrt[a + b] - Sqrt[a + c])]*Lo
g[1 - (I*Sqrt[a + c]*Tan[x])/Sqrt[a + b]] + Log[(Sqrt[a + c]*(1 - I*Tan[x])
)/(Sqrt[a + b] + Sqrt[a + c])]*Log[1 + (I*Sqrt[a + c]*Tan[x])/Sqrt[a + b]]
- Log[(Sqrt[a + c]*(1 + I*Tan[x]))/(-Sqrt[a + b] + Sqrt[a + c])]*Log[1 + (I
*Sqrt[a + c]*Tan[x])/Sqrt[a + b]] - PolyLog[2, (Sqrt[a + b] - I*Sqrt[a + c]
*Tan[x])/(Sqrt[a + b] - Sqrt[a + c])] + PolyLog[2, (Sqrt[a + b] - I*Sqrt[a
+ c]*Tan[x])/(Sqrt[a + b] + Sqrt[a + c])] - PolyLog[2, (Sqrt[a + b] + I*Sqr
t[a + c]*Tan[x])/(Sqrt[a + b] - Sqrt[a + c])] + PolyLog[2, (Sqrt[a + b] + I
```

```
*Sqrt[a + c]*Tan[x])/(Sqrt[a + b] + Sqrt[a + c])]))/(Log[1 - (I*Sqrt[a + c]
*Tan[x])/Sqrt[a + b]] - Log[1 + (I*Sqrt[a + c]*Tan[x])/Sqrt[a + b]])))/(2*S
qrt[a + b]*Sqrt[a + c])
```

fricas [B] time = 1.35, size = 2929, normalized size = 12.26

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*cos(x)^2+c*sin(x)^2),x, algorithm="fricas")
```

```
[Out] 1/16*(4*I*(b - c)*x*sqrt((a^2 + a*b + (a + b)*c)/(b^2 - 2*b*c + c^2))*log(-
1/2*((2*(2*a + b + c)*cos(x) + (4*I*a + 2*I*b + 2*I*c)*sin(x) - 4*((b - c)*
cos(x) - (-I*b + I*c)*sin(x))*sqrt((a^2 + a*b + (a + b)*c)/(b^2 - 2*b*c + c
^2)))*sqrt(-(2*(b - c)*sqrt((a^2 + a*b + (a + b)*c)/(b^2 - 2*b*c + c^2)) +
2*a + b + c)/(b - c)) - 2*b + 2*c)/(b - c)) - 4*I*(b - c)*x*sqrt((a^2 + a*b
 + (a + b)*c)/(b^2 - 2*b*c + c^2))*log(1/2*((2*(2*a + b + c)*cos(x) - (4*I*
a + 2*I*b + 2*I*c)*sin(x) - 4*((b - c)*cos(x) + (-I*b + I*c)*sin(x))*sqrt((
a^2 + a*b + (a + b)*c)/(b^2 - 2*b*c + c^2)))*sqrt(-(2*(b - c)*sqrt((a^2 + a
*b + (a + b)*c)/(b^2 - 2*b*c + c^2)) + 2*a + b + c)/(b - c)) + 2*b - 2*c)/(
b - c)) - 4*I*(b - c)*x*sqrt((a^2 + a*b + (a + b)*c)/(b^2 - 2*b*c + c^2))*l
og(-1/2*((2*(2*a + b + c)*cos(x) + (-4*I*a - 2*I*b - 2*I*c)*sin(x) - 4*((b
- c)*cos(x) - (I*b - I*c)*sin(x))*sqrt((a^2 + a*b + (a + b)*c)/(b^2 - 2*b*c
 + c^2)))*sqrt(-(2*(b - c)*sqrt((a^2 + a*b + (a + b)*c)/(b^2 - 2*b*c + c^2)
) + 2*a + b + c)/(b - c)) - 2*b + 2*c)/(b - c)) + 4*I*(b - c)*x*sqrt((a^2 +
 a*b + (a + b)*c)/(b^2 - 2*b*c + c^2))*log(1/2*((2*(2*a + b + c)*cos(x) - (
-4*I*a - 2*I*b - 2*I*c)*sin(x) - 4*((b - c)*cos(x) + (I*b - I*c)*sin(x))*sq
rt((a^2 + a*b + (a + b)*c)/(b^2 - 2*b*c + c^2)))*sqrt(-(2*(b - c)*sqrt((a^2
 + a*b + (a + b)*c)/(b^2 - 2*b*c + c^2)) + 2*a + b + c)/(b - c)) + 2*b - 2*
c)/(b - c)) - 4*I*(b - c)*x*sqrt((a^2 + a*b + (a + b)*c)/(b^2 - 2*b*c + c^2
))*log(-1/2*((2*(2*a + b + c)*cos(x) + (4*I*a + 2*I*b + 2*I*c)*sin(x) + 4*(
(b - c)*cos(x) + (I*b - I*c)*sin(x))*sqrt((a^2 + a*b + (a + b)*c)/(b^2 - 2*
b*c + c^2)))*sqrt((2*(b - c)*sqrt((a^2 + a*b + (a + b)*c)/(b^2 - 2*b*c + c^
2)) - 2*a - b - c)/(b - c)) - 2*b + 2*c)/(b - c)) + 4*I*(b - c)*x*sqrt((a^2
 + a*b + (a + b)*c)/(b^2 - 2*b*c + c^2))*log(1/2*((2*(2*a + b + c)*cos(x) -
 (4*I*a + 2*I*b + 2*I*c)*sin(x) + 4*((b - c)*cos(x) - (I*b - I*c)*sin(x))*s
qrt((a^2 + a*b + (a + b)*c)/(b^2 - 2*b*c + c^2)))*sqrt((2*(b - c)*sqrt((a^2
 + a*b + (a + b)*c)/(b^2 - 2*b*c + c^2)) - 2*a - b - c)/(b - c)) + 2*b - 2*
c)/(b - c)) + 4*I*(b - c)*x*sqrt((a^2 + a*b + (a + b)*c)/(b^2 - 2*b*c + c^2
))*log(-1/2*((2*(2*a + b + c)*cos(x) + (-4*I*a - 2*I*b - 2*I*c)*sin(x) + 4*
((b - c)*cos(x) + (-I*b + I*c)*sin(x))*sqrt((a^2 + a*b + (a + b)*c)/(b^2 -
2*b*c + c^2)))*sqrt((2*(b - c)*sqrt((a^2 + a*b + (a + b)*c)/(b^2 - 2*b*c +
c^2)) - 2*a - b - c)/(b - c)) - 2*b + 2*c)/(b - c)) - 4*I*(b - c)*x*sqrt((a
^2 + a*b + (a + b)*c)/(b^2 - 2*b*c + c^2))*log(1/2*((2*(2*a + b + c)*cos(x)
 - (-4*I*a - 2*I*b - 2*I*c)*sin(x) + 4*((b - c)*cos(x) - (-I*b + I*c)*sin(x
```

```
))*sqrt((a^2 + a*b + (a + b)*c)/(b^2 - 2*b*c + c^2)))*sqrt((2*(b - c)*sqrt(
(a^2 + a*b + (a + b)*c)/(b^2 - 2*b*c + c^2)) - 2*a - b - c)/(b - c)) + 2*b
- 2*c)/(b - c)) + 4*(b - c)*sqrt((a^2 + a*b + (a + b)*c)/(b^2 - 2*b*c + c^2
))*dilog(1/2*((2*(2*a + b + c)*cos(x) + (4*I*a + 2*I*b + 2*I*c)*sin(x) - 4*
((b - c)*cos(x) - (-I*b + I*c)*sin(x))*sqrt((a^2 + a*b + (a + b)*c)/(b^2 -
2*b*c + c^2)))*sqrt(-(2*(b - c)*sqrt((a^2 + a*b + (a + b)*c)/(b^2 - 2*b*c +
 c^2)) + 2*a + b + c)/(b - c)) - 2*b + 2*c)/(b - c) + 1) + 4*(b - c)*sqrt((
a^2 + a*b + (a + b)*c)/(b^2 - 2*b*c + c^2))*dilog(-1/2*((2*(2*a + b + c)*co
s(x) - (4*I*a + 2*I*b + 2*I*c)*sin(x) - 4*((b - c)*cos(x) + (-I*b + I*c)*si
n(x))*sqrt((a^2 + a*b + (a + b)*c)/(b^2 - 2*b*c + c^2)))*sqrt(-(2*(b - c)*s
qrt((a^2 + a*b + (a + b)*c)/(b^2 - 2*b*c + c^2)) + 2*a + b + c)/(b - c)) +
2*b - 2*c)/(b - c) + 1) + 4*(b - c)*sqrt((a^2 + a*b + (a + b)*c)/(b^2 - 2*b
*c + c^2))*dilog(1/2*((2*(2*a + b + c)*cos(x) + (-4*I*a - 2*I*b - 2*I*c)*si
n(x) - 4*((b - c)*cos(x) - (I*b - I*c)*sin(x))*sqrt((a^2 + a*b + (a + b)*c)
/(b^2 - 2*b*c + c^2)))*sqrt(-(2*(b - c)*sqrt((a^2 + a*b + (a + b)*c)/(b^2 -
 2*b*c + c^2)) + 2*a + b + c)/(b - c)) - 2*b + 2*c)/(b - c) + 1) + 4*(b - c
)*sqrt((a^2 + a*b + (a + b)*c)/(b^2 - 2*b*c + c^2))*dilog(-1/2*((2*(2*a + b
 + c)*cos(x) - (-4*I*a - 2*I*b - 2*I*c)*sin(x) - 4*((b - c)*cos(x) + (I*b -
 I*c)*sin(x))*sqrt((a^2 + a*b + (a + b)*c)/(b^2 - 2*b*c + c^2)))*sqrt(-(2*(
b - c)*sqrt((a^2 + a*b + (a + b)*c)/(b^2 - 2*b*c + c^2)) + 2*a + b + c)/(b
- c)) + 2*b - 2*c)/(b - c) + 1) - 4*(b - c)*sqrt((a^2 + a*b + (a + b)*c)/(b
^2 - 2*b*c + c^2))*dilog(1/2*((2*(2*a + b + c)*cos(x) + (4*I*a + 2*I*b + 2*
I*c)*sin(x) + 4*((b - c)*cos(x) + (I*b - I*c)*sin(x))*sqrt((a^2 + a*b + (a
+ b)*c)/(b^2 - 2*b*c + c^2)))*sqrt((2*(b - c)*sqrt((a^2 + a*b + (a + b)*c)/
(b^2 - 2*b*c + c^2)) - 2*a - b - c)/(b - c)) - 2*b + 2*c)/(b - c) + 1) - 4*
(b - c)*sqrt((a^2 + a*b + (a + b)*c)/(b^2 - 2*b*c + c^2))*dilog(-1/2*((2*(2
*a + b + c)*cos(x) - (4*I*a + 2*I*b + 2*I*c)*sin(x) + 4*((b - c)*cos(x) - (
I*b - I*c)*sin(x))*sqrt((a^2 + a*b + (a + b)*c)/(b^2 - 2*b*c + c^2)))*sqrt(
(2*(b - c)*sqrt((a^2 + a*b + (a + b)*c)/(b^2 - 2*b*c + c^2)) - 2*a - b - c)
/(b - c)) + 2*b - 2*c)/(b - c) + 1) - 4*(b - c)*sqrt((a^2 + a*b + (a + b)*c
)/(b^2 - 2*b*c + c^2))*dilog(1/2*((2*(2*a + b + c)*cos(x) + (-4*I*a - 2*I*b
 - 2*I*c)*sin(x) + 4*((b - c)*cos(x) + (-I*b + I*c)*sin(x))*sqrt((a^2 + a*b
 + (a + b)*c)/(b^2 - 2*b*c + c^2)))*sqrt((2*(b - c)*sqrt((a^2 + a*b + (a +
b)*c)/(b^2 - 2*b*c + c^2)) - 2*a - b - c)/(b - c)) - 2*b + 2*c)/(b - c) + 1
) - 4*(b - c)*sqrt((a^2 + a*b + (a + b)*c)/(b^2 - 2*b*c + c^2))*dilog(-1/2*
((2*(2*a + b + c)*cos(x) - (-4*I*a - 2*I*b - 2*I*c)*sin(x) + 4*((b - c)*cos
(x) - (-I*b + I*c)*sin(x))*sqrt((a^2 + a*b + (a + b)*c)/(b^2 - 2*b*c + c^2)
))*sqrt((2*(b - c)*sqrt((a^2 + a*b + (a + b)*c)/(b^2 - 2*b*c + c^2)) - 2*a
- b - c)/(b - c)) + 2*b - 2*c)/(b - c) + 1))/(a^2 + a*b + (a + b)*c)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{b\cos(x)^2 + c\sin(x)^2 + a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*cos(x)^2+c*sin(x)^2),x, algorithm="giac")
```

```
[Out] integrate(x/(b*cos(x)^2 + c*sin(x)^2 + a), x)
```

maple [B] time = 0.30, size = 820, normalized size = 3.43

$$-\frac{ix\ln\left(1-\frac{(b-c)e^{2ix}}{2\sqrt{(a+b)(a+c)}-2a-b-c}\right)}{2\sqrt{(a+b)(a+c)}}-\frac{x^2}{2\sqrt{(a+b)(a+c)}}-\frac{\operatorname{polylog}\left(2,\frac{(b-c)e^{2ix}}{2\sqrt{(a+b)(a+c)}-2a-b-c}\right)}{4\sqrt{(a+b)(a+c)}}-\frac{i\ln\left(1-\frac{(b-c)e^{2ix}}{-2\sqrt{(a+b)(a+c)}-2a-b-c}\right)}{-2\sqrt{(a+b)(a+c)}-2a-b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a+b*cos(x)^2+c*sin(x)^2),x)
```

```
[Out] -1/2*I/((a+b)*(a+c))^(1/2)*x*ln(1-(b-c)*exp(2*I*x)/(2*((a+b)*(a+c))^(1/2)-2
*a-b-c))-1/2/((a+b)*(a+c))^(1/2)*x^2-1/4/((a+b)*(a+c))^(1/2)*polylog(2,(b-c
)*exp(2*I*x)/(2*((a+b)*(a+c))^(1/2)-2*a-b-c))-I/(-2*((a+b)*(a+c))^(1/2)-2*a
-b-c)*ln(1-(b-c)*exp(2*I*x)/(-2*((a+b)*(a+c))^(1/2)-2*a-b-c))*x-I/((a+b)*(a
+c))^(1/2)/(-2*((a+b)*(a+c))^(1/2)-2*a-b-c)*ln(1-(b-c)*exp(2*I*x)/(-2*((a+b
)*(a+c))^(1/2)-2*a-b-c))*a*x-1/2*I/((a+b)*(a+c))^(1/2)/(-2*((a+b)*(a+c))^(1
/2)-2*a-b-c)*ln(1-(b-c)*exp(2*I*x)/(-2*((a+b)*(a+c))^(1/2)-2*a-b-c))*b*x-1/
2*I/((a+b)*(a+c))^(1/2)/(-2*((a+b)*(a+c))^(1/2)-2*a-b-c)*ln(1-(b-c)*exp(2*I
*x)/(-2*((a+b)*(a+c))^(1/2)-2*a-b-c))*c*x-1/(-2*((a+b)*(a+c))^(1/2)-2*a-b-c
)*x^2-1/((a+b)*(a+c))^(1/2)/(-2*((a+b)*(a+c))^(1/2)-2*a-b-c)*a*x^2-1/2/((a+
b)*(a+c))^(1/2)/(-2*((a+b)*(a+c))^(1/2)-2*a-b-c)*b*x^2-1/2/((a+b)*(a+c))^(1
/2)/(-2*((a+b)*(a+c))^(1/2)-2*a-b-c)*c*x^2-1/2/(-2*((a+b)*(a+c))^(1/2)-2*a-
b-c)*polylog(2,(b-c)*exp(2*I*x)/(-2*((a+b)*(a+c))^(1/2)-2*a-b-c))-1/2/((a+b
)*(a+c))^(1/2)/(-2*((a+b)*(a+c))^(1/2)-2*a-b-c)*polylog(2,(b-c)*exp(2*I*x)/
(-2*((a+b)*(a+c))^(1/2)-2*a-b-c))*a-1/4/((a+b)*(a+c))^(1/2)/(-2*((a+b)*(a+c
))^(1/2)-2*a-b-c)*polylog(2,(b-c)*exp(2*I*x)/(-2*((a+b)*(a+c))^(1/2)-2*a-b-
c))*b-1/4/((a+b)*(a+c))^(1/2)/(-2*((a+b)*(a+c))^(1/2)-2*a-b-c)*polylog(2,(b
-c)*exp(2*I*x)/(-2*((a+b)*(a+c))^(1/2)-2*a-b-c))*c
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{b\cos(x)^2+c\sin(x)^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*cos(x)^2+c*sin(x)^2),x, algorithm="maxima")
```

```
[Out] integrate(x/(b*cos(x)^2 + c*sin(x)^2 + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x}{b\cos(x)^2+c\sin(x)^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a + c*sin(x)^2 + b*cos(x)^2),x)
```

```
[Out] int(x/(a + c*sin(x)^2 + b*cos(x)^2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{a + b\cos^2(x) + c\sin^2(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*cos(x)**2+c*sin(x)**2),x)
```

```
[Out] Integral(x/(a + b*cos(x)**2 + c*sin(x)**2), x)
```

3.498 $\int \frac{x^2}{a+b\cos^2(x)+c\sin^2(x)}\,dx$

Optimal. Leaf size=365

$$-\frac{x\mathrm{Li}_2\left(-\frac{(b-c)e^{2ix}}{2a+b+c-2\sqrt{a+b}\sqrt{a+c}}\right)}{2\sqrt{a+b}\sqrt{a+c}}+\frac{x\mathrm{Li}_2\left(-\frac{(b-c)e^{2ix}}{2a+b+c+2\sqrt{a+b}\sqrt{a+c}}\right)}{2\sqrt{a+b}\sqrt{a+c}}-\frac{i\mathrm{Li}_3\left(-\frac{(b-c)e^{2ix}}{2a+b+c-2\sqrt{a+b}\sqrt{a+c}}\right)}{4\sqrt{a+b}\sqrt{a+c}}+\frac{i\mathrm{Li}_3\left(-\frac{(b-c)e^{2ix}}{2a+b+c+2\sqrt{a+b}\sqrt{a+c}}\right)}{4\sqrt{a+b}\sqrt{a+c}}$$

```
[Out] -1/2*I*x^2*ln(1+(b-c)*exp(2*I*x)/(2*a+b+c-2*(a+b)^(1/2)*(a+c)^(1/2)))/(a+b)
^(1/2)/(a+c)^(1/2)+1/2*I*x^2*ln(1+(b-c)*exp(2*I*x)/(2*a+b+c+2*(a+b)^(1/2)*(
a+c)^(1/2)))/(a+b)^(1/2)/(a+c)^(1/2)-1/2*x*polylog(2,-(b-c)*exp(2*I*x)/(2*a
+b+c-2*(a+b)^(1/2)*(a+c)^(1/2)))/(a+b)^(1/2)/(a+c)^(1/2)+1/2*x*polylog(2,-(
b-c)*exp(2*I*x)/(2*a+b+c+2*(a+b)^(1/2)*(a+c)^(1/2)))/(a+b)^(1/2)/(a+c)^(1/2
)-1/4*I*polylog(3,-(b-c)*exp(2*I*x)/(2*a+b+c-2*(a+b)^(1/2)*(a+c)^(1/2)))/(a
+b)^(1/2)/(a+c)^(1/2)+1/4*I*polylog(3,-(b-c)*exp(2*I*x)/(2*a+b+c+2*(a+b)^(1
/2)*(a+c)^(1/2)))/(a+b)^(1/2)/(a+c)^(1/2)
```

Rubi [A] time = 0.74, antiderivative size = 365, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.350, Rules used = {4587, 3321, 2264, 2190, 2531, 2282, 6589}

$$-\frac{x\mathrm{PolyLog}\left(2,-\frac{e^{2ix}(b-c)}{-2\sqrt{a+b}\sqrt{a+c}+2a+b+c}\right)}{2\sqrt{a+b}\sqrt{a+c}}+\frac{x\mathrm{PolyLog}\left(2,-\frac{e^{2ix}(b-c)}{2\sqrt{a+b}\sqrt{a+c}+2a+b+c}\right)}{2\sqrt{a+b}\sqrt{a+c}}-\frac{i\mathrm{PolyLog}\left(3,-\frac{e^{2ix}(b-c)}{-2\sqrt{a+b}\sqrt{a+c}+2a+b+c}\right)}{4\sqrt{a+b}\sqrt{a+c}}+$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(a + b*Cos[x]^2 + c*Sin[x]^2),x]
```

```
[Out] ((-I/2)*x^2*Log[1 + ((b - c)*E^((2*I)*x))/(2*a + b + c - 2*Sqrt[a + b]*Sqrt
[a + c])])/(Sqrt[a + b]*Sqrt[a + c]) + ((I/2)*x^2*Log[1 + ((b - c)*E^((2*I)
*x))/(2*a + b + c + 2*Sqrt[a + b]*Sqrt[a + c])])/(Sqrt[a + b]*Sqrt[a + c])
- (x*PolyLog[2, -(((b - c)*E^((2*I)*x))/(2*a + b + c - 2*Sqrt[a + b]*Sqrt[a
 + c]))])/(2*Sqrt[a + b]*Sqrt[a + c]) + (x*PolyLog[2, -(((b - c)*E^((2*I)*x
))/(2*a + b + c + 2*Sqrt[a + b]*Sqrt[a + c]))])/(2*Sqrt[a + b]*Sqrt[a + c])
 - ((I/4)*PolyLog[3, -(((b - c)*E^((2*I)*x))/(2*a + b + c - 2*Sqrt[a + b]*S
qrt[a + c]))])/(Sqrt[a + b]*Sqrt[a + c]) + ((I/4)*PolyLog[3, -(((b - c)*E^(
(2*I)*x))/(2*a + b + c + 2*Sqrt[a + b]*Sqrt[a + c]))])/(Sqrt[a + b]*Sqrt[a
+ c])
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
```

```
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2264

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c)/q, Int[
((f + g*x)^m*F^u)/(b - q + 2*c*F^u), x], x] - Dist[(2*c)/q, Int[((f + g*x)^
m*F^u)/(b + q + 2*c*F^u), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 3321

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(
x_)]), x_Symbol] :> Dist[2, Int[((c + d*x)^m*E^(I*Pi*(k - 1/2))*E^(I*(e + f
*x)))/(b + 2*a*E^(I*Pi*(k - 1/2))*E^(I*(e + f*x)) - b*E^(2*I*k*Pi)*E^(2*I*(
e + f*x))), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[2*k] && NeQ[
a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 4587

```
Int[((f_.) + (g_.)*(x_))^(m_.)/((a_.) + Cos[(d_.) + (e_.)*(x_)]^2*(b_.) + (
c_.)*Sin[(d_.) + (e_.)*(x_)]^2), x_Symbol] :> Dist[2, Int[(f + g*x)^m/(2*a
+ b + c + (b - c)*Cos[2*d + 2*e*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g},
 x] && IGtQ[m, 0] && NeQ[a + b, 0] && NeQ[a + c, 0]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{x^2}{a+b\cos^2(x)+c\sin^2(x)}\,dx = 2\int \frac{x^2}{2a+b+c+(b-c)\cos(2x)}\,dx$$

$$= 4\int \frac{e^{2ix}x^2}{b-c+2(2a+b+c)e^{2ix}+(b-c)e^{4ix}}\,dx$$

$$= \frac{(2(b-c))\int \frac{e^{2ix}x^2}{-4\sqrt{a+b}\,\sqrt{a+c}+2(2a+b+c)+2(b-c)e^{2ix}}\,dx}{\sqrt{a+b}\,\sqrt{a+c}} - \frac{(2(b-c))\int \frac{e^{2ix}x^2}{4\sqrt{a+b}\,\sqrt{a+c}+2(2a+b+c)}}{\sqrt{a+b}\,\sqrt{a+c}}$$

$$= -\frac{ix^2\log\left(1+\frac{(b-c)e^{2ix}}{2a+b+c-2\sqrt{a+b}\,\sqrt{a+c}}\right)}{2\sqrt{a+b}\,\sqrt{a+c}} + \frac{ix^2\log\left(1+\frac{(b-c)e^{2ix}}{2a+b+c+2\sqrt{a+b}\,\sqrt{a+c}}\right)}{2\sqrt{a+b}\,\sqrt{a+c}} + \frac{i\int x\log}{}$$

$$= -\frac{ix^2\log\left(1+\frac{(b-c)e^{2ix}}{2a+b+c-2\sqrt{a+b}\,\sqrt{a+c}}\right)}{2\sqrt{a+b}\,\sqrt{a+c}} + \frac{ix^2\log\left(1+\frac{(b-c)e^{2ix}}{2a+b+c+2\sqrt{a+b}\,\sqrt{a+c}}\right)}{2\sqrt{a+b}\,\sqrt{a+c}} - \frac{x\mathrm{Li}_2\left(-\frac{}{2}\right.}{2\sqrt{}}$$

$$= -\frac{ix^2\log\left(1+\frac{(b-c)e^{2ix}}{2a+b+c-2\sqrt{a+b}\,\sqrt{a+c}}\right)}{2\sqrt{a+b}\,\sqrt{a+c}} + \frac{ix^2\log\left(1+\frac{(b-c)e^{2ix}}{2a+b+c+2\sqrt{a+b}\,\sqrt{a+c}}\right)}{2\sqrt{a+b}\,\sqrt{a+c}} - \frac{x\mathrm{Li}_2\left(-\frac{}{2}\right.}{2\sqrt{}}$$

$$= -\frac{ix^2\log\left(1+\frac{(b-c)e^{2ix}}{2a+b+c-2\sqrt{a+b}\,\sqrt{a+c}}\right)}{2\sqrt{a+b}\,\sqrt{a+c}} + \frac{ix^2\log\left(1+\frac{(b-c)e^{2ix}}{2a+b+c+2\sqrt{a+b}\,\sqrt{a+c}}\right)}{2\sqrt{a+b}\,\sqrt{a+c}} - \frac{x\mathrm{Li}_2\left(-\frac{}{2}\right.}{2\sqrt{}}$$

Mathematica [A] time = 3.79, size = 258, normalized size = 0.71

$$-\frac{i\left(-2ix\mathrm{Li}_2\left(\frac{(c-b)e^{2ix}}{2a+b+c-2\sqrt{(a+b)(a+c)}}\right)+2ix\mathrm{Li}_2\left(\frac{(c-b)e^{2ix}}{2a+b+c+2\sqrt{(a+b)(a+c)}}\right)+\mathrm{Li}_3\left(\frac{(c-b)e^{2ix}}{2a+b+c-2\sqrt{(a+b)(a+c)}}\right)-\mathrm{Li}_3\left(\frac{(c-b)e^{2ix}}{2a+b+c+2\sqrt{(a+b)(a}}\right.\right.}{4\sqrt{(a+b)(a+c)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(a + b*Cos[x]^2 + c*Sin[x]^2),x]
```

```
[Out] ((-1/4*I)*(2*x^2*Log[1 + ((b - c)*E^((2*I)*x))/(2*a + b + c - 2*Sqrt[(a + b
)*(a + c)])] - 2*x^2*Log[1 + ((b - c)*E^((2*I)*x))/(2*a + b + c + 2*Sqrt[(a
```

```
 + b)*(a + c)])] - (2*I)*x*PolyLog[2, ((-b + c)*E^((2*I)*x))/(2*a + b + c -
 2*Sqrt[(a + b)*(a + c)])] + (2*I)*x*PolyLog[2, ((-b + c)*E^((2*I)*x))/(2*a
 + b + c + 2*Sqrt[(a + b)*(a + c)])] + PolyLog[3, ((-b + c)*E^((2*I)*x))/(2
*a + b + c - 2*Sqrt[(a + b)*(a + c)])] - PolyLog[3, ((-b + c)*E^((2*I)*x))/
(2*a + b + c + 2*Sqrt[(a + b)*(a + c)])]))/Sqrt[(a + b)*(a + c)]
```

fricas [C] time = 2.22, size = 4357, normalized size = 11.94

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a+b*cos(x)^2+c*sin(x)^2),x, algorithm="fricas")
```

```
[Out] 1/16*(4*I*(b - c)*x^2*sqrt((a^2 + a*b + (a + b)*c)/(b^2 - 2*b*c + c^2))*log
(-1/2*((2*(2*a + b + c)*cos(x) + (4*I*a + 2*I*b + 2*I*c)*sin(x) - 4*((b - c
)*cos(x) - (-I*b + I*c)*sin(x))*sqrt((a^2 + a*b + (a + b)*c)/(b^2 - 2*b*c +
 c^2)))*sqrt(-(2*(b - c)*sqrt((a^2 + a*b + (a + b)*c)/(b^2 - 2*b*c + c^2))
+ 2*a + b + c)/(b - c)) - 2*b + 2*c)/(b - c)) - 4*I*(b - c)*x^2*sqrt((a^2 +
 a*b + (a + b)*c)/(b^2 - 2*b*c + c^2))*log(1/2*((2*(2*a + b + c)*cos(x) - (
4*I*a + 2*I*b + 2*I*c)*sin(x) - 4*((b - c)*cos(x) + (-I*b + I*c)*sin(x))*sq
rt((a^2 + a*b + (a + b)*c)/(b^2 - 2*b*c + c^2)))*sqrt(-(2*(b - c)*sqrt((a^2
 + a*b + (a + b)*c)/(b^2 - 2*b*c + c^2)) + 2*a + b + c)/(b - c)) + 2*b - 2*
c)/(b - c)) - 4*I*(b - c)*x^2*sqrt((a^2 + a*b + (a + b)*c)/(b^2 - 2*b*c + c
^2))*log(-1/2*((2*(2*a + b + c)*cos(x) + (-4*I*a - 2*I*b - 2*I*c)*sin(x) -
4*((b - c)*cos(x) - (I*b - I*c)*sin(x))*sqrt((a^2 + a*b + (a + b)*c)/(b^2 -
 2*b*c + c^2)))*sqrt(-(2*(b - c)*sqrt((a^2 + a*b + (a + b)*c)/(b^2 - 2*b*c
+ c^2)) + 2*a + b + c)/(b - c)) - 2*b + 2*c)/(b - c)) + 4*I*(b - c)*x^2*sqr
t((a^2 + a*b + (a + b)*c)/(b^2 - 2*b*c + c^2))*log(1/2*((2*(2*a + b + c)*co
s(x) - (-4*I*a - 2*I*b - 2*I*c)*sin(x) - 4*((b - c)*cos(x) + (I*b - I*c)*si
n(x))*sqrt((a^2 + a*b + (a + b)*c)/(b^2 - 2*b*c + c^2)))*sqrt(-(2*(b - c)*s
qrt((a^2 + a*b + (a + b)*c)/(b^2 - 2*b*c + c^2)) + 2*a + b + c)/(b - c)) +
2*b - 2*c)/(b - c)) - 4*I*(b - c)*x^2*sqrt((a^2 + a*b + (a + b)*c)/(b^2 - 2
*b*c + c^2))*log(-1/2*((2*(2*a + b + c)*cos(x) + (4*I*a + 2*I*b + 2*I*c)*si
n(x) + 4*((b - c)*cos(x) + (I*b - I*c)*sin(x))*sqrt((a^2 + a*b + (a + b)*c)
/(b^2 - 2*b*c + c^2)))*sqrt((2*(b - c)*sqrt((a^2 + a*b + (a + b)*c)/(b^2 -
2*b*c + c^2)) - 2*a - b - c)/(b - c)) - 2*b + 2*c)/(b - c)) + 4*I*(b - c)*x
^2*sqrt((a^2 + a*b + (a + b)*c)/(b^2 - 2*b*c + c^2))*log(1/2*((2*(2*a + b +
 c)*cos(x) - (4*I*a + 2*I*b + 2*I*c)*sin(x) + 4*((b - c)*cos(x) - (I*b - I*
c)*sin(x))*sqrt((a^2 + a*b + (a + b)*c)/(b^2 - 2*b*c + c^2)))*sqrt((2*(b -
c)*sqrt((a^2 + a*b + (a + b)*c)/(b^2 - 2*b*c + c^2)) - 2*a - b - c)/(b - c)
) + 2*b - 2*c)/(b - c)) + 4*I*(b - c)*x^2*sqrt((a^2 + a*b + (a + b)*c)/(b^2
 - 2*b*c + c^2))*log(-1/2*((2*(2*a + b + c)*cos(x) + (-4*I*a - 2*I*b - 2*I*
c)*sin(x) + 4*((b - c)*cos(x) + (-I*b + I*c)*sin(x))*sqrt((a^2 + a*b + (a +
 b)*c)/(b^2 - 2*b*c + c^2)))*sqrt((2*(b - c)*sqrt((a^2 + a*b + (a + b)*c)/(
b^2 - 2*b*c + c^2)) - 2*a - b - c)/(b - c)) - 2*b + 2*c)/(b - c)) - 4*I*(b
```

```
- c)*x^2*sqrt((a^2 + a*b + (a + b)*c)/(b^2 - 2*b*c + c^2))*log(1/2*((2*(2*a
 + b + c)*cos(x) - (-4*I*a - 2*I*b - 2*I*c)*sin(x) + 4*((b - c)*cos(x) - (-
I*b + I*c)*sin(x))*sqrt((a^2 + a*b + (a + b)*c)/(b^2 - 2*b*c + c^2)))*sqrt(
(2*(b - c)*sqrt((a^2 + a*b + (a + b)*c)/(b^2 - 2*b*c + c^2)) - 2*a - b - c)
/(b - c)) + 2*b - 2*c)/(b - c)) + 8*(b - c)*x*sqrt((a^2 + a*b + (a + b)*c)/
(b^2 - 2*b*c + c^2))*dilog(1/2*((2*(2*a + b + c)*cos(x) + (4*I*a + 2*I*b +
2*I*c)*sin(x) - 4*((b - c)*cos(x) - (-I*b + I*c)*sin(x))*sqrt((a^2 + a*b +
(a + b)*c)/(b^2 - 2*b*c + c^2)))*sqrt(-(2*(b - c)*sqrt((a^2 + a*b + (a + b)
*c)/(b^2 - 2*b*c + c^2)) + 2*a + b + c)/(b - c)) - 2*b + 2*c)/(b - c) + 1)
+ 8*(b - c)*x*sqrt((a^2 + a*b + (a + b)*c)/(b^2 - 2*b*c + c^2))*dilog(-1/2*
((2*(2*a + b + c)*cos(x) - (4*I*a + 2*I*b + 2*I*c)*sin(x) - 4*((b - c)*cos(
x) + (-I*b + I*c)*sin(x))*sqrt((a^2 + a*b + (a + b)*c)/(b^2 - 2*b*c + c^2))
)*sqrt(-(2*(b - c)*sqrt((a^2 + a*b + (a + b)*c)/(b^2 - 2*b*c + c^2)) + 2*a
+ b + c)/(b - c)) + 2*b - 2*c)/(b - c) + 1) + 8*(b - c)*x*sqrt((a^2 + a*b +
 (a + b)*c)/(b^2 - 2*b*c + c^2))*dilog(1/2*((2*(2*a + b + c)*cos(x) + (-4*I
*a - 2*I*b - 2*I*c)*sin(x) - 4*((b - c)*cos(x) - (I*b - I*c)*sin(x))*sqrt((
a^2 + a*b + (a + b)*c)/(b^2 - 2*b*c + c^2)))*sqrt(-(2*(b - c)*sqrt((a^2 + a
*b + (a + b)*c)/(b^2 - 2*b*c + c^2)) + 2*a + b + c)/(b - c)) - 2*b + 2*c)/(
b - c) + 1) + 8*(b - c)*x*sqrt((a^2 + a*b + (a + b)*c)/(b^2 - 2*b*c + c^2))
*dilog(-1/2*((2*(2*a + b + c)*cos(x) - (-4*I*a - 2*I*b - 2*I*c)*sin(x) - 4*
((b - c)*cos(x) + (I*b - I*c)*sin(x))*sqrt((a^2 + a*b + (a + b)*c)/(b^2 - 2
*b*c + c^2)))*sqrt(-(2*(b - c)*sqrt((a^2 + a*b + (a + b)*c)/(b^2 - 2*b*c +
c^2)) + 2*a + b + c)/(b - c)) + 2*b - 2*c)/(b - c) + 1) - 8*(b - c)*x*sqrt(
(a^2 + a*b + (a + b)*c)/(b^2 - 2*b*c + c^2))*dilog(1/2*((2*(2*a + b + c)*co
s(x) + (4*I*a + 2*I*b + 2*I*c)*sin(x) + 4*((b - c)*cos(x) + (I*b - I*c)*sin
(x))*sqrt((a^2 + a*b + (a + b)*c)/(b^2 - 2*b*c + c^2)))*sqrt((2*(b - c)*sqr
t((a^2 + a*b + (a + b)*c)/(b^2 - 2*b*c + c^2)) - 2*a - b - c)/(b - c)) - 2*
b + 2*c)/(b - c) + 1) - 8*(b - c)*x*sqrt((a^2 + a*b + (a + b)*c)/(b^2 - 2*b
*c + c^2))*dilog(-1/2*((2*(2*a + b + c)*cos(x) - (4*I*a + 2*I*b + 2*I*c)*si
n(x) + 4*((b - c)*cos(x) - (I*b - I*c)*sin(x))*sqrt((a^2 + a*b + (a + b)*c)
/(b^2 - 2*b*c + c^2)))*sqrt((2*(b - c)*sqrt((a^2 + a*b + (a + b)*c)/(b^2 -
2*b*c + c^2)) - 2*a - b - c)/(b - c)) + 2*b - 2*c)/(b - c) + 1) - 8*(b - c)
*x*sqrt((a^2 + a*b + (a + b)*c)/(b^2 - 2*b*c + c^2))*dilog(1/2*((2*(2*a + b
 + c)*cos(x) + (-4*I*a - 2*I*b - 2*I*c)*sin(x) + 4*((b - c)*cos(x) + (-I*b
+ I*c)*sin(x))*sqrt((a^2 + a*b + (a + b)*c)/(b^2 - 2*b*c + c^2)))*sqrt((2*(
b - c)*sqrt((a^2 + a*b + (a + b)*c)/(b^2 - 2*b*c + c^2)) - 2*a - b - c)/(b
- c)) - 2*b + 2*c)/(b - c) + 1) - 8*(b - c)*x*sqrt((a^2 + a*b + (a + b)*c)/
(b^2 - 2*b*c + c^2))*dilog(-1/2*((2*(2*a + b + c)*cos(x) - (-4*I*a - 2*I*b
- 2*I*c)*sin(x) + 4*((b - c)*cos(x) - (-I*b + I*c)*sin(x))*sqrt((a^2 + a*b
+ (a + b)*c)/(b^2 - 2*b*c + c^2)))*sqrt((2*(b - c)*sqrt((a^2 + a*b + (a + b
)*c)/(b^2 - 2*b*c + c^2)) - 2*a - b - c)/(b - c)) + 2*b - 2*c)/(b - c) + 1)
 + 4*(2*I*b - 2*I*c)*sqrt((a^2 + a*b + (a + b)*c)/(b^2 - 2*b*c + c^2))*poly
log(3, -1/2*(2*(2*a + b + c)*cos(x) + (4*I*a + 2*I*b + 2*I*c)*sin(x) - 4*((
b - c)*cos(x) - (-I*b + I*c)*sin(x))*sqrt((a^2 + a*b + (a + b)*c)/(b^2 - 2*
b*c + c^2)))*sqrt(-(2*(b - c)*sqrt((a^2 + a*b + (a + b)*c)/(b^2 - 2*b*c + c
```

```
^2)) + 2*a + b + c)/(b - c))/(b - c)) + 4*(-2*I*b + 2*I*c)*sqrt((a^2 + a*b
+ (a + b)*c)/(b^2 - 2*b*c + c^2))*polylog(3, 1/2*(2*(2*a + b + c)*cos(x) -
(4*I*a + 2*I*b + 2*I*c)*sin(x) - 4*((b - c)*cos(x) + (-I*b + I*c)*sin(x))*s
qrt((a^2 + a*b + (a + b)*c)/(b^2 - 2*b*c + c^2)))*sqrt(-(2*(b - c)*sqrt((a^
2 + a*b + (a + b)*c)/(b^2 - 2*b*c + c^2)) + 2*a + b + c)/(b - c))/(b - c))
+ 4*(-2*I*b + 2*I*c)*sqrt((a^2 + a*b + (a + b)*c)/(b^2 - 2*b*c + c^2))*poly
log(3, -1/2*(2*(2*a + b + c)*cos(x) + (-4*I*a - 2*I*b - 2*I*c)*sin(x) - 4*(
(b - c)*cos(x) - (I*b - I*c)*sin(x))*sqrt((a^2 + a*b + (a + b)*c)/(b^2 - 2*
b*c + c^2)))*sqrt(-(2*(b - c)*sqrt((a^2 + a*b + (a + b)*c)/(b^2 - 2*b*c + c
^2)) + 2*a + b + c)/(b - c))/(b - c)) + 4*(2*I*b - 2*I*c)*sqrt((a^2 + a*b +
 (a + b)*c)/(b^2 - 2*b*c + c^2))*polylog(3, 1/2*(2*(2*a + b + c)*cos(x) - (
-4*I*a - 2*I*b - 2*I*c)*sin(x) - 4*((b - c)*cos(x) + (I*b - I*c)*sin(x))*sq
rt((a^2 + a*b + (a + b)*c)/(b^2 - 2*b*c + c^2)))*sqrt(-(2*(b - c)*sqrt((a^2
 + a*b + (a + b)*c)/(b^2 - 2*b*c + c^2)) + 2*a + b + c)/(b - c))/(b - c)) +
 4*(-2*I*b + 2*I*c)*sqrt((a^2 + a*b + (a + b)*c)/(b^2 - 2*b*c + c^2))*polyl
og(3, -1/2*(2*(2*a + b + c)*cos(x) + (4*I*a + 2*I*b + 2*I*c)*sin(x) + 4*((b
 - c)*cos(x) + (I*b - I*c)*sin(x))*sqrt((a^2 + a*b + (a + b)*c)/(b^2 - 2*b*
c + c^2)))*sqrt((2*(b - c)*sqrt((a^2 + a*b + (a + b)*c)/(b^2 - 2*b*c + c^2)
) - 2*a - b - c)/(b - c))/(b - c)) + 4*(2*I*b - 2*I*c)*sqrt((a^2 + a*b + (a
 + b)*c)/(b^2 - 2*b*c + c^2))*polylog(3, 1/2*(2*(2*a + b + c)*cos(x) - (4*I
*a + 2*I*b + 2*I*c)*sin(x) + 4*((b - c)*cos(x) - (I*b - I*c)*sin(x))*sqrt((
a^2 + a*b + (a + b)*c)/(b^2 - 2*b*c + c^2)))*sqrt((2*(b - c)*sqrt((a^2 + a*
b + (a + b)*c)/(b^2 - 2*b*c + c^2)) - 2*a - b - c)/(b - c))/(b - c)) + 4*(2
*I*b - 2*I*c)*sqrt((a^2 + a*b + (a + b)*c)/(b^2 - 2*b*c + c^2))*polylog(3,
-1/2*(2*(2*a + b + c)*cos(x) + (-4*I*a - 2*I*b - 2*I*c)*sin(x) + 4*((b - c)
*cos(x) + (-I*b + I*c)*sin(x))*sqrt((a^2 + a*b + (a + b)*c)/(b^2 - 2*b*c +
c^2)))*sqrt((2*(b - c)*sqrt((a^2 + a*b + (a + b)*c)/(b^2 - 2*b*c + c^2)) -
2*a - b - c)/(b - c))/(b - c)) + 4*(-2*I*b + 2*I*c)*sqrt((a^2 + a*b + (a +
b)*c)/(b^2 - 2*b*c + c^2))*polylog(3, 1/2*(2*(2*a + b + c)*cos(x) - (-4*I*a
 - 2*I*b - 2*I*c)*sin(x) + 4*((b - c)*cos(x) - (-I*b + I*c)*sin(x))*sqrt((a
^2 + a*b + (a + b)*c)/(b^2 - 2*b*c + c^2)))*sqrt((2*(b - c)*sqrt((a^2 + a*b
 + (a + b)*c)/(b^2 - 2*b*c + c^2)) - 2*a - b - c)/(b - c))/(b - c)))/(a^2 +
 a*b + (a + b)*c)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{b\cos(x)^2 + c\sin(x)^2 + a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a+b*cos(x)^2+c*sin(x)^2),x, algorithm="giac")
```

```
[Out] integrate(x^2/(b*cos(x)^2 + c*sin(x)^2 + a), x)
```

maple [B] time = 0.29, size = 1161, normalized size = 3.18

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a+b*cos(x)^2+c*sin(x)^2),x)
```

```
[Out] -1/(-2*((a+b)*(a+c))^(1/2)-2*a-b-c)*x*polylog(2,(b-c)*exp(2*I*x)/(-2*((a+b)
*(a+c))^(1/2)-2*a-b-c))-1/4*I/((a+b)*(a+c))^(1/2)/(-2*((a+b)*(a+c))^(1/2)-2
*a-b-c)*c*polylog(3,(b-c)*exp(2*I*x)/(-2*((a+b)*(a+c))^(1/2)-2*a-b-c))-1/((
a+b)*(a+c))^(1/2)/(-2*((a+b)*(a+c))^(1/2)-2*a-b-c)*a*x*polylog(2,(b-c)*exp(
2*I*x)/(-2*((a+b)*(a+c))^(1/2)-2*a-b-c))-1/2*I/((a+b)*(a+c))^(1/2)/(-2*((a+
b)*(a+c))^(1/2)-2*a-b-c)*b*x^2*ln(1-(b-c)*exp(2*I*x)/(-2*((a+b)*(a+c))^(1/2
)-2*a-b-c))-1/2/((a+b)*(a+c))^(1/2)*x*polylog(2,(b-c)*exp(2*I*x)/(2*((a+b)*
(a+c))^(1/2)-2*a-b-c))-1/4*I/((a+b)*(a+c))^(1/2)/(-2*((a+b)*(a+c))^(1/2)-2*
a-b-c)*b*polylog(3,(b-c)*exp(2*I*x)/(-2*((a+b)*(a+c))^(1/2)-2*a-b-c))-1/2/(
(a+b)*(a+c))^(1/2)/(-2*((a+b)*(a+c))^(1/2)-2*a-b-c)*c*x*polylog(2,(b-c)*exp
(2*I*x)/(-2*((a+b)*(a+c))^(1/2)-2*a-b-c))-1/2*I/((a+b)*(a+c))^(1/2)*x^2*ln(
1-(b-c)*exp(2*I*x)/(2*((a+b)*(a+c))^(1/2)-2*a-b-c))-1/2/((a+b)*(a+c))^(1/2)
/(-2*((a+b)*(a+c))^(1/2)-2*a-b-c)*b*x*polylog(2,(b-c)*exp(2*I*x)/(-2*((a+b)
*(a+c))^(1/2)-2*a-b-c))-1/4*I/((a+b)*(a+c))^(1/2)*polylog(3,(b-c)*exp(2*I*x
)/(2*((a+b)*(a+c))^(1/2)-2*a-b-c))-2/3/((a+b)*(a+c))^(1/2)/(-2*((a+b)*(a+c)
)^(1/2)-2*a-b-c)*a*x^3-1/2*I/(-2*((a+b)*(a+c))^(1/2)-2*a-b-c)*polylog(3,(b-
c)*exp(2*I*x)/(-2*((a+b)*(a+c))^(1/2)-2*a-b-c))-2/3/(-2*((a+b)*(a+c))^(1/2)
-2*a-b-c)*x^3-I/((a+b)*(a+c))^(1/2)/(-2*((a+b)*(a+c))^(1/2)-2*a-b-c)*a*x^2*
ln(1-(b-c)*exp(2*I*x)/(-2*((a+b)*(a+c))^(1/2)-2*a-b-c))-1/3/((a+b)*(a+c))^(
1/2)/(-2*((a+b)*(a+c))^(1/2)-2*a-b-c)*b*x^3-1/2*I/((a+b)*(a+c))^(1/2)/(-2*(
(a+b)*(a+c))^(1/2)-2*a-b-c)*c*x^2*ln(1-(b-c)*exp(2*I*x)/(-2*((a+b)*(a+c))^(
1/2)-2*a-b-c))-1/3/((a+b)*(a+c))^(1/2)/(-2*((a+b)*(a+c))^(1/2)-2*a-b-c)*c*x
^3-I/(-2*((a+b)*(a+c))^(1/2)-2*a-b-c)*x^2*ln(1-(b-c)*exp(2*I*x)/(-2*((a+b)*
(a+c))^(1/2)-2*a-b-c))-1/3/((a+b)*(a+c))^(1/2)*x^3-1/2*I/((a+b)*(a+c))^(1/2
)/(-2*((a+b)*(a+c))^(1/2)-2*a-b-c)*a*polylog(3,(b-c)*exp(2*I*x)/(-2*((a+b)*
(a+c))^(1/2)-2*a-b-c))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{b\cos(x)^2 + c\sin(x)^2 + a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a+b*cos(x)^2+c*sin(x)^2),x, algorithm="maxima")
```

```
[Out] integrate(x^2/(b*cos(x)^2 + c*sin(x)^2 + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^2}{b\cos(x)^2 + c\sin(x)^2 + a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a + c*sin(x)^2 + b*cos(x)^2),x)
[Out] int(x^2/(a + c*sin(x)^2 + b*cos(x)^2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{a + b\cos^2(x) + c\sin^2(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(a+b*cos(x)**2+c*sin(x)**2),x)
[Out] Integral(x**2/(a + b*cos(x)**2 + c*sin(x)**2), x)
```

3.499 $\int (a+b\sin(d+ex))\left(b^2+2ab\sin(d+ex)+a^2\sin^2(d+e$

Optimal. Leaf size=195

$$-\frac{\left(5a^2+4b^2\right)\cos(d+ex)(a\sin(d+ex)+b)^3}{20e}-\frac{b\left(17a^2+4b^2\right)\cos(d+ex)(a\sin(d+ex)+b)^2}{20e}-\frac{b\left(32a^4+69a^2b^2+\right.}{10}$$

```
[Out] 3/8*a*(a^4+12*a^2*b^2+8*b^4)*x-1/10*b*(32*a^4+69*a^2*b^2+4*b^4)*cos(e*x+d)/
e-1/40*a*(15*a^4+82*a^2*b^2+8*b^4)*cos(e*x+d)*sin(e*x+d)/e-1/20*b*(17*a^2+4
*b^2)*cos(e*x+d)*(b+a*sin(e*x+d))^2/e-1/20*(5*a^2+4*b^2)*cos(e*x+d)*(b+a*si
n(e*x+d))^3/e-1/5*b*cos(e*x+d)*(b+a*sin(e*x+d))^4/e
```

Rubi [A] time = 0.39, antiderivative size = 195, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 39, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {3288, 2753, 2734}

$$-\frac{b\left(69a^2b^2+32a^4+4b^4\right)\cos(d+ex)}{10e}-\frac{\left(5a^2+4b^2\right)\cos(d+ex)(a\sin(d+ex)+b)^3}{20e}-\frac{b\left(17a^2+4b^2\right)\cos(d+ex)(a}{20e}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[d + e*x])*(b^2 + 2*a*b*Sin[d + e*x] + a^2*Sin[d + e*x]^2)^2,
x]
```

```
[Out] (3*a*(a^4 + 12*a^2*b^2 + 8*b^4)*x)/8 - (b*(32*a^4 + 69*a^2*b^2 + 4*b^4)*Cos
[d + e*x])/(10*e) - (a*(15*a^4 + 82*a^2*b^2 + 8*b^4)*Cos[d + e*x]*Sin[d + e
*x])/(40*e) - (b*(17*a^2 + 4*b^2)*Cos[d + e*x]*(b + a*Sin[d + e*x])^2)/(20*
e) - ((5*a^2 + 4*b^2)*Cos[d + e*x]*(b + a*Sin[d + e*x])^3)/(20*e) - (b*Cos[
d + e*x]*(b + a*Sin[d + e*x])^4)/(5*e)
```

Rule 2734

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[((2*a*c + b*d)*x)/2, x] + (-Simp[((b*c + a*d)*Co
s[e + f*x])/f, x] - Simp[(b*d*Cos[e + f*x]*Sin[e + f*x])/(2*f), x]) /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2753

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[1/(m + 1), Int[(a + b*Sin[e + f*x])^(m - 1)*Simp[b*d*m
 + a*c*(m + 1) + (a*d*m + b*c*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a
, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && GtQ[m, 0]
```

```
&& IntegerQ[2*m]
```

Rule 3288

```
Int[((A_) + (B_.)*sin[(d_.) + (e_.)*(x_)])*((a_) + (b_.)*sin[(d_.) + (e_.)*
(x_)] + (c_.)*sin[(d_.) + (e_.)*(x_)]^2)^(n_), x_Symbol] :> Dist[1/(4^n*c^n
), Int[(A + B*Sin[d + e*x])*(b + 2*c*Sin[d + e*x])^(2*n), x], x] /; FreeQ[{
a, b, c, d, e, A, B}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[n]
```

Rubi steps

$$\int (a+b\sin(d+ex))\left(b^2+2ab\sin(d+ex)+a^2\sin^2(d+ex)\right)^2\,dx = \frac{\int \left(2ab+2a^2\sin(d+ex)\right)^4 (a+b\sin(d+e$$

$$= -\frac{b\cos(d+ex)(b+a\sin(d+ex))^4}{5e} + \frac{\int \left(2a\right.}{}$$

$$= -\frac{\left(5a^2+4b^2\right)\cos(d+ex)(b+a\sin(d+ex)}{20e}$$

$$= -\frac{b\left(17a^2+4b^2\right)\cos(d+ex)(b+a\sin(d+e}{20e}$$

$$= \frac{3}{8}a\left(a^4+12a^2b^2+8b^4\right)x - \frac{b\left(32a^4+69a^2\right.}{}$$

Mathematica [A] time = 0.90, size = 149, normalized size = 0.76

$$\frac{a\left(10\left(7a^3b+8ab^3\right)\cos(3(d+ex))-2a^3b\cos(5(d+ex))+5\left(a^4+4a^2b^2\right)\sin(4(d+ex))+60\left(a^4+12a^2b^2+8b\right.}{160e}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sin[d + e*x])*(b^2 + 2*a*b*Sin[d + e*x] + a^2*Sin[d + e*x]
^2)^2,x]
```

```
[Out] (-20*b*(29*a^4 + 68*a^2*b^2 + 8*b^4)*Cos[d + e*x] + a*(60*(a^4 + 12*a^2*b^2
 + 8*b^4)*(d + e*x) + 10*(7*a^3*b + 8*a*b^3)*Cos[3*(d + e*x)] - 2*a^3*b*Cos
[5*(d + e*x)] - 40*(a^4 + 10*a^2*b^2 + 4*b^4)*Sin[2*(d + e*x)] + 5*(a^4 + 4
*a^2*b^2)*Sin[4*(d + e*x)]))/(160*e)
```

fricas [A] time = 0.53, size = 150, normalized size = 0.77

$$-\frac{8\,a^4b\cos\left(ex+d\right)^5-80\left(a^4b+a^2b^3\right)\cos\left(ex+d\right)^3-15\left(a^5+12\,a^3b^2+8\,ab^4\right)ex+40\left(5\,a^4b+10\,a^2b^3+b^5\right)\mathrm{co}}{40\,e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(e*x+d))*(b^2+2*a*b*sin(e*x+d)+a^2*sin(e*x+d)^2)^2,x, alg
orithm="fricas")
```

```
[Out] -1/40*(8*a^4*b*cos(e*x + d)^5 - 80*(a^4*b + a^2*b^3)*cos(e*x + d)^3 - 15*(a
^5 + 12*a^3*b^2 + 8*a*b^4)*e*x + 40*(5*a^4*b + 10*a^2*b^3 + b^5)*cos(e*x +
d) - 5*(2*(a^5 + 4*a^3*b^2)*cos(e*x + d)^3 - (5*a^5 + 44*a^3*b^2 + 16*a*b^4
)*cos(e*x + d))*sin(e*x + d))/e
```

giac [A] time = 0.18, size = 158, normalized size = 0.81

$$-\frac{1}{80}\,a^4b\cos\left(5\,xe+5\,d\right)e^{(-1)}+\frac{1}{16}\left(7\,a^4b+8\,a^2b^3\right)\cos\left(3\,xe+3\,d\right)e^{(-1)}-\frac{1}{8}\left(29\,a^4b+68\,a^2b^3+8\,b^5\right)\cos\left(xe+d\right)e^{(-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(e*x+d))*(b^2+2*a*b*sin(e*x+d)+a^2*sin(e*x+d)^2)^2,x, alg
orithm="giac")
```

```
[Out] -1/80*a^4*b*cos(5*x*e + 5*d)*e^(-1) + 1/16*(7*a^4*b + 8*a^2*b^3)*cos(3*x*e
+ 3*d)*e^(-1) - 1/8*(29*a^4*b + 68*a^2*b^3 + 8*b^5)*cos(x*e + d)*e^(-1) + 1
/32*(a^5 + 4*a^3*b^2)*e^(-1)*sin(4*x*e + 4*d) - 1/4*(a^5 + 10*a^3*b^2 + 4*a
*b^4)*e^(-1)*sin(2*x*e + 2*d) + 3/8*(a^5 + 12*a^3*b^2 + 8*a*b^4)*x
```

maple [A] time = 0.10, size = 255, normalized size = 1.31

$$\frac{a\,b^4\left(ex+d\right)-4\cos\left(ex+d\right)a^2b^3+6a^3b^2\left(-\frac{\sin(ex+d)\cos(ex+d)}{2}+\frac{ex}{2}+\frac{d}{2}\right)-\frac{4a^4b\left(2+\sin^2(ex+d)\right)\cos(ex+d)}{3}+a^5\left(-\frac{\left(\sin^3(ex+d\right.}{}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(e*x+d))*(b^2+2*a*b*sin(e*x+d)+a^2*sin(e*x+d)^2)^2,x)
```

```
[Out] 1/e*(a*b^4*(e*x+d)-4*cos(e*x+d)*a^2*b^3+6*a^3*b^2*(-1/2*sin(e*x+d)*cos(e*x+
d)+1/2*e*x+1/2*d)-4/3*a^4*b*(2+sin(e*x+d)^2)*cos(e*x+d)+a^5*(-1/4*(sin(e*x+
d)^3+3/2*sin(e*x+d))*cos(e*x+d)+3/8*e*x+3/8*d)-cos(e*x+d)*b^5+4*a*b^4*(-1/2
*sin(e*x+d)*cos(e*x+d)+1/2*e*x+1/2*d)-2*a^2*b^3*(2+sin(e*x+d)^2)*cos(e*x+d)
+4*a^3*b^2*(-1/4*(sin(e*x+d)^3+3/2*sin(e*x+d))*cos(e*x+d)+3/8*e*x+3/8*d)-1/
5*a^4*b*(8/3+sin(e*x+d)^4+4/3*sin(e*x+d)^2)*cos(e*x+d))
```

maxima [A] time = 0.32, size = 246, normalized size = 1.26

$$\frac{15\left(12\,ex+12\,d+\sin\left(4\,ex+4\,d\right)-8\,\sin\left(2\,ex+2\,d\right)\right)a^5-32\left(3\,\cos\left(ex+d\right)^5-10\,\cos\left(ex+d\right)^3+15\,\cos\left(ex+d\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(e*x+d))*(b^2+2*a*b*sin(e*x+d)+a^2*sin(e*x+d)^2)^2,x, alg
orithm="maxima")
```

```
[Out] 1/480*(15*(12*e*x + 12*d + sin(4*e*x + 4*d) - 8*sin(2*e*x + 2*d))*a^5 - 32*
(3*cos(e*x + d)^5 - 10*cos(e*x + d)^3 + 15*cos(e*x + d))*a^4*b + 640*(cos(e
*x + d)^3 - 3*cos(e*x + d))*a^4*b + 60*(12*e*x + 12*d + sin(4*e*x + 4*d) -
8*sin(2*e*x + 2*d))*a^3*b^2 + 720*(2*e*x + 2*d - sin(2*e*x + 2*d))*a^3*b^2
+ 960*(cos(e*x + d)^3 - 3*cos(e*x + d))*a^2*b^3 + 480*(2*e*x + 2*d - sin(2*
e*x + 2*d))*a*b^4 + 480*(e*x + d)*a*b^4 - 1920*a^2*b^3*cos(e*x + d) - 480*b
^5*cos(e*x + d))/e
```

mupad [B] time = 4.49, size = 456, normalized size = 2.34

$$\frac{3\,a\,\mathrm{atan}\left(\frac{3\,a\,\tan\left(\frac{d}{2}+\frac{e\,x}{2}\right)\,\left(a^4+12\,a^2\,b^2+8\,b^4\right)}{4\,\left(\frac{3\,a^5}{4}+9\,a^3\,b^2+6\,a\,b^4\right)}\right)\,\left(a^4+12\,a^2\,b^2+8\,b^4\right)}{4\,e}-\frac{\tan\left(\frac{d}{2}+\frac{e\,x}{2}\right)\,\left(\frac{3\,a^5}{4}+9\,a^3\,b^2+4\,a\,b^4\right)-\tan\left(\frac{d}{2}+\frac{e\,x}{2}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(d + e*x))*(b^2 + a^2*sin(d + e*x)^2 + 2*a*b*sin(d + e*x))^2,
x)
```

```
[Out] (3*a*atan((3*a*tan(d/2 + (e*x)/2)*(a^4 + 8*b^4 + 12*a^2*b^2))/(4*(6*a*b^4 +
 (3*a^5)/4 + 9*a^3*b^2)))*(a^4 + 8*b^4 + 12*a^2*b^2))/(4*e) - (tan(d/2 + (e
*x)/2)*(4*a*b^4 + (3*a^5)/4 + 9*a^3*b^2) - tan(d/2 + (e*x)/2)^9*(4*a*b^4 +
(3*a^5)/4 + 9*a^3*b^2) + tan(d/2 + (e*x)/2)^3*(8*a*b^4 + (7*a^5)/2 + 26*a^3
*b^2) - tan(d/2 + (e*x)/2)^7*(8*a*b^4 + (7*a^5)/2 + 26*a^3*b^2) + tan(d/2 +
 (e*x)/2)^6*(16*a^4*b + 8*b^5 + 56*a^2*b^3) + tan(d/2 + (e*x)/2)^2*(32*a^4*
b + 8*b^5 + 72*a^2*b^3) + tan(d/2 + (e*x)/2)^4*(48*a^4*b + 12*b^5 + 104*a^2
*b^3) + (32*a^4*b)/5 + 2*b^5 + 16*a^2*b^3 + tan(d/2 + (e*x)/2)^8*(2*b^5 + 8
*a^2*b^3))/(e*(5*tan(d/2 + (e*x)/2)^2 + 10*tan(d/2 + (e*x)/2)^4 + 10*tan(d/
2 + (e*x)/2)^6 + 5*tan(d/2 + (e*x)/2)^8 + tan(d/2 + (e*x)/2)^10 + 1)) - (3*
a*(atan(tan(d/2 + (e*x)/2)) - (e*x)/2)*(a^4 + 8*b^4 + 12*a^2*b^2))/(4*e)
```

sympy [A] time = 3.00, size = 566, normalized size = 2.90

$$\begin{cases} \frac{3a^5x\sin^4(d+ex)}{8}+\frac{3a^5x\sin^2(d+ex)\cos^2(d+ex)}{4}+\frac{3a^5x\cos^4(d+ex)}{8}-\frac{5a^5\sin^3(d+ex)\cos(d+ex)}{8e}-\frac{3a^5\sin(d+ex)\cos^3(d+ex)}{8e}-\frac{a^4b\sin^4(d+ \\ x\left(a+b\sin(d)\right)\left(a^2\sin^2(d)+2ab\sin(d)+b^2\right)^2 \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(e*x+d))*(b**2+2*a*b*sin(e*x+d)+a**2*sin(e*x+d)**2)**2,x)
```

```
[Out] Piecewise((3*a**5*x*sin(d + e*x)**4/8 + 3*a**5*x*sin(d + e*x)**2*cos(d + e*
x)**2/4 + 3*a**5*x*cos(d + e*x)**4/8 - 5*a**5*sin(d + e*x)**3*cos(d + e*x)/
(8*e) - 3*a**5*sin(d + e*x)*cos(d + e*x)**3/(8*e) - a**4*b*sin(d + e*x)**4*
cos(d + e*x)/e - 4*a**4*b*sin(d + e*x)**2*cos(d + e*x)**3/(3*e) - 4*a**4*b*
sin(d + e*x)**2*cos(d + e*x)/e - 8*a**4*b*cos(d + e*x)**5/(15*e) - 8*a**4*b
*cos(d + e*x)**3/(3*e) + 3*a**3*b**2*x*sin(d + e*x)**4/2 + 3*a**3*b**2*x*si
n(d + e*x)**2*cos(d + e*x)**2 + 3*a**3*b**2*x*sin(d + e*x)**2 + 3*a**3*b**2
*x*cos(d + e*x)**4/2 + 3*a**3*b**2*x*cos(d + e*x)**2 - 5*a**3*b**2*sin(d +
e*x)**3*cos(d + e*x)/(2*e) - 3*a**3*b**2*sin(d + e*x)*cos(d + e*x)**3/(2*e)
 - 3*a**3*b**2*sin(d + e*x)*cos(d + e*x)/e - 6*a**2*b**3*sin(d + e*x)**2*co
s(d + e*x)/e - 4*a**2*b**3*cos(d + e*x)**3/e - 4*a**2*b**3*cos(d + e*x)/e +
 2*a*b**4*x*sin(d + e*x)**2 + 2*a*b**4*x*cos(d + e*x)**2 + a*b**4*x - 2*a*b
**4*sin(d + e*x)*cos(d + e*x)/e - b**5*cos(d + e*x)/e, Ne(e, 0)), (x*(a + b
*sin(d))*(a**2*sin(d)**2 + 2*a*b*sin(d) + b**2)**2, True))
```

3.500 $\int (a+b\sin(d+ex))\left(b^2+2ab\sin(d+ex)+a^2\sin^2(d+\right.$

Optimal. Leaf size=109

$$\frac{a\left(a^2-6b^2\right)\sin(d+ex)\cos(d+ex)}{6e}+\frac{1}{2}ax\left(a^2+4b^2\right)-\frac{a^2\cos(d+ex)(a+b\sin(d+ex))^2}{3be}+\frac{\left(a^4-8a^2b^2-3b^4\right)\cos}{3be}$$

```
[Out] 1/2*a*(a^2+4*b^2)*x+1/3*(a^4-8*a^2*b^2-3*b^4)*cos(e*x+d)/b/e+1/6*a*(a^2-6*b
^2)*cos(e*x+d)*sin(e*x+d)/e-1/3*a^2*cos(e*x+d)*(a+b*sin(e*x+d))^2/b/e
```

Rubi [A] time = 0.10, antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.054, Rules used = {3023, 2734}

$$\frac{\left(-8a^2b^2+a^4-3b^4\right)\cos(d+ex)}{3be}+\frac{a\left(a^2-6b^2\right)\sin(d+ex)\cos(d+ex)}{6e}+\frac{1}{2}ax\left(a^2+4b^2\right)-\frac{a^2\cos(d+ex)(a+b\sin}{3be}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[d + e*x])*(b^2 + 2*a*b*Sin[d + e*x] + a^2*Sin[d + e*x]^2),x]
```

```
[Out] (a*(a^2 + 4*b^2)*x)/2 + ((a^4 - 8*a^2*b^2 - 3*b^4)*Cos[d + e*x])/(3*b*e) +
(a*(a^2 - 6*b^2)*Cos[d + e*x]*Sin[d + e*x])/(6*e) - (a^2*Cos[d + e*x]*(a +
b*Sin[d + e*x])^2)/(3*b*e)
```

Rule 2734

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[((2*a*c + b*d)*x)/2, x] + (-Simp[((b*c + a*d)*Co
s[e + f*x])/f, x] - Simp[(b*d*Cos[e + f*x]*Sin[e + f*x])/(2*f), x]) /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
 !LtQ[m, -1]
```

Rubi steps

$$\int (a+b\sin(d+ex))\left(b^2+2ab\sin(d+ex)+a^2\sin^2(d+ex)\right)\,dx = -\frac{a^2\cos(d+ex)(a+b\sin(d+ex))^2}{3be}+\frac{\int(a+b}{}$$
$$=\frac{1}{2}a\left(a^2+4b^2\right)x+\frac{\left(a^4-8a^2b^2-3b^4\right)\cos(d+}{3be}$$

Mathematica [A] time = 0.29, size = 77, normalized size = 0.71

$$\frac{a\left(6\left(a^2+4b^2\right)(d+ex)-3\left(a^2+2b^2\right)\sin(2(d+ex))+ab\cos(3(d+ex))\right)-3b\left(11a^2+4b^2\right)\cos(d+ex)}{12e}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sin[d + e*x])*(b^2 + 2*a*b*Sin[d + e*x] + a^2*Sin[d + e*x]
^2),x]
```

```
[Out] (-3*b*(11*a^2 + 4*b^2)*Cos[d + e*x] + a*(6*(a^2 + 4*b^2)*(d + e*x) + a*b*Co
s[3*(d + e*x)] - 3*(a^2 + 2*b^2)*Sin[2*(d + e*x)]))/(12*e)
```

fricas [A] time = 0.74, size = 76, normalized size = 0.70

$$\frac{2\,a^2b\cos\left(ex+d\right)^3+3\left(a^3+4\,ab^2\right)ex-3\left(a^3+2\,ab^2\right)\cos\left(ex+d\right)\sin\left(ex+d\right)-6\left(3\,a^2b+b^3\right)\cos\left(ex+d\right)}{6\,e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(e*x+d))*(b^2+2*a*b*sin(e*x+d)+a^2*sin(e*x+d)^2),x, algor
ithm="fricas")
```

```
[Out] 1/6*(2*a^2*b*cos(e*x + d)^3 + 3*(a^3 + 4*a*b^2)*e*x - 3*(a^3 + 2*a*b^2)*cos
(e*x + d)*sin(e*x + d) - 6*(3*a^2*b + b^3)*cos(e*x + d))/e
```

giac [A] time = 0.15, size = 79, normalized size = 0.72

$$\frac{1}{12}\,a^2b\cos\left(3\,xe+3\,d\right)e^{(-1)}-\frac{1}{4}\left(11\,a^2b+4\,b^3\right)\cos\left(xe+d\right)e^{(-1)}-\frac{1}{4}\left(a^3+2\,ab^2\right)e^{(-1)}\sin\left(2\,xe+2\,d\right)+\frac{1}{2}\left(a^3+4\,ab^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(e*x+d))*(b^2+2*a*b*sin(e*x+d)+a^2*sin(e*x+d)^2),x, algor
ithm="giac")
```

```
[Out] 1/12*a^2*b*cos(3*x*e + 3*d)*e^(-1) - 1/4*(11*a^2*b + 4*b^3)*cos(x*e + d)*e^
(-1) - 1/4*(a^3 + 2*a*b^2)*e^(-1)*sin(2*x*e + 2*d) + 1/2*(a^3 + 4*a*b^2)*x
```

maple [A] time = 0.08, size = 115, normalized size = 1.06

$$\frac{-\frac{a^2b\left(2+\sin^2(ex+d)\right)\cos(ex+d)}{3} + a^3\left(-\frac{\sin(ex+d)\cos(ex+d)}{2} + \frac{ex}{2} + \frac{d}{2}\right) + 2a\,b^2\left(-\frac{\sin(ex+d)\cos(ex+d)}{2} + \frac{ex}{2} + \frac{d}{2}\right) - 2\cos(ex+d)}{e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(e*x+d))*(b^2+2*a*b*sin(e*x+d)+a^2*sin(e*x+d)^2),x)
```

```
[Out] 1/e*(-1/3*a^2*b*(2+sin(e*x+d)^2)*cos(e*x+d)+a^3*(-1/2*sin(e*x+d)*cos(e*x+d)
+1/2*e*x+1/2*d)+2*a*b^2*(-1/2*sin(e*x+d)*cos(e*x+d)+1/2*e*x+1/2*d)-2*cos(e*
x+d)*a^2*b-b^3*cos(e*x+d)+a*b^2*(e*x+d))
```

maxima [A] time = 0.32, size = 112, normalized size = 1.03

$$\frac{3\,(2\,ex + 2\,d - \sin(2\,ex + 2\,d))a^3 + 4\left(\cos(ex+d)^3 - 3\,\cos(ex+d)\right)a^2b + 6\,(2\,ex + 2\,d - \sin(2\,ex + 2\,d))ab^2 +}{12\,e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(e*x+d))*(b^2+2*a*b*sin(e*x+d)+a^2*sin(e*x+d)^2),x, algor
ithm="maxima")
```

```
[Out] 1/12*(3*(2*e*x + 2*d - sin(2*e*x + 2*d))*a^3 + 4*(cos(e*x + d)^3 - 3*cos(e*
x + d))*a^2*b + 6*(2*e*x + 2*d - sin(2*e*x + 2*d))*a*b^2 + 12*(e*x + d)*a*b
^2 - 24*a^2*b*cos(e*x + d) - 12*b^3*cos(e*x + d))/e
```

mupad [B] time = 2.90, size = 88, normalized size = 0.81

$$-\frac{6\,b^3\,\cos(d+e\,x) + \frac{3\,a^3\,\sin(2\,d+2\,e\,x)}{2} - \frac{a^2\,b\,\cos(3\,d+3\,e\,x)}{2} + 3\,a\,b^2\,\sin(2\,d+2\,e\,x) + \frac{33\,a^2\,b\,\cos(d+e\,x)}{2} - 3\,a^3\,e\,x - 12\,a\,b}{6\,e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(d + e*x))*(b^2 + a^2*sin(d + e*x)^2 + 2*a*b*sin(d + e*x)),x)
```

```
[Out] -(6*b^3*cos(d + e*x) + (3*a^3*sin(2*d + 2*e*x))/2 - (a^2*b*cos(3*d + 3*e*x)
)/2 + 3*a*b^2*sin(2*d + 2*e*x) + (33*a^2*b*cos(d + e*x))/2 - 3*a^3*e*x - 12
*a*b^2*e*x)/(6*e)
```

sympy [A] time = 0.67, size = 204, normalized size = 1.87

$$\begin{cases} \frac{a^3x\sin^2(d+ex)}{2} + \frac{a^3x\cos^2(d+ex)}{2} - \frac{a^3\sin(d+ex)\cos(d+ex)}{2e} - \frac{a^2b\sin^2(d+ex)\cos(d+ex)}{e} - \frac{2a^2b\cos^3(d+ex)}{3e} - \frac{2a^2b\cos(d+ex)}{e} + ab^2x\,\mathrm{si} \\ x\,(a+b\sin(d))\left(a^2\sin^2(d) + 2ab\sin(d) + b^2\right) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(e*x+d))*(b**2+2*a*b*sin(e*x+d)+a**2*sin(e*x+d)**2),x)
```

```
[Out] Piecewise((a**3*x*sin(d + e*x)**2/2 + a**3*x*cos(d + e*x)**2/2 - a**3*sin(d
 + e*x)*cos(d + e*x)/(2*e) - a**2*b*sin(d + e*x)**2*cos(d + e*x)/e - 2*a**2
*b*cos(d + e*x)**3/(3*e) - 2*a**2*b*cos(d + e*x)/e + a*b**2*x*sin(d + e*x)*
*2 + a*b**2*x*cos(d + e*x)**2 + a*b**2*x - a*b**2*sin(d + e*x)*cos(d + e*x)
/e - b**3*cos(d + e*x)/e, Ne(e, 0)), (x*(a + b*sin(d))*(a**2*sin(d)**2 + 2*
a*b*sin(d) + b**2), True))
```

3.501 $\int \frac{a+b\sin(d+ex)}{b^2+2ab\sin(d+ex)+a^2\sin^2(d+ex)}\,dx$

Optimal. Leaf size=23

$$-\frac{\cos(d+ex)}{e(a\sin(d+ex)+b)}$$

```
[Out] -cos(e*x+d)/e/(b+a*sin(e*x+d))
```

Rubi [A] time = 0.09, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 39, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {3288, 2754, 8}

$$-\frac{\cos(d+ex)}{e(a\sin(d+ex)+b)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[d + e*x])/(b^2 + 2*a*b*Sin[d + e*x] + a^2*Sin[d + e*x]^2),x]
```

```
[Out] -(Cos[d + e*x]/(e*(b + a*Sin[d + e*x])))
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2754

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e + f
*x])^(m + 1))/(f*(m + 1)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(a^2 - b^2)), I
nt[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m +
 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a
*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rule 3288

```
Int[((A_) + (B_.)*sin[(d_.) + (e_.)*(x_)])*((a_) + (b_.)*sin[(d_.) + (e_.)*
(x_)] + (c_.)*sin[(d_.) + (e_.)*(x_)]^2)^(n_), x_Symbol] :> Dist[1/(4^n*c^n
), Int[(A + B*Sin[d + e*x])*(b + 2*c*Sin[d + e*x])^(2*n), x], x] /; FreeQ[{
a, b, c, d, e, A, B}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[n]
```

Rubi steps

$$\int \frac{a+b\sin(d+ex)}{b^2+2ab\sin(d+ex)+a^2\sin^2(d+ex)}\,dx = \left(4a^2\right)\int \frac{a+b\sin(d+ex)}{\left(2ab+2a^2\sin(d+ex)\right)^2}\,dx$$
$$= -\frac{\cos(d+ex)}{e(b+a\sin(d+ex))}+\frac{\int 0\,dx}{a^2-b^2}$$
$$= -\frac{\cos(d+ex)}{e(b+a\sin(d+ex))}$$

Mathematica [A] time = 0.06, size = 23, normalized size = 1.00

$$-\frac{\cos(d+ex)}{e(a\sin(d+ex)+b)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sin[d + e*x])/(b^2 + 2*a*b*Sin[d + e*x] + a^2*Sin[d + e*x]
^2),x]
```

```
[Out] -(Cos[d + e*x]/(e*(b + a*Sin[d + e*x])))
```

fricas [A] time = 0.83, size = 23, normalized size = 1.00

$$-\frac{\cos(ex+d)}{ae\sin(ex+d)+be}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(e*x+d))/(b^2+2*a*b*sin(e*x+d)+a^2*sin(e*x+d)^2),x, algor
ithm="fricas")
```

```
[Out] -cos(e*x + d)/(a*e*sin(e*x + d) + b*e)
```

giac [B] time = 0.18, size = 52, normalized size = 2.26

$$-\frac{2\left(a\tan\left(\frac{1}{2}\,xe+\frac{1}{2}\,d\right)+b\right)e^{(-1)}}{\left(b\tan\left(\frac{1}{2}\,xe+\frac{1}{2}\,d\right)^2+2\,a\tan\left(\frac{1}{2}\,xe+\frac{1}{2}\,d\right)+b\right)b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(e*x+d))/(b^2+2*a*b*sin(e*x+d)+a^2*sin(e*x+d)^2),x, algor
ithm="giac")
```

```
[Out] -2*(a*tan(1/2*x*e + 1/2*d) + b)*e^(-1)/((b*tan(1/2*x*e + 1/2*d)^2 + 2*a*tan
(1/2*x*e + 1/2*d) + b)*b)
```

maple [B] time = 0.34, size = 52, normalized size = 2.26

$$\frac{-\frac{2a\tan\left(\frac{d}{2}+\frac{ex}{2}\right)}{b}-2}{e\left(b\left(\tan^2\left(\frac{d}{2}+\frac{ex}{2}\right)\right)+2a\tan\left(\frac{d}{2}+\frac{ex}{2}\right)+b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(e*x+d))/(b^2+2*a*b*sin(e*x+d)+a^2*sin(e*x+d)^2),x)
```

```
[Out] 2/e*(-a*tan(1/2*d+1/2*e*x)/b-1)/(b*tan(1/2*d+1/2*e*x)^2+2*a*tan(1/2*d+1/2*e
*x)+b)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(e*x+d))/(b^2+2*a*b*sin(e*x+d)+a^2*sin(e*x+d)^2),x, algor
ithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` for
more details)Is 4*a^2-4*b^2 positive or negative?
```

mupad [B] time = 2.84, size = 39, normalized size = 1.70

$$-\frac{a\,\sin\left(d+e\,x\right)+b\,\left(\cos\left(d+e\,x\right)+1\right)}{b\,e\,\left(b+a\,\sin\left(d+e\,x\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(d + e*x))/(b^2 + a^2*sin(d + e*x)^2 + 2*a*b*sin(d + e*x)),x)
```

```
[Out] -(a*sin(d + e*x) + b*(cos(d + e*x) + 1))/(b*e*(b + a*sin(d + e*x)))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(e*x+d))/(b**2+2*a*b*sin(e*x+d)+a**2*sin(e*x+d)**2),x)
```

```
[Out] Timed out
```

3.502 $\int \frac{a+b\sin(d+ex)}{\left(b^2+2ab\sin(d+ex)+a^2\sin^2(d+ex)\right)^2}\,dx$

Optimal. Leaf size=157

$$-\frac{\left(2a^2+b^2\right)\cos(d+ex)}{3e\left(a^2-b^2\right)^2(a\sin(d+ex)+b)}+\frac{b\cos(d+ex)}{3e\left(a^2-b^2\right)(a\sin(d+ex)+b)^2}+\frac{2ab\tanh^{-1}\left(\frac{a+b\tan\left(\frac{1}{2}(d+ex)\right)}{\sqrt{a^2-b^2}}\right)}{e\left(a^2-b^2\right)^{5/2}}-\frac{\cos(d+e}{3e(a\sin(d+ex}$$

```
[Out] 2*a*b*arctanh((a+b*tan(1/2*e*x+1/2*d))/(a^2-b^2)^(1/2))/(a^2-b^2)^(5/2)/e-1
/3*cos(e*x+d)/e/(b+a*sin(e*x+d))^3+1/3*b*cos(e*x+d)/(a^2-b^2)/e/(b+a*sin(e*
x+d))^2-1/3*(2*a^2+b^2)*cos(e*x+d)/(a^2-b^2)^2/e/(b+a*sin(e*x+d))
```

Rubi [A] time = 0.42, antiderivative size = 157, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 39, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {3288, 2754, 12, 2660, 618, 206}

$$-\frac{\left(2a^2+b^2\right)\cos(d+ex)}{3e\left(a^2-b^2\right)^2(a\sin(d+ex)+b)}+\frac{b\cos(d+ex)}{3e\left(a^2-b^2\right)(a\sin(d+ex)+b)^2}+\frac{2ab\tanh^{-1}\left(\frac{a+b\tan\left(\frac{1}{2}(d+ex)\right)}{\sqrt{a^2-b^2}}\right)}{e\left(a^2-b^2\right)^{5/2}}-\frac{\cos(d+e}{3e(a\sin(d+ex}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[d + e*x])/(b^2 + 2*a*b*Sin[d + e*x] + a^2*Sin[d + e*x]^2)^2,
x]
```

```
[Out] (2*a*b*ArcTanh[(a + b*Tan[(d + e*x)/2])/Sqrt[a^2 - b^2]])/((a^2 - b^2)^(5/2
)*e) - Cos[d + e*x]/(3*e*(b + a*Sin[d + e*x])^3) + (b*Cos[d + e*x])/(3*(a^2
 - b^2)*e*(b + a*Sin[d + e*x])^2) - ((2*a^2 + b^2)*Cos[d + e*x])/(3*(a^2 -
b^2)^2*e*(b + a*Sin[d + e*x]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2754

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e + f
*x])^(m + 1))/(f*(m + 1)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(a^2 - b^2)), I
nt[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m +
 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a
*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rule 3288

```
Int[((A_) + (B_.)*sin[(d_.) + (e_.)*(x_)])*((a_) + (b_.)*sin[(d_.) + (e_.)*
(x_)] + (c_.)*sin[(d_.) + (e_.)*(x_)]^2)^(n_), x_Symbol] :> Dist[1/(4^n*c^n
), Int[(A + B*Sin[d + e*x])*(b + 2*c*Sin[d + e*x])^(2*n), x], x] /; FreeQ[{
a, b, c, d, e, A, B}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[n]
```

Rubi steps

$$\int \frac{a+b\sin(d+ex)}{\left(b^2+2ab\sin(d+ex)+a^2\sin^2(d+ex)\right)^2}\,dx = \left(16a^4\right)\int \frac{a+b\sin(d+ex)}{\left(2ab+2a^2\sin(d+ex)\right)^4}\,dx$$

$$= -\frac{\cos(d+ex)}{3e(b+a\sin(d+ex))^3} + \frac{\left(4a^2\right)\int \frac{4a\left(a^2-b^2\right)\sin(d+ex)}{\left(2ab+2a^2\sin(d+ex)\right)^3}\,dx}{3\left(a^2-b^2\right)}$$

$$= -\frac{\cos(d+ex)}{3e(b+a\sin(d+ex))^3} + \frac{1}{3}\left(16a^3\right)\int \frac{\sin(d+ex)}{\left(2ab+2a^2\sin(d+ex)\right)^3}$$

$$= -\frac{\cos(d+ex)}{3e(b+a\sin(d+ex))^3} + \frac{b\cos(d+ex)}{3\left(a^2-b^2\right)e(b+a\sin(d+ex))^2} + \text{—}$$

$$= -\frac{\cos(d+ex)}{3e(b+a\sin(d+ex))^3} + \frac{b\cos(d+ex)}{3\left(a^2-b^2\right)e(b+a\sin(d+ex))^2} - \frac{}{3}$$

$$= -\frac{\cos(d+ex)}{3e(b+a\sin(d+ex))^3} + \frac{b\cos(d+ex)}{3\left(a^2-b^2\right)e(b+a\sin(d+ex))^2} - \frac{}{3}$$

$$= -\frac{\cos(d+ex)}{3e(b+a\sin(d+ex))^3} + \frac{b\cos(d+ex)}{3\left(a^2-b^2\right)e(b+a\sin(d+ex))^2} - \frac{}{3}$$

$$= -\frac{\cos(d+ex)}{3e(b+a\sin(d+ex))^3} + \frac{b\cos(d+ex)}{3\left(a^2-b^2\right)e(b+a\sin(d+ex))^2} - \frac{}{3}$$

$$= \frac{2ab\tanh^{-1}\left(\frac{a+b\tan\left(\frac{1}{2}(d+ex)\right)}{\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{5/2}e} - \frac{\cos(d+ex)}{3e(b+a\sin(d+ex))^3} + \frac{}{3\left(a^2-\right.}$$

Mathematica [A] time = 0.96, size = 140, normalized size = 0.89

$$-\frac{\frac{6ab\tan^{-1}\left(\frac{a+b\tan\left(\frac{1}{2}(d+ex)\right)}{\sqrt{b^2-a^2}}\right)}{\left(b^2-a^2\right)^{5/2}} + \frac{\cos(d+ex)\left(a^4+a^2\left(2a^2+b^2\right)\sin^2(d+ex)+3ab\left(a^2+b^2\right)\sin(d+ex)-a^2b^2+3b^4\right)}{(a-b)^2(a+b)^2(a\sin(d+ex)+b)^3}}{3e}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sin[d + e*x])/(b^2 + 2*a*b*Sin[d + e*x] + a^2*Sin[d + e*x]
^2)^2,x]
```

```
[Out] -1/3*((6*a*b*ArcTan[(a + b*Tan[(d + e*x)/2])/Sqrt[-a^2 + b^2]])/(-a^2 + b^2
)^(5/2) + (Cos[d + e*x]*(a^4 - a^2*b^2 + 3*b^4 + 3*a*b*(a^2 + b^2)*Sin[d +
e*x] + a^2*(2*a^2 + b^2)*Sin[d + e*x]^2))/((a - b)^2*(a + b)^2*(b + a*Sin[d
 + e*x])^3))/e
```

fricas [B] time = 1.98, size = 795, normalized size = 5.06

$$\left[-\frac{2\left(2\,a^6-a^4b^2-a^2b^4\right)\cos\left(ex+d\right)^3-6\left(a^5b-ab^5\right)\cos\left(ex+d\right)\sin\left(ex+d\right)-3\left(3\,a^3b^2\cos\left(ex+d\right)^2-3\,a^3b^2\right.}{6\left(3\left(a^8b-3\,a^6b^3+3\,a^4b^5-a^2b^7\right)e\cos\left(ex+d\right)^2-\left(\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(e*x+d))/(b^2+2*a*b*sin(e*x+d)+a^2*sin(e*x+d)^2)^2,x, alg
orithm="fricas")
```

```
[Out] [-1/6*(2*(2*a^6 - a^4*b^2 - a^2*b^4)*cos(e*x + d)^3 - 6*(a^5*b - a*b^5)*cos
(e*x + d)*sin(e*x + d) - 3*(3*a^3*b^2*cos(e*x + d)^2 - 3*a^3*b^2 - a*b^4 +
(a^4*b*cos(e*x + d)^2 - a^4*b - 3*a^2*b^3)*sin(e*x + d))*sqrt(a^2 - b^2)*lo
g(((a^2 - 2*b^2)*cos(e*x + d)^2 + 2*a*b*sin(e*x + d) + a^2 + b^2 + 2*(b*cos
(e*x + d)*sin(e*x + d) + a*cos(e*x + d))*sqrt(a^2 - b^2))/(a^2*cos(e*x + d)
^2 - 2*a*b*sin(e*x + d) - a^2 - b^2)) - 6*(a^6 - a^4*b^2 + a^2*b^4 - b^6)*c
os(e*x + d))/(3*(a^8*b - 3*a^6*b^3 + 3*a^4*b^5 - a^2*b^7)*e*cos(e*x + d)^2
- (3*a^8*b - 8*a^6*b^3 + 6*a^4*b^5 - b^9)*e + ((a^9 - 3*a^7*b^2 + 3*a^5*b^4
 - a^3*b^6)*e*cos(e*x + d)^2 - (a^9 - 6*a^5*b^4 + 8*a^3*b^6 - 3*a*b^8)*e)*s
in(e*x + d)), -1/3*((2*a^6 - a^4*b^2 - a^2*b^4)*cos(e*x + d)^3 - 3*(a^5*b -
 a*b^5)*cos(e*x + d)*sin(e*x + d) - 3*(3*a^3*b^2*cos(e*x + d)^2 - 3*a^3*b^2
 - a*b^4 + (a^4*b*cos(e*x + d)^2 - a^4*b - 3*a^2*b^3)*sin(e*x + d))*sqrt(-a
^2 + b^2)*arctan(-sqrt(-a^2 + b^2)*(b*sin(e*x + d) + a)/((a^2 - b^2)*cos(e*
x + d))) - 3*(a^6 - a^4*b^2 + a^2*b^4 - b^6)*cos(e*x + d))/(3*(a^8*b - 3*a^
6*b^3 + 3*a^4*b^5 - a^2*b^7)*e*cos(e*x + d)^2 - (3*a^8*b - 8*a^6*b^3 + 6*a^
4*b^5 - b^9)*e + ((a^9 - 3*a^7*b^2 + 3*a^5*b^4 - a^3*b^6)*e*cos(e*x + d)^2
- (a^9 - 6*a^5*b^4 + 8*a^3*b^6 - 3*a*b^8)*e)*sin(e*x + d))]
```

giac [B] time = 0.25, size = 454, normalized size = 2.89

$$-\frac{2}{3}\left(\frac{3\left(\pi\left\lfloor\frac{xe+d}{2\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}\left(b\right)+\arctan\left(\frac{b\tan\left(\frac{1}{2}\,xe+\frac{1}{2}\,d\right)+a}{\sqrt{-a^2+b^2}}\right)\right)ab}{\left(a^4-2\,a^2b^2+b^4\right)\sqrt{-a^2+b^2}}+\frac{3\,a^5b^2\tan\left(\frac{1}{2}\,xe+\frac{1}{2}\,d\right)^5-6\,a^3b^4\tan\left(\frac{1}{2}\,xe+\frac{1}{2}\,d\right)^5\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(e*x+d))/(b^2+2*a*b*sin(e*x+d)+a^2*sin(e*x+d)^2)^2,x, alg
orithm="giac")
```

```
[Out] -2/3*(3*(pi*floor(1/2*(x*e + d)/pi + 1/2)*sgn(b) + arctan((b*tan(1/2*x*e +
1/2*d) + a)/sqrt(-a^2 + b^2)))*a*b/((a^4 - 2*a^2*b^2 + b^4)*sqrt(-a^2 + b^2
)) + (3*a^5*b^2*tan(1/2*x*e + 1/2*d)^5 - 6*a^3*b^4*tan(1/2*x*e + 1/2*d)^5 +
 6*a*b^6*tan(1/2*x*e + 1/2*d)^5 + 6*a^6*b*tan(1/2*x*e + 1/2*d)^4 - 9*a^4*b^
3*tan(1/2*x*e + 1/2*d)^4 + 15*a^2*b^5*tan(1/2*x*e + 1/2*d)^4 + 3*b^7*tan(1/
2*x*e + 1/2*d)^4 + 4*a^7*tan(1/2*x*e + 1/2*d)^3 + 2*a^5*b^2*tan(1/2*x*e + 1
/2*d)^3 + 6*a^3*b^4*tan(1/2*x*e + 1/2*d)^3 + 18*a*b^6*tan(1/2*x*e + 1/2*d)^
3 + 6*a^6*b*tan(1/2*x*e + 1/2*d)^2 + 18*a^2*b^5*tan(1/2*x*e + 1/2*d)^2 + 6*
b^7*tan(1/2*x*e + 1/2*d)^2 + 3*a^5*b^2*tan(1/2*x*e + 1/2*d) + 12*a*b^6*tan(
1/2*x*e + 1/2*d) + a^4*b^3 - a^2*b^5 + 3*b^7)/((a^4*b^3 - 2*a^2*b^5 + b^7)*
(b*tan(1/2*x*e + 1/2*d)^2 + 2*a*tan(1/2*x*e + 1/2*d) + b)^3))*e^(-1)
```

maple [B] time = 0.33, size = 1297, normalized size = 8.26

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(e*x+d))/(b^2+2*a*b*sin(e*x+d)+a^2*sin(e*x+d)^2)^2,x)
```

```
[Out] -2/e/(b*tan(1/2*d+1/2*e*x)^2+2*a*tan(1/2*d+1/2*e*x)+b)^3*a^5/b/(a^4-2*a^2*b
^2+b^4)*tan(1/2*d+1/2*e*x)^5+4/e/(b*tan(1/2*d+1/2*e*x)^2+2*a*tan(1/2*d+1/2*
e*x)+b)^3*a^3*b/(a^4-2*a^2*b^2+b^4)*tan(1/2*d+1/2*e*x)^5-4/e/(b*tan(1/2*d+1
/2*e*x)^2+2*a*tan(1/2*d+1/2*e*x)+b)^3*a*b^3/(a^4-2*a^2*b^2+b^4)*tan(1/2*d+1
/2*e*x)^5-4/e/(b*tan(1/2*d+1/2*e*x)^2+2*a*tan(1/2*d+1/2*e*x)+b)^3/b^2/(a^4-
2*a^2*b^2+b^4)*tan(1/2*d+1/2*e*x)^4*a^6+6/e/(b*tan(1/2*d+1/2*e*x)^2+2*a*tan
(1/2*d+1/2*e*x)+b)^3/(a^4-2*a^2*b^2+b^4)*tan(1/2*d+1/2*e*x)^4*a^4-10/e/(b*t
an(1/2*d+1/2*e*x)^2+2*a*tan(1/2*d+1/2*e*x)+b)^3*b^2/(a^4-2*a^2*b^2+b^4)*tan
(1/2*d+1/2*e*x)^4*a^2-2/e/(b*tan(1/2*d+1/2*e*x)^2+2*a*tan(1/2*d+1/2*e*x)+b)
^3*b^4/(a^4-2*a^2*b^2+b^4)*tan(1/2*d+1/2*e*x)^4-8/3/e/(b*tan(1/2*d+1/2*e*x)
^2+2*a*tan(1/2*d+1/2*e*x)+b)^3*a^7/b^3/(a^4-2*a^2*b^2+b^4)*tan(1/2*d+1/2*e*
x)^3-4/3/e/(b*tan(1/2*d+1/2*e*x)^2+2*a*tan(1/2*d+1/2*e*x)+b)^3*a^5/b/(a^4-2
*a^2*b^2+b^4)*tan(1/2*d+1/2*e*x)^3-4/e/(b*tan(1/2*d+1/2*e*x)^2+2*a*tan(1/2*
d+1/2*e*x)+b)^3*a^3*b/(a^4-2*a^2*b^2+b^4)*tan(1/2*d+1/2*e*x)^3-12/e/(b*tan(
1/2*d+1/2*e*x)^2+2*a*tan(1/2*d+1/2*e*x)+b)^3*a*b^3/(a^4-2*a^2*b^2+b^4)*tan(
1/2*d+1/2*e*x)^3-4/e/(b*tan(1/2*d+1/2*e*x)^2+2*a*tan(1/2*d+1/2*e*x)+b)^3/b^
2/(a^4-2*a^2*b^2+b^4)*tan(1/2*d+1/2*e*x)^2*a^6-12/e/(b*tan(1/2*d+1/2*e*x)^2
+2*a*tan(1/2*d+1/2*e*x)+b)^3*b^2/(a^4-2*a^2*b^2+b^4)*tan(1/2*d+1/2*e*x)^2*a
^2-4/e/(b*tan(1/2*d+1/2*e*x)^2+2*a*tan(1/2*d+1/2*e*x)+b)^3*b^4/(a^4-2*a^2*b
^2+b^4)*tan(1/2*d+1/2*e*x)^2-2/e/(b*tan(1/2*d+1/2*e*x)^2+2*a*tan(1/2*d+1/2*
e*x)+b)^3*a^5/b/(a^4-2*a^2*b^2+b^4)*tan(1/2*d+1/2*e*x)-8/e/(b*tan(1/2*d+1/2
*e*x)^2+2*a*tan(1/2*d+1/2*e*x)+b)^3*a*b^3/(a^4-2*a^2*b^2+b^4)*tan(1/2*d+1/2
*e*x)-2/3/e/(b*tan(1/2*d+1/2*e*x)^2+2*a*tan(1/2*d+1/2*e*x)+b)^3/(a^4-2*a^2*
```

```
b^2+b^4)*a^4+2/3/e/(b*tan(1/2*d+1/2*e*x)^2+2*a*tan(1/2*d+1/2*e*x)+b)^3/(a^4
-2*a^2*b^2+b^4)*a^2*b^2-2/e/(b*tan(1/2*d+1/2*e*x)^2+2*a*tan(1/2*d+1/2*e*x)+
b)^3/(a^4-2*a^2*b^2+b^4)*b^4-2/e*a*b/(a^4-2*a^2*b^2+b^4)/(-a^2+b^2)^(1/2)*a
rctan(1/2*(2*b*tan(1/2*d+1/2*e*x)+2*a)/(-a^2+b^2)^(1/2))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(e*x+d))/(b^2+2*a*b*sin(e*x+d)+a^2*sin(e*x+d)^2)^2,x, alg
orithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` for
more details)Is 4*a^2-4*b^2 positive or negative?
```

mupad [B] time = 6.06, size = 497, normalized size = 3.17

$$\frac{2\,a\,b\,\mathrm{atanh}\left(\frac{\left(2\,a+2\,b\,\tan\left(\frac{d}{2}+\frac{e\,x}{2}\right)\right)\left(a^4-2\,a^2\,b^2+b^4\right)}{2\,{\left(a+b\right)}^{5/2}\,{\left(a-b\right)}^{5/2}}\right)}{e\,{\left(a+b\right)}^{5/2}\,{\left(a-b\right)}^{5/2}}-\frac{\frac{2\,\left(a^4-a^2\,b^2+3\,b^4\right)}{3\,\left(a^4-2\,a^2\,b^2+b^4\right)}+\frac{4\,\tan\left(\frac{d}{2}+\frac{e\,x}{2}\right)^2\,\left(a^6+3\,a^2\,b^4+b^6\right)}{b^2\,\left(a^4-2\,a^2\,b^2+b^4\right)}+\frac{2\,\tan\left(\frac{d}{2}+\frac{e\,x}{2}\right)^4\,\left(2\,a^6-3\,a^4\,b^2\dots\right.}{b^2\,\left(a^4-2\,a^2\,b^2+b\dots\right.}}{e\,\left(b^3\,\tan\left(\frac{d}{2}+\frac{e\,x}{2}\right)^6+\tan\left(\frac{d}{2}+\frac{e\,x}{2}\right)^3\,\left(8\,a^3+12\,a\,b^2\right)+\tan\left(\frac{d}{2}+\frac{e\,x}{2}\right)\dots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(d + e*x))/(b^2 + a^2*sin(d + e*x)^2 + 2*a*b*sin(d + e*x))^2,
x)
```

```
[Out] (2*a*b*atanh(((2*a + 2*b*tan(d/2 + (e*x)/2))*(a^4 + b^4 - 2*a^2*b^2))/(2*(a
 + b)^(5/2)*(a - b)^(5/2))))/(e*(a + b)^(5/2)*(a - b)^(5/2)) - ((2*(a^4 + 3
*b^4 - a^2*b^2))/(3*(a^4 + b^4 - 2*a^2*b^2)) + (4*tan(d/2 + (e*x)/2)^2*(a^6
 + b^6 + 3*a^2*b^4))/(b^2*(a^4 + b^4 - 2*a^2*b^2)) + (2*tan(d/2 + (e*x)/2)^
4*(2*a^6 + b^6 + 5*a^2*b^4 - 3*a^4*b^2))/(b^2*(a^4 + b^4 - 2*a^2*b^2)) + (2
*a*tan(d/2 + (e*x)/2)*(a^4 + 4*b^4))/(b*(a^4 + b^4 - 2*a^2*b^2)) + (2*a*tan
(d/2 + (e*x)/2)^5*(a^4 + 2*b^4 - 2*a^2*b^2))/(b*(a^4 + b^4 - 2*a^2*b^2)) +
(4*a*tan(d/2 + (e*x)/2)^3*(2*a^2 + 3*b^2)*(a^4 + 3*b^4 - a^2*b^2))/(3*b^3*(
a^4 + b^4 - 2*a^2*b^2)))/(e*(b^3*tan(d/2 + (e*x)/2)^6 + tan(d/2 + (e*x)/2)^
3*(12*a*b^2 + 8*a^3) + tan(d/2 + (e*x)/2)^2*(12*a^2*b + 3*b^3) + tan(d/2 +
(e*x)/2)^4*(12*a^2*b + 3*b^3) + b^3 + 6*a*b^2*tan(d/2 + (e*x)/2) + 6*a*b^2*
tan(d/2 + (e*x)/2)^5))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(e*x+d))/(b**2+2*a*b*sin(e*x+d)+a**2*sin(e*x+d)**2)**2,x)
[Out] Timed out
```

3.503 $\int \frac{d+e\sin(x)}{a+b\sin(x)+c\sin^2(x)}\,dx$

Optimal. Leaf size=242

$$\frac{\sqrt{2}\left(\frac{2cd-be}{\sqrt{b^2-4ac}}+e\right)\tan^{-1}\left(\frac{\tan\left(\frac{x}{2}\right)\left(b-\sqrt{b^2-4ac}\right)+2c}{\sqrt{2}\sqrt{-b\sqrt{b^2-4ac}-2c(a+c)+b^2}}\right)}{\sqrt{-b\sqrt{b^2-4ac}-2c(a+c)+b^2}}+\frac{\sqrt{2}\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\tan\left(\frac{x}{2}\right)\left(\sqrt{b^2-4ac}+b\right)+2c}{\sqrt{2}\sqrt{b\sqrt{b^2-4ac}-2c(a+c)+b^2}}\right)}{\sqrt{b\sqrt{b^2-4ac}-2c(a+c)+b^2}}$$

```
[Out] arctan(1/2*(2*c+(b-(-4*a*c+b^2)^(1/2))*tan(1/2*x))*2^(1/2)/(b^2-2*c*(a+c)-b
*(-4*a*c+b^2)^(1/2))^(1/2))*2^(1/2)*(e+(-b*e+2*c*d)/(-4*a*c+b^2)^(1/2))/(b^
2-2*c*(a+c)-b*(-4*a*c+b^2)^(1/2))^(1/2)+arctan(1/2*(2*c+(b+(-4*a*c+b^2)^(1/
2))*tan(1/2*x))*2^(1/2)/(b^2-2*c*(a+c)+b*(-4*a*c+b^2)^(1/2))^(1/2))*2^(1/2)
*(e+(b*e-2*c*d)/(-4*a*c+b^2)^(1/2))/(b^2-2*c*(a+c)+b*(-4*a*c+b^2)^(1/2))^(1
/2)
```

Rubi [A] time = 0.94, antiderivative size = 242, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {3292, 2660, 618, 204}

$$\frac{\sqrt{2}\left(\frac{2cd-be}{\sqrt{b^2-4ac}}+e\right)\tan^{-1}\left(\frac{\tan\left(\frac{x}{2}\right)\left(b-\sqrt{b^2-4ac}\right)+2c}{\sqrt{2}\sqrt{-b\sqrt{b^2-4ac}-2c(a+c)+b^2}}\right)}{\sqrt{-b\sqrt{b^2-4ac}-2c(a+c)+b^2}}+\frac{\sqrt{2}\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\tan\left(\frac{x}{2}\right)\left(\sqrt{b^2-4ac}+b\right)+2c}{\sqrt{2}\sqrt{b\sqrt{b^2-4ac}-2c(a+c)+b^2}}\right)}{\sqrt{b\sqrt{b^2-4ac}-2c(a+c)+b^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*Sin[x])/(a + b*Sin[x] + c*Sin[x]^2),x]
```

```
[Out] (Sqrt[2]*(e + (2*c*d - b*e)/Sqrt[b^2 - 4*a*c])*ArcTan[(2*c + (b - Sqrt[b^2
- 4*a*c])*Tan[x/2])/(Sqrt[2]*Sqrt[b^2 - 2*c*(a + c) - b*Sqrt[b^2 - 4*a*c]])
])/Sqrt[b^2 - 2*c*(a + c) - b*Sqrt[b^2 - 4*a*c]] + (Sqrt[2]*(e - (2*c*d - b
*e)/Sqrt[b^2 - 4*a*c])*ArcTan[(2*c + (b + Sqrt[b^2 - 4*a*c])*Tan[x/2])/(Sqr
t[2]*Sqrt[b^2 - 2*c*(a + c) + b*Sqrt[b^2 - 4*a*c]])])/Sqrt[b^2 - 2*c*(a + c
) + b*Sqrt[b^2 - 4*a*c]]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 3292

```
Int[((A_) + (B_.)*sin[(d_.) + (e_.)*(x_)])/((a_.) + (b_.)*sin[(d_.) + (e_.)
*(x_)] + (c_.)*sin[(d_.) + (e_.)*(x_)]^2), x_Symbol] :> Module[{q = Rt[b^2
- 4*a*c, 2]}, Dist[B + (b*B - 2*A*c)/q, Int[1/(b + q + 2*c*Sin[d + e*x]), x
], x] + Dist[B - (b*B - 2*A*c)/q, Int[1/(b - q + 2*c*Sin[d + e*x]), x], x]]
 /; FreeQ[{a, b, c, d, e, A, B}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{d+e\sin(x)}{a+b\sin(x)+c\sin^2(x)}\,dx = \left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\int \frac{1}{b+\sqrt{b^2-4ac}+2c\sin(x)}\,dx + \left(e+\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\int \frac{}{b-\sqrt{b^2}}$$

$$= \left(2\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\right)\text{Subst}\left(\int \frac{1}{b+\sqrt{b^2-4ac}+4cx+\left(b+\sqrt{b^2-4ac}\right)x^2}\,dx, x,\right.$$

$$= -\left(\left(4\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\right)\text{Subst}\left(\int \frac{1}{4\left(4c^2-\left(b+\sqrt{b^2-4ac}\right)^2\right)-x^2}\,dx, x, 4c+2\right.\right.$$

$$= \frac{\sqrt{2}\left(e+\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{2c+\left(b-\sqrt{b^2-4ac}\right)\tan\left(\frac{x}{2}\right)}{\sqrt{2}\sqrt{b^2-2c(a+c)-b\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-2c(a+c)-b\sqrt{b^2-4ac}}} + \frac{\sqrt{2}\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{2c}{\sqrt{2}}\right.}{\sqrt{b^2-2c(a+c)+b}}$$

Mathematica [C] time = 0.77, size = 286, normalized size = 1.18

$$\frac{\frac{\left(e\left(\sqrt{4ac-b^2}+ib\right)-2icd\right)\tan^{-1}\left(\frac{2c+\tan\left(\frac{x}{2}\right)\left(b-i\sqrt{4ac-b^2}\right)}{\sqrt{2}\sqrt{-ib\sqrt{4ac-b^2}-2c(a+c)+b^2}}\right)}{\sqrt{-ib\sqrt{4ac-b^2}-2c(a+c)+b^2}}+\frac{\left(e\left(\sqrt{4ac-b^2}-ib\right)+2icd\right)\tan^{-1}\left(\frac{2c+\tan\left(\frac{x}{2}\right)\left(b+i\sqrt{4ac-b^2}\right)}{\sqrt{2}\sqrt{ib\sqrt{4ac-b^2}-2c(a+c)+b^2}}\right)}{\sqrt{ib\sqrt{4ac-b^2}-2c(a+c)+b^2}}}{\sqrt{2ac-\frac{b^2}{2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*Sin[x])/(a + b*Sin[x] + c*Sin[x]^2),x]
```

```
[Out] ((((-2*I)*c*d + (I*b + Sqrt[-b^2 + 4*a*c])*e)*ArcTan[(2*c + (b - I*Sqrt[-b^
2 + 4*a*c])*Tan[x/2])/(Sqrt[2]*Sqrt[b^2 - 2*c*(a + c) - I*b*Sqrt[-b^2 + 4*a
*c]])])/Sqrt[b^2 - 2*c*(a + c) - I*b*Sqrt[-b^2 + 4*a*c]] + (((2*I)*c*d + ((
-I)*b + Sqrt[-b^2 + 4*a*c])*e)*ArcTan[(2*c + (b + I*Sqrt[-b^2 + 4*a*c])*Tan
[x/2])/(Sqrt[2]*Sqrt[b^2 - 2*c*(a + c) + I*b*Sqrt[-b^2 + 4*a*c]])])/Sqrt[b^
2 - 2*c*(a + c) + I*b*Sqrt[-b^2 + 4*a*c]])/Sqrt[-1/2*b^2 + 2*a*c]
```

fricas [B] time = 18.38, size = 6695, normalized size = 27.67

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*sin(x))/(a+b*sin(x)+c*sin(x)^2),x, algorithm="fricas")
```

```
[Out] 1/4*sqrt(2)*sqrt(-((b^2 - 2*a*c - 2*c^2)*d^2 - 2*(a*b - b*c)*d*e + (2*a^2 -
 b^2 + 2*a*c)*e^2 + (a^2*b^2 - b^4 - 4*a*c^3 - (8*a^2 - b^2)*c^2 - 2*(2*a^3
 - 3*a*b^2)*c)*sqrt((b^2*d^4 + b^2*e^4 - 4*(a*b + b*c)*d^3*e + 2*(2*a^2 + b
^2 + 4*a*c + 2*c^2)*d^2*e^2 - 4*(a*b + b*c)*d*e^3)/(a^4*b^2 - 2*a^2*b^4 + b
^6 - 4*a*c^5 - (16*a^2 - b^2)*c^4 - 12*(2*a^3 - a*b^2)*c^3 - 2*(8*a^4 - 11*
a^2*b^2 + b^4)*c^2 - 4*(a^5 - 3*a^3*b^2 + 2*a*b^4)*c)))/(a^2*b^2 - b^4 - 4*
a*c^3 - (8*a^2 - b^2)*c^2 - 2*(2*a^3 - 3*a*b^2)*c))*log(4*b*c^2*d^4 + 4*a*b
*c*e^4 - 4*(b^2*c + 2*a*c^2 + 2*c^3)*d^3*e + 12*(a*b*c + b*c^2)*d^2*e^2 - 4
*(2*a*c^2 + (2*a^2 + b^2)*c)*d*e^3 + 2*((4*a*c^4 + (8*a^2 - b^2)*c^3 + 2*(2
*a^3 - 3*a*b^2)*c^2 - (a^2*b^2 - b^4)*c)*d^2 + (a^2*b^3 - b^5 - 4*a*b*c^3 -
 (8*a^2*b - b^3)*c^2 - 2*(2*a^3*b - 3*a*b^3)*c)*d*e - (a^3*b^2 - a*b^4 - 4*
a^2*c^3 - (8*a^3 - a*b^2)*c^2 - 2*(2*a^4 - 3*a^2*b^2)*c)*e^2)*sqrt((b^2*d^4
 + b^2*e^4 - 4*(a*b + b*c)*d^3*e + 2*(2*a^2 + b^2 + 4*a*c + 2*c^2)*d^2*e^2
- 4*(a*b + b*c)*d*e^3)/(a^4*b^2 - 2*a^2*b^4 + b^6 - 4*a*c^5 - (16*a^2 - b^2
)*c^4 - 12*(2*a^3 - a*b^2)*c^3 - 2*(8*a^4 - 11*a^2*b^2 + b^4)*c^2 - 4*(a^5
- 3*a^3*b^2 + 2*a*b^4)*c))*sin(x) + sqrt(2)*(((a^2*b^4 - b^6 + 8*a*c^5 + 2*
(12*a^2 - b^2)*c^4 + 6*(4*a^3 - 3*a*b^2)*c^3 + (8*a^4 - 22*a^2*b^2 + 3*b^4)
*c^2 - 2*(3*a^3*b^2 - 4*a*b^4)*c)*d - (a^3*b^3 - a*b^5 + 4*a*b*c^4 + (4*a^2
*b - b^3)*c^3 - (4*a^3*b + 5*a*b^3)*c^2 - (4*a^4*b - 5*a^2*b^3 - b^5)*c)*e)
```

*sqrt((b^2*d^4 + b^2*e^4 - 4*(a*b + b*c)*d^3*e + 2*(2*a^2 + b^2 + 4*a*c + 2
*c^2)*d^2*e^2 - 4*(a*b + b*c)*d*e^3)/(a^4*b^2 - 2*a^2*b^4 + b^6 - 4*a*c^5 -
 (16*a^2 - b^2)*c^4 - 12*(2*a^3 - a*b^2)*c^3 - 2*(8*a^4 - 11*a^2*b^2 + b^4)
c^2 - 4(a^5 - 3*a^3*b^2 + 2*a*b^4)*c))*cos(x) - ((b^4 - 4*a*b^2*c)*d^3 -
3*(a*b^3 - 4*a*b*c^2 - (4*a^2*b - b^3)*c)*d^2*e + (2*a^2*b^2 + b^4 - 8*a^3*
c - 8*a*c^3 - 2*(8*a^2 - b^2)*c^2)*d*e^2 - (a*b^3 - 4*a*b*c^2 - (4*a^2*b -
b^3)*c)*e^3)*cos(x))*sqrt(-((b^2 - 2*a*c - 2*c^2)*d^2 - 2*(a*b - b*c)*d*e +
 (2*a^2 - b^2 + 2*a*c)*e^2 + (a^2*b^2 - b^4 - 4*a*c^3 - (8*a^2 - b^2)*c^2 -
 2*(2*a^3 - 3*a*b^2)*c)*sqrt((b^2*d^4 + b^2*e^4 - 4*(a*b + b*c)*d^3*e + 2*(
2*a^2 + b^2 + 4*a*c + 2*c^2)*d^2*e^2 - 4*(a*b + b*c)*d*e^3)/(a^4*b^2 - 2*a^
2*b^4 + b^6 - 4*a*c^5 - (16*a^2 - b^2)*c^4 - 12*(2*a^3 - a*b^2)*c^3 - 2*(8*
a^4 - 11*a^2*b^2 + b^4)*c^2 - 4*(a^5 - 3*a^3*b^2 + 2*a*b^4)*c)))/(a^2*b^2 -
 b^4 - 4*a*c^3 - (8*a^2 - b^2)*c^2 - 2*(2*a^3 - 3*a*b^2)*c)) + 2*(b^2*c*d^4
 + a*b^2*e^4 - (b^3 + 2*a*b*c + 2*b*c^2)*d^3*e + 3*(a*b^2 + b^2*c)*d^2*e^2
- (2*a^2*b + b^3 + 2*a*b*c)*d*e^3)*sin(x)) - 1/4*sqrt(2)*sqrt(-((b^2 - 2*a*
c - 2*c^2)*d^2 - 2*(a*b - b*c)*d*e + (2*a^2 - b^2 + 2*a*c)*e^2 - (a^2*b^2 -
 b^4 - 4*a*c^3 - (8*a^2 - b^2)*c^2 - 2*(2*a^3 - 3*a*b^2)*c)*sqrt((b^2*d^4 +
 b^2*e^4 - 4*(a*b + b*c)*d^3*e + 2*(2*a^2 + b^2 + 4*a*c + 2*c^2)*d^2*e^2 -
4*(a*b + b*c)*d*e^3)/(a^4*b^2 - 2*a^2*b^4 + b^6 - 4*a*c^5 - (16*a^2 - b^2)*
c^4 - 12*(2*a^3 - a*b^2)*c^3 - 2*(8*a^4 - 11*a^2*b^2 + b^4)*c^2 - 4*(a^5 -
3*a^3*b^2 + 2*a*b^4)*c)))/(a^2*b^2 - b^4 - 4*a*c^3 - (8*a^2 - b^2)*c^2 - 2*
(2*a^3 - 3*a*b^2)*c))*log(4*b*c^2*d^4 + 4*a*b*c*e^4 - 4*(b^2*c + 2*a*c^2 +
2*c^3)*d^3*e + 12*(a*b*c + b*c^2)*d^2*e^2 - 4*(2*a*c^2 + (2*a^2 + b^2)*c)*d
e^3 - 2((4*a*c^4 + (8*a^2 - b^2)*c^3 + 2*(2*a^3 - 3*a*b^2)*c^2 - (a^2*b^2
 - b^4)*c)*d^2 + (a^2*b^3 - b^5 - 4*a*b*c^3 - (8*a^2*b - b^3)*c^2 - 2*(2*a^
3*b - 3*a*b^3)*c)*d*e - (a^3*b^2 - a*b^4 - 4*a^2*c^3 - (8*a^3 - a*b^2)*c^2
- 2*(2*a^4 - 3*a^2*b^2)*c)*e^2)*sqrt((b^2*d^4 + b^2*e^4 - 4*(a*b + b*c)*d^3
e + 2(2*a^2 + b^2 + 4*a*c + 2*c^2)*d^2*e^2 - 4*(a*b + b*c)*d*e^3)/(a^4*b^
2 - 2*a^2*b^4 + b^6 - 4*a*c^5 - (16*a^2 - b^2)*c^4 - 12*(2*a^3 - a*b^2)*c^3
 - 2*(8*a^4 - 11*a^2*b^2 + b^4)*c^2 - 4*(a^5 - 3*a^3*b^2 + 2*a*b^4)*c))*sin
(x) + sqrt(2)*(((a^2*b^4 - b^6 + 8*a*c^5 + 2*(12*a^2 - b^2)*c^4 + 6*(4*a^3
- 3*a*b^2)*c^3 + (8*a^4 - 22*a^2*b^2 + 3*b^4)*c^2 - 2*(3*a^3*b^2 - 4*a*b^4)
*c)*d - (a^3*b^3 - a*b^5 + 4*a*b*c^4 + (4*a^2*b - b^3)*c^3 - (4*a^3*b + 5*a
*b^3)*c^2 - (4*a^4*b - 5*a^2*b^3 - b^5)*c)*e)*sqrt((b^2*d^4 + b^2*e^4 - 4*(
a*b + b*c)*d^3*e + 2*(2*a^2 + b^2 + 4*a*c + 2*c^2)*d^2*e^2 - 4*(a*b + b*c)*
d*e^3)/(a^4*b^2 - 2*a^2*b^4 + b^6 - 4*a*c^5 - (16*a^2 - b^2)*c^4 - 12*(2*a^
3 - a*b^2)*c^3 - 2*(8*a^4 - 11*a^2*b^2 + b^4)*c^2 - 4*(a^5 - 3*a^3*b^2 + 2*
a*b^4)*c))*cos(x) + ((b^4 - 4*a*b^2*c)*d^3 - 3*(a*b^3 - 4*a*b*c^2 - (4*a^2*
b - b^3)*c)*d^2*e + (2*a^2*b^2 + b^4 - 8*a^3*c - 8*a*c^3 - 2*(8*a^2 - b^2)*
c^2)*d*e^2 - (a*b^3 - 4*a*b*c^2 - (4*a^2*b - b^3)*c)*e^3)*cos(x))*sqrt(-((b
^2 - 2*a*c - 2*c^2)*d^2 - 2*(a*b - b*c)*d*e + (2*a^2 - b^2 + 2*a*c)*e^2 - (
a^2*b^2 - b^4 - 4*a*c^3 - (8*a^2 - b^2)*c^2 - 2*(2*a^3 - 3*a*b^2)*c)*sqrt((
b^2*d^4 + b^2*e^4 - 4*(a*b + b*c)*d^3*e + 2*(2*a^2 + b^2 + 4*a*c + 2*c^2)*d
^2*e^2 - 4*(a*b + b*c)*d*e^3)/(a^4*b^2 - 2*a^2*b^4 + b^6 - 4*a*c^5 - (16*a^
2 - b^2)*c^4 - 12*(2*a^3 - a*b^2)*c^3 - 2*(8*a^4 - 11*a^2*b^2 + b^4)*c^2 -

```
4*(a^5 - 3*a^3*b^2 + 2*a*b^4)*c)))/(a^2*b^2 - b^4 - 4*a*c^3 - (8*a^2 - b^2)
*c^2 - 2*(2*a^3 - 3*a*b^2)*c)) + 2*(b^2*c*d^4 + a*b^2*e^4 - (b^3 + 2*a*b*c
+ 2*b*c^2)*d^3*e + 3*(a*b^2 + b^2*c)*d^2*e^2 - (2*a^2*b + b^3 + 2*a*b*c)*d*
e^3)*sin(x)) + 1/4*sqrt(2)*sqrt(-((b^2 - 2*a*c - 2*c^2)*d^2 - 2*(a*b - b*c)
*d*e + (2*a^2 - b^2 + 2*a*c)*e^2 - (a^2*b^2 - b^4 - 4*a*c^3 - (8*a^2 - b^2)
*c^2 - 2*(2*a^3 - 3*a*b^2)*c)*sqrt((b^2*d^4 + b^2*e^4 - 4*(a*b + b*c)*d^3*e
 + 2*(2*a^2 + b^2 + 4*a*c + 2*c^2)*d^2*e^2 - 4*(a*b + b*c)*d*e^3)/(a^4*b^2
- 2*a^2*b^4 + b^6 - 4*a*c^5 - (16*a^2 - b^2)*c^4 - 12*(2*a^3 - a*b^2)*c^3 -
 2*(8*a^4 - 11*a^2*b^2 + b^4)*c^2 - 4*(a^5 - 3*a^3*b^2 + 2*a*b^4)*c)))/(a^2
*b^2 - b^4 - 4*a*c^3 - (8*a^2 - b^2)*c^2 - 2*(2*a^3 - 3*a*b^2)*c))*log(-4*b
*c^2*d^4 - 4*a*b*c*e^4 + 4*(b^2*c + 2*a*c^2 + 2*c^3)*d^3*e - 12*(a*b*c + b*
c^2)*d^2*e^2 + 4*(2*a*c^2 + (2*a^2 + b^2)*c)*d*e^3 + 2*((4*a*c^4 + (8*a^2 -
 b^2)*c^3 + 2*(2*a^3 - 3*a*b^2)*c^2 - (a^2*b^2 - b^4)*c)*d^2 + (a^2*b^3 - b
^5 - 4*a*b*c^3 - (8*a^2*b - b^3)*c^2 - 2*(2*a^3*b - 3*a*b^3)*c)*d*e - (a^3*
b^2 - a*b^4 - 4*a^2*c^3 - (8*a^3 - a*b^2)*c^2 - 2*(2*a^4 - 3*a^2*b^2)*c)*e^
2)*sqrt((b^2*d^4 + b^2*e^4 - 4*(a*b + b*c)*d^3*e + 2*(2*a^2 + b^2 + 4*a*c +
 2*c^2)*d^2*e^2 - 4*(a*b + b*c)*d*e^3)/(a^4*b^2 - 2*a^2*b^4 + b^6 - 4*a*c^5
 - (16*a^2 - b^2)*c^4 - 12*(2*a^3 - a*b^2)*c^3 - 2*(8*a^4 - 11*a^2*b^2 + b^
4)*c^2 - 4*(a^5 - 3*a^3*b^2 + 2*a*b^4)*c))*sin(x) + sqrt(2)*(((a^2*b^4 - b^
6 + 8*a*c^5 + 2*(12*a^2 - b^2)*c^4 + 6*(4*a^3 - 3*a*b^2)*c^3 + (8*a^4 - 22*
a^2*b^2 + 3*b^4)*c^2 - 2*(3*a^3*b^2 - 4*a*b^4)*c)*d - (a^3*b^3 - a*b^5 + 4*
a*b*c^4 + (4*a^2*b - b^3)*c^3 - (4*a^3*b + 5*a*b^3)*c^2 - (4*a^4*b - 5*a^2*
b^3 - b^5)*c)*e)*sqrt((b^2*d^4 + b^2*e^4 - 4*(a*b + b*c)*d^3*e + 2*(2*a^2 +
 b^2 + 4*a*c + 2*c^2)*d^2*e^2 - 4*(a*b + b*c)*d*e^3)/(a^4*b^2 - 2*a^2*b^4 +
 b^6 - 4*a*c^5 - (16*a^2 - b^2)*c^4 - 12*(2*a^3 - a*b^2)*c^3 - 2*(8*a^4 - 1
1*a^2*b^2 + b^4)*c^2 - 4*(a^5 - 3*a^3*b^2 + 2*a*b^4)*c))*cos(x) + ((b^4 - 4
*a*b^2*c)*d^3 - 3*(a*b^3 - 4*a*b*c^2 - (4*a^2*b - b^3)*c)*d^2*e + (2*a^2*b^
2 + b^4 - 8*a^3*c - 8*a*c^3 - 2*(8*a^2 - b^2)*c^2)*d*e^2 - (a*b^3 - 4*a*b*c
^2 - (4*a^2*b - b^3)*c)*e^3)*cos(x))*sqrt(-((b^2 - 2*a*c - 2*c^2)*d^2 - 2*(
a*b - b*c)*d*e + (2*a^2 - b^2 + 2*a*c)*e^2 - (a^2*b^2 - b^4 - 4*a*c^3 - (8*
a^2 - b^2)*c^2 - 2*(2*a^3 - 3*a*b^2)*c)*sqrt((b^2*d^4 + b^2*e^4 - 4*(a*b +
b*c)*d^3*e + 2*(2*a^2 + b^2 + 4*a*c + 2*c^2)*d^2*e^2 - 4*(a*b + b*c)*d*e^3)
/(a^4*b^2 - 2*a^2*b^4 + b^6 - 4*a*c^5 - (16*a^2 - b^2)*c^4 - 12*(2*a^3 - a*
b^2)*c^3 - 2*(8*a^4 - 11*a^2*b^2 + b^4)*c^2 - 4*(a^5 - 3*a^3*b^2 + 2*a*b^4)
*c)))/(a^2*b^2 - b^4 - 4*a*c^3 - (8*a^2 - b^2)*c^2 - 2*(2*a^3 - 3*a*b^2)*c)
) - 2*(b^2*c*d^4 + a*b^2*e^4 - (b^3 + 2*a*b*c + 2*b*c^2)*d^3*e + 3*(a*b^2 +
 b^2*c)*d^2*e^2 - (2*a^2*b + b^3 + 2*a*b*c)*d*e^3)*sin(x)) - 1/4*sqrt(2)*sq
rt(-((b^2 - 2*a*c - 2*c^2)*d^2 - 2*(a*b - b*c)*d*e + (2*a^2 - b^2 + 2*a*c)*
e^2 + (a^2*b^2 - b^4 - 4*a*c^3 - (8*a^2 - b^2)*c^2 - 2*(2*a^3 - 3*a*b^2)*c)
*sqrt((b^2*d^4 + b^2*e^4 - 4*(a*b + b*c)*d^3*e + 2*(2*a^2 + b^2 + 4*a*c + 2
*c^2)*d^2*e^2 - 4*(a*b + b*c)*d*e^3)/(a^4*b^2 - 2*a^2*b^4 + b^6 - 4*a*c^5 -
 (16*a^2 - b^2)*c^4 - 12*(2*a^3 - a*b^2)*c^3 - 2*(8*a^4 - 11*a^2*b^2 + b^4)
*c^2 - 4*(a^5 - 3*a^3*b^2 + 2*a*b^4)*c)))/(a^2*b^2 - b^4 - 4*a*c^3 - (8*a^2
 - b^2)*c^2 - 2*(2*a^3 - 3*a*b^2)*c))*log(-4*b*c^2*d^4 - 4*a*b*c*e^4 + 4*(b
^2*c + 2*a*c^2 + 2*c^3)*d^3*e - 12*(a*b*c + b*c^2)*d^2*e^2 + 4*(2*a*c^2 + (
```

```
2*a^2 + b^2)*c)*d*e^3 - 2*((4*a*c^4 + (8*a^2 - b^2)*c^3 + 2*(2*a^3 - 3*a*b^
2)*c^2 - (a^2*b^2 - b^4)*c)*d^2 + (a^2*b^3 - b^5 - 4*a*b*c^3 - (8*a^2*b - b
^3)*c^2 - 2*(2*a^3*b - 3*a*b^3)*c)*d*e - (a^3*b^2 - a*b^4 - 4*a^2*c^3 - (8*
a^3 - a*b^2)*c^2 - 2*(2*a^4 - 3*a^2*b^2)*c)*e^2)*sqrt((b^2*d^4 + b^2*e^4 -
4*(a*b + b*c)*d^3*e + 2*(2*a^2 + b^2 + 4*a*c + 2*c^2)*d^2*e^2 - 4*(a*b + b*
c)*d*e^3)/(a^4*b^2 - 2*a^2*b^4 + b^6 - 4*a*c^5 - (16*a^2 - b^2)*c^4 - 12*(2
*a^3 - a*b^2)*c^3 - 2*(8*a^4 - 11*a^2*b^2 + b^4)*c^2 - 4*(a^5 - 3*a^3*b^2 +
 2*a*b^4)*c))*sin(x) + sqrt(2)*(((a^2*b^4 - b^6 + 8*a*c^5 + 2*(12*a^2 - b^2
)*c^4 + 6*(4*a^3 - 3*a*b^2)*c^3 + (8*a^4 - 22*a^2*b^2 + 3*b^4)*c^2 - 2*(3*a
^3*b^2 - 4*a*b^4)*c)*d - (a^3*b^3 - a*b^5 + 4*a*b*c^4 + (4*a^2*b - b^3)*c^3
 - (4*a^3*b + 5*a*b^3)*c^2 - (4*a^4*b - 5*a^2*b^3 - b^5)*c)*e)*sqrt((b^2*d^
4 + b^2*e^4 - 4*(a*b + b*c)*d^3*e + 2*(2*a^2 + b^2 + 4*a*c + 2*c^2)*d^2*e^2
 - 4*(a*b + b*c)*d*e^3)/(a^4*b^2 - 2*a^2*b^4 + b^6 - 4*a*c^5 - (16*a^2 - b^
2)*c^4 - 12*(2*a^3 - a*b^2)*c^3 - 2*(8*a^4 - 11*a^2*b^2 + b^4)*c^2 - 4*(a^5
 - 3*a^3*b^2 + 2*a*b^4)*c))*cos(x) - ((b^4 - 4*a*b^2*c)*d^3 - 3*(a*b^3 - 4*
a*b*c^2 - (4*a^2*b - b^3)*c)*d^2*e + (2*a^2*b^2 + b^4 - 8*a^3*c - 8*a*c^3 -
 2*(8*a^2 - b^2)*c^2)*d*e^2 - (a*b^3 - 4*a*b*c^2 - (4*a^2*b - b^3)*c)*e^3)*
cos(x))*sqrt(-((b^2 - 2*a*c - 2*c^2)*d^2 - 2*(a*b - b*c)*d*e + (2*a^2 - b^2
 + 2*a*c)*e^2 + (a^2*b^2 - b^4 - 4*a*c^3 - (8*a^2 - b^2)*c^2 - 2*(2*a^3 - 3
*a*b^2)*c)*sqrt((b^2*d^4 + b^2*e^4 - 4*(a*b + b*c)*d^3*e + 2*(2*a^2 + b^2 +
 4*a*c + 2*c^2)*d^2*e^2 - 4*(a*b + b*c)*d*e^3)/(a^4*b^2 - 2*a^2*b^4 + b^6 -
 4*a*c^5 - (16*a^2 - b^2)*c^4 - 12*(2*a^3 - a*b^2)*c^3 - 2*(8*a^4 - 11*a^2*
b^2 + b^4)*c^2 - 4*(a^5 - 3*a^3*b^2 + 2*a*b^4)*c)))/(a^2*b^2 - b^4 - 4*a*c^
3 - (8*a^2 - b^2)*c^2 - 2*(2*a^3 - 3*a*b^2)*c)) - 2*(b^2*c*d^4 + a*b^2*e^4
- (b^3 + 2*a*b*c + 2*b*c^2)*d^3*e + 3*(a*b^2 + b^2*c)*d^2*e^2 - (2*a^2*b +
b^3 + 2*a*b*c)*d*e^3)*sin(x))
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*sin(x))/(a+b*sin(x)+c*sin(x)^2),x, algorithm="giac")
[Out] Timed out
```

maple [B] time = 0.18, size = 832, normalized size = 3.44

$$\frac{8a\arctan\left(\frac{2a\tan\left(\frac{x}{2}\right)+b+\sqrt{-4ac+b^2}}{\sqrt{4ac-2b^2-2b\sqrt{-4ac+b^2}+4a^2}}\right)dc}{\left(4ac-b^2\right)\sqrt{4ac-2b^2-2b\sqrt{-4ac+b^2}+4a^2}}-\frac{2\arctan\left(\frac{2a\tan\left(\frac{x}{2}\right)+b+\sqrt{-4ac+b^2}}{\sqrt{4ac-2b^2-2b\sqrt{-4ac+b^2}+4a^2}}\right)d\,b^2}{\left(4ac-b^2\right)\sqrt{4ac-2b^2-2b\sqrt{-4ac+b^2}+4a^2}}+\frac{4a\sqrt{-4ac+b^2}\ a}{\left(4ac-b^2\right)\sqrt{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d+e*sin(x))/(a+b*sin(x)+c*sin(x)^2),x)
```

```
[Out] 8*a/(4*a*c-b^2)/(4*a*c-2*b^2-2*b*(-4*a*c+b^2)^(1/2)+4*a^2)^(1/2)*arctan((2*
a*tan(1/2*x)+b+(-4*a*c+b^2)^(1/2))/(4*a*c-2*b^2-2*b*(-4*a*c+b^2)^(1/2)+4*a^
2)^(1/2))*d*c-2/(4*a*c-b^2)/(4*a*c-2*b^2-2*b*(-4*a*c+b^2)^(1/2)+4*a^2)^(1/2
)*arctan((2*a*tan(1/2*x)+b+(-4*a*c+b^2)^(1/2))/(4*a*c-2*b^2-2*b*(-4*a*c+b^2
)^(1/2)+4*a^2)^(1/2))*d*b^2+4*a*(-4*a*c+b^2)^(1/2)/(4*a*c-b^2)/(4*a*c-2*b^2
-2*b*(-4*a*c+b^2)^(1/2)+4*a^2)^(1/2)*arctan((2*a*tan(1/2*x)+b+(-4*a*c+b^2)^
(1/2))/(4*a*c-2*b^2-2*b*(-4*a*c+b^2)^(1/2)+4*a^2)^(1/2))*e-2*(-4*a*c+b^2)^(
1/2)/(4*a*c-b^2)/(4*a*c-2*b^2-2*b*(-4*a*c+b^2)^(1/2)+4*a^2)^(1/2)*arctan((2
*a*tan(1/2*x)+b+(-4*a*c+b^2)^(1/2))/(4*a*c-2*b^2-2*b*(-4*a*c+b^2)^(1/2)+4*a
^2)^(1/2))*d*b-8*a/(4*a*c-b^2)/(4*a*c-2*b^2+2*b*(-4*a*c+b^2)^(1/2)+4*a^2)^(
1/2)*arctan((-2*a*tan(1/2*x)+(-4*a*c+b^2)^(1/2)-b)/(4*a*c-2*b^2+2*b*(-4*a*c
+b^2)^(1/2)+4*a^2)^(1/2))*d*c+2/(4*a*c-b^2)/(4*a*c-2*b^2+2*b*(-4*a*c+b^2)^(
1/2)+4*a^2)^(1/2)*arctan((-2*a*tan(1/2*x)+(-4*a*c+b^2)^(1/2)-b)/(4*a*c-2*b^
2+2*b*(-4*a*c+b^2)^(1/2)+4*a^2)^(1/2))*d*b^2+4*a*(-4*a*c+b^2)^(1/2)/(4*a*c-
b^2)/(4*a*c-2*b^2+2*b*(-4*a*c+b^2)^(1/2)+4*a^2)^(1/2)*arctan((-2*a*tan(1/2*
x)+(-4*a*c+b^2)^(1/2)-b)/(4*a*c-2*b^2+2*b*(-4*a*c+b^2)^(1/2)+4*a^2)^(1/2))*
e-2*(-4*a*c+b^2)^(1/2)/(4*a*c-b^2)/(4*a*c-2*b^2+2*b*(-4*a*c+b^2)^(1/2)+4*a^
2)^(1/2)*arctan((-2*a*tan(1/2*x)+(-4*a*c+b^2)^(1/2)-b)/(4*a*c-2*b^2+2*b*(-4
*a*c+b^2)^(1/2)+4*a^2)^(1/2))*d*b
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{e\sin(x) + d}{c\sin(x)^2 + b\sin(x) + a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*sin(x))/(a+b*sin(x)+c*sin(x)^2),x, algorithm="maxima")
```

```
[Out] integrate((e*sin(x) + d)/(c*sin(x)^2 + b*sin(x) + a), x)
```

mupad [B] time = 17.11, size = 10465, normalized size = 43.24

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*sin(x))/(a + c*sin(x)^2 + b*sin(x)),x)
```

```
[Out] atan(((-(b^4*d^2 - b^4*e^2 + 8*a*c^3*d^2 + b*d^2*(-(4*a*c - b^2)^3)^(1/2) -
 8*a^3*c*e^2 + b*e^2*(-(4*a*c - b^2)^3)^(1/2) + 2*a^2*b^2*e^2 + 8*a^2*c^2*d
^2 - 8*a^2*c^2*e^2 - 2*b^2*c^2*d^2 - 2*a*b^3*d*e - 2*a*d*e*(-(4*a*c - b^2)^
3)^(1/2) + 2*b^3*c*d*e - 2*c*d*e*(-(4*a*c - b^2)^3)^(1/2) - 6*a*b^2*c*d^2 +
 6*a*b^2*c*e^2 - 8*a*b*c^2*d*e + 8*a^2*b*c*d*e)/(2*(a^2*b^4 - b^6 + 16*a^2*
c^4 + 32*a^3*c^3 + 16*a^4*c^2 + b^4*c^2 - 8*a*b^2*c^3 - 8*a^3*b^2*c - 32*a^
```

2*b^2*c^2 + 10*a*b^4*c)))^(1/2)*((-(b^4*d^2 - b^4*e^2 + 8*a*c^3*d^2 + b*d^2
*(-(4*a*c - b^2)^3)^(1/2) - 8*a^3*c*e^2 + b*e^2*(-(4*a*c - b^2)^3)^(1/2) +
2*a^2*b^2*e^2 + 8*a^2*c^2*d^2 - 8*a^2*c^2*e^2 - 2*b^2*c^2*d^2 - 2*a*b^3*d*e
 - 2*a*d*e*(-(4*a*c - b^2)^3)^(1/2) + 2*b^3*c*d*e - 2*c*d*e*(-(4*a*c - b^2)
^3)^(1/2) - 6*a*b^2*c*d^2 + 6*a*b^2*c*e^2 - 8*a*b*c^2*d*e + 8*a^2*b*c*d*e)/
(2*(a^2*b^4 - b^6 + 16*a^2*c^4 + 32*a^3*c^3 + 16*a^4*c^2 + b^4*c^2 - 8*a*b^
2*c^3 - 8*a^3*b^2*c - 32*a^2*b^2*c^2 + 10*a*b^4*c)))^(1/2)*((-(b^4*d^2 - b^
4*e^2 + 8*a*c^3*d^2 + b*d^2*(-(4*a*c - b^2)^3)^(1/2) - 8*a^3*c*e^2 + b*e^2*
(-(4*a*c - b^2)^3)^(1/2) + 2*a^2*b^2*e^2 + 8*a^2*c^2*d^2 - 8*a^2*c^2*e^2 -
2*b^2*c^2*d^2 - 2*a*b^3*d*e - 2*a*d*e*(-(4*a*c - b^2)^3)^(1/2) + 2*b^3*c*d*
e - 2*c*d*e*(-(4*a*c - b^2)^3)^(1/2) - 6*a*b^2*c*d^2 + 6*a*b^2*c*e^2 - 8*a*
b*c^2*d*e + 8*a^2*b*c*d*e)/(2*(a^2*b^4 - b^6 + 16*a^2*c^4 + 32*a^3*c^3 + 16
*a^4*c^2 + b^4*c^2 - 8*a*b^2*c^3 - 8*a^3*b^2*c - 32*a^2*b^2*c^2 + 10*a*b^4*
c)))^(1/2)*(tan(x/2)*(96*a*b^4 + 256*a^4*c - 64*a^3*b^2 + 512*a^2*c^3 + 768
*a^3*c^2 - 128*a*b^2*c^2 - 576*a^2*b^2*c) + 32*a^2*b^3 + 128*a^2*b*c^2 - 32
*a*b^3*c - 128*a^3*b*c) + tan(x/2)*(64*a^2*b^2*e - 256*a^2*c^2*e - 64*a*b^3
*d - 256*a^3*c*e + 256*a^2*b*c*d + 64*a*b^2*c*e) - 32*a^2*b^2*d + 128*a^2*c
^2*d + 32*a*b^3*e + 128*a^3*c*d - 32*a*b^2*c*d - 128*a^2*b*c*e) - tan(x/2)*
(64*a^3*e^2 + 32*a*b^2*d^2 - 64*a*b^2*e^2 - 128*a*c^2*d^2 - 64*a^2*c*d^2 +
128*a^2*c*e^2 - 64*a^2*b*d*e + 128*a*b*c*d*e) + 32*a^2*b*e^2 + 32*a*b*c*d^2
 - 128*a^2*c*d*e)*1i + (-(b^4*d^2 - b^4*e^2 + 8*a*c^3*d^2 + b*d^2*(-(4*a*c
 - b^2)^3)^(1/2) - 8*a^3*c*e^2 + b*e^2*(-(4*a*c - b^2)^3)^(1/2) + 2*a^2*b^2*
e^2 + 8*a^2*c^2*d^2 - 8*a^2*c^2*e^2 - 2*b^2*c^2*d^2 - 2*a*b^3*d*e - 2*a*d*e
*(-(4*a*c - b^2)^3)^(1/2) + 2*b^3*c*d*e - 2*c*d*e*(-(4*a*c - b^2)^3)^(1/2)
 - 6*a*b^2*c*d^2 + 6*a*b^2*c*e^2 - 8*a*b*c^2*d*e + 8*a^2*b*c*d*e)/(2*(a^2*b^
4 - b^6 + 16*a^2*c^4 + 32*a^3*c^3 + 16*a^4*c^2 + b^4*c^2 - 8*a*b^2*c^3 - 8*
a^3*b^2*c - 32*a^2*b^2*c^2 + 10*a*b^4*c)))^(1/2)*((-(b^4*d^2 - b^4*e^2 + 8*
a*c^3*d^2 + b*d^2*(-(4*a*c - b^2)^3)^(1/2) - 8*a^3*c*e^2 + b*e^2*(-(4*a*c -
 b^2)^3)^(1/2) + 2*a^2*b^2*e^2 + 8*a^2*c^2*d^2 - 8*a^2*c^2*e^2 - 2*b^2*c^2*
d^2 - 2*a*b^3*d*e - 2*a*d*e*(-(4*a*c - b^2)^3)^(1/2) + 2*b^3*c*d*e - 2*c*d*
e*(-(4*a*c - b^2)^3)^(1/2) - 6*a*b^2*c*d^2 + 6*a*b^2*c*e^2 - 8*a*b*c^2*d*e
 + 8*a^2*b*c*d*e)/(2*(a^2*b^4 - b^6 + 16*a^2*c^4 + 32*a^3*c^3 + 16*a^4*c^2 +
 b^4*c^2 - 8*a*b^2*c^3 - 8*a^3*b^2*c - 32*a^2*b^2*c^2 + 10*a*b^4*c)))^(1/2)
*((-(b^4*d^2 - b^4*e^2 + 8*a*c^3*d^2 + b*d^2*(-(4*a*c - b^2)^3)^(1/2) - 8*a
^3*c*e^2 + b*e^2*(-(4*a*c - b^2)^3)^(1/2) + 2*a^2*b^2*e^2 + 8*a^2*c^2*d^2 -
 8*a^2*c^2*e^2 - 2*b^2*c^2*d^2 - 2*a*b^3*d*e - 2*a*d*e*(-(4*a*c - b^2)^3)^(
1/2) + 2*b^3*c*d*e - 2*c*d*e*(-(4*a*c - b^2)^3)^(1/2) - 6*a*b^2*c*d^2 + 6*a
*b^2*c*e^2 - 8*a*b*c^2*d*e + 8*a^2*b*c*d*e)/(2*(a^2*b^4 - b^6 + 16*a^2*c^4
 + 32*a^3*c^3 + 16*a^4*c^2 + b^4*c^2 - 8*a*b^2*c^3 - 8*a^3*b^2*c - 32*a^2*b^
2*c^2 + 10*a*b^4*c)))^(1/2)*(tan(x/2)*(96*a*b^4 + 256*a^4*c - 64*a^3*b^2 +
512*a^2*c^3 + 768*a^3*c^2 - 128*a*b^2*c^2 - 576*a^2*b^2*c) + 32*a^2*b^3 + 1
28*a^2*b*c^2 - 32*a*b^3*c - 128*a^3*b*c) - tan(x/2)*(64*a^2*b^2*e - 256*a^2
*c^2*e - 64*a*b^3*d - 256*a^3*c*e + 256*a^2*b*c*d + 64*a*b^2*c*e) + 32*a^2*
b^2*d - 128*a^2*c^2*d - 32*a*b^3*e - 128*a^3*c*d + 32*a*b^2*c*d + 128*a^2*b
*c*e) - tan(x/2)*(64*a^3*e^2 + 32*a*b^2*d^2 - 64*a*b^2*e^2 - 128*a*c^2*d^2

- 64*a^2*c*d^2 + 128*a^2*c*e^2 - 64*a^2*b*d*e + 128*a*b*c*d*e) + 32*a^2*b*e
^2 + 32*a*b*c*d^2 - 128*a^2*c*d*e)*1i)/(2*tan(x/2)*(64*a^2*e^3 - 64*a*b*d*e
^2 + 64*a*c*d^2*e) + (-(b^4*d^2 - b^4*e^2 + 8*a*c^3*d^2 + b*d^2*(-(4*a*c -
b^2)^3)^(1/2) - 8*a^3*c*e^2 + b*e^2*(-(4*a*c - b^2)^3)^(1/2) + 2*a^2*b^2*e^
2 + 8*a^2*c^2*d^2 - 8*a^2*c^2*e^2 - 2*b^2*c^2*d^2 - 2*a*b^3*d*e - 2*a*d*e*(
-(4*a*c - b^2)^3)^(1/2) + 2*b^3*c*d*e - 2*c*d*e*(-(4*a*c - b^2)^3)^(1/2) -
6*a*b^2*c*d^2 + 6*a*b^2*c*e^2 - 8*a*b*c^2*d*e + 8*a^2*b*c*d*e)/(2*(a^2*b^4
- b^6 + 16*a^2*c^4 + 32*a^3*c^3 + 16*a^4*c^2 + b^4*c^2 - 8*a*b^2*c^3 - 8*a^
3*b^2*c - 32*a^2*b^2*c^2 + 10*a*b^4*c)))^(1/2)*((-(b^4*d^2 - b^4*e^2 + 8*a*
c^3*d^2 + b*d^2*(-(4*a*c - b^2)^3)^(1/2) - 8*a^3*c*e^2 + b*e^2*(-(4*a*c - b
^2)^3)^(1/2) + 2*a^2*b^2*e^2 + 8*a^2*c^2*d^2 - 8*a^2*c^2*e^2 - 2*b^2*c^2*d^
2 - 2*a*b^3*d*e - 2*a*d*e*(-(4*a*c - b^2)^3)^(1/2) + 2*b^3*c*d*e - 2*c*d*e*
(-(4*a*c - b^2)^3)^(1/2) - 6*a*b^2*c*d^2 + 6*a*b^2*c*e^2 - 8*a*b*c^2*d*e +
8*a^2*b*c*d*e)/(2*(a^2*b^4 - b^6 + 16*a^2*c^4 + 32*a^3*c^3 + 16*a^4*c^2 + b
^4*c^2 - 8*a*b^2*c^3 - 8*a^3*b^2*c - 32*a^2*b^2*c^2 + 10*a*b^4*c)))^(1/2)*(
(-(b^4*d^2 - b^4*e^2 + 8*a*c^3*d^2 + b*d^2*(-(4*a*c - b^2)^3)^(1/2) - 8*a^3
*c*e^2 + b*e^2*(-(4*a*c - b^2)^3)^(1/2) + 2*a^2*b^2*e^2 + 8*a^2*c^2*d^2 - 8
*a^2*c^2*e^2 - 2*b^2*c^2*d^2 - 2*a*b^3*d*e - 2*a*d*e*(-(4*a*c - b^2)^3)^(1/
2) + 2*b^3*c*d*e - 2*c*d*e*(-(4*a*c - b^2)^3)^(1/2) - 6*a*b^2*c*d^2 + 6*a*b
^2*c*e^2 - 8*a*b*c^2*d*e + 8*a^2*b*c*d*e)/(2*(a^2*b^4 - b^6 + 16*a^2*c^4 +
32*a^3*c^3 + 16*a^4*c^2 + b^4*c^2 - 8*a*b^2*c^3 - 8*a^3*b^2*c - 32*a^2*b^2*
c^2 + 10*a*b^4*c)))^(1/2)*(tan(x/2)*(96*a*b^4 + 256*a^4*c - 64*a^3*b^2 + 51
2*a^2*c^3 + 768*a^3*c^2 - 128*a*b^2*c^2 - 576*a^2*b^2*c) + 32*a^2*b^3 + 128
*a^2*b*c^2 - 32*a*b^3*c - 128*a^3*b*c) + tan(x/2)*(64*a^2*b^2*e - 256*a^2*c
^2*e - 64*a*b^3*d - 256*a^3*c*e + 256*a^2*b*c*d + 64*a*b^2*c*e) - 32*a^2*b^
2*d + 128*a^2*c^2*d + 32*a*b^3*e + 128*a^3*c*d - 32*a*b^2*c*d - 128*a^2*b*c
e) - tan(x/2)(64*a^3*e^2 + 32*a*b^2*d^2 - 64*a*b^2*e^2 - 128*a*c^2*d^2 -
64*a^2*c*d^2 + 128*a^2*c*e^2 - 64*a^2*b*d*e + 128*a*b*c*d*e) + 32*a^2*b*e^2
 + 32*a*b*c*d^2 - 128*a^2*c*d*e) - (-(b^4*d^2 - b^4*e^2 + 8*a*c^3*d^2 + b*d
^2*(-(4*a*c - b^2)^3)^(1/2) - 8*a^3*c*e^2 + b*e^2*(-(4*a*c - b^2)^3)^(1/2)
+ 2*a^2*b^2*e^2 + 8*a^2*c^2*d^2 - 8*a^2*c^2*e^2 - 2*b^2*c^2*d^2 - 2*a*b^3*d
*e - 2*a*d*e*(-(4*a*c - b^2)^3)^(1/2) + 2*b^3*c*d*e - 2*c*d*e*(-(4*a*c - b^
2)^3)^(1/2) - 6*a*b^2*c*d^2 + 6*a*b^2*c*e^2 - 8*a*b*c^2*d*e + 8*a^2*b*c*d*e
)/(2*(a^2*b^4 - b^6 + 16*a^2*c^4 + 32*a^3*c^3 + 16*a^4*c^2 + b^4*c^2 - 8*a*
b^2*c^3 - 8*a^3*b^2*c - 32*a^2*b^2*c^2 + 10*a*b^4*c)))^(1/2)*((-(b^4*d^2 -
b^4*e^2 + 8*a*c^3*d^2 + b*d^2*(-(4*a*c - b^2)^3)^(1/2) - 8*a^3*c*e^2 + b*e^
2*(-(4*a*c - b^2)^3)^(1/2) + 2*a^2*b^2*e^2 + 8*a^2*c^2*d^2 - 8*a^2*c^2*e^2
- 2*b^2*c^2*d^2 - 2*a*b^3*d*e - 2*a*d*e*(-(4*a*c - b^2)^3)^(1/2) + 2*b^3*c*
d*e - 2*c*d*e*(-(4*a*c - b^2)^3)^(1/2) - 6*a*b^2*c*d^2 + 6*a*b^2*c*e^2 - 8*
a*b*c^2*d*e + 8*a^2*b*c*d*e)/(2*(a^2*b^4 - b^6 + 16*a^2*c^4 + 32*a^3*c^3 +
16*a^4*c^2 + b^4*c^2 - 8*a*b^2*c^3 - 8*a^3*b^2*c - 32*a^2*b^2*c^2 + 10*a*b^
4*c)))^(1/2)*((-(b^4*d^2 - b^4*e^2 + 8*a*c^3*d^2 + b*d^2*(-(4*a*c - b^2)^3)
^(1/2) - 8*a^3*c*e^2 + b*e^2*(-(4*a*c - b^2)^3)^(1/2) + 2*a^2*b^2*e^2 + 8*a
^2*c^2*d^2 - 8*a^2*c^2*e^2 - 2*b^2*c^2*d^2 - 2*a*b^3*d*e - 2*a*d*e*(-(4*a*c
 - b^2)^3)^(1/2) + 2*b^3*c*d*e - 2*c*d*e*(-(4*a*c - b^2)^3)^(1/2) - 6*a*b^2

*c*d^2 + 6*a*b^2*c*e^2 - 8*a*b*c^2*d*e + 8*a^2*b*c*d*e)/(2*(a^2*b^4 - b^6 + 16*a^2*c^4 + 32*a^3*c^3 + 16*a^4*c^2 + b^4*c^2 - 8*a*b^2*c^3 - 8*a^3*b^2*c - 32*a^2*b^2*c^2 + 10*a*b^4*c)))^(1/2)*(tan(x/2)*(96*a*b^4 + 256*a^4*c - 64*a^3*b^2 + 512*a^2*c^3 + 768*a^3*c^2 - 128*a*b^2*c^2 - 576*a^2*b^2*c) + 32*a^2*b^3 + 128*a^2*b*c^2 - 32*a*b^3*c - 128*a^3*b*c) - tan(x/2)*(64*a^2*b^2*e - 256*a^2*c^2*e - 64*a*b^3*d - 256*a^3*c*e + 256*a^2*b*c*d + 64*a*b^2*c*e) + 32*a^2*b^2*d - 128*a^2*c^2*d - 32*a*b^3*e - 128*a^3*c*d + 32*a*b^2*c*d + 128*a^2*b*c*e) - tan(x/2)*(64*a^3*e^2 + 32*a*b^2*d^2 - 64*a*b^2*e^2 - 128*a*c^2*d^2 - 64*a^2*c*d^2 + 128*a^2*c*e^2 - 64*a^2*b*d*e + 128*a*b*c*d*e) + 32*a^2*b*e^2 + 32*a*b*c*d^2 - 128*a^2*c*d*e) + 64*a^2*d*e^2 + 64*a*c*d^3 - 64*a*b*d^2*e))*(-(b^4*d^2 - b^4*e^2 + 8*a*c^3*d^2 + b*d^2*(-(4*a*c - b^2)^3)^(1/2) - 8*a^3*c*e^2 + b*e^2*(-(4*a*c - b^2)^3)^(1/2) + 2*a^2*b^2*e^2 + 8*a^2*c^2*d^2 - 8*a^2*c^2*e^2 - 2*b^2*c^2*d^2 - 2*a*b^3*d*e - 2*a*d*e*(-(4*a*c - b^2)^3)^(1/2) + 2*b^3*c*d*e - 2*c*d*e*(-(4*a*c - b^2)^3)^(1/2) - 6*a*b^2*c*d^2 + 6*a*b^2*c*e^2 - 8*a*b*c^2*d*e + 8*a^2*b*c*d*e)/(2*(a^2*b^4 - b^6 + 16*a^2*c^4 + 32*a^3*c^3 + 16*a^4*c^2 + b^4*c^2 - 8*a*b^2*c^3 - 8*a^3*b^2*c - 32*a^2*b^2*c^2 + 10*a*b^4*c)))^(1/2)*2i + atan(((-(b^4*d^2 - b^4*e^2 + 8*a*c^3*d^2 - b*d^2*(-(4*a*c - b^2)^3)^(1/2) - 8*a^3*c*e^2 - b*e^2*(-(4*a*c - b^2)^3)^(1/2) + 2*a^2*b^2*e^2 + 8*a^2*c^2*d^2 - 8*a^2*c^2*e^2 - 2*b^2*c^2*d^2 - 2*a*b^3*d*e + 2*a*d*e*(-(4*a*c - b^2)^3)^(1/2) + 2*b^3*c*d*e + 2*c*d*e*(-(4*a*c - b^2)^3)^(1/2) - 6*a*b^2*c*d^2 + 6*a*b^2*c*e^2 - 8*a*b*c^2*d*e + 8*a^2*b*c*d*e)/(2*(a^2*b^4 - b^6 + 16*a^2*c^4 + 32*a^3*c^3 + 16*a^4*c^2 + b^4*c^2 - 8*a*b^2*c^3 - 8*a^3*b^2*c - 32*a^2*b^2*c^2 + 10*a*b^4*c)))^(1/2)*((-(b^4*d^2 - b^4*e^2 + 8*a*c^3*d^2 - b*d^2*(-(4*a*c - b^2)^3)^(1/2) - 8*a^3*c*e^2 - b*e^2*(-(4*a*c - b^2)^3)^(1/2) + 2*a^2*b^2*e^2 + 8*a^2*c^2*d^2 - 8*a^2*c^2*e^2 - 2*b^2*c^2*d^2 - 2*a*b^3*d*e + 2*a*d*e*(-(4*a*c - b^2)^3)^(1/2) + 2*b^3*c*d*e + 2*c*d*e*(-(4*a*c - b^2)^3)^(1/2) - 6*a*b^2*c*d^2 + 6*a*b^2*c*e^2 - 8*a*b*c^2*d*e + 8*a^2*b*c*d*e)/(2*(a^2*b^4 - b^6 + 16*a^2*c^4 + 32*a^3*c^3 + 16*a^4*c^2 + b^4*c^2 - 8*a*b^2*c^3 - 8*a^3*b^2*c - 32*a^2*b^2*c^2 + 10*a*b^4*c)))^(1/2)*((-(b^4*d^2 - b^4*e^2 + 8*a*c^3*d^2 - b*d^2*(-(4*a*c - b^2)^3)^(1/2) - 8*a^3*c*e^2 - b*e^2*(-(4*a*c - b^2)^3)^(1/2) + 2*a^2*b^2*e^2 + 8*a^2*c^2*d^2 - 8*a^2*c^2*e^2 - 2*b^2*c^2*d^2 - 2*a*b^3*d*e + 2*a*d*e*(-(4*a*c - b^2)^3)^(1/2) + 2*b^3*c*d*e + 2*c*d*e*(-(4*a*c - b^2)^3)^(1/2) - 6*a*b^2*c*d^2 + 6*a*b^2*c*e^2 - 8*a*b*c^2*d*e + 8*a^2*b*c*d*e)/(2*(a^2*b^4 - b^6 + 16*a^2*c^4 + 32*a^3*c^3 + 16*a^4*c^2 + b^4*c^2 - 8*a*b^2*c^3 - 8*a^3*b^2*c - 32*a^2*b^2*c^2 + 10*a*b^4*c)))^(1/2)*(tan(x/2)*(96*a*b^4 + 256*a^4*c - 64*a^3*b^2 + 512*a^2*c^3 + 768*a^3*c^2 - 128*a*b^2*c^2 - 576*a^2*b^2*c) + 32*a^2*b^3 + 128*a^2*b*c^2 - 32*a*b^3*c - 128*a^3*b*c) + tan(x/2)*(64*a^2*b^2*e - 256*a^2*c^2*e - 64*a*b^3*d - 256*a^3*c*e + 256*a^2*b*c*d + 64*a*b^2*c*e) - 32*a^2*b^2*d + 128*a^2*c^2*d + 32*a*b^3*e + 128*a^3*c*d - 32*a*b^2*c*d - 128*a^2*b*c*e) - tan(x/2)*(64*a^3*e^2 + 32*a*b^2*d^2 - 64*a*b^2*e^2 - 128*a*c^2*d^2 - 64*a^2*c*d^2 + 128*a^2*c*e^2 - 64*a^2*b*d*e + 128*a*b*c*d*e) + 32*a^2*b*e^2 + 32*a*b*c*d^2 - 128*a^2*c*d*e)*1i + (-(b^4*d^2 - b^4*e^2 + 8*a*c^3*d^2 - b*d^2*(-(4*a*c - b^2)^3)^(1/2) - 8*a^3*c*e^2 - b*e^2*(-(4*a*c - b^2)^3)^(1/2) + 2*a^2*b^2*e^2 + 8*a^2*c^2*d^2 - 8*a^2 *c^2*e^2 - 2*b^2*c^2*d^2 - 2*a*b^3*d*e + 2*a*d*e*(-(4*a*c - b^2)^3)^(1/2) + 2*b^3*c*d*e + 2*c*d*e*(-(4*a*c - b^2)^3)^(1/2) - 6*a*b^2*c*d^2 + 6*a*b^2*c*e^2 - 8*a*b*c^2*d*e + 8*a^2*b*c*d*e)/(2*(a^2*b^4 - b^6 + 16*a^2*c^4 + 32*a^3*c^3 + 16*a^4*c^2 + b^4*c^2 - 8*a*b^2*c^3 - 8*a^3*b^2*c - 32*a^2*b^2*c^2 + 10*a*b^4*c)))^(1/2)*((-(b^4*d^2 - b^4*e^2 + 8*a*c^3*d^2 - b*d^2*(-(4*a*c - b^2)^3)^(1/2) - 8*a^3*c*e^2 - b*e^2*(-(4*a*c - b^2)^3)^(1/2) + 2*a^2*b^2*e^2 + 8*a^2*c^2*d^2 - 8*a^2*c^2*e^2 - 2*b^2*c^2*d^2 - 2*a*b^3*d*e + 2*a*d*e*(-(4*a*c - b^2)^3)^(1/2) + 2*b^3*c*d*e + 2*c*d*e*(-(4*a*c - b^2)^3)^(1/2) - 6*a*b^2*c*d^2 + 6*a*b^2*c*e^2 - 8*a*b*c^2*d*e + 8*a^2*b*c*d*e)/(2*(a^2*b^4 - b^6 + 16*a^2*c^4 + 32*a^3*c^3 + 16*a^4*c^2 + b^4*c^2 - 8*a*b^2*c^3 - 8*a^3*b^2*c - 32*a^2*b^2*c^2 + 10*a*b^4*c)))^(1/2)*((-(b^4*d^2 - b^4*e^2 + 8*a*c^3*d^2 - b*d^2*(-(4*a*c - b^2)^3)^(1/2) - 8*a^3*c*e^2 - b*e^2*(-(4*a*c - b^2)^3)^(1/2) + 2*a^2*b^2*e^2 + 8*a^2*c^2*d^2 - 8*a^2*c^2*e^2 - 2*b^2*c^2*d^2 - 2*a*b^3*d*e + 2*a*d*e*(-(4*a*c - b^2)^3)^(1/2) + 2*b^3*c*d*e + 2*c*d*e*(-(4*a*c - b^2)^3)^(1/2) - 6*a*b^2*c*d^2 + 6*a*b^2*c*e^2 - 8*a*b*c^2*d*e + 8*a^2*b*c*d*e)/(2*(a^2*b^4 - b^6 + 16*a^2*c^4 + 32*a^3*c^3 + 16*a^4*c^2 + b^4*c^2 - 8*a*b^2*c^3 - 8*a^3*b^2*c - 32*a^2*b^2*c^2 + 10*a*b^4*c)))^(1/2)*(tan(x/2)*(96*a*b^4 + 256*a^4*c - 64*a^3*b^2 + 512*a^2*c^3 + 768*a^3*c^2 - 128*a*b^2*c^2 - 576*a^2*b^2*c) + 32*a^2*b^3 + 128*a^2*b*c^2 - 32*a*b^3*c - 128*a^3*b*c) - tan(x/2)*(64*a^2*b^2*e - 256*a^2*c^2*e - 64*a*b^3*d - 256*a^3*c*e + 256*a^2*b*c*d + 64*a*b^2*c*e) + 32*a^2*b^2*d - 128*a^2*c^2*d - 32*a*b^3*e - 128*a^3*c*d + 32*a*b^2*c*d + 128*a^2*b*c*e) - tan(x/2)*(64*a^3*e^2 + 32*a*b^2*d^2 - 64*a*b^2*e^2 - 128*a*c^2*d^2 - 64*a^2*c*d^2 + 128*a^2*c*e^2 - 64*a^2*b*d*e + 128*a*b*c*d*e) + 32*a^2*b*e^2 + 32*a*b*c*d^2 - 128*a^2*c*d*e)*1i)/(2*tan(x/2)*(64*a^2*e^3 - 64*a*b*d*e^2 + 64*a*c*d^2*e) + (-(b^4*d^2 - b^4*e^2 + 8*a*c^3*d^2 - b*d^2*(-(4*a*c - b^2)^3)^(1/2) - 8*a^3*c*e^2 - b*e^2*(-(4*a*c - b^2)^3)^(1/2) + 2*a^2*b^2*e^2 + 8*a^2*c^2*d^2 - 8*a^2*c^2*e^2 - 2*b^2*c^2*d^2 - 2*a*b^3*d*e + 2*a*d*e*(-(4*a*c - b^2)^3)^(1/2) + 2*b^3*c*d*e + 2*c*d*e*(-(4*a*c - b^2)^3)^(1/2) - 6*a*b^2*c*d^2 + 6*a*b^2*c*e^2 - 8*a*b*c^2*d*e + 8*a^2*b*c*d*e)/(2*(a^2*b^4 - b^6 + 16*a^2*c^4 + 32*a^3*c^3 + 16*a^4*c^2 + b^4*c^2 - 8*a*b^2*c^3 - 8*a^3*b^2*c - 32*a^2*b^2*c^2 + 10*a*b^4*c)))^(1/2)*((-(b^4*d^2 - b^4*e^2 + 8*a*c^3*d^2 - b*d^2*(-(4*a*c - b^2)^3)^(1/2) - 8*a^3*c*e^2 - b*e^2*(-(4*a*c - b^2)^3)^(1/2) + 2*a^2*b^2*e^2 + 8*a^2*c^2*d^2 - 8*a^2*c^2*e^2 - 2*b^2*c^2*d^2 - 2*a*b^3*d*e + 2*a*d*e*(-(4*a*c - b^2)^3)^(1/2) + 2*b^3*c*d*e + 2*c*d*e*(-(4*a*c - b^2)^3)^(1/2) - 6*a*b^2*c*d^2 + 6*a*b^2*c*e^2 - 8*a*b*c^2*d*e + 8*a^2*b*c*d*e)/(2*(a^2*b^4 - b^6 + 16*a^2*c^4 + 32*a^3*c^3 + 16*a^4*c^2 + b^4*c^2 - 8*a*b^2*c^3 - 8*a^3*b^2*c - 32*a^2*b^2*c^2 + 10*a*b^4*c)))^(1/2)*((-(b^4*d^2 - b^4*e^2 + 8*a*c^3*d^2 - b*d^2*(-(4*a*c - b^2)^3)^(1/2) - 8*a^3*c*e^2 - b*e^2*(-(4*a*c - b^2)^3)^(1/2) + 2*a^2*b^2*e^2 + 8*a^2*c^2*d^2 - 8*a^2*c^2*e^2 - 2*b^2*c^2*d^2 - 2*a*b^3*d*e + 2*a*d*e*(-(4*a*c - b^2)^3)^(1/2) + 2*b^3*c*d*e + 2*c*d*e*(-(4*a*c - b^2)^3)^(1/2) - 6*a*b^2*c*d^2 + 6*a*b^2*c*e^2 - 8*a*b*c^2*d*e + 8*a^2*b*c*d*e)/(2*(a^2*b^4 - b^6 + 16*a^2*c^4 + 32*a^3*c^3 + 16*a^4*c^2 + b^4*c^2 - 8*a*b^2*c^3 - 8*a^3*b^2*c - 32*a^2*b^2*c^2 + 10*a*b^4*c)))^(1/2)*(tan(x/2)*(96*a*b^4 + 256*a^4*c - 64*a^3*b^2 + 512*a^2*c^3 + 768*a^3*c^2 - 1

```
28*a*b^2*c^2 - 576*a^2*b^2*c) + 32*a^2*b^3 + 128*a^2*b*c^2 - 32*a*b^3*c - 1
28*a^3*b*c) + tan(x/2)*(64*a^2*b^2*e - 256*a^2*c^2*e - 64*a*b^3*d - 256*a^3
*c*e + 256*a^2*b*c*d + 64*a*b^2*c*e) - 32*a^2*b^2*d + 128*a^2*c^2*d + 32*a*
b^3*e + 128*a^3*c*d - 32*a*b^2*c*d - 128*a^2*b*c*e) - tan(x/2)*(64*a^3*e^2
+ 32*a*b^2*d^2 - 64*a*b^2*e^2 - 128*a*c^2*d^2 - 64*a^2*c*d^2 + 128*a^2*c*e^
2 - 64*a^2*b*d*e + 128*a*b*c*d*e) + 32*a^2*b*e^2 + 32*a*b*c*d^2 - 128*a^2*c
*d*e) - (-(b^4*d^2 - b^4*e^2 + 8*a*c^3*d^2 - b*d^2*(-(4*a*c - b^2)^3)^(1/2)
 - 8*a^3*c*e^2 - b*e^2*(-(4*a*c - b^2)^3)^(1/2) + 2*a^2*b^2*e^2 + 8*a^2*c^2
*d^2 - 8*a^2*c^2*e^2 - 2*b^2*c^2*d^2 - 2*a*b^3*d*e + 2*a*d*e*(-(4*a*c - b^2
)^3)^(1/2) + 2*b^3*c*d*e + 2*c*d*e*(-(4*a*c - b^2)^3)^(1/2) - 6*a*b^2*c*d^2
 + 6*a*b^2*c*e^2 - 8*a*b*c^2*d*e + 8*a^2*b*c*d*e)/(2*(a^2*b^4 - b^6 + 16*a^
2*c^4 + 32*a^3*c^3 + 16*a^4*c^2 + b^4*c^2 - 8*a*b^2*c^3 - 8*a^3*b^2*c - 32*
a^2*b^2*c^2 + 10*a*b^4*c)))^(1/2)*((-(b^4*d^2 - b^4*e^2 + 8*a*c^3*d^2 - b*d
^2*(-(4*a*c - b^2)^3)^(1/2) - 8*a^3*c*e^2 - b*e^2*(-(4*a*c - b^2)^3)^(1/2)
+ 2*a^2*b^2*e^2 + 8*a^2*c^2*d^2 - 8*a^2*c^2*e^2 - 2*b^2*c^2*d^2 - 2*a*b^3*d
*e + 2*a*d*e*(-(4*a*c - b^2)^3)^(1/2) + 2*b^3*c*d*e + 2*c*d*e*(-(4*a*c - b^
2)^3)^(1/2) - 6*a*b^2*c*d^2 + 6*a*b^2*c*e^2 - 8*a*b*c^2*d*e + 8*a^2*b*c*d*e
)/(2*(a^2*b^4 - b^6 + 16*a^2*c^4 + 32*a^3*c^3 + 16*a^4*c^2 + b^4*c^2 - 8*a*
b^2*c^3 - 8*a^3*b^2*c - 32*a^2*b^2*c^2 + 10*a*b^4*c)))^(1/2)*((-(b^4*d^2 -
b^4*e^2 + 8*a*c^3*d^2 - b*d^2*(-(4*a*c - b^2)^3)^(1/2) - 8*a^3*c*e^2 - b*e^
2*(-(4*a*c - b^2)^3)^(1/2) + 2*a^2*b^2*e^2 + 8*a^2*c^2*d^2 - 8*a^2*c^2*e^2
- 2*b^2*c^2*d^2 - 2*a*b^3*d*e + 2*a*d*e*(-(4*a*c - b^2)^3)^(1/2) + 2*b^3*c*
d*e + 2*c*d*e*(-(4*a*c - b^2)^3)^(1/2) - 6*a*b^2*c*d^2 + 6*a*b^2*c*e^2 - 8*
a*b*c^2*d*e + 8*a^2*b*c*d*e)/(2*(a^2*b^4 - b^6 + 16*a^2*c^4 + 32*a^3*c^3 +
16*a^4*c^2 + b^4*c^2 - 8*a*b^2*c^3 - 8*a^3*b^2*c - 32*a^2*b^2*c^2 + 10*a*b^
4*c)))^(1/2)*(tan(x/2)*(96*a*b^4 + 256*a^4*c - 64*a^3*b^2 + 512*a^2*c^3 + 7
68*a^3*c^2 - 128*a*b^2*c^2 - 576*a^2*b^2*c) + 32*a^2*b^3 + 128*a^2*b*c^2 -
32*a*b^3*c - 128*a^3*b*c) - tan(x/2)*(64*a^2*b^2*e - 256*a^2*c^2*e - 64*a*b
^3*d - 256*a^3*c*e + 256*a^2*b*c*d + 64*a*b^2*c*e) + 32*a^2*b^2*d - 128*a^2
*c^2*d - 32*a*b^3*e - 128*a^3*c*d + 32*a*b^2*c*d + 128*a^2*b*c*e) - tan(x/2
)*(64*a^3*e^2 + 32*a*b^2*d^2 - 64*a*b^2*e^2 - 128*a*c^2*d^2 - 64*a^2*c*d^2
+ 128*a^2*c*e^2 - 64*a^2*b*d*e + 128*a*b*c*d*e) + 32*a^2*b*e^2 + 32*a*b*c*d
^2 - 128*a^2*c*d*e) + 64*a^2*d*e^2 + 64*a*c*d^3 - 64*a*b*d^2*e))*(-(b^4*d^2
 - b^4*e^2 + 8*a*c^3*d^2 - b*d^2*(-(4*a*c - b^2)^3)^(1/2) - 8*a^3*c*e^2 - b
*e^2*(-(4*a*c - b^2)^3)^(1/2) + 2*a^2*b^2*e^2 + 8*a^2*c^2*d^2 - 8*a^2*c^2*e
^2 - 2*b^2*c^2*d^2 - 2*a*b^3*d*e + 2*a*d*e*(-(4*a*c - b^2)^3)^(1/2) + 2*b^3
*c*d*e + 2*c*d*e*(-(4*a*c - b^2)^3)^(1/2) - 6*a*b^2*c*d^2 + 6*a*b^2*c*e^2 -
 8*a*b*c^2*d*e + 8*a^2*b*c*d*e)/(2*(a^2*b^4 - b^6 + 16*a^2*c^4 + 32*a^3*c^3
 + 16*a^4*c^2 + b^4*c^2 - 8*a*b^2*c^3 - 8*a^3*b^2*c - 32*a^2*b^2*c^2 + 10*a
*b^4*c)))^(1/2)*2i
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*sin(x))/(a+b*sin(x)+c*sin(x)**2),x)
```

```
[Out] Timed out
```

3.504 $\int (a+b\sin(d+ex)) \left(b^2 + 2ab\sin(d + ex) + a^2 \sin^2(d + e$

Optimal. Leaf size=331

$$-\frac{b\cos(d + ex)\left(a^2 \sin^2(d + ex) + 2ab\sin(d + ex) + b^2\right)^{3/2}}{4e}-\frac{\left(4a^2 + 3b^2\right)\cos(d + ex)\left(a^2 \sin^2(d + ex) + 2ab\sin(d +\right.}{12e(a\sin(d + ex) + b)}$$

```
[Out] -1/4*b*cos(e*x+d)*(b^2+2*a*b*sin(e*x+d)+a^2*sin(e*x+d)^2)^(3/2)/e-1/6*(4*a^
4+28*a^2*b^2+3*b^4)*cos(e*x+d)*(b^2+2*a*b*sin(e*x+d)+a^2*sin(e*x+d)^2)^(3/2
)/e/(b+a*sin(e*x+d))^3-1/12*(4*a^2+3*b^2)*cos(e*x+d)*(b^2+2*a*b*sin(e*x+d)+
a^2*sin(e*x+d)^2)^(3/2)/e/(b+a*sin(e*x+d))+5/8*a^4*b*(3*a^2+4*b^2)*x*(b^2+2
*a*b*sin(e*x+d)+a^2*sin(e*x+d)^2)^(3/2)/(a*b+a^2*sin(e*x+d))^3-1/24*a^4*b*(
29*a^2+6*b^2)*cos(e*x+d)*sin(e*x+d)*(b^2+2*a*b*sin(e*x+d)+a^2*sin(e*x+d)^2)
^(3/2)/e/(a*b+a^2*sin(e*x+d))^3
```

Rubi [A] time = 0.32, antiderivative size = 331, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.073, Rules used = {3290, 2753, 2734}

$$\frac{5a^4bx\left(3a^2 + 4b^2\right)\left(a^2 \sin^2(d + ex) + 2ab\sin(d + ex) + b^2\right)^{3/2}}{8\left(a^2 \sin(d + ex) + ab\right)^3}-\frac{a^4b\left(29a^2 + 6b^2\right)\sin(d + ex)\cos(d + ex)\left(a^2 \sin^2(d\right.}{24e\left(a^2 \sin(d + ex) + a\right.}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[d + e*x])*(b^2 + 2*a*b*Sin[d + e*x] + a^2*Sin[d + e*x]^2)^(3
/2),x]
```

```
[Out] -(b*Cos[d + e*x]*(b^2 + 2*a*b*Sin[d + e*x] + a^2*Sin[d + e*x]^2)^(3/2))/(4*
e) - ((4*a^4 + 28*a^2*b^2 + 3*b^4)*Cos[d + e*x]*(b^2 + 2*a*b*Sin[d + e*x] +
 a^2*Sin[d + e*x]^2)^(3/2))/(6*e*(b + a*Sin[d + e*x])^3) - ((4*a^2 + 3*b^2)
*Cos[d + e*x]*(b^2 + 2*a*b*Sin[d + e*x] + a^2*Sin[d + e*x]^2)^(3/2))/(12*e*
(b + a*Sin[d + e*x])) + (5*a^4*b*(3*a^2 + 4*b^2)*x*(b^2 + 2*a*b*Sin[d + e*x
] + a^2*Sin[d + e*x]^2)^(3/2))/(8*(a*b + a^2*Sin[d + e*x])^3) - (a^4*b*(29*
a^2 + 6*b^2)*Cos[d + e*x]*Sin[d + e*x]*(b^2 + 2*a*b*Sin[d + e*x] + a^2*Sin[
d + e*x]^2)^(3/2))/(24*e*(a*b + a^2*Sin[d + e*x])^3)
```

Rule 2734

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[((2*a*c + b*d)*x)/2, x] + (-Simp[((b*c + a*d)*Co
s[e + f*x])/f, x] - Simp[(b*d*Cos[e + f*x]*Sin[e + f*x])/(2*f), x]) /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2753

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[1/(m + 1), Int[(a + b*Sin[e + f*x])^(m - 1)*Simp[b*d*m
 + a*c*(m + 1) + (a*d*m + b*c*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a
, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && GtQ[m, 0]
 && IntegerQ[2*m]
```

Rule 3290

```
Int[((A_) + (B_.)*sin[(d_.) + (e_.)*(x_)])*((a_) + (b_.)*sin[(d_.) + (e_.)*
(x_)] + (c_.)*sin[(d_.) + (e_.)*(x_)]^2)^(n_), x_Symbol] :> Dist[(a + b*Sin
[d + e*x] + c*Sin[d + e*x]^2)^n/(b + 2*c*Sin[d + e*x])^(2*n), Int[(A + B*Si
n[d + e*x])*(b + 2*c*Sin[d + e*x])^(2*n), x], x] /; FreeQ[{a, b, c, d, e, A
, B}, x] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[n]
```

Rubi steps

$$\int (a+b\sin(d+ex))\left(b^2+2ab\sin(d+ex)+a^2\sin^2(d+ex)\right)^{3/2}\,dx=\frac{\left(b^2+2ab\sin(d+ex)+a^2\sin^2(d+ex)\right)^{3/2}}{\left(2ab+2\right.}$$

$$=-\frac{b\cos(d+ex)\left(b^2+2ab\sin(d+ex)+a^2\right.}{4e}$$

$$=-\frac{b\cos(d+ex)\left(b^2+2ab\sin(d+ex)+a^2\right.}{4e}$$

$$=-\frac{b\cos(d+ex)\left(b^2+2ab\sin(d+ex)+a^2\right.}{4e}$$

Mathematica [A] time = 0.84, size = 140, normalized size = 0.42

$$\frac{\sqrt{(a\sin(d+ex)+b)^2}\left(8a\left(a^3+3ab^2\right)\cos(3(d+ex))+3ab\left(20\left(3a^2+4b^2\right)(d+ex)-8\left(4a^2+3b^2\right)\sin(2(d+e\right.\right.\right.}{96e(a\sin(d+ex)+b)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sin[d + e*x])*(b^2 + 2*a*b*Sin[d + e*x] + a^2*Sin[d + e*x]
^2)^(3/2),x]
```

```
[Out] (Sqrt[(b + a*Sin[d + e*x])^2]*(-24*(3*a^4 + 21*a^2*b^2 + 4*b^4)*Cos[d + e*x
] + 8*a*(a^3 + 3*a*b^2)*Cos[3*(d + e*x)] + 3*a*b*(20*(3*a^2 + 4*b^2)*(d + e
*x) - 8*(4*a^2 + 3*b^2)*Sin[2*(d + e*x)] + a^2*Sin[4*(d + e*x)])))/(96*e*(b
 + a*Sin[d + e*x]))
```

fricas [A] time = 0.85, size = 112, normalized size = 0.34

$$\frac{8\left(a^{4}+3\,a^{2}b^{2}\right)\cos\left(ex+d\right)^{3}+15\left(3\,a^{3}b+4\,ab^{3}\right)ex-24\left(a^{4}+6\,a^{2}b^{2}+b^{4}\right)\cos\left(ex+d\right)+3\left(2\,a^{3}b\cos\left(ex+d\right)^{3}-\right.}{24\,e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(e*x+d))*(b^2+2*a*b*sin(e*x+d)+a^2*sin(e*x+d)^2)^(3/2),x,
 algorithm="fricas")
```

```
[Out] 1/24*(8*(a^4 + 3*a^2*b^2)*cos(e*x + d)^3 + 15*(3*a^3*b + 4*a*b^3)*e*x - 24*
(a^4 + 6*a^2*b^2 + b^4)*cos(e*x + d) + 3*(2*a^3*b*cos(e*x + d)^3 - (17*a^3*
b + 12*a*b^3)*cos(e*x + d))*sin(e*x + d))/e
```

giac [A] time = 5.03, size = 239, normalized size = 0.72

$$\frac{1}{32}\,a^{3}be^{(-1)}\mathrm{sgn}\left(a\sin\left(xe+d\right)+b\right)\sin\left(4\,xe+4\,d\right)+\frac{1}{12}\left(a^{4}\mathrm{sgn}\left(a\sin\left(xe+d\right)+b\right)+3\,a^{2}b^{2}\mathrm{sgn}\left(a\sin\left(xe+d\right)+b\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(e*x+d))*(b^2+2*a*b*sin(e*x+d)+a^2*sin(e*x+d)^2)^(3/2),x,
 algorithm="giac")
```

```
[Out] 1/32*a^3*b*e^(-1)*sgn(a*sin(x*e + d) + b)*sin(4*x*e + 4*d) + 1/12*(a^4*sgn(
a*sin(x*e + d) + b) + 3*a^2*b^2*sgn(a*sin(x*e + d) + b))*cos(3*x*e + 3*d)*e
^(-1) - 1/4*(3*a^4*sgn(a*sin(x*e + d) + b) + 21*a^2*b^2*sgn(a*sin(x*e + d)
+ b) + 4*b^4*sgn(a*sin(x*e + d) + b))*cos(x*e + d)*e^(-1) - 1/4*(4*a^3*b*sg
n(a*sin(x*e + d) + b) + 3*a*b^3*sgn(a*sin(x*e + d) + b))*e^(-1)*sin(2*x*e +
 2*d) + 5/8*(3*a^3*b*sgn(a*sin(x*e + d) + b) + 4*a*b^3*sgn(a*sin(x*e + d) +
 b))*x
```

maple [A] time = 0.79, size = 269, normalized size = 0.81

$$-\frac{\left(-a^{2}\left(\cos^{2}\left(ex+d\right)\right)+2ab\sin\left(ex+d\right)+a^{2}+b^{2}\right)^{\frac{3}{2}}\left(6\left(\cos^{3}\left(ex+d\right)\right)\sin\left(ex+d\right)a^{3}b+8a^{4}\left(\cos^{3}\left(ex+d\right)\right)+24\right.}{24e\left(\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(e*x+d))*(b^2+2*a*b*sin(e*x+d)+a^2*sin(e*x+d)^2)^(3/2),x)
```

```
[Out] -1/24/e*(-a^2*cos(e*x+d)^2+2*a*b*sin(e*x+d)+a^2+b^2)^(3/2)*(6*cos(e*x+d)^3*
sin(e*x+d)*a^3*b+8*a^4*cos(e*x+d)^3+24*a^2*b^2*cos(e*x+d)^3-51*sin(e*x+d)*c
os(e*x+d)*a^3*b-36*cos(e*x+d)*sin(e*x+d)*a*b^3-24*a^4*cos(e*x+d)-144*a^2*b^
2*cos(e*x+d)-24*cos(e*x+d)*b^4+45*(e*x+d)*a^3*b+60*(e*x+d)*a*b^3-16*a^4-120
*a^2*b^2-24*b^4)/(cos(e*x+d)^2*sin(e*x+d)*a^3+3*cos(e*x+d)^2*a^2*b-a^3*sin(
e*x+d)-3*sin(e*x+d)*a*b^2-3*a^2*b-b^3)
```

maxima [A] time = 0.45, size = 556, normalized size = 1.68

$$4\left(3\left(3\,a^2b+2\,b^3\right)\arctan\left(\frac{\sin(ex+d)}{\cos(ex+d)+1}\right)-\frac{4\,a^3+18\,ab^2+\frac{9\,a^2b\sin(ex+d)}{\cos(ex+d)+1}+\frac{18\,ab^2\sin(ex+d)^4}{(\cos(ex+d)+1)^4}-\frac{9\,a^2b\sin(ex+d)^5}{(\cos(ex+d)+1)^5}+\frac{12\left(a^3+3\,ab^2\right)\sin(ex+d)^2}{(\cos(ex+d)+1)^2}}{\frac{3\,\sin(ex+d)^2}{(\cos(ex+d)+1)^2}+\frac{3\,\sin(ex+d)^4}{(\cos(ex+d)+1)^4}+\frac{\sin(ex+d)^6}{(\cos(ex+d)+1)^6}+1}\right)a+3\left(\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(e*x+d))*(b^2+2*a*b*sin(e*x+d)+a^2*sin(e*x+d)^2)^(3/2),x,
 algorithm="maxima")
```

```
[Out] 1/12*(4*(3*(3*a^2*b + 2*b^3)*arctan(sin(e*x + d)/(cos(e*x + d) + 1)) - (4*a
^3 + 18*a*b^2 + 9*a^2*b*sin(e*x + d)/(cos(e*x + d) + 1) + 18*a*b^2*sin(e*x
+ d)^4/(cos(e*x + d) + 1)^4 - 9*a^2*b*sin(e*x + d)^5/(cos(e*x + d) + 1)^5 +
 12*(a^3 + 3*a*b^2)*sin(e*x + d)^2/(cos(e*x + d) + 1)^2)/(3*sin(e*x + d)^2/
(cos(e*x + d) + 1)^2 + 3*sin(e*x + d)^4/(cos(e*x + d) + 1)^4 + sin(e*x + d)
^6/(cos(e*x + d) + 1)^6 + 1))*a + 3*(3*(a^3 + 4*a*b^2)*arctan(sin(e*x + d)/
(cos(e*x + d) + 1)) - (16*a^2*b + 8*b^3 + 8*b^3*sin(e*x + d)^6/(cos(e*x + d
) + 1)^6 + 3*(a^3 + 4*a*b^2)*sin(e*x + d)/(cos(e*x + d) + 1) + 8*(8*a^2*b +
 3*b^3)*sin(e*x + d)^2/(cos(e*x + d) + 1)^2 + (11*a^3 + 12*a*b^2)*sin(e*x +
 d)^3/(cos(e*x + d) + 1)^3 + 24*(2*a^2*b + b^3)*sin(e*x + d)^4/(cos(e*x + d
) + 1)^4 - (11*a^3 + 12*a*b^2)*sin(e*x + d)^5/(cos(e*x + d) + 1)^5 - 3*(a^3
 + 4*a*b^2)*sin(e*x + d)^7/(cos(e*x + d) + 1)^7)/(4*sin(e*x + d)^2/(cos(e*x
 + d) + 1)^2 + 6*sin(e*x + d)^4/(cos(e*x + d) + 1)^4 + 4*sin(e*x + d)^6/(co
s(e*x + d) + 1)^6 + sin(e*x + d)^8/(cos(e*x + d) + 1)^8 + 1))*b)/e
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int (a+b\,\sin(d+e\,x))\,\left(a^2\sin(d+e\,x)^2+2\,a\,b\,\sin(d+e\,x)+b^2\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(d + e*x))*(b^2 + a^2*sin(d + e*x)^2 + 2*a*b*sin(d + e*x))^(3
/2),x)
```

```
[Out] int((a + b*sin(d + e*x))*(b^2 + a^2*sin(d + e*x)^2 + 2*a*b*sin(d + e*x))^(3
/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(e*x+d))*(b**2+2*a*b*sin(e*x+d)+a**2*sin(e*x+d)**2)**(3/2
),x)
```

```
[Out] Timed out
```

3.505 $\int (a+b\sin(d+ex))\sqrt{b^2+2ab\sin(d+ex)+a^2\sin^2(d+$

Optimal. Leaf size=185

$$\frac{3a^2bx\sqrt{a^2\sin^2(d+ex)+2ab\sin(d+ex)+b^2}}{2\left(a^2\sin(d+ex)+ab\right)}-\frac{a^2b\sin(d+ex)\cos(d+ex)\sqrt{a^2\sin^2(d+ex)+2ab\sin(d+ex)+}}{2e\left(a^2\sin(d+ex)+ab\right)}$$

```
[Out] -(a^2+b^2)*cos(e*x+d)*(b^2+2*a*b*sin(e*x+d)+a^2*sin(e*x+d)^2)^(1/2)/e/(b+a*
sin(e*x+d))+3/2*a^2*b*x*(b^2+2*a*b*sin(e*x+d)+a^2*sin(e*x+d)^2)^(1/2)/(a*b+
a^2*sin(e*x+d))-1/2*a^2*b*cos(e*x+d)*sin(e*x+d)*(b^2+2*a*b*sin(e*x+d)+a^2*s
in(e*x+d)^2)^(1/2)/e/(a*b+a^2*sin(e*x+d))
```

Rubi [A] time = 0.11, antiderivative size = 185, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.049, Rules used = {3290, 2734}

$$\frac{3a^2bx\sqrt{a^2\sin^2(d+ex)+2ab\sin(d+ex)+b^2}}{2\left(a^2\sin(d+ex)+ab\right)}-\frac{a^2b\sin(d+ex)\cos(d+ex)\sqrt{a^2\sin^2(d+ex)+2ab\sin(d+ex)+}}{2e\left(a^2\sin(d+ex)+ab\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[d + e*x])*Sqrt[b^2 + 2*a*b*Sin[d + e*x] + a^2*Sin[d + e*x]^2
],x]
```

```
[Out] -(((a^2 + b^2)*Cos[d + e*x]*Sqrt[b^2 + 2*a*b*Sin[d + e*x] + a^2*Sin[d + e*x
]^2])/(e*(b + a*Sin[d + e*x]))) + (3*a^2*b*x*Sqrt[b^2 + 2*a*b*Sin[d + e*x]
+ a^2*Sin[d + e*x]^2])/(2*(a*b + a^2*Sin[d + e*x])) - (a^2*b*Cos[d + e*x]*S
in[d + e*x]*Sqrt[b^2 + 2*a*b*Sin[d + e*x] + a^2*Sin[d + e*x]^2])/(2*e*(a*b
+ a^2*Sin[d + e*x]))
```

Rule 2734

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[((2*a*c + b*d)*x)/2, x] + (-Simp[((b*c + a*d)*Co
s[e + f*x])/f, x] - Simp[(b*d*Cos[e + f*x]*Sin[e + f*x])/(2*f), x]) /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 3290

```
Int[((A_) + (B_.)*sin[(d_.) + (e_.)*(x_)])*((a_) + (b_.)*sin[(d_.) + (e_.)*
(x_)] + (c_.)*sin[(d_.) + (e_.)*(x_)]^2)^(n_), x_Symbol] :> Dist[(a + b*Sin
[d + e*x] + c*Sin[d + e*x]^2)^n/(b + 2*c*Sin[d + e*x])^(2*n), Int[(A + B*Si
n[d + e*x])*(b + 2*c*Sin[d + e*x])^(2*n), x], x] /; FreeQ[{a, b, c, d, e, A
, B}, x] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[n]
```

Rubi steps

$$\int (a+b\sin(d+ex))\sqrt{b^2+2ab\sin(d+ex)+a^2\sin^2(d+ex)}\,dx = \frac{\sqrt{b^2+2ab\sin(d+ex)+a^2\sin^2(d+ex)}\int \left(2a\right.}{2ab+2a^2\sin}$$
$$= -\frac{\left(a^2+b^2\right)\cos(d+ex)\sqrt{b^2+2ab\sin(d+ex)}}{e(b+a\sin(d+ex))}$$

Mathematica [A] time = 0.19, size = 70, normalized size = 0.38

$$-\frac{\sqrt{(a\sin(d+ex)+b)^2}\left(4\left(a^2+b^2\right)\cos(d+ex)+ab(\sin(2(d+ex))-6(d+ex))\right)}{4e(a\sin(d+ex)+b)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sin[d + e*x])*Sqrt[b^2 + 2*a*b*Sin[d + e*x] + a^2*Sin[d +
e*x]^2],x]
```

```
[Out] -1/4*(Sqrt[(b + a*Sin[d + e*x])^2]*(4*(a^2 + b^2)*Cos[d + e*x] + a*b*(-6*(d
 + e*x) + Sin[2*(d + e*x)])))/(e*(b + a*Sin[d + e*x]))
```

fricas [A] time = 0.87, size = 43, normalized size = 0.23

$$\frac{3\,abex-ab\cos\left(ex+d\right)\sin\left(ex+d\right)-2\left(a^2+b^2\right)\cos\left(ex+d\right)}{2\,e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(e*x+d))*(b^2+2*a*b*sin(e*x+d)+a^2*sin(e*x+d)^2)^(1/2),x,
 algorithm="fricas")
```

```
[Out] 1/2*(3*a*b*e*x - a*b*cos(e*x + d)*sin(e*x + d) - 2*(a^2 + b^2)*cos(e*x + d)
)/e
```

giac [A] time = 0.21, size = 98, normalized size = 0.53

$$-a^2\cos\left(xe+d\right)e^{(-1)}\mathrm{sgn}\left(a\sin\left(xe+d\right)+b\right)-b^2\cos\left(xe+d\right)e^{(-1)}\mathrm{sgn}\left(a\sin\left(xe+d\right)+b\right)-\frac{1}{4}\,abe^{(-1)}\mathrm{sgn}\left(a\sin\left(xe+\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(e*x+d))*(b^2+2*a*b*sin(e*x+d)+a^2*sin(e*x+d)^2)^(1/2),x,
 algorithm="giac")
```

```
[Out] -a^2*cos(x*e + d)*e^(-1)*sgn(a*sin(x*e + d) + b) - b^2*cos(x*e + d)*e^(-1)*
sgn(a*sin(x*e + d) + b) - 1/4*a*b*e^(-1)*sgn(a*sin(x*e + d) + b)*sin(2*x*e
+ 2*d) + 3/2*a*b*x*sgn(a*sin(x*e + d) + b)
```

maple [A] time = 0.59, size = 107, normalized size = 0.58

$$-\frac{\sqrt{-a^2\left(\cos^2\left(ex+d\right)\right)+2ab\sin\left(ex+d\right)+a^2+b^2}\,\left(\sin\left(ex+d\right)\cos\left(ex+d\right)ab+2a^2\cos\left(ex+d\right)+2\cos\left(ex+\right.\right.}{2e\left(b+a\sin\left(ex+d\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(e*x+d))*(b^2+2*a*b*sin(e*x+d)+a^2*sin(e*x+d)^2)^(1/2),x)
```

```
[Out] -1/2/e*(-a^2*cos(e*x+d)^2+2*a*b*sin(e*x+d)+a^2+b^2)^(1/2)*(sin(e*x+d)*cos(e
*x+d)*a*b+2*a^2*cos(e*x+d)+2*cos(e*x+d)*b^2-3*(e*x+d)*a*b+2*a^2+2*b^2)/(b+a
*sin(e*x+d))
```

maxima [A] time = 0.43, size = 187, normalized size = 1.01

$$\frac{2\left(b\arctan\left(\frac{\sin(ex+d)}{\cos(ex+d)+1}\right)-\frac{a}{\frac{\sin(ex+d)^2}{(\cos(ex+d)+1)^2}+1}\right)a+\left(a\arctan\left(\frac{\sin(ex+d)}{\cos(ex+d)+1}\right)-\frac{2b+\frac{a\sin(ex+d)}{\cos(ex+d)+1}+\frac{2b\sin(ex+d)^2}{(\cos(ex+d)+1)^2}-\frac{a\sin(ex+d)^3}{(\cos(ex+d)+1)^3}}{\frac{2\sin(ex+d)^2}{(\cos(ex+d)+1)^2}+\frac{\sin(ex+d)^4}{(\cos(ex+d)+1)^4}+1}\right)b}{e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(e*x+d))*(b^2+2*a*b*sin(e*x+d)+a^2*sin(e*x+d)^2)^(1/2),x,
 algorithm="maxima")
```

```
[Out] (2*(b*arctan(sin(e*x + d)/(cos(e*x + d) + 1)) - a/(sin(e*x + d)^2/(cos(e*x
+ d) + 1)^2 + 1))*a + (a*arctan(sin(e*x + d)/(cos(e*x + d) + 1)) - (2*b + a
*sin(e*x + d)/(cos(e*x + d) + 1) + 2*b*sin(e*x + d)^2/(cos(e*x + d) + 1)^2
- a*sin(e*x + d)^3/(cos(e*x + d) + 1)^3)/(2*sin(e*x + d)^2/(cos(e*x + d) +
1)^2 + sin(e*x + d)^4/(cos(e*x + d) + 1)^4 + 1))*b)/e
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int (a+b\,\sin(d+e\,x))\,\sqrt{a^2\sin(d+e\,x)^2+2\,a\,b\,\sin(d+e\,x)+b^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(d + e*x))*(b^2 + a^2*sin(d + e*x)^2 + 2*a*b*sin(d + e*x))^(1
/2),x)
```

```
[Out] int((a + b*sin(d + e*x))*(b^2 + a^2*sin(d + e*x)^2 + 2*a*b*sin(d + e*x))^(1
/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b\sin(d + ex))\sqrt{(a\sin(d + ex) + b)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(e*x+d))*(b**2+2*a*b*sin(e*x+d)+a**2*sin(e*x+d)**2)**(1/2
),x)
```

```
[Out] Integral((a + b*sin(d + e*x))*sqrt((a*sin(d + e*x) + b)**2), x)
```

3.506 $\int \frac{a+b\sin(d+ex)}{\sqrt{b^2+2ab\sin(d+ex)+a^2\sin^2(d+ex)}}\,dx$

Optimal. Leaf size=137

$$\frac{bx(a\sin(d+ex)+b)}{a\sqrt{a^2\sin^2(d+ex)+2ab\sin(d+ex)+b^2}}-\frac{2\sqrt{a^2-b^2}\,(a\sin(d+ex)+b)\tanh^{-1}\left(\frac{a+b\tan\left(\frac{1}{2}(d+ex)\right)}{\sqrt{a^2-b^2}}\right)}{ae\sqrt{a^2\sin^2(d+ex)+2ab\sin(d+ex)+b^2}}$$

```
[Out] b*x*(b+a*sin(e*x+d))/a/(b^2+2*a*b*sin(e*x+d)+a^2*sin(e*x+d)^2)^(1/2)-2*arct
anh((a+b*tan(1/2*e*x+1/2*d))/(a^2-b^2)^(1/2))*(b+a*sin(e*x+d))*(a^2-b^2)^(1
/2)/a/e/(b^2+2*a*b*sin(e*x+d)+a^2*sin(e*x+d)^2)^(1/2)
```

Rubi [A] time = 0.20, antiderivative size = 137, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.122, Rules used = {3290, 2735, 2660, 618, 206}

$$\frac{bx(a\sin(d+ex)+b)}{a\sqrt{a^2\sin^2(d+ex)+2ab\sin(d+ex)+b^2}}-\frac{2\sqrt{a^2-b^2}\,(a\sin(d+ex)+b)\tanh^{-1}\left(\frac{a+b\tan\left(\frac{1}{2}(d+ex)\right)}{\sqrt{a^2-b^2}}\right)}{ae\sqrt{a^2\sin^2(d+ex)+2ab\sin(d+ex)+b^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[d + e*x])/Sqrt[b^2 + 2*a*b*Sin[d + e*x] + a^2*Sin[d + e*x]^2
],x]
```

```
[Out] (b*x*(b + a*Sin[d + e*x]))/(a*Sqrt[b^2 + 2*a*b*Sin[d + e*x] + a^2*Sin[d + e
*x]^2]) - (2*Sqrt[a^2 - b^2]*ArcTanh[(a + b*Tan[(d + e*x)/2])/Sqrt[a^2 - b^
2]]*(b + a*Sin[d + e*x]))/(a*e*Sqrt[b^2 + 2*a*b*Sin[d + e*x] + a^2*Sin[d +
e*x]^2])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 3290

```
Int[((A_) + (B_.)*sin[(d_.) + (e_.)*(x_)])*((a_) + (b_.)*sin[(d_.) + (e_.)*
(x_)] + (c_.)*sin[(d_.) + (e_.)*(x_)]^2)^(n_), x_Symbol] :> Dist[(a + b*Sin
[d + e*x] + c*Sin[d + e*x]^2)^n/(b + 2*c*Sin[d + e*x])^(2*n), Int[(A + B*Si
n[d + e*x])*(b + 2*c*Sin[d + e*x])^(2*n), x], x] /; FreeQ[{a, b, c, d, e, A
, B}, x] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[n]
```

Rubi steps

$$\int \frac{a+b\sin(d+ex)}{\sqrt{b^2+2ab\sin(d+ex)+a^2\sin^2(d+ex)}}\,dx = \frac{\left(2ab+2a^2\sin(d+ex)\right)\int \frac{a+b\sin(d+ex)}{2ab+2a^2\sin(d+ex)}\,dx}{\sqrt{b^2+2ab\sin(d+ex)+a^2\sin^2(d+ex)}}$$

$$= \frac{bx(b+a\sin(d+ex))}{a\sqrt{b^2+2ab\sin(d+ex)+a^2\sin^2(d+ex)}} - \frac{\left(\left(-2a^3+2ab^2\right)\left(2a\right.\right.}{2a^2\sqrt{b^2+\ldots}}$$

$$= \frac{bx(b+a\sin(d+ex))}{a\sqrt{b^2+2ab\sin(d+ex)+a^2\sin^2(d+ex)}} - \frac{\left(\left(-2a^3+2ab^2\right)\left(2a\right.\right.}{\ldots}$$

$$= \frac{bx(b+a\sin(d+ex))}{a\sqrt{b^2+2ab\sin(d+ex)+a^2\sin^2(d+ex)}} + \frac{\left(2\left(-2a^3+2ab^2\right)\left(\right.\right.}{\ldots}$$

$$= \frac{bx(b+a\sin(d+ex))}{a\sqrt{b^2+2ab\sin(d+ex)+a^2\sin^2(d+ex)}} - \frac{2\sqrt{a^2-b^2}\,\tanh^{-1}\left(\ldots\right.}{ae\sqrt{b^2+2ab\ldots}}$$

Mathematica [A] time = 0.16, size = 85, normalized size = 0.62

$$\frac{(a\sin(d+ex)+b)\left(b(d+ex)-2\sqrt{b^2-a^2}\tan^{-1}\left(\frac{a+b\tan\left(\frac{1}{2}(d+ex)\right)}{\sqrt{b^2-a^2}}\right)\right)}{ae\sqrt{(a\sin(d+ex)+b)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sin[d + e*x])/Sqrt[b^2 + 2*a*b*Sin[d + e*x] + a^2*Sin[d +
e*x]^2],x]
```

```
[Out] ((b*(d + e*x) - 2*Sqrt[-a^2 + b^2]*ArcTan[(a + b*Tan[(d + e*x)/2])/Sqrt[-a^
2 + b^2]])*(b + a*Sin[d + e*x]))/(a*e*Sqrt[(b + a*Sin[d + e*x])^2])
```

fricas [A] time = 0.92, size = 204, normalized size = 1.49

$$\left[\frac{2\,bex+\sqrt{a^2-b^2}\,\log\left(-\frac{\left(a^2-2\,b^2\right)\cos(ex+d)^2+2\,ab\sin(ex+d)+a^2+b^2-2\,(b\cos(ex+d)\sin(ex+d)+a\cos(ex+d))\sqrt{a^2-b^2}}{a^2\cos(ex+d)^2-2\,ab\sin(ex+d)-a^2-b^2}\right)}{2\,ae},\ \frac{bex-\sqrt{-a^2+\cdots}}{\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(e*x+d))/(b^2+2*a*b*sin(e*x+d)+a^2*sin(e*x+d)^2)^(1/2),x,
 algorithm="fricas")
```

```
[Out] [1/2*(2*b*e*x + sqrt(a^2 - b^2)*log(-((a^2 - 2*b^2)*cos(e*x + d)^2 + 2*a*b*
sin(e*x + d) + a^2 + b^2 - 2*(b*cos(e*x + d)*sin(e*x + d) + a*cos(e*x + d))
*sqrt(a^2 - b^2))/(a^2*cos(e*x + d)^2 - 2*a*b*sin(e*x + d) - a^2 - b^2)))/(
a*e), (b*e*x - sqrt(-a^2 + b^2)*arctan(-sqrt(-a^2 + b^2)*(b*sin(e*x + d) +
a)/((a^2 - b^2)*cos(e*x + d))))/(a*e)]
```

giac [A] time = 0.38, size = 208, normalized size = 1.52

$$\left(\frac{\left(xe-2\,\pi\left\lfloor\frac{xe+d}{2\,\pi}+\frac{1}{2}\right\rfloor+d\right)b}{a\mathrm{sgn}\left(b\tan\left(\frac{1}{2}\,xe+\frac{1}{2}\,d\right)^4+2\,a\tan\left(\frac{1}{2}\,xe+\frac{1}{2}\,d\right)^3+2\,b\tan\left(\frac{1}{2}\,xe+\frac{1}{2}\,d\right)^2+2\,a\tan\left(\frac{1}{2}\,xe+\frac{1}{2}\,d\right)+b\right)}+\frac{\cdots}{\sqrt{-a^2+\cdots}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(e*x+d))/(b^2+2*a*b*sin(e*x+d)+a^2*sin(e*x+d)^2)^(1/2),x,
 algorithm="giac")
```

```
[Out] ((x*e - 2*pi*floor(1/2*(x*e + d)/pi + 1/2) + d)*b/(a*sgn(b*tan(1/2*x*e + 1/
2*d)^4 + 2*a*tan(1/2*x*e + 1/2*d)^3 + 2*b*tan(1/2*x*e + 1/2*d)^2 + 2*a*tan(
1/2*x*e + 1/2*d) + b)) + 2*(a^2 - b^2)*arctan((b*tan(1/2*x*e + 1/2*d) + a)/
sqrt(-a^2 + b^2))/(sqrt(-a^2 + b^2)*a*sgn(b*tan(1/2*x*e + 1/2*d)^4 + 2*a*ta
n(1/2*x*e + 1/2*d)^3 + 2*b*tan(1/2*x*e + 1/2*d)^2 + 2*a*tan(1/2*x*e + 1/2*d
) + b)))*e^(-1)
```

maple [A] time = 0.46, size = 176, normalized size = 1.28

$$-\frac{\left(2\arctan\left(\frac{b\cos(ex+d)-a\sin(ex+d)-b}{\sin(ex+d)\sqrt{-a^2+b^2}}\right)a^2-2\arctan\left(\frac{b\cos(ex+d)-a\sin(ex+d)-b}{\sin(ex+d)\sqrt{-a^2+b^2}}\right)b^2-b\left(ex+d\right)\sqrt{-a^2+b^2}\right)\left(b+a\sin\left(ex\right.\right.}{e\sqrt{-a^2\left(\cos^2\left(ex+d\right)\right)+2ab\sin\left(ex+d\right)+a^2+b^2}\;a\sqrt{-a^2+b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(e*x+d))/(b^2+2*a*b*sin(e*x+d)+a^2*sin(e*x+d)^2)^(1/2),x)
```

```
[Out] -1/e*(2*arctan((b*cos(e*x+d)-a*sin(e*x+d)-b)/sin(e*x+d)/(-a^2+b^2)^(1/2))*a
^2-2*arctan((b*cos(e*x+d)-a*sin(e*x+d)-b)/sin(e*x+d)/(-a^2+b^2)^(1/2))*b^2-
b*(e*x+d)*(-a^2+b^2)^(1/2))*(b+a*sin(e*x+d))/(-a^2*cos(e*x+d)^2+2*a*b*sin(e
*x+d)+a^2+b^2)^(1/2)/a/(-a^2+b^2)^(1/2)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(e*x+d))/(b^2+2*a*b*sin(e*x+d)+a^2*sin(e*x+d)^2)^(1/2),x,
 algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` for
more details)Is 4*a^2-4*b^2 positive or negative?
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{a+b\,\sin\left(d+e\,x\right)}{\sqrt{a^2\,{\sin\left(d+e\,x\right)}^2+2\,a\,b\,\sin\left(d+e\,x\right)+b^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(d + e*x))/(b^2 + a^2*sin(d + e*x)^2 + 2*a*b*sin(d + e*x))^(1
/2),x)
```

```
[Out] int((a + b*sin(d + e*x))/(b^2 + a^2*sin(d + e*x)^2 + 2*a*b*sin(d + e*x))^(1
/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a + b\sin(d + ex)}{\sqrt{(a\sin(d + ex) + b)^2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(e*x+d))/(b**2+2*a*b*sin(e*x+d)+a**2*sin(e*x+d)**2)**(1/2
),x)
```

```
[Out] Integral((a + b*sin(d + e*x))/sqrt((a*sin(d + e*x) + b)**2), x)
```

3.507 $\int \frac{a+b\sin(d+ex)}{\left(b^2+2ab\sin(d+ex)+a^2\sin^2(d+ex)\right)^{3/2}}\,dx$

Optimal. Leaf size=239

$$-\frac{\cos(d+ex)(a\sin(d+ex)+b)}{2e\left(a^2\sin^2(d+ex)+2ab\sin(d+ex)+b^2\right)^{3/2}}-\frac{\left(a^2\sin(d+ex)+ab\right)^3\tanh^{-1}\left(\frac{a+b\tan\left(\frac{1}{2}(d+ex)\right)}{\sqrt{a^2-b^2}}\right)}{a^2e\left(a^2-b^2\right)^{3/2}\left(a^2\sin^2(d+ex)+2ab\sin(d+ex)+b^2\right)^{3/2}}+\frac{}{2e\left(a\right.}$$

```
[Out] -1/2*cos(e*x+d)*(b+a*sin(e*x+d))/e/(b^2+2*a*b*sin(e*x+d)+a^2*sin(e*x+d)^2)^
(3/2)-arctanh((a+b*tan(1/2*e*x+1/2*d))/(a^2-b^2)^(1/2))*(a*b+a^2*sin(e*x+d)
)^3/a^2/(a^2-b^2)^(3/2)/e/(b^2+2*a*b*sin(e*x+d)+a^2*sin(e*x+d)^2)^(3/2)+1/2
*b*cos(e*x+d)*(a*b+a^2*sin(e*x+d))^3/(a^2-b^2)/e/(a^3*b+a^4*sin(e*x+d))/(b^
2+2*a*b*sin(e*x+d)+a^2*sin(e*x+d)^2)^(3/2)
```

Rubi [A] time = 0.27, antiderivative size = 239, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.146, Rules used = {3290, 2754, 12, 2660, 618, 206}

$$\frac{b\cos(d+ex)\left(a^2\sin(d+ex)+ab\right)^3}{2e\left(a^2-b^2\right)\left(a^3b+a^4\sin(d+ex)\right)\left(a^2\sin^2(d+ex)+2ab\sin(d+ex)+b^2\right)^{3/2}}-\frac{\cos(d+ex)(a\sin(d+ex)+}{2e\left(a^2\sin^2(d+ex)+2ab\sin(d+e\right.}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[d + e*x])/(b^2 + 2*a*b*Sin[d + e*x] + a^2*Sin[d + e*x]^2)^(3
/2),x]
```

```
[Out] -(Cos[d + e*x]*(b + a*Sin[d + e*x]))/(2*e*(b^2 + 2*a*b*Sin[d + e*x] + a^2*S
in[d + e*x]^2)^(3/2)) - (ArcTanh[(a + b*Tan[(d + e*x)/2])/Sqrt[a^2 - b^2]]*
(a*b + a^2*Sin[d + e*x])^3)/(a^2*(a^2 - b^2)^(3/2)*e*(b^2 + 2*a*b*Sin[d + e
*x] + a^2*Sin[d + e*x]^2)^(3/2)) + (b*Cos[d + e*x]*(a*b + a^2*Sin[d + e*x])
^3)/(2*(a^2 - b^2)*e*(a^3*b + a^4*Sin[d + e*x])*(b^2 + 2*a*b*Sin[d + e*x] +
 a^2*Sin[d + e*x]^2)^(3/2))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2754

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e + f
*x])^(m + 1))/(f*(m + 1)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(a^2 - b^2)), I
nt[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m +
 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a
*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rule 3290

```
Int[((A_) + (B_.)*sin[(d_.) + (e_.)*(x_)])*((a_) + (b_.)*sin[(d_.) + (e_.)*
(x_)] + (c_.)*sin[(d_.) + (e_.)*(x_)]^2)^(n_), x_Symbol] :> Dist[(a + b*Sin
[d + e*x] + c*Sin[d + e*x]^2)^n/(b + 2*c*Sin[d + e*x])^(2*n), Int[(A + B*Si
n[d + e*x])*(b + 2*c*Sin[d + e*x])^(2*n), x], x] /; FreeQ[{a, b, c, d, e, A
, B}, x] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[n]
```

Rubi steps

$$\int \frac{a+b\sin(d+ex)}{\left(b^2+2ab\sin(d+ex)+a^2\sin^2(d+ex)\right)^{3/2}}\,dx = \frac{\left(2ab+2a^2\sin(d+ex)\right)^3\int \frac{a+b\sin(d+ex)}{\left(2ab+2a^2\sin(d+ex)\right)^3}\,dx}{\left(b^2+2ab\sin(d+ex)+a^2\sin^2(d+ex)\right)^{3/2}}$$

$$= -\frac{\cos(d+ex)(b+a\sin(d+ex))}{2e\left(b^2+2ab\sin(d+ex)+a^2\sin^2(d+ex)\right)^{3/2}} + \frac{\left(2ab+\right.}{8a^2\left(a^2-b^2\right.}$$

$$= -\frac{\cos(d+ex)(b+a\sin(d+ex))}{2e\left(b^2+2ab\sin(d+ex)+a^2\sin^2(d+ex)\right)^{3/2}} + \frac{\left(2ab+2a^2\right.}{4a\left(b^2+2a\right.}$$

$$= -\frac{\cos(d+ex)(b+a\sin(d+ex))}{2e\left(b^2+2ab\sin(d+ex)+a^2\sin^2(d+ex)\right)^{3/2}} + \frac{}{2\left(a^2-b^2\right)e}$$

$$= -\frac{\cos(d+ex)(b+a\sin(d+ex))}{2e\left(b^2+2ab\sin(d+ex)+a^2\sin^2(d+ex)\right)^{3/2}} + \frac{}{2\left(a^2-b^2\right)e}$$

$$= -\frac{\cos(d+ex)(b+a\sin(d+ex))}{2e\left(b^2+2ab\sin(d+ex)+a^2\sin^2(d+ex)\right)^{3/2}} + \frac{}{2\left(a^2-b^2\right)e}$$

$$= -\frac{\cos(d+ex)(b+a\sin(d+ex))}{2e\left(b^2+2ab\sin(d+ex)+a^2\sin^2(d+ex)\right)^{3/2}} + \frac{}{2\left(a^2-b^2\right)e}$$

$$= -\frac{\cos(d+ex)(b+a\sin(d+ex))}{2e\left(b^2+2ab\sin(d+ex)+a^2\sin^2(d+ex)\right)^{3/2}} - \frac{\tanh^{-}}{a^2\left(a^2-b^2\right)}$$

<u>**Mathematica**</u> [**A**] time = 0.35, size = 144, normalized size = 0.60

$$\frac{\sqrt{b^2-a^2}\cos(d+ex)\left(a^2-ab\sin(d+ex)-2b^2\right)-2a(a\sin(d+ex)+b)^2\tan^{-1}\left(\frac{a+b\tan\left(\frac{1}{2}(d+ex)\right)}{\sqrt{b^2-a^2}}\right)}{2e(b-a)(a+b)\sqrt{b^2-a^2}\,(a\sin(d+ex)+b)\sqrt{(a\sin(d+ex)+b)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sin[d + e*x])/(b^2 + 2*a*b*Sin[d + e*x] + a^2*Sin[d + e*x]
^2)^(3/2),x]
```

```
[Out] (-2*a*ArcTan[(a + b*Tan[(d + e*x)/2])/Sqrt[-a^2 + b^2]]*(b + a*Sin[d + e*x]
)^2 + Sqrt[-a^2 + b^2]*Cos[d + e*x]*(a^2 - 2*b^2 - a*b*Sin[d + e*x]))/(2*(-
a + b)*(a + b)*Sqrt[-a^2 + b^2]*e*(b + a*Sin[d + e*x])*Sqrt[(b + a*Sin[d +
e*x])^2])
```

fricas [A] time = 1.65, size = 527, normalized size = 2.21

$$\left[-\frac{2\left(a^3b-ab^3\right)\cos\left(ex+d\right)\sin\left(ex+d\right)+\left(a^3\cos\left(ex+d\right)^2-2\,a^2b\sin\left(ex+d\right)-a^3-ab^2\right)\sqrt{a^2-b^2}\,\log\left(\frac{\left(a^2-2\,b^2\right)\cdots}{\cdots}\right.}{4\left(\left(a^6-2\,a^4b^2+a^2b^4\right)e\cos\left(ex+d\right)^2-2\left(a^5b-2\,a^3b^3\cdots\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(e*x+d))/(b^2+2*a*b*sin(e*x+d)+a^2*sin(e*x+d)^2)^(3/2),x,
 algorithm="fricas")
```

```
[Out] [-1/4*(2*(a^3*b - a*b^3)*cos(e*x + d)*sin(e*x + d) + (a^3*cos(e*x + d)^2 -
2*a^2*b*sin(e*x + d) - a^3 - a*b^2)*sqrt(a^2 - b^2)*log(((a^2 - 2*b^2)*cos(
e*x + d)^2 + 2*a*b*sin(e*x + d) + a^2 + b^2 + 2*(b*cos(e*x + d)*sin(e*x + d
) + a*cos(e*x + d))*sqrt(a^2 - b^2))/(a^2*cos(e*x + d)^2 - 2*a*b*sin(e*x +
d) - a^2 - b^2)) - 2*(a^4 - 3*a^2*b^2 + 2*b^4)*cos(e*x + d))/((a^6 - 2*a^4*
b^2 + a^2*b^4)*e*cos(e*x + d)^2 - 2*(a^5*b - 2*a^3*b^3 + a*b^5)*e*sin(e*x +
 d) - (a^6 - a^4*b^2 - a^2*b^4 + b^6)*e), -1/2*((a^3*b - a*b^3)*cos(e*x + d
)*sin(e*x + d) + (a^3*cos(e*x + d)^2 - 2*a^2*b*sin(e*x + d) - a^3 - a*b^2)*
sqrt(-a^2 + b^2)*arctan(-sqrt(-a^2 + b^2)*(b*sin(e*x + d) + a)/((a^2 - b^2)
*cos(e*x + d))) - (a^4 - 3*a^2*b^2 + 2*b^4)*cos(e*x + d))/((a^6 - 2*a^4*b^2
 + a^2*b^4)*e*cos(e*x + d)^2 - 2*(a^5*b - 2*a^3*b^3 + a*b^5)*e*sin(e*x + d)
 - (a^6 - a^4*b^2 - a^2*b^4 + b^6)*e)]
```

giac [B] time = 0.75, size = 479, normalized size = 2.00

$$\left(\frac{a\arctan\left(\frac{b\tan\left(\frac{1}{2}\cdots\right.}{\sqrt{-\cdots}}\right.}{\left(a^2\mathrm{sgn}\left(b\tan\left(\frac{1}{2}\,xe+\frac{1}{2}\,d\right)^4+2\,a\tan\left(\frac{1}{2}\,xe+\frac{1}{2}\,d\right)^3+2\,b\tan\left(\frac{1}{2}\,xe+\frac{1}{2}\,d\right)^2+2\,a\tan\left(\frac{1}{2}\,xe+\frac{1}{2}\,d\right)+b\right)-b^2\mathrm{sgn}\left(\cdots\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(e*x+d))/(b^2+2*a*b*sin(e*x+d)+a^2*sin(e*x+d)^2)^(3/2),x,
 algorithm="giac")
```

```
[Out] (a*arctan((b*tan(1/2*x*e + 1/2*d) + a)/sqrt(-a^2 + b^2))/((a^2*sgn(b*tan(1/
2*x*e + 1/2*d)^4 + 2*a*tan(1/2*x*e + 1/2*d)^3 + 2*b*tan(1/2*x*e + 1/2*d)^2
+ 2*a*tan(1/2*x*e + 1/2*d) + b) - b^2*sgn(b*tan(1/2*x*e + 1/2*d)^4 + 2*a*ta
```

```
n(1/2*x*e + 1/2*d)^3 + 2*b*tan(1/2*x*e + 1/2*d)^2 + 2*a*tan(1/2*x*e + 1/2*d
) + b))*sqrt(-a^2 + b^2)) - (2*a^3*b*tan(1/2*x*e + 1/2*d)^3 - 3*a*b^3*tan(1
/2*x*e + 1/2*d)^3 + 2*a^4*tan(1/2*x*e + 1/2*d)^2 - 3*a^2*b^2*tan(1/2*x*e +
1/2*d)^2 - 2*b^4*tan(1/2*x*e + 1/2*d)^2 + 2*a^3*b*tan(1/2*x*e + 1/2*d) - 5*
a*b^3*tan(1/2*x*e + 1/2*d) + a^2*b^2 - 2*b^4)/((a^2*b^2*sgn(b*tan(1/2*x*e +
 1/2*d)^4 + 2*a*tan(1/2*x*e + 1/2*d)^3 + 2*b*tan(1/2*x*e + 1/2*d)^2 + 2*a*t
an(1/2*x*e + 1/2*d) + b) - b^4*sgn(b*tan(1/2*x*e + 1/2*d)^4 + 2*a*tan(1/2*x
*e + 1/2*d)^3 + 2*b*tan(1/2*x*e + 1/2*d)^2 + 2*a*tan(1/2*x*e + 1/2*d) + b))
*(b*tan(1/2*x*e + 1/2*d)^2 + 2*a*tan(1/2*x*e + 1/2*d) + b)^2))*e^(-1)
```

maple [B] time = 0.41, size = 741, normalized size = 3.10

$$\frac{2\sin(ex+d)\left(\cos^2(ex+d)\right)\arctan\left(\frac{b\cos(ex+d)-a\sin(ex+d)-b}{\sin(ex+d)\sqrt{-a^2+b^2}}\right)a^4b^2+\sqrt{-a^2+b^2}\ \sin(ex+d)\left(\cos^2(ex+d)\right)a^5-2\sqrt{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(e*x+d))/(b^2+2*a*b*sin(e*x+d)+a^2*sin(e*x+d)^2)^(3/2),x)
```

```
[Out] 1/2/e*(2*sin(e*x+d)*cos(e*x+d)^2*arctan((b*cos(e*x+d)-a*sin(e*x+d)-b)/sin(e
*x+d)/(-a^2+b^2)^(1/2))*a^4*b^2+(-a^2+b^2)^(1/2)*sin(e*x+d)*cos(e*x+d)^2*a^
5-2*(-a^2+b^2)^(1/2)*sin(e*x+d)*cos(e*x+d)^2*a^3*b^2-(-a^2+b^2)^(1/2)*cos(e
*x+d)^3*a^2*b^3+6*cos(e*x+d)^2*arctan((b*cos(e*x+d)-a*sin(e*x+d)-b)/sin(e*x
+d)/(-a^2+b^2)^(1/2))*a^3*b^3-(-a^2+b^2)^(1/2)*sin(e*x+d)*cos(e*x+d)*a^3*b^
2+3*(-a^2+b^2)^(1/2)*sin(e*x+d)*cos(e*x+d)*a*b^4+3*(-a^2+b^2)^(1/2)*cos(e*x
+d)^2*a^4*b-6*(-a^2+b^2)^(1/2)*cos(e*x+d)^2*a^2*b^3-2*sin(e*x+d)*arctan((b*
cos(e*x+d)-a*sin(e*x+d)-b)/sin(e*x+d)/(-a^2+b^2)^(1/2))*a^4*b^2-6*sin(e*x+d
)*arctan((b*cos(e*x+d)-a*sin(e*x+d)-b)/sin(e*x+d)/(-a^2+b^2)^(1/2))*a^2*b^4
-(-a^2+b^2)^(1/2)*sin(e*x+d)*a^5-(-a^2+b^2)^(1/2)*sin(e*x+d)*a^3*b^2+6*(-a^
2+b^2)^(1/2)*sin(e*x+d)*a*b^4+2*(-a^2+b^2)^(1/2)*cos(e*x+d)*b^5-6*arctan((b
*cos(e*x+d)-a*sin(e*x+d)-b)/sin(e*x+d)/(-a^2+b^2)^(1/2))*a^3*b^3-2*arctan((
b*cos(e*x+d)-a*sin(e*x+d)-b)/sin(e*x+d)/(-a^2+b^2)^(1/2))*a*b^5-3*(-a^2+b^2
)^(1/2)*a^4*b+5*(-a^2+b^2)^(1/2)*a^2*b^3+2*(-a^2+b^2)^(1/2)*b^5)/(-a^2*cos(
e*x+d)^2+2*a*b*sin(e*x+d)+a^2+b^2)^(3/2)/(-a^2+b^2)^(1/2)/(a^2-b^2)/b^2
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(e*x+d))/(b^2+2*a*b*sin(e*x+d)+a^2*sin(e*x+d)^2)^(3/2),x,
 algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
```

```
elp (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` for
more details)Is 4*a^2-4*b^2 positive or negative?
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{a+b\,\sin\left(d+e\,x\right)}{{\left(a^2\,{\sin\left(d+e\,x\right)}^2+2\,a\,b\,\sin\left(d+e\,x\right)+b^2\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(d + e*x))/(b^2 + a^2*sin(d + e*x)^2 + 2*a*b*sin(d + e*x))^(3
/2),x)
```

```
[Out] int((a + b*sin(d + e*x))/(b^2 + a^2*sin(d + e*x)^2 + 2*a*b*sin(d + e*x))^(3
/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(e*x+d))/(b**2+2*a*b*sin(e*x+d)+a**2*sin(e*x+d)**2)**(3/2
),x)
```

```
[Out] Timed out
```

3.508 $\int \frac{a+b\cos(x)}{b^2+2ab\cos(x)+a^2\cos^2(x)}\,dx$

Optimal. Leaf size=11

$$\frac{\sin(x)}{a\cos(x)+b}$$

[Out] sin(x)/(b+a*cos(x))

Rubi [A] time = 0.08, antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {3289, 2754, 8}

$$\frac{\sin(x)}{a\cos(x)+b}$$

Antiderivative was successfully verified.

[In] Int[(a + b*Cos[x])/(b^2 + 2*a*b*Cos[x] + a^2*Cos[x]^2),x]

[Out] Sin[x]/(b + a*Cos[x])

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2754

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e + f
*x])^(m + 1))/(f*(m + 1)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(a^2 - b^2)), I
nt[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m +
 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a
*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rule 3289

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + cos[(d_.) + (e_.)*(x_)]^2*(c_.) + (a_)
)^(n_)*(cos[(d_.) + (e_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Dist[1/(4^n*c^n
), Int[(A + B*Cos[d + e*x])*(b + 2*c*Cos[d + e*x])^(2*n), x], x] /; FreeQ[{
a, b, c, d, e, A, B}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[n]
```

Rubi steps

$$\int \frac{a+b\cos(x)}{b^2+2ab\cos(x)+a^2\cos^2(x)}\,dx = \left(4a^2\right)\int \frac{a+b\cos(x)}{\left(2ab+2a^2\cos(x)\right)^2}\,dx$$
$$= \frac{\sin(x)}{b+a\cos(x)} + \frac{\int 0\,dx}{a^2-b^2}$$
$$= \frac{\sin(x)}{b+a\cos(x)}$$

Mathematica [A] time = 0.05, size = 11, normalized size = 1.00

$$\frac{\sin(x)}{a\cos(x)+b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[x])/(b^2 + 2*a*b*Cos[x] + a^2*Cos[x]^2),x]
```

```
[Out] Sin[x]/(b + a*Cos[x])
```

fricas [A] time = 0.78, size = 11, normalized size = 1.00

$$\frac{\sin(x)}{a\cos(x)+b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(x))/(b^2+2*a*b*cos(x)+a^2*cos(x)^2),x, algorithm="fricas
")
```

```
[Out] sin(x)/(a*cos(x) + b)
```

giac [B] time = 0.17, size = 32, normalized size = 2.91

$$-\frac{2\,\tan\left(\frac{1}{2}\,x\right)}{a\tan\left(\frac{1}{2}\,x\right)^2 - b\tan\left(\frac{1}{2}\,x\right)^2 - a - b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(x))/(b^2+2*a*b*cos(x)+a^2*cos(x)^2),x, algorithm="giac")
```

```
[Out] -2*tan(1/2*x)/(a*tan(1/2*x)^2 - b*tan(1/2*x)^2 - a - b)
```

maple [B] time = 0.09, size = 33, normalized size = 3.00

$$-\frac{2\tan\left(\frac{x}{2}\right)}{a\left(\tan^2\left(\frac{x}{2}\right)\right)-b\left(\tan^2\left(\frac{x}{2}\right)\right)-a-b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(x))/(b^2+2*a*b*cos(x)+a^2*cos(x)^2),x)
```

```
[Out] -2*tan(1/2*x)/(a*tan(1/2*x)^2-b*tan(1/2*x)^2-a-b)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(x))/(b^2+2*a*b*cos(x)+a^2*cos(x)^2),x, algorithm="maxima
")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` for
more details)Is 4*a^2-4*b^2 positive or negative?
```

mupad [B] time = 2.94, size = 24, normalized size = 2.18

$$\frac{2\tan\left(\frac{x}{2}\right)}{(b-a)\,\tan\left(\frac{x}{2}\right)^2+a+b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*cos(x))/(a^2*cos(x)^2 + b^2 + 2*a*b*cos(x)),x)
```

```
[Out] (2*tan(x/2))/(a + b - tan(x/2)^2*(a - b))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(x))/(b**2+2*a*b*cos(x)+a**2*cos(x)**2),x)
```

```
[Out] Timed out
```

3.509 $\int \frac{d+e\cos(x)}{a+b\cos(x)+c\cos^2(x)}\,dx$

Optimal. Leaf size=246

$$\frac{2\left(\frac{2cd-be}{\sqrt{b^2-4ac}}+e\right)\tan^{-1}\left(\frac{\tan\left(\frac{x}{2}\right)\sqrt{-\sqrt{b^2-4ac}+b-2c}}{\sqrt{-\sqrt{b^2-4ac}+b+2c}}\right)}{\sqrt{-\sqrt{b^2-4ac}+b-2c}\sqrt{-\sqrt{b^2-4ac}+b+2c}}+\frac{2\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\tan\left(\frac{x}{2}\right)\sqrt{\sqrt{b^2-4ac}+b-2c}}{\sqrt{\sqrt{b^2-4ac}+b+2c}}\right)}{\sqrt{\sqrt{b^2-4ac}+b-2c}\sqrt{\sqrt{b^2-4ac}+b+2c}}$$

```
[Out] 2*arctan((b-2*c-(-4*a*c+b^2)^(1/2))^(1/2)*tan(1/2*x)/(b+2*c-(-4*a*c+b^2)^(1
/2))^(1/2))*(e+(-b*e+2*c*d)/(-4*a*c+b^2)^(1/2))/(b-2*c-(-4*a*c+b^2)^(1/2))^
(1/2)/(b+2*c-(-4*a*c+b^2)^(1/2))^(1/2)+2*arctan((b-2*c+(-4*a*c+b^2)^(1/2))^
(1/2)*tan(1/2*x)/(b+2*c+(-4*a*c+b^2)^(1/2))^(1/2))*(e+(b*e-2*c*d)/(-4*a*c+b
^2)^(1/2))/(b-2*c+(-4*a*c+b^2)^(1/2))^(1/2)/(b+2*c+(-4*a*c+b^2)^(1/2))^(1/2
)
```

Rubi [A] time = 0.79, antiderivative size = 246, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3293, 2659, 205}

$$\frac{2\left(\frac{2cd-be}{\sqrt{b^2-4ac}}+e\right)\tan^{-1}\left(\frac{\tan\left(\frac{x}{2}\right)\sqrt{-\sqrt{b^2-4ac}+b-2c}}{\sqrt{-\sqrt{b^2-4ac}+b+2c}}\right)}{\sqrt{-\sqrt{b^2-4ac}+b-2c}\sqrt{-\sqrt{b^2-4ac}+b+2c}}+\frac{2\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\tan\left(\frac{x}{2}\right)\sqrt{\sqrt{b^2-4ac}+b-2c}}{\sqrt{\sqrt{b^2-4ac}+b+2c}}\right)}{\sqrt{\sqrt{b^2-4ac}+b-2c}\sqrt{\sqrt{b^2-4ac}+b+2c}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*Cos[x])/(a + b*Cos[x] + c*Cos[x]^2),x]
```

```
[Out] (2*(e + (2*c*d - b*e)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[b - 2*c - Sqrt[b^2 -
4*a*c]]*Tan[x/2])/Sqrt[b + 2*c - Sqrt[b^2 - 4*a*c]]])/(Sqrt[b - 2*c - Sqrt[
b^2 - 4*a*c]]*Sqrt[b + 2*c - Sqrt[b^2 - 4*a*c]]) + (2*(e - (2*c*d - b*e)/Sq
rt[b^2 - 4*a*c])*ArcTan[(Sqrt[b - 2*c + Sqrt[b^2 - 4*a*c]]*Tan[x/2])/Sqrt[b
 + 2*c + Sqrt[b^2 - 4*a*c]]])/(Sqrt[b - 2*c + Sqrt[b^2 - 4*a*c]]*Sqrt[b + 2
*c + Sqrt[b^2 - 4*a*c]])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
```

```
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 3293

```
Int[(cos[(d_.) + (e_.)*(x_)]*(B_.) + (A_))/((a_.) + cos[(d_.) + (e_.)*(x_)]
*(b_.) + cos[(d_.) + (e_.)*(x_)]^2*(c_.)), x_Symbol] :> Module[{q = Rt[b^2
- 4*a*c, 2]}, Dist[B + (b*B - 2*A*c)/q, Int[1/(b + q + 2*c*Cos[d + e*x]), x
], x] + Dist[B - (b*B - 2*A*c)/q, Int[1/(b - q + 2*c*Cos[d + e*x]), x], x]]
 /; FreeQ[{a, b, c, d, e, A, B}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{d+e\cos(x)}{a+b\cos(x)+c\cos^2(x)}\,dx = \left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\int \frac{1}{b+\sqrt{b^2-4ac}+2c\cos(x)}\,dx+\left(e+\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\int \frac{}{b-\sqrt{b}}$$

$$=\left(2\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\right)\text{Subst}\left(\int \frac{1}{b+2c+\sqrt{b^2-4ac}+\left(b-2c+\sqrt{b^2-4ac}\right)x^2}\,d\right.$$

$$=\frac{2\left(e+\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{b-2c-\sqrt{b^2-4ac}}\,\tan\left(\frac{x}{2}\right)}{\sqrt{b+2c-\sqrt{b^2-4ac}}}\right)}{\sqrt{b-2c-\sqrt{b^2-4ac}}\,\sqrt{b+2c-\sqrt{b^2-4ac}}}+\frac{2\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{b-2c+\sqrt{b^2}}}{\sqrt{b+2c+}}\right.}{\sqrt{b-2c+\sqrt{b^2-4ac}}\,\sqrt{b+2c+}}$$

Mathematica [A] time = 0.56, size = 241, normalized size = 0.98

$$\frac{\sqrt{2}\left(\frac{\left(e\left(\sqrt{b^2-4ac}-b\right)+2cd\right)\tanh^{-1}\left(\frac{\tan\left(\frac{x}{2}\right)\left(\sqrt{b^2-4ac}-b+2c\right)}{\sqrt{2b\sqrt{b^2-4ac}+4c(a+c)-2b^2}}\right)}{\sqrt{b\sqrt{b^2-4ac}+2c(a+c)-b^2}}-\frac{\left(e\left(\sqrt{b^2-4ac}+b\right)-2cd\right)\tanh^{-1}\left(\frac{\tan\left(\frac{x}{2}\right)\left(\sqrt{b^2-4ac}+b-2c\right)}{\sqrt{-2b\sqrt{b^2-4ac}+4c(a+c)-2b^2}}\right)}{\sqrt{-b\sqrt{b^2-4ac}+2c(a+c)-b^2}}\right)}{\sqrt{b^2-4ac}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*Cos[x])/(a + b*Cos[x] + c*Cos[x]^2),x]
```

```
[Out] (Sqrt[2]*(-(((-2*c*d + (b + Sqrt[b^2 - 4*a*c])*e)*ArcTanh[((b - 2*c + Sqrt[
b^2 - 4*a*c])*Tan[x/2])/Sqrt[-2*b^2 + 4*c*(a + c) - 2*b*Sqrt[b^2 - 4*a*c]]]
)/Sqrt[-b^2 + 2*c*(a + c) - b*Sqrt[b^2 - 4*a*c]]) + ((2*c*d + (-b + Sqrt[b^
2 - 4*a*c])*e)*ArcTanh[((-b + 2*c + Sqrt[b^2 - 4*a*c])*Tan[x/2])/Sqrt[-2*b^
```

```
2 + 4*c*(a + c) + 2*b*Sqrt[b^2 - 4*a*c]]])/Sqrt[-b^2 + 2*c*(a + c) + b*Sqrt
[b^2 - 4*a*c]]))/Sqrt[b^2 - 4*a*c]
```

fricas [B] time = 16.18, size = 6697, normalized size = 27.22

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*cos(x))/(a+b*cos(x)+c*cos(x)^2),x, algorithm="fricas")

[Out] 1/4*sqrt(2)*sqrt(-((b^2 - 2*a*c - 2*c^2)*d^2 - 2*(a*b - b*c)*d*e + (2*a^2 -
 b^2 + 2*a*c)*e^2 - (a^2*b^2 - b^4 - 4*a*c^3 - (8*a^2 - b^2)*c^2 - 2*(2*a^3
 - 3*a*b^2)*c)*sqrt((b^2*d^4 + b^2*e^4 - 4*(a*b + b*c)*d^3*e + 2*(2*a^2 + b
^2 + 4*a*c + 2*c^2)*d^2*e^2 - 4*(a*b + b*c)*d*e^3)/(a^4*b^2 - 2*a^2*b^4 + b
^6 - 4*a*c^5 - (16*a^2 - b^2)*c^4 - 12*(2*a^3 - a*b^2)*c^3 - 2*(8*a^4 - 11*
a^2*b^2 + b^4)*c^2 - 4*(a^5 - 3*a^3*b^2 + 2*a*b^4)*c)))/(a^2*b^2 - b^4 - 4*
a*c^3 - (8*a^2 - b^2)*c^2 - 2*(2*a^3 - 3*a*b^2)*c))*log(2*b*c^2*d^4 + 2*a*b
*c*e^4 - 2*(b^2*c + 2*a*c^2 + 2*c^3)*d^3*e + 6*(a*b*c + b*c^2)*d^2*e^2 - 2*
(2*a*c^2 + (2*a^2 + b^2)*c)*d*e^3 - ((4*a*c^4 + (8*a^2 - b^2)*c^3 + 2*(2*a^
3 - 3*a*b^2)*c^2 - (a^2*b^2 - b^4)*c)*d^2 + (a^2*b^3 - b^5 - 4*a*b*c^3 - (8
*a^2*b - b^3)*c^2 - 2*(2*a^3*b - 3*a*b^3)*c)*d*e - (a^3*b^2 - a*b^4 - 4*a^2
*c^3 - (8*a^3 - a*b^2)*c^2 - 2*(2*a^4 - 3*a^2*b^2)*c)*e^2)*sqrt((b^2*d^4 +
b^2*e^4 - 4*(a*b + b*c)*d^3*e + 2*(2*a^2 + b^2 + 4*a*c + 2*c^2)*d^2*e^2 - 4
*(a*b + b*c)*d*e^3)/(a^4*b^2 - 2*a^2*b^4 + b^6 - 4*a*c^5 - (16*a^2 - b^2)*c
^4 - 12*(2*a^3 - a*b^2)*c^3 - 2*(8*a^4 - 11*a^2*b^2 + b^4)*c^2 - 4*(a^5 - 3
*a^3*b^2 + 2*a*b^4)*c))*cos(x) + 1/2*sqrt(2)*(((a^2*b^4 - b^6 + 8*a*c^5 + 2
*(12*a^2 - b^2)*c^4 + 6*(4*a^3 - 3*a*b^2)*c^3 + (8*a^4 - 22*a^2*b^2 + 3*b^4
)*c^2 - 2*(3*a^3*b^2 - 4*a*b^4)*c)*d - (a^3*b^3 - a*b^5 + 4*a*b*c^4 + (4*a^
2*b - b^3)*c^3 - (4*a^3*b + 5*a*b^3)*c^2 - (4*a^4*b - 5*a^2*b^3 - b^5)*c)*e
)*sqrt((b^2*d^4 + b^2*e^4 - 4*(a*b + b*c)*d^3*e + 2*(2*a^2 + b^2 + 4*a*c +
2*c^2)*d^2*e^2 - 4*(a*b + b*c)*d*e^3)/(a^4*b^2 - 2*a^2*b^4 + b^6 - 4*a*c^5
- (16*a^2 - b^2)*c^4 - 12*(2*a^3 - a*b^2)*c^3 - 2*(8*a^4 - 11*a^2*b^2 + b^4
)*c^2 - 4*(a^5 - 3*a^3*b^2 + 2*a*b^4)*c))*sin(x) + ((b^4 - 4*a*b^2*c)*d^3 -
 3*(a*b^3 - 4*a*b*c^2 - (4*a^2*b - b^3)*c)*d^2*e + (2*a^2*b^2 + b^4 - 8*a^3
*c - 8*a*c^3 - 2*(8*a^2 - b^2)*c^2)*d*e^2 - (a*b^3 - 4*a*b*c^2 - (4*a^2*b -
 b^3)*c)*e^3)*sin(x))*sqrt(-((b^2 - 2*a*c - 2*c^2)*d^2 - 2*(a*b - b*c)*d*e
+ (2*a^2 - b^2 + 2*a*c)*e^2 - (a^2*b^2 - b^4 - 4*a*c^3 - (8*a^2 - b^2)*c^2
- 2*(2*a^3 - 3*a*b^2)*c)*sqrt((b^2*d^4 + b^2*e^4 - 4*(a*b + b*c)*d^3*e + 2*
(2*a^2 + b^2 + 4*a*c + 2*c^2)*d^2*e^2 - 4*(a*b + b*c)*d*e^3)/(a^4*b^2 - 2*a
^2*b^4 + b^6 - 4*a*c^5 - (16*a^2 - b^2)*c^4 - 12*(2*a^3 - a*b^2)*c^3 - 2*(8
*a^4 - 11*a^2*b^2 + b^4)*c^2 - 4*(a^5 - 3*a^3*b^2 + 2*a*b^4)*c)))/(a^2*b^2
- b^4 - 4*a*c^3 - (8*a^2 - b^2)*c^2 - 2*(2*a^3 - 3*a*b^2)*c)) + (b^2*c*d^4
+ a*b^2*e^4 - (b^3 + 2*a*b*c + 2*b*c^2)*d^3*e + 3*(a*b^2 + b^2*c)*d^2*e^2 -
 (2*a^2*b + b^3 + 2*a*b*c)*d*e^3)*cos(x)) - 1/4*sqrt(2)*sqrt(-((b^2 - 2*a*c
 - 2*c^2)*d^2 - 2*(a*b - b*c)*d*e + (2*a^2 - b^2 + 2*a*c)*e^2 - (a^2*b^2 -
```

b^4 - 4*a*c^3 - (8*a^2 - b^2)*c^2 - 2*(2*a^3 - 3*a*b^2)*c)*sqrt((b^2*d^4 + b^2*e^4 - 4*(a*b + b*c)*d^3*e + 2*(2*a^2 + b^2 + 4*a*c + 2*c^2)*d^2*e^2 - 4*(a*b + b*c)*d*e^3)/(a^4*b^2 - 2*a^2*b^4 + b^6 - 4*a*c^5 - (16*a^2 - b^2)*c^4 - 12*(2*a^3 - a*b^2)*c^3 - 2*(8*a^4 - 11*a^2*b^2 + b^4)*c^2 - 4*(a^5 - 3*a^3*b^2 + 2*a*b^4)*c)))/(a^2*b^2 - b^4 - 4*a*c^3 - (8*a^2 - b^2)*c^2 - 2*(2*a^3 - 3*a*b^2)*c))*log(2*b*c^2*d^4 + 2*a*b*c*e^4 - 2*(b^2*c + 2*a*c^2 + 2*c^3)*d^3*e + 6*(a*b*c + b*c^2)*d^2*e^2 - 2*(2*a*c^2 + (2*a^2 + b^2)*c)*d*e^3 - ((4*a*c^4 + (8*a^2 - b^2)*c^3 + 2*(2*a^3 - 3*a*b^2)*c^2 - (a^2*b^2 - b^4)*c)*d^2 + (a^2*b^3 - b^5 - 4*a*b*c^3 - (8*a^2*b - b^3)*c^2 - 2*(2*a^3*b - 3*a*b^3)*c)*d*e - (a^3*b^2 - a*b^4 - 4*a^2*c^3 - (8*a^3 - a*b^2)*c^2 - 2*(2*a^4 - 3*a^2*b^2)*c)*e^2)*sqrt((b^2*d^4 + b^2*e^4 - 4*(a*b + b*c)*d^3*e + 2*(2*a^2 + b^2 + 4*a*c + 2*c^2)*d^2*e^2 - 4*(a*b + b*c)*d*e^3)/(a^4*b^2 - 2*a^2*b^4 + b^6 - 4*a*c^5 - (16*a^2 - b^2)*c^4 - 12*(2*a^3 - a*b^2)*c^3 - 2*(8*a^4 - 11*a^2*b^2 + b^4)*c^2 - 4*(a^5 - 3*a^3*b^2 + 2*a*b^4)*c))*cos(x) - 1/2*sqrt(2)*(((a^2*b^4 - b^6 + 8*a*c^5 + 2*(12*a^2 - b^2)*c^4 + 6*(4*a^3 - 3*a*b^2)*c^3 + (8*a^4 - 22*a^2*b^2 + 3*b^4)*c^2 - 2*(3*a^3*b^2 - 4*a*b^4)*c)*d - (a^3*b^3 - a*b^5 + 4*a*b*c^4 + (4*a^2*b - b^3)*c^3 - (4*a^3*b + 5*a*b^3)*c^2 - (4*a^4*b - 5*a^2*b^3 - b^5)*c)*e)*sqrt((b^2*d^4 + b^2*e^4 - 4*(a*b + b*c)*d^3*e + 2*(2*a^2 + b^2 + 4*a*c + 2*c^2)*d^2*e^2 - 4*(a*b + b*c)*d*e^3)/(a^4*b^2 - 2*a^2*b^4 + b^6 - 4*a*c^5 - (16*a^2 - b^2)*c^4 - 12*(2*a^3 - a*b^2)*c^3 - 2*(8*a^4 - 11*a^2*b^2 + b^4)*c^2 - 4*(a^5 - 3*a^3*b^2 + 2*a*b^4)*c))*sin(x) + ((b^4 - 4*a*b^2*c)*d^3 - 3*(a*b^3 - 4*a*b*c^2 - (4*a^2*b - b^3)*c)*d^2*e + (2*a^2*b^2 + b^4 - 8*a^3*c - 8*a*c^3 - 2*(8*a^2 - b^2)*c^2)*d*e^2 - (a*b^3 - 4*a*b*c^2 - (4*a^2*b - b^3)*c)*e^3)*sin(x))*sqrt(-((b^2 - 2*a*c - 2*c^2)*d^2 - 2*(a*b - b*c)*d*e + (2*a^2 - b^2 + 2*a*c)*e^2 - (a^2*b^2 - b^4 - 4*a*c^3 - (8*a^2 - b^2)*c^2 - 2*(2*a^3 - 3*a*b^2)*c)*sqrt((b^2*d^4 + b^2*e^4 - 4*(a*b + b*c)*d^3*e + 2*(2*a^2 + b^2 + 4*a*c + 2*c^2)*d^2*e^2 - 4*(a*b + b*c)*d*e^3)/(a^4*b^2 - 2*a^2*b^4 + b^6 - 4*a*c^5 - (16*a^2 - b^2)*c^4 - 12*(2*a^3 - a*b^2)*c^3 - 2*(8*a^4 - 11*a^2*b^2 + b^4)*c^2 - 4*(a^5 - 3*a^3*b^2 + 2*a*b^4)*c)))/(a^2*b^2 - b^4 - 4*a*c^3 - (8*a^2 - b^2)*c^2 - 2*(2*a^3 - 3*a*b^2)*c)) + (b^2*c*d^4 + a*b^2*e^4 - (b^3 + 2*a*b*c + 2*b*c^2)*d^3*e + 3*(a*b^2 + b^2*c)*d^2*e^2 - (2*a^2*b + b^3 + 2*a*b*c)*d*e^3)*cos(x)) + 1/4*sqrt(2)*sqrt(-((b^2 - 2*a*c - 2*c^2)*d^2 - 2*(a*b - b*c)*d*e + (2*a^2 - b^2 + 2*a*c)*e^2 + (a^2*b^2 - b^4 - 4*a*c^3 - (8*a^2 - b^2)*c^2 - 2*(2*a^3 - 3*a*b^2)*c)*sqrt((b^2*d^4 + b^2*e^4 - 4*(a*b + b*c)*d^3*e + 2*(2*a^2 + b^2 + 4*a*c + 2*c^2)*d^2*e^2 - 4*(a*b + b*c)*d*e^3)/(a^4*b^2 - 2*a^2*b^4 + b^6 - 4*a*c^5 - (16*a^2 - b^2)*c^4 - 12*(2*a^3 - a*b^2)*c^3 - 2*(8*a^4 - 11*a^2*b^2 + b^4)*c^2 - 4*(a^5 - 3*a^3*b^2 + 2*a*b^4)*c)))/(a^2*b^2 - b^4 - 4*a*c^3 - (8*a^2 - b^2)*c^2 - 2*(2*a^3 - 3*a*b^2)*c))*log(-2*b*c^2*d^4 - 2*a*b*c*e^4 + 2*(b^2*c + 2*a*c^2 + 2*c^3)*d^3*e - 6*(a*b*c + b*c^2)*d^2*e^2 + 2*(2*a*c^2 + (2*a^2 + b^2)*c)*d*e^3 - ((4*a*c^4 + (8*a^2 - b^2)*c^3 + 2*(2*a^3 - 3*a*b^2)*c^2 - (a^2*b^2 - b^4)*c)*d^2 + (a^2*b^3 - b^5 - 4*a*b*c^3 - (8*a^2*b - b^3)*c^2 - 2*(2*a^3*b - 3*a*b^3)*c)*d*e - (a^3*b^2 - a*b^4 - 4*a^2*c^3 - (8*a^3 - a*b^2)*c^2 - 2*(2*a^4 - 3*a^2*b^2)*c)*e^2)*sqrt((b^2*d^4 + b^2*e^4 - 4*(a*b + b*c)*d^3*e + 2*(2*a^2 + b^2 + 4*a*c + 2*c^

2)*d^2*e^2 - 4*(a*b + b*c)*d*e^3)/(a^4*b^2 - 2*a^2*b^4 + b^6 - 4*a*c^5 - (1
6*a^2 - b^2)*c^4 - 12*(2*a^3 - a*b^2)*c^3 - 2*(8*a^4 - 11*a^2*b^2 + b^4)*c^
2 - 4*(a^5 - 3*a^3*b^2 + 2*a*b^4)*c))*cos(x) + 1/2*sqrt(2)*(((a^2*b^4 - b^6
 + 8*a*c^5 + 2*(12*a^2 - b^2)*c^4 + 6*(4*a^3 - 3*a*b^2)*c^3 + (8*a^4 - 22*a
^2*b^2 + 3*b^4)*c^2 - 2*(3*a^3*b^2 - 4*a*b^4)*c)*d - (a^3*b^3 - a*b^5 + 4*a
*b*c^4 + (4*a^2*b - b^3)*c^3 - (4*a^3*b + 5*a*b^3)*c^2 - (4*a^4*b - 5*a^2*b
^3 - b^5)*c)*e)*sqrt((b^2*d^4 + b^2*e^4 - 4*(a*b + b*c)*d^3*e + 2*(2*a^2 +
b^2 + 4*a*c + 2*c^2)*d^2*e^2 - 4*(a*b + b*c)*d*e^3)/(a^4*b^2 - 2*a^2*b^4 +
b^6 - 4*a*c^5 - (16*a^2 - b^2)*c^4 - 12*(2*a^3 - a*b^2)*c^3 - 2*(8*a^4 - 11
*a^2*b^2 + b^4)*c^2 - 4*(a^5 - 3*a^3*b^2 + 2*a*b^4)*c))*sin(x) - ((b^4 - 4*
a*b^2*c)*d^3 - 3*(a*b^3 - 4*a*b*c^2 - (4*a^2*b - b^3)*c)*d^2*e + (2*a^2*b^2
 + b^4 - 8*a^3*c - 8*a*c^3 - 2*(8*a^2 - b^2)*c^2)*d*e^2 - (a*b^3 - 4*a*b*c^
2 - (4*a^2*b - b^3)*c)*e^3)*sin(x))*sqrt(-((b^2 - 2*a*c - 2*c^2)*d^2 - 2*(a
*b - b*c)*d*e + (2*a^2 - b^2 + 2*a*c)*e^2 + (a^2*b^2 - b^4 - 4*a*c^3 - (8*a
^2 - b^2)*c^2 - 2*(2*a^3 - 3*a*b^2)*c)*sqrt((b^2*d^4 + b^2*e^4 - 4*(a*b + b
*c)*d^3*e + 2*(2*a^2 + b^2 + 4*a*c + 2*c^2)*d^2*e^2 - 4*(a*b + b*c)*d*e^3)/
(a^4*b^2 - 2*a^2*b^4 + b^6 - 4*a*c^5 - (16*a^2 - b^2)*c^4 - 12*(2*a^3 - a*b
^2)*c^3 - 2*(8*a^4 - 11*a^2*b^2 + b^4)*c^2 - 4*(a^5 - 3*a^3*b^2 + 2*a*b^4)*
c)))/(a^2*b^2 - b^4 - 4*a*c^3 - (8*a^2 - b^2)*c^2 - 2*(2*a^3 - 3*a*b^2)*c))
 - (b^2*c*d^4 + a*b^2*e^4 - (b^3 + 2*a*b*c + 2*b*c^2)*d^3*e + 3*(a*b^2 + b^
2*c)*d^2*e^2 - (2*a^2*b + b^3 + 2*a*b*c)*d*e^3)*cos(x)) - 1/4*sqrt(2)*sqrt(
-((b^2 - 2*a*c - 2*c^2)*d^2 - 2*(a*b - b*c)*d*e + (2*a^2 - b^2 + 2*a*c)*e^2
 + (a^2*b^2 - b^4 - 4*a*c^3 - (8*a^2 - b^2)*c^2 - 2*(2*a^3 - 3*a*b^2)*c)*sq
rt((b^2*d^4 + b^2*e^4 - 4*(a*b + b*c)*d^3*e + 2*(2*a^2 + b^2 + 4*a*c + 2*c^
2)*d^2*e^2 - 4*(a*b + b*c)*d*e^3)/(a^4*b^2 - 2*a^2*b^4 + b^6 - 4*a*c^5 - (1
6*a^2 - b^2)*c^4 - 12*(2*a^3 - a*b^2)*c^3 - 2*(8*a^4 - 11*a^2*b^2 + b^4)*c^
2 - 4*(a^5 - 3*a^3*b^2 + 2*a*b^4)*c)))/(a^2*b^2 - b^4 - 4*a*c^3 - (8*a^2 -
b^2)*c^2 - 2*(2*a^3 - 3*a*b^2)*c))*log(-2*b*c^2*d^4 - 2*a*b*c*e^4 + 2*(b^2*
c + 2*a*c^2 + 2*c^3)*d^3*e - 6*(a*b*c + b*c^2)*d^2*e^2 + 2*(2*a*c^2 + (2*a^
2 + b^2)*c)*d*e^3 - ((4*a*c^4 + (8*a^2 - b^2)*c^3 + 2*(2*a^3 - 3*a*b^2)*c^2
 - (a^2*b^2 - b^4)*c)*d^2 + (a^2*b^3 - b^5 - 4*a*b*c^3 - (8*a^2*b - b^3)*c^
2 - 2*(2*a^3*b - 3*a*b^3)*c)*d*e - (a^3*b^2 - a*b^4 - 4*a^2*c^3 - (8*a^3 -
a*b^2)*c^2 - 2*(2*a^4 - 3*a^2*b^2)*c)*e^2)*sqrt((b^2*d^4 + b^2*e^4 - 4*(a*b
 + b*c)*d^3*e + 2*(2*a^2 + b^2 + 4*a*c + 2*c^2)*d^2*e^2 - 4*(a*b + b*c)*d*e
^3)/(a^4*b^2 - 2*a^2*b^4 + b^6 - 4*a*c^5 - (16*a^2 - b^2)*c^4 - 12*(2*a^3 -
 a*b^2)*c^3 - 2*(8*a^4 - 11*a^2*b^2 + b^4)*c^2 - 4*(a^5 - 3*a^3*b^2 + 2*a*b
^4)*c))*cos(x) - 1/2*sqrt(2)*(((a^2*b^4 - b^6 + 8*a*c^5 + 2*(12*a^2 - b^2)*
c^4 + 6*(4*a^3 - 3*a*b^2)*c^3 + (8*a^4 - 22*a^2*b^2 + 3*b^4)*c^2 - 2*(3*a^3
*b^2 - 4*a*b^4)*c)*d - (a^3*b^3 - a*b^5 + 4*a*b*c^4 + (4*a^2*b - b^3)*c^3 -
 (4*a^3*b + 5*a*b^3)*c^2 - (4*a^4*b - 5*a^2*b^3 - b^5)*c)*e)*sqrt((b^2*d^4
+ b^2*e^4 - 4*(a*b + b*c)*d^3*e + 2*(2*a^2 + b^2 + 4*a*c + 2*c^2)*d^2*e^2 -
 4*(a*b + b*c)*d*e^3)/(a^4*b^2 - 2*a^2*b^4 + b^6 - 4*a*c^5 - (16*a^2 - b^2)
c^4 - 12(2*a^3 - a*b^2)*c^3 - 2*(8*a^4 - 11*a^2*b^2 + b^4)*c^2 - 4*(a^5 -
 3*a^3*b^2 + 2*a*b^4)*c))*sin(x) - ((b^4 - 4*a*b^2*c)*d^3 - 3*(a*b^3 - 4*a*
b*c^2 - (4*a^2*b - b^3)*c)*d^2*e + (2*a^2*b^2 + b^4 - 8*a^3*c - 8*a*c^3 - 2

```
*(8*a^2 - b^2)*c^2)*d*e^2 - (a*b^3 - 4*a*b*c^2 - (4*a^2*b - b^3)*c)*e^3)*si
n(x))*sqrt(-((b^2 - 2*a*c - 2*c^2)*d^2 - 2*(a*b - b*c)*d*e + (2*a^2 - b^2 +
 2*a*c)*e^2 + (a^2*b^2 - b^4 - 4*a*c^3 - (8*a^2 - b^2)*c^2 - 2*(2*a^3 - 3*a
*b^2)*c)*sqrt((b^2*d^4 + b^2*e^4 - 4*(a*b + b*c)*d^3*e + 2*(2*a^2 + b^2 + 4
*a*c + 2*c^2)*d^2*e^2 - 4*(a*b + b*c)*d*e^3)/(a^4*b^2 - 2*a^2*b^4 + b^6 - 4
*a*c^5 - (16*a^2 - b^2)*c^4 - 12*(2*a^3 - a*b^2)*c^3 - 2*(8*a^4 - 11*a^2*b^
2 + b^4)*c^2 - 4*(a^5 - 3*a^3*b^2 + 2*a*b^4)*c)))/(a^2*b^2 - b^4 - 4*a*c^3
- (8*a^2 - b^2)*c^2 - 2*(2*a^3 - 3*a*b^2)*c)) - (b^2*c*d^4 + a*b^2*e^4 - (b
^3 + 2*a*b*c + 2*b*c^2)*d^3*e + 3*(a*b^2 + b^2*c)*d^2*e^2 - (2*a^2*b + b^3
+ 2*a*b*c)*d*e^3)*cos(x))
```

giac [B] time = 8.40, size = 5302, normalized size = 21.55

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*cos(x))/(a+b*cos(x)+c*cos(x)^2),x, algorithm="giac")
```

```
[Out] ((2*a^2*b^3 - 2*b^5 - 8*a^3*b*c - 12*a^2*b^2*c + 20*a*b^3*c + 4*b^4*c + 48*
a^3*c^2 - 48*a^2*b*c^2 - 24*a*b^2*c^2 - 6*b^3*c^2 + 32*a^2*c^3 + 24*a*b*c^3
 + 4*b^2*c^3 - 16*a*c^4 + 3*sqrt(a^2 - a*b + b*c - c^2 + sqrt(b^2 - 4*a*c)*
(a - b + c))*a^2*b^2 - 2*sqrt(a^2 - a*b + b*c - c^2 + sqrt(b^2 - 4*a*c)*(a
- b + c))*a*b^3 - 5*sqrt(a^2 - a*b + b*c - c^2 + sqrt(b^2 - 4*a*c)*(a - b +
 c))*b^4 - 12*sqrt(a^2 - a*b + b*c - c^2 + sqrt(b^2 - 4*a*c)*(a - b + c))*a
^3*c + 8*sqrt(a^2 - a*b + b*c - c^2 + sqrt(b^2 - 4*a*c)*(a - b + c))*a^2*b*
c + 34*sqrt(a^2 - a*b + b*c - c^2 + sqrt(b^2 - 4*a*c)*(a - b + c))*a*b^2*c
+ 6*sqrt(a^2 - a*b + b*c - c^2 + sqrt(b^2 - 4*a*c)*(a - b + c))*b^3*c - 56*
sqrt(a^2 - a*b + b*c - c^2 + sqrt(b^2 - 4*a*c)*(a - b + c))*a^2*c^2 - 24*sq
rt(a^2 - a*b + b*c - c^2 + sqrt(b^2 - 4*a*c)*(a - b + c))*a*b*c^2 - 5*sqrt(
a^2 - a*b + b*c - c^2 + sqrt(b^2 - 4*a*c)*(a - b + c))*b^2*c^2 + 20*sqrt(a^
2 - a*b + b*c - c^2 + sqrt(b^2 - 4*a*c)*(a - b + c))*a*c^3 + 3*sqrt(a^2 - a
*b + b*c - c^2 + sqrt(b^2 - 4*a*c)*(a - b + c))*sqrt(b^2 - 4*a*c)*a^2*b - 2
*(b^2 - 4*a*c)*a^2*b - 2*sqrt(a^2 - a*b + b*c - c^2 + sqrt(b^2 - 4*a*c)*(a
- b + c))*sqrt(b^2 - 4*a*c)*a*b^2 - 5*sqrt(a^2 - a*b + b*c - c^2 + sqrt(b^2
 - 4*a*c)*(a - b + c))*sqrt(b^2 - 4*a*c)*b^3 + 2*(b^2 - 4*a*c)*b^3 + 6*sqrt
(a^2 - a*b + b*c - c^2 + sqrt(b^2 - 4*a*c)*(a - b + c))*sqrt(b^2 - 4*a*c)*a
^2*c + 12*(b^2 - 4*a*c)*a^2*c + 10*sqrt(a^2 - a*b + b*c - c^2 + sqrt(b^2 -
4*a*c)*(a - b + c))*sqrt(b^2 - 4*a*c)*a*b*c - 12*(b^2 - 4*a*c)*a*b*c - 4*sq
rt(a^2 - a*b + b*c - c^2 + sqrt(b^2 - 4*a*c)*(a - b + c))*sqrt(b^2 - 4*a*c)
*b^2*c - 4*(b^2 - 4*a*c)*b^2*c + 28*sqrt(a^2 - a*b + b*c - c^2 + sqrt(b^2 -
 4*a*c)*(a - b + c))*sqrt(b^2 - 4*a*c)*a*c^2 + 8*(b^2 - 4*a*c)*a*c^2 + 7*sq
rt(a^2 - a*b + b*c - c^2 + sqrt(b^2 - 4*a*c)*(a - b + c))*sqrt(b^2 - 4*a*c)
*b*c^2 + 6*(b^2 - 4*a*c)*b*c^2 - 10*sqrt(a^2 - a*b + b*c - c^2 + sqrt(b^2 -
 4*a*c)*(a - b + c))*sqrt(b^2 - 4*a*c)*c^3 - 4*(b^2 - 4*a*c)*c^3)*d*abs(a -
 b + c) - (4*a^3*b^2 - 6*a^2*b^3 - 4*a*b^4 + 6*b^5 - 16*a^4*c + 24*a^3*b*c
```

+ 40*a^2*b^2*c - 44*a*b^3*c - 8*b^4*c - 96*a^3*c^2 + 80*a^2*b*c^2 + 52*a*b^2*c^2 + 2*b^3*c^2 - 80*a^2*c^3 - 8*a*b*c^3 - 3*sqrt(a^2 - a*b + b*c - c^2 + sqrt(b^2 - 4*a*c)*(a - b + c))*a^2*b^2 + 2*sqrt(a^2 - a*b + b*c - c^2 + sqrt(b^2 - 4*a*c)*(a - b + c))*a*b^3 + 5*sqrt(a^2 - a*b + b*c - c^2 + sqrt(b^2 - 4*a*c)*(a - b + c))*b^4 + 12*sqrt(a^2 - a*b + b*c - c^2 + sqrt(b^2 - 4*a*c)*(a - b + c))*a^3*c - 8*sqrt(a^2 - a*b + b*c - c^2 + sqrt(b^2 - 4*a*c)*(a - b + c))*a^2*b*c - 34*sqrt(a^2 - a*b + b*c - c^2 + sqrt(b^2 - 4*a*c)*(a - b + c))*a*b^2*c - 6*sqrt(a^2 - a*b + b*c - c^2 + sqrt(b^2 - 4*a*c)*(a - b + c))*b^3*c + 56*sqrt(a^2 - a*b + b*c - c^2 + sqrt(b^2 - 4*a*c)*(a - b + c))*a^2*c^2 + 24*sqrt(a^2 - a*b + b*c - c^2 + sqrt(b^2 - 4*a*c)*(a - b + c))*a*b*c^2 + 5*sqrt(a^2 - a*b + b*c - c^2 + sqrt(b^2 - 4*a*c)*(a - b + c))*b^2*c^2 - 20*sqrt(a^2 - a*b + b*c - c^2 + sqrt(b^2 - 4*a*c)*(a - b + c))*a*c^3 + 6*sqrt(a^2 - a*b + b*c - c^2 + sqrt(b^2 - 4*a*c)*(a - b + c))*sqrt(b^2 - 4*a*c)*a^3 - 4*(b^2 - 4*a*c)*a^3 - sqrt(a^2 - a*b + b*c - c^2 + sqrt(b^2 - 4*a*c)*(a - b + c))*sqrt(b^2 - 4*a*c)*a^2*b + 6*(b^2 - 4*a*c)*a^2*b - 12*sqrt(a^2 - a*b + b*c - c^2 + sqrt(b^2 - 4*a*c)*(a - b + c))*sqrt(b^2 - 4*a*c)*a*b^2 + 4*(b^2 - 4*a*c)*a*b^2 - 5*sqrt(a^2 - a*b + b*c - c^2 + sqrt(b^2 - 4*a*c)*(a - b + c))*sqrt(b^2 - 4*a*c)*b^3 - 6*(b^2 - 4*a*c)*b^3 + 28*sqrt(a^2 - a*b + b*c - c^2 + sqrt(b^2 - 4*a*c)*(a - b + c))*sqrt(b^2 - 4*a*c)*a^2*c - 24*(b^2 - 4*a*c)*a^2*c + 26*sqrt(a^2 - a*b + b*c - c^2 + sqrt(b^2 - 4*a*c)*(a - b + c))*sqrt(b^2 - 4*a*c)*a*b*c + 20*(b^2 - 4*a*c)*a*b*c + 6*sqrt(a^2 - a*b + b*c - c^2 + sqrt(b^2 - 4*a*c)*(a - b + c))*sqrt(b^2 - 4*a*c)*b^2*c + 8*(b^2 - 4*a*c)*b^2*c - 10*sqrt(a^2 - a*b + b*c - c^2 + sqrt(b^2 - 4*a*c)*(a - b + c))*sqrt(b^2 - 4*a*c)*a*c^2 - 20*(b^2 - 4*a*c)*a*c^2 - 5*sqrt(a^2 - a*b + b*c - c^2 + sqrt(b^2 - 4*a*c)*(a - b + c))*sqrt(b^2 - 4*a*c)*b*c^2 - 2*(b^2 - 4*a*c)*b*c^2)*abs(a - b + c)*e)*(pi*floor(1/2*x/pi + 1/2) + arctan(2*sqrt(1/2)*tan(1/2*x)/sqrt((2*a - 2*c + sqrt(-4*(a + b + c)*(a - b + c) + 4*(a - c)^2))/(a - b + c))))/(3*a^5*b^2 - 5*a^4*b^3 - 6*a^3*b^4 + 10*a^2*b^5 + 3*a*b^6 - 5*b^7 - 12*a^6*c + 20*a^5*b*c + 47*a^4*b^2*c - 60*a^3*b^3*c - 46*a^2*b^4*c + 40*a*b^5*c + 11*b^6*c - 92*a^5*c^2 + 80*a^4*b*c^2 + 182*a^3*b^2*c^2 - 94*a^2*b^3*c^2 - 78*a*b^4*c^2 - 6*b^5*c^2 - 184*a^4*c^3 + 56*a^3*b*c^3 + 166*a^2*b^2*c^3 + 36*a*b^3*c^3 - 6*b^4*c^3 - 120*a^3*c^4 - 48*a^2*b*c^4 + 23*a*b^2*c^4 + 11*b^3*c^4 + 4*a^2*c^5 - 44*a*b*c^5 - 5*b^2*c^5 + 20*a*c^6) - ((2*a^2*b^3 - 2*b^5 - 8*a^3*b*c - 12*a^2*b^2*c + 20*a*b^3*c + 4*b^4*c + 48*a^3*c^2 - 48*a^2*b*c^2 - 24*a*b^2*c^2 - 6*b^3*c^2 + 32*a^2*c^3 + 24*a*b*c^3 + 4*b^2*c^3 - 16*a*c^4 - 3*sqrt(a^2 - a*b + b*c - c^2 - sqrt(b^2 - 4*a*c)*(a - b + c))*a^2*b^2 + 2*sqrt(a^2 - a*b + b*c - c^2 - sqrt(b^2 - 4*a*c)*(a - b + c))*a*b^3 + 5*sqrt(a^2 - a*b + b*c - c^2 - sqrt(b^2 - 4*a*c)*(a - b + c))*b^4 + 12*sqrt(a^2 - a*b + b*c - c^2 - sqrt(b^2 - 4*a*c)*(a - b + c))*a^3*c - 8*sqrt(a^2 - a*b + b*c - c^2 - sqrt(b^2 - 4*a*c)*(a - b + c))*a^2*b*c - 34*sqrt(a^2 - a*b + b*c - c^2 - sqrt(b^2 - 4*a*c)*(a - b + c))*a*b^2*c - 6*sqrt(a^2 - a*b + b*c - c^2 - sqrt(b^2 - 4*a*c)*(a - b + c))*b^3*c + 56*sqrt(a^2 - a*b + b*c - c^2 - sqrt(b^2 - 4*a*c)*(a - b + c))*a^2*c^2 + 24*sqrt(a^2 - a*b + b*c - c^2 - sqrt(b^2 - 4*a*c)*(a - b + c))*a*b*c^2 + 5*sqrt(a^2 - a*b + b*c - c^2 - sqrt(b^2 - 4*a*c)*(a - b + c)

)*b^2*c^2 - 20*sqrt(a^2 - a*b + b*c - c^2 - sqrt(b^2 - 4*a*c)*(a - b + c))*
a*c^3 + 3*sqrt(a^2 - a*b + b*c - c^2 - sqrt(b^2 - 4*a*c)*(a - b + c))*sqrt(
b^2 - 4*a*c)*a^2*b - 2*(b^2 - 4*a*c)*a^2*b - 2*sqrt(a^2 - a*b + b*c - c^2 -
sqrt(b^2 - 4*a*c)*(a - b + c))*sqrt(b^2 - 4*a*c)*a*b^2 - 5*sqrt(a^2 - a*b
+ b*c - c^2 - sqrt(b^2 - 4*a*c)*(a - b + c))*sqrt(b^2 - 4*a*c)*b^3 + 2*(b^2
- 4*a*c)*b^3 + 6*sqrt(a^2 - a*b + b*c - c^2 - sqrt(b^2 - 4*a*c)*(a - b + c
))*sqrt(b^2 - 4*a*c)*a^2*c + 12*(b^2 - 4*a*c)*a^2*c + 10*sqrt(a^2 - a*b + b
*c - c^2 - sqrt(b^2 - 4*a*c)*(a - b + c))*sqrt(b^2 - 4*a*c)*a*b*c - 12*(b^2
- 4*a*c)*a*b*c - 4*sqrt(a^2 - a*b + b*c - c^2 - sqrt(b^2 - 4*a*c)*(a - b +
c))*sqrt(b^2 - 4*a*c)*b^2*c - 4*(b^2 - 4*a*c)*b^2*c + 28*sqrt(a^2 - a*b +
b*c - c^2 - sqrt(b^2 - 4*a*c)*(a - b + c))*sqrt(b^2 - 4*a*c)*a*c^2 + 8*(b^2
- 4*a*c)*a*c^2 + 7*sqrt(a^2 - a*b + b*c - c^2 - sqrt(b^2 - 4*a*c)*(a - b +
c))*sqrt(b^2 - 4*a*c)*b*c^2 + 6*(b^2 - 4*a*c)*b*c^2 - 10*sqrt(a^2 - a*b +
b*c - c^2 - sqrt(b^2 - 4*a*c)*(a - b + c))*sqrt(b^2 - 4*a*c)*c^3 - 4*(b^2 -
4*a*c)*c^3)*d*abs(a - b + c) - (4*a^3*b^2 - 6*a^2*b^3 - 4*a*b^4 + 6*b^5 -
16*a^4*c + 24*a^3*b*c + 40*a^2*b^2*c - 44*a*b^3*c - 8*b^4*c - 96*a^3*c^2 +
80*a^2*b*c^2 + 52*a*b^2*c^2 + 2*b^3*c^2 - 80*a^2*c^3 - 8*a*b*c^3 + 3*sqrt(a
^2 - a*b + b*c - c^2 - sqrt(b^2 - 4*a*c)*(a - b + c))*a^2*b^2 - 2*sqrt(a^2
- a*b + b*c - c^2 - sqrt(b^2 - 4*a*c)*(a - b + c))*a*b^3 - 5*sqrt(a^2 - a*b
+ b*c - c^2 - sqrt(b^2 - 4*a*c)*(a - b + c))*b^4 - 12*sqrt(a^2 - a*b + b*c
- c^2 - sqrt(b^2 - 4*a*c)*(a - b + c))*a^3*c + 8*sqrt(a^2 - a*b + b*c - c^
2 - sqrt(b^2 - 4*a*c)*(a - b + c))*a^2*b*c + 34*sqrt(a^2 - a*b + b*c - c^2
- sqrt(b^2 - 4*a*c)*(a - b + c))*a*b^2*c + 6*sqrt(a^2 - a*b + b*c - c^2 - s
qrt(b^2 - 4*a*c)*(a - b + c))*b^3*c - 56*sqrt(a^2 - a*b + b*c - c^2 - sqrt(
b^2 - 4*a*c)*(a - b + c))*a^2*c^2 - 24*sqrt(a^2 - a*b + b*c - c^2 - sqrt(b^
2 - 4*a*c)*(a - b + c))*a*b*c^2 - 5*sqrt(a^2 - a*b + b*c - c^2 - sqrt(b^2 -
4*a*c)*(a - b + c))*b^2*c^2 + 20*sqrt(a^2 - a*b + b*c - c^2 - sqrt(b^2 - 4
*a*c)*(a - b + c))*a*c^3 + 6*sqrt(a^2 - a*b + b*c - c^2 - sqrt(b^2 - 4*a*c)
*(a - b + c))*sqrt(b^2 - 4*a*c)*a^3 - 4*(b^2 - 4*a*c)*a^3 - sqrt(a^2 - a*b
+ b*c - c^2 - sqrt(b^2 - 4*a*c)*(a - b + c))*sqrt(b^2 - 4*a*c)*a^2*b + 6*(b
^2 - 4*a*c)*a^2*b - 12*sqrt(a^2 - a*b + b*c - c^2 - sqrt(b^2 - 4*a*c)*(a -
b + c))*sqrt(b^2 - 4*a*c)*a*b^2 + 4*(b^2 - 4*a*c)*a*b^2 - 5*sqrt(a^2 - a*b
+ b*c - c^2 - sqrt(b^2 - 4*a*c)*(a - b + c))*sqrt(b^2 - 4*a*c)*b^3 - 6*(b^2
- 4*a*c)*b^3 + 28*sqrt(a^2 - a*b + b*c - c^2 - sqrt(b^2 - 4*a*c)*(a - b +
c))*sqrt(b^2 - 4*a*c)*a^2*c - 24*(b^2 - 4*a*c)*a^2*c + 26*sqrt(a^2 - a*b +
b*c - c^2 - sqrt(b^2 - 4*a*c)*(a - b + c))*sqrt(b^2 - 4*a*c)*a*b*c + 20*(b^
2 - 4*a*c)*a*b*c + 6*sqrt(a^2 - a*b + b*c - c^2 - sqrt(b^2 - 4*a*c)*(a - b
+ c))*sqrt(b^2 - 4*a*c)*b^2*c + 8*(b^2 - 4*a*c)*b^2*c - 10*sqrt(a^2 - a*b +
b*c - c^2 - sqrt(b^2 - 4*a*c)*(a - b + c))*sqrt(b^2 - 4*a*c)*a*c^2 - 20*(b
^2 - 4*a*c)*a*c^2 - 5*sqrt(a^2 - a*b + b*c - c^2 - sqrt(b^2 - 4*a*c)*(a - b
+ c))*sqrt(b^2 - 4*a*c)*b*c^2 - 2*(b^2 - 4*a*c)*b*c^2)*abs(a - b + c)*e)*(
pi*floor(1/2*x/pi + 1/2) + arctan(2*sqrt(1/2)*tan(1/2*x)/sqrt((2*a - 2*c -
sqrt(-4*(a + b + c)*(a - b + c) + 4*(a - c)^2))/(a - b + c))))/(3*a^5*b^2 -
5*a^4*b^3 - 6*a^3*b^4 + 10*a^2*b^5 + 3*a*b^6 - 5*b^7 - 12*a^6*c + 20*a^5*b
*c + 47*a^4*b^2*c - 60*a^3*b^3*c - 46*a^2*b^4*c + 40*a*b^5*c + 11*b^6*c - 9

```
2*a^5*c^2 + 80*a^4*b*c^2 + 182*a^3*b^2*c^2 - 94*a^2*b^3*c^2 - 78*a*b^4*c^2
- 6*b^5*c^2 - 184*a^4*c^3 + 56*a^3*b*c^3 + 166*a^2*b^2*c^3 + 36*a*b^3*c^3 -
 6*b^4*c^3 - 120*a^3*c^4 - 48*a^2*b*c^4 + 23*a*b^2*c^4 + 11*b^3*c^4 + 4*a^2
*c^5 - 44*a*b*c^5 - 5*b^2*c^5 + 20*a*c^6)
```

maple [B] time = 0.12, size = 2556, normalized size = 10.39

Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d+e*cos(x))/(a+b*cos(x)+c*cos(x)^2),x)
```

```
[Out] -2/(-4*a*c+b^2)^(1/2)/(a-b+c)/(((-4*a*c+b^2)^(1/2)+a-c)*(a-b+c))^(1/2)*arct
an((a-b+c)*tan(1/2*x)/(((-4*a*c+b^2)^(1/2)+a-c)*(a-b+c))^(1/2))*a^2*e+2/(-4
*a*c+b^2)^(1/2)/(a-b+c)/(((-4*a*c+b^2)^(1/2)-a+c)*(a-b+c))^(1/2)*arctanh((-
a+b-c)*tan(1/2*x)/(((-4*a*c+b^2)^(1/2)-a+c)*(a-b+c))^(1/2))*a^2*e-1/(-4*a*c
+b^2)^(1/2)/(a-b+c)/(((-4*a*c+b^2)^(1/2)+a-c)*(a-b+c))^(1/2)*arctan((a-b+c)
*tan(1/2*x)/(((-4*a*c+b^2)^(1/2)+a-c)*(a-b+c))^(1/2))*d*b^2-1/(-4*a*c+b^2)^
(1/2)/(a-b+c)/(((-4*a*c+b^2)^(1/2)+a-c)*(a-b+c))^(1/2)*arctan((a-b+c)*tan(1
/2*x)/(((-4*a*c+b^2)^(1/2)+a-c)*(a-b+c))^(1/2))*b^2*e+1/(-4*a*c+b^2)^(1/2)/
(a-b+c)/(((-4*a*c+b^2)^(1/2)-a+c)*(a-b+c))^(1/2)*arctanh((-a+b-c)*tan(1/2*x
)/(((-4*a*c+b^2)^(1/2)-a+c)*(a-b+c))^(1/2))*d*b^2+1/(-4*a*c+b^2)^(1/2)/(a-b
+c)/(((-4*a*c+b^2)^(1/2)-a+c)*(a-b+c))^(1/2)*arctanh((-a+b-c)*tan(1/2*x)/((
(-4*a*c+b^2)^(1/2)-a+c)*(a-b+c))^(1/2))*b^2*e-2/(-4*a*c+b^2)^(1/2)/(a-b+c)/
(((-4*a*c+b^2)^(1/2)+a-c)*(a-b+c))^(1/2)*arctan((a-b+c)*tan(1/2*x)/(((-4*a*
c+b^2)^(1/2)+a-c)*(a-b+c))^(1/2))*c^2*d+2/(-4*a*c+b^2)^(1/2)/(a-b+c)/(((-4*
a*c+b^2)^(1/2)-a+c)*(a-b+c))^(1/2)*arctanh((-a+b-c)*tan(1/2*x)/(((-4*a*c+b^
2)^(1/2)-a+c)*(a-b+c))^(1/2))*c^2*d-b/(a-b+c)/(((-4*a*c+b^2)^(1/2)-a+c)*(a-
b+c))^(1/2)*arctanh((-a+b-c)*tan(1/2*x)/(((-4*a*c+b^2)^(1/2)-a+c)*(a-b+c))^
(1/2))*d+b/(a-b+c)/(((-4*a*c+b^2)^(1/2)-a+c)*(a-b+c))^(1/2)*arctanh((-a+b-c
)*tan(1/2*x)/(((-4*a*c+b^2)^(1/2)-a+c)*(a-b+c))^(1/2))*e+c/(a-b+c)/(((-4*a*
c+b^2)^(1/2)+a-c)*(a-b+c))^(1/2)*arctan((a-b+c)*tan(1/2*x)/(((-4*a*c+b^2)^(
1/2)+a-c)*(a-b+c))^(1/2))*d-c/(a-b+c)/(((-4*a*c+b^2)^(1/2)+a-c)*(a-b+c))^(1
/2)*arctan((a-b+c)*tan(1/2*x)/(((-4*a*c+b^2)^(1/2)+a-c)*(a-b+c))^(1/2))*e+c
/(a-b+c)/(((-4*a*c+b^2)^(1/2)-a+c)*(a-b+c))^(1/2)*arctanh((-a+b-c)*tan(1/2*
x)/(((-4*a*c+b^2)^(1/2)-a+c)*(a-b+c))^(1/2))*d-c/(a-b+c)/(((-4*a*c+b^2)^(1/
2)-a+c)*(a-b+c))^(1/2)*arctanh((-a+b-c)*tan(1/2*x)/(((-4*a*c+b^2)^(1/2)-a+c
)*(a-b+c))^(1/2))*e-a/(a-b+c)/(((-4*a*c+b^2)^(1/2)+a-c)*(a-b+c))^(1/2)*arct
an((a-b+c)*tan(1/2*x)/(((-4*a*c+b^2)^(1/2)+a-c)*(a-b+c))^(1/2))*e+a/(a-b+c)
/(((-4*a*c+b^2)^(1/2)+a-c)*(a-b+c))^(1/2)*arctan((a-b+c)*tan(1/2*x)/(((-4*a
*c+b^2)^(1/2)+a-c)*(a-b+c))^(1/2))*d+a/(a-b+c)/(((-4*a*c+b^2)^(1/2)-a+c)*(a
-b+c))^(1/2)*arctanh((-a+b-c)*tan(1/2*x)/(((-4*a*c+b^2)^(1/2)-a+c)*(a-b+c))
^(1/2))*d-a/(a-b+c)/(((-4*a*c+b^2)^(1/2)-a+c)*(a-b+c))^(1/2)*arctanh((-a+b-
c)*tan(1/2*x)/(((-4*a*c+b^2)^(1/2)-a+c)*(a-b+c))^(1/2))*e-b/(a-b+c)/(((-4*a
*c+b^2)^(1/2)+a-c)*(a-b+c))^(1/2)*arctan((a-b+c)*tan(1/2*x)/(((-4*a*c+b^2)^
```

```
(1/2)+a-c)*(a-b+c))^(1/2))*d+b/(a-b+c)/(((-4*a*c+b^2)^(1/2)+a-c)*(a-b+c))^(
1/2)*arctan((a-b+c)*tan(1/2*x)/(((-4*a*c+b^2)^(1/2)+a-c)*(a-b+c))^(1/2))*e-
a/(-4*a*c+b^2)^(1/2)/(a-b+c)/(((-4*a*c+b^2)^(1/2)-a+c)*(a-b+c))^(1/2)*arcta
nh((-a+b-c)*tan(1/2*x)/(((-4*a*c+b^2)^(1/2)-a+c)*(a-b+c))^(1/2))*d*b-3*a/(-
4*a*c+b^2)^(1/2)/(a-b+c)/(((-4*a*c+b^2)^(1/2)-a+c)*(a-b+c))^(1/2)*arctanh((
-a+b-c)*tan(1/2*x)/(((-4*a*c+b^2)^(1/2)-a+c)*(a-b+c))^(1/2))*b*e+2*a/(-4*a*
c+b^2)^(1/2)/(a-b+c)/(((-4*a*c+b^2)^(1/2)-a+c)*(a-b+c))^(1/2)*arctanh((-a+b
-c)*tan(1/2*x)/(((-4*a*c+b^2)^(1/2)-a+c)*(a-b+c))^(1/2))*c*d+3*b/(-4*a*c+b^
2)^(1/2)/(a-b+c)/(((-4*a*c+b^2)^(1/2)+a-c)*(a-b+c))^(1/2)*arctan((a-b+c)*ta
n(1/2*x)/(((-4*a*c+b^2)^(1/2)+a-c)*(a-b+c))^(1/2))*c*d-3*b/(-4*a*c+b^2)^(1/
2)/(a-b+c)/(((-4*a*c+b^2)^(1/2)-a+c)*(a-b+c))^(1/2)*arctanh((-a+b-c)*tan(1/
2*x)/(((-4*a*c+b^2)^(1/2)-a+c)*(a-b+c))^(1/2))*c*d-2*c/(-4*a*c+b^2)^(1/2)/(
a-b+c)/(((-4*a*c+b^2)^(1/2)+a-c)*(a-b+c))^(1/2)*arctan((a-b+c)*tan(1/2*x)/(
((-4*a*c+b^2)^(1/2)+a-c)*(a-b+c))^(1/2))*a*e+c/(-4*a*c+b^2)^(1/2)/(a-b+c)/(
((-4*a*c+b^2)^(1/2)+a-c)*(a-b+c))^(1/2)*arctan((a-b+c)*tan(1/2*x)/(((-4*a*c
+b^2)^(1/2)+a-c)*(a-b+c))^(1/2))*b*e+2*c/(-4*a*c+b^2)^(1/2)/(a-b+c)/(((-4*a
*c+b^2)^(1/2)-a+c)*(a-b+c))^(1/2)*arctanh((-a+b-c)*tan(1/2*x)/(((-4*a*c+b^2
)^(1/2)-a+c)*(a-b+c))^(1/2))*a*e-c/(-4*a*c+b^2)^(1/2)/(a-b+c)/(((-4*a*c+b^2
)^(1/2)-a+c)*(a-b+c))^(1/2)*arctanh((-a+b-c)*tan(1/2*x)/(((-4*a*c+b^2)^(1/2
)-a+c)*(a-b+c))^(1/2))*b*e+a/(-4*a*c+b^2)^(1/2)/(a-b+c)/(((-4*a*c+b^2)^(1/2
)+a-c)*(a-b+c))^(1/2)*arctan((a-b+c)*tan(1/2*x)/(((-4*a*c+b^2)^(1/2)+a-c)*(
a-b+c))^(1/2))*d*b+3*a/(-4*a*c+b^2)^(1/2)/(a-b+c)/(((-4*a*c+b^2)^(1/2)+a-c)
*(a-b+c))^(1/2)*arctan((a-b+c)*tan(1/2*x)/(((-4*a*c+b^2)^(1/2)+a-c)*(a-b+c)
)^(1/2))*b*e-2*a/(-4*a*c+b^2)^(1/2)/(a-b+c)/(((-4*a*c+b^2)^(1/2)+a-c)*(a-b+
c))^(1/2)*arctan((a-b+c)*tan(1/2*x)/(((-4*a*c+b^2)^(1/2)+a-c)*(a-b+c))^(1/2
))*c*d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{e\cos(x)+d}{c\cos(x)^2+b\cos(x)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*cos(x))/(a+b*cos(x)+c*cos(x)^2),x, algorithm="maxima")
```

```
[Out] integrate((e*cos(x) + d)/(c*cos(x)^2 + b*cos(x) + a), x)
```

mupad [B] time = 16.77, size = 11781, normalized size = 47.89

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*cos(x))/(a + b*cos(x) + c*cos(x)^2),x)
```

[Out] - atan((((-(b^4*d^2 - b^4*e^2 + 8*a*c^3*d^2 + b*d^2*(-(4*a*c - b^2)^3)^(1/2
) - 8*a^3*c*e^2 + b*e^2*(-(4*a*c - b^2)^3)^(1/2) + 2*a^2*b^2*e^2 + 8*a^2*c^
2*d^2 - 8*a^2*c^2*e^2 - 2*b^2*c^2*d^2 - 2*a*b^3*d*e - 2*a*d*e*(-(4*a*c - b^
2)^3)^(1/2) + 2*b^3*c*d*e - 2*c*d*e*(-(4*a*c - b^2)^3)^(1/2) - 6*a*b^2*c*d^
2 + 6*a*b^2*c*e^2 - 8*a*b*c^2*d*e + 8*a^2*b*c*d*e)/(2*(a^2*b^4 - b^6 + 16*a
^2*c^4 + 32*a^3*c^3 + 16*a^4*c^2 + b^4*c^2 - 8*a*b^2*c^3 - 8*a^3*b^2*c - 32
*a^2*b^2*c^2 + 10*a*b^4*c)))^(1/2)*(256*a^2*c^2*d - 32*b^4*e - 32*a^2*b^2*d
 - 32*a^2*b^2*e - 32*b^4*d + 256*a^2*c^2*e - 32*b^2*c^2*d - 32*b^2*c^2*e +
tan(x/2)*(-(b^4*d^2 - b^4*e^2 + 8*a*c^3*d^2 + b*d^2*(-(4*a*c - b^2)^3)^(1/2
) - 8*a^3*c*e^2 + b*e^2*(-(4*a*c - b^2)^3)^(1/2) + 2*a^2*b^2*e^2 + 8*a^2*c^
2*d^2 - 8*a^2*c^2*e^2 - 2*b^2*c^2*d^2 - 2*a*b^3*d*e - 2*a*d*e*(-(4*a*c - b^
2)^3)^(1/2) + 2*b^3*c*d*e - 2*c*d*e*(-(4*a*c - b^2)^3)^(1/2) - 6*a*b^2*c*d^
2 + 6*a*b^2*c*e^2 - 8*a*b*c^2*d*e + 8*a^2*b*c*d*e)/(2*(a^2*b^4 - b^6 + 16*a
^2*c^4 + 32*a^3*c^3 + 16*a^4*c^2 + b^4*c^2 - 8*a*b^2*c^3 - 8*a^3*b^2*c - 32
*a^2*b^2*c^2 + 10*a*b^4*c)))^(1/2)*(64*a*b^4 + 256*a*c^4 - 256*a^4*c - 64*b
^4*c - 128*a^2*b^3 + 64*a^3*b^2 + 256*a^2*c^3 - 256*a^3*c^2 - 64*b^2*c^3 +
128*b^3*c^2 + 192*a*b^2*c^2 - 192*a^2*b^2*c - 512*a*b*c^3 + 512*a^3*b*c) +
64*a*b^3*d + 64*a*b^3*e + 128*a*c^3*d + 128*a^3*c*d + 128*a*c^3*e + 128*a^3
*c*e + 64*b^3*c*d + 64*b^3*c*e - 256*a*b*c^2*d + 64*a*b^2*c*d - 256*a^2*b*c
*d - 256*a*b*c^2*e + 64*a*b^2*c*e - 256*a^2*b*c*e) + tan(x/2)*(64*a^3*e^2 -
 32*b^3*d^2 - 32*b^3*e^2 + 64*c^3*d^2 + 32*a*b^2*d^2 + 96*a*b^2*e^2 - 128*a
^2*b*e^2 - 64*a^2*c*d^2 - 64*a*c^2*e^2 - 128*b*c^2*d^2 + 96*b^2*c*d^2 + 32*
b^2*c*e^2 + 64*a*b^2*d*e - 64*a^2*b*d*e + 256*a*c^2*d*e + 256*a^2*c*d*e - 6
4*b*c^2*d*e + 64*b^2*c*d*e - 384*a*b*c*d*e))*(-(b^4*d^2 - b^4*e^2 + 8*a*c^3
*d^2 + b*d^2*(-(4*a*c - b^2)^3)^(1/2) - 8*a^3*c*e^2 + b*e^2*(-(4*a*c - b^2)
^3)^(1/2) + 2*a^2*b^2*e^2 + 8*a^2*c^2*d^2 - 8*a^2*c^2*e^2 - 2*b^2*c^2*d^2 -
 2*a*b^3*d*e - 2*a*d*e*(-(4*a*c - b^2)^3)^(1/2) + 2*b^3*c*d*e - 2*c*d*e*(-(
4*a*c - b^2)^3)^(1/2) - 6*a*b^2*c*d^2 + 6*a*b^2*c*e^2 - 8*a*b*c^2*d*e + 8*a
^2*b*c*d*e)/(2*(a^2*b^4 - b^6 + 16*a^2*c^4 + 32*a^3*c^3 + 16*a^4*c^2 + b^4*
c^2 - 8*a*b^2*c^3 - 8*a^3*b^2*c - 32*a^2*b^2*c^2 + 10*a*b^4*c)))^(1/2)*1i -
 ((-(b^4*d^2 - b^4*e^2 + 8*a*c^3*d^2 + b*d^2*(-(4*a*c - b^2)^3)^(1/2) - 8*a
^3*c*e^2 + b*e^2*(-(4*a*c - b^2)^3)^(1/2) + 2*a^2*b^2*e^2 + 8*a^2*c^2*d^2 -
 8*a^2*c^2*e^2 - 2*b^2*c^2*d^2 - 2*a*b^3*d*e - 2*a*d*e*(-(4*a*c - b^2)^3)^(
1/2) + 2*b^3*c*d*e - 2*c*d*e*(-(4*a*c - b^2)^3)^(1/2) - 6*a*b^2*c*d^2 + 6*a
*b^2*c*e^2 - 8*a*b*c^2*d*e + 8*a^2*b*c*d*e)/(2*(a^2*b^4 - b^6 + 16*a^2*c^4
+ 32*a^3*c^3 + 16*a^4*c^2 + b^4*c^2 - 8*a*b^2*c^3 - 8*a^3*b^2*c - 32*a^2*b^
2*c^2 + 10*a*b^4*c)))^(1/2)*(256*a^2*c^2*d - 32*b^4*e - 32*a^2*b^2*d - 32*a
^2*b^2*e - 32*b^4*d + 256*a^2*c^2*e - 32*b^2*c^2*d - 32*b^2*c^2*e - tan(x/2
)*(-(b^4*d^2 - b^4*e^2 + 8*a*c^3*d^2 + b*d^2*(-(4*a*c - b^2)^3)^(1/2) - 8*a
^3*c*e^2 + b*e^2*(-(4*a*c - b^2)^3)^(1/2) + 2*a^2*b^2*e^2 + 8*a^2*c^2*d^2 -
 8*a^2*c^2*e^2 - 2*b^2*c^2*d^2 - 2*a*b^3*d*e - 2*a*d*e*(-(4*a*c - b^2)^3)^(
1/2) + 2*b^3*c*d*e - 2*c*d*e*(-(4*a*c - b^2)^3)^(1/2) - 6*a*b^2*c*d^2 + 6*a
*b^2*c*e^2 - 8*a*b*c^2*d*e + 8*a^2*b*c*d*e)/(2*(a^2*b^4 - b^6 + 16*a^2*c^4
+ 32*a^3*c^3 + 16*a^4*c^2 + b^4*c^2 - 8*a*b^2*c^3 - 8*a^3*b^2*c - 32*a^2*b^
2*c^2 + 10*a*b^4*c)))^(1/2)*(64*a*b^4 + 256*a*c^4 - 256*a^4*c - 64*b^4*c -

128*a^2*b^3 + 64*a^3*b^2 + 256*a^2*c^3 - 256*a^3*c^2 - 64*b^2*c^3 + 128*b^3
*c^2 + 192*a*b^2*c^2 - 192*a^2*b^2*c - 512*a*b*c^3 + 512*a^3*b*c) + 64*a*b^
3*d + 64*a*b^3*e + 128*a*c^3*d + 128*a^3*c*d + 128*a*c^3*e + 128*a^3*c*e +
64*b^3*c*d + 64*b^3*c*e - 256*a*b*c^2*d + 64*a*b^2*c*d - 256*a^2*b*c*d - 25
6*a*b*c^2*e + 64*a*b^2*c*e - 256*a^2*b*c*e) - tan(x/2)*(64*a^3*e^2 - 32*b^3
*d^2 - 32*b^3*e^2 + 64*c^3*d^2 + 32*a*b^2*d^2 + 96*a*b^2*e^2 - 128*a^2*b*e^
2 - 64*a^2*c*d^2 - 64*a*c^2*e^2 - 128*b*c^2*d^2 + 96*b^2*c*d^2 + 32*b^2*c*e
^2 + 64*a*b^2*d*e - 64*a^2*b*d*e + 256*a*c^2*d*e + 256*a^2*c*d*e - 64*b*c^2
*d*e + 64*b^2*c*d*e - 384*a*b*c*d*e))*(-(b^4*d^2 - b^4*e^2 + 8*a*c^3*d^2 +
b*d^2*(-(4*a*c - b^2)^3)^(1/2) - 8*a^3*c*e^2 + b*e^2*(-(4*a*c - b^2)^3)^(1/
2) + 2*a^2*b^2*e^2 + 8*a^2*c^2*d^2 - 8*a^2*c^2*e^2 - 2*b^2*c^2*d^2 - 2*a*b^
3*d*e - 2*a*d*e*(-(4*a*c - b^2)^3)^(1/2) + 2*b^3*c*d*e - 2*c*d*e*(-(4*a*c -
 b^2)^3)^(1/2) - 6*a*b^2*c*d^2 + 6*a*b^2*c*e^2 - 8*a*b*c^2*d*e + 8*a^2*b*c*
d*e)/(2*(a^2*b^4 - b^6 + 16*a^2*c^4 + 32*a^3*c^3 + 16*a^4*c^2 + b^4*c^2 - 8
*a*b^2*c^3 - 8*a^3*b^2*c - 32*a^2*b^2*c^2 + 10*a*b^4*c)))^(1/2)*1i)/(((-(b^
4*d^2 - b^4*e^2 + 8*a*c^3*d^2 + b*d^2*(-(4*a*c - b^2)^3)^(1/2) - 8*a^3*c*e^
2 + b*e^2*(-(4*a*c - b^2)^3)^(1/2) + 2*a^2*b^2*e^2 + 8*a^2*c^2*d^2 - 8*a^2*
c^2*e^2 - 2*b^2*c^2*d^2 - 2*a*b^3*d*e - 2*a*d*e*(-(4*a*c - b^2)^3)^(1/2) +
2*b^3*c*d*e - 2*c*d*e*(-(4*a*c - b^2)^3)^(1/2) - 6*a*b^2*c*d^2 + 6*a*b^2*c*
e^2 - 8*a*b*c^2*d*e + 8*a^2*b*c*d*e)/(2*(a^2*b^4 - b^6 + 16*a^2*c^4 + 32*a^
3*c^3 + 16*a^4*c^2 + b^4*c^2 - 8*a*b^2*c^3 - 8*a^3*b^2*c - 32*a^2*b^2*c^2 +
 10*a*b^4*c)))^(1/2)*(256*a^2*c^2*d - 32*b^4*e - 32*a^2*b^2*d - 32*a^2*b^2*
e - 32*b^4*d + 256*a^2*c^2*e - 32*b^2*c^2*d - 32*b^2*c^2*e + tan(x/2)*(-(b^
4*d^2 - b^4*e^2 + 8*a*c^3*d^2 + b*d^2*(-(4*a*c - b^2)^3)^(1/2) - 8*a^3*c*e^
2 + b*e^2*(-(4*a*c - b^2)^3)^(1/2) + 2*a^2*b^2*e^2 + 8*a^2*c^2*d^2 - 8*a^2*
c^2*e^2 - 2*b^2*c^2*d^2 - 2*a*b^3*d*e - 2*a*d*e*(-(4*a*c - b^2)^3)^(1/2) +
2*b^3*c*d*e - 2*c*d*e*(-(4*a*c - b^2)^3)^(1/2) - 6*a*b^2*c*d^2 + 6*a*b^2*c*
e^2 - 8*a*b*c^2*d*e + 8*a^2*b*c*d*e)/(2*(a^2*b^4 - b^6 + 16*a^2*c^4 + 32*a^
3*c^3 + 16*a^4*c^2 + b^4*c^2 - 8*a*b^2*c^3 - 8*a^3*b^2*c - 32*a^2*b^2*c^2 +
 10*a*b^4*c)))^(1/2)*(64*a*b^4 + 256*a*c^4 - 256*a^4*c - 64*b^4*c - 128*a^2
*b^3 + 64*a^3*b^2 + 256*a^2*c^3 - 256*a^3*c^2 - 64*b^2*c^3 + 128*b^3*c^2 +
192*a*b^2*c^2 - 192*a^2*b^2*c - 512*a*b*c^3 + 512*a^3*b*c) + 64*a*b^3*d + 6
4*a*b^3*e + 128*a*c^3*d + 128*a^3*c*d + 128*a*c^3*e + 128*a^3*c*e + 64*b^3*
c*d + 64*b^3*c*e - 256*a*b*c^2*d + 64*a*b^2*c*d - 256*a^2*b*c*d - 256*a*b*c
^2*e + 64*a*b^2*c*e - 256*a^2*b*c*e) + tan(x/2)*(64*a^3*e^2 - 32*b^3*d^2 -
32*b^3*e^2 + 64*c^3*d^2 + 32*a*b^2*d^2 + 96*a*b^2*e^2 - 128*a^2*b*e^2 - 64*
a^2*c*d^2 - 64*a*c^2*e^2 - 128*b*c^2*d^2 + 96*b^2*c*d^2 + 32*b^2*c*e^2 + 64
*a*b^2*d*e - 64*a^2*b*d*e + 256*a*c^2*d*e + 256*a^2*c*d*e - 64*b*c^2*d*e +
64*b^2*c*d*e - 384*a*b*c*d*e))*(-(b^4*d^2 - b^4*e^2 + 8*a*c^3*d^2 + b*d^2*(
-(4*a*c - b^2)^3)^(1/2) - 8*a^3*c*e^2 + b*e^2*(-(4*a*c - b^2)^3)^(1/2) + 2*
a^2*b^2*e^2 + 8*a^2*c^2*d^2 - 8*a^2*c^2*e^2 - 2*b^2*c^2*d^2 - 2*a*b^3*d*e -
 2*a*d*e*(-(4*a*c - b^2)^3)^(1/2) + 2*b^3*c*d*e - 2*c*d*e*(-(4*a*c - b^2)^3
)^(1/2) - 6*a*b^2*c*d^2 + 6*a*b^2*c*e^2 - 8*a*b*c^2*d*e + 8*a^2*b*c*d*e)/(2
*(a^2*b^4 - b^6 + 16*a^2*c^4 + 32*a^3*c^3 + 16*a^4*c^2 + b^4*c^2 - 8*a*b^2*
c^3 - 8*a^3*b^2*c - 32*a^2*b^2*c^2 + 10*a*b^4*c)))^(1/2) + ((-(b^4*d^2 - b^

4*e^2 + 8*a*c^3*d^2 + b*d^2*(-(4*a*c - b^2)^3)^(1/2) - 8*a^3*c*e^2 + b*e^2*
(-(4*a*c - b^2)^3)^(1/2) + 2*a^2*b^2*e^2 + 8*a^2*c^2*d^2 - 8*a^2*c^2*e^2 -
2*b^2*c^2*d^2 - 2*a*b^3*d*e - 2*a*d*e*(-(4*a*c - b^2)^3)^(1/2) + 2*b^3*c*d*
e - 2*c*d*e*(-(4*a*c - b^2)^3)^(1/2) - 6*a*b^2*c*d^2 + 6*a*b^2*c*e^2 - 8*a*
b*c^2*d*e + 8*a^2*b*c*d*e)/(2*(a^2*b^4 - b^6 + 16*a^2*c^4 + 32*a^3*c^3 + 16
*a^4*c^2 + b^4*c^2 - 8*a*b^2*c^3 - 8*a^3*b^2*c - 32*a^2*b^2*c^2 + 10*a*b^4*
c)))^(1/2)*(256*a^2*c^2*d - 32*b^4*e - 32*a^2*b^2*d - 32*a^2*b^2*e - 32*b^4
*d + 256*a^2*c^2*e - 32*b^2*c^2*d - 32*b^2*c^2*e - tan(x/2)*(-(b^4*d^2 - b^
4*e^2 + 8*a*c^3*d^2 + b*d^2*(-(4*a*c - b^2)^3)^(1/2) - 8*a^3*c*e^2 + b*e^2*
(-(4*a*c - b^2)^3)^(1/2) + 2*a^2*b^2*e^2 + 8*a^2*c^2*d^2 - 8*a^2*c^2*e^2 -
2*b^2*c^2*d^2 - 2*a*b^3*d*e - 2*a*d*e*(-(4*a*c - b^2)^3)^(1/2) + 2*b^3*c*d*
e - 2*c*d*e*(-(4*a*c - b^2)^3)^(1/2) - 6*a*b^2*c*d^2 + 6*a*b^2*c*e^2 - 8*a*
b*c^2*d*e + 8*a^2*b*c*d*e)/(2*(a^2*b^4 - b^6 + 16*a^2*c^4 + 32*a^3*c^3 + 16
*a^4*c^2 + b^4*c^2 - 8*a*b^2*c^3 - 8*a^3*b^2*c - 32*a^2*b^2*c^2 + 10*a*b^4*
c)))^(1/2)*(64*a*b^4 + 256*a*c^4 - 256*a^4*c - 64*b^4*c - 128*a^2*b^3 + 64*
a^3*b^2 + 256*a^2*c^3 - 256*a^3*c^2 - 64*b^2*c^3 + 128*b^3*c^2 + 192*a*b^2*
c^2 - 192*a^2*b^2*c - 512*a*b*c^3 + 512*a^3*b*c) + 64*a*b^3*d + 64*a*b^3*e
+ 128*a*c^3*d + 128*a^3*c*d + 128*a*c^3*e + 128*a^3*c*e + 64*b^3*c*d + 64*b
^3*c*e - 256*a*b*c^2*d + 64*a*b^2*c*d - 256*a^2*b*c*d - 256*a*b*c^2*e + 64*
a*b^2*c*e - 256*a^2*b*c*e) - tan(x/2)*(64*a^3*e^2 - 32*b^3*d^2 - 32*b^3*e^2
+ 64*c^3*d^2 + 32*a*b^2*d^2 + 96*a*b^2*e^2 - 128*a^2*b*e^2 - 64*a^2*c*d^2
- 64*a*c^2*e^2 - 128*b*c^2*d^2 + 96*b^2*c*d^2 + 32*b^2*c*e^2 + 64*a*b^2*d*e
- 64*a^2*b*d*e + 256*a*c^2*d*e + 256*a^2*c*d*e - 64*b*c^2*d*e + 64*b^2*c*d
*e - 384*a*b*c*d*e))*(-(b^4*d^2 - b^4*e^2 + 8*a*c^3*d^2 + b*d^2*(-(4*a*c -
b^2)^3)^(1/2) - 8*a^3*c*e^2 + b*e^2*(-(4*a*c - b^2)^3)^(1/2) + 2*a^2*b^2*e^
2 + 8*a^2*c^2*d^2 - 8*a^2*c^2*e^2 - 2*b^2*c^2*d^2 - 2*a*b^3*d*e - 2*a*d*e*(
-(4*a*c - b^2)^3)^(1/2) + 2*b^3*c*d*e - 2*c*d*e*(-(4*a*c - b^2)^3)^(1/2) -
6*a*b^2*c*d^2 + 6*a*b^2*c*e^2 - 8*a*b*c^2*d*e + 8*a^2*b*c*d*e)/(2*(a^2*b^4
- b^6 + 16*a^2*c^4 + 32*a^3*c^3 + 16*a^4*c^2 + b^4*c^2 - 8*a*b^2*c^3 - 8*a^
3*b^2*c - 32*a^2*b^2*c^2 + 10*a*b^4*c)))^(1/2) - 64*a^2*e^3 + 64*c^2*d^3 +
64*a^2*d*e^2 - 64*b^2*d*e^2 + 64*b^2*d^2*e - 64*c^2*d^2*e + 64*a*b*e^3 + 64
*a*c*d^3 - 64*a*c*e^3 - 64*b*c*d^3 - 64*a*b*d^2*e + 64*a*c*d*e^2 - 64*a*c*d
^2*e + 64*b*c*d*e^2))*(-(b^4*d^2 - b^4*e^2 + 8*a*c^3*d^2 + b*d^2*(-(4*a*c -
b^2)^3)^(1/2) - 8*a^3*c*e^2 + b*e^2*(-(4*a*c - b^2)^3)^(1/2) + 2*a^2*b^2*e
^2 + 8*a^2*c^2*d^2 - 8*a^2*c^2*e^2 - 2*b^2*c^2*d^2 - 2*a*b^3*d*e - 2*a*d*e*
(-(4*a*c - b^2)^3)^(1/2) + 2*b^3*c*d*e - 2*c*d*e*(-(4*a*c - b^2)^3)^(1/2) -
6*a*b^2*c*d^2 + 6*a*b^2*c*e^2 - 8*a*b*c^2*d*e + 8*a^2*b*c*d*e)/(2*(a^2*b^4
- b^6 + 16*a^2*c^4 + 32*a^3*c^3 + 16*a^4*c^2 + b^4*c^2 - 8*a*b^2*c^3 - 8*a
^3*b^2*c - 32*a^2*b^2*c^2 + 10*a*b^4*c)))^(1/2)*2i - atan((((-(b^4*d^2 - b^
4*e^2 + 8*a*c^3*d^2 - b*d^2*(-(4*a*c - b^2)^3)^(1/2) - 8*a^3*c*e^2 - b*e^2*
(-(4*a*c - b^2)^3)^(1/2) + 2*a^2*b^2*e^2 + 8*a^2*c^2*d^2 - 8*a^2*c^2*e^2 -
2*b^2*c^2*d^2 - 2*a*b^3*d*e + 2*a*d*e*(-(4*a*c - b^2)^3)^(1/2) + 2*b^3*c*d*
e + 2*c*d*e*(-(4*a*c - b^2)^3)^(1/2) - 6*a*b^2*c*d^2 + 6*a*b^2*c*e^2 - 8*a*
b*c^2*d*e + 8*a^2*b*c*d*e)/(2*(a^2*b^4 - b^6 + 16*a^2*c^4 + 32*a^3*c^3 + 16
*a^4*c^2 + b^4*c^2 - 8*a*b^2*c^3 - 8*a^3*b^2*c - 32*a^2*b^2*c^2 + 10*a*b^4*

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c)))^(1/2)*(256*a^2*c^2*d - 32*b^4*e - 32*a^2*b^2*d - 32*a^2*b^2*e - 32*b^4
*d + 256*a^2*c^2*e - 32*b^2*c^2*d - 32*b^2*c^2*e + tan(x/2)*(-(b^4*d^2 - b^
4*e^2 + 8*a*c^3*d^2 - b*d^2*(-(4*a*c - b^2)^3)^(1/2) - 8*a^3*c*e^2 - b*e^2*
(-(4*a*c - b^2)^3)^(1/2) + 2*a^2*b^2*e^2 + 8*a^2*c^2*d^2 - 8*a^2*c^2*e^2 -
2*b^2*c^2*d^2 - 2*a*b^3*d*e + 2*a*d*e*(-(4*a*c - b^2)^3)^(1/2) + 2*b^3*c*d*
e + 2*c*d*e*(-(4*a*c - b^2)^3)^(1/2) - 6*a*b^2*c*d^2 + 6*a*b^2*c*e^2 - 8*a*
b*c^2*d*e + 8*a^2*b*c*d*e)/(2*(a^2*b^4 - b^6 + 16*a^2*c^4 + 32*a^3*c^3 + 16
*a^4*c^2 + b^4*c^2 - 8*a*b^2*c^3 - 8*a^3*b^2*c - 32*a^2*b^2*c^2 + 10*a*b^4*
c)))^(1/2)*(64*a*b^4 + 256*a*c^4 - 256*a^4*c - 64*b^4*c - 128*a^2*b^3 + 64*
a^3*b^2 + 256*a^2*c^3 - 256*a^3*c^2 - 64*b^2*c^3 + 128*b^3*c^2 + 192*a*b^2*
c^2 - 192*a^2*b^2*c - 512*a*b*c^3 + 512*a^3*b*c) + 64*a*b^3*d + 64*a*b^3*e
+ 128*a*c^3*d + 128*a^3*c*d + 128*a*c^3*e + 128*a^3*c*e + 64*b^3*c*d + 64*b
^3*c*e - 256*a*b*c^2*d + 64*a*b^2*c*d - 256*a^2*b*c*d - 256*a*b*c^2*e + 64*
a*b^2*c*e - 256*a^2*b*c*e) + tan(x/2)*(64*a^3*e^2 - 32*b^3*d^2 - 32*b^3*e^2
 + 64*c^3*d^2 + 32*a*b^2*d^2 + 96*a*b^2*e^2 - 128*a^2*b*e^2 - 64*a^2*c*d^2
- 64*a*c^2*e^2 - 128*b*c^2*d^2 + 96*b^2*c*d^2 + 32*b^2*c*e^2 + 64*a*b^2*d*e
 - 64*a^2*b*d*e + 256*a*c^2*d*e + 256*a^2*c*d*e - 64*b*c^2*d*e + 64*b^2*c*d
*e - 384*a*b*c*d*e))*(-(b^4*d^2 - b^4*e^2 + 8*a*c^3*d^2 - b*d^2*(-(4*a*c -
b^2)^3)^(1/2) - 8*a^3*c*e^2 - b*e^2*(-(4*a*c - b^2)^3)^(1/2) + 2*a^2*b^2*e^
2 + 8*a^2*c^2*d^2 - 8*a^2*c^2*e^2 - 2*b^2*c^2*d^2 - 2*a*b^3*d*e + 2*a*d*e*(
-(4*a*c - b^2)^3)^(1/2) + 2*b^3*c*d*e + 2*c*d*e*(-(4*a*c - b^2)^3)^(1/2) -
6*a*b^2*c*d^2 + 6*a*b^2*c*e^2 - 8*a*b*c^2*d*e + 8*a^2*b*c*d*e)/(2*(a^2*b^4
- b^6 + 16*a^2*c^4 + 32*a^3*c^3 + 16*a^4*c^2 + b^4*c^2 - 8*a*b^2*c^3 - 8*a^
3*b^2*c - 32*a^2*b^2*c^2 + 10*a*b^4*c)))^(1/2)*1i - ((-(b^4*d^2 - b^4*e^2 +
 8*a*c^3*d^2 - b*d^2*(-(4*a*c - b^2)^3)^(1/2) - 8*a^3*c*e^2 - b*e^2*(-(4*a*
c - b^2)^3)^(1/2) + 2*a^2*b^2*e^2 + 8*a^2*c^2*d^2 - 8*a^2*c^2*e^2 - 2*b^2*c
^2*d^2 - 2*a*b^3*d*e + 2*a*d*e*(-(4*a*c - b^2)^3)^(1/2) + 2*b^3*c*d*e + 2*c
*d*e*(-(4*a*c - b^2)^3)^(1/2) - 6*a*b^2*c*d^2 + 6*a*b^2*c*e^2 - 8*a*b*c^2*d
*e + 8*a^2*b*c*d*e)/(2*(a^2*b^4 - b^6 + 16*a^2*c^4 + 32*a^3*c^3 + 16*a^4*c^
2 + b^4*c^2 - 8*a*b^2*c^3 - 8*a^3*b^2*c - 32*a^2*b^2*c^2 + 10*a*b^4*c)))^(1
/2)*(256*a^2*c^2*d - 32*b^4*e - 32*a^2*b^2*d - 32*a^2*b^2*e - 32*b^4*d + 25
6*a^2*c^2*e - 32*b^2*c^2*d - 32*b^2*c^2*e - tan(x/2)*(-(b^4*d^2 - b^4*e^2 +
 8*a*c^3*d^2 - b*d^2*(-(4*a*c - b^2)^3)^(1/2) - 8*a^3*c*e^2 - b*e^2*(-(4*a*
c - b^2)^3)^(1/2) + 2*a^2*b^2*e^2 + 8*a^2*c^2*d^2 - 8*a^2*c^2*e^2 - 2*b^2*c
^2*d^2 - 2*a*b^3*d*e + 2*a*d*e*(-(4*a*c - b^2)^3)^(1/2) + 2*b^3*c*d*e + 2*c
*d*e*(-(4*a*c - b^2)^3)^(1/2) - 6*a*b^2*c*d^2 + 6*a*b^2*c*e^2 - 8*a*b*c^2*d
*e + 8*a^2*b*c*d*e)/(2*(a^2*b^4 - b^6 + 16*a^2*c^4 + 32*a^3*c^3 + 16*a^4*c^
2 + b^4*c^2 - 8*a*b^2*c^3 - 8*a^3*b^2*c - 32*a^2*b^2*c^2 + 10*a*b^4*c)))^(1
/2)*(64*a*b^4 + 256*a*c^4 - 256*a^4*c - 64*b^4*c - 128*a^2*b^3 + 64*a^3*b^2
 + 256*a^2*c^3 - 256*a^3*c^2 - 64*b^2*c^3 + 128*b^3*c^2 + 192*a*b^2*c^2 - 1
92*a^2*b^2*c - 512*a*b*c^3 + 512*a^3*b*c) + 64*a*b^3*d + 64*a*b^3*e + 128*a
*c^3*d + 128*a^3*c*d + 128*a*c^3*e + 128*a^3*c*e + 64*b^3*c*d + 64*b^3*c*e
- 256*a*b*c^2*d + 64*a*b^2*c*d - 256*a^2*b*c*d - 256*a*b*c^2*e + 64*a*b^2*c
*e - 256*a^2*b*c*e) - tan(x/2)*(64*a^3*e^2 - 32*b^3*d^2 - 32*b^3*e^2 + 64*c
^3*d^2 + 32*a*b^2*d^2 + 96*a*b^2*e^2 - 128*a^2*b*e^2 - 64*a^2*c*d^2 - 64*a*
```

c^2*e^2 - 128*b*c^2*d^2 + 96*b^2*c*d^2 + 32*b^2*c*e^2 + 64*a*b^2*d*e - 64*a
^2*b*d*e + 256*a*c^2*d*e + 256*a^2*c*d*e - 64*b*c^2*d*e + 64*b^2*c*d*e - 38
4*a*b*c*d*e))*(-(b^4*d^2 - b^4*e^2 + 8*a*c^3*d^2 - b*d^2*(-(4*a*c - b^2)^3)
^(1/2) - 8*a^3*c*e^2 - b*e^2*(-(4*a*c - b^2)^3)^(1/2) + 2*a^2*b^2*e^2 + 8*a
^2*c^2*d^2 - 8*a^2*c^2*e^2 - 2*b^2*c^2*d^2 - 2*a*b^3*d*e + 2*a*d*e*(-(4*a*c
 - b^2)^3)^(1/2) + 2*b^3*c*d*e + 2*c*d*e*(-(4*a*c - b^2)^3)^(1/2) - 6*a*b^2
*c*d^2 + 6*a*b^2*c*e^2 - 8*a*b*c^2*d*e + 8*a^2*b*c*d*e)/(2*(a^2*b^4 - b^6 +
 16*a^2*c^4 + 32*a^3*c^3 + 16*a^4*c^2 + b^4*c^2 - 8*a*b^2*c^3 - 8*a^3*b^2*c
 - 32*a^2*b^2*c^2 + 10*a*b^4*c)))^(1/2)*1i)/(((-(b^4*d^2 - b^4*e^2 + 8*a*c^
3*d^2 - b*d^2*(-(4*a*c - b^2)^3)^(1/2) - 8*a^3*c*e^2 - b*e^2*(-(4*a*c - b^2
)^3)^(1/2) + 2*a^2*b^2*e^2 + 8*a^2*c^2*d^2 - 8*a^2*c^2*e^2 - 2*b^2*c^2*d^2
- 2*a*b^3*d*e + 2*a*d*e*(-(4*a*c - b^2)^3)^(1/2) + 2*b^3*c*d*e + 2*c*d*e*(-
(4*a*c - b^2)^3)^(1/2) - 6*a*b^2*c*d^2 + 6*a*b^2*c*e^2 - 8*a*b*c^2*d*e + 8*
a^2*b*c*d*e)/(2*(a^2*b^4 - b^6 + 16*a^2*c^4 + 32*a^3*c^3 + 16*a^4*c^2 + b^4
*c^2 - 8*a*b^2*c^3 - 8*a^3*b^2*c - 32*a^2*b^2*c^2 + 10*a*b^4*c)))^(1/2)*(25
6*a^2*c^2*d - 32*b^4*e - 32*a^2*b^2*d - 32*a^2*b^2*e - 32*b^4*d + 256*a^2*c
^2*e - 32*b^2*c^2*d - 32*b^2*c^2*e + tan(x/2)*(-(b^4*d^2 - b^4*e^2 + 8*a*c^
3*d^2 - b*d^2*(-(4*a*c - b^2)^3)^(1/2) - 8*a^3*c*e^2 - b*e^2*(-(4*a*c - b^2
)^3)^(1/2) + 2*a^2*b^2*e^2 + 8*a^2*c^2*d^2 - 8*a^2*c^2*e^2 - 2*b^2*c^2*d^2
- 2*a*b^3*d*e + 2*a*d*e*(-(4*a*c - b^2)^3)^(1/2) + 2*b^3*c*d*e + 2*c*d*e*(-
(4*a*c - b^2)^3)^(1/2) - 6*a*b^2*c*d^2 + 6*a*b^2*c*e^2 - 8*a*b*c^2*d*e + 8*
a^2*b*c*d*e)/(2*(a^2*b^4 - b^6 + 16*a^2*c^4 + 32*a^3*c^3 + 16*a^4*c^2 + b^4
*c^2 - 8*a*b^2*c^3 - 8*a^3*b^2*c - 32*a^2*b^2*c^2 + 10*a*b^4*c)))^(1/2)*(64
*a*b^4 + 256*a*c^4 - 256*a^4*c - 64*b^4*c - 128*a^2*b^3 + 64*a^3*b^2 + 256*
a^2*c^3 - 256*a^3*c^2 - 64*b^2*c^3 + 128*b^3*c^2 + 192*a*b^2*c^2 - 192*a^2*
b^2*c - 512*a*b*c^3 + 512*a^3*b*c) + 64*a*b^3*d + 64*a*b^3*e + 128*a*c^3*d
+ 128*a^3*c*d + 128*a*c^3*e + 128*a^3*c*e + 64*b^3*c*d + 64*b^3*c*e - 256*a
*b*c^2*d + 64*a*b^2*c*d - 256*a^2*b*c*d - 256*a*b*c^2*e + 64*a*b^2*c*e - 25
6*a^2*b*c*e) + tan(x/2)*(64*a^3*e^2 - 32*b^3*d^2 - 32*b^3*e^2 + 64*c^3*d^2
+ 32*a*b^2*d^2 + 96*a*b^2*e^2 - 128*a^2*b*e^2 - 64*a^2*c*d^2 - 64*a*c^2*e^2
 - 128*b*c^2*d^2 + 96*b^2*c*d^2 + 32*b^2*c*e^2 + 64*a*b^2*d*e - 64*a^2*b*d*
e + 256*a*c^2*d*e + 256*a^2*c*d*e - 64*b*c^2*d*e + 64*b^2*c*d*e - 384*a*b*c
*d*e))*(-(b^4*d^2 - b^4*e^2 + 8*a*c^3*d^2 - b*d^2*(-(4*a*c - b^2)^3)^(1/2)
- 8*a^3*c*e^2 - b*e^2*(-(4*a*c - b^2)^3)^(1/2) + 2*a^2*b^2*e^2 + 8*a^2*c^2*
d^2 - 8*a^2*c^2*e^2 - 2*b^2*c^2*d^2 - 2*a*b^3*d*e + 2*a*d*e*(-(4*a*c - b^2)
^3)^(1/2) + 2*b^3*c*d*e + 2*c*d*e*(-(4*a*c - b^2)^3)^(1/2) - 6*a*b^2*c*d^2
+ 6*a*b^2*c*e^2 - 8*a*b*c^2*d*e + 8*a^2*b*c*d*e)/(2*(a^2*b^4 - b^6 + 16*a^2
*c^4 + 32*a^3*c^3 + 16*a^4*c^2 + b^4*c^2 - 8*a*b^2*c^3 - 8*a^3*b^2*c - 32*a
^2*b^2*c^2 + 10*a*b^4*c)))^(1/2) + ((-(b^4*d^2 - b^4*e^2 + 8*a*c^3*d^2 - b*
d^2*(-(4*a*c - b^2)^3)^(1/2) - 8*a^3*c*e^2 - b*e^2*(-(4*a*c - b^2)^3)^(1/2)
 + 2*a^2*b^2*e^2 + 8*a^2*c^2*d^2 - 8*a^2*c^2*e^2 - 2*b^2*c^2*d^2 - 2*a*b^3*
d*e + 2*a*d*e*(-(4*a*c - b^2)^3)^(1/2) + 2*b^3*c*d*e + 2*c*d*e*(-(4*a*c - b
^2)^3)^(1/2) - 6*a*b^2*c*d^2 + 6*a*b^2*c*e^2 - 8*a*b*c^2*d*e + 8*a^2*b*c*d*
e)/(2*(a^2*b^4 - b^6 + 16*a^2*c^4 + 32*a^3*c^3 + 16*a^4*c^2 + b^4*c^2 - 8*a
*b^2*c^3 - 8*a^3*b^2*c - 32*a^2*b^2*c^2 + 10*a*b^4*c)))^(1/2)*(256*a^2*c^2*

```
d - 32*b^4*e - 32*a^2*b^2*d - 32*a^2*b^2*e - 32*b^4*d + 256*a^2*c^2*e - 32*
b^2*c^2*d - 32*b^2*c^2*e - tan(x/2)*(-(b^4*d^2 - b^4*e^2 + 8*a*c^3*d^2 - b*
d^2*(-(4*a*c - b^2)^3)^(1/2) - 8*a^3*c*e^2 - b*e^2*(-(4*a*c - b^2)^3)^(1/2)
 + 2*a^2*b^2*e^2 + 8*a^2*c^2*d^2 - 8*a^2*c^2*e^2 - 2*b^2*c^2*d^2 - 2*a*b^3*
d*e + 2*a*d*e*(-(4*a*c - b^2)^3)^(1/2) + 2*b^3*c*d*e + 2*c*d*e*(-(4*a*c - b
^2)^3)^(1/2) - 6*a*b^2*c*d^2 + 6*a*b^2*c*e^2 - 8*a*b*c^2*d*e + 8*a^2*b*c*d*
e)/(2*(a^2*b^4 - b^6 + 16*a^2*c^4 + 32*a^3*c^3 + 16*a^4*c^2 + b^4*c^2 - 8*a
*b^2*c^3 - 8*a^3*b^2*c - 32*a^2*b^2*c^2 + 10*a*b^4*c)))^(1/2)*(64*a*b^4 + 2
56*a*c^4 - 256*a^4*c - 64*b^4*c - 128*a^2*b^3 + 64*a^3*b^2 + 256*a^2*c^3 -
256*a^3*c^2 - 64*b^2*c^3 + 128*b^3*c^2 + 192*a*b^2*c^2 - 192*a^2*b^2*c - 51
2*a*b*c^3 + 512*a^3*b*c) + 64*a*b^3*d + 64*a*b^3*e + 128*a*c^3*d + 128*a^3*
c*d + 128*a*c^3*e + 128*a^3*c*e + 64*b^3*c*d + 64*b^3*c*e - 256*a*b*c^2*d +
 64*a*b^2*c*d - 256*a^2*b*c*d - 256*a*b*c^2*e + 64*a*b^2*c*e - 256*a^2*b*c*
e) - tan(x/2)*(64*a^3*e^2 - 32*b^3*d^2 - 32*b^3*e^2 + 64*c^3*d^2 + 32*a*b^2
*d^2 + 96*a*b^2*e^2 - 128*a^2*b*e^2 - 64*a^2*c*d^2 - 64*a*c^2*e^2 - 128*b*c
^2*d^2 + 96*b^2*c*d^2 + 32*b^2*c*e^2 + 64*a*b^2*d*e - 64*a^2*b*d*e + 256*a*
c^2*d*e + 256*a^2*c*d*e - 64*b*c^2*d*e + 64*b^2*c*d*e - 384*a*b*c*d*e))*(-(
b^4*d^2 - b^4*e^2 + 8*a*c^3*d^2 - b*d^2*(-(4*a*c - b^2)^3)^(1/2) - 8*a^3*c*
e^2 - b*e^2*(-(4*a*c - b^2)^3)^(1/2) + 2*a^2*b^2*e^2 + 8*a^2*c^2*d^2 - 8*a^
2*c^2*e^2 - 2*b^2*c^2*d^2 - 2*a*b^3*d*e + 2*a*d*e*(-(4*a*c - b^2)^3)^(1/2)
+ 2*b^3*c*d*e + 2*c*d*e*(-(4*a*c - b^2)^3)^(1/2) - 6*a*b^2*c*d^2 + 6*a*b^2*
c*e^2 - 8*a*b*c^2*d*e + 8*a^2*b*c*d*e)/(2*(a^2*b^4 - b^6 + 16*a^2*c^4 + 32*
a^3*c^3 + 16*a^4*c^2 + b^4*c^2 - 8*a*b^2*c^3 - 8*a^3*b^2*c - 32*a^2*b^2*c^2
 + 10*a*b^4*c)))^(1/2) - 64*a^2*e^3 + 64*c^2*d^3 + 64*a^2*d*e^2 - 64*b^2*d*
e^2 + 64*b^2*d^2*e - 64*c^2*d^2*e + 64*a*b*e^3 + 64*a*c*d^3 - 64*a*c*e^3 -
64*b*c*d^3 - 64*a*b*d^2*e + 64*a*c*d*e^2 - 64*a*c*d^2*e + 64*b*c*d*e^2))*(-
(b^4*d^2 - b^4*e^2 + 8*a*c^3*d^2 - b*d^2*(-(4*a*c - b^2)^3)^(1/2) - 8*a^3*c
*e^2 - b*e^2*(-(4*a*c - b^2)^3)^(1/2) + 2*a^2*b^2*e^2 + 8*a^2*c^2*d^2 - 8*a
^2*c^2*e^2 - 2*b^2*c^2*d^2 - 2*a*b^3*d*e + 2*a*d*e*(-(4*a*c - b^2)^3)^(1/2)
 + 2*b^3*c*d*e + 2*c*d*e*(-(4*a*c - b^2)^3)^(1/2) - 6*a*b^2*c*d^2 + 6*a*b^2
*c*e^2 - 8*a*b*c^2*d*e + 8*a^2*b*c*d*e)/(2*(a^2*b^4 - b^6 + 16*a^2*c^4 + 32
*a^3*c^3 + 16*a^4*c^2 + b^4*c^2 - 8*a*b^2*c^3 - 8*a^3*b^2*c - 32*a^2*b^2*c^
2 + 10*a*b^4*c)))^(1/2)*2i
```

sympy **[F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*cos(x))/(a+b*cos(x)+c*cos(x)**2),x)
```

```
[Out] Timed out
```

3.510 $\int (a+b\tan(d+ex))\left(b^2+2ab\tan(d+ex)+a^2\tan^2(d+\right.$

Optimal. Leaf size=144

$$-\frac{a\left(a^4-b^4\right)\tan(d+ex)}{e}+\frac{\left(a^2+b^2\right)(a\tan(d+ex)+b)^3}{3e}+\frac{b\left(a^2+b^2\right)(a\tan(d+ex)+b)^2}{2e}+\frac{b\left(3a^2-b^2\right)\left(a^2+b^2\right.}{e}$$

```
[Out] a*(a^2-3*b^2)*(a^2+b^2)*x+b*(3*a^2-b^2)*(a^2+b^2)*ln(cos(e*x+d))/e-a*(a^4-b
^4)*tan(e*x+d)/e+1/2*b*(a^2+b^2)*(b+a*tan(e*x+d))^2/e+1/3*(a^2+b^2)*(b+a*ta
n(e*x+d))^3/e+1/4*b*(b+a*tan(e*x+d))^4/e
```

Rubi [A] time = 0.27, antiderivative size = 144, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 39, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.128, Rules used = {3708, 3528, 12, 3525, 3475}

$$\frac{\left(a^2+b^2\right)(a\tan(d+ex)+b)^3}{3e}+\frac{b\left(a^2+b^2\right)(a\tan(d+ex)+b)^2}{2e}-\frac{a\left(a^4-b^4\right)\tan(d+ex)}{e}+\frac{b\left(3a^2-b^2\right)\left(a^2+b^2\right)l}{e}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Tan[d + e*x])*(b^2 + 2*a*b*Tan[d + e*x] + a^2*Tan[d + e*x]^2)^2,
x]
```

```
[Out] a*(a^2 - 3*b^2)*(a^2 + b^2)*x + (b*(3*a^2 - b^2)*(a^2 + b^2)*Log[Cos[d + e*
x]])/e - (a*(a^4 - b^4)*Tan[d + e*x])/e + (b*(a^2 + b^2)*(b + a*Tan[d + e*x
])^2)/(2*e) + ((a^2 + b^2)*(b + a*Tan[d + e*x])^3)/(3*e) + (b*(b + a*Tan[d
+ e*x])^4)/(4*e)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3525

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[(a*c - b*d)*x, x] + (Dist[b*c + a*d, Int[Tan[e +
 f*x], x], x] + Simp[(b*d*Tan[e + f*x])/f, x]) /; FreeQ[{a, b, c, d, e, f},
 x] && NeQ[b*c - a*d, 0] && NeQ[b*c + a*d, 0]
```

Rule 3528

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[(d*(a + b*Tan[e + f*x])^m)/(f*m), x] + Int
[(a + b*Tan[e + f*x])^(m - 1)*Simp[a*c - b*d + (b*c + a*d)*Tan[e + f*x], x]
, x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2,
 0] && GtQ[m, 0]
```

Rule 3708

```
Int[((A_) + (B_.)*tan[(d_.) + (e_.)*(x_)])*((a_) + (b_.)*tan[(d_.) + (e_.)*
(x_)] + (c_.)*tan[(d_.) + (e_.)*(x_)]^2)^(n_), x_Symbol] :> Dist[1/(4^n*c^n
), Int[(A + B*Tan[d + e*x])*(b + 2*c*Tan[d + e*x])^(2*n), x], x] /; FreeQ[{
a, b, c, d, e, A, B}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[n]
```

Rubi steps

$$\int (a+b\tan(d+ex))\left(b^2+2ab\tan(d+ex)+a^2\tan^2(d+ex)\right)^2\,dx = \frac{\int \left(2ab+2a^2\tan(d+ex)\right)^4 (a+b\tan(d+e$$

$$= \frac{b(b+a\tan(d+ex))^4}{4e} + \frac{\int 2a\left(a^2+b^2\right)\tan(}{}$$

$$= \frac{b(b+a\tan(d+ex))^4}{4e} + \frac{\left(a^2+b^2\right)\int \tan(d+}{}$$

$$= \frac{\left(a^2+b^2\right)(b+a\tan(d+ex))^3}{3e} + \frac{b(b+a\tan}{4e}$$

$$= \frac{b\left(a^2+b^2\right)(b+a\tan(d+ex))^2}{2e} + \frac{\left(a^2+b^2\right)}{}$$

$$= a\left(a^2-3b^2\right)\left(a^2+b^2\right)x - \frac{a\left(a^4-b^4\right)\tan(d}{e}$$

$$= a\left(a^2-3b^2\right)\left(a^2+b^2\right)x + \frac{b\left(3a^2-b^2\right)\left(a^2+\right.}{}$$

Mathematica [C] time = 2.10, size = 153, normalized size = 1.06

$$\frac{3a^4b\tan^4(d+ex)+18a^2b\left(a^2+2b^2\right)\tan^2(d+ex)+6\left(a^2+b^2\right)\left(i(a+ib)^3\log(\tan(d+ex)+i)-i(a-ib)^3\log(-\right.}{12e}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Tan[d + e*x])*(b^2 + 2*a*b*Tan[d + e*x] + a^2*Tan[d + e*x]
^2)^2,x]
```

```
[Out] (6*(a^2 + b^2)*((-I)*(a - I*b)^3*Log[I - Tan[d + e*x]] + I*(a + I*b)^3*Log[
I + Tan[d + e*x]]) - 12*a*(a^4 - 2*a^2*b^2 - 4*b^4)*Tan[d + e*x] + 18*a^2*b
*(a^2 + 2*b^2)*Tan[d + e*x]^2 + 4*a^3*(a^2 + 4*b^2)*Tan[d + e*x]^3 + 3*a^4*
b*Tan[d + e*x]^4)/(12*e)
```

fricas [A] time = 1.71, size = 149, normalized size = 1.03

$$\frac{3\,a^4b\tan\left(ex+d\right)^4+4\left(a^5+4\,a^3b^2\right)\tan\left(ex+d\right)^3+12\left(a^5-2\,a^3b^2-3\,ab^4\right)ex+18\left(a^4b+2\,a^2b^3\right)\tan\left(ex+d\right)^2}{12\,e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(e*x+d))*(b^2+2*a*b*tan(e*x+d)+a^2*tan(e*x+d)^2)^2,x, alg
orithm="fricas")
```

```
[Out] 1/12*(3*a^4*b*tan(e*x + d)^4 + 4*(a^5 + 4*a^3*b^2)*tan(e*x + d)^3 + 12*(a^5
 - 2*a^3*b^2 - 3*a*b^4)*e*x + 18*(a^4*b + 2*a^2*b^3)*tan(e*x + d)^2 + 6*(3*
a^4*b + 2*a^2*b^3 - b^5)*log(1/(tan(e*x + d)^2 + 1)) - 12*(a^5 - 2*a^3*b^2
- 4*a*b^4)*tan(e*x + d))/e
```

giac [B] time = 5.12, size = 2326, normalized size = 16.15

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(e*x+d))*(b^2+2*a*b*tan(e*x+d)+a^2*tan(e*x+d)^2)^2,x, alg
orithm="giac")
```

```
[Out] 1/12*(12*a^5*x*e*tan(x*e)^4*tan(d)^4 - 24*a^3*b^2*x*e*tan(x*e)^4*tan(d)^4 -
 36*a*b^4*x*e*tan(x*e)^4*tan(d)^4 + 18*a^4*b*log(4*(tan(x*e)^4*tan(d)^2 - 2
*tan(x*e)^3*tan(d) + tan(x*e)^2*tan(d)^2 + tan(x*e)^2 - 2*tan(x*e)*tan(d) +
 1)/(tan(d)^2 + 1))*tan(x*e)^4*tan(d)^4 + 12*a^2*b^3*log(4*(tan(x*e)^4*tan(
d)^2 - 2*tan(x*e)^3*tan(d) + tan(x*e)^2*tan(d)^2 + tan(x*e)^2 - 2*tan(x*e)*
tan(d) + 1)/(tan(d)^2 + 1))*tan(x*e)^4*tan(d)^4 - 6*b^5*log(4*(tan(x*e)^4*t
an(d)^2 - 2*tan(x*e)^3*tan(d) + tan(x*e)^2*tan(d)^2 + tan(x*e)^2 - 2*tan(x*
e)*tan(d) + 1)/(tan(d)^2 + 1))*tan(x*e)^4*tan(d)^4 - 48*a^5*x*e*tan(x*e)^3*
tan(d)^3 + 96*a^3*b^2*x*e*tan(x*e)^3*tan(d)^3 + 144*a*b^4*x*e*tan(x*e)^3*ta
n(d)^3 + 15*a^4*b*tan(x*e)^4*tan(d)^4 + 36*a^2*b^3*tan(x*e)^4*tan(d)^4 - 72
*a^4*b*log(4*(tan(x*e)^4*tan(d)^2 - 2*tan(x*e)^3*tan(d) + tan(x*e)^2*tan(d)
^2 + tan(x*e)^2 - 2*tan(x*e)*tan(d) + 1)/(tan(d)^2 + 1))*tan(x*e)^3*tan(d)^
3 - 48*a^2*b^3*log(4*(tan(x*e)^4*tan(d)^2 - 2*tan(x*e)^3*tan(d) + tan(x*e)^
2*tan(d)^2 + tan(x*e)^2 - 2*tan(x*e)*tan(d) + 1)/(tan(d)^2 + 1))*tan(x*e)^3
```

*tan(d)^3 + 24*b^5*log(4*(tan(x*e)^4*tan(d)^2 - 2*tan(x*e)^3*tan(d) + tan(x
*e)^2*tan(d)^2 + tan(x*e)^2 - 2*tan(x*e)*tan(d) + 1)/(tan(d)^2 + 1))*tan(x*
e)^3*tan(d)^3 + 12*a^5*tan(x*e)^4*tan(d)^3 - 24*a^3*b^2*tan(x*e)^4*tan(d)^3
 - 48*a*b^4*tan(x*e)^4*tan(d)^3 + 12*a^5*tan(x*e)^3*tan(d)^4 - 24*a^3*b^2*t
an(x*e)^3*tan(d)^4 - 48*a*b^4*tan(x*e)^3*tan(d)^4 + 72*a^5*x*e*tan(x*e)^2*t
an(d)^2 - 144*a^3*b^2*x*e*tan(x*e)^2*tan(d)^2 - 216*a*b^4*x*e*tan(x*e)^2*ta
n(d)^2 + 18*a^4*b*tan(x*e)^4*tan(d)^2 + 36*a^2*b^3*tan(x*e)^4*tan(d)^2 - 24
*a^4*b*tan(x*e)^3*tan(d)^3 - 72*a^2*b^3*tan(x*e)^3*tan(d)^3 + 18*a^4*b*tan(
x*e)^2*tan(d)^4 + 36*a^2*b^3*tan(x*e)^2*tan(d)^4 - 4*a^5*tan(x*e)^4*tan(d)
- 16*a^3*b^2*tan(x*e)^4*tan(d) + 108*a^4*b*log(4*(tan(x*e)^4*tan(d)^2 - 2*t
an(x*e)^3*tan(d) + tan(x*e)^2*tan(d)^2 + tan(x*e)^2 - 2*tan(x*e)*tan(d) + 1
)/(tan(d)^2 + 1))*tan(x*e)^2*tan(d)^2 + 72*a^2*b^3*log(4*(tan(x*e)^4*tan(d)
^2 - 2*tan(x*e)^3*tan(d) + tan(x*e)^2*tan(d)^2 + tan(x*e)^2 - 2*tan(x*e)*ta
n(d) + 1)/(tan(d)^2 + 1))*tan(x*e)^2*tan(d)^2 - 36*b^5*log(4*(tan(x*e)^4*ta
n(d)^2 - 2*tan(x*e)^3*tan(d) + tan(x*e)^2*tan(d)^2 + tan(x*e)^2 - 2*tan(x*e
)*tan(d) + 1)/(tan(d)^2 + 1))*tan(x*e)^2*tan(d)^2 - 48*a^5*tan(x*e)^3*tan(d
)^2 + 24*a^3*b^2*tan(x*e)^3*tan(d)^2 + 144*a*b^4*tan(x*e)^3*tan(d)^2 - 48*a
^5*tan(x*e)^2*tan(d)^3 + 24*a^3*b^2*tan(x*e)^2*tan(d)^3 + 144*a*b^4*tan(x*e
)^2*tan(d)^3 - 4*a^5*tan(x*e)*tan(d)^4 - 16*a^3*b^2*tan(x*e)*tan(d)^4 + 3*a
^4*b*tan(x*e)^4 - 48*a^5*x*e*tan(x*e)*tan(d) + 96*a^3*b^2*x*e*tan(x*e)*tan(
d) + 144*a*b^4*x*e*tan(x*e)*tan(d) - 24*a^4*b*tan(x*e)^3*tan(d) - 72*a^2*b^
3*tan(x*e)^3*tan(d) + 36*a^4*b*tan(x*e)^2*tan(d)^2 + 72*a^2*b^3*tan(x*e)^2*
tan(d)^2 - 24*a^4*b*tan(x*e)*tan(d)^3 - 72*a^2*b^3*tan(x*e)*tan(d)^3 + 3*a^
4*b*tan(d)^4 + 4*a^5*tan(x*e)^3 + 16*a^3*b^2*tan(x*e)^3 - 72*a^4*b*log(4*(t
an(x*e)^4*tan(d)^2 - 2*tan(x*e)^3*tan(d) + tan(x*e)^2*tan(d)^2 + tan(x*e)^2
 - 2*tan(x*e)*tan(d) + 1)/(tan(d)^2 + 1))*tan(x*e)*tan(d) - 48*a^2*b^3*log(
4*(tan(x*e)^4*tan(d)^2 - 2*tan(x*e)^3*tan(d) + tan(x*e)^2*tan(d)^2 + tan(x*
e)^2 - 2*tan(x*e)*tan(d) + 1)/(tan(d)^2 + 1))*tan(x*e)*tan(d) + 24*b^5*log(
4*(tan(x*e)^4*tan(d)^2 - 2*tan(x*e)^3*tan(d) + tan(x*e)^2*tan(d)^2 + tan(x*
e)^2 - 2*tan(x*e)*tan(d) + 1)/(tan(d)^2 + 1))*tan(x*e)*tan(d) + 48*a^5*tan(
x*e)^2*tan(d) - 24*a^3*b^2*tan(x*e)^2*tan(d) - 144*a*b^4*tan(x*e)^2*tan(d)
+ 48*a^5*tan(x*e)*tan(d)^2 - 24*a^3*b^2*tan(x*e)*tan(d)^2 - 144*a*b^4*tan(x
*e)*tan(d)^2 + 4*a^5*tan(d)^3 + 16*a^3*b^2*tan(d)^3 + 12*a^5*x*e - 24*a^3*b
^2*x*e - 36*a*b^4*x*e + 18*a^4*b*tan(x*e)^2 + 36*a^2*b^3*tan(x*e)^2 - 24*a^
4*b*tan(x*e)*tan(d) - 72*a^2*b^3*tan(x*e)*tan(d) + 18*a^4*b*tan(d)^2 + 36*a
^2*b^3*tan(d)^2 + 18*a^4*b*log(4*(tan(x*e)^4*tan(d)^2 - 2*tan(x*e)^3*tan(d)
 + tan(x*e)^2*tan(d)^2 + tan(x*e)^2 - 2*tan(x*e)*tan(d) + 1)/(tan(d)^2 + 1)
) + 12*a^2*b^3*log(4*(tan(x*e)^4*tan(d)^2 - 2*tan(x*e)^3*tan(d) + tan(x*e)^
2*tan(d)^2 + tan(x*e)^2 - 2*tan(x*e)*tan(d) + 1)/(tan(d)^2 + 1)) - 6*b^5*lo
g(4*(tan(x*e)^4*tan(d)^2 - 2*tan(x*e)^3*tan(d) + tan(x*e)^2*tan(d)^2 + tan(
x*e)^2 - 2*tan(x*e)*tan(d) + 1)/(tan(d)^2 + 1)) - 12*a^5*tan(x*e) + 24*a^3*
b^2*tan(x*e) + 48*a*b^4*tan(x*e) - 12*a^5*tan(d) + 24*a^3*b^2*tan(d) + 48*a
*b^4*tan(d) + 15*a^4*b + 36*a^2*b^3)/(e*tan(x*e)^4*tan(d)^4 - 4*e*tan(x*e)^
3*tan(d)^3 + 6*e*tan(x*e)^2*tan(d)^2 - 4*e*tan(x*e)*tan(d) + e)

maple [A] time = 0.01, size = 245, normalized size = 1.70

$$\frac{a^4b\left(\tan^4(ex+d)\right)}{4e}+\frac{\left(\tan^3(ex+d)\right)a^5}{3e}+\frac{4\left(\tan^3(ex+d)\right)a^3b^2}{3e}+\frac{3\left(\tan^2(ex+d)\right)a^4b}{2e}+\frac{3\left(\tan^2(ex+d)\right)a^2b^3}{e}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*tan(e*x+d))*(b^2+2*a*b*tan(e*x+d)+a^2*tan(e*x+d)^2)^2,x)
```

```
[Out] 1/4/e*a^4*b*tan(e*x+d)^4+1/3/e*tan(e*x+d)^3*a^5+4/3/e*tan(e*x+d)^3*a^3*b^2+
3/2/e*tan(e*x+d)^2*a^4*b+3/e*tan(e*x+d)^2*a^2*b^3-1/e*a^5*tan(e*x+d)+2/e*a^
3*b^2*tan(e*x+d)+4/e*a*b^4*tan(e*x+d)-3/2/e*ln(1+tan(e*x+d)^2)*a^4*b-1/e*ln
(1+tan(e*x+d)^2)*a^2*b^3+1/2/e*ln(1+tan(e*x+d)^2)*b^5+1/e*arctan(tan(e*x+d)
)*a^5-2/e*arctan(tan(e*x+d))*a^3*b^2-3/e*arctan(tan(e*x+d))*a*b^4
```

maxima [A] time = 0.40, size = 150, normalized size = 1.04

$$\frac{3\,a^4b\tan(ex+d)^4+4\left(a^5+4\,a^3b^2\right)\tan(ex+d)^3+18\left(a^4b+2\,a^2b^3\right)\tan(ex+d)^2+12\left(a^5-2\,a^3b^2-3\,ab^4\right)(ex}{12\,e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(e*x+d))*(b^2+2*a*b*tan(e*x+d)+a^2*tan(e*x+d)^2)^2,x, alg
orithm="maxima")
```

```
[Out] 1/12*(3*a^4*b*tan(e*x + d)^4 + 4*(a^5 + 4*a^3*b^2)*tan(e*x + d)^3 + 18*(a^4
*b + 2*a^2*b^3)*tan(e*x + d)^2 + 12*(a^5 - 2*a^3*b^2 - 3*a*b^4)*(e*x + d) -
 6*(3*a^4*b + 2*a^2*b^3 - b^5)*log(tan(e*x + d)^2 + 1) - 12*(a^5 - 2*a^3*b^
2 - 4*a*b^4)*tan(e*x + d))/e
```

mupad [B] time = 2.85, size = 205, normalized size = 1.42

$$\frac{\tan(d+e\,x)^3\,\left(\frac{a^5}{3}+\frac{4\,a^3\,b^2}{3}\right)}{e}+\frac{\tan(d+e\,x)\,\left(-a^5+2\,a^3\,b^2+4\,a\,b^4\right)}{e}-\frac{\ln\left(\tan(d+e\,x)^2+1\right)\,\left(\frac{3\,a^4\,b}{2}+a^2\,b^3-\frac{b^5}{2}\right)}{e}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*tan(d + e*x))*(b^2 + a^2*tan(d + e*x)^2 + 2*a*b*tan(d + e*x))^2,
x)
```

```
[Out] (tan(d + e*x)^3*(a^5/3 + (4*a^3*b^2)/3))/e + (tan(d + e*x)*(4*a*b^4 - a^5 +
 2*a^3*b^2))/e - (log(tan(d + e*x)^2 + 1)*((3*a^4*b)/2 - b^5/2 + a^2*b^3))/
e + (tan(d + e*x)^2*((3*a^4*b)/2 + 3*a^2*b^3))/e + (a^4*b*tan(d + e*x)^4)/(
4*e) - (a*atan((a*tan(d + e*x)*(a^2 - 3*b^2)*(a^2 + b^2))/(3*a*b^4 - a^5 +
2*a^3*b^2))*(a^2 - 3*b^2)*(a^2 + b^2))/e
```

sympy [A] time = 0.63, size = 248, normalized size = 1.72

$$\begin{cases} a^5x + \frac{a^5\tan^3(d+ex)}{3e} - \frac{a^5\tan(d+ex)}{e} - \frac{3a^4b\log\left(\tan^2(d+ex)+1\right)}{2e} + \frac{a^4b\tan^4(d+ex)}{4e} + \frac{3a^4b\tan^2(d+ex)}{2e} - 2a^3b^2x + \frac{4a^3b^2\tan^3(d+ex)}{3e} + \\ x\left(a+b\tan(d)\right)\left(a^2\tan^2(d)+2ab\tan(d)+b^2\right)^2 \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(e*x+d))*(b**2+2*a*b*tan(e*x+d)+a**2*tan(e*x+d)**2)**2,x)
```

```
[Out] Piecewise((a**5*x + a**5*tan(d + e*x)**3/(3*e) - a**5*tan(d + e*x)/e - 3*a*
*4*b*log(tan(d + e*x)**2 + 1)/(2*e) + a**4*b*tan(d + e*x)**4/(4*e) + 3*a**4
*b*tan(d + e*x)**2/(2*e) - 2*a**3*b**2*x + 4*a**3*b**2*tan(d + e*x)**3/(3*e
) + 2*a**3*b**2*tan(d + e*x)/e - a**2*b**3*log(tan(d + e*x)**2 + 1)/e + 3*a
**2*b**3*tan(d + e*x)**2/e - 3*a*b**4*x + 4*a*b**4*tan(d + e*x)/e + b**5*lo
g(tan(d + e*x)**2 + 1)/(2*e), Ne(e, 0)), (x*(a + b*tan(d))*(a**2*tan(d)**2
+ 2*a*b*tan(d) + b**2)**2, True))
```

3.511 $\int (a+b\tan(d+ex))\left(b^2+2ab\tan(d+ex)+a^2\tan^2(d+\right.$

Optimal. Leaf size=72

$$-\frac{b\left(a^2+b^2\right)\log(\cos(d+ex))}{e}-ax\left(a^2+b^2\right)+\frac{a^2(a+b\tan(d+ex))^2}{2be}+\frac{2ab^2\tan(d+ex)}{e}$$

```
[Out] -a*(a^2+b^2)*x-b*(a^2+b^2)*ln(cos(e*x+d))/e+2*a*b^2*tan(e*x+d)/e+1/2*a^2*(a
+b*tan(e*x+d))^2/b/e
```

Rubi [A] time = 0.08, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.081, Rules used = {3630, 3525, 3475}

$$-\frac{b\left(a^2+b^2\right)\log(\cos(d+ex))}{e}-ax\left(a^2+b^2\right)+\frac{a^2(a+b\tan(d+ex))^2}{2be}+\frac{2ab^2\tan(d+ex)}{e}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Tan[d + e*x])*(b^2 + 2*a*b*Tan[d + e*x] + a^2*Tan[d + e*x]^2),x]
```

```
[Out] -(a*(a^2 + b^2)*x) - (b*(a^2 + b^2)*Log[Cos[d + e*x]])/e + (2*a*b^2*Tan[d +
 e*x])/e + (a^2*(a + b*Tan[d + e*x])^2)/(2*b*e)
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3525

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[(a*c - b*d)*x, x] + (Dist[b*c + a*d, Int[Tan[e +
 f*x], x], x] + Simp[(b*d*Tan[e + f*x])/f, x]) /; FreeQ[{a, b, c, d, e, f},
 x] && NeQ[b*c - a*d, 0] && NeQ[b*c + a*d, 0]
```

Rule 3630

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(C*(a +
 b*Tan[e + f*x])^(m + 1))/(b*f*(m + 1)), x] + Int[(a + b*Tan[e + f*x])^m*Si
mp[A - C + B*Tan[e + f*x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
NeQ[A*b^2 - a*b*B + a^2*C, 0] &&  !LeQ[m, -1]
```

Rubi steps

$$\int (a+b\tan(d+ex))\left(b^2+2ab\tan(d+ex)+a^2\tan^2(d+ex)\right)\,dx = \frac{a^2(a+b\tan(d+ex))^2}{2be} + \int (a+b\tan(d+ex$$

$$= -a\left(a^2+b^2\right)x + \frac{2ab^2\tan(d+ex)}{e} + \frac{a^2(a+b}{}$$

$$= -a\left(a^2+b^2\right)x - \frac{b\left(a^2+b^2\right)\log(\cos(d+ex))}{e}$$

Mathematica [C] time = 0.33, size = 88, normalized size = 1.22

$$\frac{2a\left(a^2+2b^2\right)\tan(d+ex)+\left(a^2+b^2\right)\left((b+ia)\log(-\tan(d+ex)+i)+(b-ia)\log(\tan(d+ex)+i)\right)+a^2b\tan^2(d}{2e}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Tan[d + e*x])*(b^2 + 2*a*b*Tan[d + e*x] + a^2*Tan[d + e*x]
^2),x]
```

```
[Out] ((a^2 + b^2)*((I*a + b)*Log[I - Tan[d + e*x]] + ((-I)*a + b)*Log[I + Tan[d
+ e*x]]) + 2*a*(a^2 + 2*b^2)*Tan[d + e*x] + a^2*b*Tan[d + e*x]^2)/(2*e)
```

fricas [A] time = 1.12, size = 74, normalized size = 1.03

$$\frac{a^2b\tan\left(ex+d\right)^2-2\left(a^3+ab^2\right)ex-\left(a^2b+b^3\right)\log\left(\frac{1}{\tan\left(ex+d\right)^2+1}\right)+2\left(a^3+2\,ab^2\right)\tan\left(ex+d\right)}{2\,e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(e*x+d))*(b^2+2*a*b*tan(e*x+d)+a^2*tan(e*x+d)^2),x, algor
ithm="fricas")
```

```
[Out] 1/2*(a^2*b*tan(e*x + d)^2 - 2*(a^3 + a*b^2)*e*x - (a^2*b + b^3)*log(1/(tan(
e*x + d)^2 + 1)) + 2*(a^3 + 2*a*b^2)*tan(e*x + d))/e
```

giac [B] time = 1.02, size = 709, normalized size = 9.85

$$-\frac{2\,a^3xe\tan\left(xe\right)^2\tan(d)^2+2\,ab^2xe\tan\left(xe\right)^2\tan(d)^2+a^2b\log\left(\frac{4\left(\tan\left(xe\right)^4\tan(d)^2-2\,\tan\left(xe\right)^3\tan(d)+\tan\left(xe\right)^2\tan(d)^2+\tan\left(xe\right)^2}{\tan(d)^2+1}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(e*x+d))*(b^2+2*a*b*tan(e*x+d)+a^2*tan(e*x+d)^2),x, algor
ithm="giac")
```

```
[Out] -1/2*(2*a^3*x*e*tan(x*e)^2*tan(d)^2 + 2*a*b^2*x*e*tan(x*e)^2*tan(d)^2 + a^2
*b*log(4*(tan(x*e)^4*tan(d)^2 - 2*tan(x*e)^3*tan(d) + tan(x*e)^2*tan(d)^2 +
 tan(x*e)^2 - 2*tan(x*e)*tan(d) + 1)/(tan(d)^2 + 1))*tan(x*e)^2*tan(d)^2 +
b^3*log(4*(tan(x*e)^4*tan(d)^2 - 2*tan(x*e)^3*tan(d) + tan(x*e)^2*tan(d)^2
+ tan(x*e)^2 - 2*tan(x*e)*tan(d) + 1)/(tan(d)^2 + 1))*tan(x*e)^2*tan(d)^2 -
 4*a^3*x*e*tan(x*e)*tan(d) - 4*a*b^2*x*e*tan(x*e)*tan(d) - a^2*b*tan(x*e)^2
*tan(d)^2 - 2*a^2*b*log(4*(tan(x*e)^4*tan(d)^2 - 2*tan(x*e)^3*tan(d) + tan(
x*e)^2*tan(d)^2 + tan(x*e)^2 - 2*tan(x*e)*tan(d) + 1)/(tan(d)^2 + 1))*tan(x
*e)*tan(d) - 2*b^3*log(4*(tan(x*e)^4*tan(d)^2 - 2*tan(x*e)^3*tan(d) + tan(x
*e)^2*tan(d)^2 + tan(x*e)^2 - 2*tan(x*e)*tan(d) + 1)/(tan(d)^2 + 1))*tan(x*
e)*tan(d) + 2*a^3*tan(x*e)^2*tan(d) + 4*a*b^2*tan(x*e)^2*tan(d) + 2*a^3*tan
(x*e)*tan(d)^2 + 4*a*b^2*tan(x*e)*tan(d)^2 + 2*a^3*x*e + 2*a*b^2*x*e - a^2*
b*tan(x*e)^2 - a^2*b*tan(d)^2 + a^2*b*log(4*(tan(x*e)^4*tan(d)^2 - 2*tan(x*
e)^3*tan(d) + tan(x*e)^2*tan(d)^2 + tan(x*e)^2 - 2*tan(x*e)*tan(d) + 1)/(ta
n(d)^2 + 1)) + b^3*log(4*(tan(x*e)^4*tan(d)^2 - 2*tan(x*e)^3*tan(d) + tan(x
*e)^2*tan(d)^2 + tan(x*e)^2 - 2*tan(x*e)*tan(d) + 1)/(tan(d)^2 + 1)) - 2*a^
3*tan(x*e) - 4*a*b^2*tan(x*e) - 2*a^3*tan(d) - 4*a*b^2*tan(d) - a^2*b)/(e*t
an(x*e)^2*tan(d)^2 - 2*e*tan(x*e)*tan(d) + e)
```

maple [A] time = 0.01, size = 117, normalized size = 1.62

$$\frac{a^2b\left(\tan^2(ex+d)\right)}{2e}+\frac{a^3\tan(ex+d)}{e}+\frac{2a\,b^2\tan(ex+d)}{e}+\frac{\ln\left(1+\tan^2(ex+d)\right)a^2b}{2e}+\frac{\ln\left(1+\tan^2(ex+d)\right)b^3}{2e}-\text{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*tan(e*x+d))*(b^2+2*a*b*tan(e*x+d)+a^2*tan(e*x+d)^2),x)
```

```
[Out] 1/2/e*a^2*b*tan(e*x+d)^2+1/e*a^3*tan(e*x+d)+2*a*b^2*tan(e*x+d)/e+1/2/e*ln(1
+tan(e*x+d)^2)*a^2*b+1/2/e*ln(1+tan(e*x+d)^2)*b^3-1/e*arctan(tan(e*x+d))*a^
3-1/e*arctan(tan(e*x+d))*a*b^2
```

maxima [A] time = 0.41, size = 74, normalized size = 1.03

$$\frac{a^2b\tan(ex+d)^2-2\left(a^3+ab^2\right)(ex+d)+\left(a^2b+b^3\right)\log\left(\tan(ex+d)^2+1\right)+2\left(a^3+2\,ab^2\right)\tan(ex+d)}{2\,e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(e*x+d))*(b^2+2*a*b*tan(e*x+d)+a^2*tan(e*x+d)^2),x, algor
ithm="maxima")
```

```
[Out] 1/2*(a^2*b*tan(e*x + d)^2 - 2*(a^3 + a*b^2)*(e*x + d) + (a^2*b + b^3)*log(t
an(e*x + d)^2 + 1) + 2*(a^3 + 2*a*b^2)*tan(e*x + d))/e
```

mupad [B] time = 2.77, size = 105, normalized size = 1.46

$$\frac{\tan(d+e\,x)\,\left(a^3+2\,a\,b^2\right)}{e}+\frac{\ln\left(\tan(d+e\,x)^2+1\right)\,\left(\frac{a^2\,b}{2}+\frac{b^3}{2}\right)}{e}+\frac{a^2\,b\,\tan(d+e\,x)^2}{2\,e}-\frac{a\,\mathrm{atan}\left(\frac{a\,\tan(d+e\,x)\,\left(a^2+b^2\right)}{a^3+a\,b^2}\right)\,\left(a^2+\right.}{e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*tan(d + e*x))*(b^2 + a^2*tan(d + e*x)^2 + 2*a*b*tan(d + e*x)),x)
```

```
[Out] (tan(d + e*x)*(2*a*b^2 + a^3))/e + (log(tan(d + e*x)^2 + 1)*((a^2*b)/2 + b^
3/2))/e + (a^2*b*tan(d + e*x)^2)/(2*e) - (a*atan((a*tan(d + e*x)*(a^2 + b^2
))/(a*b^2 + a^3))*(a^2 + b^2))/e
```

sympy [A] time = 0.25, size = 122, normalized size = 1.69

$$\begin{cases} -a^3x+\frac{a^3\tan(d+ex)}{e}+\frac{a^2b\log\left(\tan^2(d+ex)+1\right)}{2e}+\frac{a^2b\tan^2(d+ex)}{2e}-ab^2x+\frac{2ab^2\tan(d+ex)}{e}+\frac{b^3\log\left(\tan^2(d+ex)+1\right)}{2e} & \text{for } e\neq 0 \\ x\,(a+b\tan(d))\left(a^2\tan^2(d)+2ab\tan(d)+b^2\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(e*x+d))*(b**2+2*a*b*tan(e*x+d)+a**2*tan(e*x+d)**2),x)
```

```
[Out] Piecewise((-a**3*x + a**3*tan(d + e*x)/e + a**2*b*log(tan(d + e*x)**2 + 1)/
(2*e) + a**2*b*tan(d + e*x)**2/(2*e) - a*b**2*x + 2*a*b**2*tan(d + e*x)/e +
 b**3*log(tan(d + e*x)**2 + 1)/(2*e), Ne(e, 0)), (x*(a + b*tan(d))*(a**2*ta
n(d)**2 + 2*a*b*tan(d) + b**2), True))
```

3.512 $\int \frac{a+b\tan(d+ex)}{b^2+2ab\tan(d+ex)+a^2\tan^2(d+ex)}\,dx$

Optimal. Leaf size=101

$$-\frac{a^2-b^2}{e\left(a^2+b^2\right)\left(a\tan(d+ex)+b\right)}+\frac{b\left(3a^2-b^2\right)\log(a\sin(d+ex)+b\cos(d+ex))}{e\left(a^2+b^2\right)^2}-\frac{ax\left(a^2-3b^2\right)}{\left(a^2+b^2\right)^2}$$

```
[Out] -a*(a^2-3*b^2)*x/(a^2+b^2)^2+b*(3*a^2-b^2)*ln(b*cos(e*x+d)+a*sin(e*x+d))/(a
^2+b^2)^2/e+(-a^2+b^2)/(a^2+b^2)/e/(b+a*tan(e*x+d))
```

Rubi [A] time = 0.26, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 39, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {3708, 3529, 3531, 3530}

$$-\frac{a^2-b^2}{e\left(a^2+b^2\right)\left(a\tan(d+ex)+b\right)}+\frac{b\left(3a^2-b^2\right)\log(a\sin(d+ex)+b\cos(d+ex))}{e\left(a^2+b^2\right)^2}-\frac{ax\left(a^2-3b^2\right)}{\left(a^2+b^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Tan[d + e*x])/(b^2 + 2*a*b*Tan[d + e*x] + a^2*Tan[d + e*x]^2),x]
```

```
[Out] -((a*(a^2 - 3*b^2)*x)/(a^2 + b^2)^2) + (b*(3*a^2 - b^2)*Log[b*Cos[d + e*x]
+ a*Sin[d + e*x]])/((a^2 + b^2)^2*e) - (a^2 - b^2)/((a^2 + b^2)*e*(b + a*Ta
n[d + e*x]))
```

Rule 3529

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[((b*c - a*d)*(a + b*Tan[e + f*x])^(m + 1))
/(f*(m + 1)*(a^2 + b^2)), x] + Dist[1/(a^2 + b^2), Int[(a + b*Tan[e + f*x])
^(m + 1)*Simp[a*c + b*d - (b*c - a*d)*Tan[e + f*x], x], x], x] /; FreeQ[{a,
 b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && LtQ[m, -1]
```

Rule 3530

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_) + (b_.)*tan[(e_.) + (f_.)*
(x_)]), x_Symbol] :> Simp[(c*Log[RemoveContent[a*Cos[e + f*x] + b*Sin[e + f
*x], x]])/(b*f), x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
 NeQ[a^2 + b^2, 0] && EqQ[a*c + b*d, 0]
```

Rule 3531

```
Int[((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_.) + (b_.)*tan[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[((a*c + b*d)*x)/(a^2 + b^2), x] + Dist[(b*c - a
```

```
*d)/(a^2 + b^2), Int[(b - a*Tan[e + f*x])/(a + b*Tan[e + f*x]), x], x] /; F
reeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && Ne
Q[a*c + b*d, 0]
```

Rule 3708

```
Int[((A_) + (B_.)*tan[(d_.) + (e_.)*(x_)])*((a_) + (b_.)*tan[(d_.) + (e_.)*
(x_)] + (c_.)*tan[(d_.) + (e_.)*(x_)]^2)^(n_), x_Symbol] :> Dist[1/(4^n*c^n
), Int[(A + B*Tan[d + e*x])*(b + 2*c*Tan[d + e*x])^(2*n), x], x] /; FreeQ[{
a, b, c, d, e, A, B}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[n]
```

Rubi steps

$$\int \frac{a+b\tan(d+ex)}{b^2+2ab\tan(d+ex)+a^2\tan^2(d+ex)}\,dx = \left(4a^2\right)\int \frac{a+b\tan(d+ex)}{\left(2ab+2a^2\tan(d+ex)\right)^2}\,dx$$

$$= -\frac{a^2-b^2}{\left(a^2+b^2\right)e(b+a\tan(d+ex))}+\frac{\int \frac{4a^2b-2a\left(a^2-b^2\right)\tan(d+ex)}{2ab+2a^2\tan(d+ex)}\,dx}{a^2+b^2}$$

$$= -\frac{a\left(a^2-3b^2\right)x}{\left(a^2+b^2\right)^2}-\frac{a^2-b^2}{\left(a^2+b^2\right)e(b+a\tan(d+ex))}+\frac{\left(b\left(3a^2-b^2\right)\right)}{\left(a^2\right.}$$

$$= -\frac{a\left(a^2-3b^2\right)x}{\left(a^2+b^2\right)^2}+\frac{b\left(3a^2-b^2\right)\log(b\cos(d+ex)+a\sin(d+ex))}{\left(a^2+b^2\right)^2 e}$$

Mathematica [C] time = 2.34, size = 187, normalized size = 1.85

$$\frac{\frac{b(-((a+ib)\log(-\tan(d+ex)+i))-(a-ib)\log(\tan(d+ex)+i)+2a\log(a\tan(d+ex)+b))}{a^2+b^2}+(a-b)(a+b)\left(\frac{2a\left(2b\log(a\tan(d+ex)+b)-\frac{a^2+b^2}{a\tan(d+ex)+b}\right)}{\left(a^2+b^2\right)^2}+\right.}{2ae}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Tan[d + e*x])/(b^2 + 2*a*b*Tan[d + e*x] + a^2*Tan[d + e*x]
^2),x]
```

```
[Out] ((b*(-((a + I*b)*Log[I - Tan[d + e*x]]) - (a - I*b)*Log[I + Tan[d + e*x]] +
 2*a*Log[b + a*Tan[d + e*x]]))/(a^2 + b^2) + (a - b)*(a + b)*((I*Log[I - Ta
n[d + e*x]])/(a - I*b)^2 - (I*Log[I + Tan[d + e*x]])/(a + I*b)^2 + (2*a*(2*
```

```
b*Log[b + a*Tan[d + e*x]] - (a^2 + b^2)/(b + a*Tan[d + e*x])))/(a^2 + b^2)^
2))/(2*a*e)
```

fricas [A] time = 0.72, size = 191, normalized size = 1.89

$$-\frac{2 a^4 - 2 a^2b^2 + 2 \left(a^3b - 3 ab^3\right)ex - \left(3 a^2b^2 - b^4 + \left(3 a^3b - ab^3\right)\tan\left(ex+d\right)\right)\log\left(\frac{a^2\tan(ex+d)^2+2ab\tan(ex+d)+b^2}{\tan(ex+d)^2+1}\right) -}{2\left(\left(a^5 + 2 a^3b^2 + ab^4\right)e\tan\left(ex+d\right) + \left(a^4b + 2 a^2b^3 + b^5\right)e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(e*x+d))/(b^2+2*a*b*tan(e*x+d)+a^2*tan(e*x+d)^2),x, algor
ithm="fricas")
```

```
[Out] -1/2*(2*a^4 - 2*a^2*b^2 + 2*(a^3*b - 3*a*b^3)*e*x - (3*a^2*b^2 - b^4 + (3*a
^3*b - a*b^3)*tan(e*x + d))*log((a^2*tan(e*x + d)^2 + 2*a*b*tan(e*x + d) +
b^2)/(tan(e*x + d)^2 + 1)) - 2*(a^3*b - a*b^3 - (a^4 - 3*a^2*b^2)*e*x)*tan(
e*x + d))/((a^5 + 2*a^3*b^2 + a*b^4)*e*tan(e*x + d) + (a^4*b + 2*a^2*b^3 +
b^5)*e)
```

giac [B] time = 0.89, size = 204, normalized size = 2.02

$$-\frac{1}{2}\left(\frac{2\left(a^3 - 3 ab^2\right)(xe+d)}{a^4 + 2 a^2b^2 + b^4} + \frac{\left(3 a^2b - b^3\right)\log\left(\tan\left(xe+d\right)^2 + 1\right)}{a^4 + 2 a^2b^2 + b^4} - \frac{2\left(3 a^3b - ab^3\right)\log\left(|a\tan\left(xe+d\right) + b|\right)}{a^5 + 2 a^3b^2 + ab^4} + \frac{2}{\ }\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(e*x+d))/(b^2+2*a*b*tan(e*x+d)+a^2*tan(e*x+d)^2),x, algor
ithm="giac")
```

```
[Out] -1/2*(2*(a^3 - 3*a*b^2)*(x*e + d)/(a^4 + 2*a^2*b^2 + b^4) + (3*a^2*b - b^3)
*log(tan(x*e + d)^2 + 1)/(a^4 + 2*a^2*b^2 + b^4) - 2*(3*a^3*b - a*b^3)*log(
abs(a*tan(x*e + d) + b))/(a^5 + 2*a^3*b^2 + a*b^4) + 2*(3*a^3*b*tan(x*e + d
) - a*b^3*tan(x*e + d) + a^4 + 3*a^2*b^2 - 2*b^4)/((a^4 + 2*a^2*b^2 + b^4)*
(a*tan(x*e + d) + b)))*e^(-1)
```

maple [B] time = 0.16, size = 222, normalized size = 2.20

$$-\frac{a^2}{e\left(a^2+b^2\right)\left(b+a\tan\left(ex+d\right)\right)}+\frac{b^2}{e\left(a^2+b^2\right)\left(b+a\tan\left(ex+d\right)\right)}+\frac{3b\ln\left(b+a\tan\left(ex+d\right)\right)a^2}{e\left(a^2+b^2\right)^2}-\frac{b^3\ln\left(b+a\tan\left(\right.\right.}{e\left(a^2+b^2\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*tan(e*x+d))/(b^2+2*a*b*tan(e*x+d)+a^2*tan(e*x+d)^2),x)
```

```
[Out] -1/e/(a^2+b^2)/(b+a*tan(e*x+d))*a^2+1/e/(a^2+b^2)/(b+a*tan(e*x+d))*b^2+3/e*
b/(a^2+b^2)^2*ln(b+a*tan(e*x+d))*a^2-1/e*b^3/(a^2+b^2)^2*ln(b+a*tan(e*x+d))
-3/2/e/(a^2+b^2)^2*ln(1+tan(e*x+d)^2)*a^2*b+1/2/e/(a^2+b^2)^2*ln(1+tan(e*x+
d)^2)*b^3-1/e/(a^2+b^2)^2*arctan(tan(e*x+d))*a^3+3/e/(a^2+b^2)^2*arctan(tan
(e*x+d))*a*b^2
```

maxima [A] time = 0.41, size = 161, normalized size = 1.59

$$-\frac{\frac{2\left(a^3-3ab^2\right)(ex+d)}{a^4+2a^2b^2+b^4}-\frac{2\left(3a^2b-b^3\right)\log(a\tan(ex+d)+b)}{a^4+2a^2b^2+b^4}+\frac{\left(3a^2b-b^3\right)\log\left(\tan(ex+d)^2+1\right)}{a^4+2a^2b^2+b^4}+\frac{2\left(a^2-b^2\right)}{a^2b+b^3+\left(a^3+ab^2\right)\tan(ex+d)}}{2e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(e*x+d))/(b^2+2*a*b*tan(e*x+d)+a^2*tan(e*x+d)^2),x, algor
ithm="maxima")
```

```
[Out] -1/2*(2*(a^3 - 3*a*b^2)*(e*x + d)/(a^4 + 2*a^2*b^2 + b^4) - 2*(3*a^2*b - b^
3)*log(a*tan(e*x + d) + b)/(a^4 + 2*a^2*b^2 + b^4) + (3*a^2*b - b^3)*log(ta
n(e*x + d)^2 + 1)/(a^4 + 2*a^2*b^2 + b^4) + 2*(a^2 - b^2)/(a^2*b + b^3 + (a
^3 + a*b^2)*tan(e*x + d)))/e
```

mupad [B] time = 3.09, size = 152, normalized size = 1.50

$$\frac{b\,\ln\left(b+a\tan\left(d+e\,x\right)\right)\,\left(3\,a^2-b^2\right)}{e\,{\left(a^2+b^2\right)}^2}-\frac{\ln\left(\tan\left(d+e\,x\right)+1{}\mathrm{i}\right)\,\left(a-b\,1{}\mathrm{i}\right)}{2\,e\,\left(-a^2\,1{}\mathrm{i}+2\,a\,b+b^2\,1{}\mathrm{i}\right)}-\frac{a^2-b^2}{e\,\left(a^2+b^2\right)\,\left(b+a\tan\left(d+e\,x\right)\right)}-\frac{\ln\left(\tan\left(a\right.\right.}{2\,e\,\left(a^2\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*tan(d + e*x))/(b^2 + a^2*tan(d + e*x)^2 + 2*a*b*tan(d + e*x)),x)
```

```
[Out] (b*log(b + a*tan(d + e*x))*(3*a^2 - b^2))/(e*(a^2 + b^2)^2) - (log(tan(d +
e*x) + 1i)*(a - b*1i))/(2*e*(2*a*b - a^2*1i + b^2*1i)) - (a^2 - b^2)/(e*(a^
2 + b^2)*(b + a*tan(d + e*x))) - (log(tan(d + e*x) - 1i)*(a + b*1i))/(2*e*(
2*a*b + a^2*1i - b^2*1i))
```

sympy [A] time = 1.62, size = 1358, normalized size = 13.45

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(e*x+d))/(b**2+2*a*b*tan(e*x+d)+a**2*tan(e*x+d)**2),x)
```

```
[Out] Piecewise((zoo*x*tan(d), Eq(a, 0) & Eq(b, 0) & Eq(e, 0)), (log(tan(d + e*x)
**2 + 1)/(2*b*e), Eq(a, 0)), (1/(2*b*e*tan(d + e*x)**2 + 4*I*b*e*tan(d + e*
```

```
x) - 2*b*e), Eq(a, -I*b)), (1/(2*b*e*tan(d + e*x)**2 - 4*I*b*e*tan(d + e*x)
 - 2*b*e), Eq(a, I*b)), (x*(a + b*tan(d))/(a**2*tan(d)**2 + 2*a*b*tan(d) +
b**2), Eq(e, 0)), (-2*a**4*e*x*tan(d + e*x)/(2*a**5*e*tan(d + e*x) + 2*a**4
*b*e + 4*a**3*b**2*e*tan(d + e*x) + 4*a**2*b**3*e + 2*a*b**4*e*tan(d + e*x)
 + 2*b**5*e) - 2*a**4/(2*a**5*e*tan(d + e*x) + 2*a**4*b*e + 4*a**3*b**2*e*t
an(d + e*x) + 4*a**2*b**3*e + 2*a*b**4*e*tan(d + e*x) + 2*b**5*e) - 2*a**3*
b*e*x/(2*a**5*e*tan(d + e*x) + 2*a**4*b*e + 4*a**3*b**2*e*tan(d + e*x) + 4*
a**2*b**3*e + 2*a*b**4*e*tan(d + e*x) + 2*b**5*e) + 6*a**3*b*log(tan(d + e*
x) + b/a)*tan(d + e*x)/(2*a**5*e*tan(d + e*x) + 2*a**4*b*e + 4*a**3*b**2*e*
tan(d + e*x) + 4*a**2*b**3*e + 2*a*b**4*e*tan(d + e*x) + 2*b**5*e) - 3*a**3
*b*log(tan(d + e*x)**2 + 1)*tan(d + e*x)/(2*a**5*e*tan(d + e*x) + 2*a**4*b*
e + 4*a**3*b**2*e*tan(d + e*x) + 4*a**2*b**3*e + 2*a*b**4*e*tan(d + e*x) +
2*b**5*e) + 6*a**2*b**2*e*x*tan(d + e*x)/(2*a**5*e*tan(d + e*x) + 2*a**4*b*
e + 4*a**3*b**2*e*tan(d + e*x) + 4*a**2*b**3*e + 2*a*b**4*e*tan(d + e*x) +
2*b**5*e) + 6*a**2*b**2*log(tan(d + e*x) + b/a)/(2*a**5*e*tan(d + e*x) + 2*
a**4*b*e + 4*a**3*b**2*e*tan(d + e*x) + 4*a**2*b**3*e + 2*a*b**4*e*tan(d +
e*x) + 2*b**5*e) - 3*a**2*b**2*log(tan(d + e*x)**2 + 1)/(2*a**5*e*tan(d + e
*x) + 2*a**4*b*e + 4*a**3*b**2*e*tan(d + e*x) + 4*a**2*b**3*e + 2*a*b**4*e*
tan(d + e*x) + 2*b**5*e) + 6*a*b**3*e*x/(2*a**5*e*tan(d + e*x) + 2*a**4*b*e
 + 4*a**3*b**2*e*tan(d + e*x) + 4*a**2*b**3*e + 2*a*b**4*e*tan(d + e*x) + 2
*b**5*e) - 2*a*b**3*log(tan(d + e*x) + b/a)*tan(d + e*x)/(2*a**5*e*tan(d +
e*x) + 2*a**4*b*e + 4*a**3*b**2*e*tan(d + e*x) + 4*a**2*b**3*e + 2*a*b**4*e
*tan(d + e*x) + 2*b**5*e) + a*b**3*log(tan(d + e*x)**2 + 1)*tan(d + e*x)/(2
*a**5*e*tan(d + e*x) + 2*a**4*b*e + 4*a**3*b**2*e*tan(d + e*x) + 4*a**2*b**
3*e + 2*a*b**4*e*tan(d + e*x) + 2*b**5*e) - 2*b**4*log(tan(d + e*x) + b/a)/
(2*a**5*e*tan(d + e*x) + 2*a**4*b*e + 4*a**3*b**2*e*tan(d + e*x) + 4*a**2*b
**3*e + 2*a*b**4*e*tan(d + e*x) + 2*b**5*e) + b**4*log(tan(d + e*x)**2 + 1)
/(2*a**5*e*tan(d + e*x) + 2*a**4*b*e + 4*a**3*b**2*e*tan(d + e*x) + 4*a**2*
b**3*e + 2*a*b**4*e*tan(d + e*x) + 2*b**5*e) + 2*b**4/(2*a**5*e*tan(d + e*x
) + 2*a**4*b*e + 4*a**3*b**2*e*tan(d + e*x) + 4*a**2*b**3*e + 2*a*b**4*e*ta
n(d + e*x) + 2*b**5*e), True))
```

3.513 $\int \frac{a+b\tan(d+ex)}{\left(b^2+2ab\tan(d+ex)+a^2\tan^2(d+ex)\right)^2}\,dx$

Optimal. Leaf size=197

$$-\frac{a^2-b^2}{3e\left(a^2+b^2\right)(a\tan(d+ex)+b)^3}-\frac{b\left(3a^2-b^2\right)}{2e\left(a^2+b^2\right)^2(a\tan(d+ex)+b)^2}+\frac{a^4-6a^2b^2+b^4}{e\left(a^2+b^2\right)^3(a\tan(d+ex)+b)}-\frac{b\left(5a^4-1\right.}{}$$

```
[Out] a*(a^4-10*a^2*b^2+5*b^4)*x/(a^2+b^2)^4-b*(5*a^4-10*a^2*b^2+b^4)*ln(b*cos(e*
x+d)+a*sin(e*x+d))/(a^2+b^2)^4/e+1/3*(-a^2+b^2)/(a^2+b^2)/e/(b+a*tan(e*x+d)
)^3-1/2*b*(3*a^2-b^2)/(a^2+b^2)^2/e/(b+a*tan(e*x+d))^2+(a^4-6*a^2*b^2+b^4)/
(a^2+b^2)^3/e/(b+a*tan(e*x+d))
```

Rubi [A] time = 0.54, antiderivative size = 197, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 39, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {3708, 3529, 3531, 3530}

$$-\frac{a^2-b^2}{3e\left(a^2+b^2\right)(a\tan(d+ex)+b)^3}+\frac{-6a^2b^2+a^4+b^4}{e\left(a^2+b^2\right)^3(a\tan(d+ex)+b)}-\frac{b\left(3a^2-b^2\right)}{2e\left(a^2+b^2\right)^2(a\tan(d+ex)+b)^2}-\frac{b\left(-10a^2b\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Tan[d + e*x])/(b^2 + 2*a*b*Tan[d + e*x] + a^2*Tan[d + e*x]^2)^2,
x]
```

```
[Out] (a*(a^4 - 10*a^2*b^2 + 5*b^4)*x)/(a^2 + b^2)^4 - (b*(5*a^4 - 10*a^2*b^2 + b
^4)*Log[b*Cos[d + e*x] + a*Sin[d + e*x]])/((a^2 + b^2)^4*e) - (a^2 - b^2)/(
3*(a^2 + b^2)*e*(b + a*Tan[d + e*x])^3) - (b*(3*a^2 - b^2))/(2*(a^2 + b^2)^
2*e*(b + a*Tan[d + e*x])^2) + (a^4 - 6*a^2*b^2 + b^4)/((a^2 + b^2)^3*e*(b +
 a*Tan[d + e*x]))
```

Rule 3529

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[((b*c - a*d)*(a + b*Tan[e + f*x])^(m + 1))
/(f*(m + 1)*(a^2 + b^2)), x] + Dist[1/(a^2 + b^2), Int[(a + b*Tan[e + f*x])
^(m + 1)*Simp[a*c + b*d - (b*c - a*d)*Tan[e + f*x], x], x], x] /; FreeQ[{a,
 b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && LtQ[m, -1]
```

Rule 3530

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_) + (b_.)*tan[(e_.) + (f_.)*
(x_)]), x_Symbol] :> Simp[(c*Log[RemoveContent[a*Cos[e + f*x] + b*Sin[e + f
*x], x]])/(b*f), x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
```

```
NeQ[a^2 + b^2, 0] && EqQ[a*c + b*d, 0]
```

Rule 3531

```
Int[((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_.) + (b_.)*tan[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[((a*c + b*d)*x)/(a^2 + b^2), x] + Dist[(b*c - a
*d)/(a^2 + b^2), Int[(b - a*Tan[e + f*x])/(a + b*Tan[e + f*x]), x], x] /; F
reeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && Ne
Q[a*c + b*d, 0]
```

Rule 3708

```
Int[((A_) + (B_.)*tan[(d_.) + (e_.)*(x_)])*((a_) + (b_.)*tan[(d_.) + (e_.)*
(x_)] + (c_.)*tan[(d_.) + (e_.)*(x_)]^2)^(n_), x_Symbol] :> Dist[1/(4^n*c^n
), Int[(A + B*Tan[d + e*x])*(b + 2*c*Tan[d + e*x])^(2*n), x], x] /; FreeQ[{
a, b, c, d, e, A, B}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[n]
```

Rubi steps

$$\int \frac{a+b\tan(d+ex)}{\left(b^2+2ab\tan(d+ex)+a^2\tan^2(d+ex)\right)^2}\,dx = \left(16a^4\right)\int \frac{a+b\tan(d+ex)}{\left(2ab+2a^2\tan(d+ex)\right)^4}\,dx$$

$$= -\frac{a^2-b^2}{3\left(a^2+b^2\right)e(b+a\tan(d+ex))^3} + \frac{\left(4a^2\right)\int \frac{4a^2b-2a\left(a^2-b^2\right)\tan(}{\left(2ab+2a^2\tan(d+ex\right.}}{a^2+b^2}$$

$$= -\frac{a^2-b^2}{3\left(a^2+b^2\right)e(b+a\tan(d+ex))^3} - \frac{b\left(3a^2-b^2\right)}{2\left(a^2+b^2\right)^2e(b+a\tan(}$$

$$= -\frac{a^2-b^2}{3\left(a^2+b^2\right)e(b+a\tan(d+ex))^3} - \frac{b\left(3a^2-b^2\right)}{2\left(a^2+b^2\right)^2e(b+a\tan(}$$

$$= \frac{a\left(a^4-10a^2b^2+5b^4\right)x}{\left(a^2+b^2\right)^4} - \frac{a^2-b^2}{3\left(a^2+b^2\right)e(b+a\tan(d+ex))^3} -$$

$$= \frac{a\left(a^4-10a^2b^2+5b^4\right)x}{\left(a^2+b^2\right)^4} - \frac{b\left(5a^4-10a^2b^2+b^4\right)\log(b\cos(d}{\left(a^2+b^2\right)^4e}$$

Mathematica [C] time = 5.41, size = 308, normalized size = 1.56

$$\frac{3b\left(\frac{a\left(-\frac{(a^2+b^2)(a^2+4ab\tan(d+ex)+5b^2)}{(a\tan(d+ex)+b)^2}-2(a^2-3b^2)\log(a\tan(d+ex)+b)\right)}{(a^2+b^2)^3}+\frac{\log(-\tan(d+ex)+i)}{(a-ib)^3}+\frac{\log(\tan(d+ex)+i)}{(a+ib)^3}\right)-(a-b)(a+b)\left(-\frac{6\cdots}{(a^2+b^2\cdots}\right.\cdots}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Tan[d + e*x])/(b^2 + 2*a*b*Tan[d + e*x] + a^2*Tan[d + e*x]
^2)^2,x]
```

```
[Out] (-((a - b)*(a + b)*(((3*I)*Log[I - Tan[d + e*x]])/(a - I*b)^4 - ((3*I)*Log[
I + Tan[d + e*x]])/(a + I*b)^4 + (24*a*(a - b)*b*(a + b)*Log[b + a*Tan[d +
e*x]])/(a^2 + b^2)^4 + (2*a)/((a^2 + b^2)*(b + a*Tan[d + e*x])^3) + (6*a*b)
/((a^2 + b^2)^2*(b + a*Tan[d + e*x])^2) - (6*a*(a^2 - 3*b^2))/((a^2 + b^2)^
3*(b + a*Tan[d + e*x])))) + 3*b*(Log[I - Tan[d + e*x]]/(a - I*b)^3 + Log[I
+ Tan[d + e*x]]/(a + I*b)^3 + (a*(-2*(a^2 - 3*b^2)*Log[b + a*Tan[d + e*x]]
- ((a^2 + b^2)*(a^2 + 5*b^2 + 4*a*b*Tan[d + e*x]))/(b + a*Tan[d + e*x])^2))
/(a^2 + b^2)^3))/(6*a*e)
```

fricas [B] time = 2.02, size = 580, normalized size = 2.94

$$-\frac{2a^8+7a^6b^2+66a^4b^4-27a^2b^6+\left(21a^7b-56a^5b^3+11a^3b^5-6\left(a^8-10a^6b^2+5a^4b^4\right)ex\right)\tan(ex+d)^3-6\left(\cdots\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(e*x+d))/(b^2+2*a*b*tan(e*x+d)+a^2*tan(e*x+d)^2)^2,x, alg
orithm="fricas")
```

```
[Out] -1/6*(2*a^8 + 7*a^6*b^2 + 66*a^4*b^4 - 27*a^2*b^6 + (21*a^7*b - 56*a^5*b^3
+ 11*a^3*b^5 - 6*(a^8 - 10*a^6*b^2 + 5*a^4*b^4)*e*x)*tan(e*x + d)^3 - 6*(a^
5*b^3 - 10*a^3*b^5 + 5*a*b^7)*e*x - 3*(2*a^8 - 31*a^6*b^2 + 46*a^4*b^4 - 9*
a^2*b^6 + 6*(a^7*b - 10*a^5*b^3 + 5*a^3*b^5)*e*x)*tan(e*x + d)^2 + 3*(5*a^4
*b^4 - 10*a^2*b^6 + b^8 + (5*a^7*b - 10*a^5*b^3 + a^3*b^5)*tan(e*x + d)^3 +
 3*(5*a^6*b^2 - 10*a^4*b^4 + a^2*b^6)*tan(e*x + d)^2 + 3*(5*a^5*b^3 - 10*a^
3*b^5 + a*b^7)*tan(e*x + d))*log((a^2*tan(e*x + d)^2 + 2*a*b*tan(e*x + d) +
 b^2)/(tan(e*x + d)^2 + 1)) - 3*(a^7*b - 46*a^5*b^3 + 35*a^3*b^5 - 6*a*b^7
+ 6*(a^6*b^2 - 10*a^4*b^4 + 5*a^2*b^6)*e*x)*tan(e*x + d))/((a^11 + 4*a^9*b^
2 + 6*a^7*b^4 + 4*a^5*b^6 + a^3*b^8)*e*tan(e*x + d)^3 + 3*(a^10*b + 4*a^8*b
^3 + 6*a^6*b^5 + 4*a^4*b^7 + a^2*b^9)*e*tan(e*x + d)^2 + 3*(a^9*b^2 + 4*a^7
*b^4 + 6*a^5*b^6 + 4*a^3*b^8 + a*b^10)*e*tan(e*x + d) + (a^8*b^3 + 4*a^6*b^
5 + 6*a^4*b^7 + 4*a^2*b^9 + b^11)*e)
```

giac [B] time = 1.94, size = 451, normalized size = 2.29

$$\frac{1}{6}\left(\frac{6\left(a^{5}-10\,a^{3}b^{2}+5\,ab^{4}\right)(xe+d)}{a^{8}+4\,a^{6}b^{2}+6\,a^{4}b^{4}+4\,a^{2}b^{6}+b^{8}}+\frac{3\left(5\,a^{4}b-10\,a^{2}b^{3}+b^{5}\right)\log\left(\tan\left(xe+d\right)^{2}+1\right)}{a^{8}+4\,a^{6}b^{2}+6\,a^{4}b^{4}+4\,a^{2}b^{6}+b^{8}}-\frac{6\left(5\,a^{5}b-10\,a^{3}b^{3}+a\right.}{a^{9}+4\,a^{7}b^{2}+6\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(e*x+d))/(b^2+2*a*b*tan(e*x+d)+a^2*tan(e*x+d)^2)^2,x, alg
orithm="giac")
```

```
[Out] 1/6*(6*(a^5 - 10*a^3*b^2 + 5*a*b^4)*(x*e + d)/(a^8 + 4*a^6*b^2 + 6*a^4*b^4
+ 4*a^2*b^6 + b^8) + 3*(5*a^4*b - 10*a^2*b^3 + b^5)*log(tan(x*e + d)^2 + 1)
/(a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8) - 6*(5*a^5*b - 10*a^3*b^3
+ a*b^5)*log(abs(a*tan(x*e + d) + b))/(a^9 + 4*a^7*b^2 + 6*a^5*b^4 + 4*a^3*
b^6 + a*b^8) + (55*a^7*b*tan(x*e + d)^3 - 110*a^5*b^3*tan(x*e + d)^3 + 11*a
^3*b^5*tan(x*e + d)^3 + 6*a^8*tan(x*e + d)^2 + 135*a^6*b^2*tan(x*e + d)^2 -
 360*a^4*b^4*tan(x*e + d)^2 + 39*a^2*b^6*tan(x*e + d)^2 + 3*a^7*b*tan(x*e +
 d) + 90*a^5*b^3*tan(x*e + d) - 393*a^3*b^5*tan(x*e + d) + 48*a*b^7*tan(x*e
 + d) - 2*a^8 - 7*a^6*b^2 + 10*a^4*b^4 - 139*a^2*b^6 + 22*b^8)/((a^8 + 4*a^
6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)*(a*tan(x*e + d) + b)^3))*e^(-1)
```

maple [B] time = 0.18, size = 458, normalized size = 2.32

$$-\frac{a^{2}}{3e\left(a^{2}+b^{2}\right)\left(b+a\tan\left(ex+d\right)\right)^{3}}+\frac{b^{2}}{3e\left(a^{2}+b^{2}\right)\left(b+a\tan\left(ex+d\right)\right)^{3}}-\frac{3b\,a^{2}}{2e\left(a^{2}+b^{2}\right)^{2}\left(b+a\tan\left(ex+d\right)\right)^{2}}+\frac{}{2e\left(a^{2}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*tan(e*x+d))/(b^2+2*a*b*tan(e*x+d)+a^2*tan(e*x+d)^2)^2,x)
```

```
[Out] -1/3/e/(a^2+b^2)/(b+a*tan(e*x+d))^3*a^2+1/3/e/(a^2+b^2)/(b+a*tan(e*x+d))^3*
b^2-3/2/e*b/(a^2+b^2)^2/(b+a*tan(e*x+d))^2*a^2+1/2/e*b^3/(a^2+b^2)^2/(b+a*t
an(e*x+d))^2+1/e/(a^2+b^2)^3/(b+a*tan(e*x+d))*a^4-6/e/(a^2+b^2)^3/(b+a*tan(
e*x+d))*a^2*b^2+1/e/(a^2+b^2)^3/(b+a*tan(e*x+d))*b^4-5/e*b/(a^2+b^2)^4*ln(b
+a*tan(e*x+d))*a^4+10/e*b^3/(a^2+b^2)^4*ln(b+a*tan(e*x+d))*a^2-1/e*b^5/(a^2
+b^2)^4*ln(b+a*tan(e*x+d))+5/2/e/(a^2+b^2)^4*ln(1+tan(e*x+d)^2)*a^4*b-5/e/(
a^2+b^2)^4*ln(1+tan(e*x+d)^2)*a^2*b^3+1/2/e/(a^2+b^2)^4*ln(1+tan(e*x+d)^2)*
b^5+1/e/(a^2+b^2)^4*arctan(tan(e*x+d))*a^5-10/e/(a^2+b^2)^4*arctan(tan(e*x+
d))*a^3*b^2+5/e/(a^2+b^2)^4*arctan(tan(e*x+d))*a*b^4
```

maxima [B] time = 0.43, size = 419, normalized size = 2.13

$$\frac{\frac{6\left(a^{5}-10\,a^{3}b^{2}+5\,ab^{4}\right)(ex+d)}{a^{8}+4\,a^{6}b^{2}+6\,a^{4}b^{4}+4\,a^{2}b^{6}+b^{8}}-\frac{6\left(5\,a^{4}b-10\,a^{2}b^{3}+b^{5}\right)\log(a\tan(ex+d)+b)}{a^{8}+4\,a^{6}b^{2}+6\,a^{4}b^{4}+4\,a^{2}b^{6}+b^{8}}+\frac{3\left(5\,a^{4}b-10\,a^{2}b^{3}+b^{5}\right)\log\left(\tan(ex+d)^{2}+1\right)}{a^{8}+4\,a^{6}b^{2}+6\,a^{4}b^{4}+4\,a^{2}b^{6}+b^{8}}-\frac{}{a^{6}b^{3}+3\,a^{4}b^{5}+3\,a^{2}b^{7}+b^{9}}}{6\,e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(e*x+d))/(b^2+2*a*b*tan(e*x+d)+a^2*tan(e*x+d)^2)^2,x, alg
orithm="maxima")
```

```
[Out] 1/6*(6*(a^5 - 10*a^3*b^2 + 5*a*b^4)*(e*x + d)/(a^8 + 4*a^6*b^2 + 6*a^4*b^4
+ 4*a^2*b^6 + b^8) - 6*(5*a^4*b - 10*a^2*b^3 + b^5)*log(a*tan(e*x + d) + b)
/(a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8) + 3*(5*a^4*b - 10*a^2*b^3
+ b^5)*log(tan(e*x + d)^2 + 1)/(a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b
^8) - (2*a^6 + 5*a^4*b^2 + 40*a^2*b^4 - 11*b^6 - 6*(a^6 - 6*a^4*b^2 + a^2*b
^4)*tan(e*x + d)^2 - 3*(a^5*b - 26*a^3*b^3 + 5*a*b^5)*tan(e*x + d))/(a^6*b^
3 + 3*a^4*b^5 + 3*a^2*b^7 + b^9 + (a^9 + 3*a^7*b^2 + 3*a^5*b^4 + a^3*b^6)*t
an(e*x + d)^3 + 3*(a^8*b + 3*a^6*b^3 + 3*a^4*b^5 + a^2*b^7)*tan(e*x + d)^2
+ 3*(a^7*b^2 + 3*a^5*b^4 + 3*a^3*b^6 + a*b^8)*tan(e*x + d)))/e
```

mupad [B] time = 3.26, size = 388, normalized size = 1.97

$$\frac{\frac{\tan(d+e\,x)^2\,\left(a^6-6\,a^4\,b^2+a^2\,b^4\right)}{a^6+3\,a^4\,b^2+3\,a^2\,b^4+b^6}-\frac{2\,a^6+5\,a^4\,b^2+40\,a^2\,b^4-11\,b^6}{6\,\left(a^6+3\,a^4\,b^2+3\,a^2\,b^4+b^6\right)}+\frac{\tan(d+e\,x)\,\left(a^5\,b-26\,a^3\,b^3+5\,a\,b^5\right)}{2\,\left(a^6+3\,a^4\,b^2+3\,a^2\,b^4+b^6\right)}}{e\,\left(a^3\,\tan\left(d+e\,x\right)^3+3\,a^2\,b\,\tan\left(d+e\,x\right)^2+3\,a\,b^2\,\tan\left(d+e\,x\right)+b^3\right)}-\frac{\ln\left(b+a\,\tan\left(d+e\,x\right)\right)\,\left(\frac{5\,b}{{\left(a^2+b^2\right)}^2}-\cdots\right.}{e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*tan(d + e*x))/(b^2 + a^2*tan(d + e*x)^2 + 2*a*b*tan(d + e*x))^2,
x)
```

```
[Out] ((tan(d + e*x)^2*(a^6 + a^2*b^4 - 6*a^4*b^2))/(a^6 + b^6 + 3*a^2*b^4 + 3*a^
4*b^2) - (2*a^6 - 11*b^6 + 40*a^2*b^4 + 5*a^4*b^2)/(6*(a^6 + b^6 + 3*a^2*b^
4 + 3*a^4*b^2)) + (tan(d + e*x)*(5*a*b^5 + a^5*b - 26*a^3*b^3))/(2*(a^6 + b
^6 + 3*a^2*b^4 + 3*a^4*b^2)))/(e*(b^3 + a^3*tan(d + e*x)^3 + 3*a^2*b*tan(d
+ e*x)^2 + 3*a*b^2*tan(d + e*x))) - (log(b + a*tan(d + e*x))*((5*b)/(a^2 +
b^2)^2 - (20*b^3)/(a^2 + b^2)^3 + (16*b^5)/(a^2 + b^2)^4))/e + (log(tan(d +
 e*x) - 1i)*(a + b*1i))/(2*e*(4*a^3*b - 4*a*b^3 + a^4*1i + b^4*1i - a^2*b^2
*6i)) - (log(tan(d + e*x) + 1i)*(a - b*1i))/(2*e*(4*a*b^3 - 4*a^3*b + a^4*1
i + b^4*1i - a^2*b^2*6i))
```

sympy [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: AttributeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(e*x+d))/(b**2+2*a*b*tan(e*x+d)+a**2*tan(e*x+d)**2)**2,x)
```

```
[Out] Exception raised: AttributeError
```

3.514 $\int (a+b\tan(d+ex)) \left(b^2 + 2ab\tan(d+ex) + a^2\tan^2(d+\right.$

Optimal. Leaf size=284

$$\frac{b\left(a^2\tan^2(d+ex)+2ab\tan(d+ex)+b^2\right)^{3/2}}{3e}+\frac{\left(a^2+b^2\right)\left(a^2\tan^2(d+ex)+2ab\tan(d+ex)+b^2\right)^{3/2}}{2e(a\tan(d+ex)+b)}-\frac{2a^4bx\left(a^2\right.}{}$$

```
[Out] 1/3*b*(b^2+2*a*b*tan(e*x+d)+a^2*tan(e*x+d)^2)^(3/2)/e+(a^4-b^4)*ln(cos(e*x+
d))*(b^2+2*a*b*tan(e*x+d)+a^2*tan(e*x+d)^2)^(3/2)/e/(b+a*tan(e*x+d))^3+1/2*
(a^2+b^2)*(b^2+2*a*b*tan(e*x+d)+a^2*tan(e*x+d)^2)^(3/2)/e/(b+a*tan(e*x+d))-
2*a^4*b*(a^2+b^2)*x*(b^2+2*a*b*tan(e*x+d)+a^2*tan(e*x+d)^2)^(3/2)/(a*b+a^2*
tan(e*x+d))^3+a^4*b*(a^2+b^2)*tan(e*x+d)*(b^2+2*a*b*tan(e*x+d)+a^2*tan(e*x+
d)^2)^(3/2)/e/(a*b+a^2*tan(e*x+d))^3
```

Rubi [A] time = 0.23, antiderivative size = 284, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.122, Rules used = {3710, 3528, 12, 3525, 3475}

$$-\frac{2a^4bx\left(a^2+b^2\right)\left(a^2\tan^2(d+ex)+2ab\tan(d+ex)+b^2\right)^{3/2}}{\left(a^2\tan(d+ex)+ab\right)^3}+\frac{a^4b\left(a^2+b^2\right)\tan(d+ex)\left(a^2\tan^2(d+ex)+2ab\,\mathrm{t}\right.}{e\left(a^2\tan(d+ex)+ab\right)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Tan[d + e*x])*(b^2 + 2*a*b*Tan[d + e*x] + a^2*Tan[d + e*x]^2)^(3
/2),x]
```

```
[Out] (b*(b^2 + 2*a*b*Tan[d + e*x] + a^2*Tan[d + e*x]^2)^(3/2))/(3*e) + ((a^4 - b
^4)*Log[Cos[d + e*x]]*(b^2 + 2*a*b*Tan[d + e*x] + a^2*Tan[d + e*x]^2)^(3/2)
)/(e*(b + a*Tan[d + e*x])^3) + ((a^2 + b^2)*(b^2 + 2*a*b*Tan[d + e*x] + a^2
*Tan[d + e*x]^2)^(3/2))/(2*e*(b + a*Tan[d + e*x])) - (2*a^4*b*(a^2 + b^2)*x
*(b^2 + 2*a*b*Tan[d + e*x] + a^2*Tan[d + e*x]^2)^(3/2))/(a*b + a^2*Tan[d +
e*x])^3 + (a^4*b*(a^2 + b^2)*Tan[d + e*x]*(b^2 + 2*a*b*Tan[d + e*x] + a^2*T
an[d + e*x]^2)^(3/2))/(e*(a*b + a^2*Tan[d + e*x])^3)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3525

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[(a*c - b*d)*x, x] + (Dist[b*c + a*d, Int[Tan[e +
 f*x], x], x] + Simp[(b*d*Tan[e + f*x])/f, x]) /; FreeQ[{a, b, c, d, e, f},
 x] && NeQ[b*c - a*d, 0] && NeQ[b*c + a*d, 0]
```

Rule 3528

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[(d*(a + b*Tan[e + f*x])^m)/(f*m), x] + Int
[(a + b*Tan[e + f*x])^(m - 1)*Simp[a*c - b*d + (b*c + a*d)*Tan[e + f*x], x]
, x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2,
 0] && GtQ[m, 0]
```

Rule 3710

```
Int[((A_) + (B_.)*tan[(d_.) + (e_.)*(x_)])*((a_) + (b_.)*tan[(d_.) + (e_.)*
(x_)] + (c_.)*tan[(d_.) + (e_.)*(x_)]^2)^(n_), x_Symbol] :> Dist[(a + b*Tan
[d + e*x] + c*Tan[d + e*x]^2)^n/(b + 2*c*Tan[d + e*x])^(2*n), Int[(A + B*Ta
n[d + e*x])*(b + 2*c*Tan[d + e*x])^(2*n), x], x] /; FreeQ[{a, b, c, d, e, A
, B}, x] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[n]
```

Rubi steps

$$\int (a+b\tan(d+ex))\left(b^2+2ab\tan(d+ex)+a^2\tan^2(d+ex)\right)^{3/2}\,dx = \frac{\left(b^2+2ab\tan(d+ex)+a^2\tan^2(d+ex)\right)^{3}}{\left(2ab+\right.}$$

$$= \frac{b\left(b^2+2ab\tan(d+ex)+a^2\tan^2(d+ex)\right.}{3e}$$

$$= \frac{b\left(b^2+2ab\tan(d+ex)+a^2\tan^2(d+ex)\right.}{3e}$$

$$= \frac{b\left(b^2+2ab\tan(d+ex)+a^2\tan^2(d+ex)\right.}{3e}$$

$$= \frac{b\left(b^2+2ab\tan(d+ex)+a^2\tan^2(d+ex)\right.}{3e}$$

$$= \frac{b\left(b^2+2ab\tan(d+ex)+a^2\tan^2(d+ex)\right.}{3e}$$

Mathematica [C] time = 1.33, size = 147, normalized size = 0.52

$$\frac{\sqrt{(a\tan(d+ex)+b)^2}\,\left(2a^3b\tan^3(d+ex)+3a^2\left(a^2+3b^2\right)\tan^2(d+ex)+6ab\left(2a^2+3b^2\right)\tan(d+ex)-3\left(a^2+\right.\right.}{6e(a\tan(d+ex)+b)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Tan[d + e*x])*(b^2 + 2*a*b*Tan[d + e*x] + a^2*Tan[d + e*x]
^2)^(3/2),x]
```

```
[Out] (Sqrt[(b + a*Tan[d + e*x])^2]*(-3*(a^2 + b^2)*((a - I*b)^2*Log[I - Tan[d +
e*x]] + (a + I*b)^2*Log[I + Tan[d + e*x]]) + 6*a*b*(2*a^2 + 3*b^2)*Tan[d +
e*x] + 3*a^2*(a^2 + 3*b^2)*Tan[d + e*x]^2 + 2*a^3*b*Tan[d + e*x]^3))/(6*e*(
b + a*Tan[d + e*x]))
```

fricas [A] time = 2.83, size = 102, normalized size = 0.36

$$\frac{2\,a^3b\tan(ex+d)^3-12\left(a^3b+ab^3\right)ex+3\left(a^4+3\,a^2b^2\right)\tan(ex+d)^2+3\left(a^4-b^4\right)\log\left(\frac{1}{\tan(ex+d)^2+1}\right)+6\left(2\,a^3b+\right.}{6\,e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(e*x+d))*(b^2+2*a*b*tan(e*x+d)+a^2*tan(e*x+d)^2)^(3/2),x,
 algorithm="fricas")
```

```
[Out] 1/6*(2*a^3*b*tan(e*x + d)^3 - 12*(a^3*b + a*b^3)*e*x + 3*(a^4 + 3*a^2*b^2)*
tan(e*x + d)^2 + 3*(a^4 - b^4)*log(1/(tan(e*x + d)^2 + 1)) + 6*(2*a^3*b + 3
*a*b^3)*tan(e*x + d))/e
```

giac [B] time = 2.44, size = 1751, normalized size = 6.17

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(e*x+d))*(b^2+2*a*b*tan(e*x+d)+a^2*tan(e*x+d)^2)^(3/2),x,
 algorithm="giac")
```

```
[Out] -1/6*(12*a^3*b*x*e*sgn(a*tan(x*e + d) + b)*tan(x*e)^3*tan(d)^3 + 12*a*b^3*x
*e*sgn(a*tan(x*e + d) + b)*tan(x*e)^3*tan(d)^3 - 3*a^4*log(4*(tan(x*e)^4*ta
n(d)^2 - 2*tan(x*e)^3*tan(d) + tan(x*e)^2*tan(d)^2 + tan(x*e)^2 - 2*tan(x*e
)*tan(d) + 1)/(tan(d)^2 + 1))*sgn(a*tan(x*e + d) + b)*tan(x*e)^3*tan(d)^3 +
 3*b^4*log(4*(tan(x*e)^4*tan(d)^2 - 2*tan(x*e)^3*tan(d) + tan(x*e)^2*tan(d)
^2 + tan(x*e)^2 - 2*tan(x*e)*tan(d) + 1)/(tan(d)^2 + 1))*sgn(a*tan(x*e + d)
 + b)*tan(x*e)^3*tan(d)^3 - 36*a^3*b*x*e*sgn(a*tan(x*e + d) + b)*tan(x*e)^2
*tan(d)^2 - 36*a*b^3*x*e*sgn(a*tan(x*e + d) + b)*tan(x*e)^2*tan(d)^2 - 3*a^
4*sgn(a*tan(x*e + d) + b)*tan(x*e)^3*tan(d)^3 - 9*a^2*b^2*sgn(a*tan(x*e + d
) + b)*tan(x*e)^3*tan(d)^3 + 9*a^4*log(4*(tan(x*e)^4*tan(d)^2 - 2*tan(x*e)^
3*tan(d) + tan(x*e)^2*tan(d)^2 + tan(x*e)^2 - 2*tan(x*e)*tan(d) + 1)/(tan(d
)^2 + 1))*sgn(a*tan(x*e + d) + b)*tan(x*e)^2*tan(d)^2 - 9*b^4*log(4*(tan(x*
e)^4*tan(d)^2 - 2*tan(x*e)^3*tan(d) + tan(x*e)^2*tan(d)^2 + tan(x*e)^2 - 2*
tan(x*e)*tan(d) + 1)/(tan(d)^2 + 1))*sgn(a*tan(x*e + d) + b)*tan(x*e)^2*tan
(d)^2 + 12*a^3*b*sgn(a*tan(x*e + d) + b)*tan(x*e)^3*tan(d)^2 + 18*a*b^3*sgn
(a*tan(x*e + d) + b)*tan(x*e)^3*tan(d)^2 + 12*a^3*b*sgn(a*tan(x*e + d) + b)
*tan(x*e)^2*tan(d)^3 + 18*a*b^3*sgn(a*tan(x*e + d) + b)*tan(x*e)^2*tan(d)^3
 + 36*a^3*b*x*e*sgn(a*tan(x*e + d) + b)*tan(x*e)*tan(d) + 36*a*b^3*x*e*sgn(
a*tan(x*e + d) + b)*tan(x*e)*tan(d) - 3*a^4*sgn(a*tan(x*e + d) + b)*tan(x*e
)^3*tan(d) - 9*a^2*b^2*sgn(a*tan(x*e + d) + b)*tan(x*e)^3*tan(d) + 3*a^4*sg
n(a*tan(x*e + d) + b)*tan(x*e)^2*tan(d)^2 + 9*a^2*b^2*sgn(a*tan(x*e + d) +
b)*tan(x*e)^2*tan(d)^2 - 3*a^4*sgn(a*tan(x*e + d) + b)*tan(x*e)*tan(d)^3 -
9*a^2*b^2*sgn(a*tan(x*e + d) + b)*tan(x*e)*tan(d)^3 + 2*a^3*b*sgn(a*tan(x*e
 + d) + b)*tan(x*e)^3 - 9*a^4*log(4*(tan(x*e)^4*tan(d)^2 - 2*tan(x*e)^3*tan
(d) + tan(x*e)^2*tan(d)^2 + tan(x*e)^2 - 2*tan(x*e)*tan(d) + 1)/(tan(d)^2 +
 1))*sgn(a*tan(x*e + d) + b)*tan(x*e)*tan(d) + 9*b^4*log(4*(tan(x*e)^4*tan(
d)^2 - 2*tan(x*e)^3*tan(d) + tan(x*e)^2*tan(d)^2 + tan(x*e)^2 - 2*tan(x*e)*
tan(d) + 1)/(tan(d)^2 + 1))*sgn(a*tan(x*e + d) + b)*tan(x*e)*tan(d) - 18*a^
```

```
3*b*sgn(a*tan(x*e + d) + b)*tan(x*e)^2*tan(d) - 36*a*b^3*sgn(a*tan(x*e + d)
 + b)*tan(x*e)^2*tan(d) - 18*a^3*b*sgn(a*tan(x*e + d) + b)*tan(x*e)*tan(d)^
2 - 36*a*b^3*sgn(a*tan(x*e + d) + b)*tan(x*e)*tan(d)^2 + 2*a^3*b*sgn(a*tan(
x*e + d) + b)*tan(d)^3 - 12*a^3*b*x*e*sgn(a*tan(x*e + d) + b) - 12*a*b^3*x*
e*sgn(a*tan(x*e + d) + b) + 3*a^4*sgn(a*tan(x*e + d) + b)*tan(x*e)^2 + 9*a^
2*b^2*sgn(a*tan(x*e + d) + b)*tan(x*e)^2 - 3*a^4*sgn(a*tan(x*e + d) + b)*ta
n(x*e)*tan(d) - 9*a^2*b^2*sgn(a*tan(x*e + d) + b)*tan(x*e)*tan(d) + 3*a^4*s
gn(a*tan(x*e + d) + b)*tan(d)^2 + 9*a^2*b^2*sgn(a*tan(x*e + d) + b)*tan(d)^
2 + 3*a^4*log(4*(tan(x*e)^4*tan(d)^2 - 2*tan(x*e)^3*tan(d) + tan(x*e)^2*tan
(d)^2 + tan(x*e)^2 - 2*tan(x*e)*tan(d) + 1)/(tan(d)^2 + 1))*sgn(a*tan(x*e +
 d) + b) - 3*b^4*log(4*(tan(x*e)^4*tan(d)^2 - 2*tan(x*e)^3*tan(d) + tan(x*e
)^2*tan(d)^2 + tan(x*e)^2 - 2*tan(x*e)*tan(d) + 1)/(tan(d)^2 + 1))*sgn(a*ta
n(x*e + d) + b) + 12*a^3*b*sgn(a*tan(x*e + d) + b)*tan(x*e) + 18*a*b^3*sgn(
a*tan(x*e + d) + b)*tan(x*e) + 12*a^3*b*sgn(a*tan(x*e + d) + b)*tan(d) + 18
*a*b^3*sgn(a*tan(x*e + d) + b)*tan(d) + 3*a^4*sgn(a*tan(x*e + d) + b) + 9*a
^2*b^2*sgn(a*tan(x*e + d) + b))/(e*tan(x*e)^3*tan(d)^3 - 3*e*tan(x*e)^2*tan
(d)^2 + 3*e*tan(x*e)*tan(d) - e)
```

maple [A] time = 0.31, size = 158, normalized size = 0.56

$$-\frac{\left(b^2+2ab\tan(ex+d)+a^2\left(\tan^2(ex+d)\right)\right)^{\frac{3}{2}}\left(-2\left(\tan^3(ex+d)\right)a^3b-3\left(\tan^2(ex+d)\right)a^4-9\left(\tan^2(ex+d)\right)a\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*tan(e*x+d))*(b^2+2*a*b*tan(e*x+d)+a^2*tan(e*x+d)^2)^(3/2),x)
```

```
[Out] -1/6/e*((b+a*tan(e*x+d))^2)^(3/2)*(-2*tan(e*x+d)^3*a^3*b-3*tan(e*x+d)^2*a^4
-9*tan(e*x+d)^2*a^2*b^2+3*ln(1+tan(e*x+d)^2)*a^4-3*ln(1+tan(e*x+d)^2)*b^4+1
2*arctan(tan(e*x+d))*a^3*b+12*arctan(tan(e*x+d))*a*b^3-12*tan(e*x+d)*a^3*b-
18*tan(e*x+d)*a*b^3)/(b+a*tan(e*x+d))^3
```

maxima [A] time = 0.43, size = 166, normalized size = 0.58

$$\frac{3\left(a^3\tan(ex+d)^2+6\,a^2b\tan(ex+d)-2\left(3\,a^2b-b^3\right)(ex+d)-\left(a^3-3\,ab^2\right)\log\left(\tan(ex+d)^2+1\right)\right)a+\left(2\,a^3\tan\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(e*x+d))*(b^2+2*a*b*tan(e*x+d)+a^2*tan(e*x+d)^2)^(3/2),x,
 algorithm="maxima")
```

```
[Out] 1/6*(3*(a^3*tan(e*x + d)^2 + 6*a^2*b*tan(e*x + d) - 2*(3*a^2*b - b^3)*(e*x
+ d) - (a^3 - 3*a*b^2)*log(tan(e*x + d)^2 + 1))*a + (2*a^3*tan(e*x + d)^3 +
 9*a^2*b*tan(e*x + d)^2 + 6*(a^3 - 3*a*b^2)*(e*x + d) - 3*(3*a^2*b - b^3)*l
og(tan(e*x + d)^2 + 1) - 6*(a^3 - 3*a*b^2)*tan(e*x + d))*b)/e
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int (a+b\tan(d+e\,x))\,\left(a^2\tan(d+e\,x)^2+2\,a\,b\tan(d+e\,x)+b^2\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*tan(d + e*x))*(b^2 + a^2*tan(d + e*x)^2 + 2*a*b*tan(d + e*x))^(3
/2),x)
```

```
[Out] int((a + b*tan(d + e*x))*(b^2 + a^2*tan(d + e*x)^2 + 2*a*b*tan(d + e*x))^(3
/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a+b\tan(d+ex))\left((a\tan(d+ex)+b)^2\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(e*x+d))*(b**2+2*a*b*tan(e*x+d)+a**2*tan(e*x+d)**2)**(3/2
),x)
```

```
[Out] Integral((a + b*tan(d + e*x))*((a*tan(d + e*x) + b)**2)**(3/2), x)
```

3.515 $\int (a+b\tan(d+ex))\sqrt{b^2 + 2ab\tan(d+ex) + a^2\tan^2(d +$

Optimal. Leaf size=122

$$\frac{a^2b\tan(d+ex)\sqrt{a^2\tan^2(d+ex)+2ab\tan(d+ex)+b^2}}{e\left(a^2\tan(d+ex)+ab\right)}-\frac{\left(a^2+b^2\right)\log(\cos(d+ex))\sqrt{a^2\tan^2(d+ex)+2ab\tan}}{e(a\tan(d+ex)+b)}$$

```
[Out] -(a^2+b^2)*ln(cos(e*x+d))*(b^2+2*a*b*tan(e*x+d)+a^2*tan(e*x+d)^2)^(1/2)/e/(
b+a*tan(e*x+d))+a^2*b*(b^2+2*a*b*tan(e*x+d)+a^2*tan(e*x+d)^2)^(1/2)*tan(e*x
+d)/e/(a*b+a^2*tan(e*x+d))
```

Rubi [A] time = 0.10, antiderivative size = 122, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.073, Rules used = {3710, 3525, 3475}

$$\frac{a^2b\tan(d+ex)\sqrt{a^2\tan^2(d+ex)+2ab\tan(d+ex)+b^2}}{e\left(a^2\tan(d+ex)+ab\right)}-\frac{\left(a^2+b^2\right)\log(\cos(d+ex))\sqrt{a^2\tan^2(d+ex)+2ab\tan}}{e(a\tan(d+ex)+b)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Tan[d + e*x])*Sqrt[b^2 + 2*a*b*Tan[d + e*x] + a^2*Tan[d + e*x]^2
],x]
```

```
[Out] -(((a^2 + b^2)*Log[Cos[d + e*x]]*Sqrt[b^2 + 2*a*b*Tan[d + e*x] + a^2*Tan[d
+ e*x]^2])/(e*(b + a*Tan[d + e*x]))) + (a^2*b*Tan[d + e*x]*Sqrt[b^2 + 2*a*b
*Tan[d + e*x] + a^2*Tan[d + e*x]^2])/(e*(a*b + a^2*Tan[d + e*x]))
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3525

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[(a*c - b*d)*x, x] + (Dist[b*c + a*d, Int[Tan[e +
 f*x], x], x] + Simp[(b*d*Tan[e + f*x])/f, x]) /; FreeQ[{a, b, c, d, e, f},
 x] && NeQ[b*c - a*d, 0] && NeQ[b*c + a*d, 0]
```

Rule 3710

```
Int[((A_) + (B_.)*tan[(d_.) + (e_.)*(x_)])*((a_) + (b_.)*tan[(d_.) + (e_.)*
(x_)] + (c_.)*tan[(d_.) + (e_.)*(x_)]^2)^(n_), x_Symbol] :> Dist[(a + b*Tan
[d + e*x] + c*Tan[d + e*x]^2)^n/(b + 2*c*Tan[d + e*x])^(2*n), Int[(A + B*Ta
```

```
n[d + e*x])*(b + 2*c*Tan[d + e*x])^(2*n), x], x] /; FreeQ[{a, b, c, d, e, A
, B}, x] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[n]
```

Rubi steps

$$\int (a+b\tan(d+ex))\sqrt{b^2+2ab\tan(d+ex)+a^2\tan^2(d+ex)}\,dx = \frac{\sqrt{b^2+2ab\tan(d+ex)+a^2\tan^2(d+ex)}\int\left(2\right.}{2ab+2a^2\,\mathrm{t}}$$

$$= \frac{a^2b\tan(d+ex)\sqrt{b^2+2ab\tan(d+ex)+a^2\,\mathrm{ta}}}{e\left(ab+a^2\tan(d+ex)\right)}$$

$$= -\frac{\left(a^2+b^2\right)\log(\cos(d+ex))\sqrt{b^2+2ab\tan(d}}{e(b+a\tan(d+ex))}$$

Mathematica [A] time = 0.30, size = 58, normalized size = 0.48

$$\frac{\sqrt{(a\tan(d+ex)+b)^2}\left(ab\tan(d+ex)-\left(a^2+b^2\right)\log(\cos(d+ex))\right)}{e(a\tan(d+ex)+b)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Tan[d + e*x])*Sqrt[b^2 + 2*a*b*Tan[d + e*x] + a^2*Tan[d +
e*x]^2],x]
```

```
[Out] (Sqrt[(b + a*Tan[d + e*x])^2]*(-((a^2 + b^2)*Log[Cos[d + e*x]]) + a*b*Tan[d
 + e*x]))/(e*(b + a*Tan[d + e*x]))
```

fricas [A] time = 2.63, size = 38, normalized size = 0.31

$$\frac{2\,ab\tan\left(ex+d\right)-\left(a^2+b^2\right)\log\left(\frac{1}{\tan\left(ex+d\right)^2+1}\right)}{2\,e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2+2*a*b*tan(e*x+d)+a^2*tan(e*x+d)^2)^(1/2)*(a+b*tan(e*x+d)),x,
 algorithm="fricas")
```

```
[Out] 1/2*(2*a*b*tan(e*x + d) - (a^2 + b^2)*log(1/(tan(e*x + d)^2 + 1)))/e
```

giac [B] time = 0.46, size = 395, normalized size = 3.24

$$-\frac{a^2\log\left(\frac{4\left(\tan(xe)^4\tan(d)^2-2\tan(xe)^3\tan(d)+\tan(xe)^2\tan(d)^2+\tan(xe)^2-2\tan(xe)\tan(d)+1\right)}{\tan(d)^2+1}\right)\mathrm{sgn}\left(a\tan\left(xe+d\right)+b\right)\tan\left(xe\right)\tan(a}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2+2*a*b*tan(e*x+d)+a^2*tan(e*x+d)^2)^(1/2)*(a+b*tan(e*x+d)),x,
 algorithm="giac")
```

```
[Out] -1/2*(a^2*log(4*(tan(x*e)^4*tan(d)^2 - 2*tan(x*e)^3*tan(d) + tan(x*e)^2*tan
(d)^2 + tan(x*e)^2 - 2*tan(x*e)*tan(d) + 1)/(tan(d)^2 + 1))*sgn(a*tan(x*e +
 d) + b)*tan(x*e)*tan(d) + b^2*log(4*(tan(x*e)^4*tan(d)^2 - 2*tan(x*e)^3*ta
n(d) + tan(x*e)^2*tan(d)^2 + tan(x*e)^2 - 2*tan(x*e)*tan(d) + 1)/(tan(d)^2
+ 1))*sgn(a*tan(x*e + d) + b)*tan(x*e)*tan(d) - a^2*log(4*(tan(x*e)^4*tan(d
)^2 - 2*tan(x*e)^3*tan(d) + tan(x*e)^2*tan(d)^2 + tan(x*e)^2 - 2*tan(x*e)*t
an(d) + 1)/(tan(d)^2 + 1))*sgn(a*tan(x*e + d) + b) - b^2*log(4*(tan(x*e)^4*
tan(d)^2 - 2*tan(x*e)^3*tan(d) + tan(x*e)^2*tan(d)^2 + tan(x*e)^2 - 2*tan(x
*e)*tan(d) + 1)/(tan(d)^2 + 1))*sgn(a*tan(x*e + d) + b) + 2*a*b*sgn(a*tan(x
*e + d) + b)*tan(x*e) + 2*a*b*sgn(a*tan(x*e + d) + b)*tan(d))/(e*tan(x*e)*t
an(d) - e)
```

maple [C] time = 0.32, size = 75, normalized size = 0.61

$$\frac{\operatorname{csgn}\left(b+a\tan\left(ex+d\right)\right)\left(\ln\left(a^{2}\left(\tan^{2}\left(ex+d\right)\right)+a^{2}\right)a^{2}+\ln\left(a^{2}\left(\tan^{2}\left(ex+d\right)\right)+a^{2}\right)b^{2}+2ab\tan\left(ex+d\right)+2b^{2}\right)}{2e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2+2*a*b*tan(e*x+d)+a^2*tan(e*x+d)^2)^(1/2)*(a+b*tan(e*x+d)),x)
```

```
[Out] 1/2/e*csgn(b+a*tan(e*x+d))*(ln(a^2*tan(e*x+d)^2+a^2)*a^2+ln(a^2*tan(e*x+d)^
2+a^2)*b^2+2*a*b*tan(e*x+d)+2*b^2)
```

maxima [A] time = 0.42, size = 65, normalized size = 0.53

$$\frac{\left(2\left(ex+d\right)b+a\log\left(\tan\left(ex+d\right)^{2}+1\right)\right)a-\left(2\left(ex+d\right)a-b\log\left(\tan\left(ex+d\right)^{2}+1\right)-2\,a\tan\left(ex+d\right)\right)b}{2\,e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2+2*a*b*tan(e*x+d)+a^2*tan(e*x+d)^2)^(1/2)*(a+b*tan(e*x+d)),x,
 algorithm="maxima")
```

```
[Out] 1/2*((2*(e*x + d)*b + a*log(tan(e*x + d)^2 + 1))*a - (2*(e*x + d)*a - b*log
(tan(e*x + d)^2 + 1) - 2*a*tan(e*x + d))*b)/e
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \left(a+b\tan\left(d+e\,x\right)\right)\,\sqrt{a^2\tan\left(d+e\,x\right)^2+2\,a\,b\tan\left(d+e\,x\right)+b^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*tan(d + e*x))*(b^2 + a^2*tan(d + e*x)^2 + 2*a*b*tan(d + e*x))^(1
/2),x)
```

```
[Out] int((a + b*tan(d + e*x))*(b^2 + a^2*tan(d + e*x)^2 + 2*a*b*tan(d + e*x))^(1
/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b\tan(d + ex))\sqrt{(a\tan(d + ex) + b)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2+2*a*b*tan(e*x+d)+a**2*tan(e*x+d)**2)**(1/2)*(a+b*tan(e*x+d)
),x)
```

```
[Out] Integral((a + b*tan(d + e*x))*sqrt((a*tan(d + e*x) + b)**2), x)
```

3.516 $\int \frac{a+b\tan(d+ex)}{\sqrt{b^2+2ab\tan(d+ex)+a^2\tan^2(d+ex)}}\,dx$

Optimal. Leaf size=138

$$\frac{2bx\left(a^2\tan(d+ex)+ab\right)}{\left(a^2+b^2\right)\sqrt{a^2\tan^2(d+ex)+2ab\tan(d+ex)+b^2}}+\frac{\left(a^2-b^2\right)(a\tan(d+ex)+b)\log(a\sin(d+ex)+b\cos(d+e}{e\left(a^2+b^2\right)\sqrt{a^2\tan^2(d+ex)+2ab\tan(d+ex)+b^2}}$$

```
[Out] (a^2-b^2)*ln(b*cos(e*x+d)+a*sin(e*x+d))*(b+a*tan(e*x+d))/(a^2+b^2)/e/(b^2+2
*a*b*tan(e*x+d)+a^2*tan(e*x+d)^2)^(1/2)+2*b*x*(a*b+a^2*tan(e*x+d))/(a^2+b^2
)/(b^2+2*a*b*tan(e*x+d)+a^2*tan(e*x+d)^2)^(1/2)
```

Rubi [A] time = 0.19, antiderivative size = 138, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.073, Rules used = {3710, 3531, 3530}

$$\frac{2bx\left(a^2\tan(d+ex)+ab\right)}{\left(a^2+b^2\right)\sqrt{a^2\tan^2(d+ex)+2ab\tan(d+ex)+b^2}}+\frac{\left(a^2-b^2\right)(a\tan(d+ex)+b)\log(a\sin(d+ex)+b\cos(d+e}{e\left(a^2+b^2\right)\sqrt{a^2\tan^2(d+ex)+2ab\tan(d+ex)+b^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Tan[d + e*x])/Sqrt[b^2 + 2*a*b*Tan[d + e*x] + a^2*Tan[d + e*x]^2
],x]
```

```
[Out] ((a^2 - b^2)*Log[b*Cos[d + e*x] + a*Sin[d + e*x]]*(b + a*Tan[d + e*x]))/((a
^2 + b^2)*e*Sqrt[b^2 + 2*a*b*Tan[d + e*x] + a^2*Tan[d + e*x]^2]) + (2*b*x*(
a*b + a^2*Tan[d + e*x]))/((a^2 + b^2)*Sqrt[b^2 + 2*a*b*Tan[d + e*x] + a^2*T
an[d + e*x]^2])
```

Rule 3530

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_) + (b_.)*tan[(e_.) + (f_.)*
(x_)]), x_Symbol] :> Simp[(c*Log[RemoveContent[a*Cos[e + f*x] + b*Sin[e + f
*x], x]])/(b*f), x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
 NeQ[a^2 + b^2, 0] && EqQ[a*c + b*d, 0]
```

Rule 3531

```
Int[((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_.) + (b_.)*tan[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[((a*c + b*d)*x)/(a^2 + b^2), x] + Dist[(b*c - a
*d)/(a^2 + b^2), Int[(b - a*Tan[e + f*x])/(a + b*Tan[e + f*x]), x], x] /; F
reeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && Ne
Q[a*c + b*d, 0]
```

Rule 3710

```
Int[((A_) + (B_.)*tan[(d_.) + (e_.)*(x_)])*((a_) + (b_.)*tan[(d_.) + (e_.)*
(x_)] + (c_.)*tan[(d_.) + (e_.)*(x_)]^2)^(n_), x_Symbol] :> Dist[(a + b*Tan
[d + e*x] + c*Tan[d + e*x]^2)^n/(b + 2*c*Tan[d + e*x])^(2*n), Int[(A + B*Ta
n[d + e*x])*(b + 2*c*Tan[d + e*x])^(2*n), x], x] /; FreeQ[{a, b, c, d, e, A
, B}, x] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[n]
```

Rubi steps

$$\int \frac{a+b\tan(d+ex)}{\sqrt{b^2+2ab\tan(d+ex)+a^2\tan^2(d+ex)}}\,dx = \frac{\left(2ab+2a^2\tan(d+ex)\right)\int \frac{a+b\tan(d+ex)}{2ab+2a^2\tan(d+ex)}\,dx}{\sqrt{b^2+2ab\tan(d+ex)+a^2\tan^2(d+ex)}}$$

$$= \frac{2bx\left(ab+a^2\tan(d+ex)\right)}{\left(a^2+b^2\right)\sqrt{b^2+2ab\tan(d+ex)+a^2\tan^2(d+ex)}} + \frac{\left(\left(a^2-b^2\right.\right.}{2a\left(a^2+\right.}$$

$$= \frac{\left(a^2-b^2\right)\log(b\cos(d+ex)+a\sin(d+ex))(b+a\tan(d+ex))}{\left(a^2+b^2\right)e\sqrt{b^2+2ab\tan(d+ex)+a^2\tan^2(d+ex)}}$$

Mathematica [A] time = 0.68, size = 88, normalized size = 0.64

$$\frac{(a\tan(d+ex)+b)\left(4ab\tan^{-1}(\tan(d+ex))-\left(a^2-b^2\right)\left(\log\left(\sec^2(d+ex)\right)-2\log(a\tan(d+ex)+b)\right)\right)}{2e\left(a^2+b^2\right)\sqrt{(a\tan(d+ex)+b)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Tan[d + e*x])/Sqrt[b^2 + 2*a*b*Tan[d + e*x] + a^2*Tan[d +
e*x]^2],x]
```

```
[Out] ((4*a*b*ArcTan[Tan[d + e*x]] - (a^2 - b^2)*(Log[Sec[d + e*x]^2] - 2*Log[b +
 a*Tan[d + e*x]]))*(b + a*Tan[d + e*x]))/(2*(a^2 + b^2)*e*Sqrt[(b + a*Tan[d
 + e*x])^2])
```

fricas [A] time = 0.72, size = 71, normalized size = 0.51

$$\frac{4\,abex+\left(a^2-b^2\right)\log\left(\frac{a^2\tan(ex+d)^2+2\,ab\tan(ex+d)+b^2}{\tan(ex+d)^2+1}\right)}{2\left(a^2+b^2\right)e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(e*x+d))/(b^2+2*a*b*tan(e*x+d)+a^2*tan(e*x+d)^2)^(1/2),x,
 algorithm="fricas")
```

```
[Out] 1/2*(4*a*b*e*x + (a^2 - b^2)*log((a^2*tan(e*x + d)^2 + 2*a*b*tan(e*x + d) +
 b^2)/(tan(e*x + d)^2 + 1)))/((a^2 + b^2)*e)
```

giac [B] time = 3.07, size = 554, normalized size = 4.01

$$-\frac{1}{2}\left(\frac{2\left(\pi\operatorname{sgn}\left(\tan\left(\frac{1}{2}\,xe+\frac{1}{2}\,d\right)\right)+2\,\mathrm{ar}\right.}{a^2\operatorname{sgn}\left(-b\tan\left(\frac{1}{2}\,xe+\frac{1}{2}\,d\right)^4+2\,a\tan\left(\frac{1}{2}\,xe+\frac{1}{2}\,d\right)^3+2\,b\tan\left(\frac{1}{2}\,xe+\frac{1}{2}\,d\right)^2-2\,a\tan\left(\frac{1}{2}\,xe+\frac{1}{2}\,d\right)-b\right)+b^2\mathrm{s}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(e*x+d))/(b^2+2*a*b*tan(e*x+d)+a^2*tan(e*x+d)^2)^(1/2),x,
 algorithm="giac")
```

```
[Out] -1/2*(2*(pi*sgn(tan(1/2*x*e + 1/2*d)) + 2*arctan(1/2*(tan(1/2*x*e + 1/2*d)^
2 - 1)/tan(1/2*x*e + 1/2*d)))*a*b/(a^2*sgn(-b*tan(1/2*x*e + 1/2*d)^4 + 2*a*
tan(1/2*x*e + 1/2*d)^3 + 2*b*tan(1/2*x*e + 1/2*d)^2 - 2*a*tan(1/2*x*e + 1/2
*d) - b) + b^2*sgn(-b*tan(1/2*x*e + 1/2*d)^4 + 2*a*tan(1/2*x*e + 1/2*d)^3 +
 2*b*tan(1/2*x*e + 1/2*d)^2 - 2*a*tan(1/2*x*e + 1/2*d) - b)) - (a^2 - b^2)*
log((1/tan(1/2*x*e + 1/2*d) - tan(1/2*x*e + 1/2*d))^2 + 4)/(a^2*sgn(-b*tan(
1/2*x*e + 1/2*d)^4 + 2*a*tan(1/2*x*e + 1/2*d)^3 + 2*b*tan(1/2*x*e + 1/2*d)^
2 - 2*a*tan(1/2*x*e + 1/2*d) - b) + b^2*sgn(-b*tan(1/2*x*e + 1/2*d)^4 + 2*a
*tan(1/2*x*e + 1/2*d)^3 + 2*b*tan(1/2*x*e + 1/2*d)^2 - 2*a*tan(1/2*x*e + 1/
2*d) - b)) + 2*(a^2*b - b^3)*log(abs(-b*(1/tan(1/2*x*e + 1/2*d) - tan(1/2*x
*e + 1/2*d)) - 2*a))/(a^2*b*sgn(-b*tan(1/2*x*e + 1/2*d)^4 + 2*a*tan(1/2*x*e
 + 1/2*d)^3 + 2*b*tan(1/2*x*e + 1/2*d)^2 - 2*a*tan(1/2*x*e + 1/2*d) - b) +
b^3*sgn(-b*tan(1/2*x*e + 1/2*d)^4 + 2*a*tan(1/2*x*e + 1/2*d)^3 + 2*b*tan(1/
2*x*e + 1/2*d)^2 - 2*a*tan(1/2*x*e + 1/2*d) - b)))*e^(-1)
```

maple [A] time = 0.23, size = 114, normalized size = 0.83

$$\frac{(b+a\tan(ex+d))\left(2\ln(b+a\tan(ex+d))\,a^2-2\ln(b+a\tan(ex+d))\,b^2-\ln\left(1+\tan^2(ex+d)\right)a^2+\ln\left(1+\right.\right.}{2e\sqrt{b^2+2ab\tan(ex+d)+a^2\left(\tan^2(ex+d)\right)}\,\left(a^2+b^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*tan(e*x+d))/(b^2+2*a*b*tan(e*x+d)+a^2*tan(e*x+d)^2)^(1/2),x)
```

```
[Out] 1/2/e*(b+a*tan(e*x+d))*(2*ln(b+a*tan(e*x+d))*a^2-2*ln(b+a*tan(e*x+d))*b^2-l
n(1+tan(e*x+d)^2)*a^2+ln(1+tan(e*x+d)^2)*b^2+4*a*b*arctan(tan(e*x+d)))/((b+
a*tan(e*x+d))^2)^(1/2)/(a^2+b^2)
```

maxima [A] time = 0.42, size = 137, normalized size = 0.99

$$\frac{a\left(\frac{2\,(ex+d)b}{a^2+b^2}+\frac{2\,a\log(a\tan(ex+d)+b)}{a^2+b^2}-\frac{a\log\left(\tan(ex+d)^2+1\right)}{a^2+b^2}\right)+\left(\frac{2\,(ex+d)a}{a^2+b^2}-\frac{2\,b\log(a\tan(ex+d)+b)}{a^2+b^2}+\frac{b\log\left(\tan(ex+d)^2+1\right)}{a^2+b^2}\right)b}{2\,e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(e*x+d))/(b^2+2*a*b*tan(e*x+d)+a^2*tan(e*x+d)^2)^(1/2),x,
 algorithm="maxima")
```

```
[Out] 1/2*(a*(2*(e*x + d)*b/(a^2 + b^2) + 2*a*log(a*tan(e*x + d) + b)/(a^2 + b^2)
 - a*log(tan(e*x + d)^2 + 1)/(a^2 + b^2)) + (2*(e*x + d)*a/(a^2 + b^2) - 2*
b*log(a*tan(e*x + d) + b)/(a^2 + b^2) + b*log(tan(e*x + d)^2 + 1)/(a^2 + b^
2))*b)/e
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{a+b\tan\left(d+e\,x\right)}{\sqrt{a^2\tan\left(d+e\,x\right)^2+2\,a\,b\tan\left(d+e\,x\right)+b^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*tan(d + e*x))/(b^2 + a^2*tan(d + e*x)^2 + 2*a*b*tan(d + e*x))^(1
/2),x)
```

```
[Out] int((a + b*tan(d + e*x))/(b^2 + a^2*tan(d + e*x)^2 + 2*a*b*tan(d + e*x))^(1
/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a+b\tan\left(d+ex\right)}{\sqrt{\left(a\tan\left(d+ex\right)+b\right)^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(e*x+d))/(b**2+2*a*b*tan(e*x+d)+a**2*tan(e*x+d)**2)**(1/2
),x)
```

```
[Out] Integral((a + b*tan(d + e*x))/sqrt((a*tan(d + e*x) + b)**2), x)
```

3.517 $\int \frac{a+b\tan(d+ex)}{\left(b^2+2ab\tan(d+ex)+a^2\tan^2(d+ex)\right)^{3/2}}\,dx$

Optimal. Leaf size=316

$$-\frac{\left(a^2-b^2\right)\left(a\tan(d+ex)+b\right)}{2e\left(a^2+b^2\right)\left(a^2\tan^2(d+ex)+2ab\tan(d+ex)+b^2\right)^{3/2}}-\frac{4bx\left(a^2-b^2\right)\left(a^2\tan(d+ex)+ab\right)^3}{a^2\left(a^2+b^2\right)^3\left(a^2\tan^2(d+ex)+2ab\tan(d+ex)+b^2\right)}$$

```
[Out] -1/2*(a^2-b^2)*(b+a*tan(e*x+d))/(a^2+b^2)/e/(b^2+2*a*b*tan(e*x+d)+a^2*tan(e
*x+d)^2)^(3/2)-(a^4-6*a^2*b^2+b^4)*ln(b*cos(e*x+d)+a*sin(e*x+d))*(b+a*tan(e
*x+d))^3/(a^2+b^2)^3/e/(b^2+2*a*b*tan(e*x+d)+a^2*tan(e*x+d)^2)^(3/2)-4*b*(a
^2-b^2)*x*(a*b+a^2*tan(e*x+d))^3/a^2/(a^2+b^2)^3/(b^2+2*a*b*tan(e*x+d)+a^2*
tan(e*x+d)^2)^(3/2)-b*(3*a^2-b^2)*(a*b+a^2*tan(e*x+d))^3/(a^2+b^2)^2/e/(a^3
*b+a^4*tan(e*x+d))/(b^2+2*a*b*tan(e*x+d)+a^2*tan(e*x+d)^2)^(3/2)
```

Rubi [A] time = 0.40, antiderivative size = 316, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.098, Rules used = {3710, 3529, 3531, 3530}

$$-\frac{\left(a^2-b^2\right)\left(a\tan(d+ex)+b\right)}{2e\left(a^2+b^2\right)\left(a^2\tan^2(d+ex)+2ab\tan(d+ex)+b^2\right)^{3/2}}-\frac{4bx\left(a^2-b^2\right)\left(a^2\tan(d+ex)+ab\right)^3}{a^2\left(a^2+b^2\right)^3\left(a^2\tan^2(d+ex)+2ab\tan(d+ex)+b^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Tan[d + e*x])/(b^2 + 2*a*b*Tan[d + e*x] + a^2*Tan[d + e*x]^2)^(3
/2),x]
```

```
[Out] -((a^2 - b^2)*(b + a*Tan[d + e*x]))/(2*(a^2 + b^2)*e*(b^2 + 2*a*b*Tan[d + e
*x] + a^2*Tan[d + e*x]^2)^(3/2)) - ((a^4 - 6*a^2*b^2 + b^4)*Log[b*Cos[d + e
*x] + a*Sin[d + e*x]]*(b + a*Tan[d + e*x])^3)/((a^2 + b^2)^3*e*(b^2 + 2*a*b
*Tan[d + e*x] + a^2*Tan[d + e*x]^2)^(3/2)) - (4*b*(a^2 - b^2)*x*(a*b + a^2*
Tan[d + e*x])^3)/(a^2*(a^2 + b^2)^3*(b^2 + 2*a*b*Tan[d + e*x] + a^2*Tan[d +
 e*x]^2)^(3/2)) - (b*(3*a^2 - b^2)*(a*b + a^2*Tan[d + e*x])^3)/((a^2 + b^2)
^2*e*(a^3*b + a^4*Tan[d + e*x])*(b^2 + 2*a*b*Tan[d + e*x] + a^2*Tan[d + e*x
]^2)^(3/2))
```

Rule 3529

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[((b*c - a*d)*(a + b*Tan[e + f*x])^(m + 1))
/(f*(m + 1)*(a^2 + b^2)), x] + Dist[1/(a^2 + b^2), Int[(a + b*Tan[e + f*x])
^(m + 1)*Simp[a*c + b*d - (b*c - a*d)*Tan[e + f*x], x], x], x] /; FreeQ[{a,
 b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && LtQ[m, -1]
```

Rule 3530

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_) + (b_.)*tan[(e_.) + (f_.)*
(x_)]), x_Symbol] :> Simp[(c*Log[RemoveContent[a*Cos[e + f*x] + b*Sin[e + f
*x], x]])/(b*f), x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
 NeQ[a^2 + b^2, 0] && EqQ[a*c + b*d, 0]
```

Rule 3531

```
Int[((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_.) + (b_.)*tan[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[((a*c + b*d)*x)/(a^2 + b^2), x] + Dist[(b*c - a
*d)/(a^2 + b^2), Int[(b - a*Tan[e + f*x])/(a + b*Tan[e + f*x]), x], x] /; F
reeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && Ne
Q[a*c + b*d, 0]
```

Rule 3710

```
Int[((A_) + (B_.)*tan[(d_.) + (e_.)*(x_)])*((a_) + (b_.)*tan[(d_.) + (e_.)*
(x_)] + (c_.)*tan[(d_.) + (e_.)*(x_)]^2)^(n_), x_Symbol] :> Dist[(a + b*Tan
[d + e*x] + c*Tan[d + e*x]^2)^n/(b + 2*c*Tan[d + e*x])^(2*n), Int[(A + B*Ta
n[d + e*x])*(b + 2*c*Tan[d + e*x])^(2*n), x], x] /; FreeQ[{a, b, c, d, e, A
, B}, x] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[n]
```

Rubi steps

$$\int \frac{a+b\tan(d+ex)}{\left(b^2+2ab\tan(d+ex)+a^2\tan^2(d+ex)\right)^{3/2}}\,dx = \frac{\left(2ab+2a^2\tan(d+ex)\right)^3\int\frac{a+b\tan(d+ex)}{\left(2ab+2a^2\tan(d+ex)\right)^3}\,dx}{\left(b^2+2ab\tan(d+ex)+a^2\tan^2(d+ex)\right)^{3/2}}$$

$$= -\frac{\left(a^2-b^2\right)(b+a\tan(d+ex))}{2\left(a^2+b^2\right)e\left(b^2+2ab\tan(d+ex)+a^2\tan^2(d+ex)\right)^{3/2}} +$$

$$= -\frac{\left(a^2-b^2\right)(b+a\tan(d+ex))}{2\left(a^2+b^2\right)e\left(b^2+2ab\tan(d+ex)+a^2\tan^2(d+ex)\right)^{3/2}} -$$

$$= -\frac{\left(a^2-b^2\right)(b+a\tan(d+ex))}{2\left(a^2+b^2\right)e\left(b^2+2ab\tan(d+ex)+a^2\tan^2(d+ex)\right)^{3/2}} -$$

$$= -\frac{\left(a^2-b^2\right)(b+a\tan(d+ex))}{2\left(a^2+b^2\right)e\left(b^2+2ab\tan(d+ex)+a^2\tan^2(d+ex)\right)^{3/2}} -$$

Mathematica [C] time = 3.44, size = 268, normalized size = 0.85

$$\frac{(a\tan(d+ex)+b)^3\left(b\left(\frac{2a\left(2b\log(a\tan(d+ex)+b)-\frac{a^2+b^2}{a\tan(d+ex)+b}\right)}{\left(a^2+b^2\right)^2}+\frac{i\log(-\tan(d+ex)+i)}{(a-ib)^2}-\frac{i\log(\tan(d+ex)+i)}{(a+ib)^2}\right)+(a-b)(a+b)\left(\frac{a(\cdots}{\cdots}\right.\right.}{2ae\left((a\tan(d+ex)+b)^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Tan[d + e*x])/(b^2 + 2*a*b*Tan[d + e*x] + a^2*Tan[d + e*x]
^2)^(3/2),x]
```

```
[Out] ((b + a*Tan[d + e*x])^3*(b*((I*Log[I - Tan[d + e*x]])/(a - I*b)^2 - (I*Log[
I + Tan[d + e*x]])/(a + I*b)^2 + (2*a*(2*b*Log[b + a*Tan[d + e*x]] - (a^2 +
 b^2)/(b + a*Tan[d + e*x])))/(a^2 + b^2)^2) + (a - b)*(a + b)*(Log[I - Tan[
d + e*x]]/(a - I*b)^3 + Log[I + Tan[d + e*x]]/(a + I*b)^3 + (a*(-2*(a^2 - 3
*b^2)*Log[b + a*Tan[d + e*x]] - ((a^2 + b^2)*(a^2 + 5*b^2 + 4*a*b*Tan[d + e
*x]))/(b + a*Tan[d + e*x])^2))/(a^2 + b^2)^3)))/(2*a*e*((b + a*Tan[d + e*x]
)^2)^(3/2))
```

fricas [A] time = 2.03, size = 355, normalized size = 1.12

$$-\frac{a^6+8\,a^4b^2-5\,a^2b^4+8\left(a^3b^3-ab^5\right)ex+\left(a^6-8\,a^4b^2+3\,a^2b^4+8\left(a^5b-a^3b^3\right)ex\right)\tan\left(ex+d\right)^2+\left(a^4b^2-6\,a^2b\right.}{2\left(\left(a^8+3\,a^6b^2+3\,a^4b^4+a^2b^6\right)e\,\mathrm{t}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(e*x+d))/(b^2+2*a*b*tan(e*x+d)+a^2*tan(e*x+d)^2)^(3/2),x,
 algorithm="fricas")
```

```
[Out] -1/2*(a^6 + 8*a^4*b^2 - 5*a^2*b^4 + 8*(a^3*b^3 - a*b^5)*e*x + (a^6 - 8*a^4*
b^2 + 3*a^2*b^4 + 8*(a^5*b - a^3*b^3)*e*x)*tan(e*x + d)^2 + (a^4*b^2 - 6*a^
2*b^4 + b^6 + (a^6 - 6*a^4*b^2 + a^2*b^4)*tan(e*x + d)^2 + 2*(a^5*b - 6*a^3
*b^3 + a*b^5)*tan(e*x + d))*log((a^2*tan(e*x + d)^2 + 2*a*b*tan(e*x + d) +
b^2)/(tan(e*x + d)^2 + 1)) + 4*(2*a^5*b - 3*a^3*b^3 + a*b^5 + 4*(a^4*b^2 -
a^2*b^4)*e*x)*tan(e*x + d))/((a^8 + 3*a^6*b^2 + 3*a^4*b^4 + a^2*b^6)*e*tan(
e*x + d)^2 + 2*(a^7*b + 3*a^5*b^3 + 3*a^3*b^5 + a*b^7)*e*tan(e*x + d) + (a^
6*b^2 + 3*a^4*b^4 + 3*a^2*b^6 + b^8)*e)
```

giac [B] time = 10.07, size = 1571, normalized size = 4.97

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(e*x+d))/(b^2+2*a*b*tan(e*x+d)+a^2*tan(e*x+d)^2)^(3/2),x,
 algorithm="giac")
```

```
[Out] 1/2*(4*(a^3*b - a*b^3)*(pi*sgn(tan(1/2*x*e + 1/2*d)) + 2*arctan(1/2*(tan(1/
2*x*e + 1/2*d)^2 - 1)/tan(1/2*x*e + 1/2*d)))/(a^6*sgn(-b*tan(1/2*x*e + 1/2*
d)^4 + 2*a*tan(1/2*x*e + 1/2*d)^3 + 2*b*tan(1/2*x*e + 1/2*d)^2 - 2*a*tan(1/
2*x*e + 1/2*d) - b) + 3*a^4*b^2*sgn(-b*tan(1/2*x*e + 1/2*d)^4 + 2*a*tan(1/2
*x*e + 1/2*d)^3 + 2*b*tan(1/2*x*e + 1/2*d)^2 - 2*a*tan(1/2*x*e + 1/2*d) - b
) + 3*a^2*b^4*sgn(-b*tan(1/2*x*e + 1/2*d)^4 + 2*a*tan(1/2*x*e + 1/2*d)^3 +
2*b*tan(1/2*x*e + 1/2*d)^2 - 2*a*tan(1/2*x*e + 1/2*d) - b) + b^6*sgn(-b*tan
(1/2*x*e + 1/2*d)^4 + 2*a*tan(1/2*x*e + 1/2*d)^3 + 2*b*tan(1/2*x*e + 1/2*d)
^2 - 2*a*tan(1/2*x*e + 1/2*d) - b)) - (a^4 - 6*a^2*b^2 + b^4)*log((1/tan(1/
2*x*e + 1/2*d) - tan(1/2*x*e + 1/2*d))^2 + 4)/(a^6*sgn(-b*tan(1/2*x*e + 1/2
*d)^4 + 2*a*tan(1/2*x*e + 1/2*d)^3 + 2*b*tan(1/2*x*e + 1/2*d)^2 - 2*a*tan(1
/2*x*e + 1/2*d) - b) + 3*a^4*b^2*sgn(-b*tan(1/2*x*e + 1/2*d)^4 + 2*a*tan(1/
2*x*e + 1/2*d)^3 + 2*b*tan(1/2*x*e + 1/2*d)^2 - 2*a*tan(1/2*x*e + 1/2*d) -
b) + 3*a^2*b^4*sgn(-b*tan(1/2*x*e + 1/2*d)^4 + 2*a*tan(1/2*x*e + 1/2*d)^3 +
 2*b*tan(1/2*x*e + 1/2*d)^2 - 2*a*tan(1/2*x*e + 1/2*d) - b) + b^6*sgn(-b*ta
n(1/2*x*e + 1/2*d)^4 + 2*a*tan(1/2*x*e + 1/2*d)^3 + 2*b*tan(1/2*x*e + 1/2*d
)^2 - 2*a*tan(1/2*x*e + 1/2*d) - b)) + 2*(a^4*b - 6*a^2*b^3 + b^5)*log(abs(
```

```
-b*(1/tan(1/2*x*e + 1/2*d) - tan(1/2*x*e + 1/2*d)) - 2*a))/(a^6*b*sgn(-b*ta
n(1/2*x*e + 1/2*d)^4 + 2*a*tan(1/2*x*e + 1/2*d)^3 + 2*b*tan(1/2*x*e + 1/2*d
)^2 - 2*a*tan(1/2*x*e + 1/2*d) - b) + 3*a^4*b^3*sgn(-b*tan(1/2*x*e + 1/2*d)
^4 + 2*a*tan(1/2*x*e + 1/2*d)^3 + 2*b*tan(1/2*x*e + 1/2*d)^2 - 2*a*tan(1/2*
x*e + 1/2*d) - b) + 3*a^2*b^5*sgn(-b*tan(1/2*x*e + 1/2*d)^4 + 2*a*tan(1/2*x
*e + 1/2*d)^3 + 2*b*tan(1/2*x*e + 1/2*d)^2 - 2*a*tan(1/2*x*e + 1/2*d) - b)
+ b^7*sgn(-b*tan(1/2*x*e + 1/2*d)^4 + 2*a*tan(1/2*x*e + 1/2*d)^3 + 2*b*tan(
1/2*x*e + 1/2*d)^2 - 2*a*tan(1/2*x*e + 1/2*d) - b)) - (3*a^4*b^4*(1/tan(1/2
*x*e + 1/2*d) - tan(1/2*x*e + 1/2*d))^2 - 18*a^2*b^6*(1/tan(1/2*x*e + 1/2*d
) - tan(1/2*x*e + 1/2*d))^2 + 3*b^8*(1/tan(1/2*x*e + 1/2*d) - tan(1/2*x*e +
 1/2*d))^2 + 4*a^7*b*(1/tan(1/2*x*e + 1/2*d) - tan(1/2*x*e + 1/2*d)) + 28*a
^5*b^3*(1/tan(1/2*x*e + 1/2*d) - tan(1/2*x*e + 1/2*d)) - 68*a^3*b^5*(1/tan(
1/2*x*e + 1/2*d) - tan(1/2*x*e + 1/2*d)) + 4*a*b^7*(1/tan(1/2*x*e + 1/2*d)
- tan(1/2*x*e + 1/2*d)) + 4*a^8 + 40*a^6*b^2 - 60*a^4*b^4)/((a^6*b^2*sgn(-b
*tan(1/2*x*e + 1/2*d)^4 + 2*a*tan(1/2*x*e + 1/2*d)^3 + 2*b*tan(1/2*x*e + 1/
2*d)^2 - 2*a*tan(1/2*x*e + 1/2*d) - b) + 3*a^4*b^4*sgn(-b*tan(1/2*x*e + 1/2
*d)^4 + 2*a*tan(1/2*x*e + 1/2*d)^3 + 2*b*tan(1/2*x*e + 1/2*d)^2 - 2*a*tan(1
/2*x*e + 1/2*d) - b) + 3*a^2*b^6*sgn(-b*tan(1/2*x*e + 1/2*d)^4 + 2*a*tan(1/
2*x*e + 1/2*d)^3 + 2*b*tan(1/2*x*e + 1/2*d)^2 - 2*a*tan(1/2*x*e + 1/2*d) -
b) + b^8*sgn(-b*tan(1/2*x*e + 1/2*d)^4 + 2*a*tan(1/2*x*e + 1/2*d)^3 + 2*b*t
an(1/2*x*e + 1/2*d)^2 - 2*a*tan(1/2*x*e + 1/2*d) - b))*(b*(1/tan(1/2*x*e +
1/2*d) - tan(1/2*x*e + 1/2*d)) + 2*a)^2))*e^(-1)
```

maple [B] time = 0.19, size = 622, normalized size = 1.97

$$-\frac{\left(3a^2b^4 + a^6 - 3b^6 + 6\ln\left(1+\tan^2(ex+d)\right)\left(\tan^2(ex+d)\right)a^4b^2 + 2\ln\left(b+a\tan(ex+d)\right)\left(\tan^2(ex+d)\right)a^2b^4\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*tan(e*x+d))/(b^2+2*a*b*tan(e*x+d)+a^2*tan(e*x+d)^2)^(3/2),x)
```

```
[Out] -1/2/e*(2*ln(b+a*tan(e*x+d))*tan(e*x+d)^2*a^6+3*a^2*b^4+a^6-3*b^6+6*ln(1+ta
n(e*x+d)^2)*tan(e*x+d)^2*a^4*b^2+2*ln(b+a*tan(e*x+d))*tan(e*x+d)^2*a^2*b^4-
12*ln(b+a*tan(e*x+d))*tan(e*x+d)^2*a^4*b^2-ln(1+tan(e*x+d)^2)*tan(e*x+d)^2*
a^2*b^4+8*arctan(tan(e*x+d))*tan(e*x+d)^2*a^5*b-8*arctan(tan(e*x+d))*tan(e*
x+d)^2*a^3*b^3+4*ln(b+a*tan(e*x+d))*tan(e*x+d)*a^5*b-24*ln(b+a*tan(e*x+d))*
tan(e*x+d)*a^3*b^3+4*ln(b+a*tan(e*x+d))*tan(e*x+d)*a*b^5-2*ln(1+tan(e*x+d)^
2)*tan(e*x+d)*a^5*b+12*ln(1+tan(e*x+d)^2)*tan(e*x+d)*a^3*b^3-2*ln(1+tan(e*x
+d)^2)*tan(e*x+d)*a*b^5+16*arctan(tan(e*x+d))*tan(e*x+d)*a^4*b^2-16*arctan(
tan(e*x+d))*tan(e*x+d)*a^2*b^4+2*ln(b+a*tan(e*x+d))*b^6-ln(1+tan(e*x+d)^2)*
b^6+6*tan(e*x+d)*a^5*b+4*tan(e*x+d)*a^3*b^3-2*tan(e*x+d)*a*b^5-ln(1+tan(e*x
+d)^2)*tan(e*x+d)^2*a^6+2*ln(b+a*tan(e*x+d))*a^4*b^2-12*ln(b+a*tan(e*x+d))*
a^2*b^4-ln(1+tan(e*x+d)^2)*a^4*b^2+6*ln(1+tan(e*x+d)^2)*a^2*b^4+8*arctan(ta
```

```
n(e*x+d))*a^3*b^3-8*arctan(tan(e*x+d))*a*b^5+7*a^4*b^2)*(b+a*tan(e*x+d))/(a
^2+b^2)^3/((b+a*tan(e*x+d))^2)^(3/2)
```

maxima [A] time = 0.44, size = 498, normalized size = 1.58

$$-\frac{\left(\frac{2\left(3a^2b-b^3\right)(ex+d)}{a^6+3a^4b^2+3a^2b^4+b^6}+\frac{2\left(a^3-3ab^2\right)\log(a\tan(ex+d)+b)}{a^6+3a^4b^2+3a^2b^4+b^6}-\frac{\left(a^3-3ab^2\right)\log\left(\tan(ex+d)^2+1\right)}{a^6+3a^4b^2+3a^2b^4+b^6}+\frac{4a^2b\tan(ex+d)+a^3+5a}{a^4b^2+2a^2b^4+b^6+\left(a^6+2a^4b^2+a^2b^4\right)\tan(ex+d)^2+2}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(e*x+d))/(b^2+2*a*b*tan(e*x+d)+a^2*tan(e*x+d)^2)^(3/2),x,
 algorithm="maxima")
```

```
[Out] -1/2*((2*(3*a^2*b - b^3)*(e*x + d)/(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6) + 2*
(a^3 - 3*a*b^2)*log(a*tan(e*x + d) + b)/(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)
 - (a^3 - 3*a*b^2)*log(tan(e*x + d)^2 + 1)/(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b
^6) + (4*a^2*b*tan(e*x + d) + a^3 + 5*a*b^2)/(a^4*b^2 + 2*a^2*b^4 + b^6 + (
a^6 + 2*a^4*b^2 + a^2*b^4)*tan(e*x + d)^2 + 2*(a^5*b + 2*a^3*b^3 + a*b^5)*t
an(e*x + d)))*a + (2*(a^3 - 3*a*b^2)*(e*x + d)/(a^6 + 3*a^4*b^2 + 3*a^2*b^4
 + b^6) - 2*(3*a^2*b - b^3)*log(a*tan(e*x + d) + b)/(a^6 + 3*a^4*b^2 + 3*a^
2*b^4 + b^6) + (3*a^2*b - b^3)*log(tan(e*x + d)^2 + 1)/(a^6 + 3*a^4*b^2 + 3
*a^2*b^4 + b^6) + (a^2*b - 3*b^3 + 2*(a^3 - a*b^2)*tan(e*x + d))/(a^4*b^2 +
 2*a^2*b^4 + b^6 + (a^6 + 2*a^4*b^2 + a^2*b^4)*tan(e*x + d)^2 + 2*(a^5*b +
2*a^3*b^3 + a*b^5)*tan(e*x + d)))*b)/e
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{a+b\tan(d+e\,x)}{\left(a^2\tan(d+e\,x)^2+2\,a\,b\tan(d+e\,x)+b^2\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*tan(d + e*x))/(b^2 + a^2*tan(d + e*x)^2 + 2*a*b*tan(d + e*x))^(3
/2),x)
```

```
[Out] int((a + b*tan(d + e*x))/(b^2 + a^2*tan(d + e*x)^2 + 2*a*b*tan(d + e*x))^(3
/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a+b\tan(d+ex)}{\left((a\tan(d+ex)+b)^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(e*x+d))/(b**2+2*a*b*tan(e*x+d)+a**2*tan(e*x+d)**2)**(3/2
),x)
```

```
[Out] Integral((a + b*tan(d + e*x))/((a*tan(d + e*x) + b)**2)**(3/2), x)
```

3.518 $\int (a+b\sec(d+ex)) \left(b^2 + 2ab\sec(d + ex) + a^2 \sec^2(d + e$

Optimal. Leaf size=184

$$\frac{a^2b\left(41a^2+26b^2\right)\tan(d+ex)\sec(d+ex)}{24e}+\frac{\left(4a^2+7b^2\right)\tan(d+ex)\left(a^2\sec(d+ex)+ab\right)^2}{12ae}+\frac{b\tan(d+ex)\left(a^2\sec(\right.}{4a^2e}$$

```
[Out] a*b^4*x+1/8*b*(19*a^4+56*a^2*b^2+8*b^4)*arctanh(sin(e*x+d))/e+1/6*a*(4*a^4+
50*a^2*b^2+19*b^4)*tan(e*x+d)/e+1/24*a^2*b*(41*a^2+26*b^2)*sec(e*x+d)*tan(e
*x+d)/e+1/12*(4*a^2+7*b^2)*(a*b+a^2*sec(e*x+d))^2*tan(e*x+d)/a/e+1/4*b*(a*b
+a^2*sec(e*x+d))^3*tan(e*x+d)/a^2/e
```

Rubi [A] time = 0.43, antiderivative size = 184, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 39, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.180, Rules used = {4172, 3918, 4056, 4048, 3770, 3767, 8}

$$\frac{a\left(50a^2b^2+4a^4+19b^4\right)\tan(d+ex)}{6e}+\frac{b\left(56a^2b^2+19a^4+8b^4\right)\tanh^{-1}(\sin(d+ex))}{8e}+\frac{a^2b\left(41a^2+26b^2\right)\tan(d+ex}{24e}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[d + e*x])*(b^2 + 2*a*b*Sec[d + e*x] + a^2*Sec[d + e*x]^2)^2,
x]
```

```
[Out] a*b^4*x + (b*(19*a^4 + 56*a^2*b^2 + 8*b^4)*ArcTanh[Sin[d + e*x]])/(8*e) + (
a*(4*a^4 + 50*a^2*b^2 + 19*b^4)*Tan[d + e*x])/(6*e) + (a^2*b*(41*a^2 + 26*b
^2)*Sec[d + e*x]*Tan[d + e*x])/(24*e) + ((4*a^2 + 7*b^2)*(a*b + a^2*Sec[d +
 e*x])^2*Tan[d + e*x])/(12*a*e) + (b*(a*b + a^2*Sec[d + e*x])^3*Tan[d + e*x
])/(4*a^2*e)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3918

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d
_.) + (c_)), x_Symbol] :> -Simp[(b*d*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m -
 1))/(f*m), x] + Dist[1/m, Int[(a + b*Csc[e + f*x])^(m - 2)*Simp[a^2*c*m +
(b^2*d*(m - 1) + 2*a*b*c*m + a^2*d*m)*Csc[e + f*x] + b*(b*c*m + a*d*(2*m -
1))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0] && GtQ[m, 1] && NeQ[a^2 - b^2, 0] && IntegerQ[2*m]
```

Rule 4048

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbol] :> -Simp[(b*C*Csc[e +
f*x]*Cot[e + f*x])/(2*f), x] + Dist[1/2, Int[Simp[2*A*a + (2*B*a + b*(2*A +
 C))*Csc[e + f*x] + 2*(a*C + B*b)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b
, e, f, A, B, C}, x]
```

Rule 4056

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.), x_Symbol] :> -Simp[(C*Cot[
e + f*x]*(a + b*Csc[e + f*x])^m)/(f*(m + 1)), x] + Dist[1/(m + 1), Int[(a +
 b*Csc[e + f*x])^(m - 1)*Simp[a*A*(m + 1) + ((A*b + a*B)*(m + 1) + b*C*m)*C
sc[e + f*x] + (b*B*(m + 1) + a*C*m)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a,
 b, e, f, A, B, C}, x] && NeQ[a^2 - b^2, 0] && IGtQ[2*m, 0]
```

Rule 4172

```
Int[((A_) + (B_.)*sec[(d_.) + (e_.)*(x_)])*((a_) + (b_.)*sec[(d_.) + (e_.)*
(x_)] + (c_.)*sec[(d_.) + (e_.)*(x_)]^2)^(n_), x_Symbol] :> Dist[1/(4^n*c^n
), Int[(A + B*Sec[d + e*x])*(b + 2*c*Sec[d + e*x])^(2*n), x], x] /; FreeQ[{
a, b, c, d, e, A, B}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[n]
```

Rubi steps

$$\int (a+b\sec(d+ex))\left(b^2+2ab\sec(d+ex)+a^2\sec^2(d+ex)\right)^2\,dx = \frac{\int \left(2ab+2a^2\sec(d+ex)\right)^4 (a+b\sec(d+ex}{16a^4}$$

$$= \frac{b\left(ab+a^2\sec(d+ex)\right)^3\tan(d+ex)}{4a^2e} + \frac{\int \left(2a\right.}{}$$

$$= \frac{\left(4a^2+7b^2\right)\left(ab+a^2\sec(d+ex)\right)^2\tan(d+e}{12ae}$$

$$= \frac{a^2b\left(41a^2+26b^2\right)\sec(d+ex)\tan(d+ex)}{24e} +$$

$$= ab^4x + \frac{a^2b\left(41a^2+26b^2\right)\sec(d+ex)\tan(d}{24e}$$

$$= ab^4x + \frac{b\left(19a^4+56a^2b^2+8b^4\right)\tanh^{-1}(\sin(}{8e}$$

$$= ab^4x + \frac{b\left(19a^4+56a^2b^2+8b^4\right)\tanh^{-1}(\sin(}{8e}$$

Mathematica [A] time = 0.87, size = 130, normalized size = 0.71

$$\frac{3b\left(19a^4+56a^2b^2+8b^4\right)\tanh^{-1}(\sin(d+ex))+8a^3\left(a^2+4b^2\right)\tan^3(d+ex)+3a\tan(d+ex)\left(2a^3b\sec^3(d+ex)+\right.}{24e}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sec[d + e*x])*(b^2 + 2*a*b*Sec[d + e*x] + a^2*Sec[d + e*x]
^2)^2,x]
```

```
[Out] (24*a*b^4*e*x + 3*b*(19*a^4 + 56*a^2*b^2 + 8*b^4)*ArcTanh[Sin[d + e*x]] + 3
*a*(8*(a^4 + 10*a^2*b^2 + 4*b^4) + a*b*(19*a^2 + 24*b^2)*Sec[d + e*x] + 2*a
^3*b*Sec[d + e*x]^3)*Tan[d + e*x] + 8*a^3*(a^2 + 4*b^2)*Tan[d + e*x]^3)/(24
*e)
```

fricas [A] time = 2.43, size = 198, normalized size = 1.08

$$\frac{48\,ab^4ex\cos(ex+d)^4+3\left(19\,a^4b+56\,a^2b^3+8\,b^5\right)\cos(ex+d)^4\log(\sin(ex+d)+1)-3\left(19\,a^4b+56\,a^2b^3+8\,b\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(e*x+d))*(b^2+2*a*b*sec(e*x+d)+a^2*sec(e*x+d)^2)^2,x, alg
orithm="fricas")
```

```
[Out] 1/48*(48*a*b^4*e*x*cos(e*x + d)^4 + 3*(19*a^4*b + 56*a^2*b^3 + 8*b^5)*cos(e
*x + d)^4*log(sin(e*x + d) + 1) - 3*(19*a^4*b + 56*a^2*b^3 + 8*b^5)*cos(e*x
 + d)^4*log(-sin(e*x + d) + 1) + 2*(6*a^4*b + 16*(a^5 + 13*a^3*b^2 + 6*a*b^
4)*cos(e*x + d)^3 + 3*(19*a^4*b + 24*a^2*b^3)*cos(e*x + d)^2 + 8*(a^5 + 4*a
^3*b^2)*cos(e*x + d))*sin(e*x + d))/(e*cos(e*x + d)^4)
```

giac [B] time = 0.39, size = 470, normalized size = 2.55

$$\frac{1}{24}\left(24\,(xe+d)ab^4+3\left(19\,a^4b+56\,a^2b^3+8\,b^5\right)\log\left(\left|\tan\left(\frac{1}{2}\,xe+\frac{1}{2}\,d\right)+1\right|\right)-3\left(19\,a^4b+56\,a^2b^3+8\,b^5\right)\log\left(\right|\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(e*x+d))*(b^2+2*a*b*sec(e*x+d)+a^2*sec(e*x+d)^2)^2,x, alg
orithm="giac")
```

```
[Out] 1/24*(24*(x*e + d)*a*b^4 + 3*(19*a^4*b + 56*a^2*b^3 + 8*b^5)*log(abs(tan(1/
2*x*e + 1/2*d) + 1)) - 3*(19*a^4*b + 56*a^2*b^3 + 8*b^5)*log(abs(tan(1/2*x*
e + 1/2*d) - 1)) - 2*(24*a^5*tan(1/2*x*e + 1/2*d)^7 - 63*a^4*b*tan(1/2*x*e
+ 1/2*d)^7 + 240*a^3*b^2*tan(1/2*x*e + 1/2*d)^7 - 72*a^2*b^3*tan(1/2*x*e +
1/2*d)^7 + 96*a*b^4*tan(1/2*x*e + 1/2*d)^7 - 40*a^5*tan(1/2*x*e + 1/2*d)^5
+ 39*a^4*b*tan(1/2*x*e + 1/2*d)^5 - 592*a^3*b^2*tan(1/2*x*e + 1/2*d)^5 + 72
*a^2*b^3*tan(1/2*x*e + 1/2*d)^5 - 288*a*b^4*tan(1/2*x*e + 1/2*d)^5 + 40*a^5
*tan(1/2*x*e + 1/2*d)^3 + 39*a^4*b*tan(1/2*x*e + 1/2*d)^3 + 592*a^3*b^2*tan
(1/2*x*e + 1/2*d)^3 + 72*a^2*b^3*tan(1/2*x*e + 1/2*d)^3 + 288*a*b^4*tan(1/2
*x*e + 1/2*d)^3 - 24*a^5*tan(1/2*x*e + 1/2*d) - 63*a^4*b*tan(1/2*x*e + 1/2*
d) - 240*a^3*b^2*tan(1/2*x*e + 1/2*d) - 72*a^2*b^3*tan(1/2*x*e + 1/2*d) - 9
6*a*b^4*tan(1/2*x*e + 1/2*d))/(tan(1/2*x*e + 1/2*d)^2 - 1)^4)*e^(-1)
```

maple [A] time = 0.14, size = 246, normalized size = 1.34

$$a\,b^4x+\frac{a\,b^4d}{e}+\frac{7a^2b^3\ln\left(\sec\left(ex+d\right)+\tan\left(ex+d\right)\right)}{e}+\frac{26a^3b^2\tan\left(ex+d\right)}{3e}+\frac{19a^4b\sec\left(ex+d\right)\tan\left(ex+d\right)}{8e}+\frac{19a^4b}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(e*x+d))*(b^2+2*a*b*sec(e*x+d)+a^2*sec(e*x+d)^2)^2,x)
```

```
[Out] a*b^4*x+1/e*a*b^4*d+7/e*a^2*b^3*ln(sec(e*x+d)+tan(e*x+d))+26/3/e*a^3*b^2*ta
n(e*x+d)+19/8/e*a^4*b*sec(e*x+d)*tan(e*x+d)+19/8/e*a^4*b*ln(sec(e*x+d)+tan(
e*x+d))+2/3/e*a^5*tan(e*x+d)+1/3/e*a^5*tan(e*x+d)*sec(e*x+d)^2+1/e*b^5*ln(s
ec(e*x+d)+tan(e*x+d))+4/e*a*b^4*tan(e*x+d)+3/e*a^2*b^3*sec(e*x+d)*tan(e*x+d
)+4/3/e*a^3*b^2*tan(e*x+d)*sec(e*x+d)^2+1/4/e*a^4*b*tan(e*x+d)*sec(e*x+d)^3
```

maxima [A] time = 0.34, size = 299, normalized size = 1.62

$$\frac{16\left(\tan\left(ex+d\right)^{3}+3\,\tan\left(ex+d\right)\right)a^{5}+64\left(\tan\left(ex+d\right)^{3}+3\,\tan\left(ex+d\right)\right)a^{3}b^{2}+48\,(ex+d)ab^{4}-3\,a^{4}b\left(\frac{2\left(3\,\sin(ex\cdots}{\sin(ex+d)^{4}\cdots}\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(e*x+d))*(b^2+2*a*b*sec(e*x+d)+a^2*sec(e*x+d)^2)^2,x, alg
orithm="maxima")
```

```
[Out] 1/48*(16*(tan(e*x + d)^3 + 3*tan(e*x + d))*a^5 + 64*(tan(e*x + d)^3 + 3*tan
(e*x + d))*a^3*b^2 + 48*(e*x + d)*a*b^4 - 3*a^4*b*(2*(3*sin(e*x + d)^3 - 5*
sin(e*x + d))/(sin(e*x + d)^4 - 2*sin(e*x + d)^2 + 1) - 3*log(sin(e*x + d)
+ 1) + 3*log(sin(e*x + d) - 1)) - 48*a^4*b*(2*sin(e*x + d)/(sin(e*x + d)^2
- 1) - log(sin(e*x + d) + 1) + log(sin(e*x + d) - 1)) - 72*a^2*b^3*(2*sin(e
*x + d)/(sin(e*x + d)^2 - 1) - log(sin(e*x + d) + 1) + log(sin(e*x + d) - 1
)) + 192*a^2*b^3*log(sec(e*x + d) + tan(e*x + d)) + 48*b^5*log(sec(e*x + d)
 + tan(e*x + d)) + 288*a^3*b^2*tan(e*x + d) + 192*a*b^4*tan(e*x + d))/e
```

mupad [B] time = 3.30, size = 323, normalized size = 1.76

$$\frac{2\,a^5\,\sin\left(d+e\,x\right)}{3\,e\,\cos\left(d+e\,x\right)}+\frac{a^5\,\sin\left(d+e\,x\right)}{3\,e\,{\cos\left(d+e\,x\right)}^3}+\frac{2\,a\,b^4\,\mathrm{atan}\left(\frac{\sin\left(\frac{d}{2}+\frac{e\,x}{2}\right)}{\cos\left(\frac{d}{2}+\frac{e\,x}{2}\right)}\right)}{e}+\frac{4\,a\,b^4\,\sin\left(d+e\,x\right)}{e\,\cos\left(d+e\,x\right)}+\frac{19\,a^4\,b\,\sin\left(d+e\,x\right)}{8\,e\,{\cos\left(d+e\,x\right)}^2}+\frac{a^4\,b\,\sin\left(\cdots\right.}{4\,e\,\cos\left(\cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(d + e*x))*(b^2 + a^2/cos(d + e*x)^2 + (2*a*b)/cos(d + e*x))^
2,x)
```

```
[Out] (2*a^5*sin(d + e*x))/(3*e*cos(d + e*x)) - (b^5*atan((sin(d/2 + (e*x)/2)*1i)
/cos(d/2 + (e*x)/2))*2i)/e + (a^5*sin(d + e*x))/(3*e*cos(d + e*x)^3) - (a^2
*b^3*atan((sin(d/2 + (e*x)/2)*1i)/cos(d/2 + (e*x)/2))*14i)/e + (2*a*b^4*ata
n(sin(d/2 + (e*x)/2)/cos(d/2 + (e*x)/2)))/e - (a^4*b*atan((sin(d/2 + (e*x)/
2)*1i)/cos(d/2 + (e*x)/2))*19i)/(4*e) + (4*a*b^4*sin(d + e*x))/(e*cos(d + e
*x)) + (19*a^4*b*sin(d + e*x))/(8*e*cos(d + e*x)^2) + (a^4*b*sin(d + e*x))/
(4*e*cos(d + e*x)^4) + (26*a^3*b^2*sin(d + e*x))/(3*e*cos(d + e*x)) + (3*a^
2*b^3*sin(d + e*x))/(e*cos(d + e*x)^2) + (4*a^3*b^2*sin(d + e*x))/(3*e*cos(
d + e*x)^3)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a + b\sec\left(d + ex\right)\right)\left(a\sec\left(d + ex\right) + b\right)^{4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(e*x+d))*(b**2+2*a*b*sec(e*x+d)+a**2*sec(e*x+d)**2)**2,x)
```

```
[Out] Integral((a + b*sec(d + e*x))*(a*sec(d + e*x) + b)**4, x)
```

3.519 $\int (a+b\sec(d+ex))\left(b^2+2ab\sec(d+ex)+a^2\sec^2(d+e$

Optimal. Leaf size=76

$$\frac{a\left(a^2+2b^2\right)\tan(d+ex)}{e}+\frac{b\left(5a^2+2b^2\right)\tanh^{-1}(\sin(d+ex))}{2e}+\frac{a^2b\tan(d+ex)\sec(d+ex)}{2e}+ab^2x$$

```
[Out] a*b^2*x+1/2*b*(5*a^2+2*b^2)*arctanh(sin(e*x+d))/e+a*(a^2+2*b^2)*tan(e*x+d)/
e+1/2*a^2*b*sec(e*x+d)*tan(e*x+d)/e
```

Rubi [A] time = 0.08, antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.108, Rules used = {4048, 3770, 3767, 8}

$$\frac{a\left(a^2+2b^2\right)\tan(d+ex)}{e}+\frac{b\left(5a^2+2b^2\right)\tanh^{-1}(\sin(d+ex))}{2e}+\frac{a^2b\tan(d+ex)\sec(d+ex)}{2e}+ab^2x$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[d + e*x])*(b^2 + 2*a*b*Sec[d + e*x] + a^2*Sec[d + e*x]^2),x]
```

```
[Out] a*b^2*x + (b*(5*a^2 + 2*b^2)*ArcTanh[Sin[d + e*x]])/(2*e) + (a*(a^2 + 2*b^2
)*Tan[d + e*x])/e + (a^2*b*Sec[d + e*x]*Tan[d + e*x])/(2*e)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 4048

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbol] :> -Simp[(b*C*Csc[e +
f*x]*Cot[e + f*x])/(2*f), x] + Dist[1/2, Int[Simp[2*A*a + (2*B*a + b*(2*A +
 C))*Csc[e + f*x] + 2*(a*C + B*b)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b
```

```
, e, f, A, B, C}, x]
```

Rubi steps

$$\int (a+b\sec(d+ex))\left(b^2+2ab\sec(d+ex)+a^2\sec^2(d+ex)\right)\,dx = \frac{a^2b\sec(d+ex)\tan(d+ex)}{2e}+\frac{1}{2}\int\left(2ab^2+\right.$$
$$= ab^2x+\frac{a^2b\sec(d+ex)\tan(d+ex)}{2e}+\left(a\left(a^2\right.\right.$$
$$= ab^2x+\frac{b\left(5a^2+2b^2\right)\tanh^{-1}(\sin(d+ex))}{2e}+$$
$$= ab^2x+\frac{b\left(5a^2+2b^2\right)\tanh^{-1}(\sin(d+ex))}{2e}+$$

Mathematica [A] time = 0.28, size = 64, normalized size = 0.84

$$\frac{b\left(5a^2+2b^2\right)\tanh^{-1}(\sin(d+ex))+a\tan(d+ex)\left(2a^2+ab\sec(d+ex)+4b^2\right)+2ab^2ex}{2e}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sec[d + e*x])*(b^2 + 2*a*b*Sec[d + e*x] + a^2*Sec[d + e*x]
^2),x]
```

```
[Out] (2*a*b^2*e*x + b*(5*a^2 + 2*b^2)*ArcTanh[Sin[d + e*x]] + a*(2*a^2 + 4*b^2 +
 a*b*Sec[d + e*x])*Tan[d + e*x])/(2*e)
```

fricas [A] time = 1.50, size = 125, normalized size = 1.64

$$\frac{4\,ab^2ex\cos(ex+d)^2+\left(5\,a^2b+2\,b^3\right)\cos(ex+d)^2\log(\sin(ex+d)+1)-\left(5\,a^2b+2\,b^3\right)\cos(ex+d)^2\log(-\sin}{4\,e\cos(ex+d)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(e*x+d))*(b^2+2*a*b*sec(e*x+d)+a^2*sec(e*x+d)^2),x, algor
ithm="fricas")
```

```
[Out] 1/4*(4*a*b^2*e*x*cos(e*x + d)^2 + (5*a^2*b + 2*b^3)*cos(e*x + d)^2*log(sin(
e*x + d) + 1) - (5*a^2*b + 2*b^3)*cos(e*x + d)^2*log(-sin(e*x + d) + 1) + 2
*(a^2*b + 2*(a^3 + 2*a*b^2)*cos(e*x + d))*sin(e*x + d))/(e*cos(e*x + d)^2)
```

giac [B] time = 0.25, size = 191, normalized size = 2.51

$$\frac{1}{2}\left(2\,(xe+d)ab^2+\left(5\,a^2b+2\,b^3\right)\log\left(\left|\tan\left(\frac{1}{2}\,xe+\frac{1}{2}\,d\right)+1\right|\right)-\left(5\,a^2b+2\,b^3\right)\log\left(\left|\tan\left(\frac{1}{2}\,xe+\frac{1}{2}\,d\right)-1\right|\right)-\frac{2\left(2\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(e*x+d))*(b^2+2*a*b*sec(e*x+d)+a^2*sec(e*x+d)^2),x, algor
ithm="giac")
```

```
[Out] 1/2*(2*(x*e + d)*a*b^2 + (5*a^2*b + 2*b^3)*log(abs(tan(1/2*x*e + 1/2*d) + 1
)) - (5*a^2*b + 2*b^3)*log(abs(tan(1/2*x*e + 1/2*d) - 1)) - 2*(2*a^3*tan(1/
2*x*e + 1/2*d)^3 - a^2*b*tan(1/2*x*e + 1/2*d)^3 + 4*a*b^2*tan(1/2*x*e + 1/2
*d)^3 - 2*a^3*tan(1/2*x*e + 1/2*d) - a^2*b*tan(1/2*x*e + 1/2*d) - 4*a*b^2*t
an(1/2*x*e + 1/2*d))/(tan(1/2*x*e + 1/2*d)^2 - 1)^2)*e^(-1)
```

maple [A] time = 0.09, size = 110, normalized size = 1.45

$$a\,b^2x+\frac{a\,b^2d}{e}+\frac{5a^2b\ln\left(\sec\left(ex+d\right)+\tan\left(ex+d\right)\right)}{2e}+\frac{a^3\tan\left(ex+d\right)}{e}+\frac{b^3\ln\left(\sec\left(ex+d\right)+\tan\left(ex+d\right)\right)}{e}+\frac{2a\,b^2\tan}{e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(e*x+d))*(b^2+2*a*b*sec(e*x+d)+a^2*sec(e*x+d)^2),x)
```

```
[Out] a*b^2*x+1/e*a*b^2*d+5/2/e*a^2*b*ln(sec(e*x+d)+tan(e*x+d))+1/e*a^3*tan(e*x+d
)+1/e*b^3*ln(sec(e*x+d)+tan(e*x+d))+2*a*b^2*tan(e*x+d)/e+1/2*a^2*b*sec(e*x+
d)*tan(e*x+d)/e
```

maxima [A] time = 0.34, size = 126, normalized size = 1.66

$$\frac{4\,(ex+d)ab^2-a^2b\left(\frac{2\,\sin(ex+d)}{\sin(ex+d)^2-1}-\log\left(\sin\left(ex+d\right)+1\right)+\log\left(\sin\left(ex+d\right)-1\right)\right)+8\,a^2b\log\left(\sec\left(ex+d\right)+\tan\left(ex\right.\right.}{4\,e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(e*x+d))*(b^2+2*a*b*sec(e*x+d)+a^2*sec(e*x+d)^2),x, algor
ithm="maxima")
```

```
[Out] 1/4*(4*(e*x + d)*a*b^2 - a^2*b*(2*sin(e*x + d)/(sin(e*x + d)^2 - 1) - log(s
in(e*x + d) + 1) + log(sin(e*x + d) - 1)) + 8*a^2*b*log(sec(e*x + d) + tan(
e*x + d)) + 4*b^3*log(sec(e*x + d) + tan(e*x + d)) + 4*a^3*tan(e*x + d) + 8
*a*b^2*tan(e*x + d))/e
```

mupad [B] time = 2.91, size = 160, normalized size = 2.11

$$\frac{2\,b^3\,\mathrm{atanh}\left(\frac{\sin\left(\frac{d}{2}+\frac{e\,x}{2}\right)}{\cos\left(\frac{d}{2}+\frac{e\,x}{2}\right)}\right)}{e}+\frac{a^3\,\sin\left(d+e\,x\right)}{e\,\cos\left(d+e\,x\right)}+\frac{2\,a\,b^2\,\mathrm{atan}\left(\frac{\sin\left(\frac{d}{2}+\frac{e\,x}{2}\right)}{\cos\left(\frac{d}{2}+\frac{e\,x}{2}\right)}\right)}{e}+\frac{5\,a^2\,b\,\mathrm{atanh}\left(\frac{\sin\left(\frac{d}{2}+\frac{e\,x}{2}\right)}{\cos\left(\frac{d}{2}+\frac{e\,x}{2}\right)}\right)}{e}+\frac{2\,a\,b^2\,\sin\left(d+e\,x\right)}{e\,\cos\left(d+e\,x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(d + e*x))*(b^2 + a^2/cos(d + e*x)^2 + (2*a*b)/cos(d + e*x)),
x)
```

```
[Out] (2*b^3*atanh(sin(d/2 + (e*x)/2)/cos(d/2 + (e*x)/2)))/e + (a^3*sin(d + e*x))
/(e*cos(d + e*x)) + (2*a*b^2*atan(sin(d/2 + (e*x)/2)/cos(d/2 + (e*x)/2)))/e
 + (5*a^2*b*atanh(sin(d/2 + (e*x)/2)/cos(d/2 + (e*x)/2)))/e + (2*a*b^2*sin(
d + e*x))/(e*cos(d + e*x)) + (a^2*b*sin(d + e*x))/(2*e*cos(d + e*x)^2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a+b\sec(d+ex))\,(a\sec(d+ex)+b)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(e*x+d))*(b**2+2*a*b*sec(e*x+d)+a**2*sec(e*x+d)**2),x)
```

```
[Out] Integral((a + b*sec(d + e*x))*(a*sec(d + e*x) + b)**2, x)
```

3.520 $\int \frac{a+b\sec(d+ex)}{b^2+2ab\sec(d+ex)+a^2\sec^2(d+ex)}\,dx$

Optimal. Leaf size=92

$$-\frac{a^2\tan(d+ex)}{be\left(a^2\sec(d+ex)+ab\right)}-\frac{2\sqrt{a-b}\,\sqrt{a+b}\,\tan^{-1}\left(\frac{\sqrt{a-b}\,\tan\left(\frac{1}{2}(d+ex)\right)}{\sqrt{a+b}}\right)}{b^2e}+\frac{ax}{b^2}$$

```
[Out] a*x/b^2-2*arctan((a-b)^(1/2)*tan(1/2*e*x+1/2*d)/(a+b)^(1/2))*(a-b)^(1/2)*(a
+b)^(1/2)/b^2/e-a^2*tan(e*x+d)/b/e/(a*b+a^2*sec(e*x+d))
```

Rubi [A] time = 0.30, antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 39, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {4172, 3923, 3919, 3831, 2659, 205}

$$-\frac{a^2\tan(d+ex)}{be\left(a^2\sec(d+ex)+ab\right)}-\frac{2\sqrt{a-b}\,\sqrt{a+b}\,\tan^{-1}\left(\frac{\sqrt{a-b}\,\tan\left(\frac{1}{2}(d+ex)\right)}{\sqrt{a+b}}\right)}{b^2e}+\frac{ax}{b^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[d + e*x])/(b^2 + 2*a*b*Sec[d + e*x] + a^2*Sec[d + e*x]^2),x]
```

```
[Out] (a*x)/b^2 - (2*Sqrt[a - b]*Sqrt[a + b]*ArcTan[(Sqrt[a - b]*Tan[(d + e*x)/2]
)/Sqrt[a + b]])/(b^2*e) - (a^2*Tan[d + e*x])/(b*e*(a*b + a^2*Sec[d + e*x]))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 3831

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Dist[1/b, Int[1/(1 + (a*Sin[e + f*x])/b), x], x] /; FreeQ[{a, b, e, f
}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3919

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
 (a_)), x_Symbol] :> Simp[(c*x)/a, x] - Dist[(b*c - a*d)/a, Int[Csc[e + f*x
]/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0]
```

Rule 3923

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d
_.) + (c_)), x_Symbol] :> Simp[(b*(b*c - a*d)*Cot[e + f*x]*(a + b*Csc[e + f
*x])^(m + 1))/(a*f*(m + 1)*(a^2 - b^2)), x] + Dist[1/(a*(m + 1)*(a^2 - b^2)
), Int[(a + b*Csc[e + f*x])^(m + 1)*Simp[c*(a^2 - b^2)*(m + 1) - (a*(b*c -
a*d)*(m + 1))*Csc[e + f*x] + b*(b*c - a*d)*(m + 2)*Csc[e + f*x]^2, x], x],
x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] && Ne
Q[a^2 - b^2, 0] && IntegerQ[2*m]
```

Rule 4172

```
Int[((A_) + (B_.)*sec[(d_.) + (e_.)*(x_)])*((a_) + (b_.)*sec[(d_.) + (e_.)*
(x_)] + (c_.)*sec[(d_.) + (e_.)*(x_)]^2)^(n_), x_Symbol] :> Dist[1/(4^n*c^n
), Int[(A + B*Sec[d + e*x])*(b + 2*c*Sec[d + e*x])^(2*n), x], x] /; FreeQ[{
a, b, c, d, e, A, B}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[n]
```

Rubi steps

$$\int \frac{a+b\sec(d+ex)}{b^2+2ab\sec(d+ex)+a^2\sec^2(d+ex)}\,dx = \left(4a^2\right)\int \frac{a+b\sec(d+ex)}{\left(2ab+2a^2\sec(d+ex)\right)^2}\,dx$$

$$= -\frac{a^2\tan(d+ex)}{be\left(ab+a^2\sec(d+ex)\right)} + \frac{\int \frac{4a^3\left(a^2-b^2\right)+4a^2b\left(a^2-b^2\right)\sec(d+ex)}{2ab+2a^2\sec(d+ex)}\,dx}{2ab\left(a^2-b^2\right)}$$

$$= \frac{ax}{b^2} - \frac{a^2\tan(d+ex)}{be\left(ab+a^2\sec(d+ex)\right)} - \frac{\left(2a\left(a^2-b^2\right)\right)\int \frac{\sec(d+ex)}{2ab+2a^2\sec(d+ex)}\,dx}{b^2}$$

$$= \frac{ax}{b^2} - \frac{a^2\tan(d+ex)}{be\left(ab+a^2\sec(d+ex)\right)} - \frac{\left(a^2-b^2\right)\int \frac{1}{1+\frac{b\cos(d+ex)}{a}}\,dx}{ab^2}$$

$$= \frac{ax}{b^2} - \frac{a^2\tan(d+ex)}{be\left(ab+a^2\sec(d+ex)\right)} - \frac{\left(2\left(a^2-b^2\right)\right)\text{Subst}\left(\int \frac{1}{1+\frac{b}{a}+\left(1-\frac{b}{a}\right)x}\right.}{ab^2e}$$

$$= \frac{ax}{b^2} - \frac{2\sqrt{a-b}\,\sqrt{a+b}\,\tan^{-1}\left(\frac{\sqrt{a-b}\,\tan\left(\frac{1}{2}(d+ex)\right)}{\sqrt{a+b}}\right)}{b^2e} - \frac{a^2\tan(d+e}{be\left(ab+a^2\sec(d\right.}$$

Mathematica [A] time = 0.39, size = 97, normalized size = 1.05

$$\frac{2\sqrt{b^2-a^2}\,\tanh^{-1}\left(\frac{(b-a)\tan\left(\frac{1}{2}(d+ex)\right)}{\sqrt{b^2-a^2}}\right)+\frac{a(ad+aex-b\sin(d+ex)+b(d+ex)\cos(d+ex))}{a+b\cos(d+ex)}}{b^2e}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sec[d + e*x])/(b^2 + 2*a*b*Sec[d + e*x] + a^2*Sec[d + e*x]
^2),x]
```

```
[Out] (2*Sqrt[-a^2 + b^2]*ArcTanh[((-a + b)*Tan[(d + e*x)/2])/Sqrt[-a^2 + b^2]] +
 (a*(a*d + a*e*x + b*(d + e*x)*Cos[d + e*x] - b*Sin[d + e*x]))/(a + b*Cos[d
 + e*x]))/(b^2*e)
```

fricas [A] time = 0.85, size = 279, normalized size = 3.03

$$\left[\frac{2\,abex\cos(ex+d)+2\,a^2ex-2\,ab\sin(ex+d)+\sqrt{-a^2+b^2}\,(b\cos(ex+d)+a)\log\left(\frac{2\,ab\cos(ex+d)+\left(2\,a^2-b^2\right)\cos(ex+d)^2}{b^2\cos(ex+}\right.}{2\left(b^3e\cos(ex+d)+ab^2e\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(e*x+d))/(b^2+2*a*b*sec(e*x+d)+a^2*sec(e*x+d)^2),x, algor
ithm="fricas")
```

```
[Out] [1/2*(2*a*b*e*x*cos(e*x + d) + 2*a^2*e*x - 2*a*b*sin(e*x + d) + sqrt(-a^2 +
 b^2)*(b*cos(e*x + d) + a)*log((2*a*b*cos(e*x + d) + (2*a^2 - b^2)*cos(e*x
+ d)^2 + 2*sqrt(-a^2 + b^2)*(a*cos(e*x + d) + b)*sin(e*x + d) - a^2 + 2*b^2
)/(b^2*cos(e*x + d)^2 + 2*a*b*cos(e*x + d) + a^2)))/(b^3*e*cos(e*x + d) + a
*b^2*e), (a*b*e*x*cos(e*x + d) + a^2*e*x - a*b*sin(e*x + d) - sqrt(a^2 - b^
2)*(b*cos(e*x + d) + a)*arctan(-(a*cos(e*x + d) + b)/(sqrt(a^2 - b^2)*sin(e
*x + d))))/(b^3*e*cos(e*x + d) + a*b^2*e)]
```

giac [A] time = 0.28, size = 145, normalized size = 1.58

$$\left(\frac{(xe+d)a}{b^2} - \frac{2\,a\tan\left(\frac{1}{2}\,xe + \frac{1}{2}\,d\right)}{\left(a\tan\left(\frac{1}{2}\,xe + \frac{1}{2}\,d\right)^2 - b\tan\left(\frac{1}{2}\,xe + \frac{1}{2}\,d\right)^2 + a + b\right)b} - \frac{2\left(\pi\left\lfloor\frac{xe+d}{2\,\pi} + \frac{1}{2}\right\rfloor\operatorname{sgn}\left(2\,a - 2\,b\right) + \arctan\left(\frac{a\tan}{\ }\right.\right.}{b^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(e*x+d))/(b^2+2*a*b*sec(e*x+d)+a^2*sec(e*x+d)^2),x, algor
ithm="giac")
```

```
[Out] ((x*e + d)*a/b^2 - 2*a*tan(1/2*x*e + 1/2*d)/((a*tan(1/2*x*e + 1/2*d)^2 - b*
tan(1/2*x*e + 1/2*d)^2 + a + b)*b) - 2*(pi*floor(1/2*(x*e + d)/pi + 1/2)*sg
n(2*a - 2*b) + arctan((a*tan(1/2*x*e + 1/2*d) - b*tan(1/2*x*e + 1/2*d))/sqr
t(a^2 - b^2)))*sqrt(a^2 - b^2)/b^2)*e^(-1)
```

maple [A] time = 0.29, size = 163, normalized size = 1.77

$$-\frac{2\tan\left(\frac{d}{2}+\frac{ex}{2}\right)a}{eb\left(a\left(\tan^2\left(\frac{d}{2}+\frac{ex}{2}\right)\right) - b\left(\tan^2\left(\frac{d}{2}+\frac{ex}{2}\right)\right) + a + b\right)} - \frac{2\arctan\left(\frac{\tan\left(\frac{d}{2}+\frac{ex}{2}\right)(a-b)}{\sqrt{(a+b)(a-b)}}\right)a^2}{e\,b^2\sqrt{(a+b)\,(a-b)}} + \frac{2\arctan\left(\frac{\tan\left(\frac{d}{2}+\frac{ex}{2}\right)(a-b)}{\sqrt{(a+b)(a-b)}}\right)}{e\sqrt{(a+b)\,(a-b)}} + \frac{2a}{\ }$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(e*x+d))/(b^2+2*a*b*sec(e*x+d)+a^2*sec(e*x+d)^2),x)
```

```
[Out] -2/e/b*tan(1/2*d+1/2*e*x)*a/(a*tan(1/2*d+1/2*e*x)^2-b*tan(1/2*d+1/2*e*x)^2+
a+b)-2/e/b^2/((a+b)*(a-b))^(1/2)*arctan(tan(1/2*d+1/2*e*x)*(a-b)/((a+b)*(a-
b))^(1/2))*a^2+2/e/((a+b)*(a-b))^(1/2)*arctan(tan(1/2*d+1/2*e*x)*(a-b)/((a+
b)*(a-b))^(1/2))+2/e*a/b^2*arctan(tan(1/2*d+1/2*e*x))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(e*x+d))/(b^2+2*a*b*sec(e*x+d)+a^2*sec(e*x+d)^2),x, algor
ithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more details)Is 4*b^2-4*a^2 positive or negative?
```

mupad [B] time = 3.02, size = 444, normalized size = 4.83

$$\frac{2\,\mathrm{atanh}\left(\frac{64\,a^3\tan\left(\frac{d}{2}+\frac{e\,x}{2}\right)\sqrt{b^2-a^2}}{64\,a^4-128\,a^3\,b+128\,a\,b^3-64\,b^4}-\frac{192\,a^2\tan\left(\frac{d}{2}+\frac{e\,x}{2}\right)\sqrt{b^2-a^2}}{128\,a\,b^2-128\,a^3-64\,b^3+\frac{64\,a^4}{b}}+\frac{192\,a\tan\left(\frac{d}{2}+\frac{e\,x}{2}\right)\sqrt{b^2-a^2}}{128\,a\,b-64\,b^2-\frac{128\,a^3}{b}+\frac{64\,a^4}{b^2}}-\frac{64\,b\tan\left(\frac{d}{2}+\frac{e\,x}{2}\right)\sqrt{b^2-a^2}}{128\,a\,b-64\,b^2-\frac{128\,a^3}{b}+\frac{64\,a^4}{b^2}}\right)\sqrt{b^2-}}{b^2\,e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(d + e*x))/(b^2 + a^2/cos(d + e*x)^2 + (2*a*b)/cos(d + e*x)),
x)
```

```
[Out] (2*atanh((64*a^3*tan(d/2 + (e*x)/2)*(b^2 - a^2)^(1/2))/(128*a*b^3 - 128*a^3
*b + 64*a^4 - 64*b^4) - (192*a^2*tan(d/2 + (e*x)/2)*(b^2 - a^2)^(1/2))/(128
*a*b^2 - 128*a^3 - 64*b^3 + (64*a^4)/b) + (192*a*tan(d/2 + (e*x)/2)*(b^2 -
a^2)^(1/2))/(128*a*b - 64*b^2 - (128*a^3)/b + (64*a^4)/b^2) - (64*b*tan(d/2
 + (e*x)/2)*(b^2 - a^2)^(1/2))/(128*a*b - 64*b^2 - (128*a^3)/b + (64*a^4)/b
^2))*(b^2 - a^2)^(1/2))/(b^2*e) - (2*a*atan((64*a^2*tan(d/2 + (e*x)/2))/(64
*a*b - 64*a^2 - (64*a^3)/b + (64*a^4)/b^2) + (64*a^3*tan(d/2 + (e*x)/2))/(6
4*a*b^2 - 64*a^2*b - 64*a^3 + (64*a^4)/b) - (64*a^4*tan(d/2 + (e*x)/2))/(64
*a*b^3 - 64*a^3*b + 64*a^4 - 64*a^2*b^2) - (64*a*b*tan(d/2 + (e*x)/2))/(64*
a*b - 64*a^2 - (64*a^3)/b + (64*a^4)/b^2)))/(b^2*e) - (2*a*tan(d/2 + (e*x)/
2))/(b*e*(a + b + tan(d/2 + (e*x)/2)^2*(a - b)))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a + b\sec(d + ex)}{(a\sec(d + ex) + b)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(e*x+d))/(b**2+2*a*b*sec(e*x+d)+a**2*sec(e*x+d)**2),x)
```

```
[Out] Integral((a + b*sec(d + e*x))/(a*sec(d + e*x) + b)**2, x)
```

3.521 $\int \frac{a+b\sec(d+ex)}{\left(b^2+2ab\sec(d+ex)+a^2\sec^2(d+ex)\right)^2}\,dx$

Optimal. Leaf size=230

$$-\frac{a\left(3a^2-5b^2\right)\tan(d+ex)}{6b^2e\left(a^2-b^2\right)(a\sec(d+ex)+b)^2}-\frac{\left(a^2-2b^2\right)\left(2a^4-a^2b^2+b^4\right)\tan^{-1}\left(\frac{\sqrt{a-b}\,\tan\left(\frac{1}{2}(d+ex)\right)}{\sqrt{a+b}}\right)}{b^4e(a-b)^{5/2}(a+b)^{5/2}}-\frac{a\left(6a^4-11a^2b^2+1\ldots\right.}{6b^3e\left(a^2-b^2\right)^2(a\,\mathrm{s}\ldots}$$

```
[Out] a*x/b^4-(a^2-2*b^2)*(2*a^4-a^2*b^2+b^4)*arctan((a-b)^(1/2)*tan(1/2*e*x+1/2*
d)/(a+b)^(1/2))/(a-b)^(5/2)/b^4/(a+b)^(5/2)/e-1/6*a*(3*a^2-5*b^2)*tan(e*x+d
)/b^2/(a^2-b^2)/e/(b+a*sec(e*x+d))^2-1/6*a*(6*a^4-11*a^2*b^2+11*b^4)*tan(e*
x+d)/b^3/(a^2-b^2)^2/e/(b+a*sec(e*x+d))-1/3*a^4*tan(e*x+d)/b/e/(a*b+a^2*sec
(e*x+d))^3
```

Rubi [A] time = 0.83, antiderivative size = 230, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 39, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.180, Rules used = {4172, 3923, 4060, 3919, 3831, 2659, 205}

$$-\frac{\left(a^2-2b^2\right)\left(-a^2b^2+2a^4+b^4\right)\tan^{-1}\left(\frac{\sqrt{a-b}\,\tan\left(\frac{1}{2}(d+ex)\right)}{\sqrt{a+b}}\right)}{b^4e(a-b)^{5/2}(a+b)^{5/2}}-\frac{a\left(-11a^2b^2+6a^4+11b^4\right)\tan(d+ex)}{6b^3e\left(a^2-b^2\right)^2(a\sec(d+ex)+b)}-\frac{a\left(3a^2-5\ldots\right.}{6b^2e\left(a^2-b^2\right)\ldots}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[d + e*x])/(b^2 + 2*a*b*Sec[d + e*x] + a^2*Sec[d + e*x]^2)^2,
x]
```

```
[Out] (a*x)/b^4 - ((a^2 - 2*b^2)*(2*a^4 - a^2*b^2 + b^4)*ArcTan[(Sqrt[a - b]*Tan[
(d + e*x)/2])/Sqrt[a + b]])/((a - b)^(5/2)*b^4*(a + b)^(5/2)*e) - (a*(3*a^2
 - 5*b^2)*Tan[d + e*x])/(6*b^2*(a^2 - b^2)*e*(b + a*Sec[d + e*x])^2) - (a*(
6*a^4 - 11*a^2*b^2 + 11*b^4)*Tan[d + e*x])/(6*b^3*(a^2 - b^2)^2*e*(b + a*Se
c[d + e*x])) - (a^4*Tan[d + e*x])/(3*b*e*(a*b + a^2*Sec[d + e*x])^3)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
```

```
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 3831

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Dist[1/b, Int[1/(1 + (a*Sin[e + f*x])/b), x], x] /; FreeQ[{a, b, e, f
}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3919

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
 (a_)), x_Symbol] :> Simp[(c*x)/a, x] - Dist[(b*c - a*d)/a, Int[Csc[e + f*x
]/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0]
```

Rule 3923

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d
_.) + (c_)), x_Symbol] :> Simp[(b*(b*c - a*d)*Cot[e + f*x]*(a + b*Csc[e + f
*x])^(m + 1))/(a*f*(m + 1)*(a^2 - b^2)), x] + Dist[1/(a*(m + 1)*(a^2 - b^2)
), Int[(a + b*Csc[e + f*x])^(m + 1)*Simp[c*(a^2 - b^2)*(m + 1) - (a*(b*c -
a*d)*(m + 1))*Csc[e + f*x] + b*(b*c - a*d)*(m + 2)*Csc[e + f*x]^2, x], x],
x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] && Ne
Q[a^2 - b^2, 0] && IntegerQ[2*m]
```

Rule 4060

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_Symbol] :> Simp[((A*b^2 -
 a*b*B + a^2*C)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1))/(a*f*(m + 1)*(a^
2 - b^2)), x] + Dist[1/(a*(m + 1)*(a^2 - b^2)), Int[(a + b*Csc[e + f*x])^(m
 + 1)*Simp[A*(a^2 - b^2)*(m + 1) - a*(A*b - a*B + b*C)*(m + 1)*Csc[e + f*x]
 + (A*b^2 - a*b*B + a^2*C)*(m + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a,
b, e, f, A, B, C}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1]
```

Rule 4172

```
Int[((A_) + (B_.)*sec[(d_.) + (e_.)*(x_)])*((a_) + (b_.)*sec[(d_.) + (e_.)*
(x_)] + (c_.)*sec[(d_.) + (e_.)*(x_)]^2)^(n_), x_Symbol] :> Dist[1/(4^n*c^n
), Int[(A + B*Sec[d + e*x])*(b + 2*c*Sec[d + e*x])^(2*n), x], x] /; FreeQ[{
a, b, c, d, e, A, B}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[n]
```

Rubi steps

$$\int \frac{a+b\sec(d+ex)}{\left(b^2+2ab\sec(d+ex)+a^2\sec^2(d+ex)\right)^2}\,dx = \left(16a^4\right)\int \frac{a+b\sec(d+ex)}{\left(2ab+2a^2\sec(d+ex)\right)^4}\,dx$$

$$= -\frac{a^4\tan(d+ex)}{3be\left(ab+a^2\sec(d+ex)\right)^3} + \frac{(2a)\int \frac{12a^3\left(a^2-b^2\right)+12a^2b\left(a^2-b^2\right)\sec}{\left(2ab+2a^2\sec\right.}}{3b\left(a^2-\right.}$$

$$= -\frac{a\left(3a^2-5b^2\right)\tan(d+ex)}{6b^2\left(a^2-b^2\right)e(b+a\sec(d+ex))^2} - \frac{a^4\tan(d+ex)}{3be\left(ab+a^2\sec(d+ex\right.}$$

$$= -\frac{a\left(3a^2-5b^2\right)\tan(d+ex)}{6b^2\left(a^2-b^2\right)e(b+a\sec(d+ex))^2} - \frac{a^4\tan(d+ex)}{3be\left(ab+a^2\sec(d+ex\right.}$$

$$= \frac{ax}{b^4} - \frac{a\left(3a^2-5b^2\right)\tan(d+ex)}{6b^2\left(a^2-b^2\right)e(b+a\sec(d+ex))^2} - \frac{a^4\tan(d+e}{3be\left(ab+a^2\sec(d\right.}$$

$$= \frac{ax}{b^4} - \frac{a\left(3a^2-5b^2\right)\tan(d+ex)}{6b^2\left(a^2-b^2\right)e(b+a\sec(d+ex))^2} - \frac{a^4\tan(d+e}{3be\left(ab+a^2\sec(d\right.}$$

$$= \frac{ax}{b^4} - \frac{a\left(3a^2-5b^2\right)\tan(d+ex)}{6b^2\left(a^2-b^2\right)e(b+a\sec(d+ex))^2} - \frac{a^4\tan(d+e}{3be\left(ab+a^2\sec(d\right.}$$

$$= \frac{ax}{b^4} - \frac{\left(a^2-2b^2\right)\left(2a^4-a^2b^2+b^4\right)\tan^{-1}\left(\frac{\sqrt{a-b}\,\tan\left(\frac{1}{2}(d+ex)\right)}{\sqrt{a+b}}\right)}{(a-b)^{5/2}b^4(a+b)^{5/2}e} -$$

Mathematica [A] time = 1.53, size = 276, normalized size = 1.20

$$\frac{\sec^3(d+ex)(a+b\cos(d+ex))(a+b\sec(d+ex))\left(-2a^3b\sin(d+ex)+\frac{a^2b\left(7a^2-9b^2\right)\sin(d+ex)(a+b\cos(d+ex))}{(a-b)(a+b)} - \frac{ab\left(11a^4-\right.}{}\right.}{6b^4e(a\cos(d+e}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sec[d + e*x])/(b^2 + 2*a*b*Sec[d + e*x] + a^2*Sec[d + e*x]
^2)^2,x]
```

```
[Out] ((a + b*Cos[d + e*x])*Sec[d + e*x]^3*(a + b*Sec[d + e*x])*(6*a*(d + e*x)*(a
 + b*Cos[d + e*x])^3 + (6*(-2*a^6 + 5*a^4*b^2 - 3*a^2*b^4 + 2*b^6)*ArcTanh[
((-a + b)*Tan[(d + e*x)/2])/Sqrt[-a^2 + b^2]]*(a + b*Cos[d + e*x])^3)/(-a^2
 + b^2)^(5/2) - 2*a^3*b*Sin[d + e*x] + (a^2*b*(7*a^2 - 9*b^2)*(a + b*Cos[d
+ e*x])*Sin[d + e*x])/((a - b)*(a + b)) - (a*b*(11*a^4 - 23*a^2*b^2 + 18*b^
4)*(a + b*Cos[d + e*x])^2*Sin[d + e*x])/((a - b)^2*(a + b)^2)))/(6*b^4*e*(b
 + a*Cos[d + e*x])*(b + a*Sec[d + e*x])^4)
```

fricas [B] time = 2.11, size = 1335, normalized size = 5.80

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(e*x+d))/(b^2+2*a*b*sec(e*x+d)+a^2*sec(e*x+d)^2)^2,x, alg
orithm="fricas")
```

```
[Out] [1/12*(12*(a^7*b^3 - 3*a^5*b^5 + 3*a^3*b^7 - a*b^9)*e*x*cos(e*x + d)^3 + 36
*(a^8*b^2 - 3*a^6*b^4 + 3*a^4*b^6 - a^2*b^8)*e*x*cos(e*x + d)^2 + 36*(a^9*b
 - 3*a^7*b^3 + 3*a^5*b^5 - a^3*b^7)*e*x*cos(e*x + d) + 12*(a^10 - 3*a^8*b^2
 + 3*a^6*b^4 - a^4*b^6)*e*x + 3*(2*a^9 - 5*a^7*b^2 + 3*a^5*b^4 - 2*a^3*b^6
+ (2*a^6*b^3 - 5*a^4*b^5 + 3*a^2*b^7 - 2*b^9)*cos(e*x + d)^3 + 3*(2*a^7*b^2
 - 5*a^5*b^4 + 3*a^3*b^6 - 2*a*b^8)*cos(e*x + d)^2 + 3*(2*a^8*b - 5*a^6*b^3
 + 3*a^4*b^5 - 2*a^2*b^7)*cos(e*x + d))*sqrt(-a^2 + b^2)*log((2*a*b*cos(e*x
 + d) + (2*a^2 - b^2)*cos(e*x + d)^2 + 2*sqrt(-a^2 + b^2)*(a*cos(e*x + d) +
 b)*sin(e*x + d) - a^2 + 2*b^2)/(b^2*cos(e*x + d)^2 + 2*a*b*cos(e*x + d) +
a^2)) - 2*(6*a^9*b - 17*a^7*b^3 + 22*a^5*b^5 - 11*a^3*b^7 + (11*a^7*b^3 - 3
4*a^5*b^5 + 41*a^3*b^7 - 18*a*b^9)*cos(e*x + d)^2 + 3*(5*a^8*b^2 - 15*a^6*b
^4 + 19*a^4*b^6 - 9*a^2*b^8)*cos(e*x + d))*sin(e*x + d))/((a^6*b^7 - 3*a^4*
b^9 + 3*a^2*b^11 - b^13)*e*cos(e*x + d)^3 + 3*(a^7*b^6 - 3*a^5*b^8 + 3*a^3*
b^10 - a*b^12)*e*cos(e*x + d)^2 + 3*(a^8*b^5 - 3*a^6*b^7 + 3*a^4*b^9 - a^2*
b^11)*e*cos(e*x + d) + (a^9*b^4 - 3*a^7*b^6 + 3*a^5*b^8 - a^3*b^10)*e), 1/6
*(6*(a^7*b^3 - 3*a^5*b^5 + 3*a^3*b^7 - a*b^9)*e*x*cos(e*x + d)^3 + 18*(a^8*
b^2 - 3*a^6*b^4 + 3*a^4*b^6 - a^2*b^8)*e*x*cos(e*x + d)^2 + 18*(a^9*b - 3*a
^7*b^3 + 3*a^5*b^5 - a^3*b^7)*e*x*cos(e*x + d) + 6*(a^10 - 3*a^8*b^2 + 3*a^
6*b^4 - a^4*b^6)*e*x - 3*(2*a^9 - 5*a^7*b^2 + 3*a^5*b^4 - 2*a^3*b^6 + (2*a^
6*b^3 - 5*a^4*b^5 + 3*a^2*b^7 - 2*b^9)*cos(e*x + d)^3 + 3*(2*a^7*b^2 - 5*a^
5*b^4 + 3*a^3*b^6 - 2*a*b^8)*cos(e*x + d)^2 + 3*(2*a^8*b - 5*a^6*b^3 + 3*a^
4*b^5 - 2*a^2*b^7)*cos(e*x + d))*sqrt(a^2 - b^2)*arctan(-(a*cos(e*x + d) +
b)/(sqrt(a^2 - b^2)*sin(e*x + d))) - (6*a^9*b - 17*a^7*b^3 + 22*a^5*b^5 - 1
1*a^3*b^7 + (11*a^7*b^3 - 34*a^5*b^5 + 41*a^3*b^7 - 18*a*b^9)*cos(e*x + d)^
2 + 3*(5*a^8*b^2 - 15*a^6*b^4 + 19*a^4*b^6 - 9*a^2*b^8)*cos(e*x + d))*sin(e
*x + d))/((a^6*b^7 - 3*a^4*b^9 + 3*a^2*b^11 - b^13)*e*cos(e*x + d)^3 + 3*(a
```

```
^7*b^6 - 3*a^5*b^8 + 3*a^3*b^10 - a*b^12)*e*cos(e*x + d)^2 + 3*(a^8*b^5 - 3
*a^6*b^7 + 3*a^4*b^9 - a^2*b^11)*e*cos(e*x + d) + (a^9*b^4 - 3*a^7*b^6 + 3*
a^5*b^8 - a^3*b^10)*e)]
```

giac [B] time = 0.49, size = 490, normalized size = 2.13

$$\frac{1}{3}\left(\frac{3\left(2a^6-5a^4b^2+3a^2b^4-2b^6\right)\left(\pi\left\lfloor\frac{xe+d}{2\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}\left(-2a+2b\right)+\arctan\left(-\frac{a\tan\left(\frac{1}{2}xe+\frac{1}{2}d\right)-b\tan\left(\frac{1}{2}xe+\frac{1}{2}d\right)}{\sqrt{a^2-b^2}}\right)\right)}{\left(a^4b^4-2a^2b^6+b^8\right)\sqrt{a^2-b^2}}+\frac{3(\ldots}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(e*x+d))/(b^2+2*a*b*sec(e*x+d)+a^2*sec(e*x+d)^2)^2,x, alg
orithm="giac")
```

```
[Out] 1/3*(3*(2*a^6 - 5*a^4*b^2 + 3*a^2*b^4 - 2*b^6)*(pi*floor(1/2*(x*e + d)/pi +
 1/2)*sgn(-2*a + 2*b) + arctan(-(a*tan(1/2*x*e + 1/2*d) - b*tan(1/2*x*e + 1
/2*d))/sqrt(a^2 - b^2)))/((a^4*b^4 - 2*a^2*b^6 + b^8)*sqrt(a^2 - b^2)) + 3*
(x*e + d)*a/b^4 - (6*a^7*tan(1/2*x*e + 1/2*d)^5 - 15*a^6*b*tan(1/2*x*e + 1/
2*d)^5 + 30*a^4*b^3*tan(1/2*x*e + 1/2*d)^5 - 12*a^3*b^4*tan(1/2*x*e + 1/2*d
)^5 - 27*a^2*b^5*tan(1/2*x*e + 1/2*d)^5 + 18*a*b^6*tan(1/2*x*e + 1/2*d)^5 +
 12*a^7*tan(1/2*x*e + 1/2*d)^3 - 44*a^5*b^2*tan(1/2*x*e + 1/2*d)^3 + 68*a^3
*b^4*tan(1/2*x*e + 1/2*d)^3 - 36*a*b^6*tan(1/2*x*e + 1/2*d)^3 + 6*a^7*tan(1
/2*x*e + 1/2*d) + 15*a^6*b*tan(1/2*x*e + 1/2*d) - 30*a^4*b^3*tan(1/2*x*e +
1/2*d) - 12*a^3*b^4*tan(1/2*x*e + 1/2*d) + 27*a^2*b^5*tan(1/2*x*e + 1/2*d)
+ 18*a*b^6*tan(1/2*x*e + 1/2*d))/((a^4*b^3 - 2*a^2*b^5 + b^7)*(a*tan(1/2*x*
e + 1/2*d)^2 - b*tan(1/2*x*e + 1/2*d)^2 + a + b)^3))*e^(-1)
```

maple [B] time = 0.35, size = 1118, normalized size = 4.86

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(e*x+d))/(b^2+2*a*b*sec(e*x+d)+a^2*sec(e*x+d)^2)^2,x)
```

```
[Out] -2/e/b^3/(a*tan(1/2*d+1/2*e*x)^2-b*tan(1/2*d+1/2*e*x)^2+a+b)^3*a^5/(a^2+2*a
*b+b^2)*tan(1/2*d+1/2*e*x)^5+1/e/b^2/(a*tan(1/2*d+1/2*e*x)^2-b*tan(1/2*d+1/
2*e*x)^2+a+b)^3*a^4/(a^2+2*a*b+b^2)*tan(1/2*d+1/2*e*x)^5+4/e/b/(a*tan(1/2*d
+1/2*e*x)^2-b*tan(1/2*d+1/2*e*x)^2+a+b)^3*a^3/(a^2+2*a*b+b^2)*tan(1/2*d+1/2
*e*x)^5-3/e/(a*tan(1/2*d+1/2*e*x)^2-b*tan(1/2*d+1/2*e*x)^2+a+b)^3*a^2/(a^2+
2*a*b+b^2)*tan(1/2*d+1/2*e*x)^5-6/e*b/(a*tan(1/2*d+1/2*e*x)^2-b*tan(1/2*d+1
/2*e*x)^2+a+b)^3*a/(a^2+2*a*b+b^2)*tan(1/2*d+1/2*e*x)^5-4/e/b^3/(a*tan(1/2*
d+1/2*e*x)^2-b*tan(1/2*d+1/2*e*x)^2+a+b)^3*a^5/(a+b)/(a-b)*tan(1/2*d+1/2*e*
```

```
x)^3+32/3/e/b/(a*tan(1/2*d+1/2*e*x)^2-b*tan(1/2*d+1/2*e*x)^2+a+b)^3*a^3/(a+
b)/(a-b)*tan(1/2*d+1/2*e*x)^3-12/e*b/(a*tan(1/2*d+1/2*e*x)^2-b*tan(1/2*d+1/
2*e*x)^2+a+b)^3*a/(a+b)/(a-b)*tan(1/2*d+1/2*e*x)^3-2/e/b^3/(a*tan(1/2*d+1/2
*e*x)^2-b*tan(1/2*d+1/2*e*x)^2+a+b)^3*a^5/(a^2-2*a*b+b^2)*tan(1/2*d+1/2*e*x
)-1/e/b^2/(a*tan(1/2*d+1/2*e*x)^2-b*tan(1/2*d+1/2*e*x)^2+a+b)^3*a^4/(a^2-2*
a*b+b^2)*tan(1/2*d+1/2*e*x)+4/e/b/(a*tan(1/2*d+1/2*e*x)^2-b*tan(1/2*d+1/2*e
*x)^2+a+b)^3*a^3/(a^2-2*a*b+b^2)*tan(1/2*d+1/2*e*x)+3/e/(a*tan(1/2*d+1/2*e*
x)^2-b*tan(1/2*d+1/2*e*x)^2+a+b)^3*a^2/(a^2-2*a*b+b^2)*tan(1/2*d+1/2*e*x)-6
/e*b/(a*tan(1/2*d+1/2*e*x)^2-b*tan(1/2*d+1/2*e*x)^2+a+b)^3*a/(a^2-2*a*b+b^2
)*tan(1/2*d+1/2*e*x)-2/e/b^4/(a^4-2*a^2*b^2+b^4)/((a+b)*(a-b))^(1/2)*arctan
(tan(1/2*d+1/2*e*x)*(a-b)/((a+b)*(a-b))^(1/2))*a^6+5/e/b^2/(a^4-2*a^2*b^2+b
^4)/((a+b)*(a-b))^(1/2)*arctan(tan(1/2*d+1/2*e*x)*(a-b)/((a+b)*(a-b))^(1/2)
)*a^4-3/e/(a^4-2*a^2*b^2+b^4)/((a+b)*(a-b))^(1/2)*arctan(tan(1/2*d+1/2*e*x)
*(a-b)/((a+b)*(a-b))^(1/2))*a^2+2/e*b^2/(a^4-2*a^2*b^2+b^4)/((a+b)*(a-b))^(
1/2)*arctan(tan(1/2*d+1/2*e*x)*(a-b)/((a+b)*(a-b))^(1/2))+2/e*a/b^4*arctan(
tan(1/2*d+1/2*e*x))
```

maxima [**F(-2)**] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(e*x+d))/(b^2+2*a*b*sec(e*x+d)+a^2*sec(e*x+d)^2)^2,x, alg
orithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more details)Is 4*b^2-4*a^2 positive or negative?
```

mupad [**B**] time = 11.47, size = 5469, normalized size = 23.78

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(d + e*x))/(b^2 + a^2/cos(d + e*x)^2 + (2*a*b)/cos(d + e*x))^
2,x)
```

```
[Out] - ((tan(d/2 + (e*x)/2)*(6*a*b^4 + a^4*b + 2*a^5 - 3*a^2*b^3 - 4*a^3*b^2))/(
b^5 - 2*a*b^4 + a^2*b^3) + (tan(d/2 + (e*x)/2)^5*(6*a*b^4 - a^4*b + 2*a^5 +
 3*a^2*b^3 - 4*a^3*b^2))/(b^3*(a + b)^2) + (4*tan(d/2 + (e*x)/2)^3*(9*a*b^4
 + 3*a^5 - 8*a^3*b^2))/(3*(a*b^3 - b^4)*(a + b)))/(e*(3*a*b^2 - tan(d/2 + (
e*x)/2)^4*(3*a*b^2 + 3*a^2*b - 3*a^3 - 3*b^3) - tan(d/2 + (e*x)/2)^2*(3*a*b
^2 - 3*a^2*b - 3*a^3 + 3*b^3) + 3*a^2*b + a^3 + b^3 + tan(d/2 + (e*x)/2)^6*
(3*a*b^2 - 3*a^2*b + a^3 - b^3))) - (2*a*atan(((a*((a*((8*(4*b^18 - 14*a^2*
```

b^16 - 6*a^3*b^15 + 26*a^4*b^14 + 14*a^5*b^13 - 30*a^6*b^12 - 10*a^7*b^11 + 18*a^8*b^10 + 2*a^9*b^9 - 4*a^10*b^8))/(a*b^15 + b^16 - 3*a^2*b^14 - 3*a^3*b^13 + 3*a^4*b^12 + 3*a^5*b^11 - a^6*b^10 - a^7*b^9) - (a*tan(d/2 + (e*x)/2)*(8*a*b^17 - 8*a^2*b^16 - 32*a^3*b^15 + 32*a^4*b^14 + 48*a^5*b^13 - 48*a^6*b^12 - 32*a^7*b^11 + 32*a^8*b^10 + 8*a^9*b^9 - 8*a^10*b^8)*8i)/(b^4*(a*b^12 + b^13 - 3*a^2*b^11 - 3*a^3*b^10 + 3*a^4*b^9 + 3*a^5*b^8 - a^6*b^7 - a^7*b^6)))*1i)/b^4 - (8*tan(d/2 + (e*x)/2)*(8*a^11*b - 8*a^12 - 4*b^12 + 8*a^2*b^10 + 8*a^3*b^9 - 17*a^4*b^8 - 32*a^5*b^7 + 30*a^6*b^6 + 48*a^7*b^5 - 45*a^8*b^4 - 32*a^9*b^3 + 32*a^10*b^2))/(a*b^12 + b^13 - 3*a^2*b^11 - 3*a^3*b^10 + 3*a^4*b^9 + 3*a^5*b^8 - a^6*b^7 - a^7*b^6)))/b^4 - (a*((a*((8*(4*b^18 - 14*a^2*b^16 - 6*a^3*b^15 + 26*a^4*b^14 + 14*a^5*b^13 - 30*a^6*b^12 - 10*a^7*b^11 + 18*a^8*b^10 + 2*a^9*b^9 - 4*a^10*b^8))/(a*b^15 + b^16 - 3*a^2*b^14 - 3*a^3*b^13 + 3*a^4*b^12 + 3*a^5*b^11 - a^6*b^10 - a^7*b^9) + (a*tan(d/2 + (e*x)/2)*(8*a*b^17 - 8*a^2*b^16 - 32*a^3*b^15 + 32*a^4*b^14 + 48*a^5*b^13 - 48*a^6*b^12 - 32*a^7*b^11 + 32*a^8*b^10 + 8*a^9*b^9 - 8*a^10*b^8)*8i)/(b^4*(a*b^12 + b^13 - 3*a^2*b^11 - 3*a^3*b^10 + 3*a^4*b^9 + 3*a^5*b^8 - a^6*b^7 - a^7*b^6)))*1i)/b^4 + (8*tan(d/2 + (e*x)/2)*(8*a^11*b - 8*a^12 - 4*b^12 + 8*a^2*b^10 + 8*a^3*b^9 - 17*a^4*b^8 - 32*a^5*b^7 + 30*a^6*b^6 + 48*a^7*b^5 - 45*a^8*b^4 - 32*a^9*b^3 + 32*a^10*b^2))/(a*b^12 + b^13 - 3*a^2*b^11 - 3*a^3*b^10 + 3*a^4*b^9 + 3*a^5*b^8 - a^6*b^7 - a^7*b^6)))/b^4)/((a*((a*((8*(4*b^18 - 14*a^2*b^16 - 6*a^3*b^15 + 26*a^4*b^14 + 14*a^5*b^13 - 30*a^6*b^12 - 10*a^7*b^11 + 18*a^8*b^10 + 2*a^9*b^9 - 4*a^10*b^8))/(a*b^15 + b^16 - 3*a^2*b^14 - 3*a^3*b^13 + 3*a^4*b^12 + 3*a^5*b^11 - a^6*b^10 - a^7*b^9) - (a*tan(d/2 + (e*x)/2)*(8*a*b^17 - 8*a^2*b^16 - 32*a^3*b^15 + 32*a^4*b^14 + 48*a^5*b^13 - 48*a^6*b^12 - 32*a^7*b^11 + 32*a^8*b^10 + 8*a^9*b^9 - 8*a^10*b^8)*8i)/(b^4*(a*b^12 + b^13 - 3*a^2*b^11 - 3*a^3*b^10 + 3*a^4*b^9 + 3*a^5*b^8 - a^6*b^7 - a^7*b^6)))*1i)/b^4 - (8*tan(d/2 + (e*x)/2)*(8*a^11*b - 8*a^12 - 4*b^12 + 8*a^2*b^10 + 8*a^3*b^9 - 17*a^4*b^8 - 32*a^5*b^7 + 30*a^6*b^6 + 48*a^7*b^5 - 45*a^8*b^4 - 32*a^9*b^3 + 32*a^10*b^2))/(a*b^12 + b^13 - 3*a^2*b^11 - 3*a^3*b^10 + 3*a^4*b^9 + 3*a^5*b^8 - a^6*b^7 - a^7*b^6))*1i)/b^4 - (16*(4*a*b^11 - 2*a^11*b + 4*a^12 - 4*a^2*b^10 - 8*a^3*b^9 + 14*a^4*b^8 + 15*a^5*b^7 - 26*a^6*b^6 - 12*a^7*b^5 + 30*a^8*b^4 + 7*a^9*b^3 - 18*a^10*b^2))/(a*b^15 + b^16 - 3*a^2*b^14 - 3*a^3*b^13 + 3*a^4*b^12 + 3*a^5*b^11 - a^6*b^10 - a^7*b^9) + (a*((a*((8*(4*b^18 - 14*a^2*b^16 - 6*a^3*b^15 + 26*a^4*b^14 + 14*a^5*b^13 - 30*a^6*b^12 - 10*a^7*b^11 + 18*a^8*b^10 + 2*a^9*b^9 - 4*a^10*b^8))/(a*b^15 + b^16 - 3*a^2*b^14 - 3*a^3*b^13 + 3*a^4*b^12 + 3*a^5*b^11 - a^6*b^10 - a^7*b^9) + (a*tan(d/2 + (e*x)/2)*(8*a*b^17 - 8*a^2*b^16 - 32*a^3*b^15 + 32*a^4*b^14 + 48*a^5*b^13 - 48*a^6*b^12 - 32*a^7*b^11 + 32*a^8*b^10 + 8*a^9*b^9 - 8*a^10*b^8)*8i)/(b^4*(a*b^12 + b^13 - 3*a^2*b^11 - 3*a^3*b^10 + 3*a^4*b^9 + 3*a^5*b^8 - a^6*b^7 - a^7*b^6)))*1i)/b^4 + (8*tan(d/2 + (e*x)/2)*(8*a^11*b - 8*a^12 - 4*b^12 + 8*a^2*b^10 + 8*a^3*b^9 - 17*a^4*b^8 - 32*a^5*b^7 + 30*a^6*b^6 + 48*a^7*b^5 - 45*a^8*b^4 - 32*a^9*b^3 + 32*a^10*b^2))/(a*b^12 + b^13 - 3*a^2*b^11 - 3*a^3*b^10 + 3*a^4*b^9 + 3*a^5*b^8 - a^6*b^7 - a^7*b^6))*1i)/b^4)))/(b^4*e) - (atan((((a^2 - 2*b^2)*(-(a + b)^5*(a - b)^5)^(1/2)*((8*tan(d/2 + (e*x)/2)*(8*a^11*b - 8*a^12 - 4*b^12 + 8*a

^2*b^10 + 8*a^3*b^9 - 17*a^4*b^8 - 32*a^5*b^7 + 30*a^6*b^6 + 48*a^7*b^5 - 4
5*a^8*b^4 - 32*a^9*b^3 + 32*a^10*b^2))/(a*b^12 + b^13 - 3*a^2*b^11 - 3*a^3*
b^10 + 3*a^4*b^9 + 3*a^5*b^8 - a^6*b^7 - a^7*b^6) - (((8*(4*b^18 - 14*a^2*b
^16 - 6*a^3*b^15 + 26*a^4*b^14 + 14*a^5*b^13 - 30*a^6*b^12 - 10*a^7*b^11 +
18*a^8*b^10 + 2*a^9*b^9 - 4*a^10*b^8))/(a*b^15 + b^16 - 3*a^2*b^14 - 3*a^3*
b^13 + 3*a^4*b^12 + 3*a^5*b^11 - a^6*b^10 - a^7*b^9) - (4*tan(d/2 + (e*x)/2
)*(a^2 - 2*b^2)*(-(a + b)^5*(a - b)^5)^(1/2)*(2*a^4 + b^4 - a^2*b^2)*(8*a*b
^17 - 8*a^2*b^16 - 32*a^3*b^15 + 32*a^4*b^14 + 48*a^5*b^13 - 48*a^6*b^12 -
32*a^7*b^11 + 32*a^8*b^10 + 8*a^9*b^9 - 8*a^10*b^8))/((b^14 - 5*a^2*b^12 +
10*a^4*b^10 - 10*a^6*b^8 + 5*a^8*b^6 - a^10*b^4)*(a*b^12 + b^13 - 3*a^2*b^1
1 - 3*a^3*b^10 + 3*a^4*b^9 + 3*a^5*b^8 - a^6*b^7 - a^7*b^6)))*(a^2 - 2*b^2)
(-(a + b)^5(a - b)^5)^(1/2)*(2*a^4 + b^4 - a^2*b^2))/(2*(b^14 - 5*a^2*b^1
2 + 10*a^4*b^10 - 10*a^6*b^8 + 5*a^8*b^6 - a^10*b^4)))*(2*a^4 + b^4 - a^2*b
^2)*1i)/(2*(b^14 - 5*a^2*b^12 + 10*a^4*b^10 - 10*a^6*b^8 + 5*a^8*b^6 - a^10
*b^4)) + ((a^2 - 2*b^2)*(-(a + b)^5*(a - b)^5)^(1/2)*((8*tan(d/2 + (e*x)/2)
*(8*a^11*b - 8*a^12 - 4*b^12 + 8*a^2*b^10 + 8*a^3*b^9 - 17*a^4*b^8 - 32*a^5
*b^7 + 30*a^6*b^6 + 48*a^7*b^5 - 45*a^8*b^4 - 32*a^9*b^3 + 32*a^10*b^2))/(a
*b^12 + b^13 - 3*a^2*b^11 - 3*a^3*b^10 + 3*a^4*b^9 + 3*a^5*b^8 - a^6*b^7 -
a^7*b^6) + (((8*(4*b^18 - 14*a^2*b^16 - 6*a^3*b^15 + 26*a^4*b^14 + 14*a^5*b
^13 - 30*a^6*b^12 - 10*a^7*b^11 + 18*a^8*b^10 + 2*a^9*b^9 - 4*a^10*b^8))/(a
*b^15 + b^16 - 3*a^2*b^14 - 3*a^3*b^13 + 3*a^4*b^12 + 3*a^5*b^11 - a^6*b^10
 - a^7*b^9) + (4*tan(d/2 + (e*x)/2)*(a^2 - 2*b^2)*(-(a + b)^5*(a - b)^5)^(1
/2)*(2*a^4 + b^4 - a^2*b^2)*(8*a*b^17 - 8*a^2*b^16 - 32*a^3*b^15 + 32*a^4*b
^14 + 48*a^5*b^13 - 48*a^6*b^12 - 32*a^7*b^11 + 32*a^8*b^10 + 8*a^9*b^9 - 8
*a^10*b^8))/((b^14 - 5*a^2*b^12 + 10*a^4*b^10 - 10*a^6*b^8 + 5*a^8*b^6 - a^
10*b^4)*(a*b^12 + b^13 - 3*a^2*b^11 - 3*a^3*b^10 + 3*a^4*b^9 + 3*a^5*b^8 -
a^6*b^7 - a^7*b^6)))*(a^2 - 2*b^2)*(-(a + b)^5*(a - b)^5)^(1/2)*(2*a^4 + b^
4 - a^2*b^2))/(2*(b^14 - 5*a^2*b^12 + 10*a^4*b^10 - 10*a^6*b^8 + 5*a^8*b^6
- a^10*b^4)))*(2*a^4 + b^4 - a^2*b^2)*1i)/(2*(b^14 - 5*a^2*b^12 + 10*a^4*b^
10 - 10*a^6*b^8 + 5*a^8*b^6 - a^10*b^4)))/((16*(4*a*b^11 - 2*a^11*b + 4*a^1
2 - 4*a^2*b^10 - 8*a^3*b^9 + 14*a^4*b^8 + 15*a^5*b^7 - 26*a^6*b^6 - 12*a^7*
b^5 + 30*a^8*b^4 + 7*a^9*b^3 - 18*a^10*b^2))/(a*b^15 + b^16 - 3*a^2*b^14 -
3*a^3*b^13 + 3*a^4*b^12 + 3*a^5*b^11 - a^6*b^10 - a^7*b^9) + ((a^2 - 2*b^2)
(-(a + b)^5(a - b)^5)^(1/2)*((8*tan(d/2 + (e*x)/2)*(8*a^11*b - 8*a^12 - 4
*b^12 + 8*a^2*b^10 + 8*a^3*b^9 - 17*a^4*b^8 - 32*a^5*b^7 + 30*a^6*b^6 + 48*
a^7*b^5 - 45*a^8*b^4 - 32*a^9*b^3 + 32*a^10*b^2))/(a*b^12 + b^13 - 3*a^2*b^
11 - 3*a^3*b^10 + 3*a^4*b^9 + 3*a^5*b^8 - a^6*b^7 - a^7*b^6) - (((8*(4*b^18
 - 14*a^2*b^16 - 6*a^3*b^15 + 26*a^4*b^14 + 14*a^5*b^13 - 30*a^6*b^12 - 10*
a^7*b^11 + 18*a^8*b^10 + 2*a^9*b^9 - 4*a^10*b^8))/(a*b^15 + b^16 - 3*a^2*b^
14 - 3*a^3*b^13 + 3*a^4*b^12 + 3*a^5*b^11 - a^6*b^10 - a^7*b^9) - (4*tan(d/
2 + (e*x)/2)*(a^2 - 2*b^2)*(-(a + b)^5*(a - b)^5)^(1/2)*(2*a^4 + b^4 - a^2*
b^2)*(8*a*b^17 - 8*a^2*b^16 - 32*a^3*b^15 + 32*a^4*b^14 + 48*a^5*b^13 - 48*
a^6*b^12 - 32*a^7*b^11 + 32*a^8*b^10 + 8*a^9*b^9 - 8*a^10*b^8))/((b^14 - 5*
a^2*b^12 + 10*a^4*b^10 - 10*a^6*b^8 + 5*a^8*b^6 - a^10*b^4)*(a*b^12 + b^13
- 3*a^2*b^11 - 3*a^3*b^10 + 3*a^4*b^9 + 3*a^5*b^8 - a^6*b^7 - a^7*b^6)))*(a

```
^2 - 2*b^2)*(-(a + b)^5*(a - b)^5)^(1/2)*(2*a^4 + b^4 - a^2*b^2))/(2*(b^14
- 5*a^2*b^12 + 10*a^4*b^10 - 10*a^6*b^8 + 5*a^8*b^6 - a^10*b^4)))*(2*a^4 +
b^4 - a^2*b^2))/(2*(b^14 - 5*a^2*b^12 + 10*a^4*b^10 - 10*a^6*b^8 + 5*a^8*b^
6 - a^10*b^4)) - ((a^2 - 2*b^2)*(-(a + b)^5*(a - b)^5)^(1/2)*((8*tan(d/2 +
(e*x)/2)*(8*a^11*b - 8*a^12 - 4*b^12 + 8*a^2*b^10 + 8*a^3*b^9 - 17*a^4*b^8
- 32*a^5*b^7 + 30*a^6*b^6 + 48*a^7*b^5 - 45*a^8*b^4 - 32*a^9*b^3 + 32*a^10*
b^2))/(a*b^12 + b^13 - 3*a^2*b^11 - 3*a^3*b^10 + 3*a^4*b^9 + 3*a^5*b^8 - a^
6*b^7 - a^7*b^6) + (((8*(4*b^18 - 14*a^2*b^16 - 6*a^3*b^15 + 26*a^4*b^14 +
14*a^5*b^13 - 30*a^6*b^12 - 10*a^7*b^11 + 18*a^8*b^10 + 2*a^9*b^9 - 4*a^10*
b^8))/(a*b^15 + b^16 - 3*a^2*b^14 - 3*a^3*b^13 + 3*a^4*b^12 + 3*a^5*b^11 -
a^6*b^10 - a^7*b^9) + (4*tan(d/2 + (e*x)/2)*(a^2 - 2*b^2)*(-(a + b)^5*(a -
b)^5)^(1/2)*(2*a^4 + b^4 - a^2*b^2)*(8*a*b^17 - 8*a^2*b^16 - 32*a^3*b^15 +
32*a^4*b^14 + 48*a^5*b^13 - 48*a^6*b^12 - 32*a^7*b^11 + 32*a^8*b^10 + 8*a^9
*b^9 - 8*a^10*b^8))/((b^14 - 5*a^2*b^12 + 10*a^4*b^10 - 10*a^6*b^8 + 5*a^8*
b^6 - a^10*b^4)*(a*b^12 + b^13 - 3*a^2*b^11 - 3*a^3*b^10 + 3*a^4*b^9 + 3*a^
5*b^8 - a^6*b^7 - a^7*b^6)))*(a^2 - 2*b^2)*(-(a + b)^5*(a - b)^5)^(1/2)*(2*
a^4 + b^4 - a^2*b^2))/(2*(b^14 - 5*a^2*b^12 + 10*a^4*b^10 - 10*a^6*b^8 + 5*
a^8*b^6 - a^10*b^4)))*(2*a^4 + b^4 - a^2*b^2))/(2*(b^14 - 5*a^2*b^12 + 10*a
^4*b^10 - 10*a^6*b^8 + 5*a^8*b^6 - a^10*b^4))))*(a^2 - 2*b^2)*(-(a + b)^5*(
a - b)^5)^(1/2)*(2*a^4 + b^4 - a^2*b^2)*1i)/(e*(b^14 - 5*a^2*b^12 + 10*a^4*
b^10 - 10*a^6*b^8 + 5*a^8*b^6 - a^10*b^4))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a + b\sec(d + ex)}{(a\sec(d + ex) + b)^4}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(e*x+d))/(b**2+2*a*b*sec(e*x+d)+a**2*sec(e*x+d)**2)**2,x)
[Out] Integral((a + b*sec(d + e*x))/(a*sec(d + e*x) + b)**4, x)
```

3.522 $\int (a+b\sec(d+ex))\left(b^2+2ab\sec(d+ex)+a^2\sec^2(d+e$

Optimal. Leaf size=359

$$\frac{a^5\left(3a^2+5b^2\right)\tan(d+ex)\sec(d+ex)\left(a^2\sec^2(d+ex)+2ab\sec(d+ex)+b^2\right)^{3/2}}{6e\left(a^2\sec(d+ex)+ab\right)^3}+\frac{a^4b\left(11a^2+8b^2\right)\tan(d+ex)\left(a^2\right.}{3e\left(a^2\right.}$$

```
[Out] 1/2*(a^4+9*a^2*b^2+2*b^4)*arctanh(sin(e*x+d))*(b^2+2*a*b*sec(e*x+d)+a^2*sec
(e*x+d)^2)^(3/2)/e/(b+a*sec(e*x+d))^3+a^4*b^3*x*(b^2+2*a*b*sec(e*x+d)+a^2*s
ec(e*x+d)^2)^(3/2)/(a*b+a^2*sec(e*x+d))^3+1/3*a^4*b*(11*a^2+8*b^2)*(b^2+2*a
*b*sec(e*x+d)+a^2*sec(e*x+d)^2)^(3/2)*tan(e*x+d)/e/(a*b+a^2*sec(e*x+d))^3+1
/6*a^5*(3*a^2+5*b^2)*sec(e*x+d)*(b^2+2*a*b*sec(e*x+d)+a^2*sec(e*x+d)^2)^(3/
2)*tan(e*x+d)/e/(a*b+a^2*sec(e*x+d))^3+1/3*b*(a^2*b+a^3*sec(e*x+d))^2*(b^2+
2*a*b*sec(e*x+d)+a^2*sec(e*x+d)^2)^(3/2)*tan(e*x+d)/e/(a*b+a^2*sec(e*x+d))^
3
```

Rubi [A] time = 0.29, antiderivative size = 359, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.146, Rules used = {4174, 3918, 4048, 3770, 3767, 8}

$$\frac{a^4b^3x\left(a^2\sec^2(d+ex)+2ab\sec(d+ex)+b^2\right)^{3/2}}{\left(a^2\sec(d+ex)+ab\right)^3}+\frac{a^5\left(3a^2+5b^2\right)\tan(d+ex)\sec(d+ex)\left(a^2\sec^2(d+ex)+2ab\,\mathrm{s}\right.}{6e\left(a^2\sec(d+ex)+ab\right)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[d + e*x])*(b^2 + 2*a*b*Sec[d + e*x] + a^2*Sec[d + e*x]^2)^(3
/2),x]
```

```
[Out] ((a^4 + 9*a^2*b^2 + 2*b^4)*ArcTanh[Sin[d + e*x]]*(b^2 + 2*a*b*Sec[d + e*x]
+ a^2*Sec[d + e*x]^2)^(3/2))/(2*e*(b + a*Sec[d + e*x])^3) + (a^4*b^3*x*(b^2
 + 2*a*b*Sec[d + e*x] + a^2*Sec[d + e*x]^2)^(3/2))/(a*b + a^2*Sec[d + e*x])
^3 + (a^4*b*(11*a^2 + 8*b^2)*(b^2 + 2*a*b*Sec[d + e*x] + a^2*Sec[d + e*x]^2
)^(3/2)*Tan[d + e*x])/(3*e*(a*b + a^2*Sec[d + e*x])^3) + (a^5*(3*a^2 + 5*b^
2)*Sec[d + e*x]*(b^2 + 2*a*b*Sec[d + e*x] + a^2*Sec[d + e*x]^2)^(3/2)*Tan[d
 + e*x])/(6*e*(a*b + a^2*Sec[d + e*x])^3) + (b*(a^2*b + a^3*Sec[d + e*x])^2
*(b^2 + 2*a*b*Sec[d + e*x] + a^2*Sec[d + e*x]^2)^(3/2)*Tan[d + e*x])/(3*e*(
a*b + a^2*Sec[d + e*x])^3)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3918

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d
_.) + (c_)), x_Symbol] :> -Simp[(b*d*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m -
 1))/(f*m), x] + Dist[1/m, Int[(a + b*Csc[e + f*x])^(m - 2)*Simp[a^2*c*m +
(b^2*d*(m - 1) + 2*a*b*c*m + a^2*d*m)*Csc[e + f*x] + b*(b*c*m + a*d*(2*m -
1))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0] && GtQ[m, 1] && NeQ[a^2 - b^2, 0] && IntegerQ[2*m]
```

Rule 4048

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbol] :> -Simp[(b*C*Csc[e +
f*x]*Cot[e + f*x])/(2*f), x] + Dist[1/2, Int[Simp[2*A*a + (2*B*a + b*(2*A +
 C))*Csc[e + f*x] + 2*(a*C + B*b)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b
, e, f, A, B, C}, x]
```

Rule 4174

```
Int[((A_) + (B_.)*sec[(d_.) + (e_.)*(x_)])*((a_) + (b_.)*sec[(d_.) + (e_.)*
(x_)] + (c_.)*sec[(d_.) + (e_.)*(x_)]^2)^(n_), x_Symbol] :> Dist[(a + b*Sec
[d + e*x] + c*Sec[d + e*x]^2)^n/(b + 2*c*Sec[d + e*x])^(2*n), Int[(A + B*Se
c[d + e*x])*(b + 2*c*Sec[d + e*x])^(2*n), x], x] /; FreeQ[{a, b, c, d, e, A
, B}, x] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[n]
```

Rubi steps

$$\int (a+b\sec(d+ex))\left(b^2+2ab\sec(d+ex)+a^2\sec^2(d+ex)\right)^{3/2}\,dx = \frac{\left(b^2+2ab\sec(d+ex)+a^2\sec^2(d+ex)\right)^{3/2}}{\left(2ab+2a\right.}$$

$$= \frac{b\left(a^2b+a^3\sec(d+ex)\right)^2\left(b^2+2ab\sec(d+\right.}{3e\left(ab+a^2\sec(\right.}$$

$$= \frac{a^5\left(3a^2+5b^2\right)\sec(d+ex)\left(b^2+2ab\sec(d\right.}{6e\left(ab+a^2\sec\right.}$$

$$= \frac{a^4b^3x\left(b^2+2ab\sec(d+ex)+a^2\sec^2(d+e\right.}{\left(ab+a^2\sec(d+ex)\right)^3}$$

$$= \frac{\left(a^4+9a^2b^2+2b^4\right)\tanh^{-1}(\sin(d+ex))\left(b^2\right.}{2e(b+a\sec(}$$

$$= \frac{\left(a^4+9a^2b^2+2b^4\right)\tanh^{-1}(\sin(d+ex))\left(b^2\right.}{2e(b+a\sec(}$$

Mathematica [A] time = 0.82, size = 128, normalized size = 0.36

$$\frac{\cos(d+ex)\sqrt{(a\sec(d+ex)+b)^2}\left(2a^3b\tan^3(d+ex)+3a\tan(d+ex)\left(a\left(a^2+3b^2\right)\sec(d+ex)+8a^2b+6b^3\right)+3\right.}{6e(a+b\cos(d+ex))}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sec[d + e*x])*(b^2 + 2*a*b*Sec[d + e*x] + a^2*Sec[d + e*x]
^2)^(3/2),x]
```

```
[Out] (Cos[d + e*x]*Sqrt[(b + a*Sec[d + e*x])^2]*(6*a*b^3*e*x + 3*(a^4 + 9*a^2*b^
2 + 2*b^4)*ArcTanh[Sin[d + e*x]] + 3*a*(8*a^2*b + 6*b^3 + a*(a^2 + 3*b^2)*S
ec[d + e*x])*Tan[d + e*x] + 2*a^3*b*Tan[d + e*x]^3))/(6*e*(a + b*Cos[d + e*
x]))
```

fricas [A] time = 1.58, size = 162, normalized size = 0.45

$$\frac{12\,ab^3ex\cos(ex+d)^3+3\left(a^4+9\,a^2b^2+2\,b^4\right)\cos(ex+d)^3\log(\sin(ex+d)+1)-3\left(a^4+9\,a^2b^2+2\,b^4\right)\cos(ex\right.}{12\,e\cos}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(e*x+d))*(b^2+2*a*b*sec(e*x+d)+a^2*sec(e*x+d)^2)^(3/2),x,
 algorithm="fricas")
```

```
[Out] 1/12*(12*a*b^3*e*x*cos(e*x + d)^3 + 3*(a^4 + 9*a^2*b^2 + 2*b^4)*cos(e*x + d
)^3*log(sin(e*x + d) + 1) - 3*(a^4 + 9*a^2*b^2 + 2*b^4)*cos(e*x + d)^3*log(
-sin(e*x + d) + 1) + 2*(2*a^3*b + 2*(11*a^3*b + 9*a*b^3)*cos(e*x + d)^2 + 3
*(a^4 + 3*a^2*b^2)*cos(e*x + d))*sin(e*x + d))/(e*cos(e*x + d)^3)
```

giac [A] time = 0.51, size = 652, normalized size = 1.82

$$\frac{1}{6}\left(6\,(xe+d)ab^3\mathrm{sgn}\left(b\cos\left(xe+d\right)^2+a\cos\left(xe+d\right)\right)+3\left(a^4\mathrm{sgn}\left(b\cos\left(xe+d\right)^2+a\cos\left(xe+d\right)\right)+9\,a^2b^2\mathrm{sgn}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(e*x+d))*(b^2+2*a*b*sec(e*x+d)+a^2*sec(e*x+d)^2)^(3/2),x,
 algorithm="giac")
```

```
[Out] 1/6*(6*(x*e + d)*a*b^3*sgn(b*cos(x*e + d)^2 + a*cos(x*e + d)) + 3*(a^4*sgn(
b*cos(x*e + d)^2 + a*cos(x*e + d)) + 9*a^2*b^2*sgn(b*cos(x*e + d)^2 + a*cos
(x*e + d)) + 2*b^4*sgn(b*cos(x*e + d)^2 + a*cos(x*e + d)))*log(abs(tan(1/2*
x*e + 1/2*d) + 1)) - 3*(a^4*sgn(b*cos(x*e + d)^2 + a*cos(x*e + d)) + 9*a^2*
b^2*sgn(b*cos(x*e + d)^2 + a*cos(x*e + d)) + 2*b^4*sgn(b*cos(x*e + d)^2 + a
*cos(x*e + d)))*log(abs(tan(1/2*x*e + 1/2*d) - 1)) + 2*(3*a^4*sgn(b*cos(x*e
 + d)^2 + a*cos(x*e + d))*tan(1/2*x*e + 1/2*d)^5 - 24*a^3*b*sgn(b*cos(x*e +
 d)^2 + a*cos(x*e + d))*tan(1/2*x*e + 1/2*d)^5 + 9*a^2*b^2*sgn(b*cos(x*e +
d)^2 + a*cos(x*e + d))*tan(1/2*x*e + 1/2*d)^5 - 18*a*b^3*sgn(b*cos(x*e + d)
^2 + a*cos(x*e + d))*tan(1/2*x*e + 1/2*d)^5 + 40*a^3*b*sgn(b*cos(x*e + d)^2
 + a*cos(x*e + d))*tan(1/2*x*e + 1/2*d)^3 + 36*a*b^3*sgn(b*cos(x*e + d)^2 +
 a*cos(x*e + d))*tan(1/2*x*e + 1/2*d)^3 - 3*a^4*sgn(b*cos(x*e + d)^2 + a*co
s(x*e + d))*tan(1/2*x*e + 1/2*d) - 24*a^3*b*sgn(b*cos(x*e + d)^2 + a*cos(x*
e + d))*tan(1/2*x*e + 1/2*d) - 9*a^2*b^2*sgn(b*cos(x*e + d)^2 + a*cos(x*e +
 d))*tan(1/2*x*e + 1/2*d) - 18*a*b^3*sgn(b*cos(x*e + d)^2 + a*cos(x*e + d))
*tan(1/2*x*e + 1/2*d))/(tan(1/2*x*e + 1/2*d)^2 - 1)^3)*e^(-1)
```

maple [A] time = 0.61, size = 387, normalized size = 1.08

$$\frac{\left(3\ln\left(\frac{1-\cos(ex+d)+\sin(ex+d)}{\sin(ex+d)}\right)\left(\cos^3\left(ex+d\right)\right)a^4+27\ln\left(\frac{1-\cos(ex+d)+\sin(ex+d)}{\sin(ex+d)}\right)\left(\cos^3\left(ex+d\right)\right)a^2b^2+6\ln\left(\frac{1-\cos(ex+d)}{\sin(ex}\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(e*x+d))*(b^2+2*a*b*sec(e*x+d)+a^2*sec(e*x+d)^2)^(3/2),x)
```

```
[Out] 1/6/e*(3*ln((1-cos(e*x+d)+sin(e*x+d))/sin(e*x+d))*cos(e*x+d)^3*a^4+27*ln((1
-cos(e*x+d)+sin(e*x+d))/sin(e*x+d))*cos(e*x+d)^3*a^2*b^2+6*ln((1-cos(e*x+d)
+sin(e*x+d))/sin(e*x+d))*cos(e*x+d)^3*b^4-3*ln(-(cos(e*x+d)-1+sin(e*x+d))/s
in(e*x+d))*cos(e*x+d)^3*a^4-27*ln(-(cos(e*x+d)-1+sin(e*x+d))/sin(e*x+d))*co
s(e*x+d)^3*a^2*b^2-6*ln(-(cos(e*x+d)-1+sin(e*x+d))/sin(e*x+d))*cos(e*x+d)^3
*b^4+6*cos(e*x+d)^3*(e*x+d)*a*b^3+22*sin(e*x+d)*cos(e*x+d)^2*a^3*b+18*sin(e
*x+d)*cos(e*x+d)^2*a*b^3+3*sin(e*x+d)*cos(e*x+d)*a^4+9*sin(e*x+d)*cos(e*x+d
)*a^2*b^2+2*a^3*b*sin(e*x+d))*((b*cos(e*x+d)+a)^2/cos(e*x+d)^2)^(3/2)/(b*co
s(e*x+d)+a)^3
```

maxima [A] time = 0.45, size = 440, normalized size = 1.23

$$\frac{3\left(4\,b^3\arctan\left(\frac{\sin(ex+d)}{\cos(ex+d)+1}\right)+\left(a^3+6\,ab^2\right)\log\left(\frac{\sin(ex+d)}{\cos(ex+d)+1}+1\right)-\left(a^3+6\,ab^2\right)\log\left(\frac{\sin(ex+d)}{\cos(ex+d)+1}-1\right)-\frac{2\left(\frac{\left(a^3+6\,a^2b\right)\sin(ex}{\cos(ex+d)+1}\right.}{\frac{2\,\sin(ex+d)^2}{(\cos(ex+d)+1}}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(e*x+d))*(b^2+2*a*b*sec(e*x+d)+a^2*sec(e*x+d)^2)^(3/2),x,
 algorithm="maxima")
```

```
[Out] 1/6*(3*(4*b^3*arctan(sin(e*x + d)/(cos(e*x + d) + 1)) + (a^3 + 6*a*b^2)*log
(sin(e*x + d)/(cos(e*x + d) + 1) + 1) - (a^3 + 6*a*b^2)*log(sin(e*x + d)/(c
os(e*x + d) + 1) - 1) - 2*((a^3 + 6*a^2*b)*sin(e*x + d)/(cos(e*x + d) + 1)
+ (a^3 - 6*a^2*b)*sin(e*x + d)^3/(cos(e*x + d) + 1)^3)/(2*sin(e*x + d)^2/(c
os(e*x + d) + 1)^2 - sin(e*x + d)^4/(cos(e*x + d) + 1)^4 - 1))*a + (3*(3*a^
2*b + 2*b^3)*log(sin(e*x + d)/(cos(e*x + d) + 1) + 1) - 3*(3*a^2*b + 2*b^3)
*log(sin(e*x + d)/(cos(e*x + d) + 1) - 1) - 2*(3*(2*a^3 + 3*a^2*b + 6*a*b^2
)*sin(e*x + d)/(cos(e*x + d) + 1) - 4*(a^3 + 9*a*b^2)*sin(e*x + d)^3/(cos(e
*x + d) + 1)^3 + 3*(2*a^3 - 3*a^2*b + 6*a*b^2)*sin(e*x + d)^5/(cos(e*x + d)
 + 1)^5)/(3*sin(e*x + d)^2/(cos(e*x + d) + 1)^2 - 3*sin(e*x + d)^4/(cos(e*x
 + d) + 1)^4 + sin(e*x + d)^6/(cos(e*x + d) + 1)^6 - 1))*b)/e
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \left(a+\frac{b}{\cos\left(d+e\,x\right)}\right)\,\left(b^2+\frac{a^2}{\cos\left(d+e\,x\right)^2}+\frac{2\,a\,b}{\cos\left(d+e\,x\right)}\right)^{3/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(d + e*x))*(b^2 + a^2/cos(d + e*x)^2 + (2*a*b)/cos(d + e*x))^
(3/2),x)
```

```
[Out] int((a + b/cos(d + e*x))*(b^2 + a^2/cos(d + e*x)^2 + (2*a*b)/cos(d + e*x))^
(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b\sec(d + ex))\left((a\sec(d + ex) + b)^2\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(e*x+d))*(b**2+2*a*b*sec(e*x+d)+a**2*sec(e*x+d)**2)**(3/2
),x)
```

```
[Out] Integral((a + b*sec(d + e*x))*((a*sec(d + e*x) + b)**2)**(3/2), x)
```

3.523 $\int (a+b\sec(d+ex))\sqrt{b^2+2ab\sec(d+ex)+a^2\sec^2(d+$

Optimal. Leaf size=173

$$\frac{a^2bx\sqrt{a^2\sec^2(d+ex)+2ab\sec(d+ex)+b^2}}{a^2\sec(d+ex)+ab}+\frac{a^2b\tan(d+ex)\sqrt{a^2\sec^2(d+ex)+2ab\sec(d+ex)+b^2}}{e\left(a^2\sec(d+ex)+ab\right)}+\frac{\left(a^2+b^2\right.}{}$$

```
[Out] (a^2+b^2)*arctanh(sin(e*x+d))*(b^2+2*a*b*sec(e*x+d)+a^2*sec(e*x+d)^2)^(1/2)
/e/(b+a*sec(e*x+d))+a^2*b*x*(b^2+2*a*b*sec(e*x+d)+a^2*sec(e*x+d)^2)^(1/2)/(
a*b+a^2*sec(e*x+d))+a^2*b*(b^2+2*a*b*sec(e*x+d)+a^2*sec(e*x+d)^2)^(1/2)*tan
(e*x+d)/e/(a*b+a^2*sec(e*x+d))
```

Rubi [A] time = 0.12, antiderivative size = 173, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.122, Rules used = {4174, 3914, 3767, 8, 3770}

$$\frac{a^2bx\sqrt{a^2\sec^2(d+ex)+2ab\sec(d+ex)+b^2}}{a^2\sec(d+ex)+ab}+\frac{a^2b\tan(d+ex)\sqrt{a^2\sec^2(d+ex)+2ab\sec(d+ex)+b^2}}{e\left(a^2\sec(d+ex)+ab\right)}+\frac{\left(a^2+b^2\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[d + e*x])*Sqrt[b^2 + 2*a*b*Sec[d + e*x] + a^2*Sec[d + e*x]^2
],x]
```

```
[Out] ((a^2 + b^2)*ArcTanh[Sin[d + e*x]]*Sqrt[b^2 + 2*a*b*Sec[d + e*x] + a^2*Sec[
d + e*x]^2])/(e*(b + a*Sec[d + e*x])) + (a^2*b*x*Sqrt[b^2 + 2*a*b*Sec[d + e
*x] + a^2*Sec[d + e*x]^2])/(a*b + a^2*Sec[d + e*x]) + (a^2*b*Sqrt[b^2 + 2*a
*b*Sec[d + e*x] + a^2*Sec[d + e*x]^2]*Tan[d + e*x])/(e*(a*b + a^2*Sec[d + e
*x]))
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3914

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))*(csc[(e_.) + (f_.)*(x_)]*(d_.) +
 (c_)), x_Symbol] :> Simp[a*c*x, x] + (Dist[b*d, Int[Csc[e + f*x]^2, x], x]
 + Dist[b*c + a*d, Int[Csc[e + f*x], x], x]) /; FreeQ[{a, b, c, d, e, f}, x
] && NeQ[b*c - a*d, 0] && NeQ[b*c + a*d, 0]
```

Rule 4174

```
Int[((A_) + (B_.)*sec[(d_.) + (e_.)*(x_)])*((a_) + (b_.)*sec[(d_.) + (e_.)*
(x_)] + (c_.)*sec[(d_.) + (e_.)*(x_)]^2)^(n_), x_Symbol] :> Dist[(a + b*Sec
[d + e*x] + c*Sec[d + e*x]^2)^n/(b + 2*c*Sec[d + e*x])^(2*n), Int[(A + B*Se
c[d + e*x])*(b + 2*c*Sec[d + e*x])^(2*n), x], x] /; FreeQ[{a, b, c, d, e, A
, B}, x] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[n]
```

Rubi steps

$$\int (a+b\sec(d+ex))\sqrt{b^2+2ab\sec(d+ex)+a^2\sec^2(d+ex)}\,dx = \frac{\sqrt{b^2+2ab\sec(d+ex)+a^2\sec^2(d+ex)}\int (\ldots}{2ab+2a^2\ldots}$$

$$= \frac{a^2bx\sqrt{b^2+2ab\sec(d+ex)+a^2\sec^2(d+ex\ldots}}{ab+a^2\sec(d+ex)}$$

$$= \frac{\left(a^2+b^2\right)\tanh^{-1}(\sin(d+ex))\sqrt{b^2+2ab\sec\ldots}}{e(b+a\sec(d+ex)\ldots}$$

$$= \frac{\left(a^2+b^2\right)\tanh^{-1}(\sin(d+ex))\sqrt{b^2+2ab\sec\ldots}}{e(b+a\sec(d+ex)\ldots}$$

Mathematica [A] time = 0.26, size = 67, normalized size = 0.39

$$\frac{\cos(d+ex)\sqrt{(a\sec(d+ex)+b)^2}\left(\left(a^2+b^2\right)\tanh^{-1}(\sin(d+ex))+ab(\tan(d+ex)+ex)\right)}{e(a+b\cos(d+ex))}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sec[d + e*x])*Sqrt[b^2 + 2*a*b*Sec[d + e*x] + a^2*Sec[d +
e*x]^2],x]
```

```
[Out] (Cos[d + e*x]*Sqrt[(b + a*Sec[d + e*x])^2]*((a^2 + b^2)*ArcTanh[Sin[d + e*x
]] + a*b*(e*x + Tan[d + e*x])))/(e*(a + b*Cos[d + e*x]))
```

fricas [A] time = 0.96, size = 85, normalized size = 0.49

$$\frac{2\,abex\cos\left(ex+d\right)+\left(a^{2}+b^{2}\right)\cos\left(ex+d\right)\log\left(\sin\left(ex+d\right)+1\right)-\left(a^{2}+b^{2}\right)\cos\left(ex+d\right)\log\left(-\sin\left(ex+d\right)+1\right)-}{2\,e\cos\left(ex+d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(e*x+d))*(b^2+2*a*b*sec(e*x+d)+a^2*sec(e*x+d)^2)^(1/2),x,
 algorithm="fricas")
```

```
[Out] 1/2*(2*a*b*e*x*cos(e*x + d) + (a^2 + b^2)*cos(e*x + d)*log(sin(e*x + d) + 1
) - (a^2 + b^2)*cos(e*x + d)*log(-sin(e*x + d) + 1) + 2*a*b*sin(e*x + d))/(
e*cos(e*x + d))
```

giac [A] time = 0.31, size = 224, normalized size = 1.29

$$\left((xe+d)ab\mathrm{sgn}\left(b\cos\left(xe+d\right)^{2}+a\cos\left(xe+d\right)\right)-\frac{2\,ab\mathrm{sgn}\left(b\cos\left(xe+d\right)^{2}+a\cos\left(xe+d\right)\right)\tan\left(\frac{1}{2}\,xe+\frac{1}{2}\,d\right)}{\tan\left(\frac{1}{2}\,xe+\frac{1}{2}\,d\right)^{2}-1}+\left(\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(e*x+d))*(b^2+2*a*b*sec(e*x+d)+a^2*sec(e*x+d)^2)^(1/2),x,
 algorithm="giac")
```

```
[Out] ((x*e + d)*a*b*sgn(b*cos(x*e + d)^2 + a*cos(x*e + d)) - 2*a*b*sgn(b*cos(x*e
 + d)^2 + a*cos(x*e + d))*tan(1/2*x*e + 1/2*d)/(tan(1/2*x*e + 1/2*d)^2 - 1)
 + (a^2*sgn(b*cos(x*e + d)^2 + a*cos(x*e + d)) + b^2*sgn(b*cos(x*e + d)^2 +
 a*cos(x*e + d)))*log(abs(tan(1/2*x*e + 1/2*d) + 1)) - (a^2*sgn(b*cos(x*e +
 d)^2 + a*cos(x*e + d)) + b^2*sgn(b*cos(x*e + d)^2 + a*cos(x*e + d)))*log(a
bs(tan(1/2*x*e + 1/2*d) - 1)))*e^(-1)
```

maple [A] time = 0.58, size = 211, normalized size = 1.22

$$-\frac{\left(\cos\left(ex+d\right)\ln\left(-\frac{\cos(ex+d)-1+\sin(ex+d)}{\sin(ex+d)}\right)a^{2}+\cos\left(ex+d\right)\ln\left(-\frac{\cos(ex+d)-1+\sin(ex+d)}{\sin(ex+d)}\right)b^{2}-\cos\left(ex+d\right)\ln\left(\frac{1-\cos(ex+d}{\sin(e}\right.\right.}{e\left(b\cos\left(\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(e*x+d))*(b^2+2*a*b*sec(e*x+d)+a^2*sec(e*x+d)^2)^(1/2),x)
```

```
[Out] -1/e*(cos(e*x+d)*ln(-(cos(e*x+d)-1+sin(e*x+d))/sin(e*x+d))*a^2+cos(e*x+d)*l
n(-(cos(e*x+d)-1+sin(e*x+d))/sin(e*x+d))*b^2-cos(e*x+d)*ln((1-cos(e*x+d)+si
n(e*x+d))/sin(e*x+d))*a^2-cos(e*x+d)*ln((1-cos(e*x+d)+sin(e*x+d))/sin(e*x+d
```

```
))*b^2-cos(e*x+d)*(e*x+d)*a*b-a*b*sin(e*x+d))*((b*cos(e*x+d)+a)^2/cos(e*x+d
)^2)^(1/2)/(b*cos(e*x+d)+a)
```

maxima [A] time = 0.44, size = 164, normalized size = 0.95

$$\frac{\left(2\,b\arctan\left(\frac{\sin(ex+d)}{\cos(ex+d)+1}\right)+a\log\left(\frac{\sin(ex+d)}{\cos(ex+d)+1}+1\right)-a\log\left(\frac{\sin(ex+d)}{\cos(ex+d)+1}-1\right)\right)a+\left(b\log\left(\frac{\sin(ex+d)}{\cos(ex+d)+1}+1\right)-b\log\left(\frac{s}{c}\right.\right.}{e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(e*x+d))*(b^2+2*a*b*sec(e*x+d)+a^2*sec(e*x+d)^2)^(1/2),x,
 algorithm="maxima")
```

```
[Out] ((2*b*arctan(sin(e*x + d)/(cos(e*x + d) + 1)) + a*log(sin(e*x + d)/(cos(e*x
 + d) + 1) + 1) - a*log(sin(e*x + d)/(cos(e*x + d) + 1) - 1))*a + (b*log(si
n(e*x + d)/(cos(e*x + d) + 1) + 1) - b*log(sin(e*x + d)/(cos(e*x + d) + 1)
- 1) - 2*a*sin(e*x + d)/((sin(e*x + d)^2/(cos(e*x + d) + 1)^2 - 1)*(cos(e*x
 + d) + 1)))*b)/e
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \left(a+\frac{b}{\cos\left(d+e\,x\right)}\right)\,\sqrt{b^2+\frac{a^2}{{\cos\left(d+e\,x\right)}^2}+\frac{2\,a\,b}{\cos\left(d+e\,x\right)}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(d + e*x))*(b^2 + a^2/cos(d + e*x)^2 + (2*a*b)/cos(d + e*x))^
(1/2),x)
```

```
[Out] int((a + b/cos(d + e*x))*(b^2 + a^2/cos(d + e*x)^2 + (2*a*b)/cos(d + e*x))^
(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a + b \sec{\left(d + e x \right)}\right) \sqrt{\left(a \sec{\left(d + e x \right)} + b\right)^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(e*x+d))*(b**2+2*a*b*sec(e*x+d)+a**2*sec(e*x+d)**2)**(1/2
),x)
```

```
[Out] Integral((a + b*sec(d + e*x))*sqrt((a*sec(d + e*x) + b)**2), x)
```

3.524 $\int \frac{a+b\sec(d+ex)}{\sqrt{b^2+2ab\sec(d+ex)+a^2\sec^2(d+ex)}}\,dx$

Optimal. Leaf size=142

$$\frac{x\left(a^2\sec(d+ex)+ab\right)}{b\sqrt{a^2\sec^2(d+ex)+2ab\sec(d+ex)+b^2}}-\frac{2\sqrt{a-b}\sqrt{a+b}\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(d+ex)\right)}{\sqrt{a+b}}\right)(a\sec(d+ex)+b)}{be\sqrt{a^2\sec^2(d+ex)+2ab\sec(d+ex)+b^2}}$$

```
[Out] x*(a*b+a^2*sec(e*x+d))/b/(b^2+2*a*b*sec(e*x+d)+a^2*sec(e*x+d)^2)^(1/2)-2*ar
ctan((a-b)^(1/2)*tan(1/2*e*x+1/2*d)/(a+b)^(1/2))*(b+a*sec(e*x+d))*(a-b)^(1/
2)*(a+b)^(1/2)/b/e/(b^2+2*a*b*sec(e*x+d)+a^2*sec(e*x+d)^2)^(1/2)
```

Rubi [A] time = 0.21, antiderivative size = 142, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.122, Rules used = {4174, 3919, 3831, 2659, 205}

$$\frac{x\left(a^2\sec(d+ex)+ab\right)}{b\sqrt{a^2\sec^2(d+ex)+2ab\sec(d+ex)+b^2}}-\frac{2\sqrt{a-b}\sqrt{a+b}\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(d+ex)\right)}{\sqrt{a+b}}\right)(a\sec(d+ex)+b)}{be\sqrt{a^2\sec^2(d+ex)+2ab\sec(d+ex)+b^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[d + e*x])/Sqrt[b^2 + 2*a*b*Sec[d + e*x] + a^2*Sec[d + e*x]^2
],x]
```

```
[Out] (-2*Sqrt[a - b]*Sqrt[a + b]*ArcTan[(Sqrt[a - b]*Tan[(d + e*x)/2])/Sqrt[a +
b]]*(b + a*Sec[d + e*x]))/(b*e*Sqrt[b^2 + 2*a*b*Sec[d + e*x] + a^2*Sec[d +
e*x]^2]) + (x*(a*b + a^2*Sec[d + e*x]))/(b*Sqrt[b^2 + 2*a*b*Sec[d + e*x] +
a^2*Sec[d + e*x]^2])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 3831

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Dist[1/b, Int[1/(1 + (a*Sin[e + f*x])/b), x], x] /; FreeQ[{a, b, e, f
}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3919

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
 (a_)), x_Symbol] :> Simp[(c*x)/a, x] - Dist[(b*c - a*d)/a, Int[Csc[e + f*x
]/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0]
```

Rule 4174

```
Int[((A_) + (B_.)*sec[(d_.) + (e_.)*(x_)])*((a_) + (b_.)*sec[(d_.) + (e_.)*
(x_)] + (c_.)*sec[(d_.) + (e_.)*(x_)]^2)^(n_), x_Symbol] :> Dist[(a + b*Sec
[d + e*x] + c*Sec[d + e*x]^2)^n/(b + 2*c*Sec[d + e*x])^(2*n), Int[(A + B*Se
c[d + e*x])*(b + 2*c*Sec[d + e*x])^(2*n), x], x] /; FreeQ[{a, b, c, d, e, A
, B}, x] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[n]
```

Rubi steps

$$\int \frac{a+b\sec(d+ex)}{\sqrt{b^2+2ab\sec(d+ex)+a^2\sec^2(d+ex)}}\,dx = \frac{\left(2ab+2a^2\sec(d+ex)\right)\int\frac{a+b\sec(d+ex)}{2ab+2a^2\sec(d+ex)}\,dx}{\sqrt{b^2+2ab\sec(d+ex)+a^2\sec^2(d+ex)}}$$

$$= \frac{x\left(ab+a^2\sec(d+ex)\right)}{b\sqrt{b^2+2ab\sec(d+ex)+a^2\sec^2(d+ex)}} - \frac{\left(\left(2a^3-2ab^2\right)\left(2a\right.\right.}{2ab\sqrt{b^2+}}$$

$$= \frac{x\left(ab+a^2\sec(d+ex)\right)}{b\sqrt{b^2+2ab\sec(d+ex)+a^2\sec^2(d+ex)}} - \frac{\left(\left(2a^3-2ab^2\right)\left(2a\right.\right.}{4a^3b\sqrt{b^2+2}}$$

$$= \frac{x\left(ab+a^2\sec(d+ex)\right)}{b\sqrt{b^2+2ab\sec(d+ex)+a^2\sec^2(d+ex)}} - \frac{\left(\left(2a^3-2ab^2\right)\left(2a\right.\right.}{}$$

$$= -\frac{2\sqrt{a-b}\,\sqrt{a+b}\,\tan^{-1}\left(\frac{\sqrt{a-b}\,\tan\left(\frac{1}{2}(d+ex)\right)}{\sqrt{a+b}}\right)(b+a\sec(d+ex))}{be\sqrt{b^2+2ab\sec(d+ex)+a^2\sec^2(d+ex)}}$$

Mathematica [A] time = 0.37, size = 92, normalized size = 0.65

$$\frac{\sec(d+ex)(a+b\cos(d+ex))\left(2\sqrt{b^2-a^2}\tanh^{-1}\left(\frac{(b-a)\tan\left(\frac{1}{2}(d+ex)\right)}{\sqrt{b^2-a^2}}\right)+a(d+ex)\right)}{be\sqrt{(a\sec(d+ex)+b)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sec[d + e*x])/Sqrt[b^2 + 2*a*b*Sec[d + e*x] + a^2*Sec[d +
e*x]^2],x]
```

```
[Out] ((a*(d + e*x) + 2*Sqrt[-a^2 + b^2]*ArcTanh[((-a + b)*Tan[(d + e*x)/2])/Sqrt
[-a^2 + b^2]])*(a + b*Cos[d + e*x])*Sec[d + e*x])/(b*e*Sqrt[(b + a*Sec[d +
e*x])^2])
```

fricas [A] time = 1.06, size = 184, normalized size = 1.30

$$\left[\frac{2\,aex+\sqrt{-a^2+b^2}\,\log\left(\frac{2\,ab\cos(ex+d)+\left(2\,a^2-b^2\right)\cos(ex+d)^2+2\,\sqrt{-a^2+b^2}\,(a\cos(ex+d)+b)\sin(ex+d)-a^2+2\,b^2}{b^2\cos(ex+d)^2+2\,ab\cos(ex+d)+a^2}\right)}{2\,be}, \frac{aex-\sqrt{a^2-b^2}\,\text{arct}}{b}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(e*x+d))/(b^2+2*a*b*sec(e*x+d)+a^2*sec(e*x+d)^2)^(1/2),x,
 algorithm="fricas")
```

```
[Out] [1/2*(2*a*e*x + sqrt(-a^2 + b^2)*log((2*a*b*cos(e*x + d) + (2*a^2 - b^2)*co
s(e*x + d)^2 + 2*sqrt(-a^2 + b^2)*(a*cos(e*x + d) + b)*sin(e*x + d) - a^2 +
 2*b^2)/(b^2*cos(e*x + d)^2 + 2*a*b*cos(e*x + d) + a^2)))/(b*e), (a*e*x - s
qrt(a^2 - b^2)*arctan(-(a*cos(e*x + d) + b)/(sqrt(a^2 - b^2)*sin(e*x + d)))
)/(b*e)]
```

giac [B] time = 4.83, size = 1504, normalized size = 10.59

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(e*x+d))/(b^2+2*a*b*sec(e*x+d)+a^2*sec(e*x+d)^2)^(1/2),x,
 algorithm="giac")
```

```
[Out] ((sqrt(a^2 - b^2)*abs(a - b)*abs(b)*abs(sgn(a*tan(1/2*x*e + 1/2*d)^4 - b*ta
n(1/2*x*e + 1/2*d)^4 + 2*b*tan(1/2*x*e + 1/2*d)^2 - a - b)) + sqrt(a^2 - b^
2)*(2*a + b)*abs(a - b)*sgn(a*tan(1/2*x*e + 1/2*d)^4 - b*tan(1/2*x*e + 1/2*
d)^4 + 2*b*tan(1/2*x*e + 1/2*d)^2 - a - b))*arctan(tan(1/2*x*e + 1/2*d)/sqr
```

```
t((a*sgn(a*tan(1/2*x*e + 1/2*d)^4 - b*tan(1/2*x*e + 1/2*d)^4 + 2*b*tan(1/2*
x*e + 1/2*d)^2 - a - b) + sqrt(-(a*sgn(a*tan(1/2*x*e + 1/2*d)^4 - b*tan(1/2
*x*e + 1/2*d)^4 + 2*b*tan(1/2*x*e + 1/2*d)^2 - a - b) + b*sgn(a*tan(1/2*x*e
 + 1/2*d)^4 - b*tan(1/2*x*e + 1/2*d)^4 + 2*b*tan(1/2*x*e + 1/2*d)^2 - a - b
))*(a*sgn(a*tan(1/2*x*e + 1/2*d)^4 - b*tan(1/2*x*e + 1/2*d)^4 + 2*b*tan(1/2
*x*e + 1/2*d)^2 - a - b) - b*sgn(a*tan(1/2*x*e + 1/2*d)^4 - b*tan(1/2*x*e +
 1/2*d)^4 + 2*b*tan(1/2*x*e + 1/2*d)^2 - a - b)) + a^2))/(a*sgn(a*tan(1/2*x
*e + 1/2*d)^4 - b*tan(1/2*x*e + 1/2*d)^4 + 2*b*tan(1/2*x*e + 1/2*d)^2 - a -
 b) - b*sgn(a*tan(1/2*x*e + 1/2*d)^4 - b*tan(1/2*x*e + 1/2*d)^4 + 2*b*tan(1
/2*x*e + 1/2*d)^2 - a - b))))/((a^2 - a*b)*abs(b)*abs(sgn(a*tan(1/2*x*e + 1
/2*d)^4 - b*tan(1/2*x*e + 1/2*d)^4 + 2*b*tan(1/2*x*e + 1/2*d)^2 - a - b))*s
gn(a*tan(1/2*x*e + 1/2*d)^4 - b*tan(1/2*x*e + 1/2*d)^4 + 2*b*tan(1/2*x*e +
1/2*d)^2 - a - b) + (a - b)*b^2) + (a*abs(b)*abs(sgn(a*tan(1/2*x*e + 1/2*d)
^4 - b*tan(1/2*x*e + 1/2*d)^4 + 2*b*tan(1/2*x*e + 1/2*d)^2 - a - b)) - b*ab
s(b)*abs(sgn(a*tan(1/2*x*e + 1/2*d)^4 - b*tan(1/2*x*e + 1/2*d)^4 + 2*b*tan(
1/2*x*e + 1/2*d)^2 - a - b)) - 2*a^2*sgn(a*tan(1/2*x*e + 1/2*d)^4 - b*tan(1
/2*x*e + 1/2*d)^4 + 2*b*tan(1/2*x*e + 1/2*d)^2 - a - b) + a*b*sgn(a*tan(1/2
*x*e + 1/2*d)^4 - b*tan(1/2*x*e + 1/2*d)^4 + 2*b*tan(1/2*x*e + 1/2*d)^2 - a
 - b) + b^2*sgn(a*tan(1/2*x*e + 1/2*d)^4 - b*tan(1/2*x*e + 1/2*d)^4 + 2*b*t
an(1/2*x*e + 1/2*d)^2 - a - b))*arctan(tan(1/2*x*e + 1/2*d)/sqrt((a*sgn(a*t
an(1/2*x*e + 1/2*d)^4 - b*tan(1/2*x*e + 1/2*d)^4 + 2*b*tan(1/2*x*e + 1/2*d)
^2 - a - b) - sqrt(-(a*sgn(a*tan(1/2*x*e + 1/2*d)^4 - b*tan(1/2*x*e + 1/2*d
)^4 + 2*b*tan(1/2*x*e + 1/2*d)^2 - a - b) + b*sgn(a*tan(1/2*x*e + 1/2*d)^4
- b*tan(1/2*x*e + 1/2*d)^4 + 2*b*tan(1/2*x*e + 1/2*d)^2 - a - b))*(a*sgn(a*
tan(1/2*x*e + 1/2*d)^4 - b*tan(1/2*x*e + 1/2*d)^4 + 2*b*tan(1/2*x*e + 1/2*d
)^2 - a - b) - b*sgn(a*tan(1/2*x*e + 1/2*d)^4 - b*tan(1/2*x*e + 1/2*d)^4 +
2*b*tan(1/2*x*e + 1/2*d)^2 - a - b)) + a^2))/(a*sgn(a*tan(1/2*x*e + 1/2*d)^
4 - b*tan(1/2*x*e + 1/2*d)^4 + 2*b*tan(1/2*x*e + 1/2*d)^2 - a - b) - b*sgn(
a*tan(1/2*x*e + 1/2*d)^4 - b*tan(1/2*x*e + 1/2*d)^4 + 2*b*tan(1/2*x*e + 1/2
*d)^2 - a - b))))/(a*abs(b)*abs(sgn(a*tan(1/2*x*e + 1/2*d)^4 - b*tan(1/2*x*
e + 1/2*d)^4 + 2*b*tan(1/2*x*e + 1/2*d)^2 - a - b))*sgn(a*tan(1/2*x*e + 1/2
*d)^4 - b*tan(1/2*x*e + 1/2*d)^4 + 2*b*tan(1/2*x*e + 1/2*d)^2 - a - b) - b^
2))*e^(-1)
```

maple [A] time = 0.66, size = 157, normalized size = 1.11

$$\frac{(b\cos(ex+d)+a)\left(2\arctan\left(\frac{(\cos(ex+d)-1)(a-b)}{\sin(ex+d)\sqrt{(a+b)(a-b)}}\right)a^2-2\arctan\left(\frac{(\cos(ex+d)-1)(a-b)}{\sin(ex+d)\sqrt{(a+b)(a-b)}}\right)b^2+a(ex+d)\sqrt{(a+b)(a-}\right.}{e\cos(ex+d)\sqrt{\frac{(b\cos(ex+d)+a)^2}{\cos(ex+d)^2}}\;b\sqrt{(a+b)(a-b)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(e*x+d))/(b^2+2*a*b*sec(e*x+d)+a^2*sec(e*x+d)^2)^(1/2),x)
```

```
[Out] 1/e*(b*cos(e*x+d)+a)*(2*arctan((cos(e*x+d)-1)*(a-b)/sin(e*x+d)/((a+b)*(a-b)
```

```
)^(1/2))*a^2-2*arctan((cos(e*x+d)-1)*(a-b)/sin(e*x+d)/((a+b)*(a-b))^(1/2))*
b^2+a*(e*x+d)*((a+b)*(a-b))^(1/2))/cos(e*x+d)/((b*cos(e*x+d)+a)^2/cos(e*x+d
)^2)^(1/2)/b/((a+b)*(a-b))^(1/2)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(e*x+d))/(b^2+2*a*b*sec(e*x+d)+a^2*sec(e*x+d)^2)^(1/2),x,
 algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more details)Is 4*b^2-4*a^2 positive or negative?
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{a+\frac{b}{\cos(d+e\,x)}}{\sqrt{b^2+\frac{a^2}{\cos(d+e\,x)^2}+\frac{2\,a\,b}{\cos(d+e\,x)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(d + e*x))/(b^2 + a^2/cos(d + e*x)^2 + (2*a*b)/cos(d + e*x))^
(1/2),x)
```

```
[Out] int((a + b/cos(d + e*x))/(b^2 + a^2/cos(d + e*x)^2 + (2*a*b)/cos(d + e*x))^
(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a+b\sec(d+ex)}{\sqrt{(a\sec(d+ex)+b)^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(e*x+d))/(b**2+2*a*b*sec(e*x+d)+a**2*sec(e*x+d)**2)**(1/2
),x)
```

```
[Out] Integral((a + b*sec(d + e*x))/sqrt((a*sec(d + e*x) + b)**2), x)
```

3.525 $\int \frac{a+b\sec(d+ex)}{\left(b^2+2ab\sec(d+ex)+a^2\sec^2(d+ex)\right)^{3/2}}\,dx$

Optimal. Leaf size=330

$$-\frac{\tan(d+ex)\left(a^2\sec(d+ex)+ab\right)}{2be\left(a^2\sec^2(d+ex)+2ab\sec(d+ex)+b^2\right)^{3/2}}+\frac{x\left(a^2\sec(d+ex)+ab\right)^3}{a^2b^3\left(a^2\sec^2(d+ex)+2ab\sec(d+ex)+b^2\right)^{3/2}}-\frac{\left(2a^4-3a^2b^2\right.}{b^3e(a-b)^{3/2}}$$

```
[Out] -(2*a^4-3*a^2*b^2+2*b^4)*arctan((a-b)^(1/2)*tan(1/2*e*x+1/2*d)/(a+b)^(1/2))
*(b+a*sec(e*x+d))^3/(a-b)^(3/2)/b^3/(a+b)^(3/2)/e/(b^2+2*a*b*sec(e*x+d)+a^2
*sec(e*x+d)^2)^(3/2)+x*(a*b+a^2*sec(e*x+d))^3/a^2/b^3/(b^2+2*a*b*sec(e*x+d)
+a^2*sec(e*x+d)^2)^(3/2)-1/2*(a*b+a^2*sec(e*x+d))*tan(e*x+d)/b/e/(b^2+2*a*b
*sec(e*x+d)+a^2*sec(e*x+d)^2)^(3/2)-1/2*(2*a^2-3*b^2)*(a*b+a^2*sec(e*x+d))^
3*tan(e*x+d)/b^2/(a^2-b^2)/e/(a^2*b+a^3*sec(e*x+d))/(b^2+2*a*b*sec(e*x+d)+a
^2*sec(e*x+d)^2)^(3/2)
```

Rubi [A] time = 0.57, antiderivative size = 330, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.171, Rules used = {4174, 3923, 4060, 3919, 3831, 2659, 205}

$$\frac{x\left(a^2\sec(d+ex)+ab\right)^3}{a^2b^3\left(a^2\sec^2(d+ex)+2ab\sec(d+ex)+b^2\right)^{3/2}}-\frac{\left(-3a^2b^2+2a^4+2b^4\right)\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(d+ex)\right)}{\sqrt{a+b}}\right)(a\sec(d+ex)+}{b^3e(a-b)^{3/2}(a+b)^{3/2}\left(a^2\sec^2(d+ex)+2ab\sec(d+ex)+b^2\right.}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[d + e*x])/(b^2 + 2*a*b*Sec[d + e*x] + a^2*Sec[d + e*x]^2)^(3
/2),x]
```

```
[Out] -(((2*a^4 - 3*a^2*b^2 + 2*b^4)*ArcTan[(Sqrt[a - b]*Tan[(d + e*x)/2])/Sqrt[a
 + b]]*(b + a*Sec[d + e*x])^3)/((a - b)^(3/2)*b^3*(a + b)^(3/2)*e*(b^2 + 2*
a*b*Sec[d + e*x] + a^2*Sec[d + e*x]^2)^(3/2))) + (x*(a*b + a^2*Sec[d + e*x]
)^3)/(a^2*b^3*(b^2 + 2*a*b*Sec[d + e*x] + a^2*Sec[d + e*x]^2)^(3/2)) - ((a*
b + a^2*Sec[d + e*x])*Tan[d + e*x])/(2*b*e*(b^2 + 2*a*b*Sec[d + e*x] + a^2*
Sec[d + e*x]^2)^(3/2)) - ((2*a^2 - 3*b^2)*(a*b + a^2*Sec[d + e*x])^3*Tan[d
+ e*x])/(2*b^2*(a^2 - b^2)*e*(a^2*b + a^3*Sec[d + e*x])*(b^2 + 2*a*b*Sec[d
+ e*x] + a^2*Sec[d + e*x]^2)^(3/2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 3831

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Dist[1/b, Int[1/(1 + (a*Sin[e + f*x])/b), x], x] /; FreeQ[{a, b, e, f
}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3919

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
 (a_)), x_Symbol] :> Simp[(c*x)/a, x] - Dist[(b*c - a*d)/a, Int[Csc[e + f*x
]/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0]
```

Rule 3923

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d
_.) + (c_)), x_Symbol] :> Simp[(b*(b*c - a*d)*Cot[e + f*x]*(a + b*Csc[e + f
*x])^(m + 1))/(a*f*(m + 1)*(a^2 - b^2)), x] + Dist[1/(a*(m + 1)*(a^2 - b^2)
), Int[(a + b*Csc[e + f*x])^(m + 1)*Simp[c*(a^2 - b^2)*(m + 1) - (a*(b*c -
a*d)*(m + 1))*Csc[e + f*x] + b*(b*c - a*d)*(m + 2)*Csc[e + f*x]^2, x], x],
x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] && Ne
Q[a^2 - b^2, 0] && IntegerQ[2*m]
```

Rule 4060

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_Symbol] :> Simp[((A*b^2 -
 a*b*B + a^2*C)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1))/(a*f*(m + 1)*(a^
2 - b^2)), x] + Dist[1/(a*(m + 1)*(a^2 - b^2)), Int[(a + b*Csc[e + f*x])^(m
 + 1)*Simp[A*(a^2 - b^2)*(m + 1) - a*(A*b - a*B + b*C)*(m + 1)*Csc[e + f*x]
 + (A*b^2 - a*b*B + a^2*C)*(m + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a,
b, e, f, A, B, C}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1]
```

Rule 4174

```
Int[((A_) + (B_.)*sec[(d_.) + (e_.)*(x_)])*((a_) + (b_.)*sec[(d_.) + (e_.)*
(x_)] + (c_.)*sec[(d_.) + (e_.)*(x_)]^2)^(n_), x_Symbol] :> Dist[(a + b*Sec
[d + e*x] + c*Sec[d + e*x]^2)^n/(b + 2*c*Sec[d + e*x])^(2*n), Int[(A + B*Se
```

```
c[d + e*x])*(b + 2*c*Sec[d + e*x])^(2*n), x], x] /; FreeQ[{a, b, c, d, e, A
, B}, x] && EqQ[b^2 - 4*a*c, 0] && !IntegerQ[n]
```

Rubi steps

$$\int \frac{a+b\sec(d+ex)}{\left(b^2+2ab\sec(d+ex)+a^2\sec^2(d+ex)\right)^{3/2}}\,dx = \frac{\left(2ab+2a^2\sec(d+ex)\right)^3 \int \frac{a+b\sec(d+ex)}{\left(2ab+2a^2\sec(d+ex)\right)^3}\,dx}{\left(b^2+2ab\sec(d+ex)+a^2\sec^2(d+ex)\right)^{3/2}}$$

$$= -\frac{\left(ab+a^2\sec(d+ex)\right)\tan(d+ex)}{2be\left(b^2+2ab\sec(d+ex)+a^2\sec^2(d+ex)\right)^{3/2}} + \frac{\left(2ab+2\cdots\right.}{\cdots}$$

$$= -\frac{\left(ab+a^2\sec(d+ex)\right)\tan(d+ex)}{2be\left(b^2+2ab\sec(d+ex)+a^2\sec^2(d+ex)\right)^{3/2}} - \frac{\left(2\cdots\right.}{2ab^2\left(a^2\cdots\right.}$$

$$= \frac{x\left(ab+a^2\sec(d+ex)\right)^3}{a^2b^3\left(b^2+2ab\sec(d+ex)+a^2\sec^2(d+ex)\right)^{3/2}} - \frac{\left(a\cdots\right.}{2be\left(b^2+\cdots\right.}$$

$$= \frac{x\left(ab+a^2\sec(d+ex)\right)^3}{a^2b^3\left(b^2+2ab\sec(d+ex)+a^2\sec^2(d+ex)\right)^{3/2}} - \frac{\left(a\cdots\right.}{2be\left(b^2+\cdots\right.}$$

$$= \frac{x\left(ab+a^2\sec(d+ex)\right)^3}{a^2b^3\left(b^2+2ab\sec(d+ex)+a^2\sec^2(d+ex)\right)^{3/2}} - \frac{\left(a\cdots\right.}{2be\left(b^2+\cdots\right.}$$

$$= -\frac{\left(2a^4-3a^2b^2+2b^4\right)\tan^{-1}\left(\frac{\sqrt{a-b}\,\tan\left(\frac{1}{2}(d+ex)\right)}{\sqrt{a+b}}\right)(b+a\sec(d+\cdots}{(a-b)^{3/2}b^3(a+b)^{3/2}e\left(b^2+2ab\sec(d+ex)+a^2\sec^2(d+\cdots\right.}$$

Mathematica [A] time = 1.00, size = 216, normalized size = 0.65

$$\frac{\sec^2(d+ex)(a+b\cos(d+ex))(a+b\sec(d+ex))\left(\frac{ab\left(3a^2-4b^2\right)\sin(d+ex)(a+b\cos(d+ex))}{(b-a)(a+b)} + a^2b\sin(d+ex) + \frac{2\left(2a^4-3a^2b^2\cdots\right.}{\cdots}\right.}{2b^3e(a\cos(d+ex)+b)\left((a\sec(d+ex)+b)^2\cdots\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sec[d + e*x])/(b^2 + 2*a*b*Sec[d + e*x] + a^2*Sec[d + e*x]
^2)^(3/2),x]
```

```
[Out] ((a + b*Cos[d + e*x])*Sec[d + e*x]^2*(a + b*Sec[d + e*x])*(2*a*(d + e*x)*(a
 + b*Cos[d + e*x])^2 + (2*(2*a^4 - 3*a^2*b^2 + 2*b^4)*ArcTanh[((-a + b)*Tan
[(d + e*x)/2])/Sqrt[-a^2 + b^2]]*(a + b*Cos[d + e*x])^2)/(-a^2 + b^2)^(3/2)
 + a^2*b*Sin[d + e*x] + (a*b*(3*a^2 - 4*b^2)*(a + b*Cos[d + e*x])*Sin[d + e
*x])/((-a + b)*(a + b))))/(2*b^3*e*(b + a*Cos[d + e*x])*((b + a*Sec[d + e*x
])^2)^(3/2))
```

fricas [A] time = 1.09, size = 798, normalized size = 2.42

$$\left[\frac{4\left(a^5b^2 - 2\,a^3b^4 + ab^6\right)ex\cos\left(ex+d\right)^2 + 8\left(a^6b - 2\,a^4b^3 + a^2b^5\right)ex\cos\left(ex+d\right) + 4\left(a^7 - 2\,a^5b^2 + a^3b^4\right)ex + \left(2\,a\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(e*x+d))/(b^2+2*a*b*sec(e*x+d)+a^2*sec(e*x+d)^2)^(3/2),x,
 algorithm="fricas")
```

```
[Out] [1/4*(4*(a^5*b^2 - 2*a^3*b^4 + a*b^6)*e*x*cos(e*x + d)^2 + 8*(a^6*b - 2*a^4
*b^3 + a^2*b^5)*e*x*cos(e*x + d) + 4*(a^7 - 2*a^5*b^2 + a^3*b^4)*e*x + (2*a
^6 - 3*a^4*b^2 + 2*a^2*b^4 + (2*a^4*b^2 - 3*a^2*b^4 + 2*b^6)*cos(e*x + d)^2
 + 2*(2*a^5*b - 3*a^3*b^3 + 2*a*b^5)*cos(e*x + d))*sqrt(-a^2 + b^2)*log((2*
a*b*cos(e*x + d) + (2*a^2 - b^2)*cos(e*x + d)^2 + 2*sqrt(-a^2 + b^2)*(a*cos
(e*x + d) + b)*sin(e*x + d) - a^2 + 2*b^2)/(b^2*cos(e*x + d)^2 + 2*a*b*cos(
e*x + d) + a^2)) - 2*(2*a^6*b - 5*a^4*b^3 + 3*a^2*b^5 + (3*a^5*b^2 - 7*a^3*
b^4 + 4*a*b^6)*cos(e*x + d))*sin(e*x + d))/((a^4*b^5 - 2*a^2*b^7 + b^9)*e*c
os(e*x + d)^2 + 2*(a^5*b^4 - 2*a^3*b^6 + a*b^8)*e*cos(e*x + d) + (a^6*b^3 -
 2*a^4*b^5 + a^2*b^7)*e), 1/2*(2*(a^5*b^2 - 2*a^3*b^4 + a*b^6)*e*x*cos(e*x
+ d)^2 + 4*(a^6*b - 2*a^4*b^3 + a^2*b^5)*e*x*cos(e*x + d) + 2*(a^7 - 2*a^5*
b^2 + a^3*b^4)*e*x - (2*a^6 - 3*a^4*b^2 + 2*a^2*b^4 + (2*a^4*b^2 - 3*a^2*b^
4 + 2*b^6)*cos(e*x + d)^2 + 2*(2*a^5*b - 3*a^3*b^3 + 2*a*b^5)*cos(e*x + d))
*sqrt(a^2 - b^2)*arctan(-(a*cos(e*x + d) + b)/(sqrt(a^2 - b^2)*sin(e*x + d)
)) - (2*a^6*b - 5*a^4*b^3 + 3*a^2*b^5 + (3*a^5*b^2 - 7*a^3*b^4 + 4*a*b^6)*c
os(e*x + d))*sin(e*x + d))/((a^4*b^5 - 2*a^2*b^7 + b^9)*e*cos(e*x + d)^2 +
2*(a^5*b^4 - 2*a^3*b^6 + a*b^8)*e*cos(e*x + d) + (a^6*b^3 - 2*a^4*b^5 + a^2
*b^7)*e)]
```

giac [A] time = 5.12, size = 569, normalized size = 1.72

$$\left(\frac{\left(2\,a^4 - 3\,a^2b^2 + 2\,b^4\right)\arctan\left(\frac{a\tan\left(\frac{1}{2}\,xe+\frac{1}{2}\,d\right)-b\tan\left(\frac{1}{2}\,xe\right.}{\sqrt{a^2-b^2}}\right.}{\left(a^2b^3\mathrm{sgn}\left(a\tan\left(\frac{1}{2}\,xe+\frac{1}{2}\,d\right)^4 - b\tan\left(\frac{1}{2}\,xe+\frac{1}{2}\,d\right)^4 + 2\,b\tan\left(\frac{1}{2}\,xe+\frac{1}{2}\,d\right)^2 - a - b\right) - b^5\mathrm{sgn}\left(a\tan\left(\frac{1}{2}\,xe+\frac{1}{2}\,d\right)\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(e*x+d))/(b^2+2*a*b*sec(e*x+d)+a^2*sec(e*x+d)^2)^(3/2),x,
 algorithm="giac")
```

```
[Out] ((2*a^4 - 3*a^2*b^2 + 2*b^4)*arctan((a*tan(1/2*x*e + 1/2*d) - b*tan(1/2*x*e
 + 1/2*d))/sqrt(a^2 - b^2))/((a^2*b^3*sgn(a*tan(1/2*x*e + 1/2*d)^4 - b*tan(
1/2*x*e + 1/2*d)^4 + 2*b*tan(1/2*x*e + 1/2*d)^2 - a - b) - b^5*sgn(a*tan(1/
2*x*e + 1/2*d)^4 - b*tan(1/2*x*e + 1/2*d)^4 + 2*b*tan(1/2*x*e + 1/2*d)^2 -
a - b))*sqrt(a^2 - b^2)) + (2*a^4*tan(1/2*x*e + 1/2*d)^3 - 3*a^3*b*tan(1/2*
x*e + 1/2*d)^3 - 3*a^2*b^2*tan(1/2*x*e + 1/2*d)^3 + 4*a*b^3*tan(1/2*x*e + 1
/2*d)^3 + 2*a^4*tan(1/2*x*e + 1/2*d) + 3*a^3*b*tan(1/2*x*e + 1/2*d) - 3*a^2
*b^2*tan(1/2*x*e + 1/2*d) - 4*a*b^3*tan(1/2*x*e + 1/2*d))/((a^2*b^2*sgn(a*t
an(1/2*x*e + 1/2*d)^4 - b*tan(1/2*x*e + 1/2*d)^4 + 2*b*tan(1/2*x*e + 1/2*d)
^2 - a - b) - b^4*sgn(a*tan(1/2*x*e + 1/2*d)^4 - b*tan(1/2*x*e + 1/2*d)^4 +
 2*b*tan(1/2*x*e + 1/2*d)^2 - a - b))*(a*tan(1/2*x*e + 1/2*d)^2 - b*tan(1/2
*x*e + 1/2*d)^2 + a + b)^2) - (x*e - 2*pi*floor(1/2*(x*e + d)/pi + 1/2) + d
)*a/(b^3*sgn(a*tan(1/2*x*e + 1/2*d)^4 - b*tan(1/2*x*e + 1/2*d)^4 + 2*b*tan(
1/2*x*e + 1/2*d)^2 - a - b)))*e^(-1)
```

maple [B] time = 0.61, size = 756, normalized size = 2.29

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(e*x+d))/(b^2+2*a*b*sec(e*x+d)+a^2*sec(e*x+d)^2)^(3/2),x)
```

```
[Out] 1/2/e*(b*cos(e*x+d)+a)*(4*arctan((cos(e*x+d)-1)*(a-b)/sin(e*x+d)/((a+b)*(a-
b))^(1/2))*cos(e*x+d)^2*a^4*b^2-6*arctan((cos(e*x+d)-1)*(a-b)/sin(e*x+d)/((
a+b)*(a-b))^(1/2))*cos(e*x+d)^2*a^2*b^4+4*arctan((cos(e*x+d)-1)*(a-b)/sin(e
*x+d)/((a+b)*(a-b))^(1/2))*cos(e*x+d)^2*b^6+2*cos(e*x+d)^2*((a+b)*(a-b))^(1
/2)*(e*x+d)*a^3*b^2-2*cos(e*x+d)^2*((a+b)*(a-b))^(1/2)*(e*x+d)*a*b^4+8*arct
an((cos(e*x+d)-1)*(a-b)/sin(e*x+d)/((a+b)*(a-b))^(1/2))*cos(e*x+d)*a^5*b-12
*arctan((cos(e*x+d)-1)*(a-b)/sin(e*x+d)/((a+b)*(a-b))^(1/2))*cos(e*x+d)*a^3
*b^3+8*arctan((cos(e*x+d)-1)*(a-b)/sin(e*x+d)/((a+b)*(a-b))^(1/2))*cos(e*x+
d)*a*b^5-3*cos(e*x+d)*((a+b)*(a-b))^(1/2)*sin(e*x+d)*a^3*b^2+4*cos(e*x+d)*(
(a+b)*(a-b))^(1/2)*sin(e*x+d)*a*b^4+4*cos(e*x+d)*((a+b)*(a-b))^(1/2)*(e*x+d
```

```
)*a^4*b-4*cos(e*x+d)*((a+b)*(a-b))^(1/2)*(e*x+d)*a^2*b^3+4*arctan((cos(e*x+
d)-1)*(a-b)/sin(e*x+d)/((a+b)*(a-b))^(1/2))*a^6-6*arctan((cos(e*x+d)-1)*(a-
b)/sin(e*x+d)/((a+b)*(a-b))^(1/2))*a^4*b^2+4*arctan((cos(e*x+d)-1)*(a-b)/si
n(e*x+d)/((a+b)*(a-b))^(1/2))*a^2*b^4-2*((a+b)*(a-b))^(1/2)*a^4*b*sin(e*x+d
)+3*((a+b)*(a-b))^(1/2)*a^2*b^3*sin(e*x+d)+2*((a+b)*(a-b))^(1/2)*(e*x+d)*a^
5-2*((a+b)*(a-b))^(1/2)*(e*x+d)*a^3*b^2)/cos(e*x+d)^3/((b*cos(e*x+d)+a)^2/c
os(e*x+d)^2)^(3/2)/((a+b)*(a-b))^(1/2)/(a^2-b^2)/b^3
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(e*x+d))/(b^2+2*a*b*sec(e*x+d)+a^2*sec(e*x+d)^2)^(3/2),x,
 algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more details)Is 4*b^2-4*a^2 positive or negative?
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{a+\frac{b}{\cos(d+e\,x)}}{{\left(b^2+\frac{a^2}{{\cos(d+e\,x)}^2}+\frac{2\,a\,b}{\cos(d+e\,x)}\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(d + e*x))/(b^2 + a^2/cos(d + e*x)^2 + (2*a*b)/cos(d + e*x))^
(3/2),x)
```

```
[Out] int((a + b/cos(d + e*x))/(b^2 + a^2/cos(d + e*x)^2 + (2*a*b)/cos(d + e*x))^
(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a+b\sec(d+ex)}{\left((a\sec(d+ex)+b)^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(e*x+d))/(b**2+2*a*b*sec(e*x+d)+a**2*sec(e*x+d)**2)**(3/2
),x)
```

```
[Out] Integral((a + b*sec(d + e*x))/((a*sec(d + e*x) + b)**2)**(3/2), x)
```

3.526 $\int \frac{\cos(x)-i\sin(x)}{\cos(x)+i\sin(x)}\,dx$

Optimal. Leaf size=17

$$\frac{1}{2}i(\cos(x)-i\sin(x))^2$$

```
[Out] 1/2*I*(cos(x)-I*sin(x))^2
```

Rubi [A] time = 0.04, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.048, Rules used = {4385}

$$\frac{1}{2}i(\cos(x)-i\sin(x))^2$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[x] - I*Sin[x])/(Cos[x] + I*Sin[x]),x]
```

```
[Out] (I/2)*(Cos[x] - I*Sin[x])^2
```

Rule 4385

```
Int[(u_)*(y_)^(m_.), x_Symbol] :> With[{q = DerivativeDivides[ActivateTrig[
y], ActivateTrig[u], x]}, Simp[(q*ActivateTrig[y^(m + 1)])/(m + 1), x] /;
!FalseQ[q]] /; FreeQ[m, x] && NeQ[m, -1] &&  !InertTrigFreeQ[u]
```

Rubi steps

$$\int \frac{\cos(x)-i\sin(x)}{\cos(x)+i\sin(x)}\,dx=\frac{1}{2}i(\cos(x)-i\sin(x))^2$$

Mathematica [A] time = 0.01, size = 19, normalized size = 1.12

$$\frac{1}{2}\sin(2x)+\frac{1}{2}i\cos(2x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[x] - I*Sin[x])/(Cos[x] + I*Sin[x]),x]
```

```
[Out] (I/2)*Cos[2*x] + Sin[2*x]/2
```

fricas [A] time = 0.70, size = 6, normalized size = 0.35

$$\frac{1}{2}i\,e^{(-2i\,x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((cos(x)-I*sin(x))/(cos(x)+I*sin(x)),x, algorithm="fricas")
```

```
[Out] 1/2*I*e^(-2*I*x)
```

giac [A]   time = 0.14, size = 14, normalized size = 0.82

$$-\frac{2\tan\left(\frac{1}{2}x\right)}{\left(\tan\left(\frac{1}{2}x\right)-i\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((cos(x)-I*sin(x))/(cos(x)+I*sin(x)),x, algorithm="giac")
```

```
[Out] -2*tan(1/2*x)/(tan(1/2*x) - I)^2
```

maple [A]   time = 0.20, size = 8, normalized size = 0.47

$$\frac{1}{\tan(x)-i}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(x)-I*sin(x))/(cos(x)+I*sin(x)),x)
```

```
[Out] 1/(tan(x)-I)
```

maxima [F(-2)]   time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((cos(x)-I*sin(x))/(cos(x)+I*sin(x)),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

mupad [B]   time = 2.76, size = 16, normalized size = 0.94

$$-\frac{\cos(x)}{-\sin(x)+\cos(x)\,1\mathrm{i}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(x) - sin(x)*1i)/(cos(x) + sin(x)*1i),x)
```

```
[Out] -cos(x)/(cos(x)*1i - sin(x))
```

sympy [A] time = 0.08, size = 8, normalized size = 0.47

$$\frac{ie^{-2ix}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((cos(x)-I*sin(x))/(cos(x)+I*sin(x)),x)
[Out] I*exp(-2*I*x)/2
```

3.527 $\int \frac{\cos(x)+i\sin(x)}{\cos(x)-i\sin(x)}\,dx$

Optimal. Leaf size=17

$$-\frac{i}{2(\cos(x)-i\sin(x))^2}$$

```
[Out] -1/2*I/(cos(x)-I*sin(x))^2
```

Rubi [A] time = 0.04, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.048, Rules used = {4385}

$$-\frac{i}{2(\cos(x)-i\sin(x))^2}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[x] + I*Sin[x])/(Cos[x] - I*Sin[x]),x]
```

```
[Out] (-I/2)/(Cos[x] - I*Sin[x])^2
```

Rule 4385

```
Int[(u_)*(y_)^(m_.), x_Symbol] :> With[{q = DerivativeDivides[ActivateTrig[
y], ActivateTrig[u], x]}, Simp[(q*ActivateTrig[y^(m + 1)])/(m + 1), x] /;
!FalseQ[q]] /; FreeQ[m, x] && NeQ[m, -1] &&  !InertTrigFreeQ[u]
```

Rubi steps

$$\int \frac{\cos(x)+i\sin(x)}{\cos(x)-i\sin(x)}\,dx = -\frac{i}{2(\cos(x)-i\sin(x))^2}$$

Mathematica [A] time = 0.01, size = 19, normalized size = 1.12

$$\frac{1}{2}\sin(2x)-\frac{1}{2}i\cos(2x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[x] + I*Sin[x])/(Cos[x] - I*Sin[x]),x]
```

```
[Out] (-1/2*I)*Cos[2*x] + Sin[2*x]/2
```

fricas [A] time = 0.65, size = 6, normalized size = 0.35

$$-\frac{1}{2}i\,e^{(2i\,x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((cos(x)+I*sin(x))/(cos(x)-I*sin(x)),x, algorithm="fricas")
```

```
[Out] -1/2*I*e^(2*I*x)
```

giac [A] time = 0.14, size = 14, normalized size = 0.82

$$-\frac{2\,\tan\left(\frac{1}{2}\,x\right)}{\left(\tan\left(\frac{1}{2}\,x\right)+i\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((cos(x)+I*sin(x))/(cos(x)-I*sin(x)),x, algorithm="giac")
```

```
[Out] -2*tan(1/2*x)/(tan(1/2*x) + I)^2
```

maple [A] time = 0.17, size = 8, normalized size = 0.47

$$\frac{1}{\tan(x)+i}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(x)+I*sin(x))/(cos(x)-I*sin(x)),x)
```

```
[Out] 1/(tan(x)+I)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((cos(x)+I*sin(x))/(cos(x)-I*sin(x)),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

mupad [B] time = 2.74, size = 13, normalized size = 0.76

$$\frac{\sin(x)}{\cos(x)-\sin(x)\,1\mathrm{i}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(x) + sin(x)*1i)/(cos(x) - sin(x)*1i),x)
```

```
[Out] sin(x)/(cos(x) - sin(x)*1i)
```

sympy [A] time = 0.08, size = 10, normalized size = 0.59

$$-\frac{ie^{2ix}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((cos(x)+I*sin(x))/(cos(x)-I*sin(x)),x)
```

```
[Out] -I*exp(2*I*x)/2
```

3.528 $\int \frac{\cos(x)-\sin(x)}{\cos(x)+\sin(x)}\,dx$

Optimal. Leaf size=6

$$\log(\sin(x)+\cos(x))$$

```
[Out] ln(cos(x)+sin(x))
```

Rubi [A] time = 0.02, antiderivative size = 6, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {3133}

$$\log(\sin(x)+\cos(x))$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[x] - Sin[x])/(Cos[x] + Sin[x]),x]
```

```
[Out] Log[Cos[x] + Sin[x]]
```

Rule 3133

```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])
/((a_.) + cos[(d_.) + (e_.)*(x_)]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)]), x
_Symbol] :> Simp[((b*B + c*C)*x)/(b^2 + c^2), x] + Simp[((c*B - b*C)*Log[a
+ b*Cos[d + e*x] + c*Sin[d + e*x]])/(e*(b^2 + c^2)), x] /; FreeQ[{a, b, c,
d, e, A, B, C}, x] && NeQ[b^2 + c^2, 0] && EqQ[A*(b^2 + c^2) - a*(b*B + c*C
), 0]
```

Rubi steps

$$\int \frac{\cos(x)-\sin(x)}{\cos(x)+\sin(x)}\,dx=\log(\cos(x)+\sin(x))$$

Mathematica [A] time = 0.03, size = 6, normalized size = 1.00

$$\log(\sin(x)+\cos(x))$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[x] - Sin[x])/(Cos[x] + Sin[x]),x]
```

```
[Out] Log[Cos[x] + Sin[x]]
```

fricas [A] time = 0.82, size = 11, normalized size = 1.83

$$\frac{1}{2}\,\log\left(2\,\cos(x)\sin(x)+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((cos(x)-sin(x))/(cos(x)+sin(x)),x, algorithm="fricas")
[Out] 1/2*log(2*cos(x)*sin(x) + 1)
```

giac [B] time = 0.18, size = 16, normalized size = 2.67

$$-\frac{1}{2}\,\log\left(\tan(x)^{2}+1\right)+\log\left(|\tan(x)+1|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((cos(x)-sin(x))/(cos(x)+sin(x)),x, algorithm="giac")
[Out] -1/2*log(tan(x)^2 + 1) + log(abs(tan(x) + 1))
```

maple [A] time = 0.06, size = 7, normalized size = 1.17

$$\ln\left(\cos(x)+\sin(x)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(x)-sin(x))/(cos(x)+sin(x)),x)
[Out] ln(cos(x)+sin(x))
```

maxima [A] time = 0.31, size = 6, normalized size = 1.00

$$\log\left(\cos(x)+\sin(x)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((cos(x)-sin(x))/(cos(x)+sin(x)),x, algorithm="maxima")
[Out] log(cos(x) + sin(x))
```

mupad [B] time = 2.92, size = 32, normalized size = 5.33

$$2\,\mathrm{atanh}\left(\frac{128\tan\left(\frac{x}{2}\right)+128}{16\tan\left(\frac{x}{2}\right)^{2}+32\tan\left(\frac{x}{2}\right)+48}-3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(x) - sin(x))/(cos(x) + sin(x)),x)
```

```
[Out] 2*atanh((128*tan(x/2) + 128)/(32*tan(x/2) + 16*tan(x/2)^2 + 48) - 3)
```

sympy [A] time = 0.12, size = 7, normalized size = 1.17

$$\log(\sin(x) + \cos(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((cos(x)-sin(x))/(cos(x)+sin(x)),x)
```

```
[Out] log(sin(x) + cos(x))
```

3.529 $\int \frac{B\cos(x)+C\sin(x)}{b\cos(x)+c\sin(x)}\,dx$

Optimal. Leaf size=47

$$\frac{x(bB+cC)}{b^2+c^2}+\frac{(Bc-bC)\log(b\cos(x)+c\sin(x))}{b^2+c^2}$$

```
[Out] (B*b+C*c)*x/(b^2+c^2)+(B*c-C*b)*ln(b*cos(x)+c*sin(x))/(b^2+c^2)
```

Rubi [A] time = 0.04, antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.048, Rules used = {3133}

$$\frac{x(bB+cC)}{b^2+c^2}+\frac{(Bc-bC)\log(b\cos(x)+c\sin(x))}{b^2+c^2}$$

Antiderivative was successfully verified.

```
[In] Int[(B*Cos[x] + C*Sin[x])/(b*Cos[x] + c*Sin[x]),x]
```

```
[Out] ((b*B + c*C)*x)/(b^2 + c^2) + ((B*c - b*C)*Log[b*Cos[x] + c*Sin[x]])/(b^2 +
 c^2)
```

Rule 3133

```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])
/((a_.) + cos[(d_.) + (e_.)*(x_)]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)]), x
_Symbol] :> Simp[((b*B + c*C)*x)/(b^2 + c^2), x] + Simp[((c*B - b*C)*Log[a
+ b*Cos[d + e*x] + c*Sin[d + e*x]])/(e*(b^2 + c^2)), x] /; FreeQ[{a, b, c,
d, e, A, B, C}, x] && NeQ[b^2 + c^2, 0] && EqQ[A*(b^2 + c^2) - a*(b*B + c*C
), 0]
```

Rubi steps

$$\int \frac{B\cos(x)+C\sin(x)}{b\cos(x)+c\sin(x)}\,dx=\frac{(bB+cC)x}{b^2+c^2}+\frac{(Bc-bC)\log(b\cos(x)+c\sin(x))}{b^2+c^2}$$

Mathematica [A] time = 0.14, size = 39, normalized size = 0.83

$$\frac{x(bB+cC)+(Bc-bC)\log(b\cos(x)+c\sin(x))}{b^2+c^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(B*Cos[x] + C*Sin[x])/(b*Cos[x] + c*Sin[x]),x]
```

```
[Out] ((b*B + c*C)*x + (B*c - b*C)*Log[b*Cos[x] + c*Sin[x]])/(b^2 + c^2)
```

fricas [A] time = 0.88, size = 59, normalized size = 1.26

$$\frac{2\,(Bb+Cc)x-(Cb-Bc)\log\left(2\,bc\cos(x)\sin(x)+\left(b^2-c^2\right)\cos(x)^2+c^2\right)}{2\left(b^2+c^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*cos(x)+C*sin(x))/(b*cos(x)+c*sin(x)),x, algorithm="fricas")
```

```
[Out] 1/2*(2*(B*b + C*c)*x - (C*b - B*c)*log(2*b*c*cos(x)*sin(x) + (b^2 - c^2)*co
s(x)^2 + c^2))/(b^2 + c^2)
```

giac [A] time = 0.17, size = 77, normalized size = 1.64

$$\frac{(Bb+Cc)x}{b^2+c^2}+\frac{(Cb-Bc)\log\left(\tan(x)^2+1\right)}{2\left(b^2+c^2\right)}-\frac{\left(Cbc-Bc^2\right)\log\left(|c\tan(x)+b|\right)}{b^2c+c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*cos(x)+C*sin(x))/(b*cos(x)+c*sin(x)),x, algorithm="giac")
```

```
[Out] (B*b + C*c)*x/(b^2 + c^2) + 1/2*(C*b - B*c)*log(tan(x)^2 + 1)/(b^2 + c^2) -
 (C*b*c - B*c^2)*log(abs(c*tan(x) + b))/(b^2*c + c^3)
```

maple [B] time = 0.14, size = 111, normalized size = 2.36

$$\frac{\ln\left(c\tan(x)+b\right)Bc}{b^2+c^2}-\frac{\ln\left(c\tan(x)+b\right)bC}{b^2+c^2}-\frac{\ln\left(1+\tan^2(x)\right)Bc}{2\left(b^2+c^2\right)}+\frac{\ln\left(1+\tan^2(x)\right)bC}{2b^2+2c^2}+\frac{B\arctan\left(\tan(x)\right)b}{b^2+c^2}+\frac{C\,\mathrm{arc}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*cos(x)+C*sin(x))/(b*cos(x)+c*sin(x)),x)
```

```
[Out] 1/(b^2+c^2)*ln(c*tan(x)+b)*B*c-1/(b^2+c^2)*ln(c*tan(x)+b)*b*C-1/2/(b^2+c^2)
*ln(1+tan(x)^2)*B*c+1/2/(b^2+c^2)*ln(1+tan(x)^2)*b*C+1/(b^2+c^2)*B*arctan(t
an(x))*b+1/(b^2+c^2)*C*arctan(tan(x))*c
```

maxima [B] time = 0.42, size = 181, normalized size = 3.85

$$B\left(\frac{2\,b\arctan\left(\frac{\sin(x)}{\cos(x)+1}\right)}{b^2+c^2}+\frac{c\log\left(-b-\frac{2\,c\sin(x)}{\cos(x)+1}+\frac{b\sin(x)^2}{(\cos(x)+1)^2}\right)}{b^2+c^2}-\frac{c\log\left(\frac{\sin(x)^2}{(\cos(x)+1)^2}+1\right)}{b^2+c^2}\right)+C\left(\frac{2\,c\arctan\left(\frac{\sin(x)}{\cos(x)+1}\right)}{b^2+c^2}-\cdots\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*cos(x)+C*sin(x))/(b*cos(x)+c*sin(x)),x, algorithm="maxima")
```

```
[Out] B*(2*b*arctan(sin(x)/(cos(x) + 1))/(b^2 + c^2) + c*log(-b - 2*c*sin(x)/(cos
(x) + 1) + b*sin(x)^2/(cos(x) + 1)^2)/(b^2 + c^2) - c*log(sin(x)^2/(cos(x)
+ 1)^2 + 1)/(b^2 + c^2)) + C*(2*c*arctan(sin(x)/(cos(x) + 1))/(b^2 + c^2) -
 b*log(-b - 2*c*sin(x)/(cos(x) + 1) + b*sin(x)^2/(cos(x) + 1)^2)/(b^2 + c^2
) + b*log(sin(x)^2/(cos(x) + 1)^2 + 1)/(b^2 + c^2))
```

mupad [B] time = 12.82, size = 1976, normalized size = 42.04

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*cos(x) + C*sin(x))/(b*cos(x) + c*sin(x)),x)
```

```
[Out] (log(b + 2*c*tan(x/2) - b*tan(x/2)^2)*(B*c - C*b))/(b^2 + c^2) - (log(1/(co
s(x) + 1))*(2*B*c - 2*C*b))/(2*(b^2 + c^2)) + (2*atan(((((32*B*C^2*b^2 - ((
2*B*c - 2*C*b)*(((2*B*c - 2*C*b)*(64*B*b^2*c^2 - 32*B*b^4 + 32*C*b*c^3 - 64
*C*b^3*c + ((2*B*c - 2*C*b)*(96*b^4*c + 96*b^2*c^3))/(2*(b^2 + c^2))))/(2*(
b^2 + c^2)) - 32*C^2*b^2*c - 32*B^2*b^2*c + 64*B*C*b^3 + 64*B*C*b*c^2))/(2*
(b^2 + c^2)) + ((B*b + C*c)*(((B*b + C*c)*(64*B*b^2*c^2 - 32*B*b^4 + 32*C*b
*c^3 - 64*C*b^3*c + ((2*B*c - 2*C*b)*(96*b^4*c + 96*b^2*c^3))/(2*(b^2 + c^2
))))/(b^2 + c^2) + ((B*b + C*c)*(2*B*c - 2*C*b)*(96*b^4*c + 96*b^2*c^3))/(2
*(b^2 + c^2)^2)))/(b^2 + c^2) - 32*B^2*C*b*c + ((B*b + C*c)^2*(2*B*c - 2*C*
b)*(96*b^4*c + 96*b^2*c^3))/(2*(b^2 + c^2)^3))*(12*B^2*b*c^3 - 6*B^2*b^3*c
- 6*C^2*b*c^3 + 12*C^2*b^3*c + 4*B*C*b^4 + 4*B*C*c^4 - 28*B*C*b^2*c^2))/((b
^2 + c^2)^2*(B^2*b^2 + 4*B^2*c^2 + 4*C^2*b^2 + C^2*c^2 - 6*B*C*b*c)^2) - ta
n(x/2)*(((32*B^3*b*c - 32*B^2*C*b^2 - 64*C^3*b^2 + ((2*B*c - 2*C*b)*(32*B^2
*b^3 - 96*B^2*b*c^2 + 64*C^2*b*c^2 - ((2*B*c - 2*C*b)*(32*C*b^2*c^2 - 64*C*
b^4 + 32*B*b*c^3 + 128*B*b^3*c - ((2*B*c - 2*C*b)*(96*b*c^4 + 96*b^3*c^2))/
(2*(b^2 + c^2))))/(2*(b^2 + c^2)) + 192*B*C*b^2*c))/(2*(b^2 + c^2)) + ((B*b
 + C*c)*(((B*b + C*c)*(32*C*b^2*c^2 - 64*C*b^4 + 32*B*b*c^3 + 128*B*b^3*c -
 ((2*B*c - 2*C*b)*(96*b*c^4 + 96*b^3*c^2))/(2*(b^2 + c^2))))/(b^2 + c^2) -
((B*b + C*c)*(2*B*c - 2*C*b)*(96*b*c^4 + 96*b^3*c^2))/(2*(b^2 + c^2)^2)))/(
b^2 + c^2) + 64*B*C^2*b*c - ((B*b + C*c)^2*(2*B*c - 2*C*b)*(96*b*c^4 + 96*b
^3*c^2))/(2*(b^2 + c^2)^3))*(12*B^2*b*c^3 - 6*B^2*b^3*c - 6*C^2*b*c^3 + 12*
C^2*b^3*c + 4*B*C*b^4 + 4*B*C*c^4 - 28*B*C*b^2*c^2))/((b^2 + c^2)^2*(B^2*b^
2 + 4*B^2*c^2 + 4*C^2*b^2 + C^2*c^2 - 6*B*C*b*c)^2) + ((((B*b + C*c)^3*(96*
b*c^4 + 96*b^3*c^2))/(b^2 + c^2)^3 - ((B*b + C*c)*(32*B^2*b^3 - 96*B^2*b*c^
2 + 64*C^2*b*c^2 - ((2*B*c - 2*C*b)*(32*C*b^2*c^2 - 64*C*b^4 + 32*B*b*c^3 +
 128*B*b^3*c - ((2*B*c - 2*C*b)*(96*b*c^4 + 96*b^3*c^2))/(2*(b^2 + c^2))))/
(2*(b^2 + c^2)) + 192*B*C*b^2*c))/(b^2 + c^2) + ((2*B*c - 2*C*b)*(((B*b + C
*c)*(32*C*b^2*c^2 - 64*C*b^4 + 32*B*b*c^3 + 128*B*b^3*c - ((2*B*c - 2*C*b)*
```

```
(96*b*c^4 + 96*b^3*c^2))/(2*(b^2 + c^2))))/(b^2 + c^2) - ((B*b + C*c)*(2*B*
c - 2*C*b)*(96*b*c^4 + 96*b^3*c^2))/(2*(b^2 + c^2)^2)))/(2*(b^2 + c^2)))*(B
^2*b^4 + 4*B^2*c^4 - 4*C^2*b^4 - C^2*c^4 - 13*B^2*b^2*c^2 + 13*C^2*b^2*c^2
- 18*B*C*b*c^3 + 18*B*C*b^3*c))/((b^2 + c^2)^2*(B^2*b^2 + 4*B^2*c^2 + 4*C^2
*b^2 + C^2*c^2 - 6*B*C*b*c)^2)) + ((((B*b + C*c)*(((2*B*c - 2*C*b)*(64*B*b^
2*c^2 - 32*B*b^4 + 32*C*b*c^3 - 64*C*b^3*c + ((2*B*c - 2*C*b)*(96*b^4*c + 9
6*b^2*c^3))/(2*(b^2 + c^2))))/(2*(b^2 + c^2)) - 32*C^2*b^2*c - 32*B^2*b^2*c
 + 64*B*C*b^3 + 64*B*C*b*c^2))/(b^2 + c^2) - ((B*b + C*c)^3*(96*b^4*c + 96*
b^2*c^3))/(b^2 + c^2)^3 + ((2*B*c - 2*C*b)*(((B*b + C*c)*(64*B*b^2*c^2 - 32
*B*b^4 + 32*C*b*c^3 - 64*C*b^3*c + ((2*B*c - 2*C*b)*(96*b^4*c + 96*b^2*c^3)
)/(2*(b^2 + c^2))))/(b^2 + c^2) + ((B*b + C*c)*(2*B*c - 2*C*b)*(96*b^4*c +
96*b^2*c^3))/(2*(b^2 + c^2)^2)))/(2*(b^2 + c^2)))*(B^2*b^4 + 4*B^2*c^4 - 4*
C^2*b^4 - C^2*c^4 - 13*B^2*b^2*c^2 + 13*C^2*b^2*c^2 - 18*B*C*b*c^3 + 18*B*C
*b^3*c))/((b^2 + c^2)^2*(B^2*b^2 + 4*B^2*c^2 + 4*C^2*b^2 + C^2*c^2 - 6*B*C*
b*c)^2))*(b^4 + c^4 + 2*b^2*c^2))/(32*B*b^2 + 32*C*b*c))*(B*b + C*c))/(b^2
+ c^2)
```

sympy [A] time = 0.88, size = 371, normalized size = 7.89

$$\begin{cases} \tilde{\infty}\left(B\log\left(\sin\left(x\right)\right) + Cx\right) \\ \frac{B\log\left(\sin\left(x\right)\right)+Cx}{c} \\ \frac{Bx\sin\left(x\right)}{-2ic\sin\left(x\right)-2c\cos\left(x\right)} - \frac{iBx\cos\left(x\right)}{-2ic\sin\left(x\right)-2c\cos\left(x\right)} - \frac{iB\sin\left(x\right)}{-2ic\sin\left(x\right)-2c\cos\left(x\right)} - \frac{iCx\sin\left(x\right)}{-2ic\sin\left(x\right)-2c\cos\left(x\right)} - \frac{Cx\cos\left(x\right)}{-2ic\sin\left(x\right)-2c\cos\left(x\right)} + \frac{C\sin\left(x\right)}{-2ic\sin\left(x\right)-2c\cos\left(x\right)} \\ \frac{Bx\sin\left(x\right)}{2ic\sin\left(x\right)-2c\cos\left(x\right)} + \frac{iBx\cos\left(x\right)}{2ic\sin\left(x\right)-2c\cos\left(x\right)} + \frac{iB\sin\left(x\right)}{2ic\sin\left(x\right)-2c\cos\left(x\right)} + \frac{iCx\sin\left(x\right)}{2ic\sin\left(x\right)-2c\cos\left(x\right)} - \frac{Cx\cos\left(x\right)}{2ic\sin\left(x\right)-2c\cos\left(x\right)} + \frac{C\sin\left(x\right)}{2ic\sin\left(x\right)-2c\cos\left(x\right)} \\ \frac{Bbx}{b^{2}+c^{2}} + \frac{Bc\log\left(\cos\left(x\right)+\frac{c\sin\left(x\right)}{b}\right)}{b^{2}+c^{2}} - \frac{Cb\log\left(\cos\left(x\right)+\frac{c\sin\left(x\right)}{b}\right)}{b^{2}+c^{2}} + \frac{Ccx}{b^{2}+c^{2}} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*cos(x)+C*sin(x))/(b*cos(x)+c*sin(x)),x)
```

```
[Out] Piecewise((zoo*(B*log(sin(x)) + C*x), Eq(b, 0) & Eq(c, 0)), ((B*log(sin(x))
 + C*x)/c, Eq(b, 0)), (B*x*sin(x)/(-2*I*c*sin(x) - 2*c*cos(x)) - I*B*x*cos(
x)/(-2*I*c*sin(x) - 2*c*cos(x)) - I*B*sin(x)/(-2*I*c*sin(x) - 2*c*cos(x)) -
 I*C*x*sin(x)/(-2*I*c*sin(x) - 2*c*cos(x)) - C*x*cos(x)/(-2*I*c*sin(x) - 2*
c*cos(x)) + C*sin(x)/(-2*I*c*sin(x) - 2*c*cos(x)), Eq(b, -I*c)), (B*x*sin(x
)/(2*I*c*sin(x) - 2*c*cos(x)) + I*B*x*cos(x)/(2*I*c*sin(x) - 2*c*cos(x)) +
I*B*sin(x)/(2*I*c*sin(x) - 2*c*cos(x)) + I*C*x*sin(x)/(2*I*c*sin(x) - 2*c*c
os(x)) - C*x*cos(x)/(2*I*c*sin(x) - 2*c*cos(x)) + C*sin(x)/(2*I*c*sin(x) -
2*c*cos(x)), Eq(b, I*c)), (B*b*x/(b**2 + c**2) + B*c*log(cos(x) + c*sin(x)/
b)/(b**2 + c**2) - C*b*log(cos(x) + c*sin(x)/b)/(b**2 + c**2) + C*c*x/(b**2
 + c**2), True))
```

3.530 $\int \frac{B\cos(x)+C\sin(x)}{(b\cos(x)+c\sin(x))^2}\,dx$

Optimal. Leaf size=74

$$-\frac{Bc-bC}{(b^2+c^2)(b\cos(x)+c\sin(x))}-\frac{(bB+cC)\tanh^{-1}\left(\frac{c\cos(x)-b\sin(x)}{\sqrt{b^2+c^2}}\right)}{(b^2+c^2)^{3/2}}$$

```
[Out] -(B*b+C*c)*arctanh((c*cos(x)-b*sin(x))/(b^2+c^2)^(1/2))/(b^2+c^2)^(3/2)+(-B
*c+C*b)/(b^2+c^2)/(b*cos(x)+c*sin(x))
```

Rubi [A] time = 0.07, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3153, 3074, 206}

$$-\frac{Bc-bC}{(b^2+c^2)(b\cos(x)+c\sin(x))}-\frac{(bB+cC)\tanh^{-1}\left(\frac{c\cos(x)-b\sin(x)}{\sqrt{b^2+c^2}}\right)}{(b^2+c^2)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(B*Cos[x] + C*Sin[x])/(b*Cos[x] + c*Sin[x])^2,x]
```

```
[Out] -(((b*B + c*C)*ArcTanh[(c*Cos[x] - b*Sin[x])/Sqrt[b^2 + c^2]])/(b^2 + c^2)^
(3/2)) - (B*c - b*C)/((b^2 + c^2)*(b*Cos[x] + c*Sin[x]))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3074

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x
_Symbol] :> -Dist[d^(-1), Subst[Int[1/(a^2 + b^2 - x^2), x], x, b*Cos[c + d
*x] - a*Sin[c + d*x]], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

Rule 3153

```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])
/((a_.) + cos[(d_.) + (e_.)*(x_)]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)])^2,
 x_Symbol] :> Simp[(c*B - b*C - (a*C - c*A)*Cos[d + e*x] + (a*B - b*A)*Sin[
d + e*x])/(e*(a^2 - b^2 - c^2)*(a + b*Cos[d + e*x] + c*Sin[d + e*x])), x] +
 Dist[(a*A - b*B - c*C)/(a^2 - b^2 - c^2), Int[1/(a + b*Cos[d + e*x] + c*Si
```

```
n[d + e*x]), x], x] /; FreeQ[{a, b, c, d, e, A, B, C}, x] && NeQ[a^2 - b^2
- c^2, 0] && NeQ[a*A - b*B - c*C, 0]
```

Rubi steps

$$\int \frac{B\cos(x)+C\sin(x)}{(b\cos(x)+c\sin(x))^2}\,dx = -\frac{Bc-bC}{\left(b^2+c^2\right)(b\cos(x)+c\sin(x))}+\frac{(bB+cC)\int \frac{1}{b\cos(x)+c\sin(x)}\,dx}{b^2+c^2}$$

$$= -\frac{Bc-bC}{\left(b^2+c^2\right)(b\cos(x)+c\sin(x))}-\frac{(bB+cC)\,\mathrm{Subst}\left(\int \frac{1}{b^2+c^2-x^2}\,dx,x,c\cos(x)-b\,\mathrm{s}\right.}{b^2+c^2}$$

$$= -\frac{(bB+cC)\tanh^{-1}\left(\frac{c\cos(x)-b\sin(x)}{\sqrt{b^2+c^2}}\right)}{\left(b^2+c^2\right)^{3/2}}-\frac{Bc-bC}{\left(b^2+c^2\right)(b\cos(x)+c\sin(x))}$$

Mathematica [A] time = 0.24, size = 75, normalized size = 1.01

$$\frac{bC-Bc}{\left(b^2+c^2\right)(b\cos(x)+c\sin(x))}+\frac{2(bB+cC)\tanh^{-1}\left(\frac{b\tan\left(\frac{x}{2}\right)-c}{\sqrt{b^2+c^2}}\right)}{\left(b^2+c^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(B*Cos[x] + C*Sin[x])/(b*Cos[x] + c*Sin[x])^2,x]
```

```
[Out] (2*(b*B + c*C)*ArcTanh[(-c + b*Tan[x/2])/Sqrt[b^2 + c^2]])/(b^2 + c^2)^(3/2
) + (-(B*c) + b*C)/((b^2 + c^2)*(b*Cos[x] + c*Sin[x]))
```

fricas [B] time = 1.63, size = 194, normalized size = 2.62

$$\frac{2\,Cb^3-2\,Bb^2c+2\,Cbc^2-2\,Bc^3+\sqrt{b^2+c^2}\left(\left(Bb^2+Cbc\right)\cos(x)+\left(Bbc+Cc^2\right)\sin(x)\right)\log\left(-\frac{2\,bc\cos(x)\sin(x)+\left(b^2-\right.}{2\,bc\,\mathrm{co}}\right.}{2\left(\left(b^5+2\,b^3c^2+bc^4\right)\cos(x)+\left(b^4c+2\,b^2c^3+c^5\right)\sin(x)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*cos(x)+C*sin(x))/(b*cos(x)+c*sin(x))^2,x, algorithm="fricas")
```

```
[Out] 1/2*(2*C*b^3 - 2*B*b^2*c + 2*C*b*c^2 - 2*B*c^3 + sqrt(b^2 + c^2)*((B*b^2 +
C*b*c)*cos(x) + (B*b*c + C*c^2)*sin(x))*log(-(2*b*c*cos(x)*sin(x) + (b^2 -
c^2)*cos(x)^2 - 2*b^2 - c^2 + 2*sqrt(b^2 + c^2)*(c*cos(x) - b*sin(x)))/(2*b
```

```
*c*cos(x)*sin(x) + (b^2 - c^2)*cos(x)^2 + c^2)))/((b^5 + 2*b^3*c^2 + b*c^4)
*cos(x) + (b^4*c + 2*b^2*c^3 + c^5)*sin(x))
```

giac [A] time = 0.24, size = 132, normalized size = 1.78

$$-\frac{(Bb+Cc)\log\left(\frac{\left|2\,b\tan\left(\frac{1}{2}\,x\right)-2\,c-2\,\sqrt{b^2+c^2}\right|}{\left|2\,b\tan\left(\frac{1}{2}\,x\right)-2\,c+2\,\sqrt{b^2+c^2}\right|}\right)}{\left(b^2+c^2\right)^{\frac{3}{2}}}-\frac{2\left(Cbc\tan\left(\frac{1}{2}\,x\right)-Bc^2\tan\left(\frac{1}{2}\,x\right)+Cb^2-Bbc\right)}{\left(b^3+bc^2\right)\left(b\tan\left(\frac{1}{2}\,x\right)^2-2\,c\tan\left(\frac{1}{2}\,x\right)-b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*cos(x)+C*sin(x))/(b*cos(x)+c*sin(x))^2,x, algorithm="giac")
```

```
[Out] -(B*b + C*c)*log(abs(2*b*tan(1/2*x) - 2*c - 2*sqrt(b^2 + c^2))/abs(2*b*tan(
1/2*x) - 2*c + 2*sqrt(b^2 + c^2)))/(b^2 + c^2)^(3/2) - 2*(C*b*c*tan(1/2*x)
- B*c^2*tan(1/2*x) + C*b^2 - B*b*c)/((b^3 + b*c^2)*(b*tan(1/2*x)^2 - 2*c*ta
n(1/2*x) - b))
```

maple [A] time = 0.17, size = 113, normalized size = 1.53

$$-\frac{2\left(-\frac{c(Bc-bC)\tan\left(\frac{x}{2}\right)}{b\left(b^2+c^2\right)}-\frac{Bc-bC}{b^2+c^2}\right)}{b\left(\tan^2\left(\frac{x}{2}\right)\right)-2c\tan\left(\frac{x}{2}\right)-b}+\frac{2\,(bB+Cc)\,\mathrm{arctanh}\left(\frac{2b\tan\left(\frac{x}{2}\right)-2c}{2\sqrt{b^2+c^2}}\right)}{\left(b^2+c^2\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*cos(x)+C*sin(x))/(b*cos(x)+c*sin(x))^2,x)
```

```
[Out] -2*(-c*(B*c-C*b)/b/(b^2+c^2)*tan(1/2*x)-(B*c-C*b)/(b^2+c^2))/(b*tan(1/2*x)^
2-2*c*tan(1/2*x)-b)+2*(B*b+C*c)/(b^2+c^2)^(3/2)*arctanh(1/2*(2*b*tan(1/2*x)
-2*c)/(b^2+c^2)^(1/2))
```

maxima [B] time = 0.43, size = 271, normalized size = 3.66

$$-B\left(\frac{b\log\left(\frac{c-\frac{b\sin(x)}{\cos(x)+1}+\sqrt{b^2+c^2}}{c-\frac{b\sin(x)}{\cos(x)+1}-\sqrt{b^2+c^2}}\right)}{\left(b^2+c^2\right)^{\frac{3}{2}}}+\frac{2\left(bc+\frac{c^2\sin(x)}{\cos(x)+1}\right)}{b^4+b^2c^2+\frac{2\left(b^3c+bc^3\right)\sin(x)}{\cos(x)+1}-\frac{\left(b^4+b^2c^2\right)\sin(x)^2}{(\cos(x)+1)^2}}\right)-C\left(\frac{c\log\left(\frac{c-\frac{b\sin(x)}{\cos(x)+1}+\sqrt{b^2+c^2}}{c-\frac{b\sin(x)}{\cos(x)+1}-\sqrt{b^2+c^2}}\right)}{\left(b^2+c^2\right)^{\frac{3}{2}}}-\frac{\cdots}{b^3+bc\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*cos(x)+C*sin(x))/(b*cos(x)+c*sin(x))^2,x, algorithm="maxima")
```

```
[Out] -B*(b*log((c - b*sin(x)/(cos(x) + 1) + sqrt(b^2 + c^2))/(c - b*sin(x)/(cos(
x) + 1) - sqrt(b^2 + c^2)))/(b^2 + c^2)^(3/2) + 2*(b*c + c^2*sin(x)/(cos(x)
 + 1))/(b^4 + b^2*c^2 + 2*(b^3*c + b*c^3)*sin(x)/(cos(x) + 1) - (b^4 + b^2*
c^2)*sin(x)^2/(cos(x) + 1)^2)) - C*(c*log((c - b*sin(x)/(cos(x) + 1) + sqrt
(b^2 + c^2))/(c - b*sin(x)/(cos(x) + 1) - sqrt(b^2 + c^2)))/(b^2 + c^2)^(3/
2) - 2*(b + c*sin(x)/(cos(x) + 1))/(b^3 + b*c^2 + 2*(b^2*c + c^3)*sin(x)/(c
os(x) + 1) - (b^3 + b*c^2)*sin(x)^2/(cos(x) + 1)^2))
```

mupad [B] time = 3.01, size = 129, normalized size = 1.74

$$-\frac{\frac{2\,\left(B\,c-C\,b\right)}{b^2+c^2}+\frac{2\,c\,\tan\left(\frac{x}{2}\right)\,\left(B\,c-C\,b\right)}{b\,\left(b^2+c^2\right)}}{-b\,{\tan\left(\frac{x}{2}\right)}^2+2\,c\,\tan\left(\frac{x}{2}\right)+b}+\frac{\mathrm{atan}\left(\frac{b^2\,c\,1{}\mathrm{i}+c^3\,1{}\mathrm{i}-b\,\tan\left(\frac{x}{2}\right)\,\left(b^2+c^2\right)\,1{}\mathrm{i}}{{\left(b^2+c^2\right)}^{3/2}}\right)\,\left(B\,b+C\,c\right)\,2{}\mathrm{i}}{{\left(b^2+c^2\right)}^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*cos(x) + C*sin(x))/(b*cos(x) + c*sin(x))^2,x)
```

```
[Out] (atan((b^2*c*1i + c^3*1i - b*tan(x/2)*(b^2 + c^2)*1i)/(b^2 + c^2)^(3/2))*(B
*b + C*c)*2i)/(b^2 + c^2)^(3/2) - ((2*(B*c - C*b))/(b^2 + c^2) + (2*c*tan(x
/2)*(B*c - C*b))/(b*(b^2 + c^2)))/(b + 2*c*tan(x/2) - b*tan(x/2)^2)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*cos(x)+C*sin(x))/(b*cos(x)+c*sin(x))**2,x)
```

```
[Out] Timed out
```

3.531 $\int \frac{B\cos(x)+C\sin(x)}{(b\cos(x)+c\sin(x))^3}\,dx$

Optimal. Leaf size=66

$$\frac{\sin(x)(bB+cC)}{b\left(b^2+c^2\right)(b\cos(x)+c\sin(x))}-\frac{Bc-bC}{2\left(b^2+c^2\right)(b\cos(x)+c\sin(x))^2}$$

```
[Out] 1/2*(-B*c+C*b)/(b^2+c^2)/(b*cos(x)+c*sin(x))^2+(B*b+C*c)*sin(x)/b/(b^2+c^2)
/(b*cos(x)+c*sin(x))
```

Rubi [A] time = 0.06, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3156, 12, 3075}

$$\frac{\sin(x)(bB+cC)}{b\left(b^2+c^2\right)(b\cos(x)+c\sin(x))}-\frac{Bc-bC}{2\left(b^2+c^2\right)(b\cos(x)+c\sin(x))^2}$$

Antiderivative was successfully verified.

```
[In] Int[(B*Cos[x] + C*Sin[x])/(b*Cos[x] + c*Sin[x])^3,x]
```

```
[Out] -(B*c - b*C)/(2*(b^2 + c^2)*(b*Cos[x] + c*Sin[x])^2) + ((b*B + c*C)*Sin[x])
/(b*(b^2 + c^2)*(b*Cos[x] + c*Sin[x]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3075

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-2), x
_Symbol] :> Simp[Sin[c + d*x]/(a*d*(a*Cos[c + d*x] + b*Sin[c + d*x])), x] /
; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

Rule 3156

```
Int[((a_.) + cos[(d_.) + (e_.)*(x_)]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)])
^(n_)*((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)
]), x_Symbol] :> -Simp[((c*B - b*C - (a*C - c*A)*Cos[d + e*x] + (a*B - b*A)
*Sin[d + e*x])*(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n + 1))/(e*(n + 1)*(a
^2 - b^2 - c^2)), x] + Dist[1/((n + 1)*(a^2 - b^2 - c^2)), Int[(a + b*Cos[d
 + e*x] + c*Sin[d + e*x])^(n + 1)*Simp[(n + 1)*(a*A - b*B - c*C) + (n + 2)*
(a*B - b*A)*Cos[d + e*x] + (n + 2)*(a*C - c*A)*Sin[d + e*x], x], x], x] /;
FreeQ[{a, b, c, d, e, A, B, C}, x] && LtQ[n, -1] && NeQ[a^2 - b^2 - c^2, 0]
```

```
&& NeQ[n, -2]
```

Rubi steps

$$\int \frac{B\cos(x)+C\sin(x)}{(b\cos(x)+c\sin(x))^3}\,dx = -\frac{Bc-bC}{2\left(b^2+c^2\right)(b\cos(x)+c\sin(x))^2}+\frac{\int \frac{2(bB+cC)}{(b\cos(x)+c\sin(x))^2}\,dx}{2\left(b^2+c^2\right)}$$
$$= -\frac{Bc-bC}{2\left(b^2+c^2\right)(b\cos(x)+c\sin(x))^2}+\frac{(bB+cC)\int \frac{1}{(b\cos(x)+c\sin(x))^2}\,dx}{b^2+c^2}$$
$$= -\frac{Bc-bC}{2\left(b^2+c^2\right)(b\cos(x)+c\sin(x))^2}+\frac{(bB+cC)\sin(x)}{b\left(b^2+c^2\right)(b\cos(x)+c\sin(x))}$$

Mathematica [A] time = 0.19, size = 64, normalized size = 0.97

$$\frac{C\left(b^2+c^2\right)+b\sin(2x)(bB+cC)-c\cos(2x)(bB+cC)}{2b\left(b^2+c^2\right)(b\cos(x)+c\sin(x))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(B*Cos[x] + C*Sin[x])/(b*Cos[x] + c*Sin[x])^3,x]
```

```
[Out] ((b^2 + c^2)*C - c*(b*B + c*C)*Cos[2*x] + b*(b*B + c*C)*Sin[2*x])/(2*b*(b^2
 + c^2)*(b*Cos[x] + c*Sin[x])^2)
```

fricas [B] time = 0.54, size = 152, normalized size = 2.30

$$\frac{Cb^3+Bb^2c+3\,Cbc^2-Bc^3-4\left(Bb^2c+Cbc^2\right)\cos(x)^2+2\left(Bb^3+Cb^2c-Bbc^2-Cc^3\right)\cos(x)\sin(x)}{2\left(b^4c^2+2\,b^2c^4+c^6+\left(b^6+b^4c^2-b^2c^4-c^6\right)\cos(x)^2+2\left(b^5c+2\,b^3c^3+bc^5\right)\cos(x)\sin(x)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*cos(x)+C*sin(x))/(b*cos(x)+c*sin(x))^3,x, algorithm="fricas")
```

```
[Out] 1/2*(C*b^3 + B*b^2*c + 3*C*b*c^2 - B*c^3 - 4*(B*b^2*c + C*b*c^2)*cos(x)^2 +
 2*(B*b^3 + C*b^2*c - B*b*c^2 - C*c^3)*cos(x)*sin(x))/(b^4*c^2 + 2*b^2*c^4
+ c^6 + (b^6 + b^4*c^2 - b^2*c^4 - c^6)*cos(x)^2 + 2*(b^5*c + 2*b^3*c^3 + b
*c^5)*cos(x)*sin(x))
```

giac [A] time = 0.20, size = 26, normalized size = 0.39

$$-\frac{2\,Cc\tan(x)+Cb+Bc}{2\,(c\tan(x)+b)^2c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*cos(x)+C*sin(x))/(b*cos(x)+c*sin(x))^3,x, algorithm="giac")
```

```
[Out] -1/2*(2*C*c*tan(x) + C*b + B*c)/((c*tan(x) + b)^2*c^2)
```

maple [A] time = 0.19, size = 37, normalized size = 0.56

$$-\frac{Bc-bC}{2c^{2}\left(c\tan\left(x\right)+b\right)^{2}}-\frac{C}{c^{2}\left(c\tan\left(x\right)+b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*cos(x)+C*sin(x))/(b*cos(x)+c*sin(x))^3,x)
```

```
[Out] -1/2*(B*c-C*b)/c^2/(c*tan(x)+b)^2-C/c^2/(c*tan(x)+b)
```

maxima [B] time = 0.34, size = 199, normalized size = 3.02

$$\frac{2\,B\left(\frac{b\sin\left(x\right)}{\cos\left(x\right)+1}+\frac{c\sin\left(x\right)^{2}}{\left(\cos\left(x\right)+1\right)^{2}}-\frac{b\sin\left(x\right)^{3}}{\left(\cos\left(x\right)+1\right)^{3}}\right)}{b^{4}+\frac{4\,b^{3}c\sin\left(x\right)}{\cos\left(x\right)+1}-\frac{4\,b^{3}c\sin\left(x\right)^{3}}{\left(\cos\left(x\right)+1\right)^{3}}+\frac{b^{4}\sin\left(x\right)^{4}}{\left(\cos\left(x\right)+1\right)^{4}}-\frac{2\left(b^{4}-2\,b^{2}c^{2}\right)\sin\left(x\right)^{2}}{\left(\cos\left(x\right)+1\right)^{2}}}+\frac{2\,C\sin\left(x\right)^{2}}{\left(b^{3}+\frac{4\,b^{2}c\sin\left(x\right)}{\cos\left(x\right)+1}-\frac{4\,b^{2}c\sin\left(x\right)^{3}}{\left(\cos\left(x\right)+1\right)^{3}}+\frac{b^{3}\sin\left(x\right)^{4}}{\left(\cos\left(x\right)+1\right)^{4}}-\frac{2\left(b^{3}-2\,bc^{2}\right)\sin\left(x\right)^{2}}{\left(\cos\left(x\right)+1\right)^{2}}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*cos(x)+C*sin(x))/(b*cos(x)+c*sin(x))^3,x, algorithm="maxima")
```

```
[Out] 2*B*(b*sin(x)/(cos(x) + 1) + c*sin(x)^2/(cos(x) + 1)^2 - b*sin(x)^3/(cos(x)
 + 1)^3)/(b^4 + 4*b^3*c*sin(x)/(cos(x) + 1) - 4*b^3*c*sin(x)^3/(cos(x) + 1)
^3 + b^4*sin(x)^4/(cos(x) + 1)^4 - 2*(b^4 - 2*b^2*c^2)*sin(x)^2/(cos(x) + 1
)^2) + 2*C*sin(x)^2/((b^3 + 4*b^2*c*sin(x)/(cos(x) + 1) - 4*b^2*c*sin(x)^3/
(cos(x) + 1)^3 + b^3*sin(x)^4/(cos(x) + 1)^4 - 2*(b^3 - 2*b*c^2)*sin(x)^2/(
cos(x) + 1)^2)*(cos(x) + 1)^2)
```

mupad [B] time = 2.82, size = 95, normalized size = 1.44

$$\frac{\frac{2\tan\left(\frac{x}{2}\right)^{2}\left(B\,c+C\,b\right)}{b^{2}}-\frac{2\,B\tan\left(\frac{x}{2}\right)^{3}}{b}+\frac{2\,B\tan\left(\frac{x}{2}\right)}{b}}{b^{2}-\tan\left(\frac{x}{2}\right)^{2}\left(2\,b^{2}-4\,c^{2}\right)+b^{2}\tan\left(\frac{x}{2}\right)^{4}+4\,b\,c\tan\left(\frac{x}{2}\right)-4\,b\,c\tan\left(\frac{x}{2}\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*cos(x) + C*sin(x))/(b*cos(x) + c*sin(x))^3,x)
```

```
[Out] ((2*tan(x/2)^2*(B*c + C*b))/b^2 - (2*B*tan(x/2)^3)/b + (2*B*tan(x/2))/b)/(b
^2 - tan(x/2)^2*(2*b^2 - 4*c^2) + b^2*tan(x/2)^4 + 4*b*c*tan(x/2) - 4*b*c*t
an(x/2)^3)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*cos(x)+C*sin(x))/(b*cos(x)+c*sin(x))**3,x)
[Out] Timed out
```

3.532 $\int \frac{A+B\cos(x)+C\sin(x)}{b\cos(x)+c\sin(x)}\,dx$

Optimal. Leaf size=84

$$-\frac{A\tanh^{-1}\left(\frac{c\cos(x)-b\sin(x)}{\sqrt{b^2+c^2}}\right)}{\sqrt{b^2+c^2}}+\frac{x(bB+cC)}{b^2+c^2}+\frac{(Bc-bC)\log(b\cos(x)+c\sin(x))}{b^2+c^2}$$

```
[Out] (B*b+C*c)*x/(b^2+c^2)+(B*c-C*b)*ln(b*cos(x)+c*sin(x))/(b^2+c^2)-A*arctanh((
c*cos(x)-b*sin(x))/(b^2+c^2)^(1/2))/(b^2+c^2)^(1/2)
```

Rubi [**A**] time = 0.06, antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {3136, 3074, 206}

$$-\frac{A\tanh^{-1}\left(\frac{c\cos(x)-b\sin(x)}{\sqrt{b^2+c^2}}\right)}{\sqrt{b^2+c^2}}+\frac{x(bB+cC)}{b^2+c^2}+\frac{(Bc-bC)\log(b\cos(x)+c\sin(x))}{b^2+c^2}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cos[x] + C*Sin[x])/(b*Cos[x] + c*Sin[x]),x]
```

```
[Out] ((b*B + c*C)*x)/(b^2 + c^2) - (A*ArcTanh[(c*Cos[x] - b*Sin[x])/Sqrt[b^2 + c
^2]])/Sqrt[b^2 + c^2] + ((B*c - b*C)*Log[b*Cos[x] + c*Sin[x]])/(b^2 + c^2)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3074

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x
_Symbol] :> -Dist[d^(-1), Subst[Int[1/(a^2 + b^2 - x^2), x], x, b*Cos[c + d
*x] - a*Sin[c + d*x]], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

Rule 3136

```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])
/((a_.) + cos[(d_.) + (e_.)*(x_)]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)]), x
_Symbol] :> Simp[((b*B + c*C)*x)/(b^2 + c^2), x] + (Dist[(A*(b^2 + c^2) - a
*(b*B + c*C))/(b^2 + c^2), Int[1/(a + b*Cos[d + e*x] + c*Sin[d + e*x]), x],
 x] + Simp[((c*B - b*C)*Log[a + b*Cos[d + e*x] + c*Sin[d + e*x]])/(e*(b^2 +
```

```
 c^2)), x]) /; FreeQ[{a, b, c, d, e, A, B, C}, x] && NeQ[b^2 + c^2, 0] && N
eQ[A*(b^2 + c^2) - a*(b*B + c*C), 0]
```

Rubi steps

$$\begin{aligned}\int \frac{A+B\cos(x)+C\sin(x)}{b\cos(x)+c\sin(x)}\,dx &= \frac{(bB+cC)x}{b^2+c^2}+\frac{(Bc-bC)\log(b\cos(x)+c\sin(x))}{b^2+c^2}+A\int \frac{1}{b\cos(x)+c\sin(x)}\,dx\\ &= \frac{(bB+cC)x}{b^2+c^2}+\frac{(Bc-bC)\log(b\cos(x)+c\sin(x))}{b^2+c^2}-A\,\mathrm{Subst}\left(\int \frac{1}{b^2+c^2-x^2}\,d\right.\\ &= \frac{(bB+cC)x}{b^2+c^2}-\frac{A\tanh^{-1}\left(\frac{c\cos(x)-b\sin(x)}{\sqrt{b^2+c^2}}\right)}{\sqrt{b^2+c^2}}+\frac{(Bc-bC)\log(b\cos(x)+c\sin(x))}{b^2+c^2}\end{aligned}$$

Mathematica [A] time = 0.25, size = 78, normalized size = 0.93

$$\frac{2A\sqrt{b^2+c^2}\,\tanh^{-1}\left(\frac{b\tan\left(\frac{x}{2}\right)-c}{\sqrt{b^2+c^2}}\right)+x(bB+cC)+(Bc-bC)\log(b\cos(x)+c\sin(x))}{b^2+c^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Cos[x] + C*Sin[x])/(b*Cos[x] + c*Sin[x]),x]
```

```
[Out] ((b*B + c*C)*x + 2*A*Sqrt[b^2 + c^2]*ArcTanh[(-c + b*Tan[x/2])/Sqrt[b^2 + c
^2]] + (B*c - b*C)*Log[b*Cos[x] + c*Sin[x]])/(b^2 + c^2)
```

fricas [A] time = 0.77, size = 155, normalized size = 1.85

$$\frac{\sqrt{b^2+c^2}\,A\log\left(-\frac{2\,bc\cos(x)\sin(x)+\left(b^2-c^2\right)\cos(x)^2-2\,b^2-c^2+2\,\sqrt{b^2+c^2}\,(c\cos(x)-b\sin(x))}{2\,bc\cos(x)\sin(x)+\left(b^2-c^2\right)\cos(x)^2+c^2}\right)+2\,(Bb+Cc)x-(Cb-Bc)\log\left(2\,bc\right.}{2\left(b^2+c^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(x)+C*sin(x))/(b*cos(x)+c*sin(x)),x, algorithm="fricas")
```

```
[Out] 1/2*(sqrt(b^2 + c^2)*A*log(-(2*b*c*cos(x)*sin(x) + (b^2 - c^2)*cos(x)^2 - 2
*b^2 - c^2 + 2*sqrt(b^2 + c^2)*(c*cos(x) - b*sin(x)))/(2*b*c*cos(x)*sin(x)
+ (b^2 - c^2)*cos(x)^2 + c^2)) + 2*(B*b + C*c)*x - (C*b - B*c)*log(2*b*c*co
s(x)*sin(x) + (b^2 - c^2)*cos(x)^2 + c^2))/(b^2 + c^2)
```

giac [A] time = 0.25, size = 148, normalized size = 1.76

$$-\frac{A\log\left(\frac{\left|2\,b\tan\left(\frac{1}{2}\,x\right)-2\,c-2\,\sqrt{b^2+c^2}\right|}{\left|2\,b\tan\left(\frac{1}{2}\,x\right)-2\,c+2\,\sqrt{b^2+c^2}\right|}\right)}{\sqrt{b^2+c^2}}+\frac{(Bb+Cc)x}{b^2+c^2}+\frac{(Cb-Bc)\log\left(\tan\left(\frac{1}{2}\,x\right)^2+1\right)}{b^2+c^2}-\frac{(Cb-Bc)\log\left(\left|b\tan\left(\frac{1}{2}\,x\right)^2-2\,c\right.\right.}{b^2+c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(x)+C*sin(x))/(b*cos(x)+c*sin(x)),x, algorithm="giac")
```

```
[Out] -A*log(abs(2*b*tan(1/2*x) - 2*c - 2*sqrt(b^2 + c^2))/abs(2*b*tan(1/2*x) - 2
*c + 2*sqrt(b^2 + c^2)))/sqrt(b^2 + c^2) + (B*b + C*c)*x/(b^2 + c^2) + (C*b
 - B*c)*log(tan(1/2*x)^2 + 1)/(b^2 + c^2) - (C*b - B*c)*log(abs(b*tan(1/2*x
)^2 - 2*c*tan(1/2*x) - b))/(b^2 + c^2)
```

maple [B] time = 0.14, size = 222, normalized size = 2.64

$$\frac{Bc\ln\left(b\left(\tan^2\left(\frac{x}{2}\right)\right)-2c\tan\left(\frac{x}{2}\right)-b\right)}{b^2+c^2}-\frac{bC\ln\left(b\left(\tan^2\left(\frac{x}{2}\right)\right)-2c\tan\left(\frac{x}{2}\right)-b\right)}{b^2+c^2}+\frac{2\operatorname{arctanh}\left(\frac{2b\tan\left(\frac{x}{2}\right)-2c}{2\sqrt{b^2+c^2}}\right)A\,b^2}{\left(b^2+c^2\right)^{\frac{3}{2}}}+\frac{2\operatorname{arct}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(x)+C*sin(x))/(b*cos(x)+c*sin(x)),x)
```

```
[Out] 1/(b^2+c^2)*B*c*ln(b*tan(1/2*x)^2-2*c*tan(1/2*x)-b)-1/(b^2+c^2)*b*C*ln(b*ta
n(1/2*x)^2-2*c*tan(1/2*x)-b)+2/(b^2+c^2)^(3/2)*arctanh(1/2*(2*b*tan(1/2*x)-
2*c)/(b^2+c^2)^(1/2))*A*b^2+2/(b^2+c^2)^(3/2)*arctanh(1/2*(2*b*tan(1/2*x)-2
*c)/(b^2+c^2)^(1/2))*A*c^2-B/(b^2+c^2)*c*ln(1+tan(1/2*x)^2)+C/(b^2+c^2)*b*l
n(1+tan(1/2*x)^2)+2*B/(b^2+c^2)*b*arctan(tan(1/2*x))+2*C/(b^2+c^2)*c*arctan
(tan(1/2*x))
```

maxima [B] time = 0.42, size = 243, normalized size = 2.89

$$B\left(\frac{2\,b\arctan\left(\frac{\sin(x)}{\cos(x)+1}\right)}{b^2+c^2}+\frac{c\log\left(-b-\frac{2\,c\sin(x)}{\cos(x)+1}+\frac{b\sin(x)^2}{(\cos(x)+1)^2}\right)}{b^2+c^2}-\frac{c\log\left(\frac{\sin(x)^2}{(\cos(x)+1)^2}+1\right)}{b^2+c^2}\right)+C\left(\frac{2\,c\arctan\left(\frac{\sin(x)}{\cos(x)+1}\right)}{b^2+c^2}-\frac{b}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(x)+C*sin(x))/(b*cos(x)+c*sin(x)),x, algorithm="maxima")
```

```
[Out] B*(2*b*arctan(sin(x)/(cos(x) + 1))/(b^2 + c^2) + c*log(-b - 2*c*sin(x)/(cos
(x) + 1) + b*sin(x)^2/(cos(x) + 1)^2)/(b^2 + c^2) - c*log(sin(x)^2/(cos(x)
```

```
+ 1)^2 + 1)/(b^2 + c^2)) + C*(2*c*arctan(sin(x)/(cos(x) + 1))/(b^2 + c^2) -
 b*log(-b - 2*c*sin(x)/(cos(x) + 1) + b*sin(x)^2/(cos(x) + 1)^2)/(b^2 + c^2
) + b*log(sin(x)^2/(cos(x) + 1)^2 + 1)/(b^2 + c^2)) - A*log((c - b*sin(x)/(
cos(x) + 1) + sqrt(b^2 + c^2))/(c - b*sin(x)/(cos(x) + 1) - sqrt(b^2 + c^2)
))/sqrt(b^2 + c^2)
```

mupad [B] time = 9.41, size = 1099, normalized size = 13.08

$$\ln\left(32\,A^2\,B\,b^2-32\,A\,B^2\,b^2-32\,A\,C^2\,b^2-32\,B\,C^2\,b^2+32\,b\tan\left(\frac{x}{2}\right)\,\left(c\,A^2\,B+b\,A^2\,C-2\,c\,A\,B^2-2\,c\,A\,C^2+c\,\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cos(x) + C*sin(x))/(b*cos(x) + c*sin(x)),x)
```

```
[Out] log(32*A^2*B*b^2 - 32*A*B^2*b^2 - 32*A*C^2*b^2 - 32*B*C^2*b^2 + 32*b*tan(x/
2)*(B^3*c - 2*C^3*b - 2*A*B^2*c + A^2*B*c + A^2*C*b - 2*A*C^2*c - B^2*C*b +
 2*B*C^2*c) - ((A*((b^2 + c^2)^3)^(1/2) + B*c^3 - C*b^3 + B*b^2*c - C*b*c^2
)*(32*b*tan(x/2)*(A^2*b^2 - A^2*c^2 + B^2*b^2 - 3*B^2*c^2 + 2*C^2*c^2 + 4*A
*B*c^2 - 4*A*C*b*c + 6*B*C*b*c) - 32*B^2*b^2*c - 32*C^2*b^2*c - 64*A^2*b^2*
c - 64*A*C*b^3 + 64*B*C*b^3 + ((A*((b^2 + c^2)^3)^(1/2) + B*c^3 - C*b^3 + B
*b^2*c - C*b*c^2)*(32*A*b^4 + 32*B*b^4 + 32*A*b^2*c^2 - 64*B*b^2*c^2 - 32*C
*b*c^3 + 64*C*b^3*c + 32*b*tan(x/2)*(2*A*c^3 + B*c^3 - 2*C*b^3 + 2*A*b^2*c
+ 4*B*b^2*c + C*b*c^2) + (96*b*c*(b + c*tan(x/2))*(A*((b^2 + c^2)^3)^(1/2)
+ B*c^3 - C*b^3 + B*b^2*c - C*b*c^2))/(b^2 + c^2)))/(b^2 + c^2)^2 + 64*A*B*
b^2*c + 64*B*C*b*c^2))/(b^2 + c^2)^2 - 32*A^2*C*b*c + 32*B^2*C*b*c)*((B*c -
 C*b)/(b^2 + c^2) + (A*((b^2 + c^2)^3)^(1/2))/(b^2 + c^2)^2) + log(32*A^2*B
*b^2 - 32*A*B^2*b^2 - 32*A*C^2*b^2 - 32*B*C^2*b^2 + 32*b*tan(x/2)*(B^3*c -
2*C^3*b - 2*A*B^2*c + A^2*B*c + A^2*C*b - 2*A*C^2*c - B^2*C*b + 2*B*C^2*c)
- ((A*((b^2 + c^2)^3)^(1/2) - B*c^3 + C*b^3 - B*b^2*c + C*b*c^2)*(64*A^2*b^
2*c + 32*B^2*b^2*c + 32*C^2*b^2*c - 32*b*tan(x/2)*(A^2*b^2 - A^2*c^2 + B^2*
b^2 - 3*B^2*c^2 + 2*C^2*c^2 + 4*A*B*c^2 - 4*A*C*b*c + 6*B*C*b*c) + 64*A*C*b
^3 - 64*B*C*b^3 + ((A*((b^2 + c^2)^3)^(1/2) - B*c^3 + C*b^3 - B*b^2*c + C*b
*c^2)*(32*A*b^4 + 32*B*b^4 + 32*A*b^2*c^2 - 64*B*b^2*c^2 - 32*C*b*c^3 + 64*
C*b^3*c + 32*b*tan(x/2)*(2*A*c^3 + B*c^3 - 2*C*b^3 + 2*A*b^2*c + 4*B*b^2*c
```

```
+ C*b*c^2) - (96*b*c*(b + c*tan(x/2))*(A*((b^2 + c^2)^3)^(1/2) - B*c^3 + C*
b^3 - B*b^2*c + C*b*c^2))/(b^2 + c^2)))/(b^2 + c^2)^2 - 64*A*B*b^2*c - 64*B
*C*b*c^2))/(b^2 + c^2)^2 - 32*A^2*C*b*c + 32*B^2*C*b*c)*((B*c - C*b)/(b^2 +
 c^2) - (A*((b^2 + c^2)^3)^(1/2))/(b^2 + c^2)^2) - (log(tan(x/2) + 1i)*(B -
 C*1i))/(b*1i + c) + (log(tan(x/2) - 1i)*(B + C*1i))/(b*1i - c)
```

sympy [A] time = 38.59, size = 1042, normalized size = 12.40

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(x)+C*sin(x))/(b*cos(x)+c*sin(x)),x)
```

```
[Out] Piecewise((zoo*(A*log(tan(x/2)) - B*log(tan(x/2)**2 + 1) + B*log(tan(x/2))
+ C*x), Eq(b, 0) & Eq(c, 0)), ((A*log(tan(x/2)) - B*log(tan(x/2)**2 + 1) +
B*log(tan(x/2)) + C*x)/c, Eq(b, 0)), (2*A/(-2*I*c*sin(x) - 2*c*cos(x)) + B*
x*sin(x)/(-2*I*c*sin(x) - 2*c*cos(x)) - I*B*x*cos(x)/(-2*I*c*sin(x) - 2*c*c
os(x)) - I*B*sin(x)/(-2*I*c*sin(x) - 2*c*cos(x)) - I*C*x*sin(x)/(-2*I*c*sin
(x) - 2*c*cos(x)) - C*x*cos(x)/(-2*I*c*sin(x) - 2*c*cos(x)) + C*sin(x)/(-2*
I*c*sin(x) - 2*c*cos(x)), Eq(b, -I*c)), (2*A/(2*I*c*sin(x) - 2*c*cos(x)) +
B*x*sin(x)/(2*I*c*sin(x) - 2*c*cos(x)) + I*B*x*cos(x)/(2*I*c*sin(x) - 2*c*c
os(x)) + I*B*sin(x)/(2*I*c*sin(x) - 2*c*cos(x)) + I*C*x*sin(x)/(2*I*c*sin(x
) - 2*c*cos(x)) - C*x*cos(x)/(2*I*c*sin(x) - 2*c*cos(x)) + C*sin(x)/(2*I*c*
sin(x) - 2*c*cos(x)), Eq(b, I*c)), (-A*b**2*log(tan(x/2) - c/b - sqrt(b**2
+ c**2)/b)/(b**2*sqrt(b**2 + c**2) + c**2*sqrt(b**2 + c**2)) + A*b**2*log(t
an(x/2) - c/b + sqrt(b**2 + c**2)/b)/(b**2*sqrt(b**2 + c**2) + c**2*sqrt(b*
*2 + c**2)) - A*c**2*log(tan(x/2) - c/b - sqrt(b**2 + c**2)/b)/(b**2*sqrt(b
**2 + c**2) + c**2*sqrt(b**2 + c**2)) + A*c**2*log(tan(x/2) - c/b + sqrt(b*
*2 + c**2)/b)/(b**2*sqrt(b**2 + c**2) + c**2*sqrt(b**2 + c**2)) + B*b*x*sqr
t(b**2 + c**2)/(b**2*sqrt(b**2 + c**2) + c**2*sqrt(b**2 + c**2)) - B*c*sqrt
(b**2 + c**2)*log(tan(x/2)**2 + 1)/(b**2*sqrt(b**2 + c**2) + c**2*sqrt(b**2
 + c**2)) + B*c*sqrt(b**2 + c**2)*log(tan(x/2) - c/b - sqrt(b**2 + c**2)/b)
/(b**2*sqrt(b**2 + c**2) + c**2*sqrt(b**2 + c**2)) + B*c*sqrt(b**2 + c**2)*
log(tan(x/2) - c/b + sqrt(b**2 + c**2)/b)/(b**2*sqrt(b**2 + c**2) + c**2*sq
rt(b**2 + c**2)) + C*b*sqrt(b**2 + c**2)*log(tan(x/2)**2 + 1)/(b**2*sqrt(b*
*2 + c**2) + c**2*sqrt(b**2 + c**2)) - C*b*sqrt(b**2 + c**2)*log(tan(x/2) -
 c/b - sqrt(b**2 + c**2)/b)/(b**2*sqrt(b**2 + c**2) + c**2*sqrt(b**2 + c**2
)) - C*b*sqrt(b**2 + c**2)*log(tan(x/2) - c/b + sqrt(b**2 + c**2)/b)/(b**2*
sqrt(b**2 + c**2) + c**2*sqrt(b**2 + c**2)) + C*c*x*sqrt(b**2 + c**2)/(b**2
*sqrt(b**2 + c**2) + c**2*sqrt(b**2 + c**2)), True))
```

3.533 $\int \frac{A+B\cos(x)+C\sin(x)}{(b\cos(x)+c\sin(x))^2}\,dx$

Optimal. Leaf size=85

$$-\frac{-Ab\sin(x)+Ac\cos(x)-bC+Bc}{(b^2+c^2)\,(b\cos(x)+c\sin(x))}-\frac{(bB+cC)\tanh^{-1}\left(\frac{c\cos(x)-b\sin(x)}{\sqrt{b^2+c^2}}\right)}{(b^2+c^2)^{3/2}}$$

```
[Out] -(B*b+C*c)*arctanh((c*cos(x)-b*sin(x))/(b^2+c^2)^(1/2))/(b^2+c^2)^(3/2)+(-B
*c+b*C-A*c*cos(x)+A*b*sin(x))/(b^2+c^2)/(b*cos(x)+c*sin(x))
```

Rubi [A] time = 0.06, antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {3153, 3074, 206}

$$-\frac{-Ab\sin(x)+Ac\cos(x)-bC+Bc}{(b^2+c^2)\,(b\cos(x)+c\sin(x))}-\frac{(bB+cC)\tanh^{-1}\left(\frac{c\cos(x)-b\sin(x)}{\sqrt{b^2+c^2}}\right)}{(b^2+c^2)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cos[x] + C*Sin[x])/(b*Cos[x] + c*Sin[x])^2,x]
```

```
[Out] -(((b*B + c*C)*ArcTanh[(c*Cos[x] - b*Sin[x])/Sqrt[b^2 + c^2]])/(b^2 + c^2)^
(3/2)) - (B*c - b*C + A*c*Cos[x] - A*b*Sin[x])/((b^2 + c^2)*(b*Cos[x] + c*S
in[x]))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3074

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x
_Symbol] :> -Dist[d^(-1), Subst[Int[1/(a^2 + b^2 - x^2), x], x, b*Cos[c + d
*x] - a*Sin[c + d*x]], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

Rule 3153

```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])
/((a_.) + cos[(d_.) + (e_.)*(x_)]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)])^2,
 x_Symbol] :> Simp[(c*B - b*C - (a*C - c*A)*Cos[d + e*x] + (a*B - b*A)*Sin[
d + e*x])/(e*(a^2 - b^2 - c^2)*(a + b*Cos[d + e*x] + c*Sin[d + e*x])), x] +
```

```
Dist[(a*A - b*B - c*C)/(a^2 - b^2 - c^2), Int[1/(a + b*Cos[d + e*x] + c*Si
n[d + e*x]), x], x] /; FreeQ[{a, b, c, d, e, A, B, C}, x] && NeQ[a^2 - b^2
- c^2, 0] && NeQ[a*A - b*B - c*C, 0]
```

Rubi steps

$$\int \frac{A+B\cos(x)+C\sin(x)}{(b\cos(x)+c\sin(x))^2}\,dx = -\frac{Bc-bC+Ac\cos(x)-Ab\sin(x)}{\left(b^2+c^2\right)(b\cos(x)+c\sin(x))}+\frac{(bB+cC)\int \frac{1}{b\cos(x)+c\sin(x)}\,dx}{b^2+c^2}$$

$$= -\frac{Bc-bC+Ac\cos(x)-Ab\sin(x)}{\left(b^2+c^2\right)(b\cos(x)+c\sin(x))}-\frac{(bB+cC)\,\text{Subst}\left(\int \frac{1}{b^2+c^2-x^2}\,dx,x,c\cos(x)\right.}{b^2+c^2}$$

$$= -\frac{(bB+cC)\tanh^{-1}\left(\frac{c\cos(x)-b\sin(x)}{\sqrt{b^2+c^2}}\right)}{\left(b^2+c^2\right)^{3/2}}-\frac{Bc-bC+Ac\cos(x)-Ab\sin(x)}{\left(b^2+c^2\right)(b\cos(x)+c\sin(x))}$$

Mathematica [A] time = 0.28, size = 92, normalized size = 1.08

$$\frac{A\left(b^2+c^2\right)\sin(x)+b(bC-Bc)}{b\left(b^2+c^2\right)(b\cos(x)+c\sin(x))}+\frac{2(bB+cC)\tanh^{-1}\left(\frac{b\tan\left(\frac{x}{2}\right)-c}{\sqrt{b^2+c^2}}\right)}{\left(b^2+c^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Cos[x] + C*Sin[x])/(b*Cos[x] + c*Sin[x])^2,x]
```

```
[Out] (2*(b*B + c*C)*ArcTanh[(-c + b*Tan[x/2])/Sqrt[b^2 + c^2]])/(b^2 + c^2)^(3/2
) + (b*(-(B*c) + b*C) + A*(b^2 + c^2)*Sin[x])/(b*(b^2 + c^2)*(b*Cos[x] + c*
Sin[x]))
```

fricas [B] time = 3.02, size = 226, normalized size = 2.66

$$\frac{2\,Cb^3-2\,Bb^2c+2\,Cbc^2-2\,Bc^3+\sqrt{b^2+c^2}\left(\left(Bb^2+Cbc\right)\cos(x)+\left(Bbc+Cc^2\right)\sin(x)\right)\log\left(-\frac{2\,bc\cos(x)\sin(x)+\left(b^2-c^2\right.}{2\,bc\cos}\right.}{2\left(\left(b^5+2\,b^3c^2+bc^4\right)\cos(x)+\left(b^4c+2\,b^2\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(x)+C*sin(x))/(b*cos(x)+c*sin(x))^2,x, algorithm="fricas"
)
```

```
[Out] 1/2*(2*C*b^3 - 2*B*b^2*c + 2*C*b*c^2 - 2*B*c^3 + sqrt(b^2 + c^2)*((B*b^2 +
C*b*c)*cos(x) + (B*b*c + C*c^2)*sin(x))*log(-(2*b*c*cos(x)*sin(x) + (b^2 -
c^2)*cos(x)^2 - 2*b^2 - c^2 + 2*sqrt(b^2 + c^2)*(c*cos(x) - b*sin(x)))/(2*b
*c*cos(x)*sin(x) + (b^2 - c^2)*cos(x)^2 + c^2)) - 2*(A*b^2*c + A*c^3)*cos(x
) + 2*(A*b^3 + A*b*c^2)*sin(x))/((b^5 + 2*b^3*c^2 + b*c^4)*cos(x) + (b^4*c
+ 2*b^2*c^3 + c^5)*sin(x))
```

giac [A] time = 0.23, size = 150, normalized size = 1.76

$$-\frac{(Bb+Cc)\log\left(\frac{\left|2\,b\tan\left(\frac{1}{2}\,x\right)-2\,c-2\,\sqrt{b^2+c^2}\right|}{\left|2\,b\tan\left(\frac{1}{2}\,x\right)-2\,c+2\,\sqrt{b^2+c^2}\right|}\right)}{\left(b^2+c^2\right)^{\frac{3}{2}}}-\frac{2\left(Ab^2\tan\left(\frac{1}{2}\,x\right)+Cbc\tan\left(\frac{1}{2}\,x\right)+Ac^2\tan\left(\frac{1}{2}\,x\right)-Bc^2\tan\left(\frac{1}{2}\,x\right)+C\right.}{\left(b^3+bc^2\right)\left(b\tan\left(\frac{1}{2}\,x\right)^2-2\,c\tan\left(\frac{1}{2}\,x\right)-b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(x)+C*sin(x))/(b*cos(x)+c*sin(x))^2,x, algorithm="giac")
```

```
[Out] -(B*b + C*c)*log(abs(2*b*tan(1/2*x) - 2*c - 2*sqrt(b^2 + c^2))/abs(2*b*tan(
1/2*x) - 2*c + 2*sqrt(b^2 + c^2)))/(b^2 + c^2)^(3/2) - 2*(A*b^2*tan(1/2*x)
+ C*b*c*tan(1/2*x) + A*c^2*tan(1/2*x) - B*c^2*tan(1/2*x) + C*b^2 - B*b*c)/(
(b^3 + b*c^2)*(b*tan(1/2*x)^2 - 2*c*tan(1/2*x) - b))
```

maple [A] time = 0.17, size = 124, normalized size = 1.46

$$\frac{-\frac{2\left(A\,b^2+A\,c^2-B\,c^2+Cbc\right)\tan\left(\frac{x}{2}\right)}{b\left(b^2+c^2\right)}+\frac{2(Bc-bC)}{b^2+c^2}}{b\left(\tan^2\left(\frac{x}{2}\right)\right)-2c\tan\left(\frac{x}{2}\right)-b}+\frac{2\,(bB+Cc)\,\mathrm{arctanh}\left(\frac{2b\tan\left(\frac{x}{2}\right)-2c}{2\sqrt{b^2+c^2}}\right)}{\left(b^2+c^2\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(x)+C*sin(x))/(b*cos(x)+c*sin(x))^2,x)
```

```
[Out] 2*(-(A*b^2+A*c^2-B*c^2+C*b*c)/b/(b^2+c^2)*tan(1/2*x)+(B*c-C*b)/(b^2+c^2))/(
b*tan(1/2*x)^2-2*c*tan(1/2*x)-b)+2*(B*b+C*c)/(b^2+c^2)^(3/2)*arctanh(1/2*(2
*b*tan(1/2*x)-2*c)/(b^2+c^2)^(1/2))
```

maxima [B] time = 0.42, size = 286, normalized size = 3.36

$$-B\left(\frac{b\log\left(\frac{c-\frac{b\sin(x)}{\cos(x)+1}+\sqrt{b^2+c^2}}{c-\frac{b\sin(x)}{\cos(x)+1}-\sqrt{b^2+c^2}}\right)}{\left(b^2+c^2\right)^{\frac{3}{2}}}+\frac{2\left(bc+\frac{c^2\sin(x)}{\cos(x)+1}\right)}{b^4+b^2c^2+\frac{2\left(b^3c+bc^3\right)\sin(x)}{\cos(x)+1}-\frac{\left(b^4+b^2c^2\right)\sin(x)^2}{(\cos(x)+1)^2}}\right)-C\left(\frac{c\log\left(\frac{c-\frac{b\sin(x)}{\cos(x)+1}+\sqrt{b^2+c^2}}{c-\frac{b\sin(x)}{\cos(x)+1}-\sqrt{b^2+c^2}}\right)}{\left(b^2+c^2\right)^{\frac{3}{2}}}-\frac{}{b^3+b}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(x)+C*sin(x))/(b*cos(x)+c*sin(x))^2,x, algorithm="maxima"
)
```

```
[Out] -B*(b*log((c - b*sin(x)/(cos(x) + 1) + sqrt(b^2 + c^2))/(c - b*sin(x)/(cos(
x) + 1) - sqrt(b^2 + c^2)))/(b^2 + c^2)^(3/2) + 2*(b*c + c^2*sin(x)/(cos(x)
 + 1))/(b^4 + b^2*c^2 + 2*(b^3*c + b*c^3)*sin(x)/(cos(x) + 1) - (b^4 + b^2*
c^2)*sin(x)^2/(cos(x) + 1)^2)) - C*(c*log((c - b*sin(x)/(cos(x) + 1) + sqrt
(b^2 + c^2))/(c - b*sin(x)/(cos(x) + 1) - sqrt(b^2 + c^2)))/(b^2 + c^2)^(3/
2) - 2*(b + c*sin(x)/(cos(x) + 1))/(b^3 + b*c^2 + 2*(b^2*c + c^3)*sin(x)/(c
os(x) + 1) - (b^3 + b*c^2)*sin(x)^2/(cos(x) + 1)^2)) - A/(c^2*tan(x) + b*c)
```

mupad [B] time = 3.05, size = 141, normalized size = 1.66

$$-\frac{\frac{2\,(B\,c-C\,b)}{b^2+c^2}-\frac{2\tan\left(\frac{x}{2}\right)\left(A\,b^2+A\,c^2-B\,c^2+C\,b\,c\right)}{b\left(b^2+c^2\right)}}{-b\tan\left(\frac{x}{2}\right)^2+2\,c\tan\left(\frac{x}{2}\right)+b}+\frac{\mathrm{atan}\left(\frac{b^2\,c\,1\mathrm{i}+c^3\,1\mathrm{i}-b\tan\left(\frac{x}{2}\right)\left(b^2+c^2\right)1\mathrm{i}}{\left(b^2+c^2\right)^{3/2}}\right)(B\,b+C\,c)\,2\mathrm{i}}{\left(b^2+c^2\right)^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cos(x) + C*sin(x))/(b*cos(x) + c*sin(x))^2,x)
```

```
[Out] (atan((b^2*c*1i + c^3*1i - b*tan(x/2)*(b^2 + c^2)*1i)/(b^2 + c^2)^(3/2))*(B
*b + C*c)*2i)/(b^2 + c^2)^(3/2) - ((2*(B*c - C*b))/(b^2 + c^2) - (2*tan(x/2
)*(A*b^2 + A*c^2 - B*c^2 + C*b*c))/(b*(b^2 + c^2)))/(b + 2*c*tan(x/2) - b*t
an(x/2)^2)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(x)+C*sin(x))/(b*cos(x)+c*sin(x))**2,x)
```

```
[Out] Timed out
```

3.534 $\int \frac{A+B\cos(x)+C\sin(x)}{(b\cos(x)+c\sin(x))^3}\,dx$

Optimal. Leaf size=129

$$-\frac{-Ab\sin(x)+Ac\cos(x)-bC+Bc}{2\left(b^2+c^2\right)(b\cos(x)+c\sin(x))^2}-\frac{A\tanh^{-1}\left(\frac{c\cos(x)-b\sin(x)}{\sqrt{b^2+c^2}}\right)}{2\left(b^2+c^2\right)^{3/2}}-\frac{c\cos(x)(bB+cC)-b\sin(x)(bB+cC)}{\left(b^2+c^2\right)^2(b\cos(x)+c\sin(x))}$$

```
[Out] -1/2*A*arctanh((c*cos(x)-b*sin(x))/(b^2+c^2)^(1/2))/(b^2+c^2)^(3/2)+1/2*(-B
*c+b*C-A*c*cos(x)+A*b*sin(x))/(b^2+c^2)/(b*cos(x)+c*sin(x))^2+(-c*(B*b+C*c)
*cos(x)+b*(B*b+C*c)*sin(x))/(b^2+c^2)^2/(b*cos(x)+c*sin(x))
```

Rubi [A] time = 0.12, antiderivative size = 129, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {3156, 3153, 3074, 206}

$$-\frac{-Ab\sin(x)+Ac\cos(x)-bC+Bc}{2\left(b^2+c^2\right)(b\cos(x)+c\sin(x))^2}-\frac{A\tanh^{-1}\left(\frac{c\cos(x)-b\sin(x)}{\sqrt{b^2+c^2}}\right)}{2\left(b^2+c^2\right)^{3/2}}-\frac{c\cos(x)(bB+cC)-b\sin(x)(bB+cC)}{\left(b^2+c^2\right)^2(b\cos(x)+c\sin(x))}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cos[x] + C*Sin[x])/(b*Cos[x] + c*Sin[x])^3,x]
```

```
[Out] -(A*ArcTanh[(c*Cos[x] - b*Sin[x])/Sqrt[b^2 + c^2]])/(2*(b^2 + c^2)^(3/2)) -
 (B*c - b*C + A*c*Cos[x] - A*b*Sin[x])/(2*(b^2 + c^2)*(b*Cos[x] + c*Sin[x])
^2) - (c*(b*B + c*C)*Cos[x] - b*(b*B + c*C)*Sin[x])/((b^2 + c^2)^2*(b*Cos[x
] + c*Sin[x]))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3074

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x
_Symbol] :> -Dist[d^(-1), Subst[Int[1/(a^2 + b^2 - x^2), x], x, b*Cos[c + d
*x] - a*Sin[c + d*x]], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

Rule 3153

```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])
/((a_.) + cos[(d_.) + (e_.)*(x_)]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)])^2,
```

```
 x_Symbol] :> Simp[(c*B - b*C - (a*C - c*A)*Cos[d + e*x] + (a*B - b*A)*Sin[
d + e*x])/(e*(a^2 - b^2 - c^2)*(a + b*Cos[d + e*x] + c*Sin[d + e*x])), x] +
 Dist[(a*A - b*B - c*C)/(a^2 - b^2 - c^2), Int[1/(a + b*Cos[d + e*x] + c*Si
n[d + e*x]), x], x] /; FreeQ[{a, b, c, d, e, A, B, C}, x] && NeQ[a^2 - b^2
- c^2, 0] && NeQ[a*A - b*B - c*C, 0]
```

Rule 3156

```
Int[((a_.) + cos[(d_.) + (e_.)*(x_)]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)])
^(n_)*((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)
]), x_Symbol] :> -Simp[((c*B - b*C - (a*C - c*A)*Cos[d + e*x] + (a*B - b*A)
*Sin[d + e*x])*(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n + 1))/(e*(n + 1)*(a
^2 - b^2 - c^2)), x] + Dist[1/((n + 1)*(a^2 - b^2 - c^2)), Int[(a + b*Cos[d
 + e*x] + c*Sin[d + e*x])^(n + 1)*Simp[(n + 1)*(a*A - b*B - c*C) + (n + 2)*
(a*B - b*A)*Cos[d + e*x] + (n + 2)*(a*C - c*A)*Sin[d + e*x], x], x], x] /;
FreeQ[{a, b, c, d, e, A, B, C}, x] && LtQ[n, -1] && NeQ[a^2 - b^2 - c^2, 0]
 && NeQ[n, -2]
```

Rubi steps

$$\int \frac{A+B\cos(x)+C\sin(x)}{(b\cos(x)+c\sin(x))^3}\,dx = -\frac{Bc-bC+Ac\cos(x)-Ab\sin(x)}{2\left(b^2+c^2\right)(b\cos(x)+c\sin(x))^2}+\frac{\int \frac{2(bB+cC)+Ab\cos(x)+Ac\sin(x)}{(b\cos(x)+c\sin(x))^2}\,dx}{2\left(b^2+c^2\right)}$$

$$= -\frac{Bc-bC+Ac\cos(x)-Ab\sin(x)}{2\left(b^2+c^2\right)(b\cos(x)+c\sin(x))^2}-\frac{c(bB+cC)\cos(x)-b(bB+cC)\sin(x)}{\left(b^2+c^2\right)^2(b\cos(x)+c\sin(x))}+$$

$$= -\frac{Bc-bC+Ac\cos(x)-Ab\sin(x)}{2\left(b^2+c^2\right)(b\cos(x)+c\sin(x))^2}-\frac{c(bB+cC)\cos(x)-b(bB+cC)\sin(x)}{\left(b^2+c^2\right)^2(b\cos(x)+c\sin(x))}-$$

$$= -\frac{A\tanh^{-1}\left(\frac{c\cos(x)-b\sin(x)}{\sqrt{b^2+c^2}}\right)}{2\left(b^2+c^2\right)^{3/2}}-\frac{Bc-bC+Ac\cos(x)-Ab\sin(x)}{2\left(b^2+c^2\right)(b\cos(x)+c\sin(x))^2}-\frac{c(bB+cC)\cos}{\left(b^2+c^2\right)^2}$$

Mathematica [A] time = 0.66, size = 122, normalized size = 0.95

$$\frac{Ab^2\sin(x)-Abc\cos(x)+b^2B\sin(2x)+b^2C-c\cos(2x)(bB+cC)+bcC\sin(2x)+c^2C}{2b\left(b^2+c^2\right)(b\cos(x)+c\sin(x))^2}+\frac{A\tanh^{-1}\left(\frac{b\tan\left(\frac{x}{2}\right)-c}{\sqrt{b^2+c^2}}\right)}{\left(b^2+c^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Cos[x] + C*Sin[x])/(b*Cos[x] + c*Sin[x])^3,x]
```

```
[Out] (A*ArcTanh[(-c + b*Tan[x/2])/Sqrt[b^2 + c^2]])/(b^2 + c^2)^(3/2) + (b^2*C +
 c^2*C - A*b*c*Cos[x] - c*(b*B + c*C)*Cos[2*x] + A*b^2*Sin[x] + b^2*B*Sin[2
*x] + b*c*C*Sin[2*x])/(2*b*(b^2 + c^2)*(b*Cos[x] + c*Sin[x])^2)
```

fricas [B] time = 1.69, size = 311, normalized size = 2.41

$$\frac{2\,Cb^3 + 2\,Bb^2c + 6\,Cbc^2 - 2\,Bc^3 - 8\left(Bb^2c + Cbc^2\right)\cos(x)^2 + \left(2\,Abc\cos(x)\sin(x) + Ac^2 + \left(Ab^2 - Ac^2\right)\cos(x)^2}{4\left(b^4c^2 + 2\,b^2c^4 + c^6\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(x)+C*sin(x))/(b*cos(x)+c*sin(x))^3,x, algorithm="fricas"
)
```

```
[Out] 1/4*(2*C*b^3 + 2*B*b^2*c + 6*C*b*c^2 - 2*B*c^3 - 8*(B*b^2*c + C*b*c^2)*cos(
x)^2 + (2*A*b*c*cos(x)*sin(x) + A*c^2 + (A*b^2 - A*c^2)*cos(x)^2)*sqrt(b^2
+ c^2)*log(-(2*b*c*cos(x)*sin(x) + (b^2 - c^2)*cos(x)^2 - 2*b^2 - c^2 + 2*s
qrt(b^2 + c^2)*(c*cos(x) - b*sin(x)))/(2*b*c*cos(x)*sin(x) + (b^2 - c^2)*co
s(x)^2 + c^2)) - 2*(A*b^2*c + A*c^3)*cos(x) + 2*(A*b^3 + A*b*c^2 + 2*(B*b^3
 + C*b^2*c - B*b*c^2 - C*c^3)*cos(x))*sin(x))/(b^4*c^2 + 2*b^2*c^4 + c^6 +
(b^6 + b^4*c^2 - b^2*c^4 - c^6)*cos(x)^2 + 2*(b^5*c + 2*b^3*c^3 + b*c^5)*co
s(x)*sin(x))
```

giac [B] time = 0.26, size = 270, normalized size = 2.09

$$\frac{A\log\left(\frac{\left|-2\,b\tan\left(\frac{1}{2}\,x\right)+2\,c-2\,\sqrt{b^2+c^2}\right|}{\left|-2\,b\tan\left(\frac{1}{2}\,x\right)+2\,c+2\,\sqrt{b^2+c^2}\right|}\right)}{2\left(b^2+c^2\right)^{\frac{3}{2}}} + \frac{Ab^3\tan\left(\frac{1}{2}\,x\right)^3 - 2\,Bb^3\tan\left(\frac{1}{2}\,x\right)^3 + 2\,Abc^2\tan\left(\frac{1}{2}\,x\right)^3 - 2\,Bbc^2\tan\left(\frac{1}{2}\,x\right)^3 + 2}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(x)+C*sin(x))/(b*cos(x)+c*sin(x))^3,x, algorithm="giac")
```

```
[Out] 1/2*A*log(abs(-2*b*tan(1/2*x) + 2*c - 2*sqrt(b^2 + c^2))/abs(-2*b*tan(1/2*x
) + 2*c + 2*sqrt(b^2 + c^2)))/(b^2 + c^2)^(3/2) + (A*b^3*tan(1/2*x)^3 - 2*B
*b^3*tan(1/2*x)^3 + 2*A*b*c^2*tan(1/2*x)^3 - 2*B*b*c^2*tan(1/2*x)^3 + 2*C*b
^3*tan(1/2*x)^2 + A*b^2*c*tan(1/2*x)^2 + 2*B*b^2*c*tan(1/2*x)^2 + 2*C*b*c^2
*tan(1/2*x)^2 - 2*A*c^3*tan(1/2*x)^2 + 2*B*c^3*tan(1/2*x)^2 + A*b^3*tan(1/2
*x) + 2*B*b^3*tan(1/2*x) - 2*A*b*c^2*tan(1/2*x) + 2*B*b*c^2*tan(1/2*x) - A*
b^2*c)/((b^4 + b^2*c^2)*(b*tan(1/2*x)^2 - 2*c*tan(1/2*x) - b)^2)
```

maple [A] time = 0.20, size = 218, normalized size = 1.69

$$-\frac{2\left(-\frac{\left(A\,b^2+2A\,c^2-2B\,b^2-2B\,c^2\right)\left(\tan^3\left(\frac{x}{2}\right)\right)}{2\left(b^2+c^2\right)b}-\frac{\left(A\,b^2c-2A\,c^3+2B\,b^2c+2B\,c^3+2C\,b^3+2Cb\,c^2\right)\left(\tan^2\left(\frac{x}{2}\right)\right)}{2\left(b^2+c^2\right)b^2}-\frac{\left(A\,b^2-2A\,c^2+2B\,b^2+2B\,c^2\right)\tan\left(\frac{x}{2}\right)}{2\left(b^2+c^2\right)b}+\frac{}{2b^2}\right.}{\left(b\left(\tan^2\left(\frac{x}{2}\right)\right)-2c\tan\left(\frac{x}{2}\right)-b\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(x)+C*sin(x))/(b*cos(x)+c*sin(x))^3,x)
```

```
[Out] -2*(-1/2*(A*b^2+2*A*c^2-2*B*b^2-2*B*c^2)/(b^2+c^2)/b*tan(1/2*x)^3-1/2*(A*b^
2*c-2*A*c^3+2*B*b^2*c+2*B*c^3+2*C*b^3+2*C*b*c^2)/(b^2+c^2)/b^2*tan(1/2*x)^2
-1/2*(A*b^2-2*A*c^2+2*B*b^2+2*B*c^2)/(b^2+c^2)/b*tan(1/2*x)+1/2*A*c/(b^2+c^
2))/(b*tan(1/2*x)^2-2*c*tan(1/2*x)-b)^2+A/(b^2+c^2)^(3/2)*arctanh(1/2*(2*b*
tan(1/2*x)-2*c)/(b^2+c^2)^(1/2))
```

maxima [B] time = 0.45, size = 451, normalized size = 3.50

$$-\frac{1}{2}\,A\left(\frac{2\left(b^2c-\frac{\left(b^3-2\,bc^2\right)\sin(x)}{\cos(x)+1}-\frac{\left(b^2c-2\,c^3\right)\sin(x)^2}{(\cos(x)+1)^2}-\frac{\left(b^3+2\,bc^2\right)\sin(x)^3}{(\cos(x)+1)^3}\right)}{b^6+b^4c^2+\frac{4\left(b^5c+b^3c^3\right)\sin(x)}{\cos(x)+1}-\frac{2\left(b^6-b^4c^2-2\,b^2c^4\right)\sin(x)^2}{(\cos(x)+1)^2}-\frac{4\left(b^5c+b^3c^3\right)\sin(x)^3}{(\cos(x)+1)^3}+\frac{\left(b^6+b^4c^2\right)\sin(x)^4}{(\cos(x)+1)^4}}+\frac{\log\left(\frac{c-\frac{b\sin(x)}{\cos(x)+1}+\sqrt{b^2+}}{c-\frac{b\sin(x)}{\cos(x)+1}-\sqrt{b^2+}}\right.}{\left(b^2+c^2\right)^{\frac{3}{2}}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(x)+C*sin(x))/(b*cos(x)+c*sin(x))^3,x, algorithm="maxima"
)
```

```
[Out] -1/2*A*(2*(b^2*c - (b^3 - 2*b*c^2)*sin(x)/(cos(x) + 1) - (b^2*c - 2*c^3)*si
n(x)^2/(cos(x) + 1)^2 - (b^3 + 2*b*c^2)*sin(x)^3/(cos(x) + 1)^3)/(b^6 + b^4
*c^2 + 4*(b^5*c + b^3*c^3)*sin(x)/(cos(x) + 1) - 2*(b^6 - b^4*c^2 - 2*b^2*c
^4)*sin(x)^2/(cos(x) + 1)^2 - 4*(b^5*c + b^3*c^3)*sin(x)^3/(cos(x) + 1)^3 +
 (b^6 + b^4*c^2)*sin(x)^4/(cos(x) + 1)^4) + log((c - b*sin(x)/(cos(x) + 1)
+ sqrt(b^2 + c^2))/(c - b*sin(x)/(cos(x) + 1) - sqrt(b^2 + c^2)))/(b^2 + c^
2)^(3/2)) + 2*B*(b*sin(x)/(cos(x) + 1) + c*sin(x)^2/(cos(x) + 1)^2 - b*sin(
x)^3/(cos(x) + 1)^3)/(b^4 + 4*b^3*c*sin(x)/(cos(x) + 1) - 4*b^3*c*sin(x)^3/
(cos(x) + 1)^3 + b^4*sin(x)^4/(cos(x) + 1)^4 - 2*(b^4 - 2*b^2*c^2)*sin(x)^2
/(cos(x) + 1)^2) + 2*C*sin(x)^2/((b^3 + 4*b^2*c*sin(x)/(cos(x) + 1) - 4*b^2
*c*sin(x)^3/(cos(x) + 1)^3 + b^3*sin(x)^4/(cos(x) + 1)^4 - 2*(b^3 - 2*b*c^2
)*sin(x)^2/(cos(x) + 1)^2)*(cos(x) + 1)^2)
```

mupad [B] time = 3.44, size = 264, normalized size = 2.05

$$\frac{\frac{\tan\left(\frac{x}{2}\right)\left(A\,b^2-2\,A\,c^2+2\,B\,b^2+2\,B\,c^2\right)}{b\,\left(b^2+c^2\right)}-\frac{A\,c}{b^2+c^2}+\frac{\tan\left(\frac{x}{2}\right)^2\left(2\,B\,c^3-2\,A\,c^3+2\,C\,b^3+A\,b^2\,c+2\,B\,b^2\,c+2\,C\,b\,c^2\right)}{b^2\,\left(b^2+c^2\right)}+\frac{\tan\left(\frac{x}{2}\right)^3\left(A\,b^2+2\,A\,c^2-2\,B\,b^2-2\,B\,c^2\right)}{b\,\left(b^2+c^2\right)}}{b^2-\tan\left(\frac{x}{2}\right)^2\,\left(2\,b^2-4\,c^2\right)+b^2\,\tan\left(\frac{x}{2}\right)^4+4\,b\,c\,\tan\left(\frac{x}{2}\right)-4\,b\,c\,\tan\left(\frac{x}{2}\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cos(x) + C*sin(x))/(b*cos(x) + c*sin(x))^3,x)
```

```
[Out] ((tan(x/2)*(A*b^2 - 2*A*c^2 + 2*B*b^2 + 2*B*c^2))/(b*(b^2 + c^2)) - (A*c)/(
b^2 + c^2) + (tan(x/2)^2*(2*B*c^3 - 2*A*c^3 + 2*C*b^3 + A*b^2*c + 2*B*b^2*c
 + 2*C*b*c^2))/(b^2*(b^2 + c^2)) + (tan(x/2)^3*(A*b^2 + 2*A*c^2 - 2*B*b^2 -
 2*B*c^2))/(b*(b^2 + c^2)))/(b^2 - tan(x/2)^2*(2*b^2 - 4*c^2) + b^2*tan(x/2
)^4 + 4*b*c*tan(x/2) - 4*b*c*tan(x/2)^3) + (A*atan((b^2*c*1i + c^3*1i - b*t
an(x/2)*(b^2 + c^2)*1i)/(b^2 + c^2)^(3/2))*1i)/(b^2 + c^2)^(3/2)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(x)+C*sin(x))/(b*cos(x)+c*sin(x))**3,x)
```

```
[Out] Timed out
```

3.535 $\int \frac{A+B\cos(x)}{a+b\cos(x)+c\sin(x)}\,dx$

Optimal. Leaf size=115

$$-\frac{2\left(abB-A\left(b^2+c^2\right)\right)\tan^{-1}\left(\frac{(a-b)\tan\left(\frac{x}{2}\right)+c}{\sqrt{a^2-b^2-c^2}}\right)}{\left(b^2+c^2\right)\sqrt{a^2-b^2-c^2}}+\frac{Bc\log(a+b\cos(x)+c\sin(x))}{b^2+c^2}+\frac{bBx}{b^2+c^2}$$

```
[Out] b*B*x/(b^2+c^2)+B*c*ln(a+b*cos(x)+c*sin(x))/(b^2+c^2)-2*(a*b*B-A*(b^2+c^2))
*arctan((c+(a-b)*tan(1/2*x))/(a^2-b^2-c^2)^(1/2))/(b^2+c^2)/(a^2-b^2-c^2)^(
1/2)
```

Rubi [A] time = 0.13, antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.210, Rules used = {3138, 3124, 618, 204}

$$-\frac{2\left(abB-A\left(b^2+c^2\right)\right)\tan^{-1}\left(\frac{(a-b)\tan\left(\frac{x}{2}\right)+c}{\sqrt{a^2-b^2-c^2}}\right)}{\left(b^2+c^2\right)\sqrt{a^2-b^2-c^2}}+\frac{Bc\log(a+b\cos(x)+c\sin(x))}{b^2+c^2}+\frac{bBx}{b^2+c^2}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cos[x])/(a + b*Cos[x] + c*Sin[x]),x]
```

```
[Out] (b*B*x)/(b^2 + c^2) - (2*(a*b*B - A*(b^2 + c^2))*ArcTan[(c + (a - b)*Tan[x/
2])/Sqrt[a^2 - b^2 - c^2]])/(Sqrt[a^2 - b^2 - c^2]*(b^2 + c^2)) + (B*c*Log[
a + b*Cos[x] + c*Sin[x]])/(b^2 + c^2)
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 3124

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(-1), x_Symbol] :> Module[{f = FreeFactors[Tan[(d + e*x)/2], x]}, Dist[(2*f
```

```
)/e, Subst[Int[1/(a + b + 2*c*f*x + (a - b)*f^2*x^2), x], x, Tan[(d + e*x)/
2]/f], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0]
```

Rule 3138

```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.))/((a_.) + cos[(d_.) + (e_.)*(x_)
]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)]), x_Symbol] :> Simp[(b*B*(d + e*x))
/(e*(b^2 + c^2)), x] + (Dist[(A*(b^2 + c^2) - a*b*B)/(b^2 + c^2), Int[1/(a
+ b*Cos[d + e*x] + c*Sin[d + e*x]), x], x] + Simp[(c*B*Log[a + b*Cos[d + e*
x] + c*Sin[d + e*x]])/(e*(b^2 + c^2)), x]) /; FreeQ[{a, b, c, d, e, A, B},
x] && NeQ[b^2 + c^2, 0] && NeQ[A*(b^2 + c^2) - a*b*B, 0]
```

Rubi steps

$$\int \frac{A+B\cos(x)}{a+b\cos(x)+c\sin(x)}\,dx = \frac{bBx}{b^2+c^2}+\frac{Bc\log(a+b\cos(x)+c\sin(x))}{b^2+c^2}+\left(A-\frac{abB}{b^2+c^2}\right)\int \frac{1}{a+b\cos(x)+c\sin(x)}$$

$$= \frac{bBx}{b^2+c^2}+\frac{Bc\log(a+b\cos(x)+c\sin(x))}{b^2+c^2}+\left(2\left(A-\frac{abB}{b^2+c^2}\right)\right)\text{Subst}\left(\int \frac{}{a+b+}\right.$$

$$= \frac{bBx}{b^2+c^2}+\frac{Bc\log(a+b\cos(x)+c\sin(x))}{b^2+c^2}-\left(4\left(A-\frac{abB}{b^2+c^2}\right)\right)\text{Subst}\left(\int \frac{}{-4\left(a^2\right.}\right.$$

$$= \frac{bBx}{b^2+c^2}+\frac{2\left(A-\frac{abB}{b^2+c^2}\right)\tan^{-1}\left(\frac{c+(a-b)\tan\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2-c^2}}\right)}{\sqrt{a^2-b^2-c^2}}+\frac{Bc\log(a+b\cos(x)+c\sin(x))}{b^2+c^2}$$

Mathematica [A] time = 0.29, size = 95, normalized size = 0.83

$$\frac{B(c\log(a+b\cos(x)+c\sin(x))+bx)-\frac{2\left(A\left(b^2+c^2\right)-abB\right)\tanh^{-1}\left(\frac{(a-b)\tan\left(\frac{x}{2}\right)+c}{\sqrt{-a^2+b^2+c^2}}\right)}{\sqrt{-a^2+b^2+c^2}}}{b^2+c^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Cos[x])/(a + b*Cos[x] + c*Sin[x]),x]
```

```
[Out] ((-2*(-(a*b*B) + A*(b^2 + c^2))*ArcTanh[(c + (a - b)*Tan[x/2])/Sqrt[-a^2 +
b^2 + c^2]])/Sqrt[-a^2 + b^2 + c^2] + B*(b*x + c*Log[a + b*Cos[x] + c*Sin[x
]]))/(b^2 + c^2)
```

fricas [B] time = 1.78, size = 625, normalized size = 5.43

$$\left[-\frac{\left(Bab - Ab^2 - Ac^2\right)\sqrt{-a^2 + b^2 + c^2}\,\log\left(\frac{a^2b^2 - 2\,b^4 - c^4 - \left(a^2 + 3\,b^2\right)c^2 - \left(2\,a^2b^2 - b^4 - 2\,a^2c^2 + c^4\right)\cos(x)^2 - 2\,\left(ab^3 + abc^2\right)\cos(x) - 2\,\left(ab^2c + ac^3\right.}{2\,ab\cos(x) + \left(b^2\right.}\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(x))/(a+b*cos(x)+c*sin(x)),x, algorithm="fricas")
```

```
[Out] [-1/2*((B*a*b - A*b^2 - A*c^2)*sqrt(-a^2 + b^2 + c^2)*log((a^2*b^2 - 2*b^4
- c^4 - (a^2 + 3*b^2)*c^2 - (2*a^2*b^2 - b^4 - 2*a^2*c^2 + c^4)*cos(x)^2 -
2*(a*b^3 + a*b*c^2)*cos(x) - 2*(a*b^2*c + a*c^3 - (b*c^3 - (2*a^2*b - b^3)*
c)*cos(x))*sin(x) - 2*(2*a*b*c*cos(x)^2 - a*b*c + (b^2*c + c^3)*cos(x) - (b
^3 + b*c^2 + (a*b^2 - a*c^2)*cos(x))*sin(x))*sqrt(-a^2 + b^2 + c^2))/(2*a*b
*cos(x) + (b^2 - c^2)*cos(x)^2 + a^2 + c^2 + 2*(b*c*cos(x) + a*c)*sin(x)))
- 2*(B*a^2*b - B*b^3 - B*b*c^2)*x + (B*c^3 - (B*a^2 - B*b^2)*c)*log(2*a*b*c
os(x) + (b^2 - c^2)*cos(x)^2 + a^2 + c^2 + 2*(b*c*cos(x) + a*c)*sin(x)))/(a
^2*b^2 - b^4 - c^4 + (a^2 - 2*b^2)*c^2), -1/2*(2*(B*a*b - A*b^2 - A*c^2)*sq
rt(a^2 - b^2 - c^2)*arctan(-(a*b*cos(x) + a*c*sin(x) + b^2 + c^2)*sqrt(a^2
- b^2 - c^2)/((c^3 - (a^2 - b^2)*c)*cos(x) + (a^2*b - b^3 - b*c^2)*sin(x)))
 - 2*(B*a^2*b - B*b^3 - B*b*c^2)*x + (B*c^3 - (B*a^2 - B*b^2)*c)*log(2*a*b*
cos(x) + (b^2 - c^2)*cos(x)^2 + a^2 + c^2 + 2*(b*c*cos(x) + a*c)*sin(x)))/(
a^2*b^2 - b^4 - c^4 + (a^2 - 2*b^2)*c^2)]
```

giac [A] time = 0.18, size = 178, normalized size = 1.55

$$\frac{Bbx}{b^2 + c^2} + \frac{Bc\log\left(-a\tan\left(\frac{1}{2}\,x\right)^2 + b\tan\left(\frac{1}{2}\,x\right)^2 - 2\,c\tan\left(\frac{1}{2}\,x\right) - a - b\right)}{b^2 + c^2} - \frac{Bc\log\left(\tan\left(\frac{1}{2}\,x\right)^2 + 1\right)}{b^2 + c^2} + \frac{2\left(Bab - Ab^2 - A\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(x))/(a+b*cos(x)+c*sin(x)),x, algorithm="giac")
```

```
[Out] B*b*x/(b^2 + c^2) + B*c*log(-a*tan(1/2*x)^2 + b*tan(1/2*x)^2 - 2*c*tan(1/2*
x) - a - b)/(b^2 + c^2) - B*c*log(tan(1/2*x)^2 + 1)/(b^2 + c^2) + 2*(B*a*b
- A*b^2 - A*c^2)*(pi*floor(1/2*x/pi + 1/2)*sgn(-2*a + 2*b) + arctan(-(a*tan
(1/2*x) - b*tan(1/2*x) + c)/sqrt(a^2 - b^2 - c^2)))/(sqrt(a^2 - b^2 - c^2)*
(b^2 + c^2))
```

maple [B] time = 0.12, size = 544, normalized size = 4.73

$$\frac{\ln\left(a\left(\tan^2\left(\frac{x}{2}\right)\right)-b\left(\tan^2\left(\frac{x}{2}\right)\right)+2c\tan\left(\frac{x}{2}\right)+a+b\right)aBc}{\left(b^2+c^2\right)(a-b)}-\frac{\ln\left(a\left(\tan^2\left(\frac{x}{2}\right)\right)-b\left(\tan^2\left(\frac{x}{2}\right)\right)+2c\tan\left(\frac{x}{2}\right)+a+b\right)b}{\left(b^2+c^2\right)(a-b)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(x))/(a+b*cos(x)+c*sin(x)),x)
```

```
[Out] 1/(b^2+c^2)/(a-b)*ln(a*tan(1/2*x)^2-b*tan(1/2*x)^2+2*c*tan(1/2*x)+a+b)*a*B*
c-1/(b^2+c^2)/(a-b)*ln(a*tan(1/2*x)^2-b*tan(1/2*x)^2+2*c*tan(1/2*x)+a+b)*b*
B*c+2/(b^2+c^2)/(a^2-b^2-c^2)^(1/2)*arctan(1/2*(2*(a-b)*tan(1/2*x)+2*c)/(a^
2-b^2-c^2)^(1/2))*A*b^2+2/(b^2+c^2)/(a^2-b^2-c^2)^(1/2)*arctan(1/2*(2*(a-b)
*tan(1/2*x)+2*c)/(a^2-b^2-c^2)^(1/2))*A*c^2-2/(b^2+c^2)/(a^2-b^2-c^2)^(1/2)
*arctan(1/2*(2*(a-b)*tan(1/2*x)+2*c)/(a^2-b^2-c^2)^(1/2))*a*b*B+2/(b^2+c^2)
/(a^2-b^2-c^2)^(1/2)*arctan(1/2*(2*(a-b)*tan(1/2*x)+2*c)/(a^2-b^2-c^2)^(1/2
))*B*c^2-2/(b^2+c^2)/(a^2-b^2-c^2)^(1/2)*arctan(1/2*(2*(a-b)*tan(1/2*x)+2*c
)/(a^2-b^2-c^2)^(1/2))*c^2/(a-b)*a*B+2/(b^2+c^2)/(a^2-b^2-c^2)^(1/2)*arctan
(1/2*(2*(a-b)*tan(1/2*x)+2*c)/(a^2-b^2-c^2)^(1/2))*c^2/(a-b)*b*B-B/(b^2+c^2
)*c*ln(1+tan(1/2*x)^2)+2*B/(b^2+c^2)*b*arctan(tan(1/2*x))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(x))/(a+b*cos(x)+c*sin(x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(c^2+b^2-a^2>0)', see `assume?` for
more details)Is c^2+b^2-a^2 positive or negative?
```

mupad [B] time = 25.56, size = 1709, normalized size = 14.86

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cos(x))/(a + b*cos(x) + c*sin(x)),x)
```

```
[Out] (log(32*B^3*a^2 - 32*A*B^2*a^2 + 32*A*B^2*b^2 - 32*A^2*B*b^2 - 32*B^3*a*b +
 32*A^2*B*a*b - ((B*c^3 + A*b^2*(b^2 - a^2 + c^2)^(1/2) - B*a^2*c + A*c^2*(
b^2 - a^2 + c^2)^(1/2) + B*b^2*c - B*a*b*(b^2 - a^2 + c^2)^(1/2))*(64*A^2*b
```

```
^2*c - 32*B^2*a^2*c + 32*B^2*b^2*c + 32*tan(x/2)*(a - b)*(A^2*b^2 + 2*B^2*a
^2 - A^2*c^2 + B^2*b^2 - 3*B^2*c^2 + 4*A*B*c^2 - 2*B^2*a*b - 2*A*B*a*b) + 6
4*A*B*a^2*c - 64*A*B*b^2*c - 64*A^2*a*b*c + ((B*c^3 + A*b^2*(b^2 - a^2 + c^
2)^(1/2) - B*a^2*c + A*c^2*(b^2 - a^2 + c^2)^(1/2) + B*b^2*c - B*a*b*(b^2 -
 a^2 + c^2)^(1/2))*(32*A*a^2*c^2 - 32*B*b^4 - 32*A*a^2*b^2 - 32*A*b^4 - 32*
B*a^2*b^2 - 32*A*b^2*c^2 + 32*B*a^2*c^2 + 64*B*b^2*c^2 + 64*A*a*b^3 + 64*B*
a*b^3 - 96*B*a*b*c^2 + 32*c*tan(x/2)*(a - b)*(2*A*b^2 + 2*A*c^2 + 4*B*b^2 +
 B*c^2 - 2*A*a*b - 4*B*a*b) + (32*(a - b)*(B*c^3 + A*b^2*(b^2 - a^2 + c^2)^
(1/2) - B*a^2*c + A*c^2*(b^2 - a^2 + c^2)^(1/2) + B*b^2*c - B*a*b*(b^2 - a^
2 + c^2)^(1/2))*(3*c^4*tan(x/2) + a*c^3 + 3*b*c^3 + 3*b^3*c + 2*a^2*b^2*tan
(x/2) - 2*a^2*c^2*tan(x/2) + 3*b^2*c^2*tan(x/2) - 2*a*b^3*tan(x/2) + a*b^2*
c - 4*a^2*b*c - 2*a*b*c^2*tan(x/2)))/((b^2 + c^2)*(b^2 - a^2 + c^2))))/((b^
2 + c^2)*(b^2 - a^2 + c^2))))/((b^2 + c^2)*(b^2 - a^2 + c^2)) + 32*B*c*tan(
x/2)*(A - B)^2*(a - b))*(B*c^3 + b^2*(A*(b^2 - a^2 + c^2)^(1/2) + B*c) - B*
a^2*c + A*c^2*(b^2 - a^2 + c^2)^(1/2) - B*a*b*(b^2 - a^2 + c^2)^(1/2)))/((b
^2 + c^2)*(b^2 - a^2 + c^2)) - (B*log(tan(x/2) + 1i))/(b*1i + c) - (B*log(t
an(x/2) - 1i)*1i)/(b + c*1i) - (log(32*B^3*a^2 - 32*A*B^2*a^2 + 32*A*B^2*b^
2 - 32*A^2*B*b^2 - 32*B^3*a*b + 32*A^2*B*a*b - ((B*c^3 - A*b^2*(b^2 - a^2 +
 c^2)^(1/2) - B*a^2*c - A*c^2*(b^2 - a^2 + c^2)^(1/2) + B*b^2*c + B*a*b*(b^
2 - a^2 + c^2)^(1/2))*(64*A^2*b^2*c - 32*B^2*a^2*c + 32*B^2*b^2*c + 32*tan(
x/2)*(a - b)*(A^2*b^2 + 2*B^2*a^2 - A^2*c^2 + B^2*b^2 - 3*B^2*c^2 + 4*A*B*c
^2 - 2*B^2*a*b - 2*A*B*a*b) + 64*A*B*a^2*c - 64*A*B*b^2*c - 64*A^2*a*b*c +
((B*c^3 - A*b^2*(b^2 - a^2 + c^2)^(1/2) - B*a^2*c - A*c^2*(b^2 - a^2 + c^2)
^(1/2) + B*b^2*c + B*a*b*(b^2 - a^2 + c^2)^(1/2))*(32*A*a^2*c^2 - 32*B*b^4
- 32*A*a^2*b^2 - 32*A*b^4 - 32*B*a^2*b^2 - 32*A*b^2*c^2 + 32*B*a^2*c^2 + 64
*B*b^2*c^2 + 64*A*a*b^3 + 64*B*a*b^3 - 96*B*a*b*c^2 + 32*c*tan(x/2)*(a - b)
*(2*A*b^2 + 2*A*c^2 + 4*B*b^2 + B*c^2 - 2*A*a*b - 4*B*a*b) + (32*(a - b)*(B
*c^3 - A*b^2*(b^2 - a^2 + c^2)^(1/2) - B*a^2*c - A*c^2*(b^2 - a^2 + c^2)^(1
/2) + B*b^2*c + B*a*b*(b^2 - a^2 + c^2)^(1/2))*(3*c^4*tan(x/2) + a*c^3 + 3*
b*c^3 + 3*b^3*c + 2*a^2*b^2*tan(x/2) - 2*a^2*c^2*tan(x/2) + 3*b^2*c^2*tan(x
/2) - 2*a*b^3*tan(x/2) + a*b^2*c - 4*a^2*b*c - 2*a*b*c^2*tan(x/2)))/((b^2 +
 c^2)*(b^2 - a^2 + c^2))))/((b^2 + c^2)*(b^2 - a^2 + c^2))))/((b^2 + c^2)*(
b^2 - a^2 + c^2)) + 32*B*c*tan(x/2)*(A - B)^2*(a - b))*(b^2*(A*(b^2 - a^2 +
 c^2)^(1/2) - B*c) - B*c^3 + B*a^2*c + A*c^2*(b^2 - a^2 + c^2)^(1/2) - B*a*
b*(b^2 - a^2 + c^2)^(1/2)))/((b^2 + c^2)*(b^2 - a^2 + c^2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(x))/(a+b*cos(x)+c*sin(x)),x)
```

```
[Out] Timed out
```

3.536 $\int \frac{A+B\cos(x)}{(a+b\cos(x)+c\sin(x))^2}\,dx$

Optimal. Leaf size=113

$$\frac{2(aA-bB)\tan^{-1}\left(\frac{(a-b)\tan\left(\frac{x}{2}\right)+c}{\sqrt{a^2-b^2-c^2}}\right)}{\left(a^2-b^2-c^2\right)^{3/2}}+\frac{-\sin(x)(Ab-aB)+Ac\cos(x)+Bc}{\left(a^2-b^2-c^2\right)(a+b\cos(x)+c\sin(x))}$$

```
[Out] 2*(A*a-B*b)*arctan((c+(a-b)*tan(1/2*x))/(a^2-b^2-c^2)^(1/2))/(a^2-b^2-c^2)^
(3/2)+(B*c+A*c*cos(x)-(A*b-B*a)*sin(x))/(a^2-b^2-c^2)/(a+b*cos(x)+c*sin(x))
```

Rubi [A] time = 0.11, antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.210, Rules used = {3155, 3124, 618, 204}

$$\frac{2(aA-bB)\tan^{-1}\left(\frac{(a-b)\tan\left(\frac{x}{2}\right)+c}{\sqrt{a^2-b^2-c^2}}\right)}{\left(a^2-b^2-c^2\right)^{3/2}}+\frac{-\sin(x)(Ab-aB)+Ac\cos(x)+Bc}{\left(a^2-b^2-c^2\right)(a+b\cos(x)+c\sin(x))}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cos[x])/(a + b*Cos[x] + c*Sin[x])^2,x]
```

```
[Out] (2*(a*A - b*B)*ArcTan[(c + (a - b)*Tan[x/2])/Sqrt[a^2 - b^2 - c^2]])/(a^2 -
 b^2 - c^2)^(3/2) + (B*c + A*c*Cos[x] - (A*b - a*B)*Sin[x])/((a^2 - b^2 - c
^2)*(a + b*Cos[x] + c*Sin[x]))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 3124

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(-1), x_Symbol] :> Module[{f = FreeFactors[Tan[(d + e*x)/2], x]}, Dist[(2*f
)/e, Subst[Int[1/(a + b + 2*c*f*x + (a - b)*f^2*x^2), x], x, Tan[(d + e*x)/
```

```
2]/f], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0]
```

Rule 3155

```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.))/((a_.) + cos[(d_.) + (e_.)*(x_)
]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)])^2, x_Symbol] :> Simp[(c*B + c*A*Co
s[d + e*x] + (a*B - b*A)*Sin[d + e*x])/(e*(a^2 - b^2 - c^2)*(a + b*Cos[d +
e*x] + c*Sin[d + e*x])), x] + Dist[(a*A - b*B)/(a^2 - b^2 - c^2), Int[1/(a
+ b*Cos[d + e*x] + c*Sin[d + e*x]), x], x] /; FreeQ[{a, b, c, d, e, A, B},
x] && NeQ[a^2 - b^2 - c^2, 0] && NeQ[a*A - b*B, 0]
```

Rubi steps

$$\int \frac{A+B\cos(x)}{(a+b\cos(x)+c\sin(x))^2}\,dx = \frac{Bc+Ac\cos(x)-(Ab-aB)\sin(x)}{\left(a^2-b^2-c^2\right)(a+b\cos(x)+c\sin(x))}+\frac{(aA-bB)\int \frac{1}{a+b\cos(x)+c\sin(x)}\,dx}{a^2-b^2-c^2}$$

$$= \frac{Bc+Ac\cos(x)-(Ab-aB)\sin(x)}{\left(a^2-b^2-c^2\right)(a+b\cos(x)+c\sin(x))}+\frac{(2(aA-bB))\,\mathrm{Subst}\left(\int \frac{1}{a+b+2cx+(a-b)x^2}\right.}{a^2-b^2-c^2}$$

$$= \frac{Bc+Ac\cos(x)-(Ab-aB)\sin(x)}{\left(a^2-b^2-c^2\right)(a+b\cos(x)+c\sin(x))}-\frac{(4(aA-bB))\,\mathrm{Subst}\left(\int \frac{1}{-4\left(a^2-b^2-c^2\right)-x^2}\right.}{a^2-b^2-c}$$

$$= \frac{2(aA-bB)\tan^{-1}\left(\frac{c+(a-b)\tan\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2-c^2}}\right)}{\left(a^2-b^2-c^2\right)^{3/2}}+\frac{Bc+Ac\cos(x)-(Ab-aB)\sin(x)}{\left(a^2-b^2-c^2\right)(a+b\cos(x)+c\sin(x))}$$

Mathematica [A] time = 0.33, size = 118, normalized size = 1.04

$$\frac{\sin(x)\left(A\left(b^2+c^2\right)-abB\right)+c(aA-bB)}{b\left(-a^2+b^2+c^2\right)(a+b\cos(x)+c\sin(x))}+\frac{2(aA-bB)\tanh^{-1}\left(\frac{(a-b)\tan\left(\frac{x}{2}\right)+c}{\sqrt{-a^2+b^2+c^2}}\right)}{\left(-a^2+b^2+c^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Cos[x])/(a + b*Cos[x] + c*Sin[x])^2,x]
```

```
[Out] (2*(a*A - b*B)*ArcTanh[(c + (a - b)*Tan[x/2])/Sqrt[-a^2 + b^2 + c^2]])/(-a^
2 + b^2 + c^2)^(3/2) + ((a*A - b*B)*c + (-(a*b*B) + A*(b^2 + c^2))*Sin[x])/
(b*(-a^2 + b^2 + c^2)*(a + b*Cos[x] + c*Sin[x]))
```

fricas [B] time = 1.69, size = 1277, normalized size = 11.30

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(x))/(a+b*cos(x)+c*sin(x))^2,x, algorithm="fricas")
```

```
[Out] [-1/2*(2*B*c^5 - 4*(B*a^2 - B*b^2)*c^3 + (A*a^2*b^2 - B*a*b^3 + (A*a^2 - B*
a*b)*c^2 + (A*a*b^3 - B*b^4 + (A*a*b - B*b^2)*c^2)*cos(x) + ((A*a - B*b)*c^
3 + (A*a*b^2 - B*b^3)*c)*sin(x))*sqrt(-a^2 + b^2 + c^2)*log(-(a^2*b^2 - 2*b
^4 - c^4 - (a^2 + 3*b^2)*c^2 - (2*a^2*b^2 - b^4 - 2*a^2*c^2 + c^4)*cos(x)^2
 - 2*(a*b^3 + a*b*c^2)*cos(x) - 2*(a*b^2*c + a*c^3 - (b*c^3 - (2*a^2*b - b^
3)*c)*cos(x))*sin(x) + 2*(2*a*b*c*cos(x)^2 - a*b*c + (b^2*c + c^3)*cos(x) -
 (b^3 + b*c^2 + (a*b^2 - a*c^2)*cos(x))*sin(x))*sqrt(-a^2 + b^2 + c^2))/(2*
a*b*cos(x) + (b^2 - c^2)*cos(x)^2 + a^2 + c^2 + 2*(b*c*cos(x) + a*c)*sin(x)
)) + 2*(B*a^4 - 2*B*a^2*b^2 + B*b^4)*c + 2*(A*c^5 - (A*a^2 + B*a*b - 2*A*b^
2)*c^3 + (B*a^3*b - A*a^2*b^2 - B*a*b^3 + A*b^4)*c)*cos(x) - 2*(B*a^3*b^2 -
 A*a^2*b^3 - B*a*b^4 + A*b^5 + A*b*c^4 - (A*a^2*b + B*a*b^2 - 2*A*b^3)*c^2)
*sin(x))/(a^5*b^2 - 2*a^3*b^4 + a*b^6 + a*c^6 - (2*a^3 - 3*a*b^2)*c^4 + (a^
5 - 4*a^3*b^2 + 3*a*b^4)*c^2 + (a^4*b^3 - 2*a^2*b^5 + b^7 + b*c^6 - (2*a^2*
b - 3*b^3)*c^4 + (a^4*b - 4*a^2*b^3 + 3*b^5)*c^2)*cos(x) + (c^7 - (2*a^2 -
3*b^2)*c^5 + (a^4 - 4*a^2*b^2 + 3*b^4)*c^3 + (a^4*b^2 - 2*a^2*b^4 + b^6)*c)
*sin(x)), -(B*c^5 - 2*(B*a^2 - B*b^2)*c^3 - (A*a^2*b^2 - B*a*b^3 + (A*a^2 -
 B*a*b)*c^2 + (A*a*b^3 - B*b^4 + (A*a*b - B*b^2)*c^2)*cos(x) + ((A*a - B*b)
*c^3 + (A*a*b^2 - B*b^3)*c)*sin(x))*sqrt(a^2 - b^2 - c^2)*arctan(-(a*b*cos(
x) + a*c*sin(x) + b^2 + c^2)*sqrt(a^2 - b^2 - c^2)/((c^3 - (a^2 - b^2)*c)*c
os(x) + (a^2*b - b^3 - b*c^2)*sin(x))) + (B*a^4 - 2*B*a^2*b^2 + B*b^4)*c +
(A*c^5 - (A*a^2 + B*a*b - 2*A*b^2)*c^3 + (B*a^3*b - A*a^2*b^2 - B*a*b^3 + A
*b^4)*c)*cos(x) - (B*a^3*b^2 - A*a^2*b^3 - B*a*b^4 + A*b^5 + A*b*c^4 - (A*a
^2*b + B*a*b^2 - 2*A*b^3)*c^2)*sin(x))/(a^5*b^2 - 2*a^3*b^4 + a*b^6 + a*c^6
 - (2*a^3 - 3*a*b^2)*c^4 + (a^5 - 4*a^3*b^2 + 3*a*b^4)*c^2 + (a^4*b^3 - 2*a
^2*b^5 + b^7 + b*c^6 - (2*a^2*b - 3*b^3)*c^4 + (a^4*b - 4*a^2*b^3 + 3*b^5)*
c^2)*cos(x) + (c^7 - (2*a^2 - 3*b^2)*c^5 + (a^4 - 4*a^2*b^2 + 3*b^4)*c^3 +
(a^4*b^2 - 2*a^2*b^4 + b^6)*c)*sin(x))]
```

giac [A] time = 0.19, size = 209, normalized size = 1.85

$$-\frac{2\left(\pi\left\lfloor\frac{x}{2\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}\left(-2a+2b\right)+\arctan\left(-\frac{a\tan\left(\frac{1}{2}x\right)-b\tan\left(\frac{1}{2}x\right)+c}{\sqrt{a^2-b^2-c^2}}\right)\right)(Aa-Bb)}{\left(a^2-b^2-c^2\right)^{\frac{3}{2}}}+\frac{2\left(Ba^2\tan\left(\frac{1}{2}x\right)-Aab\tan\left(\frac{1}{2}x\right)\right.}{\left(a^3-a^2b-ab^2+b^3\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(x))/(a+b*cos(x)+c*sin(x))^2,x, algorithm="giac")
```

```
[Out] -2*(pi*floor(1/2*x/pi + 1/2)*sgn(-2*a + 2*b) + arctan(-(a*tan(1/2*x) - b*ta
n(1/2*x) + c)/sqrt(a^2 - b^2 - c^2)))*(A*a - B*b)/(a^2 - b^2 - c^2)^(3/2) +
 2*(B*a^2*tan(1/2*x) - A*a*b*tan(1/2*x) - B*a*b*tan(1/2*x) + A*b^2*tan(1/2*
x) + A*c^2*tan(1/2*x) - B*c^2*tan(1/2*x) + A*a*c - B*b*c)/((a^3 - a^2*b - a
*b^2 + b^3 - a*c^2 + b*c^2)*(a*tan(1/2*x)^2 - b*tan(1/2*x)^2 + 2*c*tan(1/2*
x) + a + b))
```

maple [B] time = 0.18, size = 254, normalized size = 2.25

$$\frac{-\frac{2\left(aAb-A\,b^{2}-A\,c^{2}-a^{2}B+abB+B\,c^{2}\right)\tan\left(\frac{x}{2}\right)}{a^{3}-a^{2}b-a\,b^{2}-a\,c^{2}+b^{3}+c^{2}b}+\frac{2(aA-bB)c}{a^{3}-a^{2}b-a\,b^{2}-a\,c^{2}+b^{3}+c^{2}b}}{a\left(\tan^{2}\left(\frac{x}{2}\right)\right)-b\left(\tan^{2}\left(\frac{x}{2}\right)\right)+2c\tan\left(\frac{x}{2}\right)+a+b}+\frac{2\arctan\left(\frac{2(a-b)\tan\left(\frac{x}{2}\right)+2c}{2\sqrt{a^{2}-b^{2}-c^{2}}}\right)aA}{\left(a^{2}-b^{2}-c^{2}\right)^{\frac{3}{2}}}-\frac{2\arctan\left(\frac{2(a-b)\tan\left(\frac{x}{2}\right)+2c}{2\sqrt{a^{2}-b^{2}-c^{2}}}\right)}{\left(a^{2}-b^{2}-c^{2}\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(x))/(a+b*cos(x)+c*sin(x))^2,x)
```

```
[Out] 2*(-(A*a*b-A*b^2-A*c^2-B*a^2+B*a*b+B*c^2)/(a^3-a^2*b-a*b^2-a*c^2+b^3+b*c^2)
*tan(1/2*x)+(A*a-B*b)*c/(a^3-a^2*b-a*b^2-a*c^2+b^3+b*c^2))/(a*tan(1/2*x)^2-
b*tan(1/2*x)^2+2*c*tan(1/2*x)+a+b)+2/(a^2-b^2-c^2)^(3/2)*arctan(1/2*(2*(a-b
)*tan(1/2*x)+2*c)/(a^2-b^2-c^2)^(1/2))*a*A-2/(a^2-b^2-c^2)^(3/2)*arctan(1/2
*(2*(a-b)*tan(1/2*x)+2*c)/(a^2-b^2-c^2)^(1/2))*b*B
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(x))/(a+b*cos(x)+c*sin(x))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(c^2+b^2-a^2>0)', see `assume?` for
more details)Is c^2+b^2-a^2 positive or negative?
```

mupad [B] time = 3.25, size = 205, normalized size = 1.81

$$\frac{2\,\mathrm{atanh}\left(\frac{\tan\left(\frac{x}{2}\right)(2\,a-2\,b)+\frac{2\left(-a^{2}\,c+b^{2}\,c+c^{3}\right)}{-a^{2}+b^{2}+c^{2}}}{2\,\sqrt{-a^{2}+b^{2}+c^{2}}}\right)(A\,a-B\,b)}{\left(-a^{2}+b^{2}+c^{2}\right)^{3/2}}-\frac{\frac{2\,(A\,a\,c-B\,b\,c)}{(a-b)\left(-a^{2}+b^{2}+c^{2}\right)}+\frac{2\tan\left(\frac{x}{2}\right)\left(A\,b^{2}+B\,a^{2}+A\,c^{2}-B\,c^{2}-A\,a\,b-B\,a\,b\right)}{(a-b)\left(-a^{2}+b^{2}+c^{2}\right)}}{(a-b)\,\tan\left(\frac{x}{2}\right)^{2}+2\,c\tan\left(\frac{x}{2}\right)+a+b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cos(x))/(a + b*cos(x) + c*sin(x))^2,x)
```

```
[Out] (2*atanh((tan(x/2)*(2*a - 2*b) + (2*(b^2*c - a^2*c + c^3))/(b^2 - a^2 + c^2
))/(2*(b^2 - a^2 + c^2)^(1/2)))*(A*a - B*b))/(b^2 - a^2 + c^2)^(3/2) - ((2*
(A*a*c - B*b*c))/((a - b)*(b^2 - a^2 + c^2)) + (2*tan(x/2)*(A*b^2 + B*a^2 +
 A*c^2 - B*c^2 - A*a*b - B*a*b))/((a - b)*(b^2 - a^2 + c^2)))/(a + b + 2*c*
tan(x/2) + tan(x/2)^2*(a - b))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(x))/(a+b*cos(x)+c*sin(x))**2,x)
```

```
[Out] Timed out
```

3.537 $\int \frac{A+B\cos(x)}{(a+b\cos(x)+c\sin(x))^3}\,dx$

Optimal. Leaf size=200

$$\frac{\left(2a^2A-3abB+A\left(b^2+c^2\right)\right)\tan^{-1}\left(\frac{(a-b)\tan\left(\frac{x}{2}\right)+c}{\sqrt{a^2-b^2-c^2}}\right)}{\left(a^2-b^2-c^2\right)^{5/2}}+\frac{-\sin(x)\left(a^2(-B)+3aAb-2b^2B\right)+c\cos(x)(3aA-2bB)+a}{2\left(a^2-b^2-c^2\right)^2(a+b\cos(x)+c\sin(x))}$$

```
[Out] (2*a^2*A-3*a*b*B+A*(b^2+c^2))*arctan((c+(a-b)*tan(1/2*x))/(a^2-b^2-c^2)^(1/
2))/(a^2-b^2-c^2)^(5/2)+1/2*(B*c+A*c*cos(x)-(A*b-B*a)*sin(x))/(a^2-b^2-c^2)
/(a+b*cos(x)+c*sin(x))^2+1/2*(a*B*c+(3*A*a-2*B*b)*c*cos(x)-(3*A*a*b-B*a^2-2
*B*b^2)*sin(x))/(a^2-b^2-c^2)^2/(a+b*cos(x)+c*sin(x))
```

Rubi [A] time = 0.25, antiderivative size = 200, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {3158, 3153, 3124, 618, 204}

$$\frac{\left(2a^2A-3abB+A\left(b^2+c^2\right)\right)\tan^{-1}\left(\frac{(a-b)\tan\left(\frac{x}{2}\right)+c}{\sqrt{a^2-b^2-c^2}}\right)}{\left(a^2-b^2-c^2\right)^{5/2}}+\frac{-\sin(x)\left(a^2(-B)+3aAb-2b^2B\right)+c\cos(x)(3aA-2bB)+a}{2\left(a^2-b^2-c^2\right)^2(a+b\cos(x)+c\sin(x))}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cos[x])/(a + b*Cos[x] + c*Sin[x])^3,x]
```

```
[Out] ((2*a^2*A - 3*a*b*B + A*(b^2 + c^2))*ArcTan[(c + (a - b)*Tan[x/2])/Sqrt[a^2
 - b^2 - c^2]])/(a^2 - b^2 - c^2)^(5/2) + (B*c + A*c*Cos[x] - (A*b - a*B)*S
in[x])/(2*(a^2 - b^2 - c^2)*(a + b*Cos[x] + c*Sin[x])^2) + (a*B*c + (3*a*A
- 2*b*B)*c*Cos[x] - (3*a*A*b - a^2*B - 2*b^2*B)*Sin[x])/(2*(a^2 - b^2 - c^2
)^2*(a + b*Cos[x] + c*Sin[x]))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 3124

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(-1), x_Symbol] :> Module[{f = FreeFactors[Tan[(d + e*x)/2], x]}, Dist[(2*f
)/e, Subst[Int[1/(a + b + 2*c*f*x + (a - b)*f^2*x^2), x], x, Tan[(d + e*x)/
2]/f], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0]
```

Rule 3153

```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])
/((a_.) + cos[(d_.) + (e_.)*(x_)]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)])^2,
 x_Symbol] :> Simp[(c*B - b*C - (a*C - c*A)*Cos[d + e*x] + (a*B - b*A)*Sin[
d + e*x])/(e*(a^2 - b^2 - c^2)*(a + b*Cos[d + e*x] + c*Sin[d + e*x])), x] +
 Dist[(a*A - b*B - c*C)/(a^2 - b^2 - c^2), Int[1/(a + b*Cos[d + e*x] + c*Si
n[d + e*x]), x], x] /; FreeQ[{a, b, c, d, e, A, B, C}, x] && NeQ[a^2 - b^2
- c^2, 0] && NeQ[a*A - b*B - c*C, 0]
```

Rule 3158

```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.))*((a_.) + cos[(d_.) + (e_.)*(x_)
]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)])^(n_), x_Symbol] :> -Simp[((c*B + c
*A*Cos[d + e*x] + (a*B - b*A)*Sin[d + e*x])*(a + b*Cos[d + e*x] + c*Sin[d +
 e*x])^(n + 1))/(e*(n + 1)*(a^2 - b^2 - c^2)), x] + Dist[1/((n + 1)*(a^2 -
b^2 - c^2)), Int[(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n + 1)*Simp[(n + 1)
*(a*A - b*B) + (n + 2)*(a*B - b*A)*Cos[d + e*x] - (n + 2)*c*A*Sin[d + e*x],
 x], x], x] /; FreeQ[{a, b, c, d, e, A, B}, x] && LtQ[n, -1] && NeQ[a^2 - b
^2 - c^2, 0] && NeQ[n, -2]
```

Rubi steps

$$\int \frac{A+B\cos(x)}{(a+b\cos(x)+c\sin(x))^3}\,dx = \frac{Bc+Ac\cos(x)-(Ab-aB)\sin(x)}{2\left(a^2-b^2-c^2\right)(a+b\cos(x)+c\sin(x))^2} - \frac{\int \frac{-2(aA-bB)+(Ab-aB)\cos(x)+Ac\sin(x)}{(a+b\cos(x)+c\sin(x))^2}\,dx}{2\left(a^2-b^2-c^2\right)}$$

$$= \frac{Bc+Ac\cos(x)-(Ab-aB)\sin(x)}{2\left(a^2-b^2-c^2\right)(a+b\cos(x)+c\sin(x))^2} + \frac{aBc+(3aA-2bB)c\cos(x)-\left(3aA\right.}{2\left(a^2-b^2-c^2\right)^2(a+b\cos}$$

$$= \frac{Bc+Ac\cos(x)-(Ab-aB)\sin(x)}{2\left(a^2-b^2-c^2\right)(a+b\cos(x)+c\sin(x))^2} + \frac{aBc+(3aA-2bB)c\cos(x)-\left(3aA\right.}{2\left(a^2-b^2-c^2\right)^2(a+b\cos}$$

$$= \frac{Bc+Ac\cos(x)-(Ab-aB)\sin(x)}{2\left(a^2-b^2-c^2\right)(a+b\cos(x)+c\sin(x))^2} + \frac{aBc+(3aA-2bB)c\cos(x)-\left(3aA\right.}{2\left(a^2-b^2-c^2\right)^2(a+b\cos}$$

$$= \frac{\left(2a^2A-3abB+A\left(b^2+c^2\right)\right)\tan^{-1}\left(\frac{c+(a-b)\tan\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2-c^2}}\right)}{\left(a^2-b^2-c^2\right)^{5/2}} + \frac{Bc+Ac\cos(x)-(Ab}{2\left(a^2-b^2-c^2\right)(a+b\cos}$$

Mathematica [A] time = 0.94, size = 326, normalized size = 1.63

$$\frac{-6a^3Ac+4a^3bB\sin(x)-2bc\cos(x)\left(2a^2A-3abB+A\left(b^2+c^2\right)\right)+c\cos(2x)\left(a^2(-b)B+3aA\left(b^2+c^2\right)-2bB\left(b\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Cos[x])/(a + b*Cos[x] + c*Sin[x])^3,x]
```

```
[Out] -(((2*a^2*A - 3*a*b*B + A*(b^2 + c^2))*ArcTanh[(c + (a - b)*Tan[x/2])/Sqrt[
-a^2 + b^2 + c^2]])/(-a^2 + b^2 + c^2)^(5/2)) + (-6*a^3*A*c - 3*a*A*b^2*c +
 9*a^2*b*B*c - 3*a*A*c^3 - 2*b*c*(2*a^2*A - 3*a*b*B + A*(b^2 + c^2))*Cos[x]
 + c*(-(a^2*b*B) + 3*a*A*(b^2 + c^2) - 2*b*B*(b^2 + c^2))*Cos[2*x] - 8*a^2*
A*b^2*Sin[x] + 2*A*b^4*Sin[x] + 4*a^3*b*B*Sin[x] + 2*a*b^3*B*Sin[x] - 12*a^
2*A*c^2*Sin[x] + 2*A*b^2*c^2*Sin[x] + 8*a*b*B*c^2*Sin[x] - 3*a*A*b^3*Sin[2*
x] + a^2*b^2*B*Sin[2*x] + 2*b^4*B*Sin[2*x] - 3*a*A*b*c^2*Sin[2*x] + 2*b^2*B
*c^2*Sin[2*x])/(4*b*(-a^2 + b^2 + c^2)^2*(a + b*Cos[x] + c*Sin[x])^2)
```

fricas [B] time = 2.16, size = 3402, normalized size = 17.01

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(x))/(a+b*cos(x)+c*sin(x))^3,x, algorithm="fricas")
```

```
[Out] [1/4*(2*B*c^7 - 2*(3*B*a^2 - 3*A*a*b - B*b^2)*c^5 + 2*(3*B*a^4 - 3*A*a^3*b
- 5*B*a^2*b^2 + 6*A*a*b^3 - B*b^4)*c^3 - 4*((3*A*a*b - 2*B*b^2)*c^5 - (3*A*
a^3*b - B*a^2*b^2 - 6*A*a*b^3 + 4*B*b^4)*c^3 + (B*a^4*b^2 - 3*A*a^3*b^3 + B
*a^2*b^4 + 3*A*a*b^5 - 2*B*b^6)*c)*cos(x)^2 - (2*A*a^4*b^2 - 3*B*a^3*b^3 +
A*a^2*b^4 + A*c^6 + (3*A*a^2 - 3*B*a*b + 2*A*b^2)*c^4 + (2*A*a^4 - 3*B*a^3*
b + 4*A*a^2*b^2 - 3*B*a*b^3 + A*b^4)*c^2 + (2*A*a^2*b^4 - 3*B*a*b^5 + A*b^6
 + A*b^4*c^2 - A*c^6 - (2*A*a^2 - 3*B*a*b + A*b^2)*c^4)*cos(x)^2 + 2*(2*A*a
^3*b^3 - 3*B*a^2*b^4 + A*a*b^5 + A*a*b*c^4 + (2*A*a^3*b - 3*B*a^2*b^2 + 2*A
*a*b^3)*c^2)*cos(x) + 2*(A*a*c^5 + (2*A*a^3 - 3*B*a^2*b + 2*A*a*b^2)*c^3 +
(2*A*a^3*b^2 - 3*B*a^2*b^3 + A*a*b^4)*c + (A*b*c^5 + (2*A*a^2*b - 3*B*a*b^2
 + 2*A*b^3)*c^3 + (2*A*a^2*b^3 - 3*B*a*b^4 + A*b^5)*c)*cos(x))*sin(x))*sqrt
(-a^2 + b^2 + c^2)*log(-(a^2*b^2 - 2*b^4 - c^4 - (a^2 + 3*b^2)*c^2 - (2*a^2
*b^2 - b^4 - 2*a^2*c^2 + c^4)*cos(x)^2 - 2*(a*b^3 + a*b*c^2)*cos(x) - 2*(a*
b^2*c + a*c^3 - (b*c^3 - (2*a^2*b - b^3)*c)*cos(x))*sin(x) + 2*(2*a*b*c*cos
(x)^2 - a*b*c + (b^2*c + c^3)*cos(x) - (b^3 + b*c^2 + (a*b^2 - a*c^2)*cos(x
))*sin(x))*sqrt(-a^2 + b^2 + c^2))/(2*a*b*cos(x) + (b^2 - c^2)*cos(x)^2 + a
^2 + c^2 + 2*(b*c*cos(x) + a*c)*sin(x))) - 2*(B*a^6 - 4*B*a^4*b^2 + 3*A*a^3
*b^3 + 2*B*a^2*b^4 - 3*A*a*b^5 + B*b^6)*c + 2*(A*c^7 - (5*A*a^2 - B*a*b - 3
*A*b^2)*c^5 + (4*A*a^4 + B*a^3*b - 10*A*a^2*b^2 + 2*B*a*b^3 + 3*A*b^4)*c^3
- (2*B*a^5*b - 4*A*a^4*b^2 - B*a^3*b^3 + 5*A*a^2*b^4 - B*a*b^5 - A*b^6)*c)*
cos(x) + 2*(2*B*a^5*b^2 - 4*A*a^4*b^3 - B*a^3*b^4 + 5*A*a^2*b^5 - B*a*b^6 -
 A*b^7 - A*b*c^6 + (5*A*a^2*b - B*a*b^2 - 3*A*b^3)*c^4 - (4*A*a^4*b + B*a^3
*b^2 - 10*A*a^2*b^3 + 2*B*a*b^4 + 3*A*b^5)*c^2 + (B*a^4*b^3 - 3*A*a^3*b^4 +
 B*a^2*b^5 + 3*A*a*b^6 - 2*B*b^7 - (3*A*a - 2*B*b)*c^6 + (3*A*a^3 - B*a^2*b
 - 3*A*a*b^2 + 2*B*b^3)*c^4 - (B*a^4*b - 3*A*a*b^4 + 2*B*b^5)*c^2)*cos(x))*
sin(x))/(a^8*b^2 - 3*a^6*b^4 + 3*a^4*b^6 - a^2*b^8 - c^10 + 2*(a^2 - 2*b^2)
*c^8 + (5*a^2*b^2 - 6*b^4)*c^6 - (2*a^6 - 3*a^4*b^2 - 3*a^2*b^4 + 4*b^6)*c^
4 + (a^8 - 5*a^6*b^2 + 6*a^4*b^4 - a^2*b^6 - b^8)*c^2 + (a^6*b^4 - 3*a^4*b^
6 + 3*a^2*b^8 - b^10 + c^10 - 3*(a^2 - b^2)*c^8 + (3*a^4 - 6*a^2*b^2 + 2*b^
4)*c^6 - (a^6 - 3*a^4*b^2 + 2*b^6)*c^4 - 3*(a^4*b^4 - 2*a^2*b^6 + b^8)*c^2)
*cos(x)^2 + 2*(a^7*b^3 - 3*a^5*b^5 + 3*a^3*b^7 - a*b^9 - a*b*c^8 + (3*a^3*b
 - 4*a*b^3)*c^6 - 3*(a^5*b - 3*a^3*b^3 + 2*a*b^5)*c^4 + (a^7*b - 6*a^5*b^3
+ 9*a^3*b^5 - 4*a*b^7)*c^2)*cos(x) - 2*(a*c^9 - (3*a^3 - 4*a*b^2)*c^7 + 3*(
a^5 - 3*a^3*b^2 + 2*a*b^4)*c^5 - (a^7 - 6*a^5*b^2 + 9*a^3*b^4 - 4*a*b^6)*c^
3 - (a^7*b^2 - 3*a^5*b^4 + 3*a^3*b^6 - a*b^8)*c + (b*c^9 - (3*a^2*b - 4*b^3
)*c^7 + 3*(a^4*b - 3*a^2*b^3 + 2*b^5)*c^5 - (a^6*b - 6*a^4*b^3 + 9*a^2*b^5
- 4*b^7)*c^3 - (a^6*b^3 - 3*a^4*b^5 + 3*a^2*b^7 - b^9)*c)*cos(x))*sin(x)),
1/2*(B*c^7 - (3*B*a^2 - 3*A*a*b - B*b^2)*c^5 + (3*B*a^4 - 3*A*a^3*b - 5*B*a
^2*b^2 + 6*A*a*b^3 - B*b^4)*c^3 - 2*((3*A*a*b - 2*B*b^2)*c^5 - (3*A*a^3*b -
 B*a^2*b^2 - 6*A*a*b^3 + 4*B*b^4)*c^3 + (B*a^4*b^2 - 3*A*a^3*b^3 + B*a^2*b^
4 + 3*A*a*b^5 - 2*B*b^6)*c)*cos(x)^2 + (2*A*a^4*b^2 - 3*B*a^3*b^3 + A*a^2*b
```

```
^4 + A*c^6 + (3*A*a^2 - 3*B*a*b + 2*A*b^2)*c^4 + (2*A*a^4 - 3*B*a^3*b + 4*A
*a^2*b^2 - 3*B*a*b^3 + A*b^4)*c^2 + (2*A*a^2*b^4 - 3*B*a*b^5 + A*b^6 + A*b^
4*c^2 - A*c^6 - (2*A*a^2 - 3*B*a*b + A*b^2)*c^4)*cos(x)^2 + 2*(2*A*a^3*b^3
- 3*B*a^2*b^4 + A*a*b^5 + A*a*b*c^4 + (2*A*a^3*b - 3*B*a^2*b^2 + 2*A*a*b^3)
*c^2)*cos(x) + 2*(A*a*c^5 + (2*A*a^3 - 3*B*a^2*b + 2*A*a*b^2)*c^3 + (2*A*a^
3*b^2 - 3*B*a^2*b^3 + A*a*b^4)*c + (A*b*c^5 + (2*A*a^2*b - 3*B*a*b^2 + 2*A*
b^3)*c^3 + (2*A*a^2*b^3 - 3*B*a*b^4 + A*b^5)*c)*cos(x))*sin(x))*sqrt(a^2 -
b^2 - c^2)*arctan(-(a*b*cos(x) + a*c*sin(x) + b^2 + c^2)*sqrt(a^2 - b^2 - c
^2)/((c^3 - (a^2 - b^2)*c)*cos(x) + (a^2*b - b^3 - b*c^2)*sin(x))) - (B*a^6
 - 4*B*a^4*b^2 + 3*A*a^3*b^3 + 2*B*a^2*b^4 - 3*A*a*b^5 + B*b^6)*c + (A*c^7
- (5*A*a^2 - B*a*b - 3*A*b^2)*c^5 + (4*A*a^4 + B*a^3*b - 10*A*a^2*b^2 + 2*B
*a*b^3 + 3*A*b^4)*c^3 - (2*B*a^5*b - 4*A*a^4*b^2 - B*a^3*b^3 + 5*A*a^2*b^4
- B*a*b^5 - A*b^6)*c)*cos(x) + (2*B*a^5*b^2 - 4*A*a^4*b^3 - B*a^3*b^4 + 5*A
*a^2*b^5 - B*a*b^6 - A*b^7 - A*b*c^6 + (5*A*a^2*b - B*a*b^2 - 3*A*b^3)*c^4
- (4*A*a^4*b + B*a^3*b^2 - 10*A*a^2*b^3 + 2*B*a*b^4 + 3*A*b^5)*c^2 + (B*a^4
*b^3 - 3*A*a^3*b^4 + B*a^2*b^5 + 3*A*a*b^6 - 2*B*b^7 - (3*A*a - 2*B*b)*c^6
+ (3*A*a^3 - B*a^2*b - 3*A*a*b^2 + 2*B*b^3)*c^4 - (B*a^4*b - 3*A*a*b^4 + 2*
B*b^5)*c^2)*cos(x))*sin(x))/(a^8*b^2 - 3*a^6*b^4 + 3*a^4*b^6 - a^2*b^8 - c^
10 + 2*(a^2 - 2*b^2)*c^8 + (5*a^2*b^2 - 6*b^4)*c^6 - (2*a^6 - 3*a^4*b^2 - 3
*a^2*b^4 + 4*b^6)*c^4 + (a^8 - 5*a^6*b^2 + 6*a^4*b^4 - a^2*b^6 - b^8)*c^2 +
 (a^6*b^4 - 3*a^4*b^6 + 3*a^2*b^8 - b^10 + c^10 - 3*(a^2 - b^2)*c^8 + (3*a^
4 - 6*a^2*b^2 + 2*b^4)*c^6 - (a^6 - 3*a^4*b^2 + 2*b^6)*c^4 - 3*(a^4*b^4 - 2
*a^2*b^6 + b^8)*c^2)*cos(x)^2 + 2*(a^7*b^3 - 3*a^5*b^5 + 3*a^3*b^7 - a*b^9
- a*b*c^8 + (3*a^3*b - 4*a*b^3)*c^6 - 3*(a^5*b - 3*a^3*b^3 + 2*a*b^5)*c^4 +
 (a^7*b - 6*a^5*b^3 + 9*a^3*b^5 - 4*a*b^7)*c^2)*cos(x) - 2*(a*c^9 - (3*a^3
- 4*a*b^2)*c^7 + 3*(a^5 - 3*a^3*b^2 + 2*a*b^4)*c^5 - (a^7 - 6*a^5*b^2 + 9*a
^3*b^4 - 4*a*b^6)*c^3 - (a^7*b^2 - 3*a^5*b^4 + 3*a^3*b^6 - a*b^8)*c + (b*c^
9 - (3*a^2*b - 4*b^3)*c^7 + 3*(a^4*b - 3*a^2*b^3 + 2*b^5)*c^5 - (a^6*b - 6*
a^4*b^3 + 9*a^2*b^5 - 4*b^7)*c^3 - (a^6*b^3 - 3*a^4*b^5 + 3*a^2*b^7 - b^9)*
c)*cos(x))*sin(x))]
```

giac [B] time = 0.55, size = 1162, normalized size = 5.81

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(x))/(a+b*cos(x)+c*sin(x))^3,x, algorithm="giac")
```

```
[Out] -(2*A*a^2 - 3*B*a*b + A*b^2 + A*c^2)*(pi*floor(1/2*x/pi + 1/2)*sgn(-2*a + 2
*b) + arctan(-(a*tan(1/2*x) - b*tan(1/2*x) + c)/sqrt(a^2 - b^2 - c^2)))/((a
^4 - 2*a^2*b^2 + b^4 - 2*a^2*c^2 + 2*b^2*c^2 + c^4)*sqrt(a^2 - b^2 - c^2))
+ (2*B*a^5*tan(1/2*x)^3 - 4*A*a^4*b*tan(1/2*x)^3 - 5*B*a^4*b*tan(1/2*x)^3 +
 11*A*a^3*b^2*tan(1/2*x)^3 + 5*B*a^3*b^2*tan(1/2*x)^3 - 9*A*a^2*b^3*tan(1/2
*x)^3 - 5*B*a^2*b^3*tan(1/2*x)^3 + A*a*b^4*tan(1/2*x)^3 + 5*B*a*b^4*tan(1/2
*x)^3 + A*b^5*tan(1/2*x)^3 - 2*B*b^5*tan(1/2*x)^3 + 5*A*a^3*c^2*tan(1/2*x)^
```

```
3 - 4*B*a^3*c^2*tan(1/2*x)^3 - 7*A*a^2*b*c^2*tan(1/2*x)^3 + 4*B*a^2*b*c^2*t
an(1/2*x)^3 - A*a*b^2*c^2*tan(1/2*x)^3 + 4*B*a*b^2*c^2*tan(1/2*x)^3 + 3*A*b
^3*c^2*tan(1/2*x)^3 - 4*B*b^3*c^2*tan(1/2*x)^3 - 2*A*a*c^4*tan(1/2*x)^3 + 2
*B*a*c^4*tan(1/2*x)^3 + 2*A*b*c^4*tan(1/2*x)^3 - 2*B*b*c^4*tan(1/2*x)^3 + 4
*A*a^4*c*tan(1/2*x)^2 + 2*B*a^4*c*tan(1/2*x)^2 - 12*A*a^3*b*c*tan(1/2*x)^2
- 9*B*a^3*b*c*tan(1/2*x)^2 + 13*A*a^2*b^2*c*tan(1/2*x)^2 + 14*B*a^2*b^2*c*t
an(1/2*x)^2 - 6*A*a*b^3*c*tan(1/2*x)^2 - 9*B*a*b^3*c*tan(1/2*x)^2 + A*b^4*c
*tan(1/2*x)^2 + 2*B*b^4*c*tan(1/2*x)^2 + 7*A*a^2*c^3*tan(1/2*x)^2 - 4*B*a^2
*c^3*tan(1/2*x)^2 - 6*A*a*b*c^3*tan(1/2*x)^2 - A*b^2*c^3*tan(1/2*x)^2 + 4*B
*b^2*c^3*tan(1/2*x)^2 - 2*A*c^5*tan(1/2*x)^2 + 2*B*c^5*tan(1/2*x)^2 + 2*B*a
^5*tan(1/2*x) - 4*A*a^4*b*tan(1/2*x) - 3*B*a^4*b*tan(1/2*x) + 5*A*a^3*b^2*t
an(1/2*x) + B*a^3*b^2*tan(1/2*x) + 3*A*a^2*b^3*tan(1/2*x) + B*a^2*b^3*tan(1
/2*x) - 5*A*a*b^4*tan(1/2*x) - 3*B*a*b^4*tan(1/2*x) + A*b^5*tan(1/2*x) + 2*
B*b^5*tan(1/2*x) + 11*A*a^3*c^2*tan(1/2*x) - 4*B*a^3*c^2*tan(1/2*x) - 3*A*a
^2*b*c^2*tan(1/2*x) - 8*B*a^2*b*c^2*tan(1/2*x) - 7*A*a*b^2*c^2*tan(1/2*x) +
 8*B*a*b^2*c^2*tan(1/2*x) - A*b^3*c^2*tan(1/2*x) + 4*B*b^3*c^2*tan(1/2*x) -
 2*A*a*c^4*tan(1/2*x) + 2*B*a*c^4*tan(1/2*x) - 2*A*b*c^4*tan(1/2*x) + 2*B*b
*c^4*tan(1/2*x) + 4*A*a^4*c - 5*B*a^3*b*c - 3*A*a^2*b^2*c + 5*B*a*b^3*c - A
*b^4*c - A*a^2*c^3 + 2*B*a*b*c^3 - A*b^2*c^3)/((a^6 - 2*a^5*b - a^4*b^2 + 4
*a^3*b^3 - a^2*b^4 - 2*a*b^5 + b^6 - 2*a^4*c^2 + 4*a^3*b*c^2 - 4*a*b^3*c^2
+ 2*b^4*c^2 + a^2*c^4 - 2*a*b*c^4 + b^2*c^4)*(a*tan(1/2*x)^2 - b*tan(1/2*x)
^2 + 2*c*tan(1/2*x) + a + b)^2)
```

maple [B] time = 0.21, size = 1109, normalized size = 5.54

$$-\frac{\left(4A\,a^{3}b-7A\,a^{2}b^{2}-5A\,a^{2}c^{2}+2Aa\,b^{3}+2Aab\,c^{2}+A\,b^{4}+3A\,b^{2}c^{2}+2A\,c^{4}-2B\,a^{4}+3B\,a^{3}b-2B\,a^{2}b^{2}+4B\,a^{2}c^{2}+3Ba\,b^{3}-2B\,b^{4}-4B\,b^{2}c^{2}-2B\,c^{4}\right)\left(\tan^{3}\left(\frac{x}{2}\right)\right)}{(a-b)\left(a^{4}-2a^{2}b^{2}-2a^{2}c^{2}+b^{4}+2b^{2}c^{2}+c^{4}\right)}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(x))/(a+b*cos(x)+c*sin(x))^3,x)
```

```
[Out] 2*(-1/2*(4*A*a^3*b-7*A*a^2*b^2-5*A*a^2*c^2+2*A*a*b^3+2*A*a*b*c^2+A*b^4+3*A*
b^2*c^2+2*A*c^4-2*B*a^4+3*B*a^3*b-2*B*a^2*b^2+4*B*a^2*c^2+3*B*a*b^3-2*B*b^4
-4*B*b^2*c^2-2*B*c^4)/(a-b)/(a^4-2*a^2*b^2-2*a^2*c^2+b^4+2*b^2*c^2+c^4)*tan
(1/2*x)^3+1/2*c*(4*A*a^4-12*A*a^3*b+13*A*a^2*b^2+7*A*a^2*c^2-6*A*a*b^3-6*A*
a*b*c^2+A*b^4-A*b^2*c^2-2*A*c^4+2*B*a^4-9*B*a^3*b+14*B*a^2*b^2-4*B*a^2*c^2-
9*B*a*b^3+2*B*b^4+4*B*b^2*c^2+2*B*c^4)/(a^4-2*a^2*b^2-2*a^2*c^2+b^4+2*b^2*c
^2+c^4)/(a^2-2*a*b+b^2)*tan(1/2*x)^2-1/2*(4*A*a^4*b-5*A*a^3*b^2-11*A*a^3*c^
2-3*A*a^2*b^3+3*A*a^2*b*c^2+5*A*a*b^4+7*A*a*b^2*c^2+2*A*a*c^4-A*b^5+A*b^3*c
^2+2*A*b*c^4-2*B*a^5+3*B*a^4*b-B*a^3*b^2+4*B*a^3*c^2-B*a^2*b^3+8*B*a^2*b*c^
2+3*B*a*b^4-8*B*a*b^2*c^2-2*B*a*c^4-2*B*b^5-4*B*b^3*c^2-2*B*b*c^4)/(a^4-2*a
^2*b^2-2*a^2*c^2+b^4+2*b^2*c^2+c^4)/(a^2-2*a*b+b^2)*tan(1/2*x)+1/2*c*(4*A*a
^4-3*A*a^2*b^2-A*a^2*c^2-A*b^4-A*b^2*c^2-5*B*a^3*b+5*B*a*b^3+2*B*a*b*c^2)/(
```

```
a^4-2*a^2*b^2-2*a^2*c^2+b^4+2*b^2*c^2+c^4)/(a^2-2*a*b+b^2))/(a*tan(1/2*x)^2
-b*tan(1/2*x)^2+2*c*tan(1/2*x)+a+b)^2+2/(a^4-2*a^2*b^2-2*a^2*c^2+b^4+2*b^2*
c^2+c^4)/(a^2-b^2-c^2)^(1/2)*arctan(1/2*(2*(a-b)*tan(1/2*x)+2*c)/(a^2-b^2-c
^2)^(1/2))*a^2*A+1/(a^4-2*a^2*b^2-2*a^2*c^2+b^4+2*b^2*c^2+c^4)/(a^2-b^2-c^2
)^(1/2)*arctan(1/2*(2*(a-b)*tan(1/2*x)+2*c)/(a^2-b^2-c^2)^(1/2))*A*b^2+1/(a
^4-2*a^2*b^2-2*a^2*c^2+b^4+2*b^2*c^2+c^4)/(a^2-b^2-c^2)^(1/2)*arctan(1/2*(2
*(a-b)*tan(1/2*x)+2*c)/(a^2-b^2-c^2)^(1/2))*A*c^2-3/(a^4-2*a^2*b^2-2*a^2*c^
2+b^4+2*b^2*c^2+c^4)/(a^2-b^2-c^2)^(1/2)*arctan(1/2*(2*(a-b)*tan(1/2*x)+2*c
)/(a^2-b^2-c^2)^(1/2))*a*b*B
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(x))/(a+b*cos(x)+c*sin(x))^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(c^2+b^2-a^2>0)', see `assume?` for
more details)Is c^2+b^2-a^2 positive or negative?
```

mupad [B] time = 6.84, size = 946, normalized size = 4.73

$$-\frac{\frac{-4\,A\,a^4\,c+5\,B\,a^3\,b\,c+3\,A\,a^2\,b^2\,c+A\,a^2\,c^3-5\,B\,a\,b^3\,c-2\,B\,a\,b\,c^3+A\,b^4\,c+A\,b^2\,c^3}{(a-b)^2\,\left(a^4-2\,a^2\,b^2-2\,a^2\,c^2+b^4+2\,b^2\,c^2+c^4\right)}-\frac{\tan\left(\frac{x}{2}\right)\,\left(A\,b^5+2\,B\,a^5+2\,B\,b^5+3\,A\,a^2\,b^3+5\,A\,a^3\,b^2+11\,A\,a^3\,c^2+B\,a^2\,b^3\right.}{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cos(x))/(a + b*cos(x) + c*sin(x))^3,x)
```

```
[Out] - ((A*a^2*c^3 + A*b^2*c^3 - 4*A*a^4*c + A*b^4*c - 2*B*a*b*c^3 - 5*B*a*b^3*c
 + 5*B*a^3*b*c + 3*A*a^2*b^2*c)/((a - b)^2*(a^4 + b^4 + c^4 - 2*a^2*b^2 - 2
*a^2*c^2 + 2*b^2*c^2)) - (tan(x/2)*(A*b^5 + 2*B*a^5 + 2*B*b^5 + 3*A*a^2*b^3
 + 5*A*a^3*b^2 + 11*A*a^3*c^2 + B*a^2*b^3 + B*a^3*b^2 - A*b^3*c^2 - 4*B*a^3
*c^2 + 4*B*b^3*c^2 - 5*A*a*b^4 - 4*A*a^4*b - 2*A*a*c^4 - 3*B*a*b^4 - 3*B*a^
4*b - 2*A*b*c^4 + 2*B*a*c^4 + 2*B*b*c^4 - 7*A*a*b^2*c^2 - 3*A*a^2*b*c^2 + 8
*B*a*b^2*c^2 - 8*B*a^2*b*c^2))/((a - b)^2*(a^4 + b^4 + c^4 - 2*a^2*b^2 - 2*
a^2*c^2 + 2*b^2*c^2)) - (tan(x/2)^2*(2*B*c^5 - 2*A*c^5 + 7*A*a^2*c^3 - A*b^
2*c^3 - 4*B*a^2*c^3 + 4*B*b^2*c^3 + 4*A*a^4*c + A*b^4*c + 2*B*a^4*c + 2*B*b
^4*c - 6*A*a*b*c^3 - 6*A*a*b^3*c - 12*A*a^3*b*c - 9*B*a*b^3*c - 9*B*a^3*b*c
 + 13*A*a^2*b^2*c + 14*B*a^2*b^2*c))/((a - b)^2*(a^4 + b^4 + c^4 - 2*a^2*b^
2 - 2*a^2*c^2 + 2*b^2*c^2)) + (tan(x/2)^3*(A*b^4 - 2*B*a^4 + 2*A*c^4 - 2*B*
```

```
b^4 - 2*B*c^4 - 7*A*a^2*b^2 - 5*A*a^2*c^2 - 2*B*a^2*b^2 + 3*A*b^2*c^2 + 4*B
*a^2*c^2 - 4*B*b^2*c^2 + 2*A*a*b^3 + 4*A*a^3*b + 3*B*a*b^3 + 3*B*a^3*b + 2*
A*a*b*c^2))/((a - b)*(a^4 + b^4 + c^4 - 2*a^2*b^2 - 2*a^2*c^2 + 2*b^2*c^2))
)/(tan(x/2)^4*(a^2 - 2*a*b + b^2) + 2*a*b + tan(x/2)*(4*a*c + 4*b*c) + tan(
x/2)^3*(4*a*c - 4*b*c) + a^2 + b^2 + tan(x/2)^2*(2*a^2 - 2*b^2 + 4*c^2)) -
(atanh((2*a^4*c + 2*b^4*c + 2*c^5 - 4*a^2*c^3 + 4*b^2*c^3 - 4*a^2*b^2*c)/(2
*(b^2 - a^2 + c^2)^(5/2)) + (tan(x/2)*(2*a - 2*b)*(a^4 + b^4 + c^4 - 2*a^2*
b^2 - 2*a^2*c^2 + 2*b^2*c^2))/(2*(b^2 - a^2 + c^2)^(5/2)))*(2*A*a^2 + A*b^2
 + A*c^2 - 3*B*a*b))/(b^2 - a^2 + c^2)^(5/2)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(x))/(a+b*cos(x)+c*sin(x))**3,x)
```

```
[Out] Timed out
```

3.538 $\int \frac{A+B\cos(x)}{a+b\cos(x)+ib\sin(x)}\,dx$

Optimal. Leaf size=84

$$\frac{i\left(a^2(-B)+2aAb-b^2B\right)\log(a+ib\sin(x)+b\cos(x))}{2a^2b}+\frac{x(2aA-bB)}{2a^2}+\frac{B\sin(x)}{2a}+\frac{iB\cos(x)}{2a}$$

```
[Out] 1/2*(2*A*a-B*b)*x/a^2+1/2*I*B*cos(x)/a+1/2*I*(2*A*a*b-B*a^2-B*b^2)*ln(a+b*c
os(x)+I*b*sin(x))/a^2/b+1/2*B*sin(x)/a
```

Rubi [A] time = 0.04, antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {3132}

$$\frac{i\left(a^2(-B)+2aAb-b^2B\right)\log(a+ib\sin(x)+b\cos(x))}{2a^2b}+\frac{x(2aA-bB)}{2a^2}+\frac{B\sin(x)}{2a}+\frac{iB\cos(x)}{2a}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cos[x])/(a + b*Cos[x] + I*b*Sin[x]),x]
```

```
[Out] ((2*a*A - b*B)*x)/(2*a^2) + ((I/2)*B*Cos[x])/a + ((I/2)*(2*a*A*b - a^2*B -
b^2*B)*Log[a + b*Cos[x] + I*b*Sin[x]])/(a^2*b) + (B*Sin[x])/(2*a)
```

Rule 3132

```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.))/(cos[(d_.) + (e_.)*(x_)]*(b_.)
+ (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)]), x_Symbol] :> Simp[((2*a*A - b*B)*x
)/(2*a^2), x] + (Simp[(B*Sin[d + e*x])/(2*a*e), x] - Simp[(b*B*Cos[d + e*x]
)/(2*a*c*e), x] + Simp[((a^2*B - 2*a*b*A + b^2*B)*Log[RemoveContent[a + b*C
os[d + e*x] + c*Sin[d + e*x], x]])/(2*a^2*c*e), x]) /; FreeQ[{a, b, c, d, e
, A, B}, x] && EqQ[b^2 + c^2, 0]
```

Rubi steps

$$\int \frac{A+B\cos(x)}{a+b\cos(x)+ib\sin(x)}\,dx=\frac{(2aA-bB)x}{2a^2}+\frac{iB\cos(x)}{2a}+\frac{i\left(2aAb-a^2B-b^2B\right)\log(a+b\cos(x)+ib\sin(x))}{2a^2b}+$$

Mathematica [A] time = 0.22, size = 147, normalized size = 1.75

$$\frac{2\left(a^2B-2aAb+b^2B\right)\tan^{-1}\left(\frac{(a+b)\cot\left(\frac{x}{2}\right)}{a-b}\right)+2iaAb\log\left(a^2+2ab\cos(x)+b^2\right)-ia^2B\log\left(a^2+2ab\cos(x)+b^2\right)-}{4a^2b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Cos[x])/(a + b*Cos[x] + I*b*Sin[x]),x]
```

```
[Out] (2*a*A*b*x + a^2*B*x - b^2*B*x + 2*(-2*a*A*b + a^2*B + b^2*B)*ArcTan[((a +
b)*Cot[x/2])/(a - b)] + (2*I)*a*b*B*Cos[x] + (2*I)*a*A*b*Log[a^2 + b^2 + 2*
a*b*Cos[x]] - I*a^2*B*Log[a^2 + b^2 + 2*a*b*Cos[x]] - I*b^2*B*Log[a^2 + b^2
 + 2*a*b*Cos[x]] + 2*a*b*B*Sin[x])/(4*a^2*b)
```

fricas [A] time = 0.76, size = 72, normalized size = 0.86

$$\frac{\left(i\,Bab+\left(2\,Aab-Bb^2\right)xe^{(i\,x)}+\left(-i\,Ba^2+2i\,Aab-i\,Bb^2\right)e^{(i\,x)}\log\left(\frac{be^{(i\,x)}+a}{b}\right)\right)e^{(-i\,x)}}{2\,a^2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(x))/(a+b*cos(x)+I*b*sin(x)),x, algorithm="fricas")
```

```
[Out] 1/2*(I*B*a*b + (2*A*a*b - B*b^2)*x*e^(I*x) + (-I*B*a^2 + 2*I*A*a*b - I*B*b^
2)*e^(I*x)*log((b*e^(I*x) + a)/b))*e^(-I*x)/(a^2*b)
```

giac [B] time = 0.16, size = 168, normalized size = 2.00

$$-\frac{(-2i\,Aa+i\,Bb)\log\left(-a\tan\left(\frac{1}{2}\,x\right)^2+b\tan\left(\frac{1}{2}\,x\right)^2-2i\,a\tan\left(\frac{1}{2}\,x\right)+a+b\right)}{4\,a^2}-\frac{(2i\,Aa-i\,Bb)\log\left(\tan\left(\frac{1}{2}\,x\right)-i\right)}{2\,a^2}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(x))/(a+b*cos(x)+I*b*sin(x)),x, algorithm="giac")
```

```
[Out] -1/4*(-2*I*A*a + I*B*b)*log(-a*tan(1/2*x)^2 + b*tan(1/2*x)^2 - 2*I*a*tan(1/
2*x) + a + b)/a^2 - 1/2*(2*I*A*a - I*B*b)*log(tan(1/2*x) - I)/a^2 + 1/4*(2*
B*a^2 - 2*A*a*b + B*b^2)*(x + 2*arctan((-I*a*cos(x) - a*sin(x) - I*a)/(a*co
s(x) - I*a*sin(x) - a + 2*b)))/(a^2*b) - 1/2*(-2*I*A*a*tan(1/2*x) + I*B*b*t
an(1/2*x) - 2*A*a - 2*B*a + B*b)/(a^2*(tan(1/2*x) - I))
```

maple [B] time = 0.18, size = 153, normalized size = 1.82

$$\frac{iB\ln\left(\tan\left(\frac{x}{2}\right)+i\right)}{2b}-\frac{i\ln\left(\tan\left(\frac{x}{2}\right)-i\right)A}{a}+\frac{i\ln\left(\tan\left(\frac{x}{2}\right)-i\right)bB}{2a^2}+\frac{B}{a\left(\tan\left(\frac{x}{2}\right)-i\right)}+\frac{i\ln\left(ia+ib+a\tan\left(\frac{x}{2}\right)-b\tan\left(\right.\right.}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(x))/(a+b*cos(x)+I*b*sin(x)),x)
```

```
[Out] 1/2*I*B/b*ln(tan(1/2*x)+I)-I/a*ln(tan(1/2*x)-I)*A+1/2*I/a^2*ln(tan(1/2*x)-I
)*b*B+B/a/(tan(1/2*x)-I)+I/a*ln(I*a+I*b+a*tan(1/2*x)-b*tan(1/2*x))*A-1/2*I/
b*ln(I*a+I*b+a*tan(1/2*x)-b*tan(1/2*x))*B-1/2*I/a^2*b*ln(I*a+I*b+a*tan(1/2*
x)-b*tan(1/2*x))*B
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(x))/(a+b*cos(x)+I*b*sin(x)),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

mupad [B] time = 4.33, size = 99, normalized size = 1.18

$$\frac{B}{a\,\left(\mathrm{tan}\left(\frac{x}{2}\right)-\mathrm{i}\right)}+\frac{B\,\ln\left(\mathrm{tan}\left(\frac{x}{2}\right)+1\mathrm{i}\right)\,1\mathrm{i}}{2\,b}-\frac{\ln\left(\mathrm{tan}\left(\frac{x}{2}\right)-\mathrm{i}\right)\,\left(A\,a\,1\mathrm{i}-\frac{B\,b\,1\mathrm{i}}{2}\right)}{a^2}-\frac{\ln\left(a+b-a\,\mathrm{tan}\left(\frac{x}{2}\right)\,1\mathrm{i}+b\,\mathrm{tan}\left(\frac{x}{2}\right)\,1\mathrm{i}\right)}{2\,a^2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cos(x))/(a + b*cos(x) + b*sin(x)*1i),x)
```

```
[Out] B/(a*(tan(x/2) - 1i)) + (B*log(tan(x/2) + 1i)*1i)/(2*b) - (log(tan(x/2) - 1
i)*(A*a*1i - (B*b*1i)/2))/a^2 - (log(a + b - a*tan(x/2)*1i + b*tan(x/2)*1i)
*(B*a^2 + B*b^2 - 2*A*a*b)*1i)/(2*a^2*b)
```

sympy [A] time = 0.58, size = 95, normalized size = 1.13

$$\begin{cases} \frac{iBe^{-ix}}{2a} & \text{for}\: 2a \neq 0 \\ x\left(-\frac{2Aa-Bb}{2a^2}+\frac{2Aa+Ba-Bb}{2a^2}\right) & \text{otherwise} \end{cases} + \frac{x\,(2Aa-Bb)}{2a^2} - \frac{i\left(-2Aab+Ba^2+Bb^2\right)\log\left(\frac{a}{b}+e^{ix}\right)}{2a^2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(x))/(a+b*cos(x)+I*b*sin(x)),x)
```

```
[Out] Piecewise((I*B*exp(-I*x)/(2*a), Ne(2*a, 0)), (x*(-(2*A*a - B*b)/(2*a**2) +
(2*A*a + B*a - B*b)/(2*a**2)), True)) + x*(2*A*a - B*b)/(2*a**2) - I*(-2*A*
a*b + B*a**2 + B*b**2)*log(a/b + exp(I*x))/(2*a**2*b)
```

3.539 $\int \frac{A+B\cos(x)}{a+b\cos(x)-ib\sin(x)}\,dx$

Optimal. Leaf size=84

$$-\frac{i\left(a^2(-B)+2aAb-b^2B\right)\log(a-ib\sin(x)+b\cos(x))}{2a^2b}+\frac{x(2aA-bB)}{2a^2}+\frac{B\sin(x)}{2a}-\frac{iB\cos(x)}{2a}$$

```
[Out] 1/2*(2*A*a-B*b)*x/a^2-1/2*I*B*cos(x)/a-1/2*I*(2*A*a*b-B*a^2-B*b^2)*ln(a+b*c
os(x)-I*b*sin(x))/a^2/b+1/2*B*sin(x)/a
```

Rubi [A] time = 0.04, antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {3132}

$$-\frac{i\left(a^2(-B)+2aAb-b^2B\right)\log(a-ib\sin(x)+b\cos(x))}{2a^2b}+\frac{x(2aA-bB)}{2a^2}+\frac{B\sin(x)}{2a}-\frac{iB\cos(x)}{2a}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cos[x])/(a + b*Cos[x] - I*b*Sin[x]),x]
```

```
[Out] ((2*a*A - b*B)*x)/(2*a^2) - ((I/2)*B*Cos[x])/a - ((I/2)*(2*a*A*b - a^2*B -
b^2*B)*Log[a + b*Cos[x] - I*b*Sin[x]])/(a^2*b) + (B*Sin[x])/(2*a)
```

Rule 3132

```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.))/(cos[(d_.) + (e_.)*(x_)]*(b_.)
+ (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)]), x_Symbol] :> Simp[((2*a*A - b*B)*x
)/(2*a^2), x] + (Simp[(B*Sin[d + e*x])/(2*a*e), x] - Simp[(b*B*Cos[d + e*x]
)/(2*a*c*e), x] + Simp[((a^2*B - 2*a*b*A + b^2*B)*Log[RemoveContent[a + b*C
os[d + e*x] + c*Sin[d + e*x], x]])/(2*a^2*c*e), x]) /; FreeQ[{a, b, c, d, e
, A, B}, x] && EqQ[b^2 + c^2, 0]
```

Rubi steps

$$\int \frac{A+B\cos(x)}{a+b\cos(x)-ib\sin(x)}\,dx=\frac{(2aA-bB)x}{2a^2}-\frac{iB\cos(x)}{2a}-\frac{i\left(2aAb-a^2B-b^2B\right)\log(a+b\cos(x)-ib\sin(x))}{2a^2b}$$

Mathematica [A] time = 0.19, size = 147, normalized size = 1.75

$$\frac{2\left(a^2B-2aAb+b^2B\right)\tan^{-1}\left(\frac{(a+b)\cot\left(\frac{x}{2}\right)}{a-b}\right)-2iaAb\log\left(a^2+2ab\cos(x)+b^2\right)+ia^2B\log\left(a^2+2ab\cos(x)+b^2\right)}{4a^2b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Cos[x])/(a + b*Cos[x] - I*b*Sin[x]),x]
```

```
[Out] (2*a*A*b*x + a^2*B*x - b^2*B*x + 2*(-2*a*A*b + a^2*B + b^2*B)*ArcTan[((a +
b)*Cot[x/2])/(a - b)] - (2*I)*a*b*B*Cos[x] - (2*I)*a*A*b*Log[a^2 + b^2 + 2*
a*b*Cos[x]] + I*a^2*B*Log[a^2 + b^2 + 2*a*b*Cos[x]] + I*b^2*B*Log[a^2 + b^2
 + 2*a*b*Cos[x]] + 2*a*b*B*Sin[x])/(4*a^2*b)
```

fricas [A] time = 2.14, size = 56, normalized size = 0.67

$$\frac{Ba^2x - i\,Babe^{(i\,x)} + \left(i\,Ba^2 - 2i\,Aab + i\,Bb^2\right)\log\left(\frac{ae^{(i\,x)}+b}{a}\right)}{2\,a^2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(x))/(a+b*cos(x)-I*b*sin(x)),x, algorithm="fricas")
```

```
[Out] 1/2*(B*a^2*x - I*B*a*b*e^(I*x) + (I*B*a^2 - 2*I*A*a*b + I*B*b^2)*log((a*e^(
I*x) + b)/a))/(a^2*b)
```

giac [B] time = 0.16, size = 168, normalized size = 2.00

$$-\frac{(2i\,Aa - i\,Bb)\log\left(-a\tan\left(\frac{1}{2}\,x\right)^2 + b\tan\left(\frac{1}{2}\,x\right)^2 + 2i\,a\tan\left(\frac{1}{2}\,x\right) + a + b\right)}{4\,a^2} - \frac{(-2i\,Aa + i\,Bb)\log\left(\tan\left(\frac{1}{2}\,x\right) + i\right)}{2\,a^2} + \cdots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(x))/(a+b*cos(x)-I*b*sin(x)),x, algorithm="giac")
```

```
[Out] -1/4*(2*I*A*a - I*B*b)*log(-a*tan(1/2*x)^2 + b*tan(1/2*x)^2 + 2*I*a*tan(1/2
*x) + a + b)/a^2 - 1/2*(-2*I*A*a + I*B*b)*log(tan(1/2*x) + I)/a^2 + 1/4*(2*
B*a^2 - 2*A*a*b + B*b^2)*(x + 2*arctan((I*a*cos(x) - a*sin(x) + I*a)/(a*cos
(x) + I*a*sin(x) - a + 2*b)))/(a^2*b) - 1/2*(2*I*A*a*tan(1/2*x) - I*B*b*tan
(1/2*x) - 2*A*a - 2*B*a + B*b)/(a^2*(tan(1/2*x) + I))
```

maple [B] time = 0.18, size = 284, normalized size = 3.38

$$\frac{i\ln\left(\tan\left(\frac{x}{2}\right)+i\right)A}{a} - \frac{i\ln\left(\tan\left(\frac{x}{2}\right)+i\right)bB}{2a^2} + \frac{B}{a\left(\tan\left(\frac{x}{2}\right)+i\right)} - \frac{iB\ln\left(\tan\left(\frac{x}{2}\right)-i\right)}{2b} + \frac{i\ln\left(ia+ib-a\tan\left(\frac{x}{2}\right)+b\tan\left(\frac{x}{2}\right)\cdots\right.}{-a+b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(x))/(a+b*cos(x)-I*b*sin(x)),x)
```

```
[Out] I/a*ln(tan(1/2*x)+I)*A-1/2*I/a^2*ln(tan(1/2*x)+I)*b*B+B/a/(tan(1/2*x)+I)-1/
2*I*B/b*ln(tan(1/2*x)-I)+I/(-a+b)*ln(I*a+I*b-a*tan(1/2*x)+b*tan(1/2*x))*A-I
/a*b/(-a+b)*ln(I*a+I*b-a*tan(1/2*x)+b*tan(1/2*x))*A-1/2*I*a/b/(-a+b)*ln(I*a
+I*b-a*tan(1/2*x)+b*tan(1/2*x))*B+1/2*I/(-a+b)*ln(I*a+I*b-a*tan(1/2*x)+b*ta
n(1/2*x))*B-1/2*I/a*b/(-a+b)*ln(I*a+I*b-a*tan(1/2*x)+b*tan(1/2*x))*B+1/2*I/
a^2*b^2/(-a+b)*ln(I*a+I*b-a*tan(1/2*x)+b*tan(1/2*x))*B
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(x))/(a+b*cos(x)-I*b*sin(x)),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

mupad [B] time = 8.39, size = 584, normalized size = 6.95

$$\left(\sum_{k=1}^{3} \ln\left(-(a-b)^2 \left(4\,A^2\,a^2 - 4\,A\,B\,a\,b - B^2\,a^2 + B^2\,b^2\right) 1\mathrm{i} - \mathrm{root}\left(a^4\,b^2\,d^3\,64\mathrm{i} - A\,B\,a\,b^3\,d\,64\mathrm{i} - A\,B\,a^3\,b\,d\,32\mathrm{i} + \right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cos(x))/(a + b*cos(x) - b*sin(x)*1i),x)
```

```
[Out] symsum(log(tan(x/2)*(a - b)^2*(B*a - 2*A*a + B*b)^2 - root(a^4*b^2*d^3*64i
- A*B*a*b^3*d*64i - A*B*a^3*b*d*32i + B^2*a^2*b^2*d*16i + A^2*a^2*b^2*d*64i
 + B^2*b^4*d*16i + B^2*a^4*d*16i - 32*A^2*B*a^2*b + 32*A*B^2*a*b^2 - 8*B^3*
a^2*b + 16*A*B^2*a^3 - 8*B^3*b^3, d, k)*(4*A*a^3*(a - b)^2 - 8*root(a^4*b^2
*d^3*64i - A*B*a*b^3*d*64i - A*B*a^3*b*d*32i + B^2*a^2*b^2*d*16i + A^2*a^2*
b^2*d*64i + B^2*b^4*d*16i + B^2*a^4*d*16i - 32*A^2*B*a^2*b + 32*A*B^2*a*b^2
 - 8*B^3*a^2*b + 16*A*B^2*a^3 - 8*B^3*b^3, d, k)*a^2*(a - b)^2*(a^2*tan(x/2
) + b^2*tan(x/2) - a^2*1i + b^2*1i - a*b*tan(x/2)) + 4*a*tan(x/2)*(a - b)^2
*(A*a^2*1i + B*a^2*1i + B*b^2*1i - A*a*b*2i - B*a*b*1i)) - (a - b)^2*(4*A^2
*a^2 - B^2*a^2 + B^2*b^2 - 4*A*B*a*b)*1i)*root(a^4*b^2*d^3*64i - A*B*a*b^3*
d*64i - A*B*a^3*b*d*32i + B^2*a^2*b^2*d*16i + A^2*a^2*b^2*d*64i + B^2*b^4*d
*16i + B^2*a^4*d*16i - 32*A^2*B*a^2*b + 32*A*B^2*a*b^2 - 8*B^3*a^2*b + 16*A
*B^2*a^3 - 8*B^3*b^3, d, k), k, 1, 3) + B/(a*(tan(x/2) + 1i))
```

sympy [A] time = 0.58, size = 75, normalized size = 0.89

$$\frac{Bx}{2b} + \begin{cases} -\frac{iBe^{ix}}{2a} & \text{for}\ 2a \neq 0 \\ x\left(-\frac{B}{2b} + \frac{Ba+Bb}{2ab}\right) & \text{otherwise} \end{cases} + \frac{i\left(-2Aab + Ba^2 + Bb^2\right)\log\left(e^{ix} + \frac{b}{a}\right)}{2a^2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(x))/(a+b*cos(x)-I*b*sin(x)),x)
```

```
[Out] B*x/(2*b) + Piecewise((-I*B*exp(I*x)/(2*a), Ne(2*a, 0)), (x*(-B/(2*b) + (B*
a + B*b)/(2*a*b)), True)) + I*(-2*A*a*b + B*a**2 + B*b**2)*log(exp(I*x) + b
/a)/(2*a**2*b)
```

3.540 $\int \frac{A+C\sin(x)}{a+b\cos(x)+c\sin(x)}\,dx$

Optimal. Leaf size=116

$$\frac{2\left(A\left(b^2+c^2\right)-acC\right)\tan^{-1}\left(\frac{(a-b)\tan\left(\frac{x}{2}\right)+c}{\sqrt{a^2-b^2-c^2}}\right)}{\left(b^2+c^2\right)\sqrt{a^2-b^2-c^2}}-\frac{bC\log(a+b\cos(x)+c\sin(x))}{b^2+c^2}+\frac{cCx}{b^2+c^2}$$

```
[Out] c*C*x/(b^2+c^2)-b*C*ln(a+b*cos(x)+c*sin(x))/(b^2+c^2)+2*(A*(b^2+c^2)-a*c*C)
*arctan((c+(a-b)*tan(1/2*x))/(a^2-b^2-c^2)^(1/2))/(b^2+c^2)/(a^2-b^2-c^2)^(
1/2)
```

Rubi [A] time = 0.11, antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.210, Rules used = {3137, 3124, 618, 204}

$$\frac{2\left(A\left(b^2+c^2\right)-acC\right)\tan^{-1}\left(\frac{(a-b)\tan\left(\frac{x}{2}\right)+c}{\sqrt{a^2-b^2-c^2}}\right)}{\left(b^2+c^2\right)\sqrt{a^2-b^2-c^2}}-\frac{bC\log(a+b\cos(x)+c\sin(x))}{b^2+c^2}+\frac{cCx}{b^2+c^2}$$

Antiderivative was successfully verified.

```
[In] Int[(A + C*Sin[x])/(a + b*Cos[x] + c*Sin[x]),x]
```

```
[Out] (c*C*x)/(b^2 + c^2) + (2*(A*(b^2 + c^2) - a*c*C)*ArcTan[(c + (a - b)*Tan[x/
2])/Sqrt[a^2 - b^2 - c^2]])/(Sqrt[a^2 - b^2 - c^2]*(b^2 + c^2)) - (b*C*Log[
a + b*Cos[x] + c*Sin[x]])/(b^2 + c^2)
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 3124

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(-1), x_Symbol] :> Module[{f = FreeFactors[Tan[(d + e*x)/2], x]}, Dist[(2*f
```

```
)/e, Subst[Int[1/(a + b + 2*c*f*x + (a - b)*f^2*x^2), x], x, Tan[(d + e*x)/
2]/f], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0]
```

Rule 3137

```
Int[((A_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])/((a_.) + cos[(d_.) + (e_.)*(x_)
]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)]), x_Symbol] :> Simp[(c*C*(d + e*x))
/(e*(b^2 + c^2)), x] + (Dist[(A*(b^2 + c^2) - a*c*C)/(b^2 + c^2), Int[1/(a
+ b*Cos[d + e*x] + c*Sin[d + e*x]), x], x] - Simp[(b*C*Log[a + b*Cos[d + e*
x] + c*Sin[d + e*x]])/(e*(b^2 + c^2)), x]) /; FreeQ[{a, b, c, d, e, A, C},
x] && NeQ[b^2 + c^2, 0] && NeQ[A*(b^2 + c^2) - a*c*C, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{A+C\sin(x)}{a+b\cos(x)+c\sin(x)}\,dx &= \frac{cCx}{b^2+c^2}-\frac{bC\log(a+b\cos(x)+c\sin(x))}{b^2+c^2}+\left(A-\frac{acC}{b^2+c^2}\right)\int \frac{1}{a+b\cos(x)+c\sin\\ &= \frac{cCx}{b^2+c^2}-\frac{bC\log(a+b\cos(x)+c\sin(x))}{b^2+c^2}+\left(2\left(A-\frac{acC}{b^2+c^2}\right)\right)\text{Subst}\left(\int \frac{}{a+b+\\ &= \frac{cCx}{b^2+c^2}-\frac{bC\log(a+b\cos(x)+c\sin(x))}{b^2+c^2}-\left(4\left(A-\frac{acC}{b^2+c^2}\right)\right)\text{Subst}\left(\int \frac{}{-4\left(a^2-\right.\\ &= \frac{cCx}{b^2+c^2}+\frac{2\left(A-\frac{acC}{b^2+c^2}\right)\tan^{-1}\left(\frac{c+(a-b)\tan\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2-c^2}}\right)}{\sqrt{a^2-b^2-c^2}}-\frac{bC\log(a+b\cos(x)+c\sin(x))}{b^2+c^2}\end{aligned}$$

Mathematica [A] time = 0.29, size = 96, normalized size = 0.83

$$\frac{C(cx-b\log(a+b\cos(x)+c\sin(x)))-\frac{2\left(A\left(b^2+c^2\right)-acC\right)\tanh^{-1}\left(\frac{(a-b)\tan\left(\frac{x}{2}\right)+c}{\sqrt{-a^2+b^2+c^2}}\right)}{\sqrt{-a^2+b^2+c^2}}}{b^2+c^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + C*Sin[x])/(a + b*Cos[x] + c*Sin[x]),x]
```

```
[Out] ((-2*(A*(b^2 + c^2) - a*c*C)*ArcTanh[(c + (a - b)*Tan[x/2])/Sqrt[-a^2 + b^2
 + c^2]])/Sqrt[-a^2 + b^2 + c^2] + C*(c*x - b*Log[a + b*Cos[x] + c*Sin[x]])
)/(b^2 + c^2)
```

fricas [B] time = 2.59, size = 625, normalized size = 5.39

$$\left[\frac{\left(Ab^2 - Cac + Ac^2\right)\sqrt{-a^2 + b^2 + c^2}\,\log\left(\frac{a^2b^2 - 2\,b^4 - c^4 - \left(a^2 + 3\,b^2\right)c^2 - \left(2\,a^2b^2 - b^4 - 2\,a^2c^2 + c^4\right)\cos(x)^2 - 2\,\left(ab^3 + abc^2\right)\cos(x) - 2\,\left(ab^2c + ac^3\right.}{2\,ab\cos(x) + \left(b\right.}\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*sin(x))/(a+b*cos(x)+c*sin(x)),x, algorithm="fricas")
```

```
[Out] [1/2*((A*b^2 - C*a*c + A*c^2)*sqrt(-a^2 + b^2 + c^2)*log((a^2*b^2 - 2*b^4 -
 c^4 - (a^2 + 3*b^2)*c^2 - (2*a^2*b^2 - b^4 - 2*a^2*c^2 + c^4)*cos(x)^2 - 2
*(a*b^3 + a*b*c^2)*cos(x) - 2*(a*b^2*c + a*c^3 - (b*c^3 - (2*a^2*b - b^3)*c
)*cos(x))*sin(x) - 2*(2*a*b*c*cos(x)^2 - a*b*c + (b^2*c + c^3)*cos(x) - (b^
3 + b*c^2 + (a*b^2 - a*c^2)*cos(x))*sin(x))*sqrt(-a^2 + b^2 + c^2))/(2*a*b*
cos(x) + (b^2 - c^2)*cos(x)^2 + a^2 + c^2 + 2*(b*c*cos(x) + a*c)*sin(x))) -
 2*(C*c^3 - (C*a^2 - C*b^2)*c)*x - (C*a^2*b - C*b^3 - C*b*c^2)*log(2*a*b*co
s(x) + (b^2 - c^2)*cos(x)^2 + a^2 + c^2 + 2*(b*c*cos(x) + a*c)*sin(x)))/(a^
2*b^2 - b^4 - c^4 + (a^2 - 2*b^2)*c^2), 1/2*(2*(A*b^2 - C*a*c + A*c^2)*sqrt
(a^2 - b^2 - c^2)*arctan(-(a*b*cos(x) + a*c*sin(x) + b^2 + c^2)*sqrt(a^2 -
b^2 - c^2)/((c^3 - (a^2 - b^2)*c)*cos(x) + (a^2*b - b^3 - b*c^2)*sin(x))) -
 2*(C*c^3 - (C*a^2 - C*b^2)*c)*x - (C*a^2*b - C*b^3 - C*b*c^2)*log(2*a*b*co
s(x) + (b^2 - c^2)*cos(x)^2 + a^2 + c^2 + 2*(b*c*cos(x) + a*c)*sin(x)))/(a^
2*b^2 - b^4 - c^4 + (a^2 - 2*b^2)*c^2)]
```

giac [A] time = 0.19, size = 177, normalized size = 1.53

$$\frac{Ccx}{b^2 + c^2} - \frac{Cb\log\left(-a\tan\left(\frac{1}{2}\,x\right)^2 + b\tan\left(\frac{1}{2}\,x\right)^2 - 2\,c\tan\left(\frac{1}{2}\,x\right) - a - b\right)}{b^2 + c^2} + \frac{Cb\log\left(\tan\left(\frac{1}{2}\,x\right)^2 + 1\right)}{b^2 + c^2} - \frac{2\left(Ab^2 - Cac + \right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*sin(x))/(a+b*cos(x)+c*sin(x)),x, algorithm="giac")
```

```
[Out] C*c*x/(b^2 + c^2) - C*b*log(-a*tan(1/2*x)^2 + b*tan(1/2*x)^2 - 2*c*tan(1/2*
x) - a - b)/(b^2 + c^2) + C*b*log(tan(1/2*x)^2 + 1)/(b^2 + c^2) - 2*(A*b^2
- C*a*c + A*c^2)*(pi*floor(1/2*x/pi + 1/2)*sgn(-2*a + 2*b) + arctan(-(a*tan
(1/2*x) - b*tan(1/2*x) + c)/sqrt(a^2 - b^2 - c^2)))/(sqrt(a^2 - b^2 - c^2)*
(b^2 + c^2))
```

maple [B] time = 0.13, size = 542, normalized size = 4.67

$$-\frac{\ln\left(a\left(\tan^2\left(\frac{x}{2}\right)\right)-b\left(\tan^2\left(\frac{x}{2}\right)\right)+2c\tan\left(\frac{x}{2}\right)+a+b\right)abC}{\left(b^2+c^2\right)(a-b)}+\frac{\ln\left(a\left(\tan^2\left(\frac{x}{2}\right)\right)-b\left(\tan^2\left(\frac{x}{2}\right)\right)+2c\tan\left(\frac{x}{2}\right)+a+b\right)b}{\left(b^2+c^2\right)(a-b)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+C*sin(x))/(a+b*cos(x)+c*sin(x)),x)
```

```
[Out] -1/(b^2+c^2)/(a-b)*ln(a*tan(1/2*x)^2-b*tan(1/2*x)^2+2*c*tan(1/2*x)+a+b)*a*b
*C+1/(b^2+c^2)/(a-b)*ln(a*tan(1/2*x)^2-b*tan(1/2*x)^2+2*c*tan(1/2*x)+a+b)*b
^2*C+2/(b^2+c^2)/(a^2-b^2-c^2)^(1/2)*arctan(1/2*(2*(a-b)*tan(1/2*x)+2*c)/(a
^2-b^2-c^2)^(1/2))*A*b^2+2/(b^2+c^2)/(a^2-b^2-c^2)^(1/2)*arctan(1/2*(2*(a-b
)*tan(1/2*x)+2*c)/(a^2-b^2-c^2)^(1/2))*A*c^2-2/(b^2+c^2)/(a^2-b^2-c^2)^(1/2
)*arctan(1/2*(2*(a-b)*tan(1/2*x)+2*c)/(a^2-b^2-c^2)^(1/2))*a*c*C-2/(b^2+c^2
)/(a^2-b^2-c^2)^(1/2)*arctan(1/2*(2*(a-b)*tan(1/2*x)+2*c)/(a^2-b^2-c^2)^(1/
2))*C*b*c+2/(b^2+c^2)/(a^2-b^2-c^2)^(1/2)*arctan(1/2*(2*(a-b)*tan(1/2*x)+2*
c)/(a^2-b^2-c^2)^(1/2))*c/(a-b)*a*b*C-2/(b^2+c^2)/(a^2-b^2-c^2)^(1/2)*arcta
n(1/2*(2*(a-b)*tan(1/2*x)+2*c)/(a^2-b^2-c^2)^(1/2))*c/(a-b)*b^2*C+C/(b^2+c^
2)*b*ln(1+tan(1/2*x)^2)+2*C/(b^2+c^2)*c*arctan(tan(1/2*x))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*sin(x))/(a+b*cos(x)+c*sin(x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(c^2+b^2-a^2>0)', see `assume?` for
more details)Is c^2+b^2-a^2 positive or negative?
```

mupad [B] time = 24.86, size = 1741, normalized size = 15.01

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + C*sin(x))/(a + b*cos(x) + c*sin(x)),x)
```

```
[Out] (C*log(tan(x/2) + 1i))/(b - c*1i) + (C*log(tan(x/2) - 1i)*1i)/(b*1i - c) -
(log(32*A*C^2*a^2 + 32*A*C^2*b^2 - 64*A*C^2*a*b - 32*A^2*C*a*c + 32*A^2*C*b
*c + 32*C*tan(x/2)*(a - b)*(A^2*b + 2*C^2*a - 2*C^2*b - 2*A*C*c) + ((C*b^3
```

```
+ A*b^2*(b^2 - a^2 + c^2)^(1/2) - C*a^2*b + A*c^2*(b^2 - a^2 + c^2)^(1/2) +
 C*b*c^2 - C*a*c*(b^2 - a^2 + c^2)^(1/2))*(64*A^2*b^2*c + 32*C^2*a^2*c + 32
*C^2*b^2*c + 64*A*C*b^3 + 32*tan(x/2)*(a - b)*(A^2*b^2 - A^2*c^2 - 2*C^2*a^
2 + 2*C^2*c^2 + 2*C^2*a*b + 2*A*C*a*c - 4*A*C*b*c) - 128*A*C*a*b^2 + 64*A*C
*a^2*b - 64*A^2*a*b*c - 64*C^2*a*b*c + ((C*b^3 + A*b^2*(b^2 - a^2 + c^2)^(1
/2) - C*a^2*b + A*c^2*(b^2 - a^2 + c^2)^(1/2) + C*b*c^2 - C*a*c*(b^2 - a^2
+ c^2)^(1/2))*(32*A*b^4 + 32*A*a^2*b^2 - 32*A*a^2*c^2 + 32*A*b^2*c^2 - 32*t
an(x/2)*(a - b)*(2*A*c^3 - 2*C*b^3 + 2*A*b^2*c + 2*C*a*b^2 - 2*C*a*c^2 + C*
b*c^2 - 2*A*a*b*c) - 64*A*a*b^3 + 32*C*a*c^3 - 32*C*b*c^3 + 64*C*b^3*c - 12
8*C*a*b^2*c + 64*C*a^2*b*c + (32*(a - b)*(C*b^3 + A*b^2*(b^2 - a^2 + c^2)^(
1/2) - C*a^2*b + A*c^2*(b^2 - a^2 + c^2)^(1/2) + C*b*c^2 - C*a*c*(b^2 - a^2
 + c^2)^(1/2))*(3*c^4*tan(x/2) + a*c^3 + 3*b*c^3 + 3*b^3*c + 2*a^2*b^2*tan(
x/2) - 2*a^2*c^2*tan(x/2) + 3*b^2*c^2*tan(x/2) - 2*a*b^3*tan(x/2) + a*b^2*c
 - 4*a^2*b*c - 2*a*b*c^2*tan(x/2)))/((b^2 + c^2)*(b^2 - a^2 + c^2))))/((b^2
 + c^2)*(b^2 - a^2 + c^2))))/((b^2 + c^2)*(b^2 - a^2 + c^2)))*(C*b^3 - b*(C
*a^2 - C*c^2) + A*b^2*(b^2 - a^2 + c^2)^(1/2) + A*c^2*(b^2 - a^2 + c^2)^(1/
2) - C*a*c*(b^2 - a^2 + c^2)^(1/2)))/((b^2 + c^2)*(b^2 - a^2 + c^2)) + (log
(32*A*C^2*a^2 + 32*A*C^2*b^2 - 64*A*C^2*a*b - 32*A^2*C*a*c + 32*A^2*C*b*c +
 32*C*tan(x/2)*(a - b)*(A^2*b + 2*C^2*a - 2*C^2*b - 2*A*C*c) + ((C*b^3 - A*
b^2*(b^2 - a^2 + c^2)^(1/2) - C*a^2*b - A*c^2*(b^2 - a^2 + c^2)^(1/2) + C*b
*c^2 + C*a*c*(b^2 - a^2 + c^2)^(1/2))*(64*A^2*b^2*c + 32*C^2*a^2*c + 32*C^2
*b^2*c + 64*A*C*b^3 + 32*tan(x/2)*(a - b)*(A^2*b^2 - A^2*c^2 - 2*C^2*a^2 +
2*C^2*c^2 + 2*C^2*a*b + 2*A*C*a*c - 4*A*C*b*c) - 128*A*C*a*b^2 + 64*A*C*a^2
*b - 64*A^2*a*b*c - 64*C^2*a*b*c + ((C*b^3 - A*b^2*(b^2 - a^2 + c^2)^(1/2)
- C*a^2*b - A*c^2*(b^2 - a^2 + c^2)^(1/2) + C*b*c^2 + C*a*c*(b^2 - a^2 + c^
2)^(1/2))*(32*A*b^4 + 32*A*a^2*b^2 - 32*A*a^2*c^2 + 32*A*b^2*c^2 - 32*tan(x
/2)*(a - b)*(2*A*c^3 - 2*C*b^3 + 2*A*b^2*c + 2*C*a*b^2 - 2*C*a*c^2 + C*b*c^
2 - 2*A*a*b*c) - 64*A*a*b^3 + 32*C*a*c^3 - 32*C*b*c^3 + 64*C*b^3*c - 128*C*
a*b^2*c + 64*C*a^2*b*c + (32*(a - b)*(C*b^3 - A*b^2*(b^2 - a^2 + c^2)^(1/2)
 - C*a^2*b - A*c^2*(b^2 - a^2 + c^2)^(1/2) + C*b*c^2 + C*a*c*(b^2 - a^2 + c
^2)^(1/2))*(3*c^4*tan(x/2) + a*c^3 + 3*b*c^3 + 3*b^3*c + 2*a^2*b^2*tan(x/2)
 - 2*a^2*c^2*tan(x/2) + 3*b^2*c^2*tan(x/2) - 2*a*b^3*tan(x/2) + a*b^2*c - 4
*a^2*b*c - 2*a*b*c^2*tan(x/2)))/((b^2 + c^2)*(b^2 - a^2 + c^2))))/((b^2 + c
^2)*(b^2 - a^2 + c^2))))/((b^2 + c^2)*(b^2 - a^2 + c^2)))*(b*(C*a^2 - C*c^2
) - C*b^3 + A*b^2*(b^2 - a^2 + c^2)^(1/2) + A*c^2*(b^2 - a^2 + c^2)^(1/2) -
 C*a*c*(b^2 - a^2 + c^2)^(1/2)))/((b^2 + c^2)*(b^2 - a^2 + c^2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*sin(x))/(a+b*cos(x)+c*sin(x)),x)
```

```
[Out] Timed out
```

3.541 $\int \frac{A+C\sin(x)}{(a+b\cos(x)+c\sin(x))^2}\,dx$

Optimal. Leaf size=114

$$\frac{2(aA-cC)\tan^{-1}\left(\frac{(a-b)\tan\left(\frac{x}{2}\right)+c}{\sqrt{a^2-b^2-c^2}}\right)}{\left(a^2-b^2-c^2\right)^{3/2}}-\frac{-\cos(x)(Ac-aC)+Ab\sin(x)+bC}{\left(a^2-b^2-c^2\right)(a+b\cos(x)+c\sin(x))}$$

```
[Out] 2*(A*a-C*c)*arctan((c+(a-b)*tan(1/2*x))/(a^2-b^2-c^2)^(1/2))/(a^2-b^2-c^2)^
(3/2)+(-b*C+(A*c-C*a)*cos(x)-A*b*sin(x))/(a^2-b^2-c^2)/(a+b*cos(x)+c*sin(x)
)
```

Rubi [A] time = 0.10, antiderivative size = 114, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.210, Rules used = {3154, 3124, 618, 204}

$$\frac{2(aA-cC)\tan^{-1}\left(\frac{(a-b)\tan\left(\frac{x}{2}\right)+c}{\sqrt{a^2-b^2-c^2}}\right)}{\left(a^2-b^2-c^2\right)^{3/2}}-\frac{-\cos(x)(Ac-aC)+Ab\sin(x)+bC}{\left(a^2-b^2-c^2\right)(a+b\cos(x)+c\sin(x))}$$

Antiderivative was successfully verified.

```
[In] Int[(A + C*Sin[x])/(a + b*Cos[x] + c*Sin[x])^2,x]
```

```
[Out] (2*(a*A - c*C)*ArcTan[(c + (a - b)*Tan[x/2])/Sqrt[a^2 - b^2 - c^2]])/(a^2 -
 b^2 - c^2)^(3/2) - (b*C - (A*c - a*C)*Cos[x] + A*b*Sin[x])/((a^2 - b^2 - c
^2)*(a + b*Cos[x] + c*Sin[x]))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 3124

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(-1), x_Symbol] :> Module[{f = FreeFactors[Tan[(d + e*x)/2], x]}, Dist[(2*f
```

```
)/e, Subst[Int[1/(a + b + 2*c*f*x + (a - b)*f^2*x^2), x], x, Tan[(d + e*x)/
2]/f], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0]
```

Rule 3154

```
Int[((A_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])/((a_.) + cos[(d_.) + (e_.)*(x_)
]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)])^2, x_Symbol] :> -Simp[(b*C + (a*C
- c*A)*Cos[d + e*x] + b*A*Sin[d + e*x])/(e*(a^2 - b^2 - c^2)*(a + b*Cos[d +
 e*x] + c*Sin[d + e*x])), x] + Dist[(a*A - c*C)/(a^2 - b^2 - c^2), Int[1/(a
 + b*Cos[d + e*x] + c*Sin[d + e*x]), x], x] /; FreeQ[{a, b, c, d, e, A, C},
 x] && NeQ[a^2 - b^2 - c^2, 0] && NeQ[a*A - c*C, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{A+C\sin(x)}{(a+b\cos(x)+c\sin(x))^2}\,dx &= -\frac{bC-(Ac-aC)\cos(x)+Ab\sin(x)}{\left(a^2-b^2-c^2\right)(a+b\cos(x)+c\sin(x))}+\frac{(aA-cC)\int\frac{1}{a+b\cos(x)+c\sin(x)}\,dx}{a^2-b^2-c^2}\\
&= -\frac{bC-(Ac-aC)\cos(x)+Ab\sin(x)}{\left(a^2-b^2-c^2\right)(a+b\cos(x)+c\sin(x))}+\frac{(2(aA-cC))\,\mathrm{Subst}\left(\int\frac{1}{a+b+2cx+(a-\ldots}\right.}{a^2-b^2-c^2}\\
&= -\frac{bC-(Ac-aC)\cos(x)+Ab\sin(x)}{\left(a^2-b^2-c^2\right)(a+b\cos(x)+c\sin(x))}-\frac{(4(aA-cC))\,\mathrm{Subst}\left(\int\frac{1}{-4\left(a^2-b^2-c^2\right)\ldots}\right.}{a^2-b^2\ldots}\\
&= \frac{2(aA-cC)\tan^{-1}\left(\frac{c+(a-b)\tan\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2-c^2}}\right)}{\left(a^2-b^2-c^2\right)^{3/2}}-\frac{bC-(Ac-aC)\cos(x)+Ab\sin(x)}{\left(a^2-b^2-c^2\right)(a+b\cos(x)+c\sin(x))}
\end{aligned}
$$

Mathematica [A] time = 0.39, size = 123, normalized size = 1.08

$$
\frac{a^2(-C)+\sin(x)\left(A\left(b^2+c^2\right)-acC\right)+aAc+b^2C}{b\left(-a^2+b^2+c^2\right)(a+b\cos(x)+c\sin(x))}+\frac{2(aA-cC)\tanh^{-1}\left(\frac{(a-b)\tan\left(\frac{x}{2}\right)+c}{\sqrt{-a^2+b^2+c^2}}\right)}{\left(-a^2+b^2+c^2\right)^{3/2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + C*Sin[x])/(a + b*Cos[x] + c*Sin[x])^2,x]
```

```
[Out] (2*(a*A - c*C)*ArcTanh[(c + (a - b)*Tan[x/2])/Sqrt[-a^2 + b^2 + c^2]])/(-a^
2 + b^2 + c^2)^(3/2) + (a*A*c - a^2*C + b^2*C + (A*(b^2 + c^2) - a*c*C)*Sin
[x])/(b*(-a^2 + b^2 + c^2)*(a + b*Cos[x] + c*Sin[x]))
```

fricas [B] time = 1.24, size = 1301, normalized size = 11.41

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*sin(x))/(a+b*cos(x)+c*sin(x))^2,x, algorithm="fricas")
```

```
[Out] [1/2*(2*C*a^4*b - 4*C*a^2*b^3 + 2*C*b^5 + 2*C*b*c^4 - 4*(C*a^2*b - C*b^3)*c
^2 - (A*a^2*b^2 - C*a*b^2*c + A*a^2*c^2 - C*a*c^3 + (A*a*b^3 - C*b^3*c + A*
a*b*c^2 - C*b*c^3)*cos(x) + (A*a*b^2*c - C*b^2*c^2 + A*a*c^3 - C*c^4)*sin(x
))*sqrt(-a^2 + b^2 + c^2)*log(-(a^2*b^2 - 2*b^4 - c^4 - (a^2 + 3*b^2)*c^2 -
 (2*a^2*b^2 - b^4 - 2*a^2*c^2 + c^4)*cos(x)^2 - 2*(a*b^3 + a*b*c^2)*cos(x)
- 2*(a*b^2*c + a*c^3 - (b*c^3 - (2*a^2*b - b^3)*c)*cos(x))*sin(x) + 2*(2*a*
b*c*cos(x)^2 - a*b*c + (b^2*c + c^3)*cos(x) - (b^3 + b*c^2 + (a*b^2 - a*c^2
)*cos(x))*sin(x))*sqrt(-a^2 + b^2 + c^2))/(2*a*b*cos(x) + (b^2 - c^2)*cos(x
)^2 + a^2 + c^2 + 2*(b*c*cos(x) + a*c)*sin(x))) + 2*(C*a*c^4 - A*c^5 + (A*a
^2 - 2*A*b^2)*c^3 - (C*a^3 - C*a*b^2)*c^2 + (A*a^2*b^2 - A*b^4)*c)*cos(x) -
 2*(A*a^2*b^3 - A*b^5 + C*a*b*c^3 - A*b*c^4 + (A*a^2*b - 2*A*b^3)*c^2 - (C*
a^3*b - C*a*b^3)*c)*sin(x))/(a^5*b^2 - 2*a^3*b^4 + a*b^6 + a*c^6 - (2*a^3 -
 3*a*b^2)*c^4 + (a^5 - 4*a^3*b^2 + 3*a*b^4)*c^2 + (a^4*b^3 - 2*a^2*b^5 + b^
7 + b*c^6 - (2*a^2*b - 3*b^3)*c^4 + (a^4*b - 4*a^2*b^3 + 3*b^5)*c^2)*cos(x)
 + (c^7 - (2*a^2 - 3*b^2)*c^5 + (a^4 - 4*a^2*b^2 + 3*b^4)*c^3 + (a^4*b^2 -
2*a^2*b^4 + b^6)*c)*sin(x)), (C*a^4*b - 2*C*a^2*b^3 + C*b^5 + C*b*c^4 - 2*(
C*a^2*b - C*b^3)*c^2 + (A*a^2*b^2 - C*a*b^2*c + A*a^2*c^2 - C*a*c^3 + (A*a*
b^3 - C*b^3*c + A*a*b*c^2 - C*b*c^3)*cos(x) + (A*a*b^2*c - C*b^2*c^2 + A*a*
c^3 - C*c^4)*sin(x))*sqrt(a^2 - b^2 - c^2)*arctan(-(a*b*cos(x) + a*c*sin(x)
 + b^2 + c^2)*sqrt(a^2 - b^2 - c^2)/((c^3 - (a^2 - b^2)*c)*cos(x) + (a^2*b
- b^3 - b*c^2)*sin(x))) + (C*a*c^4 - A*c^5 + (A*a^2 - 2*A*b^2)*c^3 - (C*a^3
 - C*a*b^2)*c^2 + (A*a^2*b^2 - A*b^4)*c)*cos(x) - (A*a^2*b^3 - A*b^5 + C*a*
b*c^3 - A*b*c^4 + (A*a^2*b - 2*A*b^3)*c^2 - (C*a^3*b - C*a*b^3)*c)*sin(x))/
(a^5*b^2 - 2*a^3*b^4 + a*b^6 + a*c^6 - (2*a^3 - 3*a*b^2)*c^4 + (a^5 - 4*a^3
*b^2 + 3*a*b^4)*c^2 + (a^4*b^3 - 2*a^2*b^5 + b^7 + b*c^6 - (2*a^2*b - 3*b^3
)*c^4 + (a^4*b - 4*a^2*b^3 + 3*b^5)*c^2)*cos(x) + (c^7 - (2*a^2 - 3*b^2)*c^
5 + (a^4 - 4*a^2*b^2 + 3*b^4)*c^3 + (a^4*b^2 - 2*a^2*b^4 + b^6)*c)*sin(x))]
```

giac [A] time = 0.20, size = 206, normalized size = 1.81

$$-\frac{2\left(\pi\left\lfloor\frac{x}{2\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}\left(-2a+2b\right)+\arctan\left(-\frac{a\tan\left(\frac{1}{2}x\right)-b\tan\left(\frac{1}{2}x\right)+c}{\sqrt{a^2-b^2-c^2}}\right)\right)(Aa-Cc)}{\left(a^2-b^2-c^2\right)^{\frac{3}{2}}}-\frac{2\left(Aab\tan\left(\frac{1}{2}x\right)-Ab^2\tan\left(\frac{1}{2}x\right)+\right.}{\left(a^3-a^2b-ab^2+b^3-ac^2\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*sin(x))/(a+b*cos(x)+c*sin(x))^2,x, algorithm="giac")
```

```
[Out] -2*(pi*floor(1/2*x/pi + 1/2)*sgn(-2*a + 2*b) + arctan(-(a*tan(1/2*x) - b*ta
n(1/2*x) + c)/sqrt(a^2 - b^2 - c^2)))*(A*a - C*c)/(a^2 - b^2 - c^2)^(3/2) -
 2*(A*a*b*tan(1/2*x) - A*b^2*tan(1/2*x) + C*a*c*tan(1/2*x) - C*b*c*tan(1/2*
x) - A*c^2*tan(1/2*x) + C*a^2 - C*b^2 - A*a*c)/((a^3 - a^2*b - a*b^2 + b^3
- a*c^2 + b*c^2)*(a*tan(1/2*x)^2 - b*tan(1/2*x)^2 + 2*c*tan(1/2*x) + a + b)
)
```

maple [B] time = 0.18, size = 255, normalized size = 2.24

$$\frac{-\frac{2\left(aAb-Ab^2-Ac^2+acC-Cbc\right)\tan\left(\frac{x}{2}\right)}{a^3-a^2b-ab^2-ac^2+b^3+c^2b}+\frac{2\left(aAc-a^2C+b^2C\right)}{a^3-a^2b-ab^2-ac^2+b^3+c^2b}}{a\left(\tan^2\left(\frac{x}{2}\right)\right)-b\left(\tan^2\left(\frac{x}{2}\right)\right)+2c\tan\left(\frac{x}{2}\right)+a+b}+\frac{2\arctan\left(\frac{2(a-b)\tan\left(\frac{x}{2}\right)+2c}{2\sqrt{a^2-b^2-c^2}}\right)aA}{\left(a^2-b^2-c^2\right)^{\frac{3}{2}}}-\frac{2\arctan\left(\frac{2(a-b)\tan\left(\frac{x}{2}\right)+2c}{2\sqrt{a^2-b^2-c^2}}\right)}{\left(a^2-b^2-c^2\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+C*sin(x))/(a+b*cos(x)+c*sin(x))^2,x)
```

```
[Out] 2*(-(A*a*b-A*b^2-A*c^2+C*a*c-C*b*c)/(a^3-a^2*b-a*b^2-a*c^2+b^3+b*c^2)*tan(1
/2*x)+(A*a*c-C*a^2+C*b^2)/(a^3-a^2*b-a*b^2-a*c^2+b^3+b*c^2))/(a*tan(1/2*x)^
2-b*tan(1/2*x)^2+2*c*tan(1/2*x)+a+b)+2/(a^2-b^2-c^2)^(3/2)*arctan(1/2*(2*(a
-b)*tan(1/2*x)+2*c)/(a^2-b^2-c^2)^(1/2))*a*A-2/(a^2-b^2-c^2)^(3/2)*arctan(1
/2*(2*(a-b)*tan(1/2*x)+2*c)/(a^2-b^2-c^2)^(1/2))*C*c
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*sin(x))/(a+b*cos(x)+c*sin(x))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(c^2+b^2-a^2>0)', see `assume?` for
more details)Is c^2+b^2-a^2 positive or negative?
```

mupad [B] time = 3.19, size = 204, normalized size = 1.79

$$\frac{2\,\mathrm{atanh}\left(\frac{\tan\left(\frac{x}{2}\right)(2a-2b)+\frac{2\left(-a^2c+b^2c+c^3\right)}{-a^2+b^2+c^2}}{2\sqrt{-a^2+b^2+c^2}}\right)(A\,a-C\,c)}{\left(-a^2+b^2+c^2\right)^{3/2}}-\frac{\frac{2\left(-C\,a^2+A\,c\,a+C\,b^2\right)}{(a-b)\left(-a^2+b^2+c^2\right)}+\frac{2\tan\left(\frac{x}{2}\right)\left(A\,b^2+C\,b\,c-A\,a\,b+A\,c^2-C\,a\,c\right)}{(a-b)\left(-a^2+b^2+c^2\right)}}{(a-b)\,\tan\left(\frac{x}{2}\right)^2+2\,c\tan\left(\frac{x}{2}\right)+a+b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + C*sin(x))/(a + b*cos(x) + c*sin(x))^2,x)
```

```
[Out] (2*atanh((tan(x/2)*(2*a - 2*b) + (2*(b^2*c - a^2*c + c^3))/(b^2 - a^2 + c^2
))/(2*(b^2 - a^2 + c^2)^(1/2)))*(A*a - C*c))/(b^2 - a^2 + c^2)^(3/2) - ((2*
(C*b^2 - C*a^2 + A*a*c))/((a - b)*(b^2 - a^2 + c^2)) + (2*tan(x/2)*(A*b^2 +
 A*c^2 - A*a*b - C*a*c + C*b*c))/((a - b)*(b^2 - a^2 + c^2)))/(a + b + 2*c*
tan(x/2) + tan(x/2)^2*(a - b))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*sin(x))/(a+b*cos(x)+c*sin(x))**2,x)
```

```
[Out] Timed out
```

3.542 $\int \frac{A+C\sin(x)}{(a+b\cos(x)+c\sin(x))^3}\,dx$

Optimal. Leaf size=200

$$\frac{\left(2a^2A-3acC+A\left(b^2+c^2\right)\right)\tan^{-1}\left(\frac{(a-b)\tan\left(\frac{x}{2}\right)+c}{\sqrt{a^2-b^2-c^2}}\right)}{\left(a^2-b^2-c^2\right)^{5/2}}-\frac{-\cos(x)\left(a^2(-C)+3aAc-2c^2C\right)+b\sin(x)(3aA-2cC)+}{2\left(a^2-b^2-c^2\right)^2(a+b\cos(x)+c\sin(x))}$$

```
[Out] (2*a^2*A+A*(b^2+c^2)-3*a*c*C)*arctan((c+(a-b)*tan(1/2*x))/(a^2-b^2-c^2)^(1/
2))/(a^2-b^2-c^2)^(5/2)+1/2*(-b*C+(A*c-C*a)*cos(x)-A*b*sin(x))/(a^2-b^2-c^2
)/(a+b*cos(x)+c*sin(x))^2+1/2*(-a*b*C+(3*A*a*c-C*a^2-2*C*c^2)*cos(x)-b*(3*A
*a-2*C*c)*sin(x))/(a^2-b^2-c^2)^2/(a+b*cos(x)+c*sin(x))
```

Rubi [A] time = 0.25, antiderivative size = 200, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {3157, 3153, 3124, 618, 204}

$$\frac{\left(2a^2A-3acC+A\left(b^2+c^2\right)\right)\tan^{-1}\left(\frac{(a-b)\tan\left(\frac{x}{2}\right)+c}{\sqrt{a^2-b^2-c^2}}\right)}{\left(a^2-b^2-c^2\right)^{5/2}}-\frac{-\cos(x)\left(a^2(-C)+3aAc-2c^2C\right)+b\sin(x)(3aA-2cC)+}{2\left(a^2-b^2-c^2\right)^2(a+b\cos(x)+c\sin(x))}$$

Antiderivative was successfully verified.

```
[In] Int[(A + C*Sin[x])/(a + b*Cos[x] + c*Sin[x])^3,x]
```

```
[Out] ((2*a^2*A + A*(b^2 + c^2) - 3*a*c*C)*ArcTan[(c + (a - b)*Tan[x/2])/Sqrt[a^2
 - b^2 - c^2]])/(a^2 - b^2 - c^2)^(5/2) - (b*C - (A*c - a*C)*Cos[x] + A*b*S
in[x])/(2*(a^2 - b^2 - c^2)*(a + b*Cos[x] + c*Sin[x])^2) - (a*b*C - (3*a*A*
c - a^2*C - 2*c^2*C)*Cos[x] + b*(3*a*A - 2*c*C)*Sin[x])/(2*(a^2 - b^2 - c^2
)^2*(a + b*Cos[x] + c*Sin[x]))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 3124

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(-1), x_Symbol] :> Module[{f = FreeFactors[Tan[(d + e*x)/2], x]}, Dist[(2*f
)/e, Subst[Int[1/(a + b + 2*c*f*x + (a - b)*f^2*x^2), x], x, Tan[(d + e*x)/
2]/f], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0]
```

Rule 3153

```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])
/((a_.) + cos[(d_.) + (e_.)*(x_)]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)])^2,
 x_Symbol] :> Simp[(c*B - b*C - (a*C - c*A)*Cos[d + e*x] + (a*B - b*A)*Sin[
d + e*x])/(e*(a^2 - b^2 - c^2)*(a + b*Cos[d + e*x] + c*Sin[d + e*x])), x] +
 Dist[(a*A - b*B - c*C)/(a^2 - b^2 - c^2), Int[1/(a + b*Cos[d + e*x] + c*Si
n[d + e*x]), x], x] /; FreeQ[{a, b, c, d, e, A, B, C}, x] && NeQ[a^2 - b^2
- c^2, 0] && NeQ[a*A - b*B - c*C, 0]
```

Rule 3157

```
Int[((a_.) + cos[(d_.) + (e_.)*(x_)]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)])
^(n_)*((A_.) + (C_.)*sin[(d_.) + (e_.)*(x_)]), x_Symbol] :> Simp[((b*C + (a
*C - c*A)*Cos[d + e*x] + b*A*Sin[d + e*x])*(a + b*Cos[d + e*x] + c*Sin[d +
e*x])^(n + 1))/(e*(n + 1)*(a^2 - b^2 - c^2)), x] + Dist[1/((n + 1)*(a^2 - b
^2 - c^2)), Int[(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n + 1)*Simp[(n + 1)*
(a*A - c*C) - (n + 2)*b*A*Cos[d + e*x] + (n + 2)*(a*C - c*A)*Sin[d + e*x],
x], x], x] /; FreeQ[{a, b, c, d, e, A, C}, x] && LtQ[n, -1] && NeQ[a^2 - b^
2 - c^2, 0] && NeQ[n, -2]
```

Rubi steps

$$\int \frac{A+C\sin(x)}{(a+b\cos(x)+c\sin(x))^3}\,dx = -\frac{bC-(Ac-aC)\cos(x)+Ab\sin(x)}{2\left(a^2-b^2-c^2\right)(a+b\cos(x)+c\sin(x))^2} - \frac{\int \frac{-2(aA-cC)+Ab\cos(x)+(Ac-aC)\sin(x}{(a+b\cos(x)+c\sin(x))^2}}{2\left(a^2-b^2-c^2\right)}$$

$$= -\frac{bC-(Ac-aC)\cos(x)+Ab\sin(x)}{2\left(a^2-b^2-c^2\right)(a+b\cos(x)+c\sin(x))^2} - \frac{abC-\left(3aAc-a^2C-2c^2C\right)\cos}{2\left(a^2-b^2-c^2\right)^2(a+b}$$

$$= -\frac{bC-(Ac-aC)\cos(x)+Ab\sin(x)}{2\left(a^2-b^2-c^2\right)(a+b\cos(x)+c\sin(x))^2} - \frac{abC-\left(3aAc-a^2C-2c^2C\right)\cos}{2\left(a^2-b^2-c^2\right)^2(a+b}$$

$$= -\frac{bC-(Ac-aC)\cos(x)+Ab\sin(x)}{2\left(a^2-b^2-c^2\right)(a+b\cos(x)+c\sin(x))^2} - \frac{abC-\left(3aAc-a^2C-2c^2C\right)\cos}{2\left(a^2-b^2-c^2\right)^2(a+b}$$

$$= \frac{\left(2a^2A+A\left(b^2+c^2\right)-3acC\right)\tan^{-1}\left(\frac{c+(a-b)\tan\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2-c^2}}\right)}{\left(a^2-b^2-c^2\right)^{5/2}} - \frac{bC-(Ac-aC)\cos(}{2\left(a^2-b^2-c^2\right)(a+b\cos}$$

Mathematica [A] time = 0.93, size = 361, normalized size = 1.80

$$\frac{2a^4C-6a^3Ac+4a^3cC\sin(x)-2bc\cos(x)\left(2a^2A-3acC+A\left(b^2+c^2\right)\right)-c\cos(2x)\left(a^2cC-3aA\left(b^2+c^2\right)+2c\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + C*Sin[x])/(a + b*Cos[x] + c*Sin[x])^3,x]
```

```
[Out] -((((2*a^2*A + A*(b^2 + c^2) - 3*a*c*C)*ArcTanh[(c + (a - b)*Tan[x/2])/Sqrt[
-a^2 + b^2 + c^2]])/(-a^2 + b^2 + c^2)^(5/2)) + (-6*a^3*A*c - 3*a*A*b^2*c -
 3*a*A*c^3 + 2*a^4*C - 4*a^2*b^2*C + 2*b^4*C + 5*a^2*c^2*C + 4*b^2*c^2*C +
2*c^4*C - 2*b*c*(2*a^2*A + A*(b^2 + c^2) - 3*a*c*C)*Cos[x] - c*(-3*a*A*(b^2
 + c^2) + a^2*c*C + 2*c*(b^2 + c^2)*C)*Cos[2*x] - 8*a^2*A*b^2*Sin[x] + 2*A*
b^4*Sin[x] - 12*a^2*A*c^2*Sin[x] + 2*A*b^2*c^2*Sin[x] + 4*a^3*c*C*Sin[x] +
2*a*b^2*c*C*Sin[x] + 8*a*c^3*C*Sin[x] - 3*a*A*b^3*Sin[2*x] - 3*a*A*b*c^2*Si
n[2*x] + a^2*b*c*C*Sin[2*x] + 2*b^3*c*C*Sin[2*x] + 2*b*c^3*C*Sin[2*x])/(4*b
*(-a^2 + b^2 + c^2)^2*(a + b*Cos[x] + c*Sin[x])^2)
```

fricas [B] time = 1.89, size = 3513, normalized size = 17.56

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*sin(x))/(a+b*cos(x)+c*sin(x))^3,x, algorithm="fricas")
```

```
[Out] [1/4*(2*C*a^6*b - 6*C*a^4*b^3 + 6*C*a^2*b^5 - 2*C*b^7 + 6*A*a*b*c^5 - 6*C*b
*c^6 + 2*(4*C*a^2*b - 7*C*b^3)*c^4 - 6*(A*a^3*b - 2*A*a*b^3)*c^3 - 2*(2*C*a
^4*b - 7*C*a^2*b^3 + 5*C*b^5)*c^2 - 4*(3*A*a*b*c^5 - 2*C*b*c^6 + (C*a^2*b -
 4*C*b^3)*c^4 - 3*(A*a^3*b - 2*A*a*b^3)*c^3 + (C*a^4*b + C*a^2*b^3 - 2*C*b^
5)*c^2 - 3*(A*a^3*b^3 - A*a*b^5)*c)*cos(x)^2 - (2*A*a^4*b^2 + A*a^2*b^4 - 3
*C*a^3*b^2*c - 3*C*a*c^5 + A*c^6 + (3*A*a^2 + 2*A*b^2)*c^4 - 3*(C*a^3 + C*a
*b^2)*c^3 + (2*A*a^4 + 4*A*a^2*b^2 + A*b^4)*c^2 + (2*A*a^2*b^4 + A*b^6 - 3*
C*a*b^4*c + A*b^4*c^2 + 3*C*a*c^5 - A*c^6 - (2*A*a^2 + A*b^2)*c^4)*cos(x)^2
 + 2*(2*A*a^3*b^3 + A*a*b^5 - 3*C*a^2*b^3*c - 3*C*a^2*b*c^3 + A*a*b*c^4 + 2
*(A*a^3*b + A*a*b^3)*c^2)*cos(x) - 2*(3*C*a^2*b^2*c^2 + 3*C*a^2*c^4 - A*a*c
^5 - 2*(A*a^3 + A*a*b^2)*c^3 - (2*A*a^3*b^2 + A*a*b^4)*c + (3*C*a*b^3*c^2 +
 3*C*a*b*c^4 - A*b*c^5 - 2*(A*a^2*b + A*b^3)*c^3 - (2*A*a^2*b^3 + A*b^5)*c)
*cos(x))*sin(x))*sqrt(-a^2 + b^2 + c^2)*log(-(a^2*b^2 - 2*b^4 - c^4 - (a^2
+ 3*b^2)*c^2 - (2*a^2*b^2 - b^4 - 2*a^2*c^2 + c^4)*cos(x)^2 - 2*(a*b^3 + a*
b*c^2)*cos(x) - 2*(a*b^2*c + a*c^3 - (b*c^3 - (2*a^2*b - b^3)*c)*cos(x))*si
n(x) + 2*(2*a*b*c*cos(x)^2 - a*b*c + (b^2*c + c^3)*cos(x) - (b^3 + b*c^2 +
(a*b^2 - a*c^2)*cos(x))*sin(x))*sqrt(-a^2 + b^2 + c^2))/(2*a*b*cos(x) + (b^
2 - c^2)*cos(x)^2 + a^2 + c^2 + 2*(b*c*cos(x) + a*c)*sin(x))) - 6*(A*a^3*b^
3 - A*a*b^5)*c + 2*(C*a*c^6 + A*c^7 - (5*A*a^2 - 3*A*b^2)*c^5 + (C*a^3 + 2*
C*a*b^2)*c^4 + (4*A*a^4 - 10*A*a^2*b^2 + 3*A*b^4)*c^3 - (2*C*a^5 - C*a^3*b^
2 - C*a*b^4)*c^2 + (4*A*a^4*b^2 - 5*A*a^2*b^4 + A*b^6)*c)*cos(x) - 2*(4*A*a
^4*b^3 - 5*A*a^2*b^5 + A*b^7 + C*a*b*c^5 + A*b*c^6 - (5*A*a^2*b - 3*A*b^3)*
c^4 + (C*a^3*b + 2*C*a*b^3)*c^3 + (4*A*a^4*b - 10*A*a^2*b^3 + 3*A*b^5)*c^2
- (2*C*a^5*b - C*a^3*b^3 - C*a*b^5)*c + (3*A*a^3*b^4 - 3*A*a*b^6 - 3*A*a*b^
4*c^2 + 3*A*a*c^6 - 2*C*c^7 + (C*a^2 - 2*C*b^2)*c^5 - 3*(A*a^3 - A*a*b^2)*c
^4 + (C*a^4 + 2*C*b^4)*c^3 - (C*a^4*b^2 + C*a^2*b^4 - 2*C*b^6)*c)*cos(x))*s
in(x))/(a^8*b^2 - 3*a^6*b^4 + 3*a^4*b^6 - a^2*b^8 - c^10 + 2*(a^2 - 2*b^2)*
c^8 + (5*a^2*b^2 - 6*b^4)*c^6 - (2*a^6 - 3*a^4*b^2 - 3*a^2*b^4 + 4*b^6)*c^4
 + (a^8 - 5*a^6*b^2 + 6*a^4*b^4 - a^2*b^6 - b^8)*c^2 + (a^6*b^4 - 3*a^4*b^6
 + 3*a^2*b^8 - b^10 + c^10 - 3*(a^2 - b^2)*c^8 + (3*a^4 - 6*a^2*b^2 + 2*b^4
)*c^6 - (a^6 - 3*a^4*b^2 + 2*b^6)*c^4 - 3*(a^4*b^4 - 2*a^2*b^6 + b^8)*c^2)*
cos(x)^2 + 2*(a^7*b^3 - 3*a^5*b^5 + 3*a^3*b^7 - a*b^9 - a*b*c^8 + (3*a^3*b
- 4*a*b^3)*c^6 - 3*(a^5*b - 3*a^3*b^3 + 2*a*b^5)*c^4 + (a^7*b - 6*a^5*b^3 +
 9*a^3*b^5 - 4*a*b^7)*c^2)*cos(x) - 2*(a*c^9 - (3*a^3 - 4*a*b^2)*c^7 + 3*(a
^5 - 3*a^3*b^2 + 2*a*b^4)*c^5 - (a^7 - 6*a^5*b^2 + 9*a^3*b^4 - 4*a*b^6)*c^3
 - (a^7*b^2 - 3*a^5*b^4 + 3*a^3*b^6 - a*b^8)*c + (b*c^9 - (3*a^2*b - 4*b^3)
*c^7 + 3*(a^4*b - 3*a^2*b^3 + 2*b^5)*c^5 - (a^6*b - 6*a^4*b^3 + 9*a^2*b^5 -
 4*b^7)*c^3 - (a^6*b^3 - 3*a^4*b^5 + 3*a^2*b^7 - b^9)*c)*cos(x))*sin(x)), 1
/2*(C*a^6*b - 3*C*a^4*b^3 + 3*C*a^2*b^5 - C*b^7 + 3*A*a*b*c^5 - 3*C*b*c^6 +
 (4*C*a^2*b - 7*C*b^3)*c^4 - 3*(A*a^3*b - 2*A*a*b^3)*c^3 - (2*C*a^4*b - 7*C
*a^2*b^3 + 5*C*b^5)*c^2 - 2*(3*A*a*b*c^5 - 2*C*b*c^6 + (C*a^2*b - 4*C*b^3)*
```

```
c^4 - 3*(A*a^3*b - 2*A*a*b^3)*c^3 + (C*a^4*b + C*a^2*b^3 - 2*C*b^5)*c^2 - 3
*(A*a^3*b^3 - A*a*b^5)*c)*cos(x)^2 + (2*A*a^4*b^2 + A*a^2*b^4 - 3*C*a^3*b^2
*c - 3*C*a*c^5 + A*c^6 + (3*A*a^2 + 2*A*b^2)*c^4 - 3*(C*a^3 + C*a*b^2)*c^3
+ (2*A*a^4 + 4*A*a^2*b^2 + A*b^4)*c^2 + (2*A*a^2*b^4 + A*b^6 - 3*C*a*b^4*c
+ A*b^4*c^2 + 3*C*a*c^5 - A*c^6 - (2*A*a^2 + A*b^2)*c^4)*cos(x)^2 + 2*(2*A*
a^3*b^3 + A*a*b^5 - 3*C*a^2*b^3*c - 3*C*a^2*b*c^3 + A*a*b*c^4 + 2*(A*a^3*b
+ A*a*b^3)*c^2)*cos(x) - 2*(3*C*a^2*b^2*c^2 + 3*C*a^2*c^4 - A*a*c^5 - 2*(A*
a^3 + A*a*b^2)*c^3 - (2*A*a^3*b^2 + A*a*b^4)*c + (3*C*a*b^3*c^2 + 3*C*a*b*c
^4 - A*b*c^5 - 2*(A*a^2*b + A*b^3)*c^3 - (2*A*a^2*b^3 + A*b^5)*c)*cos(x))*s
in(x))*sqrt(a^2 - b^2 - c^2)*arctan(-(a*b*cos(x) + a*c*sin(x) + b^2 + c^2)*
sqrt(a^2 - b^2 - c^2)/((c^3 - (a^2 - b^2)*c)*cos(x) + (a^2*b - b^3 - b*c^2)
*sin(x))) - 3*(A*a^3*b^3 - A*a*b^5)*c + (C*a*c^6 + A*c^7 - (5*A*a^2 - 3*A*b
^2)*c^5 + (C*a^3 + 2*C*a*b^2)*c^4 + (4*A*a^4 - 10*A*a^2*b^2 + 3*A*b^4)*c^3
- (2*C*a^5 - C*a^3*b^2 - C*a*b^4)*c^2 + (4*A*a^4*b^2 - 5*A*a^2*b^4 + A*b^6)
*c)*cos(x) - (4*A*a^4*b^3 - 5*A*a^2*b^5 + A*b^7 + C*a*b*c^5 + A*b*c^6 - (5*
A*a^2*b - 3*A*b^3)*c^4 + (C*a^3*b + 2*C*a*b^3)*c^3 + (4*A*a^4*b - 10*A*a^2*
b^3 + 3*A*b^5)*c^2 - (2*C*a^5*b - C*a^3*b^3 - C*a*b^5)*c + (3*A*a^3*b^4 - 3
*A*a*b^6 - 3*A*a*b^4*c^2 + 3*A*a*c^6 - 2*C*c^7 + (C*a^2 - 2*C*b^2)*c^5 - 3*
(A*a^3 - A*a*b^2)*c^4 + (C*a^4 + 2*C*b^4)*c^3 - (C*a^4*b^2 + C*a^2*b^4 - 2*
C*b^6)*c)*cos(x))*sin(x))/(a^8*b^2 - 3*a^6*b^4 + 3*a^4*b^6 - a^2*b^8 - c^10
 + 2*(a^2 - 2*b^2)*c^8 + (5*a^2*b^2 - 6*b^4)*c^6 - (2*a^6 - 3*a^4*b^2 - 3*a
^2*b^4 + 4*b^6)*c^4 + (a^8 - 5*a^6*b^2 + 6*a^4*b^4 - a^2*b^6 - b^8)*c^2 + (
a^6*b^4 - 3*a^4*b^6 + 3*a^2*b^8 - b^10 + c^10 - 3*(a^2 - b^2)*c^8 + (3*a^4
- 6*a^2*b^2 + 2*b^4)*c^6 - (a^6 - 3*a^4*b^2 + 2*b^6)*c^4 - 3*(a^4*b^4 - 2*a
^2*b^6 + b^8)*c^2)*cos(x)^2 + 2*(a^7*b^3 - 3*a^5*b^5 + 3*a^3*b^7 - a*b^9 -
a*b*c^8 + (3*a^3*b - 4*a*b^3)*c^6 - 3*(a^5*b - 3*a^3*b^3 + 2*a*b^5)*c^4 + (
a^7*b - 6*a^5*b^3 + 9*a^3*b^5 - 4*a*b^7)*c^2)*cos(x) - 2*(a*c^9 - (3*a^3 -
4*a*b^2)*c^7 + 3*(a^5 - 3*a^3*b^2 + 2*a*b^4)*c^5 - (a^7 - 6*a^5*b^2 + 9*a^3
*b^4 - 4*a*b^6)*c^3 - (a^7*b^2 - 3*a^5*b^4 + 3*a^3*b^6 - a*b^8)*c + (b*c^9
- (3*a^2*b - 4*b^3)*c^7 + 3*(a^4*b - 3*a^2*b^3 + 2*b^5)*c^5 - (a^6*b - 6*a^
4*b^3 + 9*a^2*b^5 - 4*b^7)*c^3 - (a^6*b^3 - 3*a^4*b^5 + 3*a^2*b^7 - b^9)*c)
*cos(x))*sin(x))]
```

giac [B] time = 0.52, size = 1054, normalized size = 5.27

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*sin(x))/(a+b*cos(x)+c*sin(x))^3,x, algorithm="giac")
```

```
[Out] -(2*A*a^2 + A*b^2 - 3*C*a*c + A*c^2)*(pi*floor(1/2*x/pi + 1/2)*sgn(-2*a + 2
*b) + arctan(-(a*tan(1/2*x) - b*tan(1/2*x) + c)/sqrt(a^2 - b^2 - c^2)))/((a
^4 - 2*a^2*b^2 + b^4 - 2*a^2*c^2 + 2*b^2*c^2 + c^4)*sqrt(a^2 - b^2 - c^2))
- (4*A*a^4*b*tan(1/2*x)^3 - 11*A*a^3*b^2*tan(1/2*x)^3 + 9*A*a^2*b^3*tan(1/2
*x)^3 - A*a*b^4*tan(1/2*x)^3 - A*b^5*tan(1/2*x)^3 + 3*C*a^4*c*tan(1/2*x)^3
```

```
- 9*C*a^3*b*c*tan(1/2*x)^3 + 9*C*a^2*b^2*c*tan(1/2*x)^3 - 3*C*a*b^3*c*tan(1
/2*x)^3 - 5*A*a^3*c^2*tan(1/2*x)^3 + 7*A*a^2*b*c^2*tan(1/2*x)^3 + A*a*b^2*c
^2*tan(1/2*x)^3 - 3*A*b^3*c^2*tan(1/2*x)^3 + 2*A*a*c^4*tan(1/2*x)^3 - 2*A*b
*c^4*tan(1/2*x)^3 + 2*C*a^5*tan(1/2*x)^2 - 2*C*a^4*b*tan(1/2*x)^2 - 4*C*a^3
*b^2*tan(1/2*x)^2 + 4*C*a^2*b^3*tan(1/2*x)^2 + 2*C*a*b^4*tan(1/2*x)^2 - 2*C
*b^5*tan(1/2*x)^2 - 4*A*a^4*c*tan(1/2*x)^2 + 12*A*a^3*b*c*tan(1/2*x)^2 - 13
*A*a^2*b^2*c*tan(1/2*x)^2 + 6*A*a*b^3*c*tan(1/2*x)^2 - A*b^4*c*tan(1/2*x)^2
 + 5*C*a^3*c^2*tan(1/2*x)^2 - 14*C*a^2*b*c^2*tan(1/2*x)^2 + 13*C*a*b^2*c^2*
tan(1/2*x)^2 - 4*C*b^3*c^2*tan(1/2*x)^2 - 7*A*a^2*c^3*tan(1/2*x)^2 + 6*A*a*
b*c^3*tan(1/2*x)^2 + A*b^2*c^3*tan(1/2*x)^2 + 2*C*a*c^4*tan(1/2*x)^2 - 2*C*
b*c^4*tan(1/2*x)^2 + 2*A*c^5*tan(1/2*x)^2 + 4*A*a^4*b*tan(1/2*x) - 5*A*a^3*
b^2*tan(1/2*x) - 3*A*a^2*b^3*tan(1/2*x) + 5*A*a*b^4*tan(1/2*x) - A*b^5*tan(
1/2*x) + 5*C*a^4*c*tan(1/2*x) - 5*C*a^3*b*c*tan(1/2*x) - 5*C*a^2*b^2*c*tan(
1/2*x) + 5*C*a*b^3*c*tan(1/2*x) - 11*A*a^3*c^2*tan(1/2*x) + 3*A*a^2*b*c^2*t
an(1/2*x) + 7*A*a*b^2*c^2*tan(1/2*x) + A*b^3*c^2*tan(1/2*x) + 4*C*a^2*c^3*t
an(1/2*x) - 4*C*a*b*c^3*tan(1/2*x) + 2*A*a*c^4*tan(1/2*x) + 2*A*b*c^4*tan(1
/2*x) + 2*C*a^5 - 4*C*a^3*b^2 + 2*C*a*b^4 - 4*A*a^4*c + 3*A*a^2*b^2*c + A*b
^4*c + C*a^3*c^2 - C*a*b^2*c^2 + A*a^2*c^3 + A*b^2*c^3)/((a^6 - 2*a^5*b - a
^4*b^2 + 4*a^3*b^3 - a^2*b^4 - 2*a*b^5 + b^6 - 2*a^4*c^2 + 4*a^3*b*c^2 - 4*
a*b^3*c^2 + 2*b^4*c^2 + a^2*c^4 - 2*a*b*c^4 + b^2*c^4)*(a*tan(1/2*x)^2 - b*
tan(1/2*x)^2 + 2*c*tan(1/2*x) + a + b)^2)
```

maple [B] time = 0.22, size = 1088, normalized size = 5.44

$$-\frac{\left(4A\,a^{3}b-7A\,a^{2}b^{2}-5A\,a^{2}c^{2}+2Aa\,b^{3}+2Aab\,c^{2}+A\,b^{4}+3A\,b^{2}c^{2}+2A\,c^{4}+3C\,a^{3}c-6C\,a^{2}bc+3Ca\,b^{2}c\right)\left(\tan^{3}\left(\frac{x}{2}\right)\right)}{\left(a^{4}-2a^{2}b^{2}-2a^{2}c^{2}+b^{4}+2b^{2}c^{2}+c^{4}\right)(a-b)}+\frac{\left(4A\,a^{4}c-12A\,a^{3}bc+13A\,a^{2}b^{2}c+7A\,a^{2}c^{3}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+C*sin(x))/(a+b*cos(x)+c*sin(x))^3,x)
```

```
[Out] 2*(-1/2*(4*A*a^3*b-7*A*a^2*b^2-5*A*a^2*c^2+2*A*a*b^3+2*A*a*b*c^2+A*b^4+3*A*
b^2*c^2+2*A*c^4+3*C*a^3*c-6*C*a^2*b*c+3*C*a*b^2*c)/(a^4-2*a^2*b^2-2*a^2*c^2
+b^4+2*b^2*c^2+c^4)/(a-b)*tan(1/2*x)^3+1/2*(4*A*a^4*c-12*A*a^3*b*c+13*A*a^2
*b^2*c+7*A*a^2*c^3-6*A*a*b^3*c-6*A*a*b*c^3+A*b^4*c-A*b^2*c^3-2*A*c^5-2*C*a^
5+2*C*a^4*b+4*C*a^3*b^2-5*C*a^3*c^2-4*C*a^2*b^3+14*C*a^2*b*c^2-2*C*a*b^4-13
*C*a*b^2*c^2-2*C*a*c^4+2*C*b^5+4*C*b^3*c^2+2*C*b*c^4)/(a^4-2*a^2*b^2-2*a^2*
c^2+b^4+2*b^2*c^2+c^4)/(a^2-2*a*b+b^2)*tan(1/2*x)^2-1/2*(4*A*a^4*b-5*A*a^3*
b^2-11*A*a^3*c^2-3*A*a^2*b^3+3*A*a^2*b*c^2+5*A*a*b^4+7*A*a*b^2*c^2+2*A*a*c^
4-A*b^5+A*b^3*c^2+2*A*b*c^4+5*C*a^4*c-5*C*a^3*b*c-5*C*a^2*b^2*c+4*C*a^2*c^3
+5*C*a*b^3*c-4*C*a*b*c^3)/(a^4-2*a^2*b^2-2*a^2*c^2+b^4+2*b^2*c^2+c^4)/(a^2-
2*a*b+b^2)*tan(1/2*x)+1/2*(4*A*a^4*c-3*A*a^2*b^2*c-A*a^2*c^3-A*b^4*c-A*b^2*
c^3-2*C*a^5+4*C*a^3*b^2-C*a^3*c^2-2*C*a*b^4+C*a*b^2*c^2)/(a^4-2*a^2*b^2-2*a
^2*c^2+b^4+2*b^2*c^2+c^4)/(a^2-2*a*b+b^2))/(a*tan(1/2*x)^2-b*tan(1/2*x)^2+2
```

```
*c*tan(1/2*x)+a+b)^2+2/(a^4-2*a^2*b^2-2*a^2*c^2+b^4+2*b^2*c^2+c^4)/(a^2-b^2
-c^2)^(1/2)*arctan(1/2*(2*(a-b)*tan(1/2*x)+2*c)/(a^2-b^2-c^2)^(1/2))*a^2*A+
1/(a^4-2*a^2*b^2-2*a^2*c^2+b^4+2*b^2*c^2+c^4)/(a^2-b^2-c^2)^(1/2)*arctan(1/
2*(2*(a-b)*tan(1/2*x)+2*c)/(a^2-b^2-c^2)^(1/2))*A*b^2+1/(a^4-2*a^2*b^2-2*a^
2*c^2+b^4+2*b^2*c^2+c^4)/(a^2-b^2-c^2)^(1/2)*arctan(1/2*(2*(a-b)*tan(1/2*x)
+2*c)/(a^2-b^2-c^2)^(1/2))*A*c^2-3/(a^4-2*a^2*b^2-2*a^2*c^2+b^4+2*b^2*c^2+c
^4)/(a^2-b^2-c^2)^(1/2)*arctan(1/2*(2*(a-b)*tan(1/2*x)+2*c)/(a^2-b^2-c^2)^(
1/2))*a*c*C
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*sin(x))/(a+b*cos(x)+c*sin(x))^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(c^2+b^2-a^2>0)', see `assume?` for
more details)Is c^2+b^2-a^2 positive or negative?
```

mupad [B] time = 6.73, size = 912, normalized size = 4.56

$$-\frac{\frac{2\,C\,a^5-4\,A\,a^4\,c-4\,C\,a^3\,b^2+C\,a^3\,c^2+3\,A\,a^2\,b^2\,c+A\,a^2\,c^3+2\,C\,a\,b^4-C\,a\,b^2\,c^2+A\,b^4\,c+A\,b^2\,c^3}{{\left(a-b\right)}^2\,\left(a^4-2\,a^2\,b^2-2\,a^2\,c^2+b^4+2\,b^2\,c^2+c^4\right)}+\frac{\mathrm{tan}\left(\frac{x}{2}\right)\,\left(4\,A\,a^4\,b+5\,C\,a^4\,c-5\,A\,a^3\,b^2-5\,C\,a^3\,b\,c-11\,A\,a\dots\right.}{\dots}}{\dots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + C*sin(x))/(a + b*cos(x) + c*sin(x))^3,x)
```

```
[Out] - ((2*C*a^5 + A*a^2*c^3 + A*b^2*c^3 - 4*C*a^3*b^2 + C*a^3*c^2 - 4*A*a^4*c +
 A*b^4*c + 2*C*a*b^4 + 3*A*a^2*b^2*c - C*a*b^2*c^2)/((a - b)^2*(a^4 + b^4 +
 c^4 - 2*a^2*b^2 - 2*a^2*c^2 + 2*b^2*c^2)) + (tan(x/2)*(A*b^3*c^2 - 3*A*a^2
*b^3 - 5*A*a^3*b^2 - 11*A*a^3*c^2 - A*b^5 + 4*C*a^2*c^3 + 5*A*a*b^4 + 4*A*a
^4*b + 2*A*a*c^4 + 2*A*b*c^4 + 5*C*a^4*c - 4*C*a*b*c^3 + 5*C*a*b^3*c - 5*C*
a^3*b*c + 7*A*a*b^2*c^2 + 3*A*a^2*b*c^2 - 5*C*a^2*b^2*c))/((a - b)^2*(a^4 +
 b^4 + c^4 - 2*a^2*b^2 - 2*a^2*c^2 + 2*b^2*c^2)) + (tan(x/2)^2*(2*A*c^5 + 2
*C*a^5 - 2*C*b^5 - 7*A*a^2*c^3 + A*b^2*c^3 + 4*C*a^2*b^3 - 4*C*a^3*b^2 + 5*
C*a^3*c^2 - 4*C*b^3*c^2 - 4*A*a^4*c - A*b^4*c + 2*C*a*b^4 - 2*C*a^4*b + 2*C
*a*c^4 - 2*C*b*c^4 + 6*A*a*b*c^3 + 6*A*a*b^3*c + 12*A*a^3*b*c - 13*A*a^2*b^
2*c + 13*C*a*b^2*c^2 - 14*C*a^2*b*c^2))/((a - b)^2*(a^4 + b^4 + c^4 - 2*a^2
*b^2 - 2*a^2*c^2 + 2*b^2*c^2)) + (tan(x/2)^3*(A*b^4 + 2*A*c^4 - 7*A*a^2*b^2
 - 5*A*a^2*c^2 + 3*A*b^2*c^2 + 2*A*a*b^3 + 4*A*a^3*b + 3*C*a^3*c + 2*A*a*b*
```

```
c^2 + 3*C*a*b^2*c - 6*C*a^2*b*c))/((a - b)*(a^4 + b^4 + c^4 - 2*a^2*b^2 - 2
*a^2*c^2 + 2*b^2*c^2)))/(tan(x/2)^4*(a^2 - 2*a*b + b^2) + 2*a*b + tan(x/2)*
(4*a*c + 4*b*c) + tan(x/2)^3*(4*a*c - 4*b*c) + a^2 + b^2 + tan(x/2)^2*(2*a^
2 - 2*b^2 + 4*c^2)) - (atanh((2*a^4*c + 2*b^4*c + 2*c^5 - 4*a^2*c^3 + 4*b^2
*c^3 - 4*a^2*b^2*c)/(2*(b^2 - a^2 + c^2)^(5/2)) + (tan(x/2)*(2*a - 2*b)*(a^
4 + b^4 + c^4 - 2*a^2*b^2 - 2*a^2*c^2 + 2*b^2*c^2))/(2*(b^2 - a^2 + c^2)^(5
/2)))*(2*A*a^2 + A*b^2 + A*c^2 - 3*C*a*c))/(b^2 - a^2 + c^2)^(5/2)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*sin(x))/(a+b*cos(x)+c*sin(x))**3,x)
```

```
[Out] Timed out
```

3.543 $\int \frac{A+C\sin(x)}{a+b\cos(x)+ib\sin(x)}\,dx$

Optimal. Leaf size=85

$$\frac{\left(a^2(-C)+2iaAb+b^2C\right)\log(a+ib\sin(x)+b\cos(x))}{2a^2b}+\frac{x(2aA-ibC)}{2a^2}+\frac{iC\sin(x)}{2a}-\frac{C\cos(x)}{2a}$$

```
[Out] 1/2*(2*a*A-I*b*C)*x/a^2-1/2*C*cos(x)/a+1/2*(2*I*a*A*b-a^2*C+b^2*C)*ln(a+b*c
os(x)+I*b*sin(x))/a^2/b+1/2*I*C*sin(x)/a
```

Rubi [A] time = 0.05, antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {3131}

$$\frac{\left(a^2(-C)+2iaAb+b^2C\right)\log(a+ib\sin(x)+b\cos(x))}{2a^2b}+\frac{x(2aA-ibC)}{2a^2}+\frac{iC\sin(x)}{2a}-\frac{C\cos(x)}{2a}$$

Antiderivative was successfully verified.

```
[In] Int[(A + C*Sin[x])/(a + b*Cos[x] + I*b*Sin[x]),x]
```

```
[Out] ((2*a*A - I*b*C)*x)/(2*a^2) - (C*Cos[x])/(2*a) + (((2*I)*a*A*b - a^2*C + b^
2*C)*Log[a + b*Cos[x] + I*b*Sin[x]])/(2*a^2*b) + ((I/2)*C*Sin[x])/a
```

Rule 3131

```
Int[((A_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])/(cos[(d_.) + (e_.)*(x_)]*(b_.)
+ (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)]), x_Symbol] :> Simp[((2*a*A - c*C)*x
)/(2*a^2), x] + (-Simp[(C*Cos[d + e*x])/(2*a*e), x] + Simp[(c*C*Sin[d + e*x
])/(2*a*b*e), x] + Simp[((-(a^2*C) + 2*a*c*A + b^2*C)*Log[RemoveContent[a +
 b*Cos[d + e*x] + c*Sin[d + e*x], x]])/(2*a^2*b*e), x]) /; FreeQ[{a, b, c,
d, e, A, C}, x] && EqQ[b^2 + c^2, 0]
```

Rubi steps

$$\int \frac{A+C\sin(x)}{a+b\cos(x)+ib\sin(x)}\,dx=\frac{(2aA-ibC)x}{2a^2}-\frac{C\cos(x)}{2a}+\frac{\left(2iaAb-a^2C+b^2C\right)\log(a+b\cos(x)+ib\sin(x))}{2a^2b}$$

Mathematica [A] time = 0.29, size = 152, normalized size = 1.79

$$\frac{\left(-2ia^2C-4aAb+2ib^2C\right)\tan^{-1}\left(\frac{(a+b)\cot\left(\frac{x}{2}\right)}{a-b}\right)+2iaAb\log\left(a^2+2ab\cos(x)+b^2\right)-a^2C\log\left(a^2+2ab\cos(x)+b\right.}{4a^2b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + C*Sin[x])/(a + b*Cos[x] + I*b*Sin[x]),x]
```

```
[Out] (2*a*A*b*x - I*a^2*C*x - I*b^2*C*x + (-4*a*A*b - (2*I)*a^2*C + (2*I)*b^2*C)
*ArcTan[((a + b)*Cot[x/2])/(a - b)] - 2*a*b*C*Cos[x] + (2*I)*a*A*b*Log[a^2
+ b^2 + 2*a*b*Cos[x]] - a^2*C*Log[a^2 + b^2 + 2*a*b*Cos[x]] + b^2*C*Log[a^2
 + b^2 + 2*a*b*Cos[x]] + (2*I)*a*b*C*Sin[x])/(4*a^2*b)
```

fricas [A] time = 0.97, size = 71, normalized size = 0.84

$$-\frac{\left(Cab-\left(2\,Aab-i\,Cb^{2}\right)xe^{(i\,x)}+\left(Ca^{2}-2i\,Aab-Cb^{2}\right)e^{(i\,x)}\log\left(\frac{be^{(i\,x)}+a}{b}\right)\right)e^{(-i\,x)}}{2\,a^{2}b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*sin(x))/(a+b*cos(x)+I*b*sin(x)),x, algorithm="fricas")
```

```
[Out] -1/2*(C*a*b - (2*A*a*b - I*C*b^2)*x*e^(I*x) + (C*a^2 - 2*I*A*a*b - C*b^2)*e
^(I*x)*log((b*e^(I*x) + a)/b))*e^(-I*x)/(a^2*b)
```

giac [B] time = 0.16, size = 169, normalized size = 1.99

$$-\frac{(-2i\,Aa-Cb)\log\left(-a\tan\left(\frac{1}{2}\,x\right)^{2}+b\tan\left(\frac{1}{2}\,x\right)^{2}-2i\,a\tan\left(\frac{1}{2}\,x\right)+a+b\right)}{4\,a^{2}}-\frac{(2i\,Aa+Cb)\log\left(\tan\left(\frac{1}{2}\,x\right)-i\right)}{2\,a^{2}}-\frac{(2i}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*sin(x))/(a+b*cos(x)+I*b*sin(x)),x, algorithm="giac")
```

```
[Out] -1/4*(-2*I*A*a - C*b)*log(-a*tan(1/2*x)^2 + b*tan(1/2*x)^2 - 2*I*a*tan(1/2*
x) + a + b)/a^2 - 1/2*(2*I*A*a + C*b)*log(tan(1/2*x) - I)/a^2 - 1/4*(2*I*C*
a^2 + 2*A*a*b - I*C*b^2)*(x + 2*arctan((-I*a*cos(x) - a*sin(x) - I*a)/(a*co
s(x) - I*a*sin(x) - a + 2*b)))/(a^2*b) - 1/2*(-2*I*A*a*tan(1/2*x) - C*b*tan
(1/2*x) - 2*A*a - 2*I*C*a + I*C*b)/(a^2*(tan(1/2*x) - I))
```

maple [B] time = 0.18, size = 151, normalized size = 1.78

$$\frac{C\ln\left(\tan\left(\frac{x}{2}\right)+i\right)}{2b}+\frac{iC}{a\left(\tan\left(\frac{x}{2}\right)-i\right)}-\frac{i\ln\left(\tan\left(\frac{x}{2}\right)-i\right)A}{a}-\frac{\ln\left(\tan\left(\frac{x}{2}\right)-i\right)bC}{2a^{2}}-\frac{\ln\left(ia+ib+a\tan\left(\frac{x}{2}\right)-b\tan\left(\frac{x}{2}\right)\right)C}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+C*sin(x))/(a+b*cos(x)+I*b*sin(x)),x)
```

```
[Out] 1/2*C/b*ln(tan(1/2*x)+I)+I*C/a/(tan(1/2*x)-I)-I/a*ln(tan(1/2*x)-I)*A-1/2/a^
2*ln(tan(1/2*x)-I)*b*C-1/2/b*ln(I*a+I*b+a*tan(1/2*x)-b*tan(1/2*x))*C+1/2/a^
2*b*ln(I*a+I*b+a*tan(1/2*x)-b*tan(1/2*x))*C+I/a*ln(I*a+I*b+a*tan(1/2*x)-b*t
an(1/2*x))*A
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*sin(x))/(a+b*cos(x)+I*b*sin(x)),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

mupad [B] time = 4.35, size = 96, normalized size = 1.13

$$\ln\left(a+b-a\tan\left(\frac{x}{2}\right)\,1\mathrm{i}+b\tan\left(\frac{x}{2}\right)\,1\mathrm{i}\right)\left(\frac{C\,b}{2\,a^2}-\frac{C}{2\,b}+\frac{A\,1\mathrm{i}}{a}\right)+\frac{C\,1\mathrm{i}}{a\,\left(\tan\left(\frac{x}{2}\right)-\mathrm{i}\right)}+\frac{C\,\ln\left(\tan\left(\frac{x}{2}\right)+1\mathrm{i}\right)}{2\,b}-\frac{\ln\left(\tan\left(\frac{x}{2}\right)\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + C*sin(x))/(a + b*cos(x) + b*sin(x)*1i),x)
```

```
[Out] log(a + b - a*tan(x/2)*1i + b*tan(x/2)*1i)*((A*1i)/a - C/(2*b) + (C*b)/(2*a
^2)) + (C*1i)/(a*(tan(x/2) - 1i)) + (C*log(tan(x/2) + 1i))/(2*b) - (log(tan
(x/2) - 1i)*(A*a*1i + (C*b)/2))/a^2
```

sympy [A] time = 0.80, size = 104, normalized size = 1.22

$$\begin{cases} -\frac{Ce^{-ix}}{2a} & \text{for}\: 2a \neq 0 \\ x\left(-\frac{2Aa-iCb}{2a^2}-\frac{i(2iAa-Ca+Cb)}{2a^2}\right) & \text{otherwise} \end{cases} - \frac{x\left(-2Aa+iCb\right)}{2a^2} - \frac{\left(-2iAab+Ca^2-Cb^2\right)\log\left(\frac{a}{b}+e^{ix}\right)}{2a^2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*sin(x))/(a+b*cos(x)+I*b*sin(x)),x)
```

```
[Out] Piecewise((-C*exp(-I*x)/(2*a), Ne(2*a, 0)), (x*(-(2*A*a - I*C*b)/(2*a**2) -
 I*(2*I*A*a - C*a + C*b)/(2*a**2)), True)) - x*(-2*A*a + I*C*b)/(2*a**2) -
(-2*I*A*a*b + C*a**2 - C*b**2)*log(a/b + exp(I*x))/(2*a**2*b)
```

3.544 $\int \frac{A+C\sin(x)}{a+b\cos(x)-ib\sin(x)}\,dx$

Optimal. Leaf size=85

$$-\frac{\left(a^2C+2iaAb-b^2C\right)\log(a-ib\sin(x)+b\cos(x))}{2a^2b}+\frac{x(2aA+ibC)}{2a^2}-\frac{iC\sin(x)}{2a}-\frac{C\cos(x)}{2a}$$

```
[Out] 1/2*(2*a*A+I*b*C)*x/a^2-1/2*C*cos(x)/a-1/2*(2*I*a*A*b+a^2*C-b^2*C)*ln(a+b*c
os(x)-I*b*sin(x))/a^2/b-1/2*I*C*sin(x)/a
```

Rubi [A] time = 0.05, antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {3131}

$$-\frac{\left(a^2C+2iaAb-b^2C\right)\log(a-ib\sin(x)+b\cos(x))}{2a^2b}+\frac{x(2aA+ibC)}{2a^2}-\frac{iC\sin(x)}{2a}-\frac{C\cos(x)}{2a}$$

Antiderivative was successfully verified.

```
[In] Int[(A + C*Sin[x])/(a + b*Cos[x] - I*b*Sin[x]),x]
```

```
[Out] ((2*a*A + I*b*C)*x)/(2*a^2) - (C*Cos[x])/(2*a) - (((2*I)*a*A*b + a^2*C - b^
2*C)*Log[a + b*Cos[x] - I*b*Sin[x]])/(2*a^2*b) - ((I/2)*C*Sin[x])/a
```

Rule 3131

```
Int[((A_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])/(cos[(d_.) + (e_.)*(x_)]*(b_.)
+ (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)]), x_Symbol] :> Simp[((2*a*A - c*C)*x
)/(2*a^2), x] + (-Simp[(C*Cos[d + e*x])/(2*a*e), x] + Simp[(c*C*Sin[d + e*x
])/(2*a*b*e), x] + Simp[((-(a^2*C) + 2*a*c*A + b^2*C)*Log[RemoveContent[a +
 b*Cos[d + e*x] + c*Sin[d + e*x], x]])/(2*a^2*b*e), x]) /; FreeQ[{a, b, c,
d, e, A, C}, x] && EqQ[b^2 + c^2, 0]
```

Rubi steps

$$\int \frac{A+C\sin(x)}{a+b\cos(x)-ib\sin(x)}\,dx=\frac{(2aA+ibC)x}{2a^2}-\frac{C\cos(x)}{2a}-\frac{\left(2iaAb+a^2C-b^2C\right)\log(a+b\cos(x)-ib\sin(x))}{2a^2b}-$$

Mathematica [A] time = 0.26, size = 152, normalized size = 1.79

$$\frac{2i\left(a^2C+2iaAb-b^2C\right)\tan^{-1}\left(\frac{(a+b)\cot\left(\frac{x}{2}\right)}{a-b}\right)-2iaAb\log\left(a^2+2ab\cos(x)+b^2\right)-a^2C\log\left(a^2+2ab\cos(x)+b^2\right)+}{4a^2b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + C*Sin[x])/(a + b*Cos[x] - I*b*Sin[x]),x]
```

```
[Out] (2*a*A*b*x + I*a^2*C*x + I*b^2*C*x + (2*I)*((2*I)*a*A*b + a^2*C - b^2*C)*Ar
cTan[((a + b)*Cot[x/2])/(a - b)] - 2*a*b*C*Cos[x] - (2*I)*a*A*b*Log[a^2 + b
^2 + 2*a*b*Cos[x]] - a^2*C*Log[a^2 + b^2 + 2*a*b*Cos[x]] + b^2*C*Log[a^2 +
b^2 + 2*a*b*Cos[x]] - (2*I)*a*b*C*Sin[x])/(4*a^2*b)
```

fricas [A] time = 0.55, size = 57, normalized size = 0.67

$$\frac{iCa^2x - Cabe^{(ix)} - \left(Ca^2 + 2iAab - Cb^2\right)\log\left(\frac{ae^{(ix)}+b}{a}\right)}{2a^2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*sin(x))/(a+b*cos(x)-I*b*sin(x)),x, algorithm="fricas")
```

```
[Out] 1/2*(I*C*a^2*x - C*a*b*e^(I*x) - (C*a^2 + 2*I*A*a*b - C*b^2)*log((a*e^(I*x)
 + b)/a))/(a^2*b)
```

giac [B] time = 0.18, size = 169, normalized size = 1.99

$$-\frac{(2iAa - Cb)\log\left(-a\tan\left(\frac{1}{2}x\right)^2 + b\tan\left(\frac{1}{2}x\right)^2 + 2ia\tan\left(\frac{1}{2}x\right) + a + b\right)}{4a^2} - \frac{(-2iAa + Cb)\log\left(\tan\left(\frac{1}{2}x\right) + i\right)}{2a^2} - \cdots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*sin(x))/(a+b*cos(x)-I*b*sin(x)),x, algorithm="giac")
```

```
[Out] -1/4*(2*I*A*a - C*b)*log(-a*tan(1/2*x)^2 + b*tan(1/2*x)^2 + 2*I*a*tan(1/2*x
) + a + b)/a^2 - 1/2*(-2*I*A*a + C*b)*log(tan(1/2*x) + I)/a^2 - 1/4*(-2*I*C
*a^2 + 2*A*a*b + I*C*b^2)*(x + 2*arctan((I*a*cos(x) - a*sin(x) + I*a)/(a*co
s(x) + I*a*sin(x) - a + 2*b)))/(a^2*b) - 1/2*(2*I*A*a*tan(1/2*x) - C*b*tan(
1/2*x) - 2*A*a + 2*I*C*a - I*C*b)/(a^2*(tan(1/2*x) + I))
```

maple [B] time = 0.18, size = 280, normalized size = 3.29

$$-\frac{iC}{a\left(\tan\left(\frac{x}{2}\right) + i\right)} + \frac{i\ln\left(\tan\left(\frac{x}{2}\right) + i\right)A}{a} - \frac{\ln\left(\tan\left(\frac{x}{2}\right) + i\right)bC}{2a^2} + \frac{C\ln\left(\tan\left(\frac{x}{2}\right) - i\right)}{2b} + \frac{a\ln\left(ia + ib - a\tan\left(\frac{x}{2}\right) + b\tan \cdots\right.}{2b(-a + b)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+C*sin(x))/(a+b*cos(x)-I*b*sin(x)),x)
```

```
[Out] -I*C/a/(tan(1/2*x)+I)+I/a*ln(tan(1/2*x)+I)*A-1/2/a^2*ln(tan(1/2*x)+I)*b*C+1
/2*C/b*ln(tan(1/2*x)-I)+1/2*a/b/(-a+b)*ln(I*a+I*b-a*tan(1/2*x)+b*tan(1/2*x)
)*C-1/2/(-a+b)*ln(I*a+I*b-a*tan(1/2*x)+b*tan(1/2*x))*C-1/2/a*b/(-a+b)*ln(I*
a+I*b-a*tan(1/2*x)+b*tan(1/2*x))*C+1/2/a^2*b^2/(-a+b)*ln(I*a+I*b-a*tan(1/2*
x)+b*tan(1/2*x))*C+I/(-a+b)*ln(I*a+I*b-a*tan(1/2*x)+b*tan(1/2*x))*A-I/a*b/(
-a+b)*ln(I*a+I*b-a*tan(1/2*x)+b*tan(1/2*x))*A
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*sin(x))/(a+b*cos(x)-I*b*sin(x)),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

mupad [B] time = 4.34, size = 96, normalized size = 1.13

$$-\ln\left(a+b+a\tan\left(\frac{x}{2}\right)1\mathrm{i}-b\tan\left(\frac{x}{2}\right)1\mathrm{i}\right)\left(\frac{C}{2\,b}-\frac{C\,b}{2\,a^2}+\frac{A\,1\mathrm{i}}{a}\right)-\frac{C\,1\mathrm{i}}{a\,\left(\tan\left(\frac{x}{2}\right)+1\mathrm{i}\right)}+\frac{C\,\ln\left(\tan\left(\frac{x}{2}\right)-\mathrm{i}\right)}{2\,b}+\frac{\ln\left(\tan\left(\frac{x}{2}\right)\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + C*sin(x))/(a + b*cos(x) - b*sin(x)*1i),x)
```

```
[Out] (C*log(tan(x/2) - 1i))/(2*b) - (C*1i)/(a*(tan(x/2) + 1i)) - log(a + b + a*t
an(x/2)*1i - b*tan(x/2)*1i)*((A*1i)/a + C/(2*b) - (C*b)/(2*a^2)) + (log(tan
(x/2) + 1i)*(A*a*1i - (C*b)/2))/a^2
```

sympy [A] time = 0.74, size = 80, normalized size = 0.94

$$\frac{iCx}{2b}+\begin{cases}-\frac{Ce^{ix}}{2a} & \text{for } 2a\neq 0\\ x\left(-\frac{iC}{2b}+\frac{iCa-iCb}{2ab}\right) & \text{otherwise}\end{cases}-\frac{\left(2iAab+Ca^2-Cb^2\right)\log\left(e^{ix}+\frac{b}{a}\right)}{2a^2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*sin(x))/(a+b*cos(x)-I*b*sin(x)),x)
```

```
[Out] I*C*x/(2*b) + Piecewise((-C*exp(I*x)/(2*a), Ne(2*a, 0)), (x*(-I*C/(2*b) + (
I*C*a - I*C*b)/(2*a*b)), True)) - (2*I*A*a*b + C*a**2 - C*b**2)*log(exp(I*x
) + b/a)/(2*a**2*b)
```

3.545 $\int \frac{B\cos(x)+C\sin(x)}{a+b\cos(x)+c\sin(x)}\,dx$

Optimal. Leaf size=119

$$-\frac{2a(bB+cC)\tan^{-1}\left(\frac{(a-b)\tan\left(\frac{x}{2}\right)+c}{\sqrt{a^2-b^2-c^2}}\right)}{\left(b^2+c^2\right)\sqrt{a^2-b^2-c^2}}+\frac{(Bc-bC)\log(a+b\cos(x)+c\sin(x))}{b^2+c^2}+\frac{x(bB+cC)}{b^2+c^2}$$

```
[Out] (B*b+C*c)*x/(b^2+c^2)+(B*c-C*b)*ln(a+b*cos(x)+c*sin(x))/(b^2+c^2)-2*a*(B*b+
C*c)*arctan((c+(a-b)*tan(1/2*x))/(a^2-b^2-c^2)^(1/2))/(b^2+c^2)/(a^2-b^2-c^
2)^(1/2)
```

Rubi [A] time = 0.11, antiderivative size = 119, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {3136, 3124, 618, 204}

$$-\frac{2a(bB+cC)\tan^{-1}\left(\frac{(a-b)\tan\left(\frac{x}{2}\right)+c}{\sqrt{a^2-b^2-c^2}}\right)}{\left(b^2+c^2\right)\sqrt{a^2-b^2-c^2}}+\frac{(Bc-bC)\log(a+b\cos(x)+c\sin(x))}{b^2+c^2}+\frac{x(bB+cC)}{b^2+c^2}$$

Antiderivative was successfully verified.

```
[In] Int[(B*Cos[x] + C*Sin[x])/(a + b*Cos[x] + c*Sin[x]),x]
```

```
[Out] ((b*B + c*C)*x)/(b^2 + c^2) - (2*a*(b*B + c*C)*ArcTan[(c + (a - b)*Tan[x/2]
)/Sqrt[a^2 - b^2 - c^2]])/(Sqrt[a^2 - b^2 - c^2]*(b^2 + c^2)) + ((B*c - b*C
)*Log[a + b*Cos[x] + c*Sin[x]])/(b^2 + c^2)
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 3124

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(-1), x_Symbol] :> Module[{f = FreeFactors[Tan[(d + e*x)/2], x]}, Dist[(2*f
```

```
)/e, Subst[Int[1/(a + b + 2*c*f*x + (a - b)*f^2*x^2), x], x, Tan[(d + e*x)/
2]/f], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0]
```

Rule 3136

```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])
/((a_.) + cos[(d_.) + (e_.)*(x_)]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)]), x
_Symbol] :> Simp[((b*B + c*C)*x)/(b^2 + c^2), x] + (Dist[(A*(b^2 + c^2) - a
*(b*B + c*C))/(b^2 + c^2), Int[1/(a + b*Cos[d + e*x] + c*Sin[d + e*x]), x],
 x] + Simp[((c*B - b*C)*Log[a + b*Cos[d + e*x] + c*Sin[d + e*x]])/(e*(b^2 +
 c^2)), x]) /; FreeQ[{a, b, c, d, e, A, B, C}, x] && NeQ[b^2 + c^2, 0] && N
eQ[A*(b^2 + c^2) - a*(b*B + c*C), 0]
```

Rubi steps

$$\int \frac{B\cos(x)+C\sin(x)}{a+b\cos(x)+c\sin(x)}\,dx = \frac{(bB+cC)x}{b^2+c^2} + \frac{(Bc-bC)\log(a+b\cos(x)+c\sin(x))}{b^2+c^2} - \frac{(a(bB+cC))\int \frac{1}{a+b\cos(x)+c}}{b^2+c^2}$$

$$= \frac{(bB+cC)x}{b^2+c^2} + \frac{(Bc-bC)\log(a+b\cos(x)+c\sin(x))}{b^2+c^2} - \frac{(2a(bB+cC))\,\mathrm{Subst}\left(\int \frac{}{a+}\right.}{b^2}$$

$$= \frac{(bB+cC)x}{b^2+c^2} + \frac{(Bc-bC)\log(a+b\cos(x)+c\sin(x))}{b^2+c^2} + \frac{(4a(bB+cC))\,\mathrm{Subst}\left(\int \frac{}{-}\right.}{}$$

$$= \frac{(bB+cC)x}{b^2+c^2} - \frac{2a(bB+cC)\tan^{-1}\left(\frac{c+(a-b)\tan\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2-c^2}}\right)}{\sqrt{a^2-b^2-c^2}\,\left(b^2+c^2\right)} + \frac{(Bc-bC)\log(a+b\cos(x)+c}{b^2+c^2}$$

Mathematica [A] time = 0.38, size = 98, normalized size = 0.82

$$\frac{\frac{2a(bB+cC)\tanh^{-1}\left(\frac{(a-b)\tan\left(\frac{x}{2}\right)+c}{\sqrt{-a^2+b^2+c^2}}\right)}{\sqrt{-a^2+b^2+c^2}} + (Bc-bC)\log(a+b\cos(x)+c\sin(x)) + x(bB+cC)}{b^2+c^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(B*Cos[x] + C*Sin[x])/(a + b*Cos[x] + c*Sin[x]),x]
```

```
[Out] ((b*B + c*C)*x + (2*a*(b*B + c*C)*ArcTanh[(c + (a - b)*Tan[x/2])/Sqrt[-a^2
+ b^2 + c^2]])/Sqrt[-a^2 + b^2 + c^2] + (B*c - b*C)*Log[a + b*Cos[x] + c*Si
n[x]])/(b^2 + c^2)
```

fricas [B] time = 5.06, size = 687, normalized size = 5.77

$$\left[-\frac{(Bab+Cac)\sqrt{-a^2+b^2+c^2}\log\left(\frac{a^2b^2-2b^4-c^4-(a^2+3b^2)c^2-(2a^2b^2-b^4-2a^2c^2+c^4)\cos(x)^2-2(ab^3+abc^2)\cos(x)-2(ab^2c+ac^3-(bc^3-}{2ab\cos(x)+(b^2-c^2)\mathrm{co}}\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*cos(x)+C*sin(x))/(a+b*cos(x)+c*sin(x)),x, algorithm="fricas")
```

```
[Out] [-1/2*((B*a*b + C*a*c)*sqrt(-a^2 + b^2 + c^2)*log((a^2*b^2 - 2*b^4 - c^4 -
(a^2 + 3*b^2)*c^2 - (2*a^2*b^2 - b^4 - 2*a^2*c^2 + c^4)*cos(x)^2 - 2*(a*b^3
 + a*b*c^2)*cos(x) - 2*(a*b^2*c + a*c^3 - (b*c^3 - (2*a^2*b - b^3)*c)*cos(x
))*sin(x) - 2*(2*a*b*c*cos(x)^2 - a*b*c + (b^2*c + c^3)*cos(x) - (b^3 + b*c
^2 + (a*b^2 - a*c^2)*cos(x))*sin(x))*sqrt(-a^2 + b^2 + c^2))/(2*a*b*cos(x)
+ (b^2 - c^2)*cos(x)^2 + a^2 + c^2 + 2*(b*c*cos(x) + a*c)*sin(x))) - 2*(B*a
^2*b - B*b^3 - B*b*c^2 - C*c^3 + (C*a^2 - C*b^2)*c)*x + (C*a^2*b - C*b^3 -
C*b*c^2 + B*c^3 - (B*a^2 - B*b^2)*c)*log(2*a*b*cos(x) + (b^2 - c^2)*cos(x)^
2 + a^2 + c^2 + 2*(b*c*cos(x) + a*c)*sin(x)))/(a^2*b^2 - b^4 - c^4 + (a^2 -
 2*b^2)*c^2), -1/2*(2*(B*a*b + C*a*c)*sqrt(a^2 - b^2 - c^2)*arctan(-(a*b*co
s(x) + a*c*sin(x) + b^2 + c^2)*sqrt(a^2 - b^2 - c^2)/((c^3 - (a^2 - b^2)*c)
*cos(x) + (a^2*b - b^3 - b*c^2)*sin(x))) - 2*(B*a^2*b - B*b^3 - B*b*c^2 - C
*c^3 + (C*a^2 - C*b^2)*c)*x + (C*a^2*b - C*b^3 - C*b*c^2 + B*c^3 - (B*a^2 -
 B*b^2)*c)*log(2*a*b*cos(x) + (b^2 - c^2)*cos(x)^2 + a^2 + c^2 + 2*(b*c*cos
(x) + a*c)*sin(x)))/(a^2*b^2 - b^4 - c^4 + (a^2 - 2*b^2)*c^2)]
```

giac [A] time = 0.15, size = 187, normalized size = 1.57

$$\frac{(Bb+Cc)x}{b^2+c^2}-\frac{(Cb-Bc)\log\left(-a\tan\left(\frac{1}{2}x\right)^2+b\tan\left(\frac{1}{2}x\right)^2-2c\tan\left(\frac{1}{2}x\right)-a-b\right)}{b^2+c^2}+\frac{(Cb-Bc)\log\left(\tan\left(\frac{1}{2}x\right)^2+1\right.}{b^2+c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*cos(x)+C*sin(x))/(a+b*cos(x)+c*sin(x)),x, algorithm="giac")
```

```
[Out] (B*b + C*c)*x/(b^2 + c^2) - (C*b - B*c)*log(-a*tan(1/2*x)^2 + b*tan(1/2*x)^
2 - 2*c*tan(1/2*x) - a - b)/(b^2 + c^2) + (C*b - B*c)*log(tan(1/2*x)^2 + 1)
/(b^2 + c^2) + 2*(B*a*b + C*a*c)*(pi*floor(1/2*x/pi + 1/2)*sgn(-2*a + 2*b)
+ arctan(-(a*tan(1/2*x) - b*tan(1/2*x) + c)/sqrt(a^2 - b^2 - c^2)))/(sqrt(a
^2 - b^2 - c^2)*(b^2 + c^2))
```

maple [B] time = 0.12, size = 824, normalized size = 6.92

$$\frac{\ln\left(a\left(\tan^2\left(\frac{x}{2}\right)\right)-b\left(\tan^2\left(\frac{x}{2}\right)\right)+2c\tan\left(\frac{x}{2}\right)+a+b\right)aBc}{\left(b^2+c^2\right)(a-b)}-\frac{\ln\left(a\left(\tan^2\left(\frac{x}{2}\right)\right)-b\left(\tan^2\left(\frac{x}{2}\right)\right)+2c\tan\left(\frac{x}{2}\right)+a+b\right)bB}{\left(b^2+c^2\right)(a-b)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*cos(x)+C*sin(x))/(a+b*cos(x)+c*sin(x)),x)
```

```
[Out] 1/(b^2+c^2)/(a-b)*ln(a*tan(1/2*x)^2-b*tan(1/2*x)^2+2*c*tan(1/2*x)+a+b)*a*B*
c-1/(b^2+c^2)/(a-b)*ln(a*tan(1/2*x)^2-b*tan(1/2*x)^2+2*c*tan(1/2*x)+a+b)*b*
B*c-1/(b^2+c^2)/(a-b)*ln(a*tan(1/2*x)^2-b*tan(1/2*x)^2+2*c*tan(1/2*x)+a+b)*
a*b*C+1/(b^2+c^2)/(a-b)*ln(a*tan(1/2*x)^2-b*tan(1/2*x)^2+2*c*tan(1/2*x)+a+b
)*b^2*C-2/(b^2+c^2)/(a^2-b^2-c^2)^(1/2)*arctan(1/2*(2*(a-b)*tan(1/2*x)+2*c)
/(a^2-b^2-c^2)^(1/2))*a*b*B+2/(b^2+c^2)/(a^2-b^2-c^2)^(1/2)*arctan(1/2*(2*(
a-b)*tan(1/2*x)+2*c)/(a^2-b^2-c^2)^(1/2))*B*c^2-2/(b^2+c^2)/(a^2-b^2-c^2)^(
1/2)*arctan(1/2*(2*(a-b)*tan(1/2*x)+2*c)/(a^2-b^2-c^2)^(1/2))*a*c*C-2/(b^2+
c^2)/(a^2-b^2-c^2)^(1/2)*arctan(1/2*(2*(a-b)*tan(1/2*x)+2*c)/(a^2-b^2-c^2)^
(1/2))*C*b*c-2/(b^2+c^2)/(a^2-b^2-c^2)^(1/2)*arctan(1/2*(2*(a-b)*tan(1/2*x)
+2*c)/(a^2-b^2-c^2)^(1/2))*c^2/(a-b)*a*B+2/(b^2+c^2)/(a^2-b^2-c^2)^(1/2)*ar
ctan(1/2*(2*(a-b)*tan(1/2*x)+2*c)/(a^2-b^2-c^2)^(1/2))*c^2/(a-b)*b*B+2/(b^2
+c^2)/(a^2-b^2-c^2)^(1/2)*arctan(1/2*(2*(a-b)*tan(1/2*x)+2*c)/(a^2-b^2-c^2)
^(1/2))*c/(a-b)*a*b*C-2/(b^2+c^2)/(a^2-b^2-c^2)^(1/2)*arctan(1/2*(2*(a-b)*t
an(1/2*x)+2*c)/(a^2-b^2-c^2)^(1/2))*c/(a-b)*b^2*C-B/(b^2+c^2)*c*ln(1+tan(1/
2*x)^2)+C/(b^2+c^2)*b*ln(1+tan(1/2*x)^2)+2*B/(b^2+c^2)*b*arctan(tan(1/2*x))
+2*C/(b^2+c^2)*c*arctan(tan(1/2*x))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*cos(x)+C*sin(x))/(a+b*cos(x)+c*sin(x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(c^2+b^2-a^2>0)', see `assume?` for
more details)Is c^2+b^2-a^2 positive or negative?
```

mupad [B] time = 28.56, size = 1864, normalized size = 15.66

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*cos(x) + C*sin(x))/(a + b*cos(x) + c*sin(x)),x)
```

```
[Out] (log(tan(x/2) - 1i)*(B + C*1i))/(b*1i - c) - (log(tan(x/2) + 1i)*(B - C*1i)
)/(b*1i + c) - (log(32*B^3*a^2 + 32*B*C^2*a^2 + 32*B*C^2*b^2 + 32*tan(x/2)*
(a - b)*(2*C^3*a + B^3*c - 2*C^3*b + 2*B^2*C*a - B^2*C*b + 2*B*C^2*c) - 32*
B^3*a*b - 64*B*C^2*a*b + 32*B^2*C*a*c - 32*B^2*C*b*c + ((C*b^3 - B*c^3 + B*
a^2*c - C*a^2*b - B*b^2*c + C*b*c^2 + B*a*b*(b^2 - a^2 + c^2)^(1/2) + C*a*c
*(b^2 - a^2 + c^2)^(1/2))*(32*B^2*b^2*c - 32*B^2*a^2*c + 32*C^2*a^2*c + 32*
C^2*b^2*c + 32*tan(x/2)*(a - b)*(2*B^2*a^2 + B^2*b^2 - 2*C^2*a^2 - 3*B^2*c^
2 + 2*C^2*c^2 - 2*B^2*a*b + 2*C^2*a*b - 4*B*C*a*c + 6*B*C*b*c) - 128*B*C*a^
3 - 64*B*C*b^3 + 192*B*C*a^2*b + 64*B*C*a*c^2 - 64*B*C*b*c^2 - 64*C^2*a*b*c
 + ((C*b^3 - B*c^3 + B*a^2*c - C*a^2*b - B*b^2*c + C*b*c^2 + B*a*b*(b^2 - a
^2 + c^2)^(1/2) + C*a*c*(b^2 - a^2 + c^2)^(1/2))*(32*B*b^4 + 32*B*a^2*b^2 -
 32*B*a^2*c^2 - 64*B*b^2*c^2 - 32*tan(x/2)*(a - b)*(B*c^3 - 2*C*b^3 + 2*C*a
*b^2 + 4*B*b^2*c - 2*C*a*c^2 + C*b*c^2 - 4*B*a*b*c) - 64*B*a*b^3 + 32*C*a*c
^3 - 32*C*b*c^3 + 64*C*b^3*c + 96*B*a*b*c^2 - 128*C*a*b^2*c + 64*C*a^2*b*c
+ (32*(a - b)*(C*b^3 - B*c^3 + B*a^2*c - C*a^2*b - B*b^2*c + C*b*c^2 + B*a*
b*(b^2 - a^2 + c^2)^(1/2) + C*a*c*(b^2 - a^2 + c^2)^(1/2))*(3*c^4*tan(x/2)
+ a*c^3 + 3*b*c^3 + 3*b^3*c + 2*a^2*b^2*tan(x/2) - 2*a^2*c^2*tan(x/2) + 3*b
^2*c^2*tan(x/2) - 2*a*b^3*tan(x/2) + a*b^2*c - 4*a^2*b*c - 2*a*b*c^2*tan(x/
2)))/((b^2 + c^2)*(b^2 - a^2 + c^2))))/((b^2 + c^2)*(b^2 - a^2 + c^2))))/((
b^2 + c^2)*(b^2 - a^2 + c^2)))*(C*b^3 - B*c^3 + B*a^2*c - C*a^2*b - B*b^2*c
 + C*b*c^2 + B*a*b*(b^2 - a^2 + c^2)^(1/2) + C*a*c*(b^2 - a^2 + c^2)^(1/2))
)/((b^2 + c^2)*(b^2 - a^2 + c^2)) + (log(32*B^3*a^2 + 32*B*C^2*a^2 + 32*B*C
^2*b^2 + 32*tan(x/2)*(a - b)*(2*C^3*a + B^3*c - 2*C^3*b + 2*B^2*C*a - B^2*C
*b + 2*B*C^2*c) - 32*B^3*a*b - 64*B*C^2*a*b + 32*B^2*C*a*c - 32*B^2*C*b*c -
 ((B*c^3 - C*b^3 - B*a^2*c + C*a^2*b + B*b^2*c - C*b*c^2 + B*a*b*(b^2 - a^2
 + c^2)^(1/2) + C*a*c*(b^2 - a^2 + c^2)^(1/2))*(32*B^2*b^2*c - 32*B^2*a^2*c
 + 32*C^2*a^2*c + 32*C^2*b^2*c + 32*tan(x/2)*(a - b)*(2*B^2*a^2 + B^2*b^2 -
 2*C^2*a^2 - 3*B^2*c^2 + 2*C^2*c^2 - 2*B^2*a*b + 2*C^2*a*b - 4*B*C*a*c + 6*
B*C*b*c) - 128*B*C*a^3 - 64*B*C*b^3 + 192*B*C*a^2*b + 64*B*C*a*c^2 - 64*B*C
*b*c^2 - 64*C^2*a*b*c + ((B*c^3 - C*b^3 - B*a^2*c + C*a^2*b + B*b^2*c - C*b
*c^2 + B*a*b*(b^2 - a^2 + c^2)^(1/2) + C*a*c*(b^2 - a^2 + c^2)^(1/2))*(32*B
*a^2*c^2 - 32*B*a^2*b^2 - 32*B*b^4 + 64*B*b^2*c^2 + 32*tan(x/2)*(a - b)*(B*
c^3 - 2*C*b^3 + 2*C*a*b^2 + 4*B*b^2*c - 2*C*a*c^2 + C*b*c^2 - 4*B*a*b*c) +
64*B*a*b^3 - 32*C*a*c^3 + 32*C*b*c^3 - 64*C*b^3*c - 96*B*a*b*c^2 + 128*C*a*
b^2*c - 64*C*a^2*b*c + (32*(a - b)*(B*c^3 - C*b^3 - B*a^2*c + C*a^2*b + B*b
^2*c - C*b*c^2 + B*a*b*(b^2 - a^2 + c^2)^(1/2) + C*a*c*(b^2 - a^2 + c^2)^(1
/2))*(3*c^4*tan(x/2) + a*c^3 + 3*b*c^3 + 3*b^3*c + 2*a^2*b^2*tan(x/2) - 2*a
^2*c^2*tan(x/2) + 3*b^2*c^2*tan(x/2) - 2*a*b^3*tan(x/2) + a*b^2*c - 4*a^2*b
*c - 2*a*b*c^2*tan(x/2)))/((b^2 + c^2)*(b^2 - a^2 + c^2))))/((b^2 + c^2)*(b
^2 - a^2 + c^2))))/((b^2 + c^2)*(b^2 - a^2 + c^2)))*(B*c^3 - C*b^3 - B*a^2*
c + C*a^2*b + B*b^2*c - C*b*c^2 + B*a*b*(b^2 - a^2 + c^2)^(1/2) + C*a*c*(b^
2 - a^2 + c^2)^(1/2)))/((b^2 + c^2)*(b^2 - a^2 + c^2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*cos(x)+C*sin(x))/(a+b*cos(x)+c*sin(x)),x)
```

```
[Out] Timed out
```

3.546 $\int \frac{B\cos(x)+C\sin(x)}{(a+b\cos(x)+c\sin(x))^2}\,dx$

Optimal. Leaf size=110

$$\frac{aB\sin(x)-aC\cos(x)-bC+Bc}{\left(a^2-b^2-c^2\right)(a+b\cos(x)+c\sin(x))}-\frac{2(bB+cC)\tan^{-1}\left(\frac{(a-b)\tan\left(\frac{x}{2}\right)+c}{\sqrt{a^2-b^2-c^2}}\right)}{\left(a^2-b^2-c^2\right)^{3/2}}$$

```
[Out] -2*(B*b+C*c)*arctan((c+(a-b)*tan(1/2*x))/(a^2-b^2-c^2)^(1/2))/(a^2-b^2-c^2)
^(3/2)+(B*c-b*C-a*C*cos(x)+a*B*sin(x))/(a^2-b^2-c^2)/(a+b*cos(x)+c*sin(x))
```

Rubi [A] time = 0.10, antiderivative size = 110, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {3153, 3124, 618, 204}

$$\frac{aB\sin(x)-aC\cos(x)-bC+Bc}{\left(a^2-b^2-c^2\right)(a+b\cos(x)+c\sin(x))}-\frac{2(bB+cC)\tan^{-1}\left(\frac{(a-b)\tan\left(\frac{x}{2}\right)+c}{\sqrt{a^2-b^2-c^2}}\right)}{\left(a^2-b^2-c^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(B*Cos[x] + C*Sin[x])/(a + b*Cos[x] + c*Sin[x])^2,x]
```

```
[Out] (-2*(b*B + c*C)*ArcTan[(c + (a - b)*Tan[x/2])/Sqrt[a^2 - b^2 - c^2]])/(a^2
- b^2 - c^2)^(3/2) + (B*c - b*C - a*C*Cos[x] + a*B*Sin[x])/((a^2 - b^2 - c^
2)*(a + b*Cos[x] + c*Sin[x]))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 3124

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(-1), x_Symbol] :> Module[{f = FreeFactors[Tan[(d + e*x)/2], x]}, Dist[(2*f
)/e, Subst[Int[1/(a + b + 2*c*f*x + (a - b)*f^2*x^2), x], x, Tan[(d + e*x)/
```

```
2]/f], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0]
```

Rule 3153

```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])
/((a_.) + cos[(d_.) + (e_.)*(x_)]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)])^2,
 x_Symbol] :> Simp[(c*B - b*C - (a*C - c*A)*Cos[d + e*x] + (a*B - b*A)*Sin[
d + e*x])/(e*(a^2 - b^2 - c^2)*(a + b*Cos[d + e*x] + c*Sin[d + e*x])), x] +
 Dist[(a*A - b*B - c*C)/(a^2 - b^2 - c^2), Int[1/(a + b*Cos[d + e*x] + c*Si
n[d + e*x]), x], x] /; FreeQ[{a, b, c, d, e, A, B, C}, x] && NeQ[a^2 - b^2
- c^2, 0] && NeQ[a*A - b*B - c*C, 0]
```

Rubi steps

$$\int \frac{B\cos(x)+C\sin(x)}{(a+b\cos(x)+c\sin(x))^2}\,dx = \frac{Bc-bC-aC\cos(x)+aB\sin(x)}{\left(a^2-b^2-c^2\right)(a+b\cos(x)+c\sin(x))}-\frac{(bB+cC)\int \frac{1}{a+b\cos(x)+c\sin(x)}\,dx}{a^2-b^2-c^2}$$

$$= \frac{Bc-bC-aC\cos(x)+aB\sin(x)}{\left(a^2-b^2-c^2\right)(a+b\cos(x)+c\sin(x))}-\frac{(2(bB+cC))\,\text{Subst}\left(\int \frac{1}{a+b+2cx+(a-b)x^2}\right.}{a^2-b^2-c^2}$$

$$= \frac{Bc-bC-aC\cos(x)+aB\sin(x)}{\left(a^2-b^2-c^2\right)(a+b\cos(x)+c\sin(x))}+\frac{(4(bB+cC))\,\text{Subst}\left(\int \frac{1}{-4\left(a^2-b^2-c^2\right)-x^2}\right.}{a^2-b^2-c}$$

$$= -\frac{2(bB+cC)\tan^{-1}\left(\frac{c+(a-b)\tan\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2-c^2}}\right)}{\left(a^2-b^2-c^2\right)^{3/2}}+\frac{Bc-bC-aC\cos(x)+aB\sin(x)}{\left(a^2-b^2-c^2\right)(a+b\cos(x)+c\sin(x))}$$

Mathematica [A] time = 0.41, size = 116, normalized size = 1.05

$$-\frac{a^2C+a\sin(x)(bB+cC)-b^2C+bBc}{b\left(-a^2+b^2+c^2\right)(a+b\cos(x)+c\sin(x))}-\frac{2(bB+cC)\tanh^{-1}\left(\frac{(a-b)\tan\left(\frac{x}{2}\right)+c}{\sqrt{-a^2+b^2+c^2}}\right)}{\left(-a^2+b^2+c^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(B*Cos[x] + C*Sin[x])/(a + b*Cos[x] + c*Sin[x])^2,x]
```

```
[Out] (-2*(b*B + c*C)*ArcTanh[(c + (a - b)*Tan[x/2])/Sqrt[-a^2 + b^2 + c^2]])/(-a
^2 + b^2 + c^2)^(3/2) - (b*B*c + a^2*C - b^2*C + a*(b*B + c*C)*Sin[x])/(b*(
-a^2 + b^2 + c^2)*(a + b*Cos[x] + c*Sin[x]))
```

fricas [B] time = 1.02, size = 1316, normalized size = 11.96

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*cos(x)+C*sin(x))/(a+b*cos(x)+c*sin(x))^2,x, algorithm="fricas"
)
```

```
[Out] [1/2*(2*C*a^4*b - 4*C*a^2*b^3 + 2*C*b^5 + 2*C*b*c^4 - 2*B*c^5 + 4*(B*a^2 -
B*b^2)*c^3 - 4*(C*a^2*b - C*b^3)*c^2 + (B*a*b^3 + C*a*b^2*c + B*a*b*c^2 + C
*a*c^3 + (B*b^4 + C*b^3*c + B*b^2*c^2 + C*b*c^3)*cos(x) + (B*b^3*c + C*b^2*
c^2 + B*b*c^3 + C*c^4)*sin(x))*sqrt(-a^2 + b^2 + c^2)*log(-(a^2*b^2 - 2*b^4
 - c^4 - (a^2 + 3*b^2)*c^2 - (2*a^2*b^2 - b^4 - 2*a^2*c^2 + c^4)*cos(x)^2 -
 2*(a*b^3 + a*b*c^2)*cos(x) - 2*(a*b^2*c + a*c^3 - (b*c^3 - (2*a^2*b - b^3)
*c)*cos(x))*sin(x) + 2*(2*a*b*c*cos(x)^2 - a*b*c + (b^2*c + c^3)*cos(x) - (
b^3 + b*c^2 + (a*b^2 - a*c^2)*cos(x))*sin(x))*sqrt(-a^2 + b^2 + c^2))/(2*a*
b*cos(x) + (b^2 - c^2)*cos(x)^2 + a^2 + c^2 + 2*(b*c*cos(x) + a*c)*sin(x)))
 - 2*(B*a^4 - 2*B*a^2*b^2 + B*b^4)*c + 2*(B*a*b*c^3 + C*a*c^4 - (C*a^3 - C*
a*b^2)*c^2 - (B*a^3*b - B*a*b^3)*c)*cos(x) + 2*(B*a^3*b^2 - B*a*b^4 - B*a*b
^2*c^2 - C*a*b*c^3 + (C*a^3*b - C*a*b^3)*c)*sin(x))/(a^5*b^2 - 2*a^3*b^4 +
a*b^6 + a*c^6 - (2*a^3 - 3*a*b^2)*c^4 + (a^5 - 4*a^3*b^2 + 3*a*b^4)*c^2 + (
a^4*b^3 - 2*a^2*b^5 + b^7 + b*c^6 - (2*a^2*b - 3*b^3)*c^4 + (a^4*b - 4*a^2*
b^3 + 3*b^5)*c^2)*cos(x) + (c^7 - (2*a^2 - 3*b^2)*c^5 + (a^4 - 4*a^2*b^2 +
3*b^4)*c^3 + (a^4*b^2 - 2*a^2*b^4 + b^6)*c)*sin(x)), (C*a^4*b - 2*C*a^2*b^3
 + C*b^5 + C*b*c^4 - B*c^5 + 2*(B*a^2 - B*b^2)*c^3 - 2*(C*a^2*b - C*b^3)*c^
2 - (B*a*b^3 + C*a*b^2*c + B*a*b*c^2 + C*a*c^3 + (B*b^4 + C*b^3*c + B*b^2*c
^2 + C*b*c^3)*cos(x) + (B*b^3*c + C*b^2*c^2 + B*b*c^3 + C*c^4)*sin(x))*sqrt
(a^2 - b^2 - c^2)*arctan(-(a*b*cos(x) + a*c*sin(x) + b^2 + c^2)*sqrt(a^2 -
b^2 - c^2)/((c^3 - (a^2 - b^2)*c)*cos(x) + (a^2*b - b^3 - b*c^2)*sin(x))) -
 (B*a^4 - 2*B*a^2*b^2 + B*b^4)*c + (B*a*b*c^3 + C*a*c^4 - (C*a^3 - C*a*b^2)
*c^2 - (B*a^3*b - B*a*b^3)*c)*cos(x) + (B*a^3*b^2 - B*a*b^4 - B*a*b^2*c^2 -
 C*a*b*c^3 + (C*a^3*b - C*a*b^3)*c)*sin(x))/(a^5*b^2 - 2*a^3*b^4 + a*b^6 +
a*c^6 - (2*a^3 - 3*a*b^2)*c^4 + (a^5 - 4*a^3*b^2 + 3*a*b^4)*c^2 + (a^4*b^3
- 2*a^2*b^5 + b^7 + b*c^6 - (2*a^2*b - 3*b^3)*c^4 + (a^4*b - 4*a^2*b^3 + 3*
b^5)*c^2)*cos(x) + (c^7 - (2*a^2 - 3*b^2)*c^5 + (a^4 - 4*a^2*b^2 + 3*b^4)*c
^3 + (a^4*b^2 - 2*a^2*b^4 + b^6)*c)*sin(x))]
```

giac [A] time = 0.18, size = 205, normalized size = 1.86

$$\frac{2\left(\pi\left\lfloor\frac{x}{2\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}\left(-2a+2b\right)+\arctan\left(-\frac{a\tan\left(\frac{1}{2}x\right)-b\tan\left(\frac{1}{2}x\right)+c}{\sqrt{a^2-b^2-c^2}}\right)\right)(Bb+Cc)}{\left(a^2-b^2-c^2\right)^{\frac{3}{2}}}+\frac{2\left(Ba^2\tan\left(\frac{1}{2}x\right)-Bab\tan\left(\frac{1}{2}x\right)-C\right.}{\left(a^3-a^2b-ab^2+b^3-ac^2\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*cos(x)+C*sin(x))/(a+b*cos(x)+c*sin(x))^2,x, algorithm="giac")
```

```
[Out] 2*(pi*floor(1/2*x/pi + 1/2)*sgn(-2*a + 2*b) + arctan(-(a*tan(1/2*x) - b*tan
(1/2*x) + c)/sqrt(a^2 - b^2 - c^2)))*(B*b + C*c)/(a^2 - b^2 - c^2)^(3/2) +
2*(B*a^2*tan(1/2*x) - B*a*b*tan(1/2*x) - C*a*c*tan(1/2*x) + C*b*c*tan(1/2*x
) - B*c^2*tan(1/2*x) - C*a^2 + C*b^2 - B*b*c)/((a^3 - a^2*b - a*b^2 + b^3 -
 a*c^2 + b*c^2)*(a*tan(1/2*x)^2 - b*tan(1/2*x)^2 + 2*c*tan(1/2*x) + a + b))
```

maple [B] time = 0.18, size = 255, normalized size = 2.32

$$-\frac{2\left(-\frac{\left(a^{2}B-abB-B\,c^{2}-acC+Cbc\right)\tan\left(\frac{x}{2}\right)}{a^{3}-a^{2}b-a\,b^{2}-a\,c^{2}+b^{3}+c^{2}b}+\frac{bBc+a^{2}C-b^{2}C}{a^{3}-a^{2}b-a\,b^{2}-a\,c^{2}+b^{3}+c^{2}b}\right)}{a\left(\tan^{2}\left(\frac{x}{2}\right)\right)-b\left(\tan^{2}\left(\frac{x}{2}\right)\right)+2c\tan\left(\frac{x}{2}\right)+a+b}-\frac{2\arctan\left(\frac{2(a-b)\tan\left(\frac{x}{2}\right)+2c}{2\sqrt{a^{2}-b^{2}-c^{2}}}\right)bB}{\left(a^{2}-b^{2}-c^{2}\right)^{\frac{3}{2}}}-\frac{2\arctan\left(\frac{2(a-b)\tan\left(\frac{x}{2}\right)+2c}{2\sqrt{a^{2}-b^{2}-c^{2}}}\right.}{\left(a^{2}-b^{2}-c^{2}\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*cos(x)+C*sin(x))/(a+b*cos(x)+c*sin(x))^2,x)
```

```
[Out] -2*(-(B*a^2-B*a*b-B*c^2-C*a*c+C*b*c)/(a^3-a^2*b-a*b^2-a*c^2+b^3+b*c^2)*tan(
1/2*x)+(B*b*c+C*a^2-C*b^2)/(a^3-a^2*b-a*b^2-a*c^2+b^3+b*c^2))/(a*tan(1/2*x)
^2-b*tan(1/2*x)^2+2*c*tan(1/2*x)+a+b)-2/(a^2-b^2-c^2)^(3/2)*arctan(1/2*(2*(
a-b)*tan(1/2*x)+2*c)/(a^2-b^2-c^2)^(1/2))*b*B-2/(a^2-b^2-c^2)^(3/2)*arctan(
1/2*(2*(a-b)*tan(1/2*x)+2*c)/(a^2-b^2-c^2)^(1/2))*C*c
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*cos(x)+C*sin(x))/(a+b*cos(x)+c*sin(x))^2,x, algorithm="maxima"
)
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(c^2+b^2-a^2>0)', see `assume?` for
more details)Is c^2+b^2-a^2 positive or negative?
```

mupad [B] time = 3.24, size = 202, normalized size = 1.84

$$\frac{\frac{2\left(C\,a^{2}-C\,b^{2}+B\,c\,b\right)}{(a-b)\left(-a^{2}+b^{2}+c^{2}\right)}+\frac{2\tan\left(\frac{x}{2}\right)\left(-B\,a^{2}+C\,a\,c+B\,b\,a+B\,c^{2}-C\,b\,c\right)}{(a-b)\left(-a^{2}+b^{2}+c^{2}\right)}}{(a-b)\,\tan\left(\frac{x}{2}\right)^{2}+2\,c\tan\left(\frac{x}{2}\right)+a+b}-\frac{2\,\mathrm{atanh}\left(\frac{\tan\left(\frac{x}{2}\right)(2\,a-2\,b)+\frac{2\left(-a^{2}\,c+b^{2}\,c+c^{3}\right)}{-a^{2}+b^{2}+c^{2}}}{2\,\sqrt{-a^{2}+b^{2}+c^{2}}}\right)(B\,b+C\,c)}{\left(-a^{2}+b^{2}+c^{2}\right)^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*cos(x) + C*sin(x))/(a + b*cos(x) + c*sin(x))^2,x)
```

```
[Out] ((2*(C*a^2 - C*b^2 + B*b*c))/((a - b)*(b^2 - a^2 + c^2)) + (2*tan(x/2)*(B*c
^2 - B*a^2 + B*a*b + C*a*c - C*b*c))/((a - b)*(b^2 - a^2 + c^2)))/(a + b +
2*c*tan(x/2) + tan(x/2)^2*(a - b)) - (2*atanh((tan(x/2)*(2*a - 2*b) + (2*(b
^2*c - a^2*c + c^3))/(b^2 - a^2 + c^2))/(2*(b^2 - a^2 + c^2)^(1/2)))*(B*b +
 C*c))/(b^2 - a^2 + c^2)^(3/2)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*cos(x)+C*sin(x))/(a+b*cos(x)+c*sin(x))**2,x)
```

```
[Out] Timed out
```

3.547 $\int \frac{B\cos(x)+C\sin(x)}{(a+b\cos(x)+c\sin(x))^3}\,dx$

Optimal. Leaf size=197

$$-\frac{3a(bB+cC)\tan^{-1}\left(\frac{(a-b)\tan\left(\frac{x}{2}\right)+c}{\sqrt{a^2-b^2-c^2}}\right)}{\left(a^2-b^2-c^2\right)^{5/2}}+\frac{-\cos(x)\left(C\left(a^2+2c^2\right)+2bBc\right)+\sin(x)\left(a^2B+2b(bB+cC)\right)+a(Bc-bC)}{2\left(a^2-b^2-c^2\right)^2(a+b\cos(x)+c\sin(x))}+$$

```
[Out] -3*a*(B*b+C*c)*arctan((c+(a-b)*tan(1/2*x))/(a^2-b^2-c^2)^(1/2))/(a^2-b^2-c^
2)^(5/2)+1/2*(B*c-b*C-a*C*cos(x)+a*B*sin(x))/(a^2-b^2-c^2)/(a+b*cos(x)+c*si
n(x))^2+1/2*(a*(B*c-C*b)-(2*b*B*c+(a^2+2*c^2)*C)*cos(x)+(a^2*B+2*b*(B*b+C*c
))*sin(x))/(a^2-b^2-c^2)^2/(a+b*cos(x)+c*sin(x))
```

Rubi [A] time = 0.23, antiderivative size = 197, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {3156, 3153, 3124, 618, 204}

$$-\frac{3a(bB+cC)\tan^{-1}\left(\frac{(a-b)\tan\left(\frac{x}{2}\right)+c}{\sqrt{a^2-b^2-c^2}}\right)}{\left(a^2-b^2-c^2\right)^{5/2}}+\frac{-\cos(x)\left(C\left(a^2+2c^2\right)+2bBc\right)+\sin(x)\left(a^2B+2b(bB+cC)\right)+a(Bc-bC)}{2\left(a^2-b^2-c^2\right)^2(a+b\cos(x)+c\sin(x))}+$$

Antiderivative was successfully verified.

```
[In] Int[(B*Cos[x] + C*Sin[x])/(a + b*Cos[x] + c*Sin[x])^3,x]
```

```
[Out] (-3*a*(b*B + c*C)*ArcTan[(c + (a - b)*Tan[x/2])/Sqrt[a^2 - b^2 - c^2]])/(a^
2 - b^2 - c^2)^(5/2) + (B*c - b*C - a*C*Cos[x] + a*B*Sin[x])/(2*(a^2 - b^2
- c^2)*(a + b*Cos[x] + c*Sin[x])^2) + (a*(B*c - b*C) - (2*b*B*c + (a^2 + 2*
c^2)*C)*Cos[x] + (a^2*B + 2*b*(b*B + c*C))*Sin[x])/(2*(a^2 - b^2 - c^2)^2*(
a + b*Cos[x] + c*Sin[x]))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 3124

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(-1), x_Symbol] :> Module[{f = FreeFactors[Tan[(d + e*x)/2], x]}, Dist[(2*f
)/e, Subst[Int[1/(a + b + 2*c*f*x + (a - b)*f^2*x^2), x], x, Tan[(d + e*x)/
2]/f], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0]
```

Rule 3153

```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])
/((a_.) + cos[(d_.) + (e_.)*(x_)]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)])^2,
 x_Symbol] :> Simp[(c*B - b*C - (a*C - c*A)*Cos[d + e*x] + (a*B - b*A)*Sin[
d + e*x])/(e*(a^2 - b^2 - c^2)*(a + b*Cos[d + e*x] + c*Sin[d + e*x])), x] +
 Dist[(a*A - b*B - c*C)/(a^2 - b^2 - c^2), Int[1/(a + b*Cos[d + e*x] + c*Si
n[d + e*x]), x], x] /; FreeQ[{a, b, c, d, e, A, B, C}, x] && NeQ[a^2 - b^2
- c^2, 0] && NeQ[a*A - b*B - c*C, 0]
```

Rule 3156

```
Int[((a_.) + cos[(d_.) + (e_.)*(x_)]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)])
^(n_)*((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)
]), x_Symbol] :> -Simp[((c*B - b*C - (a*C - c*A)*Cos[d + e*x] + (a*B - b*A)
*Sin[d + e*x])*(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n + 1))/(e*(n + 1)*(a
^2 - b^2 - c^2)), x] + Dist[1/((n + 1)*(a^2 - b^2 - c^2)), Int[(a + b*Cos[d
 + e*x] + c*Sin[d + e*x])^(n + 1)*Simp[(n + 1)*(a*A - b*B - c*C) + (n + 2)*
(a*B - b*A)*Cos[d + e*x] + (n + 2)*(a*C - c*A)*Sin[d + e*x], x], x], x] /;
FreeQ[{a, b, c, d, e, A, B, C}, x] && LtQ[n, -1] && NeQ[a^2 - b^2 - c^2, 0]
 && NeQ[n, -2]
```

Rubi steps

$$\int \frac{B\cos(x)+C\sin(x)}{(a+b\cos(x)+c\sin(x))^3}\,dx = \frac{Bc-bC-aC\cos(x)+aB\sin(x)}{2\left(a^2-b^2-c^2\right)(a+b\cos(x)+c\sin(x))^2} - \frac{\int \frac{2(bB+cC)-aB\cos(x)-aC\sin(x)}{(a+b\cos(x)+c\sin(x))^2}\,dx}{2\left(a^2-b^2-c^2\right)}$$

$$= \frac{Bc-bC-aC\cos(x)+aB\sin(x)}{2\left(a^2-b^2-c^2\right)(a+b\cos(x)+c\sin(x))^2} + \frac{a(Bc-bC)-\left(2bBc+\left(a^2+2c^2\right)C\right.}{2\left(a^2-b^2-c^2\right)^2(a}$$

$$= \frac{Bc-bC-aC\cos(x)+aB\sin(x)}{2\left(a^2-b^2-c^2\right)(a+b\cos(x)+c\sin(x))^2} + \frac{a(Bc-bC)-\left(2bBc+\left(a^2+2c^2\right)C\right.}{2\left(a^2-b^2-c^2\right)^2(a}$$

$$= \frac{Bc-bC-aC\cos(x)+aB\sin(x)}{2\left(a^2-b^2-c^2\right)(a+b\cos(x)+c\sin(x))^2} + \frac{a(Bc-bC)-\left(2bBc+\left(a^2+2c^2\right)C\right.}{2\left(a^2-b^2-c^2\right)^2(a}$$

$$= -\frac{3a(bB+cC)\tan^{-1}\left(\frac{c+(a-b)\tan\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2-c^2}}\right)}{\left(a^2-b^2-c^2\right)^{5/2}} + \frac{Bc-bC-aC\cos(x)+aB\sin(x)}{2\left(a^2-b^2-c^2\right)(a+b\cos(x)+c\sin(x))^2}$$

Mathematica [A] time = 0.90, size = 311, normalized size = 1.58

$$\frac{3a(bB+cC)\tanh^{-1}\left(\frac{(a-b)\tan\left(\frac{x}{2}\right)+c}{\sqrt{-a^2+b^2+c^2}}\right)}{\left(-a^2+b^2+c^2\right)^{5/2}} + \frac{2a^4C+4a^3bB\sin(x)+4a^3cC\sin(x)-c\cos(2x)\left(a^2+2\left(b^2+c^2\right)\right)(bB+cC)}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(B*Cos[x] + C*Sin[x])/(a + b*Cos[x] + c*Sin[x])^3,x]
```

```
[Out] (3*a*(b*B + c*C)*ArcTanh[(c + (a - b)*Tan[x/2])/Sqrt[-a^2 + b^2 + c^2]])/(-
a^2 + b^2 + c^2)^(5/2) + (9*a^2*b*B*c + 2*a^4*C - 4*a^2*b^2*C + 2*b^4*C + 5
*a^2*c^2*C + 4*b^2*c^2*C + 2*c^4*C + 6*a*b*c*(b*B + c*C)*Cos[x] - c*(a^2 +
2*(b^2 + c^2))*(b*B + c*C)*Cos[2*x] + 4*a^3*b*B*Sin[x] + 2*a*b^3*B*Sin[x] +
 8*a*b*B*c^2*Sin[x] + 4*a^3*c*C*Sin[x] + 2*a*b^2*c*C*Sin[x] + 8*a*c^3*C*Sin
[x] + a^2*b^2*B*Sin[2*x] + 2*b^4*B*Sin[2*x] + 2*b^2*B*c^2*Sin[2*x] + a^2*b*
c*C*Sin[2*x] + 2*b^3*c*C*Sin[2*x] + 2*b*c^3*C*Sin[2*x]))/(4*b*(-a^2 + b^2 +
c^2)^2*(a + b*Cos[x] + c*Sin[x])^2)
```

fricas [B] time = 2.20, size = 3264, normalized size = 16.57

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*cos(x)+C*sin(x))/(a+b*cos(x)+c*sin(x))^3,x, algorithm="fricas"
)
```

```
[Out] [1/4*(2*C*a^6*b - 6*C*a^4*b^3 + 6*C*a^2*b^5 - 2*C*b^7 - 6*C*b*c^6 + 2*B*c^7
 - 2*(3*B*a^2 - B*b^2)*c^5 + 2*(4*C*a^2*b - 7*C*b^3)*c^4 + 2*(3*B*a^4 - 5*B
*a^2*b^2 - B*b^4)*c^3 - 2*(2*C*a^4*b - 7*C*a^2*b^3 + 5*C*b^5)*c^2 + 4*(2*B*
b^2*c^5 + 2*C*b*c^6 - (C*a^2*b - 4*C*b^3)*c^4 - (B*a^2*b^2 - 4*B*b^4)*c^3 -
 (C*a^4*b + C*a^2*b^3 - 2*C*b^5)*c^2 - (B*a^4*b^2 + B*a^2*b^4 - 2*B*b^6)*c)
*cos(x)^2 - 3*(B*a^3*b^3 + C*a^3*b^2*c + B*a*b*c^4 + C*a*c^5 + (C*a^3 + C*a
*b^2)*c^3 + (B*a^3*b + B*a*b^3)*c^2 + (B*a*b^5 + C*a*b^4*c - B*a*b*c^4 - C*
a*c^5)*cos(x)^2 + 2*(B*a^2*b^4 + C*a^2*b^3*c + B*a^2*b^2*c^2 + C*a^2*b*c^3)
*cos(x) + 2*(B*a^2*b^3*c + C*a^2*b^2*c^2 + B*a^2*b*c^3 + C*a^2*c^4 + (B*a*b
^4*c + C*a*b^3*c^2 + B*a*b^2*c^3 + C*a*b*c^4)*cos(x))*sin(x))*sqrt(-a^2 + b
^2 + c^2)*log((a^2*b^2 - 2*b^4 - c^4 - (a^2 + 3*b^2)*c^2 - (2*a^2*b^2 - b^4
 - 2*a^2*c^2 + c^4)*cos(x)^2 - 2*(a*b^3 + a*b*c^2)*cos(x) - 2*(a*b^2*c + a*
c^3 - (b*c^3 - (2*a^2*b - b^3)*c)*cos(x))*sin(x) - 2*(2*a*b*c*cos(x)^2 - a*
b*c + (b^2*c + c^3)*cos(x) - (b^3 + b*c^2 + (a*b^2 - a*c^2)*cos(x))*sin(x))
*sqrt(-a^2 + b^2 + c^2))/(2*a*b*cos(x) + (b^2 - c^2)*cos(x)^2 + a^2 + c^2 +
 2*(b*c*cos(x) + a*c)*sin(x))) - 2*(B*a^6 - 4*B*a^4*b^2 + 2*B*a^2*b^4 + B*b
^6)*c + 2*(B*a*b*c^5 + C*a*c^6 + (C*a^3 + 2*C*a*b^2)*c^4 + (B*a^3*b + 2*B*a
*b^3)*c^3 - (2*C*a^5 - C*a^3*b^2 - C*a*b^4)*c^2 - (2*B*a^5*b - B*a^3*b^3 -
B*a*b^5)*c)*cos(x) + 2*(2*B*a^5*b^2 - B*a^3*b^4 - B*a*b^6 - B*a*b^2*c^4 - C
*a*b*c^5 - (C*a^3*b + 2*C*a*b^3)*c^3 - (B*a^3*b^2 + 2*B*a*b^4)*c^2 + (2*C*a
^5*b - C*a^3*b^3 - C*a*b^5)*c + (B*a^4*b^3 + B*a^2*b^5 - 2*B*b^7 + 2*B*b*c^
6 + 2*C*c^7 - (C*a^2 - 2*C*b^2)*c^5 - (B*a^2*b - 2*B*b^3)*c^4 - (C*a^4 + 2*
C*b^4)*c^3 - (B*a^4*b + 2*B*b^5)*c^2 + (C*a^4*b^2 + C*a^2*b^4 - 2*C*b^6)*c)
*cos(x))*sin(x))/(a^8*b^2 - 3*a^6*b^4 + 3*a^4*b^6 - a^2*b^8 - c^10 + 2*(a^2
 - 2*b^2)*c^8 + (5*a^2*b^2 - 6*b^4)*c^6 - (2*a^6 - 3*a^4*b^2 - 3*a^2*b^4 +
4*b^6)*c^4 + (a^8 - 5*a^6*b^2 + 6*a^4*b^4 - a^2*b^6 - b^8)*c^2 + (a^6*b^4 -
 3*a^4*b^6 + 3*a^2*b^8 - b^10 + c^10 - 3*(a^2 - b^2)*c^8 + (3*a^4 - 6*a^2*b
^2 + 2*b^4)*c^6 - (a^6 - 3*a^4*b^2 + 2*b^6)*c^4 - 3*(a^4*b^4 - 2*a^2*b^6 +
b^8)*c^2)*cos(x)^2 + 2*(a^7*b^3 - 3*a^5*b^5 + 3*a^3*b^7 - a*b^9 - a*b*c^8 +
 (3*a^3*b - 4*a*b^3)*c^6 - 3*(a^5*b - 3*a^3*b^3 + 2*a*b^5)*c^4 + (a^7*b - 6
*a^5*b^3 + 9*a^3*b^5 - 4*a*b^7)*c^2)*cos(x) - 2*(a*c^9 - (3*a^3 - 4*a*b^2)*
c^7 + 3*(a^5 - 3*a^3*b^2 + 2*a*b^4)*c^5 - (a^7 - 6*a^5*b^2 + 9*a^3*b^4 - 4*
a*b^6)*c^3 - (a^7*b^2 - 3*a^5*b^4 + 3*a^3*b^6 - a*b^8)*c + (b*c^9 - (3*a^2*
b - 4*b^3)*c^7 + 3*(a^4*b - 3*a^2*b^3 + 2*b^5)*c^5 - (a^6*b - 6*a^4*b^3 + 9
*a^2*b^5 - 4*b^7)*c^3 - (a^6*b^3 - 3*a^4*b^5 + 3*a^2*b^7 - b^9)*c)*cos(x))*
sin(x)), 1/2*(C*a^6*b - 3*C*a^4*b^3 + 3*C*a^2*b^5 - C*b^7 - 3*C*b*c^6 + B*c
^7 - (3*B*a^2 - B*b^2)*c^5 + (4*C*a^2*b - 7*C*b^3)*c^4 + (3*B*a^4 - 5*B*a^2
*b^2 - B*b^4)*c^3 - (2*C*a^4*b - 7*C*a^2*b^3 + 5*C*b^5)*c^2 + 2*(2*B*b^2*c^
5 + 2*C*b*c^6 - (C*a^2*b - 4*C*b^3)*c^4 - (B*a^2*b^2 - 4*B*b^4)*c^3 - (C*a^
4*b + C*a^2*b^3 - 2*C*b^5)*c^2 - (B*a^4*b^2 + B*a^2*b^4 - 2*B*b^6)*c)*cos(x
```

```
)^2 - 3*(B*a^3*b^3 + C*a^3*b^2*c + B*a*b*c^4 + C*a*c^5 + (C*a^3 + C*a*b^2)*
c^3 + (B*a^3*b + B*a*b^3)*c^2 + (B*a*b^5 + C*a*b^4*c - B*a*b*c^4 - C*a*c^5)
*cos(x)^2 + 2*(B*a^2*b^4 + C*a^2*b^3*c + B*a^2*b^2*c^2 + C*a^2*b*c^3)*cos(x
) + 2*(B*a^2*b^3*c + C*a^2*b^2*c^2 + B*a^2*b*c^3 + C*a^2*c^4 + (B*a*b^4*c +
 C*a*b^3*c^2 + B*a*b^2*c^3 + C*a*b*c^4)*cos(x))*sin(x))*sqrt(a^2 - b^2 - c^
2)*arctan(-(a*b*cos(x) + a*c*sin(x) + b^2 + c^2)*sqrt(a^2 - b^2 - c^2)/((c^
3 - (a^2 - b^2)*c)*cos(x) + (a^2*b - b^3 - b*c^2)*sin(x))) - (B*a^6 - 4*B*a
^4*b^2 + 2*B*a^2*b^4 + B*b^6)*c + (B*a*b*c^5 + C*a*c^6 + (C*a^3 + 2*C*a*b^2
)*c^4 + (B*a^3*b + 2*B*a*b^3)*c^3 - (2*C*a^5 - C*a^3*b^2 - C*a*b^4)*c^2 - (
2*B*a^5*b - B*a^3*b^3 - B*a*b^5)*c)*cos(x) + (2*B*a^5*b^2 - B*a^3*b^4 - B*a
*b^6 - B*a*b^2*c^4 - C*a*b*c^5 - (C*a^3*b + 2*C*a*b^3)*c^3 - (B*a^3*b^2 + 2
*B*a*b^4)*c^2 + (2*C*a^5*b - C*a^3*b^3 - C*a*b^5)*c + (B*a^4*b^3 + B*a^2*b^
5 - 2*B*b^7 + 2*B*b*c^6 + 2*C*c^7 - (C*a^2 - 2*C*b^2)*c^5 - (B*a^2*b - 2*B*
b^3)*c^4 - (C*a^4 + 2*C*b^4)*c^3 - (B*a^4*b + 2*B*b^5)*c^2 + (C*a^4*b^2 + C
*a^2*b^4 - 2*C*b^6)*c)*cos(x))*sin(x))/(a^8*b^2 - 3*a^6*b^4 + 3*a^4*b^6 - a
^2*b^8 - c^10 + 2*(a^2 - 2*b^2)*c^8 + (5*a^2*b^2 - 6*b^4)*c^6 - (2*a^6 - 3*
a^4*b^2 - 3*a^2*b^4 + 4*b^6)*c^4 + (a^8 - 5*a^6*b^2 + 6*a^4*b^4 - a^2*b^6 -
 b^8)*c^2 + (a^6*b^4 - 3*a^4*b^6 + 3*a^2*b^8 - b^10 + c^10 - 3*(a^2 - b^2)*
c^8 + (3*a^4 - 6*a^2*b^2 + 2*b^4)*c^6 - (a^6 - 3*a^4*b^2 + 2*b^6)*c^4 - 3*(
a^4*b^4 - 2*a^2*b^6 + b^8)*c^2)*cos(x)^2 + 2*(a^7*b^3 - 3*a^5*b^5 + 3*a^3*b
^7 - a*b^9 - a*b*c^8 + (3*a^3*b - 4*a*b^3)*c^6 - 3*(a^5*b - 3*a^3*b^3 + 2*a
*b^5)*c^4 + (a^7*b - 6*a^5*b^3 + 9*a^3*b^5 - 4*a*b^7)*c^2)*cos(x) - 2*(a*c^
9 - (3*a^3 - 4*a*b^2)*c^7 + 3*(a^5 - 3*a^3*b^2 + 2*a*b^4)*c^5 - (a^7 - 6*a^
5*b^2 + 9*a^3*b^4 - 4*a*b^6)*c^3 - (a^7*b^2 - 3*a^5*b^4 + 3*a^3*b^6 - a*b^8
)*c + (b*c^9 - (3*a^2*b - 4*b^3)*c^7 + 3*(a^4*b - 3*a^2*b^3 + 2*b^5)*c^5 -
(a^6*b - 6*a^4*b^3 + 9*a^2*b^5 - 4*b^7)*c^3 - (a^6*b^3 - 3*a^4*b^5 + 3*a^2*
b^7 - b^9)*c)*cos(x))*sin(x))]
```

giac [B] time = 0.51, size = 1034, normalized size = 5.25

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*cos(x)+C*sin(x))/(a+b*cos(x)+c*sin(x))^3,x, algorithm="giac")
```

```
[Out] 3*(B*a*b + C*a*c)*(pi*floor(1/2*x/pi + 1/2)*sgn(-2*a + 2*b) + arctan(-(a*ta
n(1/2*x) - b*tan(1/2*x) + c)/sqrt(a^2 - b^2 - c^2)))/((a^4 - 2*a^2*b^2 + b^
4 - 2*a^2*c^2 + 2*b^2*c^2 + c^4)*sqrt(a^2 - b^2 - c^2)) + (2*B*a^5*tan(1/2*
x)^3 - 5*B*a^4*b*tan(1/2*x)^3 + 5*B*a^3*b^2*tan(1/2*x)^3 - 5*B*a^2*b^3*tan(
1/2*x)^3 + 5*B*a*b^4*tan(1/2*x)^3 - 2*B*b^5*tan(1/2*x)^3 - 3*C*a^4*c*tan(1/
2*x)^3 + 9*C*a^3*b*c*tan(1/2*x)^3 - 9*C*a^2*b^2*c*tan(1/2*x)^3 + 3*C*a*b^3*
c*tan(1/2*x)^3 - 4*B*a^3*c^2*tan(1/2*x)^3 + 4*B*a^2*b*c^2*tan(1/2*x)^3 + 4*
B*a*b^2*c^2*tan(1/2*x)^3 - 4*B*b^3*c^2*tan(1/2*x)^3 + 2*B*a*c^4*tan(1/2*x)^
3 - 2*B*b*c^4*tan(1/2*x)^3 - 2*C*a^5*tan(1/2*x)^2 + 2*C*a^4*b*tan(1/2*x)^2
+ 4*C*a^3*b^2*tan(1/2*x)^2 - 4*C*a^2*b^3*tan(1/2*x)^2 - 2*C*a*b^4*tan(1/2*x
```

```
)^2 + 2*C*b^5*tan(1/2*x)^2 + 2*B*a^4*c*tan(1/2*x)^2 - 9*B*a^3*b*c*tan(1/2*x
)^2 + 14*B*a^2*b^2*c*tan(1/2*x)^2 - 9*B*a*b^3*c*tan(1/2*x)^2 + 2*B*b^4*c*ta
n(1/2*x)^2 - 5*C*a^3*c^2*tan(1/2*x)^2 + 14*C*a^2*b*c^2*tan(1/2*x)^2 - 13*C*
a*b^2*c^2*tan(1/2*x)^2 + 4*C*b^3*c^2*tan(1/2*x)^2 - 4*B*a^2*c^3*tan(1/2*x)^
2 + 4*B*b^2*c^3*tan(1/2*x)^2 - 2*C*a*c^4*tan(1/2*x)^2 + 2*C*b*c^4*tan(1/2*x
)^2 + 2*B*c^5*tan(1/2*x)^2 + 2*B*a^5*tan(1/2*x) - 3*B*a^4*b*tan(1/2*x) + B*
a^3*b^2*tan(1/2*x) + B*a^2*b^3*tan(1/2*x) - 3*B*a*b^4*tan(1/2*x) + 2*B*b^5*
tan(1/2*x) - 5*C*a^4*c*tan(1/2*x) + 5*C*a^3*b*c*tan(1/2*x) + 5*C*a^2*b^2*c*
tan(1/2*x) - 5*C*a*b^3*c*tan(1/2*x) - 4*B*a^3*c^2*tan(1/2*x) - 8*B*a^2*b*c^
2*tan(1/2*x) + 8*B*a*b^2*c^2*tan(1/2*x) + 4*B*b^3*c^2*tan(1/2*x) - 4*C*a^2*
c^3*tan(1/2*x) + 4*C*a*b*c^3*tan(1/2*x) + 2*B*a*c^4*tan(1/2*x) + 2*B*b*c^4*
tan(1/2*x) - 2*C*a^5 + 4*C*a^3*b^2 - 2*C*a*b^4 - 5*B*a^3*b*c + 5*B*a*b^3*c
- C*a^3*c^2 + C*a*b^2*c^2 + 2*B*a*b*c^3)/((a^6 - 2*a^5*b - a^4*b^2 + 4*a^3*
b^3 - a^2*b^4 - 2*a*b^5 + b^6 - 2*a^4*c^2 + 4*a^3*b*c^2 - 4*a*b^3*c^2 + 2*b
^4*c^2 + a^2*c^4 - 2*a*b*c^4 + b^2*c^4)*(a*tan(1/2*x)^2 - b*tan(1/2*x)^2 +
2*c*tan(1/2*x) + a + b)^2)
```

maple [B] time = 0.22, size = 881, normalized size = 4.47

$$-\frac{2\left(-\frac{\left(2B a^4-3B a^3 b+2B a^2 b^2-4B a^2 c^2-3Ba b^3+2B b^4+4B b^2 c^2+2B c^4-3C a^3 c+6C a^2 b c-3C a b^2 c\right)\left(\tan^3\left(\frac{x}{2}\right)\right)}{2\left(a^4-2a^2b^2-2a^2c^2+b^4+2b^2c^2+c^4\right)(a-b)}-\frac{\left(2B a^4 c-9B a^3 b c+14B a^2 b^2 c-4B a^2 c\right.}{}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*cos(x)+C*sin(x))/(a+b*cos(x)+c*sin(x))^3,x)
```

```
[Out] -2*(-1/2*(2*B*a^4-3*B*a^3*b+2*B*a^2*b^2-4*B*a^2*c^2-3*B*a*b^3+2*B*b^4+4*B*b
^2*c^2+2*B*c^4-3*C*a^3*c+6*C*a^2*b*c-3*C*a*b^2*c)/(a^4-2*a^2*b^2-2*a^2*c^2+
b^4+2*b^2*c^2+c^4)/(a-b)*tan(1/2*x)^3-1/2*(2*B*a^4*c-9*B*a^3*b*c+14*B*a^2*b
^2*c-4*B*a^2*c^3-9*B*a*b^3*c+2*B*b^4*c+4*B*b^2*c^3+2*B*c^5-2*C*a^5+2*C*a^4*
b+4*C*a^3*b^2-5*C*a^3*c^2-4*C*a^2*b^3+14*C*a^2*b*c^2-2*C*a*b^4-13*C*a*b^2*c
^2-2*C*a*c^4+2*C*b^5+4*C*b^3*c^2+2*C*b*c^4)/(a^4-2*a^2*b^2-2*a^2*c^2+b^4+2*
b^2*c^2+c^4)/(a^2-2*a*b+b^2)*tan(1/2*x)^2-1/2*(2*B*a^5-3*B*a^4*b+B*a^3*b^2-
4*B*a^3*c^2+B*a^2*b^3-8*B*a^2*b*c^2-3*B*a*b^4+8*B*a*b^2*c^2+2*B*a*c^4+2*B*b
^5+4*B*b^3*c^2+2*B*b*c^4-5*C*a^4*c+5*C*a^3*b*c+5*C*a^2*b^2*c-4*C*a^2*c^3-5*
C*a*b^3*c+4*C*a*b*c^3)/(a^4-2*a^2*b^2-2*a^2*c^2+b^4+2*b^2*c^2+c^4)/(a^2-2*a
*b+b^2)*tan(1/2*x)+1/2*a*(5*B*a^2*b*c-5*B*b^3*c-2*B*b*c^3+2*C*a^4-4*C*a^2*b
^2+C*a^2*c^2+2*C*b^4-C*b^2*c^2)/(a^4-2*a^2*b^2-2*a^2*c^2+b^4+2*b^2*c^2+c^4)
/(a^2-2*a*b+b^2))/(a*tan(1/2*x)^2-b*tan(1/2*x)^2+2*c*tan(1/2*x)+a+b)^2-3/(a
^4-2*a^2*b^2-2*a^2*c^2+b^4+2*b^2*c^2+c^4)/(a^2-b^2-c^2)^(1/2)*arctan(1/2*(2
*(a-b)*tan(1/2*x)+2*c)/(a^2-b^2-c^2)^(1/2))*a*b*B-3/(a^4-2*a^2*b^2-2*a^2*c^
2+b^4+2*b^2*c^2+c^4)/(a^2-b^2-c^2)^(1/2)*arctan(1/2*(2*(a-b)*tan(1/2*x)+2*c
)/(a^2-b^2-c^2)^(1/2))*a*c*C
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*cos(x)+C*sin(x))/(a+b*cos(x)+c*sin(x))^3,x, algorithm="maxima"
)
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(c^2+b^2-a^2>0)', see `assume?` for
more details)Is c^2+b^2-a^2 positive or negative?
```

mupad [B] time = 6.36, size = 923, normalized size = 4.69

$$\frac{\frac{\tan\left(\frac{x}{2}\right)^3 \left(2\,B\,a^4-3\,B\,a^3\,b-3\,C\,a^3\,c+2\,B\,a^2\,b^2+6\,C\,a^2\,b\,c-4\,B\,a^2\,c^2-3\,B\,a\,b^3-3\,C\,a\,b^2\,c+2\,B\,b^4+4\,B\,b^2\,c^2+2\,B\,c^4\right)}{(a-b)\left(a^4-2\,a^2\,b^2-2\,a^2\,c^2+b^4+2\,b^2\,c^2+c^4\right)} - \frac{2\,C\,a^5-4\,C\,a^3\,b^2+5\,B\,a^3\,b\,c+C\,a^3\,c^2+2}{(a-b)^2\left(a^4-2\,a^2\,b^2-2\,a\right.}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*cos(x) + C*sin(x))/(a + b*cos(x) + c*sin(x))^3,x)
```

```
[Out] ((tan(x/2)^3*(2*B*a^4 + 2*B*b^4 + 2*B*c^4 + 2*B*a^2*b^2 - 4*B*a^2*c^2 + 4*B
*b^2*c^2 - 3*B*a*b^3 - 3*B*a^3*b - 3*C*a^3*c - 3*C*a*b^2*c + 6*C*a^2*b*c))/
((a - b)*(a^4 + b^4 + c^4 - 2*a^2*b^2 - 2*a^2*c^2 + 2*b^2*c^2)) - (2*C*a^5
- 4*C*a^3*b^2 + C*a^3*c^2 + 2*C*a*b^4 - 2*B*a*b*c^3 - 5*B*a*b^3*c + 5*B*a^3
*b*c - C*a*b^2*c^2)/((a - b)^2*(a^4 + b^4 + c^4 - 2*a^2*b^2 - 2*a^2*c^2 + 2
*b^2*c^2)) + (tan(x/2)^2*(2*B*c^5 - 2*C*a^5 + 2*C*b^5 - 4*B*a^2*c^3 - 4*C*a
^2*b^3 + 4*C*a^3*b^2 + 4*B*b^2*c^3 - 5*C*a^3*c^2 + 4*C*b^3*c^2 + 2*B*a^4*c
- 2*C*a*b^4 + 2*C*a^4*b + 2*B*b^4*c - 2*C*a*c^4 + 2*C*b*c^4 - 9*B*a*b^3*c -
 9*B*a^3*b*c + 14*B*a^2*b^2*c - 13*C*a*b^2*c^2 + 14*C*a^2*b*c^2))/((a - b)^
2*(a^4 + b^4 + c^4 - 2*a^2*b^2 - 2*a^2*c^2 + 2*b^2*c^2)) + (tan(x/2)*(2*B*a
^5 + 2*B*b^5 + B*a^2*b^3 + B*a^3*b^2 - 4*B*a^3*c^2 + 4*B*b^3*c^2 - 4*C*a^2*
c^3 - 3*B*a*b^4 - 3*B*a^4*b + 2*B*a*c^4 + 2*B*b*c^4 - 5*C*a^4*c + 4*C*a*b*c
^3 - 5*C*a*b^3*c + 5*C*a^3*b*c + 8*B*a*b^2*c^2 - 8*B*a^2*b*c^2 + 5*C*a^2*b^
2*c))/((a - b)^2*(a^4 + b^4 + c^4 - 2*a^2*b^2 - 2*a^2*c^2 + 2*b^2*c^2)))/(t
an(x/2)^4*(a^2 - 2*a*b + b^2) + 2*a*b + tan(x/2)*(4*a*c + 4*b*c) + tan(x/2)
^3*(4*a*c - 4*b*c) + a^2 + b^2 + tan(x/2)^2*(2*a^2 - 2*b^2 + 4*c^2)) + (3*a
*atanh((3*a*(B*b + C*c)*(tan(x/2)*(2*a - 2*b) + (2*a^4*c + 2*b^4*c + 2*c^5
- 4*a^2*c^3 + 4*b^2*c^3 - 4*a^2*b^2*c)/(a^4 + b^4 + c^4 - 2*a^2*b^2 - 2*a^2
*c^2 + 2*b^2*c^2))*(a^4 + b^4 + c^4 - 2*a^2*b^2 - 2*a^2*c^2 + 2*b^2*c^2))/(
2*(3*B*a*b + 3*C*a*c)*(b^2 - a^2 + c^2)^(5/2)))*(B*b + C*c))/(b^2 - a^2 + c
^2)^(5/2)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*cos(x)+C*sin(x))/(a+b*cos(x)+c*sin(x))**3,x)
[Out] Timed out
```

3.548 $\int \frac{B\cos(x)+C\sin(x)}{a+b\cos(x)+ib\sin(x)}\,dx$

Optimal. Leaf size=92

$$-\frac{\left(a^2(C+iB)+ib^2(B+iC)\right)\log(a+ib\sin(x)+b\cos(x))}{2a^2b}-\frac{bx(B+iC)}{2a^2}+\frac{(-C+iB)(\cos(x)-i\sin(x))}{2a}$$

```
[Out] -1/2*b*(B+I*C)*x/a^2-1/2*(I*b^2*(B+I*C)+a^2*(I*B+C))*ln(a+b*cos(x)+I*b*sin(
x))/a^2/b+1/2*(I*B-C)*(cos(x)-I*sin(x))/a
```

Rubi [A] time = 0.08, antiderivative size = 87, normalized size of antiderivative = 0.95, number of steps used = 1, number of rules used = 1, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.040, Rules used = {3130}

$$-\frac{\left(\frac{ib^2(B+iC)}{a^2}+iB+C\right)\log(a+ib\sin(x)+b\cos(x))}{2b}-\frac{bx(B+iC)}{2a^2}+\frac{(-C+iB)(\cos(x)-i\sin(x))}{2a}$$

Antiderivative was successfully verified.

```
[In] Int[(B*Cos[x] + C*Sin[x])/(a + b*Cos[x] + I*b*Sin[x]),x]
```

```
[Out] -(b*(B + I*C)*x)/(2*a^2) - ((I*B + (I*b^2*(B + I*C))/a^2 + C)*Log[a + b*Cos
[x] + I*b*Sin[x]])/(2*b) + ((I*B - C)*(Cos[x] - I*Sin[x]))/(2*a)
```

Rule 3130

```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])
/(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)]), x_
Symbol] :> Simp[((2*a*A - b*B - c*C)*x)/(2*a^2), x] + (-Simp[((b*B + c*C)*(
b*Cos[d + e*x] - c*Sin[d + e*x]))/(2*a*b*c*e), x] + Simp[((a^2*(b*B - c*C)
- 2*a*A*b^2 + b^2*(b*B + c*C))*Log[RemoveContent[a + b*Cos[d + e*x] + c*Sin
[d + e*x], x]])/(2*a^2*b*c*e), x]) /; FreeQ[{a, b, c, d, e, A, B, C}, x] &&
 EqQ[b^2 + c^2, 0]
```

Rubi steps

$$\int \frac{B\cos(x)+C\sin(x)}{a+b\cos(x)+ib\sin(x)}\,dx=-\frac{b(B+iC)x}{2a^2}-\frac{\left(iB+\frac{ib^2(B+iC)}{a^2}+C\right)\log(a+b\cos(x)+ib\sin(x))}{2b}+\frac{(iB-C)(\cos(}{2}$$

Mathematica [B] time = 0.32, size = 195, normalized size = 2.12

$$\frac{x\left(a^2B-ia^2C-b^2B-ib^2C\right)}{4a^2b}-\frac{i\left(a^2B-ia^2C+b^2B+ib^2C\right)\log\left(a^2+2ab\cos(x)+b^2\right)}{4a^2b}-\frac{\left(a^2B-ia^2C+b^2B+ib^2C\right.}{2a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(B*Cos[x] + C*Sin[x])/(a + b*Cos[x] + I*b*Sin[x]),x]
```

```
[Out] ((a^2*B - b^2*B - I*a^2*C - I*b^2*C)*x)/(4*a^2*b) - ((a^2*B + b^2*B - I*a^2
*C + I*b^2*C)*ArcTan[((a + b)*Cos[x/2])/(-(a*Sin[x/2]) + b*Sin[x/2])])/(2*a
^2*b) + ((I/2)*(B + I*C)*Cos[x])/a - ((I/4)*(a^2*B + b^2*B - I*a^2*C + I*b^
2*C)*Log[a^2 + b^2 + 2*a*b*Cos[x]])/(a^2*b) + ((B + I*C)*Sin[x])/(2*a)
```

fricas [A] time = 1.05, size = 78, normalized size = 0.85

$$-\frac{\left((B+iC)b^2xe^{(ix)}-(iB-C)ab-\left((-iB-C)a^2+(-iB+C)b^2\right)e^{(ix)}\log\left(\frac{be^{(ix)}+a}{b}\right)\right)e^{(-ix)}}{2\,a^2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*cos(x)+C*sin(x))/(a+b*cos(x)+I*b*sin(x)),x, algorithm="fricas"
)
```

```
[Out] -1/2*((B + I*C)*b^2*x*e^(I*x) - (I*B - C)*a*b - ((-I*B - C)*a^2 + (-I*B + C
)*b^2)*e^(I*x)*log((b*e^(I*x) + a)/b))*e^(-I*x)/(a^2*b)
```

giac [B] time = 0.16, size = 178, normalized size = 1.93

$$-\frac{(iBb-Cb)\log\left(-a\tan\left(\frac{1}{2}x\right)^2+b\tan\left(\frac{1}{2}x\right)^2-2ia\tan\left(\frac{1}{2}x\right)+a+b\right)}{4\,a^2}-\frac{(-iBb+Cb)\log\left(\tan\left(\frac{1}{2}x\right)-i\right)}{2\,a^2}+\frac{(2\,Ba}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*cos(x)+C*sin(x))/(a+b*cos(x)+I*b*sin(x)),x, algorithm="giac")
```

```
[Out] -1/4*(I*B*b - C*b)*log(-a*tan(1/2*x)^2 + b*tan(1/2*x)^2 - 2*I*a*tan(1/2*x)
+ a + b)/a^2 - 1/2*(-I*B*b + C*b)*log(tan(1/2*x) - I)/a^2 + 1/4*(2*B*a^2 -
2*I*C*a^2 + B*b^2 + I*C*b^2)*(x + 2*arctan((-I*a*cos(x) - a*sin(x) - I*a)/(
a*cos(x) - I*a*sin(x) - a + 2*b)))/(a^2*b) - 1/2*(I*B*b*tan(1/2*x) - C*b*ta
n(1/2*x) - 2*B*a - 2*I*C*a + B*b + I*C*b)/(a^2*(tan(1/2*x) - I))
```

maple [B] time = 0.20, size = 212, normalized size = 2.30

$$\frac{C\ln\left(\tan\left(\frac{x}{2}\right)+i\right)}{2b}+\frac{iB\ln\left(\tan\left(\frac{x}{2}\right)+i\right)}{2b}+\frac{iC}{a\left(\tan\left(\frac{x}{2}\right)-i\right)}+\frac{B}{a\left(\tan\left(\frac{x}{2}\right)-i\right)}+\frac{i\ln\left(\tan\left(\frac{x}{2}\right)-i\right)bB}{2a^2}-\frac{\ln\left(\tan\left(\frac{x}{2}\right)-i\right)b}{2a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*cos(x)+C*sin(x))/(a+b*cos(x)+I*b*sin(x)),x)
```

```
[Out] 1/2*C/b*ln(tan(1/2*x)+I)+1/2*I*B/b*ln(tan(1/2*x)+I)+I*C/a/(tan(1/2*x)-I)+B/
a/(tan(1/2*x)-I)+1/2*I/a^2*ln(tan(1/2*x)-I)*b*B-1/2/a^2*ln(tan(1/2*x)-I)*b*
C-1/2/b*ln(I*a+I*b+a*tan(1/2*x)-b*tan(1/2*x))*C+1/2/a^2*b*ln(I*a+I*b+a*tan(
1/2*x)-b*tan(1/2*x))*C-1/2*I/b*ln(I*a+I*b+a*tan(1/2*x)-b*tan(1/2*x))*B-1/2*
I/a^2*b*ln(I*a+I*b+a*tan(1/2*x)-b*tan(1/2*x))*B
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*cos(x)+C*sin(x))/(a+b*cos(x)+I*b*sin(x)),x, algorithm="maxima"
)
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

mupad [B] time = 5.32, size = 118, normalized size = 1.28

$$-\ln\left(a+b-a\tan\left(\frac{x}{2}\right)\,1\mathrm{i}+b\tan\left(\frac{x}{2}\right)\,1\mathrm{i}\right)\left(\frac{\frac{C}{2}+\frac{B\,1\mathrm{i}}{2}}{b}+\frac{-\frac{C\,b^2}{2}+\frac{B\,b^2\,1\mathrm{i}}{2}}{a^2\,b}\right)+\frac{B+C\,1\mathrm{i}}{a\,\left(\tan\left(\frac{x}{2}\right)-\mathrm{i}\right)}+\frac{\ln\left(\tan\left(\frac{x}{2}\right)+1\mathrm{i}\right)\,\left(C+B\,1\right.}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*cos(x) + C*sin(x))/(a + b*cos(x) + b*sin(x)*1i),x)
```

```
[Out] (B + C*1i)/(a*(tan(x/2) - 1i)) - log(a + b - a*tan(x/2)*1i + b*tan(x/2)*1i)
*(((B*1i)/2 + C/2)/b + ((B*b^2*1i)/2 - (C*b^2)/2)/(a^2*b)) + (log(tan(x/2)
+ 1i)*(B*1i + C))/(2*b) + (log(tan(x/2) - 1i)*(B*b*1i - C*b))/(2*a^2)
```

sympy [A] time = 0.85, size = 116, normalized size = 1.26

$$\begin{cases} -\frac{(-iB+C)e^{-ix}}{2a} & \text{for}\ 2a\neq 0 \\ x\left(-\frac{-Bb-iCb}{2a^2}-\frac{i(iBa-iBb-Ca+Cb)}{2a^2}\right) & \text{otherwise} \end{cases}-\frac{x\,(Bb+iCb)}{2a^2}-\frac{i\left(Ba^2+Bb^2-iCa^2+iCb^2\right)\log\left(\frac{a}{b}+e^{ix}\right)}{2a^2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*cos(x)+C*sin(x))/(a+b*cos(x)+I*b*sin(x)),x)
```

```
[Out] Piecewise((-(-I*B + C)*exp(-I*x)/(2*a), Ne(2*a, 0)), (x*(-(-B*b - I*C*b)/(2
*a**2) - I*(I*B*a - I*B*b - C*a + C*b)/(2*a**2)), True)) - x*(B*b + I*C*b)/
(2*a**2) - I*(B*a**2 + B*b**2 - I*C*a**2 + I*C*b**2)*log(a/b + exp(I*x))/(2
*a**2*b)
```

3.549 $\int \frac{B\cos(x)+C\sin(x)}{a+b\cos(x)-ib\sin(x)}\,dx$

Optimal. Leaf size=90

$$\frac{\left(ia^2(B+iC)+b^2(C+iB)\right)\log(a-ib\sin(x)+b\cos(x))}{2a^2b}-\frac{bx(B-iC)}{2a^2}-\frac{(C+iB)(\cos(x)+i\sin(x))}{2a}$$

```
[Out] -1/2*b*(B-I*C)*x/a^2+1/2*(I*a^2*(B+I*C)+b^2*(I*B+C))*ln(a+b*cos(x)-I*b*sin(
x))/a^2/b-1/2*(I*B+C)*(cos(x)+I*sin(x))/a
```

Rubi [A] time = 0.08, antiderivative size = 85, normalized size of antiderivative = 0.94, number of steps used = 1, number of rules used = 1, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.040, Rules used = {3130}

$$-\frac{bx(B-iC)}{2a^2}+\frac{1}{2}\left(\frac{b(C+iB)}{a^2}+\frac{i(B+iC)}{b}\right)\log(a-ib\sin(x)+b\cos(x))-\frac{(C+iB)(\cos(x)+i\sin(x))}{2a}$$

Antiderivative was successfully verified.

```
[In] Int[(B*Cos[x] + C*Sin[x])/(a + b*Cos[x] - I*b*Sin[x]),x]
```

```
[Out] -(b*(B - I*C)*x)/(2*a^2) + (((I*(B + I*C))/b + (b*(I*B + C))/a^2)*Log[a + b
*Cos[x] - I*b*Sin[x]])/2 - ((I*B + C)*(Cos[x] + I*Sin[x]))/(2*a)
```

Rule 3130

```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])
/(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)]), x_
Symbol] :> Simp[((2*a*A - b*B - c*C)*x)/(2*a^2), x] + (-Simp[((b*B + c*C)*(
b*Cos[d + e*x] - c*Sin[d + e*x]))/(2*a*b*c*e), x] + Simp[((a^2*(b*B - c*C)
- 2*a*A*b^2 + b^2*(b*B + c*C))*Log[RemoveContent[a + b*Cos[d + e*x] + c*Sin
[d + e*x], x]])/(2*a^2*b*c*e), x]) /; FreeQ[{a, b, c, d, e, A, B, C}, x] &&
 EqQ[b^2 + c^2, 0]
```

Rubi steps

$$\int \frac{B\cos(x)+C\sin(x)}{a+b\cos(x)-ib\sin(x)}\,dx=-\frac{b(B-iC)x}{2a^2}+\frac{1}{2}\left(\frac{i(B+iC)}{b}+\frac{b(iB+C)}{a^2}\right)\log(a+b\cos(x)-ib\sin(x))-\frac{(iB+$$

Mathematica [B] time = 0.29, size = 195, normalized size = 2.17

$$\frac{x\left(a^2B+ia^2C-b^2B+ib^2C\right)}{4a^2b}+\frac{i\left(a^2B+ia^2C+b^2B-ib^2C\right)\log\left(a^2+2ab\cos(x)+b^2\right)}{4a^2b}+\frac{\left(a^2B+ia^2C+b^2B-ib^2\right.}{2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(B*Cos[x] + C*Sin[x])/(a + b*Cos[x] - I*b*Sin[x]),x]
```

```
[Out] ((a^2*B - b^2*B + I*a^2*C + I*b^2*C)*x)/(4*a^2*b) + ((a^2*B + b^2*B + I*a^2
*C - I*b^2*C)*ArcTan[((a + b)*Cos[x/2])/(a*Sin[x/2] - b*Sin[x/2])])/(2*a^2*
b) - ((I/2)*(B - I*C)*Cos[x])/a + ((I/4)*(a^2*B + b^2*B + I*a^2*C - I*b^2*C
)*Log[a^2 + b^2 + 2*a*b*Cos[x]])/(a^2*b) + ((B - I*C)*Sin[x])/(2*a)
```

fricas [A] time = 0.96, size = 68, normalized size = 0.76

$$\frac{(B+iC)a^2x+(-iB-C)abe^{(ix)}+\left((iB-C)a^2+(iB+C)b^2\right)\log\left(\frac{ae^{(ix)}+b}{a}\right)}{2\,a^2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*cos(x)+C*sin(x))/(a+b*cos(x)-I*b*sin(x)),x, algorithm="fricas"
)
```

```
[Out] 1/2*((B + I*C)*a^2*x + (-I*B - C)*a*b*e^(I*x) + ((I*B - C)*a^2 + (I*B + C)*
b^2)*log((a*e^(I*x) + b)/a))/(a^2*b)
```

giac [B] time = 0.16, size = 178, normalized size = 1.98

$$-\frac{(-iBb-Cb)\log\left(-a\tan\left(\frac{1}{2}x\right)^2+b\tan\left(\frac{1}{2}x\right)^2+2ia\tan\left(\frac{1}{2}x\right)+a+b\right)}{4\,a^2}-\frac{(iBb+Cb)\log\left(\tan\left(\frac{1}{2}x\right)+i\right)}{2\,a^2}+\frac{(2\,Ba^2}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*cos(x)+C*sin(x))/(a+b*cos(x)-I*b*sin(x)),x, algorithm="giac")
```

```
[Out] -1/4*(-I*B*b - C*b)*log(-a*tan(1/2*x)^2 + b*tan(1/2*x)^2 + 2*I*a*tan(1/2*x)
 + a + b)/a^2 - 1/2*(I*B*b + C*b)*log(tan(1/2*x) + I)/a^2 + 1/4*(2*B*a^2 +
2*I*C*a^2 + B*b^2 - I*C*b^2)*(x + 2*arctan((I*a*cos(x) - a*sin(x) + I*a)/(a
*cos(x) + I*a*sin(x) - a + 2*b)))/(a^2*b) - 1/2*(-I*B*b*tan(1/2*x) - C*b*ta
n(1/2*x) - 2*B*a + 2*I*C*a + B*b - I*C*b)/(a^2*(tan(1/2*x) + I))
```

maple [B] time = 0.19, size = 388, normalized size = 4.31

$$-\frac{iC}{a\left(\tan\left(\frac{x}{2}\right)+i\right)}+\frac{B}{a\left(\tan\left(\frac{x}{2}\right)+i\right)}-\frac{i\ln\left(\tan\left(\frac{x}{2}\right)+i\right)bB}{2a^2}-\frac{\ln\left(\tan\left(\frac{x}{2}\right)+i\right)bC}{2a^2}+\frac{C\ln\left(\tan\left(\frac{x}{2}\right)-i\right)}{2b}-\frac{iB\ln\left(\tan\left(\frac{x}{2}\right)-\right.}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*cos(x)+C*sin(x))/(a+b*cos(x)-I*b*sin(x)),x)
```

```
[Out] -I*C/a/(tan(1/2*x)+I)+B/a/(tan(1/2*x)+I)-1/2*I/a^2*ln(tan(1/2*x)+I)*b*B-1/2
/a^2*ln(tan(1/2*x)+I)*b*C+1/2*C/b*ln(tan(1/2*x)-I)-1/2*I*B/b*ln(tan(1/2*x)-
I)+1/2*a/b/(-a+b)*ln(I*a+I*b-a*tan(1/2*x)+b*tan(1/2*x))*C-1/2/(-a+b)*ln(I*a
+I*b-a*tan(1/2*x)+b*tan(1/2*x))*C-1/2/a*b/(-a+b)*ln(I*a+I*b-a*tan(1/2*x)+b*
tan(1/2*x))*C+1/2/a^2*b^2/(-a+b)*ln(I*a+I*b-a*tan(1/2*x)+b*tan(1/2*x))*C-1/
2*I*a/b/(-a+b)*ln(I*a+I*b-a*tan(1/2*x)+b*tan(1/2*x))*B+1/2*I/(-a+b)*ln(I*a+
I*b-a*tan(1/2*x)+b*tan(1/2*x))*B-1/2*I/a*b/(-a+b)*ln(I*a+I*b-a*tan(1/2*x)+b
*tan(1/2*x))*B+1/2*I/a^2*b^2/(-a+b)*ln(I*a+I*b-a*tan(1/2*x)+b*tan(1/2*x))*B
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*cos(x)+C*sin(x))/(a+b*cos(x)-I*b*sin(x)),x, algorithm="maxima"
)
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

mupad [B] time = 4.51, size = 118, normalized size = 1.31

$$\ln\left(a+b+a\tan\left(\frac{x}{2}\right)\,1\mathrm{i}-b\tan\left(\frac{x}{2}\right)\,1\mathrm{i}\right)\left(\frac{-\frac{C}{2}+\frac{B\,1\mathrm{i}}{2}}{b}+\frac{\frac{C\,b^2}{2}+\frac{B\,b^2\,1\mathrm{i}}{2}}{a^2\,b}\right)+\frac{B-C\,1\mathrm{i}}{a\,\left(\tan\left(\frac{x}{2}\right)+1\mathrm{i}\right)}-\frac{\ln\left(\tan\left(\frac{x}{2}\right)+1\mathrm{i}\right)\,\left(C\,b+\right.}{2\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*cos(x) + C*sin(x))/(a + b*cos(x) - b*sin(x)*1i),x)
```

```
[Out] log(a + b + a*tan(x/2)*1i - b*tan(x/2)*1i)*(((B*1i)/2 - C/2)/b + ((B*b^2*1i
)/2 + (C*b^2)/2)/(a^2*b)) + (B - C*1i)/(a*(tan(x/2) + 1i)) - (log(tan(x/2)
+ 1i)*(B*b*1i + C*b))/(2*a^2) - (log(tan(x/2) - 1i)*(B*1i - C))/(2*b)
```

sympy [A] time = 0.81, size = 102, normalized size = 1.13

$$\begin{cases} -\frac{(iB+C)e^{ix}}{2a} & \text{for}\ 2a \neq 0 \\ x\left(-\frac{B+iC}{2b}+\frac{Ba+Bb+iCa-iCb}{2ab}\right) & \text{otherwise} \end{cases} - \frac{x\,(-B-iC)}{2b} + \frac{i\left(Ba^2+Bb^2+iCa^2-iCb^2\right)\log\left(e^{ix}+\frac{b}{a}\right)}{2a^2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*cos(x)+C*sin(x))/(a+b*cos(x)-I*b*sin(x)),x)
```

```
[Out] Piecewise((-(I*B + C)*exp(I*x)/(2*a), Ne(2*a, 0)), (x*(-(B + I*C)/(2*b) + (
B*a + B*b + I*C*a - I*C*b)/(2*a*b)), True)) - x*(-B - I*C)/(2*b) + I*(B*a**
2 + B*b**2 + I*C*a**2 - I*C*b**2)*log(exp(I*x) + b/a)/(2*a**2*b)
```

3.550 $\int \frac{A+B\cos(x)+C\sin(x)}{a+b\cos(x)+c\sin(x)}\,dx$

Optimal. Leaf size=131

$$\frac{2\tan^{-1}\left(\frac{(a-b)\tan\left(\frac{x}{2}\right)+c}{\sqrt{a^2-b^2-c^2}}\right)\left(A\left(b^2+c^2\right)-a(bB+cC)\right)}{\left(b^2+c^2\right)\sqrt{a^2-b^2-c^2}}+\frac{(Bc-bC)\log(a+b\cos(x)+c\sin(x))}{b^2+c^2}+\frac{x(bB+cC)}{b^2+c^2}$$

```
[Out] (B*b+C*c)*x/(b^2+c^2)+(B*c-C*b)*ln(a+b*cos(x)+c*sin(x))/(b^2+c^2)+2*(A*(b^2
+c^2)-a*(B*b+C*c))*arctan((c+(a-b)*tan(1/2*x))/(a^2-b^2-c^2)^(1/2))/(b^2+c^
2)/(a^2-b^2-c^2)^(1/2)
```

Rubi [A] time = 0.13, antiderivative size = 131, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {3136, 3124, 618, 204}

$$\frac{2\tan^{-1}\left(\frac{(a-b)\tan\left(\frac{x}{2}\right)+c}{\sqrt{a^2-b^2-c^2}}\right)\left(A\left(b^2+c^2\right)-a(bB+cC)\right)}{\left(b^2+c^2\right)\sqrt{a^2-b^2-c^2}}+\frac{(Bc-bC)\log(a+b\cos(x)+c\sin(x))}{b^2+c^2}+\frac{x(bB+cC)}{b^2+c^2}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cos[x] + C*Sin[x])/(a + b*Cos[x] + c*Sin[x]),x]
```

```
[Out] ((b*B + c*C)*x)/(b^2 + c^2) + (2*(A*(b^2 + c^2) - a*(b*B + c*C))*ArcTan[(c
+ (a - b)*Tan[x/2])/Sqrt[a^2 - b^2 - c^2]])/(Sqrt[a^2 - b^2 - c^2]*(b^2 + c
^2)) + ((B*c - b*C)*Log[a + b*Cos[x] + c*Sin[x]])/(b^2 + c^2)
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 3124

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(-1), x_Symbol] :> Module[{f = FreeFactors[Tan[(d + e*x)/2], x]}, Dist[(2*f
```

```
)/e, Subst[Int[1/(a + b + 2*c*f*x + (a - b)*f^2*x^2), x], x, Tan[(d + e*x)/
2]/f], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0]
```

Rule 3136

```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])
/((a_.) + cos[(d_.) + (e_.)*(x_)]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)]), x
_Symbol] :> Simp[((b*B + c*C)*x)/(b^2 + c^2), x] + (Dist[(A*(b^2 + c^2) - a
*(b*B + c*C))/(b^2 + c^2), Int[1/(a + b*Cos[d + e*x] + c*Sin[d + e*x]), x],
 x] + Simp[((c*B - b*C)*Log[a + b*Cos[d + e*x] + c*Sin[d + e*x]])/(e*(b^2 +
 c^2)), x]) /; FreeQ[{a, b, c, d, e, A, B, C}, x] && NeQ[b^2 + c^2, 0] && N
eQ[A*(b^2 + c^2) - a*(b*B + c*C), 0]
```

Rubi steps

$$\int \frac{A+B\cos(x)+C\sin(x)}{a+b\cos(x)+c\sin(x)}\,dx = \frac{(bB+cC)x}{b^2+c^2}+\frac{(Bc-bC)\log(a+b\cos(x)+c\sin(x))}{b^2+c^2}+\left(A-\frac{a(bB+cC)}{b^2+c^2}\right)\int \frac{}{a}$$

$$= \frac{(bB+cC)x}{b^2+c^2}+\frac{(Bc-bC)\log(a+b\cos(x)+c\sin(x))}{b^2+c^2}+\left(2\left(A-\frac{a(bB+cC)}{b^2+c^2}\right)\right)\mathrm{Su}$$

$$= \frac{(bB+cC)x}{b^2+c^2}+\frac{(Bc-bC)\log(a+b\cos(x)+c\sin(x))}{b^2+c^2}-\left(4\left(A-\frac{a(bB+cC)}{b^2+c^2}\right)\right)\mathrm{Su}$$

$$= \frac{(bB+cC)x}{b^2+c^2}+\frac{2\left(A-\frac{a(bB+cC)}{b^2+c^2}\right)\tan^{-1}\left(\frac{c+(a-b)\tan\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2-c^2}}\right)}{\sqrt{a^2-b^2-c^2}}+\frac{(Bc-bC)\log(a+b\cos(}{b^2+c^2}$$

Mathematica [A] time = 0.39, size = 110, normalized size = 0.84

$$\frac{\frac{2\left(a(bB+cC)-A\left(b^2+c^2\right)\right)\tanh^{-1}\left(\frac{(a-b)\tan\left(\frac{x}{2}\right)+c}{\sqrt{-a^2+b^2+c^2}}\right)}{\sqrt{-a^2+b^2+c^2}}+(Bc-bC)\log(a+b\cos(x)+c\sin(x))+x(bB+cC)}{b^2+c^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Cos[x] + C*Sin[x])/(a + b*Cos[x] + c*Sin[x]),x]
```

```
[Out] ((b*B + c*C)*x + (2*(-(A*(b^2 + c^2)) + a*(b*B + c*C))*ArcTanh[(c + (a - b)
*Tan[x/2])/Sqrt[-a^2 + b^2 + c^2]])/Sqrt[-a^2 + b^2 + c^2] + (B*c - b*C)*Lo
g[a + b*Cos[x] + c*Sin[x]])/(b^2 + c^2)
```

fricas [B] time = 1.85, size = 711, normalized size = 5.43

$$\left[-\frac{\left(Bab - Ab^2 + Cac - Ac^2\right)\sqrt{-a^2 + b^2 + c^2}\,\log\left(\frac{a^2b^2 - 2\,b^4 - c^4 - \left(a^2 + 3\,b^2\right)c^2 - \left(2\,a^2b^2 - b^4 - 2\,a^2c^2 + c^4\right)\cos(x)^2 - 2\left(ab^3 + abc^2\right)\cos(x) - 2}{2\,ab}\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(x)+C*sin(x))/(a+b*cos(x)+c*sin(x)),x, algorithm="fricas"
)
```

```
[Out] [-1/2*((B*a*b - A*b^2 + C*a*c - A*c^2)*sqrt(-a^2 + b^2 + c^2)*log((a^2*b^2
- 2*b^4 - c^4 - (a^2 + 3*b^2)*c^2 - (2*a^2*b^2 - b^4 - 2*a^2*c^2 + c^4)*cos
(x)^2 - 2*(a*b^3 + a*b*c^2)*cos(x) - 2*(a*b^2*c + a*c^3 - (b*c^3 - (2*a^2*b
 - b^3)*c)*cos(x))*sin(x) - 2*(2*a*b*c*cos(x)^2 - a*b*c + (b^2*c + c^3)*cos
(x) - (b^3 + b*c^2 + (a*b^2 - a*c^2)*cos(x))*sin(x))*sqrt(-a^2 + b^2 + c^2)
)/(2*a*b*cos(x) + (b^2 - c^2)*cos(x)^2 + a^2 + c^2 + 2*(b*c*cos(x) + a*c)*s
in(x))) - 2*(B*a^2*b - B*b^3 - B*b*c^2 - C*c^3 + (C*a^2 - C*b^2)*c)*x + (C*
a^2*b - C*b^3 - C*b*c^2 + B*c^3 - (B*a^2 - B*b^2)*c)*log(2*a*b*cos(x) + (b^
2 - c^2)*cos(x)^2 + a^2 + c^2 + 2*(b*c*cos(x) + a*c)*sin(x)))/(a^2*b^2 - b^
4 - c^4 + (a^2 - 2*b^2)*c^2), -1/2*(2*(B*a*b - A*b^2 + C*a*c - A*c^2)*sqrt(
a^2 - b^2 - c^2)*arctan(-(a*b*cos(x) + a*c*sin(x) + b^2 + c^2)*sqrt(a^2 - b
^2 - c^2)/((c^3 - (a^2 - b^2)*c)*cos(x) + (a^2*b - b^3 - b*c^2)*sin(x))) -
2*(B*a^2*b - B*b^3 - B*b*c^2 - C*c^3 + (C*a^2 - C*b^2)*c)*x + (C*a^2*b - C*
b^3 - C*b*c^2 + B*c^3 - (B*a^2 - B*b^2)*c)*log(2*a*b*cos(x) + (b^2 - c^2)*c
os(x)^2 + a^2 + c^2 + 2*(b*c*cos(x) + a*c)*sin(x)))/(a^2*b^2 - b^4 - c^4 +
(a^2 - 2*b^2)*c^2)]
```

giac [A] time = 0.18, size = 199, normalized size = 1.52

$$\frac{(Bb + Cc)x}{b^2 + c^2} - \frac{(Cb - Bc)\log\left(-a\tan\left(\frac{1}{2}\,x\right)^2 + b\tan\left(\frac{1}{2}\,x\right)^2 - 2\,c\tan\left(\frac{1}{2}\,x\right) - a - b\right)}{b^2 + c^2} + \frac{(Cb - Bc)\log\left(\tan\left(\frac{1}{2}\,x\right)^2 + 1\right)}{b^2 + c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(x)+C*sin(x))/(a+b*cos(x)+c*sin(x)),x, algorithm="giac")
```

```
[Out] (B*b + C*c)*x/(b^2 + c^2) - (C*b - B*c)*log(-a*tan(1/2*x)^2 + b*tan(1/2*x)^
2 - 2*c*tan(1/2*x) - a - b)/(b^2 + c^2) + (C*b - B*c)*log(tan(1/2*x)^2 + 1)
/(b^2 + c^2) + 2*(B*a*b - A*b^2 + C*a*c - A*c^2)*(pi*floor(1/2*x/pi + 1/2)*
sgn(-2*a + 2*b) + arctan(-(a*tan(1/2*x) - b*tan(1/2*x) + c)/sqrt(a^2 - b^2
- c^2)))/(sqrt(a^2 - b^2 - c^2)*(b^2 + c^2))
```

maple [B] time = 0.13, size = 954, normalized size = 7.28

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(x)+C*sin(x))/(a+b*cos(x)+c*sin(x)),x)
```

```
[Out] 1/(b^2+c^2)/(a-b)*ln(a*tan(1/2*x)^2-b*tan(1/2*x)^2+2*c*tan(1/2*x)+a+b)*a*B*
c-1/(b^2+c^2)/(a-b)*ln(a*tan(1/2*x)^2-b*tan(1/2*x)^2+2*c*tan(1/2*x)+a+b)*b*
B*c-1/(b^2+c^2)/(a-b)*ln(a*tan(1/2*x)^2-b*tan(1/2*x)^2+2*c*tan(1/2*x)+a+b)*
a*b*C+1/(b^2+c^2)/(a-b)*ln(a*tan(1/2*x)^2-b*tan(1/2*x)^2+2*c*tan(1/2*x)+a+b
)*b^2*C+2/(b^2+c^2)/(a^2-b^2-c^2)^(1/2)*arctan(1/2*(2*(a-b)*tan(1/2*x)+2*c)
/(a^2-b^2-c^2)^(1/2))*A*b^2+2/(b^2+c^2)/(a^2-b^2-c^2)^(1/2)*arctan(1/2*(2*(
a-b)*tan(1/2*x)+2*c)/(a^2-b^2-c^2)^(1/2))*A*c^2-2/(b^2+c^2)/(a^2-b^2-c^2)^(
1/2)*arctan(1/2*(2*(a-b)*tan(1/2*x)+2*c)/(a^2-b^2-c^2)^(1/2))*a*b*B+2/(b^2+
c^2)/(a^2-b^2-c^2)^(1/2)*arctan(1/2*(2*(a-b)*tan(1/2*x)+2*c)/(a^2-b^2-c^2)^
(1/2))*B*c^2-2/(b^2+c^2)/(a^2-b^2-c^2)^(1/2)*arctan(1/2*(2*(a-b)*tan(1/2*x)
+2*c)/(a^2-b^2-c^2)^(1/2))*a*c*C-2/(b^2+c^2)/(a^2-b^2-c^2)^(1/2)*arctan(1/2
*(2*(a-b)*tan(1/2*x)+2*c)/(a^2-b^2-c^2)^(1/2))*C*b*c-2/(b^2+c^2)/(a^2-b^2-c
^2)^(1/2)*arctan(1/2*(2*(a-b)*tan(1/2*x)+2*c)/(a^2-b^2-c^2)^(1/2))*c^2/(a-b
)*a*B+2/(b^2+c^2)/(a^2-b^2-c^2)^(1/2)*arctan(1/2*(2*(a-b)*tan(1/2*x)+2*c)/(
a^2-b^2-c^2)^(1/2))*c^2/(a-b)*b*B+2/(b^2+c^2)/(a^2-b^2-c^2)^(1/2)*arctan(1/
2*(2*(a-b)*tan(1/2*x)+2*c)/(a^2-b^2-c^2)^(1/2))*c/(a-b)*a*b*C-2/(b^2+c^2)/(
a^2-b^2-c^2)^(1/2)*arctan(1/2*(2*(a-b)*tan(1/2*x)+2*c)/(a^2-b^2-c^2)^(1/2))
*c/(a-b)*b^2*C-B/(b^2+c^2)*c*ln(1+tan(1/2*x)^2)+C/(b^2+c^2)*b*ln(1+tan(1/2*
x)^2)+2*B/(b^2+c^2)*b*arctan(tan(1/2*x))+2*C/(b^2+c^2)*c*arctan(tan(1/2*x))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(x)+C*sin(x))/(a+b*cos(x)+c*sin(x)),x, algorithm="maxima"
)
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(c^2+b^2-a^2>0)', see `assume?` for
more details)Is c^2+b^2-a^2 positive or negative?
```

mupad [B] time = 55.11, size = 2711, normalized size = 20.69

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cos(x) + C*sin(x))/(a + b*cos(x) + c*sin(x)),x)
```

```
[Out] (log(tan(x/2) - 1i)*(B + C*1i))/(b*1i - c) - (log(tan(x/2) + 1i)*(B - C*1i)
)/(b*1i + c) + (log(32*B^3*a^2 - 32*A*B^2*a^2 + 32*A*B^2*b^2 + 32*A*C^2*a^2
 - 32*A^2*B*b^2 + 32*A*C^2*b^2 + 32*B*C^2*a^2 + 32*B*C^2*b^2 + 32*tan(x/2)*
(a - b)*(2*C^3*a + B^3*c - 2*C^3*b - 2*A*B^2*c + A^2*B*c + A^2*C*b + 2*B^2*
C*a - 2*A*C^2*c - B^2*C*b + 2*B*C^2*c - 2*A*B*C*a) - 32*B^3*a*b + 32*A^2*B*
a*b - 64*A*C^2*a*b - 64*B*C^2*a*b - 32*A^2*C*a*c + 32*A^2*C*b*c + 32*B^2*C*
a*c - 32*B^2*C*b*c - ((B*c^3 - C*b^3 - A*b^2*(b^2 - a^2 + c^2)^(1/2) - B*a^
2*c + C*a^2*b - A*c^2*(b^2 - a^2 + c^2)^(1/2) + B*b^2*c - C*b*c^2 + B*a*b*(
b^2 - a^2 + c^2)^(1/2) + C*a*c*(b^2 - a^2 + c^2)^(1/2))*(64*A^2*b^2*c - 32*
B^2*a^2*c + 32*B^2*b^2*c + 32*C^2*a^2*c + 32*C^2*b^2*c + 32*tan(x/2)*(a - b
)*(A^2*b^2 + 2*B^2*a^2 - A^2*c^2 + B^2*b^2 - 2*C^2*a^2 - 3*B^2*c^2 + 2*C^2*
c^2 + 4*A*B*c^2 - 2*B^2*a*b + 2*C^2*a*b - 2*A*B*a*b + 2*A*C*a*c - 4*A*C*b*c
 - 4*B*C*a*c + 6*B*C*b*c) + 64*A*C*b^3 - 128*B*C*a^3 - 64*B*C*b^3 + 64*A*B*
a^2*c - 128*A*C*a*b^2 + 64*A*C*a^2*b - 64*A*B*b^2*c + 192*B*C*a^2*b + 64*B*
C*a*c^2 - 64*B*C*b*c^2 - 64*A^2*a*b*c - 64*C^2*a*b*c - ((B*c^3 - C*b^3 - A*
b^2*(b^2 - a^2 + c^2)^(1/2) - B*a^2*c + C*a^2*b - A*c^2*(b^2 - a^2 + c^2)^(
1/2) + B*b^2*c - C*b*c^2 + B*a*b*(b^2 - a^2 + c^2)^(1/2) + C*a*c*(b^2 - a^2
 + c^2)^(1/2))*(32*A*b^4 + 32*B*b^4 - 32*tan(x/2)*(a - b)*(2*A*c^3 + B*c^3
- 2*C*b^3 + 2*A*b^2*c + 2*C*a*b^2 + 4*B*b^2*c - 2*C*a*c^2 + C*b*c^2 - 2*A*a
*b*c - 4*B*a*b*c) + 32*A*a^2*b^2 - 32*A*a^2*c^2 + 32*B*a^2*b^2 + 32*A*b^2*c
^2 - 32*B*a^2*c^2 - 64*B*b^2*c^2 - 64*A*a*b^3 - 64*B*a*b^3 + 32*C*a*c^3 - 3
2*C*b*c^3 + 64*C*b^3*c + 96*B*a*b*c^2 - 128*C*a*b^2*c + 64*C*a^2*b*c - (32*
(a - b)*(B*c^3 - C*b^3 - A*b^2*(b^2 - a^2 + c^2)^(1/2) - B*a^2*c + C*a^2*b
- A*c^2*(b^2 - a^2 + c^2)^(1/2) + B*b^2*c - C*b*c^2 + B*a*b*(b^2 - a^2 + c^
2)^(1/2) + C*a*c*(b^2 - a^2 + c^2)^(1/2))*(3*c^4*tan(x/2) + a*c^3 + 3*b*c^3
 + 3*b^3*c + 2*a^2*b^2*tan(x/2) - 2*a^2*c^2*tan(x/2) + 3*b^2*c^2*tan(x/2) -
 2*a*b^3*tan(x/2) + a*b^2*c - 4*a^2*b*c - 2*a*b*c^2*tan(x/2)))/((b^2 + c^2)
*(b^2 - a^2 + c^2))))/((b^2 + c^2)*(b^2 - a^2 + c^2))))/((b^2 + c^2)*(b^2 -
 a^2 + c^2)))*(c*(B*b^2 - B*a^2 + C*a*(b^2 - a^2 + c^2)^(1/2)) + B*c^3 - C*
b^3 - c^2*(A*(b^2 - a^2 + c^2)^(1/2) + C*b) - A*b^2*(b^2 - a^2 + c^2)^(1/2)
 + C*a^2*b + B*a*b*(b^2 - a^2 + c^2)^(1/2)))/((b^2 + c^2)*(b^2 - a^2 + c^2)
) + (log(32*B^3*a^2 - 32*A*B^2*a^2 + 32*A*B^2*b^2 + 32*A*C^2*a^2 - 32*A^2*B
*b^2 + 32*A*C^2*b^2 + 32*B*C^2*a^2 + 32*B*C^2*b^2 + 32*tan(x/2)*(a - b)*(2*
C^3*a + B^3*c - 2*C^3*b - 2*A*B^2*c + A^2*B*c + A^2*C*b + 2*B^2*C*a - 2*A*C
^2*c - B^2*C*b + 2*B*C^2*c - 2*A*B*C*a) - 32*B^3*a*b + 32*A^2*B*a*b - 64*A*
C^2*a*b - 64*B*C^2*a*b - 32*A^2*C*a*c + 32*A^2*C*b*c + 32*B^2*C*a*c - 32*B^
2*C*b*c - ((B*c^3 - C*b^3 + A*b^2*(b^2 - a^2 + c^2)^(1/2) - B*a^2*c + C*a^2
*b + A*c^2*(b^2 - a^2 + c^2)^(1/2) + B*b^2*c - C*b*c^2 - B*a*b*(b^2 - a^2 +
 c^2)^(1/2) - C*a*c*(b^2 - a^2 + c^2)^(1/2))*(64*A^2*b^2*c - 32*B^2*a^2*c +
 32*B^2*b^2*c + 32*C^2*a^2*c + 32*C^2*b^2*c + 32*tan(x/2)*(a - b)*(A^2*b^2
+ 2*B^2*a^2 - A^2*c^2 + B^2*b^2 - 2*C^2*a^2 - 3*B^2*c^2 + 2*C^2*c^2 + 4*A*B
*c^2 - 2*B^2*a*b + 2*C^2*a*b - 2*A*B*a*b + 2*A*C*a*c - 4*A*C*b*c - 4*B*C*a*
c + 6*B*C*b*c) + 64*A*C*b^3 - 128*B*C*a^3 - 64*B*C*b^3 + 64*A*B*a^2*c - 128
```

```
*A*C*a*b^2 + 64*A*C*a^2*b - 64*A*B*b^2*c + 192*B*C*a^2*b + 64*B*C*a*c^2 - 6
4*B*C*b*c^2 - 64*A^2*a*b*c - 64*C^2*a*b*c - ((B*c^3 - C*b^3 + A*b^2*(b^2 -
a^2 + c^2)^(1/2) - B*a^2*c + C*a^2*b + A*c^2*(b^2 - a^2 + c^2)^(1/2) + B*b^
2*c - C*b*c^2 - B*a*b*(b^2 - a^2 + c^2)^(1/2) - C*a*c*(b^2 - a^2 + c^2)^(1/
2))*(32*A*b^4 + 32*B*b^4 - 32*tan(x/2)*(a - b)*(2*A*c^3 + B*c^3 - 2*C*b^3 +
 2*A*b^2*c + 2*C*a*b^2 + 4*B*b^2*c - 2*C*a*c^2 + C*b*c^2 - 2*A*a*b*c - 4*B*
a*b*c) + 32*A*a^2*b^2 - 32*A*a^2*c^2 + 32*B*a^2*b^2 + 32*A*b^2*c^2 - 32*B*a
^2*c^2 - 64*B*b^2*c^2 - 64*A*a*b^3 - 64*B*a*b^3 + 32*C*a*c^3 - 32*C*b*c^3 +
 64*C*b^3*c + 96*B*a*b*c^2 - 128*C*a*b^2*c + 64*C*a^2*b*c - (32*(a - b)*(B*
c^3 - C*b^3 + A*b^2*(b^2 - a^2 + c^2)^(1/2) - B*a^2*c + C*a^2*b + A*c^2*(b^
2 - a^2 + c^2)^(1/2) + B*b^2*c - C*b*c^2 - B*a*b*(b^2 - a^2 + c^2)^(1/2) -
C*a*c*(b^2 - a^2 + c^2)^(1/2))*(3*c^4*tan(x/2) + a*c^3 + 3*b*c^3 + 3*b^3*c
+ 2*a^2*b^2*tan(x/2) - 2*a^2*c^2*tan(x/2) + 3*b^2*c^2*tan(x/2) - 2*a*b^3*ta
n(x/2) + a*b^2*c - 4*a^2*b*c - 2*a*b*c^2*tan(x/2)))/((b^2 + c^2)*(b^2 - a^2
 + c^2))))/((b^2 + c^2)*(b^2 - a^2 + c^2))))/((b^2 + c^2)*(b^2 - a^2 + c^2)
))*(B*c^3 - c*(B*a^2 - B*b^2 + C*a*(b^2 - a^2 + c^2)^(1/2)) - C*b^3 + c^2*(
A*(b^2 - a^2 + c^2)^(1/2) - C*b) + A*b^2*(b^2 - a^2 + c^2)^(1/2) + C*a^2*b
- B*a*b*(b^2 - a^2 + c^2)^(1/2)))/((b^2 + c^2)*(b^2 - a^2 + c^2))
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(x)+C*sin(x))/(a+b*cos(x)+c*sin(x)),x)
```

```
[Out] Timed out
```

3.551 $\int \frac{A+B\cos(x)+C\sin(x)}{(a+b\cos(x)+c\sin(x))^2}\,dx$

Optimal. Leaf size=127

$$\frac{2\tan^{-1}\left(\frac{(a-b)\tan\left(\frac{x}{2}\right)+c}{\sqrt{a^2-b^2-c^2}}\right)(aA-bB-cC)}{\left(a^2-b^2-c^2\right)^{3/2}}+\frac{-\sin(x)(Ab-aB)+\cos(x)(Ac-aC)-bC+Bc}{\left(a^2-b^2-c^2\right)(a+b\cos(x)+c\sin(x))}$$

```
[Out] 2*(A*a-B*b-C*c)*arctan((c+(a-b)*tan(1/2*x))/(a^2-b^2-c^2)^(1/2))/(a^2-b^2-c
^2)^(3/2)+(B*c-b*C+(A*c-C*a)*cos(x)-(A*b-B*a)*sin(x))/(a^2-b^2-c^2)/(a+b*co
s(x)+c*sin(x))
```

Rubi [A] time = 0.12, antiderivative size = 127, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {3153, 3124, 618, 204}

$$\frac{2\tan^{-1}\left(\frac{(a-b)\tan\left(\frac{x}{2}\right)+c}{\sqrt{a^2-b^2-c^2}}\right)(aA-bB-cC)}{\left(a^2-b^2-c^2\right)^{3/2}}+\frac{-\sin(x)(Ab-aB)+\cos(x)(Ac-aC)-bC+Bc}{\left(a^2-b^2-c^2\right)(a+b\cos(x)+c\sin(x))}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cos[x] + C*Sin[x])/(a + b*Cos[x] + c*Sin[x])^2,x]
```

```
[Out] (2*(a*A - b*B - c*C)*ArcTan[(c + (a - b)*Tan[x/2])/Sqrt[a^2 - b^2 - c^2]])/
(a^2 - b^2 - c^2)^(3/2) + (B*c - b*C + (A*c - a*C)*Cos[x] - (A*b - a*B)*Sin
[x])/((a^2 - b^2 - c^2)*(a + b*Cos[x] + c*Sin[x]))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 3124

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(-1), x_Symbol] :> Module[{f = FreeFactors[Tan[(d + e*x)/2], x]}, Dist[(2*f
```

```
)/e, Subst[Int[1/(a + b + 2*c*f*x + (a - b)*f^2*x^2), x], x, Tan[(d + e*x)/
2]/f], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0]
```

Rule 3153

```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])
/((a_.) + cos[(d_.) + (e_.)*(x_)]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)])^2,
 x_Symbol] :> Simp[(c*B - b*C - (a*C - c*A)*Cos[d + e*x] + (a*B - b*A)*Sin[
d + e*x])/(e*(a^2 - b^2 - c^2)*(a + b*Cos[d + e*x] + c*Sin[d + e*x])), x] +
 Dist[(a*A - b*B - c*C)/(a^2 - b^2 - c^2), Int[1/(a + b*Cos[d + e*x] + c*Si
n[d + e*x]), x], x] /; FreeQ[{a, b, c, d, e, A, B, C}, x] && NeQ[a^2 - b^2
- c^2, 0] && NeQ[a*A - b*B - c*C, 0]
```

Rubi steps

$$\int \frac{A+B\cos(x)+C\sin(x)}{(a+b\cos(x)+c\sin(x))^2}\,dx = \frac{Bc-bC+(Ac-aC)\cos(x)-(Ab-aB)\sin(x)}{\left(a^2-b^2-c^2\right)(a+b\cos(x)+c\sin(x))} + \frac{(aA-bB-cC)\int\frac{1}{a+b\cos(x)+c}}{a^2-b^2-c^2}$$

$$= \frac{Bc-bC+(Ac-aC)\cos(x)-(Ab-aB)\sin(x)}{\left(a^2-b^2-c^2\right)(a+b\cos(x)+c\sin(x))} + \frac{(2(aA-bB-cC))\,\text{Subst}\left(\int\right.}{a^2-}$$

$$= \frac{Bc-bC+(Ac-aC)\cos(x)-(Ab-aB)\sin(x)}{\left(a^2-b^2-c^2\right)(a+b\cos(x)+c\sin(x))} - \frac{(4(aA-bB-cC))\,\text{Subst}\left(\int\right.}{}$$

$$= \frac{2(aA-bB-cC)\tan^{-1}\left(\frac{c+(a-b)\tan\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2-c^2}}\right)}{\left(a^2-b^2-c^2\right)^{3/2}} + \frac{Bc-bC+(Ac-aC)\cos(x)-(Ab-a}{\left(a^2-b^2-c^2\right)(a+b\cos(x)+c\sin}$$

Mathematica [A] time = 0.50, size = 137, normalized size = 1.08

$$\frac{a^2(-C)+\sin(x)\left(A\left(b^2+c^2\right)-a(bB+cC)\right)+aAc+b(bC-Bc)}{b\left(-a^2+b^2+c^2\right)(a+b\cos(x)+c\sin(x))}+\frac{2(aA-bB-cC)\tanh^{-1}\left(\frac{(a-b)\tan\left(\frac{x}{2}\right)+c}{\sqrt{-a^2+b^2+c^2}}\right)}{\left(-a^2+b^2+c^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Cos[x] + C*Sin[x])/(a + b*Cos[x] + c*Sin[x])^2,x]
```

```
[Out] (2*(a*A - b*B - c*C)*ArcTanh[(c + (a - b)*Tan[x/2])/Sqrt[-a^2 + b^2 + c^2]]
)/(-a^2 + b^2 + c^2)^(3/2) + (a*A*c - a^2*C + b*(-(B*c) + b*C) + (A*(b^2 +
```

```
c^2) - a*(b*B + c*C))*Sin[x])/(b*(-a^2 + b^2 + c^2)*(a + b*Cos[x] + c*Sin[x
]))
```

fricas [B] time = 2.18, size = 1556, normalized size = 12.25

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(x)+C*sin(x))/(a+b*cos(x)+c*sin(x))^2,x, algorithm="frica
s")

[Out] [1/2*(2*C*a^4*b - 4*C*a^2*b^3 + 2*C*b^5 + 2*C*b*c^4 - 2*B*c^5 + 4*(B*a^2 -
B*b^2)*c^3 - 4*(C*a^2*b - C*b^3)*c^2 - (A*a^2*b^2 - B*a*b^3 - C*a*b^2*c - C
*a*c^3 + (A*a^2 - B*a*b)*c^2 + (A*a*b^3 - B*b^4 - C*b^3*c - C*b*c^3 + (A*a*
b - B*b^2)*c^2)*cos(x) - (C*b^2*c^2 + C*c^4 - (A*a - B*b)*c^3 - (A*a*b^2 -
B*b^3)*c)*sin(x))*sqrt(-a^2 + b^2 + c^2)*log(-(a^2*b^2 - 2*b^4 - c^4 - (a^2
 + 3*b^2)*c^2 - (2*a^2*b^2 - b^4 - 2*a^2*c^2 + c^4)*cos(x)^2 - 2*(a*b^3 + a
*b*c^2)*cos(x) - 2*(a*b^2*c + a*c^3 - (b*c^3 - (2*a^2*b - b^3)*c)*cos(x))*s
in(x) + 2*(2*a*b*c*cos(x)^2 - a*b*c + (b^2*c + c^3)*cos(x) - (b^3 + b*c^2 +
 (a*b^2 - a*c^2)*cos(x))*sin(x))*sqrt(-a^2 + b^2 + c^2))/(2*a*b*cos(x) + (b
^2 - c^2)*cos(x)^2 + a^2 + c^2 + 2*(b*c*cos(x) + a*c)*sin(x))) - 2*(B*a^4 -
 2*B*a^2*b^2 + B*b^4)*c + 2*(C*a*c^4 - A*c^5 + (A*a^2 + B*a*b - 2*A*b^2)*c^
3 - (C*a^3 - C*a*b^2)*c^2 - (B*a^3*b - A*a^2*b^2 - B*a*b^3 + A*b^4)*c)*cos(
x) + 2*(B*a^3*b^2 - A*a^2*b^3 - B*a*b^4 + A*b^5 - C*a*b*c^3 + A*b*c^4 - (A*
a^2*b + B*a*b^2 - 2*A*b^3)*c^2 + (C*a^3*b - C*a*b^3)*c)*sin(x))/(a^5*b^2 -
2*a^3*b^4 + a*b^6 + a*c^6 - (2*a^3 - 3*a*b^2)*c^4 + (a^5 - 4*a^3*b^2 + 3*a*
b^4)*c^2 + (a^4*b^3 - 2*a^2*b^5 + b^7 + b*c^6 - (2*a^2*b - 3*b^3)*c^4 + (a^
4*b - 4*a^2*b^3 + 3*b^5)*c^2)*cos(x) + (c^7 - (2*a^2 - 3*b^2)*c^5 + (a^4 -
4*a^2*b^2 + 3*b^4)*c^3 + (a^4*b^2 - 2*a^2*b^4 + b^6)*c)*sin(x)), (C*a^4*b -
 2*C*a^2*b^3 + C*b^5 + C*b*c^4 - B*c^5 + 2*(B*a^2 - B*b^2)*c^3 - 2*(C*a^2*b
 - C*b^3)*c^2 + (A*a^2*b^2 - B*a*b^3 - C*a*b^2*c - C*a*c^3 + (A*a^2 - B*a*b
)*c^2 + (A*a*b^3 - B*b^4 - C*b^3*c - C*b*c^3 + (A*a*b - B*b^2)*c^2)*cos(x)
- (C*b^2*c^2 + C*c^4 - (A*a - B*b)*c^3 - (A*a*b^2 - B*b^3)*c)*sin(x))*sqrt(
a^2 - b^2 - c^2)*arctan(-(a*b*cos(x) + a*c*sin(x) + b^2 + c^2)*sqrt(a^2 - b
^2 - c^2)/((c^3 - (a^2 - b^2)*c)*cos(x) + (a^2*b - b^3 - b*c^2)*sin(x))) -
(B*a^4 - 2*B*a^2*b^2 + B*b^4)*c + (C*a*c^4 - A*c^5 + (A*a^2 + B*a*b - 2*A*b
^2)*c^3 - (C*a^3 - C*a*b^2)*c^2 - (B*a^3*b - A*a^2*b^2 - B*a*b^3 + A*b^4)*c
)*cos(x) + (B*a^3*b^2 - A*a^2*b^3 - B*a*b^4 + A*b^5 - C*a*b*c^3 + A*b*c^4 -
 (A*a^2*b + B*a*b^2 - 2*A*b^3)*c^2 + (C*a^3*b - C*a*b^3)*c)*sin(x))/(a^5*b^
2 - 2*a^3*b^4 + a*b^6 + a*c^6 - (2*a^3 - 3*a*b^2)*c^4 + (a^5 - 4*a^3*b^2 +
3*a*b^4)*c^2 + (a^4*b^3 - 2*a^2*b^5 + b^7 + b*c^6 - (2*a^2*b - 3*b^3)*c^4 +
 (a^4*b - 4*a^2*b^3 + 3*b^5)*c^2)*cos(x) + (c^7 - (2*a^2 - 3*b^2)*c^5 + (a^
4 - 4*a^2*b^2 + 3*b^4)*c^3 + (a^4*b^2 - 2*a^2*b^4 + b^6)*c)*sin(x))]
```

giac [A] time = 0.17, size = 241, normalized size = 1.90

$$-\frac{2\left(\pi\left\lfloor\frac{x}{2\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}\left(-2\,a+2\,b\right)+\arctan\left(-\frac{a\tan\left(\frac{1}{2}\,x\right)-b\tan\left(\frac{1}{2}\,x\right)+c}{\sqrt{a^2-b^2-c^2}}\right)\right)(Aa-Bb-Cc)}{\left(a^2-b^2-c^2\right)^{\frac{3}{2}}}+\frac{2\left(Ba^2\tan\left(\frac{1}{2}\,x\right)-Aab\tan\left(\frac{1}{2}\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(x)+C*sin(x))/(a+b*cos(x)+c*sin(x))^2,x, algorithm="giac"
)
```

```
[Out] -2*(pi*floor(1/2*x/pi + 1/2)*sgn(-2*a + 2*b) + arctan(-(a*tan(1/2*x) - b*ta
n(1/2*x) + c)/sqrt(a^2 - b^2 - c^2)))*(A*a - B*b - C*c)/(a^2 - b^2 - c^2)^(
3/2) + 2*(B*a^2*tan(1/2*x) - A*a*b*tan(1/2*x) - B*a*b*tan(1/2*x) + A*b^2*ta
n(1/2*x) - C*a*c*tan(1/2*x) + C*b*c*tan(1/2*x) + A*c^2*tan(1/2*x) - B*c^2*t
an(1/2*x) - C*a^2 + C*b^2 + A*a*c - B*b*c)/((a^3 - a^2*b - a*b^2 + b^3 - a*
c^2 + b*c^2)*(a*tan(1/2*x)^2 - b*tan(1/2*x)^2 + 2*c*tan(1/2*x) + a + b))
```

maple [B] time = 0.18, size = 329, normalized size = 2.59

$$\frac{-\frac{2\left(aAb-A\,b^2-A\,c^2-a^2B+abB+B\,c^2+acC-Cbc\right)\tan\left(\frac{x}{2}\right)}{a^3-a^2b-a\,b^2-a\,c^2+b^3+c^2b}+\frac{2\left(aAc-bBc-a^2C+b^2C\right)}{a^3-a^2b-a\,b^2-a\,c^2+b^3+c^2b}}{a\left(\tan^2\left(\frac{x}{2}\right)\right)-b\left(\tan^2\left(\frac{x}{2}\right)\right)+2c\tan\left(\frac{x}{2}\right)+a+b}+\frac{2\arctan\left(\frac{2(a-b)\tan\left(\frac{x}{2}\right)+2c}{2\sqrt{a^2-b^2-c^2}}\right)aA}{\left(a^2-b^2-c^2\right)^{\frac{3}{2}}}-\frac{2\arctan\left(\frac{2(a-b)}{2\sqrt{a}}\right.}{\left(a^2-b^2\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(x)+C*sin(x))/(a+b*cos(x)+c*sin(x))^2,x)
```

```
[Out] 2*(-(A*a*b-A*b^2-A*c^2-B*a^2+B*a*b+B*c^2+C*a*c-C*b*c)/(a^3-a^2*b-a*b^2-a*c^
2+b^3+b*c^2)*tan(1/2*x)+(A*a*c-B*b*c-C*a^2+C*b^2)/(a^3-a^2*b-a*b^2-a*c^2+b^
3+b*c^2))/(a*tan(1/2*x)^2-b*tan(1/2*x)^2+2*c*tan(1/2*x)+a+b)+2/(a^2-b^2-c^2
)^(3/2)*arctan(1/2*(2*(a-b)*tan(1/2*x)+2*c)/(a^2-b^2-c^2)^(1/2))*a*A-2/(a^2
-b^2-c^2)^(3/2)*arctan(1/2*(2*(a-b)*tan(1/2*x)+2*c)/(a^2-b^2-c^2)^(1/2))*b*
B-2/(a^2-b^2-c^2)^(3/2)*arctan(1/2*(2*(a-b)*tan(1/2*x)+2*c)/(a^2-b^2-c^2)^(
1/2))*C*c
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(x)+C*sin(x))/(a+b*cos(x)+c*sin(x))^2,x, algorithm="maxim
a")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(c^2+b^2-a^2>0)', see `assume?` for
more details)Is c^2+b^2-a^2 positive or negative?
```

mupad [B] time = 3.49, size = 227, normalized size = 1.79

$$\frac{\frac{2\left(C\,a^2-A\,c\,a-C\,b^2+B\,c\,b\right)}{(a-b)\left(-a^2+b^2+c^2\right)}-\frac{2\tan\left(\frac{x}{2}\right)\left(A\,b^2+B\,a^2+A\,c^2-B\,c^2-A\,a\,b-B\,a\,b-C\,a\,c+C\,b\,c\right)}{(a-b)\left(-a^2+b^2+c^2\right)}}{(a-b)\,\tan\left(\frac{x}{2}\right)^2+2\,c\tan\left(\frac{x}{2}\right)+a+b}-\frac{2\,\mathrm{atanh}\left(\frac{\tan\left(\frac{x}{2}\right)(2\,a-2\,b)+\frac{2\left(-a^2\,c+b^2\,c+c^3\right)}{-a^2+b^2+c^2}}{2\,\sqrt{-a^2+b^2+c^2}}\right)(B\,b-}{\left(-a^2+b^2+c^2\right)^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cos(x) + C*sin(x))/(a + b*cos(x) + c*sin(x))^2,x)
```

```
[Out] ((2*(C*a^2 - C*b^2 - A*a*c + B*b*c))/((a - b)*(b^2 - a^2 + c^2)) - (2*tan(x
/2)*(A*b^2 + B*a^2 + A*c^2 - B*c^2 - A*a*b - B*a*b - C*a*c + C*b*c))/((a -
b)*(b^2 - a^2 + c^2)))/(a + b + 2*c*tan(x/2) + tan(x/2)^2*(a - b)) - (2*ata
nh((tan(x/2)*(2*a - 2*b) + (2*(b^2*c - a^2*c + c^3))/(b^2 - a^2 + c^2))/(2*
(b^2 - a^2 + c^2)^(1/2)))*(B*b - A*a + C*c))/(b^2 - a^2 + c^2)^(3/2)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(x)+C*sin(x))/(a+b*cos(x)+c*sin(x))**2,x)
```

```
[Out] Timed out
```

3.552 $\int \frac{A+B\cos(x)+C\sin(x)}{(a+b\cos(x)+c\sin(x))^3}\,dx$

Optimal. Leaf size=237

$$\frac{\tan^{-1}\left(\frac{(a-b)\tan\left(\frac{x}{2}\right)+c}{\sqrt{a^2-b^2-c^2}}\right)\left(2a^2A-3a(bB+cC)+A\left(b^2+c^2\right)\right)}{\left(a^2-b^2-c^2\right)^{5/2}}+\frac{-\sin(x)\left(a^2(-B)+3aAb-2b(bB+cC)\right)+\cos(x)\left(a^2\right.}{2\left(a^2-b^2-c^2\right)^2(a+b\cos}$$

```
[Out] (2*a^2*A+A*(b^2+c^2)-3*a*(B*b+C*c))*arctan((c+(a-b)*tan(1/2*x))/(a^2-b^2-c^
2)^(1/2))/(a^2-b^2-c^2)^(5/2)+1/2*(B*c-b*C+(A*c-C*a)*cos(x)-(A*b-B*a)*sin(x
))/(a^2-b^2-c^2)/(a+b*cos(x)+c*sin(x))^2+1/2*(a*(B*c-C*b)+(3*a*A*c-a^2*C-2*
c*(B*b+C*c))*cos(x)-(3*a*A*b-a^2*B-2*b*(B*b+C*c))*sin(x))/(a^2-b^2-c^2)^2/(
a+b*cos(x)+c*sin(x))
```

Rubi [A] time = 0.28, antiderivative size = 237, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {3156, 3153, 3124, 618, 204}

$$\frac{\tan^{-1}\left(\frac{(a-b)\tan\left(\frac{x}{2}\right)+c}{\sqrt{a^2-b^2-c^2}}\right)\left(2a^2A-3a(bB+cC)+A\left(b^2+c^2\right)\right)}{\left(a^2-b^2-c^2\right)^{5/2}}+\frac{-\sin(x)\left(a^2(-B)+3aAb-2b(bB+cC)\right)+\cos(x)\left(a^2\right.}{2\left(a^2-b^2-c^2\right)^2(a+b\cos}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cos[x] + C*Sin[x])/(a + b*Cos[x] + c*Sin[x])^3,x]
```

```
[Out] ((2*a^2*A + A*(b^2 + c^2) - 3*a*(b*B + c*C))*ArcTan[(c + (a - b)*Tan[x/2])/
Sqrt[a^2 - b^2 - c^2]])/(a^2 - b^2 - c^2)^(5/2) + (B*c - b*C + (A*c - a*C)*
Cos[x] - (A*b - a*B)*Sin[x])/(2*(a^2 - b^2 - c^2)*(a + b*Cos[x] + c*Sin[x])
^2) + (a*(B*c - b*C) + (3*a*A*c - a^2*C - 2*c*(b*B + c*C))*Cos[x] - (3*a*A*
b - a^2*B - 2*b*(b*B + c*C))*Sin[x])/(2*(a^2 - b^2 - c^2)^2*(a + b*Cos[x] +
 c*Sin[x]))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
```

```
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 3124

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(-1), x_Symbol] :> Module[{f = FreeFactors[Tan[(d + e*x)/2], x]}, Dist[(2*f
)/e, Subst[Int[1/(a + b + 2*c*f*x + (a - b)*f^2*x^2), x], x, Tan[(d + e*x)/
2]/f], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0]
```

Rule 3153

```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])
/((a_.) + cos[(d_.) + (e_.)*(x_)]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)])^2,
 x_Symbol] :> Simp[(c*B - b*C - (a*C - c*A)*Cos[d + e*x] + (a*B - b*A)*Sin[
d + e*x])/(e*(a^2 - b^2 - c^2)*(a + b*Cos[d + e*x] + c*Sin[d + e*x])), x] +
 Dist[(a*A - b*B - c*C)/(a^2 - b^2 - c^2), Int[1/(a + b*Cos[d + e*x] + c*Si
n[d + e*x]), x], x] /; FreeQ[{a, b, c, d, e, A, B, C}, x] && NeQ[a^2 - b^2
- c^2, 0] && NeQ[a*A - b*B - c*C, 0]
```

Rule 3156

```
Int[((a_.) + cos[(d_.) + (e_.)*(x_)]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)])
^(n_)*((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)
]), x_Symbol] :> -Simp[((c*B - b*C - (a*C - c*A)*Cos[d + e*x] + (a*B - b*A)
*Sin[d + e*x])*(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n + 1))/(e*(n + 1)*(a
^2 - b^2 - c^2)), x] + Dist[1/((n + 1)*(a^2 - b^2 - c^2)), Int[(a + b*Cos[d
 + e*x] + c*Sin[d + e*x])^(n + 1)*Simp[(n + 1)*(a*A - b*B - c*C) + (n + 2)*
(a*B - b*A)*Cos[d + e*x] + (n + 2)*(a*C - c*A)*Sin[d + e*x], x], x], x] /;
FreeQ[{a, b, c, d, e, A, B, C}, x] && LtQ[n, -1] && NeQ[a^2 - b^2 - c^2, 0]
 && NeQ[n, -2]
```

Rubi steps

$$\int \frac{A+B\cos(x)+C\sin(x)}{(a+b\cos(x)+c\sin(x))^3}\,dx = \frac{Bc-bC+(Ac-aC)\cos(x)-(Ab-aB)\sin(x)}{2\left(a^2-b^2-c^2\right)(a+b\cos(x)+c\sin(x))^2} - \frac{\int \frac{-2(aA-bB-cC)+(Ab-aB)\cos(x)+\cdots}{(a+b\cos(x)+c\sin(x))}\cdots}{2\left(a^2-b^2-c^2\right)\cdots}$$

$$= \frac{Bc-bC+(Ac-aC)\cos(x)-(Ab-aB)\sin(x)}{2\left(a^2-b^2-c^2\right)(a+b\cos(x)+c\sin(x))^2} + \frac{a(Bc-bC)+\left(3aAc-a^2C-\cdots\right.}{2\left(a^2\cdots\right.}$$

$$= \frac{Bc-bC+(Ac-aC)\cos(x)-(Ab-aB)\sin(x)}{2\left(a^2-b^2-c^2\right)(a+b\cos(x)+c\sin(x))^2} + \frac{a(Bc-bC)+\left(3aAc-a^2C-\cdots\right.}{2\left(a^2\cdots\right.}$$

$$= \frac{Bc-bC+(Ac-aC)\cos(x)-(Ab-aB)\sin(x)}{2\left(a^2-b^2-c^2\right)(a+b\cos(x)+c\sin(x))^2} + \frac{a(Bc-bC)+\left(3aAc-a^2C-\cdots\right.}{2\left(a^2\cdots\right.}$$

$$= \frac{\left(2a^2A+A\left(b^2+c^2\right)-3a(bB+cC)\right)\tan^{-1}\left(\frac{c+(a-b)\tan\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2-c^2}}\right)}{\left(a^2-b^2-c^2\right)^{5/2}} + \frac{Bc-bC+(Ac-\cdots}{2\left(a^2-b^2-c^2\right)\cdots}$$

Mathematica [A] time = 1.28, size = 452, normalized size = 1.91

$$\frac{2a^4C-6a^3Ac+4a^3bB\sin(x)+4a^3cC\sin(x)-2bc\cos(x)\left(2a^2A-3a(bB+cC)+A\left(b^2+c^2\right)\right)-c\cos(2x)\left(a^2(b\cdots\right.}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Cos[x] + C*Sin[x])/(a + b*Cos[x] + c*Sin[x])^3,x]
```

```
[Out] -(((2*a^2*A + A*(b^2 + c^2) - 3*a*(b*B + c*C))*ArcTanh[(c + (a - b)*Tan[x/2
])/Sqrt[-a^2 + b^2 + c^2]])/(-a^2 + b^2 + c^2)^(5/2)) + (-6*a^3*A*c - 3*a*A
*b^2*c + 9*a^2*b*B*c - 3*a*A*c^3 + 2*a^4*C - 4*a^2*b^2*C + 2*b^4*C + 5*a^2*
c^2*C + 4*b^2*c^2*C + 2*c^4*C - 2*b*c*(2*a^2*A + A*(b^2 + c^2) - 3*a*(b*B +
 c*C))*Cos[x] - c*(-3*a*A*(b^2 + c^2) + a^2*(b*B + c*C) + 2*(b^2 + c^2)*(b*
B + c*C))*Cos[2*x] - 8*a^2*A*b^2*Sin[x] + 2*A*b^4*Sin[x] + 4*a^3*b*B*Sin[x]
 + 2*a*b^3*B*Sin[x] - 12*a^2*A*c^2*Sin[x] + 2*A*b^2*c^2*Sin[x] + 8*a*b*B*c^
2*Sin[x] + 4*a^3*c*C*Sin[x] + 2*a*b^2*c*C*Sin[x] + 8*a*c^3*C*Sin[x] - 3*a*A
*b^3*Sin[2*x] + a^2*b^2*B*Sin[2*x] + 2*b^4*B*Sin[2*x] - 3*a*A*b*c^2*Sin[2*x
] + 2*b^2*B*c^2*Sin[2*x] + a^2*b*c*C*Sin[2*x] + 2*b^3*c*C*Sin[2*x] + 2*b*c^
3*C*Sin[2*x])/(4*b*(-a^2 + b^2 + c^2)^2*(a + b*Cos[x] + c*Sin[x])^2)
```

fricas [B] time = 1.62, size = 4240, normalized size = 17.89

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(x)+C*sin(x))/(a+b*cos(x)+c*sin(x))^3,x, algorithm="frica
s")
```

```
[Out] [1/4*(2*C*a^6*b - 6*C*a^4*b^3 + 6*C*a^2*b^5 - 2*C*b^7 - 6*C*b*c^6 + 2*B*c^7
 - 2*(3*B*a^2 - 3*A*a*b - B*b^2)*c^5 + 2*(4*C*a^2*b - 7*C*b^3)*c^4 + 2*(3*B
*a^4 - 3*A*a^3*b - 5*B*a^2*b^2 + 6*A*a*b^3 - B*b^4)*c^3 - 2*(2*C*a^4*b - 7*
C*a^2*b^3 + 5*C*b^5)*c^2 + 4*(2*C*b*c^6 - (3*A*a*b - 2*B*b^2)*c^5 - (C*a^2*
b - 4*C*b^3)*c^4 + (3*A*a^3*b - B*a^2*b^2 - 6*A*a*b^3 + 4*B*b^4)*c^3 - (C*a
^4*b + C*a^2*b^3 - 2*C*b^5)*c^2 - (B*a^4*b^2 - 3*A*a^3*b^3 + B*a^2*b^4 + 3*
A*a*b^5 - 2*B*b^6)*c)*cos(x)^2 - (2*A*a^4*b^2 - 3*B*a^3*b^3 + A*a^2*b^4 - 3
*C*a^3*b^2*c - 3*C*a*c^5 + A*c^6 + (3*A*a^2 - 3*B*a*b + 2*A*b^2)*c^4 - 3*(C
*a^3 + C*a*b^2)*c^3 + (2*A*a^4 - 3*B*a^3*b + 4*A*a^2*b^2 - 3*B*a*b^3 + A*b^
4)*c^2 + (2*A*a^2*b^4 - 3*B*a*b^5 + A*b^6 - 3*C*a*b^4*c + A*b^4*c^2 + 3*C*a
*c^5 - A*c^6 - (2*A*a^2 - 3*B*a*b + A*b^2)*c^4)*cos(x)^2 + 2*(2*A*a^3*b^3 -
 3*B*a^2*b^4 + A*a*b^5 - 3*C*a^2*b^3*c - 3*C*a^2*b*c^3 + A*a*b*c^4 + (2*A*a
^3*b - 3*B*a^2*b^2 + 2*A*a*b^3)*c^2)*cos(x) - 2*(3*C*a^2*b^2*c^2 + 3*C*a^2*
c^4 - A*a*c^5 - (2*A*a^3 - 3*B*a^2*b + 2*A*a*b^2)*c^3 - (2*A*a^3*b^2 - 3*B*
a^2*b^3 + A*a*b^4)*c + (3*C*a*b^3*c^2 + 3*C*a*b*c^4 - A*b*c^5 - (2*A*a^2*b
- 3*B*a*b^2 + 2*A*b^3)*c^3 - (2*A*a^2*b^3 - 3*B*a*b^4 + A*b^5)*c)*cos(x))*s
in(x))*sqrt(-a^2 + b^2 + c^2)*log(-(a^2*b^2 - 2*b^4 - c^4 - (a^2 + 3*b^2)*c
^2 - (2*a^2*b^2 - b^4 - 2*a^2*c^2 + c^4)*cos(x)^2 - 2*(a*b^3 + a*b*c^2)*cos
(x) - 2*(a*b^2*c + a*c^3 - (b*c^3 - (2*a^2*b - b^3)*c)*cos(x))*sin(x) + 2*(
2*a*b*c*cos(x)^2 - a*b*c + (b^2*c + c^3)*cos(x) - (b^3 + b*c^2 + (a*b^2 - a
*c^2)*cos(x))*sin(x))*sqrt(-a^2 + b^2 + c^2))/(2*a*b*cos(x) + (b^2 - c^2)*c
os(x)^2 + a^2 + c^2 + 2*(b*c*cos(x) + a*c)*sin(x))) - 2*(B*a^6 - 4*B*a^4*b^
2 + 3*A*a^3*b^3 + 2*B*a^2*b^4 - 3*A*a*b^5 + B*b^6)*c + 2*(C*a*c^6 + A*c^7 -
 (5*A*a^2 - B*a*b - 3*A*b^2)*c^5 + (C*a^3 + 2*C*a*b^2)*c^4 + (4*A*a^4 + B*a
^3*b - 10*A*a^2*b^2 + 2*B*a*b^3 + 3*A*b^4)*c^3 - (2*C*a^5 - C*a^3*b^2 - C*a
*b^4)*c^2 - (2*B*a^5*b - 4*A*a^4*b^2 - B*a^3*b^3 + 5*A*a^2*b^4 - B*a*b^5 -
A*b^6)*c)*cos(x) + 2*(2*B*a^5*b^2 - 4*A*a^4*b^3 - B*a^3*b^4 + 5*A*a^2*b^5 -
 B*a*b^6 - A*b^7 - C*a*b*c^5 - A*b*c^6 + (5*A*a^2*b - B*a*b^2 - 3*A*b^3)*c^
4 - (C*a^3*b + 2*C*a*b^3)*c^3 - (4*A*a^4*b + B*a^3*b^2 - 10*A*a^2*b^3 + 2*B
*a*b^4 + 3*A*b^5)*c^2 + (2*C*a^5*b - C*a^3*b^3 - C*a*b^5)*c + (B*a^4*b^3 -
3*A*a^3*b^4 + B*a^2*b^5 + 3*A*a*b^6 - 2*B*b^7 + 2*C*c^7 - (3*A*a - 2*B*b)*c
^6 - (C*a^2 - 2*C*b^2)*c^5 + (3*A*a^3 - B*a^2*b - 3*A*a*b^2 + 2*B*b^3)*c^4
- (C*a^4 + 2*C*b^4)*c^3 - (B*a^4*b - 3*A*a*b^4 + 2*B*b^5)*c^2 + (C*a^4*b^2
+ C*a^2*b^4 - 2*C*b^6)*c)*cos(x))*sin(x))/(a^8*b^2 - 3*a^6*b^4 + 3*a^4*b^6
- a^2*b^8 - c^10 + 2*(a^2 - 2*b^2)*c^8 + (5*a^2*b^2 - 6*b^4)*c^6 - (2*a^6 -
 3*a^4*b^2 - 3*a^2*b^4 + 4*b^6)*c^4 + (a^8 - 5*a^6*b^2 + 6*a^4*b^4 - a^2*b^
```

6 - b^8)*c^2 + (a^6*b^4 - 3*a^4*b^6 + 3*a^2*b^8 - b^10 + c^10 - 3*(a^2 - b^2)*c^8 + (3*a^4 - 6*a^2*b^2 + 2*b^4)*c^6 - (a^6 - 3*a^4*b^2 + 2*b^6)*c^4 - 3*(a^4*b^4 - 2*a^2*b^6 + b^8)*c^2)*cos(x)^2 + 2*(a^7*b^3 - 3*a^5*b^5 + 3*a^3*b^7 - a*b^9 - a*b*c^8 + (3*a^3*b - 4*a*b^3)*c^6 - 3*(a^5*b - 3*a^3*b^3 + 2*a*b^5)*c^4 + (a^7*b - 6*a^5*b^3 + 9*a^3*b^5 - 4*a*b^7)*c^2)*cos(x) - 2*(a*c^9 - (3*a^3 - 4*a*b^2)*c^7 + 3*(a^5 - 3*a^3*b^2 + 2*a*b^4)*c^5 - (a^7 - 6*a^5*b^2 + 9*a^3*b^4 - 4*a*b^6)*c^3 - (a^7*b^2 - 3*a^5*b^4 + 3*a^3*b^6 - a*b^8)*c + (b*c^9 - (3*a^2*b - 4*b^3)*c^7 + 3*(a^4*b - 3*a^2*b^3 + 2*b^5)*c^5 - (a^6*b - 6*a^4*b^3 + 9*a^2*b^5 - 4*b^7)*c^3 - (a^6*b^3 - 3*a^4*b^5 + 3*a^2*b^7 - b^9)*c)*cos(x))*sin(x)), 1/2*(C*a^6*b - 3*C*a^4*b^3 + 3*C*a^2*b^5 - C*b^7 - 3*C*b*c^6 + B*c^7 - (3*B*a^2 - 3*A*a*b - B*b^2)*c^5 + (4*C*a^2*b - 7*C*b^3)*c^4 + (3*B*a^4 - 3*A*a^3*b - 5*B*a^2*b^2 + 6*A*a*b^3 - B*b^4)*c^3 - (2*C*a^4*b - 7*C*a^2*b^3 + 5*C*b^5)*c^2 + 2*(2*C*b*c^6 - (3*A*a*b - 2*B*b^2)*c^5 - (C*a^2*b - 4*C*b^3)*c^4 + (3*A*a^3*b - B*a^2*b^2 - 6*A*a*b^3 + 4*B*b^4)*c^3 - (C*a^4*b + C*a^2*b^3 - 2*C*b^5)*c^2 - (B*a^4*b^2 - 3*A*a^3*b^3 + B*a^2*b^4 + 3*A*a*b^5 - 2*B*b^6)*c)*cos(x)^2 + (2*A*a^4*b^2 - 3*B*a^3*b^3 + A*a^2*b^4 - 3*C*a^3*b^2*c - 3*C*a*c^5 + A*c^6 + (3*A*a^2 - 3*B*a*b + 2*A*b^2)*c^4 - 3*(C*a^3 + C*a*b^2)*c^3 + (2*A*a^4 - 3*B*a^3*b + 4*A*a^2*b^2 - 3*B*a*b^3 + A*b^4)*c^2 + (2*A*a^2*b^4 - 3*B*a*b^5 + A*b^6 - 3*C*a*b^4*c + A*b^4*c^2 + 3*C*a*c^5 - A*c^6 - (2*A*a^2 - 3*B*a*b + A*b^2)*c^4)*cos(x)^2 + 2*(2*A*a^3*b^3 - 3*B*a^2*b^4 + A*a*b^5 - 3*C*a^2*b^3*c - 3*C*a^2*b*c^3 + A*a*b*c^4 + (2*A*a^3*b - 3*B*a^2*b^2 + 2*A*a*b^3)*c^2)*cos(x) - 2*(3*C*a^2*b^2*c^2 + 3*C*a^2*c^4 - A*a*c^5 - (2*A*a^3 - 3*B*a^2*b + 2*A*a*b^2)*c^3 - (2*A*a^3*b^2 - 3*B*a^2*b^3 + A*a*b^4)*c + (3*C*a*b^3*c^2 + 3*C*a*b*c^4 - A*b*c^5 - (2*A*a^2*b - 3*B*a*b^2 + 2*A*b^3)*c^3 - (2*A*a^2*b^3 - 3*B*a*b^4 + A*b^5)*c)*cos(x))*sin(x))*sqrt(a^2 - b^2 - c^2)*arctan(-(a*b*cos(x) + a*c*sin(x) + b^2 + c^2)*sqrt(a^2 - b^2 - c^2)/((c^3 - (a^2 - b^2)*c)*cos(x) + (a^2*b - b^3 - b*c^2)*sin(x))) - (B*a^6 - 4*B*a^4*b^2 + 3*A*a^3*b^3 + 2*B*a^2*b^4 - 3*A*a*b^5 + B*b^6)*c + (C*a*c^6 + A*c^7 - (5*A*a^2 - B*a*b - 3*A*b^2)*c^5 + (C*a^3 + 2*C*a*b^2)*c^4 + (4*A*a^4 + B*a^3*b - 10*A*a^2*b^2 + 2*B*a*b^3 + 3*A*b^4)*c^3 - (2*C*a^5 - C*a^3*b^2 - C*a*b^4)*c^2 - (2*B*a^5*b - 4*A*a^4*b^2 - B*a^3*b^3 + 5*A*a^2*b^4 - B*a*b^5 - A*b^6)*c)*cos(x) + (2*B*a^5*b^2 - 4*A*a^4*b^3 - B*a^3*b^4 + 5*A*a^2*b^5 - B*a*b^6 - A*b^7 - C*a*b*c^5 - A*b*c^6 + (5*A*a^2*b - B*a*b^2 - 3*A*b^3)*c^4 - (C*a^3*b + 2*C*a*b^3)*c^3 - (4*A*a^4*b + B*a^3*b^2 - 10*A*a^2*b^3 + 2*B*a*b^4 + 3*A*b^5)*c^2 + (2*C*a^5*b - C*a^3*b^3 - C*a*b^5)*c + (B*a^4*b^3 - 3*A*a^3*b^4 + B*a^2*b^5 + 3*A*a*b^6 - 2*B*b^7 + 2*C*c^7 - (3*A*a - 2*B*b)*c^6 - (C*a^2 - 2*C*b^2)*c^5 + (3*A*a^3 - B*a^2*b - 3*A*a*b^2 + 2*B*b^3)*c^4 - (C*a^4 + 2*C*b^4)*c^3 - (B*a^4*b - 3*A*a*b^4 + 2*B*b^5)*c^2 + (C*a^4*b^2 + C*a^2*b^4 - 2*C*b^6)*c)*cos(x))*sin(x))/(a^8*b^2 - 3*a^6*b^4 + 3*a^4*b^6 - a^2*b^8 - c^10 + 2*(a^2 - 2*b^2)*c^8 + (5*a^2*b^2 - 6*b^4)*c^6 - (2*a^6 - 3*a^4*b^2 - 3*a^2*b^4 + 4*b^6)*c^4 + (a^8 - 5*a^6*b^2 + 6*a^4*b^4 - a^2*b^6 - b^8)*c^2 + (a^6*b^4 - 3*a^4*b^6 + 3*a^2*b^8 - b^10 + c^10 - 3*(a^2 - b^2)*c^8 + (3*a^4 - 6*a^2*b^2 + 2*b^4)*c^6 - (a^6 - 3*a^4*b^2 + 2*b^6)*c^4 - 3*(a^4*b^4 - 2*a^2*b^6 + b^8)*c^2)*cos(x)^2 + 2*(a^7*b^3 - 3*a^5*b^5 + 3*a^3*b^7 - a*b^9 - a*b*c^8 + (3*

```
a^3*b - 4*a*b^3)*c^6 - 3*(a^5*b - 3*a^3*b^3 + 2*a*b^5)*c^4 + (a^7*b - 6*a^5
*b^3 + 9*a^3*b^5 - 4*a*b^7)*c^2)*cos(x) - 2*(a*c^9 - (3*a^3 - 4*a*b^2)*c^7
+ 3*(a^5 - 3*a^3*b^2 + 2*a*b^4)*c^5 - (a^7 - 6*a^5*b^2 + 9*a^3*b^4 - 4*a*b^
6)*c^3 - (a^7*b^2 - 3*a^5*b^4 + 3*a^3*b^6 - a*b^8)*c + (b*c^9 - (3*a^2*b -
4*b^3)*c^7 + 3*(a^4*b - 3*a^2*b^3 + 2*b^5)*c^5 - (a^6*b - 6*a^4*b^3 + 9*a^2
*b^5 - 4*b^7)*c^3 - (a^6*b^3 - 3*a^4*b^5 + 3*a^2*b^7 - b^9)*c)*cos(x))*sin(
x))]
```

giac [B] time = 0.54, size = 1506, normalized size = 6.35

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(x)+C*sin(x))/(a+b*cos(x)+c*sin(x))^3,x, algorithm="giac"
)
```

```
[Out] -(2*A*a^2 - 3*B*a*b + A*b^2 - 3*C*a*c + A*c^2)*(pi*floor(1/2*x/pi + 1/2)*sg
n(-2*a + 2*b) + arctan(-(a*tan(1/2*x) - b*tan(1/2*x) + c)/sqrt(a^2 - b^2 -
c^2)))/((a^4 - 2*a^2*b^2 + b^4 - 2*a^2*c^2 + 2*b^2*c^2 + c^4)*sqrt(a^2 - b^
2 - c^2)) + (2*B*a^5*tan(1/2*x)^3 - 4*A*a^4*b*tan(1/2*x)^3 - 5*B*a^4*b*tan(
1/2*x)^3 + 11*A*a^3*b^2*tan(1/2*x)^3 + 5*B*a^3*b^2*tan(1/2*x)^3 - 9*A*a^2*b
^3*tan(1/2*x)^3 - 5*B*a^2*b^3*tan(1/2*x)^3 + A*a*b^4*tan(1/2*x)^3 + 5*B*a*b
^4*tan(1/2*x)^3 + A*b^5*tan(1/2*x)^3 - 2*B*b^5*tan(1/2*x)^3 - 3*C*a^4*c*tan
(1/2*x)^3 + 9*C*a^3*b*c*tan(1/2*x)^3 - 9*C*a^2*b^2*c*tan(1/2*x)^3 + 3*C*a*b
^3*c*tan(1/2*x)^3 + 5*A*a^3*c^2*tan(1/2*x)^3 - 4*B*a^3*c^2*tan(1/2*x)^3 - 7
*A*a^2*b*c^2*tan(1/2*x)^3 + 4*B*a^2*b*c^2*tan(1/2*x)^3 - A*a*b^2*c^2*tan(1/
2*x)^3 + 4*B*a*b^2*c^2*tan(1/2*x)^3 + 3*A*b^3*c^2*tan(1/2*x)^3 - 4*B*b^3*c^
2*tan(1/2*x)^3 - 2*A*a*c^4*tan(1/2*x)^3 + 2*B*a*c^4*tan(1/2*x)^3 + 2*A*b*c^
4*tan(1/2*x)^3 - 2*B*b*c^4*tan(1/2*x)^3 - 2*C*a^5*tan(1/2*x)^2 + 2*C*a^4*b*
tan(1/2*x)^2 + 4*C*a^3*b^2*tan(1/2*x)^2 - 4*C*a^2*b^3*tan(1/2*x)^2 - 2*C*a*
b^4*tan(1/2*x)^2 + 2*C*b^5*tan(1/2*x)^2 + 4*A*a^4*c*tan(1/2*x)^2 + 2*B*a^4*
c*tan(1/2*x)^2 - 12*A*a^3*b*c*tan(1/2*x)^2 - 9*B*a^3*b*c*tan(1/2*x)^2 + 13*
A*a^2*b^2*c*tan(1/2*x)^2 + 14*B*a^2*b^2*c*tan(1/2*x)^2 - 6*A*a*b^3*c*tan(1/
2*x)^2 - 9*B*a*b^3*c*tan(1/2*x)^2 + A*b^4*c*tan(1/2*x)^2 + 2*B*b^4*c*tan(1/
2*x)^2 - 5*C*a^3*c^2*tan(1/2*x)^2 + 14*C*a^2*b*c^2*tan(1/2*x)^2 - 13*C*a*b^
2*c^2*tan(1/2*x)^2 + 4*C*b^3*c^2*tan(1/2*x)^2 + 7*A*a^2*c^3*tan(1/2*x)^2 -
4*B*a^2*c^3*tan(1/2*x)^2 - 6*A*a*b*c^3*tan(1/2*x)^2 - A*b^2*c^3*tan(1/2*x)^
2 + 4*B*b^2*c^3*tan(1/2*x)^2 - 2*C*a*c^4*tan(1/2*x)^2 + 2*C*b*c^4*tan(1/2*x
)^2 - 2*A*c^5*tan(1/2*x)^2 + 2*B*c^5*tan(1/2*x)^2 + 2*B*a^5*tan(1/2*x) - 4*
A*a^4*b*tan(1/2*x) - 3*B*a^4*b*tan(1/2*x) + 5*A*a^3*b^2*tan(1/2*x) + B*a^3*
b^2*tan(1/2*x) + 3*A*a^2*b^3*tan(1/2*x) + B*a^2*b^3*tan(1/2*x) - 5*A*a*b^4*
tan(1/2*x) - 3*B*a*b^4*tan(1/2*x) + A*b^5*tan(1/2*x) + 2*B*b^5*tan(1/2*x) -
 5*C*a^4*c*tan(1/2*x) + 5*C*a^3*b*c*tan(1/2*x) + 5*C*a^2*b^2*c*tan(1/2*x) -
 5*C*a*b^3*c*tan(1/2*x) + 11*A*a^3*c^2*tan(1/2*x) - 4*B*a^3*c^2*tan(1/2*x)
- 3*A*a^2*b*c^2*tan(1/2*x) - 8*B*a^2*b*c^2*tan(1/2*x) - 7*A*a*b^2*c^2*tan(1
```

```
/2*x) + 8*B*a*b^2*c^2*tan(1/2*x) - A*b^3*c^2*tan(1/2*x) + 4*B*b^3*c^2*tan(1
/2*x) - 4*C*a^2*c^3*tan(1/2*x) + 4*C*a*b*c^3*tan(1/2*x) - 2*A*a*c^4*tan(1/2
*x) + 2*B*a*c^4*tan(1/2*x) - 2*A*b*c^4*tan(1/2*x) + 2*B*b*c^4*tan(1/2*x) -
2*C*a^5 + 4*C*a^3*b^2 - 2*C*a*b^4 + 4*A*a^4*c - 5*B*a^3*b*c - 3*A*a^2*b^2*c
 + 5*B*a*b^3*c - A*b^4*c - C*a^3*c^2 + C*a*b^2*c^2 - A*a^2*c^3 + 2*B*a*b*c^
3 - A*b^2*c^3)/((a^6 - 2*a^5*b - a^4*b^2 + 4*a^3*b^3 - a^2*b^4 - 2*a*b^5 +
b^6 - 2*a^4*c^2 + 4*a^3*b*c^2 - 4*a*b^3*c^2 + 2*b^4*c^2 + a^2*c^4 - 2*a*b*c
^4 + b^2*c^4)*(a*tan(1/2*x)^2 - b*tan(1/2*x)^2 + 2*c*tan(1/2*x) + a + b)^2)
```

maple [B] time = 0.23, size = 1422, normalized size = 6.00

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(x)+C*sin(x))/(a+b*cos(x)+c*sin(x))^3,x)
```

```
[Out] 2*(-1/2*(4*A*a^3*b-7*A*a^2*b^2-5*A*a^2*c^2+2*A*a*b^3+2*A*a*b*c^2+A*b^4+3*A*
b^2*c^2+2*A*c^4-2*B*a^4+3*B*a^3*b-2*B*a^2*b^2+4*B*a^2*c^2+3*B*a*b^3-2*B*b^4
-4*B*b^2*c^2-2*B*c^4+3*C*a^3*c-6*C*a^2*b*c+3*C*a*b^2*c)/(a-b)/(a^4-2*a^2*b^
2-2*a^2*c^2+b^4+2*b^2*c^2+c^4)*tan(1/2*x)^3+1/2*(4*A*a^4*c-12*A*a^3*b*c+13*
A*a^2*b^2*c+7*A*a^2*c^3-6*A*a*b^3*c-6*A*a*b*c^3+A*b^4*c-A*b^2*c^3-2*A*c^5+2
*B*a^4*c-9*B*a^3*b*c+14*B*a^2*b^2*c-4*B*a^2*c^3-9*B*a*b^3*c+2*B*b^4*c+4*B*b
^2*c^3+2*B*c^5-2*C*a^5+2*C*a^4*b+4*C*a^3*b^2-5*C*a^3*c^2-4*C*a^2*b^3+14*C*a
^2*b*c^2-2*C*a*b^4-13*C*a*b^2*c^2-2*C*a*c^4+2*C*b^5+4*C*b^3*c^2+2*C*b*c^4)/
(a^4-2*a^2*b^2-2*a^2*c^2+b^4+2*b^2*c^2+c^4)/(a^2-2*a*b+b^2)*tan(1/2*x)^2-1/
2*(4*A*a^4*b-5*A*a^3*b^2-11*A*a^3*c^2-3*A*a^2*b^3+3*A*a^2*b*c^2+5*A*a*b^4+7
*A*a*b^2*c^2+2*A*a*c^4-A*b^5+A*b^3*c^2+2*A*b*c^4-2*B*a^5+3*B*a^4*b-B*a^3*b^
2+4*B*a^3*c^2-B*a^2*b^3+8*B*a^2*b*c^2+3*B*a*b^4-8*B*a*b^2*c^2-2*B*a*c^4-2*B
*b^5-4*B*b^3*c^2-2*B*b*c^4+5*C*a^4*c-5*C*a^3*b*c-5*C*a^2*b^2*c+4*C*a^2*c^3+
5*C*a*b^3*c-4*C*a*b*c^3)/(a^4-2*a^2*b^2-2*a^2*c^2+b^4+2*b^2*c^2+c^4)/(a^2-2
*a*b+b^2)*tan(1/2*x)+1/2*(4*A*a^4*c-3*A*a^2*b^2*c-A*a^2*c^3-A*b^4*c-A*b^2*c
^3-5*B*a^3*b*c+5*B*a*b^3*c+2*B*a*b*c^3-2*C*a^5+4*C*a^3*b^2-C*a^3*c^2-2*C*a*
b^4+C*a*b^2*c^2)/(a^4-2*a^2*b^2-2*a^2*c^2+b^4+2*b^2*c^2+c^4)/(a^2-2*a*b+b^2
))/(a*tan(1/2*x)^2-b*tan(1/2*x)^2+2*c*tan(1/2*x)+a+b)^2+2/(a^4-2*a^2*b^2-2*
a^2*c^2+b^4+2*b^2*c^2+c^4)/(a^2-b^2-c^2)^(1/2)*arctan(1/2*(2*(a-b)*tan(1/2*
x)+2*c)/(a^2-b^2-c^2)^(1/2))*a^2*A+1/(a^4-2*a^2*b^2-2*a^2*c^2+b^4+2*b^2*c^2
+c^4)/(a^2-b^2-c^2)^(1/2)*arctan(1/2*(2*(a-b)*tan(1/2*x)+2*c)/(a^2-b^2-c^2)
^(1/2))*A*b^2+1/(a^4-2*a^2*b^2-2*a^2*c^2+b^4+2*b^2*c^2+c^4)/(a^2-b^2-c^2)^(
1/2)*arctan(1/2*(2*(a-b)*tan(1/2*x)+2*c)/(a^2-b^2-c^2)^(1/2))*A*c^2-3/(a^4-
2*a^2*b^2-2*a^2*c^2+b^4+2*b^2*c^2+c^4)/(a^2-b^2-c^2)^(1/2)*arctan(1/2*(2*(a
-b)*tan(1/2*x)+2*c)/(a^2-b^2-c^2)^(1/2))*a*b*B-3/(a^4-2*a^2*b^2-2*a^2*c^2+b
^4+2*b^2*c^2+c^4)/(a^2-b^2-c^2)^(1/2)*arctan(1/2*(2*(a-b)*tan(1/2*x)+2*c)/(
a^2-b^2-c^2)^(1/2))*a*c*C
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(x)+C*sin(x))/(a+b*cos(x)+c*sin(x))^3,x, algorithm="maxim
a")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(c^2+b^2-a^2>0)', see `assume?` for
more details)Is c^2+b^2-a^2 positive or negative?
```

mupad [B] time = 8.21, size = 1160, normalized size = 4.89

$$-\frac{\frac{2\,C\,a^5-4\,A\,a^4\,c-4\,C\,a^3\,b^2+5\,B\,a^3\,b\,c+C\,a^3\,c^2+3\,A\,a^2\,b^2\,c+A\,a^2\,c^3+2\,C\,a\,b^4-5\,B\,a\,b^3\,c-C\,a\,b^2\,c^2-2\,B\,a\,b\,c^3+A\,b^4\,c+A\,b^2\,c^3}{(a-b)^2\,\left(a^4-2\,a^2\,b^2-2\,a^2\,c^2+b^4+2\,b^2\,c^2+c^4\right)}+\frac{\tan\left(\frac{x}{2}\right)^3\,\left(A\,b^4-2\,B\,a^4+\cdots\right.}{\cdots}}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cos(x) + C*sin(x))/(a + b*cos(x) + c*sin(x))^3,x)
```

```
[Out] - ((2*C*a^5 + A*a^2*c^3 + A*b^2*c^3 - 4*C*a^3*b^2 + C*a^3*c^2 - 4*A*a^4*c +
 A*b^4*c + 2*C*a*b^4 - 2*B*a*b*c^3 - 5*B*a*b^3*c + 5*B*a^3*b*c + 3*A*a^2*b^
2*c - C*a*b^2*c^2)/((a - b)^2*(a^4 + b^4 + c^4 - 2*a^2*b^2 - 2*a^2*c^2 + 2*
b^2*c^2)) + (tan(x/2)^3*(A*b^4 - 2*B*a^4 + 2*A*c^4 - 2*B*b^4 - 2*B*c^4 - 7*
A*a^2*b^2 - 5*A*a^2*c^2 - 2*B*a^2*b^2 + 3*A*b^2*c^2 + 4*B*a^2*c^2 - 4*B*b^2
*c^2 + 2*A*a*b^3 + 4*A*a^3*b + 3*B*a*b^3 + 3*B*a^3*b + 3*C*a^3*c + 2*A*a*b*
c^2 + 3*C*a*b^2*c - 6*C*a^2*b*c))/((a - b)*(a^4 + b^4 + c^4 - 2*a^2*b^2 - 2
*a^2*c^2 + 2*b^2*c^2)) - (tan(x/2)^2*(2*B*c^5 - 2*C*a^5 - 2*A*c^5 + 2*C*b^5
 + 7*A*a^2*c^3 - A*b^2*c^3 - 4*B*a^2*c^3 - 4*C*a^2*b^3 + 4*C*a^3*b^2 + 4*B*
b^2*c^3 - 5*C*a^3*c^2 + 4*C*b^3*c^2 + 4*A*a^4*c + A*b^4*c + 2*B*a^4*c - 2*C
*a*b^4 + 2*C*a^4*b + 2*B*b^4*c - 2*C*a*c^4 + 2*C*b*c^4 - 6*A*a*b*c^3 - 6*A*
a*b^3*c - 12*A*a^3*b*c - 9*B*a*b^3*c - 9*B*a^3*b*c + 13*A*a^2*b^2*c + 14*B*
a^2*b^2*c - 13*C*a*b^2*c^2 + 14*C*a^2*b*c^2))/((a - b)^2*(a^4 + b^4 + c^4 -
 2*a^2*b^2 - 2*a^2*c^2 + 2*b^2*c^2)) - (tan(x/2)*(A*b^5 + 2*B*a^5 + 2*B*b^5
 + 3*A*a^2*b^3 + 5*A*a^3*b^2 + 11*A*a^3*c^2 + B*a^2*b^3 + B*a^3*b^2 - A*b^3
*c^2 - 4*B*a^3*c^2 + 4*B*b^3*c^2 - 4*C*a^2*c^3 - 5*A*a*b^4 - 4*A*a^4*b - 2*
A*a*c^4 - 3*B*a*b^4 - 3*B*a^4*b - 2*A*b*c^4 + 2*B*a*c^4 + 2*B*b*c^4 - 5*C*a
^4*c + 4*C*a*b*c^3 - 5*C*a*b^3*c + 5*C*a^3*b*c - 7*A*a*b^2*c^2 - 3*A*a^2*b*
c^2 + 8*B*a*b^2*c^2 - 8*B*a^2*b*c^2 + 5*C*a^2*b^2*c))/((a - b)^2*(a^4 + b^4
 + c^4 - 2*a^2*b^2 - 2*a^2*c^2 + 2*b^2*c^2)))/(tan(x/2)^4*(a^2 - 2*a*b + b^
2) + 2*a*b + tan(x/2)*(4*a*c + 4*b*c) + tan(x/2)^3*(4*a*c - 4*b*c) + a^2 +
```

```
b^2 + tan(x/2)^2*(2*a^2 - 2*b^2 + 4*c^2)) - (atanh((2*a^4*c + 2*b^4*c + 2*c
^5 - 4*a^2*c^3 + 4*b^2*c^3 - 4*a^2*b^2*c)/(2*(b^2 - a^2 + c^2)^(5/2)) + (ta
n(x/2)*(2*a - 2*b)*(a^4 + b^4 + c^4 - 2*a^2*b^2 - 2*a^2*c^2 + 2*b^2*c^2))/(
2*(b^2 - a^2 + c^2)^(5/2)))*(2*A*a^2 + A*b^2 + A*c^2 - 3*B*a*b - 3*C*a*c))/
(b^2 - a^2 + c^2)^(5/2)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(x)+C*sin(x))/(a+b*cos(x)+c*sin(x))**3,x)
```

```
[Out] Timed out
```

3.553 $\int \frac{A+B\cos(x)+C\sin(x)}{a+b\cos(x)+ib\sin(x)}\,dx$

Optimal. Leaf size=105

$$\frac{i\left(-\left(a^2(B-iC)\right)+2aAb-b^2(B+iC)\right)\log(a+ib\sin(x)+b\cos(x))}{2a^2b}+\frac{x(2aA-b(B+iC))}{2a^2}+\frac{(-C+iB)(\cos(x)-}{2a}$$

```
[Out] 1/2*(2*a*A-b*(B+I*C))*x/a^2+1/2*I*(2*a*A*b-a^2*(B-I*C)-b^2*(B+I*C))*ln(a+b*
cos(x)+I*b*sin(x))/a^2/b+1/2*(I*B-C)*(cos(x)-I*sin(x))/a
```

Rubi [A] time = 0.07, antiderivative size = 105, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.038, Rules used = {3130}

$$\frac{i\left(a^2(-(B-iC))+2aAb-b^2(B+iC)\right)\log(a+ib\sin(x)+b\cos(x))}{2a^2b}+\frac{x(2aA-b(B+iC))}{2a^2}+\frac{(-C+iB)(\cos(x)-i}{2a}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cos[x] + C*Sin[x])/(a + b*Cos[x] + I*b*Sin[x]),x]
```

```
[Out] ((2*a*A - b*(B + I*C))*x)/(2*a^2) + ((I/2)*(2*a*A*b - a^2*(B - I*C) - b^2*(
B + I*C))*Log[a + b*Cos[x] + I*b*Sin[x]])/(a^2*b) + ((I*B - C)*(Cos[x] - I*
Sin[x]))/(2*a)
```

Rule 3130

```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])
/(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)]), x_
Symbol] :> Simp[((2*a*A - b*B - c*C)*x)/(2*a^2), x] + (-Simp[((b*B + c*C)*(
b*Cos[d + e*x] - c*Sin[d + e*x]))/(2*a*b*c*e), x] + Simp[((a^2*(b*B - c*C)
- 2*a*A*b^2 + b^2*(b*B + c*C))*Log[RemoveContent[a + b*Cos[d + e*x] + c*Sin
[d + e*x], x]])/(2*a^2*b*c*e), x]) /; FreeQ[{a, b, c, d, e, A, B, C}, x] &&
 EqQ[b^2 + c^2, 0]
```

Rubi steps

$$\int \frac{A+B\cos(x)+C\sin(x)}{a+b\cos(x)+ib\sin(x)}\,dx=\frac{(2aA-b(B+iC))x}{2a^2}+\frac{i\left(2aAb-a^2(B-iC)-b^2(B+iC)\right)\log(a+b\cos(x)+i}{2a^2b}$$

Mathematica [A] time = 0.44, size = 165, normalized size = 1.57

$$\frac{x\left(a^2(B-iC)+2aAb-b^2(B+iC)\right)+\left(a^2(-C-iB)+2iaAb+b^2(C-iB)\right)\log\left(a^2+2ab\cos(x)+b^2\right)+2\left(a^2(B\right.}{4a^2b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Cos[x] + C*Sin[x])/(a + b*Cos[x] + I*b*Sin[x]),x]
```

```
[Out] ((2*a*A*b + a^2*(B - I*C) - b^2*(B + I*C))*x + 2*(-2*a*A*b + a^2*(B - I*C)
+ b^2*(B + I*C))*ArcTan[((a + b)*Cot[x/2])/(a - b)] + (2*I)*a*b*(B + I*C)*C
os[x] + ((2*I)*a*A*b + a^2*((-I)*B - C) + b^2*((-I)*B + C))*Log[a^2 + b^2 +
 2*a*b*Cos[x]] + 2*a*b*(B + I*C)*Sin[x])/(4*a^2*b)
```

fricas [A] time = 1.46, size = 89, normalized size = 0.85

$$\frac{\left((i\,B-C)ab+\left(2\,Aab-(B+i\,C)b^2\right)xe^{(i\,x)}+\left((-i\,B-C)a^2+2i\,Aab+(-i\,B+C)b^2\right)e^{(i\,x)}\log\left(\frac{be^{(i\,x)}+a}{b}\right)\right)e^{(-i\,x)}}{2\,a^2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(x)+C*sin(x))/(a+b*cos(x)+I*b*sin(x)),x, algorithm="frica
s")
```

```
[Out] 1/2*((I*B - C)*a*b + (2*A*a*b - (B + I*C)*b^2)*x*e^(I*x) + ((-I*B - C)*a^2
+ 2*I*A*a*b + (-I*B + C)*b^2)*e^(I*x)*log((b*e^(I*x) + a)/b))*e^(-I*x)/(a^2
*b)
```

giac [B] time = 0.15, size = 203, normalized size = 1.93

$$-\frac{(-2i\,Aa+i\,Bb-Cb)\log\left(-a\tan\left(\frac{1}{2}\,x\right)^2+b\tan\left(\frac{1}{2}\,x\right)^2-2i\,a\tan\left(\frac{1}{2}\,x\right)+a+b\right)}{4\,a^2}-\frac{(2i\,Aa-i\,Bb+Cb)\log\left(\tan\left(\frac{1}{2}\right.\right.}{2\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(x)+C*sin(x))/(a+b*cos(x)+I*b*sin(x)),x, algorithm="giac"
)
```

```
[Out] -1/4*(-2*I*A*a + I*B*b - C*b)*log(-a*tan(1/2*x)^2 + b*tan(1/2*x)^2 - 2*I*a*
tan(1/2*x) + a + b)/a^2 - 1/2*(2*I*A*a - I*B*b + C*b)*log(tan(1/2*x) - I)/a
^2 + 1/4*(2*B*a^2 - 2*I*C*a^2 - 2*A*a*b + B*b^2 + I*C*b^2)*(x + 2*arctan((-
I*a*cos(x) - a*sin(x) - I*a)/(a*cos(x) - I*a*sin(x) - a + 2*b)))/(a^2*b) -
1/2*(-2*I*A*a*tan(1/2*x) + I*B*b*tan(1/2*x) - C*b*tan(1/2*x) - 2*A*a - 2*B*
a - 2*I*C*a + B*b + I*C*b)/(a^2*(tan(1/2*x) - I))
```

maple [B] time = 0.18, size = 257, normalized size = 2.45

$$\frac{C\ln\left(\tan\left(\frac{x}{2}\right)+i\right)}{2b}+\frac{iB\ln\left(\tan\left(\frac{x}{2}\right)+i\right)}{2b}+\frac{iC}{a\left(\tan\left(\frac{x}{2}\right)-i\right)}+\frac{B}{a\left(\tan\left(\frac{x}{2}\right)-i\right)}-\frac{i\ln\left(\tan\left(\frac{x}{2}\right)-i\right)A}{a}+\frac{i\ln\left(\tan\left(\frac{x}{2}\right)-i\right)bB}{2a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(x)+C*sin(x))/(a+b*cos(x)+I*b*sin(x)),x)
```

```
[Out] 1/2*C/b*ln(tan(1/2*x)+I)+1/2*I*B/b*ln(tan(1/2*x)+I)+I*C/a/(tan(1/2*x)-I)+B/
a/(tan(1/2*x)-I)-I/a*ln(tan(1/2*x)-I)*A+1/2*I/a^2*ln(tan(1/2*x)-I)*b*B-1/2/
a^2*ln(tan(1/2*x)-I)*b*C-1/2/b*ln(I*a+I*b+a*tan(1/2*x)-b*tan(1/2*x))*C+1/2/
a^2*b*ln(I*a+I*b+a*tan(1/2*x)-b*tan(1/2*x))*C+I/a*ln(I*a+I*b+a*tan(1/2*x)-b
*tan(1/2*x))*A-1/2*I/b*ln(I*a+I*b+a*tan(1/2*x)-b*tan(1/2*x))*B-1/2*I/a^2*b*
ln(I*a+I*b+a*tan(1/2*x)-b*tan(1/2*x))*B
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(x)+C*sin(x))/(a+b*cos(x)+I*b*sin(x)),x, algorithm="maxim
a")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

mupad [B] time = 6.93, size = 132, normalized size = 1.26

$$-\ln\left(a+b-a\tan\left(\frac{x}{2}\right)\,1\mathrm{i}+b\tan\left(\frac{x}{2}\right)\,1\mathrm{i}\right)\left(\frac{\frac{C}{2}+\frac{B\,1\mathrm{i}}{2}}{b}-\frac{\frac{C\,b^2}{2}-\frac{B\,b^2\,1\mathrm{i}}{2}+A\,a\,b\,1\mathrm{i}}{a^2\,b}\right)+\frac{\ln\left(\tan\left(\frac{x}{2}\right)+1\mathrm{i}\right)\,\left(C+B\,1\mathrm{i}\right)}{2\,b}+\frac{\ln}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cos(x) + C*sin(x))/(a + b*cos(x) + b*sin(x)*1i),x)
```

```
[Out] (log(tan(x/2) + 1i)*(B*1i + C))/(2*b) - log(a + b - a*tan(x/2)*1i + b*tan(x
/2)*1i)*(((B*1i)/2 + C/2)/b - ((C*b^2)/2 - (B*b^2*1i)/2 + A*a*b*1i)/(a^2*b)
) + (log(tan(x/2) - 1i)*(B*b - 2*A*a + C*b*1i)*1i)/(2*a^2) + (5*B + C*5i)/(
5*a*(tan(x/2) - 1i))
```

sympy [A] time = 1.37, size = 138, normalized size = 1.31

$$\begin{cases} -\frac{(-iB+C)e^{-ix}}{2a} & \text{for } 2a \neq 0 \\ x\left(-\frac{2Aa-Bb-iCb}{2a^2}-\frac{i(2iAa+iBa-iBb-Ca+Cb)}{2a^2}\right) & \text{otherwise} \end{cases} - \frac{x\left(-2Aa+Bb+iCb\right)}{2a^2} - \frac{i\left(-2Aab+Ba^2+Bb^2-iCa^2+i\right.}{2a^2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(x)+C*sin(x))/(a+b*cos(x)+I*b*sin(x)),x)
```

```
[Out] Piecewise((-(-I*B + C)*exp(-I*x)/(2*a), Ne(2*a, 0)), (x*(-(2*A*a - B*b - I*
C*b)/(2*a**2) - I*(2*I*A*a + I*B*a - I*B*b - C*a + C*b)/(2*a**2)), True)) -
 x*(-2*A*a + B*b + I*C*b)/(2*a**2) - I*(-2*A*a*b + B*a**2 + B*b**2 - I*C*a*
*2 + I*C*b**2)*log(a/b + exp(I*x))/(2*a**2*b)
```

3.554 $\int \frac{A+B\cos(x)+C\sin(x)}{a+b\cos(x)-ib\sin(x)}\,dx$

Optimal. Leaf size=103

$$-\frac{i\left(-\left(a^2(B+iC)\right)+2aAb-b^2(B-iC)\right)\log(a-ib\sin(x)+b\cos(x))}{2a^2b}+\frac{x(2aA-bB+ibC)}{2a^2}-\frac{(C+iB)(\cos(x)+i}{2a}$$

```
[Out] 1/2*(2*a*A-b*B+I*b*C)*x/a^2-1/2*I*(2*a*A*b-b^2*(B-I*C)-a^2*(B+I*C))*ln(a+b*
cos(x)-I*b*sin(x))/a^2/b-1/2*(I*B+C)*(cos(x)+I*sin(x))/a
```

Rubi [A] time = 0.07, antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.038, Rules used = {3130}

$$-\frac{i\left(a^2(-(B+iC))+2aAb-b^2(B-iC)\right)\log(a-ib\sin(x)+b\cos(x))}{2a^2b}+\frac{x(2aA-bB+ibC)}{2a^2}-\frac{(C+iB)(\cos(x)+i\,s}{2a}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cos[x] + C*Sin[x])/(a + b*Cos[x] - I*b*Sin[x]),x]
```

```
[Out] ((2*a*A - b*B + I*b*C)*x)/(2*a^2) - ((I/2)*(2*a*A*b - b^2*(B - I*C) - a^2*(
B + I*C))*Log[a + b*Cos[x] - I*b*Sin[x]])/(a^2*b) - ((I*B + C)*(Cos[x] + I*
Sin[x]))/(2*a)
```

Rule 3130

```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])
/(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)]), x_
Symbol] :> Simp[((2*a*A - b*B - c*C)*x)/(2*a^2), x] + (-Simp[((b*B + c*C)*(
b*Cos[d + e*x] - c*Sin[d + e*x]))/(2*a*b*c*e), x] + Simp[((a^2*(b*B - c*C)
- 2*a*A*b^2 + b^2*(b*B + c*C))*Log[RemoveContent[a + b*Cos[d + e*x] + c*Sin
[d + e*x], x]])/(2*a^2*b*c*e), x]) /; FreeQ[{a, b, c, d, e, A, B, C}, x] &&
 EqQ[b^2 + c^2, 0]
```

Rubi steps

$$\int \frac{A+B\cos(x)+C\sin(x)}{a+b\cos(x)-ib\sin(x)}\,dx=\frac{(2aA-bB+ibC)x}{2a^2}-\frac{i\left(2aAb-b^2(B-iC)-a^2(B+iC)\right)\log(a+b\cos(x)-ib}{2a^2b}$$

Mathematica [A] time = 0.45, size = 167, normalized size = 1.62

$$\frac{\frac{\left(ia^2(B+iC)-2iaAb+b^2(C+iB)\right)\log\left(a^2+2ab\cos(x)+b^2\right)}{b}+\frac{2\left(a^2(B+iC)-2aAb+b^2(B-iC)\right)\tan^{-1}\left(\frac{(a+b)\cot\left(\frac{x}{2}\right)}{a-b}\right)}{b}+x\left(\frac{a^2(B+iC)}{b}+2aA-b(B-iC\right.}{4a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Cos[x] + C*Sin[x])/(a + b*Cos[x] - I*b*Sin[x]),x]
```

```
[Out] ((2*a*A - b*(B - I*C) + (a^2*(B + I*C))/b)*x + (2*(-2*a*A*b + b^2*(B - I*C)
 + a^2*(B + I*C))*ArcTan[((a + b)*Cot[x/2])/(a - b)])/b - (2*I)*a*(B - I*C)
*Cos[x] + (((-2*I)*a*A*b + I*a^2*(B + I*C) + b^2*(I*B + C))*Log[a^2 + b^2 +
 2*a*b*Cos[x]])/b + 2*a*(B - I*C)*Sin[x])/(4*a^2)
```

fricas [A] time = 1.09, size = 73, normalized size = 0.71

$$\frac{(B+iC)a^2x+(-iB-C)abe^{(ix)}+\left((iB-C)a^2-2iAab+(iB+C)b^2\right)\log\left(\frac{ae^{(ix)}+b}{a}\right)}{2a^2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(x)+C*sin(x))/(a+b*cos(x)-I*b*sin(x)),x, algorithm="frica
s")
```

```
[Out] 1/2*((B + I*C)*a^2*x + (-I*B - C)*a*b*e^(I*x) + ((I*B - C)*a^2 - 2*I*A*a*b
+ (I*B + C)*b^2)*log((a*e^(I*x) + b)/a))/(a^2*b)
```

giac [B] time = 0.15, size = 203, normalized size = 1.97

$$-\frac{(2iAa-iBb-Cb)\log\left(-a\tan\left(\frac{1}{2}x\right)^2+b\tan\left(\frac{1}{2}x\right)^2+2ia\tan\left(\frac{1}{2}x\right)+a+b\right)}{4a^2}-\frac{(-2iAa+iBb+Cb)\log\left(\tan\left(\frac{1}{2}\right.\right.}{2a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(x)+C*sin(x))/(a+b*cos(x)-I*b*sin(x)),x, algorithm="giac"
)
```

```
[Out] -1/4*(2*I*A*a - I*B*b - C*b)*log(-a*tan(1/2*x)^2 + b*tan(1/2*x)^2 + 2*I*a*t
an(1/2*x) + a + b)/a^2 - 1/2*(-2*I*A*a + I*B*b + C*b)*log(tan(1/2*x) + I)/a
^2 + 1/4*(2*B*a^2 + 2*I*C*a^2 - 2*A*a*b + B*b^2 - I*C*b^2)*(x + 2*arctan((I
*a*cos(x) - a*sin(x) + I*a)/(a*cos(x) + I*a*sin(x) - a + 2*b)))/(a^2*b) - 1
/2*(2*I*A*a*tan(1/2*x) - I*B*b*tan(1/2*x) - C*b*tan(1/2*x) - 2*A*a - 2*B*a
+ 2*I*C*a + B*b - I*C*b)/(a^2*(tan(1/2*x) + I))
```

maple [B] time = 0.19, size = 475, normalized size = 4.61

$$\frac{i\ln\left(ia+ib-a\tan\left(\frac{x}{2}\right)+b\tan\left(\frac{x}{2}\right)\right)B}{-2a+2b}+\frac{B}{a\left(\tan\left(\frac{x}{2}\right)+i\right)}-\frac{ib\ln\left(ia+ib-a\tan\left(\frac{x}{2}\right)+b\tan\left(\frac{x}{2}\right)\right)A}{a\left(-a+b\right)}-\frac{iB\ln\left(\tan\left(\frac{x}{2}\right)\right.}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(x)+C*sin(x))/(a+b*cos(x)-I*b*sin(x)),x)
```

```
[Out] 1/2*I/(-a+b)*ln(I*a+I*b-a*tan(1/2*x)+b*tan(1/2*x))*B+B/a/(tan(1/2*x)+I)-I/a
*b/(-a+b)*ln(I*a+I*b-a*tan(1/2*x)+b*tan(1/2*x))*A-1/2*I*B/b*ln(tan(1/2*x)-I
)-1/2/a^2*ln(tan(1/2*x)+I)*b*C+1/2*C/b*ln(tan(1/2*x)-I)-I*C/a/(tan(1/2*x)+I
)+1/2*a/b/(-a+b)*ln(I*a+I*b-a*tan(1/2*x)+b*tan(1/2*x))*C-1/2/(-a+b)*ln(I*a+
I*b-a*tan(1/2*x)+b*tan(1/2*x))*C-1/2/a*b/(-a+b)*ln(I*a+I*b-a*tan(1/2*x)+b*t
an(1/2*x))*C+1/2/a^2*b^2/(-a+b)*ln(I*a+I*b-a*tan(1/2*x)+b*tan(1/2*x))*C-1/2
*I/a^2*ln(tan(1/2*x)+I)*b*B-1/2*I*a/b/(-a+b)*ln(I*a+I*b-a*tan(1/2*x)+b*tan(
1/2*x))*B+I/a*ln(tan(1/2*x)+I)*A-1/2*I/a*b/(-a+b)*ln(I*a+I*b-a*tan(1/2*x)+b
*tan(1/2*x))*B+I/(-a+b)*ln(I*a+I*b-a*tan(1/2*x)+b*tan(1/2*x))*A+1/2*I/a^2*b
^2/(-a+b)*ln(I*a+I*b-a*tan(1/2*x)+b*tan(1/2*x))*B
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(x)+C*sin(x))/(a+b*cos(x)-I*b*sin(x)),x, algorithm="maxim
a")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

mupad [B] time = 6.86, size = 133, normalized size = 1.29

$$\ln\left(a+b+a\tan\left(\frac{x}{2}\right)1\mathrm{i}-b\tan\left(\frac{x}{2}\right)1\mathrm{i}\right)\left(\frac{-\frac{C}{2}+\frac{B\,1\mathrm{i}}{2}}{b}+\frac{\frac{B\,b^2\,1\mathrm{i}}{2}+\frac{C\,b^2}{2}-A\,a\,b\,1\mathrm{i}}{a^2\,b}\right)+\frac{\ln\left(\tan\left(\frac{x}{2}\right)+1\mathrm{i}\right)\left(2\,A\,a-B\,b+\right.}{2\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cos(x) + C*sin(x))/(a + b*cos(x) - b*sin(x)*1i),x)
```

```
[Out] log(a + b + a*tan(x/2)*1i - b*tan(x/2)*1i)*(((B*1i)/2 - C/2)/b + ((B*b^2*1i
)/2 + (C*b^2)/2 - A*a*b*1i)/(a^2*b)) + (log(tan(x/2) + 1i)*(2*A*a - B*b + C
*b*1i)*1i)/(2*a^2) + (5*B - C*5i)/(5*a*(tan(x/2) + 1i)) - (log(tan(x/2) - 1
i)*(B*1i - C))/(2*b)
```

sympy [A] time = 1.27, size = 109, normalized size = 1.06

$$\begin{cases} -\frac{(iB+C)e^{ix}}{2a} & \text{for } 2a \neq 0 \\ x\left(-\frac{B+iC}{2b} + \frac{Ba+Bb+iCa-iCb}{2ab}\right) & \text{otherwise} \end{cases} - \frac{x(-B-iC)}{2b} + \frac{i(-2Aab + Ba^2 + Bb^2 + iCa^2 - iCb^2)\log\left(e^{ix} + \frac{b}{a}\right)}{2a^2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(x)+C*sin(x))/(a+b*cos(x)-I*b*sin(x)),x)
```

```
[Out] Piecewise((-(I*B + C)*exp(I*x)/(2*a), Ne(2*a, 0)), (x*(-(B + I*C)/(2*b) + (
B*a + B*b + I*C*a - I*C*b)/(2*a*b)), True)) - x*(-B - I*C)/(2*b) + I*(-2*A*
a*b + B*a**2 + B*b**2 + I*C*a**2 - I*C*b**2)*log(exp(I*x) + b/a)/(2*a**2*b)
```

3.555 $\int \frac{b^2+c^2+ab\cos(x)+ac\sin(x)}{(a+b\cos(x)+c\sin(x))^2}\,dx$

Optimal. Leaf size=24

$$-\frac{c\cos(x)-b\sin(x)}{a+b\cos(x)+c\sin(x)}$$

```
[Out] (-c*cos(x)+b*sin(x))/(a+b*cos(x)+c*sin(x))
```

Rubi [B] time = 0.07, antiderivative size = 68, normalized size of antiderivative = 2.83, number of steps used = 1, number of rules used = 1, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.033, Rules used = {3150}

$$-\frac{c\cos(x)\left(a^2-b^2-c^2\right)-b\sin(x)\left(a^2-b^2-c^2\right)}{\left(a^2-b^2-c^2\right)(a+b\cos(x)+c\sin(x))}$$

Antiderivative was successfully verified.

```
[In] Int[(b^2 + c^2 + a*b*Cos[x] + a*c*Sin[x])/(a + b*Cos[x] + c*Sin[x])^2,x]
```

```
[Out] -((c*(a^2 - b^2 - c^2)*Cos[x] - b*(a^2 - b^2 - c^2)*Sin[x])/((a^2 - b^2 - c
^2)*(a + b*Cos[x] + c*Sin[x])))
```

Rule 3150

```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])
/((a_.) + cos[(d_.) + (e_.)*(x_)]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)])^2,
 x_Symbol] :> Simp[(c*B - b*C - (a*C - c*A)*Cos[d + e*x] + (a*B - b*A)*Sin[
d + e*x])/(e*(a^2 - b^2 - c^2)*(a + b*Cos[d + e*x] + c*Sin[d + e*x])), x] /
; FreeQ[{a, b, c, d, e, A, B, C}, x] && NeQ[a^2 - b^2 - c^2, 0] && EqQ[a*A
- b*B - c*C, 0]
```

Rubi steps

$$\int \frac{b^2+c^2+ab\cos(x)+ac\sin(x)}{(a+b\cos(x)+c\sin(x))^2}\,dx=-\frac{c\left(a^2-b^2-c^2\right)\cos(x)-b\left(a^2-b^2-c^2\right)\sin(x)}{\left(a^2-b^2-c^2\right)(a+b\cos(x)+c\sin(x))}$$

Mathematica [A] time = 0.09, size = 32, normalized size = 1.33

$$\frac{ac+b^2\sin(x)+c^2\sin(x)}{b(a+b\cos(x)+c\sin(x))}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b^2 + c^2 + a*b*Cos[x] + a*c*Sin[x])/(a + b*Cos[x] + c*Sin[x])^2
,x]
```

```
[Out] (a*c + b^2*Sin[x] + c^2*Sin[x])/(b*(a + b*Cos[x] + c*Sin[x]))
```

fricas [A] time = 0.89, size = 24, normalized size = 1.00

$$-\frac{c\cos(x)-b\sin(x)}{b\cos(x)+c\sin(x)+a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2+c^2+a*b*cos(x)+a*c*sin(x))/(a+b*cos(x)+c*sin(x))^2,x, algori
thm="fricas")
```

```
[Out] -(c*cos(x) - b*sin(x))/(b*cos(x) + c*sin(x) + a)
```

giac [B] time = 0.18, size = 68, normalized size = 2.83

$$\frac{2\left(ab\tan\left(\frac{1}{2}x\right)-b^2\tan\left(\frac{1}{2}x\right)-c^2\tan\left(\frac{1}{2}x\right)-ac\right)}{\left(a\tan\left(\frac{1}{2}x\right)^2-b\tan\left(\frac{1}{2}x\right)^2+2c\tan\left(\frac{1}{2}x\right)+a+b\right)(a-b)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2+c^2+a*b*cos(x)+a*c*sin(x))/(a+b*cos(x)+c*sin(x))^2,x, algori
thm="giac")
```

```
[Out] 2*(a*b*tan(1/2*x) - b^2*tan(1/2*x) - c^2*tan(1/2*x) - a*c)/((a*tan(1/2*x)^2
 - b*tan(1/2*x)^2 + 2*c*tan(1/2*x) + a + b)*(a - b))
```

maple [B] time = 0.19, size = 70, normalized size = 2.92

$$-\frac{2\left(-\frac{\left(ab-b^2-c^2\right)\tan\left(\frac{x}{2}\right)}{a-b}+\frac{ac}{a-b}\right)}{a\left(\tan^2\left(\frac{x}{2}\right)\right)-b\left(\tan^2\left(\frac{x}{2}\right)\right)+2c\tan\left(\frac{x}{2}\right)+a+b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2+c^2+a*b*cos(x)+a*c*sin(x))/(a+b*cos(x)+c*sin(x))^2,x)
```

```
[Out] -2*(-(a*b-b^2-c^2)/(a-b)*tan(1/2*x)+a*c/(a-b))/(a*tan(1/2*x)^2-b*tan(1/2*x)
^2+2*c*tan(1/2*x)+a+b)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2+c^2+a*b*cos(x)+a*c*sin(x))/(a+b*cos(x)+c*sin(x))^2,x, algori
thm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(c^2+b^2-a^2>0)', see `assume?` for
more details)Is c^2+b^2-a^2 positive or negative?
```

mupad [B] time = 3.03, size = 62, normalized size = 2.58

$$-\frac{\frac{2\,a\,c}{a-b}+\frac{2\,\tan\left(\frac{x}{2}\right)\,\left(b^2-a\,b+c^2\right)}{a-b}}{\left(a-b\right)\,\tan\left(\frac{x}{2}\right)^2+2\,c\,\tan\left(\frac{x}{2}\right)+a+b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2 + c^2 + a*c*sin(x) + a*b*cos(x))/(a + b*cos(x) + c*sin(x))^2,x)
```

```
[Out] -((2*a*c)/(a - b) + (2*tan(x/2)*(b^2 - a*b + c^2))/(a - b))/(a + b + 2*c*ta
n(x/2) + tan(x/2)^2*(a - b))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2+c**2+a*b*cos(x)+a*c*sin(x))/(a+b*cos(x)+c*sin(x))**2,x)
```

```
[Out] Timed out
```

3.556 $\int (a+b\cos(x)+c\sin(x))^{5/2}(d+be\cos(x)+ce\sin(x))\,dx$

Optimal. Leaf size=390

$$-\frac{2\left(a^2-b^2-c^2\right)\left(15a^2e+56ad+25e\left(b^2+c^2\right)\right)\sqrt{\frac{a+b\cos(x)+c\sin(x)}{a+\sqrt{b^2+c^2}}}\,F\left(\frac{1}{2}\left(x-\tan^{-1}(b,c)\right)|\frac{2\sqrt{b^2+c^2}}{a+\sqrt{b^2+c^2}}\right)}{105\sqrt{a+b\cos(x)+c\sin(x)}}+\frac{2\left(15a^3e+1\right.}{}$$

```
[Out] -2/7*(a+b*cos(x)+c*sin(x))^(5/2)*(c*e*cos(x)-b*e*sin(x))-2/35*(a+b*cos(x)+c
*sin(x))^(3/2)*(c*(5*a*e+7*d)*cos(x)-b*(5*a*e+7*d)*sin(x))-2/105*(c*(56*a*d
+15*a^2*e+25*(b^2+c^2)*e)*cos(x)-b*(56*a*d+15*a^2*e+25*(b^2+c^2)*e)*sin(x))
*(a+b*cos(x)+c*sin(x))^(1/2)+2/105*(161*a^2*d+63*(b^2+c^2)*d+15*a^3*e+145*a
*(b^2+c^2)*e)*(cos(1/2*x-1/2*arctan(b,c))^2)^(1/2)/cos(1/2*x-1/2*arctan(b,c
))*EllipticE(sin(1/2*x-1/2*arctan(b,c)),2^(1/2)*((b^2+c^2)^(1/2)/(a+(b^2+c^
2)^(1/2)))^(1/2))*(a+b*cos(x)+c*sin(x))^(1/2)/((a+b*cos(x)+c*sin(x))/(a+(b^
2+c^2)^(1/2)))^(1/2)-2/105*(a^2-b^2-c^2)*(56*a*d+15*a^2*e+25*(b^2+c^2)*e)*(
cos(1/2*x-1/2*arctan(b,c))^2)^(1/2)/cos(1/2*x-1/2*arctan(b,c))*EllipticF(si
n(1/2*x-1/2*arctan(b,c)),2^(1/2)*((b^2+c^2)^(1/2)/(a+(b^2+c^2)^(1/2)))^(1/2
))*((a+b*cos(x)+c*sin(x))/(a+(b^2+c^2)^(1/2)))^(1/2)/(a+b*cos(x)+c*sin(x))^
(1/2)
```

Rubi [A] time = 0.89, antiderivative size = 390, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {3146, 3149, 3119, 2653, 3127, 2661}

$$-\frac{2\left(a^2-b^2-c^2\right)\left(15a^2e+56ad+25e\left(b^2+c^2\right)\right)\sqrt{\frac{a+b\cos(x)+c\sin(x)}{a+\sqrt{b^2+c^2}}}\,F\left(\frac{1}{2}\left(x-\tan^{-1}(b,c)\right)|\frac{2\sqrt{b^2+c^2}}{a+\sqrt{b^2+c^2}}\right)}{105\sqrt{a+b\cos(x)+c\sin(x)}}+\frac{2\left(161a^2d+\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[x] + c*Sin[x])^(5/2)*(d + b*e*Cos[x] + c*e*Sin[x]),x]
```

```
[Out] (2*(161*a^2*d + 63*(b^2 + c^2)*d + 15*a^3*e + 145*a*(b^2 + c^2)*e)*Elliptic
E[(x - ArcTan[b, c])/2, (2*Sqrt[b^2 + c^2])/(a + Sqrt[b^2 + c^2])]*Sqrt[a +
 b*Cos[x] + c*Sin[x]])/(105*Sqrt[(a + b*Cos[x] + c*Sin[x])/(a + Sqrt[b^2 +
c^2])]) - (2*(a^2 - b^2 - c^2)*(56*a*d + 15*a^2*e + 25*(b^2 + c^2)*e)*Ellip
ticF[(x - ArcTan[b, c])/2, (2*Sqrt[b^2 + c^2])/(a + Sqrt[b^2 + c^2])]*Sqrt[
(a + b*Cos[x] + c*Sin[x])/(a + Sqrt[b^2 + c^2])])/(105*Sqrt[a + b*Cos[x] +
c*Sin[x]]) - (2*(a + b*Cos[x] + c*Sin[x])^(5/2)*(c*e*Cos[x] - b*e*Sin[x]))/
```

```
7 - (2*(a + b*Cos[x] + c*Sin[x])^(3/2)*(c*(7*d + 5*a*e)*Cos[x] - b*(7*d + 5
*a*e)*Sin[x]))/35 - (2*Sqrt[a + b*Cos[x] + c*Sin[x]]*(c*(56*a*d + 15*a^2*e
+ 25*(b^2 + c^2)*e)*Cos[x] - b*(56*a*d + 15*a^2*e + 25*(b^2 + c^2)*e)*Sin[x
]))/105
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 3119

```
Int[Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_
)]], x_Symbol] :> Dist[Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]]/Sqrt[(a +
b*Cos[d + e*x] + c*Sin[d + e*x])/(a + Sqrt[b^2 + c^2])], Int[Sqrt[a/(a + Sq
rt[b^2 + c^2]) + (Sqrt[b^2 + c^2]*Cos[d + e*x - ArcTan[b, c]])/(a + Sqrt[b^
2 + c^2])], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0]
&& NeQ[b^2 + c^2, 0] &&  !GtQ[a + Sqrt[b^2 + c^2], 0]
```

Rule 3127

```
Int[1/Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(
x_)]], x_Symbol] :> Dist[Sqrt[(a + b*Cos[d + e*x] + c*Sin[d + e*x])/(a + Sq
rt[b^2 + c^2])]/Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]], Int[1/Sqrt[a/(a
+ Sqrt[b^2 + c^2]) + (Sqrt[b^2 + c^2]*Cos[d + e*x - ArcTan[b, c]])/(a + Sqr
t[b^2 + c^2])], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2,
 0] && NeQ[b^2 + c^2, 0] &&  !GtQ[a + Sqrt[b^2 + c^2], 0]
```

Rule 3146

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(n_.)*((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)
]), x_Symbol] :> Simp[((B*c - b*C - a*C*Cos[d + e*x] + a*B*Sin[d + e*x])*(a
 + b*Cos[d + e*x] + c*Sin[d + e*x])^n)/(a*e*(n + 1)), x] + Dist[1/(a*(n + 1
)), Int[(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n - 1)*Simp[a*(b*B + c*C)*n
+ a^2*A*(n + 1) + (n*(a^2*B - B*c^2 + b*c*C) + a*b*A*(n + 1))*Cos[d + e*x]
+ (n*(b*B*c + a^2*C - b^2*C) + a*c*A*(n + 1))*Sin[d + e*x], x], x], x] /; F
reeQ[{a, b, c, d, e, A, B, C}, x] && GtQ[n, 0] && NeQ[a^2 - b^2 - c^2, 0]
```

Rule 3149

```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])
/Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)]]
, x_Symbol] :> Dist[B/b, Int[Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]], x],
 x] + Dist[(A*b - a*B)/b, Int[1/Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]],
x], x] /; FreeQ[{a, b, c, d, e, A, B, C}, x] && EqQ[B*c - b*C, 0] && NeQ[A*
b - a*B, 0]
```

Rubi steps

$$\int (a+b\cos(x)+c\sin(x))^{5/2}(d+be\cos(x)+ce\sin(x))\,dx = -\frac{2}{7}(a+b\cos(x)+c\sin(x))^{5/2}(ce\cos(x)-be\sin(x))$$

$$= -\frac{2}{7}(a+b\cos(x)+c\sin(x))^{5/2}(ce\cos(x)-be\sin(x))$$

$$= -\frac{2}{7}(a+b\cos(x)+c\sin(x))^{5/2}(ce\cos(x)-be\sin(x))$$

$$= -\frac{2}{7}(a+b\cos(x)+c\sin(x))^{5/2}(ce\cos(x)-be\sin(x))$$

$$= -\frac{2}{7}(a+b\cos(x)+c\sin(x))^{5/2}(ce\cos(x)-be\sin(x))$$

$$= \frac{2\left(161a^2d+63\left(b^2+c^2\right)d+15a^3e+145a\left(b^2+c^2\right)\right.}{105\sqrt{}}$$

Mathematica [C] time = 6.94, size = 7823, normalized size = 20.06

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*Cos[x] + c*Sin[x])^(5/2)*(d + b*e*Cos[x] + c*e*Sin[x]),x]
[Out] Result too large to show
```

fricas [F] time = 1.63, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(\left(b^{3}-3\,bc^{2}\right)e\cos(x)^{3}+2\,ac^{2}e+\left(\left(b^{2}-c^{2}\right)d+2\left(ab^{2}-ac^{2}\right)e\right)\cos(x)^{2}+\left(a^{2}+c^{2}\right)d+\left(2\,abd+\left(a^{2}b+\right.\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(x)+c*sin(x))^(5/2)*(d+b*e*cos(x)+c*e*sin(x)),x, algorith
m="fricas")
```

```
[Out] integral(((b^3 - 3*b*c^2)*e*cos(x)^3 + 2*a*c^2*e + ((b^2 - c^2)*d + 2*(a*b^
2 - a*c^2)*e)*cos(x)^2 + (a^2 + c^2)*d + (2*a*b*d + (a^2*b + 3*b*c^2)*e)*co
s(x) + ((3*b^2*c - c^3)*e*cos(x)^2 + 2*a*c*d + (a^2*c + c^3)*e + 2*(2*a*b*c
*e + b*c*d)*cos(x))*sin(x))*sqrt(b*cos(x) + c*sin(x) + a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (be\cos(x)+ce\sin(x)+d)(b\cos(x)+c\sin(x)+a)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(x)+c*sin(x))^(5/2)*(d+b*e*cos(x)+c*e*sin(x)),x, algorith
m="giac")
```

```
[Out] integrate((b*e*cos(x) + c*e*sin(x) + d)*(b*cos(x) + c*sin(x) + a)^(5/2), x)
```

maple [B] time = 1.28, size = 3502, normalized size = 8.98

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(x)+c*sin(x))^(5/2)*(d+b*e*cos(x)+c*e*sin(x)),x)
```

```
[Out] (-(-b^2*sin(x-arctan(-b,c))-c^2*sin(x-arctan(-b,c))-a*(b^2+c^2)^(1/2))*cos(
x-arctan(-b,c))^2/(b^2+c^2)^(1/2))^(1/2)/(b^2+c^2)*((b^6*e+3*b^4*c^2*e+3*b^
2*c^4*e+c^6*e)*(-2/7/(b^2+c^2)^(1/2)*sin(x-arctan(-b,c))^2*(cos(x-arctan(-b
,c))^2*((b^2+c^2)^(1/2)*sin(x-arctan(-b,c))+a))^(1/2)+12/35/(b^2+c^2)*a*sin
(x-arctan(-b,c))*(cos(x-arctan(-b,c))^2*((b^2+c^2)^(1/2)*sin(x-arctan(-b,c)
)+a))^(1/2)-2/3*(5/7+24/35/(b^2+c^2)*a^2)/(b^2+c^2)^(1/2)*(cos(x-arctan(-b,
c))^2*((b^2+c^2)^(1/2)*sin(x-arctan(-b,c))+a))^(1/2)+2*(-4/35/(b^2+c^2)*a^2
+5/21)*(1/(b^2+c^2)^(1/2)*a-1)*((-(b^2+c^2)^(1/2)*sin(x-arctan(-b,c))-a)/(-
a+(b^2+c^2)^(1/2)))^(1/2)*((-sin(x-arctan(-b,c))+1)*(b^2+c^2)^(1/2)/(a+(b^2
+c^2)^(1/2)))^(1/2)*((1+sin(x-arctan(-b,c)))*(b^2+c^2)^(1/2)/(-a+(b^2+c^2)^
(1/2)))^(1/2)/(cos(x-arctan(-b,c))^2*((b^2+c^2)^(1/2)*sin(x-arctan(-b,c))+a
))^(1/2)*EllipticF(((-(b^2+c^2)^(1/2)*sin(x-arctan(-b,c))-a)/(-a+(b^2+c^2)^
(1/2)))^(1/2),((a-(b^2+c^2)^(1/2))/(a+(b^2+c^2)^(1/2)))^(1/2))+2*(-48*a^3-4
```

4*a*b^2-44*a*c^2)/(105*(b^2+c^2)^(1/2)*b^2+105*(b^2+c^2)^(1/2)*c^2)*(1/(b^2
+c^2)^(1/2)*a-1)*((-(b^2+c^2)^(1/2)*sin(x-arctan(-b,c))-a)/(-a+(b^2+c^2)^(1
/2)))^(1/2)*((-sin(x-arctan(-b,c))+1)*(b^2+c^2)^(1/2)/(a+(b^2+c^2)^(1/2)))^
(1/2)*((1+sin(x-arctan(-b,c)))*(b^2+c^2)^(1/2)/(-a+(b^2+c^2)^(1/2)))^(1/2)/
(cos(x-arctan(-b,c))^2*((b^2+c^2)^(1/2)*sin(x-arctan(-b,c))+a))^(1/2)*((-1/
(b^2+c^2)^(1/2)*a-1)*EllipticE(((-(b^2+c^2)^(1/2)*sin(x-arctan(-b,c))-a)/(-
a+(b^2+c^2)^(1/2)))^(1/2),((a-(b^2+c^2)^(1/2))/(a+(b^2+c^2)^(1/2)))^(1/2))+
EllipticF(((-(b^2+c^2)^(1/2)*sin(x-arctan(-b,c))-a)/(-a+(b^2+c^2)^(1/2)))^(
1/2),((a-(b^2+c^2)^(1/2))/(a+(b^2+c^2)^(1/2)))^(1/2))))+(3*(b^2+c^2)^(1/2)*
a*b^4*e+6*(b^2+c^2)^(1/2)*a*b^2*c^2*e+3*(b^2+c^2)^(1/2)*a*c^4*e+(b^2+c^2)^(
1/2)*b^4*d+2*(b^2+c^2)^(1/2)*b^2*c^2*d+(b^2+c^2)^(1/2)*c^4*d)*(-2/5/(b^2+c^
2)^(1/2)*sin(x-arctan(-b,c))*(cos(x-arctan(-b,c))^2*((b^2+c^2)^(1/2)*sin(x-
arctan(-b,c))+a))^(1/2)+8/15/(b^2+c^2)*a*(cos(x-arctan(-b,c))^2*((b^2+c^2)^
(1/2)*sin(x-arctan(-b,c))+a))^(1/2)+4/15/(b^2+c^2)^(1/2)*a*(1/(b^2+c^2)^(1/
2)*a-1)*((-(b^2+c^2)^(1/2)*sin(x-arctan(-b,c))-a)/(-a+(b^2+c^2)^(1/2)))^(1/
2)*((-sin(x-arctan(-b,c))+1)*(b^2+c^2)^(1/2)/(a+(b^2+c^2)^(1/2)))^(1/2)*((1
+sin(x-arctan(-b,c)))*(b^2+c^2)^(1/2)/(-a+(b^2+c^2)^(1/2)))^(1/2)/(cos(x-ar
ctan(-b,c))^2*((b^2+c^2)^(1/2)*sin(x-arctan(-b,c))+a))^(1/2)*EllipticF(((-(
b^2+c^2)^(1/2)*sin(x-arctan(-b,c))-a)/(-a+(b^2+c^2)^(1/2)))^(1/2),((a-(b^2+
c^2)^(1/2))/(a+(b^2+c^2)^(1/2)))^(1/2))+2*(3/5+8/15/(b^2+c^2)*a^2)*(1/(b^2+
c^2)^(1/2)*a-1)*((-(b^2+c^2)^(1/2)*sin(x-arctan(-b,c))-a)/(-a+(b^2+c^2)^(1/
2)))^(1/2)*((-sin(x-arctan(-b,c))+1)*(b^2+c^2)^(1/2)/(a+(b^2+c^2)^(1/2)))^(
1/2)*((1+sin(x-arctan(-b,c)))*(b^2+c^2)^(1/2)/(-a+(b^2+c^2)^(1/2)))^(1/2)/(
cos(x-arctan(-b,c))^2*((b^2+c^2)^(1/2)*sin(x-arctan(-b,c))+a))^(1/2)*((-1/(
b^2+c^2)^(1/2)*a-1)*EllipticE(((-(b^2+c^2)^(1/2)*sin(x-arctan(-b,c))-a)/(-a
+(b^2+c^2)^(1/2)))^(1/2),((a-(b^2+c^2)^(1/2))/(a+(b^2+c^2)^(1/2)))^(1/2))+E
llipticF(((-(b^2+c^2)^(1/2)*sin(x-arctan(-b,c))-a)/(-a+(b^2+c^2)^(1/2)))^(1
/2),((a-(b^2+c^2)^(1/2))/(a+(b^2+c^2)^(1/2)))^(1/2))))+(3*a^2*b^4*e+6*a^2*b
^2*c^2*e+3*a^2*c^4*e+3*a*b^4*d+6*a*b^2*c^2*d+3*a*c^4*d)*(-2/3/(b^2+c^2)^(1/
2)*(cos(x-arctan(-b,c))^2*((b^2+c^2)^(1/2)*sin(x-arctan(-b,c))+a))^(1/2)+2/
3*(1/(b^2+c^2)^(1/2)*a-1)*((-(b^2+c^2)^(1/2)*sin(x-arctan(-b,c))-a)/(-a+(b^
2+c^2)^(1/2)))^(1/2)*((-sin(x-arctan(-b,c))+1)*(b^2+c^2)^(1/2)/(a+(b^2+c^2)
^(1/2)))^(1/2)*((1+sin(x-arctan(-b,c)))*(b^2+c^2)^(1/2)/(-a+(b^2+c^2)^(1/2)
))^(1/2)/(cos(x-arctan(-b,c))^2*((b^2+c^2)^(1/2)*sin(x-arctan(-b,c))+a))^(1
/2)*EllipticF(((-(b^2+c^2)^(1/2)*sin(x-arctan(-b,c))-a)/(-a+(b^2+c^2)^(1/2)
))^(1/2),((a-(b^2+c^2)^(1/2))/(a+(b^2+c^2)^(1/2)))^(1/2))-4/3/(b^2+c^2)^(1/
2)*a*(1/(b^2+c^2)^(1/2)*a-1)*((-(b^2+c^2)^(1/2)*sin(x-arctan(-b,c))-a)/(-a+
(b^2+c^2)^(1/2)))^(1/2)*((-sin(x-arctan(-b,c))+1)*(b^2+c^2)^(1/2)/(a+(b^2+c
^2)^(1/2)))^(1/2)*((1+sin(x-arctan(-b,c)))*(b^2+c^2)^(1/2)/(-a+(b^2+c^2)^(1
/2)))^(1/2)/(cos(x-arctan(-b,c))^2*((b^2+c^2)^(1/2)*sin(x-arctan(-b,c))+a))
^(1/2)*((-1/(b^2+c^2)^(1/2)*a-1)*EllipticE(((-(b^2+c^2)^(1/2)*sin(x-arctan(
-b,c))-a)/(-a+(b^2+c^2)^(1/2)))^(1/2),((a-(b^2+c^2)^(1/2))/(a+(b^2+c^2)^(1/
2)))^(1/2))+EllipticF(((-(b^2+c^2)^(1/2)*sin(x-arctan(-b,c))-a)/(-a+(b^2+c^
2)^(1/2)))^(1/2),((a-(b^2+c^2)^(1/2))/(a+(b^2+c^2)^(1/2)))^(1/2))))+2*((b^2
+c^2)^(1/2)*a^3*b^2*e+(b^2+c^2)^(1/2)*a^3*c^2*e+(b^2+c^2)^(3/2)*a^2*d+2*a^2

```
*b^2*d*(b^2+c^2)^(1/2)+2*a^2*c^2*d*(b^2+c^2)^(1/2))*(1/(b^2+c^2)^(1/2)*a-1)
*((-(b^2+c^2)^(1/2)*sin(x-arctan(-b,c))-a)/(-a+(b^2+c^2)^(1/2)))^(1/2)*((-s
in(x-arctan(-b,c))+1)*(b^2+c^2)^(1/2)/(a+(b^2+c^2)^(1/2)))^(1/2)*((1+sin(x-
arctan(-b,c)))*(b^2+c^2)^(1/2)/(-a+(b^2+c^2)^(1/2)))^(1/2)/(cos(x-arctan(-b
,c))^2*((b^2+c^2)^(1/2)*sin(x-arctan(-b,c))+a))^(1/2)*((-1/(b^2+c^2)^(1/2)*
a-1)*EllipticE(((-(b^2+c^2)^(1/2)*sin(x-arctan(-b,c))-a)/(-a+(b^2+c^2)^(1/2
)))^(1/2),((a-(b^2+c^2)^(1/2))/(a+(b^2+c^2)^(1/2)))^(1/2))+EllipticF(((-(b^
2+c^2)^(1/2)*sin(x-arctan(-b,c))-a)/(-a+(b^2+c^2)^(1/2)))^(1/2),((a-(b^2+c^
2)^(1/2))/(a+(b^2+c^2)^(1/2)))^(1/2)))+2*a^3*b^2*d*(1/(b^2+c^2)^(1/2)*a-1)*
((-(b^2+c^2)^(1/2)*sin(x-arctan(-b,c))-a)/(-a+(b^2+c^2)^(1/2)))^(1/2)*((-si
n(x-arctan(-b,c))+1)*(b^2+c^2)^(1/2)/(a+(b^2+c^2)^(1/2)))^(1/2)*((1+sin(x-a
rctan(-b,c)))*(b^2+c^2)^(1/2)/(-a+(b^2+c^2)^(1/2)))^(1/2)/(-(-b^2*sin(x-arc
tan(-b,c))-c^2*sin(x-arctan(-b,c))-a*(b^2+c^2)^(1/2))*cos(x-arctan(-b,c))^2
/(b^2+c^2)^(1/2))^(1/2)*EllipticF(((-(b^2+c^2)^(1/2)*sin(x-arctan(-b,c))-a)
/(-a+(b^2+c^2)^(1/2)))^(1/2),((a-(b^2+c^2)^(1/2))/(a+(b^2+c^2)^(1/2)))^(1/2
))+2*a^3*c^2*d*(1/(b^2+c^2)^(1/2)*a-1)*((-(b^2+c^2)^(1/2)*sin(x-arctan(-b,c
))-a)/(-a+(b^2+c^2)^(1/2)))^(1/2)*((-sin(x-arctan(-b,c))+1)*(b^2+c^2)^(1/2)
/(a+(b^2+c^2)^(1/2)))^(1/2)*((1+sin(x-arctan(-b,c)))*(b^2+c^2)^(1/2)/(-a+(b
^2+c^2)^(1/2)))^(1/2)/(-(-b^2*sin(x-arctan(-b,c))-c^2*sin(x-arctan(-b,c))-a
*(b^2+c^2)^(1/2))*cos(x-arctan(-b,c))^2/(b^2+c^2)^(1/2))^(1/2)*EllipticF(((
-(b^2+c^2)^(1/2)*sin(x-arctan(-b,c))-a)/(-a+(b^2+c^2)^(1/2)))^(1/2),((a-(b^
2+c^2)^(1/2))/(a+(b^2+c^2)^(1/2)))^(1/2)))/cos(x-arctan(-b,c))/((b^2*sin(x-
arctan(-b,c))+c^2*sin(x-arctan(-b,c))+a*(b^2+c^2)^(1/2))/(b^2+c^2)^(1/2))^(
1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (be\cos(x) + ce\sin(x) + d)(b\cos(x) + c\sin(x) + a)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(x)+c*sin(x))^(5/2)*(d+b*e*cos(x)+c*e*sin(x)),x, algorith
m="maxima")
```

```
[Out] integrate((b*e*cos(x) + c*e*sin(x) + d)*(b*cos(x) + c*sin(x) + a)^(5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int (a + b\,\cos(x) + c\,\sin(x))^{5/2}\,(d + b\,e\,\cos(x) + c\,e\,\sin(x))\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*cos(x) + c*sin(x))^(5/2)*(d + b*e*cos(x) + c*e*sin(x)),x)
```

```
[Out] int((a + b*cos(x) + c*sin(x))^(5/2)*(d + b*e*cos(x) + c*e*sin(x)), x)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(x)+c*sin(x))**(5/2)*(d+b*e*cos(x)+c*e*sin(x)),x)
[Out] Timed out
```

3.557 $\int (a+b\cos(x)+c\sin(x))^{3/2}(d+be\cos(x)+ce\sin(x))\,dx$

Optimal. Leaf size=294

$$-\frac{2\left(a^2-b^2-c^2\right)(3ae+5d)\sqrt{\frac{a+b\cos(x)+c\sin(x)}{a+\sqrt{b^2+c^2}}}\,F\left(\frac{1}{2}\left(x-\tan^{-1}(b,c)\right)|\frac{2\sqrt{b^2+c^2}}{a+\sqrt{b^2+c^2}}\right)}{15\sqrt{a+b\cos(x)+c\sin(x)}}+\frac{2\left(3a^2e+20ad+9e\left(b^2+c^2\right)\right)}{}$$

```
[Out] -2/5*(a+b*cos(x)+c*sin(x))^(3/2)*(c*e*cos(x)-b*e*sin(x))-2/15*(c*(3*a*e+5*d
)*cos(x)-b*(3*a*e+5*d)*sin(x))*(a+b*cos(x)+c*sin(x))^(1/2)+2/15*(20*a*d+3*a
^2*e+9*(b^2+c^2)*e)*(cos(1/2*x-1/2*arctan(b,c))^2)^(1/2)/cos(1/2*x-1/2*arct
an(b,c))*EllipticE(sin(1/2*x-1/2*arctan(b,c)),2^(1/2)*((b^2+c^2)^(1/2)/(a+(
b^2+c^2)^(1/2)))^(1/2))*(a+b*cos(x)+c*sin(x))^(1/2)/((a+b*cos(x)+c*sin(x))/
(a+(b^2+c^2)^(1/2)))^(1/2)-2/15*(a^2-b^2-c^2)*(3*a*e+5*d)*(cos(1/2*x-1/2*ar
ctan(b,c))^2)^(1/2)/cos(1/2*x-1/2*arctan(b,c))*EllipticF(sin(1/2*x-1/2*arct
an(b,c)),2^(1/2)*((b^2+c^2)^(1/2)/(a+(b^2+c^2)^(1/2)))^(1/2))*((a+b*cos(x)+
c*sin(x))/(a+(b^2+c^2)^(1/2)))^(1/2)/(a+b*cos(x)+c*sin(x))^(1/2)
```

Rubi [A] time = 0.56, antiderivative size = 294, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {3146, 3149, 3119, 2653, 3127, 2661}

$$-\frac{2\left(a^2-b^2-c^2\right)(3ae+5d)\sqrt{\frac{a+b\cos(x)+c\sin(x)}{a+\sqrt{b^2+c^2}}}\,F\left(\frac{1}{2}\left(x-\tan^{-1}(b,c)\right)|\frac{2\sqrt{b^2+c^2}}{a+\sqrt{b^2+c^2}}\right)}{15\sqrt{a+b\cos(x)+c\sin(x)}}+\frac{2\left(3a^2e+20ad+9e\left(b^2+c^2\right)\right)}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[x] + c*Sin[x])^(3/2)*(d + b*e*Cos[x] + c*e*Sin[x]),x]
```

```
[Out] (2*(20*a*d + 3*a^2*e + 9*(b^2 + c^2)*e)*EllipticE[(x - ArcTan[b, c])/2, (2*
Sqrt[b^2 + c^2])/(a + Sqrt[b^2 + c^2])]*Sqrt[a + b*Cos[x] + c*Sin[x]])/(15*
Sqrt[(a + b*Cos[x] + c*Sin[x])/(a + Sqrt[b^2 + c^2])]) - (2*(a^2 - b^2 - c^
2)*(5*d + 3*a*e)*EllipticF[(x - ArcTan[b, c])/2, (2*Sqrt[b^2 + c^2])/(a + S
qrt[b^2 + c^2])]*Sqrt[(a + b*Cos[x] + c*Sin[x])/(a + Sqrt[b^2 + c^2])])/(15
*Sqrt[a + b*Cos[x] + c*Sin[x]]) - (2*(a + b*Cos[x] + c*Sin[x])^(3/2)*(c*e*C
os[x] - b*e*Sin[x]))/5 - (2*Sqrt[a + b*Cos[x] + c*Sin[x]]*(c*(5*d + 3*a*e)*
Cos[x] - b*(5*d + 3*a*e)*Sin[x]))/15
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 3119

```
Int[Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_
)]], x_Symbol] :> Dist[Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]]/Sqrt[(a +
b*Cos[d + e*x] + c*Sin[d + e*x])/(a + Sqrt[b^2 + c^2])], Int[Sqrt[a/(a + Sq
rt[b^2 + c^2]) + (Sqrt[b^2 + c^2]*Cos[d + e*x - ArcTan[b, c]])/(a + Sqrt[b^
2 + c^2])], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0]
&& NeQ[b^2 + c^2, 0] &&  !GtQ[a + Sqrt[b^2 + c^2], 0]
```

Rule 3127

```
Int[1/Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(
x_)]], x_Symbol] :> Dist[Sqrt[(a + b*Cos[d + e*x] + c*Sin[d + e*x])/(a + Sq
rt[b^2 + c^2])]/Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]], Int[1/Sqrt[a/(a
+ Sqrt[b^2 + c^2]) + (Sqrt[b^2 + c^2]*Cos[d + e*x - ArcTan[b, c]])/(a + Sqr
t[b^2 + c^2])], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2,
 0] && NeQ[b^2 + c^2, 0] &&  !GtQ[a + Sqrt[b^2 + c^2], 0]
```

Rule 3146

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(n_.)*((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)
]), x_Symbol] :> Simp[((B*c - b*C - a*C*Cos[d + e*x] + a*B*Sin[d + e*x])*(a
 + b*Cos[d + e*x] + c*Sin[d + e*x])^n)/(a*e*(n + 1)), x] + Dist[1/(a*(n + 1
)), Int[(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n - 1)*Simp[a*(b*B + c*C)*n
+ a^2*A*(n + 1) + (n*(a^2*B - B*c^2 + b*c*C) + a*b*A*(n + 1))*Cos[d + e*x]
+ (n*(b*B*c + a^2*C - b^2*C) + a*c*A*(n + 1))*Sin[d + e*x], x], x], x] /; F
reeQ[{a, b, c, d, e, A, B, C}, x] && GtQ[n, 0] && NeQ[a^2 - b^2 - c^2, 0]
```

Rule 3149

```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])
/Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)]]
, x_Symbol] :> Dist[B/b, Int[Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]], x],
 x] + Dist[(A*b - a*B)/b, Int[1/Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]],
```

```
x], x] /; FreeQ[{a, b, c, d, e, A, B, C}, x] && EqQ[B*c - b*C, 0] && NeQ[A*
b - a*B, 0]
```

Rubi steps

$$\int (a+b\cos(x)+c\sin(x))^{3/2}(d+be\cos(x)+ce\sin(x))\,dx = -\frac{2}{5}(a+b\cos(x)+c\sin(x))^{3/2}(ce\cos(x)-be\sin(x)$$

$$= -\frac{2}{5}(a+b\cos(x)+c\sin(x))^{3/2}(ce\cos(x)-be\sin(x)$$

$$= -\frac{2}{5}(a+b\cos(x)+c\sin(x))^{3/2}(ce\cos(x)-be\sin(x)$$

$$= -\frac{2}{5}(a+b\cos(x)+c\sin(x))^{3/2}(ce\cos(x)-be\sin(x)$$

$$= \frac{2\left(20ad+3a^2e+9\left(b^2+c^2\right)e\right)E\left(\frac{1}{2}\left(x-\tan^{-1}(b,c\right.\right.}{15\sqrt{\frac{a+b\cos(x)+c\sin}{a+\sqrt{b^2+c^2}}}}$$

Mathematica [C] time = 6.58, size = 5218, normalized size = 17.75

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*Cos[x] + c*Sin[x])^(3/2)*(d + b*e*Cos[x] + c*e*Sin[x]),x]
```

```
[Out] Result too large to show
```

fricas [F] time = 1.00, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(\left(\left(b^2-c^2\right)e\cos(x)^2+c^2e+ad+(abe+bd)\cos(x)+(2\,bce\cos(x)+ace+cd)\sin(x)\right)\sqrt{b\cos(x)+c\sin}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(x)+c*sin(x))^(3/2)*(d+b*e*cos(x)+c*e*sin(x)),x, algorith
m="fricas")
```

```
[Out] integral(((b^2 - c^2)*e*cos(x)^2 + c^2*e + a*d + (a*b*e + b*d)*cos(x) + (2*
b*c*e*cos(x) + a*c*e + c*d)*sin(x))*sqrt(b*cos(x) + c*sin(x) + a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (be\cos(x) + ce\sin(x) + d)(b\cos(x) + c\sin(x) + a)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(x)+c*sin(x))^(3/2)*(d+b*e*cos(x)+c*e*sin(x)),x, algorith
m="giac")
```

```
[Out] integrate((b*e*cos(x) + c*e*sin(x) + d)*(b*cos(x) + c*sin(x) + a)^(3/2), x)
```

maple [B] time = 1.01, size = 2238, normalized size = 7.61

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(x)+c*sin(x))^(3/2)*(d+b*e*cos(x)+c*e*sin(x)),x)
```

```
[Out] (-(-b^2*sin(x-arctan(-b,c))-c^2*sin(x-arctan(-b,c))-a*(b^2+c^2)^(1/2))*cos(
x-arctan(-b,c))^2/(b^2+c^2)^(1/2))^(1/2)/(b^2+c^2)^(1/2)*((b^4*e+2*b^2*c^2*
e+c^4*e)*(-2/5/(b^2+c^2)^(1/2)*sin(x-arctan(-b,c))*(cos(x-arctan(-b,c))^2*(
(b^2+c^2)^(1/2)*sin(x-arctan(-b,c))+a))^(1/2)+8/15/(b^2+c^2)*a*(cos(x-arcta
n(-b,c))^2*((b^2+c^2)^(1/2)*sin(x-arctan(-b,c))+a))^(1/2)+4/15/(b^2+c^2)^(1
/2)*a*(1/(b^2+c^2)^(1/2)*a-1)*((-(b^2+c^2)^(1/2)*sin(x-arctan(-b,c))-a)/(-a
+(b^2+c^2)^(1/2)))^(1/2)*((-sin(x-arctan(-b,c))+1)*(b^2+c^2)^(1/2)/(a+(b^2+
c^2)^(1/2)))^(1/2)*((1+sin(x-arctan(-b,c)))*(b^2+c^2)^(1/2)/(-a+(b^2+c^2)^(
1/2)))^(1/2)/(cos(x-arctan(-b,c))^2*((b^2+c^2)^(1/2)*sin(x-arctan(-b,c))+a)
)^(1/2)*EllipticF(((-(b^2+c^2)^(1/2)*sin(x-arctan(-b,c))-a)/(-a+(b^2+c^2)^(
1/2)))^(1/2),((a-(b^2+c^2)^(1/2))/(a+(b^2+c^2)^(1/2)))^(1/2))+2*(3/5+8/15/(
b^2+c^2)*a^2)*(1/(b^2+c^2)^(1/2)*a-1)*((-(b^2+c^2)^(1/2)*sin(x-arctan(-b,c)
)-a)/(-a+(b^2+c^2)^(1/2)))^(1/2)*((-sin(x-arctan(-b,c))+1)*(b^2+c^2)^(1/2)/
(a+(b^2+c^2)^(1/2)))^(1/2)*((1+sin(x-arctan(-b,c)))*(b^2+c^2)^(1/2)/(-a+(b^
2+c^2)^(1/2)))^(1/2)/(cos(x-arctan(-b,c))^2*((b^2+c^2)^(1/2)*sin(x-arctan(-
b,c))+a))^(1/2)*((-1/(b^2+c^2)^(1/2)*a-1)*EllipticE(((-(b^2+c^2)^(1/2)*sin(
x-arctan(-b,c))-a)/(-a+(b^2+c^2)^(1/2)))^(1/2),((a-(b^2+c^2)^(1/2))/(a+(b^2
+c^2)^(1/2)))^(1/2))+EllipticF(((-(b^2+c^2)^(1/2)*sin(x-arctan(-b,c))-a)/(-
a+(b^2+c^2)^(1/2)))^(1/2),((a-(b^2+c^2)^(1/2))/(a+(b^2+c^2)^(1/2)))^(1/2)))
)+(2*(b^2+c^2)^(1/2)*a*b^2*e+2*(b^2+c^2)^(1/2)*a*c^2*e+(b^2+c^2)^(1/2)*b^2*
d+(b^2+c^2)^(1/2)*c^2*d)*(-2/3/(b^2+c^2)^(1/2)*(cos(x-arctan(-b,c))^2*((b^2
+c^2)^(1/2)*sin(x-arctan(-b,c))+a))^(1/2)+2/3*(1/(b^2+c^2)^(1/2)*a-1)*((-(b
^2+c^2)^(1/2)*sin(x-arctan(-b,c))-a)/(-a+(b^2+c^2)^(1/2)))^(1/2)*((-sin(x-a
rctan(-b,c))+1)*(b^2+c^2)^(1/2)/(a+(b^2+c^2)^(1/2)))^(1/2)*((1+sin(x-arctan
```

```
(-b,c)))*(b^2+c^2)^(1/2)/(-a+(b^2+c^2)^(1/2)))^(1/2)/(cos(x-arctan(-b,c))^2
*((b^2+c^2)^(1/2)*sin(x-arctan(-b,c))+a))^(1/2)*EllipticF(((-(b^2+c^2)^(1/2
)*sin(x-arctan(-b,c))-a)/(-a+(b^2+c^2)^(1/2)))^(1/2),((a-(b^2+c^2)^(1/2))/(
a+(b^2+c^2)^(1/2)))^(1/2))-4/3/(b^2+c^2)^(1/2)*a*(1/(b^2+c^2)^(1/2)*a-1)*((
-(b^2+c^2)^(1/2)*sin(x-arctan(-b,c))-a)/(-a+(b^2+c^2)^(1/2)))^(1/2)*((-sin(
x-arctan(-b,c))+1)*(b^2+c^2)^(1/2)/(a+(b^2+c^2)^(1/2)))^(1/2)*((1+sin(x-arc
tan(-b,c)))*(b^2+c^2)^(1/2)/(-a+(b^2+c^2)^(1/2)))^(1/2)/(cos(x-arctan(-b,c)
)^2*((b^2+c^2)^(1/2)*sin(x-arctan(-b,c))+a))^(1/2)*((-1/(b^2+c^2)^(1/2)*a-1
)*EllipticE(((-(b^2+c^2)^(1/2)*sin(x-arctan(-b,c))-a)/(-a+(b^2+c^2)^(1/2)))
^(1/2),((a-(b^2+c^2)^(1/2))/(a+(b^2+c^2)^(1/2)))^(1/2))+EllipticF(((-(b^2+c
^2)^(1/2)*sin(x-arctan(-b,c))-a)/(-a+(b^2+c^2)^(1/2)))^(1/2),((a-(b^2+c^2)^
(1/2))/(a+(b^2+c^2)^(1/2)))^(1/2))))+2*(a^2*b^2*e+a^2*c^2*e+2*a*b^2*d+2*a*c
^2*d)*(1/(b^2+c^2)^(1/2)*a-1)*((-(b^2+c^2)^(1/2)*sin(x-arctan(-b,c))-a)/(-a
+(b^2+c^2)^(1/2)))^(1/2)*((-sin(x-arctan(-b,c))+1)*(b^2+c^2)^(1/2)/(a+(b^2+
c^2)^(1/2)))^(1/2)*((1+sin(x-arctan(-b,c)))*(b^2+c^2)^(1/2)/(-a+(b^2+c^2)^(
1/2)))^(1/2)/(cos(x-arctan(-b,c))^2*((b^2+c^2)^(1/2)*sin(x-arctan(-b,c))+a)
)^(1/2)*((-1/(b^2+c^2)^(1/2)*a-1)*EllipticE(((-(b^2+c^2)^(1/2)*sin(x-arctan
(-b,c))-a)/(-a+(b^2+c^2)^(1/2)))^(1/2),((a-(b^2+c^2)^(1/2))/(a+(b^2+c^2)^(1
/2)))^(1/2))+EllipticF(((-(b^2+c^2)^(1/2)*sin(x-arctan(-b,c))-a)/(-a+(b^2+c
^2)^(1/2)))^(1/2),((a-(b^2+c^2)^(1/2))/(a+(b^2+c^2)^(1/2)))^(1/2)))+2*d*a^2
*(b^2+c^2)^(1/2)*(1/(b^2+c^2)^(1/2)*a-1)*((-(b^2+c^2)^(1/2)*sin(x-arctan(-b
,c))-a)/(-a+(b^2+c^2)^(1/2)))^(1/2)*((-sin(x-arctan(-b,c))+1)*(b^2+c^2)^(1/
2)/(a+(b^2+c^2)^(1/2)))^(1/2)*((1+sin(x-arctan(-b,c)))*(b^2+c^2)^(1/2)/(-a+
(b^2+c^2)^(1/2)))^(1/2)/(-(-b^2*sin(x-arctan(-b,c))-c^2*sin(x-arctan(-b,c))
-a*(b^2+c^2)^(1/2))*cos(x-arctan(-b,c))^2/(b^2+c^2)^(1/2))^(1/2)*EllipticF(
((-(b^2+c^2)^(1/2)*sin(x-arctan(-b,c))-a)/(-a+(b^2+c^2)^(1/2)))^(1/2),((a-(
b^2+c^2)^(1/2))/(a+(b^2+c^2)^(1/2)))^(1/2)))/cos(x-arctan(-b,c))/((b^2*sin(
x-arctan(-b,c))+c^2*sin(x-arctan(-b,c))+a*(b^2+c^2)^(1/2))/(b^2+c^2)^(1/2))
^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (be\cos(x) + ce\sin(x) + d)(b\cos(x) + c\sin(x) + a)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(x)+c*sin(x))^(3/2)*(d+b*e*cos(x)+c*e*sin(x)),x, algorith
m="maxima")
```

```
[Out] integrate((b*e*cos(x) + c*e*sin(x) + d)*(b*cos(x) + c*sin(x) + a)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int {\left(a+b\,\cos(x)+c\,\sin(x)\right)}^{3/2}\,\left(d+b\,e\,\cos(x)+c\,e\,\sin(x)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*cos(x) + c*sin(x))^(3/2)*(d + b*e*cos(x) + c*e*sin(x)),x)
[Out] int((a + b*cos(x) + c*sin(x))^(3/2)*(d + b*e*cos(x) + c*e*sin(x)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(x)+c*sin(x))**(3/2)*(d+b*e*cos(x)+c*e*sin(x)),x)
[Out] Timed out
```

3.558 $\int \sqrt{a+b\cos(x)+c\sin(x)}\,(d+be\cos(x)+ce\sin(x))\,dx$

Optimal. Leaf size=229

$$-\frac{2e\left(a^2-b^2-c^2\right)\sqrt{\frac{a+b\cos(x)+c\sin(x)}{a+\sqrt{b^2+c^2}}}\,F\left(\frac{1}{2}\left(x-\tan^{-1}(b,c)\right)|\frac{2\sqrt{b^2+c^2}}{a+\sqrt{b^2+c^2}}\right)}{3\sqrt{a+b\cos(x)+c\sin(x)}}+\frac{2(ae+3d)\sqrt{a+b\cos(x)+c\sin(x)}\,E\left(\frac{1}{2}\left(\right.\right.}{3\sqrt{\frac{a+b\cos(x)+c\sin(}{a+\sqrt{b^2+c^2}}}}$$

```
[Out] -2/3*(c*e*cos(x)-b*e*sin(x))*(a+b*cos(x)+c*sin(x))^(1/2)+2/3*(a*e+3*d)*(cos
(1/2*x-1/2*arctan(b,c))^2)^(1/2)/cos(1/2*x-1/2*arctan(b,c))*EllipticE(sin(1
/2*x-1/2*arctan(b,c)),2^(1/2)*((b^2+c^2)^(1/2)/(a+(b^2+c^2)^(1/2)))^(1/2))*
(a+b*cos(x)+c*sin(x))^(1/2)/((a+b*cos(x)+c*sin(x))/(a+(b^2+c^2)^(1/2)))^(1/
2)-2/3*(a^2-b^2-c^2)*e*(cos(1/2*x-1/2*arctan(b,c))^2)^(1/2)/cos(1/2*x-1/2*a
rctan(b,c))*EllipticF(sin(1/2*x-1/2*arctan(b,c)),2^(1/2)*((b^2+c^2)^(1/2)/(
a+(b^2+c^2)^(1/2)))^(1/2))*((a+b*cos(x)+c*sin(x))/(a+(b^2+c^2)^(1/2)))^(1/2
)/(a+b*cos(x)+c*sin(x))^(1/2)
```

Rubi [A] time = 0.33, antiderivative size = 229, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {3146, 3149, 3119, 2653, 3127, 2661}

$$-\frac{2e\left(a^2-b^2-c^2\right)\sqrt{\frac{a+b\cos(x)+c\sin(x)}{a+\sqrt{b^2+c^2}}}\,F\left(\frac{1}{2}\left(x-\tan^{-1}(b,c)\right)|\frac{2\sqrt{b^2+c^2}}{a+\sqrt{b^2+c^2}}\right)}{3\sqrt{a+b\cos(x)+c\sin(x)}}+\frac{2(ae+3d)\sqrt{a+b\cos(x)+c\sin(x)}\,E\left(\frac{1}{2}\left(\right.\right.}{3\sqrt{\frac{a+b\cos(x)+c\sin(}{a+\sqrt{b^2+c^2}}}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*Cos[x] + c*Sin[x]]*(d + b*e*Cos[x] + c*e*Sin[x]),x]
```

```
[Out] (2*(3*d + a*e)*EllipticE[(x - ArcTan[b, c])/2, (2*Sqrt[b^2 + c^2])/(a + Sqr
t[b^2 + c^2])]*Sqrt[a + b*Cos[x] + c*Sin[x]])/(3*Sqrt[(a + b*Cos[x] + c*Sin
[x])/(a + Sqrt[b^2 + c^2])]) - (2*(a^2 - b^2 - c^2)*e*EllipticF[(x - ArcTan
[b, c])/2, (2*Sqrt[b^2 + c^2])/(a + Sqrt[b^2 + c^2])]*Sqrt[(a + b*Cos[x] +
c*Sin[x])/(a + Sqrt[b^2 + c^2])])/(3*Sqrt[a + b*Cos[x] + c*Sin[x]]) - (2*Sq
rt[a + b*Cos[x] + c*Sin[x]]*(c*e*Cos[x] - b*e*Sin[x]))/3
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 3119

```
Int[Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_
)]], x_Symbol] :> Dist[Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]]/Sqrt[(a +
b*Cos[d + e*x] + c*Sin[d + e*x])/(a + Sqrt[b^2 + c^2])], Int[Sqrt[a/(a + Sq
rt[b^2 + c^2]) + (Sqrt[b^2 + c^2]*Cos[d + e*x - ArcTan[b, c]])/(a + Sqrt[b^
2 + c^2])], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0]
&& NeQ[b^2 + c^2, 0] &&  !GtQ[a + Sqrt[b^2 + c^2], 0]
```

Rule 3127

```
Int[1/Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(
x_)]], x_Symbol] :> Dist[Sqrt[(a + b*Cos[d + e*x] + c*Sin[d + e*x])/(a + Sq
rt[b^2 + c^2])]/Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]], Int[1/Sqrt[a/(a
+ Sqrt[b^2 + c^2]) + (Sqrt[b^2 + c^2]*Cos[d + e*x - ArcTan[b, c]])/(a + Sqr
t[b^2 + c^2])], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2,
 0] && NeQ[b^2 + c^2, 0] &&  !GtQ[a + Sqrt[b^2 + c^2], 0]
```

Rule 3146

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(n_.)*((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)
]), x_Symbol] :> Simp[((B*c - b*C - a*C*Cos[d + e*x] + a*B*Sin[d + e*x])*(a
 + b*Cos[d + e*x] + c*Sin[d + e*x])^n)/(a*e*(n + 1)), x] + Dist[1/(a*(n + 1
)), Int[(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n - 1)*Simp[a*(b*B + c*C)*n
+ a^2*A*(n + 1) + (n*(a^2*B - B*c^2 + b*c*C) + a*b*A*(n + 1))*Cos[d + e*x]
+ (n*(b*B*c + a^2*C - b^2*C) + a*c*A*(n + 1))*Sin[d + e*x], x], x], x] /; F
reeQ[{a, b, c, d, e, A, B, C}, x] && GtQ[n, 0] && NeQ[a^2 - b^2 - c^2, 0]
```

Rule 3149

```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])
/Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)]]
, x_Symbol] :> Dist[B/b, Int[Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]], x],
 x] + Dist[(A*b - a*B)/b, Int[1/Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]],
x], x] /; FreeQ[{a, b, c, d, e, A, B, C}, x] && EqQ[B*c - b*C, 0] && NeQ[A*
b - a*B, 0]
```

Rubi steps

$$\int \sqrt{a+b\cos(x)+c\sin(x)}\,(d+be\cos(x)+ce\sin(x))\,dx = -\frac{2}{3}\sqrt{a+b\cos(x)+c\sin(x)}\,(ce\cos(x)-be\sin(x))+$$

$$= -\frac{2}{3}\sqrt{a+b\cos(x)+c\sin(x)}\,(ce\cos(x)-be\sin(x))-$$

$$= -\frac{2}{3}\sqrt{a+b\cos(x)+c\sin(x)}\,(ce\cos(x)-be\sin(x))+$$

$$= \frac{2(3d+ae)E\left(\frac{1}{2}\left(x-\tan^{-1}(b,c)\right)|\frac{2\sqrt{b^2+c^2}}{a+\sqrt{b^2+c^2}}\right)\sqrt{a+b\,\mathrm{c}}}{3\sqrt{\frac{a+b\cos(x)+c\sin(x)}{a+\sqrt{b^2+c^2}}}}$$

Mathematica [C] time = 6.37, size = 3006, normalized size = 13.13

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[a + b*Cos[x] + c*Sin[x]]*(d + b*e*Cos[x] + c*e*Sin[x]),x]
```

```
[Out] Sqrt[a + b*Cos[x] + c*Sin[x]]*((2*b*(3*d + a*e))/(3*c) - (2*c*e*Cos[x])/3 +
 (2*b*e*Sin[x])/3) + (2*a*d*AppellF1[1/2, 1/2, 1/2, 3/2, -((a + Sqrt[1 + b^
2/c^2]*c*Sin[x + ArcTan[b/c]])/(Sqrt[1 + b^2/c^2]*(1 - a/(Sqrt[1 + b^2/c^2]
*c))*c)), -((a + Sqrt[1 + b^2/c^2]*c*Sin[x + ArcTan[b/c]])/(Sqrt[1 + b^2/c^
2]*(-1 - a/(Sqrt[1 + b^2/c^2]*c))*c))]*Sec[x + ArcTan[b/c]]*Sqrt[(c*Sqrt[(b
^2 + c^2)/c^2] - c*Sqrt[(b^2 + c^2)/c^2]*Sin[x + ArcTan[b/c]])/(a + c*Sqrt[
(b^2 + c^2)/c^2])]*Sqrt[a + c*Sqrt[(b^2 + c^2)/c^2]*Sin[x + ArcTan[b/c]]]*S
qrt[(c*Sqrt[(b^2 + c^2)/c^2] + c*Sqrt[(b^2 + c^2)/c^2]*Sin[x + ArcTan[b/c]]
)/(-a + c*Sqrt[(b^2 + c^2)/c^2])]])/(Sqrt[1 + b^2/c^2]*c) + (2*b^2*e*AppellF
1[1/2, 1/2, 1/2, 3/2, -((a + Sqrt[1 + b^2/c^2]*c*Sin[x + ArcTan[b/c]])/(Sqr
t[1 + b^2/c^2]*(1 - a/(Sqrt[1 + b^2/c^2]*c))*c)), -((a + Sqrt[1 + b^2/c^2]*
c*Sin[x + ArcTan[b/c]])/(Sqrt[1 + b^2/c^2]*(-1 - a/(Sqrt[1 + b^2/c^2]*c))*c
))]*Sec[x + ArcTan[b/c]]*Sqrt[(c*Sqrt[(b^2 + c^2)/c^2] - c*Sqrt[(b^2 + c^2)
/c^2]*Sin[x + ArcTan[b/c]])/(a + c*Sqrt[(b^2 + c^2)/c^2])]*Sqrt[a + c*Sqrt[
(b^2 + c^2)/c^2]*Sin[x + ArcTan[b/c]]]*Sqrt[(c*Sqrt[(b^2 + c^2)/c^2] + c*Sq
rt[(b^2 + c^2)/c^2]*Sin[x + ArcTan[b/c]])/(-a + c*Sqrt[(b^2 + c^2)/c^2])]])/
(3*Sqrt[1 + b^2/c^2]*c) + (2*c*e*AppellF1[1/2, 1/2, 1/2, 3/2, -((a + Sqrt[1
```

+ b^2/c^2]*c*Sin[x + ArcTan[b/c]])/(Sqrt[1 + b^2/c^2]*(1 - a/(Sqrt[1 + b^2/c^2]*c))*c)), -((a + Sqrt[1 + b^2/c^2]*c*Sin[x + ArcTan[b/c]])/(Sqrt[1 + b^2/c^2]*(-1 - a/(Sqrt[1 + b^2/c^2]*c))*c))]*Sec[x + ArcTan[b/c]]*Sqrt[(c*Sqrt[(b^2 + c^2)/c^2] - c*Sqrt[(b^2 + c^2)/c^2]*Sin[x + ArcTan[b/c]])/(a + c*Sqrt[(b^2 + c^2)/c^2])]*Sqrt[a + c*Sqrt[(b^2 + c^2)/c^2]*Sin[x + ArcTan[b/c]]]*Sqrt[(c*Sqrt[(b^2 + c^2)/c^2] + c*Sqrt[(b^2 + c^2)/c^2]*Sin[x + ArcTan[b/c]])/(-a + c*Sqrt[(b^2 + c^2)/c^2])]])/(3*Sqrt[1 + b^2/c^2]) + (b^2*d*(-((c*AppellF1[-1/2, -1/2, -1/2, 1/2, -((a + b*Sqrt[1 + c^2/b^2]*Cos[x - ArcTan[c/b]])/(b*Sqrt[1 + c^2/b^2]*(1 - a/(b*Sqrt[1 + c^2/b^2])))), -((a + b*Sqrt[1 + c^2/b^2]*Cos[x - ArcTan[c/b]])/(b*Sqrt[1 + c^2/b^2]*(-1 - a/(b*Sqrt[1 + c^2/b^2]))))]*Sin[x - ArcTan[c/b]])/(b*Sqrt[1 + c^2/b^2]*Sqrt[(b*Sqrt[(b^2 + c^2)/b^2] - b*Sqrt[(b^2 + c^2)/b^2]*Cos[x - ArcTan[c/b]])/(a + b*Sqrt[(b^2 + c^2)/b^2])]*Sqrt[a + b*Sqrt[(b^2 + c^2)/b^2]*Cos[x - ArcTan[c/b]]]*Sqrt[(b*Sqrt[(b^2 + c^2)/b^2] + b*Sqrt[(b^2 + c^2)/b^2]*Cos[x - ArcTan[c/b]])/(-a + b*Sqrt[(b^2 + c^2)/b^2])]])) - ((2*b*(a + b*Sqrt[1 + c^2/b^2]*Cos[x - ArcTan[c/b]]))/(b^2 + c^2) - (c*Sin[x - ArcTan[c/b]])/(b*Sqrt[1 + c^2/b^2]))/Sqrt[a + b*Sqrt[1 + c^2/b^2]*Cos[x - ArcTan[c/b]]]))/c + c*d*(-((c*AppellF1[-1/2, -1/2, -1/2, 1/2, -((a + b*Sqrt[1 + c^2/b^2]*Cos[x - ArcTan[c/b]])/(b*Sqrt[1 + c^2/b^2]*(1 - a/(b*Sqrt[1 + c^2/b^2])))), -((a + b*Sqrt[1 + c^2/b^2]*Cos[x - ArcTan[c/b]])/(b*Sqrt[1 + c^2/b^2]*(-1 - a/(b*Sqrt[1 + c^2/b^2]))))]*Sin[x - ArcTan[c/b]])/(b*Sqrt[1 + c^2/b^2]*Sqrt[(b*Sqrt[(b^2 + c^2)/b^2] - b*Sqrt[(b^2 + c^2)/b^2]*Cos[x - ArcTan[c/b]])/(a + b*Sqrt[(b^2 + c^2)/b^2])]*Sqrt[a + b*Sqrt[(b^2 + c^2)/b^2]*Cos[x - ArcTan[c/b]]]*Sqrt[(b*Sqrt[(b^2 + c^2)/b^2] + b*Sqrt[(b^2 + c^2)/b^2]*Cos[x - ArcTan[c/b]])/(-a + b*Sqrt[(b^2 + c^2)/b^2])]])) - ((2*b*(a + b*Sqrt[1 + c^2/b^2]*Cos[x - ArcTan[c/b]]))/(b^2 + c^2) - (c*Sin[x - ArcTan[c/b]])/(b*Sqrt[1 + c^2/b^2]))/Sqrt[a + b*Sqrt[1 + c^2/b^2]*Cos[x - ArcTan[c/b]]]) + (a*b^2*e*(-((c*AppellF1[-1/2, -1/2, -1/2, 1/2, -((a + b*Sqrt[1 + c^2/b^2]*Cos[x - ArcTan[c/b]])/(b*Sqrt[1 + c^2/b^2]*(1 - a/(b*Sqrt[1 + c^2/b^2])))), -((a + b*Sqrt[1 + c^2/b^2]*Cos[x - ArcTan[c/b]])/(b*Sqrt[1 + c^2/b^2]*(-1 - a/(b*Sqrt[1 + c^2/b^2]))))]*Sin[x - ArcTan[c/b]])/(b*Sqrt[1 + c^2/b^2]*Sqrt[(b*Sqrt[(b^2 + c^2)/b^2] - b*Sqrt[(b^2 + c^2)/b^2]*Cos[x - ArcTan[c/b]])/(a + b*Sqrt[(b^2 + c^2)/b^2])]*Sqrt[a + b*Sqrt[(b^2 + c^2)/b^2]*Cos[x - ArcTan[c/b]]]*Sqrt[(b*Sqrt[(b^2 + c^2)/b^2] + b*Sqrt[(b^2 + c^2)/b^2]*Cos[x - ArcTan[c/b]])/(-a + b*Sqrt[(b^2 + c^2)/b^2])]])) - ((2*b*(a + b*Sqrt[1 + c^2/b^2]*Cos[x - ArcTan[c/b]]))/(b^2 + c^2) - (c*Sin[x - ArcTan[c/b]])/(b*Sqrt[1 + c^2/b^2]))/Sqrt[a + b*Sqrt[1 + c^2/b^2]*Cos[x - ArcTan[c/b]]]))/(3*c) + (a*c*e*(-((c*AppellF1[-1/2, -1/2, -1/2, 1/2, -((a + b*Sqrt[1 + c^2/b^2]*Cos[x - ArcTan[c/b]])/(b*Sqrt[1 + c^2/b^2]*(1 - a/(b*Sqrt[1 + c^2/b^2])))), -((a + b*Sqrt[1 + c^2/b^2]*Cos[x - ArcTan[c/b]])/(b*Sqrt[1 + c^2/b^2]*(-1 - a/(b*Sqrt[1 + c^2/b^2]))))]*Sin[x - ArcTan[c/b]])/(b*Sqrt[1 + c^2/b^2]*Sqrt[(b*Sqrt[(b^2 + c^2)/b^2] - b*Sqrt[(b^2 + c^2)/b^2]*Cos[x - ArcTan[c/b]])/(a + b*Sqrt[(b^2 + c^2)/b^2])]*Sqrt[a + b*Sqrt[(b^2 + c^2)/b^2]*Cos[x - ArcTan[c/b]]]*Sqrt[(b*Sqrt[(b^2 + c^2)/b^2] + b*Sqrt[(b^2 + c^2)/b^2]*Cos[x - ArcTan[c/b]])/(-a + b*Sqrt[(b^2 + c^2)/b^2])]])) - ((2*b*(a + b*Sqrt[1 + c^2/b^2]*Cos[x - ArcTan[c/b]

```
]))/(b^2 + c^2) - (c*Sin[x - ArcTan[c/b]])/(b*Sqrt[1 + c^2/b^2]))/Sqrt[a +
b*Sqrt[1 + c^2/b^2]*Cos[x - ArcTan[c/b]]]))/3
```

fricas [F] time = 1.18, size = 0, normalized size = 0.00

$$\text{integral}\left((be\cos(x) + ce\sin(x) + d)\sqrt{b\cos(x) + c\sin(x) + a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(x)+c*sin(x))^(1/2)*(d+b*e*cos(x)+c*e*sin(x)),x, algorith
m="fricas")
```

```
[Out] integral((b*e*cos(x) + c*e*sin(x) + d)*sqrt(b*cos(x) + c*sin(x) + a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (be\cos(x) + ce\sin(x) + d)\sqrt{b\cos(x) + c\sin(x) + a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(x)+c*sin(x))^(1/2)*(d+b*e*cos(x)+c*e*sin(x)),x, algorith
m="giac")
```

```
[Out] integrate((b*e*cos(x) + c*e*sin(x) + d)*sqrt(b*cos(x) + c*sin(x) + a), x)
```

maple [B] time = 0.77, size = 1460, normalized size = 6.38

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(x)+c*sin(x))^(1/2)*(d+b*e*cos(x)+c*e*sin(x)),x)
```

```
[Out] (-(-b^2*sin(x-arctan(-b,c))-c^2*sin(x-arctan(-b,c))-a*(b^2+c^2)^(1/2))*cos(
x-arctan(-b,c))^2/(b^2+c^2)^(1/2))^(1/2)/(b^2+c^2)^(1/2)*(((b^2+c^2)^(1/2)*
b^2*e+(b^2+c^2)^(1/2)*c^2*e)*(-2/3/(b^2+c^2)^(1/2)*(cos(x-arctan(-b,c))^2*(
(b^2+c^2)^(1/2)*sin(x-arctan(-b,c))+a))^(1/2)+2/3*(1/(b^2+c^2)^(1/2)*a-1)*(
(-(b^2+c^2)^(1/2)*sin(x-arctan(-b,c))-a)/(-a+(b^2+c^2)^(1/2)))^(1/2)*((-sin
(x-arctan(-b,c))+1)*(b^2+c^2)^(1/2)/(a+(b^2+c^2)^(1/2)))^(1/2)*((1+sin(x-ar
ctan(-b,c)))*(b^2+c^2)^(1/2)/(-a+(b^2+c^2)^(1/2)))^(1/2)/(cos(x-arctan(-b,c
))^2*((b^2+c^2)^(1/2)*sin(x-arctan(-b,c))+a))^(1/2)*EllipticF(((-(b^2+c^2)^
(1/2)*sin(x-arctan(-b,c))-a)/(-a+(b^2+c^2)^(1/2)))^(1/2),((a-(b^2+c^2)^(1/2
))/(a+(b^2+c^2)^(1/2)))^(1/2))-4/3/(b^2+c^2)^(1/2)*a*(1/(b^2+c^2)^(1/2)*a-1
)*((-(b^2+c^2)^(1/2)*sin(x-arctan(-b,c))-a)/(-a+(b^2+c^2)^(1/2)))^(1/2)*((-
sin(x-arctan(-b,c))+1)*(b^2+c^2)^(1/2)/(a+(b^2+c^2)^(1/2)))^(1/2)*((1+sin(x
-arctan(-b,c)))*(b^2+c^2)^(1/2)/(-a+(b^2+c^2)^(1/2)))^(1/2)/(cos(x-arctan(-
```

```
b,c))^2*((b^2+c^2)^(1/2)*sin(x-arctan(-b,c))+a))^(1/2)*((-1/(b^2+c^2)^(1/2)
*a-1)*EllipticE(((-(b^2+c^2)^(1/2)*sin(x-arctan(-b,c))-a)/(-a+(b^2+c^2)^(1/
2)))^(1/2),((a-(b^2+c^2)^(1/2))/(a+(b^2+c^2)^(1/2)))^(1/2))+EllipticF(((-(b
^2+c^2)^(1/2)*sin(x-arctan(-b,c))-a)/(-a+(b^2+c^2)^(1/2)))^(1/2),((a-(b^2+c
^2)^(1/2))/(a+(b^2+c^2)^(1/2)))^(1/2))))+2*(a*b^2*e+a*c^2*e+b^2*d+c^2*d)*(1
/(b^2+c^2)^(1/2)*a-1)*((-(b^2+c^2)^(1/2)*sin(x-arctan(-b,c))-a)/(-a+(b^2+c^
2)^(1/2)))^(1/2)*((-sin(x-arctan(-b,c))+1)*(b^2+c^2)^(1/2)/(a+(b^2+c^2)^(1/
2)))^(1/2)*((1+sin(x-arctan(-b,c)))*(b^2+c^2)^(1/2)/(-a+(b^2+c^2)^(1/2)))^(
1/2)/(cos(x-arctan(-b,c))^2*((b^2+c^2)^(1/2)*sin(x-arctan(-b,c))+a))^(1/2)*
((-1/(b^2+c^2)^(1/2)*a-1)*EllipticE(((-(b^2+c^2)^(1/2)*sin(x-arctan(-b,c))-
a)/(-a+(b^2+c^2)^(1/2)))^(1/2),((a-(b^2+c^2)^(1/2))/(a+(b^2+c^2)^(1/2)))^(1
/2))+EllipticF(((-(b^2+c^2)^(1/2)*sin(x-arctan(-b,c))-a)/(-a+(b^2+c^2)^(1/2
)))^(1/2),((a-(b^2+c^2)^(1/2))/(a+(b^2+c^2)^(1/2)))^(1/2)))+2*d*a*(b^2+c^2)
^(1/2)*(1/(b^2+c^2)^(1/2)*a-1)*((-(b^2+c^2)^(1/2)*sin(x-arctan(-b,c))-a)/(-
a+(b^2+c^2)^(1/2)))^(1/2)*((-sin(x-arctan(-b,c))+1)*(b^2+c^2)^(1/2)/(a+(b^2
+c^2)^(1/2)))^(1/2)*((1+sin(x-arctan(-b,c)))*(b^2+c^2)^(1/2)/(-a+(b^2+c^2)^
(1/2)))^(1/2)/(-(-b^2*sin(x-arctan(-b,c))-c^2*sin(x-arctan(-b,c))-a*(b^2+c^
2)^(1/2))*cos(x-arctan(-b,c))^2/(b^2+c^2)^(1/2))^(1/2)*EllipticF(((-(b^2+c^
2)^(1/2)*sin(x-arctan(-b,c))-a)/(-a+(b^2+c^2)^(1/2)))^(1/2),((a-(b^2+c^2)^(
1/2))/(a+(b^2+c^2)^(1/2)))^(1/2)))/cos(x-arctan(-b,c))/((b^2*sin(x-arctan(-
b,c))+c^2*sin(x-arctan(-b,c))+a*(b^2+c^2)^(1/2))/(b^2+c^2)^(1/2))^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (be\cos(x) + ce\sin(x) + d)\sqrt{b\cos(x) + c\sin(x) + a}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(x)+c*sin(x))^(1/2)*(d+b*e*cos(x)+c*e*sin(x)),x, algorith
m="maxima")
```

```
[Out] integrate((b*e*cos(x) + c*e*sin(x) + d)*sqrt(b*cos(x) + c*sin(x) + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \sqrt{a + b\,\cos(x) + c\,\sin(x)}\,\left(d + b\,e\,\cos(x) + c\,e\,\sin(x)\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*cos(x) + c*sin(x))^(1/2)*(d + b*e*cos(x) + c*e*sin(x)),x)
```

```
[Out] int((a + b*cos(x) + c*sin(x))^(1/2)*(d + b*e*cos(x) + c*e*sin(x)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a + b\cos{\left(x \right)} + c\sin{\left(x \right)}}\,\left(be\cos{\left(x \right)} + ce\sin{\left(x \right)} + d\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(x)+c*sin(x))**(1/2)*(d+b*e*cos(x)+c*e*sin(x)),x)
[Out] Integral(sqrt(a + b*cos(x) + c*sin(x))*(b*e*cos(x) + c*e*sin(x) + d), x)
```

3.559 $\int \frac{d+be\cos(x)+ce\sin(x)}{\sqrt{a+b\cos(x)+c\sin(x)}}\,dx$

Optimal. Leaf size=180

$$\frac{2(d-ae)\sqrt{\frac{a+b\cos(x)+c\sin(x)}{a+\sqrt{b^2+c^2}}}\,F\left(\frac{1}{2}\left(x-\tan^{-1}(b,c)\right)|\frac{2\sqrt{b^2+c^2}}{a+\sqrt{b^2+c^2}}\right)}{\sqrt{a+b\cos(x)+c\sin(x)}}+\frac{2e\sqrt{a+b\cos(x)+c\sin(x)}\,E\left(\frac{1}{2}\left(x-\tan^{-1}(b,c)\right)|\frac{2\sqrt{b^2+c^2}}{a+\sqrt{b^2+c^2}}\right)}{\sqrt{\frac{a+b\cos(x)+c\sin(x)}{a+\sqrt{b^2+c^2}}}}$$

```
[Out] 2*e*(cos(1/2*x-1/2*arctan(b,c))^2)^(1/2)/cos(1/2*x-1/2*arctan(b,c))*Ellipti
cE(sin(1/2*x-1/2*arctan(b,c)),2^(1/2)*((b^2+c^2)^(1/2)/(a+(b^2+c^2)^(1/2)))
^(1/2))*(a+b*cos(x)+c*sin(x))^(1/2)/((a+b*cos(x)+c*sin(x))/(a+(b^2+c^2)^(1/
2)))^(1/2)+2*(-a*e+d)*(cos(1/2*x-1/2*arctan(b,c))^2)^(1/2)/cos(1/2*x-1/2*ar
ctan(b,c))*EllipticF(sin(1/2*x-1/2*arctan(b,c)),2^(1/2)*((b^2+c^2)^(1/2)/(a
+(b^2+c^2)^(1/2)))^(1/2))*((a+b*cos(x)+c*sin(x))/(a+(b^2+c^2)^(1/2)))^(1/2)
/(a+b*cos(x)+c*sin(x))^(1/2)
```

Rubi [A] time = 0.19, antiderivative size = 180, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {3149, 3119, 2653, 3127, 2661}

$$\frac{2(d-ae)\sqrt{\frac{a+b\cos(x)+c\sin(x)}{a+\sqrt{b^2+c^2}}}\,F\left(\frac{1}{2}\left(x-\tan^{-1}(b,c)\right)|\frac{2\sqrt{b^2+c^2}}{a+\sqrt{b^2+c^2}}\right)}{\sqrt{a+b\cos(x)+c\sin(x)}}+\frac{2e\sqrt{a+b\cos(x)+c\sin(x)}\,E\left(\frac{1}{2}\left(x-\tan^{-1}(b,c)\right)|\frac{2\sqrt{b^2+c^2}}{a+\sqrt{b^2+c^2}}\right)}{\sqrt{\frac{a+b\cos(x)+c\sin(x)}{a+\sqrt{b^2+c^2}}}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + b*e*Cos[x] + c*e*Sin[x])/Sqrt[a + b*Cos[x] + c*Sin[x]],x]
```

```
[Out] (2*e*EllipticE[(x - ArcTan[b, c])/2, (2*Sqrt[b^2 + c^2])/(a + Sqrt[b^2 + c^
2])]*Sqrt[a + b*Cos[x] + c*Sin[x]])/Sqrt[(a + b*Cos[x] + c*Sin[x])/(a + Sqr
t[b^2 + c^2])] + (2*(d - a*e)*EllipticF[(x - ArcTan[b, c])/2, (2*Sqrt[b^2 +
 c^2])/(a + Sqrt[b^2 + c^2])]*Sqrt[(a + b*Cos[x] + c*Sin[x])/(a + Sqrt[b^2
+ c^2])])/Sqrt[a + b*Cos[x] + c*Sin[x]]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 3119

```
Int[Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_
)]], x_Symbol] :> Dist[Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]]/Sqrt[(a +
b*Cos[d + e*x] + c*Sin[d + e*x])/(a + Sqrt[b^2 + c^2])], Int[Sqrt[a/(a + Sq
rt[b^2 + c^2]) + (Sqrt[b^2 + c^2]*Cos[d + e*x - ArcTan[b, c]])/(a + Sqrt[b^
2 + c^2])], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0]
&& NeQ[b^2 + c^2, 0] &&  !GtQ[a + Sqrt[b^2 + c^2], 0]
```

Rule 3127

```
Int[1/Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(
x_)]], x_Symbol] :> Dist[Sqrt[(a + b*Cos[d + e*x] + c*Sin[d + e*x])/(a + Sq
rt[b^2 + c^2])]/Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]], Int[1/Sqrt[a/(a
+ Sqrt[b^2 + c^2]) + (Sqrt[b^2 + c^2]*Cos[d + e*x - ArcTan[b, c]])/(a + Sqr
t[b^2 + c^2])], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2,
 0] && NeQ[b^2 + c^2, 0] &&  !GtQ[a + Sqrt[b^2 + c^2], 0]
```

Rule 3149

```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])
/Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)]]
, x_Symbol] :> Dist[B/b, Int[Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]], x],
 x] + Dist[(A*b - a*B)/b, Int[1/Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]],
x], x] /; FreeQ[{a, b, c, d, e, A, B, C}, x] && EqQ[B*c - b*C, 0] && NeQ[A*
b - a*B, 0]
```

Rubi steps

$$\int \frac{d+be\cos(x)+ce\sin(x)}{\sqrt{a+b\cos(x)+c\sin(x)}}\,dx = e\int \sqrt{a+b\cos(x)+c\sin(x)}\,dx + (d-ae)\int \frac{1}{\sqrt{a+b\cos(x)+c\sin(x)}}\,dx$$

$$= \frac{\left(e\sqrt{a+b\cos(x)+c\sin(x)}\right)\int \sqrt{\frac{a}{a+\sqrt{b^2+c^2}}+\frac{\sqrt{b^2+c^2}\cos\left(x-\tan^{-1}(b,c)\right)}{a+\sqrt{b^2+c^2}}}\,dx}{\sqrt{\frac{a+b\cos(x)+c\sin(x)}{a+\sqrt{b^2+c^2}}}} + \frac{\left((d-a\right.}{}$$

$$= \frac{2eE\left(\frac{1}{2}\left(x-\tan^{-1}(b,c)\right)|\frac{2\sqrt{b^2+c^2}}{a+\sqrt{b^2+c^2}}\right)\sqrt{a+b\cos(x)+c\sin(x)}}{\sqrt{\frac{a+b\cos(x)+c\sin(x)}{a+\sqrt{b^2+c^2}}}} + \frac{2(d-ae)F\left(\frac{1}{2}\left(x\right.\right.}{}$$

Mathematica [C] time = 6.31, size = 1319, normalized size = 7.33

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[(d + b*e*Cos[x] + c*e*Sin[x])/Sqrt[a + b*Cos[x] + c*Sin[x]],x]
```

```
[Out] (2*b*e*Sqrt[a + b*Cos[x] + c*Sin[x]])/c + (2*d*AppellF1[1/2, 1/2, 1/2, 3/2,
 -((a + Sqrt[1 + b^2/c^2]*c*Sin[x + ArcTan[b/c]])/(Sqrt[1 + b^2/c^2]*(1 - a
/(Sqrt[1 + b^2/c^2]*c))*c)), -((a + Sqrt[1 + b^2/c^2]*c*Sin[x + ArcTan[b/c]
])/(Sqrt[1 + b^2/c^2]*(-1 - a/(Sqrt[1 + b^2/c^2]*c))*c))]*Sec[x + ArcTan[b/
c]]*Sqrt[(c*Sqrt[(b^2 + c^2)/c^2] - c*Sqrt[(b^2 + c^2)/c^2]*Sin[x + ArcTan[
b/c]])/(a + c*Sqrt[(b^2 + c^2)/c^2])]*Sqrt[a + c*Sqrt[(b^2 + c^2)/c^2]*Sin[
x + ArcTan[b/c]]]*Sqrt[(c*Sqrt[(b^2 + c^2)/c^2] + c*Sqrt[(b^2 + c^2)/c^2]*S
in[x + ArcTan[b/c]])/(-a + c*Sqrt[(b^2 + c^2)/c^2])])/(Sqrt[1 + b^2/c^2]*c)
 + (b^2*e*(-((c*AppellF1[-1/2, -1/2, -1/2, 1/2, -((a + b*Sqrt[1 + c^2/b^2]*
Cos[x - ArcTan[c/b]])/(b*Sqrt[1 + c^2/b^2]*(1 - a/(b*Sqrt[1 + c^2/b^2])))),
 -((a + b*Sqrt[1 + c^2/b^2]*Cos[x - ArcTan[c/b]])/(b*Sqrt[1 + c^2/b^2]*(-1
- a/(b*Sqrt[1 + c^2/b^2]))))]*Sin[x - ArcTan[c/b]])/(b*Sqrt[1 + c^2/b^2]*Sq
rt[(b*Sqrt[(b^2 + c^2)/b^2] - b*Sqrt[(b^2 + c^2)/b^2]*Cos[x - ArcTan[c/b]])
/(a + b*Sqrt[(b^2 + c^2)/b^2])]*Sqrt[a + b*Sqrt[(b^2 + c^2)/b^2]*Cos[x - Ar
cTan[c/b]]]*Sqrt[(b*Sqrt[(b^2 + c^2)/b^2] + b*Sqrt[(b^2 + c^2)/b^2]*Cos[x -
 ArcTan[c/b]])/(-a + b*Sqrt[(b^2 + c^2)/b^2])])) - ((2*b*(a + b*Sqrt[1 + c^
2/b^2]*Cos[x - ArcTan[c/b]]))/(b^2 + c^2) - (c*Sin[x - ArcTan[c/b]])/(b*Sqr
t[1 + c^2/b^2]))/Sqrt[a + b*Sqrt[1 + c^2/b^2]*Cos[x - ArcTan[c/b]]]))/c + c
*e*(-((c*AppellF1[-1/2, -1/2, -1/2, 1/2, -((a + b*Sqrt[1 + c^2/b^2]*Cos[x -
 ArcTan[c/b]])/(b*Sqrt[1 + c^2/b^2]*(1 - a/(b*Sqrt[1 + c^2/b^2])))), -((a +
 b*Sqrt[1 + c^2/b^2]*Cos[x - ArcTan[c/b]])/(b*Sqrt[1 + c^2/b^2]*(-1 - a/(b*
```

```
Sqrt[1 + c^2/b^2]))))]*Sin[x - ArcTan[c/b]])/(b*Sqrt[1 + c^2/b^2]*Sqrt[(b*S
qrt[(b^2 + c^2)/b^2] - b*Sqrt[(b^2 + c^2)/b^2]*Cos[x - ArcTan[c/b]])/(a + b
*Sqrt[(b^2 + c^2)/b^2])]*Sqrt[a + b*Sqrt[(b^2 + c^2)/b^2]*Cos[x - ArcTan[c/
b]]]*Sqrt[(b*Sqrt[(b^2 + c^2)/b^2] + b*Sqrt[(b^2 + c^2)/b^2]*Cos[x - ArcTan
[c/b]])/(-a + b*Sqrt[(b^2 + c^2)/b^2])])) - ((2*b*(a + b*Sqrt[1 + c^2/b^2]*
Cos[x - ArcTan[c/b]]))/(b^2 + c^2) - (c*Sin[x - ArcTan[c/b]])/(b*Sqrt[1 + c
^2/b^2]))/Sqrt[a + b*Sqrt[1 + c^2/b^2]*Cos[x - ArcTan[c/b]]])
```

fricas [F] time = 0.96, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{be\cos(x)+ce\sin(x)+d}{\sqrt{b\cos(x)+c\sin(x)+a}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+b*e*cos(x)+c*e*sin(x))/(a+b*cos(x)+c*sin(x))^(1/2),x, algorith
m="fricas")
```

```
[Out] integral((b*e*cos(x) + c*e*sin(x) + d)/sqrt(b*cos(x) + c*sin(x) + a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{be\cos(x)+ce\sin(x)+d}{\sqrt{b\cos(x)+c\sin(x)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+b*e*cos(x)+c*e*sin(x))/(a+b*cos(x)+c*sin(x))^(1/2),x, algorith
m="giac")
```

```
[Out] integrate((b*e*cos(x) + c*e*sin(x) + d)/sqrt(b*cos(x) + c*sin(x) + a), x)
```

maple [B] time = 0.65, size = 777, normalized size = 4.32

$$\sqrt{-\frac{\left(-b^2\sin(x-\arctan(-b,c))-c^2\sin(x-\arctan(-b,c))-a\sqrt{b^2+c^2}\right)\left(\cos^2(x-\arctan(-b,c))\right)}{\sqrt{b^2+c^2}}}\left(\frac{2\left(b^2e+c^2e\right)\left(\frac{a}{\sqrt{b^2+c^2}}-1\right)\sqrt{\frac{-\sqrt{b^2+c^2}\,\sin(x-\arctan(-b,c))-a}{-a+\sqrt{b^2+c^2}}}\cdots}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d+b*e*cos(x)+c*e*sin(x))/(a+b*cos(x)+c*sin(x))^(1/2),x)
```

```
[Out] (-(-b^2*sin(x-arctan(-b,c))-c^2*sin(x-arctan(-b,c))-a*(b^2+c^2)^(1/2))*cos(
x-arctan(-b,c))^2/(b^2+c^2)^(1/2))^(1/2)/(b^2+c^2)^(1/2)*(2*(b^2*e+c^2*e)*(
```

```
1/(b^2+c^2)^(1/2)*a-1)*((-(b^2+c^2)^(1/2)*sin(x-arctan(-b,c))-a)/(-a+(b^2+c
^2)^(1/2)))^(1/2)*((-sin(x-arctan(-b,c))+1)*(b^2+c^2)^(1/2)/(a+(b^2+c^2)^(1
/2)))^(1/2)*((1+sin(x-arctan(-b,c)))*(b^2+c^2)^(1/2)/(-a+(b^2+c^2)^(1/2)))^
(1/2)/(cos(x-arctan(-b,c))^2*((b^2+c^2)^(1/2)*sin(x-arctan(-b,c))+a))^(1/2)
*((-1/(b^2+c^2)^(1/2)*a-1)*EllipticE(((-(b^2+c^2)^(1/2)*sin(x-arctan(-b,c))
-a)/(-a+(b^2+c^2)^(1/2)))^(1/2),((a-(b^2+c^2)^(1/2))/(a+(b^2+c^2)^(1/2)))^(
1/2))+EllipticF(((-(b^2+c^2)^(1/2)*sin(x-arctan(-b,c))-a)/(-a+(b^2+c^2)^(1/
2)))^(1/2),((a-(b^2+c^2)^(1/2))/(a+(b^2+c^2)^(1/2)))^(1/2)))+2*d*(b^2+c^2)^
(1/2)*(1/(b^2+c^2)^(1/2)*a-1)*((-(b^2+c^2)^(1/2)*sin(x-arctan(-b,c))-a)/(-a
+(b^2+c^2)^(1/2)))^(1/2)*((-sin(x-arctan(-b,c))+1)*(b^2+c^2)^(1/2)/(a+(b^2+
c^2)^(1/2)))^(1/2)*((1+sin(x-arctan(-b,c)))*(b^2+c^2)^(1/2)/(-a+(b^2+c^2)^(
1/2)))^(1/2)/(-(-b^2*sin(x-arctan(-b,c))-c^2*sin(x-arctan(-b,c))-a*(b^2+c^2
)^(1/2))*cos(x-arctan(-b,c))^2/(b^2+c^2)^(1/2))^(1/2)*EllipticF(((-(b^2+c^2
)^(1/2)*sin(x-arctan(-b,c))-a)/(-a+(b^2+c^2)^(1/2)))^(1/2),((a-(b^2+c^2)^(1
/2))/(a+(b^2+c^2)^(1/2)))^(1/2)))/cos(x-arctan(-b,c))/((b^2*sin(x-arctan(-b
,c))+c^2*sin(x-arctan(-b,c))+a*(b^2+c^2)^(1/2))/(b^2+c^2)^(1/2))^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{be\cos(x) + ce\sin(x) + d}{\sqrt{b\cos(x) + c\sin(x) + a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+b*e*cos(x)+c*e*sin(x))/(a+b*cos(x)+c*sin(x))^(1/2),x, algorith
m="maxima")
```

```
[Out] integrate((b*e*cos(x) + c*e*sin(x) + d)/sqrt(b*cos(x) + c*sin(x) + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{d + b\,e\,\cos(x) + c\,e\,\sin(x)}{\sqrt{a + b\,\cos(x) + c\,\sin(x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + b*e*cos(x) + c*e*sin(x))/(a + b*cos(x) + c*sin(x))^(1/2),x)
```

```
[Out] int((d + b*e*cos(x) + c*e*sin(x))/(a + b*cos(x) + c*sin(x))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{be\cos{\left(x \right)} + ce\sin{\left(x \right)} + d}{\sqrt{a + b\cos{\left(x \right)} + c\sin{\left(x \right)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+b*e*cos(x)+c*e*sin(x))/(a+b*cos(x)+c*sin(x))**(1/2),x)
[Out] Integral((b*e*cos(x) + c*e*sin(x) + d)/sqrt(a + b*cos(x) + c*sin(x)), x)
```

3.560 $\int \frac{d+be\cos(x)+ce\sin(x)}{(a+b\cos(x)+c\sin(x))^{3/2}}\,dx$

Optimal. Leaf size=250

$$\frac{2(d-ae)\sqrt{a+b\cos(x)+c\sin(x)}\,E\left(\frac{1}{2}\left(x-\tan^{-1}(b,c)\right)|\frac{2\sqrt{b^2+c^2}}{a+\sqrt{b^2+c^2}}\right)}{\left(a^2-b^2-c^2\right)\sqrt{\frac{a+b\cos(x)+c\sin(x)}{a+\sqrt{b^2+c^2}}}}+\frac{2e\sqrt{\frac{a+b\cos(x)+c\sin(x)}{a+\sqrt{b^2+c^2}}}\,F\left(\frac{1}{2}\left(x-\tan^{-1}(b,c)\right)|\frac{2\sqrt{b^2+c^2}}{a+\sqrt{b^2+c^2}}\right)}{\sqrt{a+b\cos(x)+c\sin(x)}}$$

```
[Out] 2*(c*(-a*e+d)*cos(x)-b*(-a*e+d)*sin(x))/(a^2-b^2-c^2)/(a+b*cos(x)+c*sin(x))
^(1/2)+2*(-a*e+d)*(cos(1/2*x-1/2*arctan(b,c))^2)^(1/2)/cos(1/2*x-1/2*arctan
(b,c))*EllipticE(sin(1/2*x-1/2*arctan(b,c)),2^(1/2)*((b^2+c^2)^(1/2)/(a+(b^
2+c^2)^(1/2)))^(1/2))*(a+b*cos(x)+c*sin(x))^(1/2)/(a^2-b^2-c^2)/((a+b*cos(x
)+c*sin(x))/(a+(b^2+c^2)^(1/2)))^(1/2)+2*e*(cos(1/2*x-1/2*arctan(b,c))^2)^(
1/2)/cos(1/2*x-1/2*arctan(b,c))*EllipticF(sin(1/2*x-1/2*arctan(b,c)),2^(1/2
)*((b^2+c^2)^(1/2)/(a+(b^2+c^2)^(1/2)))^(1/2))*((a+b*cos(x)+c*sin(x))/(a+(b
^2+c^2)^(1/2)))^(1/2)/(a+b*cos(x)+c*sin(x))^(1/2)
```

Rubi [A] time = 0.32, antiderivative size = 250, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {3156, 3149, 3119, 2653, 3127, 2661}

$$\frac{2(d-ae)\sqrt{a+b\cos(x)+c\sin(x)}\,E\left(\frac{1}{2}\left(x-\tan^{-1}(b,c)\right)|\frac{2\sqrt{b^2+c^2}}{a+\sqrt{b^2+c^2}}\right)}{\left(a^2-b^2-c^2\right)\sqrt{\frac{a+b\cos(x)+c\sin(x)}{a+\sqrt{b^2+c^2}}}}+\frac{2e\sqrt{\frac{a+b\cos(x)+c\sin(x)}{a+\sqrt{b^2+c^2}}}\,F\left(\frac{1}{2}\left(x-\tan^{-1}(b,c)\right)|\frac{2\sqrt{b^2+c^2}}{a+\sqrt{b^2+c^2}}\right)}{\sqrt{a+b\cos(x)+c\sin(x)}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + b*e*Cos[x] + c*e*Sin[x])/(a + b*Cos[x] + c*Sin[x])^(3/2),x]
```

```
[Out] (2*(d - a*e)*EllipticE[(x - ArcTan[b, c])/2, (2*Sqrt[b^2 + c^2])/(a + Sqrt[
b^2 + c^2])]*Sqrt[a + b*Cos[x] + c*Sin[x]])/((a^2 - b^2 - c^2)*Sqrt[(a + b*
Cos[x] + c*Sin[x])/(a + Sqrt[b^2 + c^2])]) + (2*e*EllipticF[(x - ArcTan[b,
c])/2, (2*Sqrt[b^2 + c^2])/(a + Sqrt[b^2 + c^2])]*Sqrt[(a + b*Cos[x] + c*Si
n[x])/(a + Sqrt[b^2 + c^2])])/Sqrt[a + b*Cos[x] + c*Sin[x]] + (2*(c*(d - a*
e)*Cos[x] - b*(d - a*e)*Sin[x]))/((a^2 - b^2 - c^2)*Sqrt[a + b*Cos[x] + c*S
in[x]])
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 3119

```
Int[Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_
)]], x_Symbol] :> Dist[Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]]/Sqrt[(a +
b*Cos[d + e*x] + c*Sin[d + e*x])/(a + Sqrt[b^2 + c^2])], Int[Sqrt[a/(a + Sq
rt[b^2 + c^2]) + (Sqrt[b^2 + c^2]*Cos[d + e*x - ArcTan[b, c]])/(a + Sqrt[b^
2 + c^2])], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0]
&& NeQ[b^2 + c^2, 0] &&  !GtQ[a + Sqrt[b^2 + c^2], 0]
```

Rule 3127

```
Int[1/Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(
x_)]], x_Symbol] :> Dist[Sqrt[(a + b*Cos[d + e*x] + c*Sin[d + e*x])/(a + Sq
rt[b^2 + c^2])]/Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]], Int[1/Sqrt[a/(a
+ Sqrt[b^2 + c^2]) + (Sqrt[b^2 + c^2]*Cos[d + e*x - ArcTan[b, c]])/(a + Sqr
t[b^2 + c^2])], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2,
 0] && NeQ[b^2 + c^2, 0] &&  !GtQ[a + Sqrt[b^2 + c^2], 0]
```

Rule 3149

```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])
/Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)]]
, x_Symbol] :> Dist[B/b, Int[Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]], x],
 x] + Dist[(A*b - a*B)/b, Int[1/Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]],
x], x] /; FreeQ[{a, b, c, d, e, A, B, C}, x] && EqQ[B*c - b*C, 0] && NeQ[A*
b - a*B, 0]
```

Rule 3156

```
Int[((a_.) + cos[(d_.) + (e_.)*(x_)]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)])
^(n_)*((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)
]), x_Symbol] :> -Simp[((c*B - b*C - (a*C - c*A)*Cos[d + e*x] + (a*B - b*A)
*Sin[d + e*x])*(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n + 1))/(e*(n + 1)*(a
^2 - b^2 - c^2)), x] + Dist[1/((n + 1)*(a^2 - b^2 - c^2)), Int[(a + b*Cos[d
 + e*x] + c*Sin[d + e*x])^(n + 1)*Simp[(n + 1)*(a*A - b*B - c*C) + (n + 2)*
(a*B - b*A)*Cos[d + e*x] + (n + 2)*(a*C - c*A)*Sin[d + e*x], x], x], x] /;
FreeQ[{a, b, c, d, e, A, B, C}, x] && LtQ[n, -1] && NeQ[a^2 - b^2 - c^2, 0]
 && NeQ[n, -2]
```

Rubi steps

$$\int \frac{d+be\cos(x)+ce\sin(x)}{(a+b\cos(x)+c\sin(x))^{3/2}}\,dx = \frac{2(c(d-ae)\cos(x)-b(d-ae)\sin(x))}{\left(a^2-b^2-c^2\right)\sqrt{a+b\cos(x)+c\sin(x)}} - \frac{2\int \frac{\frac{1}{2}\left(-ad+\left(b^2+c^2\right)e\right)-\frac{1}{2}b(d-ae)\cos(x)-\frac{1}{2}c(d}{\sqrt{a+b\cos(x)+c\sin(x)}}}{a^2-b^2-c^2}$$

$$= \frac{2(c(d-ae)\cos(x)-b(d-ae)\sin(x))}{\left(a^2-b^2-c^2\right)\sqrt{a+b\cos(x)+c\sin(x)}} + e\int \frac{1}{\sqrt{a+b\cos(x)+c\sin(x)}}\,dx +$$

$$= \frac{2(c(d-ae)\cos(x)-b(d-ae)\sin(x))}{\left(a^2-b^2-c^2\right)\sqrt{a+b\cos(x)+c\sin(x)}} + \frac{\left((d-ae)\sqrt{a+b\cos(x)+c\sin(x)}\right)}{\left(a^2-b^2-c^2\right)}$$

$$= \frac{2(d-ae)E\left(\frac{1}{2}\left(x-\tan^{-1}(b,c)\right)|\frac{2\sqrt{b^2+c^2}}{a+\sqrt{b^2+c^2}}\right)\sqrt{a+b\cos(x)+c\sin(x)}}{\left(a^2-b^2-c^2\right)\sqrt{\frac{a+b\cos(x)+c\sin(x)}{a+\sqrt{b^2+c^2}}}} + \frac{2eF\left(\frac{1}{2}\right.}{}$$

Mathematica [C] time = 6.57, size = 3176, normalized size = 12.70

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(d + b*e*Cos[x] + c*e*Sin[x])/(a + b*Cos[x] + c*Sin[x])^(3/2),x]
```

```
[Out] Sqrt[a + b*Cos[x] + c*Sin[x]]*((2*(b^2 + c^2)*(-d + a*e))/(b*c*(-a^2 + b^2
+ c^2)) - (2*(-(a*c*d) + a^2*c*e - b^2*d*Sin[x] - c^2*d*Sin[x] + a*b^2*e*Si
n[x] + a*c^2*e*Sin[x]))/(b*(-a^2 + b^2 + c^2)*(a + b*Cos[x] + c*Sin[x]))) -
 (2*a*d*AppellF1[1/2, 1/2, 1/2, 3/2, -((a + Sqrt[1 + b^2/c^2]*c*Sin[x + Arc
Tan[b/c]])/(Sqrt[1 + b^2/c^2]*(1 - a/(Sqrt[1 + b^2/c^2]*c))*c)), -((a + Sqr
t[1 + b^2/c^2]*c*Sin[x + ArcTan[b/c]])/(Sqrt[1 + b^2/c^2]*(-1 - a/(Sqrt[1 +
 b^2/c^2]*c))*c))]*Sec[x + ArcTan[b/c]]*Sqrt[(c*Sqrt[(b^2 + c^2)/c^2] - c*S
qrt[(b^2 + c^2)/c^2]*Sin[x + ArcTan[b/c]])/(a + c*Sqrt[(b^2 + c^2)/c^2])]*S
qrt[a + c*Sqrt[(b^2 + c^2)/c^2]*Sin[x + ArcTan[b/c]]]*Sqrt[(c*Sqrt[(b^2 + c
^2)/c^2] + c*Sqrt[(b^2 + c^2)/c^2]*Sin[x + ArcTan[b/c]])/(-a + c*Sqrt[(b^2
+ c^2)/c^2])])/(Sqrt[1 + b^2/c^2]*c*(-a^2 + b^2 + c^2)) + (2*b^2*e*AppellF1
[1/2, 1/2, 1/2, 3/2, -((a + Sqrt[1 + b^2/c^2]*c*Sin[x + ArcTan[b/c]])/(Sqrt
[1 + b^2/c^2]*(1 - a/(Sqrt[1 + b^2/c^2]*c))*c)), -((a + Sqrt[1 + b^2/c^2]*c
*Sin[x + ArcTan[b/c]])/(Sqrt[1 + b^2/c^2]*(-1 - a/(Sqrt[1 + b^2/c^2]*c))*c)
)]*Sec[x + ArcTan[b/c]]*Sqrt[(c*Sqrt[(b^2 + c^2)/c^2] - c*Sqrt[(b^2 + c^2)/
```

c^2]*Sin[x + ArcTan[b/c]])/(a + c*Sqrt[(b^2 + c^2)/c^2])]*Sqrt[a + c*Sqrt[(
b^2 + c^2)/c^2]*Sin[x + ArcTan[b/c]]]*Sqrt[(c*Sqrt[(b^2 + c^2)/c^2] + c*Sqr
t[(b^2 + c^2)/c^2]*Sin[x + ArcTan[b/c]])/(-a + c*Sqrt[(b^2 + c^2)/c^2])])/(
Sqrt[1 + b^2/c^2]*c*(-a^2 + b^2 + c^2)) + (2*c*e*AppellF1[1/2, 1/2, 1/2, 3/
2, -((a + Sqrt[1 + b^2/c^2]*c*Sin[x + ArcTan[b/c]])/(Sqrt[1 + b^2/c^2]*(1 -
 a/(Sqrt[1 + b^2/c^2]*c))*c)), -((a + Sqrt[1 + b^2/c^2]*c*Sin[x + ArcTan[b/
c]])/(Sqrt[1 + b^2/c^2]*(-1 - a/(Sqrt[1 + b^2/c^2]*c))*c))]*Sec[x + ArcTan[
b/c]]*Sqrt[(c*Sqrt[(b^2 + c^2)/c^2] - c*Sqrt[(b^2 + c^2)/c^2]*Sin[x + ArcTa
n[b/c]])/(a + c*Sqrt[(b^2 + c^2)/c^2])]*Sqrt[a + c*Sqrt[(b^2 + c^2)/c^2]*Si
n[x + ArcTan[b/c]]]*Sqrt[(c*Sqrt[(b^2 + c^2)/c^2] + c*Sqrt[(b^2 + c^2)/c^2]
*Sin[x + ArcTan[b/c]])/(-a + c*Sqrt[(b^2 + c^2)/c^2])])/(Sqrt[1 + b^2/c^2]*
(-a^2 + b^2 + c^2)) - (b^2*d*(-((c*AppellF1[-1/2, -1/2, -1/2, 1/2, -((a + b
*Sqrt[1 + c^2/b^2]*Cos[x - ArcTan[c/b]])/(b*Sqrt[1 + c^2/b^2]*(1 - a/(b*Sqr
t[1 + c^2/b^2])))), -((a + b*Sqrt[1 + c^2/b^2]*Cos[x - ArcTan[c/b]])/(b*Sqr
t[1 + c^2/b^2]*(-1 - a/(b*Sqrt[1 + c^2/b^2]))))]*Sin[x - ArcTan[c/b]])/(b*S
qrt[1 + c^2/b^2]*Sqrt[(b*Sqrt[(b^2 + c^2)/b^2] - b*Sqrt[(b^2 + c^2)/b^2]*Co
s[x - ArcTan[c/b]])/(a + b*Sqrt[(b^2 + c^2)/b^2])]*Sqrt[a + b*Sqrt[(b^2 + c
^2)/b^2]*Cos[x - ArcTan[c/b]]]*Sqrt[(b*Sqrt[(b^2 + c^2)/b^2] + b*Sqrt[(b^2
+ c^2)/b^2]*Cos[x - ArcTan[c/b]])/(-a + b*Sqrt[(b^2 + c^2)/b^2])])) - ((2*b
*(a + b*Sqrt[1 + c^2/b^2]*Cos[x - ArcTan[c/b]]))/(b^2 + c^2) - (c*Sin[x - A
rcTan[c/b]])/(b*Sqrt[1 + c^2/b^2]))/Sqrt[a + b*Sqrt[1 + c^2/b^2]*Cos[x - Ar
cTan[c/b]]]))/(c*(-a^2 + b^2 + c^2)) - (c*d*(-((c*AppellF1[-1/2, -1/2, -1/2
, 1/2, -((a + b*Sqrt[1 + c^2/b^2]*Cos[x - ArcTan[c/b]])/(b*Sqrt[1 + c^2/b^2
]*(1 - a/(b*Sqrt[1 + c^2/b^2])))), -((a + b*Sqrt[1 + c^2/b^2]*Cos[x - ArcTa
n[c/b]])/(b*Sqrt[1 + c^2/b^2]*(-1 - a/(b*Sqrt[1 + c^2/b^2]))))]*Sin[x - Arc
Tan[c/b]])/(b*Sqrt[1 + c^2/b^2]*Sqrt[(b*Sqrt[(b^2 + c^2)/b^2] - b*Sqrt[(b^2
 + c^2)/b^2]*Cos[x - ArcTan[c/b]])/(a + b*Sqrt[(b^2 + c^2)/b^2])]*Sqrt[a +
b*Sqrt[(b^2 + c^2)/b^2]*Cos[x - ArcTan[c/b]]]*Sqrt[(b*Sqrt[(b^2 + c^2)/b^2]
 + b*Sqrt[(b^2 + c^2)/b^2]*Cos[x - ArcTan[c/b]])/(-a + b*Sqrt[(b^2 + c^2)/b
^2])])) - ((2*b*(a + b*Sqrt[1 + c^2/b^2]*Cos[x - ArcTan[c/b]]))/(b^2 + c^2)
 - (c*Sin[x - ArcTan[c/b]])/(b*Sqrt[1 + c^2/b^2]))/Sqrt[a + b*Sqrt[1 + c^2/
b^2]*Cos[x - ArcTan[c/b]]]))/(-a^2 + b^2 + c^2) + (a*b^2*e*(-((c*AppellF1[-
1/2, -1/2, -1/2, 1/2, -((a + b*Sqrt[1 + c^2/b^2]*Cos[x - ArcTan[c/b]])/(b*S
qrt[1 + c^2/b^2]*(1 - a/(b*Sqrt[1 + c^2/b^2])))), -((a + b*Sqrt[1 + c^2/b^2
]*Cos[x - ArcTan[c/b]])/(b*Sqrt[1 + c^2/b^2]*(-1 - a/(b*Sqrt[1 + c^2/b^2]))
))]*Sin[x - ArcTan[c/b]])/(b*Sqrt[1 + c^2/b^2]*Sqrt[(b*Sqrt[(b^2 + c^2)/b^2
] - b*Sqrt[(b^2 + c^2)/b^2]*Cos[x - ArcTan[c/b]])/(a + b*Sqrt[(b^2 + c^2)/b
^2])]*Sqrt[a + b*Sqrt[(b^2 + c^2)/b^2]*Cos[x - ArcTan[c/b]]]*Sqrt[(b*Sqrt[(
b^2 + c^2)/b^2] + b*Sqrt[(b^2 + c^2)/b^2]*Cos[x - ArcTan[c/b]])/(-a + b*Sqr
t[(b^2 + c^2)/b^2])])) - ((2*b*(a + b*Sqrt[1 + c^2/b^2]*Cos[x - ArcTan[c/b]
]))/(b^2 + c^2) - (c*Sin[x - ArcTan[c/b]])/(b*Sqrt[1 + c^2/b^2]))/Sqrt[a +
b*Sqrt[1 + c^2/b^2]*Cos[x - ArcTan[c/b]]]))/(c*(-a^2 + b^2 + c^2)) + (a*c*e
*(-((c*AppellF1[-1/2, -1/2, -1/2, 1/2, -((a + b*Sqrt[1 + c^2/b^2]*Cos[x - A
rcTan[c/b]])/(b*Sqrt[1 + c^2/b^2]*(1 - a/(b*Sqrt[1 + c^2/b^2])))), -((a + b
*Sqrt[1 + c^2/b^2]*Cos[x - ArcTan[c/b]])/(b*Sqrt[1 + c^2/b^2]*(-1 - a/(b*Sq

```
rt[1 + c^2/b^2]))))]*Sin[x - ArcTan[c/b]])/(b*Sqrt[1 + c^2/b^2]*Sqrt[(b*Sqr
t[(b^2 + c^2)/b^2] - b*Sqrt[(b^2 + c^2)/b^2]*Cos[x - ArcTan[c/b]])/(a + b*S
qrt[(b^2 + c^2)/b^2])]*Sqrt[a + b*Sqrt[(b^2 + c^2)/b^2]*Cos[x - ArcTan[c/b]
]]*Sqrt[(b*Sqrt[(b^2 + c^2)/b^2] + b*Sqrt[(b^2 + c^2)/b^2]*Cos[x - ArcTan[c
/b]])/(-a + b*Sqrt[(b^2 + c^2)/b^2])])) - ((2*b*(a + b*Sqrt[1 + c^2/b^2]*Co
s[x - ArcTan[c/b]]))/(b^2 + c^2) - (c*Sin[x - ArcTan[c/b]])/(b*Sqrt[1 + c^2
/b^2]))/Sqrt[a + b*Sqrt[1 + c^2/b^2]*Cos[x - ArcTan[c/b]]]))/(-a^2 + b^2 +
c^2)
```

fricas [F] time = 2.94, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{(be\cos(x) + ce\sin(x) + d)\sqrt{b\cos(x) + c\sin(x) + a}}{2\,ab\cos(x) + \left(b^2 - c^2\right)\cos(x)^2 + a^2 + c^2 + 2\,(bc\cos(x) + ac)\sin(x)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+b*e*cos(x)+c*e*sin(x))/(a+b*cos(x)+c*sin(x))^(3/2),x, algorith
m="fricas")
```

```
[Out] integral((b*e*cos(x) + c*e*sin(x) + d)*sqrt(b*cos(x) + c*sin(x) + a)/(2*a*b
*cos(x) + (b^2 - c^2)*cos(x)^2 + a^2 + c^2 + 2*(b*c*cos(x) + a*c)*sin(x)),
x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{be\cos(x) + ce\sin(x) + d}{(b\cos(x) + c\sin(x) + a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+b*e*cos(x)+c*e*sin(x))/(a+b*cos(x)+c*sin(x))^(3/2),x, algorith
m="giac")
```

```
[Out] integrate((b*e*cos(x) + c*e*sin(x) + d)/(b*cos(x) + c*sin(x) + a)^(3/2), x)
```

maple [B] time = 1.07, size = 2596, normalized size = 10.38

Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d+b*e*cos(x)+c*e*sin(x))/(a+b*cos(x)+c*sin(x))^(3/2),x)
```

```
[Out] (-(-b^2*sin(x-arctan(-b,c))-c^2*sin(x-arctan(-b,c))-a*(b^2+c^2)^(1/2))*cos(
x-arctan(-b,c))^2/(b^2+c^2)^(1/2))^(1/2)/(b^2+c^2)^(1/2)*(2*(b^2+c^2)^(1/2)
*e*(1/(b^2+c^2)^(1/2)*a-1)*((-(b^2+c^2)^(1/2)*sin(x-arctan(-b,c))-a)/(-a+(b
```

```
^2+c^2)^(1/2)))^(1/2)*((-sin(x-arctan(-b,c))+1)*(b^2+c^2)^(1/2)/(a+(b^2+c^2
)^(1/2)))^(1/2)*((1+sin(x-arctan(-b,c)))*(b^2+c^2)^(1/2)/(-a+(b^2+c^2)^(1/2
)))^(1/2)/(-(-b^2*sin(x-arctan(-b,c))-c^2*sin(x-arctan(-b,c))-a*(b^2+c^2)^(
1/2))*cos(x-arctan(-b,c))^2/(b^2+c^2)^(1/2))^(1/2)*EllipticF(((-(b^2+c^2)^(
1/2)*sin(x-arctan(-b,c))-a)/(-a+(b^2+c^2)^(1/2)))^(1/2),((a-(b^2+c^2)^(1/2)
)/(a+(b^2+c^2)^(1/2)))^(1/2))+(b^2+c^2)^(1/2)*(-b^2-c^2)*cos(x-arctan(-b,c)
)^2/(a^2-b^2-c^2)*(a*e-d)/(-(-(b^2+c^2)^(1/2)*sin(x-arctan(-b,c))-a)*cos(x-
arctan(-b,c))^2*(b^2+c^2))^(1/2)-a*(b^2+c^2)*(a*e-d)/(a^2-b^2-c^2)*(1/(b^2+
c^2)^(1/2)*a-1)*((-(b^2+c^2)^(1/2)*sin(x-arctan(-b,c))-a)/(-a+(b^2+c^2)^(1/
2)))^(1/2)*((-sin(x-arctan(-b,c))+1)*(b^2+c^2)^(1/2)/(a+(b^2+c^2)^(1/2)))^(
1/2)*((1+sin(x-arctan(-b,c)))*(b^2+c^2)^(1/2)/(-a+(b^2+c^2)^(1/2)))^(1/2)/(
-(-(b^2+c^2)^(1/2)*sin(x-arctan(-b,c))-a)*cos(x-arctan(-b,c))^2*(b^2+c^2))^
(1/2)*EllipticF(((-(b^2+c^2)^(1/2)*sin(x-arctan(-b,c))-a)/(-a+(b^2+c^2)^(1/
2)))^(1/2),((a-(b^2+c^2)^(1/2))/(a+(b^2+c^2)^(1/2)))^(1/2))-2*(-1/2*(b^2+c^
2)^(3/2)*(a*e-d)/(a^2-b^2-c^2)+1/2*(b^2+c^2)^(1/2)*(2*b^2+2*c^2)/(a^2-b^2-c
^2)*(a*e-d))*(1/(b^2+c^2)^(1/2)*a-1)*((-(b^2+c^2)^(1/2)*sin(x-arctan(-b,c))
-a)/(-a+(b^2+c^2)^(1/2)))^(1/2)*((-sin(x-arctan(-b,c))+1)*(b^2+c^2)^(1/2)/(
a+(b^2+c^2)^(1/2)))^(1/2)*((1+sin(x-arctan(-b,c)))*(b^2+c^2)^(1/2)/(-a+(b^2
+c^2)^(1/2)))^(1/2)/(-(-(b^2+c^2)^(1/2)*sin(x-arctan(-b,c))-a)*cos(x-arctan
(-b,c))^2*(b^2+c^2))^(1/2)*((-1/(b^2+c^2)^(1/2)*a-1)*EllipticE(((-(b^2+c^2)
^(1/2)*sin(x-arctan(-b,c))-a)/(-a+(b^2+c^2)^(1/2)))^(1/2),((a-(b^2+c^2)^(1/
2))/(a+(b^2+c^2)^(1/2)))^(1/2))+EllipticF(((-(b^2+c^2)^(1/2)*sin(x-arctan(-
b,c))-a)/(-a+(b^2+c^2)^(1/2)))^(1/2),((a-(b^2+c^2)^(1/2))/(a+(b^2+c^2)^(1/2
)))^(1/2)))-1/2*(a*b^2*e+a*c^2*e-b^2*d-c^2*d)*(1/(b^2+c^2)^(1/2)*a-1)*((-(b
^2+c^2)^(1/2)*sin(x-arctan(-b,c))-a)/(-a+(b^2+c^2)^(1/2)))^(1/2)*((-sin(x-a
rctan(-b,c))+1)*(b^2+c^2)^(1/2)/(a+(b^2+c^2)^(1/2)))^(1/2)*((1+sin(x-arctan
(-b,c)))*(b^2+c^2)^(1/2)/(-a+(b^2+c^2)^(1/2)))^(1/2)/(-(-(b^2+c^2)^(1/2)*si
n(x-arctan(-b,c))-a)*cos(x-arctan(-b,c))^2*(b^2+c^2))^(1/2)/a*EllipticPi(((
-(b^2+c^2)^(1/2)*sin(x-arctan(-b,c))-a)/(-a+(b^2+c^2)^(1/2)))^(1/2),-1/2*(-
1/(b^2+c^2)^(1/2)*a+1)*(b^2+c^2)^(1/2)/a,((a-(b^2+c^2)^(1/2))/(a+(b^2+c^2)^
(1/2)))^(1/2))-(b^2+c^2)*cos(x-arctan(-b,c))^2*(a*e-d)/(a^2-b^2-c^2)/(-(-(b
^2+c^2)^(1/2)*sin(x-arctan(-b,c))-a)*cos(x-arctan(-b,c))^2)^(1/2)-a*(b^2+c^
2)^(1/2)*(a*e-d)/(a^2-b^2-c^2)*(1/(b^2+c^2)^(1/2)*a-1)*((-(b^2+c^2)^(1/2)*s
in(x-arctan(-b,c))-a)/(-a+(b^2+c^2)^(1/2)))^(1/2)*((-sin(x-arctan(-b,c))+1)
*(b^2+c^2)^(1/2)/(a+(b^2+c^2)^(1/2)))^(1/2)*((1+sin(x-arctan(-b,c)))*(b^2+c
^2)^(1/2)/(-a+(b^2+c^2)^(1/2)))^(1/2)/(-(-(b^2+c^2)^(1/2)*sin(x-arctan(-b,c
))-a)*cos(x-arctan(-b,c))^2)^(1/2)*EllipticF(((-(b^2+c^2)^(1/2)*sin(x-arcta
n(-b,c))-a)/(-a+(b^2+c^2)^(1/2)))^(1/2),((a-(b^2+c^2)^(1/2))/(a+(b^2+c^2)^(
1/2)))^(1/2))-(b^2+c^2)*(a*e-d)/(a^2-b^2-c^2)*(1/(b^2+c^2)^(1/2)*a-1)*((-(b
^2+c^2)^(1/2)*sin(x-arctan(-b,c))-a)/(-a+(b^2+c^2)^(1/2)))^(1/2)*((-sin(x-a
rctan(-b,c))+1)*(b^2+c^2)^(1/2)/(a+(b^2+c^2)^(1/2)))^(1/2)*((1+sin(x-arctan
(-b,c)))*(b^2+c^2)^(1/2)/(-a+(b^2+c^2)^(1/2)))^(1/2)/(-(-(b^2+c^2)^(1/2)*si
n(x-arctan(-b,c))-a)*cos(x-arctan(-b,c))^2)^(1/2)*((-1/(b^2+c^2)^(1/2)*a-1)
*EllipticE(((-(b^2+c^2)^(1/2)*sin(x-arctan(-b,c))-a)/(-a+(b^2+c^2)^(1/2)))^
(1/2),((a-(b^2+c^2)^(1/2))/(a+(b^2+c^2)^(1/2)))^(1/2))+EllipticF(((-(b^2+c^
```

```
2)^(1/2)*sin(x-arctan(-b,c))-a)/(-a+(b^2+c^2)^(1/2)))^(1/2),((a-(b^2+c^2)^(
1/2))/(a+(b^2+c^2)^(1/2)))^(1/2)))+(1/2*a*e-1/2*d)*(1/(b^2+c^2)^(1/2)*a-1)*
((-(b^2+c^2)^(1/2)*sin(x-arctan(-b,c))-a)/(-a+(b^2+c^2)^(1/2)))^(1/2)*((-si
n(x-arctan(-b,c))+1)*(b^2+c^2)^(1/2)/(a+(b^2+c^2)^(1/2)))^(1/2)*((1+sin(x-a
rctan(-b,c)))*(b^2+c^2)^(1/2)/(-a+(b^2+c^2)^(1/2)))^(1/2)/(-(-(b^2+c^2)^(1/
2)*sin(x-arctan(-b,c))-a)*cos(x-arctan(-b,c))^2)^(1/2)*(b^2+c^2)^(1/2)/a*El
lipticPi(((-(b^2+c^2)^(1/2)*sin(x-arctan(-b,c))-a)/(-a+(b^2+c^2)^(1/2)))^(1
/2),-1/2*(-1/(b^2+c^2)^(1/2)*a+1)*(b^2+c^2)^(1/2)/a,((a-(b^2+c^2)^(1/2))/(a
+(b^2+c^2)^(1/2)))^(1/2)))/cos(x-arctan(-b,c))/((b^2*sin(x-arctan(-b,c))+c^
2*sin(x-arctan(-b,c))+a*(b^2+c^2)^(1/2))/(b^2+c^2)^(1/2))^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{be\cos(x) + ce\sin(x) + d}{(b\cos(x) + c\sin(x) + a)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+b*e*cos(x)+c*e*sin(x))/(a+b*cos(x)+c*sin(x))^(3/2),x, algorith
m="maxima")
```

```
[Out] integrate((b*e*cos(x) + c*e*sin(x) + d)/(b*cos(x) + c*sin(x) + a)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{d + b\,e\,\cos(x) + c\,e\,\sin(x)}{(a + b\,\cos(x) + c\,\sin(x))^{3/2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + b*e*cos(x) + c*e*sin(x))/(a + b*cos(x) + c*sin(x))^(3/2),x)
```

```
[Out] int((d + b*e*cos(x) + c*e*sin(x))/(a + b*cos(x) + c*sin(x))^(3/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+b*e*cos(x)+c*e*sin(x))/(a+b*cos(x)+c*sin(x))**(3/2),x)
```

```
[Out] Timed out
```

3.561 $\int \frac{d+be\cos(x)+ce\sin(x)}{(a+b\cos(x)+c\sin(x))^{5/2}}\,dx$

Optimal. Leaf size=378

$$-\frac{2(d-ae)\sqrt{\frac{a+b\cos(x)+c\sin(x)}{a+\sqrt{b^2+c^2}}}F\left(\frac{1}{2}\left(x-\tan^{-1}(b,c)\right)|\frac{2\sqrt{b^2+c^2}}{a+\sqrt{b^2+c^2}}\right)}{3\left(a^2-b^2-c^2\right)\sqrt{a+b\cos(x)+c\sin(x)}}+\frac{2\left(a^2(-e)+4ad-3e\left(b^2+c^2\right)\right)\sqrt{a+b\cos(x)+c}}{3\left(a^2-b^2-c^2\right)^2\sqrt{\frac{a+b}{}}}$$

```
[Out] 2/3*(c*(-a*e+d)*cos(x)-b*(-a*e+d)*sin(x))/(a^2-b^2-c^2)/(a+b*cos(x)+c*sin(x
))^(3/2)+2/3*(c*(4*a*d-a^2*e-3*(b^2+c^2)*e)*cos(x)-b*(4*a*d-a^2*e-3*(b^2+c^
2)*e)*sin(x))/(a^2-b^2-c^2)^2/(a+b*cos(x)+c*sin(x))^(1/2)+2/3*(4*a*d-a^2*e-
3*(b^2+c^2)*e)*(cos(1/2*x-1/2*arctan(b,c))^2)^(1/2)/cos(1/2*x-1/2*arctan(b,
c))*EllipticE(sin(1/2*x-1/2*arctan(b,c)),2^(1/2)*((b^2+c^2)^(1/2)/(a+(b^2+c
^2)^(1/2)))^(1/2))*(a+b*cos(x)+c*sin(x))^(1/2)/(a^2-b^2-c^2)^2/((a+b*cos(x)
+c*sin(x))/(a+(b^2+c^2)^(1/2)))^(1/2)-2/3*(-a*e+d)*(cos(1/2*x-1/2*arctan(b,
c))^2)^(1/2)/cos(1/2*x-1/2*arctan(b,c))*EllipticF(sin(1/2*x-1/2*arctan(b,c)
),2^(1/2)*((b^2+c^2)^(1/2)/(a+(b^2+c^2)^(1/2)))^(1/2))*((a+b*cos(x)+c*sin(x
))/(a+(b^2+c^2)^(1/2)))^(1/2)/(a^2-b^2-c^2)/(a+b*cos(x)+c*sin(x))^(1/2)
```

Rubi [A] time = 0.56, antiderivative size = 378, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {3156, 3149, 3119, 2653, 3127, 2661}

$$-\frac{2(d-ae)\sqrt{\frac{a+b\cos(x)+c\sin(x)}{a+\sqrt{b^2+c^2}}}F\left(\frac{1}{2}\left(x-\tan^{-1}(b,c)\right)|\frac{2\sqrt{b^2+c^2}}{a+\sqrt{b^2+c^2}}\right)}{3\left(a^2-b^2-c^2\right)\sqrt{a+b\cos(x)+c\sin(x)}}+\frac{2\left(a^2(-e)+4ad-3e\left(b^2+c^2\right)\right)\sqrt{a+b\cos(x)+c}}{3\left(a^2-b^2-c^2\right)^2\sqrt{\frac{a+b}{}}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + b*e*Cos[x] + c*e*Sin[x])/(a + b*Cos[x] + c*Sin[x])^(5/2),x]
```

```
[Out] (2*(4*a*d - a^2*e - 3*(b^2 + c^2)*e)*EllipticE[(x - ArcTan[b, c])/2, (2*Sqr
t[b^2 + c^2])/(a + Sqrt[b^2 + c^2])]*Sqrt[a + b*Cos[x] + c*Sin[x]])/(3*(a^2
 - b^2 - c^2)^2*Sqrt[(a + b*Cos[x] + c*Sin[x])/(a + Sqrt[b^2 + c^2])]) - (2
*(d - a*e)*EllipticF[(x - ArcTan[b, c])/2, (2*Sqrt[b^2 + c^2])/(a + Sqrt[b^
2 + c^2])]*Sqrt[(a + b*Cos[x] + c*Sin[x])/(a + Sqrt[b^2 + c^2])])/(3*(a^2 -
 b^2 - c^2)*Sqrt[a + b*Cos[x] + c*Sin[x]]) + (2*(c*(d - a*e)*Cos[x] - b*(d
- a*e)*Sin[x]))/(3*(a^2 - b^2 - c^2)*(a + b*Cos[x] + c*Sin[x])^(3/2)) + (2*
(c*(4*a*d - a^2*e - 3*(b^2 + c^2)*e)*Cos[x] - b*(4*a*d - a^2*e - 3*(b^2 + c
^2)*e)*Sin[x]))/(3*(a^2 - b^2 - c^2)^2*Sqrt[a + b*Cos[x] + c*Sin[x]])
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 3119

```
Int[Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_
)]], x_Symbol] :> Dist[Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]]/Sqrt[(a +
b*Cos[d + e*x] + c*Sin[d + e*x])/(a + Sqrt[b^2 + c^2])], Int[Sqrt[a/(a + Sq
rt[b^2 + c^2]) + (Sqrt[b^2 + c^2]*Cos[d + e*x - ArcTan[b, c]])/(a + Sqrt[b^
2 + c^2])], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0]
&& NeQ[b^2 + c^2, 0] &&  !GtQ[a + Sqrt[b^2 + c^2], 0]
```

Rule 3127

```
Int[1/Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(
x_)]], x_Symbol] :> Dist[Sqrt[(a + b*Cos[d + e*x] + c*Sin[d + e*x])/(a + Sq
rt[b^2 + c^2])]/Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]], Int[1/Sqrt[a/(a
+ Sqrt[b^2 + c^2]) + (Sqrt[b^2 + c^2]*Cos[d + e*x - ArcTan[b, c]])/(a + Sqr
t[b^2 + c^2])], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2,
 0] && NeQ[b^2 + c^2, 0] &&  !GtQ[a + Sqrt[b^2 + c^2], 0]
```

Rule 3149

```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])
/Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)]]
, x_Symbol] :> Dist[B/b, Int[Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]], x],
 x] + Dist[(A*b - a*B)/b, Int[1/Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]],
x], x] /; FreeQ[{a, b, c, d, e, A, B, C}, x] && EqQ[B*c - b*C, 0] && NeQ[A*
b - a*B, 0]
```

Rule 3156

```
Int[((a_.) + cos[(d_.) + (e_.)*(x_)]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)])
^(n_)*((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)
]), x_Symbol] :> -Simp[((c*B - b*C - (a*C - c*A)*Cos[d + e*x] + (a*B - b*A)
*Sin[d + e*x])*(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n + 1))/(e*(n + 1)*(a
^2 - b^2 - c^2)), x] + Dist[1/((n + 1)*(a^2 - b^2 - c^2)), Int[(a + b*Cos[d
 + e*x] + c*Sin[d + e*x])^(n + 1)*Simp[(n + 1)*(a*A - b*B - c*C) + (n + 2)*
```

```
(a*B - b*A)*Cos[d + e*x] + (n + 2)*(a*C - c*A)*Sin[d + e*x], x], x], x] /;
FreeQ[{a, b, c, d, e, A, B, C}, x] && LtQ[n, -1] && NeQ[a^2 - b^2 - c^2, 0]
 && NeQ[n, -2]
```

Rubi steps

$$\int \frac{d+be\cos(x)+ce\sin(x)}{(a+b\cos(x)+c\sin(x))^{5/2}}\,dx = \frac{2(c(d-ae)\cos(x)-b(d-ae)\sin(x))}{3\left(a^2-b^2-c^2\right)(a+b\cos(x)+c\sin(x))^{3/2}} - \frac{2\int \frac{-\frac{3}{2}\left(ad-\left(b^2+c^2\right)e\right)+\frac{1}{2}b(d-ae)\cos(x}{(a+b\cos(x)+c\sin(x)}}{3\left(a^2-b^2-c^2\right.}$$

$$= \frac{2(c(d-ae)\cos(x)-b(d-ae)\sin(x))}{3\left(a^2-b^2-c^2\right)(a+b\cos(x)+c\sin(x))^{3/2}} + \frac{2\left(c\left(4ad-a^2e-3\left(b^2+c^2\right)e\right)\right.}{3\left(a^2-b^2-c\right.}$$

$$= \frac{2(c(d-ae)\cos(x)-b(d-ae)\sin(x))}{3\left(a^2-b^2-c^2\right)(a+b\cos(x)+c\sin(x))^{3/2}} + \frac{2\left(c\left(4ad-a^2e-3\left(b^2+c^2\right)e\right)\right.}{3\left(a^2-b^2-c\right.}$$

$$= \frac{2(c(d-ae)\cos(x)-b(d-ae)\sin(x))}{3\left(a^2-b^2-c^2\right)(a+b\cos(x)+c\sin(x))^{3/2}} + \frac{2\left(c\left(4ad-a^2e-3\left(b^2+c^2\right)e\right)\right.}{3\left(a^2-b^2-c\right.}$$

$$= \frac{2\left(4ad-a^2e-3\left(b^2+c^2\right)e\right)E\left(\frac{1}{2}\left(x-\tan^{-1}(b,c)\right)|\frac{2\sqrt{b^2+c^2}}{a+\sqrt{b^2+c^2}}\right)\sqrt{a+b\cos(x)}}{3\left(a^2-b^2-c^2\right)^2\sqrt{\frac{a+b\cos(x)+c\sin(x)}{a+\sqrt{b^2+c^2}}}}$$

Mathematica [C] time = 6.95, size = 5554, normalized size = 14.69

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(d + b*e*Cos[x] + c*e*Sin[x])/(a + b*Cos[x] + c*Sin[x])^(5/2),x]
[Out] Result too large to show
```

fricas [F] time = 0.95, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{(be\cos(x)+ce\sin(x)+d)\sqrt{b\cos(x)+c\sin(x)+a}}{\left(b^3-3\,bc^2\right)\cos(x)^3+a^3+3\,ac^2+3\left(ab^2-ac^2\right)\cos(x)^2+3\left(a^2b+bc^2\right)\cos(x)+\left(6\,abc\cos(x)+3\,a^2c\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+b*e*cos(x)+c*e*sin(x))/(a+b*cos(x)+c*sin(x))^(5/2),x, algorith
m="fricas")
```

```
[Out] integral((b*e*cos(x) + c*e*sin(x) + d)*sqrt(b*cos(x) + c*sin(x) + a)/((b^3
- 3*b*c^2)*cos(x)^3 + a^3 + 3*a*c^2 + 3*(a*b^2 - a*c^2)*cos(x)^2 + 3*(a^2*b
 + b*c^2)*cos(x) + (6*a*b*c*cos(x) + 3*a^2*c + c^3 + (3*b^2*c - c^3)*cos(x)
^2)*sin(x)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{be\cos(x) + ce\sin(x) + d}{(b\cos(x) + c\sin(x) + a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+b*e*cos(x)+c*e*sin(x))/(a+b*cos(x)+c*sin(x))^(5/2),x, algorith
m="giac")
```

```
[Out] integrate((b*e*cos(x) + c*e*sin(x) + d)/(b*cos(x) + c*sin(x) + a)^(5/2), x)
```

maple [B] time = 3.89, size = 3164, normalized size = 8.37

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d+b*e*cos(x)+c*e*sin(x))/(a+b*cos(x)+c*sin(x))^(5/2),x)
```

```
[Out] (-(-b^2*sin(x-arctan(-b,c))-c^2*sin(x-arctan(-b,c))-a*(b^2+c^2)^(1/2))*cos(
x-arctan(-b,c))^2/(b^2+c^2)^(1/2))^(1/2)/(b^2+c^2)^(1/2)*(-1/4*(a*b^2*e+a*c
^2*e-b^2*d-c^2*d)/a/(a^2-b^2-c^2)*(cos(x-arctan(-b,c))^2*(b^2+c^2)*((b^2+c^
2)^(1/2)*sin(x-arctan(-b,c))+a))^(1/2)/(b^2*sin(x-arctan(-b,c))+c^2*sin(x-a
rctan(-b,c))-a*(b^2+c^2)^(1/2))-1/3*(a*e-d)/(a^2-b^2-c^2)/(b^2+c^2)^(1/2)*(
cos(x-arctan(-b,c))^2*(b^2+c^2)*((b^2+c^2)^(1/2)*sin(x-arctan(-b,c))+a))^(1
/2)/(sin(x-arctan(-b,c))+1/(b^2+c^2)^(1/2)*a)^2+1/3*(b^2+c^2)^(1/2)*(-b^2-c
^2)*cos(x-arctan(-b,c))^2/(a^2-b^2-c^2)^2*(a^2*e+3*b^2*e+3*c^2*e-4*a*d)/(co
s(x-arctan(-b,c))^2*(b^2+c^2)*((b^2+c^2)^(1/2)*sin(x-arctan(-b,c))+a))^(1/2
)+2*(7/24*(a*b^2*e+a*c^2*e-b^2*d-c^2*d)/(a^2-b^2-c^2)-1/6*a*(b^2+c^2)*(a^2*
e+3*b^2*e+3*c^2*e-4*a*d)/(a^2-b^2-c^2)^2)*(1/(b^2+c^2)^(1/2)*a-1)*((-(b^2+c
^2)^(1/2)*sin(x-arctan(-b,c))-a)/(-a+(b^2+c^2)^(1/2)))^(1/2)*((-sin(x-arcta
n(-b,c))+1)*(b^2+c^2)^(1/2)/(a+(b^2+c^2)^(1/2)))^(1/2)*((1+sin(x-arctan(-b,
c)))*(b^2+c^2)^(1/2)/(-a+(b^2+c^2)^(1/2)))^(1/2)/(cos(x-arctan(-b,c))^2*(b^
2+c^2)*((b^2+c^2)^(1/2)*sin(x-arctan(-b,c))+a))^(1/2)*EllipticF(((-(b^2+c^2
)^(1/2)*sin(x-arctan(-b,c))-a)/(-a+(b^2+c^2)^(1/2)))^(1/2),((a-(b^2+c^2)^(1
```

/2))/(a+(b^2+c^2)^(1/2)))^(1/2))+2*(-1/8*(b^2+c^2)^(1/2)*(a*b^2*e+a*c^2*e-b
^2*d-c^2*d)/a/(a^2-b^2-c^2)+1/6*(b^2+c^2)^(3/2)*(a^2*e+3*b^2*e+3*c^2*e-4*a*
d)/(a^2-b^2-c^2)^2-1/6*(b^2+c^2)^(1/2)*(2*b^2+2*c^2)/(a^2-b^2-c^2)^2*(a^2*e
+3*b^2*e+3*c^2*e-4*a*d))*(1/(b^2+c^2)^(1/2)*a-1)*((-(b^2+c^2)^(1/2)*sin(x-a
rctan(-b,c))-a)/(-a+(b^2+c^2)^(1/2)))^(1/2)*((-sin(x-arctan(-b,c))+1)*(b^2+
c^2)^(1/2)/(a+(b^2+c^2)^(1/2)))^(1/2)*((1+sin(x-arctan(-b,c)))*(b^2+c^2)^(1
/2)/(-a+(b^2+c^2)^(1/2)))^(1/2)/(cos(x-arctan(-b,c))^2*(b^2+c^2)*((b^2+c^2)
^(1/2)*sin(x-arctan(-b,c))+a))^(1/2)*((-1/(b^2+c^2)^(1/2)*a-1)*EllipticE(((
-(b^2+c^2)^(1/2)*sin(x-arctan(-b,c))-a)/(-a+(b^2+c^2)^(1/2)))^(1/2),((a-(b^
2+c^2)^(1/2))/(a+(b^2+c^2)^(1/2)))^(1/2))+EllipticF(((-(b^2+c^2)^(1/2)*sin(
x-arctan(-b,c))-a)/(-a+(b^2+c^2)^(1/2)))^(1/2),((a-(b^2+c^2)^(1/2))/(a+(b^2
+c^2)^(1/2)))^(1/2)))-1/8*(a^3*b^2*e+a^3*c^2*e+3*a*b^4*e+6*a*b^2*c^2*e+3*a*
c^4*e-5*a^2*b^2*d-5*a^2*c^2*d+b^4*d+2*b^2*c^2*d+c^4*d)/a^2/(a^2-b^2-c^2)*(1
/(b^2+c^2)^(1/2)*a-1)*((-(b^2+c^2)^(1/2)*sin(x-arctan(-b,c))-a)/(-a+(b^2+c^
2)^(1/2)))^(1/2)*((-sin(x-arctan(-b,c))+1)*(b^2+c^2)^(1/2)/(a+(b^2+c^2)^(1/
2)))^(1/2)*((1+sin(x-arctan(-b,c)))*(b^2+c^2)^(1/2)/(-a+(b^2+c^2)^(1/2)))^(
1/2)/(cos(x-arctan(-b,c))^2*(b^2+c^2)*((b^2+c^2)^(1/2)*sin(x-arctan(-b,c))+
a))^(1/2)*EllipticPi(((-(b^2+c^2)^(1/2)*sin(x-arctan(-b,c))-a)/(-a+(b^2+c^2
)^(1/2)))^(1/2),-1/2*(-1/(b^2+c^2)^(1/2)*a+1)*(b^2+c^2)^(1/2)/a,((a-(b^2+c^
2)^(1/2))/(a+(b^2+c^2)^(1/2)))^(1/2))+1/4*(b^2+c^2)^(3/2)*(a*e-d)/a/(a^2-b^
2-c^2)*(cos(x-arctan(-b,c))^2*((b^2+c^2)^(1/2)*sin(x-arctan(-b,c))+a))^(1/2
)/(b^2*sin(x-arctan(-b,c))+c^2*sin(x-arctan(-b,c))-a*(b^2+c^2)^(1/2))-1/3*(
a*e-d)/(a^2-b^2-c^2)*(cos(x-arctan(-b,c))^2*((b^2+c^2)^(1/2)*sin(x-arctan(-
b,c))+a))^(1/2)/(sin(x-arctan(-b,c))+1/(b^2+c^2)^(1/2)*a)^2-1/3*(b^2+c^2)*c
os(x-arctan(-b,c))^2/(a^2-b^2-c^2)^2*(a^2*e+3*b^2*e+3*c^2*e-4*a*d)/(cos(x-a
rctan(-b,c))^2*((b^2+c^2)^(1/2)*sin(x-arctan(-b,c))+a))^(1/2)+2*(1/24*(b^2+
c^2)^(1/2)*(a*e-d)/(a^2-b^2-c^2)-1/6*a*(b^2+c^2)^(1/2)*(a^2*e+3*b^2*e+3*c^2
*e-4*a*d)/(a^2-b^2-c^2)^2)*(1/(b^2+c^2)^(1/2)*a-1)*((-(b^2+c^2)^(1/2)*sin(x
-arctan(-b,c))-a)/(-a+(b^2+c^2)^(1/2)))^(1/2)*((-sin(x-arctan(-b,c))+1)*(b^
2+c^2)^(1/2)/(a+(b^2+c^2)^(1/2)))^(1/2)*((1+sin(x-arctan(-b,c)))*(b^2+c^2)^
(1/2)/(-a+(b^2+c^2)^(1/2)))^(1/2)/(cos(x-arctan(-b,c))^2*((b^2+c^2)^(1/2)*s
in(x-arctan(-b,c))+a))^(1/2)*EllipticF(((-(b^2+c^2)^(1/2)*sin(x-arctan(-b,c
))-a)/(-a+(b^2+c^2)^(1/2)))^(1/2),((a-(b^2+c^2)^(1/2))/(a+(b^2+c^2)^(1/2)))
^(1/2))+2*(1/8*(a*b^2*e+a*c^2*e-b^2*d-c^2*d)/a/(a^2-b^2-c^2)-1/6*(b^2+c^2)*
(a^2*e+3*b^2*e+3*c^2*e-4*a*d)/(a^2-b^2-c^2)^2)*(1/(b^2+c^2)^(1/2)*a-1)*((-(
b^2+c^2)^(1/2)*sin(x-arctan(-b,c))-a)/(-a+(b^2+c^2)^(1/2)))^(1/2)*((-sin(x-
arctan(-b,c))+1)*(b^2+c^2)^(1/2)/(a+(b^2+c^2)^(1/2)))^(1/2)*((1+sin(x-arcta
n(-b,c)))*(b^2+c^2)^(1/2)/(-a+(b^2+c^2)^(1/2)))^(1/2)/(cos(x-arctan(-b,c))^
2*((b^2+c^2)^(1/2)*sin(x-arctan(-b,c))+a))^(1/2)*((-1/(b^2+c^2)^(1/2)*a-1)*
EllipticE(((-(b^2+c^2)^(1/2)*sin(x-arctan(-b,c))-a)/(-a+(b^2+c^2)^(1/2)))^(
1/2),((a-(b^2+c^2)^(1/2))/(a+(b^2+c^2)^(1/2)))^(1/2))+EllipticF(((-(b^2+c^2
)^(1/2)*sin(x-arctan(-b,c))-a)/(-a+(b^2+c^2)^(1/2)))^(1/2),((a-(b^2+c^2)^(1
/2))/(a+(b^2+c^2)^(1/2)))^(1/2)))+1/8*(a^3*b^2*e+a^3*c^2*e+3*a*b^4*e+6*a*b^
2*c^2*e+3*a*c^4*e-5*a^2*b^2*d-5*a^2*c^2*d+b^4*d+2*b^2*c^2*d+c^4*d)/a^2/(a^2
-b^2-c^2)/(b^2+c^2)^(1/2)*(1/(b^2+c^2)^(1/2)*a-1)*((-(b^2+c^2)^(1/2)*sin(x-

```
arctan(-b,c))-a)/(-a+(b^2+c^2)^(1/2)))^(1/2)*((-sin(x-arctan(-b,c))+1)*(b^2
+c^2)^(1/2)/(a+(b^2+c^2)^(1/2)))^(1/2)*((1+sin(x-arctan(-b,c)))*(b^2+c^2)^(
1/2)/(-a+(b^2+c^2)^(1/2)))^(1/2)/(cos(x-arctan(-b,c))^2*((b^2+c^2)^(1/2)*si
n(x-arctan(-b,c))+a))^(1/2)*EllipticPi(((-(b^2+c^2)^(1/2)*sin(x-arctan(-b,c
))-a)/(-a+(b^2+c^2)^(1/2)))^(1/2),-1/2*(-1/(b^2+c^2)^(1/2)*a+1)*(b^2+c^2)^(
1/2)/a,((a-(b^2+c^2)^(1/2))/(a+(b^2+c^2)^(1/2)))^(1/2)))/cos(x-arctan(-b,c)
)/((b^2*sin(x-arctan(-b,c))+c^2*sin(x-arctan(-b,c))+a*(b^2+c^2)^(1/2))/(b^2
+c^2)^(1/2))^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{be\cos(x) + ce\sin(x) + d}{(b\cos(x) + c\sin(x) + a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+b*e*cos(x)+c*e*sin(x))/(a+b*cos(x)+c*sin(x))^(5/2),x, algorith
m="maxima")
```

```
[Out] integrate((b*e*cos(x) + c*e*sin(x) + d)/(b*cos(x) + c*sin(x) + a)^(5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{d + b\,e\,\cos(x) + c\,e\,\sin(x)}{(a + b\,\cos(x) + c\,\sin(x))^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + b*e*cos(x) + c*e*sin(x))/(a + b*cos(x) + c*sin(x))^(5/2),x)
```

```
[Out] int((d + b*e*cos(x) + c*e*sin(x))/(a + b*cos(x) + c*sin(x))^(5/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+b*e*cos(x)+c*e*sin(x))/(a+b*cos(x)+c*sin(x))**(5/2),x)
```

```
[Out] Timed out
```

3.562 $\int \frac{A+B\cos(d+ex)+C\sin(d+ex)}{a+c\sin(d+ex)}\,dx$

Optimal. Leaf size=84

$$\frac{2(Ac-aC)\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(d+ex)\right)+c}{\sqrt{a^2-c^2}}\right)}{ce\sqrt{a^2-c^2}}+\frac{B\log(a+c\sin(d+ex))}{ce}+\frac{Cx}{c}$$

```
[Out] C*x/c+B*ln(a+c*sin(e*x+d))/c/e+2*(A*c-C*a)*arctan((c+a*tan(1/2*e*x+1/2*d))/
(a^2-c^2)^(1/2))/c/e/(a^2-c^2)^(1/2)
```

Rubi [A] time = 0.15, antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.226, Rules used = {4376, 2735, 2660, 618, 204, 2668, 31}

$$\frac{2(Ac-aC)\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(d+ex)\right)+c}{\sqrt{a^2-c^2}}\right)}{ce\sqrt{a^2-c^2}}+\frac{B\log(a+c\sin(d+ex))}{ce}+\frac{Cx}{c}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cos[d + e*x] + C*Sin[d + e*x])/(a + c*Sin[d + e*x]),x]
```

```
[Out] (C*x)/c + (2*(A*c - a*C)*ArcTan[(c + a*Tan[(d + e*x)/2])/Sqrt[a^2 - c^2]])/
(c*Sqrt[a^2 - c^2]*e) + (B*Log[a + c*Sin[d + e*x]])/(c*e)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2668

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 4376

```
Int[(u_)*((v_) + (d_.)*(F_)[(c_.)*((a_.) + (b_.)*(x_))]^(n_.)), x_Symbol] :
> With[{e = FreeFactors[Sin[c*(a + b*x)], x]}, Int[ActivateTrig[u*v], x] +
Dist[d, Int[ActivateTrig[u]*Cos[c*(a + b*x)]^n, x], x] /; FunctionOfQ[Sin[c
*(a + b*x)]/e, u, x]] /; FreeQ[{a, b, c, d}, x] &&  !FreeQ[v, x] && Integer
Q[(n - 1)/2] && NonsumQ[u] && (EqQ[F, Cos] || EqQ[F, cos])
```

Rubi steps

$$\int \frac{A+B\cos(d+ex)+C\sin(d+ex)}{a+c\sin(d+ex)}\,dx = B\int \frac{\cos(d+ex)}{a+c\sin(d+ex)}\,dx + \int \frac{A+C\sin(d+ex)}{a+c\sin(d+ex)}\,dx$$
$$= \frac{Cx}{c} - \frac{(-Ac+aC)\int \frac{1}{a+c\sin(d+ex)}\,dx}{c} + \frac{B\,\text{Subst}\left(\int \frac{1}{a+x}\,dx, x, c\sin(d+$$
$$= \frac{Cx}{c} + \frac{B\log(a+c\sin(d+ex))}{ce} + \frac{(2(Ac-aC))\,\text{Subst}\left(\int \frac{1}{a+2cx+ax^2}\,dx$$
$$= \frac{Cx}{c} + \frac{B\log(a+c\sin(d+ex))}{ce} - \frac{(4(Ac-aC))\,\text{Subst}\left(\int \frac{1}{-4(a^2-c^2)-x^2}$$
$$= \frac{Cx}{c} + \frac{2(Ac-aC)\tan^{-1}\left(\frac{c+a\tan\left(\frac{1}{2}(d+ex)\right)}{\sqrt{a^2-c^2}}\right)}{c\sqrt{a^2-c^2}\,e} + \frac{B\log(a+c\sin(d+ex))}{ce}$$

Mathematica [**A**] time = 0.25, size = 80, normalized size = 0.95

$$\frac{\frac{2(Ac-aC)\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(d+ex)\right)+c}{\sqrt{a^2-c^2}}\right)}{\sqrt{a^2-c^2}} + B\log(a+c\sin(d+ex)) + C(d+ex)}{ce}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Cos[d + e*x] + C*Sin[d + e*x])/(a + c*Sin[d + e*x]),x]
```

```
[Out] (C*(d + e*x) + (2*(A*c - a*C)*ArcTan[(c + a*Tan[(d + e*x)/2])/Sqrt[a^2 - c^
2]])/Sqrt[a^2 - c^2] + B*Log[a + c*Sin[d + e*x]])/(c*e)
```

fricas [**A**] time = 1.00, size = 346, normalized size = 4.12

$$\left[\frac{2\left(Ca^2-Cc^2\right)ex + (Ca-Ac)\sqrt{-a^2+c^2}\,\log\left(\frac{\left(2a^2-c^2\right)\cos(ex+d)^2-2ac\sin(ex+d)-a^2-c^2+2\left(a\cos(ex+d)\sin(ex+d)+c\cos(ex+d)\right)\sqrt{-}}{c^2\cos(ex+d)^2-2ac\sin(ex+d)-a^2-c^2}\right.}{2\left(a^2c-c^3\right)e}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(e*x+d)+C*sin(e*x+d))/(a+c*sin(e*x+d)),x, algorithm="fric
as")
```

```
[Out] [1/2*(2*(C*a^2 - C*c^2)*e*x + (C*a - A*c)*sqrt(-a^2 + c^2)*log(((2*a^2 - c^
2)*cos(e*x + d)^2 - 2*a*c*sin(e*x + d) - a^2 - c^2 + 2*(a*cos(e*x + d)*sin(
e*x + d) + c*cos(e*x + d))*sqrt(-a^2 + c^2))/(c^2*cos(e*x + d)^2 - 2*a*c*si
n(e*x + d) - a^2 - c^2)) + (B*a^2 - B*c^2)*log(-c^2*cos(e*x + d)^2 + 2*a*c*
sin(e*x + d) + a^2 + c^2))/((a^2*c - c^3)*e), 1/2*(2*(C*a^2 - C*c^2)*e*x +
2*(C*a - A*c)*sqrt(a^2 - c^2)*arctan(-(a*sin(e*x + d) + c)/(sqrt(a^2 - c^2)
*cos(e*x + d))) + (B*a^2 - B*c^2)*log(-c^2*cos(e*x + d)^2 + 2*a*c*sin(e*x +
 d) + a^2 + c^2))/((a^2*c - c^3)*e)]
```

giac [A] time = 0.18, size = 141, normalized size = 1.68

$$\left(\frac{(xe+d)C}{c} + \frac{B\log\left(a\tan\left(\frac{1}{2}\,xe + \frac{1}{2}\,d\right)^2 + 2\,c\tan\left(\frac{1}{2}\,xe + \frac{1}{2}\,d\right) + a\right)}{c} - \frac{B\log\left(\tan\left(\frac{1}{2}\,xe + \frac{1}{2}\,d\right)^2 + 1\right)}{c} - \frac{2\left(\pi\left\lfloor\frac{xe+d}{2\,\pi}\right.\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(e*x+d)+C*sin(e*x+d))/(a+c*sin(e*x+d)),x, algorithm="giac
")
```

```
[Out] ((x*e + d)*C/c + B*log(a*tan(1/2*x*e + 1/2*d)^2 + 2*c*tan(1/2*x*e + 1/2*d)
+ a)/c - B*log(tan(1/2*x*e + 1/2*d)^2 + 1)/c - 2*(pi*floor(1/2*(x*e + d)/pi
 + 1/2)*sgn(a) + arctan((a*tan(1/2*x*e + 1/2*d) + c)/sqrt(a^2 - c^2)))*(C*a
 - A*c)/(sqrt(a^2 - c^2)*c))*e^(-1)
```

maple [B] time = 0.25, size = 178, normalized size = 2.12

$$\frac{B\ln\left(a\left(\tan^2\left(\frac{d}{2}+\frac{ex}{2}\right)\right)+2c\tan\left(\frac{d}{2}+\frac{ex}{2}\right)+a\right)}{ec}+\frac{2\arctan\left(\frac{2a\tan\left(\frac{d}{2}+\frac{ex}{2}\right)+2c}{2\sqrt{a^2-c^2}}\right)A}{e\sqrt{a^2-c^2}}-\frac{2\arctan\left(\frac{2a\tan\left(\frac{d}{2}+\frac{ex}{2}\right)+2c}{2\sqrt{a^2-c^2}}\right)Ca}{ec\sqrt{a^2-c^2}}-\frac{B\ln\left(\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(e*x+d)+C*sin(e*x+d))/(a+c*sin(e*x+d)),x)
```

```
[Out] 1/e/c*B*ln(a*tan(1/2*d+1/2*e*x)^2+2*c*tan(1/2*d+1/2*e*x)+a)+2/e/(a^2-c^2)^(
1/2)*arctan(1/2*(2*a*tan(1/2*d+1/2*e*x)+2*c)/(a^2-c^2)^(1/2))*A-2/e/c/(a^2-
c^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*d+1/2*e*x)+2*c)/(a^2-c^2)^(1/2))*C*a-1/e
/c*B*ln(1+tan(1/2*d+1/2*e*x)^2)+2/e/c*C*arctan(tan(1/2*d+1/2*e*x))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(e*x+d)+C*sin(e*x+d))/(a+c*sin(e*x+d)),x, algorithm="maxi
ma")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*c^2-4*a^2>0)', see `assume?` for
more details)Is 4*c^2-4*a^2 positive or negative?
```

mupad [B] time = 9.63, size = 1143, normalized size = 13.61

$$\ln\left(32\,B^3\,a^2-32\,A\,B^2\,a^2+32\,A\,C^2\,a^2+32\,B\,C^2\,a^2+32\,a\,\tan\left(\frac{d}{2}+\frac{e\,x}{2}\right)\,\left(c\,A^2\,B-2\,c\,A\,B^2-2\,a\,A\,B\,C-2\,c\,A\,C\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cos(d + e*x) + C*sin(d + e*x))/(a + c*sin(d + e*x)),x)
```

```
[Out] (log(32*B^3*a^2 - 32*A*B^2*a^2 + 32*A*C^2*a^2 + 32*B*C^2*a^2 + 32*a*tan(d/2
 + (e*x)/2)*(2*C^3*a + B^3*c - 2*A*B^2*c + A^2*B*c + 2*B^2*C*a - 2*A*C^2*c
+ 2*B*C^2*c - 2*A*B*C*a) - 32*A^2*C*a*c + 32*B^2*C*a*c - ((B*a^2 - B*c^2 +
A*c*(c^2 - a^2)^(1/2) - C*a*(c^2 - a^2)^(1/2))*(32*C^2*a^2*c - 32*B^2*a^2*c
 - 128*B*C*a^3 + 32*a*tan(d/2 + (e*x)/2)*(2*B^2*a^2 - A^2*c^2 - 2*C^2*a^2 -
 3*B^2*c^2 + 2*C^2*c^2 + 4*A*B*c^2 + 2*A*C*a*c - 4*B*C*a*c) + 64*A*B*a^2*c
+ 64*B*C*a*c^2 + ((B*a^2 - B*c^2 + A*c*(c^2 - a^2)^(1/2) - C*a*(c^2 - a^2)^
(1/2))*(32*A*a^2*c^2 + 32*B*a^2*c^2 - 32*C*a*c^3 + 32*a*c^2*tan(d/2 + (e*x)
/2)*(2*A*c - 2*C*a + B*c) + (32*a*c*(a*c - 2*a^2*tan(d/2 + (e*x)/2) + 3*c^2
*tan(d/2 + (e*x)/2))*(B*a^2 - B*c^2 + A*c*(c^2 - a^2)^(1/2) - C*a*(c^2 - a^
2)^(1/2)))/(a^2 - c^2)))/(c*(a^2 - c^2))))/(c*(a^2 - c^2)))*(B*a^2 - B*c^2
+ A*c*(c^2 - a^2)^(1/2) - C*a*(c^2 - a^2)^(1/2)))/(c*e*(a^2 - c^2)) - (log(
tan(d/2 + (e*x)/2) + 1i)*(B - C*1i))/(c*e) - (log(tan(d/2 + (e*x)/2) - 1i)*
(B + C*1i))/(c*e) + (log(32*B^3*a^2 - 32*A*B^2*a^2 + 32*A*C^2*a^2 + 32*B*C^
2*a^2 + 32*a*tan(d/2 + (e*x)/2)*(2*C^3*a + B^3*c - 2*A*B^2*c + A^2*B*c + 2*
B^2*C*a - 2*A*C^2*c + 2*B*C^2*c - 2*A*B*C*a) - 32*A^2*C*a*c + 32*B^2*C*a*c
- ((B*a^2 - B*c^2 - A*c*(c^2 - a^2)^(1/2) + C*a*(c^2 - a^2)^(1/2))*(32*C^2*
a^2*c - 32*B^2*a^2*c - 128*B*C*a^3 + 32*a*tan(d/2 + (e*x)/2)*(2*B^2*a^2 - A
^2*c^2 - 2*C^2*a^2 - 3*B^2*c^2 + 2*C^2*c^2 + 4*A*B*c^2 + 2*A*C*a*c - 4*B*C*
```

```
a*c) + 64*A*B*a^2*c + 64*B*C*a*c^2 + ((B*a^2 - B*c^2 - A*c*(c^2 - a^2)^(1/2
) + C*a*(c^2 - a^2)^(1/2))*(32*A*a^2*c^2 + 32*B*a^2*c^2 - 32*C*a*c^3 + 32*a
*c^2*tan(d/2 + (e*x)/2)*(2*A*c - 2*C*a + B*c) + (32*a*c*(a*c - 2*a^2*tan(d/
2 + (e*x)/2) + 3*c^2*tan(d/2 + (e*x)/2))*(B*a^2 - B*c^2 - A*c*(c^2 - a^2)^(
1/2) + C*a*(c^2 - a^2)^(1/2)))/(a^2 - c^2)))/(c*(a^2 - c^2))))/(c*(a^2 - c^
2)))*(B*a^2 - B*c^2 - A*c*(c^2 - a^2)^(1/2) + C*a*(c^2 - a^2)^(1/2)))/(c*e*
(a^2 - c^2))
```

sympy [A] time = 29.52, size = 1110, normalized size = 13.21

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(e*x+d)+C*sin(e*x+d))/(a+c*sin(e*x+d)),x)
```

```
[Out] Piecewise((zoo*x*(A + B*cos(d) + C*sin(d))/sin(d), Eq(a, 0) & Eq(c, 0) & Eq
(e, 0)), ((A*log(tan(d/2 + e*x/2))/e - B*log(tan(d/2 + e*x/2)**2 + 1)/e + B
*log(tan(d/2 + e*x/2))/e + C*x)/c, Eq(a, 0)), (2*A/(c*e*tan(d/2 + e*x/2) -
c*e) + 2*B*log(tan(d/2 + e*x/2) - 1)*tan(d/2 + e*x/2)/(c*e*tan(d/2 + e*x/2)
 - c*e) - 2*B*log(tan(d/2 + e*x/2) - 1)/(c*e*tan(d/2 + e*x/2) - c*e) - B*lo
g(tan(d/2 + e*x/2)**2 + 1)*tan(d/2 + e*x/2)/(c*e*tan(d/2 + e*x/2) - c*e) +
B*log(tan(d/2 + e*x/2)**2 + 1)/(c*e*tan(d/2 + e*x/2) - c*e) + C*e*x*tan(d/2
 + e*x/2)/(c*e*tan(d/2 + e*x/2) - c*e) - C*e*x/(c*e*tan(d/2 + e*x/2) - c*e)
 + 2*C/(c*e*tan(d/2 + e*x/2) - c*e), Eq(a, -c)), (-2*A/(c*e*tan(d/2 + e*x/2
) + c*e) + 2*B*log(tan(d/2 + e*x/2) + 1)*tan(d/2 + e*x/2)/(c*e*tan(d/2 + e*
x/2) + c*e) + 2*B*log(tan(d/2 + e*x/2) + 1)/(c*e*tan(d/2 + e*x/2) + c*e) -
B*log(tan(d/2 + e*x/2)**2 + 1)*tan(d/2 + e*x/2)/(c*e*tan(d/2 + e*x/2) + c*e
) - B*log(tan(d/2 + e*x/2)**2 + 1)/(c*e*tan(d/2 + e*x/2) + c*e) + C*e*x*tan
(d/2 + e*x/2)/(c*e*tan(d/2 + e*x/2) + c*e) + C*e*x/(c*e*tan(d/2 + e*x/2) +
c*e) + 2*C/(c*e*tan(d/2 + e*x/2) + c*e), Eq(a, c)), ((A*x + B*sin(d + e*x)/
e - C*cos(d + e*x)/e)/a, Eq(c, 0)), (x*(A + B*cos(d) + C*sin(d))/(a + c*sin
(d)), Eq(e, 0)), (-A*c*sqrt(-a**2 + c**2)*log(tan(d/2 + e*x/2) + c/a - sqrt
(-a**2 + c**2)/a)/(a**2*c*e - c**3*e) + A*c*sqrt(-a**2 + c**2)*log(tan(d/2
+ e*x/2) + c/a + sqrt(-a**2 + c**2)/a)/(a**2*c*e - c**3*e) - B*a**2*log(tan
(d/2 + e*x/2)**2 + 1)/(a**2*c*e - c**3*e) + B*a**2*log(tan(d/2 + e*x/2) + c
/a - sqrt(-a**2 + c**2)/a)/(a**2*c*e - c**3*e) + B*a**2*log(tan(d/2 + e*x/2
) + c/a + sqrt(-a**2 + c**2)/a)/(a**2*c*e - c**3*e) + B*c**2*log(tan(d/2 +
e*x/2)**2 + 1)/(a**2*c*e - c**3*e) - B*c**2*log(tan(d/2 + e*x/2) + c/a - sq
rt(-a**2 + c**2)/a)/(a**2*c*e - c**3*e) - B*c**2*log(tan(d/2 + e*x/2) + c/a
 + sqrt(-a**2 + c**2)/a)/(a**2*c*e - c**3*e) + C*a**2*e*x/(a**2*c*e - c**3*
e) + C*a*sqrt(-a**2 + c**2)*log(tan(d/2 + e*x/2) + c/a - sqrt(-a**2 + c**2)
/a)/(a**2*c*e - c**3*e) - C*a*sqrt(-a**2 + c**2)*log(tan(d/2 + e*x/2) + c/a
 + sqrt(-a**2 + c**2)/a)/(a**2*c*e - c**3*e) - C*c**2*e*x/(a**2*c*e - c**3*
e), True))
```

3.563 $\int \frac{A+B\cos(d+ex)+C\sin(d+ex)}{(a+c\sin(d+ex))^2}\,dx$

Optimal. Leaf size=118

$$\frac{2(aA-cC)\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(d+ex)\right)+c}{\sqrt{a^2-c^2}}\right)}{e\left(a^2-c^2\right)^{3/2}}+\frac{(Ac-aC)\cos(d+ex)}{e\left(a^2-c^2\right)(a+c\sin(d+ex))}-\frac{B}{ce(a+c\sin(d+ex))}$$

```
[Out] 2*(A*a-C*c)*arctan((c+a*tan(1/2*e*x+1/2*d))/(a^2-c^2)^(1/2))/(a^2-c^2)^(3/2
)/e-B/c/e/(a+c*sin(e*x+d))+(A*c-C*a)*cos(e*x+d)/(a^2-c^2)/e/(a+c*sin(e*x+d)
)
```

Rubi [A] time = 0.16, antiderivative size = 118, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.258, Rules used = {4376, 2754, 12, 2660, 618, 204, 2668, 32}

$$\frac{2(aA-cC)\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(d+ex)\right)+c}{\sqrt{a^2-c^2}}\right)}{e\left(a^2-c^2\right)^{3/2}}+\frac{(Ac-aC)\cos(d+ex)}{e\left(a^2-c^2\right)(a+c\sin(d+ex))}-\frac{B}{ce(a+c\sin(d+ex))}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cos[d + e*x] + C*Sin[d + e*x])/(a + c*Sin[d + e*x])^2,x]
```

```
[Out] (2*(a*A - c*C)*ArcTan[(c + a*Tan[(d + e*x)/2])/Sqrt[a^2 - c^2]])/((a^2 - c^
2)^(3/2)*e) - B/(c*e*(a + c*Sin[d + e*x])) + ((A*c - a*C)*Cos[d + e*x])/((a
^2 - c^2)*e*(a + c*Sin[d + e*x]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
```

```
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2668

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rule 2754

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e + f
*x])^(m + 1))/(f*(m + 1)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(a^2 - b^2)), I
nt[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m +
 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a
*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rule 4376

```
Int[(u_)*((v_) + (d_.)*(F_)[(c_.)*((a_.) + (b_.)*(x_))]^(n_.)), x_Symbol] :
> With[{e = FreeFactors[Sin[c*(a + b*x)], x]}, Int[ActivateTrig[u*v], x] +
Dist[d, Int[ActivateTrig[u]*Cos[c*(a + b*x)]^n, x], x] /; FunctionOfQ[Sin[c
*(a + b*x)]/e, u, x]] /; FreeQ[{a, b, c, d}, x] &&  !FreeQ[v, x] && Integer
Q[(n - 1)/2] && NonsumQ[u] && (EqQ[F, Cos] || EqQ[F, cos])
```

Rubi steps

$$\int \frac{A+B\cos(d+ex)+C\sin(d+ex)}{(a+c\sin(d+ex))^2}\,dx = B\int \frac{\cos(d+ex)}{(a+c\sin(d+ex))^2}\,dx + \int \frac{A+C\sin(d+ex)}{(a+c\sin(d+ex))^2}\,dx$$

$$= \frac{(Ac-aC)\cos(d+ex)}{\left(a^2-c^2\right)e(a+c\sin(d+ex))} + \frac{\int \frac{-aA+cC}{a+c\sin(d+ex)}\,dx}{-a^2+c^2} + \frac{B\,\text{Subst}\left(\int \frac{1}{(a+x)^2}\right.}{}$$

$$= -\frac{B}{ce(a+c\sin(d+ex))} + \frac{(Ac-aC)\cos(d+ex)}{\left(a^2-c^2\right)e(a+c\sin(d+ex))} + \frac{(aA-cC)}{a}$$

$$= -\frac{B}{ce(a+c\sin(d+ex))} + \frac{(Ac-aC)\cos(d+ex)}{\left(a^2-c^2\right)e(a+c\sin(d+ex))} + \frac{(2(aA-cC}{}$$

$$= -\frac{B}{ce(a+c\sin(d+ex))} + \frac{(Ac-aC)\cos(d+ex)}{\left(a^2-c^2\right)e(a+c\sin(d+ex))} - \frac{(4(aA-cC}{}$$

$$= \frac{2(aA-cC)\tan^{-1}\left(\frac{c+a\tan\left(\frac{1}{2}(d+ex)\right)}{\sqrt{a^2-c^2}}\right)}{\left(a^2-c^2\right)^{3/2}e} - \frac{B}{ce(a+c\sin(d+ex))} + \frac{(Ac}{\left(a^2-c^2\right.}$$

Mathematica [A] time = 0.43, size = 114, normalized size = 0.97

$$\frac{\frac{B\left(a^2-c^2\right)-c(Ac-aC)\cos(d+ex)}{c(c-a)(a+c)(a+c\sin(d+ex))} + \frac{2(aA-cC)\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(d+ex)\right)+c}{\sqrt{a^2-c^2}}\right)}{\left(a^2-c^2\right)^{3/2}}}{e}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Cos[d + e*x] + C*Sin[d + e*x])/(a + c*Sin[d + e*x])^2,x]
```

```
[Out] ((2*(a*A - c*C)*ArcTan[(c + a*Tan[(d + e*x)/2])/Sqrt[a^2 - c^2]])/(a^2 - c^
2)^(3/2) + (B*(a^2 - c^2) - c*(A*c - a*C)*Cos[d + e*x])/(c*(-a + c)*(a + c)
*(a + c*Sin[d + e*x])))/e
```

fricas [A] time = 1.36, size = 458, normalized size = 3.88

$$\left[-\frac{2\,Ba^4 - 4\,Ba^2c^2 + 2\,Bc^4 + \left(Aa^2c - Cac^2 + \left(Aac^2 - Cc^3\right)\sin(ex+d)\right)\sqrt{-a^2+c^2}\,\log\left(\frac{\left(2\,a^2-c^2\right)\cos(ex+d)^2 - 2\,ac\sin(}{c^2}\right.}{2\left(\left(a^4c^2 - 2\,a^2c^4 + c^6\right)e\sin(ex+d) + \right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(e*x+d)+C*sin(e*x+d))/(a+c*sin(e*x+d))^2,x, algorithm="fr
icas")
```

```
[Out] [-1/2*(2*B*a^4 - 4*B*a^2*c^2 + 2*B*c^4 + (A*a^2*c - C*a*c^2 + (A*a*c^2 - C*
c^3)*sin(e*x + d))*sqrt(-a^2 + c^2)*log(((2*a^2 - c^2)*cos(e*x + d)^2 - 2*a
*c*sin(e*x + d) - a^2 - c^2 + 2*(a*cos(e*x + d)*sin(e*x + d) + c*cos(e*x +
d))*sqrt(-a^2 + c^2))/(c^2*cos(e*x + d)^2 - 2*a*c*sin(e*x + d) - a^2 - c^2)
) + 2*(C*a^3*c - A*a^2*c^2 - C*a*c^3 + A*c^4)*cos(e*x + d))/((a^4*c^2 - 2*a
^2*c^4 + c^6)*e*sin(e*x + d) + (a^5*c - 2*a^3*c^3 + a*c^5)*e), -(B*a^4 - 2*
B*a^2*c^2 + B*c^4 + (A*a^2*c - C*a*c^2 + (A*a*c^2 - C*c^3)*sin(e*x + d))*sq
rt(a^2 - c^2)*arctan(-(a*sin(e*x + d) + c)/(sqrt(a^2 - c^2)*cos(e*x + d)))
+ (C*a^3*c - A*a^2*c^2 - C*a*c^3 + A*c^4)*cos(e*x + d))/((a^4*c^2 - 2*a^2*c
^4 + c^6)*e*sin(e*x + d) + (a^5*c - 2*a^3*c^3 + a*c^5)*e)]
```

giac [A] time = 0.20, size = 187, normalized size = 1.58

$$2\left(\frac{\left(\pi\left\lfloor\frac{xe+d}{2\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(a)+\arctan\left(\frac{a\tan\left(\frac{1}{2}xe+\frac{1}{2}d\right)+c}{\sqrt{a^2-c^2}}\right)\right)(Aa-Cc)}{\left(a^2-c^2\right)^{\frac{3}{2}}}+\frac{Ba^2\tan\left(\frac{1}{2}xe+\frac{1}{2}d\right)-Cac\tan\left(\frac{1}{2}xe+\frac{1}{2}d\right)+Ac}{\left(a^3-ac^2\right)\left(a\tan\left(\frac{1}{2}xe+\frac{1}{2}\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(e*x+d)+C*sin(e*x+d))/(a+c*sin(e*x+d))^2,x, algorithm="gi
ac")
```

```
[Out] 2*((pi*floor(1/2*(x*e + d)/pi + 1/2)*sgn(a) + arctan((a*tan(1/2*x*e + 1/2*d
) + c)/sqrt(a^2 - c^2)))*(A*a - C*c)/(a^2 - c^2)^(3/2) + (B*a^2*tan(1/2*x*e
 + 1/2*d) - C*a*c*tan(1/2*x*e + 1/2*d) + A*c^2*tan(1/2*x*e + 1/2*d) - B*c^2
*tan(1/2*x*e + 1/2*d) - C*a^2 + A*a*c)/((a^3 - a*c^2)*(a*tan(1/2*x*e + 1/2*
d)^2 + 2*c*tan(1/2*x*e + 1/2*d) + a)))*e^(-1)
```

maple [B] time = 0.40, size = 426, normalized size = 3.61

$$\frac{2\tan\left(\frac{d}{2}+\frac{ex}{2}\right)A\,c^2}{e\left(a\left(\tan^2\left(\frac{d}{2}+\frac{ex}{2}\right)\right)+2c\tan\left(\frac{d}{2}+\frac{ex}{2}\right)+a\right)a\left(a^2-c^2\right)}+\frac{2a\tan\left(\frac{d}{2}+\frac{ex}{2}\right)B}{e\left(a\left(\tan^2\left(\frac{d}{2}+\frac{ex}{2}\right)\right)+2c\tan\left(\frac{d}{2}+\frac{ex}{2}\right)+a\right)\left(a^2-c^2\right)}-\frac{}{e\left(a\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(e*x+d)+C*sin(e*x+d))/(a+c*sin(e*x+d))^2,x)
```

```
[Out] 2/e/(a*tan(1/2*d+1/2*e*x)^2+2*c*tan(1/2*d+1/2*e*x)+a)/a/(a^2-c^2)*tan(1/2*d
+1/2*e*x)*A*c^2+2/e/(a*tan(1/2*d+1/2*e*x)^2+2*c*tan(1/2*d+1/2*e*x)+a)*a/(a^
2-c^2)*tan(1/2*d+1/2*e*x)*B-2/e/(a*tan(1/2*d+1/2*e*x)^2+2*c*tan(1/2*d+1/2*e
*x)+a)/a/(a^2-c^2)*tan(1/2*d+1/2*e*x)*B*c^2-2/e/(a*tan(1/2*d+1/2*e*x)^2+2*c
*tan(1/2*d+1/2*e*x)+a)/(a^2-c^2)*tan(1/2*d+1/2*e*x)*c*C+2/e/(a*tan(1/2*d+1/
2*e*x)^2+2*c*tan(1/2*d+1/2*e*x)+a)/(a^2-c^2)*A*c-2/e/(a*tan(1/2*d+1/2*e*x)^
2+2*c*tan(1/2*d+1/2*e*x)+a)/(a^2-c^2)*C*a+2/e/(a^2-c^2)^(3/2)*arctan(1/2*(2
*a*tan(1/2*d+1/2*e*x)+2*c)/(a^2-c^2)^(1/2))*a*A-2/e/(a^2-c^2)^(3/2)*arctan(
1/2*(2*a*tan(1/2*d+1/2*e*x)+2*c)/(a^2-c^2)^(1/2))*C*c
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(e*x+d)+C*sin(e*x+d))/(a+c*sin(e*x+d))^2,x, algorithm="ma
xima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*c^2-4*a^2>0)', see `assume?` for
more details)Is 4*c^2-4*a^2 positive or negative?
```

mupad [B] time = 3.33, size = 227, normalized size = 1.92

$$\frac{\frac{2\,(A\,c-C\,a)}{a^2-c^2}+\frac{2\tan\left(\frac{d}{2}+\frac{e\,x}{2}\right)\left(B\,a^2+A\,c^2-B\,c^2-C\,a\,c\right)}{a\,\left(a^2-c^2\right)}}{e\,\left(a\tan\left(\frac{d}{2}+\frac{e\,x}{2}\right)^2+2\,c\tan\left(\frac{d}{2}+\frac{e\,x}{2}\right)+a\right)}+\frac{2\,\mathrm{atan}\left(\frac{\left(a^2-c^2\right)\left(\frac{2\,a\tan\left(\frac{d}{2}+\frac{e\,x}{2}\right)(A\,a-C\,c)}{(a+c)^{3/2}\,(a-c)^{3/2}}+\frac{2\left(a^2\,c-c^3\right)(A\,a-C\,c)}{(a+c)^{3/2}\left(a^2-c^2\right)(a-c)^{3/2}}\right)}{2\,(A\,a-C\,c)}\right)(A\,a-C\,c)}{e\,(a+c)^{3/2}\,(a-c)^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cos(d + e*x) + C*sin(d + e*x))/(a + c*sin(d + e*x))^2,x)
```

```
[Out] ((2*(A*c - C*a))/(a^2 - c^2) + (2*tan(d/2 + (e*x)/2)*(B*a^2 + A*c^2 - B*c^2
 - C*a*c))/(a*(a^2 - c^2)))/(e*(a + 2*c*tan(d/2 + (e*x)/2) + a*tan(d/2 + (e
*x)/2)^2)) + (2*atan(((a^2 - c^2)*((2*a*tan(d/2 + (e*x)/2)*(A*a - C*c))/((a
 + c)^(3/2)*(a - c)^(3/2)) + (2*(a^2*c - c^3)*(A*a - C*c))/((a + c)^(3/2)*(
a^2 - c^2)*(a - c)^(3/2))))/(2*(A*a - C*c)))*(A*a - C*c))/(e*(a + c)^(3/2)*
(a - c)^(3/2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(e*x+d)+C*sin(e*x+d))/(a+c*sin(e*x+d))**2,x)
```

```
[Out] Timed out
```

3.564 $\int \frac{A+B\cos(d+ex)+C\sin(d+ex)}{(a+c\sin(d+ex))^3}\,dx$

Optimal. Leaf size=185

$$\frac{\left(2a^2A-3acC+Ac^2\right)\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(d+ex)\right)+c}{\sqrt{a^2-c^2}}\right)}{e\left(a^2-c^2\right)^{5/2}}+\frac{\left(a^2(-C)+3aAc-2c^2C\right)\cos(d+ex)}{2e\left(a^2-c^2\right)^2(a+c\sin(d+ex))}+\frac{(Ac-aC)\cos(d+ex}{2e\left(a^2-c^2\right)(a+c\sin(d+}$$

```
[Out] (2*A*a^2+A*c^2-3*C*a*c)*arctan((c+a*tan(1/2*e*x+1/2*d))/(a^2-c^2)^(1/2))/(a
^2-c^2)^(5/2)/e-1/2*B/c/e/(a+c*sin(e*x+d))^2+1/2*(A*c-C*a)*cos(e*x+d)/(a^2-
c^2)/e/(a+c*sin(e*x+d))^2+1/2*(3*A*a*c-C*a^2-2*C*c^2)*cos(e*x+d)/(a^2-c^2)^
2/e/(a+c*sin(e*x+d))
```

Rubi [A] time = 0.25, antiderivative size = 185, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.258, Rules used = {4376, 2754, 12, 2660, 618, 204, 2668, 32}

$$\frac{\left(2a^2A-3acC+Ac^2\right)\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(d+ex)\right)+c}{\sqrt{a^2-c^2}}\right)}{e\left(a^2-c^2\right)^{5/2}}+\frac{\left(a^2(-C)+3aAc-2c^2C\right)\cos(d+ex)}{2e\left(a^2-c^2\right)^2(a+c\sin(d+ex))}+\frac{(Ac-aC)\cos(d+ex}{2e\left(a^2-c^2\right)(a+c\sin(d+}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cos[d + e*x] + C*Sin[d + e*x])/(a + c*Sin[d + e*x])^3,x]
```

```
[Out] ((2*a^2*A + A*c^2 - 3*a*c*C)*ArcTan[(c + a*Tan[(d + e*x)/2])/Sqrt[a^2 - c^2
]])/((a^2 - c^2)^(5/2)*e) - B/(2*c*e*(a + c*Sin[d + e*x])^2) + ((A*c - a*C)
*Cos[d + e*x])/(2*(a^2 - c^2)*e*(a + c*Sin[d + e*x])^2) + ((3*a*A*c - a^2*C
 - 2*c^2*C)*Cos[d + e*x])/(2*(a^2 - c^2)^2*e*(a + c*Sin[d + e*x]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2668

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rule 2754

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e + f
*x])^(m + 1))/(f*(m + 1)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(a^2 - b^2)), I
nt[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m +
 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a
*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rule 4376

```
Int[(u_)*((v_) + (d_.)*(F_)[(c_.)*((a_.) + (b_.)*(x_))]^(n_.)), x_Symbol] :
> With[{e = FreeFactors[Sin[c*(a + b*x)], x]}, Int[ActivateTrig[u*v], x] +
Dist[d, Int[ActivateTrig[u]*Cos[c*(a + b*x)]^n, x], x] /; FunctionOfQ[Sin[c
*(a + b*x)]/e, u, x]] /; FreeQ[{a, b, c, d}, x] &&  !FreeQ[v, x] && Integer
Q[(n - 1)/2] && NonsumQ[u] && (EqQ[F, Cos] || EqQ[F, cos])
```

Rubi steps

$$\int \frac{A+B\cos(d+ex)+C\sin(d+ex)}{(a+c\sin(d+ex))^3}\,dx = B\int \frac{\cos(d+ex)}{(a+c\sin(d+ex))^3}\,dx + \int \frac{A+C\sin(d+ex)}{(a+c\sin(d+ex))^3}\,dx$$

$$= \frac{(Ac-aC)\cos(d+ex)}{2\left(a^2-c^2\right)e(a+c\sin(d+ex))^2} - \frac{\int \frac{-2(aA-cC)+(Ac-aC)\sin(d+ex)}{(a+c\sin(d+ex))^2}\,dx}{2\left(a^2-c^2\right)} + \cdots$$

$$= -\frac{B}{2ce(a+c\sin(d+ex))^2} + \frac{(Ac-aC)\cos(d+ex)}{2\left(a^2-c^2\right)e(a+c\sin(d+ex))^2} + \frac{\left(3aAc\right.}{2\left(a^2\right.}$$

$$= -\frac{B}{2ce(a+c\sin(d+ex))^2} + \frac{(Ac-aC)\cos(d+ex)}{2\left(a^2-c^2\right)e(a+c\sin(d+ex))^2} + \frac{\left(3aAc\right.}{2\left(a^2\right.}$$

$$= -\frac{B}{2ce(a+c\sin(d+ex))^2} + \frac{(Ac-aC)\cos(d+ex)}{2\left(a^2-c^2\right)e(a+c\sin(d+ex))^2} + \frac{\left(3aAc\right.}{2\left(a^2\right.}$$

$$= -\frac{B}{2ce(a+c\sin(d+ex))^2} + \frac{(Ac-aC)\cos(d+ex)}{2\left(a^2-c^2\right)e(a+c\sin(d+ex))^2} + \frac{\left(3aAc\right.}{2\left(a^2\right.}$$

$$= \frac{\left(2a^2A+Ac^2-3acC\right)\tan^{-1}\left(\frac{c+a\tan\left(\frac{1}{2}(d+ex)\right)}{\sqrt{a^2-c^2}}\right)}{\left(a^2-c^2\right)^{5/2}e} - \frac{B}{2ce(a+c\sin(d+ex)}$$

Mathematica [A] time = 0.91, size = 174, normalized size = 0.94

$$\frac{\frac{B\left(c^2-a^2\right)+c(Ac-aC)\cos(d+ex)}{c(a-c)(a+c)(a+c\sin(d+ex))^2} + \frac{2\left(2a^2A-3acC+Ac^2\right)\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(d+ex)\right)+c}{\sqrt{a^2-c^2}}\right)}{\left(a^2-c^2\right)^{5/2}} - \frac{\left(a^2C-3aAc+2c^2C\right)\cos(d+ex)}{(a-c)^2(a+c)^2(a+c\sin(d+ex))}}{2e}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Cos[d + e*x] + C*Sin[d + e*x])/(a + c*Sin[d + e*x])^3,x]
```

```
[Out] ((2*(2*a^2*A + A*c^2 - 3*a*c*C)*ArcTan[(c + a*Tan[(d + e*x)/2])/Sqrt[a^2 -
c^2]])/(a^2 - c^2)^(5/2) + (B*(-a^2 + c^2) + c*(A*c - a*C)*Cos[d + e*x])/((
a - c)*c*(a + c)*(a + c*Sin[d + e*x])^2) - ((-3*a*A*c + a^2*C + 2*c^2*C)*Co
s[d + e*x])/((a - c)^2*(a + c)^2*(a + c*Sin[d + e*x])))/(2*e)
```

fricas [B] time = 1.17, size = 880, normalized size = 4.76

$$\left[\frac{2\,Ba^6-6\,Ba^4c^2+6\,Ba^2c^4-2\,Bc^6+2\left(Ca^4c^2-3\,Aa^3c^3+Ca^2c^4+3\,Aac^5-2\,Cc^6\right)\cos\left(ex+d\right)\sin\left(ex+d\right)+\left(2\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(e*x+d)+C*sin(e*x+d))/(a+c*sin(e*x+d))^3,x, algorithm="fr
icas")
```

```
[Out] [1/4*(2*B*a^6 - 6*B*a^4*c^2 + 6*B*a^2*c^4 - 2*B*c^6 + 2*(C*a^4*c^2 - 3*A*a^
3*c^3 + C*a^2*c^4 + 3*A*a*c^5 - 2*C*c^6)*cos(e*x + d)*sin(e*x + d) + (2*A*a
^4*c - 3*C*a^3*c^2 + 3*A*a^2*c^3 - 3*C*a*c^4 + A*c^5 - (2*A*a^2*c^3 - 3*C*a
*c^4 + A*c^5)*cos(e*x + d)^2 + 2*(2*A*a^3*c^2 - 3*C*a^2*c^3 + A*a*c^4)*sin(
e*x + d))*sqrt(-a^2 + c^2)*log(((2*a^2 - c^2)*cos(e*x + d)^2 - 2*a*c*sin(e*
x + d) - a^2 - c^2 + 2*(a*cos(e*x + d)*sin(e*x + d) + c*cos(e*x + d))*sqrt(
-a^2 + c^2))/(c^2*cos(e*x + d)^2 - 2*a*c*sin(e*x + d) - a^2 - c^2)) + 2*(2*
C*a^5*c - 4*A*a^4*c^2 - C*a^3*c^3 + 5*A*a^2*c^4 - C*a*c^5 - A*c^6)*cos(e*x
+ d))/((a^6*c^3 - 3*a^4*c^5 + 3*a^2*c^7 - c^9)*e*cos(e*x + d)^2 - 2*(a^7*c^
2 - 3*a^5*c^4 + 3*a^3*c^6 - a*c^8)*e*sin(e*x + d) - (a^8*c - 2*a^6*c^3 + 2*
a^2*c^7 - c^9)*e), 1/2*(B*a^6 - 3*B*a^4*c^2 + 3*B*a^2*c^4 - B*c^6 + (C*a^4*
c^2 - 3*A*a^3*c^3 + C*a^2*c^4 + 3*A*a*c^5 - 2*C*c^6)*cos(e*x + d)*sin(e*x +
 d) + (2*A*a^4*c - 3*C*a^3*c^2 + 3*A*a^2*c^3 - 3*C*a*c^4 + A*c^5 - (2*A*a^2
*c^3 - 3*C*a*c^4 + A*c^5)*cos(e*x + d)^2 + 2*(2*A*a^3*c^2 - 3*C*a^2*c^3 + A
*a*c^4)*sin(e*x + d))*sqrt(a^2 - c^2)*arctan(-(a*sin(e*x + d) + c)/(sqrt(a^
2 - c^2)*cos(e*x + d))) + (2*C*a^5*c - 4*A*a^4*c^2 - C*a^3*c^3 + 5*A*a^2*c^
4 - C*a*c^5 - A*c^6)*cos(e*x + d))/((a^6*c^3 - 3*a^4*c^5 + 3*a^2*c^7 - c^9)
*e*cos(e*x + d)^2 - 2*(a^7*c^2 - 3*a^5*c^4 + 3*a^3*c^6 - a*c^8)*e*sin(e*x +
 d) - (a^8*c - 2*a^6*c^3 + 2*a^2*c^7 - c^9)*e)]
```

giac [B] time = 0.27, size = 596, normalized size = 3.22

$$\left(\frac{\left(2\,Aa^2-3\,Cac+Ac^2\right)\left(\pi\left\lfloor\frac{xe+d}{2\,\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(a)+\arctan\left(\frac{a\tan\left(\frac{1}{2}\,xe+\frac{1}{2}\,d\right)+c}{\sqrt{a^2-c^2}}\right)\right)}{\left(a^4-2\,a^2c^2+c^4\right)\sqrt{a^2-c^2}}+\frac{2\,Ba^5\tan\left(\frac{1}{2}\,xe+\frac{1}{2}\,d\right)^3-3\,Ca^4c\tan\left(\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(e*x+d)+C*sin(e*x+d))/(a+c*sin(e*x+d))^3,x, algorithm="gi
ac")
```

```
[Out] ((2*A*a^2 - 3*C*a*c + A*c^2)*(pi*floor(1/2*(x*e + d)/pi + 1/2)*sgn(a) + arc
tan((a*tan(1/2*x*e + 1/2*d) + c)/sqrt(a^2 - c^2)))/((a^4 - 2*a^2*c^2 + c^4)
*sqrt(a^2 - c^2)) + (2*B*a^5*tan(1/2*x*e + 1/2*d)^3 - 3*C*a^4*c*tan(1/2*x*e
 + 1/2*d)^3 + 5*A*a^3*c^2*tan(1/2*x*e + 1/2*d)^3 - 4*B*a^3*c^2*tan(1/2*x*e
+ 1/2*d)^3 - 2*A*a*c^4*tan(1/2*x*e + 1/2*d)^3 + 2*B*a*c^4*tan(1/2*x*e + 1/2
*d)^3 - 2*C*a^5*tan(1/2*x*e + 1/2*d)^2 + 4*A*a^4*c*tan(1/2*x*e + 1/2*d)^2 +
 2*B*a^4*c*tan(1/2*x*e + 1/2*d)^2 - 5*C*a^3*c^2*tan(1/2*x*e + 1/2*d)^2 + 7*
A*a^2*c^3*tan(1/2*x*e + 1/2*d)^2 - 4*B*a^2*c^3*tan(1/2*x*e + 1/2*d)^2 - 2*C
*a*c^4*tan(1/2*x*e + 1/2*d)^2 - 2*A*c^5*tan(1/2*x*e + 1/2*d)^2 + 2*B*c^5*ta
n(1/2*x*e + 1/2*d)^2 + 2*B*a^5*tan(1/2*x*e + 1/2*d) - 5*C*a^4*c*tan(1/2*x*e
 + 1/2*d) + 11*A*a^3*c^2*tan(1/2*x*e + 1/2*d) - 4*B*a^3*c^2*tan(1/2*x*e + 1
/2*d) - 4*C*a^2*c^3*tan(1/2*x*e + 1/2*d) - 2*A*a*c^4*tan(1/2*x*e + 1/2*d) +
 2*B*a*c^4*tan(1/2*x*e + 1/2*d) - 2*C*a^5 + 4*A*a^4*c - C*a^3*c^2 - A*a^2*c
^3)/((a^6 - 2*a^4*c^2 + a^2*c^4)*(a*tan(1/2*x*e + 1/2*d)^2 + 2*c*tan(1/2*x*
e + 1/2*d) + a)^2))*e^(-1)
```

maple [B] time = 0.43, size = 1891, normalized size = 10.22

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(e*x+d)+C*sin(e*x+d))/(a+c*sin(e*x+d))^3,x)
```

```
[Out] -2/e/(a*tan(1/2*d+1/2*e*x)^2+2*c*tan(1/2*d+1/2*e*x)+a)^2/(a^4-2*a^2*c^2+c^4
)*C*a^3-1/e/(a*tan(1/2*d+1/2*e*x)^2+2*c*tan(1/2*d+1/2*e*x)+a)^2/(a^4-2*a^2*
c^2+c^4)*A*c^3-2/e/(a*tan(1/2*d+1/2*e*x)^2+2*c*tan(1/2*d+1/2*e*x)+a)^2/(a^4
-2*a^2*c^2+c^4)*a^3*tan(1/2*d+1/2*e*x)^2*C+2/e/(a*tan(1/2*d+1/2*e*x)^2+2*c*
tan(1/2*d+1/2*e*x)+a)^2*a^3/(a^4-2*a^2*c^2+c^4)*tan(1/2*d+1/2*e*x)*B+4/e/(a
*tan(1/2*d+1/2*e*x)^2+2*c*tan(1/2*d+1/2*e*x)+a)^2/(a^4-2*a^2*c^2+c^4)*A*a^2
*c-1/e/(a*tan(1/2*d+1/2*e*x)^2+2*c*tan(1/2*d+1/2*e*x)+a)^2/(a^4-2*a^2*c^2+c
^4)*C*a*c^2+7/e/(a*tan(1/2*d+1/2*e*x)^2+2*c*tan(1/2*d+1/2*e*x)+a)^2/(a^4-2*
a^2*c^2+c^4)*tan(1/2*d+1/2*e*x)^2*A*c^3-4/e/(a*tan(1/2*d+1/2*e*x)^2+2*c*tan
(1/2*d+1/2*e*x)+a)^2/(a^4-2*a^2*c^2+c^4)*tan(1/2*d+1/2*e*x)^2*B*c^3-4/e/(a*
tan(1/2*d+1/2*e*x)^2+2*c*tan(1/2*d+1/2*e*x)+a)^2/(a^4-2*a^2*c^2+c^4)*tan(1/
2*d+1/2*e*x)*C*c^3+2/e/(a^4-2*a^2*c^2+c^4)/(a^2-c^2)^(1/2)*arctan(1/2*(2*a*
tan(1/2*d+1/2*e*x)+2*c)/(a^2-c^2)^(1/2))*a^2*A+1/e/(a^4-2*a^2*c^2+c^4)/(a^2
-c^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*d+1/2*e*x)+2*c)/(a^2-c^2)^(1/2))*A*c^2+
2/e/(a*tan(1/2*d+1/2*e*x)^2+2*c*tan(1/2*d+1/2*e*x)+a)^2/(a^4-2*a^2*c^2+c^4)
*a^3*tan(1/2*d+1/2*e*x)^3*B-4/e/(a*tan(1/2*d+1/2*e*x)^2+2*c*tan(1/2*d+1/2*e
*x)+a)^2/(a^4-2*a^2*c^2+c^4)*a*tan(1/2*d+1/2*e*x)^3*B*c^2+2/e/(a*tan(1/2*d+
1/2*e*x)^2+2*c*tan(1/2*d+1/2*e*x)+a)^2/(a^4-2*a^2*c^2+c^4)/a*tan(1/2*d+1/2*
e*x)^3*B*c^4-3/e/(a*tan(1/2*d+1/2*e*x)^2+2*c*tan(1/2*d+1/2*e*x)+a)^2/(a^4-2
*a^2*c^2+c^4)*a^2*tan(1/2*d+1/2*e*x)^3*C*c+4/e/(a*tan(1/2*d+1/2*e*x)^2+2*c*
tan(1/2*d+1/2*e*x)+a)^2/(a^4-2*a^2*c^2+c^4)*a^2*tan(1/2*d+1/2*e*x)^2*A*c-2/
e/(a*tan(1/2*d+1/2*e*x)^2+2*c*tan(1/2*d+1/2*e*x)+a)^2/(a^4-2*a^2*c^2+c^4)/a
```

```
^2*tan(1/2*d+1/2*e*x)^2*A*c^5+2/e/(a*tan(1/2*d+1/2*e*x)^2+2*c*tan(1/2*d+1/2
*e*x)+a)^2/(a^4-2*a^2*c^2+c^4)*a^2*tan(1/2*d+1/2*e*x)^2*B*c+2/e/(a*tan(1/2*
d+1/2*e*x)^2+2*c*tan(1/2*d+1/2*e*x)+a)^2/(a^4-2*a^2*c^2+c^4)/a^2*tan(1/2*d+
1/2*e*x)^2*B*c^5-5/e/(a*tan(1/2*d+1/2*e*x)^2+2*c*tan(1/2*d+1/2*e*x)+a)^2/(a
^4-2*a^2*c^2+c^4)*a*tan(1/2*d+1/2*e*x)^2*C*c^2-2/e/(a*tan(1/2*d+1/2*e*x)^2+
2*c*tan(1/2*d+1/2*e*x)+a)^2/(a^4-2*a^2*c^2+c^4)/a*tan(1/2*d+1/2*e*x)^2*C*c^
4+11/e/(a*tan(1/2*d+1/2*e*x)^2+2*c*tan(1/2*d+1/2*e*x)+a)^2*a/(a^4-2*a^2*c^2
+c^4)*tan(1/2*d+1/2*e*x)*A*c^2-2/e/(a*tan(1/2*d+1/2*e*x)^2+2*c*tan(1/2*d+1/
2*e*x)+a)^2/a/(a^4-2*a^2*c^2+c^4)*tan(1/2*d+1/2*e*x)*A*c^4-4/e/(a*tan(1/2*d
+1/2*e*x)^2+2*c*tan(1/2*d+1/2*e*x)+a)^2*a/(a^4-2*a^2*c^2+c^4)*tan(1/2*d+1/2
*e*x)*B*c^2+2/e/(a*tan(1/2*d+1/2*e*x)^2+2*c*tan(1/2*d+1/2*e*x)+a)^2/a/(a^4-
2*a^2*c^2+c^4)*tan(1/2*d+1/2*e*x)*B*c^4-5/e/(a*tan(1/2*d+1/2*e*x)^2+2*c*tan
(1/2*d+1/2*e*x)+a)^2*a^2/(a^4-2*a^2*c^2+c^4)*tan(1/2*d+1/2*e*x)*C*c-3/e/(a^
4-2*a^2*c^2+c^4)/(a^2-c^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*d+1/2*e*x)+2*c)/(a
^2-c^2)^(1/2))*a*c*C+5/e/(a*tan(1/2*d+1/2*e*x)^2+2*c*tan(1/2*d+1/2*e*x)+a)^
2/(a^4-2*a^2*c^2+c^4)*a*tan(1/2*d+1/2*e*x)^3*A*c^2-2/e/(a*tan(1/2*d+1/2*e*x
)^2+2*c*tan(1/2*d+1/2*e*x)+a)^2/(a^4-2*a^2*c^2+c^4)/a*tan(1/2*d+1/2*e*x)^3*
A*c^4
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(e*x+d)+C*sin(e*x+d))/(a+c*sin(e*x+d))^3,x, algorithm="ma
xima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*c^2-4*a^2>0)', see `assume?` for
more details)Is 4*c^2-4*a^2 positive or negative?
```

mupad [B] time = 5.27, size = 557, normalized size = 3.01

$$\frac{\mathrm{atan}\left(\frac{\left(\frac{\left(2\,A\,a^2-3\,C\,a\,c+A\,c^2\right)\,\left(2\,a^4\,c-4\,a^2\,c^3+2\,c^5\right)}{2\,{\left(a+c\right)}^{5/2}\,{\left(a-c\right)}^{5/2}\,\left(a^4-2\,a^2\,c^2+c^4\right)}+\frac{a\,\tan\left(\frac{d}{2}+\frac{e\,x}{2}\right)\,\left(2\,A\,a^2-3\,C\,a\,c+A\,c^2\right)}{{\left(a+c\right)}^{5/2}\,{\left(a-c\right)}^{5/2}}\right)\,\left(a^4-2\,a^2\,c^2+c^4\right)}{2\,A\,a^2-3\,C\,a\,c+A\,c^2}\right)\,\left(2\,A\,a^2-3\,C\,a\,c+A\,c^2\right)}{e\,{\left(a+c\right)}^{5/2}\,{\left(a-c\right)}^{5/2}}-\frac{\frac{2\,C\,a^3-4\,A\,a^2\,c+}{a^4-2\,a^2\,c}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cos(d + e*x) + C*sin(d + e*x))/(a + c*sin(d + e*x))^3,x)
```

```
[Out] (atan(((((2*A*a^2 + A*c^2 - 3*C*a*c)*(2*a^4*c + 2*c^5 - 4*a^2*c^3))/(2*(a +
 c)^(5/2)*(a - c)^(5/2)*(a^4 + c^4 - 2*a^2*c^2)) + (a*tan(d/2 + (e*x)/2)*(2
*A*a^2 + A*c^2 - 3*C*a*c))/((a + c)^(5/2)*(a - c)^(5/2)))*(a^4 + c^4 - 2*a^
2*c^2))/(2*A*a^2 + A*c^2 - 3*C*a*c))*(2*A*a^2 + A*c^2 - 3*C*a*c))/(e*(a + c
)^(5/2)*(a - c)^(5/2)) - ((A*c^3 + 2*C*a^3 - 4*A*a^2*c + C*a*c^2)/(a^4 + c^
4 - 2*a^2*c^2) - (tan(d/2 + (e*x)/2)^3*(2*B*a^4 - 2*A*c^4 + 2*B*c^4 + 5*A*a
^2*c^2 - 4*B*a^2*c^2 - 3*C*a^3*c))/(a*(a^4 + c^4 - 2*a^2*c^2)) + (tan(d/2 +
 (e*x)/2)*(2*A*c^4 - 2*B*a^4 - 2*B*c^4 - 11*A*a^2*c^2 + 4*B*a^2*c^2 + 4*C*a
*c^3 + 5*C*a^3*c))/(a*(a^4 + c^4 - 2*a^2*c^2)) + (tan(d/2 + (e*x)/2)^2*(2*A
*c^5 + 2*C*a^5 - 2*B*c^5 - 7*A*a^2*c^3 + 4*B*a^2*c^3 + 5*C*a^3*c^2 - 4*A*a^
4*c - 2*B*a^4*c + 2*C*a*c^4))/(a^2*(a^4 + c^4 - 2*a^2*c^2)))/(e*(tan(d/2 +
(e*x)/2)^2*(2*a^2 + 4*c^2) + a^2*tan(d/2 + (e*x)/2)^4 + a^2 + 4*a*c*tan(d/2
 + (e*x)/2)^3 + 4*a*c*tan(d/2 + (e*x)/2)))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(e*x+d)+C*sin(e*x+d))/(a+c*sin(e*x+d))**3,x)
```

```
[Out] Timed out
```

3.565 $\int \frac{A+B\cos(d+ex)+C\sin(d+ex)}{(a+c\sin(d+ex))^4}\,dx$

Optimal. Leaf size=258

$$\frac{\left(-2a^2C+5aAc-3c^2C\right)\cos(d+ex)}{6e\left(a^2-c^2\right)^2(a+c\sin(d+ex))^2}+\frac{(Ac-aC)\cos(d+ex)}{3e\left(a^2-c^2\right)(a+c\sin(d+ex))^3}+\frac{\left(2a^3A-4a^2cC+3aAc^2-c^3C\right)\tan^{-1}\left(\ldots\right)}{e\left(a^2-c^2\right)^{7/2}}$$

```
[Out] (2*A*a^3+3*A*a*c^2-4*C*a^2*c-C*c^3)*arctan((c+a*tan(1/2*e*x+1/2*d))/(a^2-c^
2)^(1/2))/(a^2-c^2)^(7/2)/e-1/3*B/c/e/(a+c*sin(e*x+d))^3+1/3*(A*c-C*a)*cos(
e*x+d)/(a^2-c^2)/e/(a+c*sin(e*x+d))^3+1/6*(5*A*a*c-2*C*a^2-3*C*c^2)*cos(e*x
+d)/(a^2-c^2)^2/e/(a+c*sin(e*x+d))^2+1/6*(11*A*a^2*c+4*A*c^3-2*C*a^3-13*C*a
*c^2)*cos(e*x+d)/(a^2-c^2)^3/e/(a+c*sin(e*x+d))
```

Rubi [A] time = 0.40, antiderivative size = 258, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 8, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.258, Rules used = {4376, 2754, 12, 2660, 618, 204, 2668, 32}

$$\frac{\left(2a^3A-4a^2cC+3aAc^2-c^3C\right)\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(d+ex)\right)+c}{\sqrt{a^2-c^2}}\right)}{e\left(a^2-c^2\right)^{7/2}}+\frac{\left(11a^2Ac-2a^3C-13ac^2C+4Ac^3\right)\cos(d+ex)}{6e\left(a^2-c^2\right)^3(a+c\sin(d+ex))}+\frac{\left(-2a^2C\ldots\right.}{6e\left(a^2\ldots\right.}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cos[d + e*x] + C*Sin[d + e*x])/(a + c*Sin[d + e*x])^4,x]
```

```
[Out] ((2*a^3*A + 3*a*A*c^2 - 4*a^2*c*C - c^3*C)*ArcTan[(c + a*Tan[(d + e*x)/2])/
Sqrt[a^2 - c^2]])/((a^2 - c^2)^(7/2)*e) - B/(3*c*e*(a + c*Sin[d + e*x])^3)
+ ((A*c - a*C)*Cos[d + e*x])/(3*(a^2 - c^2)*e*(a + c*Sin[d + e*x])^3) + ((5
*a*A*c - 2*a^2*C - 3*c^2*C)*Cos[d + e*x])/(6*(a^2 - c^2)^2*e*(a + c*Sin[d +
 e*x])^2) + ((11*a^2*A*c + 4*A*c^3 - 2*a^3*C - 13*a*c^2*C)*Cos[d + e*x])/(6
*(a^2 - c^2)^3*e*(a + c*Sin[d + e*x]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2668

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rule 2754

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e + f
*x])^(m + 1))/(f*(m + 1)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(a^2 - b^2)), I
nt[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m +
 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a
*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rule 4376

```
Int[(u_)*((v_) + (d_.)*(F_)[(c_.)*((a_.) + (b_.)*(x_))]^(n_.)), x_Symbol] :
> With[{e = FreeFactors[Sin[c*(a + b*x)], x]}, Int[ActivateTrig[u*v], x] +
Dist[d, Int[ActivateTrig[u]*Cos[c*(a + b*x)]^n, x], x] /; FunctionOfQ[Sin[c
*(a + b*x)]/e, u, x]] /; FreeQ[{a, b, c, d}, x] &&  !FreeQ[v, x] && Integer
Q[(n - 1)/2] && NonsumQ[u] && (EqQ[F, Cos] || EqQ[F, cos])
```

Rubi steps

$$\int \frac{A+B\cos(d+ex)+C\sin(d+ex)}{(a+c\sin(d+ex))^4}\,dx = B\int \frac{\cos(d+ex)}{(a+c\sin(d+ex))^4}\,dx + \int \frac{A+C\sin(d+ex)}{(a+c\sin(d+ex))^4}\,dx$$

$$= \frac{(Ac-aC)\cos(d+ex)}{3\left(a^2-c^2\right)e(a+c\sin(d+ex))^3} - \frac{\int \frac{-3(aA-cC)+2(Ac-aC)\sin(d+ex)}{(a+c\sin(d+ex))^3}\,dx}{3\left(a^2-c^2\right)} + -$$

$$= -\frac{B}{3ce(a+c\sin(d+ex))^3} + \frac{(Ac-aC)\cos(d+ex)}{3\left(a^2-c^2\right)e(a+c\sin(d+ex))^3} + \frac{(5aAc-}{6\left(a^2-\right.}$$

$$= -\frac{B}{3ce(a+c\sin(d+ex))^3} + \frac{(Ac-aC)\cos(d+ex)}{3\left(a^2-c^2\right)e(a+c\sin(d+ex))^3} + \frac{(5aAc-}{6\left(a^2-\right.}$$

$$= -\frac{B}{3ce(a+c\sin(d+ex))^3} + \frac{(Ac-aC)\cos(d+ex)}{3\left(a^2-c^2\right)e(a+c\sin(d+ex))^3} + \frac{(5aAc-}{6\left(a^2-\right.}$$

$$= -\frac{B}{3ce(a+c\sin(d+ex))^3} + \frac{(Ac-aC)\cos(d+ex)}{3\left(a^2-c^2\right)e(a+c\sin(d+ex))^3} + \frac{(5aAc-}{6\left(a^2-\right.}$$

$$= -\frac{B}{3ce(a+c\sin(d+ex))^3} + \frac{(Ac-aC)\cos(d+ex)}{3\left(a^2-c^2\right)e(a+c\sin(d+ex))^3} + \frac{(5aAc-}{6\left(a^2-\right.}$$

$$= \frac{\left(2a^3A+3aAc^2-4a^2cC-c^3C\right)\tan^{-1}\left(\frac{c+a\tan\left(\frac{1}{2}(d+ex)\right)}{\sqrt{a^2-c^2}}\right)}{\left(a^2-c^2\right)^{7/2}e} - \frac{B}{3ce(a+c\sin}$$

Mathematica [A] time = 2.64, size = 244, normalized size = 0.95

$$\frac{\frac{2B\left(c^2-a^2\right)+2c(Ac-aC)\cos(d+ex)}{c(a-c)(a+c)(a+c\sin(d+ex))^3} + \frac{\left(-2a^2C+5aAc-3c^2C\right)\cos(d+ex)}{(a-c)^2(a+c)^2(a+c\sin(d+ex))^2} + \frac{6\left(2a^3A-4a^2cC+3aAc^2-c^3C\right)\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(d+ex)\right)+c}{\sqrt{a^2-c^2}}\right)}{\left(a^2-c^2\right)^{7/2}} + \frac{\left(-2a^3C+11a^2Ac-\right.}{(a-c)^3(a+}}{6e}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Cos[d + e*x] + C*Sin[d + e*x])/(a + c*Sin[d + e*x])^4,x]
```

```
[Out] ((6*(2*a^3*A + 3*a*A*c^2 - 4*a^2*c*C - c^3*C)*ArcTan[(c + a*Tan[(d + e*x)/2
])/Sqrt[a^2 - c^2]])/(a^2 - c^2)^(7/2) + (2*B*(-a^2 + c^2) + 2*c*(A*c - a*C
```

```
)*Cos[d + e*x])/((a - c)*c*(a + c)*(a + c*Sin[d + e*x])^3) + ((5*a*A*c - 2*
a^2*C - 3*c^2*C)*Cos[d + e*x])/((a - c)^2*(a + c)^2*(a + c*Sin[d + e*x])^2)
 + ((11*a^2*A*c + 4*A*c^3 - 2*a^3*C - 13*a*c^2*C)*Cos[d + e*x])/((a - c)^3*
(a + c)^3*(a + c*Sin[d + e*x])))/(6*e)
```

fricas [B] time = 0.79, size = 1411, normalized size = 5.47

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(e*x+d)+C*sin(e*x+d))/(a+c*sin(e*x+d))^4,x, algorithm="fr
icas")
```

```
[Out] [1/12*(4*B*a^8 - 16*B*a^6*c^2 + 24*B*a^4*c^4 - 16*B*a^2*c^6 + 4*B*c^8 - 2*(
2*C*a^5*c^3 - 11*A*a^4*c^4 + 11*C*a^3*c^5 + 7*A*a^2*c^6 - 13*C*a*c^7 + 4*A*
c^8)*cos(e*x + d)^3 + 6*(2*C*a^6*c^2 - 9*A*a^5*c^3 + 7*C*a^4*c^4 + 8*A*a^3*
c^5 - 10*C*a^2*c^6 + A*a*c^7 + C*c^8)*cos(e*x + d)*sin(e*x + d) + 3*(2*A*a^
6*c - 4*C*a^5*c^2 + 9*A*a^4*c^3 - 13*C*a^3*c^4 + 9*A*a^2*c^5 - 3*C*a*c^6 -
3*(2*A*a^4*c^3 - 4*C*a^3*c^4 + 3*A*a^2*c^5 - C*a*c^6)*cos(e*x + d)^2 + (6*A
*a^5*c^2 - 12*C*a^4*c^3 + 11*A*a^3*c^4 - 7*C*a^2*c^5 + 3*A*a*c^6 - C*c^7 -
(2*A*a^3*c^4 - 4*C*a^2*c^5 + 3*A*a*c^6 - C*c^7)*cos(e*x + d)^2)*sin(e*x + d
))*sqrt(-a^2 + c^2)*log(((2*a^2 - c^2)*cos(e*x + d)^2 - 2*a*c*sin(e*x + d)
- a^2 - c^2 + 2*(a*cos(e*x + d)*sin(e*x + d) + c*cos(e*x + d))*sqrt(-a^2 +
c^2))/(c^2*cos(e*x + d)^2 - 2*a*c*sin(e*x + d) - a^2 - c^2)) + 12*(C*a^7*c
- 3*A*a^6*c^2 + C*a^5*c^3 + 2*A*a^4*c^4 - 2*C*a*c^7 + A*c^8)*cos(e*x + d))/
(3*(a^9*c^3 - 4*a^7*c^5 + 6*a^5*c^7 - 4*a^3*c^9 + a*c^11)*e*cos(e*x + d)^2
- (a^11*c - a^9*c^3 - 6*a^7*c^5 + 14*a^5*c^7 - 11*a^3*c^9 + 3*a*c^11)*e + (
(a^8*c^4 - 4*a^6*c^6 + 6*a^4*c^8 - 4*a^2*c^10 + c^12)*e*cos(e*x + d)^2 - (3
*a^10*c^2 - 11*a^8*c^4 + 14*a^6*c^6 - 6*a^4*c^8 - a^2*c^10 + c^12)*e)*sin(e
*x + d)), 1/6*(2*B*a^8 - 8*B*a^6*c^2 + 12*B*a^4*c^4 - 8*B*a^2*c^6 + 2*B*c^8
 - (2*C*a^5*c^3 - 11*A*a^4*c^4 + 11*C*a^3*c^5 + 7*A*a^2*c^6 - 13*C*a*c^7 +
4*A*c^8)*cos(e*x + d)^3 + 3*(2*C*a^6*c^2 - 9*A*a^5*c^3 + 7*C*a^4*c^4 + 8*A*
a^3*c^5 - 10*C*a^2*c^6 + A*a*c^7 + C*c^8)*cos(e*x + d)*sin(e*x + d) + 3*(2*
A*a^6*c - 4*C*a^5*c^2 + 9*A*a^4*c^3 - 13*C*a^3*c^4 + 9*A*a^2*c^5 - 3*C*a*c^
6 - 3*(2*A*a^4*c^3 - 4*C*a^3*c^4 + 3*A*a^2*c^5 - C*a*c^6)*cos(e*x + d)^2 +
(6*A*a^5*c^2 - 12*C*a^4*c^3 + 11*A*a^3*c^4 - 7*C*a^2*c^5 + 3*A*a*c^6 - C*c^
7 - (2*A*a^3*c^4 - 4*C*a^2*c^5 + 3*A*a*c^6 - C*c^7)*cos(e*x + d)^2)*sin(e*x
 + d))*sqrt(a^2 - c^2)*arctan(-(a*sin(e*x + d) + c)/(sqrt(a^2 - c^2)*cos(e*
x + d))) + 6*(C*a^7*c - 3*A*a^6*c^2 + C*a^5*c^3 + 2*A*a^4*c^4 - 2*C*a*c^7 +
 A*c^8)*cos(e*x + d))/(3*(a^9*c^3 - 4*a^7*c^5 + 6*a^5*c^7 - 4*a^3*c^9 + a*c
^11)*e*cos(e*x + d)^2 - (a^11*c - a^9*c^3 - 6*a^7*c^5 + 14*a^5*c^7 - 11*a^3
*c^9 + 3*a*c^11)*e + ((a^8*c^4 - 4*a^6*c^6 + 6*a^4*c^8 - 4*a^2*c^10 + c^12)
*e*cos(e*x + d)^2 - (3*a^10*c^2 - 11*a^8*c^4 + 14*a^6*c^6 - 6*a^4*c^8 - a^2
*c^10 + c^12)*e)*sin(e*x + d))]
```

giac [B] time = 0.31, size = 1340, normalized size = 5.19

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(e*x+d)+C*sin(e*x+d))/(a+c*sin(e*x+d))^4,x, algorithm="gi
ac")
```

```
[Out] 1/3*(3*(2*A*a^3 - 4*C*a^2*c + 3*A*a*c^2 - C*c^3)*(pi*floor(1/2*(x*e + d)/pi
 + 1/2)*sgn(a) + arctan((a*tan(1/2*x*e + 1/2*d) + c)/sqrt(a^2 - c^2)))/((a^
6 - 3*a^4*c^2 + 3*a^2*c^4 - c^6)*sqrt(a^2 - c^2)) + (6*B*a^8*tan(1/2*x*e +
1/2*d)^5 - 12*C*a^7*c*tan(1/2*x*e + 1/2*d)^5 + 27*A*a^6*c^2*tan(1/2*x*e + 1
/2*d)^5 - 18*B*a^6*c^2*tan(1/2*x*e + 1/2*d)^5 - 3*C*a^5*c^3*tan(1/2*x*e + 1
/2*d)^5 - 18*A*a^4*c^4*tan(1/2*x*e + 1/2*d)^5 + 18*B*a^4*c^4*tan(1/2*x*e +
1/2*d)^5 + 6*A*a^2*c^6*tan(1/2*x*e + 1/2*d)^5 - 6*B*a^2*c^6*tan(1/2*x*e + 1
/2*d)^5 - 6*C*a^8*tan(1/2*x*e + 1/2*d)^4 + 18*A*a^7*c*tan(1/2*x*e + 1/2*d)^
4 + 12*B*a^7*c*tan(1/2*x*e + 1/2*d)^4 - 42*C*a^6*c^2*tan(1/2*x*e + 1/2*d)^4
 + 81*A*a^5*c^3*tan(1/2*x*e + 1/2*d)^4 - 36*B*a^5*c^3*tan(1/2*x*e + 1/2*d)^
4 - 33*C*a^4*c^4*tan(1/2*x*e + 1/2*d)^4 - 36*A*a^3*c^5*tan(1/2*x*e + 1/2*d)
^4 + 36*B*a^3*c^5*tan(1/2*x*e + 1/2*d)^4 + 6*C*a^2*c^6*tan(1/2*x*e + 1/2*d)
^4 + 12*A*a*c^7*tan(1/2*x*e + 1/2*d)^4 - 12*B*a*c^7*tan(1/2*x*e + 1/2*d)^4
+ 12*B*a^8*tan(1/2*x*e + 1/2*d)^3 - 36*C*a^7*c*tan(1/2*x*e + 1/2*d)^3 + 108
*A*a^6*c^2*tan(1/2*x*e + 1/2*d)^3 - 28*B*a^6*c^2*tan(1/2*x*e + 1/2*d)^3 - 8
4*C*a^5*c^3*tan(1/2*x*e + 1/2*d)^3 + 42*A*a^4*c^4*tan(1/2*x*e + 1/2*d)^3 +
12*B*a^4*c^4*tan(1/2*x*e + 1/2*d)^3 - 34*C*a^3*c^5*tan(1/2*x*e + 1/2*d)^3 -
 8*A*a^2*c^6*tan(1/2*x*e + 1/2*d)^3 + 12*B*a^2*c^6*tan(1/2*x*e + 1/2*d)^3 +
 4*C*a*c^7*tan(1/2*x*e + 1/2*d)^3 + 8*A*c^8*tan(1/2*x*e + 1/2*d)^3 - 8*B*c^
8*tan(1/2*x*e + 1/2*d)^3 - 12*C*a^8*tan(1/2*x*e + 1/2*d)^2 + 36*A*a^7*c*tan
(1/2*x*e + 1/2*d)^2 + 12*B*a^7*c*tan(1/2*x*e + 1/2*d)^2 - 60*C*a^6*c^2*tan(
1/2*x*e + 1/2*d)^2 + 120*A*a^5*c^3*tan(1/2*x*e + 1/2*d)^2 - 36*B*a^5*c^3*ta
n(1/2*x*e + 1/2*d)^2 - 84*C*a^4*c^4*tan(1/2*x*e + 1/2*d)^2 - 18*A*a^3*c^5*t
an(1/2*x*e + 1/2*d)^2 + 36*B*a^3*c^5*tan(1/2*x*e + 1/2*d)^2 + 6*C*a^2*c^6*t
an(1/2*x*e + 1/2*d)^2 + 12*A*a*c^7*tan(1/2*x*e + 1/2*d)^2 - 12*B*a*c^7*tan(
1/2*x*e + 1/2*d)^2 + 6*B*a^8*tan(1/2*x*e + 1/2*d) - 24*C*a^7*c*tan(1/2*x*e
+ 1/2*d) + 81*A*a^6*c^2*tan(1/2*x*e + 1/2*d) - 18*B*a^6*c^2*tan(1/2*x*e + 1
/2*d) - 57*C*a^5*c^3*tan(1/2*x*e + 1/2*d) - 12*A*a^4*c^4*tan(1/2*x*e + 1/2*
d) + 18*B*a^4*c^4*tan(1/2*x*e + 1/2*d) + 6*C*a^3*c^5*tan(1/2*x*e + 1/2*d) +
 6*A*a^2*c^6*tan(1/2*x*e + 1/2*d) - 6*B*a^2*c^6*tan(1/2*x*e + 1/2*d) - 6*C*
a^8 + 18*A*a^7*c - 10*C*a^6*c^2 - 5*A*a^5*c^3 + C*a^4*c^4 + 2*A*a^3*c^5)/((
a^9 - 3*a^7*c^2 + 3*a^5*c^4 - a^3*c^6)*(a*tan(1/2*x*e + 1/2*d)^2 + 2*c*tan(
1/2*x*e + 1/2*d) + a)^3))*e^(-1)
```

maple [B] time = 0.46, size = 5051, normalized size = 19.58

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(e*x+d)+C*sin(e*x+d))/(a+c*sin(e*x+d))^4,x)
```

```
[Out] result too large to display
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(e*x+d)+C*sin(e*x+d))/(a+c*sin(e*x+d))^4,x, algorithm="ma
xima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*c^2-4*a^2>0)', see `assume?` for
more details)Is 4*c^2-4*a^2 positive or negative?
```

mupad [B] time = 6.22, size = 1085, normalized size = 4.21

$$\frac{\frac{-6\,C\,a^5+18\,A\,a^4\,c-10\,C\,a^3\,c^2-5\,A\,a^2\,c^3+C\,a\,c^4+2\,A\,c^5}{3\,\left(a^6-3\,a^4\,c^2+3\,a^2\,c^4-c^6\right)}+\frac{\tan\left(\frac{d}{2}+\frac{e\,x}{2}\right)\,\left(2\,B\,a^6+2\,A\,c^6-2\,B\,c^6-4\,A\,a^2\,c^4+27\,A\,a^4\,c^2+6\,B\,a^2\,c^4-6\,B\,a^4\,c^2-19\,C\,a^3\,c^3+2\,C}{a\,\left(a^6-3\,a^4\,c^2+3\,a^2\,c^4-c^6\right)}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cos(d + e*x) + C*sin(d + e*x))/(a + c*sin(d + e*x))^4,x)
```

```
[Out] ((2*A*c^5 - 6*C*a^5 - 5*A*a^2*c^3 - 10*C*a^3*c^2 + 18*A*a^4*c + C*a*c^4)/(3
*(a^6 - c^6 + 3*a^2*c^4 - 3*a^4*c^2)) + (tan(d/2 + (e*x)/2)*(2*B*a^6 + 2*A*
c^6 - 2*B*c^6 - 4*A*a^2*c^4 + 27*A*a^4*c^2 + 6*B*a^2*c^4 - 6*B*a^4*c^2 - 19
*C*a^3*c^3 + 2*C*a*c^5 - 8*C*a^5*c))/(a*(a^6 - c^6 + 3*a^2*c^4 - 3*a^4*c^2)
) + (2*tan(d/2 + (e*x)/2)^2*(2*A*c^7 - 2*C*a^7 - 2*B*c^7 - 3*A*a^2*c^5 + 20
*A*a^4*c^3 + 6*B*a^2*c^5 - 6*B*a^4*c^3 - 14*C*a^3*c^4 - 10*C*a^5*c^2 + 6*A*
a^6*c + 2*B*a^6*c + C*a*c^6))/(a^2*(a^6 - c^6 + 3*a^2*c^4 - 3*a^4*c^2)) + (
tan(d/2 + (e*x)/2)^4*(4*A*c^7 - 2*C*a^7 - 4*B*c^7 - 12*A*a^2*c^5 + 27*A*a^4
*c^3 + 12*B*a^2*c^5 - 12*B*a^4*c^3 - 11*C*a^3*c^4 - 14*C*a^5*c^2 + 6*A*a^6*
c + 4*B*a^6*c + 2*C*a*c^6))/(a^2*(a^6 - c^6 + 3*a^2*c^4 - 3*a^4*c^2)) - (ta
n(d/2 + (e*x)/2)^5*(2*B*c^6 - 2*A*c^6 - 2*B*a^6 + 6*A*a^2*c^4 - 9*A*a^4*c^2
 - 6*B*a^2*c^4 + 6*B*a^4*c^2 + C*a^3*c^3 + 4*C*a^5*c))/(a*(a^6 - c^6 + 3*a^
2*c^4 - 3*a^4*c^2)) + (2*tan(d/2 + (e*x)/2)^3*(3*a^2 + 2*c^2)*(2*B*a^6 + 2*
```

```
A*c^6 - 2*B*c^6 - 5*A*a^2*c^4 + 18*A*a^4*c^2 + 6*B*a^2*c^4 - 6*B*a^4*c^2 -
10*C*a^3*c^3 + C*a*c^5 - 6*C*a^5*c))/(3*a^3*(a^6 - c^6 + 3*a^2*c^4 - 3*a^4*
c^2)))/(e*(a^3*tan(d/2 + (e*x)/2)^6 + tan(d/2 + (e*x)/2)^2*(12*a*c^2 + 3*a^
3) + tan(d/2 + (e*x)/2)^4*(12*a*c^2 + 3*a^3) + tan(d/2 + (e*x)/2)^3*(12*a^2
*c + 8*c^3) + a^3 + 6*a^2*c*tan(d/2 + (e*x)/2) + 6*a^2*c*tan(d/2 + (e*x)/2)
^5)) + (atan(((((2*A*a^3 - C*c^3 + 3*A*a*c^2 - 4*C*a^2*c)*(2*a^6*c - 2*c^7
+ 6*a^2*c^5 - 6*a^4*c^3))/(2*(a + c)^(7/2)*(a - c)^(7/2)*(a^6 - c^6 + 3*a^2
*c^4 - 3*a^4*c^2)) + (a*tan(d/2 + (e*x)/2)*(2*A*a^3 - C*c^3 + 3*A*a*c^2 - 4
*C*a^2*c))/((a + c)^(7/2)*(a - c)^(7/2)))*(a^6 - c^6 + 3*a^2*c^4 - 3*a^4*c^
2))/(2*A*a^3 - C*c^3 + 3*A*a*c^2 - 4*C*a^2*c))*(2*A*a^3 - C*c^3 + 3*A*a*c^2
 - 4*C*a^2*c))/(e*(a + c)^(7/2)*(a - c)^(7/2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(e*x+d)+C*sin(e*x+d))/(a+c*sin(e*x+d))**4,x)
```

```
[Out] Timed out
```

3.566 $\int (a+b\cos(c+dx)\sin(c+dx))^m\,dx$

Optimal. Leaf size=131

$$-\frac{\cos(2c+2dx)\left(a+\frac{1}{2}b\sin(2c+2dx)\right)^m\left(\frac{2a+b\sin(2c+2dx)}{2a+b}\right)^{-m}F_1\left(\frac{1}{2};\frac{1}{2},-m;\frac{3}{2};\frac{1}{2}(1-\sin(2c+2dx)),\frac{b(1-\sin(2c+2dx))}{2a+b}\right)}{\sqrt{2}\,d\sqrt{\sin(2c+2dx)+1}}$$

```
[Out] -1/2*AppellF1(1/2,-m,1/2,3/2,b*(1-sin(2*d*x+2*c))/(2*a+b),1/2-1/2*sin(2*d*x
+2*c))*cos(2*d*x+2*c)*(a+1/2*b*sin(2*d*x+2*c))^m/d/(((2*a+b*sin(2*d*x+2*c))
/(2*a+b))^m)*2^(1/2)/(1+sin(2*d*x+2*c))^(1/2)
```

Rubi [A] time = 0.11, antiderivative size = 131, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {2666, 2665, 139, 138}

$$-\frac{\cos(2c+2dx)\left(a+\frac{1}{2}b\sin(2c+2dx)\right)^m\left(\frac{2a+b\sin(2c+2dx)}{2a+b}\right)^{-m}F_1\left(\frac{1}{2};\frac{1}{2},-m;\frac{3}{2};\frac{1}{2}(1-\sin(2c+2dx)),\frac{b(1-\sin(2c+2dx))}{2a+b}\right)}{\sqrt{2}\,d\sqrt{\sin(2c+2dx)+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x]*Sin[c + d*x])^m,x]
```

```
[Out] -((AppellF1[1/2, 1/2, -m, 3/2, (1 - Sin[2*c + 2*d*x])/2, (b*(1 - Sin[2*c +
2*d*x]))/(2*a + b)]*Cos[2*c + 2*d*x]*(a + (b*Sin[2*c + 2*d*x])/2)^m)/(Sqrt[
2]*d*Sqrt[1 + Sin[2*c + 2*d*x]]*((2*a + b*Sin[2*c + 2*d*x])/(2*a + b))^m))
```

Rule 138

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2,
-((d*(a + b*x))/(b*c - a*d)), -((f*(a + b*x))/(b*e - a*f))])/(b*(m + 1)*(b/
(b*c - a*d))^n*(b/(b*e - a*f))^p), x] /; FreeQ[{a, b, c, d, e, f, m, n, p},
 x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && GtQ[b/(b*c - a*d)
, 0] && GtQ[b/(b*e - a*f), 0] && !(GtQ[d/(d*a - c*b), 0] && GtQ[d/(d*e - c
*f), 0] && SimplerQ[c + d*x, a + b*x]) && !(GtQ[f/(f*a - e*b), 0] && GtQ[f
/(f*c - e*d), 0] && SimplerQ[e + f*x, a + b*x])
```

Rule 139

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(e + f*x)^FracPart[p]/((b/(b*e - a*f))^IntPart[p]*
((b*(e + f*x))/(b*e - a*f))^FracPart[p]), Int[(a + b*x)^m*(c + d*x)^n*((b*e
)/(b*e - a*f) + (b*f*x)/(b*e - a*f))^p, x], x] /; FreeQ[{a, b, c, d, e, f,
m, n, p}, x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && GtQ[b/(b
```

```
*c - a*d), 0] &&  !GtQ[b/(b*e - a*f), 0]
```

Rule 2665

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[Cos[c +
d*x]/(d*Sqrt[1 + Sin[c + d*x]]*Sqrt[1 - Sin[c + d*x]]), Subst[Int[(a + b*x)
^n/(Sqrt[1 + x]*Sqrt[1 - x]), x], x, Sin[c + d*x]], x] /; FreeQ[{a, b, c, d
, n}, x] && NeQ[a^2 - b^2, 0] &&  !IntegerQ[2*n]
```

Rule 2666

```
Int[((a_) + cos[(c_.) + (d_.)*(x_)]*(b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_
Symbol] :> Int[(a + (b*Sin[2*c + 2*d*x])/2)^n, x] /; FreeQ[{a, b, c, d, n},
 x]
```

Rubi steps

$$\int (a+b\cos(c+dx)\sin(c+dx))^m\,dx = \int \left(a+\frac{1}{2}b\sin(2c+2dx)\right)^m\,dx$$

$$= \frac{\cos(2c+2dx)\,\text{Subst}\left(\int \frac{\left(a+\frac{bx}{2}\right)^m}{\sqrt{1-x}\,\sqrt{1+x}}\,dx, x, \sin(2c+2dx)\right)}{2d\sqrt{1-\sin(2c+2dx)}\,\sqrt{1+\sin(2c+2dx)}}$$

$$= \frac{\left(\cos(2c+2dx)\left(a+\frac{1}{2}b\sin(2c+2dx)\right)^m\left(-\frac{a+\frac{1}{2}b\sin(2c+2dx)}{-a-\frac{b}{2}}\right)^{-m}\right)\text{Subst}\left(\int \cdots\right.}{2d\sqrt{1-\sin(2c+2dx)}\,\sqrt{1+\sin(2c+2 \cdots}}$$

$$= -\frac{F_1\left(\frac{1}{2};\frac{1}{2},-m;\frac{3}{2};\frac{1}{2}(1-\sin(2c+2dx)),\frac{b(1-\sin(2c+2dx))}{2a+b}\right)\cos(2c+2dx)\left(a+\cdots\right.}{\sqrt{2}\,d\sqrt{1+\sin(2c+2dx)}}$$

Mathematica [A] time = 0.59, size = 145, normalized size = 1.11

$$\frac{\sec(2(c+dx))\sqrt{-\frac{b(\sin(2(c+dx))-1)}{2a+b}}\,\sqrt{\frac{b(\sin(2(c+dx))+1)}{b-2a}}\,\left(a+\frac{1}{2}b\sin(2(c+dx))\right)^{m+1}F_1\left(m+1;\frac{1}{2},\frac{1}{2};m+2;\frac{2a+b\sin(2(c+dx)}{2a-b}\cdots\right.}{bd(m+1)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*Cos[c + d*x]*Sin[c + d*x])^m,x]
```

```
[Out] (AppellF1[1 + m, 1/2, 1/2, 2 + m, (2*a + b*Sin[2*(c + d*x)])/(2*a - b), (2*
a + b*Sin[2*(c + d*x)])/(2*a + b)]*Sec[2*(c + d*x)]*Sqrt[-((b*(-1 + Sin[2*(
c + d*x)]))/(2*a + b))]*Sqrt[(b*(1 + Sin[2*(c + d*x)]))/(-2*a + b)]*(a + (b
*Sin[2*(c + d*x)])/2)^(1 + m))/(b*d*(1 + m))
```

fricas [F] time = 0.91, size = 0, normalized size = 0.00

$$\text{integral}\left((b\cos(dx+c)\sin(dx+c)+a)^m, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c)*sin(d*x+c))^m,x, algorithm="fricas")
```

```
[Out] integral((b*cos(d*x + c)*sin(d*x + c) + a)^m, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int(b\cos(dx+c)\sin(dx+c)+a)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c)*sin(d*x+c))^m,x, algorithm="giac")
```

```
[Out] integrate((b*cos(d*x + c)*sin(d*x + c) + a)^m, x)
```

maple [F] time = 1.11, size = 0, normalized size = 0.00

$$\int(a+b\cos(dx+c)\sin(dx+c))^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c)*sin(d*x+c))^m,x)
```

```
[Out] int((a+b*cos(d*x+c)*sin(d*x+c))^m,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int(b\cos(dx+c)\sin(dx+c)+a)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c)*sin(d*x+c))^m,x, algorithm="maxima")
```

```
[Out] integrate((b*cos(d*x + c)*sin(d*x + c) + a)^m, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int (a+b\,\cos(c+d\,x)\,\sin(c+d\,x))^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*cos(c + d*x)*sin(c + d*x))^m,x)
[Out] int((a + b*cos(c + d*x)*sin(c + d*x))^m, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c)*sin(d*x+c))**m,x)
[Out] Timed out
```

3.567 $\int (a+b\cos(c+dx)\sin(c+dx))^3\,dx$

Optimal. Leaf size=107

$$-\frac{b\left(16a^2+b^2\right)\cos(2c+2dx)}{24d}+\frac{1}{8}ax\left(8a^2+3b^2\right)-\frac{5ab^2\sin(2c+2dx)\cos(2c+2dx)}{48d}-\frac{b\cos(2c+2dx)(2a+b\sin($$

```
[Out] 1/8*a*(8*a^2+3*b^2)*x-1/24*b*(16*a^2+b^2)*cos(2*d*x+2*c)/d-5/48*a*b^2*cos(2
*d*x+2*c)*sin(2*d*x+2*c)/d-1/48*b*cos(2*d*x+2*c)*(2*a+b*sin(2*d*x+2*c))^2/d
```

Rubi [A] time = 0.08, antiderivative size = 107, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {2666, 2656, 2734}

$$-\frac{b\left(16a^2+b^2\right)\cos(2c+2dx)}{24d}+\frac{1}{8}ax\left(8a^2+3b^2\right)-\frac{5ab^2\sin(2c+2dx)\cos(2c+2dx)}{48d}-\frac{b\cos(2c+2dx)(2a+b\sin($$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x]*Sin[c + d*x])^3,x]
```

```
[Out] (a*(8*a^2 + 3*b^2)*x)/8 - (b*(16*a^2 + b^2)*Cos[2*c + 2*d*x])/(24*d) - (5*a
*b^2*Cos[2*c + 2*d*x]*Sin[2*c + 2*d*x])/(48*d) - (b*Cos[2*c + 2*d*x]*(2*a +
 b*Sin[2*c + 2*d*x])^2)/(48*d)
```

Rule 2656

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[
c + d*x]*(a + b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[1/n, Int[(a + b*Sin
[c + d*x])^(n - 2)*Simp[a^2*n + b^2*(n - 1) + a*b*(2*n - 1)*Sin[c + d*x], x
], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[n, 1] && In
tegerQ[2*n]
```

Rule 2666

```
Int[((a_) + cos[(c_.) + (d_.)*(x_)]*(b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_
Symbol] :> Int[(a + (b*Sin[2*c + 2*d*x])/2)^n, x] /; FreeQ[{a, b, c, d, n},
 x]
```

Rule 2734

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[((2*a*c + b*d)*x)/2, x] + (-Simp[((b*c + a*d)*Co
s[e + f*x])/f, x] - Simp[(b*d*Cos[e + f*x]*Sin[e + f*x])/(2*f), x]) /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rubi steps

$$\int (a+b\cos(c+dx)\sin(c+dx))^3\,dx = \int \left(a+\frac{1}{2}b\sin(2c+2dx)\right)^3\,dx$$
$$= -\frac{b\cos(2c+2dx)(2a+b\sin(2c+2dx))^2}{48d} + \frac{1}{3}\int \left(a+\frac{1}{2}b\sin(2c+2dx)\right)\Big($$
$$= \frac{1}{8}a\left(8a^2+3b^2\right)x - \frac{b\left(16a^2+b^2\right)\cos(2c+2dx)}{24d} - \frac{5ab^2\cos(2c+2dx)\sin(}{48d}$$

Mathematica [A] time = 0.27, size = 75, normalized size = 0.70

$$\frac{-9\left(16a^2b+b^3\right)\cos(2(c+dx))+6a\left(4\left(8a^2+3b^2\right)(c+dx)-3b^2\sin(4(c+dx))\right)+b^3\cos(6(c+dx))}{192d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x]*Sin[c + d*x])^3,x]
```

```
[Out] (-9*(16*a^2*b + b^3)*Cos[2*(c + d*x)] + b^3*Cos[6*(c + d*x)] + 6*a*(4*(8*a^
2 + 3*b^2)*(c + d*x) - 3*b^2*Sin[4*(c + d*x)]))/(192*d)
```

fricas [A] time = 1.75, size = 97, normalized size = 0.91

$$\frac{4\,b^3\cos(dx+c)^6-6\,b^3\cos(dx+c)^4-36\,a^2b\cos(dx+c)^2+3\left(8\,a^3+3\,ab^2\right)dx-9\left(2\,ab^2\cos(dx+c)^3-ab^2\cos\right.}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c)*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/24*(4*b^3*cos(d*x + c)^6 - 6*b^3*cos(d*x + c)^4 - 36*a^2*b*cos(d*x + c)^2
 + 3*(8*a^3 + 3*a*b^2)*d*x - 9*(2*a*b^2*cos(d*x + c)^3 - a*b^2*cos(d*x + c)
)*sin(d*x + c))/d
```

giac [A] time = 0.19, size = 75, normalized size = 0.70

$$\frac{b^3\cos(6\,dx+6\,c)}{192\,d} - \frac{3\,ab^2\sin(4\,dx+4\,c)}{32\,d} + \frac{1}{8}\left(8\,a^3+3\,ab^2\right)x - \frac{3\left(16\,a^2b+b^3\right)\cos(2\,dx+2\,c)}{64\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c)*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/192*b^3*cos(6*d*x + 6*c)/d - 3/32*a*b^2*sin(4*d*x + 4*c)/d + 1/8*(8*a^3 +
 3*a*b^2)*x - 3/64*(16*a^2*b + b^3)*cos(2*d*x + 2*c)/d
```

maple [A] time = 0.25, size = 106, normalized size = 0.99

$$\frac{b^3\left(-\frac{\left(\sin^2(dx+c)\right)\left(\cos^4(dx+c)\right)}{6}-\frac{\left(\cos^4(dx+c)\right)}{12}\right)+3a\,b^2\left(-\frac{\sin(dx+c)\left(\cos^3(dx+c)\right)}{4}+\frac{\sin(dx+c)\cos(dx+c)}{8}+\frac{dx}{8}+\frac{c}{8}\right)-\frac{3\left(\cos^2(dx+c)\right)a}{2}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c)*sin(d*x+c))^3,x)
```

```
[Out] 1/d*(b^3*(-1/6*sin(d*x+c)^2*cos(d*x+c)^4-1/12*cos(d*x+c)^4)+3*a*b^2*(-1/4*s
in(d*x+c)*cos(d*x+c)^3+1/8*sin(d*x+c)*cos(d*x+c)+1/8*d*x+1/8*c)-3/2*cos(d*x
+c)^2*a^2*b+a^3*(d*x+c))
```

maxima [A] time = 0.38, size = 80, normalized size = 0.75

$$a^3x-\frac{3\,a^2b\cos\left(dx+c\right)^2}{2\,d}+\frac{3\,(4\,dx+4\,c-\sin\left(4\,dx+4\,c\right))ab^2}{32\,d}-\frac{\left(2\,\sin\left(dx+c\right)^6-3\,\sin\left(dx+c\right)^4\right)b^3}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c)*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] a^3*x - 3/2*a^2*b*cos(d*x + c)^2/d + 3/32*(4*d*x + 4*c - sin(4*d*x + 4*c))*
a*b^2/d - 1/12*(2*sin(d*x + c)^6 - 3*sin(d*x + c)^4)*b^3/d
```

mupad [B] time = 3.44, size = 125, normalized size = 1.17

$$a^3\,x-\frac{\tan\left(c+d\,x\right)^2\,\left(72\,a^2\,b+6\,b^3\right)+36\,a^2\,b+2\,b^3+36\,a^2\,b\tan\left(c+d\,x\right)^4-9\,a\,b^2\tan\left(c+d\,x\right)^5+9\,a\,b^2\tan\left(c+\right.}{d\,\left(24\tan\left(c+d\,x\right)^6+72\tan\left(c+d\,x\right)^4+72\tan\left(c+d\,x\right)^2+24\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*cos(c + d*x)*sin(c + d*x))^3,x)
```

```
[Out] a^3*x - (tan(c + d*x)^2*(72*a^2*b + 6*b^3) + 36*a^2*b + 2*b^3 + 36*a^2*b*ta
n(c + d*x)^4 - 9*a*b^2*tan(c + d*x)^5 + 9*a*b^2*tan(c + d*x))/(d*(72*tan(c
+ d*x)^2 + 72*tan(c + d*x)^4 + 24*tan(c + d*x)^6 + 24)) + (3*a*b^2*x)/8
```

sympy [A] time = 3.36, size = 190, normalized size = 1.78

$$\begin{cases} a^3x-\frac{3a^2b\cos^2\left(c+dx\right)}{2d}+\frac{3ab^2x\sin^4\left(c+dx\right)}{8}+\frac{3ab^2x\sin^2\left(c+dx\right)\cos^2\left(c+dx\right)}{4}+\frac{3ab^2x\cos^4\left(c+dx\right)}{8}+\frac{3ab^2\sin^3\left(c+dx\right)\cos\left(c+dx\right)}{8d}-\frac{3ab^2\sin}{} \\ x\left(a+b\sin\left(c\right)\cos\left(c\right)\right)^3 \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c)*sin(d*x+c))**3,x)
```

```
[Out] Piecewise((a**3*x - 3*a**2*b*cos(c + d*x)**2/(2*d) + 3*a*b**2*x*sin(c + d*x
)**4/8 + 3*a*b**2*x*sin(c + d*x)**2*cos(c + d*x)**2/4 + 3*a*b**2*x*cos(c +
d*x)**4/8 + 3*a*b**2*sin(c + d*x)**3*cos(c + d*x)/(8*d) - 3*a*b**2*sin(c +
d*x)*cos(c + d*x)**3/(8*d) + b**3*sin(c + d*x)**6/(12*d) + b**3*sin(c + d*x
)**4*cos(c + d*x)**2/(4*d), Ne(d, 0)), (x*(a + b*sin(c)*cos(c))**3, True))
```

3.568 $\int (a+b\cos(c+dx)\sin(c+dx))^2\,dx$

Optimal. Leaf size=61

$$\frac{1}{8}x\left(8a^2+b^2\right)-\frac{ab\cos(2c+2dx)}{2d}-\frac{b^2\sin(2c+2dx)\cos(2c+2dx)}{16d}$$

```
[Out] 1/8*(8*a^2+b^2)*x-1/2*a*b*cos(2*d*x+2*c)/d-1/16*b^2*cos(2*d*x+2*c)*sin(2*d*
x+2*c)/d
```

Rubi [A] time = 0.03, antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2666, 2644}

$$\frac{1}{8}x\left(8a^2+b^2\right)-\frac{ab\cos(2c+2dx)}{2d}-\frac{b^2\sin(2c+2dx)\cos(2c+2dx)}{16d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x]*Sin[c + d*x])^2,x]
```

```
[Out] ((8*a^2 + b^2)*x)/8 - (a*b*Cos[2*c + 2*d*x])/(2*d) - (b^2*Cos[2*c + 2*d*x]*
Sin[2*c + 2*d*x])/(16*d)
```

Rule 2644

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^2, x_Symbol] :> Simp[((2*a^2 + b
^2)*x)/2, x] + (-Simp[(2*a*b*Cos[c + d*x])/d, x] - Simp[(b^2*Cos[c + d*x]*S
in[c + d*x])/(2*d), x]) /; FreeQ[{a, b, c, d}, x]
```

Rule 2666

```
Int[((a_) + cos[(c_.) + (d_.)*(x_)]*(b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_
Symbol] :> Int[(a + (b*Sin[2*c + 2*d*x])/2)^n, x] /; FreeQ[{a, b, c, d, n},
 x]
```

Rubi steps

$$\begin{aligned}\int (a+b\cos(c+dx)\sin(c+dx))^2\,dx &= \int \left(a+\frac{1}{2}b\sin(2c+2dx)\right)^2\,dx\\ &= \frac{1}{8}\left(8a^2+b^2\right)x-\frac{ab\cos(2c+2dx)}{2d}-\frac{b^2\cos(2c+2dx)\sin(2c+2dx)}{16d}\end{aligned}$$

Mathematica [A] time = 0.15, size = 48, normalized size = 0.79

$$-\frac{-4\left(8a^2+b^2\right)(c+dx)+16ab\cos(2(c+dx))+b^2\sin(4(c+dx))}{32d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x]*Sin[c + d*x])^2,x]
```

```
[Out] -1/32*(-4*(8*a^2 + b^2)*(c + d*x) + 16*a*b*Cos[2*(c + d*x)] + b^2*Sin[4*(c
+ d*x)])/d
```

fricas [A] time = 2.40, size = 63, normalized size = 1.03

$$-\frac{8\,ab\cos\left(dx+c\right)^2-\left(8\,a^2+b^2\right)dx+\left(2\,b^2\cos\left(dx+c\right)^3-b^2\cos\left(dx+c\right)\right)\sin\left(dx+c\right)}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c)*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -1/8*(8*a*b*cos(d*x + c)^2 - (8*a^2 + b^2)*d*x + (2*b^2*cos(d*x + c)^3 - b^
2*cos(d*x + c))*sin(d*x + c))/d
```

giac [A] time = 0.16, size = 46, normalized size = 0.75

$$\frac{1}{8}\left(8\,a^2+b^2\right)x-\frac{ab\cos\left(2\,dx+2\,c\right)}{2\,d}-\frac{b^2\sin\left(4\,dx+4\,c\right)}{32\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c)*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/8*(8*a^2 + b^2)*x - 1/2*a*b*cos(2*d*x + 2*c)/d - 1/32*b^2*sin(4*d*x + 4*c
)/d
```

maple [A] time = 0.24, size = 69, normalized size = 1.13

$$\frac{b^2\left(-\frac{\sin(dx+c)\left(\cos^3(dx+c)\right)}{4}+\frac{\sin(dx+c)\cos(dx+c)}{8}+\frac{dx}{8}+\frac{c}{8}\right)-\left(\cos^2\left(dx+c\right)\right)ab+a^2\left(dx+c\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c)*sin(d*x+c))^2,x)
```

```
[Out] 1/d*(b^2*(-1/4*sin(d*x+c)*cos(d*x+c)^3+1/8*sin(d*x+c)*cos(d*x+c)+1/8*d*x+1/
8*c)-cos(d*x+c)^2*a*b+a^2*(d*x+c))
```

maxima [A] time = 0.33, size = 48, normalized size = 0.79

$$a^2x - \frac{ab\cos(dx+c)^2}{d} + \frac{(4\,dx + 4\,c - \sin(4\,dx + 4\,c))b^2}{32\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c)*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] a^2*x - a*b*cos(d*x + c)^2/d + 1/32*(4*d*x + 4*c - sin(4*d*x + 4*c))*b^2/d
```

mupad [B] time = 3.03, size = 78, normalized size = 1.28

$$x\left(a^2 + \frac{b^2}{8}\right) - \frac{-\frac{b^2\tan(c+d\,x)^3}{8} + \frac{b^2\tan(c+d\,x)}{8} + a\,b\tan(c+d\,x)^2 + a\,b}{d\left(\tan(c+d\,x)^4 + 2\tan(c+d\,x)^2 + 1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*cos(c + d*x)*sin(c + d*x))^2,x)
```

```
[Out] x*(a^2 + b^2/8) - (a*b + (b^2*tan(c + d*x))/8 - (b^2*tan(c + d*x)^3)/8 + a*
b*tan(c + d*x)^2)/(d*(2*tan(c + d*x)^2 + tan(c + d*x)^4 + 1))
```

sympy [A] time = 0.95, size = 129, normalized size = 2.11

$$\begin{cases} a^2x - \frac{ab\cos^2(c+dx)}{d} + \frac{b^2x\sin^4(c+dx)}{8} + \frac{b^2x\sin^2(c+dx)\cos^2(c+dx)}{4} + \frac{b^2x\cos^4(c+dx)}{8} + \frac{b^2\sin^3(c+dx)\cos(c+dx)}{8d} - \frac{b^2\sin(c+dx)\cos^3(c}{8d} \\ x\left(a + b\sin(c)\cos(c)\right)^2 \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c)*sin(d*x+c))**2,x)
```

```
[Out] Piecewise((a**2*x - a*b*cos(c + d*x)**2/d + b**2*x*sin(c + d*x)**4/8 + b**2
*x*sin(c + d*x)**2*cos(c + d*x)**2/4 + b**2*x*cos(c + d*x)**4/8 + b**2*sin(
c + d*x)**3*cos(c + d*x)/(8*d) - b**2*sin(c + d*x)*cos(c + d*x)**3/(8*d), N
e(d, 0)), (x*(a + b*sin(c)*cos(c))**2, True))
```

3.569 $\int (a + b\cos(c + dx)\sin(c + dx))\,dx$

Optimal. Leaf size=20

$$ax + \frac{b\sin^2(c + dx)}{2d}$$

```
[Out] a*x+1/2*b*sin(d*x+c)^2/d
```

Rubi [A] time = 0.02, antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {2564, 30}

$$ax + \frac{b\sin^2(c + dx)}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[a + b*Cos[c + d*x]*Sin[c + d*x],x]
```

```
[Out] a*x + (b*Sin[c + d*x]^2)/(2*d)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2564

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rubi steps

$$\begin{aligned}
\int (a + b\cos(c + dx)\sin(c + dx))\,dx &= ax + b\int \cos(c + dx)\sin(c + dx)\,dx \\
&= ax + \frac{b\,\text{Subst}\left(\int x\,dx, x, \sin(c + dx)\right)}{d} \\
&= ax + \frac{b\sin^2(c + dx)}{2d}
\end{aligned}$$

Mathematica [A] time = 0.01, size = 38, normalized size = 1.90

$$ax + \frac{b\sin(2c)\sin(2dx)}{4d} - \frac{b\cos(2c)\cos(2dx)}{4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[a + b*Cos[c + d*x]*Sin[c + d*x],x]
```

```
[Out] a*x - (b*Cos[2*c]*Cos[2*d*x])/(4*d) + (b*Sin[2*c]*Sin[2*d*x])/(4*d)
```

fricas [A] time = 0.75, size = 22, normalized size = 1.10

$$\frac{2\,adx - b\cos\left(dx + c\right)^2}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*cos(d*x+c)*sin(d*x+c),x, algorithm="fricas")
```

```
[Out] 1/2*(2*a*d*x - b*cos(d*x + c)^2)/d
```

giac [A] time = 0.13, size = 18, normalized size = 0.90

$$ax + \frac{b\sin\left(dx + c\right)^2}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*cos(d*x+c)*sin(d*x+c),x, algorithm="giac")
```

```
[Out] a*x + 1/2*b*sin(d*x + c)^2/d
```

maple [A] time = 0.00, size = 19, normalized size = 0.95

$$ax + \frac{b\left(\sin^2\left(dx + c\right)\right)}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(a+b*cos(d*x+c)*sin(d*x+c),x)
```

```
[Out] a*x+1/2*b*sin(d*x+c)^2/d
```

maxima [A] time = 0.48, size = 18, normalized size = 0.90

$$ax - \frac{b\cos\left(dx + c\right)^2}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*cos(d*x+c)*sin(d*x+c),x, algorithm="maxima")
```

```
[Out] a*x - 1/2*b*cos(d*x + c)^2/d
```

mupad [B] time = 2.94, size = 22, normalized size = 1.10

$$-\frac{\frac{b\cos(c+d\,x)^2}{2}-a\,d\,x}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(a + b*cos(c + d*x)*sin(c + d*x),x)
```

```
[Out] -((b*cos(c + d*x)^2)/2 - a*d*x)/d
```

sympy [A] time = 0.18, size = 26, normalized size = 1.30

$$ax+b\left(\begin{cases}-\frac{\cos^2(c+dx)}{2d} & \text{for } d\neq 0\\ x\sin(c)\cos(c) & \text{otherwise}\end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*cos(d*x+c)*sin(d*x+c),x)
```

```
[Out] a*x + b*Piecewise((-cos(c + d*x)**2/(2*d), Ne(d, 0)), (x*sin(c)*cos(c), Tru
e))
```

3.570 $\int \frac{1}{a+b\cos(c+dx)\sin(c+dx)}\,dx$

Optimal. Leaf size=48

$$\frac{2\tan^{-1}\left(\frac{2a\tan(c+dx)+b}{\sqrt{4a^2-b^2}}\right)}{d\sqrt{4a^2-b^2}}$$

```
[Out] 2*arctan((b+2*a*tan(d*x+c))/(4*a^2-b^2)^(1/2))/d/(4*a^2-b^2)^(1/2)
```

Rubi [A] time = 0.07, antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {2666, 2660, 618, 204}

$$\frac{2\tan^{-1}\left(\frac{2a\tan(c+dx)+b}{\sqrt{4a^2-b^2}}\right)}{d\sqrt{4a^2-b^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x]*Sin[c + d*x])^(-1),x]
```

```
[Out] (2*ArcTan[(b + 2*a*Tan[c + d*x])/Sqrt[4*a^2 - b^2]])/(Sqrt[4*a^2 - b^2]*d)
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2666

```
Int[((a_) + cos[(c_.) + (d_.)*(x_)]*(b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_
Symbol] :> Int[(a + (b*Sin[2*c + 2*d*x])/2)^n, x] /; FreeQ[{a, b, c, d, n},
```

x]

Rubi steps

$$\int \frac{1}{a+b\cos(c+dx)\sin(c+dx)}\,dx = \int \frac{1}{a+\frac{1}{2}b\sin(2c+2dx)}\,dx$$
$$= \frac{\text{Subst}\left(\int \frac{1}{a+bx+ax^2}\,dx, x, \tan\left(\frac{1}{2}(2c+2dx)\right)\right)}{d}$$
$$= -\frac{2\,\text{Subst}\left(\int \frac{1}{-4a^2+b^2-x^2}\,dx, x, b+2a\tan\left(\frac{1}{2}(2c+2dx)\right)\right)}{d}$$
$$= \frac{2\tan^{-1}\left(\frac{b+2a\tan(c+dx)}{\sqrt{4a^2-b^2}}\right)}{\sqrt{4a^2-b^2}\,d}$$

Mathematica [A] time = 0.08, size = 48, normalized size = 1.00

$$\frac{2\tan^{-1}\left(\frac{2a\tan(c+dx)+b}{\sqrt{4a^2-b^2}}\right)}{d\sqrt{4a^2-b^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x]*Sin[c + d*x])^(-1),x]
[Out] (2*ArcTan[(b + 2*a*Tan[c + d*x])/Sqrt[4*a^2 - b^2]])/(Sqrt[4*a^2 - b^2]*d)
```

fricas [A] time = 0.84, size = 290, normalized size = 6.04

$$\left[-\frac{\sqrt{-4\,a^2+b^2}\,\log\left(-\frac{2\left(8\,a^2-b^2\right)\cos(dx+c)^4-4\,ab\cos(dx+c)\sin(dx+c)-2\left(8\,a^2-b^2\right)\cos(dx+c)^2+2\,a^2-b^2+\left(2\,b\cos(dx+c)^2+4\left(2\,a\cos(dx+c)^3-a\right.\right.}{b^2\cos(dx+c)^4-b^2\cos(dx+c)^2-2\,ab\cos(dx+c)\sin(dx+c)-a^2}\right.}{2\left(4\,a^2-b^2\right)d}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cos(d*x+c)*sin(d*x+c)),x, algorithm="fricas")
[Out] [-1/2*sqrt(-4*a^2 + b^2)*log(-(2*(8*a^2 - b^2)*cos(d*x + c)^4 - 4*a*b*cos(d
*x + c)*sin(d*x + c) - 2*(8*a^2 - b^2)*cos(d*x + c)^2 + 2*a^2 - b^2 + (2*b*
cos(d*x + c)^2 + 4*(2*a*cos(d*x + c)^3 - a*cos(d*x + c))*sin(d*x + c) - b)*
```

```
sqrt(-4*a^2 + b^2))/(b^2*cos(d*x + c)^4 - b^2*cos(d*x + c)^2 - 2*a*b*cos(d*
x + c)*sin(d*x + c) - a^2))/((4*a^2 - b^2)*d), -arctan(-(4*a*cos(d*x + c)*s
in(d*x + c) + b)*sqrt(4*a^2 - b^2)/(2*(4*a^2 - b^2)*cos(d*x + c)^2 - 4*a^2
+ b^2))/(sqrt(4*a^2 - b^2)*d)]
```

giac [**A**] time = 0.16, size = 61, normalized size = 1.27

$$\frac{2\left(\pi\left\lfloor\frac{dx+c}{\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(a)+\arctan\left(\frac{2\,a\tan(dx+c)+b}{\sqrt{4\,a^2-b^2}}\right)\right)}{\sqrt{4\,a^2-b^2}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cos(d*x+c)*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 2*(pi*floor((d*x + c)/pi + 1/2)*sgn(a) + arctan((2*a*tan(d*x + c) + b)/sqrt
(4*a^2 - b^2)))/(sqrt(4*a^2 - b^2)*d)
```

maple [**A**] time = 0.31, size = 45, normalized size = 0.94

$$\frac{2\arctan\left(\frac{b+2a\tan(dx+c)}{\sqrt{4a^2-b^2}}\right)}{d\sqrt{4a^2-b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*cos(d*x+c)*sin(d*x+c)),x)
```

```
[Out] 2*arctan((b+2*a*tan(d*x+c))/(4*a^2-b^2)^(1/2))/d/(4*a^2-b^2)^(1/2)
```

maxima [**F(-2)**] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cos(d*x+c)*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(b^2-4*a^2>0)', see `assume?` for mo
re details)Is b^2-4*a^2 positive or negative?
```

mupad [**B**] time = 3.11, size = 44, normalized size = 0.92

$$\frac{2\,\mathrm{atan}\left(\frac{b+2\,a\tan(c+d\,x)}{\sqrt{4\,a^2-b^2}}\right)}{d\,\sqrt{4\,a^2-b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*cos(c + d*x)*sin(c + d*x)),x)
```

```
[Out] (2*atan((b + 2*a*tan(c + d*x))/(4*a^2 - b^2)^(1/2)))/(d*(4*a^2 - b^2)^(1/2)
)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cos(d*x+c)*sin(d*x+c)),x)
```

```
[Out] Timed out
```

3.571 $\int \frac{1}{(a+b\cos(c+dx)\sin(c+dx))^2}\,dx$

Optimal. Leaf size=95

$$\frac{8a\tan^{-1}\left(\frac{2a\tan(c+dx)+b}{\sqrt{4a^2-b^2}}\right)}{d\left(4a^2-b^2\right)^{3/2}}+\frac{2b\cos(2c+2dx)}{d\left(4a^2-b^2\right)(2a+b\sin(2c+2dx))}$$

```
[Out] 8*a*arctan((b+2*a*tan(d*x+c))/(4*a^2-b^2)^(1/2))/(4*a^2-b^2)^(3/2)/d+2*b*co
s(2*d*x+2*c)/(4*a^2-b^2)/d/(2*a+b*sin(2*d*x+2*c))
```

Rubi [A] time = 0.11, antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2666, 2664, 12, 2660, 618, 204}

$$\frac{8a\tan^{-1}\left(\frac{2a\tan(c+dx)+b}{\sqrt{4a^2-b^2}}\right)}{d\left(4a^2-b^2\right)^{3/2}}+\frac{2b\cos(2c+2dx)}{d\left(4a^2-b^2\right)(2a+b\sin(2c+2dx))}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x]*Sin[c + d*x])^(-2),x]
```

```
[Out] (8*a*ArcTan[(b + 2*a*Tan[c + d*x])/Sqrt[4*a^2 - b^2]])/((4*a^2 - b^2)^(3/2)
*d) + (2*b*Cos[2*c + 2*d*x])/((4*a^2 - b^2)*d*(2*a + b*Sin[2*c + 2*d*x]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2664

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[
c + d*x]*(a + b*Sin[c + d*x])^(n + 1))/(d*(n + 1)*(a^2 - b^2)), x] + Dist[1
/((n + 1)*(a^2 - b^2)), Int[(a + b*Sin[c + d*x])^(n + 1)*Simp[a*(n + 1) - b
*(n + 2)*Sin[c + d*x], x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^
2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2666

```
Int[((a_) + cos[(c_.) + (d_.)*(x_)]*(b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_
Symbol] :> Int[(a + (b*Sin[2*c + 2*d*x])/2)^n, x] /; FreeQ[{a, b, c, d, n},
 x]
```

Rubi steps

$$\int \frac{1}{(a+b\cos(c+dx)\sin(c+dx))^2}\,dx = \int \frac{1}{\left(a+\frac{1}{2}b\sin(2c+2dx)\right)^2}\,dx$$

$$= \frac{2b\cos(2c+2dx)}{\left(4a^2-b^2\right)d(2a+b\sin(2c+2dx))} + \frac{4\int \frac{a}{a+\frac{1}{2}b\sin(2c+2dx)}\,dx}{4a^2-b^2}$$

$$= \frac{2b\cos(2c+2dx)}{\left(4a^2-b^2\right)d(2a+b\sin(2c+2dx))} + \frac{(4a)\int \frac{1}{a+\frac{1}{2}b\sin(2c+2dx)}\,dx}{4a^2-b^2}$$

$$= \frac{2b\cos(2c+2dx)}{\left(4a^2-b^2\right)d(2a+b\sin(2c+2dx))} + \frac{(4a)\,\mathrm{Subst}\left(\int \frac{1}{a+bx+ax^2}\,dx, x, \tan\left(\frac{1}{2}\right.\right.}{\left(4a^2-b^2\right)d}$$

$$= \frac{2b\cos(2c+2dx)}{\left(4a^2-b^2\right)d(2a+b\sin(2c+2dx))} - \frac{(8a)\,\mathrm{Subst}\left(\int \frac{1}{-4a^2+b^2-x^2}\,dx, x, b+2\right.}{\left(4a^2-b^2\right)d}$$

$$= \frac{8a\tan^{-1}\left(\frac{b+2a\tan(c+dx)}{\sqrt{4a^2-b^2}}\right)}{\left(4a^2-b^2\right)^{3/2}d} + \frac{2b\cos(2c+2dx)}{\left(4a^2-b^2\right)d(2a+b\sin(2c+2dx))}$$

Mathematica [A] time = 0.41, size = 94, normalized size = 0.99

$$\frac{2\left(\frac{4a\tan^{-1}\left(\frac{2a\tan(c+dx)+b}{\sqrt{4a^2-b^2}}\right)}{(4a^2-b^2)^{3/2}}+\frac{b\cos(2(c+dx))}{(2a-b)(2a+b)(2a+b\sin(2(c+dx)))}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x]*Sin[c + d*x])^(-2),x]
```

```
[Out] (2*((4*a*ArcTan[(b + 2*a*Tan[c + d*x])/Sqrt[4*a^2 - b^2]])/(4*a^2 - b^2)^(3
/2) + (b*Cos[2*(c + d*x)])/((2*a - b)*(2*a + b)*(2*a + b*Sin[2*(c + d*x)]))
))/d
```

fricas [B] time = 1.45, size = 493, normalized size = 5.19

$$\left[-\frac{4\,a^2b-b^3-2\left(4\,a^2b-b^3\right)\cos\left(dx+c\right)^2-2\left(ab\cos\left(dx+c\right)\sin\left(dx+c\right)+a^2\right)\sqrt{-4\,a^2+b^2}\,\log\left(\frac{2\left(8\,a^2-b^2\right)\cos(dx}{}\right.}{\left(16\,a^4b-8\,a^2b^3+b^5\right)d\cos\left(dx+c\right)\sin}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cos(d*x+c)*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] [-(4*a^2*b - b^3 - 2*(4*a^2*b - b^3)*cos(d*x + c)^2 - 2*(a*b*cos(d*x + c)*s
in(d*x + c) + a^2)*sqrt(-4*a^2 + b^2)*log((2*(8*a^2 - b^2)*cos(d*x + c)^4 -
 4*a*b*cos(d*x + c)*sin(d*x + c) - 2*(8*a^2 - b^2)*cos(d*x + c)^2 + 2*a^2 -
 b^2 - (2*b*cos(d*x + c)^2 + 4*(2*a*cos(d*x + c)^3 - a*cos(d*x + c))*sin(d*
x + c) - b)*sqrt(-4*a^2 + b^2))/(b^2*cos(d*x + c)^4 - b^2*cos(d*x + c)^2 -
2*a*b*cos(d*x + c)*sin(d*x + c) - a^2)))/((16*a^4*b - 8*a^2*b^3 + b^5)*d*co
s(d*x + c)*sin(d*x + c) + (16*a^5 - 8*a^3*b^2 + a*b^4)*d), -(4*a^2*b - b^3
- 2*(4*a^2*b - b^3)*cos(d*x + c)^2 + 4*(a*b*cos(d*x + c)*sin(d*x + c) + a^2
)*sqrt(4*a^2 - b^2)*arctan(-(4*a*cos(d*x + c)*sin(d*x + c) + b)*sqrt(4*a^2
- b^2)/(2*(4*a^2 - b^2)*cos(d*x + c)^2 - 4*a^2 + b^2)))/((16*a^4*b - 8*a^2*
b^3 + b^5)*d*cos(d*x + c)*sin(d*x + c) + (16*a^5 - 8*a^3*b^2 + a*b^4)*d)]
```

giac [A] time = 0.18, size = 116, normalized size = 1.22

$$\frac{\frac{8\left(\pi\left\lfloor\frac{dx+c}{\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(a)+\arctan\left(\frac{2\,a\tan(dx+c)+b}{\sqrt{4\,a^2-b^2}}\right)\right)a}{\left(4\,a^2-b^2\right)^{\frac{3}{2}}}+\frac{b^2\tan(dx+c)+2\,ab}{\left(4\,a^3-ab^2\right)\left(a\tan(dx+c)^2+b\tan(dx+c)+a\right)}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cos(d*x+c)*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] (8*(pi*floor((d*x + c)/pi + 1/2)*sgn(a) + arctan((2*a*tan(d*x + c) + b)/sqr
t(4*a^2 - b^2)))*a/(4*a^2 - b^2)^(3/2) + (b^2*tan(d*x + c) + 2*a*b)/((4*a^3
 - a*b^2)*(a*tan(d*x + c)^2 + b*tan(d*x + c) + a)))/d
```

maple [A] time = 0.44, size = 139, normalized size = 1.46

$$\frac{b^2\tan(dx+c)}{d\left(\left(\tan^2(dx+c)\right)a+b\tan(dx+c)+a\right)a\left(4a^2-b^2\right)}+\frac{2b}{d\left(\left(\tan^2(dx+c)\right)a+b\tan(dx+c)+a\right)\left(4a^2-b^2\right)}+\frac{8a\,\mathrm{ar}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*cos(d*x+c)*sin(d*x+c))^2,x)
```

```
[Out] 1/d/(tan(d*x+c)^2*a+b*tan(d*x+c)+a)*b^2/a/(4*a^2-b^2)*tan(d*x+c)+2/d/(tan(d
*x+c)^2*a+b*tan(d*x+c)+a)*b/(4*a^2-b^2)+8*a*arctan((b+2*a*tan(d*x+c))/(4*a^
2-b^2)^(1/2))/(4*a^2-b^2)^(3/2)/d
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cos(d*x+c)*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(b^2-4*a^2>0)', see `assume?` for mo
re details)Is b^2-4*a^2 positive or negative?
```

mupad [B] time = 3.05, size = 181, normalized size = 1.91

$$\frac{\frac{2\,b}{4\,a^2-b^2}+\frac{b^2\tan(c+d\,x)}{a\left(4\,a^2-b^2\right)}}{d\,\left(a\tan(c+d\,x)^2+b\tan(c+d\,x)+a\right)}+\frac{8\,a\,\mathrm{atan}\left(\frac{\left(4\,a^2-b^2\right)\left(\frac{8\,a^2\tan(c+d\,x)}{(2\,a+b)^{3/2}\,(2\,a-b)^{3/2}}+\frac{4\,a\left(4\,a^2\,b-b^3\right)}{(2\,a+b)^{3/2}\left(4\,a^2-b^2\right)(2\,a-b)^{3/2}}\right)}{4\,a}\right)}{d\,(2\,a+b)^{3/2}\,(2\,a-b)^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*cos(c + d*x)*sin(c + d*x))^2,x)
```

```
[Out] ((2*b)/(4*a^2 - b^2) + (b^2*tan(c + d*x))/(a*(4*a^2 - b^2)))/(d*(a + b*tan(
c + d*x) + a*tan(c + d*x)^2)) + (8*a*atan(((4*a^2 - b^2)*((8*a^2*tan(c + d*
```

```
x))/((2*a + b)^(3/2)*(2*a - b)^(3/2)) + (4*a*(4*a^2*b - b^3))/((2*a + b)^(3
/2)*(4*a^2 - b^2)*(2*a - b)^(3/2))))/(4*a)))/(d*(2*a + b)^(3/2)*(2*a - b)^(
3/2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cos(d*x+c)*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

3.572 $\int \frac{1}{(a+b\cos(c+dx)\sin(c+dx))^3} dx$

Optimal. Leaf size=149

$$\frac{4\left(8a^2+b^2\right)\tan^{-1}\left(\frac{2a\tan(c+dx)+b}{\sqrt{4a^2-b^2}}\right)}{d\left(4a^2-b^2\right)^{5/2}}+\frac{12ab\cos(2c+2dx)}{d\left(4a^2-b^2\right)^2(2a+b\sin(2c+2dx))}+\frac{2b\cos(2c+2dx)}{d\left(4a^2-b^2\right)(2a+b\sin(2c+2dx))^2}$$

```
[Out] 4*(8*a^2+b^2)*arctan((b+2*a*tan(d*x+c))/(4*a^2-b^2)^(1/2))/(4*a^2-b^2)^(5/2
)/d+2*b*cos(2*d*x+2*c)/(4*a^2-b^2)/d/(2*a+b*sin(2*d*x+2*c))^2+12*a*b*cos(2*
d*x+2*c)/(4*a^2-b^2)^2/d/(2*a+b*sin(2*d*x+2*c))
```

Rubi [A] time = 0.18, antiderivative size = 149, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.389, Rules used = {2666, 2664, 2754, 12, 2660, 618, 204}

$$\frac{4\left(8a^2+b^2\right)\tan^{-1}\left(\frac{2a\tan(c+dx)+b}{\sqrt{4a^2-b^2}}\right)}{d\left(4a^2-b^2\right)^{5/2}}+\frac{12ab\cos(2c+2dx)}{d\left(4a^2-b^2\right)^2(2a+b\sin(2c+2dx))}+\frac{2b\cos(2c+2dx)}{d\left(4a^2-b^2\right)(2a+b\sin(2c+2dx))^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x]*Sin[c + d*x])^(-3),x]
```

```
[Out] (4*(8*a^2 + b^2)*ArcTan[(b + 2*a*Tan[c + d*x])/Sqrt[4*a^2 - b^2]])/((4*a^2
- b^2)^(5/2)*d) + (2*b*Cos[2*c + 2*d*x])/((4*a^2 - b^2)*d*(2*a + b*Sin[2*c
+ 2*d*x])^2) + (12*a*b*Cos[2*c + 2*d*x])/((4*a^2 - b^2)^2*d*(2*a + b*Sin[2*
c + 2*d*x]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
```

```
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2664

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[
c + d*x]*(a + b*Sin[c + d*x])^(n + 1))/(d*(n + 1)*(a^2 - b^2)), x] + Dist[1
/((n + 1)*(a^2 - b^2)), Int[(a + b*Sin[c + d*x])^(n + 1)*Simp[a*(n + 1) - b
*(n + 2)*Sin[c + d*x], x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^
2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2666

```
Int[((a_) + cos[(c_.) + (d_.)*(x_)]*(b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_
Symbol] :> Int[(a + (b*Sin[2*c + 2*d*x])/2)^n, x] /; FreeQ[{a, b, c, d, n},
 x]
```

Rule 2754

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e + f
*x])^(m + 1))/(f*(m + 1)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(a^2 - b^2)), I
nt[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m +
 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a
*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rubi steps

$$\int \frac{1}{(a+b\cos(c+dx)\sin(c+dx))^3}\,dx = \int \frac{1}{\left(a+\frac{1}{2}b\sin(2c+2dx)\right)^3}\,dx$$

$$= \frac{2b\cos(2c+2dx)}{\left(4a^2-b^2\right)d(2a+b\sin(2c+2dx))^2} - \frac{2\int \frac{-2a+\frac{1}{2}b\sin(2c+2dx)}{\left(a+\frac{1}{2}b\sin(2c+2dx)\right)^2}dx}{4a^2-b^2}$$

$$= \frac{2b\cos(2c+2dx)}{\left(4a^2-b^2\right)d(2a+b\sin(2c+2dx))^2} + \frac{12ab\cos(2c+2dx)}{\left(4a^2-b^2\right)^2 d(2a+b\sin(2c+2dx)}$$

$$= \frac{2b\cos(2c+2dx)}{\left(4a^2-b^2\right)d(2a+b\sin(2c+2dx))^2} + \frac{12ab\cos(2c+2dx)}{\left(4a^2-b^2\right)^2 d(2a+b\sin(2c+2dx)}$$

$$= \frac{2b\cos(2c+2dx)}{\left(4a^2-b^2\right)d(2a+b\sin(2c+2dx))^2} + \frac{12ab\cos(2c+2dx)}{\left(4a^2-b^2\right)^2 d(2a+b\sin(2c+2dx)}$$

$$= \frac{2b\cos(2c+2dx)}{\left(4a^2-b^2\right)d(2a+b\sin(2c+2dx))^2} + \frac{12ab\cos(2c+2dx)}{\left(4a^2-b^2\right)^2 d(2a+b\sin(2c+2dx)}$$

$$= \frac{4\left(8a^2+b^2\right)\tan^{-1}\left(\frac{b+2a\tan(c+dx)}{\sqrt{4a^2-b^2}}\right)}{\left(4a^2-b^2\right)^{5/2}d} + \frac{2b\cos(2c+2dx)}{\left(4a^2-b^2\right)d(2a+b\sin(2c+2dx))^2}$$

Mathematica [A] time = 0.93, size = 120, normalized size = 0.81

$$\frac{2\left(\frac{2\left(8a^2+b^2\right)\tan^{-1}\left(\frac{2a\tan(c+dx)+b}{\sqrt{4a^2-b^2}}\right)}{\left(4a^2-b^2\right)^{5/2}} + \frac{b\cos(2(c+dx))\left(16a^2+6ab\sin(2(c+dx))-b^2\right)}{\left(b^2-4a^2\right)^2(2a+b\sin(2(c+dx)))^2}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x]*Sin[c + d*x])^(-3),x]
```

```
[Out] (2*((2*(8*a^2 + b^2)*ArcTan[(b + 2*a*Tan[c + d*x])/Sqrt[4*a^2 - b^2]])/(4*a
^2 - b^2)^(5/2) + (b*Cos[2*(c + d*x)]*(16*a^2 - b^2 + 6*a*b*Sin[2*(c + d*x)
]))/((-4*a^2 + b^2)^2*(2*a + b*Sin[2*(c + d*x)])^2)))/d
```

fricas [B] time = 1.06, size = 969, normalized size = 6.50

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cos(d*x+c)*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] [1/2*(64*a^4*b - 20*a^2*b^3 + b^5 - 2*(64*a^4*b - 20*a^2*b^3 + b^5)*cos(d*x
 + c)^2 - 2*((8*a^2*b^2 + b^4)*cos(d*x + c)^4 - 8*a^4 - a^2*b^2 - (8*a^2*b^
2 + b^4)*cos(d*x + c)^2 - 2*(8*a^3*b + a*b^3)*cos(d*x + c)*sin(d*x + c))*sq
rt(-4*a^2 + b^2)*log(-(2*(8*a^2 - b^2)*cos(d*x + c)^4 - 4*a*b*cos(d*x + c)*
sin(d*x + c) - 2*(8*a^2 - b^2)*cos(d*x + c)^2 + 2*a^2 - b^2 + (2*b*cos(d*x
+ c)^2 + 4*(2*a*cos(d*x + c)^3 - a*cos(d*x + c))*sin(d*x + c) - b)*sqrt(-4*
a^2 + b^2))/(b^2*cos(d*x + c)^4 - b^2*cos(d*x + c)^2 - 2*a*b*cos(d*x + c)*s
in(d*x + c) - a^2)) - 12*(2*(4*a^3*b^2 - a*b^4)*cos(d*x + c)^3 - (4*a^3*b^2
 - a*b^4)*cos(d*x + c))*sin(d*x + c))/((64*a^6*b^2 - 48*a^4*b^4 + 12*a^2*b^
6 - b^8)*d*cos(d*x + c)^4 - (64*a^6*b^2 - 48*a^4*b^4 + 12*a^2*b^6 - b^8)*d*
cos(d*x + c)^2 - 2*(64*a^7*b - 48*a^5*b^3 + 12*a^3*b^5 - a*b^7)*d*cos(d*x +
 c)*sin(d*x + c) - (64*a^8 - 48*a^6*b^2 + 12*a^4*b^4 - a^2*b^6)*d), 1/2*(64
*a^4*b - 20*a^2*b^3 + b^5 - 2*(64*a^4*b - 20*a^2*b^3 + b^5)*cos(d*x + c)^2
- 4*((8*a^2*b^2 + b^4)*cos(d*x + c)^4 - 8*a^4 - a^2*b^2 - (8*a^2*b^2 + b^4)
*cos(d*x + c)^2 - 2*(8*a^3*b + a*b^3)*cos(d*x + c)*sin(d*x + c))*sqrt(4*a^2
 - b^2)*arctan(-(4*a*cos(d*x + c)*sin(d*x + c) + b)*sqrt(4*a^2 - b^2)/(2*(4
*a^2 - b^2)*cos(d*x + c)^2 - 4*a^2 + b^2)) - 12*(2*(4*a^3*b^2 - a*b^4)*cos(
d*x + c)^3 - (4*a^3*b^2 - a*b^4)*cos(d*x + c))*sin(d*x + c))/((64*a^6*b^2 -
 48*a^4*b^4 + 12*a^2*b^6 - b^8)*d*cos(d*x + c)^4 - (64*a^6*b^2 - 48*a^4*b^4
 + 12*a^2*b^6 - b^8)*d*cos(d*x + c)^2 - 2*(64*a^7*b - 48*a^5*b^3 + 12*a^3*b
^5 - a*b^7)*d*cos(d*x + c)*sin(d*x + c) - (64*a^8 - 48*a^6*b^2 + 12*a^4*b^4
 - a^2*b^6)*d)]
```

giac [A] time = 0.19, size = 252, normalized size = 1.69

$$\frac{\frac{8\left(\pi\left\lfloor\frac{dx+c}{\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(a)+\arctan\left(\frac{2a\tan(dx+c)+b}{\sqrt{4a^2-b^2}}\right)\right)(8a^2+b^2)}{(16a^4-8a^2b^2+b^4)\sqrt{4a^2-b^2}}+\frac{20a^3b^2\tan(dx+c)^3-2ab^4\tan(dx+c)^3+32a^4b\tan(dx+c)^2+14a^2b^3\tan(dx+c)^2-b^5\tan(dx}{(16a^6-8a^4b^2+a^2b^4)\left(a\tan(dx+c)^2+b\tan(\right.}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cos(d*x+c)*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/2*(8*(pi*floor((d*x + c)/pi + 1/2)*sgn(a) + arctan((2*a*tan(d*x + c) + b)
/sqrt(4*a^2 - b^2)))*(8*a^2 + b^2)/((16*a^4 - 8*a^2*b^2 + b^4)*sqrt(4*a^2 -
 b^2)) + (20*a^3*b^2*tan(d*x + c)^3 - 2*a*b^4*tan(d*x + c)^3 + 32*a^4*b*tan
(d*x + c)^2 + 14*a^2*b^3*tan(d*x + c)^2 - b^5*tan(d*x + c)^2 + 44*a^3*b^2*t
```

```
an(d*x + c) - 2*a*b^4*tan(d*x + c) + 32*a^4*b - 2*a^2*b^3)/((16*a^6 - 8*a^4
*b^2 + a^2*b^4)*(a*tan(d*x + c)^2 + b*tan(d*x + c) + a)^2))/d
```

maple [B] time = 0.49, size = 640, normalized size = 4.30

$$\frac{10b^2a\left(\tan^3(dx+c)\right)}{d\left(\left(\tan^2(dx+c)\right)a+b\tan(dx+c)+a\right)^2\left(16a^4-8a^2b^2+b^4\right)}-\frac{b^4\left(\tan^3(dx+c)\right)}{d\left(\left(\tan^2(dx+c)\right)a+b\tan(dx+c)+a\right)^2\left(16a^4-\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*cos(d*x+c)*sin(d*x+c))^3,x)
```

```
[Out] 10/d/(tan(d*x+c)^2*a+b*tan(d*x+c)+a)^2*b^2/(16*a^4-8*a^2*b^2+b^4)*a*tan(d*x
+c)^3-1/d/(tan(d*x+c)^2*a+b*tan(d*x+c)+a)^2*b^4/(16*a^4-8*a^2*b^2+b^4)/a*ta
n(d*x+c)^3+16/d/(tan(d*x+c)^2*a+b*tan(d*x+c)+a)^2*b/(16*a^4-8*a^2*b^2+b^4)*
a^2*tan(d*x+c)^2+7/d/(tan(d*x+c)^2*a+b*tan(d*x+c)+a)^2*b^3/(16*a^4-8*a^2*b^
2+b^4)*tan(d*x+c)^2-1/2/d/(tan(d*x+c)^2*a+b*tan(d*x+c)+a)^2*b^5/(16*a^4-8*a
^2*b^2+b^4)/a^2*tan(d*x+c)^2+22/d/(tan(d*x+c)^2*a+b*tan(d*x+c)+a)^2*b^2*a/(
16*a^4-8*a^2*b^2+b^4)*tan(d*x+c)-1/d/(tan(d*x+c)^2*a+b*tan(d*x+c)+a)^2*b^4/
a/(16*a^4-8*a^2*b^2+b^4)*tan(d*x+c)+16/d/(tan(d*x+c)^2*a+b*tan(d*x+c)+a)^2*
b/(16*a^4-8*a^2*b^2+b^4)*a^2-1/d/(tan(d*x+c)^2*a+b*tan(d*x+c)+a)^2*b^3/(16*
a^4-8*a^2*b^2+b^4)+32/d/(16*a^4-8*a^2*b^2+b^4)/(4*a^2-b^2)^(1/2)*arctan((b+
2*a*tan(d*x+c))/(4*a^2-b^2)^(1/2))*a^2+4/d/(16*a^4-8*a^2*b^2+b^4)/(4*a^2-b^
2)^(1/2)*arctan((b+2*a*tan(d*x+c))/(4*a^2-b^2)^(1/2))*b^2
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cos(d*x+c)*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(b^2-4*a^2>0)', see `assume?` for mo
re details)Is b^2-4*a^2 positive or negative?
```

mupad [B] time = 3.87, size = 396, normalized size = 2.66

$$\frac{\frac{16\,a^2\,b-b^3}{16\,a^4-8\,a^2\,b^2+b^4}+\frac{b\tan(c+d\,x)\left(22\,a^2\,b-b^3\right)}{a\left(16\,a^4-8\,a^2\,b^2+b^4\right)}+\frac{\tan(c+d\,x)^2\left(16\,a^2\,b-b^3\right)\left(2\,a^2+b^2\right)}{2\,a^2\left(16\,a^4-8\,a^2\,b^2+b^4\right)}+\frac{b\tan(c+d\,x)^3\left(10\,a^2\,b-b^3\right)}{a\left(16\,a^4-8\,a^2\,b^2+b^4\right)}}{d\,\left(\tan(c+d\,x)^2\,\left(2\,a^2+b^2\right)+a^2+a^2\tan(c+d\,x)^4+2\,a\,b\tan(c+d\,x)+2\,a\,b\tan(c+d\,x)^3\right)}+\frac{4\,\mathrm{atan}\left(\left(\frac{4\,a\tan(c+d\,x}{(2\,a+b)^{5/2}}\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*cos(c + d*x)*sin(c + d*x))^3,x)
```

```
[Out] ((16*a^2*b - b^3)/(16*a^4 + b^4 - 8*a^2*b^2) + (b*tan(c + d*x)*(22*a^2*b -
b^3))/(a*(16*a^4 + b^4 - 8*a^2*b^2)) + (tan(c + d*x)^2*(16*a^2*b - b^3)*(2*
a^2 + b^2))/(2*a^2*(16*a^4 + b^4 - 8*a^2*b^2)) + (b*tan(c + d*x)^3*(10*a^2*
b - b^3))/(a*(16*a^4 + b^4 - 8*a^2*b^2)))/(d*(tan(c + d*x)^2*(2*a^2 + b^2)
+ a^2 + a^2*tan(c + d*x)^4 + 2*a*b*tan(c + d*x) + 2*a*b*tan(c + d*x)^3)) +
(4*atan((((4*a*tan(c + d*x)*(8*a^2 + b^2))/((2*a + b)^(5/2)*(2*a - b)^(5/2)
) + (2*(8*a^2 + b^2)*(16*a^4*b + b^5 - 8*a^2*b^3))/((2*a + b)^(5/2)*(2*a -
b)^(5/2)*(16*a^4 + b^4 - 8*a^2*b^2)))*(16*a^4 + b^4 - 8*a^2*b^2))/(16*a^2 +
 2*b^2))*(8*a^2 + b^2))/(d*(2*a + b)^(5/2)*(2*a - b)^(5/2))
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cos(d*x+c)*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

3.573 $\int (a + b\cos(c + dx)\sin(c + dx))^{5/2}\,dx$

Optimal. Leaf size=265

$$-\frac{2\sqrt{2}\,a\left(4a^2-b^2\right)\sqrt{\frac{2a+b\sin(2c+2dx)}{2a+b}}\,F\left(c+dx-\frac{\pi}{4}|\frac{2b}{2a+b}\right)}{15d\sqrt{2a+b\sin(2c+2dx)}}+\frac{\left(92a^2+9b^2\right)\sqrt{2a+b\sin(2c+2dx)}\,E\left(c+dx-\frac{\pi}{4}|\frac{2b}{2a+b}\right)}{60\sqrt{2}\,d\sqrt{\frac{2a+b\sin(2c+2dx)}{2a+b}}}$$

```
[Out] -1/40*b*cos(2*d*x+2*c)*(2*a+b*sin(2*d*x+2*c))^(3/2)/d*2^(1/2)-2/15*a*b*cos(
2*d*x+2*c)*2^(1/2)*(2*a+b*sin(2*d*x+2*c))^(1/2)/d-1/120*(92*a^2+9*b^2)*(sin
(c+1/4*Pi+d*x)^2)^(1/2)/sin(c+1/4*Pi+d*x)*EllipticE(cos(c+1/4*Pi+d*x),2^(1/
2)*(b/(2*a+b))^(1/2))*(2*a+b*sin(2*d*x+2*c))^(1/2)/d*2^(1/2)/((2*a+b*sin(2*
d*x+2*c))/(2*a+b))^(1/2)+2/15*a*(4*a^2-b^2)*(sin(c+1/4*Pi+d*x)^2)^(1/2)/sin
(c+1/4*Pi+d*x)*EllipticF(cos(c+1/4*Pi+d*x),2^(1/2)*(b/(2*a+b))^(1/2))*2^(1/
2)*((2*a+b*sin(2*d*x+2*c))/(2*a+b))^(1/2)/d/(2*a+b*sin(2*d*x+2*c))^(1/2)
```

Rubi [A] time = 0.37, antiderivative size = 265, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {2666, 2656, 2753, 2752, 2663, 2661, 2655, 2653}

$$-\frac{2\sqrt{2}\,a\left(4a^2-b^2\right)\sqrt{\frac{2a+b\sin(2c+2dx)}{2a+b}}\,F\left(c+dx-\frac{\pi}{4}|\frac{2b}{2a+b}\right)}{15d\sqrt{2a+b\sin(2c+2dx)}}+\frac{\left(92a^2+9b^2\right)\sqrt{2a+b\sin(2c+2dx)}\,E\left(c+dx-\frac{\pi}{4}|\frac{2b}{2a+b}\right)}{60\sqrt{2}\,d\sqrt{\frac{2a+b\sin(2c+2dx)}{2a+b}}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x]*Sin[c + d*x])^(5/2),x]
```

```
[Out] (-2*Sqrt[2]*a*b*Cos[2*c + 2*d*x]*Sqrt[2*a + b*Sin[2*c + 2*d*x]])/(15*d) - (
b*Cos[2*c + 2*d*x]*(2*a + b*Sin[2*c + 2*d*x])^(3/2))/(20*Sqrt[2]*d) + ((92*
a^2 + 9*b^2)*EllipticE[c - Pi/4 + d*x, (2*b)/(2*a + b)]*Sqrt[2*a + b*Sin[2*
c + 2*d*x]])/(60*Sqrt[2]*d*Sqrt[(2*a + b*Sin[2*c + 2*d*x])/(2*a + b)]) - (2
*Sqrt[2]*a*(4*a^2 - b^2)*EllipticF[c - Pi/4 + d*x, (2*b)/(2*a + b)]*Sqrt[(2
*a + b*Sin[2*c + 2*d*x])/(2*a + b)])/(15*d*Sqrt[2*a + b*Sin[2*c + 2*d*x]])
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
```

```
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2656

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[
c + d*x]*(a + b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[1/n, Int[(a + b*Sin
[c + d*x])^(n - 2)*Simp[a^2*n + b^2*(n - 1) + a*b*(2*n - 1)*Sin[c + d*x], x
], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[n, 1] && In
tegerQ[2*n]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2666

```
Int[((a_) + cos[(c_.) + (d_.)*(x_)]*(b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_
Symbol] :> Int[(a + (b*Sin[2*c + 2*d*x])/2)^n, x] /; FreeQ[{a, b, c, d, n},
 x]
```

Rule 2752

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2753

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[1/(m + 1), Int[(a + b*Sin[e + f*x])^(m - 1)*Simp[b*d*m
 + a*c*(m + 1) + (a*d*m + b*c*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a
, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && GtQ[m, 0]
```

```
&& IntegerQ[2*m]
```

Rubi steps

$$\int (a+b\cos(c+dx)\sin(c+dx))^{5/2}\,dx = \int \left(a+\frac{1}{2}b\sin(2c+2dx)\right)^{5/2}\,dx$$

$$= -\frac{b\cos(2c+2dx)(2a+b\sin(2c+2dx))^{3/2}}{20\sqrt{2}\,d}+\frac{2}{5}\int\sqrt{a+\frac{1}{2}b\sin(2c+2d}$$

$$= -\frac{2\sqrt{2}\,ab\cos(2c+2dx)\sqrt{2a+b\sin(2c+2dx)}}{15d}-\frac{b\cos(2c+2dx)(2a+b}{20\sqrt{2}}$$

$$= -\frac{2\sqrt{2}\,ab\cos(2c+2dx)\sqrt{2a+b\sin(2c+2dx)}}{15d}-\frac{b\cos(2c+2dx)(2a+b}{20\sqrt{2}}$$

$$= -\frac{2\sqrt{2}\,ab\cos(2c+2dx)\sqrt{2a+b\sin(2c+2dx)}}{15d}-\frac{b\cos(2c+2dx)(2a+b}{20\sqrt{2}}$$

$$= -\frac{2\sqrt{2}\,ab\cos(2c+2dx)\sqrt{2a+b\sin(2c+2dx)}}{15d}-\frac{b\cos(2c+2dx)(2a+b}{20\sqrt{2}}$$

Mathematica [A] time = 1.85, size = 202, normalized size = 0.76

$$\frac{-32a\left(4a^2-b^2\right)\sqrt{\frac{2a+b\sin(2(c+dx))}{2a+b}}F\left(c+dx-\frac{\pi}{4}|\frac{2b}{2a+b}\right)-b\left(88a^2\cos(2(c+dx))+b\sin(4(c+dx))(28a+3b\sin(2(c\right.}{120d\sqrt{4a+2b\sin(2(c+dx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x]*Sin[c + d*x])^(5/2),x]
```

```
[Out] (2*(184*a^3 + 92*a^2*b + 18*a*b^2 + 9*b^3)*EllipticE[c - Pi/4 + d*x, (2*b)/
(2*a + b)]*Sqrt[(2*a + b*Sin[2*(c + d*x)])/(2*a + b)] - 32*a*(4*a^2 - b^2)*
EllipticF[c - Pi/4 + d*x, (2*b)/(2*a + b)]*Sqrt[(2*a + b*Sin[2*(c + d*x)])/
(2*a + b)] - b*(88*a^2*Cos[2*(c + d*x)] + b*(28*a + 3*b*Sin[2*(c + d*x)])*S
in[4*(c + d*x)]))/(120*d*Sqrt[4*a + 2*b*Sin[2*(c + d*x)]])
```

fricas [F] time = 2.71, size = 0, normalized size = 0.00

$$\text{integral}\left(-\left(b^2\cos\left(dx+c\right)^4-b^2\cos\left(dx+c\right)^2-2\,ab\cos\left(dx+c\right)\sin\left(dx+c\right)-a^2\right)\sqrt{b\cos\left(dx+c\right)\sin\left(dx+c\right)\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c)*sin(d*x+c))^(5/2),x, algorithm="fricas")
[Out] integral(-(b^2*cos(d*x + c)^4 - b^2*cos(d*x + c)^2 - 2*a*b*cos(d*x + c)*sin
(d*x + c) - a^2)*sqrt(b*cos(d*x + c)*sin(d*x + c) + a), x)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c)*sin(d*x+c))^(5/2),x, algorithm="giac")
[Out] Timed out
```

maple [B] time = 0.52, size = 1138, normalized size = 4.29

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c)*sin(d*x+c))^(5/2),x)
[Out] 1/60*(240*a^4*((2*a+b*sin(2*d*x+2*c))/(2*a-b))^(1/2)*(-(sin(2*d*x+2*c)-1)*b
/(2*a+b))^(1/2)*(-(1+sin(2*d*x+2*c))*b/(2*a-b))^(1/2)*EllipticF(((2*a+b*sin
(2*d*x+2*c))/(2*a-b))^(1/2),((2*a-b)/(2*a+b))^(1/2))+64*(-(sin(2*d*x+2*c)-1
)*b/(2*a+b))^(1/2)*(-(1+sin(2*d*x+2*c))*b/(2*a-b))^(1/2)*((2*a+b*sin(2*d*x+
2*c))/(2*a-b))^(1/2)*EllipticF(((2*a+b*sin(2*d*x+2*c))/(2*a-b))^(1/2),((2*a
-b)/(2*a+b))^(1/2))*a^3*b-24*a^2*((2*a+b*sin(2*d*x+2*c))/(2*a-b))^(1/2)*(-(
sin(2*d*x+2*c)-1)*b/(2*a+b))^(1/2)*(-(1+sin(2*d*x+2*c))*b/(2*a-b))^(1/2)*El
lipticF(((2*a+b*sin(2*d*x+2*c))/(2*a-b))^(1/2),((2*a-b)/(2*a+b))^(1/2))*b^2
-16*a*((2*a+b*sin(2*d*x+2*c))/(2*a-b))^(1/2)*(-(sin(2*d*x+2*c)-1)*b/(2*a+b)
)^(1/2)*(-(1+sin(2*d*x+2*c))*b/(2*a-b))^(1/2)*EllipticF(((2*a+b*sin(2*d*x+2
*c))/(2*a-b))^(1/2),((2*a-b)/(2*a+b))^(1/2))*b^3-9*(-(sin(2*d*x+2*c)-1)*b/(
2*a+b))^(1/2)*(-(1+sin(2*d*x+2*c))*b/(2*a-b))^(1/2)*((2*a+b*sin(2*d*x+2*c))
/(2*a-b))^(1/2)*EllipticF(((2*a+b*sin(2*d*x+2*c))/(2*a-b))^(1/2),((2*a-b)/(
2*a+b))^(1/2))*b^4-368*(-(sin(2*d*x+2*c)-1)*b/(2*a+b))^(1/2)*(-(1+sin(2*d*x
+2*c))*b/(2*a-b))^(1/2)*((2*a+b*sin(2*d*x+2*c))/(2*a-b))^(1/2)*EllipticE(((
2*a+b*sin(2*d*x+2*c))/(2*a-b))^(1/2),((2*a-b)/(2*a+b))^(1/2))*a^4+56*(-(sin
(2*d*x+2*c)-1)*b/(2*a+b))^(1/2)*(-(1+sin(2*d*x+2*c))*b/(2*a-b))^(1/2)*((2*a
+b*sin(2*d*x+2*c))/(2*a-b))^(1/2)*EllipticE(((2*a+b*sin(2*d*x+2*c))/(2*a-b)
```

```
)^(1/2),((2*a-b)/(2*a+b))^(1/2))*a^2*b^2+9*(-(sin(2*d*x+2*c)-1)*b/(2*a+b))^
(1/2)*(-(1+sin(2*d*x+2*c))*b/(2*a-b))^(1/2)*((2*a+b*sin(2*d*x+2*c))/(2*a-b)
)^(1/2)*EllipticE(((2*a+b*sin(2*d*x+2*c))/(2*a-b))^(1/2),((2*a-b)/(2*a+b))^
(1/2))*b^4+3*b^4*sin(2*d*x+2*c)^4+28*a*b^3*sin(2*d*x+2*c)^3+44*a^2*b^2*sin(
2*d*x+2*c)^2-3*b^4*sin(2*d*x+2*c)^2-28*sin(2*d*x+2*c)*a*b^3-44*a^2*b^2)/b/c
os(2*d*x+2*c)/(4*a+2*b*sin(2*d*x+2*c))^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b\cos(dx+c)\sin(dx+c)+a)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c)*sin(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((b*cos(d*x + c)*sin(d*x + c) + a)^(5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int (a+b\,\cos(c+d\,x)\,\sin(c+d\,x))^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*cos(c + d*x)*sin(c + d*x))^(5/2),x)
```

```
[Out] int((a + b*cos(c + d*x)*sin(c + d*x))^(5/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c)*sin(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

3.574 $\int (a+b\cos(c+dx)\sin(c+dx))^{3/2}\,dx$

Optimal. Leaf size=212

$$-\frac{\left(4a^2-b^2\right)\sqrt{\frac{2a+b\sin(2c+2dx)}{2a+b}}\,F\left(c+dx-\frac{\pi}{4}\middle|\frac{2b}{2a+b}\right)}{6\sqrt{2}\,d\sqrt{2a+b\sin(2c+2dx)}}-\frac{b\cos(2c+2dx)\sqrt{2a+b\sin(2c+2dx)}}{6\sqrt{2}\,d}+\frac{2\sqrt{2}\,a\sqrt{2a+b\sin(2c}}{3d\sqrt{2}}$$

```
[Out] -1/12*b*cos(2*d*x+2*c)*(2*a+b*sin(2*d*x+2*c))^(1/2)/d*2^(1/2)-2/3*a*(sin(c+
1/4*Pi+d*x)^2)^(1/2)/sin(c+1/4*Pi+d*x)*EllipticE(cos(c+1/4*Pi+d*x),2^(1/2)*
(b/(2*a+b))^(1/2))*2^(1/2)*(2*a+b*sin(2*d*x+2*c))^(1/2)/d/((2*a+b*sin(2*d*x
+2*c))/(2*a+b))^(1/2)+1/12*(4*a^2-b^2)*(sin(c+1/4*Pi+d*x)^2)^(1/2)/sin(c+1/
4*Pi+d*x)*EllipticF(cos(c+1/4*Pi+d*x),2^(1/2)*(b/(2*a+b))^(1/2))*((2*a+b*si
n(2*d*x+2*c))/(2*a+b))^(1/2)/d*2^(1/2)/(2*a+b*sin(2*d*x+2*c))^(1/2)
```

Rubi [A] time = 0.22, antiderivative size = 212, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.350, Rules used = {2666, 2656, 2752, 2663, 2661, 2655, 2653}

$$-\frac{\left(4a^2-b^2\right)\sqrt{\frac{2a+b\sin(2c+2dx)}{2a+b}}\,F\left(c+dx-\frac{\pi}{4}\middle|\frac{2b}{2a+b}\right)}{6\sqrt{2}\,d\sqrt{2a+b\sin(2c+2dx)}}-\frac{b\cos(2c+2dx)\sqrt{2a+b\sin(2c+2dx)}}{6\sqrt{2}\,d}+\frac{2\sqrt{2}\,a\sqrt{2a+b\sin(2c}}{3d\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x]*Sin[c + d*x])^(3/2),x]
```

```
[Out] -(b*Cos[2*c + 2*d*x]*Sqrt[2*a + b*Sin[2*c + 2*d*x]])/(6*Sqrt[2]*d) + (2*Sqr
t[2]*a*EllipticE[c - Pi/4 + d*x, (2*b)/(2*a + b)]*Sqrt[2*a + b*Sin[2*c + 2*
d*x]])/(3*d*Sqrt[(2*a + b*Sin[2*c + 2*d*x])/(2*a + b)]) - ((4*a^2 - b^2)*El
lipticF[c - Pi/4 + d*x, (2*b)/(2*a + b)]*Sqrt[(2*a + b*Sin[2*c + 2*d*x])/(2
*a + b)])/(6*Sqrt[2]*d*Sqrt[2*a + b*Sin[2*c + 2*d*x]])
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
```

```
0] &&  !GtQ[a + b, 0]
```

Rule 2656

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[
c + d*x]*(a + b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[1/n, Int[(a + b*Sin
[c + d*x])^(n - 2)*Simp[a^2*n + b^2*(n - 1) + a*b*(2*n - 1)*Sin[c + d*x], x
], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[n, 1] && In
tegerQ[2*n]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2666

```
Int[((a_) + cos[(c_.) + (d_.)*(x_)]*(b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_
Symbol] :> Int[(a + (b*Sin[2*c + 2*d*x])/2)^n, x] /; FreeQ[{a, b, c, d, n},
 x]
```

Rule 2752

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int (a+b\cos(c+dx)\sin(c+dx))^{3/2}\,dx = \int \left(a+\frac{1}{2}b\sin(2c+2dx)\right)^{3/2}\,dx$$

$$= -\frac{b\cos(2c+2dx)\sqrt{2a+b\sin(2c+2dx)}}{6\sqrt{2}\,d} + \frac{2}{3}\int \frac{\frac{1}{8}\left(12a^2+b^2\right)+ab\sin}{\sqrt{a+\frac{1}{2}b\sin(2c+}}$$

$$= -\frac{b\cos(2c+2dx)\sqrt{2a+b\sin(2c+2dx)}}{6\sqrt{2}\,d} + \frac{1}{3}(4a)\int \sqrt{a+\frac{1}{2}b\sin(2c+}$$

$$= -\frac{b\cos(2c+2dx)\sqrt{2a+b\sin(2c+2dx)}}{6\sqrt{2}\,d} + \frac{\left(4a\sqrt{a+\frac{1}{2}b\sin(2c+2dx)}\right.}{3\sqrt{\frac{a+\frac{1}{2}b\,s}{}}}$$

$$= -\frac{b\cos(2c+2dx)\sqrt{2a+b\sin(2c+2dx)}}{6\sqrt{2}\,d} + \frac{2\sqrt{2}\,aE\left(c-\frac{\pi}{4}+dx|\frac{2b}{2a+b}\right)}{3d\sqrt{\frac{2a+b\sin(}{2a-}}}$$

Mathematica [A] time = 1.46, size = 167, normalized size = 0.79

$$\frac{-\left(4a^2-b^2\right)\sqrt{\frac{2a+b\sin(2(c+dx))}{2a+b}}\,F\left(c+dx-\frac{\pi}{4}|\frac{2b}{2a+b}\right)-b\cos(2(c+dx))(2a+b\sin(2(c+dx)))+8a(2a+b)\sqrt{\frac{2a+b\sin}{2}}}{6d\sqrt{4a+2b\sin(2(c+dx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x]*Sin[c + d*x])^(3/2),x]
```

```
[Out] (-(b*Cos[2*(c + d*x)]*(2*a + b*Sin[2*(c + d*x)])) + 8*a*(2*a + b)*EllipticE
[c - Pi/4 + d*x, (2*b)/(2*a + b)]*Sqrt[(2*a + b*Sin[2*(c + d*x)])/(2*a + b)
] - (4*a^2 - b^2)*EllipticF[c - Pi/4 + d*x, (2*b)/(2*a + b)]*Sqrt[(2*a + b*
Sin[2*(c + d*x)])/(2*a + b)])/(6*d*Sqrt[4*a + 2*b*Sin[2*(c + d*x)]])
```

fricas [F] time = 2.98, size = 0, normalized size = 0.00

$$\text{integral}\left((b\cos(dx+c)\sin(dx+c)+a)^{\frac{3}{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c)*sin(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] integral((b*cos(d*x + c)*sin(d*x + c) + a)^(3/2), x)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c)*sin(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [B] time = 0.44, size = 844, normalized size = 3.98

$$24a^3\sqrt{\frac{2a+b\sin(2dx+2c)}{2a-b}}\,\sqrt{-\frac{(\sin(2dx+2c)-1)b}{2a+b}}\,\sqrt{-\frac{(1+\sin(2dx+2c))b}{2a-b}}\,\mathrm{EllipticF}\left(\sqrt{\frac{2a+b\sin(2dx+2c)}{2a-b}},\sqrt{\frac{2a-b}{2a+b}}\right)+4\sqrt{\frac{2a+b\sin(2dx+2c)}{2a-b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c)*sin(d*x+c))^(3/2),x)
```

```
[Out] 1/6*(24*a^3*((2*a+b*sin(2*d*x+2*c))/(2*a-b))^(1/2)*(-(sin(2*d*x+2*c)-1)*b/(
2*a+b))^(1/2)*(-(1+sin(2*d*x+2*c))*b/(2*a-b))^(1/2)*EllipticF(((2*a+b*sin(2
*d*x+2*c))/(2*a-b))^(1/2),((2*a-b)/(2*a+b))^(1/2))+4*((2*a+b*sin(2*d*x+2*c)
)/(2*a-b))^(1/2)*EllipticF(((2*a+b*sin(2*d*x+2*c))/(2*a-b))^(1/2),((2*a-b)/
(2*a+b))^(1/2))*(-(sin(2*d*x+2*c)-1)*b/(2*a+b))^(1/2)*(-(1+sin(2*d*x+2*c))*
b/(2*a-b))^(1/2)*a^2*b-6*((2*a+b*sin(2*d*x+2*c))/(2*a-b))^(1/2)*(-(sin(2*d*
x+2*c)-1)*b/(2*a+b))^(1/2)*(-(1+sin(2*d*x+2*c))*b/(2*a-b))^(1/2)*EllipticF(
((2*a+b*sin(2*d*x+2*c))/(2*a-b))^(1/2),((2*a-b)/(2*a+b))^(1/2))*b^2*a-((2*a
+b*sin(2*d*x+2*c))/(2*a-b))^(1/2)*(-(sin(2*d*x+2*c)-1)*b/(2*a+b))^(1/2)*(-(
1+sin(2*d*x+2*c))*b/(2*a-b))^(1/2)*EllipticF(((2*a+b*sin(2*d*x+2*c))/(2*a-b
))^(1/2),((2*a-b)/(2*a+b))^(1/2))*b^3-32*((2*a+b*sin(2*d*x+2*c))/(2*a-b))^(
1/2)*EllipticE(((2*a+b*sin(2*d*x+2*c))/(2*a-b))^(1/2),((2*a-b)/(2*a+b))^(1/
2))*(-(sin(2*d*x+2*c)-1)*b/(2*a+b))^(1/2)*(-(1+sin(2*d*x+2*c))*b/(2*a-b))^(
1/2)*a^3+8*((2*a+b*sin(2*d*x+2*c))/(2*a-b))^(1/2)*EllipticE(((2*a+b*sin(2*d
*x+2*c))/(2*a-b))^(1/2),((2*a-b)/(2*a+b))^(1/2))*(-(sin(2*d*x+2*c)-1)*b/(2*
a+b))^(1/2)*(-(1+sin(2*d*x+2*c))*b/(2*a-b))^(1/2)*a*b^2+b^3*sin(2*d*x+2*c)^
3+2*a*b^2*sin(2*d*x+2*c)^2-sin(2*d*x+2*c)*b^3-2*a*b^2)/b/cos(2*d*x+2*c)/(4*
a+2*b*sin(2*d*x+2*c))^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b\cos(dx+c)\sin(dx+c)+a)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c)*sin(d*x+c))^(3/2),x, algorithm="maxima")
[Out] integrate((b*cos(d*x + c)*sin(d*x + c) + a)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int (a + b\,\cos(c + d\,x)\,\sin(c + d\,x))^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*cos(c + d*x)*sin(c + d*x))^(3/2),x)
[Out] int((a + b*cos(c + d*x)*sin(c + d*x))^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b\sin(c + dx)\cos(c + dx))^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c)*sin(d*x+c))**(3/2),x)
[Out] Integral((a + b*sin(c + d*x)*cos(c + d*x))**(3/2), x)
```

3.575 $\int \sqrt{a+b\cos(c+dx)\sin(c+dx)}\,dx$

Optimal. Leaf size=76

$$\frac{\sqrt{2a+b\sin(2c+2dx)}\,E\left(c+dx-\frac{\pi}{4}|\frac{2b}{2a+b}\right)}{\sqrt{2}\,d\sqrt{\frac{2a+b\sin(2c+2dx)}{2a+b}}}$$

```
[Out] -1/2*(sin(c+1/4*Pi+d*x)^2)^(1/2)/sin(c+1/4*Pi+d*x)*EllipticE(cos(c+1/4*Pi+d
*x),2^(1/2)*(b/(2*a+b))^(1/2))*(2*a+b*sin(2*d*x+2*c))^(1/2)/d*2^(1/2)/((2*a
+b*sin(2*d*x+2*c))/(2*a+b))^(1/2)
```

Rubi [A] time = 0.06, antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {2666, 2655, 2653}

$$\frac{\sqrt{2a+b\sin(2c+2dx)}\,E\left(c+dx-\frac{\pi}{4}|\frac{2b}{2a+b}\right)}{\sqrt{2}\,d\sqrt{\frac{2a+b\sin(2c+2dx)}{2a+b}}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*Cos[c + d*x]*Sin[c + d*x]],x]
```

```
[Out] (EllipticE[c - Pi/4 + d*x, (2*b)/(2*a + b)]*Sqrt[2*a + b*Sin[2*c + 2*d*x]])
/(Sqrt[2]*d*Sqrt[(2*a + b*Sin[2*c + 2*d*x])/(2*a + b)])
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2666

```
Int[((a_) + cos[(c_.) + (d_.)*(x_)]*(b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_
Symbol] :> Int[(a + (b*Sin[2*c + 2*d*x])/2)^n, x] /; FreeQ[{a, b, c, d, n},
```

```
x]
```

Rubi steps

$$\int \sqrt{a+b\cos(c+dx)\sin(c+dx)}\,dx = \int \sqrt{a+\frac{1}{2}b\sin(2c+2dx)}\,dx$$
$$= \frac{\sqrt{a+\frac{1}{2}b\sin(2c+2dx)}\int \sqrt{\frac{a}{a+\frac{b}{2}}+\frac{b\sin(2c+2dx)}{2\left(a+\frac{b}{2}\right)}}\,dx}{\sqrt{\frac{a+\frac{1}{2}b\sin(2c+2dx)}{a+\frac{b}{2}}}}$$
$$= \frac{E\left(c-\frac{\pi}{4}+dx|\frac{2b}{2a+b}\right)\sqrt{2a+b\sin(2c+2dx)}}{\sqrt{2}\,d\sqrt{\frac{2a+b\sin(2c+2dx)}{2a+b}}}$$

Mathematica [A] time = 0.11, size = 75, normalized size = 0.99

$$\frac{(2a+b)\sqrt{\frac{2a+b\sin(2(c+dx))}{2a+b}}\,E\left(c+dx-\frac{\pi}{4}|\frac{2b}{2a+b}\right)}{d\sqrt{4a+2b\sin(2(c+dx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*Cos[c + d*x]*Sin[c + d*x]],x]
[Out] ((2*a + b)*EllipticE[c - Pi/4 + d*x, (2*b)/(2*a + b)]*Sqrt[(2*a + b*Sin[2*(
c + d*x)])/(2*a + b)])/(d*Sqrt[4*a + 2*b*Sin[2*(c + d*x)]])
```

fricas [F] time = 2.01, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\sqrt{b\cos(dx+c)\sin(dx+c)+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c)*sin(d*x+c))^(1/2),x, algorithm="fricas")
[Out] integral(sqrt(b*cos(d*x + c)*sin(d*x + c) + a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{b\cos(dx+c)\sin(dx+c)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c)*sin(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*cos(d*x + c)*sin(d*x + c) + a), x)
```

maple [B] time = 0.41, size = 312, normalized size = 4.11

$$-\frac{\sqrt{\frac{2a+b\sin(2dx+2c)}{2a-b}}\sqrt{-\frac{(\sin(2dx+2c)-1)b}{2a+b}}\sqrt{-\frac{(1+\sin(2dx+2c))b}{2a-b}}\,(2a-b)\left(2\,\mathrm{EllipticE}\left(\sqrt{\frac{2a+b\sin(2dx+2c)}{2a-b}},\sqrt{\frac{2a-b}{2a+b}}\right)a+\mathrm{Ellipt}\right.}{b\cos(2dx+2c)\sqrt{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c)*sin(d*x+c))^(1/2),x)
```

```
[Out] -((2*a+b*sin(2*d*x+2*c))/(2*a-b))^(1/2)*(-(sin(2*d*x+2*c)-1)*b/(2*a+b))^(1/
2)*(-(1+sin(2*d*x+2*c))*b/(2*a-b))^(1/2)/b*(2*a-b)*(2*EllipticE(((2*a+b*sin
(2*d*x+2*c))/(2*a-b))^(1/2),((2*a-b)/(2*a+b))^(1/2))*a+EllipticE(((2*a+b*si
n(2*d*x+2*c))/(2*a-b))^(1/2),((2*a-b)/(2*a+b))^(1/2))*b-2*a*EllipticF(((2*a
+b*sin(2*d*x+2*c))/(2*a-b))^(1/2),((2*a-b)/(2*a+b))^(1/2))-EllipticF(((2*a+
b*sin(2*d*x+2*c))/(2*a-b))^(1/2),((2*a-b)/(2*a+b))^(1/2))*b)/cos(2*d*x+2*c)
/(4*a+2*b*sin(2*d*x+2*c))^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{b\cos(dx+c)\sin(dx+c)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c)*sin(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*cos(d*x + c)*sin(d*x + c) + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \sqrt{a+b\,\cos(c+d\,x)\,\sin(c+d\,x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*cos(c + d*x)*sin(c + d*x))^(1/2),x)
```

```
[Out] int((a + b*cos(c + d*x)*sin(c + d*x))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a+b\sin(c+dx)\cos(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c)*sin(d*x+c))**(1/2),x)
```

```
[Out] Integral(sqrt(a + b*sin(c + d*x)*cos(c + d*x)), x)
```

3.576 $\int \frac{1}{\sqrt{a+b\cos(c+dx)\sin(c+dx)}}\,dx$

Optimal. Leaf size=76

$$\frac{\sqrt{2}\sqrt{\frac{2a+b\sin(2c+2dx)}{2a+b}}\,F\left(c+dx-\frac{\pi}{4}\middle|\frac{2b}{2a+b}\right)}{d\sqrt{2a+b\sin(2c+2dx)}}$$

```
[Out] -(sin(c+1/4*Pi+d*x)^2)^(1/2)/sin(c+1/4*Pi+d*x)*EllipticF(cos(c+1/4*Pi+d*x),
2^(1/2)*(b/(2*a+b))^(1/2))*2^(1/2)*((2*a+b*sin(2*d*x+2*c))/(2*a+b))^(1/2)/d
/(2*a+b*sin(2*d*x+2*c))^(1/2)
```

Rubi [A] time = 0.07, antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {2666, 2663, 2661}

$$\frac{\sqrt{2}\sqrt{\frac{2a+b\sin(2c+2dx)}{2a+b}}\,F\left(c+dx-\frac{\pi}{4}\middle|\frac{2b}{2a+b}\right)}{d\sqrt{2a+b\sin(2c+2dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[a + b*Cos[c + d*x]*Sin[c + d*x]],x]
```

```
[Out] (Sqrt[2]*EllipticF[c - Pi/4 + d*x, (2*b)/(2*a + b)]*Sqrt[(2*a + b*Sin[2*c +
 2*d*x])/(2*a + b)])/(d*Sqrt[2*a + b*Sin[2*c + 2*d*x]])
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2666

```
Int[((a_) + cos[(c_.) + (d_.)*(x_)]*(b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_
Symbol] :> Int[(a + (b*Sin[2*c + 2*d*x])/2)^n, x] /; FreeQ[{a, b, c, d, n},
 x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{\sqrt{a+b\cos(c+dx)\sin(c+dx)}}\,dx &= \int \frac{1}{\sqrt{a+\frac{1}{2}b\sin(2c+2dx)}}\,dx\\
&= \frac{\sqrt{\frac{a+\frac{1}{2}b\sin(2c+2dx)}{a+\frac{b}{2}}}\int \frac{1}{\sqrt{\frac{a}{a+\frac{b}{2}}+\frac{b\sin(2c+2dx)}{2\left(a+\frac{b}{2}\right)}}}\,dx}{\sqrt{a+\frac{1}{2}b\sin(2c+2dx)}}\\
&= \frac{\sqrt{2}F\left(c-\frac{\pi}{4}+dx\middle|\frac{2b}{2a+b}\right)\sqrt{\frac{2a+b\sin(2c+2dx)}{2a+b}}}{d\sqrt{2a+b\sin(2c+2dx)}}
\end{aligned}
$$

Mathematica [A] time = 0.13, size = 70, normalized size = 0.92

$$
\frac{\sqrt{\frac{2a+b\sin(2(c+dx))}{2a+b}}F\left(c+dx-\frac{\pi}{4}\middle|\frac{2b}{2a+b}\right)}{d\sqrt{a+\frac{1}{2}b\sin(2(c+dx))}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[a + b*Cos[c + d*x]*Sin[c + d*x]],x]
```

```
[Out] (EllipticF[c - Pi/4 + d*x, (2*b)/(2*a + b)]*Sqrt[(2*a + b*Sin[2*(c + d*x)])
/(2*a + b)])/(d*Sqrt[a + (b*Sin[2*(c + d*x)])/2])
```

fricas [F] time = 2.00, size = 0, normalized size = 0.00

$$
\mathrm{integral}\left(\frac{1}{\sqrt{b\cos(dx+c)\sin(dx+c)+a}},x\right)
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cos(d*x+c)*sin(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(1/sqrt(b*cos(d*x + c)*sin(d*x + c) + a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$
\int \frac{1}{\sqrt{b\cos(dx+c)\sin(dx+c)+a}}\,dx
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cos(d*x+c)*sin(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(b*cos(d*x + c)*sin(d*x + c) + a), x)
```

maple [A] time = 0.43, size = 165, normalized size = 2.17

$$\frac{2\left(2a-b\right)\sqrt{\frac{2a+b\sin(2dx+2c)}{2a-b}}\,\sqrt{-\frac{(\sin(2dx+2c)-1)b}{2a+b}}\,\sqrt{-\frac{(1+\sin(2dx+2c))b}{2a-b}}\,\mathrm{EllipticF}\left(\sqrt{\frac{2a+b\sin(2dx+2c)}{2a-b}},\sqrt{\frac{2a-b}{2a+b}}\right)}{b\cos\left(2dx+2c\right)\sqrt{4a+2b\sin\left(2dx+2c\right)}\;d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*cos(d*x+c)*sin(d*x+c))^(1/2),x)
```

```
[Out] 2*(2*a-b)*((2*a+b*sin(2*d*x+2*c))/(2*a-b))^(1/2)*(-(sin(2*d*x+2*c)-1)*b/(2*
a+b))^(1/2)*(-(1+sin(2*d*x+2*c))*b/(2*a-b))^(1/2)*EllipticF(((2*a+b*sin(2*d
*x+2*c))/(2*a-b))^(1/2),((2*a-b)/(2*a+b))^(1/2))/b/cos(2*d*x+2*c)/(4*a+2*b*
sin(2*d*x+2*c))^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{b\cos\left(dx+c\right)\sin\left(dx+c\right)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cos(d*x+c)*sin(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(b*cos(d*x + c)*sin(d*x + c) + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sqrt{a+b\,\cos\left(c+d\,x\right)\,\sin\left(c+d\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*cos(c + d*x)*sin(c + d*x))^(1/2),x)
```

```
[Out] int(1/(a + b*cos(c + d*x)*sin(c + d*x))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a+b\sin\left(c+dx\right)\cos\left(c+dx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cos(d*x+c)*sin(d*x+c))**(1/2),x)
```

```
[Out] Integral(1/sqrt(a + b*sin(c + d*x)*cos(c + d*x)), x)
```

3.577 $\int \frac{1}{(a+b\cos(c+dx)\sin(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=143

$$\frac{2\sqrt{2}\,b\cos(2c+2dx)}{d\left(4a^2-b^2\right)\sqrt{2a+b\sin(2c+2dx)}}+\frac{2\sqrt{2}\,\sqrt{2a+b\sin(2c+2dx)}\,E\left(c+dx-\frac{\pi}{4}\middle|\frac{2b}{2a+b}\right)}{d\left(4a^2-b^2\right)\sqrt{\frac{2a+b\sin(2c+2dx)}{2a+b}}}$$

```
[Out] 2*b*cos(2*d*x+2*c)*2^(1/2)/(4*a^2-b^2)/d/(2*a+b*sin(2*d*x+2*c))^(1/2)-2*(si
n(c+1/4*Pi+d*x)^2)^(1/2)/sin(c+1/4*Pi+d*x)*EllipticE(cos(c+1/4*Pi+d*x),2^(1
/2)*(b/(2*a+b))^(1/2))*2^(1/2)*(2*a+b*sin(2*d*x+2*c))^(1/2)/(4*a^2-b^2)/d/(
(2*a+b*sin(2*d*x+2*c))/(2*a+b))^(1/2)
```

Rubi [A] time = 0.09, antiderivative size = 143, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {2666, 2664, 21, 2655, 2653}

$$\frac{2\sqrt{2}\,b\cos(2c+2dx)}{d\left(4a^2-b^2\right)\sqrt{2a+b\sin(2c+2dx)}}+\frac{2\sqrt{2}\,\sqrt{2a+b\sin(2c+2dx)}\,E\left(c+dx-\frac{\pi}{4}\middle|\frac{2b}{2a+b}\right)}{d\left(4a^2-b^2\right)\sqrt{\frac{2a+b\sin(2c+2dx)}{2a+b}}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x]*Sin[c + d*x])^(-3/2),x]
```

```
[Out] (2*Sqrt[2]*b*Cos[2*c + 2*d*x])/((4*a^2 - b^2)*d*Sqrt[2*a + b*Sin[2*c + 2*d*
x]]) + (2*Sqrt[2]*EllipticE[c - Pi/4 + d*x, (2*b)/(2*a + b)]*Sqrt[2*a + b*S
in[2*c + 2*d*x]])/((4*a^2 - b^2)*d*Sqrt[(2*a + b*Sin[2*c + 2*d*x])/(2*a + b
)])
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2664

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[
c + d*x]*(a + b*Sin[c + d*x])^(n + 1))/(d*(n + 1)*(a^2 - b^2)), x] + Dist[1
/((n + 1)*(a^2 - b^2)), Int[(a + b*Sin[c + d*x])^(n + 1)*Simp[a*(n + 1) - b
*(n + 2)*Sin[c + d*x], x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^
2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2666

```
Int[((a_) + cos[(c_.) + (d_.)*(x_)]*(b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_
Symbol] :> Int[(a + (b*Sin[2*c + 2*d*x])/2)^n, x] /; FreeQ[{a, b, c, d, n},
 x]
```

Rubi steps

$$\int \frac{1}{(a+b\cos(c+dx)\sin(c+dx))^{3/2}}\,dx = \int \frac{1}{\left(a+\frac{1}{2}b\sin(2c+2dx)\right)^{3/2}}\,dx$$

$$= \frac{2\sqrt{2}\,b\cos(2c+2dx)}{\left(4a^2-b^2\right)d\sqrt{2a+b\sin(2c+2dx)}} - \frac{8\int \frac{-\frac{a}{2}-\frac{1}{4}b\sin(2c+2dx)}{\sqrt{a+\frac{1}{2}b\sin(2c+2dx)}}\,dx}{4a^2-b^2}$$

$$= \frac{2\sqrt{2}\,b\cos(2c+2dx)}{\left(4a^2-b^2\right)d\sqrt{2a+b\sin(2c+2dx)}} + \frac{4\int \sqrt{a+\frac{1}{2}b\sin(2c+2dx)}\,dx}{4a^2-b^2}$$

$$= \frac{2\sqrt{2}\,b\cos(2c+2dx)}{\left(4a^2-b^2\right)d\sqrt{2a+b\sin(2c+2dx)}} + \frac{\left(4\sqrt{a+\frac{1}{2}b\sin(2c+2dx)}\right)\int \sqrt{\ldots}}{\left(4a^2-b^2\right)\sqrt{\frac{a+\frac{1}{2}b\sin\ldots}{\ldots}}}$$

$$= \frac{2\sqrt{2}\,b\cos(2c+2dx)}{\left(4a^2-b^2\right)d\sqrt{2a+b\sin(2c+2dx)}} + \frac{2\sqrt{2}\,E\left(c-\frac{\pi}{4}+dx\middle|\frac{2b}{2a+b}\right)\sqrt{2a+\ldots}}{\left(4a^2-b^2\right)d\sqrt{\frac{2a+b\sin(\ldots}{2a+\ldots}}}$$

Mathematica [A] time = 0.42, size = 101, normalized size = 0.71

$$\frac{2\left((2a+b)\sqrt{\frac{2a+b\sin(2(c+dx))}{2a+b}}\,E\left(c+dx-\frac{\pi}{4}|\frac{2b}{2a+b}\right)+b\cos(2(c+dx))\right)}{d\left(4a^2-b^2\right)\sqrt{a+\frac{1}{2}b\sin(2(c+dx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x]*Sin[c + d*x])^(-3/2),x]
```

```
[Out] (2*(b*Cos[2*(c + d*x)] + (2*a + b)*EllipticE[c - Pi/4 + d*x, (2*b)/(2*a + b
)]*Sqrt[(2*a + b*Sin[2*(c + d*x)])/(2*a + b)]))/((4*a^2 - b^2)*d*Sqrt[a + (
b*Sin[2*(c + d*x)])/2])
```

fricas [F] time = 0.55, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\sqrt{b\cos\left(dx+c\right)\sin\left(dx+c\right)+a}}{b^2\cos\left(dx+c\right)^4-b^2\cos\left(dx+c\right)^2-2\,ab\cos\left(dx+c\right)\sin\left(dx+c\right)-a^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cos(d*x+c)*sin(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(b*cos(d*x + c)*sin(d*x + c) + a)/(b^2*cos(d*x + c)^4 - b^2*c
os(d*x + c)^2 - 2*a*b*cos(d*x + c)*sin(d*x + c) - a^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b\cos\left(dx+c\right)\sin\left(dx+c\right)+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cos(d*x+c)*sin(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*cos(d*x + c)*sin(d*x + c) + a)^(-3/2), x)
```

maple [B] time = 0.50, size = 570, normalized size = 3.99

$$16a^2\sqrt{\frac{2a+b\sin(2dx+2c)}{2a-b}}\,\sqrt{-\frac{(\sin(2dx+2c)-1)b}{2a+b}}\,\sqrt{-\frac{(1+\sin(2dx+2c))b}{2a-b}}\,\,\text{EllipticF}\left(\sqrt{\frac{2a+b\sin(2dx+2c)}{2a-b}},\sqrt{\frac{2a-b}{2a+b}}\right)-4\sqrt{\frac{2a+b\sin(2dx+2}{2a-b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*cos(d*x+c)*sin(d*x+c))^(3/2),x)
```

```
[Out] 4/b*(4*a^2*((2*a+b*sin(2*d*x+2*c))/(2*a-b))^(1/2)*(-(sin(2*d*x+2*c)-1)*b/(2
*a+b))^(1/2)*(-(1+sin(2*d*x+2*c))*b/(2*a-b))^(1/2)*EllipticF(((2*a+b*sin(2*
d*x+2*c))/(2*a-b))^(1/2),((2*a-b)/(2*a+b))^(1/2))-((2*a+b*sin(2*d*x+2*c))/(
2*a-b))^(1/2)*EllipticF(((2*a+b*sin(2*d*x+2*c))/(2*a-b))^(1/2),((2*a-b)/(2*
a+b))^(1/2))*(-(sin(2*d*x+2*c)-1)*b/(2*a+b))^(1/2)*(-(1+sin(2*d*x+2*c))*b/(
2*a-b))^(1/2)*b^2-4*((2*a+b*sin(2*d*x+2*c))/(2*a-b))^(1/2)*EllipticE(((2*a+
b*sin(2*d*x+2*c))/(2*a-b))^(1/2),((2*a-b)/(2*a+b))^(1/2))*(-(sin(2*d*x+2*c)
-1)*b/(2*a+b))^(1/2)*(-(1+sin(2*d*x+2*c))*b/(2*a-b))^(1/2)*a^2+((2*a+b*sin(
2*d*x+2*c))/(2*a-b))^(1/2)*EllipticE(((2*a+b*sin(2*d*x+2*c))/(2*a-b))^(1/2)
,((2*a-b)/(2*a+b))^(1/2))*(-(sin(2*d*x+2*c)-1)*b/(2*a+b))^(1/2)*(-(1+sin(2*
d*x+2*c))*b/(2*a-b))^(1/2)*b^2-b^2*sin(2*d*x+2*c)^2+b^2)/(4*a^2-b^2)/cos(2*
d*x+2*c)/(4*a+2*b*sin(2*d*x+2*c))^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(b\cos(dx+c)\sin(dx+c)+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cos(d*x+c)*sin(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*cos(d*x + c)*sin(d*x + c) + a)^(-3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{(a+b\,\cos(c+d\,x)\,\sin(c+d\,x))^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*cos(c + d*x)*sin(c + d*x))^(3/2),x)
```

```
[Out] int(1/(a + b*cos(c + d*x)*sin(c + d*x))^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a+b\sin(c+dx)\cos(c+dx))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cos(d*x+c)*sin(d*x+c))**(3/2),x)
```

```
[Out] Integral((a + b*sin(c + d*x)*cos(c + d*x))**(-3/2), x)
```

3.578 $\int \frac{1}{(a+b\cos(c+dx)\sin(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=295

$$\frac{32\sqrt{2}\,ab\cos(2c+2dx)}{3d\left(4a^2-b^2\right)^2\sqrt{2a+b\sin(2c+2dx)}}+\frac{4\sqrt{2}\,b\cos(2c+2dx)}{3d\left(4a^2-b^2\right)(2a+b\sin(2c+2dx))^{3/2}}-\frac{4\sqrt{2}\sqrt{\frac{2a+b\sin(2c+2dx)}{2a+b}}F\left(c+dx-\right.}{3d\left(4a^2-b^2\right)\sqrt{2a+b\sin(2c+}}$$

```
[Out] 4/3*b*cos(2*d*x+2*c)*2^(1/2)/(4*a^2-b^2)/d/(2*a+b*sin(2*d*x+2*c))^(3/2)+32/
3*a*b*cos(2*d*x+2*c)*2^(1/2)/(4*a^2-b^2)^2/d/(2*a+b*sin(2*d*x+2*c))^(1/2)-3
2/3*a*(sin(c+1/4*Pi+d*x)^2)^(1/2)/sin(c+1/4*Pi+d*x)*EllipticE(cos(c+1/4*Pi+
d*x),2^(1/2)*(b/(2*a+b))^(1/2))*2^(1/2)*(2*a+b*sin(2*d*x+2*c))^(1/2)/(4*a^2
-b^2)^2/d/((2*a+b*sin(2*d*x+2*c))/(2*a+b))^(1/2)+4/3*(sin(c+1/4*Pi+d*x)^2)^
(1/2)/sin(c+1/4*Pi+d*x)*EllipticF(cos(c+1/4*Pi+d*x),2^(1/2)*(b/(2*a+b))^(1/
2))*2^(1/2)*((2*a+b*sin(2*d*x+2*c))/(2*a+b))^(1/2)/(4*a^2-b^2)/d/(2*a+b*sin
(2*d*x+2*c))^(1/2)
```

Rubi [A] time = 0.30, antiderivative size = 295, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {2666, 2664, 2754, 2752, 2663, 2661, 2655, 2653}

$$\frac{32\sqrt{2}\,ab\cos(2c+2dx)}{3d\left(4a^2-b^2\right)^2\sqrt{2a+b\sin(2c+2dx)}}+\frac{4\sqrt{2}\,b\cos(2c+2dx)}{3d\left(4a^2-b^2\right)(2a+b\sin(2c+2dx))^{3/2}}-\frac{4\sqrt{2}\sqrt{\frac{2a+b\sin(2c+2dx)}{2a+b}}F\left(c+dx-\right.}{3d\left(4a^2-b^2\right)\sqrt{2a+b\sin(2c+}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x]*Sin[c + d*x])^(-5/2),x]
```

```
[Out] (4*Sqrt[2]*b*Cos[2*c + 2*d*x])/(3*(4*a^2 - b^2)*d*(2*a + b*Sin[2*c + 2*d*x]
)^(3/2)) + (32*Sqrt[2]*a*b*Cos[2*c + 2*d*x])/(3*(4*a^2 - b^2)^2*d*Sqrt[2*a
+ b*Sin[2*c + 2*d*x]]) + (32*Sqrt[2]*a*EllipticE[c - Pi/4 + d*x, (2*b)/(2*a
 + b)]*Sqrt[2*a + b*Sin[2*c + 2*d*x]])/(3*(4*a^2 - b^2)^2*d*Sqrt[(2*a + b*S
in[2*c + 2*d*x])/(2*a + b)]) - (4*Sqrt[2]*EllipticF[c - Pi/4 + d*x, (2*b)/(
2*a + b)]*Sqrt[(2*a + b*Sin[2*c + 2*d*x])/(2*a + b)])/(3*(4*a^2 - b^2)*d*Sq
rt[2*a + b*Sin[2*c + 2*d*x]])
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2664

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[
c + d*x]*(a + b*Sin[c + d*x])^(n + 1))/(d*(n + 1)*(a^2 - b^2)), x] + Dist[1
/((n + 1)*(a^2 - b^2)), Int[(a + b*Sin[c + d*x])^(n + 1)*Simp[a*(n + 1) - b
*(n + 2)*Sin[c + d*x], x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^
2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2666

```
Int[((a_) + cos[(c_.) + (d_.)*(x_)]*(b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_
Symbol] :> Int[(a + (b*Sin[2*c + 2*d*x])/2)^n, x] /; FreeQ[{a, b, c, d, n},
 x]
```

Rule 2752

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2754

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e + f
*x])^(m + 1))/(f*(m + 1)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(a^2 - b^2)), I
nt[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m +
```

```
2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a
*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rubi steps

$$\int \frac{1}{(a+b\cos(c+dx)\sin(c+dx))^{5/2}}\,dx = \int \frac{1}{\left(a+\frac{1}{2}b\sin(2c+2dx)\right)^{5/2}}\,dx$$

$$= \frac{4\sqrt{2}\,b\cos(2c+2dx)}{3\left(4a^2-b^2\right)d(2a+b\sin(2c+2dx))^{3/2}} - \frac{8\int \frac{-\frac{3a}{2}+\frac{1}{4}b\sin(2c+2dx)}{\left(a+\frac{1}{2}b\sin(2c+2dx)\right)^{3/2}}\,dx}{3\left(4a^2-b^2\right)}$$

$$= \frac{4\sqrt{2}\,b\cos(2c+2dx)}{3\left(4a^2-b^2\right)d(2a+b\sin(2c+2dx))^{3/2}} + \frac{32\sqrt{2}\,ab\cos(2c+2dx}{3\left(4a^2-b^2\right)^2 d\sqrt{2a+b\sin(2c}}$$

$$= \frac{4\sqrt{2}\,b\cos(2c+2dx)}{3\left(4a^2-b^2\right)d(2a+b\sin(2c+2dx))^{3/2}} + \frac{32\sqrt{2}\,ab\cos(2c+2dx}{3\left(4a^2-b^2\right)^2 d\sqrt{2a+b\sin(2c}}$$

$$= \frac{4\sqrt{2}\,b\cos(2c+2dx)}{3\left(4a^2-b^2\right)d(2a+b\sin(2c+2dx))^{3/2}} + \frac{32\sqrt{2}\,ab\cos(2c+2dx}{3\left(4a^2-b^2\right)^2 d\sqrt{2a+b\sin(2c}}$$

$$= \frac{4\sqrt{2}\,b\cos(2c+2dx)}{3\left(4a^2-b^2\right)d(2a+b\sin(2c+2dx))^{3/2}} + \frac{32\sqrt{2}\,ab\cos(2c+2dx}{3\left(4a^2-b^2\right)^2 d\sqrt{2a+b\sin(2c}}$$

Mathematica [A] time = 1.52, size = 201, normalized size = 0.68

$$-\frac{4\sqrt{2}\left(b\cos(2(c+dx))\left(-20a^2-8ab\sin(2(c+dx))+b^2\right)+(2a-b)(2a+b)^2\left(\frac{2a+b\sin(2(c+dx))}{2a+b}\right)^{3/2}F\left(c+dx-\frac{\pi}{4}\middle|\frac{}{2a}\right.\right.}{3d\left(b^2-4a^2\right)^2(2a+b\sin(2(c+dx)))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x]*Sin[c + d*x])^(-5/2),x]
```

```
[Out] (-4*Sqrt[2]*((-8*a*EllipticE[c - Pi/4 + d*x, (2*b)/(2*a + b)]*(2*a + b*Sin[
2*(c + d*x)])^2)/Sqrt[(2*a + b*Sin[2*(c + d*x)])/(2*a + b)] + (2*a - b)*(2*
a + b)^2*EllipticF[c - Pi/4 + d*x, (2*b)/(2*a + b)]*((2*a + b*Sin[2*(c + d*
x)])/(2*a + b))^(3/2) + b*Cos[2*(c + d*x)]*(-20*a^2 + b^2 - 8*a*b*Sin[2*(c
+ d*x)])))/(3*(-4*a^2 + b^2)^2*d*(2*a + b*Sin[2*(c + d*x)])^(3/2))
```

fricas [F] time = 1.74, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\sqrt{b\cos\left(dx+c\right)\sin\left(dx+c\right)+a}}{3\,ab^2\cos\left(dx+c\right)^4-3\,ab^2\cos\left(dx+c\right)^2-a^3+\left(b^3\cos\left(dx+c\right)^5-b^3\cos\left(dx+c\right)^3-3\,a^2b\cos\left(dx+\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cos(d*x+c)*sin(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(b*cos(d*x + c)*sin(d*x + c) + a)/(3*a*b^2*cos(d*x + c)^4 - 3
*a*b^2*cos(d*x + c)^2 - a^3 + (b^3*cos(d*x + c)^5 - b^3*cos(d*x + c)^3 - 3*
a^2*b*cos(d*x + c))*sin(d*x + c)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b\cos\left(dx+c\right)\sin\left(dx+c\right)+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cos(d*x+c)*sin(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*cos(d*x + c)*sin(d*x + c) + a)^(-5/2), x)
```

maple [B] time = 0.52, size = 1554, normalized size = 5.27

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*cos(d*x+c)*sin(d*x+c))^(5/2),x)
```

```
[Out] 8/3*(8*sin(2*d*x+2*c)*cos(2*d*x+2*c)^2*a*b^3+(b/(2*a-b)*sin(2*d*x+2*c)+2*a/
(2*a-b))^(1/2)*(-b/(2*a+b)*sin(2*d*x+2*c)+b/(2*a+b))^(1/2)*(-b/(2*a-b)*sin(
2*d*x+2*c)-b/(2*a-b))^(1/2)*b*(24*EllipticF((b/(2*a-b)*sin(2*d*x+2*c)+2*a/(
2*a-b))^(1/2),((2*a-b)/(2*a+b))^(1/2))*a^3+4*EllipticF((b/(2*a-b)*sin(2*d*x
+2*c)+2*a/(2*a-b))^(1/2),((2*a-b)/(2*a+b))^(1/2))*a^2*b-6*EllipticF((b/(2*a
-b)*sin(2*d*x+2*c)+2*a/(2*a-b))^(1/2),((2*a-b)/(2*a+b))^(1/2))*a*b^2-Ellipt
icF((b/(2*a-b)*sin(2*d*x+2*c)+2*a/(2*a-b))^(1/2),((2*a-b)/(2*a+b))^(1/2))*b
```

```
^3-32*EllipticE((b/(2*a-b)*sin(2*d*x+2*c)+2*a/(2*a-b))^(1/2),((2*a-b)/(2*a+
b))^(1/2))*a^3+8*EllipticE((b/(2*a-b)*sin(2*d*x+2*c)+2*a/(2*a-b))^(1/2),((2
*a-b)/(2*a+b))^(1/2))*a*b^2)*sin(2*d*x+2*c)+(20*a^2*b^2-b^4)*cos(2*d*x+2*c)
^2+48*(b/(2*a-b)*sin(2*d*x+2*c)+2*a/(2*a-b))^(1/2)*EllipticF((b/(2*a-b)*sin
(2*d*x+2*c)+2*a/(2*a-b))^(1/2),((2*a-b)/(2*a+b))^(1/2))*(-b/(2*a+b)*sin(2*d
*x+2*c)+b/(2*a+b))^(1/2)*(-b/(2*a-b)*sin(2*d*x+2*c)-b/(2*a-b))^(1/2)*a^4+8*
(b/(2*a-b)*sin(2*d*x+2*c)+2*a/(2*a-b))^(1/2)*EllipticF((b/(2*a-b)*sin(2*d*x
+2*c)+2*a/(2*a-b))^(1/2),((2*a-b)/(2*a+b))^(1/2))*(-b/(2*a+b)*sin(2*d*x+2*c
)+b/(2*a+b))^(1/2)*(-b/(2*a-b)*sin(2*d*x+2*c)-b/(2*a-b))^(1/2)*a^3*b-12*(b/
(2*a-b)*sin(2*d*x+2*c)+2*a/(2*a-b))^(1/2)*EllipticF((b/(2*a-b)*sin(2*d*x+2*
c)+2*a/(2*a-b))^(1/2),((2*a-b)/(2*a+b))^(1/2))*(-b/(2*a+b)*sin(2*d*x+2*c)+b
/(2*a+b))^(1/2)*(-b/(2*a-b)*sin(2*d*x+2*c)-b/(2*a-b))^(1/2)*a^2*b^2-2*(b/(2
*a-b)*sin(2*d*x+2*c)+2*a/(2*a-b))^(1/2)*EllipticF((b/(2*a-b)*sin(2*d*x+2*c)
+2*a/(2*a-b))^(1/2),((2*a-b)/(2*a+b))^(1/2))*(-b/(2*a+b)*sin(2*d*x+2*c)+b/(
2*a+b))^(1/2)*(-b/(2*a-b)*sin(2*d*x+2*c)-b/(2*a-b))^(1/2)*a*b^3-64*(b/(2*a-
b)*sin(2*d*x+2*c)+2*a/(2*a-b))^(1/2)*(-b/(2*a+b)*sin(2*d*x+2*c)+b/(2*a+b))^
(1/2)*(-b/(2*a-b)*sin(2*d*x+2*c)-b/(2*a-b))^(1/2)*EllipticE((b/(2*a-b)*sin(
2*d*x+2*c)+2*a/(2*a-b))^(1/2),((2*a-b)/(2*a+b))^(1/2))*a^4+16*(b/(2*a-b)*si
n(2*d*x+2*c)+2*a/(2*a-b))^(1/2)*(-b/(2*a+b)*sin(2*d*x+2*c)+b/(2*a+b))^(1/2)
*(-b/(2*a-b)*sin(2*d*x+2*c)-b/(2*a-b))^(1/2)*EllipticE((b/(2*a-b)*sin(2*d*x
+2*c)+2*a/(2*a-b))^(1/2),((2*a-b)/(2*a+b))^(1/2))*a^2*b^2)/(2*a+b*sin(2*d*x
+2*c))/(4*a^2-b^2)^2/b/cos(2*d*x+2*c)/(4*a+2*b*sin(2*d*x+2*c))^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(b\cos(dx+c)\sin(dx+c)+a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cos(d*x+c)*sin(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((b*cos(d*x + c)*sin(d*x + c) + a)^(-5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\left(a+b\,\cos\left(c+d\,x\right)\,\sin\left(c+d\,x\right)\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*cos(c + d*x)*sin(c + d*x))^(5/2),x)
```

```
[Out] int(1/(a + b*cos(c + d*x)*sin(c + d*x))^(5/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cos(d*x+c)*sin(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

3.579 $\int \frac{x^3}{a+b\cos(x)\sin(x)}\,dx$

Optimal. Leaf size=461

$$-\frac{3x^2\text{Li}_2\left(\frac{ibe^{2ix}}{2a-\sqrt{4a^2-b^2}}\right)}{2\sqrt{4a^2-b^2}}+\frac{3x^2\text{Li}_2\left(\frac{ibe^{2ix}}{2a+\sqrt{4a^2-b^2}}\right)}{2\sqrt{4a^2-b^2}}-\frac{3ix\text{Li}_3\left(\frac{ibe^{2ix}}{2a-\sqrt{4a^2-b^2}}\right)}{2\sqrt{4a^2-b^2}}+\frac{3ix\text{Li}_3\left(\frac{ibe^{2ix}}{2a+\sqrt{4a^2-b^2}}\right)}{2\sqrt{4a^2-b^2}}+\frac{3\text{Li}_4\left(\frac{ibe^{2ix}}{2a-\sqrt{4a^2-b^2}}\right)}{4\sqrt{4a^2-b^2}}-\frac{3\text{Li}_4}{4}$$

```
[Out] -I*x^3*ln(1-I*b*exp(2*I*x)/(2*a-(4*a^2-b^2)^(1/2)))/(4*a^2-b^2)^(1/2)+I*x^3
*ln(1-I*b*exp(2*I*x)/(2*a+(4*a^2-b^2)^(1/2)))/(4*a^2-b^2)^(1/2)-3/2*x^2*pol
ylog(2,I*b*exp(2*I*x)/(2*a-(4*a^2-b^2)^(1/2)))/(4*a^2-b^2)^(1/2)+3/2*x^2*po
lylog(2,I*b*exp(2*I*x)/(2*a+(4*a^2-b^2)^(1/2)))/(4*a^2-b^2)^(1/2)-3/2*I*x*p
olylog(3,I*b*exp(2*I*x)/(2*a-(4*a^2-b^2)^(1/2)))/(4*a^2-b^2)^(1/2)+3/2*I*x*
polylog(3,I*b*exp(2*I*x)/(2*a+(4*a^2-b^2)^(1/2)))/(4*a^2-b^2)^(1/2)+3/4*pol
ylog(4,I*b*exp(2*I*x)/(2*a-(4*a^2-b^2)^(1/2)))/(4*a^2-b^2)^(1/2)-3/4*polylo
g(4,I*b*exp(2*I*x)/(2*a+(4*a^2-b^2)^(1/2)))/(4*a^2-b^2)^(1/2)
```

Rubi [A] time = 0.63, antiderivative size = 461, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 8, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.571, Rules used = {4584, 3323, 2264, 2190, 2531, 6609, 2282, 6589}

$$-\frac{3x^2\text{PolyLog}\left(2,\frac{ibe^{2ix}}{2a-\sqrt{4a^2-b^2}}\right)}{2\sqrt{4a^2-b^2}}+\frac{3x^2\text{PolyLog}\left(2,\frac{ibe^{2ix}}{\sqrt{4a^2-b^2}+2a}\right)}{2\sqrt{4a^2-b^2}}-\frac{3ix\text{PolyLog}\left(3,\frac{ibe^{2ix}}{2a-\sqrt{4a^2-b^2}}\right)}{2\sqrt{4a^2-b^2}}+\frac{3ix\text{PolyLog}\left(3,\frac{ib}{\sqrt{4a^2-}}\right.}{2\sqrt{4a^2-b^2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/(a + b*Cos[x]*Sin[x]),x]
```

```
[Out] ((-I)*x^3*Log[1 - (I*b*E^((2*I)*x))/(2*a - Sqrt[4*a^2 - b^2])])/Sqrt[4*a^2
- b^2] + (I*x^3*Log[1 - (I*b*E^((2*I)*x))/(2*a + Sqrt[4*a^2 - b^2])])/Sqrt[
4*a^2 - b^2] - (3*x^2*PolyLog[2, (I*b*E^((2*I)*x))/(2*a - Sqrt[4*a^2 - b^2]
)])/(2*Sqrt[4*a^2 - b^2]) + (3*x^2*PolyLog[2, (I*b*E^((2*I)*x))/(2*a + Sqrt
[4*a^2 - b^2])])/(2*Sqrt[4*a^2 - b^2]) - (((3*I)/2)*x*PolyLog[3, (I*b*E^((2
*I)*x))/(2*a - Sqrt[4*a^2 - b^2])])/Sqrt[4*a^2 - b^2] + (((3*I)/2)*x*PolyLo
g[3, (I*b*E^((2*I)*x))/(2*a + Sqrt[4*a^2 - b^2])])/Sqrt[4*a^2 - b^2] + (3*P
olyLog[4, (I*b*E^((2*I)*x))/(2*a - Sqrt[4*a^2 - b^2])])/(4*Sqrt[4*a^2 - b^2
]) - (3*PolyLog[4, (I*b*E^((2*I)*x))/(2*a + Sqrt[4*a^2 - b^2])])/(4*Sqrt[4*
a^2 - b^2])
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
```

```
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2264

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c)/q, Int[
((f + g*x)^m*F^u)/(b - q + 2*c*F^u), x], x] - Dist[(2*c)/q, Int[((f + g*x)^
m*F^u)/(b + q + 2*c*F^u), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 3323

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Sy
mbol] :> Dist[2, Int[((c + d*x)^m*E^(I*(e + f*x)))/(I*b + 2*a*E^(I*(e + f*x
)) - I*b*E^(2*I*(e + f*x))), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[
a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 4584

```
Int[((e_.) + (f_.)*(x_))^(m_.)*((a_) + Cos[(c_.) + (d_.)*(x_)]*(b_.)*Sin[(c
_.) + (d_.)*(x_)])^(n_.), x_Symbol] :> Int[(e + f*x)^m*(a + (b*Sin[2*c + 2*
d*x])/2)^n, x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
```

```
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\int \frac{x^3}{a+b\cos(x)\sin(x)}\,dx = \int \frac{x^3}{a+\frac{1}{2}b\sin(2x)}\,dx$$

$$= 2\int \frac{e^{2ix}x^3}{\frac{ib}{2}+2ae^{2ix}-\frac{1}{2}ibe^{4ix}}\,dx$$

$$= -\frac{(2ib)\int \frac{e^{2ix}x^3}{2a-\sqrt{4a^2-b^2}-ibe^{2ix}}\,dx}{\sqrt{4a^2-b^2}} + \frac{(2ib)\int \frac{e^{2ix}x^3}{2a+\sqrt{4a^2-b^2}-ibe^{2ix}}\,dx}{\sqrt{4a^2-b^2}}$$

$$= -\frac{ix^3\log\left(1-\frac{ibe^{2ix}}{2a-\sqrt{4a^2-b^2}}\right)}{\sqrt{4a^2-b^2}} + \frac{ix^3\log\left(1-\frac{ibe^{2ix}}{2a+\sqrt{4a^2-b^2}}\right)}{\sqrt{4a^2-b^2}} + \frac{(3i)\int x^2\log\left(1-\frac{ibe^{2ix}}{2a-\sqrt{4a^2-b^2}}\right)}{\sqrt{4a^2-b^2}}$$

$$= -\frac{ix^3\log\left(1-\frac{ibe^{2ix}}{2a-\sqrt{4a^2-b^2}}\right)}{\sqrt{4a^2-b^2}} + \frac{ix^3\log\left(1-\frac{ibe^{2ix}}{2a+\sqrt{4a^2-b^2}}\right)}{\sqrt{4a^2-b^2}} - \frac{3x^2\text{Li}_2\left(\frac{ibe^{2ix}}{2a-\sqrt{4a^2-b^2}}\right)}{2\sqrt{4a^2-b^2}} + \frac{3x^2\text{Li}}{2}$$

$$= -\frac{ix^3\log\left(1-\frac{ibe^{2ix}}{2a-\sqrt{4a^2-b^2}}\right)}{\sqrt{4a^2-b^2}} + \frac{ix^3\log\left(1-\frac{ibe^{2ix}}{2a+\sqrt{4a^2-b^2}}\right)}{\sqrt{4a^2-b^2}} - \frac{3x^2\text{Li}_2\left(\frac{ibe^{2ix}}{2a-\sqrt{4a^2-b^2}}\right)}{2\sqrt{4a^2-b^2}} + \frac{3x^2\text{Li}}{2}$$

$$= -\frac{ix^3\log\left(1-\frac{ibe^{2ix}}{2a-\sqrt{4a^2-b^2}}\right)}{\sqrt{4a^2-b^2}} + \frac{ix^3\log\left(1-\frac{ibe^{2ix}}{2a+\sqrt{4a^2-b^2}}\right)}{\sqrt{4a^2-b^2}} - \frac{3x^2\text{Li}_2\left(\frac{ibe^{2ix}}{2a-\sqrt{4a^2-b^2}}\right)}{2\sqrt{4a^2-b^2}} + \frac{3x^2\text{Li}}{2}$$

$$= -\frac{ix^3\log\left(1-\frac{ibe^{2ix}}{2a-\sqrt{4a^2-b^2}}\right)}{\sqrt{4a^2-b^2}} + \frac{ix^3\log\left(1-\frac{ibe^{2ix}}{2a+\sqrt{4a^2-b^2}}\right)}{\sqrt{4a^2-b^2}} - \frac{3x^2\text{Li}_2\left(\frac{ibe^{2ix}}{2a-\sqrt{4a^2-b^2}}\right)}{2\sqrt{4a^2-b^2}} + \frac{3x^2\text{Li}}{2}$$

Mathematica [A] time = 0.87, size = 340, normalized size = 0.74

$$\frac{-6x^2\mathrm{Li}_2\left(-\frac{ibe^{2ix}}{\sqrt{4a^2-b^2}-2a}\right)+6x^2\mathrm{Li}_2\left(\frac{ibe^{2ix}}{2a+\sqrt{4a^2-b^2}}\right)-6ix\mathrm{Li}_3\left(-\frac{ibe^{2ix}}{\sqrt{4a^2-b^2}-2a}\right)+6ix\mathrm{Li}_3\left(\frac{ibe^{2ix}}{2a+\sqrt{4a^2-b^2}}\right)+3\mathrm{Li}_4\left(-\frac{ibe^{2ix}}{\sqrt{4a^2-b^2}-2\cdots}\right.}{4\sqrt{4a^2-b^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/(a + b*Cos[x]*Sin[x]),x]
```

```
[Out] ((-4*I)*x^3*Log[1 + (I*b*E^((2*I)*x))/(-2*a + Sqrt[4*a^2 - b^2])] + (4*I)*x
^3*Log[1 - (I*b*E^((2*I)*x))/(2*a + Sqrt[4*a^2 - b^2])] - 6*x^2*PolyLog[2,
((-I)*b*E^((2*I)*x))/(-2*a + Sqrt[4*a^2 - b^2])] + 6*x^2*PolyLog[2, (I*b*E^
((2*I)*x))/(2*a + Sqrt[4*a^2 - b^2])] - (6*I)*x*PolyLog[3, ((-I)*b*E^((2*I)
*x))/(-2*a + Sqrt[4*a^2 - b^2])] + (6*I)*x*PolyLog[3, (I*b*E^((2*I)*x))/(2*
a + Sqrt[4*a^2 - b^2])] + 3*PolyLog[4, ((-I)*b*E^((2*I)*x))/(-2*a + Sqrt[4*
a^2 - b^2])] - 3*PolyLog[4, (I*b*E^((2*I)*x))/(2*a + Sqrt[4*a^2 - b^2])])/(
4*Sqrt[4*a^2 - b^2])
```

fricas [C] time = 1.29, size = 3324, normalized size = 7.21

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a+b*cos(x)*sin(x)),x, algorithm="fricas")
```

```
[Out] -1/4*(2*b*x^3*sqrt(-(4*a^2 - b^2)/b^2)*log(1/2*((4*I*a*cos(x) + 4*a*sin(x)
- 2*(b*cos(x) - I*b*sin(x))*sqrt(-(4*a^2 - b^2)/b^2))*sqrt((b*sqrt(-(4*a^2
- b^2)/b^2) + 2*I*a)/b) + 2*b)/b) + 2*b*x^3*sqrt(-(4*a^2 - b^2)/b^2)*log(1/
2*((-4*I*a*cos(x) - 4*a*sin(x) + 2*(b*cos(x) - I*b*sin(x))*sqrt(-(4*a^2 - b
^2)/b^2))*sqrt((b*sqrt(-(4*a^2 - b^2)/b^2) + 2*I*a)/b) + 2*b)/b) - 2*b*x^3*
sqrt(-(4*a^2 - b^2)/b^2)*log(-((2*I*a*cos(x) - 2*a*sin(x) - (b*cos(x) + I*b
*sin(x))*sqrt(-(4*a^2 - b^2)/b^2))*sqrt(-(b*sqrt(-(4*a^2 - b^2)/b^2) + 2*I*
a)/b) - b)/b) - 2*b*x^3*sqrt(-(4*a^2 - b^2)/b^2)*log(-((-2*I*a*cos(x) + 2*a
*sin(x) + (b*cos(x) + I*b*sin(x))*sqrt(-(4*a^2 - b^2)/b^2))*sqrt(-(b*sqrt(-
(4*a^2 - b^2)/b^2) + 2*I*a)/b) - b)/b) + 2*b*x^3*sqrt(-(4*a^2 - b^2)/b^2)*l
og(-((2*I*a*cos(x) - 2*a*sin(x) + (b*cos(x) + I*b*sin(x))*sqrt(-(4*a^2 - b^
2)/b^2))*sqrt((b*sqrt(-(4*a^2 - b^2)/b^2) - 2*I*a)/b) - b)/b) + 2*b*x^3*sqr
t(-(4*a^2 - b^2)/b^2)*log(-((-2*I*a*cos(x) + 2*a*sin(x) - (b*cos(x) + I*b*s
in(x))*sqrt(-(4*a^2 - b^2)/b^2))*sqrt((b*sqrt(-(4*a^2 - b^2)/b^2) - 2*I*a)/
b) - b)/b) - 2*b*x^3*sqrt(-(4*a^2 - b^2)/b^2)*log(1/2*((4*I*a*cos(x) + 4*a*
sin(x) + 2*(b*cos(x) - I*b*sin(x))*sqrt(-(4*a^2 - b^2)/b^2))*sqrt(-(b*sqrt(
-(4*a^2 - b^2)/b^2) - 2*I*a)/b) + 2*b)/b) - 2*b*x^3*sqrt(-(4*a^2 - b^2)/b^2
)*log(1/2*((-4*I*a*cos(x) - 4*a*sin(x) - 2*(b*cos(x) - I*b*sin(x))*sqrt(-(4
*a^2 - b^2)/b^2))*sqrt(-(b*sqrt(-(4*a^2 - b^2)/b^2) - 2*I*a)/b) + 2*b)/b) +
```

6*I*b*x^2*sqrt(-(4*a^2 - b^2)/b^2)*dilog(-1/2*((4*I*a*cos(x) + 4*a*sin(x) - 2*(b*cos(x) - I*b*sin(x))*sqrt(-(4*a^2 - b^2)/b^2))*sqrt((b*sqrt(-(4*a^2 - b^2)/b^2) + 2*I*a)/b) + 2*b)/b + 1) + 6*I*b*x^2*sqrt(-(4*a^2 - b^2)/b^2)*dilog(-1/2*((-4*I*a*cos(x) - 4*a*sin(x) + 2*(b*cos(x) - I*b*sin(x))*sqrt(-(4*a^2 - b^2)/b^2))*sqrt((b*sqrt(-(4*a^2 - b^2)/b^2) + 2*I*a)/b) + 2*b)/b + 1) + 6*I*b*x^2*sqrt(-(4*a^2 - b^2)/b^2)*dilog(((2*I*a*cos(x) - 2*a*sin(x) - (b*cos(x) + I*b*sin(x))*sqrt(-(4*a^2 - b^2)/b^2))*sqrt(-(b*sqrt(-(4*a^2 - b^2)/b^2) + 2*I*a)/b) - b)/b + 1) + 6*I*b*x^2*sqrt(-(4*a^2 - b^2)/b^2)*dilog(((-2*I*a*cos(x) + 2*a*sin(x) + (b*cos(x) + I*b*sin(x))*sqrt(-(4*a^2 - b^2)/b^2))*sqrt(-(b*sqrt(-(4*a^2 - b^2)/b^2) + 2*I*a)/b) - b)/b + 1) - 6*I*b*x^2*sqrt(-(4*a^2 - b^2)/b^2)*dilog(((2*I*a*cos(x) - 2*a*sin(x) + (b*cos(x) + I*b*sin(x))*sqrt(-(4*a^2 - b^2)/b^2))*sqrt((b*sqrt(-(4*a^2 - b^2)/b^2) - 2*I*a)/b) - b)/b + 1) - 6*I*b*x^2*sqrt(-(4*a^2 - b^2)/b^2)*dilog(((-2*I*a*cos(x) + 2*a*sin(x) - (b*cos(x) + I*b*sin(x))*sqrt(-(4*a^2 - b^2)/b^2))*sqrt((b*sqrt(-(4*a^2 - b^2)/b^2) - 2*I*a)/b) - b)/b + 1) - 6*I*b*x^2*sqrt(-(4*a^2 - b^2)/b^2)*dilog(-1/2*((4*I*a*cos(x) + 4*a*sin(x) + 2*(b*cos(x) - I*b*sin(x))*sqrt(-(4*a^2 - b^2)/b^2))*sqrt(-(b*sqrt(-(4*a^2 - b^2)/b^2) - 2*I*a)/b) + 2*b)/b + 1) - 6*I*b*x^2*sqrt(-(4*a^2 - b^2)/b^2)*dilog(-1/2*((-4*I*a*cos(x) - 4*a*sin(x) - 2*(b*cos(x) - I*b*sin(x))*sqrt(-(4*a^2 - b^2)/b^2))*sqrt(-(b*sqrt(-(4*a^2 - b^2)/b^2) - 2*I*a)/b) + 2*b)/b + 1) + 12*b*x*sqrt(-(4*a^2 - b^2)/b^2)*polylog(3, 1/2*(4*I*a*cos(x) + 4*a*sin(x) - 2*(b*cos(x) - I*b*sin(x))*sqrt(-(4*a^2 - b^2)/b^2))*sqrt((b*sqrt(-(4*a^2 - b^2)/b^2) + 2*I*a)/b)/b) + 12*b*x*sqrt(-(4*a^2 - b^2)/b^2)*polylog(3, 1/2*(-4*I*a*cos(x) - 4*a*sin(x) + 2*(b*cos(x) - I*b*sin(x))*sqrt(-(4*a^2 - b^2)/b^2))*sqrt((b*sqrt(-(4*a^2 - b^2)/b^2) + 2*I*a)/b)/b) - 12*b*x*sqrt(-(4*a^2 - b^2)/b^2)*polylog(3, -(2*I*a*cos(x) - 2*a*sin(x) - (b*cos(x) + I*b*sin(x))*sqrt(-(4*a^2 - b^2)/b^2))*sqrt(-(b*sqrt(-(4*a^2 - b^2)/b^2) + 2*I*a)/b)/b) - 12*b*x*sqrt(-(4*a^2 - b^2)/b^2)*polylog(3, -(-2*I*a*cos(x) + 2*a*sin(x) + (b*cos(x) + I*b*sin(x))*sqrt(-(4*a^2 - b^2)/b^2))*sqrt(-(b*sqrt(-(4*a^2 - b^2)/b^2) + 2*I*a)/b)/b) + 12*b*x*sqrt(-(4*a^2 - b^2)/b^2)*polylog(3, -(2*I*a*cos(x) - 2*a*sin(x) + (b*cos(x) + I*b*sin(x))*sqrt(-(4*a^2 - b^2)/b^2))*sqrt((b*sqrt(-(4*a^2 - b^2)/b^2) - 2*I*a)/b)/b) + 12*b*x*sqrt(-(4*a^2 - b^2)/b^2)*polylog(3, -(-2*I*a*cos(x) + 2*a*sin(x) - (b*cos(x) + I*b*sin(x))*sqrt(-(4*a^2 - b^2)/b^2))*sqrt((b*sqrt(-(4*a^2 - b^2)/b^2) - 2*I*a)/b)/b) - 12*b*x*sqrt(-(4*a^2 - b^2)/b^2)*polylog(3, 1/2*(4*I*a*cos(x) + 4*a*sin(x) + 2*(b*cos(x) - I*b*sin(x))*sqrt(-(4*a^2 - b^2)/b^2))*sqrt(-(b*sqrt(-(4*a^2 - b^2)/b^2) - 2*I*a)/b)/b) - 12*b*x*sqrt(-(4*a^2 - b^2)/b^2)*polylog(3, 1/2*(-4*I*a*cos(x) - 4*a*sin(x) - 2*(b*cos(x) - I*b*sin(x))*sqrt(-(4*a^2 - b^2)/b^2))*sqrt(-(b*sqrt(-(4*a^2 - b^2)/b^2) - 2*I*a)/b)/b) - 12*I*b*sqrt(-(4*a^2 - b^2)/b^2)*polylog(4, 1/2*(4*I*a*cos(x) + 4*a*sin(x) - 2*(b*cos(x) - I*b*sin(x))*sqrt(-(4*a^2 - b^2)/b^2))*sqrt((b*sqrt(-(4*a^2 - b^2)/b^2) + 2*I*a)/b)/b) - 12*I*b*sqrt(-(4*a^2 - b^2)/b^2)*polylog(4, 1/2*(-4*I*a*cos(x) - 4*a*sin(x) + 2*(b*cos(x) - I*b*sin(x))*sqrt(-(4*a^2 - b^2)/b^2))*sqrt((b*sqrt(-(4*a^2 - b^2)/b^2) + 2*I*a)/b)/b) - 12*I*b*sqrt(-(4*a^2 - b^2)/b^2)*polylog(4, -(2*I*a*cos(x) - 2*a*sin(x) - (b*cos(x) + I*b*sin(x))*sqrt(-(4*a^2 - b^2)/b^2)

```
)*sqrt(-(b*sqrt(-(4*a^2 - b^2)/b^2) + 2*I*a)/b)/b) - 12*I*b*sqrt(-(4*a^2 -
b^2)/b^2)*polylog(4, -(-2*I*a*cos(x) + 2*a*sin(x) + (b*cos(x) + I*b*sin(x))
*sqrt(-(4*a^2 - b^2)/b^2))*sqrt(-(b*sqrt(-(4*a^2 - b^2)/b^2) + 2*I*a)/b)/b)
 + 12*I*b*sqrt(-(4*a^2 - b^2)/b^2)*polylog(4, -(2*I*a*cos(x) - 2*a*sin(x) +
 (b*cos(x) + I*b*sin(x))*sqrt(-(4*a^2 - b^2)/b^2))*sqrt((b*sqrt(-(4*a^2 - b
^2)/b^2) - 2*I*a)/b)/b) + 12*I*b*sqrt(-(4*a^2 - b^2)/b^2)*polylog(4, -(-2*I
*a*cos(x) + 2*a*sin(x) - (b*cos(x) + I*b*sin(x))*sqrt(-(4*a^2 - b^2)/b^2))*
sqrt((b*sqrt(-(4*a^2 - b^2)/b^2) - 2*I*a)/b)/b) + 12*I*b*sqrt(-(4*a^2 - b^2
)/b^2)*polylog(4, 1/2*(4*I*a*cos(x) + 4*a*sin(x) + 2*(b*cos(x) - I*b*sin(x)
)*sqrt(-(4*a^2 - b^2)/b^2))*sqrt(-(b*sqrt(-(4*a^2 - b^2)/b^2) - 2*I*a)/b)/b
) + 12*I*b*sqrt(-(4*a^2 - b^2)/b^2)*polylog(4, 1/2*(-4*I*a*cos(x) - 4*a*sin
(x) - 2*(b*cos(x) - I*b*sin(x))*sqrt(-(4*a^2 - b^2)/b^2))*sqrt(-(b*sqrt(-(4
*a^2 - b^2)/b^2) - 2*I*a)/b)/b))/(4*a^2 - b^2)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{b\cos(x)\sin(x) + a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a+b*cos(x)*sin(x)),x, algorithm="giac")
```

```
[Out] integrate(x^3/(b*cos(x)*sin(x) + a), x)
```

maple [B] time = 0.30, size = 2282, normalized size = 4.95

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(a+b*cos(x)*sin(x)),x)
```

```
[Out] 8/(8*a^2-2*b^2)/(-2*I*a-(-(2*a-b)*(2*a+b))^(1/2))*ln(1-b*exp(2*I*x)/(-2*I*a
-(-(2*a-b)*(2*a+b))^(1/2)))*a^2*x^3-2/(8*a^2-2*b^2)/(-2*I*a-(-(2*a-b)*(2*a+
b))^(1/2))*ln(1-b*exp(2*I*x)/(-2*I*a-(-(2*a-b)*(2*a+b))^(1/2)))*b^2*x^3-2/(
8*a^2-2*b^2)/(-2*I*a-(-(2*a-b)*(2*a+b))^(1/2))*(-(2*a-b)*(2*a+b))^(1/2)*a*x
^4+12/(8*a^2-2*b^2)/(-2*I*a-(-(2*a-b)*(2*a+b))^(1/2))*polylog(3,b*exp(2*I*x
)/(-2*I*a-(-(2*a-b)*(2*a+b))^(1/2)))*a^2*x-3/(8*a^2-2*b^2)/(-2*I*a-(-(2*a-b
)*(2*a+b))^(1/2))*polylog(3,b*exp(2*I*x)/(-2*I*a-(-(2*a-b)*(2*a+b))^(1/2)))
*b^2*x+3*I/(8*a^2-2*b^2)/(-2*I*a-(-(2*a-b)*(2*a+b))^(1/2))*polylog(2,b*exp(
2*I*x)/(-2*I*a-(-(2*a-b)*(2*a+b))^(1/2)))*b^2*x^2-12*I/(8*a^2-2*b^2)/(-2*I*
a-(-(2*a-b)*(2*a+b))^(1/2))*polylog(2,b*exp(2*I*x)/(-2*I*a-(-(2*a-b)*(2*a+b
))^(1/2)))*a^2*x^2-12*I/(8*a^2-2*b^2)/(-2*I*a+(-(2*a-b)*(2*a+b))^(1/2))*pol
ylog(2,b*exp(2*I*x)/(-2*I*a+(-(2*a-b)*(2*a+b))^(1/2)))*a^2*x^2+3*I/(8*a^2-2
*b^2)/(-2*I*a+(-(2*a-b)*(2*a+b))^(1/2))*polylog(2,b*exp(2*I*x)/(-2*I*a+(-(2
```

```
*a-b)*(2*a+b))^(1/2)))*b^2*x^2+6/(8*a^2-2*b^2)/(-2*I*a+(-(2*a-b)*(2*a+b))^(
1/2))*polylog(2,b*exp(2*I*x)/(-2*I*a+(-(2*a-b)*(2*a+b))^(1/2)))*(-(2*a-b)*(
2*a+b))^(1/2)*a*x^2-6/(8*a^2-2*b^2)/(-2*I*a-(-(2*a-b)*(2*a+b))^(1/2))*polyl
og(2,b*exp(2*I*x)/(-2*I*a-(-(2*a-b)*(2*a+b))^(1/2)))*(-(2*a-b)*(2*a+b))^(1/
2)*a*x^2+4*I/(8*a^2-2*b^2)/(-2*I*a+(-(2*a-b)*(2*a+b))^(1/2))*ln(1-b*exp(2*I
*x)/(-2*I*a+(-(2*a-b)*(2*a+b))^(1/2)))*(-(2*a-b)*(2*a+b))^(1/2)*a*x^3+6*I/(
8*a^2-2*b^2)/(-2*I*a+(-(2*a-b)*(2*a+b))^(1/2))*(-(2*a-b)*(2*a+b))^(1/2)*pol
ylog(3,b*exp(2*I*x)/(-2*I*a+(-(2*a-b)*(2*a+b))^(1/2)))*a*x-4*I/(8*a^2-2*b^2
)/(-2*I*a-(-(2*a-b)*(2*a+b))^(1/2))*ln(1-b*exp(2*I*x)/(-2*I*a-(-(2*a-b)*(2*
a+b))^(1/2)))*(-(2*a-b)*(2*a+b))^(1/2)*a*x^3-6*I/(8*a^2-2*b^2)/(-2*I*a-(-(2
*a-b)*(2*a+b))^(1/2))*polylog(3,b*exp(2*I*x)/(-2*I*a-(-(2*a-b)*(2*a+b))^(1/
2)))*(-(2*a-b)*(2*a+b))^(1/2)*a*x+8/(8*a^2-2*b^2)/(-2*I*a+(-(2*a-b)*(2*a+b)
)^(1/2))*ln(1-b*exp(2*I*x)/(-2*I*a+(-(2*a-b)*(2*a+b))^(1/2)))*a^2*x^3-2/(8*
a^2-2*b^2)/(-2*I*a+(-(2*a-b)*(2*a+b))^(1/2))*ln(1-b*exp(2*I*x)/(-2*I*a+(-(2
*a-b)*(2*a+b))^(1/2)))*b^2*x^3+2/(8*a^2-2*b^2)/(-2*I*a+(-(2*a-b)*(2*a+b))^(
1/2))*(-(2*a-b)*(2*a+b))^(1/2)*a*x^4+12/(8*a^2-2*b^2)/(-2*I*a+(-(2*a-b)*(2*
a+b))^(1/2))*polylog(3,b*exp(2*I*x)/(-2*I*a+(-(2*a-b)*(2*a+b))^(1/2)))*a^2*
x-3/(8*a^2-2*b^2)/(-2*I*a+(-(2*a-b)*(2*a+b))^(1/2))*polylog(3,b*exp(2*I*x)/
(-2*I*a+(-(2*a-b)*(2*a+b))^(1/2)))*b^2*x-3/(8*a^2-2*b^2)/(-2*I*a+(-(2*a-b)*
(2*a+b))^(1/2))*(-(2*a-b)*(2*a+b))^(1/2)*polylog(4,b*exp(2*I*x)/(-2*I*a+(-(
2*a-b)*(2*a+b))^(1/2)))*a+I/(8*a^2-2*b^2)/(-2*I*a+(-(2*a-b)*(2*a+b))^(1/2))
*b^2*x^4+I/(8*a^2-2*b^2)/(-2*I*a-(-(2*a-b)*(2*a+b))^(1/2))*b^2*x^4-4*I/(8*a
^2-2*b^2)/(-2*I*a+(-(2*a-b)*(2*a+b))^(1/2))*a^2*x^4+6*I/(8*a^2-2*b^2)/(-2*I
*a+(-(2*a-b)*(2*a+b))^(1/2))*polylog(4,b*exp(2*I*x)/(-2*I*a+(-(2*a-b)*(2*a+
b))^(1/2)))*a^2-3/2*I/(8*a^2-2*b^2)/(-2*I*a+(-(2*a-b)*(2*a+b))^(1/2))*polyl
og(4,b*exp(2*I*x)/(-2*I*a+(-(2*a-b)*(2*a+b))^(1/2)))*b^2-4*I/(8*a^2-2*b^2)/
(-2*I*a-(-(2*a-b)*(2*a+b))^(1/2))*a^2*x^4+6*I/(8*a^2-2*b^2)/(-2*I*a-(-(2*a-
b)*(2*a+b))^(1/2))*polylog(4,b*exp(2*I*x)/(-2*I*a-(-(2*a-b)*(2*a+b))^(1/2))
)*a^2-3/2*I/(8*a^2-2*b^2)/(-2*I*a-(-(2*a-b)*(2*a+b))^(1/2))*polylog(4,b*exp
(2*I*x)/(-2*I*a-(-(2*a-b)*(2*a+b))^(1/2)))*b^2+3/(8*a^2-2*b^2)/(-2*I*a-(-(2
*a-b)*(2*a+b))^(1/2))*polylog(4,b*exp(2*I*x)/(-2*I*a-(-(2*a-b)*(2*a+b))^(1/
2)))*(-(2*a-b)*(2*a+b))^(1/2)*a
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{b\cos(x)\sin(x) + a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a+b*cos(x)*sin(x)),x, algorithm="maxima")
```

```
[Out] integrate(x^3/(b*cos(x)*sin(x) + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^3}{a + b\cos(x)\sin(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(a + b*cos(x)*sin(x)),x)
[Out] int(x^3/(a + b*cos(x)*sin(x)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(a+b*cos(x)*sin(x)),x)
[Out] Timed out
```

3.580 $\int \frac{x^2}{a+b\cos(x)\sin(x)}\,dx$

Optimal. Leaf size=340

$$-\frac{x\text{Li}_2\left(\frac{ibe^{2ix}}{2a-\sqrt{4a^2-b^2}}\right)}{\sqrt{4a^2-b^2}}+\frac{x\text{Li}_2\left(\frac{ibe^{2ix}}{2a+\sqrt{4a^2-b^2}}\right)}{\sqrt{4a^2-b^2}}-\frac{i\text{Li}_3\left(\frac{ibe^{2ix}}{2a-\sqrt{4a^2-b^2}}\right)}{2\sqrt{4a^2-b^2}}+\frac{i\text{Li}_3\left(\frac{ibe^{2ix}}{2a+\sqrt{4a^2-b^2}}\right)}{2\sqrt{4a^2-b^2}}-\frac{ix^2\log\left(1-\frac{ibe^{2ix}}{2a-\sqrt{4a^2-b^2}}\right)}{\sqrt{4a^2-b^2}}+\frac{ix^2\log}{}$$

```
[Out] -I*x^2*ln(1-I*b*exp(2*I*x)/(2*a-(4*a^2-b^2)^(1/2)))/(4*a^2-b^2)^(1/2)+I*x^2
*ln(1-I*b*exp(2*I*x)/(2*a+(4*a^2-b^2)^(1/2)))/(4*a^2-b^2)^(1/2)-x*polylog(2
,I*b*exp(2*I*x)/(2*a-(4*a^2-b^2)^(1/2)))/(4*a^2-b^2)^(1/2)+x*polylog(2,I*b*
exp(2*I*x)/(2*a+(4*a^2-b^2)^(1/2)))/(4*a^2-b^2)^(1/2)-1/2*I*polylog(3,I*b*e
xp(2*I*x)/(2*a-(4*a^2-b^2)^(1/2)))/(4*a^2-b^2)^(1/2)+1/2*I*polylog(3,I*b*ex
p(2*I*x)/(2*a+(4*a^2-b^2)^(1/2)))/(4*a^2-b^2)^(1/2)
```

Rubi [A] time = 0.54, antiderivative size = 340, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 7, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {4584, 3323, 2264, 2190, 2531, 2282, 6589}

$$-\frac{x\text{PolyLog}\left(2,\frac{ibe^{2ix}}{2a-\sqrt{4a^2-b^2}}\right)}{\sqrt{4a^2-b^2}}+\frac{x\text{PolyLog}\left(2,\frac{ibe^{2ix}}{\sqrt{4a^2-b^2}+2a}\right)}{\sqrt{4a^2-b^2}}-\frac{i\text{PolyLog}\left(3,\frac{ibe^{2ix}}{2a-\sqrt{4a^2-b^2}}\right)}{2\sqrt{4a^2-b^2}}+\frac{i\text{PolyLog}\left(3,\frac{ibe^{2ix}}{\sqrt{4a^2-b^2}+2a}\right)}{2\sqrt{4a^2-b^2}}-$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(a + b*Cos[x]*Sin[x]),x]
```

```
[Out] ((-I)*x^2*Log[1 - (I*b*E^((2*I)*x))/(2*a - Sqrt[4*a^2 - b^2])])/Sqrt[4*a^2
- b^2] + (I*x^2*Log[1 - (I*b*E^((2*I)*x))/(2*a + Sqrt[4*a^2 - b^2])])/Sqrt[
4*a^2 - b^2] - (x*PolyLog[2, (I*b*E^((2*I)*x))/(2*a - Sqrt[4*a^2 - b^2])])/
Sqrt[4*a^2 - b^2] + (x*PolyLog[2, (I*b*E^((2*I)*x))/(2*a + Sqrt[4*a^2 - b^2
])])/Sqrt[4*a^2 - b^2] - ((I/2)*PolyLog[3, (I*b*E^((2*I)*x))/(2*a - Sqrt[4*
a^2 - b^2])])/Sqrt[4*a^2 - b^2] + ((I/2)*PolyLog[3, (I*b*E^((2*I)*x))/(2*a
+ Sqrt[4*a^2 - b^2])])/Sqrt[4*a^2 - b^2]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2264

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c)/q, Int[
((f + g*x)^m*F^u)/(b - q + 2*c*F^u), x], x] - Dist[(2*c)/q, Int[((f + g*x)^
m*F^u)/(b + q + 2*c*F^u), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 3323

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Sy
mbol] :> Dist[2, Int[((c + d*x)^m*E^(I*(e + f*x)))/(I*b + 2*a*E^(I*(e + f*x
)) - I*b*E^(2*I*(e + f*x))), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[
a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 4584

```
Int[((e_.) + (f_.)*(x_))^(m_.)*((a_) + Cos[(c_.) + (d_.)*(x_)]*(b_.)*Sin[(c
_.) + (d_.)*(x_)])^(n_.), x_Symbol] :> Int[(e + f*x)^m*(a + (b*Sin[2*c + 2*
d*x])/2)^n, x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{x^2}{a+b\cos(x)\sin(x)}\,dx = \int \frac{x^2}{a+\frac{1}{2}b\sin(2x)}\,dx$$

$$= 2\int \frac{e^{2ix}x^2}{\frac{ib}{2}+2ae^{2ix}-\frac{1}{2}ibe^{4ix}}\,dx$$

$$= -\frac{(2ib)\int \frac{e^{2ix}x^2}{2a-\sqrt{4a^2-b^2}-ibe^{2ix}}\,dx}{\sqrt{4a^2-b^2}} + \frac{(2ib)\int \frac{e^{2ix}x^2}{2a+\sqrt{4a^2-b^2}-ibe^{2ix}}\,dx}{\sqrt{4a^2-b^2}}$$

$$= -\frac{ix^2\log\left(1-\frac{ibe^{2ix}}{2a-\sqrt{4a^2-b^2}}\right)}{\sqrt{4a^2-b^2}} + \frac{ix^2\log\left(1-\frac{ibe^{2ix}}{2a+\sqrt{4a^2-b^2}}\right)}{\sqrt{4a^2-b^2}} + \frac{(2i)\int x\log\left(1-\frac{ibe^{2ix}}{2a-\sqrt{4a^2-b^2}}\right)}{\sqrt{4a^2-b^2}}$$

$$= -\frac{ix^2\log\left(1-\frac{ibe^{2ix}}{2a-\sqrt{4a^2-b^2}}\right)}{\sqrt{4a^2-b^2}} + \frac{ix^2\log\left(1-\frac{ibe^{2ix}}{2a+\sqrt{4a^2-b^2}}\right)}{\sqrt{4a^2-b^2}} - \frac{x\operatorname{Li}_2\left(\frac{ibe^{2ix}}{2a-\sqrt{4a^2-b^2}}\right)}{\sqrt{4a^2-b^2}} + \frac{x\operatorname{Li}_2\left(\right.}{\sqrt{4a^2}}$$

$$= -\frac{ix^2\log\left(1-\frac{ibe^{2ix}}{2a-\sqrt{4a^2-b^2}}\right)}{\sqrt{4a^2-b^2}} + \frac{ix^2\log\left(1-\frac{ibe^{2ix}}{2a+\sqrt{4a^2-b^2}}\right)}{\sqrt{4a^2-b^2}} - \frac{x\operatorname{Li}_2\left(\frac{ibe^{2ix}}{2a-\sqrt{4a^2-b^2}}\right)}{\sqrt{4a^2-b^2}} + \frac{x\operatorname{Li}_2\left(\right.}{\sqrt{4a^2}}$$

$$= -\frac{ix^2\log\left(1-\frac{ibe^{2ix}}{2a-\sqrt{4a^2-b^2}}\right)}{\sqrt{4a^2-b^2}} + \frac{ix^2\log\left(1-\frac{ibe^{2ix}}{2a+\sqrt{4a^2-b^2}}\right)}{\sqrt{4a^2-b^2}} - \frac{x\operatorname{Li}_2\left(\frac{ibe^{2ix}}{2a-\sqrt{4a^2-b^2}}\right)}{\sqrt{4a^2-b^2}} + \frac{x\operatorname{Li}_2\left(\right.}{\sqrt{4a^2}}$$

Mathematica [A] time = 0.72, size = 256, normalized size = 0.75

$$-\frac{i\left(-2ix\operatorname{Li}_2\left(-\frac{ibe^{2ix}}{\sqrt{4a^2-b^2}-2a}\right)+2ix\operatorname{Li}_2\left(\frac{ibe^{2ix}}{2a+\sqrt{4a^2-b^2}}\right)+\operatorname{Li}_3\left(-\frac{ibe^{2ix}}{\sqrt{4a^2-b^2}-2a}\right)-\operatorname{Li}_3\left(\frac{ibe^{2ix}}{2a+\sqrt{4a^2-b^2}}\right)+2x^2\log\left(1+\frac{ibe^{2ix}}{\sqrt{4a^2-b^2}-\right.}\right.}{2\sqrt{4a^2-b^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(a + b*Cos[x]*Sin[x]),x]
```

```
[Out] ((-1/2*I)*(2*x^2*Log[1 + (I*b*E^((2*I)*x))/(-2*a + Sqrt[4*a^2 - b^2])] - 2*
x^2*Log[1 - (I*b*E^((2*I)*x))/(2*a + Sqrt[4*a^2 - b^2])] - (2*I)*x*PolyLog[
2, ((-I)*b*E^((2*I)*x))/(-2*a + Sqrt[4*a^2 - b^2])] + (2*I)*x*PolyLog[2, (I
*b*E^((2*I)*x))/(2*a + Sqrt[4*a^2 - b^2])] + PolyLog[3, ((-I)*b*E^((2*I)*x)
)/(-2*a + Sqrt[4*a^2 - b^2])] - PolyLog[3, (I*b*E^((2*I)*x))/(2*a + Sqrt[4*
a^2 - b^2])]))/Sqrt[4*a^2 - b^2]
```

fricas [C] time = 2.28, size = 2506, normalized size = 7.37

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a+b*cos(x)*sin(x)),x, algorithm="fricas")
```

```
[Out] -1/4*(2*b*x^2*sqrt(-(4*a^2 - b^2)/b^2)*log(1/2*((4*I*a*cos(x) + 4*a*sin(x)
- 2*(b*cos(x) - I*b*sin(x))*sqrt(-(4*a^2 - b^2)/b^2))*sqrt((b*sqrt(-(4*a^2
- b^2)/b^2) + 2*I*a)/b) + 2*b)/b) + 2*b*x^2*sqrt(-(4*a^2 - b^2)/b^2)*log(1/
2*((-4*I*a*cos(x) - 4*a*sin(x) + 2*(b*cos(x) - I*b*sin(x))*sqrt(-(4*a^2 - b
^2)/b^2))*sqrt((b*sqrt(-(4*a^2 - b^2)/b^2) + 2*I*a)/b) + 2*b)/b) - 2*b*x^2*
sqrt(-(4*a^2 - b^2)/b^2)*log(-((2*I*a*cos(x) - 2*a*sin(x) - (b*cos(x) + I*b
*sin(x))*sqrt(-(4*a^2 - b^2)/b^2))*sqrt(-(b*sqrt(-(4*a^2 - b^2)/b^2) + 2*I*
a)/b) - b)/b) - 2*b*x^2*sqrt(-(4*a^2 - b^2)/b^2)*log(-((-2*I*a*cos(x) + 2*a
*sin(x) + (b*cos(x) + I*b*sin(x))*sqrt(-(4*a^2 - b^2)/b^2))*sqrt(-(b*sqrt(-
(4*a^2 - b^2)/b^2) + 2*I*a)/b) - b)/b) + 2*b*x^2*sqrt(-(4*a^2 - b^2)/b^2)*l
og(-((2*I*a*cos(x) - 2*a*sin(x) + (b*cos(x) + I*b*sin(x))*sqrt(-(4*a^2 - b^
2)/b^2))*sqrt((b*sqrt(-(4*a^2 - b^2)/b^2) - 2*I*a)/b) - b)/b) + 2*b*x^2*sqr
t(-(4*a^2 - b^2)/b^2)*log(-((-2*I*a*cos(x) + 2*a*sin(x) - (b*cos(x) + I*b*s
in(x))*sqrt(-(4*a^2 - b^2)/b^2))*sqrt((b*sqrt(-(4*a^2 - b^2)/b^2) - 2*I*a)/
b) - b)/b) - 2*b*x^2*sqrt(-(4*a^2 - b^2)/b^2)*log(1/2*((4*I*a*cos(x) + 4*a*
sin(x) + 2*(b*cos(x) - I*b*sin(x))*sqrt(-(4*a^2 - b^2)/b^2))*sqrt(-(b*sqrt(
-(4*a^2 - b^2)/b^2) - 2*I*a)/b) + 2*b)/b) - 2*b*x^2*sqrt(-(4*a^2 - b^2)/b^2
)*log(1/2*((-4*I*a*cos(x) - 4*a*sin(x) - 2*(b*cos(x) - I*b*sin(x))*sqrt(-(4
*a^2 - b^2)/b^2))*sqrt(-(b*sqrt(-(4*a^2 - b^2)/b^2) - 2*I*a)/b) + 2*b)/b) +
 4*I*b*x*sqrt(-(4*a^2 - b^2)/b^2)*dilog(-1/2*((4*I*a*cos(x) + 4*a*sin(x) -
2*(b*cos(x) - I*b*sin(x))*sqrt(-(4*a^2 - b^2)/b^2))*sqrt((b*sqrt(-(4*a^2 -
b^2)/b^2) + 2*I*a)/b) + 2*b)/b + 1) + 4*I*b*x*sqrt(-(4*a^2 - b^2)/b^2)*dilo
g(-1/2*((-4*I*a*cos(x) - 4*a*sin(x) + 2*(b*cos(x) - I*b*sin(x))*sqrt(-(4*a^
2 - b^2)/b^2))*sqrt((b*sqrt(-(4*a^2 - b^2)/b^2) + 2*I*a)/b) + 2*b)/b + 1) +
 4*I*b*x*sqrt(-(4*a^2 - b^2)/b^2)*dilog(((2*I*a*cos(x) - 2*a*sin(x) - (b*co
s(x) + I*b*sin(x))*sqrt(-(4*a^2 - b^2)/b^2))*sqrt(-(b*sqrt(-(4*a^2 - b^2)/b
^2) + 2*I*a)/b) - b)/b + 1) + 4*I*b*x*sqrt(-(4*a^2 - b^2)/b^2)*dilog(((-2*I
*a*cos(x) + 2*a*sin(x) + (b*cos(x) + I*b*sin(x))*sqrt(-(4*a^2 - b^2)/b^2))*
sqrt(-(b*sqrt(-(4*a^2 - b^2)/b^2) + 2*I*a)/b) - b)/b + 1) - 4*I*b*x*sqrt(-(
4*a^2 - b^2)/b^2)*dilog(((2*I*a*cos(x) - 2*a*sin(x) + (b*cos(x) + I*b*sin(x
))*sqrt(-(4*a^2 - b^2)/b^2))*sqrt((b*sqrt(-(4*a^2 - b^2)/b^2) - 2*I*a)/b) -
 b)/b + 1) - 4*I*b*x*sqrt(-(4*a^2 - b^2)/b^2)*dilog(((-2*I*a*cos(x) + 2*a*s
in(x) - (b*cos(x) + I*b*sin(x))*sqrt(-(4*a^2 - b^2)/b^2))*sqrt((b*sqrt(-(4*
a^2 - b^2)/b^2) - 2*I*a)/b) - b)/b + 1) - 4*I*b*x*sqrt(-(4*a^2 - b^2)/b^2)*
dilog(-1/2*((4*I*a*cos(x) + 4*a*sin(x) + 2*(b*cos(x) - I*b*sin(x))*sqrt(-(4
*a^2 - b^2)/b^2))*sqrt(-(b*sqrt(-(4*a^2 - b^2)/b^2) - 2*I*a)/b) + 2*b)/b +
1) - 4*I*b*x*sqrt(-(4*a^2 - b^2)/b^2)*dilog(-1/2*((-4*I*a*cos(x) - 4*a*sin(
x) - 2*(b*cos(x) - I*b*sin(x))*sqrt(-(4*a^2 - b^2)/b^2))*sqrt(-(b*sqrt(-(4*
a^2 - b^2)/b^2) - 2*I*a)/b) + 2*b)/b + 1) + 4*b*sqrt(-(4*a^2 - b^2)/b^2)*po
lylog(3, 1/2*(4*I*a*cos(x) + 4*a*sin(x) - 2*(b*cos(x) - I*b*sin(x))*sqrt(-(
4*a^2 - b^2)/b^2))*sqrt((b*sqrt(-(4*a^2 - b^2)/b^2) + 2*I*a)/b)/b) + 4*b*sq
```

```
rt(-(4*a^2 - b^2)/b^2)*polylog(3, 1/2*(-4*I*a*cos(x) - 4*a*sin(x) + 2*(b*co
s(x) - I*b*sin(x))*sqrt(-(4*a^2 - b^2)/b^2))*sqrt((b*sqrt(-(4*a^2 - b^2)/b^
2) + 2*I*a)/b)/b) - 4*b*sqrt(-(4*a^2 - b^2)/b^2)*polylog(3, -(2*I*a*cos(x)
- 2*a*sin(x) - (b*cos(x) + I*b*sin(x))*sqrt(-(4*a^2 - b^2)/b^2))*sqrt(-(b*s
qrt(-(4*a^2 - b^2)/b^2) + 2*I*a)/b)/b) - 4*b*sqrt(-(4*a^2 - b^2)/b^2)*polyl
og(3, -(-2*I*a*cos(x) + 2*a*sin(x) + (b*cos(x) + I*b*sin(x))*sqrt(-(4*a^2 -
 b^2)/b^2))*sqrt(-(b*sqrt(-(4*a^2 - b^2)/b^2) + 2*I*a)/b)/b) + 4*b*sqrt(-(4
*a^2 - b^2)/b^2)*polylog(3, -(2*I*a*cos(x) - 2*a*sin(x) + (b*cos(x) + I*b*s
in(x))*sqrt(-(4*a^2 - b^2)/b^2))*sqrt((b*sqrt(-(4*a^2 - b^2)/b^2) - 2*I*a)/
b)/b) + 4*b*sqrt(-(4*a^2 - b^2)/b^2)*polylog(3, -(-2*I*a*cos(x) + 2*a*sin(x
) - (b*cos(x) + I*b*sin(x))*sqrt(-(4*a^2 - b^2)/b^2))*sqrt((b*sqrt(-(4*a^2
- b^2)/b^2) - 2*I*a)/b)/b) - 4*b*sqrt(-(4*a^2 - b^2)/b^2)*polylog(3, 1/2*(4
*I*a*cos(x) + 4*a*sin(x) + 2*(b*cos(x) - I*b*sin(x))*sqrt(-(4*a^2 - b^2)/b^
2))*sqrt(-(b*sqrt(-(4*a^2 - b^2)/b^2) - 2*I*a)/b)/b) - 4*b*sqrt(-(4*a^2 - b
^2)/b^2)*polylog(3, 1/2*(-4*I*a*cos(x) - 4*a*sin(x) - 2*(b*cos(x) - I*b*sin
(x))*sqrt(-(4*a^2 - b^2)/b^2))*sqrt(-(b*sqrt(-(4*a^2 - b^2)/b^2) - 2*I*a)/b
)/b))/(4*a^2 - b^2)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{b\cos(x)\sin(x) + a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a+b*cos(x)*sin(x)),x, algorithm="giac")
```

```
[Out] integrate(x^2/(b*cos(x)*sin(x) + a), x)
```

maple [B] time = 0.23, size = 1782, normalized size = 5.24

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a+b*cos(x)*sin(x)),x)
```

```
[Out] 8/3/(8*a^2-2*b^2)/(-2*I*a+(-(2*a-b)*(2*a+b))^(1/2))*(-(2*a-b)*(2*a+b))^(1/2
)*a*x^3+2*I/(8*a^2-2*b^2)/(-2*I*a+(-(2*a-b)*(2*a+b))^(1/2))*polylog(2,b*exp
(2*I*x)/(-2*I*a+(-(2*a-b)*(2*a+b))^(1/2)))*b^2*x-8*I/(8*a^2-2*b^2)/(-2*I*a+
(-(2*a-b)*(2*a+b))^(1/2))*polylog(2,b*exp(2*I*x)/(-2*I*a+(-(2*a-b)*(2*a+b))
^(1/2)))*a^2*x+2*I/(8*a^2-2*b^2)/(-2*I*a-(-(2*a-b)*(2*a+b))^(1/2))*polylog(
2,b*exp(2*I*x)/(-2*I*a-(-(2*a-b)*(2*a+b))^(1/2)))*b^2*x+8/(8*a^2-2*b^2)/(-2
*I*a+(-(2*a-b)*(2*a+b))^(1/2))*ln(1-b*exp(2*I*x)/(-2*I*a+(-(2*a-b)*(2*a+b))
^(1/2)))*a^2*x^2-2/(8*a^2-2*b^2)/(-2*I*a+(-(2*a-b)*(2*a+b))^(1/2))*ln(1-b*e
xp(2*I*x)/(-2*I*a+(-(2*a-b)*(2*a+b))^(1/2)))*b^2*x^2+4/(8*a^2-2*b^2)/(-2*I*
```

```
a+(-(2*a-b)*(2*a+b))^(1/2))*(-(2*a-b)*(2*a+b))^(1/2)*polylog(2,b*exp(2*I*x)
/(-2*I*a+(-(2*a-b)*(2*a+b))^(1/2)))*a*x-4*I/(8*a^2-2*b^2)/(-2*I*a-(-(2*a-b)
*(2*a+b))^(1/2))*ln(1-b*exp(2*I*x)/(-2*I*a-(-(2*a-b)*(2*a+b))^(1/2)))*(-(2*
a-b)*(2*a+b))^(1/2)*a*x^2+2*I/(8*a^2-2*b^2)/(-2*I*a+(-(2*a-b)*(2*a+b))^(1/2
))*polylog(3,b*exp(2*I*x)/(-2*I*a+(-(2*a-b)*(2*a+b))^(1/2)))*(-(2*a-b)*(2*a
+b))^(1/2)*a-16/3*I/(8*a^2-2*b^2)/(-2*I*a-(-(2*a-b)*(2*a+b))^(1/2))*a^2*x^3
+4/(8*a^2-2*b^2)/(-2*I*a+(-(2*a-b)*(2*a+b))^(1/2))*polylog(3,b*exp(2*I*x)/(
-2*I*a+(-(2*a-b)*(2*a+b))^(1/2)))*a^2-1/(8*a^2-2*b^2)/(-2*I*a+(-(2*a-b)*(2*
a+b))^(1/2))*polylog(3,b*exp(2*I*x)/(-2*I*a+(-(2*a-b)*(2*a+b))^(1/2)))*b^2-
8/3/(8*a^2-2*b^2)/(-2*I*a-(-(2*a-b)*(2*a+b))^(1/2))*(-(2*a-b)*(2*a+b))^(1/2
)*a*x^3-16/3*I/(8*a^2-2*b^2)/(-2*I*a+(-(2*a-b)*(2*a+b))^(1/2))*a^2*x^3+4/3*
I/(8*a^2-2*b^2)/(-2*I*a-(-(2*a-b)*(2*a+b))^(1/2))*b^2*x^3-8*I/(8*a^2-2*b^2)
/(-2*I*a-(-(2*a-b)*(2*a+b))^(1/2))*polylog(2,b*exp(2*I*x)/(-2*I*a-(-(2*a-b)
*(2*a+b))^(1/2)))*a^2*x+8/(8*a^2-2*b^2)/(-2*I*a-(-(2*a-b)*(2*a+b))^(1/2))*l
n(1-b*exp(2*I*x)/(-2*I*a-(-(2*a-b)*(2*a+b))^(1/2)))*a^2*x^2-2/(8*a^2-2*b^2)
/(-2*I*a-(-(2*a-b)*(2*a+b))^(1/2))*ln(1-b*exp(2*I*x)/(-2*I*a-(-(2*a-b)*(2*a
+b))^(1/2)))*b^2*x^2-4/(8*a^2-2*b^2)/(-2*I*a-(-(2*a-b)*(2*a+b))^(1/2))*poly
log(2,b*exp(2*I*x)/(-2*I*a-(-(2*a-b)*(2*a+b))^(1/2)))*(-(2*a-b)*(2*a+b))^(1
/2)*a*x+4/3*I/(8*a^2-2*b^2)/(-2*I*a+(-(2*a-b)*(2*a+b))^(1/2))*b^2*x^3-2*I/(
8*a^2-2*b^2)/(-2*I*a-(-(2*a-b)*(2*a+b))^(1/2))*polylog(3,b*exp(2*I*x)/(-2*I
*a-(-(2*a-b)*(2*a+b))^(1/2)))*(-(2*a-b)*(2*a+b))^(1/2)*a+4*I/(8*a^2-2*b^2)/
(-2*I*a+(-(2*a-b)*(2*a+b))^(1/2))*(-(2*a-b)*(2*a+b))^(1/2)*ln(1-b*exp(2*I*x
)/(-2*I*a+(-(2*a-b)*(2*a+b))^(1/2)))*a*x^2+4/(8*a^2-2*b^2)/(-2*I*a-(-(2*a-b
)*(2*a+b))^(1/2))*polylog(3,b*exp(2*I*x)/(-2*I*a-(-(2*a-b)*(2*a+b))^(1/2)))
*a^2-1/(8*a^2-2*b^2)/(-2*I*a-(-(2*a-b)*(2*a+b))^(1/2))*polylog(3,b*exp(2*I*
x)/(-2*I*a-(-(2*a-b)*(2*a+b))^(1/2)))*b^2
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{b\cos(x)\sin(x) + a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a+b*cos(x)*sin(x)),x, algorithm="maxima")
```

```
[Out] integrate(x^2/(b*cos(x)*sin(x) + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^2}{a + b\,\cos(x)\,\sin(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a + b*cos(x)*sin(x)),x)
```

```
[Out] int(x^2/(a + b*cos(x)*sin(x)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(a+b*cos(x)*sin(x)),x)
```

```
[Out] Timed out
```

3.581 $\int \frac{x}{a+b\cos(x)\sin(x)}\,dx$

Optimal. Leaf size=225

$$-\frac{\text{Li}_2\left(\frac{ibe^{2ix}}{2a-\sqrt{4a^2-b^2}}\right)}{2\sqrt{4a^2-b^2}}+\frac{\text{Li}_2\left(\frac{ibe^{2ix}}{2a+\sqrt{4a^2-b^2}}\right)}{2\sqrt{4a^2-b^2}}-\frac{ix\log\left(1-\frac{ibe^{2ix}}{2a-\sqrt{4a^2-b^2}}\right)}{\sqrt{4a^2-b^2}}+\frac{ix\log\left(1-\frac{ibe^{2ix}}{\sqrt{4a^2-b^2}+2a}\right)}{\sqrt{4a^2-b^2}}$$

```
[Out] -I*x*ln(1-I*b*exp(2*I*x)/(2*a-(4*a^2-b^2)^(1/2)))/(4*a^2-b^2)^(1/2)+I*x*ln(
1-I*b*exp(2*I*x)/(2*a+(4*a^2-b^2)^(1/2)))/(4*a^2-b^2)^(1/2)-1/2*polylog(2,I
*b*exp(2*I*x)/(2*a-(4*a^2-b^2)^(1/2)))/(4*a^2-b^2)^(1/2)+1/2*polylog(2,I*b*
exp(2*I*x)/(2*a+(4*a^2-b^2)^(1/2)))/(4*a^2-b^2)^(1/2)
```

Rubi [A] time = 0.32, antiderivative size = 225, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {4584, 3323, 2264, 2190, 2279, 2391}

$$-\frac{\text{PolyLog}\left(2,\frac{ibe^{2ix}}{2a-\sqrt{4a^2-b^2}}\right)}{2\sqrt{4a^2-b^2}}+\frac{\text{PolyLog}\left(2,\frac{ibe^{2ix}}{\sqrt{4a^2-b^2}+2a}\right)}{2\sqrt{4a^2-b^2}}-\frac{ix\log\left(1-\frac{ibe^{2ix}}{2a-\sqrt{4a^2-b^2}}\right)}{\sqrt{4a^2-b^2}}+\frac{ix\log\left(1-\frac{ibe^{2ix}}{\sqrt{4a^2-b^2}+2a}\right)}{\sqrt{4a^2-b^2}}$$

Antiderivative was successfully verified.

```
[In] Int[x/(a + b*Cos[x]*Sin[x]),x]
```

```
[Out] ((-I)*x*Log[1 - (I*b*E^((2*I)*x))/(2*a - Sqrt[4*a^2 - b^2])])/Sqrt[4*a^2 -
b^2] + (I*x*Log[1 - (I*b*E^((2*I)*x))/(2*a + Sqrt[4*a^2 - b^2])])/Sqrt[4*a^
2 - b^2] - PolyLog[2, (I*b*E^((2*I)*x))/(2*a - Sqrt[4*a^2 - b^2])]/(2*Sqrt[
4*a^2 - b^2]) + PolyLog[2, (I*b*E^((2*I)*x))/(2*a + Sqrt[4*a^2 - b^2])]/(2*
Sqrt[4*a^2 - b^2])
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2264

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c)/q, Int[
((f + g*x)^m*F^u)/(b - q + 2*c*F^u), x], x] - Dist[(2*c)/q, Int[((f + g*x)^
m*F^u)/(b + q + 2*c*F^u), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
```

```
2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3323

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Sy
mbol] :> Dist[2, Int[((c + d*x)^m*E^(I*(e + f*x)))/(I*b + 2*a*E^(I*(e + f*x
)) - I*b*E^(2*I*(e + f*x))), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[
a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 4584

```
Int[((e_.) + (f_.)*(x_))^(m_.)*((a_) + Cos[(c_.) + (d_.)*(x_)]*(b_.)*Sin[(c
_.) + (d_.)*(x_)])^(n_.), x_Symbol] :> Int[(e + f*x)^m*(a + (b*Sin[2*c + 2*
d*x])/2)^n, x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int \frac{x}{a+b\cos(x)\sin(x)}\,dx = \int \frac{x}{a+\frac{1}{2}b\sin(2x)}\,dx$$
$$= 2\int \frac{e^{2ix}x}{\frac{ib}{2}+2ae^{2ix}-\frac{1}{2}ibe^{4ix}}\,dx$$
$$= -\frac{(2ib)\int \frac{e^{2ix}x}{2a-\sqrt{4a^2-b^2}-ibe^{2ix}}\,dx}{\sqrt{4a^2-b^2}} + \frac{(2ib)\int \frac{e^{2ix}x}{2a+\sqrt{4a^2-b^2}-ibe^{2ix}}\,dx}{\sqrt{4a^2-b^2}}$$
$$= -\frac{ix\log\left(1-\frac{ibe^{2ix}}{2a-\sqrt{4a^2-b^2}}\right)}{\sqrt{4a^2-b^2}} + \frac{ix\log\left(1-\frac{ibe^{2ix}}{2a+\sqrt{4a^2-b^2}}\right)}{\sqrt{4a^2-b^2}} + \frac{i\int \log\left(1-\frac{ibe^{2ix}}{2a-\sqrt{4a^2-b^2}}\right)dx}{\sqrt{4a^2-b^2}} -$$
$$= -\frac{ix\log\left(1-\frac{ibe^{2ix}}{2a-\sqrt{4a^2-b^2}}\right)}{\sqrt{4a^2-b^2}} + \frac{ix\log\left(1-\frac{ibe^{2ix}}{2a+\sqrt{4a^2-b^2}}\right)}{\sqrt{4a^2-b^2}} + \frac{\text{Subst}\left(\int \frac{\log\left(1-\frac{ibx}{2a-\sqrt{4a^2-b^2}}\right)}{x}\,dx,\right.}{2\sqrt{4a^2-b^2}}$$
$$= -\frac{ix\log\left(1-\frac{ibe^{2ix}}{2a-\sqrt{4a^2-b^2}}\right)}{\sqrt{4a^2-b^2}} + \frac{ix\log\left(1-\frac{ibe^{2ix}}{2a+\sqrt{4a^2-b^2}}\right)}{\sqrt{4a^2-b^2}} - \frac{\text{Li}_2\left(\frac{ibe^{2ix}}{2a-\sqrt{4a^2-b^2}}\right)}{2\sqrt{4a^2-b^2}} + \frac{\text{Li}_2\left(\frac{ibe}{2a+\sqrt{4}}\right.}{2\sqrt{4a^2}}$$

Mathematica [B] time = 1.41, size = 788, normalized size = 3.50

$$\frac{1}{2}\left(\frac{\pi\tan^{-1}\left(\frac{2a\tan(x)+b}{\sqrt{4a^2-b^2}}\right)}{\sqrt{4a^2-b^2}} + \frac{i\left(\text{Li}_2\left(\frac{\left(2a-i\sqrt{b^2-4a^2}\right)\left(2a+b-\sqrt{b^2-4a^2}\cot\left(x+\frac{\pi}{4}\right)\right)}{b\left(2a+b+\sqrt{b^2-4a^2}\cot\left(x+\frac{\pi}{4}\right)\right)}\right) - \text{Li}_2\left(\frac{\left(2a+i\sqrt{b^2-4a^2}\right)\left(2a+b-\sqrt{b^2-4a^2}\cot\left(x+\frac{\pi}{4}\right)\right)}{b\left(2a+b+\sqrt{b^2-4a^2}\cot\left(x+\frac{\pi}{4}\right)\right)}\right)\right.}{}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(a + b*Cos[x]*Sin[x]),x]
```

```
[Out] ((Pi*ArcTan[(b + 2*a*Tan[x])/Sqrt[4*a^2 - b^2]])/Sqrt[4*a^2 - b^2] + (2*Arc
Cos[(-2*a)/b]*ArcTanh[((2*a - b)*Cot[Pi/4 + x])/Sqrt[-4*a^2 + b^2]] + (Pi -
 4*x)*ArcTanh[((2*a + b)*Tan[Pi/4 + x])/Sqrt[-4*a^2 + b^2]] - (ArcCos[(-2*a
)/b] + (2*I)*ArcTanh[((2*a - b)*Cot[Pi/4 + x])/Sqrt[-4*a^2 + b^2]])*Log[((2
*a + b)*(-2*a + b - I*Sqrt[-4*a^2 + b^2])*(1 + I*Cot[Pi/4 + x]))/(b*(2*a +
b + Sqrt[-4*a^2 + b^2]*Cot[Pi/4 + x]))] - (ArcCos[(-2*a)/b] - (2*I)*ArcTanh
[((2*a - b)*Cot[Pi/4 + x])/Sqrt[-4*a^2 + b^2]])*Log[((2*a + b)*((2*I)*a - I
*b + Sqrt[-4*a^2 + b^2])*(I + Cot[Pi/4 + x]))/(b*(2*a + b + Sqrt[-4*a^2 + b
^2]*Cot[Pi/4 + x]))] + (ArcCos[(-2*a)/b] + (2*I)*(ArcTanh[((2*a - b)*Cot[Pi
/4 + x])/Sqrt[-4*a^2 + b^2]] + ArcTanh[((2*a + b)*Tan[Pi/4 + x])/Sqrt[-4*a^
```

```
2 + b^2]]))*Log[((-1)^(1/4)*Sqrt[-4*a^2 + b^2])/(2*Sqrt[b]*E^(I*x)*Sqrt[a +
 b*Cos[x]*Sin[x]])] + (ArcCos[(-2*a)/b] - (2*I)*ArcTanh[((2*a - b)*Cot[Pi/4
 + x])/Sqrt[-4*a^2 + b^2]] - (2*I)*ArcTanh[((2*a + b)*Tan[Pi/4 + x])/Sqrt[-
4*a^2 + b^2]])*Log[-1/2*((-1)^(3/4)*Sqrt[-4*a^2 + b^2]*E^(I*x))/(Sqrt[b]*Sq
rt[a + b*Cos[x]*Sin[x]])] + I*(PolyLog[2, ((2*a - I*Sqrt[-4*a^2 + b^2])*(2*
a + b - Sqrt[-4*a^2 + b^2]*Cot[Pi/4 + x]))/(b*(2*a + b + Sqrt[-4*a^2 + b^2]
*Cot[Pi/4 + x]))] - PolyLog[2, ((2*a + I*Sqrt[-4*a^2 + b^2])*(2*a + b - Sqr
t[-4*a^2 + b^2]*Cot[Pi/4 + x]))/(b*(2*a + b + Sqrt[-4*a^2 + b^2]*Cot[Pi/4 +
 x]))]))/Sqrt[-4*a^2 + b^2])/2
```

fricas [B] time = 2.25, size = 1688, normalized size = 7.50

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*cos(x)*sin(x)),x, algorithm="fricas")
```

```
[Out] -1/4*(2*b*x*sqrt(-(4*a^2 - b^2)/b^2)*log(1/2*((4*I*a*cos(x) + 4*a*sin(x) -
2*(b*cos(x) - I*b*sin(x))*sqrt(-(4*a^2 - b^2)/b^2))*sqrt((b*sqrt(-(4*a^2 -
b^2)/b^2) + 2*I*a)/b) + 2*b)/b) + 2*b*x*sqrt(-(4*a^2 - b^2)/b^2)*log(1/2*((
-4*I*a*cos(x) - 4*a*sin(x) + 2*(b*cos(x) - I*b*sin(x))*sqrt(-(4*a^2 - b^2)/
b^2))*sqrt((b*sqrt(-(4*a^2 - b^2)/b^2) + 2*I*a)/b) + 2*b)/b) - 2*b*x*sqrt(-
(4*a^2 - b^2)/b^2)*log(-((2*I*a*cos(x) - 2*a*sin(x) - (b*cos(x) + I*b*sin(x
))*sqrt(-(4*a^2 - b^2)/b^2))*sqrt(-(b*sqrt(-(4*a^2 - b^2)/b^2) + 2*I*a)/b)
- b)/b) - 2*b*x*sqrt(-(4*a^2 - b^2)/b^2)*log(-((-2*I*a*cos(x) + 2*a*sin(x)
+ (b*cos(x) + I*b*sin(x))*sqrt(-(4*a^2 - b^2)/b^2))*sqrt(-(b*sqrt(-(4*a^2 -
 b^2)/b^2) + 2*I*a)/b) - b)/b) + 2*b*x*sqrt(-(4*a^2 - b^2)/b^2)*log(-((2*I*
a*cos(x) - 2*a*sin(x) + (b*cos(x) + I*b*sin(x))*sqrt(-(4*a^2 - b^2)/b^2))*s
qrt((b*sqrt(-(4*a^2 - b^2)/b^2) - 2*I*a)/b) - b)/b) + 2*b*x*sqrt(-(4*a^2 -
b^2)/b^2)*log(-((-2*I*a*cos(x) + 2*a*sin(x) - (b*cos(x) + I*b*sin(x))*sqrt(
-(4*a^2 - b^2)/b^2))*sqrt((b*sqrt(-(4*a^2 - b^2)/b^2) - 2*I*a)/b) - b)/b) -
 2*b*x*sqrt(-(4*a^2 - b^2)/b^2)*log(1/2*((4*I*a*cos(x) + 4*a*sin(x) + 2*(b*
cos(x) - I*b*sin(x))*sqrt(-(4*a^2 - b^2)/b^2))*sqrt(-(b*sqrt(-(4*a^2 - b^2)
/b^2) - 2*I*a)/b) + 2*b)/b) - 2*b*x*sqrt(-(4*a^2 - b^2)/b^2)*log(1/2*((-4*I
*a*cos(x) - 4*a*sin(x) - 2*(b*cos(x) - I*b*sin(x))*sqrt(-(4*a^2 - b^2)/b^2)
)*sqrt(-(b*sqrt(-(4*a^2 - b^2)/b^2) - 2*I*a)/b) + 2*b)/b) + 2*I*b*sqrt(-(4*
a^2 - b^2)/b^2)*dilog(-1/2*((4*I*a*cos(x) + 4*a*sin(x) - 2*(b*cos(x) - I*b*
sin(x))*sqrt(-(4*a^2 - b^2)/b^2))*sqrt((b*sqrt(-(4*a^2 - b^2)/b^2) + 2*I*a)
/b) + 2*b)/b + 1) + 2*I*b*sqrt(-(4*a^2 - b^2)/b^2)*dilog(-1/2*((-4*I*a*cos(
x) - 4*a*sin(x) + 2*(b*cos(x) - I*b*sin(x))*sqrt(-(4*a^2 - b^2)/b^2))*sqrt(
(b*sqrt(-(4*a^2 - b^2)/b^2) + 2*I*a)/b) + 2*b)/b + 1) + 2*I*b*sqrt(-(4*a^2
- b^2)/b^2)*dilog(((2*I*a*cos(x) - 2*a*sin(x) - (b*cos(x) + I*b*sin(x))*sqr
t(-(4*a^2 - b^2)/b^2))*sqrt(-(b*sqrt(-(4*a^2 - b^2)/b^2) + 2*I*a)/b) - b)/b
 + 1) + 2*I*b*sqrt(-(4*a^2 - b^2)/b^2)*dilog(((-2*I*a*cos(x) + 2*a*sin(x) +
 (b*cos(x) + I*b*sin(x))*sqrt(-(4*a^2 - b^2)/b^2))*sqrt(-(b*sqrt(-(4*a^2 -
```

```
b^2)/b^2) + 2*I*a)/b) - b)/b + 1) - 2*I*b*sqrt(-(4*a^2 - b^2)/b^2)*dilog(((
2*I*a*cos(x) - 2*a*sin(x) + (b*cos(x) + I*b*sin(x))*sqrt(-(4*a^2 - b^2)/b^2
))*sqrt((b*sqrt(-(4*a^2 - b^2)/b^2) - 2*I*a)/b) - b)/b + 1) - 2*I*b*sqrt(-(
4*a^2 - b^2)/b^2)*dilog(((-2*I*a*cos(x) + 2*a*sin(x) - (b*cos(x) + I*b*sin(
x))*sqrt(-(4*a^2 - b^2)/b^2))*sqrt((b*sqrt(-(4*a^2 - b^2)/b^2) - 2*I*a)/b)
- b)/b + 1) - 2*I*b*sqrt(-(4*a^2 - b^2)/b^2)*dilog(-1/2*((4*I*a*cos(x) + 4*
a*sin(x) + 2*(b*cos(x) - I*b*sin(x))*sqrt(-(4*a^2 - b^2)/b^2))*sqrt(-(b*sqr
t(-(4*a^2 - b^2)/b^2) - 2*I*a)/b) + 2*b)/b + 1) - 2*I*b*sqrt(-(4*a^2 - b^2)
/b^2)*dilog(-1/2*((-4*I*a*cos(x) - 4*a*sin(x) - 2*(b*cos(x) - I*b*sin(x))*s
qrt(-(4*a^2 - b^2)/b^2))*sqrt(-(b*sqrt(-(4*a^2 - b^2)/b^2) - 2*I*a)/b) + 2*
b)/b + 1))/(4*a^2 - b^2)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{b\cos(x)\sin(x) + a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*cos(x)*sin(x)),x, algorithm="giac")
```

```
[Out] integrate(x/(b*cos(x)*sin(x) + a), x)
```

maple [B] time = 0.22, size = 1284, normalized size = 5.71

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a+b*cos(x)*sin(x)),x)
```

```
[Out] 4*I/(8*a^2-2*b^2)/(-2*I*a+(-(2*a-b)*(2*a+b))^(1/2))*ln(1-b*exp(2*I*x)/(-2*I
*a+(-(2*a-b)*(2*a+b))^(1/2)))*(-(2*a-b)*(2*a+b))^(1/2)*a*x-4/(8*a^2-2*b^2)/
(-2*I*a-(-(2*a-b)*(2*a+b))^(1/2))*(-(2*a-b)*(2*a+b))^(1/2)*a*x^2-2/(8*a^2-2
*b^2)/(-2*I*a+(-(2*a-b)*(2*a+b))^(1/2))*ln(1-b*exp(2*I*x)/(-2*I*a+(-(2*a-b)
*(2*a+b))^(1/2)))*b^2*x+4/(8*a^2-2*b^2)/(-2*I*a+(-(2*a-b)*(2*a+b))^(1/2))*(
-(2*a-b)*(2*a+b))^(1/2)*a*x^2-2/(8*a^2-2*b^2)/(-2*I*a-(-(2*a-b)*(2*a+b))^(1
/2))*polylog(2,b*exp(2*I*x)/(-2*I*a-(-(2*a-b)*(2*a+b))^(1/2)))*(-(2*a-b)*(2
*a+b))^(1/2)*a-8*I/(8*a^2-2*b^2)/(-2*I*a+(-(2*a-b)*(2*a+b))^(1/2))*a^2*x^2+
2*I/(8*a^2-2*b^2)/(-2*I*a+(-(2*a-b)*(2*a+b))^(1/2))*b^2*x^2-4*I/(8*a^2-2*b^
2)/(-2*I*a+(-(2*a-b)*(2*a+b))^(1/2))*polylog(2,b*exp(2*I*x)/(-2*I*a+(-(2*a-
b)*(2*a+b))^(1/2)))*a^2+I/(8*a^2-2*b^2)/(-2*I*a+(-(2*a-b)*(2*a+b))^(1/2))*p
olylog(2,b*exp(2*I*x)/(-2*I*a+(-(2*a-b)*(2*a+b))^(1/2)))*b^2-4*I/(8*a^2-2*b
^2)/(-2*I*a-(-(2*a-b)*(2*a+b))^(1/2))*ln(1-b*exp(2*I*x)/(-2*I*a-(-(2*a-b)*(
2*a+b))^(1/2)))*(-(2*a-b)*(2*a+b))^(1/2)*a*x+2/(8*a^2-2*b^2)/(-2*I*a+(-(2*a
-b)*(2*a+b))^(1/2))*polylog(2,b*exp(2*I*x)/(-2*I*a+(-(2*a-b)*(2*a+b))^(1/2)
))*(-(2*a-b)*(2*a+b))^(1/2)*a+8/(8*a^2-2*b^2)/(-2*I*a-(-(2*a-b)*(2*a+b))^(1
```

```
/2))*ln(1-b*exp(2*I*x)/(-2*I*a-(-(2*a-b)*(2*a+b))^(1/2)))*a^2*x+8/(8*a^2-2*
b^2)/(-2*I*a+(-(2*a-b)*(2*a+b))^(1/2))*ln(1-b*exp(2*I*x)/(-2*I*a+(-(2*a-b)*
(2*a+b))^(1/2)))*a^2*x-2/(8*a^2-2*b^2)/(-2*I*a-(-(2*a-b)*(2*a+b))^(1/2))*ln
(1-b*exp(2*I*x)/(-2*I*a-(-(2*a-b)*(2*a+b))^(1/2)))*b^2*x-8*I/(8*a^2-2*b^2)/
(-2*I*a-(-(2*a-b)*(2*a+b))^(1/2))*a^2*x^2-4*I/(8*a^2-2*b^2)/(-2*I*a-(-(2*a-
b)*(2*a+b))^(1/2))*polylog(2,b*exp(2*I*x)/(-2*I*a-(-(2*a-b)*(2*a+b))^(1/2))
)*a^2+I/(8*a^2-2*b^2)/(-2*I*a-(-(2*a-b)*(2*a+b))^(1/2))*polylog(2,b*exp(2*I
*x)/(-2*I*a-(-(2*a-b)*(2*a+b))^(1/2)))*b^2+2*I/(8*a^2-2*b^2)/(-2*I*a-(-(2*a
-b)*(2*a+b))^(1/2))*b^2*x^2
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{b\cos(x)\sin(x) + a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*cos(x)*sin(x)),x, algorithm="maxima")
```

```
[Out] integrate(x/(b*cos(x)*sin(x) + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x}{a + b\,\cos(x)\,\sin(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a + b*cos(x)*sin(x)),x)
```

```
[Out] int(x/(a + b*cos(x)*sin(x)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{a + b\sin(x)\cos(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*cos(x)*sin(x)),x)
```

```
[Out] Integral(x/(a + b*sin(x)*cos(x)), x)
```

3.582 $\int \frac{1}{x(a+b\cos(x)\sin(x))}\,dx$

Optimal. Leaf size=20

$$\mathrm{Int}\left(\frac{1}{x\left(a+\frac{1}{2}b\sin(2x)\right)},x\right)$$

```
[Out] Unintegrable(1/x/(a+1/2*b*sin(2*x)),x)
```

Rubi [A] time = 0.09, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{x(a+b\cos(x)\sin(x))}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[1/(x*(a + b*Cos[x]*Sin[x])),x]
```

```
[Out] Defer[Int][1/(x*(a + (b*Sin[2*x])/2)), x]
```

Rubi steps

$$\int \frac{1}{x(a+b\cos(x)\sin(x))}\,dx=\int \frac{1}{x\left(a+\frac{1}{2}b\sin(2x)\right)}\,dx$$

Mathematica [A] time = 1.58, size = 0, normalized size = 0.00

$$\int \frac{1}{x(a+b\cos(x)\sin(x))}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/(x*(a + b*Cos[x]*Sin[x])),x]
```

```
[Out] Integrate[1/(x*(a + b*Cos[x]*Sin[x])), x]
```

fricas [A] time = 0.69, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{1}{bx\cos(x)\sin(x)+ax},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b*cos(x)*sin(x)),x, algorithm="fricas")
[Out] integral(1/(b*x*cos(x)*sin(x) + a*x), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(b\cos(x)\sin(x) + a)x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b*cos(x)*sin(x)),x, algorithm="giac")
[Out] integrate(1/((b*cos(x)*sin(x) + a)*x), x)
```

maple [A] time = 0.25, size = 0, normalized size = 0.00

$$\int \frac{1}{x\,(a + b\cos(x)\sin(x))}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(a+b*cos(x)*sin(x)),x)
[Out] int(1/x/(a+b*cos(x)*sin(x)),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(b\cos(x)\sin(x) + a)x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b*cos(x)*sin(x)),x, algorithm="maxima")
[Out] integrate(1/((b*cos(x)*sin(x) + a)*x), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.05

$$\int \frac{1}{x\,\left(a + b\,\cos(x)\,\sin(x)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(a + b*cos(x)*sin(x))),x)
[Out] int(1/(x*(a + b*cos(x)*sin(x))), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x\left(a+b\sin\left(x\right)\cos\left(x\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b*cos(x)*sin(x)),x)
```

```
[Out] Integral(1/(x*(a + b*sin(x)*cos(x))), x)
```

3.583 $\int \frac{(bx)^{2-n}\sin^n(ax)}{(acx\cos(ax)-c\sin(ax))^2}\,dx$

Optimal. Leaf size=79

$$\frac{b^2(1-n)\mathrm{Int}\left((bx)^{-n}\sin^{n-2}(ax),x\right)}{a^2c^2}+\frac{b(bx)^{1-n}\sin^{n-1}(ax)}{a^2\left(ac^2x\cos(ax)-c^2\sin(ax)\right)}$$

```
[Out] b*(b*x)^(1-n)*sin(a*x)^(-1+n)/a^2/(a*c^2*x*cos(a*x)-c^2*sin(a*x))+b^2*(1-n)
*Unintegrable(sin(a*x)^(-2+n)/((b*x)^n),x)/a^2/c^2
```

Rubi [A] time = 0.16, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{(bx)^{2-n}\sin^n(ax)}{(acx\cos(ax)-c\sin(ax))^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[((b*x)^(2 - n)*Sin[a*x]^n)/(a*c*x*Cos[a*x] - c*Sin[a*x])^2,x]
```

```
[Out] (b*(b*x)^(1 - n)*Sin[a*x]^(-1 + n))/(a^2*(a*c^2*x*Cos[a*x] - c^2*Sin[a*x]))
 + (b^2*(1 - n)*Defer[Int][Sin[a*x]^(-2 + n)/(b*x)^n, x])/(a^2*c^2)
```

Rubi steps

$$\int \frac{(bx)^{2-n}\sin^n(ax)}{(acx\cos(ax)-c\sin(ax))^2}\,dx=\frac{b(bx)^{1-n}\sin^{-1+n}(ax)}{a^2\left(ac^2x\cos(ax)-c^2\sin(ax)\right)}+\frac{\left(b^2(1-n)\right)\int(bx)^{-n}\sin^{-2+n}(ax)\,dx}{a^2c^2}$$

Mathematica [A] time = 5.46, size = 0, normalized size = 0.00

$$\int \frac{(bx)^{2-n}\sin^n(ax)}{(acx\cos(ax)-c\sin(ax))^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[((b*x)^(2 - n)*Sin[a*x]^n)/(a*c*x*Cos[a*x] - c*Sin[a*x])^2,x]
```

```
[Out] Integrate[((b*x)^(2 - n)*Sin[a*x]^n)/(a*c*x*Cos[a*x] - c*Sin[a*x])^2, x]
```

fricas [A] time = 0.96, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(-\frac{(bx)^{-n+2}\sin(ax)^n}{2\,ac^2x\cos(ax)\sin(ax)-\left(a^2c^2x^2-c^2\right)\cos(ax)^2-c^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x)^(2-n)*sin(a*x)^n/(a*c*x*cos(a*x)-c*sin(a*x))^2,x, algorithm
="fricas")
```

```
[Out] integral(-(b*x)^(-n + 2)*sin(a*x)^n/(2*a*c^2*x*cos(a*x)*sin(a*x) - (a^2*c^2
*x^2 - c^2)*cos(a*x)^2 - c^2), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(bx)^{-n+2} \sin(ax)^n}{(acx\cos(ax) - c\sin(ax))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x)^(2-n)*sin(a*x)^n/(a*c*x*cos(a*x)-c*sin(a*x))^2,x, algorithm
="giac")
```

```
[Out] integrate((b*x)^(-n + 2)*sin(a*x)^n/(a*c*x*cos(a*x) - c*sin(a*x))^2, x)
```

maple [A] time = 2.19, size = 0, normalized size = 0.00

$$\int \frac{(bx)^{2-n} (\sin^n(ax))}{(acx\cos(ax) - c\sin(ax))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x)^(2-n)*sin(a*x)^n/(a*c*x*cos(a*x)-c*sin(a*x))^2,x)
```

```
[Out] int((b*x)^(2-n)*sin(a*x)^n/(a*c*x*cos(a*x)-c*sin(a*x))^2,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(bx)^{-n+2} \sin(ax)^n}{(acx\cos(ax) - c\sin(ax))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x)^(2-n)*sin(a*x)^n/(a*c*x*cos(a*x)-c*sin(a*x))^2,x, algorithm
="maxima")
```

```
[Out] integrate((b*x)^(-n + 2)*sin(a*x)^n/(a*c*x*cos(a*x) - c*sin(a*x))^2, x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sin(a\,x)^n\,(b\,x)^{2-n}}{(c\,\sin(a\,x) - a\,c\,x\,\cos(a\,x))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((sin(a*x)^n*(b*x)^(2 - n))/(c*sin(a*x) - a*c*x*cos(a*x))^2,x)
[Out] int((sin(a*x)^n*(b*x)^(2 - n))/(c*sin(a*x) - a*c*x*cos(a*x))^2, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x)**(2-n)*sin(a*x)**n/(a*c*x*cos(a*x)-c*sin(a*x))**2,x)
[Out] Timed out
```

3.584 $\int \frac{(bx)^{2-n}\cos^n(ax)}{(c\cos(ax)+acx\sin(ax))^2}\,dx$

Optimal. Leaf size=79

$$\frac{b^2(1-n)\mathrm{Int}\left((bx)^{-n}\cos^{n-2}(ax),x\right)}{a^2c^2}-\frac{b(bx)^{1-n}\cos^{n-1}(ax)}{a^2\left(ac^2x\sin(ax)+c^2\cos(ax)\right)}$$

```
[Out] -b*(b*x)^(1-n)*cos(a*x)^(-1+n)/a^2/(c^2*cos(a*x)+a*c^2*x*sin(a*x))+b^2*(1-n
)*Unintegrable(cos(a*x)^(-2+n)/((b*x)^n),x)/a^2/c^2
```

Rubi [A] time = 0.15, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{(bx)^{2-n}\cos^n(ax)}{(c\cos(ax)+acx\sin(ax))^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[((b*x)^(2 - n)*Cos[a*x]^n)/(c*Cos[a*x] + a*c*x*Sin[a*x])^2,x]
```

```
[Out] -((b*(b*x)^(1 - n)*Cos[a*x]^(-1 + n))/(a^2*(c^2*Cos[a*x] + a*c^2*x*Sin[a*x]
))) + (b^2*(1 - n)*Defer[Int][Cos[a*x]^(-2 + n)/(b*x)^n, x])/(a^2*c^2)
```

Rubi steps

$$\int \frac{(bx)^{2-n}\cos^n(ax)}{(c\cos(ax)+acx\sin(ax))^2}\,dx=-\frac{b(bx)^{1-n}\cos^{-1+n}(ax)}{a^2\left(c^2\cos(ax)+ac^2x\sin(ax)\right)}+\frac{\left(b^2(1-n)\right)\int (bx)^{-n}\cos^{-2+n}(ax)\,dx}{a^2c^2}$$

Mathematica [A] time = 4.97, size = 0, normalized size = 0.00

$$\int \frac{(bx)^{2-n}\cos^n(ax)}{(c\cos(ax)+acx\sin(ax))^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[((b*x)^(2 - n)*Cos[a*x]^n)/(c*Cos[a*x] + a*c*x*Sin[a*x])^2,x]
```

```
[Out] Integrate[((b*x)^(2 - n)*Cos[a*x]^n)/(c*Cos[a*x] + a*c*x*Sin[a*x])^2, x]
```

fricas [A] time = 0.91, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{(bx)^{-n+2}\cos(ax)^n}{a^2c^2x^2+2\,ac^2x\cos(ax)\sin(ax)-\left(a^2c^2x^2-c^2\right)\cos(ax)^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x)^(2-n)*cos(a*x)^n/(c*cos(a*x)+a*c*x*sin(a*x))^2,x, algorithm
="fricas")
```

```
[Out] integral((b*x)^(-n + 2)*cos(a*x)^n/(a^2*c^2*x^2 + 2*a*c^2*x*cos(a*x)*sin(a*
x) - (a^2*c^2*x^2 - c^2)*cos(a*x)^2), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(bx)^{-n+2}\cos(ax)^{n}}{(acx\sin(ax)+c\cos(ax))^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x)^(2-n)*cos(a*x)^n/(c*cos(a*x)+a*c*x*sin(a*x))^2,x, algorithm
="giac")
```

```
[Out] integrate((b*x)^(-n + 2)*cos(a*x)^n/(a*c*x*sin(a*x) + c*cos(a*x))^2, x)
```

maple [A] time = 2.19, size = 0, normalized size = 0.00

$$\int \frac{(bx)^{2-n}(\cos^{n}(ax))}{(c\cos(ax)+acx\sin(ax))^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x)^(2-n)*cos(a*x)^n/(c*cos(a*x)+a*c*x*sin(a*x))^2,x)
```

```
[Out] int((b*x)^(2-n)*cos(a*x)^n/(c*cos(a*x)+a*c*x*sin(a*x))^2,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(bx)^{-n+2}\cos(ax)^{n}}{(acx\sin(ax)+c\cos(ax))^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x)^(2-n)*cos(a*x)^n/(c*cos(a*x)+a*c*x*sin(a*x))^2,x, algorithm
="maxima")
```

```
[Out] integrate((b*x)^(-n + 2)*cos(a*x)^n/(a*c*x*sin(a*x) + c*cos(a*x))^2, x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos(a\,x)^{n}\,(b\,x)^{2-n}}{(c\,\cos(a\,x)+a\,c\,x\,\sin(a\,x))^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(a*x)^n*(b*x)^(2 - n))/(c*cos(a*x) + a*c*x*sin(a*x))^2,x)
[Out] int((cos(a*x)^n*(b*x)^(2 - n))/(c*cos(a*x) + a*c*x*sin(a*x))^2, x)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x)**(2-n)*cos(a*x)**n/(c*cos(a*x)+a*c*x*sin(a*x))**2,x)
[Out] Timed out
```

3.585 $\int \frac{\sin^6(ax)}{x^4(ax\cos(ax)-\sin(ax))^2}\,dx$

Optimal. Leaf size=175

$$-\frac{2}{3}a^3\text{Si}(2ax)+\frac{16}{3}a^3\text{Si}(4ax)+\frac{\sin^4(ax)}{a^2x^5}+\frac{\sin^5(ax)}{a^2x^5(ax\cos(ax)-\sin(ax))}+\frac{a^2}{x}+\frac{32a^2\sin^4(ax)}{3x}-\frac{10a^2\sin^2(ax)}{x}+\frac{\sin^3(ax)\,\text{c}}{ax^4}$$

```
[Out] a^2/x-2/3*a^3*Si(2*a*x)+16/3*a^3*Si(4*a*x)+a*cos(a*x)*sin(a*x)/x^2+sin(a*x)
^2/x^3-10*a^2*sin(a*x)^2/x+cos(a*x)*sin(a*x)^3/a/x^4-8/3*a*cos(a*x)*sin(a*x
)^3/x^2+sin(a*x)^4/a^2/x^5-4/3*sin(a*x)^4/x^3+32/3*a^2*sin(a*x)^4/x+sin(a*x
)^5/a^2/x^5/(a*x*cos(a*x)-sin(a*x))
```

Rubi [A] time = 0.30, antiderivative size = 175, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {4598, 3314, 30, 3313, 12, 3299}

$$-\frac{2}{3}a^3\text{Si}(2ax)+\frac{16}{3}a^3\text{Si}(4ax)+\frac{\sin^4(ax)}{a^2x^5}+\frac{\sin^5(ax)}{a^2x^5(ax\cos(ax)-\sin(ax))}+\frac{a^2}{x}+\frac{32a^2\sin^4(ax)}{3x}-\frac{10a^2\sin^2(ax)}{x}-\frac{4\sin^4(ax)}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a*x]^6/(x^4*(a*x*Cos[a*x] - Sin[a*x])^2),x]
```

```
[Out] a^2/x + (a*Cos[a*x]*Sin[a*x])/x^2 + Sin[a*x]^2/x^3 - (10*a^2*Sin[a*x]^2)/x
+ (Cos[a*x]*Sin[a*x]^3)/(a*x^4) - (8*a*Cos[a*x]*Sin[a*x]^3)/(3*x^2) + Sin[a
*x]^4/(a^2*x^5) - (4*Sin[a*x]^4)/(3*x^3) + (32*a^2*Sin[a*x]^4)/(3*x) + Sin[
a*x]^5/(a^2*x^5*(a*x*Cos[a*x] - Sin[a*x])) - (2*a^3*SinIntegral[2*a*x])/3 +
 (16*a^3*SinIntegral[4*a*x])/3
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3313

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> Si
mp[((c + d*x)^(m + 1)*Sin[e + f*x]^n)/(d*(m + 1)), x] - Dist[(f*n)/(d*(m +
1)), Int[ExpandTrigReduce[(c + d*x)^(m + 1), Cos[e + f*x]*Sin[e + f*x]^(n -
 1), x], x], x] /; FreeQ[{c, d, e, f, m}, x] && IGtQ[n, 1] && GeQ[m, -2] &&
 LtQ[m, -1]
```

Rule 3314

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[((c + d*x)^(m + 1)*(b*Sin[e + f*x])^n)/(d*(m + 1)), x] + (Dist[(
b^2*f^2*n*(n - 1))/(d^2*(m + 1)*(m + 2)), Int[(c + d*x)^(m + 2)*(b*Sin[e +
f*x])^(n - 2), x], x] - Dist[(f^2*n^2)/(d^2*(m + 1)*(m + 2)), Int[(c + d*x)
^(m + 2)*(b*Sin[e + f*x])^n, x], x] - Simp[(b*f*n*(c + d*x)^(m + 2)*Cos[e +
 f*x]*(b*Sin[e + f*x])^(n - 1))/(d^2*(m + 1)*(m + 2)), x]) /; FreeQ[{b, c,
d, e, f}, x] && GtQ[n, 1] && LtQ[m, -2]
```

Rule 4598

```
Int[(((b_.)*(x_))^(m_)*Sin[(a_.)*(x_)]^(n_))/(Cos[(a_.)*(x_)]*(d_.)*(x_) +
(c_.)*Sin[(a_.)*(x_)])^2, x_Symbol] :> Simp[(b*(b*x)^(m - 1)*Sin[a*x]^(n -
1))/(a*d*(c*Sin[a*x] + d*x*Cos[a*x])), x] - Dist[(b^2*(n - 1))/d^2, Int[(b*
x)^(m - 2)*Sin[a*x]^(n - 2), x], x] /; FreeQ[{a, b, c, d, m, n}, x] && EqQ[
a*c + d, 0] && EqQ[m, 2 - n]
```

Rubi steps

$$\int \frac{\sin^6(ax)}{x^4(ax\cos(ax)-\sin(ax))^2}\,dx = \frac{\sin^5(ax)}{a^2x^5(ax\cos(ax)-\sin(ax))} - \frac{5\int \frac{\sin^4(ax)}{x^6}\,dx}{a^2}$$

$$= \frac{\cos(ax)\sin^3(ax)}{ax^4} + \frac{\sin^4(ax)}{a^2x^5} + \frac{\sin^5(ax)}{a^2x^5(ax\cos(ax)-\sin(ax))} - 3\int \frac{\sin^2(ax)}{x^4}\,dx$$

$$= \frac{a\cos(ax)\sin(ax)}{x^2} + \frac{\sin^2(ax)}{x^3} + \frac{\cos(ax)\sin^3(ax)}{ax^4} - \frac{8a\cos(ax)\sin^3(ax)}{3x^2} + \cdots$$

$$= \frac{a^2}{x} + \frac{a\cos(ax)\sin(ax)}{x^2} + \frac{\sin^2(ax)}{x^3} - \frac{10a^2\sin^2(ax)}{x} + \frac{\cos(ax)\sin^3(ax)}{ax^4} - \cdots$$

$$= \frac{a^2}{x} + \frac{a\cos(ax)\sin(ax)}{x^2} + \frac{\sin^2(ax)}{x^3} - \frac{10a^2\sin^2(ax)}{x} + \frac{\cos(ax)\sin^3(ax)}{ax^4} - \cdots$$

$$= \frac{a^2}{x} + \frac{a\cos(ax)\sin(ax)}{x^2} + \frac{\sin^2(ax)}{x^3} - \frac{10a^2\sin^2(ax)}{x} + \frac{\cos(ax)\sin^3(ax)}{ax^4} - \cdots$$

Mathematica [A] time = 1.45, size = 198, normalized size = 1.13

$$\frac{-32a^3x^3\text{Si}(2ax)(ax\cos(ax)-\sin(ax))+256a^3x^3\text{Si}(4ax)(ax\cos(ax)-\sin(ax))-8a^3x^3\cos(ax)+24a^3x^3\cos(3ax}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a*x]^6/(x^4*(a*x*Cos[a*x] - Sin[a*x])^2),x]
```

```
[Out] (8*a*x*Cos[a*x] - 8*a^3*x^3*Cos[a*x] - 12*a*x*Cos[3*a*x] + 24*a^3*x^3*Cos[3
*a*x] + 4*a*x*Cos[5*a*x] + 32*a^3*x^3*Cos[5*a*x] + 10*Sin[a*x] - 12*a^2*x^2
*Sin[a*x] - 5*Sin[3*a*x] + 44*a^2*x^2*Sin[3*a*x] + Sin[5*a*x] - 24*a^2*x^2*
Sin[5*a*x] - 32*a^3*x^3*(a*x*Cos[a*x] - Sin[a*x])*SinIntegral[2*a*x] + 256*
a^3*x^3*(a*x*Cos[a*x] - Sin[a*x])*SinIntegral[4*a*x])/(48*x^3*(a*x*Cos[a*x]
 - Sin[a*x]))
```

fricas [A] time = 1.07, size = 186, normalized size = 1.06

$$\frac{4\left(8\,a^3x^3+ax\right)\cos\left(ax\right)^5-2\left(17\,a^3x^3+4\,ax\right)\cos\left(ax\right)^3+\left(16\,a^4x^4\,\text{Si}\left(4\,ax\right)-2\,a^4x^4\,\text{Si}\left(2\,ax\right)+5\,a^3x^3+4\,ax\right)\cos}{3\left(ax^4\cos\left(ax\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a*x)^6/x^4/(a*x*cos(a*x)-sin(a*x))^2,x, algorithm="fricas")
```

```
[Out] 1/3*(4*(8*a^3*x^3 + a*x)*cos(a*x)^5 - 2*(17*a^3*x^3 + 4*a*x)*cos(a*x)^3 + (
16*a^4*x^4*sin_integral(4*a*x) - 2*a^4*x^4*sin_integral(2*a*x) + 5*a^3*x^3
+ 4*a*x)*cos(a*x) - (16*a^3*x^3*sin_integral(4*a*x) - 2*a^3*x^3*sin_integra
l(2*a*x) + (24*a^2*x^2 - 1)*cos(a*x)^4 + 5*a^2*x^2 - (29*a^2*x^2 - 2)*cos(a
*x)^2 - 1)*sin(a*x))/(a*x^4*cos(a*x) - x^3*sin(a*x))
```

giac [C] time = 1.20, size = 7347, normalized size = 41.98

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a*x)^6/x^4/(a*x*cos(a*x)-sin(a*x))^2,x, algorithm="giac")
```

```
[Out] 1/12*(32*a^8*x^8*imag_part(cos_integral(4*a*x))*tan(2*a*x)^2*tan(a*x)^2*tan
(1/2*a*x)^2 - 4*a^8*x^8*imag_part(cos_integral(2*a*x))*tan(2*a*x)^2*tan(a*x
)^2*tan(1/2*a*x)^2 + 4*a^8*x^8*imag_part(cos_integral(-2*a*x))*tan(2*a*x)^2
*tan(a*x)^2*tan(1/2*a*x)^2 - 32*a^8*x^8*imag_part(cos_integral(-4*a*x))*tan
(2*a*x)^2*tan(a*x)^2*tan(1/2*a*x)^2 + 64*a^8*x^8*sin_integral(4*a*x)*tan(2*
a*x)^2*tan(a*x)^2*tan(1/2*a*x)^2 - 8*a^8*x^8*sin_integral(2*a*x)*tan(2*a*x)
^2*tan(a*x)^2*tan(1/2*a*x)^2 - 32*a^8*x^8*imag_part(cos_integral(4*a*x))*ta
```

```
n(2*a*x)^2*tan(a*x)^2 + 4*a^8*x^8*imag_part(cos_integral(2*a*x))*tan(2*a*x)
^2*tan(a*x)^2 - 4*a^8*x^8*imag_part(cos_integral(-2*a*x))*tan(2*a*x)^2*tan(
a*x)^2 + 32*a^8*x^8*imag_part(cos_integral(-4*a*x))*tan(2*a*x)^2*tan(a*x)^2
 - 64*a^8*x^8*sin_integral(4*a*x)*tan(2*a*x)^2*tan(a*x)^2 + 8*a^8*x^8*sin_i
ntegral(2*a*x)*tan(2*a*x)^2*tan(a*x)^2 + 32*a^8*x^8*imag_part(cos_integral(
4*a*x))*tan(2*a*x)^2*tan(1/2*a*x)^2 - 4*a^8*x^8*imag_part(cos_integral(2*a*
x))*tan(2*a*x)^2*tan(1/2*a*x)^2 + 4*a^8*x^8*imag_part(cos_integral(-2*a*x))
*tan(2*a*x)^2*tan(1/2*a*x)^2 - 32*a^8*x^8*imag_part(cos_integral(-4*a*x))*t
an(2*a*x)^2*tan(1/2*a*x)^2 + 64*a^8*x^8*sin_integral(4*a*x)*tan(2*a*x)^2*ta
n(1/2*a*x)^2 - 8*a^8*x^8*sin_integral(2*a*x)*tan(2*a*x)^2*tan(1/2*a*x)^2 +
32*a^8*x^8*imag_part(cos_integral(4*a*x))*tan(a*x)^2*tan(1/2*a*x)^2 - 4*a^8
*x^8*imag_part(cos_integral(2*a*x))*tan(a*x)^2*tan(1/2*a*x)^2 + 4*a^8*x^8*i
mag_part(cos_integral(-2*a*x))*tan(a*x)^2*tan(1/2*a*x)^2 - 32*a^8*x^8*imag_
part(cos_integral(-4*a*x))*tan(a*x)^2*tan(1/2*a*x)^2 + 64*a^8*x^8*sin_integ
ral(4*a*x)*tan(a*x)^2*tan(1/2*a*x)^2 - 8*a^8*x^8*sin_integral(2*a*x)*tan(a*
x)^2*tan(1/2*a*x)^2 + 64*a^7*x^7*imag_part(cos_integral(4*a*x))*tan(2*a*x)^
2*tan(a*x)^2*tan(1/2*a*x) - 8*a^7*x^7*imag_part(cos_integral(2*a*x))*tan(2*
a*x)^2*tan(a*x)^2*tan(1/2*a*x) + 8*a^7*x^7*imag_part(cos_integral(-2*a*x))*
tan(2*a*x)^2*tan(a*x)^2*tan(1/2*a*x) - 64*a^7*x^7*imag_part(cos_integral(-4
*a*x))*tan(2*a*x)^2*tan(a*x)^2*tan(1/2*a*x) + 128*a^7*x^7*sin_integral(4*a*
x)*tan(2*a*x)^2*tan(a*x)^2*tan(1/2*a*x) - 16*a^7*x^7*sin_integral(2*a*x)*ta
n(2*a*x)^2*tan(a*x)^2*tan(1/2*a*x) - 12*a^7*x^7*tan(2*a*x)^2*tan(a*x)^2*tan
(1/2*a*x)^2 - 32*a^8*x^8*imag_part(cos_integral(4*a*x))*tan(2*a*x)^2 + 4*a^
8*x^8*imag_part(cos_integral(2*a*x))*tan(2*a*x)^2 - 4*a^8*x^8*imag_part(cos
_integral(-2*a*x))*tan(2*a*x)^2 + 32*a^8*x^8*imag_part(cos_integral(-4*a*x)
)*tan(2*a*x)^2 - 64*a^8*x^8*sin_integral(4*a*x)*tan(2*a*x)^2 + 8*a^8*x^8*si
n_integral(2*a*x)*tan(2*a*x)^2 - 32*a^8*x^8*imag_part(cos_integral(4*a*x))*
tan(a*x)^2 + 4*a^8*x^8*imag_part(cos_integral(2*a*x))*tan(a*x)^2 - 4*a^8*x^
8*imag_part(cos_integral(-2*a*x))*tan(a*x)^2 + 32*a^8*x^8*imag_part(cos_int
egral(-4*a*x))*tan(a*x)^2 - 64*a^8*x^8*sin_integral(4*a*x)*tan(a*x)^2 + 8*a
^8*x^8*sin_integral(2*a*x)*tan(a*x)^2 + 32*a^8*x^8*imag_part(cos_integral(4
*a*x))*tan(1/2*a*x)^2 - 4*a^8*x^8*imag_part(cos_integral(2*a*x))*tan(1/2*a*
x)^2 + 4*a^8*x^8*imag_part(cos_integral(-2*a*x))*tan(1/2*a*x)^2 - 32*a^8*x^
8*imag_part(cos_integral(-4*a*x))*tan(1/2*a*x)^2 + 64*a^8*x^8*sin_integral(
4*a*x)*tan(1/2*a*x)^2 - 8*a^8*x^8*sin_integral(2*a*x)*tan(1/2*a*x)^2 + 64*a
^6*x^6*imag_part(cos_integral(4*a*x))*tan(2*a*x)^2*tan(a*x)^2*tan(1/2*a*x)^
2 - 8*a^6*x^6*imag_part(cos_integral(2*a*x))*tan(2*a*x)^2*tan(a*x)^2*tan(1/
2*a*x)^2 + 8*a^6*x^6*imag_part(cos_integral(-2*a*x))*tan(2*a*x)^2*tan(a*x)^
2*tan(1/2*a*x)^2 - 64*a^6*x^6*imag_part(cos_integral(-4*a*x))*tan(2*a*x)^2*
tan(a*x)^2*tan(1/2*a*x)^2 + 128*a^6*x^6*sin_integral(4*a*x)*tan(2*a*x)^2*ta
n(a*x)^2*tan(1/2*a*x)^2 - 16*a^6*x^6*sin_integral(2*a*x)*tan(2*a*x)^2*tan(a
*x)^2*tan(1/2*a*x)^2 + 12*a^7*x^7*tan(2*a*x)^2*tan(a*x)^2 + 64*a^7*x^7*imag
_part(cos_integral(4*a*x))*tan(2*a*x)^2*tan(1/2*a*x) - 8*a^7*x^7*imag_part(
cos_integral(2*a*x))*tan(2*a*x)^2*tan(1/2*a*x) + 8*a^7*x^7*imag_part(cos_in
tegral(-2*a*x))*tan(2*a*x)^2*tan(1/2*a*x) - 64*a^7*x^7*imag_part(cos_integr
```

al(-4*a*x))*tan(2*a*x)^2*tan(1/2*a*x) + 128*a^7*x^7*sin_integral(4*a*x)*tan
(2*a*x)^2*tan(1/2*a*x) - 16*a^7*x^7*sin_integral(2*a*x)*tan(2*a*x)^2*tan(1/
2*a*x) + 64*a^7*x^7*imag_part(cos_integral(4*a*x))*tan(a*x)^2*tan(1/2*a*x)
- 8*a^7*x^7*imag_part(cos_integral(2*a*x))*tan(a*x)^2*tan(1/2*a*x) + 8*a^7*
x^7*imag_part(cos_integral(-2*a*x))*tan(a*x)^2*tan(1/2*a*x) - 64*a^7*x^7*im
ag_part(cos_integral(-4*a*x))*tan(a*x)^2*tan(1/2*a*x) + 128*a^7*x^7*sin_int
egral(4*a*x)*tan(a*x)^2*tan(1/2*a*x) - 16*a^7*x^7*sin_integral(2*a*x)*tan(a
*x)^2*tan(1/2*a*x) - 20*a^7*x^7*tan(2*a*x)^2*tan(1/2*a*x)^2 + 20*a^7*x^7*ta
n(a*x)^2*tan(1/2*a*x)^2 - 32*a^8*x^8*imag_part(cos_integral(4*a*x)) + 4*a^8
*x^8*imag_part(cos_integral(2*a*x)) - 4*a^8*x^8*imag_part(cos_integral(-2*a
*x)) + 32*a^8*x^8*imag_part(cos_integral(-4*a*x)) - 64*a^8*x^8*sin_integral
(4*a*x) + 8*a^8*x^8*sin_integral(2*a*x) - 64*a^6*x^6*imag_part(cos_integral
(4*a*x))*tan(2*a*x)^2*tan(a*x)^2 + 8*a^6*x^6*imag_part(cos_integral(2*a*x))
*tan(2*a*x)^2*tan(a*x)^2 - 8*a^6*x^6*imag_part(cos_integral(-2*a*x))*tan(2*
a*x)^2*tan(a*x)^2 + 64*a^6*x^6*imag_part(cos_integral(-4*a*x))*tan(2*a*x)^2
*tan(a*x)^2 - 128*a^6*x^6*sin_integral(4*a*x)*tan(2*a*x)^2*tan(a*x)^2 + 16*
a^6*x^6*sin_integral(2*a*x)*tan(2*a*x)^2*tan(a*x)^2 - 24*a^6*x^6*tan(2*a*x)
^2*tan(a*x)^2*tan(1/2*a*x) + 64*a^6*x^6*imag_part(cos_integral(4*a*x))*tan(
2*a*x)^2*tan(1/2*a*x)^2 - 8*a^6*x^6*imag_part(cos_integral(2*a*x))*tan(2*a*
x)^2*tan(1/2*a*x)^2 + 8*a^6*x^6*imag_part(cos_integral(-2*a*x))*tan(2*a*x)^
2*tan(1/2*a*x)^2 - 64*a^6*x^6*imag_part(cos_integral(-4*a*x))*tan(2*a*x)^2*
tan(1/2*a*x)^2 + 128*a^6*x^6*sin_integral(4*a*x)*tan(2*a*x)^2*tan(1/2*a*x)^
2 - 16*a^6*x^6*sin_integral(2*a*x)*tan(2*a*x)^2*tan(1/2*a*x)^2 - 4*a^6*x^6*
tan(2*a*x)^2*tan(a*x)*tan(1/2*a*x)^2 + 64*a^6*x^6*imag_part(cos_integral(4*
a*x))*tan(a*x)^2*tan(1/2*a*x)^2 - 8*a^6*x^6*imag_part(cos_integral(2*a*x))*
tan(a*x)^2*tan(1/2*a*x)^2 + 8*a^6*x^6*imag_part(cos_integral(-2*a*x))*tan(a
*x)^2*tan(1/2*a*x)^2 - 64*a^6*x^6*imag_part(cos_integral(-4*a*x))*tan(a*x)^
2*tan(1/2*a*x)^2 + 128*a^6*x^6*sin_integral(4*a*x)*tan(a*x)^2*tan(1/2*a*x)^
2 - 16*a^6*x^6*sin_integral(2*a*x)*tan(a*x)^2*tan(1/2*a*x)^2 + 8*a^6*x^6*ta
n(2*a*x)*tan(a*x)^2*tan(1/2*a*x)^2 + 20*a^7*x^7*tan(2*a*x)^2 - 20*a^7*x^7*t
an(a*x)^2 + 64*a^7*x^7*imag_part(cos_integral(4*a*x))*tan(1/2*a*x) - 8*a^7*
x^7*imag_part(cos_integral(2*a*x))*tan(1/2*a*x) + 8*a^7*x^7*imag_part(cos_i
ntegral(-2*a*x))*tan(1/2*a*x) - 64*a^7*x^7*imag_part(cos_integral(-4*a*x))*
tan(1/2*a*x) + 128*a^7*x^7*sin_integral(4*a*x)*tan(1/2*a*x) - 16*a^7*x^7*si
n_integral(2*a*x)*tan(1/2*a*x) + 128*a^5*x^5*imag_part(cos_integral(4*a*x))
*tan(2*a*x)^2*tan(a*x)^2*tan(1/2*a*x) - 16*a^5*x^5*imag_part(cos_integral(2
*a*x))*tan(2*a*x)^2*tan(a*x)^2*tan(1/2*a*x) + 16*a^5*x^5*imag_part(cos_inte
gral(-2*a*x))*tan(2*a*x)^2*tan(a*x)^2*tan(1/2*a*x) - 128*a^5*x^5*imag_part(
cos_integral(-4*a*x))*tan(2*a*x)^2*tan(a*x)^2*tan(1/2*a*x) + 256*a^5*x^5*si
n_integral(4*a*x)*tan(2*a*x)^2*tan(a*x)^2*tan(1/2*a*x) - 32*a^5*x^5*sin_int
egral(2*a*x)*tan(2*a*x)^2*tan(a*x)^2*tan(1/2*a*x) + 12*a^7*x^7*tan(1/2*a*x)
^2 - 24*a^5*x^5*tan(2*a*x)^2*tan(a*x)^2*tan(1/2*a*x)^2 - 64*a^6*x^6*imag_pa
rt(cos_integral(4*a*x))*tan(2*a*x)^2 + 8*a^6*x^6*imag_part(cos_integral(2*a
*x))*tan(2*a*x)^2 - 8*a^6*x^6*imag_part(cos_integral(-2*a*x))*tan(2*a*x)^2
+ 64*a^6*x^6*imag_part(cos_integral(-4*a*x))*tan(2*a*x)^2 - 128*a^6*x^6*sin

```
_integral(4*a*x)*tan(2*a*x)^2 + 16*a^6*x^6*sin_integral(2*a*x)*tan(2*a*x)^2
 + 4*a^6*x^6*tan(2*a*x)^2*tan(a*x) - 64*a^6*x^6*imag_part(cos_integral(4*a*
x))*tan(a*x)^2 + 8*a^6*x^6*imag_part(cos_integral(2*a*x))*tan(a*x)^2 - 8*a^
6*x^6*imag_part(cos_integral(-2*a*x))*tan(a*x)^2 + 64*a^6*x^6*imag_part(cos
_integral(-4*a*x))*tan(a*x)^2 - 128*a^6*x^6*sin_integral(4*a*x)*tan(a*x)^2
+ 16*a^6*x^6*sin_integral(2*a*x)*tan(a*x)^2 - 8*a^6*x^6*tan(2*a*x)*tan(a*x)
^2 - 40*a^6*x^6*tan(2*a*x)^2*tan(1/2*a*x) + 40*a^6*x^6*tan(a*x)^2*tan(1/2*a
*x) + 64*a^6*x^6*imag_part(cos_integral(4*a*x))*tan(1/2*a*x)^2 - 8*a^6*x^6*
imag_part(cos_integral(2*a*x))*tan(1/2*a*x)^2 + 8*a^6*x^6*imag_part(cos_int
egral(-2*a*x))*tan(1/2*a*x)^2 - 64*a^6*x^6*imag_part(cos_integral(-4*a*x))*
tan(1/2*a*x)^2 + 128*a^6*x^6*sin_integral(4*a*x)*tan(1/2*a*x)^2 - 16*a^6*x^
6*sin_integral(2*a*x)*tan(1/2*a*x)^2 + 8*a^6*x^6*tan(2*a*x)*tan(1/2*a*x)^2
- 4*a^6*x^6*tan(a*x)*tan(1/2*a*x)^2 + 32*a^4*x^4*imag_part(cos_integral(4*a
*x))*tan(2*a*x)^2*tan(a*x)^2*tan(1/2*a*x)^2 - 4*a^4*x^4*imag_part(cos_integ
ral(2*a*x))*tan(2*a*x)^2*tan(a*x)^2*tan(1/2*a*x)^2 + 4*a^4*x^4*imag_part(co
s_integral(-2*a*x))*tan(2*a*x)^2*tan(a*x)^2*tan(1/2*a*x)^2 - 32*a^4*x^4*ima
g_part(cos_integral(-4*a*x))*tan(2*a*x)^2*tan(a*x)^2*tan(1/2*a*x)^2 + 64*a^
4*x^4*sin_integral(4*a*x)*tan(2*a*x)^2*tan(a*x)^2*tan(1/2*a*x)^2 - 8*a^4*x^
4*sin_integral(2*a*x)*tan(2*a*x)^2*tan(a*x)^2*tan(1/2*a*x)^2 - 12*a^7*x^7 +
 24*a^5*x^5*tan(2*a*x)^2*tan(a*x)^2 + 128*a^5*x^5*imag_part(cos_integral(4*
a*x))*tan(2*a*x)^2*tan(1/2*a*x) - 16*a^5*x^5*imag_part(cos_integral(2*a*x))
*tan(2*a*x)^2*tan(1/2*a*x) + 16*a^5*x^5*imag_part(cos_integral(-2*a*x))*tan
(2*a*x)^2*tan(1/2*a*x) - 128*a^5*x^5*imag_part(cos_integral(-4*a*x))*tan(2*
a*x)^2*tan(1/2*a*x) + 256*a^5*x^5*sin_integral(4*a*x)*tan(2*a*x)^2*tan(1/2*
a*x) - 32*a^5*x^5*sin_integral(2*a*x)*tan(2*a*x)^2*tan(1/2*a*x) - 8*a^5*x^5
*tan(2*a*x)^2*tan(a*x)*tan(1/2*a*x) + 128*a^5*x^5*imag_part(cos_integral(4*
a*x))*tan(a*x)^2*tan(1/2*a*x) - 16*a^5*x^5*imag_part(cos_integral(2*a*x))*t
an(a*x)^2*tan(1/2*a*x) + 16*a^5*x^5*imag_part(cos_integral(-2*a*x))*tan(a*x
)^2*tan(1/2*a*x) - 128*a^5*x^5*imag_part(cos_integral(-4*a*x))*tan(a*x)^2*t
an(1/2*a*x) + 256*a^5*x^5*sin_integral(4*a*x)*tan(a*x)^2*tan(1/2*a*x) - 32*
a^5*x^5*sin_integral(2*a*x)*tan(a*x)^2*tan(1/2*a*x) + 16*a^5*x^5*tan(2*a*x)
*tan(a*x)^2*tan(1/2*a*x) - 36*a^5*x^5*tan(2*a*x)^2*tan(1/2*a*x)^2 + 36*a^5*
x^5*tan(a*x)^2*tan(1/2*a*x)^2 - 64*a^6*x^6*imag_part(cos_integral(4*a*x)) +
 8*a^6*x^6*imag_part(cos_integral(2*a*x)) - 8*a^6*x^6*imag_part(cos_integra
l(-2*a*x)) + 64*a^6*x^6*imag_part(cos_integral(-4*a*x)) - 128*a^6*x^6*sin_i
ntegral(4*a*x) + 16*a^6*x^6*sin_integral(2*a*x) - 8*a^6*x^6*tan(2*a*x) + 4*
a^6*x^6*tan(a*x) - 32*a^4*x^4*imag_part(cos_integral(4*a*x))*tan(2*a*x)^2*t
an(a*x)^2 + 4*a^4*x^4*imag_part(cos_integral(2*a*x))*tan(2*a*x)^2*tan(a*x)^
2 - 4*a^4*x^4*imag_part(cos_integral(-2*a*x))*tan(2*a*x)^2*tan(a*x)^2 + 32*
a^4*x^4*imag_part(cos_integral(-4*a*x))*tan(2*a*x)^2*tan(a*x)^2 - 64*a^4*x^
4*sin_integral(4*a*x)*tan(2*a*x)^2*tan(a*x)^2 + 8*a^4*x^4*sin_integral(2*a*
x)*tan(2*a*x)^2*tan(a*x)^2 + 24*a^6*x^6*tan(1/2*a*x) - 48*a^4*x^4*tan(2*a*x
)^2*tan(a*x)^2*tan(1/2*a*x) + 32*a^4*x^4*imag_part(cos_integral(4*a*x))*tan
(2*a*x)^2*tan(1/2*a*x)^2 - 4*a^4*x^4*imag_part(cos_integral(2*a*x))*tan(2*a
*x)^2*tan(1/2*a*x)^2 + 4*a^4*x^4*imag_part(cos_integral(-2*a*x))*tan(2*a*x)
```

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^2*tan(1/2*a*x)^2 - 32*a^4*x^4*imag_part(cos_integral(-4*a*x))*tan(2*a*x)^2
*tan(1/2*a*x)^2 + 64*a^4*x^4*sin_integral(4*a*x)*tan(2*a*x)^2*tan(1/2*a*x)^
2 - 8*a^4*x^4*sin_integral(2*a*x)*tan(2*a*x)^2*tan(1/2*a*x)^2 - 2*a^4*x^4*t
an(2*a*x)^2*tan(a*x)*tan(1/2*a*x)^2 + 32*a^4*x^4*imag_part(cos_integral(4*a
*x))*tan(a*x)^2*tan(1/2*a*x)^2 - 4*a^4*x^4*imag_part(cos_integral(2*a*x))*t
an(a*x)^2*tan(1/2*a*x)^2 + 4*a^4*x^4*imag_part(cos_integral(-2*a*x))*tan(a*
x)^2*tan(1/2*a*x)^2 - 32*a^4*x^4*imag_part(cos_integral(-4*a*x))*tan(a*x)^2
*tan(1/2*a*x)^2 + 64*a^4*x^4*sin_integral(4*a*x)*tan(a*x)^2*tan(1/2*a*x)^2
- 8*a^4*x^4*sin_integral(2*a*x)*tan(a*x)^2*tan(1/2*a*x)^2 + 13*a^4*x^4*tan(
2*a*x)*tan(a*x)^2*tan(1/2*a*x)^2 + 36*a^5*x^5*tan(2*a*x)^2 - 36*a^5*x^5*tan
(a*x)^2 + 128*a^5*x^5*imag_part(cos_integral(4*a*x))*tan(1/2*a*x) - 16*a^5*
x^5*imag_part(cos_integral(2*a*x))*tan(1/2*a*x) + 16*a^5*x^5*imag_part(cos_
integral(-2*a*x))*tan(1/2*a*x) - 128*a^5*x^5*imag_part(cos_integral(-4*a*x)
)*tan(1/2*a*x) + 256*a^5*x^5*sin_integral(4*a*x)*tan(1/2*a*x) - 32*a^5*x^5*
sin_integral(2*a*x)*tan(1/2*a*x) + 16*a^5*x^5*tan(2*a*x)*tan(1/2*a*x) - 8*a
^5*x^5*tan(a*x)*tan(1/2*a*x) + 64*a^3*x^3*imag_part(cos_integral(4*a*x))*ta
n(2*a*x)^2*tan(a*x)^2*tan(1/2*a*x) - 8*a^3*x^3*imag_part(cos_integral(2*a*x
))*tan(2*a*x)^2*tan(a*x)^2*tan(1/2*a*x) + 8*a^3*x^3*imag_part(cos_integral(
-2*a*x))*tan(2*a*x)^2*tan(a*x)^2*tan(1/2*a*x) - 64*a^3*x^3*imag_part(cos_in
tegral(-4*a*x))*tan(2*a*x)^2*tan(a*x)^2*tan(1/2*a*x) + 128*a^3*x^3*sin_inte
gral(4*a*x)*tan(2*a*x)^2*tan(a*x)^2*tan(1/2*a*x) - 16*a^3*x^3*sin_integral(
2*a*x)*tan(2*a*x)^2*tan(a*x)^2*tan(1/2*a*x) + 24*a^5*x^5*tan(1/2*a*x)^2 - 3
*a^3*x^3*tan(2*a*x)^2*tan(a*x)^2*tan(1/2*a*x)^2 - 32*a^4*x^4*imag_part(cos_
integral(4*a*x))*tan(2*a*x)^2 + 4*a^4*x^4*imag_part(cos_integral(2*a*x))*ta
n(2*a*x)^2 - 4*a^4*x^4*imag_part(cos_integral(-2*a*x))*tan(2*a*x)^2 + 32*a^
4*x^4*imag_part(cos_integral(-4*a*x))*tan(2*a*x)^2 - 64*a^4*x^4*sin_integra
l(4*a*x)*tan(2*a*x)^2 + 8*a^4*x^4*sin_integral(2*a*x)*tan(2*a*x)^2 + 2*a^4*
x^4*tan(2*a*x)^2*tan(a*x) - 32*a^4*x^4*imag_part(cos_integral(4*a*x))*tan(a
*x)^2 + 4*a^4*x^4*imag_part(cos_integral(2*a*x))*tan(a*x)^2 - 4*a^4*x^4*ima
g_part(cos_integral(-2*a*x))*tan(a*x)^2 + 32*a^4*x^4*imag_part(cos_integral
(-4*a*x))*tan(a*x)^2 - 64*a^4*x^4*sin_integral(4*a*x)*tan(a*x)^2 + 8*a^4*x^
4*sin_integral(2*a*x)*tan(a*x)^2 - 13*a^4*x^4*tan(2*a*x)*tan(a*x)^2 - 72*a^
4*x^4*tan(2*a*x)^2*tan(1/2*a*x) + 72*a^4*x^4*tan(a*x)^2*tan(1/2*a*x) + 32*a
^4*x^4*imag_part(cos_integral(4*a*x))*tan(1/2*a*x)^2 - 4*a^4*x^4*imag_part(
cos_integral(2*a*x))*tan(1/2*a*x)^2 + 4*a^4*x^4*imag_part(cos_integral(-2*a
*x))*tan(1/2*a*x)^2 - 32*a^4*x^4*imag_part(cos_integral(-4*a*x))*tan(1/2*a*
x)^2 + 64*a^4*x^4*sin_integral(4*a*x)*tan(1/2*a*x)^2 - 8*a^4*x^4*sin_integr
al(2*a*x)*tan(1/2*a*x)^2 + 13*a^4*x^4*tan(2*a*x)*tan(1/2*a*x)^2 - 2*a^4*x^4
*tan(a*x)*tan(1/2*a*x)^2 - 24*a^5*x^5 + 3*a^3*x^3*tan(2*a*x)^2*tan(a*x)^2 +
 64*a^3*x^3*imag_part(cos_integral(4*a*x))*tan(2*a*x)^2*tan(1/2*a*x) - 8*a^
3*x^3*imag_part(cos_integral(2*a*x))*tan(2*a*x)^2*tan(1/2*a*x) + 8*a^3*x^3*
imag_part(cos_integral(-2*a*x))*tan(2*a*x)^2*tan(1/2*a*x) - 64*a^3*x^3*imag
_part(cos_integral(-4*a*x))*tan(2*a*x)^2*tan(1/2*a*x) + 128*a^3*x^3*sin_int
egral(4*a*x)*tan(2*a*x)^2*tan(1/2*a*x) - 16*a^3*x^3*sin_integral(2*a*x)*tan
(2*a*x)^2*tan(1/2*a*x) - 4*a^3*x^3*tan(2*a*x)^2*tan(a*x)*tan(1/2*a*x) + 64*
```

```
a^3*x^3*imag_part(cos_integral(4*a*x))*tan(a*x)^2*tan(1/2*a*x) - 8*a^3*x^3*
imag_part(cos_integral(2*a*x))*tan(a*x)^2*tan(1/2*a*x) + 8*a^3*x^3*imag_par
t(cos_integral(-2*a*x))*tan(a*x)^2*tan(1/2*a*x) - 64*a^3*x^3*imag_part(cos_
integral(-4*a*x))*tan(a*x)^2*tan(1/2*a*x) + 128*a^3*x^3*sin_integral(4*a*x)
*tan(a*x)^2*tan(1/2*a*x) - 16*a^3*x^3*sin_integral(2*a*x)*tan(a*x)^2*tan(1/
2*a*x) + 26*a^3*x^3*tan(2*a*x)*tan(a*x)^2*tan(1/2*a*x) - 15*a^3*x^3*tan(2*a
*x)^2*tan(1/2*a*x)^2 + 24*a^3*x^3*tan(a*x)^2*tan(1/2*a*x)^2 - 32*a^4*x^4*im
ag_part(cos_integral(4*a*x)) + 4*a^4*x^4*imag_part(cos_integral(2*a*x)) - 4
*a^4*x^4*imag_part(cos_integral(-2*a*x)) + 32*a^4*x^4*imag_part(cos_integra
l(-4*a*x)) - 64*a^4*x^4*sin_integral(4*a*x) + 8*a^4*x^4*sin_integral(2*a*x)
 - 13*a^4*x^4*tan(2*a*x) + 2*a^4*x^4*tan(a*x) + 48*a^4*x^4*tan(1/2*a*x) - 3
0*a^2*x^2*tan(2*a*x)^2*tan(a*x)^2*tan(1/2*a*x) - 10*a^2*x^2*tan(2*a*x)^2*ta
n(a*x)*tan(1/2*a*x)^2 + 5*a^2*x^2*tan(2*a*x)*tan(a*x)^2*tan(1/2*a*x)^2 + 15
*a^3*x^3*tan(2*a*x)^2 - 24*a^3*x^3*tan(a*x)^2 + 64*a^3*x^3*imag_part(cos_in
tegral(4*a*x))*tan(1/2*a*x) - 8*a^3*x^3*imag_part(cos_integral(2*a*x))*tan(
1/2*a*x) + 8*a^3*x^3*imag_part(cos_integral(-2*a*x))*tan(1/2*a*x) - 64*a^3*
x^3*imag_part(cos_integral(-4*a*x))*tan(1/2*a*x) + 128*a^3*x^3*sin_integral
(4*a*x)*tan(1/2*a*x) - 16*a^3*x^3*sin_integral(2*a*x)*tan(1/2*a*x) + 26*a^3
*x^3*tan(2*a*x)*tan(1/2*a*x) - 4*a^3*x^3*tan(a*x)*tan(1/2*a*x) + 12*a^3*x^3
*tan(1/2*a*x)^2 - 3*a*x*tan(2*a*x)^2*tan(a*x)^2*tan(1/2*a*x)^2 + 10*a^2*x^2
*tan(2*a*x)^2*tan(a*x) - 5*a^2*x^2*tan(2*a*x)*tan(a*x)^2 - 54*a^2*x^2*tan(2
*a*x)^2*tan(1/2*a*x) + 24*a^2*x^2*tan(a*x)^2*tan(1/2*a*x) + 5*a^2*x^2*tan(2
*a*x)*tan(1/2*a*x)^2 - 10*a^2*x^2*tan(a*x)*tan(1/2*a*x)^2 - 12*a^3*x^3 + 3*
a*x*tan(2*a*x)^2*tan(a*x)^2 - 20*a*x*tan(2*a*x)^2*tan(a*x)*tan(1/2*a*x) + 1
0*a*x*tan(2*a*x)*tan(a*x)^2*tan(1/2*a*x) + a*x*tan(2*a*x)^2*tan(1/2*a*x)^2
- 4*a*x*tan(a*x)^2*tan(1/2*a*x)^2 - 5*a^2*x^2*tan(2*a*x) + 10*a^2*x^2*tan(a
*x) - 6*tan(2*a*x)^2*tan(a*x)^2*tan(1/2*a*x) - a*x*tan(2*a*x)^2 + 4*a*x*tan
(a*x)^2 + 10*a*x*tan(2*a*x)*tan(1/2*a*x) - 20*a*x*tan(a*x)*tan(1/2*a*x) + 2
*tan(2*a*x)^2*tan(1/2*a*x) - 8*tan(a*x)^2*tan(1/2*a*x))/(a^5*x^8*tan(2*a*x)
^2*tan(a*x)^2*tan(1/2*a*x)^2 - a^5*x^8*tan(2*a*x)^2*tan(a*x)^2 + a^5*x^8*ta
n(2*a*x)^2*tan(1/2*a*x)^2 + a^5*x^8*tan(a*x)^2*tan(1/2*a*x)^2 + 2*a^4*x^7*t
an(2*a*x)^2*tan(a*x)^2*tan(1/2*a*x) - a^5*x^8*tan(2*a*x)^2 - a^5*x^8*tan(a*
x)^2 + a^5*x^8*tan(1/2*a*x)^2 + 2*a^3*x^6*tan(2*a*x)^2*tan(a*x)^2*tan(1/2*a
*x)^2 + 2*a^4*x^7*tan(2*a*x)^2*tan(1/2*a*x) + 2*a^4*x^7*tan(a*x)^2*tan(1/2*
a*x) - a^5*x^8 - 2*a^3*x^6*tan(2*a*x)^2*tan(a*x)^2 + 2*a^3*x^6*tan(2*a*x)^2
*tan(1/2*a*x)^2 + 2*a^3*x^6*tan(a*x)^2*tan(1/2*a*x)^2 + 2*a^4*x^7*tan(1/2*a
*x) + 4*a^2*x^5*tan(2*a*x)^2*tan(a*x)^2*tan(1/2*a*x) - 2*a^3*x^6*tan(2*a*x)
^2 - 2*a^3*x^6*tan(a*x)^2 + 2*a^3*x^6*tan(1/2*a*x)^2 + a*x^4*tan(2*a*x)^2*t
an(a*x)^2*tan(1/2*a*x)^2 + 4*a^2*x^5*tan(2*a*x)^2*tan(1/2*a*x) + 4*a^2*x^5*
tan(a*x)^2*tan(1/2*a*x) - 2*a^3*x^6 - a*x^4*tan(2*a*x)^2*tan(a*x)^2 + a*x^4
*tan(2*a*x)^2*tan(1/2*a*x)^2 + a*x^4*tan(a*x)^2*tan(1/2*a*x)^2 + 4*a^2*x^5*
tan(1/2*a*x) + 2*x^3*tan(2*a*x)^2*tan(a*x)^2*tan(1/2*a*x) - a*x^4*tan(2*a*x
)^2 - a*x^4*tan(a*x)^2 + a*x^4*tan(1/2*a*x)^2 + 2*x^3*tan(2*a*x)^2*tan(1/2*
a*x) + 2*x^3*tan(a*x)^2*tan(1/2*a*x) - a*x^4 + 2*x^3*tan(1/2*a*x))
```

maple [F(-1)] time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{\sin^6(ax)}{x^4 (ax\cos(ax) - \sin(ax))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a*x)^6/x^4/(a*x*cos(a*x)-sin(a*x))^2,x)
[Out] int(sin(a*x)^6/x^4/(a*x*cos(a*x)-sin(a*x))^2,x)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a*x)^6/x^4/(a*x*cos(a*x)-sin(a*x))^2,x, algorithm="maxima")
[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is und
efined.
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sin(a\,x)^6}{x^4\,(\sin(a\,x) - a\,x\,\cos(a\,x))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a*x)^6/(x^4*(sin(a*x) - a*x*cos(a*x))^2),x)
[Out] int(sin(a*x)^6/(x^4*(sin(a*x) - a*x*cos(a*x))^2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin^6(ax)}{x^4 (ax\cos(ax) - \sin(ax))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a*x)**6/x**4/(a*x*cos(a*x)-sin(a*x))**2,x)
[Out] Integral(sin(a*x)**6/(x**4*(a*x*cos(a*x) - sin(a*x))**2), x)
```

3.586 $\int \frac{\sin^5(ax)}{x^3(ax\cos(ax)-\sin(ax))^2}\,dx$

Optimal. Leaf size=131

$$-\frac{1}{8}a^2\text{Si}(ax)+\frac{27}{8}a^2\text{Si}(3ax)+\frac{\sin^3(ax)}{a^2x^4}+\frac{\sin^4(ax)}{a^2x^4(ax\cos(ax)-\sin(ax))}+\frac{\sin^2(ax)\cos(ax)}{ax^3}-\frac{3\sin^3(ax)}{2x^2}+\frac{\sin(ax)}{x^2}+\frac{a\cos}{}$$

```
[Out] a*cos(a*x)/x-1/8*a^2*Si(a*x)+27/8*a^2*Si(3*a*x)+sin(a*x)/x^2+cos(a*x)*sin(a
*x)^2/a/x^3-9/2*a*cos(a*x)*sin(a*x)^2/x+sin(a*x)^3/a^2/x^4-3/2*sin(a*x)^3/x
^2+sin(a*x)^4/a^2/x^4/(a*x*cos(a*x)-sin(a*x))
```

Rubi [A] time = 0.23, antiderivative size = 131, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {4598, 3314, 3297, 3299, 3312}

$$-\frac{1}{8}a^2\text{Si}(ax)+\frac{27}{8}a^2\text{Si}(3ax)+\frac{\sin^3(ax)}{a^2x^4}+\frac{\sin^4(ax)}{a^2x^4(ax\cos(ax)-\sin(ax))}-\frac{3\sin^3(ax)}{2x^2}+\frac{\sin(ax)}{x^2}+\frac{\sin^2(ax)\cos(ax)}{ax^3}+\frac{a\cos}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a*x]^5/(x^3*(a*x*Cos[a*x] - Sin[a*x])^2),x]
```

```
[Out] (a*Cos[a*x])/x + Sin[a*x]/x^2 + (Cos[a*x]*Sin[a*x]^2)/(a*x^3) - (9*a*Cos[a*
x]*Sin[a*x]^2)/(2*x) + Sin[a*x]^3/(a^2*x^4) - (3*Sin[a*x]^3)/(2*x^2) + Sin[
a*x]^4/(a^2*x^4*(a*x*Cos[a*x] - Sin[a*x])) - (a^2*SinIntegral[a*x])/8 + (27
*a^2*SinIntegral[3*a*x])/8
```

Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[((
c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 3314

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[((c + d*x)^(m + 1)*(b*Sin[e + f*x])^n)/(d*(m + 1)), x] + (Dist[(
b^2*f^2*n*(n - 1))/(d^2*(m + 1)*(m + 2)), Int[(c + d*x)^(m + 2)*(b*Sin[e +
f*x])^(n - 2), x], x] - Dist[(f^2*n^2)/(d^2*(m + 1)*(m + 2)), Int[(c + d*x)
^(m + 2)*(b*Sin[e + f*x])^n, x], x] - Simp[(b*f*n*(c + d*x)^(m + 2)*Cos[e +
 f*x]*(b*Sin[e + f*x])^(n - 1))/(d^2*(m + 1)*(m + 2)), x]) /; FreeQ[{b, c,
d, e, f}, x] && GtQ[n, 1] && LtQ[m, -2]
```

Rule 4598

```
Int[(((b_.)*(x_))^(m_)*Sin[(a_.)*(x_)]^(n_))/(Cos[(a_.)*(x_)]*(d_.)*(x_) +
(c_.)*Sin[(a_.)*(x_)])^2, x_Symbol] :> Simp[(b*(b*x)^(m - 1)*Sin[a*x]^(n -
1))/(a*d*(c*Sin[a*x] + d*x*Cos[a*x])), x] - Dist[(b^2*(n - 1))/d^2, Int[(b*
x)^(m - 2)*Sin[a*x]^(n - 2), x], x] /; FreeQ[{a, b, c, d, m, n}, x] && EqQ[
a*c + d, 0] && EqQ[m, 2 - n]
```

Rubi steps

$$\int \frac{\sin^5(ax)}{x^3(ax\cos(ax)-\sin(ax))^2}\,dx = \frac{\sin^4(ax)}{a^2x^4(ax\cos(ax)-\sin(ax))} - \frac{4\int \frac{\sin^3(ax)}{x^5}\,dx}{a^2}$$

$$= \frac{\cos(ax)\sin^2(ax)}{ax^3} + \frac{\sin^3(ax)}{a^2x^4} + \frac{\sin^4(ax)}{a^2x^4(ax\cos(ax)-\sin(ax))} - 2\int \frac{\sin(ax)}{x^3}\,dx +$$

$$= \frac{\sin(ax)}{x^2} + \frac{\cos(ax)\sin^2(ax)}{ax^3} - \frac{9a\cos(ax)\sin^2(ax)}{2x} + \frac{\sin^3(ax)}{a^2x^4} - \frac{3\sin^3(ax)}{2x^2} +$$

$$= \frac{a\cos(ax)}{x} + \frac{\sin(ax)}{x^2} + \frac{\cos(ax)\sin^2(ax)}{ax^3} - \frac{9a\cos(ax)\sin^2(ax)}{2x} + \frac{\sin^3(ax)}{a^2x^4} -$$

$$= \frac{a\cos(ax)}{x} + \frac{\sin(ax)}{x^2} + \frac{\cos(ax)\sin^2(ax)}{ax^3} - \frac{9a\cos(ax)\sin^2(ax)}{2x} + \frac{\sin^3(ax)}{a^2x^4} -$$

$$= \frac{a\cos(ax)}{x} + \frac{\sin(ax)}{x^2} + \frac{\cos(ax)\sin^2(ax)}{ax^3} - \frac{9a\cos(ax)\sin^2(ax)}{2x} + \frac{\sin^3(ax)}{a^2x^4} -$$

Mathematica [A] time = 0.97, size = 142, normalized size = 1.08

$$\frac{-2a^2x^2\text{Si}(ax)(ax\cos(ax)-\sin(ax)) + 54a^2x^2\text{Si}(3ax)(ax\cos(ax)-\sin(ax)) - a^2x^2 + 8a^2x^2\cos(2ax) + 9a^2x^2\cos(}{16x^2(ax\cos(ax)-\sin(ax))}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a*x]^5/(x^3*(a*x*Cos[a*x] - Sin[a*x])^2),x]
```

```
[Out] (3 - a^2*x^2 - 4*Cos[2*a*x] + 8*a^2*x^2*Cos[2*a*x] + Cos[4*a*x] + 9*a^2*x^2
*Cos[4*a*x] + 12*a*x*Sin[2*a*x] - 6*a*x*Sin[4*a*x] - 2*a^2*x^2*(a*x*Cos[a*x
] - Sin[a*x])*SinIntegral[a*x] + 54*a^2*x^2*(a*x*Cos[a*x] - Sin[a*x])*SinIn
tegral[3*a*x])/(16*x^2*(a*x*Cos[a*x] - Sin[a*x]))
```

fricas [A] time = 1.04, size = 142, normalized size = 1.08

$$\frac{4\left(9\,a^2x^2+1\right)\cos\left(ax\right)^4-4\left(7\,a^2x^2+2\right)\cos\left(ax\right)^2+\left(27\,a^3x^3\,\mathrm{Si}\left(3\,ax\right)-a^3x^3\,\mathrm{Si}\left(ax\right)\right)\cos\left(ax\right)-\left(24\,ax\cos\left(ax\right)^3\right.}{8\left(ax^3\cos\left(ax\right)-x^2\sin\left(ax\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a*x)^5/x^3/(a*x*cos(a*x)-sin(a*x))^2,x, algorithm="fricas")
```

```
[Out] 1/8*(4*(9*a^2*x^2 + 1)*cos(a*x)^4 - 4*(7*a^2*x^2 + 2)*cos(a*x)^2 + (27*a^3*
x^3*sin_integral(3*a*x) - a^3*x^3*sin_integral(a*x))*cos(a*x) - (24*a*x*cos
(a*x)^3 + 27*a^2*x^2*sin_integral(3*a*x) - a^2*x^2*sin_integral(a*x) - 24*a
*x*cos(a*x))*sin(a*x) + 4)/(a*x^3*cos(a*x) - x^2*sin(a*x))
```

giac [C] time = 0.81, size = 4175, normalized size = 31.87

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a*x)^5/x^3/(a*x*cos(a*x)-sin(a*x))^2,x, algorithm="giac")
```

```
[Out] 1/16*(27*a^7*x^7*imag_part(cos_integral(3*a*x))*tan(3/2*a*x)^2*tan(1/2*a*x)
^4 - a^7*x^7*imag_part(cos_integral(a*x))*tan(3/2*a*x)^2*tan(1/2*a*x)^4 + a
^7*x^7*imag_part(cos_integral(-a*x))*tan(3/2*a*x)^2*tan(1/2*a*x)^4 - 27*a^7
*x^7*imag_part(cos_integral(-3*a*x))*tan(3/2*a*x)^2*tan(1/2*a*x)^4 + 54*a^7
*x^7*sin_integral(3*a*x)*tan(3/2*a*x)^2*tan(1/2*a*x)^4 - 2*a^7*x^7*sin_inte
gral(a*x)*tan(3/2*a*x)^2*tan(1/2*a*x)^4 + 27*a^7*x^7*imag_part(cos_integral
(3*a*x))*tan(1/2*a*x)^4 - a^7*x^7*imag_part(cos_integral(a*x))*tan(1/2*a*x)
^4 + a^7*x^7*imag_part(cos_integral(-a*x))*tan(1/2*a*x)^4 - 27*a^7*x^7*imag
_part(cos_integral(-3*a*x))*tan(1/2*a*x)^4 + 54*a^7*x^7*sin_integral(3*a*x)
*tan(1/2*a*x)^4 - 2*a^7*x^7*sin_integral(a*x)*tan(1/2*a*x)^4 + 54*a^6*x^6*i
mag_part(cos_integral(3*a*x))*tan(3/2*a*x)^2*tan(1/2*a*x)^3 - 2*a^6*x^6*ima
g_part(cos_integral(a*x))*tan(3/2*a*x)^2*tan(1/2*a*x)^3 + 2*a^6*x^6*imag_pa
rt(cos_integral(-a*x))*tan(3/2*a*x)^2*tan(1/2*a*x)^3 - 54*a^6*x^6*imag_part
(cos_integral(-3*a*x))*tan(3/2*a*x)^2*tan(1/2*a*x)^3 + 108*a^6*x^6*sin_inte
gral(3*a*x)*tan(3/2*a*x)^2*tan(1/2*a*x)^3 - 4*a^6*x^6*sin_integral(a*x)*tan
(3/2*a*x)^2*tan(1/2*a*x)^3 - 16*a^6*x^6*tan(3/2*a*x)^2*tan(1/2*a*x)^4 - 27*
a^7*x^7*imag_part(cos_integral(3*a*x))*tan(3/2*a*x)^2 + a^7*x^7*imag_part(c
```

os_integral(a*x))*tan(3/2*a*x)^2 - a^7*x^7*imag_part(cos_integral(-a*x))*ta
n(3/2*a*x)^2 + 27*a^7*x^7*imag_part(cos_integral(-3*a*x))*tan(3/2*a*x)^2 -
54*a^7*x^7*sin_integral(3*a*x)*tan(3/2*a*x)^2 + 2*a^7*x^7*sin_integral(a*x)
*tan(3/2*a*x)^2 + 54*a^5*x^5*imag_part(cos_integral(3*a*x))*tan(3/2*a*x)^2*
tan(1/2*a*x)^4 - 2*a^5*x^5*imag_part(cos_integral(a*x))*tan(3/2*a*x)^2*tan(
1/2*a*x)^4 + 2*a^5*x^5*imag_part(cos_integral(-a*x))*tan(3/2*a*x)^2*tan(1/2
*a*x)^4 - 54*a^5*x^5*imag_part(cos_integral(-3*a*x))*tan(3/2*a*x)^2*tan(1/2
*a*x)^4 + 108*a^5*x^5*sin_integral(3*a*x)*tan(3/2*a*x)^2*tan(1/2*a*x)^4 - 4
*a^5*x^5*sin_integral(a*x)*tan(3/2*a*x)^2*tan(1/2*a*x)^4 + 54*a^6*x^6*imag_
part(cos_integral(3*a*x))*tan(3/2*a*x)^2*tan(1/2*a*x) - 2*a^6*x^6*imag_part
(cos_integral(a*x))*tan(3/2*a*x)^2*tan(1/2*a*x) + 2*a^6*x^6*imag_part(cos_i
ntegral(-a*x))*tan(3/2*a*x)^2*tan(1/2*a*x) - 54*a^6*x^6*imag_part(cos_integ
ral(-3*a*x))*tan(3/2*a*x)^2*tan(1/2*a*x) + 108*a^6*x^6*sin_integral(3*a*x)*
tan(3/2*a*x)^2*tan(1/2*a*x) - 4*a^6*x^6*sin_integral(a*x)*tan(3/2*a*x)^2*ta
n(1/2*a*x) - 4*a^6*x^6*tan(3/2*a*x)^2*tan(1/2*a*x)^2 + 54*a^6*x^6*imag_part
(cos_integral(3*a*x))*tan(1/2*a*x)^3 - 2*a^6*x^6*imag_part(cos_integral(a*x
))*tan(1/2*a*x)^3 + 2*a^6*x^6*imag_part(cos_integral(-a*x))*tan(1/2*a*x)^3
- 54*a^6*x^6*imag_part(cos_integral(-3*a*x))*tan(1/2*a*x)^3 + 108*a^6*x^6*s
in_integral(3*a*x)*tan(1/2*a*x)^3 - 4*a^6*x^6*sin_integral(a*x)*tan(1/2*a*x
)^3 + 20*a^6*x^6*tan(1/2*a*x)^4 - 27*a^7*x^7*imag_part(cos_integral(3*a*x))
 + a^7*x^7*imag_part(cos_integral(a*x)) - a^7*x^7*imag_part(cos_integral(-a
*x)) + 27*a^7*x^7*imag_part(cos_integral(-3*a*x)) - 54*a^7*x^7*sin_integral
(3*a*x) + 2*a^7*x^7*sin_integral(a*x) - 36*a^5*x^5*tan(3/2*a*x)^2*tan(1/2*a
*x)^3 + 54*a^5*x^5*imag_part(cos_integral(3*a*x))*tan(1/2*a*x)^4 - 2*a^5*x^
5*imag_part(cos_integral(a*x))*tan(1/2*a*x)^4 + 2*a^5*x^5*imag_part(cos_int
egral(-a*x))*tan(1/2*a*x)^4 - 54*a^5*x^5*imag_part(cos_integral(-3*a*x))*ta
n(1/2*a*x)^4 + 108*a^5*x^5*sin_integral(3*a*x)*tan(1/2*a*x)^4 - 4*a^5*x^5*s
in_integral(a*x)*tan(1/2*a*x)^4 + 12*a^5*x^5*tan(3/2*a*x)*tan(1/2*a*x)^4 +
20*a^6*x^6*tan(3/2*a*x)^2 + 54*a^6*x^6*imag_part(cos_integral(3*a*x))*tan(1
/2*a*x) - 2*a^6*x^6*imag_part(cos_integral(a*x))*tan(1/2*a*x) + 2*a^6*x^6*i
mag_part(cos_integral(-a*x))*tan(1/2*a*x) - 54*a^6*x^6*imag_part(cos_integr
al(-3*a*x))*tan(1/2*a*x) + 108*a^6*x^6*sin_integral(3*a*x)*tan(1/2*a*x) - 4
*a^6*x^6*sin_integral(a*x)*tan(1/2*a*x) - 4*a^6*x^6*tan(1/2*a*x)^2 + 108*a^
4*x^4*imag_part(cos_integral(3*a*x))*tan(3/2*a*x)^2*tan(1/2*a*x)^3 - 4*a^4*
x^4*imag_part(cos_integral(a*x))*tan(3/2*a*x)^2*tan(1/2*a*x)^3 + 4*a^4*x^4*
imag_part(cos_integral(-a*x))*tan(3/2*a*x)^2*tan(1/2*a*x)^3 - 108*a^4*x^4*i
mag_part(cos_integral(-3*a*x))*tan(3/2*a*x)^2*tan(1/2*a*x)^3 + 216*a^4*x^4*
sin_integral(3*a*x)*tan(3/2*a*x)^2*tan(1/2*a*x)^3 - 8*a^4*x^4*sin_integral(
a*x)*tan(3/2*a*x)^2*tan(1/2*a*x)^3 - 32*a^4*x^4*tan(3/2*a*x)^2*tan(1/2*a*x)
^4 - 54*a^5*x^5*imag_part(cos_integral(3*a*x))*tan(3/2*a*x)^2 + 2*a^5*x^5*i
mag_part(cos_integral(a*x))*tan(3/2*a*x)^2 - 2*a^5*x^5*imag_part(cos_integr
al(-a*x))*tan(3/2*a*x)^2 + 54*a^5*x^5*imag_part(cos_integral(-3*a*x))*tan(3
/2*a*x)^2 - 108*a^5*x^5*sin_integral(3*a*x)*tan(3/2*a*x)^2 + 4*a^5*x^5*sin_
integral(a*x)*tan(3/2*a*x)^2 - 36*a^5*x^5*tan(3/2*a*x)^2*tan(1/2*a*x) + 36*
a^5*x^5*tan(1/2*a*x)^3 + 27*a^3*x^3*imag_part(cos_integral(3*a*x))*tan(3/2*

a*x)^2*tan(1/2*a*x)^4 - a^3*x^3*imag_part(cos_integral(a*x))*tan(3/2*a*x)^2
*tan(1/2*a*x)^4 + a^3*x^3*imag_part(cos_integral(-a*x))*tan(3/2*a*x)^2*tan(
1/2*a*x)^4 - 27*a^3*x^3*imag_part(cos_integral(-3*a*x))*tan(3/2*a*x)^2*tan(
1/2*a*x)^4 + 54*a^3*x^3*sin_integral(3*a*x)*tan(3/2*a*x)^2*tan(1/2*a*x)^4 -
 2*a^3*x^3*sin_integral(a*x)*tan(3/2*a*x)^2*tan(1/2*a*x)^4 - 16*a^6*x^6 + 1
08*a^4*x^4*imag_part(cos_integral(3*a*x))*tan(3/2*a*x)^2*tan(1/2*a*x) - 4*a
^4*x^4*imag_part(cos_integral(a*x))*tan(3/2*a*x)^2*tan(1/2*a*x) + 4*a^4*x^4
*imag_part(cos_integral(-a*x))*tan(3/2*a*x)^2*tan(1/2*a*x) - 108*a^4*x^4*im
ag_part(cos_integral(-3*a*x))*tan(3/2*a*x)^2*tan(1/2*a*x) + 216*a^4*x^4*sin
_integral(3*a*x)*tan(3/2*a*x)^2*tan(1/2*a*x) - 8*a^4*x^4*sin_integral(a*x)*
tan(3/2*a*x)^2*tan(1/2*a*x) - 8*a^4*x^4*tan(3/2*a*x)^2*tan(1/2*a*x)^2 + 108
*a^4*x^4*imag_part(cos_integral(3*a*x))*tan(1/2*a*x)^3 - 4*a^4*x^4*imag_par
t(cos_integral(a*x))*tan(1/2*a*x)^3 + 4*a^4*x^4*imag_part(cos_integral(-a*x
))*tan(1/2*a*x)^3 - 108*a^4*x^4*imag_part(cos_integral(-3*a*x))*tan(1/2*a*x
)^3 + 216*a^4*x^4*sin_integral(3*a*x)*tan(1/2*a*x)^3 - 8*a^4*x^4*sin_integr
al(a*x)*tan(1/2*a*x)^3 + 24*a^4*x^4*tan(3/2*a*x)*tan(1/2*a*x)^3 + 32*a^4*x^
4*tan(1/2*a*x)^4 - 54*a^5*x^5*imag_part(cos_integral(3*a*x)) + 2*a^5*x^5*im
ag_part(cos_integral(a*x)) - 2*a^5*x^5*imag_part(cos_integral(-a*x)) + 54*a
^5*x^5*imag_part(cos_integral(-3*a*x)) - 108*a^5*x^5*sin_integral(3*a*x) +
4*a^5*x^5*sin_integral(a*x) - 12*a^5*x^5*tan(3/2*a*x) + 36*a^5*x^5*tan(1/2*
a*x) - 48*a^3*x^3*tan(3/2*a*x)^2*tan(1/2*a*x)^3 + 27*a^3*x^3*imag_part(cos_
integral(3*a*x))*tan(1/2*a*x)^4 - a^3*x^3*imag_part(cos_integral(a*x))*tan(
1/2*a*x)^4 + a^3*x^3*imag_part(cos_integral(-a*x))*tan(1/2*a*x)^4 - 27*a^3*
x^3*imag_part(cos_integral(-3*a*x))*tan(1/2*a*x)^4 + 54*a^3*x^3*sin_integra
l(3*a*x)*tan(1/2*a*x)^4 - 2*a^3*x^3*sin_integral(a*x)*tan(1/2*a*x)^4 + 16*a
^3*x^3*tan(3/2*a*x)*tan(1/2*a*x)^4 + 32*a^4*x^4*tan(3/2*a*x)^2 + 108*a^4*x^
4*imag_part(cos_integral(3*a*x))*tan(1/2*a*x) - 4*a^4*x^4*imag_part(cos_int
egral(a*x))*tan(1/2*a*x) + 4*a^4*x^4*imag_part(cos_integral(-a*x))*tan(1/2*
a*x) - 108*a^4*x^4*imag_part(cos_integral(-3*a*x))*tan(1/2*a*x) + 216*a^4*x
^4*sin_integral(3*a*x)*tan(1/2*a*x) - 8*a^4*x^4*sin_integral(a*x)*tan(1/2*a
*x) + 24*a^4*x^4*tan(3/2*a*x)*tan(1/2*a*x) - 8*a^4*x^4*tan(1/2*a*x)^2 + 54*
a^2*x^2*imag_part(cos_integral(3*a*x))*tan(3/2*a*x)^2*tan(1/2*a*x)^3 - 2*a^
2*x^2*imag_part(cos_integral(a*x))*tan(3/2*a*x)^2*tan(1/2*a*x)^3 + 2*a^2*x^
2*imag_part(cos_integral(-a*x))*tan(3/2*a*x)^2*tan(1/2*a*x)^3 - 54*a^2*x^2*
imag_part(cos_integral(-3*a*x))*tan(3/2*a*x)^2*tan(1/2*a*x)^3 + 108*a^2*x^2
*sin_integral(3*a*x)*tan(3/2*a*x)^2*tan(1/2*a*x)^3 - 4*a^2*x^2*sin_integral
(a*x)*tan(3/2*a*x)^2*tan(1/2*a*x)^3 - 32*a^2*x^2*tan(3/2*a*x)^2*tan(1/2*a*x
)^4 - 27*a^3*x^3*imag_part(cos_integral(3*a*x))*tan(3/2*a*x)^2 + a^3*x^3*im
ag_part(cos_integral(a*x))*tan(3/2*a*x)^2 - a^3*x^3*imag_part(cos_integral(
-a*x))*tan(3/2*a*x)^2 + 27*a^3*x^3*imag_part(cos_integral(-3*a*x))*tan(3/2*
a*x)^2 - 54*a^3*x^3*sin_integral(3*a*x)*tan(3/2*a*x)^2 + 2*a^3*x^3*sin_inte
gral(a*x)*tan(3/2*a*x)^2 - 80*a^3*x^3*tan(3/2*a*x)^2*tan(1/2*a*x) + 80*a^3*
x^3*tan(1/2*a*x)^3 - 32*a^4*x^4 + 54*a^2*x^2*imag_part(cos_integral(3*a*x))
*tan(3/2*a*x)^2*tan(1/2*a*x) - 2*a^2*x^2*imag_part(cos_integral(a*x))*tan(3
/2*a*x)^2*tan(1/2*a*x) + 2*a^2*x^2*imag_part(cos_integral(-a*x))*tan(3/2*a*

```
x)^2*tan(1/2*a*x) - 54*a^2*x^2*imag_part(cos_integral(-3*a*x))*tan(3/2*a*x)
^2*tan(1/2*a*x) + 108*a^2*x^2*sin_integral(3*a*x)*tan(3/2*a*x)^2*tan(1/2*a*
x) - 4*a^2*x^2*sin_integral(a*x)*tan(3/2*a*x)^2*tan(1/2*a*x) - 60*a^2*x^2*t
an(3/2*a*x)^2*tan(1/2*a*x)^2 + 54*a^2*x^2*imag_part(cos_integral(3*a*x))*ta
n(1/2*a*x)^3 - 2*a^2*x^2*imag_part(cos_integral(a*x))*tan(1/2*a*x)^3 + 2*a^
2*x^2*imag_part(cos_integral(-a*x))*tan(1/2*a*x)^3 - 54*a^2*x^2*imag_part(c
os_integral(-3*a*x))*tan(1/2*a*x)^3 + 108*a^2*x^2*sin_integral(3*a*x)*tan(1
/2*a*x)^3 - 4*a^2*x^2*sin_integral(a*x)*tan(1/2*a*x)^3 + 32*a^2*x^2*tan(3/2
*a*x)*tan(1/2*a*x)^3 - 4*a^2*x^2*tan(1/2*a*x)^4 - 27*a^3*x^3*imag_part(cos_
integral(3*a*x)) + a^3*x^3*imag_part(cos_integral(a*x)) - a^3*x^3*imag_part
(cos_integral(-a*x)) + 27*a^3*x^3*imag_part(cos_integral(-3*a*x)) - 54*a^3*
x^3*sin_integral(3*a*x) + 2*a^3*x^3*sin_integral(a*x) - 16*a^3*x^3*tan(3/2*
a*x) + 48*a^3*x^3*tan(1/2*a*x) - 12*a*x*tan(3/2*a*x)^2*tan(1/2*a*x)^3 + 4*a
*x*tan(3/2*a*x)*tan(1/2*a*x)^4 - 4*a^2*x^2*tan(3/2*a*x)^2 + 54*a^2*x^2*imag
_part(cos_integral(3*a*x))*tan(1/2*a*x) - 2*a^2*x^2*imag_part(cos_integral(
a*x))*tan(1/2*a*x) + 2*a^2*x^2*imag_part(cos_integral(-a*x))*tan(1/2*a*x) -
 54*a^2*x^2*imag_part(cos_integral(-3*a*x))*tan(1/2*a*x) + 108*a^2*x^2*sin_
integral(3*a*x)*tan(1/2*a*x) - 4*a^2*x^2*sin_integral(a*x)*tan(1/2*a*x) + 3
2*a^2*x^2*tan(3/2*a*x)*tan(1/2*a*x) - 60*a^2*x^2*tan(1/2*a*x)^2 - 44*a*x*ta
n(3/2*a*x)^2*tan(1/2*a*x) + 44*a*x*tan(1/2*a*x)^3 - 32*a^2*x^2 - 24*tan(3/2
*a*x)^2*tan(1/2*a*x)^2 + 8*tan(3/2*a*x)*tan(1/2*a*x)^3 - 4*a*x*tan(3/2*a*x)
 + 12*a*x*tan(1/2*a*x) + 8*tan(3/2*a*x)*tan(1/2*a*x) - 24*tan(1/2*a*x)^2)/(
a^5*x^7*tan(3/2*a*x)^2*tan(1/2*a*x)^4 + a^5*x^7*tan(1/2*a*x)^4 + 2*a^4*x^6*
tan(3/2*a*x)^2*tan(1/2*a*x)^3 - a^5*x^7*tan(3/2*a*x)^2 + 2*a^3*x^5*tan(3/2*
a*x)^2*tan(1/2*a*x)^4 + 2*a^4*x^6*tan(3/2*a*x)^2*tan(1/2*a*x) + 2*a^4*x^6*t
an(1/2*a*x)^3 - a^5*x^7 + 2*a^3*x^5*tan(1/2*a*x)^4 + 2*a^4*x^6*tan(1/2*a*x)
 + 4*a^2*x^4*tan(3/2*a*x)^2*tan(1/2*a*x)^3 - 2*a^3*x^5*tan(3/2*a*x)^2 + a*x
^3*tan(3/2*a*x)^2*tan(1/2*a*x)^4 + 4*a^2*x^4*tan(3/2*a*x)^2*tan(1/2*a*x) +
4*a^2*x^4*tan(1/2*a*x)^3 - 2*a^3*x^5 + a*x^3*tan(1/2*a*x)^4 + 4*a^2*x^4*tan
(1/2*a*x) + 2*x^2*tan(3/2*a*x)^2*tan(1/2*a*x)^3 - a*x^3*tan(3/2*a*x)^2 + 2*
x^2*tan(3/2*a*x)^2*tan(1/2*a*x) + 2*x^2*tan(1/2*a*x)^3 - a*x^3 + 2*x^2*tan(
1/2*a*x))
```

maple [F(-1)] time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{\sin^5(ax)}{x^3(ax\cos(ax)-\sin(ax))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a*x)^5/x^3/(a*x*cos(a*x)-sin(a*x))^2,x)
```

```
[Out] int(sin(a*x)^5/x^3/(a*x*cos(a*x)-sin(a*x))^2,x)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a*x)^5/x^3/(a*x*cos(a*x)-sin(a*x))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is und
efined.
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sin(a\,x)^5}{x^3\,\left(\sin(a\,x)-a\,x\,\cos(a\,x)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a*x)^5/(x^3*(sin(a*x) - a*x*cos(a*x))^2),x)
```

```
[Out] int(sin(a*x)^5/(x^3*(sin(a*x) - a*x*cos(a*x))^2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin^5(ax)}{x^3\,(ax\cos(ax)-\sin(ax))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a*x)**5/x**3/(a*x*cos(a*x)-sin(a*x))**2,x)
```

```
[Out] Integral(sin(a*x)**5/(x**3*(a*x*cos(a*x) - sin(a*x))**2), x)
```

3.587 $\int \frac{\sin^4(ax)}{x^2(ax\cos(ax)-\sin(ax))^2}\,dx$

Optimal. Leaf size=80

$$\frac{\sin^2(ax)}{a^2x^3}+\frac{\sin^3(ax)}{a^2x^3(ax\cos(ax)-\sin(ax))}+2a\text{Si}(2ax)+\frac{\sin(ax)\cos(ax)}{ax^2}-\frac{2\sin^2(ax)}{x}+\frac{1}{x}$$

```
[Out] 1/x+2*a*Si(2*a*x)+cos(a*x)*sin(a*x)/a/x^2+sin(a*x)^2/a^2/x^3-2*sin(a*x)^2/x
+sin(a*x)^3/a^2/x^3/(a*x*cos(a*x)-sin(a*x))
```

Rubi [A] time = 0.13, antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {4598, 3314, 30, 3313, 12, 3299}

$$\frac{\sin^2(ax)}{a^2x^3}+\frac{\sin^3(ax)}{a^2x^3(ax\cos(ax)-\sin(ax))}+2a\text{Si}(2ax)+\frac{\sin(ax)\cos(ax)}{ax^2}-\frac{2\sin^2(ax)}{x}+\frac{1}{x}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a*x]^4/(x^2*(a*x*Cos[a*x] - Sin[a*x])^2),x]
```

```
[Out] x^(-1) + (Cos[a*x]*Sin[a*x])/(a*x^2) + Sin[a*x]^2/(a^2*x^3) - (2*Sin[a*x]^2
)/x + Sin[a*x]^3/(a^2*x^3*(a*x*Cos[a*x] - Sin[a*x])) + 2*a*SinIntegral[2*a*
x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3313

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> Si
mp[((c + d*x)^(m + 1)*Sin[e + f*x]^n)/(d*(m + 1)), x] - Dist[(f*n)/(d*(m +
1)), Int[ExpandTrigReduce[(c + d*x)^(m + 1), Cos[e + f*x]*Sin[e + f*x]^(n -
```

```
1), x], x], x] /; FreeQ[{c, d, e, f, m}, x] && IGtQ[n, 1] && GeQ[m, -2] &&
LtQ[m, -1]
```

Rule 3314

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[((c + d*x)^(m + 1)*(b*Sin[e + f*x])^n)/(d*(m + 1)), x] + (Dist[(
b^2*f^2*n*(n - 1))/(d^2*(m + 1)*(m + 2)), Int[(c + d*x)^(m + 2)*(b*Sin[e +
f*x])^(n - 2), x], x] - Dist[(f^2*n^2)/(d^2*(m + 1)*(m + 2)), Int[(c + d*x)
^(m + 2)*(b*Sin[e + f*x])^n, x], x] - Simp[(b*f*n*(c + d*x)^(m + 2)*Cos[e +
 f*x]*(b*Sin[e + f*x])^(n - 1))/(d^2*(m + 1)*(m + 2)), x]) /; FreeQ[{b, c,
d, e, f}, x] && GtQ[n, 1] && LtQ[m, -2]
```

Rule 4598

```
Int[(((b_.)*(x_))^(m_)*Sin[(a_.)*(x_)]^(n_))/(Cos[(a_.)*(x_)]*(d_.)*(x_) +
(c_.)*Sin[(a_.)*(x_)])^2, x_Symbol] :> Simp[(b*(b*x)^(m - 1)*Sin[a*x]^(n -
1))/(a*d*(c*Sin[a*x] + d*x*Cos[a*x])), x] - Dist[(b^2*(n - 1))/d^2, Int[(b*
x)^(m - 2)*Sin[a*x]^(n - 2), x], x] /; FreeQ[{a, b, c, d, m, n}, x] && EqQ[
a*c + d, 0] && EqQ[m, 2 - n]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sin^4(ax)}{x^2(ax\cos(ax)-\sin(ax))^2}\,dx &= \frac{\sin^3(ax)}{a^2x^3(ax\cos(ax)-\sin(ax))} - \frac{3\int \frac{\sin^2(ax)}{x^4}\,dx}{a^2} \\
&= \frac{\cos(ax)\sin(ax)}{ax^2} + \frac{\sin^2(ax)}{a^2x^3} + \frac{\sin^3(ax)}{a^2x^3(ax\cos(ax)-\sin(ax))} + 2\int \frac{\sin^2(ax)}{x^2}\,dx \\
&= \frac{1}{x} + \frac{\cos(ax)\sin(ax)}{ax^2} + \frac{\sin^2(ax)}{a^2x^3} - \frac{2\sin^2(ax)}{x} + \frac{\sin^3(ax)}{a^2x^3(ax\cos(ax)-\sin(ax))} + \\
&= \frac{1}{x} + \frac{\cos(ax)\sin(ax)}{ax^2} + \frac{\sin^2(ax)}{a^2x^3} - \frac{2\sin^2(ax)}{x} + \frac{\sin^3(ax)}{a^2x^3(ax\cos(ax)-\sin(ax))} + \\
&= \frac{1}{x} + \frac{\cos(ax)\sin(ax)}{ax^2} + \frac{\sin^2(ax)}{a^2x^3} - \frac{2\sin^2(ax)}{x} + \frac{\sin^3(ax)}{a^2x^3(ax\cos(ax)-\sin(ax))} +
\end{aligned}$$

Mathematica [A] time = 0.82, size = 77, normalized size = 0.96

$$\frac{8ax\operatorname{Si}(2ax)(ax\cos(ax)-\sin(ax))+3\sin(ax)-\sin(3ax)+2ax\cos(ax)+2ax\cos(3ax)}{4x(ax\cos(ax)-\sin(ax))}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a*x]^4/(x^2*(a*x*Cos[a*x] - Sin[a*x])^2),x]
```

```
[Out] (2*a*x*Cos[a*x] + 2*a*x*Cos[3*a*x] + 3*Sin[a*x] - Sin[3*a*x] + 8*a*x*(a*x*C
os[a*x] - Sin[a*x])*SinIntegral[2*a*x])/(4*x*(a*x*Cos[a*x] - Sin[a*x]))
```

fricas [A] time = 1.03, size = 77, normalized size = 0.96

$$\frac{2\,ax\cos\left(ax\right)^{3}+\left(2\,a^{2}x^{2}\,\mathrm{Si}\left(2\,ax\right)-ax\right)\cos\left(ax\right)-\left(2\,ax\,\mathrm{Si}\left(2\,ax\right)+\cos\left(ax\right)^{2}-1\right)\sin\left(ax\right)}{ax^{2}\cos\left(ax\right)-x\sin\left(ax\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a*x)^4/x^2/(a*x*cos(a*x)-sin(a*x))^2,x, algorithm="fricas")
```

```
[Out] (2*a*x*cos(a*x)^3 + (2*a^2*x^2*sin_integral(2*a*x) - a*x)*cos(a*x) - (2*a*x
*sin_integral(2*a*x) + cos(a*x)^2 - 1)*sin(a*x))/(a*x^2*cos(a*x) - x*sin(a*
x))
```

giac [C] time = 0.48, size = 1033, normalized size = 12.91

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a*x)^4/x^2/(a*x*cos(a*x)-sin(a*x))^2,x, algorithm="giac")
```

```
[Out] (a^4*x^4*imag_part(cos_integral(2*a*x))*tan(a*x)^2*tan(1/2*a*x)^2 - a^4*x^4
*imag_part(cos_integral(-2*a*x))*tan(a*x)^2*tan(1/2*a*x)^2 + 2*a^4*x^4*sin_
integral(2*a*x)*tan(a*x)^2*tan(1/2*a*x)^2 - a^4*x^4*imag_part(cos_integral(
2*a*x))*tan(a*x)^2 + a^4*x^4*imag_part(cos_integral(-2*a*x))*tan(a*x)^2 - 2
*a^4*x^4*sin_integral(2*a*x)*tan(a*x)^2 + a^4*x^4*imag_part(cos_integral(2*
a*x))*tan(1/2*a*x)^2 - a^4*x^4*imag_part(cos_integral(-2*a*x))*tan(1/2*a*x)
^2 + 2*a^4*x^4*sin_integral(2*a*x)*tan(1/2*a*x)^2 + 2*a^3*x^3*imag_part(cos
_integral(2*a*x))*tan(a*x)^2*tan(1/2*a*x) - 2*a^3*x^3*imag_part(cos_integra
l(-2*a*x))*tan(a*x)^2*tan(1/2*a*x) + 4*a^3*x^3*sin_integral(2*a*x)*tan(a*x)
^2*tan(1/2*a*x) - a^3*x^3*tan(a*x)^2*tan(1/2*a*x)^2 - a^4*x^4*imag_part(cos
_integral(2*a*x)) + a^4*x^4*imag_part(cos_integral(-2*a*x)) - 2*a^4*x^4*sin
_integral(2*a*x) + a^2*x^2*imag_part(cos_integral(2*a*x))*tan(a*x)^2*tan(1/
2*a*x)^2 - a^2*x^2*imag_part(cos_integral(-2*a*x))*tan(a*x)^2*tan(1/2*a*x)^
2 + 2*a^2*x^2*sin_integral(2*a*x)*tan(a*x)^2*tan(1/2*a*x)^2 + a^3*x^3*tan(a
*x)^2 + 2*a^3*x^3*imag_part(cos_integral(2*a*x))*tan(1/2*a*x) - 2*a^3*x^3*i
mag_part(cos_integral(-2*a*x))*tan(1/2*a*x) + 4*a^3*x^3*sin_integral(2*a*x)
*tan(1/2*a*x) + a^3*x^3*tan(1/2*a*x)^2 - a^2*x^2*imag_part(cos_integral(2*a
*x))*tan(a*x)^2 + a^2*x^2*imag_part(cos_integral(-2*a*x))*tan(a*x)^2 - 2*a^
2*x^2*sin_integral(2*a*x)*tan(a*x)^2 - 2*a^2*x^2*tan(a*x)^2*tan(1/2*a*x) +
a^2*x^2*imag_part(cos_integral(2*a*x))*tan(1/2*a*x)^2 - a^2*x^2*imag_part(c
os_integral(-2*a*x))*tan(1/2*a*x)^2 + 2*a^2*x^2*sin_integral(2*a*x)*tan(1/2
```

```
*a*x)^2 + a^2*x^2*tan(a*x)*tan(1/2*a*x)^2 - a^3*x^3 + 2*a*x*imag_part(cos_i
ntegral(2*a*x))*tan(a*x)^2*tan(1/2*a*x) - 2*a*x*imag_part(cos_integral(-2*a
*x))*tan(a*x)^2*tan(1/2*a*x) + 4*a*x*sin_integral(2*a*x)*tan(a*x)^2*tan(1/2
*a*x) - a^2*x^2*imag_part(cos_integral(2*a*x)) + a^2*x^2*imag_part(cos_inte
gral(-2*a*x)) - 2*a^2*x^2*sin_integral(2*a*x) - a^2*x^2*tan(a*x) + 2*a^2*x^
2*tan(1/2*a*x) + 2*a*x*imag_part(cos_integral(2*a*x))*tan(1/2*a*x) - 2*a*x*
imag_part(cos_integral(-2*a*x))*tan(1/2*a*x) + 4*a*x*sin_integral(2*a*x)*ta
n(1/2*a*x) + 2*a*x*tan(a*x)*tan(1/2*a*x) + a*x*tan(1/2*a*x)^2 - 2*tan(a*x)^
2*tan(1/2*a*x) - a*x)/(a^3*x^4*tan(a*x)^2*tan(1/2*a*x)^2 - a^3*x^4*tan(a*x)
^2 + a^3*x^4*tan(1/2*a*x)^2 + 2*a^2*x^3*tan(a*x)^2*tan(1/2*a*x) - a^3*x^4 +
 a*x^2*tan(a*x)^2*tan(1/2*a*x)^2 + 2*a^2*x^3*tan(1/2*a*x) - a*x^2*tan(a*x)^
2 + a*x^2*tan(1/2*a*x)^2 + 2*x*tan(a*x)^2*tan(1/2*a*x) - a*x^2 + 2*x*tan(1/
2*a*x))
```

maple [F(-1)] time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{\sin^4(ax)}{x^2 (ax\cos(ax) - \sin(ax))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a*x)^4/x^2/(a*x*cos(a*x)-sin(a*x))^2,x)
```

```
[Out] int(sin(a*x)^4/x^2/(a*x*cos(a*x)-sin(a*x))^2,x)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a*x)^4/x^2/(a*x*cos(a*x)-sin(a*x))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is und
efined.
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sin(a\,x)^4}{x^2 (\sin(a\,x) - a\,x\,\cos(a\,x))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a*x)^4/(x^2*(sin(a*x) - a*x*cos(a*x))^2),x)
```

```
[Out] int(sin(a*x)^4/(x^2*(sin(a*x) - a*x*cos(a*x))^2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin^4(ax)}{x^2(ax\cos(ax)-\sin(ax))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a*x)**4/x**2/(a*x*cos(a*x)-sin(a*x))**2,x)
```

```
[Out] Integral(sin(a*x)**4/(x**2*(a*x*cos(a*x) - sin(a*x))**2), x)
```

3.588 $\int \frac{\sin^3(ax)}{x(ax\cos(ax)-\sin(ax))^2}\,dx$

Optimal. Leaf size=56

$$\frac{\sin(ax)}{a^2x^2}+\frac{\sin^2(ax)}{a^2x^2(ax\cos(ax)-\sin(ax))}+\mathrm{Si}(ax)+\frac{\cos(ax)}{ax}$$

```
[Out] cos(a*x)/a/x+Si(a*x)+sin(a*x)/a^2/x^2+sin(a*x)^2/a^2/x^2/(a*x*cos(a*x)-sin(
a*x))
```

Rubi [A] time = 0.10, antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {4598, 3297, 3299}

$$\frac{\sin(ax)}{a^2x^2}+\frac{\sin^2(ax)}{a^2x^2(ax\cos(ax)-\sin(ax))}+\mathrm{Si}(ax)+\frac{\cos(ax)}{ax}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a*x]^3/(x*(a*x*Cos[a*x] - Sin[a*x])^2),x]
```

```
[Out] Cos[a*x]/(a*x) + Sin[a*x]/(a^2*x^2) + Sin[a*x]^2/(a^2*x^2*(a*x*Cos[a*x] - S
in[a*x])) + SinIntegral[a*x]
```

Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[((
c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 4598

```
Int[(((b_.)*(x_))^(m_)*Sin[(a_.)*(x_)]^(n_))/(Cos[(a_.)*(x_)]*(d_.)*(x_) +
(c_.)*Sin[(a_.)*(x_)])^2, x_Symbol] :> Simp[(b*(b*x)^(m - 1)*Sin[a*x]^(n -
1))/(a*d*(c*Sin[a*x] + d*x*Cos[a*x])), x] - Dist[(b^2*(n - 1))/d^2, Int[(b*
x)^(m - 2)*Sin[a*x]^(n - 2), x], x] /; FreeQ[{a, b, c, d, m, n}, x] && EqQ[
a*c + d, 0] && EqQ[m, 2 - n]
```

Rubi steps

$$\int \frac{\sin^3(ax)}{x(ax\cos(ax)-\sin(ax))^2}\,dx = \frac{\sin^2(ax)}{a^2x^2(ax\cos(ax)-\sin(ax))} - \frac{2\int \frac{\sin(ax)}{x^3}\,dx}{a^2}$$
$$= \frac{\sin(ax)}{a^2x^2} + \frac{\sin^2(ax)}{a^2x^2(ax\cos(ax)-\sin(ax))} - \frac{\int \frac{\cos(ax)}{x^2}\,dx}{a}$$
$$= \frac{\cos(ax)}{ax} + \frac{\sin(ax)}{a^2x^2} + \frac{\sin^2(ax)}{a^2x^2(ax\cos(ax)-\sin(ax))} + \int \frac{\sin(ax)}{x}\,dx$$
$$= \frac{\cos(ax)}{ax} + \frac{\sin(ax)}{a^2x^2} + \frac{\sin^2(ax)}{a^2x^2(ax\cos(ax)-\sin(ax))} + \mathrm{Si}(ax)$$

Mathematica [C] time = 7.42, size = 242, normalized size = 4.32

$$-ie\mathrm{Ci}(i-ax)(ax\cos(ax)-\sin(ax)) + ie\mathrm{Ci}(ax+i)(ax\cos(ax)-\sin(ax)) - ie\mathrm{Ei}(-iax-1)\sin(ax) + ie\mathrm{Ei}(iax-1)\,\mathrm{s}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sin[a*x]^3/(x*(a*x*Cos[a*x] - Sin[a*x])^2),x]
```

```
[Out] (1 + Cos[2*a*x] + I*a*E*x*Cos[a*x]*ExpIntegralEi[-1 - I*a*x] - I*a*E*x*Cos[
a*x]*ExpIntegralEi[-1 + I*a*x] - I*E*CosIntegral[I - a*x]*(a*x*Cos[a*x] - S
in[a*x]) + I*E*CosIntegral[I + a*x]*(a*x*Cos[a*x] - Sin[a*x]) - I*E*ExpInte
gralEi[-1 - I*a*x]*Sin[a*x] + I*E*ExpIntegralEi[-1 + I*a*x]*Sin[a*x] + 2*a*
x*Cos[a*x]*SinIntegral[a*x] - 2*Sin[a*x]*SinIntegral[a*x] + a*E*x*Cos[a*x]*
SinIntegral[I - a*x] - E*Sin[a*x]*SinIntegral[I - a*x] - a*E*x*Cos[a*x]*Sin
Integral[I + a*x] + E*Sin[a*x]*SinIntegral[I + a*x])/(2*a*x*Cos[a*x] - 2*Si
n[a*x])
```

fricas [A] time = 0.87, size = 45, normalized size = 0.80

$$\frac{ax\cos(ax)\,\mathrm{Si}(ax) + \cos(ax)^2 - \sin(ax)\,\mathrm{Si}(ax)}{ax\cos(ax) - \sin(ax)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a*x)^3/x/(a*x*cos(a*x)-sin(a*x))^2,x, algorithm="fricas")
```

```
[Out] (a*x*cos(a*x)*sin_integral(a*x) + cos(a*x)^2 - sin(a*x)*sin_integral(a*x))/
(a*x*cos(a*x) - sin(a*x))
```

giac [C] time = 0.37, size = 496, normalized size = 8.86

$$\frac{a^3x^3\Im(\text{Ci}(ax))\tan\left(\frac{1}{2}ax\right)^4 - a^3x^3\Im(\text{Ci}(-ax))\tan\left(\frac{1}{2}ax\right)^4 + 2a^3x^3\text{Si}(ax)\tan\left(\frac{1}{2}ax\right)^4 + 2a^2x^2\Im(\text{Ci}(ax))\tan\left(\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a*x)^3/x/(a*x*cos(a*x)-sin(a*x))^2,x, algorithm="giac")
```

```
[Out] 1/2*(a^3*x^3*imag_part(cos_integral(a*x))*tan(1/2*a*x)^4 - a^3*x^3*imag_par
t(cos_integral(-a*x))*tan(1/2*a*x)^4 + 2*a^3*x^3*sin_integral(a*x)*tan(1/2*
a*x)^4 + 2*a^2*x^2*imag_part(cos_integral(a*x))*tan(1/2*a*x)^3 - 2*a^2*x^2*
imag_part(cos_integral(-a*x))*tan(1/2*a*x)^3 + 4*a^2*x^2*sin_integral(a*x)*
tan(1/2*a*x)^3 - 2*a^2*x^2*tan(1/2*a*x)^4 - a^3*x^3*imag_part(cos_integral(
a*x)) + a^3*x^3*imag_part(cos_integral(-a*x)) - 2*a^3*x^3*sin_integral(a*x)
 + a*x*imag_part(cos_integral(a*x))*tan(1/2*a*x)^4 - a*x*imag_part(cos_inte
gral(-a*x))*tan(1/2*a*x)^4 + 2*a*x*sin_integral(a*x)*tan(1/2*a*x)^4 + 2*a^2
*x^2*imag_part(cos_integral(a*x))*tan(1/2*a*x) - 2*a^2*x^2*imag_part(cos_in
tegral(-a*x))*tan(1/2*a*x) + 4*a^2*x^2*sin_integral(a*x)*tan(1/2*a*x) + 4*a
^2*x^2*tan(1/2*a*x)^2 - 2*a^2*x^2 + 2*imag_part(cos_integral(a*x))*tan(1/2*
a*x)^3 - 2*imag_part(cos_integral(-a*x))*tan(1/2*a*x)^3 + 4*sin_integral(a*
x)*tan(1/2*a*x)^3 - 4*tan(1/2*a*x)^4 - a*x*imag_part(cos_integral(a*x)) + a
*x*imag_part(cos_integral(-a*x)) - 2*a*x*sin_integral(a*x) + 2*imag_part(co
s_integral(a*x))*tan(1/2*a*x) - 2*imag_part(cos_integral(-a*x))*tan(1/2*a*x
) + 4*sin_integral(a*x)*tan(1/2*a*x) - 4)/(a^3*x^3*tan(1/2*a*x)^4 + 2*a^2*x
^2*tan(1/2*a*x)^3 - a^3*x^3 + a*x*tan(1/2*a*x)^4 + 2*a^2*x^2*tan(1/2*a*x) +
 2*tan(1/2*a*x)^3 - a*x + 2*tan(1/2*a*x))
```

maple [C] time = 4.28, size = 108, normalized size = 1.93

$$\frac{i\,\text{Ei}(1,-iax)}{2} + \frac{i\mathrm{e}^{iax}}{2iax-2} + \frac{i\mathrm{e}^{-iax}}{2iax+2} - \frac{i\,\text{Ei}(1,iax)}{2} + \frac{2\,\mathrm{e}^{iax}}{(ax+i)(ax-i)\left(ax\,\mathrm{e}^{2iax} + i\mathrm{e}^{2iax} + ax - i\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a*x)^3/x/(a*x*cos(a*x)-sin(a*x))^2,x)
```

```
[Out] 1/2*I*Ei(1,-I*a*x)+1/2*I*exp(I*a*x)/(-1+I*a*x)+1/2*I*exp(-I*a*x)/(I*a*x+1)-
1/2*I*Ei(1,I*a*x)+2*exp(I*a*x)/(a*x+I)/(a*x-I)/(a*x*exp(2*I*a*x)+I*exp(2*I*
a*x)+a*x-I)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a*x)^3/x/(a*x*cos(a*x)-sin(a*x))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is und
efined.
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\sin(a\,x)^3}{x\,(\sin(a\,x) - a\,x\,\cos(a\,x))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a*x)^3/(x*(sin(a*x) - a*x*cos(a*x))^2),x)
```

```
[Out] int(sin(a*x)^3/(x*(sin(a*x) - a*x*cos(a*x))^2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin^3(ax)}{x\,(ax\cos(ax) - \sin(ax))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a*x)**3/x/(a*x*cos(a*x)-sin(a*x))**2,x)
```

```
[Out] Integral(sin(a*x)**3/(x*(a*x*cos(a*x) - sin(a*x))**2), x)
```

3.589 $\int \frac{\sin^2(ax)}{(ax\cos(ax)-\sin(ax))^2}\,dx$

Optimal. Leaf size=35

$$\frac{1}{a^2x}+\frac{\sin(ax)}{a^2x(ax\cos(ax)-\sin(ax))}$$

```
[Out] 1/a^2/x+sin(a*x)/a^2/x/(a*x*cos(a*x)-sin(a*x))
```

Rubi [A] time = 0.02, antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.043, Rules used = {4596}

$$\frac{1}{a^2x}+\frac{\sin(ax)}{a^2x(ax\cos(ax)-\sin(ax))}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a*x]^2/(a*x*Cos[a*x] - Sin[a*x])^2,x]
```

```
[Out] 1/(a^2*x) + Sin[a*x]/(a^2*x*(a*x*Cos[a*x] - Sin[a*x]))
```

Rule 4596

```
Int[Sin[(a_.)*(x_)]^2/(Cos[(a_.)*(x_)]*(d_.)*(x_) + (c_.)*Sin[(a_.)*(x_)])^
2, x_Symbol] :> Simp[1/(d^2*x), x] + Simp[Sin[a*x]/(a*d*x*(d*x*Cos[a*x] + c
*Sin[a*x])), x] /; FreeQ[{a, c, d}, x] && EqQ[a*c + d, 0]
```

Rubi steps

$$\int \frac{\sin^2(ax)}{(ax\cos(ax)-\sin(ax))^2}\,dx=\frac{1}{a^2x}+\frac{\sin(ax)}{a^2x(ax\cos(ax)-\sin(ax))}$$

Mathematica [A] time = 0.28, size = 24, normalized size = 0.69

$$\frac{\cos(ax)}{a^2x\cos(ax)-a\sin(ax)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a*x]^2/(a*x*Cos[a*x] - Sin[a*x])^2,x]
```

```
[Out] Cos[a*x]/(a^2*x*Cos[a*x] - a*Sin[a*x])
```

fricas [A] time = 1.95, size = 24, normalized size = 0.69

$$\frac{\cos(ax)}{a^2x\cos(ax) - a\sin(ax)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a*x)^2/(a*x*cos(a*x)-sin(a*x))^2,x, algorithm="fricas")
```

```
[Out] cos(a*x)/(a^2*x*cos(a*x) - a*sin(a*x))
```

giac [A] time = 0.19, size = 39, normalized size = 1.11

$$\frac{\tan\left(\frac{1}{2}\,ax\right)^2 - 1}{a^2x\tan\left(\frac{1}{2}\,ax\right)^2 - a^2x + 2\,a\tan\left(\frac{1}{2}\,ax\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a*x)^2/(a*x*cos(a*x)-sin(a*x))^2,x, algorithm="giac")
```

```
[Out] (tan(1/2*a*x)^2 - 1)/(a^2*x*tan(1/2*a*x)^2 - a^2*x + 2*a*tan(1/2*a*x))
```

maple [B] time = 1.47, size = 77, normalized size = 2.20

$$\frac{\frac{\tan^4\left(\frac{ax}{2}\right)}{a} + \frac{\tan^6\left(\frac{ax}{2}\right)}{a} - \frac{1}{a} - \frac{\tan^2\left(\frac{ax}{2}\right)}{a}}{\left(1+\tan^2\left(\frac{ax}{2}\right)\right)^2\left(ax\left(\tan^2\left(\frac{ax}{2}\right)\right) - ax + 2\tan\left(\frac{ax}{2}\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a*x)^2/(a*x*cos(a*x)-sin(a*x))^2,x)
```

```
[Out] (1/a*tan(1/2*a*x)^4+1/a*tan(1/2*a*x)^6-1/a-1/a*tan(1/2*a*x)^2)/(1+tan(1/2*a
*x)^2)^2/(a*x*tan(1/2*a*x)^2-a*x+2*tan(1/2*a*x))
```

maxima [B] time = 0.32, size = 114, normalized size = 3.26

$$\frac{ax\cos(2\,ax)^2 + ax\sin(2\,ax)^2 + 2\,ax\cos(2\,ax) + ax - 2\,\sin(2\,ax)}{\left(a^2x^2 + \left(a^2x^2+1\right)\cos(2\,ax)^2 - 4\,ax\sin(2\,ax) + \left(a^2x^2+1\right)\sin(2\,ax)^2 + 2\left(a^2x^2-1\right)\cos(2\,ax) + 1\right)a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a*x)^2/(a*x*cos(a*x)-sin(a*x))^2,x, algorithm="maxima")
```

```
[Out] (a*x*cos(2*a*x)^2 + a*x*sin(2*a*x)^2 + 2*a*x*cos(2*a*x) + a*x - 2*sin(2*a*x
))/((a^2*x^2 + (a^2*x^2 + 1)*cos(2*a*x)^2 - 4*a*x*sin(2*a*x) + (a^2*x^2 + 1
)*sin(2*a*x)^2 + 2*(a^2*x^2 - 1)*cos(2*a*x) + 1)*a)
```

mupad [B] time = 3.03, size = 24, normalized size = 0.69

$$-\frac{\cos\left(a\,x\right)}{a\,\left(\sin\left(a\,x\right)-a\,x\,\cos\left(a\,x\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a*x)^2/(sin(a*x) - a*x*cos(a*x))^2,x)
```

```
[Out] -cos(a*x)/(a*(sin(a*x) - a*x*cos(a*x)))
```

sympy [A] time = 3.42, size = 20, normalized size = 0.57

$$\frac{\cos\left(ax\right)}{a^{2}x\cos\left(ax\right)-a\sin\left(ax\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a*x)**2/(a*x*cos(a*x)-sin(a*x))**2,x)
```

```
[Out] cos(a*x)/(a**2*x*cos(a*x) - a*sin(a*x))
```

3.590 $\int \frac{x\sin(ax)}{(ax\cos(ax)-\sin(ax))^2}\,dx$

Optimal. Leaf size=20

$$\frac{1}{a^2(ax\cos(ax)-\sin(ax))}$$

```
[Out] 1/a^2/(a*x*cos(a*x)-sin(a*x))
```

Rubi [A] time = 0.04, antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {6686}

$$\frac{1}{a^2(ax\cos(ax)-\sin(ax))}$$

Antiderivative was successfully verified.

```
[In] Int[(x*Sin[a*x])/(a*x*Cos[a*x] - Sin[a*x])^2,x]
[Out] 1/(a^2*(a*x*Cos[a*x] - Sin[a*x]))
```

Rule 6686

```
Int[(u_)*(y_)^(m_.), x_Symbol] :> With[{q = DerivativeDivides[y, u, x]}, Si
mp[(q*y^(m + 1))/(m + 1), x] /;  !FalseQ[q]] /; FreeQ[m, x] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{x\sin(ax)}{(ax\cos(ax)-\sin(ax))^2}\,dx=\frac{1}{a^2(ax\cos(ax)-\sin(ax))}$$

Mathematica [A] time = 0.03, size = 20, normalized size = 1.00

$$-\frac{1}{a^2(\sin(ax)-ax\cos(ax))}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*Sin[a*x])/(a*x*Cos[a*x] - Sin[a*x])^2,x]
[Out] -(1/(a^2*(-(a*x*Cos[a*x]) + Sin[a*x])))
```

fricas [A] time = 1.75, size = 21, normalized size = 1.05

$$\frac{1}{a^3x\cos(ax)-a^2\sin(ax)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(a*x)/(a*x*cos(a*x)-sin(a*x))^2,x, algorithm="fricas")
[Out] 1/(a^3*x*cos(a*x) - a^2*sin(a*x))
```

giac [B] time = 0.18, size = 42, normalized size = 2.10

$$-\frac{2\left(\tan\left(\frac{1}{2}\,ax\right)^{2}+1\right)}{a^{3}x\tan\left(\frac{1}{2}\,ax\right)^{2}-a^{3}x+2\,a^{2}\tan\left(\frac{1}{2}\,ax\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(a*x)/(a*x*cos(a*x)-sin(a*x))^2,x, algorithm="giac")
[Out] -2*(tan(1/2*a*x)^2 + 1)/(a^3*x*tan(1/2*a*x)^2 - a^3*x + 2*a^2*tan(1/2*a*x))
```

maple [A] time = 0.20, size = 21, normalized size = 1.05

$$\frac{1}{a^{2}\left(ax\cos\left(ax\right)-\sin\left(ax\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*sin(a*x)/(a*x*cos(a*x)-sin(a*x))^2,x)
[Out] 1/a^2/(a*x*cos(a*x)-sin(a*x))
```

maxima [A] time = 0.31, size = 20, normalized size = 1.00

$$\frac{1}{(ax\cos\left(ax\right)-\sin\left(ax\right))a^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(a*x)/(a*x*cos(a*x)-sin(a*x))^2,x, algorithm="maxima")
[Out] 1/((a*x*cos(a*x) - sin(a*x))*a^2)
```

mupad [B] time = 0.12, size = 23, normalized size = 1.15

$$-\frac{1}{a^{2}\,\sin\left(a\,x\right)-a^{3}\,x\,\cos\left(a\,x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*sin(a*x))/(sin(a*x) - a*x*cos(a*x))^2,x)
```

```
[Out] -1/(a^2*sin(a*x) - a^3*x*cos(a*x))
```

sympy [A] time = 3.48, size = 19, normalized size = 0.95

$$\frac{1}{a^3 x \cos (ax) - a^2 \sin (ax)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(a*x)/(a*x*cos(a*x)-sin(a*x))**2,x)
```

```
[Out] 1/(a**3*x*cos(a*x) - a**2*sin(a*x))
```

3.591 $\int \frac{x^2}{(ax\cos(ax)-\sin(ax))^2}\,dx$

Optimal. Leaf size=35

$$\frac{x\csc(ax)}{a^2(ax\cos(ax)-\sin(ax))}-\frac{\cot(ax)}{a^3}$$

```
[Out] -cot(a*x)/a^3+x*csc(a*x)/a^2/(a*x*cos(a*x)-sin(a*x))
```

Rubi [A] time = 0.04, antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {4594, 3767, 8}

$$\frac{x\csc(ax)}{a^2(ax\cos(ax)-\sin(ax))}-\frac{\cot(ax)}{a^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(a*x*Cos[a*x] - Sin[a*x])^2,x]
```

```
[Out] -(Cot[a*x]/a^3) + (x*Csc[a*x])/(a^2*(a*x*Cos[a*x] - Sin[a*x]))
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 4594

```
Int[(x_)^2/(Cos[(a_.)*(x_)]*(d_.)*(x_) + (c_.)*Sin[(a_.)*(x_)])^2, x_Symbol
] :> Simp[x/(a*d*Sin[a*x]*(c*Sin[a*x] + d*x*Cos[a*x])), x] + Dist[1/d^2, In
t[1/Sin[a*x]^2, x], x] /; FreeQ[{a, c, d}, x] && EqQ[a*c + d, 0]
```

Rubi steps

$$\int \frac{x^2}{(ax\cos(ax)-\sin(ax))^2}\,dx = \frac{x\csc(ax)}{a^2(ax\cos(ax)-\sin(ax))} + \frac{\int \csc^2(ax)\,dx}{a^2}$$
$$= \frac{x\csc(ax)}{a^2(ax\cos(ax)-\sin(ax))} - \frac{\text{Subst}(\int 1\,dx, x, \cot(ax))}{a^3}$$
$$= -\frac{\cot(ax)}{a^3} + \frac{x\csc(ax)}{a^2(ax\cos(ax)-\sin(ax))}$$

Mathematica [**A**] time = 0.46, size = 32, normalized size = 0.91

$$\frac{ax\sin(ax)+\cos(ax)}{a^3(ax\cos(ax)-\sin(ax))}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(a*x*Cos[a*x] - Sin[a*x])^2,x]
[Out] (Cos[a*x] + a*x*Sin[a*x])/(a^3*(a*x*Cos[a*x] - Sin[a*x]))
```

fricas [**A**] time = 0.82, size = 34, normalized size = 0.97

$$\frac{ax\sin(ax)+\cos(ax)}{a^4x\cos(ax)-a^3\sin(ax)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a*x*cos(a*x)-sin(a*x))^2,x, algorithm="fricas")
[Out] (a*x*sin(a*x) + cos(a*x))/(a^4*x*cos(a*x) - a^3*sin(a*x))
```

giac [**A**] time = 0.15, size = 53, normalized size = 1.51

$$-\frac{2\,ax\tan\left(\frac{1}{2}\,ax\right)-\tan\left(\frac{1}{2}\,ax\right)^2+1}{a^4x\tan\left(\frac{1}{2}\,ax\right)^2-a^4x+2\,a^3\tan\left(\frac{1}{2}\,ax\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a*x*cos(a*x)-sin(a*x))^2,x, algorithm="giac")
[Out] -(2*a*x*tan(1/2*a*x) - tan(1/2*a*x)^2 + 1)/(a^4*x*tan(1/2*a*x)^2 - a^4*x +
2*a^3*tan(1/2*a*x))
```

maple [A] time = 1.23, size = 54, normalized size = 1.54

$$\frac{\frac{\tan^2\left(\frac{ax}{2}\right)}{a^3}-\frac{1}{a^3}-\frac{2x\tan\left(\frac{ax}{2}\right)}{a^2}}{ax\left(\tan^2\left(\frac{ax}{2}\right)\right)-ax+2\tan\left(\frac{ax}{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a*x*cos(a*x)-sin(a*x))^2,x)
```

```
[Out] (1/a^3*tan(1/2*a*x)^2-1/a^3-2*x/a^2*tan(1/2*a*x))/(a*x*tan(1/2*a*x)^2-a*x+2
*tan(1/2*a*x))
```

maxima [B] time = 0.31, size = 100, normalized size = 2.86

$$\frac{2\left(2\,ax\cos\left(2\,ax\right)+\left(a^2x^2-1\right)\sin\left(2\,ax\right)\right)}{\left(a^2x^2+\left(a^2x^2+1\right)\cos\left(2\,ax\right)^2-4\,ax\sin\left(2\,ax\right)+\left(a^2x^2+1\right)\sin\left(2\,ax\right)^2+2\left(a^2x^2-1\right)\cos\left(2\,ax\right)+1\right)a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a*x*cos(a*x)-sin(a*x))^2,x, algorithm="maxima")
```

```
[Out] 2*(2*a*x*cos(2*a*x) + (a^2*x^2 - 1)*sin(2*a*x))/((a^2*x^2 + (a^2*x^2 + 1)*c
os(2*a*x)^2 - 4*a*x*sin(2*a*x) + (a^2*x^2 + 1)*sin(2*a*x)^2 + 2*(a^2*x^2 -
1)*cos(2*a*x) + 1)*a^3)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{x^2}{\left(\sin\left(a\,x\right)-a\,x\,\cos\left(a\,x\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(sin(a*x) - a*x*cos(a*x))^2,x)
```

```
[Out] int(x^2/(sin(a*x) - a*x*cos(a*x))^2, x)
```

sympy [B] time = 5.05, size = 112, normalized size = 3.20

$$-\frac{2ax\tan\left(\frac{ax}{2}\right)}{a^4x\tan^2\left(\frac{ax}{2}\right)-a^4x+2a^3\tan\left(\frac{ax}{2}\right)}+\frac{\tan^2\left(\frac{ax}{2}\right)}{a^4x\tan^2\left(\frac{ax}{2}\right)-a^4x+2a^3\tan\left(\frac{ax}{2}\right)}-\frac{1}{a^4x\tan^2\left(\frac{ax}{2}\right)-a^4x+2a^3\tan\left(\frac{ax}{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(a*x*cos(a*x)-sin(a*x))**2,x)
```

```
[Out] -2*a*x*tan(a*x/2)/(a**4*x*tan(a*x/2)**2 - a**4*x + 2*a**3*tan(a*x/2)) + tan
(a*x/2)**2/(a**4*x*tan(a*x/2)**2 - a**4*x + 2*a**3*tan(a*x/2)) - 1/(a**4*x*
tan(a*x/2)**2 - a**4*x + 2*a**3*tan(a*x/2))
```

3.592 $\int \frac{x^3 \csc(ax)}{(ax\cos(ax)-\sin(ax))^2}\,dx$

Optimal. Leaf size=104

$$\frac{i\text{Li}_2\left(-e^{iax}\right)}{a^4}-\frac{i\text{Li}_2\left(e^{iax}\right)}{a^4}-\frac{\csc(ax)}{a^4}-\frac{2x\tanh^{-1}\left(e^{iax}\right)}{a^3}-\frac{x\cot(ax)\csc(ax)}{a^3}+\frac{x^2\csc^2(ax)}{a^2(ax\cos(ax)-\sin(ax))}$$

```
[Out] -2*x*arctanh(exp(I*a*x))/a^3-csc(a*x)/a^4-x*cot(a*x)*csc(a*x)/a^3+I*polylog
(2,-exp(I*a*x))/a^4-I*polylog(2,exp(I*a*x))/a^4+x^2*csc(a*x)^2/a^2/(a*x*cos
(a*x)-sin(a*x))
```

Rubi [A] time = 0.09, antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {4600, 4185, 4183, 2279, 2391}

$$\frac{i\text{PolyLog}\left(2,-e^{iax}\right)}{a^4}-\frac{i\text{PolyLog}\left(2,e^{iax}\right)}{a^4}+\frac{x^2\csc^2(ax)}{a^2(ax\cos(ax)-\sin(ax))}-\frac{\csc(ax)}{a^4}-\frac{2x\tanh^{-1}\left(e^{iax}\right)}{a^3}-\frac{x\cot(ax)\csc(ax)}{a^3}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*Csc[a*x])/(a*x*Cos[a*x] - Sin[a*x])^2,x]
```

```
[Out] (-2*x*ArcTanh[E^(I*a*x)])/a^3 - Csc[a*x]/a^4 - (x*Cot[a*x]*Csc[a*x])/a^3 +
(I*PolyLog[2, -E^(I*a*x)])/a^4 - (I*PolyLog[2, E^(I*a*x)])/a^4 + (x^2*Csc[a
*x]^2)/(a^2*(a*x*Cos[a*x] - Sin[a*x]))
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4183

```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[(
-2*(c + d*x)^m*ArcTanh[E^(I*(e + f*x))])/f, x] + (-Dist[(d*m)/f, Int[(c + d
*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Dist[(d*m)/f, Int[(c + d*x)^
(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 4185

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_)), x_Symbol] :>
 -Simp[(b^2*(c + d*x)*Cot[e + f*x]*(b*Csc[e + f*x])^(n - 2))/(f*(n - 1)), x
] + (Dist[(b^2*(n - 2))/(n - 1), Int[(c + d*x)*(b*Csc[e + f*x])^(n - 2), x]
, x] - Simp[(b^2*d*(b*Csc[e + f*x])^(n - 2))/(f^2*(n - 1)*(n - 2)), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && NeQ[n, 2]
```

Rule 4600

```
Int[(Csc[(a_.)*(x_)]^(n_.)*((b_.)*(x_))^(m_.))/(Cos[(a_.)*(x_)]*(d_.)*(x_)
+ (c_.)*Sin[(a_.)*(x_)])^2, x_Symbol] :> Simp[(b*(b*x)^(m - 1)*Csc[a*x]^(n
+ 1))/(a*d*(c*Sin[a*x] + d*x*Cos[a*x])), x] + Dist[(b^2*(n + 1))/d^2, Int[(
b*x)^(m - 2)*Csc[a*x]^(n + 2), x], x] /; FreeQ[{a, b, c, d, m, n}, x] && Eq
Q[a*c + d, 0] && EqQ[m, n + 2]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^3\csc(ax)}{(ax\cos(ax)-\sin(ax))^2}\,dx &= \frac{x^2\csc^2(ax)}{a^2(ax\cos(ax)-\sin(ax))}+\frac{2\int x\csc^3(ax)\,dx}{a^2}\\
&= -\frac{\csc(ax)}{a^4}-\frac{x\cot(ax)\csc(ax)}{a^3}+\frac{x^2\csc^2(ax)}{a^2(ax\cos(ax)-\sin(ax))}+\frac{\int x\csc(ax)\,dx}{a^2}\\
&= -\frac{2x\tanh^{-1}\left(e^{iax}\right)}{a^3}-\frac{\csc(ax)}{a^4}-\frac{x\cot(ax)\csc(ax)}{a^3}+\frac{x^2\csc^2(ax)}{a^2(ax\cos(ax)-\sin(ax))}-\\
&= -\frac{2x\tanh^{-1}\left(e^{iax}\right)}{a^3}-\frac{\csc(ax)}{a^4}-\frac{x\cot(ax)\csc(ax)}{a^3}+\frac{x^2\csc^2(ax)}{a^2(ax\cos(ax)-\sin(ax))}+\\
&= -\frac{2x\tanh^{-1}\left(e^{iax}\right)}{a^3}-\frac{\csc(ax)}{a^4}-\frac{x\cot(ax)\csc(ax)}{a^3}+\frac{i\text{Li}_2\left(-e^{iax}\right)}{a^4}-\frac{i\text{Li}_2\left(e^{iax}\right)}{a^4}+
\end{aligned}$$

Mathematica [A] time = 1.00, size = 157, normalized size = 1.51

$$\frac{a^2x^2\csc(ax)+a^2x^2\log\left(1-e^{iax}\right)\cot(ax)-a^2x^2\log\left(1+e^{iax}\right)\cot(ax)+i\text{Li}_2\left(-e^{iax}\right)(ax\cot(ax)-1)-i\text{Li}_2\left(e^{iax}\right)}{a^4(ax\cot(ax)-1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*Csc[a*x])/(a*x*Cos[a*x] - Sin[a*x])^2,x]
```

```
[Out] (Csc[a*x] + a^2*x^2*Csc[a*x] - a*x*Log[1 - E^(I*a*x)] + a^2*x^2*Cot[a*x]*Lo
g[1 - E^(I*a*x)] + a*x*Log[1 + E^(I*a*x)] - a^2*x^2*Cot[a*x]*Log[1 + E^(I*a
```

```
*x)] + I*(-1 + a*x*Cot[a*x])*PolyLog[2, -E^(I*a*x)] - I*(-1 + a*x*Cot[a*x])
*PolyLog[2, E^(I*a*x)])/(a^4*(-1 + a*x*Cot[a*x]))
```

fricas [B] time = 2.96, size = 295, normalized size = 2.84

$$\frac{2\,a^2x^2 - (i\,ax\cos(ax) - i\,\sin(ax))\mathrm{Li}_2(\cos(ax) + i\,\sin(ax)) - (-i\,ax\cos(ax) + i\,\sin(ax))\mathrm{Li}_2(\cos(ax) - i\,\sin(ax}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*csc(a*x)/(a*x*cos(a*x)-sin(a*x))^2,x, algorithm="fricas")
```

```
[Out] 1/2*(2*a^2*x^2 - (I*a*x*cos(a*x) - I*sin(a*x))*dilog(cos(a*x) + I*sin(a*x))
 - (-I*a*x*cos(a*x) + I*sin(a*x))*dilog(cos(a*x) - I*sin(a*x)) - (I*a*x*cos
(a*x) - I*sin(a*x))*dilog(-cos(a*x) + I*sin(a*x)) - (-I*a*x*cos(a*x) + I*si
n(a*x))*dilog(-cos(a*x) - I*sin(a*x)) - (a^2*x^2*cos(a*x) - a*x*sin(a*x))*l
og(cos(a*x) + I*sin(a*x) + 1) - (a^2*x^2*cos(a*x) - a*x*sin(a*x))*log(cos(a
*x) - I*sin(a*x) + 1) + (a^2*x^2*cos(a*x) - a*x*sin(a*x))*log(-cos(a*x) + I
*sin(a*x) + 1) + (a^2*x^2*cos(a*x) - a*x*sin(a*x))*log(-cos(a*x) - I*sin(a*
x) + 1) + 2)/(a^5*x*cos(a*x) - a^4*sin(a*x))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3\csc(ax)}{(ax\cos(ax) - \sin(ax))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*csc(a*x)/(a*x*cos(a*x)-sin(a*x))^2,x, algorithm="giac")
```

```
[Out] integrate(x^3*csc(a*x)/(a*x*cos(a*x) - sin(a*x))^2, x)
```

maple [F] time = 2.47, size = 0, normalized size = 0.00

$$\int \frac{x^3\csc(ax)}{(ax\cos(ax) - \sin(ax))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*csc(a*x)/(a*x*cos(a*x)-sin(a*x))^2,x)
```

```
[Out] int(x^3*csc(a*x)/(a*x*cos(a*x)-sin(a*x))^2,x)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*csc(a*x)/(a*x*cos(a*x)-sin(a*x))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is und
efined.
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^3}{\sin(a\,x)\,\left(\sin(a\,x) - a\,x\,\cos(a\,x)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(sin(a*x)*(sin(a*x) - a*x*cos(a*x))^2),x)
```

```
[Out] int(x^3/(sin(a*x)*(sin(a*x) - a*x*cos(a*x))^2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3\csc(ax)}{(ax\cos(ax) - \sin(ax))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*csc(a*x)/(a*x*cos(a*x)-sin(a*x))**2,x)
```

```
[Out] Integral(x**3*csc(a*x)/(a*x*cos(a*x) - sin(a*x))**2, x)
```

3.593 $\int \frac{x^4 \csc^2(ax)}{(ax\cos(ax)-\sin(ax))^2}\,dx$

Optimal. Leaf size=127

$$-\frac{2i\mathrm{Li}_2\left(e^{2iax}\right)}{a^5}-\frac{\cot(ax)}{a^5}+\frac{4x\log\left(1-e^{2iax}\right)}{a^4}-\frac{x\csc^2(ax)}{a^4}-\frac{2ix^2}{a^3}-\frac{2x^2\cot(ax)}{a^3}-\frac{x^2\cot(ax)\csc^2(ax)}{a^3}+\frac{x^3\csc^3(ax)}{a^2(ax\cos(ax)-\sin(ax))}$$

```
[Out] -2*I*x^2/a^3-cot(a*x)/a^5-2*x^2*cot(a*x)/a^3-x*csc(a*x)^2/a^4-x^2*cot(a*x)*
csc(a*x)^2/a^3+4*x*ln(1-exp(2*I*a*x))/a^4-2*I*polylog(2,exp(2*I*a*x))/a^5+x
^3*csc(a*x)^3/a^2/(a*x*cos(a*x)-sin(a*x))
```

Rubi [A] time = 0.18, antiderivative size = 127, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.346, Rules used = {4600, 4186, 3767, 8, 4184, 3717, 2190, 2279, 2391}

$$-\frac{2i\text{PolyLog}\left(2,e^{2iax}\right)}{a^5}-\frac{2ix^2}{a^3}-\frac{2x^2\cot(ax)}{a^3}-\frac{x^2\cot(ax)\csc^2(ax)}{a^3}+\frac{x^3\csc^3(ax)}{a^2(ax\cos(ax)-\sin(ax))}+\frac{4x\log\left(1-e^{2iax}\right)}{a^4}-\frac{\cot(ax)}{a^5}-\frac{x\csc^2(ax)}{a^4}$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*Csc[a*x]^2)/(a*x*Cos[a*x] - Sin[a*x])^2,x]
```

```
[Out] ((-2*I)*x^2)/a^3 - Cot[a*x]/a^5 - (2*x^2*Cot[a*x])/a^3 - (x*Csc[a*x]^2)/a^4
 - (x^2*Cot[a*x]*Csc[a*x]^2)/a^3 + (4*x*Log[1 - E^((2*I)*a*x)])/a^4 - ((2*I
)*PolyLog[2, E^((2*I)*a*x)])/a^5 + (x^3*Csc[a*x]^3)/(a^2*(a*x*Cos[a*x] - Si
n[a*x]))
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3717

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] - Dist[2*I, Int[((c + d*x)^
m*E^(2*I*k*Pi)*E^(2*I*(e + f*x)))/(1 + E^(2*I*k*Pi)*E^(2*I*(e + f*x))), x],
 x] /; FreeQ[{c, d, e, f}, x] && IntegerQ[4*k] && IGtQ[m, 0]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 4184

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> -Sim
p[((c + d*x)^m*Cot[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Co
t[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 4186

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_))^(m_), x_Symbo
l] :> -Simp[(b^2*(c + d*x)^m*Cot[e + f*x]*(b*Csc[e + f*x])^(n - 2))/(f*(n -
 1)), x] + (Dist[(b^2*d^2*m*(m - 1))/(f^2*(n - 1)*(n - 2)), Int[(c + d*x)^(
m - 2)*(b*Csc[e + f*x])^(n - 2), x], x] + Dist[(b^2*(n - 2))/(n - 1), Int[(
c + d*x)^m*(b*Csc[e + f*x])^(n - 2), x], x] - Simp[(b^2*d*m*(c + d*x)^(m -
1)*(b*Csc[e + f*x])^(n - 2))/(f^2*(n - 1)*(n - 2)), x]) /; FreeQ[{b, c, d,
e, f}, x] && GtQ[n, 1] && NeQ[n, 2] && GtQ[m, 1]
```

Rule 4600

```
Int[(Csc[(a_.)*(x_)]^(n_.)*((b_.)*(x_))^(m_.))/(Cos[(a_.)*(x_)]*(d_.)*(x_)
+ (c_.)*Sin[(a_.)*(x_)])^2, x_Symbol] :> Simp[(b*(b*x)^(m - 1)*Csc[a*x]^(n
+ 1))/(a*d*(c*Sin[a*x] + d*x*Cos[a*x])), x] + Dist[(b^2*(n + 1))/d^2, Int[(
b*x)^(m - 2)*Csc[a*x]^(n + 2), x], x] /; FreeQ[{a, b, c, d, m, n}, x] && Eq
Q[a*c + d, 0] && EqQ[m, n + 2]
```

Rubi steps

$$\int \frac{x^4 \csc^2(ax)}{(ax\cos(ax)-\sin(ax))^2}\,dx = \frac{x^3\csc^3(ax)}{a^2(ax\cos(ax)-\sin(ax))}+\frac{3\int x^2\csc^4(ax)\,dx}{a^2}$$
$$= -\frac{x\csc^2(ax)}{a^4}-\frac{x^2\cot(ax)\csc^2(ax)}{a^3}+\frac{x^3\csc^3(ax)}{a^2(ax\cos(ax)-\sin(ax))}+\frac{\int\csc^2(ax)\,dx}{a^4}+$$
$$= -\frac{2x^2\cot(ax)}{a^3}-\frac{x\csc^2(ax)}{a^4}-\frac{x^2\cot(ax)\csc^2(ax)}{a^3}+\frac{x^3\csc^3(ax)}{a^2(ax\cos(ax)-\sin(ax))}-$$
$$= -\frac{2ix^2}{a^3}-\frac{\cot(ax)}{a^5}-\frac{2x^2\cot(ax)}{a^3}-\frac{x\csc^2(ax)}{a^4}-\frac{x^2\cot(ax)\csc^2(ax)}{a^3}+\frac{x^3\csc}{a^2(ax\cos(a}$$
$$= -\frac{2ix^2}{a^3}-\frac{\cot(ax)}{a^5}-\frac{2x^2\cot(ax)}{a^3}-\frac{x\csc^2(ax)}{a^4}-\frac{x^2\cot(ax)\csc^2(ax)}{a^3}+\frac{4x\log\left(1-\right.}{a^4}$$
$$= -\frac{2ix^2}{a^3}-\frac{\cot(ax)}{a^5}-\frac{2x^2\cot(ax)}{a^3}-\frac{x\csc^2(ax)}{a^4}-\frac{x^2\cot(ax)\csc^2(ax)}{a^3}+\frac{4x\log\left(1-\right.}{a^4}$$
$$= -\frac{2ix^2}{a^3}-\frac{\cot(ax)}{a^5}-\frac{2x^2\cot(ax)}{a^3}-\frac{x\csc^2(ax)}{a^4}-\frac{x^2\cot(ax)\csc^2(ax)}{a^3}+\frac{4x\log\left(1-\right.}{a^4}$$

Mathematica [A] time = 1.05, size = 102, normalized size = 0.80

$$\frac{a^3\left(-x^2\right)\cot(ax)-2ia\left(a^2x^2+\text{Li}_2\left(e^{2iax}\right)\right)+\frac{\left(a^2x^2+1\right)^2\sin(ax)}{x(ax\cos(ax)-\sin(ax))}+a^2x+4a^2x\log\left(1-e^{2iax}\right)+\frac{1}{x}}{a^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^4*Csc[a*x]^2)/(a*x*Cos[a*x] - Sin[a*x])^2,x]
```

```
[Out] (x^(-1) + a^2*x - a^3*x^2*Cot[a*x] + 4*a^2*x*Log[1 - E^((2*I)*a*x)] - (2*I)
*a*(a^2*x^2 + PolyLog[2, E^((2*I)*a*x)]) + ((1 + a^2*x^2)^2*Sin[a*x])/(x*(a
*x*Cos[a*x] - Sin[a*x])))/a^6
```

fricas [B] time = 1.00, size = 406, normalized size = 3.20

$$\frac{a^3x^3-\left(2\,a^3x^3+ax\right)\cos\left(ax\right)^2+\left(2\,a^2x^2+1\right)\cos\left(ax\right)\sin\left(ax\right)+ax+\left(-2i\,ax\cos\left(ax\right)\sin\left(ax\right)-2i\,\cos\left(ax\right)^2+2i\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*csc(a*x)^2/(a*x*cos(a*x)-sin(a*x))^2,x, algorithm="fricas")
```

```
[Out] (a^3*x^3 - (2*a^3*x^3 + a*x)*cos(a*x)^2 + (2*a^2*x^2 + 1)*cos(a*x)*sin(a*x)
 + a*x + (-2*I*a*x*cos(a*x)*sin(a*x) - 2*I*cos(a*x)^2 + 2*I)*dilog(cos(a*x)
 + I*sin(a*x)) + (2*I*a*x*cos(a*x)*sin(a*x) + 2*I*cos(a*x)^2 - 2*I)*dilog(c
os(a*x) - I*sin(a*x)) + (2*I*a*x*cos(a*x)*sin(a*x) + 2*I*cos(a*x)^2 - 2*I)*
dilog(-cos(a*x) + I*sin(a*x)) + (-2*I*a*x*cos(a*x)*sin(a*x) - 2*I*cos(a*x)^
2 + 2*I)*dilog(-cos(a*x) - I*sin(a*x)) + 2*(a^2*x^2*cos(a*x)*sin(a*x) + a*x
*cos(a*x)^2 - a*x)*log(cos(a*x) + I*sin(a*x) + 1) + 2*(a^2*x^2*cos(a*x)*sin
(a*x) + a*x*cos(a*x)^2 - a*x)*log(cos(a*x) - I*sin(a*x) + 1) + 2*(a^2*x^2*c
os(a*x)*sin(a*x) + a*x*cos(a*x)^2 - a*x)*log(-cos(a*x) + I*sin(a*x) + 1) +
2*(a^2*x^2*cos(a*x)*sin(a*x) + a*x*cos(a*x)^2 - a*x)*log(-cos(a*x) - I*sin(
a*x) + 1))/(a^6*x*cos(a*x)*sin(a*x) + a^5*cos(a*x)^2 - a^5)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4 \csc (ax)^2}{(ax \cos (ax) - \sin (ax))^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*csc(a*x)^2/(a*x*cos(a*x)-sin(a*x))^2,x, algorithm="giac")
```

```
[Out] integrate(x^4*csc(a*x)^2/(a*x*cos(a*x) - sin(a*x))^2, x)
```

maple [A] time = 1.10, size = 172, normalized size = 1.35

$$-\frac{2i\left(2ia^2x^2e^{2iax} + 2x^3a^3 - 2ia^2x^2 - ax\,e^{2iax} + ie^{2iax} + ax - i\right)}{\left(e^{2iax} - 1\right)\left(ax\,e^{2iax} + ie^{2iax} + ax - i\right)a^5} - \frac{4ix^2}{a^3} + \frac{4x\ln\left(e^{iax} + 1\right)}{a^4} - \frac{4i\,\mathrm{polylog}\left(2, -e^{iax}\right)}{a^5} + \frac{4x}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*csc(a*x)^2/(a*x*cos(a*x)-sin(a*x))^2,x)
```

```
[Out] -2*I*(2*I*a^2*x^2*exp(2*I*a*x)+2*x^3*a^3-2*I*a^2*x^2-a*x*exp(2*I*a*x)+I*exp
(2*I*a*x)+a*x-I)/(exp(2*I*a*x)-1)/(a*x*exp(2*I*a*x)+I*exp(2*I*a*x)+a*x-I)/a
^5-4*I/a^3*x^2+4/a^4*x*ln(exp(I*a*x)+1)-4*I/a^5*polylog(2,-exp(I*a*x))+4/a^
4*x*ln(1-exp(I*a*x))-4*I/a^5*polylog(2,exp(I*a*x))
```

maxima [B] time = 0.38, size = 608, normalized size = 4.79

$$-\frac{2\,ax + \left(4\,a^2x^2 + 8i\,ax\cos(2\,ax) - 8\,ax\sin(2\,ax) - 4i\,ax - \left(4\,a^2x^2 + 4i\,ax\right)\cos(4\,ax) + 4\left(-i\,a^2x^2 + ax\right)\sin(4\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*csc(a*x)^2/(a*x*cos(a*x)-sin(a*x))^2,x, algorithm="maxima")
```

```
[Out] -(2*a*x + (4*a^2*x^2 + 8*I*a*x*cos(2*a*x) - 8*a*x*sin(2*a*x) - 4*I*a*x - (4
*a^2*x^2 + 4*I*a*x)*cos(4*a*x) + 4*(-I*a^2*x^2 + a*x)*sin(4*a*x))*arctan2(s
in(a*x), cos(a*x) + 1) - (4*a^2*x^2 + 8*I*a*x*cos(2*a*x) - 8*a*x*sin(2*a*x)
 - 4*I*a*x - (4*a^2*x^2 + 4*I*a*x)*cos(4*a*x) - 4*(I*a^2*x^2 - a*x)*sin(4*a
*x))*arctan2(sin(a*x), -cos(a*x) + 1) + 4*(a^3*x^3 + I*a^2*x^2)*cos(4*a*x)
- (4*I*a^2*x^2 + 2*a*x - 2*I)*cos(2*a*x) - (4*a*x - (4*a*x + 4*I)*cos(4*a*x
) - 4*(I*a*x - 1)*sin(4*a*x) + 8*I*cos(2*a*x) - 8*sin(2*a*x) - 4*I)*dilog(-
e^(I*a*x)) - (4*a*x - (4*a*x + 4*I)*cos(4*a*x) - 4*(I*a*x - 1)*sin(4*a*x) +
 8*I*cos(2*a*x) - 8*sin(2*a*x) - 4*I)*dilog(e^(I*a*x)) - (2*I*a^2*x^2 - 4*a
*x*cos(2*a*x) - 4*I*a*x*sin(2*a*x) + 2*a*x - 2*(I*a^2*x^2 - a*x)*cos(4*a*x)
 + (2*a^2*x^2 + 2*I*a*x)*sin(4*a*x))*log(cos(a*x)^2 + sin(a*x)^2 + 2*cos(a*
x) + 1) - (2*I*a^2*x^2 - 4*a*x*cos(2*a*x) - 4*I*a*x*sin(2*a*x) + 2*a*x - 2*
(I*a^2*x^2 - a*x)*cos(4*a*x) + (2*a^2*x^2 + 2*I*a*x)*sin(4*a*x))*log(cos(a*
x)^2 + sin(a*x)^2 - 2*cos(a*x) + 1) - (-4*I*a^3*x^3 + 4*a^2*x^2)*sin(4*a*x)
 + (4*a^2*x^2 - 2*I*a*x - 2)*sin(2*a*x) - 2*I)/((I*a*x + (-I*a*x + 1)*cos(4
*a*x) + (a*x + I)*sin(4*a*x) - 2*cos(2*a*x) - 2*I*sin(2*a*x) + 1)*a^5)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^4}{\sin (a\,x)^2\,(\sin (a\,x) - a\,x\,\cos (a\,x))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(sin(a*x)^2*(sin(a*x) - a*x*cos(a*x))^2),x)
```

```
[Out] int(x^4/(sin(a*x)^2*(sin(a*x) - a*x*cos(a*x))^2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4 \csc^2 (ax)}{(ax \cos (ax) - \sin (ax))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*csc(a*x)**2/(a*x*cos(a*x)-sin(a*x))**2,x)
```

```
[Out] Integral(x**4*csc(a*x)**2/(a*x*cos(a*x) - sin(a*x))**2, x)
```

3.594 $\int \frac{\cos^6(ax)}{x^4(\cos(ax)+ax\sin(ax))^2} dx$

Optimal. Leaf size=176

$$\frac{2}{3}a^3\text{Si}(2ax)+\frac{16}{3}a^3\text{Si}(4ax)+\frac{\cos^4(ax)}{a^2x^5}-\frac{\cos^5(ax)}{a^2x^5(ax\sin(ax)+\cos(ax))}+\frac{a^2}{x}+\frac{32a^2\cos^4(ax)}{3x}-\frac{10a^2\cos^2(ax)}{x}-\frac{\sin(ax)\,\mathrm{c}}{ax}$$

```
[Out] a^2/x+cos(a*x)^2/x^3-10*a^2*cos(a*x)^2/x+cos(a*x)^4/a^2/x^5-4/3*cos(a*x)^4/
x^3+32/3*a^2*cos(a*x)^4/x+2/3*a^3*Si(2*a*x)+16/3*a^3*Si(4*a*x)-a*cos(a*x)*s
in(a*x)/x^2-cos(a*x)^3*sin(a*x)/a/x^4+8/3*a*cos(a*x)^3*sin(a*x)/x^2-cos(a*x
)^5/a^2/x^5/(cos(a*x)+a*x*sin(a*x))
```

Rubi [A] time = 0.30, antiderivative size = 176, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {4599, 3314, 30, 3313, 12, 3299}

$$\frac{2}{3}a^3\text{Si}(2ax)+\frac{16}{3}a^3\text{Si}(4ax)+\frac{\cos^4(ax)}{a^2x^5}-\frac{\cos^5(ax)}{a^2x^5(ax\sin(ax)+\cos(ax))}+\frac{a^2}{x}+\frac{32a^2\cos^4(ax)}{3x}-\frac{10a^2\cos^2(ax)}{x}-\frac{4\cos^4(a}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a*x]^6/(x^4*(Cos[a*x] + a*x*Sin[a*x])^2),x]
```

```
[Out] a^2/x + Cos[a*x]^2/x^3 - (10*a^2*Cos[a*x]^2)/x + Cos[a*x]^4/(a^2*x^5) - (4*
Cos[a*x]^4)/(3*x^3) + (32*a^2*Cos[a*x]^4)/(3*x) - (a*Cos[a*x]*Sin[a*x])/x^2
 - (Cos[a*x]^3*Sin[a*x])/(a*x^4) + (8*a*Cos[a*x]^3*Sin[a*x])/(3*x^2) - Cos[
a*x]^5/(a^2*x^5*(Cos[a*x] + a*x*Sin[a*x])) + (2*a^3*SinIntegral[2*a*x])/3 +
 (16*a^3*SinIntegral[4*a*x])/3
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3313

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> Si
mp[((c + d*x)^(m + 1)*Sin[e + f*x]^n)/(d*(m + 1)), x] - Dist[(f*n)/(d*(m +
1)), Int[ExpandTrigReduce[(c + d*x)^(m + 1), Cos[e + f*x]*Sin[e + f*x]^(n -
 1), x], x], x] /; FreeQ[{c, d, e, f, m}, x] && IGtQ[n, 1] && GeQ[m, -2] &&
 LtQ[m, -1]
```

Rule 3314

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[((c + d*x)^(m + 1)*(b*Sin[e + f*x])^n)/(d*(m + 1)), x] + (Dist[(
b^2*f^2*n*(n - 1))/(d^2*(m + 1)*(m + 2)), Int[(c + d*x)^(m + 2)*(b*Sin[e +
f*x])^(n - 2), x], x] - Dist[(f^2*n^2)/(d^2*(m + 1)*(m + 2)), Int[(c + d*x)
^(m + 2)*(b*Sin[e + f*x])^n, x], x] - Simp[(b*f*n*(c + d*x)^(m + 2)*Cos[e +
 f*x]*(b*Sin[e + f*x])^(n - 1))/(d^2*(m + 1)*(m + 2)), x]) /; FreeQ[{b, c,
d, e, f}, x] && GtQ[n, 1] && LtQ[m, -2]
```

Rule 4599

```
Int[(Cos[(a_.)*(x_)]^(n_)*((b_.)*(x_))^(m_))/(Cos[(a_.)*(x_)]*(c_.) + (d_.)
*(x_)*Sin[(a_.)*(x_)])^2, x_Symbol] :> -Simp[(b*(b*x)^(m - 1)*Cos[a*x]^(n -
 1))/(a*d*(c*Cos[a*x] + d*x*Sin[a*x])), x] - Dist[(b^2*(n - 1))/d^2, Int[(b
*x)^(m - 2)*Cos[a*x]^(n - 2), x], x] /; FreeQ[{a, b, c, d, m, n}, x] && EqQ
[a*c - d, 0] && EqQ[m, 2 - n]
```

Rubi steps

$$\begin{aligned}\int \frac{\cos^6(ax)}{x^4(\cos(ax)+ax\sin(ax))^2}\,dx &= -\frac{\cos^5(ax)}{a^2x^5(\cos(ax)+ax\sin(ax))} - \frac{5\int \frac{\cos^4(ax)}{x^6}\,dx}{a^2}\\
&= \frac{\cos^4(ax)}{a^2x^5} - \frac{\cos^3(ax)\sin(ax)}{ax^4} - \frac{\cos^5(ax)}{a^2x^5(\cos(ax)+ax\sin(ax))} - 3\int \frac{\cos^2(ax)}{x^4}\,dx\\
&= \frac{\cos^2(ax)}{x^3} + \frac{\cos^4(ax)}{a^2x^5} - \frac{4\cos^4(ax)}{3x^3} - \frac{a\cos(ax)\sin(ax)}{x^2} - \frac{\cos^3(ax)\sin(ax)}{ax^4} + \\
&= \frac{a^2}{x} + \frac{\cos^2(ax)}{x^3} - \frac{10a^2\cos^2(ax)}{x} + \frac{\cos^4(ax)}{a^2x^5} - \frac{4\cos^4(ax)}{3x^3} + \frac{32a^2\cos^4(ax)}{3x} - \\
&= \frac{a^2}{x} + \frac{\cos^2(ax)}{x^3} - \frac{10a^2\cos^2(ax)}{x} + \frac{\cos^4(ax)}{a^2x^5} - \frac{4\cos^4(ax)}{3x^3} + \frac{32a^2\cos^4(ax)}{3x} - \\
&= \frac{a^2}{x} + \frac{\cos^2(ax)}{x^3} - \frac{10a^2\cos^2(ax)}{x} + \frac{\cos^4(ax)}{a^2x^5} - \frac{4\cos^4(ax)}{3x^3} + \frac{32a^2\cos^4(ax)}{3x} - \end{aligned}$$

Mathematica [A] time = 1.22, size = 194, normalized size = 1.10

$$\frac{32a^3x^3\mathrm{Si}(2ax)(ax\sin(ax)+\cos(ax))+256a^3x^3\mathrm{Si}(4ax)(ax\sin(ax)+\cos(ax))-8a^3x^3\sin(ax)-24a^3x^3\sin(3ax}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a*x]^6/(x^4*(Cos[a*x] + a*x*Sin[a*x])^2),x]
```

```
[Out] (-10*Cos[a*x] + 12*a^2*x^2*Cos[a*x] - 5*Cos[3*a*x] + 44*a^2*x^2*Cos[3*a*x]
- Cos[5*a*x] + 24*a^2*x^2*Cos[5*a*x] + 8*a*x*Sin[a*x] - 8*a^3*x^3*Sin[a*x]
+ 12*a*x*Sin[3*a*x] - 24*a^3*x^3*Sin[3*a*x] + 4*a*x*Sin[5*a*x] + 32*a^3*x^3
*Sin[5*a*x] + 32*a^3*x^3*(Cos[a*x] + a*x*Sin[a*x])*SinIntegral[2*a*x] + 256
*a^3*x^3*(Cos[a*x] + a*x*Sin[a*x])*SinIntegral[4*a*x])/(48*x^3*(Cos[a*x] +
a*x*Sin[a*x]))
```

fricas [A] time = 0.92, size = 162, normalized size = 0.92

$$-\frac{19\,a^2x^2\cos(ax)^3-\left(24\,a^2x^2-1\right)\cos(ax)^5-2\left(8\,a^3x^3\,\mathrm{Si}(4\,ax)+a^3x^3\,\mathrm{Si}(2\,ax)\right)\cos(ax)-\left(16\,a^4x^4\,\mathrm{Si}(4\,ax)+\right.}{3\left(ax^4\sin(ax)+x^3\cos(ax)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(a*x)^6/x^4/(cos(a*x)+a*x*sin(a*x))^2,x, algorithm="fricas")
```

```
[Out] -1/3*(19*a^2*x^2*cos(a*x)^3 - (24*a^2*x^2 - 1)*cos(a*x)^5 - 2*(8*a^3*x^3*si
n_integral(4*a*x) + a^3*x^3*sin_integral(2*a*x))*cos(a*x) - (16*a^4*x^4*sin
_integral(4*a*x) + 2*a^4*x^4*sin_integral(2*a*x) - 30*a^3*x^3*cos(a*x)^2 +
3*a^3*x^3 + 4*(8*a^3*x^3 + a*x)*cos(a*x)^4)*sin(a*x))/(a*x^4*sin(a*x) + x^3
*cos(a*x))
```

giac [C] time = 1.21, size = 7279, normalized size = 41.36

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(a*x)^6/x^4/(cos(a*x)+a*x*sin(a*x))^2,x, algorithm="giac")
```

```
[Out] 1/12*(64*a^8*x^8*imag_part(cos_integral(4*a*x))*tan(2*a*x)^2*tan(a*x)^2*tan
(1/2*a*x) + 8*a^8*x^8*imag_part(cos_integral(2*a*x))*tan(2*a*x)^2*tan(a*x)^
2*tan(1/2*a*x) - 8*a^8*x^8*imag_part(cos_integral(-2*a*x))*tan(2*a*x)^2*tan
(a*x)^2*tan(1/2*a*x) - 64*a^8*x^8*imag_part(cos_integral(-4*a*x))*tan(2*a*x
)^2*tan(a*x)^2*tan(1/2*a*x) + 128*a^8*x^8*sin_integral(4*a*x)*tan(2*a*x)^2*
tan(a*x)^2*tan(1/2*a*x) + 16*a^8*x^8*sin_integral(2*a*x)*tan(2*a*x)^2*tan(a
*x)^2*tan(1/2*a*x) - 32*a^7*x^7*imag_part(cos_integral(4*a*x))*tan(2*a*x)^2
```

*tan(a*x)^2*tan(1/2*a*x)^2 - 4*a^7*x^7*imag_part(cos_integral(2*a*x))*tan(2
*a*x)^2*tan(a*x)^2*tan(1/2*a*x)^2 + 4*a^7*x^7*imag_part(cos_integral(-2*a*x
))*tan(2*a*x)^2*tan(a*x)^2*tan(1/2*a*x)^2 + 32*a^7*x^7*imag_part(cos_integr
al(-4*a*x))*tan(2*a*x)^2*tan(a*x)^2*tan(1/2*a*x)^2 - 64*a^7*x^7*sin_integra
l(4*a*x)*tan(2*a*x)^2*tan(a*x)^2*tan(1/2*a*x)^2 - 8*a^7*x^7*sin_integral(2*
a*x)*tan(2*a*x)^2*tan(a*x)^2*tan(1/2*a*x)^2 + 64*a^8*x^8*imag_part(cos_inte
gral(4*a*x))*tan(2*a*x)^2*tan(1/2*a*x) + 8*a^8*x^8*imag_part(cos_integral(2
*a*x))*tan(2*a*x)^2*tan(1/2*a*x) - 8*a^8*x^8*imag_part(cos_integral(-2*a*x)
)*tan(2*a*x)^2*tan(1/2*a*x) - 64*a^8*x^8*imag_part(cos_integral(-4*a*x))*ta
n(2*a*x)^2*tan(1/2*a*x) + 128*a^8*x^8*sin_integral(4*a*x)*tan(2*a*x)^2*tan(
1/2*a*x) + 16*a^8*x^8*sin_integral(2*a*x)*tan(2*a*x)^2*tan(1/2*a*x) + 64*a^
8*x^8*imag_part(cos_integral(4*a*x))*tan(a*x)^2*tan(1/2*a*x) + 8*a^8*x^8*im
ag_part(cos_integral(2*a*x))*tan(a*x)^2*tan(1/2*a*x) - 8*a^8*x^8*imag_part(
cos_integral(-2*a*x))*tan(a*x)^2*tan(1/2*a*x) - 64*a^8*x^8*imag_part(cos_in
tegral(-4*a*x))*tan(a*x)^2*tan(1/2*a*x) + 128*a^8*x^8*sin_integral(4*a*x)*t
an(a*x)^2*tan(1/2*a*x) + 16*a^8*x^8*sin_integral(2*a*x)*tan(a*x)^2*tan(1/2*
a*x) + 32*a^7*x^7*imag_part(cos_integral(4*a*x))*tan(2*a*x)^2*tan(a*x)^2 +
4*a^7*x^7*imag_part(cos_integral(2*a*x))*tan(2*a*x)^2*tan(a*x)^2 - 4*a^7*x^
7*imag_part(cos_integral(-2*a*x))*tan(2*a*x)^2*tan(a*x)^2 - 32*a^7*x^7*imag
_part(cos_integral(-4*a*x))*tan(2*a*x)^2*tan(a*x)^2 + 64*a^7*x^7*sin_integr
al(4*a*x)*tan(2*a*x)^2*tan(a*x)^2 + 8*a^7*x^7*sin_integral(2*a*x)*tan(2*a*x
)^2*tan(a*x)^2 - 40*a^7*x^7*tan(2*a*x)^2*tan(a*x)^2*tan(1/2*a*x) - 32*a^7*x
^7*imag_part(cos_integral(4*a*x))*tan(2*a*x)^2*tan(1/2*a*x)^2 - 4*a^7*x^7*i
mag_part(cos_integral(2*a*x))*tan(2*a*x)^2*tan(1/2*a*x)^2 + 4*a^7*x^7*imag_
part(cos_integral(-2*a*x))*tan(2*a*x)^2*tan(1/2*a*x)^2 + 32*a^7*x^7*imag_pa
rt(cos_integral(-4*a*x))*tan(2*a*x)^2*tan(1/2*a*x)^2 - 64*a^7*x^7*sin_integ
ral(4*a*x)*tan(2*a*x)^2*tan(1/2*a*x)^2 - 8*a^7*x^7*sin_integral(2*a*x)*tan(
2*a*x)^2*tan(1/2*a*x)^2 - 32*a^7*x^7*imag_part(cos_integral(4*a*x))*tan(a*x
)^2*tan(1/2*a*x)^2 - 4*a^7*x^7*imag_part(cos_integral(2*a*x))*tan(a*x)^2*ta
n(1/2*a*x)^2 + 4*a^7*x^7*imag_part(cos_integral(-2*a*x))*tan(a*x)^2*tan(1/2
*a*x)^2 + 32*a^7*x^7*imag_part(cos_integral(-4*a*x))*tan(a*x)^2*tan(1/2*a*x
)^2 - 64*a^7*x^7*sin_integral(4*a*x)*tan(a*x)^2*tan(1/2*a*x)^2 - 8*a^7*x^7*
sin_integral(2*a*x)*tan(a*x)^2*tan(1/2*a*x)^2 + 64*a^8*x^8*imag_part(cos_in
tegral(4*a*x))*tan(1/2*a*x) + 8*a^8*x^8*imag_part(cos_integral(2*a*x))*tan(
1/2*a*x) - 8*a^8*x^8*imag_part(cos_integral(-2*a*x))*tan(1/2*a*x) - 64*a^8*
x^8*imag_part(cos_integral(-4*a*x))*tan(1/2*a*x) + 128*a^8*x^8*sin_integral
(4*a*x)*tan(1/2*a*x) + 16*a^8*x^8*sin_integral(2*a*x)*tan(1/2*a*x) + 128*a^
6*x^6*imag_part(cos_integral(4*a*x))*tan(2*a*x)^2*tan(a*x)^2*tan(1/2*a*x) +
 16*a^6*x^6*imag_part(cos_integral(2*a*x))*tan(2*a*x)^2*tan(a*x)^2*tan(1/2*
a*x) - 16*a^6*x^6*imag_part(cos_integral(-2*a*x))*tan(2*a*x)^2*tan(a*x)^2*t
an(1/2*a*x) - 128*a^6*x^6*imag_part(cos_integral(-4*a*x))*tan(2*a*x)^2*tan(
a*x)^2*tan(1/2*a*x) + 256*a^6*x^6*sin_integral(4*a*x)*tan(2*a*x)^2*tan(a*x)
^2*tan(1/2*a*x) + 32*a^6*x^6*sin_integral(2*a*x)*tan(2*a*x)^2*tan(a*x)^2*ta
n(1/2*a*x) + 20*a^6*x^6*tan(2*a*x)^2*tan(a*x)^2*tan(1/2*a*x)^2 + 32*a^7*x^7
*imag_part(cos_integral(4*a*x))*tan(2*a*x)^2 + 4*a^7*x^7*imag_part(cos_inte

gral(2*a*x))*tan(2*a*x)^2 - 4*a^7*x^7*imag_part(cos_integral(-2*a*x))*tan(2
*a*x)^2 - 32*a^7*x^7*imag_part(cos_integral(-4*a*x))*tan(2*a*x)^2 + 64*a^7*
x^7*sin_integral(4*a*x)*tan(2*a*x)^2 + 8*a^7*x^7*sin_integral(2*a*x)*tan(2*
a*x)^2 + 32*a^7*x^7*imag_part(cos_integral(4*a*x))*tan(a*x)^2 + 4*a^7*x^7*i
mag_part(cos_integral(2*a*x))*tan(a*x)^2 - 4*a^7*x^7*imag_part(cos_integral
(-2*a*x))*tan(a*x)^2 - 32*a^7*x^7*imag_part(cos_integral(-4*a*x))*tan(a*x)^
2 + 64*a^7*x^7*sin_integral(4*a*x)*tan(a*x)^2 + 8*a^7*x^7*sin_integral(2*a*
x)*tan(a*x)^2 - 24*a^7*x^7*tan(2*a*x)^2*tan(1/2*a*x) + 24*a^7*x^7*tan(a*x)^
2*tan(1/2*a*x) - 32*a^7*x^7*imag_part(cos_integral(4*a*x))*tan(1/2*a*x)^2 -
 4*a^7*x^7*imag_part(cos_integral(2*a*x))*tan(1/2*a*x)^2 + 4*a^7*x^7*imag_p
art(cos_integral(-2*a*x))*tan(1/2*a*x)^2 + 32*a^7*x^7*imag_part(cos_integra
l(-4*a*x))*tan(1/2*a*x)^2 - 64*a^7*x^7*sin_integral(4*a*x)*tan(1/2*a*x)^2 -
 8*a^7*x^7*sin_integral(2*a*x)*tan(1/2*a*x)^2 - 64*a^5*x^5*imag_part(cos_in
tegral(4*a*x))*tan(2*a*x)^2*tan(a*x)^2*tan(1/2*a*x)^2 - 8*a^5*x^5*imag_part
(cos_integral(2*a*x))*tan(2*a*x)^2*tan(a*x)^2*tan(1/2*a*x)^2 + 8*a^5*x^5*im
ag_part(cos_integral(-2*a*x))*tan(2*a*x)^2*tan(a*x)^2*tan(1/2*a*x)^2 + 64*a
^5*x^5*imag_part(cos_integral(-4*a*x))*tan(2*a*x)^2*tan(a*x)^2*tan(1/2*a*x)
^2 - 128*a^5*x^5*sin_integral(4*a*x)*tan(2*a*x)^2*tan(a*x)^2*tan(1/2*a*x)^2
 - 16*a^5*x^5*sin_integral(2*a*x)*tan(2*a*x)^2*tan(a*x)^2*tan(1/2*a*x)^2 -
20*a^6*x^6*tan(2*a*x)^2*tan(a*x)^2 + 128*a^6*x^6*imag_part(cos_integral(4*a
*x))*tan(2*a*x)^2*tan(1/2*a*x) + 16*a^6*x^6*imag_part(cos_integral(2*a*x))*
tan(2*a*x)^2*tan(1/2*a*x) - 16*a^6*x^6*imag_part(cos_integral(-2*a*x))*tan(
2*a*x)^2*tan(1/2*a*x) - 128*a^6*x^6*imag_part(cos_integral(-4*a*x))*tan(2*a
*x)^2*tan(1/2*a*x) + 256*a^6*x^6*sin_integral(4*a*x)*tan(2*a*x)^2*tan(1/2*a
*x) + 32*a^6*x^6*sin_integral(2*a*x)*tan(2*a*x)^2*tan(1/2*a*x) + 8*a^6*x^6*
tan(2*a*x)^2*tan(a*x)*tan(1/2*a*x) + 128*a^6*x^6*imag_part(cos_integral(4*a
*x))*tan(a*x)^2*tan(1/2*a*x) + 16*a^6*x^6*imag_part(cos_integral(2*a*x))*ta
n(a*x)^2*tan(1/2*a*x) - 16*a^6*x^6*imag_part(cos_integral(-2*a*x))*tan(a*x)
^2*tan(1/2*a*x) - 128*a^6*x^6*imag_part(cos_integral(-4*a*x))*tan(a*x)^2*ta
n(1/2*a*x) + 256*a^6*x^6*sin_integral(4*a*x)*tan(a*x)^2*tan(1/2*a*x) + 32*a
^6*x^6*sin_integral(2*a*x)*tan(a*x)^2*tan(1/2*a*x) + 16*a^6*x^6*tan(2*a*x)*
tan(a*x)^2*tan(1/2*a*x) + 12*a^6*x^6*tan(2*a*x)^2*tan(1/2*a*x)^2 - 12*a^6*x
^6*tan(a*x)^2*tan(1/2*a*x)^2 + 32*a^7*x^7*imag_part(cos_integral(4*a*x)) +
4*a^7*x^7*imag_part(cos_integral(2*a*x)) - 4*a^7*x^7*imag_part(cos_integral
(-2*a*x)) - 32*a^7*x^7*imag_part(cos_integral(-4*a*x)) + 64*a^7*x^7*sin_int
egral(4*a*x) + 8*a^7*x^7*sin_integral(2*a*x) + 64*a^5*x^5*imag_part(cos_int
egral(4*a*x))*tan(2*a*x)^2*tan(a*x)^2 + 8*a^5*x^5*imag_part(cos_integral(2*
a*x))*tan(2*a*x)^2*tan(a*x)^2 - 8*a^5*x^5*imag_part(cos_integral(-2*a*x))*t
an(2*a*x)^2*tan(a*x)^2 - 64*a^5*x^5*imag_part(cos_integral(-4*a*x))*tan(2*a
*x)^2*tan(a*x)^2 + 128*a^5*x^5*sin_integral(4*a*x)*tan(2*a*x)^2*tan(a*x)^2
+ 16*a^5*x^5*sin_integral(2*a*x)*tan(2*a*x)^2*tan(a*x)^2 + 40*a^7*x^7*tan(1
/2*a*x) - 72*a^5*x^5*tan(2*a*x)^2*tan(a*x)^2*tan(1/2*a*x) - 64*a^5*x^5*imag
_part(cos_integral(4*a*x))*tan(2*a*x)^2*tan(1/2*a*x)^2 - 8*a^5*x^5*imag_par
t(cos_integral(2*a*x))*tan(2*a*x)^2*tan(1/2*a*x)^2 + 8*a^5*x^5*imag_part(co
s_integral(-2*a*x))*tan(2*a*x)^2*tan(1/2*a*x)^2 + 64*a^5*x^5*imag_part(cos_

integral(-4*a*x))*tan(2*a*x)^2*tan(1/2*a*x)^2 - 128*a^5*x^5*sin_integral(4*a*x)*tan(2*a*x)^2*tan(1/2*a*x)^2 - 16*a^5*x^5*sin_integral(2*a*x)*tan(2*a*x)^2*tan(1/2*a*x)^2 - 4*a^5*x^5*tan(2*a*x)^2*tan(a*x)*tan(1/2*a*x)^2 - 64*a^5*x^5*imag_part(cos_integral(4*a*x))*tan(a*x)^2*tan(1/2*a*x)^2 - 8*a^5*x^5*imag_part(cos_integral(2*a*x))*tan(a*x)^2*tan(1/2*a*x)^2 + 8*a^5*x^5*imag_part(cos_integral(-2*a*x))*tan(a*x)^2*tan(1/2*a*x)^2 + 64*a^5*x^5*imag_part(cos_integral(-4*a*x))*tan(a*x)^2*tan(1/2*a*x)^2 - 128*a^5*x^5*sin_integral(4*a*x)*tan(a*x)^2*tan(1/2*a*x)^2 - 16*a^5*x^5*sin_integral(2*a*x)*tan(a*x)^2*tan(1/2*a*x)^2 - 8*a^5*x^5*tan(2*a*x)*tan(a*x)^2*tan(1/2*a*x)^2 - 12*a^6*x^6*tan(2*a*x)^2 + 12*a^6*x^6*tan(a*x)^2 + 128*a^6*x^6*imag_part(cos_integral(4*a*x))*tan(1/2*a*x) + 16*a^6*x^6*imag_part(cos_integral(2*a*x))*tan(1/2*a*x) - 16*a^6*x^6*imag_part(cos_integral(-2*a*x))*tan(1/2*a*x) - 128*a^6*x^6*imag_part(cos_integral(-4*a*x))*tan(1/2*a*x) + 256*a^6*x^6*sin_integral(4*a*x)*tan(1/2*a*x) + 32*a^6*x^6*sin_integral(2*a*x)*tan(1/2*a*x) + 16*a^6*x^6*tan(2*a*x)*tan(1/2*a*x) + 8*a^6*x^6*tan(a*x)*tan(1/2*a*x) + 64*a^4*x^4*imag_part(cos_integral(4*a*x))*tan(2*a*x)^2*tan(a*x)^2*tan(1/2*a*x) + 8*a^4*x^4*imag_part(cos_integral(2*a*x))*tan(2*a*x)^2*tan(a*x)^2*tan(1/2*a*x) - 8*a^4*x^4*imag_part(cos_integral(-2*a*x))*tan(2*a*x)^2*tan(a*x)^2*tan(1/2*a*x) - 64*a^4*x^4*imag_part(cos_integral(-4*a*x))*tan(2*a*x)^2*tan(a*x)^2*tan(1/2*a*x) + 128*a^4*x^4*sin_integral(4*a*x)*tan(2*a*x)^2*tan(a*x)^2*tan(1/2*a*x) + 16*a^4*x^4*sin_integral(2*a*x)*tan(2*a*x)^2*tan(a*x)^2*tan(1/2*a*x) - 20*a^6*x^6*tan(1/2*a*x)^2 + 36*a^4*x^4*tan(2*a*x)^2*tan(a*x)^2*tan(1/2*a*x)^2 + 64*a^5*x^5*imag_part(cos_integral(4*a*x))*tan(2*a*x)^2 + 8*a^5*x^5*imag_part(cos_integral(2*a*x))*tan(2*a*x)^2 - 8*a^5*x^5*imag_part(cos_integral(-2*a*x))*tan(2*a*x)^2 - 64*a^5*x^5*imag_part(cos_integral(-4*a*x))*tan(2*a*x)^2 + 128*a^5*x^5*sin_integral(4*a*x)*tan(2*a*x)^2 + 16*a^5*x^5*sin_integral(2*a*x)*tan(2*a*x)^2 + 4*a^5*x^5*tan(2*a*x)^2*tan(a*x) + 64*a^5*x^5*imag_part(cos_integral(4*a*x))*tan(a*x)^2 + 8*a^5*x^5*imag_part(cos_integral(2*a*x))*tan(a*x)^2 - 8*a^5*x^5*imag_part(cos_integral(-2*a*x))*tan(a*x)^2 - 64*a^5*x^5*imag_part(cos_integral(-4*a*x))*tan(a*x)^2 + 128*a^5*x^5*sin_integral(4*a*x)*tan(a*x)^2 + 16*a^5*x^5*sin_integral(2*a*x)*tan(a*x)^2 + 8*a^5*x^5*tan(2*a*x)*tan(a*x)^2 - 48*a^5*x^5*tan(2*a*x)^2*tan(1/2*a*x) + 48*a^5*x^5*tan(a*x)^2*tan(1/2*a*x) - 64*a^5*x^5*imag_part(cos_integral(4*a*x))*tan(1/2*a*x)^2 - 8*a^5*x^5*imag_part(cos_integral(2*a*x))*tan(1/2*a*x)^2 + 8*a^5*x^5*imag_part(cos_integral(-2*a*x))*tan(1/2*a*x)^2 + 64*a^5*x^5*imag_part(cos_integral(-4*a*x))*tan(1/2*a*x)^2 - 128*a^5*x^5*sin_integral(4*a*x)*tan(1/2*a*x)^2 - 16*a^5*x^5*sin_integral(2*a*x)*tan(1/2*a*x)^2 - 8*a^5*x^5*tan(2*a*x)*tan(1/2*a*x)^2 - 4*a^5*x^5*tan(a*x)*tan(1/2*a*x)^2 - 32*a^3*x^3*imag_part(cos_integral(4*a*x))*tan(2*a*x)^2*tan(a*x)^2*tan(1/2*a*x)^2 - 4*a^3*x^3*imag_part(cos_integral(2*a*x))*tan(2*a*x)^2*tan(a*x)^2*tan(1/2*a*x)^2 + 4*a^3*x^3*imag_part(cos_integral(-2*a*x))*tan(2*a*x)^2*tan(a*x)^2*tan(1/2*a*x)^2 + 32*a^3*x^3*imag_part(cos_integral(-4*a*x))*tan(2*a*x)^2*tan(a*x)^2*tan(1/2*a*x)^2 - 64*a^3*x^3*sin_integral(4*a*x)*tan(2*a*x)^2*tan(a*x)^2*tan(1/2*a*x)^2 - 8*a^3*x^3*sin_integral(2*a*x)*tan(2*a*x)^2*tan(a*x)^2*tan(1/2*a*x)^2 + 20*a^6*x^6 - 36*a^4*x^4*tan(2*a*x)^2*tan(a*x)^2 + 64*a^4*x^4 *imag_part(cos_integral(4*a*x))*tan(2*a*x)^2*tan(1/2*a*x) + 8*a^4*x^4*imag_part(cos_integral(2*a*x))*tan(2*a*x)^2*tan(1/2*a*x) - 8*a^4*x^4*imag_part(cos_integral(-2*a*x))*tan(2*a*x)^2*tan(1/2*a*x) - 64*a^4*x^4*imag_part(cos_integral(-4*a*x))*tan(2*a*x)^2*tan(1/2*a*x) + 128*a^4*x^4*sin_integral(4*a*x)*tan(2*a*x)^2*tan(1/2*a*x) + 16*a^4*x^4*sin_integral(2*a*x)*tan(2*a*x)^2*tan(1/2*a*x) + 4*a^4*x^4*tan(2*a*x)^2*tan(a*x)*tan(1/2*a*x) + 64*a^4*x^4*imag_part(cos_integral(4*a*x))*tan(a*x)^2*tan(1/2*a*x) + 8*a^4*x^4*imag_part(cos_integral(2*a*x))*tan(a*x)^2*tan(1/2*a*x) - 8*a^4*x^4*imag_part(cos_integral(-2*a*x))*tan(a*x)^2*tan(1/2*a*x) - 64*a^4*x^4*imag_part(cos_integral(-4*a*x))*tan(a*x)^2*tan(1/2*a*x) + 128*a^4*x^4*sin_integral(4*a*x)*tan(a*x)^2*tan(1/2*a*x) + 16*a^4*x^4*sin_integral(2*a*x)*tan(a*x)^2*tan(1/2*a*x) + 26*a^4*x^4*tan(2*a*x)*tan(a*x)^2*tan(1/2*a*x) + 24*a^4*x^4*tan(2*a*x)^2*tan(1/2*a*x)^2 - 24*a^4*x^4*tan(a*x)^2*tan(1/2*a*x)^2 + 64*a^5*x^5*imag_part(cos_integral(4*a*x)) + 8*a^5*x^5*imag_part(cos_integral(2*a*x)) - 8*a^5*x^5*imag_part(cos_integral(-2*a*x)) - 64*a^5*x^5*imag_part(cos_integral(-4*a*x)) + 128*a^5*x^5*sin_integral(4*a*x) + 16*a^5*x^5*sin_integral(2*a*x) + 8*a^5*x^5*tan(2*a*x) + 4*a^5*x^5*tan(a*x) + 32*a^3*x^3*imag_part(cos_integral(4*a*x))*tan(2*a*x)^2*tan(a*x)^2 + 4*a^3*x^3*imag_part(cos_integral(2*a*x))*tan(2*a*x)^2*tan(a*x)^2 - 4*a^3*x^3*imag_part(cos_integral(-2*a*x))*tan(2*a*x)^2*tan(a*x)^2 - 32*a^3*x^3*imag_part(cos_integral(-4*a*x))*tan(2*a*x)^2*tan(a*x)^2 + 64*a^3*x^3*sin_integral(4*a*x)*tan(2*a*x)^2*tan(a*x)^2 + 8*a^3*x^3*sin_integral(2*a*x)*tan(2*a*x)^2*tan(a*x)^2 + 72*a^5*x^5*tan(1/2*a*x) - 30*a^3*x^3*tan(2*a*x)^2*tan(a*x)^2*tan(1/2*a*x) - 32*a^3*x^3*imag_part(cos_integral(4*a*x))*tan(2*a*x)^2*tan(1/2*a*x)^2 - 4*a^3*x^3*imag_part(cos_integral(2*a*x))*tan(2*a*x)^2*tan(1/2*a*x)^2 + 4*a^3*x^3*imag_part(cos_integral(-2*a*x))*tan(2*a*x)^2*tan(1/2*a*x)^2 + 32*a^3*x^3*imag_part(cos_integral(-4*a*x))*tan(2*a*x)^2*tan(1/2*a*x)^2 - 64*a^3*x^3*sin_integral(4*a*x)*tan(2*a*x)^2*tan(1/2*a*x)^2 - 8*a^3*x^3*sin_integral(2*a*x)*tan(2*a*x)^2*tan(1/2*a*x)^2 - 2*a^3*x^3*tan(2*a*x)^2*tan(a*x)*tan(1/2*a*x)^2 - 32*a^3*x^3*imag_part(cos_integral(4*a*x))*tan(a*x)^2*tan(1/2*a*x)^2 - 4*a^3*x^3*imag_part(cos_integral(2*a*x))*tan(a*x)^2*tan(1/2*a*x)^2 + 4*a^3*x^3*imag_part(cos_integral(-2*a*x))*tan(a*x)^2*tan(1/2*a*x)^2 + 32*a^3*x^3*imag_part(cos_integral(-4*a*x))*tan(a*x)^2*tan(1/2*a*x)^2 - 64*a^3*x^3*sin_integral(4*a*x)*tan(a*x)^2*tan(1/2*a*x)^2 - 8*a^3*x^3*sin_integral(2*a*x)*tan(a*x)^2*tan(1/2*a*x)^2 - 13*a^3*x^3*tan(2*a*x)*tan(a*x)^2*tan(1/2*a*x)^2 - 24*a^4*x^4*tan(2*a*x)^2 + 24*a^4*x^4*tan(a*x)^2 + 64*a^4*x^4*imag_part(cos_integral(4*a*x))*tan(1/2*a*x) + 8*a^4*x^4*imag_part(cos_integral(2*a*x))*tan(1/2*a*x) - 8*a^4*x^4*imag_part(cos_integral(-2*a*x))*tan(1/2*a*x) - 64*a^4*x^4*imag_part(cos_integral(-4*a*x))*tan(1/2*a*x) + 128*a^4*x^4*sin_integral(4*a*x)*tan(1/2*a*x) + 16*a^4*x^4*sin_integral(2*a*x)*tan(1/2*a*x) + 26*a^4*x^4*tan(2*a*x)*tan(1/2*a*x) + 4*a^4*x^4*tan(a*x)*tan(1/2*a*x) - 36*a^4*x^4*tan(1/2*a*x)^2 + 27*a^2*x^2*tan(2*a*x)^2*tan(a*x)^2*tan(1/2*a*x)^2 + 32*a^3*x^3*imag_part(cos_integral(4*a*x))*tan(2*a*x)^2 + 4*a^3*x^3*imag_part(cos_integral(2*a*x))*tan(2*a*x)^2 - 4*a^3*x^3*imag_part(cos_integral(-2*a*x))*tan(2*a*x)^2 - 32*a^3*x^3*imag_part(cos_integral(-4*a*x))*tan(2*a*x)^2 + 64*a^3*x^3*sin_integr

```
al(4*a*x)*tan(2*a*x)^2 + 8*a^3*x^3*sin_integral(2*a*x)*tan(2*a*x)^2 + 2*a^3
*x^3*tan(2*a*x)^2*tan(a*x) + 32*a^3*x^3*imag_part(cos_integral(4*a*x))*tan(
a*x)^2 + 4*a^3*x^3*imag_part(cos_integral(2*a*x))*tan(a*x)^2 - 4*a^3*x^3*im
ag_part(cos_integral(-2*a*x))*tan(a*x)^2 - 32*a^3*x^3*imag_part(cos_integra
l(-4*a*x))*tan(a*x)^2 + 64*a^3*x^3*sin_integral(4*a*x)*tan(a*x)^2 + 8*a^3*x
^3*sin_integral(2*a*x)*tan(a*x)^2 + 13*a^3*x^3*tan(2*a*x)*tan(a*x)^2 - 6*a^
3*x^3*tan(2*a*x)^2*tan(1/2*a*x) + 24*a^3*x^3*tan(a*x)^2*tan(1/2*a*x) - 32*a
^3*x^3*imag_part(cos_integral(4*a*x))*tan(1/2*a*x)^2 - 4*a^3*x^3*imag_part(
cos_integral(2*a*x))*tan(1/2*a*x)^2 + 4*a^3*x^3*imag_part(cos_integral(-2*a
*x))*tan(1/2*a*x)^2 + 32*a^3*x^3*imag_part(cos_integral(-4*a*x))*tan(1/2*a*
x)^2 - 64*a^3*x^3*sin_integral(4*a*x)*tan(1/2*a*x)^2 - 8*a^3*x^3*sin_integr
al(2*a*x)*tan(1/2*a*x)^2 - 13*a^3*x^3*tan(2*a*x)*tan(1/2*a*x)^2 - 2*a^3*x^3
*tan(a*x)*tan(1/2*a*x)^2 + 36*a^4*x^4 - 27*a^2*x^2*tan(2*a*x)^2*tan(a*x)^2
+ 20*a^2*x^2*tan(2*a*x)^2*tan(a*x)*tan(1/2*a*x) + 10*a^2*x^2*tan(2*a*x)*tan
(a*x)^2*tan(1/2*a*x) + 15*a^2*x^2*tan(2*a*x)^2*tan(1/2*a*x)^2 + 32*a^3*x^3*
imag_part(cos_integral(4*a*x)) + 4*a^3*x^3*imag_part(cos_integral(2*a*x)) -
 4*a^3*x^3*imag_part(cos_integral(-2*a*x)) - 32*a^3*x^3*imag_part(cos_integ
ral(-4*a*x)) + 64*a^3*x^3*sin_integral(4*a*x) + 8*a^3*x^3*sin_integral(2*a*
x) + 13*a^3*x^3*tan(2*a*x) + 2*a^3*x^3*tan(a*x) + 48*a^3*x^3*tan(1/2*a*x) +
 2*a*x*tan(2*a*x)^2*tan(a*x)^2*tan(1/2*a*x) - 10*a*x*tan(2*a*x)^2*tan(a*x)*
tan(1/2*a*x)^2 - 5*a*x*tan(2*a*x)*tan(a*x)^2*tan(1/2*a*x)^2 - 15*a^2*x^2*ta
n(2*a*x)^2 + 10*a^2*x^2*tan(2*a*x)*tan(1/2*a*x) + 20*a^2*x^2*tan(a*x)*tan(1
/2*a*x) - 12*a^2*x^2*tan(1/2*a*x)^2 - tan(2*a*x)^2*tan(a*x)^2*tan(1/2*a*x)^
2 + 10*a*x*tan(2*a*x)^2*tan(a*x) + 5*a*x*tan(2*a*x)*tan(a*x)^2 - 6*a*x*tan(
2*a*x)^2*tan(1/2*a*x) - 5*a*x*tan(2*a*x)*tan(1/2*a*x)^2 - 10*a*x*tan(a*x)*t
an(1/2*a*x)^2 + 12*a^2*x^2 + tan(2*a*x)^2*tan(a*x)^2 + 3*tan(2*a*x)^2*tan(1
/2*a*x)^2 + 5*a*x*tan(2*a*x) + 10*a*x*tan(a*x) - 8*a*x*tan(1/2*a*x) - 3*tan
(2*a*x)^2 + 4*tan(1/2*a*x)^2 - 4)/(2*a^5*x^8*tan(2*a*x)^2*tan(a*x)^2*tan(1/
2*a*x) - a^4*x^7*tan(2*a*x)^2*tan(a*x)^2*tan(1/2*a*x)^2 + 2*a^5*x^8*tan(2*a
*x)^2*tan(1/2*a*x) + 2*a^5*x^8*tan(a*x)^2*tan(1/2*a*x) + a^4*x^7*tan(2*a*x)
^2*tan(a*x)^2 - a^4*x^7*tan(2*a*x)^2*tan(1/2*a*x)^2 - a^4*x^7*tan(a*x)^2*ta
n(1/2*a*x)^2 + 2*a^5*x^8*tan(1/2*a*x) + 4*a^3*x^6*tan(2*a*x)^2*tan(a*x)^2*t
an(1/2*a*x) + a^4*x^7*tan(2*a*x)^2 + a^4*x^7*tan(a*x)^2 - a^4*x^7*tan(1/2*a
*x)^2 - 2*a^2*x^5*tan(2*a*x)^2*tan(a*x)^2*tan(1/2*a*x)^2 + 4*a^3*x^6*tan(2*
a*x)^2*tan(1/2*a*x) + 4*a^3*x^6*tan(a*x)^2*tan(1/2*a*x) + a^4*x^7 + 2*a^2*x
^5*tan(2*a*x)^2*tan(a*x)^2 - 2*a^2*x^5*tan(2*a*x)^2*tan(1/2*a*x)^2 - 2*a^2*
x^5*tan(a*x)^2*tan(1/2*a*x)^2 + 4*a^3*x^6*tan(1/2*a*x) + 2*a*x^4*tan(2*a*x)
^2*tan(a*x)^2*tan(1/2*a*x) + 2*a^2*x^5*tan(2*a*x)^2 + 2*a^2*x^5*tan(a*x)^2
- 2*a^2*x^5*tan(1/2*a*x)^2 - x^3*tan(2*a*x)^2*tan(a*x)^2*tan(1/2*a*x)^2 + 2
*a*x^4*tan(2*a*x)^2*tan(1/2*a*x) + 2*a*x^4*tan(a*x)^2*tan(1/2*a*x) + 2*a^2*
x^5 + x^3*tan(2*a*x)^2*tan(a*x)^2 - x^3*tan(2*a*x)^2*tan(1/2*a*x)^2 - x^3*t
an(a*x)^2*tan(1/2*a*x)^2 + 2*a*x^4*tan(1/2*a*x) + x^3*tan(2*a*x)^2 + x^3*ta
n(a*x)^2 - x^3*tan(1/2*a*x)^2 + x^3)
```

maple [F(-1)] time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{\cos^6(ax)}{x^4(\cos(ax) + ax\sin(ax))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a*x)^6/x^4/(cos(a*x)+a*x*sin(a*x))^2,x)
[Out] int(cos(a*x)^6/x^4/(cos(a*x)+a*x*sin(a*x))^2,x)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(a*x)^6/x^4/(cos(a*x)+a*x*sin(a*x))^2,x, algorithm="maxima")
[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is und
efined.
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos(a\,x)^6}{x^4\,(\cos(a\,x) + a\,x\,\sin(a\,x))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a*x)^6/(x^4*(cos(a*x) + a*x*sin(a*x))^2),x)
[Out] int(cos(a*x)^6/(x^4*(cos(a*x) + a*x*sin(a*x))^2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos^6(ax)}{x^4(ax\sin(ax) + \cos(ax))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(a*x)**6/x**4/(cos(a*x)+a*x*sin(a*x))**2,x)
[Out] Integral(cos(a*x)**6/(x**4*(a*x*sin(a*x) + cos(a*x))**2), x)
```

3.595 $\int \frac{\cos^5(ax)}{x^3(\cos(ax)+ax\sin(ax))^2}\,dx$

Optimal. Leaf size=132

$$-\frac{1}{8}a^2\text{Ci}(ax)-\frac{27}{8}a^2\text{Ci}(3ax)+\frac{\cos^3(ax)}{a^2x^4}-\frac{\cos^4(ax)}{a^2x^4(ax\sin(ax)+\cos(ax))}-\frac{\sin(ax)\cos^2(ax)}{ax^3}-\frac{3\cos^3(ax)}{2x^2}+\frac{\cos(ax)}{x^2}-\frac{a\sin}{x}$$

```
[Out] -1/8*a^2*Ci(a*x)-27/8*a^2*Ci(3*a*x)+cos(a*x)/x^2+cos(a*x)^3/a^2/x^4-3/2*cos
(a*x)^3/x^2-a*sin(a*x)/x-cos(a*x)^2*sin(a*x)/a/x^3+9/2*a*cos(a*x)^2*sin(a*x
)/x-cos(a*x)^4/a^2/x^4/(cos(a*x)+a*x*sin(a*x))
```

Rubi [A] time = 0.23, antiderivative size = 132, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {4599, 3314, 3297, 3302, 3312}

$$-\frac{1}{8}a^2\text{CosIntegral}(ax)-\frac{27}{8}a^2\text{CosIntegral}(3ax)+\frac{\cos^3(ax)}{a^2x^4}-\frac{\cos^4(ax)}{a^2x^4(ax\sin(ax)+\cos(ax))}-\frac{3\cos^3(ax)}{2x^2}+\frac{\cos(ax)}{x^2}-\frac{\sin}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a*x]^5/(x^3*(Cos[a*x] + a*x*Sin[a*x])^2),x]
```

```
[Out] Cos[a*x]/x^2 + Cos[a*x]^3/(a^2*x^4) - (3*Cos[a*x]^3)/(2*x^2) - (a^2*CosInte
gral[a*x])/8 - (27*a^2*CosIntegral[3*a*x])/8 - (a*Sin[a*x])/x - (Cos[a*x]^2
*Sin[a*x])/(a*x^3) + (9*a*Cos[a*x]^2*Sin[a*x])/(2*x) - Cos[a*x]^4/(a^2*x^4*
(Cos[a*x] + a*x*Sin[a*x]))
```

Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[((
c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3302

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
```

```
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 3314

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[((c + d*x)^(m + 1)*(b*Sin[e + f*x])^n)/(d*(m + 1)), x] + (Dist[(
b^2*f^2*n*(n - 1))/(d^2*(m + 1)*(m + 2)), Int[(c + d*x)^(m + 2)*(b*Sin[e +
f*x])^(n - 2), x], x] - Dist[(f^2*n^2)/(d^2*(m + 1)*(m + 2)), Int[(c + d*x)
^(m + 2)*(b*Sin[e + f*x])^n, x], x] - Simp[(b*f*n*(c + d*x)^(m + 2)*Cos[e +
 f*x]*(b*Sin[e + f*x])^(n - 1))/(d^2*(m + 1)*(m + 2)), x]) /; FreeQ[{b, c,
d, e, f}, x] && GtQ[n, 1] && LtQ[m, -2]
```

Rule 4599

```
Int[(Cos[(a_.)*(x_)]^(n_)*((b_.)*(x_))^(m_))/(Cos[(a_.)*(x_)]*(c_.) + (d_.)
*(x_)*Sin[(a_.)*(x_)])^2, x_Symbol] :> -Simp[(b*(b*x)^(m - 1)*Cos[a*x]^(n -
 1))/(a*d*(c*Cos[a*x] + d*x*Sin[a*x])), x] - Dist[(b^2*(n - 1))/d^2, Int[(b
*x)^(m - 2)*Cos[a*x]^(n - 2), x], x] /; FreeQ[{a, b, c, d, m, n}, x] && EqQ
[a*c - d, 0] && EqQ[m, 2 - n]
```

Rubi steps

$$\begin{aligned}\int \frac{\cos^5(ax)}{x^3(\cos(ax)+ax\sin(ax))^2}\,dx &= -\frac{\cos^4(ax)}{a^2x^4(\cos(ax)+ax\sin(ax))}-\frac{4\int \frac{\cos^3(ax)}{x^5}\,dx}{a^2}\\ &= \frac{\cos^3(ax)}{a^2x^4}-\frac{\cos^2(ax)\sin(ax)}{ax^3}-\frac{\cos^4(ax)}{a^2x^4(\cos(ax)+ax\sin(ax))}-2\int \frac{\cos(ax)}{x^3}\,dx\\ &= \frac{\cos(ax)}{x^2}+\frac{\cos^3(ax)}{a^2x^4}-\frac{3\cos^3(ax)}{2x^2}-\frac{\cos^2(ax)\sin(ax)}{ax^3}+\frac{9a\cos^2(ax)\sin(ax)}{2x}\\ &= \frac{\cos(ax)}{x^2}+\frac{\cos^3(ax)}{a^2x^4}-\frac{3\cos^3(ax)}{2x^2}+9a^2\mathrm{Ci}(ax)-\frac{a\sin(ax)}{x}-\frac{\cos^2(ax)\sin(ax)}{ax^3}\\ &= \frac{\cos(ax)}{x^2}+\frac{\cos^3(ax)}{a^2x^4}-\frac{3\cos^3(ax)}{2x^2}+10a^2\mathrm{Ci}(ax)-\frac{a\sin(ax)}{x}-\frac{\cos^2(ax)\sin(a}{ax^3}\\ &= \frac{\cos(ax)}{x^2}+\frac{\cos^3(ax)}{a^2x^4}-\frac{3\cos^3(ax)}{2x^2}-\frac{1}{8}a^2\mathrm{Ci}(ax)-\frac{27}{8}a^2\mathrm{Ci}(3ax)-\frac{a\sin(ax)}{x}-\end{aligned}$$

Mathematica [A] time = 0.80, size = 136, normalized size = 1.03

$$-\frac{2a^2x^2\mathrm{Ci}(ax)(ax\sin(ax)+\cos(ax))+54a^2x^2\mathrm{Ci}(3ax)(ax\sin(ax)+\cos(ax))-a^2x^2-8a^2x^2\cos(2ax)+9a^2x^2\,\mathrm{c}}{16x^2(ax\sin(ax)+\cos(ax))}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a*x]^5/(x^3*(Cos[a*x] + a*x*Sin[a*x])^2),x]
```

```
[Out] -1/16*(3 - a^2*x^2 + 4*Cos[2*a*x] - 8*a^2*x^2*Cos[2*a*x] + Cos[4*a*x] + 9*a
^2*x^2*Cos[4*a*x] + 2*a^2*x^2*CosIntegral[a*x]*(Cos[a*x] + a*x*Sin[a*x]) +
54*a^2*x^2*CosIntegral[3*a*x]*(Cos[a*x] + a*x*Sin[a*x]) - 12*a*x*Sin[2*a*x]
 - 6*a*x*Sin[4*a*x])/(x^2*(Cos[a*x] + a*x*Sin[a*x]))
```

fricas [A] time = 0.92, size = 185, normalized size = 1.40

$$\frac{88\,a^2x^2\cos(ax)^2 - 8\left(9\,a^2x^2+1\right)\cos(ax)^4 - 16\,a^2x^2 - \left(27\,a^2x^2\,\mathrm{Ci}(3\,ax) + a^2x^2\,\mathrm{Ci}(ax) + a^2x^2\,\mathrm{Ci}(-ax) + 27\,a^2x}{16\left(ax^3\sin\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(a*x)^5/x^3/(cos(a*x)+a*x*sin(a*x))^2,x, algorithm="fricas")
```

```
[Out] 1/16*(88*a^2*x^2*cos(a*x)^2 - 8*(9*a^2*x^2 + 1)*cos(a*x)^4 - 16*a^2*x^2 - (
27*a^2*x^2*cos_integral(3*a*x) + a^2*x^2*cos_integral(a*x) + a^2*x^2*cos_in
tegral(-a*x) + 27*a^2*x^2*cos_integral(-3*a*x))*cos(a*x) - (27*a^3*x^3*cos_
integral(3*a*x) + a^3*x^3*cos_integral(a*x) + a^3*x^3*cos_integral(-a*x) +
27*a^3*x^3*cos_integral(-3*a*x) - 48*a*x*cos(a*x)^3)*sin(a*x))/(a*x^3*sin(a
*x) + x^2*cos(a*x))
```

giac [C] time = 0.69, size = 3130, normalized size = 23.71

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(a*x)^5/x^3/(cos(a*x)+a*x*sin(a*x))^2,x, algorithm="giac")
```

```
[Out] -1/16*(54*a^7*x^7*real_part(cos_integral(3*a*x))*tan(3/2*a*x)^2*tan(1/2*a*x
)^3 + 2*a^7*x^7*real_part(cos_integral(a*x))*tan(3/2*a*x)^2*tan(1/2*a*x)^3
+ 2*a^7*x^7*real_part(cos_integral(-a*x))*tan(3/2*a*x)^2*tan(1/2*a*x)^3 + 5
4*a^7*x^7*real_part(cos_integral(-3*a*x))*tan(3/2*a*x)^2*tan(1/2*a*x)^3 - 2
7*a^6*x^6*real_part(cos_integral(3*a*x))*tan(3/2*a*x)^2*tan(1/2*a*x)^4 - a^
6*x^6*real_part(cos_integral(a*x))*tan(3/2*a*x)^2*tan(1/2*a*x)^4 - a^6*x^6*
real_part(cos_integral(-a*x))*tan(3/2*a*x)^2*tan(1/2*a*x)^4 - 27*a^6*x^6*re
al_part(cos_integral(-3*a*x))*tan(3/2*a*x)^2*tan(1/2*a*x)^4 + 54*a^7*x^7*re
al_part(cos_integral(3*a*x))*tan(3/2*a*x)^2*tan(1/2*a*x) + 2*a^7*x^7*real_p
art(cos_integral(a*x))*tan(3/2*a*x)^2*tan(1/2*a*x) + 2*a^7*x^7*real_part(co
s_integral(-a*x))*tan(3/2*a*x)^2*tan(1/2*a*x) + 54*a^7*x^7*real_part(cos_in
tegral(-3*a*x))*tan(3/2*a*x)^2*tan(1/2*a*x) + 54*a^7*x^7*real_part(cos_inte
gral(3*a*x))*tan(1/2*a*x)^3 + 2*a^7*x^7*real_part(cos_integral(a*x))*tan(1/
```

2*a*x)^3 + 2*a^7*x^7*real_part(cos_integral(-a*x))*tan(1/2*a*x)^3 + 54*a^7*x^7*real_part(cos_integral(-3*a*x))*tan(1/2*a*x)^3 - 27*a^6*x^6*real_part(cos_integral(3*a*x))*tan(1/2*a*x)^4 - a^6*x^6*real_part(cos_integral(a*x))*tan(1/2*a*x)^4 - a^6*x^6*real_part(cos_integral(-a*x))*tan(1/2*a*x)^4 - 27*a^6*x^6*real_part(cos_integral(-3*a*x))*tan(1/2*a*x)^4 + 54*a^7*x^7*real_part(cos_integral(3*a*x))*tan(1/2*a*x) + 2*a^7*x^7*real_part(cos_integral(a*x))*tan(1/2*a*x) + 2*a^7*x^7*real_part(cos_integral(-a*x))*tan(1/2*a*x) + 54*a^7*x^7*real_part(cos_integral(-3*a*x))*tan(1/2*a*x) - 8*a^6*x^6*tan(3/2*a*x)^2*tan(1/2*a*x)^2 - 72*a^6*x^6*tan(3/2*a*x)*tan(1/2*a*x)^3 + 108*a^5*x^5*real_part(cos_integral(3*a*x))*tan(3/2*a*x)^2*tan(1/2*a*x)^3 + 4*a^5*x^5*real_part(cos_integral(a*x))*tan(3/2*a*x)^2*tan(1/2*a*x)^3 + 4*a^5*x^5*real_part(cos_integral(-a*x))*tan(3/2*a*x)^2*tan(1/2*a*x)^3 + 108*a^5*x^5*real_part(cos_integral(-3*a*x))*tan(3/2*a*x)^2*tan(1/2*a*x)^3 + 27*a^6*x^6*real_part(cos_integral(3*a*x))*tan(3/2*a*x)^2 + a^6*x^6*real_part(cos_integral(a*x))*tan(3/2*a*x)^2 + a^6*x^6*real_part(cos_integral(-a*x))*tan(3/2*a*x)^2 + 27*a^6*x^6*real_part(cos_integral(-3*a*x))*tan(3/2*a*x)^2 - 12*a^5*x^5*tan(3/2*a*x)^2*tan(1/2*a*x)^3 + 36*a^5*x^5*tan(3/2*a*x)*tan(1/2*a*x)^4 - 54*a^4*x^4*real_part(cos_integral(3*a*x))*tan(3/2*a*x)^2*tan(1/2*a*x)^4 - 2*a^4*x^4*real_part(cos_integral(a*x))*tan(3/2*a*x)^2*tan(1/2*a*x)^4 - 2*a^4*x^4*real_part(cos_integral(-a*x))*tan(3/2*a*x)^2*tan(1/2*a*x)^4 - 54*a^4*x^4*real_part(cos_integral(-3*a*x))*tan(3/2*a*x)^2*tan(1/2*a*x)^4 - 72*a^6*x^6*tan(3/2*a*x)*tan(1/2*a*x) + 108*a^5*x^5*real_part(cos_integral(3*a*x))*tan(3/2*a*x)^2*tan(1/2*a*x) + 4*a^5*x^5*real_part(cos_integral(a*x))*tan(3/2*a*x)^2*tan(1/2*a*x) + 4*a^5*x^5*real_part(cos_integral(-a*x))*tan(3/2*a*x)^2*tan(1/2*a*x) + 108*a^5*x^5*real_part(cos_integral(-3*a*x))*tan(3/2*a*x)^2*tan(1/2*a*x) - 8*a^6*x^6*tan(1/2*a*x)^2 + 108*a^5*x^5*real_part(cos_integral(3*a*x))*tan(1/2*a*x)^3 + 4*a^5*x^5*real_part(cos_integral(a*x))*tan(1/2*a*x)^3 + 4*a^5*x^5*real_part(cos_integral(-a*x))*tan(1/2*a*x)^3 + 108*a^5*x^5*real_part(cos_integral(-3*a*x))*tan(1/2*a*x)^3 + 8*a^4*x^4*tan(3/2*a*x)^2*tan(1/2*a*x)^4 + 27*a^6*x^6*real_part(cos_integral(3*a*x)) + a^6*x^6*real_part(cos_integral(a*x)) + a^6*x^6*real_part(cos_integral(-a*x)) + 27*a^6*x^6*real_part(cos_integral(-3*a*x)) - 12*a^5*x^5*tan(3/2*a*x)^2*tan(1/2*a*x) + 12*a^5*x^5*tan(1/2*a*x)^3 - 54*a^4*x^4*real_part(cos_integral(3*a*x))*tan(1/2*a*x)^4 - 2*a^4*x^4*real_part(cos_integral(a*x))*tan(1/2*a*x)^4 - 2*a^4*x^4*real_part(cos_integral(-a*x))*tan(1/2*a*x)^4 - 54*a^4*x^4*real_part(cos_integral(-3*a*x))*tan(1/2*a*x)^4 + 108*a^5*x^5*real_part(cos_integral(3*a*x))*tan(1/2*a*x) + 4*a^5*x^5*real_part(cos_integral(a*x))*tan(1/2*a*x) + 4*a^5*x^5*real_part(cos_integral(-a*x))*tan(1/2*a*x) + 108*a^5*x^5*real_part(cos_integral(-3*a*x))*tan(1/2*a*x) - 4*a^4*x^4*tan(3/2*a*x)^2*tan(1/2*a*x)^2 - 128*a^4*x^4*tan(3/2*a*x)*tan(1/2*a*x)^3 + 54*a^3*x^3*real_part(cos_integral(3*a*x))*tan(3/2*a*x)^2*tan(1/2*a*x)^3 + 2*a^3*x^3*real_part(cos_integral(a*x))*tan(3/2*a*x)^2*tan(1/2*a*x)^3 + 2*a^3*x^3*real_part(cos_integral(-a*x))*tan(3/2*a*x)^2*tan(1/2*a*x)^3 + 54*a^3*x^3*real_part(cos_integral(-3*a*x))*tan(3/2*a*x)^2*tan(1/2*a*x)^3 - 4*a^4*x^4*tan(1/2*a*x)^4 - 36*a^5*x^5*tan(3/2*a*x) + 54*a^4*x^4*real_part(cos_integral(3*a*x))*tan(3/2*a*x)^2 + 2*a

```
^4*x^4*real_part(cos_integral(a*x))*tan(3/2*a*x)^2 + 2*a^4*x^4*real_part(co
s_integral(-a*x))*tan(3/2*a*x)^2 + 54*a^4*x^4*real_part(cos_integral(-3*a*x
))*tan(3/2*a*x)^2 + 12*a^5*x^5*tan(1/2*a*x) + 64*a^3*x^3*tan(3/2*a*x)*tan(1
/2*a*x)^4 - 27*a^2*x^2*real_part(cos_integral(3*a*x))*tan(3/2*a*x)^2*tan(1/
2*a*x)^4 - a^2*x^2*real_part(cos_integral(a*x))*tan(3/2*a*x)^2*tan(1/2*a*x)
^4 - a^2*x^2*real_part(cos_integral(-a*x))*tan(3/2*a*x)^2*tan(1/2*a*x)^4 -
27*a^2*x^2*real_part(cos_integral(-3*a*x))*tan(3/2*a*x)^2*tan(1/2*a*x)^4 -
4*a^4*x^4*tan(3/2*a*x)^2 - 128*a^4*x^4*tan(3/2*a*x)*tan(1/2*a*x) + 54*a^3*x
^3*real_part(cos_integral(3*a*x))*tan(3/2*a*x)^2*tan(1/2*a*x) + 2*a^3*x^3*r
eal_part(cos_integral(a*x))*tan(3/2*a*x)^2*tan(1/2*a*x) + 2*a^3*x^3*real_pa
rt(cos_integral(-a*x))*tan(3/2*a*x)^2*tan(1/2*a*x) + 54*a^3*x^3*real_part(c
os_integral(-3*a*x))*tan(3/2*a*x)^2*tan(1/2*a*x) - 4*a^4*x^4*tan(1/2*a*x)^2
 + 54*a^3*x^3*real_part(cos_integral(3*a*x))*tan(1/2*a*x)^3 + 2*a^3*x^3*rea
l_part(cos_integral(a*x))*tan(1/2*a*x)^3 + 2*a^3*x^3*real_part(cos_integral
(-a*x))*tan(1/2*a*x)^3 + 54*a^3*x^3*real_part(cos_integral(-3*a*x))*tan(1/2
*a*x)^3 + 32*a^2*x^2*tan(3/2*a*x)^2*tan(1/2*a*x)^4 + 54*a^4*x^4*real_part(c
os_integral(3*a*x)) + 2*a^4*x^4*real_part(cos_integral(a*x)) + 2*a^4*x^4*re
al_part(cos_integral(-a*x)) + 54*a^4*x^4*real_part(cos_integral(-3*a*x)) -
32*a^3*x^3*tan(3/2*a*x)^2*tan(1/2*a*x) + 32*a^3*x^3*tan(1/2*a*x)^3 - 27*a^2
*x^2*real_part(cos_integral(3*a*x))*tan(1/2*a*x)^4 - a^2*x^2*real_part(cos_
integral(a*x))*tan(1/2*a*x)^4 - a^2*x^2*real_part(cos_integral(-a*x))*tan(1
/2*a*x)^4 - 27*a^2*x^2*real_part(cos_integral(-3*a*x))*tan(1/2*a*x)^4 + 8*a
^4*x^4 + 54*a^3*x^3*real_part(cos_integral(3*a*x))*tan(1/2*a*x) + 2*a^3*x^3
*real_part(cos_integral(a*x))*tan(1/2*a*x) + 2*a^3*x^3*real_part(cos_integr
al(-a*x))*tan(1/2*a*x) + 54*a^3*x^3*real_part(cos_integral(-3*a*x))*tan(1/2
*a*x) + 24*a^2*x^2*tan(3/2*a*x)^2*tan(1/2*a*x)^2 - 56*a^2*x^2*tan(3/2*a*x)*
tan(1/2*a*x)^3 + 16*a^2*x^2*tan(1/2*a*x)^4 - 64*a^3*x^3*tan(3/2*a*x) + 27*a
^2*x^2*real_part(cos_integral(3*a*x))*tan(3/2*a*x)^2 + a^2*x^2*real_part(co
s_integral(a*x))*tan(3/2*a*x)^2 + a^2*x^2*real_part(cos_integral(-a*x))*tan
(3/2*a*x)^2 + 27*a^2*x^2*real_part(cos_integral(-3*a*x))*tan(3/2*a*x)^2 + 1
2*a*x*tan(3/2*a*x)^2*tan(1/2*a*x)^3 + 28*a*x*tan(3/2*a*x)*tan(1/2*a*x)^4 +
16*a^2*x^2*tan(3/2*a*x)^2 - 56*a^2*x^2*tan(3/2*a*x)*tan(1/2*a*x) + 24*a^2*x
^2*tan(1/2*a*x)^2 + 8*tan(3/2*a*x)^2*tan(1/2*a*x)^4 + 27*a^2*x^2*real_part(
cos_integral(3*a*x)) + a^2*x^2*real_part(cos_integral(a*x)) + a^2*x^2*real_
part(cos_integral(-a*x)) + 27*a^2*x^2*real_part(cos_integral(-3*a*x)) - 20*
a*x*tan(3/2*a*x)^2*tan(1/2*a*x) + 20*a*x*tan(1/2*a*x)^3 + 32*a^2*x^2 - 12*t
an(3/2*a*x)^2*tan(1/2*a*x)^2 + 4*tan(1/2*a*x)^4 - 28*a*x*tan(3/2*a*x) - 12*
a*x*tan(1/2*a*x) + 4*tan(3/2*a*x)^2 - 12*tan(1/2*a*x)^2 + 8)/(2*a^5*x^7*tan
(3/2*a*x)^2*tan(1/2*a*x)^3 - a^4*x^6*tan(3/2*a*x)^2*tan(1/2*a*x)^4 + 2*a^5*
x^7*tan(3/2*a*x)^2*tan(1/2*a*x) + 2*a^5*x^7*tan(1/2*a*x)^3 - a^4*x^6*tan(1/
2*a*x)^4 + 2*a^5*x^7*tan(1/2*a*x) + 4*a^3*x^5*tan(3/2*a*x)^2*tan(1/2*a*x)^3
 + a^4*x^6*tan(3/2*a*x)^2 - 2*a^2*x^4*tan(3/2*a*x)^2*tan(1/2*a*x)^4 + 4*a^3
*x^5*tan(3/2*a*x)^2*tan(1/2*a*x) + 4*a^3*x^5*tan(1/2*a*x)^3 + a^4*x^6 - 2*a
^2*x^4*tan(1/2*a*x)^4 + 4*a^3*x^5*tan(1/2*a*x) + 2*a*x^3*tan(3/2*a*x)^2*tan
(1/2*a*x)^3 + 2*a^2*x^4*tan(3/2*a*x)^2 - x^2*tan(3/2*a*x)^2*tan(1/2*a*x)^4
```

```
+ 2*a*x^3*tan(3/2*a*x)^2*tan(1/2*a*x) + 2*a*x^3*tan(1/2*a*x)^3 + 2*a^2*x^4
- x^2*tan(1/2*a*x)^4 + 2*a*x^3*tan(1/2*a*x) + x^2*tan(3/2*a*x)^2 + x^2)
```

maple [F(-1)] time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{\cos^5(ax)}{x^3(\cos(ax) + ax\sin(ax))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a*x)^5/x^3/(cos(a*x)+a*x*sin(a*x))^2,x)
```

```
[Out] int(cos(a*x)^5/x^3/(cos(a*x)+a*x*sin(a*x))^2,x)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(a*x)^5/x^3/(cos(a*x)+a*x*sin(a*x))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is und
efined.
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos(a\,x)^5}{x^3\,(\cos(a\,x) + a\,x\,\sin(a\,x))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a*x)^5/(x^3*(cos(a*x) + a*x*sin(a*x))^2),x)
```

```
[Out] int(cos(a*x)^5/(x^3*(cos(a*x) + a*x*sin(a*x))^2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos^5(ax)}{x^3(ax\sin(ax) + \cos(ax))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(a*x)**5/x**3/(cos(a*x)+a*x*sin(a*x))**2,x)
```

```
[Out] Integral(cos(a*x)**5/(x**3*(a*x*sin(a*x) + cos(a*x))**2), x)
```

3.596 $\int \frac{\cos^4(ax)}{x^2(\cos(ax)+ax\sin(ax))^2}\,dx$

Optimal. Leaf size=80

$$\frac{\cos^2(ax)}{a^2x^3}-\frac{\cos^3(ax)}{a^2x^3(ax\sin(ax)+\cos(ax))}-2a\mathrm{Si}(2ax)-\frac{\sin(ax)\cos(ax)}{ax^2}-\frac{2\cos^2(ax)}{x}+\frac{1}{x}$$

```
[Out] 1/x+cos(a*x)^2/a^2/x^3-2*cos(a*x)^2/x-2*a*Si(2*a*x)-cos(a*x)*sin(a*x)/a/x^2
-cos(a*x)^3/a^2/x^3/(cos(a*x)+a*x*sin(a*x))
```

Rubi [A] time = 0.13, antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {4599, 3314, 30, 3313, 12, 3299}

$$\frac{\cos^2(ax)}{a^2x^3}-\frac{\cos^3(ax)}{a^2x^3(ax\sin(ax)+\cos(ax))}-2a\mathrm{Si}(2ax)-\frac{\sin(ax)\cos(ax)}{ax^2}-\frac{2\cos^2(ax)}{x}+\frac{1}{x}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a*x]^4/(x^2*(Cos[a*x] + a*x*Sin[a*x])^2),x]
```

```
[Out] x^(-1) + Cos[a*x]^2/(a^2*x^3) - (2*Cos[a*x]^2)/x - (Cos[a*x]*Sin[a*x])/(a*x
^2) - Cos[a*x]^3/(a^2*x^3*(Cos[a*x] + a*x*Sin[a*x])) - 2*a*SinIntegral[2*a*
x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3313

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> Si
mp[((c + d*x)^(m + 1)*Sin[e + f*x]^n)/(d*(m + 1)), x] - Dist[(f*n)/(d*(m +
1)), Int[ExpandTrigReduce[(c + d*x)^(m + 1), Cos[e + f*x]*Sin[e + f*x]^(n -
```

```
1), x], x], x] /; FreeQ[{c, d, e, f, m}, x] && IGtQ[n, 1] && GeQ[m, -2] &&
LtQ[m, -1]
```

Rule 3314

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[((c + d*x)^(m + 1)*(b*Sin[e + f*x])^n)/(d*(m + 1)), x] + (Dist[(
b^2*f^2*n*(n - 1))/(d^2*(m + 1)*(m + 2)), Int[(c + d*x)^(m + 2)*(b*Sin[e +
f*x])^(n - 2), x], x] - Dist[(f^2*n^2)/(d^2*(m + 1)*(m + 2)), Int[(c + d*x)
^(m + 2)*(b*Sin[e + f*x])^n, x], x] - Simp[(b*f*n*(c + d*x)^(m + 2)*Cos[e +
 f*x]*(b*Sin[e + f*x])^(n - 1))/(d^2*(m + 1)*(m + 2)), x]) /; FreeQ[{b, c,
d, e, f}, x] && GtQ[n, 1] && LtQ[m, -2]
```

Rule 4599

```
Int[(Cos[(a_.)*(x_)]^(n_)*((b_.)*(x_))^(m_))/(Cos[(a_.)*(x_)]*(c_.) + (d_.)
*(x_)*Sin[(a_.)*(x_)])^2, x_Symbol] :> -Simp[(b*(b*x)^(m - 1)*Cos[a*x]^(n -
 1))/(a*d*(c*Cos[a*x] + d*x*Sin[a*x])), x] - Dist[(b^2*(n - 1))/d^2, Int[(b
*x)^(m - 2)*Cos[a*x]^(n - 2), x], x] /; FreeQ[{a, b, c, d, m, n}, x] && EqQ
[a*c - d, 0] && EqQ[m, 2 - n]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\cos^4(ax)}{x^2(\cos(ax)+ax\sin(ax))^2}\,dx &= -\frac{\cos^3(ax)}{a^2x^3(\cos(ax)+ax\sin(ax))}-\frac{3\int\frac{\cos^2(ax)}{x^4}\,dx}{a^2}\\
&= \frac{\cos^2(ax)}{a^2x^3}-\frac{\cos(ax)\sin(ax)}{ax^2}-\frac{\cos^3(ax)}{a^2x^3(\cos(ax)+ax\sin(ax))}+2\int\frac{\cos^2(ax)}{x^2}\,dx\\
&= \frac{1}{x}+\frac{\cos^2(ax)}{a^2x^3}-\frac{2\cos^2(ax)}{x}-\frac{\cos(ax)\sin(ax)}{ax^2}-\frac{\cos^3(ax)}{a^2x^3(\cos(ax)+ax\sin(ax))}\\
&= \frac{1}{x}+\frac{\cos^2(ax)}{a^2x^3}-\frac{2\cos^2(ax)}{x}-\frac{\cos(ax)\sin(ax)}{ax^2}-\frac{\cos^3(ax)}{a^2x^3(\cos(ax)+ax\sin(ax))}\\
&= \frac{1}{x}+\frac{\cos^2(ax)}{a^2x^3}-\frac{2\cos^2(ax)}{x}-\frac{\cos(ax)\sin(ax)}{ax^2}-\frac{\cos^3(ax)}{a^2x^3(\cos(ax)+ax\sin(ax))}
\end{aligned}
$$

Mathematica [A] time = 0.68, size = 71, normalized size = 0.89

$$
-\frac{8ax\text{Si}(2ax)(ax\sin(ax)+\cos(ax))-2ax\sin(ax)+2ax\sin(3ax)+3\cos(ax)+\cos(3ax)}{4x(ax\sin(ax)+\cos(ax))}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a*x]^4/(x^2*(Cos[a*x] + a*x*Sin[a*x])^2),x]
```

```
[Out] -1/4*(3*Cos[a*x] + Cos[3*a*x] - 2*a*x*Sin[a*x] + 2*a*x*Sin[3*a*x] + 8*a*x*(
Cos[a*x] + a*x*Sin[a*x])*SinIntegral[2*a*x])/(x*(Cos[a*x] + a*x*Sin[a*x]))
```

fricas [A] time = 0.90, size = 73, normalized size = 0.91

$$-\frac{2\,ax\cos\left(ax\right)\mathrm{Si}\left(2\,ax\right)+\cos\left(ax\right)^{3}+\left(2\,a^{2}x^{2}\,\mathrm{Si}\left(2\,ax\right)+2\,ax\cos\left(ax\right)^{2}-ax\right)\sin\left(ax\right)}{ax^{2}\sin\left(ax\right)+x\cos\left(ax\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(a*x)^4/x^2/(cos(a*x)+a*x*sin(a*x))^2,x, algorithm="fricas")
```

```
[Out] -(2*a*x*cos(a*x)*sin_integral(2*a*x) + cos(a*x)^3 + (2*a^2*x^2*sin_integral
(2*a*x) + 2*a*x*cos(a*x)^2 - a*x)*sin(a*x))/(a*x^2*sin(a*x) + x*cos(a*x))
```

giac [C] time = 0.49, size = 997, normalized size = 12.46

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(a*x)^4/x^2/(cos(a*x)+a*x*sin(a*x))^2,x, algorithm="giac")
```

```
[Out] -(2*a^4*x^4*imag_part(cos_integral(2*a*x))*tan(a*x)^2*tan(1/2*a*x) - 2*a^4*
x^4*imag_part(cos_integral(-2*a*x))*tan(a*x)^2*tan(1/2*a*x) + 4*a^4*x^4*sin
_integral(2*a*x)*tan(a*x)^2*tan(1/2*a*x) - a^3*x^3*imag_part(cos_integral(2
*a*x))*tan(a*x)^2*tan(1/2*a*x)^2 + a^3*x^3*imag_part(cos_integral(-2*a*x))*
tan(a*x)^2*tan(1/2*a*x)^2 - 2*a^3*x^3*sin_integral(2*a*x)*tan(a*x)^2*tan(1/
2*a*x)^2 + 2*a^4*x^4*imag_part(cos_integral(2*a*x))*tan(1/2*a*x) - 2*a^4*x^
4*imag_part(cos_integral(-2*a*x))*tan(1/2*a*x) + 4*a^4*x^4*sin_integral(2*a
*x)*tan(1/2*a*x) + a^3*x^3*imag_part(cos_integral(2*a*x))*tan(a*x)^2 - a^3*
x^3*imag_part(cos_integral(-2*a*x))*tan(a*x)^2 + 2*a^3*x^3*sin_integral(2*a
*x)*tan(a*x)^2 - 2*a^3*x^3*tan(a*x)^2*tan(1/2*a*x) - a^3*x^3*imag_part(cos_
integral(2*a*x))*tan(1/2*a*x)^2 + a^3*x^3*imag_part(cos_integral(-2*a*x))*t
an(1/2*a*x)^2 - 2*a^3*x^3*sin_integral(2*a*x)*tan(1/2*a*x)^2 + 2*a^2*x^2*im
ag_part(cos_integral(2*a*x))*tan(a*x)^2*tan(1/2*a*x) - 2*a^2*x^2*imag_part(
cos_integral(-2*a*x))*tan(a*x)^2*tan(1/2*a*x) + 4*a^2*x^2*sin_integral(2*a*
x)*tan(a*x)^2*tan(1/2*a*x) + a^2*x^2*tan(a*x)^2*tan(1/2*a*x)^2 + a^3*x^3*im
ag_part(cos_integral(2*a*x)) - a^3*x^3*imag_part(cos_integral(-2*a*x)) + 2*
a^3*x^3*sin_integral(2*a*x) + 2*a^3*x^3*tan(1/2*a*x) - a*x*imag_part(cos_in
tegral(2*a*x))*tan(a*x)^2*tan(1/2*a*x)^2 + a*x*imag_part(cos_integral(-2*a*
x))*tan(a*x)^2*tan(1/2*a*x)^2 - 2*a*x*sin_integral(2*a*x)*tan(a*x)^2*tan(1/
2*a*x)^2 - a^2*x^2*tan(a*x)^2 + 2*a^2*x^2*imag_part(cos_integral(2*a*x))*ta
n(1/2*a*x) - 2*a^2*x^2*imag_part(cos_integral(-2*a*x))*tan(1/2*a*x) + 4*a^2
*x^2*sin_integral(2*a*x)*tan(1/2*a*x) + 2*a^2*x^2*tan(a*x)*tan(1/2*a*x) - a
```

```
^2*x^2*tan(1/2*a*x)^2 + a*x*imag_part(cos_integral(2*a*x))*tan(a*x)^2 - a*x
*imag_part(cos_integral(-2*a*x))*tan(a*x)^2 + 2*a*x*sin_integral(2*a*x)*tan
(a*x)^2 - 2*a*x*tan(a*x)^2*tan(1/2*a*x) - a*x*imag_part(cos_integral(2*a*x)
)*tan(1/2*a*x)^2 + a*x*imag_part(cos_integral(-2*a*x))*tan(1/2*a*x)^2 - 2*a
*x*sin_integral(2*a*x)*tan(1/2*a*x)^2 - a*x*tan(a*x)*tan(1/2*a*x)^2 + a^2*x
^2 + a*x*imag_part(cos_integral(2*a*x)) - a*x*imag_part(cos_integral(-2*a*x
)) + 2*a*x*sin_integral(2*a*x) + a*x*tan(a*x) - tan(1/2*a*x)^2 + 1)/(2*a^3*
x^4*tan(a*x)^2*tan(1/2*a*x) - a^2*x^3*tan(a*x)^2*tan(1/2*a*x)^2 + 2*a^3*x^4
*tan(1/2*a*x) + a^2*x^3*tan(a*x)^2 - a^2*x^3*tan(1/2*a*x)^2 + 2*a*x^2*tan(a
*x)^2*tan(1/2*a*x) + a^2*x^3 - x*tan(a*x)^2*tan(1/2*a*x)^2 + 2*a*x^2*tan(1/
2*a*x) + x*tan(a*x)^2 - x*tan(1/2*a*x)^2 + x)
```

maple [**F(-1)**] time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{\cos^4(ax)}{x^2(\cos(ax)+ax\sin(ax))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a*x)^4/x^2/(cos(a*x)+a*x*sin(a*x))^2,x)
[Out] int(cos(a*x)^4/x^2/(cos(a*x)+a*x*sin(a*x))^2,x)
```

maxima [**F(-2)**] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(a*x)^4/x^2/(cos(a*x)+a*x*sin(a*x))^2,x, algorithm="maxima")
[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is und
efined.
```

mupad [**F**] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos(a\,x)^4}{x^2\,(\cos(a\,x)+a\,x\,\sin(a\,x))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a*x)^4/(x^2*(cos(a*x) + a*x*sin(a*x))^2),x)
[Out] int(cos(a*x)^4/(x^2*(cos(a*x) + a*x*sin(a*x))^2), x)
```

sympy [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos^4(ax)}{x^2(ax\sin(ax)+\cos(ax))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(a*x)**4/x**2/(cos(a*x)+a*x*sin(a*x))**2,x)
```

```
[Out] Integral(cos(a*x)**4/(x**2*(a*x*sin(a*x) + cos(a*x))**2), x)
```

3.597 $\int \frac{\cos^3(ax)}{x(\cos(ax)+ax\sin(ax))^2} dx$

Optimal. Leaf size=56

$$\frac{\cos(ax)}{a^2x^2} - \frac{\cos^2(ax)}{a^2x^2(ax\sin(ax)+\cos(ax))} + \text{Ci}(ax) - \frac{\sin(ax)}{ax}$$

```
[Out] Ci(a*x)+cos(a*x)/a^2/x^2-sin(a*x)/a/x-cos(a*x)^2/a^2/x^2/(cos(a*x)+a*x*sin(
a*x))
```

Rubi [A] time = 0.09, antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {4599, 3297, 3302}

$$\frac{\cos(ax)}{a^2x^2} - \frac{\cos^2(ax)}{a^2x^2(ax\sin(ax)+\cos(ax))} + \text{CosIntegral}(ax) - \frac{\sin(ax)}{ax}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a*x]^3/(x*(Cos[a*x] + a*x*Sin[a*x])^2),x]
```

```
[Out] Cos[a*x]/(a^2*x^2) + CosIntegral[a*x] - Sin[a*x]/(a*x) - Cos[a*x]^2/(a^2*x^
2*(Cos[a*x] + a*x*Sin[a*x]))
```

Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[((
c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3302

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 4599

```
Int[(Cos[(a_.)*(x_)]^(n_)*((b_.)*(x_))^(m_))/(Cos[(a_.)*(x_)]*(c_.) + (d_.)
*(x_)*Sin[(a_.)*(x_)])^2, x_Symbol] :> -Simp[(b*(b*x)^(m - 1)*Cos[a*x]^(n -
 1))/(a*d*(c*Cos[a*x] + d*x*Sin[a*x])), x] - Dist[(b^2*(n - 1))/d^2, Int[(b
*x)^(m - 2)*Cos[a*x]^(n - 2), x], x] /; FreeQ[{a, b, c, d, m, n}, x] && EqQ
[a*c - d, 0] && EqQ[m, 2 - n]
```

Rubi steps

$$\int \frac{\cos^3(ax)}{x(\cos(ax)+ax\sin(ax))^2}\,dx = -\frac{\cos^2(ax)}{a^2x^2(\cos(ax)+ax\sin(ax))} - \frac{2\int \frac{\cos(ax)}{x^3}\,dx}{a^2}$$
$$= \frac{\cos(ax)}{a^2x^2} - \frac{\cos^2(ax)}{a^2x^2(\cos(ax)+ax\sin(ax))} + \frac{\int \frac{\sin(ax)}{x^2}\,dx}{a}$$
$$= \frac{\cos(ax)}{a^2x^2} - \frac{\sin(ax)}{ax} - \frac{\cos^2(ax)}{a^2x^2(\cos(ax)+ax\sin(ax))} + \int \frac{\cos(ax)}{x}\,dx$$
$$= \frac{\cos(ax)}{a^2x^2} + \mathrm{Ci}(ax) - \frac{\sin(ax)}{ax} - \frac{\cos^2(ax)}{a^2x^2(\cos(ax)+ax\sin(ax))}$$

Mathematica [C] time = 7.40, size = 237, normalized size = 4.23

$$-eax\mathrm{Ci}(ax+i)\sin(ax) - e\mathrm{Ci}(ax+i)\cos(ax) + 2\mathrm{Ci}(ax)(ax\sin(ax)+\cos(ax)) - e\mathrm{Ci}(i-ax)(ax\sin(ax)+\cos(ax))$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[a*x]^3/(x*(Cos[a*x] + a*x*Sin[a*x])^2),x]
```

```
[Out] (-1 + Cos[2*a*x] - E*Cos[a*x]*CosIntegral[I + a*x] + E*Cos[a*x]*ExpIntegral
Ei[-1 - I*a*x] + E*Cos[a*x]*ExpIntegralEi[-1 + I*a*x] - a*E*x*CosIntegral[I
 + a*x]*Sin[a*x] + a*E*x*ExpIntegralEi[-1 - I*a*x]*Sin[a*x] + a*E*x*ExpInte
gralEi[-1 + I*a*x]*Sin[a*x] + 2*CosIntegral[a*x]*(Cos[a*x] + a*x*Sin[a*x])
- E*CosIntegral[I - a*x]*(Cos[a*x] + a*x*Sin[a*x]) - I*E*Cos[a*x]*SinIntegr
al[I - a*x] - I*a*E*x*Sin[a*x]*SinIntegral[I - a*x] - I*E*Cos[a*x]*SinInteg
ral[I + a*x] - I*a*E*x*Sin[a*x]*SinIntegral[I + a*x])/(2*(Cos[a*x] + a*x*Si
n[a*x]))
```

fricas [A] time = 0.95, size = 62, normalized size = 1.11

$$\frac{(\mathrm{Ci}\,(ax) + \mathrm{Ci}\,(-ax))\cos\,(ax) + 2\,\cos\,(ax)^2 + (ax\,\mathrm{Ci}\,(ax) + ax\,\mathrm{Ci}\,(-ax))\sin\,(ax) - 2}{2\,(ax\sin\,(ax) + \cos\,(ax))}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(a*x)^3/x/(cos(a*x)+a*x*sin(a*x))^2,x, algorithm="fricas")
```

```
[Out] 1/2*((cos_integral(a*x) + cos_integral(-a*x))*cos(a*x) + 2*cos(a*x)^2 + (a*
x*cos_integral(a*x) + a*x*cos_integral(-a*x))*sin(a*x) - 2)/(a*x*sin(a*x) +
 cos(a*x))
```

giac [C] time = 0.35, size = 366, normalized size = 6.54

$$\frac{2\,a^3x^3\Re\left(\mathrm{Ci}\left(ax\right)\right)\tan\left(\frac{1}{2}\,ax\right)^3+2\,a^3x^3\Re\left(\mathrm{Ci}\left(-ax\right)\right)\tan\left(\frac{1}{2}\,ax\right)^3-a^2x^2\Re\left(\mathrm{Ci}\left(ax\right)\right)\tan\left(\frac{1}{2}\,ax\right)^4-a^2x^2\Re\left(\mathrm{Ci}\left(-ax\right)\right)}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(a*x)^3/x/(cos(a*x)+a*x*sin(a*x))^2,x, algorithm="giac")
```

```
[Out] 1/2*(2*a^3*x^3*real_part(cos_integral(a*x))*tan(1/2*a*x)^3 + 2*a^3*x^3*real
_part(cos_integral(-a*x))*tan(1/2*a*x)^3 - a^2*x^2*real_part(cos_integral(a
*x))*tan(1/2*a*x)^4 - a^2*x^2*real_part(cos_integral(-a*x))*tan(1/2*a*x)^4
+ 2*a^3*x^3*real_part(cos_integral(a*x))*tan(1/2*a*x) + 2*a^3*x^3*real_part
(cos_integral(-a*x))*tan(1/2*a*x) - 8*a^2*x^2*tan(1/2*a*x)^2 + 2*a*x*real_p
art(cos_integral(a*x))*tan(1/2*a*x)^3 + 2*a*x*real_part(cos_integral(-a*x))
*tan(1/2*a*x)^3 + a^2*x^2*real_part(cos_integral(a*x)) + a^2*x^2*real_part(
cos_integral(-a*x)) - real_part(cos_integral(a*x))*tan(1/2*a*x)^4 - real_pa
rt(cos_integral(-a*x))*tan(1/2*a*x)^4 + 2*a*x*real_part(cos_integral(a*x))*
tan(1/2*a*x) + 2*a*x*real_part(cos_integral(-a*x))*tan(1/2*a*x) - 2*tan(1/2
*a*x)^4 - 12*tan(1/2*a*x)^2 + real_part(cos_integral(a*x)) + real_part(cos_
integral(-a*x)) - 2)/(2*a^3*x^3*tan(1/2*a*x)^3 - a^2*x^2*tan(1/2*a*x)^4 + 2
*a^3*x^3*tan(1/2*a*x) + 2*a*x*tan(1/2*a*x)^3 + a^2*x^2 - tan(1/2*a*x)^4 + 2
*a*x*tan(1/2*a*x) + 1)
```

maple [C] time = 5.14, size = 106, normalized size = 1.89

$$-\frac{\mathrm{Ei}\left(1,-iax\right)}{2}-\frac{\mathrm{e}^{iax}}{2\left(iax-1\right)}+\frac{\mathrm{e}^{-iax}}{2iax+2}-\frac{\mathrm{Ei}\left(1,iax\right)}{2}-\frac{2i\mathrm{e}^{iax}}{\left(ax+i\right)\left(ax-i\right)\left(ax\,\mathrm{e}^{2iax}-ax+i\mathrm{e}^{2iax}+i\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a*x)^3/x/(cos(a*x)+a*x*sin(a*x))^2,x)
```

```
[Out] -1/2*Ei(1,-I*a*x)-1/2*exp(I*a*x)/(-1+I*a*x)+1/2*exp(-I*a*x)/(I*a*x+1)-1/2*E
i(1,I*a*x)-2*I*exp(I*a*x)/(a*x+I)/(a*x-I)/(a*x*exp(2*I*a*x)-a*x+I*exp(2*I*a
*x)+I)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(a*x)^3/x/(cos(a*x)+a*x*sin(a*x))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is und
efined.
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\cos(a\,x)^3}{x\,(\cos(a\,x)+a\,x\,\sin(a\,x))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a*x)^3/(x*(cos(a*x) + a*x*sin(a*x))^2),x)
```

```
[Out] int(cos(a*x)^3/(x*(cos(a*x) + a*x*sin(a*x))^2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos^3(ax)}{x\,(ax\sin(ax)+\cos(ax))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(a*x)**3/x/(cos(a*x)+a*x*sin(a*x))**2,x)
```

```
[Out] Integral(cos(a*x)**3/(x*(a*x*sin(a*x) + cos(a*x))**2), x)
```

3.598 $\int \frac{\cos^2(ax)}{(\cos(ax)+ax\sin(ax))^2}\,dx$

Optimal. Leaf size=34

$$\frac{1}{a^2x}-\frac{\cos(ax)}{a^2x(ax\sin(ax)+\cos(ax))}$$

```
[Out] 1/a^2/x-cos(a*x)/a^2/x/(cos(a*x)+a*x*sin(a*x))
```

Rubi [A] time = 0.02, antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.048, Rules used = {4597}

$$\frac{1}{a^2x}-\frac{\cos(ax)}{a^2x(ax\sin(ax)+\cos(ax))}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a*x]^2/(Cos[a*x] + a*x*Sin[a*x])^2,x]
```

```
[Out] 1/(a^2*x) - Cos[a*x]/(a^2*x*(Cos[a*x] + a*x*Sin[a*x]))
```

Rule 4597

```
Int[Cos[(a_.)*(x_)]^2/(Cos[(a_.)*(x_)]*(c_.) + (d_.)*(x_)*Sin[(a_.)*(x_)])^
2, x_Symbol] :> Simp[1/(d^2*x), x] - Simp[Cos[a*x]/(a*d*x*(d*x*Sin[a*x] + c
*Cos[a*x])), x] /; FreeQ[{a, c, d}, x] && EqQ[a*c - d, 0]
```

Rubi steps

$$\int \frac{\cos^2(ax)}{(\cos(ax)+ax\sin(ax))^2}\,dx=\frac{1}{a^2x}-\frac{\cos(ax)}{a^2x(\cos(ax)+ax\sin(ax))}$$

Mathematica [A] time = 0.21, size = 22, normalized size = 0.65

$$\frac{\sin(ax)}{a(ax\sin(ax)+\cos(ax))}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a*x]^2/(Cos[a*x] + a*x*Sin[a*x])^2,x]
```

```
[Out] Sin[a*x]/(a*(Cos[a*x] + a*x*Sin[a*x]))
```

fricas [A] time = 3.57, size = 23, normalized size = 0.68

$$\frac{\sin(ax)}{a^2x\sin(ax) + a\cos(ax)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(a*x)^2/(cos(a*x)+a*x*sin(a*x))^2,x, algorithm="fricas")
```

```
[Out] sin(a*x)/(a^2*x*sin(a*x) + a*cos(a*x))
```

giac [A] time = 0.17, size = 32, normalized size = 0.94

$$\frac{2\tan\left(\frac{1}{2}ax\right)}{2a^2x\tan\left(\frac{1}{2}ax\right) - a\tan\left(\frac{1}{2}ax\right)^2 + a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(a*x)^2/(cos(a*x)+a*x*sin(a*x))^2,x, algorithm="giac")
```

```
[Out] 2*tan(1/2*a*x)/(2*a^2*x*tan(1/2*a*x) - a*tan(1/2*a*x)^2 + a)
```

maple [B] time = 1.79, size = 70, normalized size = 2.06

$$\frac{\frac{2\tan\left(\frac{ax}{2}\right)}{a} + \frac{4\left(\tan^3\left(\frac{ax}{2}\right)\right)}{a} + \frac{2\left(\tan^5\left(\frac{ax}{2}\right)\right)}{a}}{\left(1+\tan^2\left(\frac{ax}{2}\right)\right)^2\left(2\tan\left(\frac{ax}{2}\right)xa - \left(\tan^2\left(\frac{ax}{2}\right)\right) + 1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a*x)^2/(cos(a*x)+a*x*sin(a*x))^2,x)
```

```
[Out] (2/a*tan(1/2*a*x)+4/a*tan(1/2*a*x)^3+2/a*tan(1/2*a*x)^5)/(1+tan(1/2*a*x)^2)
^2/(2*tan(1/2*a*x)*x*a-tan(1/2*a*x)^2+1)
```

maxima [B] time = 0.33, size = 114, normalized size = 3.35

$$\frac{ax\cos(2ax)^2 + ax\sin(2ax)^2 - 2ax\cos(2ax) + ax + 2\sin(2ax)}{\left(a^2x^2 + \left(a^2x^2+1\right)\cos(2ax)^2 + 4ax\sin(2ax) + \left(a^2x^2+1\right)\sin(2ax)^2 - 2\left(a^2x^2-1\right)\cos(2ax) + 1\right)a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(a*x)^2/(cos(a*x)+a*x*sin(a*x))^2,x, algorithm="maxima")
```

```
[Out] (a*x*cos(2*a*x)^2 + a*x*sin(2*a*x)^2 - 2*a*x*cos(2*a*x) + a*x + 2*sin(2*a*x
))/((a^2*x^2 + (a^2*x^2 + 1)*cos(2*a*x)^2 + 4*a*x*sin(2*a*x) + (a^2*x^2 + 1
)*sin(2*a*x)^2 - 2*(a^2*x^2 - 1)*cos(2*a*x) + 1)*a)
```

mupad [B] time = 0.16, size = 22, normalized size = 0.65

$$\frac{\sin(a\,x)}{a\,\left(\cos(a\,x)+a\,x\,\sin(a\,x)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a*x)^2/(cos(a*x) + a*x*sin(a*x))^2,x)
[Out] sin(a*x)/(a*(cos(a*x) + a*x*sin(a*x)))
```

sympy [A] time = 3.00, size = 20, normalized size = 0.59

$$\frac{\sin(ax)}{a^2x\sin(ax)+a\cos(ax)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(a*x)**2/(cos(a*x)+a*x*sin(a*x))**2,x)
[Out] sin(a*x)/(a**2*x*sin(a*x) + a*cos(a*x))
```

3.599 $\int \frac{x\cos(ax)}{(\cos(ax)+ax\sin(ax))^2}\,dx$

Optimal. Leaf size=19

$$-\frac{1}{a^2(ax\sin(ax)+\cos(ax))}$$

```
[Out] -1/a^2/(cos(a*x)+a*x*sin(a*x))
```

Rubi [A] time = 0.06, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.050, Rules used = {6686}

$$-\frac{1}{a^2(ax\sin(ax)+\cos(ax))}$$

Antiderivative was successfully verified.

```
[In] Int[(x*Cos[a*x])/(Cos[a*x] + a*x*Sin[a*x])^2,x]
```

```
[Out] -(1/(a^2*(Cos[a*x] + a*x*Sin[a*x])))
```

Rule 6686

```
Int[(u_)*(y_)^(m_.), x_Symbol] :> With[{q = DerivativeDivides[y, u, x]}, Si
mp[(q*y^(m + 1))/(m + 1), x] /;  !FalseQ[q]] /; FreeQ[m, x] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{x\cos(ax)}{(\cos(ax)+ax\sin(ax))^2}\,dx=-\frac{1}{a^2(\cos(ax)+ax\sin(ax))}$$

Mathematica [A] time = 0.02, size = 19, normalized size = 1.00

$$-\frac{1}{a^2(ax\sin(ax)+\cos(ax))}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*Cos[a*x])/(Cos[a*x] + a*x*Sin[a*x])^2,x]
```

```
[Out] -(1/(a^2*(Cos[a*x] + a*x*Sin[a*x])))
```

fricas [A] time = 0.86, size = 22, normalized size = 1.16

$$-\frac{1}{a^3x\sin(ax)+a^2\cos(ax)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(a*x)/(cos(a*x)+a*x*sin(a*x))^2,x, algorithm="fricas")
```

```
[Out] -1/(a^3*x*sin(a*x) + a^2*cos(a*x))
```

giac [B] time = 0.18, size = 40, normalized size = 2.11

$$-\frac{2\left(\tan\left(\frac{1}{2}\,ax\right)^{2}+1\right)}{2\,a^{3}x\tan\left(\frac{1}{2}\,ax\right)-a^{2}\tan\left(\frac{1}{2}\,ax\right)^{2}+a^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(a*x)/(cos(a*x)+a*x*sin(a*x))^2,x, algorithm="giac")
```

```
[Out] -2*(tan(1/2*a*x)^2 + 1)/(2*a^3*x*tan(1/2*a*x) - a^2*tan(1/2*a*x)^2 + a^2)
```

maple [A] time = 0.24, size = 20, normalized size = 1.05

$$-\frac{1}{a^{2}\left(\cos\left(ax\right)+ax\sin\left(ax\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*cos(a*x)/(cos(a*x)+a*x*sin(a*x))^2,x)
```

```
[Out] -1/a^2/(cos(a*x)+a*x*sin(a*x))
```

maxima [A] time = 0.31, size = 19, normalized size = 1.00

$$-\frac{1}{\left(ax\sin\left(ax\right)+\cos\left(ax\right)\right)a^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(a*x)/(cos(a*x)+a*x*sin(a*x))^2,x, algorithm="maxima")
```

```
[Out] -1/((a*x*sin(a*x) + cos(a*x))*a^2)
```

mupad [B] time = 0.09, size = 22, normalized size = 1.16

$$-\frac{1}{a^{2}\,\cos\left(a\,x\right)+a^{3}\,x\,\sin\left(a\,x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*cos(a*x))/(cos(a*x) + a*x*sin(a*x))^2,x)
```

```
[Out] -1/(a^2*cos(a*x) + a^3*x*sin(a*x))
```

sympy [A] time = 3.01, size = 20, normalized size = 1.05

$$-\frac{1}{a^3 x \sin(ax) + a^2 \cos(ax)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(a*x)/(cos(a*x)+a*x*sin(a*x))**2,x)
```

```
[Out] -1/(a**3*x*sin(a*x) + a**2*cos(a*x))
```

3.600 $\int \frac{x^2}{(\cos(ax)+ax\sin(ax))^2} dx$

Optimal. Leaf size=33

$$\frac{\tan(ax)}{a^3} - \frac{x\sec(ax)}{a^2(ax\sin(ax)+\cos(ax))}$$

```
[Out] -x*sec(a*x)/a^2/(cos(a*x)+a*x*sin(a*x))+tan(a*x)/a^3
```

Rubi [A] time = 0.04, antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {4595, 3767, 8}

$$\frac{\tan(ax)}{a^3} - \frac{x\sec(ax)}{a^2(ax\sin(ax)+\cos(ax))}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(Cos[a*x] + a*x*Sin[a*x])^2,x]
```

```
[Out] -((x*Sec[a*x])/(a^2*(Cos[a*x] + a*x*Sin[a*x]))) + Tan[a*x]/a^3
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 4595

```
Int[(x_)^2/(Cos[(a_.)*(x_)]*(c_.) + (d_.)*(x_)*Sin[(a_.)*(x_)])^2, x_Symbol
] :> -Simp[x/(a*d*Cos[a*x]*(c*Cos[a*x] + d*x*Sin[a*x])), x] + Dist[1/d^2, I
nt[1/Cos[a*x]^2, x], x] /; FreeQ[{a, c, d}, x] && EqQ[a*c - d, 0]
```

Rubi steps

$$\int \frac{x^2}{(\cos(ax)+ax\sin(ax))^2}\,dx = -\frac{x\sec(ax)}{a^2(\cos(ax)+ax\sin(ax))} + \frac{\int \sec^2(ax)\,dx}{a^2}$$
$$= -\frac{x\sec(ax)}{a^2(\cos(ax)+ax\sin(ax))} - \frac{\text{Subst}(\int 1\,dx, x, -\tan(ax))}{a^3}$$
$$= -\frac{x\sec(ax)}{a^2(\cos(ax)+ax\sin(ax))} + \frac{\tan(ax)}{a^3}$$

Mathematica [**A**] time = 0.41, size = 31, normalized size = 0.94

$$\frac{\sin(ax)-ax\cos(ax)}{a^3(ax\sin(ax)+\cos(ax))}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(Cos[a*x] + a*x*Sin[a*x])^2,x]
[Out] (-(a*x*Cos[a*x]) + Sin[a*x])/(a^3*(Cos[a*x] + a*x*Sin[a*x]))
```

fricas [**A**] time = 1.85, size = 36, normalized size = 1.09

$$-\frac{ax\cos(ax)-\sin(ax)}{a^4x\sin(ax)+a^3\cos(ax)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(cos(a*x)+a*x*sin(a*x))^2,x, algorithm="fricas")
[Out] -(a*x*cos(a*x) - sin(a*x))/(a^4*x*sin(a*x) + a^3*cos(a*x))
```

giac [**A**] time = 0.15, size = 52, normalized size = 1.58

$$\frac{ax\tan\left(\frac{1}{2}ax\right)^2-ax+2\tan\left(\frac{1}{2}ax\right)}{2a^4x\tan\left(\frac{1}{2}ax\right)-a^3\tan\left(\frac{1}{2}ax\right)^2+a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(cos(a*x)+a*x*sin(a*x))^2,x, algorithm="giac")
[Out] (a*x*tan(1/2*a*x)^2 - a*x + 2*tan(1/2*a*x))/(2*a^4*x*tan(1/2*a*x) - a^3*tan
(1/2*a*x)^2 + a^3)
```

maple [A] time = 1.19, size = 53, normalized size = 1.61

$$\frac{\frac{x\left(\tan^{2}\left(\frac{ax}{2}\right)\right)}{a^{2}}-\frac{x}{a^{2}}+\frac{2\tan\left(\frac{ax}{2}\right)}{a^{3}}}{2\tan\left(\frac{ax}{2}\right)xa-\left(\tan^{2}\left(\frac{ax}{2}\right)\right)+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(cos(a*x)+a*x*sin(a*x))^2,x)
```

```
[Out] (x/a^2*tan(1/2*a*x)^2-x/a^2+2/a^3*tan(1/2*a*x))/(2*tan(1/2*a*x)*x*a-tan(1/2
*a*x)^2+1)
```

maxima [B] time = 0.31, size = 100, normalized size = 3.03

$$-\frac{2\left(2\,ax\cos\left(2\,ax\right)+\left(a^{2}x^{2}-1\right)\sin\left(2\,ax\right)\right)}{\left(a^{2}x^{2}+\left(a^{2}x^{2}+1\right)\cos\left(2\,ax\right)^{2}+4\,ax\sin\left(2\,ax\right)+\left(a^{2}x^{2}+1\right)\sin\left(2\,ax\right)^{2}-2\left(a^{2}x^{2}-1\right)\cos\left(2\,ax\right)+1\right)a^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(cos(a*x)+a*x*sin(a*x))^2,x, algorithm="maxima")
```

```
[Out] -2*(2*a*x*cos(2*a*x) + (a^2*x^2 - 1)*sin(2*a*x))/((a^2*x^2 + (a^2*x^2 + 1)*
cos(2*a*x)^2 + 4*a*x*sin(2*a*x) + (a^2*x^2 + 1)*sin(2*a*x)^2 - 2*(a^2*x^2 -
 1)*cos(2*a*x) + 1)*a^3)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{x^2}{{\left(\cos\left(a\,x\right)+a\,x\,\sin\left(a\,x\right)\right)}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(cos(a*x) + a*x*sin(a*x))^2,x)
```

```
[Out] int(x^2/(cos(a*x) + a*x*sin(a*x))^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{2}}{\left(ax\sin\left(ax\right)+\cos\left(ax\right)\right)^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(cos(a*x)+a*x*sin(a*x))**2,x)
```

```
[Out] Integral(x**2/(a*x*sin(a*x) + cos(a*x))**2, x)
```

3.601 $\int \frac{x^3 \sec(ax)}{(\cos(ax)+ax\sin(ax))^2}\,dx$

Optimal. Leaf size=110

$$\frac{i\text{Li}_2\left(-ie^{iax}\right)}{a^4}-\frac{i\text{Li}_2\left(ie^{iax}\right)}{a^4}-\frac{\sec(ax)}{a^4}-\frac{2ix\tan^{-1}\left(e^{iax}\right)}{a^3}+\frac{x\tan(ax)\sec(ax)}{a^3}-\frac{x^2\sec^2(ax)}{a^2(ax\sin(ax)+\cos(ax))}$$

```
[Out] -2*I*x*arctan(exp(I*a*x))/a^3+I*polylog(2,-I*exp(I*a*x))/a^4-I*polylog(2,I*
exp(I*a*x))/a^4-sec(a*x)/a^4-x^2*sec(a*x)^2/a^2/(cos(a*x)+a*x*sin(a*x))+x*s
ec(a*x)*tan(a*x)/a^3
```

Rubi [A] time = 0.09, antiderivative size = 110, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {4601, 4185, 4181, 2279, 2391}

$$\frac{i\text{PolyLog}\left(2,-ie^{iax}\right)}{a^4}-\frac{i\text{PolyLog}\left(2,ie^{iax}\right)}{a^4}-\frac{x^2\sec^2(ax)}{a^2(ax\sin(ax)+\cos(ax))}-\frac{2ix\tan^{-1}\left(e^{iax}\right)}{a^3}-\frac{\sec(ax)}{a^4}+\frac{x\tan(ax)\sec(ax)}{a^3}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*Sec[a*x])/(Cos[a*x] + a*x*Sin[a*x])^2,x]
```

```
[Out] ((-2*I)*x*ArcTan[E^(I*a*x)])/a^3 + (I*PolyLog[2, (-I)*E^(I*a*x)])/a^4 - (I*
PolyLog[2, I*E^(I*a*x)])/a^4 - Sec[a*x]/a^4 - (x^2*Sec[a*x]^2)/(a^2*(Cos[a*
x] + a*x*Sin[a*x])) + (x*Sec[a*x]*Tan[a*x])/a^3
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4181

```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))])/f, x] + (-Di
st[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x))
], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 4185

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_)), x_Symbol] :>
 -Simp[(b^2*(c + d*x)*Cot[e + f*x]*(b*Csc[e + f*x])^(n - 2))/(f*(n - 1)), x
] + (Dist[(b^2*(n - 2))/(n - 1), Int[(c + d*x)*(b*Csc[e + f*x])^(n - 2), x]
, x] - Simp[(b^2*d*(b*Csc[e + f*x])^(n - 2))/(f^2*(n - 1)*(n - 2)), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && NeQ[n, 2]
```

Rule 4601

```
Int[(((b_.)*(x_))^(m_.)*Sec[(a_.)*(x_)]^(n_.))/(Cos[(a_.)*(x_)]*(c_.) + (d_
.)*(x_)*Sin[(a_.)*(x_)])^2, x_Symbol] :> -Simp[(b*(b*x)^(m - 1)*Sec[a*x]^(n
 + 1))/(a*d*(c*Cos[a*x] + d*x*Sin[a*x])), x] + Dist[(b^2*(n + 1))/d^2, Int[
(b*x)^(m - 2)*Sec[a*x]^(n + 2), x], x] /; FreeQ[{a, b, c, d, m, n}, x] && E
qQ[a*c - d, 0] && EqQ[m, n + 2]
```

Rubi steps

$$\int \frac{x^3\sec(ax)}{(\cos(ax)+ax\sin(ax))^2}\,dx = -\frac{x^2\sec^2(ax)}{a^2(\cos(ax)+ax\sin(ax))}+\frac{2\int x\sec^3(ax)\,dx}{a^2}$$
$$= -\frac{\sec(ax)}{a^4}-\frac{x^2\sec^2(ax)}{a^2(\cos(ax)+ax\sin(ax))}+\frac{x\sec(ax)\tan(ax)}{a^3}+\frac{\int x\sec(ax)\,dx}{a^2}$$
$$= -\frac{2ix\tan^{-1}\left(e^{iax}\right)}{a^3}-\frac{\sec(ax)}{a^4}-\frac{x^2\sec^2(ax)}{a^2(\cos(ax)+ax\sin(ax))}+\frac{x\sec(ax)\tan(ax)}{a^3}-$$
$$= -\frac{2ix\tan^{-1}\left(e^{iax}\right)}{a^3}-\frac{\sec(ax)}{a^4}-\frac{x^2\sec^2(ax)}{a^2(\cos(ax)+ax\sin(ax))}+\frac{x\sec(ax)\tan(ax)}{a^3}+$$
$$= -\frac{2ix\tan^{-1}\left(e^{iax}\right)}{a^3}+\frac{i\mathrm{Li}_2\left(-ie^{iax}\right)}{a^4}-\frac{i\mathrm{Li}_2\left(ie^{iax}\right)}{a^4}-\frac{\sec(ax)}{a^4}-\frac{x^2\sec^2(ax)}{a^2(\cos(ax)+ax\sin}$$

Mathematica [A] time = 1.11, size = 176, normalized size = 1.60

$$-\frac{a^2x^2\sec(ax)-a^2x^2\log\left(1-ie^{iax}\right)\tan(ax)+a^2x^2\log\left(1+ie^{iax}\right)\tan(ax)-i\mathrm{Li}_2\left(-ie^{iax}\right)(ax\tan(ax)+1)+i\mathrm{Li}_2\left(\right.}{a^4(ax\tan(ax)+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*Sec[a*x])/(Cos[a*x] + a*x*Sin[a*x])^2,x]
```

```
[Out] -((-(a*x*Log[1 - I*E^(I*a*x)]) + a*x*Log[1 + I*E^(I*a*x)] + Sec[a*x] + a^2*
x^2*Sec[a*x] - a^2*x^2*Log[1 - I*E^(I*a*x)]*Tan[a*x] + a^2*x^2*Log[1 + I*E^
```

```
(I*a*x)]*Tan[a*x] - I*PolyLog[2, (-I)*E^(I*a*x)]*(1 + a*x*Tan[a*x]) + I*Pol
yLog[2, I*E^(I*a*x)]*(1 + a*x*Tan[a*x]))/(a^4*(1 + a*x*Tan[a*x])))
```

fricas [B] time = 0.99, size = 290, normalized size = 2.64

$$-\frac{2\,a^2x^2 - (-i\,ax\sin(ax) - i\,\cos(ax))\mathrm{Li}_2\,(i\,\cos(ax) + \sin(ax)) - (-i\,ax\sin(ax) - i\,\cos(ax))\mathrm{Li}_2\,(i\,\cos(ax) - \sin}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*sec(a*x)/(cos(a*x)+a*x*sin(a*x))^2,x, algorithm="fricas")
```

```
[Out] -1/2*(2*a^2*x^2 - (-I*a*x*sin(a*x) - I*cos(a*x))*dilog(I*cos(a*x) + sin(a*x
)) - (-I*a*x*sin(a*x) - I*cos(a*x))*dilog(I*cos(a*x) - sin(a*x)) - (I*a*x*s
in(a*x) + I*cos(a*x))*dilog(-I*cos(a*x) + sin(a*x)) - (I*a*x*sin(a*x) + I*c
os(a*x))*dilog(-I*cos(a*x) - sin(a*x)) - (a^2*x^2*sin(a*x) + a*x*cos(a*x))*
log(I*cos(a*x) + sin(a*x) + 1) + (a^2*x^2*sin(a*x) + a*x*cos(a*x))*log(I*co
s(a*x) - sin(a*x) + 1) - (a^2*x^2*sin(a*x) + a*x*cos(a*x))*log(-I*cos(a*x)
+ sin(a*x) + 1) + (a^2*x^2*sin(a*x) + a*x*cos(a*x))*log(-I*cos(a*x) - sin(a
*x) + 1) + 2)/(a^5*x*sin(a*x) + a^4*cos(a*x))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3 \sec(ax)}{(ax\sin(ax) + \cos(ax))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*sec(a*x)/(cos(a*x)+a*x*sin(a*x))^2,x, algorithm="giac")
```

```
[Out] integrate(x^3*sec(a*x)/(a*x*sin(a*x) + cos(a*x))^2, x)
```

maple [F] time = 2.54, size = 0, normalized size = 0.00

$$\int \frac{x^3 \sec(ax)}{(\cos(ax) + ax\sin(ax))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*sec(a*x)/(cos(a*x)+a*x*sin(a*x))^2,x)
```

```
[Out] int(x^3*sec(a*x)/(cos(a*x)+a*x*sin(a*x))^2,x)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*sec(a*x)/(cos(a*x)+a*x*sin(a*x))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is und
efined.
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^3}{\cos(a\,x)\,\left(\cos(a\,x)+a\,x\,\sin(a\,x)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(cos(a*x)*(cos(a*x) + a*x*sin(a*x))^2),x)
```

```
[Out] int(x^3/(cos(a*x)*(cos(a*x) + a*x*sin(a*x))^2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3 \sec(ax)}{(ax\sin(ax)+\cos(ax))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*sec(a*x)/(cos(a*x)+a*x*sin(a*x))**2,x)
```

```
[Out] Integral(x**3*sec(a*x)/(a*x*sin(a*x) + cos(a*x))**2, x)
```

3.602 $\int \frac{x^4 \sec^2(ax)}{(\cos(ax)+ax\sin(ax))^2} dx$

Optimal. Leaf size=124

$$-\frac{2i\text{Li}_2\left(-e^{2iax}\right)}{a^5}+\frac{\tan(ax)}{a^5}+\frac{4x\log\left(1+e^{2iax}\right)}{a^4}-\frac{x\sec^2(ax)}{a^4}-\frac{2ix^2}{a^3}+\frac{2x^2\tan(ax)}{a^3}+\frac{x^2\tan(ax)\sec^2(ax)}{a^3}-\frac{x^3\sec^3}{a^2(ax\sin(ax)}$$

```
[Out] -2*I*x^2/a^3+4*x*ln(1+exp(2*I*a*x))/a^4-2*I*polylog(2,-exp(2*I*a*x))/a^5-x*
sec(a*x)^2/a^4-x^3*sec(a*x)^3/a^2/(cos(a*x)+a*x*sin(a*x))+tan(a*x)/a^5+2*x^
2*tan(a*x)/a^3+x^2*sec(a*x)^2*tan(a*x)/a^3
```

Rubi [A] time = 0.18, antiderivative size = 124, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {4601, 4186, 3767, 8, 4184, 3719, 2190, 2279, 2391}

$$-\frac{2i\text{PolyLog}\left(2,-e^{2iax}\right)}{a^5}-\frac{2ix^2}{a^3}+\frac{2x^2\tan(ax)}{a^3}+\frac{x^2\tan(ax)\sec^2(ax)}{a^3}-\frac{x^3\sec^3(ax)}{a^2(ax\sin(ax)+\cos(ax))}+\frac{4x\log\left(1+e^{2iax}\right)}{a^4}+\cdots$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*Sec[a*x]^2)/(Cos[a*x] + a*x*Sin[a*x])^2,x]
```

```
[Out] ((-2*I)*x^2)/a^3 + (4*x*Log[1 + E^((2*I)*a*x)])/a^4 - ((2*I)*PolyLog[2, -E^
((2*I)*a*x)])/a^5 - (x*Sec[a*x]^2)/a^4 - (x^3*Sec[a*x]^3)/(a^2*(Cos[a*x] +
a*x*Sin[a*x])) + Tan[a*x]/a^5 + (2*x^2*Tan[a*x])/a^3 + (x^2*Sec[a*x]^2*Tan[
a*x])/a^3
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3719

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
I*(c + d*x)^(m + 1))/(d*(m + 1)), x] - Dist[2*I, Int[((c + d*x)^m*E^(2*I*(e
 + f*x)))/(1 + E^(2*I*(e + f*x))), x], x] /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 4184

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> -Sim
p[((c + d*x)^m*Cot[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Co
t[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 4186

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_))^(m_), x_Symbo
l] :> -Simp[(b^2*(c + d*x)^m*Cot[e + f*x]*(b*Csc[e + f*x])^(n - 2))/(f*(n -
 1)), x] + (Dist[(b^2*d^2*m*(m - 1))/(f^2*(n - 1)*(n - 2)), Int[(c + d*x)^(
m - 2)*(b*Csc[e + f*x])^(n - 2), x], x] + Dist[(b^2*(n - 2))/(n - 1), Int[(
c + d*x)^m*(b*Csc[e + f*x])^(n - 2), x], x] - Simp[(b^2*d*m*(c + d*x)^(m -
1)*(b*Csc[e + f*x])^(n - 2))/(f^2*(n - 1)*(n - 2)), x]) /; FreeQ[{b, c, d,
e, f}, x] && GtQ[n, 1] && NeQ[n, 2] && GtQ[m, 1]
```

Rule 4601

```
Int[(((b_.)*(x_))^(m_.)*Sec[(a_.)*(x_)]^(n_.))/(Cos[(a_.)*(x_)]*(c_.) + (d_
.)*(x_)*Sin[(a_.)*(x_)])^2, x_Symbol] :> -Simp[(b*(b*x)^(m - 1)*Sec[a*x]^(n
 + 1))/(a*d*(c*Cos[a*x] + d*x*Sin[a*x])), x] + Dist[(b^2*(n + 1))/d^2, Int[
(b*x)^(m - 2)*Sec[a*x]^(n + 2), x], x] /; FreeQ[{a, b, c, d, m, n}, x] && E
qQ[a*c - d, 0] && EqQ[m, n + 2]
```

Rubi steps

$$\int \frac{x^4 \sec^2(ax)}{(\cos(ax) + ax\sin(ax))^2}\,dx = -\frac{x^3 \sec^3(ax)}{a^2(\cos(ax) + ax\sin(ax))} + \frac{3\int x^2 \sec^4(ax)\,dx}{a^2}$$
$$= -\frac{x \sec^2(ax)}{a^4} - \frac{x^3 \sec^3(ax)}{a^2(\cos(ax) + ax\sin(ax))} + \frac{x^2 \sec^2(ax)\tan(ax)}{a^3} + \frac{\int \sec^2(ax)\,dx}{a^4}$$
$$= -\frac{x \sec^2(ax)}{a^4} - \frac{x^3 \sec^3(ax)}{a^2(\cos(ax) + ax\sin(ax))} + \frac{2x^2 \tan(ax)}{a^3} + \frac{x^2 \sec^2(ax)\tan(ax)}{a^3} -$$
$$= -\frac{2ix^2}{a^3} - \frac{x \sec^2(ax)}{a^4} - \frac{x^3 \sec^3(ax)}{a^2(\cos(ax) + ax\sin(ax))} + \frac{\tan(ax)}{a^5} + \frac{2x^2 \tan(ax)}{a^3} +$$
$$= -\frac{2ix^2}{a^3} + \frac{4x\log\left(1 + e^{2iax}\right)}{a^4} - \frac{x \sec^2(ax)}{a^4} - \frac{x^3 \sec^3(ax)}{a^2(\cos(ax) + ax\sin(ax))} + \frac{\tan(ax)}{a^5} +$$
$$= -\frac{2ix^2}{a^3} + \frac{4x\log\left(1 + e^{2iax}\right)}{a^4} - \frac{x \sec^2(ax)}{a^4} - \frac{x^3 \sec^3(ax)}{a^2(\cos(ax) + ax\sin(ax))} + \frac{\tan(ax)}{a^5} +$$
$$= -\frac{2ix^2}{a^3} + \frac{4x\log\left(1 + e^{2iax}\right)}{a^4} - \frac{2i\text{Li}_2\left(-e^{2iax}\right)}{a^5} - \frac{x \sec^2(ax)}{a^4} - \frac{x^3 \sec^3(ax)}{a^2(\cos(ax) + ax\sin(a}$$

Mathematica [A] time = 1.07, size = 130, normalized size = 1.05

$$\frac{a^3x^3\tan^2(ax) - ax\left(a^2x^2 + 2iax - 4\log\left(1 + e^{2iax}\right) + 1\right) + \left(-2ia^3x^3 + 2a^2x^2 + 4a^2x^2\log\left(1 + e^{2iax}\right) + 1\right)\tan(ax) -}{a^5(ax\tan(ax) + 1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^4*Sec[a*x]^2)/(Cos[a*x] + a*x*Sin[a*x])^2,x]
```

```
[Out] (-(a*x*(1 + (2*I)*a*x + a^2*x^2 - 4*Log[1 + E^((2*I)*a*x)])) + (1 + 2*a^2*x
^2 - (2*I)*a^3*x^3 + 4*a^2*x^2*Log[1 + E^((2*I)*a*x)])*Tan[a*x] + a^3*x^3*T
an[a*x]^2 - (2*I)*PolyLog[2, -E^((2*I)*a*x)]*(1 + a*x*Tan[a*x]))/(a^5*(1 +
a*x*Tan[a*x]))
```

fricas [B] time = 1.88, size = 378, normalized size = 3.05

$$\frac{a^3x^3 - \left(2\,a^3x^3 + ax\right)\cos(ax)^2 + \left(2\,a^2x^2 + 1\right)\cos(ax)\sin(ax) + \left(2i\,ax\cos(ax)\sin(ax) + 2i\,\cos(ax)^2\right)\text{Li}_2\left(i\,\cos(\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*sec(a*x)^2/(cos(a*x)+a*x*sin(a*x))^2,x, algorithm="fricas")
```

```
[Out] (a^3*x^3 - (2*a^3*x^3 + a*x)*cos(a*x)^2 + (2*a^2*x^2 + 1)*cos(a*x)*sin(a*x)
 + (2*I*a*x*cos(a*x)*sin(a*x) + 2*I*cos(a*x)^2)*dilog(I*cos(a*x) + sin(a*x)
) + (-2*I*a*x*cos(a*x)*sin(a*x) - 2*I*cos(a*x)^2)*dilog(I*cos(a*x) - sin(a*
x)) + (-2*I*a*x*cos(a*x)*sin(a*x) - 2*I*cos(a*x)^2)*dilog(-I*cos(a*x) + sin
(a*x)) + (2*I*a*x*cos(a*x)*sin(a*x) + 2*I*cos(a*x)^2)*dilog(-I*cos(a*x) - s
in(a*x)) + 2*(a^2*x^2*cos(a*x)*sin(a*x) + a*x*cos(a*x)^2)*log(I*cos(a*x) +
sin(a*x) + 1) + 2*(a^2*x^2*cos(a*x)*sin(a*x) + a*x*cos(a*x)^2)*log(I*cos(a*
x) - sin(a*x) + 1) + 2*(a^2*x^2*cos(a*x)*sin(a*x) + a*x*cos(a*x)^2)*log(-I*
cos(a*x) + sin(a*x) + 1) + 2*(a^2*x^2*cos(a*x)*sin(a*x) + a*x*cos(a*x)^2)*l
og(-I*cos(a*x) - sin(a*x) + 1))/(a^6*x*cos(a*x)*sin(a*x) + a^5*cos(a*x)^2)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4 \sec(ax)^2}{(ax\sin(ax)+\cos(ax))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*sec(a*x)^2/(cos(a*x)+a*x*sin(a*x))^2,x, algorithm="giac")
```

```
[Out] integrate(x^4*sec(a*x)^2/(a*x*sin(a*x) + cos(a*x))^2, x)
```

maple [A] time = 1.24, size = 141, normalized size = 1.14

$$-\frac{2i\left(-2ia^2x^2e^{2iax}+2x^3a^3-2ia^2x^2+ax\,e^{2iax}-ie^{2iax}+ax-i\right)}{\left(1+e^{2iax}\right)\left(ax\,e^{2iax}-ax+ie^{2iax}+i\right)a^5}-\frac{4ix^2}{a^3}+\frac{4x\ln\left(1+e^{2iax}\right)}{a^4}-\frac{2i\,\mathrm{polylog}\left(2,-e^{2iax}\right)}{a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*sec(a*x)^2/(cos(a*x)+a*x*sin(a*x))^2,x)
```

```
[Out] -2*I*(-2*I*a^2*x^2*exp(2*I*a*x)+2*x^3*a^3-2*I*a^2*x^2+a*x*exp(2*I*a*x)-I*ex
p(2*I*a*x)+a*x-I)/(1+exp(2*I*a*x))/(a*x*exp(2*I*a*x)-a*x+I*exp(2*I*a*x)+I)/
a^5-4*I/a^3*x^2+4*x*ln(1+exp(2*I*a*x))/a^4-2*I*polylog(2,-exp(2*I*a*x))/a^5
```

maxima [B] time = 0.46, size = 381, normalized size = 3.07

$$-\frac{2\,ax+\left(4\,a^2x^2-8i\,ax\cos(2\,ax)+8\,ax\sin(2\,ax)-4i\,ax-\left(4\,a^2x^2+4i\,ax\right)\cos(4\,ax)+4\left(-i\,a^2x^2+ax\right)\sin(4\,$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*sec(a*x)^2/(cos(a*x)+a*x*sin(a*x))^2,x, algorithm="maxima")
```

```
[Out] -(2*a*x + (4*a^2*x^2 - 8*I*a*x*cos(2*a*x) + 8*a*x*sin(2*a*x) - 4*I*a*x - (4
*a^2*x^2 + 4*I*a*x)*cos(4*a*x) + 4*(-I*a^2*x^2 + a*x)*sin(4*a*x))*arctan2(s
```

```
in(2*a*x), cos(2*a*x) + 1) + 4*(a^3*x^3 + I*a^2*x^2)*cos(4*a*x) - (-4*I*a^2
*x^2 - 2*a*x + 2*I)*cos(2*a*x) - (2*a*x - (2*a*x + 2*I)*cos(4*a*x) - 2*(I*a
*x - 1)*sin(4*a*x) - 4*I*cos(2*a*x) + 4*sin(2*a*x) - 2*I)*dilog(-e^(2*I*a*x
)) - (2*I*a^2*x^2 + 4*a*x*cos(2*a*x) + 4*I*a*x*sin(2*a*x) + 2*a*x - 2*(I*a^
2*x^2 - a*x)*cos(4*a*x) + (2*a^2*x^2 + 2*I*a*x)*sin(4*a*x))*log(cos(2*a*x)^
2 + sin(2*a*x)^2 + 2*cos(2*a*x) + 1) - (-4*I*a^3*x^3 + 4*a^2*x^2)*sin(4*a*x
) - (4*a^2*x^2 - 2*I*a*x - 2)*sin(2*a*x) - 2*I)/((I*a*x + (-I*a*x + 1)*cos(
4*a*x) + (a*x + I)*sin(4*a*x) + 2*cos(2*a*x) + 2*I*sin(2*a*x) + 1)*a^5)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^4}{\cos(a\,x)^2\,\left(\cos(a\,x)+a\,x\,\sin(a\,x)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(cos(a*x)^2*(cos(a*x) + a*x*sin(a*x))^2),x)
[Out] int(x^4/(cos(a*x)^2*(cos(a*x) + a*x*sin(a*x))^2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4\sec^2(ax)}{(ax\sin(ax)+\cos(ax))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*sec(a*x)**2/(cos(a*x)+a*x*sin(a*x))**2,x)
[Out] Integral(x**4*sec(a*x)**2/(a*x*sin(a*x) + cos(a*x))**2, x)
```

3.603 $\int \sec^4(2(a+bx))\sqrt{c\tan(a+bx)\tan(2(a+bx))}\,dx$

Optimal. Leaf size=157

$$\frac{c\tan(2a+2bx)\sec^3(2a+2bx)}{7b\sqrt{c\sec(2a+2bx)-c}}-\frac{6\tan(2a+2bx)(c\sec(2a+2bx)-c)^{3/2}}{35bc}-\frac{4\tan(2a+2bx)\sqrt{c\sec(2a+2bx)-c}}{35b}$$

```
[Out] -6/35*(-c+c*sec(2*b*x+2*a))^(3/2)*tan(2*b*x+2*a)/b/c-2/5*c*tan(2*b*x+2*a)/b
/(-c+c*sec(2*b*x+2*a))^(1/2)+1/7*c*sec(2*b*x+2*a)^3*tan(2*b*x+2*a)/b/(-c+c*
sec(2*b*x+2*a))^(1/2)-4/35*(-c+c*sec(2*b*x+2*a))^(1/2)*tan(2*b*x+2*a)/b
```

Rubi [A] time = 0.45, antiderivative size = 157, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.161, Rules used = {4397, 3803, 3800, 4001, 3792}

$$\frac{c\tan(2a+2bx)\sec^3(2a+2bx)}{7b\sqrt{c\sec(2a+2bx)-c}}-\frac{6\tan(2a+2bx)(c\sec(2a+2bx)-c)^{3/2}}{35bc}-\frac{4\tan(2a+2bx)\sqrt{c\sec(2a+2bx)-c}}{35b}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[2*(a + b*x)]^4*Sqrt[c*Tan[a + b*x]*Tan[2*(a + b*x)]],x]
```

```
[Out] (-2*c*Tan[2*a + 2*b*x])/(5*b*Sqrt[-c + c*Sec[2*a + 2*b*x]]) + (c*Sec[2*a +
2*b*x]^3*Tan[2*a + 2*b*x])/(7*b*Sqrt[-c + c*Sec[2*a + 2*b*x]]) - (4*Sqrt[-c
 + c*Sec[2*a + 2*b*x]]*Tan[2*a + 2*b*x])/(35*b) - (6*(-c + c*Sec[2*a + 2*b*
x])^(3/2)*Tan[2*a + 2*b*x])/(35*b*c)
```

Rule 3792

```
Int[csc[(e_.) + (f_.)*(x_)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[(-2*b*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e + f*x]]), x] /; Free
Q[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3800

```
Int[csc[(e_.) + (f_.)*(x_)]^3*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_),
x_Symbol] :> -Simp[(Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1))/(b*f*(m + 2)
), x] + Dist[1/(b*(m + 2)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m*(b*(m +
 1) - a*Csc[e + f*x]), x], x] /; FreeQ[{a, b, e, f, m}, x] && EqQ[a^2 - b^2
, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 3803

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[(-2*b*d*Cot[e + f*x]*(d*Csc[e + f*x])^(n - 1))/
```

```
(f*(2*n - 1)*Sqrt[a + b*Csc[e + f*x]]), x] + Dist[(2*a*d*(n - 1))/(b*(2*n -
 1)), Int[Sqrt[a + b*Csc[e + f*x]]*(d*Csc[e + f*x])^(n - 1), x], x] /; Free
Q[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[n, 1] && IntegerQ[2*n]
```

Rule 4001

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(B*Cot[e + f*x]*(a
+ b*Csc[e + f*x])^m)/(f*(m + 1)), x] + Dist[(a*B*m + A*b*(m + 1))/(b*(m + 1
)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m, x], x] /; FreeQ[{a, b, A, B, e
, f, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && NeQ[a*B*m + A*b*(m
 + 1), 0] &&  !LtQ[m, -2^(-1)]
```

Rule 4397

```
Int[u_, x_Symbol] :> Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]
```

Rubi steps

$$
\begin{aligned}
\int \sec^4(2(a+bx))\sqrt{c\tan(a+bx)\tan(2(a+bx))}\,dx &= \int \sec^4(2a+2bx)\sqrt{-c+c\sec(2a+2bx)}\,dx \\
&= \frac{c\sec^3(2a+2bx)\tan(2a+2bx)}{7b\sqrt{-c+c\sec(2a+2bx)}} - \frac{6}{7}\int \sec^3(2a+2bx)\sqrt{-c \\
&= \frac{c\sec^3(2a+2bx)\tan(2a+2bx)}{7b\sqrt{-c+c\sec(2a+2bx)}} - \frac{6(-c+c\sec(2a+2bx))^3}{35bc} \\
&= \frac{c\sec^3(2a+2bx)\tan(2a+2bx)}{7b\sqrt{-c+c\sec(2a+2bx)}} - \frac{4\sqrt{-c+c\sec(2a+2bx)}}{35b} \\
&= -\frac{2c\tan(2a+2bx)}{5b\sqrt{-c+c\sec(2a+2bx)}} + \frac{c\sec^3(2a+2bx)\tan(2a+2b}{7b\sqrt{-c+c\sec(2a+2bx)}}
\end{aligned}
$$

Mathematica [A] time = 0.22, size = 64, normalized size = 0.41

$$
-\frac{(7\cos(3(a+bx))+2\cos(7(a+bx)))\csc(a+bx)\sec^3(2(a+bx))\sqrt{c\tan(a+bx)\tan(2(a+bx))}}{35b}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[2*(a + b*x)]^4*Sqrt[c*Tan[a + b*x]*Tan[2*(a + b*x)]],x]
```

```
[Out] -1/35*((7*Cos[3*(a + b*x)] + 2*Cos[7*(a + b*x)])*Csc[a + b*x]*Sec[2*(a + b*
x)]^3*Sqrt[c*Tan[a + b*x]*Tan[2*(a + b*x)]])/b
```

fricas [A] time = 0.88, size = 106, normalized size = 0.68

$$-\frac{\sqrt{2}\left(35\tan\left(bx+a\right)^{6}-35\tan\left(bx+a\right)^{4}+49\tan\left(bx+a\right)^{2}-9\right)\sqrt{-\frac{c\tan(bx+a)^{2}}{\tan(bx+a)^{2}-1}}}{35\left(b\tan\left(bx+a\right)^{7}-3\,b\tan\left(bx+a\right)^{5}+3\,b\tan\left(bx+a\right)^{3}-b\tan\left(bx+a\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(2*b*x+2*a)^4*(c*tan(b*x+a)*tan(2*b*x+2*a))^(1/2),x, algorithm
="fricas")
```

```
[Out] -1/35*sqrt(2)*(35*tan(b*x + a)^6 - 35*tan(b*x + a)^4 + 49*tan(b*x + a)^2 -
9)*sqrt(-c*tan(b*x + a)^2/(tan(b*x + a)^2 - 1))/(b*tan(b*x + a)^7 - 3*b*tan
(b*x + a)^5 + 3*b*tan(b*x + a)^3 - b*tan(b*x + a))
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(2*b*x+2*a)^4*(c*tan(b*x+a)*tan(2*b*x+2*a))^(1/2),x, algorithm
="giac")
```

```
[Out] Timed out
```

maple [A] time = 1.59, size = 98, normalized size = 0.62

$$-\frac{\sqrt{2}\,\cos\left(bx+a\right)\sqrt{\frac{c\left(\sin^{2}(bx+a)\right)}{2\left(\cos^{2}(bx+a)\right)-1}}\,\left(128\left(\cos^{6}\left(bx+a\right)\right)-224\left(\cos^{4}\left(bx+a\right)\right)+140\left(\cos^{2}\left(bx+a\right)\right)-35\right)\sqrt{4}}{70b\sin\left(bx+a\right)\left(2\left(\cos^{2}\left(bx+a\right)\right)-1\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(2*b*x+2*a)^4*(c*tan(b*x+a)*tan(2*b*x+2*a))^(1/2),x)
```

```
[Out] -1/70*2^(1/2)/b*cos(b*x+a)*(c*sin(b*x+a)^2/(2*cos(b*x+a)^2-1))^(1/2)*(128*c
os(b*x+a)^6-224*cos(b*x+a)^4+140*cos(b*x+a)^2-35)/sin(b*x+a)/(2*cos(b*x+a)^
2-1)^3*4^(1/2)
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(2*b*x+2*a)^4*(c*tan(b*x+a)*tan(2*b*x+2*a))^(1/2),x, algorithm
="maxima")
```

```
[Out] Timed out
```

mupad [B] time = 8.88, size = 463, normalized size = 2.95

$$-\frac{e^{a\,2i+b\,x\,2i}\sqrt{\frac{c\,\left(e^{a\,2i+b\,x\,2i}\,1i-i\right)\left(e^{a\,4i+b\,x\,4i}\,1i-i\right)}{\left(e^{a\,2i+b\,x\,2i}+1\right)\left(e^{a\,4i+b\,x\,4i}+1\right)}}\;16i}{35\,b\,\left(e^{a\,2i+b\,x\,2i}-1\right)}+\frac{\left(\frac{8i}{7\,b}-\frac{e^{a\,2i+b\,x\,2i}\,8i}{7\,b}\right)\sqrt{\frac{c\,\left(e^{a\,2i+b\,x\,2i}\,1i-i\right)\left(e^{a\,4i+b\,x\,4i}\,1i-i\right)}{\left(e^{a\,2i+b\,x\,2i}+1\right)\left(e^{a\,4i+b\,x\,4i}+1\right)}}}{\left(e^{a\,2i+b\,x\,2i}-1\right)\left(e^{a\,4i+b\,x\,4i}+1\right)^3}-\frac{\left(\frac{8i}{5\,b}-\frac{e^{a\,2i+b\,x\,2i}\,64i}{35\,b}\right)\ \cdots}{\left(e^{a\,2i+b\,x\,2i}\ \cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*tan(a + b*x)*tan(2*a + 2*b*x))^(1/2)/cos(2*a + 2*b*x)^4,x)
```

```
[Out] ((8i/(7*b) - (exp(a*2i + b*x*2i)*8i)/(7*b))*((c*(exp(a*2i + b*x*2i)*1i - 1i
)*(exp(a*4i + b*x*4i)*1i - 1i))/((exp(a*2i + b*x*2i) + 1)*(exp(a*4i + b*x*4
i) + 1)))^(1/2))/((exp(a*2i + b*x*2i) - 1)*(exp(a*4i + b*x*4i) + 1)^3) - (e
xp(a*2i + b*x*2i)*((c*(exp(a*2i + b*x*2i)*1i - 1i)*(exp(a*4i + b*x*4i)*1i -
 1i))/((exp(a*2i + b*x*2i) + 1)*(exp(a*4i + b*x*4i) + 1)))^(1/2)*16i)/(35*b
*(exp(a*2i + b*x*2i) - 1)) - ((8i/(5*b) - (exp(a*2i + b*x*2i)*64i)/(35*b))*
((c*(exp(a*2i + b*x*2i)*1i - 1i)*(exp(a*4i + b*x*4i)*1i - 1i))/((exp(a*2i +
 b*x*2i) + 1)*(exp(a*4i + b*x*4i) + 1)))^(1/2))/((exp(a*2i + b*x*2i) - 1)*(
exp(a*4i + b*x*4i) + 1)^2) - (exp(a*2i + b*x*2i)*((c*(exp(a*2i + b*x*2i)*1i
 - 1i)*(exp(a*4i + b*x*4i)*1i - 1i))/((exp(a*2i + b*x*2i) + 1)*(exp(a*4i +
b*x*4i) + 1)))^(1/2)*8i)/(35*b*(exp(a*2i + b*x*2i) - 1)*(exp(a*4i + b*x*4i)
 + 1))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(2*b*x+2*a)**4*(c*tan(b*x+a)*tan(2*b*x+2*a))**(1/2),x)
```

```
[Out] Timed out
```

3.604 $\int \sec^3(2(a+bx))\sqrt{c\tan(a+bx)\tan(2(a+bx))}\,dx$

Optimal. Leaf size=110

$$\frac{\tan(2a+2bx)(c\sec(2a+2bx)-c)^{3/2}}{5bc}+\frac{2\tan(2a+2bx)\sqrt{c\sec(2a+2bx)-c}}{15b}+\frac{7c\tan(2a+2bx)}{15b\sqrt{c\sec(2a+2bx)-c}}$$

```
[Out] 1/5*(-c+c*sec(2*b*x+2*a))^(3/2)*tan(2*b*x+2*a)/b/c+7/15*c*tan(2*b*x+2*a)/b/
(-c+c*sec(2*b*x+2*a))^(1/2)+2/15*(-c+c*sec(2*b*x+2*a))^(1/2)*tan(2*b*x+2*a)
/b
```

Rubi [A] time = 0.28, antiderivative size = 110, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.129, Rules used = {4397, 3800, 4001, 3792}

$$\frac{\tan(2a+2bx)(c\sec(2a+2bx)-c)^{3/2}}{5bc}+\frac{2\tan(2a+2bx)\sqrt{c\sec(2a+2bx)-c}}{15b}+\frac{7c\tan(2a+2bx)}{15b\sqrt{c\sec(2a+2bx)-c}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[2*(a + b*x)]^3*Sqrt[c*Tan[a + b*x]*Tan[2*(a + b*x)]],x]
```

```
[Out] (7*c*Tan[2*a + 2*b*x])/(15*b*Sqrt[-c + c*Sec[2*a + 2*b*x]]) + (2*Sqrt[-c +
c*Sec[2*a + 2*b*x]]*Tan[2*a + 2*b*x])/(15*b) + ((-c + c*Sec[2*a + 2*b*x])^(
3/2)*Tan[2*a + 2*b*x])/(5*b*c)
```

Rule 3792

```
Int[csc[(e_.) + (f_.)*(x_)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[(-2*b*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e + f*x]]), x] /; Free
Q[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3800

```
Int[csc[(e_.) + (f_.)*(x_)]^3*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_),
x_Symbol] :> -Simp[(Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1))/(b*f*(m + 2)
), x] + Dist[1/(b*(m + 2)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m*(b*(m +
 1) - a*Csc[e + f*x]), x], x] /; FreeQ[{a, b, e, f, m}, x] && EqQ[a^2 - b^2
, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 4001

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(B*Cot[e + f*x]*(a
+ b*Csc[e + f*x])^m)/(f*(m + 1)), x] + Dist[(a*B*m + A*b*(m + 1))/(b*(m + 1
```

```
)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m, x], x] /; FreeQ[{a, b, A, B, e
, f, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && NeQ[a*B*m + A*b*(m
 + 1), 0] &&  !LtQ[m, -2^(-1)]
```

Rule 4397

```
Int[u_, x_Symbol] :> Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]
```

Rubi steps

$$\int \sec^3(2(a+bx))\sqrt{c\tan(a+bx)\tan(2(a+bx))}\,dx = \int \sec^3(2a+2bx)\sqrt{-c+c\sec(2a+2bx)}\,dx$$
$$= \frac{(-c+c\sec(2a+2bx))^{3/2}\tan(2a+2bx)}{5bc} + \frac{2\int \sec(2a+2b}{}$$
$$= \frac{2\sqrt{-c+c\sec(2a+2bx)}\,\tan(2a+2bx)}{15b} + \frac{(-c+c\sec(2a+}{}$$
$$= \frac{7c\tan(2a+2bx)}{15b\sqrt{-c+c\sec(2a+2bx)}} + \frac{2\sqrt{-c+c\sec(2a+2bx)}\,\tan(}{15b}$$

Mathematica [A] time = 0.18, size = 62, normalized size = 0.56

$$\frac{(5\cos(a+bx)+2\cos(5(a+bx)))\csc(a+bx)\sec^2(2(a+bx))\sqrt{c\tan(a+bx)\tan(2(a+bx))}}{15b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[2*(a + b*x)]^3*Sqrt[c*Tan[a + b*x]*Tan[2*(a + b*x)]],x]
```

```
[Out] ((5*Cos[a + b*x] + 2*Cos[5*(a + b*x)])*Csc[a + b*x]*Sec[2*(a + b*x)]^2*Sqrt
[c*Tan[a + b*x]*Tan[2*(a + b*x)]])/(15*b)
```

fricas [A] time = 1.50, size = 84, normalized size = 0.76

$$\frac{\sqrt{2}\left(15\,\tan(bx+a)^4-10\,\tan(bx+a)^2+7\right)\sqrt{-\frac{c\tan(bx+a)^2}{\tan(bx+a)^2-1}}}{15\left(b\tan(bx+a)^5-2\,b\tan(bx+a)^3+b\tan(bx+a)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(2*b*x+2*a)^3*(c*tan(b*x+a)*tan(2*b*x+2*a))^(1/2),x, algorithm
="fricas")
```

```
[Out] 1/15*sqrt(2)*(15*tan(b*x + a)^4 - 10*tan(b*x + a)^2 + 7)*sqrt(-c*tan(b*x +
a)^2/(tan(b*x + a)^2 - 1))/(b*tan(b*x + a)^5 - 2*b*tan(b*x + a)^3 + b*tan(b
*x + a))
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(2*b*x+2*a)^3*(c*tan(b*x+a)*tan(2*b*x+2*a))^(1/2),x, algorithm
="giac")
```

```
[Out] Timed out
```

maple [A] time = 1.17, size = 88, normalized size = 0.80

$$\frac{\sqrt{2}\,\sqrt{\frac{c\left(\sin^2(bx+a)\right)}{2\left(\cos^2(bx+a)\right)-1}}\,\cos\left(bx+a\right)\left(32\left(\cos^4\left(bx+a\right)\right)-40\left(\cos^2\left(bx+a\right)\right)+15\right)\sqrt{4}}{30b\sin\left(bx+a\right)\left(2\left(\cos^2\left(bx+a\right)\right)-1\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(2*b*x+2*a)^3*(c*tan(b*x+a)*tan(2*b*x+2*a))^(1/2),x)
```

```
[Out] 1/30*2^(1/2)/b*(c*sin(b*x+a)^2/(2*cos(b*x+a)^2-1))^(1/2)*cos(b*x+a)*(32*cos
(b*x+a)^4-40*cos(b*x+a)^2+15)/sin(b*x+a)/(2*cos(b*x+a)^2-1)^2*4^(1/2)
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(2*b*x+2*a)^3*(c*tan(b*x+a)*tan(2*b*x+2*a))^(1/2),x, algorithm
="maxima")
```

```
[Out] Timed out
```

mupad [B] time = 11.03, size = 148, normalized size = 1.35

$$\frac{4\,\left(\mathrm{e}^{a\,4\mathrm{i}+b\,x\,4\mathrm{i}}\,5\mathrm{i}+\mathrm{e}^{a\,6\mathrm{i}+b\,x\,6\mathrm{i}}\,5\mathrm{i}+\mathrm{e}^{a\,10\mathrm{i}+b\,x\,10\mathrm{i}}\,2\mathrm{i}+2\mathrm{i}\right)\,\sqrt{\frac{c\,\left(\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)\,\left(\mathrm{e}^{a\,4\mathrm{i}+b\,x\,4\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)}{\left(\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}+1\right)\,\left(\mathrm{e}^{a\,4\mathrm{i}+b\,x\,4\mathrm{i}}+1\right)}}}{15\,b\,\left(\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}-1\right)\,{\left(\mathrm{e}^{a\,4\mathrm{i}+b\,x\,4\mathrm{i}}+1\right)}^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*tan(a + b*x)*tan(2*a + 2*b*x))^(1/2)/cos(2*a + 2*b*x)^3,x)
```

```
[Out] (4*(exp(a*4i + b*x*4i)*5i + exp(a*6i + b*x*6i)*5i + exp(a*10i + b*x*10i)*2i
 + 2i)*((c*(exp(a*2i + b*x*2i)*1i - 1i)*(exp(a*4i + b*x*4i)*1i - 1i))/((exp
(a*2i + b*x*2i) + 1)*(exp(a*4i + b*x*4i) + 1)))^(1/2))/(15*b*(exp(a*2i + b*
x*2i) - 1)*(exp(a*4i + b*x*4i) + 1)^2)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(2*b*x+2*a)**3*(c*tan(b*x+a)*tan(2*b*x+2*a))**(1/2),x)
```

```
[Out] Timed out
```

3.605 $\int \sec^2(2(a+bx))\sqrt{c\tan(a+bx)\tan(2(a+bx))}\,dx$

Optimal. Leaf size=72

$$\frac{\tan(2a+2bx)\sqrt{c\sec(2a+2bx)-c}}{3b}-\frac{c\tan(2a+2bx)}{3b\sqrt{c\sec(2a+2bx)-c}}$$

```
[Out] -1/3*c*tan(2*b*x+2*a)/b/(-c+c*sec(2*b*x+2*a))^(1/2)+1/3*(-c+c*sec(2*b*x+2*a
))^(1/2)*tan(2*b*x+2*a)/b
```

Rubi [A] time = 0.20, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.097, Rules used = {4397, 3798, 3792}

$$\frac{\tan(2a+2bx)\sqrt{c\sec(2a+2bx)-c}}{3b}-\frac{c\tan(2a+2bx)}{3b\sqrt{c\sec(2a+2bx)-c}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[2*(a + b*x)]^2*Sqrt[c*Tan[a + b*x]*Tan[2*(a + b*x)]],x]
```

```
[Out] -(c*Tan[2*a + 2*b*x])/(3*b*Sqrt[-c + c*Sec[2*a + 2*b*x]]) + (Sqrt[-c + c*Se
c[2*a + 2*b*x]]*Tan[2*a + 2*b*x])/(3*b)
```

Rule 3792

```
Int[csc[(e_.) + (f_.)*(x_)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[(-2*b*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e + f*x]]), x] /; Free
Q[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3798

```
Int[csc[(e_.) + (f_.)*(x_)]^2*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_),
x_Symbol] :> -Simp[(Cot[e + f*x]*(a + b*Csc[e + f*x])^m)/(f*(m + 1)), x] +
Dist[(a*m)/(b*(m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m, x], x] /;
FreeQ[{a, b, e, f, m}, x] && EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 4397

```
Int[u_, x_Symbol] :> Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]
```

Rubi steps

$$\int \sec^2(2(a+bx))\sqrt{c\tan(a+bx)\tan(2(a+bx))}\,dx = \int \sec^2(2a+2bx)\sqrt{-c+c\sec(2a+2bx)}\,dx$$
$$= \frac{\sqrt{-c+c\sec(2a+2bx)}\,\tan(2a+2bx)}{3b} - \frac{1}{3}\int \sec(2a+2bx$$
$$= -\frac{c\tan(2a+2bx)}{3b\sqrt{-c+c\sec(2a+2bx)}} + \frac{\sqrt{-c+c\sec(2a+2bx)}\,\tan(2}{3b}$$

Mathematica [A] time = 0.18, size = 44, normalized size = 0.61

$$\frac{\sqrt{c\tan(a+bx)\tan(2(a+bx))}\,(\tan(2(a+bx))-\cot(a+bx))}{3b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[2*(a + b*x)]^2*Sqrt[c*Tan[a + b*x]*Tan[2*(a + b*x)]],x]
```

```
[Out] (Sqrt[c*Tan[a + b*x]*Tan[2*(a + b*x)]]*(-Cot[a + b*x] + Tan[2*(a + b*x)]))/
(3*b)
```

fricas [A] time = 0.91, size = 64, normalized size = 0.89

$$-\frac{\sqrt{2}\sqrt{-\frac{c\tan(bx+a)^2}{\tan(bx+a)^2-1}}\left(3\tan(bx+a)^2-1\right)}{3\left(b\tan(bx+a)^3-b\tan(bx+a)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(2*b*x+2*a)^2*(c*tan(b*x+a)*tan(2*b*x+2*a))^(1/2),x, algorithm
="fricas")
```

```
[Out] -1/3*sqrt(2)*sqrt(-c*tan(b*x + a)^2/(tan(b*x + a)^2 - 1))*(3*tan(b*x + a)^2
 - 1)/(b*tan(b*x + a)^3 - b*tan(b*x + a))
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(2*b*x+2*a)^2*(c*tan(b*x+a)*tan(2*b*x+2*a))^(1/2),x, algorithm
="giac")
```

```
[Out] Timed out
```

maple [A] time = 1.10, size = 78, normalized size = 1.08

$$-\frac{\sqrt{2}\,\sqrt{\frac{c\left(\sin^2(bx+a)\right)}{2\left(\cos^2(bx+a)\right)-1}}\,\cos\left(bx+a\right)\left(4\left(\cos^2\left(bx+a\right)\right)-3\right)\sqrt{4}}{6b\sin\left(bx+a\right)\left(2\left(\cos^2\left(bx+a\right)\right)-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(2*b*x+2*a)^2*(c*tan(b*x+a)*tan(2*b*x+2*a))^(1/2),x)
```

```
[Out] -1/6*2^(1/2)/b*(c*sin(b*x+a)^2/(2*cos(b*x+a)^2-1))^(1/2)*cos(b*x+a)*(4*cos(
b*x+a)^2-3)/sin(b*x+a)/(2*cos(b*x+a)^2-1)*4^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(2*b*x+2*a)^2*(c*tan(b*x+a)*tan(2*b*x+2*a))^(1/2),x, algorithm
="maxima")
```

```
[Out] -2/3*(6*(cos(4*b*x + 4*a)^2 + sin(4*b*x + 4*a)^2 + 2*cos(4*b*x + 4*a) + 1)^
(3/4)*b*sqrt(c)*integrate(-(((cos(12*b*x + 12*a)*cos(4*b*x + 4*a) + 2*cos(8
*b*x + 8*a)*cos(4*b*x + 4*a) + cos(4*b*x + 4*a)^2 + sin(12*b*x + 12*a)*sin(
4*b*x + 4*a) + 2*sin(8*b*x + 8*a)*sin(4*b*x + 4*a) + sin(4*b*x + 4*a)^2)*co
s(1/2*arctan2(sin(4*b*x + 4*a), -cos(4*b*x + 4*a) - 1)) + (cos(4*b*x + 4*a)
*sin(12*b*x + 12*a) + 2*cos(4*b*x + 4*a)*sin(8*b*x + 8*a) - cos(12*b*x + 12
*a)*sin(4*b*x + 4*a) - 2*cos(8*b*x + 8*a)*sin(4*b*x + 4*a))*sin(1/2*arctan2
(sin(4*b*x + 4*a), -cos(4*b*x + 4*a) - 1)))*cos(3/2*arctan2(sin(4*b*x + 4*a
), cos(4*b*x + 4*a))) + ((cos(4*b*x + 4*a)*sin(12*b*x + 12*a) + 2*cos(4*b*x
 + 4*a)*sin(8*b*x + 8*a) - cos(12*b*x + 12*a)*sin(4*b*x + 4*a) - 2*cos(8*b*
x + 8*a)*sin(4*b*x + 4*a))*cos(1/2*arctan2(sin(4*b*x + 4*a), -cos(4*b*x + 4
*a) - 1)) - (cos(12*b*x + 12*a)*cos(4*b*x + 4*a) + 2*cos(8*b*x + 8*a)*cos(4
*b*x + 4*a) + cos(4*b*x + 4*a)^2 + sin(12*b*x + 12*a)*sin(4*b*x + 4*a) + 2*
sin(8*b*x + 8*a)*sin(4*b*x + 4*a) + sin(4*b*x + 4*a)^2)*sin(1/2*arctan2(sin
(4*b*x + 4*a), -cos(4*b*x + 4*a) - 1)))*sin(3/2*arctan2(sin(4*b*x + 4*a), c
os(4*b*x + 4*a))))/(((2*(2*cos(8*b*x + 8*a) + cos(4*b*x + 4*a))*cos(12*b*x
+ 12*a) + cos(12*b*x + 12*a)^2 + 4*cos(8*b*x + 8*a)^2 + 4*cos(8*b*x + 8*a)*
cos(4*b*x + 4*a) + cos(4*b*x + 4*a)^2 + 2*(2*sin(8*b*x + 8*a) + sin(4*b*x +
 4*a))*sin(12*b*x + 12*a) + sin(12*b*x + 12*a)^2 + 4*sin(8*b*x + 8*a)^2 + 4
*sin(8*b*x + 8*a)*sin(4*b*x + 4*a) + sin(4*b*x + 4*a)^2)*cos(1/2*arctan2(si
n(4*b*x + 4*a), -cos(4*b*x + 4*a) - 1))^2 + (2*(2*cos(8*b*x + 8*a) + cos(4*
b*x + 4*a))*cos(12*b*x + 12*a) + cos(12*b*x + 12*a)^2 + 4*cos(8*b*x + 8*a)^
2 + 4*cos(8*b*x + 8*a)*cos(4*b*x + 4*a) + cos(4*b*x + 4*a)^2 + 2*(2*sin(8*b
```

```
*x + 8*a) + sin(4*b*x + 4*a))*sin(12*b*x + 12*a) + sin(12*b*x + 12*a)^2 + 4
*sin(8*b*x + 8*a)^2 + 4*sin(8*b*x + 8*a)*sin(4*b*x + 4*a) + sin(4*b*x + 4*a
)^2)*sin(1/2*arctan2(sin(4*b*x + 4*a), -cos(4*b*x + 4*a) - 1))^2)*(cos(4*b*
x + 4*a)^2 + sin(4*b*x + 4*a)^2 + 2*cos(4*b*x + 4*a) + 1)^(1/4)), x) + sqrt
(c)*sin(3/2*arctan2(sin(4*b*x + 4*a), -cos(4*b*x + 4*a) - 1)))/((cos(4*b*x
+ 4*a)^2 + sin(4*b*x + 4*a)^2 + 2*cos(4*b*x + 4*a) + 1)^(3/4)*b)
```

mupad [B] time = 7.34, size = 129, normalized size = 1.79

$$-\frac{2\,\left({\mathrm{e}}^{a\,6{}\mathrm{i}+b\,x\,6{}\mathrm{i}}\,1{}\mathrm{i}+1{}\mathrm{i}\right)\,\sqrt{\frac{c\,\left({\mathrm{e}}^{a\,2{}\mathrm{i}+b\,x\,2{}\mathrm{i}}\,1{}\mathrm{i}-\mathrm{i}\right)\,\left({\mathrm{e}}^{a\,4{}\mathrm{i}+b\,x\,4{}\mathrm{i}}\,1{}\mathrm{i}-\mathrm{i}\right)}{\left({\mathrm{e}}^{a\,2{}\mathrm{i}+b\,x\,2{}\mathrm{i}}+1\right)\,\left({\mathrm{e}}^{a\,4{}\mathrm{i}+b\,x\,4{}\mathrm{i}}+1\right)}}}{3\,b\,\left({\mathrm{e}}^{a\,2{}\mathrm{i}+b\,x\,2{}\mathrm{i}}-{\mathrm{e}}^{a\,4{}\mathrm{i}+b\,x\,4{}\mathrm{i}}+{\mathrm{e}}^{a\,6{}\mathrm{i}+b\,x\,6{}\mathrm{i}}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*tan(a + b*x)*tan(2*a + 2*b*x))^(1/2)/cos(2*a + 2*b*x)^2,x)
```

```
[Out] -(2*(exp(a*6i + b*x*6i)*1i + 1i)*((c*(exp(a*2i + b*x*2i)*1i - 1i)*(exp(a*4i
 + b*x*4i)*1i - 1i))/((exp(a*2i + b*x*2i) + 1)*(exp(a*4i + b*x*4i) + 1)))^(
1/2))/(3*b*(exp(a*2i + b*x*2i) - exp(a*4i + b*x*4i) + exp(a*6i + b*x*6i) -
1))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(2*b*x+2*a)**2*(c*tan(b*x+a)*tan(2*b*x+2*a))**(1/2),x)
```

```
[Out] Timed out
```

3.606 $\int \sec(2(a+bx))\sqrt{c\tan(a+bx)\tan(2(a+bx))}\,dx$

Optimal. Leaf size=33

$$\frac{c\tan(2a+2bx)}{b\sqrt{c\sec(2a+2bx)-c}}$$

```
[Out] c*tan(2*b*x+2*a)/b/(-c+c*sec(2*b*x+2*a))^(1/2)
```

Rubi [A] time = 0.06, antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.069, Rules used = {4397, 3792}

$$\frac{c\tan(2a+2bx)}{b\sqrt{c\sec(2a+2bx)-c}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[2*(a + b*x)]*Sqrt[c*Tan[a + b*x]*Tan[2*(a + b*x)]],x]
```

```
[Out] (c*Tan[2*a + 2*b*x])/(b*Sqrt[-c + c*Sec[2*a + 2*b*x]])
```

Rule 3792

```
Int[csc[(e_.) + (f_.)*(x_)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[(-2*b*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e + f*x]]), x] /; Free
Q[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 4397

```
Int[u_, x_Symbol] :> Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]
```

Rubi steps

$$\begin{aligned}\int \sec(2(a+bx))\sqrt{c\tan(a+bx)\tan(2(a+bx))}\,dx &= \int \sec(2a+2bx)\sqrt{-c+c\sec(2a+2bx)}\,dx\\ &= \frac{c\tan(2a+2bx)}{b\sqrt{-c+c\sec(2a+2bx)}}\end{aligned}$$

Mathematica [A] time = 0.08, size = 30, normalized size = 0.91

$$\frac{\cot(a+bx)\sqrt{c\tan(a+bx)\tan(2(a+bx))}}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[2*(a + b*x)]*Sqrt[c*Tan[a + b*x]*Tan[2*(a + b*x)]],x]
```

```
[Out] (Cot[a + b*x]*Sqrt[c*Tan[a + b*x]*Tan[2*(a + b*x)]])/b
```

fricas [**A**] time = 0.43, size = 40, normalized size = 1.21

$$\frac{\sqrt{2}\sqrt{-\frac{c\tan(bx+a)^2}{\tan(bx+a)^2-1}}}{b\tan(bx+a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(2*b*x+2*a)*(c*tan(b*x+a)*tan(2*b*x+2*a))^(1/2),x, algorithm="
fricas")
```

```
[Out] sqrt(2)*sqrt(-c*tan(b*x + a)^2/(tan(b*x + a)^2 - 1))/(b*tan(b*x + a))
```

giac [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(2*b*x+2*a)*(c*tan(b*x+a)*tan(2*b*x+2*a))^(1/2),x, algorithm="
giac")
```

```
[Out] Timed out
```

maple [**A**] time = 0.97, size = 52, normalized size = 1.58

$$\frac{\sqrt{2}\sqrt{\frac{c\left(\sin^2(bx+a)\right)}{2\left(\cos^2(bx+a)\right)-1}}\cos(bx+a)\sqrt{4}}{2b\sin(bx+a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(2*b*x+2*a)*(c*tan(b*x+a)*tan(2*b*x+2*a))^(1/2),x)
```

```
[Out] 1/2*2^(1/2)/b*(c*sin(b*x+a)^2/(2*cos(b*x+a)^2-1))^(1/2)*cos(b*x+a)/sin(b*x+
a)*4^(1/2)
```

maxima [**F**] time = 0.00, size = 0, normalized size = 0.00

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(2*b*x+2*a)*(c*tan(b*x+a)*tan(2*b*x+2*a))^(1/2),x, algorithm="
maxima")
```

```
[Out] (2*(cos(4*b*x + 4*a)^2 + sin(4*b*x + 4*a)^2 + 2*cos(4*b*x + 4*a) + 1)^(1/4)
*b*sqrt(c)*integrate(-(((cos(8*b*x + 8*a)*cos(4*b*x + 4*a) + cos(4*b*x + 4*
a)^2 + sin(8*b*x + 8*a)*sin(4*b*x + 4*a) + sin(4*b*x + 4*a)^2)*cos(1/2*arct
an2(sin(4*b*x + 4*a), -cos(4*b*x + 4*a) - 1)) + (cos(4*b*x + 4*a)*sin(8*b*x
 + 8*a) - cos(8*b*x + 8*a)*sin(4*b*x + 4*a))*sin(1/2*arctan2(sin(4*b*x + 4*
a), -cos(4*b*x + 4*a) - 1)))*cos(1/2*arctan2(sin(4*b*x + 4*a), cos(4*b*x +
4*a))) + ((cos(4*b*x + 4*a)*sin(8*b*x + 8*a) - cos(8*b*x + 8*a)*sin(4*b*x +
 4*a))*cos(1/2*arctan2(sin(4*b*x + 4*a), -cos(4*b*x + 4*a) - 1)) - (cos(8*b
*x + 8*a)*cos(4*b*x + 4*a) + cos(4*b*x + 4*a)^2 + sin(8*b*x + 8*a)*sin(4*b*
x + 4*a) + sin(4*b*x + 4*a)^2)*sin(1/2*arctan2(sin(4*b*x + 4*a), -cos(4*b*x
 + 4*a) - 1)))*sin(1/2*arctan2(sin(4*b*x + 4*a), cos(4*b*x + 4*a))))/(((cos
(8*b*x + 8*a)^2 + 2*cos(8*b*x + 8*a)*cos(4*b*x + 4*a) + cos(4*b*x + 4*a)^2
+ sin(8*b*x + 8*a)^2 + 2*sin(8*b*x + 8*a)*sin(4*b*x + 4*a) + sin(4*b*x + 4*
a)^2)*cos(1/2*arctan2(sin(4*b*x + 4*a), -cos(4*b*x + 4*a) - 1))^2 + (cos(8*
b*x + 8*a)^2 + 2*cos(8*b*x + 8*a)*cos(4*b*x + 4*a) + cos(4*b*x + 4*a)^2 + s
in(8*b*x + 8*a)^2 + 2*sin(8*b*x + 8*a)*sin(4*b*x + 4*a) + sin(4*b*x + 4*a)^
2)*sin(1/2*arctan2(sin(4*b*x + 4*a), -cos(4*b*x + 4*a) - 1))^2)*(cos(4*b*x
+ 4*a)^2 + sin(4*b*x + 4*a)^2 + 2*cos(4*b*x + 4*a) + 1)^(1/4)), x) - sqrt(c
)*sin(1/2*arctan2(sin(4*b*x + 4*a), -cos(4*b*x + 4*a) - 1)))/((cos(4*b*x +
4*a)^2 + sin(4*b*x + 4*a)^2 + 2*cos(4*b*x + 4*a) + 1)^(1/4)*b)
```

mupad [B] time = 3.68, size = 87, normalized size = 2.64

$$-\frac{\sin\left(2\,a+2\,b\,x\right)\,\sqrt{\frac{c\,\left(\cos(2\,a+2\,b\,x)-\cos(6\,a+6\,b\,x)\right)}{3\,\cos(2\,a+2\,b\,x)+2\,\cos(4\,a+4\,b\,x)+\cos(6\,a+6\,b\,x)+2}}}{b\,\left(\cos\left(2\,a+2\,b\,x\right)-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*tan(a + b*x)*tan(2*a + 2*b*x))^(1/2)/cos(2*a + 2*b*x),x)
```

```
[Out] -(sin(2*a + 2*b*x)*((c*(cos(2*a + 2*b*x) - cos(6*a + 6*b*x)))/(3*cos(2*a +
2*b*x) + 2*cos(4*a + 4*b*x) + cos(6*a + 6*b*x) + 2))^(1/2))/(b*(cos(2*a + 2
*b*x) - 1))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(2*b*x+2*a)*(c*tan(b*x+a)*tan(2*b*x+2*a))**(1/2),x)
```

```
[Out] Timed out
```

3.607 $\int \sqrt{c\tan(a+bx)\tan(2(a+bx))}\,dx$

Optimal. Leaf size=45

$$-\frac{\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c}\tan(2a+2bx)}{\sqrt{c\sec(2a+2bx)-c}}\right)}{b}$$

```
[Out] -arctanh(c^(1/2)*tan(2*b*x+2*a)/(-c+c*sec(2*b*x+2*a))^(1/2))*c^(1/2)/b
```

Rubi [A] time = 0.04, antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {4397, 3774, 207}

$$-\frac{\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c}\tan(2a+2bx)}{\sqrt{c\sec(2a+2bx)-c}}\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c*Tan[a + b*x]*Tan[2*(a + b*x)]],x]
```

```
[Out] -((Sqrt[c]*ArcTanh[(Sqrt[c]*Tan[2*a + 2*b*x])/Sqrt[-c + c*Sec[2*a + 2*b*x]]
])/b)
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 3774

```
Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[(-2*b)/d,
 Subst[Int[1/(a + x^2), x], x, (b*Cot[c + d*x])/Sqrt[a + b*Csc[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 4397

```
Int[u_, x_Symbol] :> Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]
```

Rubi steps

$$\int \sqrt{c\tan(a+bx)\tan(2(a+bx))}\,dx = \int \sqrt{-c+c\sec(2a+2bx)}\,dx$$
$$= -\frac{c\,\text{Subst}\left(\int \frac{1}{-c+x^2}\,dx, x, -\frac{c\tan(2a+2bx)}{\sqrt{-c+c\sec(2a+2bx)}}\right)}{b}$$
$$= -\frac{\sqrt{c}\,\tanh^{-1}\left(\frac{\sqrt{c}\,\tan(2a+2bx)}{\sqrt{-c+c\sec(2a+2bx)}}\right)}{b}$$

Mathematica [A] time = 0.13, size = 73, normalized size = 1.62

$$-\frac{\sqrt{\cos(2(a+bx))}\,\csc(a+bx)\sqrt{c\tan(a+bx)\tan(2(a+bx))}\,\tanh^{-1}\left(\frac{\sqrt{2}\cos(a+bx)}{\sqrt{\cos(2(a+bx))}}\right)}{\sqrt{2}\,b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c*Tan[a + b*x]*Tan[2*(a + b*x)]],x]
```

```
[Out] -((ArcTanh[(Sqrt[2]*Cos[a + b*x])/Sqrt[Cos[2*(a + b*x)]]]*Sqrt[Cos[2*(a + b
*x)]]*Csc[a + b*x]*Sqrt[c*Tan[a + b*x]*Tan[2*(a + b*x)]])/(Sqrt[2]*b))
```

fricas [A] time = 1.46, size = 201, normalized size = 4.47

$$\left[\frac{\sqrt{c}\,\log\left(-\frac{c\tan(bx+a)^5-14\,c\tan(bx+a)^3-4\,\sqrt{2}\left(\tan(bx+a)^4-4\,\tan(bx+a)^2+3\right)\sqrt{-\frac{c\tan(bx+a)^2}{\tan(bx+a)^2-1}}\,\sqrt{c}+17\,c\tan(bx+a)}{\tan(bx+a)^5+2\,\tan(bx+a)^3+\tan(bx+a)}\right)}{4\,b}, \frac{\sqrt{-c}\,\arctan\left(\frac{2\sqrt{2}\sqrt{\cdots}}{\cdots}\right.}{\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*tan(b*x+a)*tan(2*b*x+2*a))^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*sqrt(c)*log(-(c*tan(b*x + a)^5 - 14*c*tan(b*x + a)^3 - 4*sqrt(2)*(tan(
b*x + a)^4 - 4*tan(b*x + a)^2 + 3)*sqrt(-c*tan(b*x + a)^2/(tan(b*x + a)^2 -
 1))*sqrt(c) + 17*c*tan(b*x + a))/(tan(b*x + a)^5 + 2*tan(b*x + a)^3 + tan(
b*x + a)))/b, 1/2*sqrt(-c)*arctan(2*sqrt(2)*sqrt(-c*tan(b*x + a)^2/(tan(b*x
 + a)^2 - 1))*(tan(b*x + a)^2 - 1)*sqrt(-c)/(c*tan(b*x + a)^3 - 3*c*tan(b*x
 + a)))/b]
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*tan(b*x+a)*tan(2*b*x+2*a))^(1/2),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [B] time = 0.94, size = 136, normalized size = 3.02

$$-\frac{\sqrt{\frac{c\left(1-\left(\cos^{2}(bx+a)\right)\right)}{2\left(\cos^{2}(bx+a)\right)-1}}\,\sin\left(bx+a\right)\sqrt{\frac{2\left(\cos^{2}(bx+a)\right)-1}{\left(\cos(bx+a)+1\right)^{2}}}\,\operatorname{arctanh}\left(\frac{\cos(bx+a)\sqrt{4}\,\left(-1+\cos(bx+a)\right)\sqrt{2}}{2\sin(bx+a)^{2}\sqrt{\frac{2\left(\cos^{2}(bx+a)\right)-1}{\left(\cos(bx+a)+1\right)^{2}}}}\right)\sqrt{4}}{2b\left(-1+\cos\left(bx+a\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*tan(b*x+a)*tan(2*b*x+2*a))^(1/2),x)
```

```
[Out] -1/2/b*(c*(1-cos(b*x+a)^2)/(2*cos(b*x+a)^2-1))^(1/2)*sin(b*x+a)*((2*cos(b*x
+a)^2-1)/(cos(b*x+a)+1)^2)^(1/2)*arctanh(1/2*cos(b*x+a)*4^(1/2)*(-1+cos(b*x
+a))/sin(b*x+a)^2/((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(1/2)*2^(1/2))/(-1+
cos(b*x+a))*4^(1/2)
```

maxima [B] time = 0.50, size = 430, normalized size = 9.56

$$\sqrt{c}\left(\log\left(4\sqrt{\cos\left(4\,bx+4\,a\right)^{2}+\sin\left(4\,bx+4\,a\right)^{2}+2\,\cos\left(4\,bx+4\,a\right)+1}\,\cos\left(\frac{1}{2}\,\arctan\left(\sin\left(4\,bx+4\,a\right),\cos\left(4\,b\right.\right.\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*tan(b*x+a)*tan(2*b*x+2*a))^(1/2),x, algorithm="maxima")
```

```
[Out] 1/4*sqrt(c)*(log(4*sqrt(cos(4*b*x + 4*a)^2 + sin(4*b*x + 4*a)^2 + 2*cos(4*b
*x + 4*a) + 1)*cos(1/2*arctan2(sin(4*b*x + 4*a), cos(4*b*x + 4*a) + 1))^2 +
 4*sqrt(cos(4*b*x + 4*a)^2 + sin(4*b*x + 4*a)^2 + 2*cos(4*b*x + 4*a) + 1)*s
in(1/2*arctan2(sin(4*b*x + 4*a), cos(4*b*x + 4*a) + 1))^2 + 8*(cos(4*b*x +
4*a)^2 + sin(4*b*x + 4*a)^2 + 2*cos(4*b*x + 4*a) + 1)^(1/4)*cos(1/2*arctan2
(sin(4*b*x + 4*a), cos(4*b*x + 4*a) + 1)) + 4) - log(cos(2*b*x + 2*a)^2 + s
in(2*b*x + 2*a)^2 + sqrt(cos(4*b*x + 4*a)^2 + sin(4*b*x + 4*a)^2 + 2*cos(4*
b*x + 4*a) + 1)*(cos(1/2*arctan2(sin(4*b*x + 4*a), cos(4*b*x + 4*a) + 1))^2
 + sin(1/2*arctan2(sin(4*b*x + 4*a), cos(4*b*x + 4*a) + 1))^2) + 2*(cos(4*b
```

```
*x + 4*a)^2 + sin(4*b*x + 4*a)^2 + 2*cos(4*b*x + 4*a) + 1)^(1/4)*(cos(2*b*x
 + 2*a)*cos(1/2*arctan2(sin(4*b*x + 4*a), cos(4*b*x + 4*a) + 1)) + sin(2*b*
x + 2*a)*sin(1/2*arctan2(sin(4*b*x + 4*a), cos(4*b*x + 4*a) + 1)))))/b
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \sqrt{c\tan\left(a+b\,x\right)\,\tan\left(2\,a+2\,b\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*tan(a + b*x)*tan(2*a + 2*b*x))^(1/2),x)
[Out] int((c*tan(a + b*x)*tan(2*a + 2*b*x))^(1/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*tan(b*x+a)*tan(2*b*x+2*a))**(1/2),x)
[Out] Timed out
```

3.608 $\int \cos(2(a+bx))\sqrt{c\tan(a+bx)\tan(2(a+bx))}\,dx$

Optimal. Leaf size=84

$$\frac{\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c}\tan(2a+2bx)}{\sqrt{c\sec(2a+2bx)-c}}\right)}{2b}-\frac{c\sin(2a+2bx)}{2b\sqrt{c\sec(2a+2bx)-c}}$$

```
[Out] 1/2*arctanh(c^(1/2)*tan(2*b*x+2*a)/(-c+c*sec(2*b*x+2*a))^(1/2))*c^(1/2)/b-1
/2*c*sin(2*b*x+2*a)/b/(-c+c*sec(2*b*x+2*a))^(1/2)
```

Rubi [A] time = 0.14, antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.138, Rules used = {4397, 3805, 3774, 207}

$$\frac{\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c}\tan(2a+2bx)}{\sqrt{c\sec(2a+2bx)-c}}\right)}{2b}-\frac{c\sin(2a+2bx)}{2b\sqrt{c\sec(2a+2bx)-c}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[2*(a + b*x)]*Sqrt[c*Tan[a + b*x]*Tan[2*(a + b*x)]],x]
```

```
[Out] (Sqrt[c]*ArcTanh[(Sqrt[c]*Tan[2*a + 2*b*x])/Sqrt[-c + c*Sec[2*a + 2*b*x]]])
/(2*b) - (c*Sin[2*a + 2*b*x])/(2*b*Sqrt[-c + c*Sec[2*a + 2*b*x]])
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 3774

```
Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[(-2*b)/d,
 Subst[Int[1/(a + x^2), x], x, (b*Cot[c + d*x])/Sqrt[a + b*Csc[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3805

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[(a*Cot[e + f*x]*(d*Csc[e + f*x])^n)/(f*n*Sqrt[a
 + b*Csc[e + f*x]]), x] + Dist[(a*(2*n + 1))/(2*b*d*n), Int[Sqrt[a + b*Csc[
e + f*x]]*(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f}, x] &&
EqQ[a^2 - b^2, 0] && LtQ[n, -2^(-1)] && IntegerQ[2*n]
```

Rule 4397

```
Int[u_, x_Symbol] :> Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]
```

Rubi steps

$$\int \cos(2(a+bx))\sqrt{c\tan(a+bx)\tan(2(a+bx))}\,dx = \int \cos(2a+2bx)\sqrt{-c+c\sec(2a+2bx)}\,dx$$
$$= -\frac{c\sin(2a+2bx)}{2b\sqrt{-c+c\sec(2a+2bx)}} - \frac{1}{2}\int \sqrt{-c+c\sec(2a+2bx)}$$
$$= -\frac{c\sin(2a+2bx)}{2b\sqrt{-c+c\sec(2a+2bx)}} + \frac{c\,\text{Subst}\left(\int \frac{1}{-c+x^2}\,dx, x, -\frac{c}{\sqrt{-c}}\right.}{2b}$$
$$= \frac{\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c}\tan(2a+2bx)}{\sqrt{-c+c\sec(2a+2bx)}}\right)}{2b} - \frac{c\sin(2a+2bx)}{2b\sqrt{-c+c\sec(2a+2bx)}}$$

Mathematica [A] time = 0.24, size = 92, normalized size = 1.10

$$-\frac{\csc(a+bx)\sqrt{c\tan(a+bx)\tan(2(a+bx))}\left(\cos(a+bx)+\cos(3(a+bx))-\sqrt{2}\sqrt{\cos(2(a+bx))}\tanh^{-1}\left(\frac{\sqrt{2}\cos}{\sqrt{\cos(2}}\right.\right.}{4b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[2*(a + b*x)]*Sqrt[c*Tan[a + b*x]*Tan[2*(a + b*x)]],x]
```

```
[Out] -1/4*((Cos[a + b*x] - Sqrt[2]*ArcTanh[(Sqrt[2]*Cos[a + b*x])/Sqrt[Cos[2*(a
+ b*x)]]]*Sqrt[Cos[2*(a + b*x)]] + Cos[3*(a + b*x)])*Csc[a + b*x]*Sqrt[c*Ta
n[a + b*x]*Tan[2*(a + b*x)]])/b
```

fricas [B] time = 2.28, size = 351, normalized size = 4.18

$$\left[\frac{\left(\tan(bx+a)^3+\tan(bx+a)\right)\sqrt{c}\log\left(-\frac{c\tan(bx+a)^5-14c\tan(bx+a)^3+4\sqrt{2}\left(\tan(bx+a)^4-4\tan(bx+a)^2+3\right)\sqrt{-\frac{c\tan(bx+a)^2}{\tan(bx+a)^2-1}}\sqrt{c}+17c}{\tan(bx+a)^5+2\tan(bx+a)^3+\tan(bx+a)}\right.}{8\left(b\tan(bx+a)^3+b\tan(bx+a)\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(2*b*x+2*a)*(c*tan(b*x+a)*tan(2*b*x+2*a))^(1/2),x, algorithm="
fricas")
```

```
[Out] [1/8*((tan(b*x + a)^3 + tan(b*x + a))*sqrt(c)*log(-(c*tan(b*x + a)^5 - 14*c
*tan(b*x + a)^3 + 4*sqrt(2)*(tan(b*x + a)^4 - 4*tan(b*x + a)^2 + 3)*sqrt(-c
*tan(b*x + a)^2/(tan(b*x + a)^2 - 1))*sqrt(c) + 17*c*tan(b*x + a))/(tan(b*x
 + a)^5 + 2*tan(b*x + a)^3 + tan(b*x + a))) + 4*sqrt(2)*sqrt(-c*tan(b*x + a
)^2/(tan(b*x + a)^2 - 1))*(tan(b*x + a)^2 - 1))/(b*tan(b*x + a)^3 + b*tan(b
*x + a)), -1/4*((tan(b*x + a)^3 + tan(b*x + a))*sqrt(-c)*arctan(2*sqrt(2)*s
qrt(-c*tan(b*x + a)^2/(tan(b*x + a)^2 - 1))*(tan(b*x + a)^2 - 1)*sqrt(-c)/(
c*tan(b*x + a)^3 - 3*c*tan(b*x + a))) - 2*sqrt(2)*sqrt(-c*tan(b*x + a)^2/(t
an(b*x + a)^2 - 1))*(tan(b*x + a)^2 - 1))/(b*tan(b*x + a)^3 + b*tan(b*x + a
))]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{c\tan(2\,bx + 2\,a)\tan(bx + a)}\,\cos(2\,bx + 2\,a)\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(2*b*x+2*a)*(c*tan(b*x+a)*tan(2*b*x+2*a))^(1/2),x, algorithm="
giac")
```

```
[Out] integrate(sqrt(c*tan(2*b*x + 2*a)*tan(b*x + a))*cos(2*b*x + 2*a), x)
```

maple [B] time = 1.14, size = 391, normalized size = 4.65

$$\frac{\sqrt{\frac{c\left(1-\left(\cos^2(bx+a)\right)\right)}{2\left(\cos^2(bx+a)\right)-1}}\;\sin(bx+a)\sqrt{\frac{2\left(\cos^2(bx+a)\right)-1}{(\cos(bx+a)+1)^2}}\;\operatorname{arctanh}\left(\frac{\cos(bx+a)\sqrt{4}\;(-1+\cos(bx+a))\sqrt{2}}{2\sin(bx+a)^2\sqrt{\frac{2\left(\cos^2(bx+a)\right)-1}{(\cos(bx+a)+1)^2}}}\right)\sqrt{4}}{2b\,(-1+\cos(bx+a))} - \frac{\sqrt{2}\;\sin(bx+a)\sqrt{\frac{c(1-}{2(c}}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(2*b*x+2*a)*(c*tan(b*x+a)*tan(2*b*x+2*a))^(1/2),x)
```

```
[Out] 1/2/b*(c*(1-cos(b*x+a)^2)/(2*cos(b*x+a)^2-1))^(1/2)*sin(b*x+a)*((2*cos(b*x+
a)^2-1)/(cos(b*x+a)+1)^2)^(1/2)*arctanh(1/2*cos(b*x+a)*4^(1/2)*(-1+cos(b*x+
a))/sin(b*x+a)^2/((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(1/2)*2^(1/2))/(-1+c
os(b*x+a))*4^(1/2)-1/8*2^(1/2)/b*sin(b*x+a)*(c*(1-cos(b*x+a)^2)/(2*cos(b*x+
a)^2-1))^(1/2)*(2^(1/2)*((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(1/2)*arctanh
(1/2*cos(b*x+a)*4^(1/2)*(-1+cos(b*x+a))/sin(b*x+a)^2/((2*cos(b*x+a)^2-1)/(c
os(b*x+a)+1)^2)^(1/2)*2^(1/2))*cos(b*x+a)+2^(1/2)*((2*cos(b*x+a)^2-1)/(cos(
b*x+a)+1)^2)^(1/2)*arctanh(1/2*cos(b*x+a)*4^(1/2)*(-1+cos(b*x+a))/sin(b*x+a
```

```
)^2/((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(1/2)*2^(1/2))-4*cos(b*x+a)^3+2*c
os(b*x+a))/(-1+cos(b*x+a)^2)*4^(1/2)
```

maxima [B] time = 0.61, size = 1049, normalized size = 12.49

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(2*b*x+2*a)*(c*tan(b*x+a)*tan(2*b*x+2*a))^(1/2),x, algorithm="
maxima")
```

```
[Out] 1/16*(4*(cos(4*b*x + 4*a)^2 + sin(4*b*x + 4*a)^2 + 2*cos(4*b*x + 4*a) + 1)^
(1/4)*(cos(1/2*arctan2(sin(4*b*x + 4*a), -cos(4*b*x + 4*a) - 1))*sin(2*b*x
+ 2*a) + (cos(2*b*x + 2*a) + 1)*sin(1/2*arctan2(sin(4*b*x + 4*a), -cos(4*b*
x + 4*a) - 1)))*sqrt(c) - sqrt(c)*(log(sqrt(cos(4*b*x + 4*a)^2 + sin(4*b*x
+ 4*a)^2 + 2*cos(4*b*x + 4*a) + 1)*cos(1/2*arctan2(sin(4*b*x + 4*a), -cos(4
*b*x + 4*a) - 1))^2 + sqrt(cos(4*b*x + 4*a)^2 + sin(4*b*x + 4*a)^2 + 2*cos(
4*b*x + 4*a) + 1)*sin(1/2*arctan2(sin(4*b*x + 4*a), -cos(4*b*x + 4*a) - 1))
^2 + 2*(cos(4*b*x + 4*a)^2 + sin(4*b*x + 4*a)^2 + 2*cos(4*b*x + 4*a) + 1)^(
1/4)*sin(1/2*arctan2(sin(4*b*x + 4*a), -cos(4*b*x + 4*a) - 1)) + 1) - log(s
qrt(cos(4*b*x + 4*a)^2 + sin(4*b*x + 4*a)^2 + 2*cos(4*b*x + 4*a) + 1)*cos(1
/2*arctan2(sin(4*b*x + 4*a), -cos(4*b*x + 4*a) - 1))^2 + sqrt(cos(4*b*x + 4
*a)^2 + sin(4*b*x + 4*a)^2 + 2*cos(4*b*x + 4*a) + 1)*sin(1/2*arctan2(sin(4*
b*x + 4*a), -cos(4*b*x + 4*a) - 1))^2 - 2*(cos(4*b*x + 4*a)^2 + sin(4*b*x +
 4*a)^2 + 2*cos(4*b*x + 4*a) + 1)^(1/4)*sin(1/2*arctan2(sin(4*b*x + 4*a), -
cos(4*b*x + 4*a) - 1)) + 1) + log(((cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2
)*cos(1/2*arctan2(sin(4*b*x + 4*a), -cos(4*b*x + 4*a) - 1))^2 + (cos(2*b*x
+ 2*a)^2 + sin(2*b*x + 2*a)^2)*sin(1/2*arctan2(sin(4*b*x + 4*a), -cos(4*b*x
 + 4*a) - 1))^2)*sqrt(cos(4*b*x + 4*a)^2 + sin(4*b*x + 4*a)^2 + 2*cos(4*b*x
 + 4*a) + 1) + 2*(cos(4*b*x + 4*a)^2 + sin(4*b*x + 4*a)^2 + 2*cos(4*b*x + 4
*a) + 1)^(1/4)*(cos(1/2*arctan2(sin(4*b*x + 4*a), -cos(4*b*x + 4*a) - 1))*s
in(2*b*x + 2*a) + cos(2*b*x + 2*a)*sin(1/2*arctan2(sin(4*b*x + 4*a), -cos(4
*b*x + 4*a) - 1))) + 1) - log(((cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2)*co
s(1/2*arctan2(sin(4*b*x + 4*a), -cos(4*b*x + 4*a) - 1))^2 + (cos(2*b*x + 2*
a)^2 + sin(2*b*x + 2*a)^2)*sin(1/2*arctan2(sin(4*b*x + 4*a), -cos(4*b*x + 4
*a) - 1))^2)*sqrt(cos(4*b*x + 4*a)^2 + sin(4*b*x + 4*a)^2 + 2*cos(4*b*x + 4
*a) + 1) - 2*(cos(4*b*x + 4*a)^2 + sin(4*b*x + 4*a)^2 + 2*cos(4*b*x + 4*a)
+ 1)^(1/4)*(cos(1/2*arctan2(sin(4*b*x + 4*a), -cos(4*b*x + 4*a) - 1))*sin(2
*b*x + 2*a) + cos(2*b*x + 2*a)*sin(1/2*arctan2(sin(4*b*x + 4*a), -cos(4*b*x
 + 4*a) - 1))) + 1)))/b
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \cos (2\,a+2\,b\,x)\,\sqrt{c\,\tan (a+b\,x)\,\tan (2\,a+2\,b\,x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(2*a + 2*b*x)*(c*tan(a + b*x)*tan(2*a + 2*b*x))^(1/2),x)
```

```
[Out] int(cos(2*a + 2*b*x)*(c*tan(a + b*x)*tan(2*a + 2*b*x))^(1/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(2*b*x+2*a)*(c*tan(b*x+a)*tan(2*b*x+2*a))**(1/2),x)
```

```
[Out] Timed out
```

3.609 $\int \cos^2(2(a+bx))\sqrt{c\tan(a+bx)\tan(2(a+bx))}\,dx$

Optimal. Leaf size=129

$$\frac{3c\sin(2a+2bx)}{8b\sqrt{c\sec(2a+2bx)-c}}-\frac{c\sin(2a+2bx)\cos(2a+2bx)}{4b\sqrt{c\sec(2a+2bx)-c}}-\frac{3\sqrt{c}\,\tanh^{-1}\left(\frac{\sqrt{c}\,\tan(2a+2bx)}{\sqrt{c\sec(2a+2bx)-c}}\right)}{8b}$$

```
[Out] -3/8*arctanh(c^(1/2)*tan(2*b*x+2*a)/(-c+c*sec(2*b*x+2*a))^(1/2))*c^(1/2)/b+
3/8*c*sin(2*b*x+2*a)/b/(-c+c*sec(2*b*x+2*a))^(1/2)-1/4*c*cos(2*b*x+2*a)*sin
(2*b*x+2*a)/b/(-c+c*sec(2*b*x+2*a))^(1/2)
```

Rubi [A] time = 0.21, antiderivative size = 129, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.129, Rules used = {4397, 3805, 3774, 207}

$$\frac{3c\sin(2a+2bx)}{8b\sqrt{c\sec(2a+2bx)-c}}-\frac{c\sin(2a+2bx)\cos(2a+2bx)}{4b\sqrt{c\sec(2a+2bx)-c}}-\frac{3\sqrt{c}\,\tanh^{-1}\left(\frac{\sqrt{c}\,\tan(2a+2bx)}{\sqrt{c\sec(2a+2bx)-c}}\right)}{8b}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[2*(a + b*x)]^2*Sqrt[c*Tan[a + b*x]*Tan[2*(a + b*x)]],x]
```

```
[Out] (-3*Sqrt[c]*ArcTanh[(Sqrt[c]*Tan[2*a + 2*b*x])/Sqrt[-c + c*Sec[2*a + 2*b*x]
]])/(8*b) + (3*c*Sin[2*a + 2*b*x])/(8*b*Sqrt[-c + c*Sec[2*a + 2*b*x]]) - (c
*Cos[2*a + 2*b*x]*Sin[2*a + 2*b*x])/(4*b*Sqrt[-c + c*Sec[2*a + 2*b*x]])
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 3774

```
Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[(-2*b)/d,
 Subst[Int[1/(a + x^2), x], x, (b*Cot[c + d*x])/Sqrt[a + b*Csc[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3805

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[(a*Cot[e + f*x]*(d*Csc[e + f*x])^n)/(f*n*Sqrt[a
 + b*Csc[e + f*x]]), x] + Dist[(a*(2*n + 1))/(2*b*d*n), Int[Sqrt[a + b*Csc[
e + f*x]]*(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f}, x] &&
```

```
EqQ[a^2 - b^2, 0] && LtQ[n, -2^(-1)] && IntegerQ[2*n]
```

Rule 4397

```
Int[u_, x_Symbol] :> Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]
```

Rubi steps

$$\int \cos^2(2(a+bx))\sqrt{c\tan(a+bx)\tan(2(a+bx))}\,dx = \int \cos^2(2a+2bx)\sqrt{-c+c\sec(2a+2bx)}\,dx$$

$$= -\frac{c\cos(2a+2bx)\sin(2a+2bx)}{4b\sqrt{-c+c\sec(2a+2bx)}} - \frac{3}{4}\int \cos(2a+2bx)\sqrt{-c$$

$$= \frac{3c\sin(2a+2bx)}{8b\sqrt{-c+c\sec(2a+2bx)}} - \frac{c\cos(2a+2bx)\sin(2a+2bx)}{4b\sqrt{-c+c\sec(2a+2bx)}}$$

$$= \frac{3c\sin(2a+2bx)}{8b\sqrt{-c+c\sec(2a+2bx)}} - \frac{c\cos(2a+2bx)\sin(2a+2bx)}{4b\sqrt{-c+c\sec(2a+2bx)}}$$

$$= -\frac{3\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c}\tan(2a+2bx)}{\sqrt{-c+c\sec(2a+2bx)}}\right)}{8b} + \frac{3c\sin(2a+2bx)}{8b\sqrt{-c+c\sec(2a+2b}$$

Mathematica [A] time = 0.25, size = 105, normalized size = 0.81

$$\frac{\sqrt{c\tan(a+bx)\tan(2(a+bx))}\left(2(-\sin(2(a+bx))+\sin(4(a+bx))+\cot(a+bx))-3\sqrt{2}\sqrt{\cos(2(a+bx))}\csc(a+\right.}{16b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[2*(a + b*x)]^2*Sqrt[c*Tan[a + b*x]*Tan[2*(a + b*x)]],x]
```

```
[Out] ((-3*Sqrt[2]*ArcTanh[(Sqrt[2]*Cos[a + b*x])/Sqrt[Cos[2*(a + b*x)]]]*Sqrt[Co
s[2*(a + b*x)]]*Csc[a + b*x] + 2*(Cot[a + b*x] - Sin[2*(a + b*x)] + Sin[4*(
a + b*x)]))*Sqrt[c*Tan[a + b*x]*Tan[2*(a + b*x)]])/(16*b)
```

fricas [A] time = 0.95, size = 419, normalized size = 3.25

$$\left[\frac{3\left(\tan(bx+a)^5+2\tan(bx+a)^3+\tan(bx+a)\right)\sqrt{c}\log\left(-\frac{c\tan(bx+a)^5-14c\tan(bx+a)^3-4\sqrt{2}\left(\tan(bx+a)^4-4\tan(bx+a)^2+3\right)}{\tan(bx+a)^5+2\tan(bx+a)^3+\tan(bx+a)}\right)}{32\left(b\tan(bx+a)^5+2b\tan(bx+a)^3\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(2*b*x+2*a)^2*(c*tan(b*x+a)*tan(2*b*x+2*a))^(1/2),x, algorithm
="fricas")
```

```
[Out] [1/32*(3*(tan(b*x + a)^5 + 2*tan(b*x + a)^3 + tan(b*x + a))*sqrt(c)*log(-(c
*tan(b*x + a)^5 - 14*c*tan(b*x + a)^3 - 4*sqrt(2)*(tan(b*x + a)^4 - 4*tan(b
*x + a)^2 + 3)*sqrt(-c*tan(b*x + a)^2/(tan(b*x + a)^2 - 1))*sqrt(c) + 17*c*
tan(b*x + a))/(tan(b*x + a)^5 + 2*tan(b*x + a)^3 + tan(b*x + a))) - 4*sqrt(
2)*(5*tan(b*x + a)^4 - 4*tan(b*x + a)^2 - 1)*sqrt(-c*tan(b*x + a)^2/(tan(b*
x + a)^2 - 1)))/(b*tan(b*x + a)^5 + 2*b*tan(b*x + a)^3 + b*tan(b*x + a)), 1
/16*(3*(tan(b*x + a)^5 + 2*tan(b*x + a)^3 + tan(b*x + a))*sqrt(-c)*arctan(2
*sqrt(2)*sqrt(-c*tan(b*x + a)^2/(tan(b*x + a)^2 - 1))*(tan(b*x + a)^2 - 1)*
sqrt(-c)/(c*tan(b*x + a)^3 - 3*c*tan(b*x + a))) - 2*sqrt(2)*(5*tan(b*x + a)
^4 - 4*tan(b*x + a)^2 - 1)*sqrt(-c*tan(b*x + a)^2/(tan(b*x + a)^2 - 1)))/(b
*tan(b*x + a)^5 + 2*b*tan(b*x + a)^3 + b*tan(b*x + a))]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{c\tan(2bx+2a)\tan(bx+a)}\cos(2bx+2a)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(2*b*x+2*a)^2*(c*tan(b*x+a)*tan(2*b*x+2*a))^(1/2),x, algorithm
="giac")
```

```
[Out] integrate(sqrt(c*tan(2*b*x + 2*a)*tan(b*x + a))*cos(2*b*x + 2*a)^2, x)
```

maple [B] time = 1.22, size = 657, normalized size = 5.09

$$-\frac{\sqrt{\frac{c\left(1-\left(\cos^2(bx+a)\right)\right)}{2\left(\cos^2(bx+a)\right)-1}}\sin(bx+a)\sqrt{\frac{2\left(\cos^2(bx+a)\right)-1}{(\cos(bx+a)+1)^2}}\operatorname{arctanh}\left(\frac{\cos(bx+a)\sqrt{4}\,(-1+\cos(bx+a))\sqrt{2}}{2\sin(bx+a)^2\sqrt{\frac{2\left(\cos^2(bx+a)\right)-1}{(\cos(bx+a)+1)^2}}}\right)\sqrt{4}\,\sqrt{2}\,\sin(bx+a)\sqrt{}}{2b(-1+\cos(bx+a))}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(2*b*x+2*a)^2*(c*tan(b*x+a)*tan(2*b*x+2*a))^(1/2),x)
```

```
[Out] -1/2/b*(c*(1-cos(b*x+a)^2)/(2*cos(b*x+a)^2-1))^(1/2)*sin(b*x+a)*((2*cos(b*x
+a)^2-1)/(cos(b*x+a)+1)^2)^(1/2)*arctanh(1/2*cos(b*x+a)*4^(1/2)*(-1+cos(b*x
+a))/sin(b*x+a)^2/((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(1/2)*2^(1/2))/(-1+
cos(b*x+a))*4^(1/2)+1/4*2^(1/2)/b*sin(b*x+a)*(c*(1-cos(b*x+a)^2)/(2*cos(b*x
+a)^2-1))^(1/2)*(2^(1/2)*((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(1/2)*arctan
h(1/2*cos(b*x+a)*4^(1/2)*(-1+cos(b*x+a))/sin(b*x+a)^2/((2*cos(b*x+a)^2-1)/(
cos(b*x+a)+1)^2)^(1/2)*2^(1/2))*cos(b*x+a)+2^(1/2)*((2*cos(b*x+a)^2-1)/(cos
(b*x+a)+1)^2)^(1/2)*arctanh(1/2*cos(b*x+a)*4^(1/2)*(-1+cos(b*x+a))/sin(b*x+
a)^2/((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(1/2)*2^(1/2))-4*cos(b*x+a)^3+2*
cos(b*x+a))/(-1+cos(b*x+a)^2)*4^(1/2)-1/32*2^(1/2)/b*sin(b*x+a)*(c*(1-cos(b
*x+a)^2)/(2*cos(b*x+a)^2-1))^(1/2)*(-16*cos(b*x+a)^5+3*2^(1/2)*((2*cos(b*x+
a)^2-1)/(cos(b*x+a)+1)^2)^(1/2)*arctanh(1/2*cos(b*x+a)*4^(1/2)*(-1+cos(b*x+
a))/sin(b*x+a)^2/((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(1/2)*2^(1/2))*cos(b
*x+a)+3*2^(1/2)*((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(1/2)*arctanh(1/2*cos
(b*x+a)*4^(1/2)*(-1+cos(b*x+a))/sin(b*x+a)^2/((2*cos(b*x+a)^2-1)/(cos(b*x+a
)+1)^2)^(1/2)*2^(1/2))-4*cos(b*x+a)^3+6*cos(b*x+a))/(-1+cos(b*x+a)^2)*4^(1/
2)
```

maxima [B] time = 0.72, size = 1421, normalized size = 11.02

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(2*b*x+2*a)^2*(c*tan(b*x+a)*tan(2*b*x+2*a))^(1/2),x, algorithm
="maxima")
```

```
[Out] -1/64*(4*(cos(4*b*x + 4*a)^2 + sin(4*b*x + 4*a)^2 + 2*cos(4*b*x + 4*a) + 1)
^(1/4)*((cos(1/2*arctan2(sin(4*b*x + 4*a), -cos(4*b*x + 4*a) - 1))*sin(4*b*
x + 4*a) - (cos(4*b*x + 4*a) - 2)*sin(1/2*arctan2(sin(4*b*x + 4*a), -cos(4*
b*x + 4*a) - 1)))*cos(1/2*arctan2(sin(4*b*x + 4*a), cos(4*b*x + 4*a))) - co
s(1/2*arctan2(sin(4*b*x + 4*a), -cos(4*b*x + 4*a) - 1))*sin(4*b*x + 4*a) -
(cos(4*b*x + 4*a) - 2)*sin(1/2*arctan2(sin(4*b*x + 4*a), -cos(4*b*x + 4*a)
- 1)) - ((cos(4*b*x + 4*a) - 2)*cos(1/2*arctan2(sin(4*b*x + 4*a), -cos(4*b*
x + 4*a) - 1)) + sin(4*b*x + 4*a)*sin(1/2*arctan2(sin(4*b*x + 4*a), -cos(4*
b*x + 4*a) - 1)))*sin(1/2*arctan2(sin(4*b*x + 4*a), cos(4*b*x + 4*a))))*sqr
t(c) - 3*sqrt(c)*(log(sqrt(cos(4*b*x + 4*a)^2 + sin(4*b*x + 4*a)^2 + 2*cos(
4*b*x + 4*a) + 1)*cos(1/2*arctan2(sin(4*b*x + 4*a), -cos(4*b*x + 4*a) - 1))
^2 + sqrt(cos(4*b*x + 4*a)^2 + sin(4*b*x + 4*a)^2 + 2*cos(4*b*x + 4*a) + 1)
*sin(1/2*arctan2(sin(4*b*x + 4*a), -cos(4*b*x + 4*a) - 1))^2 + 2*(cos(4*b*x
 + 4*a)^2 + sin(4*b*x + 4*a)^2 + 2*cos(4*b*x + 4*a) + 1)^(1/4)*sin(1/2*arct
an2(sin(4*b*x + 4*a), -cos(4*b*x + 4*a) - 1)) + 1) - log(sqrt(cos(4*b*x + 4
```

```
*a)^2 + sin(4*b*x + 4*a)^2 + 2*cos(4*b*x + 4*a) + 1)*cos(1/2*arctan2(sin(4*
b*x + 4*a), -cos(4*b*x + 4*a) - 1))^2 + sqrt(cos(4*b*x + 4*a)^2 + sin(4*b*x
 + 4*a)^2 + 2*cos(4*b*x + 4*a) + 1)*sin(1/2*arctan2(sin(4*b*x + 4*a), -cos(
4*b*x + 4*a) - 1))^2 - 2*(cos(4*b*x + 4*a)^2 + sin(4*b*x + 4*a)^2 + 2*cos(4
*b*x + 4*a) + 1)^(1/4)*sin(1/2*arctan2(sin(4*b*x + 4*a), -cos(4*b*x + 4*a)
- 1)) + 1) + log(((cos(1/2*arctan2(sin(4*b*x + 4*a), -cos(4*b*x + 4*a) - 1)
)^2 + sin(1/2*arctan2(sin(4*b*x + 4*a), -cos(4*b*x + 4*a) - 1))^2)*cos(1/2*
arctan2(sin(4*b*x + 4*a), cos(4*b*x + 4*a)))^2 + (cos(1/2*arctan2(sin(4*b*x
 + 4*a), -cos(4*b*x + 4*a) - 1))^2 + sin(1/2*arctan2(sin(4*b*x + 4*a), -cos
(4*b*x + 4*a) - 1))^2)*sin(1/2*arctan2(sin(4*b*x + 4*a), cos(4*b*x + 4*a)))
^2)*sqrt(cos(4*b*x + 4*a)^2 + sin(4*b*x + 4*a)^2 + 2*cos(4*b*x + 4*a) + 1)
+ 2*(cos(4*b*x + 4*a)^2 + sin(4*b*x + 4*a)^2 + 2*cos(4*b*x + 4*a) + 1)^(1/4
)*(cos(1/2*arctan2(sin(4*b*x + 4*a), cos(4*b*x + 4*a)))*sin(1/2*arctan2(sin
(4*b*x + 4*a), -cos(4*b*x + 4*a) - 1)) + cos(1/2*arctan2(sin(4*b*x + 4*a),
-cos(4*b*x + 4*a) - 1))*sin(1/2*arctan2(sin(4*b*x + 4*a), cos(4*b*x + 4*a))
)) + 1) - log(((cos(1/2*arctan2(sin(4*b*x + 4*a), -cos(4*b*x + 4*a) - 1))^2
 + sin(1/2*arctan2(sin(4*b*x + 4*a), -cos(4*b*x + 4*a) - 1))^2)*cos(1/2*arc
tan2(sin(4*b*x + 4*a), cos(4*b*x + 4*a)))^2 + (cos(1/2*arctan2(sin(4*b*x +
4*a), -cos(4*b*x + 4*a) - 1))^2 + sin(1/2*arctan2(sin(4*b*x + 4*a), -cos(4*
b*x + 4*a) - 1))^2)*sin(1/2*arctan2(sin(4*b*x + 4*a), cos(4*b*x + 4*a)))^2)
*sqrt(cos(4*b*x + 4*a)^2 + sin(4*b*x + 4*a)^2 + 2*cos(4*b*x + 4*a) + 1) - 2
*(cos(4*b*x + 4*a)^2 + sin(4*b*x + 4*a)^2 + 2*cos(4*b*x + 4*a) + 1)^(1/4)*(
cos(1/2*arctan2(sin(4*b*x + 4*a), cos(4*b*x + 4*a)))*sin(1/2*arctan2(sin(4*
b*x + 4*a), -cos(4*b*x + 4*a) - 1)) + cos(1/2*arctan2(sin(4*b*x + 4*a), -co
s(4*b*x + 4*a) - 1))*sin(1/2*arctan2(sin(4*b*x + 4*a), cos(4*b*x + 4*a))))
+ 1)))/b
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \cos(2a+2bx)^2 \sqrt{c\tan(a+bx)\tan(2a+2bx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(2*a + 2*b*x)^2*(c*tan(a + b*x)*tan(2*a + 2*b*x))^(1/2),x)
[Out] int(cos(2*a + 2*b*x)^2*(c*tan(a + b*x)*tan(2*a + 2*b*x))^(1/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(2*b*x+2*a)**2*(c*tan(b*x+a)*tan(2*b*x+2*a))**(1/2),x)
[Out] Timed out
```

3.610 $\int \cos^3(2(a+bx))\sqrt{c\tan(a+bx)\tan(2(a+bx))}\,dx$

Optimal. Leaf size=176

$$-\frac{5c\sin(2a+2bx)}{16b\sqrt{c\sec(2a+2bx)-c}}-\frac{c\sin(2a+2bx)\cos^2(2a+2bx)}{6b\sqrt{c\sec(2a+2bx)-c}}+\frac{5c\sin(2a+2bx)\cos(2a+2bx)}{24b\sqrt{c\sec(2a+2bx)-c}}+\frac{5\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c}\,\mathrm{ta}}{\sqrt{c\,\mathrm{sec}}}\right.}{16b}$$

```
[Out] 5/16*arctanh(c^(1/2)*tan(2*b*x+2*a)/(-c+c*sec(2*b*x+2*a))^(1/2))*c^(1/2)/b-
5/16*c*sin(2*b*x+2*a)/b/(-c+c*sec(2*b*x+2*a))^(1/2)+5/24*c*cos(2*b*x+2*a)*s
in(2*b*x+2*a)/b/(-c+c*sec(2*b*x+2*a))^(1/2)-1/6*c*cos(2*b*x+2*a)^2*sin(2*b*
x+2*a)/b/(-c+c*sec(2*b*x+2*a))^(1/2)
```

Rubi [A] time = 0.29, antiderivative size = 176, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.129, Rules used = {4397, 3805, 3774, 207}

$$-\frac{5c\sin(2a+2bx)}{16b\sqrt{c\sec(2a+2bx)-c}}-\frac{c\sin(2a+2bx)\cos^2(2a+2bx)}{6b\sqrt{c\sec(2a+2bx)-c}}+\frac{5c\sin(2a+2bx)\cos(2a+2bx)}{24b\sqrt{c\sec(2a+2bx)-c}}+\frac{5\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c}\,\mathrm{ta}}{\sqrt{c\,\mathrm{sec}}}\right.}{16b}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[2*(a + b*x)]^3*Sqrt[c*Tan[a + b*x]*Tan[2*(a + b*x)]],x]
```

```
[Out] (5*Sqrt[c]*ArcTanh[(Sqrt[c]*Tan[2*a + 2*b*x])/Sqrt[-c + c*Sec[2*a + 2*b*x]]
])/(16*b) - (5*c*Sin[2*a + 2*b*x])/(16*b*Sqrt[-c + c*Sec[2*a + 2*b*x]]) + (
5*c*Cos[2*a + 2*b*x]*Sin[2*a + 2*b*x])/(24*b*Sqrt[-c + c*Sec[2*a + 2*b*x]])
 - (c*Cos[2*a + 2*b*x]^2*Sin[2*a + 2*b*x])/(6*b*Sqrt[-c + c*Sec[2*a + 2*b*x
]])
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 3774

```
Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[(-2*b)/d,
 Subst[Int[1/(a + x^2), x], x, (b*Cot[c + d*x])/Sqrt[a + b*Csc[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3805

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[(a*Cot[e + f*x]*(d*Csc[e + f*x])^n)/(f*n*Sqrt[a
```

```
 + b*Csc[e + f*x]]), x] + Dist[(a*(2*n + 1))/(2*b*d*n), Int[Sqrt[a + b*Csc[
e + f*x]]*(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f}, x] &&
EqQ[a^2 - b^2, 0] && LtQ[n, -2^(-1)] && IntegerQ[2*n]
```

Rule 4397

```
Int[u_, x_Symbol] :> Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]
```

Rubi steps

$$\int \cos^3(2(a+bx))\sqrt{c\tan(a+bx)\tan(2(a+bx))}\,dx = \int \cos^3(2a+2bx)\sqrt{-c+c\sec(2a+2bx)}\,dx$$

$$= -\frac{c\cos^2(2a+2bx)\sin(2a+2bx)}{6b\sqrt{-c+c\sec(2a+2bx)}} - \frac{5}{6}\int \cos^2(2a+2bx)\sqrt{\ldots}$$

$$= \frac{5c\cos(2a+2bx)\sin(2a+2bx)}{24b\sqrt{-c+c\sec(2a+2bx)}} - \frac{c\cos^2(2a+2bx)\sin(2\ldots}{6b\sqrt{-c+c\sec(2a+\ldots}}$$

$$= -\frac{5c\sin(2a+2bx)}{16b\sqrt{-c+c\sec(2a+2bx)}} + \frac{5c\cos(2a+2bx)\sin(2a+\ldots}{24b\sqrt{-c+c\sec(2a+2\ldots}}$$

$$= -\frac{5c\sin(2a+2bx)}{16b\sqrt{-c+c\sec(2a+2bx)}} + \frac{5c\cos(2a+2bx)\sin(2a+\ldots}{24b\sqrt{-c+c\sec(2a+2\ldots}}$$

$$= \frac{5\sqrt{c}\,\tanh^{-1}\left(\frac{\sqrt{c}\,\tan(2a+2bx)}{\sqrt{-c+c\sec(2a+2bx)}}\right)}{16b} - \frac{5c\sin(2a+2bx)}{16b\sqrt{-c+c\sec(2a+2\ldots}}$$

Mathematica [A] time = 0.29, size = 116, normalized size = 0.66

$$\frac{\sqrt{c\tan(a+bx)\tan(2(a+bx))}\,\Big(30\sin(2(a+bx)) - 2\sin(4(a+bx)) + 4\sin(6(a+bx)) - 26\cot(a+bx) + 15\sqrt{2}\ldots}{96b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[2*(a + b*x)]^3*Sqrt[c*Tan[a + b*x]*Tan[2*(a + b*x)]],x]
```

```
[Out] ((-26*Cot[a + b*x] + 15*Sqrt[2]*ArcTanh[(Sqrt[2]*Cos[a + b*x])/Sqrt[Cos[2*(
a + b*x)]]]*Sqrt[Cos[2*(a + b*x)]]*Csc[a + b*x] + 30*Sin[2*(a + b*x)] - 2*S
in[4*(a + b*x)] + 4*Sin[6*(a + b*x)])*Sqrt[c*Tan[a + b*x]*Tan[2*(a + b*x)]]
)/(96*b)
```

fricas [A] time = 2.22, size = 481, normalized size = 2.73

$$\left[\frac{15\left(\tan\left(bx+a\right)^{7}+3\,\tan\left(bx+a\right)^{5}+3\,\tan\left(bx+a\right)^{3}+\tan\left(bx+a\right)\right)\sqrt{c}\,\log\left(-\frac{c\tan(bx+a)^{5}-14\,c\tan(bx+a)^{3}+4\,\sqrt{2}\left(\tan(bx\right.}{\tan(bx+a)^{5}}\right.}{192\left(b\tan\left(bx+a\right)^{7}+3\,b\tan\left(\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(2*b*x+2*a)^3*(c*tan(b*x+a)*tan(2*b*x+2*a))^(1/2),x, algorithm
="fricas")
```

```
[Out] [1/192*(15*(tan(b*x + a)^7 + 3*tan(b*x + a)^5 + 3*tan(b*x + a)^3 + tan(b*x
+ a))*sqrt(c)*log(-(c*tan(b*x + a)^5 - 14*c*tan(b*x + a)^3 + 4*sqrt(2)*(tan
(b*x + a)^4 - 4*tan(b*x + a)^2 + 3)*sqrt(-c*tan(b*x + a)^2/(tan(b*x + a)^2
- 1))*sqrt(c) + 17*c*tan(b*x + a))/(tan(b*x + a)^5 + 2*tan(b*x + a)^3 + tan
(b*x + a))) + 4*sqrt(2)*(33*tan(b*x + a)^6 - 19*tan(b*x + a)^4 - tan(b*x +
a)^2 - 13)*sqrt(-c*tan(b*x + a)^2/(tan(b*x + a)^2 - 1)))/(b*tan(b*x + a)^7
+ 3*b*tan(b*x + a)^5 + 3*b*tan(b*x + a)^3 + b*tan(b*x + a)), -1/96*(15*(tan
(b*x + a)^7 + 3*tan(b*x + a)^5 + 3*tan(b*x + a)^3 + tan(b*x + a))*sqrt(-c)*
arctan(2*sqrt(2)*sqrt(-c*tan(b*x + a)^2/(tan(b*x + a)^2 - 1))*(tan(b*x + a)
^2 - 1)*sqrt(-c)/(c*tan(b*x + a)^3 - 3*c*tan(b*x + a))) - 2*sqrt(2)*(33*tan
(b*x + a)^6 - 19*tan(b*x + a)^4 - tan(b*x + a)^2 - 13)*sqrt(-c*tan(b*x + a)
^2/(tan(b*x + a)^2 - 1)))/(b*tan(b*x + a)^7 + 3*b*tan(b*x + a)^5 + 3*b*tan(
b*x + a)^3 + b*tan(b*x + a))]
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(2*b*x+2*a)^3*(c*tan(b*x+a)*tan(2*b*x+2*a))^(1/2),x, algorithm
="giac")
```

```
[Out] Timed out
```

maple [B] time = 1.17, size = 933, normalized size = 5.30

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(2*b*x+2*a)^3*(c*tan(b*x+a)*tan(2*b*x+2*a))^(1/2),x)
```

```
[Out] 1/2/b*(c*(1-cos(b*x+a)^2)/(2*cos(b*x+a)^2-1))^(1/2)*sin(b*x+a)*((2*cos(b*x+
a)^2-1)/(cos(b*x+a)+1)^2)^(1/2)*arctanh(1/2*cos(b*x+a)*4^(1/2)*(-1+cos(b*x+
a))/sin(b*x+a)^2/((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(1/2)*2^(1/2))/(-1+c
os(b*x+a))*4^(1/2)-3/8*2^(1/2)/b*sin(b*x+a)*(c*(1-cos(b*x+a)^2)/(2*cos(b*x+
a)^2-1))^(1/2)*(2^(1/2)*((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(1/2)*arctanh
(1/2*cos(b*x+a)*4^(1/2)*(-1+cos(b*x+a))/sin(b*x+a)^2/((2*cos(b*x+a)^2-1)/(c
os(b*x+a)+1)^2)^(1/2)*2^(1/2))*cos(b*x+a)+2^(1/2)*((2*cos(b*x+a)^2-1)/(cos(
b*x+a)+1)^2)^(1/2)*arctanh(1/2*cos(b*x+a)*4^(1/2)*(-1+cos(b*x+a))/sin(b*x+a
)^2/((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(1/2)*2^(1/2))-4*cos(b*x+a)^3+2*c
os(b*x+a))/(-1+cos(b*x+a)^2)*4^(1/2)+3/32*2^(1/2)/b*sin(b*x+a)*(c*(1-cos(b*
x+a)^2)/(2*cos(b*x+a)^2-1))^(1/2)*(-16*cos(b*x+a)^5+3*2^(1/2)*((2*cos(b*x+a
)^2-1)/(cos(b*x+a)+1)^2)^(1/2)*arctanh(1/2*cos(b*x+a)*4^(1/2)*(-1+cos(b*x+a
))/sin(b*x+a)^2/((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(1/2)*2^(1/2))*cos(b*
x+a)+3*2^(1/2)*((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(1/2)*arctanh(1/2*cos(
b*x+a)*4^(1/2)*(-1+cos(b*x+a))/sin(b*x+a)^2/((2*cos(b*x+a)^2-1)/(cos(b*x+a)
+1)^2)^(1/2)*2^(1/2))-4*cos(b*x+a)^3+6*cos(b*x+a))/(-1+cos(b*x+a)^2)*4^(1/2
)-1/192*2^(1/2)/b*sin(b*x+a)*(c*(1-cos(b*x+a)^2)/(2*cos(b*x+a)^2-1))^(1/2)*
(-128*cos(b*x+a)^7-16*cos(b*x+a)^5+15*2^(1/2)*((2*cos(b*x+a)^2-1)/(cos(b*x+
a)+1)^2)^(1/2)*arctanh(1/2*cos(b*x+a)*4^(1/2)*(-1+cos(b*x+a))/sin(b*x+a)^2/
((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(1/2)*2^(1/2))*cos(b*x+a)+15*2^(1/2)*
((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(1/2)*arctanh(1/2*cos(b*x+a)*4^(1/2)*
(-1+cos(b*x+a))/sin(b*x+a)^2/((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(1/2)*2^
(1/2))-20*cos(b*x+a)^3+30*cos(b*x+a))/(-1+cos(b*x+a)^2)*4^(1/2)
```

maxima [B] time = 1.13, size = 2333, normalized size = 13.26

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(2*b*x+2*a)^3*(c*tan(b*x+a)*tan(2*b*x+2*a))^(1/2),x, algorithm
="maxima")
```

```
[Out] -1/384*(8*(cos(2/3*arctan2(sin(6*b*x + 6*a), cos(6*b*x + 6*a)))^2 + sin(2/3
*arctan2(sin(6*b*x + 6*a), cos(6*b*x + 6*a)))^2 + 2*cos(2/3*arctan2(sin(6*b
*x + 6*a), cos(6*b*x + 6*a))) + 1)^(3/4)*(cos(3/2*arctan2(sin(2/3*arctan2(s
in(6*b*x + 6*a), cos(6*b*x + 6*a))), -cos(2/3*arctan2(sin(6*b*x + 6*a), cos
(6*b*x + 6*a))) - 1))*sin(6*b*x + 6*a) + (cos(6*b*x + 6*a) + 1)*sin(3/2*arc
tan2(sin(2/3*arctan2(sin(6*b*x + 6*a), cos(6*b*x + 6*a))), -cos(2/3*arctan2
(sin(6*b*x + 6*a), cos(6*b*x + 6*a))) - 1)))*sqrt(c) + 12*(cos(2/3*arctan2(
sin(6*b*x + 6*a), cos(6*b*x + 6*a)))^2 + sin(2/3*arctan2(sin(6*b*x + 6*a),
cos(6*b*x + 6*a)))^2 + 2*cos(2/3*arctan2(sin(6*b*x + 6*a), cos(6*b*x + 6*a)
)) + 1)^(1/4)*((sin(2/3*arctan2(sin(6*b*x + 6*a), cos(6*b*x + 6*a))) - 5*si
n(1/3*arctan2(sin(6*b*x + 6*a), cos(6*b*x + 6*a))))*cos(1/2*arctan2(sin(2/3
```

```
*arctan2(sin(6*b*x + 6*a), cos(6*b*x + 6*a))), -cos(2/3*arctan2(sin(6*b*x +
 6*a), cos(6*b*x + 6*a))) - 1)) + (cos(2/3*arctan2(sin(6*b*x + 6*a), cos(6*
b*x + 6*a))) - 3*cos(1/3*arctan2(sin(6*b*x + 6*a), cos(6*b*x + 6*a))) - 4)*
sin(1/2*arctan2(sin(2/3*arctan2(sin(6*b*x + 6*a), cos(6*b*x + 6*a))), -cos(
2/3*arctan2(sin(6*b*x + 6*a), cos(6*b*x + 6*a))) - 1)))*sqrt(c) + 15*sqrt(c
)*(log(sqrt(cos(2/3*arctan2(sin(6*b*x + 6*a), cos(6*b*x + 6*a)))^2 + sin(2/
3*arctan2(sin(6*b*x + 6*a), cos(6*b*x + 6*a)))^2 + 2*cos(2/3*arctan2(sin(6*
b*x + 6*a), cos(6*b*x + 6*a))) + 1)*cos(1/2*arctan2(sin(2/3*arctan2(sin(6*b
*x + 6*a), cos(6*b*x + 6*a))), -cos(2/3*arctan2(sin(6*b*x + 6*a), cos(6*b*x
 + 6*a))) - 1))^2 + sqrt(cos(2/3*arctan2(sin(6*b*x + 6*a), cos(6*b*x + 6*a)
))^2 + sin(2/3*arctan2(sin(6*b*x + 6*a), cos(6*b*x + 6*a)))^2 + 2*cos(2/3*a
rctan2(sin(6*b*x + 6*a), cos(6*b*x + 6*a))) + 1)*sin(1/2*arctan2(sin(2/3*ar
ctan2(sin(6*b*x + 6*a), cos(6*b*x + 6*a))), -cos(2/3*arctan2(sin(6*b*x + 6*
a), cos(6*b*x + 6*a))) - 1))^2 + 2*(cos(2/3*arctan2(sin(6*b*x + 6*a), cos(6
*b*x + 6*a)))^2 + sin(2/3*arctan2(sin(6*b*x + 6*a), cos(6*b*x + 6*a)))^2 +
2*cos(2/3*arctan2(sin(6*b*x + 6*a), cos(6*b*x + 6*a))) + 1)^(1/4)*sin(1/2*a
rctan2(sin(2/3*arctan2(sin(6*b*x + 6*a), cos(6*b*x + 6*a))), -cos(2/3*arcta
n2(sin(6*b*x + 6*a), cos(6*b*x + 6*a))) - 1)) + 1) - log(sqrt(cos(2/3*arcta
n2(sin(6*b*x + 6*a), cos(6*b*x + 6*a)))^2 + sin(2/3*arctan2(sin(6*b*x + 6*a
), cos(6*b*x + 6*a)))^2 + 2*cos(2/3*arctan2(sin(6*b*x + 6*a), cos(6*b*x + 6
*a))) + 1)*cos(1/2*arctan2(sin(2/3*arctan2(sin(6*b*x + 6*a), cos(6*b*x + 6*
a))), -cos(2/3*arctan2(sin(6*b*x + 6*a), cos(6*b*x + 6*a))) - 1))^2 + sqrt(
cos(2/3*arctan2(sin(6*b*x + 6*a), cos(6*b*x + 6*a)))^2 + sin(2/3*arctan2(si
n(6*b*x + 6*a), cos(6*b*x + 6*a)))^2 + 2*cos(2/3*arctan2(sin(6*b*x + 6*a),
cos(6*b*x + 6*a))) + 1)*sin(1/2*arctan2(sin(2/3*arctan2(sin(6*b*x + 6*a), c
os(6*b*x + 6*a))), -cos(2/3*arctan2(sin(6*b*x + 6*a), cos(6*b*x + 6*a))) -
1))^2 - 2*(cos(2/3*arctan2(sin(6*b*x + 6*a), cos(6*b*x + 6*a)))^2 + sin(2/3
*arctan2(sin(6*b*x + 6*a), cos(6*b*x + 6*a)))^2 + 2*cos(2/3*arctan2(sin(6*b
*x + 6*a), cos(6*b*x + 6*a))) + 1)^(1/4)*sin(1/2*arctan2(sin(2/3*arctan2(si
n(6*b*x + 6*a), cos(6*b*x + 6*a))), -cos(2/3*arctan2(sin(6*b*x + 6*a), cos(
6*b*x + 6*a))) - 1)) + 1) + log(((cos(1/3*arctan2(sin(6*b*x + 6*a), cos(6*b
*x + 6*a)))^2 + sin(1/3*arctan2(sin(6*b*x + 6*a), cos(6*b*x + 6*a)))^2)*cos
(1/2*arctan2(sin(2/3*arctan2(sin(6*b*x + 6*a), cos(6*b*x + 6*a))), -cos(2/3
*arctan2(sin(6*b*x + 6*a), cos(6*b*x + 6*a))) - 1))^2 + (cos(1/3*arctan2(si
n(6*b*x + 6*a), cos(6*b*x + 6*a)))^2 + sin(1/3*arctan2(sin(6*b*x + 6*a), co
s(6*b*x + 6*a)))^2)*sin(1/2*arctan2(sin(2/3*arctan2(sin(6*b*x + 6*a), cos(6
*b*x + 6*a))), -cos(2/3*arctan2(sin(6*b*x + 6*a), cos(6*b*x + 6*a))) - 1))^
2)*sqrt(cos(2/3*arctan2(sin(6*b*x + 6*a), cos(6*b*x + 6*a)))^2 + sin(2/3*ar
ctan2(sin(6*b*x + 6*a), cos(6*b*x + 6*a)))^2 + 2*cos(2/3*arctan2(sin(6*b*x
+ 6*a), cos(6*b*x + 6*a))) + 1) + 2*(cos(2/3*arctan2(sin(6*b*x + 6*a), cos(
6*b*x + 6*a)))^2 + sin(2/3*arctan2(sin(6*b*x + 6*a), cos(6*b*x + 6*a)))^2 +
 2*cos(2/3*arctan2(sin(6*b*x + 6*a), cos(6*b*x + 6*a))) + 1)^(1/4)*(cos(1/2
*arctan2(sin(2/3*arctan2(sin(6*b*x + 6*a), cos(6*b*x + 6*a))), -cos(2/3*arc
tan2(sin(6*b*x + 6*a), cos(6*b*x + 6*a))) - 1))*sin(1/3*arctan2(sin(6*b*x +
 6*a), cos(6*b*x + 6*a))) + cos(1/3*arctan2(sin(6*b*x + 6*a), cos(6*b*x + 6
```

```
*a)))*sin(1/2*arctan2(sin(2/3*arctan2(sin(6*b*x + 6*a), cos(6*b*x + 6*a))),
 -cos(2/3*arctan2(sin(6*b*x + 6*a), cos(6*b*x + 6*a))) - 1))) + 1) - log(((
cos(1/3*arctan2(sin(6*b*x + 6*a), cos(6*b*x + 6*a)))^2 + sin(1/3*arctan2(si
n(6*b*x + 6*a), cos(6*b*x + 6*a)))^2)*cos(1/2*arctan2(sin(2/3*arctan2(sin(6
*b*x + 6*a), cos(6*b*x + 6*a))), -cos(2/3*arctan2(sin(6*b*x + 6*a), cos(6*b
*x + 6*a))) - 1))^2 + (cos(1/3*arctan2(sin(6*b*x + 6*a), cos(6*b*x + 6*a)))
^2 + sin(1/3*arctan2(sin(6*b*x + 6*a), cos(6*b*x + 6*a)))^2)*sin(1/2*arctan
2(sin(2/3*arctan2(sin(6*b*x + 6*a), cos(6*b*x + 6*a))), -cos(2/3*arctan2(si
n(6*b*x + 6*a), cos(6*b*x + 6*a))) - 1))^2)*sqrt(cos(2/3*arctan2(sin(6*b*x
+ 6*a), cos(6*b*x + 6*a)))^2 + sin(2/3*arctan2(sin(6*b*x + 6*a), cos(6*b*x
+ 6*a)))^2 + 2*cos(2/3*arctan2(sin(6*b*x + 6*a), cos(6*b*x + 6*a))) + 1) -
2*(cos(2/3*arctan2(sin(6*b*x + 6*a), cos(6*b*x + 6*a)))^2 + sin(2/3*arctan2
(sin(6*b*x + 6*a), cos(6*b*x + 6*a)))^2 + 2*cos(2/3*arctan2(sin(6*b*x + 6*a
), cos(6*b*x + 6*a))) + 1)^(1/4)*(cos(1/2*arctan2(sin(2/3*arctan2(sin(6*b*x
 + 6*a), cos(6*b*x + 6*a))), -cos(2/3*arctan2(sin(6*b*x + 6*a), cos(6*b*x +
 6*a))) - 1))*sin(1/3*arctan2(sin(6*b*x + 6*a), cos(6*b*x + 6*a))) + cos(1/
3*arctan2(sin(6*b*x + 6*a), cos(6*b*x + 6*a)))*sin(1/2*arctan2(sin(2/3*arct
an2(sin(6*b*x + 6*a), cos(6*b*x + 6*a))), -cos(2/3*arctan2(sin(6*b*x + 6*a)
, cos(6*b*x + 6*a))) - 1))) + 1)))/b
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \cos(2a+2bx)^3 \sqrt{c\tan(a+bx)\tan(2a+2bx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(2*a + 2*b*x)^3*(c*tan(a + b*x)*tan(2*a + 2*b*x))^(1/2),x)
[Out] int(cos(2*a + 2*b*x)^3*(c*tan(a + b*x)*tan(2*a + 2*b*x))^(1/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(2*b*x+2*a)**3*(c*tan(b*x+a)*tan(2*b*x+2*a))**(1/2),x)
[Out] Timed out
```

3.611 $\int \sec^4(2(a+bx))(c\tan(a+bx)\tan(2(a+bx)))^{3/2}\,dx$

Optimal. Leaf size=208

$$\frac{c^2\tan(2a+2bx)\sec^4(2a+2bx)}{9b\sqrt{c\sec(2a+2bx)-c}}-\frac{17c^2\tan(2a+2bx)\sec^3(2a+2bx)}{63b\sqrt{c\sec(2a+2bx)-c}}+\frac{34c^2\tan(2a+2bx)}{45b\sqrt{c\sec(2a+2bx)-c}}+34\tan(2a+2b$$

```
[Out] 34/105*(-c+c*sec(2*b*x+2*a))^(3/2)*tan(2*b*x+2*a)/b+34/45*c^2*tan(2*b*x+2*a
)/b/(-c+c*sec(2*b*x+2*a))^(1/2)-17/63*c^2*sec(2*b*x+2*a)^3*tan(2*b*x+2*a)/b
/(-c+c*sec(2*b*x+2*a))^(1/2)+1/9*c^2*sec(2*b*x+2*a)^4*tan(2*b*x+2*a)/b/(-c+
c*sec(2*b*x+2*a))^(1/2)+68/315*c*(-c+c*sec(2*b*x+2*a))^(1/2)*tan(2*b*x+2*a)
/b
```

Rubi [A] time = 0.53, antiderivative size = 208, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.226, Rules used = {4397, 3814, 21, 3803, 3800, 4001, 3792}

$$\frac{c^2\tan(2a+2bx)\sec^4(2a+2bx)}{9b\sqrt{c\sec(2a+2bx)-c}}-\frac{17c^2\tan(2a+2bx)\sec^3(2a+2bx)}{63b\sqrt{c\sec(2a+2bx)-c}}+\frac{34c^2\tan(2a+2bx)}{45b\sqrt{c\sec(2a+2bx)-c}}+34\tan(2a+2b$$

Antiderivative was successfully verified.

```
[In] Int[Sec[2*(a + b*x)]^4*(c*Tan[a + b*x]*Tan[2*(a + b*x)])^(3/2),x]
```

```
[Out] (34*c^2*Tan[2*a + 2*b*x])/(45*b*Sqrt[-c + c*Sec[2*a + 2*b*x]]) - (17*c^2*Se
c[2*a + 2*b*x]^3*Tan[2*a + 2*b*x])/(63*b*Sqrt[-c + c*Sec[2*a + 2*b*x]]) + (
c^2*Sec[2*a + 2*b*x]^4*Tan[2*a + 2*b*x])/(9*b*Sqrt[-c + c*Sec[2*a + 2*b*x]]
) + (68*c*Sqrt[-c + c*Sec[2*a + 2*b*x]]*Tan[2*a + 2*b*x])/(315*b) + (34*(-c
 + c*Sec[2*a + 2*b*x])^(3/2)*Tan[2*a + 2*b*x])/(105*b)
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 3792

```
Int[csc[(e_.) + (f_.)*(x_)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[(-2*b*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e + f*x]]), x] /; Free
Q[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3800

```
Int[csc[(e_.) + (f_.)*(x_)]^3*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_),
x_Symbol] :> -Simp[(Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1))/(b*f*(m + 2)
), x] + Dist[1/(b*(m + 2)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m*(b*(m +
 1) - a*Csc[e + f*x]), x], x] /; FreeQ[{a, b, e, f, m}, x] && EqQ[a^2 - b^2
, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 3803

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[(-2*b*d*Cot[e + f*x]*(d*Csc[e + f*x])^(n - 1))/
(f*(2*n - 1)*Sqrt[a + b*Csc[e + f*x]]), x] + Dist[(2*a*d*(n - 1))/(b*(2*n -
 1)), Int[Sqrt[a + b*Csc[e + f*x]]*(d*Csc[e + f*x])^(n - 1), x], x] /; Free
Q[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[n, 1] && IntegerQ[2*n]
```

Rule 3814

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> -Simp[(b^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 2
)*(d*Csc[e + f*x])^n)/(f*(m + n - 1)), x] + Dist[b/(m + n - 1), Int[(a + b*
Csc[e + f*x])^(m - 2)*(d*Csc[e + f*x])^n*(b*(m + 2*n - 1) + a*(3*m + 2*n -
4)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2,
0] && GtQ[m, 1] && NeQ[m + n - 1, 0] && IntegerQ[2*m]
```

Rule 4001

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(B*Cot[e + f*x]*(a
+ b*Csc[e + f*x])^m)/(f*(m + 1)), x] + Dist[(a*B*m + A*b*(m + 1))/(b*(m + 1
)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m, x], x] /; FreeQ[{a, b, A, B, e
, f, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && NeQ[a*B*m + A*b*(m
 + 1), 0] &&  !LtQ[m, -2^(-1)]
```

Rule 4397

```
Int[u_, x_Symbol] :> Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]
```

Rubi steps

$$\int \sec^4(2(a+bx))(c\tan(a+bx)\tan(2(a+bx)))^{3/2}\,dx = \int \sec^4(2a+2bx)(-c+c\sec(2a+2bx))^{3/2}\,dx$$

$$= \frac{c^2\sec^4(2a+2bx)\tan(2a+2bx)}{9b\sqrt{-c+c\sec(2a+2bx)}} + \frac{1}{9}(2c)\int \frac{\sec^4(2a+2b}{\sqrt{-}}$$

$$= \frac{c^2\sec^4(2a+2bx)\tan(2a+2bx)}{9b\sqrt{-c+c\sec(2a+2bx)}} - \frac{1}{9}(17c)\int \sec^4(2a+2$$

$$= -\frac{17c^2\sec^3(2a+2bx)\tan(2a+2bx)}{63b\sqrt{-c+c\sec(2a+2bx)}} + \frac{c^2\sec^4(2a+2bx)}{9b\sqrt{-c+c\sec}}$$

$$= -\frac{17c^2\sec^3(2a+2bx)\tan(2a+2bx)}{63b\sqrt{-c+c\sec(2a+2bx)}} + \frac{c^2\sec^4(2a+2bx)}{9b\sqrt{-c+c\sec}}$$

$$= -\frac{17c^2\sec^3(2a+2bx)\tan(2a+2bx)}{63b\sqrt{-c+c\sec(2a+2bx)}} + \frac{c^2\sec^4(2a+2bx)}{9b\sqrt{-c+c\sec}}$$

$$= \frac{34c^2\tan(2a+2bx)}{45b\sqrt{-c+c\sec(2a+2bx)}} - \frac{17c^2\sec^3(2a+2bx)\tan(2a}{63b\sqrt{-c+c\sec(2a+2}}$$

Mathematica [A] time = 0.35, size = 85, normalized size = 0.41

$$\frac{\cot(a+bx)(c\tan(a+bx)\tan(2(a+bx)))^{3/2}\left(188\cot(a+bx)\cot(2(a+bx))+35\sec^3(2(a+bx))-50\sec^2(2(a+\right.}{315b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[2*(a + b*x)]^4*(c*Tan[a + b*x]*Tan[2*(a + b*x)])^(3/2),x]
```

```
[Out] (Cot[a + b*x]*(-84 + 188*Cot[a + b*x]*Cot[2*(a + b*x)] + 52*Sec[2*(a + b*x)
] - 50*Sec[2*(a + b*x)]^2 + 35*Sec[2*(a + b*x)]^3)*(c*Tan[a + b*x]*Tan[2*(a
 + b*x)])^(3/2))/(315*b)
```

fricas [A] time = 1.05, size = 132, normalized size = 0.63

$$\frac{2\sqrt{2}\left(315\,c\tan\left(bx+a\right)^8-525\,c\tan\left(bx+a\right)^6+819\,c\tan\left(bx+a\right)^4-423\,c\tan\left(bx+a\right)^2+94\,c\right)\sqrt{-\frac{c\tan(bx+a)^2}{\tan(bx+a)^2-1}}}{315\left(b\tan\left(bx+a\right)^9-4\,b\tan\left(bx+a\right)^7+6\,b\tan\left(bx+a\right)^5-4\,b\tan\left(bx+a\right)^3+b\tan\left(bx+a\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(2*b*x+2*a)^4*(c*tan(b*x+a)*tan(2*b*x+2*a))^(3/2),x, algorithm
="fricas")
```

```
[Out] 2/315*sqrt(2)*(315*c*tan(b*x + a)^8 - 525*c*tan(b*x + a)^6 + 819*c*tan(b*x
+ a)^4 - 423*c*tan(b*x + a)^2 + 94*c)*sqrt(-c*tan(b*x + a)^2/(tan(b*x + a)^
2 - 1))/(b*tan(b*x + a)^9 - 4*b*tan(b*x + a)^7 + 6*b*tan(b*x + a)^5 - 4*b*t
an(b*x + a)^3 + b*tan(b*x + a))
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(2*b*x+2*a)^4*(c*tan(b*x+a)*tan(2*b*x+2*a))^(3/2),x, algorithm
="giac")
```

```
[Out] Timed out
```

maple [A] time = 1.12, size = 105, normalized size = 0.50

$$\frac{2\sqrt{2}\,\left(2176\left(\cos^{8}\left(bx+a\right)\right)-4896\left(\cos^{6}\left(bx+a\right)\right)+4284\left(\cos^{4}\left(bx+a\right)\right)-1785\left(\cos^{2}\left(bx+a\right)\right)+315\right)\cos\left(bx\right.}{315b\left(2\left(\cos^{2}\left(bx+a\right)\right)-1\right)^{3}\sin\left(bx+a\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(2*b*x+2*a)^4*(c*tan(b*x+a)*tan(2*b*x+2*a))^(3/2),x)
```

```
[Out] 2/315*2^(1/2)/b*(2176*cos(b*x+a)^8-4896*cos(b*x+a)^6+4284*cos(b*x+a)^4-1785
*cos(b*x+a)^2+315)*cos(b*x+a)*(c*sin(b*x+a)^2/(2*cos(b*x+a)^2-1))^(3/2)/(2*
cos(b*x+a)^2-1)^3/sin(b*x+a)^3
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(2*b*x+2*a)^4*(c*tan(b*x+a)*tan(2*b*x+2*a))^(3/2),x, algorithm
="maxima")
```

```
[Out] Timed out
```

mupad [B] time = 10.18, size = 594, normalized size = 2.86

$$\frac{\left(\frac{c\,16\mathrm{i}}{9\,b}+\frac{c\,\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}\,16\mathrm{i}}{9\,b}\right)\sqrt{\frac{c\,\left(\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)\left(\mathrm{e}^{a\,4\mathrm{i}+b\,x\,4\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)}{\left(\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}+1\right)\left(\mathrm{e}^{a\,4\mathrm{i}+b\,x\,4\mathrm{i}}+1\right)}}}{\left(\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}-1\right)\left(\mathrm{e}^{a\,4\mathrm{i}+b\,x\,4\mathrm{i}}+1\right)^{4}}-\frac{\left(\frac{c\,40\mathrm{i}}{7\,b}+\frac{c\,\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}\,88\mathrm{i}}{63\,b}\right)\sqrt{\frac{c\,\left(\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)\left(\mathrm{e}^{a\,4\mathrm{i}+b\,x\,4\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)}{\left(\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}+1\right)\left(\mathrm{e}^{a\,4\mathrm{i}+b\,x\,4\mathrm{i}}+1\right)}}}{\left(\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}-1\right)\left(\mathrm{e}^{a\,4\mathrm{i}+b\,x\,4\mathrm{i}}+1\right)^{3}}+\frac{\left(\frac{c\,24\mathrm{i}}{5\,b}-\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*tan(a + b*x)*tan(2*a + 2*b*x))^(3/2)/cos(2*a + 2*b*x)^4,x)
```

```
[Out] (((c*16i)/(9*b) + (c*exp(a*2i + b*x*2i)*16i)/(9*b))*((c*(exp(a*2i + b*x*2i)
*1i - 1i)*(exp(a*4i + b*x*4i)*1i - 1i))/((exp(a*2i + b*x*2i) + 1)*(exp(a*4i
 + b*x*4i) + 1)))^(1/2))/((exp(a*2i + b*x*2i) - 1)*(exp(a*4i + b*x*4i) + 1)
^4) - (((c*40i)/(7*b) + (c*exp(a*2i + b*x*2i)*88i)/(63*b))*((c*(exp(a*2i +
b*x*2i)*1i - 1i)*(exp(a*4i + b*x*4i)*1i - 1i))/((exp(a*2i + b*x*2i) + 1)*(e
xp(a*4i + b*x*4i) + 1)))^(1/2))/((exp(a*2i + b*x*2i) - 1)*(exp(a*4i + b*x*4
i) + 1)^3) + (((c*24i)/(5*b) - (c*exp(a*2i + b*x*2i)*176i)/(105*b))*((c*(ex
p(a*2i + b*x*2i)*1i - 1i)*(exp(a*4i + b*x*4i)*1i - 1i))/((exp(a*2i + b*x*2i
) + 1)*(exp(a*4i + b*x*4i) + 1)))^(1/2))/((exp(a*2i + b*x*2i) - 1)*(exp(a*4
i + b*x*4i) + 1)^2) + (c*exp(a*2i + b*x*2i)*((c*(exp(a*2i + b*x*2i)*1i - 1i
)*(exp(a*4i + b*x*4i)*1i - 1i))/((exp(a*2i + b*x*2i) + 1)*(exp(a*4i + b*x*4
i) + 1)))^(1/2)*272i)/(315*b*(exp(a*2i + b*x*2i) - 1)) + (c*exp(a*2i + b*x*
2i)*((c*(exp(a*2i + b*x*2i)*1i - 1i)*(exp(a*4i + b*x*4i)*1i - 1i))/((exp(a*
2i + b*x*2i) + 1)*(exp(a*4i + b*x*4i) + 1)))^(1/2)*136i)/(315*b*(exp(a*2i +
 b*x*2i) - 1)*(exp(a*4i + b*x*4i) + 1))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(2*b*x+2*a)**4*(c*tan(b*x+a)*tan(2*b*x+2*a))**(3/2),x)
```

```
[Out] Timed out
```

3.612 $\int \sec^3(2(a+bx))(c\tan(a+bx)\tan(2(a+bx)))^{3/2}\,dx$

Optimal. Leaf size=148

$$-\frac{76c^2\tan(2a+2bx)}{105b\sqrt{c\sec(2a+2bx)-c}}+\frac{\tan(2a+2bx)(c\sec(2a+2bx)-c)^{5/2}}{7bc}+\frac{2\tan(2a+2bx)(c\sec(2a+2bx)-c)^{3/2}}{35b}+$$

```
[Out] 2/35*(-c+c*sec(2*b*x+2*a))^(3/2)*tan(2*b*x+2*a)/b+1/7*(-c+c*sec(2*b*x+2*a))
^(5/2)*tan(2*b*x+2*a)/b/c-76/105*c^2*tan(2*b*x+2*a)/b/(-c+c*sec(2*b*x+2*a))
^(1/2)+19/105*c*(-c+c*sec(2*b*x+2*a))^(1/2)*tan(2*b*x+2*a)/b
```

Rubi [A] time = 0.35, antiderivative size = 148, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.161, Rules used = {4397, 3800, 4001, 3793, 3792}

$$-\frac{76c^2\tan(2a+2bx)}{105b\sqrt{c\sec(2a+2bx)-c}}+\frac{\tan(2a+2bx)(c\sec(2a+2bx)-c)^{5/2}}{7bc}+\frac{2\tan(2a+2bx)(c\sec(2a+2bx)-c)^{3/2}}{35b}+$$

Antiderivative was successfully verified.

```
[In] Int[Sec[2*(a + b*x)]^3*(c*Tan[a + b*x]*Tan[2*(a + b*x)])^(3/2),x]
```

```
[Out] (-76*c^2*Tan[2*a + 2*b*x])/(105*b*Sqrt[-c + c*Sec[2*a + 2*b*x]]) + (19*c*Sq
rt[-c + c*Sec[2*a + 2*b*x]]*Tan[2*a + 2*b*x])/(105*b) + (2*(-c + c*Sec[2*a
+ 2*b*x])^(3/2)*Tan[2*a + 2*b*x])/(35*b) + ((-c + c*Sec[2*a + 2*b*x])^(5/2)
*Tan[2*a + 2*b*x])/(7*b*c)
```

Rule 3792

```
Int[csc[(e_.) + (f_.)*(x_)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[(-2*b*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e + f*x]]), x] /; Free
Q[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3793

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_
Symbol] :> -Simp[(b*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 1))/(f*m), x] +
Dist[(a*(2*m - 1))/m, Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m - 1), x], x]
 /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, 1/2] && IntegerQ[
2*m]
```

Rule 3800

```
Int[csc[(e_.) + (f_.)*(x_)]^3*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_),
x_Symbol] :> -Simp[(Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1))/(b*f*(m + 2)
```

```
), x] + Dist[1/(b*(m + 2)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m*(b*(m +
 1) - a*Csc[e + f*x]), x], x] /; FreeQ[{a, b, e, f, m}, x] && EqQ[a^2 - b^2
, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 4001

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(B*Cot[e + f*x]*(a
+ b*Csc[e + f*x])^m)/(f*(m + 1)), x] + Dist[(a*B*m + A*b*(m + 1))/(b*(m + 1
)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m, x], x] /; FreeQ[{a, b, A, B, e
, f, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && NeQ[a*B*m + A*b*(m
 + 1), 0] &&  !LtQ[m, -2^(-1)]
```

Rule 4397

```
Int[u_, x_Symbol] :> Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]
```

Rubi steps

$$
\begin{aligned}
\int \sec^3(2(a+bx))(c\tan(a+bx)\tan(2(a+bx)))^{3/2}\,dx &= \int \sec^3(2a+2bx)(-c+c\sec(2a+2bx))^{3/2}\,dx \\
&= \frac{(-c+c\sec(2a+2bx))^{5/2}\tan(2a+2bx)}{7bc} + \frac{2\int \sec(2a+2
\\
&= \frac{2(-c+c\sec(2a+2bx))^{3/2}\tan(2a+2bx)}{35b} + \frac{(-c+c\sec(2
\\
&= \frac{19c\sqrt{-c+c\sec(2a+2bx)}\,\tan(2a+2bx)}{105b} + \frac{2(-c+c\sec
\\
&= -\frac{76c^2\tan(2a+2bx)}{105b\sqrt{-c+c\sec(2a+2bx)}} + \frac{19c\sqrt{-c+c\sec(2a+2b
}{105b}
\end{aligned}
$$

Mathematica [A] time = 0.21, size = 73, normalized size = 0.49

$$
-\frac{\cot(a+bx)(c\tan(a+bx)\tan(2(a+bx)))^{3/2}\left(76\cot(a+bx)\cot(2(a+bx))-15\sec^2(2(a+bx))+24\sec(2(a+b
}{105b}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[2*(a + b*x)]^3*(c*Tan[a + b*x]*Tan[2*(a + b*x)])^(3/2),x]
```

```
[Out] -1/105*(Cot[a + b*x]*(-28 + 76*Cot[a + b*x]*Cot[2*(a + b*x)] + 24*Sec[2*(a
+ b*x)] - 15*Sec[2*(a + b*x)]^2)*(c*Tan[a + b*x]*Tan[2*(a + b*x)])^(3/2))/b
```

fricas [A] time = 0.97, size = 111, normalized size = 0.75

$$-\frac{2\sqrt{2}\left(105\,c\tan\left(bx+a\right)^{6}-140\,c\tan\left(bx+a\right)^{4}+133\,c\tan\left(bx+a\right)^{2}-38\,c\right)\sqrt{-\frac{c\tan(bx+a)^{2}}{\tan(bx+a)^{2}-1}}}{105\left(b\tan\left(bx+a\right)^{7}-3\,b\tan\left(bx+a\right)^{5}+3\,b\tan\left(bx+a\right)^{3}-b\tan\left(bx+a\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(2*b*x+2*a)^3*(c*tan(b*x+a)*tan(2*b*x+2*a))^(3/2),x, algorithm
="fricas")
```

```
[Out] -2/105*sqrt(2)*(105*c*tan(b*x + a)^6 - 140*c*tan(b*x + a)^4 + 133*c*tan(b*x
 + a)^2 - 38*c)*sqrt(-c*tan(b*x + a)^2/(tan(b*x + a)^2 - 1))/(b*tan(b*x + a
)^7 - 3*b*tan(b*x + a)^5 + 3*b*tan(b*x + a)^3 - b*tan(b*x + a))
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(2*b*x+2*a)^3*(c*tan(b*x+a)*tan(2*b*x+2*a))^(3/2),x, algorithm
="giac")
```

```
[Out] Timed out
```

maple [A] time = 0.94, size = 95, normalized size = 0.64

$$-\frac{2\sqrt{2}\,\left(416\left(\cos^{6}\left(bx+a\right)\right)-728\left(\cos^{4}\left(bx+a\right)\right)+455\left(\cos^{2}\left(bx+a\right)\right)-105\right)\cos\left(bx+a\right)\left(\frac{c\left(\sin^{2}(bx+a)\right)}{2\left(\cos^{2}(bx+a)\right)-1}\right)^{\frac{3}{2}}}{105b\left(2\left(\cos^{2}\left(bx+a\right)\right)-1\right)^{2}\sin\left(bx+a\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(2*b*x+2*a)^3*(c*tan(b*x+a)*tan(2*b*x+2*a))^(3/2),x)
```

```
[Out] -2/105*2^(1/2)/b*(416*cos(b*x+a)^6-728*cos(b*x+a)^4+455*cos(b*x+a)^2-105)*c
os(b*x+a)*(c*sin(b*x+a)^2/(2*cos(b*x+a)^2-1))^(3/2)/(2*cos(b*x+a)^2-1)^2/si
n(b*x+a)^3
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(2*b*x+2*a)^3*(c*tan(b*x+a)*tan(2*b*x+2*a))^(3/2),x, algorithm
="maxima")
```

```
[Out] Timed out
```

mupad [B] time = 9.26, size = 479, normalized size = 3.24

$$\frac{\left(\frac{c\,8\mathrm{i}}{7\,b}-\frac{c\,\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}\,8\mathrm{i}}{7\,b}\right)\sqrt{\frac{c\,\left(\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)\,\left(\mathrm{e}^{a\,4\mathrm{i}+b\,x\,4\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)}{\left(\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}+1\right)\,\left(\mathrm{e}^{a\,4\mathrm{i}+b\,x\,4\mathrm{i}}+1\right)}}}{\left(\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}-1\right)\,{\left(\mathrm{e}^{a\,4\mathrm{i}+b\,x\,4\mathrm{i}}+1\right)}^3}-\frac{\left(\frac{c\,4\mathrm{i}}{5\,b}-\frac{c\,\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}\,92\mathrm{i}}{35\,b}\right)\sqrt{\frac{c\,\left(\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)\,\left(\mathrm{e}^{a\,4\mathrm{i}+b\,x\,4\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)}{\left(\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}+1\right)\,\left(\mathrm{e}^{a\,4\mathrm{i}+b\,x\,4\mathrm{i}}+1\right)}}}{\left(\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}-1\right)\,{\left(\mathrm{e}^{a\,4\mathrm{i}+b\,x\,4\mathrm{i}}+1\right)}^2}-\frac{\left(\frac{c\,4\mathrm{i}}{3\,b}+\frac{c\,\mathrm{e}^{a\,2\mathrm{i}+}}{10}\right.}{\left(\mathrm{e}^{a}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*tan(a + b*x)*tan(2*a + 2*b*x))^(3/2)/cos(2*a + 2*b*x)^3,x)
```

```
[Out] (((c*8i)/(7*b) - (c*exp(a*2i + b*x*2i)*8i)/(7*b))*((c*(exp(a*2i + b*x*2i)*1
i - 1i)*(exp(a*4i + b*x*4i)*1i - 1i))/((exp(a*2i + b*x*2i) + 1)*(exp(a*4i +
 b*x*4i) + 1)))^(1/2))/((exp(a*2i + b*x*2i) - 1)*(exp(a*4i + b*x*4i) + 1)^3
) - (((c*4i)/(5*b) - (c*exp(a*2i + b*x*2i)*92i)/(35*b))*((c*(exp(a*2i + b*x
*2i)*1i - 1i)*(exp(a*4i + b*x*4i)*1i - 1i))/((exp(a*2i + b*x*2i) + 1)*(exp(
a*4i + b*x*4i) + 1)))^(1/2))/((exp(a*2i + b*x*2i) - 1)*(exp(a*4i + b*x*4i)
+ 1)^2) - (((c*4i)/(3*b) + (c*exp(a*2i + b*x*2i)*52i)/(105*b))*((c*(exp(a*2
i + b*x*2i)*1i - 1i)*(exp(a*4i + b*x*4i)*1i - 1i))/((exp(a*2i + b*x*2i) + 1
)*(exp(a*4i + b*x*4i) + 1)))^(1/2))/((exp(a*2i + b*x*2i) - 1)*(exp(a*4i + b
*x*4i) + 1)) - (c*exp(a*2i + b*x*2i)*((c*(exp(a*2i + b*x*2i)*1i - 1i)*(exp(
a*4i + b*x*4i)*1i - 1i))/((exp(a*2i + b*x*2i) + 1)*(exp(a*4i + b*x*4i) + 1)
))^(1/2)*104i)/(105*b*(exp(a*2i + b*x*2i) - 1))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(2*b*x+2*a)**3*(c*tan(b*x+a)*tan(2*b*x+2*a))**(3/2),x)
```

```
[Out] Timed out
```

3.613 $\int \sec^2(2(a+bx))(c\tan(a+bx)\tan(2(a+bx)))^{3/2}\,dx$

Optimal. Leaf size=110

$$\frac{4c^2\tan(2a+2bx)}{5b\sqrt{c\sec(2a+2bx)-c}}-\frac{c\tan(2a+2bx)\sqrt{c\sec(2a+2bx)-c}}{5b}+\frac{\tan(2a+2bx)(c\sec(2a+2bx)-c)^{3/2}}{5b}$$

```
[Out] 1/5*(-c+c*sec(2*b*x+2*a))^(3/2)*tan(2*b*x+2*a)/b+4/5*c^2*tan(2*b*x+2*a)/b/(
-c+c*sec(2*b*x+2*a))^(1/2)-1/5*c*(-c+c*sec(2*b*x+2*a))^(1/2)*tan(2*b*x+2*a)
/b
```

Rubi [A] time = 0.27, antiderivative size = 110, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.129, Rules used = {4397, 3798, 3793, 3792}

$$\frac{4c^2\tan(2a+2bx)}{5b\sqrt{c\sec(2a+2bx)-c}}-\frac{c\tan(2a+2bx)\sqrt{c\sec(2a+2bx)-c}}{5b}+\frac{\tan(2a+2bx)(c\sec(2a+2bx)-c)^{3/2}}{5b}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[2*(a + b*x)]^2*(c*Tan[a + b*x]*Tan[2*(a + b*x)])^(3/2),x]
```

```
[Out] (4*c^2*Tan[2*a + 2*b*x])/(5*b*Sqrt[-c + c*Sec[2*a + 2*b*x]]) - (c*Sqrt[-c +
 c*Sec[2*a + 2*b*x]]*Tan[2*a + 2*b*x])/(5*b) + ((-c + c*Sec[2*a + 2*b*x])^(
3/2)*Tan[2*a + 2*b*x])/(5*b)
```

Rule 3792

```
Int[csc[(e_.) + (f_.)*(x_)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[(-2*b*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e + f*x]]), x] /; Free
Q[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3793

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_
Symbol] :> -Simp[(b*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 1))/(f*m), x] +
Dist[(a*(2*m - 1))/m, Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m - 1), x], x]
 /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, 1/2] && IntegerQ[
2*m]
```

Rule 3798

```
Int[csc[(e_.) + (f_.)*(x_)]^2*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_),
x_Symbol] :> -Simp[(Cot[e + f*x]*(a + b*Csc[e + f*x])^m)/(f*(m + 1)), x] +
Dist[(a*m)/(b*(m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m, x], x] /;
```

```
FreeQ[{a, b, e, f, m}, x] && EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 4397

```
Int[u_, x_Symbol] :> Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]
```

Rubi steps

$$\int \sec^2(2(a+bx))(c\tan(a+bx)\tan(2(a+bx)))^{3/2}\,dx = \int \sec^2(2a+2bx)(-c+c\sec(2a+2bx))^{3/2}\,dx$$
$$= \frac{(-c+c\sec(2a+2bx))^{3/2}\tan(2a+2bx)}{5b} - \frac{3}{5}\int \sec(2a+$$
$$= -\frac{c\sqrt{-c+c\sec(2a+2bx)}\,\tan(2a+2bx)}{5b} + \frac{(-c+c\sec(2a$$
$$= \frac{4c^2\tan(2a+2bx)}{5b\sqrt{-c+c\sec(2a+2bx)}} - \frac{c\sqrt{-c+c\sec(2a+2bx)}\,\tan}{5b}$$

Mathematica [A] time = 0.23, size = 59, normalized size = 0.54

$$\frac{\cot(a+bx)(c\tan(a+bx)\tan(2(a+bx)))^{3/2}(4\cot(a+bx)\cot(2(a+bx))+\sec(2(a+bx))-2)}{5b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[2*(a + b*x)]^2*(c*Tan[a + b*x]*Tan[2*(a + b*x)])^(3/2),x]
```

```
[Out] (Cot[a + b*x]*(-2 + 4*Cot[a + b*x]*Cot[2*(a + b*x)] + Sec[2*(a + b*x)])*(c*
Tan[a + b*x]*Tan[2*(a + b*x)])^(3/2))/(5*b)
```

fricas [A] time = 0.90, size = 88, normalized size = 0.80

$$\frac{2\sqrt{2}\left(5\,c\tan\left(bx+a\right)^4-5\,c\tan\left(bx+a\right)^2+2\,c\right)\sqrt{-\frac{c\tan(bx+a)^2}{\tan(bx+a)^2-1}}}{5\left(b\tan\left(bx+a\right)^5-2\,b\tan\left(bx+a\right)^3+b\tan\left(bx+a\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(2*b*x+2*a)^2*(c*tan(b*x+a)*tan(2*b*x+2*a))^(3/2),x, algorithm
="fricas")
```

```
[Out] 2/5*sqrt(2)*(5*c*tan(b*x + a)^4 - 5*c*tan(b*x + a)^2 + 2*c)*sqrt(-c*tan(b*x
 + a)^2/(tan(b*x + a)^2 - 1))/(b*tan(b*x + a)^5 - 2*b*tan(b*x + a)^3 + b*ta
n(b*x + a))
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(2*b*x+2*a)^2*(c*tan(b*x+a)*tan(2*b*x+2*a))^(3/2),x, algorithm
="giac")
```

```
[Out] Timed out
```

maple [A] time = 0.86, size = 85, normalized size = 0.77

$$\frac{2\sqrt{2}\,\left(12\left(\cos^4\left(bx+a\right)\right)-15\left(\cos^2\left(bx+a\right)\right)+5\right)\cos\left(bx+a\right)\left(\frac{c\left(\sin^2(bx+a)\right)}{2\left(\cos^2(bx+a)\right)-1}\right)^{\frac{3}{2}}}{5b\left(2\left(\cos^2\left(bx+a\right)\right)-1\right)\sin\left(bx+a\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(2*b*x+2*a)^2*(c*tan(b*x+a)*tan(2*b*x+2*a))^(3/2),x)
```

```
[Out] 2/5*2^(1/2)/b*(12*cos(b*x+a)^4-15*cos(b*x+a)^2+5)*cos(b*x+a)*(c*sin(b*x+a)^
2/(2*cos(b*x+a)^2-1))^(3/2)/(2*cos(b*x+a)^2-1)/sin(b*x+a)^3
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(2*b*x+2*a)^2*(c*tan(b*x+a)*tan(2*b*x+2*a))^(3/2),x, algorithm
="maxima")
```

```
[Out] Timed out
```

mupad [B] time = 11.04, size = 149, normalized size = 1.35

$$\frac{2\,c\,\left(\mathrm{e}^{a\,4\mathrm{i}+b\,x\,4\mathrm{i}}\,5\mathrm{i}+\mathrm{e}^{a\,6\mathrm{i}+b\,x\,6\mathrm{i}}\,5\mathrm{i}+\mathrm{e}^{a\,10\mathrm{i}+b\,x\,10\mathrm{i}}\,3\mathrm{i}+3\mathrm{i}\right)\,\sqrt{\frac{c\,\left(\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)\,\left(\mathrm{e}^{a\,4\mathrm{i}+b\,x\,4\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)}{\left(\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}+1\right)\,\left(\mathrm{e}^{a\,4\mathrm{i}+b\,x\,4\mathrm{i}}+1\right)}}}{5\,b\,\left(\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}-1\right)\,{\left(\mathrm{e}^{a\,4\mathrm{i}+b\,x\,4\mathrm{i}}+1\right)}^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*tan(a + b*x)*tan(2*a + 2*b*x))^(3/2)/cos(2*a + 2*b*x)^2,x)
```

```
[Out] (2*c*(exp(a*4i + b*x*4i)*5i + exp(a*6i + b*x*6i)*5i + exp(a*10i + b*x*10i)*
3i + 3i)*((c*(exp(a*2i + b*x*2i)*1i - 1i)*(exp(a*4i + b*x*4i)*1i - 1i))/((e
xp(a*2i + b*x*2i) + 1)*(exp(a*4i + b*x*4i) + 1)))^(1/2))/(5*b*(exp(a*2i + b
*x*2i) - 1)*(exp(a*4i + b*x*4i) + 1)^2)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(2*b*x+2*a)**2*(c*tan(b*x+a)*tan(2*b*x+2*a))**(3/2),x)
```

```
[Out] Timed out
```

3.614 $\int \sec(2(a+bx))(c\tan(a+bx)\tan(2(a+bx)))^{3/2}\,dx$

Optimal. Leaf size=75

$$\frac{c\tan(2a+2bx)\sqrt{c\sec(2a+2bx)-c}}{3b}-\frac{4c^2\tan(2a+2bx)}{3b\sqrt{c\sec(2a+2bx)-c}}$$

```
[Out] -4/3*c^2*tan(2*b*x+2*a)/b/(-c+c*sec(2*b*x+2*a))^(1/2)+1/3*c*(-c+c*sec(2*b*x
+2*a))^(1/2)*tan(2*b*x+2*a)/b
```

Rubi [A] time = 0.11, antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {4397, 3793, 3792}

$$\frac{c\tan(2a+2bx)\sqrt{c\sec(2a+2bx)-c}}{3b}-\frac{4c^2\tan(2a+2bx)}{3b\sqrt{c\sec(2a+2bx)-c}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[2*(a + b*x)]*(c*Tan[a + b*x]*Tan[2*(a + b*x)])^(3/2),x]
```

```
[Out] (-4*c^2*Tan[2*a + 2*b*x])/(3*b*Sqrt[-c + c*Sec[2*a + 2*b*x]]) + (c*Sqrt[-c
+ c*Sec[2*a + 2*b*x]]*Tan[2*a + 2*b*x])/(3*b)
```

Rule 3792

```
Int[csc[(e_.) + (f_.)*(x_)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[(-2*b*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e + f*x]]), x] /; Free
Q[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3793

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_
Symbol] :> -Simp[(b*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 1))/(f*m), x] +
Dist[(a*(2*m - 1))/m, Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m - 1), x], x]
 /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, 1/2] && IntegerQ[
2*m]
```

Rule 4397

```
Int[u_, x_Symbol] :> Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]
```

Rubi steps

$$\int \sec(2(a+bx))(c\tan(a+bx)\tan(2(a+bx)))^{3/2}\,dx = \int \sec(2a+2bx)(-c+c\sec(2a+2bx))^{3/2}\,dx$$
$$= \frac{c\sqrt{-c+c\sec(2a+2bx)}\,\tan(2a+2bx)}{3b} - \frac{1}{3}(4c)\int \sec(2a$$
$$= -\frac{4c^2\tan(2a+2bx)}{3b\sqrt{-c+c\sec(2a+2bx)}} + \frac{c\sqrt{-c+c\sec(2a+2bx)}\,\tan}{3b}$$

Mathematica [A] time = 0.16, size = 51, normalized size = 0.68

$$-\frac{\cot(a+bx)(4\cot(a+bx)\cot(2(a+bx))-1)(c\tan(a+bx)\tan(2(a+bx)))^{3/2}}{3b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[2*(a + b*x)]*(c*Tan[a + b*x]*Tan[2*(a + b*x)])^(3/2),x]
```

```
[Out] -1/3*(Cot[a + b*x]*(-1 + 4*Cot[a + b*x]*Cot[2*(a + b*x)])*(c*Tan[a + b*x]*T
an[2*(a + b*x)])^(3/2))/b
```

fricas [A] time = 0.88, size = 67, normalized size = 0.89

$$-\frac{2\sqrt{2}\left(3\,c\tan\left(bx+a\right)^2-2\,c\right)\sqrt{-\frac{c\tan\left(bx+a\right)^2}{\tan\left(bx+a\right)^2-1}}}{3\left(b\tan\left(bx+a\right)^3-b\tan\left(bx+a\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(2*b*x+2*a)*(c*tan(b*x+a)*tan(2*b*x+2*a))^(3/2),x, algorithm="
fricas")
```

```
[Out] -2/3*sqrt(2)*(3*c*tan(b*x + a)^2 - 2*c)*sqrt(-c*tan(b*x + a)^2/(tan(b*x + a
)^2 - 1))/(b*tan(b*x + a)^3 - b*tan(b*x + a))
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(2*b*x+2*a)*(c*tan(b*x+a)*tan(2*b*x+2*a))^(3/2),x, algorithm="
giac")
```

```
[Out] Timed out
```

maple [A] time = 0.83, size = 61, normalized size = 0.81

$$-\frac{2\sqrt{2}\,\left(5\left(\cos^{2}\left(bx+a\right)\right)-3\right)\cos\left(bx+a\right)\left(\frac{c\left(\sin^{2}\left(bx+a\right)\right)}{2\left(\cos^{2}\left(bx+a\right)\right)-1}\right)^{\frac{3}{2}}}{3b\sin\left(bx+a\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(2*b*x+2*a)*(c*tan(b*x+a)*tan(2*b*x+2*a))^(3/2),x)
```

```
[Out] -2/3*2^(1/2)/b*(5*cos(b*x+a)^2-3)*cos(b*x+a)*(c*sin(b*x+a)^2/(2*cos(b*x+a)^
2-1))^(3/2)/sin(b*x+a)^3
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(2*b*x+2*a)*(c*tan(b*x+a)*tan(2*b*x+2*a))^(3/2),x, algorithm="
maxima")
```

```
[Out] Timed out
```

mupad [B] time = 7.87, size = 158, normalized size = 2.11

$$-\frac{c\,\left({\mathrm{e}}^{a\,2{}\mathrm{i}+b\,x\,2{}\mathrm{i}}\,3{}\mathrm{i}+{\mathrm{e}}^{a\,4{}\mathrm{i}+b\,x\,4{}\mathrm{i}}\,3{}\mathrm{i}+{\mathrm{e}}^{a\,6{}\mathrm{i}+b\,x\,6{}\mathrm{i}}\,5{}\mathrm{i}+5{}\mathrm{i}\right)\,\sqrt{\frac{c\,\left({\mathrm{e}}^{a\,2{}\mathrm{i}+b\,x\,2{}\mathrm{i}}\,1{}\mathrm{i}-\mathrm{i}\right)\,\left({\mathrm{e}}^{a\,4{}\mathrm{i}+b\,x\,4{}\mathrm{i}}\,1{}\mathrm{i}-\mathrm{i}\right)}{\left({\mathrm{e}}^{a\,2{}\mathrm{i}+b\,x\,2{}\mathrm{i}}+1\right)\,\left({\mathrm{e}}^{a\,4{}\mathrm{i}+b\,x\,4{}\mathrm{i}}+1\right)}}}{3\,b\,\left({\mathrm{e}}^{a\,2{}\mathrm{i}+b\,x\,2{}\mathrm{i}}-{\mathrm{e}}^{a\,4{}\mathrm{i}+b\,x\,4{}\mathrm{i}}+{\mathrm{e}}^{a\,6{}\mathrm{i}+b\,x\,6{}\mathrm{i}}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*tan(a + b*x)*tan(2*a + 2*b*x))^(3/2)/cos(2*a + 2*b*x),x)
```

```
[Out] -(c*(exp(a*2i + b*x*2i)*3i + exp(a*4i + b*x*4i)*3i + exp(a*6i + b*x*6i)*5i
+ 5i)*((c*(exp(a*2i + b*x*2i)*1i - 1i)*(exp(a*4i + b*x*4i)*1i - 1i))/((exp(
a*2i + b*x*2i) + 1)*(exp(a*4i + b*x*4i) + 1)))^(1/2))/(3*b*(exp(a*2i + b*x*
2i) - exp(a*4i + b*x*4i) + exp(a*6i + b*x*6i) - 1))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(2*b*x+2*a)*(c*tan(b*x+a)*tan(2*b*x+2*a))**(3/2),x)
```

```
[Out] Timed out
```

3.615 $\int (c\tan(a+bx)\tan(2(a+bx)))^{3/2}\,dx$

Optimal. Leaf size=80

$$\frac{c^{3/2}\tanh^{-1}\left(\frac{\sqrt{c}\tan(2a+2bx)}{\sqrt{c\sec(2a+2bx)-c}}\right)}{b}+\frac{c^2\tan(2a+2bx)}{b\sqrt{c\sec(2a+2bx)-c}}$$

```
[Out] c^(3/2)*arctanh(c^(1/2)*tan(2*b*x+2*a)/(-c+c*sec(2*b*x+2*a))^(1/2))/b+c^2*t
an(2*b*x+2*a)/b/(-c+c*sec(2*b*x+2*a))^(1/2)
```

Rubi [A] time = 0.06, antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {4397, 3775, 21, 3774, 207}

$$\frac{c^2\tan(2a+2bx)}{b\sqrt{c\sec(2a+2bx)-c}}+\frac{c^{3/2}\tanh^{-1}\left(\frac{\sqrt{c}\tan(2a+2bx)}{\sqrt{c\sec(2a+2bx)-c}}\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[(c*Tan[a + b*x]*Tan[2*(a + b*x)])^(3/2),x]
```

```
[Out] (c^(3/2)*ArcTanh[(Sqrt[c]*Tan[2*a + 2*b*x])/Sqrt[-c + c*Sec[2*a + 2*b*x]]])
/b + (c^2*Tan[2*a + 2*b*x])/(b*Sqrt[-c + c*Sec[2*a + 2*b*x]])
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 3774

```
Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[(-2*b)/d,
 Subst[Int[1/(a + x^2), x], x, (b*Cot[c + d*x])/Sqrt[a + b*Csc[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3775

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_), x_Symbol] :> -Simp[(b^2*Co
t[c + d*x]*(a + b*Csc[c + d*x])^(n - 2))/(d*(n - 1)), x] + Dist[a/(n - 1),
Int[(a + b*Csc[c + d*x])^(n - 2)*(a*(n - 1) + b*(3*n - 4)*Csc[c + d*x]), x]
, x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && GtQ[n, 1] && Integer
Q[2*n]
```

Rule 4397

```
Int[u_, x_Symbol] :> Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]
```

Rubi steps

$$\begin{aligned}
\int (c\tan(a+bx)\tan(2(a+bx)))^{3/2}\,dx &= \int (-c+c\sec(2a+2bx))^{3/2}\,dx \\
&= \frac{c^2\tan(2a+2bx)}{b\sqrt{-c+c\sec(2a+2bx)}} - (2c)\int \frac{-\frac{c}{2}+\frac{1}{2}c\sec(2a+2bx)}{\sqrt{-c+c\sec(2a+2bx)}}\,dx \\
&= \frac{c^2\tan(2a+2bx)}{b\sqrt{-c+c\sec(2a+2bx)}} - c\int \sqrt{-c+c\sec(2a+2bx)}\,dx \\
&= \frac{c^2\tan(2a+2bx)}{b\sqrt{-c+c\sec(2a+2bx)}} + \frac{c^2\,\text{Subst}\left(\int \frac{1}{-c+x^2}\,dx, x, -\frac{c\tan(2a+2bx)}{\sqrt{-c+c\sec(2a+2bx)}}\right)}{b} \\
&= \frac{c^{3/2}\tanh^{-1}\left(\frac{\sqrt{c}\,\tan(2a+2bx)}{\sqrt{-c+c\sec(2a+2bx)}}\right)}{b} + \frac{c^2\tan(2a+2bx)}{b\sqrt{-c+c\sec(2a+2bx)}}
\end{aligned}$$

Mathematica [A] time = 0.15, size = 86, normalized size = 1.08

$$\frac{c\sqrt{c\tan(a+bx)\tan(2(a+bx))}\left(2\cot(a+bx)+\sqrt{2}\sqrt{\cos(2(a+bx))}\,\csc(a+bx)\tanh^{-1}\left(\frac{\sqrt{2}\,\cos(a+bx)}{\sqrt{\cos(2(a+bx))}}\right)\right)}{2b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*Tan[a + b*x]*Tan[2*(a + b*x)])^(3/2),x]
```

```
[Out] (c*(2*Cot[a + b*x] + Sqrt[2]*ArcTanh[(Sqrt[2]*Cos[a + b*x])/Sqrt[Cos[2*(a +
 b*x)]]]*Sqrt[Cos[2*(a + b*x)]]*Csc[a + b*x])*Sqrt[c*Tan[a + b*x]*Tan[2*(a
+ b*x)]])/(2*b)
```

fricas [A] time = 1.03, size = 296, normalized size = 3.70

$$\left[\frac{c^{\frac{3}{2}}\log\left(-\frac{c\tan(bx+a)^5-14\,c\tan(bx+a)^3+4\,\sqrt{2}\left(\tan(bx+a)^4-4\,\tan(bx+a)^2+3\right)\sqrt{-\frac{c\tan(bx+a)^2}{\tan(bx+a)^2-1}}\,\sqrt{c}+17\,c\tan(bx+a)}{\tan(bx+a)^5+2\,\tan(bx+a)^3+\tan(bx+a)}\right)\tan\left(bx+a\right)+4\,\sqrt{2}\,\sqrt{-}}{4\,b\tan\left(bx+a\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*tan(b*x+a)*tan(2*b*x+2*a))^(3/2),x, algorithm="fricas")
```

```
[Out] [1/4*(c^(3/2)*log(-(c*tan(b*x + a)^5 - 14*c*tan(b*x + a)^3 + 4*sqrt(2)*(tan
(b*x + a)^4 - 4*tan(b*x + a)^2 + 3)*sqrt(-c*tan(b*x + a)^2/(tan(b*x + a)^2
- 1))*sqrt(c) + 17*c*tan(b*x + a))/(tan(b*x + a)^5 + 2*tan(b*x + a)^3 + tan
(b*x + a)))*tan(b*x + a) + 4*sqrt(2)*sqrt(-c*tan(b*x + a)^2/(tan(b*x + a)^2
 - 1))*c)/(b*tan(b*x + a)), -1/2*(sqrt(-c)*c*arctan(2*sqrt(2)*sqrt(-c*tan(b
*x + a)^2/(tan(b*x + a)^2 - 1))*(tan(b*x + a)^2 - 1)*sqrt(-c)/(c*tan(b*x +
a)^3 - 3*c*tan(b*x + a)))*tan(b*x + a) - 2*sqrt(2)*sqrt(-c*tan(b*x + a)^2/(
tan(b*x + a)^2 - 1))*c)/(b*tan(b*x + a))]
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*tan(b*x+a)*tan(2*b*x+2*a))^(3/2),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [B] time = 0.95, size = 253, normalized size = 3.16

$$\frac{\sqrt{2}\,\left(2\left(\cos^2\left(bx+a\right)\right)-1\right)\left(\sqrt{2}\,\sqrt{\frac{2\left(\cos^2(bx+a)\right)-1}{(\cos(bx+a)+1)^2}}\,\operatorname{arctanh}\left(\frac{\cos(bx+a)\sqrt{4}\,(-1+\cos(bx+a))\sqrt{2}}{2\sin(bx+a)^2\sqrt{\frac{2\left(\cos^2(bx+a)\right)-1}{(\cos(bx+a)+1)^2}}}\right)\cos\left(bx+a\right)+\sqrt{2}\,\sqrt{\frac{2(\cos^2}{(\cos(}}\right.}{b\sin\left(bx+a\right)^3\left(2+\sqrt{2}\right)\left(\sqrt{2}-2\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*tan(b*x+a)*tan(2*b*x+2*a))^(3/2),x)
```

```
[Out] 2^(1/2)/b*(2*cos(b*x+a)^2-1)*(2^(1/2)*((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)
^(1/2)*arctanh(1/2*cos(b*x+a)*4^(1/2)*(-1+cos(b*x+a))/sin(b*x+a)^2/((2*cos(
b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(1/2)*2^(1/2))*cos(b*x+a)+2^(1/2)*((2*cos(b*x
+a)^2-1)/(cos(b*x+a)+1)^2)^(1/2)*arctanh(1/2*cos(b*x+a)*4^(1/2)*(-1+cos(b*x
+a))/sin(b*x+a)^2/((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(1/2)*2^(1/2))-2*co
s(b*x+a))*(c*sin(b*x+a)^2/(2*cos(b*x+a)^2-1))^(3/2)/sin(b*x+a)^3/(2+2^(1/2)
)/(2^(1/2)-2)
```

maxima [B] time = 0.89, size = 1317, normalized size = 16.46

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*tan(b*x+a)*tan(2*b*x+2*a))^(3/2),x, algorithm="maxima")
```

```
[Out] -1/8*((cos(4*b*x + 4*a)^2 + sin(4*b*x + 4*a)^2 + 2*cos(4*b*x + 4*a) + 1)^(1
/4)*(c*log(sqrt(cos(4*b*x + 4*a)^2 + sin(4*b*x + 4*a)^2 + 2*cos(4*b*x + 4*a
) + 1)*cos(1/2*arctan2(sin(4*b*x + 4*a), -cos(4*b*x + 4*a) - 1))^2 + sqrt(c
os(4*b*x + 4*a)^2 + sin(4*b*x + 4*a)^2 + 2*cos(4*b*x + 4*a) + 1)*sin(1/2*ar
ctan2(sin(4*b*x + 4*a), -cos(4*b*x + 4*a) - 1))^2 + 2*(cos(4*b*x + 4*a)^2 +
 sin(4*b*x + 4*a)^2 + 2*cos(4*b*x + 4*a) + 1)^(1/4)*sin(1/2*arctan2(sin(4*b
*x + 4*a), -cos(4*b*x + 4*a) - 1)) + 1) - c*log(sqrt(cos(4*b*x + 4*a)^2 + s
in(4*b*x + 4*a)^2 + 2*cos(4*b*x + 4*a) + 1)*cos(1/2*arctan2(sin(4*b*x + 4*a
), -cos(4*b*x + 4*a) - 1))^2 + sqrt(cos(4*b*x + 4*a)^2 + sin(4*b*x + 4*a)^2
 + 2*cos(4*b*x + 4*a) + 1)*sin(1/2*arctan2(sin(4*b*x + 4*a), -cos(4*b*x + 4
*a) - 1))^2 - 2*(cos(4*b*x + 4*a)^2 + sin(4*b*x + 4*a)^2 + 2*cos(4*b*x + 4*
a) + 1)^(1/4)*sin(1/2*arctan2(sin(4*b*x + 4*a), -cos(4*b*x + 4*a) - 1)) + 1
) + c*log(((cos(1/2*arctan2(sin(4*b*x + 4*a), -cos(4*b*x + 4*a) - 1))^2 + s
in(1/2*arctan2(sin(4*b*x + 4*a), -cos(4*b*x + 4*a) - 1))^2)*cos(1/2*arctan2
(sin(4*b*x + 4*a), cos(4*b*x + 4*a)))^2 + (cos(1/2*arctan2(sin(4*b*x + 4*a)
, -cos(4*b*x + 4*a) - 1))^2 + sin(1/2*arctan2(sin(4*b*x + 4*a), -cos(4*b*x
+ 4*a) - 1))^2)*sin(1/2*arctan2(sin(4*b*x + 4*a), cos(4*b*x + 4*a)))^2)*sqr
t(cos(4*b*x + 4*a)^2 + sin(4*b*x + 4*a)^2 + 2*cos(4*b*x + 4*a) + 1) + 2*(co
s(4*b*x + 4*a)^2 + sin(4*b*x + 4*a)^2 + 2*cos(4*b*x + 4*a) + 1)^(1/4)*(cos(
1/2*arctan2(sin(4*b*x + 4*a), cos(4*b*x + 4*a)))*sin(1/2*arctan2(sin(4*b*x
+ 4*a), -cos(4*b*x + 4*a) - 1)) + cos(1/2*arctan2(sin(4*b*x + 4*a), -cos(4*
b*x + 4*a) - 1))*sin(1/2*arctan2(sin(4*b*x + 4*a), cos(4*b*x + 4*a)))) + 1)
 - c*log(((cos(1/2*arctan2(sin(4*b*x + 4*a), -cos(4*b*x + 4*a) - 1))^2 + si
n(1/2*arctan2(sin(4*b*x + 4*a), -cos(4*b*x + 4*a) - 1))^2)*cos(1/2*arctan2(
sin(4*b*x + 4*a), cos(4*b*x + 4*a)))^2 + (cos(1/2*arctan2(sin(4*b*x + 4*a),
 -cos(4*b*x + 4*a) - 1))^2 + sin(1/2*arctan2(sin(4*b*x + 4*a), -cos(4*b*x +
 4*a) - 1))^2)*sin(1/2*arctan2(sin(4*b*x + 4*a), cos(4*b*x + 4*a)))^2)*sqrt
(cos(4*b*x + 4*a)^2 + sin(4*b*x + 4*a)^2 + 2*cos(4*b*x + 4*a) + 1) - 2*(cos
(4*b*x + 4*a)^2 + sin(4*b*x + 4*a)^2 + 2*cos(4*b*x + 4*a) + 1)^(1/4)*(cos(1
/2*arctan2(sin(4*b*x + 4*a), cos(4*b*x + 4*a)))*sin(1/2*arctan2(sin(4*b*x +
```

```
 4*a), -cos(4*b*x + 4*a) - 1)) + cos(1/2*arctan2(sin(4*b*x + 4*a), -cos(4*b
*x + 4*a) - 1))*sin(1/2*arctan2(sin(4*b*x + 4*a), cos(4*b*x + 4*a)))) + 1))
*sqrt(c) + 8*(c*cos(1/2*arctan2(sin(4*b*x + 4*a), cos(4*b*x + 4*a)))*sin(1/
2*arctan2(sin(4*b*x + 4*a), -cos(4*b*x + 4*a) - 1)) + c*cos(1/2*arctan2(sin
(4*b*x + 4*a), -cos(4*b*x + 4*a) - 1))*sin(1/2*arctan2(sin(4*b*x + 4*a), co
s(4*b*x + 4*a))) + c*sin(1/2*arctan2(sin(4*b*x + 4*a), -cos(4*b*x + 4*a) -
1)))*sqrt(c))/((cos(4*b*x + 4*a)^2 + sin(4*b*x + 4*a)^2 + 2*cos(4*b*x + 4*a
) + 1)^(1/4)*b)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int (c\tan(a+bx)\ \tan(2a+2bx))^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*tan(a + b*x)*tan(2*a + 2*b*x))^(3/2),x)
```

```
[Out] int((c*tan(a + b*x)*tan(2*a + 2*b*x))^(3/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*tan(b*x+a)*tan(2*b*x+2*a))**(3/2),x)
```

```
[Out] Timed out
```

3.616 $\int \cos(2(a+bx))(c\tan(a+bx)\tan(2(a+bx)))^{3/2}\,dx$

Optimal. Leaf size=86

$$\frac{c^2\sin(2a+2bx)}{2b\sqrt{c\sec(2a+2bx)-c}}-\frac{3c^{3/2}\tanh^{-1}\left(\frac{\sqrt{c}\tan(2a+2bx)}{\sqrt{c\sec(2a+2bx)-c}}\right)}{2b}$$

```
[Out] -3/2*c^(3/2)*arctanh(c^(1/2)*tan(2*b*x+2*a)/(-c+c*sec(2*b*x+2*a))^(1/2))/b+
1/2*c^2*sin(2*b*x+2*a)/b/(-c+c*sec(2*b*x+2*a))^(1/2)
```

Rubi [A] time = 0.22, antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.207, Rules used = {4397, 3814, 21, 3805, 3774, 207}

$$\frac{c^2\sin(2a+2bx)}{2b\sqrt{c\sec(2a+2bx)-c}}-\frac{3c^{3/2}\tanh^{-1}\left(\frac{\sqrt{c}\tan(2a+2bx)}{\sqrt{c\sec(2a+2bx)-c}}\right)}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[2*(a + b*x)]*(c*Tan[a + b*x]*Tan[2*(a + b*x)])^(3/2),x]
```

```
[Out] (-3*c^(3/2)*ArcTanh[(Sqrt[c]*Tan[2*a + 2*b*x])/Sqrt[-c + c*Sec[2*a + 2*b*x]
]])/(2*b) + (c^2*Sin[2*a + 2*b*x])/(2*b*Sqrt[-c + c*Sec[2*a + 2*b*x]])
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 3774

```
Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[(-2*b)/d,
 Subst[Int[1/(a + x^2), x], x, (b*Cot[c + d*x])/Sqrt[a + b*Csc[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3805

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[(a*Cot[e + f*x]*(d*Csc[e + f*x])^n)/(f*n*Sqrt[a
 + b*Csc[e + f*x]]), x] + Dist[(a*(2*n + 1))/(2*b*d*n), Int[Sqrt[a + b*Csc[
e + f*x]]*(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f}, x] &&
EqQ[a^2 - b^2, 0] && LtQ[n, -2^(-1)] && IntegerQ[2*n]
```

Rule 3814

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> -Simp[(b^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 2
)*(d*Csc[e + f*x])^n)/(f*(m + n - 1)), x] + Dist[b/(m + n - 1), Int[(a + b*
Csc[e + f*x])^(m - 2)*(d*Csc[e + f*x])^n*(b*(m + 2*n - 1) + a*(3*m + 2*n -
4)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2,
0] && GtQ[m, 1] && NeQ[m + n - 1, 0] && IntegerQ[2*m]
```

Rule 4397

```
Int[u_, x_Symbol] :> Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]
```

Rubi steps

$$\int \cos(2(a+bx))(c\tan(a+bx)\tan(2(a+bx)))^{3/2}\,dx = \int \cos(2a+2bx)(-c+c\sec(2a+2bx))^{3/2}\,dx$$

$$= -\frac{c^2\sin(2a+2bx)}{b\sqrt{-c+c\sec(2a+2bx)}} - (2c)\int \frac{\cos(2a+2bx)\left(-\frac{3c}{2}+\right.}{\sqrt{-c+c\sec(}}$$

$$= -\frac{c^2\sin(2a+2bx)}{b\sqrt{-c+c\sec(2a+2bx)}} - (3c)\int \cos(2a+2bx)\sqrt{-c+}$$

$$= \frac{c^2\sin(2a+2bx)}{2b\sqrt{-c+c\sec(2a+2bx)}} + \frac{1}{2}(3c)\int \sqrt{-c+c\sec(2a+2}$$

$$= \frac{c^2\sin(2a+2bx)}{2b\sqrt{-c+c\sec(2a+2bx)}} - \frac{\left(3c^2\right)\mathrm{Subst}\left(\int \frac{1}{-c+x^2}\,dx, x, -\right.}{2b}$$

$$= -\frac{3c^{3/2}\tanh^{-1}\left(\frac{\sqrt{c}\tan(2a+2bx)}{\sqrt{-c+c\sec(2a+2bx)}}\right)}{2b} + \frac{c^2\sin(2a+2bx)}{2b\sqrt{-c+c\sec(2a+2}}$$

Mathematica [A] time = 0.24, size = 93, normalized size = 1.08

$$\frac{c\csc(a+bx)\sqrt{c\tan(a+bx)\tan(2(a+bx))}\left(\cos(a+bx)+\cos(3(a+bx))-3\sqrt{2}\sqrt{\cos(2(a+bx))}\tanh^{-1}\left(\frac{\sqrt{2}\cos}{\sqrt{\cos(2(}}\right.\right.}{4b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[2*(a + b*x)]*(c*Tan[a + b*x]*Tan[2*(a + b*x)])^(3/2),x]
```

```
[Out] (c*(Cos[a + b*x] - 3*Sqrt[2]*ArcTanh[(Sqrt[2]*Cos[a + b*x])/Sqrt[Cos[2*(a +
 b*x)]]]*Sqrt[Cos[2*(a + b*x)]] + Cos[3*(a + b*x)])*Csc[a + b*x]*Sqrt[c*Tan
[a + b*x]*Tan[2*(a + b*x)]])/(4*b)
```

fricas [B] time = 2.09, size = 369, normalized size = 4.29

$$\left[\frac{3\left(c\tan\left(bx+a\right)^{3}+c\tan\left(bx+a\right)\right)\sqrt{c}\,\log\left(-\frac{c\tan(bx+a)^{5}-14\,c\tan(bx+a)^{3}-4\,\sqrt{2}\left(\tan(bx+a)^{4}-4\,\tan(bx+a)^{2}+3\right)\sqrt{-\frac{c\tan(bx+a)^{2}}{\tan(bx+a)^{2}-1}}\,\sqrt{c}\cdots}{\tan(bx+a)^{5}+2\,\tan(bx+a)^{3}+\tan(bx+a)}\right.\cdots}{8\left(b\tan\left(bx+a\right)^{3}+b\tan\left(bx+a\right)\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(2*b*x+2*a)*(c*tan(b*x+a)*tan(2*b*x+2*a))^(3/2),x, algorithm="
fricas")
```

```
[Out] [1/8*(3*(c*tan(b*x + a)^3 + c*tan(b*x + a))*sqrt(c)*log(-(c*tan(b*x + a)^5
- 14*c*tan(b*x + a)^3 - 4*sqrt(2)*(tan(b*x + a)^4 - 4*tan(b*x + a)^2 + 3)*s
qrt(-c*tan(b*x + a)^2/(tan(b*x + a)^2 - 1))*sqrt(c) + 17*c*tan(b*x + a))/(t
an(b*x + a)^5 + 2*tan(b*x + a)^3 + tan(b*x + a))) - 4*sqrt(2)*(c*tan(b*x +
a)^2 - c)*sqrt(-c*tan(b*x + a)^2/(tan(b*x + a)^2 - 1)))/(b*tan(b*x + a)^3 +
 b*tan(b*x + a)), 1/4*(3*(c*tan(b*x + a)^3 + c*tan(b*x + a))*sqrt(-c)*arcta
n(2*sqrt(2)*sqrt(-c*tan(b*x + a)^2/(tan(b*x + a)^2 - 1))*(tan(b*x + a)^2 -
1)*sqrt(-c)/(c*tan(b*x + a)^3 - 3*c*tan(b*x + a))) - 2*sqrt(2)*(c*tan(b*x +
 a)^2 - c)*sqrt(-c*tan(b*x + a)^2/(tan(b*x + a)^2 - 1)))/(b*tan(b*x + a)^3
+ b*tan(b*x + a))]
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(2*b*x+2*a)*(c*tan(b*x+a)*tan(2*b*x+2*a))^(3/2),x, algorithm="
giac")
```

```
[Out] Timed out
```

maple [B] time = 1.09, size = 518, normalized size = 6.02

$$-\frac{\sqrt{2}\,\left(2\left(\cos^2(bx+a)\right)-1\right)\left(\sqrt{2}\,\sqrt{\frac{2\left(\cos^2(bx+a)\right)-1}{(\cos(bx+a)+1)^2}}\,\operatorname{arctanh}\left(\frac{\cos(bx+a)\sqrt{4}\,(-1+\cos(bx+a))\sqrt{2}}{2\sin(bx+a)^2\sqrt{\frac{2\left(\cos^2(bx+a)\right)-1}{(\cos(bx+a)+1)^2}}}\right)\cos(bx+a)+\sqrt{2}\,\sqrt{\frac{2(\mathrm{co}}{(\mathrm{cos}}}}{b\sin(bx+a)^3\left(2+\sqrt{2}\right)\left(\sqrt{2}-\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(2*b*x+2*a)*(c*tan(b*x+a)*tan(2*b*x+2*a))^(3/2),x)
```

```
[Out] -2^(1/2)/b*(2*cos(b*x+a)^2-1)*(2^(1/2)*((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2
)^(1/2)*arctanh(1/2*cos(b*x+a)*4^(1/2)*(-1+cos(b*x+a))/sin(b*x+a)^2/((2*cos
(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(1/2)*2^(1/2))*cos(b*x+a)+2^(1/2)*((2*cos(b*
x+a)^2-1)/(cos(b*x+a)+1)^2)^(1/2)*arctanh(1/2*cos(b*x+a)*4^(1/2)*(-1+cos(b*
x+a))/sin(b*x+a)^2/((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(1/2)*2^(1/2))-2*c
os(b*x+a))*(c*sin(b*x+a)^2/(2*cos(b*x+a)^2-1))^(3/2)/sin(b*x+a)^3/(2+2^(1/2
))/(2^(1/2)-2)-2*2^(1/2)/b*(2*cos(b*x+a)^2-1)*(2^(1/2)*((2*cos(b*x+a)^2-1)/
(cos(b*x+a)+1)^2)^(1/2)*arctanh(1/2*cos(b*x+a)*4^(1/2)*(-1+cos(b*x+a))/sin(
b*x+a)^2/((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(1/2)*2^(1/2))*cos(b*x+a)+2^
(1/2)*((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(1/2)*arctanh(1/2*cos(b*x+a)*4^
(1/2)*(-1+cos(b*x+a))/sin(b*x+a)^2/((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(1
/2)*2^(1/2))+4*cos(b*x+a)^3+2*cos(b*x+a))*(c*sin(b*x+a)^2/(2*cos(b*x+a)^2-1
))^(3/2)/sin(b*x+a)^3/(2+2^(1/2))^3/(2^(1/2)-2)^3
```

maxima [B] time = 1.15, size = 1058, normalized size = 12.30

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(2*b*x+2*a)*(c*tan(b*x+a)*tan(2*b*x+2*a))^(3/2),x, algorithm="
maxima")
```

```
[Out] -1/16*(4*(c*cos(1/2*arctan2(sin(4*b*x + 4*a), -cos(4*b*x + 4*a) - 1))*sin(2
*b*x + 2*a) + (c*cos(2*b*x + 2*a) + c)*sin(1/2*arctan2(sin(4*b*x + 4*a), -c
os(4*b*x + 4*a) - 1)))*(cos(4*b*x + 4*a)^2 + sin(4*b*x + 4*a)^2 + 2*cos(4*b
*x + 4*a) + 1)^(1/4)*sqrt(c) - 3*(c*log(sqrt(cos(4*b*x + 4*a)^2 + sin(4*b*x
 + 4*a)^2 + 2*cos(4*b*x + 4*a) + 1)*cos(1/2*arctan2(sin(4*b*x + 4*a), -cos(
4*b*x + 4*a) - 1))^2 + sqrt(cos(4*b*x + 4*a)^2 + sin(4*b*x + 4*a)^2 + 2*cos
(4*b*x + 4*a) + 1)*sin(1/2*arctan2(sin(4*b*x + 4*a), -cos(4*b*x + 4*a) - 1)
)^2 + 2*(cos(4*b*x + 4*a)^2 + sin(4*b*x + 4*a)^2 + 2*cos(4*b*x + 4*a) + 1)^
(1/4)*sin(1/2*arctan2(sin(4*b*x + 4*a), -cos(4*b*x + 4*a) - 1)) + 1) - c*lo
g(sqrt(cos(4*b*x + 4*a)^2 + sin(4*b*x + 4*a)^2 + 2*cos(4*b*x + 4*a) + 1)*co
```

```
s(1/2*arctan2(sin(4*b*x + 4*a), -cos(4*b*x + 4*a) - 1))^2 + sqrt(cos(4*b*x
+ 4*a)^2 + sin(4*b*x + 4*a)^2 + 2*cos(4*b*x + 4*a) + 1)*sin(1/2*arctan2(sin
(4*b*x + 4*a), -cos(4*b*x + 4*a) - 1))^2 - 2*(cos(4*b*x + 4*a)^2 + sin(4*b*
x + 4*a)^2 + 2*cos(4*b*x + 4*a) + 1)^(1/4)*sin(1/2*arctan2(sin(4*b*x + 4*a)
, -cos(4*b*x + 4*a) - 1)) + 1) + c*log(((cos(2*b*x + 2*a)^2 + sin(2*b*x + 2
*a)^2)*cos(1/2*arctan2(sin(4*b*x + 4*a), -cos(4*b*x + 4*a) - 1))^2 + (cos(2
*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2)*sin(1/2*arctan2(sin(4*b*x + 4*a), -cos(
4*b*x + 4*a) - 1))^2)*sqrt(cos(4*b*x + 4*a)^2 + sin(4*b*x + 4*a)^2 + 2*cos(
4*b*x + 4*a) + 1) + 2*(cos(4*b*x + 4*a)^2 + sin(4*b*x + 4*a)^2 + 2*cos(4*b*
x + 4*a) + 1)^(1/4)*(cos(1/2*arctan2(sin(4*b*x + 4*a), -cos(4*b*x + 4*a) -
1))*sin(2*b*x + 2*a) + cos(2*b*x + 2*a)*sin(1/2*arctan2(sin(4*b*x + 4*a), -
cos(4*b*x + 4*a) - 1))) + 1) - c*log(((cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a
)^2)*cos(1/2*arctan2(sin(4*b*x + 4*a), -cos(4*b*x + 4*a) - 1))^2 + (cos(2*b
*x + 2*a)^2 + sin(2*b*x + 2*a)^2)*sin(1/2*arctan2(sin(4*b*x + 4*a), -cos(4*
b*x + 4*a) - 1))^2)*sqrt(cos(4*b*x + 4*a)^2 + sin(4*b*x + 4*a)^2 + 2*cos(4*
b*x + 4*a) + 1) - 2*(cos(4*b*x + 4*a)^2 + sin(4*b*x + 4*a)^2 + 2*cos(4*b*x
+ 4*a) + 1)^(1/4)*(cos(1/2*arctan2(sin(4*b*x + 4*a), -cos(4*b*x + 4*a) - 1)
)*sin(2*b*x + 2*a) + cos(2*b*x + 2*a)*sin(1/2*arctan2(sin(4*b*x + 4*a), -co
s(4*b*x + 4*a) - 1))) + 1))*sqrt(c))/b
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \cos(2a+2bx)\,(c\tan(a+bx)\,\tan(2a+2bx))^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(2*a + 2*b*x)*(c*tan(a + b*x)*tan(2*a + 2*b*x))^(3/2),x)
[Out] int(cos(2*a + 2*b*x)*(c*tan(a + b*x)*tan(2*a + 2*b*x))^(3/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(2*b*x+2*a)*(c*tan(b*x+a)*tan(2*b*x+2*a))**(3/2),x)
[Out] Timed out
```

3.617 $\int \cos^2(2(a+bx))(c\tan(a+bx)\tan(2(a+bx)))^{3/2}\,dx$

Optimal. Leaf size=133

$$\frac{7c^{3/2}\tanh^{-1}\left(\frac{\sqrt{c}\tan(2a+2bx)}{\sqrt{c\sec(2a+2bx)-c}}\right)}{8b}-\frac{7c^2\sin(2a+2bx)}{8b\sqrt{c\sec(2a+2bx)-c}}+\frac{c^2\sin(2a+2bx)\cos(2a+2bx)}{4b\sqrt{c\sec(2a+2bx)-c}}$$

```
[Out] 7/8*c^(3/2)*arctanh(c^(1/2)*tan(2*b*x+2*a)/(-c+c*sec(2*b*x+2*a))^(1/2))/b-7
/8*c^2*sin(2*b*x+2*a)/b/(-c+c*sec(2*b*x+2*a))^(1/2)+1/4*c^2*cos(2*b*x+2*a)*
sin(2*b*x+2*a)/b/(-c+c*sec(2*b*x+2*a))^(1/2)
```

Rubi [A] time = 0.26, antiderivative size = 133, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.194, Rules used = {4397, 3813, 21, 3805, 3774, 207}

$$-\frac{7c^2\sin(2a+2bx)}{8b\sqrt{c\sec(2a+2bx)-c}}+\frac{c^2\sin(2a+2bx)\cos(2a+2bx)}{4b\sqrt{c\sec(2a+2bx)-c}}+\frac{7c^{3/2}\tanh^{-1}\left(\frac{\sqrt{c}\tan(2a+2bx)}{\sqrt{c\sec(2a+2bx)-c}}\right)}{8b}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[2*(a + b*x)]^2*(c*Tan[a + b*x]*Tan[2*(a + b*x)])^(3/2),x]
```

```
[Out] (7*c^(3/2)*ArcTanh[(Sqrt[c]*Tan[2*a + 2*b*x])/Sqrt[-c + c*Sec[2*a + 2*b*x]]
])/(8*b) - (7*c^2*Sin[2*a + 2*b*x])/(8*b*Sqrt[-c + c*Sec[2*a + 2*b*x]]) + (
c^2*Cos[2*a + 2*b*x]*Sin[2*a + 2*b*x])/(4*b*Sqrt[-c + c*Sec[2*a + 2*b*x]])
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 3774

```
Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[(-2*b)/d,
 Subst[Int[1/(a + x^2), x], x, (b*Cot[c + d*x])/Sqrt[a + b*Csc[c + d*x]]],
```

```
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3805

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[(a*Cot[e + f*x]*(d*Csc[e + f*x])^n)/(f*n*Sqrt[a
 + b*Csc[e + f*x]]), x] + Dist[(a*(2*n + 1))/(2*b*d*n), Int[Sqrt[a + b*Csc[
e + f*x]]*(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f}, x] &&
EqQ[a^2 - b^2, 0] && LtQ[n, -2^(-1)] && IntegerQ[2*n]
```

Rule 3813

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(b^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 2)
*(d*Csc[e + f*x])^n)/(f*n), x] - Dist[a/(d*n), Int[(a + b*Csc[e + f*x])^(m
- 2)*(d*Csc[e + f*x])^(n + 1)*(b*(m - 2*n - 2) - a*(m + 2*n - 1)*Csc[e + f*
x]), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, 1]
&& (LtQ[n, -1] || (EqQ[m, 3/2] && EqQ[n, -2^(-1)])) && IntegerQ[2*m]
```

Rule 4397

```
Int[u_, x_Symbol] :> Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]
```

Rubi steps

$$\int \cos^2(2(a+bx))(c\tan(a+bx)\tan(2(a+bx)))^{3/2}\,dx = \int \cos^2(2a+2bx)(-c+c\sec(2a+2bx))^{3/2}\,dx$$

$$= \frac{c^2\cos(2a+2bx)\sin(2a+2bx)}{4b\sqrt{-c+c\sec(2a+2bx)}} - \frac{1}{2}c\int \frac{\cos(2a+2bx)\,(}{\sqrt{-c+}}$$

$$= \frac{c^2\cos(2a+2bx)\sin(2a+2bx)}{4b\sqrt{-c+c\sec(2a+2bx)}} + \frac{1}{4}(7c)\int \cos(2a+2b$$

$$= -\frac{7c^2\sin(2a+2bx)}{8b\sqrt{-c+c\sec(2a+2bx)}} + \frac{c^2\cos(2a+2bx)\sin(2a}{4b\sqrt{-c+c\sec(2a+2}}$$

$$= -\frac{7c^2\sin(2a+2bx)}{8b\sqrt{-c+c\sec(2a+2bx)}} + \frac{c^2\cos(2a+2bx)\sin(2a}{4b\sqrt{-c+c\sec(2a+2}}$$

$$= \frac{7c^{3/2}\tanh^{-1}\left(\frac{\sqrt{c}\,\tan(2a+2bx)}{\sqrt{-c+c\sec(2a+2bx)}}\right)}{8b} - \frac{7c^2\sin(2a+2bx)}{8b\sqrt{-c+c\sec(2a+2}}$$

Mathematica [A] time = 0.27, size = 105, normalized size = 0.79

$$\frac{c\csc(a+bx)\sqrt{c\tan(a+bx)\tan(2(a+bx))}\left(-5\cos(a+bx)-6\cos(3(a+bx))+\cos(5(a+bx))+7\sqrt{2}\sqrt{\cos(2(a}\right.}{16b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[2*(a + b*x)]^2*(c*Tan[a + b*x]*Tan[2*(a + b*x)])^(3/2),x]
```

```
[Out] (c*(-5*Cos[a + b*x] + 7*Sqrt[2]*ArcTanh[(Sqrt[2]*Cos[a + b*x])/Sqrt[Cos[2*(
a + b*x)]]]*Sqrt[Cos[2*(a + b*x)]] - 6*Cos[3*(a + b*x)] + Cos[5*(a + b*x)])
*Csc[a + b*x]*Sqrt[c*Tan[a + b*x]*Tan[2*(a + b*x)]])/(16*b)
```

fricas [A] time = 1.04, size = 437, normalized size = 3.29

$$\left[\frac{7\left(c\tan(bx+a)^5+2\,c\tan(bx+a)^3+c\tan(bx+a)\right)\sqrt{c}\,\log\left(-\frac{c\tan(bx+a)^5-14\,c\tan(bx+a)^3+4\,\sqrt{2}\left(\tan(bx+a)^4-4\,\tan(bx+a)^2-\right.}{\tan(bx+a)^5+2\,\tan(bx+a)^3+\tan}\right.}{32\left(b\tan(bx+a)^5+2\,b\tan(bx+a\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(2*b*x+2*a)^2*(c*tan(b*x+a)*tan(2*b*x+2*a))^(3/2),x, algorithm
="fricas")
```

```
[Out] [1/32*(7*(c*tan(b*x + a)^5 + 2*c*tan(b*x + a)^3 + c*tan(b*x + a))*sqrt(c)*l
og(-(c*tan(b*x + a)^5 - 14*c*tan(b*x + a)^3 + 4*sqrt(2)*(tan(b*x + a)^4 - 4
*tan(b*x + a)^2 + 3)*sqrt(-c*tan(b*x + a)^2/(tan(b*x + a)^2 - 1))*sqrt(c) +
 17*c*tan(b*x + a))/(tan(b*x + a)^5 + 2*tan(b*x + a)^3 + tan(b*x + a))) + 4
*sqrt(2)*(9*c*tan(b*x + a)^4 - 4*c*tan(b*x + a)^2 - 5*c)*sqrt(-c*tan(b*x +
a)^2/(tan(b*x + a)^2 - 1)))/(b*tan(b*x + a)^5 + 2*b*tan(b*x + a)^3 + b*tan(
b*x + a)), -1/16*(7*(c*tan(b*x + a)^5 + 2*c*tan(b*x + a)^3 + c*tan(b*x + a)
)*sqrt(-c)*arctan(2*sqrt(2)*sqrt(-c*tan(b*x + a)^2/(tan(b*x + a)^2 - 1))*(t
an(b*x + a)^2 - 1)*sqrt(-c)/(c*tan(b*x + a)^3 - 3*c*tan(b*x + a))) - 2*sqrt
(2)*(9*c*tan(b*x + a)^4 - 4*c*tan(b*x + a)^2 - 5*c)*sqrt(-c*tan(b*x + a)^2/
(tan(b*x + a)^2 - 1)))/(b*tan(b*x + a)^5 + 2*b*tan(b*x + a)^3 + b*tan(b*x +
 a))]
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(2*b*x+2*a)^2*(c*tan(b*x+a)*tan(2*b*x+2*a))^(3/2),x, algorithm
="giac")
```

```
[Out] Timed out
```

maple [B] time = 1.02, size = 792, normalized size = 5.95

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(2*b*x+2*a)^2*(c*tan(b*x+a)*tan(2*b*x+2*a))^(3/2),x)
```

```
[Out] 2^(1/2)/b*(2*cos(b*x+a)^2-1)*(2^(1/2)*((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)
^(1/2)*arctanh(1/2*cos(b*x+a)*4^(1/2)*(-1+cos(b*x+a))/sin(b*x+a)^2/((2*cos(
b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(1/2)*2^(1/2))*cos(b*x+a)+2^(1/2)*((2*cos(b*x
+a)^2-1)/(cos(b*x+a)+1)^2)^(1/2)*arctanh(1/2*cos(b*x+a)*4^(1/2)*(-1+cos(b*x
+a))/sin(b*x+a)^2/((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(1/2)*2^(1/2))-2*co
s(b*x+a))*(c*sin(b*x+a)^2/(2*cos(b*x+a)^2-1))^(3/2)/sin(b*x+a)^3/(2+2^(1/2)
)/(2^(1/2)-2)+4*2^(1/2)/b*(2*cos(b*x+a)^2-1)*(2^(1/2)*((2*cos(b*x+a)^2-1)/(
cos(b*x+a)+1)^2)^(1/2)*arctanh(1/2*cos(b*x+a)*4^(1/2)*(-1+cos(b*x+a))/sin(b
*x+a)^2/((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(1/2)*2^(1/2))*cos(b*x+a)+2^(
1/2)*((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(1/2)*arctanh(1/2*cos(b*x+a)*4^(
1/2)*(-1+cos(b*x+a))/sin(b*x+a)^2/((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(1/
2)*2^(1/2))+4*cos(b*x+a)^3+2*cos(b*x+a))*(c*sin(b*x+a)^2/(2*cos(b*x+a)^2-1)
)^(3/2)/sin(b*x+a)^3/(2+2^(1/2))^3/(2^(1/2)-2)^3-2*2^(1/2)/b*(2*cos(b*x+a)^
2-1)*(16*cos(b*x+a)^5+9*2^(1/2)*((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(1/2)
*arctanh(1/2*cos(b*x+a)*4^(1/2)*(-1+cos(b*x+a))/sin(b*x+a)^2/((2*cos(b*x+a)
^2-1)/(cos(b*x+a)+1)^2)^(1/2)*2^(1/2))*cos(b*x+a)-12*cos(b*x+a)^3+9*2^(1/2)
*((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(1/2)*arctanh(1/2*cos(b*x+a)*4^(1/2)
*(-1+cos(b*x+a))/sin(b*x+a)^2/((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(1/2)*2
^(1/2))+18*cos(b*x+a))*(c*sin(b*x+a)^2/(2*cos(b*x+a)^2-1))^(3/2)/sin(b*x+a)
^3/(2+2^(1/2))^5/(2^(1/2)-2)^5
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(2*b*x+2*a)^2*(c*tan(b*x+a)*tan(2*b*x+2*a))^(3/2),x, algorithm
="maxima")
```

```
[Out] Timed out
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \cos (2a + 2bx)^2 (c \tan (a + bx) \tan (2a + 2bx))^{3/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(2*a + 2*b*x)^2*(c*tan(a + b*x)*tan(2*a + 2*b*x))^(3/2),x)
[Out] int(cos(2*a + 2*b*x)^2*(c*tan(a + b*x)*tan(2*a + 2*b*x))^(3/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(2*b*x+2*a)**2*(c*tan(b*x+a)*tan(2*b*x+2*a))**(3/2),x)
[Out] Timed out
```

3.618 $\int \cos^3(2(a+bx))(c\tan(a+bx)\tan(2(a+bx)))^{3/2}\,dx$

Optimal. Leaf size=182

$$-\frac{11c^{3/2}\tanh^{-1}\left(\frac{\sqrt{c}\tan(2a+2bx)}{\sqrt{c\sec(2a+2bx)-c}}\right)}{16b}+\frac{11c^2\sin(2a+2bx)}{16b\sqrt{c\sec(2a+2bx)-c}}+\frac{c^2\sin(2a+2bx)\cos^2(2a+2bx)}{6b\sqrt{c\sec(2a+2bx)-c}}-\frac{11c^2\sin(2a+2b\ldots}{24b\sqrt{c\sec(2\ldots}}$$

```
[Out] -11/16*c^(3/2)*arctanh(c^(1/2)*tan(2*b*x+2*a)/(-c+c*sec(2*b*x+2*a))^(1/2))/
b+11/16*c^2*sin(2*b*x+2*a)/b/(-c+c*sec(2*b*x+2*a))^(1/2)-11/24*c^2*cos(2*b*
x+2*a)*sin(2*b*x+2*a)/b/(-c+c*sec(2*b*x+2*a))^(1/2)+1/6*c^2*cos(2*b*x+2*a)^
2*sin(2*b*x+2*a)/b/(-c+c*sec(2*b*x+2*a))^(1/2)
```

Rubi [A] time = 0.31, antiderivative size = 182, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.194, Rules used = {4397, 3813, 21, 3805, 3774, 207}

$$\frac{11c^2\sin(2a+2bx)}{16b\sqrt{c\sec(2a+2bx)-c}}+\frac{c^2\sin(2a+2bx)\cos^2(2a+2bx)}{6b\sqrt{c\sec(2a+2bx)-c}}-\frac{11c^2\sin(2a+2bx)\cos(2a+2bx)}{24b\sqrt{c\sec(2a+2bx)-c}}-\frac{11c^{3/2}\tanh^{-1}\left(\ldots\right)}{16\ldots}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[2*(a + b*x)]^3*(c*Tan[a + b*x]*Tan[2*(a + b*x)])^(3/2),x]
```

```
[Out] (-11*c^(3/2)*ArcTanh[(Sqrt[c]*Tan[2*a + 2*b*x])/Sqrt[-c + c*Sec[2*a + 2*b*x
]]])/(16*b) + (11*c^2*Sin[2*a + 2*b*x])/(16*b*Sqrt[-c + c*Sec[2*a + 2*b*x]]
) - (11*c^2*Cos[2*a + 2*b*x]*Sin[2*a + 2*b*x])/(24*b*Sqrt[-c + c*Sec[2*a +
2*b*x]]) + (c^2*Cos[2*a + 2*b*x]^2*Sin[2*a + 2*b*x])/(6*b*Sqrt[-c + c*Sec[2
*a + 2*b*x]])
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 3774

```
Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[(-2*b)/d,
 Subst[Int[1/(a + x^2), x], x, (b*Cot[c + d*x])/Sqrt[a + b*Csc[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3805

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[(a*Cot[e + f*x]*(d*Csc[e + f*x])^n)/(f*n*Sqrt[a
 + b*Csc[e + f*x]]), x] + Dist[(a*(2*n + 1))/(2*b*d*n), Int[Sqrt[a + b*Csc[
e + f*x]]*(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f}, x] &&
EqQ[a^2 - b^2, 0] && LtQ[n, -2^(-1)] && IntegerQ[2*n]
```

Rule 3813

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(b^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 2)
*(d*Csc[e + f*x])^n)/(f*n), x] - Dist[a/(d*n), Int[(a + b*Csc[e + f*x])^(m
- 2)*(d*Csc[e + f*x])^(n + 1)*(b*(m - 2*n - 2) - a*(m + 2*n - 1)*Csc[e + f*
x]), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, 1]
&& (LtQ[n, -1] || (EqQ[m, 3/2] && EqQ[n, -2^(-1)])) && IntegerQ[2*m]
```

Rule 4397

```
Int[u_, x_Symbol] :> Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]
```

Rubi steps

$$\int \cos^3(2(a+bx))(c\tan(a+bx)\tan(2(a+bx)))^{3/2}\,dx = \int \cos^3(2a+2bx)(-c+c\sec(2a+2bx))^{3/2}\,dx$$

$$= \frac{c^2\cos^2(2a+2bx)\sin(2a+2bx)}{6b\sqrt{-c+c\sec(2a+2bx)}} - \frac{1}{3}c\int \frac{\cos^2(2a+2bx}{\sqrt{-c}$$

$$= \frac{c^2\cos^2(2a+2bx)\sin(2a+2bx)}{6b\sqrt{-c+c\sec(2a+2bx)}} + \frac{1}{6}(11c)\int \cos^2(2a+$$

$$= -\frac{11c^2\cos(2a+2bx)\sin(2a+2bx)}{24b\sqrt{-c+c\sec(2a+2bx)}} + \frac{c^2\cos^2(2a+2bx)}{6b\sqrt{-c+c\sec}}$$

$$= \frac{11c^2\sin(2a+2bx)}{16b\sqrt{-c+c\sec(2a+2bx)}} - \frac{11c^2\cos(2a+2bx)\sin(2a}{24b\sqrt{-c+c\sec(2a+}}$$

$$= \frac{11c^2\sin(2a+2bx)}{16b\sqrt{-c+c\sec(2a+2bx)}} - \frac{11c^2\cos(2a+2bx)\sin(2a}{24b\sqrt{-c+c\sec(2a+}}$$

$$= -\frac{11c^{3/2}\tanh^{-1}\left(\frac{\sqrt{c}\tan(2a+2bx)}{\sqrt{-c+c\sec(2a+2bx)}}\right)}{16b} + \frac{11c^2\sin(2a+}{16b\sqrt{-c+c\sec(2a}}$$

Mathematica [A] time = 0.23, size = 117, normalized size = 0.64

$$\frac{c\sqrt{c\tan(a+bx)\tan(2(a+bx))}\,\Big(-42\sin(2(a+bx))+14\sin(4(a+bx))-4\sin(6(a+bx))+38\cot(a+bx)-33}{96b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[2*(a + b*x)]^3*(c*Tan[a + b*x]*Tan[2*(a + b*x)])^(3/2),x]
```

```
[Out] (c*(38*Cot[a + b*x] - 33*Sqrt[2]*ArcTanh[(Sqrt[2]*Cos[a + b*x])/Sqrt[Cos[2*
(a + b*x)]]]*Sqrt[Cos[2*(a + b*x)]]*Csc[a + b*x] - 42*Sin[2*(a + b*x)] + 14
*Sin[4*(a + b*x)] - 4*Sin[6*(a + b*x)])*Sqrt[c*Tan[a + b*x]*Tan[2*(a + b*x)
]])/(96*b)
```

fricas [A] time = 0.73, size = 503, normalized size = 2.76

$$\left[\frac{33\left(c\tan(bx+a)^7+3c\tan(bx+a)^5+3c\tan(bx+a)^3+c\tan(bx+a)\right)\sqrt{c}\,\log\left(-\frac{c\tan(bx+a)^5-14c\tan(bx+a)^3-4\sqrt{2}}{\tan}\right.}{192\left(b\tan(bx+a)^7+\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(2*b*x+2*a)^3*(c*tan(b*x+a)*tan(2*b*x+2*a))^(3/2),x, algorithm
="fricas")
```

```
[Out] [1/192*(33*(c*tan(b*x + a)^7 + 3*c*tan(b*x + a)^5 + 3*c*tan(b*x + a)^3 + c*
tan(b*x + a))*sqrt(c)*log(-(c*tan(b*x + a)^5 - 14*c*tan(b*x + a)^3 - 4*sqrt
(2)*(tan(b*x + a)^4 - 4*tan(b*x + a)^2 + 3)*sqrt(-c*tan(b*x + a)^2/(tan(b*x
 + a)^2 - 1))*sqrt(c) + 17*c*tan(b*x + a))/(tan(b*x + a)^5 + 2*tan(b*x + a)
^3 + tan(b*x + a))) - 4*sqrt(2)*(63*c*tan(b*x + a)^6 - 13*c*tan(b*x + a)^4
- 31*c*tan(b*x + a)^2 - 19*c)*sqrt(-c*tan(b*x + a)^2/(tan(b*x + a)^2 - 1)))
/(b*tan(b*x + a)^7 + 3*b*tan(b*x + a)^5 + 3*b*tan(b*x + a)^3 + b*tan(b*x +
a)), 1/96*(33*(c*tan(b*x + a)^7 + 3*c*tan(b*x + a)^5 + 3*c*tan(b*x + a)^3 +
 c*tan(b*x + a))*sqrt(-c)*arctan(2*sqrt(2)*sqrt(-c*tan(b*x + a)^2/(tan(b*x
+ a)^2 - 1))*(tan(b*x + a)^2 - 1)*sqrt(-c)/(c*tan(b*x + a)^3 - 3*c*tan(b*x
+ a))) - 2*sqrt(2)*(63*c*tan(b*x + a)^6 - 13*c*tan(b*x + a)^4 - 31*c*tan(b*
x + a)^2 - 19*c)*sqrt(-c*tan(b*x + a)^2/(tan(b*x + a)^2 - 1)))/(b*tan(b*x +
 a)^7 + 3*b*tan(b*x + a)^5 + 3*b*tan(b*x + a)^3 + b*tan(b*x + a))]
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(2*b*x+2*a)^3*(c*tan(b*x+a)*tan(2*b*x+2*a))^(3/2),x, algorithm
="giac")
```

```
[Out] Timed out
```

maple [B] time = 1.11, size = 1078, normalized size = 5.92

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(2*b*x+2*a)^3*(c*tan(b*x+a)*tan(2*b*x+2*a))^(3/2),x)
```

```
[Out] -2^(1/2)/b*(2*cos(b*x+a)^2-1)*(2^(1/2)*((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2
)^(1/2)*arctanh(1/2*cos(b*x+a)*4^(1/2)*(-1+cos(b*x+a))/sin(b*x+a)^2/((2*cos
(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(1/2)*2^(1/2))*cos(b*x+a)+2^(1/2)*((2*cos(b*
x+a)^2-1)/(cos(b*x+a)+1)^2)^(1/2)*arctanh(1/2*cos(b*x+a)*4^(1/2)*(-1+cos(b*
x+a))/sin(b*x+a)^2/((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(1/2)*2^(1/2))-2*c
os(b*x+a))*(c*sin(b*x+a)^2/(2*cos(b*x+a)^2-1))^(3/2)/sin(b*x+a)^3/(2+2^(1/2
))/(2^(1/2)-2)-6*2^(1/2)/b*(2*cos(b*x+a)^2-1)*(2^(1/2)*((2*cos(b*x+a)^2-1)/
(cos(b*x+a)+1)^2)^(1/2)*arctanh(1/2*cos(b*x+a)*4^(1/2)*(-1+cos(b*x+a))/sin(
b*x+a)^2/((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(1/2)*2^(1/2))*cos(b*x+a)+2^
```

```
(1/2)*((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(1/2)*arctanh(1/2*cos(b*x+a)*4^
(1/2)*(-1+cos(b*x+a))/sin(b*x+a)^2/((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(1
/2)*2^(1/2))+4*cos(b*x+a)^3+2*cos(b*x+a))*(c*sin(b*x+a)^2/(2*cos(b*x+a)^2-1
))^(3/2)/sin(b*x+a)^3/(2+2^(1/2))^3/(2^(1/2)-2)^3+6*2^(1/2)/b*(2*cos(b*x+a)
^2-1)*(16*cos(b*x+a)^5+9*2^(1/2)*((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(1/2
)*arctanh(1/2*cos(b*x+a)*4^(1/2)*(-1+cos(b*x+a))/sin(b*x+a)^2/((2*cos(b*x+a
)^2-1)/(cos(b*x+a)+1)^2)^(1/2)*2^(1/2))*cos(b*x+a)-12*cos(b*x+a)^3+9*2^(1/2
)*((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(1/2)*arctanh(1/2*cos(b*x+a)*4^(1/2
)*(-1+cos(b*x+a))/sin(b*x+a)^2/((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(1/2)*
2^(1/2))+18*cos(b*x+a))*(c*sin(b*x+a)^2/(2*cos(b*x+a)^2-1))^(3/2)/sin(b*x+a
)^3/(2+2^(1/2))^5/(2^(1/2)-2)^5-4/3*2^(1/2)/b*(2*cos(b*x+a)^2-1)*(128*cos(b
*x+a)^7-80*cos(b*x+a)^5+75*2^(1/2)*((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(1
/2)*arctanh(1/2*cos(b*x+a)*4^(1/2)*(-1+cos(b*x+a))/sin(b*x+a)^2/((2*cos(b*x
+a)^2-1)/(cos(b*x+a)+1)^2)^(1/2)*2^(1/2))*cos(b*x+a)+75*2^(1/2)*((2*cos(b*x
+a)^2-1)/(cos(b*x+a)+1)^2)^(1/2)*arctanh(1/2*cos(b*x+a)*4^(1/2)*(-1+cos(b*x
+a))/sin(b*x+a)^2/((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(1/2)*2^(1/2))-100*
cos(b*x+a)^3+150*cos(b*x+a))*(c*sin(b*x+a)^2/(2*cos(b*x+a)^2-1))^(3/2)/sin(
b*x+a)^3/(2+2^(1/2))^7/(2^(1/2)-2)^7
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(2*b*x+2*a)^3*(c*tan(b*x+a)*tan(2*b*x+2*a))^(3/2),x, algorithm
="maxima")
```

```
[Out] Timed out
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \cos\left(2\,a+2\,b\,x\right)^3\,{\left(c\,\tan\left(a+b\,x\right)\,\tan\left(2\,a+2\,b\,x\right)\right)}^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(2*a + 2*b*x)^3*(c*tan(a + b*x)*tan(2*a + 2*b*x))^(3/2),x)
```

```
[Out] int(cos(2*a + 2*b*x)^3*(c*tan(a + b*x)*tan(2*a + 2*b*x))^(3/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(2*b*x+2*a)**3*(c*tan(b*x+a)*tan(2*b*x+2*a))**(3/2),x)
```

```
[Out] Timed out
```

3.619 $\int \frac{\sec^4(2(a+bx))}{\sqrt{c\tan(a+bx)\tan(2(a+bx))}}\,dx$

Optimal. Leaf size=175

$$\frac{\tan(2a+2bx)\sec^2(2a+2bx)}{5b\sqrt{c\sec(2a+2bx)-c}}+\frac{\tan(2a+2bx)\sqrt{c\sec(2a+2bx)-c}}{15bc}+\frac{14\tan(2a+2bx)}{15b\sqrt{c\sec(2a+2bx)-c}}-\frac{\tanh^{-1}\left(\frac{\sqrt{c}\,\tan(}{\sqrt{2}\sqrt{c\sec(}}\right.}{\sqrt{2}\,b\sqrt{c}}$$

```
[Out] -1/2*arctanh(1/2*c^(1/2)*tan(2*b*x+2*a)*2^(1/2)/(-c+c*sec(2*b*x+2*a))^(1/2)
)/b*2^(1/2)/c^(1/2)+14/15*tan(2*b*x+2*a)/b/(-c+c*sec(2*b*x+2*a))^(1/2)+1/5*
sec(2*b*x+2*a)^2*tan(2*b*x+2*a)/b/(-c+c*sec(2*b*x+2*a))^(1/2)+1/15*(-c+c*se
c(2*b*x+2*a))^(1/2)*tan(2*b*x+2*a)/b/c
```

Rubi [A] time = 0.60, antiderivative size = 175, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.194, Rules used = {4397, 3822, 4010, 4001, 3795, 207}

$$\frac{\tan(2a+2bx)\sec^2(2a+2bx)}{5b\sqrt{c\sec(2a+2bx)-c}}+\frac{\tan(2a+2bx)\sqrt{c\sec(2a+2bx)-c}}{15bc}+\frac{14\tan(2a+2bx)}{15b\sqrt{c\sec(2a+2bx)-c}}-\frac{\tanh^{-1}\left(\frac{\sqrt{c}\,\tan(}{\sqrt{2}\sqrt{c\sec(}}\right.}{\sqrt{2}\,b\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[2*(a + b*x)]^4/Sqrt[c*Tan[a + b*x]*Tan[2*(a + b*x)]],x]
```

```
[Out] -(ArcTanh[(Sqrt[c]*Tan[2*a + 2*b*x])/(Sqrt[2]*Sqrt[-c + c*Sec[2*a + 2*b*x]]
)]/(Sqrt[2]*b*Sqrt[c])) + (14*Tan[2*a + 2*b*x])/(15*b*Sqrt[-c + c*Sec[2*a +
 2*b*x]]) + (Sec[2*a + 2*b*x]^2*Tan[2*a + 2*b*x])/(5*b*Sqrt[-c + c*Sec[2*a
+ 2*b*x]]) + (Sqrt[-c + c*Sec[2*a + 2*b*x]]*Tan[2*a + 2*b*x])/(15*b*c)
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 3795

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Dist[-2/f, Subst[Int[1/(2*a + x^2), x], x, (b*Cot[e + f*x])/Sqrt[
a + b*Csc[e + f*x]]], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3822

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[(-2*d^2*Cot[e + f*x]*(d*Csc[e + f*x])^(n - 2))/
```

```
(f*(2*n - 3)*Sqrt[a + b*Csc[e + f*x]]), x] + Dist[d^2/(b*(2*n - 3)), Int[((
d*Csc[e + f*x])^(n - 2)*(2*b*(n - 2) - a*Csc[e + f*x]))/Sqrt[a + b*Csc[e +
f*x]], x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[n, 2
] && IntegerQ[2*n]
```

Rule 4001

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(B*Cot[e + f*x]*(a
+ b*Csc[e + f*x])^m)/(f*(m + 1)), x] + Dist[(a*B*m + A*b*(m + 1))/(b*(m + 1
)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m, x], x] /; FreeQ[{a, b, A, B, e
, f, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && NeQ[a*B*m + A*b*(m
 + 1), 0] &&  !LtQ[m, -2^(-1)]
```

Rule 4010

```
Int[csc[(e_.) + (f_.)*(x_)]^2*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(
csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(B*Cot[e + f*x]*(
a + b*Csc[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m + 2)), Int[Cs
c[e + f*x]*(a + b*Csc[e + f*x])^m*Simp[b*B*(m + 1) + (A*b*(m + 2) - a*B)*Cs
c[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, m}, x] && NeQ[A*b - a*B,
 0] &&  !LtQ[m, -1]
```

Rule 4397

```
Int[u_, x_Symbol] :> Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]
```

Rubi steps

$$\int \frac{\sec^4(2(a+bx))}{\sqrt{c\tan(a+bx)\tan(2(a+bx))}}\,dx = \int \frac{\sec^4(2a+2bx)}{\sqrt{-c+c\sec(2a+2bx)}}\,dx$$

$$= \frac{\sec^2(2a+2bx)\tan(2a+2bx)}{5b\sqrt{-c+c\sec(2a+2bx)}} + \frac{\int \frac{\sec^2(2a+2bx)(4c+c\sec(2a+2bx))}{\sqrt{-c+c\sec(2a+2bx)}}\,dx}{5c}$$

$$= \frac{\sec^2(2a+2bx)\tan(2a+2bx)}{5b\sqrt{-c+c\sec(2a+2bx)}} + \frac{\sqrt{-c+c\sec(2a+2bx)}\,\tan(2a+2bx)}{15bc} +$$

$$= \frac{14\tan(2a+2bx)}{15b\sqrt{-c+c\sec(2a+2bx)}} + \frac{\sec^2(2a+2bx)\tan(2a+2bx)}{5b\sqrt{-c+c\sec(2a+2bx)}} + \frac{\sqrt{-c+c\,\mathrm{se}}}{}$$

$$= \frac{14\tan(2a+2bx)}{15b\sqrt{-c+c\sec(2a+2bx)}} + \frac{\sec^2(2a+2bx)\tan(2a+2bx)}{5b\sqrt{-c+c\sec(2a+2bx)}} + \frac{\sqrt{-c+c\,\mathrm{se}}}{}$$

$$= -\frac{\tanh^{-1}\left(\frac{\sqrt{c}\,\tan(2a+2bx)}{\sqrt{2}\,\sqrt{-c+c\sec(2a+2bx)}}\right)}{\sqrt{2}\,b\sqrt{c}} + \frac{14\tan(2a+2bx)}{15b\sqrt{-c+c\sec(2a+2bx)}} + \frac{\sec^2(2a+}{5b\sqrt{-c+}}$$

Mathematica [A] time = 0.67, size = 112, normalized size = 0.64

$$\frac{\sin(a+bx)\cos(a+bx)\sec^3(2(a+bx))\left(4\cos(2(a+bx))+26\cos(4(a+bx))+30\cos^2(2(a+bx))\tan^{-1}\left(\sqrt{\tan^2(}\right.\right.}{30b\sqrt{c\tan(a+bx)\tan(2(a+bx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[2*(a + b*x)]^4/Sqrt[c*Tan[a + b*x]*Tan[2*(a + b*x)]],x]
```

```
[Out] (Cos[a + b*x]*Sec[2*(a + b*x)]^3*Sin[a + b*x]*(38 + 4*Cos[2*(a + b*x)] + 26
*Cos[4*(a + b*x)] + 30*ArcTan[Sqrt[-1 + Tan[a + b*x]^2]]*Cos[2*(a + b*x)]^2
*Sqrt[-1 + Tan[a + b*x]^2]))/(30*b*Sqrt[c*Tan[a + b*x]*Tan[2*(a + b*x)]])
```

fricas [A] time = 0.51, size = 380, normalized size = 2.17

$$\left[\frac{4\sqrt{2}\left(15\tan(bx+a)^4-20\tan(bx+a)^2+17\right)\sqrt{-\frac{c\tan(bx+a)^2}{\tan(bx+a)^2-1}}+\frac{15\sqrt{2}\left(c\tan(bx+a)^5-2c\tan(bx+a)^3+c\tan(bx+a)\right)\log\left(\frac{\tan(b}{}\right.}{\sqrt{c}}}{60\left(bc\tan(bx+a)^5-2bc\tan(bx+a)^3+bc\tan(bx+a)\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(2*b*x+2*a)^4/(c*tan(b*x+a)*tan(2*b*x+2*a))^(1/2),x, algorithm
="fricas")
```

```
[Out] [1/60*(4*sqrt(2)*(15*tan(b*x + a)^4 - 20*tan(b*x + a)^2 + 17)*sqrt(-c*tan(b
*x + a)^2/(tan(b*x + a)^2 - 1)) + 15*sqrt(2)*(c*tan(b*x + a)^5 - 2*c*tan(b*
x + a)^3 + c*tan(b*x + a))*log((tan(b*x + a)^3 - 2*sqrt(-c*tan(b*x + a)^2/(
tan(b*x + a)^2 - 1))*(tan(b*x + a)^2 - 1)/sqrt(c) - 2*tan(b*x + a))/tan(b*x
 + a)^3)/sqrt(c))/(b*c*tan(b*x + a)^5 - 2*b*c*tan(b*x + a)^3 + b*c*tan(b*x
+ a)), -1/30*(15*sqrt(2)*(c*tan(b*x + a)^5 - 2*c*tan(b*x + a)^3 + c*tan(b*x
 + a))*sqrt(-1/c)*arctan(sqrt(-c*tan(b*x + a)^2/(tan(b*x + a)^2 - 1))*(tan(
b*x + a)^2 - 1)*sqrt(-1/c)/tan(b*x + a)) - 2*sqrt(2)*(15*tan(b*x + a)^4 - 2
0*tan(b*x + a)^2 + 17)*sqrt(-c*tan(b*x + a)^2/(tan(b*x + a)^2 - 1)))/(b*c*t
an(b*x + a)^5 - 2*b*c*tan(b*x + a)^3 + b*c*tan(b*x + a))]
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(2*b*x+2*a)^4/(c*tan(b*x+a)*tan(2*b*x+2*a))^(1/2),x, algorithm
="giac")
```

```
[Out] Timed out
```

maple [B] time = 1.36, size = 984, normalized size = 5.62

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(2*b*x+2*a)^4/(c*tan(b*x+a)*tan(2*b*x+2*a))^(1/2),x)
```

```
[Out] 1/120*2^(1/2)/b*(-1+cos(b*x+a))*(208*((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^
(1/2)*cos(b*x+a)^6+120*arctanh(1/2*4^(1/2)*(2*cos(b*x+a)^2-3*cos(b*x+a)+1)/
((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(1/2)/sin(b*x+a)^2)*cos(b*x+a)^6+120*
ln(-2*(cos(b*x+a)^2*((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(1/2)-2*cos(b*x+a
)^2+cos(b*x+a)-((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(1/2)+1)/sin(b*x+a)^2)
*cos(b*x+a)^6+208*((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(1/2)*cos(b*x+a)^5-
200*((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(1/2)*cos(b*x+a)^4-180*arctanh(1/
2*4^(1/2)*(2*cos(b*x+a)^2-3*cos(b*x+a)+1)/((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1
)^2)^(1/2)/sin(b*x+a)^2)*cos(b*x+a)^4-180*ln(-2*(cos(b*x+a)^2*((2*cos(b*x+a
)^2-1)/(cos(b*x+a)+1)^2)^(1/2)-2*cos(b*x+a)^2+cos(b*x+a)-((2*cos(b*x+a)^2-1
)/(cos(b*x+a)+1)^2)^(1/2)+1)/sin(b*x+a)^2)*cos(b*x+a)^4-200*((2*cos(b*x+a)^
2-1)/(cos(b*x+a)+1)^2)^(1/2)*cos(b*x+a)^3+60*cos(b*x+a)^2*((2*cos(b*x+a)^2-
1)/(cos(b*x+a)+1)^2)^(1/2)+90*arctanh(1/2*4^(1/2)*(2*cos(b*x+a)^2-3*cos(b*x
+a)+1)/((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(1/2)/sin(b*x+a)^2)*cos(b*x+a)
^2+90*ln(-2*(cos(b*x+a)^2*((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(1/2)-2*cos
(b*x+a)^2+cos(b*x+a)-((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(1/2)+1)/sin(b*x
+a)^2)*cos(b*x+a)^2+60*cos(b*x+a)*((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(1/
2)-15*arctanh(1/2*4^(1/2)*(2*cos(b*x+a)^2-3*cos(b*x+a)+1)/((2*cos(b*x+a)^2-
1)/(cos(b*x+a)+1)^2)^(1/2)/sin(b*x+a)^2)-15*ln(-2*(cos(b*x+a)^2*((2*cos(b*x
+a)^2-1)/(cos(b*x+a)+1)^2)^(1/2)-2*cos(b*x+a)^2+cos(b*x+a)-((2*cos(b*x+a)^2
-1)/(cos(b*x+a)+1)^2)^(1/2)+1)/sin(b*x+a)^2))/(2*cos(b*x+a)^2-1)^3/((2*cos(
b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(1/2)/(c*(1-cos(b*x+a)^2)/(2*cos(b*x+a)^2-1))
^(1/2)/sin(b*x+a)*4^(1/2)/(-3+2*2^(1/2))^3/(3+2*2^(1/2))^3
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec(2\,bx+2\,a)^4}{\sqrt{c\tan(2\,bx+2\,a)\tan(bx+a)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(2*b*x+2*a)^4/(c*tan(b*x+a)*tan(2*b*x+2*a))^(1/2),x, algorithm
="maxima")
```

```
[Out] integrate(sec(2*b*x + 2*a)^4/sqrt(c*tan(2*b*x + 2*a)*tan(b*x + a)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\cos(2\,a+2\,b\,x)^4\,\sqrt{c\tan(a+b\,x)\,\tan(2\,a+2\,b\,x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(2*a + 2*b*x)^4*(c*tan(a + b*x)*tan(2*a + 2*b*x))^(1/2)),x)
```

```
[Out] int(1/(cos(2*a + 2*b*x)^4*(c*tan(a + b*x)*tan(2*a + 2*b*x))^(1/2)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(2*b*x+2*a)**4/(c*tan(b*x+a)*tan(2*b*x+2*a))**(1/2),x)
[Out] Timed out
```

3.620 $\int \frac{\sec^3(2(a+bx))}{\sqrt{c\tan(a+bx)\tan(2(a+bx))}}\,dx$

Optimal. Leaf size=129

$$\frac{\tan(2a+2bx)\sqrt{c\sec(2a+2bx)-c}}{3bc}+\frac{2\tan(2a+2bx)}{3b\sqrt{c\sec(2a+2bx)-c}}-\frac{\tanh^{-1}\left(\frac{\sqrt{c}\tan(2a+2bx)}{\sqrt{2}\sqrt{c\sec(2a+2bx)-c}}\right)}{\sqrt{2}\,b\sqrt{c}}$$

```
[Out] -1/2*arctanh(1/2*c^(1/2)*tan(2*b*x+2*a)*2^(1/2)/(-c+c*sec(2*b*x+2*a))^(1/2)
)/b*2^(1/2)/c^(1/2)+2/3*tan(2*b*x+2*a)/b/(-c+c*sec(2*b*x+2*a))^(1/2)+1/3*(-
c+c*sec(2*b*x+2*a))^(1/2)*tan(2*b*x+2*a)/b/c
```

Rubi [A] time = 0.36, antiderivative size = 129, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.161, Rules used = {4397, 3800, 4001, 3795, 207}

$$\frac{\tan(2a+2bx)\sqrt{c\sec(2a+2bx)-c}}{3bc}+\frac{2\tan(2a+2bx)}{3b\sqrt{c\sec(2a+2bx)-c}}-\frac{\tanh^{-1}\left(\frac{\sqrt{c}\tan(2a+2bx)}{\sqrt{2}\sqrt{c\sec(2a+2bx)-c}}\right)}{\sqrt{2}\,b\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[2*(a + b*x)]^3/Sqrt[c*Tan[a + b*x]*Tan[2*(a + b*x)]],x]
```

```
[Out] -(ArcTanh[(Sqrt[c]*Tan[2*a + 2*b*x])/(Sqrt[2]*Sqrt[-c + c*Sec[2*a + 2*b*x]]
)]/(Sqrt[2]*b*Sqrt[c])) + (2*Tan[2*a + 2*b*x])/(3*b*Sqrt[-c + c*Sec[2*a + 2
*b*x]]) + (Sqrt[-c + c*Sec[2*a + 2*b*x]]*Tan[2*a + 2*b*x])/(3*b*c)
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 3795

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Dist[-2/f, Subst[Int[1/(2*a + x^2), x], x, (b*Cot[e + f*x])/Sqrt[
a + b*Csc[e + f*x]]], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3800

```
Int[csc[(e_.) + (f_.)*(x_)]^3*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_),
x_Symbol] :> -Simp[(Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1))/(b*f*(m + 2)
), x] + Dist[1/(b*(m + 2)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m*(b*(m +
```

```
 1) - a*Csc[e + f*x]), x], x] /; FreeQ[{a, b, e, f, m}, x] && EqQ[a^2 - b^2
, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 4001

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(B*Cot[e + f*x]*(a
+ b*Csc[e + f*x])^m)/(f*(m + 1)), x] + Dist[(a*B*m + A*b*(m + 1))/(b*(m + 1
)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m, x], x] /; FreeQ[{a, b, A, B, e
, f, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && NeQ[a*B*m + A*b*(m
 + 1), 0] &&  !LtQ[m, -2^(-1)]
```

Rule 4397

```
Int[u_, x_Symbol] :> Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]
```

Rubi steps

$$\int \frac{\sec^3(2(a+bx))}{\sqrt{c\tan(a+bx)\tan(2(a+bx))}}\,dx = \int \frac{\sec^3(2a+2bx)}{\sqrt{-c+c\sec(2a+2bx)}}\,dx$$

$$= \frac{\sqrt{-c+c\sec(2a+2bx)}\,\tan(2a+2bx)}{3bc} + \frac{2\int \frac{\sec(2a+2bx)\left(\frac{c}{2}+c\sec(2a+2bx)\right)}{\sqrt{-c+c\sec(2a+2bx)}}\,dx}{3c}$$

$$= \frac{2\tan(2a+2bx)}{3b\sqrt{-c+c\sec(2a+2bx)}} + \frac{\sqrt{-c+c\sec(2a+2bx)}\,\tan(2a+2bx)}{3bc} + \int$$

$$= \frac{2\tan(2a+2bx)}{3b\sqrt{-c+c\sec(2a+2bx)}} + \frac{\sqrt{-c+c\sec(2a+2bx)}\,\tan(2a+2bx)}{3bc} - \frac{\text{Su}}{}$$

$$= -\frac{\tanh^{-1}\left(\frac{\sqrt{c}\,\tan(2a+2bx)}{\sqrt{2}\,\sqrt{-c+c\sec(2a+2bx)}}\right)}{\sqrt{2}\,b\sqrt{c}} + \frac{2\tan(2a+2bx)}{3b\sqrt{-c+c\sec(2a+2bx)}} + \frac{\sqrt{-c+c\,\text{se}}}{}$$

Mathematica [A] time = 0.39, size = 89, normalized size = 0.69

$$\frac{\cos^2(a+bx)\csc(2(a+bx))\sqrt{c\tan(a+bx)\tan(2(a+bx))}\left(3\sqrt{\tan^2(a+bx)-1}\,\tan^{-1}\left(\sqrt{\tan^2(a+bx)-1}\right)+2\,\text{s}\right.}{3bc}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[2*(a + b*x)]^3/Sqrt[c*Tan[a + b*x]*Tan[2*(a + b*x)]],x]
```

```
[Out] (Cos[a + b*x]^2*Csc[2*(a + b*x)]*(2 + 2*Sec[2*(a + b*x)] + 3*ArcTan[Sqrt[-1
 + Tan[a + b*x]^2]]*Sqrt[-1 + Tan[a + b*x]^2])*Sqrt[c*Tan[a + b*x]*Tan[2*(a
 + b*x)]])/(3*b*c)
```

fricas [A] time = 2.20, size = 294, normalized size = 2.28

$$\left[\frac{\frac{3\sqrt{2}\left(c\tan(bx+a)^3-c\tan(bx+a)\right)\log\left(\frac{\tan(bx+a)^3-\frac{2\sqrt{-\frac{c\tan(bx+a)^2}{\tan(bx+a)^2-1}}\left(\tan(bx+a)^2-1\right)}{\sqrt{c}}-2\tan(bx+a)}{\tan(bx+a)^3}\right)}{\sqrt{c}}-8\sqrt{2}\sqrt{-\frac{c\tan(bx+a)^2}{\tan(bx+a)^2-1}}}{12\left(bc\tan(bx+a)^3-bc\tan(bx+a)\right)}, -\frac{3\sqrt{2}\left(c\tan(bx\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(2*b*x+2*a)^3/(c*tan(b*x+a)*tan(2*b*x+2*a))^(1/2),x, algorithm
="fricas")
```

```
[Out] [1/12*(3*sqrt(2)*(c*tan(b*x + a)^3 - c*tan(b*x + a))*log((tan(b*x + a)^3 -
2*sqrt(-c*tan(b*x + a)^2/(tan(b*x + a)^2 - 1))*(tan(b*x + a)^2 - 1)/sqrt(c)
 - 2*tan(b*x + a))/tan(b*x + a)^3)/sqrt(c) - 8*sqrt(2)*sqrt(-c*tan(b*x + a)
^2/(tan(b*x + a)^2 - 1)))/(b*c*tan(b*x + a)^3 - b*c*tan(b*x + a)), -1/6*(3*
sqrt(2)*(c*tan(b*x + a)^3 - c*tan(b*x + a))*sqrt(-1/c)*arctan(sqrt(-c*tan(b
*x + a)^2/(tan(b*x + a)^2 - 1))*(tan(b*x + a)^2 - 1)*sqrt(-1/c)/tan(b*x + a
)) + 4*sqrt(2)*sqrt(-c*tan(b*x + a)^2/(tan(b*x + a)^2 - 1)))/(b*c*tan(b*x +
 a)^3 - b*c*tan(b*x + a))]
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(2*b*x+2*a)^3/(c*tan(b*x+a)*tan(2*b*x+2*a))^(1/2),x, algorithm
="giac")
```

```
[Out] Timed out
```

maple [B] time = 1.21, size = 677, normalized size = 5.25

$$-\frac{\sqrt{2}\,\left(-1+\cos\left(bx+a\right)\right)\left(12\,\mathrm{arctanh}\left(\frac{\sqrt{4}\,\left(2\left(\cos^{2}(bx+a)\right)-3\cos(bx+a)+1\right)}{2\sqrt{\frac{2\left(\cos^{2}(bx+a)\right)-1}{(\cos(bx+a)+1)^{2}}}\,\sin(bx+a)^{2}}\right)\left(\cos^{4}\left(bx+a\right)\right)+12\ln\left(-\frac{2\left(\cos^{2}(bx+a)\right)\sqrt{\frac{2(\cos}{(\cos}}}{}\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(2*b*x+2*a)^3/(c*tan(b*x+a)*tan(2*b*x+2*a))^(1/2),x)
```

```
[Out] -1/24*2^(1/2)/b*(-1+cos(b*x+a))*(12*arctanh(1/2*4^(1/2)*(2*cos(b*x+a)^2-3*c
os(b*x+a)+1)/((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(1/2)/sin(b*x+a)^2)*cos(
b*x+a)^4+12*ln(-2*(cos(b*x+a)^2*((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(1/2)
-2*cos(b*x+a)^2+cos(b*x+a)-((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(1/2)+1)/s
in(b*x+a)^2)*cos(b*x+a)^4+8*((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(1/2)*cos
(b*x+a)^4+8*((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(1/2)*cos(b*x+a)^3-12*arc
tanh(1/2*4^(1/2)*(2*cos(b*x+a)^2-3*cos(b*x+a)+1)/((2*cos(b*x+a)^2-1)/(cos(b
*x+a)+1)^2)^(1/2)/sin(b*x+a)^2)*cos(b*x+a)^2-12*ln(-2*(cos(b*x+a)^2*((2*cos
(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(1/2)-2*cos(b*x+a)^2+cos(b*x+a)-((2*cos(b*x+
a)^2-1)/(cos(b*x+a)+1)^2)^(1/2)+1)/sin(b*x+a)^2)*cos(b*x+a)^2+3*arctanh(1/2
*4^(1/2)*(2*cos(b*x+a)^2-3*cos(b*x+a)+1)/((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)
^2)^(1/2)/sin(b*x+a)^2)+3*ln(-2*(cos(b*x+a)^2*((2*cos(b*x+a)^2-1)/(cos(b*x+
a)+1)^2)^(1/2)-2*cos(b*x+a)^2+cos(b*x+a)-((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)
^2)^(1/2)+1)/sin(b*x+a)^2))/(2*cos(b*x+a)^2-1)^2/((2*cos(b*x+a)^2-1)/(cos(b
*x+a)+1)^2)^(1/2)/(c*(1-cos(b*x+a)^2)/(2*cos(b*x+a)^2-1))^(1/2)/sin(b*x+a)*
4^(1/2)/(-3+2*2^(1/2))^2/(3+2*2^(1/2))^2
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec\left(2\,bx+2\,a\right)^{3}}{\sqrt{c\tan\left(2\,bx+2\,a\right)\tan\left(bx+a\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(2*b*x+2*a)^3/(c*tan(b*x+a)*tan(2*b*x+2*a))^(1/2),x, algorithm
="maxima")
```

```
[Out] integrate(sec(2*b*x + 2*a)^3/sqrt(c*tan(2*b*x + 2*a)*tan(b*x + a)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\cos\left(2\,a+2\,b\,x\right)^{3}\,\sqrt{c\tan\left(a+b\,x\right)\,\tan\left(2\,a+2\,b\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(2*a + 2*b*x)^3*(c*tan(a + b*x)*tan(2*a + 2*b*x))^(1/2)),x)
```

```
[Out] int(1/(cos(2*a + 2*b*x)^3*(c*tan(a + b*x)*tan(2*a + 2*b*x))^(1/2)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(2*b*x+2*a)**3/(c*tan(b*x+a)*tan(2*b*x+2*a))**(1/2),x)
```

```
[Out] Timed out
```

3.621 $\int \frac{\sec^2(2(a+bx))}{\sqrt{c\tan(a+bx)\tan(2(a+bx))}}\,dx$

Optimal. Leaf size=88

$$\frac{\tan(2a+2bx)}{b\sqrt{c\sec(2a+2bx)-c}}-\frac{\tanh^{-1}\left(\frac{\sqrt{c}\tan(2a+2bx)}{\sqrt{2}\sqrt{c\sec(2a+2bx)-c}}\right)}{\sqrt{2}\,b\sqrt{c}}$$

```
[Out] -1/2*arctanh(1/2*c^(1/2)*tan(2*b*x+2*a)*2^(1/2)/(-c+c*sec(2*b*x+2*a))^(1/2)
)/b*2^(1/2)/c^(1/2)+tan(2*b*x+2*a)/b/(-c+c*sec(2*b*x+2*a))^(1/2)
```

Rubi [A] time = 0.24, antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.129, Rules used = {4397, 3798, 3795, 207}

$$\frac{\tan(2a+2bx)}{b\sqrt{c\sec(2a+2bx)-c}}-\frac{\tanh^{-1}\left(\frac{\sqrt{c}\tan(2a+2bx)}{\sqrt{2}\sqrt{c\sec(2a+2bx)-c}}\right)}{\sqrt{2}\,b\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[2*(a + b*x)]^2/Sqrt[c*Tan[a + b*x]*Tan[2*(a + b*x)]],x]
```

```
[Out] -(ArcTanh[(Sqrt[c]*Tan[2*a + 2*b*x])/(Sqrt[2]*Sqrt[-c + c*Sec[2*a + 2*b*x]]
)]/(Sqrt[2]*b*Sqrt[c])) + Tan[2*a + 2*b*x]/(b*Sqrt[-c + c*Sec[2*a + 2*b*x]]
)
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 3795

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Dist[-2/f, Subst[Int[1/(2*a + x^2), x], x, (b*Cot[e + f*x])/Sqrt[
a + b*Csc[e + f*x]]], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3798

```
Int[csc[(e_.) + (f_.)*(x_)]^2*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_),
x_Symbol] :> -Simp[(Cot[e + f*x]*(a + b*Csc[e + f*x])^m)/(f*(m + 1)), x] +
Dist[(a*m)/(b*(m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m, x], x] /;
```

```
FreeQ[{a, b, e, f, m}, x] && EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 4397

```
Int[u_, x_Symbol] :> Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sec^2(2(a+bx))}{\sqrt{c\tan(a+bx)\tan(2(a+bx))}}\,dx &= \int \frac{\sec^2(2a+2bx)}{\sqrt{-c+c\sec(2a+2bx)}}\,dx\\
&= \frac{\tan(2a+2bx)}{b\sqrt{-c+c\sec(2a+2bx)}}+\int \frac{\sec(2a+2bx)}{\sqrt{-c+c\sec(2a+2bx)}}\,dx\\
&= \frac{\tan(2a+2bx)}{b\sqrt{-c+c\sec(2a+2bx)}}-\frac{\text{Subst}\left(\int \frac{1}{-2c+x^2}\,dx,x,-\frac{c\tan(2a+2bx)}{\sqrt{-c+c\sec(2a+2bx)}}\right)}{b}\\
&= -\frac{\tanh^{-1}\left(\frac{\sqrt{c}\,\tan(2a+2bx)}{\sqrt{2}\,\sqrt{-c+c\sec(2a+2bx)}}\right)}{\sqrt{2}\,b\sqrt{c}}+\frac{\tan(2a+2bx)}{b\sqrt{-c+c\sec(2a+2bx)}}
\end{aligned}
$$

Mathematica [A] time = 0.24, size = 67, normalized size = 0.76

$$
\frac{\left(\sqrt{\tan^2(a+bx)-1}\,\tan^{-1}\left(\sqrt{\tan^2(a+bx)-1}\right)+2\right)\tan(2(a+bx))}{2b\sqrt{c\tan(a+bx)\tan(2(a+bx))}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[2*(a + b*x)]^2/Sqrt[c*Tan[a + b*x]*Tan[2*(a + b*x)]],x]
```

```
[Out] ((2 + ArcTan[Sqrt[-1 + Tan[a + b*x]^2]]*Sqrt[-1 + Tan[a + b*x]^2])*Tan[2*(a
 + b*x)])/(2*b*Sqrt[c*Tan[a + b*x]*Tan[2*(a + b*x)]])
```

fricas [A] time = 1.63, size = 245, normalized size = 2.78

$$
\left[\frac{\sqrt{2}\sqrt{c}\,\log\left(\frac{\tan(bx+a)^3-\frac{2\sqrt{-\frac{c\tan(bx+a)^2}{\tan(bx+a)^2-1}}\left(\tan(bx+a)^2-1\right)}{\sqrt{c}}-2\tan(bx+a)}{\tan(bx+a)^3}\right)\tan(bx+a)+4\sqrt{2}\sqrt{-\frac{c\tan(bx+a)^2}{\tan(bx+a)^2-1}}}{4bc\tan(bx+a)}, -\frac{\sqrt{2}\,c\sqrt{-\frac{1}{c}}\,\text{arcta}}{}\right.
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(2*b*x+2*a)^2/(c*tan(b*x+a)*tan(2*b*x+2*a))^(1/2),x, algorithm
="fricas")
```

```
[Out] [1/4*(sqrt(2)*sqrt(c)*log((tan(b*x + a)^3 - 2*sqrt(-c*tan(b*x + a)^2/(tan(b
*x + a)^2 - 1))*(tan(b*x + a)^2 - 1)/sqrt(c) - 2*tan(b*x + a))/tan(b*x + a)
^3)*tan(b*x + a) + 4*sqrt(2)*sqrt(-c*tan(b*x + a)^2/(tan(b*x + a)^2 - 1)))/
(b*c*tan(b*x + a)), -1/2*(sqrt(2)*c*sqrt(-1/c)*arctan(sqrt(-c*tan(b*x + a)^
2/(tan(b*x + a)^2 - 1))*(tan(b*x + a)^2 - 1)*sqrt(-1/c)/tan(b*x + a))*tan(b
*x + a) - 2*sqrt(2)*sqrt(-c*tan(b*x + a)^2/(tan(b*x + a)^2 - 1)))/(b*c*tan(
b*x + a))]
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(2*b*x+2*a)^2/(c*tan(b*x+a)*tan(2*b*x+2*a))^(1/2),x, algorithm
="giac")
```

```
[Out] Timed out
```

maple [B] time = 1.19, size = 478, normalized size = 5.43

$$\frac{\sqrt{2}\left(\cos\left(bx+a\right)\ln\left(-\frac{2\left(\left(\cos^{2}(bx+a)\right)\sqrt{\frac{2\left(\cos^{2}(bx+a)\right)-1}{(\cos(bx+a)+1)^{2}}}-2\left(\cos^{2}(bx+a)\right)+\cos(bx+a)-\sqrt{\frac{2\left(\cos^{2}(bx+a)\right)-1}{(\cos(bx+a)+1)^{2}}}+1\right)}{\sin(bx+a)^{2}}\right)\sqrt{\frac{2\left(\cos^{2}(bx+a)\right)-1}{(\cos(bx+a)+1)^{2}}}+\cos\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(2*b*x+2*a)^2/(c*tan(b*x+a)*tan(2*b*x+2*a))^(1/2),x)
```

```
[Out] 1/4*2^(1/2)/b*(cos(b*x+a)*ln(-2*(cos(b*x+a)^2*((2*cos(b*x+a)^2-1)/(cos(b*x+
a)+1)^2)^(1/2)-2*cos(b*x+a)^2+cos(b*x+a)-((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)
^2)^(1/2)+1)/sin(b*x+a)^2)*((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(1/2)+cos(
b*x+a)*((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(1/2)*arctanh(1/2*4^(1/2)*(2*c
os(b*x+a)^2-3*cos(b*x+a)+1)/((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(1/2)/sin
(b*x+a)^2)+ln(-2*(cos(b*x+a)^2*((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(1/2)-
2*cos(b*x+a)^2+cos(b*x+a)-((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(1/2)+1)/si
n(b*x+a)^2)*((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(1/2)+arctanh(1/2*4^(1/2)
*(2*cos(b*x+a)^2-3*cos(b*x+a)+1)/((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(1/2
```

```
)/sin(b*x+a)^2)*((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(1/2)+4*cos(b*x+a))*s
in(b*x+a)/(2*cos(b*x+a)^2-1)/(c*sin(b*x+a)^2/(2*cos(b*x+a)^2-1))^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec(2\,bx+2\,a)^2}{\sqrt{c\tan(2\,bx+2\,a)\tan(bx+a)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(2*b*x+2*a)^2/(c*tan(b*x+a)*tan(2*b*x+2*a))^(1/2),x, algorithm
="maxima")
```

```
[Out] integrate(sec(2*b*x + 2*a)^2/sqrt(c*tan(2*b*x + 2*a)*tan(b*x + a)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\cos(2\,a+2\,b\,x)^2\,\sqrt{c\tan(a+b\,x)\,\tan(2\,a+2\,b\,x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(2*a + 2*b*x)^2*(c*tan(a + b*x)*tan(2*a + 2*b*x))^(1/2)),x)
```

```
[Out] int(1/(cos(2*a + 2*b*x)^2*(c*tan(a + b*x)*tan(2*a + 2*b*x))^(1/2)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(2*b*x+2*a)**2/(c*tan(b*x+a)*tan(2*b*x+2*a))**(1/2),x)
```

```
[Out] Timed out
```

3.622 $\int \frac{\sec(2(a+bx))}{\sqrt{c\tan(a+bx)\tan(2(a+bx))}}\,dx$

Optimal. Leaf size=55

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{c}\,\tan(2a+2bx)}{\sqrt{2}\,\sqrt{c\sec(2a+2bx)-c}}\right)}{\sqrt{2}\,b\sqrt{c}}$$

```
[Out] -1/2*arctanh(1/2*c^(1/2)*tan(2*b*x+2*a)*2^(1/2)/(-c+c*sec(2*b*x+2*a))^(1/2)
)/b*2^(1/2)/c^(1/2)
```

Rubi [A] time = 0.08, antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {4397, 3795, 207}

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{c}\,\tan(2a+2bx)}{\sqrt{2}\,\sqrt{c\sec(2a+2bx)-c}}\right)}{\sqrt{2}\,b\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[2*(a + b*x)]/Sqrt[c*Tan[a + b*x]*Tan[2*(a + b*x)]],x]
```

```
[Out] -(ArcTanh[(Sqrt[c]*Tan[2*a + 2*b*x])/(Sqrt[2]*Sqrt[-c + c*Sec[2*a + 2*b*x]]
)]/(Sqrt[2]*b*Sqrt[c]))
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 3795

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Dist[-2/f, Subst[Int[1/(2*a + x^2), x], x, (b*Cot[e + f*x])/Sqrt[
a + b*Csc[e + f*x]]], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 4397

```
Int[u_, x_Symbol] :> Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]
```

Rubi steps

$$\int \frac{\sec(2(a+bx))}{\sqrt{c\tan(a+bx)\tan(2(a+bx))}}\,dx=\int \frac{\sec(2a+2bx)}{\sqrt{-c+c\sec(2a+2bx)}}\,dx$$
$$=-\frac{\text{Subst}\left(\int \frac{1}{-2c+x^2}\,dx,x,-\frac{c\tan(2a+2bx)}{\sqrt{-c+c\sec(2a+2bx)}}\right)}{b}$$
$$=-\frac{\tanh^{-1}\left(\frac{\sqrt{c}\tan(2a+2bx)}{\sqrt{2}\sqrt{-c+c\sec(2a+2bx)}}\right)}{\sqrt{2}\,b\sqrt{c}}$$

Mathematica [A] time = 0.15, size = 64, normalized size = 1.16

$$\frac{\tan^{-1}\left(\sqrt{\tan^2(a+bx)-1}\right)\sqrt{\tan^2(a+bx)-1}\,\tan(2(a+bx))}{2b\sqrt{c\tan(a+bx)\tan(2(a+bx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[2*(a + b*x)]/Sqrt[c*Tan[a + b*x]*Tan[2*(a + b*x)]],x]
```

```
[Out] (ArcTan[Sqrt[-1 + Tan[a + b*x]^2]]*Sqrt[-1 + Tan[a + b*x]^2]*Tan[2*(a + b*x
)])/(2*b*Sqrt[c*Tan[a + b*x]*Tan[2*(a + b*x)]])
```

fricas [A] time = 2.00, size = 146, normalized size = 2.65

$$\left[\frac{\sqrt{2}\,\log\left(\frac{\tan(bx+a)^3-\frac{2\sqrt{-\frac{c\tan(bx+a)^2}{\tan(bx+a)^2-1}}\left(\tan(bx+a)^2-1\right)}{\sqrt{c}}-2\tan(bx+a)}{\tan(bx+a)^3}\right)}{4\,b\sqrt{c}},-\frac{\sqrt{2}\sqrt{-\frac{1}{c}}\,\arctan\left(\frac{\sqrt{-\frac{c\tan(bx+a)^2}{\tan(bx+a)^2-1}}\left(\tan(bx+a)^2-1\right)\sqrt{-\frac{1}{c}}}{\tan(bx+a)}\right)}{2\,b}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(2*b*x+2*a)/(c*tan(b*x+a)*tan(2*b*x+2*a))^(1/2),x, algorithm="
fricas")
```

```
[Out] [1/4*sqrt(2)*log((tan(b*x + a)^3 - 2*sqrt(-c*tan(b*x + a)^2/(tan(b*x + a)^2
 - 1))*(tan(b*x + a)^2 - 1)/sqrt(c) - 2*tan(b*x + a))/tan(b*x + a)^3)/(b*sq
```

```
rt(c)), -1/2*sqrt(2)*sqrt(-1/c)*arctan(sqrt(-c*tan(b*x + a)^2/(tan(b*x + a)
^2 - 1))*(tan(b*x + a)^2 - 1)*sqrt(-1/c)/tan(b*x + a))/b]
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(2*b*x+2*a)/(c*tan(b*x+a)*tan(2*b*x+2*a))^(1/2),x, algorithm="
giac")
```

```
[Out] Timed out
```

maple [B] time = 1.07, size = 236, normalized size = 4.29

$$\frac{\sqrt{2}\,\left(\cos\left(bx+a\right)+1\right)\sqrt{\frac{2\left(\cos^{2}(bx+a)\right)-1}{(\cos(bx+a)+1)^{2}}}\,\sqrt{\frac{c\left(1-\left(\cos^{2}(bx+a)\right)\right)}{2\left(\cos^{2}(bx+a)\right)-1}}\,\left(\ln\left(-\frac{2\left(\left(\cos^{2}(bx+a)\right)\sqrt{\frac{2\left(\cos^{2}(bx+a)\right)-1}{(\cos(bx+a)+1)^{2}}}-2\left(\cos^{2}(bx+a)\right)+\cos(bx+a)-\cdots}{\sin(bx+a)^{2}}\right.\right.}{8b\sin\left(bx+a\right)c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(2*b*x+2*a)/(c*tan(b*x+a)*tan(2*b*x+2*a))^(1/2),x)
```

```
[Out] 1/8*2^(1/2)/b*(cos(b*x+a)+1)*((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(1/2)*(c
*(1-cos(b*x+a)^2)/(2*cos(b*x+a)^2-1))^(1/2)*(ln(-2*(cos(b*x+a)^2*((2*cos(b*
x+a)^2-1)/(cos(b*x+a)+1)^2)^(1/2)-2*cos(b*x+a)^2+cos(b*x+a)-((2*cos(b*x+a)^
2-1)/(cos(b*x+a)+1)^2)^(1/2)+1)/sin(b*x+a)^2)+arctanh(1/2*4^(1/2)*(2*cos(b*
x+a)^2-3*cos(b*x+a)+1)/((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(1/2)/sin(b*x+
a)^2))/sin(b*x+a)/c*4^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec\left(2\,bx+2\,a\right)}{\sqrt{c\tan\left(2\,bx+2\,a\right)\tan\left(bx+a\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(2*b*x+2*a)/(c*tan(b*x+a)*tan(2*b*x+2*a))^(1/2),x, algorithm="
maxima")
```

```
[Out] integrate(sec(2*b*x + 2*a)/sqrt(c*tan(2*b*x + 2*a)*tan(b*x + a)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{\cos\left(2\,a+2\,b\,x\right)\,\sqrt{c\,\tan\left(a+b\,x\right)\,\tan\left(2\,a+2\,b\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(2*a + 2*b*x)*(c*tan(a + b*x)*tan(2*a + 2*b*x))^(1/2)),x)
```

```
[Out] int(1/(cos(2*a + 2*b*x)*(c*tan(a + b*x)*tan(2*a + 2*b*x))^(1/2)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(2*b*x+2*a)/(c*tan(b*x+a)*tan(2*b*x+2*a))**(1/2),x)
```

```
[Out] Timed out
```

3.623 $\int \frac{1}{\sqrt{c\tan(a+bx)\tan(2(a+bx))}}\,dx$

Optimal. Leaf size=100

$$\frac{\tanh^{-1}\left(\frac{\sqrt{c}\tan(2a+2bx)}{\sqrt{c\sec(2a+2bx)-c}}\right)}{b\sqrt{c}}-\frac{\tanh^{-1}\left(\frac{\sqrt{c}\tan(2a+2bx)}{\sqrt{2}\sqrt{c\sec(2a+2bx)-c}}\right)}{\sqrt{2}b\sqrt{c}}$$

```
[Out] arctanh(c^(1/2)*tan(2*b*x+2*a)/(-c+c*sec(2*b*x+2*a))^(1/2))/b/c^(1/2)-1/2*a
rctanh(1/2*c^(1/2)*tan(2*b*x+2*a)*2^(1/2)/(-c+c*sec(2*b*x+2*a))^(1/2))/b*2^
(1/2)/c^(1/2)
```

Rubi [A] time = 0.09, antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {4397, 3776, 3774, 207, 3795}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{c}\tan(2a+2bx)}{\sqrt{c\sec(2a+2bx)-c}}\right)}{b\sqrt{c}}-\frac{\tanh^{-1}\left(\frac{\sqrt{c}\tan(2a+2bx)}{\sqrt{2}\sqrt{c\sec(2a+2bx)-c}}\right)}{\sqrt{2}b\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[c*Tan[a + b*x]*Tan[2*(a + b*x)]],x]
```

```
[Out] ArcTanh[(Sqrt[c]*Tan[2*a + 2*b*x])/Sqrt[-c + c*Sec[2*a + 2*b*x]]]/(b*Sqrt[c
]) - ArcTanh[(Sqrt[c]*Tan[2*a + 2*b*x])/(Sqrt[2]*Sqrt[-c + c*Sec[2*a + 2*b*
x]])]/(Sqrt[2]*b*Sqrt[c])
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 3774

```
Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[(-2*b)/d,
 Subst[Int[1/(a + x^2), x], x, (b*Cot[c + d*x])/Sqrt[a + b*Csc[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3776

```
Int[1/Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[1/a, In
t[Sqrt[a + b*Csc[c + d*x]], x], x] - Dist[b/a, Int[Csc[c + d*x]/Sqrt[a + b*
Csc[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3795

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Dist[-2/f, Subst[Int[1/(2*a + x^2), x], x, (b*Cot[e + f*x])/Sqrt[
a + b*Csc[e + f*x]]], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 4397

```
Int[u_, x_Symbol] :> Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{\sqrt{c\tan(a+bx)\tan(2(a+bx))}}\,dx &= \int \frac{1}{\sqrt{-c+c\sec(2a+2bx)}}\,dx \\
&= -\frac{\int \sqrt{-c+c\sec(2a+2bx)}\,dx}{c} + \int \frac{\sec(2a+2bx)}{\sqrt{-c+c\sec(2a+2bx)}}\,dx \\
&= -\frac{\text{Subst}\left(\int \frac{1}{-2c+x^2}\,dx, x, -\frac{c\tan(2a+2bx)}{\sqrt{-c+c\sec(2a+2bx)}}\right)}{b} + \frac{\text{Subst}\left(\int \frac{1}{-c+x^2}\,dx, x, -\frac{c\,\text{ta}}{\sqrt{-c+c}}\right.}{b} \\
&= \frac{\tanh^{-1}\left(\frac{\sqrt{c}\tan(2a+2bx)}{\sqrt{-c+c\sec(2a+2bx)}}\right)}{b\sqrt{c}} - \frac{\tanh^{-1}\left(\frac{\sqrt{c}\tan(2a+2bx)}{\sqrt{2}\sqrt{-c+c\sec(2a+2bx)}}\right)}{\sqrt{2}\,b\sqrt{c}}
\end{aligned}
$$

Mathematica [A] time = 0.31, size = 94, normalized size = 0.94

$$
\frac{\tan(a+bx)\left(2\tanh^{-1}\left(\frac{1}{2}\sqrt{2-2\tan^2(a+bx)}\right)-\sqrt{2}\tanh^{-1}\left(\sqrt{1-\tan^2(a+bx)}\right)\right)}{b\sqrt{2-2\tan^2(a+bx)}\sqrt{c\tan(a+bx)\tan(2(a+bx))}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[c*Tan[a + b*x]*Tan[2*(a + b*x)]],x]
```

```
[Out] ((2*ArcTanh[Sqrt[2 - 2*Tan[a + b*x]^2]/2] - Sqrt[2]*ArcTanh[Sqrt[1 - Tan[a
 + b*x]^2]])*Tan[a + b*x])/(b*Sqrt[2 - 2*Tan[a + b*x]^2]*Sqrt[c*Tan[a + b*x]
*Tan[2*(a + b*x)]])
```

fricas [A] time = 1.90, size = 309, normalized size = 3.09

$$\left[\frac{\sqrt{2}\sqrt{c}\log\left(\frac{c\tan(bx+a)^3-2\sqrt{-\frac{c\tan(bx+a)^2}{\tan(bx+a)^2-1}}\left(\tan(bx+a)^2-1\right)\sqrt{c}-2\,c\tan(bx+a)}{\tan(bx+a)^3}\right)+2\sqrt{c}\log\left(\frac{c\tan(bx+a)^3+2\sqrt{2}\sqrt{-\frac{c\tan(bx+a)^2}{\tan(bx+a)^2-1}}\left(\tan(bx\right.}{\tan(bx+a)^3+\tan(bx+}\right.}{4\,bc}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*tan(b*x+a)*tan(2*b*x+2*a))^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*(sqrt(2)*sqrt(c)*log((c*tan(b*x + a)^3 - 2*sqrt(-c*tan(b*x + a)^2/(tan
(b*x + a)^2 - 1))*(tan(b*x + a)^2 - 1)*sqrt(c) - 2*c*tan(b*x + a))/tan(b*x
+ a)^3) + 2*sqrt(c)*log((c*tan(b*x + a)^3 + 2*sqrt(2)*sqrt(-c*tan(b*x + a)^
2/(tan(b*x + a)^2 - 1))*(tan(b*x + a)^2 - 1)*sqrt(c) - 3*c*tan(b*x + a))/(t
an(b*x + a)^3 + tan(b*x + a))))/(b*c), -1/2*(sqrt(2)*sqrt(-c)*arctan(sqrt(-
c*tan(b*x + a)^2/(tan(b*x + a)^2 - 1))*(tan(b*x + a)^2 - 1)*sqrt(-c)/(c*tan
(b*x + a))) - 2*sqrt(-c)*arctan(1/2*sqrt(2)*sqrt(-c*tan(b*x + a)^2/(tan(b*x
 + a)^2 - 1))*(tan(b*x + a)^2 - 1)*sqrt(-c)/(c*tan(b*x + a))))/(b*c)]
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*tan(b*x+a)*tan(2*b*x+2*a))^(1/2),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [B] time = 1.01, size = 301, normalized size = 3.01

$$-\frac{\sqrt{2}\,\left(\cos\left(bx+a\right)+1\right)\sqrt{\frac{2\left(\cos^2(bx+a)\right)-1}{(\cos(bx+a)+1)^2}}\,\sqrt{\frac{c\left(1-\left(\cos^2(bx+a)\right)\right)}{2\left(\cos^2(bx+a)\right)-1}}\,\left(2\sqrt{2}\,\operatorname{arctanh}\left(\frac{\cos(bx+a)\sqrt{4}\,(-1+\cos(bx+a))\sqrt{2}}{2\sin(bx+a)^2\sqrt{\frac{2\left(\cos^2(bx+a)\right)-1}{(\cos(bx+a)+1)^2}}}\right)-\ln\left(\cdots\right.\right.}{8b\sin(\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*tan(b*x+a)*tan(2*b*x+2*a))^(1/2),x)
```

```
[Out] -1/8*2^(1/2)/b*(cos(b*x+a)+1)*((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(1/2)*(
c*(1-cos(b*x+a)^2)/(2*cos(b*x+a)^2-1))^(1/2)*(2*2^(1/2)*arctanh(1/2*cos(b*x
+a)*4^(1/2)*(-1+cos(b*x+a))/sin(b*x+a)^2/((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)
^2)^(1/2)*2^(1/2))-ln(-2*(cos(b*x+a)^2*((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2
)^(1/2)-2*cos(b*x+a)^2+cos(b*x+a)-((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(1/
2)+1)/sin(b*x+a)^2)-arctanh(1/2*4^(1/2)*(2*cos(b*x+a)^2-3*cos(b*x+a)+1)/((2
*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(1/2)/sin(b*x+a)^2))/sin(b*x+a)/c*4^(1/2
)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*tan(b*x+a)*tan(2*b*x+2*a))^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 sign: argument cannot be imaginary; found %i
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sqrt{c\tan\left(a+b\,x\right)\,\tan\left(2\,a+2\,b\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*tan(a + b*x)*tan(2*a + 2*b*x))^(1/2),x)
```

```
[Out] int(1/(c*tan(a + b*x)*tan(2*a + 2*b*x))^(1/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*tan(b*x+a)*tan(2*b*x+2*a))**(1/2),x)
```

```
[Out] Timed out
```

3.624 $\int \frac{\cos(2(a+bx))}{\sqrt{c\tan(a+bx)\tan(2(a+bx))}}\,dx$

Optimal. Leaf size=138

$$\frac{\sin(2a+2bx)}{2b\sqrt{c\sec(2a+2bx)-c}}+\frac{\tanh^{-1}\left(\frac{\sqrt{c}\tan(2a+2bx)}{\sqrt{c\sec(2a+2bx)-c}}\right)}{2b\sqrt{c}}-\frac{\tanh^{-1}\left(\frac{\sqrt{c}\tan(2a+2bx)}{\sqrt{2}\sqrt{c\sec(2a+2bx)-c}}\right)}{\sqrt{2}\,b\sqrt{c}}$$

```
[Out] 1/2*arctanh(c^(1/2)*tan(2*b*x+2*a)/(-c+c*sec(2*b*x+2*a))^(1/2))/b/c^(1/2)-1
/2*arctanh(1/2*c^(1/2)*tan(2*b*x+2*a)*2^(1/2)/(-c+c*sec(2*b*x+2*a))^(1/2))/
b*2^(1/2)/c^(1/2)+1/2*sin(2*b*x+2*a)/b/(-c+c*sec(2*b*x+2*a))^(1/2)
```

Rubi [A] time = 0.28, antiderivative size = 138, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.207, Rules used = {4397, 3823, 3904, 3887, 481, 206}

$$\frac{\sin(2a+2bx)}{2b\sqrt{c\sec(2a+2bx)-c}}+\frac{\tanh^{-1}\left(\frac{\sqrt{c}\tan(2a+2bx)}{\sqrt{c\sec(2a+2bx)-c}}\right)}{2b\sqrt{c}}-\frac{\tanh^{-1}\left(\frac{\sqrt{c}\tan(2a+2bx)}{\sqrt{2}\sqrt{c\sec(2a+2bx)-c}}\right)}{\sqrt{2}\,b\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[2*(a + b*x)]/Sqrt[c*Tan[a + b*x]*Tan[2*(a + b*x)]],x]
```

```
[Out] ArcTanh[(Sqrt[c]*Tan[2*a + 2*b*x])/Sqrt[-c + c*Sec[2*a + 2*b*x]]]/(2*b*Sqrt
[c]) - ArcTanh[(Sqrt[c]*Tan[2*a + 2*b*x])/(Sqrt[2]*Sqrt[-c + c*Sec[2*a + 2*
b*x]])]/(Sqrt[2]*b*Sqrt[c]) + Sin[2*a + 2*b*x]/(2*b*Sqrt[-c + c*Sec[2*a + 2
*b*x]])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 481

```
Int[((e_.)*(x_))^(m_.)/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(n_))),
 x_Symbol] :> -Dist[(a*e^n)/(b*c - a*d), Int[(e*x)^(m - n)/(a + b*x^n), x],
 x] + Dist[(c*e^n)/(b*c - a*d), Int[(e*x)^(m - n)/(c + d*x^n), x], x] /; Fr
eeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LeQ[n, m,
2*n - 1]
```

Rule 3823

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[(Cot[e + f*x]*(d*Csc[e + f*x])^n)/(f*n*Sqrt[a +
 b*Csc[e + f*x]]), x] + Dist[1/(2*b*d*n), Int[((d*Csc[e + f*x])^(n + 1)*(a
+ b*(2*n + 1)*Csc[e + f*x]))/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a,
b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && LtQ[n, 0] && IntegerQ[2*n]
```

Rule 3887

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Dist[(-2*a^(m/2 + n + 1/2))/d, Subst[Int[(x^m*(2 + a*x^2)
^(m/2 + n - 1/2))/(1 + a*x^2), x], x, Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]]
], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[m/2] && In
tegerQ[n - 1/2]
```

Rule 3904

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Dist[(-(a*c))^m, Int[Cot[e + f*x]^(2*m)*(c
 + d*Csc[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && Eq
Q[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && RationalQ[n] &&  !(I
ntegerQ[n] && GtQ[m - n, 0])
```

Rule 4397

```
Int[u_, x_Symbol] :> Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]
```

Rubi steps

$$\int \frac{\cos(2(a+bx))}{\sqrt{c\tan(a+bx)\tan(2(a+bx))}}\,dx = \int \frac{\cos(2a+2bx)}{\sqrt{-c+c\sec(2a+2bx)}}\,dx$$

$$= \frac{\sin(2a+2bx)}{2b\sqrt{-c+c\sec(2a+2bx)}} - \frac{\int \frac{-c-c\sec(2a+2bx)}{\sqrt{-c+c\sec(2a+2bx)}}\,dx}{2c}$$

$$= \frac{\sin(2a+2bx)}{2b\sqrt{-c+c\sec(2a+2bx)}} + \frac{1}{2}c\int \frac{\tan^2(2a+2bx)}{(-c+c\sec(2a+2bx))^{3/2}}\,dx$$

$$= \frac{\sin(2a+2bx)}{2b\sqrt{-c+c\sec(2a+2bx)}} - \frac{c\,\text{Subst}\left(\int \frac{x^2}{(1-cx^2)(2-cx^2)}\,dx, x, -\frac{\tan(2a+2bx}{\sqrt{-c+c\sec(2a+}}\right.}{2b}$$

$$= \frac{\sin(2a+2bx)}{2b\sqrt{-c+c\sec(2a+2bx)}} - \frac{\text{Subst}\left(\int \frac{1}{1-cx^2}\,dx, x, -\frac{\tan(2a+2bx)}{\sqrt{-c+c\sec(2a+2bx)}}\right)}{2b} + \frac{\text{S}}{}$$

$$= \frac{\tanh^{-1}\left(\frac{\sqrt{c}\tan(2a+2bx)}{\sqrt{-c+c\sec(2a+2bx)}}\right)}{2b\sqrt{c}} - \frac{\tanh^{-1}\left(\frac{\sqrt{c}\tan(2a+2bx)}{\sqrt{2}\sqrt{-c+c\sec(2a+2bx)}}\right)}{\sqrt{2}\,b\sqrt{c}} + \frac{\sin(2a}{2b\sqrt{-c+c\,\text{s}}}$$

Mathematica [A] time = 2.32, size = 166, normalized size = 1.20

$$\frac{\tan(a+bx)\left(\sqrt{2}\tanh^{-1}\left(\frac{1}{2}\sqrt{2-2\tan^2(a+bx)}\right)-\tanh^{-1}\left(\sqrt{1-\tan^2(a+bx)}\right)+\sqrt{2}\cos^2(a+bx)\sqrt{\frac{1}{\sec(2(a+bx))}}\right.}{2b\sqrt{1-\tan^2(a+bx)}\sqrt{c\tan(a+bx)\tan(2(a+}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[2*(a + b*x)]/Sqrt[c*Tan[a + b*x]*Tan[2*(a + b*x)]],x]
```

```
[Out] (Tan[a + b*x]*(Sqrt[2]*ArcTanh[Sqrt[2 - 2*Tan[a + b*x]^2]/2] - ArcTanh[Sqrt
[1 - Tan[a + b*x]^2]] + Sqrt[2]*Cos[a + b*x]^2*Sqrt[(1 + Sec[2*(a + b*x)])^
(-1)]*(2 + ArcTan[Sqrt[-1 + Tan[a + b*x]^2]]*Sec[2*(a + b*x)]*Sqrt[-1 + Tan
[a + b*x]^2])))/(2*b*Sqrt[1 - Tan[a + b*x]^2]*Sqrt[c*Tan[a + b*x]*Tan[2*(a
+ b*x)]])
```

fricas [A] time = 0.96, size = 481, normalized size = 3.49

$$\left[\frac{\sqrt{2}\left(\tan(bx+a)^3+\tan(bx+a)\right)\sqrt{c}\log\left(\frac{c\tan(bx+a)^3-2\sqrt{-\frac{c\tan(bx+a)^2}{\tan(bx+a)^2-1}}\left(\tan(bx+a)^2-1\right)\sqrt{c}-2c\tan(bx+a)}{\tan(bx+a)^3}\right)+\left(\tan(bx+a)^3\right.}{4\left(bc\tan(bx\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(2*b*x+2*a)/(c*tan(b*x+a)*tan(2*b*x+2*a))^(1/2),x, algorithm="
fricas")
```

```
[Out] [1/4*(sqrt(2)*(tan(b*x + a)^3 + tan(b*x + a))*sqrt(c)*log((c*tan(b*x + a)^3
 - 2*sqrt(-c*tan(b*x + a)^2/(tan(b*x + a)^2 - 1))*(tan(b*x + a)^2 - 1)*sqrt
(c) - 2*c*tan(b*x + a))/tan(b*x + a)^3) + (tan(b*x + a)^3 + tan(b*x + a))*s
qrt(c)*log((c*tan(b*x + a)^3 + 2*sqrt(2)*sqrt(-c*tan(b*x + a)^2/(tan(b*x +
a)^2 - 1))*(tan(b*x + a)^2 - 1)*sqrt(c) - 3*c*tan(b*x + a))/(tan(b*x + a)^3
 + tan(b*x + a))) - 2*sqrt(2)*sqrt(-c*tan(b*x + a)^2/(tan(b*x + a)^2 - 1))*
(tan(b*x + a)^2 - 1))/(b*c*tan(b*x + a)^3 + b*c*tan(b*x + a)), -1/2*(sqrt(2
)*(tan(b*x + a)^3 + tan(b*x + a))*sqrt(-c)*arctan(sqrt(-c*tan(b*x + a)^2/(t
an(b*x + a)^2 - 1))*(tan(b*x + a)^2 - 1)*sqrt(-c)/(c*tan(b*x + a))) - (tan(
b*x + a)^3 + tan(b*x + a))*sqrt(-c)*arctan(1/2*sqrt(2)*sqrt(-c*tan(b*x + a)
^2/(tan(b*x + a)^2 - 1))*(tan(b*x + a)^2 - 1)*sqrt(-c)/(c*tan(b*x + a))) +
sqrt(2)*sqrt(-c*tan(b*x + a)^2/(tan(b*x + a)^2 - 1))*(tan(b*x + a)^2 - 1))/
(b*c*tan(b*x + a)^3 + b*c*tan(b*x + a))]
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(2*b*x+2*a)/(c*tan(b*x+a)*tan(2*b*x+2*a))^(1/2),x, algorithm="
giac")
```

```
[Out] Timed out
```

maple [B] time = 1.17, size = 1030, normalized size = 7.46

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(2*b*x+2*a)/(c*tan(b*x+a)*tan(2*b*x+2*a))^(1/2),x)
```

```
[Out] 1/16*2^(1/2)/b*(-1+cos(b*x+a))^2*(cos(b*x+a)^3*4^(1/2)*((2*cos(b*x+a)^2-1)/
(cos(b*x+a)+1)^2)^(3/2)+4*cos(b*x+a)^2*4^(1/2)*((2*cos(b*x+a)^2-1)/(cos(b*x
+a)+1)^2)^(3/2)+5*cos(b*x+a)*4^(1/2)*((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^
(3/2)+2*4^(1/2)*((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(3/2)-6*cos(b*x+a)*2^
(1/2)*arctanh(1/2*cos(b*x+a)*4^(1/2)*(-1+cos(b*x+a))/sin(b*x+a)^2/((2*cos(b
*x+a)^2-1)/(cos(b*x+a)+1)^2)^(1/2)*2^(1/2))+6*cos(b*x+a)*((2*cos(b*x+a)^2-1
)/(cos(b*x+a)+1)^2)^(1/2)+4*cos(b*x+a)*ln(-2*(cos(b*x+a)^2*((2*cos(b*x+a)^2
-1)/(cos(b*x+a)+1)^2)^(1/2)-2*cos(b*x+a)^2+cos(b*x+a)-((2*cos(b*x+a)^2-1)/(
cos(b*x+a)+1)^2)^(1/2)+1)/sin(b*x+a)^2)+4*cos(b*x+a)*arctanh(1/2*4^(1/2)*(2
```

```
*cos(b*x+a)^2-3*cos(b*x+a)+1)/((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(1/2)/s
in(b*x+a)^2)-6*2^(1/2)*arctanh(1/2*cos(b*x+a)*4^(1/2)*(-1+cos(b*x+a))/sin(b
*x+a)^2/((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(1/2)*2^(1/2))+4*((2*cos(b*x+
a)^2-1)/(cos(b*x+a)+1)^2)^(1/2)+4*ln(-2*(cos(b*x+a)^2*((2*cos(b*x+a)^2-1)/(
cos(b*x+a)+1)^2)^(1/2)-2*cos(b*x+a)^2+cos(b*x+a)-((2*cos(b*x+a)^2-1)/(cos(b
*x+a)+1)^2)^(1/2)+1)/sin(b*x+a)^2)+4*arctanh(1/2*4^(1/2)*(2*cos(b*x+a)^2-3*
cos(b*x+a)+1)/((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(1/2)/sin(b*x+a)^2))/((
2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(1/2)/(c*sin(b*x+a)^2/(2*cos(b*x+a)^2-1
))^(1/2)/sin(b*x+a)^3*4^(1/2)+1/8*2^(1/2)/b*(cos(b*x+a)+1)*((2*cos(b*x+a)^2
-1)/(cos(b*x+a)+1)^2)^(1/2)*(c*(1-cos(b*x+a)^2)/(2*cos(b*x+a)^2-1))^(1/2)*(
2*2^(1/2)*arctanh(1/2*cos(b*x+a)*4^(1/2)*(-1+cos(b*x+a))/sin(b*x+a)^2/((2*c
os(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(1/2)*2^(1/2))-ln(-2*(cos(b*x+a)^2*((2*cos
(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(1/2)-2*cos(b*x+a)^2+cos(b*x+a)-((2*cos(b*x+
a)^2-1)/(cos(b*x+a)+1)^2)^(1/2)+1)/sin(b*x+a)^2)-arctanh(1/2*4^(1/2)*(2*cos
(b*x+a)^2-3*cos(b*x+a)+1)/((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(1/2)/sin(b
*x+a)^2))/sin(b*x+a)/c*4^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos(2\,bx+2\,a)}{\sqrt{c\tan(2\,bx+2\,a)\tan(bx+a)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(2*b*x+2*a)/(c*tan(b*x+a)*tan(2*b*x+2*a))^(1/2),x, algorithm="
maxima")
```

```
[Out] integrate(cos(2*b*x + 2*a)/sqrt(c*tan(2*b*x + 2*a)*tan(b*x + a)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos(2\,a+2\,b\,x)}{\sqrt{c\tan(a+b\,x)\,\tan(2\,a+2\,b\,x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(2*a + 2*b*x)/(c*tan(a + b*x)*tan(2*a + 2*b*x))^(1/2),x)
```

```
[Out] int(cos(2*a + 2*b*x)/(c*tan(a + b*x)*tan(2*a + 2*b*x))^(1/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(2*b*x+2*a)/(c*tan(b*x+a)*tan(2*b*x+2*a))**(1/2),x)
```

```
[Out] Timed out
```

3.625 $\int \frac{\cos^2(2(a+bx))}{\sqrt{c\tan(a+bx)\tan(2(a+bx))}}\,dx$

Optimal. Leaf size=182

$$\frac{\sin(2a+2bx)}{8b\sqrt{c\sec(2a+2bx)-c}}+\frac{\sin(2a+2bx)\cos(2a+2bx)}{4b\sqrt{c\sec(2a+2bx)-c}}+\frac{7\tanh^{-1}\left(\frac{\sqrt{c}\tan(2a+2bx)}{\sqrt{c\sec(2a+2bx)-c}}\right)}{8b\sqrt{c}}-\frac{\tanh^{-1}\left(\frac{\sqrt{c}\tan(2a+2bx)}{\sqrt{2}\sqrt{c\sec(2a+2bx)-c}}\right)}{\sqrt{2}\,b\sqrt{c}}$$

```
[Out] 7/8*arctanh(c^(1/2)*tan(2*b*x+2*a)/(-c+c*sec(2*b*x+2*a))^(1/2))/b/c^(1/2)-1
/2*arctanh(1/2*c^(1/2)*tan(2*b*x+2*a)*2^(1/2)/(-c+c*sec(2*b*x+2*a))^(1/2))/
b*2^(1/2)/c^(1/2)+1/8*sin(2*b*x+2*a)/b/(-c+c*sec(2*b*x+2*a))^(1/2)+1/4*cos(
2*b*x+2*a)*sin(2*b*x+2*a)/b/(-c+c*sec(2*b*x+2*a))^(1/2)
```

Rubi [A] time = 0.37, antiderivative size = 182, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.226, Rules used = {4397, 3823, 4022, 3920, 3774, 207, 3795}

$$\frac{\sin(2a+2bx)}{8b\sqrt{c\sec(2a+2bx)-c}}+\frac{\sin(2a+2bx)\cos(2a+2bx)}{4b\sqrt{c\sec(2a+2bx)-c}}+\frac{7\tanh^{-1}\left(\frac{\sqrt{c}\tan(2a+2bx)}{\sqrt{c\sec(2a+2bx)-c}}\right)}{8b\sqrt{c}}-\frac{\tanh^{-1}\left(\frac{\sqrt{c}\tan(2a+2bx)}{\sqrt{2}\sqrt{c\sec(2a+2bx)-c}}\right)}{\sqrt{2}\,b\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[2*(a + b*x)]^2/Sqrt[c*Tan[a + b*x]*Tan[2*(a + b*x)]],x]
```

```
[Out] (7*ArcTanh[(Sqrt[c]*Tan[2*a + 2*b*x])/Sqrt[-c + c*Sec[2*a + 2*b*x]]])/(8*b*
Sqrt[c]) - ArcTanh[(Sqrt[c]*Tan[2*a + 2*b*x])/(Sqrt[2]*Sqrt[-c + c*Sec[2*a
+ 2*b*x]])]/(Sqrt[2]*b*Sqrt[c]) + Sin[2*a + 2*b*x]/(8*b*Sqrt[-c + c*Sec[2*a
 + 2*b*x]]) + (Cos[2*a + 2*b*x]*Sin[2*a + 2*b*x])/(4*b*Sqrt[-c + c*Sec[2*a
+ 2*b*x]])
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 3774

```
Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[(-2*b)/d,
 Subst[Int[1/(a + x^2), x], x, (b*Cot[c + d*x])/Sqrt[a + b*Csc[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3795

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Dist[-2/f, Subst[Int[1/(2*a + x^2), x], x, (b*Cot[e + f*x])/Sqrt[
a + b*Csc[e + f*x]]], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3823

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[(Cot[e + f*x]*(d*Csc[e + f*x])^n)/(f*n*Sqrt[a +
 b*Csc[e + f*x]]), x] + Dist[1/(2*b*d*n), Int[((d*Csc[e + f*x])^(n + 1)*(a
+ b*(2*n + 1)*Csc[e + f*x]))/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a,
b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && LtQ[n, 0] && IntegerQ[2*n]
```

Rule 3920

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)], x_Symbol] :> Dist[c/a, Int[Sqrt[a + b*Csc[e + f*x]], x], x] - D
ist[(b*c - a*d)/a, Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] /; Fre
eQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 4022

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*Cot[
e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(f*n), x] - Dist[1/(b*d
*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1)*Simp[a*A*m - b*B*n
 - A*b*(m + n + 1)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B,
m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[n, 0]
```

Rule 4397

```
Int[u_, x_Symbol] :> Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]
```

Rubi steps

$$\int \frac{\cos^2(2(a+bx))}{\sqrt{c\tan(a+bx)\tan(2(a+bx))}}\,dx = \int \frac{\cos^2(2a+2bx)}{\sqrt{-c+c\sec(2a+2bx)}}\,dx$$

$$= \frac{\cos(2a+2bx)\sin(2a+2bx)}{4b\sqrt{-c+c\sec(2a+2bx)}} - \frac{\int \frac{\cos(2a+2bx)(-c-3c\sec(2a+2bx))}{\sqrt{-c+c\sec(2a+2bx)}}\,dx}{4c}$$

$$= \frac{\sin(2a+2bx)}{8b\sqrt{-c+c\sec(2a+2bx)}} + \frac{\cos(2a+2bx)\sin(2a+2bx)}{4b\sqrt{-c+c\sec(2a+2bx)}} - \frac{\int \frac{-\frac{7c^2}{2}-\frac{1}{2}c^2\sec(}{\sqrt{-c+c\sec(2}}}{4c^2}$$

$$= \frac{\sin(2a+2bx)}{8b\sqrt{-c+c\sec(2a+2bx)}} + \frac{\cos(2a+2bx)\sin(2a+2bx)}{4b\sqrt{-c+c\sec(2a+2bx)}} - \frac{7\int\sqrt{-c+c\,s}}{}$$

$$= \frac{\sin(2a+2bx)}{8b\sqrt{-c+c\sec(2a+2bx)}} + \frac{\cos(2a+2bx)\sin(2a+2bx)}{4b\sqrt{-c+c\sec(2a+2bx)}} + \frac{7\,\mathrm{Subst}\left(\int -\right.}{}$$

$$= \frac{7\tanh^{-1}\left(\frac{\sqrt{c}\tan(2a+2bx)}{\sqrt{-c+c\sec(2a+2bx)}}\right)}{8b\sqrt{c}} - \frac{\tanh^{-1}\left(\frac{\sqrt{c}\tan(2a+2bx)}{\sqrt{2}\sqrt{-c+c\sec(2a+2bx)}}\right)}{\sqrt{2}\,b\sqrt{c}} + \frac{\sin(2a}{8b\sqrt{-c+c\,s}}$$

Mathematica [A] time = 2.90, size = 186, normalized size = 1.02

$$\frac{\tan(a+bx)\left(7\sqrt{2}\tanh^{-1}\left(\frac{1}{2}\sqrt{2-2\tan^2(a+bx)}\right)-7\tanh^{-1}\left(\sqrt{1-\tan^2(a+bx)}\right)+\sqrt{2}\cos^2(a+bx)\sec(2(a+\right.}{8b\sqrt{1-\tan^2(a+bx)}\sqrt{c\tan}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[2*(a + b*x)]^2/Sqrt[c*Tan[a + b*x]*Tan[2*(a + b*x)]],x]
```

```
[Out] (Tan[a + b*x]*(7*Sqrt[2]*ArcTanh[Sqrt[2 - 2*Tan[a + b*x]^2]/2] - 7*ArcTanh[
Sqrt[1 - Tan[a + b*x]^2]] + Sqrt[2]*Cos[a + b*x]^2*Sec[2*(a + b*x)]*Sqrt[(1
 + Sec[2*(a + b*x)])^(-1)]*(2*(1 + Cos[2*(a + b*x)] + Cos[4*(a + b*x)]) + A
rcTan[Sqrt[-1 + Tan[a + b*x]^2]]*Sqrt[-1 + Tan[a + b*x]^2])))/(8*b*Sqrt[1 -
 Tan[a + b*x]^2]*Sqrt[c*Tan[a + b*x]*Tan[2*(a + b*x)]])
```

fricas [A] time = 1.95, size = 569, normalized size = 3.13

$$\left[\frac{4\sqrt{2}\left(\tan\left(bx+a\right)^{5}+2\,\tan\left(bx+a\right)^{3}+\tan\left(bx+a\right)\right)\sqrt{c}\,\log\left(\frac{c\tan(bx+a)^{3}-2\sqrt{-\frac{c\tan(bx+a)^{2}}{\tan(bx+a)^{2}-1}}\left(\tan(bx+a)^{2}-1\right)\sqrt{c}-2\,c\tan(bx+}{\tan(bx+a)^{3}}\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(2*b*x+2*a)^2/(c*tan(b*x+a)*tan(2*b*x+2*a))^(1/2),x, algorithm
="fricas")
```

```
[Out] [1/16*(4*sqrt(2)*(tan(b*x + a)^5 + 2*tan(b*x + a)^3 + tan(b*x + a))*sqrt(c)
*log((c*tan(b*x + a)^3 - 2*sqrt(-c*tan(b*x + a)^2/(tan(b*x + a)^2 - 1))*(ta
n(b*x + a)^2 - 1)*sqrt(c) - 2*c*tan(b*x + a))/tan(b*x + a)^3) + 7*(tan(b*x
+ a)^5 + 2*tan(b*x + a)^3 + tan(b*x + a))*sqrt(c)*log((c*tan(b*x + a)^3 + 2
*sqrt(2)*sqrt(-c*tan(b*x + a)^2/(tan(b*x + a)^2 - 1))*(tan(b*x + a)^2 - 1)*
sqrt(c) - 3*c*tan(b*x + a))/(tan(b*x + a)^3 + tan(b*x + a))) + 2*sqrt(2)*(t
an(b*x + a)^4 - 4*tan(b*x + a)^2 + 3)*sqrt(-c*tan(b*x + a)^2/(tan(b*x + a)^
2 - 1)))/(b*c*tan(b*x + a)^5 + 2*b*c*tan(b*x + a)^3 + b*c*tan(b*x + a)), -1
/8*(4*sqrt(2)*(tan(b*x + a)^5 + 2*tan(b*x + a)^3 + tan(b*x + a))*sqrt(-c)*a
rctan(sqrt(-c*tan(b*x + a)^2/(tan(b*x + a)^2 - 1))*(tan(b*x + a)^2 - 1)*sqr
t(-c)/(c*tan(b*x + a))) - 7*(tan(b*x + a)^5 + 2*tan(b*x + a)^3 + tan(b*x +
a))*sqrt(-c)*arctan(1/2*sqrt(2)*sqrt(-c*tan(b*x + a)^2/(tan(b*x + a)^2 - 1)
)*(tan(b*x + a)^2 - 1)*sqrt(-c)/(c*tan(b*x + a))) - sqrt(2)*(tan(b*x + a)^4
 - 4*tan(b*x + a)^2 + 3)*sqrt(-c*tan(b*x + a)^2/(tan(b*x + a)^2 - 1)))/(b*c
*tan(b*x + a)^5 + 2*b*c*tan(b*x + a)^3 + b*c*tan(b*x + a))]
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(2*b*x+2*a)^2/(c*tan(b*x+a)*tan(2*b*x+2*a))^(1/2),x, algorithm
="giac")
```

```
[Out] Timed out
```

maple [B] time = 1.10, size = 1835, normalized size = 10.08

Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(2*b*x+2*a)^2/(c*tan(b*x+a)*tan(2*b*x+2*a))^(1/2),x)
```

```
[Out] -1/8*2^(1/2)/b*(cos(b*x+a)+1)*((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(1/2)*(
c*(1-cos(b*x+a)^2)/(2*cos(b*x+a)^2-1))^(1/2)*(2*2^(1/2)*arctanh(1/2*cos(b*x
+a)*4^(1/2)*(-1+cos(b*x+a))/sin(b*x+a)^2/((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)
^2)^(1/2)*2^(1/2))-ln(-2*(cos(b*x+a)^2*((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2
)^(1/2)-2*cos(b*x+a)^2+cos(b*x+a)-((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(1/
2)+1)/sin(b*x+a)^2)-arctanh(1/2*4^(1/2)*(2*cos(b*x+a)^2-3*cos(b*x+a)+1)/((2
*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(1/2)/sin(b*x+a)^2))/sin(b*x+a)/c*4^(1/2
)-1/8*2^(1/2)/b*(-1+cos(b*x+a))^2*(cos(b*x+a)^3*4^(1/2)*((2*cos(b*x+a)^2-1)
/(cos(b*x+a)+1)^2)^(3/2)+4*cos(b*x+a)^2*4^(1/2)*((2*cos(b*x+a)^2-1)/(cos(b*
x+a)+1)^2)^(3/2)+5*cos(b*x+a)*4^(1/2)*((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)
^(3/2)+2*4^(1/2)*((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(3/2)-6*cos(b*x+a)*2
^(1/2)*arctanh(1/2*cos(b*x+a)*4^(1/2)*(-1+cos(b*x+a))/sin(b*x+a)^2/((2*cos(
b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(1/2)*2^(1/2))+6*cos(b*x+a)*((2*cos(b*x+a)^2-
1)/(cos(b*x+a)+1)^2)^(1/2)+4*cos(b*x+a)*ln(-2*(cos(b*x+a)^2*((2*cos(b*x+a)^
2-1)/(cos(b*x+a)+1)^2)^(1/2)-2*cos(b*x+a)^2+cos(b*x+a)-((2*cos(b*x+a)^2-1)/
(cos(b*x+a)+1)^2)^(1/2)+1)/sin(b*x+a)^2)+4*cos(b*x+a)*arctanh(1/2*4^(1/2)*(
2*cos(b*x+a)^2-3*cos(b*x+a)+1)/((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(1/2)/
sin(b*x+a)^2)-6*2^(1/2)*arctanh(1/2*cos(b*x+a)*4^(1/2)*(-1+cos(b*x+a))/sin(
b*x+a)^2/((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(1/2)*2^(1/2))+4*((2*cos(b*x
+a)^2-1)/(cos(b*x+a)+1)^2)^(1/2)+4*ln(-2*(cos(b*x+a)^2*((2*cos(b*x+a)^2-1)/
(cos(b*x+a)+1)^2)^(1/2)-2*cos(b*x+a)^2+cos(b*x+a)-((2*cos(b*x+a)^2-1)/(cos(
b*x+a)+1)^2)^(1/2)+1)/sin(b*x+a)^2)+4*arctanh(1/2*4^(1/2)*(2*cos(b*x+a)^2-3
*cos(b*x+a)+1)/((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(1/2)/sin(b*x+a)^2))/(
(2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(1/2)/(c*sin(b*x+a)^2/(2*cos(b*x+a)^2-
1))^(1/2)/sin(b*x+a)^3*4^(1/2)-1/64*2^(1/2)/b*(-1+cos(b*x+a))^2*(-4*4^(1/2)
*((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(3/2)*cos(b*x+a)^5-16*4^(1/2)*((2*co
s(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(3/2)*cos(b*x+a)^4-33*cos(b*x+a)^3*4^(1/2)*
((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(3/2)-52*cos(b*x+a)^2*4^(1/2)*((2*cos
(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(3/2)-49*cos(b*x+a)*4^(1/2)*((2*cos(b*x+a)^2
-1)/(cos(b*x+a)+1)^2)^(3/2)-18*4^(1/2)*((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2
)^(3/2)+46*cos(b*x+a)*2^(1/2)*arctanh(1/2*cos(b*x+a)*4^(1/2)*(-1+cos(b*x+a)
)/sin(b*x+a)^2/((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(1/2)*2^(1/2))+46*2^(1
/2)*arctanh(1/2*cos(b*x+a)*4^(1/2)*(-1+cos(b*x+a))/sin(b*x+a)^2/((2*cos(b*x
+a)^2-1)/(cos(b*x+a)+1)^2)^(1/2)*2^(1/2))-54*cos(b*x+a)*((2*cos(b*x+a)^2-1)
/(cos(b*x+a)+1)^2)^(1/2)-32*cos(b*x+a)*ln(-2*(cos(b*x+a)^2*((2*cos(b*x+a)^2
-1)/(cos(b*x+a)+1)^2)^(1/2)-2*cos(b*x+a)^2+cos(b*x+a)-((2*cos(b*x+a)^2-1)/(
cos(b*x+a)+1)^2)^(1/2)+1)/sin(b*x+a)^2)-32*cos(b*x+a)*arctanh(1/2*4^(1/2)*(
2*cos(b*x+a)^2-3*cos(b*x+a)+1)/((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(1/2)/
sin(b*x+a)^2)-36*((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(1/2)-32*ln(-2*(cos(
b*x+a)^2*((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(1/2)-2*cos(b*x+a)^2+cos(b*x
+a)-((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(1/2)+1)/sin(b*x+a)^2)-32*arctanh
```

```
(1/2*4^(1/2)*(2*cos(b*x+a)^2-3*cos(b*x+a)+1)/((2*cos(b*x+a)^2-1)/(cos(b*x+a
)+1)^2)^(1/2)/sin(b*x+a)^2))/((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(1/2)/(c
*sin(b*x+a)^2/(2*cos(b*x+a)^2-1))^(1/2)/sin(b*x+a)^3*4^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos(2\,bx+2\,a)^2}{\sqrt{c\tan(2\,bx+2\,a)\tan(bx+a)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(2*b*x+2*a)^2/(c*tan(b*x+a)*tan(2*b*x+2*a))^(1/2),x, algorithm
="maxima")
```

```
[Out] integrate(cos(2*b*x + 2*a)^2/sqrt(c*tan(2*b*x + 2*a)*tan(b*x + a)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos(2\,a+2\,b\,x)^2}{\sqrt{c\tan(a+b\,x)\,\tan(2\,a+2\,b\,x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(2*a + 2*b*x)^2/(c*tan(a + b*x)*tan(2*a + 2*b*x))^(1/2),x)
```

```
[Out] int(cos(2*a + 2*b*x)^2/(c*tan(a + b*x)*tan(2*a + 2*b*x))^(1/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(2*b*x+2*a)**2/(c*tan(b*x+a)*tan(2*b*x+2*a))**(1/2),x)
```

```
[Out] Timed out
```

3.626 $\int \frac{\sec^4(2(a+bx))}{(c\tan(a+bx)\tan(2(a+bx)))^{3/2}}\,dx$

Optimal. Leaf size=180

$$-\frac{11\tanh^{-1}\left(\frac{\sqrt{c}\tan(2a+2bx)}{\sqrt{2}\sqrt{c\sec(2a+2bx)-c}}\right)}{4\sqrt{2}\,bc^{3/2}}+\frac{7\tan(2a+2bx)\sqrt{c\sec(2a+2bx)-c}}{12bc^2}-\frac{\tan(2a+2bx)\sec^2(2a+2bx)}{4b(c\sec(2a+2bx)-c)^{3/2}}+\frac{13\tan}{6bc\sqrt{c\sec}}$$

```
[Out] -11/8*arctanh(1/2*c^(1/2)*tan(2*b*x+2*a)*2^(1/2)/(-c+c*sec(2*b*x+2*a))^(1/2
))/b/c^(3/2)*2^(1/2)-1/4*sec(2*b*x+2*a)^2*tan(2*b*x+2*a)/b/(-c+c*sec(2*b*x+
2*a))^(3/2)+13/6*tan(2*b*x+2*a)/b/c/(-c+c*sec(2*b*x+2*a))^(1/2)+7/12*(-c+c*
sec(2*b*x+2*a))^(1/2)*tan(2*b*x+2*a)/b/c^2
```

Rubi [A] time = 0.51, antiderivative size = 180, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.194, Rules used = {4397, 3816, 4010, 4001, 3795, 207}

$$\frac{7\tan(2a+2bx)\sqrt{c\sec(2a+2bx)-c}}{12bc^2}-\frac{11\tanh^{-1}\left(\frac{\sqrt{c}\tan(2a+2bx)}{\sqrt{2}\sqrt{c\sec(2a+2bx)-c}}\right)}{4\sqrt{2}\,bc^{3/2}}-\frac{\tan(2a+2bx)\sec^2(2a+2bx)}{4b(c\sec(2a+2bx)-c)^{3/2}}+\frac{13\tan(}{6bc\sqrt{c\sec(}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[2*(a + b*x)]^4/(c*Tan[a + b*x]*Tan[2*(a + b*x)])^(3/2),x]
```

```
[Out] (-11*ArcTanh[(Sqrt[c]*Tan[2*a + 2*b*x])/(Sqrt[2]*Sqrt[-c + c*Sec[2*a + 2*b*
x]])])/(4*Sqrt[2]*b*c^(3/2)) - (Sec[2*a + 2*b*x]^2*Tan[2*a + 2*b*x])/(4*b*(
-c + c*Sec[2*a + 2*b*x])^(3/2)) + (13*Tan[2*a + 2*b*x])/(6*b*c*Sqrt[-c + c*
Sec[2*a + 2*b*x]]) + (7*Sqrt[-c + c*Sec[2*a + 2*b*x]]*Tan[2*a + 2*b*x])/(12
*b*c^2)
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 3795

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Dist[-2/f, Subst[Int[1/(2*a + x^2), x], x, (b*Cot[e + f*x])/Sqrt[
a + b*Csc[e + f*x]]], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3816

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> -Simp[(d^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*C
sc[e + f*x])^(n - 2))/(f*(2*m + 1)), x] + Dist[d^2/(a*b*(2*m + 1)), Int[(a
+ b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 2)*(b*(n - 2) + a*(m - n +
2)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
&& LtQ[m, -1] && GtQ[n, 2] && (IntegersQ[2*m, 2*n] || IntegerQ[m])
```

Rule 4001

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(B*Cot[e + f*x]*(a
+ b*Csc[e + f*x])^m)/(f*(m + 1)), x] + Dist[(a*B*m + A*b*(m + 1))/(b*(m + 1
)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m, x], x] /; FreeQ[{a, b, A, B, e
, f, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && NeQ[a*B*m + A*b*(m
 + 1), 0] &&  !LtQ[m, -2^(-1)]
```

Rule 4010

```
Int[csc[(e_.) + (f_.)*(x_)]^2*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(
csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(B*Cot[e + f*x]*(
a + b*Csc[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m + 2)), Int[Cs
c[e + f*x]*(a + b*Csc[e + f*x])^m*Simp[b*B*(m + 1) + (A*b*(m + 2) - a*B)*Cs
c[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, m}, x] && NeQ[A*b - a*B,
 0] &&  !LtQ[m, -1]
```

Rule 4397

```
Int[u_, x_Symbol] :> Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]
```

Rubi steps

$$\int \frac{\sec^4(2(a+bx))}{(c\tan(a+bx)\tan(2(a+bx)))^{3/2}}\,dx = \int \frac{\sec^4(2a+2bx)}{(-c+c\sec(2a+2bx))^{3/2}}\,dx$$

$$= -\frac{\sec^2(2a+2bx)\tan(2a+2bx)}{4b(-c+c\sec(2a+2bx))^{3/2}} + \frac{\int \frac{\sec^2(2a+2bx)\left(2c+\frac{7}{2}c\sec(2a+2bx)\right)}{\sqrt{-c+c\sec(2a+2bx)}}\,dx}{2c^2}$$

$$= -\frac{\sec^2(2a+2bx)\tan(2a+2bx)}{4b(-c+c\sec(2a+2bx))^{3/2}} + \frac{7\sqrt{-c+c\sec(2a+2bx)}\,\tan(2a+2b}{12bc^2}$$

$$= -\frac{\sec^2(2a+2bx)\tan(2a+2bx)}{4b(-c+c\sec(2a+2bx))^{3/2}} + \frac{13\tan(2a+2bx)}{6bc\sqrt{-c+c\sec(2a+2bx)}} + \frac{7\sqrt{-c+}}{}$$

$$= -\frac{\sec^2(2a+2bx)\tan(2a+2bx)}{4b(-c+c\sec(2a+2bx))^{3/2}} + \frac{13\tan(2a+2bx)}{6bc\sqrt{-c+c\sec(2a+2bx)}} + \frac{7\sqrt{-c+}}{}$$

$$= -\frac{11\tanh^{-1}\left(\frac{\sqrt{c}\,\tan(2a+2bx)}{\sqrt{2}\,\sqrt{-c+c\sec(2a+2bx)}}\right)}{4\sqrt{2}\,bc^{3/2}} - \frac{\sec^2(2a+2bx)\tan(2a+2bx)}{4b(-c+c\sec(2a+2bx))^{3/2}} + \frac{}{6bc}$$

Mathematica [A] time = 1.30, size = 100, normalized size = 0.56

$$-\frac{\cot(a+bx)\sqrt{c\tan(a+bx)\tan(2(a+bx))}\,\left(\csc^2(a+bx)((19\cos(4(a+bx))+11)\sec(2(a+bx))-24)-66\tan^{-}\right.}{48bc^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[2*(a + b*x)]^4/(c*Tan[a + b*x]*Tan[2*(a + b*x)])^(3/2),x]
```

```
[Out] -1/48*(Cot[a + b*x]*(Csc[a + b*x]^2*(-24 + (11 + 19*Cos[4*(a + b*x)])*Sec[2
*(a + b*x)]) - 66*ArcTan[Sqrt[-1 + Tan[a + b*x]^2]]*Sqrt[-1 + Tan[a + b*x]^
2])*Sqrt[c*Tan[a + b*x]*Tan[2*(a + b*x)]])/(b*c^2)
```

fricas [A] time = 1.15, size = 350, normalized size = 1.94

$$\left[\frac{33\sqrt{2}\left(\tan(bx+a)^5-\tan(bx+a)^3\right)\sqrt{c}\,\log\left(\frac{c\tan(bx+a)^3-2\sqrt{-\frac{c\tan(bx+a)^2}{\tan(bx+a)^2-1}}\left(\tan(bx+a)^2-1\right)\sqrt{c}-2c\tan(bx+a)}{\tan(bx+a)^3}\right)+2\sqrt{2}\left(27\tan\right.}{48\left(bc^2\tan(bx+a)^5-bc^2\tan(bx+a)^3\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(2*b*x+2*a)^4/(c*tan(b*x+a)*tan(2*b*x+2*a))^(3/2),x, algorithm ="fricas")

[Out] [1/48*(33*sqrt(2)*(tan(b*x + a)^5 - tan(b*x + a)^3)*sqrt(c)*log((c*tan(b*x + a)^3 - 2*sqrt(-c*tan(b*x + a)^2/(tan(b*x + a)^2 - 1))*(tan(b*x + a)^2 - 1)*sqrt(c) - 2*c*tan(b*x + a))/tan(b*x + a)^3) + 2*sqrt(2)*(27*tan(b*x + a)^ 4 - 46*tan(b*x + a)^2 + 3)*sqrt(-c*tan(b*x + a)^2/(tan(b*x + a)^2 - 1)))/(b *c^2*tan(b*x + a)^5 - b*c^2*tan(b*x + a)^3), -1/24*(33*sqrt(2)*(tan(b*x + a)^5 - tan(b*x + a)^3)*sqrt(-c)*arctan(sqrt(-c*tan(b*x + a)^2/(tan(b*x + a)^ 2 - 1))*(tan(b*x + a)^2 - 1)*sqrt(-c)/(c*tan(b*x + a))) - sqrt(2)*(27*tan(b *x + a)^4 - 46*tan(b*x + a)^2 + 3)*sqrt(-c*tan(b*x + a)^2/(tan(b*x + a)^2 - 1)))/(b*c^2*tan(b*x + a)^5 - b*c^2*tan(b*x + a)^3)]

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(2*b*x+2*a)^4/(c*tan(b*x+a)*tan(2*b*x+2*a))^(3/2),x, algorithm ="giac")

[Out] Timed out

maple [B] time = 1.13, size = 1211, normalized size = 6.73

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sec(2*b*x+2*a)^4/(c*tan(b*x+a)*tan(2*b*x+2*a))^(3/2),x)

[Out] 1/96*2^(1/2)/b*(-1+cos(b*x+a))^2*(152*((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2) ^(1/2)*cos(b*x+a)^5+132*cos(b*x+a)^5*ln(-2*(cos(b*x+a)^2*((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(1/2)-2*cos(b*x+a)^2+cos(b*x+a)-((2*cos(b*x+a)^2-1)/(co s(b*x+a)+1)^2)^(1/2)+1)/sin(b*x+a)^2)+132*cos(b*x+a)^5*arctanh(1/2*4^(1/2)* (2*cos(b*x+a)^2-3*cos(b*x+a)+1)/((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(1/2) /sin(b*x+a)^2)-132*ln(-2*(cos(b*x+a)^2*((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(1/2)-2*cos(b*x+a)^2+cos(b*x+a)-((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(1/ 2)+1)/sin(b*x+a)^2)*cos(b*x+a)^4-132*arctanh(1/2*4^(1/2)*(2*cos(b*x+a)^2-3* cos(b*x+a)+1)/((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(1/2)/sin(b*x+a)^2)*cos (b*x+a)^4-200*((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(1/2)*cos(b*x+a)^3-132* cos(b*x+a)^3*ln(-2*(cos(b*x+a)^2*((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(1/2)-2*cos(b*x+a)^2+cos(b*x+a)-((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(1/2)+1)/ sin(b*x+a)^2)-132*cos(b*x+a)^3*arctanh(1/2*4^(1/2)*(2*cos(b*x+a)^2-3*cos(b*

```
x+a)+1)/((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(1/2)/sin(b*x+a)^2)+132*ln(-2
*(cos(b*x+a)^2*((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(1/2)-2*cos(b*x+a)^2+c
os(b*x+a)-((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(1/2)+1)/sin(b*x+a)^2)*cos(
b*x+a)^2+132*arctanh(1/2*4^(1/2)*(2*cos(b*x+a)^2-3*cos(b*x+a)+1)/((2*cos(b*
x+a)^2-1)/(cos(b*x+a)+1)^2)^(1/2)/sin(b*x+a)^2)*cos(b*x+a)^2+54*cos(b*x+a)*
((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(1/2)+33*cos(b*x+a)*ln(-2*(cos(b*x+a)
^2*((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(1/2)-2*cos(b*x+a)^2+cos(b*x+a)-((
2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(1/2)+1)/sin(b*x+a)^2)+33*cos(b*x+a)*ar
ctanh(1/2*4^(1/2)*(2*cos(b*x+a)^2-3*cos(b*x+a)+1)/((2*cos(b*x+a)^2-1)/(cos(
b*x+a)+1)^2)^(1/2)/sin(b*x+a)^2)-33*ln(-2*(cos(b*x+a)^2*((2*cos(b*x+a)^2-1)
/(cos(b*x+a)+1)^2)^(1/2)-2*cos(b*x+a)^2+cos(b*x+a)-((2*cos(b*x+a)^2-1)/(cos
(b*x+a)+1)^2)^(1/2)+1)/sin(b*x+a)^2)-33*arctanh(1/2*4^(1/2)*(2*cos(b*x+a)^2
-3*cos(b*x+a)+1)/((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(1/2)/sin(b*x+a)^2))
/(2*cos(b*x+a)^2-1)^2/(c*sin(b*x+a)^2/(2*cos(b*x+a)^2-1))^(3/2)/sin(b*x+a)^
3/((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(3/2)*4^(1/2)/(-3+2*2^(1/2))^3/(3+2
*2^(1/2))^3
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec\left(2\,bx+2\,a\right)^{4}}{\left(c\tan\left(2\,bx+2\,a\right)\tan\left(bx+a\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(2*b*x+2*a)^4/(c*tan(b*x+a)*tan(2*b*x+2*a))^(3/2),x, algorithm
="maxima")
```

```
[Out] integrate(sec(2*b*x + 2*a)^4/(c*tan(2*b*x + 2*a)*tan(b*x + a))^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{{\cos\left(2\,a+2\,b\,x\right)}^4\,{\left(c\,\mathrm{tan}\left(a+b\,x\right)\,\mathrm{tan}\left(2\,a+2\,b\,x\right)\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(2*a + 2*b*x)^4*(c*tan(a + b*x)*tan(2*a + 2*b*x))^(3/2)),x)
```

```
[Out] int(1/(cos(2*a + 2*b*x)^4*(c*tan(a + b*x)*tan(2*a + 2*b*x))^(3/2)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(2*b*x+2*a)**4/(c*tan(b*x+a)*tan(2*b*x+2*a))**(3/2),x)
```

```
[Out] Timed out
```

3.627 $\int \frac{\sec^3(2(a+bx))}{(c\tan(a+bx)\tan(2(a+bx)))^{3/2}}\,dx$

Optimal. Leaf size=128

$$-\frac{7\tanh^{-1}\left(\frac{\sqrt{c}\,\tan(2a+2bx)}{\sqrt{2}\,\sqrt{c\sec(2a+2bx)-c}}\right)}{4\sqrt{2}\,bc^{3/2}}+\frac{\tan(2a+2bx)}{bc\sqrt{c\sec(2a+2bx)-c}}-\frac{\tan(2a+2bx)}{4b(c\sec(2a+2bx)-c)^{3/2}}$$

```
[Out] -7/8*arctanh(1/2*c^(1/2)*tan(2*b*x+2*a)*2^(1/2)/(-c+c*sec(2*b*x+2*a))^(1/2)
)/b/c^(3/2)*2^(1/2)-1/4*tan(2*b*x+2*a)/b/(-c+c*sec(2*b*x+2*a))^(3/2)+tan(2*
b*x+2*a)/b/c/(-c+c*sec(2*b*x+2*a))^(1/2)
```

Rubi [A] time = 0.31, antiderivative size = 128, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.161, Rules used = {4397, 3799, 4001, 3795, 207}

$$-\frac{7\tanh^{-1}\left(\frac{\sqrt{c}\,\tan(2a+2bx)}{\sqrt{2}\,\sqrt{c\sec(2a+2bx)-c}}\right)}{4\sqrt{2}\,bc^{3/2}}+\frac{\tan(2a+2bx)}{bc\sqrt{c\sec(2a+2bx)-c}}-\frac{\tan(2a+2bx)}{4b(c\sec(2a+2bx)-c)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[2*(a + b*x)]^3/(c*Tan[a + b*x]*Tan[2*(a + b*x)])^(3/2),x]
```

```
[Out] (-7*ArcTanh[(Sqrt[c]*Tan[2*a + 2*b*x])/(Sqrt[2]*Sqrt[-c + c*Sec[2*a + 2*b*x
]]])])/(4*Sqrt[2]*b*c^(3/2)) - Tan[2*a + 2*b*x]/(4*b*(-c + c*Sec[2*a + 2*b*x
])^(3/2)) + Tan[2*a + 2*b*x]/(b*c*Sqrt[-c + c*Sec[2*a + 2*b*x]])
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 3795

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Dist[-2/f, Subst[Int[1/(2*a + x^2), x], x, (b*Cot[e + f*x])/Sqrt[
a + b*Csc[e + f*x]]], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3799

```
Int[csc[(e_.) + (f_.)*(x_)]^3*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_),
x_Symbol] :> Simp[(b*Cot[e + f*x]*(a + b*Csc[e + f*x])^m)/(a*f*(2*m + 1)),
x] - Dist[1/(a^2*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*
```

```
(a*m - b*(2*m + 1)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a
^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rule 4001

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(B*Cot[e + f*x]*(a
+ b*Csc[e + f*x])^m)/(f*(m + 1)), x] + Dist[(a*B*m + A*b*(m + 1))/(b*(m + 1
)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m, x], x] /; FreeQ[{a, b, A, B, e
, f, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && NeQ[a*B*m + A*b*(m
 + 1), 0] &&  !LtQ[m, -2^(-1)]
```

Rule 4397

```
Int[u_, x_Symbol] :> Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]
```

Rubi steps

$$\int \frac{\sec^3(2(a+bx))}{(c\tan(a+bx)\tan(2(a+bx)))^{3/2}}\,dx = \int \frac{\sec^3(2a+2bx)}{(-c+c\sec(2a+2bx))^{3/2}}\,dx$$

$$= -\frac{\tan(2a+2bx)}{4b(-c+c\sec(2a+2bx))^{3/2}} + \frac{\int \frac{\sec(2a+2bx)\left(\frac{3c}{2}+2c\sec(2a+2bx)\right)}{\sqrt{-c+c\sec(2a+2bx)}}\,dx}{2c^2}$$

$$= -\frac{\tan(2a+2bx)}{4b(-c+c\sec(2a+2bx))^{3/2}} + \frac{\tan(2a+2bx)}{bc\sqrt{-c+c\sec(2a+2bx)}} + \frac{7\int \frac{\sec(2}{\sqrt{-c+c\,\mathrm{se}}}}{4}$$

$$= -\frac{\tan(2a+2bx)}{4b(-c+c\sec(2a+2bx))^{3/2}} + \frac{\tan(2a+2bx)}{bc\sqrt{-c+c\sec(2a+2bx)}} - \frac{7\,\mathrm{Subst}\left(\int\right.}{}$$

$$= -\frac{7\tanh^{-1}\left(\frac{\sqrt{c}\tan(2a+2bx)}{\sqrt{2}\sqrt{-c+c\sec(2a+2bx)}}\right)}{4\sqrt{2}\,bc^{3/2}} - \frac{\tan(2a+2bx)}{4b(-c+c\sec(2a+2bx))^{3/2}} + \frac{\mathrm{t}}{bc\sqrt{-c}}$$

Mathematica [A] time = 0.60, size = 94, normalized size = 0.73

$$\frac{\tan(2(a+bx))\left(4\sec(2(a+bx))+7\sin^2(a+bx)\tan^{-1}\left(\sqrt{\tan^2(a+bx)-1}\right)\sqrt{\tan^2(a+bx)-1}\,\sec(2(a+bx))-5\right.}{4b(c\tan(a+bx)\tan(2(a+bx)))^{3/2}}$$

Antiderivative was successfully verified.

[In] Integrate[Sec[2*(a + b*x)]^3/(c*Tan[a + b*x]*Tan[2*(a + b*x)])^(3/2),x]

[Out] ((-5 + 4*Sec[2*(a + b*x)] + 7*ArcTan[Sqrt[-1 + Tan[a + b*x]^2]]*Sec[2*(a + b*x)]*Sin[a + b*x]^2*Sqrt[-1 + Tan[a + b*x]^2])*Tan[2*(a + b*x)])/(4*b*(c*Tan[a + b*x]*Tan[2*(a + b*x)])^(3/2))

fricas [A] time = 2.00, size = 276, normalized size = 2.16

$$\left[\frac{7\sqrt{2}\sqrt{c}\log\left(\frac{c\tan(bx+a)^3-2\sqrt{-\frac{c\tan(bx+a)^2}{\tan(bx+a)^2-1}}\left(\tan(bx+a)^2-1\right)\sqrt{c}-2c\tan(bx+a)}{\tan(bx+a)^3}\right)\tan(bx+a)^3+2\sqrt{2}\sqrt{-\frac{c\tan(bx+a)^2}{\tan(bx+a)^2-1}}\left(9\tan(\right.}{16bc^2\tan(bx+a)^3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(2*b*x+2*a)^3/(c*tan(b*x+a)*tan(2*b*x+2*a))^(3/2),x, algorithm="fricas")

[Out] [1/16*(7*sqrt(2)*sqrt(c)*log((c*tan(b*x + a)^3 - 2*sqrt(-c*tan(b*x + a)^2/(tan(b*x + a)^2 - 1))*(tan(b*x + a)^2 - 1)*sqrt(c) - 2*c*tan(b*x + a))/tan(b*x + a)^3)*tan(b*x + a)^3 + 2*sqrt(2)*sqrt(-c*tan(b*x + a)^2/(tan(b*x + a)^2 - 1))*(9*tan(b*x + a)^2 - 1))/(b*c^2*tan(b*x + a)^3), -1/8*(7*sqrt(2)*sqrt(-c)*arctan(sqrt(-c*tan(b*x + a)^2/(tan(b*x + a)^2 - 1))*(tan(b*x + a)^2 - 1)*sqrt(-c)/(c*tan(b*x + a)))*tan(b*x + a)^3 - sqrt(2)*sqrt(-c*tan(b*x + a)^2/(tan(b*x + a)^2 - 1))*(9*tan(b*x + a)^2 - 1))/(b*c^2*tan(b*x + a)^3)]

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(2*b*x+2*a)^3/(c*tan(b*x+a)*tan(2*b*x+2*a))^(3/2),x, algorithm="giac")

[Out] Timed out

maple [B] time = 1.15, size = 930, normalized size = 7.27

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sec(2*b*x+2*a)^3/(c*tan(b*x+a)*tan(2*b*x+2*a))^(3/2),x)

```
[Out] -1/16*2^(1/2)/b*(7*ln(-2*(cos(b*x+a)^2*((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2
)^(1/2)-2*cos(b*x+a)^2+cos(b*x+a)-((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(1/
2)+1)/sin(b*x+a)^2)*((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(1/2)*cos(b*x+a)^
3+7*((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(1/2)*arctanh(1/2*4^(1/2)*(2*cos(
b*x+a)^2-3*cos(b*x+a)+1)/((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(1/2)/sin(b*
x+a)^2)*cos(b*x+a)^3+7*cos(b*x+a)^2*ln(-2*(cos(b*x+a)^2*((2*cos(b*x+a)^2-1)
/(cos(b*x+a)+1)^2)^(1/2)-2*cos(b*x+a)^2+cos(b*x+a)-((2*cos(b*x+a)^2-1)/(cos
(b*x+a)+1)^2)^(1/2)+1)/sin(b*x+a)^2)*((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^
(1/2)+7*cos(b*x+a)^2*((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(1/2)*arctanh(1/
2*4^(1/2)*(2*cos(b*x+a)^2-3*cos(b*x+a)+1)/((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1
)^2)^(1/2)/sin(b*x+a)^2)-7*cos(b*x+a)*ln(-2*(cos(b*x+a)^2*((2*cos(b*x+a)^2-
1)/(cos(b*x+a)+1)^2)^(1/2)-2*cos(b*x+a)^2+cos(b*x+a)-((2*cos(b*x+a)^2-1)/(c
os(b*x+a)+1)^2)^(1/2)+1)/sin(b*x+a)^2)*((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2
)^(1/2)-7*cos(b*x+a)*((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(1/2)*arctanh(1/
2*4^(1/2)*(2*cos(b*x+a)^2-3*cos(b*x+a)+1)/((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1
)^2)^(1/2)/sin(b*x+a)^2)+20*cos(b*x+a)^3-7*ln(-2*(cos(b*x+a)^2*((2*cos(b*x+
a)^2-1)/(cos(b*x+a)+1)^2)^(1/2)-2*cos(b*x+a)^2+cos(b*x+a)-((2*cos(b*x+a)^2-
1)/(cos(b*x+a)+1)^2)^(1/2)+1)/sin(b*x+a)^2)*((2*cos(b*x+a)^2-1)/(cos(b*x+a)
+1)^2)^(1/2)-7*arctanh(1/2*4^(1/2)*(2*cos(b*x+a)^2-3*cos(b*x+a)+1)/((2*cos(
b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(1/2)/sin(b*x+a)^2)*((2*cos(b*x+a)^2-1)/(cos(
b*x+a)+1)^2)^(1/2)-18*cos(b*x+a))*sin(b*x+a)/(c*sin(b*x+a)^2/(2*cos(b*x+a)^
2-1))^(3/2)/(2*cos(b*x+a)^2-1)^2
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec(2\,bx+2\,a)^3}{(c\tan(2\,bx+2\,a)\tan(bx+a))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(2*b*x+2*a)^3/(c*tan(b*x+a)*tan(2*b*x+2*a))^(3/2),x, algorithm
="maxima")
```

```
[Out] integrate(sec(2*b*x + 2*a)^3/(c*tan(2*b*x + 2*a)*tan(b*x + a))^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\cos(2\,a+2\,b\,x)^3\,(c\tan(a+b\,x)\,\tan(2\,a+2\,b\,x))^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(2*a + 2*b*x)^3*(c*tan(a + b*x)*tan(2*a + 2*b*x))^(3/2)),x)
```

```
[Out] int(1/(cos(2*a + 2*b*x)^3*(c*tan(a + b*x)*tan(2*a + 2*b*x))^(3/2)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(2*b*x+2*a)**3/(c*tan(b*x+a)*tan(2*b*x+2*a))**(3/2),x)
[Out] Timed out
```

3.628 $\int \frac{\sec^2(2(a+bx))}{(c\tan(a+bx)\tan(2(a+bx)))^{3/2}}\,dx$

Optimal. Leaf size=93

$$-\frac{3\tanh^{-1}\left(\frac{\sqrt{c}\,\tan(2a+2bx)}{\sqrt{2}\,\sqrt{c\sec(2a+2bx)-c}}\right)}{4\sqrt{2}\,bc^{3/2}}-\frac{\tan(2a+2bx)}{4b(c\sec(2a+2bx)-c)^{3/2}}$$

```
[Out] -3/8*arctanh(1/2*c^(1/2)*tan(2*b*x+2*a)*2^(1/2)/(-c+c*sec(2*b*x+2*a))^(1/2)
)/b/c^(3/2)*2^(1/2)-1/4*tan(2*b*x+2*a)/b/(-c+c*sec(2*b*x+2*a))^(3/2)
```

Rubi [A] time = 0.24, antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.129, Rules used = {4397, 3797, 3795, 207}

$$-\frac{3\tanh^{-1}\left(\frac{\sqrt{c}\,\tan(2a+2bx)}{\sqrt{2}\,\sqrt{c\sec(2a+2bx)-c}}\right)}{4\sqrt{2}\,bc^{3/2}}-\frac{\tan(2a+2bx)}{4b(c\sec(2a+2bx)-c)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[2*(a + b*x)]^2/(c*Tan[a + b*x]*Tan[2*(a + b*x)])^(3/2),x]
```

```
[Out] (-3*ArcTanh[(Sqrt[c]*Tan[2*a + 2*b*x])/(Sqrt[2]*Sqrt[-c + c*Sec[2*a + 2*b*x
]])])/(4*Sqrt[2]*b*c^(3/2)) - Tan[2*a + 2*b*x]/(4*b*(-c + c*Sec[2*a + 2*b*x
])^(3/2))
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 3795

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Dist[-2/f, Subst[Int[1/(2*a + x^2), x], x, (b*Cot[e + f*x])/Sqrt[
a + b*Csc[e + f*x]]], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3797

```
Int[csc[(e_.) + (f_.)*(x_)]^2*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_),
x_Symbol] :> -Simp[(Cot[e + f*x]*(a + b*Csc[e + f*x])^m)/(f*(2*m + 1)), x]
+ Dist[m/(b*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1), x],
x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rule 4397

```
Int[u_, x_Symbol] :> Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]
```

Rubi steps

$$\int \frac{\sec^2(2(a+bx))}{(c\tan(a+bx)\tan(2(a+bx)))^{3/2}}\,dx = \int \frac{\sec^2(2a+2bx)}{(-c+c\sec(2a+2bx))^{3/2}}\,dx$$

$$= -\frac{\tan(2a+2bx)}{4b(-c+c\sec(2a+2bx))^{3/2}} + \frac{3\int \frac{\sec(2a+2bx)}{\sqrt{-c+c\sec(2a+2bx)}}\,dx}{4c}$$

$$= -\frac{\tan(2a+2bx)}{4b(-c+c\sec(2a+2bx))^{3/2}} - \frac{3\,\text{Subst}\left(\int \frac{1}{-2c+x^2}\,dx, x, -\frac{c\tan(2a+2bx)}{\sqrt{-c+c\sec(2a+2}}\right.}{4bc}$$

$$= -\frac{3\tanh^{-1}\left(\frac{\sqrt{c}\,\tan(2a+2bx)}{\sqrt{2}\,\sqrt{-c+c\sec(2a+2bx)}}\right)}{4\sqrt{2}\,bc^{3/2}} - \frac{\tan(2a+2bx)}{4b(-c+c\sec(2a+2bx))^{3/2}}$$

Mathematica [A] time = 0.59, size = 84, normalized size = 0.90

$$\frac{\tan(2(a+bx))\left(3\sin^2(a+bx)\tan^{-1}\left(\sqrt{\tan^2(a+bx)-1}\right)\sqrt{\tan^2(a+bx)-1}\,\sec(2(a+bx))-1\right)}{4b(c\tan(a+bx)\tan(2(a+bx)))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[2*(a + b*x)]^2/(c*Tan[a + b*x]*Tan[2*(a + b*x)])^(3/2),x]
```

```
[Out] ((-1 + 3*ArcTan[Sqrt[-1 + Tan[a + b*x]^2]]*Sec[2*(a + b*x)]*Sin[a + b*x]^2*
Sqrt[-1 + Tan[a + b*x]^2])*Tan[2*(a + b*x)])/(4*b*(c*Tan[a + b*x]*Tan[2*(a
+ b*x)])^(3/2))
```

fricas [A] time = 1.03, size = 272, normalized size = 2.92

$$\left[\frac{3\sqrt{2}\sqrt{c}\log\left(\frac{c\tan(bx+a)^3-2\sqrt{-\frac{c\tan(bx+a)^2}{\tan(bx+a)^2-1}}\left(\tan(bx+a)^2-1\right)\sqrt{c}-2c\tan(bx+a)}{\tan(bx+a)^3}\right)\tan(bx+a)^3+2\sqrt{2}\sqrt{-\frac{c\tan(bx+a)^2}{\tan(bx+a)^2-1}}\left(\tan(bx\right.}{16\,bc^2\tan(bx+a)^3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(2*b*x+2*a)^2/(c*tan(b*x+a)*tan(2*b*x+2*a))^(3/2),x, algorithm
="fricas")
```

```
[Out] [1/16*(3*sqrt(2)*sqrt(c)*log((c*tan(b*x + a)^3 - 2*sqrt(-c*tan(b*x + a)^2/(
tan(b*x + a)^2 - 1))*(tan(b*x + a)^2 - 1)*sqrt(c) - 2*c*tan(b*x + a))/tan(b
*x + a)^3)*tan(b*x + a)^3 + 2*sqrt(2)*sqrt(-c*tan(b*x + a)^2/(tan(b*x + a)^
2 - 1))*(tan(b*x + a)^2 - 1))/(b*c^2*tan(b*x + a)^3), -1/8*(3*sqrt(2)*sqrt(
-c)*arctan(sqrt(-c*tan(b*x + a)^2/(tan(b*x + a)^2 - 1))*(tan(b*x + a)^2 - 1
)*sqrt(-c)/(c*tan(b*x + a)))*tan(b*x + a)^3 - sqrt(2)*sqrt(-c*tan(b*x + a)^
2/(tan(b*x + a)^2 - 1))*(tan(b*x + a)^2 - 1))/(b*c^2*tan(b*x + a)^3)]
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(2*b*x+2*a)^2/(c*tan(b*x+a)*tan(2*b*x+2*a))^(3/2),x, algorithm
="giac")
```

```
[Out] Timed out
```

maple [B] time = 1.04, size = 433, normalized size = 4.66

$$-\frac{\sqrt{2}\ (-1+\cos(bx+a))^2\left(2\cos(bx+a)\sqrt{\frac{2\left(\cos^2(bx+a)\right)-1}{(\cos(bx+a)+1)^2}}+3\cos(bx+a)\ln\left(-\frac{2\left(\left(\cos^2(bx+a)\right)\sqrt{\frac{2\left(\cos^2(bx+a)\right)-1}{(\cos(bx+a)+1)^2}}-2\left(\cos^2(b\cdots\right.\right.}{\sin(bx\cdots}\right.\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(2*b*x+2*a)^2/(c*tan(b*x+a)*tan(2*b*x+2*a))^(3/2),x)
```

```
[Out] -1/32*2^(1/2)/b*(-1+cos(b*x+a))^2*(2*cos(b*x+a)*((2*cos(b*x+a)^2-1)/(cos(b*
x+a)+1)^2)^(1/2)+3*cos(b*x+a)*ln(-2*(cos(b*x+a)^2*((2*cos(b*x+a)^2-1)/(cos(
b*x+a)+1)^2)^(1/2)-2*cos(b*x+a)^2+cos(b*x+a)-((2*cos(b*x+a)^2-1)/(cos(b*x+a
)+1)^2)^(1/2)+1)/sin(b*x+a)^2)+3*cos(b*x+a)*arctanh(1/2*4^(1/2)*(2*cos(b*x+
a)^2-3*cos(b*x+a)+1)/((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(1/2)/sin(b*x+a)
^2)-3*ln(-2*(cos(b*x+a)^2*((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(1/2)-2*cos
(b*x+a)^2+cos(b*x+a)-((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(1/2)+1)/sin(b*x
+a)^2)-3*arctanh(1/2*4^(1/2)*(2*cos(b*x+a)^2-3*cos(b*x+a)+1)/((2*cos(b*x+a)
^2-1)/(cos(b*x+a)+1)^2)^(1/2)/sin(b*x+a)^2))/(c*sin(b*x+a)^2/(2*cos(b*x+a)^
```

```
2-1))^(3/2)/sin(b*x+a)^3/((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(3/2)*4^(1/2
)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec(2\,bx+2\,a)^2}{(c\tan(2\,bx+2\,a)\tan(bx+a))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(2*b*x+2*a)^2/(c*tan(b*x+a)*tan(2*b*x+2*a))^(3/2),x, algorithm
="maxima")
```

```
[Out] integrate(sec(2*b*x + 2*a)^2/(c*tan(2*b*x + 2*a)*tan(b*x + a))^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\cos(2\,a+2\,b\,x)^2\,(c\tan(a+b\,x)\,\tan(2\,a+2\,b\,x))^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(2*a + 2*b*x)^2*(c*tan(a + b*x)*tan(2*a + 2*b*x))^(3/2)),x)
```

```
[Out] int(1/(cos(2*a + 2*b*x)^2*(c*tan(a + b*x)*tan(2*a + 2*b*x))^(3/2)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(2*b*x+2*a)**2/(c*tan(b*x+a)*tan(2*b*x+2*a))**(3/2),x)
```

```
[Out] Timed out
```

3.629 $\int \frac{\sec(2(a+bx))}{(c\tan(a+bx)\tan(2(a+bx)))^{3/2}}\,dx$

Optimal. Leaf size=93

$$\frac{\tanh^{-1}\left(\frac{\sqrt{c}\,\tan(2a+2bx)}{\sqrt{2}\,\sqrt{c\sec(2a+2bx)-c}}\right)}{4\sqrt{2}\,bc^{3/2}}-\frac{\tan(2a+2bx)}{4b(c\sec(2a+2bx)-c)^{3/2}}$$

```
[Out] 1/8*arctanh(1/2*c^(1/2)*tan(2*b*x+2*a)*2^(1/2)/(-c+c*sec(2*b*x+2*a))^(1/2))
/b/c^(3/2)*2^(1/2)-1/4*tan(2*b*x+2*a)/b/(-c+c*sec(2*b*x+2*a))^(3/2)
```

Rubi [A] time = 0.12, antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.138, Rules used = {4397, 3796, 3795, 207}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{c}\,\tan(2a+2bx)}{\sqrt{2}\,\sqrt{c\sec(2a+2bx)-c}}\right)}{4\sqrt{2}\,bc^{3/2}}-\frac{\tan(2a+2bx)}{4b(c\sec(2a+2bx)-c)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[2*(a + b*x)]/(c*Tan[a + b*x]*Tan[2*(a + b*x)])^(3/2),x]
```

```
[Out] ArcTanh[(Sqrt[c]*Tan[2*a + 2*b*x])/(Sqrt[2]*Sqrt[-c + c*Sec[2*a + 2*b*x]])]
/(4*Sqrt[2]*b*c^(3/2)) - Tan[2*a + 2*b*x]/(4*b*(-c + c*Sec[2*a + 2*b*x])^(3
/2))
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 3795

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Dist[-2/f, Subst[Int[1/(2*a + x^2), x], x, (b*Cot[e + f*x])/Sqrt[
a + b*Csc[e + f*x]]], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3796

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_
Symbol] :> Simp[(b*Cot[e + f*x]*(a + b*Csc[e + f*x])^m)/(a*f*(2*m + 1)), x]
 + Dist[(m + 1)/(a*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1
), x], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

```
&& IntegerQ[2*m]
```

Rule 4397

```
Int[u_, x_Symbol] :> Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]
```

Rubi steps

$$\int \frac{\sec(2(a+bx))}{(c\tan(a+bx)\tan(2(a+bx)))^{3/2}}\,dx = \int \frac{\sec(2a+2bx)}{(-c+c\sec(2a+2bx))^{3/2}}\,dx$$
$$= -\frac{\tan(2a+2bx)}{4b(-c+c\sec(2a+2bx))^{3/2}} - \frac{\int \frac{\sec(2a+2bx)}{\sqrt{-c+c\sec(2a+2bx)}}\,dx}{4c}$$
$$= -\frac{\tan(2a+2bx)}{4b(-c+c\sec(2a+2bx))^{3/2}} + \frac{\text{Subst}\left(\int \frac{1}{-2c+x^2}\,dx, x, -\frac{c\tan(2a+2bx)}{\sqrt{-c+c\sec(2a+2bx)}}\right)}{4bc}$$
$$= \frac{\tanh^{-1}\left(\frac{\sqrt{c}\tan(2a+2bx)}{\sqrt{2}\sqrt{-c+c\sec(2a+2bx)}}\right)}{4\sqrt{2}\,bc^{3/2}} - \frac{\tan(2a+2bx)}{4b(-c+c\sec(2a+2bx))^{3/2}}$$

Mathematica [A] time = 0.62, size = 83, normalized size = 0.89

$$-\frac{\tan(2(a+bx))\left(\sin^2(a+bx)\tan^{-1}\left(\sqrt{\tan^2(a+bx)-1}\right)\sqrt{\tan^2(a+bx)-1}\,\sec(2(a+bx))+1\right)}{4b(c\tan(a+bx)\tan(2(a+bx)))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[2*(a + b*x)]/(c*Tan[a + b*x]*Tan[2*(a + b*x)])^(3/2),x]
```

```
[Out] -1/4*((1 + ArcTan[Sqrt[-1 + Tan[a + b*x]^2]]*Sec[2*(a + b*x)]*Sin[a + b*x]^
2*Sqrt[-1 + Tan[a + b*x]^2])*Tan[2*(a + b*x)])/(b*(c*Tan[a + b*x]*Tan[2*(a
+ b*x)])^(3/2))
```

fricas [A] time = 1.21, size = 269, normalized size = 2.89

$$\left[\frac{\sqrt{2}\sqrt{c}\log\left(\frac{c\tan(bx+a)^3+2\sqrt{-\frac{c\tan(bx+a)^2}{\tan(bx+a)^2-1}}\left(\tan(bx+a)^2-1\right)\sqrt{c}-2c\tan(bx+a)}{\tan(bx+a)^3}\right)\tan(bx+a)^3+2\sqrt{2}\sqrt{-\frac{c\tan(bx+a)^2}{\tan(bx+a)^2-1}}\left(\tan(bx+\right.}{16bc^2\tan(bx+a)^3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(2*b*x+2*a)/(c*tan(b*x+a)*tan(2*b*x+2*a))^(3/2),x, algorithm="
fricas")
```

```
[Out] [1/16*(sqrt(2)*sqrt(c)*log((c*tan(b*x + a)^3 + 2*sqrt(-c*tan(b*x + a)^2/(ta
n(b*x + a)^2 - 1))*(tan(b*x + a)^2 - 1)*sqrt(c) - 2*c*tan(b*x + a))/tan(b*x
 + a)^3)*tan(b*x + a)^3 + 2*sqrt(2)*sqrt(-c*tan(b*x + a)^2/(tan(b*x + a)^2
- 1))*(tan(b*x + a)^2 - 1))/(b*c^2*tan(b*x + a)^3), 1/8*(sqrt(2)*sqrt(-c)*a
rctan(sqrt(-c*tan(b*x + a)^2/(tan(b*x + a)^2 - 1))*(tan(b*x + a)^2 - 1)*sqr
t(-c)/(c*tan(b*x + a)))*tan(b*x + a)^3 + sqrt(2)*sqrt(-c*tan(b*x + a)^2/(ta
n(b*x + a)^2 - 1))*(tan(b*x + a)^2 - 1))/(b*c^2*tan(b*x + a)^3)]
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(2*b*x+2*a)/(c*tan(b*x+a)*tan(2*b*x+2*a))^(3/2),x, algorithm="
giac")
```

```
[Out] Timed out
```

maple [B] time = 1.04, size = 599, normalized size = 6.44

$$\frac{\sqrt{2}\ (-1+\cos(bx+a))^3 \left(2\left(\cos^2(bx+a)\right)\sqrt{4}\ \left(\frac{2\left(\cos^2(bx+a)\right)-1}{(\cos(bx+a)+1)^2}\right)^{\frac{3}{2}} + 4\cos(bx+a)\sqrt{4}\ \left(\frac{2\left(\cos^2(bx+a)\right)-1}{(\cos(bx+a)+1)^2}\right)^{\frac{3}{2}} + 2\sqrt{4}\ \left(\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(2*b*x+2*a)/(c*tan(b*x+a)*tan(2*b*x+2*a))^(3/2),x)
```

```
[Out] 1/32*2^(1/2)/b*(-1+cos(b*x+a))^3*(2*cos(b*x+a)^2*4^(1/2)*((2*cos(b*x+a)^2-1
)/(cos(b*x+a)+1)^2)^(3/2)+4*cos(b*x+a)*4^(1/2)*((2*cos(b*x+a)^2-1)/(cos(b*x
+a)+1)^2)^(3/2)+2*4^(1/2)*((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(3/2)-6*cos
(b*x+a)^2*((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(1/2)-ln(-2*(cos(b*x+a)^2*(
(2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(1/2)-2*cos(b*x+a)^2+cos(b*x+a)-((2*co
s(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(1/2)+1)/sin(b*x+a)^2)*cos(b*x+a)^2-arctanh
(1/2*4^(1/2)*(2*cos(b*x+a)^2-3*cos(b*x+a)+1)/((2*cos(b*x+a)^2-1)/(cos(b*x+a
)+1)^2)^(1/2)/sin(b*x+a)^2)*cos(b*x+a)^2+2*cos(b*x+a)*((2*cos(b*x+a)^2-1)/(
cos(b*x+a)+1)^2)^(1/2)+4*((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(1/2)+ln(-2*
```

```
(cos(b*x+a)^2*((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(1/2)-2*cos(b*x+a)^2+co
s(b*x+a)-((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(1/2)+1)/sin(b*x+a)^2)+arcta
nh(1/2*4^(1/2)*(2*cos(b*x+a)^2-3*cos(b*x+a)+1)/((2*cos(b*x+a)^2-1)/(cos(b*x
+a)+1)^2)^(1/2)/sin(b*x+a)^2))/(c*sin(b*x+a)^2/(2*cos(b*x+a)^2-1))^(3/2)/si
n(b*x+a)^5/((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(3/2)*4^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec(2\,bx+2\,a)}{(c\tan(2\,bx+2\,a)\tan(bx+a))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(2*b*x+2*a)/(c*tan(b*x+a)*tan(2*b*x+2*a))^(3/2),x, algorithm="
maxima")
[Out] integrate(sec(2*b*x + 2*a)/(c*tan(2*b*x + 2*a)*tan(b*x + a))^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\cos(2\,a+2\,b\,x)\,\left(c\tan(a+b\,x)\,\tan(2\,a+2\,b\,x)\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(2*a + 2*b*x)*(c*tan(a + b*x)*tan(2*a + 2*b*x))^(3/2)),x)
[Out] int(1/(cos(2*a + 2*b*x)*(c*tan(a + b*x)*tan(2*a + 2*b*x))^(3/2)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(2*b*x+2*a)/(c*tan(b*x+a)*tan(2*b*x+2*a))**(3/2),x)
[Out] Timed out
```

3.630 $\int \frac{1}{(c\tan(a+bx)\tan(2(a+bx)))^{3/2}}\,dx$

Optimal. Leaf size=138

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{c}\tan(2a+2bx)}{\sqrt{c\sec(2a+2bx)-c}}\right)}{bc^{3/2}}+\frac{5\tanh^{-1}\left(\frac{\sqrt{c}\tan(2a+2bx)}{\sqrt{2}\sqrt{c\sec(2a+2bx)-c}}\right)}{4\sqrt{2}\,bc^{3/2}}-\frac{\tan(2a+2bx)}{4b(c\sec(2a+2bx)-c)^{3/2}}$$

```
[Out] -arctanh(c^(1/2)*tan(2*b*x+2*a)/(-c+c*sec(2*b*x+2*a))^(1/2))/b/c^(3/2)+5/8*
arctanh(1/2*c^(1/2)*tan(2*b*x+2*a)*2^(1/2)/(-c+c*sec(2*b*x+2*a))^(1/2))/b/c
^(3/2)*2^(1/2)-1/4*tan(2*b*x+2*a)/b/(-c+c*sec(2*b*x+2*a))^(3/2)
```

Rubi [A] time = 0.14, antiderivative size = 138, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {4397, 3777, 3920, 3774, 207, 3795}

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{c}\tan(2a+2bx)}{\sqrt{c\sec(2a+2bx)-c}}\right)}{bc^{3/2}}+\frac{5\tanh^{-1}\left(\frac{\sqrt{c}\tan(2a+2bx)}{\sqrt{2}\sqrt{c\sec(2a+2bx)-c}}\right)}{4\sqrt{2}\,bc^{3/2}}-\frac{\tan(2a+2bx)}{4b(c\sec(2a+2bx)-c)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(c*Tan[a + b*x]*Tan[2*(a + b*x)])^(-3/2),x]
```

```
[Out] -(ArcTanh[(Sqrt[c]*Tan[2*a + 2*b*x])/Sqrt[-c + c*Sec[2*a + 2*b*x]]]/(b*c^(3
/2))) + (5*ArcTanh[(Sqrt[c]*Tan[2*a + 2*b*x])/(Sqrt[2]*Sqrt[-c + c*Sec[2*a
+ 2*b*x]])])/(4*Sqrt[2]*b*c^(3/2)) - Tan[2*a + 2*b*x]/(4*b*(-c + c*Sec[2*a
+ 2*b*x])^(3/2))
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 3774

```
Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[(-2*b)/d,
 Subst[Int[1/(a + x^2), x], x, (b*Cot[c + d*x])/Sqrt[a + b*Csc[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3777

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_), x_Symbol] :> -Simp[(Cot[c
+ d*x]*(a + b*Csc[c + d*x])^n)/(d*(2*n + 1)), x] + Dist[1/(a^2*(2*n + 1)),
```

```
Int[(a + b*Csc[c + d*x])^(n + 1)*(a*(2*n + 1) - b*(n + 1)*Csc[c + d*x]), x]
, x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && LeQ[n, -1] && Intege
rQ[2*n]
```

Rule 3795

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Dist[-2/f, Subst[Int[1/(2*a + x^2), x], x, (b*Cot[e + f*x])/Sqrt[
a + b*Csc[e + f*x]]], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3920

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)], x_Symbol] :> Dist[c/a, Int[Sqrt[a + b*Csc[e + f*x]], x], x] - D
ist[(b*c - a*d)/a, Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] /; Fre
eQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 4397

```
Int[u_, x_Symbol] :> Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]
```

Rubi steps

$$\int \frac{1}{(c\tan(a+bx)\tan(2(a+bx)))^{3/2}}\,dx = \int \frac{1}{(-c+c\sec(2a+2bx))^{3/2}}\,dx$$

$$= -\frac{\tan(2a+2bx)}{4b(-c+c\sec(2a+2bx))^{3/2}} - \frac{\int \frac{2c+\frac{1}{2}c\sec(2a+2bx)}{\sqrt{-c+c\sec(2a+2bx)}}\,dx}{2c^2}$$

$$= -\frac{\tan(2a+2bx)}{4b(-c+c\sec(2a+2bx))^{3/2}} + \frac{\int \sqrt{-c+c\sec(2a+2bx)}\,dx}{c^2} - \frac{5\int \frac{}{\sqrt{-}}$$

$$= -\frac{\tan(2a+2bx)}{4b(-c+c\sec(2a+2bx))^{3/2}} - \frac{\text{Subst}\left(\int \frac{1}{-c+x^2}\,dx, x, -\frac{c\tan(2a+2bx)}{\sqrt{-c+c\sec(2a+2bx)}}\right.}{bc}$$

$$= -\frac{\tanh^{-1}\left(\frac{\sqrt{c}\tan(2a+2bx)}{\sqrt{-c+c\sec(2a+2bx)}}\right)}{bc^{3/2}} + \frac{5\tanh^{-1}\left(\frac{\sqrt{c}\tan(2a+2bx)}{\sqrt{2}\sqrt{-c+c\sec(2a+2bx)}}\right)}{4\sqrt{2}\,bc^{3/2}} - \frac{}{4b(-c}$$

Mathematica [A] time = 3.70, size = 196, normalized size = 1.42

$$-\frac{\cot(a+bx)\sqrt{c\tan(a+bx)\tan(2(a+bx))}\left(\tan^{-1}\left(\sqrt{\tan^2(a+bx)-1}\right)\sqrt{-\left(\tan^2(a+bx)-1\right)^2}+\cot^2(a+bx)\left(c\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c*Tan[a + b*x]*Tan[2*(a + b*x)])^(-3/2),x]
```

```
[Out] -1/8*(Cot[a + b*x]*(4*Sqrt[2]*ArcTanh[Sqrt[2 - 2*Tan[a + b*x]^2]/2]*Cos[2*(
a + b*x)]*Sec[a + b*x]^2 - 4*ArcTanh[Sqrt[1 - Tan[a + b*x]^2]]*Cos[2*(a + b
*x)]*Sec[a + b*x]^2 + Cot[a + b*x]^2*(Cos[2*(a + b*x)]*Sec[a + b*x]^2)^(3/2
) + ArcTan[Sqrt[-1 + Tan[a + b*x]^2]]*Sqrt[-(-1 + Tan[a + b*x]^2)^2])*Sqrt[
c*Tan[a + b*x]*Tan[2*(a + b*x)]])/(b*c^2*Sqrt[1 - Tan[a + b*x]^2])
```

fricas [A] time = 0.67, size = 438, normalized size = 3.17

$$\left[\frac{5\sqrt{2}\sqrt{c}\log\left(\frac{c\tan(bx+a)^3+2\sqrt{-\frac{c\tan(bx+a)^2}{\tan(bx+a)^2-1}}\left(\tan(bx+a)^2-1\right)\sqrt{c}-2c\tan(bx+a)}{\tan(bx+a)^3}\right)\tan(bx+a)^3+8\sqrt{c}\log\left(\frac{c\tan(bx+a)^3-2\sqrt{2}\sqrt{-\frac{c\tan\ldots}{\tan\ldots}}}{\tan\ldots}\right.}{16\,bc^2\tan(bx+a)^3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*tan(b*x+a)*tan(2*b*x+2*a))^(3/2),x, algorithm="fricas")
```

```
[Out] [1/16*(5*sqrt(2)*sqrt(c)*log((c*tan(b*x + a)^3 + 2*sqrt(-c*tan(b*x + a)^2/(
tan(b*x + a)^2 - 1))*(tan(b*x + a)^2 - 1)*sqrt(c) - 2*c*tan(b*x + a))/tan(b
*x + a)^3)*tan(b*x + a)^3 + 8*sqrt(c)*log((c*tan(b*x + a)^3 - 2*sqrt(2)*sqr
t(-c*tan(b*x + a)^2/(tan(b*x + a)^2 - 1))*(tan(b*x + a)^2 - 1)*sqrt(c) - 3*
c*tan(b*x + a))/(tan(b*x + a)^3 + tan(b*x + a)))*tan(b*x + a)^3 + 2*sqrt(2)
*sqrt(-c*tan(b*x + a)^2/(tan(b*x + a)^2 - 1))*(tan(b*x + a)^2 - 1))/(b*c^2*
tan(b*x + a)^3), 1/8*(5*sqrt(2)*sqrt(-c)*arctan(sqrt(-c*tan(b*x + a)^2/(tan
(b*x + a)^2 - 1))*(tan(b*x + a)^2 - 1)*sqrt(-c)/(c*tan(b*x + a)))*tan(b*x +
 a)^3 - 8*sqrt(-c)*arctan(1/2*sqrt(2)*sqrt(-c*tan(b*x + a)^2/(tan(b*x + a)^
2 - 1))*(tan(b*x + a)^2 - 1)*sqrt(-c)/(c*tan(b*x + a)))*tan(b*x + a)^3 + sq
rt(2)*sqrt(-c*tan(b*x + a)^2/(tan(b*x + a)^2 - 1))*(tan(b*x + a)^2 - 1))/(b
*c^2*tan(b*x + a)^3)]
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*tan(b*x+a)*tan(2*b*x+2*a))^(3/2),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [B] time = 1.02, size = 561, normalized size = 4.07

$$-\frac{\sqrt{2}\,\left(-1+\cos\left(bx+a\right)\right)^{2}\left(8\cos\left(bx+a\right)\sqrt{2}\,\operatorname{arctanh}\left(\frac{\cos(bx+a)\sqrt{4}\,\left(-1+\cos(bx+a)\right)\sqrt{2}}{2\sin(bx+a)^{2}\sqrt{\frac{2\left(\cos^{2}(bx+a)\right)-1}{\left(\cos(bx+a)+1\right)^{2}}}}\right)+2\cos\left(bx+a\right)\sqrt{\frac{2\left(\cos^{2}(bx+\right.}{\left(\cos(bx+a)\right.}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*tan(b*x+a)*tan(2*b*x+2*a))^(3/2),x)
```

```
[Out] -1/32*2^(1/2)/b*(-1+cos(b*x+a))^2*(8*cos(b*x+a)*2^(1/2)*arctanh(1/2*cos(b*x
+a)*4^(1/2)*(-1+cos(b*x+a))/sin(b*x+a)^2/((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)
^2)^(1/2)*2^(1/2))+2*cos(b*x+a)*((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(1/2)
-5*cos(b*x+a)*ln(-2*(cos(b*x+a)^2*((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(1/
2)-2*cos(b*x+a)^2+cos(b*x+a)-((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(1/2)+1)
/sin(b*x+a)^2)-5*cos(b*x+a)*arctanh(1/2*4^(1/2)*(2*cos(b*x+a)^2-3*cos(b*x+a
)+1)/((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(1/2)/sin(b*x+a)^2)-8*2^(1/2)*ar
ctanh(1/2*cos(b*x+a)*4^(1/2)*(-1+cos(b*x+a))/sin(b*x+a)^2/((2*cos(b*x+a)^2-
1)/(cos(b*x+a)+1)^2)^(1/2)*2^(1/2))+5*ln(-2*(cos(b*x+a)^2*((2*cos(b*x+a)^2-
1)/(cos(b*x+a)+1)^2)^(1/2)-2*cos(b*x+a)^2+cos(b*x+a)-((2*cos(b*x+a)^2-1)/(c
os(b*x+a)+1)^2)^(1/2)+1)/sin(b*x+a)^2)+5*arctanh(1/2*4^(1/2)*(2*cos(b*x+a)^
2-3*cos(b*x+a)+1)/((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(1/2)/sin(b*x+a)^2)
)/(c*sin(b*x+a)^2/(2*cos(b*x+a)^2-1))^(3/2)/sin(b*x+a)^3/((2*cos(b*x+a)^2-1
)/(cos(b*x+a)+1)^2)^(3/2)*4^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(c\tan\left(2\,bx+2\,a\right)\tan\left(bx+a\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*tan(b*x+a)*tan(2*b*x+2*a))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((c*tan(2*b*x + 2*a)*tan(b*x + a))^(-3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\left(c\tan\left(a+b\,x\right)\,\tan\left(2\,a+2\,b\,x\right)\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*tan(a + b*x)*tan(2*a + 2*b*x))^(3/2),x)
[Out] int(1/(c*tan(a + b*x)*tan(2*a + 2*b*x))^(3/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*tan(b*x+a)*tan(2*b*x+2*a))**(3/2),x)
[Out] Timed out
```

3.631 $\int \frac{\cos(2(a+bx))}{(c\tan(a+bx)\tan(2(a+bx)))^{3/2}}\,dx$

Optimal. Leaf size=178

$$-\frac{3\tanh^{-1}\left(\frac{\sqrt{c}\tan(2a+2bx)}{\sqrt{c\sec(2a+2bx)-c}}\right)}{2bc^{3/2}}+\frac{9\tanh^{-1}\left(\frac{\sqrt{c}\tan(2a+2bx)}{\sqrt{2}\sqrt{c\sec(2a+2bx)-c}}\right)}{4\sqrt{2}bc^{3/2}}-\frac{3\sin(2a+2bx)}{4bc\sqrt{c\sec(2a+2bx)-c}}-\frac{\sin(2a+2bx)}{4b(c\sec(2a+2bx)-c)^{3/2}}$$

```
[Out] -3/2*arctanh(c^(1/2)*tan(2*b*x+2*a)/(-c+c*sec(2*b*x+2*a))^(1/2))/b/c^(3/2)-
1/4*sin(2*b*x+2*a)/b/(-c+c*sec(2*b*x+2*a))^(3/2)+9/8*arctanh(1/2*c^(1/2)*ta
n(2*b*x+2*a)*2^(1/2)/(-c+c*sec(2*b*x+2*a))^(1/2))/b/c^(3/2)*2^(1/2)-3/4*sin
(2*b*x+2*a)/b/c/(-c+c*sec(2*b*x+2*a))^(1/2)
```

Rubi [A] time = 0.32, antiderivative size = 178, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.241, Rules used = {4397, 3817, 4022, 3920, 3774, 207, 3795}

$$-\frac{3\tanh^{-1}\left(\frac{\sqrt{c}\tan(2a+2bx)}{\sqrt{c\sec(2a+2bx)-c}}\right)}{2bc^{3/2}}+\frac{9\tanh^{-1}\left(\frac{\sqrt{c}\tan(2a+2bx)}{\sqrt{2}\sqrt{c\sec(2a+2bx)-c}}\right)}{4\sqrt{2}bc^{3/2}}-\frac{3\sin(2a+2bx)}{4bc\sqrt{c\sec(2a+2bx)-c}}-\frac{\sin(2a+2bx)}{4b(c\sec(2a+2bx)-c)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[2*(a + b*x)]/(c*Tan[a + b*x]*Tan[2*(a + b*x)])^(3/2),x]
```

```
[Out] (-3*ArcTanh[(Sqrt[c]*Tan[2*a + 2*b*x])/Sqrt[-c + c*Sec[2*a + 2*b*x]]])/(2*b
*c^(3/2)) + (9*ArcTanh[(Sqrt[c]*Tan[2*a + 2*b*x])/(Sqrt[2]*Sqrt[-c + c*Sec[
2*a + 2*b*x]])])/(4*Sqrt[2]*b*c^(3/2)) - Sin[2*a + 2*b*x]/(4*b*(-c + c*Sec[
2*a + 2*b*x])^(3/2)) - (3*Sin[2*a + 2*b*x])/(4*b*c*Sqrt[-c + c*Sec[2*a + 2*
b*x]])
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 3774

```
Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[(-2*b)/d,
 Subst[Int[1/(a + x^2), x], x, (b*Cot[c + d*x])/Sqrt[a + b*Csc[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3795

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Dist[-2/f, Subst[Int[1/(2*a + x^2), x], x, (b*Cot[e + f*x])/Sqrt[
a + b*Csc[e + f*x]]], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3817

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> -Simp[(Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e
 + f*x])^n)/(f*(2*m + 1)), x] + Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e +
f*x])^(m + 1)*(d*Csc[e + f*x])^n*(a*(2*m + n + 1) - b*(m + n + 1)*Csc[e + f
*x]), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && LtQ[m,
 -1] && (IntegersQ[2*m, 2*n] || IntegerQ[m])
```

Rule 3920

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)], x_Symbol] :> Dist[c/a, Int[Sqrt[a + b*Csc[e + f*x]], x], x] - D
ist[(b*c - a*d)/a, Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] /; Fre
eQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 4022

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*Cot[
e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(f*n), x] - Dist[1/(b*d
*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1)*Simp[a*A*m - b*B*n
 - A*b*(m + n + 1)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B,
m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[n, 0]
```

Rule 4397

```
Int[u_, x_Symbol] :> Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]
```

Rubi steps

$$\int \frac{\cos(2(a+bx))}{(c\tan(a+bx)\tan(2(a+bx)))^{3/2}}\,dx = \int \frac{\cos(2a+2bx)}{(-c+c\sec(2a+2bx))^{3/2}}\,dx$$

$$= -\frac{\sin(2a+2bx)}{4b(-c+c\sec(2a+2bx))^{3/2}} - \frac{\int \frac{\cos(2a+2bx)\left(3c+\frac{3}{2}c\sec(2a+2bx)\right)}{\sqrt{-c+c\sec(2a+2bx)}}\,dx}{2c^2}$$

$$= -\frac{\sin(2a+2bx)}{4b(-c+c\sec(2a+2bx))^{3/2}} - \frac{3\sin(2a+2bx)}{4bc\sqrt{-c+c\sec(2a+2bx)}} - \frac{\int \frac{3c^2+\frac{3}{2}c^2}{\sqrt{-c+c}}}{}$$

$$= -\frac{\sin(2a+2bx)}{4b(-c+c\sec(2a+2bx))^{3/2}} - \frac{3\sin(2a+2bx)}{4bc\sqrt{-c+c\sec(2a+2bx)}} + \frac{3\int\sqrt{-c}}{}$$

$$= -\frac{\sin(2a+2bx)}{4b(-c+c\sec(2a+2bx))^{3/2}} - \frac{3\sin(2a+2bx)}{4bc\sqrt{-c+c\sec(2a+2bx)}} - \frac{3\,\mathrm{Subst}(}{}$$

$$= -\frac{3\tanh^{-1}\left(\frac{\sqrt{c}\tan(2a+2bx)}{\sqrt{-c+c\sec(2a+2bx)}}\right)}{2bc^{3/2}} + \frac{9\tanh^{-1}\left(\frac{\sqrt{c}\tan(2a+2bx)}{\sqrt{2}\sqrt{-c+c\sec(2a+2bx)}}\right)}{4\sqrt{2}bc^{3/2}} - \frac{}{4b(-c}$$

Mathematica [A] time = 6.20, size = 342, normalized size = 1.92

$$\frac{\tan^2(a+bx)\tan^2(2(a+bx))\left(\frac{1}{2}\sin(2(a+bx)) - \frac{1}{4}\cot(a+bx) - \frac{1}{8}\cot(a+bx)\csc^2(a+bx)\right)}{b(c\tan(a+bx)\tan(2(a+bx)))^{3/2}} - \frac{3\tan^{\frac{3}{2}}(a+bx)\tan}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[2*(a + b*x)]/(c*Tan[a + b*x]*Tan[2*(a + b*x)])^(3/2),x]
```

```
[Out] (-3*Tan[a + b*x]^(3/2)*((ArcTan[Sqrt[-1 + Tan[a + b*x]^2]]*Csc[a + b*x]^2*S
ec[a + b*x]^2*Tan[a + b*x]^(3/2)*Sqrt[-1 + Tan[a + b*x]^2]*Sqrt[Tan[2*(a +
b*x)]])/(1 + Tan[a + b*x]^2)^2 + (Sqrt[2]*(2*ArcTanh[Sqrt[2 - 2*Tan[a + b*x
]^2]/2] - Sqrt[2]*ArcTanh[Sqrt[1 - Tan[a + b*x]^2]])*Cos[2*(a + b*x)]*Csc[a
 + b*x]^2*Sec[a + b*x]^2*Tan[a + b*x]^(3/2)*Sqrt[Tan[2*(a + b*x)]])/(Sqrt[1
 - Tan[a + b*x]^2]*(1 + Tan[a + b*x]^2)))*Tan[2*(a + b*x)]^(3/2))/(8*b*(c*T
an[a + b*x]*Tan[2*(a + b*x)])^(3/2)) + ((-1/4*Cot[a + b*x] - (Cot[a + b*x]*
Csc[a + b*x]^2)/8 + Sin[2*(a + b*x)]/2)*Tan[a + b*x]^2*Tan[2*(a + b*x)]^2)/
(b*(c*Tan[a + b*x]*Tan[2*(a + b*x)])^(3/2))
```

fricas [A] time = 1.05, size = 528, normalized size = 2.97

$$\left[\frac{9\sqrt{2}\left(\tan\left(bx+a\right)^{5}+\tan\left(bx+a\right)^{3}\right)\sqrt{c}\log\left(\frac{c\tan(bx+a)^{3}+2\sqrt{-\frac{c\tan(bx+a)^{2}}{\tan(bx+a)^{2}-1}}\left(\tan(bx+a)^{2}-1\right)\sqrt{c}-2\,c\tan(bx+a)}{\tan(bx+a)^{3}}\right)+12\left(\tan\left(bx+\right.\right.}{16\left(\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(2*b*x+2*a)/(c*tan(b*x+a)*tan(2*b*x+2*a))^(3/2),x, algorithm="
fricas")
```

```
[Out] [1/16*(9*sqrt(2)*(tan(b*x + a)^5 + tan(b*x + a)^3)*sqrt(c)*log((c*tan(b*x +
 a)^3 + 2*sqrt(-c*tan(b*x + a)^2/(tan(b*x + a)^2 - 1))*(tan(b*x + a)^2 - 1)
*sqrt(c) - 2*c*tan(b*x + a))/tan(b*x + a)^3) + 12*(tan(b*x + a)^5 + tan(b*x
 + a)^3)*sqrt(c)*log((c*tan(b*x + a)^3 - 2*sqrt(2)*sqrt(-c*tan(b*x + a)^2/(
tan(b*x + a)^2 - 1))*(tan(b*x + a)^2 - 1)*sqrt(c) - 3*c*tan(b*x + a))/(tan(
b*x + a)^3 + tan(b*x + a))) + 2*sqrt(2)*(5*tan(b*x + a)^4 - 4*tan(b*x + a)^
2 - 1)*sqrt(-c*tan(b*x + a)^2/(tan(b*x + a)^2 - 1)))/(b*c^2*tan(b*x + a)^5
+ b*c^2*tan(b*x + a)^3), 1/8*(9*sqrt(2)*(tan(b*x + a)^5 + tan(b*x + a)^3)*s
qrt(-c)*arctan(sqrt(-c*tan(b*x + a)^2/(tan(b*x + a)^2 - 1))*(tan(b*x + a)^2
 - 1)*sqrt(-c)/(c*tan(b*x + a))) - 12*(tan(b*x + a)^5 + tan(b*x + a)^3)*sqr
t(-c)*arctan(1/2*sqrt(2)*sqrt(-c*tan(b*x + a)^2/(tan(b*x + a)^2 - 1))*(tan(
b*x + a)^2 - 1)*sqrt(-c)/(c*tan(b*x + a))) + sqrt(2)*(5*tan(b*x + a)^4 - 4*
tan(b*x + a)^2 - 1)*sqrt(-c*tan(b*x + a)^2/(tan(b*x + a)^2 - 1)))/(b*c^2*ta
n(b*x + a)^5 + b*c^2*tan(b*x + a)^3)]
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(2*b*x+2*a)/(c*tan(b*x+a)*tan(2*b*x+2*a))^(3/2),x, algorithm="
giac")
```

```
[Out] Timed out
```

maple [B] time = 1.07, size = 1157, normalized size = 6.50

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(2*b*x+2*a)/(c*tan(b*x+a)*tan(2*b*x+2*a))^(3/2),x)
```

```
[Out] 1/32*2^(1/2)/b*(-1+cos(b*x+a))^2*(8*cos(b*x+a)*2^(1/2)*arctanh(1/2*cos(b*x+
a)*4^(1/2)*(-1+cos(b*x+a))/sin(b*x+a)^2/((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^
2)^(1/2)*2^(1/2))+2*cos(b*x+a)*((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(1/2)-
5*cos(b*x+a)*ln(-2*(cos(b*x+a)^2*((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(1/2
)-2*cos(b*x+a)^2+cos(b*x+a)-((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(1/2)+1)/
sin(b*x+a)^2)-5*cos(b*x+a)*arctanh(1/2*4^(1/2)*(2*cos(b*x+a)^2-3*cos(b*x+a)
+1)/((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(1/2)/sin(b*x+a)^2)-8*2^(1/2)*arc
tanh(1/2*cos(b*x+a)*4^(1/2)*(-1+cos(b*x+a))/sin(b*x+a)^2/((2*cos(b*x+a)^2-1
)/(cos(b*x+a)+1)^2)^(1/2)*2^(1/2))+5*ln(-2*(cos(b*x+a)^2*((2*cos(b*x+a)^2-1
)/(cos(b*x+a)+1)^2)^(1/2)-2*cos(b*x+a)^2+cos(b*x+a)-((2*cos(b*x+a)^2-1)/(co
s(b*x+a)+1)^2)^(1/2)+1)/sin(b*x+a)^2)+5*arctanh(1/2*4^(1/2)*(2*cos(b*x+a)^2
-3*cos(b*x+a)+1)/((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(1/2)/sin(b*x+a)^2))
/(c*sin(b*x+a)^2/(2*cos(b*x+a)^2-1))^(3/2)/sin(b*x+a)^3/((2*cos(b*x+a)^2-1)
/(cos(b*x+a)+1)^2)^(3/2)*4^(1/2)+1/16*2^(1/2)/b*(-1+cos(b*x+a))^2*(4*((2*co
s(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(1/2)*cos(b*x+a)^3-10*cos(b*x+a)*2^(1/2)*ar
ctanh(1/2*cos(b*x+a)*4^(1/2)*(-1+cos(b*x+a))/sin(b*x+a)^2/((2*cos(b*x+a)^2-
1)/(cos(b*x+a)+1)^2)^(1/2)*2^(1/2))+7*cos(b*x+a)*arctanh(1/2*4^(1/2)*(2*cos
(b*x+a)^2-3*cos(b*x+a)+1)/((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(1/2)/sin(b
*x+a)^2)+7*cos(b*x+a)*ln(-2*(cos(b*x+a)^2*((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1
)^2)^(1/2)-2*cos(b*x+a)^2+cos(b*x+a)-((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^
(1/2)+1)/sin(b*x+a)^2)-6*cos(b*x+a)*((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(
1/2)+10*2^(1/2)*arctanh(1/2*cos(b*x+a)*4^(1/2)*(-1+cos(b*x+a))/sin(b*x+a)^2
/((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(1/2)*2^(1/2))-7*arctanh(1/2*4^(1/2)
*(2*cos(b*x+a)^2-3*cos(b*x+a)+1)/((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(1/2
)/sin(b*x+a)^2)-7*ln(-2*(cos(b*x+a)^2*((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)
^(1/2)-2*cos(b*x+a)^2+cos(b*x+a)-((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(1/2
)+1)/sin(b*x+a)^2))/((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(3/2)/(c*sin(b*x+
a)^2/(2*cos(b*x+a)^2-1))^(3/2)/sin(b*x+a)^3*4^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos(2\,bx + 2\,a)}{(c\tan(2\,bx + 2\,a)\tan(bx + a))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(2*b*x+2*a)/(c*tan(b*x+a)*tan(2*b*x+2*a))^(3/2),x, algorithm="
maxima")
```

```
[Out] integrate(cos(2*b*x + 2*a)/(c*tan(2*b*x + 2*a)*tan(b*x + a))^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos(2\,a + 2\,b\,x)}{(c\tan(a + b\,x)\,\tan(2\,a + 2\,b\,x))^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(2*a + 2*b*x)/(c*tan(a + b*x)*tan(2*a + 2*b*x))^(3/2),x)
```

```
[Out] int(cos(2*a + 2*b*x)/(c*tan(a + b*x)*tan(2*a + 2*b*x))^(3/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(2*b*x+2*a)/(c*tan(b*x+a)*tan(2*b*x+2*a))**(3/2),x)
```

```
[Out] Timed out
```

3.632 $\int \frac{\cos^2(2(a+bx))}{(c\tan(a+bx)\tan(2(a+bx)))^{3/2}}\,dx$

Optimal. Leaf size=234

$$-\frac{19\tanh^{-1}\left(\frac{\sqrt{c}\tan(2a+2bx)}{\sqrt{c\sec(2a+2bx)-c}}\right)}{8bc^{3/2}}+\frac{13\tanh^{-1}\left(\frac{\sqrt{c}\tan(2a+2bx)}{\sqrt{2}\sqrt{c\sec(2a+2bx)-c}}\right)}{4\sqrt{2}\,bc^{3/2}}-\frac{7\sin(2a+2bx)}{8bc\sqrt{c\sec(2a+2bx)-c}}-\frac{\sin(2a+2bx)\cos(2a}{2bc\sqrt{c\sec(2a+2bx)}}$$

```
[Out] -19/8*arctanh(c^(1/2)*tan(2*b*x+2*a)/(-c+c*sec(2*b*x+2*a))^(1/2))/b/c^(3/2)
-1/4*cos(2*b*x+2*a)*sin(2*b*x+2*a)/b/(-c+c*sec(2*b*x+2*a))^(3/2)+13/8*arcta
nh(1/2*c^(1/2)*tan(2*b*x+2*a)*2^(1/2)/(-c+c*sec(2*b*x+2*a))^(1/2))/b/c^(3/2
)*2^(1/2)-7/8*sin(2*b*x+2*a)/b/c/(-c+c*sec(2*b*x+2*a))^(1/2)-1/2*cos(2*b*x+
2*a)*sin(2*b*x+2*a)/b/c/(-c+c*sec(2*b*x+2*a))^(1/2)
```

Rubi [A] time = 0.50, antiderivative size = 234, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.226, Rules used = {4397, 3817, 4022, 3920, 3774, 207, 3795}

$$-\frac{19\tanh^{-1}\left(\frac{\sqrt{c}\tan(2a+2bx)}{\sqrt{c\sec(2a+2bx)-c}}\right)}{8bc^{3/2}}+\frac{13\tanh^{-1}\left(\frac{\sqrt{c}\tan(2a+2bx)}{\sqrt{2}\sqrt{c\sec(2a+2bx)-c}}\right)}{4\sqrt{2}\,bc^{3/2}}-\frac{7\sin(2a+2bx)}{8bc\sqrt{c\sec(2a+2bx)-c}}-\frac{\sin(2a+2bx)\cos(2a}{2bc\sqrt{c\sec(2a+2bx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[2*(a + b*x)]^2/(c*Tan[a + b*x]*Tan[2*(a + b*x)])^(3/2),x]
```

```
[Out] (-19*ArcTanh[(Sqrt[c]*Tan[2*a + 2*b*x])/Sqrt[-c + c*Sec[2*a + 2*b*x]]])/(8*
b*c^(3/2)) + (13*ArcTanh[(Sqrt[c]*Tan[2*a + 2*b*x])/(Sqrt[2]*Sqrt[-c + c*Se
c[2*a + 2*b*x]])])/(4*Sqrt[2]*b*c^(3/2)) - (Cos[2*a + 2*b*x]*Sin[2*a + 2*b*
x])/(4*b*(-c + c*Sec[2*a + 2*b*x])^(3/2)) - (7*Sin[2*a + 2*b*x])/(8*b*c*Sqr
t[-c + c*Sec[2*a + 2*b*x]]) - (Cos[2*a + 2*b*x]*Sin[2*a + 2*b*x])/(2*b*c*Sq
rt[-c + c*Sec[2*a + 2*b*x]])
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 3774

```
Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[(-2*b)/d,
 Subst[Int[1/(a + x^2), x], x, (b*Cot[c + d*x])/Sqrt[a + b*Csc[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3795

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Dist[-2/f, Subst[Int[1/(2*a + x^2), x], x, (b*Cot[e + f*x])/Sqrt[
a + b*Csc[e + f*x]]], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3817

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> -Simp[(Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e
 + f*x])^n)/(f*(2*m + 1)), x] + Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e +
f*x])^(m + 1)*(d*Csc[e + f*x])^n*(a*(2*m + n + 1) - b*(m + n + 1)*Csc[e + f
*x]), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && LtQ[m,
 -1] && (IntegersQ[2*m, 2*n] || IntegerQ[m])
```

Rule 3920

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)], x_Symbol] :> Dist[c/a, Int[Sqrt[a + b*Csc[e + f*x]], x], x] - D
ist[(b*c - a*d)/a, Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] /; Fre
eQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 4022

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*Cot[
e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(f*n), x] - Dist[1/(b*d
*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1)*Simp[a*A*m - b*B*n
 - A*b*(m + n + 1)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B,
m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[n, 0]
```

Rule 4397

```
Int[u_, x_Symbol] :> Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]
```

Rubi steps

$$\int \frac{\cos^2(2(a+bx))}{(c\tan(a+bx)\tan(2(a+bx)))^{3/2}}\,dx = \int \frac{\cos^2(2a+2bx)}{(-c+c\sec(2a+2bx))^{3/2}}\,dx$$

$$= -\frac{\cos(2a+2bx)\sin(2a+2bx)}{4b(-c+c\sec(2a+2bx))^{3/2}} - \frac{\int \frac{\cos^2(2a+2bx)\left(4c+\frac{5}{2}c\sec(2a+2bx)\right)}{\sqrt{-c+c\sec(2a+2bx)}}\,dx}{2c^2}$$

$$= -\frac{\cos(2a+2bx)\sin(2a+2bx)}{4b(-c+c\sec(2a+2bx))^{3/2}} - \frac{\cos(2a+2bx)\sin(2a+2bx)}{2bc\sqrt{-c+c\sec(2a+2bx)}} - \frac{\int \frac{\cos(}{}}{}$$

$$= -\frac{\cos(2a+2bx)\sin(2a+2bx)}{4b(-c+c\sec(2a+2bx))^{3/2}} - \frac{7\sin(2a+2bx)}{8bc\sqrt{-c+c\sec(2a+2bx)}} - \frac{\cos(2a}{2bc\sqrt{-}}$$

$$= -\frac{\cos(2a+2bx)\sin(2a+2bx)}{4b(-c+c\sec(2a+2bx))^{3/2}} - \frac{7\sin(2a+2bx)}{8bc\sqrt{-c+c\sec(2a+2bx)}} - \frac{\cos(2a}{2bc\sqrt{-}}$$

$$= -\frac{\cos(2a+2bx)\sin(2a+2bx)}{4b(-c+c\sec(2a+2bx))^{3/2}} - \frac{7\sin(2a+2bx)}{8bc\sqrt{-c+c\sec(2a+2bx)}} - \frac{\cos(2a}{2bc\sqrt{-}}$$

$$= -\frac{19\tanh^{-1}\left(\frac{\sqrt{c}\tan(2a+2bx)}{\sqrt{-c+c\sec(2a+2bx)}}\right)}{8bc^{3/2}} + \frac{13\tanh^{-1}\left(\frac{\sqrt{c}\tan(2a+2bx)}{\sqrt{2}\sqrt{-c+c\sec(2a+2bx)}}\right)}{4\sqrt{2}\,bc^{3/2}} - \frac{\cos}{4b}$$

Mathematica [A] time = 6.21, size = 356, normalized size = 1.52

$$\frac{\tan^2(a+bx)\tan^2(2(a+bx))\left(\frac{7}{8}\sin(2(a+bx))+\frac{1}{8}\sin(4(a+bx))-\frac{5}{8}\cot(a+bx)-\frac{1}{8}\cot(a+bx)\csc^2(a+bx)\right)}{b(c\tan(a+bx)\tan(2(a+bx)))^{3/2}}+$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[2*(a + b*x)]^2/(c*Tan[a + b*x]*Tan[2*(a + b*x)])^(3/2),x]
```

```
[Out] (Tan[a + b*x]^(3/2)*((-7*ArcTan[Sqrt[-1 + Tan[a + b*x]^2]]*Csc[a + b*x]^2*S
ec[a + b*x]^2*Tan[a + b*x]^(3/2)*Sqrt[-1 + Tan[a + b*x]^2]*Sqrt[Tan[2*(a +
b*x)]])/(1 + Tan[a + b*x]^2)^2 - (19*(2*ArcTanh[Sqrt[2 - 2*Tan[a + b*x]^2]/
2] - Sqrt[2]*ArcTanh[Sqrt[1 - Tan[a + b*x]^2]])*Cos[2*(a + b*x)]*Csc[a + b*
x]^2*Sec[a + b*x]^2*Tan[a + b*x]^(3/2)*Sqrt[Tan[2*(a + b*x)]])/(Sqrt[2]*Sqr
t[1 - Tan[a + b*x]^2]*(1 + Tan[a + b*x]^2)))*Tan[2*(a + b*x)]^(3/2))/(16*b*
(c*Tan[a + b*x]*Tan[2*(a + b*x)])^(3/2)) + (((-5*Cot[a + b*x])/8 - (Cot[a +
```

```
b*x]*Csc[a + b*x]^2)/8 + (7*Sin[2*(a + b*x)])/8 + Sin[4*(a + b*x)]/8)*Tan[
a + b*x]^2*Tan[2*(a + b*x)]^2)/(b*(c*Tan[a + b*x]*Tan[2*(a + b*x)])^(3/2))
```

fricas [A] time = 0.98, size = 616, normalized size = 2.63

$$\left[\frac{13\sqrt{2}\left(\tan(bx+a)^7 + 2\tan(bx+a)^5 + \tan(bx+a)^3\right)\sqrt{c}\log\left(\frac{c\tan(bx+a)^3 + 2\sqrt{-\frac{c\tan(bx+a)^2}{\tan(bx+a)^2-1}}\left(\tan(bx+a)^2-1\right)\sqrt{c} - 2c\tan(bx+}{\tan(bx+a)^3}\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(2*b*x+2*a)^2/(c*tan(b*x+a)*tan(2*b*x+2*a))^(3/2),x, algorithm
="fricas")
```

```
[Out] [1/16*(13*sqrt(2)*(tan(b*x + a)^7 + 2*tan(b*x + a)^5 + tan(b*x + a)^3)*sqrt
(c)*log((c*tan(b*x + a)^3 + 2*sqrt(-c*tan(b*x + a)^2/(tan(b*x + a)^2 - 1))*
(tan(b*x + a)^2 - 1)*sqrt(c) - 2*c*tan(b*x + a))/tan(b*x + a)^3) + 19*(tan(
b*x + a)^7 + 2*tan(b*x + a)^5 + tan(b*x + a)^3)*sqrt(c)*log((c*tan(b*x + a)
^3 - 2*sqrt(2)*sqrt(-c*tan(b*x + a)^2/(tan(b*x + a)^2 - 1))*(tan(b*x + a)^2
 - 1)*sqrt(c) - 3*c*tan(b*x + a))/(tan(b*x + a)^3 + tan(b*x + a))) + 2*sqrt
(2)*(4*tan(b*x + a)^6 + 5*tan(b*x + a)^4 - 8*tan(b*x + a)^2 - 1)*sqrt(-c*ta
n(b*x + a)^2/(tan(b*x + a)^2 - 1)))/(b*c^2*tan(b*x + a)^7 + 2*b*c^2*tan(b*x
 + a)^5 + b*c^2*tan(b*x + a)^3), 1/8*(13*sqrt(2)*(tan(b*x + a)^7 + 2*tan(b*
x + a)^5 + tan(b*x + a)^3)*sqrt(-c)*arctan(sqrt(-c*tan(b*x + a)^2/(tan(b*x
+ a)^2 - 1))*(tan(b*x + a)^2 - 1)*sqrt(-c)/(c*tan(b*x + a))) - 19*(tan(b*x
+ a)^7 + 2*tan(b*x + a)^5 + tan(b*x + a)^3)*sqrt(-c)*arctan(1/2*sqrt(2)*sqr
t(-c*tan(b*x + a)^2/(tan(b*x + a)^2 - 1))*(tan(b*x + a)^2 - 1)*sqrt(-c)/(c*
tan(b*x + a))) + sqrt(2)*(4*tan(b*x + a)^6 + 5*tan(b*x + a)^4 - 8*tan(b*x +
 a)^2 - 1)*sqrt(-c*tan(b*x + a)^2/(tan(b*x + a)^2 - 1)))/(b*c^2*tan(b*x + a
)^7 + 2*b*c^2*tan(b*x + a)^5 + b*c^2*tan(b*x + a)^3)]
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(2*b*x+2*a)^2/(c*tan(b*x+a)*tan(2*b*x+2*a))^(3/2),x, algorithm
="giac")
```

```
[Out] Timed out
```

maple [B] time = 1.02, size = 1787, normalized size = 7.64

Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(2*b*x+2*a)^2/(c*tan(b*x+a)*tan(2*b*x+2*a))^(3/2),x)
```

```
[Out] -1/32*2^(1/2)/b*(-1+cos(b*x+a))^2*(-8*((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)
^(1/2)*cos(b*x+a)^5-14*((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(1/2)*cos(b*x+
a)^3+51*cos(b*x+a)*2^(1/2)*arctanh(1/2*cos(b*x+a)*4^(1/2)*(-1+cos(b*x+a))/s
in(b*x+a)^2/((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(1/2)*2^(1/2))-51*2^(1/2)
*arctanh(1/2*cos(b*x+a)*4^(1/2)*(-1+cos(b*x+a))/sin(b*x+a)^2/((2*cos(b*x+a)
^2-1)/(cos(b*x+a)+1)^2)^(1/2)*2^(1/2))-36*cos(b*x+a)*arctanh(1/2*4^(1/2)*(2
*cos(b*x+a)^2-3*cos(b*x+a)+1)/((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(1/2)/s
in(b*x+a)^2)-36*cos(b*x+a)*ln(-2*(cos(b*x+a)^2*((2*cos(b*x+a)^2-1)/(cos(b*x
+a)+1)^2)^(1/2)-2*cos(b*x+a)^2+cos(b*x+a)-((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1
)^2)^(1/2)+1)/sin(b*x+a)^2)+30*cos(b*x+a)*((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1
)^2)^(1/2)+36*arctanh(1/2*4^(1/2)*(2*cos(b*x+a)^2-3*cos(b*x+a)+1)/((2*cos(b
*x+a)^2-1)/(cos(b*x+a)+1)^2)^(1/2)/sin(b*x+a)^2)+36*ln(-2*(cos(b*x+a)^2*((2
*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(1/2)-2*cos(b*x+a)^2+cos(b*x+a)-((2*cos(
b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(1/2)+1)/sin(b*x+a)^2))/((2*cos(b*x+a)^2-1)/(
cos(b*x+a)+1)^2)^(3/2)/(c*sin(b*x+a)^2/(2*cos(b*x+a)^2-1))^(3/2)/sin(b*x+a)
^3*4^(1/2)-1/32*2^(1/2)/b*(-1+cos(b*x+a))^2*(8*cos(b*x+a)*2^(1/2)*arctanh(1
/2*cos(b*x+a)*4^(1/2)*(-1+cos(b*x+a))/sin(b*x+a)^2/((2*cos(b*x+a)^2-1)/(cos
(b*x+a)+1)^2)^(1/2)*2^(1/2))+2*cos(b*x+a)*((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1
)^2)^(1/2)-5*cos(b*x+a)*ln(-2*(cos(b*x+a)^2*((2*cos(b*x+a)^2-1)/(cos(b*x+a)
+1)^2)^(1/2)-2*cos(b*x+a)^2+cos(b*x+a)-((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2
)^(1/2)+1)/sin(b*x+a)^2)-5*cos(b*x+a)*arctanh(1/2*4^(1/2)*(2*cos(b*x+a)^2-3
*cos(b*x+a)+1)/((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(1/2)/sin(b*x+a)^2)-8*
2^(1/2)*arctanh(1/2*cos(b*x+a)*4^(1/2)*(-1+cos(b*x+a))/sin(b*x+a)^2/((2*cos
(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(1/2)*2^(1/2))+5*ln(-2*(cos(b*x+a)^2*((2*cos
(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(1/2)-2*cos(b*x+a)^2+cos(b*x+a)-((2*cos(b*x+
a)^2-1)/(cos(b*x+a)+1)^2)^(1/2)+1)/sin(b*x+a)^2)+5*arctanh(1/2*4^(1/2)*(2*c
os(b*x+a)^2-3*cos(b*x+a)+1)/((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(1/2)/sin
(b*x+a)^2))/(c*sin(b*x+a)^2/(2*cos(b*x+a)^2-1))^(3/2)/sin(b*x+a)^3/((2*cos(
b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(3/2)*4^(1/2)-1/8*2^(1/2)/b*(-1+cos(b*x+a))^2
*(4*((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(1/2)*cos(b*x+a)^3-10*cos(b*x+a)*
2^(1/2)*arctanh(1/2*cos(b*x+a)*4^(1/2)*(-1+cos(b*x+a))/sin(b*x+a)^2/((2*cos
(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(1/2)*2^(1/2))+7*cos(b*x+a)*arctanh(1/2*4^(1
/2)*(2*cos(b*x+a)^2-3*cos(b*x+a)+1)/((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(
1/2)/sin(b*x+a)^2)+7*cos(b*x+a)*ln(-2*(cos(b*x+a)^2*((2*cos(b*x+a)^2-1)/(co
s(b*x+a)+1)^2)^(1/2)-2*cos(b*x+a)^2+cos(b*x+a)-((2*cos(b*x+a)^2-1)/(cos(b*x
+a)+1)^2)^(1/2)+1)/sin(b*x+a)^2)-6*cos(b*x+a)*((2*cos(b*x+a)^2-1)/(cos(b*x+
a)+1)^2)^(1/2)+10*2^(1/2)*arctanh(1/2*cos(b*x+a)*4^(1/2)*(-1+cos(b*x+a))/si
```

```
n(b*x+a)^2/((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(1/2)*2^(1/2))-7*arctanh(1
/2*4^(1/2)*(2*cos(b*x+a)^2-3*cos(b*x+a)+1)/((2*cos(b*x+a)^2-1)/(cos(b*x+a)+
1)^2)^(1/2)/sin(b*x+a)^2)-7*ln(-2*(cos(b*x+a)^2*((2*cos(b*x+a)^2-1)/(cos(b*
x+a)+1)^2)^(1/2)-2*cos(b*x+a)^2+cos(b*x+a)-((2*cos(b*x+a)^2-1)/(cos(b*x+a)+
1)^2)^(1/2)+1)/sin(b*x+a)^2))/((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(3/2)/(
c*sin(b*x+a)^2/(2*cos(b*x+a)^2-1))^(3/2)/sin(b*x+a)^3*4^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos(2\,bx + 2\,a)^2}{(c\tan(2\,bx + 2\,a)\tan(bx + a))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(2*b*x+2*a)^2/(c*tan(b*x+a)*tan(2*b*x+2*a))^(3/2),x, algorithm
="maxima")
```

```
[Out] integrate(cos(2*b*x + 2*a)^2/(c*tan(2*b*x + 2*a)*tan(b*x + a))^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cos(2\,a + 2\,b\,x)^2}{(c\tan(a + b\,x)\,\tan(2\,a + 2\,b\,x))^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(2*a + 2*b*x)^2/(c*tan(a + b*x)*tan(2*a + 2*b*x))^(3/2),x)
```

```
[Out] int(cos(2*a + 2*b*x)^2/(c*tan(a + b*x)*tan(2*a + 2*b*x))^(3/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(2*b*x+2*a)**2/(c*tan(b*x+a)*tan(2*b*x+2*a))**(3/2),x)
```

```
[Out] Timed out
```

3.633 $\int \frac{\cot(x)\csc(x)}{\sqrt{\sin(2x)}}\,dx$

Optimal. Leaf size=16

$$-\frac{2\cos(x)\cot(x)}{3\sqrt{\sin(2x)}}$$

```
[Out] -2/3*cos(x)*cot(x)/sin(2*x)^(1/2)
```

Rubi [A] time = 0.09, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {4390, 30}

$$-\frac{2\cos(x)\cot(x)}{3\sqrt{\sin(2x)}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cot[x]*Csc[x])/Sqrt[Sin[2*x]],x]
```

```
[Out] (-2*Cos[x]*Cot[x])/(3*Sqrt[Sin[2*x]])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 4390

```
Int[(u_)*((c_.)*sin[v_])^(m_), x_Symbol] :> With[{w = FunctionOfTrig[(u*Sin
[v/2]^(2*m))/(c*Tan[v/2])^m, x]}, Dist[((c*Sin[v])^m*(c*Tan[v/2])^m)/Sin[v/
2]^(2*m), Int[(u*Sin[v/2]^(2*m))/(c*Tan[v/2])^m, x], x] /;  !FalseQ[w] && F
unctionOfQ[NonfreeFactors[Tan[w], x], (u*Sin[v/2]^(2*m))/(c*Tan[v/2])^m, x]
] /; FreeQ[c, x] && LinearQ[v, x] && IntegerQ[m + 1/2] &&  !SumQ[u] && Inve
rseFunctionFreeQ[u, x]
```

Rubi steps

$$\int \frac{\cot(x)\csc(x)}{\sqrt{\sin(2x)}}\,dx = \frac{\sin(x)\int \frac{\csc^2(x)}{\sqrt{\tan(x)}}\,dx}{\sqrt{\sin(2x)}\sqrt{\tan(x)}}$$
$$= \frac{\sin(x)\,\text{Subst}\left(\int \frac{1}{x^{5/2}}\,dx, x, \tan(x)\right)}{\sqrt{\sin(2x)}\sqrt{\tan(x)}}$$
$$= -\frac{2\cos(x)\cot(x)}{3\sqrt{\sin(2x)}}$$

Mathematica [A] time = 0.03, size = 16, normalized size = 1.00

$$-\frac{1}{3}\sqrt{\sin(2x)}\cot(x)\csc(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cot[x]*Csc[x])/Sqrt[Sin[2*x]],x]
[Out] -1/3*(Cot[x]*Csc[x]*Sqrt[Sin[2*x]])
```

fricas [B] time = 1.87, size = 29, normalized size = 1.81

$$\frac{\sqrt{2}\sqrt{\cos(x)\sin(x)}\cos(x) + \cos(x)^2 - 1}{3\left(\cos(x)^2 - 1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)*csc(x)/sin(2*x)^(1/2),x, algorithm="fricas")
[Out] 1/3*(sqrt(2)*sqrt(cos(x)*sin(x))*cos(x) + cos(x)^2 - 1)/(cos(x)^2 - 1)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cot(x)\csc(x)}{\sqrt{\sin(2x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)*csc(x)/sin(2*x)^(1/2),x, algorithm="giac")
[Out] integrate(cot(x)*csc(x)/sqrt(sin(2*x)), x)
```

maple [C] time = 0.21, size = 119, normalized size = 7.44

$$\frac{\sqrt{-\frac{\tan\left(\frac{x}{2}\right)}{\tan^2\left(\frac{x}{2}\right)-1}}\,\left(\tan^2\left(\frac{x}{2}\right)-1\right)\left(4\sqrt{1+\tan\left(\frac{x}{2}\right)}\,\sqrt{-2\tan\left(\frac{x}{2}\right)+2}\,\sqrt{-\tan\left(\frac{x}{2}\right)}\,\operatorname{EllipticF}\left(\sqrt{1+\tan\left(\frac{x}{2}\right)},\frac{\sqrt{2}}{2}\right)\tan\left(\frac{x}{2}\right.\right.}{6\tan\left(\frac{x}{2}\right)\sqrt{\left(\tan^2\left(\frac{x}{2}\right)-1\right)\tan\left(\frac{x}{2}\right)}\,\sqrt{\tan^3\left(\frac{x}{2}\right)-\tan\left(\frac{x}{2}\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(x)*csc(x)/sin(2*x)^(1/2),x)
```

```
[Out] 1/6*(-tan(1/2*x)/(tan(1/2*x)^2-1))^(1/2)*(tan(1/2*x)^2-1)/tan(1/2*x)*(4*(1+
tan(1/2*x))^(1/2)*(-2*tan(1/2*x)+2)^(1/2)*(-tan(1/2*x))^(1/2)*EllipticF((1+
tan(1/2*x))^(1/2),1/2*2^(1/2))*tan(1/2*x)+tan(1/2*x)^4-1)/((tan(1/2*x)^2-1)
*tan(1/2*x))^(1/2)/(tan(1/2*x)^3-tan(1/2*x))^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cot(x)\csc(x)}{\sqrt{\sin(2x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)*csc(x)/sin(2*x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(cot(x)*csc(x)/sqrt(sin(2*x)), x)
```

mupad [B] time = 3.10, size = 14, normalized size = 0.88

$$-\frac{\sqrt{\sin(2x)}\,\cos(x)}{3\sin(x)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(x)/(sin(2*x)^(1/2)*sin(x)),x)
```

```
[Out] -(sin(2*x)^(1/2)*cos(x))/(3*sin(x)^2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cot(x)\csc(x)}{\sqrt{\sin(2x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)*csc(x)/sin(2*x)**(1/2),x)
```

```
[Out] Integral(cot(x)*csc(x)/sqrt(sin(2*x)), x)
```

3.634 $\int \frac{\csc^2(x)\sec(x)}{\sqrt{\sin(2x)}\,(-2+\tan(x))}\,dx$

Optimal. Leaf size=69

$$\frac{\cos(x)}{2\sqrt{\sin(2x)}}-\frac{5\sin(x)\tanh^{-1}\left(\frac{\sqrt{\tan(x)}}{\sqrt{2}}\right)}{2\sqrt{2}\,\sqrt{\sin(2x)}\,\sqrt{\tan(x)}}+\frac{\cos(x)\cot(x)}{3\sqrt{\sin(2x)}}$$

```
[Out] 1/2*cos(x)/sin(2*x)^(1/2)+1/3*cos(x)*cot(x)/sin(2*x)^(1/2)-5/4*arctanh(1/2*
tan(x)^(1/2)*2^(1/2))*sin(x)*2^(1/2)/sin(2*x)^(1/2)/tan(x)^(1/2)
```

Rubi [A] time = 0.36, antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {4390, 898, 1262, 207}

$$\frac{\cos(x)}{2\sqrt{\sin(2x)}}-\frac{5\sin(x)\tanh^{-1}\left(\frac{\sqrt{\tan(x)}}{\sqrt{2}}\right)}{2\sqrt{2}\,\sqrt{\sin(2x)}\,\sqrt{\tan(x)}}+\frac{\cos(x)\cot(x)}{3\sqrt{\sin(2x)}}$$

Antiderivative was successfully verified.

```
[In] Int[(Csc[x]^2*Sec[x])/(Sqrt[Sin[2*x]]*(-2 + Tan[x])),x]
```

```
[Out] Cos[x]/(2*Sqrt[Sin[2*x]]) + (Cos[x]*Cot[x])/(3*Sqrt[Sin[2*x]]) - (5*ArcTanh
[Sqrt[Tan[x]]/Sqrt[2]]*Sin[x])/(2*Sqrt[2]*Sqrt[Sin[2*x]]*Sqrt[Tan[x]])
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 898

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> With[{q = Denominator[m]}, Dist[q/e, Subst[Int[x^(q*
(m + 1) - 1)*((e*f - d*g)/e + (g*x^q)/e)^n*((c*d^2 + a*e^2)/e^2 - (2*c*d*x^
q)/e^2 + (c*x^(2*q))/e^2)^p, x], x, (d + e*x)^(1/q)], x]] /; FreeQ[{a, c, d
, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a*e^2, 0] && IntegersQ[n
, p] && FractionQ[m]
```

Rule 1262

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*(a + c*x^4)^p,
```

```
x], x] /; FreeQ[{a, c, d, e, f, m, q}, x] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rule 4390

```
Int[(u_)*((c_.)*sin[v_])^(m_), x_Symbol] :> With[{w = FunctionOfTrig[(u*Sin
[v/2]^(2*m))/(c*Tan[v/2])^m, x]}, Dist[((c*Sin[v])^m*(c*Tan[v/2])^m)/Sin[v/
2]^(2*m), Int[(u*Sin[v/2]^(2*m))/(c*Tan[v/2])^m, x], x] /;  !FalseQ[w] && F
unctionOfQ[NonfreeFactors[Tan[w], x], (u*Sin[v/2]^(2*m))/(c*Tan[v/2])^m, x]
] /; FreeQ[c, x] && LinearQ[v, x] && IntegerQ[m + 1/2] &&  !SumQ[u] && Inve
rseFunctionFreeQ[u, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{\csc^2(x)\sec(x)}{\sqrt{\sin(2x)}\,(-2+\tan(x))}\,dx &= \frac{\sin(x)\int \frac{\csc^3(x)\sec(x)\sqrt{\tan(x)}}{-2+\tan(x)}\,dx}{\sqrt{\sin(2x)}\sqrt{\tan(x)}}\\
&= \frac{\sin(x)\,\text{Subst}\left(\int \frac{1+x^2}{(-2+x)x^{5/2}}\,dx,x,\tan(x)\right)}{\sqrt{\sin(2x)}\sqrt{\tan(x)}}\\
&= \frac{(2\sin(x))\,\text{Subst}\left(\int \frac{1+x^4}{x^4(-2+x^2)}\,dx,x,\sqrt{\tan(x)}\right)}{\sqrt{\sin(2x)}\sqrt{\tan(x)}}\\
&= \frac{(2\sin(x))\,\text{Subst}\left(\int \left(-\frac{1}{2x^4}-\frac{1}{4x^2}+\frac{5}{4(-2+x^2)}\right)dx,x,\sqrt{\tan(x)}\right)}{\sqrt{\sin(2x)}\sqrt{\tan(x)}}\\
&= \frac{\cos(x)}{2\sqrt{\sin(2x)}}+\frac{\cos(x)\cot(x)}{3\sqrt{\sin(2x)}}+\frac{(5\sin(x))\,\text{Subst}\left(\int \frac{1}{-2+x^2}\,dx,x,\sqrt{\tan(x)}\right)}{2\sqrt{\sin(2x)}\sqrt{\tan(x)}}\\
&= \frac{\cos(x)}{2\sqrt{\sin(2x)}}+\frac{\cos(x)\cot(x)}{3\sqrt{\sin(2x)}}-\frac{5\tanh^{-1}\left(\frac{\sqrt{\tan(x)}}{\sqrt{2}}\right)\sin(x)}{2\sqrt{2}\sqrt{\sin(2x)}\sqrt{\tan(x)}}
\end{aligned}$$

Mathematica [C] time = 5.95, size = 119, normalized size = 1.72

$$\frac{1}{4}\sqrt{\sin(2x)}\left(\left(\frac{2\cot(x)}{3}+1\right)\csc(x)+5\sqrt{\frac{\cos(x)}{2\cos(x)-2}}\sqrt{\tan\left(\frac{x}{2}\right)}\sec(x)\left(F\left(\sin^{-1}\left(\frac{1}{\sqrt{\tan\left(\frac{x}{2}\right)}}\right)\middle|-1\right)-\Pi\left(-\frac{2}{-1+\right.\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(Csc[x]^2*Sec[x])/(Sqrt[Sin[2*x]]*(-2 + Tan[x])),x]
```

```
[Out] (Sqrt[Sin[2*x]]*((1 + (2*Cot[x])/3)*Csc[x] + 5*Sqrt[Cos[x]/(-2 + 2*Cos[x])]
*(EllipticF[ArcSin[1/Sqrt[Tan[x/2]]], -1] - EllipticPi[-2/(-1 + Sqrt[5]), A
rcSin[1/Sqrt[Tan[x/2]]], -1] - EllipticPi[(-1 + Sqrt[5])/2, ArcSin[1/Sqrt[T
an[x/2]]], -1])*Sec[x]*Sqrt[Tan[x/2]]))/4
```

fricas [B] time = 2.70, size = 120, normalized size = 1.74

$$-\frac{4\sqrt{2}\sqrt{\cos(x)\sin(x)}\,(2\cos(x)+3\sin(x))-4\cos(x)^2-15\left(\cos(x)^2-1\right)\log\left(-\frac{1}{2}\sqrt{2}\sqrt{\cos(x)\sin(x)}\,(4\cos(x)+\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)^2*sec(x)/sin(2*x)^(1/2)/(-2+tan(x)),x, algorithm="fricas")
```

```
[Out] -1/48*(4*sqrt(2)*sqrt(cos(x)*sin(x))*(2*cos(x) + 3*sin(x)) - 4*cos(x)^2 - 1
5*(cos(x)^2 - 1)*log(-1/2*sqrt(2)*sqrt(cos(x)*sin(x))*(4*cos(x) + 3*sin(x))
 + 1/2*cos(x)^2 + 7/2*cos(x)*sin(x) + 1/2) + 15*(cos(x)^2 - 1)*log(1/2*cos(
x)^2 + 1/2*sqrt(2)*sqrt(cos(x)*sin(x))*sin(x) - 1/2*cos(x)*sin(x) + 1/2) +
4)/(cos(x)^2 - 1)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc(x)^2\sec(x)}{(\tan(x)-2)\sqrt{\sin(2x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)^2*sec(x)/sin(2*x)^(1/2)/(-2+tan(x)),x, algorithm="giac")
```

```
[Out] integrate(csc(x)^2*sec(x)/((tan(x) - 2)*sqrt(sin(2*x))), x)
```

maple [C] time = 0.31, size = 397, normalized size = 5.75

$$\frac{\sqrt{-\frac{\tan\left(\frac{x}{2}\right)}{\tan^2\left(\frac{x}{2}\right)-1}}\left(140\sqrt{1+\tan\left(\frac{x}{2}\right)}\sqrt{-\tan\left(\frac{x}{2}\right)}\sqrt{-2\tan\left(\frac{x}{2}\right)+2}\ \mathrm{EllipticF}\left(\sqrt{1+\tan\left(\frac{x}{2}\right)},\frac{\sqrt{2}}{2}\right)\sqrt{\left(\tan^2\left(\frac{x}{2}\right)-1\right)\tan\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(x)^2*sec(x)/sin(2*x)^(1/2)/(-2+tan(x)),x)
```

```
[Out] 1/480*(-tan(1/2*x)/(tan(1/2*x)^2-1))^(1/2)/tan(1/2*x)^2*(140*(1+tan(1/2*x))
^(1/2)*(-tan(1/2*x))^(1/2)*(-2*tan(1/2*x)+2)^(1/2)*EllipticF((1+tan(1/2*x))
```

```
^(1/2),1/2*2^(1/2))*((tan(1/2*x)^2-1)*tan(1/2*x))^(1/2)*tan(1/2*x)-240*(1+t
an(1/2*x))^(1/2)*(-tan(1/2*x))^(1/2)*(-2*tan(1/2*x)+2)^(1/2)*EllipticE((1+t
an(1/2*x))^(1/2),1/2*2^(1/2))*((tan(1/2*x)^2-1)*tan(1/2*x))^(1/2)*tan(1/2*x
)-(tan(1/2*x)^3-tan(1/2*x))^(1/2)*((tan(1/2*x)^2-1)*tan(1/2*x))^(1/2)*2^(1/
2)*sum((14*_alpha^3+3*_alpha^2+14*_alpha-11)*(_alpha^3+2*_alpha-3)*(1+tan(1
/2*x))^(1/2)*(1-tan(1/2*x))^(1/2)*(-tan(1/2*x))^(1/2)/((tan(1/2*x)^2-1)*tan
(1/2*x))^(1/2)*EllipticPi((1+tan(1/2*x))^(1/2),-1/4*_alpha^3-1/2*_alpha+3/4
,1/2*2^(1/2)),_alpha=RootOf(_Z^4+_Z^3+2*_Z^2-_Z+1))*tan(1/2*x)-40*((tan(1/2
*x)^2-1)*tan(1/2*x))^(1/2)*tan(1/2*x)^4-120*tan(1/2*x)^3*(tan(1/2*x)^3-tan(
1/2*x))^(1/2)+120*(tan(1/2*x)^3-tan(1/2*x))^(1/2)*tan(1/2*x)+40*((tan(1/2*x
)^2-1)*tan(1/2*x))^(1/2))/(tan(1/2*x)^3-tan(1/2*x))^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc(x)^2 \sec(x)}{(\tan(x)-2)\sqrt{\sin(2x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)^2*sec(x)/sin(2*x)^(1/2)/(-2+tan(x)),x, algorithm="maxima")
[Out] integrate(csc(x)^2*sec(x)/((tan(x) - 2)*sqrt(sin(2*x))), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sqrt{\sin(2x)}\,\cos(x)\sin(x)^2\,(\tan(x)-2)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(2*x)^(1/2)*cos(x)*sin(x)^2*(tan(x) - 2)),x)
[Out] int(1/(sin(2*x)^(1/2)*cos(x)*sin(x)^2*(tan(x) - 2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc^2(x)\sec(x)}{(\tan(x)-2)\sqrt{\sin(2x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)**2*sec(x)/sin(2*x)**(1/2)/(-2+tan(x)),x)
[Out] Integral(csc(x)**2*sec(x)/((tan(x) - 2)*sqrt(sin(2*x))), x)
```

3.635 $\int \frac{\cos^2(x)\sin(x)}{\left(\sin^2(x)-\sin(2x)\right)\sin^{\frac{5}{2}}(2x)}\,dx$

Optimal. Leaf size=79

$$\frac{\sin(x)\cos^4(x)}{3\sin^{\frac{5}{2}}(2x)}+\frac{\sin^2(x)\cos^3(x)}{2\sin^{\frac{5}{2}}(2x)}-\frac{5\sin^5(x)\tanh^{-1}\left(\frac{\sqrt{\tan(x)}}{\sqrt{2}}\right)}{2\sqrt{2}\,\sin^{\frac{5}{2}}(2x)\tan^{\frac{5}{2}}(x)}$$

```
[Out] 1/3*cos(x)^4*sin(x)/sin(2*x)^(5/2)+1/2*cos(x)^3*sin(x)^2/sin(2*x)^(5/2)-5/4
*arctanh(1/2*tan(x)^(1/2)*2^(1/2))*sin(x)^5/sin(2*x)^(5/2)*2^(1/2)/tan(x)^(
5/2)
```

Rubi [A] time = 0.57, antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {4390, 898, 1262, 207}

$$\frac{\sin^2(x)\cos^3(x)}{2\sin^{\frac{5}{2}}(2x)}+\frac{\sin(x)\cos^4(x)}{3\sin^{\frac{5}{2}}(2x)}-\frac{5\sin^5(x)\tanh^{-1}\left(\frac{\sqrt{\tan(x)}}{\sqrt{2}}\right)}{2\sqrt{2}\,\sin^{\frac{5}{2}}(2x)\tan^{\frac{5}{2}}(x)}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[x]^2*Sin[x])/((Sin[x]^2 - Sin[2*x])*Sin[2*x]^(5/2)),x]
```

```
[Out] (Cos[x]^4*Sin[x])/(3*Sin[2*x]^(5/2)) + (Cos[x]^3*Sin[x]^2)/(2*Sin[2*x]^(5/2
)) - (5*ArcTanh[Sqrt[Tan[x]]/Sqrt[2]]*Sin[x]^5)/(2*Sqrt[2]*Sin[2*x]^(5/2)*T
an[x]^(5/2))
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 898

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> With[{q = Denominator[m]}, Dist[q/e, Subst[Int[x^(q*
(m + 1) - 1)*((e*f - d*g)/e + (g*x^q)/e)^n*((c*d^2 + a*e^2)/e^2 - (2*c*d*x^
q)/e^2 + (c*x^(2*q))/e^2)^p, x], x, (d + e*x)^(1/q)], x]] /; FreeQ[{a, c, d
, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a*e^2, 0] && IntegersQ[n
, p] && FractionQ[m]
```

Rule 1262

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*(a + c*x^4)^p,
x], x] /; FreeQ[{a, c, d, e, f, m, q}, x] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rule 4390

```
Int[(u_)*((c_.)*sin[v_])^(m_), x_Symbol] :> With[{w = FunctionOfTrig[(u*Sin
[v/2]^(2*m))/(c*Tan[v/2])^m, x]}, Dist[((c*Sin[v])^m*(c*Tan[v/2])^m)/Sin[v/
2]^(2*m), Int[(u*Sin[v/2]^(2*m))/(c*Tan[v/2])^m, x], x] /;  !FalseQ[w] && F
unctionOfQ[NonfreeFactors[Tan[w], x], (u*Sin[v/2]^(2*m))/(c*Tan[v/2])^m, x]
] /; FreeQ[c, x] && LinearQ[v, x] && IntegerQ[m + 1/2] &&  !SumQ[u] && Inve
rseFunctionFreeQ[u, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{\cos^2(x)\sin(x)}{\left(\sin^2(x)-\sin(2x)\right)\sin^{\frac{5}{2}}(2x)}\,dx &= \frac{\sin^5(x)\int \frac{\csc^2(x)\sqrt{\tan(x)}}{\sin^2(x)-\sin(2x)}\,dx}{\sin^{\frac{5}{2}}(2x)\tan^{\frac{5}{2}}(x)}\\
&= \frac{\sin^5(x)\,\mathrm{Subst}\left(\int \frac{-1-x^2}{(2-x)x^{5/2}}\,dx, x, \tan(x)\right)}{\sin^{\frac{5}{2}}(2x)\tan^{\frac{5}{2}}(x)}\\
&= \frac{\left(2\sin^5(x)\right)\mathrm{Subst}\left(\int \frac{-1-x^4}{x^4\left(2-x^2\right)}\,dx, x, \sqrt{\tan(x)}\right)}{\sin^{\frac{5}{2}}(2x)\tan^{\frac{5}{2}}(x)}\\
&= \frac{\left(2\sin^5(x)\right)\mathrm{Subst}\left(\int \left(-\frac{1}{2x^4}-\frac{1}{4x^2}+\frac{5}{4\left(-2+x^2\right)}\right)dx, x, \sqrt{\tan(x)}\right)}{\sin^{\frac{5}{2}}(2x)\tan^{\frac{5}{2}}(x)}\\
&= \frac{\cos^4(x)\sin(x)}{3\sin^{\frac{5}{2}}(2x)}+\frac{\cos^3(x)\sin^2(x)}{2\sin^{\frac{5}{2}}(2x)}+\frac{\left(5\sin^5(x)\right)\mathrm{Subst}\left(\int \frac{1}{-2+x^2}\,dx, x, \sqrt{\tan(x)}\right)}{2\sin^{\frac{5}{2}}(2x)\tan^{\frac{5}{2}}(x)}\\
&= \frac{\cos^4(x)\sin(x)}{3\sin^{\frac{5}{2}}(2x)}+\frac{\cos^3(x)\sin^2(x)}{2\sin^{\frac{5}{2}}(2x)}-\frac{5\tanh^{-1}\left(\frac{\sqrt{\tan(x)}}{\sqrt{2}}\right)\sin^5(x)}{2\sqrt{2}\,\sin^{\frac{5}{2}}(2x)\tan^{\frac{5}{2}}(x)}
\end{aligned}$$

Mathematica [C] time = 4.96, size = 183, normalized size = 2.32

$$\frac{1}{96}\sqrt{\sin(2x)}\sec(x)\left(2\cot^2(x)+6\cot(x)+2\csc^2(x)+15\sqrt{2}\sqrt{\frac{\cos(x)}{\cos(x)-1}}\sqrt{\tan\left(\frac{x}{2}\right)}F\left(\sin^{-1}\left(\frac{1}{\sqrt{\tan\left(\frac{x}{2}\right)}}\right)\middle|-1\right)\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[x]^2*Sin[x])/((Sin[x]^2 - Sin[2*x])*Sin[2*x]^(5/2)),x]
```

```
[Out] (Sec[x]*Sqrt[Sin[2*x]]*(-2 + 6*Cot[x] + 2*Cot[x]^2 + 2*Csc[x]^2 + 15*Sqrt[2
]*Sqrt[Cos[x]/(-1 + Cos[x])]*EllipticF[ArcSin[1/Sqrt[Tan[x/2]]], -1]*Sqrt[T
an[x/2]] - 15*Sqrt[2]*Sqrt[Cos[x]/(-1 + Cos[x])]*EllipticPi[-2/(-1 + Sqrt[5
]), ArcSin[1/Sqrt[Tan[x/2]]], -1]*Sqrt[Tan[x/2]] - 15*Sqrt[2]*Sqrt[Cos[x]/(
-1 + Cos[x])]*EllipticPi[(-1 + Sqrt[5])/2, ArcSin[1/Sqrt[Tan[x/2]]], -1]*Sq
rt[Tan[x/2]]))/96
```

fricas [A] time = 1.04, size = 120, normalized size = 1.52

$$-\frac{4\sqrt{2}\sqrt{\cos(x)\sin(x)}\left(2\cos(x)+3\sin(x)\right)-4\cos(x)^2-15\left(\cos(x)^2-1\right)\log\left(-\frac{1}{2}\sqrt{2}\sqrt{\cos(x)\sin(x)}\left(4\cos(x)+\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^2*sin(x)/(sin(x)^2-sin(2*x))/sin(2*x)^(5/2),x, algorithm="
fricas")
```

```
[Out] -1/192*(4*sqrt(2)*sqrt(cos(x)*sin(x))*(2*cos(x) + 3*sin(x)) - 4*cos(x)^2 -
15*(cos(x)^2 - 1)*log(-1/2*sqrt(2)*sqrt(cos(x)*sin(x))*(4*cos(x) + 3*sin(x)
) + 1/2*cos(x)^2 + 7/2*cos(x)*sin(x) + 1/2) + 15*(cos(x)^2 - 1)*log(1/2*cos
(x)^2 + 1/2*sqrt(2)*sqrt(cos(x)*sin(x))*sin(x) - 1/2*cos(x)*sin(x) + 1/2) +
 4)/(cos(x)^2 - 1)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos(x)^2\sin(x)}{\left(\sin(x)^2-\sin(2x)\right)\sin(2x)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^2*sin(x)/(sin(x)^2-sin(2*x))/sin(2*x)^(5/2),x, algorithm="
giac")
```

```
[Out] integrate(cos(x)^2*sin(x)/((sin(x)^2 - sin(2*x))*sin(2*x)^(5/2)), x)
```

maple [C] time = 0.28, size = 397, normalized size = 5.03

$$\frac{\sqrt{-\frac{\tan\left(\frac{x}{2}\right)}{\tan^2\left(\frac{x}{2}\right)-1}}\left(140\sqrt{1+\tan\left(\frac{x}{2}\right)}\,\sqrt{-\tan\left(\frac{x}{2}\right)}\,\sqrt{-2\tan\left(\frac{x}{2}\right)+2}\,\,\mathrm{EllipticF}\left(\sqrt{1+\tan\left(\frac{x}{2}\right)},\frac{\sqrt{2}}{2}\right)\sqrt{\left(\tan^2\left(\frac{x}{2}\right)-1\right)\mathrm{ta}}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)^2*sin(x)/(sin(x)^2-sin(2*x))/sin(2*x)^(5/2),x)
```

```
[Out] 1/1920*(-tan(1/2*x)/(tan(1/2*x)^2-1))^(1/2)/tan(1/2*x)^2*(140*(1+tan(1/2*x)
)^(1/2)*(-tan(1/2*x))^(1/2)*(-2*tan(1/2*x)+2)^(1/2)*EllipticF((1+tan(1/2*x)
)^(1/2),1/2*2^(1/2))*((tan(1/2*x)^2-1)*tan(1/2*x))^(1/2)*tan(1/2*x)-240*(1+
tan(1/2*x))^(1/2)*(-tan(1/2*x))^(1/2)*(-2*tan(1/2*x)+2)^(1/2)*EllipticE((1+
tan(1/2*x))^(1/2),1/2*2^(1/2))*((tan(1/2*x)^2-1)*tan(1/2*x))^(1/2)*tan(1/2*
x)-(tan(1/2*x)^3-tan(1/2*x))^(1/2)*((tan(1/2*x)^2-1)*tan(1/2*x))^(1/2)*2^(1
/2)*sum((14*_alpha^3+3*_alpha^2+14*_alpha-11)*(_alpha^3+2*_alpha-3)*(1+tan(
1/2*x))^(1/2)*(1-tan(1/2*x))^(1/2)*(-tan(1/2*x))^(1/2)/((tan(1/2*x)^2-1)*ta
n(1/2*x))^(1/2)*EllipticPi((1+tan(1/2*x))^(1/2),-1/4*_alpha^3-1/2*_alpha+3/
4,1/2*2^(1/2)),_alpha=RootOf(_Z^4+_Z^3+2*_Z^2-_Z+1))*tan(1/2*x)-40*((tan(1/
2*x)^2-1)*tan(1/2*x))^(1/2)*tan(1/2*x)^4-120*tan(1/2*x)^3*(tan(1/2*x)^3-tan
(1/2*x))^(1/2)+120*(tan(1/2*x)^3-tan(1/2*x))^(1/2)*tan(1/2*x)+40*((tan(1/2*
x)^2-1)*tan(1/2*x))^(1/2))/(tan(1/2*x)^3-tan(1/2*x))^(1/2)
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^2*sin(x)/(sin(x)^2-sin(2*x))/sin(2*x)^(5/2),x, algorithm="
maxima")
```

```
[Out] Timed out
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$-\int \frac{\cos(x)^2\,\sin(x)}{\sin(2\,x)^{5/2}\,\left(\sin(2\,x)-\sin(x)^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(cos(x)^2*sin(x))/(sin(2*x)^(5/2)*(sin(2*x) - sin(x)^2)),x)
```

```
[Out] -int((cos(x)^2*sin(x))/(sin(2*x)^(5/2)*(sin(2*x) - sin(x)^2)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)**2*sin(x)/(sin(x)**2-sin(2*x))/sin(2*x)**(5/2),x)
[Out] Timed out
```

3.636 $\int \frac{\cos^3(x)\cos(2x)}{(\sin^2(x)-\sin(2x))\sin^{\frac{5}{2}}(2x)}\,dx$

Optimal. Leaf size=95

$$\frac{\cos^5(x)}{5\sin^{\frac{5}{2}}(2x)}+\frac{\sin(x)\cos^4(x)}{6\sin^{\frac{5}{2}}(2x)}-\frac{3\sin^2(x)\cos^3(x)}{4\sin^{\frac{5}{2}}(2x)}+\frac{3\sin^5(x)\tanh^{-1}\left(\frac{\sqrt{\tan(x)}}{\sqrt{2}}\right)}{4\sqrt{2}\,\sin^{\frac{5}{2}}(2x)\tan^{\frac{5}{2}}(x)}$$

```
[Out] 1/5*cos(x)^5/sin(2*x)^(5/2)+1/6*cos(x)^4*sin(x)/sin(2*x)^(5/2)-3/4*cos(x)^3
*sin(x)^2/sin(2*x)^(5/2)+3/8*arctanh(1/2*tan(x)^(1/2)*2^(1/2))*sin(x)^5/sin
(2*x)^(5/2)*2^(1/2)/tan(x)^(5/2)
```

Rubi [A] time = 0.58, antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {4390, 898, 1262, 207}

$$\frac{\cos^5(x)}{5\sin^{\frac{5}{2}}(2x)}+\frac{\sin(x)\cos^4(x)}{6\sin^{\frac{5}{2}}(2x)}-\frac{3\sin^2(x)\cos^3(x)}{4\sin^{\frac{5}{2}}(2x)}+\frac{3\sin^5(x)\tanh^{-1}\left(\frac{\sqrt{\tan(x)}}{\sqrt{2}}\right)}{4\sqrt{2}\,\sin^{\frac{5}{2}}(2x)\tan^{\frac{5}{2}}(x)}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[x]^3*Cos[2*x])/((Sin[x]^2 - Sin[2*x])*Sin[2*x]^(5/2)),x]
```

```
[Out] Cos[x]^5/(5*Sin[2*x]^(5/2)) + (Cos[x]^4*Sin[x])/(6*Sin[2*x]^(5/2)) - (3*Cos
[x]^3*Sin[x]^2)/(4*Sin[2*x]^(5/2)) + (3*ArcTanh[Sqrt[Tan[x]]/Sqrt[2]]*Sin[x
]^5)/(4*Sqrt[2]*Sin[2*x]^(5/2)*Tan[x]^(5/2))
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 898

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> With[{q = Denominator[m]}, Dist[q/e, Subst[Int[x^(q*
(m + 1) - 1)*((e*f - d*g)/e + (g*x^q)/e)^n*((c*d^2 + a*e^2)/e^2 - (2*c*d*x^
q)/e^2 + (c*x^(2*q))/e^2)^p, x], x, (d + e*x)^(1/q)], x]] /; FreeQ[{a, c, d
, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a*e^2, 0] && IntegersQ[n
, p] && FractionQ[m]
```

Rule 1262

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*(a + c*x^4)^p,
x], x] /; FreeQ[{a, c, d, e, f, m, q}, x] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rule 4390

```
Int[(u_)*((c_.)*sin[v_])^(m_), x_Symbol] :> With[{w = FunctionOfTrig[(u*Sin
[v/2]^(2*m))/(c*Tan[v/2])^m, x]}, Dist[((c*Sin[v])^m*(c*Tan[v/2])^m)/Sin[v/
2]^(2*m), Int[(u*Sin[v/2]^(2*m))/(c*Tan[v/2])^m, x], x] /;  !FalseQ[w] && F
unctionOfQ[NonfreeFactors[Tan[w], x], (u*Sin[v/2]^(2*m))/(c*Tan[v/2])^m, x]
] /; FreeQ[c, x] && LinearQ[v, x] && IntegerQ[m + 1/2] &&  !SumQ[u] && Inve
rseFunctionFreeQ[u, x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\cos^3(x)\cos(2x)}{\left(\sin^2(x)-\sin(2x)\right)\sin^{\frac{5}{2}}(2x)}\,dx &= \frac{\sin^5(x)\int \frac{\cos(2x)\csc^2(x)}{\left(\sin^2(x)-\sin(2x)\right)\sqrt{\tan(x)}}\,dx}{\sin^{\frac{5}{2}}(2x)\tan^{\frac{5}{2}}(x)}\\
&= \frac{\sin^5(x)\,\text{Subst}\left(\int \frac{-1+x^2}{(2-x)x^{7/2}}\,dx,x,\tan(x)\right)}{\sin^{\frac{5}{2}}(2x)\tan^{\frac{5}{2}}(x)}\\
&= \frac{\left(2\sin^5(x)\right)\text{Subst}\left(\int \frac{-1+x^4}{x^6\left(2-x^2\right)}\,dx,x,\sqrt{\tan(x)}\right)}{\sin^{\frac{5}{2}}(2x)\tan^{\frac{5}{2}}(x)}\\
&= \frac{\left(2\sin^5(x)\right)\text{Subst}\left(\int \left(-\frac{1}{2x^6}-\frac{1}{4x^4}+\frac{3}{8x^2}-\frac{3}{8\left(-2+x^2\right)}\right)dx,x,\sqrt{\tan(x)}\right)}{\sin^{\frac{5}{2}}(2x)\tan^{\frac{5}{2}}(x)}\\
&= \frac{\cos^5(x)}{5\sin^{\frac{5}{2}}(2x)}+\frac{\cos^4(x)\sin(x)}{6\sin^{\frac{5}{2}}(2x)}-\frac{3\cos^3(x)\sin^2(x)}{4\sin^{\frac{5}{2}}(2x)}-\frac{\left(3\sin^5(x)\right)\text{Subst}\left(\int \frac{1}{-2+\cdots}\right.}{4\sin^{\frac{5}{2}}(2x)\,\text{ta}\cdots}\\
&= \frac{\cos^5(x)}{5\sin^{\frac{5}{2}}(2x)}+\frac{\cos^4(x)\sin(x)}{6\sin^{\frac{5}{2}}(2x)}-\frac{3\cos^3(x)\sin^2(x)}{4\sin^{\frac{5}{2}}(2x)}+\frac{3\tanh^{-1}\left(\frac{\sqrt{\tan(x)}}{\sqrt{2}}\right)\sin^5(\cdots}{4\sqrt{2}\,\sin^{\frac{5}{2}}(2x)\tan^{\frac{5}{2}}(x)}
\end{aligned}
$$

Mathematica [C] time = 15.72, size = 184, normalized size = 1.94

$$\frac{1}{960}\sqrt{\sin(2x)}\sec(x)\left(20\cot^2(x)-114\cot(x)+24\cot(x)\csc^2(x)-45\sqrt{2}\sqrt{\frac{\cos(x)}{\cos(x)-1}}\sqrt{\tan\left(\frac{x}{2}\right)}F\left(\sin^{-1}\left(\frac{}{\sqrt{\text{ta}}}\right.\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[x]^3*Cos[2*x])/((Sin[x]^2 - Sin[2*x])*Sin[2*x]^(5/2)),x]
```

```
[Out] (Sec[x]*Sqrt[Sin[2*x]]*(-114*Cot[x] + 20*Cot[x]^2 + 24*Cot[x]*Csc[x]^2 - 45
*Sqrt[2]*Sqrt[Cos[x]/(-1 + Cos[x])]*EllipticF[ArcSin[1/Sqrt[Tan[x/2]]], -1]
*Sqrt[Tan[x/2]] + 45*Sqrt[2]*Sqrt[Cos[x]/(-1 + Cos[x])]*EllipticPi[-2/(-1 +
 Sqrt[5]), ArcSin[1/Sqrt[Tan[x/2]]], -1]*Sqrt[Tan[x/2]] + 45*Sqrt[2]*Sqrt[C
os[x]/(-1 + Cos[x])]*EllipticPi[(-1 + Sqrt[5])/2, ArcSin[1/Sqrt[Tan[x/2]]],
 -1]*Sqrt[Tan[x/2]]))/960
```

fricas [A] time = 0.47, size = 136, normalized size = 1.43

$$-\frac{45\left(\cos(x)^2-1\right)\log\left(-\frac{1}{2}\sqrt{2}\sqrt{\cos(x)\sin(x)}\left(4\cos(x)+3\sin(x)\right)+\frac{1}{2}\cos(x)^2+\frac{7}{2}\cos(x)\sin(x)+\frac{1}{2}\right)\sin(x)-}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^3*cos(2*x)/(sin(x)^2-sin(2*x))/sin(2*x)^(5/2),x, algorithm
="fricas")
```

```
[Out] -1/1920*(45*(cos(x)^2 - 1)*log(-1/2*sqrt(2)*sqrt(cos(x)*sin(x))*(4*cos(x) +
 3*sin(x)) + 1/2*cos(x)^2 + 7/2*cos(x)*sin(x) + 1/2)*sin(x) - 45*(cos(x)^2
- 1)*log(1/2*cos(x)^2 + 1/2*sqrt(2)*sqrt(cos(x)*sin(x))*sin(x) - 1/2*cos(x)
*sin(x) + 1/2)*sin(x) + 4*sqrt(2)*(57*cos(x)^2 + 10*cos(x)*sin(x) - 45)*sqr
t(cos(x)*sin(x)) + 268*(cos(x)^2 - 1)*sin(x))/((cos(x)^2 - 1)*sin(x))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos(2x)\cos(x)^3}{\left(\sin(x)^2-\sin(2x)\right)\sin(2x)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^3*cos(2*x)/(sin(x)^2-sin(2*x))/sin(2*x)^(5/2),x, algorithm
="giac")
```

```
[Out] integrate(cos(2*x)*cos(x)^3/((sin(x)^2 - sin(2*x))*sin(2*x)^(5/2)), x)
```

maple [C] time = 0.38, size = 761, normalized size = 8.01

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)^3*cos(2*x)/(sin(x)^2-sin(2*x))/sin(2*x)^(5/2),x)
```

```
[Out] 1/3840*(-tan(1/2*x)/(tan(1/2*x)^2-1))^(1/2)/tan(1/2*x)^3*(1772*((tan(1/2*x)
-1)*(1+tan(1/2*x))*tan(1/2*x))^(1/2)*(-2*tan(1/2*x)+2)^(1/2)*EllipticF((1+t
an(1/2*x))^(1/2),1/2*2^(1/2))*((tan(1/2*x)^2-1)*tan(1/2*x))^(1/2)*(1+tan(1/
2*x))^(1/2)*(-tan(1/2*x))^(1/2)*tan(1/2*x)^2-4464*((tan(1/2*x)-1)*(1+tan(1/
2*x))*tan(1/2*x))^(1/2)*(-2*tan(1/2*x)+2)^(1/2)*EllipticE((1+tan(1/2*x))^(1
/2),1/2*2^(1/2))*((tan(1/2*x)^2-1)*tan(1/2*x))^(1/2)*(1+tan(1/2*x))^(1/2)*(
-tan(1/2*x))^(1/2)*tan(1/2*x)^2+24*((tan(1/2*x)-1)*(1+tan(1/2*x))*tan(1/2*x
))^(1/2)*((tan(1/2*x)^2-1)*tan(1/2*x))^(1/2)*tan(1/2*x)^6+3*((tan(1/2*x)-1)
*(1+tan(1/2*x))*tan(1/2*x))^(1/2)*(tan(1/2*x)^3-tan(1/2*x))^(1/2)*sum((6*_a
lpha^3+7*_alpha^2+6*_alpha+1)*(_alpha^3+2*_alpha-3)*(1+tan(1/2*x))^(1/2)*(1
-tan(1/2*x))^(1/2)*(-tan(1/2*x))^(1/2)*EllipticPi((1+tan(1/2*x))^(1/2),-1/4
*_alpha^3-1/2*_alpha+3/4,1/2*2^(1/2))/((tan(1/2*x)^2-1)*tan(1/2*x))^(1/2),_
alpha=RootOf(_Z^4+_Z^3+2*_Z^2-_Z+1))*((tan(1/2*x)^2-1)*tan(1/2*x))^(1/2)*2^
(1/2)*tan(1/2*x)^2-40*((tan(1/2*x)-1)*(1+tan(1/2*x))*tan(1/2*x))^(1/2)*((ta
n(1/2*x)^2-1)*tan(1/2*x))^(1/2)*tan(1/2*x)^5-1272*tan(1/2*x)^4*((tan(1/2*x)
-1)*(1+tan(1/2*x))*tan(1/2*x))^(1/2)*(tan(1/2*x)^3-tan(1/2*x))^(1/2)-24*tan
(1/2*x)^4*((tan(1/2*x)^2-1)*tan(1/2*x))^(1/2)*((tan(1/2*x)-1)*(1+tan(1/2*x)
)*tan(1/2*x))^(1/2)-1920*tan(1/2*x)^4*((tan(1/2*x)^2-1)*tan(1/2*x))^(1/2)*(
tan(1/2*x)^3-tan(1/2*x))^(1/2)+1272*((tan(1/2*x)-1)*(1+tan(1/2*x))*tan(1/2*
x))^(1/2)*(tan(1/2*x)^3-tan(1/2*x))^(1/2)*tan(1/2*x)^2-24*((tan(1/2*x)-1)*(
1+tan(1/2*x))*tan(1/2*x))^(1/2)*((tan(1/2*x)^2-1)*tan(1/2*x))^(1/2)*tan(1/2
*x)^2+40*((tan(1/2*x)^2-1)*tan(1/2*x))^(1/2)*((tan(1/2*x)-1)*(1+tan(1/2*x))
*tan(1/2*x))^(1/2)*tan(1/2*x)+24*((tan(1/2*x)^2-1)*tan(1/2*x))^(1/2)*((tan(
1/2*x)-1)*(1+tan(1/2*x))*tan(1/2*x))^(1/2))/((tan(1/2*x)-1)*(1+tan(1/2*x))*
tan(1/2*x))^(1/2)/(tan(1/2*x)^3-tan(1/2*x))^(1/2)
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^3*cos(2*x)/(sin(x)^2-sin(2*x))/sin(2*x)^(5/2),x, algorithm
="maxima")
```

```
[Out] Timed out
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int -\frac{\cos(2x)\cos(x)^3}{\sin(2x)^{5/2}\left(\sin(2x)-\sin(x)^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(cos(2*x)*cos(x)^3)/(sin(2*x)^(5/2)*(sin(2*x) - sin(x)^2)),x)
[Out] int(-(cos(2*x)*cos(x)^3)/(sin(2*x)^(5/2)*(sin(2*x) - sin(x)^2)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)**3*cos(2*x)/(sin(x)**2-sin(2*x))/sin(2*x)**(5/2),x)
[Out] Timed out
```

3.637 $\int (b\sec(c+dx)+a\sin(c+dx))^n(a\cos(c+dx)+b\sec(c+dx)\tan(c+dx))\,dx$

Optimal. Leaf size=30

$$\frac{(a\sin(c+dx)+b\sec(c+dx))^{n+1}}{d(n+1)}$$

```
[Out] (b*sec(d*x+c)+a*sin(d*x+c))^(1+n)/d/(1+n)
```

Rubi [A] time = 0.06, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.023, Rules used = {4385}

$$\frac{(a\sin(c+dx)+b\sec(c+dx))^{n+1}}{d(n+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Sec[c + d*x] + a*Sin[c + d*x])^n*(a*Cos[c + d*x] + b*Sec[c + d*x]*Ta
n[c + d*x]),x]
```

```
[Out] (b*Sec[c + d*x] + a*Sin[c + d*x])^(1 + n)/(d*(1 + n))
```

Rule 4385

```
Int[(u_)*(y_)^(m_.), x_Symbol] :> With[{q = DerivativeDivides[ActivateTrig[
y], ActivateTrig[u], x]}, Simp[(q*ActivateTrig[y^(m + 1)])/(m + 1), x] /;
!FalseQ[q]] /; FreeQ[m, x] && NeQ[m, -1] &&  !InertTrigFreeQ[u]
```

Rubi steps

$$\int (b\sec(c+dx)+a\sin(c+dx))^n(a\cos(c+dx)+b\sec(c+dx)\tan(c+dx))\,dx=\frac{(b\sec(c+dx)+a\sin(c+dx))^{1}}{d(1+n)}$$

Mathematica [A] time = 1.22, size = 51, normalized size = 1.70

$$\frac{\sec(c+dx)(a\sin(2(c+dx))+2b)(a\sin(c+dx)+b\sec(c+dx))^n}{2d(n+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Sec[c + d*x] + a*Sin[c + d*x])^n*(a*Cos[c + d*x] + b*Sec[c + d
*x]*Tan[c + d*x]),x]
```

```
[Out] (Sec[c + d*x]*(b*Sec[c + d*x] + a*Sin[c + d*x])^n*(2*b + a*Sin[2*(c + d*x)]
))/(2*d*(1 + n))
```

fricas [A] time = 3.05, size = 59, normalized size = 1.97

$$\frac{(a\cos(dx+c)\sin(dx+c)+b)\left(\frac{a\cos(dx+c)\sin(dx+c)+b}{\cos(dx+c)}\right)^n}{(dn+d)\cos(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(d*x+c)+a*sin(d*x+c))^n*(a*cos(d*x+c)+b*sec(d*x+c)*tan(d*x+
c)),x, algorithm="fricas")
```

```
[Out] (a*cos(d*x + c)*sin(d*x + c) + b)*((a*cos(d*x + c)*sin(d*x + c) + b)/cos(d*
x + c))^n/((d*n + d)*cos(d*x + c))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b\sec(dx+c)\tan(dx+c)+a\cos(dx+c))(b\sec(dx+c)+a\sin(dx+c))^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(d*x+c)+a*sin(d*x+c))^n*(a*cos(d*x+c)+b*sec(d*x+c)*tan(d*x+
c)),x, algorithm="giac")
```

```
[Out] integrate((b*sec(d*x + c)*tan(d*x + c) + a*cos(d*x + c))*(b*sec(d*x + c) +
a*sin(d*x + c))^n, x)
```

maple [A] time = 0.46, size = 31, normalized size = 1.03

$$\frac{(b\sec(dx+c)+a\sin(dx+c))^{n+1}}{d(n+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*sec(d*x+c)+a*sin(d*x+c))^n*(a*cos(d*x+c)+b*sec(d*x+c)*tan(d*x+c)),x)
```

```
[Out] (b*sec(d*x+c)+a*sin(d*x+c))^(n+1)/d/(n+1)
```

maxima [A] time = 0.31, size = 30, normalized size = 1.00

$$\frac{(b\sec(dx+c)+a\sin(dx+c))^{n+1}}{d(n+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(d*x+c)+a*sin(d*x+c))^n*(a*cos(d*x+c)+b*sec(d*x+c)*tan(d*x+
c)),x, algorithm="maxima")
```

```
[Out] (b*sec(d*x + c) + a*sin(d*x + c))^(n + 1)/(d*(n + 1))
```

mupad [B] time = 5.54, size = 63, normalized size = 2.10

$$\begin{cases} \frac{\ln\left(a\,\sin(c+d\,x)+\frac{b}{\cos(c+d\,x)}\right)}{d} & \text{if } n=-1 \\ \frac{\left(a\,\sin(c+d\,x)+\frac{b}{\cos(c+d\,x)}\right)^{n+1}}{d\,(n+1)} & \text{if } n\neq -1 \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*sin(c + d*x) + b/cos(c + d*x))^n*(a*cos(c + d*x) + (b*tan(c + d*x))/
cos(c + d*x)),x)
```

```
[Out] piecewise(n == -1, log(a*sin(c + d*x) + b/cos(c + d*x))/d, n ~= -1, (a*sin(
c + d*x) + b/cos(c + d*x))^(n + 1)/(d*(n + 1)))
```

sympy [A] time = 74.07, size = 138, normalized size = 4.60

$$\begin{cases} \frac{x(a\cos(c)+b\tan(c)\sec(c))}{a\sin(c)+b\sec(c)} & \text{for } d=0 \wedge n=-1 \\ x\,(a\sin(c)+b\sec(c))^n\,(a\cos(c)+b\tan(c)\sec(c)) & \text{for } d=0 \\ \frac{\log\left(\sin(c+dx)+\frac{b\sec(c+dx)}{a}\right)}{d} & \text{for } n=-1 \\ \frac{a(a\sin(c+dx)+b\sec(c+dx))^n\sin(c+dx)}{dn+d}+\frac{b(a\sin(c+dx)+b\sec(c+dx))^n\sec(c+dx)}{dn+d} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(d*x+c)+a*sin(d*x+c))**n*(a*cos(d*x+c)+b*sec(d*x+c)*tan(d*x
+c)),x)
```

```
[Out] Piecewise((x*(a*cos(c) + b*tan(c)*sec(c))/(a*sin(c) + b*sec(c)), Eq(d, 0) &
 Eq(n, -1)), (x*(a*sin(c) + b*sec(c))**n*(a*cos(c) + b*tan(c)*sec(c)), Eq(d
, 0)), (log(sin(c + d*x) + b*sec(c + d*x)/a)/d, Eq(n, -1)), (a*(a*sin(c + d
*x) + b*sec(c + d*x))**n*sin(c + d*x)/(d*n + d) + b*(a*sin(c + d*x) + b*sec
(c + d*x))**n*sec(c + d*x)/(d*n + d), True))
```

3.638 $\int (b\sec(c+dx)+a\sin(c+dx))^3(a\cos(c+dx)+b\sec(c+dx)\tan(c+dx))\,dx$

Optimal. Leaf size=26

$$\frac{(a\sin(c+dx)+b\sec(c+dx))^4}{4d}$$

```
[Out] 1/4*(b*sec(d*x+c)+a*sin(d*x+c))^4/d
```

Rubi [A] time = 0.04, antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.023, Rules used = {4385}

$$\frac{(a\sin(c+dx)+b\sec(c+dx))^4}{4d}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Sec[c + d*x] + a*Sin[c + d*x])^3*(a*Cos[c + d*x] + b*Sec[c + d*x]*Ta
n[c + d*x]),x]
```

```
[Out] (b*Sec[c + d*x] + a*Sin[c + d*x])^4/(4*d)
```

Rule 4385

```
Int[(u_)*(y_)^(m_.), x_Symbol] :> With[{q = DerivativeDivides[ActivateTrig[
y], ActivateTrig[u], x]}, Simp[(q*ActivateTrig[y^(m + 1)])/(m + 1), x] /;
!FalseQ[q]] /; FreeQ[m, x] && NeQ[m, -1] &&  !InertTrigFreeQ[u]
```

Rubi steps

$$\int (b\sec(c+dx)+a\sin(c+dx))^3(a\cos(c+dx)+b\sec(c+dx)\tan(c+dx))\,dx=\frac{(b\sec(c+dx)+a\sin(c+dx)}{4d}$$

Mathematica [B] time = 6.54, size = 938, normalized size = 36.08

$$\frac{a^4\cos(4c)\cos(4dx)(b\sec(c+dx)+a\sin(c+dx))^3(a\cos(c+dx)+b\sec(c+dx)\tan(c+dx))\cos^5(c+dx)}{d(3a\cos(c+dx)+a\cos(3c+3dx)+4b\sin(c+dx))(2b+a\sin(2c+2dx))^3}-\frac{4a^3}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Sec[c + d*x] + a*Sin[c + d*x])^3*(a*Cos[c + d*x] + b*Sec[c + d
*x]*Tan[c + d*x]),x]
```

```
[Out] (8*b^4*Cos[c + d*x]*(b*Sec[c + d*x] + a*Sin[c + d*x])^3*(a*Cos[c + d*x] + b
*Sec[c + d*x]*Tan[c + d*x]))/(d*(3*a*Cos[c + d*x] + a*Cos[3*c + 3*d*x] + 4*
b*Sin[c + d*x])*(2*b + a*Sin[2*c + 2*d*x])^3) + (a^4*Cos[4*c]*Cos[4*d*x]*Co
s[c + d*x]^5*(b*Sec[c + d*x] + a*Sin[c + d*x])^3*(a*Cos[c + d*x] + b*Sec[c
+ d*x]*Tan[c + d*x]))/(d*(3*a*Cos[c + d*x] + a*Cos[3*c + 3*d*x] + 4*b*Sin[c
 + d*x])*(2*b + a*Sin[2*c + 2*d*x])^3) + (16*a*b^2*Cos[c + d*x]^3*Sec[c]*(3
*a*Cos[c] + 2*b*Sin[c])*(b*Sec[c + d*x] + a*Sin[c + d*x])^3*(a*Cos[c + d*x]
 + b*Sec[c + d*x]*Tan[c + d*x]))/(d*(3*a*Cos[c + d*x] + a*Cos[3*c + 3*d*x]
+ 4*b*Sin[c + d*x])*(2*b + a*Sin[2*c + 2*d*x])^3) - (4*a^3*Cos[2*d*x]*Cos[c
 + d*x]^5*(a*Cos[2*c] + 4*b*Sin[2*c])*(b*Sec[c + d*x] + a*Sin[c + d*x])^3*(
a*Cos[c + d*x] + b*Sec[c + d*x]*Tan[c + d*x]))/(d*(3*a*Cos[c + d*x] + a*Cos
[3*c + 3*d*x] + 4*b*Sin[c + d*x])*(2*b + a*Sin[2*c + 2*d*x])^3) + (32*a*b^3
*Cos[c + d*x]^2*Sec[c]*Sin[d*x]*(b*Sec[c + d*x] + a*Sin[c + d*x])^3*(a*Cos[
c + d*x] + b*Sec[c + d*x]*Tan[c + d*x]))/(d*(3*a*Cos[c + d*x] + a*Cos[3*c +
 3*d*x] + 4*b*Sin[c + d*x])*(2*b + a*Sin[2*c + 2*d*x])^3) + (32*a^3*b*Cos[c
 + d*x]^4*Sec[c]*Sin[d*x]*(b*Sec[c + d*x] + a*Sin[c + d*x])^3*(a*Cos[c + d*
x] + b*Sec[c + d*x]*Tan[c + d*x]))/(d*(3*a*Cos[c + d*x] + a*Cos[3*c + 3*d*x
] + 4*b*Sin[c + d*x])*(2*b + a*Sin[2*c + 2*d*x])^3) + (4*a^3*Cos[c + d*x]^5
*(-4*b*Cos[2*c] + a*Sin[2*c])*Sin[2*d*x]*(b*Sec[c + d*x] + a*Sin[c + d*x])^
3*(a*Cos[c + d*x] + b*Sec[c + d*x]*Tan[c + d*x]))/(d*(3*a*Cos[c + d*x] + a*
Cos[3*c + 3*d*x] + 4*b*Sin[c + d*x])*(2*b + a*Sin[2*c + 2*d*x])^3) - (a^4*C
os[c + d*x]^5*Sin[4*c]*Sin[4*d*x]*(b*Sec[c + d*x] + a*Sin[c + d*x])^3*(a*Co
s[c + d*x] + b*Sec[c + d*x]*Tan[c + d*x]))/(d*(3*a*Cos[c + d*x] + a*Cos[3*c
 + 3*d*x] + 4*b*Sin[c + d*x])*(2*b + a*Sin[2*c + 2*d*x])^3)
```

fricas [B] time = 0.69, size = 122, normalized size = 4.69

$$\frac{8\,a^4\cos\left(dx+c\right)^8-16\,a^4\cos\left(dx+c\right)^6+5\,a^4\cos\left(dx+c\right)^4+48\,a^2b^2\cos\left(dx+c\right)^2+8\,b^4-32\,\left(a^3b\cos\left(dx+c\right)^5-\right.}{32\,d\cos\left(dx+c\right)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(d*x+c)+a*sin(d*x+c))^3*(a*cos(d*x+c)+b*sec(d*x+c)*tan(d*x+
c)),x, algorithm="fricas")
```

```
[Out] 1/32*(8*a^4*cos(d*x + c)^8 - 16*a^4*cos(d*x + c)^6 + 5*a^4*cos(d*x + c)^4 +
 48*a^2*b^2*cos(d*x + c)^2 + 8*b^4 - 32*(a^3*b*cos(d*x + c)^5 - a^3*b*cos(d
*x + c)^3 - a*b^3*cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c)^4)
```

giac [B] time = 1.95, size = 142, normalized size = 5.46

$$\frac{b^4\tan\left(dx+c\right)^4+4\,ab^3\tan\left(dx+c\right)^3+6\,a^2b^2\tan\left(dx+c\right)^2+2\,b^4\tan\left(dx+c\right)^2+4\,a^3b\tan\left(dx+c\right)+4\,ab^3\tan\left(d\right.}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(d*x+c)+a*sin(d*x+c))^3*(a*cos(d*x+c)+b*sec(d*x+c)*tan(d*x+
c)),x, algorithm="giac")
```

```
[Out] 1/4*(b^4*tan(d*x + c)^4 + 4*a*b^3*tan(d*x + c)^3 + 6*a^2*b^2*tan(d*x + c)^2
 + 2*b^4*tan(d*x + c)^2 + 4*a^3*b*tan(d*x + c) + 4*a*b^3*tan(d*x + c) - (4*
a^3*b*tan(d*x + c)^3 + 2*a^4*tan(d*x + c)^2 + 4*a^3*b*tan(d*x + c) + a^4)/(
tan(d*x + c)^2 + 1)^2)/d
```

maple [B] time = 0.61, size = 137, normalized size = 5.27

$$\frac{a^4\left(\sin^4(dx+c)\right)}{4d}+\frac{a^3b\left(\sin^5(dx+c)\right)}{d\cos(dx+c)}+\frac{a^3b\cos(dx+c)\left(\sin^3(dx+c)\right)}{d}+\frac{3a^2b^2\left(\tan^2(dx+c)\right)}{2d}+\frac{a\,b^3\left(\sin^3(dx+\right.}{d\cos(dx+c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*sec(d*x+c)+a*sin(d*x+c))^3*(a*cos(d*x+c)+b*sec(d*x+c)*tan(d*x+c)),x)
```

```
[Out] 1/4/d*a^4*sin(d*x+c)^4+1/d*a^3*b*sin(d*x+c)^5/cos(d*x+c)+1/d*a^3*b*cos(d*x+
c)*sin(d*x+c)^3+3/2/d*a^2*b^2*tan(d*x+c)^2+1/d*a*b^3*sin(d*x+c)^3/cos(d*x+c
)^3+1/d*a*b^3*tan(d*x+c)+1/4/d*b^4/cos(d*x+c)^4
```

maxima [A] time = 0.31, size = 24, normalized size = 0.92

$$\frac{(b\sec(dx+c)+a\sin(dx+c))^4}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(d*x+c)+a*sin(d*x+c))^3*(a*cos(d*x+c)+b*sec(d*x+c)*tan(d*x+
c)),x, algorithm="maxima")
```

```
[Out] 1/4*(b*sec(d*x + c) + a*sin(d*x + c))^4/d
```

mupad [B] time = 3.57, size = 185, normalized size = 7.12

$$\frac{a^4\cos(2\,c+2\,d\,x)^4-2\,a^4\cos(2\,c+2\,d\,x)^2+a^4-8\,\sin(2\,c+2\,d\,x)\,a^3\,b\cos(2\,c+2\,d\,x)^2+8\,\sin(2\,c+2\,d\,x)\,a}{d\,\left(16\cos(2\,c+2\,d\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*sin(c + d*x) + b/cos(c + d*x))^3*(a*cos(c + d*x) + (b*tan(c + d*x))/
cos(c + d*x)),x)
```

```
[Out] (a^4*cos(2*c + 2*d*x)^4 - 2*a^4*cos(2*c + 2*d*x)^2 - 4*b^4*cos(2*c + 2*d*x)
^2 + a^4 + 12*b^4 + 24*a^2*b^2 - 8*b^4*cos(2*c + 2*d*x) - 24*a^2*b^2*cos(2*
c + 2*d*x)^2 + 32*a*b^3*sin(2*c + 2*d*x) + 8*a^3*b*sin(2*c + 2*d*x) - 8*a^3
```

```
*b*cos(2*c + 2*d*x)^2*sin(2*c + 2*d*x))/(d*(32*cos(2*c + 2*d*x) + 16*cos(2*
c + 2*d*x)^2 + 16))
```

sympy [A] time = 35.72, size = 129, normalized size = 4.96

$$\begin{cases} \frac{a^4\sin^4(c+dx)}{4d} + \frac{a^3b\sin^3(c+dx)\sec(c+dx)}{d} + \frac{3a^2b^2\sin^2(c+dx)\sec^2(c+dx)}{2d} + \frac{ab^3\sin(c+dx)\sec^3(c+dx)}{d} + \frac{b^4\sec^4(c+dx)}{4d} & \text{for } d \neq 0 \\ x(a\sin(c) + b\sec(c))^3(a\cos(c) + b\tan(c)\sec(c)) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(d*x+c)+a*sin(d*x+c))**3*(a*cos(d*x+c)+b*sec(d*x+c)*tan(d*x
+c)),x)
```

```
[Out] Piecewise((a**4*sin(c + d*x)**4/(4*d) + a**3*b*sin(c + d*x)**3*sec(c + d*x)
/d + 3*a**2*b**2*sin(c + d*x)**2*sec(c + d*x)**2/(2*d) + a*b**3*sin(c + d*x
)*sec(c + d*x)**3/d + b**4*sec(c + d*x)**4/(4*d), Ne(d, 0)), (x*(a*sin(c) +
 b*sec(c))**3*(a*cos(c) + b*tan(c)*sec(c)), True))
```

3.639 $\int (b\sec(c+dx)+a\sin(c+dx))^2(a\cos(c+dx)+b\sec(c+dx)\tan(c+dx))\,dx$

Optimal. Leaf size=26

$$\frac{(a\sin(c+dx)+b\sec(c+dx))^3}{3d}$$

```
[Out] 1/3*(b*sec(d*x+c)+a*sin(d*x+c))^3/d
```

Rubi [A] time = 0.04, antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.023, Rules used = {4385}

$$\frac{(a\sin(c+dx)+b\sec(c+dx))^3}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Sec[c + d*x] + a*Sin[c + d*x])^2*(a*Cos[c + d*x] + b*Sec[c + d*x]*Ta
n[c + d*x]),x]
```

```
[Out] (b*Sec[c + d*x] + a*Sin[c + d*x])^3/(3*d)
```

Rule 4385

```
Int[(u_)*(y_)^(m_.), x_Symbol] :> With[{q = DerivativeDivides[ActivateTrig[
y], ActivateTrig[u], x]}, Simp[(q*ActivateTrig[y^(m + 1)])/(m + 1), x] /;
!FalseQ[q]] /; FreeQ[m, x] && NeQ[m, -1] &&  !InertTrigFreeQ[u]
```

Rubi steps

$$\int (b\sec(c+dx)+a\sin(c+dx))^2(a\cos(c+dx)+b\sec(c+dx)\tan(c+dx))\,dx=\frac{(b\sec(c+dx)+a\sin(c+dx)}{3d}$$

Mathematica [A] time = 1.22, size = 31, normalized size = 1.19

$$\frac{\sec^3(c+dx)(a\sin(2(c+dx))+2b)^3}{24d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Sec[c + d*x] + a*Sin[c + d*x])^2*(a*Cos[c + d*x] + b*Sec[c + d
*x]*Tan[c + d*x]),x]
```

```
[Out] (Sec[c + d*x]^3*(2*b + a*Sin[2*(c + d*x)])^3)/(24*d)
```

fricas [B] time = 0.98, size = 92, normalized size = 3.54

$$-\frac{3\,a^{2}b\cos\left(dx+c\right)^{4}-3\,a^{2}b\cos\left(dx+c\right)^{2}-b^{3}+\left(a^{3}\cos\left(dx+c\right)^{5}-a^{3}\cos\left(dx+c\right)^{3}-3\,ab^{2}\cos\left(dx+c\right)\right)\sin\left(dx+c\right)}{3\,d\cos\left(dx+c\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(d*x+c)+a*sin(d*x+c))^2*(a*cos(d*x+c)+b*sec(d*x+c)*tan(d*x+
c)),x, algorithm="fricas")
```

```
[Out] -1/3*(3*a^2*b*cos(d*x + c)^4 - 3*a^2*b*cos(d*x + c)^2 - b^3 + (a^3*cos(d*x
+ c)^5 - a^3*cos(d*x + c)^3 - 3*a*b^2*cos(d*x + c))*sin(d*x + c))/(d*cos(d*
x + c)^3)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(d*x+c)+a*sin(d*x+c))^2*(a*cos(d*x+c)+b*sec(d*x+c)*tan(d*x+
c)),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [B] time = 0.56, size = 118, normalized size = 4.54

$$\frac{a^{3}\left(\sin^{3}\left(dx+c\right)\right)}{3d}+\frac{a^{2}b\left(\sin^{4}\left(dx+c\right)\right)}{d\cos\left(dx+c\right)}+\frac{a^{2}b\left(\sin^{2}\left(dx+c\right)\right)\cos\left(dx+c\right)}{d}+\frac{a\,b^{2}\left(\sin^{3}\left(dx+c\right)\right)}{d\cos\left(dx+c\right)^{2}}+\frac{a\,b^{2}\sin\left(dx+c\right)}{d}+\frac{b^{3}}{3d\cos\left(dx+c\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*sec(d*x+c)+a*sin(d*x+c))^2*(a*cos(d*x+c)+b*sec(d*x+c)*tan(d*x+c)),x)
```

```
[Out] 1/3/d*a^3*sin(d*x+c)^3+1/d*a^2*b*sin(d*x+c)^4/cos(d*x+c)+1/d*a^2*b*sin(d*x+
c)^2*cos(d*x+c)+1/d*a*b^2*sin(d*x+c)^3/cos(d*x+c)^2+1/d*a*b^2*sin(d*x+c)+1/
3/d*b^3/cos(d*x+c)^3
```

maxima [A] time = 0.31, size = 24, normalized size = 0.92

$$\frac{\left(b\sec\left(dx+c\right)+a\sin\left(dx+c\right)\right)^{3}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(d*x+c)+a*sin(d*x+c))^2*(a*cos(d*x+c)+b*sec(d*x+c)*tan(d*x+
c)),x, algorithm="maxima")
```

```
[Out] 1/3*(b*sec(d*x + c) + a*sin(d*x + c))^3/d
```

mupad [B] time = 3.22, size = 100, normalized size = 3.85

$$\frac{a^3\sin(c+dx)}{3d}+\frac{a^2b\cos(c+dx)^2+\sin(c+dx)\,ab^2\cos(c+dx)+\frac{b^3}{3}}{d\cos(c+dx)^3}-\frac{a^3\cos(c+dx)^2\sin(c+dx)}{3d}-\frac{a^2b\,\mathrm{c}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*sin(c + d*x) + b/cos(c + d*x))^2*(a*cos(c + d*x) + (b*tan(c + d*x))/
cos(c + d*x)),x)
```

```
[Out] (a^3*sin(c + d*x))/(3*d) + (b^3/3 + a^2*b*cos(c + d*x)^2 + a*b^2*cos(c + d*
x)*sin(c + d*x))/(d*cos(c + d*x)^3) - (a^3*cos(c + d*x)^2*sin(c + d*x))/(3*
d) - (a^2*b*cos(c + d*x))/d
```

sympy [A] time = 11.16, size = 100, normalized size = 3.85

$$\begin{cases}\frac{a^3\sin^3(c+dx)}{3d}+\frac{a^2b\sin^2(c+dx)\sec(c+dx)}{d}+\frac{ab^2\sin(c+dx)\sec^2(c+dx)}{d}+\frac{b^3\sec^3(c+dx)}{3d} & \text{for } d\neq 0\\ x\left(a\sin(c)+b\sec(c)\right)^2\left(a\cos(c)+b\tan(c)\sec(c)\right) & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(d*x+c)+a*sin(d*x+c))**2*(a*cos(d*x+c)+b*sec(d*x+c)*tan(d*x
+c)),x)
```

```
[Out] Piecewise((a**3*sin(c + d*x)**3/(3*d) + a**2*b*sin(c + d*x)**2*sec(c + d*x)
/d + a*b**2*sin(c + d*x)*sec(c + d*x)**2/d + b**3*sec(c + d*x)**3/(3*d), Ne
(d, 0)), (x*(a*sin(c) + b*sec(c))**2*(a*cos(c) + b*tan(c)*sec(c)), True))
```

3.640 $\int (b\sec(c+dx)+a\sin(c+dx))(a\cos(c+dx)+b\sec(c+dx)\tan(c+dx))\,dx$

Optimal. Leaf size=26

$$\frac{(a\sin(c+dx)+b\sec(c+dx))^2}{2d}$$

```
[Out] 1/2*(b*sec(d*x+c)+a*sin(d*x+c))^2/d
```

Rubi [A] time = 0.03, antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.024, Rules used = {4385}

$$\frac{(a\sin(c+dx)+b\sec(c+dx))^2}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Sec[c + d*x] + a*Sin[c + d*x])*(a*Cos[c + d*x] + b*Sec[c + d*x]*Tan[
c + d*x]),x]
```

```
[Out] (b*Sec[c + d*x] + a*Sin[c + d*x])^2/(2*d)
```

Rule 4385

```
Int[(u_)*(y_)^(m_.), x_Symbol] :> With[{q = DerivativeDivides[ActivateTrig[
y], ActivateTrig[u], x]}, Simp[(q*ActivateTrig[y^(m + 1)])/(m + 1), x] /;
!FalseQ[q]] /; FreeQ[m, x] && NeQ[m, -1] &&  !InertTrigFreeQ[u]
```

Rubi steps

$$\int (b\sec(c+dx)+a\sin(c+dx))(a\cos(c+dx)+b\sec(c+dx)\tan(c+dx))\,dx=\frac{(b\sec(c+dx)+a\sin(c+dx))^2}{2d}$$

Mathematica [B] time = 0.04, size = 67, normalized size = 2.58

$$-\frac{a^2\cos^2(c+dx)}{2d}-\frac{ab\tan^{-1}(\tan(c+dx))}{d}+\frac{ab\tan(c+dx)}{d}+abx+\frac{b^2\sec^2(c+dx)}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Sec[c + d*x] + a*Sin[c + d*x])*(a*Cos[c + d*x] + b*Sec[c + d*x
]*Tan[c + d*x]),x]
```

```
[Out] a*b*x - (a*b*ArcTan[Tan[c + d*x]])/d - (a^2*Cos[c + d*x]^2)/(2*d) + (b^2*Se
c[c + d*x]^2)/(2*d) + (a*b*Tan[c + d*x])/d
```

fricas [B] time = 0.98, size = 61, normalized size = 2.35

$$-\frac{2\,a^2\cos\left(dx+c\right)^4-a^2\cos\left(dx+c\right)^2-4\,ab\cos\left(dx+c\right)\sin\left(dx+c\right)-2\,b^2}{4\,d\cos\left(dx+c\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(d*x+c)+a*sin(d*x+c))*(a*cos(d*x+c)+b*sec(d*x+c)*tan(d*x+c)
),x, algorithm="fricas")
```

```
[Out] -1/4*(2*a^2*cos(d*x + c)^4 - a^2*cos(d*x + c)^2 - 4*a*b*cos(d*x + c)*sin(d*
x + c) - 2*b^2)/(d*cos(d*x + c)^2)
```

giac [A] time = 2.67, size = 45, normalized size = 1.73

$$\frac{b^2\tan\left(dx+c\right)^2+2\,ab\tan\left(dx+c\right)-\frac{a^2}{\tan(dx+c)^2+1}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(d*x+c)+a*sin(d*x+c))*(a*cos(d*x+c)+b*sec(d*x+c)*tan(d*x+c)
),x, algorithm="giac")
```

```
[Out] 1/2*(b^2*tan(d*x + c)^2 + 2*a*b*tan(d*x + c) - a^2/(tan(d*x + c)^2 + 1))/d
```

maple [B] time = 0.46, size = 57, normalized size = 2.19

$$\frac{-\frac{\left(\cos^2(dx+c)\right)a^2}{2}+ab\left(\tan\left(dx+c\right)-dx-c\right)+\left(dx+c\right)ab+\frac{b^2}{2\cos(dx+c)^2}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*sec(d*x+c)+a*sin(d*x+c))*(a*cos(d*x+c)+b*sec(d*x+c)*tan(d*x+c)),x)
```

```
[Out] 1/d*(-1/2*cos(d*x+c)^2*a^2+a*b*(tan(d*x+c)-d*x-c)+(d*x+c)*a*b+1/2*b^2/cos(d
*x+c)^2)
```

maxima [A] time = 0.55, size = 24, normalized size = 0.92

$$\frac{\left(b\sec\left(dx+c\right)+a\sin\left(dx+c\right)\right)^2}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(d*x+c)+a*sin(d*x+c))*(a*cos(d*x+c)+b*sec(d*x+c)*tan(d*x+c)
),x, algorithm="maxima")
```

```
[Out] 1/2*(b*sec(d*x + c) + a*sin(d*x + c))^2/d
```

mupad [B] time = 3.18, size = 61, normalized size = 2.35

$$-\frac{\frac{a^2\left(2\sin(2c+2dx)^2-1\right)}{16}+\frac{a^2}{16}+\frac{b^2}{2}+\frac{ab\sin(2c+2dx)}{2}}{d\left(\sin\left(c+dx\right)^2-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*sin(c + d*x) + b/cos(c + d*x))*(a*cos(c + d*x) + (b*tan(c + d*x))/co
s(c + d*x)),x)
```

```
[Out] -((a^2*(2*sin(2*c + 2*d*x)^2 - 1))/16 + a^2/16 + b^2/2 + (a*b*sin(2*c + 2*d
*x))/2)/(d*(sin(c + d*x)^2 - 1))
```

sympy [A] time = 3.46, size = 73, normalized size = 2.81

$$\begin{cases} \frac{a^2\sin^2\left(c+dx\right)}{2d}+\frac{ab\sin\left(c+dx\right)\sec\left(c+dx\right)}{d}+\frac{b^2\sec^2\left(c+dx\right)}{2d} & \text{for } d\neq 0 \\ x\left(a\sin\left(c\right)+b\sec\left(c\right)\right)\left(a\cos\left(c\right)+b\tan\left(c\right)\sec\left(c\right)\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(d*x+c)+a*sin(d*x+c))*(a*cos(d*x+c)+b*sec(d*x+c)*tan(d*x+c)
),x)
```

```
[Out] Piecewise((a**2*sin(c + d*x)**2/(2*d) + a*b*sin(c + d*x)*sec(c + d*x)/d + b
**2*sec(c + d*x)**2/(2*d), Ne(d, 0)), (x*(a*sin(c) + b*sec(c))*(a*cos(c) +
b*tan(c)*sec(c)), True))
```

3.641 $\int \frac{a\cos(c+dx)+b\sec(c+dx)\tan(c+dx)}{b\sec(c+dx)+a\sin(c+dx)}\,dx$

Optimal. Leaf size=22

$$\frac{\log(a\sin(c+dx)+b\sec(c+dx))}{d}$$

```
[Out] ln(b*sec(d*x+c)+a*sin(d*x+c))/d
```

Rubi [A] time = 0.05, antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.023, Rules used = {4383}

$$\frac{\log(a\sin(c+dx)+b\sec(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Cos[c + d*x] + b*Sec[c + d*x]*Tan[c + d*x])/(b*Sec[c + d*x] + a*Sin[
c + d*x]),x]
```

```
[Out] Log[b*Sec[c + d*x] + a*Sin[c + d*x]]/d
```

Rule 4383

```
Int[(u_)/(y_), x_Symbol] :> With[{q = DerivativeDivides[ActivateTrig[y], Ac
tivateTrig[u], x]}, Simp[q*Log[RemoveContent[ActivateTrig[y], x]], x] /;  !
FalseQ[q]] /;  !InertTrigFreeQ[u]
```

Rubi steps

$$\int \frac{a\cos(c+dx)+b\sec(c+dx)\tan(c+dx)}{b\sec(c+dx)+a\sin(c+dx)}\,dx = \frac{\log(b\sec(c+dx)+a\sin(c+dx))}{d}$$

Mathematica [A] time = 0.46, size = 29, normalized size = 1.32

$$\frac{\log(a\sin(2(c+dx))+2b)-\log(\cos(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Cos[c + d*x] + b*Sec[c + d*x]*Tan[c + d*x])/(b*Sec[c + d*x] +
a*Sin[c + d*x]),x]
```

```
[Out] (-Log[Cos[c + d*x]] + Log[2*b + a*Sin[2*(c + d*x)]])/d
```

fricas [A] time = 1.13, size = 33, normalized size = 1.50

$$\frac{\log\left(a\cos\left(dx+c\right)\sin\left(dx+c\right)+b\right)-\log\left(-\cos\left(dx+c\right)\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cos(d*x+c)+b*sec(d*x+c)*tan(d*x+c))/(b*sec(d*x+c)+a*sin(d*x+c)
),x, algorithm="fricas")
```

```
[Out] (log(a*cos(d*x + c)*sin(d*x + c) + b) - log(-cos(d*x + c)))/d
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cos(d*x+c)+b*sec(d*x+c)*tan(d*x+c))/(b*sec(d*x+c)+a*sin(d*x+c)
),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Evaluation time: 0.56gen.cc:simplify/tmp.typ
e!=_EXT Error: Bad Argument Value
```

maple [A] time = 0.52, size = 23, normalized size = 1.05

$$\frac{\ln\left(b\sec\left(dx+c\right)+a\sin\left(dx+c\right)\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*cos(d*x+c)+b*sec(d*x+c)*tan(d*x+c))/(b*sec(d*x+c)+a*sin(d*x+c)),x)
```

```
[Out] ln(b*sec(d*x+c)+a*sin(d*x+c))/d
```

maxima [A] time = 0.32, size = 22, normalized size = 1.00

$$\frac{\log\left(b\sec\left(dx+c\right)+a\sin\left(dx+c\right)\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cos(d*x+c)+b*sec(d*x+c)*tan(d*x+c))/(b*sec(d*x+c)+a*sin(d*x+c)
),x, algorithm="maxima")
```

```
[Out] log(b*sec(d*x + c) + a*sin(d*x + c))/d
```

mupad [B] time = 4.86, size = 133, normalized size = 6.05

$$\frac{\operatorname{atan}\left(\frac{-\cos(c+d\,x)\,a^6+8\,\cos(c+d\,x)\,a^4\,b^2-16\,\cos(c+d\,x)\,a^2\,b^4+\frac{\sin(2\,c+2\,d\,x)\,a\,b^5}{2}+b^6}{1\mathrm{i}\,\cos(c+d\,x)\,a^6-8\mathrm{i}\,\cos(c+d\,x)\,a^4\,b^2+16\mathrm{i}\,\cos(c+d\,x)\,a^2\,b^4+\frac{1\mathrm{i}\,\sin(2\,c+2\,d\,x)\,a\,b^5}{2}+b^6\,1\mathrm{i}}\right)\,2\mathrm{i}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*cos(c + d*x) + (b*tan(c + d*x))/cos(c + d*x))/(a*sin(c + d*x) + b/co
s(c + d*x)),x)
```

```
[Out] (atan((b^6 - a^6*cos(c + d*x) - 16*a^2*b^4*cos(c + d*x) + 8*a^4*b^2*cos(c +
 d*x) + (a*b^5*sin(2*c + 2*d*x))/2)/(a^6*cos(c + d*x)*1i + b^6*1i + a^2*b^4
*cos(c + d*x)*16i - a^4*b^2*cos(c + d*x)*8i + (a*b^5*sin(2*c + 2*d*x)*1i)/2
))*2i)/d
```

sympy [A] time = 7.48, size = 63, normalized size = 2.86

$$\begin{cases} x \tan{\left(c \right)} & \text{for}\: a = 0 \wedge d = 0 \\ \frac{\log{\left(\tan^{2}{\left(c + d x \right)} + 1 \right)}}{2 d} & \text{for}\: a = 0 \\ \frac{x \left(a \cos{\left(c \right)} + b \tan{\left(c \right)} \sec{\left(c \right)}\right)}{a \sin{\left(c \right)} + b \sec{\left(c \right)}} & \text{for}\: d = 0 \\ \frac{\log{\left(\sin{\left(c + d x \right)} + \frac{b \sec{\left(c + d x \right)}}{a} \right)}}{d} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cos(d*x+c)+b*sec(d*x+c)*tan(d*x+c))/(b*sec(d*x+c)+a*sin(d*x+c)
),x)
```

```
[Out] Piecewise((x*tan(c), Eq(a, 0) & Eq(d, 0)), (log(tan(c + d*x)**2 + 1)/(2*d),
 Eq(a, 0)), (x*(a*cos(c) + b*tan(c)*sec(c))/(a*sin(c) + b*sec(c)), Eq(d, 0)
), (log(sin(c + d*x) + b*sec(c + d*x)/a)/d, True))
```

3.642 $\int \frac{a\cos(c+dx)+b\sec(c+dx)\tan(c+dx)}{(b\sec(c+dx)+a\sin(c+dx))^2}\,dx$

Optimal. Leaf size=24

$$-\frac{1}{d(a\sin(c+dx)+b\sec(c+dx))}$$

```
[Out] -1/d/(b*sec(d*x+c)+a*sin(d*x+c))
```

Rubi [A] time = 0.04, antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.023, Rules used = {4385}

$$-\frac{1}{d(a\sin(c+dx)+b\sec(c+dx))}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Cos[c + d*x] + b*Sec[c + d*x]*Tan[c + d*x])/(b*Sec[c + d*x] + a*Sin[
c + d*x])^2,x]
```

```
[Out] -(1/(d*(b*Sec[c + d*x] + a*Sin[c + d*x])))
```

Rule 4385

```
Int[(u_)*(y_)^(m_.), x_Symbol] :> With[{q = DerivativeDivides[ActivateTrig[
y], ActivateTrig[u], x]}, Simp[(q*ActivateTrig[y^(m + 1)])/(m + 1), x] /;
!FalseQ[q]] /; FreeQ[m, x] && NeQ[m, -1] &&  !InertTrigFreeQ[u]
```

Rubi steps

$$\int \frac{a\cos(c+dx)+b\sec(c+dx)\tan(c+dx)}{(b\sec(c+dx)+a\sin(c+dx))^2}\,dx=-\frac{1}{d(b\sec(c+dx)+a\sin(c+dx))}$$

Mathematica [A] time = 0.30, size = 27, normalized size = 1.12

$$-\frac{2\cos(c+dx)}{d(a\sin(2(c+dx))+2b)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Cos[c + d*x] + b*Sec[c + d*x]*Tan[c + d*x])/(b*Sec[c + d*x] +
a*Sin[c + d*x])^2,x]
```

```
[Out] (-2*Cos[c + d*x])/(d*(2*b + a*Sin[2*(c + d*x)]))
```

fricas [A] time = 0.90, size = 29, normalized size = 1.21

$$-\frac{\cos(dx+c)}{ad\cos(dx+c)\sin(dx+c)+bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cos(d*x+c)+b*sec(d*x+c)*tan(d*x+c))/(b*sec(d*x+c)+a*sin(d*x+c)
)^2,x, algorithm="fricas")
```

```
[Out] -cos(d*x + c)/(a*d*cos(d*x + c)*sin(d*x + c) + b*d)
```

giac [B] time = 0.73, size = 108, normalized size = 4.50

$$\frac{2\left(a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}-b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-b\right)}{\left(b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{4}-2\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}+2\,b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+2\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+b\right)bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cos(d*x+c)+b*sec(d*x+c)*tan(d*x+c))/(b*sec(d*x+c)+a*sin(d*x+c)
)^2,x, algorithm="giac")
```

```
[Out] 2*(a*tan(1/2*d*x + 1/2*c)^3 - b*tan(1/2*d*x + 1/2*c)^2 - a*tan(1/2*d*x + 1/
2*c) - b)/((b*tan(1/2*d*x + 1/2*c)^4 - 2*a*tan(1/2*d*x + 1/2*c)^3 + 2*b*tan
(1/2*d*x + 1/2*c)^2 + 2*a*tan(1/2*d*x + 1/2*c) + b)*b*d)
```

maple [A] time = 0.63, size = 25, normalized size = 1.04

$$-\frac{1}{d\left(b\sec(dx+c)+a\sin(dx+c)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*cos(d*x+c)+b*sec(d*x+c)*tan(d*x+c))/(b*sec(d*x+c)+a*sin(d*x+c))^2,x)
```

```
[Out] -1/d/(b*sec(d*x+c)+a*sin(d*x+c))
```

maxima [A] time = 0.43, size = 24, normalized size = 1.00

$$-\frac{1}{\left(b\sec(dx+c)+a\sin(dx+c)\right)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cos(d*x+c)+b*sec(d*x+c)*tan(d*x+c))/(b*sec(d*x+c)+a*sin(d*x+c)
)^2,x, algorithm="maxima")
```

```
[Out] -1/((b*sec(d*x + c) + a*sin(d*x + c))*d)
```

mupad [B] time = 3.24, size = 47, normalized size = 1.96

$$-\frac{b\,\left(\cos\left(c+d\,x\right)+1\right)+\frac{a\,\sin(2\,c+2\,d\,x)}{2}}{b\,d\,\left(b+\frac{a\,\sin(2\,c+2\,d\,x)}{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*cos(c + d*x) + (b*tan(c + d*x))/cos(c + d*x))/(a*sin(c + d*x) + b/co
s(c + d*x))^2,x)
```

```
[Out] -(b*(cos(c + d*x) + 1) + (a*sin(2*c + 2*d*x))/2)/(b*d*(b + (a*sin(2*c + 2*d
*x))/2))
```

sympy [A] time = 21.18, size = 49, normalized size = 2.04

$$\begin{cases} -\frac{1}{ad\sin\left(c+dx\right)+bd\sec\left(c+dx\right)} & \text{for}\ d\neq 0 \\ \frac{x(a\cos\left(c\right)+b\tan\left(c\right)\sec\left(c\right))}{\left(a\sin\left(c\right)+b\sec\left(c\right)\right)^2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cos(d*x+c)+b*sec(d*x+c)*tan(d*x+c))/(b*sec(d*x+c)+a*sin(d*x+c)
)**2,x)
```

```
[Out] Piecewise((-1/(a*d*sin(c + d*x) + b*d*sec(c + d*x)), Ne(d, 0)), (x*(a*cos(c
) + b*tan(c)*sec(c))/(a*sin(c) + b*sec(c))**2, True))
```

3.643 $\int \frac{a\cos(c+dx)+b\sec(c+dx)\tan(c+dx)}{(b\sec(c+dx)+a\sin(c+dx))^3}\,dx$

Optimal. Leaf size=26

$$-\frac{1}{2d(a\sin(c+dx)+b\sec(c+dx))^2}$$

```
[Out] -1/2/d/(b*sec(d*x+c)+a*sin(d*x+c))^2
```

Rubi [A] time = 0.05, antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.023, Rules used = {4385}

$$-\frac{1}{2d(a\sin(c+dx)+b\sec(c+dx))^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Cos[c + d*x] + b*Sec[c + d*x]*Tan[c + d*x])/(b*Sec[c + d*x] + a*Sin[
c + d*x])^3,x]
```

```
[Out] -1/(2*d*(b*Sec[c + d*x] + a*Sin[c + d*x])^2)
```

Rule 4385

```
Int[(u_)*(y_)^(m_.), x_Symbol] :> With[{q = DerivativeDivides[ActivateTrig[
y], ActivateTrig[u], x]}, Simp[(q*ActivateTrig[y^(m + 1)])/(m + 1), x] /;
!FalseQ[q]] /; FreeQ[m, x] && NeQ[m, -1] &&  !InertTrigFreeQ[u]
```

Rubi steps

$$\int \frac{a\cos(c+dx)+b\sec(c+dx)\tan(c+dx)}{(b\sec(c+dx)+a\sin(c+dx))^3}\,dx=-\frac{1}{2d(b\sec(c+dx)+a\sin(c+dx))^2}$$

Mathematica [A] time = 0.71, size = 29, normalized size = 1.12

$$-\frac{2\cos^2(c+dx)}{d(a\sin(2(c+dx))+2b)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Cos[c + d*x] + b*Sec[c + d*x]*Tan[c + d*x])/(b*Sec[c + d*x] +
a*Sin[c + d*x])^3,x]
```

```
[Out] (-2*Cos[c + d*x]^2)/(d*(2*b + a*Sin[2*(c + d*x)])^2)
```

fricas [B] time = 1.91, size = 63, normalized size = 2.42

$$\frac{\cos(dx+c)^2}{2\left(a^2d\cos(dx+c)^4 - a^2d\cos(dx+c)^2 - 2\,abd\cos(dx+c)\sin(dx+c) - b^2d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cos(d*x+c)+b*sec(d*x+c)*tan(d*x+c))/(b*sec(d*x+c)+a*sin(d*x+c)
)^3,x, algorithm="fricas")
```

```
[Out] 1/2*cos(d*x + c)^2/(a^2*d*cos(d*x + c)^4 - a^2*d*cos(d*x + c)^2 - 2*a*b*d*c
os(d*x + c)*sin(d*x + c) - b^2*d)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cos(d*x+c)+b*sec(d*x+c)*tan(d*x+c))/(b*sec(d*x+c)+a*sin(d*x+c)
)^3,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Evaluation time: 52.12gen.cc:simplify/tmp.ty
pe!=_EXT Error: Bad Argument Value
```

maple [A] time = 0.66, size = 25, normalized size = 0.96

$$-\frac{1}{2d\left(b\sec(dx+c)+a\sin(dx+c)\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*cos(d*x+c)+b*sec(d*x+c)*tan(d*x+c))/(b*sec(d*x+c)+a*sin(d*x+c))^3,x)
```

```
[Out] -1/2/d/(b*sec(d*x+c)+a*sin(d*x+c))^2
```

maxima [A] time = 0.31, size = 24, normalized size = 0.92

$$-\frac{1}{2\left(b\sec(dx+c)+a\sin(dx+c)\right)^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cos(d*x+c)+b*sec(d*x+c)*tan(d*x+c))/(b*sec(d*x+c)+a*sin(d*x+c)
)^3,x, algorithm="maxima")
```

```
[Out] -1/2/((b*sec(d*x + c) + a*sin(d*x + c))^2*d)
```

mupad [B] time = 6.31, size = 291, normalized size = 11.19

$$\frac{\frac{2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2\left(a^2+b^2\right)}{b^2}+\frac{2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^6\left(a^2+b^2\right)}{b^2}+\frac{2a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{b}-\frac{4\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4\left(a^2-\right.}{b^2}}{d\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2\left(4a^2+4b^2\right)+\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^6\left(4a^2+4b^2\right)-\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4\left(8a^2-6b^2\right)+b^2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^8+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*cos(c + d*x) + (b*tan(c + d*x))/cos(c + d*x))/(a*sin(c + d*x) + b/co
s(c + d*x))^3,x)
```

```
[Out] ((2*tan(c/2 + (d*x)/2)^2*(a^2 + b^2))/b^2 + (2*tan(c/2 + (d*x)/2)^6*(a^2 +
b^2))/b^2 + (2*a*tan(c/2 + (d*x)/2))/b - (4*tan(c/2 + (d*x)/2)^4*(a^2 - b^2
))/b^2 + (2*a*tan(c/2 + (d*x)/2)^3)/b - (2*a*tan(c/2 + (d*x)/2)^5)/b - (2*a
*tan(c/2 + (d*x)/2)^7)/b)/(d*(tan(c/2 + (d*x)/2)^2*(4*a^2 + 4*b^2) + tan(c/
2 + (d*x)/2)^6*(4*a^2 + 4*b^2) - tan(c/2 + (d*x)/2)^4*(8*a^2 - 6*b^2) + b^2
*tan(c/2 + (d*x)/2)^8 + b^2 + 4*a*b*tan(c/2 + (d*x)/2)^3 - 4*a*b*tan(c/2 +
(d*x)/2)^5 - 4*a*b*tan(c/2 + (d*x)/2)^7 + 4*a*b*tan(c/2 + (d*x)/2)))
```

sympy [A] time = 44.09, size = 80, normalized size = 3.08

$$\begin{cases} -\frac{1}{2a^2d\sin^2(c+dx)+4abd\sin(c+dx)\sec(c+dx)+2b^2d\sec^2(c+dx)} & \text{for } d \neq 0 \\ \frac{x(a\cos(c)+b\tan(c)\sec(c))}{(a\sin(c)+b\sec(c))^3} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cos(d*x+c)+b*sec(d*x+c)*tan(d*x+c))/(b*sec(d*x+c)+a*sin(d*x+c)
)**3,x)
```

```
[Out] Piecewise((-1/(2*a**2*d*sin(c + d*x)**2 + 4*a*b*d*sin(c + d*x)*sec(c + d*x)
 + 2*b**2*d*sec(c + d*x)**2), Ne(d, 0)), (x*(a*cos(c) + b*tan(c)*sec(c))/(a
*sin(c) + b*sec(c))**3, True))
```

3.644 $\int F(c,d,\cos(a+bx),r,s)\sin(a+bx)\,dx$

Optimal. Leaf size=21

$$\text{Int}(\sin(a+bx)F(c,d,\cos(a+bx),r,s),x)$$

```
[Out] CannotIntegrate(F(c,d,cos(b*x+a),r,s)*sin(b*x+a),x)
```

Rubi [A] time = 0.01, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int F(c,d,\cos(a+bx),r,s)\sin(a+bx)\,dx$$

Verification is Not applicable to the result.

```
[In] Int[F[c, d, Cos[a + b*x], r, s]*Sin[a + b*x],x]
[Out] -(Defer[Subst][Defer[Int][F[c, d, x, r, s], x], x, Cos[a + b*x]]/b)
```

Rubi steps

$$\int F(c,d,\cos(a+bx),r,s)\sin(a+bx)\,dx = -\frac{\text{Subst}(\int F(c,d,x,r,s)\,dx,x,\cos(a+bx))}{b}$$

Mathematica [A] time = 0.05, size = 0, normalized size = 0.00

$$\int F(c,d,\cos(a+bx),r,s)\sin(a+bx)\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[F[c, d, Cos[a + b*x], r, s]*Sin[a + b*x],x]
[Out] Integrate[F[c, d, Cos[a + b*x], r, s]*Sin[a + b*x], x]
```

fricas [A] time = 0.86, size = 0, normalized size = 0.00

$$\text{integral}\left(F\left(c,d,\cos\left(bx+a\right),r,s\right)\sin\left(bx+a\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F(c,d,cos(b*x+a),r,s)*sin(b*x+a),x, algorithm="fricas")
```

```
[Out] integral(F(c, d, cos(b*x + a), r, s)*sin(b*x + a), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int F\left(c,d,\cos\left(bx+a\right),r,s\right)\sin\left(bx+a\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F(c,d,cos(b*x+a),r,s)*sin(b*x+a),x, algorithm="giac")
[Out] integrate(F(c, d, cos(b*x + a), r, s)*sin(b*x + a), x)
```

maple [A] time = 0.05, size = 0, normalized size = 0.00

$$\int F\left(c,d,\cos\left(bx+a\right),r,s\right)\sin\left(bx+a\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(F(c,d,cos(b*x+a),r,s)*sin(b*x+a),x)
[Out] int(F(c,d,cos(b*x+a),r,s)*sin(b*x+a),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int F\left(c,d,\cos\left(bx+a\right),r,s\right)\sin\left(bx+a\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F(c,d,cos(b*x+a),r,s)*sin(b*x+a),x, algorithm="maxima")
[Out] integrate(F(c, d, cos(b*x + a), r, s)*sin(b*x + a), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.05

$$\int \sin\left(a+b\,x\right)\,F\left(c,d,\cos\left(a+b\,x\right),r,s\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b*x)*F(c, d, cos(a + b*x), r, s),x)
[Out] int(sin(a + b*x)*F(c, d, cos(a + b*x), r, s), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int F(c,d,\cos\left(a+bx\right),r,s)\sin\left(a+bx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F(c,d,cos(b*x+a),r,s)*sin(b*x+a),x)
```

```
[Out] Integral(F(c, d, cos(a + b*x), r, s)*sin(a + b*x), x)
```

3.645 $\int \cos(a+bx)F(c,d,\sin(a+bx),r,s)\,dx$

Optimal. Leaf size=21

$$\text{Int}(\cos(a+bx)F(c,d,\sin(a+bx),r,s),x)$$

```
[Out] CannotIntegrate(cos(b*x+a)*F(c,d,sin(b*x+a),r,s),x)
```

Rubi [A] time = 0.01, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \cos(a+bx)F(c,d,\sin(a+bx),r,s)\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Cos[a + b*x]*F[c, d, Sin[a + b*x], r, s],x]
[Out] Defer[Subst][Defer[Int][F[c, d, x, r, s], x], x, Sin[a + b*x]]/b
```

Rubi steps

$$\int \cos(a+bx)F(c,d,\sin(a+bx),r,s)\,dx = \frac{\text{Subst}\left(\int F(c,d,x,r,s)\,dx,x,\sin(a+bx)\right)}{b}$$

Mathematica [A] time = 0.03, size = 0, normalized size = 0.00

$$\int \cos(a+bx)F(c,d,\sin(a+bx),r,s)\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Cos[a + b*x]*F[c, d, Sin[a + b*x], r, s],x]
[Out] Integrate[Cos[a + b*x]*F[c, d, Sin[a + b*x], r, s], x]
```

fricas [A] time = 0.90, size = 0, normalized size = 0.00

$$\text{integral}\left(F\left(c,d,\sin\left(bx+a\right),r,s\right)\cos\left(bx+a\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*F(c,d,sin(b*x+a),r,s),x, algorithm="fricas")
```

```
[Out] integral(F(c, d, sin(b*x + a), r, s)*cos(b*x + a), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int F(c,d,\sin(bx+a),r,s)\cos(bx+a)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*F(c,d,sin(b*x+a),r,s),x, algorithm="giac")
[Out] integrate(F(c, d, sin(b*x + a), r, s)*cos(b*x + a), x)
```

maple [A] time = 0.03, size = 0, normalized size = 0.00

$$\int \cos(bx+a)\,F(c,d,\sin(bx+a),r,s)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)*F(c,d,sin(b*x+a),r,s),x)
[Out] int(cos(b*x+a)*F(c,d,sin(b*x+a),r,s),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int F(c,d,\sin(bx+a),r,s)\cos(bx+a)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*F(c,d,sin(b*x+a),r,s),x, algorithm="maxima")
[Out] integrate(F(c, d, sin(b*x + a), r, s)*cos(b*x + a), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.05

$$\int \cos(a+b\,x)\;F(c,d,\sin(a+b\,x),r,s)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)*F(c, d, sin(a + b*x), r, s),x)
[Out] int(cos(a + b*x)*F(c, d, sin(a + b*x), r, s), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int F(c,d,\sin(a+bx),r,s)\cos(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*F(c,d,sin(b*x+a),r,s),x)
```

```
[Out] Integral(F(c, d, sin(a + b*x), r, s)*cos(a + b*x), x)
```

3.646 $\int F(c,d,\tan(a+bx),r,s)\sec^2(a+bx)\,dx$

Optimal. Leaf size=23

$$\text{Int}\left(\sec^2(a+bx)F(c,d,\tan(a+bx),r,s),x\right)$$

```
[Out] CannotIntegrate(F(c,d,tan(b*x+a),r,s)*sec(b*x+a)^2,x)
```

Rubi [A] time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int F(c,d,\tan(a+bx),r,s)\sec^2(a+bx)\,dx$$

Verification is Not applicable to the result.

```
[In] Int[F[c, d, Tan[a + b*x], r, s]*Sec[a + b*x]^2,x]
```

```
[Out] Defer[Subst][Defer[Int][F[c, d, x, r, s], x], x, Tan[a + b*x]]/b
```

Rubi steps

$$\int F(c,d,\tan(a+bx),r,s)\sec^2(a+bx)\,dx = \frac{\text{Subst}(\int F(c,d,x,r,s)\,dx,x,\tan(a+bx))}{b}$$

Mathematica [A] time = 0.08, size = 0, normalized size = 0.00

$$\int F(c,d,\tan(a+bx),r,s)\sec^2(a+bx)\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[F[c, d, Tan[a + b*x], r, s]*Sec[a + b*x]^2,x]
```

```
[Out] Integrate[F[c, d, Tan[a + b*x], r, s]*Sec[a + b*x]^2, x]
```

fricas [A] time = 0.86, size = 0, normalized size = 0.00

$$\text{integral}\left(F\left(c,d,\tan\left(bx+a\right),r,s\right)\sec\left(bx+a\right)^2,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F(c,d,tan(b*x+a),r,s)*sec(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] integral(F(c, d, tan(b*x + a), r, s)*sec(b*x + a)^2, x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int F(c,d,\tan(bx+a),r,s)\sec(bx+a)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F(c,d,tan(b*x+a),r,s)*sec(b*x+a)^2,x, algorithm="giac")
[Out] integrate(F(c, d, tan(b*x + a), r, s)*sec(b*x + a)^2, x)
```

maple [A] time = 0.06, size = 0, normalized size = 0.00

$$\int F(c,d,\tan(bx+a),r,s)\left(\sec^2(bx+a)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(F(c,d,tan(b*x+a),r,s)*sec(b*x+a)^2,x)
[Out] int(F(c,d,tan(b*x+a),r,s)*sec(b*x+a)^2,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int F(c,d,\tan(bx+a),r,s)\sec(bx+a)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F(c,d,tan(b*x+a),r,s)*sec(b*x+a)^2,x, algorithm="maxima")
[Out] integrate(F(c, d, tan(b*x + a), r, s)*sec(b*x + a)^2, x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{F(c,d,\tan(a+b\,x),r,s)}{\cos(a+b\,x)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(F(c, d, tan(a + b*x), r, s)/cos(a + b*x)^2,x)
[Out] int(F(c, d, tan(a + b*x), r, s)/cos(a + b*x)^2, x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int F(c,d,\tan(a+bx),r,s)\sec^2(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F(c,d,tan(b*x+a),r,s)*sec(b*x+a)**2,x)
```

```
[Out] Integral(F(c, d, tan(a + b*x), r, s)*sec(a + b*x)**2, x)
```

3.647 $\int \csc^2(a+bx)F(c,d,\cot(a+bx),r,s)\,dx$

Optimal. Leaf size=23

$$\text{Int}\left(\csc^2(a+bx)F(c,d,\cot(a+bx),r,s),x\right)$$

```
[Out] CannotIntegrate(csc(b*x+a)^2*F(c,d,cot(b*x+a),r,s),x)
```

Rubi [A] time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \csc^2(a+bx)F(c,d,\cot(a+bx),r,s)\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Csc[a + b*x]^2*F[c, d, Cot[a + b*x], r, s],x]
```

```
[Out] -(Defer[Subst][Defer[Int][F[c, d, x, r, s], x], x, Cot[a + b*x]]/b)
```

Rubi steps

$$\int \csc^2(a+bx)F(c,d,\cot(a+bx),r,s)\,dx = -\frac{\text{Subst}\left(\int F(c,d,x,r,s)\,dx,x,\cot(a+bx)\right)}{b}$$

Mathematica [A] time = 0.07, size = 0, normalized size = 0.00

$$\int \csc^2(a+bx)F(c,d,\cot(a+bx),r,s)\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Csc[a + b*x]^2*F[c, d, Cot[a + b*x], r, s],x]
```

```
[Out] Integrate[Csc[a + b*x]^2*F[c, d, Cot[a + b*x], r, s], x]
```

fricas [A] time = 1.97, size = 0, normalized size = 0.00

$$\text{integral}\left(F\left(c,d,\cot\left(bx+a\right),r,s\right)\csc\left(bx+a\right)^2,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^2*F(c,d,cot(b*x+a),r,s),x, algorithm="fricas")
```

```
[Out] integral(F(c, d, cot(b*x + a), r, s)*csc(b*x + a)^2, x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int F(c,d,\cot(bx+a),r,s)\csc(bx+a)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^2*F(c,d,cot(b*x+a),r,s),x, algorithm="giac")
[Out] integrate(F(c, d, cot(b*x + a), r, s)*csc(b*x + a)^2, x)
```

maple [A] time = 0.05, size = 0, normalized size = 0.00

$$\int \left(\csc^2(bx+a)\right)F(c,d,\cot(bx+a),r,s)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(b*x+a)^2*F(c,d,cot(b*x+a),r,s),x)
[Out] int(csc(b*x+a)^2*F(c,d,cot(b*x+a),r,s),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int F(c,d,\cot(bx+a),r,s)\csc(bx+a)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^2*F(c,d,cot(b*x+a),r,s),x, algorithm="maxima")
[Out] integrate(F(c, d, cot(b*x + a), r, s)*csc(b*x + a)^2, x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{F(c,d,\cot(a+b\,x),r,s)}{\sin(a+b\,x)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(F(c, d, cot(a + b*x), r, s)/sin(a + b*x)^2,x)
[Out] int(F(c, d, cot(a + b*x), r, s)/sin(a + b*x)^2, x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int F(c,d,\cot(a+bx),r,s)\csc^2(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)**2*F(c,d,cot(b*x+a),r,s),x)
```

```
[Out] Integral(F(c, d, cot(a + b*x), r, s)*csc(a + b*x)**2, x)
```

3.648 $\int \frac{\sin(x)}{a+b\cos(x)}\,dx$

Optimal. Leaf size=12

$$-\frac{\log(a+b\cos(x))}{b}$$

[Out] -ln(a+b*cos(x))/b

Rubi [A] time = 0.02, antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {2668, 31}

$$-\frac{\log(a+b\cos(x))}{b}$$

Antiderivative was successfully verified.

[In] Int[Sin[x]/(a + b*Cos[x]),x]

[Out] -(Log[a + b*Cos[x]]/b)

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x, 
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2668

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\sin(x)}{a+b\cos(x)}\,dx &= -\frac{\text{Subst}\left(\int \frac{1}{a+x}\,dx, x, b\cos(x)\right)}{b}\\ &= -\frac{\log(a+b\cos(x))}{b}\end{aligned}$$

Mathematica [A] time = 0.02, size = 12, normalized size = 1.00

$$-\frac{\log(a+b\cos(x))}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[x]/(a + b*Cos[x]),x]
[Out] -(Log[a + b*Cos[x]]/b)
```

fricas [A] time = 0.73, size = 15, normalized size = 1.25

$$-\frac{\log\left(-b\cos\left(x\right)-a\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)/(a+b*cos(x)),x, algorithm="fricas")
[Out] -log(-b*cos(x) - a)/b
```

giac [A] time = 0.13, size = 13, normalized size = 1.08

$$-\frac{\log\left(|b\cos\left(x\right)+a|\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)/(a+b*cos(x)),x, algorithm="giac")
[Out] -log(abs(b*cos(x) + a))/b
```

maple [A] time = 0.03, size = 13, normalized size = 1.08

$$-\frac{\ln\left(a+b\cos\left(x\right)\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)/(a+b*cos(x)),x)
[Out] -ln(a+b*cos(x))/b
```

maxima [A] time = 0.31, size = 12, normalized size = 1.00

$$-\frac{\log\left(b\cos\left(x\right)+a\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)/(a+b*cos(x)),x, algorithm="maxima")
[Out] -log(b*cos(x) + a)/b
```

mupad [B] time = 0.06, size = 12, normalized size = 1.00

$$-\frac{\ln\left(a+b\,\cos(x)\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)/(a + b*cos(x)),x)
[Out] -log(a + b*cos(x))/b
```

sympy [A] time = 0.33, size = 17, normalized size = 1.42

$$\begin{cases} -\frac{\log\left(\frac{a}{b}+\cos\left(x\right)\right)}{b} & \text{for}\ b \neq 0 \\ -\frac{\cos\left(x\right)}{a} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)/(a+b*cos(x)),x)
[Out] Piecewise((-log(a/b + cos(x))/b, Ne(b, 0)), (-cos(x)/a, True))
```

3.649 $\int (a+b\cos(x))^n \sin(x)\,dx$

Optimal. Leaf size=20

$$-\frac{(a+b\cos(x))^{n+1}}{b(n+1)}$$

[Out] -(a+b*cos(x))^(1+n)/b/(1+n)

Rubi [A] time = 0.02, antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {2668, 32}

$$-\frac{(a+b\cos(x))^{n+1}}{b(n+1)}$$

Antiderivative was successfully verified.

[In] Int[(a + b*Cos[x])^n*Sin[x],x]

[Out] -((a + b*Cos[x])^(1 + n)/(b*(1 + n)))

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 2668

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}\int (a+b\cos(x))^n \sin(x)\,dx &= -\frac{\text{Subst}\left(\int (a+x)^n\,dx, x, b\cos(x)\right)}{b}\\ &= -\frac{(a+b\cos(x))^{1+n}}{b(1+n)}\end{aligned}$$

Mathematica [A] time = 0.03, size = 19, normalized size = 0.95

$$-\frac{(a+b\cos(x))^{n+1}}{bn+b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[x])^n*Sin[x],x]
```

```
[Out] -((a + b*Cos[x])^(1 + n)/(b + b*n))
```

fricas [A] time = 0.63, size = 23, normalized size = 1.15

$$-\frac{(b\cos(x)+a)(b\cos(x)+a)^n}{bn+b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(x))^n*sin(x),x, algorithm="fricas")
```

```
[Out] -(b*cos(x) + a)*(b*cos(x) + a)^n/(b*n + b)
```

giac [A] time = 0.14, size = 20, normalized size = 1.00

$$-\frac{(b\cos(x)+a)^{n+1}}{b(n+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(x))^n*sin(x),x, algorithm="giac")
```

```
[Out] -(b*cos(x) + a)^(n + 1)/(b*(n + 1))
```

maple [A] time = 0.03, size = 21, normalized size = 1.05

$$-\frac{(a+b\cos(x))^{n+1}}{b(n+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(x))^n*sin(x),x)
```

```
[Out] -(a+b*cos(x))^(n+1)/b/(n+1)
```

maxima [A] time = 0.38, size = 20, normalized size = 1.00

$$-\frac{(b\cos(x)+a)^{n+1}}{b(n+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(x))^n*sin(x),x, algorithm="maxima")
```

```
[Out] -(b*cos(x) + a)^(n + 1)/(b*(n + 1))
```

mupad [B] time = 3.15, size = 20, normalized size = 1.00

$$-\frac{(a+b\,\cos(x))^{n+1}}{b\,(n+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)*(a + b*cos(x))^n,x)
[Out] -(a + b*cos(x))^(n + 1)/(b*(n + 1))
```

sympy [A] time = 2.02, size = 63, normalized size = 3.15

$$\begin{cases} -\frac{\cos(x)}{a} & \text{for } b=0 \wedge n=-1 \\ -a^n\cos(x) & \text{for } b=0 \\ -\frac{\log\left(\frac{a}{b}+\cos(x)\right)}{b} & \text{for } n=-1 \\ -\frac{a(a+b\cos(x))^n}{bn+b}-\frac{b(a+b\cos(x))^n\cos(x)}{bn+b} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(x))**n*sin(x),x)
[Out] Piecewise((-cos(x)/a, Eq(b, 0) & Eq(n, -1)), (-a**n*cos(x), Eq(b, 0)), (-lo
g(a/b + cos(x))/b, Eq(n, -1)), (-a*(a + b*cos(x))**n/(b*n + b) - b*(a + b*c
os(x))**n*cos(x)/(b*n + b), True))
```

3.650 $\int \frac{\sin(x)}{\sqrt{1+\cos^2(x)}}\,dx$

Optimal. Leaf size=5

$$-\sinh^{-1}(\cos(x))$$

```
[Out] -arcsinh(cos(x))
```

Rubi [A] time = 0.02, antiderivative size = 5, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {3190, 215}

$$-\sinh^{-1}(\cos(x))$$

Antiderivative was successfully verified.

```
[In] Int[Sin[x]/Sqrt[1 + Cos[x]^2],x]
```

```
[Out] -ArcSinh[Cos[x]]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 3190

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Su
bst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]/
ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\begin{aligned}\int \frac{\sin(x)}{\sqrt{1+\cos^2(x)}}\,dx &= -\text{Subst}\left(\int \frac{1}{\sqrt{1+x^2}}\,dx, x, \cos(x)\right)\\ &= -\sinh^{-1}(\cos(x))\end{aligned}$$

Mathematica [A] time = 0.02, size = 5, normalized size = 1.00

$$-\sinh^{-1}(\cos(x))$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[x]/Sqrt[1 + Cos[x]^2],x]
```

```
[Out] -ArcSinh[Cos[x]]
```

fricas [B] time = 0.96, size = 36, normalized size = 7.20

$$\frac{1}{4}\log\left(8\cos(x)^4+8\cos(x)^2-4\left(2\cos(x)^3+\cos(x)\right)\sqrt{\cos(x)^2+1}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)/(1+cos(x)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/4*log(8*cos(x)^4 + 8*cos(x)^2 - 4*(2*cos(x)^3 + cos(x))*sqrt(cos(x)^2 + 1
) + 1)
```

giac [B] time = 0.13, size = 14, normalized size = 2.80

$$\log\left(\sqrt{\cos(x)^2+1}-\cos(x)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)/(1+cos(x)^2)^(1/2),x, algorithm="giac")
```

```
[Out] log(sqrt(cos(x)^2 + 1) - cos(x))
```

maple [A] time = 0.06, size = 6, normalized size = 1.20

$$-\operatorname{arcsinh}(\cos(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)/(1+cos(x)^2)^(1/2),x)
```

```
[Out] -arcsinh(cos(x))
```

maxima [A] time = 1.41, size = 5, normalized size = 1.00

$$-\operatorname{arsinh}(\cos(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)/(1+cos(x)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] -arcsinh(cos(x))
```

mupad [B] time = 3.08, size = 5, normalized size = 1.00

$$-\operatorname{asinh}(\cos(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)/(cos(x)^2 + 1)^(1/2),x)
```

```
[Out] -asinh(cos(x))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin(x)}{\sqrt{\cos^2(x) + 1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)/(1+cos(x)**2)**(1/2),x)
```

```
[Out] Integral(sin(x)/sqrt(cos(x)**2 + 1), x)
```

3.651 $\int \cos(\cos(x))\sin(x)\,dx$

Optimal. Leaf size=5

$$-\sin(\cos(x))$$

```
[Out] -sin(cos(x))
```

Rubi [A] time = 0.01, antiderivative size = 5, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {4335, 2637}

$$-\sin(\cos(x))$$

Antiderivative was successfully verified.

```
[In] Int[Cos[Cos[x]]*Sin[x],x]
```

```
[Out] -Sin[Cos[x]]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 4335

```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{d = FreeFacto
rs[Cos[c*(a + b*x)], x]}, -Dist[d/(b*c), Subst[Int[SubstFor[1, Cos[c*(a + b
*x)]/d, u, x], x], x, Cos[c*(a + b*x)]/d], x] /; FunctionOfQ[Cos[c*(a + b*x
)]/d, u, x, True]] /; FreeQ[{a, b, c}, x] && (EqQ[F, Sin] || EqQ[F, sin])
```

Rubi steps

$$\begin{aligned}\int \cos(\cos(x))\sin(x)\,dx &= -\text{Subst}\left(\int \cos(x)\,dx, x, \cos(x)\right)\\ &= -\sin(\cos(x))\end{aligned}$$

Mathematica [A] time = 2.57, size = 5, normalized size = 1.00

$$-\sin(\cos(x))$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[Cos[x]]*Sin[x],x]
```

[Out] -Sin[Cos[x]]

fricas [B] time = 0.79, size = 20, normalized size = 4.00

$$\sin\left(\frac{\tan\left(\frac{1}{2}x\right)^2-1}{\tan\left(\frac{1}{2}x\right)^2+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(cos(x))*sin(x),x, algorithm="fricas")
[Out] sin((tan(1/2*x)^2 - 1)/(tan(1/2*x)^2 + 1))
```

giac [A] time = 0.14, size = 5, normalized size = 1.00

$$-\sin(\cos(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(cos(x))*sin(x),x, algorithm="giac")
[Out] -sin(cos(x))
```

maple [A] time = 0.01, size = 6, normalized size = 1.20

$$-\sin(\cos(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(cos(x))*sin(x),x)
[Out] -sin(cos(x))
```

maxima [A] time = 0.55, size = 5, normalized size = 1.00

$$-\sin(\cos(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(cos(x))*sin(x),x, algorithm="maxima")
[Out] -sin(cos(x))
```

mupad [B] time = 0.09, size = 5, normalized size = 1.00

$$-\sin(\cos(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(cos(x))*sin(x),x)
```

```
[Out] -sin(cos(x))
```

sympy [A] time = 0.44, size = 5, normalized size = 1.00

$$-\sin(\cos(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(cos(x))*sin(x),x)
```

```
[Out] -sin(cos(x))
```

3.652 $\int \cos(x)\cos(\cos(x))\sin(x)\sin(\cos(x))\,dx$

Optimal. Leaf size=28

$$\frac{\cos(x)}{4}-\frac{1}{2}\cos(x)\sin^2(\cos(x))-\frac{1}{4}\cos(\cos(x))\sin(\cos(x))$$

```
[Out] 1/4*cos(x)-1/4*cos(cos(x))*sin(cos(x))-1/2*cos(x)*sin(cos(x))^2
```

Rubi [A] time = 0.03, antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {4335, 3443, 2635, 8}

$$\frac{\cos(x)}{4}-\frac{1}{2}\cos(x)\sin^2(\cos(x))-\frac{1}{4}\cos(\cos(x))\sin(\cos(x))$$

Antiderivative was successfully verified.

```
[In] Int[Cos[x]*Cos[Cos[x]]*Sin[x]*Sin[Cos[x]],x]
```

```
[Out] Cos[x]/4 - (Cos[Cos[x]]*Sin[Cos[x]])/4 - (Cos[x]*Sin[Cos[x]]^2)/2
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 3443

```
Int[Cos[(a_.) + (b_.)*(x_)^(n_.)]*(x_)^(m_.)*Sin[(a_.) + (b_.)*(x_)^(n_.)]^
(p_.), x_Symbol] :> Simp[(x^(m - n + 1)*Sin[a + b*x^n]^(p + 1))/(b*n*(p + 1
)), x] - Dist[(m - n + 1)/(b*n*(p + 1)), Int[x^(m - n)*Sin[a + b*x^n]^(p +
1), x], x] /; FreeQ[{a, b, p}, x] && LtQ[0, n, m + 1] && NeQ[p, -1]
```

Rule 4335

```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{d = FreeFacto
rs[Cos[c*(a + b*x)], x]}, -Dist[d/(b*c), Subst[Int[SubstFor[1, Cos[c*(a + b
*x)]/d, u, x], x], x, Cos[c*(a + b*x)]/d], x] /; FunctionOfQ[Cos[c*(a + b*x
)]/d, u, x, True]] /; FreeQ[{a, b, c}, x] && (EqQ[F, Sin] || EqQ[F, sin])
```

Rubi steps

$$\int \cos(x)\cos(\cos(x))\sin(x)\sin(\cos(x))\,dx = -\text{Subst}(\int x\cos(x)\sin(x)\,dx, x, \cos(x))$$
$$= -\frac{1}{2}\cos(x)\sin^2(\cos(x)) + \frac{1}{2}\text{Subst}\left(\int \sin^2(x)\,dx, x, \cos(x)\right)$$
$$= -\frac{1}{4}\cos(\cos(x))\sin(\cos(x)) - \frac{1}{2}\cos(x)\sin^2(\cos(x)) + \frac{1}{4}\text{Subst}(\int$$
$$= \frac{\cos(x)}{4} - \frac{1}{4}\cos(\cos(x))\sin(\cos(x)) - \frac{1}{2}\cos(x)\sin^2(\cos(x))$$

Mathematica [A] time = 1.48, size = 21, normalized size = 0.75

$$\frac{1}{4}\cos(x)\cos(2\cos(x)) - \frac{1}{8}\sin(2\cos(x))$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[x]*Cos[Cos[x]]*Sin[x]*Sin[Cos[x]],x]
[Out] (Cos[x]*Cos[2*Cos[x]])/4 - Sin[2*Cos[x]]/8
```

fricas [B] time = 1.00, size = 73, normalized size = 2.61

$$\frac{1}{2}\cos(x)\cos\left(\frac{\tan\left(\frac{1}{2}x\right)^2 - 1}{\tan\left(\frac{1}{2}x\right)^2 + 1}\right)^2 + \frac{1}{4}\cos\left(\frac{\tan\left(\frac{1}{2}x\right)^2 - 1}{\tan\left(\frac{1}{2}x\right)^2 + 1}\right)\sin\left(\frac{\tan\left(\frac{1}{2}x\right)^2 - 1}{\tan\left(\frac{1}{2}x\right)^2 + 1}\right) - \frac{1}{4}\cos(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*cos(cos(x))*sin(x)*sin(cos(x)),x, algorithm="fricas")
[Out] 1/2*cos(x)*cos((tan(1/2*x)^2 - 1)/(tan(1/2*x)^2 + 1))^2 + 1/4*cos((tan(1/2*
x)^2 - 1)/(tan(1/2*x)^2 + 1))*sin((tan(1/2*x)^2 - 1)/(tan(1/2*x)^2 + 1)) -
1/4*cos(x)
```

giac [A] time = 0.14, size = 17, normalized size = 0.61

$$\frac{1}{4}\cos(x)\cos(2\cos(x)) - \frac{1}{8}\sin(2\cos(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*cos(cos(x))*sin(x)*sin(cos(x)),x, algorithm="giac")
```

```
[Out] 1/4*cos(x)*cos(2*cos(x)) - 1/8*sin(2*cos(x))
```

maple [A] time = 0.01, size = 23, normalized size = 0.82

$$\frac{\left(\cos^2\left(\cos(x)\right)\right)\cos(x)}{2}-\frac{\cos\left(\cos(x)\right)\sin\left(\cos(x)\right)}{4}-\frac{\cos(x)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)*cos(cos(x))*sin(x)*sin(cos(x)),x)
```

```
[Out] 1/2*cos(cos(x))^2*cos(x)-1/4*cos(cos(x))*sin(cos(x))-1/4*cos(x)
```

maxima [A] time = 0.59, size = 17, normalized size = 0.61

$$\frac{1}{4}\cos(x)\cos\left(2\cos(x)\right)-\frac{1}{8}\sin\left(2\cos(x)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*cos(cos(x))*sin(x)*sin(cos(x)),x, algorithm="maxima")
```

```
[Out] 1/4*cos(x)*cos(2*cos(x)) - 1/8*sin(2*cos(x))
```

mupad [B] time = 3.05, size = 22, normalized size = 0.79

$$\frac{\cos(x)\cos\left(\cos(x)\right)^2}{2}-\frac{\sin\left(\cos(x)\right)\cos\left(\cos(x)\right)}{4}-\frac{\cos(x)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(cos(x))*sin(cos(x))*cos(x)*sin(x),x)
```

```
[Out] (cos(cos(x))^2*cos(x))/2 - cos(x)/4 - (cos(cos(x))*sin(cos(x)))/4
```

sympy [A] time = 5.57, size = 34, normalized size = 1.21

$$-\frac{\sin^2\left(\cos\left(x\right)\right)\cos\left(x\right)}{4}-\frac{\sin\left(\cos\left(x\right)\right)\cos\left(\cos\left(x\right)\right)}{4}+\frac{\cos\left(x\right)\cos^2\left(\cos\left(x\right)\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*cos(cos(x))*sin(x)*sin(cos(x)),x)
```

```
[Out] -sin(cos(x))**2*cos(x)/4 - sin(cos(x))*cos(cos(x))/4 + cos(x)*cos(cos(x))**
2/4
```

3.653 $\int \cos(\cos(x))\sin(x)\sin^2(6\cos(x))\,dx$

Optimal. Leaf size=26

$$-\frac{1}{2}\sin(\cos(x))+\frac{1}{44}\sin(11\cos(x))+\frac{1}{52}\sin(13\cos(x))$$

```
[Out] -1/2*sin(cos(x))+1/44*sin(11*cos(x))+1/52*sin(13*cos(x))
```

Rubi [A] time = 0.05, antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {4335, 4354, 2637}

$$-\frac{1}{2}\sin(\cos(x))+\frac{1}{44}\sin(11\cos(x))+\frac{1}{52}\sin(13\cos(x))$$

Antiderivative was successfully verified.

```
[In] Int[Cos[Cos[x]]*Sin[x]*Sin[6*Cos[x]]^2,x]
```

```
[Out] -Sin[Cos[x]]/2 + Sin[11*Cos[x]]/44 + Sin[13*Cos[x]]/52
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 4335

```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{d = FreeFacto
rs[Cos[c*(a + b*x)], x]}, -Dist[d/(b*c), Subst[Int[SubstFor[1, Cos[c*(a + b
*x)]/d, u, x], x], x, Cos[c*(a + b*x)]/d], x] /; FunctionOfQ[Cos[c*(a + b*x
)]/d, u, x, True]] /; FreeQ[{a, b, c}, x] && (EqQ[F, Sin] || EqQ[F, sin])
```

Rule 4354

```
Int[(F_)[(a_.) + (b_.)*(x_)]^(p_.)*(G_)[(c_.) + (d_.)*(x_)]^(q_.), x_Symbol
] :> Int[ExpandTrigReduce[ActivateTrig[F[a + b*x]^p*G[c + d*x]^q], x], x] /
; FreeQ[{a, b, c, d}, x] && (EqQ[F, sin] || EqQ[F, cos]) && (EqQ[G, sin] ||
 EqQ[G, cos]) && IGtQ[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\int \cos(\cos(x))\sin(x)\sin^2(6\cos(x))\,dx = -\operatorname{Subst}\left(\int \cos(x)\sin^2(6x)\,dx, x, \cos(x)\right)$$
$$= -\operatorname{Subst}\left(\int \left(\frac{\cos(x)}{2} - \frac{1}{4}\cos(11x) - \frac{1}{4}\cos(13x)\right)dx, x, \cos(x)\right)$$
$$= \frac{1}{4}\operatorname{Subst}(\int \cos(11x)\,dx, x, \cos(x)) + \frac{1}{4}\operatorname{Subst}(\int \cos(13x)\,dx, x, \cos(x))$$
$$= -\frac{1}{2}\sin(\cos(x)) + \frac{1}{44}\sin(11\cos(x)) + \frac{1}{52}\sin(13\cos(x))$$

Mathematica [**A**] time = 4.50, size = 26, normalized size = 1.00

$$-\frac{1}{2}\sin(\cos(x)) + \frac{1}{44}\sin(11\cos(x)) + \frac{1}{52}\sin(13\cos(x))$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[Cos[x]]*Sin[x]*Sin[6*Cos[x]]^2,x]
```

```
[Out] -1/2*Sin[Cos[x]] + Sin[11*Cos[x]]/44 + Sin[13*Cos[x]]/52
```

fricas [**B**] time = 1.92, size = 168, normalized size = 6.46

$$-\frac{4}{143}\left(2816\cos\left(\frac{\tan\left(\frac{1}{2}x\right)^2-1}{\tan\left(\frac{1}{2}x\right)^2+1}\right)^{12} - 6912\cos\left(\frac{\tan\left(\frac{1}{2}x\right)^2-1}{\tan\left(\frac{1}{2}x\right)^2+1}\right)^{10} + 6048\cos\left(\frac{\tan\left(\frac{1}{2}x\right)^2-1}{\tan\left(\frac{1}{2}x\right)^2+1}\right)^{8} - 2240\cos\left(\cdots\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(cos(x))*sin(x)*sin(6*cos(x))^2,x, algorithm="fricas")
```

```
[Out] -4/143*(2816*cos((tan(1/2*x)^2 - 1)/(tan(1/2*x)^2 + 1))^12 - 6912*cos((tan(
1/2*x)^2 - 1)/(tan(1/2*x)^2 + 1))^10 + 6048*cos((tan(1/2*x)^2 - 1)/(tan(1/2
*x)^2 + 1))^8 - 2240*cos((tan(1/2*x)^2 - 1)/(tan(1/2*x)^2 + 1))^6 + 315*cos
((tan(1/2*x)^2 - 1)/(tan(1/2*x)^2 + 1))^4 - 9*cos((tan(1/2*x)^2 - 1)/(tan(1
/2*x)^2 + 1))^2 - 18)*sin((tan(1/2*x)^2 - 1)/(tan(1/2*x)^2 + 1))
```

giac [**A**] time = 0.15, size = 20, normalized size = 0.77

$$\frac{1}{52}\sin\left(13\cos(x)\right) + \frac{1}{44}\sin\left(11\cos(x)\right) - \frac{1}{2}\sin\left(\cos(x)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(cos(x))*sin(x)*sin(6*cos(x))^2,x, algorithm="giac")
```

```
[Out] 1/52*sin(13*cos(x)) + 1/44*sin(11*cos(x)) - 1/2*sin(cos(x))
```

maple [A] time = 0.16, size = 21, normalized size = 0.81

$$-\frac{\sin(\cos(x))}{2}+\frac{\sin(11\cos(x))}{44}+\frac{\sin(13\cos(x))}{52}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(cos(x))*sin(x)*sin(6*cos(x))^2,x)
```

```
[Out] -1/2*sin(cos(x))+1/44*sin(11*cos(x))+1/52*sin(13*cos(x))
```

maxima [A] time = 0.33, size = 20, normalized size = 0.77

$$\frac{1}{52}\sin(13\cos(x))+\frac{1}{44}\sin(11\cos(x))-\frac{1}{2}\sin(\cos(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(cos(x))*sin(x)*sin(6*cos(x))^2,x, algorithm="maxima")
```

```
[Out] 1/52*sin(13*cos(x)) + 1/44*sin(11*cos(x)) - 1/2*sin(cos(x))
```

mupad [B] time = 3.13, size = 20, normalized size = 0.77

$$\frac{\sin(11\cos(x))}{44}-\frac{\sin(\cos(x))}{2}+\frac{\sin(13\cos(x))}{52}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(cos(x))*sin(6*cos(x))^2*sin(x),x)
```

```
[Out] sin(11*cos(x))/44 - sin(cos(x))/2 + sin(13*cos(x))/52
```

sympy [B] time = 47.65, size = 54, normalized size = 2.08

$$-\frac{71\sin(\cos(x))\sin^2(6\cos(x))}{143}-\frac{72\sin(\cos(x))\cos^2(6\cos(x))}{143}+\frac{12\sin(6\cos(x))\cos(\cos(x))\cos(6\cos(x))}{143}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(cos(x))*sin(x)*sin(6*cos(x))**2,x)
```

```
[Out] -71*sin(cos(x))*sin(6*cos(x))**2/143 - 72*sin(cos(x))*cos(6*cos(x))**2/143
+ 12*sin(6*cos(x))*cos(cos(x))*cos(6*cos(x))/143
```

3.654 $\int \cos^3(x) \left(a+b\cos^2(x)\right)^3 \sin(x)\,dx$

Optimal. Leaf size=36

$$\frac{a\left(a+b\cos^2(x)\right)^4}{8b^2}-\frac{\left(a+b\cos^2(x)\right)^5}{10b^2}$$

```
[Out] 1/8*a*(a+b*cos(x)^2)^4/b^2-1/10*(a+b*cos(x)^2)^5/b^2
```

Rubi [A] time = 0.09, antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {4335, 266, 43}

$$\frac{a\left(a+b\cos^2(x)\right)^4}{8b^2}-\frac{\left(a+b\cos^2(x)\right)^5}{10b^2}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[x]^3*(a + b*Cos[x]^2)^3*Sin[x],x]
```

```
[Out] (a*(a + b*Cos[x]^2)^4)/(8*b^2) - (a + b*Cos[x]^2)^5/(10*b^2)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 4335

```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{d = FreeFacto
rs[Cos[c*(a + b*x)], x]}, -Dist[d/(b*c), Subst[Int[SubstFor[1, Cos[c*(a + b
*x)]/d, u, x], x], x, Cos[c*(a + b*x)]/d], x] /; FunctionOfQ[Cos[c*(a + b*x
)]/d, u, x, True]] /; FreeQ[{a, b, c}, x] && (EqQ[F, Sin] || EqQ[F, sin])
```

Rubi steps

$$
\begin{aligned}
\int \cos^3(x)\left(a+b\cos^2(x)\right)^3\sin(x)\,dx &= -\text{Subst}\left(\int x^3\left(a+bx^2\right)^3\,dx,x,\cos(x)\right)\\
&= -\left(\frac{1}{2}\text{Subst}\left(\int x(a+bx)^3\,dx,x,\cos^2(x)\right)\right)\\
&= -\left(\frac{1}{2}\text{Subst}\left(\int\left(-\frac{a(a+bx)^3}{b}+\frac{(a+bx)^4}{b}\right)dx,x,\cos^2(x)\right)\right)\\
&= \frac{a\left(a+b\cos^2(x)\right)^4}{8b^2}-\frac{\left(a+b\cos^2(x)\right)^5}{10b^2}
\end{aligned}
$$

Mathematica [B] time = 0.28, size = 137, normalized size = 3.81

$$
\frac{1}{32}\left(-4a^3\cos(2x)-a^3\cos(4x)-12a^2b\cos^4(x)-4a^2b\cos(3x)\cos^3(x)-8ab^2\cos^6(x)-\frac{1}{32}ab^2(48\cos(2x)+36\cos
$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[x]^3*(a + b*Cos[x]^2)^3*Sin[x],x]
```

```
[Out] (-12*a^2*b*Cos[x]^4 - 8*a*b^2*Cos[x]^6 - 2*b^3*Cos[x]^8 - 4*a^3*Cos[2*x] -
4*a^2*b*Cos[x]^3*Cos[3*x] - a^3*Cos[4*x] - (a*b^2*(48*Cos[2*x] + 36*Cos[4*x
] + 16*Cos[6*x] + 3*Cos[8*x]))/32 - (b^3*(140*Cos[2*x] + 100*Cos[4*x] + 50*
Cos[6*x] + 15*Cos[8*x] + 2*Cos[10*x]))/320)/32
```

fricas [A] time = 0.92, size = 39, normalized size = 1.08

$$
-\frac{1}{10}\,b^3\cos(x)^{10}-\frac{3}{8}\,ab^2\cos(x)^8-\frac{1}{2}\,a^2b\cos(x)^6-\frac{1}{4}\,a^3\cos(x)^4
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^3*(a+b*cos(x)^2)^3*sin(x),x, algorithm="fricas")
```

```
[Out] -1/10*b^3*cos(x)^10 - 3/8*a*b^2*cos(x)^8 - 1/2*a^2*b*cos(x)^6 - 1/4*a^3*cos
(x)^4
```

giac [A] time = 0.13, size = 39, normalized size = 1.08

$$
-\frac{1}{10}\,b^3\cos(x)^{10}-\frac{3}{8}\,ab^2\cos(x)^8-\frac{1}{2}\,a^2b\cos(x)^6-\frac{1}{4}\,a^3\cos(x)^4
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^3*(a+b*cos(x)^2)^3*sin(x),x, algorithm="giac")
```

```
[Out] -1/10*b^3*cos(x)^10 - 3/8*a*b^2*cos(x)^8 - 1/2*a^2*b*cos(x)^6 - 1/4*a^3*cos
(x)^4
```

maple [A] time = 0.01, size = 40, normalized size = 1.11

$$-\frac{b^3\left(\cos^{10}(x)\right)}{10}-\frac{3a\,b^2\left(\cos^{8}(x)\right)}{8}-\frac{a^2b\left(\cos^{6}(x)\right)}{2}-\frac{a^3\left(\cos^{4}(x)\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)^3*(a+b*cos(x)^2)^3*sin(x),x)
```

```
[Out] -1/10*b^3*cos(x)^10-3/8*a*b^2*cos(x)^8-1/2*a^2*b*cos(x)^6-1/4*a^3*cos(x)^4
```

maxima [B] time = 0.39, size = 103, normalized size = 2.86

$$\frac{1}{10}\,b^3\sin(x)^{10}-\frac{1}{8}\left(3\,ab^2+4\,b^3\right)\sin(x)^8+\frac{1}{2}\left(a^2b+3\,ab^2+2\,b^3\right)\sin(x)^6-\frac{1}{4}\left(a^3+6\,a^2b+9\,ab^2+4\,b^3\right)\sin(x)^4+\frac{1}{2}\,($$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^3*(a+b*cos(x)^2)^3*sin(x),x, algorithm="maxima")
```

```
[Out] 1/10*b^3*sin(x)^10 - 1/8*(3*a*b^2 + 4*b^3)*sin(x)^8 + 1/2*(a^2*b + 3*a*b^2
+ 2*b^3)*sin(x)^6 - 1/4*(a^3 + 6*a^2*b + 9*a*b^2 + 4*b^3)*sin(x)^4 + 1/2*(a
^3 + 3*a^2*b + 3*a*b^2 + b^3)*sin(x)^2
```

mupad [B] time = 0.09, size = 39, normalized size = 1.08

$$-\frac{a^3\cos(x)^4}{4}-\frac{a^2\,b\cos(x)^6}{2}-\frac{3\,a\,b^2\cos(x)^8}{8}-\frac{b^3\cos(x)^{10}}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)^3*sin(x)*(a + b*cos(x)^2)^3,x)
```

```
[Out] - (a^3*cos(x)^4)/4 - (b^3*cos(x)^10)/10 - (a^2*b*cos(x)^6)/2 - (3*a*b^2*cos
(x)^8)/8
```

sympy [A] time = 11.65, size = 46, normalized size = 1.28

$$-\frac{a^3\cos^4(x)}{4}-\frac{a^2b\cos^6(x)}{2}-\frac{3ab^2\cos^8(x)}{8}-\frac{b^3\cos^{10}(x)}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)**3*(a+b*cos(x)**2)**3*sin(x),x)
```

```
[Out] -a**3*cos(x)**4/4 - a**2*b*cos(x)**6/2 - 3*a*b**2*cos(x)**8/8 - b**3*cos(x)
**10/10
```

3.655 $\int \sin(3x)\sin(\cos(3x))\,dx$

Optimal. Leaf size=9

$$\frac{1}{3}\cos(\cos(3x))$$

```
[Out] 1/3*cos(cos(3*x))
```

Rubi [A] time = 0.01, antiderivative size = 9, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {4335, 2638}

$$\frac{1}{3}\cos(\cos(3x))$$

Antiderivative was successfully verified.

```
[In] Int[Sin[3*x]*Sin[Cos[3*x]],x]
```

```
[Out] Cos[Cos[3*x]]/3
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 4335

```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{d = FreeFacto
rs[Cos[c*(a + b*x)], x]}, -Dist[d/(b*c), Subst[Int[SubstFor[1, Cos[c*(a + b
*x)]/d, u, x], x], x, Cos[c*(a + b*x)]/d], x] /; FunctionOfQ[Cos[c*(a + b*x
)]/d, u, x, True]] /; FreeQ[{a, b, c}, x] && (EqQ[F, Sin] || EqQ[F, sin])
```

Rubi steps

$$\begin{aligned}\int \sin(3x)\sin(\cos(3x))\,dx &= -\left(\frac{1}{3}\text{Subst}\left(\int \sin(x)\,dx, x, \cos(3x)\right)\right)\\ &= \frac{1}{3}\cos(\cos(3x))\end{aligned}$$

Mathematica [A] time = 2.68, size = 9, normalized size = 1.00

$$\frac{1}{3}\cos(\cos(3x))$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[3*x]*Sin[Cos[3*x]],x]
```

```
[Out] Cos[Cos[3*x]]/3
```

fricas [B] time = 0.95, size = 22, normalized size = 2.44

$$\frac{1}{3} \cos \left(\frac{\tan \left(\frac{3}{2} x \right)^2 - 1}{\tan \left(\frac{3}{2} x \right)^2 + 1} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(3*x)*sin(cos(3*x)),x, algorithm="fricas")
```

```
[Out] 1/3*cos((tan(3/2*x)^2 - 1)/(tan(3/2*x)^2 + 1))
```

giac [A] time = 0.16, size = 7, normalized size = 0.78

$$\frac{1}{3} \cos \left(\cos \left(3 x \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(3*x)*sin(cos(3*x)),x, algorithm="giac")
```

```
[Out] 1/3*cos(cos(3*x))
```

maple [A] time = 0.01, size = 8, normalized size = 0.89

$$\frac{\cos \left(\cos \left(3x \right) \right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(3*x)*sin(cos(3*x)),x)
```

```
[Out] 1/3*cos(cos(3*x))
```

maxima [A] time = 0.32, size = 7, normalized size = 0.78

$$\frac{1}{3} \cos \left(\cos \left(3 x \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(3*x)*sin(cos(3*x)),x, algorithm="maxima")
```

```
[Out] 1/3*cos(cos(3*x))
```

mupad [B] time = 3.01, size = 7, normalized size = 0.78

$$\frac{\cos(\cos(3x))}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(3*x)*sin(cos(3*x)),x)
```

```
[Out] cos(cos(3*x))/3
```

sympy [A] time = 0.42, size = 7, normalized size = 0.78

$$\frac{\cos(\cos(3x))}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(3*x)*sin(cos(3*x)),x)
```

```
[Out] cos(cos(3*x))/3
```

3.656 $\int e^{\cos(1+3x)} \cos(1+3x) \sin(1+3x) \, dx$

Optimal. Leaf size=31

$$\frac{1}{3} e^{\cos(3x+1)} - \frac{1}{3} e^{\cos(3x+1)} \cos(3x+1)$$

```
[Out] 1/3*exp(cos(1+3*x))-1/3*exp(cos(1+3*x))*cos(1+3*x)
```

Rubi [A] time = 0.02, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {4335, 2176, 2194}

$$\frac{1}{3} e^{\cos(3x+1)} - \frac{1}{3} e^{\cos(3x+1)} \cos(3x+1)$$

Antiderivative was successfully verified.

```
[In] Int[E^Cos[1 + 3*x]*Cos[1 + 3*x]*Sin[1 + 3*x],x]
```

```
[Out] E^Cos[1 + 3*x]/3 - (E^Cos[1 + 3*x]*Cos[1 + 3*x])/3
```

Rule 2176

```
Int[((b_.)*(F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m
_.), x_Symbol] :> Simp[((c + d*x)^m*(b*F^(g*(e + f*x)))^n)/(f*g*n*Log[F]),
x] - Dist[(d*m)/(f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*(b*F^(g*(e + f*x)))^n
, x], x] /; FreeQ[{F, b, c, d, e, f, g, n}, x] && GtQ[m, 0] && IntegerQ[2*m
] &&  !$UseGamma === True
```

Rule 2194

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rule 4335

```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{d = FreeFacto
rs[Cos[c*(a + b*x)], x]}, -Dist[d/(b*c), Subst[Int[SubstFor[1, Cos[c*(a + b
*x)]/d, u, x], x], x, Cos[c*(a + b*x)]/d], x] /; FunctionOfQ[Cos[c*(a + b*x
)]/d, u, x, True]] /; FreeQ[{a, b, c}, x] && (EqQ[F, Sin] || EqQ[F, sin])
```

Rubi steps

$$\int e^{\cos(1+3x)}\cos(1+3x)\sin(1+3x)\,dx = -\left(\frac{1}{3}\operatorname{Subst}\left(\int e^x x\,dx, x, \cos(1+3x)\right)\right)$$
$$= -\frac{1}{3}e^{\cos(1+3x)}\cos(1+3x) + \frac{1}{3}\operatorname{Subst}\left(\int e^x\,dx, x, \cos(1+3x)\right)$$
$$= \frac{1}{3}e^{\cos(1+3x)} - \frac{1}{3}e^{\cos(1+3x)}\cos(1+3x)$$

Mathematica [A] time = 0.12, size = 24, normalized size = 0.77

$$\frac{2}{3}\sin^2\left(\frac{1}{2}(3x+1)\right)e^{\cos(3x+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^Cos[1 + 3*x]*Cos[1 + 3*x]*Sin[1 + 3*x],x]
[Out] (2*E^Cos[1 + 3*x]*Sin[(1 + 3*x)/2]^2)/3
```

fricas [A] time = 0.84, size = 17, normalized size = 0.55

$$-\frac{1}{3}\left(\cos\left(3\,x+1\right)-1\right)e^{\left(\cos\left(3\,x+1\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(cos(1+3*x))*cos(1+3*x)*sin(1+3*x),x, algorithm="fricas")
[Out] -1/3*(cos(3*x + 1) - 1)*e^(cos(3*x + 1))
```

giac [A] time = 0.15, size = 17, normalized size = 0.55

$$-\frac{1}{3}\left(\cos\left(3\,x+1\right)-1\right)e^{\left(\cos\left(3\,x+1\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(cos(1+3*x))*cos(1+3*x)*sin(1+3*x),x, algorithm="giac")
[Out] -1/3*(cos(3*x + 1) - 1)*e^(cos(3*x + 1))
```

maple [A] time = 0.01, size = 26, normalized size = 0.84

$$\frac{e^{\cos(1+3x)}}{3} - \frac{e^{\cos(1+3x)}\cos\left(1+3x\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(cos(1+3*x))*cos(1+3*x)*sin(1+3*x),x)
```

```
[Out] 1/3*exp(cos(1+3*x))-1/3*exp(cos(1+3*x))*cos(1+3*x)
```

maxima [A] time = 0.32, size = 17, normalized size = 0.55

$$-\frac{1}{3}\left(\cos\left(3\,x+1\right)-1\right)e^{\left(\cos\left(3\,x+1\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(cos(1+3*x))*cos(1+3*x)*sin(1+3*x),x, algorithm="maxima")
```

```
[Out] -1/3*(cos(3*x + 1) - 1)*e^(cos(3*x + 1))
```

mupad [B] time = 0.12, size = 17, normalized size = 0.55

$$-\frac{\mathrm{e}^{\cos\left(3\,x+1\right)}\,\left(\cos\left(3\,x+1\right)-1\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(cos(3*x + 1))*cos(3*x + 1)*sin(3*x + 1),x)
```

```
[Out] -(exp(cos(3*x + 1))*(cos(3*x + 1) - 1))/3
```

sympy [A] time = 0.67, size = 26, normalized size = 0.84

$$-\frac{e^{\cos\left(3x+1\right)}\cos\left(3x+1\right)}{3}+\frac{e^{\cos\left(3x+1\right)}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(cos(1+3*x))*cos(1+3*x)*sin(1+3*x),x)
```

```
[Out] -exp(cos(3*x + 1))*cos(3*x + 1)/3 + exp(cos(3*x + 1))/3
```

3.657 $\int \frac{\cos^2(x)\sin(x)}{\sqrt{1-\cos^6(x)}}\,dx$

Optimal. Leaf size=9

$$-\frac{1}{3}\sin^{-1}\left(\cos^3(x)\right)$$

```
[Out] -1/3*arcsin(cos(x)^3)
```

Rubi [A] time = 0.07, antiderivative size = 9, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {4335, 275, 216}

$$-\frac{1}{3}\sin^{-1}\left(\cos^3(x)\right)$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[x]^2*Sin[x])/Sqrt[1 - Cos[x]^6],x]
```

```
[Out] -ArcSin[Cos[x]^3]/3
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 4335

```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{d = FreeFacto
rs[Cos[c*(a + b*x)], x]}, -Dist[d/(b*c), Subst[Int[SubstFor[1, Cos[c*(a + b
*x)]/d, u, x], x], x, Cos[c*(a + b*x)]/d], x] /; FunctionOfQ[Cos[c*(a + b*x
)]/d, u, x, True]] /; FreeQ[{a, b, c}, x] && (EqQ[F, Sin] || EqQ[F, sin])
```

Rubi steps

$$\int \frac{\cos^2(x)\sin(x)}{\sqrt{1-\cos^6(x)}}\,dx = -\text{Subst}\left(\int \frac{x^2}{\sqrt{1-x^6}}\,dx, x, \cos(x)\right)$$
$$= -\left(\frac{1}{3}\text{Subst}\left(\int \frac{1}{\sqrt{1-x^2}}\,dx, x, \cos^3(x)\right)\right)$$
$$= -\frac{1}{3}\sin^{-1}\left(\cos^3(x)\right)$$

Mathematica [C] time = 2.22, size = 162, normalized size = 18.00

$$-\frac{i\sin(x)\cos^2(x)\sqrt{1-\frac{2i\tan^2(x)}{\sqrt{3}-3i}}\sqrt{1+\frac{2i\tan^2(x)}{\sqrt{3}+3i}}\,\Pi\left(\frac{3}{2}+\frac{i\sqrt{3}}{2};i\sinh^{-1}\left(\sqrt{-\frac{2i}{-3i+\sqrt{3}}}\tan(x)\right)|\frac{3i-\sqrt{3}}{3i+\sqrt{3}}\right)}{\sqrt{2}\sqrt{-\frac{i}{\sqrt{3}-3i}}\sqrt{1-\cos^6(x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[x]^2*Sin[x])/Sqrt[1 - Cos[x]^6],x]
```

```
[Out] ((-I)*Cos[x]^2*EllipticPi[3/2 + (I/2)*Sqrt[3], I*ArcSinh[Sqrt[(-2*I)/(-3*I
+ Sqrt[3])]*Tan[x]], (3*I - Sqrt[3])/(3*I + Sqrt[3])]*Sin[x]*Sqrt[1 - ((2*I
)*Tan[x]^2)/(-3*I + Sqrt[3])]*Sqrt[1 + ((2*I)*Tan[x]^2)/(3*I + Sqrt[3])])/(
Sqrt[2]*Sqrt[(-I)/(-3*I + Sqrt[3])]*Sqrt[1 - Cos[x]^6])
```

fricas [B] time = 1.11, size = 29, normalized size = 3.22

$$\frac{1}{6}\arctan\left(\frac{2\sqrt{-\cos(x)^6+1}\cos(x)^3}{2\cos(x)^6-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^2*sin(x)/(1-cos(x)^6)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/6*arctan(2*sqrt(-cos(x)^6 + 1)*cos(x)^3/(2*cos(x)^6 - 1))
```

giac [A] time = 0.15, size = 7, normalized size = 0.78

$$-\frac{1}{3}\arcsin\left(\cos(x)^3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^2*sin(x)/(1-cos(x)^6)^(1/2),x, algorithm="giac")
```

```
[Out] -1/3*arcsin(cos(x)^3)
```

maple [F] time = 1.26, size = 0, normalized size = 0.00

$$\int \frac{\left(\cos^2(x)\right)\sin(x)}{\sqrt{1-\left(\cos^6(x)\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)^2*sin(x)/(1-cos(x)^6)^(1/2),x)
[Out] int(cos(x)^2*sin(x)/(1-cos(x)^6)^(1/2),x)
```

maxima [B] time = 0.60, size = 18, normalized size = 2.00

$$\frac{1}{3}\arctan\left(\frac{\sqrt{-\cos(x)^6+1}}{\cos(x)^3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^2*sin(x)/(1-cos(x)^6)^(1/2),x, algorithm="maxima")
[Out] 1/3*arctan(sqrt(-cos(x)^6 + 1)/cos(x)^3)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.11

$$\int \frac{\cos(x)^2\,\sin(x)}{\sqrt{1-\cos(x)^6}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(x)^2*sin(x))/(1 - cos(x)^6)^(1/2),x)
[Out] int((cos(x)^2*sin(x))/(1 - cos(x)^6)^(1/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)**2*sin(x)/(1-cos(x)**6)**(1/2),x)
[Out] Timed out
```

3.658 $\int \frac{\sin^5(x)}{\sqrt{1-5\cos(x)}}\,dx$

Optimal. Leaf size=71

$$\frac{2(1-5\cos(x))^{9/2}}{28125}-\frac{8(1-5\cos(x))^{7/2}}{21875}-\frac{88(1-5\cos(x))^{5/2}}{15625}+\frac{64(1-5\cos(x))^{3/2}}{3125}+\frac{1152\sqrt{1-5\cos(x)}}{3125}$$

```
[Out] 64/3125*(1-5*cos(x))^(3/2)-88/15625*(1-5*cos(x))^(5/2)-8/21875*(1-5*cos(x))
^(7/2)+2/28125*(1-5*cos(x))^(9/2)+1152/3125*(1-5*cos(x))^(1/2)
```

Rubi [A] time = 0.07, antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {2668, 697}

$$\frac{2(1-5\cos(x))^{9/2}}{28125}-\frac{8(1-5\cos(x))^{7/2}}{21875}-\frac{88(1-5\cos(x))^{5/2}}{15625}+\frac{64(1-5\cos(x))^{3/2}}{3125}+\frac{1152\sqrt{1-5\cos(x)}}{3125}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[x]^5/Sqrt[1 - 5*Cos[x]],x]
```

```
[Out] (1152*Sqrt[1 - 5*Cos[x]])/3125 + (64*(1 - 5*Cos[x])^(3/2))/3125 - (88*(1 -
5*Cos[x])^(5/2))/15625 - (8*(1 - 5*Cos[x])^(7/2))/21875 + (2*(1 - 5*Cos[x])
^(9/2))/28125
```

Rule 697

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m},
 x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0]
```

Rule 2668

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\sin^5(x)}{\sqrt{1-5\cos(x)}}\,dx = \frac{\text{Subst}\left(\int \frac{(25-x^2)^2}{\sqrt{1+x}}\,dx, x, -5\cos(x)\right)}{3125}$$
$$= \frac{\text{Subst}\left(\int \left(\frac{576}{\sqrt{1+x}} + 96\sqrt{1+x} - 44(1+x)^{3/2} - 4(1+x)^{5/2} + (1+x)^{7/2}\right)dx, x, -5\cos(x)\right)}{3125}$$
$$= \frac{1152\sqrt{1-5\cos(x)}}{3125} + \frac{64(1-5\cos(x))^{3/2}}{3125} - \frac{88(1-5\cos(x))^{5/2}}{15625} - \frac{8(1-5\cos(x))^{7/2}}{21875} + \frac{2(}{}$$

Mathematica [A] time = 0.16, size = 59, normalized size = 0.83

$$\frac{180607\left(\sqrt{1-5\cos(x)}-1\right)}{562500}+\sqrt{1-5\cos(x)}\left(-\frac{6772\cos(x)}{196875}-\frac{2227\cos(2x)}{39375}+\frac{4\cos(3x)}{1575}+\frac{1}{180}\cos(4x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[x]^5/Sqrt[1 - 5*Cos[x]],x]
```

```
[Out] (180607*(-1 + Sqrt[1 - 5*Cos[x]]))/562500 + Sqrt[1 - 5*Cos[x]]*((-6772*Cos[
x])/196875 - (2227*Cos[2*x])/39375 + (4*Cos[3*x])/1575 + Cos[4*x]/180)
```

fricas [A] time = 1.68, size = 34, normalized size = 0.48

$$\frac{2}{984375}\left(21875\cos(x)^4+5000\cos(x)^3-77550\cos(x)^2-20680\cos(x)+188603\right)\sqrt{-5\cos(x)+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^5/(1-5*cos(x))^(1/2),x, algorithm="fricas")
```

```
[Out] 2/984375*(21875*cos(x)^4 + 5000*cos(x)^3 - 77550*cos(x)^2 - 20680*cos(x) +
188603)*sqrt(-5*cos(x) + 1)
```

giac [A] time = 0.14, size = 75, normalized size = 1.06

$$\frac{2}{28125}(5\cos(x)-1)^4\sqrt{-5\cos(x)+1}+\frac{8}{21875}(5\cos(x)-1)^3\sqrt{-5\cos(x)+1}-\frac{88}{15625}(5\cos(x)-1)^2\sqrt{-5\cos(}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^5/(1-5*cos(x))^(1/2),x, algorithm="giac")
```

[Out] 2/28125*(5*cos(x) - 1)^4*sqrt(-5*cos(x) + 1) + 8/21875*(5*cos(x) - 1)^3*sqr
t(-5*cos(x) + 1) - 88/15625*(5*cos(x) - 1)^2*sqrt(-5*cos(x) + 1) + 64/3125*
(-5*cos(x) + 1)^(3/2) + 1152/3125*sqrt(-5*cos(x) + 1)

maple [A] time = 0.17, size = 49, normalized size = 0.69

$$\frac{32\sqrt{10\left(\sin^2\left(\frac{x}{2}\right)\right)-4}\,\left(21875\left(\sin^8\left(\frac{x}{2}\right)\right)-46250\left(\sin^6\left(\frac{x}{2}\right)\right)+17175\left(\sin^4\left(\frac{x}{2}\right)\right)+9160\left(\sin^2\left(\frac{x}{2}\right)\right)+7328\right)}{984375}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sin(x)^5/(1-5*cos(x))^(1/2),x)

[Out] 32/984375*(10*sin(1/2*x)^2-4)^(1/2)*(21875*sin(1/2*x)^8-46250*sin(1/2*x)^6+
17175*sin(1/2*x)^4+9160*sin(1/2*x)^2+7328)

maxima [A] time = 0.32, size = 51, normalized size = 0.72

$$\frac{2}{28125}\,(-5\,\cos(x)+1)^{\frac{9}{2}}-\frac{8}{21875}\,(-5\,\cos(x)+1)^{\frac{7}{2}}-\frac{88}{15625}\,(-5\,\cos(x)+1)^{\frac{5}{2}}+\frac{64}{3125}\,(-5\,\cos(x)+1)^{\frac{3}{2}}+\frac{1152}{3125}\,\sqrt{-5\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(x)^5/(1-5*cos(x))^(1/2),x, algorithm="maxima")

[Out] 2/28125*(-5*cos(x) + 1)^(9/2) - 8/21875*(-5*cos(x) + 1)^(7/2) - 88/15625*(-
5*cos(x) + 1)^(5/2) + 64/3125*(-5*cos(x) + 1)^(3/2) + 1152/3125*sqrt(-5*cos
(x) + 1)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sin(x)^5}{\sqrt{1-5\,\cos(x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sin(x)^5/(1 - 5*cos(x))^(1/2),x)

[Out] int(sin(x)^5/(1 - 5*cos(x))^(1/2), x)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(x)**5/(1-5*cos(x))**(1/2),x)

[Out] Timed out

3.659 $\int e^{n\cos(a+bx)}\sin(a+bx)\,dx$

Optimal. Leaf size=18

$$-\frac{e^{n\cos(a+bx)}}{bn}$$

```
[Out] -exp(n*cos(b*x+a))/b/n
```

Rubi [A] time = 0.01, antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {4335, 2194}

$$-\frac{e^{n\cos(a+bx)}}{bn}$$

Antiderivative was successfully verified.

```
[In] Int[E^(n*Cos[a + b*x])*Sin[a + b*x],x]
```

```
[Out] -(E^(n*Cos[a + b*x])/(b*n))
```

Rule 2194

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rule 4335

```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{d = FreeFacto
rs[Cos[c*(a + b*x)], x]}, -Dist[d/(b*c), Subst[Int[SubstFor[1, Cos[c*(a + b
*x)]/d, u, x], x], x, Cos[c*(a + b*x)]/d], x] /; FunctionOfQ[Cos[c*(a + b*x
)]/d, u, x, True]] /; FreeQ[{a, b, c}, x] && (EqQ[F, Sin] || EqQ[F, sin])
```

Rubi steps

$$\begin{aligned}\int e^{n\cos(a+bx)}\sin(a+bx)\,dx &= -\frac{\text{Subst}\left(\int e^{nx}\,dx,x,\cos(a+bx)\right)}{b}\\ &= -\frac{e^{n\cos(a+bx)}}{bn}\end{aligned}$$

Mathematica [A] time = 0.05, size = 18, normalized size = 1.00

$$-\frac{e^{n\cos(a+bx)}}{bn}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(n*Cos[a + b*x])*Sin[a + b*x],x]
[Out] -(E^(n*Cos[a + b*x])/(b*n))
```

fricas [A] time = 1.04, size = 17, normalized size = 0.94

$$-\frac{e^{(n\cos(bx+a))}}{bn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*cos(b*x+a))*sin(b*x+a),x, algorithm="fricas")
[Out] -e^(n*cos(b*x + a))/(b*n)
```

giac [A] time = 0.14, size = 17, normalized size = 0.94

$$-\frac{e^{(n\cos(bx+a))}}{bn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*cos(b*x+a))*sin(b*x+a),x, algorithm="giac")
[Out] -e^(n*cos(b*x + a))/(b*n)
```

maple [A] time = 0.01, size = 18, normalized size = 1.00

$$-\frac{e^{n\cos(bx+a)}}{bn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*cos(b*x+a))*sin(b*x+a),x)
[Out] -exp(n*cos(b*x+a))/b/n
```

maxima [A] time = 0.31, size = 17, normalized size = 0.94

$$-\frac{e^{(n\cos(bx+a))}}{bn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*cos(b*x+a))*sin(b*x+a),x, algorithm="maxima")
[Out] -e^(n*cos(b*x + a))/(b*n)
```

mupad [B] time = 0.10, size = 17, normalized size = 0.94

$$-\frac{\mathrm{e}^{n\,\cos(a+b\,x)}}{b\,n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*cos(a + b*x))*sin(a + b*x),x)
```

```
[Out] -exp(n*cos(a + b*x))/(b*n)
```

sympy [A] time = 0.64, size = 39, normalized size = 2.17

$$\begin{cases} x\sin(a) & \text{for}\ b = 0 \wedge n = 0 \\ xe^{n\cos(a)}\sin(a) & \text{for}\ b = 0 \\ -\frac{\cos(a+bx)}{b} & \text{for}\ n = 0 \\ -\frac{e^{n\cos(a+bx)}}{bn} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*cos(b*x+a))*sin(b*x+a),x)
```

```
[Out] Piecewise((x*sin(a), Eq(b, 0) & Eq(n, 0)), (x*exp(n*cos(a))*sin(a), Eq(b, 0
)), (-cos(a + b*x)/b, Eq(n, 0)), (-exp(n*cos(a + b*x))/(b*n), True))
```

3.660 $\int e^{n\cos(ac+bcx)}\sin(c(a+bx))\,dx$

Optimal. Leaf size=23

$$-\frac{e^{n\cos(c(a+bx))}}{bcn}$$

```
[Out] -exp(n*cos(c*(b*x+a)))/b/c/n
```

Rubi [A] time = 0.01, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {4335, 2194}

$$-\frac{e^{n\cos(c(a+bx))}}{bcn}$$

Antiderivative was successfully verified.

```
[In] Int[E^(n*Cos[a*c + b*c*x])*Sin[c*(a + b*x)],x]
```

```
[Out] -(E^(n*Cos[c*(a + b*x)])/(b*c*n))
```

Rule 2194

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rule 4335

```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{d = FreeFacto
rs[Cos[c*(a + b*x)], x]}, -Dist[d/(b*c), Subst[Int[SubstFor[1, Cos[c*(a + b
*x)]/d, u, x], x], x, Cos[c*(a + b*x)]/d], x] /; FunctionOfQ[Cos[c*(a + b*x
)]/d, u, x, True]] /; FreeQ[{a, b, c}, x] && (EqQ[F, Sin] || EqQ[F, sin])
```

Rubi steps

$$\begin{aligned}\int e^{n\cos(ac+bcx)}\sin(c(a+bx))\,dx &= -\frac{\text{Subst}\left(\int e^{nx}\,dx, x, \cos(c(a+bx))\right)}{bc}\\ &= -\frac{e^{n\cos(c(a+bx))}}{bcn}\end{aligned}$$

Mathematica [A] time = 0.23, size = 23, normalized size = 1.00

$$-\frac{e^{n\cos(c(a+bx))}}{bcn}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(n*Cos[a*c + b*c*x])*Sin[c*(a + b*x)],x]
```

```
[Out] -(E^(n*Cos[c*(a + b*x)])/(b*c*n))
```

fricas [A] time = 1.40, size = 23, normalized size = 1.00

$$-\frac{e^{(n\cos(bcx+ac))}}{bcn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*cos(b*c*x+a*c))*sin(c*(b*x+a)),x, algorithm="fricas")
```

```
[Out] -e^(n*cos(b*c*x + a*c))/(b*c*n)
```

giac [A] time = 0.15, size = 23, normalized size = 1.00

$$-\frac{e^{(n\cos(bcx+ac))}}{bcn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*cos(b*c*x+a*c))*sin(c*(b*x+a)),x, algorithm="giac")
```

```
[Out] -e^(n*cos(b*c*x + a*c))/(b*c*n)
```

maple [A] time = 0.06, size = 24, normalized size = 1.04

$$-\frac{e^{n\cos(bcx+ac)}}{bcn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*cos(b*c*x+a*c))*sin(c*(b*x+a)),x)
```

```
[Out] -exp(n*cos(b*c*x+a*c))/b/c/n
```

maxima [A] time = 0.37, size = 23, normalized size = 1.00

$$-\frac{e^{(n\cos(bcx+ac))}}{bcn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*cos(b*c*x+a*c))*sin(c*(b*x+a)),x, algorithm="maxima")
```

```
[Out] -e^(n*cos(b*c*x + a*c))/(b*c*n)
```

mupad [B] time = 3.16, size = 23, normalized size = 1.00

$$-\frac{e^{n\,\cos(a\,c+b\,c\,x)}}{b\,c\,n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c*(a + b*x))*exp(n*cos(a*c + b*c*x)),x)
```

```
[Out] -exp(n*cos(a*c + b*c*x))/(b*c*n)
```

sympy [A] time = 9.84, size = 54, normalized size = 2.35

$$\begin{cases} xe^{n\cos(ac)}\sin(ac) & \text{for } b=0 \\ 0 & \text{for } c=0 \\ \begin{cases} x\sin(ac) & \text{for } b=0 \\ 0 & \text{for } c=0 \\ -\frac{\cos(ac+bcx)}{bc} & \text{otherwise} \end{cases} & \text{for } n=0 \\ -\frac{e^{n\cos(ac+bcx)}}{bcn} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*cos(b*c*x+a*c))*sin(c*(b*x+a)),x)
```

```
[Out] Piecewise((x*exp(n*cos(a*c))*sin(a*c), Eq(b, 0)), (0, Eq(c, 0)), (Piecewise
((x*sin(a*c), Eq(b, 0)), (0, Eq(c, 0)), (-cos(a*c + b*c*x)/(b*c), True)), E
q(n, 0)), (-exp(n*cos(a*c + b*c*x))/(b*c*n), True))
```

3.661 $\int e^{n\cos(c(a+bx))}\sin(ac+bcx)\,dx$

Optimal. Leaf size=24

$$-\frac{e^{n\cos(ac+bcx)}}{bcn}$$

```
[Out] -exp(n*cos(b*c*x+a*c))/b/c/n
```

Rubi [A] time = 0.01, antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {4335, 2194}

$$-\frac{e^{n\cos(ac+bcx)}}{bcn}$$

Antiderivative was successfully verified.

```
[In] Int[E^(n*Cos[c*(a + b*x)])*Sin[a*c + b*c*x],x]
```

```
[Out] -(E^(n*Cos[a*c + b*c*x])/(b*c*n))
```

Rule 2194

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rule 4335

```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{d = FreeFacto
rs[Cos[c*(a + b*x)], x]}, -Dist[d/(b*c), Subst[Int[SubstFor[1, Cos[c*(a + b
*x)]/d, u, x], x], x, Cos[c*(a + b*x)]/d], x] /; FunctionOfQ[Cos[c*(a + b*x
)]/d, u, x, True]] /; FreeQ[{a, b, c}, x] && (EqQ[F, Sin] || EqQ[F, sin])
```

Rubi steps

$$\begin{aligned}\int e^{n\cos(c(a+bx))}\sin(ac+bcx)\,dx &= -\frac{\text{Subst}\left(\int e^{nx}\,dx, x, \cos(ac+bcx)\right)}{bc}\\ &= -\frac{e^{n\cos(ac+bcx)}}{bcn}\end{aligned}$$

Mathematica [A] time = 0.04, size = 23, normalized size = 0.96

$$-\frac{e^{n\cos(c(a+bx))}}{bcn}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(n*Cos[c*(a + b*x)])*Sin[a*c + b*c*x],x]
[Out] -(E^(n*Cos[c*(a + b*x)])/(b*c*n))
```

fricas [A] time = 0.90, size = 23, normalized size = 0.96

$$-\frac{e^{(n\cos(bcx+ac))}}{bcn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*cos(c*(b*x+a)))*sin(b*c*x+a*c),x, algorithm="fricas")
[Out] -e^(n*cos(b*c*x + a*c))/(b*c*n)
```

giac [A] time = 0.15, size = 23, normalized size = 0.96

$$-\frac{e^{(n\cos(bcx+ac))}}{bcn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*cos(c*(b*x+a)))*sin(b*c*x+a*c),x, algorithm="giac")
[Out] -e^(n*cos(b*c*x + a*c))/(b*c*n)
```

maple [A] time = 0.03, size = 24, normalized size = 1.00

$$-\frac{e^{n\cos(bcx+ac)}}{bcn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*cos(c*(b*x+a)))*sin(b*c*x+a*c),x)
[Out] -exp(n*cos(b*c*x+a*c))/b/c/n
```

maxima [A] time = 0.32, size = 23, normalized size = 0.96

$$-\frac{e^{(n\cos(bcx+ac))}}{bcn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*cos(c*(b*x+a)))*sin(b*c*x+a*c),x, algorithm="maxima")
[Out] -e^(n*cos(b*c*x + a*c))/(b*c*n)
```

mupad [B] time = 3.02, size = 23, normalized size = 0.96

$$-\frac{\mathrm{e}^{n\,\cos(a\,c+b\,c\,x)}}{b\,c\,n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*cos(c*(a + b*x)))*sin(a*c + b*c*x),x)
```

```
[Out] -exp(n*cos(a*c + b*c*x))/(b*c*n)
```

sympy [A] time = 2.23, size = 54, normalized size = 2.25

$$\begin{cases} 0 & \text{for}\: c = 0 \wedge \left(b = 0 \vee c = 0\right) \wedge \left(c = 0 \vee n = 0\right) \\ x e^{n \cos{\left(a c \right)}} \sin{\left(a c \right)} & \text{for}\: b = 0 \\ - \frac{\cos{\left(a c + b c x \right)}}{b c} & \text{for}\: n = 0 \\ - \frac{e^{n \cos{\left(a c + b c x \right)}}}{b c n} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*cos(c*(b*x+a)))*sin(b*c*x+a*c),x)
```

```
[Out] Piecewise((0, Eq(c, 0) & (Eq(b, 0) | Eq(c, 0)) & (Eq(c, 0) | Eq(n, 0))), (x
*exp(n*cos(a*c))*sin(a*c), Eq(b, 0)), (-cos(a*c + b*c*x)/(b*c), Eq(n, 0)),
(-exp(n*cos(a*c + b*c*x))/(b*c*n), True))
```

3.662 $\int e^{n\cos(a+bx)}\tan(a+bx)\,dx$

Optimal. Leaf size=14

$$-\frac{\text{Ei}(n\cos(a+bx))}{b}$$

```
[Out] -Ei(n*cos(b*x+a))/b
```

Rubi [A] time = 0.02, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {4339, 2178}

$$-\frac{\text{Ei}(n\cos(a+bx))}{b}$$

Antiderivative was successfully verified.

```
[In] Int[E^(n*Cos[a + b*x])*Tan[a + b*x],x]
```

```
[Out] -(ExpIntegralEi[n*Cos[a + b*x]]/b)
```

Rule 2178

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - (c*f)/d))*ExpIntegralEi[(f*g*(c + d*x)*Log[F])/d])/d, x] /; F
reeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 4339

```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{d = FreeFacto
rs[Cos[c*(a + b*x)], x]}, -Dist[(b*c)^(-1), Subst[Int[SubstFor[1/x, Cos[c*(
a + b*x)]/d, u, x], x], x, Cos[c*(a + b*x)]/d], x] /; FunctionOfQ[Cos[c*(a
+ b*x)]/d, u, x, True]] /; FreeQ[{a, b, c}, x] && (EqQ[F, Tan] || EqQ[F, ta
n])
```

Rubi steps

$$\begin{aligned}\int e^{n\cos(a+bx)}\tan(a+bx)\,dx &= -\frac{\text{Subst}\left(\int \frac{e^{nx}}{x}\,dx, x, \cos(a+bx)\right)}{b}\\ &= -\frac{\text{Ei}(n\cos(a+bx))}{b}\end{aligned}$$

Mathematica [A] time = 0.04, size = 14, normalized size = 1.00

$$-\frac{\text{Ei}(n\cos(a+bx))}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(n*Cos[a + b*x])*Tan[a + b*x],x]
[Out] -(ExpIntegralEi[n*Cos[a + b*x]]/b)
```

fricas [A] time = 1.49, size = 14, normalized size = 1.00

$$-\frac{\text{Ei}\left(n\cos\left(bx+a\right)\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*cos(b*x+a))*tan(b*x+a),x, algorithm="fricas")
[Out] -Ei(n*cos(b*x + a))/b
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int e^{(n\cos(bx+a))}\tan\left(bx+a\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*cos(b*x+a))*tan(b*x+a),x, algorithm="giac")
[Out] integrate(e^(n*cos(b*x + a))*tan(b*x + a), x)
```

maple [A] time = 0.03, size = 16, normalized size = 1.14

$$\frac{\text{Ei}\left(1,-n\cos\left(bx+a\right)\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*cos(b*x+a))*tan(b*x+a),x)
[Out] 1/b*Ei(1,-n*cos(b*x+a))
```

maxima [A] time = 0.37, size = 14, normalized size = 1.00

$$-\frac{\text{Ei}\left(n\cos\left(bx+a\right)\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*cos(b*x+a))*tan(b*x+a),x, algorithm="maxima")
```

```
[Out] -Ei(n*cos(b*x + a))/b
```

mupad [F] time = 0.00, size = -1, normalized size = -0.07

$$\int e^{n \cos(a+b x)} \tan(a+b x) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*cos(a + b*x))*tan(a + b*x),x)
```

```
[Out] int(exp(n*cos(a + b*x))*tan(a + b*x), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int e^{n \cos(a+bx)} \tan(a+bx) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*cos(b*x+a))*tan(b*x+a),x)
```

```
[Out] Integral(exp(n*cos(a + b*x))*tan(a + b*x), x)
```

3.663 $\int e^{n\cos(ac+bcx)}\tan(c(a+bx))\,dx$

Optimal. Leaf size=19

$$-\frac{\text{Ei}(n\cos(c(a+bx)))}{bc}$$

```
[Out] -Ei(n*cos(c*(b*x+a)))/b/c
```

Rubi [A] time = 0.02, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {4339, 2178}

$$-\frac{\text{Ei}(n\cos(c(a+bx)))}{bc}$$

Antiderivative was successfully verified.

```
[In] Int[E^(n*Cos[a*c + b*c*x])*Tan[c*(a + b*x)],x]
```

```
[Out] -(ExpIntegralEi[n*Cos[c*(a + b*x)]]/(b*c))
```

Rule 2178

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - (c*f)/d))*ExpIntegralEi[(f*g*(c + d*x)*Log[F])/d])/d, x] /; F
reeQ[{F, c, d, e, f, g}, x] && !$UseGamma === True
```

Rule 4339

```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{d = FreeFacto
rs[Cos[c*(a + b*x)], x]}, -Dist[(b*c)^(-1), Subst[Int[SubstFor[1/x, Cos[c*(
a + b*x)]/d, u, x], x], x, Cos[c*(a + b*x)]/d], x] /; FunctionOfQ[Cos[c*(a
+ b*x)]/d, u, x, True]] /; FreeQ[{a, b, c}, x] && (EqQ[F, Tan] || EqQ[F, ta
n])
```

Rubi steps

$$\begin{aligned}\int e^{n\cos(ac+bcx)}\tan(c(a+bx))\,dx&=-\frac{\text{Subst}\left(\int\frac{e^{nx}}{x}\,dx,x,\cos(c(a+bx))\right)}{bc}\\&=-\frac{\text{Ei}(n\cos(c(a+bx)))}{bc}\end{aligned}$$

Mathematica [A] time = 0.06, size = 19, normalized size = 1.00

$$-\frac{\text{Ei}(n\cos(c(a+bx)))}{bc}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(n*Cos[a*c + b*c*x])*Tan[c*(a + b*x)],x]
[Out] -(ExpIntegralEi[n*Cos[c*(a + b*x)]]/(b*c))
```

fricas [A] time = 0.89, size = 20, normalized size = 1.05

$$-\frac{\text{Ei}\left(n\cos\left(bcx+ac\right)\right)}{bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*cos(b*c*x+a*c))*tan(c*(b*x+a)),x, algorithm="fricas")
[Out] -Ei(n*cos(b*c*x + a*c))/(b*c)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int e^{(n\cos(bcx+ac))}\tan\left((bx+a)c\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*cos(b*c*x+a*c))*tan(c*(b*x+a)),x, algorithm="giac")
[Out] integrate(e^(n*cos(b*c*x + a*c))*tan((b*x + a)*c), x)
```

maple [A] time = 0.07, size = 22, normalized size = 1.16

$$\frac{\text{Ei}\left(1,-n\cos\left(bcx+ac\right)\right)}{cb}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*cos(b*c*x+a*c))*tan(c*(b*x+a)),x)
[Out] 1/c/b*Ei(1,-n*cos(b*c*x+a*c))
```

maxima [A] time = 0.38, size = 20, normalized size = 1.05

$$-\frac{\text{Ei}\left(n\cos\left(bcx+ac\right)\right)}{bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*cos(b*c*x+a*c))*tan(c*(b*x+a)),x, algorithm="maxima")
[Out] -Ei(n*cos(b*c*x + a*c))/(b*c)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.05

$$\int \tan \left(c\ (a+b\,x)\right)\,\mathrm{e}^{n\,\cos(a\,c+b\,c\,x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c*(a + b*x))*exp(n*cos(a*c + b*c*x)),x)
[Out] int(tan(c*(a + b*x))*exp(n*cos(a*c + b*c*x)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int e^{n\cos{(ac+bcx)}}\tan{(ac+bcx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*cos(b*c*x+a*c))*tan(c*(b*x+a)),x)
[Out] Integral(exp(n*cos(a*c + b*c*x))*tan(a*c + b*c*x), x)
```

3.664 $\int e^{n\cos(c(a+bx))}\tan(ac+bcx)\,dx$

Optimal. Leaf size=20

$$-\frac{\text{Ei}(n\cos(ac+bxc))}{bc}$$

```
[Out] -Ei(n*cos(b*c*x+a*c))/b/c
```

Rubi [A] time = 0.02, antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {4339, 2178}

$$-\frac{\text{Ei}(n\cos(ac+bxc))}{bc}$$

Antiderivative was successfully verified.

```
[In] Int[E^(n*Cos[c*(a + b*x)])*Tan[a*c + b*c*x],x]
```

```
[Out] -(ExpIntegralEi[n*Cos[a*c + b*c*x]]/(b*c))
```

Rule 2178

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - (c*f)/d))*ExpIntegralEi[(f*g*(c + d*x)*Log[F])/d])/d, x] /; F
reeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 4339

```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{d = FreeFacto
rs[Cos[c*(a + b*x)], x]}, -Dist[(b*c)^(-1), Subst[Int[SubstFor[1/x, Cos[c*(
a + b*x)]/d, u, x], x], x, Cos[c*(a + b*x)]/d], x] /; FunctionOfQ[Cos[c*(a
+ b*x)]/d, u, x, True]] /; FreeQ[{a, b, c}, x] && (EqQ[F, Tan] || EqQ[F, ta
n])
```

Rubi steps

$$\begin{aligned}\int e^{n\cos(c(a+bx))}\tan(ac+bcx)\,dx &= -\frac{\text{Subst}\left(\int\frac{e^{nx}}{x}\,dx,x,\cos(ac+bcx)\right)}{bc}\\ &= -\frac{\text{Ei}(n\cos(ac+bcx))}{bc}\end{aligned}$$

Mathematica [A] time = 0.06, size = 19, normalized size = 0.95

$$-\frac{\text{Ei}(n\cos(c(a+bx)))}{bc}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(n*Cos[c*(a + b*x)])*Tan[a*c + b*c*x],x]
[Out] -(ExpIntegralEi[n*Cos[c*(a + b*x)]]/(b*c))
```

fricas [A] time = 0.77, size = 20, normalized size = 1.00

$$-\frac{\text{Ei}\left(n\cos\left(bcx+ac\right)\right)}{bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*cos(c*(b*x+a)))*tan(b*c*x+a*c),x, algorithm="fricas")
[Out] -Ei(n*cos(b*c*x + a*c))/(b*c)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int e^{(n\cos((bx+a)c))}\tan\left(bcx+ac\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*cos(c*(b*x+a)))*tan(b*c*x+a*c),x, algorithm="giac")
[Out] integrate(e^(n*cos((b*x + a)*c))*tan(b*c*x + a*c), x)
```

maple [A] time = 0.05, size = 22, normalized size = 1.10

$$\frac{\text{Ei}\left(1,-n\cos\left(bcx+ac\right)\right)}{cb}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*cos(c*(b*x+a)))*tan(b*c*x+a*c),x)
[Out] 1/c/b*Ei(1,-n*cos(b*c*x+a*c))
```

maxima [A] time = 0.44, size = 20, normalized size = 1.00

$$-\frac{\text{Ei}\left(n\cos\left(bcx+ac\right)\right)}{bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*cos(c*(b*x+a)))*tan(b*c*x+a*c),x, algorithm="maxima")
```

```
[Out] -Ei(n*cos(b*c*x + a*c))/(b*c)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.05

$$\int \mathrm{e}^{n\cos(c\,(a+b\,x))}\tan\left(a\,c+b\,c\,x\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*cos(c*(a + b*x)))*tan(a*c + b*c*x),x)
```

```
[Out] int(exp(n*cos(c*(a + b*x)))*tan(a*c + b*c*x), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int e^{n\cos(ac+bcx)}\tan\left(ac+bcx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*cos(c*(b*x+a)))*tan(b*c*x+a*c),x)
```

```
[Out] Integral(exp(n*cos(a*c + b*c*x))*tan(a*c + b*c*x), x)
```

3.665 $\int \frac{\cos(x)}{a+b\sin(x)}\,dx$

Optimal. Leaf size=11

$$\frac{\log(a+b\sin(x))}{b}$$

```
[Out] ln(a+b*sin(x))/b
```

Rubi [A] time = 0.02, antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {2668, 31}

$$\frac{\log(a+b\sin(x))}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[x]/(a + b*Sin[x]),x]
```

```
[Out] Log[a + b*Sin[x]]/b
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2668

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\cos(x)}{a+b\sin(x)}\,dx &= \frac{\text{Subst}\left(\int \frac{1}{a+x}\,dx, x, b\sin(x)\right)}{b}\\ &= \frac{\log(a+b\sin(x))}{b}\end{aligned}$$

Mathematica [A] time = 0.01, size = 11, normalized size = 1.00

$$\frac{\log(a+b\sin(x))}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[x]/(a + b*Sin[x]),x]
[Out] Log[a + b*Sin[x]]/b
```

fricas [A] time = 1.97, size = 11, normalized size = 1.00

$$\frac{\log\left(b\sin(x)+a\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)/(a+b*sin(x)),x, algorithm="fricas")
[Out] log(b*sin(x) + a)/b
```

giac [A] time = 0.14, size = 12, normalized size = 1.09

$$\frac{\log\left(|b\sin(x)+a|\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)/(a+b*sin(x)),x, algorithm="giac")
[Out] log(abs(b*sin(x) + a))/b
```

maple [A] time = 0.03, size = 12, normalized size = 1.09

$$\frac{\ln\left(a+b\sin(x)\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)/(a+b*sin(x)),x)
[Out] ln(a+b*sin(x))/b
```

maxima [A] time = 0.32, size = 11, normalized size = 1.00

$$\frac{\log\left(b\sin(x)+a\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)/(a+b*sin(x)),x, algorithm="maxima")
[Out] log(b*sin(x) + a)/b
```

mupad [B] time = 0.03, size = 11, normalized size = 1.00

$$\frac{\ln\left(a+b\,\sin(x)\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)/(a + b*sin(x)),x)
```

```
[Out] log(a + b*sin(x))/b
```

sympy [A] time = 0.32, size = 14, normalized size = 1.27

$$\begin{cases} \frac{\log\left(\frac{a}{b}+\sin\left(x\right)\right)}{b} & \text{for}\ b\neq 0 \\ \frac{\sin\left(x\right)}{a} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)/(a+b*sin(x)),x)
```

```
[Out] Piecewise((log(a/b + sin(x))/b, Ne(b, 0)), (sin(x)/a, True))
```

3.666 $\int \cos(x)(a+b\sin(x))^n\,dx$

Optimal. Leaf size=19

$$\frac{(a+b\sin(x))^{n+1}}{b(n+1)}$$

```
[Out] (a+b*sin(x))^(1+n)/b/(1+n)
```

Rubi [A] time = 0.02, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {2668, 32}

$$\frac{(a+b\sin(x))^{n+1}}{b(n+1)}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[x]*(a + b*Sin[x])^n,x]
```

```
[Out] (a + b*Sin[x])^(1 + n)/(b*(1 + n))
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 2668

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}\int \cos(x)(a+b\sin(x))^n\,dx &= \frac{\text{Subst}\left(\int (a+x)^n\,dx, x, b\sin(x)\right)}{b}\\ &= \frac{(a+b\sin(x))^{1+n}}{b(1+n)}\end{aligned}$$

Mathematica [A] time = 0.02, size = 18, normalized size = 0.95

$$\frac{(a+b\sin(x))^{n+1}}{bn+b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[x]*(a + b*Sin[x])^n,x]
[Out] (a + b*Sin[x])^(1 + n)/(b + b*n)
```

fricas [A] time = 0.57, size = 22, normalized size = 1.16

$$\frac{(b\sin(x)+a)(b\sin(x)+a)^n}{bn+b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*(a+b*sin(x))^n,x, algorithm="fricas")
[Out] (b*sin(x) + a)*(b*sin(x) + a)^n/(b*n + b)
```

giac [A] time = 0.15, size = 19, normalized size = 1.00

$$\frac{(b\sin(x)+a)^{n+1}}{b(n+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*(a+b*sin(x))^n,x, algorithm="giac")
[Out] (b*sin(x) + a)^(n + 1)/(b*(n + 1))
```

maple [A] time = 0.03, size = 20, normalized size = 1.05

$$\frac{(a+b\sin(x))^{n+1}}{b\,(n+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)*(a+b*sin(x))^n,x)
[Out] (a+b*sin(x))^(n+1)/b/(n+1)
```

maxima [A] time = 0.32, size = 19, normalized size = 1.00

$$\frac{(b\sin(x)+a)^{n+1}}{b(n+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*(a+b*sin(x))^n,x, algorithm="maxima")
```

```
[Out] (b*sin(x) + a)^(n + 1)/(b*(n + 1))
```

mupad [B] time = 3.13, size = 19, normalized size = 1.00

$$\frac{(a+b\,\sin(x))^{n+1}}{b\,(n+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)*(a + b*sin(x))^n,x)
```

```
[Out] (a + b*sin(x))^(n + 1)/(b*(n + 1))
```

sympy [A] time = 1.88, size = 56, normalized size = 2.95

$$\begin{cases} \frac{\sin{(x)}}{a} & \text{for}\ b = 0 \wedge n = -1 \\ a^{n} \sin{(x)} & \text{for}\ b = 0 \\ \frac{\log{\left(\frac{a}{b} + \sin{(x)} \right)}}{b} & \text{for}\ n = -1 \\ \frac{a(a+b\sin{(x)})^{n}}{bn+b} + \frac{b(a+b\sin{(x)})^{n}\sin{(x)}}{bn+b} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*(a+b*sin(x))**n,x)
```

```
[Out] Piecewise((sin(x)/a, Eq(b, 0) & Eq(n, -1)), (a**n*sin(x), Eq(b, 0)), (log(a
/b + sin(x))/b, Eq(n, -1)), (a*(a + b*sin(x))**n/(b*n + b) + b*(a + b*sin(x
))**n*sin(x)/(b*n + b), True))
```

3.667 $\int \frac{\cos(x)}{\sqrt{1+\sin^2(x)}} dx$

Optimal. Leaf size=3

$$\sinh^{-1}(\sin(x))$$

```
[Out] arcsinh(sin(x))
```

Rubi [A] time = 0.02, antiderivative size = 3, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {3190, 215}

$$\sinh^{-1}(\sin(x))$$

Antiderivative was successfully verified.

```
[In] Int[Cos[x]/Sqrt[1 + Sin[x]^2],x]
```

```
[Out] ArcSinh[Sin[x]]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 3190

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Su
bst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]/
ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\begin{aligned}\int \frac{\cos(x)}{\sqrt{1+\sin^2(x)}}\,dx &= \text{Subst}\left(\int \frac{1}{\sqrt{1+x^2}}\,dx, x, \sin(x)\right)\\ &= \sinh^{-1}(\sin(x))\end{aligned}$$

Mathematica [A] time = 0.01, size = 3, normalized size = 1.00

$$\sinh^{-1}(\sin(x))$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[x]/Sqrt[1 + Sin[x]^2],x]
```

```
[Out] ArcSinh[Sin[x]]
```

fricas [B] time = 1.19, size = 39, normalized size = 13.00

$$\frac{1}{4}\log\left(8\cos(x)^4-4\left(2\cos(x)^2-3\right)\sqrt{-\cos(x)^2+2}\sin(x)-24\cos(x)^2+17\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)/(1+sin(x)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/4*log(8*cos(x)^4 - 4*(2*cos(x)^2 - 3)*sqrt(-cos(x)^2 + 2)*sin(x) - 24*cos
(x)^2 + 17)
```

giac [B] time = 0.13, size = 16, normalized size = 5.33

$$-\log\left(\sqrt{\sin(x)^2+1}-\sin(x)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)/(1+sin(x)^2)^(1/2),x, algorithm="giac")
```

```
[Out] -log(sqrt(sin(x)^2 + 1) - sin(x))
```

maple [A] time = 0.06, size = 4, normalized size = 1.33

$$\operatorname{arcsinh}(\sin(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)/(1+sin(x)^2)^(1/2),x)
```

```
[Out] arcsinh(sin(x))
```

maxima [A] time = 0.41, size = 3, normalized size = 1.00

$$\operatorname{arsinh}(\sin(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)/(1+sin(x)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] arcsinh(sin(x))
```

mupad [B] time = 0.02, size = 9, normalized size = 3.00

$$-\operatorname{asin}(\sin(x)\,1\mathrm{i})\,1\mathrm{i}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)/(sin(x)^2 + 1)^(1/2),x)
```

```
[Out] -asin(sin(x)*1i)*1i
```

sympy **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos(x)}{\sqrt{\sin^2(x)+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)/(1+sin(x)**2)**(1/2),x)
```

```
[Out] Integral(cos(x)/sqrt(sin(x)**2 + 1), x)
```

3.668 $\int \frac{\cos(x)}{\sqrt{4-\sin^2(x)}}\,dx$

Optimal. Leaf size=7

$$\sin^{-1}\left(\frac{\sin(x)}{2}\right)$$

```
[Out] arcsin(1/2*sin(x))
```

Rubi [A] time = 0.02, antiderivative size = 7, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {3190, 216}

$$\sin^{-1}\left(\frac{\sin(x)}{2}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Cos[x]/Sqrt[4 - Sin[x]^2],x]
```

```
[Out] ArcSin[Sin[x]/2]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 3190

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Su
bst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]/
ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\begin{aligned}\int \frac{\cos(x)}{\sqrt{4-\sin^2(x)}}\,dx &= \text{Subst}\left(\int \frac{1}{\sqrt{4-x^2}}\,dx, x, \sin(x)\right)\\ &= \sin^{-1}\left(\frac{\sin(x)}{2}\right)\end{aligned}$$

Mathematica [A] time = 0.01, size = 7, normalized size = 1.00

$$\sin^{-1}\left(\frac{\sin(x)}{2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[x]/Sqrt[4 - Sin[x]^2],x]
```

```
[Out] ArcSin[Sin[x]/2]
```

fricas [B] time = 3.02, size = 53, normalized size = 7.57

$$\frac{1}{2}\arctan\left(\frac{\sqrt{\cos(x)^2+3}\left(\cos(x)^2+1\right)\sin(x)-4\cos(x)\sin(x)}{\cos(x)^4+6\cos(x)^2-3}\right)+\frac{1}{2}\arctan\left(\frac{\sin(x)}{\cos(x)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)/(4-sin(x)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*arctan((sqrt(cos(x)^2 + 3)*(cos(x)^2 + 1)*sin(x) - 4*cos(x)*sin(x))/(co
s(x)^4 + 6*cos(x)^2 - 3)) + 1/2*arctan(sin(x)/cos(x))
```

giac [A] time = 0.15, size = 5, normalized size = 0.71

$$\arcsin\left(\frac{1}{2}\sin(x)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)/(4-sin(x)^2)^(1/2),x, algorithm="giac")
```

```
[Out] arcsin(1/2*sin(x))
```

maple [A] time = 0.08, size = 6, normalized size = 0.86

$$\arcsin\left(\frac{\sin(x)}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)/(4-sin(x)^2)^(1/2),x)
```

```
[Out] arcsin(1/2*sin(x))
```

maxima [A] time = 0.42, size = 5, normalized size = 0.71

$$\arcsin\left(\frac{1}{2}\sin(x)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)/(4-sin(x)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] arcsin(1/2*sin(x))
```

mupad [B] time = 2.98, size = 5, normalized size = 0.71

$$\mathrm{asin}\left(\frac{\sin(x)}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)/(4 - sin(x)^2)^(1/2),x)
```

```
[Out] asin(sin(x)/2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos(x)}{\sqrt{-(\sin(x)-2)(\sin(x)+2)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)/(4-sin(x)**2)**(1/2),x)
```

```
[Out] Integral(cos(x)/sqrt(-(sin(x) - 2)*(sin(x) + 2)), x)
```

3.669 $\int \frac{\cos(3x)}{\sqrt{4-\sin^2(3x)}}\,dx$

Optimal. Leaf size=13

$$\frac{1}{3}\sin^{-1}\left(\frac{1}{2}\sin(3x)\right)$$

```
[Out] 1/3*arcsin(1/2*sin(3*x))
```

Rubi [A] time = 0.03, antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {3190, 216}

$$\frac{1}{3}\sin^{-1}\left(\frac{1}{2}\sin(3x)\right)$$

Antiderivative was successfully verified.

```
[In] Int[Cos[3*x]/Sqrt[4 - Sin[3*x]^2],x]
```

```
[Out] ArcSin[Sin[3*x]/2]/3
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 3190

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Su
bst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]/
ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\begin{aligned}\int \frac{\cos(3x)}{\sqrt{4-\sin^2(3x)}}\,dx &= \frac{1}{3}\operatorname{Subst}\left(\int \frac{1}{\sqrt{4-x^2}}\,dx, x, \sin(3x)\right)\\ &= \frac{1}{3}\sin^{-1}\left(\frac{1}{2}\sin(3x)\right)\end{aligned}$$

Mathematica [A] time = 0.03, size = 13, normalized size = 1.00

$$\frac{1}{3}\sin^{-1}\left(\frac{1}{2}\sin(3x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[3*x]/Sqrt[4 - Sin[3*x]^2],x]
```

```
[Out] ArcSin[Sin[3*x]/2]/3
```

fricas [B] time = 0.60, size = 71, normalized size = 5.46

$$\frac{1}{6}\arctan\left(\frac{\sqrt{\cos(3x)^2+3}\left(\cos(3x)^2+1\right)\sin(3x)-4\cos(3x)\sin(3x)}{\cos(3x)^4+6\cos(3x)^2-3}\right)+\frac{1}{6}\arctan\left(\frac{\sin(3x)}{\cos(3x)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(3*x)/(4-sin(3*x)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/6*arctan((sqrt(cos(3*x)^2 + 3)*(cos(3*x)^2 + 1)*sin(3*x) - 4*cos(3*x)*sin
(3*x))/(cos(3*x)^4 + 6*cos(3*x)^2 - 3)) + 1/6*arctan(sin(3*x)/cos(3*x))
```

giac [A] time = 0.22, size = 9, normalized size = 0.69

$$\frac{1}{3}\arcsin\left(\frac{1}{2}\sin(3x)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(3*x)/(4-sin(3*x)^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/3*arcsin(1/2*sin(3*x))
```

maple [A] time = 0.08, size = 10, normalized size = 0.77

$$\frac{\arcsin\left(\frac{\sin(3x)}{2}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(3*x)/(4-sin(3*x)^2)^(1/2),x)
```

```
[Out] 1/3*arcsin(1/2*sin(3*x))
```

maxima [A] time = 0.42, size = 9, normalized size = 0.69

$$\frac{1}{3}\,\arcsin\left(\frac{1}{2}\,\sin\left(3\,x\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(3*x)/(4-sin(3*x)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/3*arcsin(1/2*sin(3*x))
```

mupad [B] time = 2.98, size = 9, normalized size = 0.69

$$\frac{\mathrm{asin}\left(\frac{\sin(3\,x)}{2}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(3*x)/(4 - sin(3*x)^2)^(1/2),x)
```

```
[Out] asin(sin(3*x)/2)/3
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos\left(3x\right)}{\sqrt{-\left(\sin\left(3x\right)-2\right)\left(\sin\left(3x\right)+2\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(3*x)/(4-sin(3*x)**2)**(1/2),x)
```

```
[Out] Integral(cos(3*x)/sqrt(-(sin(3*x) - 2)*(sin(3*x) + 2)), x)
```

3.670 $\int \cos(x)\sqrt{1+\csc(x)}\, dx$

Optimal. Leaf size=21

$$\sin(x)\sqrt{\csc(x)+1} + \tanh^{-1}\left(\sqrt{\csc(x)+1}\right)$$

```
[Out] arctanh((1+csc(x))^(1/2))+sin(x)*(1+csc(x))^(1/2)
```

Rubi [A] time = 0.02, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {3873, 47, 63, 207}

$$\sin(x)\sqrt{\csc(x)+1} + \tanh^{-1}\left(\sqrt{\csc(x)+1}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Cos[x]*Sqrt[1 + Csc[x]],x]
```

```
[Out] ArcTanh[Sqrt[1 + Csc[x]]] + Sqrt[1 + Csc[x]]*Sin[x]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 3873

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m
_), x_Symbol] :> -Dist[(f*b^(p - 1))^(-1), Subst[Int[((-a + b*x)^((p - 1)/2
)*(a + b*x)^(m + (p - 1)/2))/x^(p + 1), x], x, Csc[e + f*x]], x] /; FreeQ[{
a, b, e, f, m}, x] && IntegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}\int \cos(x)\sqrt{1+\csc(x)}\,dx &= -\text{Subst}\left(\int \frac{\sqrt{1+x}}{x^2}\,dx, x, \csc(x)\right)\\ &= \sqrt{1+\csc(x)}\,\sin(x) - \frac{1}{2}\text{Subst}\left(\int \frac{1}{x\sqrt{1+x}}\,dx, x, \csc(x)\right)\\ &= \sqrt{1+\csc(x)}\,\sin(x) - \text{Subst}\left(\int \frac{1}{-1+x^2}\,dx, x, \sqrt{1+\csc(x)}\right)\\ &= \tanh^{-1}\left(\sqrt{1+\csc(x)}\right) + \sqrt{1+\csc(x)}\,\sin(x)\end{aligned}$$

Mathematica [A] time = 0.01, size = 21, normalized size = 1.00

$$\sin(x)\sqrt{\csc(x)+1} + \tanh^{-1}\left(\sqrt{\csc(x)+1}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[x]*Sqrt[1 + Csc[x]],x]
```

```
[Out] ArcTanh[Sqrt[1 + Csc[x]]] + Sqrt[1 + Csc[x]]*Sin[x]
```

fricas [B] time = 0.99, size = 79, normalized size = 3.76

$$\sqrt{\frac{\sin(x)+1}{\sin(x)}}\,\sin(x)+\frac{1}{2}\,\log\left(\frac{2\left(\sqrt{\frac{\sin(x)+1}{\sin(x)}}\,\sin(x)+\sin(x)+1\right)}{\cos(x)+\sin(x)+1}\right)-\frac{1}{2}\,\log\left(-\frac{2\left(\sqrt{\frac{\sin(x)+1}{\sin(x)}}\,\sin(x)-\sin(x)-1\right)}{\cos(x)+\sin(x)+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*(1+csc(x))^(1/2),x, algorithm="fricas")
```

```
[Out] sqrt((sin(x) + 1)/sin(x))*sin(x) + 1/2*log(2*(sqrt((sin(x) + 1)/sin(x))*sin
(x) + sin(x) + 1)/(cos(x) + sin(x) + 1)) - 1/2*log(-2*(sqrt((sin(x) + 1)/si
n(x))*sin(x) - sin(x) - 1)/(cos(x) + sin(x) + 1))
```

giac [B] time = 0.15, size = 38, normalized size = 1.81

$$\frac{1}{2}\left(2\sqrt{\sin(x)^2+\sin(x)}-\log\left(\left|2\sqrt{\sin(x)^2+\sin(x)}-2\sin(x)-1\right|\right)\right)\operatorname{sgn}(\sin(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*(1+csc(x))^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*(2*sqrt(sin(x)^2 + sin(x)) - log(abs(2*sqrt(sin(x)^2 + sin(x)) - 2*sin(
x) - 1)))*sgn(sin(x))
```

maple [B] time = 0.08, size = 48, normalized size = 2.29

$$\frac{1}{2\sqrt{1+\csc(x)}-2}-\frac{\ln\left(\sqrt{1+\csc(x)}-1\right)}{2}+\frac{1}{2\sqrt{1+\csc(x)}+2}+\frac{\ln\left(\sqrt{1+\csc(x)}+1\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)*(1+csc(x))^(1/2),x)
```

```
[Out] 1/2/((1+csc(x))^(1/2)-1)-1/2*ln((1+csc(x))^(1/2)-1)+1/2/((1+csc(x))^(1/2)+1
)+1/2*ln((1+csc(x))^(1/2)+1)
```

maxima [B] time = 0.33, size = 38, normalized size = 1.81

$$\sqrt{\frac{1}{\sin(x)}+1}\,\sin(x)+\frac{1}{2}\log\left(\sqrt{\frac{1}{\sin(x)}+1}+1\right)-\frac{1}{2}\log\left(\sqrt{\frac{1}{\sin(x)}+1}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*(1+csc(x))^(1/2),x, algorithm="maxima")
```

```
[Out] sqrt(1/sin(x) + 1)*sin(x) + 1/2*log(sqrt(1/sin(x) + 1) + 1) - 1/2*log(sqrt(
1/sin(x) + 1) - 1)
```

mupad [B] time = 3.10, size = 47, normalized size = 2.24

$$\sin(x)\sqrt{\frac{1}{\sin(x)}+1}+\frac{\ln\left(\sin(x)+\sqrt{\sin(x)^2+\sin(x)}+\frac{1}{2}\right)\sin(x)\sqrt{\frac{1}{\sin(x)}+1}}{2\sqrt{\sin(x)^2+\sin(x)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)*(1/sin(x) + 1)^(1/2),x)
```

```
[Out] sin(x)*(1/sin(x) + 1)^(1/2) + (log(sin(x) + (sin(x) + sin(x)^2)^(1/2) + 1/2
)*sin(x)*(1/sin(x) + 1)^(1/2))/(2*(sin(x) + sin(x)^2)^(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{\csc(x) + 1} \cos(x)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*(1+csc(x))**(1/2),x)
```

```
[Out] Integral(sqrt(csc(x) + 1)*cos(x), x)
```

3.671 $\int \cos(x)\sqrt{4-\sin^2(x)}\,dx$

Optimal. Leaf size=28

$$2\sin^{-1}\left(\frac{\sin(x)}{2}\right)+\frac{1}{2}\sin(x)\sqrt{4-\sin^2(x)}$$

```
[Out] 2*arcsin(1/2*sin(x))+1/2*sin(x)*(4-sin(x)^2)^(1/2)
```

Rubi [A] time = 0.03, antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {3190, 195, 216}

$$2\sin^{-1}\left(\frac{\sin(x)}{2}\right)+\frac{1}{2}\sin(x)\sqrt{4-\sin^2(x)}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[x]*Sqrt[4 - Sin[x]^2],x]
```

```
[Out] 2*ArcSin[Sin[x]/2] + (Sin[x]*Sqrt[4 - Sin[x]^2])/2
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 3190

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Su
bst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]/
ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\int \cos(x)\sqrt{4-\sin^2(x)}\,dx = \text{Subst}\left(\int \sqrt{4-x^2}\,dx, x, \sin(x)\right)$$
$$= \frac{1}{2}\sin(x)\sqrt{4-\sin^2(x)} + 2\,\text{Subst}\left(\int \frac{1}{\sqrt{4-x^2}}\,dx, x, \sin(x)\right)$$
$$= 2\sin^{-1}\left(\frac{\sin(x)}{2}\right) + \frac{1}{2}\sin(x)\sqrt{4-\sin^2(x)}$$

Mathematica [A] time = 0.02, size = 28, normalized size = 1.00

$$2\sin^{-1}\left(\frac{\sin(x)}{2}\right) + \frac{1}{2}\sin(x)\sqrt{4-\sin^2(x)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[x]*Sqrt[4 - Sin[x]^2],x]
[Out] 2*ArcSin[Sin[x]/2] + (Sin[x]*Sqrt[4 - Sin[x]^2])/2
```

fricas [B] time = 1.07, size = 61, normalized size = 2.18

$$\frac{1}{2}\sqrt{\cos(x)^2+3}\,\sin(x) + \arctan\left(\frac{\sqrt{\cos(x)^2+3}\left(\cos(x)^2+1\right)\sin(x) - 4\cos(x)\sin(x)}{\cos(x)^4+6\cos(x)^2-3}\right) + \arctan\left(\frac{\sin(x)}{\cos(x)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*(4-sin(x)^2)^(1/2),x, algorithm="fricas")
[Out] 1/2*sqrt(cos(x)^2 + 3)*sin(x) + arctan((sqrt(cos(x)^2 + 3)*(cos(x)^2 + 1)*s
in(x) - 4*cos(x)*sin(x))/(cos(x)^4 + 6*cos(x)^2 - 3)) + arctan(sin(x)/cos(x
))
```

giac [A] time = 0.14, size = 22, normalized size = 0.79

$$\frac{1}{2}\sqrt{-\sin(x)^2+4}\,\sin(x) + 2\arcsin\left(\frac{1}{2}\sin(x)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*(4-sin(x)^2)^(1/2),x, algorithm="giac")
[Out] 1/2*sqrt(-sin(x)^2 + 4)*sin(x) + 2*arcsin(1/2*sin(x))
```

maple [A] time = 0.07, size = 23, normalized size = 0.82

$$2\arcsin\left(\frac{\sin(x)}{2}\right)+\frac{\sin(x)\sqrt{4-\left(\sin^2(x)\right)}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)*(4-sin(x)^2)^(1/2),x)
[Out] 2*arcsin(1/2*sin(x))+1/2*sin(x)*(4-sin(x)^2)^(1/2)
```

maxima [A] time = 0.46, size = 22, normalized size = 0.79

$$\frac{1}{2}\sqrt{-\sin(x)^2+4}\,\sin(x)+2\,\arcsin\left(\frac{1}{2}\,\sin(x)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*(4-sin(x)^2)^(1/2),x, algorithm="maxima")
[Out] 1/2*sqrt(-sin(x)^2 + 4)*sin(x) + 2*arcsin(1/2*sin(x))
```

mupad [B] time = 2.97, size = 20, normalized size = 0.71

$$2\,\mathrm{asin}\left(\frac{\sin(x)}{2}\right)+\frac{\sin(x)\,\sqrt{\cos(x)^2+3}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)*(4 - sin(x)^2)^(1/2),x)
[Out] 2*asin(sin(x)/2) + (sin(x)*(cos(x)^2 + 3)^(1/2))/2
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{-(\sin(x)-2)(\sin(x)+2)}\,\cos(x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*(4-sin(x)**2)**(1/2),x)
[Out] Integral(sqrt(-(sin(x) - 2)*(sin(x) + 2))*cos(x), x)
```

3.672 $\int \cos(x)\sin(x)\sqrt{1+\sin^2(x)}\,dx$

Optimal. Leaf size=14

$$\frac{1}{3}\left(\sin^2(x)+1\right)^{3/2}$$

```
[Out] 1/3*(1+sin(x)^2)^(3/2)
```

Rubi [A] time = 0.03, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {3198, 261}

$$\frac{1}{3}\left(\sin^2(x)+1\right)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[x]*Sin[x]*Sqrt[1 + Sin[x]^2],x]
```

```
[Out] (1 + Sin[x]^2)^(3/2)/3
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 3198

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a
_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.), x_Symbol] :> With[{ff = FreeFa
ctors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(d*ff*x)^n*(1 - ff^2*x^2)^((m
 - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, d
, e, f, n, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\begin{aligned}
\int \cos(x)\sin(x)\sqrt{1+\sin^2(x)}\,dx &= \text{Subst}\left(\int x\sqrt{1+x^2}\,dx, x, \sin(x)\right)\\
&= \frac{1}{3}\left(1+\sin^2(x)\right)^{3/2}
\end{aligned}$$

Mathematica [A] time = 0.01, size = 14, normalized size = 1.00

$$\frac{1}{3}\left(\sin^2(x)+1\right)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[x]*Sin[x]*Sqrt[1 + Sin[x]^2],x]
```

```
[Out] (1 + Sin[x]^2)^(3/2)/3
```

fricas [A] time = 0.88, size = 12, normalized size = 0.86

$$\frac{1}{3}\left(-\cos(x)^2+2\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*sin(x)*(1+sin(x)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/3*(-cos(x)^2 + 2)^(3/2)
```

giac [A] time = 0.13, size = 10, normalized size = 0.71

$$\frac{1}{3}\left(\sin(x)^2+1\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*sin(x)*(1+sin(x)^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/3*(sin(x)^2 + 1)^(3/2)
```

maple [A] time = 0.01, size = 11, normalized size = 0.79

$$\frac{\left(1+\sin^2(x)\right)^{\frac{3}{2}}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)*sin(x)*(1+sin(x)^2)^(1/2),x)
```

```
[Out] 1/3*(1+sin(x)^2)^(3/2)
```

maxima [A] time = 0.37, size = 10, normalized size = 0.71

$$\frac{1}{3}\left(\sin(x)^2+1\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*sin(x)*(1+sin(x)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/3*(sin(x)^2 + 1)^(3/2)
```

mupad [B] time = 0.10, size = 10, normalized size = 0.71

$$\frac{\left(\sin(x)^2+1\right)^{3/2}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)*sin(x)*(sin(x)^2 + 1)^(1/2),x)
[Out] (sin(x)^2 + 1)^(3/2)/3
```

sympy [B] time = 0.73, size = 27, normalized size = 1.93

$$\frac{\sqrt{\sin^2(x)+1}\,\sin^2(x)}{3}+\frac{\sqrt{\sin^2(x)+1}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*sin(x)*(1+sin(x)**2)**(1/2),x)
[Out] sqrt(sin(x)**2 + 1)*sin(x)**2/3 + sqrt(sin(x)**2 + 1)/3
```

3.673 $\int \frac{\cos(x)}{\sqrt{2\sin(x)+\sin^2(x)}}\,dx$

Optimal. Leaf size=19

$$2\tanh^{-1}\left(\frac{\sin(x)}{\sqrt{\sin^2(x)+2\sin(x)}}\right)$$

```
[Out] 2*arctanh(sin(x)/(2*sin(x)+sin(x)^2)^(1/2))
```

Rubi [A] time = 0.03, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {3258, 620, 206}

$$2\tanh^{-1}\left(\frac{\sin(x)}{\sqrt{\sin^2(x)+2\sin(x)}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Cos[x]/Sqrt[2*Sin[x] + Sin[x]^2],x]
```

```
[Out] 2*ArcTanh[Sin[x]/Sqrt[2*Sin[x] + Sin[x]^2]]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 620

```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[Int[1/(1
- c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

Rule 3258

```
Int[cos[(d_.) + (e_.)*(x_)]^(m_.)*((a_.) + (b_.)*((f_.)*sin[(d_.) + (e_.)*(
x_)])^(n_.) + (c_.)*((f_.)*sin[(d_.) + (e_.)*(x_)])^(n2_.))^(p_.), x_Symbol
] :> Module[{g = FreeFactors[Sin[d + e*x], x]}, Dist[g/e, Subst[Int[(1 - g^
2*x^2)^((m - 1)/2)*(a + b*(f*g*x)^n + c*(f*g*x)^(2*n))^p, x], x, Sin[d + e*
x]/g], x]] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && EqQ[n2, 2*n] && Integer
Q[(m - 1)/2]
```

Rubi steps

$$\int \frac{\cos(x)}{\sqrt{2\sin(x)+\sin^2(x)}}\,dx = \text{Subst}\left(\int \frac{1}{\sqrt{2x+x^2}}\,dx, x, \sin(x)\right)$$
$$= 2\,\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \frac{\sin(x)}{\sqrt{2\sin(x)+\sin^2(x)}}\right)$$
$$= 2\tanh^{-1}\left(\frac{\sin(x)}{\sqrt{2\sin(x)+\sin^2(x)}}\right)$$

Mathematica [B] time = 0.02, size = 40, normalized size = 2.11

$$\frac{2\sqrt{\sin(x)}\sqrt{\sin(x)+2}\,\sinh^{-1}\left(\frac{\sqrt{\sin(x)}}{\sqrt{2}}\right)}{\sqrt{\sin(x)(\sin(x)+2)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[x]/Sqrt[2*Sin[x] + Sin[x]^2],x]
```

```
[Out] (2*ArcSinh[Sqrt[Sin[x]]/Sqrt[2]]*Sqrt[Sin[x]]*Sqrt[2 + Sin[x]])/Sqrt[Sin[x]
*(2 + Sin[x])]
```

fricas [B] time = 2.96, size = 35, normalized size = 1.84

$$\frac{1}{2}\log\left(-2\cos(x)^2 + 2\sqrt{-\cos(x)^2 + 2\sin(x) + 1}\,(\sin(x)+1) + 4\sin(x) + 3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)/(2*sin(x)+sin(x)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*log(-2*cos(x)^2 + 2*sqrt(-cos(x)^2 + 2*sin(x) + 1)*(sin(x) + 1) + 4*sin
(x) + 3)
```

giac [A] time = 0.16, size = 20, normalized size = 1.05

$$-\log\left(-\sqrt{\sin(x)^2 + 2\sin(x)} + \sin(x) + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)/(2*sin(x)+sin(x)^2)^(1/2),x, algorithm="giac")
```

```
[Out] -log(-sqrt(sin(x)^2 + 2*sin(x)) + sin(x) + 1)
```

maple [A] time = 0.10, size = 17, normalized size = 0.89

$$\ln\left(\sin(x)+1+\sqrt{2\sin(x)+\sin^{2}(x)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)/(2*sin(x)+sin(x)^2)^(1/2),x)
[Out] ln(sin(x)+1+(2*sin(x)+sin(x)^2)^(1/2))
```

maxima [A] time = 0.31, size = 20, normalized size = 1.05

$$\log\left(2\sqrt{\sin(x)^{2}+2\sin(x)}+2\sin(x)+2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)/(2*sin(x)+sin(x)^2)^(1/2),x, algorithm="maxima")
[Out] log(2*sqrt(sin(x)^2 + 2*sin(x)) + 2*sin(x) + 2)
```

mupad [B] time = 3.17, size = 14, normalized size = 0.74

$$\ln\left(\sin(x)+\sqrt{\sin(x)\,(\sin(x)+2)}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)/(2*sin(x) + sin(x)^2)^(1/2),x)
[Out] log(sin(x) + (sin(x)*(sin(x) + 2))^(1/2) + 1)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\cos(x)}{\sqrt{(\sin(x)+2)\sin(x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)/(2*sin(x)+sin(x)**2)**(1/2),x)
[Out] Integral(cos(x)/sqrt((sin(x) + 2)*sin(x)), x)
```

3.674 $\int \cos(x)\cos(\sin(x))\,dx$

Optimal. Leaf size=3

$$\sin(\sin(x))$$

```
[Out] sin(sin(x))
```

Rubi [A] time = 0.01, antiderivative size = 3, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {4334, 2637}

$$\sin(\sin(x))$$

Antiderivative was successfully verified.

```
[In] Int[Cos[x]*Cos[Sin[x]],x]
```

```
[Out] Sin[Sin[x]]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 4334

```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{d = FreeFacto
rs[Sin[c*(a + b*x)], x]}, Dist[d/(b*c), Subst[Int[SubstFor[1, Sin[c*(a + b*
x)]/d, u, x], x], x, Sin[c*(a + b*x)]/d], x] /; FunctionOfQ[Sin[c*(a + b*x)
]/d, u, x, True]] /; FreeQ[{a, b, c}, x] && (EqQ[F, Cos] || EqQ[F, cos])
```

Rubi steps

$$\begin{aligned}\int \cos(x)\cos(\sin(x))\,dx &= \text{Subst}\left(\int \cos(x)\,dx, x, \sin(x)\right)\\ &= \sin(\sin(x))\end{aligned}$$

Mathematica [A] time = 1.47, size = 3, normalized size = 1.00

$$\sin(\sin(x))$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[x]*Cos[Sin[x]],x]
```

```
[Out] Sin[Sin[x]]
```

fricas [B] time = 1.64, size = 17, normalized size = 5.67

$$\sin\left(\frac{2\,\tan\left(\frac{1}{2}\,x\right)}{\tan\left(\frac{1}{2}\,x\right)^{2}+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*cos(sin(x)),x, algorithm="fricas")
[Out] sin(2*tan(1/2*x)/(tan(1/2*x)^2 + 1))
```

giac [A] time = 0.14, size = 3, normalized size = 1.00

$$\sin\left(\sin(x)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*cos(sin(x)),x, algorithm="giac")
[Out] sin(sin(x))
```

maple [A] time = 0.01, size = 4, normalized size = 1.33

$$\sin\left(\sin(x)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)*cos(sin(x)),x)
[Out] sin(sin(x))
```

maxima [A] time = 0.32, size = 3, normalized size = 1.00

$$\sin\left(\sin(x)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*cos(sin(x)),x, algorithm="maxima")
[Out] sin(sin(x))
```

mupad [B] time = 2.95, size = 3, normalized size = 1.00

$$\sin\left(\sin(x)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(sin(x))*cos(x),x)
```

```
[Out] sin(sin(x))
```

sympy [A] time = 0.41, size = 3, normalized size = 1.00

$$\sin(\sin(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*cos(sin(x)),x)
```

```
[Out] sin(sin(x))
```

3.675 $\int \cos(x)\cos(\sin(x))\cos(\sin(\sin(x)))\,dx$

Optimal. Leaf size=4

$$\sin(\sin(\sin(x)))$$

[Out] sin(sin(sin(x)))

Rubi [A] time = 0.02, antiderivative size = 4, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {4334, 2637}

$$\sin(\sin(\sin(x)))$$

Antiderivative was successfully verified.

[In] Int[Cos[x]*Cos[Sin[x]]*Cos[Sin[Sin[x]]],x]

[Out] Sin[Sin[Sin[x]]]

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 4334

```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{d = FreeFacto
rs[Sin[c*(a + b*x)], x]}, Dist[d/(b*c), Subst[Int[SubstFor[1, Sin[c*(a + b*
x)]/d, u, x], x], x, Sin[c*(a + b*x)]/d], x] /; FunctionOfQ[Sin[c*(a + b*x)
]/d, u, x, True]] /; FreeQ[{a, b, c}, x] && (EqQ[F, Cos] || EqQ[F, cos])
```

Rubi steps

$$\begin{aligned}\int \cos(x)\cos(\sin(x))\cos(\sin(\sin(x)))\,dx &= \text{Subst}\left(\int \cos(x)\cos(\sin(x))\,dx, x, \sin(x)\right)\\ &= \text{Subst}\left(\int \cos(x)\,dx, x, \sin(\sin(x))\right)\\ &= \sin(\sin(\sin(x)))\end{aligned}$$

Mathematica [A] time = 8.96, size = 4, normalized size = 1.00

$$\sin(\sin(\sin(x)))$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[x]*Cos[Sin[x]]*Cos[Sin[Sin[x]]],x]
```

```
[Out] Sin[Sin[Sin[x]]]
```

fricas [B] time = 0.67, size = 41, normalized size = 10.25

$$\sin\left(\frac{2\,\tan\left(\frac{\tan\left(\frac{1}{2}\,x\right)}{\tan\left(\frac{1}{2}\,x\right)^{2}+1}\right)}{\tan\left(\frac{\tan\left(\frac{1}{2}\,x\right)}{\tan\left(\frac{1}{2}\,x\right)^{2}+1}\right)^{2}+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*cos(sin(x))*cos(sin(sin(x))),x, algorithm="fricas")
```

```
[Out] sin(2*tan(tan(1/2*x)/(tan(1/2*x)^2 + 1))/(tan(tan(1/2*x)/(tan(1/2*x)^2 + 1)
)^2 + 1))
```

giac [A] time = 0.13, size = 4, normalized size = 1.00

$$\sin\left(\sin\left(\sin\left(x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*cos(sin(x))*cos(sin(sin(x))),x, algorithm="giac")
```

```
[Out] sin(sin(sin(x)))
```

maple [A] time = 0.01, size = 5, normalized size = 1.25

$$\sin\left(\sin\left(\sin\left(x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)*cos(sin(x))*cos(sin(sin(x))),x)
```

```
[Out] sin(sin(sin(x)))
```

maxima [A] time = 0.32, size = 4, normalized size = 1.00

$$\sin\left(\sin\left(\sin\left(x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*cos(sin(x))*cos(sin(sin(x))),x, algorithm="maxima")
```

```
[Out] sin(sin(sin(x)))
```

mupad [B] time = 3.00, size = 4, normalized size = 1.00

$$\sin(\sin(\sin(x)))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(sin(x))*cos(sin(sin(x)))*cos(x),x)
[Out] sin(sin(sin(x)))
```

sympy [A] time = 8.85, size = 5, normalized size = 1.25

$$\sin(\sin(\sin(x)))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*cos(sin(x))*cos(sin(sin(x))),x)
[Out] sin(sin(sin(x)))
```

3.676 $\int \cos(x)\sec(\sin(x))\,dx$

Optimal. Leaf size=4

$$\tanh^{-1}(\sin(\sin(x)))$$

```
[Out] arctanh(sin(sin(x)))
```

Rubi [A] time = 0.01, antiderivative size = 4, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {4334, 3770}

$$\tanh^{-1}(\sin(\sin(x)))$$

Antiderivative was successfully verified.

```
[In] Int[Cos[x]*Sec[Sin[x]],x]
```

```
[Out] ArcTanh[Sin[Sin[x]]]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 4334

```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{d = FreeFacto
rs[Sin[c*(a + b*x)], x]}, Dist[d/(b*c), Subst[Int[SubstFor[1, Sin[c*(a + b*
x)]/d, u, x], x], x, Sin[c*(a + b*x)]/d], x] /; FunctionOfQ[Sin[c*(a + b*x)
]/d, u, x, True]] /; FreeQ[{a, b, c}, x] && (EqQ[F, Cos] || EqQ[F, cos])
```

Rubi steps

$$\begin{aligned}\int \cos(x)\sec(\sin(x))\,dx &= \text{Subst}\left(\int \sec(x)\,dx, x, \sin(x)\right)\\ &= \tanh^{-1}(\sin(\sin(x)))\end{aligned}$$

Mathematica [A] time = 0.01, size = 4, normalized size = 1.00

$$\tanh^{-1}(\sin(\sin(x)))$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[x]*Sec[Sin[x]],x]
```

```
[Out] ArcTanh[Sin[Sin[x]]]
```

fricas [B] time = 0.88, size = 47, normalized size = 11.75

$$\frac{1}{2} \log\left(\sin\left(\frac{2 \tan\left(\frac{1}{2} x\right)}{\tan\left(\frac{1}{2} x\right)^2 + 1}\right) + 1\right) - \frac{1}{2} \log\left(-\sin\left(\frac{2 \tan\left(\frac{1}{2} x\right)}{\tan\left(\frac{1}{2} x\right)^2 + 1}\right) + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*sec(sin(x)),x, algorithm="fricas")
```

```
[Out] 1/2*log(sin(2*tan(1/2*x)/(tan(1/2*x)^2 + 1)) + 1) - 1/2*log(-sin(2*tan(1/2*
x)/(tan(1/2*x)^2 + 1)) + 1)
```

giac [B] time = 0.14, size = 29, normalized size = 7.25

$$\frac{1}{4} \log\left(\left|\frac{1}{\sin(\sin(x))} + \sin(\sin(x)) + 2\right|\right) - \frac{1}{4} \log\left(\left|\frac{1}{\sin(\sin(x))} + \sin(\sin(x)) - 2\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*sec(sin(x)),x, algorithm="giac")
```

```
[Out] 1/4*log(abs(1/sin(sin(x)) + sin(sin(x)) + 2)) - 1/4*log(abs(1/sin(sin(x)) +
 sin(sin(x)) - 2))
```

maple [A] time = 0.01, size = 9, normalized size = 2.25

$$\ln(\sec(\sin(x)) + \tan(\sin(x)))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)*sec(sin(x)),x)
```

```
[Out] ln(sec(sin(x))+tan(sin(x)))
```

maxima [A] time = 0.33, size = 8, normalized size = 2.00

$$\log(\sec(\sin(x)) + \tan(\sin(x)))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*sec(sin(x)),x, algorithm="maxima")
```

```
[Out] log(sec(sin(x)) + tan(sin(x)))
```

mupad [B] time = 3.24, size = 21, normalized size = 5.25

$$-\operatorname{atan}\left(e^{-\frac{e^{-x\,1i}}{2}}\,e^{\frac{e^{x\,1i}}{2}}\right)2i$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)/cos(sin(x)),x)
[Out] -atan(exp(-exp(-x*1i)/2)*exp(exp(x*1i)/2))*2i
```

sympy [A] time = 1.33, size = 10, normalized size = 2.50

$$\log\left(\tan\left(\sin\left(x\right)\right)+\sec\left(\sin\left(x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*sec(sin(x)),x)
[Out] log(tan(sin(x)) + sec(sin(x)))
```

3.677 $\int \cos(x)\sin^3(x)\left(a+b\sin^2(x)\right)^3\,dx$

Optimal. Leaf size=36

$$\frac{\left(a+b\sin^2(x)\right)^5}{10b^2}-\frac{a\left(a+b\sin^2(x)\right)^4}{8b^2}$$

```
[Out] -1/8*a*(a+b*sin(x)^2)^4/b^2+1/10*(a+b*sin(x)^2)^5/b^2
```

Rubi [A] time = 0.08, antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {3198, 266, 43}

$$\frac{\left(a+b\sin^2(x)\right)^5}{10b^2}-\frac{a\left(a+b\sin^2(x)\right)^4}{8b^2}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[x]*Sin[x]^3*(a + b*Sin[x]^2)^3,x]
```

```
[Out] -(a*(a + b*Sin[x]^2)^4)/(8*b^2) + (a + b*Sin[x]^2)^5/(10*b^2)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 3198

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a
_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.), x_Symbol] :> With[{ff = FreeFa
ctors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(d*ff*x)^n*(1 - ff^2*x^2)^((m
 - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, d
, e, f, n, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\int \cos(x)\sin^3(x)\left(a+b\sin^2(x)\right)^3\,dx=\text{Subst}\left(\int x^3\left(a+bx^2\right)^3\,dx,x,\sin(x)\right)$$
$$=\frac{1}{2}\text{Subst}\left(\int x(a+bx)^3\,dx,x,\sin^2(x)\right)$$
$$=\frac{1}{2}\text{Subst}\left(\int\left(-\frac{a(a+bx)^3}{b}+\frac{(a+bx)^4}{b}\right)dx,x,\sin^2(x)\right)$$
$$=-\frac{a\left(a+b\sin^2(x)\right)^4}{8b^2}+\frac{\left(a+b\sin^2(x)\right)^5}{10b^2}$$

Mathematica [B] time = 0.35, size = 128, normalized size = 3.56

$$\frac{-20\left(64a^3+24ab^2+7b^3\right)\cos(2x)+20\left(16a^3+18ab^2+5b^3\right)\cos(4x)+b\left(3840a^2\sin^4(x)-1280a^2\sin(3x)\sin^3\right.}{10240}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[x]*Sin[x]^3*(a + b*Sin[x]^2)^3,x]
```

```
[Out] (-20*(64*a^3 + 24*a*b^2 + 7*b^3)*Cos[2*x] + 20*(16*a^3 + 18*a*b^2 + 5*b^3)*
Cos[4*x] + b*(-10*b*(16*a + 5*b)*Cos[6*x] + 15*b*(2*a + b)*Cos[8*x] - 2*b^2
*Cos[10*x] + 3840*a^2*Sin[x]^4 + 2560*a*b*Sin[x]^6 + 640*b^2*Sin[x]^8 - 128
0*a^2*Sin[x]^3*Sin[3*x]))/10240
```

fricas [B] time = 1.19, size = 103, normalized size = 2.86

$$-\frac{1}{10}b^3\cos(x)^{10}+\frac{1}{8}\left(3ab^2+4b^3\right)\cos(x)^8-\frac{1}{2}\left(a^2b+3ab^2+2b^3\right)\cos(x)^6+\frac{1}{4}\left(a^3+6a^2b+9ab^2+4b^3\right)\cos(x)^4-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*sin(x)^3*(a+b*sin(x)^2)^3,x, algorithm="fricas")
```

```
[Out] -1/10*b^3*cos(x)^10 + 1/8*(3*a*b^2 + 4*b^3)*cos(x)^8 - 1/2*(a^2*b + 3*a*b^2
 + 2*b^3)*cos(x)^6 + 1/4*(a^3 + 6*a^2*b + 9*a*b^2 + 4*b^3)*cos(x)^4 - 1/2*(
a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cos(x)^2
```

giac [A] time = 0.12, size = 39, normalized size = 1.08

$$\frac{1}{10}b^3\sin(x)^{10}+\frac{3}{8}ab^2\sin(x)^8+\frac{1}{2}a^2b\sin(x)^6+\frac{1}{4}a^3\sin(x)^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*sin(x)^3*(a+b*sin(x)^2)^3,x, algorithm="giac")
```

```
[Out] 1/10*b^3*sin(x)^10 + 3/8*a*b^2*sin(x)^8 + 1/2*a^2*b*sin(x)^6 + 1/4*a^3*sin(
x)^4
```

maple [A] time = 0.01, size = 40, normalized size = 1.11

$$\frac{b^3\left(\sin^{10}(x)\right)}{10}+\frac{3a\,b^2\left(\sin^{8}(x)\right)}{8}+\frac{a^2b\left(\sin^{6}(x)\right)}{2}+\frac{a^3\left(\sin^{4}(x)\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)*sin(x)^3*(a+b*sin(x)^2)^3,x)
```

```
[Out] 1/10*b^3*sin(x)^10+3/8*a*b^2*sin(x)^8+1/2*a^2*b*sin(x)^6+1/4*a^3*sin(x)^4
```

maxima [A] time = 0.33, size = 39, normalized size = 1.08

$$\frac{1}{10}\,b^3\sin(x)^{10}+\frac{3}{8}\,ab^2\sin(x)^8+\frac{1}{2}\,a^2b\sin(x)^6+\frac{1}{4}\,a^3\sin(x)^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*sin(x)^3*(a+b*sin(x)^2)^3,x, algorithm="maxima")
```

```
[Out] 1/10*b^3*sin(x)^10 + 3/8*a*b^2*sin(x)^8 + 1/2*a^2*b*sin(x)^6 + 1/4*a^3*sin(
x)^4
```

mupad [B] time = 0.07, size = 73, normalized size = 2.03

$$\frac{b^2\cos(x)^8\,\left(3\,a+4\,b\right)}{8}-\frac{b^3\cos(x)^{10}}{10}-\frac{\cos(x)^2\,{\left(a+b\right)}^3}{2}-\frac{b\cos(x)^6\,\left(a^2+3\,a\,b+2\,b^2\right)}{2}+\frac{\cos(x)^4\,{\left(a+b\right)}^2\,\left(a+4\,b\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)*sin(x)^3*(a + b*sin(x)^2)^3,x)
```

```
[Out] (b^2*cos(x)^8*(3*a + 4*b))/8 - (b^3*cos(x)^10)/10 - (cos(x)^2*(a + b)^3)/2
- (b*cos(x)^6*(3*a*b + a^2 + 2*b^2))/2 + (cos(x)^4*(a + b)^2*(a + 4*b))/4
```

sympy [A] time = 11.38, size = 44, normalized size = 1.22

$$\frac{a^3\sin^4{(x)}}{4}+\frac{a^2b\sin^6{(x)}}{2}+\frac{3ab^2\sin^8{(x)}}{8}+\frac{b^3\sin^{10}{(x)}}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*sin(x)**3*(a+b*sin(x)**2)**3,x)
```

```
[Out] a**3*sin(x)**4/4 + a**2*b*sin(x)**6/2 + 3*a*b**2*sin(x)**8/8 + b**3*sin(x)*
*10/10
```

3.678 $\int e^{\sin(x)}\cos(x)\sin(x)\,dx$

Optimal. Leaf size=14

$$e^{\sin(x)}\sin(x)-e^{\sin(x)}$$

```
[Out] -exp(sin(x))+exp(sin(x))*sin(x)
```

Rubi [A] time = 0.02, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {4334, 2176, 2194}

$$e^{\sin(x)}\sin(x)-e^{\sin(x)}$$

Antiderivative was successfully verified.

```
[In] Int[E^Sin[x]*Cos[x]*Sin[x],x]
```

```
[Out] -E^Sin[x] + E^Sin[x]*Sin[x]
```

Rule 2176

```
Int[((b_.)*(F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m
_.), x_Symbol] :> Simp[((c + d*x)^m*(b*F^(g*(e + f*x)))^n)/(f*g*n*Log[F]),
x] - Dist[(d*m)/(f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*(b*F^(g*(e + f*x)))^n
, x], x] /; FreeQ[{F, b, c, d, e, f, g, n}, x] && GtQ[m, 0] && IntegerQ[2*m
] &&  !$UseGamma === True
```

Rule 2194

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rule 4334

```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{d = FreeFacto
rs[Sin[c*(a + b*x)], x]}, Dist[d/(b*c), Subst[Int[SubstFor[1, Sin[c*(a + b*
x)]/d, u, x], x], x, Sin[c*(a + b*x)]/d], x] /; FunctionOfQ[Sin[c*(a + b*x)
]/d, u, x, True]] /; FreeQ[{a, b, c}, x] && (EqQ[F, Cos] || EqQ[F, cos])
```

Rubi steps

$$\int e^{\sin(x)} \cos(x) \sin(x)\, dx = \text{Subst}\left(\int e^x x\, dx, x, \sin(x)\right)$$
$$= e^{\sin(x)} \sin(x) - \text{Subst}\left(\int e^x\, dx, x, \sin(x)\right)$$
$$= -e^{\sin(x)} + e^{\sin(x)} \sin(x)$$

Mathematica [**A**] time = 0.01, size = 9, normalized size = 0.64

$$e^{\sin(x)}(\sin(x) - 1)$$

Antiderivative was successfully verified.

```
[In] Integrate[E^Sin[x]*Cos[x]*Sin[x],x]
[Out] E^Sin[x]*(-1 + Sin[x])
```

fricas [**A**] time = 0.86, size = 8, normalized size = 0.57

$$(\sin(x) - 1)e^{\sin(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(sin(x))*cos(x)*sin(x),x, algorithm="fricas")
[Out] (sin(x) - 1)*e^sin(x)
```

giac [**A**] time = 0.14, size = 8, normalized size = 0.57

$$(\sin(x) - 1)e^{\sin(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(sin(x))*cos(x)*sin(x),x, algorithm="giac")
[Out] (sin(x) - 1)*e^sin(x)
```

maple [**A**] time = 0.00, size = 13, normalized size = 0.93

$$-\mathrm{e}^{\sin(x)} + \mathrm{e}^{\sin(x)} \sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(sin(x))*cos(x)*sin(x),x)
```

```
[Out] -exp(sin(x))+exp(sin(x))*sin(x)
```

maxima [A] time = 0.33, size = 8, normalized size = 0.57

$$(\sin(x) - 1)e^{\sin(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(sin(x))*cos(x)*sin(x),x, algorithm="maxima")
[Out] (sin(x) - 1)*e^sin(x)
```

mupad [B] time = 2.91, size = 8, normalized size = 0.57

$$\mathrm{e}^{\sin(x)}\,(\sin(x) - 1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(sin(x))*cos(x)*sin(x),x)
[Out] exp(sin(x))*(sin(x) - 1)
```

sympy [A] time = 0.64, size = 12, normalized size = 0.86

$$e^{\sin{(x)}}\sin{(x)} - e^{\sin{(x)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(sin(x))*cos(x)*sin(x),x)
[Out] exp(sin(x))*sin(x) - exp(sin(x))
```

3.679 $\int \frac{\cos^3(x)}{\sqrt{\sin^3(x)}}\,dx$

Optimal. Leaf size=25

$$-\frac{2\sin(x)}{\sqrt{\sin^3(x)}}-\frac{2}{3}\sqrt{\sin^3(x)}$$

[Out] -2*sin(x)/(sin(x)^3)^(1/2)-2/3*(sin(x)^3)^(1/2)

Rubi [A] time = 0.05, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {3207, 2564, 14}

$$-\frac{2\sin(x)}{\sqrt{\sin^3(x)}}-\frac{2}{3}\sqrt{\sin^3(x)}$$

Antiderivative was successfully verified.

[In] Int[Cos[x]^3/Sqrt[Sin[x]^3],x]

[Out] (-2*Sin[x])/Sqrt[Sin[x]^3] - (2*Sqrt[Sin[x]^3])/3

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2564

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 3207

```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Dist[((b*ff^n)^IntPart[p]*(b*Sin[e + f*x]^
n)^FracPart[p])/(Sin[e + f*x]/ff)^(n*FracPart[p]), Int[ActivateTrig[u]*(Sin
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$
\begin{aligned}
\int \frac{\cos^3(x)}{\sqrt{\sin^3(x)}}\,dx &= \frac{\sin^{\frac{3}{2}}(x)\int \frac{\cos^3(x)}{\sin^{\frac{3}{2}}(x)}\,dx}{\sqrt{\sin^3(x)}} \\
&= \frac{\sin^{\frac{3}{2}}(x)\,\text{Subst}\left(\int \frac{1-x^2}{x^{3/2}}\,dx, x, \sin(x)\right)}{\sqrt{\sin^3(x)}} \\
&= \frac{\sin^{\frac{3}{2}}(x)\,\text{Subst}\left(\int \left(\frac{1}{x^{3/2}}-\sqrt{x}\right)dx, x, \sin(x)\right)}{\sqrt{\sin^3(x)}} \\
&= -\frac{2\sin(x)}{\sqrt{\sin^3(x)}}-\frac{2}{3}\sqrt{\sin^3(x)}
\end{aligned}
$$

Mathematica [A] time = 0.02, size = 20, normalized size = 0.80

$$\frac{\sin(x)(\cos(2x)-7)}{3\sqrt{\sin^3(x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[x]^3/Sqrt[Sin[x]^3],x]
[Out] ((-7 + Cos[2*x])*Sin[x])/(3*Sqrt[Sin[x]^3])
```

fricas [A] time = 0.89, size = 28, normalized size = 1.12

$$-\frac{2\left(\cos(x)^2-4\right)\sqrt{-\left(\cos(x)^2-1\right)\sin(x)}}{3\left(\cos(x)^2-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^3/(sin(x)^3)^(1/2),x, algorithm="fricas")
[Out] -2/3*(cos(x)^2 - 4)*sqrt(-(cos(x)^2 - 1)*sin(x))/(cos(x)^2 - 1)
```

giac [A] time = 0.13, size = 13, normalized size = 0.52

$$-\frac{2}{3}\sin(x)^{\frac{3}{2}}-\frac{2}{\sqrt{\sin(x)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^3/(sin(x)^3)^(1/2),x, algorithm="giac")
[Out] -2/3*sin(x)^(3/2) - 2/sqrt(sin(x))
```

maple [A] time = 0.16, size = 14, normalized size = 0.56

$$-\frac{2\left(\sin^{\frac{3}{2}}(x)\right)}{3}-\frac{2}{\sqrt{\sin(x)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)^3/(sin(x)^3)^(1/2),x)
[Out] -2/3*sin(x)^(3/2)-2/sin(x)^(1/2)
```

maxima [A] time = 0.32, size = 19, normalized size = 0.76

$$-\frac{2}{3}\sqrt{\sin(x)^3}-\frac{2\sin(x)}{\sqrt{\sin(x)^3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^3/(sin(x)^3)^(1/2),x, algorithm="maxima")
[Out] -2/3*sqrt(sin(x)^3) - 2*sin(x)/sqrt(sin(x)^3)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.04

$$\int\frac{\cos(x)^3}{\sqrt{\sin(x)^3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)^3/(sin(x)^3)^(1/2),x)
[Out] int(cos(x)^3/(sin(x)^3)^(1/2), x)
```

sympy [A] time = 2.07, size = 36, normalized size = 1.44

$$-\frac{8\sin^3(x)}{3\sqrt{\sin^3(x)}}-\frac{2\sin(x)\cos^2(x)}{\sqrt{\sin^3(x)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)**3/(sin(x)**3)**(1/2),x)
[Out] -8*sin(x)**3/(3*sqrt(sin(x)**3)) - 2*sin(x)*cos(x)**2/sqrt(sin(x)**3)
```

3.680 $\int \frac{e^{\sqrt{\sin(x)}} \cos(x)}{\sqrt{\sin(x)}} \, dx$

Optimal. Leaf size=10

$$2e^{\sqrt{\sin(x)}}$$

```
[Out] 2*exp(sin(x)^(1/2))
```

Rubi [A] time = 0.03, antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {4334, 2209}

$$2e^{\sqrt{\sin(x)}}$$

Antiderivative was successfully verified.

```
[In] Int[(E^Sqrt[Sin[x]]*Cos[x])/Sqrt[Sin[x]],x]
```

```
[Out] 2*E^Sqrt[Sin[x]]
```

Rule 2209

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[((e + f*x)^n*F^(a + b*(c + d*x)^n))/(b*f*n*(c + d*x)^
n*Log[F]), x] /; FreeQ[{F, a, b, c, d, e, f, n}, x] && EqQ[m, n - 1] && EqQ
[d*e - c*f, 0]
```

Rule 4334

```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{d = FreeFacto
rs[Sin[c*(a + b*x)], x]}, Dist[d/(b*c), Subst[Int[SubstFor[1, Sin[c*(a + b*
x)]/d, u, x], x], x, Sin[c*(a + b*x)]/d], x] /; FunctionOfQ[Sin[c*(a + b*x)
]/d, u, x, True]] /; FreeQ[{a, b, c}, x] && (EqQ[F, Cos] || EqQ[F, cos])
```

Rubi steps

$$\begin{aligned} \int \frac{e^{\sqrt{\sin(x)}} \cos(x)}{\sqrt{\sin(x)}} \, dx &= \text{Subst}\left(\int \frac{e^{\sqrt{x}}}{\sqrt{x}} \, dx, x, \sin(x)\right) \\ &= 2e^{\sqrt{\sin(x)}} \end{aligned}$$

Mathematica [A] time = 0.01, size = 10, normalized size = 1.00

$$2e^{\sqrt{\sin(x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^Sqrt[Sin[x]]*Cos[x])/Sqrt[Sin[x]],x]
[Out] 2*E^Sqrt[Sin[x]]
```

fricas [A]   time = 1.18, size = 7, normalized size = 0.70

$$2e^{\sqrt{\sin(x)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(sin(x)^(1/2))*cos(x)/sin(x)^(1/2),x, algorithm="fricas")
[Out] 2*e^sqrt(sin(x))
```

giac [A]   time = 0.15, size = 7, normalized size = 0.70

$$2e^{\sqrt{\sin(x)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(sin(x)^(1/2))*cos(x)/sin(x)^(1/2),x, algorithm="giac")
[Out] 2*e^sqrt(sin(x))
```

maple [A]   time = 0.02, size = 8, normalized size = 0.80

$$2\,\mathrm{e}^{\sqrt{\sin}(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(sin(x)^(1/2))*cos(x)/sin(x)^(1/2),x)
[Out] 2*exp(sin(x)^(1/2))
```

maxima [A]   time = 0.32, size = 7, normalized size = 0.70

$$2e^{\sqrt{\sin(x)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(sin(x)^(1/2))*cos(x)/sin(x)^(1/2),x, algorithm="maxima")
[Out] 2*e^sqrt(sin(x))
```

mupad [B]   time = 3.00, size = 7, normalized size = 0.70

$$2\,\mathrm{e}^{\sqrt{\sin(x)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((exp(sin(x)^(1/2))*cos(x))/sin(x)^(1/2),x)
```

```
[Out] 2*exp(sin(x)^(1/2))
```

sympy [A] time = 0.46, size = 8, normalized size = 0.80

$$2e^{\sqrt{\sin(x)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(sin(x)**(1/2))*cos(x)/sin(x)**(1/2),x)
```

```
[Out] 2*exp(sqrt(sin(x)))
```

3.681 $\int e^{4+\sin(x)} \cos(x)\,dx$

Optimal. Leaf size=6

$$e^{\sin(x)+4}$$

```
[Out] exp(4+sin(x))
```

Rubi [A] time = 0.01, antiderivative size = 6, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {4334, 2194}

$$e^{\sin(x)+4}$$

Antiderivative was successfully verified.

```
[In] Int[E^(4 + Sin[x])*Cos[x],x]
```

```
[Out] E^(4 + Sin[x])
```

Rule 2194

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rule 4334

```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{d = FreeFacto
rs[Sin[c*(a + b*x)], x]}, Dist[d/(b*c), Subst[Int[SubstFor[1, Sin[c*(a + b*
x)]/d, u, x], x], x, Sin[c*(a + b*x)]/d], x] /; FunctionOfQ[Sin[c*(a + b*x)
]/d, u, x, True]] /; FreeQ[{a, b, c}, x] && (EqQ[F, Cos] || EqQ[F, cos])
```

Rubi steps

$$\begin{aligned}\int e^{4+\sin(x)} \cos(x)\,dx &= \text{Subst}\left(\int e^{4+x}\,dx, x, \sin(x)\right)\\ &= e^{4+\sin(x)}\end{aligned}$$

Mathematica [A] time = 0.01, size = 6, normalized size = 1.00

$$e^{\sin(x)+4}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(4 + Sin[x])*Cos[x],x]
```

```
[Out] E^(4 + Sin[x])
```

fricas [A] time = 0.94, size = 5, normalized size = 0.83

$$e^{(\sin(x)+4)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(4+sin(x))*cos(x),x, algorithm="fricas")
[Out] e^(sin(x) + 4)
```

giac [A] time = 0.12, size = 5, normalized size = 0.83

$$e^{(\sin(x)+4)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(4+sin(x))*cos(x),x, algorithm="giac")
[Out] e^(sin(x) + 4)
```

maple [A] time = 0.03, size = 6, normalized size = 1.00

$$e^{4+\sin(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(4+sin(x))*cos(x),x)
[Out] exp(4+sin(x))
```

maxima [A] time = 0.32, size = 5, normalized size = 0.83

$$e^{(\sin(x)+4)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(4+sin(x))*cos(x),x, algorithm="maxima")
[Out] e^(sin(x) + 4)
```

mupad [B] time = 2.91, size = 6, normalized size = 1.00

$$e^{\sin(x)}\,e^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(sin(x) + 4)*cos(x),x)
```

```
[Out] exp(sin(x))*exp(4)
```

sympy [A] time = 0.54, size = 7, normalized size = 1.17

$$e^4 e^{\sin(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(4+sin(x))*cos(x),x)
```

```
[Out] exp(4)*exp(sin(x))
```

3.682 $\int e^{\cos(x)\sin(x)}\cos(2x)\,dx$

Optimal. Leaf size=10

$$e^{\frac{1}{2}\sin(2x)}$$

[Out] exp(1/2*sin(2*x))

Rubi [A] time = 0.01, antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {4356, 2194}

$$e^{\frac{1}{2}\sin(2x)}$$

Antiderivative was successfully verified.

[In] Int[E^(Cos[x]*Sin[x])*Cos[2*x],x]

[Out] E^(Sin[2*x]/2)

Rule 2194

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rule 4356

```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{d = FreeFacto
rs[Sin[c*(a + b*x)], x]}, Dist[d/(b*c), Subst[Int[SubstFor[1, Sin[c*(a + b*
x)]/d, u, x], x], x, Sin[c*(a + b*x)]/d], x] /; FunctionOfQ[Sin[c*(a + b*x)
]/d, u, x]] /; FreeQ[{a, b, c}, x] && (EqQ[F, Cos] || EqQ[F, cos])
```

Rubi steps

$$\begin{aligned}\int e^{\cos(x)\sin(x)}\cos(2x)\,dx &= \frac{1}{2}\text{Subst}\left(\int e^{x/2}\,dx, x, \sin(2x)\right)\\ &= e^{\frac{1}{2}\sin(2x)}\end{aligned}$$

Mathematica [A] time = 0.03, size = 7, normalized size = 0.70

$$e^{\sin(x)\cos(x)}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(Cos[x]*Sin[x])*Cos[2*x],x]
[Out] E^(Cos[x]*Sin[x])
```

fricas [A] time = 0.79, size = 6, normalized size = 0.60

$$e^{(\cos(x)\sin(x))}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(cos(x)*sin(x))*cos(2*x),x, algorithm="fricas")
[Out] e^(cos(x)*sin(x))
```

giac [A] time = 0.15, size = 12, normalized size = 1.20

$$e^{\left(\frac{\tan(x)}{\tan(x)^2+1}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(cos(x)*sin(x))*cos(2*x),x, algorithm="giac")
[Out] e^(tan(x)/(tan(x)^2 + 1))
```

maple [A] time = 0.08, size = 7, normalized size = 0.70

$$e^{\cos(x)\sin(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(cos(x)*sin(x))*cos(2*x),x)
[Out] exp(cos(x)*sin(x))
```

maxima [A] time = 0.95, size = 7, normalized size = 0.70

$$e^{\left(\frac{1}{2}\sin(2x)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(cos(x)*sin(x))*cos(2*x),x, algorithm="maxima")
[Out] e^(1/2*sin(2*x))
```

mupad [B] time = 2.99, size = 7, normalized size = 0.70

$$e^{\frac{\sin(2x)}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(2*x)*exp(cos(x)*sin(x)),x)
```

```
[Out] exp(sin(2*x)/2)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(cos(x)*sin(x))*cos(2*x),x)
```

```
[Out] Timed out
```

3.683 $\int e^{\cos\left(\frac{x}{2}\right)\sin\left(\frac{x}{2}\right)}\cos(x)\,dx$

Optimal. Leaf size=10

$$2e^{\frac{\sin(x)}{2}}$$

```
[Out] 2*exp(1/2*sin(x))
```

Rubi [A] time = 0.01, antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {4356, 2194}

$$2e^{\frac{\sin(x)}{2}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(Cos[x/2]*Sin[x/2])*Cos[x],x]
```

```
[Out] 2*E^(Sin[x]/2)
```

Rule 2194

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rule 4356

```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{d = FreeFacto
rs[Sin[c*(a + b*x)], x]}, Dist[d/(b*c), Subst[Int[SubstFor[1, Sin[c*(a + b*
x)]/d, u, x], x], x, Sin[c*(a + b*x)]/d], x] /; FunctionOfQ[Sin[c*(a + b*x)
]/d, u, x]] /; FreeQ[{a, b, c}, x] && (EqQ[F, Cos] || EqQ[F, cos])
```

Rubi steps

$$\begin{aligned}\int e^{\cos\left(\frac{x}{2}\right)\sin\left(\frac{x}{2}\right)}\cos(x)\,dx &= \text{Subst}\left(\int e^{x/2}\,dx, x, \sin(x)\right)\\ &= 2e^{\frac{\sin(x)}{2}}\end{aligned}$$

Mathematica [A] time = 0.01, size = 10, normalized size = 1.00

$$2e^{\frac{\sin(x)}{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(Cos[x/2]*Sin[x/2])*Cos[x],x]
[Out] 2*E^(Sin[x]/2)
```

fricas [A] time = 0.72, size = 12, normalized size = 1.20

$$2 e^{\left(\cos\left(\frac{1}{2} x\right) \sin\left(\frac{1}{2} x\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(cos(1/2*x)*sin(1/2*x))*cos(x),x, algorithm="fricas")
[Out] 2*e^(cos(1/2*x)*sin(1/2*x))
```

giac [B] time = 0.14, size = 18, normalized size = 1.80

$$2 e^{\left(\frac{\tan\left(\frac{1}{2} x\right)}{\tan\left(\frac{1}{2} x\right)^{2}+1}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(cos(1/2*x)*sin(1/2*x))*cos(x),x, algorithm="giac")
[Out] 2*e^(tan(1/2*x)/(tan(1/2*x)^2 + 1))
```

maple [A] time = 0.08, size = 13, normalized size = 1.30

$$2 \mathrm{e}^{\cos\left(\frac{x}{2}\right) \sin\left(\frac{x}{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(cos(1/2*x)*sin(1/2*x))*cos(x),x)
[Out] 2*exp(cos(1/2*x)*sin(1/2*x))
```

maxima [A] time = 0.33, size = 7, normalized size = 0.70

$$2 e^{\left(\frac{1}{2} \sin\left(x\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(cos(1/2*x)*sin(1/2*x))*cos(x),x, algorithm="maxima")
[Out] 2*e^(1/2*sin(x))
```

mupad [B] time = 2.95, size = 7, normalized size = 0.70

$$2\,\mathrm{e}^{\frac{\sin(x)}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(cos(x/2)*sin(x/2))*cos(x),x)
```

```
[Out] 2*exp(sin(x)/2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int e^{\sin\left(\frac{x}{2}\right)\cos\left(\frac{x}{2}\right)}\cos\left(x\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(cos(1/2*x)*sin(1/2*x))*cos(x),x)
```

```
[Out] Integral(exp(sin(x/2)*cos(x/2))*cos(x), x)
```

3.684 $\int e^{n\sin(a+bx)}\cos(a+bx)\,dx$

Optimal. Leaf size=17

$$\frac{e^{n\sin(a+bx)}}{bn}$$

```
[Out] exp(n*sin(b*x+a))/b/n
```

Rubi [A] time = 0.01, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {4334, 2194}

$$\frac{e^{n\sin(a+bx)}}{bn}$$

Antiderivative was successfully verified.

```
[In] Int[E^(n*Sin[a + b*x])*Cos[a + b*x],x]
```

```
[Out] E^(n*Sin[a + b*x])/(b*n)
```

Rule 2194

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rule 4334

```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{d = FreeFacto
rs[Sin[c*(a + b*x)], x]}, Dist[d/(b*c), Subst[Int[SubstFor[1, Sin[c*(a + b*
x)]/d, u, x], x], x, Sin[c*(a + b*x)]/d], x] /; FunctionOfQ[Sin[c*(a + b*x)
]/d, u, x, True]] /; FreeQ[{a, b, c}, x] && (EqQ[F, Cos] || EqQ[F, cos])
```

Rubi steps

$$\begin{aligned}\int e^{n\sin(a+bx)}\cos(a+bx)\,dx &= \frac{\text{Subst}\left(\int e^{nx}\,dx, x, \sin(a+bx)\right)}{b}\\ &= \frac{e^{n\sin(a+bx)}}{bn}\end{aligned}$$

Mathematica [A] time = 0.02, size = 17, normalized size = 1.00

$$\frac{e^{n\sin(a+bx)}}{bn}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(n*Sin[a + b*x])*Cos[a + b*x],x]
```

```
[Out] E^(n*Sin[a + b*x])/(b*n)
```

fricas [A] time = 0.78, size = 16, normalized size = 0.94

$$\frac{e^{\left(n \sin\left(bx+a\right)\right)}}{bn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*sin(b*x+a))*cos(b*x+a),x, algorithm="fricas")
```

```
[Out] e^(n*sin(b*x + a))/(b*n)
```

giac [A] time = 0.13, size = 16, normalized size = 0.94

$$\frac{e^{\left(n \sin\left(bx+a\right)\right)}}{bn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*sin(b*x+a))*cos(b*x+a),x, algorithm="giac")
```

```
[Out] e^(n*sin(b*x + a))/(b*n)
```

maple [A] time = 0.01, size = 17, normalized size = 1.00

$$\frac{\mathrm{e}^{n \sin\left(bx+a\right)}}{bn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*sin(b*x+a))*cos(b*x+a),x)
```

```
[Out] exp(n*sin(b*x+a))/b/n
```

maxima [A] time = 0.31, size = 16, normalized size = 0.94

$$\frac{e^{\left(n \sin\left(bx+a\right)\right)}}{bn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*sin(b*x+a))*cos(b*x+a),x, algorithm="maxima")
```

```
[Out] e^(n*sin(b*x + a))/(b*n)
```

mupad [B] time = 0.11, size = 16, normalized size = 0.94

$$\frac{e^{n \sin(a+b x)}}{b n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)*exp(n*sin(a + b*x)),x)
```

```
[Out] exp(n*sin(a + b*x))/(b*n)
```

sympy [A] time = 0.43, size = 36, normalized size = 2.12

$$\begin{cases} x \cos(a) & \text{for } b = 0 \wedge n = 0 \\ x e^{n \sin(a)} \cos(a) & \text{for } b = 0 \\ \frac{\sin(a+bx)}{b} & \text{for } n = 0 \\ \frac{e^{n \sin(a+bx)}}{bn} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*sin(b*x+a))*cos(b*x+a),x)
```

```
[Out] Piecewise((x*cos(a), Eq(b, 0) & Eq(n, 0)), (x*exp(n*sin(a))*cos(a), Eq(b, 0
)), (sin(a + b*x)/b, Eq(n, 0)), (exp(n*sin(a + b*x))/(b*n), True))
```

3.685 $\int e^{n\sin(ac+bcx)}\cos(c(a+bx))\,dx$

Optimal. Leaf size=22

$$\frac{e^{n\sin(c(a+bx))}}{bcn}$$

[Out] exp(n*sin(c*(b*x+a)))/b/c/n

Rubi [A] time = 0.01, antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {4334, 2194}

$$\frac{e^{n\sin(c(a+bx))}}{bcn}$$

Antiderivative was successfully verified.

[In] Int[E^(n*Sin[a*c + b*c*x])*Cos[c*(a + b*x)],x]

[Out] E^(n*Sin[c*(a + b*x)])/(b*c*n)

Rule 2194

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rule 4334

```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{d = FreeFacto
rs[Sin[c*(a + b*x)], x]}, Dist[d/(b*c), Subst[Int[SubstFor[1, Sin[c*(a + b*
x)]/d, u, x], x], x, Sin[c*(a + b*x)]/d], x] /; FunctionOfQ[Sin[c*(a + b*x)
]/d, u, x, True]] /; FreeQ[{a, b, c}, x] && (EqQ[F, Cos] || EqQ[F, cos])
```

Rubi steps

$$\begin{aligned}\int e^{n\sin(ac+bcx)}\cos(c(a+bx))\,dx &= \frac{\text{Subst}\left(\int e^{nx}\,dx, x, \sin(c(a+bx))\right)}{bc}\\ &= \frac{e^{n\sin(c(a+bx))}}{bcn}\end{aligned}$$

Mathematica [A] time = 0.14, size = 23, normalized size = 1.05

$$\frac{e^{n\sin(ac+bcx)}}{bcn}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(n*Sin[a*c + b*c*x])*Cos[c*(a + b*x)],x]
```

```
[Out] E^(n*Sin[a*c + b*c*x])/(b*c*n)
```

fricas [A] time = 1.05, size = 22, normalized size = 1.00

$$\frac{e^{(n\sin(bcx+ac))}}{bcn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*sin(b*c*x+a*c))*cos(c*(b*x+a)),x, algorithm="fricas")
```

```
[Out] e^(n*sin(b*c*x + a*c))/(b*c*n)
```

giac [A] time = 0.13, size = 22, normalized size = 1.00

$$\frac{e^{(n\sin(bcx+ac))}}{bcn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*sin(b*c*x+a*c))*cos(c*(b*x+a)),x, algorithm="giac")
```

```
[Out] e^(n*sin(b*c*x + a*c))/(b*c*n)
```

maple [A] time = 0.06, size = 23, normalized size = 1.05

$$\frac{e^{n\sin(bcx+ac)}}{bcn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*sin(b*c*x+a*c))*cos(c*(b*x+a)),x)
```

```
[Out] exp(n*sin(b*c*x+a*c))/b/c/n
```

maxima [A] time = 0.32, size = 22, normalized size = 1.00

$$\frac{e^{(n\sin(bcx+ac))}}{bcn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*sin(b*c*x+a*c))*cos(c*(b*x+a)),x, algorithm="maxima")
```

```
[Out] e^(n*sin(b*c*x + a*c))/(b*c*n)
```

mupad [B] time = 3.08, size = 22, normalized size = 1.00

$$\frac{e^{n\,\sin(a\,c+b\,c\,x)}}{b\,c\,n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c*(a + b*x))*exp(n*sin(a*c + b*c*x)),x)
```

```
[Out] exp(n*sin(a*c + b*c*x))/(b*c*n)
```

sympy [A] time = 9.32, size = 51, normalized size = 2.32

$$\begin{cases} xe^{n\sin(ac)}\cos(ac) & \text{for } b = 0 \\ x & \text{for } c = 0 \\ \begin{cases} x\cos(ac) & \text{for } b = 0 \\ x & \text{for } c = 0 \\ \frac{\sin(ac+bcx)}{bc} & \text{otherwise} \end{cases} & \text{for } n = 0 \\ \frac{e^{n\sin(ac+bcx)}}{bcn} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*sin(b*c*x+a*c))*cos(c*(b*x+a)),x)
```

```
[Out] Piecewise((x*exp(n*sin(a*c))*cos(a*c), Eq(b, 0)), (x, Eq(c, 0)), (Piecewise
((x*cos(a*c), Eq(b, 0)), (x, Eq(c, 0)), (sin(a*c + b*c*x)/(b*c), True)), Eq
(n, 0)), (exp(n*sin(a*c + b*c*x))/(b*c*n), True))
```

3.686 $\int e^{n\sin(c(a+bx))}\cos(ac+bcx)\,dx$

Optimal. Leaf size=23

$$\frac{e^{n\sin(ac+bcx)}}{bcn}$$

```
[Out] exp(n*sin(b*c*x+a*c))/b/c/n
```

Rubi [A] time = 0.01, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {4334, 2194}

$$\frac{e^{n\sin(ac+bcx)}}{bcn}$$

Antiderivative was successfully verified.

```
[In] Int[E^(n*Sin[c*(a + b*x)])*Cos[a*c + b*c*x],x]
```

```
[Out] E^(n*Sin[a*c + b*c*x])/(b*c*n)
```

Rule 2194

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rule 4334

```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{d = FreeFacto
rs[Sin[c*(a + b*x)], x]}, Dist[d/(b*c), Subst[Int[SubstFor[1, Sin[c*(a + b*
x)]/d, u, x], x], x, Sin[c*(a + b*x)]/d], x] /; FunctionOfQ[Sin[c*(a + b*x)
]/d, u, x, True]] /; FreeQ[{a, b, c}, x] && (EqQ[F, Cos] || EqQ[F, cos])
```

Rubi steps

$$\begin{aligned}\int e^{n\sin(c(a+bx))}\cos(ac+bcx)\,dx &= \frac{\text{Subst}\left(\int e^{nx}\,dx, x, \sin(ac+bcx)\right)}{bc}\\ &= \frac{e^{n\sin(ac+bcx)}}{bcn}\end{aligned}$$

Mathematica [A] time = 0.04, size = 23, normalized size = 1.00

$$\frac{e^{n\sin(ac+bcx)}}{bcn}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(n*Sin[c*(a + b*x)])*Cos[a*c + b*c*x],x]
```

```
[Out] E^(n*Sin[a*c + b*c*x])/(b*c*n)
```

fricas [A] time = 0.80, size = 22, normalized size = 0.96

$$\frac{e^{(n \sin(bcx+ac))}}{bcn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*sin(c*(b*x+a)))*cos(b*c*x+a*c),x, algorithm="fricas")
```

```
[Out] e^(n*sin(b*c*x + a*c))/(b*c*n)
```

giac [A] time = 0.15, size = 22, normalized size = 0.96

$$\frac{e^{(n \sin(bcx+ac))}}{bcn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*sin(c*(b*x+a)))*cos(b*c*x+a*c),x, algorithm="giac")
```

```
[Out] e^(n*sin(b*c*x + a*c))/(b*c*n)
```

maple [A] time = 0.03, size = 23, normalized size = 1.00

$$\frac{e^{n \sin(bcx+ac)}}{bcn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*sin(c*(b*x+a)))*cos(b*c*x+a*c),x)
```

```
[Out] exp(n*sin(b*c*x+a*c))/b/c/n
```

maxima [A] time = 0.31, size = 22, normalized size = 0.96

$$\frac{e^{(n \sin(bcx+ac))}}{bcn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*sin(c*(b*x+a)))*cos(b*c*x+a*c),x, algorithm="maxima")
```

```
[Out] e^(n*sin(b*c*x + a*c))/(b*c*n)
```

mupad [B] time = 2.99, size = 22, normalized size = 0.96

$$\frac{e^{n\sin(ac+bcx)}}{bcn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*sin(c*(a + b*x)))*cos(a*c + b*c*x),x)
```

```
[Out] exp(n*sin(a*c + b*c*x))/(b*c*n)
```

sympy [A] time = 2.27, size = 51, normalized size = 2.22

$$\begin{cases} x & \text{for } c = 0 \wedge (b = 0 \vee c = 0) \wedge (c = 0 \vee n = 0) \\ xe^{n\sin(ac)}\cos(ac) & \text{for } b = 0 \\ \frac{\sin(ac+bcx)}{bc} & \text{for } n = 0 \\ \frac{e^{n\sin(ac+bcx)}}{bcn} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*sin(c*(b*x+a)))*cos(b*c*x+a*c),x)
```

```
[Out] Piecewise((x, Eq(c, 0) & (Eq(b, 0) | Eq(c, 0)) & (Eq(c, 0) | Eq(n, 0))), (x
*exp(n*sin(a*c))*cos(a*c), Eq(b, 0)), (sin(a*c + b*c*x)/(b*c), Eq(n, 0)), (
exp(n*sin(a*c + b*c*x))/(b*c*n), True))
```

3.687 $\int e^{n\sin(a+bx)}\cot(a+bx)\,dx$

Optimal. Leaf size=13

$$\frac{\text{Ei}(n\sin(a+bx))}{b}$$

```
[Out] Ei(n*sin(b*x+a))/b
```

Rubi [A] time = 0.02, antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {4338, 2178}

$$\frac{\text{Ei}(n\sin(a+bx))}{b}$$

Antiderivative was successfully verified.

```
[In] Int[E^(n*Sin[a + b*x])*Cot[a + b*x],x]
```

```
[Out] ExpIntegralEi[n*Sin[a + b*x]]/b
```

Rule 2178

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - (c*f)/d))*ExpIntegralEi[(f*g*(c + d*x)*Log[F])/d])/d, x] /; F
reeQ[{F, c, d, e, f, g}, x] && !$UseGamma === True
```

Rule 4338

```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{d = FreeFacto
rs[Sin[c*(a + b*x)], x]}, Dist[1/(b*c), Subst[Int[SubstFor[1/x, Sin[c*(a +
b*x)]/d, u, x], x], x, Sin[c*(a + b*x)]/d], x] /; FunctionOfQ[Sin[c*(a + b*
x)]/d, u, x, True]] /; FreeQ[{a, b, c}, x] && (EqQ[F, Cot] || EqQ[F, cot])
```

Rubi steps

$$\begin{aligned}\int e^{n\sin(a+bx)}\cot(a+bx)\,dx &= \frac{\text{Subst}\left(\int \frac{e^{nx}}{x}\,dx, x, \sin(a+bx)\right)}{b}\\ &= \frac{\text{Ei}(n\sin(a+bx))}{b}\end{aligned}$$

Mathematica [A] time = 0.04, size = 13, normalized size = 1.00

$$\frac{\text{Ei}(n\sin(a+bx))}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(n*Sin[a + b*x])*Cot[a + b*x],x]
```

```
[Out] ExpIntegralEi[n*Sin[a + b*x]]/b
```

fricas [A] time = 1.00, size = 13, normalized size = 1.00

$$\frac{\mathrm{Ei}\left(n\sin\left(bx+a\right)\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*sin(b*x+a))*cot(b*x+a),x, algorithm="fricas")
```

```
[Out] Ei(n*sin(b*x + a))/b
```

giac [A] time = 0.14, size = 13, normalized size = 1.00

$$\frac{\mathrm{Ei}\left(n\sin\left(bx+a\right)\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*sin(b*x+a))*cot(b*x+a),x, algorithm="giac")
```

```
[Out] Ei(n*sin(b*x + a))/b
```

maple [A] time = 0.03, size = 17, normalized size = 1.31

$$-\frac{\mathrm{Ei}\left(1,-n\sin\left(bx+a\right)\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*sin(b*x+a))*cot(b*x+a),x)
```

```
[Out] -1/b*Ei(1,-n*sin(b*x+a))
```

maxima [A] time = 0.37, size = 13, normalized size = 1.00

$$\frac{\mathrm{Ei}\left(n\sin\left(bx+a\right)\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*sin(b*x+a))*cot(b*x+a),x, algorithm="maxima")
```

```
[Out] Ei(n*sin(b*x + a))/b
```

mupad [F] time = 0.00, size = -1, normalized size = -0.08

$$\int \cot(a + b\,x)\ \mathrm{e}^{n\ \sin(a+b\,x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(a + b*x)*exp(n*sin(a + b*x)),x)
[Out] int(cot(a + b*x)*exp(n*sin(a + b*x)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int e^{n\sin(a+bx)}\cot(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*sin(b*x+a))*cot(b*x+a),x)
[Out] Integral(exp(n*sin(a + b*x))*cot(a + b*x), x)
```

3.688 $\int e^{n \sin(ac+bcx)} \cot(c(a+bx))\,dx$

Optimal. Leaf size=18

$$\frac{\text{Ei}(n\sin(c(a+bx)))}{bc}$$

```
[Out] Ei(n*sin(c*(b*x+a)))/b/c
```

Rubi [A] time = 0.02, antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {4338, 2178}

$$\frac{\text{Ei}(n\sin(c(a+bx)))}{bc}$$

Antiderivative was successfully verified.

```
[In] Int[E^(n*Sin[a*c + b*c*x])*Cot[c*(a + b*x)],x]
```

```
[Out] ExpIntegralEi[n*Sin[c*(a + b*x)]]/(b*c)
```

Rule 2178

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - (c*f)/d))*ExpIntegralEi[(f*g*(c + d*x)*Log[F])/d])/d, x] /; F
reeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 4338

```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{d = FreeFacto
rs[Sin[c*(a + b*x)], x]}, Dist[1/(b*c), Subst[Int[SubstFor[1/x, Sin[c*(a +
b*x)]/d, u, x], x], x, Sin[c*(a + b*x)]/d], x] /; FunctionOfQ[Sin[c*(a + b*
x)]/d, u, x, True]] /; FreeQ[{a, b, c}, x] && (EqQ[F, Cot] || EqQ[F, cot])
```

Rubi steps

$$\begin{aligned}\int e^{n\sin(ac+bcx)}\cot(c(a+bx))\,dx &= \frac{\text{Subst}\left(\int \frac{e^{nx}}{x}\,dx, x, \sin(c(a+bx))\right)}{bc}\\ &= \frac{\text{Ei}(n\sin(c(a+bx)))}{bc}\end{aligned}$$

Mathematica [A] time = 0.07, size = 18, normalized size = 1.00

$$\frac{\text{Ei}(n\sin(c(a+bx)))}{bc}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(n*Sin[a*c + b*c*x])*Cot[c*(a + b*x)],x]
```

```
[Out] ExpIntegralEi[n*Sin[c*(a + b*x)]]/(b*c)
```

fricas [A] time = 0.67, size = 19, normalized size = 1.06

$$\frac{\mathrm{Ei}\left(n\sin\left(bcx+ac\right)\right)}{bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*sin(b*c*x+a*c))*cot(c*(b*x+a)),x, algorithm="fricas")
```

```
[Out] Ei(n*sin(b*c*x + a*c))/(b*c)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \cot\left((bx+a)c\right)e^{\left(n\sin\left(bcx+ac\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*sin(b*c*x+a*c))*cot(c*(b*x+a)),x, algorithm="giac")
```

```
[Out] integrate(cot((b*x + a)*c)*e^(n*sin(b*c*x + a*c)), x)
```

maple [A] time = 0.10, size = 23, normalized size = 1.28

$$-\frac{\mathrm{Ei}\left(1,-n\sin\left(bcx+ac\right)\right)}{cb}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*sin(b*c*x+a*c))*cot(c*(b*x+a)),x)
```

```
[Out] -1/c/b*Ei(1,-n*sin(b*c*x+a*c))
```

maxima [A] time = 0.38, size = 19, normalized size = 1.06

$$\frac{\mathrm{Ei}\left(n\sin\left(bcx+ac\right)\right)}{bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*sin(b*c*x+a*c))*cot(c*(b*x+a)),x, algorithm="maxima")
```

```
[Out] Ei(n*sin(b*c*x + a*c))/(b*c)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.06

$$\int \cot\left(c\ (a+b\,x)\right)\,\mathrm{e}^{n\ \sin(a\,c+b\,c\,x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c*(a + b*x))*exp(n*sin(a*c + b*c*x)),x)
[Out] int(cot(c*(a + b*x))*exp(n*sin(a*c + b*c*x)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int e^{n\sin\left(ac+bcx\right)}\cot\left(ac+bcx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*sin(b*c*x+a*c))*cot(c*(b*x+a)),x)
[Out] Integral(exp(n*sin(a*c + b*c*x))*cot(a*c + b*c*x), x)
```

3.689 $\int e^{n\sin(c(a+bx))}\cot(ac+bcx)\,dx$

Optimal. Leaf size=19

$$\frac{\text{Ei}(n\sin(ac+bxc))}{bc}$$

[Out] Ei(n*sin(b*c*x+a*c))/b/c

Rubi [A] time = 0.02, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {4338, 2178}

$$\frac{\text{Ei}(n\sin(ac+bxc))}{bc}$$

Antiderivative was successfully verified.

[In] Int[E^(n*Sin[c*(a + b*x)])*Cot[a*c + b*c*x],x]

[Out] ExpIntegralEi[n*Sin[a*c + b*c*x]]/(b*c)

Rule 2178

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - (c*f)/d))*ExpIntegralEi[(f*g*(c + d*x)*Log[F])/d])/d, x] /; F
reeQ[{F, c, d, e, f, g}, x] && !$UseGamma === True
```

Rule 4338

```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{d = FreeFacto
rs[Sin[c*(a + b*x)], x]}, Dist[1/(b*c), Subst[Int[SubstFor[1/x, Sin[c*(a +
b*x)]/d, u, x], x], x, Sin[c*(a + b*x)]/d], x] /; FunctionOfQ[Sin[c*(a + b*
x)]/d, u, x, True]] /; FreeQ[{a, b, c}, x] && (EqQ[F, Cot] || EqQ[F, cot])
```

Rubi steps

$$\begin{aligned}\int e^{n\sin(c(a+bx))}\cot(ac+bcx)\,dx&=\frac{\text{Subst}\left(\int\frac{e^{nx}}{x}\,dx,x,\sin(ac+bcx)\right)}{bc}\\&=\frac{\text{Ei}(n\sin(ac+bcx))}{bc}\end{aligned}$$

Mathematica [A] time = 0.06, size = 18, normalized size = 0.95

$$\frac{\text{Ei}(n\sin(c(a+bx)))}{bc}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(n*Sin[c*(a + b*x)])*Cot[a*c + b*c*x],x]
```

```
[Out] ExpIntegralEi[n*Sin[c*(a + b*x)]]/(b*c)
```

fricas [A] time = 0.92, size = 19, normalized size = 1.00

$$\frac{\mathrm{Ei}\left(n\sin\left(bcx+ac\right)\right)}{bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*sin(c*(b*x+a)))*cot(b*c*x+a*c),x, algorithm="fricas")
```

```
[Out] Ei(n*sin(b*c*x + a*c))/(b*c)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \cot\left(bcx+ac\right)e^{\left(n\sin\left((bx+a)c\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*sin(c*(b*x+a)))*cot(b*c*x+a*c),x, algorithm="giac")
```

```
[Out] integrate(cot(b*c*x + a*c)*e^(n*sin((b*x + a)*c)), x)
```

maple [A] time = 0.07, size = 23, normalized size = 1.21

$$-\frac{\mathrm{Ei}\left(1,-n\sin\left(bcx+ac\right)\right)}{cb}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*sin(c*(b*x+a)))*cot(b*c*x+a*c),x)
```

```
[Out] -1/c/b*Ei(1,-n*sin(b*c*x+a*c))
```

maxima [A] time = 0.41, size = 19, normalized size = 1.00

$$\frac{\mathrm{Ei}\left(n\sin\left(bcx+ac\right)\right)}{bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*sin(c*(b*x+a)))*cot(b*c*x+a*c),x, algorithm="maxima")
```

```
[Out] Ei(n*sin(b*c*x + a*c))/(b*c)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.05

$$\int e^{n\sin(c(a+bx))}\cot(ac+bcx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*sin(c*(a + b*x)))*cot(a*c + b*c*x),x)
[Out] int(exp(n*sin(c*(a + b*x)))*cot(a*c + b*c*x), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int e^{n\sin(ac+bcx)}\cot(ac+bcx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*sin(c*(b*x+a)))*cot(b*c*x+a*c),x)
[Out] Integral(exp(n*sin(a*c + b*c*x))*cot(a*c + b*c*x), x)
```

3.690 $\int \frac{\sec^2(x)}{a+b\tan(x)}\,dx$

Optimal. Leaf size=11

$$\frac{\log(a+b\tan(x))}{b}$$

```
[Out] ln(a+b*tan(x))/b
```

Rubi [A] time = 0.03, antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {3506, 31}

$$\frac{\log(a+b\tan(x))}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[x]^2/(a + b*Tan[x]),x]
```

```
[Out] Log[a + b*Tan[x]]/b
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 3506

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(b*f), Subst[Int[(a + x)^n*(1 + x^2/b^2)^(m/2 - 1),
x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && NeQ[a^2 + b^2, 0
] && IntegerQ[m/2]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sec^2(x)}{a+b\tan(x)}\,dx &= \frac{\text{Subst}\left(\int \frac{1}{a+x}\,dx, x, b\tan(x)\right)}{b}\\
&= \frac{\log(a+b\tan(x))}{b}
\end{aligned}$$

Mathematica [A] time = 0.06, size = 20, normalized size = 1.82

$$\frac{\log(a\cos(x)+b\sin(x))-\log(\cos(x))}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[x]^2/(a + b*Tan[x]),x]
```

```
[Out] (-Log[Cos[x]] + Log[a*Cos[x] + b*Sin[x]])/b
```

fricas [B] time = 1.82, size = 40, normalized size = 3.64

$$\frac{\log\left(2\,ab\cos(x)\sin(x) + \left(a^2 - b^2\right)\cos(x)^2 + b^2\right) - \log\left(\cos(x)^2\right)}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)^2/(a+b*tan(x)),x, algorithm="fricas")
```

```
[Out] 1/2*(log(2*a*b*cos(x)*sin(x) + (a^2 - b^2)*cos(x)^2 + b^2) - log(cos(x)^2))
/b
```

giac [A] time = 0.12, size = 12, normalized size = 1.09

$$\frac{\log\left(|b\tan(x) + a|\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)^2/(a+b*tan(x)),x, algorithm="giac")
```

```
[Out] log(abs(b*tan(x) + a))/b
```

maple [A] time = 0.07, size = 12, normalized size = 1.09

$$\frac{\ln\left(a + b\tan(x)\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(x)^2/(a+b*tan(x)),x)
```

```
[Out] ln(a+b*tan(x))/b
```

maxima [A] time = 0.31, size = 11, normalized size = 1.00

$$\frac{\log\left(b\tan(x) + a\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)^2/(a+b*tan(x)),x, algorithm="maxima")
```

```
[Out] log(b*tan(x) + a)/b
```

mupad [B] time = 3.03, size = 11, normalized size = 1.00

$$\frac{\ln\left(a+b\tan(x)\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(x)^2*(a + b*tan(x))),x)
```

```
[Out] log(a + b*tan(x))/b
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec^2\left(x\right)}{a+b\tan\left(x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)**2/(a+b*tan(x)),x)
```

```
[Out] Integral(sec(x)**2/(a + b*tan(x)), x)
```

3.691 $\int \frac{\sec^2(x)}{1-\tan^2(x)} dx$

Optimal. Leaf size=11

$$\frac{1}{2}\tanh^{-1}(2\sin(x)\cos(x))$$

```
[Out] 1/2*arctanh(2*cos(x)*sin(x))
```

Rubi [A] time = 0.03, antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {3675, 206}

$$\frac{1}{2}\tanh^{-1}(2\sin(x)\cos(x))$$

Antiderivative was successfully verified.

```
[In] Int[Sec[x]^2/(1 - Tan[x]^2),x]
```

```
[Out] ArcTanh[2*Cos[x]*Sin[x]]/2
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3675

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/(c^(m - 1)*f), Subst[Int[(c^2 + ff^2*x^2)^(m/2 - 1)*(a + b*(ff*x)^n)^p
, x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && In
tegerQ[m/2] && (IntegersQ[n, p] || IGtQ[m, 0] || IGtQ[p, 0] || EqQ[n^2, 4]
|| EqQ[n^2, 16])
```

Rubi steps

$$\begin{aligned}\int \frac{\sec^2(x)}{1-\tan^2(x)}\,dx &= \text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \tan(x)\right)\\ &= \frac{1}{2}\tanh^{-1}(2\cos(x)\sin(x))\end{aligned}$$

Mathematica [B] time = 0.01, size = 23, normalized size = 2.09

$$\frac{1}{2}\log(\sin(x)+\cos(x))-\frac{1}{2}\log(\cos(x)-\sin(x))$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[x]^2/(1 - Tan[x]^2),x]
[Out] -1/2*Log[Cos[x] - Sin[x]] + Log[Cos[x] + Sin[x]]/2
```

fricas [B] time = 0.57, size = 23, normalized size = 2.09

$$\frac{1}{4}\log\left(2\cos(x)\sin(x)+1\right)-\frac{1}{4}\log\left(-2\cos(x)\sin(x)+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)^2/(1-tan(x)^2),x, algorithm="fricas")
[Out] 1/4*log(2*cos(x)*sin(x) + 1) - 1/4*log(-2*cos(x)*sin(x) + 1)
```

giac [A] time = 0.16, size = 17, normalized size = 1.55

$$\frac{1}{2}\log\left(|\tan(x)+1|\right)-\frac{1}{2}\log\left(|\tan(x)-1|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)^2/(1-tan(x)^2),x, algorithm="giac")
[Out] 1/2*log(abs(tan(x) + 1)) - 1/2*log(abs(tan(x) - 1))
```

maple [A] time = 0.10, size = 4, normalized size = 0.36

$$\operatorname{arctanh}\left(\tan(x)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(x)^2/(1-tan(x)^2),x)
[Out] arctanh(tan(x))
```

maxima [A] time = 0.31, size = 15, normalized size = 1.36

$$\frac{1}{2}\log\left(\tan(x)+1\right)-\frac{1}{2}\log\left(\tan(x)-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)^2/(1-tan(x)^2),x, algorithm="maxima")
```

```
[Out] 1/2*log(tan(x) + 1) - 1/2*log(tan(x) - 1)
```

mupad [B] time = 3.08, size = 3, normalized size = 0.27

$$\operatorname{atanh}(\tan(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-1/(cos(x)^2*(tan(x)^2 - 1)),x)
```

```
[Out] atanh(tan(x))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{\sec^2(x)}{\tan^2(x)-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)**2/(1-tan(x)**2),x)
```

```
[Out] -Integral(sec(x)**2/(tan(x)**2 - 1), x)
```

3.692 $\int \frac{\sec^2(x)}{9+\tan^2(x)} dx$

Optimal. Leaf size=27

$$\frac{x}{3} - \frac{1}{3}\tan^{-1}\left(\frac{2\sin(x)\cos(x)}{2\cos^2(x)+1}\right)$$

```
[Out] 1/3*x-1/3*arctan(2*cos(x)*sin(x)/(1+2*cos(x)^2))
```

Rubi [A] time = 0.03, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {3675, 203}

$$\frac{x}{3} - \frac{1}{3}\tan^{-1}\left(\frac{2\sin(x)\cos(x)}{2\cos^2(x)+1}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sec[x]^2/(9 + Tan[x]^2),x]
```

```
[Out] x/3 - ArcTan[(2*Cos[x]*Sin[x])/(1 + 2*Cos[x]^2)]/3
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 3675

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/(c^(m - 1)*f), Subst[Int[(c^2 + ff^2*x^2)^(m/2 - 1)*(a + b*(ff*x)^n)^p
, x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && In
tegerQ[m/2] && (IntegersQ[n, p] || IGtQ[m, 0] || IGtQ[p, 0] || EqQ[n^2, 4]
|| EqQ[n^2, 16])
```

Rubi steps

$$\begin{aligned}
\int \frac{\sec^2(x)}{9+\tan^2(x)} dx &= \text{Subst}\left(\int \frac{1}{9+x^2} dx, x, \tan(x)\right)\\
&= \frac{x}{3} - \frac{1}{3}\tan^{-1}\left(\frac{2\cos(x)\sin(x)}{1+2\cos^2(x)}\right)
\end{aligned}$$

Mathematica [A] time = 0.02, size = 9, normalized size = 0.33

$$-\frac{1}{3}\tan^{-1}(3\cot(x))$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[x]^2/(9 + Tan[x]^2),x]
[Out] -1/3*ArcTan[3*Cot[x]]
```

fricas [A] time = 0.55, size = 21, normalized size = 0.78

$$-\frac{1}{6}\arctan\left(\frac{10\cos(x)^2-1}{6\cos(x)\sin(x)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)^2/(9+tan(x)^2),x, algorithm="fricas")
[Out] -1/6*arctan(1/6*(10*cos(x)^2 - 1)/(cos(x)*sin(x)))
```

giac [A] time = 0.15, size = 7, normalized size = 0.26

$$\frac{1}{3}\arctan\left(\frac{1}{3}\tan(x)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)^2/(9+tan(x)^2),x, algorithm="giac")
[Out] 1/3*arctan(1/3*tan(x))
```

maple [A] time = 0.10, size = 8, normalized size = 0.30

$$\frac{\arctan\left(\frac{\tan(x)}{3}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(x)^2/(9+tan(x)^2),x)
[Out] 1/3*arctan(1/3*tan(x))
```

maxima [A] time = 0.41, size = 7, normalized size = 0.26

$$\frac{1}{3}\arctan\left(\frac{1}{3}\tan(x)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)^2/(9+tan(x)^2),x, algorithm="maxima")
```

```
[Out] 1/3*arctan(1/3*tan(x))
```

mupad [B] time = 2.88, size = 7, normalized size = 0.26

$$\frac{\operatorname{atan}\left(\frac{\tan(x)}{3}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(x)^2*(tan(x)^2 + 9)),x)
```

```
[Out] atan(tan(x)/3)/3
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec^2(x)}{\tan^2(x)+9}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)**2/(9+tan(x)**2),x)
```

```
[Out] Integral(sec(x)**2/(tan(x)**2 + 9), x)
```

3.693 $\int \sec^2(x)(a+b\tan(x))^n\,dx$

Optimal. Leaf size=19

$$\frac{(a+b\tan(x))^{n+1}}{b(n+1)}$$

```
[Out] (a+b*tan(x))^(1+n)/b/(1+n)
```

Rubi [A] time = 0.04, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {3506, 32}

$$\frac{(a+b\tan(x))^{n+1}}{b(n+1)}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[x]^2*(a + b*Tan[x])^n,x]
```

```
[Out] (a + b*Tan[x])^(1 + n)/(b*(1 + n))
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 3506

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(b*f), Subst[Int[(a + x)^n*(1 + x^2/b^2)^(m/2 - 1),
x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && NeQ[a^2 + b^2, 0
] && IntegerQ[m/2]
```

Rubi steps

$$\begin{aligned}\int \sec^2(x)(a+b\tan(x))^n\,dx &= \frac{\text{Subst}\left(\int (a+x)^n\,dx, x, b\tan(x)\right)}{b}\\ &= \frac{(a+b\tan(x))^{1+n}}{b(1+n)}\end{aligned}$$

Mathematica [A] time = 0.19, size = 18, normalized size = 0.95

$$\frac{(a+b\tan(x))^{n+1}}{bn+b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[x]^2*(a + b*Tan[x])^n,x]
```

```
[Out] (a + b*Tan[x])^(1 + n)/(b + b*n)
```

fricas [A]   time = 1.06, size = 37, normalized size = 1.95

$$\frac{(a\cos(x)+b\sin(x))\left(\frac{a\cos(x)+b\sin(x)}{\cos(x)}\right)^{n}}{(bn+b)\cos(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)^2*(a+b*tan(x))^n,x, algorithm="fricas")
```

```
[Out] (a*cos(x) + b*sin(x))*((a*cos(x) + b*sin(x))/cos(x))^n/((b*n + b)*cos(x))
```

giac [F(-2)]   time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)^2*(a+b*tan(x))^n,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Unable to divide, perhaps due to rounding er
ror%%%{1,[0,1,0]%%%} / %%%{1,[0,0,1]%%%} Error: Bad Argument Value
```

maple [A]   time = 0.08, size = 20, normalized size = 1.05

$$\frac{(a+b\tan(x))^{n+1}}{b\,(n+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(x)^2*(a+b*tan(x))^n,x)
```

```
[Out] (a+b*tan(x))^(n+1)/b/(n+1)
```

maxima [A]   time = 0.31, size = 19, normalized size = 1.00

$$\frac{(b\tan(x)+a)^{n+1}}{b(n+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)^2*(a+b*tan(x))^n,x, algorithm="maxima")
```

```
[Out] (b*tan(x) + a)^(n + 1)/(b*(n + 1))
```

mupad [B] time = 3.56, size = 37, normalized size = 1.95

$$\begin{cases} \frac{\ln(a+b\tan(x))}{b} & \text{if } n=-1 \\ \frac{(a+b\tan(x))^{n+1}}{b\,(n+1)} & \text{if } n\neq -1 \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*tan(x))^n/cos(x)^2,x)
```

```
[Out] piecewise(n == -1, log(a + b*tan(x))/b, n ~= -1, (a + b*tan(x))^(n + 1)/(b*
(n + 1)))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a+b\tan(x))^n \sec^2(x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)**2*(a+b*tan(x))**n,x)
```

```
[Out] Integral((a + b*tan(x))**n*sec(x)**2, x)
```

3.694 $\int \sec^2(x)\left(1+\frac{1}{1+\tan^2(x)}\right)\,dx$

Optimal. Leaf size=4

$$x+\tan(x)$$

[Out] x+tan(x)

Rubi [A] time = 0.04, antiderivative size = 4, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {203}

$$x+\tan(x)$$

Antiderivative was successfully verified.

```
[In] Int[Sec[x]^2*(1 + (1 + Tan[x]^2)^(-1)),x]
```

```
[Out] x + Tan[x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \sec^2(x)\left(1+\frac{1}{1+\tan^2(x)}\right)\,dx &= \text{Subst}\left(\int\left(1+\frac{1}{1+x^2}\right)\,dx,x,\tan(x)\right)\\ &= \tan(x)+\text{Subst}\left(\int\frac{1}{1+x^2}\,dx,x,\tan(x)\right)\\ &= x+\tan(x)\end{aligned}$$

Mathematica [A] time = 0.01, size = 4, normalized size = 1.00

$$x+\tan(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[x]^2*(1 + (1 + Tan[x]^2)^(-1)),x]
```

```
[Out] x + Tan[x]
```

fricas [B] time = 0.89, size = 12, normalized size = 3.00

$$\frac{x\cos(x)+\sin(x)}{\cos(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)^2*(1+1/(1+tan(x)^2)),x, algorithm="fricas")
[Out] (x*cos(x) + sin(x))/cos(x)
```

giac [A] time = 0.13, size = 4, normalized size = 1.00

$$x+\tan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)^2*(1+1/(1+tan(x)^2)),x, algorithm="giac")
[Out] x + tan(x)
```

maple [A] time = 0.12, size = 5, normalized size = 1.25

$$x+\tan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(x)^2*(1+1/(1+tan(x)^2)),x)
[Out] x+tan(x)
```

maxima [A] time = 0.41, size = 4, normalized size = 1.00

$$x+\tan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)^2*(1+1/(1+tan(x)^2)),x, algorithm="maxima")
[Out] x + tan(x)
```

mupad [B] time = 2.94, size = 4, normalized size = 1.00

$$x+\tan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/(tan(x)^2 + 1) + 1)/cos(x)^2,x)
```

```
[Out] x + tan(x)
```

sympy [B] time = 0.74, size = 27, normalized size = 6.75

$$\frac{x\sec^2(x)}{\tan^2(x)+1}+\frac{\tan(x)\sec^2(x)}{\tan^2(x)+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)**2*(1+1/(1+tan(x)**2)),x)
```

```
[Out] x*sec(x)**2/(tan(x)**2 + 1) + tan(x)*sec(x)**2/(tan(x)**2 + 1)
```

3.695 $\int \frac{\sec^2(x)(2+\tan^2(x))}{1+\tan^2(x)}\,dx$

Optimal. Leaf size=4

$$x+\tan(x)$$

[Out] x+tan(x)

Rubi [A] time = 0.06, antiderivative size = 4, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {3657, 3473, 8}

$$x+\tan(x)$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[x]^2*(2 + Tan[x]^2))/(1 + Tan[x]^2),x]
```

```
[Out] x + Tan[x]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3657

```
Int[(u_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*sec[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a, b]
```

Rubi steps

$$\int \frac{\sec^2(x)\left(2+\tan^2(x)\right)}{1+\tan^2(x)}\,dx = \int \left(2+\tan^2(x)\right)\,dx$$
$$= 2x + \int \tan^2(x)\,dx$$
$$= 2x + \tan(x) - \int 1\,dx$$
$$= x + \tan(x)$$

Mathematica [A] time = 0.00, size = 4, normalized size = 1.00

$$x + \tan(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[x]^2*(2 + Tan[x]^2))/(1 + Tan[x]^2),x]
[Out] x + Tan[x]
```

fricas [B] time = 0.89, size = 12, normalized size = 3.00

$$\frac{x\cos(x) + \sin(x)}{\cos(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)^2*(2+tan(x)^2)/(1+tan(x)^2),x, algorithm="fricas")
[Out] (x*cos(x) + sin(x))/cos(x)
```

giac [A] time = 0.12, size = 4, normalized size = 1.00

$$x + \tan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)^2*(2+tan(x)^2)/(1+tan(x)^2),x, algorithm="giac")
[Out] x + tan(x)
```

maple [A] time = 0.11, size = 5, normalized size = 1.25

$$x + \tan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(x)^2*(2+tan(x)^2)/(1+tan(x)^2),x)
```

```
[Out] x+tan(x)
```

maxima [A] time = 0.50, size = 4, normalized size = 1.00

$$x + \tan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)^2*(2+tan(x)^2)/(1+tan(x)^2),x, algorithm="maxima")
```

```
[Out] x + tan(x)
```

mupad [B] time = 3.00, size = 4, normalized size = 1.00

$$x + \tan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((tan(x)^2 + 2)/(cos(x)^2*(tan(x)^2 + 1)),x)
```

```
[Out] x + tan(x)
```

sympy [B] time = 0.74, size = 27, normalized size = 6.75

$$\frac{x\sec^2(x)}{\tan^2(x)+1} + \frac{\tan(x)\sec^2(x)}{\tan^2(x)+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)**2*(2+tan(x)**2)/(1+tan(x)**2),x)
```

```
[Out] x*sec(x)**2/(tan(x)**2 + 1) + tan(x)*sec(x)**2/(tan(x)**2 + 1)
```

3.696 $\int \frac{\sec^2(x)}{2+2\tan(x)+\tan^2(x)}\,dx$

Optimal. Leaf size=33

$$x-\tan^{-1}\left(\frac{-2\cos^2(x)+\sin(x)\cos(x)+1}{\cos^2(x)+2\sin(x)\cos(x)+2}\right)$$

```
[Out] x-arctan((1-2*cos(x)^2+cos(x)*sin(x))/(2+cos(x)^2+2*cos(x)*sin(x)))
```

Rubi [A] time = 0.04, antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {4342, 617, 204}

$$x-\tan^{-1}\left(\frac{-2\cos^2(x)+\sin(x)\cos(x)+1}{\cos^2(x)+2\sin(x)\cos(x)+2}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sec[x]^2/(2 + 2*Tan[x] + Tan[x]^2),x]
```

```
[Out] x - ArcTan[(1 - 2*Cos[x]^2 + Cos[x]*Sin[x])/(2 + Cos[x]^2 + 2*Cos[x]*Sin[x]
)]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 4342

```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))]^2, x_Symbol] :> With[{d = FreeFac
tors[Tan[c*(a + b*x)], x]}, Dist[d/(b*c), Subst[Int[SubstFor[1, Tan[c*(a +
b*x)]/d, u, x], x], x, Tan[c*(a + b*x)]/d], x] /; FunctionOfQ[Tan[c*(a + b*
x)]/d, u, x, True]] /; FreeQ[{a, b, c}, x] && NonsumQ[u] && (EqQ[F, Sec] ||
 EqQ[F, sec])
```

Rubi steps

$$\int \frac{\sec^2(x)}{2+2\tan(x)+\tan^2(x)}\,dx = \text{Subst}\left(\int \frac{1}{2+2x+x^2}\,dx, x, \tan(x)\right)$$
$$= -\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1+\tan(x)\right)$$
$$= x - \tan^{-1}\left(\frac{1-2\cos^2(x)+\cos(x)\sin(x)}{2+\cos^2(x)+2\cos(x)\sin(x)}\right)$$

Mathematica [A] time = 0.03, size = 31, normalized size = 0.94

$$2\left(\frac{1}{4}\tan^{-1}(\sec(x)(\sin(x)+\cos(x)))-\frac{1}{4}\tan^{-1}\left(\frac{\cos(x)}{\sin(x)+\cos(x)}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[x]^2/(2 + 2*Tan[x] + Tan[x]^2),x]
```

```
[Out] 2*(-1/4*ArcTan[Cos[x]/(Cos[x] + Sin[x])] + ArcTan[Sec[x]*(Cos[x] + Sin[x])]
/4)
```

fricas [A] time = 0.86, size = 35, normalized size = 1.06

$$-\frac{1}{2}\arctan\left(-\frac{3\cos(x)^2+6\cos(x)\sin(x)+1}{2\left(2\cos(x)^2-\cos(x)\sin(x)-1\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)^2/(2+2*tan(x)+tan(x)^2),x, algorithm="fricas")
```

```
[Out] -1/2*arctan(-1/2*(3*cos(x)^2 + 6*cos(x)*sin(x) + 1)/(2*cos(x)^2 - cos(x)*si
n(x) - 1))
```

giac [A] time = 0.13, size = 5, normalized size = 0.15

$$\arctan(\tan(x)+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)^2/(2+2*tan(x)+tan(x)^2),x, algorithm="giac")
```

```
[Out] arctan(tan(x) + 1)
```

maple [A] time = 0.14, size = 6, normalized size = 0.18

$$\arctan\left(1+\tan(x)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(x)^2/(2+2*tan(x)+tan(x)^2),x)
[Out] arctan(1+tan(x))
```

maxima [A] time = 0.41, size = 5, normalized size = 0.15

$$\arctan\left(\tan(x)+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)^2/(2+2*tan(x)+tan(x)^2),x, algorithm="maxima")
[Out] arctan(tan(x) + 1)
```

mupad [B] time = 3.12, size = 5, normalized size = 0.15

$$\operatorname{atan}\left(\tan(x)+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(x)^2*(2*tan(x) + tan(x)^2 + 2)),x)
[Out] atan(tan(x) + 1)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sec^{2}\left(x\right)}{\tan^{2}\left(x\right)+2\tan\left(x\right)+2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)**2/(2+2*tan(x)+tan(x)**2),x)
[Out] Integral(sec(x)**2/(tan(x)**2 + 2*tan(x) + 2), x)
```

3.697 $\int \frac{\sec^2(x)}{\tan^2(x)+\tan^3(x)} \, dx$

Optimal. Leaf size=10

$$\log(\cot(x)+1)-\cot(x)$$

```
[Out] -cot(x)+ln(1+cot(x))
```

Rubi [A] time = 0.05, antiderivative size = 15, normalized size of antiderivative = 1.50, number of steps used = 3, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {4342, 44}

$$-\cot(x)-\log(\tan(x))+\log(\tan(x)+1)$$

Antiderivative was successfully verified.

```
[In] Int[Sec[x]^2/(Tan[x]^2 + Tan[x]^3),x]
```

```
[Out] -Cot[x] - Log[Tan[x]] + Log[1 + Tan[x]]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 4342

```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))]^2, x_Symbol] :> With[{d = FreeFac
tors[Tan[c*(a + b*x)], x]}, Dist[d/(b*c), Subst[Int[SubstFor[1, Tan[c*(a +
b*x)]/d, u, x], x], x, Tan[c*(a + b*x)]/d], x] /; FunctionOfQ[Tan[c*(a + b*
x)]/d, u, x, True]] /; FreeQ[{a, b, c}, x] && NonsumQ[u] && (EqQ[F, Sec] ||
 EqQ[F, sec])
```

Rubi steps

$$\begin{aligned}
\int \frac{\sec^2(x)}{\tan^2(x)+\tan^3(x)} \, dx &= \text{Subst}\left(\int \frac{1}{x^2(1+x)} \, dx, x, \tan(x)\right)\\
&= \text{Subst}\left(\int \left(\frac{1}{x^2}-\frac{1}{x}+\frac{1}{1+x}\right) dx, x, \tan(x)\right)\\
&= -\cot(x)-\log(\tan(x))+\log(1+\tan(x))
\end{aligned}$$

Mathematica [A] time = 0.04, size = 16, normalized size = 1.60

$$-\cot(x) - \log(\sin(x)) + \log(\sin(x) + \cos(x))$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[x]^2/(Tan[x]^2 + Tan[x]^3),x]
[Out] -Cot[x] - Log[Sin[x]] + Log[Cos[x] + Sin[x]]
```

fricas [B] time = 1.29, size = 36, normalized size = 3.60

$$-\frac{\log\left(-\frac{1}{4}\cos(x)^2 + \frac{1}{4}\right)\sin(x) - \log\left(2\cos(x)\sin(x) + 1\right)\sin(x) + 2\cos(x)}{2\sin(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)^2/(tan(x)^2+tan(x)^3),x, algorithm="fricas")
[Out] -1/2*(log(-1/4*cos(x)^2 + 1/4)*sin(x) - log(2*cos(x)*sin(x) + 1)*sin(x) + 2
*cos(x))/sin(x)
```

giac [A] time = 0.14, size = 19, normalized size = 1.90

$$-\frac{1}{\tan(x)} + \log\left(|\tan(x) + 1|\right) - \log\left(|\tan(x)|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)^2/(tan(x)^2+tan(x)^3),x, algorithm="giac")
[Out] -1/tan(x) + log(abs(tan(x) + 1)) - log(abs(tan(x)))
```

maple [A] time = 0.13, size = 18, normalized size = 1.80

$$-\frac{1}{\tan(x)} - \ln\left(\tan(x)\right) + \ln\left(1 + \tan(x)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(x)^2/(tan(x)^2+tan(x)^3),x)
[Out] -1/tan(x)-ln(tan(x))+ln(1+tan(x))
```

maxima [A] time = 0.31, size = 17, normalized size = 1.70

$$-\frac{1}{\tan(x)} + \log\left(\tan(x) + 1\right) - \log\left(\tan(x)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)^2/(tan(x)^2+tan(x)^3),x, algorithm="maxima")
```

```
[Out] -1/tan(x) + log(tan(x) + 1) - log(tan(x))
```

mupad [B] time = 3.10, size = 16, normalized size = 1.60

$$2\,\mathrm{atanh}\left(2\tan(x)+1\right)-\frac{1}{\tan(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(x)^2*(tan(x)^2 + tan(x)^3)),x)
```

```
[Out] 2*atanh(2*tan(x) + 1) - 1/tan(x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec^2(x)}{(\tan(x)+1)\tan^2(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)**2/(tan(x)**2+tan(x)**3),x)
```

```
[Out] Integral(sec(x)**2/((tan(x) + 1)*tan(x)**2), x)
```

3.698 $\int \frac{\sec^2(x)}{-\tan^2(x)+\tan^3(x)}\,dx$

Optimal. Leaf size=10

$$\cot(x)+\log(1-\cot(x))$$

```
[Out] cot(x)+ln(1-cot(x))
```

Rubi [A] time = 0.05, antiderivative size = 15, normalized size of antiderivative = 1.50, number of steps used = 3, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {4342, 44}

$$\cot(x)+\log(1-\tan(x))-\log(\tan(x))$$

Antiderivative was successfully verified.

```
[In] Int[Sec[x]^2/(-Tan[x]^2 + Tan[x]^3),x]
```

```
[Out] Cot[x] + Log[1 - Tan[x]] - Log[Tan[x]]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 4342

```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))]^2, x_Symbol] :> With[{d = FreeFac
tors[Tan[c*(a + b*x)], x]}, Dist[d/(b*c), Subst[Int[SubstFor[1, Tan[c*(a +
b*x)]/d, u, x], x], x, Tan[c*(a + b*x)]/d], x] /; FunctionOfQ[Tan[c*(a + b*
x)]/d, u, x, True]] /; FreeQ[{a, b, c}, x] && NonsumQ[u] && (EqQ[F, Sec] ||
 EqQ[F, sec])
```

Rubi steps

$$\begin{aligned}
\int \frac{\sec^2(x)}{-\tan^2(x)+\tan^3(x)}\,dx &= \text{Subst}\left(\int \frac{1}{(-1+x)x^2}\,dx, x, \tan(x)\right)\\
&= \text{Subst}\left(\int \left(\frac{1}{-1+x}-\frac{1}{x^2}-\frac{1}{x}\right)dx, x, \tan(x)\right)\\
&= \cot(x)+\log(1-\tan(x))-\log(\tan(x))
\end{aligned}$$

Mathematica [A] time = 0.03, size = 16, normalized size = 1.60

$$\cot(x)-\log(\sin(x))+\log(\cos(x)-\sin(x))$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[x]^2/(-Tan[x]^2 + Tan[x]^3),x]
[Out] Cot[x] + Log[Cos[x] - Sin[x]] - Log[Sin[x]]
```

fricas [B] time = 0.87, size = 36, normalized size = 3.60

$$-\frac{\log\left(-\frac{1}{4}\cos(x)^2+\frac{1}{4}\right)\sin(x)-\log\left(-2\cos(x)\sin(x)+1\right)\sin(x)-2\cos(x)}{2\sin(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)^2/(-tan(x)^2+tan(x)^3),x, algorithm="fricas")
[Out] -1/2*(log(-1/4*cos(x)^2 + 1/4)*sin(x) - log(-2*cos(x)*sin(x) + 1)*sin(x) -
2*cos(x))/sin(x)
```

giac [A] time = 0.16, size = 17, normalized size = 1.70

$$\frac{1}{\tan(x)}+\log\left(|\tan(x)-1|\right)-\log\left(|\tan(x)|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)^2/(-tan(x)^2+tan(x)^3),x, algorithm="giac")
[Out] 1/tan(x) + log(abs(tan(x) - 1)) - log(abs(tan(x)))
```

maple [A] time = 0.14, size = 16, normalized size = 1.60

$$\frac{1}{\tan(x)}-\ln\left(\tan(x)\right)+\ln\left(\tan(x)-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(x)^2/(-tan(x)^2+tan(x)^3),x)
[Out] 1/tan(x)-ln(tan(x))+ln(tan(x)-1)
```

maxima [A] time = 0.31, size = 15, normalized size = 1.50

$$\frac{1}{\tan(x)}+\log\left(\tan(x)-1\right)-\log\left(\tan(x)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)^2/(-tan(x)^2+tan(x)^3),x, algorithm="maxima")
```

```
[Out] 1/tan(x) + log(tan(x) - 1) - log(tan(x))
```

mupad [B] time = 2.98, size = 14, normalized size = 1.40

$$\frac{1}{\tan(x)} - 2\,\mathrm{atanh}\left(2\tan(x) - 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-1/(cos(x)^2*(tan(x)^2 - tan(x)^3)),x)
```

```
[Out] 1/tan(x) - 2*atanh(2*tan(x) - 1)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec^2(x)}{(\tan(x) - 1)\tan^2(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)**2/(-tan(x)**2+tan(x)**3),x)
```

```
[Out] Integral(sec(x)**2/((tan(x) - 1)*tan(x)**2), x)
```

3.699 $\int \frac{\sec^2(x)}{3-4\tan^3(x)}\,dx$

Optimal. Leaf size=176

$$\frac{x}{3\ 2^{2/3}\sqrt[6]{3}}+\frac{\log\left(2\sqrt[3]{2}\tan^2(x)+2^{2/3}\sqrt[3]{3}\tan(x)+3^{2/3}\right)}{6\ 6^{2/3}}-\frac{\log\left(\sqrt[3]{3}-2^{2/3}\tan(x)\right)}{3\ 6^{2/3}}-\frac{\tan^{-1}\left(\frac{-2\ 6^{2/3}\cos^2(x)+2\left(3-2\sqrt[3]{6}\right)\sin\ldots}{\left(6-4\sqrt[3]{6}\right)\cos^2(x)+2\ 6^{2/3}\sin(x)\cos\ldots}\right)}{3\ 2^{2/3}\sqrt[6]{3}}$$

```
[Out] 1/18*x*2^(1/3)*3^(5/6)-1/18*arctan((6^(2/3)-2*6^(2/3)*cos(x)^2+2*(3-2*6^(1/
3))*cos(x)*sin(x))/(3*2^(2/3)*3^(1/6)+4*6^(1/3)+(6-4*6^(1/3))*cos(x)^2+2*6^
(2/3)*cos(x)*sin(x)))*2^(1/3)*3^(5/6)-1/18*ln(3^(1/3)-2^(2/3)*tan(x))*6^(1/
3)+1/36*ln(3^(2/3)+2^(2/3)*3^(1/3)*tan(x)+2*2^(1/3)*tan(x)^2)*6^(1/3)
```

Rubi [A] time = 0.14, antiderivative size = 176, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.467, Rules used = {3675, 200, 31, 634, 617, 204, 628}

$$\frac{x}{3\ 2^{2/3}\sqrt[6]{3}}+\frac{\log\left(2\sqrt[3]{2}\tan^2(x)+2^{2/3}\sqrt[3]{3}\tan(x)+3^{2/3}\right)}{6\ 6^{2/3}}-\frac{\log\left(\sqrt[3]{3}-2^{2/3}\tan(x)\right)}{3\ 6^{2/3}}-\frac{\tan^{-1}\left(\frac{-2\ 6^{2/3}\cos^2(x)+2\left(3-2\sqrt[3]{6}\right)\sin\ldots}{\left(6-4\sqrt[3]{6}\right)\cos^2(x)+2\ 6^{2/3}\sin(x)\cos\ldots}\right)}{3\ 2^{2/3}\sqrt[6]{3}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[x]^2/(3 - 4*Tan[x]^3),x]
```

```
[Out] x/(3*2^(2/3)*3^(1/6)) - ArcTan[(6^(2/3) - 2*6^(2/3)*Cos[x]^2 + 2*(3 - 2*6^(
1/3))*Cos[x]*Sin[x])/(3*2^(2/3)*3^(1/6) + 4*6^(1/3) + (6 - 4*6^(1/3))*Cos[x
]^2 + 2*6^(2/3)*Cos[x]*Sin[x])]/(3*2^(2/3)*3^(1/6)) - Log[3^(1/3) - 2^(2/3)
*Tan[x]]/(3*6^(2/3)) + Log[3^(2/3) + 2^(2/3)*3^(1/3)*Tan[x] + 2*2^(1/3)*Tan
[x]^2]/(6*6^(2/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 3675

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/(c^(m - 1)*f), Subst[Int[(c^2 + ff^2*x^2)^(m/2 - 1)*(a + b*(ff*x)^n)^p
, x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && In
tegerQ[m/2] && (IntegersQ[n, p] || IGtQ[m, 0] || IGtQ[p, 0] || EqQ[n^2, 4]
|| EqQ[n^2, 16])
```

Rubi steps

$$\int \frac{\sec^2(x)}{3-4\tan^3(x)}\,dx = \text{Subst}\left(\int \frac{1}{3-4x^3}\,dx, x, \tan(x)\right)$$
$$= \frac{\text{Subst}\left(\int \frac{1}{\sqrt[3]{3}-2^{2/3}x}\,dx, x, \tan(x)\right)}{3\ 3^{2/3}} + \frac{\text{Subst}\left(\int \frac{2\sqrt[3]{3}+2^{2/3}x}{3^{2/3}+2^{2/3}\sqrt[3]{3}x+2\sqrt[3]{2}x^2}\,dx, x, \tan(x)\right)}{3\ 3^{2/3}}$$
$$= -\frac{\log\left(\sqrt[3]{3}-2^{2/3}\tan(x)\right)}{3\ 6^{2/3}} + \frac{\text{Subst}\left(\int \frac{1}{3^{2/3}+2^{2/3}\sqrt[3]{3}x+2\sqrt[3]{2}x^2}\,dx, x, \tan(x)\right)}{2\sqrt[3]{3}} + \frac{\text{Subst}\left(\int \frac{2^{2/3}\sqrt[3]{3}\ldots}{3^{2/3}+2^{2/3}\ldots}\right.}{\ldots}$$
$$= -\frac{\log\left(\sqrt[3]{3}-2^{2/3}\tan(x)\right)}{3\ 6^{2/3}} + \frac{\log\left(3^{2/3}+2^{2/3}\sqrt[3]{3}\tan(x)+2\sqrt[3]{2}\tan^2(x)\right)}{6\ 6^{2/3}} - \frac{\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx\ldots\right.}{6\ldots}$$
$$= \frac{x}{3\ 2^{2/3}\sqrt[6]{3}} - \frac{\tan^{-1}\left(\frac{6^{2/3}-2\ 6^{2/3}\cos^2(x)+2\left(3-2\sqrt[3]{6}\right)\cos(x)\sin(x)}{3\ 2^{2/3}\sqrt[6]{3}+4\sqrt[3]{6}+2\left(3-2\sqrt[3]{6}\right)\cos^2(x)+2\ 6^{2/3}\cos(x)\sin(x)}\right)}{3\ 2^{2/3}\sqrt[6]{3}} - \frac{\log\left(\sqrt[3]{3}-2^{2/3}\tan(x)\right)}{3\ 6^{2/3}}$$

Mathematica [A] time = 0.12, size = 74, normalized size = 0.42

$$\frac{2\sqrt{3}\tan^{-1}\left(\frac{2\ 6^{2/3}\tan(x)+3}{3\sqrt{3}}\right)+\log\left(2\sqrt[3]{6}\tan^2(x)+6^{2/3}\tan(x)+3\right)-2\log\left(3-6^{2/3}\tan(x)\right)}{6\ 6^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[x]^2/(3 - 4*Tan[x]^3),x]
```

```
[Out] (2*Sqrt[3]*ArcTan[(3 + 2*6^(2/3)*Tan[x])/(3*Sqrt[3])] - 2*Log[3 - 6^(2/3)*T
an[x]] + Log[3 + 6^(2/3)*Tan[x] + 2*6^(1/3)*Tan[x]^2])/(6*6^(2/3))
```

fricas [B] time = 1.10, size = 441, normalized size = 2.51

$$-\frac{1}{36}\cdot 36^{\frac{1}{6}}\sqrt{3}\,(-1)^{\frac{1}{3}}\arctan\left(-\frac{36^{\frac{1}{6}}\left(28\left(36^{\frac{2}{3}}\sqrt{3}\,(-1)^{\frac{2}{3}}-9\sqrt{3}\,(-1)^{\frac{1}{3}}\right)\cos(x)^6-4\left(14\cdot 36^{\frac{2}{3}}\sqrt{3}\,(-1)^{\frac{2}{3}}+36\cdot 36^{\frac{1}{3}}\sqrt{3}\ldots\right.\right.}{\ldots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)^2/(3-4*tan(x)^3),x, algorithm="fricas")
```

```
[Out] -1/36*36^(1/6)*sqrt(3)*(-1)^(1/3)*arctan(-1/108*36^(1/6)*(28*(36^(2/3)*sqrt
(3)*(-1)^(2/3) - 9*sqrt(3)*(-1)^(1/3))*cos(x)^6 - 4*(14*36^(2/3)*sqrt(3)*(-
```

```
1)^(2/3) + 36*36^(1/3)*sqrt(3) - 63*sqrt(3)*(-1)^(1/3))*cos(x)^4 + (37*36^(
2/3)*sqrt(3)*(-1)^(2/3) + 144*36^(1/3)*sqrt(3) + 144*sqrt(3)*(-1)^(1/3))*co
s(x)^2 - 6*(16*(36^(2/3)*sqrt(3)*(-1)^(2/3) - 9*sqrt(3)*(-1)^(1/3))*cos(x)^
5 - (24*36^(2/3)*sqrt(3)*(-1)^(2/3) - 7*36^(1/3)*sqrt(3) - 72*sqrt(3)*(-1)^
(1/3))*cos(x)^3 + 4*(36^(2/3)*sqrt(3)*(-1)^(2/3) - 4*36^(1/3)*sqrt(3) + 9*s
qrt(3)*(-1)^(1/3))*cos(x))*sin(x) - 18*36^(1/3)*sqrt(3) - 144*sqrt(3)*(-1)^
(1/3))/(48*cos(x)^6 - 72*cos(x)^4 + 18*cos(x)^2 + 14*(cos(x)^5 - cos(x)^3)*
sin(x) + 3)) - 1/432*36^(2/3)*(-1)^(1/3)*log(-3*(2*36^(2/3)*(-1)^(1/3) - 8*
36^(1/3)*(-1)^(2/3) + 25)*cos(x)^4 + 3*(3*36^(2/3)*(-1)^(1/3) - 4*36^(1/3)*
(-1)^(2/3) + 32)*cos(x)^2 - 2*((4*36^(2/3)*(-1)^(1/3) + 9*36^(1/3)*(-1)^(2/
3))*cos(x)^3 - 4*(36^(2/3)*(-1)^(1/3) - 9)*cos(x))*sin(x) - 12*36^(1/3)*(-1
)^(2/3) - 48) + 1/216*36^(2/3)*(-1)^(1/3)*log(3*(2*36^(2/3)*(-1)^(1/3) + 8*
36^(1/3)*(-1)^(2/3) - 7)*cos(x)^2 + 2*(4*36^(2/3)*(-1)^(1/3) - 9*36^(1/3)*(
-1)^(2/3) + 36)*cos(x)*sin(x) - 3*36^(2/3)*(-1)^(1/3) - 12*36^(1/3)*(-1)^(2
/3) + 48)
```

giac [A] time = 0.17, size = 61, normalized size = 0.35

$$\frac{1}{9}\sqrt{3}\left(\frac{3}{4}\right)^{\frac{1}{3}}\arctan\left(\frac{4}{9}\sqrt{3}\left(\frac{3}{4}\right)^{\frac{2}{3}}\left(\left(\frac{3}{4}\right)^{\frac{1}{3}}+2\tan(x)\right)\right)+\frac{1}{36}\cdot 6^{\frac{1}{3}}\log\left(\tan(x)^2+\left(\frac{3}{4}\right)^{\frac{1}{3}}\tan(x)+\left(\frac{3}{4}\right)^{\frac{2}{3}}\right)-\frac{1}{9}\left(\frac{3}{4}\right)^{\frac{1}{3}}\log\left(\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)^2/(3-4*tan(x)^3),x, algorithm="giac")
```

```
[Out] 1/9*sqrt(3)*(3/4)^(1/3)*arctan(4/9*sqrt(3)*(3/4)^(2/3)*((3/4)^(1/3) + 2*tan
(x))) + 1/36*6^(1/3)*log(tan(x)^2 + (3/4)^(1/3)*tan(x) + (3/4)^(2/3)) - 1/9
*(3/4)^(1/3)*log(abs(-(3/4)^(1/3) + tan(x)))
```

maple [A] time = 0.12, size = 80, normalized size = 0.45

$$-\frac{3^{\frac{1}{3}}4^{\frac{2}{3}}\ln\left(\tan(x)-\frac{3^{\frac{1}{3}}4^{\frac{2}{3}}}{4}\right)}{36}+\frac{3^{\frac{1}{3}}4^{\frac{2}{3}}\ln\left(\tan^2(x)+\frac{3^{\frac{1}{3}}4^{\frac{2}{3}}\tan(x)}{4}+\frac{3^{\frac{2}{3}}4^{\frac{1}{3}}}{4}\right)}{72}+\frac{3^{\frac{5}{6}}4^{\frac{2}{3}}\arctan\left(\frac{\sqrt{3}\left(\frac{2\,3^{\frac{2}{3}}4^{\frac{1}{3}}\tan(x)}{3}+1\right)}{3}\right)}{36}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(x)^2/(3-4*tan(x)^3),x)
```

```
[Out] -1/36*3^(1/3)*4^(2/3)*ln(tan(x)-1/4*3^(1/3)*4^(2/3))+1/72*3^(1/3)*4^(2/3)*l
n(tan(x)^2+1/4*3^(1/3)*4^(2/3)*tan(x)+1/4*3^(2/3)*4^(1/3))+1/36*3^(5/6)*4^(
2/3)*arctan(1/3*3^(1/2)*(2/3*3^(2/3)*4^(1/3)*tan(x)+1))
```

maxima [A] time = 0.43, size = 89, normalized size = 0.51

$$\frac{1}{36}\cdot 4^{\frac{2}{3}}3^{\frac{5}{6}}\arctan\left(\frac{1}{12}\cdot 4^{\frac{2}{3}}3^{\frac{1}{6}}\left(2\cdot 4^{\frac{2}{3}}\tan(x)+4^{\frac{1}{3}}3^{\frac{1}{3}}\right)\right)+\frac{1}{72}\cdot 4^{\frac{2}{3}}3^{\frac{1}{3}}\log\left(4^{\frac{2}{3}}\tan(x)^2+4^{\frac{1}{3}}3^{\frac{1}{3}}\tan(x)+3^{\frac{2}{3}}\right)-\frac{1}{36}\cdot 4^{\frac{2}{3}}3^{\frac{1}{3}}\log$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)^2/(3-4*tan(x)^3),x, algorithm="maxima")
```

```
[Out] 1/36*4^(2/3)*3^(5/6)*arctan(1/12*4^(2/3)*3^(1/6)*(2*4^(2/3)*tan(x) + 4^(1/3
)*3^(1/3))) + 1/72*4^(2/3)*3^(1/3)*log(4^(2/3)*tan(x)^2 + 4^(1/3)*3^(1/3)*t
an(x) + 3^(2/3)) - 1/36*4^(2/3)*3^(1/3)*log(1/4*4^(2/3)*(4^(1/3)*tan(x) - 3
^(1/3)))
```

mupad [B] time = 3.31, size = 75, normalized size = 0.43

$$-\frac{6^{1/3}\ln\left(\tan(x)-\frac{6^{1/3}}{2}\right)}{18}-\frac{6^{1/3}\ln\left(\tan(x)-\frac{6^{1/3}\left(-1+\sqrt{3}\,1\mathrm{i}\right)}{4}\right)\left(-1+\sqrt{3}\,1\mathrm{i}\right)}{36}+\frac{6^{1/3}\ln\left(\tan(x)+\frac{6^{1/3}\left(1+\sqrt{3}\,1\mathrm{i}\right)}{4}\right)\left(1+\sqrt{3}\right.}{36}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-1/(cos(x)^2*(4*tan(x)^3 - 3)),x)
```

```
[Out] (6^(1/3)*log(tan(x) + (6^(1/3)*(3^(1/2)*1i + 1))/4)*(3^(1/2)*1i + 1))/36 -
(6^(1/3)*log(tan(x) - (6^(1/3)*(3^(1/2)*1i - 1))/4)*(3^(1/2)*1i - 1))/36 -
(6^(1/3)*log(tan(x) - 6^(1/3)/2))/18
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int\frac{\sec^2(x)}{4\tan^3(x)-3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)**2/(3-4*tan(x)**3),x)
```

```
[Out] -Integral(sec(x)**2/(4*tan(x)**3 - 3), x)
```

3.700 $\int \frac{\sec^2(x)}{11-5\tan(x)+5\tan^2(x)}\,dx$

Optimal. Leaf size=53

$$\frac{2x}{\sqrt{195}}-\frac{2\tan^{-1}\left(\frac{10\cos^2(x)+12\sin(x)\cos(x)-5}{12\cos^2(x)-10\sin(x)\cos(x)+\sqrt{195}+10}\right)}{\sqrt{195}}$$

```
[Out] 2/195*x*195^(1/2)-2/195*arctan((-5+10*cos(x)^2+12*cos(x)*sin(x))/(10+12*cos
(x)^2-10*cos(x)*sin(x)+195^(1/2)))*195^(1/2)
```

Rubi [A] time = 0.07, antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {4342, 618, 204}

$$\frac{2x}{\sqrt{195}}-\frac{2\tan^{-1}\left(\frac{10\cos^2(x)+12\sin(x)\cos(x)-5}{12\cos^2(x)-10\sin(x)\cos(x)+\sqrt{195}+10}\right)}{\sqrt{195}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[x]^2/(11 - 5*Tan[x] + 5*Tan[x]^2),x]
```

```
[Out] (2*x)/Sqrt[195] - (2*ArcTan[(-5 + 10*Cos[x]^2 + 12*Cos[x]*Sin[x])/(10 + Sqr
t[195] + 12*Cos[x]^2 - 10*Cos[x]*Sin[x])])/Sqrt[195]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 4342

```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))]^2, x_Symbol] :> With[{d = FreeFac
tors[Tan[c*(a + b*x)], x]}, Dist[d/(b*c), Subst[Int[SubstFor[1, Tan[c*(a +
b*x)]/d, u, x], x], x, Tan[c*(a + b*x)]/d], x] /; FunctionOfQ[Tan[c*(a + b*
x)]/d, u, x, True]] /; FreeQ[{a, b, c}, x] && NonsumQ[u] && (EqQ[F, Sec] ||
 EqQ[F, sec])
```

Rubi steps

$$\int \frac{\sec^2(x)}{11-5\tan(x)+5\tan^2(x)}\,dx = \text{Subst}\left(\int \frac{1}{11-5x+5x^2}\,dx,x,\tan(x)\right)$$
$$= -\left(2\,\text{Subst}\left(\int \frac{1}{-195-x^2}\,dx,x,-5+10\tan(x)\right)\right)$$
$$= \frac{2x}{\sqrt{195}} + \frac{2\tan^{-1}\left(\frac{5-10\cos^2(x)-12\cos(x)\sin(x)}{10+\sqrt{195}+12\cos^2(x)-10\cos(x)\sin(x)}\right)}{\sqrt{195}}$$

Mathematica [A] time = 0.05, size = 22, normalized size = 0.42

$$-\frac{2\tan^{-1}\left(\sqrt{\frac{5}{39}}\,(1-2\tan(x))\right)}{\sqrt{195}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[x]^2/(11 - 5*Tan[x] + 5*Tan[x]^2),x]
[Out] (-2*ArcTan[Sqrt[5/39]*(1 - 2*Tan[x])])/Sqrt[195]
```

fricas [A] time = 0.98, size = 48, normalized size = 0.91

$$\frac{1}{195}\sqrt{195}\,\arctan\left(-\frac{192\sqrt{195}\,\cos(x)^2-160\sqrt{195}\,\cos(x)\sin(x)-35\sqrt{195}}{195\left(10\,\cos(x)^2+12\,\cos(x)\sin(x)-5\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)^2/(11-5*tan(x)+5*tan(x)^2),x, algorithm="fricas")
[Out] 1/195*sqrt(195)*arctan(-1/195*(192*sqrt(195)*cos(x)^2 - 160*sqrt(195)*cos(x
)*sin(x) - 35*sqrt(195))/(10*cos(x)^2 + 12*cos(x)*sin(x) - 5))
```

giac [A] time = 0.13, size = 17, normalized size = 0.32

$$\frac{2}{195}\sqrt{195}\,\arctan\left(\frac{1}{39}\sqrt{195}\,(2\,\tan(x)-1)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)^2/(11-5*tan(x)+5*tan(x)^2),x, algorithm="giac")
```

```
[Out] 2/195*sqrt(195)*arctan(1/39*sqrt(195)*(2*tan(x) - 1))
```

maple [A] time = 0.15, size = 18, normalized size = 0.34

$$\frac{2\sqrt{195}\ \arctan\left(\frac{(10\tan(x)-5)\sqrt{195}}{195}\right)}{195}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(x)^2/(11-5*tan(x)+5*tan(x)^2),x)
[Out] 2/195*195^(1/2)*arctan(1/195*(10*tan(x)-5)*195^(1/2))
```

maxima [A] time = 0.45, size = 17, normalized size = 0.32

$$\frac{2}{195}\sqrt{195}\arctan\left(\frac{1}{39}\sqrt{195}\left(2\tan(x)-1\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)^2/(11-5*tan(x)+5*tan(x)^2),x, algorithm="maxima")
[Out] 2/195*sqrt(195)*arctan(1/39*sqrt(195)*(2*tan(x) - 1))
```

mupad [B] time = 3.11, size = 17, normalized size = 0.32

$$\frac{2\sqrt{195}\ \mathrm{atan}\left(\frac{\sqrt{195}\ (2\tan(x)-1)}{39}\right)}{195}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(x)^2*(5*tan(x)^2 - 5*tan(x) + 11)),x)
[Out] (2*195^(1/2)*atan((195^(1/2)*(2*tan(x) - 1))/39))/195
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sec^2(x)}{5\tan^2(x)-5\tan(x)+11}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)**2/(11-5*tan(x)+5*tan(x)**2),x)
[Out] Integral(sec(x)**2/(5*tan(x)**2 - 5*tan(x) + 11), x)
```

3.701 $\int \frac{\sec^2(x)(a+b\tan(x))}{c+d\tan(x)}\,dx$

Optimal. Leaf size=28

$$\frac{b\tan(x)}{d}-\frac{(bc-ad)\log(c+d\tan(x))}{d^2}$$

```
[Out] -(-a*d+b*c)*ln(c+d*tan(x))/d^2+b*tan(x)/d
```

Rubi [A] time = 0.09, antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {4342, 43}

$$\frac{b\tan(x)}{d}-\frac{(bc-ad)\log(c+d\tan(x))}{d^2}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[x]^2*(a + b*Tan[x]))/(c + d*Tan[x]),x]
```

```
[Out] -(((b*c - a*d)*Log[c + d*Tan[x]])/d^2) + (b*Tan[x])/d
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 4342

```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))]^2, x_Symbol] :> With[{d = FreeFac
tors[Tan[c*(a + b*x)], x]}, Dist[d/(b*c), Subst[Int[SubstFor[1, Tan[c*(a +
b*x)]/d, u, x], x], x, Tan[c*(a + b*x)]/d], x] /; FunctionOfQ[Tan[c*(a + b*
x)]/d, u, x, True]] /; FreeQ[{a, b, c}, x] && NonsumQ[u] && (EqQ[F, Sec] ||
 EqQ[F, sec])
```

Rubi steps

$$\int \frac{\sec^2(x)(a+b\tan(x))}{c+d\tan(x)}\,dx = \text{Subst}\left(\int \frac{a+bx}{c+dx}\,dx, x, \tan(x)\right)$$
$$= \text{Subst}\left(\int \left(\frac{b}{d}+\frac{-bc+ad}{d(c+dx)}\right)dx, x, \tan(x)\right)$$
$$= -\frac{(bc-ad)\log(c+d\tan(x))}{d^2}+\frac{b\tan(x)}{d}$$

Mathematica [A] time = 0.38, size = 54, normalized size = 1.93

$$\frac{\cos(x)(a+b\tan(x))((bc-ad)(\log(\cos(x))-\log(c\cos(x)+d\sin(x)))+bd\tan(x))}{d^2(a\cos(x)+b\sin(x))}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[x]^2*(a + b*Tan[x]))/(c + d*Tan[x]),x]
```

```
[Out] (Cos[x]*(a + b*Tan[x])*((b*c - a*d)*(Log[Cos[x]] - Log[c*Cos[x] + d*Sin[x]]
) + b*d*Tan[x]))/(d^2*(a*Cos[x] + b*Sin[x]))
```

fricas [B] time = 0.91, size = 71, normalized size = 2.54

$$-\frac{(bc-ad)\cos(x)\log\left(2\,cd\cos(x)\sin(x)+\left(c^2-d^2\right)\cos(x)^2+d^2\right)-(bc-ad)\cos(x)\log\left(\cos(x)^2\right)-2\,bd\sin(x)}{2\,d^2\cos(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)^2*(a+b*tan(x))/(c+d*tan(x)),x, algorithm="fricas")
```

```
[Out] -1/2*((b*c - a*d)*cos(x)*log(2*c*d*cos(x)*sin(x) + (c^2 - d^2)*cos(x)^2 + d
^2) - (b*c - a*d)*cos(x)*log(cos(x)^2) - 2*b*d*sin(x))/(d^2*cos(x))
```

giac [A] time = 0.14, size = 29, normalized size = 1.04

$$\frac{b\tan(x)}{d}-\frac{(bc-ad)\log(|d\tan(x)+c|)}{d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)^2*(a+b*tan(x))/(c+d*tan(x)),x, algorithm="giac")
```

```
[Out] b*tan(x)/d - (b*c - a*d)*log(abs(d*tan(x) + c))/d^2
```

maple [A] time = 0.12, size = 35, normalized size = 1.25

$$\frac{b\tan(x)}{d}+\frac{\ln(c+d\tan(x))\,a}{d}-\frac{\ln(c+d\tan(x))\,cb}{d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(x)^2*(a+b*tan(x))/(c+d*tan(x)),x)
```

```
[Out] b*tan(x)/d+1/d*ln(c+d*tan(x))*a-1/d^2*ln(c+d*tan(x))*c*b
```

maxima [A] time = 0.32, size = 28, normalized size = 1.00

$$\frac{b\tan(x)}{d} - \frac{(bc-ad)\log\left(d\tan(x)+c\right)}{d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)^2*(a+b*tan(x))/(c+d*tan(x)),x, algorithm="maxima")
```

```
[Out] b*tan(x)/d - (b*c - a*d)*log(d*tan(x) + c)/d^2
```

mupad [B] time = 3.09, size = 27, normalized size = 0.96

$$\frac{b\tan(x)}{d} + \frac{\ln\left(c+d\tan(x)\right)\,\left(a\,d-b\,c\right)}{d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*tan(x))/(cos(x)^2*(c + d*tan(x))),x)
```

```
[Out] (b*tan(x))/d + (log(c + d*tan(x))*(a*d - b*c))/d^2
```

sympy [A] time = 4.91, size = 29, normalized size = 1.04

$$\frac{b\tan{(x)}}{d} + \frac{(ad-bc)\left(\begin{cases}\frac{\tan{(x)}}{c} & \text{for}\ d=0\\ \frac{\log{(c+d\tan{(x)})}}{d} & \text{otherwise}\end{cases}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)**2*(a+b*tan(x))/(c+d*tan(x)),x)
```

```
[Out] b*tan(x)/d + (a*d - b*c)*Piecewise((tan(x)/c, Eq(d, 0)), (log(c + d*tan(x))
/d, True))/d
```

3.702 $\int \frac{\sec^2(x)(a+b\tan(x))^2}{c+d\tan(x)}\,dx$

Optimal. Leaf size=53

$$\frac{(bc-ad)^2\log(c+d\tan(x))}{d^3}-\frac{b\tan(x)(bc-ad)}{d^2}+\frac{(a+b\tan(x))^2}{2d}$$

```
[Out] (-a*d+b*c)^2*ln(c+d*tan(x))/d^3-b*(-a*d+b*c)*tan(x)/d^2+1/2*(a+b*tan(x))^2/
d
```

Rubi [A] time = 0.14, antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {4342, 43}

$$-\frac{b\tan(x)(bc-ad)}{d^2}+\frac{(bc-ad)^2\log(c+d\tan(x))}{d^3}+\frac{(a+b\tan(x))^2}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[x]^2*(a + b*Tan[x])^2)/(c + d*Tan[x]),x]
```

```
[Out] ((b*c - a*d)^2*Log[c + d*Tan[x]])/d^3 - (b*(b*c - a*d)*Tan[x])/d^2 + (a + b
*Tan[x])^2/(2*d)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 4342

```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))]^2, x_Symbol] :> With[{d = FreeFac
tors[Tan[c*(a + b*x)], x]}, Dist[d/(b*c), Subst[Int[SubstFor[1, Tan[c*(a +
b*x)]/d, u, x], x], x, Tan[c*(a + b*x)]/d], x] /; FunctionOfQ[Tan[c*(a + b*
x)]/d, u, x, True]] /; FreeQ[{a, b, c}, x] && NonsumQ[u] && (EqQ[F, Sec] ||
 EqQ[F, sec])
```

Rubi steps

$$\int \frac{\sec^2(x)(a+b\tan(x))^2}{c+d\tan(x)}\,dx = \operatorname{Subst}\left(\int \frac{(a+bx)^2}{c+dx}\,dx, x, \tan(x)\right)$$
$$= \operatorname{Subst}\left(\int \left(-\frac{b(bc-ad)}{d^2}+\frac{b(a+bx)}{d}+\frac{(-bc+ad)^2}{d^2(c+dx)}\right)dx, x, \tan(x)\right)$$
$$= \frac{(bc-ad)^2\log(c+d\tan(x))}{d^3}-\frac{b(bc-ad)\tan(x)}{d^2}+\frac{(a+b\tan(x))^2}{2d}$$

Mathematica [A] time = 0.59, size = 62, normalized size = 1.17

$$\frac{b^2d^2\sec^2(x)-2\left(bd\tan(x)(bc-2ad)+(bc-ad)^2(\log(\cos(x))-\log(c\cos(x)+d\sin(x)))\right)}{2d^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[x]^2*(a + b*Tan[x])^2)/(c + d*Tan[x]),x]
```

```
[Out] (b^2*d^2*Sec[x]^2 - 2*((b*c - a*d)^2*(Log[Cos[x]] - Log[c*Cos[x] + d*Sin[x]
]) + b*d*(b*c - 2*a*d)*Tan[x]))/(2*d^3)
```

fricas [B] time = 1.63, size = 122, normalized size = 2.30

$$\frac{b^2d^2+\left(b^2c^2-2\,abcd+a^2d^2\right)\cos(x)^2\log\left(2\,cd\cos(x)\sin(x)+\left(c^2-d^2\right)\cos(x)^2+d^2\right)-\left(b^2c^2-2\,abcd+a^2d^2\right)\mathrm{co}}{2\,d^3\cos(x)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)^2*(a+b*tan(x))^2/(c+d*tan(x)),x, algorithm="fricas")
```

```
[Out] 1/2*(b^2*d^2 + (b^2*c^2 - 2*a*b*c*d + a^2*d^2)*cos(x)^2*log(2*c*d*cos(x)*si
n(x) + (c^2 - d^2)*cos(x)^2 + d^2) - (b^2*c^2 - 2*a*b*c*d + a^2*d^2)*cos(x)
^2*log(cos(x)^2) - 2*(b^2*c*d - 2*a*b*d^2)*cos(x)*sin(x))/(d^3*cos(x)^2)
```

giac [A] time = 0.15, size = 64, normalized size = 1.21

$$\frac{b^2d\tan(x)^2-2\,b^2c\tan(x)+4\,abd\tan(x)}{2\,d^2}+\frac{\left(b^2c^2-2\,abcd+a^2d^2\right)\log\left(|d\tan(x)+c|\right)}{d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)^2*(a+b*tan(x))^2/(c+d*tan(x)),x, algorithm="giac")
```

```
[Out] 1/2*(b^2*d*tan(x)^2 - 2*b^2*c*tan(x) + 4*a*b*d*tan(x))/d^2 + (b^2*c^2 - 2*a
*b*c*d + a^2*d^2)*log(abs(d*tan(x) + c))/d^3
```

maple [A] time = 0.13, size = 80, normalized size = 1.51

$$\frac{b^2\left(\tan^2(x)\right)}{2d}+\frac{2ba\tan(x)}{d}-\frac{b^2\tan(x)c}{d^2}+\frac{\ln(c+d\tan(x))\,a^2}{d}-\frac{2\ln(c+d\tan(x))\,abc}{d^2}+\frac{\ln(c+d\tan(x))\,b^2c^2}{d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(x)^2*(a+b*tan(x))^2/(c+d*tan(x)),x)
```

```
[Out] 1/2*b^2/d*tan(x)^2+2*b/d*a*tan(x)-b^2/d^2*tan(x)*c+1/d*ln(c+d*tan(x))*a^2-2
/d^2*ln(c+d*tan(x))*a*b*c+1/d^3*ln(c+d*tan(x))*b^2*c^2
```

maxima [A] time = 0.35, size = 63, normalized size = 1.19

$$\frac{b^2d\tan(x)^2-2\left(b^2c-2\,abd\right)\tan(x)}{2\,d^2}+\frac{\left(b^2c^2-2\,abcd+a^2d^2\right)\log\left(d\tan(x)+c\right)}{d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)^2*(a+b*tan(x))^2/(c+d*tan(x)),x, algorithm="maxima")
```

```
[Out] 1/2*(b^2*d*tan(x)^2 - 2*(b^2*c - 2*a*b*d)*tan(x))/d^2 + (b^2*c^2 - 2*a*b*c*
d + a^2*d^2)*log(d*tan(x) + c)/d^3
```

mupad [B] time = 2.98, size = 65, normalized size = 1.23

$$\frac{\ln\left(c+d\tan(x)\right)\left(a^2\,d^2-2\,a\,b\,c\,d+b^2\,c^2\right)}{d^3}-\tan(x)\left(\frac{b^2\,c}{d^2}-\frac{2\,a\,b}{d}\right)+\frac{b^2\,\tan(x)^2}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*tan(x))^2/(cos(x)^2*(c + d*tan(x))),x)
```

```
[Out] (log(c + d*tan(x))*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d))/d^3 - tan(x)*((b^2*c)/d
^2 - (2*a*b)/d) + (b^2*tan(x)^2)/(2*d)
```

sympy [A] time = 7.63, size = 56, normalized size = 1.06

$$\frac{b^2\tan^2(x)}{2d}+\frac{(ad-bc)^2\left(\begin{cases}\frac{\tan(x)}{c} & \text{for } d=0\\ \frac{\log(c+d\tan(x))}{d} & \text{otherwise}\end{cases}\right)}{d^2}+\frac{\left(2abd-b^2c\right)\tan(x)}{d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)**2*(a+b*tan(x))**2/(c+d*tan(x)),x)
```

```
[Out] b**2*tan(x)**2/(2*d) + (a*d - b*c)**2*Piecewise((tan(x)/c, Eq(d, 0)), (log(
c + d*tan(x))/d, True))/d**2 + (2*a*b*d - b**2*c)*tan(x)/d**2
```

3.703 $\int \frac{\sec^2(x)(a+b\tan(x))^3}{c+d\tan(x)}\,dx$

Optimal. Leaf size=78

$$-\frac{(bc-ad)^3\log(c+d\tan(x))}{d^4}+\frac{b\tan(x)(bc-ad)^2}{d^3}-\frac{(bc-ad)(a+b\tan(x))^2}{2d^2}+\frac{(a+b\tan(x))^3}{3d}$$

```
[Out] -(-a*d+b*c)^3*ln(c+d*tan(x))/d^4+b*(-a*d+b*c)^2*tan(x)/d^3-1/2*(-a*d+b*c)*(
a+b*tan(x))^2/d^2+1/3*(a+b*tan(x))^3/d
```

Rubi [A] time = 0.15, antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {4342, 43}

$$\frac{b\tan(x)(bc-ad)^2}{d^3}-\frac{(bc-ad)(a+b\tan(x))^2}{2d^2}-\frac{(bc-ad)^3\log(c+d\tan(x))}{d^4}+\frac{(a+b\tan(x))^3}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[x]^2*(a + b*Tan[x])^3)/(c + d*Tan[x]),x]
```

```
[Out] -(((b*c - a*d)^3*Log[c + d*Tan[x]])/d^4) + (b*(b*c - a*d)^2*Tan[x])/d^3 - (
(b*c - a*d)*(a + b*Tan[x])^2)/(2*d^2) + (a + b*Tan[x])^3/(3*d)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 4342

```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))]^2, x_Symbol] :> With[{d = FreeFac
tors[Tan[c*(a + b*x)], x]}, Dist[d/(b*c), Subst[Int[SubstFor[1, Tan[c*(a +
b*x)]/d, u, x], x], x, Tan[c*(a + b*x)]/d], x] /; FunctionOfQ[Tan[c*(a + b*
x)]/d, u, x, True]] /; FreeQ[{a, b, c}, x] && NonsumQ[u] && (EqQ[F, Sec] ||
 EqQ[F, sec])
```

Rubi steps

$$\int \frac{\sec^2(x)(a+b\tan(x))^3}{c+d\tan(x)}\,dx = \text{Subst}\left(\int \frac{(a+bx)^3}{c+dx}\,dx, x, \tan(x)\right)$$
$$= \text{Subst}\left(\int \left(\frac{b(bc-ad)^2}{d^3} - \frac{b(bc-ad)(a+bx)}{d^2} + \frac{b(a+bx)^2}{d} + \frac{(-bc+ad)^3}{d^3(c+dx)}\right)dx, x,$$
$$= -\frac{(bc-ad)^3\log(c+d\tan(x))}{d^4} + \frac{b(bc-ad)^2\tan(x)}{d^3} - \frac{(bc-ad)(a+b\tan(x))^2}{2d^2} +$$

Mathematica [A] time = 0.96, size = 133, normalized size = 1.71

$$\frac{(a+b\tan(x))^3(c\cos(x)+d\sin(x))\left(bd^2(9a\sin(2x)(ad-bc)+b(9ad-3bc+2bd\tan(x)))+6\cos^2(x)(bc-ad)^3(\right.}{6d^4(c+d\tan(x))(a\cos(x)+b\sin(x))^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[x]^2*(a + b*Tan[x])^3)/(c + d*Tan[x]),x]
```

```
[Out] ((c*Cos[x] + d*Sin[x])*(a + b*Tan[x])^3*(6*(b*c - a*d)^3*Cos[x]^2*(Log[Cos[
x]] - Log[c*Cos[x] + d*Sin[x]]) - b^3*d*(-3*c^2 + d^2)*Sin[2*x] + b*d^2*(9*
a*(-(b*c) + a*d)*Sin[2*x] + b*(-3*b*c + 9*a*d + 2*b*d*Tan[x]))))/(6*d^4*(a*
Cos[x] + b*Sin[x])^3*(c + d*Tan[x]))
```

fricas [B] time = 2.11, size = 201, normalized size = 2.58

$$-\frac{3\left(b^3c^3-3\,ab^2c^2d+3\,a^2bcd^2-a^3d^3\right)\cos(x)^3\log\left(2\,cd\cos(x)\sin(x)+\left(c^2-d^2\right)\cos(x)^2+d^2\right)-3\left(b^3c^3-3\,ab^2\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)^2*(a+b*tan(x))^3/(c+d*tan(x)),x, algorithm="fricas")
```

```
[Out] -1/6*(3*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*cos(x)^3*log(2*
c*d*cos(x)*sin(x) + (c^2 - d^2)*cos(x)^2 + d^2) - 3*(b^3*c^3 - 3*a*b^2*c^2*
d + 3*a^2*b*c*d^2 - a^3*d^3)*cos(x)^3*log(cos(x)^2) + 3*(b^3*c*d^2 - 3*a*b^
2*d^3)*cos(x) - 2*(b^3*d^3 + (3*b^3*c^2*d - 9*a*b^2*c*d^2 + (9*a^2*b - b^3)
*d^3)*cos(x)^2)*sin(x))/(d^4*cos(x)^3)
```

giac [A] time = 0.15, size = 123, normalized size = 1.58

$$\frac{2\,b^3d^2\tan(x)^3-3\,b^3cd\tan(x)^2+9\,ab^2d^2\tan(x)^2+6\,b^3c^2\tan(x)-18\,ab^2cd\tan(x)+18\,a^2bd^2\tan(x)}{6\,d^3}-\frac{\left(b^3c^3-3\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)^2*(a+b*tan(x))^3/(c+d*tan(x)),x, algorithm="giac")
```

```
[Out] 1/6*(2*b^3*d^2*tan(x)^3 - 3*b^3*c*d*tan(x)^2 + 9*a*b^2*d^2*tan(x)^2 + 6*b^3
*c^2*tan(x) - 18*a*b^2*c*d*tan(x) + 18*a^2*b*d^2*tan(x))/d^3 - (b^3*c^3 - 3
*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*log(abs(d*tan(x) + c))/d^4
```

maple [A] time = 0.16, size = 143, normalized size = 1.83

$$\frac{b^3\left(\tan^3(x)\right)}{3d}+\frac{3b^2\left(\tan^2(x)\right)a}{2d}-\frac{b^3\left(\tan^2(x)\right)c}{2d^2}+\frac{3b\,a^2\tan(x)}{d}-\frac{3b^2ac\tan(x)}{d^2}+\frac{b^3c^2\tan(x)}{d^3}+\frac{\ln\left(c+d\tan(x)\right)a^3}{d}-\frac{3\ln}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(x)^2*(a+b*tan(x))^3/(c+d*tan(x)),x)
```

```
[Out] 1/3*b^3/d*tan(x)^3+3/2*b^2/d*tan(x)^2*a-1/2*b^3/d^2*tan(x)^2*c+3*b/d*a^2*ta
n(x)-3*b^2/d^2*a*c*tan(x)+b^3/d^3*c^2*tan(x)+1/d*ln(c+d*tan(x))*a^3-3/d^2*l
n(c+d*tan(x))*a^2*b*c+3/d^3*ln(c+d*tan(x))*a*b^2*c^2-1/d^4*ln(c+d*tan(x))*b
^3*c^3
```

maxima [A] time = 0.52, size = 118, normalized size = 1.51

$$\frac{2\,b^3d^2\tan(x)^3-3\left(b^3cd-3\,ab^2d^2\right)\tan(x)^2+6\left(b^3c^2-3\,ab^2cd+3\,a^2bd^2\right)\tan(x)}{6\,d^3}-\frac{\left(b^3c^3-3\,ab^2c^2d+3\,a^2bcd^2-a\right.}{d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)^2*(a+b*tan(x))^3/(c+d*tan(x)),x, algorithm="maxima")
```

```
[Out] 1/6*(2*b^3*d^2*tan(x)^3 - 3*(b^3*c*d - 3*a*b^2*d^2)*tan(x)^2 + 6*(b^3*c^2 -
 3*a*b^2*c*d + 3*a^2*b*d^2)*tan(x))/d^3 - (b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*
b*c*d^2 - a^3*d^3)*log(d*tan(x) + c)/d^4
```

mupad [B] time = 2.96, size = 122, normalized size = 1.56

$$\tan(x)\left(\frac{3\,a^2\,b}{d}-\frac{c\,\left(\frac{3\,a\,b^2}{d}-\frac{b^3\,c}{d^2}\right)}{d}\right)+\tan(x)^2\,\left(\frac{3\,a\,b^2}{2\,d}-\frac{b^3\,c}{2\,d^2}\right)+\frac{b^3\,\tan(x)^3}{3\,d}+\frac{\ln\left(c+d\tan(x)\right)\,\left(a^3\,d^3-3\,a^2\,b\,c\,d^2+3\,a\right.}{d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*tan(x))^3/(cos(x)^2*(c + d*tan(x))),x)
```

```
[Out] tan(x)*((3*a^2*b)/d - (c*((3*a*b^2)/d - (b^3*c)/d^2))/d) + tan(x)^2*((3*a*b
^2)/(2*d) - (b^3*c)/(2*d^2)) + (b^3*tan(x)^3)/(3*d) + (log(c + d*tan(x))*(a
^3*d^3 - b^3*c^3 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2))/d^4
```

sympy [A] time = 10.57, size = 95, normalized size = 1.22

$$\frac{b^3\tan^3(x)}{3d}+\frac{\left(3ab^2d-b^3c\right)\tan^2(x)}{2d^2}+\frac{(ad-bc)^3\left(\begin{cases}\frac{\tan(x)}{c} & \text{for } d=0\\ \frac{\log(c+d\tan(x))}{d} & \text{otherwise}\end{cases}\right)}{d^3}+\frac{\left(3a^2bd^2-3ab^2cd+b^3c^2\right)\tan(x)}{d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)**2*(a+b*tan(x))**3/(c+d*tan(x)),x)
```

```
[Out] b**3*tan(x)**3/(3*d) + (3*a*b**2*d - b**3*c)*tan(x)**2/(2*d**2) + (a*d - b*
c)**3*Piecewise((tan(x)/c, Eq(d, 0)), (log(c + d*tan(x))/d, True))/d**3 + (
3*a**2*b*d**2 - 3*a*b**2*c*d + b**3*c**2)*tan(x)/d**3
```

3.704 $\int \frac{\sec^2(x)\tan^2(x)}{\left(2+\tan^3(x)\right)^2}\,dx$

Optimal. Leaf size=12

$$-\frac{1}{3\left(\tan^3(x)+2\right)}$$

```
[Out] -1/3/(2+tan(x)^3)
```

Rubi [A] time = 0.08, antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {4342, 261}

$$-\frac{1}{3\left(\tan^3(x)+2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[x]^2*Tan[x]^2)/(2 + Tan[x]^3)^2,x]
```

```
[Out] -1/(3*(2 + Tan[x]^3))
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 4342

```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))]^2, x_Symbol] :> With[{d = FreeFac
tors[Tan[c*(a + b*x)], x]}, Dist[d/(b*c), Subst[Int[SubstFor[1, Tan[c*(a +
b*x)]/d, u, x], x], x, Tan[c*(a + b*x)]/d], x] /; FunctionOfQ[Tan[c*(a + b*
x)]/d, u, x, True]] /; FreeQ[{a, b, c}, x] && NonsumQ[u] && (EqQ[F, Sec] ||
 EqQ[F, sec])
```

Rubi steps

$$\int \frac{\sec^2(x)\tan^2(x)}{\left(2+\tan^3(x)\right)^2}\,dx = \text{Subst}\left(\int \frac{x^2}{\left(2+x^3\right)^2}\,dx, x, \tan(x)\right)$$
$$= -\frac{1}{3\left(2+\tan^3(x)\right)}$$

Mathematica [A] time = 0.03, size = 12, normalized size = 1.00

$$-\frac{1}{3\left(\tan^3(x)+2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[x]^2*Tan[x]^2)/(2 + Tan[x]^3)^2,x]
```

```
[Out] -1/3*1/(2 + Tan[x]^3)
```

fricas [B] time = 0.85, size = 36, normalized size = 3.00

$$-\frac{\cos(x)^3+2\left(\cos(x)^2-1\right)\sin(x)}{15\left(2\cos(x)^3-\left(\cos(x)^2-1\right)\sin(x)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)^2*tan(x)^2/(2+tan(x)^3)^2,x, algorithm="fricas")
```

```
[Out] -1/15*(cos(x)^3 + 2*(cos(x)^2 - 1)*sin(x))/(2*cos(x)^3 - (cos(x)^2 - 1)*sin
(x))
```

giac [A] time = 0.15, size = 10, normalized size = 0.83

$$-\frac{1}{3\left(\tan(x)^3+2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)^2*tan(x)^2/(2+tan(x)^3)^2,x, algorithm="giac")
```

```
[Out] -1/3/(tan(x)^3 + 2)
```

maple [A] time = 0.13, size = 11, normalized size = 0.92

$$-\frac{1}{3\left(2+\tan^3(x)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(x)^2*tan(x)^2/(2+tan(x)^3)^2,x)
```

```
[Out] -1/3/(2+tan(x)^3)
```

maxima [A] time = 0.32, size = 10, normalized size = 0.83

$$-\frac{1}{3\left(\tan(x)^3+2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)^2*tan(x)^2/(2+tan(x)^3)^2,x, algorithm="maxima")
```

```
[Out] -1/3/(tan(x)^3 + 2)
```

mupad [B] time = 2.93, size = 12, normalized size = 1.00

$$-\frac{1}{3\left(\tan(x)^3+2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(x)^2/(cos(x)^2*(tan(x)^3 + 2)^2),x)
```

```
[Out] -1/(3*(tan(x)^3 + 2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)**2*tan(x)**2/(2+tan(x)**3)**2,x)
```

```
[Out] Timed out
```

3.705 $\int \sec^2(x)\tan^6(x)\left(1+\tan^2(x)\right)^3\,dx$

Optimal. Leaf size=33

$$\frac{\tan^{13}(x)}{13}+\frac{3\tan^{11}(x)}{11}+\frac{\tan^9(x)}{3}+\frac{\tan^7(x)}{7}$$

```
[Out] 1/7*tan(x)^7+1/3*tan(x)^9+3/11*tan(x)^11+1/13*tan(x)^13
```

Rubi [A] time = 0.09, antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {3657, 2607, 270}

$$\frac{\tan^{13}(x)}{13}+\frac{3\tan^{11}(x)}{11}+\frac{\tan^9(x)}{3}+\frac{\tan^7(x)}{7}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[x]^2*Tan[x]^6*(1 + Tan[x]^2)^3,x]
```

```
[Out] Tan[x]^7/7 + Tan[x]^9/3 + (3*Tan[x]^11)/11 + Tan[x]^13/13
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 3657

```
Int[(u_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*sec[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a, b]
```

Rubi steps

$$\int \sec^2(x)\tan^6(x)\left(1+\tan^2(x)\right)^3\,dx = \int \sec^8(x)\tan^6(x)\,dx$$
$$= \text{Subst}\left(\int x^6\left(1+x^2\right)^3\,dx, x, \tan(x)\right)$$
$$= \text{Subst}\left(\int \left(x^6+3x^8+3x^{10}+x^{12}\right)dx, x, \tan(x)\right)$$
$$= \frac{\tan^7(x)}{7}+\frac{\tan^9(x)}{3}+\frac{3\tan^{11}(x)}{11}+\frac{\tan^{13}(x)}{13}$$

Mathematica [B] time = 0.02, size = 67, normalized size = 2.03

$$-\frac{16\tan(x)}{3003}+\frac{1}{13}\tan(x)\sec^{12}(x)-\frac{27}{143}\tan(x)\sec^{10}(x)+\frac{53}{429}\tan(x)\sec^8(x)-\frac{5\tan(x)\sec^6(x)}{3003}-\frac{2\tan(x)\sec^4(x)}{1001}-\frac{8\,\mathrm{ta}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[x]^2*Tan[x]^6*(1 + Tan[x]^2)^3,x]
```

```
[Out] (-16*Tan[x])/3003 - (8*Sec[x]^2*Tan[x])/3003 - (2*Sec[x]^4*Tan[x])/1001 - (
5*Sec[x]^6*Tan[x])/3003 + (53*Sec[x]^8*Tan[x])/429 - (27*Sec[x]^10*Tan[x])/
143 + (Sec[x]^12*Tan[x])/13
```

fricas [A] time = 0.86, size = 46, normalized size = 1.39

$$-\frac{\left(16\cos(x)^{12}+8\cos(x)^{10}+6\cos(x)^8+5\cos(x)^6-371\cos(x)^4+567\cos(x)^2-231\right)\sin(x)}{3003\cos(x)^{13}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)^2*tan(x)^6*(1+tan(x)^2)^3,x, algorithm="fricas")
```

```
[Out] -1/3003*(16*cos(x)^12 + 8*cos(x)^10 + 6*cos(x)^8 + 5*cos(x)^6 - 371*cos(x)^
4 + 567*cos(x)^2 - 231)*sin(x)/cos(x)^13
```

giac [A] time = 0.14, size = 25, normalized size = 0.76

$$\frac{1}{13}\tan(x)^{13}+\frac{3}{11}\tan(x)^{11}+\frac{1}{3}\tan(x)^9+\frac{1}{7}\tan(x)^7$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)^2*tan(x)^6*(1+tan(x)^2)^3,x, algorithm="giac")
```

```
[Out] 1/13*tan(x)^13 + 3/11*tan(x)^11 + 1/3*tan(x)^9 + 1/7*tan(x)^7
```

maple [A] time = 0.06, size = 42, normalized size = 1.27

$$\frac{\sin^7(x)}{7\cos(x)^7}+\frac{\sin^9(x)}{3\cos(x)^9}+\frac{3\left(\sin^{11}(x)\right)}{11\cos(x)^{11}}+\frac{\sin^{13}(x)}{13\cos(x)^{13}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(x)^2*tan(x)^6*(1+tan(x)^2)^3,x)
```

```
[Out] 1/7*sin(x)^7/cos(x)^7+1/3*sin(x)^9/cos(x)^9+3/11*sin(x)^11/cos(x)^11+1/13*s
in(x)^13/cos(x)^13
```

maxima [A] time = 0.39, size = 25, normalized size = 0.76

$$\frac{1}{13}\tan(x)^{13}+\frac{3}{11}\tan(x)^{11}+\frac{1}{3}\tan(x)^9+\frac{1}{7}\tan(x)^7$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)^2*tan(x)^6*(1+tan(x)^2)^3,x, algorithm="maxima")
```

```
[Out] 1/13*tan(x)^13 + 3/11*tan(x)^11 + 1/3*tan(x)^9 + 1/7*tan(x)^7
```

mupad [B] time = 2.92, size = 25, normalized size = 0.76

$$\frac{\tan(x)^{13}}{13}+\frac{3\tan(x)^{11}}{11}+\frac{\tan(x)^9}{3}+\frac{\tan(x)^7}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((tan(x)^6*(tan(x)^2 + 1)^3)/cos(x)^2,x)
```

```
[Out] tan(x)^7/7 + tan(x)^9/3 + (3*tan(x)^11)/11 + tan(x)^13/13
```

sympy [A] time = 19.28, size = 27, normalized size = 0.82

$$\frac{\tan^{13}(x)}{13}+\frac{3\tan^{11}(x)}{11}+\frac{\tan^9(x)}{3}+\frac{\tan^7(x)}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)**2*tan(x)**6*(1+tan(x)**2)**3,x)
```

```
[Out] tan(x)**13/13 + 3*tan(x)**11/11 + tan(x)**9/3 + tan(x)**7/7
```

3.706 $\int \frac{\sec^2(x)\left(2+\tan^2(x)\right)}{1+\tan^3(x)}\,dx$

Optimal. Leaf size=46

$$\frac{2x}{\sqrt{3}}+\log(\tan(x)+1)+\frac{2\tan^{-1}\left(\frac{1-2\cos^2(x)}{-2\sin(x)\cos(x)+\sqrt{3}+2}\right)}{\sqrt{3}}$$

```
[Out] ln(1+tan(x))+2/3*x*3^(1/2)+2/3*arctan((1-2*cos(x)^2)/(2-2*cos(x)*sin(x)+3^(
1/2)))*3^(1/2)
```

Rubi [A] time = 0.09, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {4342, 1863, 31, 618, 204}

$$\frac{2x}{\sqrt{3}}+\log(\tan(x)+1)+\frac{2\tan^{-1}\left(\frac{1-2\cos^2(x)}{-2\sin(x)\cos(x)+\sqrt{3}+2}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[x]^2*(2 + Tan[x]^2))/(1 + Tan[x]^3),x]
```

```
[Out] (2*x)/Sqrt[3] + (2*ArcTan[(1 - 2*Cos[x]^2)/(2 + Sqrt[3] - 2*Cos[x]*Sin[x])]
)/Sqrt[3] + Log[1 + Tan[x]]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1863

```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, With[{q = a^(1/3)/b^(1/3)}, Dist[C
/b, Int[1/(q + x), x], x] + Dist[(B + C*q)/b, Int[1/(q^2 - q*x + x^2), x],
x]] /; EqQ[A*b^(2/3) - a^(1/3)*b^(1/3)*B - 2*a^(2/3)*C, 0]] /; FreeQ[{a, b}
, x] && PolyQ[P2, x, 2]
```

Rule 4342

```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))]^2, x_Symbol] :> With[{d = FreeFac
tors[Tan[c*(a + b*x)], x]}, Dist[d/(b*c), Subst[Int[SubstFor[1, Tan[c*(a +
b*x)]/d, u, x], x], x, Tan[c*(a + b*x)]/d], x] /; FunctionOfQ[Tan[c*(a + b*
x)]/d, u, x, True]] /; FreeQ[{a, b, c}, x] && NonsumQ[u] && (EqQ[F, Sec] ||
 EqQ[F, sec])
```

Rubi steps

$$\begin{aligned}\int \frac{\sec^2(x)\left(2+\tan^2(x)\right)}{1+\tan^3(x)}\,dx &= \text{Subst}\left(\int \frac{2+x^2}{1+x^3}\,dx, x, \tan(x)\right)\\ &= \text{Subst}\left(\int \frac{1}{1+x}\,dx, x, \tan(x)\right)+\text{Subst}\left(\int \frac{1}{1-x+x^2}\,dx, x, \tan(x)\right)\\ &= \log(1+\tan(x))-2\,\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, -1+2\tan(x)\right)\\ &= \frac{2x}{\sqrt{3}}+\frac{2\tan^{-1}\left(\frac{1-2\cos^2(x)}{2+\sqrt{3}-2\cos(x)\sin(x)}\right)}{\sqrt{3}}+\log(1+\tan(x))\end{aligned}$$

Mathematica [A] time = 0.22, size = 32, normalized size = 0.70

$$-\frac{2\tan^{-1}\left(\frac{1-2\tan(x)}{\sqrt{3}}\right)}{\sqrt{3}}-\log(\cos(x))+\log(\sin(x)+\cos(x))$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[x]^2*(2 + Tan[x]^2))/(1 + Tan[x]^3),x]
```

```
[Out] (-2*ArcTan[(1 - 2*Tan[x])/Sqrt[3]])/Sqrt[3] - Log[Cos[x]] + Log[Cos[x] + Si
n[x]]
```

fricas [A] time = 2.51, size = 52, normalized size = 1.13

$$\frac{1}{3}\sqrt{3}\,\arctan\left(\frac{4\sqrt{3}\cos(x)\sin(x)-\sqrt{3}}{3\left(2\cos(x)^2-1\right)}\right)-\frac{1}{2}\log\left(\cos(x)^2\right)+\frac{1}{2}\log\left(2\cos(x)\sin(x)+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)^2*(2+tan(x)^2)/(1+tan(x)^3),x, algorithm="fricas")
```

```
[Out] 1/3*sqrt(3)*arctan(1/3*(4*sqrt(3)*cos(x)*sin(x) - sqrt(3))/(2*cos(x)^2 - 1)
) - 1/2*log(cos(x)^2) + 1/2*log(2*cos(x)*sin(x) + 1)
```

giac [A] time = 0.16, size = 24, normalized size = 0.52

$$\frac{2}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\,(2\tan(x)-1)\right)+\log\left(|\tan(x)+1|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)^2*(2+tan(x)^2)/(1+tan(x)^3),x, algorithm="giac")
```

```
[Out] 2/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*tan(x) - 1)) + log(abs(tan(x) + 1))
```

maple [A] time = 0.20, size = 24, normalized size = 0.52

$$\frac{2\sqrt{3}\arctan\left(\frac{(-1+2\tan(x))\sqrt{3}}{3}\right)}{3}+\ln\left(1+\tan(x)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(x)^2*(2+tan(x)^2)/(1+tan(x)^3),x)
```

```
[Out] 2/3*3^(1/2)*arctan(1/3*(-1+2*tan(x))*3^(1/2))+ln(1+tan(x))
```

maxima [A] time = 0.41, size = 23, normalized size = 0.50

$$\frac{2}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\,(2\tan(x)-1)\right)+\log\left(\tan(x)+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)^2*(2+tan(x)^2)/(1+tan(x)^3),x, algorithm="maxima")
```

```
[Out] 2/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*tan(x) - 1)) + log(tan(x) + 1)
```

mupad [B] time = 2.97, size = 30, normalized size = 0.65

$$\ln\left(\tan(x)+1\right)-\frac{2\sqrt{3}\,\mathrm{atan}\left(\frac{\sqrt{3}-\sqrt{3}\tan(x)}{\tan(x)+1}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((tan(x)^2 + 2)/(cos(x)^2*(tan(x)^3 + 1)),x)
```

```
[Out] log(tan(x) + 1) - (2*3^(1/2)*atan((3^(1/2) - 3^(1/2)*tan(x))/(tan(x) + 1)))
/3
```

sympy [A] time = 9.25, size = 41, normalized size = 0.89

$$\frac{2\sqrt{3}\left(\operatorname{atan}\left(\frac{2\sqrt{3}\left(\tan\left(x\right)-\frac{1}{2}\right)}{3}\right)+\pi\left\lfloor\frac{x-\frac{\pi}{2}}{\pi}\right\rfloor\right)}{3}+\log\left(\tan\left(x\right)+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)**2*(2+tan(x)**2)/(1+tan(x)**3),x)
```

```
[Out] 2*sqrt(3)*(atan(2*sqrt(3)*(tan(x) - 1/2)/3) + pi*floor((x - pi/2)/pi))/3 +
log(tan(x) + 1)
```

3.707 $\int \left(1+\cos^2(x)\right) \sec^2(x) \, dx$

Optimal. Leaf size=4

$$x+\tan(x)$$

```
[Out] x+tan(x)
```

Rubi [A] time = 0.02, antiderivative size = 4, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {3012, 8}

$$x+\tan(x)$$

Antiderivative was successfully verified.

```
[In] Int[(1 + Cos[x]^2)*Sec[x]^2,x]
```

```
[Out] x + Tan[x]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3012

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_) + (C_.)*sin[(e_.) + (f_.)*(x
_)]^2), x_Symbol] :> Simp[(A*Cos[e + f*x]*(b*Sin[e + f*x])^(m + 1))/(b*f*(m
 + 1)), x] + Dist[(A*(m + 2) + C*(m + 1))/(b^2*(m + 1)), Int[(b*Sin[e + f*x
])^(m + 2), x], x] /; FreeQ[{b, e, f, A, C}, x] && LtQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\int \left(1+\cos^2(x)\right) \sec^2(x) \, dx &= \tan(x)+\int 1 \, dx \\
&= x+\tan(x)
\end{aligned}$$

Mathematica [A] time = 0.00, size = 4, normalized size = 1.00

$$x+\tan(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + Cos[x]^2)*Sec[x]^2,x]
```

```
[Out] x + Tan[x]
```

fricas [B] time = 0.41, size = 12, normalized size = 3.00

$$\frac{x\cos(x)+\sin(x)}{\cos(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+cos(x)^2)*sec(x)^2,x, algorithm="fricas")
[Out] (x*cos(x) + sin(x))/cos(x)
```

giac [B] time = 0.19, size = 15, normalized size = 3.75

$$-\pi\left\lfloor\frac{x}{\pi}+\frac{1}{2}\right\rfloor+x+\tan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+cos(x)^2)*sec(x)^2,x, algorithm="giac")
[Out] -pi*floor(x/pi + 1/2) + x + tan(x)
```

maple [A] time = 0.08, size = 5, normalized size = 1.25

$$x+\tan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+cos(x)^2)*sec(x)^2,x)
[Out] x+tan(x)
```

maxima [A] time = 0.46, size = 4, normalized size = 1.00

$$x+\tan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+cos(x)^2)*sec(x)^2,x, algorithm="maxima")
[Out] x + tan(x)
```

mupad [B] time = 2.88, size = 4, normalized size = 1.00

$$x+\tan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(x)^2 + 1)/cos(x)^2,x)
```

```
[Out] x + tan(x)
```

sympy **[A]** time = 5.38, size = 3, normalized size = 0.75

$$x + \tan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+cos(x)**2)*sec(x)**2,x)
```

```
[Out] x + tan(x)
```

3.708 $\int \frac{\sec^2(x)}{1+\sec^2(x)-3\tan(x)}\,dx$

Optimal. Leaf size=21

$$\log(2\cos(x)-\sin(x))-\log(\cos(x)-\sin(x))$$

```
[Out] -ln(cos(x)-sin(x))+ln(2*cos(x)-sin(x))
```

Rubi [A] time = 0.12, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {616, 31}

$$\log(2\cos(x)-\sin(x))-\log(\cos(x)-\sin(x))$$

Antiderivative was successfully verified.

```
[In] Int[Sec[x]^2/(1 + Sec[x]^2 - 3*Tan[x]),x]
```

```
[Out] -Log[Cos[x] - Sin[x]] + Log[2*Cos[x] - Sin[x]]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 616

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[c/q, Int[1/Simp[b/2 - q/2 + c*x, x], x], x] - Dist[c/q, Int[1/Simp[b/2 + q/2 + c*x, x], x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[b^2 - 4*a*c] && PerfectSquareQ[b^2 - 4*a*c]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sec^2(x)}{1+\sec^2(x)-3\tan(x)}\,dx &= \text{Subst}\left(\int \frac{1}{2-3x+x^2}\,dx, x, \tan(x)\right)\\
&= \text{Subst}\left(\int \frac{1}{-2+x}\,dx, x, \tan(x)\right)-\text{Subst}\left(\int \frac{1}{-1+x}\,dx, x, \tan(x)\right)\\
&= -\log(1-\tan(x))+\log(2-\tan(x))
\end{aligned}$$

Mathematica [A] time = 0.03, size = 29, normalized size = 1.38

$$2\left(\frac{1}{2}\log(2\cos(x)-\sin(x))-\frac{1}{2}\log(\cos(x)-\sin(x))\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[x]^2/(1 + Sec[x]^2 - 3*Tan[x]),x]
```

```
[Out] 2*(-1/2*Log[Cos[x] - Sin[x]] + Log[2*Cos[x] - Sin[x]]/2)
```

fricas [A] time = 0.58, size = 29, normalized size = 1.38

$$\frac{1}{2}\log\left(\frac{3}{4}\cos(x)^2 - \cos(x)\sin(x) + \frac{1}{4}\right) - \frac{1}{2}\log\left(-2\cos(x)\sin(x) + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)^2/(1+sec(x)^2-3*tan(x)),x, algorithm="fricas")
```

```
[Out] 1/2*log(3/4*cos(x)^2 - cos(x)*sin(x) + 1/4) - 1/2*log(-2*cos(x)*sin(x) + 1)
```

giac [A] time = 0.16, size = 15, normalized size = 0.71

$$-\log\left(|\tan(x) - 1|\right) + \log\left(|\tan(x) - 2|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)^2/(1+sec(x)^2-3*tan(x)),x, algorithm="giac")
```

```
[Out] -log(abs(tan(x) - 1)) + log(abs(tan(x) - 2))
```

maple [A] time = 0.13, size = 14, normalized size = 0.67

$$\ln\left(-2 + \tan(x)\right) - \ln\left(\tan(x) - 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(x)^2/(1+sec(x)^2-3*tan(x)),x)
```

```
[Out] ln(-2+tan(x))-ln(tan(x)-1)
```

maxima [A] time = 0.32, size = 13, normalized size = 0.62

$$-\log\left(\tan(x) - 1\right) + \log\left(\tan(x) - 2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)^2/(1+sec(x)^2-3*tan(x)),x, algorithm="maxima")
```

```
[Out] -log(tan(x) - 1) + log(tan(x) - 2)
```

mupad [B] time = 3.51, size = 9, normalized size = 0.43

$$-2\,\mathrm{atanh}\left(2\tan(x) - 3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(x)^2*(1/cos(x)^2 - 3*tan(x) + 1)),x)
```

```
[Out] -2*atanh(2*tan(x) - 3)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec^2(x)}{-3\tan(x) + \sec^2(x) + 1}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)**2/(1+sec(x)**2-3*tan(x)),x)
```

```
[Out] Integral(sec(x)**2/(-3*tan(x) + sec(x)**2 + 1), x)
```

3.709 $\int \frac{\sec^2(x)}{\sqrt{4-\sec^2(x)}}\,dx$

Optimal. Leaf size=9

$$\sin^{-1}\left(\frac{\tan(x)}{\sqrt{3}}\right)$$

```
[Out] arcsin(1/3*tan(x)*3^(1/2))
```

Rubi [A] time = 0.05, antiderivative size = 9, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {4146, 216}

$$\sin^{-1}\left(\frac{\tan(x)}{\sqrt{3}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sec[x]^2/Sqrt[4 - Sec[x]^2],x]
```

```
[Out] ArcSin[Tan[x]/Sqrt[3]]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 4146

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_)
)^(p_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, S
ubst[Int[(1 + ff^2*x^2)^(m/2 - 1)*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x
]^p, x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[
m/2] && IntegerQ[n/2]
```

Rubi steps

$$\begin{aligned}\int \frac{\sec^2(x)}{\sqrt{4-\sec^2(x)}}\,dx &= \text{Subst}\left(\int \frac{1}{\sqrt{3-x^2}}\,dx, x, \tan(x)\right)\\ &= \sin^{-1}\left(\frac{\tan(x)}{\sqrt{3}}\right)\end{aligned}$$

Mathematica [B] time = 0.04, size = 43, normalized size = 4.78

$$\frac{\sqrt{2\cos(2x)+1}\,\sec(x)\tan^{-1}\left(\frac{\sin(x)}{\sqrt{3-4\sin^2(x)}}\right)}{\sqrt{4-\sec^2(x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[x]^2/Sqrt[4 - Sec[x]^2],x]
```

```
[Out] (ArcTan[Sin[x]/Sqrt[3 - 4*Sin[x]^2]]*Sqrt[1 + 2*Cos[2*x]]*Sec[x])/Sqrt[4 -
Sec[x]^2]
```

fricas [B] time = 0.83, size = 25, normalized size = 2.78

$$-\arctan\left(\frac{\sqrt{\frac{4\cos(x)^2-1}{\cos(x)^2}}\cos(x)}{\sin(x)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)^2/(4-sec(x)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] -arctan(sqrt((4*cos(x)^2 - 1)/cos(x)^2)*cos(x)/sin(x))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sec(x)^2}{\sqrt{-\sec(x)^2+4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)^2/(4-sec(x)^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sec(x)^2/sqrt(-sec(x)^2 + 4), x)
```

maple [C] time = 0.28, size = 103, normalized size = 11.44

$$-\frac{\sqrt{2}\sqrt{\frac{2\cos(x)-1}{1+\cos(x)}}\sqrt{6}\sqrt{\frac{1+2\cos(x)}{1+\cos(x)}}\left(\mathrm{EllipticF}\left(\frac{\sqrt{3}\,(-1+\cos(x))}{\sin(x)},\frac{1}{3}\right)-2\,\mathrm{EllipticPi}\left(\frac{\sqrt{3}\,(-1+\cos(x))}{\sin(x)},\frac{1}{3},\frac{1}{3}\right)\right)\left(\sin^2(x)\right)\sqrt{3}}{9\sqrt{\frac{4\left(\cos^2(x)\right)-1}{\cos(x)^2}}\cos(x)\,(-1+\cos(x))}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(x)^2/(4-sec(x)^2)^(1/2),x)
```

```
[Out] -1/9*2^(1/2)*((2*cos(x)-1)/(1+cos(x)))^(1/2)*6^(1/2)*((1+2*cos(x))/(1+cos(x
)))^(1/2)*(EllipticF(3^(1/2)*(-1+cos(x))/sin(x),1/3)-2*EllipticPi(3^(1/2)*(
-1+cos(x))/sin(x),1/3,1/3))*sin(x)^2/((4*cos(x)^2-1)/cos(x)^2)^(1/2)/cos(x)
/(-1+cos(x))*3^(1/2)
```

maxima [A] time = 0.43, size = 8, normalized size = 0.89

$$\arcsin\left(\frac{1}{3}\sqrt{3}\tan(x)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)^2/(4-sec(x)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] arcsin(1/3*sqrt(3)*tan(x))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.11

$$\int \frac{1}{\cos(x)^2\sqrt{4-\frac{1}{\cos(x)^2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(x)^2*(4 - 1/cos(x)^2)^(1/2)),x)
```

```
[Out] int(1/(cos(x)^2*(4 - 1/cos(x)^2)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec^2(x)}{\sqrt{-(\sec(x)-2)(\sec(x)+2)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)**2/(4-sec(x)**2)**(1/2),x)
```

```
[Out] Integral(sec(x)**2/sqrt(-(sec(x) - 2)*(sec(x) + 2)), x)
```

3.710 $\int \frac{\sec^2(x)}{\sqrt{1-4\tan^2(x)}}\,dx$

Optimal. Leaf size=9

$$\frac{1}{2}\sin^{-1}(2\tan(x))$$

```
[Out] 1/2*arcsin(2*tan(x))
```

Rubi [A] time = 0.05, antiderivative size = 9, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {3675, 216}

$$\frac{1}{2}\sin^{-1}(2\tan(x))$$

Antiderivative was successfully verified.

```
[In] Int[Sec[x]^2/Sqrt[1 - 4*Tan[x]^2],x]
```

```
[Out] ArcSin[2*Tan[x]]/2
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 3675

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/(c^(m - 1)*f), Subst[Int[(c^2 + ff^2*x^2)^(m/2 - 1)*(a + b*(ff*x)^n)^p
, x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && In
tegerQ[m/2] && (IntegersQ[n, p] || IGtQ[m, 0] || IGtQ[p, 0] || EqQ[n^2, 4]
|| EqQ[n^2, 16])
```

Rubi steps

$$\begin{aligned}\int \frac{\sec^2(x)}{\sqrt{1-4\tan^2(x)}}\,dx &= \text{Subst}\left(\int \frac{1}{\sqrt{1-4x^2}}\,dx, x, \tan(x)\right)\\ &= \frac{1}{2}\sin^{-1}(2\tan(x))\end{aligned}$$

Mathematica [B] time = 0.06, size = 47, normalized size = 5.22

$$\frac{\sqrt{5\cos(2x)-3}\ \sec(x)\tan^{-1}\left(\frac{2\sin(x)}{\sqrt{1-5\sin^2(x)}}\right)}{2\sqrt{2-8\tan^2(x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[x]^2/Sqrt[1 - 4*Tan[x]^2],x]
```

```
[Out] (ArcTan[(2*Sin[x])/Sqrt[1 - 5*Sin[x]^2]]*Sqrt[-3 + 5*Cos[2*x]]*Sec[x])/(2*S
qrt[2 - 8*Tan[x]^2])
```

fricas [B] time = 0.83, size = 45, normalized size = 5.00

$$-\frac{1}{4}\arctan\left(\frac{\left(9\cos(x)^3-8\cos(x)\right)\sqrt{\frac{5\cos(x)^2-4}{\cos(x)^2}}}{4\left(5\cos(x)^2-4\right)\sin(x)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)^2/(1-4*tan(x)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/4*arctan(1/4*(9*cos(x)^3 - 8*cos(x))*sqrt((5*cos(x)^2 - 4)/cos(x)^2)/((5
*cos(x)^2 - 4)*sin(x)))
```

giac [A] time = 0.17, size = 7, normalized size = 0.78

$$\frac{1}{2}\arcsin\left(2\tan(x)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)^2/(1-4*tan(x)^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*arcsin(2*tan(x))
```

maple [C] time = 0.69, size = 171, normalized size = 19.00

$$-\frac{\sqrt{2}\sqrt{\frac{2\cos(x)\sqrt{5}+5\cos(x)-2\sqrt{5}-4}{1+\cos(x)}}\sqrt{-\frac{2\left(2\cos(x)\sqrt{5}-5\cos(x)-2\sqrt{5}+4\right)}{1+\cos(x)}}\left(\mathrm{EllipticF}\left(\frac{(-1+\cos(x))\left(\sqrt{5}+2\right)}{\sin(x)},9-4\sqrt{5}\right)-2\,\mathrm{Elliptic}\right.}{\sqrt{\frac{5\left(\cos^2(x)\right)-4}{\cos(x)^2}}\ \cos(x)\left(-1+\cos(x)\right)\sqrt{9+4\sqrt{5}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(x)^2/(1-4*tan(x)^2)^(1/2),x)
```

```
[Out] -2^(1/2)*((2*cos(x)*5^(1/2)+5*cos(x)-2*5^(1/2)-4)/(1+cos(x)))^(1/2)*(-2*(2*
cos(x)*5^(1/2)-5*cos(x)-2*5^(1/2)+4)/(1+cos(x)))^(1/2)*(EllipticF((-1+cos(x
))*(5^(1/2)+2)/sin(x),9-4*5^(1/2))-2*EllipticPi((9+4*5^(1/2))^(1/2)*(-1+cos
(x))/sin(x),1/(9+4*5^(1/2)),(9-4*5^(1/2))^(1/2)/(9+4*5^(1/2))^(1/2)))*sin(x
)^2/((5*cos(x)^2-4)/cos(x)^2)^(1/2)/cos(x)/(-1+cos(x))/(9+4*5^(1/2))^(1/2)
```

maxima [A] time = 0.42, size = 7, normalized size = 0.78

$$\frac{1}{2}\arcsin\left(2\tan(x)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)^2/(1-4*tan(x)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*arcsin(2*tan(x))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.11

$$\int \frac{1}{\cos(x)^2\sqrt{1-4\tan(x)^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(x)^2*(1 - 4*tan(x)^2)^(1/2)),x)
```

```
[Out] int(1/(cos(x)^2*(1 - 4*tan(x)^2)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec^2(x)}{\sqrt{-(2\tan(x)-1)(2\tan(x)+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)**2/(1-4*tan(x)**2)**(1/2),x)
```

```
[Out] Integral(sec(x)**2/sqrt(-(2*tan(x) - 1)*(2*tan(x) + 1)), x)
```

3.711 $\int \frac{\sec^2(x)}{\sqrt{-4+\tan^2(x)}}\,dx$

Optimal. Leaf size=14

$$\tanh^{-1}\left(\frac{\tan(x)}{\sqrt{\tan^2(x)-4}}\right)$$

```
[Out] arctanh(tan(x)/(-4+tan(x)^2)^(1/2))
```

Rubi [A] time = 0.04, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {3675, 217, 206}

$$\tanh^{-1}\left(\frac{\tan(x)}{\sqrt{\tan^2(x)-4}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sec[x]^2/Sqrt[-4 + Tan[x]^2],x]
```

```
[Out] ArcTanh[Tan[x]/Sqrt[-4 + Tan[x]^2]]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 3675

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/(c^(m - 1)*f), Subst[Int[(c^2 + ff^2*x^2)^(m/2 - 1)*(a + b*(ff*x)^n)^p
, x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && In
tegerQ[m/2] && (IntegersQ[n, p] || IGtQ[m, 0] || IGtQ[p, 0] || EqQ[n^2, 4]
|| EqQ[n^2, 16])
```

Rubi steps

$$\int \frac{\sec^2(x)}{\sqrt{-4+\tan^2(x)}}\,dx = \text{Subst}\left(\int \frac{1}{\sqrt{-4+x^2}}\,dx, x, \tan(x)\right)$$
$$= \text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \frac{\tan(x)}{\sqrt{-4+\tan^2(x)}}\right)$$
$$= \tanh^{-1}\left(\frac{\tan(x)}{\sqrt{-4+\tan^2(x)}}\right)$$

Mathematica [B] time = 0.05, size = 46, normalized size = 3.29

$$\frac{\sqrt{5\cos(2x)+3}\,\sec(x)\tan^{-1}\left(\frac{\sin(x)}{\sqrt{4-5\sin^2(x)}}\right)}{\sqrt{2}\sqrt{\tan^2(x)-4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[x]^2/Sqrt[-4 + Tan[x]^2],x]
```

```
[Out] (ArcTan[Sin[x]/Sqrt[4 - 5*Sin[x]^2]]*Sqrt[3 + 5*Cos[2*x]]*Sec[x])/(Sqrt[2]*
Sqrt[-4 + Tan[x]^2])
```

fricas [B] time = 1.00, size = 67, normalized size = 4.79

$$\frac{1}{4}\log\left(\frac{1}{2}\sqrt{-\frac{5\cos(x)^2-1}{\cos(x)^2}}\cos(x)\sin(x)-\frac{3}{2}\cos(x)^2+\frac{1}{2}\right)-\frac{1}{4}\log\left(-\frac{1}{2}\sqrt{-\frac{5\cos(x)^2-1}{\cos(x)^2}}\cos(x)\sin(x)-\frac{3}{2}\cos\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)^2/(-4+tan(x)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/4*log(1/2*sqrt(-(5*cos(x)^2 - 1)/cos(x)^2)*cos(x)*sin(x) - 3/2*cos(x)^2 +
 1/2) - 1/4*log(-1/2*sqrt(-(5*cos(x)^2 - 1)/cos(x)^2)*cos(x)*sin(x) - 3/2*c
os(x)^2 + 1/2)
```

giac [A] time = 0.18, size = 17, normalized size = 1.21

$$-\log\left(\left|\sqrt{\tan(x)^2-4}-\tan(x)\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)^2/(-4+tan(x)^2)^(1/2),x, algorithm="giac")
```

```
[Out] -log(abs(sqrt(tan(x)^2 - 4) - tan(x)))
```

maple [C] time = 0.72, size = 171, normalized size = 12.21

$$-\frac{\sqrt{-\frac{2\left(\cos(x)\sqrt{5}-5\cos(x)-\sqrt{5}+1\right)}{1+\cos(x)}}\,\sqrt{2}\,\sqrt{\frac{\cos(x)\sqrt{5}-\sqrt{5}+5\cos(x)-1}{1+\cos(x)}}\left(\mathrm{EllipticF}\left(\frac{(-1+\cos(x))\left(\sqrt{5}-1\right)}{2\sin(x)},\frac{3}{2}+\frac{\sqrt{5}}{2}\right)-2\,\mathrm{EllipticPi}\left(\cdots\right.\right.}{4\sqrt{-\frac{5\left(\cos^2(x)\right)-1}{\cos(x)^2}}\,\cos(x)\,(-1+\cos(x))\,\sqrt{\frac{3}{2}-\frac{\sqrt{5}}{2}}\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(x)^2/(-4+tan(x)^2)^(1/2),x)
```

```
[Out] -1/4*(-2*(cos(x)*5^(1/2)-5*cos(x)-5^(1/2)+1)/(1+cos(x)))^(1/2)*2^(1/2)*((co
s(x)*5^(1/2)-5^(1/2)+5*cos(x)-1)/(1+cos(x)))^(1/2)*(EllipticF(1/2*(-1+cos(x
))*(5^(1/2)-1)/sin(x),3/2+1/2*5^(1/2))-2*EllipticPi((3/2-1/2*5^(1/2))^(1/2)
*(-1+cos(x))/sin(x),-2/(5^(1/2)-3),(3/2+1/2*5^(1/2))^(1/2)/(3/2-1/2*5^(1/2)
)^(1/2)))*sin(x)^2/(-(5*cos(x)^2-1)/cos(x)^2)^(1/2)/cos(x)/(-1+cos(x))/(3/2
-1/2*5^(1/2))^(1/2)
```

maxima [A] time = 0.31, size = 16, normalized size = 1.14

$$\log\left(2\sqrt{\tan(x)^2-4}+2\tan(x)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)^2/(-4+tan(x)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] log(2*sqrt(tan(x)^2 - 4) + 2*tan(x))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.07

$$\int \frac{1}{\cos(x)^2\sqrt{\tan(x)^2-4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(x)^2*(tan(x)^2 - 4)^(1/2)),x)
```

```
[Out] int(1/(cos(x)^2*(tan(x)^2 - 4)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec^2(x)}{\sqrt{(\tan(x)-2)(\tan(x)+2)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)**2/(-4+tan(x)**2)**(1/2),x)
```

```
[Out] Integral(sec(x)**2/sqrt((tan(x) - 2)*(tan(x) + 2)), x)
```

3.712 $\int \sqrt{1-\cot^2(x)} \sec^2(x) \, dx$

Optimal. Leaf size=19

$$\tan(x)\sqrt{1-\cot^2(x)} + \sin^{-1}(\cot(x))$$

[Out] arcsin(cot(x))+(1-cot(x)^2)^(1/2)*tan(x)

Rubi [A] time = 0.05, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {3663, 277, 216}

$$\tan(x)\sqrt{1-\cot^2(x)} + \sin^{-1}(\cot(x))$$

Antiderivative was successfully verified.

[In] Int[Sqrt[1 - Cot[x]^2]*Sec[x]^2,x]

[Out] ArcSin[Cot[x]] + Sqrt[1 - Cot[x]^2]*Tan[x]

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 277

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 3663

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[(c*ff^(m + 1))/f, Subst[Int[(x^m*(a + b*(ff*x)^n)^p)/(c^2 + ff^2*x^2)^(m/
2 + 1), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x]
 && IntegerQ[m/2]
```

Rubi steps

$$\int \sqrt{1-\cot^2(x)} \sec^2(x)\,dx = -\text{Subst}\left(\int \frac{\sqrt{1-x^2}}{x^2}\,dx, x, \cot(x)\right)$$
$$= \sqrt{1-\cot^2(x)} \tan(x) + \text{Subst}\left(\int \frac{1}{\sqrt{1-x^2}}\,dx, x, \cot(x)\right)$$
$$= \sin^{-1}(\cot(x)) + \sqrt{1-\cot^2(x)} \tan(x)$$

Mathematica [B] time = 0.49, size = 52, normalized size = 2.74

$$\tan(x)\sqrt{1-\cot^2(x)} \sec(2x)\left(\cos(2x) - \cos(x)\sqrt{-\cos(2x)} \tan^{-1}\left(\frac{\cos(x)}{\sqrt{-\cos(2x)}}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[1 - Cot[x]^2]*Sec[x]^2,x]
```

```
[Out] (-(ArcTan[Cos[x]/Sqrt[-Cos[2*x]]]*Cos[x]*Sqrt[-Cos[2*x]]) + Cos[2*x])*Sqrt[
1 - Cot[x]^2]*Sec[2*x]*Tan[x]
```

fricas [B] time = 0.97, size = 78, normalized size = 4.11

$$-\frac{\arctan\left(\frac{\left(3\cos(x)^2-1\right)\sqrt{\frac{2\cos(x)^2-1}{\cos(x)^2-1}}\sin(x)}{2\left(2\cos(x)^3-\cos(x)\right)}\right)\cos(x) - 2\sqrt{\frac{2\cos(x)^2-1}{\cos(x)^2-1}}\sin(x)}{2\cos(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)^2*(1-cot(x)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/2*(arctan(1/2*(3*cos(x)^2 - 1)*sqrt((2*cos(x)^2 - 1)/(cos(x)^2 - 1))*sin
(x)/(2*cos(x)^3 - cos(x)))*cos(x) - 2*sqrt((2*cos(x)^2 - 1)/(cos(x)^2 - 1))
*sin(x))/cos(x)
```

giac [C] time = 0.20, size = 142, normalized size = 7.47

$$-\frac{1}{2}(\pi + 2\arctan(-i) + 2i)\text{sgn}(\sin(x)) + \frac{1}{4}\left(2\pi\text{sgn}(\cos(x)) + \sqrt{2}\left(\frac{\sqrt{2}\sqrt{-2\cos(x)^2+1} - \sqrt{2}}{\cos(x)} - \frac{4\,\text{co}}{\sqrt{2}\sqrt{-2\cos(}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)^2*(1-cot(x)^2)^(1/2),x, algorithm="giac")
```

```
[Out] -1/2*(pi + 2*arctan(-I) + 2*I)*sgn(sin(x)) + 1/4*(2*pi*sgn(cos(x)) + sqrt(2
)*((sqrt(2)*sqrt(-2*cos(x)^2 + 1) - sqrt(2))/cos(x) - 4*cos(x)/(sqrt(2)*sqr
t(-2*cos(x)^2 + 1) - sqrt(2))) + 4*arctan(-1/4*sqrt(2)*((sqrt(2)*sqrt(-2*co
s(x)^2 + 1) - sqrt(2))^2/cos(x)^2 - 4)*cos(x)/(sqrt(2)*sqrt(-2*cos(x)^2 + 1
) - sqrt(2))))*sgn(sin(x))
```

maple [C] time = 0.61, size = 223, normalized size = 11.74

$$-\frac{(-1+\cos(x))\left(4i\cos(x)\ln\left(\frac{4(-1+\cos(x))\left(2i\cos(x)-\cos(x)\sqrt{-\frac{2(\cos^2(x))-1}{(1+\cos(x))^2}}+i-\sqrt{-\frac{2(\cos^2(x))-1}{(1+\cos(x))^2}}\right)}{\sin(x)^2}\right)-3\cos(x)\arcsin\left(\frac{(1+2\cos(x))\sqrt{2}}{2+2\cos(x)}\right.\right.}{2\cos(x)\sin(x)\sqrt{-\frac{2(\cos^2(x))-1}{(1+\cos(x))^2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(x)^2*(1-cot(x)^2)^(1/2),x)
```

```
[Out] -1/2*(-1+cos(x))*(4*I*cos(x)*ln(4*(-1+cos(x))*(2*I*cos(x)-cos(x)*(-(2*cos(x
)^2-1)/(1+cos(x))^2)^(1/2)+I-(-(2*cos(x)^2-1)/(1+cos(x))^2)^(1/2))/sin(x)^2
)-3*cos(x)*arcsin(1/2*(1+2*cos(x))/(1+cos(x))*2^(1/2))-cos(x)*arctan((2*cos
(x)^2-3*cos(x)+1)/(-(2*cos(x)^2-1)/(1+cos(x))^2)^(1/2)/sin(x)^2)+2*cos(x)*(
-(2*cos(x)^2-1)/(1+cos(x))^2)^(1/2)+2*(-(2*cos(x)^2-1)/(1+cos(x))^2)^(1/2))
*((2*cos(x)^2-1)/(-1+cos(x)^2))^(1/2)/cos(x)/sin(x)/(-(2*cos(x)^2-1)/(1+cos
(x))^2)^(1/2)
```

maxima [A] time = 0.41, size = 30, normalized size = 1.58

$$\sqrt{-\frac{1}{\tan(x)^2}+1}\,\tan(x)-\arctan\left(\sqrt{-\frac{1}{\tan(x)^2}+1}\,\tan(x)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)^2*(1-cot(x)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] sqrt(-1/tan(x)^2 + 1)*tan(x) - arctan(sqrt(-1/tan(x)^2 + 1)*tan(x))
```

mupad [B] time = 3.06, size = 19, normalized size = 1.00

$$\mathrm{asin}\left(\cot(x)\right)+\frac{\sqrt{1-\cot(x)^2}}{\cot(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1 - cot(x)^2)^(1/2)/cos(x)^2,x)
```

```
[Out] asin(cot(x)) + (1 - cot(x)^2)^(1/2)/cot(x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{-(\cot(x)-1)(\cot(x)+1)}\sec^2(x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)**2*(1-cot(x)**2)**(1/2),x)
```

```
[Out] Integral(sqrt(-(cot(x) - 1)*(cot(x) + 1))*sec(x)**2, x)
```

3.713 $\int \sec^2(x)\sqrt{1-\tan^2(x)}\,dx$

Optimal. Leaf size=26

$$\frac{1}{2}\tan(x)\sqrt{1-\tan^2(x)}+\frac{1}{2}\sin^{-1}(\tan(x))$$

```
[Out] 1/2*arcsin(tan(x))+1/2*(1-tan(x)^2)^(1/2)*tan(x)
```

Rubi [A] time = 0.05, antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {3675, 195, 216}

$$\frac{1}{2}\tan(x)\sqrt{1-\tan^2(x)}+\frac{1}{2}\sin^{-1}(\tan(x))$$

Antiderivative was successfully verified.

```
[In] Int[Sec[x]^2*Sqrt[1 - Tan[x]^2],x]
```

```
[Out] ArcSin[Tan[x]]/2 + (Tan[x]*Sqrt[1 - Tan[x]^2])/2
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 3675

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/(c^(m - 1)*f), Subst[Int[(c^2 + ff^2*x^2)^(m/2 - 1)*(a + b*(ff*x)^n)^p
, x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && In
tegerQ[m/2] && (IntegersQ[n, p] || IGtQ[m, 0] || IGtQ[p, 0] || EqQ[n^2, 4]
|| EqQ[n^2, 16])
```

Rubi steps

$$\begin{aligned}\int \sec^2(x)\sqrt{1-\tan^2(x)}\,dx &= \text{Subst}\left(\int \sqrt{1-x^2}\,dx, x, \tan(x)\right)\\ &= \frac{1}{2}\tan(x)\sqrt{1-\tan^2(x)} + \frac{1}{2}\text{Subst}\left(\int \frac{1}{\sqrt{1-x^2}}\,dx, x, \tan(x)\right)\\ &= \frac{1}{2}\sin^{-1}(\tan(x)) + \frac{1}{2}\tan(x)\sqrt{1-\tan^2(x)}\end{aligned}$$

Mathematica [B] time = 0.12, size = 63, normalized size = 2.42

$$\frac{\cos(2x)\tan(x) + \sqrt{\cos^2(x)}\cos(x)\sqrt{1-\tan^2(x)}\sin^{-1}\left(\frac{\sin(x)}{\sqrt{\cos^2(x)}}\right)}{2\sqrt{\cos^2(x)}\sqrt{\cos(2x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[x]^2*Sqrt[1 - Tan[x]^2],x]
```

```
[Out] (Cos[2*x]*Tan[x] + ArcSin[Sin[x]/Sqrt[Cos[x]^2]]*Cos[x]*Sqrt[Cos[x]^2]*Sqrt
[1 - Tan[x]^2])/(2*Sqrt[Cos[x]^2]*Sqrt[Cos[2*x]])
```

fricas [B] time = 1.05, size = 72, normalized size = 2.77

$$-\frac{\arctan\left(\frac{\left(3\cos(x)^3-2\cos(x)\right)\sqrt{\frac{2\cos(x)^2-1}{\cos(x)^2}}}{2\left(2\cos(x)^2-1\right)\sin(x)}\right)\cos(x) - 2\sqrt{\frac{2\cos(x)^2-1}{\cos(x)^2}}\sin(x)}{4\cos(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)^2*(1-tan(x)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/4*(arctan(1/2*(3*cos(x)^3 - 2*cos(x))*sqrt((2*cos(x)^2 - 1)/cos(x)^2)/((
2*cos(x)^2 - 1)*sin(x)))*cos(x) - 2*sqrt((2*cos(x)^2 - 1)/cos(x)^2)*sin(x))
/cos(x)
```

giac [A] time = 0.15, size = 20, normalized size = 0.77

$$\frac{1}{2}\sqrt{-\tan(x)^2+1}\,\tan(x) + \frac{1}{2}\arcsin\left(\tan(x)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)^2*(1-tan(x)^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(-tan(x)^2 + 1)*tan(x) + 1/2*arcsin(tan(x))
```

maple [C] time = 0.43, size = 492, normalized size = 18.92

$$\frac{\sin(x)\left(2\left(\cos^2(x)\right)\sin(x)\sqrt{2}\sqrt{\frac{\cos(x)\sqrt{2}-\sqrt{2}+2\cos(x)-1}{1+\cos(x)}}\sqrt{-\frac{2\left(\cos(x)\sqrt{2}-\sqrt{2}-2\cos(x)+1\right)}{1+\cos(x)}}\ \mathrm{EllipticPi}\left(\frac{\sqrt{3+2\sqrt{2}}\ (-1+\cos(x))}{\sin(x)},\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(x)^2*(1-tan(x)^2)^(1/2),x)
```

```
[Out] 1/2*sin(x)*(2*cos(x)^2*sin(x)*2^(1/2)*((cos(x)*2^(1/2)-2^(1/2)+2*cos(x)-1)/
(1+cos(x)))^(1/2)*(-2*(cos(x)*2^(1/2)-2^(1/2)-2*cos(x)+1)/(1+cos(x)))^(1/2)
*EllipticPi((3+2*2^(1/2))^(1/2)*(-1+cos(x))/sin(x),1/(3+2*2^(1/2)),(3-2*2^(
1/2))^(1/2)/(3+2*2^(1/2))^(1/2))-cos(x)^2*sin(x)*2^(1/2)*((cos(x)*2^(1/2)-2
^(1/2)+2*cos(x)-1)/(1+cos(x)))^(1/2)*(-2*(cos(x)*2^(1/2)-2^(1/2)-2*cos(x)+1
)/(1+cos(x)))^(1/2)*EllipticF((1+2^(1/2))*(-1+cos(x))/sin(x),3-2*2^(1/2))+4
*cos(x)^2*sin(x)*((cos(x)*2^(1/2)-2^(1/2)+2*cos(x)-1)/(1+cos(x)))^(1/2)*(-2
*(cos(x)*2^(1/2)-2^(1/2)-2*cos(x)+1)/(1+cos(x)))^(1/2)*EllipticPi((3+2*2^(1
/2))^(1/2)*(-1+cos(x))/sin(x),1/(3+2*2^(1/2)),(3-2*2^(1/2))^(1/2)/(3+2*2^(1
/2))^(1/2))-2*cos(x)^2*sin(x)*((cos(x)*2^(1/2)-2^(1/2)+2*cos(x)-1)/(1+cos(x
)))^(1/2)*(-2*(cos(x)*2^(1/2)-2^(1/2)-2*cos(x)+1)/(1+cos(x)))^(1/2)*Ellipti
cF((1+2^(1/2))*(-1+cos(x))/sin(x),3-2*2^(1/2))+4*cos(x)^3*2^(1/2)-4*cos(x)^
2*2^(1/2)+6*cos(x)^3-2*cos(x)*2^(1/2)-6*cos(x)^2+2*2^(1/2)-3*cos(x)+3)*((2*
cos(x)^2-1)/cos(x)^2)^(1/2)/(-1+cos(x))/(2*cos(x)^2-1)/cos(x)/(1+2^(1/2))/(
3+2*2^(1/2))^(1/2)
```

maxima [A] time = 0.41, size = 20, normalized size = 0.77

$$\frac{1}{2}\sqrt{-\tan(x)^2+1}\ \tan(x)+\frac{1}{2}\ \arcsin\left(\tan(x)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)^2*(1-tan(x)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*sqrt(-tan(x)^2 + 1)*tan(x) + 1/2*arcsin(tan(x))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.04

$$\int\frac{\sqrt{1-\tan(x)^2}}{\cos(x)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1 - tan(x)^2)^(1/2)/cos(x)^2,x)
```

```
[Out] int((1 - tan(x)^2)^(1/2)/cos(x)^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{-\left(\tan \left(x\right)-1\right)\left(\tan \left(x\right)+1\right)}\,\sec ^{2}\left(x\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)**2*(1-tan(x)**2)**(1/2),x)
```

```
[Out] Integral(sqrt(-(tan(x) - 1)*(tan(x) + 1))*sec(x)**2, x)
```

3.714 $\int e^{\tan(x)} \sec^2(x)\, dx$

Optimal. Leaf size=4

$$e^{\tan(x)}$$

[Out] exp(tan(x))

Rubi [A] time = 0.01, antiderivative size = 4, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {4342, 2194}

$$e^{\tan(x)}$$

Antiderivative was successfully verified.

[In] Int[E^Tan[x]*Sec[x]^2,x]

[Out] E^Tan[x]

Rule 2194

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rule 4342

```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))]^2, x_Symbol] :> With[{d = FreeFac
tors[Tan[c*(a + b*x)], x]}, Dist[d/(b*c), Subst[Int[SubstFor[1, Tan[c*(a +
b*x)]/d, u, x], x], x, Tan[c*(a + b*x)]/d], x] /; FunctionOfQ[Tan[c*(a + b*
x)]/d, u, x, True]] /; FreeQ[{a, b, c}, x] && NonsumQ[u] && (EqQ[F, Sec] ||
 EqQ[F, sec])
```

Rubi steps

$$\begin{aligned} \int e^{\tan(x)} \sec^2(x)\, dx &= \text{Subst}\left(\int e^x\, dx, x, \tan(x)\right)\\ &= e^{\tan(x)} \end{aligned}$$

Mathematica [A] time = 0.06, size = 4, normalized size = 1.00

$$e^{\tan(x)}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^Tan[x]*Sec[x]^2,x]
[Out] E^Tan[x]
```

fricas [B] time = 0.52, size = 8, normalized size = 2.00

$$e^{\frac{\sin(x)}{\cos(x)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(tan(x))*sec(x)^2,x, algorithm="fricas")
[Out] e^(sin(x)/cos(x))
```

giac [A] time = 0.13, size = 3, normalized size = 0.75

$$e^{\tan(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(tan(x))*sec(x)^2,x, algorithm="giac")
[Out] e^tan(x)
```

maple [A] time = 0.05, size = 4, normalized size = 1.00

$$\mathrm{e}^{\tan(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(tan(x))*sec(x)^2,x)
[Out] exp(tan(x))
```

maxima [A] time = 0.32, size = 3, normalized size = 0.75

$$e^{\tan(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(tan(x))*sec(x)^2,x, algorithm="maxima")
[Out] e^tan(x)
```

mupad [B] time = 3.10, size = 3, normalized size = 0.75

$$\mathrm{e}^{\tan(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(tan(x))/cos(x)^2,x)
```

```
[Out] exp(tan(x))
```

sympy [A] time = 0.95, size = 3, normalized size = 0.75

$$e^{\tan(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(tan(x))*sec(x)**2,x)
```

```
[Out] exp(tan(x))
```

3.715 $\int \sec^4(x)\left(-1+\sec^2(x)\right)^2\tan(x)\,dx$

Optimal. Leaf size=17

$$\frac{\tan^8(x)}{8}+\frac{\tan^6(x)}{6}$$

```
[Out] 1/6*tan(x)^6+1/8*tan(x)^8
```

Rubi [A] time = 0.07, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {4120, 2607, 14}

$$\frac{\tan^8(x)}{8}+\frac{\tan^6(x)}{6}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[x]^4*(-1 + Sec[x]^2)^2*Tan[x],x]
```

```
[Out] Tan[x]^6/6 + Tan[x]^8/8
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 4120

```
Int[(u_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Dist[
b^p, Int[ActivateTrig[u*tan[e + f*x]^(2*p)], x], x] /; FreeQ[{a, b, e, f, p
}, x] && EqQ[a + b, 0] && IntegerQ[p]
```

Rubi steps

$$
\begin{aligned}
\int \sec^4(x)\left(-1+\sec^2(x)\right)^2\tan(x)\,dx &= \int \sec^4(x)\tan^5(x)\,dx\\
&= \text{Subst}\left(\int x^5\left(1+x^2\right)\,dx, x, \tan(x)\right)\\
&= \text{Subst}\left(\int \left(x^5+x^7\right)\,dx, x, \tan(x)\right)\\
&= \frac{\tan^6(x)}{6}+\frac{\tan^8(x)}{8}
\end{aligned}
$$

Mathematica [A] time = 0.02, size = 25, normalized size = 1.47

$$\frac{\sec^8(x)}{8}-\frac{\sec^6(x)}{3}+\frac{\sec^4(x)}{4}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[x]^4*(-1 + Sec[x]^2)^2*Tan[x],x]
```

```
[Out] Sec[x]^4/4 - Sec[x]^6/3 + Sec[x]^8/8
```

fricas [A] time = 0.81, size = 20, normalized size = 1.18

$$\frac{6\,\cos(x)^4-8\,\cos(x)^2+3}{24\,\cos(x)^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)^4*(-1+sec(x)^2)^2*tan(x),x, algorithm="fricas")
```

```
[Out] 1/24*(6*cos(x)^4 - 8*cos(x)^2 + 3)/cos(x)^8
```

giac [A] time = 0.15, size = 20, normalized size = 1.18

$$\frac{6\,\cos(x)^4-8\,\cos(x)^2+3}{24\,\cos(x)^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)^4*(-1+sec(x)^2)^2*tan(x),x, algorithm="giac")
```

```
[Out] 1/24*(6*cos(x)^4 - 8*cos(x)^2 + 3)/cos(x)^8
```

maple [A] time = 0.05, size = 20, normalized size = 1.18

$$\frac{\left(\sec^8(x)\right)}{8}-\frac{\left(\sec^6(x)\right)}{3}+\frac{\left(\sec^4(x)\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(x)^4*(-1+sec(x)^2)^2*tan(x),x)
```

```
[Out] 1/8*sec(x)^8-1/3*sec(x)^6+1/4*sec(x)^4
```

maxima [B] time = 0.31, size = 42, normalized size = 2.47

$$\frac{6\sin(x)^4 - 4\sin(x)^2 + 1}{24\left(\sin(x)^8 - 4\sin(x)^6 + 6\sin(x)^4 - 4\sin(x)^2 + 1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)^4*(-1+sec(x)^2)^2*tan(x),x, algorithm="maxima")
```

```
[Out] 1/24*(6*sin(x)^4 - 4*sin(x)^2 + 1)/(sin(x)^8 - 4*sin(x)^6 + 6*sin(x)^4 - 4*
sin(x)^2 + 1)
```

mupad [B] time = 2.92, size = 14, normalized size = 0.82

$$\frac{\tan(x)^6\left(3\tan(x)^2 + 4\right)}{24}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((tan(x)*(1/cos(x)^2 - 1)^2)/cos(x)^4,x)
```

```
[Out] (tan(x)^6*(3*tan(x)^2 + 4))/24
```

sympy [A] time = 6.97, size = 19, normalized size = 1.12

$$\frac{\sec^8(x)}{8} - \frac{\sec^6(x)}{3} + \frac{\sec^4(x)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)**4*(-1+sec(x)**2)**2*tan(x),x)
```

```
[Out] sec(x)**8/8 - sec(x)**6/3 + sec(x)**4/4
```

3.716 $\int \frac{\csc^2(x)}{a+b\cot(x)}\,dx$

Optimal. Leaf size=12

$$-\frac{\log(a+b\cot(x))}{b}$$

```
[Out] -ln(a+b*cot(x))/b
```

Rubi [A] time = 0.04, antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {3506, 31}

$$-\frac{\log(a+b\cot(x))}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[x]^2/(a + b*Cot[x]),x]
```

```
[Out] -(Log[a + b*Cot[x]]/b)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 3506

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Dist[1/(b*f), Subst[Int[(a + x)^n*(1 + x^2/b^2)^(m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && NeQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rubi steps

$$\begin{aligned}\int \frac{\csc^2(x)}{a+b\cot(x)}\,dx &= -\frac{\text{Subst}\left(\int \frac{1}{a+x}\,dx, x, b\cot(x)\right)}{b}\\ &= -\frac{\log(a+b\cot(x))}{b}\end{aligned}$$

Mathematica [A] time = 0.06, size = 20, normalized size = 1.67

$$\frac{\log(\sin(x))-\log(a\sin(x)+b\cos(x))}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[x]^2/(a + b*Cot[x]),x]
```

```
[Out] (Log[Sin[x]] - Log[b*Cos[x] + a*Sin[x]])/b
```

fricas [B] time = 1.44, size = 45, normalized size = 3.75

$$-\frac{\log\left(2\,ab\cos(x)\sin(x) - \left(a^2 - b^2\right)\cos(x)^2 + a^2\right) - \log\left(-\frac{1}{4}\,\cos(x)^2 + \frac{1}{4}\right)}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)^2/(a+b*cot(x)),x, algorithm="fricas")
```

```
[Out] -1/2*(log(2*a*b*cos(x)*sin(x) - (a^2 - b^2)*cos(x)^2 + a^2) - log(-1/4*cos(
x)^2 + 1/4))/b
```

giac [A] time = 0.17, size = 22, normalized size = 1.83

$$-\frac{\log\left(|a\tan(x) + b|\right)}{b} + \frac{\log\left(|\tan(x)|\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)^2/(a+b*cot(x)),x, algorithm="giac")
```

```
[Out] -log(abs(a*tan(x) + b))/b + log(abs(tan(x)))/b
```

maple [A] time = 0.07, size = 13, normalized size = 1.08

$$-\frac{\ln\left(a + b\cot(x)\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(x)^2/(a+b*cot(x)),x)
```

```
[Out] -ln(a+b*cot(x))/b
```

maxima [A] time = 0.33, size = 12, normalized size = 1.00

$$-\frac{\log\left(b\cot(x) + a\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)^2/(a+b*cot(x)),x, algorithm="maxima")
```

```
[Out] -log(b*cot(x) + a)/b
```

mupad [B] time = 3.01, size = 16, normalized size = 1.33

$$-\frac{2\,\mathrm{atanh}\left(\frac{2\,a\,\tan(x)}{b}+1\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(x)^2*(a + b*cot(x))),x)
[Out] -(2*atanh((2*a*tan(x))/b + 1))/b
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc^2(x)}{a+b\cot(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)**2/(a+b*cot(x)),x)
[Out] Integral(csc(x)**2/(a + b*cot(x)), x)
```

3.717 $\int (a+b\cot(x))^n \csc^2(x)\,dx$

Optimal. Leaf size=20

$$-\frac{(a+b\cot(x))^{n+1}}{b(n+1)}$$

```
[Out] -(a+b*cot(x))^(1+n)/b/(1+n)
```

Rubi [A] time = 0.04, antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {3506, 32}

$$-\frac{(a+b\cot(x))^{n+1}}{b(n+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cot[x])^n*Csc[x]^2,x]
```

```
[Out] -((a + b*Cot[x])^(1 + n)/(b*(1 + n)))
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m + 1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 3506

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Dist[1/(b*f), Subst[Int[(a + x)^n*(1 + x^2/b^2)^(m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && NeQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rubi steps

$$\begin{aligned}\int (a+b\cot(x))^n \csc^2(x)\,dx &= -\frac{\text{Subst}\left(\int (a+x)^n\,dx, x, b\cot(x)\right)}{b}\\ &= -\frac{(a+b\cot(x))^{1+n}}{b(1+n)}\end{aligned}$$

Mathematica [A] time = 0.20, size = 19, normalized size = 0.95

$$-\frac{(a+b\cot(x))^{n+1}}{bn+b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cot[x])^n*Csc[x]^2,x]
```

```
[Out] -((a + b*Cot[x])^(1 + n)/(b + b*n))
```

fricas [A] time = 0.65, size = 38, normalized size = 1.90

$$-\frac{(b\cos(x) + a\sin(x))\left(\frac{b\cos(x)+a\sin(x)}{\sin(x)}\right)^n}{(bn + b)\sin(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cot(x))^n*csc(x)^2,x, algorithm="fricas")
```

```
[Out] -(b*cos(x) + a*sin(x))*((b*cos(x) + a*sin(x))/sin(x))^n/((b*n + b)*sin(x))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b\cot(x) + a)^n \csc(x)^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cot(x))^n*csc(x)^2,x, algorithm="giac")
```

```
[Out] integrate((b*cot(x) + a)^n*csc(x)^2, x)
```

maple [A] time = 0.08, size = 21, normalized size = 1.05

$$-\frac{(a + b\cot(x))^{n+1}}{b\,(n+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cot(x))^n*csc(x)^2,x)
```

```
[Out] -(a+b*cot(x))^(n+1)/b/(n+1)
```

maxima [A] time = 0.32, size = 20, normalized size = 1.00

$$-\frac{(b\cot(x) + a)^{n+1}}{b(n+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cot(x))^n*csc(x)^2,x, algorithm="maxima")
```

```
[Out] -(b*cot(x) + a)^(n + 1)/(b*(n + 1))
```

mupad [B] time = 3.19, size = 43, normalized size = 2.15

$$\begin{cases} -\frac{\ln\left(a+\frac{b}{\tan(x)}\right)}{b} & \text{if } n = -1 \\ -\frac{\left(a+\frac{b}{\tan(x)}\right)^{n+1}}{b\,(n+1)} & \text{if } n \neq -1 \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*cot(x))^n/sin(x)^2,x)
[Out] piecewise(n == -1, -log(a + b/tan(x))/b, n ~= -1, -(a + b/tan(x))^(n + 1)/(
b*(n + 1)))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b\cot(x))^n \csc^2(x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cot(x))**n*csc(x)**2,x)
[Out] Integral((a + b*cot(x))**n*csc(x)**2, x)
```

3.718 $\int \csc^2(x) \left(1+\sin^2(x)\right) \, dx$

Optimal. Leaf size=6

$$x-\cot(x)$$

[Out] x-cot(x)

Rubi [A] time = 0.02, antiderivative size = 6, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {3012, 8}

$$x-\cot(x)$$

Antiderivative was successfully verified.

[In] Int[Csc[x]^2*(1 + Sin[x]^2),x]

[Out] x - Cot[x]

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3012

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_) + (C_.)*sin[(e_.) + (f_.)*(x
_)]^2), x_Symbol] :> Simp[(A*Cos[e + f*x]*(b*Sin[e + f*x])^(m + 1))/(b*f*(m
 + 1)), x] + Dist[(A*(m + 2) + C*(m + 1))/(b^2*(m + 1)), Int[(b*Sin[e + f*x
])^(m + 2), x], x] /; FreeQ[{b, e, f, A, C}, x] && LtQ[m, -1]
```

Rubi steps

$$\begin{aligned} \int \csc^2(x) \left(1+\sin^2(x)\right) \, dx &= -\cot(x)+\int 1 \, dx \\ &= x-\cot(x) \end{aligned}$$

Mathematica [A] time = 0.00, size = 6, normalized size = 1.00

$$x-\cot(x)$$

Antiderivative was successfully verified.

[In] Integrate[Csc[x]^2*(1 + Sin[x]^2),x]

[Out] x - Cot[x]

fricas [B] time = 0.67, size = 14, normalized size = 2.33

$$\frac{x\sin(x)-\cos(x)}{\sin(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)^2*(1+sin(x)^2),x, algorithm="fricas")
[Out] (x*sin(x) - cos(x))/sin(x)
```

giac [B] time = 0.15, size = 16, normalized size = 2.67

$$x-\frac{1}{2\tan\left(\frac{1}{2}x\right)}+\frac{1}{2}\tan\left(\frac{1}{2}x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)^2*(1+sin(x)^2),x, algorithm="giac")
[Out] x - 1/2/tan(1/2*x) + 1/2*tan(1/2*x)
```

maple [A] time = 0.06, size = 7, normalized size = 1.17

$$x-\cot(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(x)^2*(1+sin(x)^2),x)
[Out] x-cot(x)
```

maxima [A] time = 0.41, size = 8, normalized size = 1.33

$$x-\frac{1}{\tan(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)^2*(1+sin(x)^2),x, algorithm="maxima")
[Out] x - 1/tan(x)
```

mupad [B] time = 2.93, size = 6, normalized size = 1.00

$$x-\cot(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((sin(x)^2 + 1)/sin(x)^2,x)
```

```
[Out] x - cot(x)
```

sympy **[A]** time = 4.21, size = 3, normalized size = 0.50

$$x - \cot(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)**2*(1+sin(x)**2),x)
```

```
[Out] x - cot(x)
```

3.719 $\int \left(1+\frac{1}{1+\cot^2(x)}\right)\csc^2(x)\,dx$

Optimal. Leaf size=6

$$x-\cot(x)$$

```
[Out] x-cot(x)
```

Rubi [A] time = 0.05, antiderivative size = 6, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {14, 203}

$$x-\cot(x)$$

Antiderivative was successfully verified.

```
[In] Int[(1 + (1 + Cot[x]^2)^(-1))*Csc[x]^2,x]
```

```
[Out] x - Cot[x]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \left(1+\frac{1}{1+\cot^2(x)}\right)\csc^2(x)\,dx &= \text{Subst}\left(\int \frac{1+\frac{1}{1+\frac{1}{x^2}}}{x^2}\,dx,x,\tan(x)\right)\\
&= \text{Subst}\left(\int \left(\frac{1}{x^2}+\frac{1}{1+x^2}\right)dx,x,\tan(x)\right)\\
&= -\cot(x)+\text{Subst}\left(\int \frac{1}{1+x^2}\,dx,x,\tan(x)\right)\\
&= x-\cot(x)
\end{aligned}$$

Mathematica [A]   time = 0.01, size = 6, normalized size = 1.00

$$x-\cot(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + (1 + Cot[x]^2)^(-1))*Csc[x]^2,x]
```

```
[Out] x - Cot[x]
```

fricas [B]   time = 0.66, size = 14, normalized size = 2.33

$$\frac{x\sin(x)-\cos(x)}{\sin(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+1/(1+cot(x)^2))*csc(x)^2,x, algorithm="fricas")
```

```
[Out] (x*sin(x) - cos(x))/sin(x)
```

giac [B]   time = 0.14, size = 16, normalized size = 2.67

$$x-\frac{1}{2\tan\left(\frac{1}{2}x\right)}+\frac{1}{2}\tan\left(\frac{1}{2}x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+1/(1+cot(x)^2))*csc(x)^2,x, algorithm="giac")
```

```
[Out] x - 1/2/tan(1/2*x) + 1/2*tan(1/2*x)
```

maple [A]   time = 0.09, size = 7, normalized size = 1.17

$$x-\cot(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+1/(1+cot(x)^2))*csc(x)^2,x)
```

```
[Out] x-cot(x)
```

maxima [A]   time = 0.41, size = 8, normalized size = 1.33

$$x-\frac{1}{\tan(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+1/(1+cot(x)^2))*csc(x)^2,x, algorithm="maxima")
```

```
[Out] x - 1/tan(x)
```

mupad [B] time = 2.94, size = 6, normalized size = 1.00

$$x - \cot(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/(cot(x)^2 + 1) + 1)/sin(x)^2,x)
```

```
[Out] x - cot(x)
```

sympy [B] time = 0.70, size = 27, normalized size = 4.50

$$\frac{x\csc^2(x)}{\cot^2(x)+1} - \frac{\cot(x)\csc^2(x)}{\cot^2(x)+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+1/(1+cot(x)**2))*csc(x)**2,x)
```

```
[Out] x*csc(x)**2/(cot(x)**2 + 1) - cot(x)*csc(x)**2/(cot(x)**2 + 1)
```

3.720 $\int \frac{(a+b\cot(x))\csc^2(x)}{c+d\cot(x)}\,dx$

Optimal. Leaf size=28

$$\frac{(bc-ad)\log(c+d\cot(x))}{d^2}-\frac{b\cot(x)}{d}$$

```
[Out] -b*cot(x)/d+(-a*d+b*c)*ln(c+d*cot(x))/d^2
```

Rubi [A] time = 0.08, antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {4344, 43}

$$\frac{(bc-ad)\log(c+d\cot(x))}{d^2}-\frac{b\cot(x)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*Cot[x])*Csc[x]^2)/(c + d*Cot[x]),x]
```

```
[Out] -((b*Cot[x])/d) + ((b*c - a*d)*Log[c + d*Cot[x]])/d^2
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 4344

```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))]^2, x_Symbol] :> With[{d = FreeFac
tors[Cot[c*(a + b*x)], x]}, -Dist[d/(b*c), Subst[Int[SubstFor[1, Cot[c*(a +
 b*x)]/d, u, x], x], x, Cot[c*(a + b*x)]/d], x] /; FunctionOfQ[Cot[c*(a + b
*x)]/d, u, x, True]] /; FreeQ[{a, b, c}, x] && NonsumQ[u] && (EqQ[F, Csc] |
| EqQ[F, csc])
```

Rubi steps

$$\int \frac{(a+b\cot(x))\csc^2(x)}{c+d\cot(x)}\,dx = -\operatorname{Subst}\left(\int \frac{a+bx}{c+dx}\,dx, x, \cot(x)\right)$$
$$= -\operatorname{Subst}\left(\int \left(\frac{b}{d}+\frac{-bc+ad}{d(c+dx)}\right)dx, x, \cot(x)\right)$$
$$= -\frac{b\cot(x)}{d}+\frac{(bc-ad)\log(c+d\cot(x))}{d^2}$$

Mathematica [A] time = 0.38, size = 56, normalized size = 2.00

$$\frac{\sin(x)(a+b\cot(x))(-(bc-ad)(\log(\sin(x))-\log(c\sin(x)+d\cos(x)))-bd\cot(x))}{d^2(a\sin(x)+b\cos(x))}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*Cot[x])*Csc[x]^2)/(c + d*Cot[x]),x]
```

```
[Out] ((a + b*Cot[x])*(-(b*d*Cot[x]) - (b*c - a*d)*(Log[Sin[x]] - Log[d*Cos[x] +
c*Sin[x]]))*Sin[x])/(d^2*(b*Cos[x] + a*Sin[x]))
```

fricas [B] time = 1.17, size = 76, normalized size = 2.71

$$-\frac{2\,bd\cos(x)-(bc-ad)\log\left(2\,cd\cos(x)\sin(x)-\left(c^2-d^2\right)\cos(x)^2+c^2\right)\sin(x)+(bc-ad)\log\left(-\frac{1}{4}\cos(x)^2+\frac{1}{4}\right.}{2\,d^2\sin(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cot(x))*csc(x)^2/(c+d*cot(x)),x, algorithm="fricas")
```

```
[Out] -1/2*(2*b*d*cos(x) - (b*c - a*d)*log(2*c*d*cos(x)*sin(x) - (c^2 - d^2)*cos(
x)^2 + c^2)*sin(x) + (b*c - a*d)*log(-1/4*cos(x)^2 + 1/4)*sin(x))/(d^2*sin(
x))
```

giac [B] time = 0.15, size = 68, normalized size = 2.43

$$-\frac{(bc-ad)\log(|\tan(x)|)}{d^2}+\frac{\left(bc^2-acd\right)\log(|c\tan(x)+d|)}{cd^2}+\frac{bc\tan(x)-ad\tan(x)-bd}{d^2\tan(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cot(x))*csc(x)^2/(c+d*cot(x)),x, algorithm="giac")
```

```
[Out] -(b*c - a*d)*log(abs(tan(x)))/d^2 + (b*c^2 - a*c*d)*log(abs(c*tan(x) + d))/
(c*d^2) + (b*c*tan(x) - a*d*tan(x) - b*d)/(d^2*tan(x))
```

maple [A] time = 0.10, size = 56, normalized size = 2.00

$$-\frac{b}{d\tan(x)}+\frac{\ln(\tan(x))\,a}{d}-\frac{\ln(\tan(x))\,cb}{d^2}-\frac{\ln(c\tan(x)+d)\,a}{d}+\frac{\ln(c\tan(x)+d)\,cb}{d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cot(x))*csc(x)^2/(c+d*cot(x)),x)
```

```
[Out] -b/d/tan(x)+1/d*ln(tan(x))*a-1/d^2*ln(tan(x))*c*b-1/d*ln(c*tan(x)+d)*a+1/d^
2*ln(c*tan(x)+d)*c*b
```

maxima [A] time = 0.32, size = 46, normalized size = 1.64

$$\frac{(bc-ad)\log(c\tan(x)+d)}{d^2}-\frac{(bc-ad)\log(\tan(x))}{d^2}-\frac{b}{d\tan(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cot(x))*csc(x)^2/(c+d*cot(x)),x, algorithm="maxima")
```

```
[Out] (b*c - a*d)*log(c*tan(x) + d)/d^2 - (b*c - a*d)*log(tan(x))/d^2 - b/(d*tan(
x))
```

mupad [B] time = 3.07, size = 35, normalized size = 1.25

$$-\frac{b}{d\tan(x)}-\frac{2\,\mathrm{atanh}\left(\frac{2\,c\tan(x)}{d}+1\right)(a\,d-b\,c)}{d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*cot(x))/(sin(x)^2*(c + d*cot(x))),x)
```

```
[Out] - b/(d*tan(x)) - (2*atanh((2*c*tan(x))/d + 1)*(a*d - b*c))/d^2
```

sympy [A] time = 26.52, size = 31, normalized size = 1.11

$$-\frac{b\cot(x)}{d}-\frac{(ad-bc)\left(\begin{cases}\frac{\cot(x)}{c} & \text{for } d=0\\ \frac{\log(c+d\cot(x))}{d} & \text{otherwise}\end{cases}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cot(x))*csc(x)**2/(c+d*cot(x)),x)
```

```
[Out] -b*cot(x)/d - (a*d - b*c)*Piecewise((cot(x)/c, Eq(d, 0)), (log(c + d*cot(x)
)/d, True))/d
```

3.721 $\int \frac{(a+b\cot(x))^2\csc^2(x)}{c+d\cot(x)}\,dx$

Optimal. Leaf size=53

$$-\frac{(bc-ad)^2\log(c+d\cot(x))}{d^3}+\frac{b\cot(x)(bc-ad)}{d^2}-\frac{(a+b\cot(x))^2}{2d}$$

```
[Out] b*(-a*d+b*c)*cot(x)/d^2-1/2*(a+b*cot(x))^2/d-(-a*d+b*c)^2*ln(c+d*cot(x))/d^
3
```

Rubi [A] time = 0.14, antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {4344, 43}

$$\frac{b\cot(x)(bc-ad)}{d^2}-\frac{(bc-ad)^2\log(c+d\cot(x))}{d^3}-\frac{(a+b\cot(x))^2}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*Cot[x])^2*Csc[x]^2)/(c + d*Cot[x]),x]
```

```
[Out] (b*(b*c - a*d)*Cot[x])/d^2 - (a + b*Cot[x])^2/(2*d) - ((b*c - a*d)^2*Log[c
 + d*Cot[x]])/d^3
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 4344

```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))]^2, x_Symbol] :> With[{d = FreeFac
tors[Cot[c*(a + b*x)], x]}, -Dist[d/(b*c), Subst[Int[SubstFor[1, Cot[c*(a +
 b*x)]/d, u, x], x], x, Cot[c*(a + b*x)]/d], x] /; FunctionOfQ[Cot[c*(a + b
*x)]/d, u, x, True]] /; FreeQ[{a, b, c}, x] && NonsumQ[u] && (EqQ[F, Csc] |
| EqQ[F, csc])
```

Rubi steps

$$\int \frac{(a+b\cot(x))^2\csc^2(x)}{c+d\cot(x)}\,dx = -\operatorname{Subst}\left(\int \frac{(a+bx)^2}{c+dx}\,dx, x, \cot(x)\right)$$
$$= -\operatorname{Subst}\left(\int \left(-\frac{b(bc-ad)}{d^2}+\frac{b(a+bx)}{d}+\frac{(-bc+ad)^2}{d^2(c+dx)}\right)dx, x, \cot(x)\right)$$
$$= \frac{b(bc-ad)\cot(x)}{d^2}-\frac{(a+b\cot(x))^2}{2d}-\frac{(bc-ad)^2\log(c+d\cot(x))}{d^3}$$

Mathematica [A] time = 0.56, size = 62, normalized size = 1.17

$$\frac{2bd\cot(x)(bc-2ad)+2(bc-ad)^2(\log(\sin(x))-\log(c\sin(x)+d\cos(x)))-b^2d^2\csc^2(x)}{2d^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*Cot[x])^2*Csc[x]^2)/(c + d*Cot[x]),x]
```

```
[Out] (2*b*d*(b*c - 2*a*d)*Cot[x] - b^2*d^2*Csc[x]^2 + 2*(b*c - a*d)^2*(Log[Sin[x
]] - Log[d*Cos[x] + c*Sin[x]]))/(2*d^3)
```

fricas [B] time = 1.47, size = 182, normalized size = 3.43

$$\frac{b^2d^2-2\left(b^2cd-2\,abd^2\right)\cos(x)\sin(x)+\left(b^2c^2-2\,abcd+a^2d^2-\left(b^2c^2-2\,abcd+a^2d^2\right)\cos(x)^2\right)\log\left(2\,cd\cos(x)\,s\right.}{2\left(d^3\cos(x\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cot(x))^2*csc(x)^2/(c+d*cot(x)),x, algorithm="fricas")
```

```
[Out] 1/2*(b^2*d^2 - 2*(b^2*c*d - 2*a*b*d^2)*cos(x)*sin(x) + (b^2*c^2 - 2*a*b*c*d
 + a^2*d^2 - (b^2*c^2 - 2*a*b*c*d + a^2*d^2)*cos(x)^2)*log(2*c*d*cos(x)*sin
(x) - (c^2 - d^2)*cos(x)^2 + c^2) - (b^2*c^2 - 2*a*b*c*d + a^2*d^2 - (b^2*c
^2 - 2*a*b*c*d + a^2*d^2)*cos(x)^2)*log(-1/4*cos(x)^2 + 1/4))/(d^3*cos(x)^2
 - d^3)
```

giac [B] time = 0.16, size = 139, normalized size = 2.62

$$\frac{\left(b^2c^2-2\,abcd+a^2d^2\right)\log\left(|\tan(x)|\right)}{d^3}-\frac{\left(b^2c^3-2\,abc^2d+a^2cd^2\right)\log\left(|c\tan(x)+d|\right)}{cd^3}-\frac{3\,b^2c^2\tan(x)^2-6\,abcd\tan(x)}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cot(x))^2*csc(x)^2/(c+d*cot(x)),x, algorithm="giac")
```

[Out] (b^2*c^2 - 2*a*b*c*d + a^2*d^2)*log(abs(tan(x)))/d^3 - (b^2*c^3 - 2*a*b*c^2
*d + a^2*c*d^2)*log(abs(c*tan(x) + d))/(c*d^3) - 1/2*(3*b^2*c^2*tan(x)^2 -
6*a*b*c*d*tan(x)^2 + 3*a^2*d^2*tan(x)^2 - 2*b^2*c*d*tan(x) + 4*a*b*d^2*tan(
x) + b^2*d^2)/(d^3*tan(x)^2)

maple [B] time = 0.13, size = 119, normalized size = 2.25

$$-\frac{b^2}{2d\tan(x)^2}+\frac{\ln(\tan(x))\,a^2}{d}-\frac{2\ln(\tan(x))\,abc}{d^2}+\frac{\ln(\tan(x))\,b^2c^2}{d^3}-\frac{2ba}{d\tan(x)}+\frac{b^2c}{d^2\tan(x)}-\frac{\ln(c\tan(x)+d)\,a^2}{d}+\frac{2\ln}{}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b*cot(x))^2*csc(x)^2/(c+d*cot(x)),x)

[Out] -1/2*b^2/d/tan(x)^2+1/d*ln(tan(x))*a^2-2/d^2*ln(tan(x))*a*b*c+1/d^3*ln(tan(
x))*b^2*c^2-2*b/d/tan(x)*a+b^2/d^2/tan(x)*c-1/d*ln(c*tan(x)+d)*a^2+2/d^2*ln
(c*tan(x)+d)*a*b*c-1/d^3*ln(c*tan(x)+d)*b^2*c^2

maxima [A] time = 0.32, size = 92, normalized size = 1.74

$$-\frac{\left(b^2c^2-2\,abcd+a^2d^2\right)\log\left(c\tan(x)+d\right)}{d^3}+\frac{\left(b^2c^2-2\,abcd+a^2d^2\right)\log\left(\tan(x)\right)}{d^3}-\frac{b^2d-2\left(b^2c-2\,abd\right)\tan(x)}{2\,d^2\tan(x)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*cot(x))^2*csc(x)^2/(c+d*cot(x)),x, algorithm="maxima")

[Out] -(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*log(c*tan(x) + d)/d^3 + (b^2*c^2 - 2*a*b*c
*d + a^2*d^2)*log(tan(x))/d^3 - 1/2*(b^2*d - 2*(b^2*c - 2*a*b*d)*tan(x))/(d
^2*tan(x)^2)

mupad [B] time = 3.07, size = 92, normalized size = 1.74

$$-\frac{\frac{b^2}{2\,d}+\frac{b\tan(x)\,(2\,a\,d-b\,c)}{d^2}}{\tan(x)^2}-\frac{2\,\mathrm{atanh}\left(\frac{(d+2\,c\tan(x))\,(a\,d-b\,c)^2}{d\left(a^2\,d^2-2\,a\,b\,c\,d+b^2\,c^2\right)}\right)\,(a\,d-b\,c)^2}{d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b*cot(x))^2/(sin(x)^2*(c + d*cot(x))),x)

[Out] - (b^2/(2*d) + (b*tan(x)*(2*a*d - b*c))/d^2)/tan(x)^2 - (2*atanh(((d + 2*c*
tan(x))*(a*d - b*c)^2)/(d*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d)))*(a*d - b*c)^2)/
d^3

sympy [A] time = 55.64, size = 58, normalized size = 1.09

$$-\frac{b^2\cot^2(x)}{2d}-\frac{(ad-bc)^2\left(\begin{cases}\frac{\cot(x)}{c} & \text{for } d=0\\ \frac{\log(c+d\cot(x))}{d} & \text{otherwise}\end{cases}\right)}{d^2}-\frac{\left(2abd-b^2c\right)\cot(x)}{d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cot(x))**2*csc(x)**2/(c+d*cot(x)),x)
```

```
[Out] -b**2*cot(x)**2/(2*d) - (a*d - b*c)**2*Piecewise((cot(x)/c, Eq(d, 0)), (log
(c + d*cot(x))/d, True))/d**2 - (2*a*b*d - b**2*c)*cot(x)/d**2
```

3.722 $\int \frac{(a+b\cot(x))^3\csc^2(x)}{c+d\cot(x)}\,dx$

Optimal. Leaf size=78

$$\frac{(bc-ad)^3\log(c+d\cot(x))}{d^4}-\frac{b\cot(x)(bc-ad)^2}{d^3}+\frac{(bc-ad)(a+b\cot(x))^2}{2d^2}-\frac{(a+b\cot(x))^3}{3d}$$

```
[Out] -b*(-a*d+b*c)^2*cot(x)/d^3+1/2*(-a*d+b*c)*(a+b*cot(x))^2/d^2-1/3*(a+b*cot(x
))^3/d+(-a*d+b*c)^3*ln(c+d*cot(x))/d^4
```

Rubi [A] time = 0.14, antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {4344, 43}

$$-\frac{b\cot(x)(bc-ad)^2}{d^3}+\frac{(bc-ad)(a+b\cot(x))^2}{2d^2}+\frac{(bc-ad)^3\log(c+d\cot(x))}{d^4}-\frac{(a+b\cot(x))^3}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*Cot[x])^3*Csc[x]^2)/(c + d*Cot[x]),x]
```

```
[Out] -((b*(b*c - a*d)^2*Cot[x])/d^3) + ((b*c - a*d)*(a + b*Cot[x])^2)/(2*d^2) -
(a + b*Cot[x])^3/(3*d) + ((b*c - a*d)^3*Log[c + d*Cot[x]])/d^4
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 4344

```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))]^2, x_Symbol] :> With[{d = FreeFac
tors[Cot[c*(a + b*x)], x]}, -Dist[d/(b*c), Subst[Int[SubstFor[1, Cot[c*(a +
 b*x)]/d, u, x], x], x, Cot[c*(a + b*x)]/d], x] /; FunctionOfQ[Cot[c*(a + b
*x)]/d, u, x, True]] /; FreeQ[{a, b, c}, x] && NonsumQ[u] && (EqQ[F, Csc] |
| EqQ[F, csc])
```

Rubi steps

$$\int \frac{(a+b\cot(x))^3\csc^2(x)}{c+d\cot(x)}\,dx = -\text{Subst}\left(\int \frac{(a+bx)^3}{c+dx}\,dx, x, \cot(x)\right)$$
$$= -\text{Subst}\left(\int \left(\frac{b(bc-ad)^2}{d^3} - \frac{b(bc-ad)(a+bx)}{d^2} + \frac{b(a+bx)^2}{d} + \frac{(-bc+ad)^3}{d^3(c+dx)}\right)dx, x,$$
$$= -\frac{b(bc-ad)^2\cot(x)}{d^3} + \frac{(bc-ad)(a+b\cot(x))^2}{2d^2} - \frac{(a+b\cot(x))^3}{3d} + \frac{(bc-ad)^3\log(}{d^4}$$

Mathematica [A] time = 1.30, size = 135, normalized size = 1.73

$$\frac{(a+b\cot(x))^3(c\sin(x)+d\cos(x))\left(bd\left(\sin(2x)\left(-9a^2d^2+9abcd+b^2\left(d^2-3c^2\right)\right)+3bd(bc-3ad)\right)-6\sin^2(x)(bc}{6d^4(c+d\cot(x))(a\sin(x)+b\cos(x))^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*Cot[x])^3*Csc[x]^2)/(c + d*Cot[x]),x]
```

```
[Out] ((a + b*Cot[x])^3*(d*Cos[x] + c*Sin[x])*(-2*b^3*d^3*Cot[x] - 6*(b*c - a*d)^
3*(Log[Sin[x]] - Log[d*Cos[x] + c*Sin[x]])*Sin[x]^2 + b*d*(3*b*d*(b*c - 3*a
*d) + (9*a*b*c*d - 9*a^2*d^2 + b^2*(-3*c^2 + d^2))*Sin[2*x])))/(6*d^4*(c +
d*Cot[x])*(b*Cos[x] + a*Sin[x])^3)
```

fricas [B] time = 1.62, size = 320, normalized size = 4.10

$$-\frac{2\left(3\,b^3c^2d-9\,ab^2cd^2+\left(9\,a^2b-b^3\right)d^3\right)\cos(x)^3+3\left(b^3c^3-3\,ab^2c^2d+3\,a^2bcd^2-a^3d^3-\left(b^3c^3-3\,ab^2c^2d+3\,a^2b\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cot(x))^3*csc(x)^2/(c+d*cot(x)),x, algorithm="fricas")
```

```
[Out] -1/6*(2*(3*b^3*c^2*d - 9*a*b^2*c*d^2 + (9*a^2*b - b^3)*d^3)*cos(x)^3 + 3*(b
^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3 - (b^3*c^3 - 3*a*b^2*c^2*d
 + 3*a^2*b*c*d^2 - a^3*d^3)*cos(x)^2)*log(2*c*d*cos(x)*sin(x) - (c^2 - d^2)
*cos(x)^2 + c^2)*sin(x) - 3*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*
d^3 - (b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*cos(x)^2)*log(-1/
4*cos(x)^2 + 1/4)*sin(x) - 6*(b^3*c^2*d - 3*a*b^2*c*d^2 + 3*a^2*b*d^3)*cos(
x) + 3*(b^3*c*d^2 - 3*a*b^2*d^3)*sin(x))/((d^4*cos(x)^2 - d^4)*sin(x))
```

giac [B] time = 0.17, size = 232, normalized size = 2.97

$$-\frac{\left(b^3c^3-3\,ab^2c^2d+3\,a^2bcd^2-a^3d^3\right)\log\left(|\tan(x)|\right)}{d^4}+\frac{\left(b^3c^4-3\,ab^2c^3d+3\,a^2bc^2d^2-a^3cd^3\right)\log\left(|c\tan(x)+d|\right)}{cd^4}+\frac{11}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cot(x))^3*csc(x)^2/(c+d*cot(x)),x, algorithm="giac")
```

```
[Out] -(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*log(abs(tan(x)))/d^4 +
 (b^3*c^4 - 3*a*b^2*c^3*d + 3*a^2*b*c^2*d^2 - a^3*c*d^3)*log(abs(c*tan(x) +
 d))/(c*d^4) + 1/6*(11*b^3*c^3*tan(x)^3 - 33*a*b^2*c^2*d*tan(x)^3 + 33*a^2*
b*c*d^2*tan(x)^3 - 11*a^3*d^3*tan(x)^3 - 6*b^3*c^2*d*tan(x)^2 + 18*a*b^2*c*
d^2*tan(x)^2 - 18*a^2*b*d^3*tan(x)^2 + 3*b^3*c*d^2*tan(x) - 9*a*b^2*d^3*tan
(x) - 2*b^3*d^3)/(d^4*tan(x)^3)
```

maple [B] time = 0.16, size = 202, normalized size = 2.59

$$-\frac{b^3}{3d\tan(x)^3}+\frac{\ln(\tan(x))\,a^3}{d}-\frac{3\ln(\tan(x))\,a^2bc}{d^2}+\frac{3\ln(\tan(x))\,a\,b^2c^2}{d^3}-\frac{\ln(\tan(x))\,b^3c^3}{d^4}-\frac{3b\,a^2}{d\tan(x)}+\frac{3b^2ac}{d^2\tan(x)}-\frac{b}{d^3 \mathrm{t}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cot(x))^3*csc(x)^2/(c+d*cot(x)),x)
```

```
[Out] -1/3*b^3/d/tan(x)^3+1/d*ln(tan(x))*a^3-3/d^2*ln(tan(x))*a^2*b*c+3/d^3*ln(ta
n(x))*a*b^2*c^2-1/d^4*ln(tan(x))*b^3*c^3-3*b/d/tan(x)*a^2+3*b^2/d^2/tan(x)*
a*c-b^3/d^3/tan(x)*c^2-3/2*b^2/d/tan(x)^2*a+1/2*b^3/d^2/tan(x)^2*c-1/d*ln(c
*tan(x)+d)*a^3+3/d^2*ln(c*tan(x)+d)*a^2*b*c-3/d^3*ln(c*tan(x)+d)*a*b^2*c^2+
1/d^4*ln(c*tan(x)+d)*b^3*c^3
```

maxima [B] time = 0.32, size = 161, normalized size = 2.06

$$\frac{\left(b^3c^3-3\,ab^2c^2d+3\,a^2bcd^2-a^3d^3\right)\log\left(c\tan(x)+d\right)}{d^4}-\frac{\left(b^3c^3-3\,ab^2c^2d+3\,a^2bcd^2-a^3d^3\right)\log\left(\tan(x)\right)}{d^4}-\frac{2\,b^3d^2}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cot(x))^3*csc(x)^2/(c+d*cot(x)),x, algorithm="maxima")
```

```
[Out] (b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*log(c*tan(x) + d)/d^4 -
 (b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*log(tan(x))/d^4 - 1/6*
(2*b^3*d^2 + 6*(b^3*c^2 - 3*a*b^2*c*d + 3*a^2*b*d^2)*tan(x)^2 - 3*(b^3*c*d
- 3*a*b^2*d^2)*tan(x))/(d^3*tan(x)^3)
```

mupad [B] time = 3.08, size = 141, normalized size = 1.81

$$-\frac{\frac{b^3}{3\,d}+\frac{b^2\tan(x)\,(3\,a\,d-b\,c)}{2\,d^2}+\frac{b\tan(x)^2\left(3\,a^2\,d^2-3\,a\,b\,c\,d+b^2\,c^2\right)}{d^3}}{\tan(x)^3}-\frac{2\,\mathrm{atanh}\left(\frac{(d+2\,c\tan(x))\,(a\,d-b\,c)^3}{d\left(a^3\,d^3-3\,a^2\,b\,c\,d^2+3\,a\,b^2\,c^2\,d-b^3\,c^3\right)}\right)\,(a\,d-b\,c)^3}{d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*cot(x))^3/(sin(x)^2*(c + d*cot(x))),x)
```

```
[Out] - (b^3/(3*d) + (b^2*tan(x)*(3*a*d - b*c))/(2*d^2) + (b*tan(x)^2*(3*a^2*d^2
+ b^2*c^2 - 3*a*b*c*d))/d^3)/tan(x)^3 - (2*atanh(((d + 2*c*tan(x))*(a*d - b
*c)^3)/(d*(a^3*d^3 - b^3*c^3 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2)))*(a*d - b*c)
^3)/d^4
```

sympy [A] time = 69.99, size = 97, normalized size = 1.24

$$-\frac{b^3\cot^3(x)}{3d}-\frac{\left(3ab^2d-b^3c\right)\cot^2(x)}{2d^2}-\frac{(ad-bc)^3\left(\begin{cases}\frac{\cot(x)}{c} & \text{for } d=0\\ \frac{\log(c+d\cot(x))}{d} & \text{otherwise}\end{cases}\right)}{d^3}-\frac{\left(3a^2bd^2-3ab^2cd+b^3c^2\right)\cot(x)}{d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cot(x))**3*csc(x)**2/(c+d*cot(x)),x)
```

```
[Out] -b**3*cot(x)**3/(3*d) - (3*a*b**2*d - b**3*c)*cot(x)**2/(2*d**2) - (a*d - b
*c)**3*Piecewise((cot(x)/c, Eq(d, 0)), (log(c + d*cot(x))/d, True))/d**3 -
(3*a**2*b*d**2 - 3*a*b**2*c*d + b**3*c**2)*cot(x)/d**3
```

3.723 $\int e^{-\cot(x)} \csc^2(x) \, dx$

Optimal. Leaf size=6

$$e^{-\cot(x)}$$

```
[Out] exp(-cot(x))
```

Rubi [A] time = 0.01, antiderivative size = 6, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {4344, 2194}

$$e^{-\cot(x)}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[x]^2/E^Cot[x],x]
```

```
[Out] E^(-Cot[x])
```

Rule 2194

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rule 4344

```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))]^2, x_Symbol] :> With[{d = FreeFac
tors[Cot[c*(a + b*x)], x]}, -Dist[d/(b*c), Subst[Int[SubstFor[1, Cot[c*(a +
 b*x)]/d, u, x], x], x, Cot[c*(a + b*x)]/d], x] /; FunctionOfQ[Cot[c*(a + b
*x)]/d, u, x, True]] /; FreeQ[{a, b, c}, x] && NonsumQ[u] && (EqQ[F, Csc] |
| EqQ[F, csc])
```

Rubi steps

$$\begin{aligned} \int e^{-\cot(x)} \csc^2(x) \, dx &= -\text{Subst}\left(\int e^{-x} \, dx, x, \cot(x)\right) \\ &= e^{-\cot(x)} \end{aligned}$$

Mathematica [A] time = 0.07, size = 6, normalized size = 1.00

$$e^{-\cot(x)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[x]^2/E^Cot[x],x]
[Out] E^(-Cot[x])
```

fricas [A] time = 0.64, size = 9, normalized size = 1.50

$$e^{\left(-\frac{\cos(x)}{\sin(x)}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)^2/exp(cot(x)),x, algorithm="fricas")
[Out] e^(-cos(x)/sin(x))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \csc(x)^2 e^{(-\cot(x))}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)^2/exp(cot(x)),x, algorithm="giac")
[Out] integrate(csc(x)^2*e^(-cot(x)), x)
```

maple [A] time = 0.05, size = 6, normalized size = 1.00

$$\mathrm{e}^{-\cot(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(x)^2/exp(cot(x)),x)
[Out] 1/exp(cot(x))
```

maxima [A] time = 0.32, size = 5, normalized size = 0.83

$$e^{(-\cot(x))}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)^2/exp(cot(x)),x, algorithm="maxima")
[Out] e^(-cot(x))
```

mupad [B] time = 2.94, size = 7, normalized size = 1.17

$$\mathrm{e}^{-\frac{1}{\tan(x)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(-cot(x))/sin(x)^2,x)
```

```
[Out] exp(-1/tan(x))
```

sympy [A] time = 18.87, size = 5, normalized size = 0.83

$$e^{-\cot(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)**2/exp(cot(x)),x)
```

```
[Out] exp(-cot(x))
```

3.724 $\int \frac{\sec(x)\tan(x)}{a+b\sec(x)}\,dx$

Optimal. Leaf size=11

$$\frac{\log(a+b\sec(x))}{b}$$

[Out] ln(a+b*sec(x))/b

Rubi [A] time = 0.05, antiderivative size = 20, normalized size of antiderivative = 1.82, number of steps used = 4, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {4339, 36, 29, 31}

$$\frac{\log(a\cos(x)+b)}{b}-\frac{\log(\cos(x))}{b}$$

Antiderivative was successfully verified.

[In] Int[(Sec[x]*Tan[x])/(a + b*Sec[x]),x]

[Out] -(Log[Cos[x]]/b) + Log[b + a*Cos[x]]/b

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 4339

```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{d = FreeFacto
rs[Cos[c*(a + b*x)], x]}, -Dist[(b*c)^(-1), Subst[Int[SubstFor[1/x, Cos[c*(
a + b*x)]/d, u, x], x], x, Cos[c*(a + b*x)]/d], x] /; FunctionOfQ[Cos[c*(a
+ b*x)]/d, u, x, True]] /; FreeQ[{a, b, c}, x] && (EqQ[F, Tan] || EqQ[F, ta
n])
```

Rubi steps

$$\int \frac{\sec(x)\tan(x)}{a+b\sec(x)}\,dx = -\text{Subst}\left(\int \frac{1}{x(b+ax)}\,dx, x, \cos(x)\right)$$
$$= -\frac{\text{Subst}\left(\int \frac{1}{x}\,dx, x, \cos(x)\right)}{b} + \frac{a\,\text{Subst}\left(\int \frac{1}{b+ax}\,dx, x, \cos(x)\right)}{b}$$
$$= -\frac{\log(\cos(x))}{b} + \frac{\log(b+a\cos(x))}{b}$$

Mathematica [A] time = 0.02, size = 20, normalized size = 1.82

$$\frac{\log(a\cos(x)+b)}{b} - \frac{\log(\cos(x))}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[x]*Tan[x])/(a + b*Sec[x]),x]
[Out] -(Log[Cos[x]]/b) + Log[b + a*Cos[x]]/b
```

fricas [A] time = 0.67, size = 19, normalized size = 1.73

$$\frac{\log\left(a\cos(x)+b\right) - \log\left(-\cos(x)\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)*tan(x)/(a+b*sec(x)),x, algorithm="fricas")
[Out] (log(a*cos(x) + b) - log(-cos(x)))/b
```

giac [A] time = 0.14, size = 22, normalized size = 2.00

$$\frac{\log\left(|a\cos(x)+b|\right)}{b} - \frac{\log\left(|\cos(x)|\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)*tan(x)/(a+b*sec(x)),x, algorithm="giac")
[Out] log(abs(a*cos(x) + b))/b - log(abs(cos(x)))/b
```

maple [A] time = 0.04, size = 12, normalized size = 1.09

$$\frac{\ln\left(a+b\sec(x)\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(x)*tan(x)/(a+b*sec(x)),x)
```

```
[Out] ln(a+b*sec(x))/b
```

maxima [A] time = 0.31, size = 11, normalized size = 1.00

$$\frac{\log\left(b\sec(x)+a\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)*tan(x)/(a+b*sec(x)),x, algorithm="maxima")
```

```
[Out] log(b*sec(x) + a)/b
```

mupad [B] time = 3.25, size = 48, normalized size = 4.36

$$\frac{\operatorname{atan}\left(\frac{b\sin\left(\frac{x}{2}\right)^2}{a\cos\left(\frac{x}{2}\right)^2 1\mathrm{i}+b\cos\left(\frac{x}{2}\right)^2 1\mathrm{i}-a\sin\left(\frac{x}{2}\right)^2 1\mathrm{i}}\right)2\mathrm{i}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(x)/(cos(x)*(a + b/cos(x))),x)
```

```
[Out] (atan((b*sin(x/2)^2)/(a*cos(x/2)^2*1i + b*cos(x/2)^2*1i - a*sin(x/2)^2*1i))
*2i)/b
```

sympy [A] time = 0.46, size = 14, normalized size = 1.27

$$\begin{cases}\frac{\log\left(\frac{a}{b}+\sec(x)\right)}{b} & \text{for}\ b\neq 0\\ \frac{\sec(x)}{a} & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)*tan(x)/(a+b*sec(x)),x)
```

```
[Out] Piecewise((log(a/b + sec(x))/b, Ne(b, 0)), (sec(x)/a, True))
```

3.725 $\int \frac{\sec(x)\tan(x)}{1+\sec^2(x)}\,dx$

Optimal. Leaf size=5

$$-\tan^{-1}(\cos(x))$$

[Out] -arctan(cos(x))

Rubi [A] time = 0.03, antiderivative size = 5, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {4339, 203}

$$-\tan^{-1}(\cos(x))$$

Antiderivative was successfully verified.

[In] Int[(Sec[x]*Tan[x])/(1 + Sec[x]^2),x]

[Out] -ArcTan[Cos[x]]

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 4339

```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{d = FreeFacto
rs[Cos[c*(a + b*x)], x]}, -Dist[(b*c)^(-1), Subst[Int[SubstFor[1/x, Cos[c*(
a + b*x)]/d, u, x], x], x, Cos[c*(a + b*x)]/d], x] /; FunctionOfQ[Cos[c*(a
+ b*x)]/d, u, x, True]] /; FreeQ[{a, b, c}, x] && (EqQ[F, Tan] || EqQ[F, ta
n])
```

Rubi steps

$$\begin{aligned}\int \frac{\sec(x)\tan(x)}{1+\sec^2(x)}\,dx &= -\operatorname{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \cos(x)\right)\\ &= -\tan^{-1}(\cos(x))\end{aligned}$$

Mathematica [A] time = 0.02, size = 5, normalized size = 1.00

$$-\tan^{-1}(\cos(x))$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[x]*Tan[x])/(1 + Sec[x]^2),x]
[Out] -ArcTan[Cos[x]]
```

fricas [A] time = 4.94, size = 5, normalized size = 1.00

$$-\arctan\left(\cos(x)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)*tan(x)/(1+sec(x)^2),x, algorithm="fricas")
[Out] -arctan(cos(x))
```

giac [A] time = 0.14, size = 5, normalized size = 1.00

$$-\arctan\left(\cos(x)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)*tan(x)/(1+sec(x)^2),x, algorithm="giac")
[Out] -arctan(cos(x))
```

maple [A] time = 0.07, size = 4, normalized size = 0.80

$$\arctan\left(\sec(x)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(x)*tan(x)/(1+sec(x)^2),x)
[Out] arctan(sec(x))
```

maxima [A] time = 0.42, size = 5, normalized size = 1.00

$$-\arctan\left(\cos(x)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)*tan(x)/(1+sec(x)^2),x, algorithm="maxima")
[Out] -arctan(cos(x))
```

mupad [B] time = 2.95, size = 7, normalized size = 1.40

$$\mathrm{atan}\left(\tan\left(\frac{x}{2}\right)^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(x)/(cos(x)*(1/cos(x)^2 + 1)),x)
```

```
[Out] atan(tan(x/2)^2)
```

sympy [A] time = 0.21, size = 3, normalized size = 0.60

$$\operatorname{atan}\left(\sec\left(x\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)*tan(x)/(1+sec(x)**2),x)
```

```
[Out] atan(sec(x))
```

3.726 $\int \frac{\sec(x)\tan(x)}{9+4\sec^2(x)}\,dx$

Optimal. Leaf size=11

$$-\frac{1}{6}\tan^{-1}\left(\frac{3\cos(x)}{2}\right)$$

```
[Out] -1/6*arctan(3/2*cos(x))
```

Rubi [A] time = 0.03, antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {4339, 203}

$$-\frac{1}{6}\tan^{-1}\left(\frac{3\cos(x)}{2}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[x]*Tan[x])/(9 + 4*Sec[x]^2),x]
```

```
[Out] -ArcTan[(3*Cos[x])/2]/6
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 4339

```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{d = FreeFacto
rs[Cos[c*(a + b*x)], x]}, -Dist[(b*c)^(-1), Subst[Int[SubstFor[1/x, Cos[c*(
a + b*x)]/d, u, x], x], x, Cos[c*(a + b*x)]/d], x] /; FunctionOfQ[Cos[c*(a
+ b*x)]/d, u, x, True]] /; FreeQ[{a, b, c}, x] && (EqQ[F, Tan] || EqQ[F, ta
n])
```

Rubi steps

$$\begin{aligned}\int \frac{\sec(x)\tan(x)}{9+4\sec^2(x)}\,dx &= -\text{Subst}\left(\int \frac{1}{4+9x^2}\,dx, x, \cos(x)\right)\\ &= -\frac{1}{6}\tan^{-1}\left(\frac{3\cos(x)}{2}\right)\end{aligned}$$

Mathematica [A] time = 0.03, size = 11, normalized size = 1.00

$$-\frac{1}{6}\tan^{-1}\left(\frac{3\cos(x)}{2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[x]*Tan[x])/(9 + 4*Sec[x]^2),x]
[Out] -1/6*ArcTan[(3*Cos[x])/2]
```

fricas [A] time = 0.97, size = 7, normalized size = 0.64

$$-\frac{1}{6}\arctan\left(\frac{3}{2}\cos(x)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)*tan(x)/(9+4*sec(x)^2),x, algorithm="fricas")
[Out] -1/6*arctan(3/2*cos(x))
```

giac [A] time = 0.13, size = 7, normalized size = 0.64

$$-\frac{1}{6}\arctan\left(\frac{3}{2}\cos(x)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)*tan(x)/(9+4*sec(x)^2),x, algorithm="giac")
[Out] -1/6*arctan(3/2*cos(x))
```

maple [A] time = 0.06, size = 8, normalized size = 0.73

$$\frac{\arctan\left(\frac{2\sec(x)}{3}\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(x)*tan(x)/(9+4*sec(x)^2),x)
[Out] 1/6*arctan(2/3*sec(x))
```

maxima [A] time = 0.44, size = 7, normalized size = 0.64

$$-\frac{1}{6}\arctan\left(\frac{3}{2}\cos(x)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)*tan(x)/(9+4*sec(x)^2),x, algorithm="maxima")
```

```
[Out] -1/6*arctan(3/2*cos(x))
```

mupad [B] time = 3.04, size = 13, normalized size = 1.18

$$\frac{\operatorname{atan}\left(\frac{13\tan\left(\frac{x}{2}\right)^2}{12}-\frac{5}{12}\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(x)/(cos(x)*(4/cos(x)^2 + 9)),x)
```

```
[Out] atan((13*tan(x/2)^2)/12 - 5/12)/6
```

sympy [A] time = 0.23, size = 8, normalized size = 0.73

$$\frac{\operatorname{atan}\left(\frac{2\sec\left(x\right)}{3}\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)*tan(x)/(9+4*sec(x)**2),x)
```

```
[Out] atan(2*sec(x)/3)/6
```

3.727 $\int \frac{\sec(x)\tan(x)}{\sec(x)+\sec^2(x)}\,dx$

Optimal. Leaf size=7

$$-\log(\cos(x)+1)$$

```
[Out] -ln(1+cos(x))
```

Rubi [A] time = 0.03, antiderivative size = 7, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {4339, 31}

$$-\log(\cos(x)+1)$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[x]*Tan[x])/(Sec[x] + Sec[x]^2),x]
```

```
[Out] -Log[1 + Cos[x]]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 4339

```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{d = FreeFacto
rs[Cos[c*(a + b*x)], x]}, -Dist[(b*c)^(-1), Subst[Int[SubstFor[1/x, Cos[c*(
a + b*x)]/d, u, x], x], x, Cos[c*(a + b*x)]/d], x] /; FunctionOfQ[Cos[c*(a
+ b*x)]/d, u, x, True]] /; FreeQ[{a, b, c}, x] && (EqQ[F, Tan] || EqQ[F, ta
n])
```

Rubi steps

$$\begin{aligned}\int \frac{\sec(x)\tan(x)}{\sec(x)+\sec^2(x)}\,dx &= -\text{Subst}\left(\int \frac{1}{1+x}\,dx, x, \cos(x)\right)\\ &= -\log(1+\cos(x))\end{aligned}$$

Mathematica [A] time = 0.00, size = 9, normalized size = 1.29

$$-2\log\left(\cos\left(\frac{x}{2}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[x]*Tan[x])/(Sec[x] + Sec[x]^2),x]
[Out] -2*Log[Cos[x/2]]
```

fricas [A] time = 0.84, size = 9, normalized size = 1.29

$$-\log\left(\frac{1}{2}\cos(x)+\frac{1}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)*tan(x)/(sec(x)+sec(x)^2),x, algorithm="fricas")
[Out] -log(1/2*cos(x) + 1/2)
```

giac [A] time = 0.15, size = 7, normalized size = 1.00

$$-\log\left(\cos(x)+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)*tan(x)/(sec(x)+sec(x)^2),x, algorithm="giac")
[Out] -log(cos(x) + 1)
```

maple [A] time = 0.08, size = 12, normalized size = 1.71

$$\ln\left(\sec(x)\right)-\ln\left(1+\sec(x)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(x)*tan(x)/(sec(x)+sec(x)^2),x)
[Out] ln(sec(x))-ln(1+sec(x))
```

maxima [A] time = 0.31, size = 7, normalized size = 1.00

$$-\log\left(\cos(x)+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)*tan(x)/(sec(x)+sec(x)^2),x, algorithm="maxima")
[Out] -log(cos(x) + 1)
```

mupad [B] time = 3.07, size = 9, normalized size = 1.29

$$\ln\left(\tan\left(\frac{x}{2}\right)^2+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(x)/(cos(x)*(1/cos(x) + 1/cos(x)^2)),x)
```

```
[Out] log(tan(x/2)^2 + 1)
```

sympy [B] time = 0.22, size = 15, normalized size = 2.14

$$\frac{\log\left(\tan^{2}(x)+1\right)}{2}-\log(\sec(x)+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)*tan(x)/(sec(x)+sec(x)**2),x)
```

```
[Out] log(tan(x)**2 + 1)/2 - log(sec(x) + 1)
```

3.728 $\int \frac{\sec(x)\tan(x)}{\sqrt{4+\sec^2(x)}}\,dx$

Optimal. Leaf size=5

$$\operatorname{csch}^{-1}(2\cos(x))$$

```
[Out] arccsch(2*cos(x))
```

Rubi [A] time = 0.04, antiderivative size = 5, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {4339, 335, 215}

$$\operatorname{csch}^{-1}(2\cos(x))$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[x]*Tan[x])/Sqrt[4 + Sec[x]^2],x]
```

```
[Out] ArcCsch[2*Cos[x]]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 335

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Rule 4339

```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{d = FreeFacto
rs[Cos[c*(a + b*x)], x]}, -Dist[(b*c)^(-1), Subst[Int[SubstFor[1/x, Cos[c*(
a + b*x)]/d, u, x], x], x, Cos[c*(a + b*x)]/d], x] /; FunctionOfQ[Cos[c*(a
+ b*x)]/d, u, x, True]] /; FreeQ[{a, b, c}, x] && (EqQ[F, Tan] || EqQ[F, ta
n])
```

Rubi steps

$$\int \frac{\sec(x)\tan(x)}{\sqrt{4+\sec^2(x)}}\,dx = -\operatorname{Subst}\left(\int \frac{1}{\sqrt{4+\frac{1}{x^2}}\,x^2}\,dx, x, \cos(x)\right)$$
$$= \operatorname{Subst}\left(\int \frac{1}{\sqrt{4+x^2}}\,dx, x, \sec(x)\right)$$
$$= \sinh^{-1}\left(\frac{\sec(x)}{2}\right)$$

Mathematica [B] time = 0.03, size = 38, normalized size = 7.60

$$\frac{\sqrt{2\cos(2x)+3}\,\sec(x)\tanh^{-1}\left(\sqrt{4\cos^2(x)+1}\right)}{\sqrt{\sec^2(x)+4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[x]*Tan[x])/Sqrt[4 + Sec[x]^2],x]
[Out] (ArcTanh[Sqrt[1 + 4*Cos[x]^2]]*Sqrt[3 + 2*Cos[2*x]]*Sec[x])/Sqrt[4 + Sec[x]
^2]
```

fricas [B] time = 0.95, size = 27, normalized size = 5.40

$$\log\left(-\frac{\sqrt{\frac{4\,\cos(x)^2+1}{\cos(x)^2}}\,\cos(x)+1}{\cos(x)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)*tan(x)/(4+sec(x)^2)^(1/2),x, algorithm="fricas")
[Out] log(-(sqrt((4*cos(x)^2 + 1)/cos(x)^2)*cos(x) + 1)/cos(x))
```

giac [B] time = 0.15, size = 36, normalized size = 7.20

$$\frac{\log\left(\sqrt{4\,\cos(x)^2+1}+1\right)-\log\left(\sqrt{4\,\cos(x)^2+1}-1\right)}{2\,\operatorname{sgn}\left(\cos(x)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)*tan(x)/(4+sec(x)^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*(log(sqrt(4*cos(x)^2 + 1) + 1) - log(sqrt(4*cos(x)^2 + 1) - 1))/sgn(cos
(x))
```

maple [A] time = 0.10, size = 6, normalized size = 1.20

$$\operatorname{arcsinh}\left(\frac{\sec(x)}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(x)*tan(x)/(4+sec(x)^2)^(1/2),x)
```

```
[Out] arcsinh(1/2*sec(x))
```

maxima [B] time = 0.31, size = 33, normalized size = 6.60

$$\frac{1}{2}\log\left(\sqrt{\frac{1}{\cos(x)^2}+4}\cos(x)+1\right)-\frac{1}{2}\log\left(\sqrt{\frac{1}{\cos(x)^2}+4}\cos(x)-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)*tan(x)/(4+sec(x)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*log(sqrt(1/cos(x)^2 + 4)*cos(x) + 1) - 1/2*log(sqrt(1/cos(x)^2 + 4)*cos
(x) - 1)
```

mupad [B] time = 3.10, size = 7, normalized size = 1.40

$$\operatorname{asinh}\left(\frac{1}{2\cos(x)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(x)/(cos(x)*(1/cos(x)^2 + 4)^(1/2)),x)
```

```
[Out] asinh(1/(2*cos(x)))
```

sympy [A] time = 0.73, size = 5, normalized size = 1.00

$$\operatorname{asinh}\left(\frac{\sec(x)}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)*tan(x)/(4+sec(x)**2)**(1/2),x)
```

```
[Out] asinh(sec(x)/2)
```

3.729 $\int \frac{\sec(x)\tan(x)}{\sqrt{1+\cos^2(x)}}\,dx$

Optimal. Leaf size=13

$$\sqrt{\cos^2(x)+1}\,\sec(x)$$

```
[Out] sec(x)*(1+cos(x)^2)^(1/2)
```

Rubi [A] time = 0.08, antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {264}

$$\sqrt{\cos^2(x)+1}\,\sec(x)$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[x]*Tan[x])/Sqrt[1 + Cos[x]^2],x]
```

```
[Out] Sqrt[1 + Cos[x]^2]*Sec[x]
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{\sec(x)\tan(x)}{\sqrt{1+\cos^2(x)}}\,dx &= -\text{Subst}\left(\int \frac{1}{x^2\sqrt{1+x^2}}\,dx, x, \cos(x)\right)\\ &= \sqrt{1+\cos^2(x)}\,\sec(x)\end{aligned}$$

Mathematica [A] time = 0.02, size = 13, normalized size = 1.00

$$\sqrt{\cos^2(x)+1}\,\sec(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[x]*Tan[x])/Sqrt[1 + Cos[x]^2],x]
```

```
[Out] Sqrt[1 + Cos[x]^2]*Sec[x]
```

fricas [A] time = 0.67, size = 16, normalized size = 1.23

$$\frac{\sqrt{\cos(x)^2+1}+\cos(x)}{\cos(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)*tan(x)/(1+cos(x)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] (sqrt(cos(x)^2 + 1) + cos(x))/cos(x)
```

giac [A] time = 0.15, size = 21, normalized size = 1.62

$$-\frac{2}{\left(\sqrt{\cos(x)^2+1}-\cos(x)\right)^2-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)*tan(x)/(1+cos(x)^2)^(1/2),x, algorithm="giac")
```

```
[Out] -2/((sqrt(cos(x)^2 + 1) - cos(x))^2 - 1)
```

maple [B] time = 0.11, size = 25, normalized size = 1.92

$$\frac{1+\sec^2(x)}{\sqrt{\frac{1+\sec^2(x)}{\sec(x)^2}}\,\sec(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(x)*tan(x)/(1+cos(x)^2)^(1/2),x)
```

```
[Out] 1/((1+sec(x)^2)/sec(x)^2)^(1/2)/sec(x)*(1+sec(x)^2)
```

maxima [A] time = 0.42, size = 13, normalized size = 1.00

$$\frac{\sqrt{\cos(x)^2+1}}{\cos(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)*tan(x)/(1+cos(x)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] sqrt(cos(x)^2 + 1)/cos(x)
```

mupad [B] time = 0.11, size = 13, normalized size = 1.00

$$\frac{\sqrt{\cos(x)^2+1}}{\cos(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(x)/(cos(x)*(cos(x)^2 + 1)^(1/2)),x)
```

```
[Out] (cos(x)^2 + 1)^(1/2)/cos(x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan(x)\sec(x)}{\sqrt{\cos^2(x)+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)*tan(x)/(1+cos(x)**2)**(1/2),x)
```

```
[Out] Integral(tan(x)*sec(x)/sqrt(cos(x)**2 + 1), x)
```

3.730 $\int e^{\sec(x)} \sec(x) \tan(x) \, dx$

Optimal. Leaf size=4

$$e^{\sec(x)}$$

```
[Out] exp(sec(x))
```

Rubi [A] time = 0.02, antiderivative size = 4, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {4339, 2209}

$$e^{\sec(x)}$$

Antiderivative was successfully verified.

```
[In] Int[E^Sec[x]*Sec[x]*Tan[x],x]
```

```
[Out] E^Sec[x]
```

Rule 2209

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[((e + f*x)^n*F^(a + b*(c + d*x)^n))/(b*f*n*(c + d*x)^
n*Log[F]), x] /; FreeQ[{F, a, b, c, d, e, f, n}, x] && EqQ[m, n - 1] && EqQ
[d*e - c*f, 0]
```

Rule 4339

```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{d = FreeFacto
rs[Cos[c*(a + b*x)], x]}, -Dist[(b*c)^(-1), Subst[Int[SubstFor[1/x, Cos[c*(
a + b*x)]/d, u, x], x], x, Cos[c*(a + b*x)]/d], x] /; FunctionOfQ[Cos[c*(a
+ b*x)]/d, u, x, True]] /; FreeQ[{a, b, c}, x] && (EqQ[F, Tan] || EqQ[F, ta
n])
```

Rubi steps

$$\begin{aligned} \int e^{\sec(x)} \sec(x) \tan(x) \, dx &= -\text{Subst}\left(\int \frac{e^{\frac{1}{x}}}{x^2} \, dx, x, \cos(x)\right) \\ &= e^{\sec(x)} \end{aligned}$$

Mathematica [A] time = 0.01, size = 4, normalized size = 1.00

$$e^{\sec(x)}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^Sec[x]*Sec[x]*Tan[x],x]
```

```
[Out] E^Sec[x]
```

fricas [A] time = 0.92, size = 5, normalized size = 1.25

$$e^{\left(\frac{1}{\cos(x)}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(sec(x))*sec(x)*tan(x),x, algorithm="fricas")
```

```
[Out] e^(1/cos(x))
```

giac [A] time = 0.14, size = 5, normalized size = 1.25

$$e^{\left(\frac{1}{\cos(x)}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(sec(x))*sec(x)*tan(x),x, algorithm="giac")
```

```
[Out] e^(1/cos(x))
```

maple [A] time = 0.02, size = 4, normalized size = 1.00

$$e^{\sec(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(sec(x))*sec(x)*tan(x),x)
```

```
[Out] exp(sec(x))
```

maxima [A] time = 0.33, size = 3, normalized size = 0.75

$$e^{\sec(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(sec(x))*sec(x)*tan(x),x, algorithm="maxima")
```

```
[Out] e^sec(x)
```

mupad [B] time = 3.09, size = 5, normalized size = 1.25

$$e^{\frac{1}{\cos(x)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((exp(1/cos(x))*tan(x))/cos(x),x)
```

```
[Out] exp(1/cos(x))
```

sympy [**A**] time = 0.61, size = 3, normalized size = 0.75

$$e^{\sec(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(sec(x))*sec(x)*tan(x),x)
```

```
[Out] exp(sec(x))
```

3.731 $\int 2^{\sec(x)} \sec(x) \tan(x) \, dx$

Optimal. Leaf size=9

$$\frac{2^{\sec(x)}}{\log(2)}$$

[Out] 2^sec(x)/ln(2)

Rubi [A] time = 0.02, antiderivative size = 9, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {4339, 2209}

$$\frac{2^{\sec(x)}}{\log(2)}$$

Antiderivative was successfully verified.

[In] Int[2^Sec[x]*Sec[x]*Tan[x],x]

[Out] 2^Sec[x]/Log[2]

Rule 2209

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[((e + f*x)^n*F^(a + b*(c + d*x)^n))/(b*f*n*(c + d*x)^
n*Log[F]), x] /; FreeQ[{F, a, b, c, d, e, f, n}, x] && EqQ[m, n - 1] && EqQ
[d*e - c*f, 0]
```

Rule 4339

```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{d = FreeFacto
rs[Cos[c*(a + b*x)], x]}, -Dist[(b*c)^(-1), Subst[Int[SubstFor[1/x, Cos[c*(
a + b*x)]/d, u, x], x], x, Cos[c*(a + b*x)]/d], x] /; FunctionOfQ[Cos[c*(a
+ b*x)]/d, u, x, True]] /; FreeQ[{a, b, c}, x] && (EqQ[F, Tan] || EqQ[F, ta
n])
```

Rubi steps

$$\begin{aligned}
\int 2^{\sec(x)} \sec(x) \tan(x) \, dx &= -\text{Subst}\left(\int \frac{2^{\frac{1}{x}}}{x^2} \, dx, x, \cos(x)\right) \\
&= \frac{2^{\sec(x)}}{\log(2)}
\end{aligned}$$

Mathematica [A] time = 0.01, size = 9, normalized size = 1.00

$$\frac{2^{\sec(x)}}{\log(2)}$$

Antiderivative was successfully verified.

```
[In] Integrate[2^Sec[x]*Sec[x]*Tan[x],x]
```

```
[Out] 2^Sec[x]/Log[2]
```

fricas [A] time = 0.47, size = 11, normalized size = 1.22

$$\frac{2^{\left(\frac{1}{\cos(x)}\right)}}{\log(2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(2^sec(x)*sec(x)*tan(x),x, algorithm="fricas")
```

```
[Out] 2^(1/cos(x))/log(2)
```

giac [A] time = 0.15, size = 11, normalized size = 1.22

$$\frac{2^{\left(\frac{1}{\cos(x)}\right)}}{\log(2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(2^sec(x)*sec(x)*tan(x),x, algorithm="giac")
```

```
[Out] 2^(1/cos(x))/log(2)
```

maple [A] time = 0.02, size = 10, normalized size = 1.11

$$\frac{2^{\sec(x)}}{\ln(2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(2^sec(x)*sec(x)*tan(x),x)
```

```
[Out] 2^sec(x)/ln(2)
```

maxima [A] time = 0.31, size = 9, normalized size = 1.00

$$\frac{2^{\sec(x)}}{\log(2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(2^sec(x)*sec(x)*tan(x),x, algorithm="maxima")
```

```
[Out] 2^sec(x)/log(2)
```

mupad [B] time = 3.12, size = 11, normalized size = 1.22

$$\frac{2^{\frac{1}{\cos(x)}}}{\ln(2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2^(1/cos(x))*tan(x))/cos(x),x)
```

```
[Out] 2^(1/cos(x))/log(2)
```

sympy [A] time = 0.65, size = 7, normalized size = 0.78

$$\frac{2^{\sec(x)}}{\log(2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(2**sec(x)*sec(x)*tan(x),x)
```

```
[Out] 2**sec(x)/log(2)
```

3.732 $\int \frac{\sec(2x)\tan(2x)}{(1+\sec(2x))^{3/2}}\,dx$

Optimal. Leaf size=12

$$-\frac{1}{\sqrt{\sec(2x)+1}}$$

```
[Out] -1/(1+sec(2*x))^(1/2)
```

Rubi [A] time = 0.05, antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {4339, 261}

$$-\frac{1}{\sqrt{\sec(2x)+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[2*x]*Tan[2*x])/(1 + Sec[2*x])^(3/2),x]
```

```
[Out] -(1/Sqrt[1 + Sec[2*x]])
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 4339

```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{d = FreeFacto
rs[Cos[c*(a + b*x)], x]}, -Dist[(b*c)^(-1), Subst[Int[SubstFor[1/x, Cos[c*(
a + b*x)]/d, u, x], x], x, Cos[c*(a + b*x)]/d], x] /; FunctionOfQ[Cos[c*(a
+ b*x)]/d, u, x, True]] /; FreeQ[{a, b, c}, x] && (EqQ[F, Tan] || EqQ[F, ta
n])
```

Rubi steps

$$\begin{aligned}\int \frac{\sec(2x)\tan(2x)}{(1+\sec(2x))^{3/2}}\,dx &= -\left(\frac{1}{2}\,\text{Subst}\left(\int \frac{1}{\left(1+\frac{1}{x}\right)^{3/2}x^2}\,dx, x, \cos(2x)\right)\right)\\ &= -\frac{1}{\sqrt{1+\sec(2x)}}\end{aligned}$$

Mathematica [A] time = 0.07, size = 20, normalized size = 1.67

$$-\frac{2\cos^2(x)\sec(2x)}{(\sec(2x)+1)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[2*x]*Tan[2*x])/(1 + Sec[2*x])^(3/2),x]
[Out] (-2*Cos[x]^2*Sec[2*x])/(1 + Sec[2*x])^(3/2)
```

fricas [B] time = 0.76, size = 29, normalized size = 2.42

$$-\frac{\sqrt{\frac{\cos(2x)+1}{\cos(2x)}}\cos(2x)}{\cos(2x)+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(2*x)*tan(2*x)/(1+sec(2*x))^(3/2),x, algorithm="fricas")
[Out] -sqrt((cos(2*x) + 1)/cos(2*x))*cos(2*x)/(cos(2*x) + 1)
```

giac [B] time = 0.17, size = 37, normalized size = 3.08

$$\frac{1}{\left(\sqrt{\cos(2x)^2+\cos(2x)}-\cos(2x)-1\right)\operatorname{sgn}(\cos(2x))}+\operatorname{sgn}(\cos(2x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(2*x)*tan(2*x)/(1+sec(2*x))^(3/2),x, algorithm="giac")
[Out] 1/((sqrt(cos(2*x)^2 + cos(2*x)) - cos(2*x) - 1)*sgn(cos(2*x))) + sgn(cos(2*
x))
```

maple [A] time = 0.08, size = 11, normalized size = 0.92

$$-\frac{1}{\sqrt{1+\sec(2x)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(2*x)*tan(2*x)/(1+sec(2*x))^(3/2),x)
[Out] -1/(1+sec(2*x))^(1/2)
```

maxima [A] time = 0.32, size = 10, normalized size = 0.83

$$-\frac{1}{\sqrt{\sec\left(2\,x\right)+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(2*x)*tan(2*x)/(1+sec(2*x))^(3/2),x, algorithm="maxima")
```

```
[Out] -1/sqrt(sec(2*x) + 1)
```

mupad [B] time = 3.11, size = 18, normalized size = 1.50

$$-\frac{1}{\sqrt{\cos\left(2\,x\right)+1}\,\sqrt{\frac{1}{\cos(2\,x)}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(2*x)/(cos(2*x)*(1/cos(2*x) + 1)^(3/2)),x)
```

```
[Out] -1/((cos(2*x) + 1)^(1/2)*(1/cos(2*x))^(1/2))
```

sympy [A] time = 1.02, size = 12, normalized size = 1.00

$$-\frac{1}{\sqrt{\sec\left(2x\right)+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(2*x)*tan(2*x)/(1+sec(2*x))**(3/2),x)
```

```
[Out] -1/sqrt(sec(2*x) + 1)
```

3.733 $\int \sqrt{1+5\cos^2(3x)}\ \sec(3x)\tan(3x)\,dx$

Optimal. Leaf size=43

$$\frac{1}{3}\sqrt{5\cos^2(3x)+1}\ \sec(3x)-\frac{1}{3}\sqrt{5}\ \sinh^{-1}\left(\sqrt{5}\ \cos(3x)\right)$$

```
[Out] -1/3*arcsinh(cos(3*x)*5^(1/2))*5^(1/2)+1/3*sec(3*x)*(1+5*cos(3*x)^2)^(1/2)
```

Rubi [A] time = 0.09, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {277, 215}

$$\frac{1}{3}\sqrt{5\cos^2(3x)+1}\ \sec(3x)-\frac{1}{3}\sqrt{5}\ \sinh^{-1}\left(\sqrt{5}\ \cos(3x)\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[1 + 5*Cos[3*x]^2]*Sec[3*x]*Tan[3*x],x]
```

```
[Out] -(Sqrt[5]*ArcSinh[Sqrt[5]*Cos[3*x]])/3 + (Sqrt[1 + 5*Cos[3*x]^2]*Sec[3*x])/
3
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 277

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\begin{aligned}
\int \sqrt{1+5\cos^2(3x)}\ \sec(3x)\tan(3x)\,dx &= -\left(\frac{1}{3}\operatorname{Subst}\left(\int \frac{\sqrt{1+5x^2}}{x^2}\,dx,x,\cos(3x)\right)\right)\\
&= \frac{1}{3}\sqrt{1+5\cos^2(3x)}\ \sec(3x)-\frac{5}{3}\operatorname{Subst}\left(\int \frac{1}{\sqrt{1+5x^2}}\,dx,x,\cos(3x)\right)\\
&= -\frac{1}{3}\sqrt{5}\ \sinh^{-1}\left(\sqrt{5}\ \cos(3x)\right)+\frac{1}{3}\sqrt{1+5\cos^2(3x)}\ \sec(3x)
\end{aligned}$$

Mathematica [A] time = 0.05, size = 43, normalized size = 1.00

$$\frac{1}{3}\sqrt{5\cos^2(3x)+1}\ \sec(3x)-\frac{1}{3}\sqrt{5}\ \sinh^{-1}\left(\sqrt{5}\ \cos(3x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[1 + 5*Cos[3*x]^2]*Sec[3*x]*Tan[3*x],x]
```

```
[Out] -1/3*(Sqrt[5]*ArcSinh[Sqrt[5]*Cos[3*x]]) + (Sqrt[1 + 5*Cos[3*x]^2]*Sec[3*x]
)/3
```

fricas [B] time = 0.95, size = 122, normalized size = 2.84

$$\frac{\sqrt{5}\ \cos\left(3\,x\right)\log\left(80000\ \cos\left(3\,x\right)^{8}+32000\ \cos\left(3\,x\right)^{6}+4000\ \cos\left(3\,x\right)^{4}+160\ \cos\left(3\,x\right)^{2}-8\left(2000\ \sqrt{5}\ \cos\left(3\,x\right)^{7}\right.\right.}{24\ \cos\left(3\,x\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(3*x)*(1+5*cos(3*x)^2)^(1/2)*tan(3*x),x, algorithm="fricas")
```

```
[Out] 1/24*(sqrt(5)*cos(3*x)*log(80000*cos(3*x)^8 + 32000*cos(3*x)^6 + 4000*cos(3
*x)^4 + 160*cos(3*x)^2 - 8*(2000*sqrt(5)*cos(3*x)^7 + 600*sqrt(5)*cos(3*x)^
5 + 50*sqrt(5)*cos(3*x)^3 + sqrt(5)*cos(3*x))*sqrt(5*cos(3*x)^2 + 1) + 1) +
 8*sqrt(5*cos(3*x)^2 + 1))/cos(3*x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{5\ \cos\left(3\,x\right)^{2}+1}\ \sec\left(3\,x\right)\tan\left(3\,x\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(3*x)*(1+5*cos(3*x)^2)^(1/2)*tan(3*x),x, algorithm="giac")
```

```
[Out] integrate(sqrt(5*cos(3*x)^2 + 1)*sec(3*x)*tan(3*x), x)
```

maple [A] time = 0.12, size = 65, normalized size = 1.51

$$\frac{\sqrt{\frac{\sec^2(3x)+5}{\sec(3x)^2}}\ \sec\left(3x\right)\left(\sqrt{\sec^2\left(3x\right)+5}-\sqrt{5}\ \operatorname{arctanh}\left(\frac{\sqrt{5}}{\sqrt{\sec^2(3x)+5}}\right)\right)}{3\sqrt{\sec^2\left(3x\right)+5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(3*x)*(1+5*cos(3*x)^2)^(1/2)*tan(3*x),x)
```

```
[Out] 1/3*((sec(3*x)^2+5)/sec(3*x)^2)^(1/2)*sec(3*x)/(sec(3*x)^2+5)^(1/2)*((sec(3
*x)^2+5)^(1/2)-5^(1/2)*arctanh(5^(1/2)/(sec(3*x)^2+5)^(1/2)))
```

maxima [A] time = 0.43, size = 35, normalized size = 0.81

$$-\frac{1}{3}\sqrt{5}\,\operatorname{arsinh}\left(\sqrt{5}\cos\left(3x\right)\right)+\frac{\sqrt{5\cos\left(3x\right)^{2}+1}}{3\cos\left(3x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(3*x)*(1+5*cos(3*x)^2)^(1/2)*tan(3*x),x, algorithm="maxima")
[Out] -1/3*sqrt(5)*arcsinh(sqrt(5)*cos(3*x)) + 1/3*sqrt(5*cos(3*x)^2 + 1)/cos(3*x
)
```

mupad [B] time = 3.26, size = 36, normalized size = 0.84

$$\frac{\sqrt{\frac{5\cos(6x)}{2}+\frac{7}{2}}}{3\cos\left(3x\right)}+\frac{\sqrt{5}\,\operatorname{asin}\left(\sqrt{5}\,\cos\left(3x\right)\,1\mathrm{i}\right)\,1\mathrm{i}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((tan(3*x)*(5*cos(3*x)^2 + 1)^(1/2))/cos(3*x),x)
[Out] (5^(1/2)*asin(5^(1/2)*cos(3*x)*1i)*1i)/3 + ((5*cos(6*x))/2 + 7/2)^(1/2)/(3*
cos(3*x))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{5\cos^{2}\left(3x\right)+1}\,\tan\left(3x\right)\sec\left(3x\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(3*x)*(1+5*cos(3*x)**2)**(1/2)*tan(3*x),x)
[Out] Integral(sqrt(5*cos(3*x)**2 + 1)*tan(3*x)*sec(3*x), x)
```

3.734 $\int \frac{\sec(3x)\tan(3x)}{\sqrt{1+5\cos^2(3x)}}\,dx$

Optimal. Leaf size=22

$$\frac{1}{3}\sqrt{5\cos^2(3x)+1}\ \sec(3x)$$

```
[Out] 1/3*sec(3*x)*(1+5*cos(3*x)^2)^(1/2)
```

Rubi [A] time = 0.09, antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.043, Rules used = {264}

$$\frac{1}{3}\sqrt{5\cos^2(3x)+1}\ \sec(3x)$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[3*x]*Tan[3*x])/Sqrt[1 + 5*Cos[3*x]^2],x]
```

```
[Out] (Sqrt[1 + 5*Cos[3*x]^2]*Sec[3*x])/3
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{\sec(3x)\tan(3x)}{\sqrt{1+5\cos^2(3x)}}\,dx &= -\left(\frac{1}{3}\text{Subst}\left(\int \frac{1}{x^2\sqrt{1+5x^2}}\,dx, x, \cos(3x)\right)\right)\\ &= \frac{1}{3}\sqrt{1+5\cos^2(3x)}\ \sec(3x)\end{aligned}$$

Mathematica [A] time = 0.03, size = 22, normalized size = 1.00

$$\frac{1}{3}\sqrt{5\cos^2(3x)+1}\ \sec(3x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[3*x]*Tan[3*x])/Sqrt[1 + 5*Cos[3*x]^2],x]
```

```
[Out] (Sqrt[1 + 5*Cos[3*x]^2]*Sec[3*x])/3
```

fricas [A] time = 0.67, size = 20, normalized size = 0.91

$$\frac{\sqrt{5 \cos\left(3 x\right)^{2} + 1}}{3 \cos\left(3 x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(3*x)*tan(3*x)/(1+5*cos(3*x)^2)^(1/2),x, algorithm="fricas")
[Out] 1/3*sqrt(5*cos(3*x)^2 + 1)/cos(3*x)
```

giac [A] time = 0.15, size = 34, normalized size = 1.55

$$-\frac{2 \sqrt{5}}{3 \left(\left(\sqrt{5} \cos\left(3 x\right) - \sqrt{5 \cos\left(3 x\right)^{2} + 1}\right)^{2} - 1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(3*x)*tan(3*x)/(1+5*cos(3*x)^2)^(1/2),x, algorithm="giac")
[Out] -2/3*sqrt(5)/((sqrt(5)*cos(3*x) - sqrt(5*cos(3*x)^2 + 1))^2 - 1)
```

maple [A] time = 0.11, size = 34, normalized size = 1.55

$$\frac{\sec^{2}\left(3 x\right)+5}{3 \sqrt{\frac{\sec^{2}\left(3 x\right)+5}{\sec\left(3 x\right)^{2}}}\, \sec\left(3 x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(3*x)*tan(3*x)/(1+5*cos(3*x)^2)^(1/2),x)
[Out] 1/3/((sec(3*x)^2+5)/sec(3*x)^2)^(1/2)/sec(3*x)*(sec(3*x)^2+5)
```

maxima [A] time = 0.42, size = 20, normalized size = 0.91

$$\frac{\sqrt{5 \cos\left(3 x\right)^{2} + 1}}{3 \cos\left(3 x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(3*x)*tan(3*x)/(1+5*cos(3*x)^2)^(1/2),x, algorithm="maxima")
[Out] 1/3*sqrt(5*cos(3*x)^2 + 1)/cos(3*x)
```

mupad [B] time = 3.03, size = 18, normalized size = 0.82

$$\frac{\sqrt{\frac{5\cos(6x)}{2}+\frac{7}{2}}}{3\cos(3x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(3*x)/(cos(3*x)*(5*cos(3*x)^2 + 1)^(1/2)),x)
[Out] ((5*cos(6*x))/2 + 7/2)^(1/2)/(3*cos(3*x))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan(3x)\sec(3x)}{\sqrt{5\cos^2(3x)+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(3*x)*tan(3*x)/(1+5*cos(3*x)**2)**(1/2),x)
[Out] Integral(tan(3*x)*sec(3*x)/sqrt(5*cos(3*x)**2 + 1), x)
```

3.735 $\int \frac{\cot(x)\csc(x)}{a+b\csc(x)}\,dx$

Optimal. Leaf size=12

$$-\frac{\log(a+b\csc(x))}{b}$$

```
[Out] -ln(a+b*csc(x))/b
```

Rubi [A] time = 0.04, antiderivative size = 20, normalized size of antiderivative = 1.67, number of steps used = 4, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {4338, 36, 29, 31}

$$\frac{\log(\sin(x))}{b}-\frac{\log(a\sin(x)+b)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[(Cot[x]*Csc[x])/(a + b*Csc[x]),x]
```

```
[Out] Log[Sin[x]]/b - Log[b + a*Sin[x]]/b
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 4338

```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{d = FreeFacto
rs[Sin[c*(a + b*x)], x]}, Dist[1/(b*c), Subst[Int[SubstFor[1/x, Sin[c*(a +
b*x)]/d, u, x], x], x, Sin[c*(a + b*x)]/d], x] /; FunctionOfQ[Sin[c*(a + b*
x)]/d, u, x, True]] /; FreeQ[{a, b, c}, x] && (EqQ[F, Cot] || EqQ[F, cot])
```

Rubi steps

$$
\begin{aligned}
\int \frac{\cot(x)\csc(x)}{a+b\csc(x)}\,dx &= \text{Subst}\left(\int \frac{1}{x(b+ax)}\,dx, x, \sin(x)\right) \\
&= \frac{\text{Subst}\left(\int \frac{1}{x}\,dx, x, \sin(x)\right)}{b} - \frac{a\,\text{Subst}\left(\int \frac{1}{b+ax}\,dx, x, \sin(x)\right)}{b} \\
&= \frac{\log(\sin(x))}{b} - \frac{\log(b+a\sin(x))}{b}
\end{aligned}
$$

Mathematica [A] time = 0.02, size = 20, normalized size = 1.67

$$\frac{\log(\sin(x))}{b} - \frac{\log(a\sin(x)+b)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cot[x]*Csc[x])/(a + b*Csc[x]),x]
```

```
[Out] Log[Sin[x]]/b - Log[b + a*Sin[x]]/b
```

fricas [A] time = 0.69, size = 20, normalized size = 1.67

$$-\frac{\log\left(a\sin(x)+b\right) - \log\left(-\frac{1}{2}\,\sin(x)\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)*csc(x)/(a+b*csc(x)),x, algorithm="fricas")
```

```
[Out] -(log(a*sin(x) + b) - log(-1/2*sin(x)))/b
```

giac [A] time = 0.13, size = 22, normalized size = 1.83

$$-\frac{\log\left(|a\sin(x)+b|\right)}{b} + \frac{\log\left(|\sin(x)|\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)*csc(x)/(a+b*csc(x)),x, algorithm="giac")
```

```
[Out] -log(abs(a*sin(x) + b))/b + log(abs(sin(x)))/b
```

maple [A] time = 0.05, size = 13, normalized size = 1.08

$$-\frac{\ln\left(a+b\csc(x)\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(x)*csc(x)/(a+b*csc(x)),x)
```

```
[Out] -ln(a+b*csc(x))/b
```

maxima [A] time = 0.31, size = 12, normalized size = 1.00

$$-\frac{\log\left(b\csc(x)+a\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)*csc(x)/(a+b*csc(x)),x, algorithm="maxima")
```

```
[Out] -log(b*csc(x) + a)/b
```

mupad [B] time = 3.18, size = 31, normalized size = 2.58

$$-\frac{\ln\left(b\tan\left(\frac{x}{2}\right)^2+2\,a\tan\left(\frac{x}{2}\right)+b\right)-\ln\left(\tan\left(\frac{x}{2}\right)\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(x)/(sin(x)*(a + b/sin(x))),x)
```

```
[Out] -(log(b + 2*a*tan(x/2) + b*tan(x/2)^2) - log(tan(x/2)))/b
```

sympy [A] time = 0.39, size = 17, normalized size = 1.42

$$\begin{cases} -\frac{\log\left(\frac{a}{b}+\csc\left(x\right)\right)}{b} & \text{for}\: b \neq 0 \\ -\frac{\csc\left(x\right)}{a} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)*csc(x)/(a+b*csc(x)),x)
```

```
[Out] Piecewise((-log(a/b + csc(x))/b, Ne(b, 0)), (-csc(x)/a, True))
```

3.736 $\int 5^{\csc(3x)}\cot(3x)\csc(3x)\,dx$

Optimal. Leaf size=14

$$-\frac{5^{\csc(3x)}}{3\log(5)}$$

```
[Out] -1/3*5^csc(3*x)/ln(5)
```

Rubi [A] time = 0.02, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {4338, 2209}

$$-\frac{5^{\csc(3x)}}{3\log(5)}$$

Antiderivative was successfully verified.

```
[In] Int[5^Csc[3*x]*Cot[3*x]*Csc[3*x],x]
```

```
[Out] -5^Csc[3*x]/(3*Log[5])
```

Rule 2209

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[((e + f*x)^n*F^(a + b*(c + d*x)^n))/(b*f*n*(c + d*x)^
n*Log[F]), x] /; FreeQ[{F, a, b, c, d, e, f, n}, x] && EqQ[m, n - 1] && EqQ
[d*e - c*f, 0]
```

Rule 4338

```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{d = FreeFacto
rs[Sin[c*(a + b*x)], x]}, Dist[1/(b*c), Subst[Int[SubstFor[1/x, Sin[c*(a +
b*x)]/d, u, x], x], x, Sin[c*(a + b*x)]/d], x] /; FunctionOfQ[Sin[c*(a + b*
x)]/d, u, x, True]] /; FreeQ[{a, b, c}, x] && (EqQ[F, Cot] || EqQ[F, cot])
```

Rubi steps

$$\begin{aligned}\int 5^{\csc(3x)}\cot(3x)\csc(3x)\,dx &= \frac{1}{3}\text{Subst}\left(\int \frac{5^{\frac{1}{x}}}{x^2}\,dx, x, \sin(3x)\right)\\ &= -\frac{5^{\csc(3x)}}{3\log(5)}\end{aligned}$$

Mathematica [A] time = 0.02, size = 14, normalized size = 1.00

$$-\frac{5^{\csc(3x)}}{3\log(5)}$$

Antiderivative was successfully verified.

```
[In] Integrate[5^Csc[3*x]*Cot[3*x]*Csc[3*x],x]
[Out] -1/3*5^Csc[3*x]/Log[5]
```

fricas [A] time = 1.14, size = 14, normalized size = 1.00

$$-\frac{5^{\left(\frac{1}{\sin(3x)}\right)}}{3\log(5)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(5^csc(3*x)*cot(3*x)*csc(3*x),x, algorithm="fricas")
[Out] -1/3*5^(1/sin(3*x))/log(5)
```

giac [A] time = 0.25, size = 14, normalized size = 1.00

$$-\frac{5^{\left(\frac{1}{\sin(3x)}\right)}}{3\log(5)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(5^csc(3*x)*cot(3*x)*csc(3*x),x, algorithm="giac")
[Out] -1/3*5^(1/sin(3*x))/log(5)
```

maple [A] time = 0.04, size = 13, normalized size = 0.93

$$-\frac{5^{\csc(3x)}}{3\ln(5)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(5^csc(3*x)*cot(3*x)*csc(3*x),x)
[Out] -1/3*5^csc(3*x)/ln(5)
```

maxima [A] time = 0.30, size = 12, normalized size = 0.86

$$-\frac{5^{\csc(3x)}}{3\log(5)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(5^csc(3*x)*cot(3*x)*csc(3*x),x, algorithm="maxima")
```

```
[Out] -1/3*5^csc(3*x)/log(5)
```

mupad [B] time = 2.97, size = 14, normalized size = 1.00

$$-\frac{5^{\frac{1}{\sin(3\,x)}}}{3\,\ln(5)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5^(1/sin(3*x))*cot(3*x))/sin(3*x),x)
```

```
[Out] -5^(1/sin(3*x))/(3*log(5))
```

sympy [A] time = 0.61, size = 12, normalized size = 0.86

$$-\frac{5^{\csc(3x)}}{3\log(5)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(5**csc(3*x)*cot(3*x)*csc(3*x),x)
```

```
[Out] -5**csc(3*x)/(3*log(5))
```

3.737 $\int \frac{\cot(x)\csc(x)}{1+\csc^2(x)}\,dx$

Optimal. Leaf size=3

$$\tan^{-1}(\sin(x))$$

```
[Out] arctan(sin(x))
```

Rubi [A] time = 0.03, antiderivative size = 3, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {4338, 203}

$$\tan^{-1}(\sin(x))$$

Antiderivative was successfully verified.

```
[In] Int[(Cot[x]*Csc[x])/(1 + Csc[x]^2),x]
```

```
[Out] ArcTan[Sin[x]]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 4338

```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{d = FreeFacto
rs[Sin[c*(a + b*x)], x]}, Dist[1/(b*c), Subst[Int[SubstFor[1/x, Sin[c*(a +
b*x)]/d, u, x], x], x, Sin[c*(a + b*x)]/d], x] /; FunctionOfQ[Sin[c*(a + b*
x)]/d, u, x, True]] /; FreeQ[{a, b, c}, x] && (EqQ[F, Cot] || EqQ[F, cot])
```

Rubi steps

$$\begin{aligned}\int \frac{\cot(x)\csc(x)}{1+\csc^2(x)}\,dx &= \text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \sin(x)\right)\\ &= \tan^{-1}(\sin(x))\end{aligned}$$

Mathematica [A] time = 0.01, size = 3, normalized size = 1.00

$$\tan^{-1}(\sin(x))$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cot[x]*Csc[x])/(1 + Csc[x]^2),x]
[Out] ArcTan[Sin[x]]
```

fricas [A] time = 0.71, size = 3, normalized size = 1.00

$$\arctan(\sin(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)*csc(x)/(1+csc(x)^2),x, algorithm="fricas")
[Out] arctan(sin(x))
```

giac [A] time = 0.15, size = 3, normalized size = 1.00

$$\arctan(\sin(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)*csc(x)/(1+csc(x)^2),x, algorithm="giac")
[Out] arctan(sin(x))
```

maple [A] time = 0.05, size = 6, normalized size = 2.00

$$-\arctan(\csc(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(x)*csc(x)/(1+csc(x)^2),x)
[Out] -arctan(csc(x))
```

maxima [A] time = 0.40, size = 3, normalized size = 1.00

$$\arctan(\sin(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)*csc(x)/(1+csc(x)^2),x, algorithm="maxima")
[Out] arctan(sin(x))
```

mupad [B] time = 3.22, size = 26, normalized size = 8.67

$$\operatorname{atan}\left(\frac{\tan\left(\frac{x}{2}\right)^3}{2}+\frac{5\tan\left(\frac{x}{2}\right)}{2}\right)-\operatorname{atan}\left(\frac{\tan\left(\frac{x}{2}\right)}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(x)/(sin(x)*(1/sin(x)^2 + 1)),x)
```

```
[Out] atan((5*tan(x/2))/2 + tan(x/2)^3/2) - atan(tan(x/2)/2)
```

sympy [A] time = 0.19, size = 5, normalized size = 1.67

$$-\operatorname{atan}(\csc(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)*csc(x)/(1+csc(x)**2),x)
```

```
[Out] -atan(csc(x))
```

3.738 $\int \frac{\cot(6x)\csc(6x)}{\left(5-11\csc^2(6x)\right)^2}\,dx$

Optimal. Leaf size=43

$$\frac{\sin(6x)}{60\left(11-5\sin^2(6x)\right)}-\frac{\tanh^{-1}\left(\sqrt{\frac{5}{11}}\sin(6x)\right)}{60\sqrt{55}}$$

```
[Out] 1/60*sin(6*x)/(11-5*sin(6*x)^2)-1/3300*arctanh(1/11*sin(6*x)*55^(1/2))*55^(
1/2)
```

Rubi [A] time = 0.06, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {4338, 288, 206}

$$\frac{\sin(6x)}{60\left(11-5\sin^2(6x)\right)}-\frac{\tanh^{-1}\left(\sqrt{\frac{5}{11}}\sin(6x)\right)}{60\sqrt{55}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cot[6*x]*Csc[6*x])/(5 - 11*Csc[6*x]^2)^2,x]
```

```
[Out] -ArcTanh[Sqrt[5/11]*Sin[6*x]]/(60*Sqrt[55]) + Sin[6*x]/(60*(11 - 5*Sin[6*x]
^2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 4338

```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{d = FreeFacto
rs[Sin[c*(a + b*x)], x]}, Dist[1/(b*c), Subst[Int[SubstFor[1/x, Sin[c*(a +
```

```
b*x)]/d, u, x], x], x, Sin[c*(a + b*x)]/d], x] /; FunctionOfQ[Sin[c*(a + b*
x)]/d, u, x, True]] /; FreeQ[{a, b, c}, x] && (EqQ[F, Cot] || EqQ[F, cot])
```

Rubi steps

$$\begin{aligned}\int \frac{\cot(6x)\csc(6x)}{\left(5-11\csc^2(6x)\right)^2}\,dx &= \frac{1}{6}\,\text{Subst}\left(\int \frac{x^2}{\left(11-5x^2\right)^2}\,dx, x, \sin(6x)\right)\\ &= \frac{\sin(6x)}{60\left(11-5\sin^2(6x)\right)}-\frac{1}{60}\,\text{Subst}\left(\int \frac{1}{11-5x^2}\,dx, x, \sin(6x)\right)\\ &= -\frac{\tanh^{-1}\left(\sqrt{\frac{5}{11}}\sin(6x)\right)}{60\sqrt{55}}+\frac{\sin(6x)}{60\left(11-5\sin^2(6x)\right)}\end{aligned}$$

Mathematica [A] time = 0.66, size = 41, normalized size = 0.95

$$\frac{\sin(6x)}{30(5\cos(12x)+17)}-\frac{\tanh^{-1}\left(\sqrt{\frac{5}{11}}\sin(6x)\right)}{60\sqrt{55}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cot[6*x]*Csc[6*x])/(5 - 11*Csc[6*x]^2)^2,x]
```

```
[Out] -1/60*ArcTanh[Sqrt[5/11]*Sin[6*x]]/Sqrt[55] + Sin[6*x]/(30*(17 + 5*Cos[12*x
]))
```

fricas [B] time = 0.64, size = 73, normalized size = 1.70

$$\frac{\left(5\sqrt{55}\cos\left(6x\right)^2+6\sqrt{55}\right)\log\left(-\frac{5\cos(6x)^2+2\sqrt{55}\sin(6x)-16}{5\cos(6x)^2+6}\right)+110\sin\left(6x\right)}{6600\left(5\cos\left(6x\right)^2+6\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(6*x)*csc(6*x)/(5-11*csc(6*x)^2)^2,x, algorithm="fricas")
```

```
[Out] 1/6600*((5*sqrt(55)*cos(6*x)^2 + 6*sqrt(55))*log(-(5*cos(6*x)^2 + 2*sqrt(55
)*sin(6*x) - 16)/(5*cos(6*x)^2 + 6)) + 110*sin(6*x))/(5*cos(6*x)^2 + 6)
```

giac [A] time = 0.24, size = 48, normalized size = 1.12

$$\frac{1}{6600}\sqrt{55}\log\left(\frac{\sqrt{55}-5\sin\left(6x\right)}{\sqrt{55}+5\sin\left(6x\right)}\right)-\frac{\sin\left(6x\right)}{60\left(5\sin\left(6x\right)^2-11\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(6*x)*csc(6*x)/(5-11*csc(6*x)^2)^2,x, algorithm="giac")
```

```
[Out] 1/6600*sqrt(55)*log((sqrt(55) - 5*sin(6*x))/(sqrt(55) + 5*sin(6*x))) - 1/60
*sin(6*x)/(5*sin(6*x)^2 - 11)
```

maple [A] time = 0.08, size = 35, normalized size = 0.81

$$\frac{\csc(6x)}{660\left(\csc^2(6x)\right)-300}-\frac{\sqrt{55}\ \operatorname{arctanh}\left(\frac{\csc(6x)\sqrt{55}}{5}\right)}{3300}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(6*x)*csc(6*x)/(5-11*csc(6*x)^2)^2,x)
```

```
[Out] 1/60*csc(6*x)/(11*csc(6*x)^2-5)-1/3300*55^(1/2)*arctanh(1/5*csc(6*x)*55^(1/
2))
```

maxima [A] time = 0.40, size = 49, normalized size = 1.14

$$\frac{1}{6600}\sqrt{55}\log\left(-\frac{\sqrt{55}-5\sin(6x)}{\sqrt{55}+5\sin(6x)}\right)-\frac{\sin(6x)}{60\left(5\sin(6x)^2-11\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(6*x)*csc(6*x)/(5-11*csc(6*x)^2)^2,x, algorithm="maxima")
```

```
[Out] 1/6600*sqrt(55)*log(-(sqrt(55) - 5*sin(6*x))/(sqrt(55) + 5*sin(6*x))) - 1/6
0*sin(6*x)/(5*sin(6*x)^2 - 11)
```

mupad [B] time = 3.15, size = 57, normalized size = 1.33

$$-\frac{55\sin(6x)-11\sqrt{55}\ \operatorname{atanh}\left(\frac{\sqrt{55}\ \sin(6x)}{11}\right)+5\sqrt{55}\ \sin(6x)^2\operatorname{atanh}\left(\frac{\sqrt{55}\ \sin(6x)}{11}\right)}{16500\sin(6x)^2-36300}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(6*x)/(sin(6*x)*(11/sin(6*x)^2 - 5)^2),x)
```

```
[Out] -(55*sin(6*x) - 11*55^(1/2)*atanh((55^(1/2)*sin(6*x))/11) + 5*55^(1/2)*sin(
6*x)^2*atanh((55^(1/2)*sin(6*x))/11))/(16500*sin(6*x)^2 - 36300)
```

sympy [B] time = 1.72, size = 151, normalized size = 3.51

$$\frac{11\sqrt{55}\log\left(\csc(6x)-\frac{\sqrt{55}}{11}\right)\csc^2(6x)}{72600\csc^2(6x)-33000}-\frac{5\sqrt{55}\log\left(\csc(6x)-\frac{\sqrt{55}}{11}\right)}{72600\csc^2(6x)-33000}-\frac{11\sqrt{55}\log\left(\csc(6x)+\frac{\sqrt{55}}{11}\right)\csc^2(6x)}{72600\csc^2(6x)-33000}+\frac{5\sqrt{5}}{72\ldots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(6*x)*csc(6*x)/(5-11*csc(6*x)**2)**2,x)
```

```
[Out] 11*sqrt(55)*log(csc(6*x) - sqrt(55)/11)*csc(6*x)**2/(72600*csc(6*x)**2 - 33
000) - 5*sqrt(55)*log(csc(6*x) - sqrt(55)/11)/(72600*csc(6*x)**2 - 33000) -
 11*sqrt(55)*log(csc(6*x) + sqrt(55)/11)*csc(6*x)**2/(72600*csc(6*x)**2 - 3
3000) + 5*sqrt(55)*log(csc(6*x) + sqrt(55)/11)/(72600*csc(6*x)**2 - 33000)
+ 110*csc(6*x)/(72600*csc(6*x)**2 - 33000)
```

3.739 $\int \frac{\cot(x)\csc(x)}{\sqrt{1+\sin^2(x)}}\,dx$

Optimal. Leaf size=14

$$\sqrt{\sin^2(x)+1}\,(-\csc(x))$$

```
[Out] -csc(x)*(1+sin(x)^2)^(1/2)
```

Rubi [A] time = 0.08, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {264}

$$\sqrt{\sin^2(x)+1}\,(-\csc(x))$$

Antiderivative was successfully verified.

```
[In] Int[(Cot[x]*Csc[x])/Sqrt[1 + Sin[x]^2],x]
```

```
[Out] -(Csc[x]*Sqrt[1 + Sin[x]^2])
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{\cot(x)\csc(x)}{\sqrt{1+\sin^2(x)}}\,dx &= \text{Subst}\left(\int \frac{1}{x^2\sqrt{1+x^2}}\,dx, x, \sin(x)\right)\\ &= -\csc(x)\sqrt{1+\sin^2(x)}\end{aligned}$$

Mathematica [A] time = 0.02, size = 14, normalized size = 1.00

$$\sqrt{\sin^2(x)+1}\,(-\csc(x))$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cot[x]*Csc[x])/Sqrt[1 + Sin[x]^2],x]
```

```
[Out] -(Csc[x]*Sqrt[1 + Sin[x]^2])
```

fricas [A] time = 1.07, size = 21, normalized size = 1.50

$$-\frac{\sqrt{-\cos(x)^2+2}-\sin(x)}{\sin(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)*csc(x)/(1+sin(x)^2)^(1/2),x, algorithm="fricas")
[Out] -(sqrt(-cos(x)^2 + 2) - sin(x))/sin(x)
```

giac [A] time = 0.15, size = 21, normalized size = 1.50

$$\frac{2}{\left(\sqrt{\sin(x)^2+1}-\sin(x)\right)^2-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)*csc(x)/(1+sin(x)^2)^(1/2),x, algorithm="giac")
[Out] 2/((sqrt(sin(x)^2 + 1) - sin(x))^2 - 1)
```

maple [A] time = 0.16, size = 15, normalized size = 1.07

$$-\frac{\sqrt{1+\sin^2(x)}}{\sin(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(x)*csc(x)/(1+sin(x)^2)^(1/2),x)
[Out] -1/sin(x)*(1+sin(x)^2)^(1/2)
```

maxima [A] time = 0.40, size = 14, normalized size = 1.00

$$-\frac{\sqrt{\sin(x)^2+1}}{\sin(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)*csc(x)/(1+sin(x)^2)^(1/2),x, algorithm="maxima")
[Out] -sqrt(sin(x)^2 + 1)/sin(x)
```

mupad [B] time = 3.11, size = 34, normalized size = 2.43

$$-\frac{\sqrt{\frac{1}{\sin(x)^2}+1}}{\sin(x)\left(\sqrt{\frac{1}{\sin(x)^2}+1}+1\right)\sqrt{\sin(x)^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(x)/(sin(x)*(sin(x)^2 + 1)^(1/2)),x)
[Out] -(1/sin(x)^2 + 1)^(1/2)/(sin(x)*((1/sin(x)^2 + 1)^(1/2) + 1)*(sin(x)^2 + 1)
^(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cot(x)\csc(x)}{\sqrt{\sin^2(x)+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)*csc(x)/(1+sin(x)**2)**(1/2),x)
[Out] Integral(cot(x)*csc(x)/sqrt(sin(x)**2 + 1), x)
```

3.740 $\int \frac{\cot(5x)\csc^3(5x)}{\sqrt{1+\sin^2(5x)}}\,dx$

Optimal. Leaf size=43

$$\frac{2}{15}\sqrt{\sin^2(5x)+1}\,\csc(5x)-\frac{1}{15}\sqrt{\sin^2(5x)+1}\,\csc^3(5x)$$

```
[Out] 2/15*csc(5*x)*(1+sin(5*x)^2)^(1/2)-1/15*csc(5*x)^3*(1+sin(5*x)^2)^(1/2)
```

Rubi [A] time = 0.11, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {271, 264}

$$\frac{2}{15}\sqrt{\sin^2(5x)+1}\,\csc(5x)-\frac{1}{15}\sqrt{\sin^2(5x)+1}\,\csc^3(5x)$$

Antiderivative was successfully verified.

```
[In] Int[(Cot[5*x]*Csc[5*x]^3)/Sqrt[1 + Sin[5*x]^2],x]
```

```
[Out] (2*Csc[5*x]*Sqrt[1 + Sin[5*x]^2])/15 - (Csc[5*x]^3*Sqrt[1 + Sin[5*x]^2])/15
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 271

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x^(m + 1)*(
a + b*x^n)^(p + 1))/(a*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*(m +
1)), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{\cot(5x)\csc^3(5x)}{\sqrt{1+\sin^2(5x)}}\,dx = \frac{1}{5}\,\text{Subst}\left(\int \frac{1}{x^4\sqrt{1+x^2}}\,dx, x, \sin(5x)\right)$$
$$= -\frac{1}{15}\csc^3(5x)\sqrt{1+\sin^2(5x)} - \frac{2}{15}\,\text{Subst}\left(\int \frac{1}{x^2\sqrt{1+x^2}}\,dx, x, \sin(5x)\right)$$
$$= \frac{2}{15}\csc(5x)\sqrt{1+\sin^2(5x)} - \frac{1}{15}\csc^3(5x)\sqrt{1+\sin^2(5x)}$$

Mathematica [**A**] time = 0.05, size = 28, normalized size = 0.65

$$-\frac{1}{15}\sqrt{\sin^2(5x)+1}\,\csc(5x)\left(\csc^2(5x)-2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cot[5*x]*Csc[5*x]^3)/Sqrt[1 + Sin[5*x]^2],x]
```

```
[Out] -1/15*(Csc[5*x]*(-2 + Csc[5*x]^2)*Sqrt[1 + Sin[5*x]^2])
```

fricas [**A**] time = 0.68, size = 57, normalized size = 1.33

$$-\frac{2\left(\cos\left(5\,x\right)^2-1\right)\sin\left(5\,x\right)-\left(2\,\cos\left(5\,x\right)^2-1\right)\sqrt{-\cos\left(5\,x\right)^2+2}}{15\left(\cos\left(5\,x\right)^2-1\right)\sin\left(5\,x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(5*x)*csc(5*x)^3/(1+sin(5*x)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/15*(2*(cos(5*x)^2 - 1)*sin(5*x) - (2*cos(5*x)^2 - 1)*sqrt(-cos(5*x)^2 +
2))/((cos(5*x)^2 - 1)*sin(5*x))
```

giac [**A**] time = 0.51, size = 48, normalized size = 1.12

$$\frac{4\left(3\left(\sqrt{\sin\left(5\,x\right)^2+1}-\sin\left(5\,x\right)\right)^2-1\right)}{15\left(\left(\sqrt{\sin\left(5\,x\right)^2+1}-\sin\left(5\,x\right)\right)^2-1\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(5*x)*csc(5*x)^3/(1+sin(5*x)^2)^(1/2),x, algorithm="giac")
```

```
[Out] 4/15*(3*(sqrt(sin(5*x)^2 + 1) - sin(5*x))^2 - 1)/((sqrt(sin(5*x)^2 + 1) - s
in(5*x))^2 - 1)^3
```

maple [A] time = 0.24, size = 38, normalized size = 0.88

$$-\frac{\sqrt{1+\sin^2(5x)}}{15\sin(5x)^3}+\frac{2\sqrt{1+\sin^2(5x)}}{15\sin(5x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(5*x)*csc(5*x)^3/(1+sin(5*x)^2)^(1/2),x)
[Out] -1/15/sin(5*x)^3*(1+sin(5*x)^2)^(1/2)+2/15/sin(5*x)*(1+sin(5*x)^2)^(1/2)
```

maxima [A] time = 0.40, size = 37, normalized size = 0.86

$$\frac{2\sqrt{\sin(5x)^2+1}}{15\sin(5x)}-\frac{\sqrt{\sin(5x)^2+1}}{15\sin(5x)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(5*x)*csc(5*x)^3/(1+sin(5*x)^2)^(1/2),x, algorithm="maxima")
[Out] 2/15*sqrt(sin(5*x)^2 + 1)/sin(5*x) - 1/15*sqrt(sin(5*x)^2 + 1)/sin(5*x)^3
```

mupad [B] time = 3.14, size = 28, normalized size = 0.65

$$\frac{\sqrt{\sin(5x)^2+1}\left(2\sin(5x)^2-1\right)}{15\sin(5x)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(5*x)/(sin(5*x)^3*(sin(5*x)^2 + 1)^(1/2)),x)
[Out] ((sin(5*x)^2 + 1)^(1/2)*(2*sin(5*x)^2 - 1))/(15*sin(5*x)^3)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\cot(5x)\csc^3(5x)}{\sqrt{\sin^2(5x)+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(5*x)*csc(5*x)**3/(1+sin(5*x)**2)**(1/2),x)
[Out] Integral(cot(5*x)*csc(5*x)**3/sqrt(sin(5*x)**2 + 1), x)
```

3.741 $\int e^{n\sin(a+bx)}\sin(2a+2bx)\,dx$

Optimal. Leaf size=43

$$\frac{2\sin(a+bx)e^{n\sin(a+bx)}}{bn}-\frac{2e^{n\sin(a+bx)}}{bn^2}$$

```
[Out] -2*exp(n*sin(b*x+a))/b/n^2+2*exp(n*sin(b*x+a))*sin(b*x+a)/b/n
```

Rubi [A] time = 0.04, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {12, 2176, 2194}

$$\frac{2\sin(a+bx)e^{n\sin(a+bx)}}{bn}-\frac{2e^{n\sin(a+bx)}}{bn^2}$$

Antiderivative was successfully verified.

```
[In] Int[E^(n*Sin[a + b*x])*Sin[2*a + 2*b*x],x]
```

```
[Out] (-2*E^(n*Sin[a + b*x]))/(b*n^2) + (2*E^(n*Sin[a + b*x])*Sin[a + b*x])/(b*n)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2176

```
Int[((b_.)*(F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m
_.), x_Symbol] :> Simp[((c + d*x)^m*(b*F^(g*(e + f*x)))^n)/(f*g*n*Log[F]),
x] - Dist[(d*m)/(f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*(b*F^(g*(e + f*x)))^n
, x], x] /; FreeQ[{F, b, c, d, e, f, g, n}, x] && GtQ[m, 0] && IntegerQ[2*m
] &&  !$UseGamma === True
```

Rule 2194

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rubi steps

$$\int e^{n\sin(a+bx)}\sin(2a+2bx)\,dx = \frac{\mathrm{Subst}\left(\int 2e^{nx}x\,dx,x,\sin(a+bx)\right)}{b}$$
$$= \frac{2\,\mathrm{Subst}\left(\int e^{nx}x\,dx,x,\sin(a+bx)\right)}{b}$$
$$= \frac{2e^{n\sin(a+bx)}\sin(a+bx)}{bn} - \frac{2\,\mathrm{Subst}\left(\int e^{nx}\,dx,x,\sin(a+bx)\right)}{bn}$$
$$= -\frac{2e^{n\sin(a+bx)}}{bn^2} + \frac{2e^{n\sin(a+bx)}\sin(a+bx)}{bn}$$

Mathematica [A] time = 0.06, size = 28, normalized size = 0.65

$$\frac{2e^{n\sin(a+bx)}(n\sin(a+bx)-1)}{bn^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(n*Sin[a + b*x])*Sin[2*a + 2*b*x],x]
[Out] (2*E^(n*Sin[a + b*x])*(-1 + n*Sin[a + b*x]))/(b*n^2)
```

fricas [A] time = 0.60, size = 27, normalized size = 0.63

$$\frac{2\left(n\sin\left(bx+a\right)-1\right)e^{\left(n\sin\left(bx+a\right)\right)}}{bn^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*sin(b*x+a))*sin(2*b*x+2*a),x, algorithm="fricas")
[Out] 2*(n*sin(b*x + a) - 1)*e^(n*sin(b*x + a))/(b*n^2)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int e^{\left(n\sin\left(bx+a\right)\right)}\sin\left(2\,bx+2\,a\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*sin(b*x+a))*sin(2*b*x+2*a),x, algorithm="giac")
[Out] integrate(e^(n*sin(b*x + a))*sin(2*b*x + 2*a), x)
```

maple [C] time = 0.14, size = 104, normalized size = 2.42

$$-\frac{i\mathrm{e}^{n\sin(bx)\cos(a)+n\cos(bx)\sin(a)}\mathrm{e}^{ibx}\mathrm{e}^{ia}}{nb}+\frac{i\mathrm{e}^{n\sin(bx)\cos(a)+n\cos(bx)\sin(a)}\mathrm{e}^{-ibx}\mathrm{e}^{-ia}}{nb}-\frac{2\,\mathrm{e}^{n(\sin(bx)\cos(a)+\cos(bx)\sin(a))}}{n^2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*sin(b*x+a))*sin(2*b*x+2*a),x)
```

```
[Out] -I/n/b*exp(n*sin(b*x)*cos(a)+n*cos(b*x)*sin(a))*exp(I*b*x)*exp(I*a)+I/n/b*e
xp(n*sin(b*x)*cos(a)+n*cos(b*x)*sin(a))*exp(-I*b*x)*exp(-I*a)-2/n^2/b*exp(n
*(sin(b*x)*cos(a)+cos(b*x)*sin(a)))
```

maxima [A] time = 0.34, size = 37, normalized size = 0.86

$$\frac{2\left(ne^{(n\sin(bx+a))}\sin\left(bx+a\right)-e^{(n\sin(bx+a))}\right)}{bn^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*sin(b*x+a))*sin(2*b*x+2*a),x, algorithm="maxima")
```

```
[Out] 2*(n*e^(n*sin(b*x + a))*sin(b*x + a) - e^(n*sin(b*x + a)))/(b*n^2)
```

mupad [B] time = 3.18, size = 27, normalized size = 0.63

$$\frac{2\,\mathrm{e}^{n\,\sin(a+b\,x)}\,\left(n\,\sin\left(a+b\,x\right)-1\right)}{b\,n^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*sin(a + b*x))*sin(2*a + 2*b*x),x)
```

```
[Out] (2*exp(n*sin(a + b*x))*(n*sin(a + b*x) - 1))/(b*n^2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int e^{n\sin\left(a+bx\right)}\sin\left(2a+2bx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*sin(b*x+a))*sin(2*b*x+2*a),x)
```

```
[Out] Integral(exp(n*sin(a + b*x))*sin(2*a + 2*b*x), x)
```

3.742 $\int e^{n \sin(a+bx)} \sin(2(a+bx)) \, dx$

Optimal. Leaf size=43

$$\frac{2 \sin(a+bx) e^{n \sin(a+bx)}}{bn} - \frac{2 e^{n \sin(a+bx)}}{bn^2}$$

```
[Out] -2*exp(n*sin(b*x+a))/b/n^2+2*exp(n*sin(b*x+a))*sin(b*x+a)/b/n
```

Rubi [A] time = 0.03, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {12, 2176, 2194}

$$\frac{2 \sin(a+bx) e^{n \sin(a+bx)}}{bn} - \frac{2 e^{n \sin(a+bx)}}{bn^2}$$

Antiderivative was successfully verified.

```
[In] Int[E^(n*Sin[a + b*x])*Sin[2*(a + b*x)],x]
```

```
[Out] (-2*E^(n*Sin[a + b*x]))/(b*n^2) + (2*E^(n*Sin[a + b*x])*Sin[a + b*x])/(b*n)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2176

```
Int[((b_.)*(F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m
_.), x_Symbol] :> Simp[((c + d*x)^m*(b*F^(g*(e + f*x)))^n)/(f*g*n*Log[F]),
x] - Dist[(d*m)/(f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*(b*F^(g*(e + f*x)))^n
, x], x] /; FreeQ[{F, b, c, d, e, f, g, n}, x] && GtQ[m, 0] && IntegerQ[2*m
] &&  !$UseGamma === True
```

Rule 2194

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rubi steps

$$\begin{aligned}\int e^{n\sin(a+bx)}\sin(2(a+bx))\,dx &= \frac{\text{Subst}\left(\int 2e^{nx}x\,dx, x, \sin(a+bx)\right)}{b}\\ &= \frac{2\,\text{Subst}\left(\int e^{nx}x\,dx, x, \sin(a+bx)\right)}{b}\\ &= \frac{2e^{n\sin(a+bx)}\sin(a+bx)}{bn} - \frac{2\,\text{Subst}\left(\int e^{nx}\,dx, x, \sin(a+bx)\right)}{bn}\\ &= -\frac{2e^{n\sin(a+bx)}}{bn^2} + \frac{2e^{n\sin(a+bx)}\sin(a+bx)}{bn}\end{aligned}$$

Mathematica [A] time = 0.03, size = 28, normalized size = 0.65

$$\frac{2e^{n\sin(a+bx)}(n\sin(a+bx)-1)}{bn^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(n*Sin[a + b*x])*Sin[2*(a + b*x)],x]
[Out] (2*E^(n*Sin[a + b*x])*(-1 + n*Sin[a + b*x]))/(b*n^2)
```

fricas [A] time = 0.65, size = 27, normalized size = 0.63

$$\frac{2\,(n\sin(bx+a)-1)e^{(n\sin(bx+a))}}{bn^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*sin(b*x+a))*sin(2*b*x+2*a),x, algorithm="fricas")
[Out] 2*(n*sin(b*x + a) - 1)*e^(n*sin(b*x + a))/(b*n^2)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int e^{(n\sin(bx+a))}\sin(2\,bx+2\,a)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*sin(b*x+a))*sin(2*b*x+2*a),x, algorithm="giac")
[Out] integrate(e^(n*sin(b*x + a))*sin(2*b*x + 2*a), x)
```

maple [C] time = 0.00, size = 104, normalized size = 2.42

$$-\frac{i e^{n \sin(bx) \cos(a)+n \cos(bx) \sin(a)} e^{ibx} e^{ia}}{nb}+\frac{i e^{n \sin(bx) \cos(a)+n \cos(bx) \sin(a)} e^{-ibx} e^{-ia}}{nb}-\frac{2 e^{n(\sin(bx) \cos(a)+\cos(bx) \sin(a))}}{n^2 b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*sin(b*x+a))*sin(2*b*x+2*a),x)
```

```
[Out] -I/n/b*exp(n*sin(b*x)*cos(a)+n*cos(b*x)*sin(a))*exp(I*b*x)*exp(I*a)+I/n/b*e
xp(n*sin(b*x)*cos(a)+n*cos(b*x)*sin(a))*exp(-I*b*x)*exp(-I*a)-2/n^2/b*exp(n
*(sin(b*x)*cos(a)+cos(b*x)*sin(a)))
```

maxima [A] time = 0.35, size = 37, normalized size = 0.86

$$\frac{2 \left(n e^{(n \sin(bx+a))} \sin(bx+a) - e^{(n \sin(bx+a))}\right)}{bn^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*sin(b*x+a))*sin(2*b*x+2*a),x, algorithm="maxima")
```

```
[Out] 2*(n*e^(n*sin(b*x + a))*sin(b*x + a) - e^(n*sin(b*x + a)))/(b*n^2)
```

mupad [B] time = 0.00, size = 27, normalized size = 0.63

$$\frac{2 e^{n \sin(a+b x)} (n \sin(a+b x)-1)}{b n^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*sin(a + b*x))*sin(2*a + 2*b*x),x)
```

```
[Out] (2*exp(n*sin(a + b*x))*(n*sin(a + b*x) - 1))/(b*n^2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int e^{n \sin(a+bx)} \sin(2a+2bx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*sin(b*x+a))*sin(2*b*x+2*a),x)
```

```
[Out] Integral(exp(n*sin(a + b*x))*sin(2*a + 2*b*x), x)
```

3.743 $\int e^{n \sin\left(\frac{a}{2}+\frac{bx}{2}\right)} \sin(a+bx)\,dx$

Optimal. Leaf size=64

$$\frac{4\sin\left(\frac{a}{2}+\frac{bx}{2}\right)e^{n\sin\left(\frac{a}{2}+\frac{bx}{2}\right)}}{bn}-\frac{4e^{n\sin\left(\frac{a}{2}+\frac{bx}{2}\right)}}{bn^2}$$

```
[Out] -4*exp(n*sin(1/2*b*x+1/2*a))/b/n^2+4*exp(n*sin(1/2*b*x+1/2*a))*sin(1/2*b*x+
1/2*a)/b/n
```

Rubi [A] time = 0.04, antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {12, 2176, 2194}

$$\frac{4\sin\left(\frac{a}{2}+\frac{bx}{2}\right)e^{n\sin\left(\frac{a}{2}+\frac{bx}{2}\right)}}{bn}-\frac{4e^{n\sin\left(\frac{a}{2}+\frac{bx}{2}\right)}}{bn^2}$$

Antiderivative was successfully verified.

```
[In] Int[E^(n*Sin[a/2 + (b*x)/2])*Sin[a + b*x],x]
```

```
[Out] (-4*E^(n*Sin[a/2 + (b*x)/2]))/(b*n^2) + (4*E^(n*Sin[a/2 + (b*x)/2])*Sin[a/2
 + (b*x)/2])/(b*n)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2176

```
Int[((b_.)*(F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m
_.), x_Symbol] :> Simp[((c + d*x)^m*(b*F^(g*(e + f*x)))^n)/(f*g*n*Log[F]),
x] - Dist[(d*m)/(f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*(b*F^(g*(e + f*x)))^n
, x], x] /; FreeQ[{F, b, c, d, e, f, g, n}, x] && GtQ[m, 0] && IntegerQ[2*m
] &&  !$UseGamma === True
```

Rule 2194

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rubi steps

$$\int e^{n\sin\left(\frac{a}{2}+\frac{bx}{2}\right)}\sin(a+bx)\,dx = \frac{2\,\text{Subst}\left(\int 2e^{nx}x\,dx, x, \sin\left(\frac{a}{2}+\frac{bx}{2}\right)\right)}{b}$$
$$= \frac{4\,\text{Subst}\left(\int e^{nx}x\,dx, x, \sin\left(\frac{a}{2}+\frac{bx}{2}\right)\right)}{b}$$
$$= \frac{4e^{n\sin\left(\frac{a}{2}+\frac{bx}{2}\right)}\sin\left(\frac{a}{2}+\frac{bx}{2}\right)}{bn} - \frac{4\,\text{Subst}\left(\int e^{nx}\,dx, x, \sin\left(\frac{a}{2}+\frac{bx}{2}\right)\right)}{bn}$$
$$= -\frac{4e^{n\sin\left(\frac{a}{2}+\frac{bx}{2}\right)}}{bn^2} + \frac{4e^{n\sin\left(\frac{a}{2}+\frac{bx}{2}\right)}\sin\left(\frac{a}{2}+\frac{bx}{2}\right)}{bn}$$

Mathematica [A] time = 0.06, size = 36, normalized size = 0.56

$$\frac{4e^{n\sin\left(\frac{1}{2}(a+bx)\right)}\left(n\sin\left(\frac{1}{2}(a+bx)\right)-1\right)}{bn^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(n*Sin[a/2 + (b*x)/2])*Sin[a + b*x],x]
[Out] (4*E^(n*Sin[(a + b*x)/2])*(-1 + n*Sin[(a + b*x)/2]))/(b*n^2)
```

fricas [A] time = 0.98, size = 33, normalized size = 0.52

$$\frac{4\left(n\sin\left(\frac{1}{2}\,bx+\frac{1}{2}\,a\right)-1\right)e^{\left(n\sin\left(\frac{1}{2}\,bx+\frac{1}{2}\,a\right)\right)}}{bn^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*sin(1/2*a+1/2*b*x))*sin(b*x+a),x, algorithm="fricas")
[Out] 4*(n*sin(1/2*b*x + 1/2*a) - 1)*e^(n*sin(1/2*b*x + 1/2*a))/(b*n^2)
```

giac [B] time = 0.23, size = 138, normalized size = 2.16

$$\frac{4\left(2\,ne^{\left(\frac{2\,n\tan\left(\frac{1}{4}\,bx+\frac{1}{4}\,a\right)}{\tan\left(\frac{1}{4}\,bx+\frac{1}{4}\,a\right)^2+1}\right)}\tan\left(\frac{1}{4}\,bx+\frac{1}{4}\,a\right)-e^{\left(\frac{2\,n\tan\left(\frac{1}{4}\,bx+\frac{1}{4}\,a\right)}{\tan\left(\frac{1}{4}\,bx+\frac{1}{4}\,a\right)^2+1}\right)}\tan\left(\frac{1}{4}\,bx+\frac{1}{4}\,a\right)^2-e^{\left(\frac{2\,n\tan\left(\frac{1}{4}\,bx+\frac{1}{4}\,a\right)}{\tan\left(\frac{1}{4}\,bx+\frac{1}{4}\,a\right)^2+1}\right)}\right)}{bn^2\tan\left(\frac{1}{4}\,bx+\frac{1}{4}\,a\right)^2+bn^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*sin(1/2*a+1/2*b*x))*sin(b*x+a),x, algorithm="giac")
```

```
[Out] 4*(2*n*e^(2*n*tan(1/4*b*x + 1/4*a)/(tan(1/4*b*x + 1/4*a)^2 + 1))*tan(1/4*b*
x + 1/4*a) - e^(2*n*tan(1/4*b*x + 1/4*a)/(tan(1/4*b*x + 1/4*a)^2 + 1))*tan(
1/4*b*x + 1/4*a)^2 - e^(2*n*tan(1/4*b*x + 1/4*a)/(tan(1/4*b*x + 1/4*a)^2 +
1)))/(b*n^2*tan(1/4*b*x + 1/4*a)^2 + b*n^2)
```

maple [C] time = 0.13, size = 122, normalized size = 1.91

$$\frac{2ie^{n\sin\left(\frac{bx}{2}\right)\cos\left(\frac{a}{2}\right)+n\cos\left(\frac{bx}{2}\right)\sin\left(\frac{a}{2}\right)}\mathrm{e}^{-\frac{ibx}{2}}\mathrm{e}^{-\frac{ia}{2}}}{nb}-\frac{2ie^{n\sin\left(\frac{bx}{2}\right)\cos\left(\frac{a}{2}\right)+n\cos\left(\frac{bx}{2}\right)\sin\left(\frac{a}{2}\right)}\mathrm{e}^{\frac{ibx}{2}}\mathrm{e}^{\frac{ia}{2}}}{nb}-\frac{4\,\mathrm{e}^{n\left(\sin\left(\frac{bx}{2}\right)\cos\left(\frac{a}{2}\right)+\cos\left(\frac{bx}{2}\right)\sin\left(\frac{a}{2}\right)\right)}}{n^{2}b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*sin(1/2*b*x+1/2*a))*sin(b*x+a),x)
```

```
[Out] 2*I/n/b*exp(n*sin(1/2*b*x)*cos(1/2*a)+n*cos(1/2*b*x)*sin(1/2*a))*exp(-1/2*I
*b*x)*exp(-1/2*I*a)-2*I/n/b*exp(n*sin(1/2*b*x)*cos(1/2*a)+n*cos(1/2*b*x)*si
n(1/2*a))*exp(1/2*I*b*x)*exp(1/2*I*a)-4/n^2/b*exp(n*(sin(1/2*b*x)*cos(1/2*a
)+cos(1/2*b*x)*sin(1/2*a)))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int e^{\left(n\sin\left(\frac{1}{2}\,bx+\frac{1}{2}\,a\right)\right)}\sin\left(bx+a\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*sin(1/2*a+1/2*b*x))*sin(b*x+a),x, algorithm="maxima")
```

```
[Out] integrate(e^(n*sin(1/2*b*x + 1/2*a))*sin(b*x + a), x)
```

mupad [B] time = 3.18, size = 33, normalized size = 0.52

$$\frac{4\,\mathrm{e}^{n\,\sin\left(\frac{a}{2}+\frac{b\,x}{2}\right)}\,\left(n\,\sin\left(\frac{a}{2}+\frac{b\,x}{2}\right)-1\right)}{b\,n^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*sin(a/2 + (b*x)/2))*sin(a + b*x),x)
```

```
[Out] (4*exp(n*sin(a/2 + (b*x)/2))*(n*sin(a/2 + (b*x)/2) - 1))/(b*n^2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int e^{n\sin\left(\frac{a}{2}+\frac{bx}{2}\right)}\sin\left(a+bx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*sin(1/2*a+1/2*b*x))*sin(b*x+a),x)
```

```
[Out] Integral(exp(n*sin(a/2 + b*x/2))*sin(a + b*x), x)
```

3.744 $\int e^{n\sin\left(\frac{1}{2}(a+bx)\right)}\sin(a+bx)\,dx$

Optimal. Leaf size=64

$$\frac{4\sin\left(\frac{a}{2}+\frac{bx}{2}\right)e^{n\sin\left(\frac{a}{2}+\frac{bx}{2}\right)}}{bn}-\frac{4e^{n\sin\left(\frac{a}{2}+\frac{bx}{2}\right)}}{bn^2}$$

```
[Out] -4*exp(n*sin(1/2*b*x+1/2*a))/b/n^2+4*exp(n*sin(1/2*b*x+1/2*a))*sin(1/2*b*x+
1/2*a)/b/n
```

Rubi [A] time = 0.04, antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {12, 2176, 2194}

$$\frac{4\sin\left(\frac{a}{2}+\frac{bx}{2}\right)e^{n\sin\left(\frac{a}{2}+\frac{bx}{2}\right)}}{bn}-\frac{4e^{n\sin\left(\frac{a}{2}+\frac{bx}{2}\right)}}{bn^2}$$

Antiderivative was successfully verified.

```
[In] Int[E^(n*Sin[(a + b*x)/2])*Sin[a + b*x],x]
```

```
[Out] (-4*E^(n*Sin[a/2 + (b*x)/2]))/(b*n^2) + (4*E^(n*Sin[a/2 + (b*x)/2])*Sin[a/2
 + (b*x)/2])/(b*n)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2176

```
Int[((b_.)*(F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m
_.), x_Symbol] :> Simp[((c + d*x)^m*(b*F^(g*(e + f*x)))^n)/(f*g*n*Log[F]),
x] - Dist[(d*m)/(f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*(b*F^(g*(e + f*x)))^n
, x], x] /; FreeQ[{F, b, c, d, e, f, g, n}, x] && GtQ[m, 0] && IntegerQ[2*m
] &&  !$UseGamma === True
```

Rule 2194

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rubi steps

$$\int e^{n\sin\left(\frac{1}{2}(a+bx)\right)}\sin(a+bx)\,dx = \frac{2\,\text{Subst}\left(\int 2e^{nx}x\,dx, x, \sin\left(\frac{a}{2}+\frac{bx}{2}\right)\right)}{b}$$
$$= \frac{4\,\text{Subst}\left(\int e^{nx}x\,dx, x, \sin\left(\frac{a}{2}+\frac{bx}{2}\right)\right)}{b}$$
$$= \frac{4e^{n\sin\left(\frac{a}{2}+\frac{bx}{2}\right)}\sin\left(\frac{a}{2}+\frac{bx}{2}\right)}{bn} - \frac{4\,\text{Subst}\left(\int e^{nx}\,dx, x, \sin\left(\frac{a}{2}+\frac{bx}{2}\right)\right)}{bn}$$
$$= -\frac{4e^{n\sin\left(\frac{a}{2}+\frac{bx}{2}\right)}}{bn^2} + \frac{4e^{n\sin\left(\frac{a}{2}+\frac{bx}{2}\right)}\sin\left(\frac{a}{2}+\frac{bx}{2}\right)}{bn}$$

Mathematica [A] time = 0.03, size = 36, normalized size = 0.56

$$\frac{4e^{n\sin\left(\frac{1}{2}(a+bx)\right)}\left(n\sin\left(\frac{1}{2}(a+bx)\right)-1\right)}{bn^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(n*Sin[(a + b*x)/2])*Sin[a + b*x],x]
[Out] (4*E^(n*Sin[(a + b*x)/2])*(-1 + n*Sin[(a + b*x)/2]))/(b*n^2)
```

fricas [A] time = 0.69, size = 33, normalized size = 0.52

$$\frac{4\left(n\sin\left(\frac{1}{2}\,bx+\frac{1}{2}\,a\right)-1\right)e^{\left(n\sin\left(\frac{1}{2}\,bx+\frac{1}{2}\,a\right)\right)}}{bn^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*sin(1/2*a+1/2*b*x))*sin(b*x+a),x, algorithm="fricas")
[Out] 4*(n*sin(1/2*b*x + 1/2*a) - 1)*e^(n*sin(1/2*b*x + 1/2*a))/(b*n^2)
```

giac [B] time = 0.24, size = 138, normalized size = 2.16

$$\frac{4\left(2\,ne^{\left(\frac{2\,n\tan\left(\frac{1}{4}\,bx+\frac{1}{4}\,a\right)}{\tan\left(\frac{1}{4}\,bx+\frac{1}{4}\,a\right)^2+1}\right)}\tan\left(\frac{1}{4}\,bx+\frac{1}{4}\,a\right)-e^{\left(\frac{2\,n\tan\left(\frac{1}{4}\,bx+\frac{1}{4}\,a\right)}{\tan\left(\frac{1}{4}\,bx+\frac{1}{4}\,a\right)^2+1}\right)}\tan\left(\frac{1}{4}\,bx+\frac{1}{4}\,a\right)^2-e^{\left(\frac{2\,n\tan\left(\frac{1}{4}\,bx+\frac{1}{4}\,a\right)}{\tan\left(\frac{1}{4}\,bx+\frac{1}{4}\,a\right)^2+1}\right)}\right)}{bn^2\tan\left(\frac{1}{4}\,bx+\frac{1}{4}\,a\right)^2+bn^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*sin(1/2*a+1/2*b*x))*sin(b*x+a),x, algorithm="giac")
```

```
[Out] 4*(2*n*e^(2*n*tan(1/4*b*x + 1/4*a)/(tan(1/4*b*x + 1/4*a)^2 + 1))*tan(1/4*b*
x + 1/4*a) - e^(2*n*tan(1/4*b*x + 1/4*a)/(tan(1/4*b*x + 1/4*a)^2 + 1))*tan(
1/4*b*x + 1/4*a)^2 - e^(2*n*tan(1/4*b*x + 1/4*a)/(tan(1/4*b*x + 1/4*a)^2 +
1)))/(b*n^2*tan(1/4*b*x + 1/4*a)^2 + b*n^2)
```

maple [C] time = 0.00, size = 122, normalized size = 1.91

$$\frac{2i\mathrm{e}^{n\sin\left(\frac{bx}{2}\right)\cos\left(\frac{a}{2}\right)+n\cos\left(\frac{bx}{2}\right)\sin\left(\frac{a}{2}\right)}\mathrm{e}^{-\frac{ibx}{2}}\mathrm{e}^{-\frac{ia}{2}}}{nb}-\frac{2i\mathrm{e}^{n\sin\left(\frac{bx}{2}\right)\cos\left(\frac{a}{2}\right)+n\cos\left(\frac{bx}{2}\right)\sin\left(\frac{a}{2}\right)}\mathrm{e}^{\frac{ibx}{2}}\mathrm{e}^{\frac{ia}{2}}}{nb}-\frac{4\,\mathrm{e}^{n\left(\sin\left(\frac{bx}{2}\right)\cos\left(\frac{a}{2}\right)+\cos\left(\frac{bx}{2}\right)\sin\left(\frac{a}{2}\right)\right)}}{n^{2}b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*sin(1/2*b*x+1/2*a))*sin(b*x+a),x)
```

```
[Out] 2*I/n/b*exp(n*sin(1/2*b*x)*cos(1/2*a)+n*cos(1/2*b*x)*sin(1/2*a))*exp(-1/2*I
*b*x)*exp(-1/2*I*a)-2*I/n/b*exp(n*sin(1/2*b*x)*cos(1/2*a)+n*cos(1/2*b*x)*si
n(1/2*a))*exp(1/2*I*b*x)*exp(1/2*I*a)-4/n^2/b*exp(n*(sin(1/2*b*x)*cos(1/2*a
)+cos(1/2*b*x)*sin(1/2*a)))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int e^{\left(n\sin\left(\frac{1}{2}\,bx+\frac{1}{2}\,a\right)\right)}\sin\left(bx+a\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*sin(1/2*a+1/2*b*x))*sin(b*x+a),x, algorithm="maxima")
```

```
[Out] integrate(e^(n*sin(1/2*b*x + 1/2*a))*sin(b*x + a), x)
```

mupad [B] time = 0.00, size = 33, normalized size = 0.52

$$\frac{4\,\mathrm{e}^{n\,\sin\left(\frac{a}{2}+\frac{b\,x}{2}\right)}\,\left(n\,\sin\left(\frac{a}{2}+\frac{b\,x}{2}\right)-1\right)}{b\,n^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*sin(a/2 + (b*x)/2))*sin(a + b*x),x)
```

```
[Out] (4*exp(n*sin(a/2 + (b*x)/2))*(n*sin(a/2 + (b*x)/2) - 1))/(b*n^2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int e^{n\sin\left(\frac{a}{2}+\frac{bx}{2}\right)}\sin\left(a+bx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*sin(1/2*a+1/2*b*x))*sin(b*x+a),x)
```

```
[Out] Integral(exp(n*sin(a/2 + b*x/2))*sin(a + b*x), x)
```

3.745 $\int e^{n\cos(a+bx)}\sin(2a+2bx)\,dx$

Optimal. Leaf size=43

$$\frac{2e^{n\cos(a+bx)}}{bn^2}-\frac{2\cos(a+bx)e^{n\cos(a+bx)}}{bn}$$

```
[Out] 2*exp(n*cos(b*x+a))/b/n^2-2*exp(n*cos(b*x+a))*cos(b*x+a)/b/n
```

Rubi [A] time = 0.04, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {12, 2176, 2194}

$$\frac{2e^{n\cos(a+bx)}}{bn^2}-\frac{2\cos(a+bx)e^{n\cos(a+bx)}}{bn}$$

Antiderivative was successfully verified.

```
[In] Int[E^(n*Cos[a + b*x])*Sin[2*a + 2*b*x],x]
```

```
[Out] (2*E^(n*Cos[a + b*x]))/(b*n^2) - (2*E^(n*Cos[a + b*x])*Cos[a + b*x])/(b*n)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2176

```
Int[((b_.)*(F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m
_.), x_Symbol] :> Simp[((c + d*x)^m*(b*F^(g*(e + f*x)))^n)/(f*g*n*Log[F]),
x] - Dist[(d*m)/(f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*(b*F^(g*(e + f*x)))^n
, x], x] /; FreeQ[{F, b, c, d, e, f, g, n}, x] && GtQ[m, 0] && IntegerQ[2*m
] &&  !$UseGamma === True
```

Rule 2194

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rubi steps

$$\begin{aligned}\int e^{n\cos(a+bx)}\sin(2a+2bx)\,dx &= -\frac{\text{Subst}\left(\int 2e^{nx}x\,dx,x,\cos(a+bx)\right)}{b}\\ &= -\frac{2\,\text{Subst}\left(\int e^{nx}x\,dx,x,\cos(a+bx)\right)}{b}\\ &= -\frac{2e^{n\cos(a+bx)}\cos(a+bx)}{bn}+\frac{2\,\text{Subst}\left(\int e^{nx}\,dx,x,\cos(a+bx)\right)}{bn}\\ &= \frac{2e^{n\cos(a+bx)}}{bn^2}-\frac{2e^{n\cos(a+bx)}\cos(a+bx)}{bn}\end{aligned}$$

Mathematica [A] time = 0.14, size = 28, normalized size = 0.65

$$-\frac{2e^{n\cos(a+bx)}(n\cos(a+bx)-1)}{bn^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(n*Cos[a + b*x])*Sin[2*a + 2*b*x],x]
[Out] (-2*E^(n*Cos[a + b*x])*(-1 + n*Cos[a + b*x]))/(b*n^2)
```

fricas [A] time = 0.54, size = 27, normalized size = 0.63

$$-\frac{2\,(n\cos{(bx+a)}-1)e^{(n\cos(bx+a))}}{bn^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*cos(b*x+a))*sin(2*b*x+2*a),x, algorithm="fricas")
[Out] -2*(n*cos(b*x + a) - 1)*e^(n*cos(b*x + a))/(b*n^2)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int e^{(n\cos(bx+a))}\sin{(2\,bx+2\,a)}\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*cos(b*x+a))*sin(2*b*x+2*a),x, algorithm="giac")
[Out] integrate(e^(n*cos(b*x + a))*sin(2*b*x + 2*a), x)
```

maple [C] time = 0.11, size = 105, normalized size = 2.44

$$-\frac{\mathrm{e}^{n\cos(bx)\cos(a)-n\sin(bx)\sin(a)}\mathrm{e}^{ibx}\mathrm{e}^{ia}}{bn}-\frac{\mathrm{e}^{n\cos(bx)\cos(a)-n\sin(bx)\sin(a)}\mathrm{e}^{-ibx}\mathrm{e}^{-ia}}{bn}+\frac{2\,\mathrm{e}^{n(\cos(bx)\cos(a)-\sin(bx)\sin(a))}}{b\,n^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*cos(b*x+a))*sin(2*b*x+2*a),x)
```

```
[Out] -1/b/n*exp(n*cos(b*x)*cos(a)-n*sin(b*x)*sin(a))*exp(I*b*x)*exp(I*a)-1/b/n*e
xp(n*cos(b*x)*cos(a)-n*sin(b*x)*sin(a))*exp(-I*b*x)*exp(-I*a)+2/b/n^2*exp(n
*(cos(b*x)*cos(a)-sin(b*x)*sin(a)))
```

maxima [A] time = 0.34, size = 37, normalized size = 0.86

$$-\frac{2\left(n\cos\left(bx+a\right)e^{\left(n\cos(bx+a)\right)}-e^{\left(n\cos(bx+a)\right)}\right)}{bn^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*cos(b*x+a))*sin(2*b*x+2*a),x, algorithm="maxima")
```

```
[Out] -2*(n*cos(b*x + a)*e^(n*cos(b*x + a)) - e^(n*cos(b*x + a)))/(b*n^2)
```

mupad [B] time = 3.21, size = 27, normalized size = 0.63

$$-\frac{2\,\mathrm{e}^{n\,\cos(a+b\,x)}\,\left(n\,\cos\left(a+b\,x\right)-1\right)}{b\,n^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*cos(a + b*x))*sin(2*a + 2*b*x),x)
```

```
[Out] -(2*exp(n*cos(a + b*x))*(n*cos(a + b*x) - 1))/(b*n^2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int e^{n\cos\left(a+bx\right)}\sin\left(2a+2bx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*cos(b*x+a))*sin(2*b*x+2*a),x)
```

```
[Out] Integral(exp(n*cos(a + b*x))*sin(2*a + 2*b*x), x)
```

3.746 $\int e^{n\cos(a+bx)}\sin(2(a+bx))\,dx$

Optimal. Leaf size=43

$$\frac{2e^{n\cos(a+bx)}}{bn^2}-\frac{2\cos(a+bx)e^{n\cos(a+bx)}}{bn}$$

```
[Out] 2*exp(n*cos(b*x+a))/b/n^2-2*exp(n*cos(b*x+a))*cos(b*x+a)/b/n
```

Rubi [A] time = 0.03, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}=0.158$, Rules used = {12, 2176, 2194}

$$\frac{2e^{n\cos(a+bx)}}{bn^2}-\frac{2\cos(a+bx)e^{n\cos(a+bx)}}{bn}$$

Antiderivative was successfully verified.

```
[In] Int[E^(n*Cos[a + b*x])*Sin[2*(a + b*x)],x]
```

```
[Out] (2*E^(n*Cos[a + b*x]))/(b*n^2) - (2*E^(n*Cos[a + b*x])*Cos[a + b*x])/(b*n)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2176

```
Int[((b_.)*(F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m
_.), x_Symbol] :> Simp[((c + d*x)^m*(b*F^(g*(e + f*x)))^n)/(f*g*n*Log[F]),
x] - Dist[(d*m)/(f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*(b*F^(g*(e + f*x)))^n
, x], x] /; FreeQ[{F, b, c, d, e, f, g, n}, x] && GtQ[m, 0] && IntegerQ[2*m
] &&  !$UseGamma === True
```

Rule 2194

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rubi steps

$$\int e^{n\cos(a+bx)}\sin(2(a+bx))\,dx = -\frac{\text{Subst}\left(\int 2e^{nx}x\,dx,x,\cos(a+bx)\right)}{b}$$
$$= -\frac{2\,\text{Subst}\left(\int e^{nx}x\,dx,x,\cos(a+bx)\right)}{b}$$
$$= -\frac{2e^{n\cos(a+bx)}\cos(a+bx)}{bn}+\frac{2\,\text{Subst}\left(\int e^{nx}\,dx,x,\cos(a+bx)\right)}{bn}$$
$$= \frac{2e^{n\cos(a+bx)}}{bn^2}-\frac{2e^{n\cos(a+bx)}\cos(a+bx)}{bn}$$

Mathematica [A] time = 0.03, size = 28, normalized size = 0.65

$$-\frac{2e^{n\cos(a+bx)}(n\cos(a+bx)-1)}{bn^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(n*Cos[a + b*x])*Sin[2*(a + b*x)],x]
[Out] (-2*E^(n*Cos[a + b*x])*(-1 + n*Cos[a + b*x]))/(b*n^2)
```

fricas [A] time = 0.72, size = 27, normalized size = 0.63

$$-\frac{2\,(n\cos\,(bx+a)-1)e^{(n\cos(bx+a))}}{bn^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*cos(b*x+a))*sin(2*b*x+2*a),x, algorithm="fricas")
[Out] -2*(n*cos(b*x + a) - 1)*e^(n*cos(b*x + a))/(b*n^2)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int e^{(n\cos(bx+a))}\sin\,(2\,bx+2\,a)\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*cos(b*x+a))*sin(2*b*x+2*a),x, algorithm="giac")
[Out] integrate(e^(n*cos(b*x + a))*sin(2*b*x + 2*a), x)
```

maple [C] time = 0.00, size = 105, normalized size = 2.44

$$-\frac{e^{n\cos(bx)\cos(a)-n\sin(bx)\sin(a)}e^{ibx}e^{ia}}{bn}-\frac{e^{n\cos(bx)\cos(a)-n\sin(bx)\sin(a)}e^{-ibx}e^{-ia}}{bn}+\frac{2\,e^{n(\cos(bx)\cos(a)-\sin(bx)\sin(a))}}{b\,n^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*cos(b*x+a))*sin(2*b*x+2*a),x)
```

```
[Out] -1/b/n*exp(n*cos(b*x)*cos(a)-n*sin(b*x)*sin(a))*exp(I*b*x)*exp(I*a)-1/b/n*e
xp(n*cos(b*x)*cos(a)-n*sin(b*x)*sin(a))*exp(-I*b*x)*exp(-I*a)+2/b/n^2*exp(n
*(cos(b*x)*cos(a)-sin(b*x)*sin(a)))
```

maxima [A] time = 0.34, size = 37, normalized size = 0.86

$$-\frac{2\left(n\cos\left(bx+a\right)e^{\left(n\cos(bx+a)\right)}-e^{\left(n\cos(bx+a)\right)}\right)}{bn^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*cos(b*x+a))*sin(2*b*x+2*a),x, algorithm="maxima")
```

```
[Out] -2*(n*cos(b*x + a)*e^(n*cos(b*x + a)) - e^(n*cos(b*x + a)))/(b*n^2)
```

mupad [B] time = 0.00, size = 27, normalized size = 0.63

$$-\frac{2\,e^{n\,\cos(a+b\,x)}\,\left(n\,\cos\left(a+b\,x\right)-1\right)}{b\,n^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*cos(a + b*x))*sin(2*a + 2*b*x),x)
```

```
[Out] -(2*exp(n*cos(a + b*x))*(n*cos(a + b*x) - 1))/(b*n^2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int e^{n\cos\left(a+bx\right)}\sin\left(2a+2bx\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*cos(b*x+a))*sin(2*b*x+2*a),x)
```

```
[Out] Integral(exp(n*cos(a + b*x))*sin(2*a + 2*b*x), x)
```

3.747 $\int e^{n\cos\left(\frac{a}{2}+\frac{bx}{2}\right)}\sin(a+bx)\,dx$

Optimal. Leaf size=64

$$\frac{4e^{n\cos\left(\frac{a}{2}+\frac{bx}{2}\right)}}{bn^2}-\frac{4\cos\left(\frac{a}{2}+\frac{bx}{2}\right)e^{n\cos\left(\frac{a}{2}+\frac{bx}{2}\right)}}{bn}$$

```
[Out] 4*exp(n*cos(1/2*b*x+1/2*a))/b/n^2-4*exp(n*cos(1/2*b*x+1/2*a))*cos(1/2*b*x+1
/2*a)/b/n
```

Rubi [A] time = 0.04, antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {12, 2176, 2194}

$$\frac{4e^{n\cos\left(\frac{a}{2}+\frac{bx}{2}\right)}}{bn^2}-\frac{4\cos\left(\frac{a}{2}+\frac{bx}{2}\right)e^{n\cos\left(\frac{a}{2}+\frac{bx}{2}\right)}}{bn}$$

Antiderivative was successfully verified.

```
[In] Int[E^(n*Cos[a/2 + (b*x)/2])*Sin[a + b*x],x]
```

```
[Out] (4*E^(n*Cos[a/2 + (b*x)/2]))/(b*n^2) - (4*E^(n*Cos[a/2 + (b*x)/2])*Cos[a/2
+ (b*x)/2])/(b*n)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2176

```
Int[((b_.)*(F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m
_.), x_Symbol] :> Simp[((c + d*x)^m*(b*F^(g*(e + f*x)))^n)/(f*g*n*Log[F]),
x] - Dist[(d*m)/(f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*(b*F^(g*(e + f*x)))^n
, x], x] /; FreeQ[{F, b, c, d, e, f, g, n}, x] && GtQ[m, 0] && IntegerQ[2*m
] &&  !$UseGamma === True
```

Rule 2194

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rubi steps

$$\int e^{n\cos\left(\frac{a}{2}+\frac{bx}{2}\right)}\sin(a+bx)\,dx = -\frac{2\,\mathrm{Subst}\left(\int 2e^{nx}x\,dx, x, \cos\left(\frac{a}{2}+\frac{bx}{2}\right)\right)}{b}$$
$$= -\frac{4\,\mathrm{Subst}\left(\int e^{nx}x\,dx, x, \cos\left(\frac{a}{2}+\frac{bx}{2}\right)\right)}{b}$$
$$= -\frac{4e^{n\cos\left(\frac{a}{2}+\frac{bx}{2}\right)}\cos\left(\frac{a}{2}+\frac{bx}{2}\right)}{bn} + \frac{4\,\mathrm{Subst}\left(\int e^{nx}\,dx, x, \cos\left(\frac{a}{2}+\frac{bx}{2}\right)\right)}{bn}$$
$$= \frac{4e^{n\cos\left(\frac{a}{2}+\frac{bx}{2}\right)}}{bn^2} - \frac{4e^{n\cos\left(\frac{a}{2}+\frac{bx}{2}\right)}\cos\left(\frac{a}{2}+\frac{bx}{2}\right)}{bn}$$

Mathematica [A] time = 0.17, size = 36, normalized size = 0.56

$$-\frac{4e^{n\cos\left(\frac{1}{2}(a+bx)\right)}\left(n\cos\left(\frac{1}{2}(a+bx)\right)-1\right)}{bn^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(n*Cos[a/2 + (b*x)/2])*Sin[a + b*x],x]
[Out] (-4*E^(n*Cos[(a + b*x)/2])*(-1 + n*Cos[(a + b*x)/2]))/(b*n^2)
```

fricas [A] time = 2.00, size = 33, normalized size = 0.52

$$-\frac{4\left(n\cos\left(\frac{1}{2}\,bx+\frac{1}{2}\,a\right)-1\right)e^{\left(n\cos\left(\frac{1}{2}\,bx+\frac{1}{2}\,a\right)\right)}}{bn^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*cos(1/2*a+1/2*b*x))*sin(b*x+a),x, algorithm="fricas")
[Out] -4*(n*cos(1/2*b*x + 1/2*a) - 1)*e^(n*cos(1/2*b*x + 1/2*a))/(b*n^2)
```

giac [B] time = 0.22, size = 195, normalized size = 3.05

$$\frac{4\left(ne^{\left(-\frac{n\tan\left(\frac{1}{4}\,bx+\frac{1}{4}\,a\right)^2-n}{\tan\left(\frac{1}{4}\,bx+\frac{1}{4}\,a\right)^2+1}\right)}\tan\left(\frac{1}{4}\,bx+\frac{1}{4}\,a\right)^2+e^{\left(-\frac{n\tan\left(\frac{1}{4}\,bx+\frac{1}{4}\,a\right)^2-n}{\tan\left(\frac{1}{4}\,bx+\frac{1}{4}\,a\right)^2+1}\right)}\tan\left(\frac{1}{4}\,bx+\frac{1}{4}\,a\right)^2-ne^{\left(-\frac{n\tan\left(\frac{1}{4}\,bx+\frac{1}{4}\,a\right)^2-n}{\tan\left(\frac{1}{4}\,bx+\frac{1}{4}\,a\right)^2+1}\right)}+e^{\left(-\frac{n\tan\left(\frac{1}{4}\,bx\right.}{\tan\left(\frac{1}{4}\,bx+\right.}\right.}}{bn^2\tan\left(\frac{1}{4}\,bx+\frac{1}{4}\,a\right)^2+bn^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*cos(1/2*a+1/2*b*x))*sin(b*x+a),x, algorithm="giac")
```

```
[Out] 4*(n*e^(-(n*tan(1/4*b*x + 1/4*a)^2 - n)/(tan(1/4*b*x + 1/4*a)^2 + 1))*tan(1
/4*b*x + 1/4*a)^2 + e^(-(n*tan(1/4*b*x + 1/4*a)^2 - n)/(tan(1/4*b*x + 1/4*a
)^2 + 1))*tan(1/4*b*x + 1/4*a)^2 - n*e^(-(n*tan(1/4*b*x + 1/4*a)^2 - n)/(ta
n(1/4*b*x + 1/4*a)^2 + 1)) + e^(-(n*tan(1/4*b*x + 1/4*a)^2 - n)/(tan(1/4*b*
x + 1/4*a)^2 + 1)))/(b*n^2*tan(1/4*b*x + 1/4*a)^2 + b*n^2)
```

maple [C] time = 0.12, size = 123, normalized size = 1.92

$$-\frac{2\,{\mathrm e}^{n \cos\left(\frac{b x}{2}\right) \cos\left(\frac{a}{2}\right)-n \sin\left(\frac{b x}{2}\right) \sin\left(\frac{a}{2}\right)} {\mathrm e}^{\frac{i b x}{2}} {\mathrm e}^{\frac{i a}{2}}}{b n}-\frac{2\,{\mathrm e}^{n \cos\left(\frac{b x}{2}\right) \cos\left(\frac{a}{2}\right)-n \sin\left(\frac{b x}{2}\right) \sin\left(\frac{a}{2}\right)} {\mathrm e}^{-\frac{i b x}{2}} {\mathrm e}^{-\frac{i a}{2}}}{b n}+\frac{4\,{\mathrm e}^{n \left(\cos\left(\frac{b x}{2}\right) \cos\left(\frac{a}{2}\right)-\sin\left(\frac{b x}{2}\right) \sin\left(\frac{a}{2}\right)\right)}}{b\,n^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*cos(1/2*b*x+1/2*a))*sin(b*x+a),x)
```

```
[Out] -2/b/n*exp(n*cos(1/2*b*x)*cos(1/2*a)-n*sin(1/2*b*x)*sin(1/2*a))*exp(1/2*I*b
*x)*exp(1/2*I*a)-2/b/n*exp(n*cos(1/2*b*x)*cos(1/2*a)-n*sin(1/2*b*x)*sin(1/2
*a))*exp(-1/2*I*b*x)*exp(-1/2*I*a)+4/b/n^2*exp(n*(cos(1/2*b*x)*cos(1/2*a)-s
in(1/2*b*x)*sin(1/2*a)))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int e^{\left(n \cos\left(\frac{1}{2}\,b x+\frac{1}{2}\,a\right)\right)} \sin\left(b x+a\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*cos(1/2*a+1/2*b*x))*sin(b*x+a),x, algorithm="maxima")
```

```
[Out] integrate(e^(n*cos(1/2*b*x + 1/2*a))*sin(b*x + a), x)
```

mupad [B] time = 3.17, size = 33, normalized size = 0.52

$$-\frac{4\,{\mathrm{e}}^{n\,\cos\left(\frac{a}{2}+\frac{b\,x}{2}\right)}\,\left(n\,\cos\left(\frac{a}{2}+\frac{b\,x}{2}\right)-1\right)}{b\,n^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*cos(a/2 + (b*x)/2))*sin(a + b*x),x)
```

```
[Out] -(4*exp(n*cos(a/2 + (b*x)/2))*(n*cos(a/2 + (b*x)/2) - 1))/(b*n^2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int e^{n\cos\left(\frac{a}{2}+\frac{bx}{2}\right)}\sin\left(a+bx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*cos(1/2*a+1/2*b*x))*sin(b*x+a),x)
[Out] Integral(exp(n*cos(a/2 + b*x/2))*sin(a + b*x), x)
```

3.748 $\int e^{n\cos\left(\frac{1}{2}(a+bx)\right)}\sin(a+bx)\,dx$

Optimal. Leaf size=64

$$\frac{4e^{n\cos\left(\frac{a}{2}+\frac{bx}{2}\right)}}{bn^2}-\frac{4\cos\left(\frac{a}{2}+\frac{bx}{2}\right)e^{n\cos\left(\frac{a}{2}+\frac{bx}{2}\right)}}{bn}$$

```
[Out] 4*exp(n*cos(1/2*b*x+1/2*a))/b/n^2-4*exp(n*cos(1/2*b*x+1/2*a))*cos(1/2*b*x+1
/2*a)/b/n
```

Rubi [A] time = 0.04, antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {12, 2176, 2194}

$$\frac{4e^{n\cos\left(\frac{a}{2}+\frac{bx}{2}\right)}}{bn^2}-\frac{4\cos\left(\frac{a}{2}+\frac{bx}{2}\right)e^{n\cos\left(\frac{a}{2}+\frac{bx}{2}\right)}}{bn}$$

Antiderivative was successfully verified.

```
[In] Int[E^(n*Cos[(a + b*x)/2])*Sin[a + b*x],x]
```

```
[Out] (4*E^(n*Cos[a/2 + (b*x)/2]))/(b*n^2) - (4*E^(n*Cos[a/2 + (b*x)/2])*Cos[a/2
+ (b*x)/2])/(b*n)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2176

```
Int[((b_.)*(F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m
_.), x_Symbol] :> Simp[((c + d*x)^m*(b*F^(g*(e + f*x)))^n)/(f*g*n*Log[F]),
x] - Dist[(d*m)/(f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*(b*F^(g*(e + f*x)))^n
, x], x] /; FreeQ[{F, b, c, d, e, f, g, n}, x] && GtQ[m, 0] && IntegerQ[2*m
] &&  !$UseGamma === True
```

Rule 2194

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rubi steps

$$\int e^{n\cos\left(\frac{1}{2}(a+bx)\right)}\sin(a+bx)\,dx = -\frac{2\,\text{Subst}\left(\int 2e^{nx}x\,dx, x, \cos\left(\frac{a}{2}+\frac{bx}{2}\right)\right)}{b}$$
$$= -\frac{4\,\text{Subst}\left(\int e^{nx}x\,dx, x, \cos\left(\frac{a}{2}+\frac{bx}{2}\right)\right)}{b}$$
$$= -\frac{4e^{n\cos\left(\frac{a}{2}+\frac{bx}{2}\right)}\cos\left(\frac{a}{2}+\frac{bx}{2}\right)}{bn}+\frac{4\,\text{Subst}\left(\int e^{nx}\,dx, x, \cos\left(\frac{a}{2}+\frac{bx}{2}\right)\right)}{bn}$$
$$= \frac{4e^{n\cos\left(\frac{a}{2}+\frac{bx}{2}\right)}}{bn^2}-\frac{4e^{n\cos\left(\frac{a}{2}+\frac{bx}{2}\right)}\cos\left(\frac{a}{2}+\frac{bx}{2}\right)}{bn}$$

Mathematica [A] time = 0.03, size = 36, normalized size = 0.56

$$-\frac{4e^{n\cos\left(\frac{1}{2}(a+bx)\right)}\left(n\cos\left(\frac{1}{2}(a+bx)\right)-1\right)}{bn^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(n*Cos[(a + b*x)/2])*Sin[a + b*x],x]
[Out] (-4*E^(n*Cos[(a + b*x)/2])*(-1 + n*Cos[(a + b*x)/2]))/(b*n^2)
```

fricas [A] time = 0.50, size = 33, normalized size = 0.52

$$-\frac{4\left(n\cos\left(\frac{1}{2}\,bx+\frac{1}{2}\,a\right)-1\right)e^{\left(n\cos\left(\frac{1}{2}\,bx+\frac{1}{2}\,a\right)\right)}}{bn^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*cos(1/2*a+1/2*b*x))*sin(b*x+a),x, algorithm="fricas")
[Out] -4*(n*cos(1/2*b*x + 1/2*a) - 1)*e^(n*cos(1/2*b*x + 1/2*a))/(b*n^2)
```

giac [B] time = 0.22, size = 195, normalized size = 3.05

$$\frac{4\left(ne^{\left(-\frac{n\tan\left(\frac{1}{4}\,bx+\frac{1}{4}\,a\right)^2-n}{\tan\left(\frac{1}{4}\,bx+\frac{1}{4}\,a\right)^2+1}\right)}\tan\left(\frac{1}{4}\,bx+\frac{1}{4}\,a\right)^2+e^{\left(-\frac{n\tan\left(\frac{1}{4}\,bx+\frac{1}{4}\,a\right)^2-n}{\tan\left(\frac{1}{4}\,bx+\frac{1}{4}\,a\right)^2+1}\right)}\tan\left(\frac{1}{4}\,bx+\frac{1}{4}\,a\right)^2-ne^{\left(-\frac{n\tan\left(\frac{1}{4}\,bx+\frac{1}{4}\,a\right)^2-n}{\tan\left(\frac{1}{4}\,bx+\frac{1}{4}\,a\right)^2+1}\right)}+e^{\left(-\frac{n\tan\left(\frac{1}{4}\,bx-\right.}{\tan\left(\frac{1}{4}\,bx+\right.}\right.}\right.}{bn^2\tan\left(\frac{1}{4}\,bx+\frac{1}{4}\,a\right)^2+bn^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*cos(1/2*a+1/2*b*x))*sin(b*x+a),x, algorithm="giac")
```

```
[Out] 4*(n*e^(-(n*tan(1/4*b*x + 1/4*a)^2 - n)/(tan(1/4*b*x + 1/4*a)^2 + 1))*tan(1
/4*b*x + 1/4*a)^2 + e^(-(n*tan(1/4*b*x + 1/4*a)^2 - n)/(tan(1/4*b*x + 1/4*a
)^2 + 1))*tan(1/4*b*x + 1/4*a)^2 - n*e^(-(n*tan(1/4*b*x + 1/4*a)^2 - n)/(ta
n(1/4*b*x + 1/4*a)^2 + 1)) + e^(-(n*tan(1/4*b*x + 1/4*a)^2 - n)/(tan(1/4*b*
x + 1/4*a)^2 + 1)))/(b*n^2*tan(1/4*b*x + 1/4*a)^2 + b*n^2)
```

maple [C] time = 0.00, size = 123, normalized size = 1.92

$$-\frac{2\,\mathrm{e}^{n\cos\left(\frac{bx}{2}\right)\cos\left(\frac{a}{2}\right)-n\sin\left(\frac{bx}{2}\right)\sin\left(\frac{a}{2}\right)}\mathrm{e}^{\frac{ibx}{2}}\mathrm{e}^{\frac{ia}{2}}}{bn}-\frac{2\,\mathrm{e}^{n\cos\left(\frac{bx}{2}\right)\cos\left(\frac{a}{2}\right)-n\sin\left(\frac{bx}{2}\right)\sin\left(\frac{a}{2}\right)}\mathrm{e}^{-\frac{ibx}{2}}\mathrm{e}^{-\frac{ia}{2}}}{bn}+\frac{4\,\mathrm{e}^{n\left(\cos\left(\frac{bx}{2}\right)\cos\left(\frac{a}{2}\right)-\sin\left(\frac{bx}{2}\right)\sin\left(\frac{a}{2}\right)\right)}}{b\,n^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*cos(1/2*b*x+1/2*a))*sin(b*x+a),x)
```

```
[Out] -2/b/n*exp(n*cos(1/2*b*x)*cos(1/2*a)-n*sin(1/2*b*x)*sin(1/2*a))*exp(1/2*I*b
*x)*exp(1/2*I*a)-2/b/n*exp(n*cos(1/2*b*x)*cos(1/2*a)-n*sin(1/2*b*x)*sin(1/2
*a))*exp(-1/2*I*b*x)*exp(-1/2*I*a)+4/b/n^2*exp(n*(cos(1/2*b*x)*cos(1/2*a)-s
in(1/2*b*x)*sin(1/2*a)))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int e^{\left(n\cos\left(\frac{1}{2}\,bx+\frac{1}{2}\,a\right)\right)}\sin\left(bx+a\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*cos(1/2*a+1/2*b*x))*sin(b*x+a),x, algorithm="maxima")
```

```
[Out] integrate(e^(n*cos(1/2*b*x + 1/2*a))*sin(b*x + a), x)
```

mupad [B] time = 0.00, size = 33, normalized size = 0.52

$$-\frac{4\,\mathrm{e}^{n\,\cos\left(\frac{a}{2}+\frac{b\,x}{2}\right)}\,\left(n\,\cos\left(\frac{a}{2}+\frac{b\,x}{2}\right)-1\right)}{b\,n^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*cos(a/2 + (b*x)/2))*sin(a + b*x),x)
```

```
[Out] -(4*exp(n*cos(a/2 + (b*x)/2))*(n*cos(a/2 + (b*x)/2) - 1))/(b*n^2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int e^{n\cos\left(\frac{a}{2}+\frac{bx}{2}\right)}\sin\left(a+bx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*cos(1/2*a+1/2*b*x))*sin(b*x+a),x)
```

```
[Out] Integral(exp(n*cos(a/2 + b*x/2))*sin(a + b*x), x)
```

3.749 $\int \csc(x)\log(\tan(x))\sec(x)\,dx$

Optimal. Leaf size=9

$$\frac{1}{2}\log^2(\tan(x))$$

```
[Out] 1/2*ln(tan(x))^2
```

Rubi [A] time = 0.02, antiderivative size = 9, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 3, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {2620, 29, 6686}

$$\frac{1}{2}\log^2(\tan(x))$$

Antiderivative was successfully verified.

```
[In] Int[Csc[x]*Log[Tan[x]]*Sec[x],x]
```

```
[Out] Log[Tan[x]]^2/2
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 2620

```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Dist[1/f, Subst[Int[(1 + x^2)^((m + n)/2 - 1)/x^m, x], x, Tan[e + f*x]],
 x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n)/2]
```

Rule 6686

```
Int[(u_)*(y_)^(m_.), x_Symbol] :> With[{q = DerivativeDivides[y, u, x]}, Si
mp[(q*y^(m + 1))/(m + 1), x] /;  !FalseQ[q]] /; FreeQ[m, x] && NeQ[m, -1]
```

Rubi steps

$$\int \csc(x)\log(\tan(x))\sec(x)\,dx = \frac{1}{2}\log^2(\tan(x))$$

Mathematica [A] time = 0.01, size = 9, normalized size = 1.00

$$\frac{1}{2}\log^2(\tan(x))$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[x]*Log[Tan[x]]*Sec[x],x]
```

```
[Out] Log[Tan[x]]^2/2
```

fricas [A] time = 0.61, size = 12, normalized size = 1.33

$$\frac{1}{2}\,\log\left(\frac{\sin(x)}{\cos(x)}\right)^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)*log(tan(x))*sec(x),x, algorithm="fricas")
```

```
[Out] 1/2*log(sin(x)/cos(x))^2
```

giac [A] time = 0.14, size = 7, normalized size = 0.78

$$\frac{1}{2}\,\log\left(\tan(x)\right)^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)*log(tan(x))*sec(x),x, algorithm="giac")
```

```
[Out] 1/2*log(tan(x))^2
```

maple [A] time = 0.08, size = 8, normalized size = 0.89

$$\frac{\ln\left(\tan(x)\right)^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(x)*ln(tan(x))*sec(x),x)
```

```
[Out] 1/2*ln(tan(x))^2
```

maxima [A] time = 0.32, size = 7, normalized size = 0.78

$$\frac{1}{2}\,\log\left(\tan(x)\right)^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)*log(tan(x))*sec(x),x, algorithm="maxima")
```

```
[Out] 1/2*log(tan(x))^2
```

mupad [B] time = 5.27, size = 27, normalized size = 3.00

$$\frac{\ln\left(-\frac{e^{x\,2i}\,1i-i}{e^{x\,2i}+1}\right)^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(tan(x))/(cos(x)*sin(x)),x)
```

```
[Out] log(-(exp(x*2i)*1i - 1i)/(exp(x*2i) + 1))^2/2
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \log(\tan(x))\csc(x)\sec(x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)*ln(tan(x))*sec(x),x)
```

```
[Out] Integral(log(tan(x))*csc(x)*sec(x), x)
```

3.750 $\int \csc(2x)\log(\tan(x))\,dx$

Optimal. Leaf size=9

$$\frac{1}{4}\log^2(\tan(x))$$

[Out] 1/4*ln(tan(x))^2

Rubi [A] time = 0.02, antiderivative size = 9, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 2, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {3770, 6686}

$$\frac{1}{4}\log^2(\tan(x))$$

Antiderivative was successfully verified.

[In] Int[Csc[2*x]*Log[Tan[x]],x]

[Out] Log[Tan[x]]^2/4

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 6686

```
Int[(u_)*(y_)^(m_.), x_Symbol] :> With[{q = DerivativeDivides[y, u, x]}, Si
mp[(q*y^(m + 1))/(m + 1), x] /;  !FalseQ[q]] /; FreeQ[m, x] && NeQ[m, -1]
```

Rubi steps

$$\int \csc(2x)\log(\tan(x))\,dx = \frac{1}{4}\log^2(\tan(x))$$

Mathematica [A] time = 0.01, size = 9, normalized size = 1.00

$$\frac{1}{4}\log^2(\tan(x))$$

Antiderivative was successfully verified.

[In] Integrate[Csc[2*x]*Log[Tan[x]],x]

[Out] Log[Tan[x]]^2/4

fricas [A] time = 0.51, size = 7, normalized size = 0.78

$$\frac{1}{4}\log\left(\tan(x)\right)^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(2*x)*log(tan(x)),x, algorithm="fricas")
```

```
[Out] 1/4*log(tan(x))^2
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \csc\left(2\,x\right)\log\left(\tan(x)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(2*x)*log(tan(x)),x, algorithm="giac")
```

```
[Out] integrate(csc(2*x)*log(tan(x)), x)
```

maple [A] time = 0.06, size = 8, normalized size = 0.89

$$\frac{\ln\left(\tan(x)\right)^2}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(2*x)*ln(tan(x)),x)
```

```
[Out] 1/4*ln(tan(x))^2
```

maxima [B] time = 0.46, size = 265, normalized size = 29.44

$$\frac{1}{4}\left(\pi-2\,\arctan\left(\sin(x),\cos(x)+1\right)-2\,\arctan\left(\sin(x),\cos(x)-1\right)\right)\arctan\left(\sin\left(2\,x\right),\cos\left(2\,x\right)+1\right)+\frac{1}{4}\,\arctan\left(\mathrm{si}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(2*x)*log(tan(x)),x, algorithm="maxima")
```

```
[Out] 1/4*(pi - 2*arctan2(sin(x), cos(x) + 1) - 2*arctan2(sin(x), cos(x) - 1))*ar
ctan2(sin(2*x), cos(2*x) + 1) + 1/4*arctan2(sin(2*x), cos(2*x) + 1)^2 - 1/4
*(pi - 2*arctan2(sin(x), cos(x) - 1))*arctan2(sin(x), cos(x) + 1) + 1/4*arc
tan2(sin(x), cos(x) + 1)^2 - 1/4*pi*arctan2(sin(x), cos(x) - 1) + 1/4*arcta
n2(sin(x), cos(x) - 1)^2 + 1/8*(log(cos(x)^2 + sin(x)^2 + 2*cos(x) + 1) + l
og(cos(x)^2 + sin(x)^2 - 2*cos(x) + 1))*log(cos(2*x)^2 + sin(2*x)^2 + 2*cos
```

```
(2*x) + 1) - 1/16*log(cos(2*x)^2 + sin(2*x)^2 + 2*cos(2*x) + 1)^2 - 1/16*lo
g(cos(x)^2 + sin(x)^2 + 2*cos(x) + 1)^2 - 1/8*log(cos(x)^2 + sin(x)^2 + 2*c
os(x) + 1)*log(cos(x)^2 + sin(x)^2 - 2*cos(x) + 1) - 1/16*log(cos(x)^2 + si
n(x)^2 - 2*cos(x) + 1)^2 - 1/2*log(cot(2*x) + csc(2*x))*log(tan(x))
```

mupad [B] time = 3.53, size = 27, normalized size = 3.00

$$\frac{\ln\left(-\frac{e^{x\,2i}\,1i-i}{e^{x\,2i}+1}\right)^2}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(tan(x))/sin(2*x),x)
[Out] log(-(exp(x*2i)*1i - 1i)/(exp(x*2i) + 1))^2/4
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(2*x)*ln(tan(x)),x)
[Out] Timed out
```

3.751 $\int e^{\cos^2(x)+\sin^2(x)}\,dx$

Optimal. Leaf size=3

$$ex$$

```
[Out] E*x
```

Rubi [A] time = 0.01, antiderivative size = 3, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {12, 203}

$$ex$$

Antiderivative was successfully verified.

```
[In] Int[E^(Cos[x]^2 + Sin[x]^2),x]
```

```
[Out] E*x
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int e^{\cos^2(x)+\sin^2(x)}\,dx &= \text{Subst}\left(\int \frac{e}{1+x^2}\,dx, x, \tan(x)\right)\\ &= e\,\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \tan(x)\right)\\ &= ex\end{aligned}$$

Mathematica [A] time = 0.00, size = 3, normalized size = 1.00

$$ex$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(Cos[x]^2 + Sin[x]^2),x]
[Out] E*x
```

fricas [C] time = 0.55, size = 4, normalized size = 1.33

$$xe$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(cos(x)^2+sin(x)^2),x, algorithm="fricas")
[Out] x*e
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$sage_0x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(cos(x)^2+sin(x)^2),x, algorithm="giac")
[Out] sage0*x
```

maple [C] time = 0.06, size = 5, normalized size = 1.67

$$\mathrm{e}x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(cos(x)^2+sin(x)^2),x)
[Out] exp(1)*x
```

maxima [C] time = 0.43, size = 4, normalized size = 1.33

$$xe$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(cos(x)^2+sin(x)^2),x, algorithm="maxima")
[Out] x*e
```

mupad [B] time = 0.03, size = 4, normalized size = 1.33

$$x\,\mathrm{e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(cos(x)^2 + sin(x)^2),x)
```

```
[Out] x*exp(1)
```

sympy [B] time = 0.14, size = 14, normalized size = 4.67

$$xe^{\sin^2(x)}e^{\cos^2(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(cos(x)**2+sin(x)**2),x)
```

```
[Out] x*exp(sin(x)**2)*exp(cos(x)**2)
```

3.752 $\int x \sec^2(x)\, dx$

Optimal. Leaf size=8

$$x \tan(x) + \log(\cos(x))$$

```
[Out] ln(cos(x))+x*tan(x)
```

Rubi [A] time = 0.02, antiderivative size = 8, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {4184, 3475}

$$x \tan(x) + \log(\cos(x))$$

Antiderivative was successfully verified.

```
[In] Int[x*Sec[x]^2,x]
```

```
[Out] Log[Cos[x]] + x*Tan[x]
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 4184

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> -Sim
p[((c + d*x)^m*Cot[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Co
t[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rubi steps

$$\begin{aligned}\int x \sec^2(x)\, dx &= x \tan(x) - \int \tan(x)\, dx\\ &= \log(\cos(x)) + x \tan(x)\end{aligned}$$

Mathematica [A] time = 0.00, size = 8, normalized size = 1.00

$$x \tan(x) + \log(\cos(x))$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Sec[x]^2,x]
```

```
[Out] Log[Cos[x]] + x*Tan[x]
```

fricas [B] time = 0.53, size = 18, normalized size = 2.25

$$\frac{\cos(x)\log(-\cos(x)) + x\sin(x)}{\cos(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sec(x)^2,x, algorithm="fricas")
```

```
[Out] (cos(x)*log(-cos(x)) + x*sin(x))/cos(x)
```

giac [B] time = 0.16, size = 103, normalized size = 12.88

$$\frac{\log\left(\frac{4\left(\tan\left(\frac{1}{2}x\right)^4-2\tan\left(\frac{1}{2}x\right)^2+1\right)}{\tan\left(\frac{1}{2}x\right)^4+2\tan\left(\frac{1}{2}x\right)^2+1}\right)\tan\left(\frac{1}{2}x\right)^2-4x\tan\left(\frac{1}{2}x\right)-\log\left(\frac{4\left(\tan\left(\frac{1}{2}x\right)^4-2\tan\left(\frac{1}{2}x\right)^2+1\right)}{\tan\left(\frac{1}{2}x\right)^4+2\tan\left(\frac{1}{2}x\right)^2+1}\right)}{2\left(\tan\left(\frac{1}{2}x\right)^2-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sec(x)^2,x, algorithm="giac")
```

```
[Out] 1/2*(log(4*(tan(1/2*x)^4 - 2*tan(1/2*x)^2 + 1)/(tan(1/2*x)^4 + 2*tan(1/2*x)
^2 + 1))*tan(1/2*x)^2 - 4*x*tan(1/2*x) - log(4*(tan(1/2*x)^4 - 2*tan(1/2*x)
^2 + 1)/(tan(1/2*x)^4 + 2*tan(1/2*x)^2 + 1)))/(tan(1/2*x)^2 - 1)
```

maple [A] time = 0.02, size = 9, normalized size = 1.12

$$\ln(\cos(x)) + x\tan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*sec(x)^2,x)
```

```
[Out] ln(cos(x))+x*tan(x)
```

maxima [B] time = 0.42, size = 74, normalized size = 9.25

$$\frac{\left(\cos(2x)^2+\sin(2x)^2+2\cos(2x)+1\right)\log\left(\cos(2x)^2+\sin(2x)^2+2\cos(2x)+1\right)+4x\sin(2x)}{2\left(\cos(2x)^2+\sin(2x)^2+2\cos(2x)+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sec(x)^2,x, algorithm="maxima")
```

```
[Out] 1/2*((cos(2*x)^2 + sin(2*x)^2 + 2*cos(2*x) + 1)*log(cos(2*x)^2 + sin(2*x)^2
 + 2*cos(2*x) + 1) + 4*x*sin(2*x))/(cos(2*x)^2 + sin(2*x)^2 + 2*cos(2*x) +
1)
```

mupad [B] time = 0.02, size = 8, normalized size = 1.00

$$\ln(\cos(x)) + x\tan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/cos(x)^2,x)
[Out] log(cos(x)) + x*tan(x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x\sec^2(x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sec(x)**2,x)
[Out] Integral(x*sec(x)**2, x)
```

3.753 $\int x\cos^4\left(x^2\right)\,dx$

Optimal. Leaf size=34

$$\frac{3x^2}{16}+\frac{1}{8}\sin\left(x^2\right)\cos^3\left(x^2\right)+\frac{3}{16}\sin\left(x^2\right)\cos\left(x^2\right)$$

```
[Out] 3/16*x^2+3/16*cos(x^2)*sin(x^2)+1/8*cos(x^2)^3*sin(x^2)
```

Rubi [A] time = 0.02, antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {3380, 2635, 8}

$$\frac{3x^2}{16}+\frac{1}{8}\sin\left(x^2\right)\cos^3\left(x^2\right)+\frac{3}{16}\sin\left(x^2\right)\cos\left(x^2\right)$$

Antiderivative was successfully verified.

```
[In] Int[x*Cos[x^2]^4,x]
```

```
[Out] (3*x^2)/16 + (3*Cos[x^2]*Sin[x^2])/16 + (Cos[x^2]^3*Sin[x^2])/8
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 3380

```
Int[((a_.) + Cos[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Cos[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rubi steps

$$\begin{aligned}\int x\cos^4\left(x^2\right)\,dx &= \frac{1}{2}\text{Subst}\left(\int \cos^4(x)\,dx,x,x^2\right)\\ &= \frac{1}{8}\cos^3\left(x^2\right)\sin\left(x^2\right)+\frac{3}{8}\text{Subst}\left(\int \cos^2(x)\,dx,x,x^2\right)\\ &= \frac{3}{16}\cos\left(x^2\right)\sin\left(x^2\right)+\frac{1}{8}\cos^3\left(x^2\right)\sin\left(x^2\right)+\frac{3}{16}\text{Subst}\left(\int 1\,dx,x,x^2\right)\\ &= \frac{3x^2}{16}+\frac{3}{16}\cos\left(x^2\right)\sin\left(x^2\right)+\frac{1}{8}\cos^3\left(x^2\right)\sin\left(x^2\right)\end{aligned}$$

Mathematica [A] time = 0.02, size = 28, normalized size = 0.82

$$\frac{3x^2}{16}+\frac{1}{8}\sin\left(2x^2\right)+\frac{1}{64}\sin\left(4x^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Cos[x^2]^4,x]
[Out] (3*x^2)/16 + Sin[2*x^2]/8 + Sin[4*x^2]/64
```

fricas [A] time = 2.30, size = 27, normalized size = 0.79

$$\frac{3}{16}\,x^2+\frac{1}{16}\left(2\,\cos\left(x^2\right)^3+3\,\cos\left(x^2\right)\right)\sin\left(x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(x^2)^4,x, algorithm="fricas")
[Out] 3/16*x^2 + 1/16*(2*cos(x^2)^3 + 3*cos(x^2))*sin(x^2)
```

giac [A] time = 0.13, size = 22, normalized size = 0.65

$$\frac{3}{16}\,x^2+\frac{1}{64}\,\sin\left(4\,x^2\right)+\frac{1}{8}\,\sin\left(2\,x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(x^2)^4,x, algorithm="giac")
[Out] 3/16*x^2 + 1/64*sin(4*x^2) + 1/8*sin(2*x^2)
```

maple [A] time = 0.07, size = 26, normalized size = 0.76

$$\frac{\left(\cos^3\left(x^2\right)+\frac{3\cos\left(x^2\right)}{2}\right)\sin\left(x^2\right)}{8}+\frac{3x^2}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*cos(x^2)^4,x)
```

```
[Out] 1/8*(cos(x^2)^3+3/2*cos(x^2))*sin(x^2)+3/16*x^2
```

maxima [A] time = 0.32, size = 22, normalized size = 0.65

$$\frac{3}{16}x^2 + \frac{1}{64}\sin\left(4x^2\right) + \frac{1}{8}\sin\left(2x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(x^2)^4,x, algorithm="maxima")
```

```
[Out] 3/16*x^2 + 1/64*sin(4*x^2) + 1/8*sin(2*x^2)
```

mupad [B] time = 2.97, size = 22, normalized size = 0.65

$$\frac{\sin\left(2x^2\right)}{8} + \frac{\sin\left(4x^2\right)}{64} + \frac{3x^2}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*cos(x^2)^4,x)
```

```
[Out] sin(2*x^2)/8 + sin(4*x^2)/64 + (3*x^2)/16
```

sympy [B] time = 0.96, size = 76, normalized size = 2.24

$$\frac{3x^2\sin^4\left(x^2\right)}{16} + \frac{3x^2\sin^2\left(x^2\right)\cos^2\left(x^2\right)}{8} + \frac{3x^2\cos^4\left(x^2\right)}{16} + \frac{3\sin^3\left(x^2\right)\cos\left(x^2\right)}{16} + \frac{5\sin\left(x^2\right)\cos^3\left(x^2\right)}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(x**2)**4,x)
```

```
[Out] 3*x**2*sin(x**2)**4/16 + 3*x**2*sin(x**2)**2*cos(x**2)**2/8 + 3*x**2*cos(x*
*2)**4/16 + 3*sin(x**2)**3*cos(x**2)/16 + 5*sin(x**2)*cos(x**2)**3/16
```

3.754 $\int \sqrt{\cos(x)}\sin(x)\,dx$

Optimal. Leaf size=10

$$-\frac{2}{3}\cos^{\frac{3}{2}}(x)$$

```
[Out] -2/3*cos(x)^(3/2)
```

Rubi [A] time = 0.01, antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {2565, 30}

$$-\frac{2}{3}\cos^{\frac{3}{2}}(x)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Cos[x]]*Sin[x],x]
```

```
[Out] (-2*Cos[x]^(3/2))/3
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2565

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> -Dist[(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rubi steps

$$\begin{aligned}\int \sqrt{\cos(x)}\sin(x)\,dx &= -\text{Subst}\left(\int \sqrt{x}\,dx, x, \cos(x)\right)\\ &= -\frac{2}{3}\cos^{\frac{3}{2}}(x)\end{aligned}$$

Mathematica [A] time = 0.00, size = 10, normalized size = 1.00

$$-\frac{2}{3}\cos^{\frac{3}{2}}(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Cos[x]]*Sin[x],x]
[Out] (-2*Cos[x]^(3/2))/3
```

fricas [A] time = 0.59, size = 6, normalized size = 0.60

$$-\frac{2}{3}\,\cos(x)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)*cos(x)^(1/2),x, algorithm="fricas")
[Out] -2/3*cos(x)^(3/2)
```

giac [A] time = 0.14, size = 6, normalized size = 0.60

$$-\frac{2}{3}\,\cos(x)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)*cos(x)^(1/2),x, algorithm="giac")
[Out] -2/3*cos(x)^(3/2)
```

maple [A] time = 0.02, size = 7, normalized size = 0.70

$$-\frac{2\left(\cos^{\frac{3}{2}}(x)\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)*cos(x)^(1/2),x)
[Out] -2/3*cos(x)^(3/2)
```

maxima [A] time = 0.32, size = 6, normalized size = 0.60

$$-\frac{2}{3}\,\cos(x)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)*cos(x)^(1/2),x, algorithm="maxima")
[Out] -2/3*cos(x)^(3/2)
```

mupad [B] time = 0.07, size = 6, normalized size = 0.60

$$-\frac{2\cos(x)^{3/2}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)^(1/2)*sin(x),x)
[Out] -(2*cos(x)^(3/2))/3
```

sympy [A] time = 0.25, size = 10, normalized size = 1.00

$$-\frac{2\cos^{\frac{3}{2}}(x)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)*cos(x)**(1/2),x)
[Out] -2*cos(x)**(3/2)/3
```

3.755 $\int e^{-2x} \tan\left(e^{-2x}\right) dx$

Optimal. Leaf size=11

$$\frac{1}{2}\log\left(\cos\left(e^{-2x}\right)\right)$$

```
[Out] 1/2*ln(cos(exp(-2*x)))
```

Rubi [A] time = 0.01, antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {2282, 3475}

$$\frac{1}{2}\log\left(\cos\left(e^{-2x}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Int[Tan[E^(-2*x)]/E^(2*x),x]
```

```
[Out] Log[Cos[E^(-2*x)]]/2
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\int e^{-2x} \tan\left(e^{-2x}\right) dx &= -\left(\frac{1}{2}\operatorname{Subst}\left(\int \tan(x)\,dx, x, e^{-2x}\right)\right)\\
&= \frac{1}{2}\log\left(\cos\left(e^{-2x}\right)\right)
\end{aligned}$$

Mathematica [A] time = 0.01, size = 11, normalized size = 1.00

$$\frac{1}{2}\log\left(\cos\left(e^{-2x}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[E^(-2*x)]/E^(2*x),x]
```

```
[Out] Log[Cos[E^(-2*x)]]/2
```

fricas [A] time = 1.38, size = 14, normalized size = 1.27

$$\frac{1}{4}\,\log\left(\frac{1}{\tan\left(e^{(-2\,x)}\right)^2+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(exp(-2*x))/exp(2*x),x, algorithm="fricas")
```

```
[Out] 1/4*log(1/(tan(e^(-2*x))^2 + 1))
```

giac [A] time = 0.13, size = 9, normalized size = 0.82

$$\frac{1}{2}\,\log\left(\left|\cos\left(e^{(-2\,x)}\right)\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(exp(-2*x))/exp(2*x),x, algorithm="giac")
```

```
[Out] 1/2*log(abs(cos(e^(-2*x))))
```

maple [A] time = 0.05, size = 9, normalized size = 0.82

$$\frac{\ln\left(\cos\left(\mathrm{e}^{-2x}\right)\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(exp(-2*x))/exp(2*x),x)
```

```
[Out] 1/2*ln(cos(exp(-2*x)))
```

maxima [A] time = 0.31, size = 8, normalized size = 0.73

$$-\frac{1}{2}\,\log\left(\sec\left(e^{(-2\,x)}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(exp(-2*x))/exp(2*x),x, algorithm="maxima")
```

```
[Out] -1/2*log(sec(e^(-2*x)))
```

mupad [B] time = 3.54, size = 12, normalized size = 1.09

$$-\frac{\ln\left(\tan\left(\mathrm{e}^{-2\,x}\right)^2+1\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(-2*x)*tan(exp(-2*x)),x)
[Out] -log(tan(exp(-2*x))^2 + 1)/4
```

sympy [A] time = 0.31, size = 15, normalized size = 1.36

$$-\frac{\log\left(\tan^2\left(e^{-2x}\right)+1\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(exp(-2*x))/exp(2*x),x)
[Out] -log(tan(exp(-2*x))**2 + 1)/4
```

3.756 $\int \frac{\sec(x)\sin(2x)}{1+\cos(x)}\,dx$

Optimal. Leaf size=7

$$-2\log(\cos(x)+1)$$

```
[Out] -2*ln(1+cos(x))
```

Rubi [A] time = 0.04, antiderivative size = 7, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {12, 31}

$$-2\log(\cos(x)+1)$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[x]*Sin[2*x])/(1 + Cos[x]),x]
```

```
[Out] -2*Log[1 + Cos[x]]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sec(x)\sin(2x)}{1+\cos(x)}\,dx &= -\text{Subst}\left(\int \frac{2}{1+x}\,dx, x, \cos(x)\right)\\
&= -\left(2\,\text{Subst}\left(\int \frac{1}{1+x}\,dx, x, \cos(x)\right)\right)\\
&= -2\log(1+\cos(x))
\end{aligned}$$

Mathematica [A] time = 0.01, size = 9, normalized size = 1.29

$$-4\log\left(\cos\left(\frac{x}{2}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[x]*Sin[2*x])/(1 + Cos[x]),x]
[Out] -4*Log[Cos[x/2]]
```

fricas [A] time = 0.71, size = 9, normalized size = 1.29

$$-2 \log\left(\frac{1}{2} \cos(x) + \frac{1}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)*sin(2*x)/(1+cos(x)),x, algorithm="fricas")
[Out] -2*log(1/2*cos(x) + 1/2)
```

giac [A] time = 0.14, size = 7, normalized size = 1.00

$$-2 \log(\cos(x) + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)*sin(2*x)/(1+cos(x)),x, algorithm="giac")
[Out] -2*log(cos(x) + 1)
```

maple [A] time = 0.08, size = 8, normalized size = 1.14

$$-2 \ln(1 + \cos(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(x)*sin(2*x)/(1+cos(x)),x)
[Out] -2*ln(1+cos(x))
```

maxima [A] time = 0.32, size = 7, normalized size = 1.00

$$-2 \log(\cos(x) + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)*sin(2*x)/(1+cos(x)),x, algorithm="maxima")
[Out] -2*log(cos(x) + 1)
```

mupad [B] time = 2.92, size = 7, normalized size = 1.00

$$-2 \ln(\cos(x) + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(2*x)/(cos(x)*(cos(x) + 1)),x)
```

```
[Out] -2*log(cos(x) + 1)
```

sympy [A] time = 2.17, size = 8, normalized size = 1.14

$$-2\log\left(\cos\left(x\right)+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)*sin(2*x)/(1+cos(x)),x)
```

```
[Out] -2*log(cos(x) + 1)
```

3.757 $\int x \sec^2(3x)\,dx$

Optimal. Leaf size=19

$$\frac{1}{3}x\tan(3x)+\frac{1}{9}\log(\cos(3x))$$

```
[Out] 1/9*ln(cos(3*x))+1/3*x*tan(3*x)
```

Rubi [A] time = 0.02, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {4184, 3475}

$$\frac{1}{3}x\tan(3x)+\frac{1}{9}\log(\cos(3x))$$

Antiderivative was successfully verified.

```
[In] Int[x*Sec[3*x]^2,x]
```

```
[Out] Log[Cos[3*x]]/9 + (x*Tan[3*x])/3
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 4184

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> -Sim
p[((c + d*x)^m*Cot[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Co
t[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rubi steps

$$\begin{aligned}\int x\sec^2(3x)\,dx &= \frac{1}{3}x\tan(3x)-\frac{1}{3}\int\tan(3x)\,dx\\ &= \frac{1}{9}\log(\cos(3x))+\frac{1}{3}x\tan(3x)\end{aligned}$$

Mathematica [A] time = 0.01, size = 19, normalized size = 1.00

$$\frac{1}{3}x\tan(3x)+\frac{1}{9}\log(\cos(3x))$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Sec[3*x]^2,x]
```

```
[Out] Log[Cos[3*x]]/9 + (x*Tan[3*x])/3
```

fricas [A] time = 0.71, size = 28, normalized size = 1.47

$$\frac{\cos(3x)\log(-\cos(3x)) + 3x\sin(3x)}{9\cos(3x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sec(3*x)^2,x, algorithm="fricas")
```

```
[Out] 1/9*(cos(3*x)*log(-cos(3*x)) + 3*x*sin(3*x))/cos(3*x)
```

giac [B] time = 0.18, size = 103, normalized size = 5.42

$$\frac{\log\left(\frac{4\left(\tan\left(\frac{3}{2}x\right)^4 - 2\tan\left(\frac{3}{2}x\right)^2 + 1\right)}{\tan\left(\frac{3}{2}x\right)^4 + 2\tan\left(\frac{3}{2}x\right)^2 + 1}\right)\tan\left(\frac{3}{2}x\right)^2 - 12x\tan\left(\frac{3}{2}x\right) - \log\left(\frac{4\left(\tan\left(\frac{3}{2}x\right)^4 - 2\tan\left(\frac{3}{2}x\right)^2 + 1\right)}{\tan\left(\frac{3}{2}x\right)^4 + 2\tan\left(\frac{3}{2}x\right)^2 + 1}\right)}{18\left(\tan\left(\frac{3}{2}x\right)^2 - 1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sec(3*x)^2,x, algorithm="giac")
```

```
[Out] 1/18*(log(4*(tan(3/2*x)^4 - 2*tan(3/2*x)^2 + 1)/(tan(3/2*x)^4 + 2*tan(3/2*x
)^2 + 1))*tan(3/2*x)^2 - 12*x*tan(3/2*x) - log(4*(tan(3/2*x)^4 - 2*tan(3/2*
x)^2 + 1)/(tan(3/2*x)^4 + 2*tan(3/2*x)^2 + 1)))/(tan(3/2*x)^2 - 1)
```

maple [A] time = 0.02, size = 16, normalized size = 0.84

$$\frac{\ln(\cos(3x))}{9} + \frac{x\tan(3x)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*sec(3*x)^2,x)
```

```
[Out] 1/9*ln(cos(3*x))+1/3*x*tan(3*x)
```

maxima [B] time = 0.41, size = 74, normalized size = 3.89

$$\frac{\left(\cos(6x)^2 + \sin(6x)^2 + 2\cos(6x) + 1\right)\log\left(\cos(6x)^2 + \sin(6x)^2 + 2\cos(6x) + 1\right) + 12x\sin(6x)}{18\left(\cos(6x)^2 + \sin(6x)^2 + 2\cos(6x) + 1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sec(3*x)^2,x, algorithm="maxima")
```

```
[Out] 1/18*((cos(6*x)^2 + sin(6*x)^2 + 2*cos(6*x) + 1)*log(cos(6*x)^2 + sin(6*x)^
2 + 2*cos(6*x) + 1) + 12*x*sin(6*x))/(cos(6*x)^2 + sin(6*x)^2 + 2*cos(6*x)
+ 1)
```

mupad [B] time = 2.90, size = 15, normalized size = 0.79

$$\frac{\ln\left(\cos\left(3\,x\right)\right)}{9}+\frac{x\,\tan\left(3\,x\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/cos(3*x)^2,x)
```

```
[Out] log(cos(3*x))/9 + (x*tan(3*x))/3
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x\sec^{2}\left(3x\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sec(3*x)**2,x)
```

```
[Out] Integral(x*sec(3*x)**2, x)
```

3.758 $\int e^{-2\pi x}\cos(2\pi x)\,dx$

Optimal. Leaf size=37

$$\frac{e^{-2\pi x}\sin(2\pi x)}{4\pi}-\frac{e^{-2\pi x}\cos(2\pi x)}{4\pi}$$

```
[Out] -1/4*cos(2*Pi*x)/exp(2*Pi*x)/Pi+1/4*sin(2*Pi*x)/exp(2*Pi*x)/Pi
```

Rubi [A] time = 0.01, antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {4433}

$$\frac{e^{-2\pi x}\sin(2\pi x)}{4\pi}-\frac{e^{-2\pi x}\cos(2\pi x)}{4\pi}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[2*Pi*x]/E^(2*Pi*x),x]
```

```
[Out] -Cos[2*Pi*x]/(4*E^(2*Pi*x)*Pi) + Sin[2*Pi*x]/(4*E^(2*Pi*x)*Pi)
```

Rule 4433

```
Int[Cos[(d_.) + (e_.)*(x_)]*(F_)^((c_.)*((a_.) + (b_.)*(x_))), x_Symbol] :>
 Simp[(b*c*Log[F]*F^(c*(a + b*x))*Cos[d + e*x])/(e^2 + b^2*c^2*Log[F]^2), x
] + Simp[(e*F^(c*(a + b*x))*Sin[d + e*x])/(e^2 + b^2*c^2*Log[F]^2), x] /; F
reeQ[{F, a, b, c, d, e}, x] && NeQ[e^2 + b^2*c^2*Log[F]^2, 0]
```

Rubi steps

$$\int e^{-2\pi x}\cos(2\pi x)\,dx=-\frac{e^{-2\pi x}\cos(2\pi x)}{4\pi}+\frac{e^{-2\pi x}\sin(2\pi x)}{4\pi}$$

Mathematica [A] time = 0.03, size = 26, normalized size = 0.70

$$\frac{e^{-2\pi x}(\sin(2\pi x)-\cos(2\pi x))}{4\pi}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[2*Pi*x]/E^(2*Pi*x),x]
```

```
[Out] (-Cos[2*Pi*x] + Sin[2*Pi*x])/(4*E^(2*Pi*x)*Pi)
```

fricas [A] time = 0.70, size = 29, normalized size = 0.78

$$-\frac{\cos(2\,\pi x)\,e^{(-2\,\pi x)} - e^{(-2\,\pi x)}\sin(2\,\pi x)}{4\,\pi}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(2*pi*x)/exp(2*pi*x),x, algorithm="fricas")
[Out] -1/4*(cos(2*pi*x)*e^(-2*pi*x) - e^(-2*pi*x)*sin(2*pi*x))/pi
```

giac [A] time = 0.13, size = 27, normalized size = 0.73

$$-\frac{1}{4}\left(\frac{\cos(2\,\pi x)}{\pi} - \frac{\sin(2\,\pi x)}{\pi}\right)e^{(-2\,\pi x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(2*pi*x)/exp(2*pi*x),x, algorithm="giac")
[Out] -1/4*(cos(2*pi*x)/pi - sin(2*pi*x)/pi)*e^(-2*pi*x)
```

maple [A] time = 0.06, size = 31, normalized size = 0.84

$$\frac{-\frac{e^{-2\pi x}\cos(2\pi x)}{2} + \frac{e^{-2\pi x}\sin(2\pi x)}{2}}{2\pi}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(2*Pi*x)/exp(2*Pi*x),x)
[Out] 1/2/Pi*(-1/2*exp(-2*Pi*x)*cos(2*Pi*x)+1/2*exp(-2*Pi*x)*sin(2*Pi*x))
```

maxima [A] time = 0.32, size = 26, normalized size = 0.70

$$-\frac{(\pi\cos(2\,\pi x) - \pi\sin(2\,\pi x))e^{(-2\,\pi x)}}{4\,\pi^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(2*pi*x)/exp(2*pi*x),x, algorithm="maxima")
[Out] -1/4*(pi*cos(2*pi*x) - pi*sin(2*pi*x))*e^(-2*pi*x)/pi^2
```

mupad [B] time = 2.90, size = 25, normalized size = 0.68

$$-\frac{e^{-2\,\Pi\,x}\,\left(2\,\cos(2\,\Pi\,x) - 2\,\sin(2\,\Pi\,x)\right)}{8\,\Pi}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(-2*Pi*x)*cos(2*Pi*x),x)
```

```
[Out] -(exp(-2*Pi*x)*(2*cos(2*Pi*x) - 2*sin(2*Pi*x)))/(8*Pi)
```

sympy [A] time = 0.42, size = 32, normalized size = 0.86

$$\frac{e^{-2\pi x}\sin(2\pi x)}{4\pi}-\frac{e^{-2\pi x}\cos(2\pi x)}{4\pi}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(2*pi*x)/exp(2*pi*x),x)
```

```
[Out] exp(-2*pi*x)*sin(2*pi*x)/(4*pi) - exp(-2*pi*x)*cos(2*pi*x)/(4*pi)
```

3.759 $\int \left(\cos^{12}(x)\sin^{10}(x)-\cos^{10}(x)\sin^{12}(x)\right) dx$

Optimal. Leaf size=12

$$\frac{1}{11}\sin^{11}(x)\cos^{11}(x)$$

```
[Out] 1/11*cos(x)^11*sin(x)^11
```

Rubi [B] time = 0.32, antiderivative size = 129, normalized size of antiderivative = 10.75, number of steps used = 25, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {2568, 2635, 8}

$$-\frac{1}{22}\sin^9(x)\cos^{13}(x)-\frac{9}{440}\sin^7(x)\cos^{13}(x)-\frac{7}{880}\sin^5(x)\cos^{13}(x)-\frac{7\sin^3(x)\cos^{13}(x)}{2816}+\frac{1}{22}\sin^{11}(x)\cos^{11}(x)+\frac{1}{40}\text{s}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[x]^12*Sin[x]^10 - Cos[x]^10*Sin[x]^12,x]
```

```
[Out] (3*Cos[x]^11*Sin[x])/5632 - (3*Cos[x]^13*Sin[x])/5632 + (Cos[x]^11*Sin[x]^3
)/512 - (7*Cos[x]^13*Sin[x]^3)/2816 + (7*Cos[x]^11*Sin[x]^5)/1280 - (7*Cos[
x]^13*Sin[x]^5)/880 + (Cos[x]^11*Sin[x]^7)/80 - (9*Cos[x]^13*Sin[x]^7)/440
+ (Cos[x]^11*Sin[x]^9)/40 - (Cos[x]^13*Sin[x]^9)/22 + (Cos[x]^11*Sin[x]^11)
/22
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2568

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> -Simp[(a*(b*Cos[e + f*x])^(n + 1)*(a*Sin[e + f*x])^(m - 1)
)/(b*f*(m + n)), x] + Dist[(a^2*(m - 1))/(m + n), Int[(b*Cos[e + f*x])^n*(a
*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1] &&
 NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rubi steps

$$\int \left(\cos^{12}(x)\sin^{10}(x) - \cos^{10}(x)\sin^{12}(x)\right) dx = \int \cos^{12}(x)\sin^{10}(x)\,dx - \int \cos^{10}(x)\sin^{12}(x)\,dx$$
$$= -\frac{1}{22}\cos^{13}(x)\sin^{9}(x) + \frac{1}{22}\cos^{11}(x)\sin^{11}(x) + \frac{9}{22}\int \cos^{12}(x)\,\mathrm{si}$$
$$= -\frac{9}{440}\cos^{13}(x)\sin^{7}(x) + \frac{1}{40}\cos^{11}(x)\sin^{9}(x) - \frac{1}{22}\cos^{13}(x)\sin^{9}($$
$$= -\frac{7}{880}\cos^{13}(x)\sin^{5}(x) + \frac{1}{80}\cos^{11}(x)\sin^{7}(x) - \frac{9}{440}\cos^{13}(x)\sin$$
$$= -\frac{7\cos^{13}(x)\sin^{3}(x)}{2816} + \frac{7\cos^{11}(x)\sin^{5}(x)}{1280} - \frac{7}{880}\cos^{13}(x)\sin^{5}(x)$$
$$= -\frac{3\cos^{13}(x)\sin(x)}{5632} + \frac{1}{512}\cos^{11}(x)\sin^{3}(x) - \frac{7\cos^{13}(x)\sin^{3}(x)}{2816}$$
$$= \frac{3\cos^{11}(x)\sin(x)}{5632} - \frac{3\cos^{13}(x)\sin(x)}{5632} + \frac{1}{512}\cos^{11}(x)\sin^{3}(x) -$$

Mathematica [B] time = 0.03, size = 49, normalized size = 4.08

$$\frac{21\sin(2x)}{1048576} - \frac{15\sin(6x)}{1048576} + \frac{15\sin(10x)}{2097152} - \frac{5\sin(14x)}{2097152} + \frac{\sin(18x)}{2097152} - \frac{\sin(22x)}{23068672}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[x]^12*Sin[x]^10 - Cos[x]^10*Sin[x]^12,x]
```

```
[Out] (21*Sin[2*x])/1048576 - (15*Sin[6*x])/1048576 + (15*Sin[10*x])/2097152 - (5
*Sin[14*x])/2097152 + Sin[18*x]/2097152 - Sin[22*x]/23068672
```

fricas [B] time = 1.53, size = 39, normalized size = 3.25

$$-\frac{1}{11}\left(\cos(x)^{21} - 5\cos(x)^{19} + 10\cos(x)^{17} - 10\cos(x)^{15} + 5\cos(x)^{13} - \cos(x)^{11}\right)\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^12*sin(x)^10-cos(x)^10*sin(x)^12,x, algorithm="fricas")
```

```
[Out] -1/11*(cos(x)^21 - 5*cos(x)^19 + 10*cos(x)^17 - 10*cos(x)^15 + 5*cos(x)^13
- cos(x)^11)*sin(x)
```

giac [B] time = 0.15, size = 37, normalized size = 3.08

$$-\frac{1}{23068672}\sin(22x) + \frac{1}{2097152}\sin(18x) - \frac{5}{2097152}\sin(14x) + \frac{15}{2097152}\sin(10x) - \frac{15}{1048576}\sin(6x) + \frac{21}{1048}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^12*sin(x)^10-cos(x)^10*sin(x)^12,x, algorithm="giac")
```

```
[Out] -1/23068672*sin(22*x) + 1/2097152*sin(18*x) - 5/2097152*sin(14*x) + 15/2097
152*sin(10*x) - 15/1048576*sin(6*x) + 21/1048576*sin(2*x)
```

maple [B] time = 0.20, size = 176, normalized size = 14.67

$$-\frac{\left(\cos^{13}(x)\right)\left(\sin^{9}(x)\right)}{22}-\frac{9\left(\sin^{7}(x)\right)\left(\cos^{13}(x)\right)}{440}-\frac{7\left(\sin^{5}(x)\right)\left(\cos^{13}(x)\right)}{880}-\frac{7\left(\sin^{3}(x)\right)\left(\cos^{13}(x)\right)}{2816}-\frac{3\sin(x)\left(\cos^{13}(x)\right)}{5632}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)^12*sin(x)^10-cos(x)^10*sin(x)^12,x)
```

```
[Out] -1/22*cos(x)^13*sin(x)^9-9/440*sin(x)^7*cos(x)^13-7/880*sin(x)^5*cos(x)^13-
7/2816*sin(x)^3*cos(x)^13-3/5632*sin(x)*cos(x)^13+1/22528*(cos(x)^11+11/10*
cos(x)^9+99/80*cos(x)^7+231/160*cos(x)^5+231/128*cos(x)^3+693/256*cos(x))*s
in(x)+1/22*cos(x)^11*sin(x)^11+1/40*sin(x)^9*cos(x)^11+1/80*sin(x)^7*cos(x)
^11+7/1280*sin(x)^5*cos(x)^11+1/512*sin(x)^3*cos(x)^11+1/2048*sin(x)*cos(x)
^11-1/20480*(cos(x)^9+9/8*cos(x)^7+21/16*cos(x)^5+105/64*cos(x)^3+315/128*c
os(x))*sin(x)
```

maxima [A] time = 0.34, size = 8, normalized size = 0.67

$$\frac{1}{22528}\sin\left(2\,x\right)^{11}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^12*sin(x)^10-cos(x)^10*sin(x)^12,x, algorithm="maxima")
```

```
[Out] 1/22528*sin(2*x)^11
```

mupad [B] time = 2.99, size = 49, normalized size = 4.08

$$-\frac{\sin(x)\cos(x)^{21}}{11}+\frac{5\ \sin(x)\cos(x)^{19}}{11}-\frac{10\ \sin(x)\cos(x)^{17}}{11}+\frac{10\ \sin(x)\cos(x)^{15}}{11}-\frac{5\ \sin(x)\cos(x)^{13}}{11}+\frac{\sin(x)\cos(x)^{11}}{11}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)^12*sin(x)^10 - cos(x)^10*sin(x)^12,x)
```

```
[Out] (cos(x)^11*sin(x))/11 - (5*cos(x)^13*sin(x))/11 + (10*cos(x)^15*sin(x))/11
- (10*cos(x)^17*sin(x))/11 + (5*cos(x)^19*sin(x))/11 - (cos(x)^21*sin(x))/1
1
```

sympy [B] time = 0.08, size = 236, normalized size = 19.67

$$-\frac{\sin^{21}(x)\cos(x)}{22}+\frac{89\sin^{19}(x)\cos(x)}{440}-\frac{301\sin^{17}(x)\cos(x)}{880}+\frac{3683\sin^{15}(x)\cos(x)}{14080}-\frac{433\sin^{13}(x)\cos(x)}{5632}+\frac{\sin^{1}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)**12*sin(x)**10-cos(x)**10*sin(x)**12,x)
```

```
[Out] -sin(x)**21*cos(x)/22 + 89*sin(x)**19*cos(x)/440 - 301*sin(x)**17*cos(x)/88
0 + 3683*sin(x)**15*cos(x)/14080 - 433*sin(x)**13*cos(x)/5632 + sin(x)**11*
cos(x)/22528 + sin(x)**9*cos(x)/20480 + 9*sin(x)**7*cos(x)/163840 + 21*sin(
x)**5*cos(x)/327680 + 21*sin(x)**3*cos(x)/262144 - sin(x)*cos(x)**21/22 + 8
9*sin(x)*cos(x)**19/440 - 301*sin(x)*cos(x)**17/880 + 3683*sin(x)*cos(x)**1
5/14080 - 433*sin(x)*cos(x)**13/5632 + sin(x)*cos(x)**11/22528 + sin(x)*cos
(x)**9/20480 + 9*sin(x)*cos(x)**7/163840 + 21*sin(x)*cos(x)**5/327680 + 21*
sin(x)*cos(x)**3/262144 + 63*sin(x)*cos(x)/262144
```

3.760 $\int x \cot(x^2) \, dx$

Optimal. Leaf size=9

$$\frac{1}{2} \log(\sin(x^2))$$

```
[Out] 1/2*ln(sin(x^2))
```

Rubi [A] time = 0.01, antiderivative size = 9, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3748, 3475}

$$\frac{1}{2} \log(\sin(x^2))$$

Antiderivative was successfully verified.

```
[In] Int[x*Cot[x^2],x]
```

```
[Out] Log[Sin[x^2]]/2
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3748

```
Int[((a_.) + Cot[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Cot[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IGtQ[Simplify[(m +
1)/n], 0] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned} \int x \cot(x^2) \, dx &= \frac{1}{2} \text{Subst}\left(\int \cot(x) \, dx, x, x^2\right) \\ &= \frac{1}{2} \log(\sin(x^2)) \end{aligned}$$

Mathematica [B] time = 0.01, size = 19, normalized size = 2.11

$$\frac{1}{2} \log(\tan(x^2)) + \frac{1}{2} \log(\cos(x^2))$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Cot[x^2],x]
```

```
[Out] Log[Cos[x^2]]/2 + Log[Tan[x^2]]/2
```

fricas [A] time = 0.65, size = 13, normalized size = 1.44

$$\frac{1}{4}\,\log\left(-\frac{1}{2}\,\cos\left(2\,x^2\right)+\frac{1}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cot(x^2),x, algorithm="fricas")
```

```
[Out] 1/4*log(-1/2*cos(2*x^2) + 1/2)
```

giac [A] time = 0.15, size = 8, normalized size = 0.89

$$\frac{1}{2}\,\log\left(\left|\sin\left(x^2\right)\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cot(x^2),x, algorithm="giac")
```

```
[Out] 1/2*log(abs(sin(x^2)))
```

maple [A] time = 0.00, size = 8, normalized size = 0.89

$$\frac{\ln\left(\sin\left(x^2\right)\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*cot(x^2),x)
```

```
[Out] 1/2*ln(sin(x^2))
```

maxima [A] time = 0.32, size = 7, normalized size = 0.78

$$\frac{1}{2}\,\log\left(\sin\left(x^2\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cot(x^2),x, algorithm="maxima")
```

```
[Out] 1/2*log(sin(x^2))
```

mupad [B] time = 0.12, size = 19, normalized size = 2.11

$$\frac{\ln\left(\mathrm{e}^{x^2\,2\mathrm{i}}-1\right)}{2}-\frac{x^2\,1\mathrm{i}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*cot(x^2),x)
```

```
[Out] log(exp(x^2*2i) - 1)/2 - (x^2*1i)/2
```

sympy [B] time = 0.14, size = 19, normalized size = 2.11

$$-\frac{\log\left(\tan^2\left(x^2\right)+1\right)}{4}+\frac{\log\left(\tan\left(x^2\right)\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cot(x**2),x)
```

```
[Out] -log(tan(x**2)**2 + 1)/4 + log(tan(x**2))/2
```

3.761 $\int x \sec^2\left(x^2\right)\,dx$

Optimal. Leaf size=8

$$\frac{\tan\left(x^2\right)}{2}$$

```
[Out] 1/2*tan(x^2)
```

Rubi [A] time = 0.01, antiderivative size = 8, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {4204, 3767, 8}

$$\frac{\tan\left(x^2\right)}{2}$$

Antiderivative was successfully verified.

```
[In] Int[x*Sec[x^2]^2,x]
```

```
[Out] Tan[x^2]/2
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 4204

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sec[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sec[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IGtQ[Simplify[(m +
1)/n], 0] && IntegerQ[p]
```

Rubi steps

$$\int x\sec^2\left(x^2\right)\,dx = \frac{1}{2}\,\text{Subst}\left(\int \sec^2(x)\,dx, x, x^2\right)$$
$$= -\left(\frac{1}{2}\,\text{Subst}\left(\int 1\,dx, x, -\tan\left(x^2\right)\right)\right)$$
$$= \frac{\tan\left(x^2\right)}{2}$$

Mathematica [A] time = 0.02, size = 8, normalized size = 1.00

$$\frac{\tan\left(x^2\right)}{2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Sec[x^2]^2,x]
[Out] Tan[x^2]/2
```

fricas [A] time = 1.04, size = 12, normalized size = 1.50

$$\frac{\sin\left(x^2\right)}{2\,\cos\left(x^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sec(x^2)^2,x, algorithm="fricas")
[Out] 1/2*sin(x^2)/cos(x^2)
```

giac [A] time = 0.12, size = 6, normalized size = 0.75

$$\frac{1}{2}\,\tan\left(x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sec(x^2)^2,x, algorithm="giac")
[Out] 1/2*tan(x^2)
```

maple [A] time = 0.03, size = 7, normalized size = 0.88

$$\frac{\tan\left(x^2\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*sec(x^2)^2,x)
```

```
[Out] 1/2*tan(x^2)
```

maxima [B] time = 0.32, size = 35, normalized size = 4.38

$$\frac{\sin\left(2\,x^2\right)}{\cos\left(2\,x^2\right)^2+\sin\left(2\,x^2\right)^2+2\,\cos\left(2\,x^2\right)+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sec(x^2)^2,x, algorithm="maxima")
```

```
[Out] sin(2*x^2)/(cos(2*x^2)^2 + sin(2*x^2)^2 + 2*cos(2*x^2) + 1)
```

mupad [B] time = 0.10, size = 14, normalized size = 1.75

$$\frac{1\mathrm{i}}{\mathrm{e}^{x^2\,2\mathrm{i}}+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/cos(x^2)^2,x)
```

```
[Out] 1i/(exp(x^2*2i) + 1)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x\sec^2\left(x^2\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sec(x**2)**2,x)
```

```
[Out] Integral(x*sec(x**2)**2, x)
```

3.762 $\int \frac{\sin(8x)}{9+\sin^4(4x)}\,dx$

Optimal. Leaf size=15

$$\frac{1}{12}\tan^{-1}\left(\frac{1}{3}\sin^2(4x)\right)$$

```
[Out] 1/12*arctan(1/3*sin(4*x)^2)
```

Rubi [A] time = 0.03, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {12, 275, 203}

$$\frac{1}{12}\tan^{-1}\left(\frac{1}{3}\sin^2(4x)\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sin[8*x]/(9 + Sin[4*x]^4),x]
```

```
[Out] ArcTan[Sin[4*x]^2/3]/12
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}\int \frac{\sin(8x)}{9+\sin^4(4x)}\,dx &= \frac{1}{4}\,\text{Subst}\left(\int \frac{2x}{9+x^4}\,dx, x, \sin(4x)\right)\\ &= \frac{1}{2}\,\text{Subst}\left(\int \frac{x}{9+x^4}\,dx, x, \sin(4x)\right)\\ &= \frac{1}{4}\,\text{Subst}\left(\int \frac{1}{9+x^2}\,dx, x, \sin^2(4x)\right)\\ &= \frac{1}{12}\tan^{-1}\left(\frac{1}{3}\sin^2(4x)\right)\end{aligned}$$

Mathematica [A] time = 0.02, size = 15, normalized size = 1.00

$$\frac{1}{12}\tan^{-1}\left(\frac{1}{3}\sin^2(4x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[8*x]/(9 + Sin[4*x]^4),x]
```

```
[Out] ArcTan[Sin[4*x]^2/3]/12
```

fricas [A] time = 0.66, size = 13, normalized size = 0.87

$$-\frac{1}{12}\arctan\left(\frac{1}{3}\cos\left(4\,x\right)^2-\frac{1}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(8*x)/(9+sin(4*x)^4),x, algorithm="fricas")
```

```
[Out] -1/12*arctan(1/3*cos(4*x)^2 - 1/3)
```

giac [A] time = 0.66, size = 15, normalized size = 1.00

$$\frac{1}{12}\arctan\left(\frac{3}{\cos\left(4\,x\right)^2-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(8*x)/(9+sin(4*x)^4),x, algorithm="giac")
```

```
[Out] 1/12*arctan(3/(cos(4*x)^2 - 1))
```

maple [A] time = 0.17, size = 12, normalized size = 0.80

$$\frac{\arctan\left(\frac{\left(\sin^{2}(4x)\right)}{3}\right)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(8*x)/(9+sin(4*x)^4),x)
```

```
[Out] 1/12*arctan(1/3*sin(4*x)^2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin\left(8\,x\right)}{\sin\left(4\,x\right)^{4}+9}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(8*x)/(9+sin(4*x)^4),x, algorithm="maxima")
```

```
[Out] integrate(sin(8*x)/(sin(4*x)^4 + 9), x)
```

mupad [B] time = 2.96, size = 13, normalized size = 0.87

$$\frac{\mathrm{atan}\left(\frac{10\tan\left(4\,x\right)^{2}}{3}+3\right)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(8*x)/(sin(4*x)^4 + 9),x)
```

```
[Out] atan((10*tan(4*x)^2)/3 + 3)/12
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(8*x)/(9+sin(4*x)**4),x)
```

```
[Out] Timed out
```

3.763 $\int \frac{\cos(2x)}{8+\sin^2(2x)}\,dx$

Optimal. Leaf size=23

$$\frac{\tan^{-1}\left(\frac{\sin(2x)}{2\sqrt{2}}\right)}{4\sqrt{2}}$$

```
[Out] 1/8*arctan(1/4*sin(2*x)*2^(1/2))*2^(1/2)
```

Rubi [A] time = 0.02, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {3190, 203}

$$\frac{\tan^{-1}\left(\frac{\sin(2x)}{2\sqrt{2}}\right)}{4\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[2*x]/(8 + Sin[2*x]^2),x]
```

```
[Out] ArcTan[Sin[2*x]/(2*Sqrt[2])]/(4*Sqrt[2])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 3190

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Su
bst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]/
ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\begin{aligned}\int \frac{\cos(2x)}{8+\sin^2(2x)}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{1}{8+x^2}\,dx, x, \sin(2x)\right)\\ &= \frac{\tan^{-1}\left(\frac{\sin(2x)}{2\sqrt{2}}\right)}{4\sqrt{2}}\end{aligned}$$

Mathematica [A] time = 0.01, size = 20, normalized size = 0.87

$$\frac{\tan^{-1}\left(\frac{\sin(x)\cos(x)}{\sqrt{2}}\right)}{4\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[2*x]/(8 + Sin[2*x]^2),x]
```

```
[Out] ArcTan[(Cos[x]*Sin[x])/Sqrt[2]]/(4*Sqrt[2])
```

fricas [A] time = 1.58, size = 15, normalized size = 0.65

$$\frac{1}{8}\sqrt{2}\arctan\left(\frac{1}{4}\sqrt{2}\sin\left(2x\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(2*x)/(8+sin(2*x)^2),x, algorithm="fricas")
```

```
[Out] 1/8*sqrt(2)*arctan(1/4*sqrt(2)*sin(2*x))
```

giac [A] time = 0.15, size = 15, normalized size = 0.65

$$\frac{1}{8}\sqrt{2}\arctan\left(\frac{1}{4}\sqrt{2}\sin\left(2x\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(2*x)/(8+sin(2*x)^2),x, algorithm="giac")
```

```
[Out] 1/8*sqrt(2)*arctan(1/4*sqrt(2)*sin(2*x))
```

maple [A] time = 0.04, size = 16, normalized size = 0.70

$$\frac{\arctan\left(\frac{\sin(2x)\sqrt{2}}{4}\right)\sqrt{2}}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(2*x)/(8+sin(2*x)^2),x)
```

```
[Out] 1/8*arctan(1/4*sin(2*x)*2^(1/2))*2^(1/2)
```

maxima [A] time = 0.43, size = 15, normalized size = 0.65

$$\frac{1}{8}\sqrt{2}\arctan\left(\frac{1}{4}\sqrt{2}\sin\left(2x\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(2*x)/(8+sin(2*x)^2),x, algorithm="maxima")
```

```
[Out] 1/8*sqrt(2)*arctan(1/4*sqrt(2)*sin(2*x))
```

mupad [B] time = 0.07, size = 15, normalized size = 0.65

$$\frac{\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}\,\sin(2\,x)}{4}\right)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(2*x)/(sin(2*x)^2 + 8),x)
```

```
[Out] (2^(1/2)*atan((2^(1/2)*sin(2*x))/4))/8
```

sympy [A] time = 0.27, size = 19, normalized size = 0.83

$$\frac{\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}\,\sin(2x)}{4}\right)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(2*x)/(8+sin(2*x)**2),x)
```

```
[Out] sqrt(2)*atan(sqrt(2)*sin(2*x)/4)/8
```

3.764 $\int x\left(\cos^3\left(x^2\right)-\sin^3\left(x^2\right)\right)dx$

Optimal. Leaf size=37

$$-\frac{1}{6}\sin^3\left(x^2\right)+\frac{\sin\left(x^2\right)}{2}-\frac{1}{6}\cos^3\left(x^2\right)+\frac{\cos\left(x^2\right)}{2}$$

```
[Out] 1/2*cos(x^2)-1/6*cos(x^2)^3+1/2*sin(x^2)-1/6*sin(x^2)^3
```

Rubi [A] time = 0.03, antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {14, 3380, 2633, 3379}

$$-\frac{1}{6}\sin^3\left(x^2\right)+\frac{\sin\left(x^2\right)}{2}-\frac{1}{6}\cos^3\left(x^2\right)+\frac{\cos\left(x^2\right)}{2}$$

Antiderivative was successfully verified.

```
[In] Int[x*(Cos[x^2]^3 - Sin[x^2]^3),x]
```

```
[Out] Cos[x^2]/2 - Cos[x^2]^3/6 + Sin[x^2]/2 - Sin[x^2]^3/6
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 3379

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rule 3380

```
Int[((a_.) + Cos[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Cos[c + d*x])^p
```

```
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rubi steps

$$\begin{aligned}
\int x\left(\cos^3\left(x^2\right)-\sin^3\left(x^2\right)\right)\,dx &= \int \left(x\cos^3\left(x^2\right)-x\sin^3\left(x^2\right)\right)\,dx\\
&= \int x\cos^3\left(x^2\right)\,dx-\int x\sin^3\left(x^2\right)\,dx\\
&= \frac{1}{2}\text{Subst}\left(\int \cos^3(x)\,dx,x,x^2\right)-\frac{1}{2}\text{Subst}\left(\int \sin^3(x)\,dx,x,x^2\right)\\
&= \frac{1}{2}\text{Subst}\left(\int \left(1-x^2\right)\,dx,x,\cos\left(x^2\right)\right)-\frac{1}{2}\text{Subst}\left(\int \left(1-x^2\right)\,dx,x,-\sin\left(x^2\right)\right)\\
&= \frac{\cos\left(x^2\right)}{2}-\frac{1}{6}\cos^3\left(x^2\right)+\frac{\sin\left(x^2\right)}{2}-\frac{1}{6}\sin^3\left(x^2\right)
\end{aligned}$$

Mathematica [A] time = 0.02, size = 37, normalized size = 1.00

$$-\frac{1}{6}\sin^3\left(x^2\right)+\frac{\sin\left(x^2\right)}{2}+\frac{3\cos\left(x^2\right)}{8}-\frac{1}{24}\cos\left(3x^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(Cos[x^2]^3 - Sin[x^2]^3),x]
[Out] (3*Cos[x^2])/8 - Cos[3*x^2]/24 + Sin[x^2]/2 - Sin[x^2]^3/6
```

fricas [A] time = 1.96, size = 29, normalized size = 0.78

$$-\frac{1}{6}\cos\left(x^2\right)^3+\frac{1}{6}\left(\cos\left(x^2\right)^2+2\right)\sin\left(x^2\right)+\frac{1}{2}\cos\left(x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(cos(x^2)^3-sin(x^2)^3),x, algorithm="fricas")
[Out] -1/6*cos(x^2)^3 + 1/6*(cos(x^2)^2 + 2)*sin(x^2) + 1/2*cos(x^2)
```

giac [A] time = 0.13, size = 29, normalized size = 0.78

$$-\frac{1}{6}\cos\left(x^2\right)^3-\frac{1}{6}\sin\left(x^2\right)^3+\frac{1}{2}\cos\left(x^2\right)+\frac{1}{2}\sin\left(x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(cos(x^2)^3-sin(x^2)^3),x, algorithm="giac")
```

```
[Out] -1/6*cos(x^2)^3 - 1/6*sin(x^2)^3 + 1/2*cos(x^2) + 1/2*sin(x^2)
```

maple [A] time = 0.17, size = 30, normalized size = 0.81

$$\frac{\left(2+\cos^2\left(x^2\right)\right)\sin\left(x^2\right)}{6}+\frac{\left(2+\sin^2\left(x^2\right)\right)\cos\left(x^2\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(cos(x^2)^3-sin(x^2)^3),x)
```

```
[Out] 1/6*(2+cos(x^2)^2)*sin(x^2)+1/6*(2+sin(x^2)^2)*cos(x^2)
```

maxima [A] time = 0.33, size = 29, normalized size = 0.78

$$-\frac{1}{24}\cos\left(3\,x^2\right)+\frac{3}{8}\cos\left(x^2\right)+\frac{1}{24}\sin\left(3\,x^2\right)+\frac{3}{8}\sin\left(x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(cos(x^2)^3-sin(x^2)^3),x, algorithm="maxima")
```

```
[Out] -1/24*cos(3*x^2) + 3/8*cos(x^2) + 1/24*sin(3*x^2) + 3/8*sin(x^2)
```

mupad [B] time = 2.97, size = 33, normalized size = 0.89

$$-\frac{\cos\left(x^2\right)^3}{6}+\frac{\sin\left(x^2\right)\,\cos\left(x^2\right)^2}{6}+\frac{\cos\left(x^2\right)}{2}+\frac{\sin\left(x^2\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(cos(x^2)^3 - sin(x^2)^3),x)
```

```
[Out] cos(x^2)/2 + sin(x^2)/3 + (cos(x^2)^2*sin(x^2))/6 - cos(x^2)^3/6
```

sympy [A] time = 0.52, size = 42, normalized size = 1.14

$$\frac{\sin^3\left(x^2\right)}{3}+\frac{\sin^2\left(x^2\right)\cos\left(x^2\right)}{2}+\frac{\sin\left(x^2\right)\cos^2\left(x^2\right)}{2}+\frac{\cos^3\left(x^2\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(cos(x**2)**3-sin(x**2)**3),x)
```

```
[Out] sin(x**2)**3/3 + sin(x**2)**2*cos(x**2)/2 + sin(x**2)*cos(x**2)**2/2 + cos(
x**2)**3/3
```

3.765 $\int \frac{\cos(x)\sin(x)}{1-\cos(x)}\,dx$

Optimal. Leaf size=10

$$\cos(x) + \log(1 - \cos(x))$$

```
[Out] cos(x)+ln(1-cos(x))
```

Rubi [A] time = 0.03, antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {2833, 43}

$$\cos(x) + \log(1 - \cos(x))$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[x]*Sin[x])/(1 - Cos[x]),x]
```

```
[Out] Cos[x] + Log[1 - Cos[x]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2833

```
Int[cos[(e_.) + (f_.)*(x_)]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((
c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b*f), Sub
st[Int[(a + x)^m*(c + (d*x)/b)^n, x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b
, c, d, e, f, m, n}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{\cos(x)\sin(x)}{1-\cos(x)}\,dx &= -\text{Subst}\left(\int \frac{x}{1+x}\,dx, x, -\cos(x)\right)\\
&= -\text{Subst}\left(\int \left(1+\frac{1}{-1-x}\right)dx, x, -\cos(x)\right)\\
&= \cos(x) + \log(1-\cos(x))
\end{aligned}$$

Mathematica [A] time = 0.02, size = 12, normalized size = 1.20

$$\cos(x) + 2\log\left(\sin\left(\frac{x}{2}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[x]*Sin[x])/(1 - Cos[x]),x]
[Out] Cos[x] + 2*Log[Sin[x/2]]
```

fricas [A] time = 1.38, size = 10, normalized size = 1.00

$$\cos(x) + \log\left(-\frac{1}{2}\cos(x) + \frac{1}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*sin(x)/(1-cos(x)),x, algorithm="fricas")
[Out] cos(x) + log(-1/2*cos(x) + 1/2)
```

giac [A] time = 0.14, size = 10, normalized size = 1.00

$$\cos(x) + \log\left(-\cos(x) + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*sin(x)/(1-cos(x)),x, algorithm="giac")
[Out] cos(x) + log(-cos(x) + 1)
```

maple [A] time = 0.04, size = 9, normalized size = 0.90

$$\cos(x) + \ln\left(-1 + \cos(x)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)*sin(x)/(1-cos(x)),x)
[Out] cos(x)+ln(-1+cos(x))
```

maxima [A] time = 0.32, size = 8, normalized size = 0.80

$$\cos(x) + \log\left(\cos(x) - 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*sin(x)/(1-cos(x)),x, algorithm="maxima")
```

```
[Out] cos(x) + log(cos(x) - 1)
```

mupad [B] time = 2.92, size = 8, normalized size = 0.80

$$\ln(\cos(x) - 1) + \cos(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(cos(x)*sin(x))/(cos(x) - 1),x)
[Out] log(cos(x) - 1) + cos(x)
```

sympy [A] time = 0.17, size = 8, normalized size = 0.80

$$\log(\cos(x) - 1) + \cos(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*sin(x)/(1-cos(x)),x)
[Out] log(cos(x) - 1) + cos(x)
```

3.766 $\int x \cos\left(x^2\right) dx$

Optimal. Leaf size=8

$$\frac{\sin\left(x^2\right)}{2}$$

```
[Out] 1/2*sin(x^2)
```

Rubi [A] time = 0.01, antiderivative size = 8, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3380, 2637}

$$\frac{\sin\left(x^2\right)}{2}$$

Antiderivative was successfully verified.

```
[In] Int[x*Cos[x^2],x]
```

```
[Out] Sin[x^2]/2
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3380

```
Int[((a_.) + Cos[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Cos[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rubi steps

$$\begin{aligned} \int x \cos\left(x^2\right) dx &= \frac{1}{2} \text{Subst}\left(\int \cos(x)\, dx, x, x^2\right)\\ &= \frac{\sin\left(x^2\right)}{2} \end{aligned}$$

Mathematica [A] time = 0.00, size = 8, normalized size = 1.00

$$\frac{\sin\left(x^2\right)}{2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Cos[x^2],x]
```

```
[Out] Sin[x^2]/2
```

fricas [A] time = 0.53, size = 6, normalized size = 0.75

$$\frac{1}{2} \sin\left(x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(x^2),x, algorithm="fricas")
```

```
[Out] 1/2*sin(x^2)
```

giac [A] time = 0.13, size = 6, normalized size = 0.75

$$\frac{1}{2} \sin\left(x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(x^2),x, algorithm="giac")
```

```
[Out] 1/2*sin(x^2)
```

maple [A] time = 0.04, size = 7, normalized size = 0.88

$$\frac{\sin\left(x^2\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*cos(x^2),x)
```

```
[Out] 1/2*sin(x^2)
```

maxima [A] time = 0.32, size = 6, normalized size = 0.75

$$\frac{1}{2} \sin\left(x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(x^2),x, algorithm="maxima")
```

```
[Out] 1/2*sin(x^2)
```

mupad [B] time = 0.06, size = 6, normalized size = 0.75

$$\frac{\sin\left(x^2\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*cos(x^2),x)
[Out] sin(x^2)/2
```

sympy [A] time = 0.15, size = 5, normalized size = 0.62

$$\frac{\sin\left(x^2\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(x**2),x)
[Out] sin(x**2)/2
```

3.767 $\int x^2 \cos\left(4x^3\right)\,dx$

Optimal. Leaf size=10

$$\frac{1}{12}\sin\left(4x^3\right)$$

```
[Out] 1/12*sin(4*x^3)
```

Rubi [A] time = 0.01, antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {3380, 2637}

$$\frac{1}{12}\sin\left(4x^3\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^2*Cos[4*x^3],x]
```

```
[Out] Sin[4*x^3]/12
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3380

```
Int[((a_.) + Cos[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Cos[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rubi steps

$$\begin{aligned}\int x^2\cos\left(4x^3\right)\,dx &= \frac{1}{3}\operatorname{Subst}\left(\int \cos(4x)\,dx, x, x^3\right)\\ &= \frac{1}{12}\sin\left(4x^3\right)\end{aligned}$$

Mathematica [A] time = 0.00, size = 10, normalized size = 1.00

$$\frac{1}{12}\sin\left(4x^3\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*Cos[4*x^3],x]
```

```
[Out] Sin[4*x^3]/12
```

fricas [A] time = 0.77, size = 8, normalized size = 0.80

$$\frac{1}{12}\,\sin\left(4\,x^{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*cos(4*x^3),x, algorithm="fricas")
```

```
[Out] 1/12*sin(4*x^3)
```

giac [A] time = 0.12, size = 8, normalized size = 0.80

$$\frac{1}{12}\,\sin\left(4\,x^{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*cos(4*x^3),x, algorithm="giac")
```

```
[Out] 1/12*sin(4*x^3)
```

maple [A] time = 0.04, size = 9, normalized size = 0.90

$$\frac{\sin\left(4x^{3}\right)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*cos(4*x^3),x)
```

```
[Out] 1/12*sin(4*x^3)
```

maxima [A] time = 0.32, size = 8, normalized size = 0.80

$$\frac{1}{12}\,\sin\left(4\,x^{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*cos(4*x^3),x, algorithm="maxima")
```

```
[Out] 1/12*sin(4*x^3)
```

mupad [B] time = 0.06, size = 8, normalized size = 0.80

$$\frac{\sin\left(4\,x^3\right)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*cos(4*x^3),x)
```

```
[Out] sin(4*x^3)/12
```

sympy [A] time = 0.28, size = 7, normalized size = 0.70

$$\frac{\sin\left(4x^3\right)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*cos(4*x**3),x)
```

```
[Out] sin(4*x**3)/12
```

3.768 $\int x^3 \cos\left(x^4\right) dx$

Optimal. Leaf size=8

$$\frac{\sin\left(x^4\right)}{4}$$

[Out] 1/4*sin(x^4)

Rubi [A] time = 0.01, antiderivative size = 8, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {3380, 2637}

$$\frac{\sin\left(x^4\right)}{4}$$

Antiderivative was successfully verified.

[In] Int[x^3*Cos[x^4],x]

[Out] Sin[x^4]/4

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3380

```
Int[((a_.) + Cos[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Cos[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rubi steps

$$\begin{aligned} \int x^3 \cos\left(x^4\right) dx &= \frac{1}{4}\text{Subst}\left(\int \cos(x)\, dx, x, x^4\right)\\ &= \frac{\sin\left(x^4\right)}{4} \end{aligned}$$

Mathematica [A] time = 0.00, size = 8, normalized size = 1.00

$$\frac{\sin\left(x^4\right)}{4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*Cos[x^4],x]
```

```
[Out] Sin[x^4]/4
```

fricas [A] time = 0.67, size = 6, normalized size = 0.75

$$\frac{1}{4}\sin\left(x^4\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*cos(x^4),x, algorithm="fricas")
```

```
[Out] 1/4*sin(x^4)
```

giac [A] time = 0.14, size = 6, normalized size = 0.75

$$\frac{1}{4}\sin\left(x^4\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*cos(x^4),x, algorithm="giac")
```

```
[Out] 1/4*sin(x^4)
```

maple [A] time = 0.04, size = 7, normalized size = 0.88

$$\frac{\sin\left(x^4\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*cos(x^4),x)
```

```
[Out] 1/4*sin(x^4)
```

maxima [A] time = 0.31, size = 6, normalized size = 0.75

$$\frac{1}{4}\sin\left(x^4\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*cos(x^4),x, algorithm="maxima")
```

```
[Out] 1/4*sin(x^4)
```

mupad [B] time = 0.07, size = 6, normalized size = 0.75

$$\frac{\sin\left(x^4\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*cos(x^4),x)
[Out] sin(x^4)/4
```

sympy [A] time = 0.48, size = 5, normalized size = 0.62

$$\frac{\sin\left(x^4\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*cos(x**4),x)
[Out] sin(x**4)/4
```

3.769 $\int x\sin\left(\frac{x^2}{2}\right)\,dx$

Optimal. Leaf size=10

$$-\cos\left(\frac{x^2}{2}\right)$$

```
[Out] -cos(1/2*x^2)
```

Rubi [A] time = 0.01, antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {3379, 2638}

$$-\cos\left(\frac{x^2}{2}\right)$$

Antiderivative was successfully verified.

```
[In] Int[x*Sin[x^2/2],x]
```

```
[Out] -Cos[x^2/2]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3379

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rubi steps

$$\begin{aligned}\int x\sin\left(\frac{x^2}{2}\right)\,dx &= \frac{1}{2}\text{Subst}\left(\int \sin\left(\frac{x}{2}\right)\,dx,x,x^2\right)\\ &= -\cos\left(\frac{x^2}{2}\right)\end{aligned}$$

Mathematica [A] time = 0.01, size = 10, normalized size = 1.00

$$-\cos\left(\frac{x^2}{2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Sin[x^2/2],x]
[Out] -Cos[x^2/2]
```

fricas [A] time = 0.49, size = 8, normalized size = 0.80

$$-\cos\left(\frac{1}{2}\,x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(1/2*x^2),x, algorithm="fricas")
[Out] -cos(1/2*x^2)
```

giac [A] time = 0.12, size = 8, normalized size = 0.80

$$-\cos\left(\frac{1}{2}\,x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(1/2*x^2),x, algorithm="giac")
[Out] -cos(1/2*x^2)
```

maple [A] time = 0.00, size = 9, normalized size = 0.90

$$-\cos\left(\frac{x^2}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*sin(1/2*x^2),x)
[Out] -cos(1/2*x^2)
```

maxima [A] time = 0.33, size = 8, normalized size = 0.80

$$-\cos\left(\frac{1}{2}\,x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(1/2*x^2),x, algorithm="maxima")
```

```
[Out] -cos(1/2*x^2)
```

mupad [B] time = 2.92, size = 8, normalized size = 0.80

$$-\cos\left(\frac{x^2}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*sin(x^2/2),x)
```

```
[Out] -cos(x^2/2)
```

sympy [A] time = 0.15, size = 7, normalized size = 0.70

$$-\cos\left(\frac{x^2}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(1/2*x**2),x)
```

```
[Out] -cos(x**2/2)
```

3.770 $\int x \sec \left(x^2\right) \tan \left(x^2\right) \, dx$

Optimal. Leaf size=8

$$\frac{\sec \left(x^2\right)}{2}$$

```
[Out] 1/2*sec(x^2)
```

Rubi [A] time = 0.06, antiderivative size = 8, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {6715, 2606, 8}

$$\frac{\sec \left(x^2\right)}{2}$$

Antiderivative was successfully verified.

```
[In] Int[x*Sec[x^2]*Tan[x^2],x]
```

```
[Out] Sec[x^2]/2
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 6715

```
Int[(u_)*(x_)^(m_.), x_Symbol] :> Dist[1/(m + 1), Subst[Int[SubstFor[x^(m +
 1), u, x], x], x, x^(m + 1)], x] /; FreeQ[m, x] && NeQ[m, -1] && FunctionO
fQ[x^(m + 1), u, x]
```

Rubi steps

$$\int x\sec\left(x^2\right)\tan\left(x^2\right)\,dx = \frac{1}{2}\,\text{Subst}\left(\int \sec(x)\tan(x)\,dx, x, x^2\right)$$
$$= \frac{1}{2}\,\text{Subst}\left(\int 1\,dx, x, \sec\left(x^2\right)\right)$$
$$= \frac{\sec\left(x^2\right)}{2}$$

Mathematica [A] time = 0.01, size = 8, normalized size = 1.00

$$\frac{\sec\left(x^2\right)}{2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Sec[x^2]*Tan[x^2],x]
[Out] Sec[x^2]/2
```

fricas [A] time = 0.77, size = 8, normalized size = 1.00

$$\frac{1}{2\cos\left(x^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sec(x^2)*tan(x^2),x, algorithm="fricas")
[Out] 1/2/cos(x^2)
```

giac [A] time = 0.12, size = 8, normalized size = 1.00

$$\frac{1}{2\cos\left(x^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sec(x^2)*tan(x^2),x, algorithm="giac")
[Out] 1/2/cos(x^2)
```

maple [A] time = 0.03, size = 7, normalized size = 0.88

$$\frac{\sec\left(x^2\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*sec(x^2)*tan(x^2),x)
```

```
[Out] 1/2*sec(x^2)
```

maxima [B] time = 0.33, size = 56, normalized size = 7.00

$$\frac{\cos\left(2\,x^2\right)\cos\left(x^2\right)+\sin\left(2\,x^2\right)\sin\left(x^2\right)+\cos\left(x^2\right)}{\cos\left(2\,x^2\right)^2+\sin\left(2\,x^2\right)^2+2\,\cos\left(2\,x^2\right)+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sec(x^2)*tan(x^2),x, algorithm="maxima")
```

```
[Out] (cos(2*x^2)*cos(x^2) + sin(2*x^2)*sin(x^2) + cos(x^2))/(cos(2*x^2)^2 + sin(
2*x^2)^2 + 2*cos(2*x^2) + 1)
```

mupad [B] time = 0.07, size = 8, normalized size = 1.00

$$\frac{1}{2\,\cos\left(x^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*tan(x^2))/cos(x^2),x)
```

```
[Out] 1/(2*cos(x^2))
```

sympy [A] time = 0.28, size = 5, normalized size = 0.62

$$\frac{\sec\left(x^2\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sec(x**2)*tan(x**2),x)
```

```
[Out] sec(x**2)/2
```

3.771 $\int \frac{\tan^2\left(\frac{1}{x}\right)}{x^2} \, dx$

Optimal. Leaf size=10

$$\frac{1}{x} - \tan\left(\frac{1}{x}\right)$$

```
[Out] 1/x-tan(1/x)
```

Rubi [A] time = 0.02, antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {3747, 3473, 8}

$$\frac{1}{x} - \tan\left(\frac{1}{x}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Tan[x^(-1)]^2/x^2,x]
```

```
[Out] x^(-1) - Tan[x^(-1)]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3747

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Tan[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Tan[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IGtQ[Simplify[(m +
1)/n], 0] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}\int \frac{\tan^2\left(\frac{1}{x}\right)}{x^2}\,dx &= -\,\mathrm{Subst}\left(\int \tan^2(x)\,dx, x, \frac{1}{x}\right)\\ &= -\tan\left(\frac{1}{x}\right) + \mathrm{Subst}\left(\int 1\,dx, x, \frac{1}{x}\right)\\ &= \frac{1}{x} - \tan\left(\frac{1}{x}\right)\end{aligned}$$

Mathematica [A] time = 0.02, size = 12, normalized size = 1.20

$$\tan^{-1}\left(\tan\left(\frac{1}{x}\right)\right) - \tan\left(\frac{1}{x}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[x^(-1)]^2/x^2,x]
[Out] ArcTan[Tan[x^(-1)]] - Tan[x^(-1)]
```

fricas [A] time = 0.72, size = 13, normalized size = 1.30

$$-\frac{x\tan\left(\frac{1}{x}\right) - 1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(1/x)^2/x^2,x, algorithm="fricas")
[Out] -(x*tan(1/x) - 1)/x
```

giac [A] time = 0.14, size = 10, normalized size = 1.00

$$\frac{1}{x} - \tan\left(\frac{1}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(1/x)^2/x^2,x, algorithm="giac")
[Out] 1/x - tan(1/x)
```

maple [A] time = 0.01, size = 11, normalized size = 1.10

$$\frac{1}{x} - \tan\left(\frac{1}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(1/x)^2/x^2,x)
```

```
[Out] 1/x-tan(1/x)
```

maxima [B] time = 0.33, size = 67, normalized size = 6.70

$$\frac{\cos\left(\frac{2}{x}\right)^{2}-2\,x\sin\left(\frac{2}{x}\right)+\sin\left(\frac{2}{x}\right)^{2}+2\,\cos\left(\frac{2}{x}\right)+1}{\left(\cos\left(\frac{2}{x}\right)^{2}+\sin\left(\frac{2}{x}\right)^{2}+2\,\cos\left(\frac{2}{x}\right)+1\right)x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(1/x)^2/x^2,x, algorithm="maxima")
```

```
[Out] (cos(2/x)^2 - 2*x*sin(2/x) + sin(2/x)^2 + 2*cos(2/x) + 1)/((cos(2/x)^2 + si
n(2/x)^2 + 2*cos(2/x) + 1)*x)
```

mupad [B] time = 2.92, size = 10, normalized size = 1.00

$$\frac{1}{x}-\tan\left(\frac{1}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(1/x)^2/x^2,x)
```

```
[Out] 1/x - tan(1/x)
```

sympy [A] time = 0.22, size = 7, normalized size = 0.70

$$-\tan\left(\frac{1}{x}\right)+\frac{1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(1/x)**2/x**2,x)
```

```
[Out] -tan(1/x) + 1/x
```

3.772 $\int x \tan\left(1+x^2\right)\,dx$

Optimal. Leaf size=11

$$-\frac{1}{2}\log\left(\cos\left(x^2+1\right)\right)$$

```
[Out] -1/2*ln(cos(x^2+1))
```

Rubi [A] time = 0.01, antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {3747, 3475}

$$-\frac{1}{2}\log\left(\cos\left(x^2+1\right)\right)$$

Antiderivative was successfully verified.

```
[In] Int[x*Tan[1 + x^2],x]
```

```
[Out] -Log[Cos[1 + x^2]]/2
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3747

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Tan[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Tan[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IGtQ[Simplify[(m +
1)/n], 0] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}\int x \tan\left(1+x^2\right)\,dx &= \frac{1}{2}\text{Subst}\left(\int \tan(1+x)\,dx, x, x^2\right)\\ &= -\frac{1}{2}\log\left(\cos\left(1+x^2\right)\right)\end{aligned}$$

Mathematica [A] time = 0.02, size = 11, normalized size = 1.00

$$-\frac{1}{2}\log\left(\cos\left(x^2+1\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Tan[1 + x^2],x]
```

```
[Out] -1/2*Log[Cos[1 + x^2]]
```

fricas [A] time = 0.79, size = 15, normalized size = 1.36

$$-\frac{1}{4}\log\left(\frac{1}{\tan\left(x^2+1\right)^2+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*tan(x^2+1),x, algorithm="fricas")
```

```
[Out] -1/4*log(1/(tan(x^2 + 1)^2 + 1))
```

giac [A] time = 0.17, size = 10, normalized size = 0.91

$$-\frac{1}{2}\log\left(\left|\cos\left(x^2+1\right)\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*tan(x^2+1),x, algorithm="giac")
```

```
[Out] -1/2*log(abs(cos(x^2 + 1)))
```

maple [A] time = 0.00, size = 10, normalized size = 0.91

$$-\frac{\ln\left(\cos\left(x^2+1\right)\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*tan(x^2+1),x)
```

```
[Out] -1/2*ln(cos(x^2+1))
```

maxima [A] time = 0.33, size = 9, normalized size = 0.82

$$\frac{1}{2}\log\left(\sec\left(x^2+1\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*tan(x^2+1),x, algorithm="maxima")
```

```
[Out] 1/2*log(sec(x^2 + 1))
```

mupad [B] time = 0.28, size = 13, normalized size = 1.18

$$\frac{\ln\left(\tan\left(x^2+1\right)^2+1\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*tan(x^2 + 1),x)
[Out] log(tan(x^2 + 1)^2 + 1)/4
```

sympy [A] time = 0.12, size = 12, normalized size = 1.09

$$\frac{\log\left(\tan^2\left(x^2+1\right)+1\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*tan(x**2+1),x)
[Out] log(tan(x**2 + 1)**2 + 1)/4
```

3.773 $\int \sin(\pi(1+2x))\,dx$

Optimal. Leaf size=12

$$\frac{\cos(2\pi x)}{2\pi}$$

```
[Out] 1/2*cos(2*Pi*x)/Pi
```

Rubi [A] time = 0.00, antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {2638}

$$\frac{\cos(2\pi x)}{2\pi}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[Pi*(1 + 2*x)],x]
```

```
[Out] Cos[2*Pi*x]/(2*Pi)
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rubi steps

$$\int \sin(\pi(1+2x))\,dx = \frac{\cos(2\pi x)}{2\pi}$$

Mathematica [A] time = 0.01, size = 12, normalized size = 1.00

$$\frac{\cos(2\pi x)}{2\pi}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[Pi*(1 + 2*x)],x]
```

```
[Out] Cos[2*Pi*x]/(2*Pi)
```

fricas [A] time = 0.57, size = 12, normalized size = 1.00

$$-\frac{\cos(\pi + 2\pi x)}{2\pi}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(pi*(1+2*x)),x, algorithm="fricas")
```

```
[Out] -1/2*cos(pi + 2*pi*x)/pi
```

giac [A] time = 0.14, size = 10, normalized size = 0.83

$$\frac{\cos\left(2\,\pi x\right)}{2\,\pi}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(pi*(1+2*x)),x, algorithm="giac")
```

```
[Out] 1/2*cos(2*pi*x)/pi
```

maple [A] time = 0.04, size = 11, normalized size = 0.92

$$\frac{\cos\left(2\pi x\right)}{2\pi}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(Pi*(1+2*x)),x)
```

```
[Out] 1/2*cos(2*Pi*x)/Pi
```

maxima [A] time = 0.32, size = 10, normalized size = 0.83

$$\frac{\cos\left(2\,\pi x\right)}{2\,\pi}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(pi*(1+2*x)),x, algorithm="maxima")
```

```
[Out] 1/2*cos(2*pi*x)/pi
```

mupad [B] time = 2.92, size = 13, normalized size = 1.08

$$-\frac{\cos\left(\Pi\,\left(2\,x+1\right)\right)}{2\,\Pi}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(Pi*(2*x + 1)),x)
```

```
[Out] -cos(Pi*(2*x + 1))/(2*Pi)
```

sympy [A] time = 0.79, size = 12, normalized size = 1.00

$$-\frac{\cos\left(\pi\left(2x+1\right)\right)}{2\pi}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(pi*(1+2*x)),x)
```

```
[Out] -cos(pi*(2*x + 1))/(2*pi)
```

3.774 $\int \frac{\cot(x)+\csc^2(x)}{1-\cos^2(x)}\,dx$

Optimal. Leaf size=21

$$-\frac{1}{3}\cot^3(x)-\frac{\cot^2(x)}{2}-\cot(x)$$

```
[Out] -cot(x)-1/2*cot(x)^2-1/3*cot(x)^3
```

Rubi [A] time = 0.06, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 1, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.056, Rules used = {14}

$$-\frac{1}{3}\cot^3(x)-\frac{\cot^2(x)}{2}-\cot(x)$$

Antiderivative was successfully verified.

```
[In] Int[(Cot[x] + Csc[x]^2)/(1 - Cos[x]^2),x]
```

```
[Out] -Cot[x] - Cot[x]^2/2 - Cot[x]^3/3
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\int \frac{\cot(x)+\csc^2(x)}{1-\cos^2(x)}\,dx &= \text{Subst}\left(\int \frac{1+x+x^2}{x^4}\,dx,x,\tan(x)\right)\\ &= \text{Subst}\left(\int \left(\frac{1}{x^4}+\frac{1}{x^3}+\frac{1}{x^2}\right)dx,x,\tan(x)\right)\\ &= -\cot(x)-\frac{\cot^2(x)}{2}-\frac{\cot^3(x)}{3}\end{aligned}$$

Mathematica [A] time = 0.02, size = 25, normalized size = 1.19

$$-\frac{2\cot(x)}{3}-\frac{\csc^2(x)}{2}-\frac{1}{3}\cot(x)\csc^2(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cot[x] + Csc[x]^2)/(1 - Cos[x]^2),x]
[Out] (-2*Cot[x])/3 - Csc[x]^2/2 - (Cot[x]*Csc[x]^2)/3
```

fricas [A] time = 0.67, size = 29, normalized size = 1.38

$$-\frac{4\cos(x)^3 - 6\cos(x) - 3\sin(x)}{6\left(\cos(x)^2 - 1\right)\sin(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((cot(x)+csc(x)^2)/(1-cos(x)^2),x, algorithm="fricas")
[Out] -1/6*(4*cos(x)^3 - 6*cos(x) - 3*sin(x))/((cos(x)^2 - 1)*sin(x))
```

giac [A] time = 0.15, size = 18, normalized size = 0.86

$$-\frac{6\tan(x)^2 + 3\tan(x) + 2}{6\tan(x)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((cot(x)+csc(x)^2)/(1-cos(x)^2),x, algorithm="giac")
[Out] -1/6*(6*tan(x)^2 + 3*tan(x) + 2)/tan(x)^3
```

maple [A] time = 0.15, size = 20, normalized size = 0.95

$$-\frac{1}{2\tan(x)^2} - \frac{1}{\tan(x)} - \frac{1}{3\tan(x)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cot(x)+csc(x)^2)/(1-cos(x)^2),x)
[Out] -1/2/tan(x)^2-1/tan(x)-1/3/tan(x)^3
```

maxima [A] time = 0.33, size = 18, normalized size = 0.86

$$-\frac{6\tan(x)^2 + 3\tan(x) + 2}{6\tan(x)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((cot(x)+csc(x)^2)/(1-cos(x)^2),x, algorithm="maxima")
[Out] -1/6*(6*tan(x)^2 + 3*tan(x) + 2)/tan(x)^3
```

mupad [B] time = 3.00, size = 16, normalized size = 0.76

$$-\frac{\cot(x)\,\left(2\cot(x)^2+3\cot(x)+6\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(cot(x) + 1/sin(x)^2)/(cos(x)^2 - 1),x)
```

```
[Out] -(cot(x)*(3*cot(x) + 2*cot(x)^2 + 6))/6
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int\frac{\cot(x)}{\cos^2(x)-1}\,dx-\int\frac{\csc^2(x)}{\cos^2(x)-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((cot(x)+csc(x)**2)/(1-cos(x)**2),x)
```

```
[Out] -Integral(cot(x)/(cos(x)**2 - 1), x) - Integral(csc(x)**2/(cos(x)**2 - 1),
x)
```

3.775 $\int x^2 \cos(4x^3)\cos(5x^3)\,dx$

Optimal. Leaf size=19

$$\frac{\sin(x^3)}{6}+\frac{1}{54}\sin(9x^3)$$

```
[Out] 1/6*sin(x^3)+1/54*sin(9*x^3)
```

Rubi [A] time = 0.04, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {4572, 3380, 2637}

$$\frac{\sin(x^3)}{6}+\frac{1}{54}\sin(9x^3)$$

Antiderivative was successfully verified.

```
[In] Int[x^2*Cos[4*x^3]*Cos[5*x^3],x]
```

```
[Out] Sin[x^3]/6 + Sin[9*x^3]/54
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3380

```
Int[((a_.) + Cos[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Cos[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rule 4572

```
Int[Cos[v_]^(p_.)*Cos[w_]^(q_.)*(x_)^(m_.), x_Symbol] :> Int[ExpandTrigRedu
ce[x^m, Cos[v]^p*Cos[w]^q, x], x] /; IGtQ[m, 0] && IGtQ[p, 0] && IGtQ[q, 0]
 && ((PolynomialQ[v, x] && PolynomialQ[w, x]) || (BinomialQ[{v, w}, x] && I
ndependentQ[Cancel[v/w], x]))
```

Rubi steps

$$\begin{aligned}
\int x^2\cos\left(4x^3\right)\cos\left(5x^3\right)\,dx &= \int \left(\frac{1}{2}x^2\cos\left(x^3\right)+\frac{1}{2}x^2\cos\left(9x^3\right)\right)dx\\
&= \frac{1}{2}\int x^2\cos\left(x^3\right)\,dx+\frac{1}{2}\int x^2\cos\left(9x^3\right)\,dx\\
&= \frac{1}{6}\operatorname{Subst}\left(\int \cos(x)\,dx,x,x^3\right)+\frac{1}{6}\operatorname{Subst}\left(\int \cos(9x)\,dx,x,x^3\right)\\
&= \frac{\sin\left(x^3\right)}{6}+\frac{1}{54}\sin\left(9x^3\right)
\end{aligned}$$

Mathematica [A] time = 0.01, size = 19, normalized size = 1.00

$$\frac{\sin\left(x^3\right)}{6}+\frac{1}{54}\sin\left(9x^3\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*Cos[4*x^3]*Cos[5*x^3],x]
```

```
[Out] Sin[x^3]/6 + Sin[9*x^3]/54
```

fricas [B] time = 0.50, size = 40, normalized size = 2.11

$$\frac{1}{27}\left(128\,\cos\left(x^3\right)^8-224\,\cos\left(x^3\right)^6+120\,\cos\left(x^3\right)^4-20\,\cos\left(x^3\right)^2+5\right)\sin\left(x^3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*cos(4*x^3)*cos(5*x^3),x, algorithm="fricas")
```

```
[Out] 1/27*(128*cos(x^3)^8 - 224*cos(x^3)^6 + 120*cos(x^3)^4 - 20*cos(x^3)^2 + 5)
*sin(x^3)
```

giac [B] time = 0.14, size = 39, normalized size = 2.05

$$\frac{128}{27}\,\sin\left(x^3\right)^9-\frac{32}{3}\,\sin\left(x^3\right)^7+8\,\sin\left(x^3\right)^5-\frac{20}{9}\,\sin\left(x^3\right)^3+\frac{1}{3}\,\sin\left(x^3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*cos(4*x^3)*cos(5*x^3),x, algorithm="giac")
```

```
[Out] 128/27*sin(x^3)^9 - 32/3*sin(x^3)^7 + 8*sin(x^3)^5 - 20/9*sin(x^3)^3 + 1/3*
sin(x^3)
```

maple [**A**] time = 0.23, size = 16, normalized size = 0.84

$$\frac{\sin\left(x^3\right)}{6}+\frac{\sin\left(9x^3\right)}{54}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*cos(4*x^3)*cos(5*x^3),x)
```

```
[Out] 1/6*sin(x^3)+1/54*sin(9*x^3)
```

maxima [**A**] time = 0.33, size = 15, normalized size = 0.79

$$\frac{1}{54}\,\sin\left(9\,x^3\right)+\frac{1}{6}\,\sin\left(x^3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*cos(4*x^3)*cos(5*x^3),x, algorithm="maxima")
```

```
[Out] 1/54*sin(9*x^3) + 1/6*sin(x^3)
```

mupad [**B**] time = 3.03, size = 15, normalized size = 0.79

$$\frac{\sin\left(x^3\right)}{6}+\frac{\sin\left(9\,x^3\right)}{54}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*cos(4*x^3)*cos(5*x^3),x)
```

```
[Out] sin(x^3)/6 + sin(9*x^3)/54
```

sympy [**B**] time = 5.25, size = 32, normalized size = 1.68

$$-\frac{4\sin\left(4x^3\right)\cos\left(5x^3\right)}{27}+\frac{5\sin\left(5x^3\right)\cos\left(4x^3\right)}{27}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*cos(4*x**3)*cos(5*x**3),x)
```

```
[Out] -4*sin(4*x**3)*cos(5*x**3)/27 + 5*sin(5*x**3)*cos(4*x**3)/27
```

3.776 $\int x^{14} \sin\left(x^3\right) dx$

Optimal. Leaf size=47

$$-8x^3 \sin\left(x^3\right) - 8\cos\left(x^3\right) - \frac{1}{3}x^{12}\cos\left(x^3\right) + \frac{4}{3}x^9 \sin\left(x^3\right) + 4x^6 \cos\left(x^3\right)$$

```
[Out] -8*cos(x^3)+4*x^6*cos(x^3)-1/3*x^12*cos(x^3)-8*x^3*sin(x^3)+4/3*x^9*sin(x^3
)
```

Rubi [A] time = 0.06, antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {3379, 3296, 2638}

$$\frac{4}{3}x^9 \sin\left(x^3\right) - 8x^3 \sin\left(x^3\right) - \frac{1}{3}x^{12}\cos\left(x^3\right) + 4x^6 \cos\left(x^3\right) - 8\cos\left(x^3\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^14*Sin[x^3],x]
```

```
[Out] -8*Cos[x^3] + 4*x^6*Cos[x^3] - (x^12*Cos[x^3])/3 - 8*x^3*Sin[x^3] + (4*x^9*
Sin[x^3])/3
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3379

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rubi steps

$$\begin{aligned}\int x^{14}\sin\left(x^3\right)\,dx &= \frac{1}{3}\,\text{Subst}\left(\int x^4\sin(x)\,dx,x,x^3\right)\\ &= -\frac{1}{3}x^{12}\cos\left(x^3\right)+\frac{4}{3}\,\text{Subst}\left(\int x^3\cos(x)\,dx,x,x^3\right)\\ &= -\frac{1}{3}x^{12}\cos\left(x^3\right)+\frac{4}{3}x^9\sin\left(x^3\right)-4\,\text{Subst}\left(\int x^2\sin(x)\,dx,x,x^3\right)\\ &= 4x^6\cos\left(x^3\right)-\frac{1}{3}x^{12}\cos\left(x^3\right)+\frac{4}{3}x^9\sin\left(x^3\right)-8\,\text{Subst}\left(\int x\cos(x)\,dx,x,x^3\right)\\ &= 4x^6\cos\left(x^3\right)-\frac{1}{3}x^{12}\cos\left(x^3\right)-8x^3\sin\left(x^3\right)+\frac{4}{3}x^9\sin\left(x^3\right)+8\,\text{Subst}\left(\int \sin(x)\,dx,x,x^3\right)\\ &= -8\cos\left(x^3\right)+4x^6\cos\left(x^3\right)-\frac{1}{3}x^{12}\cos\left(x^3\right)-8x^3\sin\left(x^3\right)+\frac{4}{3}x^9\sin\left(x^3\right)\end{aligned}$$

Mathematica [A] time = 0.03, size = 35, normalized size = 0.74

$$\frac{4}{3}x^3\left(x^6-6\right)\sin\left(x^3\right)-\frac{1}{3}\left(x^{12}-12x^6+24\right)\cos\left(x^3\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^14*Sin[x^3],x]
[Out] -1/3*((24 - 12*x^6 + x^12)*Cos[x^3]) + (4*x^3*(-6 + x^6)*Sin[x^3])/3
```

fricas [A] time = 0.71, size = 32, normalized size = 0.68

$$-\frac{1}{3}\left(x^{12}-12\,x^6+24\right)\cos\left(x^3\right)+\frac{4}{3}\left(x^9-6\,x^3\right)\sin\left(x^3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^14*sin(x^3),x, algorithm="fricas")
[Out] -1/3*(x^12 - 12*x^6 + 24)*cos(x^3) + 4/3*(x^9 - 6*x^3)*sin(x^3)
```

giac [A] time = 0.12, size = 32, normalized size = 0.68

$$-\frac{1}{3}\left(x^{12}-12\,x^6+24\right)\cos\left(x^3\right)+\frac{4}{3}\left(x^9-6\,x^3\right)\sin\left(x^3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^14*sin(x^3),x, algorithm="giac")
[Out] -1/3*(x^12 - 12*x^6 + 24)*cos(x^3) + 4/3*(x^9 - 6*x^3)*sin(x^3)
```

maple [A] time = 0.03, size = 33, normalized size = 0.70

$$\left(-\frac{1}{3}x^{12}+4x^{6}-8\right)\cos\left(x^{3}\right)+\frac{4x^{3}\left(x^{6}-6\right)\sin\left(x^{3}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^14*sin(x^3),x)
```

```
[Out] (-1/3*x^12+4*x^6-8)*cos(x^3)+4/3*x^3*(x^6-6)*sin(x^3)
```

maxima [A] time = 0.33, size = 32, normalized size = 0.68

$$-\frac{1}{3}\left(x^{12}-12\,x^{6}+24\right)\cos\left(x^{3}\right)+\frac{4}{3}\left(x^{9}-6\,x^{3}\right)\sin\left(x^{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^14*sin(x^3),x, algorithm="maxima")
```

```
[Out] -1/3*(x^12 - 12*x^6 + 24)*cos(x^3) + 4/3*(x^9 - 6*x^3)*sin(x^3)
```

mupad [B] time = 3.00, size = 43, normalized size = 0.91

$$4\,x^{6}\,\cos\left(x^{3}\right)-8\,\cos\left(x^{3}\right)-\frac{x^{12}\,\cos\left(x^{3}\right)}{3}-8\,x^{3}\,\sin\left(x^{3}\right)+\frac{4\,x^{9}\,\sin\left(x^{3}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^14*sin(x^3),x)
```

```
[Out] 4*x^6*cos(x^3) - 8*cos(x^3) - (x^12*cos(x^3))/3 - 8*x^3*sin(x^3) + (4*x^9*s
in(x^3))/3
```

sympy [A] time = 71.97, size = 48, normalized size = 1.02

$$-\frac{x^{12}\cos\left(x^{3}\right)}{3}+\frac{4x^{9}\sin\left(x^{3}\right)}{3}+4x^{6}\cos\left(x^{3}\right)-8x^{3}\sin\left(x^{3}\right)-8\cos\left(x^{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**14*sin(x**3),x)
```

```
[Out] -x**12*cos(x**3)/3 + 4*x**9*sin(x**3)/3 + 4*x**6*cos(x**3) - 8*x**3*sin(x**
3) - 8*cos(x**3)
```

3.777 $\int e^{-3x^3} x^2 \sin\left(2x^3\right) dx$

Optimal. Leaf size=35

$$-\frac{1}{13} e^{-3x^3} \sin\left(2x^3\right) - \frac{2}{39} e^{-3x^3} \cos\left(2x^3\right)$$

```
[Out] -2/39*cos(2*x^3)/exp(3*x^3)-1/13*sin(2*x^3)/exp(3*x^3)
```

Rubi [A] time = 0.16, antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {6715, 4432}

$$-\frac{1}{13} e^{-3x^3} \sin\left(2x^3\right) - \frac{2}{39} e^{-3x^3} \cos\left(2x^3\right)$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*Sin[2*x^3])/E^(3*x^3),x]
```

```
[Out] (-2*Cos[2*x^3])/(39*E^(3*x^3)) - Sin[2*x^3]/(13*E^(3*x^3))
```

Rule 4432

```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sin[(d_.) + (e_.)*(x_)], x_Symbol] :>
 Simp[(b*c*Log[F]*F^(c*(a + b*x))*Sin[d + e*x])/(e^2 + b^2*c^2*Log[F]^2), x
] - Simp[(e*F^(c*(a + b*x))*Cos[d + e*x])/(e^2 + b^2*c^2*Log[F]^2), x] /; F
reeQ[{F, a, b, c, d, e}, x] && NeQ[e^2 + b^2*c^2*Log[F]^2, 0]
```

Rule 6715

```
Int[(u_)*(x_)^(m_.), x_Symbol] :> Dist[1/(m + 1), Subst[Int[SubstFor[x^(m +
 1), u, x], x], x, x^(m + 1)], x] /; FreeQ[m, x] && NeQ[m, -1] && FunctionO
fQ[x^(m + 1), u, x]
```

Rubi steps

$$\begin{aligned} \int e^{-3x^3} x^2 \sin\left(2x^3\right) dx &= \frac{1}{3} \text{Subst}\left(\int e^{-3x} \sin(2x)\, dx, x, x^3\right) \\ &= -\frac{2}{39} e^{-3x^3} \cos\left(2x^3\right) - \frac{1}{13} e^{-3x^3} \sin\left(2x^3\right) \end{aligned}$$

Mathematica [A] time = 0.04, size = 28, normalized size = 0.80

$$-\frac{1}{39} e^{-3x^3} \left(3 \sin\left(2x^3\right) + 2 \cos\left(2x^3\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*Sin[2*x^3])/E^(3*x^3),x]
[Out] -1/39*(2*Cos[2*x^3] + 3*Sin[2*x^3])/E^(3*x^3)
```

fricas [A] time = 0.75, size = 29, normalized size = 0.83

$$-\frac{2}{39}\cos\left(2x^3\right)e^{\left(-3x^3\right)}-\frac{1}{13}e^{\left(-3x^3\right)}\sin\left(2x^3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*sin(2*x^3)/exp(3*x^3),x, algorithm="fricas")
[Out] -2/39*cos(2*x^3)*e^(-3*x^3) - 1/13*e^(-3*x^3)*sin(2*x^3)
```

giac [A] time = 0.15, size = 25, normalized size = 0.71

$$-\frac{1}{39}\left(2\cos\left(2x^3\right)+3\sin\left(2x^3\right)\right)e^{\left(-3x^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*sin(2*x^3)/exp(3*x^3),x, algorithm="giac")
[Out] -1/39*(2*cos(2*x^3) + 3*sin(2*x^3))*e^(-3*x^3)
```

maple [A] time = 0.03, size = 36, normalized size = 1.03

$$\frac{\left(-\frac{2}{39}+\frac{2\left(\tan^2\left(x^3\right)\right)}{39}-\frac{2\tan\left(x^3\right)}{13}\right)\mathrm{e}^{-3x^3}}{1+\tan^2\left(x^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*sin(2*x^3)/exp(3*x^3),x)
[Out] (-2/39+2/39*tan(x^3)^2-2/13*tan(x^3))/(1+tan(x^3)^2)/exp(3*x^3)
```

maxima [A] time = 0.33, size = 25, normalized size = 0.71

$$-\frac{1}{39}\left(2\cos\left(2x^3\right)+3\sin\left(2x^3\right)\right)e^{\left(-3x^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*sin(2*x^3)/exp(3*x^3),x, algorithm="maxima")
```

```
[Out] -1/39*(2*cos(2*x^3) + 3*sin(2*x^3))*e^(-3*x^3)
```

mupad [B] time = 2.97, size = 25, normalized size = 0.71

$$-\frac{e^{-3\,x^3}\,\left(2\,\cos\left(2\,x^3\right)+3\,\sin\left(2\,x^3\right)\right)}{39}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*exp(-3*x^3)*sin(2*x^3),x)
[Out] -(exp(-3*x^3)*(2*cos(2*x^3) + 3*sin(2*x^3)))/39
```

sympy [A] time = 1.82, size = 32, normalized size = 0.91

$$-\frac{e^{-3x^3}\sin\left(2x^3\right)}{13}-\frac{2e^{-3x^3}\cos\left(2x^3\right)}{39}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*sin(2*x**3)/exp(3*x**3),x)
[Out] -exp(-3*x**3)*sin(2*x**3)/13 - 2*exp(-3*x**3)*cos(2*x**3)/39
```

3.778 $\int 2x\cos\left(x^2\right)\,dx$

Optimal. Leaf size=4

$$\sin\left(x^2\right)$$

```
[Out] sin(x^2)
```

Rubi [A] time = 0.01, antiderivative size = 4, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {12, 3380, 2637}

$$\sin\left(x^2\right)$$

Antiderivative was successfully verified.

```
[In] Int[2*x*Cos[x^2],x]
```

```
[Out] Sin[x^2]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3380

```
Int[((a_.) + Cos[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Cos[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rubi steps

$$\begin{aligned}
\int 2x\cos\left(x^2\right)\,dx &= 2\int x\cos\left(x^2\right)\,dx\\
&= \text{Subst}\left(\int \cos(x)\,dx, x, x^2\right)\\
&= \sin\left(x^2\right)
\end{aligned}$$

Mathematica [A] time = 0.00, size = 4, normalized size = 1.00

$$\sin\left(x^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[2*x*Cos[x^2],x]
[Out] Sin[x^2]
```

fricas [A] time = 0.79, size = 4, normalized size = 1.00

$$\sin\left(x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(2*x*cos(x^2),x, algorithm="fricas")
[Out] sin(x^2)
```

giac [A] time = 0.14, size = 4, normalized size = 1.00

$$\sin\left(x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(2*x*cos(x^2),x, algorithm="giac")
[Out] sin(x^2)
```

maple [A] time = 0.00, size = 5, normalized size = 1.25

$$\sin\left(x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(2*x*cos(x^2),x)
[Out] sin(x^2)
```

maxima [A] time = 0.33, size = 4, normalized size = 1.00

$$\sin\left(x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(2*x*cos(x^2),x, algorithm="maxima")
```

```
[Out] sin(x^2)
```

mupad [B] time = 2.92, size = 4, normalized size = 1.00

$$\sin\left(x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(2*x*cos(x^2),x)
```

```
[Out] sin(x^2)
```

sympy [A] time = 0.15, size = 3, normalized size = 0.75

$$\sin\left(x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(2*x*cos(x**2),x)
```

```
[Out] sin(x**2)
```

3.779 $\int 3x^2 \cos\left(7+x^3\right)\,dx$

Optimal. Leaf size=6

$$\sin\left(x^3+7\right)$$

```
[Out] sin(x^3+7)
```

Rubi [A] time = 0.01, antiderivative size = 6, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {12, 3380, 2637}

$$\sin\left(x^3+7\right)$$

Antiderivative was successfully verified.

```
[In] Int[3*x^2*Cos[7 + x^3],x]
```

```
[Out] Sin[7 + x^3]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3380

```
Int[((a_.) + Cos[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Cos[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rubi steps

$$\begin{aligned}
\int 3x^2\cos\left(7+x^3\right)\,dx &= 3\int x^2\cos\left(7+x^3\right)\,dx\\
&= \text{Subst}\left(\int \cos(7+x)\,dx, x, x^3\right)\\
&= \sin\left(7+x^3\right)
\end{aligned}$$

Mathematica [A] time = 0.00, size = 6, normalized size = 1.00

$$\sin\left(x^3+7\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[3*x^2*Cos[7 + x^3],x]
[Out] Sin[7 + x^3]
```

fricas [A] time = 0.65, size = 6, normalized size = 1.00

$$\sin\left(x^3+7\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(3*x^2*cos(x^3+7),x, algorithm="fricas")
[Out] sin(x^3 + 7)
```

giac [A] time = 0.12, size = 6, normalized size = 1.00

$$\sin\left(x^3+7\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(3*x^2*cos(x^3+7),x, algorithm="giac")
[Out] sin(x^3 + 7)
```

maple [A] time = 0.04, size = 7, normalized size = 1.17

$$\sin\left(x^3+7\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(3*x^2*cos(x^3+7),x)
[Out] sin(x^3+7)
```

maxima [A] time = 0.32, size = 6, normalized size = 1.00

$$\sin\left(x^3+7\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(3*x^2*cos(x^3+7),x, algorithm="maxima")
```

```
[Out] sin(x^3 + 7)
```

mupad [B] time = 0.06, size = 6, normalized size = 1.00

$$\sin\left(x^3+7\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(3*x^2*cos(x^3 + 7),x)
```

```
[Out] sin(x^3 + 7)
```

sympy [A] time = 0.27, size = 5, normalized size = 0.83

$$\sin\left(x^3+7\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(3*x**2*cos(x**3+7),x)
```

```
[Out] sin(x**3 + 7)
```

3.780 $\int \left(\frac{1}{1+x^2}+\sin(x)\right)\,dx$

Optimal. Leaf size=7

$$\tan^{-1}(x)-\cos(x)$$

```
[Out] arctan(x)-cos(x)
```

Rubi [A] time = 0.00, antiderivative size = 7, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {203, 2638}

$$\tan^{-1}(x)-\cos(x)$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x^2)^(-1) + Sin[x],x]
```

```
[Out] ArcTan[x] - Cos[x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \left(\frac{1}{1+x^2}+\sin(x)\right)\,dx &= \int \frac{1}{1+x^2}\,dx+\int \sin(x)\,dx\\ &= \tan^{-1}(x)-\cos(x)\end{aligned}$$

Mathematica [A] time = 0.01, size = 7, normalized size = 1.00

$$\tan^{-1}(x)-\cos(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x^2)^(-1) + Sin[x],x]
```

```
[Out] ArcTan[x] - Cos[x]
```

fricas [A] time = 0.84, size = 7, normalized size = 1.00

$$\arctan(x) - \cos(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2+1)+sin(x),x, algorithm="fricas")
[Out] arctan(x) - cos(x)
```

giac [A] time = 0.12, size = 7, normalized size = 1.00

$$\arctan(x) - \cos(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2+1)+sin(x),x, algorithm="giac")
[Out] arctan(x) - cos(x)
```

maple [A] time = 0.00, size = 8, normalized size = 1.14

$$\arctan(x) - \cos(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2+1)+sin(x),x)
[Out] arctan(x)-cos(x)
```

maxima [A] time = 0.43, size = 7, normalized size = 1.00

$$\arctan(x) - \cos(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2+1)+sin(x),x, algorithm="maxima")
[Out] arctan(x) - cos(x)
```

mupad [B] time = 0.05, size = 7, normalized size = 1.00

$$\mathrm{atan}(x) - \cos(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x) + 1/(x^2 + 1),x)
```

```
[Out] atan(x) - cos(x)
```

sympy [**A**] time = 0.08, size = 5, normalized size = 0.71

$$-\cos(x) + \operatorname{atan}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**2+1)+sin(x),x)
[Out] -cos(x) + atan(x)
```

3.781 $\int x \sin(1+x^2)\,dx$

Optimal. Leaf size=10

$$-\frac{1}{2}\cos\left(x^2+1\right)$$

```
[Out] -1/2*cos(x^2+1)
```

Rubi [A] time = 0.01, antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {3379, 2638}

$$-\frac{1}{2}\cos\left(x^2+1\right)$$

Antiderivative was successfully verified.

```
[In] Int[x*Sin[1 + x^2],x]
```

```
[Out] -Cos[1 + x^2]/2
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3379

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rubi steps

$$\begin{aligned}\int x \sin\left(1+x^2\right)\,dx &= \frac{1}{2}\text{Subst}\left(\int \sin(1+x)\,dx, x, x^2\right)\\ &= -\frac{1}{2}\cos\left(1+x^2\right)\end{aligned}$$

Mathematica [B] time = 0.01, size = 21, normalized size = 2.10

$$\frac{1}{2}\sin(1)\sin\left(x^2\right)-\frac{1}{2}\cos(1)\cos\left(x^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Sin[1 + x^2],x]
[Out] -1/2*(Cos[1]*Cos[x^2]) + (Sin[1]*Sin[x^2])/2
```

fricas [A] time = 0.71, size = 8, normalized size = 0.80

$$-\frac{1}{2}\cos\left(x^2+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(x^2+1),x, algorithm="fricas")
[Out] -1/2*cos(x^2 + 1)
```

giac [A] time = 0.12, size = 8, normalized size = 0.80

$$-\frac{1}{2}\cos\left(x^2+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(x^2+1),x, algorithm="giac")
[Out] -1/2*cos(x^2 + 1)
```

maple [A] time = 0.00, size = 9, normalized size = 0.90

$$-\frac{\cos\left(x^2+1\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*sin(x^2+1),x)
[Out] -1/2*cos(x^2+1)
```

maxima [A] time = 0.32, size = 8, normalized size = 0.80

$$-\frac{1}{2}\cos\left(x^2+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(x^2+1),x, algorithm="maxima")
[Out] -1/2*cos(x^2 + 1)
```

mupad [B] time = 0.04, size = 8, normalized size = 0.80

$$-\frac{\cos\left(x^2+1\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*sin(x^2 + 1),x)
[Out] -cos(x^2 + 1)/2
```

sympy [A] time = 0.15, size = 8, normalized size = 0.80

$$-\frac{\cos\left(x^2+1\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(x**2+1),x)
[Out] -cos(x**2 + 1)/2
```

3.782 $\int x\cos\left(1+x^2\right)\,dx$

Optimal. Leaf size=10

$$\frac{1}{2}\sin\left(x^2+1\right)$$

```
[Out] 1/2*sin(x^2+1)
```

Rubi [A] time = 0.01, antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {3380, 2637}

$$\frac{1}{2}\sin\left(x^2+1\right)$$

Antiderivative was successfully verified.

```
[In] Int[x*Cos[1 + x^2],x]
```

```
[Out] Sin[1 + x^2]/2
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3380

```
Int[((a_.) + Cos[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Cos[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rubi steps

$$\begin{aligned}\int x\cos\left(1+x^2\right)\,dx &= \frac{1}{2}\text{Subst}\left(\int\cos(1+x)\,dx,x,x^2\right)\\ &= \frac{1}{2}\sin\left(1+x^2\right)\end{aligned}$$

Mathematica [A] time = 0.00, size = 10, normalized size = 1.00

$$\frac{1}{2}\sin\left(x^2+1\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Cos[1 + x^2],x]
```

```
[Out] Sin[1 + x^2]/2
```

fricas [A] time = 0.58, size = 8, normalized size = 0.80

$$\frac{1}{2}\sin\left(x^2+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(x^2+1),x, algorithm="fricas")
```

```
[Out] 1/2*sin(x^2 + 1)
```

giac [A] time = 0.14, size = 8, normalized size = 0.80

$$\frac{1}{2}\sin\left(x^2+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(x^2+1),x, algorithm="giac")
```

```
[Out] 1/2*sin(x^2 + 1)
```

maple [A] time = 0.04, size = 9, normalized size = 0.90

$$\frac{\sin\left(x^2+1\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*cos(x^2+1),x)
```

```
[Out] 1/2*sin(x^2+1)
```

maxima [A] time = 0.32, size = 8, normalized size = 0.80

$$\frac{1}{2}\sin\left(x^2+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(x^2+1),x, algorithm="maxima")
```

```
[Out] 1/2*sin(x^2 + 1)
```

mupad [B] time = 2.98, size = 8, normalized size = 0.80

$$\frac{\sin\left(x^2+1\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*cos(x^2 + 1),x)
```

```
[Out] sin(x^2 + 1)/2
```

sympy [A] time = 0.15, size = 7, normalized size = 0.70

$$\frac{\sin\left(x^2+1\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(x**2+1),x)
```

```
[Out] sin(x**2 + 1)/2
```

3.783 $\int \left(1+x^2 \cos \left(x^3\right)\right) \, dx$

Optimal. Leaf size=10

$$\frac{\sin \left(x^3\right)}{3}+x$$

```
[Out] x+1/3*sin(x^3)
```

Rubi [A] time = 0.01, antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {3380, 2637}

$$\frac{\sin \left(x^3\right)}{3}+x$$

Antiderivative was successfully verified.

```
[In] Int[1 + x^2*Cos[x^3],x]
```

```
[Out] x + Sin[x^3]/3
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3380

```
Int[((a_.) + Cos[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Cos[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rubi steps

$$\begin{aligned}
\int \left(1+x^2 \cos \left(x^3\right)\right) \, dx &= x+\int x^2 \cos \left(x^3\right) \, dx\\
&= x+\frac{1}{3} \text{Subst}\left(\int \cos (x) \, dx,x,x^3\right)\\
&= x+\frac{\sin \left(x^3\right)}{3}
\end{aligned}$$

Mathematica [A] time = 0.00, size = 10, normalized size = 1.00

$$\frac{\sin\left(x^3\right)}{3}+x$$

Antiderivative was successfully verified.

```
[In] Integrate[1 + x^2*Cos[x^3],x]
[Out] x + Sin[x^3]/3
```

fricas [A] time = 0.62, size = 8, normalized size = 0.80

$$x+\frac{1}{3}\sin\left(x^3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1+x^2*cos(x^3),x, algorithm="fricas")
[Out] x + 1/3*sin(x^3)
```

giac [A] time = 0.14, size = 8, normalized size = 0.80

$$x+\frac{1}{3}\sin\left(x^3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1+x^2*cos(x^3),x, algorithm="giac")
[Out] x + 1/3*sin(x^3)
```

maple [A] time = 0.00, size = 9, normalized size = 0.90

$$x+\frac{\sin\left(x^3\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1+x^2*cos(x^3),x)
[Out] x+1/3*sin(x^3)
```

maxima [A] time = 0.32, size = 8, normalized size = 0.80

$$x+\frac{1}{3}\sin\left(x^3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1+x^2*cos(x^3),x, algorithm="maxima")
```

```
[Out] x + 1/3*sin(x^3)
```

mupad [B] time = 0.05, size = 8, normalized size = 0.80

$$x + \frac{\sin\left(x^3\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*cos(x^3) + 1,x)
```

```
[Out] x + sin(x^3)/3
```

sympy [A] time = 0.26, size = 7, normalized size = 0.70

$$x + \frac{\sin\left(x^3\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1+x**2*cos(x**3),x)
```

```
[Out] x + sin(x**3)/3
```

3.784 $\int x^2 \sin\left(1+x^3\right)\,dx$

Optimal. Leaf size=10

$$-\frac{1}{3}\cos\left(x^3+1\right)$$

```
[Out] -1/3*cos(x^3+1)
```

Rubi [A] time = 0.01, antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {3379, 2638}

$$-\frac{1}{3}\cos\left(x^3+1\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^2*Sin[1 + x^3],x]
```

```
[Out] -Cos[1 + x^3]/3
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3379

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rubi steps

$$\begin{aligned}\int x^2 \sin\left(1+x^3\right)\,dx &= \frac{1}{3}\text{Subst}\left(\int \sin(1+x)\,dx, x, x^3\right)\\ &= -\frac{1}{3}\cos\left(1+x^3\right)\end{aligned}$$

Mathematica [B] time = 0.01, size = 21, normalized size = 2.10

$$\frac{1}{3}\sin(1)\sin\left(x^3\right)-\frac{1}{3}\cos(1)\cos\left(x^3\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*Sin[1 + x^3],x]
```

```
[Out] -1/3*(Cos[1]*Cos[x^3]) + (Sin[1]*Sin[x^3])/3
```

fricas [A] time = 0.96, size = 8, normalized size = 0.80

$$-\frac{1}{3}\cos\left(x^3+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*sin(x^3+1),x, algorithm="fricas")
```

```
[Out] -1/3*cos(x^3 + 1)
```

giac [A] time = 0.14, size = 8, normalized size = 0.80

$$-\frac{1}{3}\cos\left(x^3+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*sin(x^3+1),x, algorithm="giac")
```

```
[Out] -1/3*cos(x^3 + 1)
```

maple [A] time = 0.00, size = 9, normalized size = 0.90

$$-\frac{\cos\left(x^3+1\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*sin(x^3+1),x)
```

```
[Out] -1/3*cos(x^3+1)
```

maxima [A] time = 0.32, size = 8, normalized size = 0.80

$$-\frac{1}{3}\cos\left(x^3+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*sin(x^3+1),x, algorithm="maxima")
```

```
[Out] -1/3*cos(x^3 + 1)
```

mupad [B] time = 0.05, size = 8, normalized size = 0.80

$$-\frac{\cos\left(x^3+1\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*sin(x^3 + 1),x)
[Out] -cos(x^3 + 1)/3
```

sympy [A] time = 0.26, size = 8, normalized size = 0.80

$$-\frac{\cos\left(x^3+1\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*sin(x**3+1),x)
[Out] -cos(x**3 + 1)/3
```

3.785 $\int 12x^2 \cos\left(x^3\right)\,dx$

Optimal. Leaf size=6

$$4\sin\left(x^3\right)$$

```
[Out] 4*sin(x^3)
```

Rubi [A] time = 0.01, antiderivative size = 6, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {12, 3380, 2637}

$$4\sin\left(x^3\right)$$

Antiderivative was successfully verified.

```
[In] Int[12*x^2*Cos[x^3],x]
```

```
[Out] 4*Sin[x^3]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3380

```
Int[((a_.) + Cos[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Cos[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rubi steps

$$\begin{aligned}\int 12x^2\cos\left(x^3\right)\,dx &= 12\int x^2\cos\left(x^3\right)\,dx\\ &= 4\,\text{Subst}\left(\int \cos(x)\,dx, x, x^3\right)\\ &= 4\sin\left(x^3\right)\end{aligned}$$

Mathematica [A] time = 0.00, size = 6, normalized size = 1.00

$$4\sin\left(x^3\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[12*x^2*Cos[x^3],x]
[Out] 4*Sin[x^3]
```

fricas [A] time = 0.66, size = 6, normalized size = 1.00

$$4 \sin\left(x^3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(12*x^2*cos(x^3),x, algorithm="fricas")
[Out] 4*sin(x^3)
```

giac [A] time = 0.14, size = 6, normalized size = 1.00

$$4 \sin\left(x^3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(12*x^2*cos(x^3),x, algorithm="giac")
[Out] 4*sin(x^3)
```

maple [A] time = 0.00, size = 7, normalized size = 1.17

$$4\sin\left(x^3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(12*x^2*cos(x^3),x)
[Out] 4*sin(x^3)
```

maxima [A] time = 0.32, size = 6, normalized size = 1.00

$$4 \sin\left(x^3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(12*x^2*cos(x^3),x, algorithm="maxima")
```

```
[Out] 4*sin(x^3)
```

mupad [B] time = 0.05, size = 6, normalized size = 1.00

$$4\ \sin\left(x^3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(12*x^2*cos(x^3),x)
[Out] 4*sin(x^3)
```

sympy [A] time = 0.27, size = 5, normalized size = 0.83

$$4\sin\left(x^3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(12*x**2*cos(x**3),x)
[Out] 4*sin(x**3)
```

3.786 $\int (1+x)\sin(1+x)\,dx$

Optimal. Leaf size=14

$$\sin(x+1)-(x+1)\cos(x+1)$$

```
[Out] -(1+x)*cos(1+x)+sin(1+x)
```

Rubi [A] time = 0.01, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {3296, 2637}

$$\sin(x+1)-(x+1)\cos(x+1)$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x)*Sin[1 + x],x]
```

```
[Out] -((1 + x)*Cos[1 + x]) + Sin[1 + x]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rubi steps

$$\begin{aligned}\int (1+x)\sin(1+x)\,dx &= -(1+x)\cos(1+x)+\int \cos(1+x)\,dx\\ &= -(1+x)\cos(1+x)+\sin(1+x)\end{aligned}$$

Mathematica [A] time = 0.03, size = 14, normalized size = 1.00

$$\sin(x+1)-(x+1)\cos(x+1)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x)*Sin[1 + x],x]
```

```
[Out] -((1 + x)*Cos[1 + x]) + Sin[1 + x]
```

fricas [A] time = 0.53, size = 14, normalized size = 1.00

$$-(x+1)\cos(x+1)+\sin(x+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)*sin(1+x),x, algorithm="fricas")
[Out] -(x + 1)*cos(x + 1) + sin(x + 1)
```

giac [A] time = 0.14, size = 14, normalized size = 1.00

$$-(x+1)\cos(x+1)+\sin(x+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)*sin(1+x),x, algorithm="giac")
[Out] -(x + 1)*cos(x + 1) + sin(x + 1)
```

maple [A] time = 0.03, size = 15, normalized size = 1.07

$$-(1+x)\cos(1+x)+\sin(1+x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+x)*sin(1+x),x)
[Out] -(1+x)*cos(1+x)+sin(1+x)
```

maxima [A] time = 0.32, size = 14, normalized size = 1.00

$$-(x+1)\cos(x+1)+\sin(x+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)*sin(1+x),x, algorithm="maxima")
[Out] -(x + 1)*cos(x + 1) + sin(x + 1)
```

mupad [B] time = 2.95, size = 14, normalized size = 1.00

$$\sin(x+1)-\cos(x+1)\,(x+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x + 1)*(x + 1),x)
```

```
[Out] sin(x + 1) - cos(x + 1)*(x + 1)
```

sympy [A] time = 0.15, size = 15, normalized size = 1.07

$$-x\cos(x+1)+\sin(x+1)-\cos(x+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)*sin(1+x),x)
```

```
[Out] -x*cos(x + 1) + sin(x + 1) - cos(x + 1)
```

3.787 $\int x^5 \cos(x^3)\, dx$

Optimal. Leaf size=20

$$\frac{1}{3}x^3\sin(x^3)+\frac{\cos(x^3)}{3}$$

```
[Out] 1/3*cos(x^3)+1/3*x^3*sin(x^3)
```

Rubi [A] time = 0.02, antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {3380, 3296, 2638}

$$\frac{1}{3}x^3\sin(x^3)+\frac{\cos(x^3)}{3}$$

Antiderivative was successfully verified.

```
[In] Int[x^5*Cos[x^3],x]
```

```
[Out] Cos[x^3]/3 + (x^3*Sin[x^3])/3
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3380

```
Int[((a_.) + Cos[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Cos[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rubi steps

$$\int x^5 \cos\left(x^3\right) dx = \frac{1}{3}\text{Subst}\left(\int x\cos(x)\,dx, x, x^3\right)$$
$$= \frac{1}{3}x^3 \sin\left(x^3\right) - \frac{1}{3}\text{Subst}\left(\int \sin(x)\,dx, x, x^3\right)$$
$$= \frac{\cos\left(x^3\right)}{3} + \frac{1}{3}x^3 \sin\left(x^3\right)$$

Mathematica [**A**] time = 0.01, size = 20, normalized size = 1.00

$$\frac{1}{3}x^3 \sin\left(x^3\right) + \frac{\cos\left(x^3\right)}{3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5*Cos[x^3],x]
[Out] Cos[x^3]/3 + (x^3*Sin[x^3])/3
```

fricas [**A**] time = 0.59, size = 16, normalized size = 0.80

$$\frac{1}{3}\,x^3 \sin\left(x^3\right) + \frac{1}{3}\,\cos\left(x^3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*cos(x^3),x, algorithm="fricas")
[Out] 1/3*x^3*sin(x^3) + 1/3*cos(x^3)
```

giac [**A**] time = 0.14, size = 16, normalized size = 0.80

$$\frac{1}{3}\,x^3 \sin\left(x^3\right) + \frac{1}{3}\,\cos\left(x^3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*cos(x^3),x, algorithm="giac")
[Out] 1/3*x^3*sin(x^3) + 1/3*cos(x^3)
```

maple [**A**] time = 0.05, size = 17, normalized size = 0.85

$$\frac{\cos\left(x^3\right)}{3} + \frac{x^3 \sin\left(x^3\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*cos(x^3),x)
```

```
[Out] 1/3*cos(x^3)+1/3*x^3*sin(x^3)
```

maxima [A] time = 0.33, size = 16, normalized size = 0.80

$$\frac{1}{3}x^3\sin\left(x^3\right)+\frac{1}{3}\cos\left(x^3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*cos(x^3),x, algorithm="maxima")
```

```
[Out] 1/3*x^3*sin(x^3) + 1/3*cos(x^3)
```

mupad [B] time = 2.96, size = 16, normalized size = 0.80

$$\frac{\cos\left(x^3\right)}{3}+\frac{x^3\sin\left(x^3\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*cos(x^3),x)
```

```
[Out] cos(x^3)/3 + (x^3*sin(x^3))/3
```

sympy [A] time = 1.51, size = 15, normalized size = 0.75

$$\frac{x^3\sin\left(x^3\right)}{3}+\frac{\cos\left(x^3\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*cos(x**3),x)
```

```
[Out] x**3*sin(x**3)/3 + cos(x**3)/3
```

3.788 $\int e^{-3x}\cos(x)\,dx$

Optimal. Leaf size=23

$$\frac{1}{10}e^{-3x}\sin(x)-\frac{3}{10}e^{-3x}\cos(x)$$

```
[Out] -3/10*cos(x)/exp(3*x)+1/10*sin(x)/exp(3*x)
```

Rubi [A] time = 0.01, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {4433}

$$\frac{1}{10}e^{-3x}\sin(x)-\frac{3}{10}e^{-3x}\cos(x)$$

Antiderivative was successfully verified.

```
[In] Int[Cos[x]/E^(3*x),x]
[Out] (-3*Cos[x])/(10*E^(3*x)) + Sin[x]/(10*E^(3*x))
```

Rule 4433

```
Int[Cos[(d_.) + (e_.)*(x_)]*(F_)^((c_.)*((a_.) + (b_.)*(x_))), x_Symbol] :>
 Simp[(b*c*Log[F]*F^(c*(a + b*x))*Cos[d + e*x])/(e^2 + b^2*c^2*Log[F]^2), x
] + Simp[(e*F^(c*(a + b*x))*Sin[d + e*x])/(e^2 + b^2*c^2*Log[F]^2), x] /; F
reeQ[{F, a, b, c, d, e}, x] && NeQ[e^2 + b^2*c^2*Log[F]^2, 0]
```

Rubi steps

$$\int e^{-3x}\cos(x)\,dx=-\frac{3}{10}e^{-3x}\cos(x)+\frac{1}{10}e^{-3x}\sin(x)$$

Mathematica [A] time = 0.01, size = 16, normalized size = 0.70

$$\frac{1}{10}e^{-3x}(\sin(x)-3\cos(x))$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[x]/E^(3*x),x]
[Out] (-3*Cos[x] + Sin[x])/(10*E^(3*x))
```

fricas [A] time = 0.69, size = 17, normalized size = 0.74

$$-\frac{3}{10}\cos(x)e^{(-3x)}+\frac{1}{10}e^{(-3x)}\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)/exp(3*x),x, algorithm="fricas")
[Out] -3/10*cos(x)*e^(-3*x) + 1/10*e^(-3*x)*sin(x)
```

giac [A] time = 0.14, size = 15, normalized size = 0.65

$$-\frac{1}{10}(3\cos(x)-\sin(x))e^{(-3x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)/exp(3*x),x, algorithm="giac")
[Out] -1/10*(3*cos(x) - sin(x))*e^(-3*x)
```

maple [A] time = 0.02, size = 18, normalized size = 0.78

$$-\frac{3\,e^{-3x}\cos(x)}{10}+\frac{e^{-3x}\sin(x)}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)/exp(3*x),x)
[Out] -3/10*exp(-3*x)*cos(x)+1/10*exp(-3*x)*sin(x)
```

maxima [A] time = 0.33, size = 15, normalized size = 0.65

$$-\frac{1}{10}(3\cos(x)-\sin(x))e^{(-3x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)/exp(3*x),x, algorithm="maxima")
[Out] -1/10*(3*cos(x) - sin(x))*e^(-3*x)
```

mupad [B] time = 0.02, size = 15, normalized size = 0.65

$$-\frac{e^{-3x}\,(3\cos(x)-\sin(x))}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(-3*x)*cos(x),x)
```

```
[Out] -(exp(-3*x)*(3*cos(x) - sin(x)))/10
```

sympy [A] time = 0.40, size = 20, normalized size = 0.87

$$\frac{e^{-3x}\sin{(x)}}{10} - \frac{3e^{-3x}\cos{(x)}}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)/exp(3*x),x)
```

```
[Out] exp(-3*x)*sin(x)/10 - 3*exp(-3*x)*cos(x)/10
```

3.789 $\int x^3 \sin\left(x^2\right)\,dx$

Optimal. Leaf size=20

$$\frac{\sin\left(x^2\right)}{2}-\frac{1}{2}x^2\cos\left(x^2\right)$$

```
[Out] -1/2*x^2*cos(x^2)+1/2*sin(x^2)
```

Rubi [A] time = 0.02, antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {3379, 3296, 2637}

$$\frac{\sin\left(x^2\right)}{2}-\frac{1}{2}x^2\cos\left(x^2\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^3*Sin[x^2],x]
```

```
[Out] -(x^2*Cos[x^2])/2 + Sin[x^2]/2
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3379

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rubi steps

$$\int x^3 \sin\left(x^2\right)\, dx = \frac{1}{2}\operatorname{Subst}\left(\int x\sin(x)\, dx, x, x^2\right)$$
$$= -\frac{1}{2}x^2\cos\left(x^2\right) + \frac{1}{2}\operatorname{Subst}\left(\int \cos(x)\, dx, x, x^2\right)$$
$$= -\frac{1}{2}x^2\cos\left(x^2\right) + \frac{\sin\left(x^2\right)}{2}$$

Mathematica [A] time = 0.00, size = 20, normalized size = 1.00

$$\frac{\sin\left(x^2\right)}{2} - \frac{1}{2}x^2\cos\left(x^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*Sin[x^2],x]
[Out] -1/2*(x^2*Cos[x^2]) + Sin[x^2]/2
```

fricas [A] time = 0.53, size = 16, normalized size = 0.80

$$-\frac{1}{2}\,x^2\cos\left(x^2\right) + \frac{1}{2}\,\sin\left(x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*sin(x^2),x, algorithm="fricas")
[Out] -1/2*x^2*cos(x^2) + 1/2*sin(x^2)
```

giac [A] time = 0.14, size = 16, normalized size = 0.80

$$-\frac{1}{2}\,x^2\cos\left(x^2\right) + \frac{1}{2}\,\sin\left(x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*sin(x^2),x, algorithm="giac")
[Out] -1/2*x^2*cos(x^2) + 1/2*sin(x^2)
```

maple [A] time = 0.00, size = 17, normalized size = 0.85

$$-\frac{x^2\cos\left(x^2\right)}{2} + \frac{\sin\left(x^2\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*sin(x^2),x)
```

```
[Out] -1/2*x^2*cos(x^2)+1/2*sin(x^2)
```

maxima [A] time = 0.33, size = 16, normalized size = 0.80

$$-\frac{1}{2}x^2\cos\left(x^2\right)+\frac{1}{2}\sin\left(x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*sin(x^2),x, algorithm="maxima")
```

```
[Out] -1/2*x^2*cos(x^2) + 1/2*sin(x^2)
```

mupad [B] time = 2.96, size = 16, normalized size = 0.80

$$\frac{\sin\left(x^2\right)}{2}-\frac{x^2\cos\left(x^2\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*sin(x^2),x)
```

```
[Out] sin(x^2)/2 - (x^2*cos(x^2))/2
```

sympy [A] time = 0.48, size = 15, normalized size = 0.75

$$-\frac{x^2\cos\left(x^2\right)}{2}+\frac{\sin\left(x^2\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*sin(x**2),x)
```

```
[Out] -x**2*cos(x**2)/2 + sin(x**2)/2
```

3.790 $\int x^3 \cos(x^2)\, dx$

Optimal. Leaf size=20

$$\frac{1}{2}x^2 \sin\left(x^2\right) + \frac{\cos\left(x^2\right)}{2}$$

```
[Out] 1/2*cos(x^2)+1/2*x^2*sin(x^2)
```

Rubi [A] time = 0.02, antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {3380, 3296, 2638}

$$\frac{1}{2}x^2 \sin\left(x^2\right) + \frac{\cos\left(x^2\right)}{2}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*Cos[x^2],x]
```

```
[Out] Cos[x^2]/2 + (x^2*Sin[x^2])/2
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3380

```
Int[((a_.) + Cos[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Cos[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rubi steps

$$\int x^3 \cos\left(x^2\right) dx = \frac{1}{2}\text{Subst}\left(\int x\cos(x)\,dx, x, x^2\right)$$
$$= \frac{1}{2}x^2\sin\left(x^2\right) - \frac{1}{2}\text{Subst}\left(\int \sin(x)\,dx, x, x^2\right)$$
$$= \frac{\cos\left(x^2\right)}{2} + \frac{1}{2}x^2\sin\left(x^2\right)$$

Mathematica [**A**] time = 0.01, size = 20, normalized size = 1.00

$$\frac{1}{2}x^2\sin\left(x^2\right) + \frac{\cos\left(x^2\right)}{2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*Cos[x^2],x]
[Out] Cos[x^2]/2 + (x^2*Sin[x^2])/2
```

fricas [**A**] time = 1.52, size = 16, normalized size = 0.80

$$\frac{1}{2}\,x^2\sin\left(x^2\right) + \frac{1}{2}\,\cos\left(x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*cos(x^2),x, algorithm="fricas")
[Out] 1/2*x^2*sin(x^2) + 1/2*cos(x^2)
```

giac [**A**] time = 0.14, size = 16, normalized size = 0.80

$$\frac{1}{2}\,x^2\sin\left(x^2\right) + \frac{1}{2}\,\cos\left(x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*cos(x^2),x, algorithm="giac")
[Out] 1/2*x^2*sin(x^2) + 1/2*cos(x^2)
```

maple [**A**] time = 0.04, size = 17, normalized size = 0.85

$$\frac{\cos\left(x^2\right)}{2} + \frac{x^2\sin\left(x^2\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*cos(x^2),x)
```

```
[Out] 1/2*cos(x^2)+1/2*x^2*sin(x^2)
```

maxima [A] time = 0.33, size = 16, normalized size = 0.80

$$\frac{1}{2}\,x^2\sin\left(x^2\right)+\frac{1}{2}\,\cos\left(x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*cos(x^2),x, algorithm="maxima")
```

```
[Out] 1/2*x^2*sin(x^2) + 1/2*cos(x^2)
```

mupad [B] time = 0.05, size = 16, normalized size = 0.80

$$\frac{\cos\left(x^2\right)}{2}+\frac{x^2\,\sin\left(x^2\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*cos(x^2),x)
```

```
[Out] cos(x^2)/2 + (x^2*sin(x^2))/2
```

sympy [A] time = 0.48, size = 15, normalized size = 0.75

$$\frac{x^2\sin\left(x^2\right)}{2}+\frac{\cos\left(x^2\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*cos(x**2),x)
```

```
[Out] x**2*sin(x**2)/2 + cos(x**2)/2
```

3.791 $\int \cos(x)\cos(2\sin(x))\,dx$

Optimal. Leaf size=9

$$\frac{1}{2}\sin(2\sin(x))$$

```
[Out] 1/2*sin(2*sin(x))
```

Rubi [A] time = 0.01, antiderivative size = 9, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {4334, 2637}

$$\frac{1}{2}\sin(2\sin(x))$$

Antiderivative was successfully verified.

```
[In] Int[Cos[x]*Cos[2*Sin[x]],x]
```

```
[Out] Sin[2*Sin[x]]/2
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 4334

```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{d = FreeFacto
rs[Sin[c*(a + b*x)], x]}, Dist[d/(b*c), Subst[Int[SubstFor[1, Sin[c*(a + b*
x)]/d, u, x], x], x, Sin[c*(a + b*x)]/d], x] /; FunctionOfQ[Sin[c*(a + b*x)
]/d, u, x, True]] /; FreeQ[{a, b, c}, x] && (EqQ[F, Cos] || EqQ[F, cos])
```

Rubi steps

$$\begin{aligned}\int \cos(x)\cos(2\sin(x))\,dx &= \text{Subst}\left(\int \cos(2x)\,dx, x, \sin(x)\right)\\ &= \frac{1}{2}\sin(2\sin(x))\end{aligned}$$

Mathematica [A] time = 1.38, size = 9, normalized size = 1.00

$$\frac{1}{2}\sin(2\sin(x))$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[x]*Cos[2*Sin[x]],x]
[Out] Sin[2*Sin[x]]/2
```

fricas [B] time = 0.79, size = 19, normalized size = 2.11

$$\frac{1}{2}\,\sin\left(\frac{4\,\tan\left(\frac{1}{2}\,x\right)}{\tan\left(\frac{1}{2}\,x\right)^{2}+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*cos(2*sin(x)),x, algorithm="fricas")
[Out] 1/2*sin(4*tan(1/2*x)/(tan(1/2*x)^2 + 1))
```

giac [A] time = 0.14, size = 7, normalized size = 0.78

$$\frac{1}{2}\,\sin\left(2\,\sin(x)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*cos(2*sin(x)),x, algorithm="giac")
[Out] 1/2*sin(2*sin(x))
```

maple [A] time = 0.03, size = 8, normalized size = 0.89

$$\frac{\sin\left(2\sin(x)\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)*cos(2*sin(x)),x)
[Out] 1/2*sin(2*sin(x))
```

maxima [A] time = 0.33, size = 7, normalized size = 0.78

$$\frac{1}{2}\,\sin\left(2\,\sin(x)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*cos(2*sin(x)),x, algorithm="maxima")
[Out] 1/2*sin(2*sin(x))
```

mupad [B] time = 0.07, size = 7, normalized size = 0.78

$$\frac{\sin\left(2\ \sin(x)\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(2*sin(x))*cos(x),x)
```

```
[Out] sin(2*sin(x))/2
```

sympy [A] time = 0.42, size = 7, normalized size = 0.78

$$\frac{\sin\left(2\sin\left(x\right)\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*cos(2*sin(x)),x)
```

```
[Out] sin(2*sin(x))/2
```

3.792 $\int \frac{\cos(x)\sin(x)}{1+\cos^2(x)}\,dx$

Optimal. Leaf size=11

$$-\frac{1}{2}\log\left(\cos^2(x)+1\right)$$

```
[Out] -1/2*ln(1+cos(x)^2)
```

Rubi [A] time = 0.03, antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {4335, 260}

$$-\frac{1}{2}\log\left(\cos^2(x)+1\right)$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[x]*Sin[x])/(1 + Cos[x]^2),x]
```

```
[Out] -Log[1 + Cos[x]^2]/2
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 4335

```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{d = FreeFacto
rs[Cos[c*(a + b*x)], x]}, -Dist[d/(b*c), Subst[Int[SubstFor[1, Cos[c*(a + b
*x)]/d, u, x], x], x, Cos[c*(a + b*x)]/d], x] /; FunctionOfQ[Cos[c*(a + b*x
)]/d, u, x, True]] /; FreeQ[{a, b, c}, x] && (EqQ[F, Sin] || EqQ[F, sin])
```

Rubi steps

$$\begin{aligned}\int \frac{\cos(x)\sin(x)}{1+\cos^2(x)}\,dx &= -\text{Subst}\left(\int \frac{x}{1+x^2}\,dx, x, \cos(x)\right)\\ &= -\frac{1}{2}\log\left(1+\cos^2(x)\right)\end{aligned}$$

Mathematica [A] time = 0.03, size = 11, normalized size = 1.00

$$-\frac{1}{2}\log(\cos(2x)+3)$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[x]*Sin[x])/(1 + Cos[x]^2),x]
```

```
[Out] -1/2*Log[3 + Cos[2*x]]
```

fricas [A] time = 0.69, size = 11, normalized size = 1.00

$$-\frac{1}{2}\log\left(\frac{1}{2}\cos(x)^2+\frac{1}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*sin(x)/(1+cos(x)^2),x, algorithm="fricas")
```

```
[Out] -1/2*log(1/2*cos(x)^2 + 1/2)
```

giac [A] time = 0.12, size = 9, normalized size = 0.82

$$-\frac{1}{2}\log\left(\cos(x)^2+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*sin(x)/(1+cos(x)^2),x, algorithm="giac")
```

```
[Out] -1/2*log(cos(x)^2 + 1)
```

maple [A] time = 0.03, size = 10, normalized size = 0.91

$$-\frac{\ln\left(1+\cos^2(x)\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)*sin(x)/(1+cos(x)^2),x)
```

```
[Out] -1/2*ln(1+cos(x)^2)
```

maxima [A] time = 0.33, size = 9, normalized size = 0.82

$$-\frac{1}{2}\log\left(\cos(x)^2+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*sin(x)/(1+cos(x)^2),x, algorithm="maxima")
```

```
[Out] -1/2*log(cos(x)^2 + 1)
```

mupad [B] time = 3.01, size = 17, normalized size = 1.55

$$-\operatorname{atanh}\left(\frac{16}{3\left(12\tan(x)^2+16\right)}-\frac{1}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(x)*sin(x))/(cos(x)^2 + 1),x)
```

```
[Out] -atanh(16/(3*(12*tan(x)^2 + 16)) - 1/3)
```

sympy [A] time = 0.17, size = 10, normalized size = 0.91

$$-\frac{\log\left(\cos^2(x)+1\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*sin(x)/(1+cos(x)**2),x)
```

```
[Out] -log(cos(x)**2 + 1)/2
```

3.793 $\int (1+\cos(x))(x+\sin(x))^3\,dx$

Optimal. Leaf size=10

$$\frac{1}{4}(x+\sin(x))^4$$

```
[Out] 1/4*(x+sin(x))^4
```

Rubi [A] time = 0.04, antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {6686}

$$\frac{1}{4}(x+\sin(x))^4$$

Antiderivative was successfully verified.

```
[In] Int[(1 + Cos[x])*(x + Sin[x])^3,x]
```

```
[Out] (x + Sin[x])^4/4
```

Rule 6686

```
Int[(u_)*(y_)^(m_.), x_Symbol] :> With[{q = DerivativeDivides[y, u, x]}, Si
mp[(q*y^(m + 1))/(m + 1), x] /;  !FalseQ[q]] /; FreeQ[m, x] && NeQ[m, -1]
```

Rubi steps

$$\int (1+\cos(x))(x+\sin(x))^3\,dx=\frac{1}{4}(x+\sin(x))^4$$

Mathematica [A] time = 0.02, size = 10, normalized size = 1.00

$$\frac{1}{4}(x+\sin(x))^4$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + Cos[x])*(x + Sin[x])^3,x]
```

```
[Out] (x + Sin[x])^4/4
```

fricas [B] time = 0.62, size = 45, normalized size = 4.50

$$\frac{1}{4}\,x^4+\frac{1}{4}\,\cos(x)^4-\frac{1}{2}\left(3\,x^2+1\right)\cos(x)^2+\frac{3}{2}\,x^2+\left(x^3-x\cos(x)^2+x\right)\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+cos(x))*(x+sin(x))^3,x, algorithm="fricas")
```

```
[Out] 1/4*x^4 + 1/4*cos(x)^4 - 1/2*(3*x^2 + 1)*cos(x)^2 + 3/2*x^2 + (x^3 - x*cos(
x)^2 + x)*sin(x)
```

giac [B] time = 0.15, size = 61, normalized size = 6.10

$$\frac{1}{4}x^4+\frac{3}{4}x^2-\frac{1}{4}\left(3x^2-1\right)\cos\left(2x\right)-\frac{1}{4}x\sin\left(3x\right)+\frac{1}{4}\left(4x^3-21x\right)\sin(x)+6x\sin(x)+\frac{1}{32}\cos\left(4x\right)-\frac{3}{8}\cos\left(2x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+cos(x))*(x+sin(x))^3,x, algorithm="giac")
```

```
[Out] 1/4*x^4 + 3/4*x^2 - 1/4*(3*x^2 - 1)*cos(2*x) - 1/4*x*sin(3*x) + 1/4*(4*x^3
- 21*x)*sin(x) + 6*x*sin(x) + 1/32*cos(4*x) - 3/8*cos(2*x)
```

maple [B] time = 0.09, size = 65, normalized size = 6.50

$$x^3\sin(x)-\frac{3\left(\cos^2(x)\right)x^2}{2}+3x\left(\frac{\cos(x)\sin(x)}{2}+\frac{x}{2}\right)-\frac{3x^2}{2}+x\left(\sin^3(x)\right)+\frac{\left(\sin^4(x)\right)}{4}+\frac{x^4}{4}+3x\left(-\frac{\cos(x)\sin(x)}{2}+\frac{x}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+cos(x))*(x+sin(x))^3,x)
```

```
[Out] x^3*sin(x)-3/2*cos(x)^2*x^2+3*x*(1/2*cos(x)*sin(x)+1/2*x)-3/2*x^2+x*sin(x)^
3+1/4*sin(x)^4+1/4*x^4+3*x*(-1/2*cos(x)*sin(x)+1/2*x)
```

maxima [A] time = 0.33, size = 8, normalized size = 0.80

$$\frac{1}{4}\left(x+\sin(x)\right)^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+cos(x))*(x+sin(x))^3,x, algorithm="maxima")
```

```
[Out] 1/4*(x + sin(x))^4
```

mupad [B] time = 3.15, size = 8, normalized size = 0.80

$$\frac{\left(x+\sin(x)\right)^4}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(x) + 1)*(x + sin(x))^3,x)
```

```
[Out] (x + sin(x))^4/4
```

sympy [B] time = 0.52, size = 36, normalized size = 3.60

$$\frac{x^4}{4} + x^3 \sin(x) + \frac{3x^2 \sin^2(x)}{2} + x \sin^3(x) + \frac{\sin^4(x)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+cos(x))*(x+sin(x))**3,x)
```

```
[Out] x**4/4 + x**3*sin(x) + 3*x**2*sin(x)**2/2 + x*sin(x)**3 + sin(x)**4/4
```

3.794 $\int (1+\cos(x))\csc^2(x)\,dx$

Optimal. Leaf size=9

$$-\cot(x)-\csc(x)$$

[Out] -cot(x)-csc(x)

Rubi [A] time = 0.03, antiderivative size = 9, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2669, 3767, 8}

$$-\cot(x)-\csc(x)$$

Antiderivative was successfully verified.

[In] Int[(1 + Cos[x])*Csc[x]^2,x]

[Out] -Cot[x] - Csc[x]

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2669

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1))/(f*g*(p + 1)), x] + D
ist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] && (I
ntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\begin{aligned}\int (1+\cos(x))\csc^2(x)\,dx &= -\csc(x)+\int \csc^2(x)\,dx\\ &= -\csc(x)-\text{Subst}\left(\int 1\,dx,x,\cot(x)\right)\\ &= -\cot(x)-\csc(x)\end{aligned}$$

Mathematica [A] time = 0.00, size = 9, normalized size = 1.00

$$-\cot(x) - \csc(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + Cos[x])*Csc[x]^2,x]
[Out] -Cot[x] - Csc[x]
```

fricas [A] time = 0.63, size = 10, normalized size = 1.11

$$-\frac{\cos(x) + 1}{\sin(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+cos(x))*csc(x)^2,x, algorithm="fricas")
[Out] -(cos(x) + 1)/sin(x)
```

giac [A] time = 0.14, size = 8, normalized size = 0.89

$$-\frac{1}{\tan\left(\frac{1}{2}\,x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+cos(x))*csc(x)^2,x, algorithm="giac")
[Out] -1/tan(1/2*x)
```

maple [A] time = 0.06, size = 12, normalized size = 1.33

$$-\frac{1}{\sin(x)} - \cot(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+cos(x))*csc(x)^2,x)
[Out] -1/sin(x)-cot(x)
```

maxima [A] time = 0.33, size = 13, normalized size = 1.44

$$-\frac{1}{\sin(x)} - \frac{1}{\tan(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+cos(x))*csc(x)^2,x, algorithm="maxima")
```

```
[Out] -1/sin(x) - 1/tan(x)
```

mupad [B] time = 2.93, size = 6, normalized size = 0.67

$$-\cot\left(\frac{x}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(x) + 1)/sin(x)^2,x)
```

```
[Out] -cot(x/2)
```

sympy [A] time = 1.80, size = 8, normalized size = 0.89

$$-\cot(x) - \frac{1}{\sin(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+cos(x))*csc(x)**2,x)
```

```
[Out] -cot(x) - 1/sin(x)
```

3.795 $\int \sin(x)\tan^2(x)\,dx$

Optimal. Leaf size=5

$$\cos(x)+\sec(x)$$

```
[Out] cos(x)+sec(x)
```

Rubi [A] time = 0.02, antiderivative size = 5, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {2590, 14}

$$\cos(x)+\sec(x)$$

Antiderivative was successfully verified.

```
[In] Int[Sin[x]*Tan[x]^2,x]
```

```
[Out] Cos[x] + Sec[x]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2590

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> -Dist[f^(-1), Subst[Int[(1 - x^2)^((m + n - 1)/2)/x^n, x], x, Cos[e + f*
x]], x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n - 1)/2]
```

Rubi steps

$$\begin{aligned}\int \sin(x)\tan^2(x)\,dx &= -\operatorname{Subst}\left(\int \frac{1-x^2}{x^2}\,dx,x,\cos(x)\right)\\ &= -\operatorname{Subst}\left(\int \left(-1+\frac{1}{x^2}\right)dx,x,\cos(x)\right)\\ &= \cos(x)+\sec(x)\end{aligned}$$

Mathematica [A] time = 0.01, size = 5, normalized size = 1.00

$$\cos(x)+\sec(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[x]*Tan[x]^2,x]
[Out] Cos[x] + Sec[x]
```

fricas [B] time = 0.70, size = 11, normalized size = 2.20

$$\frac{\cos(x)^2 + 1}{\cos(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)*tan(x)^2,x, algorithm="fricas")
[Out] (cos(x)^2 + 1)/cos(x)
```

giac [A] time = 0.14, size = 7, normalized size = 1.40

$$\frac{1}{\cos(x)} + \cos(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)*tan(x)^2,x, algorithm="giac")
[Out] 1/cos(x) + cos(x)
```

maple [B] time = 0.04, size = 20, normalized size = 4.00

$$\frac{\sin^4(x)}{\cos(x)} + \left(2 + \sin^2(x)\right)\cos(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)*tan(x)^2,x)
[Out] sin(x)^4/cos(x)+(2+sin(x)^2)*cos(x)
```

maxima [A] time = 0.31, size = 7, normalized size = 1.40

$$\frac{1}{\cos(x)} + \cos(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)*tan(x)^2,x, algorithm="maxima")
[Out] 1/cos(x) + cos(x)
```

mupad [**B**] time = 3.00, size = 7, normalized size = 1.40

$$\cos(x) + \frac{1}{\cos(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)*tan(x)^2,x)
[Out] cos(x) + 1/cos(x)
```

sympy [**A**] time = 0.07, size = 7, normalized size = 1.40

$$\cos(x) + \frac{1}{\cos(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)*tan(x)**2,x)
[Out] cos(x) + 1/cos(x)
```

3.796 $\int e^{\sin(x)} \sec^2(x) \left(x \cos^3(x) - \sin(x)\right) dx$

Optimal. Leaf size=13

$$e^{\sin(x)}(x\cos(x) - 1)\sec(x)$$

```
[Out] exp(sin(x))*(-1+x*cos(x))*sec(x)
```

Rubi [F] time = 0.64, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int e^{\sin(x)} \sec^2(x) \left(x \cos^3(x) - \sin(x)\right) dx$$

Verification is Not applicable to the result.

```
[In] Int[E^Sin[x]*Sec[x]^2*(x*Cos[x]^3 - Sin[x]),x]
[Out] Defer[Int][E^Sin[x]*x*Cos[x], x] - Defer[Int][E^Sin[x]*Sec[x]*Tan[x], x]
```

Rubi steps

$$\begin{aligned}\int e^{\sin(x)} \sec^2(x) \left(x \cos^3(x) - \sin(x)\right) dx &= \int \left(e^{\sin(x)} x \cos(x) - e^{\sin(x)} \sec(x) \tan(x)\right) dx\\ &= \int e^{\sin(x)} x \cos(x)\, dx - \int e^{\sin(x)} \sec(x) \tan(x)\, dx\end{aligned}$$

Mathematica [A] time = 0.29, size = 13, normalized size = 1.00

$$e^{\sin(x)}(x\cos(x) - 1)\sec(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[E^Sin[x]*Sec[x]^2*(x*Cos[x]^3 - Sin[x]),x]
[Out] E^Sin[x]*(-1 + x*Cos[x])*Sec[x]
```

fricas [A] time = 0.68, size = 14, normalized size = 1.08

$$\frac{(x\cos(x) - 1)e^{\sin(x)}}{\cos(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(sin(x))*sec(x)^2*(x*cos(x)^3-sin(x)),x, algorithm="fricas")
```

```
[Out] (x*cos(x) - 1)*e^sin(x)/cos(x)
```

giac [B] time = 0.20, size = 794, normalized size = 61.08

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(sin(x))*sec(x)^2*(x*cos(x)^3-sin(x)),x, algorithm="giac")
```

```
[Out] (x*e^(2*tan(1/2*x)/(tan(1/2*x)^2 + 1))*tan(3/2*x)^2*tan(1/2*x)^8 + e^(2*tan
(1/2*x)/(tan(1/2*x)^2 + 1))*tan(3/2*x)^2*tan(1/2*x)^8 - 16*x*e^(2*tan(1/2*x
)/(tan(1/2*x)^2 + 1))*tan(3/2*x)^2*tan(1/2*x)^6 + 12*x*e^(2*tan(1/2*x)/(tan
(1/2*x)^2 + 1))*tan(3/2*x)*tan(1/2*x)^7 - x*e^(2*tan(1/2*x)/(tan(1/2*x)^2 +
 1))*tan(1/2*x)^8 - 14*e^(2*tan(1/2*x)/(tan(1/2*x)^2 + 1))*tan(3/2*x)^2*tan
(1/2*x)^6 + 12*e^(2*tan(1/2*x)/(tan(1/2*x)^2 + 1))*tan(3/2*x)*tan(1/2*x)^7
- e^(2*tan(1/2*x)/(tan(1/2*x)^2 + 1))*tan(1/2*x)^8 + 30*x*e^(2*tan(1/2*x)/(
tan(1/2*x)^2 + 1))*tan(3/2*x)^2*tan(1/2*x)^4 - 52*x*e^(2*tan(1/2*x)/(tan(1/
2*x)^2 + 1))*tan(3/2*x)*tan(1/2*x)^5 + 16*x*e^(2*tan(1/2*x)/(tan(1/2*x)^2 +
 1))*tan(1/2*x)^6 - 28*e^(2*tan(1/2*x)/(tan(1/2*x)^2 + 1))*tan(3/2*x)*tan(1
/2*x)^5 + 14*e^(2*tan(1/2*x)/(tan(1/2*x)^2 + 1))*tan(1/2*x)^6 - 16*x*e^(2*t
an(1/2*x)/(tan(1/2*x)^2 + 1))*tan(3/2*x)^2*tan(1/2*x)^2 + 52*x*e^(2*tan(1/2
*x)/(tan(1/2*x)^2 + 1))*tan(3/2*x)*tan(1/2*x)^3 - 30*x*e^(2*tan(1/2*x)/(tan
(1/2*x)^2 + 1))*tan(1/2*x)^4 + 14*e^(2*tan(1/2*x)/(tan(1/2*x)^2 + 1))*tan(3
/2*x)^2*tan(1/2*x)^2 - 28*e^(2*tan(1/2*x)/(tan(1/2*x)^2 + 1))*tan(3/2*x)*ta
n(1/2*x)^3 + x*e^(2*tan(1/2*x)/(tan(1/2*x)^2 + 1))*tan(3/2*x)^2 - 12*x*e^(2
*tan(1/2*x)/(tan(1/2*x)^2 + 1))*tan(3/2*x)*tan(1/2*x) + 16*x*e^(2*tan(1/2*x
)/(tan(1/2*x)^2 + 1))*tan(1/2*x)^2 - e^(2*tan(1/2*x)/(tan(1/2*x)^2 + 1))*ta
n(3/2*x)^2 + 12*e^(2*tan(1/2*x)/(tan(1/2*x)^2 + 1))*tan(3/2*x)*tan(1/2*x) -
 14*e^(2*tan(1/2*x)/(tan(1/2*x)^2 + 1))*tan(1/2*x)^2 - x*e^(2*tan(1/2*x)/(t
an(1/2*x)^2 + 1)) + e^(2*tan(1/2*x)/(tan(1/2*x)^2 + 1)))/(tan(3/2*x)^2*tan(
1/2*x)^8 + 2*tan(3/2*x)^2*tan(1/2*x)^6 + tan(1/2*x)^8 + 2*tan(1/2*x)^6 - 2*
tan(3/2*x)^2*tan(1/2*x)^2 - tan(3/2*x)^2 - 2*tan(1/2*x)^2 - 1)
```

maple [C] time = 0.39, size = 30, normalized size = 2.31

$$\frac{\left(x\,{\mathrm e}^{2 i x}+x-2\,{\mathrm e}^{i x}\right) {\mathrm e}^{\sin \left(x \right)}}{{\mathrm e}^{2 i x}+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(sin(x))*sec(x)^2*(x*cos(x)^3-sin(x)),x)
```

```
[Out] (x*exp(2*I*x)+x-2*exp(I*x))/(exp(2*I*x)+1)*exp(sin(x))
```

maxima [B] time = 0.77, size = 88, normalized size = 6.77

$$\frac{x\cos(2x)^2 e^{\sin(x)} + xe^{\sin(x)}\sin(2x)^2 - 2e^{\sin(x)}\sin(2x)\sin(x) + 2\left(xe^{\sin(x)} - \cos(x)e^{\sin(x)}\right)\cos(2x) + xe^{\sin(x)} - 2}{\cos(2x)^2 + \sin(2x)^2 + 2\cos(2x) + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(sin(x))*sec(x)^2*(x*cos(x)^3-sin(x)),x, algorithm="maxima")
```

```
[Out] (x*cos(2*x)^2*e^sin(x) + x*e^sin(x)*sin(2*x)^2 - 2*e^sin(x)*sin(2*x)*sin(x)
 + 2*(x*e^sin(x) - cos(x)*e^sin(x))*cos(2*x) + x*e^sin(x) - 2*cos(x)*e^sin(
x))/(cos(2*x)^2 + sin(2*x)^2 + 2*cos(2*x) + 1)
```

mupad [B] time = 3.12, size = 14, normalized size = 1.08

$$\frac{e^{\sin(x)}\,(x\,\cos(x) - 1)}{\cos(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(exp(sin(x))*(sin(x) - x*cos(x)^3))/cos(x)^2,x)
```

```
[Out] (exp(sin(x))*(x*cos(x) - 1))/cos(x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(sin(x))*sec(x)**2*(x*cos(x)**3-sin(x)),x)
```

```
[Out] Timed out
```

3.797 $\int x \csc^2(x)\,dx$

Optimal. Leaf size=9

$$\log(\sin(x)) - x\cot(x)$$

```
[Out] -x*cot(x)+ln(sin(x))
```

Rubi [A] time = 0.02, antiderivative size = 9, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {4184, 3475}

$$\log(\sin(x)) - x\cot(x)$$

Antiderivative was successfully verified.

```
[In] Int[x*Csc[x]^2,x]
```

```
[Out] -(x*Cot[x]) + Log[Sin[x]]
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 4184

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> -Sim
p[((c + d*x)^m*Cot[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Co
t[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rubi steps

$$\begin{aligned}\int x\csc^2(x)\,dx &= -x\cot(x) + \int \cot(x)\,dx\\ &= -x\cot(x) + \log(\sin(x))\end{aligned}$$

Mathematica [A] time = 0.02, size = 9, normalized size = 1.00

$$\log(\sin(x)) - x\cot(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Csc[x]^2,x]
```

[Out] -(x*Cot[x]) + Log[Sin[x]]

fricas [B] time = 0.62, size = 20, normalized size = 2.22

$$-\frac{x\cos(x)-\log\left(\frac{1}{2}\sin(x)\right)\sin(x)}{\sin(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*csc(x)^2,x, algorithm="fricas")

[Out] -(x*cos(x) - log(1/2*sin(x))*sin(x))/sin(x)

giac [B] time = 0.19, size = 52, normalized size = 5.78

$$\frac{x\tan\left(\frac{1}{2}x\right)^2+\log\left(\frac{16\tan\left(\frac{1}{2}x\right)^2}{\tan\left(\frac{1}{2}x\right)^4+2\tan\left(\frac{1}{2}x\right)^2+1}\right)\tan\left(\frac{1}{2}x\right)-x}{2\tan\left(\frac{1}{2}x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*csc(x)^2,x, algorithm="giac")

[Out] 1/2*(x*tan(1/2*x)^2 + log(16*tan(1/2*x)^2/(tan(1/2*x)^4 + 2*tan(1/2*x)^2 + 1))*tan(1/2*x) - x)/tan(1/2*x)

maple [A] time = 0.03, size = 10, normalized size = 1.11

$$-x\cot(x)+\ln(\sin(x))$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x*csc(x)^2,x)

[Out] -x*cot(x)+ln(sin(x))

maxima [B] time = 0.32, size = 104, normalized size = 11.56

$$\frac{\left(\cos(2x)^2+\sin(2x)^2-2\cos(2x)+1\right)\log\left(\cos(x)^2+\sin(x)^2+2\cos(x)+1\right)+\left(\cos(2x)^2+\sin(2x)^2-2\cos\right.}{2\left(\cos(2x)^2+\sin(2x)^2-2\cos(2x)+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*csc(x)^2,x, algorithm="maxima")

```
[Out] 1/2*((cos(2*x)^2 + sin(2*x)^2 - 2*cos(2*x) + 1)*log(cos(x)^2 + sin(x)^2 + 2
*cos(x) + 1) + (cos(2*x)^2 + sin(2*x)^2 - 2*cos(2*x) + 1)*log(cos(x)^2 + si
n(x)^2 - 2*cos(x) + 1) - 4*x*sin(2*x))/(cos(2*x)^2 + sin(2*x)^2 - 2*cos(2*x
) + 1)
```

mupad [B] time = 0.03, size = 9, normalized size = 1.00

$$\ln(\sin(x)) - x\cot(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/sin(x)^2,x)
```

```
[Out] log(sin(x)) - x*cot(x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x\csc^2(x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*csc(x)**2,x)
```

```
[Out] Integral(x*csc(x)**2, x)
```

3.798 $\int \cos(x)\sin\left(\frac{\pi}{6}+x\right)\,dx$

Optimal. Leaf size=20

$$\frac{x}{4}-\frac{1}{4}\cos\left(2x+\frac{\pi}{6}\right)$$

```
[Out] 1/4*x-1/4*cos(1/6*Pi+2*x)
```

Rubi [A] time = 0.02, antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {4574, 2638}

$$\frac{x}{4}-\frac{1}{4}\cos\left(2x+\frac{\pi}{6}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Cos[x]*Sin[Pi/6 + x],x]
```

```
[Out] x/4 - Cos[Pi/6 + 2*x]/4
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 4574

```
Int[Cos[w_]^(q_.)*Sin[v_]^(p_.), x_Symbol] :> Int[ExpandTrigReduce[Sin[v]^p
*Cos[w]^q, x], x] /; IGtQ[p, 0] && IGtQ[q, 0] && ((PolynomialQ[v, x] && Pol
ynomialQ[w, x]) || (BinomialQ[{v, w}, x] && IndependentQ[Cancel[v/w], x]))
```

Rubi steps

$$\begin{aligned}\int \cos(x)\sin\left(\frac{\pi}{6}+x\right)\,dx &= \int \left(\frac{1}{4}+\frac{1}{2}\sin\left(\frac{\pi}{6}+2x\right)\right)\,dx\\ &= \frac{x}{4}+\frac{1}{2}\int \sin\left(\frac{\pi}{6}+2x\right)\,dx\\ &= \frac{x}{4}-\frac{1}{4}\cos\left(\frac{\pi}{6}+2x\right)\end{aligned}$$

Mathematica [A] time = 0.01, size = 20, normalized size = 1.00

$$\frac{x}{4}-\frac{1}{4}\cos\left(2x+\frac{\pi}{6}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[x]*Sin[Pi/6 + x],x]
```

```
[Out] x/4 - Cos[Pi/6 + 2*x]/4
```

fricas [B] time = 1.74, size = 31, normalized size = 1.55

$$-\frac{1}{4}\sqrt{3}\cos\left(\frac{1}{6}\pi + x\right)^2 - \frac{1}{4}\cos\left(\frac{1}{6}\pi + x\right)\sin\left(\frac{1}{6}\pi + x\right) + \frac{1}{4}x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*sin(1/6*pi+x),x, algorithm="fricas")
```

```
[Out] -1/4*sqrt(3)*cos(1/6*pi + x)^2 - 1/4*cos(1/6*pi + x)*sin(1/6*pi + x) + 1/4*
x
```

giac [A] time = 0.16, size = 14, normalized size = 0.70

$$\frac{1}{4}x - \frac{1}{4}\cos\left(\frac{1}{6}\pi + 2x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*sin(1/6*pi+x),x, algorithm="giac")
```

```
[Out] 1/4*x - 1/4*cos(1/6*pi + 2*x)
```

maple [A] time = 0.15, size = 15, normalized size = 0.75

$$\frac{x}{4} - \frac{\cos\left(\frac{\pi}{6} + 2x\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)*sin(1/6*Pi+x),x)
```

```
[Out] 1/4*x-1/4*cos(1/6*Pi+2*x)
```

maxima [A] time = 0.31, size = 14, normalized size = 0.70

$$\frac{1}{4}x - \frac{1}{4}\cos\left(\frac{1}{6}\pi + 2x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*sin(1/6*pi+x),x, algorithm="maxima")
```

```
[Out] 1/4*x - 1/4*cos(1/6*pi + 2*x)
```

mupad [B] time = 0.03, size = 18, normalized size = 0.90

$$\frac{x\,\sin\left(\frac{\Pi}{6}\right)}{2}-\frac{\cos\left(\frac{\Pi}{6}+2\,x\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)*sin(Pi/6 + x),x)
```

```
[Out] (x*sin(Pi/6))/2 - cos(Pi/6 + 2*x)/4
```

sympy [B] time = 0.46, size = 37, normalized size = 1.85

$$-\frac{x\sin\left(x\right)\cos\left(x+\frac{\pi}{6}\right)}{2}+\frac{x\sin\left(x+\frac{\pi}{6}\right)\cos\left(x\right)}{2}+\frac{\sin\left(x\right)\sin\left(x+\frac{\pi}{6}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*sin(1/6*pi+x),x)
```

```
[Out] -x*sin(x)*cos(x + pi/6)/2 + x*sin(x + pi/6)*cos(x)/2 + sin(x)*sin(x + pi/6)
/2
```

3.799 $\int x \sin^3\left(x^2\right)\,dx$

Optimal. Leaf size=19

$$\frac{1}{6}\cos^3\left(x^2\right)-\frac{\cos\left(x^2\right)}{2}$$

```
[Out] -1/2*cos(x^2)+1/6*cos(x^2)^3
```

Rubi [A] time = 0.01, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {3379, 2633}

$$\frac{1}{6}\cos^3\left(x^2\right)-\frac{\cos\left(x^2\right)}{2}$$

Antiderivative was successfully verified.

```
[In] Int[x*Sin[x^2]^3,x]
```

```
[Out] -Cos[x^2]/2 + Cos[x^2]^3/6
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 3379

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rubi steps

$$\begin{aligned}
\int x\sin^3\left(x^2\right)\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \sin^3(x)\,dx,x,x^2\right)\\
&= -\left(\frac{1}{2}\operatorname{Subst}\left(\int \left(1-x^2\right)\,dx,x,\cos\left(x^2\right)\right)\right)\\
&= -\frac{1}{2}\cos\left(x^2\right)+\frac{1}{6}\cos^3\left(x^2\right)
\end{aligned}$$

Mathematica [A] time = 0.01, size = 19, normalized size = 1.00

$$\frac{1}{24}\cos\left(3x^2\right)-\frac{3\cos\left(x^2\right)}{8}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Sin[x^2]^3,x]
```

```
[Out] (-3*Cos[x^2])/8 + Cos[3*x^2]/24
```

fricas [A] time = 0.68, size = 15, normalized size = 0.79

$$\frac{1}{6}\cos\left(x^2\right)^3-\frac{1}{2}\cos\left(x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(x^2)^3,x, algorithm="fricas")
```

```
[Out] 1/6*cos(x^2)^3 - 1/2*cos(x^2)
```

giac [A] time = 0.14, size = 15, normalized size = 0.79

$$\frac{1}{6}\cos\left(x^2\right)^3-\frac{1}{2}\cos\left(x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(x^2)^3,x, algorithm="giac")
```

```
[Out] 1/6*cos(x^2)^3 - 1/2*cos(x^2)
```

maple [A] time = 0.02, size = 15, normalized size = 0.79

$$-\frac{\left(2+\sin^2\left(x^2\right)\right)\cos\left(x^2\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*sin(x^2)^3,x)
```

```
[Out] -1/6*(2+sin(x^2)^2)*cos(x^2)
```

maxima [A] time = 0.32, size = 15, normalized size = 0.79

$$\frac{1}{24}\cos\left(3\,x^2\right)-\frac{3}{8}\cos\left(x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(x^2)^3,x, algorithm="maxima")
```

```
[Out] 1/24*cos(3*x^2) - 3/8*cos(x^2)
```

mupad [**B**] time = 2.95, size = 14, normalized size = 0.74

$$\frac{\cos\left(x^2\right)\left(\cos\left(x^2\right)^2-3\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*sin(x^2)^3,x)
```

```
[Out] (cos(x^2)*(cos(x^2)^2 - 3))/6
```

sympy [**A**] time = 0.48, size = 22, normalized size = 1.16

$$-\frac{\sin^2\left(x^2\right)\cos\left(x^2\right)}{2}-\frac{\cos^3\left(x^2\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(x**2)**3,x)
```

```
[Out] -sin(x**2)**2*cos(x**2)/2 - cos(x**2)**3/3
```

3.800 $\int \sin^2(x)\tan(x)\,dx$

Optimal. Leaf size=14

$$\frac{\cos^2(x)}{2}-\log(\cos(x))$$

```
[Out] 1/2*cos(x)^2-ln(cos(x))
```

Rubi [A] time = 0.01, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {2590, 14}

$$\frac{\cos^2(x)}{2}-\log(\cos(x))$$

Antiderivative was successfully verified.

```
[In] Int[Sin[x]^2*Tan[x],x]
```

```
[Out] Cos[x]^2/2 - Log[Cos[x]]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2590

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> -Dist[f^(-1), Subst[Int[(1 - x^2)^((m + n - 1)/2)/x^n, x], x, Cos[e + f*
x]], x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n - 1)/2]
```

Rubi steps

$$\begin{aligned}\int \sin^2(x)\tan(x)\,dx &= -\text{Subst}\left(\int \frac{1-x^2}{x}\,dx,x,\cos(x)\right)\\ &= -\text{Subst}\left(\int \left(\frac{1}{x}-x\right)\,dx,x,\cos(x)\right)\\ &= \frac{\cos^2(x)}{2}-\log(\cos(x))\end{aligned}$$

Mathematica [A] time = 0.00, size = 14, normalized size = 1.00

$$\frac{\cos^2(x)}{2} - \log(\cos(x))$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[x]^2*Tan[x],x]
[Out] Cos[x]^2/2 - Log[Cos[x]]
```

fricas [A] time = 0.80, size = 14, normalized size = 1.00

$$\frac{1}{2}\cos(x)^2 - \log(-\cos(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^2*tan(x),x, algorithm="fricas")
[Out] 1/2*cos(x)^2 - log(-cos(x))
```

giac [A] time = 0.13, size = 18, normalized size = 1.29

$$-\frac{1}{2}\sin(x)^2 - \frac{1}{2}\log\left(-\sin(x)^2 + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^2*tan(x),x, algorithm="giac")
[Out] -1/2*sin(x)^2 - 1/2*log(-sin(x)^2 + 1)
```

maple [A] time = 0.04, size = 13, normalized size = 0.93

$$-\frac{\left(\sin^2(x)\right)}{2} - \ln(\cos(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)^2*tan(x),x)
[Out] -1/2*sin(x)^2-ln(cos(x))
```

maxima [A] time = 0.30, size = 16, normalized size = 1.14

$$-\frac{1}{2}\sin(x)^2 - \frac{1}{2}\log\left(\sin(x)^2 - 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^2*tan(x),x, algorithm="maxima")
[Out] -1/2*sin(x)^2 - 1/2*log(sin(x)^2 - 1)
```

mupad [B] time = 2.95, size = 16, normalized size = 1.14

$$\frac{\cos(x)^2}{2} + \frac{\ln\left(\tan(x)^2 + 1\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)^2*tan(x),x)
[Out] log(tan(x)^2 + 1)/2 + cos(x)^2/2
```

sympy [A] time = 0.07, size = 10, normalized size = 0.71

$$-\log\left(\cos\left(x\right)\right) + \frac{\cos^2\left(x\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)**2*tan(x),x)
[Out] -log(cos(x)) + cos(x)**2/2
```

3.801 $\int \cos^2(x) \cot^3(x) \, dx$

Optimal. Leaf size=22

$$\frac{\sin^2(x)}{2} - \frac{1}{2}\csc^2(x) - 2\log(\sin(x))$$

```
[Out] -1/2*csc(x)^2-2*ln(sin(x))+1/2*sin(x)^2
```

Rubi [A] time = 0.03, antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2590, 266, 43}

$$\frac{\sin^2(x)}{2} - \frac{1}{2}\csc^2(x) - 2\log(\sin(x))$$

Antiderivative was successfully verified.

```
[In] Int[Cos[x]^2*Cot[x]^3,x]
```

```
[Out] -Csc[x]^2/2 - 2*Log[Sin[x]] + Sin[x]^2/2
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 2590

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> -Dist[f^(-1), Subst[Int[(1 - x^2)^((m + n - 1)/2)/x^n, x], x, Cos[e + f*
x]], x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n - 1)/2]
```

Rubi steps

$$\int \cos^2(x)\cot^3(x)\,dx = \text{Subst}\left(\int \frac{(1-x^2)^2}{x^3}\,dx, x, -\sin(x)\right)$$
$$= \frac{1}{2}\text{Subst}\left(\int \frac{(1-x)^2}{x^2}\,dx, x, \sin^2(x)\right)$$
$$= \frac{1}{2}\text{Subst}\left(\int \left(1+\frac{1}{x^2}-\frac{2}{x}\right)dx, x, \sin^2(x)\right)$$
$$= -\frac{1}{2}\csc^2(x) - 2\log(\sin(x)) + \frac{\sin^2(x)}{2}$$

Mathematica [A] time = 0.02, size = 20, normalized size = 0.91

$$\frac{1}{2}\left(\sin^2(x) - \csc^2(x) - 4\log(\sin(x))\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[x]^2*Cot[x]^3,x]
```

```
[Out] (-Csc[x]^2 - 4*Log[Sin[x]] + Sin[x]^2)/2
```

fricas [B] time = 0.60, size = 37, normalized size = 1.68

$$-\frac{2\cos(x)^4 - 3\cos(x)^2 + 8\left(\cos(x)^2-1\right)\log\left(\frac{1}{2}\sin(x)\right) - 1}{4\left(\cos(x)^2-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^2*cot(x)^3,x, algorithm="fricas")
```

```
[Out] -1/4*(2*cos(x)^4 - 3*cos(x)^2 + 8*(cos(x)^2 - 1)*log(1/2*sin(x)) - 1)/(cos(
x)^2 - 1)
```

giac [A] time = 0.13, size = 28, normalized size = 1.27

$$-\frac{1}{2}\cos(x)^2 + \frac{1}{2\left(\cos(x)^2-1\right)} - \log\left(-\cos(x)^2+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^2*cot(x)^3,x, algorithm="giac")
```

```
[Out] -1/2*cos(x)^2 + 1/2/(cos(x)^2 - 1) - log(-cos(x)^2 + 1)
```

maple [A] time = 0.06, size = 29, normalized size = 1.32

$$-\frac{\cos^6(x)}{2\sin(x)^2}-\frac{\left(\cos^4(x)\right)}{2}-\left(\cos^2(x)\right)-2\ln\left(\sin(x)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)^2*cot(x)^3,x)
[Out] -1/2/sin(x)^2*cos(x)^6-1/2*cos(x)^4-cos(x)^2-2*ln(sin(x))
```

maxima [A] time = 0.32, size = 20, normalized size = 0.91

$$\frac{1}{2}\sin(x)^2-\frac{1}{2\sin(x)^2}-\log\left(\sin(x)^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^2*cot(x)^3,x, algorithm="maxima")
[Out] 1/2*sin(x)^2 - 1/2/sin(x)^2 - log(sin(x)^2)
```

mupad [B] time = 2.97, size = 32, normalized size = 1.45

$$\ln\left(\tan(x)^2+1\right)-2\,\ln\left(\tan(x)\right)-\frac{\tan(x)^2+\frac{1}{2}}{\tan(x)^4+\tan(x)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)^2*cot(x)^3,x)
[Out] log(tan(x)^2 + 1) - 2*log(tan(x)) - (tan(x)^2 + 1/2)/(tan(x)^2 + tan(x)^4)
```

sympy [A] time = 0.08, size = 20, normalized size = 0.91

$$-2\log\left(\sin\left(x\right)\right)+\frac{\sin^2\left(x\right)}{2}-\frac{1}{2\sin^2\left(x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)**2*cot(x)**3,x)
[Out] -2*log(sin(x)) + sin(x)**2/2 - 1/(2*sin(x)**2)
```

3.802 $\int \sec(x)(1-\sin(x))\,dx$

Optimal. Leaf size=5

$$\log(\sin(x)+1)$$

```
[Out] ln(1+sin(x))
```

Rubi [A] time = 0.01, antiderivative size = 5, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {2667, 31}

$$\log(\sin(x)+1)$$

Antiderivative was successfully verified.

```
[In] Int[Sec[x]*(1 - Sin[x]),x]
```

```
[Out] Log[1 + Sin[x]]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2667

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rubi steps

$$\begin{aligned}\int \sec(x)(1-\sin(x))\,dx &= -\text{Subst}\left(\int \frac{1}{1-x}\,dx, x, -\sin(x)\right)\\ &= \log(1+\sin(x))\end{aligned}$$

Mathematica [B] time = 0.01, size = 36, normalized size = 7.20

$$\log(\cos(x))-\log\left(\cos\left(\frac{x}{2}\right)-\sin\left(\frac{x}{2}\right)\right)+\log\left(\sin\left(\frac{x}{2}\right)+\cos\left(\frac{x}{2}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[x]*(1 - Sin[x]),x]
[Out] Log[Cos[x]] - Log[Cos[x/2] - Sin[x/2]] + Log[Cos[x/2] + Sin[x/2]]
```

fricas [A] time = 1.44, size = 5, normalized size = 1.00

$$\log(\sin(x) + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)*(1-sin(x)),x, algorithm="fricas")
[Out] log(sin(x) + 1)
```

giac [A] time = 0.14, size = 5, normalized size = 1.00

$$\log(\sin(x) + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)*(1-sin(x)),x, algorithm="giac")
[Out] log(sin(x) + 1)
```

maple [A] time = 0.05, size = 6, normalized size = 1.20

$$\ln(1 + \sin(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(x)*(1-sin(x)),x)
[Out] ln(1+sin(x))
```

maxima [A] time = 0.31, size = 5, normalized size = 1.00

$$\log(\sin(x) + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)*(1-sin(x)),x, algorithm="maxima")
[Out] log(sin(x) + 1)
```

mupad [B] time = 2.94, size = 5, normalized size = 1.00

$$\ln(\sin(x) + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(sin(x) - 1)/cos(x),x)
```

```
[Out] log(sin(x) + 1)
```

sympy [B] time = 2.04, size = 12, normalized size = 2.40

$$\log(\tan(x) + \sec(x)) + \log(\cos(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)*(1-sin(x)),x)
```

```
[Out] log(tan(x) + sec(x)) + log(cos(x))
```

3.803 $\int (1+\cos(x))\csc(x)\,dx$

Optimal. Leaf size=7

$$\log(1-\cos(x))$$

```
[Out] ln(1-cos(x))
```

Rubi [A] time = 0.02, antiderivative size = 7, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {2667, 31}

$$\log(1-\cos(x))$$

Antiderivative was successfully verified.

```
[In] Int[(1 + Cos[x])*Csc[x],x]
```

```
[Out] Log[1 - Cos[x]]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2667

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2])
```

Rubi steps

$$\begin{aligned}\int (1+\cos(x))\csc(x)\,dx &= -\text{Subst}\left(\int \frac{1}{1-x}\,dx, x, \cos(x)\right)\\ &= \log(1-\cos(x))\end{aligned}$$

Mathematica [B] time = 0.01, size = 20, normalized size = 2.86

$$\log\left(\sin\left(\frac{x}{2}\right)\right)+\log(\sin(x))-\log\left(\cos\left(\frac{x}{2}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + Cos[x])*Csc[x],x]
[Out] -Log[Cos[x/2]] + Log[Sin[x/2]] + Log[Sin[x]]
```

fricas [A] time = 0.74, size = 7, normalized size = 1.00

$$\log\left(-\frac{1}{2}\cos(x)+\frac{1}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+cos(x))*csc(x),x, algorithm="fricas")
[Out] log(-1/2*cos(x) + 1/2)
```

giac [A] time = 0.13, size = 7, normalized size = 1.00

$$\log\left(-\cos(x)+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+cos(x))*csc(x),x, algorithm="giac")
[Out] log(-cos(x) + 1)
```

maple [A] time = 0.05, size = 6, normalized size = 0.86

$$\ln\left(-1+\cos(x)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+cos(x))*csc(x),x)
[Out] ln(-1+cos(x))
```

maxima [A] time = 0.31, size = 5, normalized size = 0.71

$$\log\left(\cos(x)-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+cos(x))*csc(x),x, algorithm="maxima")
[Out] log(cos(x) - 1)
```

mupad [B] time = 2.91, size = 5, normalized size = 0.71

$$\ln\left(\cos(x)-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(x) + 1)/sin(x),x)
```

```
[Out] log(cos(x) - 1)
```

sympy [B] time = 1.89, size = 12, normalized size = 1.71

$$-\log\left(\cot\left(x\right)+\csc\left(x\right)\right)+\log\left(\sin\left(x\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+cos(x))*csc(x),x)
```

```
[Out] -log(cot(x) + csc(x)) + log(sin(x))
```

3.804 $\int \cos^2(x)\left(1-\tan^2(x)\right)\,dx$

Optimal. Leaf size=5

$$\sin(x)\cos(x)$$

```
[Out] cos(x)*sin(x)
```

Rubi [A] time = 0.02, antiderivative size = 5, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {3675, 383}

$$\sin(x)\cos(x)$$

Antiderivative was successfully verified.

```
[In] Int[Cos[x]^2*(1 - Tan[x]^2),x]
```

```
[Out] Cos[x]*Sin[x]
```

Rule 383

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> S
imp[(c*x*(a + b*x^n)^(p + 1))/a, x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[
b*c - a*d, 0] && EqQ[a*d - b*c*(n*(p + 1) + 1), 0]
```

Rule 3675

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/(c^(m - 1)*f), Subst[Int[(c^2 + ff^2*x^2)^(m/2 - 1)*(a + b*(ff*x)^n)^p
, x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && In
tegerQ[m/2] && (IntegersQ[n, p] || IGtQ[m, 0] || IGtQ[p, 0] || EqQ[n^2, 4]
|| EqQ[n^2, 16])
```

Rubi steps

$$\begin{aligned}\int \cos^2(x)\left(1-\tan^2(x)\right)\,dx &= \text{Subst}\left(\int \frac{1-x^2}{\left(1+x^2\right)^2}\,dx, x, \tan(x)\right)\\ &= \cos(x)\sin(x)\end{aligned}$$

Mathematica [A] time = 0.00, size = 8, normalized size = 1.60

$$\frac{1}{2}\sin(2x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[x]^2*(1 - Tan[x]^2),x]
[Out] Sin[2*x]/2
```

fricas [A] time = 0.53, size = 5, normalized size = 1.00

$$\cos(x)\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^2*(1-tan(x)^2),x, algorithm="fricas")
[Out] cos(x)*sin(x)
```

giac [A] time = 0.15, size = 9, normalized size = 1.80

$$\frac{1}{\frac{1}{\tan(x)}+\tan(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^2*(1-tan(x)^2),x, algorithm="giac")
[Out] 1/(1/tan(x) + tan(x))
```

maple [A] time = 0.05, size = 6, normalized size = 1.20

$$\cos(x)\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)^2*(1-tan(x)^2),x)
[Out] cos(x)*sin(x)
```

maxima [B] time = 0.31, size = 11, normalized size = 2.20

$$\frac{\tan(x)}{\tan(x)^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^2*(1-tan(x)^2),x, algorithm="maxima")
[Out] tan(x)/(tan(x)^2 + 1)
```

mupad [B] time = 2.89, size = 6, normalized size = 1.20

$$\frac{\sin(2x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-cos(x)^2*(tan(x)^2 - 1),x)
```

```
[Out] sin(2*x)/2
```

sympy [B] time = 3.11, size = 14, normalized size = 2.80

$$\frac{\sin(x)\cos(x)}{2} + \frac{\sin(2x)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)**2*(1-tan(x)**2),x)
```

```
[Out] sin(x)*cos(x)/2 + sin(2*x)/4
```

3.805 $\int \csc(2x)(\cos(x) + \sin(x))\,dx$

Optimal. Leaf size=15

$$\frac{1}{2}\tanh^{-1}(\sin(x)) - \frac{1}{2}\tanh^{-1}(\cos(x))$$

```
[Out] -1/2*arctanh(cos(x))+1/2*arctanh(sin(x))
```

Rubi [A] time = 0.05, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {4401, 4287, 3770, 4288}

$$\frac{1}{2}\tanh^{-1}(\sin(x)) - \frac{1}{2}\tanh^{-1}(\cos(x))$$

Antiderivative was successfully verified.

```
[In] Int[Csc[2*x]*(Cos[x] + Sin[x]),x]
```

```
[Out] -ArcTanh[Cos[x]]/2 + ArcTanh[Sin[x]]/2
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 4287

```
Int[(cos[(a_.) + (b_.)*(x_)]*(e_.))^(m_.)*sin[(c_.) + (d_.)*(x_)]^(p_.), x_
Symbol] :> Dist[2^p/e^p, Int[(e*Cos[a + b*x])^(m + p)*Sin[a + b*x]^p, x], x
] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && In
tegerQ[p]
```

Rule 4288

```
Int[((f_.)*sin[(a_.) + (b_.)*(x_)])^(n_.)*sin[(c_.) + (d_.)*(x_)]^(p_.), x_
Symbol] :> Dist[2^p/f^p, Int[Cos[a + b*x]^p*(f*Sin[a + b*x])^(n + p), x], x
] /; FreeQ[{a, b, c, d, f, n}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && In
tegerQ[p]
```

Rule 4401

```
Int[u_, x_Symbol] :> With[{v = ExpandTrig[u, x]}, Int[v, x] /; SumQ[v]] /;
 !InertTrigFreeQ[u]
```

Rubi steps

$$\begin{aligned}\int \csc(2x)(\cos(x)+\sin(x))\,dx &= \int (\cos(x)\csc(2x)+\csc(2x)\sin(x))\,dx\\ &= \int \cos(x)\csc(2x)\,dx+\int \csc(2x)\sin(x)\,dx\\ &= \frac{1}{2}\int \csc(x)\,dx+\frac{1}{2}\int \sec(x)\,dx\\ &= -\frac{1}{2}\tanh^{-1}(\cos(x))+\frac{1}{2}\tanh^{-1}(\sin(x))\end{aligned}$$

Mathematica [B] time = 0.01, size = 61, normalized size = 4.07

$$\frac{1}{2}\log\left(\sin\left(\frac{x}{2}\right)\right)-\frac{1}{2}\log\left(\cos\left(\frac{x}{2}\right)\right)-\frac{1}{2}\log\left(\cos\left(\frac{x}{2}\right)-\sin\left(\frac{x}{2}\right)\right)+\frac{1}{2}\log\left(\sin\left(\frac{x}{2}\right)+\cos\left(\frac{x}{2}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[2*x]*(Cos[x] + Sin[x]),x]
```

```
[Out] -1/2*Log[Cos[x/2]] - Log[Cos[x/2] - Sin[x/2]]/2 + Log[Sin[x/2]]/2 + Log[Cos
[x/2] + Sin[x/2]]/2
```

fricas [B] time = 0.75, size = 35, normalized size = 2.33

$$-\frac{1}{4}\log\left(-\frac{1}{2}(\cos(x)+1)\sin(x)+\frac{1}{2}\cos(x)+\frac{1}{2}\right)+\frac{1}{4}\log\left(-\frac{1}{2}(\cos(x)-1)\sin(x)-\frac{1}{2}\cos(x)+\frac{1}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(2*x)*(cos(x)+sin(x)),x, algorithm="fricas")
```

```
[Out] -1/4*log(-1/2*(cos(x) + 1)*sin(x) + 1/2*cos(x) + 1/2) + 1/4*log(-1/2*(cos(x
) - 1)*sin(x) - 1/2*cos(x) + 1/2)
```

giac [B] time = 0.15, size = 29, normalized size = 1.93

$$\frac{1}{2}\log\left(\left|\tan\left(\frac{1}{2}x\right)+1\right|\right)-\frac{1}{2}\log\left(\left|\tan\left(\frac{1}{2}x\right)-1\right|\right)+\frac{1}{2}\log\left(\left|\tan\left(\frac{1}{2}x\right)\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(2*x)*(cos(x)+sin(x)),x, algorithm="giac")
```

[Out] 1/2*log(abs(tan(1/2*x) + 1)) - 1/2*log(abs(tan(1/2*x) - 1)) + 1/2*log(abs(t
an(1/2*x)))

maple [A] time = 0.26, size = 20, normalized size = 1.33

$$\frac{\ln\left(\sec(x)+\tan(x)\right)}{2}+\frac{\ln\left(\csc(x)-\cot(x)\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(csc(2*x)*(cos(x)+sin(x)),x)

[Out] 1/2*ln(sec(x)+tan(x))+1/2*ln(csc(x)-cot(x))

maxima [B] time = 0.42, size = 69, normalized size = 4.60

$$-\frac{1}{4}\log\left(\cos(x)^2+\sin(x)^2+2\cos(x)+1\right)+\frac{1}{4}\log\left(\cos(x)^2+\sin(x)^2-2\cos(x)+1\right)+\frac{1}{4}\log\left(\cos(x)^2+\sin(x)^2\right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(2*x)*(cos(x)+sin(x)),x, algorithm="maxima")

[Out] -1/4*log(cos(x)^2 + sin(x)^2 + 2*cos(x) + 1) + 1/4*log(cos(x)^2 + sin(x)^2
- 2*cos(x) + 1) + 1/4*log(cos(x)^2 + sin(x)^2 + 2*sin(x) + 1) - 1/4*log(cos
(x)^2 + sin(x)^2 - 2*sin(x) + 1)

mupad [B] time = 3.11, size = 24, normalized size = 1.60

$$\frac{\ln\left({\tan\left(\frac{x}{2}\right)}^2+\tan\left(\frac{x}{2}\right)\right)}{2}-\frac{\ln\left(\tan\left(\frac{x}{2}\right)-1\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((cos(x) + sin(x))/sin(2*x),x)

[Out] log(tan(x/2) + tan(x/2)^2)/2 - log(tan(x/2) - 1)/2

sympy [B] time = 1.86, size = 32, normalized size = 2.13

$$-\frac{\log\left(\sin\left(x\right)-1\right)}{4}+\frac{\log\left(\sin\left(x\right)+1\right)}{4}+\frac{\log\left(\cos\left(x\right)-1\right)}{4}-\frac{\log\left(\cos\left(x\right)+1\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(2*x)*(cos(x)+sin(x)),x)

[Out] -log(sin(x) - 1)/4 + log(sin(x) + 1)/4 + log(cos(x) - 1)/4 - log(cos(x) + 1
)/4

3.806 $\int \frac{\cos(x)(-3+2\sin(x))}{2-3\sin(x)+\sin^2(x)}\,dx$

Optimal. Leaf size=11

$$\log\left(\sin^2(x)-3\sin(x)+2\right)$$

```
[Out] ln(2-3*sin(x)+sin(x)^2)
```

Rubi [A] time = 0.05, antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {4334, 628}

$$\log\left(\sin^2(x)-3\sin(x)+2\right)$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[x]*(-3 + 2*Sin[x]))/(2 - 3*Sin[x] + Sin[x]^2),x]
```

```
[Out] Log[2 - 3*Sin[x] + Sin[x]^2]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 4334

```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{d = FreeFacto
rs[Sin[c*(a + b*x)], x]}, Dist[d/(b*c), Subst[Int[SubstFor[1, Sin[c*(a + b*
x)]/d, u, x], x], x, Sin[c*(a + b*x)]/d], x] /; FunctionOfQ[Sin[c*(a + b*x)
]/d, u, x, True]] /; FreeQ[{a, b, c}, x] && (EqQ[F, Cos] || EqQ[F, cos])
```

Rubi steps

$$\begin{aligned}\int \frac{\cos(x)(-3+2\sin(x))}{2-3\sin(x)+\sin^2(x)}\,dx &= \text{Subst}\left(\int \frac{-3+2x}{2-3x+x^2}\,dx, x, \sin(x)\right)\\ &= \log\left(2-3\sin(x)+\sin^2(x)\right)\end{aligned}$$

Mathematica [B] time = 0.09, size = 26, normalized size = 2.36

$$\log(2-\sin(x))+2\log\left(\cos\left(\frac{x}{2}\right)-\sin\left(\frac{x}{2}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[x]*(-3 + 2*Sin[x]))/(2 - 3*Sin[x] + Sin[x]^2),x]
[Out] 2*Log[Cos[x/2] - Sin[x/2]] + Log[2 - Sin[x]]
```

fricas [A] time = 0.70, size = 15, normalized size = 1.36

$$\log\left(-\frac{1}{2}\sin(x)+1\right)+\log\left(-\sin(x)+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*(-3+2*sin(x))/(2-3*sin(x)+sin(x)^2),x, algorithm="fricas")
[Out] log(-1/2*sin(x) + 1) + log(-sin(x) + 1)
```

giac [A] time = 0.13, size = 15, normalized size = 1.36

$$\log\left(-\sin(x)+2\right)+\log\left(-\sin(x)+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*(-3+2*sin(x))/(2-3*sin(x)+sin(x)^2),x, algorithm="giac")
[Out] log(-sin(x) + 2) + log(-sin(x) + 1)
```

maple [A] time = 0.05, size = 12, normalized size = 1.09

$$\ln\left(2-3\sin(x)+\sin^2(x)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)*(-3+2*sin(x))/(2-3*sin(x)+sin(x)^2),x)
[Out] ln(2-3*sin(x)+sin(x)^2)
```

maxima [A] time = 0.31, size = 11, normalized size = 1.00

$$\log\left(\sin(x)^2-3\sin(x)+2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*(-3+2*sin(x))/(2-3*sin(x)+sin(x)^2),x, algorithm="maxima")
[Out] log(sin(x)^2 - 3*sin(x) + 2)
```

mupad [B] time = 0.08, size = 11, normalized size = 1.00

$$\ln\left(\sin(x)^2-3\sin(x)+2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(x)*(2*sin(x) - 3))/(sin(x)^2 - 3*sin(x) + 2),x)
```

```
[Out] log(sin(x)^2 - 3*sin(x) + 2)
```

sympy [A] time = 0.19, size = 12, normalized size = 1.09

$$\log\left(\sin\left(x\right)-2\right)+\log\left(\sin\left(x\right)-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*(-3+2*sin(x))/(2-3*sin(x)+sin(x)**2),x)
```

```
[Out] log(sin(x) - 2) + log(sin(x) - 1)
```

3.807 $\int \frac{\cos^2(x)\sin(x)}{5+\cos^2(x)}\,dx$

Optimal. Leaf size=20

$$\sqrt{5}\,\tan^{-1}\left(\frac{\cos(x)}{\sqrt{5}}\right)-\cos(x)$$

```
[Out] -cos(x)+arctan(1/5*cos(x)*5^(1/2))*5^(1/2)
```

Rubi [A] time = 0.05, antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {4335, 321, 203}

$$\sqrt{5}\,\tan^{-1}\left(\frac{\cos(x)}{\sqrt{5}}\right)-\cos(x)$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[x]^2*Sin[x])/(5 + Cos[x]^2),x]
```

```
[Out] Sqrt[5]*ArcTan[Cos[x]/Sqrt[5]] - Cos[x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 4335

```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{d = FreeFacto
rs[Cos[c*(a + b*x)], x]}, -Dist[d/(b*c), Subst[Int[SubstFor[1, Cos[c*(a + b
*x)]/d, u, x], x], x, Cos[c*(a + b*x)]/d], x] /; FunctionOfQ[Cos[c*(a + b*x
)]/d, u, x, True]] /; FreeQ[{a, b, c}, x] && (EqQ[F, Sin] || EqQ[F, sin])
```

Rubi steps

$$\int \frac{\cos^2(x)\sin(x)}{5+\cos^2(x)}\,dx = -\operatorname{Subst}\left(\int \frac{x^2}{5+x^2}\,dx, x, \cos(x)\right)$$
$$= -\cos(x) + 5\operatorname{Subst}\left(\int \frac{1}{5+x^2}\,dx, x, \cos(x)\right)$$
$$= \sqrt{5}\tan^{-1}\left(\frac{\cos(x)}{\sqrt{5}}\right) - \cos(x)$$

Mathematica [B] time = 0.17, size = 82, normalized size = 4.10

$$\frac{1}{20}\left(-20\cos(x) + 21\sqrt{5}\tan^{-1}\left(\frac{1}{\sqrt{5}} - \sqrt{\frac{6}{5}}\tan\left(\frac{x}{2}\right)\right) + 21\sqrt{5}\tan^{-1}\left(\sqrt{\frac{6}{5}}\tan\left(\frac{x}{2}\right) + \frac{1}{\sqrt{5}}\right) - \sqrt{5}\tan^{-1}\left(\frac{\cos(x)}{\sqrt{5}}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[x]^2*Sin[x])/(5 + Cos[x]^2),x]
[Out] (-(Sqrt[5]*ArcTan[Cos[x]/Sqrt[5]]) + 21*Sqrt[5]*ArcTan[1/Sqrt[5] - Sqrt[6/5
]*Tan[x/2]] + 21*Sqrt[5]*ArcTan[1/Sqrt[5] + Sqrt[6/5]*Tan[x/2]] - 20*Cos[x]
)/20
```

fricas [A] time = 0.62, size = 17, normalized size = 0.85

$$\sqrt{5}\arctan\left(\frac{1}{5}\sqrt{5}\cos(x)\right) - \cos(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^2*sin(x)/(5+cos(x)^2),x, algorithm="fricas")
[Out] sqrt(5)*arctan(1/5*sqrt(5)*cos(x)) - cos(x)
```

giac [A] time = 0.12, size = 17, normalized size = 0.85

$$\sqrt{5}\arctan\left(\frac{1}{5}\sqrt{5}\cos(x)\right) - \cos(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^2*sin(x)/(5+cos(x)^2),x, algorithm="giac")
[Out] sqrt(5)*arctan(1/5*sqrt(5)*cos(x)) - cos(x)
```

maple [A] time = 0.04, size = 18, normalized size = 0.90

$$-\cos(x)+\arctan\left(\frac{\cos(x)\sqrt{5}}{5}\right)\sqrt{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)^2*sin(x)/(5+cos(x)^2),x)
[Out] -cos(x)+arctan(1/5*cos(x)*5^(1/2))*5^(1/2)
```

maxima [A] time = 0.43, size = 17, normalized size = 0.85

$$\sqrt{5}\arctan\left(\frac{1}{5}\sqrt{5}\cos(x)\right)-\cos(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^2*sin(x)/(5+cos(x)^2),x, algorithm="maxima")
[Out] sqrt(5)*arctan(1/5*sqrt(5)*cos(x)) - cos(x)
```

mupad [B] time = 2.90, size = 17, normalized size = 0.85

$$\sqrt{5}\,\mathrm{atan}\left(\frac{\sqrt{5}\,\cos(x)}{5}\right)-\cos(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(x)^2*sin(x))/(cos(x)^2 + 5),x)
[Out] 5^(1/2)*atan((5^(1/2)*cos(x))/5) - cos(x)
```

sympy [A] time = 0.43, size = 19, normalized size = 0.95

$$-\cos(x)+\sqrt{5}\,\mathrm{atan}\left(\frac{\sqrt{5}\cos(x)}{5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)**2*sin(x)/(5+cos(x)**2),x)
[Out] -cos(x) + sqrt(5)*atan(sqrt(5)*cos(x)/5)
```

3.808 $\int \frac{\cos(x)}{\sin(x)+\sin^2(x)}\,dx$

Optimal. Leaf size=11

$$\log(\sin(x)) - \log(\sin(x) + 1)$$

```
[Out] ln(sin(x))-ln(1+sin(x))
```

Rubi [A] time = 0.02, antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3258, 615}

$$\log(\sin(x)) - \log(\sin(x) + 1)$$

Antiderivative was successfully verified.

```
[In] Int[Cos[x]/(Sin[x] + Sin[x]^2),x]
```

```
[Out] Log[Sin[x]] - Log[1 + Sin[x]]
```

Rule 615

```
Int[((b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[Log[x]/b, x] - Simp
[Log[RemoveContent[b + c*x, x]]/b, x] /; FreeQ[{b, c}, x]
```

Rule 3258

```
Int[cos[(d_.) + (e_.)*(x_)]^(m_.)*((a_.) + (b_.)*((f_.)*sin[(d_.) + (e_.)*(
x_)])^(n_.) + (c_.)*((f_.)*sin[(d_.) + (e_.)*(x_)])^(n2_.))^(p_.), x_Symbol
] :> Module[{g = FreeFactors[Sin[d + e*x], x]}, Dist[g/e, Subst[Int[(1 - g^
2*x^2)^((m - 1)/2)*(a + b*(f*g*x)^n + c*(f*g*x)^(2*n))^p, x], x, Sin[d + e*
x]/g], x]] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && EqQ[n2, 2*n] && Integer
Q[(m - 1)/2]
```

Rubi steps

$$\begin{aligned}\int \frac{\cos(x)}{\sin(x)+\sin^2(x)}\,dx &= \text{Subst}\left(\int \frac{1}{x+x^2}\,dx, x, \sin(x)\right)\\ &= \log(\sin(x)) - \log(1+\sin(x))\end{aligned}$$

Mathematica [A] time = 0.01, size = 11, normalized size = 1.00

$$\log(\sin(x)) - \log(\sin(x) + 1)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[x]/(Sin[x] + Sin[x]^2),x]
[Out] Log[Sin[x]] - Log[1 + Sin[x]]
```

fricas [A] time = 0.60, size = 13, normalized size = 1.18

$$\log\left(\frac{1}{2}\sin(x)\right) - \log(\sin(x) + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)/(sin(x)+sin(x)^2),x, algorithm="fricas")
[Out] log(1/2*sin(x)) - log(sin(x) + 1)
```

giac [A] time = 0.15, size = 12, normalized size = 1.09

$$-\log(\sin(x) + 1) + \log(|\sin(x)|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)/(sin(x)+sin(x)^2),x, algorithm="giac")
[Out] -log(sin(x) + 1) + log(abs(sin(x)))
```

maple [A] time = 0.08, size = 12, normalized size = 1.09

$$\ln(\sin(x)) - \ln(1 + \sin(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)/(sin(x)+sin(x)^2),x)
[Out] ln(sin(x))-ln(1+sin(x))
```

maxima [A] time = 0.31, size = 11, normalized size = 1.00

$$-\log(\sin(x) + 1) + \log(\sin(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)/(sin(x)+sin(x)^2),x, algorithm="maxima")
[Out] -log(sin(x) + 1) + log(sin(x))
```

mupad [B] time = 2.98, size = 9, normalized size = 0.82

$$-2\,\mathrm{atanh}(2\,\sin(x) + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)/(sin(x) + sin(x)^2),x)
```

```
[Out] -2*atanh(2*sin(x) + 1)
```

sympy [A] time = 0.18, size = 10, normalized size = 0.91

$$-\log(\sin(x)+1)+\log(\sin(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)/(sin(x)+sin(x)**2),x)
```

```
[Out] -log(sin(x) + 1) + log(sin(x))
```

3.809 $\int \frac{\cos(x)}{\sin(x)+\sin^{\sqrt{2}}(x)}\,dx$

Optimal. Leaf size=26

$$\log(\sin(x)) - \left(1+\sqrt{2}\right)\log\left(\sin^{\sqrt{2}-1}(x)+1\right)$$

```
[Out] ln(sin(x))-ln(1+sin(x)^(2^(1/2)-1))*(1+2^(1/2))
```

Rubi [A] time = 0.05, antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312, Rules used = {4334, 266, 36, 29, 31}

$$\log(\sin(x)) - \left(1+\sqrt{2}\right)\log\left(\sin^{\sqrt{2}-1}(x)+1\right)$$

Antiderivative was successfully verified.

```
[In] Int[Cos[x]/(Sin[x] + Sin[x]^Sqrt[2]),x]
```

```
[Out] Log[Sin[x]] - (1 + Sqrt[2])*Log[1 + Sin[x]^(-1 + Sqrt[2])]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 4334

```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{d = FreeFacto
rs[Sin[c*(a + b*x)], x]}, Dist[d/(b*c), Subst[Int[SubstFor[1, Sin[c*(a + b*
x)]/d, u, x], x], x, Sin[c*(a + b*x)]/d], x] /; FunctionOfQ[Sin[c*(a + b*x)
]/d, u, x, True]] /; FreeQ[{a, b, c}, x] && (EqQ[F, Cos] || EqQ[F, cos])
```

Rubi steps

$$\begin{aligned}\int \frac{\cos(x)}{\sin(x)+\sin^{\sqrt{2}}(x)}\,dx &= \text{Subst}\left(\int \frac{1}{x\left(1+x^{-1+\sqrt{2}}\right)}\,dx, x, \sin(x)\right)\\ &= \left(1+\sqrt{2}\right)\text{Subst}\left(\int \frac{1}{x(1+x)}\,dx, x, \sin^{-1+\sqrt{2}}(x)\right)\\ &= \left(-1-\sqrt{2}\right)\text{Subst}\left(\int \frac{1}{1+x}\,dx, x, \sin^{-1+\sqrt{2}}(x)\right)+\left(1+\sqrt{2}\right)\text{Subst}\left(\int \frac{1}{x}\,dx, x, \sin^{-1+\sqrt{2}}\right.\\ &= \log(\sin(x))-\left(1+\sqrt{2}\right)\log\left(1+\sin^{-1+\sqrt{2}}(x)\right)\end{aligned}$$

Mathematica [A] time = 0.04, size = 26, normalized size = 1.00

$$\log(\sin(x))-\left(1+\sqrt{2}\right)\log\left(\sin^{\sqrt{2}-1}(x)+1\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[x]/(Sin[x] + Sin[x]^Sqrt[2]),x]
[Out] Log[Sin[x]] - (1 + Sqrt[2])*Log[1 + Sin[x]^(-1 + Sqrt[2])]
```

fricas [A] time = 1.39, size = 27, normalized size = 1.04

$$-\left(\sqrt{2}+1\right)\log\left(\sin(x)^{\left(\sqrt{2}\right)}+\sin(x)\right)+\left(\sqrt{2}+2\right)\log\left(\sin(x)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)/(sin(x)+sin(x)^(2^(1/2))),x, algorithm="fricas")
[Out] -(sqrt(2) + 1)*log(sin(x)^sqrt(2) + sin(x)) + (sqrt(2) + 2)*log(sin(x))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos(x)}{\sin(x)^{\left(\sqrt{2}\right)}+\sin(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)/(sin(x)+sin(x)^(2^(1/2))),x, algorithm="giac")
```

```
[Out] integrate(cos(x)/(sin(x)^sqrt(2) + sin(x)), x)
```

maple [C] time = 0.75, size = 1856, normalized size = 71.38

Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)/(sin(x)+sin(x)^(2^(1/2))),x)
```

```
[Out] -1/2*I*2^(1/2)*csgn(I*(exp(I*x)-1)*(1+exp(I*x)))*csgn(I*(exp(I*x)-1))*csgn(
I*(1+exp(I*x)))*Pi+1/2*I*2^(1/2)*csgn(I*(exp(I*x)-1)*(1+exp(I*x)))*csgn(I*(
1+exp(I*x))*(-1+exp(-I*x)))*csgn(I*exp(-I*x))*Pi-2*ln(2)-I*Pi+1/2*I*2^(1/2)
*csgn(I*(exp(I*x)-1)*(1+exp(I*x)))^2*csgn(I*(exp(I*x)-1))*Pi+1/2*I*2^(1/2)*
csgn(I*(exp(I*x)-1)*(1+exp(I*x)))^2*csgn(I*(1+exp(I*x)))*Pi+1/2*I*2^(1/2)*c
sgn(I*(exp(I*x)-1)*(1+exp(I*x)))*csgn(I*(1+exp(I*x))*(-1+exp(-I*x)))^2*Pi+1
/2*I*2^(1/2)*csgn(I*(1+exp(I*x))*(-1+exp(-I*x)))^2*csgn(I*exp(-I*x))*Pi-1/2
*I*2^(1/2)*csgn(I*(1+exp(I*x))*(-1+exp(-I*x)))*csgn((1+exp(I*x))*(-1+exp(-I
*x)))^2*Pi-1/2*I*2^(1/2)*csgn(I*(1+exp(I*x))*(-1+exp(-I*x)))*csgn((1+exp(I*
x))*(-1+exp(-I*x)))*Pi+I*csgn(I*(exp(I*x)-1)*(1+exp(I*x)))*csgn(I*(1+exp(I*
x))*(-1+exp(-I*x)))*csgn(I*exp(-I*x))*Pi-I*csgn(I*(exp(I*x)-1)*(1+exp(I*x))
)*csgn(I*(exp(I*x)-1))*csgn(I*(1+exp(I*x)))*Pi-1/2*I*2^(1/2)*Pi-I*csgn(I*(1
+exp(I*x))*(-1+exp(-I*x)))*csgn((1+exp(I*x))*(-1+exp(-I*x)))^2*Pi-I*csgn(I*
(1+exp(I*x))*(-1+exp(-I*x)))*csgn((1+exp(I*x))*(-1+exp(-I*x)))*Pi-I*csgn(I*
(exp(I*x)-1)*(1+exp(I*x)))^3*Pi+I*csgn(I*(1+exp(I*x))*(-1+exp(-I*x)))^3*Pi+
I*csgn((1+exp(I*x))*(-1+exp(-I*x)))^3*Pi+I*csgn((1+exp(I*x))*(-1+exp(-I*x))
)^2*Pi-ln(exp(-1/2*2^(1/2)*(I*csgn(I*(exp(I*x)-1)*(1+exp(I*x)))*csgn(I*(exp
(I*x)-1))*csgn(I*(1+exp(I*x)))*Pi+I*csgn(I*(1+exp(I*x))*(-1+exp(-I*x)))*csg
n((1+exp(I*x))*(-1+exp(-I*x)))^2*Pi+I*csgn(I*(exp(I*x)-1)*(1+exp(I*x)))^3*P
i-I*csgn(I*(exp(I*x)-1)*(1+exp(I*x)))*csgn(I*(1+exp(I*x))*(-1+exp(-I*x)))*c
sgn(I*exp(-I*x))*Pi-I*csgn(I*(exp(I*x)-1)*(1+exp(I*x)))^2*csgn(I*(1+exp(I*x
)))*Pi+I*csgn(I*(1+exp(I*x))*(-1+exp(-I*x)))*csgn((1+exp(I*x))*(-1+exp(-I*x
)))*Pi-I*csgn(I*(exp(I*x)-1)*(1+exp(I*x)))^2*csgn(I*(exp(I*x)-1))*Pi-I*csgn
(I*(1+exp(I*x))*(-1+exp(-I*x)))^2*csgn(I*exp(-I*x))*Pi+I*Pi-I*csgn((1+exp(I
*x))*(-1+exp(-I*x)))^2*Pi-I*csgn(I*(exp(I*x)-1)*(1+exp(I*x)))*csgn(I*(1+exp
(I*x))*(-1+exp(-I*x)))^2*Pi-I*csgn(I*(1+exp(I*x))*(-1+exp(-I*x)))^3*Pi-I*cs
gn((1+exp(I*x))*(-1+exp(-I*x)))^3*Pi+2*ln(exp(I*x))-2*ln(exp(I*x)-1)-2*ln(1
+exp(I*x))+2*ln(2)))+sin(x))+2*ln(1+exp(I*x))-ln(exp(-1/2*2^(1/2)*(I*csgn(I
*(exp(I*x)-1)*(1+exp(I*x)))*csgn(I*(exp(I*x)-1))*csgn(I*(1+exp(I*x)))*Pi+I*
csgn(I*(1+exp(I*x))*(-1+exp(-I*x)))*csgn((1+exp(I*x))*(-1+exp(-I*x)))^2*Pi+
I*csgn(I*(exp(I*x)-1)*(1+exp(I*x)))^3*Pi-I*csgn(I*(exp(I*x)-1)*(1+exp(I*x))
)*csgn(I*(1+exp(I*x))*(-1+exp(-I*x)))*csgn(I*exp(-I*x))*Pi-I*csgn(I*(exp(I*
```

```
x)-1)*(1+exp(I*x)))^2*csgn(I*(1+exp(I*x)))*Pi+I*csgn(I*(1+exp(I*x))*(-1+exp
(-I*x)))*csgn((1+exp(I*x))*(-1+exp(-I*x)))*Pi-I*csgn(I*(exp(I*x)-1)*(1+exp(
I*x)))^2*csgn(I*(exp(I*x)-1))*Pi-I*csgn(I*(1+exp(I*x))*(-1+exp(-I*x)))^2*cs
gn(I*exp(-I*x))*Pi+I*Pi-I*csgn((1+exp(I*x))*(-1+exp(-I*x)))^2*Pi-I*csgn(I*(
exp(I*x)-1)*(1+exp(I*x)))*csgn(I*(1+exp(I*x))*(-1+exp(-I*x)))^2*Pi-I*csgn(I
*(1+exp(I*x))*(-1+exp(-I*x)))^3*Pi-I*csgn((1+exp(I*x))*(-1+exp(-I*x)))^3*Pi
+2*ln(exp(I*x))-2*ln(exp(I*x)-1)-2*ln(1+exp(I*x))+2*ln(2)))+sin(x))*2^(1/2)
-1/2*I*2^(1/2)*csgn(I*(exp(I*x)-1)*(1+exp(I*x)))^3*Pi+1/2*I*2^(1/2)*csgn(I*
(1+exp(I*x))*(-1+exp(-I*x)))^3*Pi+1/2*I*2^(1/2)*csgn((1+exp(I*x))*(-1+exp(-
I*x)))^3*Pi+1/2*I*2^(1/2)*csgn((1+exp(I*x))*(-1+exp(-I*x)))^2*Pi-2*ln(exp(I
*x))+I*csgn(I*(exp(I*x)-1)*(1+exp(I*x)))^2*csgn(I*(exp(I*x)-1))*Pi+I*csgn(I
*(exp(I*x)-1)*(1+exp(I*x)))^2*csgn(I*(1+exp(I*x)))*Pi+I*csgn(I*(exp(I*x)-1)
*(1+exp(I*x)))*csgn(I*(1+exp(I*x))*(-1+exp(-I*x)))^2*Pi+I*csgn(I*(1+exp(I*x
))*(-1+exp(-I*x)))^2*csgn(I*exp(-I*x))*Pi+2*ln(exp(I*x)-1)-2^(1/2)*ln(exp(I
*x))+2^(1/2)*ln(exp(I*x)-1)+2^(1/2)*ln(1+exp(I*x))-2^(1/2)*ln(2)
```

maxima [A] time = 0.42, size = 34, normalized size = 1.31

$$\frac{\sqrt{2}\log\left(\sin(x)\right)}{\sqrt{2}-1}-\frac{\log\left(\sin(x)^{\left(\sqrt{2}\right)}+\sin(x)\right)}{\sqrt{2}-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)/(sin(x)+sin(x)^(2^(1/2))),x, algorithm="maxima")
```

```
[Out] sqrt(2)*log(sin(x))/(sqrt(2) - 1) - log(sin(x)^sqrt(2) + sin(x))/(sqrt(2) -
 1)
```

mupad [B] time = 3.08, size = 29, normalized size = 1.12

$$\ln\left(\sin(x)\right)\left(\sqrt{2}+2\right)-\frac{\ln\left(\sin(x)+\sin(x)^{\sqrt{2}}\right)}{\sqrt{2}-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)/(sin(x) + sin(x)^(2^(1/2))),x)
```

```
[Out] log(sin(x))*(2^(1/2) + 2) - log(sin(x) + sin(x)^(2^(1/2)))/(2^(1/2) - 1)
```

sympy [B] time = 1.07, size = 82, normalized size = 3.15

$$\frac{\sqrt{2}\log\left(\sin\left(x\right)+\sin^{\sqrt{2}}\left(x\right)\right)}{-3+2\sqrt{2}}-\frac{\log\left(\sin\left(x\right)+\sin^{\sqrt{2}}\left(x\right)\right)}{-3+2\sqrt{2}}+\frac{\sqrt{2}\log\left(\sin\left(x\right)\right)}{-3+2\sqrt{2}}-\frac{2\log\left(\sin\left(x\right)\right)}{-3+2\sqrt{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)/(sin(x)+sin(x)**(2**(1/2))),x)
```

```
[Out] sqrt(2)*log(sin(x) + sin(x)**(sqrt(2)))/(-3 + 2*sqrt(2)) - log(sin(x) + sin
(x)**(sqrt(2)))/(-3 + 2*sqrt(2)) + sqrt(2)*log(sin(x))/(-3 + 2*sqrt(2)) - 2
*log(sin(x))/(-3 + 2*sqrt(2))
```

3.810 $\int \frac{1}{2\sin(x)+\sin(2x)}\,dx$

Optimal. Leaf size=24

$$\frac{1}{8}\tan^2\left(\frac{x}{2}\right)+\frac{1}{4}\log\left(\tan\left(\frac{x}{2}\right)\right)$$

```
[Out] 1/4*ln(tan(1/2*x))+1/8*tan(1/2*x)^2
```

Rubi [A] time = 0.03, antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {12, 14}

$$\frac{1}{8}\tan^2\left(\frac{x}{2}\right)+\frac{1}{4}\log\left(\tan\left(\frac{x}{2}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Int[(2*Sin[x] + Sin[2*x])^(-1),x]
```

```
[Out] Log[Tan[x/2]]/4 + Tan[x/2]^2/8
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{2\sin(x)+\sin(2x)}\,dx &= 2\,\text{Subst}\left(\int \frac{1+x^2}{8x}\,dx, x, \tan\left(\frac{x}{2}\right)\right)\\
&= \frac{1}{4}\,\text{Subst}\left(\int \frac{1+x^2}{x}\,dx, x, \tan\left(\frac{x}{2}\right)\right)\\
&= \frac{1}{4}\,\text{Subst}\left(\int \left(\frac{1}{x}+x\right)dx, x, \tan\left(\frac{x}{2}\right)\right)\\
&= \frac{1}{4}\log\left(\tan\left(\frac{x}{2}\right)\right)+\frac{1}{8}\tan^2\left(\frac{x}{2}\right)
\end{aligned}$$

Mathematica [A] time = 0.03, size = 39, normalized size = 1.62

$$\frac{1-2\cos^2\left(\frac{x}{2}\right)\left(\log\left(\cos\left(\frac{x}{2}\right)\right)-\log\left(\sin\left(\frac{x}{2}\right)\right)\right)}{4(\cos(x)+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2*Sin[x] + Sin[2*x])^(-1),x]
```

```
[Out] (1 - 2*Cos[x/2]^2*(Log[Cos[x/2]] - Log[Sin[x/2]]))/(4*(1 + Cos[x]))
```

fricas [B] time = 0.60, size = 35, normalized size = 1.46

$$-\frac{(\cos(x)+1)\log\left(\frac{1}{2}\cos(x)+\frac{1}{2}\right)-(\cos(x)+1)\log\left(-\frac{1}{2}\cos(x)+\frac{1}{2}\right)-2}{8(\cos(x)+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*sin(x)+sin(2*x)),x, algorithm="fricas")
```

```
[Out] -1/8*((cos(x) + 1)*log(1/2*cos(x) + 1/2) - (cos(x) + 1)*log(-1/2*cos(x) + 1
/2) - 2)/(cos(x) + 1)
```

giac [A] time = 0.14, size = 28, normalized size = 1.17

$$-\frac{\cos(x)-1}{8(\cos(x)+1)}+\frac{1}{8}\log\left(-\frac{\cos(x)-1}{\cos(x)+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*sin(x)+sin(2*x)),x, algorithm="giac")
```

```
[Out] -1/8*(cos(x) - 1)/(cos(x) + 1) + 1/8*log(-(cos(x) - 1)/(cos(x) + 1))
```

maple [A] time = 0.24, size = 24, normalized size = 1.00

$$\frac{\ln(-1+\cos(x))}{8}+\frac{1}{4+4\cos(x)}-\frac{\ln(1+\cos(x))}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2*sin(x)+sin(2*x)),x)
```

```
[Out] 1/8*ln(-1+cos(x))+1/4/(1+cos(x))-1/8*ln(1+cos(x))
```

maxima [B] time = 0.33, size = 220, normalized size = 9.17

$$\frac{4\cos(2x)\cos(x) + 8\cos(x)^2 - \left(2(2\cos(x)+1)\cos(2x) + \cos(2x)^2 + 4\cos(x)^2 + \sin(2x)^2 + 4\sin(2x)\sin(x\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*sin(x)+sin(2*x)),x, algorithm="maxima")
```

```
[Out] 1/8*(4*cos(2*x)*cos(x) + 8*cos(x)^2 - (2*(2*cos(x) + 1)*cos(2*x) + cos(2*x)
^2 + 4*cos(x)^2 + sin(2*x)^2 + 4*sin(2*x)*sin(x) + 4*sin(x)^2 + 4*cos(x) +
1)*log(cos(x)^2 + sin(x)^2 + 2*cos(x) + 1) + (2*(2*cos(x) + 1)*cos(2*x) + c
os(2*x)^2 + 4*cos(x)^2 + sin(2*x)^2 + 4*sin(2*x)*sin(x) + 4*sin(x)^2 + 4*co
s(x) + 1)*log(cos(x)^2 + sin(x)^2 - 2*cos(x) + 1) + 4*sin(2*x)*sin(x) + 8*s
in(x)^2 + 4*cos(x))/(2*(2*cos(x) + 1)*cos(2*x) + cos(2*x)^2 + 4*cos(x)^2 +
sin(2*x)^2 + 4*sin(2*x)*sin(x) + 4*sin(x)^2 + 4*cos(x) + 1)
```

mupad [B] time = 3.07, size = 16, normalized size = 0.67

$$\frac{\ln\left(\tan\left(\frac{x}{2}\right)\right)}{4} + \frac{\tan\left(\frac{x}{2}\right)^2}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(2*x) + 2*sin(x)),x)
```

```
[Out] log(tan(x/2))/4 + tan(x/2)^2/8
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{2\sin(x) + \sin(2x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*sin(x)+sin(2*x)),x)
```

```
[Out] Integral(1/(2*sin(x) + sin(2*x)), x)
```

3.811 $\int \left(-3+4x+x^2\right)\sin(2x)\,dx$

Optimal. Leaf size=40

$$-\frac{1}{2}x^2\cos(2x)+\frac{1}{2}x\sin(2x)+\sin(2x)-2x\cos(2x)+\frac{7}{4}\cos(2x)$$

```
[Out] 7/4*cos(2*x)-2*x*cos(2*x)-1/2*x^2*cos(2*x)+sin(2*x)+1/2*x*sin(2*x)
```

Rubi [A] time = 0.07, antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {6742, 2638, 3296, 2637}

$$-\frac{1}{2}x^2\cos(2x)+\frac{1}{2}x\sin(2x)+\sin(2x)-2x\cos(2x)+\frac{7}{4}\cos(2x)$$

Antiderivative was successfully verified.

```
[In] Int[(-3 + 4*x + x^2)*Sin[2*x],x]
```

```
[Out] (7*Cos[2*x])/4 - 2*x*Cos[2*x] - (x^2*Cos[2*x])/2 + Sin[2*x] + (x*Sin[2*x])/
2
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int \left(-3+4x+x^2\right)\sin(2x)\,dx = \int \left(-3\sin(2x)+4x\sin(2x)+x^2\sin(2x)\right)\,dx$$
$$= -(3\int \sin(2x)\,dx) + 4\int x\sin(2x)\,dx + \int x^2\sin(2x)\,dx$$
$$= \frac{3}{2}\cos(2x) - 2x\cos(2x) - \frac{1}{2}x^2\cos(2x) + 2\int \cos(2x)\,dx + \int x\cos(2x)\,dx$$
$$= \frac{3}{2}\cos(2x) - 2x\cos(2x) - \frac{1}{2}x^2\cos(2x) + \sin(2x) + \frac{1}{2}x\sin(2x) - \frac{1}{2}\int \sin(2x)\,dx$$
$$= \frac{7}{4}\cos(2x) - 2x\cos(2x) - \frac{1}{2}x^2\cos(2x) + \sin(2x) + \frac{1}{2}x\sin(2x)$$

Mathematica [**A**] time = 0.04, size = 29, normalized size = 0.72

$$\frac{1}{4}\left(\left(-2x^2-8x+7\right)\cos(2x)+2(x+2)\sin(2x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-3 + 4*x + x^2)*Sin[2*x],x]
[Out] ((7 - 8*x - 2*x^2)*Cos[2*x] + 2*(2 + x)*Sin[2*x])/4
```

fricas [**A**] time = 0.55, size = 26, normalized size = 0.65

$$-\frac{1}{4}\left(2\,x^2+8\,x-7\right)\cos\left(2\,x\right)+\frac{1}{2}\left(x+2\right)\sin\left(2\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+4*x-3)*sin(2*x),x, algorithm="fricas")
[Out] -1/4*(2*x^2 + 8*x - 7)*cos(2*x) + 1/2*(x + 2)*sin(2*x)
```

giac [**A**] time = 0.13, size = 26, normalized size = 0.65

$$-\frac{1}{4}\left(2\,x^2+8\,x-7\right)\cos\left(2\,x\right)+\frac{1}{2}\left(x+2\right)\sin\left(2\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+4*x-3)*sin(2*x),x, algorithm="giac")
[Out] -1/4*(2*x^2 + 8*x - 7)*cos(2*x) + 1/2*(x + 2)*sin(2*x)
```

maple [A] time = 0.04, size = 35, normalized size = 0.88

$$\frac{7\cos(2x)}{4} - 2x\cos(2x) - \frac{x^2\cos(2x)}{2} + \sin(2x) + \frac{x\sin(2x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+4*x-3)*sin(2*x),x)
```

```
[Out] 7/4*cos(2*x)-2*x*cos(2*x)-1/2*x^2*cos(2*x)+sin(2*x)+1/2*x*sin(2*x)
```

maxima [A] time = 0.33, size = 38, normalized size = 0.95

$$-\frac{1}{4}\left(2x^2-1\right)\cos(2x) - 2x\cos(2x) + \frac{1}{2}x\sin(2x) + \frac{3}{2}\cos(2x) + \sin(2x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+4*x-3)*sin(2*x),x, algorithm="maxima")
```

```
[Out] -1/4*(2*x^2 - 1)*cos(2*x) - 2*x*cos(2*x) + 1/2*x*sin(2*x) + 3/2*cos(2*x) +
sin(2*x)
```

mupad [B] time = 2.90, size = 34, normalized size = 0.85

$$\frac{7\cos(2x)}{4} + \sin(2x) - 2x\cos(2x) + \frac{x\sin(2x)}{2} - \frac{x^2\cos(2x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(2*x)*(4*x + x^2 - 3),x)
```

```
[Out] (7*cos(2*x))/4 + sin(2*x) - 2*x*cos(2*x) + (x*sin(2*x))/2 - (x^2*cos(2*x))/
2
```

sympy [A] time = 0.29, size = 39, normalized size = 0.98

$$-\frac{x^2\cos(2x)}{2} + \frac{x\sin(2x)}{2} - 2x\cos(2x) + \sin(2x) + \frac{7\cos(2x)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+4*x-3)*sin(2*x),x)
```

```
[Out] -x**2*cos(2*x)/2 + x*sin(2*x)/2 - 2*x*cos(2*x) + sin(2*x) + 7*cos(2*x)/4
```

3.812 $\int e^{-3x} \cos(4x)\,dx$

Optimal. Leaf size=27

$$\frac{4}{25}e^{-3x}\sin(4x) - \frac{3}{25}e^{-3x}\cos(4x)$$

```
[Out] -3/25*cos(4*x)/exp(3*x)+4/25*sin(4*x)/exp(3*x)
```

Rubi [A] time = 0.01, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {4433}

$$\frac{4}{25}e^{-3x}\sin(4x) - \frac{3}{25}e^{-3x}\cos(4x)$$

Antiderivative was successfully verified.

```
[In] Int[Cos[4*x]/E^(3*x),x]
```

```
[Out] (-3*Cos[4*x])/(25*E^(3*x)) + (4*Sin[4*x])/(25*E^(3*x))
```

Rule 4433

```
Int[Cos[(d_.) + (e_.)*(x_)]*(F_)^((c_.)*((a_.) + (b_.)*(x_))), x_Symbol] :>
 Simp[(b*c*Log[F]*F^(c*(a + b*x))*Cos[d + e*x])/(e^2 + b^2*c^2*Log[F]^2), x
] + Simp[(e*F^(c*(a + b*x))*Sin[d + e*x])/(e^2 + b^2*c^2*Log[F]^2), x] /; F
reeQ[{F, a, b, c, d, e}, x] && NeQ[e^2 + b^2*c^2*Log[F]^2, 0]
```

Rubi steps

$$\int e^{-3x}\cos(4x)\,dx = -\frac{3}{25}e^{-3x}\cos(4x) + \frac{4}{25}e^{-3x}\sin(4x)$$

Mathematica [A] time = 0.03, size = 22, normalized size = 0.81

$$\frac{1}{25}e^{-3x}(4\sin(4x) - 3\cos(4x))$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[4*x]/E^(3*x),x]
```

```
[Out] (-3*Cos[4*x] + 4*Sin[4*x])/(25*E^(3*x))
```

fricas [A] time = 0.60, size = 21, normalized size = 0.78

$$-\frac{3}{25}\cos\left(4\,x\right)e^{\left(-3\,x\right)}+\frac{4}{25}\,e^{\left(-3\,x\right)}\sin\left(4\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(4*x)/exp(3*x),x, algorithm="fricas")
[Out] -3/25*cos(4*x)*e^(-3*x) + 4/25*e^(-3*x)*sin(4*x)
```

giac [A] time = 0.14, size = 19, normalized size = 0.70

$$-\frac{1}{25}\left(3\,\cos\left(4\,x\right)-4\,\sin\left(4\,x\right)\right)e^{\left(-3\,x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(4*x)/exp(3*x),x, algorithm="giac")
[Out] -1/25*(3*cos(4*x) - 4*sin(4*x))*e^(-3*x)
```

maple [A] time = 0.03, size = 22, normalized size = 0.81

$$-\frac{3\,\mathrm{e}^{-3x}\cos\left(4x\right)}{25}+\frac{4\,\mathrm{e}^{-3x}\sin\left(4x\right)}{25}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(4*x)/exp(3*x),x)
[Out] -3/25*exp(-3*x)*cos(4*x)+4/25*exp(-3*x)*sin(4*x)
```

maxima [A] time = 0.33, size = 19, normalized size = 0.70

$$-\frac{1}{25}\left(3\,\cos\left(4\,x\right)-4\,\sin\left(4\,x\right)\right)e^{\left(-3\,x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(4*x)/exp(3*x),x, algorithm="maxima")
[Out] -1/25*(3*cos(4*x) - 4*sin(4*x))*e^(-3*x)
```

mupad [B] time = 0.03, size = 19, normalized size = 0.70

$$-\frac{\mathrm{e}^{-3\,x}\,\left(3\,\cos\left(4\,x\right)-4\,\sin\left(4\,x\right)\right)}{25}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(4*x)*exp(-3*x),x)
```

```
[Out] -(exp(-3*x)*(3*cos(4*x) - 4*sin(4*x)))/25
```

sympy [A] time = 0.40, size = 26, normalized size = 0.96

$$\frac{4e^{-3x}\sin(4x)}{25} - \frac{3e^{-3x}\cos(4x)}{25}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(4*x)/exp(3*x),x)
```

```
[Out] 4*exp(-3*x)*sin(4*x)/25 - 3*exp(-3*x)*cos(4*x)/25
```

3.813 $\int \frac{\cos(x)\sin(x)}{\sqrt{1+\sin(x)}}\,dx$

Optimal. Leaf size=23

$$\frac{2}{3}(\sin(x)+1)^{3/2}-2\sqrt{\sin(x)+1}$$

```
[Out] 2/3*(1+sin(x))^(3/2)-2*(1+sin(x))^(1/2)
```

Rubi [A] time = 0.04, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {2833, 43}

$$\frac{2}{3}(\sin(x)+1)^{3/2}-2\sqrt{\sin(x)+1}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[x]*Sin[x])/Sqrt[1 + Sin[x]],x]
```

```
[Out] -2*Sqrt[1 + Sin[x]] + (2*(1 + Sin[x])^(3/2))/3
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2833

```
Int[cos[(e_.) + (f_.)*(x_)]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((
c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b*f), Sub
st[Int[(a + x)^m*(c + (d*x)/b)^n, x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b
, c, d, e, f, m, n}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\cos(x)\sin(x)}{\sqrt{1+\sin(x)}}\,dx &= \text{Subst}\left(\int \frac{x}{\sqrt{1+x}}\,dx, x, \sin(x)\right)\\ &= \text{Subst}\left(\int \left(-\frac{1}{\sqrt{1+x}}+\sqrt{1+x}\right)dx, x, \sin(x)\right)\\ &= -2\sqrt{1+\sin(x)}+\frac{2}{3}(1+\sin(x))^{3/2}\end{aligned}$$

Mathematica [A] time = 0.02, size = 31, normalized size = 1.35

$$\frac{2(\sin(x)-2)\left(\sin\left(\frac{x}{2}\right)+\cos\left(\frac{x}{2}\right)\right)^2}{3\sqrt{\sin(x)+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[x]*Sin[x])/Sqrt[1 + Sin[x]],x]
[Out] (2*(Cos[x/2] + Sin[x/2])^2*(-2 + Sin[x]))/(3*Sqrt[1 + Sin[x]])
```

fricas [A] time = 0.71, size = 12, normalized size = 0.52

$$\frac{2}{3}\sqrt{\sin(x)+1}\,(\sin(x)-2)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*sin(x)/(1+sin(x))^(1/2),x, algorithm="fricas")
[Out] 2/3*sqrt(sin(x) + 1)*(sin(x) - 2)
```

giac [A] time = 0.13, size = 17, normalized size = 0.74

$$\frac{2}{3}(\sin(x)+1)^{\frac{3}{2}}-2\sqrt{\sin(x)+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*sin(x)/(1+sin(x))^(1/2),x, algorithm="giac")
[Out] 2/3*(sin(x) + 1)^(3/2) - 2*sqrt(sin(x) + 1)
```

maple [A] time = 0.03, size = 18, normalized size = 0.78

$$\frac{2\,(1+\sin(x))^{\frac{3}{2}}}{3}-2\sqrt{1+\sin(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)*sin(x)/(1+sin(x))^(1/2),x)
[Out] 2/3*(1+sin(x))^(3/2)-2*(1+sin(x))^(1/2)
```

maxima [A] time = 0.32, size = 17, normalized size = 0.74

$$\frac{2}{3}(\sin(x)+1)^{\frac{3}{2}}-2\sqrt{\sin(x)+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*sin(x)/(1+sin(x))^(1/2),x, algorithm="maxima")
```

```
[Out] 2/3*(sin(x) + 1)^(3/2) - 2*sqrt(sin(x) + 1)
```

mupad [B] time = 0.10, size = 12, normalized size = 0.52

$$\frac{2\sqrt{\sin(x)+1}\ (\sin(x)-2)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(x)*sin(x))/(sin(x) + 1)^(1/2),x)
```

```
[Out] (2*(sin(x) + 1)^(1/2)*(sin(x) - 2))/3
```

sympy [A] time = 0.29, size = 26, normalized size = 1.13

$$\frac{2\sqrt{\sin{(x)}+1}\ \sin{(x)}}{3} - \frac{4\sqrt{\sin{(x)}+1}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*sin(x)/(1+sin(x))**(1/2),x)
```

```
[Out] 2*sqrt(sin(x) + 1)*sin(x)/3 - 4*sqrt(sin(x) + 1)/3
```

3.814 $\int \left(x+60\cos^5(x)\sin^4(x)\right)\,dx$

Optimal. Leaf size=30

$$\frac{x^2}{2}+\frac{20\sin^9(x)}{3}-\frac{120\sin^7(x)}{7}+12\sin^5(x)$$

```
[Out] 1/2*x^2+12*sin(x)^5-120/7*sin(x)^7+20/3*sin(x)^9
```

Rubi [A] time = 0.03, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {2564, 270}

$$\frac{x^2}{2}+\frac{20\sin^9(x)}{3}-\frac{120\sin^7(x)}{7}+12\sin^5(x)$$

Antiderivative was successfully verified.

```
[In] Int[x + 60*Cos[x]^5*Sin[x]^4,x]
```

```
[Out] x^2/2 + 12*Sin[x]^5 - (120*Sin[x]^7)/7 + (20*Sin[x]^9)/3
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2564

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rubi steps

$$
\begin{aligned}
\int \left(x+60\cos^5(x)\sin^4(x)\right)dx &= \frac{x^2}{2}+60\int \cos^5(x)\sin^4(x)\,dx\\
&= \frac{x^2}{2}+60\,\text{Subst}\left(\int x^4\left(1-x^2\right)^2\,dx,x,\sin(x)\right)\\
&= \frac{x^2}{2}+60\,\text{Subst}\left(\int \left(x^4-2x^6+x^8\right)dx,x,\sin(x)\right)\\
&= \frac{x^2}{2}+12\sin^5(x)-\frac{120\sin^7(x)}{7}+\frac{20\sin^9(x)}{3}
\end{aligned}
$$

Mathematica [A] time = 0.02, size = 46, normalized size = 1.53

$$
\frac{x^2}{2}+\frac{45\sin(x)}{32}-\frac{5}{16}\sin(3x)-\frac{3}{16}\sin(5x)+\frac{15}{448}\sin(7x)+\frac{5}{192}\sin(9x)
$$

Antiderivative was successfully verified.

```
[In] Integrate[x + 60*Cos[x]^5*Sin[x]^4,x]
```

```
[Out] x^2/2 + (45*Sin[x])/32 - (5*Sin[3*x])/16 - (3*Sin[5*x])/16 + (15*Sin[7*x])/
448 + (5*Sin[9*x])/192
```

fricas [A] time = 0.59, size = 36, normalized size = 1.20

$$
\frac{1}{2}x^2+\frac{4}{21}\left(35\cos(x)^8-50\cos(x)^6+3\cos(x)^4+4\cos(x)^2+8\right)\sin(x)
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x+60*cos(x)^5*sin(x)^4,x, algorithm="fricas")
```

```
[Out] 1/2*x^2 + 4/21*(35*cos(x)^8 - 50*cos(x)^6 + 3*cos(x)^4 + 4*cos(x)^2 + 8)*si
n(x)
```

giac [A] time = 0.14, size = 24, normalized size = 0.80

$$
\frac{20}{3}\sin(x)^9-\frac{120}{7}\sin(x)^7+12\sin(x)^5+\frac{1}{2}x^2
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x+60*cos(x)^5*sin(x)^4,x, algorithm="giac")
```

```
[Out] 20/3*sin(x)^9 - 120/7*sin(x)^7 + 12*sin(x)^5 + 1/2*x^2
```

maple [A] time = 0.01, size = 41, normalized size = 1.37

$$\frac{x^2}{2}-\frac{20\left(\cos^6(x)\right)\left(\sin^3(x)\right)}{3}-\frac{20\sin(x)\left(\cos^6(x)\right)}{7}+\frac{4\left(\frac{8}{3}+\cos^4(x)+\frac{4\left(\cos^2(x)\right)}{3}\right)\sin(x)}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x+60*cos(x)^5*sin(x)^4,x)
```

```
[Out] 1/2*x^2-20/3*cos(x)^6*sin(x)^3-20/7*sin(x)*cos(x)^6+4/7*(8/3+cos(x)^4+4/3*c
os(x)^2)*sin(x)
```

maxima [A] time = 0.33, size = 24, normalized size = 0.80

$$\frac{20}{3}\sin(x)^9-\frac{120}{7}\sin(x)^7+12\sin(x)^5+\frac{1}{2}x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x+60*cos(x)^5*sin(x)^4,x, algorithm="maxima")
```

```
[Out] 20/3*sin(x)^9 - 120/7*sin(x)^7 + 12*sin(x)^5 + 1/2*x^2
```

mupad [B] time = 3.00, size = 24, normalized size = 0.80

$$\frac{x^2}{2}+\frac{20\sin(x)^9}{3}-\frac{120\sin(x)^7}{7}+12\sin(x)^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x + 60*cos(x)^5*sin(x)^4,x)
```

```
[Out] 12*sin(x)^5 - (120*sin(x)^7)/7 + (20*sin(x)^9)/3 + x^2/2
```

sympy [A] time = 0.06, size = 27, normalized size = 0.90

$$\frac{x^2}{2}+\frac{20\sin^9(x)}{3}-\frac{120\sin^7(x)}{7}+12\sin^5(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x+60*cos(x)**5*sin(x)**4,x)
```

```
[Out] x**2/2 + 20*sin(x)**9/3 - 120*sin(x)**7/7 + 12*sin(x)**5
```

3.815 $\int \cos(x)(\sec(x)+\tan(x))\,dx$

Optimal. Leaf size=6

$$x-\cos(x)$$

```
[Out] x-cos(x)
```

Rubi [A] time = 0.01, antiderivative size = 6, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {3161, 2638}

$$x-\cos(x)$$

Antiderivative was successfully verified.

```
[In] Int[Cos[x]*(Sec[x] + Tan[x]),x]
```

```
[Out] x - Cos[x]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3161

```
Int[cos[(d_.) + (e_.)*(x_)]^(n_.)*((a_.) + (b_.)*sec[(d_.) + (e_.)*(x_)] +
(c_.)*tan[(d_.) + (e_.)*(x_)])^(n_.), x_Symbol] :> Int[(b + a*Cos[d + e*x]
+ c*Sin[d + e*x])^n, x] /; FreeQ[{a, b, c, d, e}, x] && IntegerQ[n]
```

Rubi steps

$$\begin{aligned}\int \cos(x)(\sec(x)+\tan(x))\,dx &= \int (1+\sin(x))\,dx\\ &= x+\int \sin(x)\,dx\\ &= x-\cos(x)\end{aligned}$$

Mathematica [A] time = 0.00, size = 6, normalized size = 1.00

$$x-\cos(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[x]*(Sec[x] + Tan[x]),x]
```

```
[Out] x - Cos[x]
```

fricas [A] time = 0.53, size = 6, normalized size = 1.00

$$x - \cos(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*(sec(x)+tan(x)),x, algorithm="fricas")
[Out] x - cos(x)
```

giac [B] time = 0.13, size = 14, normalized size = 2.33

$$x - \frac{2}{\tan\left(\frac{1}{2}\,x\right)^2 + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*(sec(x)+tan(x)),x, algorithm="giac")
[Out] x - 2/(tan(1/2*x)^2 + 1)
```

maple [A] time = 0.08, size = 7, normalized size = 1.17

$$x - \cos(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)*(sec(x)+tan(x)),x)
[Out] x-cos(x)
```

maxima [A] time = 0.33, size = 6, normalized size = 1.00

$$x - \cos(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*(sec(x)+tan(x)),x, algorithm="maxima")
[Out] x - cos(x)
```

mupad [B] time = 2.95, size = 6, normalized size = 1.00

$$x - \cos(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)*(tan(x) + 1/cos(x)),x)
```

```
[Out] x - cos(x)
```

sympy [A] time = 1.13, size = 3, normalized size = 0.50

$$x - \cos(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*(sec(x)+tan(x)),x)
```

```
[Out] x - cos(x)
```

3.816 $\int \cos(x)\left(\sec^3(x)+\tan(x)\right)\,dx$

Optimal. Leaf size=7

$$\tan(x)-\cos(x)$$

```
[Out] -cos(x)+tan(x)
```

Rubi [A] time = 0.04, antiderivative size = 7, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {4401, 3767, 8, 2638}

$$\tan(x)-\cos(x)$$

Antiderivative was successfully verified.

```
[In] Int[Cos[x]*(Sec[x]^3 + Tan[x]),x]
```

```
[Out] -Cos[x] + Tan[x]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 4401

```
Int[u_, x_Symbol] :> With[{v = ExpandTrig[u, x]}, Int[v, x] /; SumQ[v]] /;
 !InertTrigFreeQ[u]
```

Rubi steps

$$\int \cos(x)\left(\sec^3(x)+\tan(x)\right)\,dx = \int \left(\sec^2(x)+\sin(x)\right)\,dx$$
$$= \int \sec^2(x)\,dx + \int \sin(x)\,dx$$
$$= -\cos(x) - \text{Subst}\left(\int 1\,dx, x, -\tan(x)\right)$$
$$= -\cos(x) + \tan(x)$$

Mathematica [A] time = 0.00, size = 7, normalized size = 1.00

$$\tan(x) - \cos(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[x]*(Sec[x]^3 + Tan[x]),x]
[Out] -Cos[x] + Tan[x]
```

fricas [B] time = 0.74, size = 15, normalized size = 2.14

$$-\frac{\cos(x)^2 - \sin(x)}{\cos(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*(sec(x)^3+tan(x)),x, algorithm="fricas")
[Out] -(cos(x)^2 - sin(x))/cos(x)
```

giac [B] time = 0.14, size = 30, normalized size = 4.29

$$-\frac{2\left(\tan\left(\frac{1}{2}\,x\right)^3 + \tan\left(\frac{1}{2}\,x\right)^2 + \tan\left(\frac{1}{2}\,x\right) - 1\right)}{\tan\left(\frac{1}{2}\,x\right)^4 - 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*(sec(x)^3+tan(x)),x, algorithm="giac")
[Out] -2*(tan(1/2*x)^3 + tan(1/2*x)^2 + tan(1/2*x) - 1)/(tan(1/2*x)^4 - 1)
```

maple [A] time = 0.10, size = 8, normalized size = 1.14

$$-\cos(x) + \tan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)*(sec(x)^3+tan(x)),x)
```

```
[Out] -cos(x)+tan(x)
```

maxima [A] time = 0.33, size = 7, normalized size = 1.00

$$-\cos(x) + \tan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*(sec(x)^3+tan(x)),x, algorithm="maxima")
```

```
[Out] -cos(x) + tan(x)
```

mupad [B] time = 2.97, size = 12, normalized size = 1.71

$$\frac{\sin(x)}{\cos(x)} - \cos(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)*(tan(x) + 1/cos(x)^3),x)
```

```
[Out] sin(x)/cos(x) - cos(x)
```

sympy [A] time = 4.81, size = 8, normalized size = 1.14

$$\frac{\sin{(x)}}{\cos{(x)}} - \cos{(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*(sec(x)**3+tan(x)),x)
```

```
[Out] sin(x)/cos(x) - cos(x)
```

3.817 $\int \frac{1}{2}\left(-\cot(x)\csc(x)+\csc^2(x)\right)\,dx$

Optimal. Leaf size=13

$$\frac{\csc(x)}{2}-\frac{\cot(x)}{2}$$

```
[Out] -1/2*cot(x)+1/2*csc(x)
```

Rubi [A] time = 0.01, antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {12, 2606, 8, 3767}

$$\frac{\csc(x)}{2}-\frac{\cot(x)}{2}$$

Antiderivative was successfully verified.

```
[In] Int[(-(Cot[x]*Csc[x]) + Csc[x]^2)/2,x]
```

```
[Out] -Cot[x]/2 + Csc[x]/2
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{2}\left(-\cot(x)\csc(x)+\csc^2(x)\right)\,dx &= \frac{1}{2}\int\left(-\cot(x)\csc(x)+\csc^2(x)\right)\,dx\\ &= -\left(\frac{1}{2}\int\cot(x)\csc(x)\,dx\right)+\frac{1}{2}\int\csc^2(x)\,dx\\ &= -\left(\frac{1}{2}\text{Subst}(\int 1\,dx,x,\cot(x))\right)+\frac{1}{2}\text{Subst}(\int 1\,dx,x,\csc(x))\\ &= -\frac{\cot(x)}{2}+\frac{\csc(x)}{2}\end{aligned}$$

Mathematica [A] time = 0.00, size = 10, normalized size = 0.77

$$\frac{1}{2}\tan\left(\frac{x}{2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-(Cot[x]*Csc[x]) + Csc[x]^2)/2,x]
[Out] Tan[x/2]/2
```

fricas [A] time = 1.08, size = 10, normalized size = 0.77

$$\frac{\sin(x)}{2\,(\cos(x)+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-1/2*cot(x)*csc(x)+1/2*csc(x)^2,x, algorithm="fricas")
[Out] 1/2*sin(x)/(cos(x) + 1)
```

giac [A] time = 0.15, size = 13, normalized size = 1.00

$$\frac{1}{2\,\sin(x)}-\frac{1}{2\,\tan(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-1/2*cot(x)*csc(x)+1/2*csc(x)^2,x, algorithm="giac")
[Out] 1/2/sin(x) - 1/2/tan(x)
```

maple [A] time = 0.04, size = 10, normalized size = 0.77

$$-\frac{\cot(x)}{2}+\frac{\csc(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-1/2*cot(x)*csc(x)+1/2*csc(x)^2,x)
```

```
[Out] -1/2*cot(x)+1/2*csc(x)
```

maxima [A] time = 0.32, size = 13, normalized size = 1.00

$$\frac{1}{2\sin(x)} - \frac{1}{2\tan(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-1/2*cot(x)*csc(x)+1/2*csc(x)^2,x, algorithm="maxima")
```

```
[Out] 1/2/sin(x) - 1/2/tan(x)
```

mupad [B] time = 2.93, size = 6, normalized size = 0.46

$$\frac{\tan\left(\frac{x}{2}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2*sin(x)^2) - cot(x)/(2*sin(x)),x)
```

```
[Out] tan(x/2)/2
```

sympy [A] time = 0.06, size = 14, normalized size = 1.08

$$-\frac{\cos(x)}{2\sin(x)} + \frac{1}{2\sin(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-1/2*cot(x)*csc(x)+1/2*csc(x)**2,x)
```

```
[Out] -cos(x)/(2*sin(x)) + 1/(2*sin(x))
```

3.818 $\int \left(-\csc^2(x)+\sin(2x)\right)\,dx$

Optimal. Leaf size=11

$$\cot(x)-\frac{1}{2}\cos(2x)$$

```
[Out] -1/2*cos(2*x)+cot(x)
```

Rubi [A] time = 0.01, antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {3767, 8, 2638}

$$\cot(x)-\frac{1}{2}\cos(2x)$$

Antiderivative was successfully verified.

```
[In] Int[-Csc[x]^2 + Sin[2*x],x]
```

```
[Out] -Cos[2*x]/2 + Cot[x]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\begin{aligned}\int \left(-\csc^2(x)+\sin(2x)\right)\,dx &= -\int \csc^2(x)\,dx+\int \sin(2x)\,dx\\ &= -\frac{1}{2}\cos(2x)+\text{Subst}\left(\int 1\,dx,x,\cot(x)\right)\\ &= -\frac{1}{2}\cos(2x)+\cot(x)\end{aligned}$$

Mathematica [A] time = 0.01, size = 11, normalized size = 1.00

$$\cot(x)-\frac{1}{2}\cos(2x)$$

Antiderivative was successfully verified.

```
[In] Integrate[-Csc[x]^2 + Sin[2*x],x]
[Out] -1/2*Cos[2*x] + Cot[x]
```

fricas [B] time = 0.59, size = 22, normalized size = 2.00

$$-\frac{\left(2\cos(x)^2-1\right)\sin(x)-2\cos(x)}{2\sin(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-csc(x)^2+sin(2*x),x, algorithm="fricas")
[Out] -1/2*((2*cos(x)^2 - 1)*sin(x) - 2*cos(x))/sin(x)
```

giac [A] time = 0.13, size = 11, normalized size = 1.00

$$\frac{1}{\tan(x)}-\frac{1}{2}\cos(2x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-csc(x)^2+sin(2*x),x, algorithm="giac")
[Out] 1/tan(x) - 1/2*cos(2*x)
```

maple [A] time = 0.03, size = 10, normalized size = 0.91

$$-\frac{\cos(2x)}{2}+\cot(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-csc(x)^2+sin(2*x),x)
[Out] -1/2*cos(2*x)+cot(x)
```

maxima [A] time = 0.33, size = 11, normalized size = 1.00

$$\frac{1}{\tan(x)}-\frac{1}{2}\cos(2x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-csc(x)^2+sin(2*x),x, algorithm="maxima")
```

```
[Out] 1/tan(x) - 1/2*cos(2*x)
```

mupad [B] time = 2.92, size = 14, normalized size = 1.27

$$\frac{\cos(x)}{\sin(x)} - \cos(x)^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(2*x) - 1/sin(x)^2,x)
```

```
[Out] cos(x)/sin(x) - cos(x)^2
```

sympy [A] time = 0.06, size = 12, normalized size = 1.09

$$-\frac{\cos(2x)}{2} + \frac{\cos(x)}{\sin(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-csc(x)**2+sin(2*x),x)
```

```
[Out] -cos(2*x)/2 + cos(x)/sin(x)
```

3.819 $\int (2\cot(2x) - 3\sin(3x))\,dx$

Optimal. Leaf size=10

$$\cos(3x) + \log(\sin(2x))$$

[Out] cos(3*x)+ln(sin(2*x))

Rubi [A] time = 0.01, antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {3475, 2638}

$$\cos(3x) + \log(\sin(2x))$$

Antiderivative was successfully verified.

[In] Int[2*Cot[2*x] - 3*Sin[3*x],x]

[Out] Cos[3*x] + Log[Sin[2*x]]

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int (2\cot(2x) - 3\sin(3x))\,dx &= 2\int \cot(2x)\,dx - 3\int \sin(3x)\,dx\\ &= \cos(3x) + \log(\sin(2x))\end{aligned}$$

Mathematica [A] time = 0.01, size = 10, normalized size = 1.00

$$\cos(3x) + \log(\sin(2x))$$

Antiderivative was successfully verified.

[In] Integrate[2*Cot[2*x] - 3*Sin[3*x],x]

[Out] Cos[3*x] + Log[Sin[2*x]]

fricas [A] time = 0.91, size = 18, normalized size = 1.80

$$4 \cos(x)^3 - 3 \cos(x) + \log\left(-\frac{1}{2} \cos(x) \sin(x)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(2*cot(2*x)-3*sin(3*x),x, algorithm="fricas")
[Out] 4*cos(x)^3 - 3*cos(x) + log(-1/2*cos(x)*sin(x))
```

giac [A] time = 0.15, size = 11, normalized size = 1.10

$$\cos(3x) + \log(|\sin(2x)|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(2*cot(2*x)-3*sin(3*x),x, algorithm="giac")
[Out] cos(3*x) + log(abs(sin(2*x)))
```

maple [A] time = 0.03, size = 17, normalized size = 1.70

$$-\frac{\ln\left(\cot^2(2x) + 1\right)}{2} + \cos(3x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(2*cot(2*x)-3*sin(3*x),x)
[Out] -1/2*ln(cot(2*x)^2+1)+cos(3*x)
```

maxima [A] time = 0.32, size = 10, normalized size = 1.00

$$\cos(3x) + \log(\sin(2x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(2*cot(2*x)-3*sin(3*x),x, algorithm="maxima")
[Out] cos(3*x) + log(sin(2*x))
```

mupad [B] time = 3.08, size = 24, normalized size = 2.40

$$\cos(3x) + \ln\left(\cos\left(\frac{x}{2}\right)\left(\sin\left(\frac{x}{2}\right) - 2\sin\left(\frac{x}{2}\right)^3\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(2*cot(2*x) - 3*sin(3*x),x)
```

```
[Out] cos(3*x) + log(cos(x/2)*(sin(x/2) - 2*sin(x/2)^3))
```

sympy [A] time = 0.06, size = 10, normalized size = 1.00

$$\log(\sin(2x)) + \cos(3x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(2*cot(2*x)-3*sin(3*x),x)
```

```
[Out] log(sin(2*x)) + cos(3*x)
```

3.820 $\int x \sin(2x^2)\, dx$

Optimal. Leaf size=10

$$-\frac{1}{4}\cos\left(2x^2\right)$$

[Out] -1/4*cos(2*x^2)

Rubi [A] time = 0.01, antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {3379, 2638}

$$-\frac{1}{4}\cos\left(2x^2\right)$$

Antiderivative was successfully verified.

[In] Int[x*Sin[2*x^2],x]

[Out] -Cos[2*x^2]/4

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3379

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rubi steps

$$\begin{aligned}\int x \sin\left(2x^2\right)\, dx &= \frac{1}{2}\text{Subst}\left(\int \sin(2x)\, dx, x, x^2\right)\\ &= -\frac{1}{4}\cos\left(2x^2\right)\end{aligned}$$

Mathematica [A] time = 0.01, size = 10, normalized size = 1.00

$$-\frac{1}{4}\cos\left(2x^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Sin[2*x^2],x]
```

```
[Out] -1/4*Cos[2*x^2]
```

fricas [A] time = 0.70, size = 8, normalized size = 0.80

$$-\frac{1}{4}\cos\left(2\,x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(2*x^2),x, algorithm="fricas")
```

```
[Out] -1/4*cos(2*x^2)
```

giac [A] time = 0.14, size = 8, normalized size = 0.80

$$-\frac{1}{4}\cos\left(2\,x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(2*x^2),x, algorithm="giac")
```

```
[Out] -1/4*cos(2*x^2)
```

maple [A] time = 0.00, size = 9, normalized size = 0.90

$$-\frac{\cos\left(2x^2\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*sin(2*x^2),x)
```

```
[Out] -1/4*cos(2*x^2)
```

maxima [A] time = 0.32, size = 8, normalized size = 0.80

$$-\frac{1}{4}\cos\left(2\,x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(2*x^2),x, algorithm="maxima")
```

```
[Out] -1/4*cos(2*x^2)
```

mupad [B] time = 0.05, size = 8, normalized size = 0.80

$$\frac{\sin\left(x^2\right)^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*sin(2*x^2),x)
```

```
[Out] sin(x^2)^2/2
```

sympy [A] time = 0.15, size = 8, normalized size = 0.80

$$-\frac{\cos\left(2x^2\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(2*x**2),x)
```

```
[Out] -cos(2*x**2)/4
```

3.821 $\int -\cos(1-x)\sin(1-x)\sqrt{1+\sin^2(1-x)}\,dx$

Optimal. Leaf size=18

$$\frac{1}{3}\left(\sin^2(1-x)+1\right)^{3/2}$$

```
[Out] 1/3*(1+sin(x-1)^2)^(3/2)
```

Rubi [A] time = 0.04, antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {3198, 261}

$$\frac{1}{3}\left(\sin^2(1-x)+1\right)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Int[-(Cos[1 - x]*Sin[1 - x]*Sqrt[1 + Sin[1 - x]^2]),x]
```

```
[Out] (1 + Sin[1 - x]^2)^(3/2)/3
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 3198

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a
_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.), x_Symbol] :> With[{ff = FreeFa
ctors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(d*ff*x)^n*(1 - ff^2*x^2)^((m
 - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, d
, e, f, n, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\begin{aligned}\int -\cos(1-x)\sin(1-x)\sqrt{1+\sin^2(1-x)}\,dx &= \text{Subst}\left(\int x\sqrt{1+x^2}\,dx, x, \sin(1-x)\right)\\ &= \frac{1}{3}\left(1+\sin^2(1-x)\right)^{3/2}\end{aligned}$$

Mathematica [A] time = 0.03, size = 18, normalized size = 1.00

$$\frac{1}{3}\left(\sin^2(1-x)+1\right)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Integrate[-(Cos[1 - x]*Sin[1 - x]*Sqrt[1 + Sin[1 - x]^2]),x]
[Out] (1 + Sin[1 - x]^2)^(3/2)/3
```

fricas [A] time = 0.63, size = 14, normalized size = 0.78

$$\frac{1}{3}\left(-\cos\left(x-1\right)^2+2\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(-1+x)*sin(-1+x)*(1+sin(-1+x)^2)^(1/2),x, algorithm="fricas")
[Out] 1/3*(-cos(x - 1)^2 + 2)^(3/2)
```

giac [A] time = 0.13, size = 12, normalized size = 0.67

$$\frac{1}{3}\left(\sin\left(x-1\right)^2+1\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(-1+x)*sin(-1+x)*(1+sin(-1+x)^2)^(1/2),x, algorithm="giac")
[Out] 1/3*(sin(x - 1)^2 + 1)^(3/2)
```

maple [A] time = 0.07, size = 13, normalized size = 0.72

$$\frac{\left(1+\sin^2\left(-1+x\right)\right)^{\frac{3}{2}}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(-1+x)*sin(-1+x)*(1+sin(-1+x)^2)^(1/2),x)
[Out] 1/3*(1+sin(-1+x)^2)^(3/2)
```

maxima [A] time = 0.33, size = 12, normalized size = 0.67

$$\frac{1}{3}\left(\sin\left(x-1\right)^2+1\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(-1+x)*sin(-1+x)*(1+sin(-1+x)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/3*(sin(x - 1)^2 + 1)^(3/2)
```

mupad [B] time = 2.99, size = 12, normalized size = 0.67

$$\frac{\left(\sin\left(x-1\right)^2+1\right)^{3/2}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x - 1)*sin(x - 1)*(sin(x - 1)^2 + 1)^(1/2),x)
```

```
[Out] (sin(x - 1)^2 + 1)^(3/2)/3
```

sympy [B] time = 0.75, size = 32, normalized size = 1.78

$$\frac{\sqrt{\sin^2\left(x-1\right)+1}\,\sin^2\left(x-1\right)}{3}+\frac{\sqrt{\sin^2\left(x-1\right)+1}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(-1+x)*sin(-1+x)*(1+sin(-1+x)**2)**(1/2),x)
```

```
[Out] sqrt(sin(x - 1)**2 + 1)*sin(x - 1)**2/3 + sqrt(sin(x - 1)**2 + 1)/3
```

3.822 $\int \frac{\cos\left(\frac{1}{x}\right)\sin\left(\frac{1}{x}\right)}{x^2}\,dx$

Optimal. Leaf size=10

$$-\frac{1}{2}\sin^2\left(\frac{1}{x}\right)$$

```
[Out] -1/2*sin(1/x)^2
```

Rubi [A] time = 0.01, antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {3441}

$$-\frac{1}{2}\sin^2\left(\frac{1}{x}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[x^(-1)]*Sin[x^(-1)])/x^2,x]
```

```
[Out] -Sin[x^(-1)]^2/2
```

Rule 3441

```
Int[Cos[(a_.) + (b_.)*(x_)^(n_.)]*(x_)^(m_.)*Sin[(a_.) + (b_.)*(x_)^(n_.)]^
(p_.), x_Symbol] :> Simp[Sin[a + b*x^n]^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[
{a, b, m, n, p}, x] && EqQ[m, n - 1] && NeQ[p, -1]
```

Rubi steps

$$\int \frac{\cos\left(\frac{1}{x}\right)\sin\left(\frac{1}{x}\right)}{x^2}\,dx = -\frac{1}{2}\sin^2\left(\frac{1}{x}\right)$$

Mathematica [A] time = 0.01, size = 10, normalized size = 1.00

$$\frac{1}{2}\cos^2\left(\frac{1}{x}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[x^(-1)]*Sin[x^(-1)])/x^2,x]
```

```
[Out] Cos[x^(-1)]^2/2
```

fricas [**A**] time = 0.60, size = 8, normalized size = 0.80

$$\frac{1}{2}\,\cos\left(\frac{1}{x}\right)^{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(1/x)*sin(1/x)/x^2,x, algorithm="fricas")
[Out] 1/2*cos(1/x)^2
```

giac [**A**] time = 0.13, size = 8, normalized size = 0.80

$$\frac{1}{2}\,\cos\left(\frac{1}{x}\right)^{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(1/x)*sin(1/x)/x^2,x, algorithm="giac")
[Out] 1/2*cos(1/x)^2
```

maple [**A**] time = 0.00, size = 9, normalized size = 0.90

$$\frac{\left(\cos^{2}\left(\frac{1}{x}\right)\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(1/x)*sin(1/x)/x^2,x)
[Out] 1/2*cos(1/x)^2
```

maxima [**A**] time = 0.33, size = 8, normalized size = 0.80

$$\frac{1}{2}\,\cos\left(\frac{1}{x}\right)^{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(1/x)*sin(1/x)/x^2,x, algorithm="maxima")
[Out] 1/2*cos(1/x)^2
```

mupad [**B**] time = 2.92, size = 8, normalized size = 0.80

$$\frac{\cos\left(\frac{1}{x}\right)^{2}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(1/x)*sin(1/x))/x^2,x)
```

```
[Out] cos(1/x)^2/2
```

sympy [**B**] time = 1.28, size = 31, normalized size = 3.10

$$-\frac{2\tan^{2}\left(\frac{1}{2x}\right)}{\tan^{4}\left(\frac{1}{2x}\right)+2\tan^{2}\left(\frac{1}{2x}\right)+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(1/x)*sin(1/x)/x**2,x)
```

```
[Out] -2*tan(1/(2*x))**2/(tan(1/(2*x))**4 + 2*tan(1/(2*x))**2 + 1)
```

3.823 $\int \cos\left(\frac{1}{2}(1+3x)\right)\sin^3\left(\frac{1}{2}(1+3x)\right)\,dx$

Optimal. Leaf size=16

$$\frac{1}{6}\sin^4\left(\frac{3x}{2}+\frac{1}{2}\right)$$

[Out] 1/6*sin(1/2+3/2*x)^4

Rubi [A] time = 0.02, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {2564, 30}

$$\frac{1}{6}\sin^4\left(\frac{3x}{2}+\frac{1}{2}\right)$$

Antiderivative was successfully verified.

[In] Int[Cos[(1 + 3*x)/2]*Sin[(1 + 3*x)/2]^3,x]

[Out] Sin[1/2 + (3*x)/2]^4/6

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2564

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rubi steps

$$\begin{aligned}\int \cos\left(\frac{1}{2}(1+3x)\right)\sin^3\left(\frac{1}{2}(1+3x)\right)\,dx &= \frac{2}{3}\operatorname{Subst}\left(\int x^3\,dx, x, \sin\left(\frac{1}{2}+\frac{3x}{2}\right)\right)\\ &= \frac{1}{6}\sin^4\left(\frac{1}{2}+\frac{3x}{2}\right)\end{aligned}$$

Mathematica [A] time = 0.02, size = 25, normalized size = 1.56

$$\frac{1}{2}\left(\frac{1}{24}\cos(6x+2)-\frac{1}{6}\cos(3x+1)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[(1 + 3*x)/2]*Sin[(1 + 3*x)/2]^3,x]
```

```
[Out] (-1/6*Cos[1 + 3*x] + Cos[2 + 6*x]/24)/2
```

fricas [B] time = 0.75, size = 21, normalized size = 1.31

$$\frac{1}{6}\cos\left(\frac{3}{2}x+\frac{1}{2}\right)^{4}-\frac{1}{3}\cos\left(\frac{3}{2}x+\frac{1}{2}\right)^{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(1/2+3/2*x)*sin(1/2+3/2*x)^3,x, algorithm="fricas")
```

```
[Out] 1/6*cos(3/2*x + 1/2)^4 - 1/3*cos(3/2*x + 1/2)^2
```

giac [A] time = 0.20, size = 10, normalized size = 0.62

$$\frac{1}{6}\sin\left(\frac{3}{2}x+\frac{1}{2}\right)^{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(1/2+3/2*x)*sin(1/2+3/2*x)^3,x, algorithm="giac")
```

```
[Out] 1/6*sin(3/2*x + 1/2)^4
```

maple [A] time = 0.04, size = 11, normalized size = 0.69

$$\frac{\left(\sin^{4}\left(\frac{1}{2}+\frac{3x}{2}\right)\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(1/2+3/2*x)*sin(1/2+3/2*x)^3,x)
```

```
[Out] 1/6*sin(1/2+3/2*x)^4
```

maxima [A] time = 0.33, size = 10, normalized size = 0.62

$$\frac{1}{6}\sin\left(\frac{3}{2}x+\frac{1}{2}\right)^{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(1/2+3/2*x)*sin(1/2+3/2*x)^3,x, algorithm="maxima")
```

```
[Out] 1/6*sin(3/2*x + 1/2)^4
```

mupad [B] time = 0.08, size = 14, normalized size = 0.88

$$\frac{\left(\frac{\cos(3\,x+1)}{2}-\frac{1}{2}\right)^2}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos((3*x)/2 + 1/2)*sin((3*x)/2 + 1/2)^3,x)
[Out] (cos(3*x + 1)/2 - 1/2)^2/6
```

sympy [A] time = 0.46, size = 12, normalized size = 0.75

$$\frac{\sin^4\left(\frac{3x}{2}+\frac{1}{2}\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(1/2+3/2*x)*sin(1/2+3/2*x)**3,x)
[Out] sin(3*x/2 + 1/2)**4/6
```

3.824 $\int 4x\tan\left(x^2\right)\,dx$

Optimal. Leaf size=7

$$-2\log\left(\cos\left(x^2\right)\right)$$

```
[Out] -2*ln(cos(x^2))
```

Rubi [A] time = 0.01, antiderivative size = 7, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {12, 3747, 3475}

$$-2\log\left(\cos\left(x^2\right)\right)$$

Antiderivative was successfully verified.

```
[In] Int[4*x*Tan[x^2],x]
```

```
[Out] -2*Log[Cos[x^2]]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3747

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Tan[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Tan[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IGtQ[Simplify[(m +
1)/n], 0] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}\int 4x\tan\left(x^2\right)\,dx &= 4\int x\tan\left(x^2\right)\,dx\\ &= 2\,\text{Subst}\left(\int \tan(x)\,dx, x, x^2\right)\\ &= -2\log\left(\cos\left(x^2\right)\right)\end{aligned}$$

Mathematica [A] time = 0.01, size = 7, normalized size = 1.00

$$-2\log\left(\cos\left(x^2\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[4*x*Tan[x^2],x]
[Out] -2*Log[Cos[x^2]]
```

fricas [A] time = 0.61, size = 13, normalized size = 1.86

$$-\log\left(\frac{1}{\tan\left(x^2\right)^2+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(4*x*tan(x^2),x, algorithm="fricas")
[Out] -log(1/(tan(x^2)^2 + 1))
```

giac [A] time = 0.14, size = 9, normalized size = 1.29

$$\log\left(\tan\left(x^2\right)^2+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(4*x*tan(x^2),x, algorithm="giac")
[Out] log(tan(x^2)^2 + 1)
```

maple [A] time = 0.00, size = 8, normalized size = 1.14

$$-2\ln\left(\cos\left(x^2\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(4*x*tan(x^2),x)
[Out] -2*ln(cos(x^2))
```

maxima [A] time = 0.33, size = 7, normalized size = 1.00

$$2\log\left(\sec\left(x^2\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(4*x*tan(x^2),x, algorithm="maxima")
```

```
[Out] 2*log(sec(x^2))
```

mupad [B] time = 0.07, size = 9, normalized size = 1.29

$$\ln\left(\tan\left(x^2\right)^2+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(4*x*tan(x^2),x)
```

```
[Out] log(tan(x^2)^2 + 1)
```

sympy [A] time = 0.12, size = 8, normalized size = 1.14

$$\log\left(\tan^2\left(x^2\right)+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(4*x*tan(x**2),x)
```

```
[Out] log(tan(x**2)**2 + 1)
```

3.825 $\int x \sec(5-x^2)\,dx$

Optimal. Leaf size=13

$$-\frac{1}{2}\tanh^{-1}\left(\sin\left(5-x^2\right)\right)$$

```
[Out] 1/2*arctanh(sin(x^2-5))
```

Rubi [A] time = 0.01, antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {4204, 3770}

$$-\frac{1}{2}\tanh^{-1}\left(\sin\left(5-x^2\right)\right)$$

Antiderivative was successfully verified.

```
[In] Int[x*Sec[5 - x^2],x]
```

```
[Out] -ArcTanh[Sin[5 - x^2]]/2
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 4204

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sec[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sec[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IGtQ[Simplify[(m +
1)/n], 0] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}\int x\sec\left(5-x^2\right)\,dx &= \frac{1}{2}\text{Subst}\left(\int \sec(5-x)\,dx, x, x^2\right)\\ &= -\frac{1}{2}\tanh^{-1}\left(\sin\left(5-x^2\right)\right)\end{aligned}$$

Mathematica [A] time = 0.02, size = 13, normalized size = 1.00

$$-\frac{1}{2}\tanh^{-1}\left(\sin\left(5-x^2\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Sec[5 - x^2],x]
```

```
[Out] -1/2*ArcTanh[Sin[5 - x^2]]
```

fricas [B] time = 0.69, size = 25, normalized size = 1.92

$$\frac{1}{4}\log\left(\sin\left(x^2-5\right)+1\right)-\frac{1}{4}\log\left(-\sin\left(x^2-5\right)+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sec(x^2-5),x, algorithm="fricas")
```

```
[Out] 1/4*log(sin(x^2 - 5) + 1) - 1/4*log(-sin(x^2 - 5) + 1)
```

giac [B] time = 0.16, size = 41, normalized size = 3.15

$$\frac{1}{8}\log\left(\left|\frac{1}{\sin\left(x^2-5\right)}+\sin\left(x^2-5\right)+2\right|\right)-\frac{1}{8}\log\left(\left|\frac{1}{\sin\left(x^2-5\right)}+\sin\left(x^2-5\right)-2\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sec(x^2-5),x, algorithm="giac")
```

```
[Out] 1/8*log(abs(1/sin(x^2 - 5) + sin(x^2 - 5) + 2)) - 1/8*log(abs(1/sin(x^2 - 5
) + sin(x^2 - 5) - 2))
```

maple [A] time = 0.00, size = 17, normalized size = 1.31

$$\frac{\ln\left(\sec\left(x^2-5\right)+\tan\left(x^2-5\right)\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*sec(x^2-5),x)
```

```
[Out] 1/2*ln(sec(x^2-5)+tan(x^2-5))
```

maxima [A] time = 0.32, size = 16, normalized size = 1.23

$$\frac{1}{2}\log\left(\sec\left(x^2-5\right)+\tan\left(x^2-5\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sec(x^2-5),x, algorithm="maxima")
```

```
[Out] 1/2*log(sec(x^2 - 5) + tan(x^2 - 5))
```

mupad [**B**] time = 3.53, size = 15, normalized size = 1.15

$$-\mathrm{atan}\left(\mathrm{e}^{-5\mathrm{i}}\,\mathrm{e}^{x^2\,1\mathrm{i}}\right)\,1\mathrm{i}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/cos(x^2 - 5),x)
[Out] -atan(exp(-5i)*exp(x^2*1i))*1i
```

sympy [**A**] time = 1.03, size = 15, normalized size = 1.15

$$\frac{\log\left(\tan\left(x^2-5\right)+\sec\left(x^2-5\right)\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sec(x**2-5),x)
[Out] log(tan(x**2 - 5) + sec(x**2 - 5))/2
```

3.826 $\int \frac{\csc\left(\frac{1}{x}\right)}{x^2} \, dx$

Optimal. Leaf size=5

$$\tanh^{-1}\left(\cos\left(\frac{1}{x}\right)\right)$$

```
[Out] arctanh(cos(1/x))
```

Rubi [A] time = 0.01, antiderivative size = 5, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {4205, 3770}

$$\tanh^{-1}\left(\cos\left(\frac{1}{x}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Int[Csc[x^(-1)]/x^2,x]
```

```
[Out] ArcTanh[Cos[x^(-1)]]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 4205

```
Int[((a_.) + Csc[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Csc[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IGtQ[Simplify[(m +
1)/n], 0] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned} \int \frac{\csc\left(\frac{1}{x}\right)}{x^2} \, dx &= -\text{Subst}\left(\int \csc(x) \, dx, x, \frac{1}{x}\right) \\ &= \tanh^{-1}\left(\cos\left(\frac{1}{x}\right)\right) \end{aligned}$$

Mathematica [B] time = 0.01, size = 21, normalized size = 4.20

$$\log\left(\cos\left(\frac{1}{2x}\right)\right)-\log\left(\sin\left(\frac{1}{2x}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[x^(-1)]/x^2,x]
[Out] Log[Cos[1/(2*x)]] - Log[Sin[1/(2*x)]]
```

fricas [B] time = 0.65, size = 23, normalized size = 4.60

$$\frac{1}{2}\log\left(\frac{1}{2}\cos\left(\frac{1}{x}\right)+\frac{1}{2}\right)-\frac{1}{2}\log\left(-\frac{1}{2}\cos\left(\frac{1}{x}\right)+\frac{1}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(1/x)/x^2,x, algorithm="fricas")
[Out] 1/2*log(1/2*cos(1/x) + 1/2) - 1/2*log(-1/2*cos(1/x) + 1/2)
```

giac [B] time = 0.14, size = 43, normalized size = 8.60

$$-\frac{1}{2}\log\left(\frac{4\tan\left(\frac{1}{2x}\right)^{2}}{\tan\left(\frac{1}{2x}\right)^{2}+1}\right)+\frac{1}{2}\log\left(\frac{4}{\tan\left(\frac{1}{2x}\right)^{2}+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(1/x)/x^2,x, algorithm="giac")
[Out] -1/2*log(4*tan(1/2/x)^2/(tan(1/2/x)^2 + 1)) + 1/2*log(4/(tan(1/2/x)^2 + 1))
```

maple [A] time = 0.00, size = 11, normalized size = 2.20

$$\ln\left(\csc\left(\frac{1}{x}\right)+\cot\left(\frac{1}{x}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(1/x)/x^2,x)
[Out] ln(csc(1/x)+cot(1/x))
```

maxima [A] time = 0.33, size = 10, normalized size = 2.00

$$\log\left(\cot\left(\frac{1}{x}\right)+\csc\left(\frac{1}{x}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(1/x)/x^2,x, algorithm="maxima")
```

```
[Out] log(cot(1/x) + csc(1/x))
```

mupad [B] time = 3.68, size = 31, normalized size = 6.20

$$\ln\left(-e^{1i/x}\,2i-2i\right)-\ln\left(-e^{1i/x}\,2i+2i\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*sin(1/x)),x)
```

```
[Out] log(- exp(1i/x)*2i - 2i) - log(2i - exp(1i/x)*2i)
```

sympy [A] time = 1.26, size = 10, normalized size = 2.00

$$\log\left(\cot\left(\frac{1}{x}\right)+\csc\left(\frac{1}{x}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(1/x)/x**2,x)
```

```
[Out] log(cot(1/x) + csc(1/x))
```

3.827 $\int (\csc(x) - \sec(x))(\cos(x) + \sin(x))\,dx$

Optimal. Leaf size=7

$$\log(\sin(x)) + \log(\cos(x))$$

```
[Out] ln(cos(x))+ln(sin(x))
```

Rubi [A] time = 0.05, antiderivative size = 9, normalized size of antiderivative = 1.29, number of steps used = 4, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {446, 72}

$$\log(\tan(x)) + 2\log(\cos(x))$$

Antiderivative was successfully verified.

```
[In] Int[(Csc[x] - Sec[x])*(Cos[x] + Sin[x]),x]
```

```
[Out] 2*Log[Cos[x]] + Log[Tan[x]]
```

Rule 72

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int (\csc(x) - \sec(x))(\cos(x) + \sin(x))\,dx &= \text{Subst}\left(\int \frac{1-x^2}{x\left(1+x^2\right)}\,dx, x, \tan(x)\right)\\
&= \frac{1}{2}\text{Subst}\left(\int \frac{1-x}{x(1+x)}\,dx, x, \tan^2(x)\right)\\
&= \frac{1}{2}\text{Subst}\left(\int \left(\frac{1}{x} - \frac{2}{1+x}\right)dx, x, \tan^2(x)\right)\\
&= 2\log(\cos(x)) + \log(\tan(x))
\end{aligned}$$

Mathematica [A] time = 0.01, size = 7, normalized size = 1.00

$$\log(\sin(x)) + \log(\cos(x))$$

Antiderivative was successfully verified.

```
[In] Integrate[(Csc[x] - Sec[x])*(Cos[x] + Sin[x]),x]
```

```
[Out] Log[Cos[x]] + Log[Sin[x]]
```

fricas [A] time = 0.62, size = 7, normalized size = 1.00

$$\log\left(-\frac{1}{2}\cos(x)\sin(x)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((csc(x)-sec(x))*(cos(x)+sin(x)),x, algorithm="fricas")
```

```
[Out] log(-1/2*cos(x)*sin(x))
```

giac [B] time = 0.15, size = 16, normalized size = 2.29

$$\frac{1}{2}\log\left(-\cos(x)^2 + 1\right) + \log\left(|\cos(x)|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((csc(x)-sec(x))*(cos(x)+sin(x)),x, algorithm="giac")
```

```
[Out] 1/2*log(-cos(x)^2 + 1) + log(abs(cos(x)))
```

maple [A] time = 0.12, size = 8, normalized size = 1.14

$$\ln\left(\cos(x)\right) + \ln\left(\sin(x)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((csc(x)-sec(x))*(cos(x)+sin(x)),x)
```

```
[Out] ln(cos(x))+ln(sin(x))
```

maxima [B] time = 0.33, size = 15, normalized size = 2.14

$$\frac{1}{2}\log\left(-\sin(x)^2 + 1\right) + \log\left(\sin(x)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((csc(x)-sec(x))*(cos(x)+sin(x)),x, algorithm="maxima")
[Out] 1/2*log(-sin(x)^2 + 1) + log(sin(x))
```

mupad [B] time = 3.25, size = 26, normalized size = 3.71

$$\ln\left(\tan\left(\frac{x}{2}\right)^3-\tan\left(\frac{x}{2}\right)\right)-2\,\ln\left(\tan\left(\frac{x}{2}\right)^2+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(cos(x) + sin(x))*(1/cos(x) - 1/sin(x)),x)
[Out] log(tan(x/2)^3 - tan(x/2)) - 2*log(tan(x/2)^2 + 1)
```

sympy [A] time = 2.31, size = 8, normalized size = 1.14

$$\log\left(\sin\left(x\right)\right)+\log\left(\cos\left(x\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((csc(x)-sec(x))*(cos(x)+sin(x)),x)
[Out] log(sin(x)) + log(cos(x))
```

3.828 $\int (-\cos(3x)\sin(2x)+\cos(2x)\sin(3x))\,dx$

Optimal. Leaf size=4

$$-\cos(x)$$

```
[Out] -cos(x)
```

Rubi [A] time = 0.02, antiderivative size = 4, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.050, Rules used = {4284}

$$-\cos(x)$$

Antiderivative was successfully verified.

```
[In] Int[-(Cos[3*x]*Sin[2*x]) + Cos[2*x]*Sin[3*x],x]
```

```
[Out] -Cos[x]
```

Rule 4284

```
Int[cos[(c_.) + (d_.)*(x_)]*sin[(a_.) + (b_.)*(x_)], x_Symbol] :> -Simp[Cos
[a - c + (b - d)*x]/(2*(b - d)), x] - Simp[Cos[a + c + (b + d)*x]/(2*(b + d
)), x] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - d^2, 0]
```

Rubi steps

$$\begin{aligned}\int (-\cos(3x)\sin(2x)+\cos(2x)\sin(3x))\,dx &= -\int \cos(3x)\sin(2x)\,dx+\int \cos(2x)\sin(3x)\,dx\\ &= -\cos(x)\end{aligned}$$

Mathematica [A] time = 0.00, size = 4, normalized size = 1.00

$$-\cos(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[-(Cos[3*x]*Sin[2*x]) + Cos[2*x]*Sin[3*x],x]
```

```
[Out] -Cos[x]
```

fricas [A] time = 0.69, size = 4, normalized size = 1.00

$$-\cos(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-cos(3*x)*sin(2*x)+cos(2*x)*sin(3*x),x, algorithm="fricas")
[Out] -cos(x)
```

giac [A] time = 0.14, size = 4, normalized size = 1.00

$$-\cos(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-cos(3*x)*sin(2*x)+cos(2*x)*sin(3*x),x, algorithm="giac")
[Out] -cos(x)
```

maple [A] time = 0.16, size = 5, normalized size = 1.25

$$-\cos(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-cos(3*x)*sin(2*x)+cos(2*x)*sin(3*x),x)
[Out] -cos(x)
```

maxima [A] time = 0.32, size = 4, normalized size = 1.00

$$-\cos(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-cos(3*x)*sin(2*x)+cos(2*x)*sin(3*x),x, algorithm="maxima")
[Out] -cos(x)
```

mupad [B] time = 3.07, size = 4, normalized size = 1.00

$$-\cos(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(2*x)*sin(3*x) - cos(3*x)*sin(2*x),x)
[Out] -cos(x)
```

sympy [B] time = 0.69, size = 20, normalized size = 5.00

$$-\sin(2x)\sin(3x) - \cos(2x)\cos(3x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-cos(3*x)*sin(2*x)+cos(2*x)*sin(3*x),x)
[Out] -sin(2*x)*sin(3*x) - cos(2*x)*cos(3*x)
```

3.829 $\int 4x\sec^2(2x)\,dx$

Optimal. Leaf size=13

$$2x\tan(2x)+\log(\cos(2x))$$

```
[Out] ln(cos(2*x))+2*x*tan(2*x)
```

Rubi [A] time = 0.02, antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {12, 4184, 3475}

$$2x\tan(2x)+\log(\cos(2x))$$

Antiderivative was successfully verified.

```
[In] Int[4*x*Sec[2*x]^2,x]
```

```
[Out] Log[Cos[2*x]] + 2*x*Tan[2*x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 4184

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> -Sim
p[((c + d*x)^m*Cot[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Co
t[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rubi steps

$$\begin{aligned}\int 4x\sec^2(2x)\,dx &= 4\int x\sec^2(2x)\,dx\\ &= 2x\tan(2x)-2\int\tan(2x)\,dx\\ &= \log(\cos(2x))+2x\tan(2x)\end{aligned}$$

Mathematica [A] time = 0.01, size = 21, normalized size = 1.62

$$4\left(\frac{1}{2}x\tan(2x)+\frac{1}{4}\log(\cos(2x))\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[4*x*Sec[2*x]^2,x]
```

```
[Out] 4*(Log[Cos[2*x]]/4 + (x*Tan[2*x])/2)
```

fricas [B] time = 1.32, size = 27, normalized size = 2.08

$$\frac{\cos(2x)\log(-\cos(2x))+2x\sin(2x)}{\cos(2x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(4*x*sec(2*x)^2,x, algorithm="fricas")
```

```
[Out] (cos(2*x)*log(-cos(2*x)) + 2*x*sin(2*x))/cos(2*x)
```

giac [B] time = 0.17, size = 81, normalized size = 6.23

$$\frac{\log\left(\frac{4\left(\tan(x)^4-2\tan(x)^2+1\right)}{\tan(x)^4+2\tan(x)^2+1}\right)\tan(x)^2-8x\tan(x)-\log\left(\frac{4\left(\tan(x)^4-2\tan(x)^2+1\right)}{\tan(x)^4+2\tan(x)^2+1}\right)}{2\left(\tan(x)^2-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(4*x*sec(2*x)^2,x, algorithm="giac")
```

```
[Out] 1/2*(log(4*(tan(x)^4 - 2*tan(x)^2 + 1)/(tan(x)^4 + 2*tan(x)^2 + 1))*tan(x)^
2 - 8*x*tan(x) - log(4*(tan(x)^4 - 2*tan(x)^2 + 1)/(tan(x)^4 + 2*tan(x)^2 +
 1)))/(tan(x)^2 - 1)
```

maple [A] time = 0.03, size = 14, normalized size = 1.08

$$\ln(\cos(2x))+2x\tan(2x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(4*x*sec(2*x)^2,x)
```

```
[Out] ln(cos(2*x))+2*x*tan(2*x)
```

maxima [B] time = 0.44, size = 74, normalized size = 5.69

$$\frac{\left(\cos\left(4\,x\right)^{2}+\sin\left(4\,x\right)^{2}+2\,\cos\left(4\,x\right)+1\right)\log\left(\cos\left(4\,x\right)^{2}+\sin\left(4\,x\right)^{2}+2\,\cos\left(4\,x\right)+1\right)+8\,x\sin\left(4\,x\right)}{2\left(\cos\left(4\,x\right)^{2}+\sin\left(4\,x\right)^{2}+2\,\cos\left(4\,x\right)+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(4*x*sec(2*x)^2,x, algorithm="maxima")
```

```
[Out] 1/2*((cos(4*x)^2 + sin(4*x)^2 + 2*cos(4*x) + 1)*log(cos(4*x)^2 + sin(4*x)^2
 + 2*cos(4*x) + 1) + 8*x*sin(4*x))/(cos(4*x)^2 + sin(4*x)^2 + 2*cos(4*x) +
1)
```

mupad [B] time = 2.99, size = 13, normalized size = 1.00

$$\ln\left(\cos\left(2\,x\right)\right)+2\,x\tan\left(2\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((4*x)/cos(2*x)^2,x)
```

```
[Out] log(cos(2*x)) + 2*x*tan(2*x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$4\int x\sec^{2}(2x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(4*x*sec(2*x)**2,x)
```

```
[Out] 4*Integral(x*sec(2*x)**2, x)
```

3.830 $\int 4\sin^2(x)\tan^2(x)\,dx$

Optimal. Leaf size=16

$$-6x + 6\tan(x) - 2\sin^2(x)\tan(x)$$

```
[Out] -6*x+6*tan(x)-2*sin(x)^2*tan(x)
```

Rubi [A] time = 0.03, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {12, 2591, 288, 321, 203}

$$-6x + 6\tan(x) - 2\sin^2(x)\tan(x)$$

Antiderivative was successfully verified.

```
[In] Int[4*Sin[x]^2*Tan[x]^2,x]
```

```
[Out] -6*x + 6*Tan[x] - 2*Sin[x]^2*Tan[x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2591

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[(b*ff)/f, Subst[In
t[(ff*x)^(m + n)/(b^2 + ff^2*x^2)^(m/2 + 1), x], x, (b*Tan[e + f*x])/ff], x
]] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2]
```

Rubi steps

$$
\begin{aligned}
\int 4\sin^2(x)\tan^2(x)\,dx &= 4\int \sin^2(x)\tan^2(x)\,dx\\
&= 4\,\mathrm{Subst}\left(\int \frac{x^4}{\left(1+x^2\right)^2}\,dx, x, \tan(x)\right)\\
&= -2\sin^2(x)\tan(x)+6\,\mathrm{Subst}\left(\int \frac{x^2}{1+x^2}\,dx, x, \tan(x)\right)\\
&= 6\tan(x)-2\sin^2(x)\tan(x)-6\,\mathrm{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \tan(x)\right)\\
&= -6x+6\tan(x)-2\sin^2(x)\tan(x)
\end{aligned}
$$

Mathematica [A] time = 0.02, size = 18, normalized size = 1.12

$$4\left(-\frac{3x}{2}+\frac{1}{4}\sin(2x)+\tan(x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[4*Sin[x]^2*Tan[x]^2,x]
[Out] 4*((-3*x)/2 + Sin[2*x]/4 + Tan[x])
```

fricas [A] time = 0.59, size = 22, normalized size = 1.38

$$-\frac{2\left(3\,x\cos(x)-\left(\cos(x)^2+2\right)\sin(x)\right)}{\cos(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(4*sin(x)^2*tan(x)^2,x, algorithm="fricas")
[Out] -2*(3*x*cos(x) - (cos(x)^2 + 2)*sin(x))/cos(x)
```

giac [A] time = 0.14, size = 20, normalized size = 1.25

$$-6\,x + \frac{2\,\tan(x)}{\tan(x)^2 + 1} + 4\,\tan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(4*sin(x)^2*tan(x)^2,x, algorithm="giac")
[Out] -6*x + 2*tan(x)/(tan(x)^2 + 1) + 4*tan(x)
```

maple [A] time = 0.04, size = 28, normalized size = 1.75

$$\frac{4\left(\sin^5(x)\right)}{\cos(x)} + 4\left(\sin^3(x) + \frac{3\sin(x)}{2}\right)\cos(x) - 6x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(4*sin(x)^2*tan(x)^2,x)
[Out] 4*sin(x)^5/cos(x)+4*(sin(x)^3+3/2*sin(x))*cos(x)-6*x
```

maxima [A] time = 0.43, size = 20, normalized size = 1.25

$$-6\,x + \frac{2\,\tan(x)}{\tan(x)^2 + 1} + 4\,\tan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(4*sin(x)^2*tan(x)^2,x, algorithm="maxima")
[Out] -6*x + 2*tan(x)/(tan(x)^2 + 1) + 4*tan(x)
```

mupad [B] time = 2.96, size = 18, normalized size = 1.12

$$2\,\cos(x)\,\sin(x) - 6\,x + \frac{4\,\sin(x)}{\cos(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(4*sin(x)^2*tan(x)^2,x)
[Out] 2*cos(x)*sin(x) - 6*x + (4*sin(x))/cos(x)
```

sympy [A] time = 0.05, size = 20, normalized size = 1.25

$$-6x + \frac{4\sin^3{(x)}}{\cos{(x)}} + 6\sin{(x)}\cos{(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(4*sin(x)**2*tan(x)**2,x)
```

```
[Out] -6*x + 4*sin(x)**3/cos(x) + 6*sin(x)*cos(x)
```

3.831 $\int \cos^4(x)\cot^2(x)\,dx$

Optimal. Leaf size=32

$$-\frac{15x}{8}-\frac{15\cot(x)}{8}+\frac{1}{4}\cos^4(x)\cot(x)+\frac{5}{8}\cos^2(x)\cot(x)$$

```
[Out] -15/8*x-15/8*cot(x)+5/8*cos(x)^2*cot(x)+1/4*cos(x)^4*cot(x)
```

Rubi [A] time = 0.03, antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.444, Rules used = {2591, 288, 321, 203}

$$-\frac{15x}{8}-\frac{15\cot(x)}{8}+\frac{1}{4}\cos^4(x)\cot(x)+\frac{5}{8}\cos^2(x)\cot(x)$$

Antiderivative was successfully verified.

```
[In] Int[Cos[x]^4*Cot[x]^2,x]
```

```
[Out] (-15*x)/8 - (15*Cot[x])/8 + (5*Cos[x]^2*Cot[x])/8 + (Cos[x]^4*Cot[x])/4
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2591

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[(b*ff)/f, Subst[In
t[(ff*x)^(m + n)/(b^2 + ff^2*x^2)^(m/2 + 1), x], x, (b*Tan[e + f*x])/ff], x
]] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2]
```

Rubi steps

$$\begin{aligned}
\int \cos^4(x)\cot^2(x)\,dx &= -\text{Subst}\left(\int \frac{x^6}{\left(1+x^2\right)^3}\,dx, x, \cot(x)\right)\\
&= \frac{1}{4}\cos^4(x)\cot(x) - \frac{5}{4}\text{Subst}\left(\int \frac{x^4}{\left(1+x^2\right)^2}\,dx, x, \cot(x)\right)\\
&= \frac{5}{8}\cos^2(x)\cot(x) + \frac{1}{4}\cos^4(x)\cot(x) - \frac{15}{8}\text{Subst}\left(\int \frac{x^2}{1+x^2}\,dx, x, \cot(x)\right)\\
&= -\frac{15\cot(x)}{8} + \frac{5}{8}\cos^2(x)\cot(x) + \frac{1}{4}\cos^4(x)\cot(x) + \frac{15}{8}\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \cot(x)\right)\\
&= -\frac{15x}{8} - \frac{15\cot(x)}{8} + \frac{5}{8}\cos^2(x)\cot(x) + \frac{1}{4}\cos^4(x)\cot(x)
\end{aligned}$$

Mathematica [A] time = 0.02, size = 26, normalized size = 0.81

$$-\frac{15x}{8} - \frac{1}{2}\sin(2x) - \frac{1}{32}\sin(4x) - \cot(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[x]^4*Cot[x]^2,x]
[Out] (-15*x)/8 - Cot[x] - Sin[2*x]/2 - Sin[4*x]/32
```

fricas [A] time = 1.63, size = 28, normalized size = 0.88

$$\frac{2\cos(x)^5 + 5\cos(x)^3 - 15x\sin(x) - 15\cos(x)}{8\sin(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^4*cot(x)^2,x, algorithm="fricas")
[Out] 1/8*(2*cos(x)^5 + 5*cos(x)^3 - 15*x*sin(x) - 15*cos(x))/sin(x)
```

giac [A] time = 0.13, size = 31, normalized size = 0.97

$$-\frac{15}{8}\,x-\frac{7\,\tan\left(x\right)^{3}+9\,\tan\left(x\right)}{8\,{\left(\tan\left(x\right)^{2}+1\right)}^{2}}-\frac{1}{\tan\left(x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^4*cot(x)^2,x, algorithm="giac")
```

```
[Out] -15/8*x - 1/8*(7*tan(x)^3 + 9*tan(x))/(tan(x)^2 + 1)^2 - 1/tan(x)
```

maple [A] time = 0.04, size = 34, normalized size = 1.06

$$-\frac{\cos^{7}\left(x\right)}{\sin\left(x\right)}-\left(\cos^{5}\left(x\right)+\frac{5\left(\cos^{3}\left(x\right)\right)}{4}+\frac{15\cos\left(x\right)}{8}\right)\sin\left(x\right)-\frac{15x}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)^4*cot(x)^2,x)
```

```
[Out] -1/sin(x)*cos(x)^7-(cos(x)^5+5/4*cos(x)^3+15/8*cos(x))*sin(x)-15/8*x
```

maxima [A] time = 0.43, size = 35, normalized size = 1.09

$$-\frac{15}{8}\,x-\frac{15\,\tan\left(x\right)^{4}+25\,\tan\left(x\right)^{2}+8}{8\,{\left(\tan\left(x\right)^{5}+2\,\tan\left(x\right)^{3}+\tan\left(x\right)\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^4*cot(x)^2,x, algorithm="maxima")
```

```
[Out] -15/8*x - 1/8*(15*tan(x)^4 + 25*tan(x)^2 + 8)/(tan(x)^5 + 2*tan(x)^3 + tan(
x))
```

mupad [B] time = 3.00, size = 26, normalized size = 0.81

$$\frac{\frac{{\cos\left(x\right)}^5}{4}+\frac{5\,{\cos\left(x\right)}^3}{8}-\frac{15\,\cos\left(x\right)}{8}}{\sin\left(x\right)}-\frac{15\,x}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)^4*cot(x)^2,x)
```

```
[Out] ((5*cos(x)^3)/8 - (15*cos(x))/8 + cos(x)^5/4)/sin(x) - (15*x)/8
```

sympy [A] time = 0.06, size = 36, normalized size = 1.12

$$-\frac{15x}{8}-\frac{5\sin(x)\cos^3(x)}{4}-\frac{15\sin(x)\cos(x)}{8}-\frac{\cos^5(x)}{\sin(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)**4*cot(x)**2,x)
[Out] -15*x/8 - 5*sin(x)*cos(x)**3/4 - 15*sin(x)*cos(x)/8 - cos(x)**5/sin(x)
```

3.832 $\int 16\cos^2(x)\sin^2(x)\,dx$

Optimal. Leaf size=18

$$2x - 4\sin(x)\cos^3(x) + 2\sin(x)\cos(x)$$

```
[Out] 2*x+2*cos(x)*sin(x)-4*cos(x)^3*sin(x)
```

Rubi [A] time = 0.03, antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {12, 2568, 2635, 8}

$$2x - 4\sin(x)\cos^3(x) + 2\sin(x)\cos(x)$$

Antiderivative was successfully verified.

```
[In] Int[16*Cos[x]^2*Sin[x]^2,x]
```

```
[Out] 2*x + 2*Cos[x]*Sin[x] - 4*Cos[x]^3*Sin[x]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2568

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> -Simp[(a*(b*Cos[e + f*x])^(n + 1)*(a*Sin[e + f*x])^(m - 1)
)/(b*f*(m + n)), x] + Dist[(a^2*(m - 1))/(m + n), Int[(b*Cos[e + f*x])^n*(a
*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1] &&
 NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rubi steps

$$
\begin{aligned}
\int 16\cos^2(x)\sin^2(x)\,dx &= 16\int \cos^2(x)\sin^2(x)\,dx \\
&= -4\cos^3(x)\sin(x) + 4\int \cos^2(x)\,dx \\
&= 2\cos(x)\sin(x) - 4\cos^3(x)\sin(x) + 2\int 1\,dx \\
&= 2x + 2\cos(x)\sin(x) - 4\cos^3(x)\sin(x)
\end{aligned}
$$

Mathematica [A] time = 0.01, size = 16, normalized size = 0.89

$$4\left(\frac{x}{2} - \frac{1}{8}\sin(4x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[16*Cos[x]^2*Sin[x]^2,x]
[Out] 4*(x/2 - Sin[4*x]/8)
```

fricas [A] time = 1.56, size = 19, normalized size = 1.06

$$-2\left(2\cos(x)^3 - \cos(x)\right)\sin(x) + 2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(16*cos(x)^2*sin(x)^2,x, algorithm="fricas")
[Out] -2*(2*cos(x)^3 - cos(x))*sin(x) + 2*x
```

giac [A] time = 0.14, size = 10, normalized size = 0.56

$$2x - \frac{1}{2}\sin(4x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(16*cos(x)^2*sin(x)^2,x, algorithm="giac")
[Out] 2*x - 1/2*sin(4*x)
```

maple [A] time = 0.01, size = 19, normalized size = 1.06

$$2x + 2\cos(x)\sin(x) - 4\left(\cos^3(x)\right)\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(16*cos(x)^2*sin(x)^2,x)
```

```
[Out] 2*x+2*cos(x)*sin(x)-4*cos(x)^3*sin(x)
```

maxima [A] time = 0.33, size = 10, normalized size = 0.56

$$2x - \frac{1}{2}\sin(4x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(16*cos(x)^2*sin(x)^2,x, algorithm="maxima")
```

```
[Out] 2*x - 1/2*sin(4*x)
```

mupad [B] time = 0.05, size = 18, normalized size = 1.00

$$4\cos(x)\sin(x)^3 - 2\cos(x)\sin(x) + 2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(16*cos(x)^2*sin(x)^2,x)
```

```
[Out] 2*x - 2*cos(x)*sin(x) + 4*cos(x)*sin(x)^3
```

sympy [A] time = 0.06, size = 12, normalized size = 0.67

$$2x - \sin(2x)\cos(2x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(16*cos(x)**2*sin(x)**2,x)
```

```
[Out] 2*x - sin(2*x)*cos(2*x)
```

3.833 $\int 8\cos^2(x)\sin^4(x)\,dx$

Optimal. Leaf size=34

$$\frac{x}{2}-\frac{4}{3}\sin^3(x)\cos^3(x)-\sin(x)\cos^3(x)+\frac{1}{2}\sin(x)\cos(x)$$

```
[Out] 1/2*x+1/2*cos(x)*sin(x)-cos(x)^3*sin(x)-4/3*cos(x)^3*sin(x)^3
```

Rubi [A] time = 0.05, antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {12, 2568, 2635, 8}

$$\frac{x}{2}-\frac{4}{3}\sin^3(x)\cos^3(x)-\sin(x)\cos^3(x)+\frac{1}{2}\sin(x)\cos(x)$$

Antiderivative was successfully verified.

```
[In] Int[8*Cos[x]^2*Sin[x]^4,x]
```

```
[Out] x/2 + (Cos[x]*Sin[x])/2 - Cos[x]^3*Sin[x] - (4*Cos[x]^3*Sin[x]^3)/3
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2568

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> -Simp[(a*(b*Cos[e + f*x])^(n + 1)*(a*Sin[e + f*x])^(m - 1)
)/(b*f*(m + n)), x] + Dist[(a^2*(m - 1))/(m + n), Int[(b*Cos[e + f*x])^n*(a
*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1] &&
 NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rubi steps

$$
\begin{aligned}
\int 8\cos^2(x)\sin^4(x)\,dx &= 8\int \cos^2(x)\sin^4(x)\,dx\\
&= -\frac{4}{3}\cos^3(x)\sin^3(x)+4\int \cos^2(x)\sin^2(x)\,dx\\
&= -\cos^3(x)\sin(x)-\frac{4}{3}\cos^3(x)\sin^3(x)+\int \cos^2(x)\,dx\\
&= \frac{1}{2}\cos(x)\sin(x)-\cos^3(x)\sin(x)-\frac{4}{3}\cos^3(x)\sin^3(x)+\frac{\int 1\,dx}{2}\\
&= \frac{x}{2}+\frac{1}{2}\cos(x)\sin(x)-\cos^3(x)\sin(x)-\frac{4}{3}\cos^3(x)\sin^3(x)
\end{aligned}
$$

Mathematica [A] time = 0.01, size = 32, normalized size = 0.94

$$8\left(\frac{x}{16}-\frac{1}{64}\sin(2x)-\frac{1}{64}\sin(4x)+\frac{1}{192}\sin(6x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[8*Cos[x]^2*Sin[x]^4,x]
[Out] 8*(x/16 - Sin[2*x]/64 - Sin[4*x]/64 + Sin[6*x]/192)
```

fricas [A] time = 2.39, size = 25, normalized size = 0.74

$$\frac{1}{6}\left(8\cos(x)^5-14\cos(x)^3+3\cos(x)\right)\sin(x)+\frac{1}{2}x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(8*cos(x)^2*sin(x)^4,x, algorithm="fricas")
[Out] 1/6*(8*cos(x)^5 - 14*cos(x)^3 + 3*cos(x))*sin(x) + 1/2*x
```

giac [A] time = 0.12, size = 22, normalized size = 0.65

$$\frac{1}{2}x+\frac{1}{24}\sin(6x)-\frac{1}{8}\sin(4x)-\frac{1}{8}\sin(2x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(8*cos(x)^2*sin(x)^4,x, algorithm="giac")
[Out] 1/2*x + 1/24*sin(6*x) - 1/8*sin(4*x) - 1/8*sin(2*x)
```

maple [A] time = 0.01, size = 29, normalized size = 0.85

$$\frac{x}{2}+\frac{\cos(x)\sin(x)}{2}-\left(\cos^{3}(x)\right)\sin(x)-\frac{4\left(\cos^{3}(x)\right)\left(\sin^{3}(x)\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(8*cos(x)^2*sin(x)^4,x)
[Out] 1/2*x+1/2*cos(x)*sin(x)-cos(x)^3*sin(x)-4/3*cos(x)^3*sin(x)^3
```

maxima [A] time = 0.33, size = 18, normalized size = 0.53

$$-\frac{1}{6}\,\sin\left(2\,x\right)^{3}+\frac{1}{2}\,x-\frac{1}{8}\,\sin\left(4\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(8*cos(x)^2*sin(x)^4,x, algorithm="maxima")
[Out] -1/6*sin(2*x)^3 + 1/2*x - 1/8*sin(4*x)
```

mupad [B] time = 0.05, size = 24, normalized size = 0.71

$$\frac{4\,\cos(x)\sin(x)^{5}}{3}+\frac{x}{2}-\frac{\sin\left(2\,x\right)}{3}+\frac{\sin\left(4\,x\right)}{24}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(8*cos(x)^2*sin(x)^4,x)
[Out] x/2 - sin(2*x)/3 + sin(4*x)/24 + (4*cos(x)*sin(x)^5)/3
```

sympy [A] time = 0.05, size = 32, normalized size = 0.94

$$\frac{x}{2}+\frac{4\sin^{5}\left(x\right)\cos\left(x\right)}{3}-\frac{\sin^{3}\left(x\right)\cos\left(x\right)}{3}-\frac{\sin\left(x\right)\cos\left(x\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(8*cos(x)**2*sin(x)**4,x)
[Out] x/2 + 4*sin(x)**5*cos(x)/3 - sin(x)**3*cos(x)/3 - sin(x)*cos(x)/2
```

3.834 $\int 35\cos^3(x)\sin^4(x)\,dx$

Optimal. Leaf size=13

$$7\sin^5(x)-5\sin^7(x)$$

[Out] 7*sin(x)^5-5*sin(x)^7

Rubi [A] time = 0.02, antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {12, 2564, 14}

$$7\sin^5(x)-5\sin^7(x)$$

Antiderivative was successfully verified.

[In] Int[35*Cos[x]^3*Sin[x]^4,x]

[Out] 7*Sin[x]^5 - 5*Sin[x]^7

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2564

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rubi steps

$$\begin{aligned}\int 35\cos^3(x)\sin^4(x)\,dx &= 35\int \cos^3(x)\sin^4(x)\,dx\\ &= 35\,\text{Subst}\left(\int x^4\left(1-x^2\right)\,dx, x, \sin(x)\right)\\ &= 35\,\text{Subst}\left(\int \left(x^4-x^6\right)\,dx, x, \sin(x)\right)\\ &= 7\sin^5(x)-5\sin^7(x)\end{aligned}$$

Mathematica [B] time = 0.01, size = 33, normalized size = 2.54

$$35\left(\frac{3\sin(x)}{64}-\frac{1}{64}\sin(3x)-\frac{1}{320}\sin(5x)+\frac{1}{448}\sin(7x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[35*Cos[x]^3*Sin[x]^4,x]
[Out] 35*((3*Sin[x])/64 - Sin[3*x]/64 - Sin[5*x]/320 + Sin[7*x]/448)
```

fricas [A] time = 0.47, size = 21, normalized size = 1.62

$$\left(5\,\cos(x)^6-8\,\cos(x)^4+\cos(x)^2+2\right)\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(35*cos(x)^3*sin(x)^4,x, algorithm="fricas")
[Out] (5*cos(x)^6 - 8*cos(x)^4 + cos(x)^2 + 2)*sin(x)
```

giac [A] time = 0.13, size = 13, normalized size = 1.00

$$-5\,\sin(x)^7+7\,\sin(x)^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(35*cos(x)^3*sin(x)^4,x, algorithm="giac")
[Out] -5*sin(x)^7 + 7*sin(x)^5
```

maple [B] time = 0.01, size = 29, normalized size = 2.23

$$-5\left(\cos^4(x)\right)\left(\sin^3(x)\right)-3\sin(x)\left(\cos^4(x)\right)+\left(2+\cos^2(x)\right)\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(35*cos(x)^3*sin(x)^4,x)
```

```
[Out] -5*cos(x)^4*sin(x)^3-3*sin(x)*cos(x)^4+(2+cos(x)^2)*sin(x)
```

maxima [A] time = 0.32, size = 13, normalized size = 1.00

$$-5\,\sin(x)^7+7\,\sin(x)^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(35*cos(x)^3*sin(x)^4,x, algorithm="maxima")
```

```
[Out] -5*sin(x)^7 + 7*sin(x)^5
```

mupad [B] time = 0.04, size = 13, normalized size = 1.00

$$7\sin(x)^5-5\sin(x)^7$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(35*cos(x)^3*sin(x)^4,x)
```

```
[Out] 7*sin(x)^5 - 5*sin(x)^7
```

sympy [A] time = 0.05, size = 12, normalized size = 0.92

$$-5\sin^7(x)+7\sin^5(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(35*cos(x)**3*sin(x)**4,x)
```

```
[Out] -5*sin(x)**7 + 7*sin(x)**5
```

3.835 $\int 4\cos^4(x)\sin^4(x)\,dx$

Optimal. Leaf size=46

$$\frac{3x}{32}-\frac{1}{2}\sin^3(x)\cos^5(x)-\frac{1}{4}\sin(x)\cos^5(x)+\frac{1}{16}\sin(x)\cos^3(x)+\frac{3}{32}\sin(x)\cos(x)$$

```
[Out] 3/32*x+3/32*cos(x)*sin(x)+1/16*cos(x)^3*sin(x)-1/4*cos(x)^5*sin(x)-1/2*cos(
x)^5*sin(x)^3
```

Rubi [A] time = 0.06, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {12, 2568, 2635, 8}

$$\frac{3x}{32}-\frac{1}{2}\sin^3(x)\cos^5(x)-\frac{1}{4}\sin(x)\cos^5(x)+\frac{1}{16}\sin(x)\cos^3(x)+\frac{3}{32}\sin(x)\cos(x)$$

Antiderivative was successfully verified.

```
[In] Int[4*Cos[x]^4*Sin[x]^4,x]
```

```
[Out] (3*x)/32 + (3*Cos[x]*Sin[x])/32 + (Cos[x]^3*Sin[x])/16 - (Cos[x]^5*Sin[x])/
4 - (Cos[x]^5*Sin[x]^3)/2
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2568

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> -Simp[(a*(b*Cos[e + f*x])^(n + 1)*(a*Sin[e + f*x])^(m - 1)
)/(b*f*(m + n)), x] + Dist[(a^2*(m - 1))/(m + n), Int[(b*Cos[e + f*x])^n*(a
*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1] &&
 NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
```

]

Rubi steps

$$\begin{aligned}\int 4\cos^4(x)\sin^4(x)\,dx &= 4\int \cos^4(x)\sin^4(x)\,dx\\ &= -\frac{1}{2}\cos^5(x)\sin^3(x)+\frac{3}{2}\int \cos^4(x)\sin^2(x)\,dx\\ &= -\frac{1}{4}\cos^5(x)\sin(x)-\frac{1}{2}\cos^5(x)\sin^3(x)+\frac{1}{4}\int \cos^4(x)\,dx\\ &= \frac{1}{16}\cos^3(x)\sin(x)-\frac{1}{4}\cos^5(x)\sin(x)-\frac{1}{2}\cos^5(x)\sin^3(x)+\frac{3}{16}\int \cos^2(x)\,dx\\ &= \frac{3}{32}\cos(x)\sin(x)+\frac{1}{16}\cos^3(x)\sin(x)-\frac{1}{4}\cos^5(x)\sin(x)-\frac{1}{2}\cos^5(x)\sin^3(x)+\frac{3\int 1\,dx}{32}\\ &= \frac{3x}{32}+\frac{3}{32}\cos(x)\sin(x)+\frac{1}{16}\cos^3(x)\sin(x)-\frac{1}{4}\cos^5(x)\sin(x)-\frac{1}{2}\cos^5(x)\sin^3(x)\end{aligned}$$

Mathematica [A] time = 0.01, size = 24, normalized size = 0.52

$$4\left(\frac{3x}{128}-\frac{1}{128}\sin(4x)+\frac{\sin(8x)}{1024}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[4*Cos[x]^4*Sin[x]^4,x]
[Out] 4*((3*x)/128 - Sin[4*x]/128 + Sin[8*x]/1024)
```

fricas [A] time = 1.97, size = 31, normalized size = 0.67

$$\frac{1}{32}\left(16\,\cos(x)^7-24\,\cos(x)^5+2\,\cos(x)^3+3\,\cos(x)\right)\sin(x)+\frac{3}{32}\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(4*cos(x)^4*sin(x)^4,x, algorithm="fricas")
[Out] 1/32*(16*cos(x)^7 - 24*cos(x)^5 + 2*cos(x)^3 + 3*cos(x))*sin(x) + 3/32*x
```

giac [A] time = 0.14, size = 16, normalized size = 0.35

$$\frac{3}{32}\,x+\frac{1}{256}\,\sin\left(8\,x\right)-\frac{1}{32}\,\sin\left(4\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(4*cos(x)^4*sin(x)^4,x, algorithm="giac")
```

```
[Out] 3/32*x + 1/256*sin(8*x) - 1/32*sin(4*x)
```

maple [A] time = 0.01, size = 36, normalized size = 0.78

$$-\frac{\left(\cos^5(x)\right)\left(\sin^3(x)\right)}{2}-\frac{\left(\cos^5(x)\right)\sin(x)}{4}+\frac{\left(\cos^3(x)+\frac{3\cos(x)}{2}\right)\sin(x)}{16}+\frac{3x}{32}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(4*cos(x)^4*sin(x)^4,x)
```

```
[Out] -1/2*cos(x)^5*sin(x)^3-1/4*cos(x)^5*sin(x)+1/16*(cos(x)^3+3/2*cos(x))*sin(x
)+3/32*x
```

maxima [A] time = 0.33, size = 16, normalized size = 0.35

$$\frac{3}{32}\,x+\frac{1}{256}\,\sin\left(8\,x\right)-\frac{1}{32}\,\sin\left(4\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(4*cos(x)^4*sin(x)^4,x, algorithm="maxima")
```

```
[Out] 3/32*x + 1/256*sin(8*x) - 1/32*sin(4*x)
```

mupad [B] time = 0.04, size = 33, normalized size = 0.72

$$\frac{3\,x}{32}-\frac{\sin\left(2\,x\right)}{16}+\frac{\sin\left(4\,x\right)}{128}+4\,\sin(x)^5\,\left(\frac{\cos(x)^3}{8}+\frac{\cos(x)}{16}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(4*cos(x)^4*sin(x)^4,x)
```

```
[Out] (3*x)/32 - sin(2*x)/16 + sin(4*x)/128 + 4*sin(x)^5*(cos(x)/16 + cos(x)^3/8)
```

sympy [A] time = 0.06, size = 31, normalized size = 0.67

$$\frac{3x}{32}-\frac{\sin^3\left(2x\right)\cos\left(2x\right)}{32}-\frac{3\sin\left(2x\right)\cos\left(2x\right)}{64}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(4*cos(x)**4*sin(x)**4,x)
```

```
[Out] 3*x/32 - sin(2*x)**3*cos(2*x)/32 - 3*sin(2*x)*cos(2*x)/64
```

3.836 $\int \frac{\cos(x)}{-\sin(x)+\sin^3(x)}\,dx$

Optimal. Leaf size=9

$$\log(\cos(x)) - \log(\sin(x))$$

```
[Out] ln(cos(x))-ln(sin(x))
```

Rubi [A] time = 0.03, antiderivative size = 9, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {4334, 266, 36, 31, 29}

$$\log(\cos(x)) - \log(\sin(x))$$

Antiderivative was successfully verified.

```
[In] Int[Cos[x]/(-Sin[x] + Sin[x]^3),x]
```

```
[Out] Log[Cos[x]] - Log[Sin[x]]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 4334

```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{d = FreeFacto
rs[Sin[c*(a + b*x)], x]}, Dist[d/(b*c), Subst[Int[SubstFor[1, Sin[c*(a + b*
x)]/d, u, x], x], x, Sin[c*(a + b*x)]/d], x] /; FunctionOfQ[Sin[c*(a + b*x)
```

```
]/d, u, x, True]] /; FreeQ[{a, b, c}, x] && (EqQ[F, Cos] || EqQ[F, cos])
```

Rubi steps

$$\begin{aligned}\int \frac{\cos(x)}{-\sin(x)+\sin^3(x)}\,dx &= \text{Subst}\left(\int \frac{1}{x\left(-1+x^2\right)}\,dx, x, \sin(x)\right)\\ &= \frac{1}{2}\,\text{Subst}\left(\int \frac{1}{(-1+x)x}\,dx, x, \sin^2(x)\right)\\ &= \frac{1}{2}\,\text{Subst}\left(\int \frac{1}{-1+x}\,dx, x, \sin^2(x)\right)-\frac{1}{2}\,\text{Subst}\left(\int \frac{1}{x}\,dx, x, \sin^2(x)\right)\\ &= \log(\cos(x))-\log(\sin(x))\end{aligned}$$

Mathematica [A] time = 0.00, size = 9, normalized size = 1.00

$$\log(\cos(x))-\log(\sin(x))$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[x]/(-Sin[x] + Sin[x]^3),x]
[Out] Log[Cos[x]] - Log[Sin[x]]
```

fricas [B] time = 0.57, size = 19, normalized size = 2.11

$$\frac{1}{2}\,\log\left(\cos(x)^2\right)-\frac{1}{2}\,\log\left(-\frac{1}{4}\,\cos(x)^2+\frac{1}{4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)/(-sin(x)+sin(x)^3),x, algorithm="fricas")
[Out] 1/2*log(cos(x)^2) - 1/2*log(-1/4*cos(x)^2 + 1/4)
```

giac [A] time = 0.13, size = 18, normalized size = 2.00

$$\frac{1}{2}\,\log\left(-\sin(x)^2+1\right)-\log\left(|\sin(x)|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)/(-sin(x)+sin(x)^3),x, algorithm="giac")
[Out] 1/2*log(-sin(x)^2 + 1) - log(abs(sin(x)))
```

maple [B] time = 0.08, size = 21, normalized size = 2.33

$$-\ln\left(\sin(x)\right)+\frac{\ln\left(\sin(x)-1\right)}{2}+\frac{\ln\left(1+\sin(x)\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)/(-sin(x)+sin(x)^3),x)
[Out] -ln(sin(x))+1/2*ln(sin(x)-1)+1/2*ln(1+sin(x))
```

maxima [B] time = 0.32, size = 20, normalized size = 2.22

$$\frac{1}{2}\log\left(\sin(x)+1\right)+\frac{1}{2}\log\left(\sin(x)-1\right)-\log\left(\sin(x)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)/(-sin(x)+sin(x)^3),x, algorithm="maxima")
[Out] 1/2*log(sin(x) + 1) + 1/2*log(sin(x) - 1) - log(sin(x))
```

mupad [B] time = 3.01, size = 13, normalized size = 1.44

$$\frac{\ln\left(\cos(x)^2\right)}{2}-\ln\left(\sin(x)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-cos(x)/(sin(x) - sin(x)^3),x)
[Out] log(cos(x)^2)/2 - log(sin(x))
```

sympy [B] time = 0.32, size = 20, normalized size = 2.22

$$\frac{\log\left(\sin\left(x\right)-1\right)}{2}+\frac{\log\left(\sin\left(x\right)+1\right)}{2}-\log\left(\sin\left(x\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)/(-sin(x)+sin(x)**3),x)
[Out] log(sin(x) - 1)/2 + log(sin(x) + 1)/2 - log(sin(x))
```

3.837 $\int \left(-1+2\cos^2(x)+\cos(x)\sin(x)\right)\,dx$

Optimal. Leaf size=14

$$\frac{\sin^2(x)}{2}+\sin(x)\cos(x)$$

```
[Out] cos(x)*sin(x)+1/2*sin(x)^2
```

Rubi [A] time = 0.02, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {2635, 8, 2564, 30}

$$\frac{\sin^2(x)}{2}+\sin(x)\cos(x)$$

Antiderivative was successfully verified.

```
[In] Int[-1 + 2*Cos[x]^2 + Cos[x]*Sin[x],x]
```

```
[Out] Cos[x]*Sin[x] + Sin[x]^2/2
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2564

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] && !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rubi steps

$$\int \left(-1+2\cos^2(x)+\cos(x)\sin(x)\right)\,dx = -x+2\int \cos^2(x)\,dx+\int \cos(x)\sin(x)\,dx$$
$$= -x+\cos(x)\sin(x)+\int 1\,dx+\text{Subst}(\int x\,dx,x,\sin(x))$$
$$= \cos(x)\sin(x)+\frac{\sin^2(x)}{2}$$

Mathematica [A] time = 0.01, size = 17, normalized size = 1.21

$$\frac{1}{2}\sin(2x)-\frac{\cos^2(x)}{2}$$

Antiderivative was successfully verified.

```
[In] Integrate[-1 + 2*Cos[x]^2 + Cos[x]*Sin[x],x]
[Out] -1/2*Cos[x]^2 + Sin[2*x]/2
```

fricas [A] time = 0.71, size = 12, normalized size = 0.86

$$-\frac{1}{2}\cos(x)^2+\cos(x)\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-1+2*cos(x)^2+cos(x)*sin(x),x, algorithm="fricas")
[Out] -1/2*cos(x)^2 + cos(x)*sin(x)
```

giac [A] time = 0.14, size = 13, normalized size = 0.93

$$-\frac{1}{2}\cos(x)^2+\frac{1}{2}\sin(2x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-1+2*cos(x)^2+cos(x)*sin(x),x, algorithm="giac")
[Out] -1/2*cos(x)^2 + 1/2*sin(2*x)
```

maple [A] time = 0.02, size = 13, normalized size = 0.93

$$\cos(x)\sin(x)+\frac{\left(\sin^2(x)\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-1+2*cos(x)^2+cos(x)*sin(x),x)
```

```
[Out] cos(x)*sin(x)+1/2*sin(x)^2
```

maxima [A] time = 0.32, size = 13, normalized size = 0.93

$$-\frac{1}{2}\cos(x)^2+\frac{1}{2}\sin\left(2\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-1+2*cos(x)^2+cos(x)*sin(x),x, algorithm="maxima")
```

```
[Out] -1/2*cos(x)^2 + 1/2*sin(2*x)
```

mupad [B] time = 2.97, size = 11, normalized size = 0.79

$$-\frac{\cos(x)\,\left(\cos(x)-2\,\sin(x)\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)*sin(x) + 2*cos(x)^2 - 1,x)
```

```
[Out] -(cos(x)*(cos(x) - 2*sin(x)))/2
```

sympy [A] time = 0.05, size = 12, normalized size = 0.86

$$\frac{\sin^2\left(x\right)}{2}+\sin\left(x\right)\cos\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-1+2*cos(x)**2+cos(x)*sin(x),x)
```

```
[Out] sin(x)**2/2 + sin(x)*cos(x)
```

3.838 $\int \left(\cos^2(x)+\sin^2(x)\right) dx$

Optimal. Leaf size=1

$$x$$

[Out] x

Rubi [A] time = 0.01, antiderivative size = 1, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {2635, 8}

$$x$$

Antiderivative was successfully verified.

[In] Int[Cos[x]^2 + Sin[x]^2,x]

[Out] x

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rubi steps

$$\begin{aligned}\int \left(\cos^2(x)+\sin^2(x)\right) dx &= \int \cos^2(x)\,dx+\int \sin^2(x)\,dx\\ &= 2\frac{\int 1\,dx}{2}\\ &= x\end{aligned}$$

Mathematica [A] time = 0.00, size = 1, normalized size = 1.00

$$x$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[x]^2 + Sin[x]^2,x]
```

```
[Out] x
```

fricas [A] time = 1.35, size = 1, normalized size = 1.00

$$x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^2+sin(x)^2,x, algorithm="fricas")
```

```
[Out] x
```

giac [A] time = 0.14, size = 1, normalized size = 1.00

$$x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^2+sin(x)^2,x, algorithm="giac")
```

```
[Out] x
```

maple [A] time = 0.02, size = 2, normalized size = 2.00

$$x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)^2+sin(x)^2,x)
```

```
[Out] x
```

maxima [A] time = 0.33, size = 1, normalized size = 1.00

$$x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^2+sin(x)^2,x, algorithm="maxima")
```

```
[Out] x
```

mupad [B] time = 2.93, size = 1, normalized size = 1.00

$$x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)^2 + sin(x)^2,x)
```

```
[Out] x
```

sympy [A] time = 0.06, size = 0, normalized size = 0.00

$$x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)**2+sin(x)**2,x)
```

```
[Out] x
```

3.839 $\int \left(-\cos^2(x)+\sin^2(x)\right) \, dx$

Optimal. Leaf size=6

$$\sin(x)(-\cos(x))$$

```
[Out] -cos(x)*sin(x)
```

Rubi [A] time = 0.01, antiderivative size = 6, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {2635, 8}

$$\sin(x)(-\cos(x))$$

Antiderivative was successfully verified.

```
[In] Int[-Cos[x]^2 + Sin[x]^2,x]
```

```
[Out] -(Cos[x]*Sin[x])
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rubi steps

$$\begin{aligned} \int \left(-\cos^2(x)+\sin^2(x)\right) \, dx &= -\int \cos^2(x) \, dx+\int \sin^2(x) \, dx \\ &= -\cos(x) \sin(x) \end{aligned}$$

Mathematica [A] time = 0.00, size = 8, normalized size = 1.33

$$-\frac{1}{2} \sin(2x)$$

Antiderivative was successfully verified.

```
[In] Integrate[-Cos[x]^2 + Sin[x]^2,x]
```

```
[Out] -1/2*Sin[2*x]
```

fricas [A] time = 0.41, size = 6, normalized size = 1.00

$$-\cos(x)\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-cos(x)^2+sin(x)^2,x, algorithm="fricas")
[Out] -cos(x)*sin(x)
```

giac [A] time = 0.12, size = 6, normalized size = 1.00

$$-\frac{1}{2}\sin(2x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-cos(x)^2+sin(x)^2,x, algorithm="giac")
[Out] -1/2*sin(2*x)
```

maple [A] time = 0.00, size = 7, normalized size = 1.17

$$-\cos(x)\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-cos(x)^2+sin(x)^2,x)
[Out] -cos(x)*sin(x)
```

maxima [A] time = 0.32, size = 6, normalized size = 1.00

$$-\frac{1}{2}\sin(2x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-cos(x)^2+sin(x)^2,x, algorithm="maxima")
[Out] -1/2*sin(2*x)
```

mupad [B] time = 2.94, size = 6, normalized size = 1.00

$$-\frac{\sin(2x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)^2 - cos(x)^2,x)
```

```
[Out] -sin(2*x)/2
```

sympy [A] time = 0.06, size = 7, normalized size = 1.17

$$-\sin(x)\cos(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-cos(x)**2+sin(x)**2,x)
```

```
[Out] -sin(x)*cos(x)
```

3.840 $\int 2^{\sin(x)} \cos(x) \, dx$

Optimal. Leaf size=9

$$\frac{2^{\sin(x)}}{\log(2)}$$

[Out] 2^sin(x)/ln(2)

Rubi [A] time = 0.01, antiderivative size = 9, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {4334, 2194}

$$\frac{2^{\sin(x)}}{\log(2)}$$

Antiderivative was successfully verified.

[In] Int[2^Sin[x]*Cos[x],x]

[Out] 2^Sin[x]/Log[2]

Rule 2194

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rule 4334

```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{d = FreeFacto
rs[Sin[c*(a + b*x)], x]}, Dist[d/(b*c), Subst[Int[SubstFor[1, Sin[c*(a + b*
x)]/d, u, x], x], x, Sin[c*(a + b*x)]/d], x] /; FunctionOfQ[Sin[c*(a + b*x)
]/d, u, x, True]] /; FreeQ[{a, b, c}, x] && (EqQ[F, Cos] || EqQ[F, cos])
```

Rubi steps

$$\begin{aligned} \int 2^{\sin(x)} \cos(x) \, dx &= \text{Subst}\left(\int 2^x \, dx, x, \sin(x)\right) \\ &= \frac{2^{\sin(x)}}{\log(2)} \end{aligned}$$

Mathematica [A] time = 0.01, size = 9, normalized size = 1.00

$$\frac{2^{\sin(x)}}{\log(2)}$$

Antiderivative was successfully verified.

```
[In] Integrate[2^Sin[x]*Cos[x],x]
```

```
[Out] 2^Sin[x]/Log[2]
```

fricas [A] time = 0.56, size = 9, normalized size = 1.00

$$\frac{2^{\sin(x)}}{\log(2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(2^sin(x)*cos(x),x, algorithm="fricas")
```

```
[Out] 2^sin(x)/log(2)
```

giac [A] time = 0.13, size = 9, normalized size = 1.00

$$\frac{2^{\sin(x)}}{\log(2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(2^sin(x)*cos(x),x, algorithm="giac")
```

```
[Out] 2^sin(x)/log(2)
```

maple [A] time = 0.03, size = 10, normalized size = 1.11

$$\frac{2^{\sin(x)}}{\ln(2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(2^sin(x)*cos(x),x)
```

```
[Out] 2^sin(x)/ln(2)
```

maxima [A] time = 0.31, size = 9, normalized size = 1.00

$$\frac{2^{\sin(x)}}{\log(2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(2^sin(x)*cos(x),x, algorithm="maxima")
```

```
[Out] 2^sin(x)/log(2)
```

mupad [B] time = 0.07, size = 9, normalized size = 1.00

$$\frac{2^{\sin(x)}}{\ln(2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(2^sin(x)*cos(x),x)
[Out] 2^sin(x)/log(2)
```

sympy [A] time = 0.25, size = 7, normalized size = 0.78

$$\frac{2^{\sin(x)}}{\log(2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(2**sin(x)*cos(x),x)
[Out] 2**sin(x)/log(2)
```

3.841 $\int \left(\tan^3(x)+\tan^5(x)\right)\,dx$

Optimal. Leaf size=8

$$\frac{\tan^4(x)}{4}$$

```
[Out] 1/4*tan(x)^4
```

Rubi [A] time = 0.02, antiderivative size = 8, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {3473, 3475}

$$\frac{\tan^4(x)}{4}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[x]^3 + Tan[x]^5,x]
```

```
[Out] Tan[x]^4/4
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\int \left(\tan^3(x)+\tan^5(x)\right)\,dx &= \int \tan^3(x)\,dx+\int \tan^5(x)\,dx\\
&= \frac{\tan^2(x)}{2}+\frac{\tan^4(x)}{4}-\int \tan(x)\,dx-\int \tan^3(x)\,dx\\
&= \log(\cos(x))+\frac{\tan^4(x)}{4}+\int \tan(x)\,dx\\
&= \frac{\tan^4(x)}{4}
\end{aligned}$$

Mathematica [A] time = 0.00, size = 8, normalized size = 1.00

$$\frac{\tan^4(x)}{4}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[x]^3 + Tan[x]^5,x]
[Out] Tan[x]^4/4
```

fricas [A] time = 1.47, size = 6, normalized size = 0.75

$$\frac{1}{4}\tan(x)^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)^3+tan(x)^5,x, algorithm="fricas")
[Out] 1/4*tan(x)^4
```

giac [A] time = 0.14, size = 6, normalized size = 0.75

$$\frac{1}{4}\tan(x)^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)^3+tan(x)^5,x, algorithm="giac")
[Out] 1/4*tan(x)^4
```

maple [A] time = 0.00, size = 7, normalized size = 0.88

$$\frac{\left(\tan^4(x)\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(x)^3+tan(x)^5,x)
[Out] 1/4*tan(x)^4
```

maxima [B] time = 0.32, size = 35, normalized size = 4.38

$$\frac{4\sin(x)^2-3}{4\left(\sin(x)^4-2\sin(x)^2+1\right)}-\frac{1}{2\left(\sin(x)^2-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)^3+tan(x)^5,x, algorithm="maxima")
[Out] 1/4*(4*sin(x)^2 - 3)/(sin(x)^4 - 2*sin(x)^2 + 1) - 1/2/(sin(x)^2 - 1)
```

mupad [B] time = 2.95, size = 6, normalized size = 0.75

$$\frac{\tan(x)^4}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(x)^3 + tan(x)^5,x)
[Out] tan(x)^4/4
```

sympy [B] time = 0.12, size = 22, normalized size = 2.75

$$-\frac{4\cos^2(x)-1}{4\cos^4(x)}+\frac{1}{2\cos^2(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)**3+tan(x)**5,x)
[Out] -(4*cos(x)**2 - 1)/(4*cos(x)**4) + 1/(2*cos(x)**2)
```

3.842 $\int x\sec(x)(2+x\tan(x))\,dx$

Optimal. Leaf size=6

$$x^2\sec(x)$$

[Out] x^2*sec(x)

Rubi [A] time = 0.18, antiderivative size = 6, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 5, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {6742, 4181, 2279, 2391, 3757}

$$x^2\sec(x)$$

Antiderivative was successfully verified.

[In] Int[x*Sec[x]*(2 + x*Tan[x]),x]

[Out] x^2*Sec[x]

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3757

```
Int[(x_)^(m_.)*Sec[(a_.) + (b_.)*(x_)^(n_.)]^(p_.)*Tan[(a_.) + (b_.)*(x_)^(
n_.)]^(q_.), x_Symbol] :> Simp[(x^(m - n + 1)*Sec[a + b*x^n]^p)/(b*n*p), x]
 - Dist[(m - n + 1)/(b*n*p), Int[x^(m - n)*Sec[a + b*x^n]^p, x], x] /; Free
Q[{a, b, p}, x] && IntegerQ[n] && GeQ[m, n] && EqQ[q, 1]
```

Rule 4181

```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))])/f, x] + (-Di
st[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x))
], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$
\begin{aligned}
\int x\sec(x)(2+x\tan(x))\,dx &= \int \left(2x\sec(x)+x^2\sec(x)\tan(x)\right)\,dx\\
&= 2\int x\sec(x)\,dx+\int x^2\sec(x)\tan(x)\,dx\\
&= -4ix\tan^{-1}\left(e^{ix}\right)+x^2\sec(x)-2\int \log\left(1-ie^{ix}\right)\,dx+2\int \log\left(1+ie^{ix}\right)\,dx-2\int x\\
&= x^2\sec(x)+2i\,\text{Subst}\left(\int \frac{\log(1-ix)}{x}\,dx,x,e^{ix}\right)-2i\,\text{Subst}\left(\int \frac{\log(1+ix)}{x}\,dx,x,e^{ix}\right)\\
&= 2i\text{Li}_2\left(-ie^{ix}\right)-2i\text{Li}_2\left(ie^{ix}\right)+x^2\sec(x)-2i\,\text{Subst}\left(\int \frac{\log(1-ix)}{x}\,dx,x,e^{ix}\right)+2i\,\text{Sul}\\
&= x^2\sec(x)
\end{aligned}
$$

Mathematica [A] time = 0.02, size = 6, normalized size = 1.00

$$x^2\sec(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Sec[x]*(2 + x*Tan[x]),x]
```

```
[Out] x^2*Sec[x]
```

fricas [A] time = 2.14, size = 8, normalized size = 1.33

$$\frac{x^2}{\cos(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sec(x)*(2+x*tan(x)),x, algorithm="fricas")
```

```
[Out] x^2/cos(x)
```

giac [B] time = 0.15, size = 26, normalized size = 4.33

$$-\frac{x^2\tan\left(\frac{1}{2}x\right)^2+x^2}{\tan\left(\frac{1}{2}x\right)^2-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sec(x)*(2+x*tan(x)),x, algorithm="giac")
```

```
[Out] -(x^2*tan(1/2*x)^2 + x^2)/(tan(1/2*x)^2 - 1)
```

maple [A] time = 0.02, size = 9, normalized size = 1.50

$$\frac{x^{2}}{\cos(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*sec(x)*(2+x*tan(x)),x)
```

```
[Out] x^2/cos(x)
```

maxima [B] time = 0.48, size = 51, normalized size = 8.50

$$\frac{2\left(x^{2}\cos\left(2\,x\right)\cos(x)+x^{2}\sin\left(2\,x\right)\sin(x)+x^{2}\cos(x)\right)}{\cos\left(2\,x\right)^{2}+\sin\left(2\,x\right)^{2}+2\,\cos\left(2\,x\right)+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sec(x)*(2+x*tan(x)),x, algorithm="maxima")
```

```
[Out] 2*(x^2*cos(2*x)*cos(x) + x^2*sin(2*x)*sin(x) + x^2*cos(x))/(cos(2*x)^2 + si
n(2*x)^2 + 2*cos(2*x) + 1)
```

mupad [B] time = 0.08, size = 8, normalized size = 1.33

$$\frac{x^{2}}{\cos(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(x*tan(x) + 2))/cos(x),x)
```

```
[Out] x^2/cos(x)
```

sympy [A] time = 0.48, size = 5, normalized size = 0.83

$$x^{2}\sec\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sec(x)*(2+x*tan(x)),x)
```

```
[Out] x**2*sec(x)
```

3.843 $\int \frac{\cot(\sqrt{x})\csc(\sqrt{x})}{\sqrt{x}}\,dx$

Optimal. Leaf size=8

$$-2\csc\left(\sqrt{x}\right)$$

```
[Out] -2*csc(x^(1/2))
```

Rubi [A] time = 0.20, antiderivative size = 8, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {6715, 2606, 8}

$$-2\csc\left(\sqrt{x}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(Cot[Sqrt[x]]*Csc[Sqrt[x]])/Sqrt[x],x]
```

```
[Out] -2*Csc[Sqrt[x]]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 6715

```
Int[(u_)*(x_)^(m_.), x_Symbol] :> Dist[1/(m + 1), Subst[Int[SubstFor[x^(m +
 1), u, x], x], x, x^(m + 1)], x] /; FreeQ[m, x] && NeQ[m, -1] && FunctionO
fQ[x^(m + 1), u, x]
```

Rubi steps

$$\int \frac{\cot\left(\sqrt{x}\right)\csc\left(\sqrt{x}\right)}{\sqrt{x}}\,dx = 2\,\text{Subst}\left(\int \cot(x)\csc(x)\,dx, x, \sqrt{x}\right)$$
$$= -\left(2\,\text{Subst}\left(\int 1\,dx, x, \csc\left(\sqrt{x}\right)\right)\right)$$
$$= -2\csc\left(\sqrt{x}\right)$$

Mathematica [**A**] time = 0.02, size = 8, normalized size = 1.00

$$-2\csc\left(\sqrt{x}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cot[Sqrt[x]]*Csc[Sqrt[x]])/Sqrt[x],x]
[Out] -2*Csc[Sqrt[x]]
```

fricas [**A**] time = 1.44, size = 8, normalized size = 1.00

$$-\frac{2}{\sin\left(\sqrt{x}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x^(1/2))*csc(x^(1/2))/x^(1/2),x, algorithm="fricas")
[Out] -2/sin(sqrt(x))
```

giac [**A**] time = 0.12, size = 8, normalized size = 1.00

$$-\frac{2}{\sin\left(\sqrt{x}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x^(1/2))*csc(x^(1/2))/x^(1/2),x, algorithm="giac")
[Out] -2/sin(sqrt(x))
```

maple [**A**] time = 0.08, size = 7, normalized size = 0.88

$$-2\csc\left(\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(x^(1/2))*csc(x^(1/2))/x^(1/2),x)
[Out] -2*csc(x^(1/2))
```

maxima [A] time = 0.31, size = 8, normalized size = 1.00

$$-\frac{2}{\sin\left(\sqrt{x}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x^(1/2))*csc(x^(1/2))/x^(1/2),x, algorithm="maxima")
[Out] -2/sin(sqrt(x))
```

mupad [B] time = 3.07, size = 8, normalized size = 1.00

$$-\frac{2}{\sin\left(\sqrt{x}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(x^(1/2))/(x^(1/2)*sin(x^(1/2))),x)
[Out] -2/sin(x^(1/2))
```

sympy [A] time = 0.28, size = 8, normalized size = 1.00

$$-2\csc\left(\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x**(1/2))*csc(x**(1/2))/x**(1/2),x)
[Out] -2*csc(sqrt(x))
```

3.844 $\int \frac{\cos(\sqrt{x})\sin(\sqrt{x})}{\sqrt{x}}\,dx$

Optimal. Leaf size=8

$$\sin^2\left(\sqrt{x}\right)$$

```
[Out] sin(x^(1/2))^2
```

Rubi [A] time = 0.01, antiderivative size = 8, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.056, Rules used = {3441}

$$\sin^2\left(\sqrt{x}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[Sqrt[x]]*Sin[Sqrt[x]])/Sqrt[x],x]
```

```
[Out] Sin[Sqrt[x]]^2
```

Rule 3441

```
Int[Cos[(a_.) + (b_.)*(x_)^(n_.)]*(x_)^(m_.)*Sin[(a_.) + (b_.)*(x_)^(n_.)]^
(p_.), x_Symbol] :> Simp[Sin[a + b*x^n]^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[
{a, b, m, n, p}, x] && EqQ[m, n - 1] && NeQ[p, -1]
```

Rubi steps

$$\int \frac{\cos\left(\sqrt{x}\right)\sin\left(\sqrt{x}\right)}{\sqrt{x}}\,dx = \sin^2\left(\sqrt{x}\right)$$

Mathematica [A] time = 0.01, size = 12, normalized size = 1.50

$$-\frac{1}{2}\cos\left(2\sqrt{x}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[Sqrt[x]]*Sin[Sqrt[x]])/Sqrt[x],x]
```

```
[Out] -1/2*Cos[2*Sqrt[x]]
```

fricas [A] time = 0.73, size = 8, normalized size = 1.00

$$-\cos\left(\sqrt{x}\right)^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x^(1/2))*sin(x^(1/2))/x^(1/2),x, algorithm="fricas")
[Out] -cos(sqrt(x))^2
```

giac [A] time = 0.12, size = 6, normalized size = 0.75

$$\sin\left(\sqrt{x}\right)^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x^(1/2))*sin(x^(1/2))/x^(1/2),x, algorithm="giac")
[Out] sin(sqrt(x))^2
```

maple [A] time = 0.04, size = 9, normalized size = 1.12

$$-\left(\cos^2\left(\sqrt{x}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x^(1/2))*sin(x^(1/2))/x^(1/2),x)
[Out] -cos(x^(1/2))^2
```

maxima [A] time = 0.31, size = 8, normalized size = 1.00

$$-\cos\left(\sqrt{x}\right)^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x^(1/2))*sin(x^(1/2))/x^(1/2),x, algorithm="maxima")
[Out] -cos(sqrt(x))^2
```

mupad [B] time = 3.05, size = 8, normalized size = 1.00

$$-\cos\left(\sqrt{x}\right)^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(x^(1/2))*sin(x^(1/2)))/x^(1/2),x)
[Out] -cos(x^(1/2))^2
```

sympy [A] time = 0.27, size = 8, normalized size = 1.00

$$-\cos^2\left(\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x**(1/2))*sin(x**(1/2))/x**(1/2),x)
```

```
[Out] -cos(sqrt(x))**2
```

3.845 $\int \frac{\sec\left(\sqrt{x}\right)\tan\left(\sqrt{x}\right)}{\sqrt{x}}\,dx$

Optimal. Leaf size=8

$$2\sec\left(\sqrt{x}\right)$$

```
[Out] 2*sec(x^(1/2))
```

Rubi [A] time = 0.18, antiderivative size = 8, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {6715, 2606, 8}

$$2\sec\left(\sqrt{x}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[Sqrt[x]]*Tan[Sqrt[x]])/Sqrt[x],x]
```

```
[Out] 2*Sec[Sqrt[x]]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 6715

```
Int[(u_)*(x_)^(m_.), x_Symbol] :> Dist[1/(m + 1), Subst[Int[SubstFor[x^(m +
 1), u, x], x], x, x^(m + 1)], x] /; FreeQ[m, x] && NeQ[m, -1] && FunctionO
fQ[x^(m + 1), u, x]
```

Rubi steps

$$\int \frac{\sec\left(\sqrt{x}\right)\tan\left(\sqrt{x}\right)}{\sqrt{x}}\,dx = 2\,\text{Subst}\left(\int \sec(x)\tan(x)\,dx, x, \sqrt{x}\right)$$
$$= 2\,\text{Subst}\left(\int 1\,dx, x, \sec\left(\sqrt{x}\right)\right)$$
$$= 2\sec\left(\sqrt{x}\right)$$

Mathematica [A] time = 0.02, size = 8, normalized size = 1.00

$$2\sec\left(\sqrt{x}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[Sqrt[x]]*Tan[Sqrt[x]])/Sqrt[x],x]
[Out] 2*Sec[Sqrt[x]]
```

fricas [A] time = 0.57, size = 8, normalized size = 1.00

$$\frac{2}{\cos\left(\sqrt{x}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x^(1/2))*tan(x^(1/2))/x^(1/2),x, algorithm="fricas")
[Out] 2/cos(sqrt(x))
```

giac [A] time = 0.14, size = 8, normalized size = 1.00

$$\frac{2}{\cos\left(\sqrt{x}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x^(1/2))*tan(x^(1/2))/x^(1/2),x, algorithm="giac")
[Out] 2/cos(sqrt(x))
```

maple [A] time = 0.06, size = 7, normalized size = 0.88

$$2\sec\left(\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(x^(1/2))*tan(x^(1/2))/x^(1/2),x)
```

```
[Out] 2*sec(x^(1/2))
```

maxima [A] time = 0.31, size = 8, normalized size = 1.00

$$\frac{2}{\cos\left(\sqrt{x}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x^(1/2))*tan(x^(1/2))/x^(1/2),x, algorithm="maxima")
```

```
[Out] 2/cos(sqrt(x))
```

mupad [B] time = 3.00, size = 8, normalized size = 1.00

$$\frac{2}{\cos\left(\sqrt{x}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(x^(1/2))/(x^(1/2)*cos(x^(1/2))),x)
```

```
[Out] 2/cos(x^(1/2))
```

sympy [A] time = 0.29, size = 7, normalized size = 0.88

$$2\sec\left(\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x**(1/2))*tan(x**(1/2))/x**(1/2),x)
```

```
[Out] 2*sec(sqrt(x))
```

3.846 $\int \frac{\sin^2(x)}{a+b\sin(2x)}\,dx$

Optimal. Leaf size=55

$$\frac{\tan^{-1}\left(\frac{a\tan(x)+b}{\sqrt{a^2-b^2}}\right)}{2\sqrt{a^2-b^2}}-\frac{\log(a+b\sin(2x))}{4b}$$

```
[Out] -1/4*ln(a+b*sin(2*x))/b+1/2*arctan((b+a*tan(x))/(a^2-b^2)^(1/2))/(a^2-b^2)^
(1/2)
```

Rubi [A] time = 0.17, antiderivative size = 70, normalized size of antiderivative = 1.27, number of steps used = 9, number of rules used = 7, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.467, Rules used = {1075, 12, 634, 618, 204, 628, 260}

$$\frac{\tan^{-1}\left(\frac{a\tan(x)+b}{\sqrt{a^2-b^2}}\right)}{2\sqrt{a^2-b^2}}-\frac{\log\left(a\tan^2(x)+a+2b\tan(x)\right)}{4b}-\frac{\log(\cos(x))}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[x]^2/(a + b*Sin[2*x]),x]
```

```
[Out] ArcTan[(b + a*Tan[x])/Sqrt[a^2 - b^2]]/(2*Sqrt[a^2 - b^2]) - Log[Cos[x]]/(2
*b) - Log[a + 2*b*Tan[x] + a*Tan[x]^2]/(4*b)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
```

```
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1075

```
Int[((A_.) + (C_.)*(x_)^2)/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*((d_) + (f_.
)*(x_)^2)), x_Symbol] :> With[{q = c^2*d^2 + b^2*d*f - 2*a*c*d*f + a^2*f^2}
, Dist[1/q, Int[(A*c^2*d - a*c*C*d + A*b^2*f - a*A*c*f + a^2*C*f + c*(-(b*C
*d) + A*b*f)*x)/(a + b*x + c*x^2), x], x] + Dist[1/q, Int[(c*C*d^2 - A*c*d*
f - a*C*d*f + a*A*f^2 - f*(-(b*C*d) + A*b*f)*x)/(d + f*x^2), x], x] /; NeQ[
q, 0]] /; FreeQ[{a, b, c, d, f, A, C}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{\sin^2(x)}{a+b\sin(2x)}\,dx = \text{Subst}\left(\int \frac{x^2}{(1+x^2)(a+2bx+ax^2)}\,dx, x, \tan(x)\right)$$
$$= \frac{\text{Subst}\left(\int \frac{2bx}{1+x^2}\,dx, x, \tan(x)\right)}{4b^2} + \frac{\text{Subst}\left(\int -\frac{2abx}{a+2bx+ax^2}\,dx, x, \tan(x)\right)}{4b^2}$$
$$= \frac{\text{Subst}\left(\int \frac{x}{1+x^2}\,dx, x, \tan(x)\right)}{2b} - \frac{a\,\text{Subst}\left(\int \frac{x}{a+2bx+ax^2}\,dx, x, \tan(x)\right)}{2b}$$
$$= -\frac{\log(\cos(x))}{2b} + \frac{1}{2}\text{Subst}\left(\int \frac{1}{a+2bx+ax^2}\,dx, x, \tan(x)\right) - \frac{\text{Subst}\left(\int \frac{2b+2ax}{a+2bx+ax^2}\,dx, x, \tan(\right.}{4b}$$
$$= -\frac{\log(\cos(x))}{2b} - \frac{\log\left(a+2b\tan(x)+a\tan^2(x)\right)}{4b} - \text{Subst}\left(\int \frac{1}{-4(a^2-b^2)-x^2}\,dx, x, 2b+\right.$$
$$= \frac{\tan^{-1}\left(\frac{2b+2a\tan(x)}{2\sqrt{a^2-b^2}}\right)}{2\sqrt{a^2-b^2}} - \frac{\log(\cos(x))}{2b} - \frac{\log\left(a+2b\tan(x)+a\tan^2(x)\right)}{4b}$$

Mathematica [A] time = 0.08, size = 55, normalized size = 1.00

$$\frac{\tan^{-1}\left(\frac{a\tan(x)+b}{\sqrt{a^2-b^2}}\right)}{2\sqrt{a^2-b^2}} - \frac{\log(a+b\sin(2x))}{4b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[x]^2/(a + b*Sin[2*x]),x]
```

```
[Out] ArcTan[(b + a*Tan[x])/Sqrt[a^2 - b^2]]/(2*Sqrt[a^2 - b^2]) - Log[a + b*Sin[
2*x]]/(4*b)
```

fricas [B] time = 1.91, size = 320, normalized size = 5.82

$$\left[-\frac{\sqrt{-a^2+b^2}\,b\log\left(-\frac{4\left(2a^2-b^2\right)\cos(x)^4-4ab\cos(x)\sin(x)-4\left(2a^2-b^2\right)\cos(x)^2+a^2-2b^2+2\left(2b\cos(x)^2+2\left(2a\cos(x)^3-a\cos(x)\right)\sin(x)-b\right)\sqrt{-}}{4b^2\cos(x)^4-4b^2\cos(x)^2-4ab\cos(x)\sin(x)-a^2}\right.}{8\left(a^2b-b^3\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^2/(a+b*sin(2*x)),x, algorithm="fricas")
```

```
[Out] [-1/8*(sqrt(-a^2 + b^2)*b*log(-(4*(2*a^2 - b^2)*cos(x)^4 - 4*a*b*cos(x)*sin
(x) - 4*(2*a^2 - b^2)*cos(x)^2 + a^2 - 2*b^2 + 2*(2*b*cos(x)^2 + 2*(2*a*cos
(x)^3 - a*cos(x))*sin(x) - b)*sqrt(-a^2 + b^2))/(4*b^2*cos(x)^4 - 4*b^2*cos
(x)^2 - 4*a*b*cos(x)*sin(x) - a^2)) + (a^2 - b^2)*log(-4*b^2*cos(x)^4 + 4*b
^2*cos(x)^2 + 4*a*b*cos(x)*sin(x) + a^2))/(a^2*b - b^3), -1/8*(2*sqrt(a^2 -
 b^2)*b*arctan(-(2*a*cos(x)*sin(x) + b)*sqrt(a^2 - b^2)/(2*(a^2 - b^2)*cos(
x)^2 - a^2 + b^2)) + (a^2 - b^2)*log(-4*b^2*cos(x)^4 + 4*b^2*cos(x)^2 + 4*a
*b*cos(x)*sin(x) + a^2))/(a^2*b - b^3)]
```

giac [A] time = 0.16, size = 77, normalized size = 1.40

$$\frac{\pi\left\lfloor\frac{x}{\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(a)+\arctan\left(\frac{a\tan(x)+b}{\sqrt{a^2-b^2}}\right)}{2\sqrt{a^2-b^2}}-\frac{\log\left(a\tan(x)^2+2\,b\tan(x)+a\right)}{4\,b}+\frac{\log\left(\tan(x)^2+1\right)}{4\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^2/(a+b*sin(2*x)),x, algorithm="giac")
```

```
[Out] 1/2*(pi*floor(x/pi + 1/2)*sgn(a) + arctan((a*tan(x) + b)/sqrt(a^2 - b^2)))/
sqrt(a^2 - b^2) - 1/4*log(a*tan(x)^2 + 2*b*tan(x) + a)/b + 1/4*log(tan(x)^2
 + 1)/b
```

maple [A] time = 0.19, size = 69, normalized size = 1.25

$$-\frac{\ln\left(a+2b\tan(x)+a\left(\tan^2(x)\right)\right)}{4b}+\frac{\arctan\left(\frac{2a\tan(x)+2b}{2\sqrt{a^2-b^2}}\right)}{2\sqrt{a^2-b^2}}+\frac{\ln\left(1+\tan^2(x)\right)}{4b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)^2/(a+b*sin(2*x)),x)
```

```
[Out] -1/4*ln(a+2*b*tan(x)+a*tan(x)^2)/b+1/2/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(
x)+2*b)/(a^2-b^2)^(1/2))+1/4/b*ln(1+tan(x)^2)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^2/(a+b*sin(2*x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more details)Is 4*b^2-4*a^2 positive or negative?
```

mupad [B] time = 4.18, size = 1108, normalized size = 20.15

$$\frac{\ln\left(\tan(x)^2+1\right)}{4\,b}+\frac{\mathrm{atan}\left(\frac{2\tan(x)\left(a^2-b^2\right)^{3/2}\left(\frac{\left(4\,a^2\,b-2\,b^3\right)\left(2\,a\,b-\frac{\frac{8\,a\,b^3+\frac{\left(8\,a^2\,b-8\,b^3\right)\left(96\,a\,b^4-64\,a^3\,b^2\right)}{2\left(16\,b^4-16\,a^2\,b^2\right)}}{4\,\sqrt{a^2-b^2}}+\frac{\left(8\,a^2\,b-8\,b^3\right)\left(96\,a\,b^4-64\,a^3\,b^2\right)}{8\,\sqrt{a^2-b^2}\,\left(16\,b^4-16\,a^2\,b^2\right)}}{4\,\sqrt{a^2-b^2}}+\frac{\left(8\,a^2\,b-8\,b^3\right)\left(4\,a\cdots\right.}{\cdots}\right)}{a^3\left(4\,a^2-3\,b^2\cdots\right.}\cdots\right)}{\cdots}\right)}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)^2/(a + b*sin(2*x)),x)
```

```
[Out] log(tan(x)^2 + 1)/(4*b) + atan((2*tan(x)*(a^2 - b^2)^(3/2)*(((4*a^2*b - 2*b
^3)*(2*a*b - ((8*a*b^3 + ((8*a^2*b - 8*b^3)*(96*a*b^4 - 64*a^3*b^2))/(2*(16
*b^4 - 16*a^2*b^2)))/(4*(a^2 - b^2)^(1/2)) + ((8*a^2*b - 8*b^3)*(96*a*b^4 -
 64*a^3*b^2))/(8*(a^2 - b^2)^(1/2)*(16*b^4 - 16*a^2*b^2))))/(4*(a^2 - b^2)^(
1/2)) + ((8*a^2*b - 8*b^3)*(4*a^3 - 16*a*b^2 + ((8*a^2*b - 8*b^3)*(8*a*b^3
+ ((8*a^2*b - 8*b^3)*(96*a*b^4 - 64*a^3*b^2))/(2*(16*b^4 - 16*a^2*b^2))))/(
2*(16*b^4 - 16*a^2*b^2))))/(2*(16*b^4 - 16*a^2*b^2)) - ((8*a^2*b - 8*b^3)*(
96*a*b^4 - 64*a^3*b^2))/(32*(a^2 - b^2)*(16*b^4 - 16*a^2*b^2))))/(a^3*(4*a^
2 - 3*b^2)^2) - ((4*a^4 + 2*b^4 - 5*a^2*b^2)*((4*a^3 - 16*a*b^2 + ((8*a^2*b
 - 8*b^3)*(8*a*b^3 + ((8*a^2*b - 8*b^3)*(96*a*b^4 - 64*a^3*b^2))/(2*(16*b^4
```

```
 - 16*a^2*b^2))))/(2*(16*b^4 - 16*a^2*b^2)))/(4*(a^2 - b^2)^(1/2)) - (96*a*
b^4 - 64*a^3*b^2)/(64*(a^2 - b^2)^(3/2)) + ((8*a^2*b - 8*b^3)*((8*a*b^3 + (
(8*a^2*b - 8*b^3)*(96*a*b^4 - 64*a^3*b^2))/(2*(16*b^4 - 16*a^2*b^2)))/(4*(a
^2 - b^2)^(1/2)) + ((8*a^2*b - 8*b^3)*(96*a*b^4 - 64*a^3*b^2))/(8*(a^2 - b^
2)^(1/2)*(16*b^4 - 16*a^2*b^2))))/(2*(16*b^4 - 16*a^2*b^2))))/(a^3*(a^2 - b
^2)^(1/2)*(4*a^2 - 3*b^2)^2)))/a + (2*(a^2 - b^2)*((6*a^2*b - (8*a^2*b^3*(8
*a^2*b - 8*b^3)^2)/(16*b^4 - 16*a^2*b^2)^2)/(4*(a^2 - b^2)^(1/2)) + (a^2*b^
3)/(2*(a^2 - b^2)^(3/2)) - (4*a^2*b^3*(8*a^2*b - 8*b^3)^2)/((a^2 - b^2)^(1/
2)*(16*b^4 - 16*a^2*b^2)^2))*(4*a^4 + 2*b^4 - 5*a^2*b^2))/(a^4*(4*a^2 - 3*b
^2)^2) - (2*(4*a^2*b - 2*b^3)*(a^2 - b^2)^(3/2)*(((8*a^2*b - 8*b^3)*(6*a^2*
b - (8*a^2*b^3*(8*a^2*b - 8*b^3)^2)/(16*b^4 - 16*a^2*b^2)^2))/(2*(16*b^4 -
16*a^2*b^2)) - a^2 + (3*a^2*b^3*(8*a^2*b - 8*b^3))/((a^2 - b^2)*(16*b^4 - 1
6*a^2*b^2))))/(a^4*(4*a^2 - 3*b^2)^2))/(2*(a^2 - b^2)^(1/2)) + (log(a + a*t
an(x)^2 + 2*b*tan(x))*(8*a^2*b - 8*b^3))/(2*(16*b^4 - 16*a^2*b^2))
```

sympy [A] time = 9.31, size = 155, normalized size = 2.82

$$-\begin{cases} \frac{\log\left(\frac{a}{b}+\sin(2x)\right)}{4b} & \text{for } b \neq 0 \\ \frac{\sin(2x)}{4a} & \text{otherwise} \end{cases} + \begin{cases} \frac{\sqrt{b^2}}{2b^2\tan(x)-2b\sqrt{b^2}} & \text{for } a = -\sqrt{b^2} \\ -\frac{\sqrt{b^2}}{2b^2\tan(x)+2b\sqrt{b^2}} & \text{for } a = \sqrt{b^2} \\ \frac{\log(\tan(x))}{4b} & \text{for } a = 0 \\ \frac{\log\left(\tan(x)+\frac{b}{a}-\frac{\sqrt{-a^2+b^2}}{a}\right)}{4\sqrt{-a^2+b^2}} - \frac{\log\left(\tan(x)+\frac{b}{a}+\frac{\sqrt{-a^2+b^2}}{a}\right)}{4\sqrt{-a^2+b^2}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)**2/(a+b*sin(2*x)),x)
```

```
[Out] -Piecewise((log(a/b + sin(2*x))/(4*b), Ne(b, 0)), (sin(2*x)/(4*a), True)) +
 Piecewise((sqrt(b**2)/(2*b**2*tan(x) - 2*b*sqrt(b**2)), Eq(a, -sqrt(b**2))
), (-sqrt(b**2)/(2*b**2*tan(x) + 2*b*sqrt(b**2)), Eq(a, sqrt(b**2))), (log(
tan(x))/(4*b), Eq(a, 0)), (log(tan(x) + b/a - sqrt(-a**2 + b**2)/a)/(4*sqrt
(-a**2 + b**2)) - log(tan(x) + b/a + sqrt(-a**2 + b**2)/a)/(4*sqrt(-a**2 +
b**2)), True))
```

3.847 $\int \frac{\cos^2(x)}{a+b\sin(2x)}\,dx$

Optimal. Leaf size=55

$$\frac{\tan^{-1}\left(\frac{a\tan(x)+b}{\sqrt{a^2-b^2}}\right)}{2\sqrt{a^2-b^2}}+\frac{\log(a+b\sin(2x))}{4b}$$

```
[Out] 1/4*ln(a+b*sin(2*x))/b+1/2*arctan((b+a*tan(x))/(a^2-b^2)^(1/2))/(a^2-b^2)^(
1/2)
```

Rubi [A] time = 0.13, antiderivative size = 70, normalized size of antiderivative = 1.27, number of steps used = 8, number of rules used = 7, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.467, Rules used = {981, 634, 618, 204, 628, 12, 260}

$$\frac{\tan^{-1}\left(\frac{a\tan(x)+b}{\sqrt{a^2-b^2}}\right)}{2\sqrt{a^2-b^2}}+\frac{\log\left(a\tan^2(x)+a+2b\tan(x)\right)}{4b}+\frac{\log(\cos(x))}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[x]^2/(a + b*Sin[2*x]),x]
```

```
[Out] ArcTan[(b + a*Tan[x])/Sqrt[a^2 - b^2]]/(2*Sqrt[a^2 - b^2]) + Log[Cos[x]]/(2
*b) + Log[a + 2*b*Tan[x] + a*Tan[x]^2]/(4*b)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
```

```
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 981

```
Int[1/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*((d_) + (f_.)*(x_)^2)), x_Symbol]
 :> With[{q = c^2*d^2 + b^2*d*f - 2*a*c*d*f + a^2*f^2}, Dist[1/q, Int[(c^2*
d + b^2*f - a*c*f + b*c*f*x)/(a + b*x + c*x^2), x], x] - Dist[1/q, Int[(c*d
*f - a*f^2 + b*f^2*x)/(d + f*x^2), x], x] /; NeQ[q, 0]] /; FreeQ[{a, b, c,
d, f}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{\cos^2(x)}{a+b\sin(2x)}\,dx = \text{Subst}\left(\int \frac{1}{\left(1+x^2\right)\left(a+2bx+ax^2\right)}\,dx,x,\tan(x)\right)$$

$$= -\frac{\text{Subst}\left(\int \frac{2bx}{1+x^2}\,dx,x,\tan(x)\right)}{4b^2}+\frac{\text{Subst}\left(\int \frac{4b^2+2abx}{a+2bx+ax^2}\,dx,x,\tan(x)\right)}{4b^2}$$

$$= \frac{1}{2}\text{Subst}\left(\int \frac{1}{a+2bx+ax^2}\,dx,x,\tan(x)\right)+\frac{\text{Subst}\left(\int \frac{2b+2ax}{a+2bx+ax^2}\,dx,x,\tan(x)\right)}{4b}-\frac{\text{Subst}\left(\int \frac{}{1\cdot}\right.}{}$$

$$= \frac{\log(\cos(x))}{2b}+\frac{\log\left(a+2b\tan(x)+a\tan^2(x)\right)}{4b}-\text{Subst}\left(\int \frac{1}{-4\left(a^2-b^2\right)-x^2}\,dx,x,2b+2a\right.$$

$$= \frac{\tan^{-1}\left(\frac{2b+2a\tan(x)}{2\sqrt{a^2-b^2}}\right)}{2\sqrt{a^2-b^2}}+\frac{\log(\cos(x))}{2b}+\frac{\log\left(a+2b\tan(x)+a\tan^2(x)\right)}{4b}$$

Mathematica [A] time = 0.06, size = 54, normalized size = 0.98

$$\frac{1}{4}\left(\frac{2\tan^{-1}\left(\frac{a\tan(x)+b}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}}+\frac{\log(a+b\sin(2x))}{b}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[x]^2/(a + b*Sin[2*x]),x]
```

```
[Out] ((2*ArcTan[(b + a*Tan[x])/Sqrt[a^2 - b^2]])/Sqrt[a^2 - b^2] + Log[a + b*Sin
[2*x]]/b)/4
```

fricas [B] time = 1.28, size = 322, normalized size = 5.85

$$\left[-\frac{\sqrt{-a^2+b^2}\,b\log\left(-\frac{4\left(2a^2-b^2\right)\cos(x)^4-4ab\cos(x)\sin(x)-4\left(2a^2-b^2\right)\cos(x)^2+a^2-2b^2+2\left(2b\cos(x)^2+2\left(2a\cos(x)^3-a\cos(x)\right)\sin(x)-b\right)\sqrt{-}}{4b^2\cos(x)^4-4b^2\cos(x)^2-4ab\cos(x)\sin(x)-a^2}\right.}{8\left(a^2b-b^3\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^2/(a+b*sin(2*x)),x, algorithm="fricas")
```

```
[Out] [-1/8*(sqrt(-a^2 + b^2)*b*log(-(4*(2*a^2 - b^2)*cos(x)^4 - 4*a*b*cos(x)*sin
(x) - 4*(2*a^2 - b^2)*cos(x)^2 + a^2 - 2*b^2 + 2*(2*b*cos(x)^2 + 2*(2*a*cos
(x)^3 - a*cos(x))*sin(x) - b)*sqrt(-a^2 + b^2))/(4*b^2*cos(x)^4 - 4*b^2*cos
(x)^2 - 4*a*b*cos(x)*sin(x) - a^2)) - (a^2 - b^2)*log(-4*b^2*cos(x)^4 + 4*b
^2*cos(x)^2 + 4*a*b*cos(x)*sin(x) + a^2))/(a^2*b - b^3), -1/8*(2*sqrt(a^2 -
 b^2)*b*arctan(-(2*a*cos(x)*sin(x) + b)*sqrt(a^2 - b^2)/(2*(a^2 - b^2)*cos(
x)^2 - a^2 + b^2)) - (a^2 - b^2)*log(-4*b^2*cos(x)^4 + 4*b^2*cos(x)^2 + 4*a
*b*cos(x)*sin(x) + a^2))/(a^2*b - b^3)]
```

giac [A] time = 0.16, size = 77, normalized size = 1.40

$$\frac{\pi\left\lfloor\frac{x}{\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(a)+\arctan\left(\frac{a\tan(x)+b}{\sqrt{a^2-b^2}}\right)}{2\sqrt{a^2-b^2}}+\frac{\log\left(a\tan(x)^2+2b\tan(x)+a\right)}{4b}-\frac{\log\left(\tan(x)^2+1\right)}{4b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^2/(a+b*sin(2*x)),x, algorithm="giac")
```

```
[Out] 1/2*(pi*floor(x/pi + 1/2)*sgn(a) + arctan((a*tan(x) + b)/sqrt(a^2 - b^2)))/
sqrt(a^2 - b^2) + 1/4*log(a*tan(x)^2 + 2*b*tan(x) + a)/b - 1/4*log(tan(x)^2
 + 1)/b
```

maple [A] time = 0.17, size = 69, normalized size = 1.25

$$\frac{\ln\left(a+2b\tan(x)+a\left(\tan^2(x)\right)\right)}{4b}+\frac{\arctan\left(\frac{2a\tan(x)+2b}{2\sqrt{a^2-b^2}}\right)}{2\sqrt{a^2-b^2}}-\frac{\ln\left(1+\tan^2(x)\right)}{4b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)^2/(a+b*sin(2*x)),x)
```

```
[Out] 1/4*ln(a+2*b*tan(x)+a*tan(x)^2)/b+1/2/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(x
)+2*b)/(a^2-b^2)^(1/2))-1/4/b*ln(1+tan(x)^2)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^2/(a+b*sin(2*x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more details)Is 4*b^2-4*a^2 positive or negative?
```

mupad [B] time = 3.44, size = 1374, normalized size = 24.98

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)^2/(a + b*sin(2*x)),x)
```

```
[Out] - log(tan(x)^2 + 1)/(4*b) - atan((2*tan(x)*((6*b*(2*a^2 - 3*b^2)*(((24*a*b^
3 - ((8*a^2*b - 8*b^3)*(96*a*b^4 - 64*a^3*b^2))/(2*(16*b^4 - 16*a^2*b^2)))/
(4*(a^2 - b^2)^(1/2)) - ((8*a^2*b - 8*b^3)*(96*a*b^4 - 64*a^3*b^2))/(8*(a^2
 - b^2)^(1/2)*(16*b^4 - 16*a^2*b^2)))/(4*(a^2 - b^2)^(1/2)) + ((8*a^2*b - 8
*b^3)*(4*a^3 - ((8*a^2*b - 8*b^3)*(24*a*b^3 - ((8*a^2*b - 8*b^3)*(96*a*b^4
- 64*a^3*b^2))/(2*(16*b^4 - 16*a^2*b^2))))/(2*(16*b^4 - 16*a^2*b^2))))/(2*(
16*b^4 - 16*a^2*b^2)) - ((8*a^2*b - 8*b^3)*(96*a*b^4 - 64*a^3*b^2))/(32*(a^
2 - b^2)*(16*b^4 - 16*a^2*b^2))))/(a^3*(4*a^2 - 3*b^2)^2) - (((96*a*b^4 - 6
4*a^3*b^2)/(64*(a^2 - b^2)^(3/2)) - (4*a^3 - ((8*a^2*b - 8*b^3)*(24*a*b^3 -
 ((8*a^2*b - 8*b^3)*(96*a*b^4 - 64*a^3*b^2))/(2*(16*b^4 - 16*a^2*b^2))))/(2
*(16*b^4 - 16*a^2*b^2)))/(4*(a^2 - b^2)^(1/2)) + ((8*a^2*b - 8*b^3)*((24*a*
b^3 - ((8*a^2*b - 8*b^3)*(96*a*b^4 - 64*a^3*b^2))/(2*(16*b^4 - 16*a^2*b^2))
)/(4*(a^2 - b^2)^(1/2)) - ((8*a^2*b - 8*b^3)*(96*a*b^4 - 64*a^3*b^2))/(8*(a
```

```
^2 - b^2)^(1/2)*(16*b^4 - 16*a^2*b^2))))/(2*(16*b^4 - 16*a^2*b^2)))*(4*a^4
+ 18*b^4 - 21*a^2*b^2))/(a^3*(a^2 - b^2)^(1/2)*(4*a^2 - 3*b^2)^2))*(a^2 - b
^2)^(3/2))/a - (2*(a^2 - b^2)*((a^2*b^3)/(2*(a^2 - b^2)^(3/2)) - (2*a^2*b -
 ((8*a^2*b - 8*b^3)*(16*a^2*b^2 - (16*a^2*b^3*(8*a^2*b - 8*b^3))/(16*b^4 -
16*a^2*b^2)))/(2*(16*b^4 - 16*a^2*b^2)))/(4*(a^2 - b^2)^(1/2)) + ((8*a^2*b
- 8*b^3)*((16*a^2*b^2 - (16*a^2*b^3*(8*a^2*b - 8*b^3))/(16*b^4 - 16*a^2*b^2
))/(4*(a^2 - b^2)^(1/2)) - (4*a^2*b^3*(8*a^2*b - 8*b^3))/((a^2 - b^2)^(1/2)
*(16*b^4 - 16*a^2*b^2))))/(2*(16*b^4 - 16*a^2*b^2)))*(4*a^4 + 18*b^4 - 21*a
^2*b^2))/(a^4*(4*a^2 - 3*b^2)^2) + (12*b*(a^2 - b^2)^(3/2)*(2*a^2 - 3*b^2)*
(((16*a^2*b^2 - (16*a^2*b^3*(8*a^2*b - 8*b^3))/(16*b^4 - 16*a^2*b^2))/(4*(a
^2 - b^2)^(1/2)) - (4*a^2*b^3*(8*a^2*b - 8*b^3))/((a^2 - b^2)^(1/2)*(16*b^4
 - 16*a^2*b^2)))/(4*(a^2 - b^2)^(1/2)) + ((8*a^2*b - 8*b^3)*(2*a^2*b - ((8*
a^2*b - 8*b^3)*(16*a^2*b^2 - (16*a^2*b^3*(8*a^2*b - 8*b^3))/(16*b^4 - 16*a^
2*b^2)))/(2*(16*b^4 - 16*a^2*b^2))))/(2*(16*b^4 - 16*a^2*b^2)) - (a^2*b^3*(
8*a^2*b - 8*b^3))/((a^2 - b^2)*(16*b^4 - 16*a^2*b^2))))/(a^4*(4*a^2 - 3*b^2
)^2))/(2*(a^2 - b^2)^(1/2)) - (log(a + a*tan(x)^2 + 2*b*tan(x))*(8*a^2*b -
8*b^3))/(2*(16*b^4 - 16*a^2*b^2))
```

sympy [A] time = 9.45, size = 155, normalized size = 2.82

$$\begin{cases} \frac{\log\left(\frac{a}{b}+\sin(2x)\right)}{4b} & \text{for } b \neq 0 \\ \frac{\sin(2x)}{4a} & \text{otherwise} \end{cases} + \begin{cases} \frac{\sqrt{b^2}}{2b^2\tan(x)-2b\sqrt{b^2}} & \text{for } a = -\sqrt{b^2} \\ -\frac{\sqrt{b^2}}{2b^2\tan(x)+2b\sqrt{b^2}} & \text{for } a = \sqrt{b^2} \\ \frac{\log(\tan(x))}{4b} & \text{for } a = 0 \\ \frac{\log\left(\tan(x)+\frac{b}{a}-\frac{\sqrt{-a^2+b^2}}{a}\right)}{4\sqrt{-a^2+b^2}} - \frac{\log\left(\tan(x)+\frac{b}{a}+\frac{\sqrt{-a^2+b^2}}{a}\right)}{4\sqrt{-a^2+b^2}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)**2/(a+b*sin(2*x)),x)
```

```
[Out] Piecewise((log(a/b + sin(2*x))/(4*b), Ne(b, 0)), (sin(2*x)/(4*a), True)) +
Piecewise((sqrt(b**2)/(2*b**2*tan(x) - 2*b*sqrt(b**2)), Eq(a, -sqrt(b**2)))
, (-sqrt(b**2)/(2*b**2*tan(x) + 2*b*sqrt(b**2)), Eq(a, sqrt(b**2))), (log(t
an(x))/(4*b), Eq(a, 0)), (log(tan(x) + b/a - sqrt(-a**2 + b**2)/a)/(4*sqrt(
-a**2 + b**2)) - log(tan(x) + b/a + sqrt(-a**2 + b**2)/a)/(4*sqrt(-a**2 + b
**2)), True))
```

3.848 $\int \frac{\sin^2(x)}{a+b\cos(2x)}\,dx$

Optimal. Leaf size=52

$$\frac{\sqrt{a+b}\,\tan^{-1}\left(\frac{\sqrt{a-b}\,\tan(x)}{\sqrt{a+b}}\right)}{2b\sqrt{a-b}}-\frac{x}{2b}$$

```
[Out] -1/2*x/b+1/2*arctan((a-b)^(1/2)*tan(x)/(a+b)^(1/2))*(a+b)^(1/2)/b/(a-b)^(1/
2)
```

Rubi [A] time = 0.12, antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {1130, 205}

$$\frac{\sqrt{a+b}\,\tan^{-1}\left(\frac{\sqrt{a-b}\,\tan(x)}{\sqrt{a+b}}\right)}{2b\sqrt{a-b}}-\frac{x}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[x]^2/(a + b*Cos[2*x]),x]
```

```
[Out] -x/(2*b) + (Sqrt[a + b]*ArcTan[(Sqrt[a - b]*Tan[x])/Sqrt[a + b]])/(2*Sqrt[a
 - b]*b)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1130

```
Int[((d_.)*(x_))^(m_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> Wi
th[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(d^2*(b/q + 1))/2, Int[(d*x)^(m - 2)/(b/2
 + q/2 + c*x^2), x], x] - Dist[(d^2*(b/q - 1))/2, Int[(d*x)^(m - 2)/(b/2 -
q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - 4*a*c, 0] && G
eQ[m, 2]
```

Rubi steps

$$\int \frac{\sin^2(x)}{a+b\cos(2x)}\,dx = \text{Subst}\left(\int \frac{x^2}{a+b+2ax^2+(a-b)x^4}\,dx, x, \tan(x)\right)$$
$$= -\left(\frac{1}{2}\left(-1+\frac{a}{b}\right)\text{Subst}\left(\int \frac{1}{a-b+(a-b)x^2}\,dx, x, \tan(x)\right)\right) + \frac{(a+b)\,\text{Subst}\left(\int \frac{1}{a+b+(a-b)x^2}\, d\right.}{2b}$$
$$= -\frac{x}{2b} + \frac{\sqrt{a+b}\,\tan^{-1}\left(\frac{\sqrt{a-b}\,\tan(x)}{\sqrt{a+b}}\right)}{2\sqrt{a-b}\,b}$$

Mathematica [A] time = 0.09, size = 48, normalized size = 0.92

$$-\frac{\frac{(a+b)\tanh^{-1}\left(\frac{(a-b)\tan(x)}{\sqrt{b^2-a^2}}\right)}{\sqrt{b^2-a^2}}+x}{2b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[x]^2/(a + b*Cos[2*x]),x]
```

```
[Out] -1/2*(x + ((a + b)*ArcTanh[((a - b)*Tan[x])/Sqrt[-a^2 + b^2]])/Sqrt[-a^2 +
b^2])/b
```

fricas [A] time = 1.22, size = 225, normalized size = 4.33

$$\left[\frac{\sqrt{-\frac{a+b}{a-b}}\,\log\left(\frac{4\left(2a^2-b^2\right)\cos(x)^4-4\left(2a^2-ab-b^2\right)\cos(x)^2-4\left(2\left(a^2-ab\right)\cos(x)^3-\left(a^2-2ab+b^2\right)\cos(x)\right)\sqrt{-\frac{a+b}{a-b}}\,\sin(x)+a^2-2ab+b^2}{4b^2\cos(x)^4+4\left(ab-b^2\right)\cos(x)^2+a^2-2ab+b^2}\right)-4x}{8b}, -\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^2/(a+b*cos(2*x)),x, algorithm="fricas")
```

```
[Out] [1/8*(sqrt(-(a + b)/(a - b))*log((4*(2*a^2 - b^2)*cos(x)^4 - 4*(2*a^2 - a*b
 - b^2)*cos(x)^2 - 4*(2*(a^2 - a*b)*cos(x)^3 - (a^2 - 2*a*b + b^2)*cos(x))*
sqrt(-(a + b)/(a - b))*sin(x) + a^2 - 2*a*b + b^2)/(4*b^2*cos(x)^4 + 4*(a*b
 - b^2)*cos(x)^2 + a^2 - 2*a*b + b^2)) - 4*x)/b, -1/4*(sqrt((a + b)/(a - b)
)*arctan(1/2*(2*a*cos(x)^2 - a + b)*sqrt((a + b)/(a - b))/((a + b)*cos(x)*s
in(x))) + 2*x)/b]
```

giac [B] time = 0.17, size = 141, normalized size = 2.71

$$\frac{\sqrt{a^2-b^2}\left(\pi\left\lfloor\frac{x}{\pi}+\frac{1}{2}\right\rfloor+\arctan\left(\frac{2\tan(x)}{\sqrt{\frac{4a+\sqrt{-16(a+b)(a-b)+16a^2}}{a-b}}}\right)\right)|a-b|}{2\left(a^2-2ab+b^2\right)|b|}-\frac{\pi\left\lfloor\frac{x}{\pi}+\frac{1}{2}\right\rfloor+\arctan\left(\frac{2\tan(x)}{\sqrt{\frac{4a-\sqrt{-16(a+b)(a-b)+16a^2}}{a-b}}}\right)}{2|b|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^2/(a+b*cos(2*x)),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(a^2 - b^2)*(pi*floor(x/pi + 1/2) + arctan(2*tan(x)/sqrt((4*a + sqr
t(-16*(a + b)*(a - b) + 16*a^2))/(a - b))))*abs(a - b)/((a^2 - 2*a*b + b^2)
*abs(b)) - 1/2*(pi*floor(x/pi + 1/2) + arctan(2*tan(x)/sqrt((4*a - sqrt(-16
*(a + b)*(a - b) + 16*a^2))/(a - b))))/abs(b)
```

maple [A] time = 0.13, size = 80, normalized size = 1.54

$$\frac{\arctan\left(\frac{\tan(x)(a-b)}{\sqrt{(a+b)(a-b)}}\right)a}{2b\sqrt{(a+b)(a-b)}}+\frac{\arctan\left(\frac{\tan(x)(a-b)}{\sqrt{(a+b)(a-b)}}\right)}{2\sqrt{(a+b)(a-b)}}-\frac{\arctan(\tan(x))}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)^2/(a+b*cos(2*x)),x)
```

```
[Out] 1/2/b/((a+b)*(a-b))^(1/2)*arctan(tan(x)*(a-b)/((a+b)*(a-b))^(1/2))*a+1/2/((
a+b)*(a-b))^(1/2)*arctan(tan(x)*(a-b)/((a+b)*(a-b))^(1/2))-1/2/b*arctan(tan
(x))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^2/(a+b*cos(2*x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` for
more details)Is 4*a^2-4*b^2 positive or negative?
```

mupad [B] time = 3.43, size = 108, normalized size = 2.08

$$\frac{\mathrm{atan}\left(\frac{2b^3\tan(x)}{2a^2b-2b^3}-\frac{2a^2b\tan(x)}{2a^2b-2b^3}\right)}{2b}+\frac{\mathrm{atanh}\left(\frac{a\tan(x)}{\sqrt{b^2-a^2}}-\frac{b\tan(x)}{\sqrt{b^2-a^2}}\right)\sqrt{b^2-a^2}}{2\left(ab-b^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)^2/(a + b*cos(2*x)),x)
```

```
[Out] atan((2*b^3*tan(x))/(2*a^2*b - 2*b^3) - (2*a^2*b*tan(x))/(2*a^2*b - 2*b^3))
/(2*b) + (atanh((a*tan(x))/(b^2 - a^2)^(1/2) - (b*tan(x))/(b^2 - a^2)^(1/2)
)*(b^2 - a^2)^(1/2))/(2*(a*b - b^2))
```

sympy [B] time = 35.27, size = 432, normalized size = 8.31

$$\begin{cases} \tilde{\infty}\left(-\frac{\log(\tan(x)-1)}{2}+\frac{\log(\tan(x)+1)}{2}\right) & \text{for } a=0 \wedge b=0 \\ \frac{1}{4b\tan(x)} & \text{for } a=-b \\ \frac{\tan(x)}{4b} & \text{for } a=b \\ \frac{\log\left(-\sqrt{-\frac{a}{a-b}-\frac{b}{a-b}}+\tan(x)\right)}{4a\sqrt{-\frac{a}{a-b}-\frac{b}{a-b}}-4b\sqrt{-\frac{a}{a-b}-\frac{b}{a-b}}}-\frac{\log\left(\sqrt{-\frac{a}{a-b}-\frac{b}{a-b}}+\tan(x)\right)}{4a\sqrt{-\frac{a}{a-b}-\frac{b}{a-b}}-4b\sqrt{-\frac{a}{a-b}-\frac{b}{a-b}}} & \text{otherwise} \end{cases} - \begin{cases} \tilde{\infty}x \\ \frac{x}{2b}-\frac{\tan(x)}{4b} \\ \frac{x}{2b}+\frac{1}{4b\tan(x)} \\ \frac{\sin(2x)}{4a} \\ \frac{2ax\sqrt{-\frac{a}{a-b}-\frac{b}{a-b}}}{4ab\sqrt{-\frac{a}{a-b}-\frac{b}{a-b}}-4b^2\sqrt{-\frac{a}{a-b}-\frac{b}{a-b}}}-\frac{a\log}{4ab\sqrt{-}} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)**2/(a+b*cos(2*x)),x)
```

```
[Out] Piecewise((zoo*(-log(tan(x) - 1)/2 + log(tan(x) + 1)/2), Eq(a, 0) & Eq(b, 0
)), (1/(4*b*tan(x)), Eq(a, -b)), (tan(x)/(4*b), Eq(a, b)), (log(-sqrt(-a/(a
 - b) - b/(a - b)) + tan(x))/(4*a*sqrt(-a/(a - b) - b/(a - b)) - 4*b*sqrt(-
a/(a - b) - b/(a - b))) - log(sqrt(-a/(a - b) - b/(a - b)) + tan(x))/(4*a*s
qrt(-a/(a - b) - b/(a - b)) - 4*b*sqrt(-a/(a - b) - b/(a - b))), True)) - P
iecewise((zoo*x, Eq(a, 0) & Eq(b, 0)), (x/(2*b) - tan(x)/(4*b), Eq(a, b)),
(x/(2*b) + 1/(4*b*tan(x)), Eq(a, -b)), (sin(2*x)/(4*a), Eq(b, 0)), (2*a*x*s
qrt(-a/(a - b) - b/(a - b))/(4*a*b*sqrt(-a/(a - b) - b/(a - b)) - 4*b**2*sq
rt(-a/(a - b) - b/(a - b))) - a*log(-sqrt(-a/(a - b) - b/(a - b)) + tan(x))
/(4*a*b*sqrt(-a/(a - b) - b/(a - b)) - 4*b**2*sqrt(-a/(a - b) - b/(a - b)))
 + a*log(sqrt(-a/(a - b) - b/(a - b)) + tan(x))/(4*a*b*sqrt(-a/(a - b) - b/
(a - b)) - 4*b**2*sqrt(-a/(a - b) - b/(a - b))) - 2*b*x*sqrt(-a/(a - b) - b
/(a - b))/(4*a*b*sqrt(-a/(a - b) - b/(a - b)) - 4*b**2*sqrt(-a/(a - b) - b/
(a - b))), True))
```

3.849 $\int \frac{\cos^2(x)}{a+b\cos(2x)}\,dx$

Optimal. Leaf size=52

$$\frac{x}{2b}-\frac{\sqrt{a-b}\,\tan^{-1}\left(\frac{\sqrt{a-b}\,\tan(x)}{\sqrt{a+b}}\right)}{2b\sqrt{a+b}}$$

```
[Out] 1/2*x/b-1/2*arctan((a-b)^(1/2)*tan(x)/(a+b)^(1/2))*(a-b)^(1/2)/b/(a+b)^(1/2
)
```

Rubi [A] time = 0.09, antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {1093, 205}

$$\frac{x}{2b}-\frac{\sqrt{a-b}\,\tan^{-1}\left(\frac{\sqrt{a-b}\,\tan(x)}{\sqrt{a+b}}\right)}{2b\sqrt{a+b}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[x]^2/(a + b*Cos[2*x]),x]
```

```
[Out] x/(2*b) - (Sqrt[a - b]*ArcTan[(Sqrt[a - b]*Tan[x])/Sqrt[a + b]])/(2*b*Sqrt[
a + b])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1093

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(-1), x_Symbol] :> With[{q = Rt[b^
2 - 4*a*c, 2]}, Dist[c/q, Int[1/(b/2 - q/2 + c*x^2), x], x] - Dist[c/q, Int
[1/(b/2 + q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c,
0] && PosQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{\cos^2(x)}{a+b\cos(2x)}\,dx = \text{Subst}\left(\int \frac{1}{a+b+2ax^2+(a-b)x^4}\,dx, x, \tan(x)\right)$$
$$= \frac{(a-b)\,\text{Subst}\left(\int \frac{1}{a-b+(a-b)x^2}\,dx, x, \tan(x)\right)}{2b} - \frac{(a-b)\,\text{Subst}\left(\int \frac{1}{a+b+(a-b)x^2}\,dx, x, \tan(x)\right)}{2b}$$
$$= \frac{x}{2b} - \frac{\sqrt{a-b}\,\tan^{-1}\left(\frac{\sqrt{a-b}\,\tan(x)}{\sqrt{a+b}}\right)}{2b\sqrt{a+b}}$$

Mathematica [A] time = 0.05, size = 50, normalized size = 0.96

$$\frac{\frac{(a-b)\tanh^{-1}\left(\frac{(a-b)\tan(x)}{\sqrt{b^2-a^2}}\right)}{\sqrt{b^2-a^2}}+x}{2b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[x]^2/(a + b*Cos[2*x]),x]
```

```
[Out] (x + ((a - b)*ArcTanh[((a - b)*Tan[x])/Sqrt[-a^2 + b^2]])/Sqrt[-a^2 + b^2])
/(2*b)
```

fricas [A] time = 1.11, size = 224, normalized size = 4.31

$$\left[\frac{\sqrt{-\frac{a-b}{a+b}}\,\log\left(\frac{4\left(2a^2-b^2\right)\cos(x)^4-4\left(2a^2-ab-b^2\right)\cos(x)^2+4\left(2\left(a^2+ab\right)\cos(x)^3-\left(a^2-b^2\right)\cos(x)\right)\sqrt{-\frac{a-b}{a+b}}\,\sin(x)+a^2-2ab+b^2}{4b^2\cos(x)^4+4\left(ab-b^2\right)\cos(x)^2+a^2-2ab+b^2}\right)+4x}{8b}, -\frac{\sqrt{\frac{a-b}{a+b}}}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^2/(a+b*cos(2*x)),x, algorithm="fricas")
```

```
[Out] [1/8*(sqrt(-(a - b)/(a + b))*log((4*(2*a^2 - b^2)*cos(x)^4 - 4*(2*a^2 - a*b
 - b^2)*cos(x)^2 + 4*(2*(a^2 + a*b)*cos(x)^3 - (a^2 - b^2)*cos(x))*sqrt(-(a
 - b)/(a + b))*sin(x) + a^2 - 2*a*b + b^2)/(4*b^2*cos(x)^4 + 4*(a*b - b^2)*
cos(x)^2 + a^2 - 2*a*b + b^2)) + 4*x)/b, -1/4*(sqrt((a - b)/(a + b))*arctan
(-1/2*(2*a*cos(x)^2 - a + b)*sqrt((a - b)/(a + b))/((a - b)*cos(x)*sin(x)))
 - 2*x)/b]
```

giac [B] time = 0.15, size = 159, normalized size = 3.06

$$-\frac{\sqrt{a^2-b^2}\left(\pi\left\lfloor\frac{x}{\pi}+\frac{1}{2}\right\rfloor+\arctan\left(\frac{2\tan(x)}{\sqrt{\frac{4a+\sqrt{-16(a+b)(a-b)+16a^2}}{a-b}}}\right)\right)|a-b|}{2\left((a-b)b^2+\left(a^2-ab\right)|b|\right)}-\frac{\left(\pi\left\lfloor\frac{x}{\pi}+\frac{1}{2}\right\rfloor+\arctan\left(\frac{2\tan(x)}{\sqrt{\frac{4a-\sqrt{-16(a+b)(a-b)+16a^2}}{a-b}}}\right)\right)(a-}{2\left(b^2-a|b|\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^2/(a+b*cos(2*x)),x, algorithm="giac")
```

```
[Out] -1/2*sqrt(a^2 - b^2)*(pi*floor(x/pi + 1/2) + arctan(2*tan(x)/sqrt((4*a + sq
rt(-16*(a + b)*(a - b) + 16*a^2))/(a - b))))*abs(a - b)/((a - b)*b^2 + (a^2
 - a*b)*abs(b)) - 1/2*(pi*floor(x/pi + 1/2) + arctan(2*tan(x)/sqrt((4*a - s
qrt(-16*(a + b)*(a - b) + 16*a^2))/(a - b))))*(a - b)/(b^2 - a*abs(b))
```

maple [A] time = 0.13, size = 80, normalized size = 1.54

$$-\frac{\arctan\left(\frac{\tan(x)(a-b)}{\sqrt{(a+b)(a-b)}}\right)a}{2b\sqrt{(a+b)(a-b)}}+\frac{\arctan\left(\frac{\tan(x)(a-b)}{\sqrt{(a+b)(a-b)}}\right)}{2\sqrt{(a+b)(a-b)}}+\frac{\arctan(\tan(x))}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)^2/(a+b*cos(2*x)),x)
```

```
[Out] -1/2/b/((a+b)*(a-b))^(1/2)*arctan(tan(x)*(a-b)/((a+b)*(a-b))^(1/2))*a+1/2/(
(a+b)*(a-b))^(1/2)*arctan(tan(x)*(a-b)/((a+b)*(a-b))^(1/2))+1/2/b*arctan(ta
n(x))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^2/(a+b*cos(2*x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` for
more details)Is 4*a^2-4*b^2 positive or negative?
```

mupad [B] time = 3.32, size = 684, normalized size = 13.15

$$\frac{\operatorname{atan}\left(\frac{2\,a^2\tan(x)}{2\,a^2-4\,a\,b+2\,b^2}+\frac{2\,b^2\tan(x)}{2\,a^2-4\,a\,b+2\,b^2}-\frac{4\,a\,b\tan(x)}{2\,a^2-4\,a\,b+2\,b^2}\right)}{2\,b}+\frac{\operatorname{atan}\left(\frac{\left(\frac{\tan(x)\left(4\,a^3-12\,a^2\,b+12\,a\,b^2-4\,b^3\right)}{4}+\frac{\sqrt{b^2-a^2}\left(4\,b^4-8\,a\,b^3+4\,a^2\,b^2+\frac{\tan(x)\,\sqrt{b^2-a^2}\,(}{}\right.}{4\left(b^2+a\,b\right)}\right)}{b^2+a\,b}}{\frac{\left(\frac{\tan(x)\left(4\,a^3-12\,a^2\,b+12\,a\,b^2-4\,b^3\right)}{4}+\frac{\sqrt{b^2-a^2}\left(4\,b^4-8\,a\,b^3+4\,a^2\,b^2+\frac{\tan(x)\,\sqrt{b^2-a^2}}{}\right.}{4\left(b^2+a\,b\right)}\right)}{b^2+a\,b}}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)^2/(a + b*cos(2*x)),x)
```

```
[Out] atan((2*a^2*tan(x))/(2*a^2 - 4*a*b + 2*b^2) + (2*b^2*tan(x))/(2*a^2 - 4*a*b
 + 2*b^2) - (4*a*b*tan(x))/(2*a^2 - 4*a*b + 2*b^2))/(2*b) + (atan(((((tan(x
)*(12*a*b^2 - 12*a^2*b + 4*a^3 - 4*b^3))/4 + ((b^2 - a^2)^(1/2)*(4*b^4 - 8*
a*b^3 + 4*a^2*b^2 + (tan(x)*(b^2 - a^2)^(1/2)*(64*a*b^4 - 128*a^2*b^3 + 64*
a^3*b^2))/(16*(a*b + b^2))))/(4*(a*b + b^2)))*(b^2 - a^2)^(1/2)*1i)/(a*b +
b^2) + (((tan(x)*(12*a*b^2 - 12*a^2*b + 4*a^3 - 4*b^3))/4 + ((b^2 - a^2)^(1
/2)*(8*a*b^3 - 4*b^4 - 4*a^2*b^2 + (tan(x)*(b^2 - a^2)^(1/2)*(64*a*b^4 - 12
8*a^2*b^3 + 64*a^3*b^2))/(16*(a*b + b^2))))/(4*(a*b + b^2)))*(b^2 - a^2)^(1
/2)*1i)/(a*b + b^2))/((((tan(x)*(12*a*b^2 - 12*a^2*b + 4*a^3 - 4*b^3))/4 +
((b^2 - a^2)^(1/2)*(4*b^4 - 8*a*b^3 + 4*a^2*b^2 + (tan(x)*(b^2 - a^2)^(1/2)
*(64*a*b^4 - 128*a^2*b^3 + 64*a^3*b^2))/(16*(a*b + b^2))))/(4*(a*b + b^2)))
*(b^2 - a^2)^(1/2))/(a*b + b^2) - (((tan(x)*(12*a*b^2 - 12*a^2*b + 4*a^3 -
4*b^3))/4 + ((b^2 - a^2)^(1/2)*(8*a*b^3 - 4*b^4 - 4*a^2*b^2 + (tan(x)*(b^2
- a^2)^(1/2)*(64*a*b^4 - 128*a^2*b^3 + 64*a^3*b^2))/(16*(a*b + b^2))))/(4*(
a*b + b^2)))*(b^2 - a^2)^(1/2))/(a*b + b^2)))*(b^2 - a^2)^(1/2)*1i)/(2*(a*b
 + b^2))
```

sympy [B] time = 35.33, size = 432, normalized size = 8.31

$$\begin{cases} \tilde{\infty}\left(-\frac{\log(\tan(x)-1)}{2}+\frac{\log(\tan(x)+1)}{2}\right) & \text{for } a=0\wedge b=0 \\ \frac{1}{4b\tan(x)} & \text{for } a=-b \\ \frac{\tan(x)}{4b} & \text{for } a=b \\ \frac{\log\left(-\sqrt{-\frac{a}{a-b}-\frac{b}{a-b}}+\tan(x)\right)}{4a\sqrt{-\frac{a}{a-b}-\frac{b}{a-b}}-4b\sqrt{-\frac{a}{a-b}-\frac{b}{a-b}}}-\frac{\log\left(\sqrt{-\frac{a}{a-b}-\frac{b}{a-b}}+\tan(x)\right)}{4a\sqrt{-\frac{a}{a-b}-\frac{b}{a-b}}-4b\sqrt{-\frac{a}{a-b}-\frac{b}{a-b}}} & \text{otherwise} \end{cases} + \begin{cases} \tilde{\infty}x \\ \frac{x}{2b}-\frac{\tan(x)}{4b} \\ \frac{x}{2b}+\frac{1}{4b\tan(x)} \\ \frac{\sin(2x)}{4a} \\ \frac{2ax\sqrt{-\frac{a}{a-b}-\frac{b}{a-b}}}{4ab\sqrt{-\frac{a}{a-b}-\frac{b}{a-b}}-4b^2\sqrt{-\frac{a}{a-b}-\frac{b}{a-b}}}-\frac{a\log\left(\right.}{4ab\sqrt{-\right.} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)**2/(a+b*cos(2*x)),x)
```

```
[Out] Piecewise((zoo*(-log(tan(x) - 1)/2 + log(tan(x) + 1)/2), Eq(a, 0) & Eq(b, 0
)), (1/(4*b*tan(x)), Eq(a, -b)), (tan(x)/(4*b), Eq(a, b)), (log(-sqrt(-a/(a
 - b) - b/(a - b)) + tan(x))/(4*a*sqrt(-a/(a - b) - b/(a - b)) - 4*b*sqrt(-
a/(a - b) - b/(a - b))) - log(sqrt(-a/(a - b) - b/(a - b)) + tan(x))/(4*a*s
qrt(-a/(a - b) - b/(a - b)) - 4*b*sqrt(-a/(a - b) - b/(a - b))), True)) + P
iecewise((zoo*x, Eq(a, 0) & Eq(b, 0)), (x/(2*b) - tan(x)/(4*b), Eq(a, b)),
(x/(2*b) + 1/(4*b*tan(x)), Eq(a, -b)), (sin(2*x)/(4*a), Eq(b, 0)), (2*a*x*s
qrt(-a/(a - b) - b/(a - b))/(4*a*b*sqrt(-a/(a - b) - b/(a - b)) - 4*b**2*sq
rt(-a/(a - b) - b/(a - b))) - a*log(-sqrt(-a/(a - b) - b/(a - b)) + tan(x))
/(4*a*b*sqrt(-a/(a - b) - b/(a - b)) - 4*b**2*sqrt(-a/(a - b) - b/(a - b)))
 + a*log(sqrt(-a/(a - b) - b/(a - b)) + tan(x))/(4*a*b*sqrt(-a/(a - b) - b/
(a - b)) - 4*b**2*sqrt(-a/(a - b) - b/(a - b))) - 2*b*x*sqrt(-a/(a - b) - b
/(a - b))/(4*a*b*sqrt(-a/(a - b) - b/(a - b)) - 4*b**2*sqrt(-a/(a - b) - b/
(a - b))), True))
```

3.850 $\int \frac{\tan(c+dx)}{\sqrt{a\sin^2(c+dx)}}\,dx$

Optimal. Leaf size=30

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a\sin^2(c+dx)}}{\sqrt{a}}\right)}{\sqrt{a}\,d}$$

```
[Out] arctanh((a*sin(d*x+c)^2)^(1/2)/a^(1/2))/d/a^(1/2)
```

Rubi [A] time = 0.04, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3205, 63, 206}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a\sin^2(c+dx)}}{\sqrt{a}}\right)}{\sqrt{a}\,d}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]/Sqrt[a*Sin[c + d*x]^2],x]
```

```
[Out] ArcTanh[Sqrt[a*Sin[c + d*x]^2]/Sqrt[a]]/(Sqrt[a]*d)
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3205

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_)]^(m_.
), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x]^2, x]}, Dist[ff^((m + 1
)/2)/(2*f), Subst[Int[(x^((m - 1)/2)*(b*ff^(n/2)*x^(n/2))^p)/(1 - ff*x)^((m
 + 1)/2), x], x, Sin[e + f*x]^2/ff], x]] /; FreeQ[{b, e, f, p}, x] && Integ
```

```
erQ[(m - 1)/2] && IntegerQ[n/2]
```

Rubi steps

$$\int \frac{\tan(c+dx)}{\sqrt{a\sin^2(c+dx)}}\,dx = \frac{\text{Subst}\left(\int \frac{1}{(1-x)\sqrt{ax}}\,dx, x, \sin^2(c+dx)\right)}{2d}$$
$$= \frac{\text{Subst}\left(\int \frac{1}{1-\frac{x^2}{a}}\,dx, x, \sqrt{a\sin^2(c+dx)}\right)}{ad}$$
$$= \frac{\tanh^{-1}\left(\frac{\sqrt{a\sin^2(c+dx)}}{\sqrt{a}}\right)}{\sqrt{a}\,d}$$

Mathematica [A] time = 0.04, size = 31, normalized size = 1.03

$$\frac{\sin(c+dx)\tanh^{-1}(\sin(c+dx))}{d\sqrt{a\sin^2(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]/Sqrt[a*Sin[c + d*x]^2],x]
[Out] (ArcTanh[Sin[c + d*x]]*Sin[c + d*x])/(d*Sqrt[a*Sin[c + d*x]^2])
```

fricas [A] time = 0.89, size = 91, normalized size = 3.03

$$\left[\frac{\sqrt{-a\cos\left(dx+c\right)^2+a}\,\log\left(-\frac{\sin(dx+c)+1}{\sin(dx+c)-1}\right)}{2\,ad\sin\left(dx+c\right)}, -\frac{\sqrt{-a}\,\arctan\left(\frac{\sqrt{-a\cos(dx+c)^2+a}\,\sqrt{-a}}{a}\right)}{ad}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)/(a*sin(d*x+c)^2)^(1/2),x, algorithm="fricas")
[Out] [1/2*sqrt(-a*cos(d*x + c)^2 + a)*log(-(sin(d*x + c) + 1)/(sin(d*x + c) - 1)
)/(a*d*sin(d*x + c)), -sqrt(-a)*arctan(sqrt(-a*cos(d*x + c)^2 + a)*sqrt(-a)
/a)/(a*d)]
```

giac [B] time = 0.35, size = 61, normalized size = 2.03

$$\frac{\frac{\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)}{\operatorname{sgn}\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}-\frac{\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)}{\operatorname{sgn}\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}}{\sqrt{a}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)/(a*sin(d*x+c)^2)^(1/2),x, algorithm="giac")
```

```
[Out] (log(abs(tan(1/2*d*x + 1/2*c) + 1))/sgn(tan(1/2*d*x + 1/2*c)) - log(abs(tan
(1/2*d*x + 1/2*c) - 1))/sgn(tan(1/2*d*x + 1/2*c)))/(sqrt(a)*d)
```

maple [A] time = 0.15, size = 30, normalized size = 1.00

$$\frac{\sin\left(dx+c\right)\operatorname{arctanh}\left(\sin\left(dx+c\right)\right)}{\sqrt{a\left(\sin^{2}\left(dx+c\right)\right)}\;d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)/(a*sin(d*x+c)^2)^(1/2),x)
```

```
[Out] 1/(a*sin(d*x+c)^2)^(1/2)*sin(d*x+c)*arctanh(sin(d*x+c))/d
```

maxima [B] time = 0.43, size = 76, normalized size = 2.53

$$\frac{\frac{(-1)^{2\,a\sin(dx+c)}\log\left(-\frac{a\sin(dx+c)}{\sin(dx+c)+1}\right)}{\sqrt{a}}+\frac{(-1)^{2\,a\sin(dx+c)}\log\left(-\frac{a\sin(dx+c)}{\sin(dx+c)-1}\right)}{\sqrt{a}}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)/(a*sin(d*x+c)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*((-1)^(2*a*sin(d*x + c))*log(-a*sin(d*x + c)/(sin(d*x + c) + 1))/sqrt(a
) + (-1)^(2*a*sin(d*x + c))*log(-a*sin(d*x + c)/(sin(d*x + c) - 1))/sqrt(a)
)/d
```

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{\tan\left(c+d\,x\right)}{\sqrt{a\,{\sin\left(c+d\,x\right)}^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)/(a*sin(c + d*x)^2)^(1/2),x)
```

```
[Out] int(tan(c + d*x)/(a*sin(c + d*x)^2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan(c+dx)}{\sqrt{a\sin^2(c+dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)/(a*sin(d*x+c)**2)**(1/2),x)
```

```
[Out] Integral(tan(c + d*x)/sqrt(a*sin(c + d*x)**2), x)
```

3.851 $\int \frac{\cot(c+dx)}{\sqrt{a\cos^2(c+dx)}}\,dx$

Optimal. Leaf size=31

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{a\cos^2(c+dx)}}{\sqrt{a}}\right)}{\sqrt{a}\,d}$$

```
[Out] -arctanh((a*cos(d*x+c)^2)^(1/2)/a^(1/2))/d/a^(1/2)
```

Rubi [A] time = 0.03, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3205, 63, 206}

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{a\cos^2(c+dx)}}{\sqrt{a}}\right)}{\sqrt{a}\,d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]/Sqrt[a*Cos[c + d*x]^2],x]
```

```
[Out] -(ArcTanh[Sqrt[a*Cos[c + d*x]^2]/Sqrt[a]]/(Sqrt[a]*d))
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3205

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_)]^(m_.
), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x]^2, x]}, Dist[ff^((m + 1
)/2)/(2*f), Subst[Int[(x^((m - 1)/2)*(b*ff^(n/2)*x^(n/2))^p)/(1 - ff*x)^((m
 + 1)/2), x], x, Sin[e + f*x]^2/ff], x]] /; FreeQ[{b, e, f, p}, x] && Integ
erQ[(m - 1)/2] && IntegerQ[n/2]
```

Rubi steps

$$\int \frac{\cot(c+dx)}{\sqrt{a\cos^2(c+dx)}}\,dx = -\frac{\operatorname{Subst}\left(\int \frac{1}{(1-x)\sqrt{ax}}\,dx, x, \cos^2(c+dx)\right)}{2d}$$
$$= -\frac{\operatorname{Subst}\left(\int \frac{1}{1-\frac{x^2}{a}}\,dx, x, \sqrt{a\cos^2(c+dx)}\right)}{ad}$$
$$= -\frac{\tanh^{-1}\left(\frac{\sqrt{a\cos^2(c+dx)}}{\sqrt{a}}\right)}{\sqrt{a}\,d}$$

Mathematica [A] time = 0.06, size = 49, normalized size = 1.58

$$\frac{\cos(c+dx)\left(\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)-\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)\right)}{d\sqrt{a\cos^2(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]/Sqrt[a*Cos[c + d*x]^2],x]
```

```
[Out] (Cos[c + d*x]*(-Log[Cos[(c + d*x)/2]] + Log[Sin[(c + d*x)/2]]))/(d*Sqrt[a*C
os[c + d*x]^2])
```

fricas [A] time = 1.21, size = 84, normalized size = 2.71

$$\left[-\frac{\sqrt{a\cos\left(dx+c\right)^2}\,\log\left(-\frac{\cos(dx+c)+1}{\cos(dx+c)-1}\right)}{2\,ad\cos\left(dx+c\right)}, \frac{\sqrt{-a}\,\arctan\left(\frac{\sqrt{a\cos(dx+c)^2}\,\sqrt{-a}}{a}\right)}{ad}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)/(a*cos(d*x+c)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/2*sqrt(a*cos(d*x + c)^2)*log(-(cos(d*x + c) + 1)/(cos(d*x + c) - 1))/(a
*d*cos(d*x + c)), sqrt(-a)*arctan(sqrt(a*cos(d*x + c)^2)*sqrt(-a)/a)/(a*d)]
```

giac [A] time = 0.25, size = 31, normalized size = 1.00

$$\frac{\arctan\left(\frac{\sqrt{-a\sin(dx+c)^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)/(a*cos(d*x+c)^2)^(1/2),x, algorithm="giac")
```

```
[Out] arctan(sqrt(-a*sin(d*x + c)^2 + a)/sqrt(-a))/(sqrt(-a)*d)
```

maple [A] time = 0.16, size = 31, normalized size = 1.00

$$-\frac{\cos\left(dx+c\right)\operatorname{arctanh}\left(\cos\left(dx+c\right)\right)}{\sqrt{a\left(\cos^{2}\left(dx+c\right)\right)}\, d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)/(a*cos(d*x+c)^2)^(1/2),x)
```

```
[Out] -1/(a*cos(d*x+c)^2)^(1/2)*cos(d*x+c)*arctanh(cos(d*x+c))/d
```

maxima [B] time = 0.32, size = 51, normalized size = 1.65

$$-\frac{\log\left(\frac{2\sqrt{-a\sin\left(dx+c\right)^{2}+a}\,\sqrt{a}}{\left|\sin\left(dx+c\right)\right|}+\frac{2\,a}{\left|\sin\left(dx+c\right)\right|}\right)}{\sqrt{a}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)/(a*cos(d*x+c)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] -log(2*sqrt(-a*sin(d*x + c)^2 + a)*sqrt(a)/abs(sin(d*x + c)) + 2*a/abs(sin(
d*x + c)))/(sqrt(a)*d)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{\cot\left(c+d\,x\right)}{\sqrt{a\cos\left(c+d\,x\right)^{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)/(a*cos(c + d*x)^2)^(1/2),x)
```

```
[Out] int(cot(c + d*x)/(a*cos(c + d*x)^2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cot\left(c+dx\right)}{\sqrt{a\cos^{2}\left(c+dx\right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)/(a*cos(d*x+c)**2)**(1/2),x)
[Out] Integral(cot(c + d*x)/sqrt(a*cos(c + d*x)**2), x)
```

3.852 $\int \frac{x\cos(x^2)}{\sqrt{\sin(x^2)}}\,dx$

Optimal. Leaf size=8

$$\sqrt{\sin\left(x^2\right)}$$

```
[Out] sin(x^2)^(1/2)
```

Rubi [A] time = 0.01, antiderivative size = 8, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {3441}

$$\sqrt{\sin\left(x^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(x*Cos[x^2])/Sqrt[Sin[x^2]],x]
```

```
[Out] Sqrt[Sin[x^2]]
```

Rule 3441

```
Int[Cos[(a_.) + (b_.)*(x_)^(n_.)]*(x_)^(m_.)*Sin[(a_.) + (b_.)*(x_)^(n_.)]^
(p_.), x_Symbol] :> Simp[Sin[a + b*x^n]^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[
{a, b, m, n, p}, x] && EqQ[m, n - 1] && NeQ[p, -1]
```

Rubi steps

$$\int \frac{x\cos\left(x^2\right)}{\sqrt{\sin\left(x^2\right)}}\,dx = \sqrt{\sin\left(x^2\right)}$$

Mathematica [A] time = 0.00, size = 8, normalized size = 1.00

$$\sqrt{\sin\left(x^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*Cos[x^2])/Sqrt[Sin[x^2]],x]
```

```
[Out] Sqrt[Sin[x^2]]
```

fricas [A] time = 2.08, size = 6, normalized size = 0.75

$$\sqrt{\sin\left(x^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(x^2)/sin(x^2)^(1/2),x, algorithm="fricas")
[Out] sqrt(sin(x^2))
```

giac [A] time = 0.14, size = 6, normalized size = 0.75

$$\sqrt{\sin\left(x^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(x^2)/sin(x^2)^(1/2),x, algorithm="giac")
[Out] sqrt(sin(x^2))
```

maple [A] time = 0.02, size = 7, normalized size = 0.88

$$\sqrt{\sin}\,\left(x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*cos(x^2)/sin(x^2)^(1/2),x)
[Out] sin(x^2)^(1/2)
```

maxima [A] time = 0.31, size = 6, normalized size = 0.75

$$\sqrt{\sin\left(x^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(x^2)/sin(x^2)^(1/2),x, algorithm="maxima")
[Out] sqrt(sin(x^2))
```

mupad [B] time = 3.19, size = 6, normalized size = 0.75

$$\sqrt{\sin\left(x^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*cos(x^2))/sin(x^2)^(1/2),x)
```

```
[Out] sin(x^2)^(1/2)
```

sympy [A] time = 0.29, size = 7, normalized size = 0.88

$$\sqrt{\sin\left(x^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(x**2)/sin(x**2)**(1/2),x)
```

```
[Out] sqrt(sin(x**2))
```

3.853 $\int \frac{\cos(x)}{\sqrt{1-\cos(2x)}}\,dx$

Optimal. Leaf size=19

$$\frac{\sin(x)\log(\sin(x))}{\sqrt{2}\sqrt{\sin^2(x)}}$$

```
[Out] 1/2*ln(sin(x))*sin(x)*2^(1/2)/(sin(x)^2)^(1/2)
```

Rubi [A] time = 0.03, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {4356, 12, 15, 29}

$$\frac{\sin(x)\log(\sin(x))}{\sqrt{2}\sqrt{\sin^2(x)}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[x]/Sqrt[1 - Cos[2*x]],x]
```

```
[Out] (Log[Sin[x]]*Sin[x])/(Sqrt[2]*Sqrt[Sin[x]^2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 4356

```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{d = FreeFacto
rs[Sin[c*(a + b*x)], x]}, Dist[d/(b*c), Subst[Int[SubstFor[1, Sin[c*(a + b*
x)]/d, u, x], x], x, Sin[c*(a + b*x)]/d], x] /; FunctionOfQ[Sin[c*(a + b*x)
]/d, u, x]] /; FreeQ[{a, b, c}, x] && (EqQ[F, Cos] || EqQ[F, cos])
```

Rubi steps

$$\int \frac{\cos(x)}{\sqrt{1-\cos(2x)}}\,dx = \text{Subst}\left(\int \frac{1}{\sqrt{2}\sqrt{x^2}}\,dx, x, \sin(x)\right)$$
$$= \frac{\text{Subst}\left(\int \frac{1}{\sqrt{x^2}}\,dx, x, \sin(x)\right)}{\sqrt{2}}$$
$$= \frac{\sin(x)\,\text{Subst}\left(\int \frac{1}{x}\,dx, x, \sin(x)\right)}{\sqrt{2}\sqrt{\sin^2(x)}}$$
$$= \frac{\log(\sin(x))\sin(x)}{\sqrt{2}\sqrt{\sin^2(x)}}$$

Mathematica [A] time = 0.01, size = 18, normalized size = 0.95

$$\frac{\sin(x)\log(\sin(x))}{\sqrt{1-\cos(2x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[x]/Sqrt[1 - Cos[2*x]],x]
[Out] (Log[Sin[x]]*Sin[x])/Sqrt[1 - Cos[2*x]]
```

fricas [A] time = 1.33, size = 21, normalized size = 1.11

$$\frac{\sqrt{-2\cos(x)^2+2}\,\log\left(\frac{1}{2}\sin(x)\right)}{2\sin(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)/(1-cos(2*x))^(1/2),x, algorithm="fricas")
[Out] 1/2*sqrt(-2*cos(x)^2 + 2)*log(1/2*sin(x))/sin(x)
```

giac [A] time = 0.16, size = 14, normalized size = 0.74

$$\frac{\sqrt{2}\log(|\sin(x)|)}{2\,\text{sgn}(\sin(x))}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)/(1-cos(2*x))^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(2)*log(abs(sin(x)))/sgn(sin(x))
```

maple [A] time = 0.45, size = 25, normalized size = 1.32

$$\frac{\sin(x)\left(\ln\left(-1+\cos(x)\right)+\ln\left(1+\cos(x)\right)\right)\sqrt{2}}{2\sqrt{2-2\cos\left(2x\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)/(1-cos(2*x))^(1/2),x)
```

```
[Out] 1/4*sin(x)*(ln(-1+cos(x))+ln(1+cos(x)))*2^(1/2)/(sin(x)^2)^(1/2)
```

maxima [B] time = 0.42, size = 41, normalized size = 2.16

$$\frac{1}{4}\sqrt{2}\,\log\left(\cos(x)^2+\sin(x)^2+2\,\cos(x)+1\right)+\frac{1}{4}\sqrt{2}\,\log\left(\cos(x)^2+\sin(x)^2-2\,\cos(x)+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)/(1-cos(2*x))^(1/2),x, algorithm="maxima")
```

```
[Out] 1/4*sqrt(2)*log(cos(x)^2 + sin(x)^2 + 2*cos(x) + 1) + 1/4*sqrt(2)*log(cos(x
)^2 + sin(x)^2 - 2*cos(x) + 1)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.05

$$\int \frac{\cos(x)}{\sqrt{1-\cos\left(2\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)/(1 - cos(2*x))^(1/2),x)
```

```
[Out] int(cos(x)/(1 - cos(2*x))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos\left(x\right)}{\sqrt{1-\cos\left(2x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)/(1-cos(2*x))**(1/2),x)
```

```
[Out] Integral(cos(x)/sqrt(1 - cos(2*x)), x)
```

3.854 $\int \frac{\cos^2(\log(x))\sin^2(\log(x))}{x}\,dx$

Optimal. Leaf size=29

$$\frac{\log(x)}{8}-\frac{1}{4}\sin(\log(x))\cos^3(\log(x))+\frac{1}{8}\sin(\log(x))\cos(\log(x))$$

```
[Out] 1/8*ln(x)+1/8*cos(ln(x))*sin(ln(x))-1/4*cos(ln(x))^3*sin(ln(x))
```

Rubi [A] time = 0.06, antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {2568, 2635, 8}

$$\frac{\log(x)}{8}-\frac{1}{4}\sin(\log(x))\cos^3(\log(x))+\frac{1}{8}\sin(\log(x))\cos(\log(x))$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[Log[x]]^2*Sin[Log[x]]^2)/x,x]
```

```
[Out] Log[x]/8 + (Cos[Log[x]]*Sin[Log[x]])/8 - (Cos[Log[x]]^3*Sin[Log[x]])/4
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2568

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> -Simp[(a*(b*Cos[e + f*x])^(n + 1)*(a*Sin[e + f*x])^(m - 1)
)/(b*f*(m + n)), x] + Dist[(a^2*(m - 1))/(m + n), Int[(b*Cos[e + f*x])^n*(a
*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1] &&
 NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rubi steps

$$\int \frac{\cos^2(\log(x))\sin^2(\log(x))}{x}\,dx = \text{Subst}\left(\int \cos^2(x)\sin^2(x)\,dx, x, \log(x)\right)$$
$$= -\frac{1}{4}\cos^3(\log(x))\sin(\log(x)) + \frac{1}{4}\text{Subst}\left(\int \cos^2(x)\,dx, x, \log(x)\right)$$
$$= \frac{1}{8}\cos(\log(x))\sin(\log(x)) - \frac{1}{4}\cos^3(\log(x))\sin(\log(x)) + \frac{1}{8}\text{Subst}(\int 1\,dx, x, \text{lo}$$
$$= \frac{\log(x)}{8} + \frac{1}{8}\cos(\log(x))\sin(\log(x)) - \frac{1}{4}\cos^3(\log(x))\sin(\log(x))$$

Mathematica [A] time = 0.02, size = 16, normalized size = 0.55

$$\frac{\log(x)}{8} - \frac{1}{32}\sin(4\log(x))$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[Log[x]]^2*Sin[Log[x]]^2)/x,x]
[Out] Log[x]/8 - Sin[4*Log[x]]/32
```

fricas [A] time = 0.41, size = 23, normalized size = 0.79

$$-\frac{1}{8}\left(2\cos\left(\log(x)\right)^3 - \cos\left(\log(x)\right)\right)\sin\left(\log(x)\right) + \frac{1}{8}\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(log(x))^2*sin(log(x))^2/x,x, algorithm="fricas")
[Out] -1/8*(2*cos(log(x))^3 - cos(log(x)))*sin(log(x)) + 1/8*log(x)
```

giac [A] time = 0.13, size = 12, normalized size = 0.41

$$\frac{1}{8}\log(x) - \frac{1}{32}\sin\left(4\log(x)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(log(x))^2*sin(log(x))^2/x,x, algorithm="giac")
[Out] 1/8*log(x) - 1/32*sin(4*log(x))
```

maple [A] time = 0.04, size = 24, normalized size = 0.83

$$\frac{\ln(x)}{8} + \frac{\cos(\ln(x))\sin(\ln(x))}{8} - \frac{\left(\cos^3(\ln(x))\right)\sin(\ln(x))}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(ln(x))^2*sin(ln(x))^2/x,x)
```

```
[Out] 1/8*ln(x)+1/8*cos(ln(x))*sin(ln(x))-1/4*cos(ln(x))^3*sin(ln(x))
```

maxima [A] time = 0.30, size = 12, normalized size = 0.41

$$\frac{1}{8}\log(x) - \frac{1}{32}\sin\left(4\log(x)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(log(x))^2*sin(log(x))^2/x,x, algorithm="maxima")
```

```
[Out] 1/8*log(x) - 1/32*sin(4*log(x))
```

mupad [B] time = 3.20, size = 12, normalized size = 0.41

$$\frac{\ln(x)}{8} - \frac{\sin\left(4\ln(x)\right)}{32}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(log(x))^2*sin(log(x))^2)/x,x)
```

```
[Out] log(x)/8 - sin(4*log(x))/32
```

sympy [B] time = 20.22, size = 476, normalized size = 16.41

$$\frac{\log{(x)}\tan^{8}{\left(\frac{\log{(x)}}{2}\right)}}{8\tan^{8}{\left(\frac{\log{(x)}}{2}\right)} + 32\tan^{6}{\left(\frac{\log{(x)}}{2}\right)} + 48\tan^{4}{\left(\frac{\log{(x)}}{2}\right)} + 32\tan^{2}{\left(\frac{\log{(x)}}{2}\right)} + 8} + \frac{4\log{(x)}\,\mathrm{ta}}{8\tan^{8}{\left(\frac{\log{(x)}}{2}\right)} + 32\tan^{6}{\left(\frac{\log{(x)}}{2}\right)} + 4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(ln(x))**2*sin(ln(x))**2/x,x)
```

```
[Out] log(x)*tan(log(x)/2)**8/(8*tan(log(x)/2)**8 + 32*tan(log(x)/2)**6 + 48*tan(
log(x)/2)**4 + 32*tan(log(x)/2)**2 + 8) + 4*log(x)*tan(log(x)/2)**6/(8*tan(
log(x)/2)**8 + 32*tan(log(x)/2)**6 + 48*tan(log(x)/2)**4 + 32*tan(log(x)/2)
**2 + 8) + 6*log(x)*tan(log(x)/2)**4/(8*tan(log(x)/2)**8 + 32*tan(log(x)/2)
**6 + 48*tan(log(x)/2)**4 + 32*tan(log(x)/2)**2 + 8) + 4*log(x)*tan(log(x)/
2)**2/(8*tan(log(x)/2)**8 + 32*tan(log(x)/2)**6 + 48*tan(log(x)/2)**4 + 32*
tan(log(x)/2)**2 + 8) + log(x)/(8*tan(log(x)/2)**8 + 32*tan(log(x)/2)**6 +
48*tan(log(x)/2)**4 + 32*tan(log(x)/2)**2 + 8) + 2*tan(log(x)/2)**7/(8*tan(
log(x)/2)**8 + 32*tan(log(x)/2)**6 + 48*tan(log(x)/2)**4 + 32*tan(log(x)/2)
```

```
**2 + 8) - 14*tan(log(x)/2)**5/(8*tan(log(x)/2)**8 + 32*tan(log(x)/2)**6 +
48*tan(log(x)/2)**4 + 32*tan(log(x)/2)**2 + 8) + 14*tan(log(x)/2)**3/(8*tan
(log(x)/2)**8 + 32*tan(log(x)/2)**6 + 48*tan(log(x)/2)**4 + 32*tan(log(x)/2
)**2 + 8) - 2*tan(log(x)/2)/(8*tan(log(x)/2)**8 + 32*tan(log(x)/2)**6 + 48*
tan(log(x)/2)**4 + 32*tan(log(x)/2)**2 + 8)
```

3.855 $\int \frac{\sin^3(x)}{\cos^3(x)+\sin^3(x)}\,dx$

Optimal. Leaf size=29

$$\frac{x}{2}+\frac{1}{3}\log(2-\sin(2x))-\frac{1}{6}\log(\sin(x)+\cos(x))$$

```
[Out] 1/2*x-1/6*ln(cos(x)+sin(x))+1/3*ln(2-sin(2*x))
```

Rubi [A] time = 0.13, antiderivative size = 37, normalized size of antiderivative = 1.28, number of steps used = 7, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312, Rules used = {2074, 635, 203, 260, 628}

$$\frac{x}{2}+\frac{1}{3}\log\left(\tan^2(x)-\tan(x)+1\right)-\frac{1}{6}\log(\tan(x)+1)+\frac{1}{2}\log(\cos(x))$$

Antiderivative was successfully verified.

```
[In] Int[Sin[x]^3/(Cos[x]^3 + Sin[x]^3),x]
```

```
[Out] x/2 + Log[Cos[x]]/2 - Log[1 + Tan[x]]/6 + Log[1 - Tan[x] + Tan[x]^2]/3
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 2074

```
Int[(P_)^(p_)*(Q_)^(q_.), x_Symbol] :> With[{PP = Factor[P]}, Int[ExpandInt
egrand[PP^p*Q^q, x], x] /;  !SumQ[NonfreeFactors[PP, x]]] /; FreeQ[q, x] &&
 PolyQ[P, x] && PolyQ[Q, x] && IntegerQ[p] && NeQ[P, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sin^3(x)}{\cos^3(x)+\sin^3(x)}\,dx &= \text{Subst}\left(\int \frac{x^3}{1+x^2+x^3+x^5}\,dx,x,\tan(x)\right)\\
&= \text{Subst}\left(\int \left(-\frac{1}{6(1+x)}+\frac{1-x}{2\left(1+x^2\right)}+\frac{-1+2x}{3\left(1-x+x^2\right)}\right)dx,x,\tan(x)\right)\\
&= -\frac{1}{6}\log(1+\tan(x))+\frac{1}{3}\text{Subst}\left(\int \frac{-1+2x}{1-x+x^2}\,dx,x,\tan(x)\right)+\frac{1}{2}\text{Subst}\left(\int \frac{1-x}{1+x^2}\,dx,x\right.\\
&= -\frac{1}{6}\log(1+\tan(x))+\frac{1}{3}\log\left(1-\tan(x)+\tan^2(x)\right)+\frac{1}{2}\text{Subst}\left(\int \frac{1}{1+x^2}\,dx,x,\tan(x)\right)\\
&= \frac{x}{2}+\frac{1}{2}\log(\cos(x))-\frac{1}{6}\log(1+\tan(x))+\frac{1}{3}\log\left(1-\tan(x)+\tan^2(x)\right)
\end{aligned}$$

Mathematica [A] time = 0.10, size = 29, normalized size = 1.00

$$\frac{x}{2}+\frac{1}{3}\log(2-\sin(2x))-\frac{1}{6}\log(\sin(x)+\cos(x))$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[x]^3/(Cos[x]^3 + Sin[x]^3),x]
[Out] x/2 - Log[Cos[x] + Sin[x]]/6 + Log[2 - Sin[2*x]]/3
```

fricas [A] time = 2.01, size = 26, normalized size = 0.90

$$\frac{1}{2}\,x-\frac{1}{12}\,\log\left(2\,\cos(x)\sin(x)+1\right)+\frac{1}{3}\,\log\left(-\cos(x)\sin(x)+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^3/(cos(x)^3+sin(x)^3),x, algorithm="fricas")
[Out] 1/2*x - 1/12*log(2*cos(x)*sin(x) + 1) + 1/3*log(-cos(x)*sin(x) + 1)
```

giac [A] time = 0.17, size = 34, normalized size = 1.17

$$\frac{1}{2}\,x+\frac{1}{3}\,\log\left(\tan(x)^2-\tan(x)+1\right)-\frac{1}{4}\,\log\left(\tan(x)^2+1\right)-\frac{1}{6}\,\log\left(|\tan(x)+1|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^3/(cos(x)^3+sin(x)^3),x, algorithm="giac")
```

```
[Out] 1/2*x + 1/3*log(tan(x)^2 - tan(x) + 1) - 1/4*log(tan(x)^2 + 1) - 1/6*log(ab
s(tan(x) + 1))
```

maple [A] time = 0.14, size = 34, normalized size = 1.17

$$\frac{\ln\left(1-\tan(x)+\tan^{2}(x)\right)}{3}-\frac{\ln\left(1+\tan^{2}(x)\right)}{4}-\frac{\ln\left(1+\tan(x)\right)}{6}+\frac{x}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)^3/(cos(x)^3+sin(x)^3),x)
```

```
[Out] 1/3*ln(1-tan(x)+tan(x)^2)-1/4*ln(1+tan(x)^2)-1/6*ln(1+tan(x))+1/2*x
```

maxima [B] time = 0.41, size = 103, normalized size = 3.55

$$\arctan\left(\frac{\sin(x)}{\cos(x)+1}\right)+\frac{1}{3}\log\left(-\frac{2\sin(x)}{\cos(x)+1}+\frac{2\sin(x)^{2}}{(\cos(x)+1)^{2}}+\frac{2\sin(x)^{3}}{(\cos(x)+1)^{3}}+\frac{\sin(x)^{4}}{(\cos(x)+1)^{4}}+1\right)-\frac{1}{6}\log\left(-\frac{2\sin(x)}{\cos(x)+1}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^3/(cos(x)^3+sin(x)^3),x, algorithm="maxima")
```

```
[Out] arctan(sin(x)/(cos(x) + 1)) + 1/3*log(-2*sin(x)/(cos(x) + 1) + 2*sin(x)^2/(
cos(x) + 1)^2 + 2*sin(x)^3/(cos(x) + 1)^3 + sin(x)^4/(cos(x) + 1)^4 + 1) -
1/6*log(-2*sin(x)/(cos(x) + 1) + sin(x)^2/(cos(x) + 1)^2 - 1) - 1/2*log(sin
(x)^2/(cos(x) + 1)^2 + 1)
```

mupad [B] time = 3.31, size = 45, normalized size = 1.55

$$\frac{x}{2}-\frac{\ln\left(\frac{1}{\cos\left(\frac{x}{2}\right)^{2}}\right)}{2}+\frac{\ln\left(\frac{\sin(2\,x)-2}{\cos\left(\frac{x}{2}\right)^{4}}\right)}{3}-\frac{\ln\left(\frac{\sin\left(x+\frac{\pi}{4}\right)}{\cos\left(\frac{x}{2}\right)^{2}}\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)^3/(cos(x)^3 + sin(x)^3),x)
```

```
[Out] x/2 - log(1/cos(x/2)^2)/2 + log((sin(2*x) - 2)/cos(x/2)^4)/3 - log(sin(x +
pi/4)/cos(x/2)^2)/6
```

sympy [A] time = 0.37, size = 32, normalized size = 1.10

$$\frac{x}{2}-\frac{\log\left(\sin\left(x\right)+\cos\left(x\right)\right)}{6}+\frac{\log\left(\sin^{2}\left(x\right)-\sin\left(x\right)\cos\left(x\right)+\cos^{2}\left(x\right)\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)**3/(cos(x)**3+sin(x)**3),x)
```

```
[Out] x/2 - log(sin(x) + cos(x))/6 + log(sin(x)**2 - sin(x)*cos(x) + cos(x)**2)/3
```

3.856 $\int \frac{\cos^3(x)}{\cos^3(x)+\sin^3(x)} dx$

Optimal. Leaf size=29

$$\frac{x}{2} - \frac{1}{3}\log(2 - \sin(2x)) + \frac{1}{6}\log(\sin(x) + \cos(x))$$

```
[Out] 1/2*x+1/6*ln(cos(x)+sin(x))-1/3*ln(2-sin(2*x))
```

Rubi [A] time = 0.09, antiderivative size = 37, normalized size of antiderivative = 1.28, number of steps used = 7, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312, Rules used = {2058, 635, 203, 260, 628}

$$\frac{x}{2} - \frac{1}{3}\log\left(\tan^2(x) - \tan(x) + 1\right) + \frac{1}{6}\log(\tan(x) + 1) - \frac{1}{2}\log(\cos(x))$$

Antiderivative was successfully verified.

```
[In] Int[Cos[x]^3/(Cos[x]^3 + Sin[x]^3),x]
```

```
[Out] x/2 - Log[Cos[x]]/2 + Log[1 + Tan[x]]/6 - Log[1 - Tan[x] + Tan[x]^2]/3
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 2058

```
Int[(P_)^(p_), x_Symbol] :> With[{u = Factor[P]}, Int[ExpandIntegrand[u^p,
x], x] /;  !SumQ[NonfreeFactors[u, x]]] /; PolyQ[P, x] && ILtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\cos^3(x)}{\cos^3(x)+\sin^3(x)}\,dx &= \text{Subst}\left(\int \frac{1}{1+x^2+x^3+x^5}\,dx, x, \tan(x)\right)\\ &= \text{Subst}\left(\int \left(\frac{1}{6(1+x)}+\frac{1+x}{2\left(1+x^2\right)}+\frac{1-2x}{3\left(1-x+x^2\right)}\right)dx, x, \tan(x)\right)\\ &= \frac{1}{6}\log(1+\tan(x))+\frac{1}{3}\text{Subst}\left(\int \frac{1-2x}{1-x+x^2}\,dx, x, \tan(x)\right)+\frac{1}{2}\text{Subst}\left(\int \frac{1+x}{1+x^2}\,dx, x, \text{t}\right.\\ &= \frac{1}{6}\log(1+\tan(x))-\frac{1}{3}\log\left(1-\tan(x)+\tan^2(x)\right)+\frac{1}{2}\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \tan(x)\right)+\\ &= \frac{x}{2}-\frac{1}{2}\log(\cos(x))+\frac{1}{6}\log(1+\tan(x))-\frac{1}{3}\log\left(1-\tan(x)+\tan^2(x)\right)\end{aligned}$$

Mathematica [A] time = 0.07, size = 29, normalized size = 1.00

$$\frac{x}{2}-\frac{1}{3}\log(2-\sin(2x))+\frac{1}{6}\log(\sin(x)+\cos(x))$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[x]^3/(Cos[x]^3 + Sin[x]^3),x]
[Out] x/2 + Log[Cos[x] + Sin[x]]/6 - Log[2 - Sin[2*x]]/3
```

fricas [A] time = 1.01, size = 26, normalized size = 0.90

$$\frac{1}{2}x+\frac{1}{12}\log\left(2\cos(x)\sin(x)+1\right)-\frac{1}{3}\log\left(-\cos(x)\sin(x)+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^3/(cos(x)^3+sin(x)^3),x, algorithm="fricas")
[Out] 1/2*x + 1/12*log(2*cos(x)*sin(x) + 1) - 1/3*log(-cos(x)*sin(x) + 1)
```

giac [A] time = 0.17, size = 34, normalized size = 1.17

$$\frac{1}{2}x-\frac{1}{3}\log\left(\tan(x)^2-\tan(x)+1\right)+\frac{1}{4}\log\left(\tan(x)^2+1\right)+\frac{1}{6}\log\left(|\tan(x)+1|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^3/(cos(x)^3+sin(x)^3),x, algorithm="giac")
```

```
[Out] 1/2*x - 1/3*log(tan(x)^2 - tan(x) + 1) + 1/4*log(tan(x)^2 + 1) + 1/6*log(ab
s(tan(x) + 1))
```

maple [A] time = 0.14, size = 34, normalized size = 1.17

$$-\frac{\ln\left(1-\tan(x)+\tan^{2}(x)\right)}{3}+\frac{\ln\left(1+\tan^{2}(x)\right)}{4}+\frac{\ln\left(1+\tan(x)\right)}{6}+\frac{x}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)^3/(cos(x)^3+sin(x)^3),x)
```

```
[Out] -1/3*ln(1-tan(x)+tan(x)^2)+1/4*ln(1+tan(x)^2)+1/6*ln(1+tan(x))+1/2*x
```

maxima [B] time = 0.42, size = 103, normalized size = 3.55

$$\arctan\left(\frac{\sin(x)}{\cos(x)+1}\right)-\frac{1}{3}\log\left(-\frac{2\sin(x)}{\cos(x)+1}+\frac{2\sin(x)^{2}}{(\cos(x)+1)^{2}}+\frac{2\sin(x)^{3}}{(\cos(x)+1)^{3}}+\frac{\sin(x)^{4}}{(\cos(x)+1)^{4}}+1\right)+\frac{1}{6}\log\left(-\frac{2\sin}{\cos(}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^3/(cos(x)^3+sin(x)^3),x, algorithm="maxima")
```

```
[Out] arctan(sin(x)/(cos(x) + 1)) - 1/3*log(-2*sin(x)/(cos(x) + 1) + 2*sin(x)^2/(
cos(x) + 1)^2 + 2*sin(x)^3/(cos(x) + 1)^3 + sin(x)^4/(cos(x) + 1)^4 + 1) +
1/6*log(-2*sin(x)/(cos(x) + 1) + sin(x)^2/(cos(x) + 1)^2 - 1) + 1/2*log(sin
(x)^2/(cos(x) + 1)^2 + 1)
```

mupad [B] time = 3.19, size = 45, normalized size = 1.55

$$\frac{x}{2}+\frac{\ln\left(\frac{1}{\cos\left(\frac{x}{2}\right)^{2}}\right)}{2}-\frac{\ln\left(\frac{\sin(2\,x)-2}{\cos\left(\frac{x}{2}\right)^{4}}\right)}{3}+\frac{\ln\left(\frac{\sin\left(x+\frac{\pi}{4}\right)}{\cos\left(\frac{x}{2}\right)^{2}}\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)^3/(cos(x)^3 + sin(x)^3),x)
```

```
[Out] x/2 + log(1/cos(x/2)^2)/2 - log((sin(2*x) - 2)/cos(x/2)^4)/3 + log(sin(x +
pi/4)/cos(x/2)^2)/6
```

sympy [A] time = 0.37, size = 32, normalized size = 1.10

$$\frac{x}{2}+\frac{\log\left(\sin\left(x\right)+\cos\left(x\right)\right)}{6}-\frac{\log\left(\sin^{2}\left(x\right)-\sin\left(x\right)\cos\left(x\right)+\cos^{2}\left(x\right)\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)**3/(cos(x)**3+sin(x)**3),x)
```

```
[Out] x/2 + log(sin(x) + cos(x))/6 - log(sin(x)**2 - sin(x)*cos(x) + cos(x)**2)/3
```

3.857 $\int \frac{\sec(x)}{-5+\cos^2(x)+4\sin(x)}\,dx$

Optimal. Leaf size=44

$$\frac{1}{3(2-\sin(x))}+\frac{1}{2}\log(1-\sin(x))-\frac{4}{9}\log(2-\sin(x))-\frac{1}{18}\log(\sin(x)+1)$$

```
[Out] 1/2*ln(1-sin(x))-4/9*ln(2-sin(x))-1/18*ln(1+sin(x))+1/3/(2-sin(x))
```

Rubi [A] time = 0.06, antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {710, 801}

$$\frac{1}{3(2-\sin(x))}+\frac{1}{2}\log(1-\sin(x))-\frac{4}{9}\log(2-\sin(x))-\frac{1}{18}\log(\sin(x)+1)$$

Antiderivative was successfully verified.

```
[In] Int[Sec[x]/(-5 + Cos[x]^2 + 4*Sin[x]),x]
```

```
[Out] Log[1 - Sin[x]]/2 - (4*Log[2 - Sin[x]])/9 - Log[1 + Sin[x]]/18 + 1/(3*(2 -
Sin[x]))
```

Rule 710

```
Int[((d_) + (e_.)*(x_))^(m_)/((a_) + (c_.)*(x_)^2), x_Symbol] :> Simp[(e*(d
 + e*x)^(m + 1))/((m + 1)*(c*d^2 + a*e^2)), x] + Dist[c/(c*d^2 + a*e^2), In
t[((d + e*x)^(m + 1)*(d - e*x))/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e, m
}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[m, -1]
```

Rule 801

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a + c*x^2), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rubi steps

$$\int \frac{\sec(x)}{-5+\cos^2(x)+4\sin(x)}\,dx = \text{Subst}\left(\int \frac{1}{(2-x)^2\left(-1+x^2\right)}\,dx, x, \sin(x)\right)$$
$$= \frac{1}{3(2-\sin(x))} + \frac{1}{3}\text{Subst}\left(\int \frac{2+x}{(2-x)\left(-1+x^2\right)}\,dx, x, \sin(x)\right)$$
$$= \frac{1}{3(2-\sin(x))} + \frac{1}{3}\text{Subst}\left(\int \left(-\frac{4}{3(-2+x)} + \frac{3}{2(-1+x)} - \frac{1}{6(1+x)}\right)dx, x, \sin(x)\right)$$
$$= \frac{1}{2}\log(1-\sin(x)) - \frac{4}{9}\log(2-\sin(x)) - \frac{1}{18}\log(1+\sin(x)) + \frac{1}{3(2-\sin(x))}$$

Mathematica [A] time = 0.08, size = 38, normalized size = 0.86

$$\frac{1}{18}\left(-\frac{6}{\sin(x)-2} + 9\log(1-\sin(x)) - 8\log(2-\sin(x)) - \log(\sin(x)+1)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[x]/(-5 + Cos[x]^2 + 4*Sin[x]),x]
```

```
[Out] (9*Log[1 - Sin[x]] - 8*Log[2 - Sin[x]] - Log[1 + Sin[x]] - 6/(-2 + Sin[x]))
/18
```

fricas [A] time = 1.17, size = 46, normalized size = 1.05

$$-\frac{(\sin(x)-2)\log(\sin(x)+1) + 8(\sin(x)-2)\log\left(-\frac{1}{2}\sin(x)+1\right) - 9(\sin(x)-2)\log(-\sin(x)+1) + 6}{18(\sin(x)-2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)/(-5+cos(x)^2+4*sin(x)),x, algorithm="fricas")
```

```
[Out] -1/18*((sin(x) - 2)*log(sin(x) + 1) + 8*(sin(x) - 2)*log(-1/2*sin(x) + 1) -
 9*(sin(x) - 2)*log(-sin(x) + 1) + 6)/(sin(x) - 2)
```

giac [A] time = 0.12, size = 34, normalized size = 0.77

$$-\frac{1}{3(\sin(x)-2)} - \frac{1}{18}\log(\sin(x)+1) - \frac{4}{9}\log(-\sin(x)+2) + \frac{1}{2}\log(-\sin(x)+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)/(-5+cos(x)^2+4*sin(x)),x, algorithm="giac")
```

```
[Out] -1/3/(sin(x) - 2) - 1/18*log(sin(x) + 1) - 4/9*log(-sin(x) + 2) + 1/2*log(-
sin(x) + 1)
```

maple [A] time = 0.16, size = 31, normalized size = 0.70

$$-\frac{1}{3\left(\sin(x)-2\right)}-\frac{4\ln\left(\sin(x)-2\right)}{9}+\frac{\ln\left(\sin(x)-1\right)}{2}-\frac{\ln\left(1+\sin(x)\right)}{18}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(x)/(-5+cos(x)^2+4*sin(x)),x)
```

```
[Out] -1/3/(sin(x)-2)-4/9*ln(sin(x)-2)+1/2*ln(sin(x)-1)-1/18*ln(1+sin(x))
```

maxima [A] time = 0.31, size = 30, normalized size = 0.68

$$-\frac{1}{3\left(\sin(x)-2\right)}-\frac{1}{18}\log\left(\sin(x)+1\right)+\frac{1}{2}\log\left(\sin(x)-1\right)-\frac{4}{9}\log\left(\sin(x)-2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)/(-5+cos(x)^2+4*sin(x)),x, algorithm="maxima")
```

```
[Out] -1/3/(sin(x) - 2) - 1/18*log(sin(x) + 1) + 1/2*log(sin(x) - 1) - 4/9*log(si
n(x) - 2)
```

mupad [B] time = 0.07, size = 32, normalized size = 0.73

$$\frac{\ln\left(\sin(x)-1\right)}{2}-\frac{\ln\left(\sin(x)+1\right)}{18}-\frac{4\ \ln\left(\sin(x)-2\right)}{9}-\frac{1}{3\ \left(\sin(x)-2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(x)*(4*sin(x) + cos(x)^2 - 5)),x)
```

```
[Out] log(sin(x) - 1)/2 - log(sin(x) + 1)/18 - (4*log(sin(x) - 2))/9 - 1/(3*(sin(
x) - 2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sec\left(x\right)}{4\sin\left(x\right)+\cos^{2}\left(x\right)-5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)/(-5+cos(x)**2+4*sin(x)),x)
```

```
[Out] Integral(sec(x)/(4*sin(x) + cos(x)**2 - 5), x)
```

3.858 $\int \frac{1}{\cos^{\frac{3}{2}}(x)\sqrt{3\cos(x)+\sin(x)}}\,dx$

Optimal. Leaf size=19

$$\frac{2\sqrt{\sin(x)+3\cos(x)}}{\sqrt{\cos(x)}}$$

```
[Out] 2*(3*cos(x)+sin(x))^(1/2)/cos(x)^(1/2)
```

Rubi [B] time = 2.24, antiderivative size = 88, normalized size of antiderivative = 4.63, number of steps used = 5, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {6719, 1063, 8}

$$\frac{2\cos^2\left(\frac{x}{2}\right)\left(-3\tan^2\left(\frac{x}{2}\right)+2\tan\left(\frac{x}{2}\right)+3\right)}{\sqrt{\cos^2\left(\frac{x}{2}\right)\left(-3\tan^2\left(\frac{x}{2}\right)+2\tan\left(\frac{x}{2}\right)+3\right)}\sqrt{\cos^2\left(\frac{x}{2}\right)\left(1-\tan^2\left(\frac{x}{2}\right)\right)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Cos[x]^(3/2)*Sqrt[3*Cos[x] + Sin[x]]),x]
```

```
[Out] (2*Cos[x/2]^2*(3 + 2*Tan[x/2] - 3*Tan[x/2]^2))/(Sqrt[Cos[x/2]^2*(3 + 2*Tan[
x/2] - 3*Tan[x/2]^2)]*Sqrt[Cos[x/2]^2*(1 - Tan[x/2]^2)])
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 1063

```
Int[((a_) + (c_.)*(x_)^2)^(p_)*((A_.) + (C_.)*(x_)^2)*((d_) + (e_.)*(x_) +
(f_.)*(x_)^2)^(q_), x_Symbol] :> Simp[((a + c*x^2)^(p + 1)*(d + e*x + f*x^2
)^(q + 1)*((A*c - a*C)*(2*a*c*e) + c*(A*(2*c^2*d - c*(2*a*f)) + C*(-2*a*(c*
d - a*f)))*x))/((-4*a*c)*(a*c*e^2 + (c*d - a*f)^2)*(p + 1)), x] + Dist[1/((
-4*a*c)*(a*c*e^2 + (c*d - a*f)^2)*(p + 1)), Int[(a + c*x^2)^(p + 1)*(d + e*
x + f*x^2)^q*Simp[(-2*A*c - 2*a*C)*((c*d - a*f)^2 - (-(a*e))*(c*e))*(p + 1)
 + (2*(A*c*(c*d - a*f) - a*(c*C*d - a*C*f)))*(a*f*(p + 1) - c*d*(p + 2)) -
e*((A*c - a*C)*(2*a*c*e))*(p + q + 2) - (2*f*((A*c - a*C)*(2*a*c*e))*(p + q
 + 2) - (2*(A*c*(c*d - a*f) - a*(c*C*d - a*C*f)))*(-(c*e*(2*p + q + 4))))*x
 - c*f*(2*(A*c*(c*d - a*f) - a*(c*C*d - a*C*f)))*(2*p + 2*q + 5)*x^2, x], x
], x] /; FreeQ[{a, c, d, e, f, A, C, q}, x] && NeQ[e^2 - 4*d*f, 0] && LtQ[p
, -1] && NeQ[a*c*e^2 + (c*d - a*f)^2, 0] &&  !( !IntegerQ[p] && ILtQ[q, -1]
) &&  !IGtQ[q, 0]
```

Rule 6719

```
Int[(u_.)*((a_.)*(v_)^(m_.)*(w_)^(n_.))^(p_), x_Symbol] :> Dist[(a^IntPart[
p]*(a*v^m*w^n)^FracPart[p])/(v^(m*FracPart[p])*w^(n*FracPart[p])), Int[u*v^
(m*p)*w^(n*p), x], x] /; FreeQ[{a, m, n, p}, x] &&  !IntegerQ[p] &&  !FreeQ
[v, x] &&  !FreeQ[w, x]
```

Rubi steps

$$\int \frac{1}{\cos^{\frac{3}{2}}(x)\sqrt{3\cos(x)+\sin(x)}}\,dx = 2\,\text{Subst}\left(\int \frac{1}{\left(1-x^2\right)\sqrt{\frac{3+2x-3x^2}{1+x^2}}\sqrt{\frac{1-x^2}{1+x^2}}}\,dx, x, \tan\left(\frac{x}{2}\right)\right)$$

$$= \frac{\left(2\sqrt{3+2\tan\left(\frac{x}{2}\right)-3\tan^2\left(\frac{x}{2}\right)}\right)\text{Subst}\left(\int \frac{\sqrt{1+x^2}}{\sqrt{3+2x-3x^2}\left(1-x^2\right)\sqrt{\frac{1-x^2}{1+x^2}}}\,dx, x, \tan\left(\frac{x}{2}\right.\right.}{\sqrt{\sec^2\left(\frac{x}{2}\right)}\sqrt{\cos^2\left(\frac{x}{2}\right)\left(3+2\tan\left(\frac{x}{2}\right)-3\tan^2\left(\frac{x}{2}\right)\right)}}$$

$$= \frac{\left(2\cos^2\left(\frac{x}{2}\right)\sqrt{3+2\tan\left(\frac{x}{2}\right)-3\tan^2\left(\frac{x}{2}\right)}\sqrt{1-\tan^2\left(\frac{x}{2}\right)}\right)\text{Subst}\left(\int \frac{1+}{\sqrt{3+2x-3x^2}}\right.}{\sqrt{\cos^2\left(\frac{x}{2}\right)\left(3+2\tan\left(\frac{x}{2}\right)-3\tan^2\left(\frac{x}{2}\right)\right)}\sqrt{\cos^2\left(\frac{x}{2}\right)\left(1-\tan\right.}}$$

$$= \frac{2\cos^2\left(\frac{x}{2}\right)\left(3+2\tan\left(\frac{x}{2}\right)-3\tan^2\left(\frac{x}{2}\right)\right)}{\sqrt{\cos^2\left(\frac{x}{2}\right)\left(3+2\tan\left(\frac{x}{2}\right)-3\tan^2\left(\frac{x}{2}\right)\right)}\sqrt{\cos^2\left(\frac{x}{2}\right)\left(1-\tan^2\left(\frac{x}{2}\right)\right)}} + \frac{\left(\cos^2\left(\right.\right.}{}$$

$$= \frac{2\cos^2\left(\frac{x}{2}\right)\left(3+2\tan\left(\frac{x}{2}\right)-3\tan^2\left(\frac{x}{2}\right)\right)}{\sqrt{\cos^2\left(\frac{x}{2}\right)\left(3+2\tan\left(\frac{x}{2}\right)-3\tan^2\left(\frac{x}{2}\right)\right)}\sqrt{\cos^2\left(\frac{x}{2}\right)\left(1-\tan^2\left(\frac{x}{2}\right)\right)}}$$

Mathematica [A] time = 0.06, size = 19, normalized size = 1.00

$$\frac{2\sqrt{\sin(x)+3\cos(x)}}{\sqrt{\cos(x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Cos[x]^(3/2)*Sqrt[3*Cos[x] + Sin[x]]),x]
[Out] (2*Sqrt[3*Cos[x] + Sin[x]])/Sqrt[Cos[x]]
```

fricas [A] time = 0.87, size = 15, normalized size = 0.79

$$\frac{2\sqrt{3\cos(x)+\sin(x)}}{\sqrt{\cos(x)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(x)^(3/2)/(3*cos(x)+sin(x))^(1/2),x, algorithm="fricas")
```

```
[Out] 2*sqrt(3*cos(x) + sin(x))/sqrt(cos(x))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{3 \cos(x) + \sin(x)} \cos(x)^{\frac{3}{2}}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(x)^(3/2)/(3*cos(x)+sin(x))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(3*cos(x) + sin(x))*cos(x)^(3/2)), x)
```

maple [A] time = 0.38, size = 16, normalized size = 0.84

$$\frac{2\sqrt{3\cos(x) + \sin(x)}}{\sqrt{\cos(x)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/cos(x)^(3/2)/(3*cos(x)+sin(x))^(1/2),x)
```

```
[Out] 2*(3*cos(x)+sin(x))^(1/2)/cos(x)^(1/2)
```

maxima [B] time = 0.47, size = 145, normalized size = 7.63

$$\frac{2\left(\frac{2 \sin(x)}{\cos(x)+1} - \frac{6 \sin(x)^2}{(\cos(x)+1)^2} - \frac{2 \sin(x)^3}{(\cos(x)+1)^3} + \frac{3 \sin(x)^4}{(\cos(x)+1)^4} + 3\right)\left(\frac{\sin(x)^2}{(\cos(x)+1)^2} + 1\right)^2}{\sqrt{\frac{2 \sin(x)}{\cos(x)+1} - \frac{3 \sin(x)^2}{(\cos(x)+1)^2} + 3}\left(\frac{\sin(x)}{\cos(x)+1} + 1\right)^{\frac{3}{2}}\left(-\frac{\sin(x)}{\cos(x)+1} + 1\right)^{\frac{3}{2}}\left(\frac{2 \sin(x)^2}{(\cos(x)+1)^2} + \frac{\sin(x)^4}{(\cos(x)+1)^4} + 1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(x)^(3/2)/(3*cos(x)+sin(x))^(1/2),x, algorithm="maxima")
```

```
[Out] 2*(2*sin(x)/(cos(x) + 1) - 6*sin(x)^2/(cos(x) + 1)^2 - 2*sin(x)^3/(cos(x) +
 1)^3 + 3*sin(x)^4/(cos(x) + 1)^4 + 3)*(sin(x)^2/(cos(x) + 1)^2 + 1)^2/(sqr
t(2*sin(x)/(cos(x) + 1) - 3*sin(x)^2/(cos(x) + 1)^2 + 3)*(sin(x)/(cos(x) +
1) + 1)^(3/2)*(-sin(x)/(cos(x) + 1) + 1)^(3/2)*(2*sin(x)^2/(cos(x) + 1)^2 +
 sin(x)^4/(cos(x) + 1)^4 + 1))
```

mupad [B] time = 3.79, size = 15, normalized size = 0.79

$$\frac{2\sqrt{3 \cos(x) + \sin(x)}}{\sqrt{\cos(x)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(x)^(3/2)*(3*cos(x) + sin(x))^(1/2)),x)
```

```
[Out] (2*(3*cos(x) + sin(x))^(1/2))/cos(x)^(1/2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{\sin(x) + 3\cos(x)}\,\cos^{\frac{3}{2}}(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(x)**(3/2)/(3*cos(x)+sin(x))**(1/2),x)
```

```
[Out] Integral(1/(sqrt(sin(x) + 3*cos(x))*cos(x)**(3/2)), x)
```

3.859 $\int \frac{\csc(x)\sqrt{\cos(x)+\sin(x)}}{\cos^{\frac{3}{2}}(x)}\,dx$

Optimal. Leaf size=44

$$-\log(\sin(x))+\frac{2\sqrt{\sin(x)+\cos(x)}}{\sqrt{\cos(x)}}+2\log\left(\sqrt{\sin(x)+\cos(x)}-\sqrt{\cos(x)}\right)$$

```
[Out] -ln(sin(x))+2*ln(-cos(x)^(1/2)+(cos(x)+sin(x))^(1/2))+2*(cos(x)+sin(x))^(1/
2)/cos(x)^(1/2)
```

Rubi [F] time = 2.57, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\csc(x)\sqrt{\cos(x)+\sin(x)}}{\cos^{\frac{3}{2}}(x)}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(Csc[x]*Sqrt[Cos[x] + Sin[x]])/Cos[x]^(3/2),x]
```

```
[Out] Defer[Int][(Csc[x]*Sqrt[Cos[x] + Sin[x]])/Cos[x]^(3/2), x]
```

Rubi steps

$$\int \frac{\csc(x)\sqrt{\cos(x)+\sin(x)}}{\cos^{\frac{3}{2}}(x)}\,dx=\int \frac{\csc(x)\sqrt{\cos(x)+\sin(x)}}{\cos^{\frac{3}{2}}(x)}\,dx$$

Mathematica [A] time = 0.40, size = 68, normalized size = 1.55

$$\frac{2\left(\sin(x)+\cos(x)-\sqrt{\cos(x)}\sqrt{\sqrt{\sin^2(x)}+\cos(x)}\coth^{-1}\left(\frac{\sqrt{\sqrt{\sin^2(x)}+\cos(x)}}{\sqrt{\cos(x)}}\right)\right)}{\sqrt{\cos(x)}\sqrt{\sin(x)+\cos(x)}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Csc[x]*Sqrt[Cos[x] + Sin[x]])/Cos[x]^(3/2),x]
```

```
[Out] (2*(Cos[x] + Sin[x] - ArcCoth[Sqrt[Cos[x] + Sqrt[Sin[x]^2]]/Sqrt[Cos[x]]]*S
qrt[Cos[x]]*Sqrt[Cos[x] + Sqrt[Sin[x]^2]]))/(Sqrt[Cos[x]]*Sqrt[Cos[x] + Sin
[x]])
```

fricas [B] time = 0.89, size = 96, normalized size = 2.18

$$-\frac{\cos(x)\log\left((2\cos(x)+\sin(x))\sqrt{\cos(x)+\sin(x)}\sqrt{\cos(x)}+\frac{7}{4}\cos(x)^2+2\cos(x)\sin(x)+\frac{1}{4}\right)-\cos(x)\log\left(-(\right.}{4\,\mathrm{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)*(cos(x)+sin(x))^(1/2)/cos(x)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/4*(cos(x)*log((2*cos(x) + sin(x))*sqrt(cos(x) + sin(x))*sqrt(cos(x)) + 7
/4*cos(x)^2 + 2*cos(x)*sin(x) + 1/4) - cos(x)*log(-(2*cos(x) + sin(x))*sqrt
(cos(x) + sin(x))*sqrt(cos(x)) + 7/4*cos(x)^2 + 2*cos(x)*sin(x) + 1/4) - 8*
sqrt(cos(x) + sin(x))*sqrt(cos(x)))/cos(x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\cos(x)+\sin(x)}\,\csc(x)}{\cos(x)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)*(cos(x)+sin(x))^(1/2)/cos(x)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(cos(x) + sin(x))*csc(x)/cos(x)^(3/2), x)
```

maple [C] time = 0.77, size = 917, normalized size = 20.84

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(x)*(cos(x)+sin(x))^(1/2)/cos(x)^(3/2),x)
```

```
[Out] (-1+cos(x))^2*(1+cos(x))^2*(-EllipticF(1/2*((sin(x)-1)/cos(x)*(2+2^(1/2))*2
^(1/2))^(1/2)*2^(1/2),I/(2+2^(1/2))*((2-2^(1/2))*(2+2^(1/2)))^(1/2))*((cos(
x)*2^(1/2)-sin(x)*2^(1/2)+2*sin(x)+2^(1/2)-2)/cos(x)*2^(1/2))^(1/2)*((cos(x
)*2^(1/2)-sin(x)*2^(1/2)-2*sin(x)+2^(1/2)+2)/cos(x)*2^(1/2))^(1/2)*((sin(x)
-1)/cos(x)*(2+2^(1/2))*2^(1/2))^(1/2)*sin(x)+EllipticPi(1/2*((sin(x)-1)/cos
(x)*(2+2^(1/2))*2^(1/2))^(1/2)*2^(1/2),2^(1/2)/(2+2^(1/2)),I/(2+2^(1/2))*((
2-2^(1/2))*(2+2^(1/2)))^(1/2))*((cos(x)*2^(1/2)-sin(x)*2^(1/2)+2*sin(x)+2^(
1/2)-2)/cos(x)*2^(1/2))^(1/2)*((cos(x)*2^(1/2)-sin(x)*2^(1/2)-2*sin(x)+2^(1
/2)+2)/cos(x)*2^(1/2))^(1/2)*((sin(x)-1)/cos(x)*(2+2^(1/2))*2^(1/2))^(1/2)*
sin(x)+EllipticPi(1/2*((sin(x)-1)/cos(x)*(2+2^(1/2))*2^(1/2))^(1/2)*2^(1/2)
,-2^(1/2)/(2+2^(1/2)),I/(2+2^(1/2))*((2-2^(1/2))*(2+2^(1/2)))^(1/2))*((cos(
x)*2^(1/2)-sin(x)*2^(1/2)+2*sin(x)+2^(1/2)-2)/cos(x)*2^(1/2))^(1/2)*((cos(x
```

```
)*2^(1/2)-sin(x)*2^(1/2)-2*sin(x)+2^(1/2)+2)/cos(x)*2^(1/2))^(1/2)*((sin(x)
-1)/cos(x)*(2+2^(1/2))*2^(1/2))^(1/2)*sin(x)-((cos(x)*2^(1/2)-sin(x)*2^(1/2
)+2*sin(x)+2^(1/2)-2)/cos(x)*2^(1/2))^(1/2)*((cos(x)*2^(1/2)-sin(x)*2^(1/2)
-2*sin(x)+2^(1/2)+2)/cos(x)*2^(1/2))^(1/2)*EllipticF(1/2*((sin(x)-1)/cos(x)
*(2+2^(1/2))*2^(1/2))^(1/2)*2^(1/2),I/(2+2^(1/2))*((2-2^(1/2))*(2+2^(1/2)))
^(1/2))*((sin(x)-1)/cos(x)*(2+2^(1/2))*2^(1/2))^(1/2)+((cos(x)*2^(1/2)-sin(
x)*2^(1/2)+2*sin(x)+2^(1/2)-2)/cos(x)*2^(1/2))^(1/2)*((cos(x)*2^(1/2)-sin(x
)*2^(1/2)-2*sin(x)+2^(1/2)+2)/cos(x)*2^(1/2))^(1/2)*EllipticPi(1/2*((sin(x)
-1)/cos(x)*(2+2^(1/2))*2^(1/2))^(1/2)*2^(1/2),2^(1/2)/(2+2^(1/2)),I/(2+2^(1
/2))*((2-2^(1/2))*(2+2^(1/2)))^(1/2))*((sin(x)-1)/cos(x)*(2+2^(1/2))*2^(1/2
))^(1/2)+((cos(x)*2^(1/2)-sin(x)*2^(1/2)+2*sin(x)+2^(1/2)-2)/cos(x)*2^(1/2)
)^(1/2)*((cos(x)*2^(1/2)-sin(x)*2^(1/2)-2*sin(x)+2^(1/2)+2)/cos(x)*2^(1/2))
^(1/2)*EllipticPi(1/2*((sin(x)-1)/cos(x)*(2+2^(1/2))*2^(1/2))^(1/2)*2^(1/2)
,-2^(1/2)/(2+2^(1/2)),I/(2+2^(1/2))*((2-2^(1/2))*(2+2^(1/2)))^(1/2))*((sin(
x)-1)/cos(x)*(2+2^(1/2))*2^(1/2))^(1/2)+2*cos(x)*2^(1/2)+2*sin(x)*2^(1/2)+4
*cos(x)+4*sin(x))/sin(x)^4/(cos(x)+sin(x))^(1/2)/cos(x)^(1/2)/(2+2^(1/2))
```

maxima [B] time = 0.76, size = 518, normalized size = 11.77

$$\frac{4\left((2\cos(2x)+\sin(2x))\cos\left(\frac{1}{2}\arctan(-\cos(4x)+\sin(4x)+2\sin(2x)+1,\cos(4x)+2\cos(2x)+\sin(4x\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)*(cos(x)+sin(x))^(1/2)/cos(x)^(3/2),x, algorithm="maxima")
```

```
[Out] 4*((2*cos(2*x) + sin(2*x))*cos(1/2*arctan2(-cos(4*x) + sin(4*x) + 2*sin(2*x
) + 1, cos(4*x) + 2*cos(2*x) + sin(4*x) + 1))^3 + (2*cos(2*x) + sin(2*x))*c
os(1/2*arctan2(-cos(4*x) + sin(4*x) + 2*sin(2*x) + 1, cos(4*x) + 2*cos(2*x)
 + sin(4*x) + 1))*sin(1/2*arctan2(-cos(4*x) + sin(4*x) + 2*sin(2*x) + 1, co
s(4*x) + 2*cos(2*x) + sin(4*x) + 1))^2 - (cos(2*x) - 2*sin(2*x) + 1)*sin(1/
2*arctan2(-cos(4*x) + sin(4*x) + 2*sin(2*x) + 1, cos(4*x) + 2*cos(2*x) + si
n(4*x) + 1))^3 - (cos(2*x) - sin(2*x) - 1)*cos(1/2*arctan2(-cos(4*x) + sin(
4*x) + 2*sin(2*x) + 1, cos(4*x) + 2*cos(2*x) + sin(4*x) + 1)) - ((cos(2*x)
- 2*sin(2*x) + 1)*cos(1/2*arctan2(-cos(4*x) + sin(4*x) + 2*sin(2*x) + 1, co
s(4*x) + 2*cos(2*x) + sin(4*x) + 1))^2 + cos(2*x) + sin(2*x) - 1)*sin(1/2*a
rctan2(-cos(4*x) + sin(4*x) + 2*sin(2*x) + 1, cos(4*x) + 2*cos(2*x) + sin(4
*x) + 1)))/((4*(cos(2*x) - sin(2*x))*cos(4*x) + 2*cos(4*x)^2 + 4*cos(2*x)^2
 + 4*(cos(2*x) + sin(2*x) + 1)*sin(4*x) + 2*sin(4*x)^2 + 4*sin(2*x)^2 + 4*c
os(2*x) + 4*sin(2*x) + 2)^(1/4)*(cos(1/2*arctan2(-cos(4*x) + sin(4*x) + 2*s
in(2*x) + 1, cos(4*x) + 2*cos(2*x) + sin(4*x) + 1))^2 + sin(1/2*arctan2(-co
s(4*x) + sin(4*x) + 2*sin(2*x) + 1, cos(4*x) + 2*cos(2*x) + sin(4*x) + 1))^
2))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\sqrt{\cos(x)+\sin(x)}}{\cos(x)^{3/2}\,\sin(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(x) + sin(x))^(1/2)/(cos(x)^(3/2)*sin(x)),x)
[Out] int((cos(x) + sin(x))^(1/2)/(cos(x)^(3/2)*sin(x)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\sin{\left(x \right)}+\cos{\left(x \right)}}\,\csc{\left(x \right)}}{\cos^{\frac{3}{2}}{\left(x \right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)*(cos(x)+sin(x))**(1/2)/cos(x)**(3/2),x)
[Out] Integral(sqrt(sin(x) + cos(x))*csc(x)/cos(x)**(3/2), x)
```

3.860 $\int \frac{\cos(x)+\sin(x)}{\sqrt{1+\sin(2x)}}\,dx$

Optimal. Leaf size=19

$$\frac{x\sqrt{\sin(2x)+1}}{\sin(x)+\cos(x)}$$

```
[Out] x*(1+sin(2*x))^(1/2)/(cos(x)+sin(x))
```

Rubi [B] time = 1.71, antiderivative size = 72, normalized size of antiderivative = 3.79, number of steps used = 17, number of rules used = 9, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.562, Rules used = {4401, 6719, 1075, 628, 635, 203, 260, 12, 1023}

$$\frac{2\cos^2\left(\frac{x}{2}\right)\tan^{-1}\left(\tan\left(\frac{x}{2}\right)\right)\left(-\tan^2\left(\frac{x}{2}\right)+2\tan\left(\frac{x}{2}\right)+1\right)}{\sqrt{\cos^4\left(\frac{x}{2}\right)\left(-\tan^2\left(\frac{x}{2}\right)+2\tan\left(\frac{x}{2}\right)+1\right)^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[x] + Sin[x])/Sqrt[1 + Sin[2*x]],x]
```

```
[Out] (2*ArcTan[Tan[x/2]]*Cos[x/2]^2*(1 + 2*Tan[x/2] - Tan[x/2]^2))/Sqrt[Cos[x/2]
^4*(1 + 2*Tan[x/2] - Tan[x/2]^2)^2]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 1023

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*((d_) + (f_.)*
(x_)^2)), x_Symbol] :> With[{q = Simplify[c^2*d^2 + b^2*d*f - 2*a*c*d*f + a
^2*f^2]}, Dist[1/q, Int[Simp[g*c^2*d + g*b^2*f - a*b*h*f - a*g*c*f + c*(h*c
*d + g*b*f - a*h*f)*x, x]/(a + b*x + c*x^2), x], x] + Dist[1/q, Int[Simp[b*
h*d*f - g*c*d*f + a*g*f^2 - f*(h*c*d + g*b*f - a*h*f)*x, x]/(d + f*x^2), x]
, x] /; NeQ[q, 0]] /; FreeQ[{a, b, c, d, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0
]
```

Rule 1075

```
Int[((A_.) + (C_.)*(x_)^2)/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*((d_) + (f_.
)*(x_)^2)), x_Symbol] :> With[{q = c^2*d^2 + b^2*d*f - 2*a*c*d*f + a^2*f^2}
, Dist[1/q, Int[(A*c^2*d - a*c*C*d + A*b^2*f - a*A*c*f + a^2*C*f + c*(-(b*C
*d) + A*b*f)*x)/(a + b*x + c*x^2), x], x] + Dist[1/q, Int[(c*C*d^2 - A*c*d*
f - a*C*d*f + a*A*f^2 - f*(-(b*C*d) + A*b*f)*x)/(d + f*x^2), x], x] /; NeQ[
q, 0]] /; FreeQ[{a, b, c, d, f, A, C}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 4401

```
Int[u_, x_Symbol] :> With[{v = ExpandTrig[u, x]}, Int[v, x] /; SumQ[v]] /;
 !InertTrigFreeQ[u]
```

Rule 6719

```
Int[(u_.)*((a_.)*(v_)^(m_.)*(w_)^(n_.))^(p_), x_Symbol] :> Dist[(a^IntPart[
p]*(a*v^m*w^n)^FracPart[p])/(v^(m*FracPart[p])*w^(n*FracPart[p])), Int[u*v^
(m*p)*w^(n*p), x], x] /; FreeQ[{a, m, n, p}, x] &&  !IntegerQ[p] &&  !FreeQ
[v, x] &&  !FreeQ[w, x]
```

Rubi steps

$$\int \frac{\cos(x)+\sin(x)}{\sqrt{1+\sin(2x)}}\,dx = \int \left(\frac{\cos(x)}{\sqrt{1+\sin(2x)}} + \frac{\sin(x)}{\sqrt{1+\sin(2x)}}\right)dx$$

$$= \int \frac{\cos(x)}{\sqrt{1+\sin(2x)}}\,dx + \int \frac{\sin(x)}{\sqrt{1+\sin(2x)}}\,dx$$

$$= 2\,\text{Subst}\left(\int \frac{2x}{\left(1+x^2\right)^2\sqrt{\frac{\left(-1-2x+x^2\right)^2}{\left(1+x^2\right)^2}}}\,dx, x, \tan\left(\frac{x}{2}\right)\right) + 2\,\text{Subst}\left(\int \frac{1-x^2}{\left(1+x^2\right)^2\sqrt{\frac{\left(-1-2x+x^2\right)^2}{\left(1+x^2\right)^2}}}\right.$$

$$= 4\,\text{Subst}\left(\int \frac{x}{\left(1+x^2\right)^2\sqrt{\frac{\left(-1-2x+x^2\right)^2}{\left(1+x^2\right)^2}}}\,dx, x, \tan\left(\frac{x}{2}\right)\right) + \frac{\left(2\cos^2\left(\frac{x}{2}\right)\left(-1-2\tan\left(\frac{x}{2}\right)+\tan^2\right.\right.}{\sqrt{\cos^4\left(\frac{x}{2}\right)\left(-1\right.}}$$

$$= \frac{\left(\cos^2\left(\frac{x}{2}\right)\left(-1-2\tan\left(\frac{x}{2}\right)+\tan^2\left(\frac{x}{2}\right)\right)\right)\text{Subst}\left(\int \frac{-4+4x}{1+x^2}\,dx, x, \tan\left(\frac{x}{2}\right)\right)}{4\sqrt{\cos^4\left(\frac{x}{2}\right)\left(-1-2\tan\left(\frac{x}{2}\right)+\tan^2\left(\frac{x}{2}\right)\right)^2}} + \frac{\left(\cos^2\left(\frac{x}{2}\right)\left(-1-\right.\right.}{4}$$

$$= \frac{\cos^2\left(\frac{x}{2}\right)\log\left(1+2\tan\left(\frac{x}{2}\right)-\tan^2\left(\frac{x}{2}\right)\right)\left(1+2\tan\left(\frac{x}{2}\right)-\tan^2\left(\frac{x}{2}\right)\right)}{2\sqrt{\cos^4\left(\frac{x}{2}\right)\left(1+2\tan\left(\frac{x}{2}\right)-\tan^2\left(\frac{x}{2}\right)\right)^2}} + \frac{\left(\cos^2\left(\frac{x}{2}\right)\left(-1-2\,\text{ta}\right.\right.}{2\sqrt{\text{co}}}$$

$$= \frac{x\cos^2\left(\frac{x}{2}\right)\left(1+2\tan\left(\frac{x}{2}\right)-\tan^2\left(\frac{x}{2}\right)\right)}{2\sqrt{\cos^4\left(\frac{x}{2}\right)\left(1+2\tan\left(\frac{x}{2}\right)-\tan^2\left(\frac{x}{2}\right)\right)^2}} + \frac{\cos^2\left(\frac{x}{2}\right)\log\left(\cos\left(\frac{x}{2}\right)\right)\left(1+2\tan\left(\frac{x}{2}\right)-\tan^2\left(\right.\right.}{\sqrt{\cos^4\left(\frac{x}{2}\right)\left(1+2\tan\left(\frac{x}{2}\right)-\tan^2\left(\frac{x}{2}\right)\right)^2}}$$

$$= \frac{x\cos^2\left(\frac{x}{2}\right)\left(1+2\tan\left(\frac{x}{2}\right)-\tan^2\left(\frac{x}{2}\right)\right)}{\sqrt{\cos^4\left(\frac{x}{2}\right)\left(1+2\tan\left(\frac{x}{2}\right)-\tan^2\left(\frac{x}{2}\right)\right)^2}}$$

Mathematica [A] time = 0.01, size = 17, normalized size = 0.89

$$\frac{x(\sin(x)+\cos(x))}{\sqrt{\sin(2x)+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[x] + Sin[x])/Sqrt[1 + Sin[2*x]],x]
```

```
[Out] (x*(Cos[x] + Sin[x]))/Sqrt[1 + Sin[2*x]]
```

fricas [A] time = 1.39, size = 3, normalized size = 0.16

$$-x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((cos(x)+sin(x))/(1+sin(2*x))^(1/2),x, algorithm="fricas")
```

```
[Out] -x
```

giac [B] time = 0.17, size = 42, normalized size = 2.21

$$\frac{2\,\pi \left\lfloor \frac{x}{2\,\pi} + \frac{1}{2} \right\rfloor - x}{\mathrm{sgn}\left(\tan\left(\frac{1}{2}\,x\right)^{4} - 2\,\tan\left(\frac{1}{2}\,x\right)^{3} - 2\,\tan\left(\frac{1}{2}\,x\right) - 1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((cos(x)+sin(x))/(1+sin(2*x))^(1/2),x, algorithm="giac")
```

```
[Out] (2*pi*floor(1/2*x/pi + 1/2) - x)/sgn(tan(1/2*x)^4 - 2*tan(1/2*x)^3 - 2*tan(
1/2*x) - 1)
```

maple [C] time = 0.42, size = 12372, normalized size = 651.16

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(x)+sin(x))/(1+sin(2*x))^(1/2),x)
```

```
[Out] result too large to display
```

maxima [B] time = 0.50, size = 329, normalized size = 17.32

$$\frac{1}{16}\,\sqrt{2}\left(2\,\sqrt{2}\,\arctan\left(\sin\left(2\,x\right)+1, \cos\left(2\,x\right)\right) + \sqrt{2}\,\log\left(\cos\left(2\,x\right)^{2} + \sin\left(2\,x\right)^{2} + 2\,\sin\left(2\,x\right) + 1\right) + 4\,\left(\cos\left(4\,x\right)^{2}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((cos(x)+sin(x))/(1+sin(2*x))^(1/2),x, algorithm="maxima")
```

```
[Out] 1/16*sqrt(2)*(2*sqrt(2)*arctan2(sin(2*x) + 1, cos(2*x)) + sqrt(2)*log(cos(2
*x)^2 + sin(2*x)^2 + 2*sin(2*x) + 1) + 4*(cos(4*x)^2 + 4*cos(2*x)^2 + 4*cos
(2*x)*sin(4*x) + sin(4*x)^2 - 4*cos(4*x)*sin(2*x) + 4*sin(2*x)^2)^(1/4)*(co
```

```
s(1/2*arctan2(cos(4*x) - 2*sin(2*x), 2*cos(2*x) + sin(4*x)))*sin(2*x) + cos
(2*x)*sin(1/2*arctan2(cos(4*x) - 2*sin(2*x), 2*cos(2*x) + sin(4*x))))) + 1/
16*sqrt(2)*(2*sqrt(2)*arctan2(sin(2*x) + 1, cos(2*x)) - sqrt(2)*log(cos(2*x
)^2 + sin(2*x)^2 + 2*sin(2*x) + 1) - 4*(cos(4*x)^2 + 4*cos(2*x)^2 + 4*cos(2
*x)*sin(4*x) + sin(4*x)^2 - 4*cos(4*x)*sin(2*x) + 4*sin(2*x)^2)^(1/4)*(cos(
2*x)*cos(1/2*arctan2(cos(4*x) - 2*sin(2*x), 2*cos(2*x) + sin(4*x))) - sin(2
*x)*sin(1/2*arctan2(cos(4*x) - 2*sin(2*x), 2*cos(2*x) + sin(4*x)))))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.05

$$\int \frac{\cos(x)+\sin(x)}{\sqrt{\sin(2\,x)+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(x) + sin(x))/(sin(2*x) + 1)^(1/2),x)
```

```
[Out] int((cos(x) + sin(x))/(sin(2*x) + 1)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin(x)+\cos(x)}{\sqrt{\sin(2x)+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((cos(x)+sin(x))/(1+sin(2*x))**(1/2),x)
```

```
[Out] Integral((sin(x) + cos(x))/sqrt(sin(2*x) + 1), x)
```

3.861 $\int \sec(x)\sqrt{\sec(x)+\tan(x)}\,dx$

Optimal. Leaf size=13

$$2\sqrt{(\sin(x)+1)\sec(x)}$$

```
[Out] 2*(sec(x)*(1+sin(x)))^(1/2)
```

Rubi [A] time = 0.15, antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {4397, 4400, 2705, 2671}

$$2\sqrt{(\sin(x)+1)\sec(x)}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[x]*Sqrt[Sec[x] + Tan[x]],x]
```

```
[Out] 2*Sqrt[Sec[x]*(1 + Sin[x])]
```

Rule 2671

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
])^m)/(a*f*g*m), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0]
 && EqQ[Simplify[m + p + 1], 0] && !ILtQ[p, 0]
```

Rule 2705

```
Int[((g_.)*sec[(e_.) + (f_.)*(x_)])^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.), x_Symbol] :> Dist[g^(2*IntPart[p])*(g*Cos[e + f*x])^FracPart[p]
*(g*Sec[e + f*x])^FracPart[p], Int[(a + b*Sin[e + f*x])^m/(g*Cos[e + f*x])^
p, x], x] /; FreeQ[{a, b, e, f, g, m, p}, x] && !IntegerQ[p]
```

Rule 4397

```
Int[u_, x_Symbol] :> Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]
```

Rule 4400

```
Int[(u_.)*((v_)^(m_.)*(w_)^(n_.))^(p_), x_Symbol] :> With[{uu = ActivateTri
g[u], vv = ActivateTrig[v], ww = ActivateTrig[w]}, Dist[(vv^m*ww^n)^FracPar
t[p]/(vv^(m*FracPart[p])*ww^(n*FracPart[p])), Int[uu*vv^(m*p)*ww^(n*p), x],
 x]] /; FreeQ[{m, n, p}, x] && !IntegerQ[p] && ( !InertTrigFreeQ[v] || !I
nertTrigFreeQ[w])
```

Rubi steps

$$
\begin{aligned}
\int \sec(x)\sqrt{\sec(x)+\tan(x)}\,dx &= \int \sec(x)\sqrt{\sec(x)(1+\sin(x))}\,dx\\
&= \frac{\sqrt{\sec(x)(1+\sin(x))}\int \sec^{\frac{3}{2}}(x)\sqrt{1+\sin(x)}\,dx}{\sqrt{\sec(x)}\sqrt{1+\sin(x)}}\\
&= \frac{\left(\sqrt{\cos(x)}\sqrt{\sec(x)(1+\sin(x))}\right)\int \frac{\sqrt{1+\sin(x)}}{\cos^{\frac{3}{2}}(x)}\,dx}{\sqrt{1+\sin(x)}}\\
&= 2\sqrt{\sec(x)(1+\sin(x))}
\end{aligned}
$$

Mathematica [B] time = 0.04, size = 37, normalized size = 2.85

$$2\sqrt{\frac{\sin\left(\frac{x}{2}\right)+\cos\left(\frac{x}{2}\right)}{\cos\left(\frac{x}{2}\right)-\sin\left(\frac{x}{2}\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[x]*Sqrt[Sec[x] + Tan[x]],x]
[Out] 2*Sqrt[(Cos[x/2] + Sin[x/2])/(Cos[x/2] - Sin[x/2])]
```

fricas [A] time = 0.89, size = 21, normalized size = 1.62

$$2\sqrt{\frac{\cos(x)+\sin(x)+1}{\cos(x)-\sin(x)+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)*(sec(x)+tan(x))^(1/2),x, algorithm="fricas")
[Out] 2*sqrt((cos(x) + sin(x) + 1)/(cos(x) - sin(x) + 1))
```

giac [B] time = 0.32, size = 55, normalized size = 4.23

$$-\frac{4\,\mathrm{sgn}\left(-\tan\left(\frac{1}{2}\,x\right)^{3}-\tan\left(\frac{1}{2}\,x\right)^{2}-\tan\left(\frac{1}{2}\,x\right)-1\right)\mathrm{sgn}\left(\cos(x)\right)}{\frac{\sqrt{-\tan\left(\frac{1}{2}\,x\right)^{2}+1}-1}{\tan\left(\frac{1}{2}\,x\right)}+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)*(sec(x)+tan(x))^(1/2),x, algorithm="giac")
```

```
[Out] -4*sgn(-tan(1/2*x)^3 - tan(1/2*x)^2 - tan(1/2*x) - 1)*sgn(cos(x))/((sqrt(-t
an(1/2*x)^2 + 1) - 1)/tan(1/2*x) + 1)
```

maple [A] time = 0.14, size = 10, normalized size = 0.77

$$2\sqrt{\sec(x)+\tan(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(x)*(sec(x)+tan(x))^(1/2),x)
```

```
[Out] 2*(sec(x)+tan(x))^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{\sec(x)+\tan(x)}\,\sec(x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)*(sec(x)+tan(x))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(sec(x) + tan(x))*sec(x), x)
```

mupad [B] time = 0.29, size = 14, normalized size = 1.08

$$2\sqrt{\frac{1}{\cos(x)}}\,\sqrt{\sin(x)+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((tan(x) + 1/cos(x))^(1/2)/cos(x),x)
```

```
[Out] 2*(1/cos(x))^(1/2)*(sin(x) + 1)^(1/2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{\tan\left(x\right)+\sec\left(x\right)}\,\sec\left(x\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)*(sec(x)+tan(x))**(1/2),x)
```

```
[Out] Integral(sqrt(tan(x) + sec(x))*sec(x), x)
```

3.862 $\int \sec(x)\sqrt{4+3\sec(x)}\,\tan(x)\,dx$

Optimal. Leaf size=14

$$\frac{2}{9}(3\sec(x)+4)^{3/2}$$

```
[Out] 2/9*(4+3*sec(x))^(3/2)
```

Rubi [A] time = 0.04, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {4339, 261}

$$\frac{2}{9}(3\sec(x)+4)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[x]*Sqrt[4 + 3*Sec[x]]*Tan[x],x]
```

```
[Out] (2*(4 + 3*Sec[x])^(3/2))/9
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 4339

```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{d = FreeFacto
rs[Cos[c*(a + b*x)], x]}, -Dist[(b*c)^(-1), Subst[Int[SubstFor[1/x, Cos[c*(
a + b*x)]/d, u, x], x], x, Cos[c*(a + b*x)]/d], x] /; FunctionOfQ[Cos[c*(a
+ b*x)]/d, u, x, True]] /; FreeQ[{a, b, c}, x] && (EqQ[F, Tan] || EqQ[F, ta
n])
```

Rubi steps

$$\begin{aligned}\int \sec(x)\sqrt{4+3\sec(x)}\,\tan(x)\,dx &= -\text{Subst}\left(\int \frac{\sqrt{4+\frac{3}{x}}}{x^2}\,dx, x, \cos(x)\right)\\ &= \frac{2}{9}(4+3\sec(x))^{3/2}\end{aligned}$$

Mathematica [A] time = 0.05, size = 14, normalized size = 1.00

$$\frac{2}{9}(3\sec(x)+4)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[x]*Sqrt[4 + 3*Sec[x]]*Tan[x],x]
[Out] (2*(4 + 3*Sec[x])^(3/2))/9
```

fricas [B] time = 1.26, size = 25, normalized size = 1.79

$$\frac{2\sqrt{\frac{4\cos(x)+3}{\cos(x)}}(4\cos(x)+3)}{9\cos(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)*(4+3*sec(x))^(1/2)*tan(x),x, algorithm="fricas")
[Out] 2/9*sqrt((4*cos(x) + 3)/cos(x))*(4*cos(x) + 3)/cos(x)
```

giac [B] time = 0.16, size = 68, normalized size = 4.86

$$\frac{2\left(4\left(\sqrt{4\cos(x)^2+3\cos(x)}-2\cos(x)\right)^2-6\sqrt{4\cos(x)^2+3\cos(x)}+12\cos(x)+3\right)\operatorname{sgn}(\cos(x))}{\left(\sqrt{4\cos(x)^2+3\cos(x)}-2\cos(x)\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)*(4+3*sec(x))^(1/2)*tan(x),x, algorithm="giac")
[Out] 2*(4*(sqrt(4*cos(x)^2 + 3*cos(x)) - 2*cos(x))^2 - 6*sqrt(4*cos(x)^2 + 3*cos
(x)) + 12*cos(x) + 3)*sgn(cos(x))/(sqrt(4*cos(x)^2 + 3*cos(x)) - 2*cos(x))^
3
```

maple [A] time = 0.06, size = 11, normalized size = 0.79

$$\frac{2(4+3\sec(x))^{\frac{3}{2}}}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(x)*(4+3*sec(x))^(1/2)*tan(x),x)
[Out] 2/9*(4+3*sec(x))^(3/2)
```

maxima [A] time = 0.32, size = 10, normalized size = 0.71

$$\frac{2}{9}\,(3\,\sec(x)+4)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)*(4+3*sec(x))^(1/2)*tan(x),x, algorithm="maxima")
```

```
[Out] 2/9*(3*sec(x) + 4)^(3/2)
```

mupad [B] time = 3.23, size = 29, normalized size = 2.07

$$\frac{8\,\sqrt{\frac{3}{\cos(x)}+4}}{9}+\frac{2\,\sqrt{\frac{3}{\cos(x)}+4}}{3\,\cos(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((tan(x)*(3/cos(x) + 4)^(1/2))/cos(x),x)
```

```
[Out] (8*(3/cos(x) + 4)^(1/2))/9 + (2*(3/cos(x) + 4)^(1/2))/(3*cos(x))
```

sympy [B] time = 0.68, size = 29, normalized size = 2.07

$$\frac{2\sqrt{3\sec{(x)}+4}\,\sec{(x)}}{3}+\frac{8\sqrt{3\sec{(x)}+4}}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)*(4+3*sec(x))**(1/2)*tan(x),x)
```

```
[Out] 2*sqrt(3*sec(x) + 4)*sec(x)/3 + 8*sqrt(3*sec(x) + 4)/9
```

3.863 $\int \sec(x)\sqrt{1+\sec(x)}\,\tan^3(x)\,dx$

Optimal. Leaf size=25

$$\frac{2}{7}(\sec(x)+1)^{7/2}-\frac{4}{5}(\sec(x)+1)^{5/2}$$

```
[Out] -4/5*(1+sec(x))^(5/2)+2/7*(1+sec(x))^(7/2)
```

Rubi [A] time = 0.09, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {4373, 1570, 1469, 627, 43}

$$\frac{2}{7}(\sec(x)+1)^{7/2}-\frac{4}{5}(\sec(x)+1)^{5/2}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[x]*Sqrt[1 + Sec[x]]*Tan[x]^3,x]
```

```
[Out] (-4*(1 + Sec[x])^(5/2))/5 + (2*(1 + Sec[x])^(7/2))/7
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 627

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int
[(d + e*x)^(m + p)*(a/d + (c*x)/e)^p, x] /; FreeQ[{a, c, d, e, m, p}, x] &&
 EqQ[c*d^2 + a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && Intege
rQ[m + p]))
```

Rule 1469

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q
_.), x_Symbol] :> Dist[1/n, Subst[Int[(d + e*x)^q*(a + c*x^2)^p, x], x, x^n
], x] /; FreeQ[{a, c, d, e, m, n, p, q}, x] && EqQ[n2, 2*n] && EqQ[Simplify
[m - n + 1], 0]
```

Rule 1570

```
Int[(x_)^(m_.)*((a_.) + (c_.)*(x_)^(mn2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_.))
^(q_.), x_Symbol] :> Int[x^(m - 2*n*p)*(d + e*x^n)^q*(c + a*x^(2*n))^p, x]
```

```
/; FreeQ[{a, c, d, e, m, n, q}, x] && EqQ[mn2, -2*n] && IntegerQ[p]
```

Rule 4373

```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))]^(n_), x_Symbol] :> With[{d = Free
Factors[Cos[c*(a + b*x)], x]}, -Dist[(b*c*d^(n - 1))^(-1), Subst[Int[SubstF
or[(1 - d^2*x^2)^((n - 1)/2)/x^n, Cos[c*(a + b*x)]/d, u, x], x], x, Cos[c*(
a + b*x)]/d], x] /; FunctionOfQ[Cos[c*(a + b*x)]/d, u, x]] /; FreeQ[{a, b,
c}, x] && IntegerQ[(n - 1)/2] && NonsumQ[u] && (EqQ[F, Tan] || EqQ[F, tan])
```

Rubi steps

$$\begin{aligned}\int \sec(x)\sqrt{1+\sec(x)}\,\tan^3(x)\,dx &= -\text{Subst}\left(\int \frac{\sqrt{1+\frac{1}{x}}\,(1-x^2)}{x^4}\,dx, x, \cos(x)\right)\\ &= -\text{Subst}\left(\int \frac{\left(-1+\frac{1}{x^2}\right)\sqrt{1+\frac{1}{x}}}{x^2}\,dx, x, \cos(x)\right)\\ &= \text{Subst}\left(\int \sqrt{1+x}\,(-1+x^2)\,dx, x, \sec(x)\right)\\ &= \text{Subst}\left(\int (-1+x)(1+x)^{3/2}\,dx, x, \sec(x)\right)\\ &= \text{Subst}\left(\int \left(-2(1+x)^{3/2}+(1+x)^{5/2}\right)dx, x, \sec(x)\right)\\ &= -\frac{4}{5}(1+\sec(x))^{5/2}+\frac{2}{7}(1+\sec(x))^{7/2}\end{aligned}$$

Mathematica [A] time = 0.19, size = 30, normalized size = 1.20

$$-\frac{8}{35}\cos^4\left(\frac{x}{2}\right)(9\cos(x)-5)\sec^3(x)\sqrt{\sec(x)+1}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[x]*Sqrt[1 + Sec[x]]*Tan[x]^3,x]
[Out] (-8*Cos[x/2]^4*(-5 + 9*Cos[x])*Sec[x]^3*Sqrt[1 + Sec[x]])/35
```

fricas [B] time = 0.87, size = 35, normalized size = 1.40

$$-\frac{2\left(9\cos(x)^3+13\cos(x)^2-\cos(x)-5\right)\sqrt{\frac{\cos(x)+1}{\cos(x)}}}{35\cos(x)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)*(1+sec(x))^(1/2)*tan(x)^3,x, algorithm="fricas")
```

```
[Out] -2/35*(9*cos(x)^3 + 13*cos(x)^2 - cos(x) - 5)*sqrt((cos(x) + 1)/cos(x))/cos
(x)^3
```

giac [B] time = 0.17, size = 128, normalized size = 5.12

$$-\frac{2\left(35\left(\sqrt{\cos(x)^2+\cos(x)}-\cos(x)\right)^6-35\left(\sqrt{\cos(x)^2+\cos(x)}-\cos(x)\right)^5-35\left(\sqrt{\cos(x)^2+\cos(x)}-\cos(x)\right)^4}{35\left(\sqrt{c}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)*(1+sec(x))^(1/2)*tan(x)^3,x, algorithm="giac")
```

```
[Out] -2/35*(35*(sqrt(cos(x)^2 + cos(x)) - cos(x))^6 - 35*(sqrt(cos(x)^2 + cos(x)
) - cos(x))^5 - 35*(sqrt(cos(x)^2 + cos(x)) - cos(x))^4 + 105*(sqrt(cos(x)^
2 + cos(x)) - cos(x))^3 - 91*(sqrt(cos(x)^2 + cos(x)) - cos(x))^2 + 35*sqrt
(cos(x)^2 + cos(x)) - 35*cos(x) - 5)*sgn(cos(x))/(sqrt(cos(x)^2 + cos(x)) -
 cos(x))^7
```

maple [A] time = 0.17, size = 34, normalized size = 1.36

$$-\frac{2\left(9\cos(x)-5\right)\sqrt{\frac{1+\cos(x)}{\cos(x)}}\,\left(\sin^4(x)\right)}{35\left(-1+\cos(x)\right)^2\cos(x)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(x)*(1+sec(x))^(1/2)*tan(x)^3,x)
```

```
[Out] -2/35*(9*cos(x)-5)*((1+cos(x))/cos(x))^(1/2)*sin(x)^4/(-1+cos(x))^2/cos(x)^
3
```

maxima [A] time = 0.33, size = 21, normalized size = 0.84

$$\frac{2}{7}\left(\frac{1}{\cos(x)}+1\right)^{\frac{7}{2}}-\frac{4}{5}\left(\frac{1}{\cos(x)}+1\right)^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)*(1+sec(x))^(1/2)*tan(x)^3,x, algorithm="maxima")
```

```
[Out] 2/7*(1/cos(x) + 1)^(7/2) - 4/5*(1/cos(x) + 1)^(5/2)
```

mupad [B] time = 3.33, size = 24, normalized size = 0.96

$$-\frac{2\,{\left(\cos(x)+1\right)}^{5/2}\,\sqrt{\frac{1}{\cos(x)}}\,\left(9\,\cos(x)-5\right)}{35\,{\cos(x)}^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((tan(x)^3*(1/cos(x) + 1)^(1/2))/cos(x),x)
[Out] -(2*(cos(x) + 1)^(5/2)*(1/cos(x))^(1/2)*(9*cos(x) - 5))/(35*cos(x)^3)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{\sec(x)+1}\,\tan^3(x)\sec(x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)*(1+sec(x))**(1/2)*tan(x)**3,x)
[Out] Integral(sqrt(sec(x) + 1)*tan(x)**3*sec(x), x)
```

3.864 $\int \cot^3(x)\csc(x)\sqrt{1+\csc(x)}\,dx$

Optimal. Leaf size=25

$$\frac{4}{5}(\csc(x)+1)^{5/2}-\frac{2}{7}(\csc(x)+1)^{7/2}$$

```
[Out] 4/5*(1+csc(x))^(5/2)-2/7*(1+csc(x))^(7/2)
```

Rubi [A] time = 0.08, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {4372, 1570, 1469, 627, 43}

$$\frac{4}{5}(\csc(x)+1)^{5/2}-\frac{2}{7}(\csc(x)+1)^{7/2}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[x]^3*Csc[x]*Sqrt[1 + Csc[x]],x]
[Out] (4*(1 + Csc[x])^(5/2))/5 - (2*(1 + Csc[x])^(7/2))/7
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 627

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int
[(d + e*x)^(m + p)*(a/d + (c*x)/e)^p, x] /; FreeQ[{a, c, d, e, m, p}, x] &&
 EqQ[c*d^2 + a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && Intege
rQ[m + p]))
```

Rule 1469

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q
_.), x_Symbol] :> Dist[1/n, Subst[Int[(d + e*x)^q*(a + c*x^2)^p, x], x, x^n
], x] /; FreeQ[{a, c, d, e, m, n, p, q}, x] && EqQ[n2, 2*n] && EqQ[Simplify
[m - n + 1], 0]
```

Rule 1570

```
Int[(x_)^(m_.)*((a_.) + (c_.)*(x_)^(mn2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_.))
^(q_.), x_Symbol] :> Int[x^(m - 2*n*p)*(d + e*x^n)^q*(c + a*x^(2*n))^p, x]
```

```
/; FreeQ[{a, c, d, e, m, n, q}, x] && EqQ[mn2, -2*n] && IntegerQ[p]
```

Rule 4372

```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))]^(n_), x_Symbol] :> With[{d = Free
Factors[Sin[c*(a + b*x)], x]}, Dist[1/(b*c*d^(n - 1)), Subst[Int[SubstFor[(
1 - d^2*x^2)^((n - 1)/2)/x^n, Sin[c*(a + b*x)]/d, u, x], x], x, Sin[c*(a +
b*x)]/d], x] /; FunctionOfQ[Sin[c*(a + b*x)]/d, u, x]] /; FreeQ[{a, b, c},
x] && IntegerQ[(n - 1)/2] && NonsumQ[u] && (EqQ[F, Cot] || EqQ[F, cot])
```

Rubi steps

$$\begin{aligned}
\int \cot^3(x)\csc(x)\sqrt{1+\csc(x)}\,dx &= \text{Subst}\left(\int \frac{\sqrt{1+\frac{1}{x}}\,(1-x^2)}{x^4}\,dx, x, \sin(x)\right)\\
&= \text{Subst}\left(\int \frac{\left(-1+\frac{1}{x^2}\right)\sqrt{1+\frac{1}{x}}}{x^2}\,dx, x, \sin(x)\right)\\
&= -\text{Subst}\left(\int \sqrt{1+x}\,(-1+x^2)\,dx, x, \csc(x)\right)\\
&= -\text{Subst}\left(\int (-1+x)(1+x)^{3/2}\,dx, x, \csc(x)\right)\\
&= -\text{Subst}\left(\int \left(-2(1+x)^{3/2}+(1+x)^{5/2}\right)dx, x, \csc(x)\right)\\
&= \frac{4}{5}(1+\csc(x))^{5/2}-\frac{2}{7}(1+\csc(x))^{7/2}
\end{aligned}$$

Mathematica [A] time = 0.04, size = 18, normalized size = 0.72

$$-\frac{2}{35}(\csc(x)+1)^{5/2}(5\csc(x)-9)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[x]^3*Csc[x]*Sqrt[1 + Csc[x]],x]
```

```
[Out] (-2*(1 + Csc[x])^(5/2)*(-9 + 5*Csc[x]))/35
```

fricas [B] time = 0.41, size = 44, normalized size = 1.76

$$\frac{2\left(13\cos(x)^2+\left(9\cos(x)^2-8\right)\sin(x)-8\right)\sqrt{\frac{\sin(x)+1}{\sin(x)}}}{35\left(\cos(x)^2-1\right)\sin(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)^3*csc(x)*(1+csc(x))^(1/2),x, algorithm="fricas")
```

```
[Out] 2/35*(13*cos(x)^2 + (9*cos(x)^2 - 8)*sin(x) - 8)*sqrt((sin(x) + 1)/sin(x))/
((cos(x)^2 - 1)*sin(x))
```

giac [B] time = 0.14, size = 128, normalized size = 5.12

$$\frac{2\left(35\left(\sqrt{\sin(x)^2+\sin(x)}-\sin(x)\right)^6-35\left(\sqrt{\sin(x)^2+\sin(x)}-\sin(x)\right)^5-35\left(\sqrt{\sin(x)^2+\sin(x)}-\sin(x)\right)^4+1\right.}{35\left(\sqrt{\sin(x\right.}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)^3*csc(x)*(1+csc(x))^(1/2),x, algorithm="giac")
```

```
[Out] 2/35*(35*(sqrt(sin(x)^2 + sin(x)) - sin(x))^6 - 35*(sqrt(sin(x)^2 + sin(x))
 - sin(x))^5 - 35*(sqrt(sin(x)^2 + sin(x)) - sin(x))^4 + 105*(sqrt(sin(x)^2
 + sin(x)) - sin(x))^3 - 91*(sqrt(sin(x)^2 + sin(x)) - sin(x))^2 + 35*sqrt(
sin(x)^2 + sin(x)) - 35*sin(x) - 5)*sgn(sin(x))/(sqrt(sin(x)^2 + sin(x)) -
sin(x))^7
```

maple [B] time = 0.18, size = 38, normalized size = 1.52

$$-\frac{2\left(9\left(\cos^2(x)\right)\sin(x)+13\left(\cos^2(x)\right)-8\sin(x)-8\right)\sqrt{\frac{1+\sin(x)}{\sin(x)}}}{35\sin(x)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(x)^3*csc(x)*(1+csc(x))^(1/2),x)
```

```
[Out] -2/35*(9*cos(x)^2*sin(x)+13*cos(x)^2-8*sin(x)-8)*((1+sin(x))/sin(x))^(1/2)/
sin(x)^3
```

maxima [A] time = 0.32, size = 21, normalized size = 0.84

$$-\frac{2}{7}\left(\frac{1}{\sin(x)}+1\right)^{\frac{7}{2}}+\frac{4}{5}\left(\frac{1}{\sin(x)}+1\right)^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)^3*csc(x)*(1+csc(x))^(1/2),x, algorithm="maxima")
```

```
[Out] -2/7*(1/sin(x) + 1)^(7/2) + 4/5*(1/sin(x) + 1)^(5/2)
```

mupad [B] time = 3.42, size = 24, normalized size = 0.96

$$\frac{2\,{\left(\sin(x)+1\right)}^{5/2}\,\sqrt{\frac{1}{\sin(x)}}\,\left(9\,\sin(x)-5\right)}{35\,{\sin(x)}^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cot(x)^3*(1/sin(x) + 1)^(1/2))/sin(x),x)
```

```
[Out] (2*(sin(x) + 1)^(5/2)*(1/sin(x))^(1/2)*(9*sin(x) - 5))/(35*sin(x)^3)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{\csc (x)+1}\, \cot^3 (x) \csc (x)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)**3*csc(x)*(1+csc(x))**(1/2),x)
```

```
[Out] Integral(sqrt(csc(x) + 1)*cot(x)**3*csc(x), x)
```

3.865 $\int \sqrt{\csc(x)}\,(x\cos(x)-4\sec(x)\tan(x))\,dx$

Optimal. Leaf size=20

$$\frac{2x}{\sqrt{\csc(x)}}-\frac{4\sec(x)}{\csc^{\frac{3}{2}}(x)}$$

[Out] -4*sec(x)/csc(x)^(3/2)+2*x/csc(x)^(1/2)

Rubi [A] time = 0.15, antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {6742, 4213, 3771, 2639, 2626}

$$\frac{2x}{\sqrt{\csc(x)}}-\frac{4\sec(x)}{\csc^{\frac{3}{2}}(x)}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[Csc[x]]*(x*Cos[x] - 4*Sec[x]*Tan[x]),x]

[Out] (2*x)/Sqrt[Csc[x]] - (4*Sec[x])/Csc[x]^(3/2)

Rule 2626

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*((b_.)*sec[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(a*b*(a*Csc[e + f*x])^(m - 1)*(b*Sec[e + f*x])^(n -
1))/(f*(n - 1)), x] + Dist[(b^2*(m + n - 2))/(n - 1), Int[(a*Csc[e + f*x])^
m*(b*Sec[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1
] && IntegersQ[2*m, 2*n]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 4213

```
Int[Cos[(a_.) + (b_.)*(x_)^(n_.)]*Csc[(a_.) + (b_.)*(x_)^(n_.)]^(p_)*(x_)^(
m_.), x_Symbol] :> -Simp[(x^(m - n + 1)*Csc[a + b*x^n]^(p - 1))/(b*n*(p - 1
)), x] + Dist[(m - n + 1)/(b*n*(p - 1)), Int[x^(m - n)*Csc[a + b*x^n]^(p -
```

```
1), x], x] /; FreeQ[{a, b, p}, x] && IntegerQ[n] && GeQ[m - n, 0] && NeQ[p,
 1]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\begin{aligned}\int \sqrt{\csc(x)}\,(x\cos(x)-4\sec(x)\tan(x))\,dx &= \int \left(x\cos(x)\sqrt{\csc(x)}-\frac{4\sec^2(x)}{\sqrt{\csc(x)}}\right)dx\\ &= -\left(4\int \frac{\sec^2(x)}{\sqrt{\csc(x)}}\,dx\right)+\int x\cos(x)\sqrt{\csc(x)}\,dx\\ &= \frac{2x}{\sqrt{\csc(x)}}-\frac{4\sec(x)}{\csc^{\frac{3}{2}}(x)}\end{aligned}$$

Mathematica [A] time = 0.44, size = 17, normalized size = 0.85

$$\frac{2(x\csc(x)-2\sec(x))}{\csc^{\frac{3}{2}}(x)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Csc[x]]*(x*Cos[x] - 4*Sec[x]*Tan[x]),x]
```

```
[Out] (2*(x*Csc[x] - 2*Sec[x]))/Csc[x]^(3/2)
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)^(1/2)*(x*cos(x)-4*sec(x)*tan(x)),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (x\cos(x)-4\,\sec(x)\tan(x))\sqrt{\csc(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)^(1/2)*(x*cos(x)-4*sec(x)*tan(x)),x, algorithm="giac")
[Out] integrate((x*cos(x) - 4*sec(x)*tan(x))*sqrt(csc(x)), x)
```

maple [F] time = 0.50, size = 0, normalized size = 0.00

$$\int \left(\sqrt{\csc}\,(x)\right)(x\cos(x) - 4\sec(x)\tan(x))\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(x)^(1/2)*(x*cos(x)-4*sec(x)*tan(x)),x)
[Out] int(csc(x)^(1/2)*(x*cos(x)-4*sec(x)*tan(x)),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (x\cos(x) - 4\sec(x)\tan(x))\sqrt{\csc(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)^(1/2)*(x*cos(x)-4*sec(x)*tan(x)),x, algorithm="maxima")
[Out] integrate((x*cos(x) - 4*sec(x)*tan(x))*sqrt(csc(x)), x)
```

mupad [B] time = 3.46, size = 77, normalized size = 3.85

$$\frac{\left(4\cos(x)^3 - 4\cos(x) + 2x\cos(x)^2\sin(x) - \sin(x)\,4i - x\cos(x)^3\,2i + \cos(x)^2\sin(x)\,4i + x\cos(x)\,2i\right)1i}{\cos(x)\sin(x)\sqrt{\frac{1}{\sin(x)}}\,(-\sin(x) + \cos(x)\,1i)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(1/sin(x))^(1/2)*((4*tan(x))/cos(x) - x*cos(x)),x)
[Out] ((4*cos(x)^3 - sin(x)*4i - x*cos(x)^3*2i - 4*cos(x) + cos(x)^2*sin(x)*4i +
x*cos(x)*2i + 2*x*cos(x)^2*sin(x))*1i)/(cos(x)*sin(x)*(1/sin(x))^(1/2)*(cos
(x)*1i - sin(x)))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (x\cos(x) - 4\tan(x)\sec(x))\sqrt{\csc(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)**(1/2)*(x*cos(x)-4*sec(x)*tan(x)),x)
[Out] Integral((x*cos(x) - 4*tan(x)*sec(x))*sqrt(csc(x)), x)
```

3.866 $\int \cot(x)\sqrt{-1+\csc^2(x)}\left(1-\sin^2(x)\right)^3\,dx$

Optimal. Leaf size=76

$$-\frac{35}{16}\sqrt{\cot^2(x)}+\frac{1}{6}\cos^6(x)\sqrt{\cot^2(x)}+\frac{7}{24}\cos^4(x)\sqrt{\cot^2(x)}+\frac{35}{48}\cos^2(x)\sqrt{\cot^2(x)}-\frac{35}{16}x\tan(x)\sqrt{\cot^2(x)}$$

```
[Out] -35/16*(cot(x)^2)^(1/2)+35/48*cos(x)^2*(cot(x)^2)^(1/2)+7/24*cos(x)^4*(cot(
x)^2)^(1/2)+1/6*cos(x)^6*(cot(x)^2)^(1/2)-35/16*x*(cot(x)^2)^(1/2)*tan(x)
```

Rubi [A] time = 0.16, antiderivative size = 84, normalized size of antiderivative = 1.11, number of steps used = 10, number of rules used = 8, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.348, Rules used = {3175, 4360, 25, 266, 47, 50, 63, 203}

$$-\frac{35}{16}\sqrt{\csc^2(x)-1}+\frac{1}{6}\sin^6(x)\left(\csc^2(x)-1\right)^{7/2}+\frac{7}{24}\sin^4(x)\left(\csc^2(x)-1\right)^{5/2}+\frac{35}{48}\sin^2(x)\left(\csc^2(x)-1\right)^{3/2}+\frac{35}{16}\tan^{-1}\Big($$

Antiderivative was successfully verified.

```
[In] Int[Cot[x]*Sqrt[-1 + Csc[x]^2]*(1 - Sin[x]^2)^3,x]
```

```
[Out] (35*ArcTan[Sqrt[-1 + Csc[x]^2]])/16 - (35*Sqrt[-1 + Csc[x]^2])/16 + (35*(-1
 + Csc[x]^2)^(3/2)*Sin[x]^2)/48 + (7*(-1 + Csc[x]^2)^(5/2)*Sin[x]^4)/24 + (
(-1 + Csc[x]^2)^(7/2)*Sin[x]^6)/6
```

Rule 25

```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_.))^(m_.)*((c_) + (d_.)*(x_)^(q_.))^(p_.),
x_Symbol] :> Dist[(d/a)^p, Int[(u*(a + b*x^n)^(m + p))/x^(n*p), x], x] /; F
reeQ[{a, b, c, d, m, n}, x] && EqQ[q, -n] && IntegerQ[p] && EqQ[a*c - b*d,
0] &&  !(IntegerQ[m] && NegQ[n])
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
```

```
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 3175

```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Dist[
a^p, Int[ActivateTrig[u*cos[e + f*x]^(2*p)], x], x] /; FreeQ[{a, b, e, f, p
}, x] && EqQ[a + b, 0] && IntegerQ[p]
```

Rule 4360

```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{d = FreeFacto
rs[Sin[c*(a + b*x)], x]}, Dist[1/(b*c), Subst[Int[SubstFor[1/x, Sin[c*(a +
b*x)]/d, u, x], x], x, Sin[c*(a + b*x)]/d], x] /; FunctionOfQ[Sin[c*(a + b*
x)]/d, u, x]] /; FreeQ[{a, b, c}, x] && (EqQ[F, Cot] || EqQ[F, cot])
```

Rubi steps

$$\int \cot(x)\sqrt{-1+\csc^2(x)}\,\left(1-\sin^2(x)\right)^3\,dx = \int \cos^6(x)\cot(x)\sqrt{-1+\csc^2(x)}\,dx$$
$$= \text{Subst}\left(\int \frac{\sqrt{-1+\frac{1}{x^2}}\,\left(1-x^2\right)^3}{x}\,dx, x, \sin(x)\right)$$
$$= \text{Subst}\left(\int \left(-1+\frac{1}{x^2}\right)^{7/2} x^5\,dx, x, \sin(x)\right)$$
$$= -\left(\frac{1}{2}\,\text{Subst}\left(\int \frac{(-1+x)^{7/2}}{x^4}\,dx, x, \csc^2(x)\right)\right)$$
$$= \frac{1}{6}\cot^2(x)^{7/2}\sin^6(x) - \frac{7}{12}\,\text{Subst}\left(\int \frac{(-1+x)^{5/2}}{x^3}\,dx, x, \csc^2(x)\right)$$
$$= \frac{7}{24}\cot^2(x)^{5/2}\sin^4(x) + \frac{1}{6}\cot^2(x)^{7/2}\sin^6(x) - \frac{35}{48}\,\text{Subst}\left(\int \frac{(-1+x)^{3}}{x^2}\right.$$
$$= \frac{35}{48}\cot^2(x)^{3/2}\sin^2(x) + \frac{7}{24}\cot^2(x)^{5/2}\sin^4(x) + \frac{1}{6}\cot^2(x)^{7/2}\sin^6(x) -$$
$$= -\frac{35}{16}\sqrt{\cot^2(x)} + \frac{35}{48}\cot^2(x)^{3/2}\sin^2(x) + \frac{7}{24}\cot^2(x)^{5/2}\sin^4(x) + \frac{1}{6}\,\text{co}$$
$$= -\frac{35}{16}\sqrt{\cot^2(x)} + \frac{35}{48}\cot^2(x)^{3/2}\sin^2(x) + \frac{7}{24}\cot^2(x)^{5/2}\sin^4(x) + \frac{1}{6}\,\text{co}$$
$$= \frac{35}{16}\tan^{-1}\left(\sqrt{\cot^2(x)}\right) - \frac{35}{16}\sqrt{\cot^2(x)} + \frac{35}{48}\cot^2(x)^{3/2}\sin^2(x) + \frac{7}{24}\,\text{co}$$

Mathematica [A] time = 0.09, size = 40, normalized size = 0.53

$$\frac{1}{384}\sqrt{\cot^2(x)}\,\sec(x)(-840x\sin(x) - 525\cos(x) + 126\cos(3x) + 14\cos(5x) + \cos(7x))$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[x]*Sqrt[-1 + Csc[x]^2]*(1 - Sin[x]^2)^3,x]
```

```
[Out] (Sqrt[Cot[x]^2]*Sec[x]*(-525*Cos[x] + 126*Cos[3*x] + 14*Cos[5*x] + Cos[7*x]
 - 840*x*Sin[x]))/384
```

fricas [A] time = 1.92, size = 34, normalized size = 0.45

$$-\frac{8\cos(x)^7 + 14\cos(x)^5 + 35\cos(x)^3 - 105\,x\sin(x) - 105\cos(x)}{48\sin(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)*(1-sin(x)^2)^3*(-1+csc(x)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/48*(8*cos(x)^7 + 14*cos(x)^5 + 35*cos(x)^3 - 105*x*sin(x) - 105*cos(x))/
sin(x)
```

giac [A] time = 0.15, size = 97, normalized size = 1.28

$$-\frac{1}{48}\left(\left(2\left(4\sin(x)^2-19\right)\sin(x)^2+87\right)\sqrt{-\sin(x)^2+1}\,\sin(x)-105\left(\pi\left\lfloor\frac{x}{\pi}+\frac{1}{2}\right\rfloor-x\right)(-1)^{\left\lfloor\frac{x}{\pi}+\frac{1}{2}\right\rfloor}+\frac{24\left(\sqrt{-\sin(x)^2\right.}{\sin(x}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)*(1-sin(x)^2)^3*(-1+csc(x)^2)^(1/2),x, algorithm="giac")
```

```
[Out] -1/48*((2*(4*sin(x)^2 - 19)*sin(x)^2 + 87)*sqrt(-sin(x)^2 + 1)*sin(x) - 105
*(pi*floor(x/pi + 1/2) - x)*(-1)^floor(x/pi + 1/2) + 24*(sqrt(-sin(x)^2 + 1
) - 1)/sin(x) - 24*sin(x)/(sqrt(-sin(x)^2 + 1) - 1))*sgn(sin(x))
```

maple [A] time = 0.38, size = 54, normalized size = 0.71

$$-\frac{\left(-8\left(\cos^7(x)\right)-14\left(\cos^5(x)\right)-35\left(\cos^3(x)\right)+105x\sin(x)+105\cos(x)\right)\sqrt{-\frac{\cos^2(x)}{-1+\cos^2(x)}}\,\sqrt{4}}{96\cos(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(x)*(1-sin(x)^2)^3*(-1+csc(x)^2)^(1/2),x)
```

```
[Out] -1/96*(-8*cos(x)^7-14*cos(x)^5-35*cos(x)^3+105*x*sin(x)+105*cos(x))*(-cos(x
)^2/(-1+cos(x)^2))^(1/2)/cos(x)*4^(1/2)
```

maxima [B] time = 0.71, size = 136, normalized size = 1.79

$$-\frac{3}{2}\sqrt{\frac{1}{\sin(x)^2}-1}\,\sin(x)^2-\sqrt{\frac{1}{\sin(x)^2}-1}+\frac{3\left(\frac{1}{\sin(x)^2}-1\right)^{\frac{5}{2}}+8\left(\frac{1}{\sin(x)^2}-1\right)^{\frac{3}{2}}-3\sqrt{\frac{1}{\sin(x)^2}-1}}{48\left(\left(\frac{1}{\sin(x)^2}-1\right)^3+3\left(\frac{1}{\sin(x)^2}-1\right)^2+\frac{3}{\sin(x)^2}-2\right)}-\frac{3\left(\left(\frac{1}{\sin(x)^2}-1\right)^{\frac{3}{2}}-\right.}{8\left(\left(\frac{1}{\sin(x)^2}-1\right)^2\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)*(1-sin(x)^2)^3*(-1+csc(x)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] -3/2*sqrt(1/sin(x)^2 - 1)*sin(x)^2 - sqrt(1/sin(x)^2 - 1) + 1/48*(3*(1/sin(
x)^2 - 1)^(5/2) + 8*(1/sin(x)^2 - 1)^(3/2) - 3*sqrt(1/sin(x)^2 - 1))/((1/si
```

```
n(x)^2 - 1)^3 + 3*(1/sin(x)^2 - 1)^2 + 3/sin(x)^2 - 2) - 3/8*((1/sin(x)^2 -
 1)^(3/2) - sqrt(1/sin(x)^2 - 1))/((1/sin(x)^2 - 1)^2 + 2/sin(x)^2 - 1) + 3
5/16*arctan(sqrt(1/sin(x)^2 - 1))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int -\cot(x)\sqrt{\frac{1}{\sin(x)^2}-1}\,\left(\sin(x)^2-1\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-cot(x)*(1/sin(x)^2 - 1)^(1/2)*(sin(x)^2 - 1)^3,x)
```

```
[Out] int(-cot(x)*(1/sin(x)^2 - 1)^(1/2)*(sin(x)^2 - 1)^3, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)*(1-sin(x)**2)**3*(-1+csc(x)**2)**(1/2),x)
```

```
[Out] Timed out
```

3.867 $\int \cos(x)\sqrt{-1+\csc^2(x)}\left(1-\sin^2(x)\right)^3\,dx$

Optimal. Leaf size=81

$$\sin(x)\sqrt{\cot^2(x)}+\frac{1}{7}\sin(x)\cos^6(x)\sqrt{\cot^2(x)}+\frac{1}{5}\sin(x)\cos^4(x)\sqrt{\cot^2(x)}+\frac{1}{3}\sin(x)\cos^2(x)\sqrt{\cot^2(x)}-\tan(x)\sqrt{\text{co}}$$

```
[Out] sin(x)*(cot(x)^2)^(1/2)+1/3*cos(x)^2*sin(x)*(cot(x)^2)^(1/2)+1/5*cos(x)^4*s
in(x)*(cot(x)^2)^(1/2)+1/7*cos(x)^6*sin(x)*(cot(x)^2)^(1/2)-arctanh(cos(x))
*(cot(x)^2)^(1/2)*tan(x)
```

Rubi [A] time = 0.16, antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {3175, 4121, 3658, 2592, 302, 206}

$$\sin(x)\sqrt{\cot^2(x)}+\frac{1}{7}\sin(x)\cos^6(x)\sqrt{\cot^2(x)}+\frac{1}{5}\sin(x)\cos^4(x)\sqrt{\cot^2(x)}+\frac{1}{3}\sin(x)\cos^2(x)\sqrt{\cot^2(x)}-\tan(x)\sqrt{\text{co}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[x]*Sqrt[-1 + Csc[x]^2]*(1 - Sin[x]^2)^3,x]
```

```
[Out] Sqrt[Cot[x]^2]*Sin[x] + (Cos[x]^2*Sqrt[Cot[x]^2]*Sin[x])/3 + (Cos[x]^4*Sqrt
[Cot[x]^2]*Sin[x])/5 + (Cos[x]^6*Sqrt[Cot[x]^2]*Sin[x])/7 - ArcTanh[Cos[x]]
*Sqrt[Cot[x]^2]*Tan[x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 2592

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(
ff*x)^(m + n)/(a^2 - ff^2*x^2)^((n + 1)/2), x], x, (a*Sin[e + f*x])/ff], x]
] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2]
```

Rule 3175

```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Dist[
a^p, Int[ActivateTrig[u*cos[e + f*x]^(2*p)], x], x] /; FreeQ[{a, b, e, f, p
}, x] && EqQ[a + b, 0] && IntegerQ[p]
```

Rule 3658

```
Int[(u_.)*((b_.)*tan[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Tan[e + f*x], x]}, Dist[((b*ff^n)^IntPart[p]*(b*Tan[e + f*x]^
n)^FracPart[p])/(Tan[e + f*x]/ff)^(n*FracPart[p]), Int[ActivateTrig[u]*(Tan
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rule 4121

```
Int[(u_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(b*tan[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a + b, 0]
```

Rubi steps

$$\begin{aligned}
\int \cos(x)\sqrt{-1+\csc^2(x)}\,\left(1-\sin^2(x)\right)^3\,dx &= \int \cos^7(x)\sqrt{-1+\csc^2(x)}\,dx\\
&= \int \cos^7(x)\sqrt{\cot^2(x)}\,dx\\
&= \left(\sqrt{\cot^2(x)}\,\tan(x)\right)\int \cos^7(x)\cot(x)\,dx\\
&= -\left(\left(\sqrt{\cot^2(x)}\,\tan(x)\right)\text{Subst}\left(\int \frac{x^8}{1-x^2}\,dx,x,\cos(x)\right)\right)\\
&= -\left(\left(\sqrt{\cot^2(x)}\,\tan(x)\right)\text{Subst}\left(\int \left(-1-x^2-x^4-x^6+\frac{1}{1-x^2}\right)dx,x,\right.\right.\\
&= \sqrt{\cot^2(x)}\,\sin(x)+\frac{1}{3}\cos^2(x)\sqrt{\cot^2(x)}\,\sin(x)+\frac{1}{5}\cos^4(x)\sqrt{\cot^2(x)}\,\\
&= \sqrt{\cot^2(x)}\,\sin(x)+\frac{1}{3}\cos^2(x)\sqrt{\cot^2(x)}\,\sin(x)+\frac{1}{5}\cos^4(x)\sqrt{\cot^2(x)}\,
\end{aligned}$$

Mathematica [A] time = 0.06, size = 55, normalized size = 0.68

$$\frac{\tan(x)\sqrt{\cot^2(x)}\,\left(9765\cos(x)+1295\cos(3x)+189\cos(5x)+15\cos(7x)+6720\log\left(\sin\left(\frac{x}{2}\right)\right)-6720\log\left(\cos\left(\frac{x}{2}\right.\right.\right.}{6720}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[x]*Sqrt[-1 + Csc[x]^2]*(1 - Sin[x]^2)^3,x]
```

```
[Out] (Sqrt[Cot[x]^2]*(9765*Cos[x] + 1295*Cos[3*x] + 189*Cos[5*x] + 15*Cos[7*x] -
 6720*Log[Cos[x/2]] + 6720*Log[Sin[x/2]])*Tan[x])/6720
```

fricas [A] time = 0.93, size = 41, normalized size = 0.51

$$-\frac{1}{7}\cos(x)^7 - \frac{1}{5}\cos(x)^5 - \frac{1}{3}\cos(x)^3 - \cos(x) + \frac{1}{2}\log\left(\frac{1}{2}\cos(x) + \frac{1}{2}\right) - \frac{1}{2}\log\left(-\frac{1}{2}\cos(x) + \frac{1}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*(1-sin(x)^2)^3*(-1+csc(x)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/7*cos(x)^7 - 1/5*cos(x)^5 - 1/3*cos(x)^3 - cos(x) + 1/2*log(1/2*cos(x) +
 1/2) - 1/2*log(-1/2*cos(x) + 1/2)
```

giac [A] time = 0.15, size = 44, normalized size = 0.54

$$\frac{1}{210}\left(30\cos(x)^7 + 42\cos(x)^5 + 70\cos(x)^3 + 210\cos(x) - 105\log(\cos(x) + 1) + 105\log(-\cos(x) + 1)\right)\operatorname{sgn}($$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*(1-sin(x)^2)^3*(-1+csc(x)^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/210*(30*cos(x)^7 + 42*cos(x)^5 + 70*cos(x)^3 + 210*cos(x) - 105*log(cos(x
) + 1) + 105*log(-cos(x) + 1))*sgn(sin(x))
```

maple [A] time = 0.27, size = 65, normalized size = 0.80

$$\frac{\left(15\left(\cos^7(x)\right) + 21\left(\cos^5(x)\right) + 35\left(\cos^3(x)\right) + 105\cos(x) + 105\ln\left(-\frac{-1+\cos(x)}{\sin(x)}\right) + 176\right)\sin(x)\sqrt{-\frac{\cos^2(x)}{-1+\cos^2(x)}}\ \sqrt{4}}{210\cos(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)*(1-sin(x)^2)^3*(-1+csc(x)^2)^(1/2),x)
```

```
[Out] 1/210*(15*cos(x)^7+21*cos(x)^5+35*cos(x)^3+105*cos(x)+105*ln(-(-1+cos(x))/s
in(x))+176)*sin(x)*(-cos(x)^2/(-1+cos(x)^2))^(1/2)/cos(x)*4^(1/2)
```

maxima [A] time = 0.93, size = 86, normalized size = 1.06

$$\frac{1}{7}\left(\frac{1}{\sin(x)^2} - 1\right)^{\frac{7}{2}}\sin(x)^7 + \frac{1}{5}\left(\frac{1}{\sin(x)^2} - 1\right)^{\frac{5}{2}}\sin(x)^5 + \frac{1}{3}\left(\frac{1}{\sin(x)^2} - 1\right)^{\frac{3}{2}}\sin(x)^3 + \sqrt{\frac{1}{\sin(x)^2} - 1}\ \sin(x) - \frac{1}{2}\log\left(\sqrt{\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*(1-sin(x)^2)^3*(-1+csc(x)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/7*(1/sin(x)^2 - 1)^(7/2)*sin(x)^7 + 1/5*(1/sin(x)^2 - 1)^(5/2)*sin(x)^5 +
 1/3*(1/sin(x)^2 - 1)^(3/2)*sin(x)^3 + sqrt(1/sin(x)^2 - 1)*sin(x) - 1/2*lo
g(sqrt(1/sin(x)^2 - 1)*sin(x) + 1) + 1/2*log(sqrt(1/sin(x)^2 - 1)*sin(x) -
1)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$-\int \cos(x)\sqrt{\frac{1}{\sin(x)^2}-1}\,\left(\sin(x)^2-1\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-cos(x)*(1/sin(x)^2 - 1)^(1/2)*(sin(x)^2 - 1)^3,x)
```

```
[Out] -int(cos(x)*(1/sin(x)^2 - 1)^(1/2)*(sin(x)^2 - 1)^3, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*(1-sin(x)**2)**3*(-1+csc(x)**2)**(1/2),x)
```

```
[Out] Timed out
```

3.868 $\int \frac{x \csc(x) \sec(x)}{\sqrt{a \sec^2(x)}} \, dx$

Optimal. Leaf size=76

$$\frac{i\text{Li}_2\left(-e^{ix}\right)\sec(x)}{\sqrt{a\sec^2(x)}} - \frac{i\text{Li}_2\left(e^{ix}\right)\sec(x)}{\sqrt{a\sec^2(x)}} - \frac{2x\sec(x)\tanh^{-1}\left(e^{ix}\right)}{\sqrt{a\sec^2(x)}}$$

```
[Out] -2*x*arctanh(exp(I*x))*sec(x)/(a*sec(x)^2)^(1/2)+I*polylog(2,-exp(I*x))*sec
(x)/(a*sec(x)^2)^(1/2)-I*polylog(2,exp(I*x))*sec(x)/(a*sec(x)^2)^(1/2)
```

Rubi [A] time = 0.53, antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {6720, 4183, 2279, 2391}

$$\frac{i\sec(x)\text{PolyLog}\left(2,-e^{ix}\right)}{\sqrt{a\sec^2(x)}} - \frac{i\sec(x)\text{PolyLog}\left(2,e^{ix}\right)}{\sqrt{a\sec^2(x)}} - \frac{2x\sec(x)\tanh^{-1}\left(e^{ix}\right)}{\sqrt{a\sec^2(x)}}$$

Antiderivative was successfully verified.

```
[In] Int[(x*Csc[x]*Sec[x])/Sqrt[a*Sec[x]^2],x]
```

```
[Out] (-2*x*ArcTanh[E^(I*x)]*Sec[x])/Sqrt[a*Sec[x]^2] + (I*PolyLog[2, -E^(I*x)]*S
ec[x])/Sqrt[a*Sec[x]^2] - (I*PolyLog[2, E^(I*x)]*Sec[x])/Sqrt[a*Sec[x]^2]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4183

```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[(
-2*(c + d*x)^m*ArcTanh[E^(I*(e + f*x))])/f, x] + (-Dist[(d*m)/f, Int[(c + d
*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Dist[(d*m)/f, Int[(c + d*x)^
(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 6720

```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Dist[(a^IntPart[p]*(a*v^m)^
FracPart[p])/v^(m*FracPart[p]), Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p}, x
] && !IntegerQ[p] && !FreeQ[v, x] && !(EqQ[a, 1] && EqQ[m, 1]) && !(EqQ
[v, x] && EqQ[m, 1])
```

Rubi steps

$$\begin{aligned}\int \frac{x\csc(x)\sec(x)}{\sqrt{a\sec^2(x)}}\,dx &= \frac{\sec(x)\int x\csc(x)\,dx}{\sqrt{a\sec^2(x)}}\\ &= -\frac{2x\tanh^{-1}\left(e^{ix}\right)\sec(x)}{\sqrt{a\sec^2(x)}} - \frac{\sec(x)\int \log\left(1-e^{ix}\right)\,dx}{\sqrt{a\sec^2(x)}} + \frac{\sec(x)\int \log\left(1+e^{ix}\right)\,dx}{\sqrt{a\sec^2(x)}}\\ &= -\frac{2x\tanh^{-1}\left(e^{ix}\right)\sec(x)}{\sqrt{a\sec^2(x)}} + \frac{(i\sec(x))\,\text{Subst}\left(\int \frac{\log(1-x)}{x}\,dx,x,e^{ix}\right)}{\sqrt{a\sec^2(x)}} - \frac{(i\sec(x))\,\text{Subst}\left(\int \frac{\log(1}{x}\right.}{\sqrt{a\sec^2(x)}}\\ &= -\frac{2x\tanh^{-1}\left(e^{ix}\right)\sec(x)}{\sqrt{a\sec^2(x)}} + \frac{i\text{Li}_2\left(-e^{ix}\right)\sec(x)}{\sqrt{a\sec^2(x)}} - \frac{i\text{Li}_2\left(e^{ix}\right)\sec(x)}{\sqrt{a\sec^2(x)}}\end{aligned}$$

Mathematica [A] time = 0.06, size = 69, normalized size = 0.91

$$\frac{\sec(x)\left(i\text{Li}_2\left(-e^{ix}\right) - i\text{Li}_2\left(e^{ix}\right) + x\left(\log\left(1-e^{ix}\right) - \log\left(1+e^{ix}\right)\right)\right)}{\sqrt{a\sec^2(x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*Csc[x]*Sec[x])/Sqrt[a*Sec[x]^2],x]
```

```
[Out] ((x*(Log[1 - E^(I*x)] - Log[1 + E^(I*x)]) + I*PolyLog[2, -E^(I*x)] - I*Poly
Log[2, E^(I*x)])*Sec[x])/Sqrt[a*Sec[x]^2]
```

fricas [B] time = 1.27, size = 124, normalized size = 1.63

$$-\frac{\left(x\cos(x)\log\left(\cos(x)+i\,\sin(x)+1\right) + x\cos(x)\log\left(\cos(x)-i\,\sin(x)+1\right) - x\cos(x)\log\left(-\cos(x)+i\,\sin(x)+1\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*csc(x)*sec(x)/(a*sec(x)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/2*(x*cos(x)*log(cos(x) + I*sin(x) + 1) + x*cos(x)*log(cos(x) - I*sin(x)
+ 1) - x*cos(x)*log(-cos(x) + I*sin(x) + 1) - x*cos(x)*log(-cos(x) - I*sin(
```

```
x) + 1) + I*cos(x)*dilog(cos(x) + I*sin(x)) - I*cos(x)*dilog(cos(x) - I*sin
(x)) + I*cos(x)*dilog(-cos(x) + I*sin(x)) - I*cos(x)*dilog(-cos(x) - I*sin(
x)))*sqrt(a/cos(x)^2)/a
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x\csc(x)\sec(x)}{\sqrt{a\sec(x)^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*csc(x)*sec(x)/(a*sec(x)^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x*csc(x)*sec(x)/sqrt(a*sec(x)^2), x)
```

maple [A] time = 0.23, size = 98, normalized size = 1.29

$$-\frac{2i\left(-\frac{i e^{ix} x\ln\left(1+e^{ix}\right)}{2}-\frac{e^{ix}\operatorname{polylog}\left(2,-e^{ix}\right)}{2}+\frac{i e^{ix} x\ln\left(1-e^{ix}\right)}{2}+\frac{e^{ix}\operatorname{polylog}\left(2,e^{ix}\right)}{2}\right)}{\sqrt{\frac{a\,e^{2ix}}{\left(e^{2ix}+1\right)^2}}\,\left(e^{2ix}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*csc(x)*sec(x)/(a*sec(x)^2)^(1/2),x)
```

```
[Out] -2*I/(a*exp(2*I*x)/(exp(2*I*x)+1)^2)^(1/2)/(exp(2*I*x)+1)*(-1/2*I*exp(I*x)*
x*ln(1+exp(I*x))-1/2*exp(I*x)*polylog(2,-exp(I*x))+1/2*I*exp(I*x)*x*ln(1-ex
p(I*x))+1/2*exp(I*x)*polylog(2,exp(I*x)))
```

maxima [A] time = 0.85, size = 79, normalized size = 1.04

$$-\frac{2i\,x\arctan\left(\sin(x),\cos(x)+1\right)+2i\,x\arctan\left(\sin(x),-\cos(x)+1\right)+x\log\left(\cos(x)^2+\sin(x)^2+2\,\cos(x)+1\right)-}{2\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*csc(x)*sec(x)/(a*sec(x)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/2*(2*I*x*arctan2(sin(x), cos(x) + 1) + 2*I*x*arctan2(sin(x), -cos(x) + 1
) + x*log(cos(x)^2 + sin(x)^2 + 2*cos(x) + 1) - x*log(cos(x)^2 + sin(x)^2 -
 2*cos(x) + 1) - 2*I*dilog(-e^(I*x)) + 2*I*dilog(e^(I*x)))/sqrt(a)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x}{\cos(x)\,\sin(x)\,\sqrt{\frac{a}{\cos(x)^2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(cos(x)*sin(x)*(a/cos(x)^2)^(1/2)),x)
```

```
[Out] int(x/(cos(x)*sin(x)*(a/cos(x)^2)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x \csc(x) \sec(x)}{\sqrt{a \sec^2(x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*csc(x)*sec(x)/(a*sec(x)**2)**(1/2),x)
```

```
[Out] Integral(x*csc(x)*sec(x)/sqrt(a*sec(x)**2), x)
```

3.869 $\int \frac{x^2 \csc(x) \sec(x)}{\sqrt{a \sec^2(x)}} \, dx$

Optimal. Leaf size=128

$$\frac{2ix\text{Li}_2\left(-e^{ix}\right)\sec(x)}{\sqrt{a\sec^2(x)}}-\frac{2ix\text{Li}_2\left(e^{ix}\right)\sec(x)}{\sqrt{a\sec^2(x)}}-\frac{2\text{Li}_3\left(-e^{ix}\right)\sec(x)}{\sqrt{a\sec^2(x)}}+\frac{2\text{Li}_3\left(e^{ix}\right)\sec(x)}{\sqrt{a\sec^2(x)}}-\frac{2x^2\sec(x)\tanh^{-1}\left(e^{ix}\right)}{\sqrt{a\sec^2(x)}}$$

```
[Out] -2*x^2*arctanh(exp(I*x))*sec(x)/(a*sec(x)^2)^(1/2)+2*I*x*polylog(2,-exp(I*x
))*sec(x)/(a*sec(x)^2)^(1/2)-2*I*x*polylog(2,exp(I*x))*sec(x)/(a*sec(x)^2)^
(1/2)-2*polylog(3,-exp(I*x))*sec(x)/(a*sec(x)^2)^(1/2)+2*polylog(3,exp(I*x)
)*sec(x)/(a*sec(x)^2)^(1/2)
```

Rubi [A] time = 0.59, antiderivative size = 128, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {6720, 4183, 2531, 2282, 6589}

$$\frac{2ix\sec(x)\text{PolyLog}\left(2,-e^{ix}\right)}{\sqrt{a\sec^2(x)}}-\frac{2ix\sec(x)\text{PolyLog}\left(2,e^{ix}\right)}{\sqrt{a\sec^2(x)}}-\frac{2\sec(x)\text{PolyLog}\left(3,-e^{ix}\right)}{\sqrt{a\sec^2(x)}}+\frac{2\sec(x)\text{PolyLog}\left(3,e^{ix}\right)}{\sqrt{a\sec^2(x)}}-$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*Csc[x]*Sec[x])/Sqrt[a*Sec[x]^2],x]
```

```
[Out] (-2*x^2*ArcTanh[E^(I*x)]*Sec[x])/Sqrt[a*Sec[x]^2] + ((2*I)*x*PolyLog[2, -E^
(I*x)]*Sec[x])/Sqrt[a*Sec[x]^2] - ((2*I)*x*PolyLog[2, E^(I*x)]*Sec[x])/Sqrt
[a*Sec[x]^2] - (2*PolyLog[3, -E^(I*x)]*Sec[x])/Sqrt[a*Sec[x]^2] + (2*PolyLo
g[3, E^(I*x)]*Sec[x])/Sqrt[a*Sec[x]^2]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 4183

```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[(
-2*(c + d*x)^m*ArcTanh[E^(I*(e + f*x))])/f, x] + (-Dist[(d*m)/f, Int[(c + d
*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Dist[(d*m)/f, Int[(c + d*x)^
(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6720

```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Dist[(a^IntPart[p]*(a*v^m)^
FracPart[p])/v^(m*FracPart[p]), Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p}, x
] && !IntegerQ[p] && !FreeQ[v, x] && !(EqQ[a, 1] && EqQ[m, 1]) && !(EqQ
[v, x] && EqQ[m, 1])
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^2 \csc(x)\sec(x)}{\sqrt{a\sec^2(x)}}\,dx &= \frac{\sec(x)\int x^2\csc(x)\,dx}{\sqrt{a\sec^2(x)}}\\
&= -\frac{2x^2\tanh^{-1}\left(e^{ix}\right)\sec(x)}{\sqrt{a\sec^2(x)}} - \frac{(2\sec(x))\int x\log\left(1-e^{ix}\right)\,dx}{\sqrt{a\sec^2(x)}} + \frac{(2\sec(x))\int x\log\left(1+e^{ix}\right)\,d}{\sqrt{a\sec^2(x)}}\\
&= -\frac{2x^2\tanh^{-1}\left(e^{ix}\right)\sec(x)}{\sqrt{a\sec^2(x)}} + \frac{2ix\mathrm{Li}_2\left(-e^{ix}\right)\sec(x)}{\sqrt{a\sec^2(x)}} - \frac{2ix\mathrm{Li}_2\left(e^{ix}\right)\sec(x)}{\sqrt{a\sec^2(x)}} - \frac{(2i\sec(x))\int \mathrm{Li}_2}{\sqrt{a\sec^2(}}\\
&= -\frac{2x^2\tanh^{-1}\left(e^{ix}\right)\sec(x)}{\sqrt{a\sec^2(x)}} + \frac{2ix\mathrm{Li}_2\left(-e^{ix}\right)\sec(x)}{\sqrt{a\sec^2(x)}} - \frac{2ix\mathrm{Li}_2\left(e^{ix}\right)\sec(x)}{\sqrt{a\sec^2(x)}} - \frac{(2\sec(x))\,\mathrm{Subst}}{\sqrt{a\,s}}\\
&= -\frac{2x^2\tanh^{-1}\left(e^{ix}\right)\sec(x)}{\sqrt{a\sec^2(x)}} + \frac{2ix\mathrm{Li}_2\left(-e^{ix}\right)\sec(x)}{\sqrt{a\sec^2(x)}} - \frac{2ix\mathrm{Li}_2\left(e^{ix}\right)\sec(x)}{\sqrt{a\sec^2(x)}} - \frac{2\mathrm{Li}_3\left(-e^{ix}\right)\sec(}{\sqrt{a\sec^2(x)}}
\end{aligned}
$$

Mathematica [A] time = 0.07, size = 99, normalized size = 0.77

$$
\frac{\sec(x)\left(2ix\mathrm{Li}_2\left(-e^{ix}\right)-2ix\mathrm{Li}_2\left(e^{ix}\right)-2\mathrm{Li}_3\left(-e^{ix}\right)+2\mathrm{Li}_3\left(e^{ix}\right)+x^2\log\left(1-e^{ix}\right)-x^2\log\left(1+e^{ix}\right)\right)}{\sqrt{a\sec^2(x)}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*Csc[x]*Sec[x])/Sqrt[a*Sec[x]^2],x]
```

```
[Out] ((x^2*Log[1 - E^(I*x)] - x^2*Log[1 + E^(I*x)] + (2*I)*x*PolyLog[2, -E^(I*x)
] - (2*I)*x*PolyLog[2, E^(I*x)] - 2*PolyLog[3, -E^(I*x)] + 2*PolyLog[3, E^(
I*x)])*Sec[x])/Sqrt[a*Sec[x]^2]
```

fricas [C] time = 1.62, size = 227, normalized size = 1.77

$$\frac{2\sqrt{\frac{a}{\cos(x)^2}}\cos(x)\mathrm{polylog}(3,\cos(x)+i\sin(x))+2\sqrt{\frac{a}{\cos(x)^2}}\cos(x)\mathrm{polylog}(3,\cos(x)-i\sin(x))-2\sqrt{\frac{a}{\cos(x)^2}}\,\mathrm{co}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*csc(x)*sec(x)/(a*sec(x)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*(2*sqrt(a/cos(x)^2)*cos(x)*polylog(3, cos(x) + I*sin(x)) + 2*sqrt(a/cos
(x)^2)*cos(x)*polylog(3, cos(x) - I*sin(x)) - 2*sqrt(a/cos(x)^2)*cos(x)*pol
ylog(3, -cos(x) + I*sin(x)) - 2*sqrt(a/cos(x)^2)*cos(x)*polylog(3, -cos(x)
- I*sin(x)) - (x^2*cos(x)*log(cos(x) + I*sin(x) + 1) + x^2*cos(x)*log(cos(x
) - I*sin(x) + 1) - x^2*cos(x)*log(-cos(x) + I*sin(x) + 1) - x^2*cos(x)*log
(-cos(x) - I*sin(x) + 1) + 2*I*x*cos(x)*dilog(cos(x) + I*sin(x)) - 2*I*x*co
s(x)*dilog(cos(x) - I*sin(x)) + 2*I*x*cos(x)*dilog(-cos(x) + I*sin(x)) - 2*
I*x*cos(x)*dilog(-cos(x) - I*sin(x)))*sqrt(a/cos(x)^2))/a
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2\csc(x)\sec(x)}{\sqrt{a\sec(x)^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*csc(x)*sec(x)/(a*sec(x)^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^2*csc(x)*sec(x)/sqrt(a*sec(x)^2), x)
```

maple [A] time = 0.20, size = 132, normalized size = 1.03

$$-\frac{2\left(\frac{e^{ix}x^2\ln\left(1+e^{ix}\right)}{2}-ie^{ix}x\,\mathrm{polylog}\left(2,-e^{ix}\right)+e^{ix}\,\mathrm{polylog}\left(3,-e^{ix}\right)-\frac{e^{ix}x^2\ln\left(1-e^{ix}\right)}{2}+ie^{ix}x\,\mathrm{polylog}\left(2,e^{ix}\right)-e^{ix}\,\mathrm{po}\right.}{\sqrt{\frac{a\,e^{2ix}}{\left(e^{2ix}+1\right)^2}}\left(e^{2ix}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*csc(x)*sec(x)/(a*sec(x)^2)^(1/2),x)
```

```
[Out] -2/(a*exp(2*I*x)/(exp(2*I*x)+1)^2)^(1/2)/(exp(2*I*x)+1)*(1/2*exp(I*x)*x^2*l
n(1+exp(I*x))-I*exp(I*x)*x*polylog(2,-exp(I*x))+exp(I*x)*polylog(3,-exp(I*x
))-1/2*exp(I*x)*x^2*ln(1-exp(I*x))+I*exp(I*x)*x*polylog(2,exp(I*x))-exp(I*x
)*polylog(3,exp(I*x)))
```

maxima [A] time = 0.68, size = 107, normalized size = 0.84

$$-\frac{2i\,x^2\arctan\left(\sin(x),\cos(x)+1\right)+2i\,x^2\arctan\left(\sin(x),-\cos(x)+1\right)+x^2\log\left(\cos(x)^2+\sin(x)^2+2\,\cos(x)+1\right)}{2\,\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*csc(x)*sec(x)/(a*sec(x)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/2*(2*I*x^2*arctan2(sin(x), cos(x) + 1) + 2*I*x^2*arctan2(sin(x), -cos(x)
 + 1) + x^2*log(cos(x)^2 + sin(x)^2 + 2*cos(x) + 1) - x^2*log(cos(x)^2 + si
n(x)^2 - 2*cos(x) + 1) - 4*I*x*dilog(-e^(I*x)) + 4*I*x*dilog(e^(I*x)) + 4*p
olylog(3, -e^(I*x)) - 4*polylog(3, e^(I*x)))/sqrt(a)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^2}{\cos(x)\,\sin(x)\,\sqrt{\frac{a}{\cos(x)^2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(cos(x)*sin(x)*(a/cos(x)^2)^(1/2)),x)
```

```
[Out] int(x^2/(cos(x)*sin(x)*(a/cos(x)^2)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2\csc\left(x\right)\sec\left(x\right)}{\sqrt{a\sec^2\left(x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*csc(x)*sec(x)/(a*sec(x)**2)**(1/2),x)
```

```
[Out] Integral(x**2*csc(x)*sec(x)/sqrt(a*sec(x)**2), x)
```

3.870 $\int \frac{x^3 \csc(x)\sec(x)}{\sqrt{a\sec^2(x)}}\,dx$

Optimal. Leaf size=186

$$\frac{3ix^2\text{Li}_2\left(-e^{ix}\right)\sec(x)}{\sqrt{a\sec^2(x)}}-\frac{3ix^2\text{Li}_2\left(e^{ix}\right)\sec(x)}{\sqrt{a\sec^2(x)}}-\frac{6x\text{Li}_3\left(-e^{ix}\right)\sec(x)}{\sqrt{a\sec^2(x)}}+\frac{6x\text{Li}_3\left(e^{ix}\right)\sec(x)}{\sqrt{a\sec^2(x)}}-\frac{6i\text{Li}_4\left(-e^{ix}\right)\sec(x)}{\sqrt{a\sec^2(x)}}+\frac{6i\text{Li}_4\left(\right.}{\sqrt{a\,\text{s}}}$$

```
[Out] -2*x^3*arctanh(exp(I*x))*sec(x)/(a*sec(x)^2)^(1/2)+3*I*x^2*polylog(2,-exp(I
*x))*sec(x)/(a*sec(x)^2)^(1/2)-3*I*x^2*polylog(2,exp(I*x))*sec(x)/(a*sec(x)
^2)^(1/2)-6*x*polylog(3,-exp(I*x))*sec(x)/(a*sec(x)^2)^(1/2)+6*x*polylog(3,
exp(I*x))*sec(x)/(a*sec(x)^2)^(1/2)-6*I*polylog(4,-exp(I*x))*sec(x)/(a*sec(
x)^2)^(1/2)+6*I*polylog(4,exp(I*x))*sec(x)/(a*sec(x)^2)^(1/2)
```

Rubi [A] time = 0.57, antiderivative size = 186, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 6, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {6720, 4183, 2531, 6609, 2282, 6589}

$$\frac{3ix^2\sec(x)\text{PolyLog}\left(2,-e^{ix}\right)}{\sqrt{a\sec^2(x)}}-\frac{3ix^2\sec(x)\text{PolyLog}\left(2,e^{ix}\right)}{\sqrt{a\sec^2(x)}}-\frac{6x\sec(x)\text{PolyLog}\left(3,-e^{ix}\right)}{\sqrt{a\sec^2(x)}}+\frac{6x\sec(x)\text{PolyLog}\left(3,\right.}{\sqrt{a\sec^2(x)}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*Csc[x]*Sec[x])/Sqrt[a*Sec[x]^2],x]
```

```
[Out] (-2*x^3*ArcTanh[E^(I*x)]*Sec[x])/Sqrt[a*Sec[x]^2] + ((3*I)*x^2*PolyLog[2, -
E^(I*x)]*Sec[x])/Sqrt[a*Sec[x]^2] - ((3*I)*x^2*PolyLog[2, E^(I*x)]*Sec[x])/
Sqrt[a*Sec[x]^2] - (6*x*PolyLog[3, -E^(I*x)]*Sec[x])/Sqrt[a*Sec[x]^2] + (6*
x*PolyLog[3, E^(I*x)]*Sec[x])/Sqrt[a*Sec[x]^2] - ((6*I)*PolyLog[4, -E^(I*x)
]*Sec[x])/Sqrt[a*Sec[x]^2] + ((6*I)*PolyLog[4, E^(I*x)]*Sec[x])/Sqrt[a*Sec[
x]^2]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
```

```
1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 4183

```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[(
-2*(c + d*x)^m*ArcTanh[E^(I*(e + f*x))])/f, x] + (-Dist[(d*m)/f, Int[(c + d
*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Dist[(d*m)/f, Int[(c + d*x)^
(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rule 6720

```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Dist[(a^IntPart[p]*(a*v^m)^
FracPart[p])/v^(m*FracPart[p]), Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p}, x
] && !IntegerQ[p] && !FreeQ[v, x] && !(EqQ[a, 1] && EqQ[m, 1]) && !(EqQ
[v, x] && EqQ[m, 1])
```

Rubi steps

$$\int \frac{x^3 \csc(x)\sec(x)}{\sqrt{a\sec^2(x)}}\,dx = \frac{\sec(x)\int x^3\csc(x)\,dx}{\sqrt{a\sec^2(x)}}$$
$$= -\frac{2x^3\tanh^{-1}\left(e^{ix}\right)\sec(x)}{\sqrt{a\sec^2(x)}} - \frac{(3\sec(x))\int x^2\log\left(1-e^{ix}\right)\,dx}{\sqrt{a\sec^2(x)}} + \frac{(3\sec(x))\int x^2\log\left(1+e^{i}\right.}{\sqrt{a\sec^2(x)}}$$
$$= -\frac{2x^3\tanh^{-1}\left(e^{ix}\right)\sec(x)}{\sqrt{a\sec^2(x)}} + \frac{3ix^2\text{Li}_2\left(-e^{ix}\right)\sec(x)}{\sqrt{a\sec^2(x)}} - \frac{3ix^2\text{Li}_2\left(e^{ix}\right)\sec(x)}{\sqrt{a\sec^2(x)}} - \frac{(6i\sec(x))\int}{\sqrt{a\,\text{se}}}$$
$$= -\frac{2x^3\tanh^{-1}\left(e^{ix}\right)\sec(x)}{\sqrt{a\sec^2(x)}} + \frac{3ix^2\text{Li}_2\left(-e^{ix}\right)\sec(x)}{\sqrt{a\sec^2(x)}} - \frac{3ix^2\text{Li}_2\left(e^{ix}\right)\sec(x)}{\sqrt{a\sec^2(x)}} - \frac{6x\text{Li}_3\left(-e^{ix}\right)}{\sqrt{a\sec^2(}}$$
$$= -\frac{2x^3\tanh^{-1}\left(e^{ix}\right)\sec(x)}{\sqrt{a\sec^2(x)}} + \frac{3ix^2\text{Li}_2\left(-e^{ix}\right)\sec(x)}{\sqrt{a\sec^2(x)}} - \frac{3ix^2\text{Li}_2\left(e^{ix}\right)\sec(x)}{\sqrt{a\sec^2(x)}} - \frac{6x\text{Li}_3\left(-e^{ix}\right)}{\sqrt{a\sec^2(}}$$
$$= -\frac{2x^3\tanh^{-1}\left(e^{ix}\right)\sec(x)}{\sqrt{a\sec^2(x)}} + \frac{3ix^2\text{Li}_2\left(-e^{ix}\right)\sec(x)}{\sqrt{a\sec^2(x)}} - \frac{3ix^2\text{Li}_2\left(e^{ix}\right)\sec(x)}{\sqrt{a\sec^2(x)}} - \frac{6x\text{Li}_3\left(-e^{ix}\right)}{\sqrt{a\sec^2(}}$$

Mathematica [A] time = 0.10, size = 147, normalized size = 0.79

$$-\frac{i\sec(x)\left(-24x^2\text{Li}_2\left(e^{-ix}\right)-24x^2\text{Li}_2\left(-e^{ix}\right)+48ix\text{Li}_3\left(e^{-ix}\right)-48ix\text{Li}_3\left(-e^{ix}\right)+48\text{Li}_4\left(e^{-ix}\right)+48\text{Li}_4\left(-e^{ix}\right)-2x^4\right.}{8\sqrt{a\sec^2(x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*Csc[x]*Sec[x])/Sqrt[a*Sec[x]^2],x]
```

```
[Out] ((-1/8*I)*(Pi^4 - 2*x^4 + (8*I)*x^3*Log[1 - E^((-I)*x)] - (8*I)*x^3*Log[1 +
 E^(I*x)] - 24*x^2*PolyLog[2, E^((-I)*x)] - 24*x^2*PolyLog[2, -E^(I*x)] + (
48*I)*x*PolyLog[3, E^((-I)*x)] - (48*I)*x*PolyLog[3, -E^(I*x)] + 48*PolyLog
[4, E^((-I)*x)] + 48*PolyLog[4, -E^(I*x)])*Sec[x])/Sqrt[a*Sec[x]^2]
```

fricas [C] time = 1.31, size = 327, normalized size = 1.76

$$\frac{6x\sqrt{\frac{a}{\cos(x)^2}}\cos(x)\text{polylog}(3,\cos(x)+i\sin(x))+6x\sqrt{\frac{a}{\cos(x)^2}}\cos(x)\text{polylog}(3,\cos(x)-i\sin(x))-6x\sqrt{\frac{a}{\cos(x}}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*csc(x)*sec(x)/(a*sec(x)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*(6*x*sqrt(a/cos(x)^2)*cos(x)*polylog(3, cos(x) + I*sin(x)) + 6*x*sqrt(a
/cos(x)^2)*cos(x)*polylog(3, cos(x) - I*sin(x)) - 6*x*sqrt(a/cos(x)^2)*cos(
```

```
x)*polylog(3, -cos(x) + I*sin(x)) - 6*x*sqrt(a/cos(x)^2)*cos(x)*polylog(3,
-cos(x) - I*sin(x)) + 6*I*sqrt(a/cos(x)^2)*cos(x)*polylog(4, cos(x) + I*sin
(x)) - 6*I*sqrt(a/cos(x)^2)*cos(x)*polylog(4, cos(x) - I*sin(x)) + 6*I*sqrt
(a/cos(x)^2)*cos(x)*polylog(4, -cos(x) + I*sin(x)) - 6*I*sqrt(a/cos(x)^2)*c
os(x)*polylog(4, -cos(x) - I*sin(x)) - (x^3*cos(x)*log(cos(x) + I*sin(x) +
1) + x^3*cos(x)*log(cos(x) - I*sin(x) + 1) - x^3*cos(x)*log(-cos(x) + I*sin
(x) + 1) - x^3*cos(x)*log(-cos(x) - I*sin(x) + 1) + 3*I*x^2*cos(x)*dilog(co
s(x) + I*sin(x)) - 3*I*x^2*cos(x)*dilog(cos(x) - I*sin(x)) + 3*I*x^2*cos(x)
*dilog(-cos(x) + I*sin(x)) - 3*I*x^2*cos(x)*dilog(-cos(x) - I*sin(x)))*sqrt
(a/cos(x)^2))/a
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3 \csc(x) \sec(x)}{\sqrt{a \sec(x)^2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*csc(x)*sec(x)/(a*sec(x)^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^3*csc(x)*sec(x)/sqrt(a*sec(x)^2), x)
```

maple [A] time = 0.19, size = 172, normalized size = 0.92

$$\frac{2i\left(\frac{ie^{ix}x^3\ln\left(1+e^{ix}\right)}{2}+\frac{3\,e^{ix}x^2\operatorname{polylog}\left(2,-e^{ix}\right)}{2}+3ie^{ix}x\operatorname{polylog}\left(3,-e^{ix}\right)-3\,e^{ix}\operatorname{polylog}\left(4,-e^{ix}\right)-\frac{ie^{ix}x^3\ln\left(1-e^{ix}\right)}{2}-\frac{3\,e^{ix}x^2\operatorname{po}}{}\right.}{\sqrt{\frac{a\,e^{2ix}}{\left(e^{2ix}+1\right)^2}}\,\left(e^{2ix}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*csc(x)*sec(x)/(a*sec(x)^2)^(1/2),x)
```

```
[Out] 2*I/(a*exp(2*I*x)/(exp(2*I*x)+1)^2)^(1/2)/(exp(2*I*x)+1)*(1/2*I*exp(I*x)*x^
3*ln(1+exp(I*x))+3/2*exp(I*x)*x^2*polylog(2,-exp(I*x))+3*I*exp(I*x)*x*polyl
og(3,-exp(I*x))-3*exp(I*x)*polylog(4,-exp(I*x))-1/2*I*exp(I*x)*x^3*ln(1-exp
(I*x))-3/2*exp(I*x)*x^2*polylog(2,exp(I*x))-3*I*exp(I*x)*x*polylog(3,exp(I*
x))+3*exp(I*x)*polylog(4,exp(I*x)))
```

maxima [A] time = 0.82, size = 131, normalized size = 0.70

$$-\frac{2i\,x^3\arctan\left(\sin(x),\cos(x)+1\right)+2i\,x^3\arctan\left(\sin(x),-\cos(x)+1\right)+x^3\log\left(\cos(x)^2+\sin(x)^2+2\,\cos(x)+1\right)}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*csc(x)*sec(x)/(a*sec(x)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/2*(2*I*x^3*arctan2(sin(x), cos(x) + 1) + 2*I*x^3*arctan2(sin(x), -cos(x)
 + 1) + x^3*log(cos(x)^2 + sin(x)^2 + 2*cos(x) + 1) - x^3*log(cos(x)^2 + si
n(x)^2 - 2*cos(x) + 1) - 6*I*x^2*dilog(-e^(I*x)) + 6*I*x^2*dilog(e^(I*x)) +
 12*x*polylog(3, -e^(I*x)) - 12*x*polylog(3, e^(I*x)) + 12*I*polylog(4, -e^
(I*x)) - 12*I*polylog(4, e^(I*x)))/sqrt(a)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^3}{\cos(x)\,\sin(x)\,\sqrt{\frac{a}{\cos(x)^2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(cos(x)*sin(x)*(a/cos(x)^2)^(1/2)),x)
```

```
[Out] int(x^3/(cos(x)*sin(x)*(a/cos(x)^2)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3 \csc(x) \sec(x)}{\sqrt{a \sec^2(x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*csc(x)*sec(x)/(a*sec(x)**2)**(1/2),x)
```

```
[Out] Integral(x**3*csc(x)*sec(x)/sqrt(a*sec(x)**2), x)
```

3.871 $\int \frac{x \csc(x) \sec(x)}{\sqrt{a \sec^4(x)}} \, dx$

Optimal. Leaf size=81

$$-\frac{i\text{Li}_2\left(e^{2ix}\right)\sec^2(x)}{2\sqrt{a\sec^4(x)}}-\frac{ix^2\sec^2(x)}{2\sqrt{a\sec^4(x)}}+\frac{x\log\left(1-e^{2ix}\right)\sec^2(x)}{\sqrt{a\sec^4(x)}}$$

```
[Out] -1/2*I*x^2*sec(x)^2/(a*sec(x)^4)^(1/2)+x*ln(1-exp(2*I*x))*sec(x)^2/(a*sec(x
)^4)^(1/2)-1/2*I*polylog(2,exp(2*I*x))*sec(x)^2/(a*sec(x)^4)^(1/2)
```

Rubi [A] time = 0.49, antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312, Rules used = {6720, 3717, 2190, 2279, 2391}

$$-\frac{i\sec^2(x)\text{PolyLog}\left(2,e^{2ix}\right)}{2\sqrt{a\sec^4(x)}}-\frac{ix^2\sec^2(x)}{2\sqrt{a\sec^4(x)}}+\frac{x\log\left(1-e^{2ix}\right)\sec^2(x)}{\sqrt{a\sec^4(x)}}$$

Antiderivative was successfully verified.

```
[In] Int[(x*Csc[x]*Sec[x])/Sqrt[a*Sec[x]^4],x]
```

```
[Out] ((-I/2)*x^2*Sec[x]^2)/Sqrt[a*Sec[x]^4] + (x*Log[1 - E^((2*I)*x)]*Sec[x]^2)/
Sqrt[a*Sec[x]^4] - ((I/2)*PolyLog[2, E^((2*I)*x)]*Sec[x]^2)/Sqrt[a*Sec[x]^4
]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3717

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] - Dist[2*I, Int[((c + d*x)^
m*E^(2*I*k*Pi)*E^(2*I*(e + f*x)))/(1 + E^(2*I*k*Pi)*E^(2*I*(e + f*x))), x],
 x] /; FreeQ[{c, d, e, f}, x] && IntegerQ[4*k] && IGtQ[m, 0]
```

Rule 6720

```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Dist[(a^IntPart[p]*(a*v^m)^
FracPart[p])/v^(m*FracPart[p]), Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p}, x
] && !IntegerQ[p] && !FreeQ[v, x] && !(EqQ[a, 1] && EqQ[m, 1]) && !(EqQ
[v, x] && EqQ[m, 1])
```

Rubi steps

$$
\begin{aligned}
\int \frac{x\csc(x)\sec(x)}{\sqrt{a\sec^4(x)}}\,dx &= \frac{\sec^2(x)\int x\cot(x)\,dx}{\sqrt{a\sec^4(x)}} \\
&= -\frac{ix^2\sec^2(x)}{2\sqrt{a\sec^4(x)}} - \frac{\left(2i\sec^2(x)\right)\int \frac{e^{2ix}x}{1-e^{2ix}}\,dx}{\sqrt{a\sec^4(x)}} \\
&= -\frac{ix^2\sec^2(x)}{2\sqrt{a\sec^4(x)}} + \frac{x\log\left(1-e^{2ix}\right)\sec^2(x)}{\sqrt{a\sec^4(x)}} - \frac{\sec^2(x)\int \log\left(1-e^{2ix}\right)\,dx}{\sqrt{a\sec^4(x)}} \\
&= -\frac{ix^2\sec^2(x)}{2\sqrt{a\sec^4(x)}} + \frac{x\log\left(1-e^{2ix}\right)\sec^2(x)}{\sqrt{a\sec^4(x)}} + \frac{\left(i\sec^2(x)\right)\text{Subst}\left(\int \frac{\log(1-x)}{x}\,dx, x, e^{2ix}\right)}{2\sqrt{a\sec^4(x)}} \\
&= -\frac{ix^2\sec^2(x)}{2\sqrt{a\sec^4(x)}} + \frac{x\log\left(1-e^{2ix}\right)\sec^2(x)}{\sqrt{a\sec^4(x)}} - \frac{i\text{Li}_2\left(e^{2ix}\right)\sec^2(x)}{2\sqrt{a\sec^4(x)}}
\end{aligned}
$$

Mathematica [A] time = 0.04, size = 50, normalized size = 0.62

$$
-\frac{i\sec^2(x)\left(\text{Li}_2\left(e^{2ix}\right)+x\left(x+2i\log\left(1-e^{2ix}\right)\right)\right)}{2\sqrt{a\sec^4(x)}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*Csc[x]*Sec[x])/Sqrt[a*Sec[x]^4],x]
```

```
[Out] ((-1/2*I)*(x*(x + (2*I)*Log[1 - E^((2*I)*x)]) + PolyLog[2, E^((2*I)*x)])*Se
c[x]^2)/Sqrt[a*Sec[x]^4]
```

fricas [B] time = 2.10, size = 138, normalized size = 1.70

$$\frac{\left(x\cos(x)^2\log\left(\cos(x)+i\,\sin(x)+1\right)+x\cos(x)^2\log\left(\cos(x)-i\,\sin(x)+1\right)+x\cos(x)^2\log\left(-\cos(x)+i\,\sin(x)+\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*csc(x)*sec(x)/(a*sec(x)^4)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*(x*cos(x)^2*log(cos(x) + I*sin(x) + 1) + x*cos(x)^2*log(cos(x) - I*sin(
x) + 1) + x*cos(x)^2*log(-cos(x) + I*sin(x) + 1) + x*cos(x)^2*log(-cos(x) -
 I*sin(x) + 1) - I*cos(x)^2*dilog(cos(x) + I*sin(x)) + I*cos(x)^2*dilog(cos
(x) - I*sin(x)) + I*cos(x)^2*dilog(-cos(x) + I*sin(x)) - I*cos(x)^2*dilog(-
cos(x) - I*sin(x)))*sqrt(a/cos(x)^4)/a
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x\csc(x)\sec(x)}{\sqrt{a\sec(x)^4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*csc(x)*sec(x)/(a*sec(x)^4)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x*csc(x)*sec(x)/sqrt(a*sec(x)^4), x)
```

maple [B] time = 0.20, size = 147, normalized size = 1.81

$$\frac{i\mathrm{e}^{2ix}x^2}{2\sqrt{\frac{a\,\mathrm{e}^{4ix}}{\left(\mathrm{e}^{2ix}+1\right)^4}}\,\left(\mathrm{e}^{2ix}+1\right)^2}-\frac{2i\left(\frac{\mathrm{e}^{2ix}x^2}{2}+\frac{i\mathrm{e}^{2ix}x\ln\left(1+\mathrm{e}^{ix}\right)}{2}+\frac{\mathrm{e}^{2ix}\operatorname{polylog}\left(2,-\mathrm{e}^{ix}\right)}{2}+\frac{i\mathrm{e}^{2ix}x\ln\left(1-\mathrm{e}^{ix}\right)}{2}+\frac{\mathrm{e}^{2ix}\operatorname{polylog}\left(2,\mathrm{e}^{ix}\right)}{2}\right)}{\sqrt{\frac{a\,\mathrm{e}^{4ix}}{\left(\mathrm{e}^{2ix}+1\right)^4}}\,\left(\mathrm{e}^{2ix}+1\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*csc(x)*sec(x)/(a*sec(x)^4)^(1/2),x)
```

```
[Out] 1/2*I/(a*exp(4*I*x)/(exp(2*I*x)+1)^4)^(1/2)/(exp(2*I*x)+1)^2*exp(2*I*x)*x^2
-2*I/(a*exp(4*I*x)/(exp(2*I*x)+1)^4)^(1/2)/(exp(2*I*x)+1)^2*(1/2*exp(2*I*x)
*x^2+1/2*I*exp(2*I*x)*x*ln(1+exp(I*x))+1/2*exp(2*I*x)*polylog(2,-exp(I*x))+
1/2*I*exp(2*I*x)*x*ln(1-exp(I*x))+1/2*exp(2*I*x)*polylog(2,exp(I*x)))
```

maxima [A] time = 1.92, size = 83, normalized size = 1.02

$$\frac{-i\,x^2+2i\,x\arctan\left(\sin(x),\cos(x)+1\right)-2i\,x\arctan\left(\sin(x),-\cos(x)+1\right)+x\log\left(\cos(x)^2+\sin(x)^2+2\,\cos(x)+\right.}{2\,\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*csc(x)*sec(x)/(a*sec(x)^4)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*(-I*x^2 + 2*I*x*arctan2(sin(x), cos(x) + 1) - 2*I*x*arctan2(sin(x), -co
s(x) + 1) + x*log(cos(x)^2 + sin(x)^2 + 2*cos(x) + 1) + x*log(cos(x)^2 + si
n(x)^2 - 2*cos(x) + 1) - 2*I*dilog(-e^(I*x)) - 2*I*dilog(e^(I*x)))/sqrt(a)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x}{\cos(x)\,\sin(x)\,\sqrt{\frac{a}{\cos(x)^4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(cos(x)*sin(x)*(a/cos(x)^4)^(1/2)),x)
```

```
[Out] int(x/(cos(x)*sin(x)*(a/cos(x)^4)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x\csc(x)\sec(x)}{\sqrt{a\sec^4(x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*csc(x)*sec(x)/(a*sec(x)**4)**(1/2),x)
```

```
[Out] Integral(x*csc(x)*sec(x)/sqrt(a*sec(x)**4), x)
```

3.872 $\int \frac{x^2 \csc(x) \sec(x)}{\sqrt{a \sec^4(x)}} dx$

Optimal. Leaf size=109

$$-\frac{ix\text{Li}_2\left(e^{2ix}\right)\sec^2(x)}{\sqrt{a\sec^4(x)}}+\frac{\text{Li}_3\left(e^{2ix}\right)\sec^2(x)}{2\sqrt{a\sec^4(x)}}-\frac{ix^3\sec^2(x)}{3\sqrt{a\sec^4(x)}}+\frac{x^2\log\left(1-e^{2ix}\right)\sec^2(x)}{\sqrt{a\sec^4(x)}}$$

```
[Out] -1/3*I*x^3*sec(x)^2/(a*sec(x)^4)^(1/2)+x^2*ln(1-exp(2*I*x))*sec(x)^2/(a*sec
(x)^4)^(1/2)-I*x*polylog(2,exp(2*I*x))*sec(x)^2/(a*sec(x)^4)^(1/2)+1/2*poly
log(3,exp(2*I*x))*sec(x)^2/(a*sec(x)^4)^(1/2)
```

Rubi [A] time = 0.57, antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {6720, 3717, 2190, 2531, 2282, 6589}

$$-\frac{ix\sec^2(x)\text{PolyLog}\left(2,e^{2ix}\right)}{\sqrt{a\sec^4(x)}}+\frac{\sec^2(x)\text{PolyLog}\left(3,e^{2ix}\right)}{2\sqrt{a\sec^4(x)}}-\frac{ix^3\sec^2(x)}{3\sqrt{a\sec^4(x)}}+\frac{x^2\log\left(1-e^{2ix}\right)\sec^2(x)}{\sqrt{a\sec^4(x)}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*Csc[x]*Sec[x])/Sqrt[a*Sec[x]^4],x]
```

```
[Out] ((-I/3)*x^3*Sec[x]^2)/Sqrt[a*Sec[x]^4] + (x^2*Log[1 - E^((2*I)*x)]*Sec[x]^2
)/Sqrt[a*Sec[x]^4] - (I*x*PolyLog[2, E^((2*I)*x)]*Sec[x]^2)/Sqrt[a*Sec[x]^4
] + (PolyLog[3, E^((2*I)*x)]*Sec[x]^2)/(2*Sqrt[a*Sec[x]^4])
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 3717

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] - Dist[2*I, Int[((c + d*x)^
m*E^(2*I*k*Pi)*E^(2*I*(e + f*x)))/(1 + E^(2*I*k*Pi)*E^(2*I*(e + f*x))), x],
 x] /; FreeQ[{c, d, e, f}, x] && IntegerQ[4*k] && IGtQ[m, 0]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6720

```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Dist[(a^IntPart[p]*(a*v^m)^
FracPart[p])/v^(m*FracPart[p]), Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p}, x
] && !IntegerQ[p] && !FreeQ[v, x] && !(EqQ[a, 1] && EqQ[m, 1]) && !(EqQ
[v, x] && EqQ[m, 1])
```

Rubi steps

$$\int \frac{x^2 \csc(x)\sec(x)}{\sqrt{a\sec^4(x)}}\,dx = \frac{\sec^2(x)\int x^2\cot(x)\,dx}{\sqrt{a\sec^4(x)}}$$

$$= -\frac{ix^3\sec^2(x)}{3\sqrt{a\sec^4(x)}} - \frac{\left(2i\sec^2(x)\right)\int \frac{e^{2ix}x^2}{1-e^{2ix}}\,dx}{\sqrt{a\sec^4(x)}}$$

$$= -\frac{ix^3\sec^2(x)}{3\sqrt{a\sec^4(x)}} + \frac{x^2\log\left(1-e^{2ix}\right)\sec^2(x)}{\sqrt{a\sec^4(x)}} - \frac{\left(2\sec^2(x)\right)\int x\log\left(1-e^{2ix}\right)\,dx}{\sqrt{a\sec^4(x)}}$$

$$= -\frac{ix^3\sec^2(x)}{3\sqrt{a\sec^4(x)}} + \frac{x^2\log\left(1-e^{2ix}\right)\sec^2(x)}{\sqrt{a\sec^4(x)}} - \frac{ix\text{Li}_2\left(e^{2ix}\right)\sec^2(x)}{\sqrt{a\sec^4(x)}} + \frac{\left(i\sec^2(x)\right)\int \text{Li}_2\left(e^{2ix}\right)}{\sqrt{a\sec^4(x)}}$$

$$= -\frac{ix^3\sec^2(x)}{3\sqrt{a\sec^4(x)}} + \frac{x^2\log\left(1-e^{2ix}\right)\sec^2(x)}{\sqrt{a\sec^4(x)}} - \frac{ix\text{Li}_2\left(e^{2ix}\right)\sec^2(x)}{\sqrt{a\sec^4(x)}} + \frac{\sec^2(x)\,\text{Subst}\left(\int \frac{\text{Li}_2(x)}{x}\right.}{2\sqrt{a\sec^4(x)}}$$

$$= -\frac{ix^3\sec^2(x)}{3\sqrt{a\sec^4(x)}} + \frac{x^2\log\left(1-e^{2ix}\right)\sec^2(x)}{\sqrt{a\sec^4(x)}} - \frac{ix\text{Li}_2\left(e^{2ix}\right)\sec^2(x)}{\sqrt{a\sec^4(x)}} + \frac{\text{Li}_3\left(e^{2ix}\right)\sec^2(x)}{2\sqrt{a\sec^4(x)}}$$

Mathematica [A] time = 0.06, size = 75, normalized size = 0.69

$$\frac{\sec^2(x)\left(24ix\text{Li}_2\left(e^{-2ix}\right) + 12\text{Li}_3\left(e^{-2ix}\right) + 8ix^3 + 24x^2\log\left(1-e^{-2ix}\right) - i\pi^3\right)}{24\sqrt{a\sec^4(x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*Csc[x]*Sec[x])/Sqrt[a*Sec[x]^4],x]
```

```
[Out] (((-I)*Pi^3 + (8*I)*x^3 + 24*x^2*Log[1 - E^((-2*I)*x)] + (24*I)*x*PolyLog[2
, E^((-2*I)*x)] + 12*PolyLog[3, E^((-2*I)*x)])*Sec[x]^2)/(24*Sqrt[a*Sec[x]^
4])
```

fricas [C] time = 2.50, size = 248, normalized size = 2.28

$$\frac{2\sqrt{\frac{a}{\cos(x)^4}}\cos(x)^2\text{polylog}\left(3,\cos(x)+i\sin(x)\right) + 2\sqrt{\frac{a}{\cos(x)^4}}\cos(x)^2\text{polylog}\left(3,\cos(x)-i\sin(x)\right) + 2\sqrt{\frac{a}{\cos(x)^4}}\ \text{c}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*csc(x)*sec(x)/(a*sec(x)^4)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*(2*sqrt(a/cos(x)^4)*cos(x)^2*polylog(3, cos(x) + I*sin(x)) + 2*sqrt(a/c
os(x)^4)*cos(x)^2*polylog(3, cos(x) - I*sin(x)) + 2*sqrt(a/cos(x)^4)*cos(x)
```

```
^2*polylog(3, -cos(x) + I*sin(x)) + 2*sqrt(a/cos(x)^4)*cos(x)^2*polylog(3,
-cos(x) - I*sin(x)) + (x^2*cos(x)^2*log(cos(x) + I*sin(x) + 1) + x^2*cos(x)
^2*log(cos(x) - I*sin(x) + 1) + x^2*cos(x)^2*log(-cos(x) + I*sin(x) + 1) +
x^2*cos(x)^2*log(-cos(x) - I*sin(x) + 1) - 2*I*x*cos(x)^2*dilog(cos(x) + I*
sin(x)) + 2*I*x*cos(x)^2*dilog(cos(x) - I*sin(x)) + 2*I*x*cos(x)^2*dilog(-c
os(x) + I*sin(x)) - 2*I*x*cos(x)^2*dilog(-cos(x) - I*sin(x)))*sqrt(a/cos(x)
^4))/a
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2 \csc(x)\sec(x)}{\sqrt{a\sec(x)^4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*csc(x)*sec(x)/(a*sec(x)^4)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^2*csc(x)*sec(x)/sqrt(a*sec(x)^4), x)
```

maple [B] time = 0.18, size = 183, normalized size = 1.68

$$\frac{i e^{2ix} x^3}{3\sqrt{\frac{a\,e^{4ix}}{\left(e^{2ix}+1\right)^4}}\,\left(e^{2ix}+1\right)^2} - \frac{2\left(\frac{i e^{2ix} x^3}{3} - \frac{e^{2ix} x^2 \ln\left(1+e^{ix}\right)}{2} + i e^{2ix} x \operatorname{polylog}\left(2,-e^{ix}\right) - e^{2ix} \operatorname{polylog}\left(3,-e^{ix}\right) - \frac{e^{2ix} x^2 \ln\left(1-e\right.}{2}\right.}{\sqrt{\frac{a\,e^{4ix}}{\left(e^{2ix}+1\right)^4}}\,\left(e^{2ix}+1\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*csc(x)*sec(x)/(a*sec(x)^4)^(1/2),x)
```

```
[Out] 1/3*I/(a*exp(4*I*x)/(exp(2*I*x)+1)^4)^(1/2)/(exp(2*I*x)+1)^2*exp(2*I*x)*x^3
-2/(a*exp(4*I*x)/(exp(2*I*x)+1)^4)^(1/2)/(exp(2*I*x)+1)^2*(1/3*I*exp(2*I*x)
*x^3-1/2*exp(2*I*x)*x^2*ln(1+exp(I*x))+I*exp(2*I*x)*x*polylog(2,-exp(I*x))-
exp(2*I*x)*polylog(3,-exp(I*x))-1/2*exp(2*I*x)*x^2*ln(1-exp(I*x))+I*exp(2*I
*x)*x*polylog(2,exp(I*x))-exp(2*I*x)*polylog(3,exp(I*x)))
```

maxima [A] time = 1.10, size = 113, normalized size = 1.04

$$\frac{-2i x^3 + 6i x^2 \arctan\left(\sin(x), \cos(x)+1\right) - 6i x^2 \arctan\left(\sin(x), -\cos(x)+1\right) + 3x^2 \log\left(\cos(x)^2 + \sin(x)^2 + 2\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*csc(x)*sec(x)/(a*sec(x)^4)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/6*(-2*I*x^3 + 6*I*x^2*arctan2(sin(x), cos(x) + 1) - 6*I*x^2*arctan2(sin(x
), -cos(x) + 1) + 3*x^2*log(cos(x)^2 + sin(x)^2 + 2*cos(x) + 1) + 3*x^2*log
(cos(x)^2 + sin(x)^2 - 2*cos(x) + 1) - 12*I*x*dilog(-e^(I*x)) - 12*I*x*dilo
g(e^(I*x)) + 12*polylog(3, -e^(I*x)) + 12*polylog(3, e^(I*x)))/sqrt(a)
```

mupad **[F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^2}{\cos(x)\,\sin(x)\,\sqrt{\frac{a}{\cos(x)^4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(cos(x)*sin(x)*(a/cos(x)^4)^(1/2)),x)
```

```
[Out] int(x^2/(cos(x)*sin(x)*(a/cos(x)^4)^(1/2)), x)
```

sympy **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2 \csc(x) \sec(x)}{\sqrt{a \sec^4(x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*csc(x)*sec(x)/(a*sec(x)**4)**(1/2),x)
```

```
[Out] Integral(x**2*csc(x)*sec(x)/sqrt(a*sec(x)**4), x)
```

3.873 $\int \frac{x^3 \csc(x)\sec(x)}{\sqrt{a\sec^4(x)}}\,dx$

Optimal. Leaf size=143

$$-\frac{3ix^2\mathrm{Li}_2\left(e^{2ix}\right)\sec^2(x)}{2\sqrt{a\sec^4(x)}}+\frac{3x\mathrm{Li}_3\left(e^{2ix}\right)\sec^2(x)}{2\sqrt{a\sec^4(x)}}+\frac{3i\mathrm{Li}_4\left(e^{2ix}\right)\sec^2(x)}{4\sqrt{a\sec^4(x)}}-\frac{ix^4\sec^2(x)}{4\sqrt{a\sec^4(x)}}+\frac{x^3\log\left(1-e^{2ix}\right)\sec^2(x)}{\sqrt{a\sec^4(x)}}$$

```
[Out] -1/4*I*x^4*sec(x)^2/(a*sec(x)^4)^(1/2)+x^3*ln(1-exp(2*I*x))*sec(x)^2/(a*sec
(x)^4)^(1/2)-3/2*I*x^2*polylog(2,exp(2*I*x))*sec(x)^2/(a*sec(x)^4)^(1/2)+3/
2*x*polylog(3,exp(2*I*x))*sec(x)^2/(a*sec(x)^4)^(1/2)+3/4*I*polylog(4,exp(2
*I*x))*sec(x)^2/(a*sec(x)^4)^(1/2)
```

Rubi [A] time = 0.61, antiderivative size = 143, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.389, Rules used = {6720, 3717, 2190, 2531, 6609, 2282, 6589}

$$-\frac{3ix^2\sec^2(x)\mathrm{PolyLog}\left(2,e^{2ix}\right)}{2\sqrt{a\sec^4(x)}}+\frac{3x\sec^2(x)\mathrm{PolyLog}\left(3,e^{2ix}\right)}{2\sqrt{a\sec^4(x)}}+\frac{3i\sec^2(x)\mathrm{PolyLog}\left(4,e^{2ix}\right)}{4\sqrt{a\sec^4(x)}}-\frac{ix^4\sec^2(x)}{4\sqrt{a\sec^4(x)}}+\frac{x^3\log}{}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*Csc[x]*Sec[x])/Sqrt[a*Sec[x]^4],x]
```

```
[Out] ((-I/4)*x^4*Sec[x]^2)/Sqrt[a*Sec[x]^4] + (x^3*Log[1 - E^((2*I)*x)]*Sec[x]^2
)/Sqrt[a*Sec[x]^4] - (((3*I)/2)*x^2*PolyLog[2, E^((2*I)*x)]*Sec[x]^2)/Sqrt[
a*Sec[x]^4] + (3*x*PolyLog[3, E^((2*I)*x)]*Sec[x]^2)/(2*Sqrt[a*Sec[x]^4]) +
 (((3*I)/4)*PolyLog[4, E^((2*I)*x)]*Sec[x]^2)/Sqrt[a*Sec[x]^4]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 3717

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] - Dist[2*I, Int[((c + d*x)^
m*E^(2*I*k*Pi)*E^(2*I*(e + f*x)))/(1 + E^(2*I*k*Pi)*E^(2*I*(e + f*x))), x],
 x] /; FreeQ[{c, d, e, f}, x] && IntegerQ[4*k] && IGtQ[m, 0]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rule 6720

```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Dist[(a^IntPart[p]*(a*v^m)^
FracPart[p])/v^(m*FracPart[p]), Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p}, x
] &&  !IntegerQ[p] &&  !FreeQ[v, x] &&  !(EqQ[a, 1] && EqQ[m, 1]) &&  !(EqQ
[v, x] && EqQ[m, 1])
```

Rubi steps

$$\int \frac{x^3\csc(x)\sec(x)}{\sqrt{a\sec^4(x)}}\,dx = \frac{\sec^2(x)\int x^3\cot(x)\,dx}{\sqrt{a\sec^4(x)}}$$

$$= -\frac{ix^4\sec^2(x)}{4\sqrt{a\sec^4(x)}} - \frac{\left(2i\sec^2(x)\right)\int \frac{e^{2ix}x^3}{1-e^{2ix}}\,dx}{\sqrt{a\sec^4(x)}}$$

$$= -\frac{ix^4\sec^2(x)}{4\sqrt{a\sec^4(x)}} + \frac{x^3\log\left(1-e^{2ix}\right)\sec^2(x)}{\sqrt{a\sec^4(x)}} - \frac{\left(3\sec^2(x)\right)\int x^2\log\left(1-e^{2ix}\right)\,dx}{\sqrt{a\sec^4(x)}}$$

$$= -\frac{ix^4\sec^2(x)}{4\sqrt{a\sec^4(x)}} + \frac{x^3\log\left(1-e^{2ix}\right)\sec^2(x)}{\sqrt{a\sec^4(x)}} - \frac{3ix^2\text{Li}_2\left(e^{2ix}\right)\sec^2(x)}{2\sqrt{a\sec^4(x)}} + \frac{\left(3i\sec^2(x)\right)\int x\text{Li}_2}{\sqrt{a\sec^4(x}}$$

$$= -\frac{ix^4\sec^2(x)}{4\sqrt{a\sec^4(x)}} + \frac{x^3\log\left(1-e^{2ix}\right)\sec^2(x)}{\sqrt{a\sec^4(x)}} - \frac{3ix^2\text{Li}_2\left(e^{2ix}\right)\sec^2(x)}{2\sqrt{a\sec^4(x)}} + \frac{3x\text{Li}_3\left(e^{2ix}\right)\sec^2(x}{2\sqrt{a\sec^4(x)}}$$

$$= -\frac{ix^4\sec^2(x)}{4\sqrt{a\sec^4(x)}} + \frac{x^3\log\left(1-e^{2ix}\right)\sec^2(x)}{\sqrt{a\sec^4(x)}} - \frac{3ix^2\text{Li}_2\left(e^{2ix}\right)\sec^2(x)}{2\sqrt{a\sec^4(x)}} + \frac{3x\text{Li}_3\left(e^{2ix}\right)\sec^2(x}{2\sqrt{a\sec^4(x)}}$$

$$= -\frac{ix^4\sec^2(x)}{4\sqrt{a\sec^4(x)}} + \frac{x^3\log\left(1-e^{2ix}\right)\sec^2(x)}{\sqrt{a\sec^4(x)}} - \frac{3ix^2\text{Li}_2\left(e^{2ix}\right)\sec^2(x)}{2\sqrt{a\sec^4(x)}} + \frac{3x\text{Li}_3\left(e^{2ix}\right)\sec^2(x}{2\sqrt{a\sec^4(x)}}$$

Mathematica [A] time = 0.07, size = 87, normalized size = 0.61

$$-\frac{i\sec^2(x)\left(-96x^2\text{Li}_2\left(e^{-2ix}\right)+96ix\text{Li}_3\left(e^{-2ix}\right)+48\text{Li}_4\left(e^{-2ix}\right)-16x^4+64ix^3\log\left(1-e^{-2ix}\right)+\pi^4\right)}{64\sqrt{a\sec^4(x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*Csc[x]*Sec[x])/Sqrt[a*Sec[x]^4],x]
```

```
[Out] ((-1/64*I)*(Pi^4 - 16*x^4 + (64*I)*x^3*Log[1 - E^((-2*I)*x)] - 96*x^2*PolyL
og[2, E^((-2*I)*x)] + (96*I)*x*PolyLog[3, E^((-2*I)*x)] + 48*PolyLog[4, E^(
(-2*I)*x)])*Sec[x]^2)/Sqrt[a*Sec[x]^4]
```

fricas [C] time = 2.05, size = 356, normalized size = 2.49

$$6\,x\sqrt{\frac{a}{\cos(x)^4}}\,\cos(x)^2\text{polylog}\left(3,\cos(x)+i\,\sin(x)\right)+6\,x\sqrt{\frac{a}{\cos(x)^4}}\,\cos(x)^2\text{polylog}\left(3,\cos(x)-i\,\sin(x)\right)+6\,x\sqrt{\frac{}{\cos}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*csc(x)*sec(x)/(a*sec(x)^4)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*(6*x*sqrt(a/cos(x)^4)*cos(x)^2*polylog(3, cos(x) + I*sin(x)) + 6*x*sqrt
(a/cos(x)^4)*cos(x)^2*polylog(3, cos(x) - I*sin(x)) + 6*x*sqrt(a/cos(x)^4)*
cos(x)^2*polylog(3, -cos(x) + I*sin(x)) + 6*x*sqrt(a/cos(x)^4)*cos(x)^2*pol
ylog(3, -cos(x) - I*sin(x)) + 6*I*sqrt(a/cos(x)^4)*cos(x)^2*polylog(4, cos(
x) + I*sin(x)) - 6*I*sqrt(a/cos(x)^4)*cos(x)^2*polylog(4, cos(x) - I*sin(x)
) - 6*I*sqrt(a/cos(x)^4)*cos(x)^2*polylog(4, -cos(x) + I*sin(x)) + 6*I*sqrt
(a/cos(x)^4)*cos(x)^2*polylog(4, -cos(x) - I*sin(x)) + (x^3*cos(x)^2*log(co
s(x) + I*sin(x) + 1) + x^3*cos(x)^2*log(cos(x) - I*sin(x) + 1) + x^3*cos(x)
^2*log(-cos(x) + I*sin(x) + 1) + x^3*cos(x)^2*log(-cos(x) - I*sin(x) + 1) -
 3*I*x^2*cos(x)^2*dilog(cos(x) + I*sin(x)) + 3*I*x^2*cos(x)^2*dilog(cos(x)
- I*sin(x)) + 3*I*x^2*cos(x)^2*dilog(-cos(x) + I*sin(x)) - 3*I*x^2*cos(x)^2
*dilog(-cos(x) - I*sin(x)))*sqrt(a/cos(x)^4))/a
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3 \csc(x) \sec(x)}{\sqrt{a \sec(x)^4}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*csc(x)*sec(x)/(a*sec(x)^4)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^3*csc(x)*sec(x)/sqrt(a*sec(x)^4), x)
```

maple [A] time = 0.19, size = 221, normalized size = 1.55

$$\frac{i e^{2ix} x^4}{4\sqrt{\frac{a e^{4ix}}{\left(e^{2ix}+1\right)^4}}\,\left(e^{2ix}+1\right)^2}+\frac{2i\left(-\frac{e^{2ix} x^4}{4}-\frac{i e^{2ix} x^3 \ln\left(1+e^{ix}\right)}{2}-\frac{3 e^{2ix} x^2 \operatorname{polylog}\left(2,-e^{ix}\right)}{2}-3i e^{2ix} x \operatorname{polylog}\left(3,-e^{ix}\right)+3 e^{2ix} \operatorname{poly}\right.}{\sqrt{\frac{a}{\left(e^{2\right.}}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*csc(x)*sec(x)/(a*sec(x)^4)^(1/2),x)
```

```
[Out] 1/4*I/(a*exp(4*I*x)/(exp(2*I*x)+1)^4)^(1/2)/(exp(2*I*x)+1)^2*exp(2*I*x)*x^4
+2*I/(a*exp(4*I*x)/(exp(2*I*x)+1)^4)^(1/2)/(exp(2*I*x)+1)^2*(-1/4*exp(2*I*x
)*x^4-1/2*I*exp(2*I*x)*x^3*ln(1+exp(I*x))-3/2*exp(2*I*x)*x^2*polylog(2,-exp
(I*x))-3*I*exp(2*I*x)*x*polylog(3,-exp(I*x))+3*exp(2*I*x)*polylog(4,-exp(I*
x))-1/2*I*exp(2*I*x)*x^3*ln(1-exp(I*x))-3/2*exp(2*I*x)*x^2*polylog(2,exp(I*
x))-3*I*exp(2*I*x)*x*polylog(3,exp(I*x))+3*exp(2*I*x)*polylog(4,exp(I*x)))
```

maxima [A] time = 0.55, size = 137, normalized size = 0.96

$$\frac{-i x^4 + 4i x^3 \arctan\left(\sin(x), \cos(x)+1\right) - 4i x^3 \arctan\left(\sin(x), -\cos(x)+1\right) + 2 x^3 \log\left(\cos(x)^2 + \sin(x)^2 + 2 \cos\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*csc(x)*sec(x)/(a*sec(x)^4)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/4*(-I*x^4 + 4*I*x^3*arctan2(sin(x), cos(x) + 1) - 4*I*x^3*arctan2(sin(x),
 -cos(x) + 1) + 2*x^3*log(cos(x)^2 + sin(x)^2 + 2*cos(x) + 1) + 2*x^3*log(c
os(x)^2 + sin(x)^2 - 2*cos(x) + 1) - 12*I*x^2*dilog(-e^(I*x)) - 12*I*x^2*di
log(e^(I*x)) + 24*x*polylog(3, -e^(I*x)) + 24*x*polylog(3, e^(I*x)) + 24*I*
polylog(4, -e^(I*x)) + 24*I*polylog(4, e^(I*x)))/sqrt(a)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^3}{\cos(x)\,\sin(x)\,\sqrt{\frac{a}{\cos(x)^4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(cos(x)*sin(x)*(a/cos(x)^4)^(1/2)),x)
```

```
[Out] int(x^3/(cos(x)*sin(x)*(a/cos(x)^4)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3 \csc(x) \sec(x)}{\sqrt{a \sec^4(x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*csc(x)*sec(x)/(a*sec(x)**4)**(1/2),x)
```

```
[Out] Integral(x**3*csc(x)*sec(x)/sqrt(a*sec(x)**4), x)
```

3.874 $\int x \csc(x) \sec(x) \sqrt{a \sec^2(x)}\, dx$

Optimal. Leaf size=105

$$i\text{Li}_2\left(-e^{ix}\right)\cos(x)\sqrt{a\sec^2(x)} - i\text{Li}_2\left(e^{ix}\right)\cos(x)\sqrt{a\sec^2(x)} + x\sqrt{a\sec^2(x)} - 2x\cos(x)\tanh^{-1}\left(e^{ix}\right)\sqrt{a\sec^2(x)} - \cos(x$$

```
[Out] x*(a*sec(x)^2)^(1/2)-2*x*arctanh(exp(I*x))*cos(x)*(a*sec(x)^2)^(1/2)-arctan
h(sin(x))*cos(x)*(a*sec(x)^2)^(1/2)+I*cos(x)*polylog(2,-exp(I*x))*(a*sec(x)
^2)^(1/2)-I*cos(x)*polylog(2,exp(I*x))*(a*sec(x)^2)^(1/2)
```

Rubi [A] time = 0.34, antiderivative size = 105, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 10, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.625, Rules used = {6720, 2622, 321, 207, 4420, 6271, 4183, 2279, 2391, 3770}

$$i\cos(x)\text{PolyLog}\left(2,-e^{ix}\right)\sqrt{a\sec^2(x)} - i\cos(x)\text{PolyLog}\left(2,e^{ix}\right)\sqrt{a\sec^2(x)} + x\sqrt{a\sec^2(x)} - 2x\cos(x)\tanh^{-1}\left(e^{ix}\right)\sqrt{}$$

Antiderivative was successfully verified.

```
[In] Int[x*Csc[x]*Sec[x]*Sqrt[a*Sec[x]^2],x]
```

```
[Out] x*Sqrt[a*Sec[x]^2] - 2*x*ArcTanh[E^(I*x)]*Cos[x]*Sqrt[a*Sec[x]^2] - ArcTanh
[Sin[x]]*Cos[x]*Sqrt[a*Sec[x]^2] + I*Cos[x]*PolyLog[2, -E^(I*x)]*Sqrt[a*Sec
[x]^2] - I*Cos[x]*PolyLog[2, E^(I*x)]*Sqrt[a*Sec[x]^2]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2622

```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Dist[1/(f*a^n), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/2
), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)
/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 4183

```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[(
-2*(c + d*x)^m*ArcTanh[E^(I*(e + f*x))])/f, x] + (-Dist[(d*m)/f, Int[(c + d
*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Dist[(d*m)/f, Int[(c + d*x)^
(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 4420

```
Int[Csc[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sec[(a_.) + (b
_.)*(x_)]^(p_.), x_Symbol] :> Module[{u = IntHide[Csc[a + b*x]^n*Sec[a + b*
x]^p, x]}, Dist[(c + d*x)^m, u, x] - Dist[d*m, Int[(c + d*x)^(m - 1)*u, x],
 x]] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p] && GtQ[m, 0] && NeQ[n, p]
```

Rule 6271

```
Int[ArcTanh[u_], x_Symbol] :> Simp[x*ArcTanh[u], x] - Int[SimplifyIntegrand
[(x*D[u, x])/(1 - u^2), x], x] /; InverseFunctionFreeQ[u, x]
```

Rule 6720

```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Dist[(a^IntPart[p]*(a*v^m)^
FracPart[p])/v^(m*FracPart[p]), Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p}, x
] &&  !IntegerQ[p] &&  !FreeQ[v, x] &&  !(EqQ[a, 1] && EqQ[m, 1]) &&  !(EqQ
[v, x] && EqQ[m, 1])
```

Rubi steps

$$\int x\csc(x)\sec(x)\sqrt{a\sec^2(x)}\,dx = \left(\cos(x)\sqrt{a\sec^2(x)}\right)\int x\csc(x)\sec^2(x)\,dx$$

$$= x\sqrt{a\sec^2(x)} - x\tanh^{-1}(\cos(x))\cos(x)\sqrt{a\sec^2(x)} - \left(\cos(x)\sqrt{a\sec^2(x)}\right)\int (-$$

$$= x\sqrt{a\sec^2(x)} - x\tanh^{-1}(\cos(x))\cos(x)\sqrt{a\sec^2(x)} + \left(\cos(x)\sqrt{a\sec^2(x)}\right)\int \mathrm{ta}$$

$$= x\sqrt{a\sec^2(x)} - \tanh^{-1}(\sin(x))\cos(x)\sqrt{a\sec^2(x)} + \left(\cos(x)\sqrt{a\sec^2(x)}\right)\int x\,\mathrm{cs}$$

$$= x\sqrt{a\sec^2(x)} - 2x\tanh^{-1}\left(e^{ix}\right)\cos(x)\sqrt{a\sec^2(x)} - \tanh^{-1}(\sin(x))\cos(x)\sqrt{a\,\mathrm{se}}$$

$$= x\sqrt{a\sec^2(x)} - 2x\tanh^{-1}\left(e^{ix}\right)\cos(x)\sqrt{a\sec^2(x)} - \tanh^{-1}(\sin(x))\cos(x)\sqrt{a\,\mathrm{se}}$$

$$= x\sqrt{a\sec^2(x)} - 2x\tanh^{-1}\left(e^{ix}\right)\cos(x)\sqrt{a\sec^2(x)} - \tanh^{-1}(\sin(x))\cos(x)\sqrt{a\,\mathrm{se}}$$

Mathematica [A] time = 0.08, size = 108, normalized size = 1.03

$$\sqrt{a\sec^2(x)}\left(i\left(\mathrm{Li}_2\left(-e^{ix}\right) - \mathrm{Li}_2\left(e^{ix}\right)\right)\cos(x) + x + x\left(\log\left(1-e^{ix}\right) - \log\left(1+e^{ix}\right)\right)\cos(x) + \cos(x)\log\left(\cos\left(\frac{x}{2}\right) - \mathrm{s}\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Csc[x]*Sec[x]*Sqrt[a*Sec[x]^2],x]
```

```
[Out] (x + x*Cos[x]*(Log[1 - E^(I*x)] - Log[1 + E^(I*x)]) + Cos[x]*Log[Cos[x/2] -
 Sin[x/2]] - Cos[x]*Log[Cos[x/2] + Sin[x/2]] + I*Cos[x]*(PolyLog[2, -E^(I*x
)] - PolyLog[2, E^(I*x)]))*Sqrt[a*Sec[x]^2]
```

fricas [A] time = 0.85, size = 140, normalized size = 1.33

$$-\frac{1}{2}\left(x\cos(x)\log\left(\cos(x) + i\sin(x) + 1\right) + x\cos(x)\log\left(\cos(x) - i\sin(x) + 1\right) - x\cos(x)\log\left(-\cos(x) + i\sin(x) \right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*csc(x)*sec(x)*(a*sec(x)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/2*(x*cos(x)*log(cos(x) + I*sin(x) + 1) + x*cos(x)*log(cos(x) - I*sin(x)
+ 1) - x*cos(x)*log(-cos(x) + I*sin(x) + 1) - x*cos(x)*log(-cos(x) - I*sin(
x) + 1) + I*cos(x)*dilog(cos(x) + I*sin(x)) - I*cos(x)*dilog(cos(x) - I*sin
(x)) + I*cos(x)*dilog(-cos(x) + I*sin(x)) - I*cos(x)*dilog(-cos(x) - I*sin(
x)) + cos(x)*log(-(sin(x) + 1)/(sin(x) - 1)) - 2*x)*sqrt(a/cos(x)^2)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a\sec(x)^2}\,x\csc(x)\sec(x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*csc(x)*sec(x)*(a*sec(x)^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(a*sec(x)^2)*x*csc(x)*sec(x), x)
```

maple [A] time = 0.25, size = 86, normalized size = 0.82

$$2\sqrt{\frac{a\,e^{2ix}}{\left(e^{2ix}+1\right)^2}}\,x+4\sqrt{\frac{a\,e^{2ix}}{\left(e^{2ix}+1\right)^2}}\left(i\arctan\left(e^{ix}\right)+\frac{i\,\mathrm{dilog}\left(1+e^{ix}\right)}{2}-\frac{x\ln\left(1+e^{ix}\right)}{2}+\frac{i\,\mathrm{dilog}\left(e^{ix}\right)}{2}\right)\cos(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*csc(x)*sec(x)*(a*sec(x)^2)^(1/2),x)
```

```
[Out] 2*(a*exp(2*I*x)/(exp(2*I*x)+1)^2)^(1/2)*x+4*(a*exp(2*I*x)/(exp(2*I*x)+1)^2)
^(1/2)*(I*arctan(exp(I*x))+1/2*I*dilog(1+exp(I*x))-1/2*x*ln(1+exp(I*x))+1/2
*I*dilog(exp(I*x)))*cos(x)
```

maxima [B] time = 0.70, size = 299, normalized size = 2.85

$$\frac{\left((2\cos(2x)+2i\sin(2x)+2)\arctan(\cos(x),\sin(x)+1)+(2\cos(2x)+2i\sin(2x)+2)\arctan(\cos(x),-\sin\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*csc(x)*sec(x)*(a*sec(x)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] ((2*cos(2*x) + 2*I*sin(2*x) + 2)*arctan2(cos(x), sin(x) + 1) + (2*cos(2*x)
+ 2*I*sin(2*x) + 2)*arctan2(cos(x), -sin(x) + 1) - 2*(x*cos(2*x) + I*x*sin(
2*x) + x)*arctan2(sin(x), cos(x) + 1) - 2*(x*cos(2*x) + I*x*sin(2*x) + x)*a
rctan2(sin(x), -cos(x) + 1) - 4*I*x*cos(x) + (2*cos(2*x) + 2*I*sin(2*x) + 2
)*dilog(-e^(I*x)) - (2*cos(2*x) + 2*I*sin(2*x) + 2)*dilog(e^(I*x)) - (-I*x*
cos(2*x) + x*sin(2*x) - I*x)*log(cos(x)^2 + sin(x)^2 + 2*cos(x) + 1) - (I*x
*cos(2*x) - x*sin(2*x) + I*x)*log(cos(x)^2 + sin(x)^2 - 2*cos(x) + 1) - (-I
*cos(2*x) + sin(2*x) - I)*log(cos(x)^2 + sin(x)^2 + 2*sin(x) + 1) - (I*cos(
2*x) - sin(2*x) + I)*log(cos(x)^2 + sin(x)^2 - 2*sin(x) + 1) + 4*x*sin(x))*
sqrt(a)/(-2*I*cos(2*x) + 2*sin(2*x) - 2*I)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x\sqrt{\frac{a}{\cos(x)^2}}}{\cos(x)\,\sin(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(a/cos(x)^2)^(1/2))/(cos(x)*sin(x)),x)
```

```
[Out] int((x*(a/cos(x)^2)^(1/2))/(cos(x)*sin(x)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x\sqrt{a\sec^2(x)}\,\csc(x)\sec(x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*csc(x)*sec(x)*(a*sec(x)**2)**(1/2),x)
```

```
[Out] Integral(x*sqrt(a*sec(x)**2)*csc(x)*sec(x), x)
```

3.875 $\int x^2 \csc(x) \sec(x) \sqrt{a \sec^2(x)}\, dx$

Optimal. Leaf size=225

$$2ix\text{Li}_2\left(-e^{ix}\right)\cos(x)\sqrt{a\sec^2(x)}-2ix\text{Li}_2\left(e^{ix}\right)\cos(x)\sqrt{a\sec^2(x)}-2i\text{Li}_2\left(-ie^{ix}\right)\cos(x)\sqrt{a\sec^2(x)}+2i\text{Li}_2\left(ie^{ix}\right)\cos($$

```
[Out] x^2*(a*sec(x)^2)^(1/2)+4*I*x*arctan(exp(I*x))*cos(x)*(a*sec(x)^2)^(1/2)-2*x
^2*arctanh(exp(I*x))*cos(x)*(a*sec(x)^2)^(1/2)+2*I*x*cos(x)*polylog(2,-exp(
I*x))*(a*sec(x)^2)^(1/2)-2*I*cos(x)*polylog(2,-I*exp(I*x))*(a*sec(x)^2)^(1/
2)+2*I*cos(x)*polylog(2,I*exp(I*x))*(a*sec(x)^2)^(1/2)-2*I*x*cos(x)*polylog
(2,exp(I*x))*(a*sec(x)^2)^(1/2)-2*cos(x)*polylog(3,-exp(I*x))*(a*sec(x)^2)^
(1/2)+2*cos(x)*polylog(3,exp(I*x))*(a*sec(x)^2)^(1/2)
```

Rubi [A] time = 0.53, antiderivative size = 225, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 14, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.778, Rules used = {6720, 2622, 321, 207, 4420, 14, 6273, 4183, 2531, 2282, 6589, 4181, 2279, 2391}

$$2ix\cos(x)\text{PolyLog}\left(2,-e^{ix}\right)\sqrt{a\sec^2(x)}-2ix\cos(x)\text{PolyLog}\left(2,e^{ix}\right)\sqrt{a\sec^2(x)}-2i\cos(x)\text{PolyLog}\left(2,-ie^{ix}\right)\sqrt{a$$

Antiderivative was successfully verified.

```
[In] Int[x^2*Csc[x]*Sec[x]*Sqrt[a*Sec[x]^2],x]
```

```
[Out] x^2*Sqrt[a*Sec[x]^2] + (4*I)*x*ArcTan[E^(I*x)]*Cos[x]*Sqrt[a*Sec[x]^2] - 2*
x^2*ArcTanh[E^(I*x)]*Cos[x]*Sqrt[a*Sec[x]^2] + (2*I)*x*Cos[x]*PolyLog[2, -E
^(I*x)]*Sqrt[a*Sec[x]^2] - (2*I)*Cos[x]*PolyLog[2, (-I)*E^(I*x)]*Sqrt[a*Sec
[x]^2] + (2*I)*Cos[x]*PolyLog[2, I*E^(I*x)]*Sqrt[a*Sec[x]^2] - (2*I)*x*Cos[
x]*PolyLog[2, E^(I*x)]*Sqrt[a*Sec[x]^2] - 2*Cos[x]*PolyLog[3, -E^(I*x)]*Sqr
t[a*Sec[x]^2] + 2*Cos[x]*PolyLog[3, E^(I*x)]*Sqrt[a*Sec[x]^2]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2622

```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Dist[1/(f*a^n), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/2
), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)
/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 4181

```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))])/f, x] + (-Di
st[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
```

```
x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x))
], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 4183

```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[(
-2*(c + d*x)^m*ArcTanh[E^(I*(e + f*x))])/f, x] + (-Dist[(d*m)/f, Int[(c + d
*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Dist[(d*m)/f, Int[(c + d*x)^
(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 4420

```
Int[Csc[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sec[(a_.) + (b
_.)*(x_)]^(p_.), x_Symbol] :> Module[{u = IntHide[Csc[a + b*x]^n*Sec[a + b*
x]^p, x]}, Dist[(c + d*x)^m, u, x] - Dist[d*m, Int[(c + d*x)^(m - 1)*u, x],
 x]] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p] && GtQ[m, 0] && NeQ[n, p]
```

Rule 6273

```
Int[((a_.) + ArcTanh[u_]*(b_.))*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Si
mp[((c + d*x)^(m + 1)*(a + b*ArcTanh[u]))/(d*(m + 1)), x] - Dist[b/(d*(m +
1)), Int[SimplifyIntegrand[((c + d*x)^(m + 1)*D[u, x])/(1 - u^2), x], x], x
] /; FreeQ[{a, b, c, d, m}, x] && NeQ[m, -1] && InverseFunctionFreeQ[u, x]
&&  !FunctionOfQ[(c + d*x)^(m + 1), u, x] && FalseQ[PowerVariableExpn[u, m
+ 1, x]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6720

```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Dist[(a^IntPart[p]*(a*v^m)^
FracPart[p])/v^(m*FracPart[p]), Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p}, x
] &&  !IntegerQ[p] &&  !FreeQ[v, x] &&  !(EqQ[a, 1] && EqQ[m, 1]) &&  !(EqQ
[v, x] && EqQ[m, 1])
```

Rubi steps

$$\int x^2\csc(x)\sec(x)\sqrt{a\sec^2(x)}\,dx = \left(\cos(x)\sqrt{a\sec^2(x)}\right)\int x^2\csc(x)\sec^2(x)\,dx$$
$$= x^2\sqrt{a\sec^2(x)} - x^2\tanh^{-1}(\cos(x))\cos(x)\sqrt{a\sec^2(x)} - \left(2\cos(x)\sqrt{a\sec^2(x)}\right)$$
$$= x^2\sqrt{a\sec^2(x)} - x^2\tanh^{-1}(\cos(x))\cos(x)\sqrt{a\sec^2(x)} - \left(2\cos(x)\sqrt{a\sec^2(x)}\right)$$
$$= x^2\sqrt{a\sec^2(x)} - x^2\tanh^{-1}(\cos(x))\cos(x)\sqrt{a\sec^2(x)} + \left(2\cos(x)\sqrt{a\sec^2(x)}\right)$$
$$= x^2\sqrt{a\sec^2(x)} + 4ix\tan^{-1}\left(e^{ix}\right)\cos(x)\sqrt{a\sec^2(x)} + \left(\cos(x)\sqrt{a\sec^2(x)}\right)\int x^2$$
$$= x^2\sqrt{a\sec^2(x)} + 4ix\tan^{-1}\left(e^{ix}\right)\cos(x)\sqrt{a\sec^2(x)} - 2x^2\tanh^{-1}\left(e^{ix}\right)\cos(x)\sqrt{a}$$
$$= x^2\sqrt{a\sec^2(x)} + 4ix\tan^{-1}\left(e^{ix}\right)\cos(x)\sqrt{a\sec^2(x)} - 2x^2\tanh^{-1}\left(e^{ix}\right)\cos(x)\sqrt{a}$$
$$= x^2\sqrt{a\sec^2(x)} + 4ix\tan^{-1}\left(e^{ix}\right)\cos(x)\sqrt{a\sec^2(x)} - 2x^2\tanh^{-1}\left(e^{ix}\right)\cos(x)\sqrt{a}$$
$$= x^2\sqrt{a\sec^2(x)} + 4ix\tan^{-1}\left(e^{ix}\right)\cos(x)\sqrt{a\sec^2(x)} - 2x^2\tanh^{-1}\left(e^{ix}\right)\cos(x)\sqrt{a}$$

Mathematica [A] time = 0.12, size = 174, normalized size = 0.77

$$\sqrt{a\sec^2(x)}\left(2ix\left(\text{Li}_2\left(-e^{ix}\right)-\text{Li}_2\left(e^{ix}\right)\right)\cos(x)+2\left(\text{Li}_3\left(e^{ix}\right)-\text{Li}_3\left(-e^{ix}\right)\right)\cos(x)-2\cos(x)\left(i\left(\text{Li}_2\left(-ie^{ix}\right)-\text{Li}_2\left(ie\right.\right.\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*Csc[x]*Sec[x]*Sqrt[a*Sec[x]^2],x]
```

```
[Out] (x^2 + x^2*Cos[x]*(Log[1 - E^(I*x)] - Log[1 + E^(I*x)]) - 2*Cos[x]*(x*(Log[
1 - I*E^(I*x)] - Log[1 + I*E^(I*x)]) + I*(PolyLog[2, (-I)*E^(I*x)] - PolyLo
g[2, I*E^(I*x)])) + (2*I)*x*Cos[x]*(PolyLog[2, -E^(I*x)] - PolyLog[2, E^(I*
x)]) + 2*Cos[x]*(-PolyLog[3, -E^(I*x)] + PolyLog[3, E^(I*x)]))*Sqrt[a*Sec[x
]^2]
```

fricas [C] time = 0.81, size = 337, normalized size = 1.50

$$\sqrt{\frac{a}{\cos(x)^2}}\cos(x)\text{polylog}(3,\cos(x)+i\sin(x))+\sqrt{\frac{a}{\cos(x)^2}}\cos(x)\text{polylog}(3,\cos(x)-i\sin(x))-\sqrt{\frac{a}{\cos(x)^2}}\cos($$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*csc(x)*sec(x)*(a*sec(x)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] sqrt(a/cos(x)^2)*cos(x)*polylog(3, cos(x) + I*sin(x)) + sqrt(a/cos(x)^2)*co
s(x)*polylog(3, cos(x) - I*sin(x)) - sqrt(a/cos(x)^2)*cos(x)*polylog(3, -co
```

```
s(x) + I*sin(x)) - sqrt(a/cos(x)^2)*cos(x)*polylog(3, -cos(x) - I*sin(x)) -
 1/2*(x^2*cos(x)*log(cos(x) + I*sin(x) + 1) + x^2*cos(x)*log(cos(x) - I*sin
(x) + 1) - x^2*cos(x)*log(-cos(x) + I*sin(x) + 1) - x^2*cos(x)*log(-cos(x)
- I*sin(x) + 1) + 2*I*x*cos(x)*dilog(cos(x) + I*sin(x)) - 2*I*x*cos(x)*dilo
g(cos(x) - I*sin(x)) + 2*I*x*cos(x)*dilog(-cos(x) + I*sin(x)) - 2*I*x*cos(x
)*dilog(-cos(x) - I*sin(x)) + 2*x*cos(x)*log(I*cos(x) + sin(x) + 1) - 2*x*c
os(x)*log(I*cos(x) - sin(x) + 1) + 2*x*cos(x)*log(-I*cos(x) + sin(x) + 1) -
 2*x*cos(x)*log(-I*cos(x) - sin(x) + 1) - 2*x^2 - 2*I*cos(x)*dilog(I*cos(x)
 + sin(x)) - 2*I*cos(x)*dilog(I*cos(x) - sin(x)) + 2*I*cos(x)*dilog(-I*cos(
x) + sin(x)) + 2*I*cos(x)*dilog(-I*cos(x) - sin(x)))*sqrt(a/cos(x)^2)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a \sec(x)^2}\, x^2 \csc(x) \sec(x)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*csc(x)*sec(x)*(a*sec(x)^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(a*sec(x)^2)*x^2*csc(x)*sec(x), x)
```

maple [A] time = 0.30, size = 200, normalized size = 0.89

$$2\sqrt{\frac{a\,\mathrm{e}^{2ix}}{\left(\mathrm{e}^{2ix}+1\right)^{2}}}\, x^{2}-4i\sqrt{\frac{a\,\mathrm{e}^{2ix}}{\left(\mathrm{e}^{2ix}+1\right)^{2}}}\left(2i\left(\frac{x\ln\left(1+i\mathrm{e}^{ix}\right)}{2}-\frac{x\ln\left(1-i\mathrm{e}^{ix}\right)}{2}-\frac{i\,\mathrm{dilog}\left(1+i\mathrm{e}^{ix}\right)}{2}+\frac{i\,\mathrm{dilog}\left(1-i\mathrm{e}^{ix}\right)}{2}\right)\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*csc(x)*sec(x)*(a*sec(x)^2)^(1/2),x)
```

```
[Out] 2*(a*exp(2*I*x)/(exp(2*I*x)+1)^2)^(1/2)*x^2-4*I*(a*exp(2*I*x)/(exp(2*I*x)+1
)^2)^(1/2)*(2*I*(1/2*x*ln(1+I*exp(I*x))-1/2*x*ln(1-I*exp(I*x))-1/2*I*dilog(
1+I*exp(I*x))+1/2*I*dilog(1-I*exp(I*x)))-1/2*I*(-1/3*I*x^3+x^2*ln(1+exp(I*x
))-2*I*x*polylog(2,-exp(I*x))+2*polylog(3,-exp(I*x)))-1/2*I*(1/3*I*x^3-x^2*
ln(1-exp(I*x))+2*I*x*polylog(2,exp(I*x))-2*polylog(3,exp(I*x))))*cos(x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*csc(x)*sec(x)*(a*sec(x)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] -(4*I*x^2*cos(x) - 4*x^2*sin(x) + 2*(x^2*cos(2*x) + I*x^2*sin(2*x) + x^2)*a
rctan2(sin(x), cos(x) + 1) + 2*(x^2*cos(2*x) + I*x^2*sin(2*x) + x^2)*arctan
```

```
2(sin(x), -cos(x) + 1) - 4*(x*cos(2*x) + I*x*sin(2*x) + x)*dilog(-e^(I*x))
+ 4*(x*cos(2*x) + I*x*sin(2*x) + x)*dilog(e^(I*x)) + (-8*I*cos(2*x) + 8*sin
(2*x) - 8*I)*integrate((x*cos(2*x)*cos(x) + x*sin(2*x)*sin(x) + x*cos(x))/(
cos(2*x)^2 + sin(2*x)^2 + 2*cos(2*x) + 1), x) - (8*cos(2*x) + 8*I*sin(2*x)
+ 8)*integrate((x*cos(x)*sin(2*x) - x*cos(2*x)*sin(x) - x*sin(x))/(cos(2*x)
^2 + sin(2*x)^2 + 2*cos(2*x) + 1), x) + (-I*x^2*cos(2*x) + x^2*sin(2*x) - I
*x^2)*log(cos(x)^2 + sin(x)^2 + 2*cos(x) + 1) + (I*x^2*cos(2*x) - x^2*sin(2
*x) + I*x^2)*log(cos(x)^2 + sin(x)^2 - 2*cos(x) + 1) + (-4*I*cos(2*x) + 4*s
in(2*x) - 4*I)*polylog(3, -e^(I*x)) + (4*I*cos(2*x) - 4*sin(2*x) + 4*I)*pol
ylog(3, e^(I*x)))*sqrt(a)/(-2*I*cos(2*x) + 2*sin(2*x) - 2*I)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^2 \sqrt{\frac{a}{\cos(x)^2}}}{\cos(x) \sin(x)} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(a/cos(x)^2)^(1/2))/(cos(x)*sin(x)),x)
[Out] int((x^2*(a/cos(x)^2)^(1/2))/(cos(x)*sin(x)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2 \sqrt{a \sec^2(x)} \csc(x) \sec(x) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*csc(x)*sec(x)*(a*sec(x)**2)**(1/2),x)
[Out] Integral(x**2*sqrt(a*sec(x)**2)*csc(x)*sec(x), x)
```

3.876 $\int x^3 \csc(x) \sec(x) \sqrt{a \sec^2(x)}\, dx$

Optimal. Leaf size=341

$$3ix^2\text{Li}_2\left(-e^{ix}\right)\cos(x)\sqrt{a\sec^2(x)} - 3ix^2\text{Li}_2\left(e^{ix}\right)\cos(x)\sqrt{a\sec^2(x)} - 6ix\text{Li}_2\left(-ie^{ix}\right)\cos(x)\sqrt{a\sec^2(x)} + 6ix\text{Li}_2\left(ie^{ix}\right)$$

```
[Out] x^3*(a*sec(x)^2)^(1/2)+6*I*x^2*arctan(exp(I*x))*cos(x)*(a*sec(x)^2)^(1/2)-2
*x^3*arctanh(exp(I*x))*cos(x)*(a*sec(x)^2)^(1/2)+3*I*x^2*cos(x)*polylog(2,-
exp(I*x))*(a*sec(x)^2)^(1/2)-6*I*x*cos(x)*polylog(2,-I*exp(I*x))*(a*sec(x)^
2)^(1/2)+6*I*x*cos(x)*polylog(2,I*exp(I*x))*(a*sec(x)^2)^(1/2)-3*I*x^2*cos(
x)*polylog(2,exp(I*x))*(a*sec(x)^2)^(1/2)-6*x*cos(x)*polylog(3,-exp(I*x))*(
a*sec(x)^2)^(1/2)+6*cos(x)*polylog(3,-I*exp(I*x))*(a*sec(x)^2)^(1/2)-6*cos(
x)*polylog(3,I*exp(I*x))*(a*sec(x)^2)^(1/2)+6*x*cos(x)*polylog(3,exp(I*x))*
(a*sec(x)^2)^(1/2)-6*I*cos(x)*polylog(4,-exp(I*x))*(a*sec(x)^2)^(1/2)+6*I*c
os(x)*polylog(4,exp(I*x))*(a*sec(x)^2)^(1/2)
```

Rubi [A] time = 0.63, antiderivative size = 341, normalized size of antiderivative = 1.00, number of steps used = 21, number of rules used = 13, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.722, Rules used = {6720, 2622, 321, 207, 4420, 14, 6273, 4183, 2531, 6609, 2282, 6589, 4181}

$$3ix^2\cos(x)\text{PolyLog}\left(2,-e^{ix}\right)\sqrt{a\sec^2(x)} - 3ix^2\cos(x)\text{PolyLog}\left(2,e^{ix}\right)\sqrt{a\sec^2(x)} - 6ix\cos(x)\text{PolyLog}\left(2,-ie^{ix}\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^3*Csc[x]*Sec[x]*Sqrt[a*Sec[x]^2],x]
```

```
[Out] x^3*Sqrt[a*Sec[x]^2] + (6*I)*x^2*ArcTan[E^(I*x)]*Cos[x]*Sqrt[a*Sec[x]^2] -
2*x^3*ArcTanh[E^(I*x)]*Cos[x]*Sqrt[a*Sec[x]^2] + (3*I)*x^2*Cos[x]*PolyLog[2
, -E^(I*x)]*Sqrt[a*Sec[x]^2] - (6*I)*x*Cos[x]*PolyLog[2, (-I)*E^(I*x)]*Sqrt
[a*Sec[x]^2] + (6*I)*x*Cos[x]*PolyLog[2, I*E^(I*x)]*Sqrt[a*Sec[x]^2] - (3*I
)*x^2*Cos[x]*PolyLog[2, E^(I*x)]*Sqrt[a*Sec[x]^2] - 6*x*Cos[x]*PolyLog[3, -
E^(I*x)]*Sqrt[a*Sec[x]^2] + 6*Cos[x]*PolyLog[3, (-I)*E^(I*x)]*Sqrt[a*Sec[x]
^2] - 6*Cos[x]*PolyLog[3, I*E^(I*x)]*Sqrt[a*Sec[x]^2] + 6*x*Cos[x]*PolyLog[
3, E^(I*x)]*Sqrt[a*Sec[x]^2] - (6*I)*Cos[x]*PolyLog[4, -E^(I*x)]*Sqrt[a*Sec
[x]^2] + (6*I)*Cos[x]*PolyLog[4, E^(I*x)]*Sqrt[a*Sec[x]^2]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2622

```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Dist[1/(f*a^n), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/2
), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)
/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 4181

```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))])/f, x] + (-Di
st[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x))
], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 4183

```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[(
-2*(c + d*x)^m*ArcTanh[E^(I*(e + f*x))])/f, x] + (-Dist[(d*m)/f, Int[(c + d
*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Dist[(d*m)/f, Int[(c + d*x)^
(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 4420

```
Int[Csc[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sec[(a_.) + (b
_.)*(x_)]^(p_.), x_Symbol] :> Module[{u = IntHide[Csc[a + b*x]^n*Sec[a + b*
x]^p, x]}, Dist[(c + d*x)^m, u, x] - Dist[d*m, Int[(c + d*x)^(m - 1)*u, x],
 x]] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p] && GtQ[m, 0] && NeQ[n, p]
```

Rule 6273

```
Int[((a_.) + ArcTanh[u_]*(b_.))*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Si
mp[((c + d*x)^(m + 1)*(a + b*ArcTanh[u]))/(d*(m + 1)), x] - Dist[b/(d*(m +
1)), Int[SimplifyIntegrand[((c + d*x)^(m + 1)*D[u, x])/(1 - u^2), x], x], x
] /; FreeQ[{a, b, c, d, m}, x] && NeQ[m, -1] && InverseFunctionFreeQ[u, x]
&&  !FunctionOfQ[(c + d*x)^(m + 1), u, x] && FalseQ[PowerVariableExpn[u, m
+ 1, x]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rule 6720

```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Dist[(a^IntPart[p]*(a*v^m)^
FracPart[p])/v^(m*FracPart[p]), Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p}, x
] &&  !IntegerQ[p] &&  !FreeQ[v, x] &&  !(EqQ[a, 1] && EqQ[m, 1]) &&  !(EqQ
[v, x] && EqQ[m, 1])
```

Rubi steps

$$\int x^3 \csc(x)\sec(x)\sqrt{a\sec^2(x)}\,dx = \left(\cos(x)\sqrt{a\sec^2(x)}\right)\int x^3\csc(x)\sec^2(x)\,dx$$
$$= x^3\sqrt{a\sec^2(x)} - x^3\tanh^{-1}(\cos(x))\cos(x)\sqrt{a\sec^2(x)} - \left(3\cos(x)\sqrt{a\sec^2(x)}\right)$$
$$= x^3\sqrt{a\sec^2(x)} - x^3\tanh^{-1}(\cos(x))\cos(x)\sqrt{a\sec^2(x)} - \left(3\cos(x)\sqrt{a\sec^2(x)}\right)$$
$$= x^3\sqrt{a\sec^2(x)} - x^3\tanh^{-1}(\cos(x))\cos(x)\sqrt{a\sec^2(x)} + \left(3\cos(x)\sqrt{a\sec^2(x)}\right)$$
$$= x^3\sqrt{a\sec^2(x)} + 6ix^2\tan^{-1}\left(e^{ix}\right)\cos(x)\sqrt{a\sec^2(x)} + \left(\cos(x)\sqrt{a\sec^2(x)}\right)\int x^3$$
$$= x^3\sqrt{a\sec^2(x)} + 6ix^2\tan^{-1}\left(e^{ix}\right)\cos(x)\sqrt{a\sec^2(x)} - 2x^3\tanh^{-1}\left(e^{ix}\right)\cos(x)\sqrt{}$$
$$= x^3\sqrt{a\sec^2(x)} + 6ix^2\tan^{-1}\left(e^{ix}\right)\cos(x)\sqrt{a\sec^2(x)} - 2x^3\tanh^{-1}\left(e^{ix}\right)\cos(x)\sqrt{}$$
$$= x^3\sqrt{a\sec^2(x)} + 6ix^2\tan^{-1}\left(e^{ix}\right)\cos(x)\sqrt{a\sec^2(x)} - 2x^3\tanh^{-1}\left(e^{ix}\right)\cos(x)\sqrt{}$$
$$= x^3\sqrt{a\sec^2(x)} + 6ix^2\tan^{-1}\left(e^{ix}\right)\cos(x)\sqrt{a\sec^2(x)} - 2x^3\tanh^{-1}\left(e^{ix}\right)\cos(x)\sqrt{}$$
$$= x^3\sqrt{a\sec^2(x)} + 6ix^2\tan^{-1}\left(e^{ix}\right)\cos(x)\sqrt{a\sec^2(x)} - 2x^3\tanh^{-1}\left(e^{ix}\right)\cos(x)\sqrt{}$$

Mathematica [A] time = 0.41, size = 290, normalized size = 0.85

$$\frac{1}{8}\sqrt{a\sec^2(x)}\left(24ix^2\text{Li}_2\left(e^{-ix}\right)\cos(x) + 24ix^2\text{Li}_2\left(-e^{ix}\right)\cos(x) - 48ix\text{Li}_2\left(-ie^{ix}\right)\cos(x) + 48ix\text{Li}_2\left(ie^{ix}\right)\cos(x) + 48\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*Csc[x]*Sec[x]*Sqrt[a*Sec[x]^2],x]
```

```
[Out] ((8*x^3 - I*Pi^4*Cos[x] + (2*I)*x^4*Cos[x] + 8*x^3*Cos[x]*Log[1 - E^((-I)*x
)] - 24*x^2*Cos[x]*Log[1 - I*E^(I*x)] + 24*x^2*Cos[x]*Log[1 + I*E^(I*x)] -
8*x^3*Cos[x]*Log[1 + E^(I*x)] + (24*I)*x^2*Cos[x]*PolyLog[2, E^((-I)*x)] +
(24*I)*x^2*Cos[x]*PolyLog[2, -E^(I*x)] - (48*I)*x*Cos[x]*PolyLog[2, (-I)*E^
(I*x)] + (48*I)*x*Cos[x]*PolyLog[2, I*E^(I*x)] + 48*x*Cos[x]*PolyLog[3, E^(
(-I)*x)] - 48*x*Cos[x]*PolyLog[3, -E^(I*x)] + 48*Cos[x]*PolyLog[3, (-I)*E^(
I*x)] - 48*Cos[x]*PolyLog[3, I*E^(I*x)] - (48*I)*Cos[x]*PolyLog[4, E^((-I)*
x)] - (48*I)*Cos[x]*PolyLog[4, -E^(I*x)])*Sqrt[a*Sec[x]^2])/8
```

fricas [C] time = 1.15, size = 539, normalized size = 1.58

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*csc(x)*sec(x)*(a*sec(x)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 3*x*sqrt(a/cos(x)^2)*cos(x)*polylog(3, cos(x) + I*sin(x)) + 3*x*sqrt(a/cos(
x)^2)*cos(x)*polylog(3, cos(x) - I*sin(x)) - 3*x*sqrt(a/cos(x)^2)*cos(x)*po
lylog(3, -cos(x) + I*sin(x)) - 3*x*sqrt(a/cos(x)^2)*cos(x)*polylog(3, -cos(
x) - I*sin(x)) + 3*I*sqrt(a/cos(x)^2)*cos(x)*polylog(4, cos(x) + I*sin(x))
- 3*I*sqrt(a/cos(x)^2)*cos(x)*polylog(4, cos(x) - I*sin(x)) + 3*I*sqrt(a/co
s(x)^2)*cos(x)*polylog(4, -cos(x) + I*sin(x)) - 3*I*sqrt(a/cos(x)^2)*cos(x)
*polylog(4, -cos(x) - I*sin(x)) + 3*sqrt(a/cos(x)^2)*cos(x)*polylog(3, I*co
s(x) + sin(x)) - 3*sqrt(a/cos(x)^2)*cos(x)*polylog(3, I*cos(x) - sin(x)) +
3*sqrt(a/cos(x)^2)*cos(x)*polylog(3, -I*cos(x) + sin(x)) - 3*sqrt(a/cos(x)^
2)*cos(x)*polylog(3, -I*cos(x) - sin(x)) - 1/2*(x^3*cos(x)*log(cos(x) + I*s
in(x) + 1) + x^3*cos(x)*log(cos(x) - I*sin(x) + 1) - x^3*cos(x)*log(-cos(x)
 + I*sin(x) + 1) - x^3*cos(x)*log(-cos(x) - I*sin(x) + 1) + 3*I*x^2*cos(x)*
dilog(cos(x) + I*sin(x)) - 3*I*x^2*cos(x)*dilog(cos(x) - I*sin(x)) + 3*I*x^
2*cos(x)*dilog(-cos(x) + I*sin(x)) - 3*I*x^2*cos(x)*dilog(-cos(x) - I*sin(x
)) + 3*x^2*cos(x)*log(I*cos(x) + sin(x) + 1) - 3*x^2*cos(x)*log(I*cos(x) -
sin(x) + 1) + 3*x^2*cos(x)*log(-I*cos(x) + sin(x) + 1) - 3*x^2*cos(x)*log(-
I*cos(x) - sin(x) + 1) - 2*x^3 - 6*I*x*cos(x)*dilog(I*cos(x) + sin(x)) - 6*
I*x*cos(x)*dilog(I*cos(x) - sin(x)) + 6*I*x*cos(x)*dilog(-I*cos(x) + sin(x)
) + 6*I*x*cos(x)*dilog(-I*cos(x) - sin(x)))*sqrt(a/cos(x)^2)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a \sec(x)^2}\, x^3 \csc(x) \sec(x)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*csc(x)*sec(x)*(a*sec(x)^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(a*sec(x)^2)*x^3*csc(x)*sec(x), x)
```

maple [A] time = 0.42, size = 250, normalized size = 0.73

$$2\sqrt{\frac{a\,e^{2ix}}{\left(e^{2ix}+1\right)^2}}\,x^3+4\sqrt{\frac{a\,e^{2ix}}{\left(e^{2ix}+1\right)^2}}\left(\frac{3x^2\ln\left(1+ie^{ix}\right)}{2}-3ix\,\mathrm{polylog}\left(2,-ie^{ix}\right)+3\,\mathrm{polylog}\left(3,-ie^{ix}\right)-\frac{3x^2\ln\left(1\right.}{2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*csc(x)*sec(x)*(a*sec(x)^2)^(1/2),x)
```

```
[Out] 2*(a*exp(2*I*x)/(exp(2*I*x)+1)^2)^(1/2)*x^3+4*(a*exp(2*I*x)/(exp(2*I*x)+1)^
2)^(1/2)*(3/2*x^2*ln(1+I*exp(I*x))-3*I*x*polylog(2,-I*exp(I*x))+3*polylog(3
,-I*exp(I*x))-3/2*x^2*ln(1-I*exp(I*x))+3*I*x*polylog(2,I*exp(I*x))-3*polylo
g(3,I*exp(I*x))+1/2*I*(1/4*x^4+I*x^3*ln(1+exp(I*x))+3*x^2*polylog(2,-exp(I*
```

```
x))+6*I*x*polylog(3,-exp(I*x))-6*polylog(4,-exp(I*x)))+1/2*I*(-1/4*x^4-I*x^
3*ln(1-exp(I*x))-3*x^2*polylog(2,exp(I*x))-6*I*x*polylog(3,exp(I*x))+6*poly
log(4,exp(I*x))))*cos(x)
```

maxima [B] time = 0.48, size = 568, normalized size = 1.67

$$-\frac{\left(4i\,x^3\cos(x)-4\,x^3\sin(x)-6\left(x^2\cos\left(2\,x\right)+i\,x^2\sin\left(2\,x\right)+x^2\right)\arctan\left(\cos(x),\sin(x)+1\right)-6\left(x^2\cos\left(2\,x\right)+i\,x\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*csc(x)*sec(x)*(a*sec(x)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] -(4*I*x^3*cos(x) - 4*x^3*sin(x) - 6*(x^2*cos(2*x) + I*x^2*sin(2*x) + x^2)*a
rctan2(cos(x), sin(x) + 1) - 6*(x^2*cos(2*x) + I*x^2*sin(2*x) + x^2)*arctan
2(cos(x), -sin(x) + 1) + 2*(x^3*cos(2*x) + I*x^3*sin(2*x) + x^3)*arctan2(si
n(x), cos(x) + 1) + 2*(x^3*cos(2*x) + I*x^3*sin(2*x) + x^3)*arctan2(sin(x),
 -cos(x) + 1) - 12*(x*cos(2*x) + I*x*sin(2*x) + x)*dilog(I*e^(I*x)) + 12*(x
*cos(2*x) + I*x*sin(2*x) + x)*dilog(-I*e^(I*x)) - 6*(x^2*cos(2*x) + I*x^2*s
in(2*x) + x^2)*dilog(-e^(I*x)) + 6*(x^2*cos(2*x) + I*x^2*sin(2*x) + x^2)*di
log(e^(I*x)) + (-I*x^3*cos(2*x) + x^3*sin(2*x) - I*x^3)*log(cos(x)^2 + sin(
x)^2 + 2*cos(x) + 1) + (I*x^3*cos(2*x) - x^3*sin(2*x) + I*x^3)*log(cos(x)^2
 + sin(x)^2 - 2*cos(x) + 1) + (-3*I*x^2*cos(2*x) + 3*x^2*sin(2*x) - 3*I*x^2
)*log(cos(x)^2 + sin(x)^2 + 2*sin(x) + 1) + (3*I*x^2*cos(2*x) - 3*x^2*sin(2
*x) + 3*I*x^2)*log(cos(x)^2 + sin(x)^2 - 2*sin(x) + 1) + (12*cos(2*x) + 12*
I*sin(2*x) + 12)*polylog(4, -e^(I*x)) - (12*cos(2*x) + 12*I*sin(2*x) + 12)*
polylog(4, e^(I*x)) + (-12*I*cos(2*x) + 12*sin(2*x) - 12*I)*polylog(3, I*e^
(I*x)) + (12*I*cos(2*x) - 12*sin(2*x) + 12*I)*polylog(3, -I*e^(I*x)) + (-12
*I*x*cos(2*x) + 12*x*sin(2*x) - 12*I*x)*polylog(3, -e^(I*x)) + (12*I*x*cos(
2*x) - 12*x*sin(2*x) + 12*I*x)*polylog(3, e^(I*x)))*sqrt(a)/(-2*I*cos(2*x)
+ 2*sin(2*x) - 2*I)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^3\,\sqrt{\frac{a}{{\cos(x)}^2}}}{\cos(x)\,\sin(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(a/cos(x)^2)^(1/2))/(cos(x)*sin(x)),x)
```

```
[Out] int((x^3*(a/cos(x)^2)^(1/2))/(cos(x)*sin(x)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^3\sqrt{a\sec^2\left(x\right)}\,\csc\left(x\right)\sec\left(x\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*csc(x)*sec(x)*(a*sec(x)**2)**(1/2),x)
```

```
[Out] Integral(x**3*sqrt(a*sec(x)**2)*csc(x)*sec(x), x)
```

3.877 $\int x\csc(x)\sec(x)\sqrt{a\sec^4(x)}\,dx$

Optimal. Leaf size=142

$$\frac{1}{2}i\text{Li}_2\left(-e^{2ix}\right)\cos^2(x)\sqrt{a\sec^4(x)}-\frac{1}{2}i\text{Li}_2\left(e^{2ix}\right)\cos^2(x)\sqrt{a\sec^4(x)}+\frac{1}{2}x\cos^2(x)\sqrt{a\sec^4(x)}+\frac{1}{2}x\sin^2(x)\sqrt{a\sec^4(x)}-$$

```
[Out] 1/2*x*cos(x)^2*(a*sec(x)^4)^(1/2)-2*x*arctanh(exp(2*I*x))*cos(x)^2*(a*sec(x
)^4)^(1/2)+1/2*I*cos(x)^2*polylog(2,-exp(2*I*x))*(a*sec(x)^4)^(1/2)-1/2*I*c
os(x)^2*polylog(2,exp(2*I*x))*(a*sec(x)^4)^(1/2)-1/2*cos(x)*sin(x)*(a*sec(x
)^4)^(1/2)+1/2*x*sin(x)^2*(a*sec(x)^4)^(1/2)
```

Rubi [A] time = 0.40, antiderivative size = 142, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 11, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.688, Rules used = {6720, 2620, 14, 4420, 2548, 4419, 4183, 2279, 2391, 3473, 8}

$$\frac{1}{2}i\cos^2(x)\text{PolyLog}\left(2,-e^{2ix}\right)\sqrt{a\sec^4(x)}-\frac{1}{2}i\cos^2(x)\text{PolyLog}\left(2,e^{2ix}\right)\sqrt{a\sec^4(x)}+\frac{1}{2}x\cos^2(x)\sqrt{a\sec^4(x)}+\frac{1}{2}x\sin$$

Antiderivative was successfully verified.

```
[In] Int[x*Csc[x]*Sec[x]*Sqrt[a*Sec[x]^4],x]
```

```
[Out] (x*Cos[x]^2*Sqrt[a*Sec[x]^4])/2 - 2*x*ArcTanh[E^((2*I)*x)]*Cos[x]^2*Sqrt[a*
Sec[x]^4] + (I/2)*Cos[x]^2*PolyLog[2, -E^((2*I)*x)]*Sqrt[a*Sec[x]^4] - (I/2
)*Cos[x]^2*PolyLog[2, E^((2*I)*x)]*Sqrt[a*Sec[x]^4] - (Cos[x]*Sqrt[a*Sec[x]
^4]*Sin[x])/2 + (x*Sqrt[a*Sec[x]^4]*Sin[x]^2)/2
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2548

```
Int[Log[u_], x_Symbol] :> Simp[x*Log[u], x] - Int[SimplifyIntegrand[(x*D[u,
 x])/u, x], x] /; InverseFunctionFreeQ[u, x]
```

Rule 2620

```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Dist[1/f, Subst[Int[(1 + x^2)^((m + n)/2 - 1)/x^m, x], x, Tan[e + f*x]],
 x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n)/2]
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 4183

```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[(
-2*(c + d*x)^m*ArcTanh[E^(I*(e + f*x))])/f, x] + (-Dist[(d*m)/f, Int[(c + d
*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Dist[(d*m)/f, Int[(c + d*x)^
(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 4419

```
Int[Csc[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sec[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Dist[2^n, Int[(c + d*x)^m*Csc[2*a + 2*b*x]^n,
 x], x] /; FreeQ[{a, b, c, d, m}, x] && IntegerQ[n] && RationalQ[m]
```

Rule 4420

```
Int[Csc[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sec[(a_.) + (b
_.)*(x_)]^(p_.), x_Symbol] :> Module[{u = IntHide[Csc[a + b*x]^n*Sec[a + b*
x]^p, x]}, Dist[(c + d*x)^m, u, x] - Dist[d*m, Int[(c + d*x)^(m - 1)*u, x],
 x]] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p] && GtQ[m, 0] && NeQ[n, p]
```

Rule 6720

```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Dist[(a^IntPart[p]*(a*v^m)^
FracPart[p])/v^(m*FracPart[p]), Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p}, x
```

```
] && !IntegerQ[p] && !FreeQ[v, x] && !(EqQ[a, 1] && EqQ[m, 1]) && !(EqQ
[v, x] && EqQ[m, 1])
```

Rubi steps

$$\int x\csc(x)\sec(x)\sqrt{a\sec^4(x)}\,dx = \left(\cos^2(x)\sqrt{a\sec^4(x)}\right)\int x\csc(x)\sec^3(x)\,dx$$
$$= x\cos^2(x)\log(\tan(x))\sqrt{a\sec^4(x)} + \frac{1}{2}x\sqrt{a\sec^4(x)}\,\sin^2(x) - \left(\cos^2(x)\sqrt{a\sec^4(x)}\right.$$
$$= x\cos^2(x)\log(\tan(x))\sqrt{a\sec^4(x)} + \frac{1}{2}x\sqrt{a\sec^4(x)}\,\sin^2(x) - \frac{1}{2}\left(\cos^2(x)\sqrt{a\sec^4}\right.$$
$$= -\frac{1}{2}\cos(x)\sqrt{a\sec^4(x)}\,\sin(x) + \frac{1}{2}x\sqrt{a\sec^4(x)}\,\sin^2(x) + \frac{1}{2}\left(\cos^2(x)\sqrt{a\sec^4(x)}\right)$$
$$= \frac{1}{2}x\cos^2(x)\sqrt{a\sec^4(x)} - \frac{1}{2}\cos(x)\sqrt{a\sec^4(x)}\,\sin(x) + \frac{1}{2}x\sqrt{a\sec^4(x)}\,\sin^2(x) +$$
$$= \frac{1}{2}x\cos^2(x)\sqrt{a\sec^4(x)} - 2x\tanh^{-1}\left(e^{2ix}\right)\cos^2(x)\sqrt{a\sec^4(x)} - \frac{1}{2}\cos(x)\sqrt{a\sec^4}$$
$$= \frac{1}{2}x\cos^2(x)\sqrt{a\sec^4(x)} - 2x\tanh^{-1}\left(e^{2ix}\right)\cos^2(x)\sqrt{a\sec^4(x)} - \frac{1}{2}\cos(x)\sqrt{a\sec^4}$$
$$= \frac{1}{2}x\cos^2(x)\sqrt{a\sec^4(x)} - 2x\tanh^{-1}\left(e^{2ix}\right)\cos^2(x)\sqrt{a\sec^4(x)} + \frac{1}{2}i\cos^2(x)\mathrm{Li}_2\left(-\right.$$

Mathematica [A] time = 0.23, size = 85, normalized size = 0.60

$$\frac{1}{2}\cos^2(x)\sqrt{a\sec^4(x)}\left(i\mathrm{Li}_2\left(-e^{2ix}\right) - i\mathrm{Li}_2\left(e^{2ix}\right) + 2x\log\left(1-e^{2ix}\right) - 2x\log\left(1+e^{2ix}\right) - \tan(x) + x\sec^2(x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Csc[x]*Sec[x]*Sqrt[a*Sec[x]^4],x]
```

```
[Out] (Cos[x]^2*Sqrt[a*Sec[x]^4]*(2*x*Log[1 - E^((2*I)*x)] - 2*x*Log[1 + E^((2*I)
*x)] + I*PolyLog[2, -E^((2*I)*x)] - I*PolyLog[2, E^((2*I)*x)] + x*Sec[x]^2
- Tan[x]))/2
```

fricas [B] time = 0.76, size = 270, normalized size = 1.90

$$\frac{1}{2}\left(x\cos(x)^2\log(\cos(x) + i\sin(x) + 1) + x\cos(x)^2\log(\cos(x) - i\sin(x) + 1) - x\cos(x)^2\log(i\cos(x) + \sin(x) + \right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*csc(x)*sec(x)*(a*sec(x)^4)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*(x*cos(x)^2*log(cos(x) + I*sin(x) + 1) + x*cos(x)^2*log(cos(x) - I*sin(
x) + 1) - x*cos(x)^2*log(I*cos(x) + sin(x) + 1) - x*cos(x)^2*log(I*cos(x) -
 sin(x) + 1) - x*cos(x)^2*log(-I*cos(x) + sin(x) + 1) - x*cos(x)^2*log(-I*c
os(x) - sin(x) + 1) + x*cos(x)^2*log(-cos(x) + I*sin(x) + 1) + x*cos(x)^2*l
og(-cos(x) - I*sin(x) + 1) - I*cos(x)^2*dilog(cos(x) + I*sin(x)) + I*cos(x)
^2*dilog(cos(x) - I*sin(x)) - I*cos(x)^2*dilog(I*cos(x) + sin(x)) + I*cos(x
)^2*dilog(I*cos(x) - sin(x)) + I*cos(x)^2*dilog(-I*cos(x) + sin(x)) - I*cos
(x)^2*dilog(-I*cos(x) - sin(x)) + I*cos(x)^2*dilog(-cos(x) + I*sin(x)) - I*
cos(x)^2*dilog(-cos(x) - I*sin(x)) - cos(x)*sin(x) + x)*sqrt(a/cos(x)^4)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a\sec(x)^4}\, x\csc(x)\sec(x)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*csc(x)*sec(x)*(a*sec(x)^4)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(a*sec(x)^4)*x*csc(x)*sec(x), x)
```

maple [A] time = 0.20, size = 165, normalized size = 1.16

$$\sqrt{\frac{a\,e^{4ix}}{\left(e^{2ix}+1\right)^4}}\,\left(-i+2x-ie^{-2ix}\right)-4i\sqrt{\frac{a\,e^{4ix}}{\left(e^{2ix}+1\right)^4}}\,\left(e^{2ix}+1\right)^2\left(\frac{ie^{-2ix}x\ln\left(1+e^{ix}\right)}{4}+\frac{e^{-2ix}\operatorname{polylog}\left(2,-e^{ix}\right)}{4}-\frac{ie}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*csc(x)*sec(x)*(a*sec(x)^4)^(1/2),x)
```

```
[Out] (a*exp(4*I*x)/(exp(2*I*x)+1)^4)^(1/2)*(-I+2*x-I*exp(-2*I*x))-4*I*(a*exp(4*I
*x)/(exp(2*I*x)+1)^4)^(1/2)*(exp(2*I*x)+1)^2*(1/4*I*exp(-2*I*x)*x*ln(1+exp(
I*x))+1/4*exp(-2*I*x)*polylog(2,-exp(I*x))-1/4*I*exp(-2*I*x)*x*ln(exp(2*I*x
)+1)-1/8*exp(-2*I*x)*polylog(2,-exp(2*I*x))+1/4*I*exp(-2*I*x)*x*ln(1-exp(I*
x))+1/4*exp(-2*I*x)*polylog(2,exp(I*x)))
```

maxima [B] time = 0.50, size = 432, normalized size = 3.04

$$-\frac{\left((2\,x\cos\left(4\,x\right)+4\,x\cos\left(2\,x\right)+2i\,x\sin\left(4\,x\right)+4i\,x\sin\left(2\,x\right)+2\,x\right)\arctan\left(\sin\left(2\,x\right),\cos\left(2\,x\right)+1\right)-\left(2\,x\cos\left(4\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*csc(x)*sec(x)*(a*sec(x)^4)^(1/2),x, algorithm="maxima")
```

```
[Out] -((2*x*cos(4*x) + 4*x*cos(2*x) + 2*I*x*sin(4*x) + 4*I*x*sin(2*x) + 2*x)*arc
tan2(sin(2*x), cos(2*x) + 1) - (2*x*cos(4*x) + 4*x*cos(2*x) + 2*I*x*sin(4*x
) + 4*I*x*sin(2*x) + 2*x)*arctan2(sin(x), cos(x) + 1) + (2*x*cos(4*x) + 4*x
*cos(2*x) + 2*I*x*sin(4*x) + 4*I*x*sin(2*x) + 2*x)*arctan2(sin(x), -cos(x)
+ 1) - 2*(-2*I*x - 1)*cos(2*x) - (cos(4*x) + 2*cos(2*x) + I*sin(4*x) + 2*I*
sin(2*x) + 1)*dilog(-e^(2*I*x)) + (2*cos(4*x) + 4*cos(2*x) + 2*I*sin(4*x) +
 4*I*sin(2*x) + 2)*dilog(-e^(I*x)) + (2*cos(4*x) + 4*cos(2*x) + 2*I*sin(4*x
) + 4*I*sin(2*x) + 2)*dilog(e^(I*x)) + (-I*x*cos(4*x) - 2*I*x*cos(2*x) + x*
sin(4*x) + 2*x*sin(2*x) - I*x)*log(cos(2*x)^2 + sin(2*x)^2 + 2*cos(2*x) + 1
) + (I*x*cos(4*x) + 2*I*x*cos(2*x) - x*sin(4*x) - 2*x*sin(2*x) + I*x)*log(c
os(x)^2 + sin(x)^2 + 2*cos(x) + 1) + (I*x*cos(4*x) + 2*I*x*cos(2*x) - x*sin
(4*x) - 2*x*sin(2*x) + I*x)*log(cos(x)^2 + sin(x)^2 - 2*cos(x) + 1) - (4*x
- 2*I)*sin(2*x) + 2)*sqrt(a)/(-2*I*cos(4*x) - 4*I*cos(2*x) + 2*sin(4*x) + 4
*sin(2*x) - 2*I)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x\sqrt{\frac{a}{\cos(x)^4}}}{\cos(x)\,\sin(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(a/cos(x)^4)^(1/2))/(cos(x)*sin(x)),x)
[Out] int((x*(a/cos(x)^4)^(1/2))/(cos(x)*sin(x)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x\sqrt{a\sec^4{(x)}}\csc{(x)}\sec{(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*csc(x)*sec(x)*(a*sec(x)**4)**(1/2),x)
[Out] Integral(x*sqrt(a*sec(x)**4)*csc(x)*sec(x), x)
```

3.878 $\int x^2 \csc(x) \sec(x) \sqrt{a \sec^4(x)}\, dx$

Optimal. Leaf size=220

$$ix\text{Li}_2\left(-e^{2ix}\right)\cos^2(x)\sqrt{a\sec^4(x)}-ix\text{Li}_2\left(e^{2ix}\right)\cos^2(x)\sqrt{a\sec^4(x)}-\frac{1}{2}\text{Li}_3\left(-e^{2ix}\right)\cos^2(x)\sqrt{a\sec^4(x)}+\frac{1}{2}\text{Li}_3\left(e^{2ix}\right)\text{co}$$

```
[Out] 1/2*x^2*cos(x)^2*(a*sec(x)^4)^(1/2)-2*x^2*arctanh(exp(2*I*x))*cos(x)^2*(a*s
ec(x)^4)^(1/2)-cos(x)^2*ln(cos(x))*(a*sec(x)^4)^(1/2)+I*x*cos(x)^2*polylog(
2,-exp(2*I*x))*(a*sec(x)^4)^(1/2)-I*x*cos(x)^2*polylog(2,exp(2*I*x))*(a*sec
(x)^4)^(1/2)-1/2*cos(x)^2*polylog(3,-exp(2*I*x))*(a*sec(x)^4)^(1/2)+1/2*cos
(x)^2*polylog(3,exp(2*I*x))*(a*sec(x)^4)^(1/2)-x*cos(x)*sin(x)*(a*sec(x)^4)
^(1/2)+1/2*x^2*sin(x)^2*(a*sec(x)^4)^(1/2)
```

Rubi [A] time = 0.54, antiderivative size = 220, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 13, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.722, Rules used = {6720, 2620, 14, 4420, 2551, 4419, 4183, 2531, 2282, 6589, 3720, 3475, 30}

$$ix\cos^2(x)\text{PolyLog}\left(2,-e^{2ix}\right)\sqrt{a\sec^4(x)}-ix\cos^2(x)\text{PolyLog}\left(2,e^{2ix}\right)\sqrt{a\sec^4(x)}-\frac{1}{2}\cos^2(x)\text{PolyLog}\left(3,-e^{2ix}\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^2*Csc[x]*Sec[x]*Sqrt[a*Sec[x]^4],x]
```

```
[Out] (x^2*Cos[x]^2*Sqrt[a*Sec[x]^4])/2 - 2*x^2*ArcTanh[E^((2*I)*x)]*Cos[x]^2*Sqr
t[a*Sec[x]^4] - Cos[x]^2*Log[Cos[x]]*Sqrt[a*Sec[x]^4] + I*x*Cos[x]^2*PolyLo
g[2, -E^((2*I)*x)]*Sqrt[a*Sec[x]^4] - I*x*Cos[x]^2*PolyLog[2, E^((2*I)*x)]*
Sqrt[a*Sec[x]^4] - (Cos[x]^2*PolyLog[3, -E^((2*I)*x)]*Sqrt[a*Sec[x]^4])/2 +
 (Cos[x]^2*PolyLog[3, E^((2*I)*x)]*Sqrt[a*Sec[x]^4])/2 - x*Cos[x]*Sqrt[a*Se
c[x]^4]*Sin[x] + (x^2*Sqrt[a*Sec[x]^4]*Sin[x]^2)/2
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2551

```
Int[Log[u_]*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[((a + b*x)^(m + 1
)*Log[u])/(b*(m + 1)), x] - Dist[1/(b*(m + 1)), Int[SimplifyIntegrand[((a +
 b*x)^(m + 1)*D[u, x])/u, x], x], x] /; FreeQ[{a, b, m}, x] && InverseFunct
ionFreeQ[u, x] && NeQ[m, -1]
```

Rule 2620

```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Dist[1/f, Subst[Int[(1 + x^2)^((m + n)/2 - 1)/x^m, x], x, Tan[e + f*x]],
 x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n)/2]
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3720

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symb
ol] :> Simp[(b*(c + d*x)^m*(b*Tan[e + f*x])^(n - 1))/(f*(n - 1)), x] + (-Di
st[(b*d*m)/(f*(n - 1)), Int[(c + d*x)^(m - 1)*(b*Tan[e + f*x])^(n - 1), x],
 x] - Dist[b^2, Int[(c + d*x)^m*(b*Tan[e + f*x])^(n - 2), x], x]) /; FreeQ[
{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 0]
```

Rule 4183

```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[(
-2*(c + d*x)^m*ArcTanh[E^(I*(e + f*x))])/f, x] + (-Dist[(d*m)/f, Int[(c + d
*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Dist[(d*m)/f, Int[(c + d*x)^
```

```
(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 4419

```
Int[Csc[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sec[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Dist[2^n, Int[(c + d*x)^m*Csc[2*a + 2*b*x]^n,
 x], x] /; FreeQ[{a, b, c, d, m}, x] && IntegerQ[n] && RationalQ[m]
```

Rule 4420

```
Int[Csc[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sec[(a_.) + (b
_.)*(x_)]^(p_.), x_Symbol] :> Module[{u = IntHide[Csc[a + b*x]^n*Sec[a + b*
x]^p, x]}, Dist[(c + d*x)^m, u, x] - Dist[d*m, Int[(c + d*x)^(m - 1)*u, x],
 x]] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p] && GtQ[m, 0] && NeQ[n, p]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6720

```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Dist[(a^IntPart[p]*(a*v^m)^
FracPart[p])/v^(m*FracPart[p]), Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p}, x
] &&  !IntegerQ[p] &&  !FreeQ[v, x] &&  !(EqQ[a, 1] && EqQ[m, 1]) &&  !(EqQ
[v, x] && EqQ[m, 1])
```

Rubi steps

$$\int x^2 \csc(x)\sec(x)\sqrt{a\sec^4(x)}\,dx = \left(\cos^2(x)\sqrt{a\sec^4(x)}\right)\int x^2\csc(x)\sec^3(x)\,dx$$

$$= x^2\cos^2(x)\log(\tan(x))\sqrt{a\sec^4(x)} + \frac{1}{2}x^2\sqrt{a\sec^4(x)}\,\sin^2(x) - \left(2\cos^2(x)\sqrt{a\,\mathrm{se}}\right.$$

$$= x^2\cos^2(x)\log(\tan(x))\sqrt{a\sec^4(x)} + \frac{1}{2}x^2\sqrt{a\sec^4(x)}\,\sin^2(x) - \left(2\cos^2(x)\sqrt{a\,\mathrm{se}}\right.$$

$$= x^2\cos^2(x)\log(\tan(x))\sqrt{a\sec^4(x)} + \frac{1}{2}x^2\sqrt{a\sec^4(x)}\,\sin^2(x) - \left(\cos^2(x)\sqrt{a\,\mathrm{sec}}\right.$$

$$= -x\cos(x)\sqrt{a\sec^4(x)}\,\sin(x) + \frac{1}{2}x^2\sqrt{a\sec^4(x)}\,\sin^2(x) + \left(\cos^2(x)\sqrt{a\sec^4(x)}\right)$$

$$= \frac{1}{2}x^2\cos^2(x)\sqrt{a\sec^4(x)} - \cos^2(x)\log(\cos(x))\sqrt{a\sec^4(x)} - x\cos(x)\sqrt{a\sec^4(x}$$

$$= \frac{1}{2}x^2\cos^2(x)\sqrt{a\sec^4(x)} - 2x^2\tanh^{-1}\left(e^{2ix}\right)\cos^2(x)\sqrt{a\sec^4(x)} - \cos^2(x)\log(c$$

$$= \frac{1}{2}x^2\cos^2(x)\sqrt{a\sec^4(x)} - 2x^2\tanh^{-1}\left(e^{2ix}\right)\cos^2(x)\sqrt{a\sec^4(x)} - \cos^2(x)\log(c$$

$$= \frac{1}{2}x^2\cos^2(x)\sqrt{a\sec^4(x)} - 2x^2\tanh^{-1}\left(e^{2ix}\right)\cos^2(x)\sqrt{a\sec^4(x)} - \cos^2(x)\log(c$$

$$= \frac{1}{2}x^2\cos^2(x)\sqrt{a\sec^4(x)} - 2x^2\tanh^{-1}\left(e^{2ix}\right)\cos^2(x)\sqrt{a\sec^4(x)} - \cos^2(x)\log(c$$

Mathematica [A] time = 0.63, size = 138, normalized size = 0.63

$$\frac{1}{24}\cos^2(x)\sqrt{a\sec^4(x)}\left(24ix\mathrm{Li}_2\left(e^{-2ix}\right) + 24ix\mathrm{Li}_2\left(-e^{2ix}\right) + 12\mathrm{Li}_3\left(e^{-2ix}\right) - 12\mathrm{Li}_3\left(-e^{2ix}\right) + 16ix^3 + 24x^2\log\left(1 - e\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*Csc[x]*Sec[x]*Sqrt[a*Sec[x]^4],x]
```

```
[Out] (Cos[x]^2*Sqrt[a*Sec[x]^4]*((-I)*Pi^3 + (16*I)*x^3 + 24*x^2*Log[1 - E^((-2*
I)*x)] - 24*x^2*Log[1 + E^((2*I)*x)] - 24*Log[Cos[x]] + (24*I)*x*PolyLog[2,
 E^((-2*I)*x)] + (24*I)*x*PolyLog[2, -E^((2*I)*x)] + 12*PolyLog[3, E^((-2*I
)*x)] - 12*PolyLog[3, -E^((2*I)*x)] + 12*x^2*Sec[x]^2 - 24*x*Tan[x]))/24
```

fricas [C] time = 2.05, size = 550, normalized size = 2.50

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*csc(x)*sec(x)*(a*sec(x)^4)^(1/2),x, algorithm="fricas")
```

```
[Out] sqrt(a/cos(x)^4)*cos(x)^2*polylog(3, cos(x) + I*sin(x)) + sqrt(a/cos(x)^4)*
cos(x)^2*polylog(3, cos(x) - I*sin(x)) - sqrt(a/cos(x)^4)*cos(x)^2*polylog(
3, I*cos(x) + sin(x)) - sqrt(a/cos(x)^4)*cos(x)^2*polylog(3, I*cos(x) - sin
(x)) - sqrt(a/cos(x)^4)*cos(x)^2*polylog(3, -I*cos(x) + sin(x)) - sqrt(a/co
s(x)^4)*cos(x)^2*polylog(3, -I*cos(x) - sin(x)) + sqrt(a/cos(x)^4)*cos(x)^2
*polylog(3, -cos(x) + I*sin(x)) + sqrt(a/cos(x)^4)*cos(x)^2*polylog(3, -cos
(x) - I*sin(x)) + 1/2*(x^2*cos(x)^2*log(cos(x) + I*sin(x) + 1) + x^2*cos(x)
^2*log(cos(x) - I*sin(x) + 1) - x^2*cos(x)^2*log(I*cos(x) + sin(x) + 1) - x
^2*cos(x)^2*log(I*cos(x) - sin(x) + 1) - x^2*cos(x)^2*log(-I*cos(x) + sin(x
) + 1) - x^2*cos(x)^2*log(-I*cos(x) - sin(x) + 1) + x^2*cos(x)^2*log(-cos(x
) + I*sin(x) + 1) + x^2*cos(x)^2*log(-cos(x) - I*sin(x) + 1) - 2*I*x*cos(x)
^2*dilog(cos(x) + I*sin(x)) + 2*I*x*cos(x)^2*dilog(cos(x) - I*sin(x)) - 2*I
*x*cos(x)^2*dilog(I*cos(x) + sin(x)) + 2*I*x*cos(x)^2*dilog(I*cos(x) - sin(
x)) + 2*I*x*cos(x)^2*dilog(-I*cos(x) + sin(x)) - 2*I*x*cos(x)^2*dilog(-I*co
s(x) - sin(x)) + 2*I*x*cos(x)^2*dilog(-cos(x) + I*sin(x)) - 2*I*x*cos(x)^2*
dilog(-cos(x) - I*sin(x)) - cos(x)^2*log(cos(x) + I*sin(x) + I) - cos(x)^2*
log(cos(x) - I*sin(x) + I) - cos(x)^2*log(-cos(x) + I*sin(x) + I) - cos(x)^
2*log(-cos(x) - I*sin(x) + I) - 2*x*cos(x)*sin(x) + x^2)*sqrt(a/cos(x)^4)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a\sec(x)^4}\, x^2 \csc(x)\sec(x)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*csc(x)*sec(x)*(a*sec(x)^4)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(a*sec(x)^4)*x^2*csc(x)*sec(x), x)
```

maple [C] time = 0.23, size = 254, normalized size = 1.15

$$2\sqrt{\frac{a\,e^{4ix}}{\left(e^{2ix}+1\right)^4}}\, x\left(x-i-ie^{-2ix}\right)+2\sqrt{\frac{a\,e^{4ix}}{\left(e^{2ix}+1\right)^4}}\,\left(e^{2ix}+1\right)^2\left(-\frac{e^{-2ix}\ln\left(e^{2ix}+1\right)}{2}-e^{-2ix}\Im(x)+e^{-2ix}\ln\left(e^{i\Re(x)}\right)\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*csc(x)*sec(x)*(a*sec(x)^4)^(1/2),x)
```

```
[Out] 2*(a*exp(4*I*x)/(exp(2*I*x)+1)^4)^(1/2)*x*(x-I-I*exp(-2*I*x))+2*(a*exp(4*I*
x)/(exp(2*I*x)+1)^4)^(1/2)*(exp(2*I*x)+1)^2*(-1/2*exp(-2*I*x)*ln(exp(2*I*x)
+1)-exp(-2*I*x)*Im(x)+exp(-2*I*x)*ln(exp(I*Re(x)))+1/2*exp(-2*I*x)*x^2*ln(1
+exp(I*x))-I*exp(-2*I*x)*x*polylog(2,-exp(I*x))+exp(-2*I*x)*polylog(3,-exp(
I*x))-1/2*exp(-2*I*x)*x^2*ln(exp(2*I*x)+1)+1/2*I*exp(-2*I*x)*x*polylog(2,-e
xp(2*I*x))-1/4*exp(-2*I*x)*polylog(3,-exp(2*I*x))+1/2*exp(-2*I*x)*x^2*ln(1-
```

```
exp(I*x))-I*exp(-2*I*x)*x*polylog(2,exp(I*x))+exp(-2*I*x)*polylog(3,exp(I*x
)))
```

maxima [B] time = 0.52, size = 653, normalized size = 2.97

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*csc(x)*sec(x)*(a*sec(x)^4)^(1/2),x, algorithm="maxima")
```

```
[Out] -((2*x^2 + 2*(x^2 + 1)*cos(4*x) + 4*(x^2 + 1)*cos(2*x) + (2*I*x^2 + 2*I)*si
n(4*x) + (4*I*x^2 + 4*I)*sin(2*x) + 2)*arctan2(sin(2*x), cos(2*x) + 1) - (2
*x^2*cos(4*x) + 4*x^2*cos(2*x) + 2*I*x^2*sin(4*x) + 4*I*x^2*sin(2*x) + 2*x^
2)*arctan2(sin(x), cos(x) + 1) + (2*x^2*cos(4*x) + 4*x^2*cos(2*x) + 2*I*x^2
*sin(4*x) + 4*I*x^2*sin(2*x) + 2*x^2)*arctan2(sin(x), -cos(x) + 1) - 4*x*co
s(4*x) + (4*I*x^2 - 4*x)*cos(2*x) - (2*x*cos(4*x) + 4*x*cos(2*x) + 2*I*x*si
n(4*x) + 4*I*x*sin(2*x) + 2*x)*dilog(-e^(2*I*x)) + (4*x*cos(4*x) + 8*x*cos(
2*x) + 4*I*x*sin(4*x) + 8*I*x*sin(2*x) + 4*x)*dilog(-e^(I*x)) + (4*x*cos(4*
x) + 8*x*cos(2*x) + 4*I*x*sin(4*x) + 8*I*x*sin(2*x) + 4*x)*dilog(e^(I*x)) +
 (-I*x^2 + (-I*x^2 - I)*cos(4*x) + (-2*I*x^2 - 2*I)*cos(2*x) + (x^2 + 1)*si
n(4*x) + 2*(x^2 + 1)*sin(2*x) - I)*log(cos(2*x)^2 + sin(2*x)^2 + 2*cos(2*x)
 + 1) + (I*x^2*cos(4*x) + 2*I*x^2*cos(2*x) - x^2*sin(4*x) - 2*x^2*sin(2*x)
+ I*x^2)*log(cos(x)^2 + sin(x)^2 + 2*cos(x) + 1) + (I*x^2*cos(4*x) + 2*I*x^
2*cos(2*x) - x^2*sin(4*x) - 2*x^2*sin(2*x) + I*x^2)*log(cos(x)^2 + sin(x)^2
 - 2*cos(x) + 1) + (-I*cos(4*x) - 2*I*cos(2*x) + sin(4*x) + 2*sin(2*x) - I)
*polylog(3, -e^(2*I*x)) + (4*I*cos(4*x) + 8*I*cos(2*x) - 4*sin(4*x) - 8*sin
(2*x) + 4*I)*polylog(3, -e^(I*x)) + (4*I*cos(4*x) + 8*I*cos(2*x) - 4*sin(4*
x) - 8*sin(2*x) + 4*I)*polylog(3, e^(I*x)) - 4*I*x*sin(4*x) - 4*(x^2 + I*x)
*sin(2*x))*sqrt(a)/(-2*I*cos(4*x) - 4*I*cos(2*x) + 2*sin(4*x) + 4*sin(2*x)
- 2*I)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^2 \sqrt{\frac{a}{\cos(x)^4}}}{\cos(x)\,\sin(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(a/cos(x)^4)^(1/2))/(cos(x)*sin(x)),x)
```

```
[Out] int((x^2*(a/cos(x)^4)^(1/2))/(cos(x)*sin(x)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2 \sqrt{a \sec^4(x)}\, \csc(x) \sec(x)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*csc(x)*sec(x)*(a*sec(x)**4)**(1/2),x)
```

```
[Out] Integral(x**2*sqrt(a*sec(x)**4)*csc(x)*sec(x), x)
```

3.879 $\int x^3 \csc(x)\sec(x)\sqrt{a\sec^4(x)}\,dx$

Optimal. Leaf size=356

$$\frac{3}{2}ix^2\text{Li}_2\left(-e^{2ix}\right)\cos^2(x)\sqrt{a\sec^4(x)}-\frac{3}{2}ix^2\text{Li}_2\left(e^{2ix}\right)\cos^2(x)\sqrt{a\sec^4(x)}-\frac{3}{2}x\text{Li}_3\left(-e^{2ix}\right)\cos^2(x)\sqrt{a\sec^4(x)}+\frac{3}{2}x\text{Li}_3\left(e\right.$$

```
[Out] 3/2*I*x^2*cos(x)^2*(a*sec(x)^4)^(1/2)+1/2*x^3*cos(x)^2*(a*sec(x)^4)^(1/2)-2
*x^3*arctanh(exp(2*I*x))*cos(x)^2*(a*sec(x)^4)^(1/2)-3*x*cos(x)^2*ln(1+exp(
2*I*x))*(a*sec(x)^4)^(1/2)+3/2*I*cos(x)^2*polylog(2,-exp(2*I*x))*(a*sec(x)^
4)^(1/2)+3/2*I*x^2*cos(x)^2*polylog(2,-exp(2*I*x))*(a*sec(x)^4)^(1/2)-3/2*I
*x^2*cos(x)^2*polylog(2,exp(2*I*x))*(a*sec(x)^4)^(1/2)-3/2*x*cos(x)^2*polyl
og(3,-exp(2*I*x))*(a*sec(x)^4)^(1/2)+3/2*x*cos(x)^2*polylog(3,exp(2*I*x))*(
a*sec(x)^4)^(1/2)-3/4*I*cos(x)^2*polylog(4,-exp(2*I*x))*(a*sec(x)^4)^(1/2)+
3/4*I*cos(x)^2*polylog(4,exp(2*I*x))*(a*sec(x)^4)^(1/2)-3/2*x^2*cos(x)*sin(
x)*(a*sec(x)^4)^(1/2)+1/2*x^3*sin(x)^2*(a*sec(x)^4)^(1/2)
```

Rubi [A] time = 0.64, antiderivative size = 356, normalized size of antiderivative = 1.00, number of steps used = 21, number of rules used = 17, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.944, Rules used = {6720, 2620, 14, 4420, 2551, 4419, 4183, 2531, 6609, 2282, 6589, 3720, 3719, 2190, 2279, 2391, 30}

$$\frac{3}{2}ix^2\cos^2(x)\text{PolyLog}\left(2,-e^{2ix}\right)\sqrt{a\sec^4(x)}-\frac{3}{2}ix^2\cos^2(x)\text{PolyLog}\left(2,e^{2ix}\right)\sqrt{a\sec^4(x)}-\frac{3}{2}x\cos^2(x)\text{PolyLog}\left(3,-e\right.$$

Antiderivative was successfully verified.

```
[In] Int[x^3*Csc[x]*Sec[x]*Sqrt[a*Sec[x]^4],x]
```

```
[Out] ((3*I)/2)*x^2*Cos[x]^2*Sqrt[a*Sec[x]^4] + (x^3*Cos[x]^2*Sqrt[a*Sec[x]^4])/2
 - 2*x^3*ArcTanh[E^((2*I)*x)]*Cos[x]^2*Sqrt[a*Sec[x]^4] - 3*x*Cos[x]^2*Log[
1 + E^((2*I)*x)]*Sqrt[a*Sec[x]^4] + ((3*I)/2)*Cos[x]^2*PolyLog[2, -E^((2*I)
*x)]*Sqrt[a*Sec[x]^4] + ((3*I)/2)*x^2*Cos[x]^2*PolyLog[2, -E^((2*I)*x)]*Sqr
t[a*Sec[x]^4] - ((3*I)/2)*x^2*Cos[x]^2*PolyLog[2, E^((2*I)*x)]*Sqrt[a*Sec[x
]^4] - (3*x*Cos[x]^2*PolyLog[3, -E^((2*I)*x)]*Sqrt[a*Sec[x]^4])/2 + (3*x*Co
s[x]^2*PolyLog[3, E^((2*I)*x)]*Sqrt[a*Sec[x]^4])/2 - ((3*I)/4)*Cos[x]^2*Pol
yLog[4, -E^((2*I)*x)]*Sqrt[a*Sec[x]^4] + ((3*I)/4)*Cos[x]^2*PolyLog[4, E^((
2*I)*x)]*Sqrt[a*Sec[x]^4] - (3*x^2*Cos[x]*Sqrt[a*Sec[x]^4]*Sin[x])/2 + (x^3
*Sqrt[a*Sec[x]^4]*Sin[x]^2)/2
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2551

```
Int[Log[u_]*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[((a + b*x)^(m + 1
)*Log[u])/(b*(m + 1)), x] - Dist[1/(b*(m + 1)), Int[SimplifyIntegrand[((a +
 b*x)^(m + 1)*D[u, x])/u, x], x], x] /; FreeQ[{a, b, m}, x] && InverseFunct
```

```
ionFreeQ[u, x] && NeQ[m, -1]
```

Rule 2620

```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Dist[1/f, Subst[Int[(1 + x^2)^((m + n)/2 - 1)/x^m, x], x, Tan[e + f*x]],
 x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n)/2]
```

Rule 3719

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
I*(c + d*x)^(m + 1))/(d*(m + 1)), x] - Dist[2*I, Int[((c + d*x)^m*E^(2*I*(e
 + f*x)))/(1 + E^(2*I*(e + f*x))), x], x] /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 3720

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symb
ol] :> Simp[(b*(c + d*x)^m*(b*Tan[e + f*x])^(n - 1))/(f*(n - 1)), x] + (-Di
st[(b*d*m)/(f*(n - 1)), Int[(c + d*x)^(m - 1)*(b*Tan[e + f*x])^(n - 1), x],
 x] - Dist[b^2, Int[(c + d*x)^m*(b*Tan[e + f*x])^(n - 2), x], x]) /; FreeQ[
{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 0]
```

Rule 4183

```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[(
-2*(c + d*x)^m*ArcTanh[E^(I*(e + f*x))])/f, x] + (-Dist[(d*m)/f, Int[(c + d
*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Dist[(d*m)/f, Int[(c + d*x)^
(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 4419

```
Int[Csc[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sec[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Dist[2^n, Int[(c + d*x)^m*Csc[2*a + 2*b*x]^n,
 x], x] /; FreeQ[{a, b, c, d, m}, x] && IntegerQ[n] && RationalQ[m]
```

Rule 4420

```
Int[Csc[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sec[(a_.) + (b
_.)*(x_)]^(p_.), x_Symbol] :> Module[{u = IntHide[Csc[a + b*x]^n*Sec[a + b*
x]^p, x]}, Dist[(c + d*x)^m, u, x] - Dist[d*m, Int[(c + d*x)^(m - 1)*u, x],
 x]] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p] && GtQ[m, 0] && NeQ[n, p]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rule 6720

```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Dist[(a^IntPart[p]*(a*v^m)^
FracPart[p])/v^(m*FracPart[p]), Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p}, x
] && !IntegerQ[p] && !FreeQ[v, x] && !(EqQ[a, 1] && EqQ[m, 1]) && !(EqQ
[v, x] && EqQ[m, 1])
```

Rubi steps

$$\int x^3 \csc(x)\sec(x)\sqrt{a\sec^4(x)}\,dx = \left(\cos^2(x)\sqrt{a\sec^4(x)}\right)\int x^3\csc(x)\sec^3(x)\,dx$$
$$= x^3\cos^2(x)\log(\tan(x))\sqrt{a\sec^4(x)} + \frac{1}{2}x^3\sqrt{a\sec^4(x)}\sin^2(x) - \left(3\cos^2(x)\sqrt{a\,\mathrm{se}\right.$$
$$= x^3\cos^2(x)\log(\tan(x))\sqrt{a\sec^4(x)} + \frac{1}{2}x^3\sqrt{a\sec^4(x)}\sin^2(x) - \left(3\cos^2(x)\sqrt{a\,\mathrm{se}\right.$$
$$= x^3\cos^2(x)\log(\tan(x))\sqrt{a\sec^4(x)} + \frac{1}{2}x^3\sqrt{a\sec^4(x)}\sin^2(x) - \frac{1}{2}\left(3\cos^2(x)\sqrt{a\right.$$
$$= -\frac{3}{2}x^2\cos(x)\sqrt{a\sec^4(x)}\sin(x) + \frac{1}{2}x^3\sqrt{a\sec^4(x)}\sin^2(x) + \left(\cos^2(x)\sqrt{a\sec^4(x}\right.$$
$$= \frac{3}{2}ix^2\cos^2(x)\sqrt{a\sec^4(x)} + \frac{1}{2}x^3\cos^2(x)\sqrt{a\sec^4(x)} - \frac{3}{2}x^2\cos(x)\sqrt{a\sec^4(x)}\,\mathrm{sin}$$
$$= \frac{3}{2}ix^2\cos^2(x)\sqrt{a\sec^4(x)} + \frac{1}{2}x^3\cos^2(x)\sqrt{a\sec^4(x)} - 2x^3\tanh^{-1}\left(e^{2ix}\right)\cos^2(x)$$
$$= \frac{3}{2}ix^2\cos^2(x)\sqrt{a\sec^4(x)} + \frac{1}{2}x^3\cos^2(x)\sqrt{a\sec^4(x)} - 2x^3\tanh^{-1}\left(e^{2ix}\right)\cos^2(x)$$
$$= \frac{3}{2}ix^2\cos^2(x)\sqrt{a\sec^4(x)} + \frac{1}{2}x^3\cos^2(x)\sqrt{a\sec^4(x)} - 2x^3\tanh^{-1}\left(e^{2ix}\right)\cos^2(x)$$
$$= \frac{3}{2}ix^2\cos^2(x)\sqrt{a\sec^4(x)} + \frac{1}{2}x^3\cos^2(x)\sqrt{a\sec^4(x)} - 2x^3\tanh^{-1}\left(e^{2ix}\right)\cos^2(x)$$
$$= \frac{3}{2}ix^2\cos^2(x)\sqrt{a\sec^4(x)} + \frac{1}{2}x^3\cos^2(x)\sqrt{a\sec^4(x)} - 2x^3\tanh^{-1}\left(e^{2ix}\right)\cos^2(x)$$

Mathematica [A] time = 1.07, size = 191, normalized size = 0.54

$$\frac{1}{64}\cos^2(x)\sqrt{a\sec^4(x)}\left(96ix^2\mathrm{Li}_2\left(e^{-2ix}\right) + 96i\left(x^2+1\right)\mathrm{Li}_2\left(-e^{2ix}\right) + 96x\mathrm{Li}_3\left(e^{-2ix}\right) - 96x\mathrm{Li}_3\left(-e^{2ix}\right) - 48i\mathrm{Li}_4\left(e^{-2ix}\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*Csc[x]*Sec[x]*Sqrt[a*Sec[x]^4],x]
```

```
[Out] (Cos[x]^2*Sqrt[a*Sec[x]^4]*((-I)*Pi^4 + (96*I)*x^2 + (32*I)*x^4 + 64*x^3*Lo
g[1 - E^((-2*I)*x)] - 192*x*Log[1 + E^((2*I)*x)] - 64*x^3*Log[1 + E^((2*I)*
x)] + (96*I)*x^2*PolyLog[2, E^((-2*I)*x)] + (96*I)*(1 + x^2)*PolyLog[2, -E^
((2*I)*x)] + 96*x*PolyLog[3, E^((-2*I)*x)] - 96*x*PolyLog[3, -E^((2*I)*x)]
- (48*I)*PolyLog[4, E^((-2*I)*x)] - (48*I)*PolyLog[4, -E^((2*I)*x)] + 32*x^
3*Sec[x]^2 - 96*x^2*Tan[x]))/64
```

fricas [C] time = 1.08, size = 736, normalized size = 2.07

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*csc(x)*sec(x)*(a*sec(x)^4)^(1/2),x, algorithm="fricas")
```

```
[Out] 3*x*sqrt(a/cos(x)^4)*cos(x)^2*polylog(3, cos(x) + I*sin(x)) + 3*x*sqrt(a/co
s(x)^4)*cos(x)^2*polylog(3, cos(x) - I*sin(x)) - 3*x*sqrt(a/cos(x)^4)*cos(x
)^2*polylog(3, I*cos(x) + sin(x)) - 3*x*sqrt(a/cos(x)^4)*cos(x)^2*polylog(3
, I*cos(x) - sin(x)) - 3*x*sqrt(a/cos(x)^4)*cos(x)^2*polylog(3, -I*cos(x) +
 sin(x)) - 3*x*sqrt(a/cos(x)^4)*cos(x)^2*polylog(3, -I*cos(x) - sin(x)) + 3
*x*sqrt(a/cos(x)^4)*cos(x)^2*polylog(3, -cos(x) + I*sin(x)) + 3*x*sqrt(a/co
s(x)^4)*cos(x)^2*polylog(3, -cos(x) - I*sin(x)) + 3*I*sqrt(a/cos(x)^4)*cos(
x)^2*polylog(4, cos(x) + I*sin(x)) - 3*I*sqrt(a/cos(x)^4)*cos(x)^2*polylog(
4, cos(x) - I*sin(x)) + 3*I*sqrt(a/cos(x)^4)*cos(x)^2*polylog(4, I*cos(x) +
 sin(x)) - 3*I*sqrt(a/cos(x)^4)*cos(x)^2*polylog(4, I*cos(x) - sin(x)) - 3*
I*sqrt(a/cos(x)^4)*cos(x)^2*polylog(4, -I*cos(x) + sin(x)) + 3*I*sqrt(a/cos
(x)^4)*cos(x)^2*polylog(4, -I*cos(x) - sin(x)) - 3*I*sqrt(a/cos(x)^4)*cos(x
)^2*polylog(4, -cos(x) + I*sin(x)) + 3*I*sqrt(a/cos(x)^4)*cos(x)^2*polylog(
4, -cos(x) - I*sin(x)) + 1/2*(x^3*cos(x)^2*log(cos(x) + I*sin(x) + 1) + x^3
*cos(x)^2*log(cos(x) - I*sin(x) + 1) + x^3*cos(x)^2*log(-cos(x) + I*sin(x)
+ 1) + x^3*cos(x)^2*log(-cos(x) - I*sin(x) + 1) - 3*I*x^2*cos(x)^2*dilog(co
s(x) + I*sin(x)) + 3*I*x^2*cos(x)^2*dilog(cos(x) - I*sin(x)) + 3*I*x^2*cos(
x)^2*dilog(-cos(x) + I*sin(x)) - 3*I*x^2*cos(x)^2*dilog(-cos(x) - I*sin(x))
 + (-3*I*x^2 - 3*I)*cos(x)^2*dilog(I*cos(x) + sin(x)) + (3*I*x^2 + 3*I)*cos
(x)^2*dilog(I*cos(x) - sin(x)) + (3*I*x^2 + 3*I)*cos(x)^2*dilog(-I*cos(x) +
 sin(x)) + (-3*I*x^2 - 3*I)*cos(x)^2*dilog(-I*cos(x) - sin(x)) - (x^3 + 3*x
)*cos(x)^2*log(I*cos(x) + sin(x) + 1) - (x^3 + 3*x)*cos(x)^2*log(I*cos(x) -
 sin(x) + 1) - (x^3 + 3*x)*cos(x)^2*log(-I*cos(x) + sin(x) + 1) - (x^3 + 3*
x)*cos(x)^2*log(-I*cos(x) - sin(x) + 1) - 3*x^2*cos(x)*sin(x) + x^3)*sqrt(a
/cos(x)^4)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a \sec(x)^4}\, x^3 \csc(x) \sec(x)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*csc(x)*sec(x)*(a*sec(x)^4)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(a*sec(x)^4)*x^3*csc(x)*sec(x), x)
```

maple [A] time = 0.21, size = 324, normalized size = 0.91

$$\sqrt{\frac{a\,\mathrm{e}^{4ix}}{\left(\mathrm{e}^{2ix}+1\right)^4}}\; x^2 \left(2x-3i-3i\mathrm{e}^{-2ix}\right) - 2i\sqrt{\frac{a\,\mathrm{e}^{4ix}}{\left(\mathrm{e}^{2ix}+1\right)^4}}\; \left(\mathrm{e}^{2ix}+1\right)^2 \left(-\frac{3\,\mathrm{e}^{-2ix}x^2}{2} - \frac{3i\mathrm{e}^{-2ix}x\ln\left(\mathrm{e}^{2ix}+1\right)}{2} - \frac{3\,\mathrm{e}^{-2ix}\,\mathrm{p}\ldots}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*csc(x)*sec(x)*(a*sec(x)^4)^(1/2),x)
```

```
[Out] (a*exp(4*I*x)/(exp(2*I*x)+1)^4)^(1/2)*x^2*(2*x-3*I-3*I*exp(-2*I*x))-2*I*(a*
exp(4*I*x)/(exp(2*I*x)+1)^4)^(1/2)*(exp(2*I*x)+1)^2*(-3/2*exp(-2*I*x)*x^2-3
/2*I*exp(-2*I*x)*x*ln(exp(2*I*x)+1)-3/4*exp(-2*I*x)*polylog(2,-exp(2*I*x))+
1/2*I*exp(-2*I*x)*x^3*ln(1+exp(I*x))+3/2*exp(-2*I*x)*x^2*polylog(2,-exp(I*x
))+3*I*exp(-2*I*x)*x*polylog(3,-exp(I*x))-3*exp(-2*I*x)*polylog(4,-exp(I*x)
)-1/2*I*exp(-2*I*x)*x^3*ln(exp(2*I*x)+1)-3/4*exp(-2*I*x)*x^2*polylog(2,-exp
(2*I*x))-3/4*I*exp(-2*I*x)*x*polylog(3,-exp(2*I*x))+3/8*exp(-2*I*x)*polylog
(4,-exp(2*I*x))+1/2*I*exp(-2*I*x)*x^3*ln(1-exp(I*x))+3/2*exp(-2*I*x)*x^2*po
lylog(2,exp(I*x))+3*I*exp(-2*I*x)*x*polylog(3,exp(I*x))-3*exp(-2*I*x)*polyl
og(4,exp(I*x)))
```

maxima [B] time = 1.40, size = 870, normalized size = 2.44

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*csc(x)*sec(x)*(a*sec(x)^4)^(1/2),x, algorithm="maxima")
```

```
[Out] (18*x^2*cos(4*x) + 18*I*x^2*sin(4*x) - (8*x^3 + 2*(4*x^3 + 9*x)*cos(4*x) +
4*(4*x^3 + 9*x)*cos(2*x) + (8*I*x^3 + 18*I*x)*sin(4*x) + (16*I*x^3 + 36*I*x
)*sin(2*x) + 18*x)*arctan2(sin(2*x), cos(2*x) + 1) + (6*x^3*cos(4*x) + 12*x
^3*cos(2*x) + 6*I*x^3*sin(4*x) + 12*I*x^3*sin(2*x) + 6*x^3)*arctan2(sin(x),
 cos(x) + 1) - (6*x^3*cos(4*x) + 12*x^3*cos(2*x) + 6*I*x^3*sin(4*x) + 12*I*
x^3*sin(2*x) + 6*x^3)*arctan2(sin(x), -cos(x) + 1) - (12*I*x^3 - 18*x^2)*co
s(2*x) + (12*x^2 + 3*(4*x^2 + 3)*cos(4*x) + 6*(4*x^2 + 3)*cos(2*x) - (-12*I
*x^2 - 9*I)*sin(4*x) - (-24*I*x^2 - 18*I)*sin(2*x) + 9)*dilog(-e^(2*I*x)) -
 (18*x^2*cos(4*x) + 36*x^2*cos(2*x) + 18*I*x^2*sin(4*x) + 36*I*x^2*sin(2*x)
 + 18*x^2)*dilog(-e^(I*x)) - (18*x^2*cos(4*x) + 36*x^2*cos(2*x) + 18*I*x^2*
sin(4*x) + 36*I*x^2*sin(2*x) + 18*x^2)*dilog(e^(I*x)) - (-4*I*x^3 + (-4*I*x
^3 - 9*I*x)*cos(4*x) + (-8*I*x^3 - 18*I*x)*cos(2*x) + (4*x^3 + 9*x)*sin(4*x
) + 2*(4*x^3 + 9*x)*sin(2*x) - 9*I*x)*log(cos(2*x)^2 + sin(2*x)^2 + 2*cos(2
*x) + 1) - (3*I*x^3*cos(4*x) + 6*I*x^3*cos(2*x) - 3*x^3*sin(4*x) - 6*x^3*si
n(2*x) + 3*I*x^3)*log(cos(x)^2 + sin(x)^2 + 2*cos(x) + 1) - (3*I*x^3*cos(4*
x) + 6*I*x^3*cos(2*x) - 3*x^3*sin(4*x) - 6*x^3*sin(2*x) + 3*I*x^3)*log(cos(
x)^2 + sin(x)^2 - 2*cos(x) + 1) - (6*cos(4*x) + 12*cos(2*x) + 6*I*sin(4*x)
+ 12*I*sin(2*x) + 6)*polylog(4, -e^(2*I*x)) + (36*cos(4*x) + 72*cos(2*x) +
36*I*sin(4*x) + 72*I*sin(2*x) + 36)*polylog(4, -e^(I*x)) + (36*cos(4*x) + 7
2*cos(2*x) + 36*I*sin(4*x) + 72*I*sin(2*x) + 36)*polylog(4, e^(I*x)) - (-12
*I*x*cos(4*x) - 24*I*x*cos(2*x) + 12*x*sin(4*x) + 24*x*sin(2*x) - 12*I*x)*p
olylog(3, -e^(2*I*x)) - (36*I*x*cos(4*x) + 72*I*x*cos(2*x) - 36*x*sin(4*x)
- 72*x*sin(2*x) + 36*I*x)*polylog(3, -e^(I*x)) - (36*I*x*cos(4*x) + 72*I*x*
cos(2*x) - 36*x*sin(4*x) - 72*x*sin(2*x) + 36*I*x)*polylog(3, e^(I*x)) + 6*
(2*x^3 + 3*I*x^2)*sin(2*x))*sqrt(a)/(-6*I*cos(4*x) - 12*I*cos(2*x) + 6*sin(
4*x) + 12*sin(2*x) - 6*I)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^3 \sqrt{\frac{a}{\cos(x)^4}}}{\cos(x)\ \sin(x)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(a/cos(x)^4)^(1/2))/(cos(x)*sin(x)),x)
[Out] int((x^3*(a/cos(x)^4)^(1/2))/(cos(x)*sin(x)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^3 \sqrt{a \sec^4{(x)}} \csc{(x)} \sec{(x)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*csc(x)*sec(x)*(a*sec(x)**4)**(1/2),x)
[Out] Integral(x**3*sqrt(a*sec(x)**4)*csc(x)*sec(x), x)
```

3.880 $\int \sin(x)\sin(2x)\sin(3x)\,dx$

Optimal. Leaf size=25

$$-\frac{1}{8}\cos(2x)-\frac{1}{16}\cos(4x)+\frac{1}{24}\cos(6x)$$

```
[Out] -1/8*cos(2*x)-1/16*cos(4*x)+1/24*cos(6*x)
```

Rubi [A] time = 0.03, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {4355, 2638}

$$-\frac{1}{8}\cos(2x)-\frac{1}{16}\cos(4x)+\frac{1}{24}\cos(6x)$$

Antiderivative was successfully verified.

```
[In] Int[Sin[x]*Sin[2*x]*Sin[3*x],x]
```

```
[Out] -Cos[2*x]/8 - Cos[4*x]/16 + Cos[6*x]/24
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 4355

```
Int[(F_)[(a_.) + (b_.)*(x_)]^(p_.)*(G_)[(c_.) + (d_.)*(x_)]^(q_.)*(H_)[(e_.
) + (f_.)*(x_)]^(r_.), x_Symbol] :> Int[ExpandTrigReduce[ActivateTrig[F[a +
 b*x]^p*G[c + d*x]^q*H[e + f*x]^r], x], x] /; FreeQ[{a, b, c, d, e, f}, x]
&& (EqQ[F, sin] || EqQ[F, cos]) && (EqQ[G, sin] || EqQ[G, cos]) && (EqQ[H,
sin] || EqQ[H, cos]) && IGtQ[p, 0] && IGtQ[q, 0] && IGtQ[r, 0]
```

Rubi steps

$$\begin{aligned}\int \sin(x)\sin(2x)\sin(3x)\,dx &= \int \left(\frac{1}{4}\sin(2x)+\frac{1}{4}\sin(4x)-\frac{1}{4}\sin(6x)\right)dx\\ &= \frac{1}{4}\int \sin(2x)\,dx+\frac{1}{4}\int \sin(4x)\,dx-\frac{1}{4}\int \sin(6x)\,dx\\ &= -\frac{1}{8}\cos(2x)-\frac{1}{16}\cos(4x)+\frac{1}{24}\cos(6x)\end{aligned}$$

Mathematica [A] time = 0.01, size = 25, normalized size = 1.00

$$-\frac{1}{8}\cos(2x)-\frac{1}{16}\cos(4x)+\frac{1}{24}\cos(6x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[x]*Sin[2*x]*Sin[3*x],x]
[Out] -1/8*Cos[2*x] - Cos[4*x]/16 + Cos[6*x]/24
```

fricas [A] time = 0.91, size = 17, normalized size = 0.68

$$\frac{4}{3}\cos(x)^6-\frac{5}{2}\cos(x)^4+\cos(x)^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)*sin(2*x)*sin(3*x),x, algorithm="fricas")
[Out] 4/3*cos(x)^6 - 5/2*cos(x)^4 + cos(x)^2
```

giac [A] time = 0.12, size = 13, normalized size = 0.52

$$-\frac{4}{3}\sin(x)^6+\frac{3}{2}\sin(x)^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)*sin(2*x)*sin(3*x),x, algorithm="giac")
[Out] -4/3*sin(x)^6 + 3/2*sin(x)^4
```

maple [A] time = 0.13, size = 20, normalized size = 0.80

$$-\frac{\cos(2x)}{8}-\frac{\cos(4x)}{16}+\frac{\cos(6x)}{24}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)*sin(2*x)*sin(3*x),x)
[Out] -1/8*cos(2*x)-1/16*cos(4*x)+1/24*cos(6*x)
```

maxima [A] time = 0.32, size = 19, normalized size = 0.76

$$\frac{1}{24}\cos(6x)-\frac{1}{16}\cos(4x)-\frac{1}{8}\cos(2x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)*sin(2*x)*sin(3*x),x, algorithm="maxima")
```

```
[Out] 1/24*cos(6*x) - 1/16*cos(4*x) - 1/8*cos(2*x)
```

mupad [B] time = 2.94, size = 14, normalized size = 0.56

$$-\frac{\sin(x)^4\,\left(8\sin(x)^2-9\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(2*x)*sin(3*x)*sin(x),x)
```

```
[Out] -(sin(x)^4*(8*sin(x)^2 - 9))/6
```

sympy [B] time = 10.55, size = 114, normalized size = 4.56

$$\frac{x\sin(x)\sin(2x)\sin(3x)}{4}+\frac{x\sin(x)\cos(2x)\cos(3x)}{4}+\frac{x\sin(2x)\cos(x)\cos(3x)}{4}-\frac{x\sin(3x)\cos(x)\cos(2x)}{4}-\frac{3\sin}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)*sin(2*x)*sin(3*x),x)
```

```
[Out] x*sin(x)*sin(2*x)*sin(3*x)/4 + x*sin(x)*cos(2*x)*cos(3*x)/4 + x*sin(2*x)*co
s(x)*cos(3*x)/4 - x*sin(3*x)*cos(x)*cos(2*x)/4 - 3*sin(x)*sin(2*x)*cos(3*x)
/8 + sin(x)*sin(3*x)*cos(2*x)/6 + sin(2*x)*sin(3*x)*cos(x)/24
```

3.881 $\int \cos(x)\cos(2x)\cos(3x)\,dx$

Optimal. Leaf size=30

$$\frac{x}{4}+\frac{1}{8}\sin(2x)+\frac{1}{16}\sin(4x)+\frac{1}{24}\sin(6x)$$

```
[Out] 1/4*x+1/8*sin(2*x)+1/16*sin(4*x)+1/24*sin(6*x)
```

Rubi [A] time = 0.03, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {4355, 2637}

$$\frac{x}{4}+\frac{1}{8}\sin(2x)+\frac{1}{16}\sin(4x)+\frac{1}{24}\sin(6x)$$

Antiderivative was successfully verified.

```
[In] Int[Cos[x]*Cos[2*x]*Cos[3*x],x]
```

```
[Out] x/4 + Sin[2*x]/8 + Sin[4*x]/16 + Sin[6*x]/24
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 4355

```
Int[(F_)[(a_.) + (b_.)*(x_)]^(p_.)*(G_)[(c_.) + (d_.)*(x_)]^(q_.)*(H_)[(e_.
) + (f_.)*(x_)]^(r_.), x_Symbol] :> Int[ExpandTrigReduce[ActivateTrig[F[a +
 b*x]^p*G[c + d*x]^q*H[e + f*x]^r], x], x] /; FreeQ[{a, b, c, d, e, f}, x]
&& (EqQ[F, sin] || EqQ[F, cos]) && (EqQ[G, sin] || EqQ[G, cos]) && (EqQ[H,
sin] || EqQ[H, cos]) && IGtQ[p, 0] && IGtQ[q, 0] && IGtQ[r, 0]
```

Rubi steps

$$\begin{aligned}
\int \cos(x)\cos(2x)\cos(3x)\,dx &= \int \left(\frac{1}{4}+\frac{1}{4}\cos(2x)+\frac{1}{4}\cos(4x)+\frac{1}{4}\cos(6x)\right)dx\\
&= \frac{x}{4}+\frac{1}{4}\int \cos(2x)\,dx+\frac{1}{4}\int \cos(4x)\,dx+\frac{1}{4}\int \cos(6x)\,dx\\
&= \frac{x}{4}+\frac{1}{8}\sin(2x)+\frac{1}{16}\sin(4x)+\frac{1}{24}\sin(6x)
\end{aligned}$$

Mathematica [A] time = 0.01, size = 30, normalized size = 1.00

$$\frac{x}{4}+\frac{1}{8}\sin(2x)+\frac{1}{16}\sin(4x)+\frac{1}{24}\sin(6x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[x]*Cos[2*x]*Cos[3*x],x]
[Out] x/4 + Sin[2*x]/8 + Sin[4*x]/16 + Sin[6*x]/24
```

fricas [A] time = 0.87, size = 25, normalized size = 0.83

$$\frac{1}{12}\left(16\cos(x)^5-10\cos(x)^3+3\cos(x)\right)\sin(x)+\frac{1}{4}x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*cos(2*x)*cos(3*x),x, algorithm="fricas")
[Out] 1/12*(16*cos(x)^5 - 10*cos(x)^3 + 3*cos(x))*sin(x) + 1/4*x
```

giac [A] time = 0.15, size = 22, normalized size = 0.73

$$\frac{1}{4}x+\frac{1}{24}\sin(6x)+\frac{1}{16}\sin(4x)+\frac{1}{8}\sin(2x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*cos(2*x)*cos(3*x),x, algorithm="giac")
[Out] 1/4*x + 1/24*sin(6*x) + 1/16*sin(4*x) + 1/8*sin(2*x)
```

maple [A] time = 0.14, size = 23, normalized size = 0.77

$$\frac{x}{4}+\frac{\sin(2x)}{8}+\frac{\sin(4x)}{16}+\frac{\sin(6x)}{24}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)*cos(2*x)*cos(3*x),x)
[Out] 1/4*x+1/8*sin(2*x)+1/16*sin(4*x)+1/24*sin(6*x)
```

maxima [A] time = 0.31, size = 22, normalized size = 0.73

$$\frac{1}{4}x+\frac{1}{24}\sin(6x)+\frac{1}{16}\sin(4x)+\frac{1}{8}\sin(2x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*cos(2*x)*cos(3*x),x, algorithm="maxima")
```

```
[Out] 1/4*x + 1/24*sin(6*x) + 1/16*sin(4*x) + 1/8*sin(2*x)
```

mupad [B] time = 3.04, size = 22, normalized size = 0.73

$$\frac{x}{4} + \frac{\sin(2x)}{8} + \frac{\sin(4x)}{16} + \frac{\sin(6x)}{24}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(2*x)*cos(3*x)*cos(x),x)
```

```
[Out] x/4 + sin(2*x)/8 + sin(4*x)/16 + sin(6*x)/24
```

sympy [B] time = 10.54, size = 116, normalized size = 3.87

$$-\frac{x\sin(x)\sin(2x)\cos(3x)}{4} + \frac{x\sin(x)\sin(3x)\cos(2x)}{4} + \frac{x\sin(2x)\sin(3x)\cos(x)}{4} + \frac{x\cos(x)\cos(2x)\cos(3x)}{4} + \frac{3}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*cos(2*x)*cos(3*x),x)
```

```
[Out] -x*sin(x)*sin(2*x)*cos(3*x)/4 + x*sin(x)*sin(3*x)*cos(2*x)/4 + x*sin(2*x)*s
in(3*x)*cos(x)/4 + x*cos(x)*cos(2*x)*cos(3*x)/4 + 3*sin(x)*sin(2*x)*sin(3*x
)/8 + sin(x)*cos(2*x)*cos(3*x)/3 + 5*sin(2*x)*cos(x)*cos(3*x)/24
```

3.882 $\int \cos(x)\sin(2x)\sin(3x)\,dx$

Optimal. Leaf size=30

$$\frac{x}{4}+\frac{1}{8}\sin(2x)-\frac{1}{16}\sin(4x)-\frac{1}{24}\sin(6x)$$

```
[Out] 1/4*x+1/8*sin(2*x)-1/16*sin(4*x)-1/24*sin(6*x)
```

Rubi [A] time = 0.03, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {4355, 2637}

$$\frac{x}{4}+\frac{1}{8}\sin(2x)-\frac{1}{16}\sin(4x)-\frac{1}{24}\sin(6x)$$

Antiderivative was successfully verified.

```
[In] Int[Cos[x]*Sin[2*x]*Sin[3*x],x]
```

```
[Out] x/4 + Sin[2*x]/8 - Sin[4*x]/16 - Sin[6*x]/24
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 4355

```
Int[(F_)[(a_.) + (b_.)*(x_)]^(p_.)*(G_)[(c_.) + (d_.)*(x_)]^(q_.)*(H_)[(e_.
) + (f_.)*(x_)]^(r_.), x_Symbol] :> Int[ExpandTrigReduce[ActivateTrig[F[a +
 b*x]^p*G[c + d*x]^q*H[e + f*x]^r], x], x] /; FreeQ[{a, b, c, d, e, f}, x]
&& (EqQ[F, sin] || EqQ[F, cos]) && (EqQ[G, sin] || EqQ[G, cos]) && (EqQ[H,
sin] || EqQ[H, cos]) && IGtQ[p, 0] && IGtQ[q, 0] && IGtQ[r, 0]
```

Rubi steps

$$\begin{aligned}\int \cos(x)\sin(2x)\sin(3x)\,dx &= \int \left(\frac{1}{4}+\frac{1}{4}\cos(2x)-\frac{1}{4}\cos(4x)-\frac{1}{4}\cos(6x)\right)dx\\ &= \frac{x}{4}+\frac{1}{4}\int \cos(2x)\,dx-\frac{1}{4}\int \cos(4x)\,dx-\frac{1}{4}\int \cos(6x)\,dx\\ &= \frac{x}{4}+\frac{1}{8}\sin(2x)-\frac{1}{16}\sin(4x)-\frac{1}{24}\sin(6x)\end{aligned}$$

Mathematica [A] time = 0.01, size = 30, normalized size = 1.00

$$\frac{x}{4}+\frac{1}{8}\sin(2x)-\frac{1}{16}\sin(4x)-\frac{1}{24}\sin(6x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[x]*Sin[2*x]*Sin[3*x],x]
[Out] x/4 + Sin[2*x]/8 - Sin[4*x]/16 - Sin[6*x]/24
```

fricas [A] time = 0.71, size = 25, normalized size = 0.83

$$-\frac{1}{12}\left(16\cos(x)^{5}-10\cos(x)^{3}-3\cos(x)\right)\sin(x)+\frac{1}{4}x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*sin(2*x)*sin(3*x),x, algorithm="fricas")
[Out] -1/12*(16*cos(x)^5 - 10*cos(x)^3 - 3*cos(x))*sin(x) + 1/4*x
```

giac [A] time = 0.13, size = 22, normalized size = 0.73

$$\frac{1}{4}x-\frac{1}{24}\sin(6x)-\frac{1}{16}\sin(4x)+\frac{1}{8}\sin(2x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*sin(2*x)*sin(3*x),x, algorithm="giac")
[Out] 1/4*x - 1/24*sin(6*x) - 1/16*sin(4*x) + 1/8*sin(2*x)
```

maple [A] time = 0.09, size = 23, normalized size = 0.77

$$\frac{x}{4}+\frac{\sin(2x)}{8}-\frac{\sin(4x)}{16}-\frac{\sin(6x)}{24}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)*sin(2*x)*sin(3*x),x)
[Out] 1/4*x+1/8*sin(2*x)-1/16*sin(4*x)-1/24*sin(6*x)
```

maxima [A] time = 0.32, size = 22, normalized size = 0.73

$$\frac{1}{4}x-\frac{1}{24}\sin(6x)-\frac{1}{16}\sin(4x)+\frac{1}{8}\sin(2x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*sin(2*x)*sin(3*x),x, algorithm="maxima")
```

```
[Out] 1/4*x - 1/24*sin(6*x) - 1/16*sin(4*x) + 1/8*sin(2*x)
```

mupad [B] time = 3.01, size = 22, normalized size = 0.73

$$\frac{x}{4}+\frac{\sin\left(2\,x\right)}{8}-\frac{\sin\left(4\,x\right)}{16}-\frac{\sin\left(6\,x\right)}{24}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(2*x)*sin(3*x)*cos(x),x)
```

```
[Out] x/4 + sin(2*x)/8 - sin(4*x)/16 - sin(6*x)/24
```

sympy [B] time = 10.41, size = 114, normalized size = 3.80

$$-\frac{x\sin(x)\sin(2x)\cos(3x)}{4}+\frac{x\sin(x)\sin(3x)\cos(2x)}{4}+\frac{x\sin(2x)\sin(3x)\cos(x)}{4}+\frac{x\cos(x)\cos(2x)\cos(3x)}{4}+\text{si}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*sin(2*x)*sin(3*x),x)
```

```
[Out] -x*sin(x)*sin(2*x)*cos(3*x)/4 + x*sin(x)*sin(3*x)*cos(2*x)/4 + x*sin(2*x)*s
in(3*x)*cos(x)/4 + x*cos(x)*cos(2*x)*cos(3*x)/4 + sin(x)*sin(2*x)*sin(3*x)/
8 + sin(x)*cos(2*x)*cos(3*x)/6 - 5*sin(2*x)*cos(x)*cos(3*x)/24
```

3.883 $\int \cos(2x)\cos(3x)\sin(x)\,dx$

Optimal. Leaf size=25

$$-\frac{1}{8}\cos(2x)+\frac{1}{16}\cos(4x)-\frac{1}{24}\cos(6x)$$

```
[Out] -1/8*cos(2*x)+1/16*cos(4*x)-1/24*cos(6*x)
```

Rubi [A] time = 0.03, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {4355, 2638}

$$-\frac{1}{8}\cos(2x)+\frac{1}{16}\cos(4x)-\frac{1}{24}\cos(6x)$$

Antiderivative was successfully verified.

```
[In] Int[Cos[2*x]*Cos[3*x]*Sin[x],x]
```

```
[Out] -Cos[2*x]/8 + Cos[4*x]/16 - Cos[6*x]/24
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 4355

```
Int[(F_)[(a_.) + (b_.)*(x_)]^(p_.)*(G_)[(c_.) + (d_.)*(x_)]^(q_.)*(H_)[(e_.
) + (f_.)*(x_)]^(r_.), x_Symbol] :> Int[ExpandTrigReduce[ActivateTrig[F[a +
 b*x]^p*G[c + d*x]^q*H[e + f*x]^r], x], x] /; FreeQ[{a, b, c, d, e, f}, x]
&& (EqQ[F, sin] || EqQ[F, cos]) && (EqQ[G, sin] || EqQ[G, cos]) && (EqQ[H,
sin] || EqQ[H, cos]) && IGtQ[p, 0] && IGtQ[q, 0] && IGtQ[r, 0]
```

Rubi steps

$$\begin{aligned}\int \cos(2x)\cos(3x)\sin(x)\,dx &= \int \left(\frac{1}{4}\sin(2x)-\frac{1}{4}\sin(4x)+\frac{1}{4}\sin(6x)\right)dx\\ &= \frac{1}{4}\int \sin(2x)\,dx-\frac{1}{4}\int \sin(4x)\,dx+\frac{1}{4}\int \sin(6x)\,dx\\ &= -\frac{1}{8}\cos(2x)+\frac{1}{16}\cos(4x)-\frac{1}{24}\cos(6x)\end{aligned}$$

Mathematica [A] time = 0.01, size = 25, normalized size = 1.00

$$-\frac{1}{8}\cos(2x)+\frac{1}{16}\cos(4x)-\frac{1}{24}\cos(6x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[2*x]*Cos[3*x]*Sin[x],x]
[Out] -1/8*Cos[2*x] + Cos[4*x]/16 - Cos[6*x]/24
```

fricas [A] time = 0.88, size = 19, normalized size = 0.76

$$-\frac{4}{3}\cos(x)^6+\frac{5}{2}\cos(x)^4-\frac{3}{2}\cos(x)^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(2*x)*cos(3*x)*sin(x),x, algorithm="fricas")
[Out] -4/3*cos(x)^6 + 5/2*cos(x)^4 - 3/2*cos(x)^2
```

giac [A] time = 0.13, size = 19, normalized size = 0.76

$$\frac{4}{3}\sin(x)^6-\frac{3}{2}\sin(x)^4+\frac{1}{2}\sin(x)^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(2*x)*cos(3*x)*sin(x),x, algorithm="giac")
[Out] 4/3*sin(x)^6 - 3/2*sin(x)^4 + 1/2*sin(x)^2
```

maple [A] time = 0.08, size = 20, normalized size = 0.80

$$-\frac{\cos(2x)}{8}+\frac{\cos(4x)}{16}-\frac{\cos(6x)}{24}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(2*x)*cos(3*x)*sin(x),x)
[Out] -1/8*cos(2*x)+1/16*cos(4*x)-1/24*cos(6*x)
```

maxima [A] time = 0.31, size = 19, normalized size = 0.76

$$-\frac{1}{24}\cos(6x)+\frac{1}{16}\cos(4x)-\frac{1}{8}\cos(2x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(2*x)*cos(3*x)*sin(x),x, algorithm="maxima")
```

```
[Out] -1/24*cos(6*x) + 1/16*cos(4*x) - 1/8*cos(2*x)
```

mupad [B] time = 3.18, size = 19, normalized size = 0.76

$$\frac{4\sin(x)^6}{3} - \frac{3\sin(x)^4}{2} + \frac{\sin(x)^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(2*x)*cos(3*x)*sin(x),x)
```

```
[Out] sin(x)^2/2 - (3*sin(x)^4)/2 + (4*sin(x)^6)/3
```

sympy [B] time = 10.45, size = 112, normalized size = 4.48

$$\frac{x\sin(x)\sin(2x)\sin(3x)}{4} + \frac{x\sin(x)\cos(2x)\cos(3x)}{4} + \frac{x\sin(2x)\cos(x)\cos(3x)}{4} - \frac{x\sin(3x)\cos(x)\cos(2x)}{4} - \sin$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(2*x)*cos(3*x)*sin(x),x)
```

```
[Out] x*sin(x)*sin(2*x)*sin(3*x)/4 + x*sin(x)*cos(2*x)*cos(3*x)/4 + x*sin(2*x)*co
s(x)*cos(3*x)/4 - x*sin(3*x)*cos(x)*cos(2*x)/4 - sin(x)*sin(2*x)*cos(3*x)/8
 + sin(x)*sin(3*x)*cos(2*x)/3 - sin(2*x)*sin(3*x)*cos(x)/24
```

3.884 $\int x \sin\left(x^2\right) dx$

Optimal. Leaf size=8

$$-\frac{1}{2}\cos\left(x^2\right)$$

```
[Out] -1/2*cos(x^2)
```

Rubi [A] time = 0.01, antiderivative size = 8, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3379, 2638}

$$-\frac{1}{2}\cos\left(x^2\right)$$

Antiderivative was successfully verified.

```
[In] Int[x*Sin[x^2],x]
```

```
[Out] -Cos[x^2]/2
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3379

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rubi steps

$$\begin{aligned}\int x \sin\left(x^2\right) dx &= \frac{1}{2}\text{Subst}\left(\int \sin(x)\, dx, x, x^2\right)\\ &= -\frac{1}{2}\cos\left(x^2\right)\end{aligned}$$

Mathematica [A] time = 0.01, size = 8, normalized size = 1.00

$$-\frac{1}{2}\cos\left(x^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Sin[x^2],x]
```

```
[Out] -1/2*Cos[x^2]
```

fricas [A] time = 0.83, size = 6, normalized size = 0.75

$$-\frac{1}{2}\cos\left(x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(x^2),x, algorithm="fricas")
```

```
[Out] -1/2*cos(x^2)
```

giac [A] time = 0.13, size = 6, normalized size = 0.75

$$-\frac{1}{2}\cos\left(x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(x^2),x, algorithm="giac")
```

```
[Out] -1/2*cos(x^2)
```

maple [A] time = 0.00, size = 7, normalized size = 0.88

$$-\frac{\cos\left(x^2\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*sin(x^2),x)
```

```
[Out] -1/2*cos(x^2)
```

maxima [A] time = 0.31, size = 6, normalized size = 0.75

$$-\frac{1}{2}\cos\left(x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(x^2),x, algorithm="maxima")
```

```
[Out] -1/2*cos(x^2)
```

mupad [B] time = 0.05, size = 6, normalized size = 0.75

$$-\frac{\cos\left(x^2\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*sin(x^2),x)
```

```
[Out] -cos(x^2)/2
```

sympy [A] time = 0.16, size = 7, normalized size = 0.88

$$-\frac{\cos\left(x^2\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(x**2),x)
```

```
[Out] -cos(x**2)/2
```

3.885 $\int (-\cos(x)+\sin(x))(\cos(x)+\sin(x))^5\,dx$

Optimal. Leaf size=11

$$-\frac{1}{6}(\sin(x)+\cos(x))^6$$

```
[Out] -1/6*(cos(x)+sin(x))^6
```

Rubi [A] time = 0.02, antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {3145}

$$-\frac{1}{6}(\sin(x)+\cos(x))^6$$

Antiderivative was successfully verified.

```
[In] Int[(-Cos[x] + Sin[x])*(Cos[x] + Sin[x])^5,x]
```

```
[Out] -(Cos[x] + Sin[x])^6/6
```

Rule 3145

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)])^(n_.)*(
cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)]), x_Symbol] :
> Simp[((c*B - b*C)*(b*Cos[d + e*x] + c*Sin[d + e*x])^(n + 1))/(e*(n + 1)*(
b^2 + c^2)), x] /; FreeQ[{b, c, d, e, B, C}, x] && NeQ[n, -1] && NeQ[b^2 +
c^2, 0] && EqQ[b*B + c*C, 0]
```

Rubi steps

$$\int (-\cos(x)+\sin(x))(\cos(x)+\sin(x))^5\,dx=-\frac{1}{6}(\cos(x)+\sin(x))^6$$

Mathematica [B] time = 0.08, size = 25, normalized size = 2.27

$$-\frac{5}{8}\sin(2x)+\frac{1}{24}\sin(6x)+\frac{1}{4}\cos(4x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-Cos[x] + Sin[x])*(Cos[x] + Sin[x])^5,x]
```

```
[Out] Cos[4*x]/4 - (5*Sin[2*x])/8 + Sin[6*x]/24
```

fricas [B] time = 0.63, size = 34, normalized size = 3.09

$$2\cos(x)^4 - 2\cos(x)^2 + \frac{1}{3}\left(4\cos(x)^5 - 4\cos(x)^3 - 3\cos(x)\right)\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-cos(x)+sin(x))*(cos(x)+sin(x))^5,x, algorithm="fricas")
```

```
[Out] 2*cos(x)^4 - 2*cos(x)^2 + 1/3*(4*cos(x)^5 - 4*cos(x)^3 - 3*cos(x))*sin(x)
```

giac [B] time = 0.13, size = 19, normalized size = 1.73

$$\frac{1}{4}\cos(4x) + \frac{1}{24}\sin(6x) - \frac{5}{8}\sin(2x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-cos(x)+sin(x))*(cos(x)+sin(x))^5,x, algorithm="giac")
```

```
[Out] 1/4*cos(4*x) + 1/24*sin(6*x) - 5/8*sin(2*x)
```

maple [B] time = 0.08, size = 97, normalized size = 8.82

$$-\frac{\left(\sin^5(x) + \frac{5\left(\sin^3(x)\right)}{4} + \frac{15\sin(x)}{8}\right)\cos(x)}{6} + \frac{2\left(\sin^6(x)\right)}{3} - \frac{5\left(\cos^3(x)\right)\left(\sin^3(x)\right)}{6} - \frac{5\left(\cos^3(x)\right)\sin(x)}{8} + \frac{5\cos(x)\sin(x)}{16} +$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-cos(x)+sin(x))*(cos(x)+sin(x))^5,x)
```

```
[Out] -1/6*(sin(x)^5+5/4*sin(x)^3+15/8*sin(x))*cos(x)+2/3*sin(x)^6-5/6*cos(x)^3*s
in(x)^3-5/8*cos(x)^3*sin(x)+5/16*cos(x)*sin(x)+5/6*cos(x)^5*sin(x)-5/24*(co
s(x)^3+3/2*cos(x))*sin(x)+2/3*cos(x)^6-1/6*(cos(x)^5+5/4*cos(x)^3+15/8*cos(
x))*sin(x)
```

maxima [A] time = 0.33, size = 9, normalized size = 0.82

$$-\frac{1}{6}\left(\cos(x) + \sin(x)\right)^6$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-cos(x)+sin(x))*(cos(x)+sin(x))^5,x, algorithm="maxima")
```

```
[Out] -1/6*(cos(x) + sin(x))^6
```

mupad [B] time = 3.19, size = 20, normalized size = 1.82

$$-\frac{\sin \left(2\,x\right)\,\left({\sin \left(2\,x\right)}^2+3\,\sin \left(2\,x\right)+3\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(cos(x) + sin(x))^5*(cos(x) - sin(x)),x)
```

```
[Out] -(sin(2*x)*(3*sin(2*x) + sin(2*x)^2 + 3))/6
```

sympy [B] time = 1.66, size = 46, normalized size = 4.18

$$\frac{2\sin^{6}\left(x\right)}{3}-\sin^{5}\left(x\right)\cos\left(x\right)-\frac{10\sin^{3}\left(x\right)\cos^{3}\left(x\right)}{3}-\sin\left(x\right)\cos^{5}\left(x\right)+\frac{2\cos^{6}\left(x\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-cos(x)+sin(x))*(cos(x)+sin(x))**5,x)
```

```
[Out] 2*sin(x)**6/3 - sin(x)**5*cos(x) - 10*sin(x)**3*cos(x)**3/3 - sin(x)*cos(x)
**5 + 2*cos(x)**6/3
```

3.886 $\int 2x\sec^2(x)\tan(x)\,dx$

Optimal. Leaf size=11

$$x\sec^2(x) - \tan(x)$$

[Out] x*sec(x)^2-tan(x)

Rubi [A] time = 0.02, antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.444, Rules used = {12, 3757, 3767, 8}

$$x\sec^2(x) - \tan(x)$$

Antiderivative was successfully verified.

[In] Int[2*x*Sec[x]^2*Tan[x],x]

[Out] x*Sec[x]^2 - Tan[x]

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3757

```
Int[(x_)^(m_.)*Sec[(a_.) + (b_.)*(x_)^(n_.)]^(p_.)*Tan[(a_.) + (b_.)*(x_)^(
n_.)]^(q_.), x_Symbol] :> Simp[(x^(m - n + 1)*Sec[a + b*x^n]^p)/(b*n*p), x]
 - Dist[(m - n + 1)/(b*n*p), Int[x^(m - n)*Sec[a + b*x^n]^p, x], x] /; Free
Q[{a, b, p}, x] && IntegerQ[n] && GeQ[m, n] && EqQ[q, 1]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\begin{aligned}\int 2x\sec^2(x)\tan(x)\,dx &= 2\int x\sec^2(x)\tan(x)\,dx\\ &= x\sec^2(x)-\int \sec^2(x)\,dx\\ &= x\sec^2(x)+\text{Subst}\left(\int 1\,dx,x,-\tan(x)\right)\\ &= x\sec^2(x)-\tan(x)\end{aligned}$$

Mathematica [A] time = 0.01, size = 18, normalized size = 1.64

$$2\left(\frac{1}{2}x\sec^2(x)-\frac{\tan(x)}{2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[2*x*Sec[x]^2*Tan[x],x]
```

```
[Out] 2*((x*Sec[x]^2)/2 - Tan[x]/2)
```

fricas [A] time = 0.81, size = 15, normalized size = 1.36

$$-\frac{\cos(x)\sin(x)-x}{\cos(x)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(2*x*sec(x)^2*tan(x),x, algorithm="fricas")
```

```
[Out] -(cos(x)*sin(x) - x)/cos(x)^2
```

giac [B] time = 0.13, size = 52, normalized size = 4.73

$$\frac{x\tan\left(\frac{1}{2}x\right)^4+2x\tan\left(\frac{1}{2}x\right)^2+2\tan\left(\frac{1}{2}x\right)^3+x-2\tan\left(\frac{1}{2}x\right)}{\tan\left(\frac{1}{2}x\right)^4-2\tan\left(\frac{1}{2}x\right)^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(2*x*sec(x)^2*tan(x),x, algorithm="giac")
```

```
[Out] (x*tan(1/2*x)^4 + 2*x*tan(1/2*x)^2 + 2*tan(1/2*x)^3 + x - 2*tan(1/2*x))/(ta
n(1/2*x)^4 - 2*tan(1/2*x)^2 + 1)
```

maple [A] time = 0.04, size = 12, normalized size = 1.09

$$\frac{x}{\cos(x)^2} - \tan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(2*x*sec(x)^2*tan(x),x)
```

```
[Out] x/cos(x)^2-tan(x)
```

maxima [B] time = 0.32, size = 133, normalized size = 12.09

$$\frac{2\left(4\,x\cos\left(2\,x\right)^2 + 4\,x\sin\left(2\,x\right)^2 + \left(2\,x\cos\left(2\,x\right) + \sin\left(2\,x\right)\right)\cos\left(4\,x\right) + 2\,x\cos\left(2\,x\right) + \left(2\,x\sin\left(2\,x\right) - \cos\left(2\,x\right) - 1\right)}{2\left(2\,\cos\left(2\,x\right) + 1\right)\cos\left(4\,x\right) + \cos\left(4\,x\right)^2 + 4\,\cos\left(2\,x\right)^2 + \sin\left(4\,x\right)^2 + 4\,\sin\left(4\,x\right)\sin\left(2\,x\right) + 4\,\sin\left(2\,x\right)^2 + 4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(2*x*sec(x)^2*tan(x),x, algorithm="maxima")
```

```
[Out] 2*(4*x*cos(2*x)^2 + 4*x*sin(2*x)^2 + (2*x*cos(2*x) + sin(2*x))*cos(4*x) + 2
*x*cos(2*x) + (2*x*sin(2*x) - cos(2*x) - 1)*sin(4*x) - sin(2*x))/(2*(2*cos(
2*x) + 1)*cos(4*x) + cos(4*x)^2 + 4*cos(2*x)^2 + sin(4*x)^2 + 4*sin(4*x)*si
n(2*x) + 4*sin(2*x)^2 + 4*cos(2*x) + 1)
```

mupad [B] time = 3.09, size = 16, normalized size = 1.45

$$\frac{2\,x - \sin\left(2\,x\right)}{2\cos(x)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x*tan(x))/cos(x)^2,x)
```

```
[Out] (2*x - sin(2*x))/(2*cos(x)^2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$2\int x\tan\left(x\right)\sec^2\left(x\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(2*x*sec(x)**2*tan(x),x)
```

```
[Out] 2*Integral(x*tan(x)*sec(x)**2, x)
```

3.887 $\int \frac{1+\cos^2(x)}{1+\cos(2x)}\,dx$

Optimal. Leaf size=12

$$\frac{x}{2}+\frac{\tan(x)}{2}$$

```
[Out] 1/2*x+1/2*tan(x)
```

Rubi [A] time = 0.05, antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {388, 203}

$$\frac{x}{2}+\frac{\tan(x)}{2}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + Cos[x]^2)/(1 + Cos[2*x]),x]
```

```
[Out] x/2 + Tan[x]/2
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1+\cos^2(x)}{1+\cos(2x)}\,dx &= \text{Subst}\left(\int \frac{2+x^2}{2+2x^2}\,dx, x, \tan(x)\right)\\ &= \frac{\tan(x)}{2}+\text{Subst}\left(\int \frac{1}{2+2x^2}\,dx, x, \tan(x)\right)\\ &= \frac{x}{2}+\frac{\tan(x)}{2}\end{aligned}$$

Mathematica [A] time = 0.02, size = 12, normalized size = 1.00

$$\frac{x}{2}+\frac{\tan(x)}{2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + Cos[x]^2)/(1 + Cos[2*x]),x]
```

```
[Out] x/2 + Tan[x]/2
```

fricas [A] time = 1.09, size = 13, normalized size = 1.08

$$\frac{x\cos(x)+\sin(x)}{2\,\cos(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+cos(x)^2)/(1+cos(2*x)),x, algorithm="fricas")
```

```
[Out] 1/2*(x*cos(x) + sin(x))/cos(x)
```

giac [A] time = 0.15, size = 8, normalized size = 0.67

$$\frac{1}{2}\,x+\frac{1}{2}\,\tan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+cos(x)^2)/(1+cos(2*x)),x, algorithm="giac")
```

```
[Out] 1/2*x + 1/2*tan(x)
```

maple [A] time = 0.12, size = 9, normalized size = 0.75

$$\frac{x}{2}+\frac{\tan(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+cos(x)^2)/(1+cos(2*x)),x)
```

```
[Out] 1/2*x+1/2*tan(x)
```

maxima [B] time = 0.31, size = 18, normalized size = 1.50

$$\frac{1}{2}\,x+\frac{\sin\left(2\,x\right)}{2\left(\cos\left(2\,x\right)+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+cos(x)^2)/(1+cos(2*x)),x, algorithm="maxima")
```

```
[Out] 1/2*x + 1/2*sin(2*x)/(cos(2*x) + 1)
```

mupad [B] time = 2.93, size = 8, normalized size = 0.67

$$\frac{x}{2}+\frac{\tan(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(x)^2 + 1)/(cos(2*x) + 1),x)
```

```
[Out] x/2 + tan(x)/2
```

sympy [A] time = 1.41, size = 7, normalized size = 0.58

$$\frac{x}{2}+\frac{\tan\left(x\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+cos(x)**2)/(1+cos(2*x)),x)
```

```
[Out] x/2 + tan(x)/2
```

3.888 $\int \frac{\sin(x)}{\cos^3(x)-\cos^5(x)}\,dx$

Optimal. Leaf size=12

$$\frac{\tan^2(x)}{2}+\log(\tan(x))$$

[Out] ln(tan(x))+1/2*tan(x)^2

Rubi [A] time = 0.04, antiderivative size = 17, normalized size of antiderivative = 1.42, number of steps used = 4, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {4335, 266, 44}

$$\frac{\sec^2(x)}{2}+\log(\sin(x))-\log(\cos(x))$$

Antiderivative was successfully verified.

[In] Int[Sin[x]/(Cos[x]^3 - Cos[x]^5),x]

[Out] -Log[Cos[x]] + Log[Sin[x]] + Sec[x]^2/2

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 4335

```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{d = FreeFacto
rs[Cos[c*(a + b*x)], x]}, -Dist[d/(b*c), Subst[Int[SubstFor[1, Cos[c*(a + b
*x)]/d, u, x], x], x, Cos[c*(a + b*x)]/d], x] /; FunctionOfQ[Cos[c*(a + b*x
)]/d, u, x, True]] /; FreeQ[{a, b, c}, x] && (EqQ[F, Sin] || EqQ[F, sin])
```

Rubi steps

$$\int \frac{\sin(x)}{\cos^3(x)-\cos^5(x)}\,dx = -\text{Subst}\left(\int \frac{1}{x^3\left(1-x^2\right)}\,dx, x, \cos(x)\right)$$
$$= -\left(\frac{1}{2}\,\text{Subst}\left(\int \frac{1}{(1-x)x^2}\,dx, x, \cos^2(x)\right)\right)$$
$$= -\left(\frac{1}{2}\,\text{Subst}\left(\int \left(\frac{1}{1-x}+\frac{1}{x^2}+\frac{1}{x}\right)dx, x, \cos^2(x)\right)\right)$$
$$= -\log(\cos(x))+\log(\sin(x))+\frac{\sec^2(x)}{2}$$

Mathematica [A] time = 0.01, size = 17, normalized size = 1.42

$$\frac{\sec^2(x)}{2}+\log(\sin(x))-\log(\cos(x))$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[x]/(Cos[x]^3 - Cos[x]^5),x]
[Out] -Log[Cos[x]] + Log[Sin[x]] + Sec[x]^2/2
```

fricas [B] time = 0.93, size = 33, normalized size = 2.75

$$-\frac{\cos(x)^2\log\left(\cos(x)^2\right)-\cos(x)^2\log\left(-\frac{1}{4}\,\cos(x)^2+\frac{1}{4}\right)-1}{2\,\cos(x)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)/(cos(x)^3-cos(x)^5),x, algorithm="fricas")
[Out] -1/2*(cos(x)^2*log(cos(x)^2) - cos(x)^2*log(-1/4*cos(x)^2 + 1/4) - 1)/cos(x
)^2
```

giac [B] time = 0.14, size = 24, normalized size = 2.00

$$\frac{1}{2\,\cos(x)^2}+\frac{1}{2}\,\log\left(-\cos(x)^2+1\right)-\log\left(|\cos(x)|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)/(cos(x)^3-cos(x)^5),x, algorithm="giac")
[Out] 1/2/cos(x)^2 + 1/2*log(-cos(x)^2 + 1) - log(abs(cos(x)))
```

maple [B] time = 0.08, size = 27, normalized size = 2.25

$$\frac{1}{2\cos(x)^2}-\ln(\cos(x))+\frac{\ln(-1+\cos(x))}{2}+\frac{\ln(1+\cos(x))}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)/(cos(x)^3-cos(x)^5),x)
```

```
[Out] 1/2/cos(x)^2-ln(cos(x))+1/2*ln(-1+cos(x))+1/2*ln(1+cos(x))
```

maxima [B] time = 0.31, size = 26, normalized size = 2.17

$$\frac{1}{2\cos(x)^2}+\frac{1}{2}\log(\cos(x)+1)+\frac{1}{2}\log(\cos(x)-1)-\log(\cos(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)/(cos(x)^3-cos(x)^5),x, algorithm="maxima")
```

```
[Out] 1/2/cos(x)^2 + 1/2*log(cos(x) + 1) + 1/2*log(cos(x) - 1) - log(cos(x))
```

mupad [B] time = 0.09, size = 19, normalized size = 1.58

$$\frac{\ln\left(\sin(x)^2\right)}{2}-\ln(\cos(x))+\frac{1}{2\cos(x)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)/(cos(x)^3 - cos(x)^5),x)
```

```
[Out] log(sin(x)^2)/2 - log(cos(x)) + 1/(2*cos(x)^2)
```

sympy [B] time = 1.49, size = 29, normalized size = 2.42

$$\frac{\log(\cos(x)-1)}{2}+\frac{\log(\cos(x)+1)}{2}-\log(\cos(x))+\frac{1}{2\cos^2(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)/(cos(x)**3-cos(x)**5),x)
```

```
[Out] log(cos(x) - 1)/2 + log(cos(x) + 1)/2 - log(cos(x)) + 1/(2*cos(x)**2)
```

3.889 $\int \sec(x)\left(5-11\sec^5(x)\right)^2\tan(x)\,dx$

Optimal. Leaf size=19

$$11\sec^{11}(x)-\frac{55\sec^6(x)}{3}+25\sec(x)$$

[Out] 25*sec(x)-55/3*sec(x)^6+11*sec(x)^11

Rubi [A] time = 0.04, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {4339, 270}

$$11\sec^{11}(x)-\frac{55\sec^6(x)}{3}+25\sec(x)$$

Antiderivative was successfully verified.

[In] Int[Sec[x]*(5 - 11*Sec[x]^5)^2*Tan[x],x]

[Out] 25*Sec[x] - (55*Sec[x]^6)/3 + 11*Sec[x]^11

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 4339

```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{d = FreeFacto
rs[Cos[c*(a + b*x)], x]}, -Dist[(b*c)^(-1), Subst[Int[SubstFor[1/x, Cos[c*(
a + b*x)]/d, u, x], x], x, Cos[c*(a + b*x)]/d], x] /; FunctionOfQ[Cos[c*(a
+ b*x)]/d, u, x, True]] /; FreeQ[{a, b, c}, x] && (EqQ[F, Tan] || EqQ[F, ta
n])
```

Rubi steps

$$\int \sec(x)\left(5-11\sec^5(x)\right)^2\tan(x)\,dx = -\operatorname{Subst}\left(\int \frac{\left(11-5x^5\right)^2}{x^{12}}\,dx,x,\cos(x)\right)$$
$$= -\operatorname{Subst}\left(\int \left(\frac{121}{x^{12}}-\frac{110}{x^7}+\frac{25}{x^2}\right)dx,x,\cos(x)\right)$$
$$= 25\sec(x)-\frac{55\sec^6(x)}{3}+11\sec^{11}(x)$$

Mathematica [**A**] time = 0.01, size = 19, normalized size = 1.00

$$11\sec^{11}(x)-\frac{55\sec^6(x)}{3}+25\sec(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[x]*(5 - 11*Sec[x]^5)^2*Tan[x],x]
[Out] 25*Sec[x] - (55*Sec[x]^6)/3 + 11*Sec[x]^11
```

fricas [**A**] time = 1.07, size = 20, normalized size = 1.05

$$\frac{75\,\cos(x)^{10}-55\,\cos(x)^5+33}{3\,\cos(x)^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)*(5-11*sec(x)^5)^2*tan(x),x, algorithm="fricas")
[Out] 1/3*(75*cos(x)^10 - 55*cos(x)^5 + 33)/cos(x)^11
```

giac [**A**] time = 0.14, size = 20, normalized size = 1.05

$$\frac{75\,\cos(x)^{10}-55\,\cos(x)^5+33}{3\,\cos(x)^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)*(5-11*sec(x)^5)^2*tan(x),x, algorithm="giac")
[Out] 1/3*(75*cos(x)^10 - 55*cos(x)^5 + 33)/cos(x)^11
```

maple [**A**] time = 0.05, size = 18, normalized size = 0.95

$$25\sec(x)-\frac{55\left(\sec^6(x)\right)}{3}+11\left(\sec^{11}(x)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(x)*(5-11*sec(x)^5)^2*tan(x),x)
```

```
[Out] 25*sec(x)-55/3*sec(x)^6+11*sec(x)^11
```

maxima [A] time = 0.31, size = 20, normalized size = 1.05

$$\frac{75\,\cos(x)^{10}-55\,\cos(x)^{5}+33}{3\,\cos(x)^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)*(5-11*sec(x)^5)^2*tan(x),x, algorithm="maxima")
```

```
[Out] 1/3*(75*cos(x)^10 - 55*cos(x)^5 + 33)/cos(x)^11
```

mupad [B] time = 3.67, size = 19, normalized size = 1.00

$$\frac{25\cos(x)^{10}-\frac{55\cos(x)^{5}}{3}+11}{\cos(x)^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((tan(x)*(11/cos(x)^5 - 5)^2)/cos(x),x)
```

```
[Out] (25*cos(x)^10 - (55*cos(x)^5)/3 + 11)/cos(x)^11
```

sympy [A] time = 24.47, size = 19, normalized size = 1.00

$$11\sec^{11}(x)-\frac{55\sec^{6}(x)}{3}+25\sec(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)*(5-11*sec(x)**5)**2*tan(x),x)
```

```
[Out] 11*sec(x)**11 - 55*sec(x)**6/3 + 25*sec(x)
```

3.890 $\int \sin^3(5x)\tan^3(5x)\,dx$

Optimal. Leaf size=44

$$\frac{1}{6}\sin^3(5x)+\frac{1}{2}\sin(5x)+\frac{1}{10}\sin^3(5x)\tan^2(5x)-\frac{1}{2}\tanh^{-1}(\sin(5x))$$

```
[Out] -1/2*arctanh(sin(5*x))+1/2*sin(5*x)+1/6*sin(5*x)^3+1/10*sin(5*x)^3*tan(5*x)
^2
```

Rubi [A] time = 0.04, antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {2592, 288, 302, 206}

$$\frac{1}{6}\sin^3(5x)+\frac{1}{2}\sin(5x)+\frac{1}{10}\sin^3(5x)\tan^2(5x)-\frac{1}{2}\tanh^{-1}(\sin(5x))$$

Antiderivative was successfully verified.

```
[In] Int[Sin[5*x]^3*Tan[5*x]^3,x]
```

```
[Out] -ArcTanh[Sin[5*x]]/2 + Sin[5*x]/2 + Sin[5*x]^3/6 + (Sin[5*x]^3*Tan[5*x]^2)/
10
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 2592

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(
ff*x)^(m + n)/(a^2 - ff^2*x^2)^((n + 1)/2), x], x, (a*Sin[e + f*x])/ff], x]
] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2]
```

Rubi steps

$$\begin{aligned}\int \sin^3(5x)\tan^3(5x)\,dx &= \frac{1}{5}\operatorname{Subst}\left(\int \frac{x^6}{\left(1-x^2\right)^2}\,dx, x, \sin(5x)\right)\\ &= \frac{1}{10}\sin^3(5x)\tan^2(5x) - \frac{1}{2}\operatorname{Subst}\left(\int \frac{x^4}{1-x^2}\,dx, x, \sin(5x)\right)\\ &= \frac{1}{10}\sin^3(5x)\tan^2(5x) - \frac{1}{2}\operatorname{Subst}\left(\int \left(-1-x^2+\frac{1}{1-x^2}\right)dx, x, \sin(5x)\right)\\ &= \frac{1}{2}\sin(5x)+\frac{1}{6}\sin^3(5x)+\frac{1}{10}\sin^3(5x)\tan^2(5x) - \frac{1}{2}\operatorname{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \sin(5x)\right)\\ &= -\frac{1}{2}\tanh^{-1}(\sin(5x))+\frac{1}{2}\sin(5x)+\frac{1}{6}\sin^3(5x)+\frac{1}{10}\sin^3(5x)\tan^2(5x)\end{aligned}$$

Mathematica [A] time = 0.04, size = 52, normalized size = 1.18

$$-\frac{1}{15}\sin^3(5x)\tan^2(5x)-\frac{1}{3}\sin(5x)\tan^2(5x)-\frac{1}{2}\tanh^{-1}(\sin(5x))+\frac{1}{2}\tan(5x)\sec(5x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[5*x]^3*Tan[5*x]^3,x]
```

```
[Out] -1/2*ArcTanh[Sin[5*x]] + (Sec[5*x]*Tan[5*x])/2 - (Sin[5*x]*Tan[5*x]^2)/3 -
(Sin[5*x]^3*Tan[5*x]^2)/15
```

fricas [A] time = 0.92, size = 65, normalized size = 1.48

$$-\frac{15\,\cos\left(5\,x\right)^2\log\left(\sin\left(5\,x\right)+1\right)-15\,\cos\left(5\,x\right)^2\log\left(-\sin\left(5\,x\right)+1\right)+2\left(2\,\cos\left(5\,x\right)^4-14\,\cos\left(5\,x\right)^2-3\right)\sin\left(5\right.}{60\,\cos\left(5\,x\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(5*x)^3*tan(5*x)^3,x, algorithm="fricas")
```

```
[Out] -1/60*(15*cos(5*x)^2*log(sin(5*x) + 1) - 15*cos(5*x)^2*log(-sin(5*x) + 1) +
 2*(2*cos(5*x)^4 - 14*cos(5*x)^2 - 3)*sin(5*x))/cos(5*x)^2
```

giac [A] time = 0.30, size = 51, normalized size = 1.16

$$\frac{1}{15}\sin(5x)^3 - \frac{\sin(5x)}{10\left(\sin(5x)^2 - 1\right)} - \frac{1}{4}\log(\sin(5x) + 1) + \frac{1}{4}\log(-\sin(5x) + 1) + \frac{2}{5}\sin(5x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(5*x)^3*tan(5*x)^3,x, algorithm="giac")
```

```
[Out] 1/15*sin(5*x)^3 - 1/10*sin(5*x)/(sin(5*x)^2 - 1) - 1/4*log(sin(5*x) + 1) +
1/4*log(-sin(5*x) + 1) + 2/5*sin(5*x)
```

maple [A] time = 0.11, size = 50, normalized size = 1.14

$$\frac{\sin^7(5x)}{10\cos(5x)^2} + \frac{\left(\sin^5(5x)\right)}{10} + \frac{\left(\sin^3(5x)\right)}{6} + \frac{\sin(5x)}{2} - \frac{\ln(\sec(5x) + \tan(5x))}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(5*x)^3*tan(5*x)^3,x)
```

```
[Out] 1/10*sin(5*x)^7/cos(5*x)^2+1/10*sin(5*x)^5+1/6*sin(5*x)^3+1/2*sin(5*x)-1/2*
ln(sec(5*x)+tan(5*x))
```

maxima [A] time = 0.31, size = 49, normalized size = 1.11

$$\frac{1}{15}\sin(5x)^3 - \frac{\sin(5x)}{10\left(\sin(5x)^2 - 1\right)} - \frac{1}{4}\log(\sin(5x) + 1) + \frac{1}{4}\log(\sin(5x) - 1) + \frac{2}{5}\sin(5x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(5*x)^3*tan(5*x)^3,x, algorithm="maxima")
```

```
[Out] 1/15*sin(5*x)^3 - 1/10*sin(5*x)/(sin(5*x)^2 - 1) - 1/4*log(sin(5*x) + 1) +
1/4*log(sin(5*x) - 1) + 2/5*sin(5*x)
```

mupad [B] time = 3.11, size = 69, normalized size = 1.57

$$\frac{5\tan\left(\frac{5x}{2}\right)^9 + \frac{20\tan\left(\frac{5x}{2}\right)^7}{3} - \frac{22\tan\left(\frac{5x}{2}\right)^5}{3} + \frac{20\tan\left(\frac{5x}{2}\right)^3}{3} + 5\tan\left(\frac{5x}{2}\right)}{5\left(\tan\left(\frac{5x}{2}\right)^2 - 1\right)^2\left(\tan\left(\frac{5x}{2}\right)^2 + 1\right)^3} - \operatorname{atanh}\left(\tan\left(\frac{5x}{2}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(5*x)^3*tan(5*x)^3,x)
```

```
[Out] (5*tan((5*x)/2) + (20*tan((5*x)/2)^3)/3 - (22*tan((5*x)/2)^5)/3 + (20*tan((
5*x)/2)^7)/3 + 5*tan((5*x)/2)^9)/(5*(tan((5*x)/2)^2 - 1)^2*(tan((5*x)/2)^2
+ 1)^3) - atanh(tan((5*x)/2))
```

sympy [**A**] time = 0.11, size = 51, normalized size = 1.16

$$\frac{\log\left(\sin\left(5x\right)-1\right)}{4}-\frac{\log\left(\sin\left(5x\right)+1\right)}{4}+\frac{\sin^{3}\left(5x\right)}{15}+\frac{2\sin\left(5x\right)}{5}-\frac{\sin\left(5x\right)}{5\left(2\sin^{2}\left(5x\right)-2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(5*x)**3*tan(5*x)**3,x)
```

```
[Out] log(sin(5*x) - 1)/4 - log(sin(5*x) + 1)/4 + sin(5*x)**3/15 + 2*sin(5*x)/5 -
 sin(5*x)/(5*(2*sin(5*x)**2 - 2))
```

3.891 $\int \sin^3(5x)\tan^4(5x)\,dx$

Optimal. Leaf size=37

$$\frac{1}{15}\cos^3(5x)-\frac{3}{5}\cos(5x)+\frac{1}{15}\sec^3(5x)-\frac{3}{5}\sec(5x)$$

```
[Out] -3/5*cos(5*x)+1/15*cos(5*x)^3-3/5*sec(5*x)+1/15*sec(5*x)^3
```

Rubi [A] time = 0.03, antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {2590, 270}

$$\frac{1}{15}\cos^3(5x)-\frac{3}{5}\cos(5x)+\frac{1}{15}\sec^3(5x)-\frac{3}{5}\sec(5x)$$

Antiderivative was successfully verified.

```
[In] Int[Sin[5*x]^3*Tan[5*x]^4,x]
```

```
[Out] (-3*Cos[5*x])/5 + Cos[5*x]^3/15 - (3*Sec[5*x])/5 + Sec[5*x]^3/15
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2590

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> -Dist[f^(-1), Subst[Int[(1 - x^2)^((m + n - 1)/2)/x^n, x], x, Cos[e + f*
x]], x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n - 1)/2]
```

Rubi steps

$$\begin{aligned}
\int \sin^3(5x)\tan^4(5x)\,dx &= -\left(\frac{1}{5}\text{Subst}\left(\int \frac{\left(1-x^2\right)^3}{x^4}\,dx,x,\cos(5x)\right)\right)\\
&= -\left(\frac{1}{5}\text{Subst}\left(\int \left(3+\frac{1}{x^4}-\frac{3}{x^2}-x^2\right)dx,x,\cos(5x)\right)\right)\\
&= -\frac{3}{5}\cos(5x)+\frac{1}{15}\cos^3(5x)-\frac{3}{5}\sec(5x)+\frac{1}{15}\sec^3(5x)
\end{aligned}$$

Mathematica [A] time = 0.03, size = 35, normalized size = 0.95

$$-\frac{11}{20}\cos(5x)+\frac{1}{60}\cos(15x)+\frac{1}{15}\sec^3(5x)-\frac{3}{5}\sec(5x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[5*x]^3*Tan[5*x]^4,x]
```

```
[Out] (-11*Cos[5*x])/20 + Cos[15*x]/60 - (3*Sec[5*x])/5 + Sec[5*x]^3/15
```

fricas [A] time = 1.85, size = 32, normalized size = 0.86

$$\frac{\cos(5x)^6-9\cos(5x)^4-9\cos(5x)^2+1}{15\cos(5x)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(5*x)^3*tan(5*x)^4,x, algorithm="fricas")
```

```
[Out] 1/15*(cos(5*x)^6 - 9*cos(5*x)^4 - 9*cos(5*x)^2 + 1)/cos(5*x)^3
```

giac [A] time = 0.76, size = 33, normalized size = 0.89

$$\frac{1}{15}\cos(5x)^3-\frac{9\cos(5x)^2-1}{15\cos(5x)^3}-\frac{3}{5}\cos(5x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(5*x)^3*tan(5*x)^4,x, algorithm="giac")
```

```
[Out] 1/15*cos(5*x)^3 - 1/15*(9*cos(5*x)^2 - 1)/cos(5*x)^3 - 3/5*cos(5*x)
```

maple [B] time = 0.11, size = 60, normalized size = 1.62

$$\frac{\sin^8(5x)}{15\cos(5x)^3}-\frac{\sin^8(5x)}{3\cos(5x)}-\frac{\left(\frac{16}{5}+\sin^6(5x)+\frac{6\left(\sin^4(5x)\right)}{5}+\frac{8\left(\sin^2(5x)\right)}{5}\right)\cos(5x)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(5*x)^3*tan(5*x)^4,x)
```

```
[Out] 1/15*sin(5*x)^8/cos(5*x)^3-1/3*sin(5*x)^8/cos(5*x)-1/3*(16/5+sin(5*x)^6+6/5
*sin(5*x)^4+8/5*sin(5*x)^2)*cos(5*x)
```

maxima [A] time = 0.31, size = 33, normalized size = 0.89

$$\frac{1}{15}\cos(5x)^3 - \frac{9\cos(5x)^2 - 1}{15\cos(5x)^3} - \frac{3}{5}\cos(5x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(5*x)^3*tan(5*x)^4,x, algorithm="maxima")
```

```
[Out] 1/15*cos(5*x)^3 - 1/15*(9*cos(5*x)^2 - 1)/cos(5*x)^3 - 3/5*cos(5*x)
```

mupad [B] time = 3.10, size = 30, normalized size = 0.81

$$\frac{(\cos(5x)+1)^4\left(\cos(5x)^2 - 4\cos(5x) + 1\right)}{15\cos(5x)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(5*x)^3*tan(5*x)^4,x)
```

```
[Out] ((cos(5*x) + 1)^4*(cos(5*x)^2 - 4*cos(5*x) + 1))/(15*cos(5*x)^3)
```

sympy [A] time = 0.09, size = 34, normalized size = 0.92

$$\frac{1 - 9\cos^2(5x)}{15\cos^3(5x)} + \frac{\cos^3(5x)}{15} - \frac{3\cos(5x)}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(5*x)**3*tan(5*x)**4,x)
```

```
[Out] (1 - 9*cos(5*x)**2)/(15*cos(5*x)**3) + cos(5*x)**3/15 - 3*cos(5*x)/5
```

3.892 $\int \sin^5(6x)\tan^3(6x)\,dx$

Optimal. Leaf size=54

$$\frac{7}{60}\sin^5(6x)+\frac{7}{36}\sin^3(6x)+\frac{7}{12}\sin(6x)+\frac{1}{12}\sin^5(6x)\tan^2(6x)-\frac{7}{12}\tanh^{-1}(\sin(6x))$$

```
[Out] -7/12*arctanh(sin(6*x))+7/12*sin(6*x)+7/36*sin(6*x)^3+7/60*sin(6*x)^5+1/12*
sin(6*x)^5*tan(6*x)^2
```

Rubi [A] time = 0.04, antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {2592, 288, 302, 206}

$$\frac{7}{60}\sin^5(6x)+\frac{7}{36}\sin^3(6x)+\frac{7}{12}\sin(6x)+\frac{1}{12}\sin^5(6x)\tan^2(6x)-\frac{7}{12}\tanh^{-1}(\sin(6x))$$

Antiderivative was successfully verified.

```
[In] Int[Sin[6*x]^5*Tan[6*x]^3,x]
```

```
[Out] (-7*ArcTanh[Sin[6*x]])/12 + (7*Sin[6*x])/12 + (7*Sin[6*x]^3)/36 + (7*Sin[6*
x]^5)/60 + (Sin[6*x]^5*Tan[6*x]^2)/12
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 2592

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(
ff*x)^(m + n)/(a^2 - ff^2*x^2)^((n + 1)/2), x], x, (a*Sin[e + f*x])/ff], x]
] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2]
```

Rubi steps

$$\begin{aligned}\int \sin^5(6x)\tan^3(6x)\,dx &= \frac{1}{6}\operatorname{Subst}\left(\int \frac{x^8}{\left(1-x^2\right)^2}\,dx, x, \sin(6x)\right)\\ &= \frac{1}{12}\sin^5(6x)\tan^2(6x) - \frac{7}{12}\operatorname{Subst}\left(\int \frac{x^6}{1-x^2}\,dx, x, \sin(6x)\right)\\ &= \frac{1}{12}\sin^5(6x)\tan^2(6x) - \frac{7}{12}\operatorname{Subst}\left(\int \left(-1-x^2-x^4+\frac{1}{1-x^2}\right)dx, x, \sin(6x)\right)\\ &= \frac{7}{12}\sin(6x) + \frac{7}{36}\sin^3(6x) + \frac{7}{60}\sin^5(6x) + \frac{1}{12}\sin^5(6x)\tan^2(6x) - \frac{7}{12}\operatorname{Subst}\left(\int \frac{1}{1-x^2}\right.\\ &= -\frac{7}{12}\tanh^{-1}(\sin(6x)) + \frac{7}{12}\sin(6x) + \frac{7}{36}\sin^3(6x) + \frac{7}{60}\sin^5(6x) + \frac{1}{12}\sin^5(6x)\tan^2(6x\end{aligned}$$

Mathematica [A] time = 0.10, size = 68, normalized size = 1.26

$$-\frac{1}{30}\sin^5(6x)\tan^2(6x)-\frac{7}{90}\sin^3(6x)\tan^2(6x)-\frac{7}{18}\sin(6x)\tan^2(6x)-\frac{7}{12}\tanh^{-1}(\sin(6x))+\frac{7}{12}\tan(6x)\sec(6x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[6*x]^5*Tan[6*x]^3,x]
```

```
[Out] (-7*ArcTanh[Sin[6*x]])/12 + (7*Sec[6*x]*Tan[6*x])/12 - (7*Sin[6*x]*Tan[6*x]
^2)/18 - (7*Sin[6*x]^3*Tan[6*x]^2)/90 - (Sin[6*x]^5*Tan[6*x]^2)/30
```

fricas [A] time = 0.93, size = 73, normalized size = 1.35

$$-\frac{105\cos(6x)^2\log(\sin(6x)+1)-105\cos(6x)^2\log(-\sin(6x)+1)-2\left(6\cos(6x)^6-32\cos(6x)^4+116\cos\right.}{360\cos(6x)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(6*x)^5*tan(6*x)^3,x, algorithm="fricas")
```

```
[Out] -1/360*(105*cos(6*x)^2*log(sin(6*x) + 1) - 105*cos(6*x)^2*log(-sin(6*x) + 1
) - 2*(6*cos(6*x)^6 - 32*cos(6*x)^4 + 116*cos(6*x)^2 + 15)*sin(6*x))/cos(6*
x)^2
```

giac [A] time = 0.42, size = 59, normalized size = 1.09

$$\frac{1}{30}\sin(6x)^5+\frac{1}{9}\sin(6x)^3-\frac{\sin(6x)}{12\left(\sin(6x)^2-1\right)}-\frac{7}{24}\log(\sin(6x)+1)+\frac{7}{24}\log(-\sin(6x)+1)+\frac{1}{2}\sin(6x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(6*x)^5*tan(6*x)^3,x, algorithm="giac")
```

```
[Out] 1/30*sin(6*x)^5 + 1/9*sin(6*x)^3 - 1/12*sin(6*x)/(sin(6*x)^2 - 1) - 7/24*lo
g(sin(6*x) + 1) + 7/24*log(-sin(6*x) + 1) + 1/2*sin(6*x)
```

maple [A] time = 0.12, size = 58, normalized size = 1.07

$$\frac{\sin^9(6x)}{12\cos(6x)^2}+\frac{\left(\sin^7(6x)\right)}{12}+\frac{7\left(\sin^5(6x)\right)}{60}+\frac{7\left(\sin^3(6x)\right)}{36}+\frac{7\sin(6x)}{12}-\frac{7\ln(\sec(6x)+\tan(6x))}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(6*x)^5*tan(6*x)^3,x)
```

```
[Out] 1/12*sin(6*x)^9/cos(6*x)^2+1/12*sin(6*x)^7+7/60*sin(6*x)^5+7/36*sin(6*x)^3+
7/12*sin(6*x)-7/12*ln(sec(6*x)+tan(6*x))
```

maxima [A] time = 0.31, size = 57, normalized size = 1.06

$$\frac{1}{30}\sin(6x)^5+\frac{1}{9}\sin(6x)^3-\frac{\sin(6x)}{12\left(\sin(6x)^2-1\right)}-\frac{7}{24}\log(\sin(6x)+1)+\frac{7}{24}\log(\sin(6x)-1)+\frac{1}{2}\sin(6x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(6*x)^5*tan(6*x)^3,x, algorithm="maxima")
```

```
[Out] 1/30*sin(6*x)^5 + 1/9*sin(6*x)^3 - 1/12*sin(6*x)/(sin(6*x)^2 - 1) - 7/24*lo
g(sin(6*x) + 1) + 7/24*log(sin(6*x) - 1) + 1/2*sin(6*x)
```

mupad [B] time = 7.21, size = 85, normalized size = 1.57

$$\frac{7\tan(3x)^{13}+\frac{70\tan(3x)^{11}}{3}+\frac{77\tan(3x)^9}{5}-\frac{412\tan(3x)^7}{15}+\frac{77\tan(3x)^5}{5}+\frac{70\tan(3x)^3}{3}+7\tan(3x)}{6\left(\tan(3x)^2-1\right)^2\left(\tan(3x)^2+1\right)^5}-\frac{7\,\mathrm{atanh}(\tan(3x))}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(6*x)^5*tan(6*x)^3,x)
```

```
[Out] (7*tan(3*x) + (70*tan(3*x)^3)/3 + (77*tan(3*x)^5)/5 - (412*tan(3*x)^7)/15 +
 (77*tan(3*x)^9)/5 + (70*tan(3*x)^11)/3 + 7*tan(3*x)^13)/(6*(tan(3*x)^2 - 1
)^2*(tan(3*x)^2 + 1)^5) - (7*atanh(tan(3*x)))/6
```

sympy [A] time = 0.11, size = 61, normalized size = 1.13

$$\frac{7\log(\sin(6x)-1)}{24}-\frac{7\log(\sin(6x)+1)}{24}+\frac{\sin^5(6x)}{30}+\frac{\sin^3(6x)}{9}+\frac{\sin(6x)}{2}-\frac{\sin(6x)}{6\left(2\sin^2(6x)-2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(6*x)**5*tan(6*x)**3,x)
```

```
[Out] 7*log(sin(6*x) - 1)/24 - 7*log(sin(6*x) + 1)/24 + sin(6*x)**5/30 + sin(6*x)
**3/9 + sin(6*x)/2 - sin(6*x)/(6*(2*sin(6*x)**2 - 2))
```

3.893 $\int \left(-1+\sec^2(2x)\right)^3 \sin(2x)\,dx$

Optimal. Leaf size=37

$$\frac{1}{2}\cos(2x)+\frac{1}{10}\sec^5(2x)-\frac{1}{2}\sec^3(2x)+\frac{3}{2}\sec(2x)$$

```
[Out] 1/2*cos(2*x)+3/2*sec(2*x)-1/2*sec(2*x)^3+1/10*sec(2*x)^5
```

Rubi [A] time = 0.04, antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {4120, 2590, 270}

$$\frac{1}{2}\cos(2x)+\frac{1}{10}\sec^5(2x)-\frac{1}{2}\sec^3(2x)+\frac{3}{2}\sec(2x)$$

Antiderivative was successfully verified.

```
[In] Int[(-1 + Sec[2*x]^2)^3*Sin[2*x],x]
```

```
[Out] Cos[2*x]/2 + (3*Sec[2*x])/2 - Sec[2*x]^3/2 + Sec[2*x]^5/10
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2590

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> -Dist[f^(-1), Subst[Int[(1 - x^2)^((m + n - 1)/2)/x^n, x], x, Cos[e + f*
x]], x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n - 1)/2]
```

Rule 4120

```
Int[(u_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Dist[
b^p, Int[ActivateTrig[u*tan[e + f*x]^(2*p)], x], x] /; FreeQ[{a, b, e, f, p
}, x] && EqQ[a + b, 0] && IntegerQ[p]
```

Rubi steps

$$\int \left(-1+\sec^2(2x)\right)^3 \sin(2x)\,dx = \int \sin(2x)\tan^6(2x)\,dx$$
$$= -\left(\frac{1}{2}\operatorname{Subst}\left(\int \frac{\left(1-x^2\right)^3}{x^6}\,dx, x, \cos(2x)\right)\right)$$
$$= -\left(\frac{1}{2}\operatorname{Subst}\left(\int \left(-1+\frac{1}{x^6}-\frac{3}{x^4}+\frac{3}{x^2}\right)dx, x, \cos(2x)\right)\right)$$
$$= \frac{1}{2}\cos(2x)+\frac{3}{2}\sec(2x)-\frac{1}{2}\sec^3(2x)+\frac{1}{10}\sec^5(2x)$$

Mathematica [**A**] time = 0.03, size = 37, normalized size = 1.00

$$\frac{1}{2}\cos(2x)+\frac{1}{10}\sec^5(2x)-\frac{1}{2}\sec^3(2x)+\frac{3}{2}\sec(2x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-1 + Sec[2*x]^2)^3*Sin[2*x],x]
[Out] Cos[2*x]/2 + (3*Sec[2*x])/2 - Sec[2*x]^3/2 + Sec[2*x]^5/10
```

fricas [**A**] time = 0.41, size = 34, normalized size = 0.92

$$\frac{5\cos(2x)^6+15\cos(2x)^4-5\cos(2x)^2+1}{10\cos(2x)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+sec(2*x)^2)^3*sin(2*x),x, algorithm="fricas")
[Out] 1/10*(5*cos(2*x)^6 + 15*cos(2*x)^4 - 5*cos(2*x)^2 + 1)/cos(2*x)^5
```

giac [**A**] time = 0.14, size = 33, normalized size = 0.89

$$\frac{15\cos(2x)^4-5\cos(2x)^2+1}{10\cos(2x)^5}+\frac{1}{2}\cos(2x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+sec(2*x)^2)^3*sin(2*x),x, algorithm="giac")
[Out] 1/10*(15*cos(2*x)^4 - 5*cos(2*x)^2 + 1)/cos(2*x)^5 + 1/2*cos(2*x)
```

maple [A] time = 0.10, size = 32, normalized size = 0.86

$$\frac{1}{10\cos\left(2x\right)^{5}}-\frac{1}{2\cos\left(2x\right)^{3}}+\frac{3}{2\cos\left(2x\right)}+\frac{\cos\left(2x\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-1+sec(2*x)^2)^3*sin(2*x),x)
```

```
[Out] 1/10/cos(2*x)^5-1/2/cos(2*x)^3+3/2/cos(2*x)+1/2*cos(2*x)
```

maxima [A] time = 0.30, size = 31, normalized size = 0.84

$$\frac{3}{2\cos\left(2x\right)}-\frac{1}{2\cos\left(2x\right)^{3}}+\frac{1}{10\cos\left(2x\right)^{5}}+\frac{1}{2}\cos\left(2x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+sec(2*x)^2)^3*sin(2*x),x, algorithm="maxima")
```

```
[Out] 3/2/cos(2*x) - 1/2/cos(2*x)^3 + 1/10/cos(2*x)^5 + 1/2*cos(2*x)
```

mupad [B] time = 2.94, size = 33, normalized size = 0.89

$$\frac{\cos\left(2x\right)}{2}+\frac{3\cos\left(2x\right)^{4}-\cos\left(2x\right)^{2}+\frac{1}{5}}{2\cos\left(2x\right)^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(2*x)*(1/cos(2*x)^2 - 1)^3,x)
```

```
[Out] cos(2*x)/2 + (3*cos(2*x)^4 - cos(2*x)^2 + 1/5)/(2*cos(2*x)^5)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+sec(2*x)**2)**3*sin(2*x),x)
```

```
[Out] Timed out
```

3.894 $\int \sin(x)\tan^5(x)\,dx$

Optimal. Leaf size=34

$$-\frac{15\sin(x)}{8}+\frac{1}{4}\sin(x)\tan^4(x)-\frac{5}{8}\sin(x)\tan^2(x)+\frac{15}{8}\tanh^{-1}(\sin(x))$$

```
[Out] 15/8*arctanh(sin(x))-15/8*sin(x)-5/8*sin(x)*tan(x)^2+1/4*sin(x)*tan(x)^4
```

Rubi [A] time = 0.02, antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.571, Rules used = {2592, 288, 321, 206}

$$-\frac{15\sin(x)}{8}+\frac{1}{4}\sin(x)\tan^4(x)-\frac{5}{8}\sin(x)\tan^2(x)+\frac{15}{8}\tanh^{-1}(\sin(x))$$

Antiderivative was successfully verified.

```
[In] Int[Sin[x]*Tan[x]^5,x]
```

```
[Out] (15*ArcTanh[Sin[x]])/8 - (15*Sin[x])/8 - (5*Sin[x]*Tan[x]^2)/8 + (Sin[x]*Ta
n[x]^4)/4
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2592

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(
ff*x)^(m + n)/(a^2 - ff^2*x^2)^((n + 1)/2), x], x, (a*Sin[e + f*x])/ff], x]
] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2]
```

Rubi steps

$$\begin{aligned}
\int \sin(x)\tan^5(x)\,dx &= \text{Subst}\left(\int \frac{x^6}{\left(1-x^2\right)^3}\,dx, x, \sin(x)\right)\\
&= \frac{1}{4}\sin(x)\tan^4(x) - \frac{5}{4}\text{Subst}\left(\int \frac{x^4}{\left(1-x^2\right)^2}\,dx, x, \sin(x)\right)\\
&= -\frac{5}{8}\sin(x)\tan^2(x) + \frac{1}{4}\sin(x)\tan^4(x) + \frac{15}{8}\text{Subst}\left(\int \frac{x^2}{1-x^2}\,dx, x, \sin(x)\right)\\
&= -\frac{15\sin(x)}{8} - \frac{5}{8}\sin(x)\tan^2(x) + \frac{1}{4}\sin(x)\tan^4(x) + \frac{15}{8}\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \sin(x)\right)\\
&= \frac{15}{8}\tanh^{-1}(\sin(x)) - \frac{15\sin(x)}{8} - \frac{5}{8}\sin(x)\tan^2(x) + \frac{1}{4}\sin(x)\tan^4(x)
\end{aligned}$$

Mathematica [A] time = 0.01, size = 42, normalized size = 1.24

$$-\sin(x)\tan^4(x) + \frac{15}{8}\tanh^{-1}(\sin(x)) - \frac{15}{4}\tan(x)\sec^3(x) + 5\tan^3(x)\sec(x) + \frac{15}{8}\tan(x)\sec(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[x]*Tan[x]^5,x]
```

```
[Out] (15*ArcTanh[Sin[x]])/8 + (15*Sec[x]*Tan[x])/8 - (15*Sec[x]^3*Tan[x])/4 + 5*
Sec[x]*Tan[x]^3 - Sin[x]*Tan[x]^4
```

fricas [A] time = 0.97, size = 49, normalized size = 1.44

$$\frac{15\,\cos(x)^4\log\left(\sin(x)+1\right) - 15\,\cos(x)^4\log\left(-\sin(x)+1\right) - 2\left(8\,\cos(x)^4 + 9\,\cos(x)^2 - 2\right)\sin(x)}{16\,\cos(x)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)*tan(x)^5,x, algorithm="fricas")
```

```
[Out] 1/16*(15*cos(x)^4*log(sin(x) + 1) - 15*cos(x)^4*log(-sin(x) + 1) - 2*(8*cos
(x)^4 + 9*cos(x)^2 - 2)*sin(x))/cos(x)^4
```

giac [A] time = 0.15, size = 42, normalized size = 1.24

$$\frac{9 \sin(x)^3 - 7 \sin(x)}{8 \left(\sin(x)^2 - 1\right)^2} + \frac{15}{16} \log\left(\sin(x) + 1\right) - \frac{15}{16} \log\left(-\sin(x) + 1\right) - \sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)*tan(x)^5,x, algorithm="giac")
```

```
[Out] 1/8*(9*sin(x)^3 - 7*sin(x))/(sin(x)^2 - 1)^2 + 15/16*log(sin(x) + 1) - 15/1
6*log(-sin(x) + 1) - sin(x)
```

maple [A] time = 0.06, size = 46, normalized size = 1.35

$$\frac{\sin^7(x)}{4 \cos(x)^4} - \frac{3 \left(\sin^7(x)\right)}{8 \cos(x)^2} - \frac{3 \left(\sin^5(x)\right)}{8} - \frac{5 \left(\sin^3(x)\right)}{8} - \frac{15 \sin(x)}{8} + \frac{15 \ln\left(\sec(x) + \tan(x)\right)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)*tan(x)^5,x)
```

```
[Out] 1/4*sin(x)^7/cos(x)^4-3/8*sin(x)^7/cos(x)^2-3/8*sin(x)^5-5/8*sin(x)^3-15/8*
sin(x)+15/8*ln(sec(x)+tan(x))
```

maxima [A] time = 0.31, size = 46, normalized size = 1.35

$$\frac{9 \sin(x)^3 - 7 \sin(x)}{8 \left(\sin(x)^4 - 2 \sin(x)^2 + 1\right)} + \frac{15}{16} \log\left(\sin(x) + 1\right) - \frac{15}{16} \log\left(\sin(x) - 1\right) - \sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)*tan(x)^5,x, algorithm="maxima")
```

```
[Out] 1/8*(9*sin(x)^3 - 7*sin(x))/(sin(x)^4 - 2*sin(x)^2 + 1) + 15/16*log(sin(x)
+ 1) - 15/16*log(sin(x) - 1) - sin(x)
```

mupad [B] time = 3.04, size = 69, normalized size = 2.03

$$\frac{15 \operatorname{atanh}\left(\tan\left(\frac{x}{2}\right)\right)}{4} - \frac{\frac{15 \tan\left(\frac{x}{2}\right)^9}{4} - 10 \tan\left(\frac{x}{2}\right)^7 + \frac{9 \tan\left(\frac{x}{2}\right)^5}{2} - 10 \tan\left(\frac{x}{2}\right)^3 + \frac{15 \tan\left(\frac{x}{2}\right)}{4}}{\left(\tan\left(\frac{x}{2}\right)^2 - 1\right)^4 \left(\tan\left(\frac{x}{2}\right)^2 + 1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)*tan(x)^5,x)
```

```
[Out] (15*atanh(tan(x/2)))/4 - ((15*tan(x/2))/4 - 10*tan(x/2)^3 + (9*tan(x/2)^5)/
2 - 10*tan(x/2)^7 + (15*tan(x/2)^9)/4)/((tan(x/2)^2 - 1)^4*(tan(x/2)^2 + 1)
)
```

sympy [A] time = 0.15, size = 49, normalized size = 1.44

$$-\frac{-9\sin^{3}(x)+7\sin(x)}{8\sin^{4}(x)-16\sin^{2}(x)+8}-\frac{15\log(\sin(x)-1)}{16}+\frac{15\log(\sin(x)+1)}{16}-\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)*tan(x)**5,x)
```

```
[Out] -(-9*sin(x)**3 + 7*sin(x))/(8*sin(x)**4 - 16*sin(x)**2 + 8) - 15*log(sin(x)
 - 1)/16 + 15*log(sin(x) + 1)/16 - sin(x)
```

3.895 $\int \cos^5(2x)\cot^4(2x)\,dx$

Optimal. Leaf size=43

$$\frac{1}{10}\sin^5(2x)-\frac{2}{3}\sin^3(2x)+3\sin(2x)-\frac{1}{6}\csc^3(2x)+2\csc(2x)$$

```
[Out] 2*csc(2*x)-1/6*csc(2*x)^3+3*sin(2*x)-2/3*sin(2*x)^3+1/10*sin(2*x)^5
```

Rubi [A] time = 0.04, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {2590, 270}

$$\frac{1}{10}\sin^5(2x)-\frac{2}{3}\sin^3(2x)+3\sin(2x)-\frac{1}{6}\csc^3(2x)+2\csc(2x)$$

Antiderivative was successfully verified.

```
[In] Int[Cos[2*x]^5*Cot[2*x]^4,x]
```

```
[Out] 2*Csc[2*x] - Csc[2*x]^3/6 + 3*Sin[2*x] - (2*Sin[2*x]^3)/3 + Sin[2*x]^5/10
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2590

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> -Dist[f^(-1), Subst[Int[(1 - x^2)^((m + n - 1)/2)/x^n, x], x, Cos[e + f*
x]], x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n - 1)/2]
```

Rubi steps

$$\begin{aligned}\int \cos^5(2x)\cot^4(2x)\,dx &= -\left(\frac{1}{2}\operatorname{Subst}\left(\int \frac{\left(1-x^2\right)^4}{x^4}\,dx,x,-\sin(2x)\right)\right)\\ &= -\left(\frac{1}{2}\operatorname{Subst}\left(\int \left(6+\frac{1}{x^4}-\frac{4}{x^2}-4x^2+x^4\right)dx,x,-\sin(2x)\right)\right)\\ &= 2\csc(2x)-\frac{1}{6}\csc^3(2x)+3\sin(2x)-\frac{2}{3}\sin^3(2x)+\frac{1}{10}\sin^5(2x)\end{aligned}$$

Mathematica [A] time = 0.03, size = 43, normalized size = 1.00

$$\frac{1}{10}\sin^5(2x)-\frac{2}{3}\sin^3(2x)+3\sin(2x)-\frac{1}{6}\csc^3(2x)+2\csc(2x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[2*x]^5*Cot[2*x]^4,x]
[Out] 2*Csc[2*x] - Csc[2*x]^3/6 + 3*Sin[2*x] - (2*Sin[2*x]^3)/3 + Sin[2*x]^5/10
```

fricas [A] time = 0.96, size = 52, normalized size = 1.21

$$-\frac{3\cos(2x)^8+8\cos(2x)^6+48\cos(2x)^4-192\cos(2x)^2+128}{30\left(\cos(2x)^2-1\right)\sin(2x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(2*x)^5*cot(2*x)^4,x, algorithm="fricas")
[Out] -1/30*(3*cos(2*x)^8 + 8*cos(2*x)^6 + 48*cos(2*x)^4 - 192*cos(2*x)^2 + 128)/
((cos(2*x)^2 - 1)*sin(2*x))
```

giac [A] time = 0.16, size = 41, normalized size = 0.95

$$\frac{1}{10}\sin(2x)^5-\frac{2}{3}\sin(2x)^3+\frac{12\sin(2x)^2-1}{6\sin(2x)^3}+3\sin(2x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(2*x)^5*cot(2*x)^4,x, algorithm="giac")
[Out] 1/10*sin(2*x)^5 - 2/3*sin(2*x)^3 + 1/6*(12*sin(2*x)^2 - 1)/sin(2*x)^3 + 3*s
in(2*x)
```

maple [A] time = 0.17, size = 68, normalized size = 1.58

$$-\frac{\cos^{10}(2x)}{6\sin(2x)^3}+\frac{7\left(\cos^{10}(2x)\right)}{6\sin(2x)}+\frac{7\left(\frac{128}{35}+\cos^8(2x)+\frac{8\left(\cos^6(2x)\right)}{7}+\frac{48\left(\cos^4(2x)\right)}{35}+\frac{64\left(\cos^2(2x)\right)}{35}\right)\sin(2x)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(2*x)^5*cot(2*x)^4,x)
[Out] -1/6/sin(2*x)^3*cos(2*x)^10+7/6/sin(2*x)*cos(2*x)^10+7/6*(128/35+cos(2*x)^8
+8/7*cos(2*x)^6+48/35*cos(2*x)^4+64/35*cos(2*x)^2)*sin(2*x)
```

maxima [A]   time = 0.31, size = 41, normalized size = 0.95

$$\frac{1}{10}\,\sin\left(2\,x\right)^{5}-\frac{2}{3}\,\sin\left(2\,x\right)^{3}+\frac{12\,\sin\left(2\,x\right)^{2}-1}{6\,\sin\left(2\,x\right)^{3}}+3\,\sin\left(2\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(2*x)^5*cot(2*x)^4,x, algorithm="maxima")
```

```
[Out] 1/10*sin(2*x)^5 - 2/3*sin(2*x)^3 + 1/6*(12*sin(2*x)^2 - 1)/sin(2*x)^3 + 3*s
in(2*x)
```

mupad [B]   time = 3.06, size = 42, normalized size = 0.98

$$\frac{3\sin\left(2\,x\right)^{8}-20\sin\left(2\,x\right)^{6}+90\sin\left(2\,x\right)^{4}+60\sin\left(2\,x\right)^{2}-5}{30\sin\left(2\,x\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(2*x)^5*cot(2*x)^4,x)
```

```
[Out] (60*sin(2*x)^2 + 90*sin(2*x)^4 - 20*sin(2*x)^6 + 3*sin(2*x)^8 - 5)/(30*sin(
2*x)^3)
```

sympy [A]   time = 0.09, size = 42, normalized size = 0.98

$$\frac{12\sin^{2}\left(2x\right)-1}{6\sin^{3}\left(2x\right)}+\frac{\sin^{5}\left(2x\right)}{10}-\frac{2\sin^{3}\left(2x\right)}{3}+3\sin\left(2x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(2*x)**5*cot(2*x)**4,x)
```

```
[Out] (12*sin(2*x)**2 - 1)/(6*sin(2*x)**3) + sin(2*x)**5/10 - 2*sin(2*x)**3/3 + 3
*sin(2*x)
```

3.896 $\int \cos(3x)\left(-1+\csc^2(3x)\right)^3\left(1-\sin^2(3x)\right)^5\,dx$

Optimal. Leaf size=87

$$\frac{1}{33}\sin^{11}(3x)-\frac{8}{27}\sin^9(3x)+\frac{4}{3}\sin^7(3x)-\frac{56}{15}\sin^5(3x)+\frac{70}{9}\sin^3(3x)-\frac{56}{3}\sin(3x)-\frac{1}{15}\csc^5(3x)+\frac{8}{9}\csc^3(3x)-\frac{28}{3}\csc($$

```
[Out] -28/3*csc(3*x)+8/9*csc(3*x)^3-1/15*csc(3*x)^5-56/3*sin(3*x)+70/9*sin(3*x)^3
-56/15*sin(3*x)^5+4/3*sin(3*x)^7-8/27*sin(3*x)^9+1/33*sin(3*x)^11
```

Rubi [A] time = 0.13, antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {3175, 4120, 2590, 270}

$$\frac{1}{33}\sin^{11}(3x)-\frac{8}{27}\sin^9(3x)+\frac{4}{3}\sin^7(3x)-\frac{56}{15}\sin^5(3x)+\frac{70}{9}\sin^3(3x)-\frac{56}{3}\sin(3x)-\frac{1}{15}\csc^5(3x)+\frac{8}{9}\csc^3(3x)-\frac{28}{3}\csc($$

Antiderivative was successfully verified.

```
[In] Int[Cos[3*x]*(-1 + Csc[3*x]^2)^3*(1 - Sin[3*x]^2)^5,x]
```

```
[Out] (-28*Csc[3*x])/3 + (8*Csc[3*x]^3)/9 - Csc[3*x]^5/15 - (56*Sin[3*x])/3 + (70
*Sin[3*x]^3)/9 - (56*Sin[3*x]^5)/15 + (4*Sin[3*x]^7)/3 - (8*Sin[3*x]^9)/27
+ Sin[3*x]^11/33
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2590

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> -Dist[f^(-1), Subst[Int[(1 - x^2)^((m + n - 1)/2)/x^n, x], x, Cos[e + f*
x]], x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n - 1)/2]
```

Rule 3175

```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Dist[
a^p, Int[ActivateTrig[u*cos[e + f*x]^(2*p)], x], x] /; FreeQ[{a, b, e, f, p
}, x] && EqQ[a + b, 0] && IntegerQ[p]
```

Rule 4120

```
Int[(u_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Dist[
b^p, Int[ActivateTrig[u*tan[e + f*x]^(2*p)], x], x] /; FreeQ[{a, b, e, f, p
}, x] && EqQ[a + b, 0] && IntegerQ[p]
```

Rubi steps

$$\int \cos(3x)\left(-1+\csc^2(3x)\right)^3\left(1-\sin^2(3x)\right)^5\,dx = \int \cos^{11}(3x)\left(-1+\csc^2(3x)\right)^3\,dx$$
$$= \int \cos^{11}(3x)\cot^6(3x)\,dx$$
$$= -\left(\frac{1}{3}\operatorname{Subst}\left(\int \frac{\left(1-x^2\right)^8}{x^6}\,dx, x, -\sin(3x)\right)\right)$$
$$= -\left(\frac{1}{3}\operatorname{Subst}\left(\int \left(-56+\frac{1}{x^6}-\frac{8}{x^4}+\frac{28}{x^2}+70x^2-56x^4+28x^6-8\right.\right.\right.$$
$$= -\frac{28}{3}\csc(3x)+\frac{8}{9}\csc^3(3x)-\frac{1}{15}\csc^5(3x)-\frac{56}{3}\sin(3x)+\frac{70}{9}\sin$$

Mathematica [A] time = 0.06, size = 87, normalized size = 1.00

$$\frac{1}{33}\sin^{11}(3x)-\frac{8}{27}\sin^9(3x)+\frac{4}{3}\sin^7(3x)-\frac{56}{15}\sin^5(3x)+\frac{70}{9}\sin^3(3x)-\frac{56}{3}\sin(3x)-\frac{1}{15}\csc^5(3x)+\frac{8}{9}\csc^3(3x)-\frac{28}{3}\csc(3$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[3*x]*(-1 + Csc[3*x]^2)^3*(1 - Sin[3*x]^2)^5,x]
```

```
[Out] (-28*Csc[3*x])/3 + (8*Csc[3*x]^3)/9 - Csc[3*x]^5/15 - (56*Sin[3*x])/3 + (70
*Sin[3*x]^3)/9 - (56*Sin[3*x]^5)/15 + (4*Sin[3*x]^7)/3 - (8*Sin[3*x]^9)/27
+ Sin[3*x]^11/33
```

fricas [A] time = 1.05, size = 92, normalized size = 1.06

$$\frac{45\cos(3x)^{16}+80\cos(3x)^{14}+160\cos(3x)^{12}+384\cos(3x)^{10}+1280\cos(3x)^8+10240\cos(3x)^6-61440}{1485\left(\cos(3x)^4-2\cos(3x)^2+1\right)\sin(3x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(3*x)*(-1+csc(3*x)^2)^3*(1-sin(3*x)^2)^5,x, algorithm="fricas"
)
```

```
[Out] 1/1485*(45*cos(3*x)^16 + 80*cos(3*x)^14 + 160*cos(3*x)^12 + 384*cos(3*x)^10
 + 1280*cos(3*x)^8 + 10240*cos(3*x)^6 - 61440*cos(3*x)^4 + 81920*cos(3*x)^2
 - 32768)/((cos(3*x)^4 - 2*cos(3*x)^2 + 1)*sin(3*x))
```

giac [A] time = 0.49, size = 73, normalized size = 0.84

$$\frac{1}{33}\sin(3x)^{11}-\frac{8}{27}\sin(3x)^{9}+\frac{4}{3}\sin(3x)^{7}-\frac{56}{15}\sin(3x)^{5}+\frac{70}{9}\sin(3x)^{3}-\frac{420\sin(3x)^{4}-40\sin(3x)^{2}+3}{45\sin(3x)^{5}}-\frac{56}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(3*x)*(-1+csc(3*x)^2)^3*(1-sin(3*x)^2)^5,x, algorithm="giac")
```

```
[Out] 1/33*sin(3*x)^11 - 8/27*sin(3*x)^9 + 4/3*sin(3*x)^7 - 56/15*sin(3*x)^5 + 70
/9*sin(3*x)^3 - 1/45*(420*sin(3*x)^4 - 40*sin(3*x)^2 + 3)/sin(3*x)^5 - 56/3
*sin(3*x)
```

maple [A] time = 0.16, size = 72, normalized size = 0.83

$$\frac{\left(\sin^{11}(3x)\right)}{33}-\frac{8\left(\sin^{9}(3x)\right)}{27}+\frac{4\left(\sin^{7}(3x)\right)}{3}-\frac{56\left(\sin^{5}(3x)\right)}{15}+\frac{70\left(\sin^{3}(3x)\right)}{9}-\frac{56\sin(3x)}{3}-\frac{28}{3\sin(3x)}+\frac{8}{9\sin(3x)^{3}}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(3*x)*(-1+csc(3*x)^2)^3*(1-sin(3*x)^2)^5,x)
```

```
[Out] 1/33*sin(3*x)^11-8/27*sin(3*x)^9+4/3*sin(3*x)^7-56/15*sin(3*x)^5+70/9*sin(3
*x)^3-56/3*sin(3*x)-28/3/sin(3*x)+8/9/sin(3*x)^3-1/15/sin(3*x)^5
```

maxima [A] time = 0.31, size = 73, normalized size = 0.84

$$\frac{1}{33}\sin(3x)^{11}-\frac{8}{27}\sin(3x)^{9}+\frac{4}{3}\sin(3x)^{7}-\frac{56}{15}\sin(3x)^{5}+\frac{70}{9}\sin(3x)^{3}-\frac{420\sin(3x)^{4}-40\sin(3x)^{2}+3}{45\sin(3x)^{5}}-\frac{56}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(3*x)*(-1+csc(3*x)^2)^3*(1-sin(3*x)^2)^5,x, algorithm="maxima"
)
```

```
[Out] 1/33*sin(3*x)^11 - 8/27*sin(3*x)^9 + 4/3*sin(3*x)^7 - 56/15*sin(3*x)^5 + 70
/9*sin(3*x)^3 - 1/45*(420*sin(3*x)^4 - 40*sin(3*x)^2 + 3)/sin(3*x)^5 - 56/3
*sin(3*x)
```

mupad [B] time = 2.97, size = 74, normalized size = 0.85

$$-\frac{-45\sin(3x)^{16}+440\sin(3x)^{14}-1980\sin(3x)^{12}+5544\sin(3x)^{10}-11550\sin(3x)^{8}+27720\sin(3x)^{6}+13}{1485\sin(3x)^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-cos(3*x)*(1/sin(3*x)^2 - 1)^3*(sin(3*x)^2 - 1)^5,x)
```

```
[Out] -(13860*sin(3*x)^4 - 1320*sin(3*x)^2 + 27720*sin(3*x)^6 - 11550*sin(3*x)^8
+ 5544*sin(3*x)^10 - 1980*sin(3*x)^12 + 440*sin(3*x)^14 - 45*sin(3*x)^16 +
99)/(1485*sin(3*x)^5)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(3*x)*(-1+csc(3*x)**2)**3*(1-sin(3*x)**2)**5,x)
```

```
[Out] Timed out
```

3.897 $\int \cot(2x)\left(-1+\csc^2(2x)\right)^2\left(1-\sin^2(2x)\right)^2\,dx$

Optimal. Leaf size=42

$$\frac{1}{8}\sin^4(2x)-\sin^2(2x)-\frac{1}{8}\csc^4(2x)+\csc^2(2x)+3\log(\sin(2x))$$

```
[Out] csc(2*x)^2-1/8*csc(2*x)^4+3*ln(sin(2*x))-sin(2*x)^2+1/8*sin(2*x)^4
```

Rubi [A] time = 0.12, antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {3175, 4360, 266, 43}

$$\frac{1}{8}\sin^4(2x)-\sin^2(2x)-\frac{1}{8}\csc^4(2x)+\csc^2(2x)+3\log(\sin(2x))$$

Antiderivative was successfully verified.

```
[In] Int[Cot[2*x]*(-1 + Csc[2*x]^2)^2*(1 - Sin[2*x]^2)^2,x]
```

```
[Out] Csc[2*x]^2 - Csc[2*x]^4/8 + 3*Log[Sin[2*x]] - Sin[2*x]^2 + Sin[2*x]^4/8
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 3175

```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Dist[
a^p, Int[ActivateTrig[u*cos[e + f*x]^(2*p)], x], x] /; FreeQ[{a, b, e, f, p
}, x] && EqQ[a + b, 0] && IntegerQ[p]
```

Rule 4360

```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{d = FreeFacto
rs[Sin[c*(a + b*x)], x]}, Dist[1/(b*c), Subst[Int[SubstFor[1/x, Sin[c*(a +
b*x)]/d, u, x], x], x, Sin[c*(a + b*x)]/d], x] /; FunctionOfQ[Sin[c*(a + b*
```

```
x)]/d, u, x]] /; FreeQ[{a, b, c}, x] && (EqQ[F, Cot] || EqQ[F, cot])
```

Rubi steps

$$
\begin{aligned}
\int \cot(2x)\left(-1+\csc^2(2x)\right)^2\left(1-\sin^2(2x)\right)^2\,dx &= \int \cos^4(2x)\cot(2x)\left(-1+\csc^2(2x)\right)^2\,dx\\
&= \frac{1}{2}\,\text{Subst}\left(\int \frac{\left(1-x^2\right)^4}{x^5}\,dx, x, \sin(2x)\right)\\
&= \frac{1}{4}\,\text{Subst}\left(\int \frac{(1-x)^4}{x^3}\,dx, x, \sin^2(2x)\right)\\
&= \frac{1}{4}\,\text{Subst}\left(\int \left(-4+\frac{1}{x^3}-\frac{4}{x^2}+\frac{6}{x}+x\right)dx, x, \sin^2(2x)\right)\\
&= \csc^2(2x)-\frac{1}{8}\csc^4(2x)+3\log(\sin(2x))-\sin^2(2x)+\frac{1}{8}\sin^4(2x)
\end{aligned}
$$

Mathematica [A] time = 0.04, size = 42, normalized size = 1.00

$$\frac{1}{8}\sin^4(2x)-\sin^2(2x)-\frac{1}{8}\csc^4(2x)+\csc^2(2x)+3\log(\sin(2x))$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[2*x]*(-1 + Csc[2*x]^2)^2*(1 - Sin[2*x]^2)^2,x]
```

```
[Out] Csc[2*x]^2 - Csc[2*x]^4/8 + 3*Log[Sin[2*x]] - Sin[2*x]^2 + Sin[2*x]^4/8
```

fricas [B] time = 0.90, size = 79, normalized size = 1.88

$$\frac{8\cos(2x)^8+32\cos(2x)^6-115\cos(2x)^4+38\cos(2x)^2+192\left(\cos(2x)^4-2\cos(2x)^2+1\right)\log\left(\frac{1}{2}\sin(2x)\right)}{64\left(\cos(2x)^4-2\cos(2x)^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(2*x)*(-1+csc(2*x)^2)^2*(1-sin(2*x)^2)^2,x, algorithm="fricas"
)
```

```
[Out] 1/64*(8*cos(2*x)^8 + 32*cos(2*x)^6 - 115*cos(2*x)^4 + 38*cos(2*x)^2 + 192*(
cos(2*x)^4 - 2*cos(2*x)^2 + 1)*log(1/2*sin(2*x)) + 29)/(cos(2*x)^4 - 2*cos(
2*x)^2 + 1)
```

giac [A] time = 0.14, size = 52, normalized size = 1.24

$$\frac{1}{8}\cos(2x)^4+\frac{3}{4}\cos(2x)^2-\frac{8\cos(2x)^2-7}{8\left(\cos(2x)^2-1\right)^2}+\frac{3}{2}\log\left(-\cos(2x)^2+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(2*x)*(-1+csc(2*x)^2)^2*(1-sin(2*x)^2)^2,x, algorithm="giac")
```

```
[Out] 1/8*cos(2*x)^4 + 3/4*cos(2*x)^2 - 1/8*(8*cos(2*x)^2 - 7)/(cos(2*x)^2 - 1)^2
 + 3/2*log(-cos(2*x)^2 + 1)
```

maple [A] time = 0.16, size = 37, normalized size = 0.88

$$\frac{\left(\sin^4(2x)\right)}{8}+\cos^2(2x)+3\ln(\sin(2x))+\frac{1}{\sin(2x)^2}-\frac{1}{8\sin(2x)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(2*x)*(-1+csc(2*x)^2)^2*(1-sin(2*x)^2)^2,x)
```

```
[Out] 1/8*sin(2*x)^4+cos(2*x)^2+3*ln(sin(2*x))+1/sin(2*x)^2-1/8/sin(2*x)^4
```

maxima [A] time = 0.30, size = 44, normalized size = 1.05

$$\frac{1}{8}\sin(2x)^4-\sin(2x)^2+\frac{8\sin(2x)^2-1}{8\sin(2x)^4}+\frac{3}{2}\log\left(\sin(2x)^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(2*x)*(-1+csc(2*x)^2)^2*(1-sin(2*x)^2)^2,x, algorithm="maxima"
)
```

```
[Out] 1/8*sin(2*x)^4 - sin(2*x)^2 + 1/8*(8*sin(2*x)^2 - 1)/sin(2*x)^4 + 3/2*log(s
in(2*x)^2)
```

mupad [B] time = 3.16, size = 71, normalized size = 1.69

$$3\ln(\tan(2x))-\frac{3\ln\left(\tan(2x)^2+1\right)}{2}+\frac{3\tan(2x)^6+\frac{9\tan(2x)^4}{2}+\tan(2x)^2-\frac{1}{4}}{2\left(\tan(2x)^8+2\tan(2x)^6+\tan(2x)^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(2*x)*(1/sin(2*x)^2 - 1)^2*(sin(2*x)^2 - 1)^2,x)
```

```
[Out] 3*log(tan(2*x)) - (3*log(tan(2*x)^2 + 1))/2 + (tan(2*x)^2 + (9*tan(2*x)^4)/
2 + 3*tan(2*x)^6 - 1/4)/(2*(tan(2*x)^4 + 2*tan(2*x)^6 + tan(2*x)^8))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(2*x)*(-1+csc(2*x)**2)**2*(1-sin(2*x)**2)**2,x)
```

```
[Out] Timed out
```

3.898 $\int \cos(2x)\left(-1+\csc^2(2x)\right)^4\left(1-\sin^2(2x)\right)^2 dx$

Optimal. Leaf size=63

$$\frac{1}{10}\sin^5(2x)-\sin^3(2x)+\frac{15}{2}\sin(2x)-\frac{1}{14}\csc^7(2x)+\frac{3}{5}\csc^5(2x)-\frac{5}{2}\csc^3(2x)+10\csc(2x)$$

```
[Out] 10*csc(2*x)-5/2*csc(2*x)^3+3/5*csc(2*x)^5-1/14*csc(2*x)^7+15/2*sin(2*x)-sin
(2*x)^3+1/10*sin(2*x)^5
```

Rubi [A] time = 0.12, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {3175, 4120, 2590, 270}

$$\frac{1}{10}\sin^5(2x)-\sin^3(2x)+\frac{15}{2}\sin(2x)-\frac{1}{14}\csc^7(2x)+\frac{3}{5}\csc^5(2x)-\frac{5}{2}\csc^3(2x)+10\csc(2x)$$

Antiderivative was successfully verified.

```
[In] Int[Cos[2*x]*(-1 + Csc[2*x]^2)^4*(1 - Sin[2*x]^2)^2,x]
```

```
[Out] 10*Csc[2*x] - (5*Csc[2*x]^3)/2 + (3*Csc[2*x]^5)/5 - Csc[2*x]^7/14 + (15*Sin
[2*x])/2 - Sin[2*x]^3 + Sin[2*x]^5/10
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2590

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> -Dist[f^(-1), Subst[Int[(1 - x^2)^((m + n - 1)/2)/x^n, x], x, Cos[e + f*
x]], x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n - 1)/2]
```

Rule 3175

```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Dist[
a^p, Int[ActivateTrig[u*cos[e + f*x]^(2*p)], x], x] /; FreeQ[{a, b, e, f, p
}, x] && EqQ[a + b, 0] && IntegerQ[p]
```

Rule 4120

```
Int[(u_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Dist[
b^p, Int[ActivateTrig[u*tan[e + f*x]^(2*p)], x], x] /; FreeQ[{a, b, e, f, p
```

```
}, x] && EqQ[a + b, 0] && IntegerQ[p]
```

Rubi steps

$$\int \cos(2x)\left(-1+\csc^2(2x)\right)^4\left(1-\sin^2(2x)\right)^2\,dx = \int \cos^5(2x)\left(-1+\csc^2(2x)\right)^4\,dx$$
$$= \int \cos^5(2x)\cot^8(2x)\,dx$$
$$= -\left(\frac{1}{2}\text{Subst}\left(\int \frac{\left(1-x^2\right)^6}{x^8}\,dx, x, -\sin(2x)\right)\right)$$
$$= -\left(\frac{1}{2}\text{Subst}\left(\int \left(15+\frac{1}{x^8}-\frac{6}{x^6}+\frac{15}{x^4}-\frac{20}{x^2}-6x^2+x^4\right)dx, x, -\text{s}\right.\right.$$
$$= 10\csc(2x)-\frac{5}{2}\csc^3(2x)+\frac{3}{5}\csc^5(2x)-\frac{1}{14}\csc^7(2x)+\frac{15}{2}\sin(2x$$

Mathematica [A] time = 0.03, size = 63, normalized size = 1.00

$$\frac{1}{10}\sin^5(2x)-\sin^3(2x)+\frac{15}{2}\sin(2x)-\frac{1}{14}\csc^7(2x)+\frac{3}{5}\csc^5(2x)-\frac{5}{2}\csc^3(2x)+10\csc(2x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[2*x]*(-1 + Csc[2*x]^2)^4*(1 - Sin[2*x]^2)^2,x]
```

```
[Out] 10*Csc[2*x] - (5*Csc[2*x]^3)/2 + (3*Csc[2*x]^5)/5 - Csc[2*x]^7/14 + (15*Sin
[2*x])/2 - Sin[2*x]^3 + Sin[2*x]^5/10
```

fricas [A] time = 2.93, size = 84, normalized size = 1.33

$$-\frac{7\cos(2x)^{12}+28\cos(2x)^{10}+280\cos(2x)^{8}-2240\cos(2x)^{6}+4480\cos(2x)^{4}-3584\cos(2x)^{2}+1024}{70\left(\cos(2x)^{6}-3\cos(2x)^{4}+3\cos(2x)^{2}-1\right)\sin(2x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(2*x)*(-1+csc(2*x)^2)^4*(1-sin(2*x)^2)^2,x, algorithm="fricas"
)
```

```
[Out] -1/70*(7*cos(2*x)^12 + 28*cos(2*x)^10 + 280*cos(2*x)^8 - 2240*cos(2*x)^6 +
4480*cos(2*x)^4 - 3584*cos(2*x)^2 + 1024)/((cos(2*x)^6 - 3*cos(2*x)^4 + 3*c
os(2*x)^2 - 1)*sin(2*x))
```

giac [A] time = 0.16, size = 57, normalized size = 0.90

$$\frac{1}{10}\sin(2x)^5 - \sin(2x)^3 + \frac{700\sin(2x)^6 - 175\sin(2x)^4 + 42\sin(2x)^2 - 5}{70\sin(2x)^7} + \frac{15}{2}\sin(2x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(2*x)*(-1+csc(2*x)^2)^4*(1-sin(2*x)^2)^2,x, algorithm="giac")
```

```
[Out] 1/10*sin(2*x)^5 - sin(2*x)^3 + 1/70*(700*sin(2*x)^6 - 175*sin(2*x)^4 + 42*s
in(2*x)^2 - 5)/sin(2*x)^7 + 15/2*sin(2*x)
```

maple [A] time = 0.15, size = 56, normalized size = 0.89

$$\frac{\left(\sin^5(2x)\right)}{10} - \left(\sin^3(2x)\right) + \frac{15\sin(2x)}{2} + \frac{10}{\sin(2x)} - \frac{5}{2\sin(2x)^3} + \frac{3}{5\sin(2x)^5} - \frac{1}{14\sin(2x)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(2*x)*(-1+csc(2*x)^2)^4*(1-sin(2*x)^2)^2,x)
```

```
[Out] 1/10*sin(2*x)^5-sin(2*x)^3+15/2*sin(2*x)+10/sin(2*x)-5/2/sin(2*x)^3+3/5/sin
(2*x)^5-1/14/sin(2*x)^7
```

maxima [A] time = 0.31, size = 57, normalized size = 0.90

$$\frac{1}{10}\sin(2x)^5 - \sin(2x)^3 + \frac{700\sin(2x)^6 - 175\sin(2x)^4 + 42\sin(2x)^2 - 5}{70\sin(2x)^7} + \frac{15}{2}\sin(2x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(2*x)*(-1+csc(2*x)^2)^4*(1-sin(2*x)^2)^2,x, algorithm="maxima"
)
```

```
[Out] 1/10*sin(2*x)^5 - sin(2*x)^3 + 1/70*(700*sin(2*x)^6 - 175*sin(2*x)^4 + 42*s
in(2*x)^2 - 5)/sin(2*x)^7 + 15/2*sin(2*x)
```

mupad [B] time = 2.97, size = 57, normalized size = 0.90

$$\frac{\frac{\sin(2x)^{12}}{10} - \sin(2x)^{10} + \frac{15\sin(2x)^8}{2} + 10\sin(2x)^6 - \frac{5\sin(2x)^4}{2} + \frac{3\sin(2x)^2}{5} - \frac{1}{14}}{\sin(2x)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(2*x)*(1/sin(2*x)^2 - 1)^4*(sin(2*x)^2 - 1)^2,x)
```

```
[Out] ((3*sin(2*x)^2)/5 - (5*sin(2*x)^4)/2 + 10*sin(2*x)^6 + (15*sin(2*x)^8)/2 -
sin(2*x)^10 + sin(2*x)^12/10 - 1/14)/sin(2*x)^7
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(2*x)*(-1+csc(2*x)**2)**4*(1-sin(2*x)**2)**2,x)
```

```
[Out] Timed out
```

3.899 $\int \cot(3x)\left(-1+\csc^2(3x)\right)^3\left(1-\sin^2(3x)\right)^2\,dx$

Optimal. Leaf size=60

$$-\frac{1}{12}\sin^4(3x)+\frac{5}{6}\sin^2(3x)-\frac{1}{18}\csc^6(3x)+\frac{5}{12}\csc^4(3x)-\frac{5}{3}\csc^2(3x)-\frac{10}{3}\log(\sin(3x))$$

```
[Out] -5/3*csc(3*x)^2+5/12*csc(3*x)^4-1/18*csc(3*x)^6-10/3*ln(sin(3*x))+5/6*sin(3
*x)^2-1/12*sin(3*x)^4
```

Rubi [A] time = 0.13, antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {3175, 4360, 266, 43}

$$-\frac{1}{12}\sin^4(3x)+\frac{5}{6}\sin^2(3x)-\frac{1}{18}\csc^6(3x)+\frac{5}{12}\csc^4(3x)-\frac{5}{3}\csc^2(3x)-\frac{10}{3}\log(\sin(3x))$$

Antiderivative was successfully verified.

```
[In] Int[Cot[3*x]*(-1 + Csc[3*x]^2)^3*(1 - Sin[3*x]^2)^2,x]
```

```
[Out] (-5*Csc[3*x]^2)/3 + (5*Csc[3*x]^4)/12 - Csc[3*x]^6/18 - (10*Log[Sin[3*x]])/
3 + (5*Sin[3*x]^2)/6 - Sin[3*x]^4/12
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 3175

```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Dist[
a^p, Int[ActivateTrig[u*cos[e + f*x]^(2*p)], x], x] /; FreeQ[{a, b, e, f, p
}, x] && EqQ[a + b, 0] && IntegerQ[p]
```

Rule 4360

```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{d = FreeFacto
rs[Sin[c*(a + b*x)], x]}, Dist[1/(b*c), Subst[Int[SubstFor[1/x, Sin[c*(a +
b*x)]/d, u, x], x], x, Sin[c*(a + b*x)]/d], x] /; FunctionOfQ[Sin[c*(a + b*
x)]/d, u, x]] /; FreeQ[{a, b, c}, x] && (EqQ[F, Cot] || EqQ[F, cot])
```

Rubi steps

$$\begin{aligned}
\int \cot(3x)\left(-1+\csc^2(3x)\right)^3\left(1-\sin^2(3x)\right)^2\,dx &= \int \cos^4(3x)\cot(3x)\left(-1+\csc^2(3x)\right)^3\,dx\\
&= \frac{1}{3}\text{Subst}\left(\int \frac{\left(1-x^2\right)^5}{x^7}\,dx, x, \sin(3x)\right)\\
&= \frac{1}{6}\text{Subst}\left(\int \frac{(1-x)^5}{x^4}\,dx, x, \sin^2(3x)\right)\\
&= \frac{1}{6}\text{Subst}\left(\int \left(5+\frac{1}{x^4}-\frac{5}{x^3}+\frac{10}{x^2}-\frac{10}{x}-x\right)dx, x, \sin^2(3x)\right)\\
&= -\frac{5}{3}\csc^2(3x)+\frac{5}{12}\csc^4(3x)-\frac{1}{18}\csc^6(3x)-\frac{10}{3}\log(\sin(3x))+\frac{5}{6}
\end{aligned}$$

Mathematica [A] time = 0.13, size = 52, normalized size = 0.87

$$\frac{1}{36}\left(-3\sin^4(3x)+30\sin^2(3x)-2\csc^6(3x)+15\csc^4(3x)-60\csc^2(3x)-120\log(\sin(3x))\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[3*x]*(-1 + Csc[3*x]^2)^3*(1 - Sin[3*x]^2)^2,x]
```

```
[Out] (-60*Csc[3*x]^2 + 15*Csc[3*x]^4 - 2*Csc[3*x]^6 - 120*Log[Sin[3*x]] + 30*Sin
[3*x]^2 - 3*Sin[3*x]^4)/36
```

fricas [B] time = 0.84, size = 103, normalized size = 1.72

$$-\frac{24\cos(3x)^{10}+120\cos(3x)^8-609\cos(3x)^6+387\cos(3x)^4+333\cos(3x)^2+960\left(\cos(3x)^6-3\cos(3x\right.}{288\left(\cos(3x)^6-3\cos(3x)^4+3\cos(3x)^2-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(3*x)*(-1+csc(3*x)^2)^3*(1-sin(3*x)^2)^2,x, algorithm="fricas"
)
```

```
[Out] -1/288*(24*cos(3*x)^10 + 120*cos(3*x)^8 - 609*cos(3*x)^6 + 387*cos(3*x)^4 +
 333*cos(3*x)^2 + 960*(cos(3*x)^6 - 3*cos(3*x)^4 + 3*cos(3*x)^2 - 1)*log(1/
2*sin(3*x)) - 271)/(cos(3*x)^6 - 3*cos(3*x)^4 + 3*cos(3*x)^2 - 1)
```

giac [A] time = 0.25, size = 60, normalized size = 1.00

$$-\frac{1}{12}\sin(3x)^4+\frac{5}{6}\sin(3x)^2+\frac{110\sin(3x)^6-60\sin(3x)^4+15\sin(3x)^2-2}{36\sin(3x)^6}-\frac{5}{3}\log\left(\sin(3x)^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(3*x)*(-1+csc(3*x)^2)^3*(1-sin(3*x)^2)^2,x, algorithm="giac")
```

```
[Out] -1/12*sin(3*x)^4 + 5/6*sin(3*x)^2 + 1/36*(110*sin(3*x)^6 - 60*sin(3*x)^4 +
15*sin(3*x)^2 - 2)/sin(3*x)^6 - 5/3*log(sin(3*x)^2)
```

maple [A] time = 0.16, size = 49, normalized size = 0.82

$$-\frac{\left(\sin^4(3x)\right)}{12}-\frac{5\left(\cos^2(3x)\right)}{6}-\frac{10\ln(\sin(3x))}{3}-\frac{5}{3\sin(3x)^2}+\frac{5}{12\sin(3x)^4}-\frac{1}{18\sin(3x)^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(3*x)*(-1+csc(3*x)^2)^3*(1-sin(3*x)^2)^2,x)
```

```
[Out] -1/12*sin(3*x)^4-5/6*cos(3*x)^2-10/3*ln(sin(3*x))-5/3/sin(3*x)^2+5/12/sin(3
*x)^4-1/18/sin(3*x)^6
```

maxima [A] time = 0.32, size = 52, normalized size = 0.87

$$-\frac{1}{12}\sin(3x)^4+\frac{5}{6}\sin(3x)^2-\frac{60\sin(3x)^4-15\sin(3x)^2+2}{36\sin(3x)^6}-\frac{5}{3}\log\left(\sin(3x)^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(3*x)*(-1+csc(3*x)^2)^3*(1-sin(3*x)^2)^2,x, algorithm="maxima"
)
```

```
[Out] -1/12*sin(3*x)^4 + 5/6*sin(3*x)^2 - 1/36*(60*sin(3*x)^4 - 15*sin(3*x)^2 + 2
)/sin(3*x)^6 - 5/3*log(sin(3*x)^2)
```

mupad [B] time = 4.67, size = 84, normalized size = 1.40

$$\frac{\ln\left(\left(\tan(3x)^2+1\right)^5\right)}{3}-\frac{10\ln(\tan(3x))}{3}-\frac{5\tan(3x)^8+\frac{15\tan(3x)^6}{2}+\frac{5\tan(3x)^4}{3}-\frac{5\tan(3x)^2}{12}+\frac{1}{6}}{3\left(\tan(3x)^{10}+2\tan(3x)^8+\tan(3x)^6\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(3*x)*(1/sin(3*x)^2 - 1)^3*(sin(3*x)^2 - 1)^2,x)
```

```
[Out] log((tan(3*x)^2 + 1)^5)/3 - (10*log(tan(3*x)))/3 - ((5*tan(3*x)^4)/3 - (5*t
an(3*x)^2)/12 + (15*tan(3*x)^6)/2 + 5*tan(3*x)^8 + 1/6)/(3*(tan(3*x)^6 + 2*
tan(3*x)^8 + tan(3*x)^10))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(3*x)*(-1+csc(3*x)**2)**3*(1-sin(3*x)**2)**2,x)
```

```
[Out] Timed out
```

3.900 $\int \left(1+\cot^2(9x)\right)^2 \left(1+\tan^2(9x)\right)^3 \, dx$

Optimal. Leaf size=47

$$\frac{1}{45}\tan^5(9x)+\frac{4}{27}\tan^3(9x)+\frac{2}{3}\tan(9x)-\frac{1}{27}\cot^3(9x)-\frac{4}{9}\cot(9x)$$

```
[Out] -4/9*cot(9*x)-1/27*cot(9*x)^3+2/3*tan(9*x)+4/27*tan(9*x)^3+1/45*tan(9*x)^5
```

Rubi [A] time = 0.10, antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3657, 2620, 270}

$$\frac{1}{45}\tan^5(9x)+\frac{4}{27}\tan^3(9x)+\frac{2}{3}\tan(9x)-\frac{1}{27}\cot^3(9x)-\frac{4}{9}\cot(9x)$$

Antiderivative was successfully verified.

```
[In] Int[(1 + Cot[9*x]^2)^2*(1 + Tan[9*x]^2)^3,x]
```

```
[Out] (-4*Cot[9*x])/9 - Cot[9*x]^3/27 + (2*Tan[9*x])/3 + (4*Tan[9*x]^3)/27 + Tan[
9*x]^5/45
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2620

```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Dist[1/f, Subst[Int[(1 + x^2)^((m + n)/2 - 1)/x^m, x], x, Tan[e + f*x]],
 x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n)/2]
```

Rule 3657

```
Int[(u_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*sec[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a, b]
```

Rubi steps

$$
\begin{aligned}
\int \left(1+\cot^2(9x)\right)^2 \left(1+\tan^2(9x)\right)^3 dx &= \int \left(1+\cot^2(9x)\right)^2 \sec^6(9x)\, dx \\
&= \int \csc^4(9x)\sec^6(9x)\, dx \\
&= \frac{1}{9}\text{Subst}\left(\int \frac{\left(1+x^2\right)^4}{x^4}\, dx, x, \tan(9x)\right) \\
&= \frac{1}{9}\text{Subst}\left(\int \left(6+\frac{1}{x^4}+\frac{4}{x^2}+4x^2+x^4\right) dx, x, \tan(9x)\right) \\
&= -\frac{4}{9}\cot(9x)-\frac{1}{27}\cot^3(9x)+\frac{2}{3}\tan(9x)+\frac{4}{27}\tan^3(9x)+\frac{1}{45}\tan^5(9x)
\end{aligned}
$$

Mathematica [**A**] time = 0.05, size = 59, normalized size = 1.26

$$
\frac{73}{135}\tan(9x)-\frac{11}{27}\cot(9x)-\frac{1}{27}\cot(9x)\csc^2(9x)+\frac{1}{45}\tan(9x)\sec^4(9x)+\frac{14}{135}\tan(9x)\sec^2(9x)
$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + Cot[9*x]^2)^2*(1 + Tan[9*x]^2)^3,x]
```

```
[Out] (-11*Cot[9*x])/27 - (Cot[9*x]*Csc[9*x]^2)/27 + (73*Tan[9*x])/135 + (14*Sec[
9*x]^2*Tan[9*x])/135 + (Sec[9*x]^4*Tan[9*x])/45
```

fricas [**A**] time = 0.63, size = 42, normalized size = 0.89

$$
\frac{3\,\tan(9\,x)^8+20\,\tan(9\,x)^6+90\,\tan(9\,x)^4-60\,\tan(9\,x)^2-5}{135\,\tan(9\,x)^3}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+cot(9*x)^2)^2*(1+tan(9*x)^2)^3,x, algorithm="fricas")
```

```
[Out] 1/135*(3*tan(9*x)^8 + 20*tan(9*x)^6 + 90*tan(9*x)^4 - 60*tan(9*x)^2 - 5)/ta
n(9*x)^3
```

giac [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+cot(9*x)^2)^2*(1+tan(9*x)^2)^3,x, algorithm="giac")
```

```
[Out] Timed out
```

maple [A] time = 0.14, size = 38, normalized size = 0.81

$$-\frac{4\cot(9x)}{9}-\frac{\left(\cot^3(9x)\right)}{27}+\frac{2\tan(9x)}{3}+\frac{4\left(\tan^3(9x)\right)}{27}+\frac{\left(\tan^5(9x)\right)}{45}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+cot(9*x)^2)^2*(1+tan(9*x)^2)^3,x)
```

```
[Out] -4/9*cot(9*x)-1/27*cot(9*x)^3+2/3*tan(9*x)+4/27*tan(9*x)^3+1/45*tan(9*x)^5
```

maxima [A] time = 0.33, size = 41, normalized size = 0.87

$$\frac{1}{45}\tan(9x)^5+\frac{4}{27}\tan(9x)^3-\frac{12\tan(9x)^2+1}{27\tan(9x)^3}+\frac{2}{3}\tan(9x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+cot(9*x)^2)^2*(1+tan(9*x)^2)^3,x, algorithm="maxima")
```

```
[Out] 1/45*tan(9*x)^5 + 4/27*tan(9*x)^3 - 1/27*(12*tan(9*x)^2 + 1)/tan(9*x)^3 + 2
/3*tan(9*x)
```

mupad [B] time = 5.38, size = 42, normalized size = 0.89

$$\frac{3\tan(9x)^8+20\tan(9x)^6+90\tan(9x)^4-60\tan(9x)^2-5}{135\tan(9x)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((tan(9*x)^2 + 1)^3*(cot(9*x)^2 + 1)^2,x)
```

```
[Out] (90*tan(9*x)^4 - 60*tan(9*x)^2 + 20*tan(9*x)^6 + 3*tan(9*x)^8 - 5)/(135*tan
(9*x)^3)
```

sympy [A] time = 5.20, size = 44, normalized size = 0.94

$$\frac{\tan^5(9x)}{45}+\frac{4\tan^3(9x)}{27}+\frac{2\tan(9x)}{3}-\frac{4}{9\tan(9x)}-\frac{1}{27\tan^3(9x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+cot(9*x)**2)**2*(1+tan(9*x)**2)**3,x)
```

```
[Out] tan(9*x)**5/45 + 4*tan(9*x)**3/27 + 2*tan(9*x)/3 - 4/(9*tan(9*x)) - 1/(27*t
an(9*x)**3)
```

3.901 $\int \frac{\cos(x)\left(9-7\sin^3(x)\right)^2}{1-\sin^2(x)}\,dx$

Optimal. Leaf size=43

$$-\frac{49}{5}\sin^5(x)-\frac{49\sin^3(x)}{3}+63\sin^2(x)-49\sin(x)-2\log(1-\sin(x))+128\log(\sin(x)+1)$$

```
[Out] -2*ln(1-sin(x))+128*ln(1+sin(x))-49*sin(x)+63*sin(x)^2-49/3*sin(x)^3-49/5*s
in(x)^5
```

Rubi [A] time = 0.12, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {3175, 3223, 1810, 633, 31}

$$-\frac{49}{5}\sin^5(x)-\frac{49\sin^3(x)}{3}+63\sin^2(x)-49\sin(x)-2\log(1-\sin(x))+128\log(\sin(x)+1)$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[x]*(9 - 7*Sin[x]^3)^2)/(1 - Sin[x]^2),x]
```

```
[Out] -2*Log[1 - Sin[x]] + 128*Log[1 + Sin[x]] - 49*Sin[x] + 63*Sin[x]^2 - (49*Si
n[x]^3)/3 - (49*Sin[x]^5)/5
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 633

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> With[{q = Rt[-(
a*c), 2]}, Dist[e/2 + (c*d)/(2*q), Int[1/(-q + c*x), x], x] + Dist[e/2 - (c
*d)/(2*q), Int[1/(q + c*x), x], x]] /; FreeQ[{a, c, d, e}, x] && NiceSqrtQ[
-(a*c)]
```

Rule 1810

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[Pq*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 3175

```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Dist[
a^p, Int[ActivateTrig[u*cos[e + f*x]^(2*p)], x], x] /; FreeQ[{a, b, e, f, p
```

```
}, x] && EqQ[a + b, 0] && IntegerQ[p]
```

Rule 3223

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*((c_.)*sin[(e_.) + (f_.)*(x
_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Di
st[ff/f, Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*(c*ff*x)^n)^p, x], x,
Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[(m - 1
)/2] && (EqQ[n, 4] || GtQ[m, 0] || IGtQ[p, 0] || IntegersQ[m, p])
```

Rubi steps

$$\int \frac{\cos(x)\left(9-7\sin^3(x)\right)^2}{1-\sin^2(x)}\,dx = \int \sec(x)\left(9-7\sin^3(x)\right)^2\,dx$$

$$= \text{Subst}\left(\int \frac{\left(9-7x^3\right)^2}{1-x^2}\,dx, x, \sin(x)\right)$$

$$= \text{Subst}\left(\int \left(-49+126x-49x^2-49x^4+\frac{2(65-63x)}{1-x^2}\right)dx, x, \sin(x)\right)$$

$$= -49\sin(x)+63\sin^2(x)-\frac{49\sin^3(x)}{3}-\frac{49\sin^5(x)}{5}+2\,\text{Subst}\left(\int \frac{65-63x}{1-x^2}\,dx, x,\right.$$

$$= -49\sin(x)+63\sin^2(x)-\frac{49\sin^3(x)}{3}-\frac{49\sin^5(x)}{5}+2\,\text{Subst}\left(\int \frac{1}{1-x}\,dx, x, \sin\right.$$

$$= -2\log(1-\sin(x))+128\log(1+\sin(x))-49\sin(x)+63\sin^2(x)-\frac{49\sin^3(x)}{3}-$$

Mathematica [A] time = 0.02, size = 71, normalized size = 1.65

$$-\frac{49}{5}\sin^5(x)-\frac{49\sin^3(x)}{3}-49\sin(x)-63\cos^2(x)+49\tanh^{-1}(\sin(x))+126\log(\cos(x))-81\log\left(\cos\left(\frac{x}{2}\right)-\sin\left(\frac{x}{2}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[x]*(9 - 7*Sin[x]^3)^2)/(1 - Sin[x]^2),x]
```

```
[Out] 49*ArcTanh[Sin[x]] - 63*Cos[x]^2 + 126*Log[Cos[x]] - 81*Log[Cos[x/2] - Sin[
x/2]] + 81*Log[Cos[x/2] + Sin[x/2]] - 49*Sin[x] - (49*Sin[x]^3)/3 - (49*Sin
[x]^5)/5
```

fricas [A] time = 1.95, size = 41, normalized size = 0.95

$$-63\cos(x)^2-\frac{49}{15}\left(3\cos(x)^4-11\cos(x)^2+23\right)\sin(x)+128\log(\sin(x)+1)-2\log(-\sin(x)+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*(9-7*sin(x)^3)^2/(1-sin(x)^2),x, algorithm="fricas")
```

```
[Out] -63*cos(x)^2 - 49/15*(3*cos(x)^4 - 11*cos(x)^2 + 23)*sin(x) + 128*log(sin(x
) + 1) - 2*log(-sin(x) + 1)
```

giac [A] time = 6.00, size = 39, normalized size = 0.91

$$-\frac{49}{5}\sin(x)^5-\frac{49}{3}\sin(x)^3+63\sin(x)^2+128\log(\sin(x)+1)-2\log(-\sin(x)+1)-49\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*(9-7*sin(x)^3)^2/(1-sin(x)^2),x, algorithm="giac")
```

```
[Out] -49/5*sin(x)^5 - 49/3*sin(x)^3 + 63*sin(x)^2 + 128*log(sin(x) + 1) - 2*log(
-sin(x) + 1) - 49*sin(x)
```

maple [A] time = 0.06, size = 38, normalized size = 0.88

$$-\frac{49\left(\sin^5(x)\right)}{5}-\frac{49\left(\sin^3(x)\right)}{3}+63\left(\sin^2(x)\right)-49\sin(x)-2\ln(\sin(x)-1)+128\ln(1+\sin(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)*(9-7*sin(x)^3)^2/(1-sin(x)^2),x)
```

```
[Out] -49/5*sin(x)^5-49/3*sin(x)^3+63*sin(x)^2-49*sin(x)-2*ln(sin(x)-1)+128*ln(1+
sin(x))
```

maxima [A] time = 0.32, size = 37, normalized size = 0.86

$$-\frac{49}{5}\sin(x)^5-\frac{49}{3}\sin(x)^3+63\sin(x)^2+128\log(\sin(x)+1)-2\log(\sin(x)-1)-49\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*(9-7*sin(x)^3)^2/(1-sin(x)^2),x, algorithm="maxima")
```

```
[Out] -49/5*sin(x)^5 - 49/3*sin(x)^3 + 63*sin(x)^2 + 128*log(sin(x) + 1) - 2*log(
sin(x) - 1) - 49*sin(x)
```

mupad [B] time = 0.08, size = 37, normalized size = 0.86

$$128\ \ln\left(\sin(x)+1\right)-2\ \ln\left(\sin(x)-1\right)-49\ \sin(x)+63\sin(x)^2-\frac{49\sin(x)^3}{3}-\frac{49\sin(x)^5}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(cos(x)*(7*sin(x)^3 - 9)^2)/(sin(x)^2 - 1),x)
```

```
[Out] 128*log(sin(x) + 1) - 2*log(sin(x) - 1) - 49*sin(x) + 63*sin(x)^2 - (49*sin
(x)^3)/3 - (49*sin(x)^5)/5
```

sympy [A] time = 2.95, size = 44, normalized size = 1.02

$$-2\log\left(\sin\left(x\right)-1\right)+128\log\left(\sin\left(x\right)+1\right)-\frac{49\sin^5\left(x\right)}{5}-\frac{49\sin^3\left(x\right)}{3}+63\sin^2\left(x\right)-49\sin\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*(9-7*sin(x)**3)**2/(1-sin(x)**2),x)
```

```
[Out] -2*log(sin(x) - 1) + 128*log(sin(x) + 1) - 49*sin(x)**5/5 - 49*sin(x)**3/3
+ 63*sin(x)**2 - 49*sin(x)
```

3.902 $\int \cos^4(2x)\cot^5(2x)\,dx$

Optimal. Leaf size=42

$$\frac{1}{8}\sin^4(2x)-\sin^2(2x)-\frac{1}{8}\csc^4(2x)+\csc^2(2x)+3\log(\sin(2x))$$

```
[Out] csc(2*x)^2-1/8*csc(2*x)^4+3*ln(sin(2*x))-sin(2*x)^2+1/8*sin(2*x)^4
```

Rubi [A] time = 0.04, antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {2590, 266, 43}

$$\frac{1}{8}\sin^4(2x)-\sin^2(2x)-\frac{1}{8}\csc^4(2x)+\csc^2(2x)+3\log(\sin(2x))$$

Antiderivative was successfully verified.

```
[In] Int[Cos[2*x]^4*Cot[2*x]^5,x]
```

```
[Out] Csc[2*x]^2 - Csc[2*x]^4/8 + 3*Log[Sin[2*x]] - Sin[2*x]^2 + Sin[2*x]^4/8
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 2590

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> -Dist[f^(-1), Subst[Int[(1 - x^2)^((m + n - 1)/2)/x^n, x], x, Cos[e + f*
x]], x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n - 1)/2]
```

Rubi steps

$$\int \cos^4(2x)\cot^5(2x)\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{\left(1-x^2\right)^4}{x^5}\,dx, x, -\sin(2x)\right)$$
$$= \frac{1}{4}\text{Subst}\left(\int \frac{(1-x)^4}{x^3}\,dx, x, \sin^2(2x)\right)$$
$$= \frac{1}{4}\text{Subst}\left(\int \left(-4+\frac{1}{x^3}-\frac{4}{x^2}+\frac{6}{x}+x\right)dx, x, \sin^2(2x)\right)$$
$$= \csc^2(2x)-\frac{1}{8}\csc^4(2x)+3\log(\sin(2x))-\sin^2(2x)+\frac{1}{8}\sin^4(2x)$$

Mathematica [A] time = 0.03, size = 42, normalized size = 1.00

$$\frac{1}{8}\sin^4(2x)-\sin^2(2x)-\frac{1}{8}\csc^4(2x)+\csc^2(2x)+3\log(\sin(2x))$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[2*x]^4*Cot[2*x]^5,x]
[Out] Csc[2*x]^2 - Csc[2*x]^4/8 + 3*Log[Sin[2*x]] - Sin[2*x]^2 + Sin[2*x]^4/8
```

fricas [B] time = 1.96, size = 79, normalized size = 1.88

$$\frac{8\cos(2x)^8+32\cos(2x)^6-115\cos(2x)^4+38\cos(2x)^2+192\left(\cos(2x)^4-2\cos(2x)^2+1\right)\log\left(\frac{1}{2}\sin(2x\right.}{64\left(\cos(2x)^4-2\cos(2x)^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(2*x)^4*cot(2*x)^5,x, algorithm="fricas")
[Out] 1/64*(8*cos(2*x)^8 + 32*cos(2*x)^6 - 115*cos(2*x)^4 + 38*cos(2*x)^2 + 192*(
cos(2*x)^4 - 2*cos(2*x)^2 + 1)*log(1/2*sin(2*x)) + 29)/(cos(2*x)^4 - 2*cos(
2*x)^2 + 1)
```

giac [A] time = 0.15, size = 52, normalized size = 1.24

$$\frac{1}{8}\cos(2x)^4+\frac{3}{4}\cos(2x)^2-\frac{8\cos(2x)^2-7}{8\left(\cos(2x)^2-1\right)^2}+\frac{3}{2}\log\left(-\cos(2x)^2+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(2*x)^4*cot(2*x)^5,x, algorithm="giac")
```

```
[Out] 1/8*cos(2*x)^4 + 3/4*cos(2*x)^2 - 1/8*(8*cos(2*x)^2 - 7)/(cos(2*x)^2 - 1)^2
 + 3/2*log(-cos(2*x)^2 + 1)
```

maple [A] time = 0.11, size = 69, normalized size = 1.64

$$-\frac{\cos^{10}(2x)}{8\sin(2x)^4}+\frac{3\left(\cos^{10}(2x)\right)}{8\sin(2x)^2}+\frac{3\left(\cos^{8}(2x)\right)}{8}+\frac{\left(\cos^{6}(2x)\right)}{2}+\frac{3\left(\cos^{4}(2x)\right)}{4}+\frac{3\left(\cos^{2}(2x)\right)}{2}+3\ln(\sin(2x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(2*x)^4*cot(2*x)^5,x)
```

```
[Out] -1/8/sin(2*x)^4*cos(2*x)^10+3/8/sin(2*x)^2*cos(2*x)^10+3/8*cos(2*x)^8+1/2*c
os(2*x)^6+3/4*cos(2*x)^4+3/2*cos(2*x)^2+3*ln(sin(2*x))
```

maxima [A] time = 0.34, size = 44, normalized size = 1.05

$$\frac{1}{8}\sin(2x)^4-\sin(2x)^2+\frac{8\sin(2x)^2-1}{8\sin(2x)^4}+\frac{3}{2}\log\left(\sin(2x)^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(2*x)^4*cot(2*x)^5,x, algorithm="maxima")
```

```
[Out] 1/8*sin(2*x)^4 - sin(2*x)^2 + 1/8*(8*sin(2*x)^2 - 1)/sin(2*x)^4 + 3/2*log(s
in(2*x)^2)
```

mupad [B] time = 3.05, size = 71, normalized size = 1.69

$$3\ln(\tan(2x))-\frac{3\ln\left(\tan(2x)^2+1\right)}{2}+\frac{3\tan(2x)^6+\frac{9\tan(2x)^4}{2}+\tan(2x)^2-\frac{1}{4}}{2\left(\tan(2x)^8+2\tan(2x)^6+\tan(2x)^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(2*x)^4*cot(2*x)^5,x)
```

```
[Out] 3*log(tan(2*x)) - (3*log(tan(2*x)^2 + 1))/2 + (tan(2*x)^2 + (9*tan(2*x)^4)/
2 + 3*tan(2*x)^6 - 1/4)/(2*(tan(2*x)^4 + 2*tan(2*x)^6 + tan(2*x)^8))
```

sympy [A] time = 0.10, size = 41, normalized size = 0.98

$$\frac{8\sin^2(2x)-1}{8\sin^4(2x)}+3\log(\sin(2x))+\frac{\sin^4(2x)}{8}-\sin^2(2x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(2*x)**4*cot(2*x)**5,x)
```

```
[Out] (8*sin(2*x)**2 - 1)/(8*sin(2*x)**4) + 3*log(sin(2*x)) + sin(2*x)**4/8 - sin
(2*x)**2
```

3.903 $\int \frac{\sec(x)\tan^2(x)}{4+3\sec(x)}\,dx$

Optimal. Leaf size=74

$$\frac{\tan(x)}{3}-\frac{4}{9}\tanh^{-1}(\sin(x))-\frac{1}{9}\sqrt{7}\log\left(\sqrt{7}\cos\left(\frac{x}{2}\right)-\sin\left(\frac{x}{2}\right)\right)+\frac{1}{9}\sqrt{7}\log\left(\sin\left(\frac{x}{2}\right)+\sqrt{7}\cos\left(\frac{x}{2}\right)\right)$$

```
[Out] -4/9*arctanh(sin(x))-1/9*ln(-sin(1/2*x)+cos(1/2*x)*7^(1/2))*7^(1/2)+1/9*ln(
sin(1/2*x)+cos(1/2*x)*7^(1/2))*7^(1/2)+1/3*tan(x)
```

Rubi [A] time = 0.25, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.467, Rules used = {4397, 2723, 3056, 3001, 3770, 2659, 206}

$$\frac{\tan(x)}{3}-\frac{4}{9}\tanh^{-1}(\sin(x))-\frac{1}{9}\sqrt{7}\log\left(\sqrt{7}\cos\left(\frac{x}{2}\right)-\sin\left(\frac{x}{2}\right)\right)+\frac{1}{9}\sqrt{7}\log\left(\sin\left(\frac{x}{2}\right)+\sqrt{7}\cos\left(\frac{x}{2}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[x]*Tan[x]^2)/(4 + 3*Sec[x]),x]
```

```
[Out] (-4*ArcTanh[Sin[x]])/9 - (Sqrt[7]*Log[Sqrt[7]*Cos[x/2] - Sin[x/2]])/9 + (Sq
rt[7]*Log[Sqrt[7]*Cos[x/2] + Sin[x/2]])/9 + Tan[x]/3
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 2723

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)/tan[(e_.) + (f_.)*(x_)]^2,
x_Symbol] :> Int[((a + b*Sin[e + f*x])^m*(1 - Sin[e + f*x]^2))/Sin[e + f*x]
^2, x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3001

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A*b
- a*B)/(b*c - a*d), Int[1/(a + b*Sin[e + f*x]), x], x] + Dist[(B*c - A*d)/(
b*c - a*d), Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f,
A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3056

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
-Simp[((A*b^2 + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin
[e + f*x])^(n + 1))/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2)), x] + Dist[1/((m +
1)*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e
+ f*x])^n*Simp[a*(m + 1)*(b*c - a*d)*(A + C) + d*(A*b^2 + a^2*C)*(m + n + 2
) - (c*(A*b^2 + a^2*C) + b*(m + 1)*(b*c - a*d)*(A + C))*Sin[e + f*x] - d*(A
*b^2 + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d,
e, f, A, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d
^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(
IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0]
)))
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 4397

```
Int[u_, x_Symbol] :> Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]
```

Rubi steps

$$\int \frac{\sec(x)\tan^2(x)}{4+3\sec(x)}\,dx = \int \frac{\tan^2(x)}{3+4\cos(x)}\,dx$$
$$= \int \frac{\left(1-\cos^2(x)\right)\sec^2(x)}{3+4\cos(x)}\,dx$$
$$= \frac{\tan(x)}{3} + \frac{1}{3}\int \frac{(-4-3\cos(x))\sec(x)}{3+4\cos(x)}\,dx$$
$$= \frac{\tan(x)}{3} - \frac{4}{9}\int \sec(x)\,dx + \frac{7}{9}\int \frac{1}{3+4\cos(x)}\,dx$$
$$= -\frac{4}{9}\tanh^{-1}(\sin(x)) + \frac{\tan(x)}{3} + \frac{14}{9}\text{Subst}\left(\int \frac{1}{7-x^2}\,dx, x, \tan\left(\frac{x}{2}\right)\right)$$
$$= -\frac{4}{9}\tanh^{-1}(\sin(x)) - \frac{1}{9}\sqrt{7}\log\left(\sqrt{7}\cos\left(\frac{x}{2}\right) - \sin\left(\frac{x}{2}\right)\right) + \frac{1}{9}\sqrt{7}\log\left(\sqrt{7}\cos\left(\frac{x}{2}\right) + \sin\left(\frac{x}{2}\right.\right.$$

Mathematica [A] time = 0.08, size = 63, normalized size = 0.85

$$\frac{1}{9}\left(3\tan(x) + 2\sqrt{7}\tanh^{-1}\left(\frac{\tan\left(\frac{x}{2}\right)}{\sqrt{7}}\right) + 4\log\left(\cos\left(\frac{x}{2}\right) - \sin\left(\frac{x}{2}\right)\right) - 4\log\left(\sin\left(\frac{x}{2}\right) + \cos\left(\frac{x}{2}\right)\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[x]*Tan[x]^2)/(4 + 3*Sec[x]),x]
```

```
[Out] (2*Sqrt[7]*ArcTanh[Tan[x/2]/Sqrt[7]] + 4*Log[Cos[x/2] - Sin[x/2]] - 4*Log[C
os[x/2] + Sin[x/2]] + 3*Tan[x])/9
```

fricas [A] time = 0.83, size = 82, normalized size = 1.11

$$\frac{\sqrt{7}\cos(x)\log\left(\frac{2\cos(x)^2+2\left(3\sqrt{7}\cos(x)+4\sqrt{7}\right)\sin(x)+24\cos(x)+23}{16\cos(x)^2+24\cos(x)+9}\right) - 4\cos(x)\log(\sin(x)+1) + 4\cos(x)\log(-\sin(x)+}{18\cos(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)*tan(x)^2/(4+3*sec(x)),x, algorithm="fricas")
```

```
[Out] 1/18*(sqrt(7)*cos(x)*log((2*cos(x)^2 + 2*(3*sqrt(7)*cos(x) + 4*sqrt(7))*sin
(x) + 24*cos(x) + 23)/(16*cos(x)^2 + 24*cos(x) + 9)) - 4*cos(x)*log(sin(x)
+ 1) + 4*cos(x)*log(-sin(x) + 1) + 6*sin(x))/cos(x)
```

giac [A] time = 0.23, size = 72, normalized size = 0.97

$$-\frac{1}{9}\sqrt{7}\log\left(\frac{\left|-2\sqrt{7}+2\tan\left(\frac{1}{2}x\right)\right|}{\left|2\sqrt{7}+2\tan\left(\frac{1}{2}x\right)\right|}\right)-\frac{2\tan\left(\frac{1}{2}x\right)}{3\left(\tan\left(\frac{1}{2}x\right)^2-1\right)}-\frac{4}{9}\log\left(\left|\tan\left(\frac{1}{2}x\right)+1\right|\right)+\frac{4}{9}\log\left(\left|\tan\left(\frac{1}{2}x\right)-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)*tan(x)^2/(4+3*sec(x)),x, algorithm="giac")
```

```
[Out] -1/9*sqrt(7)*log(abs(-2*sqrt(7) + 2*tan(1/2*x))/abs(2*sqrt(7) + 2*tan(1/2*x
))) - 2/3*tan(1/2*x)/(tan(1/2*x)^2 - 1) - 4/9*log(abs(tan(1/2*x) + 1)) + 4/
9*log(abs(tan(1/2*x) - 1))
```

maple [A] time = 0.07, size = 55, normalized size = 0.74

$$\frac{2\sqrt{7}\ \operatorname{arctanh}\left(\frac{\tan\left(\frac{x}{2}\right)\sqrt{7}}{7}\right)}{9}-\frac{1}{3\left(\tan\left(\frac{x}{2}\right)-1\right)}+\frac{4\ln\left(\tan\left(\frac{x}{2}\right)-1\right)}{9}-\frac{1}{3\left(1+\tan\left(\frac{x}{2}\right)\right)}-\frac{4\ln\left(1+\tan\left(\frac{x}{2}\right)\right)}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(x)*tan(x)^2/(4+3*sec(x)),x)
```

```
[Out] 2/9*7^(1/2)*arctanh(1/7*tan(1/2*x)*7^(1/2))-1/3/(tan(1/2*x)-1)+4/9*ln(tan(1
/2*x)-1)-1/3/(1+tan(1/2*x))-4/9*ln(1+tan(1/2*x))
```

maxima [A] time = 1.08, size = 91, normalized size = 1.23

$$-\frac{1}{9}\sqrt{7}\log\left(-\frac{\sqrt{7}-\frac{\sin(x)}{\cos(x)+1}}{\sqrt{7}+\frac{\sin(x)}{\cos(x)+1}}\right)-\frac{2\sin(x)}{3\left(\frac{\sin(x)^2}{(\cos(x)+1)^2}-1\right)(\cos(x)+1)}-\frac{4}{9}\log\left(\frac{\sin(x)}{\cos(x)+1}+1\right)+\frac{4}{9}\log\left(\frac{\sin(x)}{\cos(x)+1}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)*tan(x)^2/(4+3*sec(x)),x, algorithm="maxima")
```

```
[Out] -1/9*sqrt(7)*log(-(sqrt(7) - sin(x)/(cos(x) + 1))/(sqrt(7) + sin(x)/(cos(x)
 + 1))) - 2/3*sin(x)/((sin(x)^2/(cos(x) + 1)^2 - 1)*(cos(x) + 1)) - 4/9*log
(sin(x)/(cos(x) + 1) + 1) + 4/9*log(sin(x)/(cos(x) + 1) - 1)
```

mupad [B] time = 3.14, size = 41, normalized size = 0.55

$$\frac{2\sqrt{7}\ \operatorname{atanh}\left(\frac{\sqrt{7}\tan\left(\frac{x}{2}\right)}{7}\right)}{9}-\frac{2\tan\left(\frac{x}{2}\right)}{3\left(\tan\left(\frac{x}{2}\right)^2-1\right)}-\frac{8\operatorname{atanh}\left(\tan\left(\frac{x}{2}\right)\right)}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(x)^2/(cos(x)*(3/cos(x) + 4)),x)
```

```
[Out] (2*7^(1/2)*atanh((7^(1/2)*tan(x/2))/7))/9 - (2*tan(x/2))/(3*(tan(x/2)^2 - 1
)) - (8*atanh(tan(x/2)))/9
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan^2(x)\sec(x)}{3\sec(x)+4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)*tan(x)**2/(4+3*sec(x)),x)
```

```
[Out] Integral(tan(x)**2*sec(x)/(3*sec(x) + 4), x)
```

3.904 $\int x \sec(1+x)\tan(1+x)\,dx$

Optimal. Leaf size=14

$$x\sec(x+1)-\tanh^{-1}(\sin(x+1))$$

```
[Out] -arctanh(sin(1+x))+x*sec(1+x)
```

Rubi [A] time = 0.01, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {3757, 3770}

$$x\sec(x+1)-\tanh^{-1}(\sin(x+1))$$

Antiderivative was successfully verified.

```
[In] Int[x*Sec[1 + x]*Tan[1 + x],x]
```

```
[Out] -ArcTanh[Sin[1 + x]] + x*Sec[1 + x]
```

Rule 3757

```
Int[(x_)^(m_.)*Sec[(a_.) + (b_.)*(x_)^(n_.)]^(p_.)*Tan[(a_.) + (b_.)*(x_)^(
n_.)]^(q_.), x_Symbol] :> Simp[(x^(m - n + 1)*Sec[a + b*x^n]^p)/(b*n*p), x]
 - Dist[(m - n + 1)/(b*n*p), Int[x^(m - n)*Sec[a + b*x^n]^p, x], x] /; Free
Q[{a, b, p}, x] && IntegerQ[n] && GeQ[m, n] && EqQ[q, 1]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int x\sec(1+x)\tan(1+x)\,dx &= x\sec(1+x)-\int \sec(1+x)\,dx\\ &= -\tanh^{-1}(\sin(1+x))+x\sec(1+x)\end{aligned}$$

Mathematica [B] time = 0.04, size = 47, normalized size = 3.36

$$x\sec(x+1)+\log\left(\cos\left(\frac{x+1}{2}\right)-\sin\left(\frac{x+1}{2}\right)\right)-\log\left(\sin\left(\frac{x+1}{2}\right)+\cos\left(\frac{x+1}{2}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Sec[1 + x]*Tan[1 + x],x]
```

```
[Out] Log[Cos[(1 + x)/2] - Sin[(1 + x)/2]] - Log[Cos[(1 + x)/2] + Sin[(1 + x)/2]]
 + x*Sec[1 + x]
```

fricas [B] time = 0.67, size = 39, normalized size = 2.79

$$-\frac{\cos(x+1)\log(\sin(x+1)+1)-\cos(x+1)\log(-\sin(x+1)+1)-2x}{2\cos(x+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sec(1+x)*tan(1+x),x, algorithm="fricas")
```

```
[Out] -1/2*(cos(x + 1)*log(sin(x + 1) + 1) - cos(x + 1)*log(-sin(x + 1) + 1) - 2*
x)/cos(x + 1)
```

giac [B] time = 0.52, size = 1179, normalized size = 84.21

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sec(1+x)*tan(1+x),x, algorithm="giac")
```

```
[Out] 1/2*(2*x*tan(1/2)^2*tan(1/2*x)^2 + log(2*(tan(1/2)^2*tan(1/2*x)^4 + 2*tan(1
/2)^2*tan(1/2*x)^3 + 2*tan(1/2)*tan(1/2*x)^4 + 2*tan(1/2)^2*tan(1/2*x)^2 +
tan(1/2*x)^4 + 2*tan(1/2)^2*tan(1/2*x) - 2*tan(1/2*x)^3 + tan(1/2)^2 + 2*ta
n(1/2*x)^2 - 2*tan(1/2) - 2*tan(1/2*x) + 1)/(tan(1/2)^2 + 1))*tan(1/2)^2*ta
n(1/2*x)^2 - log(2*(tan(1/2)^2*tan(1/2*x)^4 - 2*tan(1/2)^2*tan(1/2*x)^3 - 2
*tan(1/2)*tan(1/2*x)^4 + 2*tan(1/2)^2*tan(1/2*x)^2 + tan(1/2*x)^4 - 2*tan(1
/2)^2*tan(1/2*x) + 2*tan(1/2*x)^3 + tan(1/2)^2 + 2*tan(1/2*x)^2 + 2*tan(1/2
) + 2*tan(1/2*x) + 1)/(tan(1/2)^2 + 1))*tan(1/2)^2*tan(1/2*x)^2 + 2*x*tan(1
/2)^2 - log(2*(tan(1/2)^2*tan(1/2*x)^4 + 2*tan(1/2)^2*tan(1/2*x)^3 + 2*tan(
1/2)*tan(1/2*x)^4 + 2*tan(1/2)^2*tan(1/2*x)^2 + tan(1/2*x)^4 + 2*tan(1/2)^2
*tan(1/2*x) - 2*tan(1/2*x)^3 + tan(1/2)^2 + 2*tan(1/2*x)^2 - 2*tan(1/2) - 2
*tan(1/2*x) + 1)/(tan(1/2)^2 + 1))*tan(1/2)^2 + log(2*(tan(1/2)^2*tan(1/2*x
)^4 - 2*tan(1/2)^2*tan(1/2*x)^3 - 2*tan(1/2)*tan(1/2*x)^4 + 2*tan(1/2)^2*ta
n(1/2*x)^2 + tan(1/2*x)^4 - 2*tan(1/2)^2*tan(1/2*x) + 2*tan(1/2*x)^3 + tan(
1/2)^2 + 2*tan(1/2*x)^2 + 2*tan(1/2) + 2*tan(1/2*x) + 1)/(tan(1/2)^2 + 1))*
tan(1/2)^2 - 4*log(2*(tan(1/2)^2*tan(1/2*x)^4 + 2*tan(1/2)^2*tan(1/2*x)^3 +
 2*tan(1/2)*tan(1/2*x)^4 + 2*tan(1/2)^2*tan(1/2*x)^2 + tan(1/2*x)^4 + 2*tan
(1/2)^2*tan(1/2*x) - 2*tan(1/2*x)^3 + tan(1/2)^2 + 2*tan(1/2*x)^2 - 2*tan(1
/2) - 2*tan(1/2*x) + 1)/(tan(1/2)^2 + 1))*tan(1/2)*tan(1/2*x) + 4*log(2*(ta
n(1/2)^2*tan(1/2*x)^4 - 2*tan(1/2)^2*tan(1/2*x)^3 - 2*tan(1/2)*tan(1/2*x)^4
 + 2*tan(1/2)^2*tan(1/2*x)^2 + tan(1/2*x)^4 - 2*tan(1/2)^2*tan(1/2*x) + 2*t
an(1/2*x)^3 + tan(1/2)^2 + 2*tan(1/2*x)^2 + 2*tan(1/2) + 2*tan(1/2*x) + 1)/
```

```
(tan(1/2)^2 + 1))*tan(1/2)*tan(1/2*x) + 2*x*tan(1/2*x)^2 - log(2*(tan(1/2)^
2*tan(1/2*x)^4 + 2*tan(1/2)^2*tan(1/2*x)^3 + 2*tan(1/2)*tan(1/2*x)^4 + 2*ta
n(1/2)^2*tan(1/2*x)^2 + tan(1/2*x)^4 + 2*tan(1/2)^2*tan(1/2*x) - 2*tan(1/2*
x)^3 + tan(1/2)^2 + 2*tan(1/2*x)^2 - 2*tan(1/2) - 2*tan(1/2*x) + 1)/(tan(1/
2)^2 + 1))*tan(1/2*x)^2 + log(2*(tan(1/2)^2*tan(1/2*x)^4 - 2*tan(1/2)^2*tan
(1/2*x)^3 - 2*tan(1/2)*tan(1/2*x)^4 + 2*tan(1/2)^2*tan(1/2*x)^2 + tan(1/2*x
)^4 - 2*tan(1/2)^2*tan(1/2*x) + 2*tan(1/2*x)^3 + tan(1/2)^2 + 2*tan(1/2*x)^
2 + 2*tan(1/2) + 2*tan(1/2*x) + 1)/(tan(1/2)^2 + 1))*tan(1/2*x)^2 + 2*x + l
og(2*(tan(1/2)^2*tan(1/2*x)^4 + 2*tan(1/2)^2*tan(1/2*x)^3 + 2*tan(1/2)*tan(
1/2*x)^4 + 2*tan(1/2)^2*tan(1/2*x)^2 + tan(1/2*x)^4 + 2*tan(1/2)^2*tan(1/2*
x) - 2*tan(1/2*x)^3 + tan(1/2)^2 + 2*tan(1/2*x)^2 - 2*tan(1/2) - 2*tan(1/2*
x) + 1)/(tan(1/2)^2 + 1)) - log(2*(tan(1/2)^2*tan(1/2*x)^4 - 2*tan(1/2)^2*t
an(1/2*x)^3 - 2*tan(1/2)*tan(1/2*x)^4 + 2*tan(1/2)^2*tan(1/2*x)^2 + tan(1/2
*x)^4 - 2*tan(1/2)^2*tan(1/2*x) + 2*tan(1/2*x)^3 + tan(1/2)^2 + 2*tan(1/2*x
)^2 + 2*tan(1/2) + 2*tan(1/2*x) + 1)/(tan(1/2)^2 + 1)))/(tan(1/2)^2*tan(1/2
*x)^2 - tan(1/2)^2 - 4*tan(1/2)*tan(1/2*x) - tan(1/2*x)^2 + 1)
```

maple [B] time = 0.04, size = 32, normalized size = 2.29

$$\frac{1+x}{\cos\left(1+x\right)}-\ln\left(\sec\left(1+x\right)+\tan\left(1+x\right)\right)-\frac{1}{\cos\left(1+x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*sec(1+x)*tan(1+x),x)
```

```
[Out] (1+x)/cos(1+x)-ln(sec(1+x)+tan(1+x))-1/cos(1+x)
```

maxima [B] time = 1.32, size = 176, normalized size = 12.57

$$\frac{4\,(x+1)\cos\left(2\,x+2\right)\cos\left(x+1\right)+4\,(x+1)\sin\left(2\,x+2\right)\sin\left(x+1\right)+4\,(x+1)\cos\left(x+1\right)-\left(\cos\left(2\,x+2\right)^2+\mathrm{si}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sec(1+x)*tan(1+x),x, algorithm="maxima")
```

```
[Out] 1/2*(4*(x + 1)*cos(2*x + 2)*cos(x + 1) + 4*(x + 1)*sin(2*x + 2)*sin(x + 1)
+ 4*(x + 1)*cos(x + 1) - (cos(2*x + 2)^2 + sin(2*x + 2)^2 + 2*cos(2*x + 2)
+ 1)*log(cos(x + 1)^2 + sin(x + 1)^2 + 2*sin(x + 1) + 1) + (cos(2*x + 2)^2
+ sin(2*x + 2)^2 + 2*cos(2*x + 2) + 1)*log(cos(x + 1)^2 + sin(x + 1)^2 - 2*
sin(x + 1) + 1))/(cos(2*x + 2)^2 + sin(2*x + 2)^2 + 2*cos(2*x + 2) + 1) - 1
/cos(x + 1)
```

mupad [B] time = 3.17, size = 34, normalized size = 2.43

$$\frac{2\,x\,\cos\left(x+1\right)}{\cos\left(2\,x+2\right)+1}+\mathrm{atan}\left(\cos\left(x+1\right)+\sin\left(x+1\right)\,1\mathrm{i}\right)\,2\mathrm{i}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*tan(x + 1))/cos(x + 1),x)
```

```
[Out] atan(cos(x + 1) + sin(x + 1)*1i)*2i + (2*x*cos(x + 1))/(cos(2*x + 2) + 1)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x\tan\left(x+1\right)\sec\left(x+1\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sec(1+x)*tan(1+x),x)
```

```
[Out] Integral(x*tan(x + 1)*sec(x + 1), x)
```

3.905 $\int \frac{\sin(2x)}{\sqrt{9-\sin^2(x)}}\,dx$

Optimal. Leaf size=14

$$-2\sqrt{9-\sin^2(x)}$$

```
[Out] -2*(9-sin(x)^2)^(1/2)
```

Rubi [A] time = 0.04, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {12, 261}

$$-2\sqrt{9-\sin^2(x)}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[2*x]/Sqrt[9 - Sin[x]^2],x]
```

```
[Out] -2*Sqrt[9 - Sin[x]^2]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sin(2x)}{\sqrt{9-\sin^2(x)}}\,dx &= \text{Subst}\left(\int \frac{2x}{\sqrt{9-x^2}}\,dx, x, \sin(x)\right)\\
&= 2\,\text{Subst}\left(\int \frac{x}{\sqrt{9-x^2}}\,dx, x, \sin(x)\right)\\
&= -2\sqrt{9-\sin^2(x)}
\end{aligned}$$

Mathematica [A] time = 0.02, size = 14, normalized size = 1.00

$$-2\sqrt{9-\sin^2(x)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[2*x]/Sqrt[9 - Sin[x]^2],x]
[Out] -2*Sqrt[9 - Sin[x]^2]
```

fricas [A] time = 0.77, size = 10, normalized size = 0.71

$$-2\sqrt{\cos(x)^2+8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(2*x)/(9-sin(x)^2)^(1/2),x, algorithm="fricas")
[Out] -2*sqrt(cos(x)^2 + 8)
```

giac [A] time = 0.14, size = 12, normalized size = 0.86

$$-2\sqrt{-\sin(x)^2+9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(2*x)/(9-sin(x)^2)^(1/2),x, algorithm="giac")
[Out] -2*sqrt(-sin(x)^2 + 9)
```

maple [A] time = 0.08, size = 13, normalized size = 0.93

$$-2\sqrt{9-\left(\sin^2(x)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(2*x)/(9-sin(x)^2)^(1/2),x)
[Out] -2*(9-sin(x)^2)^(1/2)
```

maxima [A] time = 0.33, size = 12, normalized size = 0.86

$$-2\sqrt{-\sin(x)^2+9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(2*x)/(9-sin(x)^2)^(1/2),x, algorithm="maxima")
[Out] -2*sqrt(-sin(x)^2 + 9)
```

mupad [B] time = 0.17, size = 10, normalized size = 0.71

$$-2\sqrt{\cos(x)^2+8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(2*x)/(9 - sin(x)^2)^(1/2),x)
```

```
[Out] -2*(cos(x)^2 + 8)^(1/2)
```

sympy [A] time = 1.43, size = 12, normalized size = 0.86

$$-2\sqrt{9-\sin^2(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(2*x)/(9-sin(x)**2)**(1/2),x)
```

```
[Out] -2*sqrt(9 - sin(x)**2)
```

3.906 $\int \frac{\sin(2x)}{\sqrt{9-\cos^4(x)}} \, dx$

Optimal. Leaf size=11

$$-\sin^{-1}\left(\frac{\cos^2(x)}{3}\right)$$

```
[Out] -arcsin(1/3*cos(x)^2)
```

Rubi [A] time = 0.05, antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {12, 1107, 619, 216}

$$-\sin^{-1}\left(\frac{\cos^2(x)}{3}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sin[2*x]/Sqrt[9 - Cos[x]^4],x]
```

```
[Out] -ArcSin[Cos[x]^2/3]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 1107

```
Int[(x_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dist[1/2,
 Subst[Int[(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\sin(2x)}{\sqrt{9-\cos^4(x)}}\,dx &= \text{Subst}\left(\int \frac{2x}{\sqrt{8+2x^2-x^4}}\,dx, x, \sin(x)\right)\\ &= 2\,\text{Subst}\left(\int \frac{x}{\sqrt{8+2x^2-x^4}}\,dx, x, \sin(x)\right)\\ &= \text{Subst}\left(\int \frac{1}{\sqrt{8+2x-x^2}}\,dx, x, \sin^2(x)\right)\\ &= -\left(\frac{1}{6}\,\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{x^2}{36}}}\,dx, x, 2\cos^2(x)\right)\right)\\ &= -\sin^{-1}\left(\frac{\cos^2(x)}{3}\right)\end{aligned}$$

Mathematica [A] time = 0.02, size = 11, normalized size = 1.00

$$-\sin^{-1}\left(\frac{\cos^2(x)}{3}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[2*x]/Sqrt[9 - Cos[x]^4],x]
[Out] -ArcSin[Cos[x]^2/3]
```

fricas [B] time = 1.31, size = 24, normalized size = 2.18

$$\arctan\left(\frac{\sqrt{-\cos(x)^4+9}\,\cos(x)^2}{\cos(x)^4-9}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(2*x)/(9-cos(x)^4)^(1/2),x, algorithm="fricas")
[Out] arctan(sqrt(-cos(x)^4 + 9)*cos(x)^2/(cos(x)^4 - 9))
```

giac [A] time = 0.15, size = 9, normalized size = 0.82

$$-\arcsin\left(\frac{1}{3}\,\cos(x)^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(2*x)/(9-cos(x)^4)^(1/2),x, algorithm="giac")
```

```
[Out] -arcsin(1/3*cos(x)^2)
```

maple [A] time = 0.11, size = 10, normalized size = 0.91

$$-\arcsin\left(\frac{\left(\cos^2(x)\right)}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(2*x)/(9-cos(x)^4)^(1/2),x)
```

```
[Out] -arcsin(1/3*cos(x)^2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin(2x)}{\sqrt{-\cos(x)^4+9}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(2*x)/(9-cos(x)^4)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sin(2*x)/sqrt(-cos(x)^4 + 9), x)
```

mupad [B] time = 3.14, size = 18, normalized size = 1.64

$$-\mathrm{atan}\left(\frac{\cos(x)^2}{\sqrt{9-\cos(x)^4}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(2*x)/(9 - cos(x)^4)^(1/2),x)
```

```
[Out] -atan(cos(x)^2/(9 - cos(x)^4)^(1/2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(2*x)/(9-cos(x)**4)**(1/2),x)
```

```
[Out] Timed out
```

3.907 $\int \frac{\cos\left(\frac{1}{x}\right)}{x^5}\,dx$

Optimal. Leaf size=34

$$-\frac{\sin\left(\frac{1}{x}\right)}{x^3}-\frac{3\cos\left(\frac{1}{x}\right)}{x^2}+\frac{6\sin\left(\frac{1}{x}\right)}{x}+6\cos\left(\frac{1}{x}\right)$$

```
[Out] 6*cos(1/x)-3*cos(1/x)/x^2-sin(1/x)/x^3+6*sin(1/x)/x
```

Rubi [A] time = 0.05, antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {3380, 3296, 2638}

$$-\frac{\sin\left(\frac{1}{x}\right)}{x^3}-\frac{3\cos\left(\frac{1}{x}\right)}{x^2}+\frac{6\sin\left(\frac{1}{x}\right)}{x}+6\cos\left(\frac{1}{x}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Cos[x^(-1)]/x^5,x]
```

```
[Out] 6*Cos[x^(-1)] - (3*Cos[x^(-1)])/x^2 - Sin[x^(-1)]/x^3 + (6*Sin[x^(-1)])/x
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3380

```
Int[((a_.) + Cos[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Cos[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rubi steps

$$\begin{aligned}\int \frac{\cos\left(\frac{1}{x}\right)}{x^5}\,dx &= -\text{Subst}\left(\int x^3\cos(x)\,dx, x, \frac{1}{x}\right)\\ &= -\frac{\sin\left(\frac{1}{x}\right)}{x^3} + 3\,\text{Subst}\left(\int x^2\sin(x)\,dx, x, \frac{1}{x}\right)\\ &= -\frac{3\cos\left(\frac{1}{x}\right)}{x^2} - \frac{\sin\left(\frac{1}{x}\right)}{x^3} + 6\,\text{Subst}\left(\int x\cos(x)\,dx, x, \frac{1}{x}\right)\\ &= -\frac{3\cos\left(\frac{1}{x}\right)}{x^2} - \frac{\sin\left(\frac{1}{x}\right)}{x^3} + \frac{6\sin\left(\frac{1}{x}\right)}{x} - 6\,\text{Subst}\left(\int \sin(x)\,dx, x, \frac{1}{x}\right)\\ &= 6\cos\left(\frac{1}{x}\right) - \frac{3\cos\left(\frac{1}{x}\right)}{x^2} - \frac{\sin\left(\frac{1}{x}\right)}{x^3} + \frac{6\sin\left(\frac{1}{x}\right)}{x}\end{aligned}$$

Mathematica [**A**] time = 0.03, size = 32, normalized size = 0.94

$$\frac{3\left(2x^2-1\right)\cos\left(\frac{1}{x}\right)}{x^2} + \frac{\left(6x^2-1\right)\sin\left(\frac{1}{x}\right)}{x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[x^(-1)]/x^5,x]
[Out] (3*(-1 + 2*x^2)*Cos[x^(-1)])/x^2 + ((-1 + 6*x^2)*Sin[x^(-1)])/x^3
```

fricas [**A**] time = 0.87, size = 32, normalized size = 0.94

$$\frac{3\left(2\,x^3-x\right)\cos\left(\frac{1}{x}\right) + \left(6\,x^2-1\right)\sin\left(\frac{1}{x}\right)}{x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(1/x)/x^5,x, algorithm="fricas")
[Out] (3*(2*x^3 - x)*cos(1/x) + (6*x^2 - 1)*sin(1/x))/x^3
```

giac [**A**] time = 0.14, size = 34, normalized size = 1.00

$$\frac{6\,\sin\left(\frac{1}{x}\right)}{x} - \frac{3\,\cos\left(\frac{1}{x}\right)}{x^2} - \frac{\sin\left(\frac{1}{x}\right)}{x^3} + 6\,\cos\left(\frac{1}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(1/x)/x^5,x, algorithm="giac")
[Out] 6*sin(1/x)/x - 3*cos(1/x)/x^2 - sin(1/x)/x^3 + 6*cos(1/x)
```

maple [A] time = 0.06, size = 35, normalized size = 1.03

$$6 \cos\left(\frac{1}{x}\right)-\frac{3 \cos\left(\frac{1}{x}\right)}{x^{2}}-\frac{\sin\left(\frac{1}{x}\right)}{x^{3}}+\frac{6 \sin\left(\frac{1}{x}\right)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(1/x)/x^5,x)
[Out] 6*cos(1/x)-3*cos(1/x)/x^2-sin(1/x)/x^3+6*sin(1/x)/x
```

maxima [C] time = 0.37, size = 19, normalized size = 0.56

$$\frac{1}{2}\,\Gamma\left(4,\frac{i}{x}\right)+\frac{1}{2}\,\Gamma\left(4,-\frac{i}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(1/x)/x^5,x, algorithm="maxima")
[Out] 1/2*gamma(4, I/x) + 1/2*gamma(4, -I/x)
```

mupad [B] time = 3.00, size = 33, normalized size = 0.97

$$6\ \cos\left(\frac{1}{x}\right)-\frac{\sin\left(\frac{1}{x}\right)+3\,x\ \cos\left(\frac{1}{x}\right)-6\,x^2\ \sin\left(\frac{1}{x}\right)}{x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(1/x)/x^5,x)
[Out] 6*cos(1/x) - (sin(1/x) + 3*x*cos(1/x) - 6*x^2*sin(1/x))/x^3
```

sympy [A] time = 3.68, size = 32, normalized size = 0.94

$$6 \cos\left(\frac{1}{x}\right)+\frac{6 \sin\left(\frac{1}{x}\right)}{x}-\frac{3 \cos\left(\frac{1}{x}\right)}{x^{2}}-\frac{\sin\left(\frac{1}{x}\right)}{x^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(1/x)/x**5,x)
[Out] 6*cos(1/x) + 6*sin(1/x)/x - 3*cos(1/x)/x**2 - sin(1/x)/x**3
```

3.908 $\int \cos^3(1+x)\sin^3(1+x)\,dx$

Optimal. Leaf size=21

$$\frac{1}{4}\sin^4(x+1)-\frac{1}{6}\sin^6(x+1)$$

```
[Out] 1/4*sin(1+x)^4-1/6*sin(1+x)^6
```

Rubi [A] time = 0.03, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {2564, 14}

$$\frac{1}{4}\sin^4(x+1)-\frac{1}{6}\sin^6(x+1)$$

Antiderivative was successfully verified.

```
[In] Int[Cos[1 + x]^3*Sin[1 + x]^3,x]
```

```
[Out] Sin[1 + x]^4/4 - Sin[1 + x]^6/6
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2564

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rubi steps

$$\begin{aligned}\int \cos^3(1+x)\sin^3(1+x)\,dx &= \text{Subst}\left(\int x^3\left(1-x^2\right)\,dx,x,\sin(1+x)\right)\\ &= \text{Subst}\left(\int \left(x^3-x^5\right)\,dx,x,\sin(1+x)\right)\\ &= \frac{1}{4}\sin^4(1+x)-\frac{1}{6}\sin^6(1+x)\end{aligned}$$

Mathematica [A] time = 0.01, size = 25, normalized size = 1.19

$$\frac{1}{8}\left(\frac{1}{24}\cos(6(x+1))-\frac{3}{8}\cos(2(x+1))\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[1 + x]^3*Sin[1 + x]^3,x]
[Out] ((-3*Cos[2*(1 + x)])/8 + Cos[6*(1 + x)]/24)/8
```

fricas [A] time = 0.82, size = 17, normalized size = 0.81

$$\frac{1}{6}\cos(x+1)^{6}-\frac{1}{4}\cos(x+1)^{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(1+x)^3*sin(1+x)^3,x, algorithm="fricas")
[Out] 1/6*cos(x + 1)^6 - 1/4*cos(x + 1)^4
```

giac [A] time = 0.14, size = 17, normalized size = 0.81

$$-\frac{1}{6}\sin(x+1)^{6}+\frac{1}{4}\sin(x+1)^{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(1+x)^3*sin(1+x)^3,x, algorithm="giac")
[Out] -1/6*sin(x + 1)^6 + 1/4*sin(x + 1)^4
```

maple [A] time = 0.08, size = 24, normalized size = 1.14

$$-\frac{\left(\cos^{4}(1+x)\right)\left(\sin^{2}(1+x)\right)}{6}-\frac{\left(\cos^{4}(1+x)\right)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(1+x)^3*sin(1+x)^3,x)
[Out] -1/6*cos(1+x)^4*sin(1+x)^2-1/12*cos(1+x)^4
```

maxima [A] time = 0.56, size = 17, normalized size = 0.81

$$-\frac{1}{6}\sin(x+1)^{6}+\frac{1}{4}\sin(x+1)^{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(1+x)^3*sin(1+x)^3,x, algorithm="maxima")
```

```
[Out] -1/6*sin(x + 1)^6 + 1/4*sin(x + 1)^4
```

mupad [B] time = 0.07, size = 18, normalized size = 0.86

$$-\frac{\sin\left(x+1\right)^4\,\left(2\,\sin\left(x+1\right)^2-3\right)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x + 1)^3*sin(x + 1)^3,x)
```

```
[Out] -(sin(x + 1)^4*(2*sin(x + 1)^2 - 3))/12
```

sympy [A] time = 1.78, size = 24, normalized size = 1.14

$$-\frac{\sin^{2}\left(x+1\right)\cos^{4}\left(x+1\right)}{4}-\frac{\cos^{6}\left(x+1\right)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(1+x)**3*sin(1+x)**3,x)
```

```
[Out] -sin(x + 1)**2*cos(x + 1)**4/4 - cos(x + 1)**6/12
```

3.909 $\int (1+2x)^3 \sin^2(1+2x)\,dx$

Optimal. Leaf size=99

$$-\frac{3x^2}{4}+\frac{1}{16}(2x+1)^4-\frac{3x}{4}+\frac{3}{8}(2x+1)^2\sin^2(2x+1)-\frac{3}{16}\sin^2(2x+1)-\frac{1}{4}(2x+1)^3\sin(2x+1)\cos(2x+1)+\frac{3}{8}(2x+1)\sin(2x$$

```
[Out] -3/4*x-3/4*x^2+1/16*(1+2*x)^4+3/8*(1+2*x)*cos(1+2*x)*sin(1+2*x)-1/4*(1+2*x)
^3*cos(1+2*x)*sin(1+2*x)-3/16*sin(1+2*x)^2+3/8*(1+2*x)^2*sin(1+2*x)^2
```

Rubi [A] time = 0.06, antiderivative size = 99, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {3311, 32, 3310}

$$-\frac{3x^2}{4}+\frac{1}{16}(2x+1)^4-\frac{3x}{4}+\frac{3}{8}(2x+1)^2\sin^2(2x+1)-\frac{3}{16}\sin^2(2x+1)-\frac{1}{4}(2x+1)^3\sin(2x+1)\cos(2x+1)+\frac{3}{8}(2x+1)\sin(2x$$

Antiderivative was successfully verified.

```
[In] Int[(1 + 2*x)^3*Sin[1 + 2*x]^2,x]
```

```
[Out] (-3*x)/4 - (3*x^2)/4 + (1 + 2*x)^4/16 + (3*(1 + 2*x)*Cos[1 + 2*x]*Sin[1 + 2
*x])/8 - ((1 + 2*x)^3*Cos[1 + 2*x]*Sin[1 + 2*x])/4 - (3*Sin[1 + 2*x]^2)/16
+ (3*(1 + 2*x)^2*Sin[1 + 2*x]^2)/8
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 3310

```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[(d*(b*Sin[e + f*x])^n)/(f^2*n^2), x] + (Dist[(b^2*(n - 1))/n, Int[(c
+ d*x)*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[(b*(c + d*x)*Cos[e + f*x]*(b
*Sin[e + f*x])^(n - 1))/(f*n), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1
]
```

Rule 3311

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[(d*m*(c + d*x)^(m - 1)*(b*Sin[e + f*x])^n)/(f^2*n^2), x] + (Dist
[(b^2*(n - 1))/n, Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Dist[(
d^2*m*(m - 1))/(f^2*n^2), Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x]
- Simp[(b*(c + d*x)^m*Cos[e + f*x]*(b*Sin[e + f*x])^(n - 1))/(f*n), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

Rubi steps

$$\int (1+2x)^3 \sin^2(1+2x)\,dx = -\frac{1}{4}(1+2x)^3\cos(1+2x)\sin(1+2x)+\frac{3}{8}(1+2x)^2\sin^2(1+2x)+\frac{1}{2}\int (1+2x)^3\,dx$$
$$= \frac{1}{16}(1+2x)^4+\frac{3}{8}(1+2x)\cos(1+2x)\sin(1+2x)-\frac{1}{4}(1+2x)^3\cos(1+2x)\sin(1+2$$
$$= -\frac{3x}{4}-\frac{3x^2}{4}+\frac{1}{16}(1+2x)^4+\frac{3}{8}(1+2x)\cos(1+2x)\sin(1+2x)-\frac{1}{4}(1+2x)^3\cos(1$$

Mathematica [A] time = 0.23, size = 55, normalized size = 0.56

$$\frac{1}{32}\left(2(2x+1)\left(\left(-8x^2-8x+1\right)\sin(4x+2)+(2x+1)^3\right)-3\left(8x^2+8x+1\right)\cos(4x+2)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + 2*x)^3*Sin[1 + 2*x]^2,x]
```

```
[Out] (-3*(1 + 8*x + 8*x^2)*Cos[2 + 4*x] + 2*(1 + 2*x)*((1 + 2*x)^3 + (1 - 8*x -
8*x^2)*Sin[2 + 4*x]))/32
```

fricas [A] time = 1.09, size = 66, normalized size = 0.67

$$x^4+2x^3-\frac{3}{16}\left(8x^2+8x+1\right)\cos(2x+1)^2-\frac{1}{8}\left(16x^3+24x^2+6x-1\right)\cos(2x+1)\sin(2x+1)+\frac{9}{4}x^2+\frac{5}{4}x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)^3*sin(1+2*x)^2,x, algorithm="fricas")
```

```
[Out] x^4 + 2*x^3 - 3/16*(8*x^2 + 8*x + 1)*cos(2*x + 1)^2 - 1/8*(16*x^3 + 24*x^2
+ 6*x - 1)*cos(2*x + 1)*sin(2*x + 1) + 9/4*x^2 + 5/4*x
```

giac [A] time = 0.13, size = 58, normalized size = 0.59

$$x^4+2x^3+\frac{3}{2}x^2-\frac{3}{32}\left(8x^2+8x+1\right)\cos(4x+2)-\frac{1}{16}\left(16x^3+24x^2+6x-1\right)\sin(4x+2)+\frac{1}{2}x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)^3*sin(1+2*x)^2,x, algorithm="giac")
```

```
[Out] x^4 + 2*x^3 + 3/2*x^2 - 3/32*(8*x^2 + 8*x + 1)*cos(4*x + 2) - 1/16*(16*x^3
+ 24*x^2 + 6*x - 1)*sin(4*x + 2) + 1/2*x
```

maple [A] time = 0.04, size = 97, normalized size = 0.98

$$\frac{(1+2x)^3\left(-\frac{\sin(1+2x)\cos(1+2x)}{2}+x+\frac{1}{2}\right)}{2}-\frac{3\left(\cos^2(1+2x)\right)(1+2x)^2}{8}+\frac{3(1+2x)\left(\frac{\sin(1+2x)\cos(1+2x)}{2}+x+\frac{1}{2}\right)}{4}-\frac{3(1-}{1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+2*x)^3*sin(1+2*x)^2,x)
```

```
[Out] 1/2*(1+2*x)^3*(-1/2*sin(1+2*x)*cos(1+2*x)+x+1/2)-3/8*cos(1+2*x)^2*(1+2*x)^2
+3/4*(1+2*x)*(1/2*sin(1+2*x)*cos(1+2*x)+x+1/2)-3/16*(1+2*x)^2-3/16*sin(1+2*
x)^2-3/16*(1+2*x)^4
```

maxima [A] time = 0.40, size = 51, normalized size = 0.52

$$\frac{1}{16}(2x+1)^4-\frac{3}{32}\left(2(2x+1)^2-1\right)\cos(4x+2)-\frac{1}{16}\left(2(2x+1)^3-6x-3\right)\sin(4x+2)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)^3*sin(1+2*x)^2,x, algorithm="maxima")
```

```
[Out] 1/16*(2*x + 1)^4 - 3/32*(2*(2*x + 1)^2 - 1)*cos(4*x + 2) - 1/16*(2*(2*x + 1
)^3 - 6*x - 3)*sin(4*x + 2)
```

mupad [B] time = 3.06, size = 69, normalized size = 0.70

$$\frac{3\sin(4x+2)(2x+1)}{16}-\frac{3\sin(2x+1)^2}{16}+\frac{(2x+1)^4}{16}-\frac{\sin(4x+2)(2x+1)^3}{8}+\frac{3(2x+1)^2\left(2\sin(2x+1)^2-1\right)}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(2*x + 1)^2*(2*x + 1)^3,x)
```

```
[Out] (3*sin(4*x + 2)*(2*x + 1))/16 - (3*sin(2*x + 1)^2)/16 + (2*x + 1)^4/16 - (s
in(4*x + 2)*(2*x + 1)^3)/8 + (3*(2*x + 1)^2*(2*sin(2*x + 1)^2 - 1))/16
```

sympy [B] time = 1.30, size = 189, normalized size = 1.91

$$x^4\sin^2(2x+1)+x^4\cos^2(2x+1)+2x^3\sin^2(2x+1)-2x^3\sin(2x+1)\cos(2x+1)+2x^3\cos^2(2x+1)+\frac{9x^2\sin^2(2}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)**3*sin(1+2*x)**2,x)
```

```
[Out] x**4*sin(2*x + 1)**2 + x**4*cos(2*x + 1)**2 + 2*x**3*sin(2*x + 1)**2 - 2*x*
*3*sin(2*x + 1)*cos(2*x + 1) + 2*x**3*cos(2*x + 1)**2 + 9*x**2*sin(2*x + 1)
```

```
**2/4 - 3*x**2*sin(2*x + 1)*cos(2*x + 1) + 3*x**2*cos(2*x + 1)**2/4 + 5*x*s
in(2*x + 1)**2/4 - 3*x*sin(2*x + 1)*cos(2*x + 1)/4 - x*cos(2*x + 1)**2/4 +
3*sin(2*x + 1)**2/16 + sin(2*x + 1)*cos(2*x + 1)/8
```

3.910 $\int \frac{-1+\sec(x)}{1-\tan(x)}\,dx$

Optimal. Leaf size=37

$$-\frac{x}{2}+\frac{1}{2}\log(\cos(x)-\sin(x))+\frac{\tanh^{-1}\left(\frac{\cos(x)(\tan(x)+1)}{\sqrt{2}}\right)}{\sqrt{2}}$$

```
[Out] -1/2*x+1/2*ln(cos(x)-sin(x))+1/2*arctanh(1/2*cos(x)*(1+tan(x))*2^(1/2))*2^(
1/2)
```

Rubi [A] time = 0.09, antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {4401, 3484, 3530, 3509, 206}

$$-\frac{x}{2}+\frac{1}{2}\log(\cos(x)-\sin(x))+\frac{\tanh^{-1}\left(\frac{\cos(x)(\tan(x)+1)}{\sqrt{2}}\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[(-1 + Sec[x])/(1 - Tan[x]),x]
```

```
[Out] -x/2 + ArcTanh[(Cos[x]*(1 + Tan[x]))/Sqrt[2]]/Sqrt[2] + Log[Cos[x] - Sin[x]
]/2
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3484

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> Simp[(a*x)/(a
^2 + b^2), x] + Dist[b/(a^2 + b^2), Int[(b - a*Tan[c + d*x])/(a + b*Tan[c +
 d*x]), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

Rule 3509

```
Int[sec[(e_.) + (f_.)*(x_)]/((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbo
l] :> -Dist[f^(-1), Subst[Int[1/(a^2 + b^2 - x^2), x], x, (b - a*Tan[e + f*
x])/Sec[e + f*x]], x] /; FreeQ[{a, b, e, f}, x] && NeQ[a^2 + b^2, 0]
```

Rule 3530

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_) + (b_.)*tan[(e_.) + (f_.)*
(x_)]), x_Symbol] :> Simp[(c*Log[RemoveContent[a*Cos[e + f*x] + b*Sin[e + f
*x], x]])/(b*f), x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
 NeQ[a^2 + b^2, 0] && EqQ[a*c + b*d, 0]
```

Rule 4401

```
Int[u_, x_Symbol] :> With[{v = ExpandTrig[u, x]}, Int[v, x] /; SumQ[v]] /;
 !InertTrigFreeQ[u]
```

Rubi steps

$$\begin{aligned}\int \frac{-1+\sec(x)}{1-\tan(x)}\,dx &= \int \left(\frac{1}{-1+\tan(x)}-\frac{\sec(x)}{-1+\tan(x)}\right)dx\\ &= \int \frac{1}{-1+\tan(x)}\,dx-\int \frac{\sec(x)}{-1+\tan(x)}\,dx\\ &= -\frac{x}{2}+\frac{1}{2}\int \frac{1+\tan(x)}{-1+\tan(x)}\,dx+\text{Subst}\left(\int \frac{1}{2-x^2}\,dx,x,\cos(x)(1+\tan(x))\right)\\ &= -\frac{x}{2}+\frac{\tanh^{-1}\left(\frac{\cos(x)(1+\tan(x))}{\sqrt{2}}\right)}{\sqrt{2}}+\frac{1}{2}\log(\cos(x)-\sin(x))\end{aligned}$$

Mathematica [C] time = 0.06, size = 40, normalized size = 1.08

$$\frac{1}{2}\left(-x+(2-2i)\sqrt[4]{-1}\,\tanh^{-1}\left(\frac{\tan\left(\frac{x}{2}\right)+1}{\sqrt{2}}\right)+\log(\cos(x)-\sin(x))\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-1 + Sec[x])/(1 - Tan[x]),x]
```

```
[Out] (-x + (2 - 2*I)*(-1)^(1/4)*ArcTanh[(1 + Tan[x/2])/Sqrt[2]] + Log[Cos[x] - S
in[x]])/2
```

fricas [A] time = 1.41, size = 51, normalized size = 1.38

$$\frac{1}{4}\sqrt{2}\log\left(\frac{2\left(\sqrt{2}+\cos(x)\right)\sin(x)+2\sqrt{2}\cos(x)+3}{2\cos(x)\sin(x)-1}\right)-\frac{1}{2}x+\frac{1}{4}\log\left(-2\cos(x)\sin(x)+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+sec(x))/(1-tan(x)),x, algorithm="fricas")
```

```
[Out] 1/4*sqrt(2)*log((2*(sqrt(2) + cos(x))*sin(x) + 2*sqrt(2)*cos(x) + 3)/(2*cos
(x)*sin(x) - 1)) - 1/2*x + 1/4*log(-2*cos(x)*sin(x) + 1)
```

giac [B] time = 0.20, size = 70, normalized size = 1.89

$$-\frac{1}{2}\sqrt{2}\log\left(\frac{\left|-2\sqrt{2}+2\tan\left(\frac{1}{2}x\right)+2\right|}{\left|2\sqrt{2}+2\tan\left(\frac{1}{2}x\right)+2\right|}\right)-\frac{1}{2}x-\frac{1}{2}\log\left(\tan\left(\frac{1}{2}x\right)^{2}+1\right)+\frac{1}{2}\log\left(\left|\tan\left(\frac{1}{2}x\right)^{2}+2\tan\left(\frac{1}{2}x\right)-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+sec(x))/(1-tan(x)),x, algorithm="giac")
```

```
[Out] -1/2*sqrt(2)*log(abs(-2*sqrt(2) + 2*tan(1/2*x) + 2)/abs(2*sqrt(2) + 2*tan(1
/2*x) + 2)) - 1/2*x - 1/2*log(tan(1/2*x)^2 + 1) + 1/2*log(abs(tan(1/2*x)^2
+ 2*tan(1/2*x) - 1))
```

maple [A] time = 0.11, size = 51, normalized size = 1.38

$$\frac{\ln\left(\tan^{2}\left(\frac{x}{2}\right)+2\tan\left(\frac{x}{2}\right)-1\right)}{2}+\sqrt{2}\ \mathrm{arctanh}\left(\frac{\left(2\tan\left(\frac{x}{2}\right)+2\right)\sqrt{2}}{4}\right)-\frac{\ln\left(1+\tan^{2}\left(\frac{x}{2}\right)\right)}{2}-\frac{x}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-1+sec(x))/(1-tan(x)),x)
```

```
[Out] 1/2*ln(tan(1/2*x)^2+2*tan(1/2*x)-1)+2^(1/2)*arctanh(1/4*(2*tan(1/2*x)+2)*2^
(1/2))-1/2*ln(1+tan(1/2*x)^2)-1/2*x
```

maxima [A] time = 0.58, size = 59, normalized size = 1.59

$$-\frac{1}{2}\sqrt{2}\log\left(-\frac{\sqrt{2}-\frac{\sin(x)}{\cos(x)+1}-1}{\sqrt{2}+\frac{\sin(x)}{\cos(x)+1}+1}\right)-\frac{1}{2}x-\frac{1}{4}\log\left(\tan(x)^{2}+1\right)+\frac{1}{2}\log\left(\tan(x)-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+sec(x))/(1-tan(x)),x, algorithm="maxima")
```

```
[Out] -1/2*sqrt(2)*log(-(sqrt(2) - sin(x)/(cos(x) + 1) - 1)/(sqrt(2) + sin(x)/(co
s(x) + 1) + 1)) - 1/2*x - 1/4*log(tan(x)^2 + 1) + 1/2*log(tan(x) - 1)
```

mupad [B] time = 3.13, size = 64, normalized size = 1.73

$$\ln\left(\tan\left(\frac{x}{2}\right)+\sqrt{2}+1\right)\left(\frac{\sqrt{2}}{2}+\frac{1}{2}\right)-\ln\left(\tan\left(\frac{x}{2}\right)-\sqrt{2}+1\right)\left(\frac{\sqrt{2}}{2}-\frac{1}{2}\right)+\ln\left(\tan\left(\frac{x}{2}\right)-\mathrm{i}\right)\left(-\frac{1}{2}+\frac{1}{2}\mathrm{i}\right)+\ln\left(\tan\left(\frac{x}{2}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(1/cos(x) - 1)/(tan(x) - 1),x)
```

```
[Out] log(tan(x/2) + 2^(1/2) + 1)*(2^(1/2)/2 + 1/2) - log(tan(x/2) + 1i)*(1/2 + 1
i/2) - log(tan(x/2) - 2^(1/2) + 1)*(2^(1/2)/2 - 1/2) - log(tan(x/2) - 1i)*(
1/2 - 1i/2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{\sec(x)}{\tan(x) - 1}\,dx - \int \left(-\frac{1}{\tan(x) - 1}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+sec(x))/(1-tan(x)),x)
```

```
[Out] -Integral(sec(x)/(tan(x) - 1), x) - Integral(-1/(tan(x) - 1), x)
```

3.911 $\int x^2 \cos(3x)\cos(5x)\,dx$

Optimal. Leaf size=57

$$\frac{1}{4}x^2\sin(2x)+\frac{1}{16}x^2\sin(8x)-\frac{1}{8}\sin(2x)-\frac{1}{512}\sin(8x)+\frac{1}{4}x\cos(2x)+\frac{1}{64}x\cos(8x)$$

```
[Out] 1/4*x*cos(2*x)+1/64*x*cos(8*x)-1/8*sin(2*x)+1/4*x^2*sin(2*x)-1/512*sin(8*x)
+1/16*x^2*sin(8*x)
```

Rubi [A] time = 0.07, antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {4429, 3296, 2637}

$$\frac{1}{4}x^2\sin(2x)+\frac{1}{16}x^2\sin(8x)-\frac{1}{8}\sin(2x)-\frac{1}{512}\sin(8x)+\frac{1}{4}x\cos(2x)+\frac{1}{64}x\cos(8x)$$

Antiderivative was successfully verified.

```
[In] Int[x^2*Cos[3*x]*Cos[5*x],x]
```

```
[Out] (x*Cos[2*x])/4 + (x*Cos[8*x])/64 - Sin[2*x]/8 + (x^2*Sin[2*x])/4 - Sin[8*x]
/512 + (x^2*Sin[8*x])/16
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 4429

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*Cos[(c_.) + (d_.)*(x_)]^(q_.)*((e_.) + (f
_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandTrigReduce[(e + f*x)^m, Cos[a + b*x
]^p*Cos[c + d*x]^q, x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[p, 0] &&
 IGtQ[q, 0] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}\int x^2\cos(3x)\cos(5x)\,dx &= \int\left(\frac{1}{2}x^2\cos(2x)+\frac{1}{2}x^2\cos(8x)\right)dx\\ &= \frac{1}{2}\int x^2\cos(2x)\,dx+\frac{1}{2}\int x^2\cos(8x)\,dx\\ &= \frac{1}{4}x^2\sin(2x)+\frac{1}{16}x^2\sin(8x)-\frac{1}{8}\int x\sin(8x)\,dx-\frac{1}{2}\int x\sin(2x)\,dx\\ &= \frac{1}{4}x\cos(2x)+\frac{1}{64}x\cos(8x)+\frac{1}{4}x^2\sin(2x)+\frac{1}{16}x^2\sin(8x)-\frac{1}{64}\int\cos(8x)\,dx-\frac{1}{4}\int \text{co}\\ &= \frac{1}{4}x\cos(2x)+\frac{1}{64}x\cos(8x)-\frac{1}{8}\sin(2x)+\frac{1}{4}x^2\sin(2x)-\frac{1}{512}\sin(8x)+\frac{1}{16}x^2\sin(8x)\end{aligned}$$

Mathematica [A] time = 0.10, size = 49, normalized size = 0.86

$$\frac{1}{512}\left(128x^2\sin(2x)+32x^2\sin(8x)-64\sin(2x)-\sin(8x)+128x\cos(2x)+8x\cos(8x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*Cos[3*x]*Cos[5*x],x]
```

```
[Out] (128*x*Cos[2*x] + 8*x*Cos[8*x] - 64*Sin[2*x] + 128*x^2*Sin[2*x] - Sin[8*x]
+ 32*x^2*Sin[8*x])/512
```

fricas [A] time = 1.13, size = 73, normalized size = 1.28

$$2\,x\cos(x)^8-4\,x\cos(x)^6+\frac{5}{2}\,x\cos(x)^4+\frac{1}{64}\left(16\left(32\,x^2-1\right)\cos(x)^7-24\left(32\,x^2-1\right)\cos(x)^5+10\left(32\,x^2-1\right)\cos(x)^3\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*cos(3*x)*cos(5*x),x, algorithm="fricas")
```

```
[Out] 2*x*cos(x)^8 - 4*x*cos(x)^6 + 5/2*x*cos(x)^4 + 1/64*(16*(32*x^2 - 1)*cos(x)
^7 - 24*(32*x^2 - 1)*cos(x)^5 + 10*(32*x^2 - 1)*cos(x)^3 - 15*cos(x))*sin(x
) - 15/64*x
```

giac [A] time = 0.12, size = 41, normalized size = 0.72

$$\frac{1}{64}\,x\cos\left(8\,x\right)+\frac{1}{4}\,x\cos\left(2\,x\right)+\frac{1}{512}\left(32\,x^2-1\right)\sin\left(8\,x\right)+\frac{1}{8}\left(2\,x^2-1\right)\sin\left(2\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*cos(3*x)*cos(5*x),x, algorithm="giac")
```

```
[Out] 1/64*x*cos(8*x) + 1/4*x*cos(2*x) + 1/512*(32*x^2 - 1)*sin(8*x) + 1/8*(2*x^2
 - 1)*sin(2*x)
```

maple [A] time = 0.14, size = 46, normalized size = 0.81

$$\frac{x\cos(2x)}{4}+\frac{x\cos(8x)}{64}-\frac{\sin(2x)}{8}+\frac{x^2\sin(2x)}{4}-\frac{\sin(8x)}{512}+\frac{x^2\sin(8x)}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*cos(3*x)*cos(5*x),x)
```

```
[Out] 1/4*x*cos(2*x)+1/64*x*cos(8*x)-1/8*sin(2*x)+1/4*x^2*sin(2*x)-1/512*sin(8*x)
+1/16*x^2*sin(8*x)
```

maxima [A] time = 0.33, size = 41, normalized size = 0.72

$$\frac{1}{64}x\cos(8x)+\frac{1}{4}x\cos(2x)+\frac{1}{512}\left(32x^2-1\right)\sin(8x)+\frac{1}{8}\left(2x^2-1\right)\sin(2x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*cos(3*x)*cos(5*x),x, algorithm="maxima")
```

```
[Out] 1/64*x*cos(8*x) + 1/4*x*cos(2*x) + 1/512*(32*x^2 - 1)*sin(8*x) + 1/8*(2*x^2
 - 1)*sin(2*x)
```

mupad [B] time = 3.05, size = 45, normalized size = 0.79

$$\frac{x\cos(2x)}{4}-\frac{\sin(8x)}{512}-\frac{\sin(2x)}{8}+\frac{x\cos(8x)}{64}+\frac{x^2\sin(2x)}{4}+\frac{x^2\sin(8x)}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*cos(3*x)*cos(5*x),x)
```

```
[Out] (x*cos(2*x))/4 - sin(8*x)/512 - sin(2*x)/8 + (x*cos(8*x))/64 + (x^2*sin(2*x
))/4 + (x^2*sin(8*x))/16
```

sympy [A] time = 6.02, size = 90, normalized size = 1.58

$$-\frac{3x^2\sin(3x)\cos(5x)}{16}+\frac{5x^2\sin(5x)\cos(3x)}{16}+\frac{15x\sin(3x)\sin(5x)}{64}+\frac{17x\cos(3x)\cos(5x)}{64}+\frac{63\sin(3x)\cos(5x)}{512}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*cos(3*x)*cos(5*x),x)
```

```
[Out] -3*x**2*sin(3*x)*cos(5*x)/16 + 5*x**2*sin(5*x)*cos(3*x)/16 + 15*x*sin(3*x)*
sin(5*x)/64 + 17*x*cos(3*x)*cos(5*x)/64 + 63*sin(3*x)*cos(5*x)/512 - 65*sin
(5*x)*cos(3*x)/512
```

3.912 $\int \frac{\cos(x)+\sin(x)}{\sqrt{\cos(x)}\sqrt{\sin(x)}}\,dx$

Optimal. Leaf size=57

$$\sqrt{2}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{\sin(x)}}{\sqrt{\cos(x)}}+1\right)-\sqrt{2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{\sin(x)}}{\sqrt{\cos(x)}}\right)$$

```
[Out] -arctan(1-2^(1/2)*sin(x)^(1/2)/cos(x)^(1/2))*2^(1/2)+arctan(1+2^(1/2)*sin(x
)^(1/2)/cos(x)^(1/2))*2^(1/2)
```

Rubi [B] time = 0.21, antiderivative size = 243, normalized size of antiderivative = 4.26, number of steps used = 22, number of rules used = 9, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {3107, 2575, 297, 1162, 617, 204, 1165, 628, 2574}

$$\frac{\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{\cos(x)}}{\sqrt{\sin(x)}}\right)}{\sqrt{2}}-\frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt{\cos(x)}}{\sqrt{\sin(x)}}+1\right)}{\sqrt{2}}-\frac{\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{\sin(x)}}{\sqrt{\cos(x)}}\right)}{\sqrt{2}}+\frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt{\sin(x)}}{\sqrt{\cos(x)}}+1\right)}{\sqrt{2}}+\frac{\log\left(\tan(x)-\frac{\sqrt{2}\sqrt{\text{c}}}{\sqrt{\text{c}}}\right.}{2\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[x] + Sin[x])/(Sqrt[Cos[x]]*Sqrt[Sin[x]]),x]
```

```
[Out] ArcTan[1 - (Sqrt[2]*Sqrt[Cos[x]])/Sqrt[Sin[x]]]/Sqrt[2] - ArcTan[1 + (Sqrt[
2]*Sqrt[Cos[x]])/Sqrt[Sin[x]]]/Sqrt[2] - ArcTan[1 - (Sqrt[2]*Sqrt[Sin[x]])/
Sqrt[Cos[x]]]/Sqrt[2] + ArcTan[1 + (Sqrt[2]*Sqrt[Sin[x]])/Sqrt[Cos[x]]]/Sqr
t[2] - Log[1 + Cot[x] - (Sqrt[2]*Sqrt[Cos[x]])/Sqrt[Sin[x]]]/(2*Sqrt[2]) +
Log[1 + Cot[x] + (Sqrt[2]*Sqrt[Cos[x]])/Sqrt[Sin[x]]]/(2*Sqrt[2]) + Log[1 -
 (Sqrt[2]*Sqrt[Sin[x]])/Sqrt[Cos[x]] + Tan[x]]/(2*Sqrt[2]) - Log[1 + (Sqrt[
2]*Sqrt[Sin[x]])/Sqrt[Cos[x]] + Tan[x]]/(2*Sqrt[2])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 2574

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> With[{k = Denominator[m]}, Dist[(k*a*b)/f, Subst[Int[x^(k*
(m + 1) - 1)/(a^2 + b^2*x^(2*k)), x], x, (a*Sin[e + f*x])^(1/k)/(b*Cos[e +
f*x])^(1/k)], x]] /; FreeQ[{a, b, e, f}, x] && EqQ[m + n, 0] && GtQ[m, 0] &
& LtQ[m, 1]
```

Rule 2575

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> With[{k = Denominator[m]}, -Dist[(k*a*b)/f, Subst[Int[x^(k
*(m + 1) - 1)/(a^2 + b^2*x^(2*k)), x], x, (a*Cos[e + f*x])^(1/k)/(b*Sin[e +
 f*x])^(1/k)], x]] /; FreeQ[{a, b, e, f}, x] && EqQ[m + n, 0] && GtQ[m, 0]
&& LtQ[m, 1]
```

Rule 3107

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*sin[(c_.) + (d_.)*(x_)]^(n_.)*(cos[(c_.)
+ (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(p_.), x_Symbol] :> In
```

```
t[ExpandTrig[cos[c + d*x]^m*sin[c + d*x]^n*(a*cos[c + d*x] + b*sin[c + d*x]
)^p, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\cos(x)+\sin(x)}{\sqrt{\cos(x)}\sqrt{\sin(x)}}\,dx &= \int \left(\frac{\sqrt{\cos(x)}}{\sqrt{\sin(x)}}+\frac{\sqrt{\sin(x)}}{\sqrt{\cos(x)}}\right)dx\\
&= \int \frac{\sqrt{\cos(x)}}{\sqrt{\sin(x)}}\,dx+\int \frac{\sqrt{\sin(x)}}{\sqrt{\cos(x)}}\,dx\\
&= -\left(2\,\text{Subst}\left(\int \frac{x^2}{1+x^4}\,dx,x,\frac{\sqrt{\cos(x)}}{\sqrt{\sin(x)}}\right)\right)+2\,\text{Subst}\left(\int \frac{x^2}{1+x^4}\,dx,x,\frac{\sqrt{\sin(x)}}{\sqrt{\cos(x)}}\right)\\
&= \text{Subst}\left(\int \frac{1-x^2}{1+x^4}\,dx,x,\frac{\sqrt{\cos(x)}}{\sqrt{\sin(x)}}\right)-\text{Subst}\left(\int \frac{1-x^2}{1+x^4}\,dx,x,\frac{\sqrt{\sin(x)}}{\sqrt{\cos(x)}}\right)-\text{Subst}\left(\int \frac{1+\dots}{1+\dots}\right.\\
&= -\left(\frac{1}{2}\,\text{Subst}\left(\int \frac{1}{1-\sqrt{2}\,x+x^2}\,dx,x,\frac{\sqrt{\cos(x)}}{\sqrt{\sin(x)}}\right)\right)+\frac{1}{2}\,\text{Subst}\left(\int \frac{1}{1-\sqrt{2}\,x+x^2}\,dx,x,\frac{\sqrt{\text{s}\dots}}{\sqrt{\text{c}\dots}}\right.\\
&= -\frac{\log\left(1+\cot(x)-\frac{\sqrt{2}\sqrt{\cos(x)}}{\sqrt{\sin(x)}}\right)}{2\sqrt{2}}+\frac{\log\left(1+\cot(x)+\frac{\sqrt{2}\sqrt{\cos(x)}}{\sqrt{\sin(x)}}\right)}{2\sqrt{2}}+\frac{\log\left(1-\frac{\sqrt{2}\sqrt{\sin(x)}}{\sqrt{\cos(x)}}+\dots\right.}{2\sqrt{2}}\\
&= \frac{\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{\cos(x)}}{\sqrt{\sin(x)}}\right)}{\sqrt{2}}-\frac{\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt{\cos(x)}}{\sqrt{\sin(x)}}\right)}{\sqrt{2}}-\frac{\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{\sin(x)}}{\sqrt{\cos(x)}}\right)}{\sqrt{2}}+\frac{\tan^{-1}\left(1+\dots\right.}{\sqrt{2}}
\end{aligned}$$

Mathematica [C] time = 0.06, size = 68, normalized size = 1.19

$$\frac{2\sqrt{\sin(x)}\,\sqrt[4]{\cos^2(x)}\left(\sin(x)\sqrt{\cos^2(x)}\,{}_2F_1\left(\frac{3}{4},\frac{3}{4};\frac{7}{4};\sin^2(x)\right)+3\cos(x)\,{}_2F_1\left(\frac{1}{4},\frac{1}{4};\frac{5}{4};\sin^2(x)\right)\right)}{3\cos^{\frac{3}{2}}(x)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[x] + Sin[x])/(Sqrt[Cos[x]]*Sqrt[Sin[x]]),x]
```

```
[Out] (2*(Cos[x]^2)^(1/4)*Sqrt[Sin[x]]*(3*Cos[x]*Hypergeometric2F1[1/4, 1/4, 5/4,
 Sin[x]^2] + Sqrt[Cos[x]^2]*Hypergeometric2F1[3/4, 3/4, 7/4, Sin[x]^2]*Sin[
x]))/(3*Cos[x]^(3/2))
```

fricas [B] time = 0.98, size = 85, normalized size = 1.49

$$-\frac{1}{4}\sqrt{2}\,\arctan\left(-\frac{\left(32\sqrt{2}\cos(x)^4-32\sqrt{2}\cos(x)^2+16\sqrt{2}\cos(x)\sin(x)-\sqrt{2}\right)\sqrt{\cos(x)}\sqrt{\sin(x)}}{8\left(4\cos(x)^5-3\cos(x)^3-\left(4\cos(x)^4-5\cos(x)^2\right)\sin(x)-\cos(x)\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((cos(x)+sin(x))/cos(x)^(1/2)/sin(x)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/4*sqrt(2)*arctan(-1/8*(32*sqrt(2)*cos(x)^4 - 32*sqrt(2)*cos(x)^2 + 16*sq
rt(2)*cos(x)*sin(x) - sqrt(2))*sqrt(cos(x))*sqrt(sin(x))/(4*cos(x)^5 - 3*co
s(x)^3 - (4*cos(x)^4 - 5*cos(x)^2)*sin(x) - cos(x)))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos(x) + \sin(x)}{\sqrt{\cos(x)}\,\sqrt{\sin(x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((cos(x)+sin(x))/cos(x)^(1/2)/sin(x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((cos(x) + sin(x))/(sqrt(cos(x))*sqrt(sin(x))), x)
```

maple [C] time = 0.26, size = 134, normalized size = 2.35

$$\frac{\sqrt{\frac{1-\cos(x)+\sin(x)}{\sin(x)}}\,\sqrt{2}\,\sqrt{\frac{\cos(x)-1+\sin(x)}{\sin(x)}}\,\sqrt{\frac{-1+\cos(x)}{\sin(x)}}\,\left(\sin^{\frac{3}{2}}(x)\right)\left(i\,\mathrm{EllipticPi}\left(\sqrt{\frac{1-\cos(x)+\sin(x)}{\sin(x)}},\frac{1}{2}+\frac{i}{2},\frac{\sqrt{2}}{2}\right)-i\,\mathrm{Elliptic}\right.}{\sqrt{\cos(x)}\,\left(-1+\cos(x)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(x)+sin(x))/cos(x)^(1/2)/sin(x)^(1/2),x)
```

```
[Out] ((1-cos(x)+sin(x))/sin(x))^(1/2)*2^(1/2)*((cos(x)-1+sin(x))/sin(x))^(1/2)*(
(-1+cos(x))/sin(x))^(1/2)*sin(x)^(3/2)*(I*EllipticPi(((1-cos(x)+sin(x))/sin
(x))^(1/2),1/2+1/2*I,1/2*2^(1/2))-I*EllipticPi(((1-cos(x)+sin(x))/sin(x))^(
1/2),1/2-1/2*I,1/2*2^(1/2))+EllipticF(((1-cos(x)+sin(x))/sin(x))^(1/2),1/2*
2^(1/2)))/cos(x)^(1/2)/(-1+cos(x))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos(x) + \sin(x)}{\sqrt{\cos(x)}\,\sqrt{\sin(x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((cos(x)+sin(x))/cos(x)^(1/2)/sin(x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((cos(x) + sin(x))/(sqrt(cos(x))*sqrt(sin(x))), x)
```

mupad [B] time = 4.62, size = 51, normalized size = 0.89

$$-\frac{2\sqrt{\cos(x)}\,\sin(x)^{3/2}\,{}_2F_1\left(\frac{1}{4},\frac{1}{4};\ \frac{5}{4};\ \cos(x)^2\right)}{\left(\sin(x)^2\right)^{3/4}}-\frac{2\cos(x)^{3/2}\,\sqrt{\sin(x)}\,{}_2F_1\left(\frac{3}{4},\frac{3}{4};\ \frac{7}{4};\ \cos(x)^2\right)}{3\left(\sin(x)^2\right)^{1/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(x) + sin(x))/(cos(x)^(1/2)*sin(x)^(1/2)),x)
```

```
[Out] - (2*cos(x)^(1/2)*sin(x)^(3/2)*hypergeom([1/4, 1/4], 5/4, cos(x)^2))/(sin(x
)^2)^(3/4) - (2*cos(x)^(3/2)*sin(x)^(1/2)*hypergeom([3/4, 3/4], 7/4, cos(x)
^2))/(3*(sin(x)^2)^(1/4))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sin(x)+\cos(x)}{\sqrt{\sin(x)}\,\sqrt{\cos(x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((cos(x)+sin(x))/cos(x)**(1/2)/sin(x)**(1/2),x)
```

```
[Out] Integral((sin(x) + cos(x))/(sqrt(sin(x))*sqrt(cos(x))), x)
```

3.913 $\int \sec^2(x)(1+\sin(x))\,dx$

Optimal. Leaf size=5

$$\tan(x)+\sec(x)$$

```
[Out] sec(x)+tan(x)
```

Rubi [A] time = 0.02, antiderivative size = 5, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2669, 3767, 8}

$$\tan(x)+\sec(x)$$

Antiderivative was successfully verified.

```
[In] Int[Sec[x]^2*(1 + Sin[x]),x]
```

```
[Out] Sec[x] + Tan[x]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2669

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1))/(f*g*(p + 1)), x] + D
ist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] && (I
ntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\begin{aligned}\int \sec^2(x)(1+\sin(x))\,dx &= \sec(x)+\int \sec^2(x)\,dx\\ &= \sec(x)-\text{Subst}\left(\int 1\,dx,x,-\tan(x)\right)\\ &= \sec(x)+\tan(x)\end{aligned}$$

Mathematica [**A**] time = 0.00, size = 5, normalized size = 1.00

$$\tan(x) + \sec(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[x]^2*(1 + Sin[x]),x]
```

```
[Out] Sec[x] + Tan[x]
```

fricas [**B**] time = 0.85, size = 17, normalized size = 3.40

$$\frac{\cos(x) + \sin(x) + 1}{\cos(x) - \sin(x) + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)^2*(1+sin(x)),x, algorithm="fricas")
```

```
[Out] (cos(x) + sin(x) + 1)/(cos(x) - sin(x) + 1)
```

giac [**A**] time = 0.13, size = 10, normalized size = 2.00

$$-\frac{2}{\tan\left(\frac{1}{2}\,x\right) - 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)^2*(1+sin(x)),x, algorithm="giac")
```

```
[Out] -2/(tan(1/2*x) - 1)
```

maple [**A**] time = 0.06, size = 8, normalized size = 1.60

$$\tan(x) + \frac{1}{\cos(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(x)^2*(1+sin(x)),x)
```

```
[Out] tan(x)+1/cos(x)
```

maxima [**A**] time = 0.32, size = 7, normalized size = 1.40

$$\frac{1}{\cos(x)} + \tan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)^2*(1+sin(x)),x, algorithm="maxima")
```

```
[Out] 1/cos(x) + tan(x)
```

mupad [B] time = 2.97, size = 10, normalized size = 2.00

$$-\frac{2}{\tan\left(\frac{x}{2}\right)-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((sin(x) + 1)/cos(x)^2,x)
```

```
[Out] -2/(tan(x/2) - 1)
```

sympy [A] time = 2.18, size = 7, normalized size = 1.40

$$\tan\left(x\right)+\frac{1}{\cos\left(x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)**2*(1+sin(x)),x)
```

```
[Out] tan(x) + 1/cos(x)
```

3.914 $\int \left(10x^9 \cos\left(x^5 \log(x)\right) - x^{10}\left(x^4 + 5x^4 \log(x)\right) \sin\left(x^5 \mathrm{l}\right.\right.$

Optimal. Leaf size=11

$$x^{10} \cos\left(x^5 \log(x)\right)$$

```
[Out] x^10*cos(x^5*ln(x))
```

Rubi [F] time = 0.28, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \left(10x^9 \cos\left(x^5 \log(x)\right) - x^{10}\left(x^4 + 5x^4 \log(x)\right) \sin\left(x^5 \log(x)\right)\right) dx$$

Verification is Not applicable to the result.

```
[In] Int[10*x^9*Cos[x^5*Log[x]] - x^10*(x^4 + 5*x^4*Log[x])*Sin[x^5*Log[x]],x]
```

```
[Out] 10*Defer[Int][x^9*Cos[x^5*Log[x]], x] - Defer[Int][x^14*Sin[x^5*Log[x]], x]
 - 5*Defer[Int][x^14*Log[x]*Sin[x^5*Log[x]], x]
```

Rubi steps

$$\begin{aligned}
\int \left(10x^9 \cos\left(x^5 \log(x)\right) - x^{10}\left(x^4 + 5x^4 \log(x)\right) \sin\left(x^5 \log(x)\right)\right) dx &= 10 \int x^9 \cos\left(x^5 \log(x)\right) dx - \int x^{10}\left(x^4 + 5\right. \\
&= 10 \int x^9 \cos\left(x^5 \log(x)\right) dx - \int x^{14}(1 + 5\,\mathrm{l} \\
&= 10 \int x^9 \cos\left(x^5 \log(x)\right) dx - \int \left(x^{14} \sin\left(x\right.\right. \\
&= -\left(5 \int x^{14} \log(x) \sin\left(x^5 \log(x)\right) dx\right) + 10
\end{aligned}$$

Mathematica [A] time = 0.31, size = 11, normalized size = 1.00

$$x^{10} \cos\left(x^5 \log(x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[10*x^9*Cos[x^5*Log[x]] - x^10*(x^4 + 5*x^4*Log[x])*Sin[x^5*Log[x]
],x]
```

```
[Out] x^10*Cos[x^5*Log[x]]
```

fricas [A] time = 0.74, size = 11, normalized size = 1.00

$$x^{10} \cos\left(x^5 \log(x)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(10*x^9*cos(x^5*log(x))-x^10*(x^4+5*x^4*log(x))*sin(x^5*log(x)),x,
 algorithm="fricas")
```

```
[Out] x^10*cos(x^5*log(x))
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(10*x^9*cos(x^5*log(x))-x^10*(x^4+5*x^4*log(x))*sin(x^5*log(x)),x,
 algorithm="giac")
```

```
[Out] Timed out
```

maple [C] time = 0.19, size = 30, normalized size = 2.73

$$\frac{x^{10}x^{ix^5}}{2}+\frac{x^{10}x^{-ix^5}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(10*x^9*cos(x^5*ln(x))-x^10*(x^4+5*x^4*ln(x))*sin(x^5*ln(x)),x)
```

```
[Out] 1/2*x^10*x^(I*x^5)+1/2*x^10/(x^(I*x^5))
```

maxima [A] time = 0.74, size = 11, normalized size = 1.00

$$x^{10}\cos\left(x^5\log(x)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(10*x^9*cos(x^5*log(x))-x^10*(x^4+5*x^4*log(x))*sin(x^5*log(x)),x,
 algorithm="maxima")
```

```
[Out] x^10*cos(x^5*log(x))
```

mupad [B] time = 3.16, size = 11, normalized size = 1.00

$$x^{10}\,\cos\left(x^5\,\ln(x)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(10*x^9*cos(x^5*log(x)) - x^10*sin(x^5*log(x))*(5*x^4*log(x) + x^4),x)
```

```
[Out] x^10*cos(x^5*log(x))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \left(-10x^9 \cos \left(x^5 \log (x)\right)\right)\, dx - \int x^{14} \sin \left(x^5 \log (x)\right) dx - \int 5x^{14} \log (x) \sin \left(x^5 \log (x)\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(10*x**9*cos(x**5*ln(x))-x**10*(x**4+5*x**4*ln(x))*sin(x**5*ln(x))
,x)
```

```
[Out] -Integral(-10*x**9*cos(x**5*log(x)), x) - Integral(x**14*sin(x**5*log(x)),
x) - Integral(5*x**14*log(x)*sin(x**5*log(x)), x)
```

3.915 $\int \cos^2\left(\frac{x}{2}\right)\tan\left(\frac{\pi}{4}+\frac{x}{2}\right)\,dx$

Optimal. Leaf size=27

$$\frac{x}{2}-\frac{\cos(x)}{2}-\log\left(\cos\left(\frac{x}{2}+\frac{\pi}{4}\right)\right)$$

```
[Out] 1/2*x-1/2*cos(x)-ln(cos(1/4*Pi+1/2*x))
```

Rubi [F] time = 0.06, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \cos^2\left(\frac{x}{2}\right)\tan\left(\frac{\pi}{4}+\frac{x}{2}\right)\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Cos[x/2]^2*Tan[Pi/4 + x/2],x]
[Out] Defer[Int][Cos[x/2]^2*Tan[Pi/4 + x/2], x]
```

Rubi steps

$$\int \cos^2\left(\frac{x}{2}\right)\tan\left(\frac{\pi}{4}+\frac{x}{2}\right)\,dx=\int \cos^2\left(\frac{x}{2}\right)\tan\left(\frac{\pi}{4}+\frac{x}{2}\right)\,dx$$

Mathematica [A] time = 0.17, size = 24, normalized size = 0.89

$$\frac{1}{2}\left(x-\cos(x)-\log(\cos(x))+2\tanh^{-1}\left(\cot\left(\frac{x}{2}\right)\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[x/2]^2*Tan[Pi/4 + x/2],x]
[Out] (x + 2*ArcTanh[Cot[x/2]] - Cos[x] - Log[Cos[x]])/2
```

fricas [A] time = 1.58, size = 27, normalized size = 1.00

$$-\cos\left(\frac{1}{2}\,x\right)^2+\frac{1}{2}\,x-\frac{1}{2}\,\log\left(-2\,\cos\left(\frac{1}{2}\,x\right)\sin\left(\frac{1}{2}\,x\right)+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(1/2*x)^2*tan(1/4*pi+1/2*x),x, algorithm="fricas")
```

```
[Out] -cos(1/2*x)^2 + 1/2*x - 1/2*log(-2*cos(1/2*x)*sin(1/2*x) + 1)
```

giac [B] time = 3.12, size = 93, normalized size = 3.44

$$\frac{x\tan\left(\frac{1}{2}x\right)^{2}-\log\left(\frac{2\left(\tan\left(\frac{1}{2}x\right)^{2}-2\tan\left(\frac{1}{2}x\right)+1\right)}{\tan\left(\frac{1}{2}x\right)^{2}+1}\right)\tan\left(\frac{1}{2}x\right)^{2}+\tan\left(\frac{1}{2}x\right)^{2}+x-\log\left(\frac{2\left(\tan\left(\frac{1}{2}x\right)^{2}-2\tan\left(\frac{1}{2}x\right)+1\right)}{\tan\left(\frac{1}{2}x\right)^{2}+1}\right)-1}{2\left(\tan\left(\frac{1}{2}x\right)^{2}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(1/2*x)^2*tan(1/4*pi+1/2*x),x, algorithm="giac")
```

```
[Out] 1/2*(x*tan(1/2*x)^2 - log(2*(tan(1/2*x)^2 - 2*tan(1/2*x) + 1)/(tan(1/2*x)^2
 + 1))*tan(1/2*x)^2 + tan(1/2*x)^2 + x - log(2*(tan(1/2*x)^2 - 2*tan(1/2*x)
 + 1)/(tan(1/2*x)^2 + 1)) - 1)/(tan(1/2*x)^2 + 1)
```

maple [A] time = 0.36, size = 22, normalized size = 0.81

$$\frac{x}{2}-\frac{\cos(x)}{2}+\frac{\ln\left(\sec(x)+\tan(x)\right)}{2}-\frac{\ln\left(\cos(x)\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(1/2*x)^2*tan(1/4*Pi+1/2*x),x)
```

```
[Out] 1/2*x-1/2*cos(x)+1/2*ln(sec(x)+tan(x))-1/2*ln(cos(x))
```

maxima [B] time = 0.64, size = 74, normalized size = 2.74

$$\frac{2\,x\cos(x)^{2}+2\,x\sin(x)^{2}-\cos\left(2\,x\right)\cos(x)-2\left(\cos(x)^{2}+\sin(x)^{2}\right)\log\left(\cos(x)^{2}+\sin(x)^{2}-2\,\sin(x)+1\right)-\sin\left(2\right.}{4\left(\cos(x)^{2}+\sin(x)^{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(1/2*x)^2*tan(1/4*pi+1/2*x),x, algorithm="maxima")
```

```
[Out] 1/4*(2*x*cos(x)^2 + 2*x*sin(x)^2 - cos(2*x)*cos(x) - 2*(cos(x)^2 + sin(x)^2
)*log(cos(x)^2 + sin(x)^2 - 2*sin(x) + 1) - sin(2*x)*sin(x) - cos(x))/(cos(
x)^2 + sin(x)^2)
```

mupad [B] time = 0.48, size = 38, normalized size = 1.41

$$-2\,\ln\left(\mathrm{e}^{\frac{\Pi\,1\mathrm{i}}{2}}\,\mathrm{e}^{x\,1\mathrm{i}}+1\right)\,\sin\left(\frac{\Pi}{4}\right)^{2}+x\,\mathrm{e}^{\frac{\Pi\,1\mathrm{i}}{4}}\,\sin\left(\frac{\Pi}{4}\right)-\frac{\cos(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x/2)^2*tan(Pi/4 + x/2),x)
```

```
[Out] x*sin(Pi/4)*exp((Pi*1i)/4) - 2*sin(Pi/4)^2*log(exp((Pi*1i)/2)*exp(x*1i) + 1
) - cos(x)/2
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \cos^2\left(\frac{x}{2}\right)\tan\left(\frac{x}{2}+\frac{\pi}{4}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(1/2*x)**2*tan(1/4*pi+1/2*x),x)
```

```
[Out] Integral(cos(x/2)**2*tan(x/2 + pi/4), x)
```

3.916 $\int (2+3x)^2 \sin^3(x)\,dx$

Optimal. Leaf size=65

$$\frac{2}{3}(3x+2)\sin^3(x)+4(3x+2)\sin(x)-\frac{2}{3}\cos^3(x)-\frac{2}{3}(3x+2)^2\cos(x)+14\cos(x)-\frac{1}{3}(3x+2)^2\sin^2(x)\cos(x)$$

```
[Out] 14*cos(x)-2/3*(2+3*x)^2*cos(x)-2/3*cos(x)^3+4*(2+3*x)*sin(x)-1/3*(2+3*x)^2*
cos(x)*sin(x)^2+2/3*(2+3*x)*sin(x)^3
```

Rubi [A] time = 0.07, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3311, 3296, 2638, 2633}

$$\frac{2}{3}(3x+2)\sin^3(x)+4(3x+2)\sin(x)-\frac{2}{3}\cos^3(x)-\frac{2}{3}(3x+2)^2\cos(x)+14\cos(x)-\frac{1}{3}(3x+2)^2\sin^2(x)\cos(x)$$

Antiderivative was successfully verified.

```
[In] Int[(2 + 3*x)^2*Sin[x]^3,x]
```

```
[Out] 14*Cos[x] - (2*(2 + 3*x)^2*Cos[x])/3 - (2*Cos[x]^3)/3 + 4*(2 + 3*x)*Sin[x]
- ((2 + 3*x)^2*Cos[x]*Sin[x]^2)/3 + (2*(2 + 3*x)*Sin[x]^3)/3
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3311

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[(d*m*(c + d*x)^(m - 1)*(b*Sin[e + f*x])^n)/(f^2*n^2), x] + (Dist
[(b^2*(n - 1))/n, Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Dist[(
```

```
d^2*m*(m - 1))/(f^2*n^2), Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x]
- Simp[(b*(c + d*x)^m*Cos[e + f*x]*(b*Sin[e + f*x])^(n - 1))/(f*n), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

Rubi steps

$$\int (2+3x)^2\sin^3(x)\,dx = -\frac{1}{3}(2+3x)^2\cos(x)\sin^2(x)+\frac{2}{3}(2+3x)\sin^3(x)+\frac{2}{3}\int (2+3x)^2\sin(x)\,dx-2\int \sin^3(x$$
$$= -\frac{2}{3}(2+3x)^2\cos(x)-\frac{1}{3}(2+3x)^2\cos(x)\sin^2(x)+\frac{2}{3}(2+3x)\sin^3(x)+2\,\mathrm{Subst}\left(\int \left(1-\right.\right.$$
$$= 2\cos(x)-\frac{2}{3}(2+3x)^2\cos(x)-\frac{2\cos^3(x)}{3}+4(2+3x)\sin(x)-\frac{1}{3}(2+3x)^2\cos(x)\sin^2(x$$
$$= 14\cos(x)-\frac{2}{3}(2+3x)^2\cos(x)-\frac{2\cos^3(x)}{3}+4(2+3x)\sin(x)-\frac{1}{3}(2+3x)^2\cos(x)\sin^2($$

Mathematica [A] time = 0.09, size = 50, normalized size = 0.77

$$\frac{1}{12}\left(-9\left(9x^2+12x-14\right)\cos(x)+\left(9x^2+12x+2\right)\cos(3x)-2(3x+2)(\sin(3x)-27\sin(x))\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + 3*x)^2*Sin[x]^3,x]
```

```
[Out] (-9*(-14 + 12*x + 9*x^2)*Cos[x] + (2 + 12*x + 9*x^2)*Cos[3*x] - 2*(2 + 3*x)
*(-27*Sin[x] + Sin[3*x]))/12
```

fricas [A] time = 1.65, size = 50, normalized size = 0.77

$$\frac{1}{3}\left(9\,x^2+12\,x+2\right)\cos(x)^3-\left(9\,x^2+12\,x-10\right)\cos(x)-\frac{2}{3}\left((3\,x+2)\cos(x)^2-21\,x-14\right)\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+3*x)^2*sin(x)^3,x, algorithm="fricas")
```

```
[Out] 1/3*(9*x^2 + 12*x + 2)*cos(x)^3 - (9*x^2 + 12*x - 10)*cos(x) - 2/3*((3*x +
2)*cos(x)^2 - 21*x - 14)*sin(x)
```

giac [A] time = 1.98, size = 51, normalized size = 0.78

$$\frac{1}{12}\left(9\,x^2+12\,x+2\right)\cos\left(3\,x\right)-\frac{3}{4}\left(9\,x^2+12\,x-14\right)\cos(x)-\frac{1}{6}\left(3\,x+2\right)\sin\left(3\,x\right)+\frac{9}{2}\left(3\,x+2\right)\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+3*x)^2*sin(x)^3,x, algorithm="giac")
```

```
[Out] 1/12*(9*x^2 + 12*x + 2)*cos(3*x) - 3/4*(9*x^2 + 12*x - 14)*cos(x) - 1/6*(3*
x + 2)*sin(3*x) + 9/2*(3*x + 2)*sin(x)
```

maple [A] time = 0.03, size = 62, normalized size = 0.95

$$-3x^2\left(2+\sin^2(x)\right)\cos(x)+12\cos(x)+12x\sin(x)+2\left(\sin^3(x)\right)x-\frac{2\left(2+\sin^2(x)\right)\cos(x)}{3}-4x\left(2+\sin^2(x)\right)\cos(x)+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2+3*x)^2*sin(x)^3,x)
```

```
[Out] -3*x^2*(2+sin(x)^2)*cos(x)+12*cos(x)+12*x*sin(x)+2*sin(x)^3*x-2/3*(2+sin(x)
^2)*cos(x)-4*x*(2+sin(x)^2)*cos(x)+4/3*sin(x)^3+8*sin(x)
```

maxima [A] time = 0.33, size = 66, normalized size = 1.02

$$\frac{4}{3}\cos(x)^3+\frac{1}{12}\left(9x^2-2\right)\cos(3x)+x\cos(3x)-\frac{27}{4}\left(x^2-2\right)\cos(x)-9x\cos(x)-\frac{1}{2}x\sin(3x)+\frac{27}{2}x\sin(x)-4\cos(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+3*x)^2*sin(x)^3,x, algorithm="maxima")
```

```
[Out] 4/3*cos(x)^3 + 1/12*(9*x^2 - 2)*cos(3*x) + x*cos(3*x) - 27/4*(x^2 - 2)*cos(
x) - 9*x*cos(x) - 1/2*x*sin(3*x) + 27/2*x*sin(x) - 4*cos(x) - 1/3*sin(3*x)
+ 9*sin(x)
```

mupad [B] time = 3.03, size = 65, normalized size = 1.00

$$10\cos(x)+\frac{28\sin(x)}{3}-9x^2\cos(x)+4x\cos(x)^3+\frac{2\cos(x)^3}{3}+3x^2\cos(x)^3-\frac{4\cos(x)^2\sin(x)}{3}-12x\cos(x)+14x\sin($$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)^3*(3*x + 2)^2,x)
```

```
[Out] 10*cos(x) + (28*sin(x))/3 - 9*x^2*cos(x) + 4*x*cos(x)^3 + (2*cos(x)^3)/3 +
3*x^2*cos(x)^3 - (4*cos(x)^2*sin(x))/3 - 12*x*cos(x) + 14*x*sin(x) - 2*x*co
s(x)^2*sin(x)
```

sympy [A] time = 1.22, size = 100, normalized size = 1.54

$$-9x^2\sin^2(x)\cos(x)-6x^2\cos^3(x)+14x\sin^3(x)-12x\sin^2(x)\cos(x)+12x\sin(x)\cos^2(x)-8x\cos^3(x)+\frac{28\sin^3(x)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+3*x)**2*sin(x)**3,x)
```

```
[Out] -9*x**2*sin(x)**2*cos(x) - 6*x**2*cos(x)**3 + 14*x*sin(x)**3 - 12*x*sin(x)*
*2*cos(x) + 12*x*sin(x)*cos(x)**2 - 8*x*cos(x)**3 + 28*sin(x)**3/3 + 10*sin
(x)**2*cos(x) + 8*sin(x)*cos(x)**2 + 32*cos(x)**3/3
```

3.917 $\int \sec^{1+m}(x)\sin(x)\,dx$

Optimal. Leaf size=8

$$\frac{\sec^m(x)}{m}$$

```
[Out] sec(x)^m/m
```

Rubi [A] time = 0.02, antiderivative size = 8, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {2622, 30}

$$\frac{\sec^m(x)}{m}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[x]^(1 + m)*Sin[x],x]
```

```
[Out] Sec[x]^m/m
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2622

```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Dist[1/(f*a^n), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/2
), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)
/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rubi steps

$$\begin{aligned}\int \sec^{1+m}(x)\sin(x)\,dx &= \text{Subst}\left(\int x^{-1+m}\,dx, x, \sec(x)\right)\\ &= \frac{\sec^m(x)}{m}\end{aligned}$$

Mathematica [A] time = 0.01, size = 8, normalized size = 1.00

$$\frac{\sec^m(x)}{m}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[x]^(1 + m)*Sin[x],x]
```

```
[Out] Sec[x]^m/m
```

fricas [A] time = 0.92, size = 14, normalized size = 1.75

$$\frac{\left(\frac{1}{\cos(x)}\right)^{m+1}\cos(x)}{m}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)^(1+m)*sin(x),x, algorithm="fricas")
```

```
[Out] (1/cos(x))^(m + 1)*cos(x)/m
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sec(x)^{m+1}\sin(x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)^(1+m)*sin(x),x, algorithm="giac")
```

```
[Out] integrate(sec(x)^(m + 1)*sin(x), x)
```

maple [A] time = 0.08, size = 11, normalized size = 1.38

$$\frac{\left(\frac{1}{\cos(x)}\right)^{m}}{m}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(x)^(1+m)*sin(x),x)
```

```
[Out] 1/m*(1/cos(x))^m
```

maxima [A] time = 0.31, size = 10, normalized size = 1.25

$$\frac{\cos(x)^{-m}}{m}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)^(1+m)*sin(x),x, algorithm="maxima")
```

```
[Out] cos(x)^(-m)/m
```

mupad [B] time = 0.14, size = 10, normalized size = 1.25

$$\frac{\left(\frac{1}{\cos(x)}\right)^m}{m}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)*(1/cos(x))^(m + 1),x)
```

```
[Out] (1/cos(x))^m/m
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sin(x)\sec^{m+1}(x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)**(1+m)*sin(x),x)
```

```
[Out] Integral(sin(x)*sec(x)**(m + 1), x)
```

3.918 $\int \cos^n(a+bx)\sin^{-2-n}(a+bx)\,dx$

Optimal. Leaf size=32

$$-\frac{\sin^{-n-1}(a+bx)\cos^{n+1}(a+bx)}{b(n+1)}$$

```
[Out] -cos(b*x+a)^(1+n)*sin(b*x+a)^(-1-n)/b/(1+n)
```

Rubi [A] time = 0.04, antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.048, Rules used = {2563}

$$-\frac{\sin^{-n-1}(a+bx)\cos^{n+1}(a+bx)}{b(n+1)}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x]^n*Sin[a + b*x]^(-2 - n),x]
```

```
[Out] -((Cos[a + b*x]^(1 + n)*Sin[a + b*x]^(-1 - n))/(b*(1 + n)))
```

Rule 2563

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(
m_.), x_Symbol] :> Simp[((a*Sin[e + f*x])^(m + 1)*(b*Cos[e + f*x])^(n + 1))
/(a*b*f*(m + 1)), x] /; FreeQ[{a, b, e, f, m, n}, x] && EqQ[m + n + 2, 0] &
& NeQ[m, -1]
```

Rubi steps

$$\int \cos^n(a+bx)\sin^{-2-n}(a+bx)\,dx = -\frac{\cos^{1+n}(a+bx)\sin^{-1-n}(a+bx)}{b(1+n)}$$

Mathematica [A] time = 0.08, size = 32, normalized size = 1.00

$$-\frac{\sin^{-n-1}(a+bx)\cos^{n+1}(a+bx)}{b(n+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x]^n*Sin[a + b*x]^(-2 - n),x]
```

```
[Out] -((Cos[a + b*x]^(1 + n)*Sin[a + b*x]^(-1 - n))/(b*(1 + n)))
```

fricas [A] time = 0.95, size = 41, normalized size = 1.28

$$-\frac{\cos\left(bx+a\right)^{n}\sin\left(bx+a\right)^{-n-2}\cos\left(bx+a\right)\sin\left(bx+a\right)}{bn+b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^n*sin(b*x+a)^(-2-n),x, algorithm="fricas")
```

```
[Out] -cos(b*x + a)^n*sin(b*x + a)^(-n - 2)*cos(b*x + a)*sin(b*x + a)/(b*n + b)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \cos\left(bx+a\right)^{n}\sin\left(bx+a\right)^{-n-2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^n*sin(b*x+a)^(-2-n),x, algorithm="giac")
```

```
[Out] integrate(cos(b*x + a)^n*sin(b*x + a)^(-n - 2), x)
```

maple [F] time = 0.37, size = 0, normalized size = 0.00

$$\int \left(\cos^{n}\left(bx+a\right)\right)\left(\sin^{-2-n}\left(bx+a\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)^n*sin(b*x+a)^(-2-n),x)
```

```
[Out] int(cos(b*x+a)^n*sin(b*x+a)^(-2-n),x)
```

maxima [B] time = 0.44, size = 125, normalized size = 3.91

$$\frac{2\left(\frac{\sin(bx+a)^{2}}{(\cos(bx+a)+1)^{2}}-1\right)\left(\cos\left(bx+a\right)+1\right)e^{\left(n\log\left(\frac{\sin(bx+a)}{\cos(bx+a)+1}+1\right)-n\log\left(\frac{\sin(bx+a)}{\cos(bx+a)+1}\right)+n\log\left(-\frac{\sin(bx+a)}{\cos(bx+a)+1}+1\right)\right)}}{\left(2^{n+2}n+2^{n+2}\right)b\sin\left(bx+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^n*sin(b*x+a)^(-2-n),x, algorithm="maxima")
```

```
[Out] 2*(sin(b*x + a)^2/(cos(b*x + a) + 1)^2 - 1)*(cos(b*x + a) + 1)*e^(n*log(sin
(b*x + a)/(cos(b*x + a) + 1) + 1) - n*log(sin(b*x + a)/(cos(b*x + a) + 1))
+ n*log(-sin(b*x + a)/(cos(b*x + a) + 1) + 1))/((2^(n + 2)*n + 2^(n + 2))*b
*sin(b*x + a))
```

mupad [B] time = 3.43, size = 45, normalized size = 1.41

$$-\frac{\cos\left(a+b\,x\right)^n\,\sin\left(2\,a+2\,b\,x\right)}{2\,b\sin\left(a+b\,x\right)^n\sin\left(a+b\,x\right)^2\,\left(n+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)^n/sin(a + b*x)^(n + 2),x)
```

```
[Out] -(cos(a + b*x)^n*sin(2*a + 2*b*x))/(2*b*sin(a + b*x)^n*sin(a + b*x)^2*(n +
1))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sin^{-n-2}\left(a+bx\right)\cos^{n}\left(a+bx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)**n*sin(b*x+a)**(-2-n),x)
```

```
[Out] Integral(sin(a + b*x)**(-n - 2)*cos(a + b*x)**n, x)
```

3.919 $\int \frac{1}{\sec(x)+\sin(x)\tan(x)}\,dx$

Optimal. Leaf size=3

$$\tan^{-1}(\sin(x))$$

```
[Out] arctan(sin(x))
```

Rubi [A] time = 0.03, antiderivative size = 3, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {4397, 3190, 203}

$$\tan^{-1}(\sin(x))$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[x] + Sin[x]*Tan[x])^(-1),x]
```

```
[Out] ArcTan[Sin[x]]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 3190

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Su
bst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]/
ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 4397

```
Int[u_, x_Symbol] :> Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]
```

Rubi steps

$$\begin{aligned} \int \frac{1}{\sec(x)+\sin(x)\tan(x)}\,dx &= \int \frac{\cos(x)}{1+\sin^2(x)}\,dx \\ &= \text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \sin(x)\right) \\ &= \tan^{-1}(\sin(x)) \end{aligned}$$

Mathematica [A] time = 0.02, size = 3, normalized size = 1.00

$$\tan^{-1}(\sin(x))$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[x] + Sin[x]*Tan[x])^(-1),x]
[Out] ArcTan[Sin[x]]
```

fricas [A] time = 1.42, size = 3, normalized size = 1.00

$$\arctan(\sin(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sec(x)+sin(x)*tan(x)),x, algorithm="fricas")
[Out] arctan(sin(x))
```

giac [A] time = 0.12, size = 3, normalized size = 1.00

$$\arctan(\sin(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sec(x)+sin(x)*tan(x)),x, algorithm="giac")
[Out] arctan(sin(x))
```

maple [A] time = 0.15, size = 4, normalized size = 1.33

$$\arctan(\sin(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sec(x)+sin(x)*tan(x)),x)
[Out] arctan(sin(x))
```

maxima [B] time = 0.33, size = 45, normalized size = 15.00

$$\frac{1}{2}\arctan(\sin(2x) + 2\sin(x), \cos(2x) + 2\cos(x) - 1) - \frac{1}{2}\arctan(\sin(2x) - 2\sin(x), \cos(2x) - 2\cos(x) - 1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sec(x)+sin(x)*tan(x)),x, algorithm="maxima")
```

```
[Out] 1/2*arctan2(sin(2*x) + 2*sin(x), cos(2*x) + 2*cos(x) - 1) - 1/2*arctan2(sin
(2*x) - 2*sin(x), cos(2*x) - 2*cos(x) - 1)
```

mupad [B] time = 3.19, size = 26, normalized size = 8.67

$$\operatorname{atan}\left(\frac{\tan\left(\frac{x}{2}\right)^3}{2}+\frac{5\tan\left(\frac{x}{2}\right)}{2}\right)-\operatorname{atan}\left(\frac{\tan\left(\frac{x}{2}\right)}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(x)*tan(x) + 1/cos(x)),x)
```

```
[Out] atan((5*tan(x/2))/2 + tan(x/2)^3/2) - atan(tan(x/2)/2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sin(x)\tan(x)+\sec(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sec(x)+sin(x)*tan(x)),x)
```

```
[Out] Integral(1/(sin(x)*tan(x) + sec(x)), x)
```

3.920 $\int \left(a+bx+cx^2\right)\sin(x)\,dx$

Optimal. Leaf size=35

$$-a\cos(x)+b\sin(x)-bx\cos(x)-cx^2\cos(x)+2cx\sin(x)+2c\cos(x)$$

```
[Out] -a*cos(x)+2*c*cos(x)-b*x*cos(x)-c*x^2*cos(x)+b*sin(x)+2*c*x*sin(x)
```

Rubi [A] time = 0.07, antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {6742, 2638, 3296, 2637}

$$-a\cos(x)+b\sin(x)-bx\cos(x)-cx^2\cos(x)+2cx\sin(x)+2c\cos(x)$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x + c*x^2)*Sin[x],x]
```

```
[Out] -(a*Cos[x]) + 2*c*Cos[x] - b*x*Cos[x] - c*x^2*Cos[x] + b*Sin[x] + 2*c*x*Sin
[x]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\begin{aligned}\int \left(a+bx+cx^2\right)\sin(x)\,dx &= \int \left(a\sin(x)+bx\sin(x)+cx^2\sin(x)\right)\,dx\\ &= a\int \sin(x)\,dx + b\int x\sin(x)\,dx + c\int x^2\sin(x)\,dx\\ &= -a\cos(x)-bx\cos(x)-cx^2\cos(x)+b\int \cos(x)\,dx+(2c)\int x\cos(x)\,dx\\ &= -a\cos(x)-bx\cos(x)-cx^2\cos(x)+b\sin(x)+2cx\sin(x)-(2c)\int \sin(x)\,dx\\ &= -a\cos(x)+2c\cos(x)-bx\cos(x)-cx^2\cos(x)+b\sin(x)+2cx\sin(x)\end{aligned}$$

Mathematica [A] time = 0.04, size = 32, normalized size = 0.91

$$-a\cos(x)+b\sin(x)-bx\cos(x)-c\left(x^2-2\right)\cos(x)+2cx\sin(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x + c*x^2)*Sin[x],x]
[Out] -(a*Cos[x]) - b*x*Cos[x] - c*(-2 + x^2)*Cos[x] + b*Sin[x] + 2*c*x*Sin[x]
```

fricas [A] time = 0.59, size = 27, normalized size = 0.77

$$-\left(cx^2+bx+a-2\,c\right)\cos(x)+(2\,cx+b)\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)*sin(x),x, algorithm="fricas")
[Out] -(c*x^2 + b*x + a - 2*c)*cos(x) + (2*c*x + b)*sin(x)
```

giac [A] time = 0.12, size = 27, normalized size = 0.77

$$-\left(cx^2+bx+a-2\,c\right)\cos(x)+(2\,cx+b)\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)*sin(x),x, algorithm="giac")
[Out] -(c*x^2 + b*x + a - 2*c)*cos(x) + (2*c*x + b)*sin(x)
```

maple [A] time = 0.03, size = 36, normalized size = 1.03

$$c\left(-x^2\cos(x)+2\cos(x)+2x\sin(x)\right)+b\left(\sin(x)-x\cos(x)\right)-a\cos(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)*sin(x),x)
```

```
[Out] c*(-x^2*cos(x)+2*cos(x)+2*x*sin(x))+b*(sin(x)-x*cos(x))-a*cos(x)
```

maxima [A] time = 0.32, size = 35, normalized size = 1.00

$$-(x\cos(x)-\sin(x))b-\left(\left(x^2-2\right)\cos(x)-2\,x\sin(x)\right)c-a\cos(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)*sin(x),x, algorithm="maxima")
```

```
[Out] -(x*cos(x) - sin(x))*b - ((x^2 - 2)*cos(x) - 2*x*sin(x))*c - a*cos(x)
```

mupad [B] time = 0.06, size = 34, normalized size = 0.97

$$b\,\sin(x)-\cos(x)\,\left(a-2\,c\right)-b\,x\,\cos(x)+2\,c\,x\,\sin(x)-c\,x^2\,\cos(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)*(a + b*x + c*x^2),x)
```

```
[Out] b*sin(x) - cos(x)*(a - 2*c) - b*x*cos(x) + 2*c*x*sin(x) - c*x^2*cos(x)
```

sympy [A] time = 0.33, size = 39, normalized size = 1.11

$$-a\cos{\left(x\right)}-bx\cos{\left(x\right)}+b\sin{\left(x\right)}-cx^{2}\cos{\left(x\right)}+2cx\sin{\left(x\right)}+2c\cos{\left(x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)*sin(x),x)
```

```
[Out] -a*cos(x) - b*x*cos(x) + b*sin(x) - c*x**2*cos(x) + 2*c*x*sin(x) + 2*c*cos(
x)
```

3.921 $\int \frac{\sin(x^5)}{x} dx$

Optimal. Leaf size=8

$$\frac{\text{Si}\left(x^5\right)}{5}$$

```
[Out] 1/5*Si(x^5)
```

Rubi [A] time = 0.01, antiderivative size = 8, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {3375}

$$\frac{\text{Si}\left(x^5\right)}{5}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[x^5]/x,x]
```

```
[Out] SinIntegral[x^5]/5
```

Rule 3375

```
Int[Sin[(d_.)*(x_)^(n_)]/(x_), x_Symbol] :> Simp[SinIntegral[d*x^n]/n, x] /
; FreeQ[{d, n}, x]
```

Rubi steps

$$\int \frac{\sin\left(x^5\right)}{x} dx = \frac{\text{Si}\left(x^5\right)}{5}$$

Mathematica [A] time = 0.00, size = 8, normalized size = 1.00

$$\frac{\text{Si}\left(x^5\right)}{5}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[x^5]/x,x]
```

```
[Out] SinIntegral[x^5]/5
```

fricas [A] time = 0.73, size = 6, normalized size = 0.75

$$\frac{1}{5}\operatorname{Si}\left(x^{5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x^5)/x,x, algorithm="fricas")
```

```
[Out] 1/5*sin_integral(x^5)
```

giac [A] time = 0.13, size = 6, normalized size = 0.75

$$\frac{1}{5}\operatorname{Si}\left(x^{5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x^5)/x,x, algorithm="giac")
```

```
[Out] 1/5*sin_integral(x^5)
```

maple [A] time = 0.03, size = 7, normalized size = 0.88

$$\frac{\operatorname{Si}\left(x^{5}\right)}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x^5)/x,x)
```

```
[Out] 1/5*Si(x^5)
```

maxima [C] time = 0.36, size = 17, normalized size = 2.12

$$-\frac{1}{10}i\operatorname{Ei}\left(i\,x^{5}\right)+\frac{1}{10}i\operatorname{Ei}\left(-i\,x^{5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x^5)/x,x, algorithm="maxima")
```

```
[Out] -1/10*I*Ei(I*x^5) + 1/10*I*Ei(-I*x^5)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.12

$$\frac{\operatorname{sinint}\left(x^{5}\right)}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x^5)/x,x)
```

```
[Out] sinint(x^5)/5
```

sympy [A] time = 0.64, size = 5, normalized size = 0.62

$$\frac{\mathrm{Si}\left(x^5\right)}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x**5)/x,x)
```

```
[Out] Si(x**5)/5
```

3.922 $\int \frac{\sin(2^x)}{1+2^x} \, dx$

Optimal. Leaf size=37

$$\frac{\sin(1)\text{Ci}\left(1+2^x\right)}{\log(2)}+\frac{\text{Si}\left(2^x\right)}{\log(2)}-\frac{\cos(1)\text{Si}\left(1+2^x\right)}{\log(2)}$$

```
[Out] Si(2^x)/ln(2)-cos(1)*Si(1+2^x)/ln(2)+Ci(1+2^x)*sin(1)/ln(2)
```

Rubi [A] time = 0.17, antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {2282, 6742, 3299, 3303, 3302}

$$\frac{\sin(1)\text{CosIntegral}\left(2^x+1\right)}{\log(2)}+\frac{\text{Si}\left(2^x\right)}{\log(2)}-\frac{\cos(1)\text{Si}\left(1+2^x\right)}{\log(2)}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[2^x]/(1 + 2^x),x]
```

```
[Out] (CosIntegral[1 + 2^x]*Sin[1])/Log[2] + SinIntegral[2^x]/Log[2] - (Cos[1]*Si
nIntegral[1 + 2^x])/Log[2]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3302

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
```

```
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sin(2^x)}{1+2^x}\,dx &= \frac{\text{Subst}\left(\int \frac{\sin(x)}{x(1+x)}\,dx, x, 2^x\right)}{\log(2)}\\
&= \frac{\text{Subst}\left(\int \left(\frac{\sin(x)}{x}-\frac{\sin(x)}{1+x}\right)dx, x, 2^x\right)}{\log(2)}\\
&= \frac{\text{Subst}\left(\int \frac{\sin(x)}{x}\,dx, x, 2^x\right)}{\log(2)}-\frac{\text{Subst}\left(\int \frac{\sin(x)}{1+x}\,dx, x, 2^x\right)}{\log(2)}\\
&= \frac{\text{Si}(2^x)}{\log(2)}-\frac{\cos(1)\,\text{Subst}\left(\int \frac{\sin(1+x)}{1+x}\,dx, x, 2^x\right)}{\log(2)}+\frac{\sin(1)\,\text{Subst}\left(\int \frac{\cos(1+x)}{1+x}\,dx, x, 2^x\right)}{\log(2)}\\
&= \frac{\text{Ci}(1+2^x)\sin(1)}{\log(2)}+\frac{\text{Si}(2^x)}{\log(2)}-\frac{\cos(1)\text{Si}(1+2^x)}{\log(2)}
\end{aligned}$$

Mathematica [A] time = 0.07, size = 29, normalized size = 0.78

$$\frac{\sin(1)\text{Ci}(1+2^x)+\text{Si}(2^x)-\cos(1)\text{Si}(1+2^x)}{\log(2)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[2^x]/(1 + 2^x),x]
```

```
[Out] (CosIntegral[1 + 2^x]*Sin[1] + SinIntegral[2^x] - Cos[1]*SinIntegral[1 + 2^
x])/Log[2]
```

fricas [A] time = 0.85, size = 43, normalized size = 1.16

$$\frac{\text{Ci}(2^x+1)\sin(1)+\text{Ci}(-2^x-1)\sin(1)-2\,\cos(1)\,\text{Si}(2^x+1)+2\,\text{Si}(2^x)}{2\,\log(2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(2^x)/(1+2^x),x, algorithm="fricas")
```

```
[Out] 1/2*(cos_integral(2^x + 1)*sin(1) + cos_integral(-2^x - 1)*sin(1) - 2*cos(1
)*sin_integral(2^x + 1) + 2*sin_integral(2^x))/log(2)
```

giac [A] time = 0.13, size = 29, normalized size = 0.78

$$\frac{\mathrm{Ci}\left(2^{x}+1\right)\sin(1)-\cos(1)\,\mathrm{Si}\left(2^{x}+1\right)+\mathrm{Si}\left(2^{x}\right)}{\log(2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(2^x)/(1+2^x),x, algorithm="giac")
```

```
[Out] (cos_integral(2^x + 1)*sin(1) - cos(1)*sin_integral(2^x + 1) + sin_integral
(2^x))/log(2)
```

maple [A] time = 0.03, size = 38, normalized size = 1.03

$$\frac{\mathrm{Si}\left(2^{x}\right)}{\ln(2)}-\frac{\cos(1)\,\mathrm{Si}\left(1+2^{x}\right)}{\ln(2)}+\frac{\mathrm{Ci}\left(1+2^{x}\right)\sin(1)}{\ln(2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(2^x)/(1+2^x),x)
```

```
[Out] Si(2^x)/ln(2)-cos(1)*Si(1+2^x)/ln(2)+Ci(1+2^x)*sin(1)/ln(2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin\left(2^{x}\right)}{2^{x}+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(2^x)/(1+2^x),x, algorithm="maxima")
```

```
[Out] integrate(sin(2^x)/(2^x + 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{\sin\left(2^{x}\right)}{2^{x}+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(2^x)/(2^x + 1),x)
```

```
[Out] int(sin(2^x)/(2^x + 1), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin(2^x)}{2^x+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(2**x)/(1+2**x),x)
```

```
[Out] Integral(sin(2**x)/(2**x + 1), x)
```

3.923 $\int x \cos\left(2x^2\right) \sin^{\frac{3}{4}}\left(2x^2\right) dx$

Optimal. Leaf size=14

$$\frac{1}{7} \sin^{\frac{7}{4}}\left(2x^2\right)$$

```
[Out] 1/7*sin(2*x^2)^(7/4)
```

Rubi [A] time = 0.01, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.056, Rules used = {3441}

$$\frac{1}{7} \sin^{\frac{7}{4}}\left(2x^2\right)$$

Antiderivative was successfully verified.

```
[In] Int[x*Cos[2*x^2]*Sin[2*x^2]^(3/4),x]
```

```
[Out] Sin[2*x^2]^(7/4)/7
```

Rule 3441

```
Int[Cos[(a_.) + (b_.)*(x_)^(n_.)]*(x_)^(m_.)*Sin[(a_.) + (b_.)*(x_)^(n_.)]^
(p_.), x_Symbol] :> Simp[Sin[a + b*x^n]^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[
{a, b, m, n, p}, x] && EqQ[m, n - 1] && NeQ[p, -1]
```

Rubi steps

$$\int x \cos\left(2x^2\right) \sin^{\frac{3}{4}}\left(2x^2\right) dx = \frac{1}{7} \sin^{\frac{7}{4}}\left(2x^2\right)$$

Mathematica [A] time = 0.01, size = 14, normalized size = 1.00

$$\frac{1}{7} \sin^{\frac{7}{4}}\left(2x^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Cos[2*x^2]*Sin[2*x^2]^(3/4),x]
```

```
[Out] Sin[2*x^2]^(7/4)/7
```

fricas [A] time = 0.87, size = 10, normalized size = 0.71

$$\frac{1}{7} \sin\left(2\,x^2\right)^{\frac{7}{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(2*x^2)*sin(2*x^2)^(3/4),x, algorithm="fricas")
```

```
[Out] 1/7*sin(2*x^2)^(7/4)
```

giac [A] time = 0.16, size = 10, normalized size = 0.71

$$\frac{1}{7} \sin\left(2 x^2\right)^{\frac{7}{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(2*x^2)*sin(2*x^2)^(3/4),x, algorithm="giac")
```

```
[Out] 1/7*sin(2*x^2)^(7/4)
```

maple [A] time = 0.02, size = 11, normalized size = 0.79

$$\frac{\left(\sin^{\frac{7}{4}}\left(2x^2\right)\right)}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*cos(2*x^2)*sin(2*x^2)^(3/4),x)
```

```
[Out] 1/7*sin(2*x^2)^(7/4)
```

maxima [A] time = 0.32, size = 10, normalized size = 0.71

$$\frac{1}{7} \sin\left(2 x^2\right)^{\frac{7}{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(2*x^2)*sin(2*x^2)^(3/4),x, algorithm="maxima")
```

```
[Out] 1/7*sin(2*x^2)^(7/4)
```

mupad [B] time = 3.14, size = 41, normalized size = 2.93

$$-\frac{\cos\left(2 x^2\right)^2 \sin\left(2 x^2\right)^{7/4} {}_2F_1\left(\frac{1}{8}, 1;\ 2;\ \cos\left(2 x^2\right)^2\right)}{8 \left(\sin\left(2 x^2\right)^2\right)^{7/8}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*cos(2*x^2)*sin(2*x^2)^(3/4),x)
```

```
[Out] -(cos(2*x^2)^2*sin(2*x^2)^(7/4)*hypergeom([1/8, 1], 2, cos(2*x^2)^2))/(8*(s
in(2*x^2)^2)^(7/8))
```

sympy [A] time = 84.15, size = 10, normalized size = 0.71

$$\frac{\sin^{\frac{7}{4}}\left(2x^2\right)}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(2*x**2)*sin(2*x**2)**(3/4),x)
```

```
[Out] sin(2*x**2)**(7/4)/7
```

3.924 $\int x \sec^2(x^2) \tan^2(x^2) \, dx$

Optimal. Leaf size=10

$$\frac{1}{6}\tan^3\left(x^2\right)$$

[Out] 1/6*tan(x^2)^3

Rubi [A] time = 0.04, antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {6686}

$$\frac{1}{6}\tan^3\left(x^2\right)$$

Antiderivative was successfully verified.

[In] Int[x*Sec[x^2]^2*Tan[x^2]^2,x]

[Out] Tan[x^2]^3/6

Rule 6686

```
Int[(u_)*(y_)^(m_.), x_Symbol] :> With[{q = DerivativeDivides[y, u, x]}, Si
mp[(q*y^(m + 1))/(m + 1), x] /;  !FalseQ[q]] /; FreeQ[m, x] && NeQ[m, -1]
```

Rubi steps

$$\int x \sec^2\left(x^2\right) \tan^2\left(x^2\right) \, dx = \frac{1}{6}\tan^3\left(x^2\right)$$

Mathematica [A] time = 0.00, size = 10, normalized size = 1.00

$$\frac{1}{6}\tan^3\left(x^2\right)$$

Antiderivative was successfully verified.

[In] Integrate[x*Sec[x^2]^2*Tan[x^2]^2,x]

[Out] Tan[x^2]^3/6

fricas [B] time = 0.43, size = 20, normalized size = 2.00

$$-\frac{\left(\cos\left(x^2\right)^2 - 1\right)\sin\left(x^2\right)}{6\cos\left(x^2\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sec(x^2)^2*tan(x^2)^2,x, algorithm="fricas")
[Out] -1/6*(cos(x^2)^2 - 1)*sin(x^2)/cos(x^2)^3
```

giac [A] time = 0.14, size = 8, normalized size = 0.80

$$\frac{1}{6}\tan\left(x^2\right)^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sec(x^2)^2*tan(x^2)^2,x, algorithm="giac")
[Out] 1/6*tan(x^2)^3
```

maple [A] time = 0.10, size = 15, normalized size = 1.50

$$\frac{\sin^3\left(x^2\right)}{6\cos\left(x^2\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*sec(x^2)^2*tan(x^2)^2,x)
[Out] 1/6*sin(x^2)^3/cos(x^2)^3
```

maxima [A] time = 0.32, size = 8, normalized size = 0.80

$$\frac{1}{6}\tan\left(x^2\right)^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sec(x^2)^2*tan(x^2)^2,x, algorithm="maxima")
[Out] 1/6*tan(x^2)^3
```

mupad [B] time = 3.09, size = 19, normalized size = 1.90

$$\frac{\tan\left(x^2\right)}{6\cos\left(x^2\right)^2}-\frac{\tan\left(x^2\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*tan(x^2)^2)/cos(x^2)^2,x)
```

```
[Out] tan(x^2)/(6*cos(x^2)^2) - tan(x^2)/6
```

sympy [A] time = 1.04, size = 7, normalized size = 0.70

$$\frac{\tan^3(x^2)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sec(x**2)**2*tan(x**2)**2,x)
```

```
[Out] tan(x**2)**3/6
```

3.925 $\int x^2 \cos^7\left(a+bx^3\right)\sin\left(a+bx^3\right)\,dx$

Optimal. Leaf size=17

$$-\frac{\cos^8\left(a+bx^3\right)}{24b}$$

```
[Out] -1/24*cos(b*x^3+a)^8/b
```

Rubi [A] time = 0.02, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {3442}

$$-\frac{\cos^8\left(a+bx^3\right)}{24b}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*Cos[a + b*x^3]^7*Sin[a + b*x^3],x]
```

```
[Out] -Cos[a + b*x^3]^8/(24*b)
```

Rule 3442

```
Int[Cos[(a_.) + (b_.)*(x_)^(n_.)]^(p_.)*(x_)^(m_.)*Sin[(a_.) + (b_.)*(x_)^(
n_.)], x_Symbol] :> -Simp[Cos[a + b*x^n]^(p + 1)/(b*n*(p + 1)), x] /; FreeQ
[{a, b, m, n, p}, x] && EqQ[m, n - 1] && NeQ[p, -1]
```

Rubi steps

$$\int x^2\cos^7\left(a+bx^3\right)\sin\left(a+bx^3\right)\,dx=-\frac{\cos^8\left(a+bx^3\right)}{24b}$$

Mathematica [A] time = 0.02, size = 17, normalized size = 1.00

$$-\frac{\cos^8\left(a+bx^3\right)}{24b}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*Cos[a + b*x^3]^7*Sin[a + b*x^3],x]
```

```
[Out] -1/24*Cos[a + b*x^3]^8/b
```

fricas [A] time = 1.70, size = 15, normalized size = 0.88

$$-\frac{\cos\left(bx^3+a\right)^8}{24\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*cos(b*x^3+a)^7*sin(b*x^3+a),x, algorithm="fricas")
[Out] -1/24*cos(b*x^3 + a)^8/b
```

giac [A] time = 0.18, size = 15, normalized size = 0.88

$$-\frac{\cos\left(bx^3+a\right)^8}{24\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*cos(b*x^3+a)^7*sin(b*x^3+a),x, algorithm="giac")
[Out] -1/24*cos(b*x^3 + a)^8/b
```

maple [A] time = 0.05, size = 16, normalized size = 0.94

$$-\frac{\cos^8\left(b\,x^3+a\right)}{24b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*cos(b*x^3+a)^7*sin(b*x^3+a),x)
[Out] -1/24*cos(b*x^3+a)^8/b
```

maxima [A] time = 0.34, size = 15, normalized size = 0.88

$$-\frac{\cos\left(bx^3+a\right)^8}{24\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*cos(b*x^3+a)^7*sin(b*x^3+a),x, algorithm="maxima")
[Out] -1/24*cos(b*x^3 + a)^8/b
```

mupad [B] time = 3.26, size = 56, normalized size = 3.29

$$-\frac{56\,\cos\left(2\,b\,x^3+2\,a\right)+28\,\cos\left(4\,b\,x^3+4\,a\right)+8\,\cos\left(6\,b\,x^3+6\,a\right)+\cos\left(8\,b\,x^3+8\,a\right)}{3072\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*cos(a + b*x^3)^7*sin(a + b*x^3),x)
```

```
[Out] -(56*cos(2*a + 2*b*x^3) + 28*cos(4*a + 4*b*x^3) + 8*cos(6*a + 6*b*x^3) + co
s(8*a + 8*b*x^3))/(3072*b)
```

sympy [A] time = 20.86, size = 27, normalized size = 1.59

$$\begin{cases} -\frac{\cos^{8}\left(a+bx^{3}\right)}{24b} & \text{for } b \neq 0 \\ \frac{x^{3}\sin\left(a\right)\cos^{7}\left(a\right)}{3} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*cos(b*x**3+a)**7*sin(b*x**3+a),x)
```

```
[Out] Piecewise((-cos(a + b*x**3)**8/(24*b), Ne(b, 0)), (x**3*sin(a)*cos(a)**7/3,
 True))
```

3.926 $\int x^5 \cos^7\left(a+bx^3\right)\sin\left(a+bx^3\right)\,dx$

Optimal. Leaf size=129

$$\frac{\sin\left(a+bx^3\right)\cos^7\left(a+bx^3\right)}{192b^2}+\frac{7\sin\left(a+bx^3\right)\cos^5\left(a+bx^3\right)}{1152b^2}+\frac{35\sin\left(a+bx^3\right)\cos^3\left(a+bx^3\right)}{4608b^2}+\frac{35\sin\left(a+bx^3\right)\mathrm{co}}{3072b^2}$$

```
[Out] 35/3072*x^3/b-1/24*x^3*cos(b*x^3+a)^8/b+35/3072*cos(b*x^3+a)*sin(b*x^3+a)/b
^2+35/4608*cos(b*x^3+a)^3*sin(b*x^3+a)/b^2+7/1152*cos(b*x^3+a)^5*sin(b*x^3+
a)/b^2+1/192*cos(b*x^3+a)^7*sin(b*x^3+a)/b^2
```

Rubi [A] time = 0.14, antiderivative size = 129, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {3444, 3380, 2635, 8}

$$\frac{\sin\left(a+bx^3\right)\cos^7\left(a+bx^3\right)}{192b^2}+\frac{7\sin\left(a+bx^3\right)\cos^5\left(a+bx^3\right)}{1152b^2}+\frac{35\sin\left(a+bx^3\right)\cos^3\left(a+bx^3\right)}{4608b^2}+\frac{35\sin\left(a+bx^3\right)\mathrm{co}}{3072b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^5*Cos[a + b*x^3]^7*Sin[a + b*x^3],x]
```

```
[Out] (35*x^3)/(3072*b) - (x^3*Cos[a + b*x^3]^8)/(24*b) + (35*Cos[a + b*x^3]*Sin[
a + b*x^3])/(3072*b^2) + (35*Cos[a + b*x^3]^3*Sin[a + b*x^3])/(4608*b^2) +
(7*Cos[a + b*x^3]^5*Sin[a + b*x^3])/(1152*b^2) + (Cos[a + b*x^3]^7*Sin[a +
b*x^3])/(192*b^2)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 3380

```
Int[((a_.) + Cos[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Cos[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rule 3444

```
Int[Cos[(a_.) + (b_.)*(x_)^(n_.)]^(p_.)*(x_)^(m_.)*Sin[(a_.) + (b_.)*(x_)^(
n_.)], x_Symbol] :> -Simp[(x^(m - n + 1)*Cos[a + b*x^n]^(p + 1))/(b*n*(p +
1)), x] + Dist[(m - n + 1)/(b*n*(p + 1)), Int[x^(m - n)*Cos[a + b*x^n]^(p +
 1), x], x] /; FreeQ[{a, b, p}, x] && LtQ[0, n, m + 1] && NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int x^5\cos^7\left(a+bx^3\right)\sin\left(a+bx^3\right)dx &= -\frac{x^3\cos^8\left(a+bx^3\right)}{24b}+\frac{\int x^2\cos^8\left(a+bx^3\right)dx}{8b}\\ &= -\frac{x^3\cos^8\left(a+bx^3\right)}{24b}+\frac{\text{Subst}\left(\int\cos^8(a+bx)\,dx,x,x^3\right)}{24b}\\ &= -\frac{x^3\cos^8\left(a+bx^3\right)}{24b}+\frac{\cos^7\left(a+bx^3\right)\sin\left(a+bx^3\right)}{192b^2}+\frac{7\,\text{Subst}\left(\int\cos^6(a\right.}{192b}\\ &= -\frac{x^3\cos^8\left(a+bx^3\right)}{24b}+\frac{7\cos^5\left(a+bx^3\right)\sin\left(a+bx^3\right)}{1152b^2}+\frac{\cos^7\left(a+bx^3\right)\sin}{192b^2}\\ &= -\frac{x^3\cos^8\left(a+bx^3\right)}{24b}+\frac{35\cos^3\left(a+bx^3\right)\sin\left(a+bx^3\right)}{4608b^2}+\frac{7\cos^5\left(a+bx^3\right)}{1152}\\ &= -\frac{x^3\cos^8\left(a+bx^3\right)}{24b}+\frac{35\cos\left(a+bx^3\right)\sin\left(a+bx^3\right)}{3072b^2}+\frac{35\cos^3\left(a+bx^3\right)}{4608}\\ &= \frac{35x^3}{3072b}-\frac{x^3\cos^8\left(a+bx^3\right)}{24b}+\frac{35\cos\left(a+bx^3\right)\sin\left(a+bx^3\right)}{3072b^2}+\frac{35\cos^3\left(\right.}{}\end{aligned}$$

Mathematica [A] time = 0.57, size = 120, normalized size = 0.93

$$\frac{672\sin\left(2\left(a+bx^3\right)\right)+168\sin\left(4\left(a+bx^3\right)\right)+32\sin\left(6\left(a+bx^3\right)\right)+3\sin\left(8\left(a+bx^3\right)\right)-1344bx^3\cos\left(2\left(a+\right.\right.}{73728b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5*Cos[a + b*x^3]^7*Sin[a + b*x^3],x]
```

```
[Out] (-1344*b*x^3*Cos[2*(a + b*x^3)] - 672*b*x^3*Cos[4*(a + b*x^3)] - 192*b*x^3*
Cos[6*(a + b*x^3)] - 24*b*x^3*Cos[8*(a + b*x^3)] + 672*Sin[2*(a + b*x^3)] +
 168*Sin[4*(a + b*x^3)] + 32*Sin[6*(a + b*x^3)] + 3*Sin[8*(a + b*x^3)])/(73
728*b^2)
```

fricas [A] time = 1.09, size = 85, normalized size = 0.66

$$-\frac{384\,bx^3\cos\left(bx^3+a\right)^8-105\,bx^3-\left(48\,\cos\left(bx^3+a\right)^7+56\,\cos\left(bx^3+a\right)^5+70\,\cos\left(bx^3+a\right)^3+105\,\cos\left(bx^3\right.\right.}{9216\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*cos(b*x^3+a)^7*sin(b*x^3+a),x, algorithm="fricas")
```

```
[Out] -1/9216*(384*b*x^3*cos(b*x^3 + a)^8 - 105*b*x^3 - (48*cos(b*x^3 + a)^7 + 56
*cos(b*x^3 + a)^5 + 70*cos(b*x^3 + a)^3 + 105*cos(b*x^3 + a))*sin(b*x^3 + a
))/b^2
```

giac [A] time = 0.44, size = 126, normalized size = 0.98

$$-\frac{24\,bx^3\cos\left(8\,bx^3+8\,a\right)+192\,bx^3\cos\left(6\,bx^3+6\,a\right)+672\,bx^3\cos\left(4\,bx^3+4\,a\right)+1344\,bx^3\cos\left(2\,bx^3+2\,a\right)-3}{73728\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*cos(b*x^3+a)^7*sin(b*x^3+a),x, algorithm="giac")
```

```
[Out] -1/73728*(24*b*x^3*cos(8*b*x^3 + 8*a) + 192*b*x^3*cos(6*b*x^3 + 6*a) + 672*
b*x^3*cos(4*b*x^3 + 4*a) + 1344*b*x^3*cos(2*b*x^3 + 2*a) - 3*sin(8*b*x^3 +
8*a) - 32*sin(6*b*x^3 + 6*a) - 168*sin(4*b*x^3 + 4*a) - 672*sin(2*b*x^3 + 2
*a))/b^2
```

maple [B] time = 0.95, size = 403, normalized size = 3.12

$$\frac{-\frac{4x^3}{3b}+\frac{4\tan\left(b\,x^3+a\right)}{3b^2}+\frac{4x^3\left(\tan^2\left(b\,x^3+a\right)\right)}{3b}}{128+128\left(\tan^2\left(b\,x^3+a\right)\right)}+\frac{\frac{\tan\left(b\,x^3+a\right)}{b^2}-\frac{x^3}{b}-\frac{\tan^3\left(b\,x^3+a\right)}{b^2}+\frac{6x^3\left(\tan^2\left(b\,x^3+a\right)\right)}{b}-\frac{x^3\left(\tan^4\left(b\,x^3+a\right)\right)}{b}}{128\left(1+\tan^2\left(b\,x^3+a\right)\right)^2}+\frac{-6x^3b\left(\cos^2\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*cos(b*x^3+a)^7*sin(b*x^3+a),x)
```

```
[Out] 1/128*(-4/3*x^3/b+4/3/b^2*tan(b*x^3+a)+4/3*x^3/b*tan(b*x^3+a)^2)/(1+tan(b*x
^3+a)^2)+1/128*(1/b^2*tan(b*x^3+a)-x^3/b-1/b^2*tan(b*x^3+a)^3+6*x^3/b*tan(b
*x^3+a)^2-x^3/b*tan(b*x^3+a)^4)/(1+tan(b*x^3+a)^2)^2+1/1152*(-6*x^3*b*cos(3
*b*x^3+3*a)^2-18*cos(b*x^3+a)^2*b*x^3+12*b*x^3+sin(3*b*x^3+3*a)*cos(3*b*x^3
+3*a)+9*cos(b*x^3+a)*sin(b*x^3+a))/b^2+1/128*(-1/6*x^3/b+1/12/b^2*tan(2*b*x
^3+2*a)+1/6*x^3/b*tan(2*b*x^3+2*a)^2)/(1+tan(2*b*x^3+2*a)^2)+1/128*(-1/24*x
^3/b+1/48/b^2*tan(2*b*x^3+2*a)-1/48/b^2*tan(2*b*x^3+2*a)^3+1/4*x^3/b*tan(2*
b*x^3+2*a)^2-1/24*x^3/b*tan(2*b*x^3+2*a)^4)/(1+tan(2*b*x^3+2*a)^2)^2
```

maxima [A] time = 0.34, size = 126, normalized size = 0.98

$$-\frac{24\,bx^3\cos\left(8\,bx^3+8\,a\right)+192\,bx^3\cos\left(6\,bx^3+6\,a\right)+672\,bx^3\cos\left(4\,bx^3+4\,a\right)+1344\,bx^3\cos\left(2\,bx^3+2\,a\right)-3}{73728\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*cos(b*x^3+a)^7*sin(b*x^3+a),x, algorithm="maxima")
```

```
[Out] -1/73728*(24*b*x^3*cos(8*b*x^3 + 8*a) + 192*b*x^3*cos(6*b*x^3 + 6*a) + 672*
b*x^3*cos(4*b*x^3 + 4*a) + 1344*b*x^3*cos(2*b*x^3 + 2*a) - 3*sin(8*b*x^3 +
8*a) - 32*sin(6*b*x^3 + 6*a) - 168*sin(4*b*x^3 + 4*a) - 672*sin(2*b*x^3 + 2
*a))/b^2
```

mupad [B] time = 3.45, size = 147, normalized size = 1.14

$$\frac{168\sin\left(2bx^3+2a\right)+42\sin\left(4bx^3+4a\right)+8\sin\left(6bx^3+6a\right)+\frac{3\sin\left(8bx^3+8a\right)}{4}+336bx^3\left(2\sin\left(bx^3+a\right)^2\right.}{1843}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*cos(a + b*x^3)^7*sin(a + b*x^3),x)
```

```
[Out] (168*sin(2*a + 2*b*x^3) + 42*sin(4*a + 4*b*x^3) + 8*sin(6*a + 6*b*x^3) + (3
*sin(8*a + 8*b*x^3))/4 + 336*b*x^3*(2*sin(a + b*x^3)^2 - 1) + 168*b*x^3*(2*
sin(2*a + 2*b*x^3)^2 - 1) + 48*b*x^3*(2*sin(3*a + 3*b*x^3)^2 - 1) + 6*b*x^3
*(2*sin(4*a + 4*b*x^3)^2 - 1))/(18432*b^2)
```

sympy [A] time = 74.97, size = 241, normalized size = 1.87

$$\begin{cases} \frac{35x^3\sin^8\left(a+bx^3\right)}{3072b}+\frac{35x^3\sin^6\left(a+bx^3\right)\cos^2\left(a+bx^3\right)}{768b}+\frac{35x^3\sin^4\left(a+bx^3\right)\cos^4\left(a+bx^3\right)}{512b}+\frac{35x^3\sin^2\left(a+bx^3\right)\cos^6\left(a+bx^3\right)}{768b}-\frac{31x^3\cos^8\left(a+bx^3\right.}{1024b} \\ \frac{x^6\sin(a)\cos^7(a)}{6} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*cos(b*x**3+a)**7*sin(b*x**3+a),x)
```

```
[Out] Piecewise((35*x**3*sin(a + b*x**3)**8/(3072*b) + 35*x**3*sin(a + b*x**3)**6
*cos(a + b*x**3)**2/(768*b) + 35*x**3*sin(a + b*x**3)**4*cos(a + b*x**3)**4
/(512*b) + 35*x**3*sin(a + b*x**3)**2*cos(a + b*x**3)**6/(768*b) - 31*x**3*
cos(a + b*x**3)**8/(1024*b) + 35*sin(a + b*x**3)**7*cos(a + b*x**3)/(3072*b
**2) + 385*sin(a + b*x**3)**5*cos(a + b*x**3)**3/(9216*b**2) + 511*sin(a +
b*x**3)**3*cos(a + b*x**3)**5/(9216*b**2) + 31*sin(a + b*x**3)*cos(a + b*x*
*3)**7/(1024*b**2), Ne(b, 0)), (x**6*sin(a)*cos(a)**7/6, True))
```

3.927 $\int x^5 \sec^7\left(a+bx^3\right)\tan\left(a+bx^3\right)\,dx$

Optimal. Leaf size=110

$$-\frac{5\tanh^{-1}\left(\sin\left(a+bx^3\right)\right)}{336b^2}-\frac{\tan\left(a+bx^3\right)\sec^5\left(a+bx^3\right)}{126b^2}-\frac{5\tan\left(a+bx^3\right)\sec^3\left(a+bx^3\right)}{504b^2}-\frac{5\tan\left(a+bx^3\right)\sec\left(a+\right.}{336b^2}$$

```
[Out] -5/336*arctanh(sin(b*x^3+a))/b^2+1/21*x^3*sec(b*x^3+a)^7/b-5/336*sec(b*x^3+
a)*tan(b*x^3+a)/b^2-5/504*sec(b*x^3+a)^3*tan(b*x^3+a)/b^2-1/126*sec(b*x^3+a
)^5*tan(b*x^3+a)/b^2
```

Rubi [A] time = 0.11, antiderivative size = 110, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {3757, 4204, 3768, 3770}

$$-\frac{5\tanh^{-1}\left(\sin\left(a+bx^3\right)\right)}{336b^2}-\frac{\tan\left(a+bx^3\right)\sec^5\left(a+bx^3\right)}{126b^2}-\frac{5\tan\left(a+bx^3\right)\sec^3\left(a+bx^3\right)}{504b^2}-\frac{5\tan\left(a+bx^3\right)\sec\left(a+\right.}{336b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^5*Sec[a + b*x^3]^7*Tan[a + b*x^3],x]
```

```
[Out] (-5*ArcTanh[Sin[a + b*x^3]])/(336*b^2) + (x^3*Sec[a + b*x^3]^7)/(21*b) - (5
*Sec[a + b*x^3]*Tan[a + b*x^3])/(336*b^2) - (5*Sec[a + b*x^3]^3*Tan[a + b*x
^3])/(504*b^2) - (Sec[a + b*x^3]^5*Tan[a + b*x^3])/(126*b^2)
```

Rule 3757

```
Int[(x_)^(m_.)*Sec[(a_.) + (b_.)*(x_)^(n_.)]^(p_.)*Tan[(a_.) + (b_.)*(x_)^(
n_.)]^(q_.), x_Symbol] :> Simp[(x^(m - n + 1)*Sec[a + b*x^n]^p)/(b*n*p), x]
 - Dist[(m - n + 1)/(b*n*p), Int[x^(m - n)*Sec[a + b*x^n]^p, x], x] /; Free
Q[{a, b, p}, x] && IntegerQ[n] && GeQ[m, n] && EqQ[q, 1]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 4204

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sec[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sec[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IGtQ[Simplify[(m +
1)/n], 0] && IntegerQ[p]
```

Rubi steps

$$\int x^5 \sec^7\left(a+bx^3\right)\tan\left(a+bx^3\right)\,dx = \frac{x^3\sec^7\left(a+bx^3\right)}{21b} - \frac{\int x^2\sec^7\left(a+bx^3\right)\,dx}{7b}$$
$$= \frac{x^3\sec^7\left(a+bx^3\right)}{21b} - \frac{\text{Subst}\left(\int \sec^7(a+bx)\,dx, x, x^3\right)}{21b}$$
$$= \frac{x^3\sec^7\left(a+bx^3\right)}{21b} - \frac{\sec^5\left(a+bx^3\right)\tan\left(a+bx^3\right)}{126b^2} - \frac{5\,\text{Subst}\left(\int \sec^5(a+\right.}{126b}$$
$$= \frac{x^3\sec^7\left(a+bx^3\right)}{21b} - \frac{5\sec^3\left(a+bx^3\right)\tan\left(a+bx^3\right)}{504b^2} - \frac{\sec^5\left(a+bx^3\right)\tan}{126b^2}$$
$$= \frac{x^3\sec^7\left(a+bx^3\right)}{21b} - \frac{5\sec\left(a+bx^3\right)\tan\left(a+bx^3\right)}{336b^2} - \frac{5\sec^3\left(a+bx^3\right)\tan}{504b^2}$$
$$= -\frac{5\tanh^{-1}\left(\sin\left(a+bx^3\right)\right)}{336b^2} + \frac{x^3\sec^7\left(a+bx^3\right)}{21b} - \frac{5\sec\left(a+bx^3\right)\tan\left(a+\right.}{336b^2}$$

Mathematica [B] time = 0.87, size = 352, normalized size = 3.20

$$\frac{\sec^7\left(a+bx^3\right)\left(-566\sin\left(2\left(a+bx^3\right)\right)-200\sin\left(4\left(a+bx^3\right)\right)-30\sin\left(6\left(a+bx^3\right)\right)+105\cos\left(5\left(a+bx^3\right)\right)\log\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5*Sec[a + b*x^3]^7*Tan[a + b*x^3],x]
```

```
[Out] (Sec[a + b*x^3]^7*(3072*b*x^3 + 105*Cos[5*(a + b*x^3)]*Log[Cos[(a + b*x^3)/
2] - Sin[(a + b*x^3)/2]] + 15*Cos[7*(a + b*x^3)]*Log[Cos[(a + b*x^3)/2] - S
in[(a + b*x^3)/2]] + 525*Cos[a + b*x^3]*(Log[Cos[(a + b*x^3)/2] - Sin[(a +
b*x^3)/2]] - Log[Cos[(a + b*x^3)/2] + Sin[(a + b*x^3)/2]]) + 315*Cos[3*(a +
 b*x^3)]*(Log[Cos[(a + b*x^3)/2] - Sin[(a + b*x^3)/2]] - Log[Cos[(a + b*x^3
)/2] + Sin[(a + b*x^3)/2]]) - 105*Cos[5*(a + b*x^3)]*Log[Cos[(a + b*x^3)/2]
 + Sin[(a + b*x^3)/2]] - 15*Cos[7*(a + b*x^3)]*Log[Cos[(a + b*x^3)/2] + Sin
[(a + b*x^3)/2]] - 566*Sin[2*(a + b*x^3)] - 200*Sin[4*(a + b*x^3)] - 30*Sin
[6*(a + b*x^3)]))/(64512*b^2)
```

fricas [A] time = 0.96, size = 115, normalized size = 1.05

$$-\frac{15\cos\left(bx^3+a\right)^7\log\left(\sin\left(bx^3+a\right)+1\right)-15\cos\left(bx^3+a\right)^7\log\left(-\sin\left(bx^3+a\right)+1\right)-96\,bx^3+2\left(15\cos\left(bx\right.\right.}{2016\,b^2\cos\left(bx^3+a\right)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*sec(b*x^3+a)^7*tan(b*x^3+a),x, algorithm="fricas")
```

```
[Out] -1/2016*(15*cos(b*x^3 + a)^7*log(sin(b*x^3 + a) + 1) - 15*cos(b*x^3 + a)^7*
log(-sin(b*x^3 + a) + 1) - 96*b*x^3 + 2*(15*cos(b*x^3 + a)^5 + 10*cos(b*x^3
 + a)^3 + 8*cos(b*x^3 + a))*sin(b*x^3 + a))/(b^2*cos(b*x^3 + a)^7)
```

giac [B] time = 1.64, size = 1455, normalized size = 13.23

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*sec(b*x^3+a)^7*tan(b*x^3+a),x, algorithm="giac")
```

```
[Out] -1/2016*(96*(b*x^3 + a)*tan(1/2*b*x^3 + 1/2*a)^14 - 96*a*tan(1/2*b*x^3 + 1/
2*a)^14 + 15*log(2*(tan(1/2*b*x^3 + 1/2*a)^2 + 2*tan(1/2*b*x^3 + 1/2*a) + 1
)/(tan(1/2*b*x^3 + 1/2*a)^2 + 1))*tan(1/2*b*x^3 + 1/2*a)^14 - 15*log(2*(tan
(1/2*b*x^3 + 1/2*a)^2 - 2*tan(1/2*b*x^3 + 1/2*a) + 1)/(tan(1/2*b*x^3 + 1/2*
a)^2 + 1))*tan(1/2*b*x^3 + 1/2*a)^14 + 672*(b*x^3 + a)*tan(1/2*b*x^3 + 1/2*
a)^12 - 672*a*tan(1/2*b*x^3 + 1/2*a)^12 - 105*log(2*(tan(1/2*b*x^3 + 1/2*a)
^2 + 2*tan(1/2*b*x^3 + 1/2*a) + 1)/(tan(1/2*b*x^3 + 1/2*a)^2 + 1))*tan(1/2*
b*x^3 + 1/2*a)^12 + 105*log(2*(tan(1/2*b*x^3 + 1/2*a)^2 - 2*tan(1/2*b*x^3 +
 1/2*a) + 1)/(tan(1/2*b*x^3 + 1/2*a)^2 + 1))*tan(1/2*b*x^3 + 1/2*a)^12 + 13
2*tan(1/2*b*x^3 + 1/2*a)^13 + 2016*(b*x^3 + a)*tan(1/2*b*x^3 + 1/2*a)^10 -
2016*a*tan(1/2*b*x^3 + 1/2*a)^10 + 315*log(2*(tan(1/2*b*x^3 + 1/2*a)^2 + 2*
tan(1/2*b*x^3 + 1/2*a) + 1)/(tan(1/2*b*x^3 + 1/2*a)^2 + 1))*tan(1/2*b*x^3 +
 1/2*a)^10 - 315*log(2*(tan(1/2*b*x^3 + 1/2*a)^2 - 2*tan(1/2*b*x^3 + 1/2*a)
 + 1)/(tan(1/2*b*x^3 + 1/2*a)^2 + 1))*tan(1/2*b*x^3 + 1/2*a)^10 - 112*tan(1
/2*b*x^3 + 1/2*a)^11 + 3360*(b*x^3 + a)*tan(1/2*b*x^3 + 1/2*a)^8 - 3360*a*t
an(1/2*b*x^3 + 1/2*a)^8 - 525*log(2*(tan(1/2*b*x^3 + 1/2*a)^2 + 2*tan(1/2*b
*x^3 + 1/2*a) + 1)/(tan(1/2*b*x^3 + 1/2*a)^2 + 1))*tan(1/2*b*x^3 + 1/2*a)^8
 + 525*log(2*(tan(1/2*b*x^3 + 1/2*a)^2 - 2*tan(1/2*b*x^3 + 1/2*a) + 1)/(tan
(1/2*b*x^3 + 1/2*a)^2 + 1))*tan(1/2*b*x^3 + 1/2*a)^8 + 340*tan(1/2*b*x^3 +
1/2*a)^9 + 3360*(b*x^3 + a)*tan(1/2*b*x^3 + 1/2*a)^6 - 3360*a*tan(1/2*b*x^3
 + 1/2*a)^6 + 525*log(2*(tan(1/2*b*x^3 + 1/2*a)^2 + 2*tan(1/2*b*x^3 + 1/2*a
) + 1)/(tan(1/2*b*x^3 + 1/2*a)^2 + 1))*tan(1/2*b*x^3 + 1/2*a)^6 - 525*log(2
*(tan(1/2*b*x^3 + 1/2*a)^2 - 2*tan(1/2*b*x^3 + 1/2*a) + 1)/(tan(1/2*b*x^3 +
 1/2*a)^2 + 1))*tan(1/2*b*x^3 + 1/2*a)^6 + 2016*(b*x^3 + a)*tan(1/2*b*x^3 +
```

```
 1/2*a)^4 - 2016*a*tan(1/2*b*x^3 + 1/2*a)^4 - 315*log(2*(tan(1/2*b*x^3 + 1/
2*a)^2 + 2*tan(1/2*b*x^3 + 1/2*a) + 1)/(tan(1/2*b*x^3 + 1/2*a)^2 + 1))*tan(
1/2*b*x^3 + 1/2*a)^4 + 315*log(2*(tan(1/2*b*x^3 + 1/2*a)^2 - 2*tan(1/2*b*x^
3 + 1/2*a) + 1)/(tan(1/2*b*x^3 + 1/2*a)^2 + 1))*tan(1/2*b*x^3 + 1/2*a)^4 -
340*tan(1/2*b*x^3 + 1/2*a)^5 + 96*b*x^3 + 672*(b*x^3 + a)*tan(1/2*b*x^3 + 1
/2*a)^2 - 672*a*tan(1/2*b*x^3 + 1/2*a)^2 + 105*log(2*(tan(1/2*b*x^3 + 1/2*a
)^2 + 2*tan(1/2*b*x^3 + 1/2*a) + 1)/(tan(1/2*b*x^3 + 1/2*a)^2 + 1))*tan(1/2
*b*x^3 + 1/2*a)^2 - 105*log(2*(tan(1/2*b*x^3 + 1/2*a)^2 - 2*tan(1/2*b*x^3 +
 1/2*a) + 1)/(tan(1/2*b*x^3 + 1/2*a)^2 + 1))*tan(1/2*b*x^3 + 1/2*a)^2 + 112
*tan(1/2*b*x^3 + 1/2*a)^3 - 15*log(2*(tan(1/2*b*x^3 + 1/2*a)^2 + 2*tan(1/2*
b*x^3 + 1/2*a) + 1)/(tan(1/2*b*x^3 + 1/2*a)^2 + 1)) + 15*log(2*(tan(1/2*b*x
^3 + 1/2*a)^2 - 2*tan(1/2*b*x^3 + 1/2*a) + 1)/(tan(1/2*b*x^3 + 1/2*a)^2 + 1
)) - 132*tan(1/2*b*x^3 + 1/2*a))/((tan(1/2*b*x^3 + 1/2*a)^14 - 7*tan(1/2*b*
x^3 + 1/2*a)^12 + 21*tan(1/2*b*x^3 + 1/2*a)^10 - 35*tan(1/2*b*x^3 + 1/2*a)^
8 + 35*tan(1/2*b*x^3 + 1/2*a)^6 - 21*tan(1/2*b*x^3 + 1/2*a)^4 + 7*tan(1/2*b
*x^3 + 1/2*a)^2 - 1)*b^2)
```

maple [C] time = 0.31, size = 160, normalized size = 1.45

$$\frac{i\left(15\,e^{13i\left(b\,x^{3}+a\right)}-3072ib\,x^{3}e^{7i\left(b\,x^{3}+a\right)}+100\,e^{11i\left(b\,x^{3}+a\right)}+283\,e^{9i\left(b\,x^{3}+a\right)}-283\,e^{5i\left(b\,x^{3}+a\right)}-100\,e^{3i\left(b\,x^{3}+a\right)}-15\,e^{i\left(b\,x^{3}\right.}\right.}{504b^{2}\left(e^{2i\left(b\,x^{3}+a\right)}+1\right)^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*sec(b*x^3+a)^7*tan(b*x^3+a),x)
```

```
[Out] 1/504*I/b^2/(exp(2*I*(b*x^3+a))+1)^7*(15*exp(13*I*(b*x^3+a))-3072*I*b*x^3*e
xp(7*I*(b*x^3+a))+100*exp(11*I*(b*x^3+a))+283*exp(9*I*(b*x^3+a))-283*exp(5*
I*(b*x^3+a))-100*exp(3*I*(b*x^3+a))-15*exp(I*(b*x^3+a)))+5/336/b^2*ln(exp(I
*(b*x^3+a))-I)-5/336/b^2*ln(exp(I*(b*x^3+a))+I)
```

maxima [B] time = 0.86, size = 3830, normalized size = 34.82

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*sec(b*x^3+a)^7*tan(b*x^3+a),x, algorithm="maxima")
```

```
[Out] 1/2016*(4*(3072*b*x^3*cos(7*b*x^3 + 7*a) - 15*sin(13*b*x^3 + 13*a) - 100*si
n(11*b*x^3 + 11*a) - 283*sin(9*b*x^3 + 9*a) + 283*sin(5*b*x^3 + 5*a) + 100*
sin(3*b*x^3 + 3*a) + 15*sin(b*x^3 + a))*cos(14*b*x^3 + 14*a) + 420*(sin(12*
b*x^3 + 12*a) + 3*sin(10*b*x^3 + 10*a) + 5*sin(8*b*x^3 + 8*a) + 5*sin(6*b*x
^3 + 6*a) + 3*sin(4*b*x^3 + 4*a) + sin(2*b*x^3 + 2*a))*cos(13*b*x^3 + 13*a)
 + 28*(3072*b*x^3*cos(7*b*x^3 + 7*a) - 100*sin(11*b*x^3 + 11*a) - 283*sin(9
```

```
*b*x^3 + 9*a) + 283*sin(5*b*x^3 + 5*a) + 100*sin(3*b*x^3 + 3*a) + 15*sin(b*
x^3 + a))*cos(12*b*x^3 + 12*a) + 2800*(3*sin(10*b*x^3 + 10*a) + 5*sin(8*b*x
^3 + 8*a) + 5*sin(6*b*x^3 + 6*a) + 3*sin(4*b*x^3 + 4*a) + sin(2*b*x^3 + 2*a
))*cos(11*b*x^3 + 11*a) + 84*(3072*b*x^3*cos(7*b*x^3 + 7*a) - 283*sin(9*b*x
^3 + 9*a) + 283*sin(5*b*x^3 + 5*a) + 100*sin(3*b*x^3 + 3*a) + 15*sin(b*x^3
+ a))*cos(10*b*x^3 + 10*a) + 7924*(5*sin(8*b*x^3 + 8*a) + 5*sin(6*b*x^3 + 6
*a) + 3*sin(4*b*x^3 + 4*a) + sin(2*b*x^3 + 2*a))*cos(9*b*x^3 + 9*a) + 140*(
3072*b*x^3*cos(7*b*x^3 + 7*a) + 283*sin(5*b*x^3 + 5*a) + 100*sin(3*b*x^3 +
3*a) + 15*sin(b*x^3 + a))*cos(8*b*x^3 + 8*a) + 12288*(35*b*x^3*cos(6*b*x^3
+ 6*a) + 21*b*x^3*cos(4*b*x^3 + 4*a) + 7*b*x^3*cos(2*b*x^3 + 2*a) + b*x^3)*
cos(7*b*x^3 + 7*a) + 140*(283*sin(5*b*x^3 + 5*a) + 100*sin(3*b*x^3 + 3*a) +
 15*sin(b*x^3 + a))*cos(6*b*x^3 + 6*a) - 7924*(3*sin(4*b*x^3 + 4*a) + sin(2
*b*x^3 + 2*a))*cos(5*b*x^3 + 5*a) + 420*(20*sin(3*b*x^3 + 3*a) + 3*sin(b*x^
3 + a))*cos(4*b*x^3 + 4*a) + 15*(2*(7*cos(12*b*x^3 + 12*a) + 21*cos(10*b*x^
3 + 10*a) + 35*cos(8*b*x^3 + 8*a) + 35*cos(6*b*x^3 + 6*a) + 21*cos(4*b*x^3
+ 4*a) + 7*cos(2*b*x^3 + 2*a) + 1)*cos(14*b*x^3 + 14*a) + cos(14*b*x^3 + 14
*a)^2 + 14*(21*cos(10*b*x^3 + 10*a) + 35*cos(8*b*x^3 + 8*a) + 35*cos(6*b*x^
3 + 6*a) + 21*cos(4*b*x^3 + 4*a) + 7*cos(2*b*x^3 + 2*a) + 1)*cos(12*b*x^3 +
 12*a) + 49*cos(12*b*x^3 + 12*a)^2 + 42*(35*cos(8*b*x^3 + 8*a) + 35*cos(6*b
*x^3 + 6*a) + 21*cos(4*b*x^3 + 4*a) + 7*cos(2*b*x^3 + 2*a) + 1)*cos(10*b*x^
3 + 10*a) + 441*cos(10*b*x^3 + 10*a)^2 + 70*(35*cos(6*b*x^3 + 6*a) + 21*cos
(4*b*x^3 + 4*a) + 7*cos(2*b*x^3 + 2*a) + 1)*cos(8*b*x^3 + 8*a) + 1225*cos(8
*b*x^3 + 8*a)^2 + 70*(21*cos(4*b*x^3 + 4*a) + 7*cos(2*b*x^3 + 2*a) + 1)*cos
(6*b*x^3 + 6*a) + 1225*cos(6*b*x^3 + 6*a)^2 + 42*(7*cos(2*b*x^3 + 2*a) + 1)
*cos(4*b*x^3 + 4*a) + 441*cos(4*b*x^3 + 4*a)^2 + 49*cos(2*b*x^3 + 2*a)^2 +
14*(sin(12*b*x^3 + 12*a) + 3*sin(10*b*x^3 + 10*a) + 5*sin(8*b*x^3 + 8*a) +
5*sin(6*b*x^3 + 6*a) + 3*sin(4*b*x^3 + 4*a) + sin(2*b*x^3 + 2*a))*sin(14*b*
x^3 + 14*a) + sin(14*b*x^3 + 14*a)^2 + 98*(3*sin(10*b*x^3 + 10*a) + 5*sin(8
*b*x^3 + 8*a) + 5*sin(6*b*x^3 + 6*a) + 3*sin(4*b*x^3 + 4*a) + sin(2*b*x^3 +
 2*a))*sin(12*b*x^3 + 12*a) + 49*sin(12*b*x^3 + 12*a)^2 + 294*(5*sin(8*b*x^
3 + 8*a) + 5*sin(6*b*x^3 + 6*a) + 3*sin(4*b*x^3 + 4*a) + sin(2*b*x^3 + 2*a)
)*sin(10*b*x^3 + 10*a) + 441*sin(10*b*x^3 + 10*a)^2 + 490*(5*sin(6*b*x^3 +
6*a) + 3*sin(4*b*x^3 + 4*a) + sin(2*b*x^3 + 2*a))*sin(8*b*x^3 + 8*a) + 1225
*sin(8*b*x^3 + 8*a)^2 + 490*(3*sin(4*b*x^3 + 4*a) + sin(2*b*x^3 + 2*a))*sin
(6*b*x^3 + 6*a) + 1225*sin(6*b*x^3 + 6*a)^2 + 441*sin(4*b*x^3 + 4*a)^2 + 29
4*sin(4*b*x^3 + 4*a)*sin(2*b*x^3 + 2*a) + 49*sin(2*b*x^3 + 2*a)^2 + 14*cos(
2*b*x^3 + 2*a) + 1)*log((cos(b*x^3 + 2*a)^2 + cos(a)^2 - 2*cos(a)*sin(b*x^3
 + 2*a) + sin(b*x^3 + 2*a)^2 + 2*cos(b*x^3 + 2*a)*sin(a) + sin(a)^2)/(cos(b
*x^3 + 2*a)^2 + cos(a)^2 + 2*cos(a)*sin(b*x^3 + 2*a) + sin(b*x^3 + 2*a)^2 -
 2*cos(b*x^3 + 2*a)*sin(a) + sin(a)^2)) + 4*(3072*b*x^3*sin(7*b*x^3 + 7*a)
+ 15*cos(13*b*x^3 + 13*a) + 100*cos(11*b*x^3 + 11*a) + 283*cos(9*b*x^3 + 9*
a) - 283*cos(5*b*x^3 + 5*a) - 100*cos(3*b*x^3 + 3*a) - 15*cos(b*x^3 + a))*s
in(14*b*x^3 + 14*a) - 60*(7*cos(12*b*x^3 + 12*a) + 21*cos(10*b*x^3 + 10*a)
+ 35*cos(8*b*x^3 + 8*a) + 35*cos(6*b*x^3 + 6*a) + 21*cos(4*b*x^3 + 4*a) + 7
*cos(2*b*x^3 + 2*a) + 1)*sin(13*b*x^3 + 13*a) + 28*(3072*b*x^3*sin(7*b*x^3
```

+ 7*a) + 100*cos(11*b*x^3 + 11*a) + 283*cos(9*b*x^3 + 9*a) - 283*cos(5*b*x^
3 + 5*a) - 100*cos(3*b*x^3 + 3*a) - 15*cos(b*x^3 + a))*sin(12*b*x^3 + 12*a)
 - 400*(21*cos(10*b*x^3 + 10*a) + 35*cos(8*b*x^3 + 8*a) + 35*cos(6*b*x^3 +
6*a) + 21*cos(4*b*x^3 + 4*a) + 7*cos(2*b*x^3 + 2*a) + 1)*sin(11*b*x^3 + 11*
a) + 84*(3072*b*x^3*sin(7*b*x^3 + 7*a) + 283*cos(9*b*x^3 + 9*a) - 283*cos(5
*b*x^3 + 5*a) - 100*cos(3*b*x^3 + 3*a) - 15*cos(b*x^3 + a))*sin(10*b*x^3 +
10*a) - 1132*(35*cos(8*b*x^3 + 8*a) + 35*cos(6*b*x^3 + 6*a) + 21*cos(4*b*x^
3 + 4*a) + 7*cos(2*b*x^3 + 2*a) + 1)*sin(9*b*x^3 + 9*a) + 140*(3072*b*x^3*s
in(7*b*x^3 + 7*a) - 283*cos(5*b*x^3 + 5*a) - 100*cos(3*b*x^3 + 3*a) - 15*co
s(b*x^3 + a))*sin(8*b*x^3 + 8*a) + 86016*(5*b*x^3*sin(6*b*x^3 + 6*a) + 3*b*
x^3*sin(4*b*x^3 + 4*a) + b*x^3*sin(2*b*x^3 + 2*a))*sin(7*b*x^3 + 7*a) - 140
*(283*cos(5*b*x^3 + 5*a) + 100*cos(3*b*x^3 + 3*a) + 15*cos(b*x^3 + a))*sin(
6*b*x^3 + 6*a) + 1132*(21*cos(4*b*x^3 + 4*a) + 7*cos(2*b*x^3 + 2*a) + 1)*si
n(5*b*x^3 + 5*a) - 420*(20*cos(3*b*x^3 + 3*a) + 3*cos(b*x^3 + a))*sin(4*b*x
^3 + 4*a) + 400*(7*cos(2*b*x^3 + 2*a) + 1)*sin(3*b*x^3 + 3*a) - 2800*cos(3*
b*x^3 + 3*a)*sin(2*b*x^3 + 2*a) - 420*cos(b*x^3 + a)*sin(2*b*x^3 + 2*a) + 4
20*cos(2*b*x^3 + 2*a)*sin(b*x^3 + a) + 60*sin(b*x^3 + a))/(b^2*cos(14*b*x^3
 + 14*a)^2 + 49*b^2*cos(12*b*x^3 + 12*a)^2 + 441*b^2*cos(10*b*x^3 + 10*a)^2
 + 1225*b^2*cos(8*b*x^3 + 8*a)^2 + 1225*b^2*cos(6*b*x^3 + 6*a)^2 + 441*b^2*
cos(4*b*x^3 + 4*a)^2 + 49*b^2*cos(2*b*x^3 + 2*a)^2 + b^2*sin(14*b*x^3 + 14*
a)^2 + 49*b^2*sin(12*b*x^3 + 12*a)^2 + 441*b^2*sin(10*b*x^3 + 10*a)^2 + 122
5*b^2*sin(8*b*x^3 + 8*a)^2 + 1225*b^2*sin(6*b*x^3 + 6*a)^2 + 441*b^2*sin(4*
b*x^3 + 4*a)^2 + 294*b^2*sin(4*b*x^3 + 4*a)*sin(2*b*x^3 + 2*a) + 49*b^2*sin
(2*b*x^3 + 2*a)^2 + 14*b^2*cos(2*b*x^3 + 2*a) + b^2 + 2*(7*b^2*cos(12*b*x^3
 + 12*a) + 21*b^2*cos(10*b*x^3 + 10*a) + 35*b^2*cos(8*b*x^3 + 8*a) + 35*b^2
*cos(6*b*x^3 + 6*a) + 21*b^2*cos(4*b*x^3 + 4*a) + 7*b^2*cos(2*b*x^3 + 2*a)
+ b^2)*cos(14*b*x^3 + 14*a) + 14*(21*b^2*cos(10*b*x^3 + 10*a) + 35*b^2*cos(
8*b*x^3 + 8*a) + 35*b^2*cos(6*b*x^3 + 6*a) + 21*b^2*cos(4*b*x^3 + 4*a) + 7*
b^2*cos(2*b*x^3 + 2*a) + b^2)*cos(12*b*x^3 + 12*a) + 42*(35*b^2*cos(8*b*x^3
 + 8*a) + 35*b^2*cos(6*b*x^3 + 6*a) + 21*b^2*cos(4*b*x^3 + 4*a) + 7*b^2*cos
(2*b*x^3 + 2*a) + b^2)*cos(10*b*x^3 + 10*a) + 70*(35*b^2*cos(6*b*x^3 + 6*a)
 + 21*b^2*cos(4*b*x^3 + 4*a) + 7*b^2*cos(2*b*x^3 + 2*a) + b^2)*cos(8*b*x^3
+ 8*a) + 70*(21*b^2*cos(4*b*x^3 + 4*a) + 7*b^2*cos(2*b*x^3 + 2*a) + b^2)*co
s(6*b*x^3 + 6*a) + 42*(7*b^2*cos(2*b*x^3 + 2*a) + b^2)*cos(4*b*x^3 + 4*a) +
 14*(b^2*sin(12*b*x^3 + 12*a) + 3*b^2*sin(10*b*x^3 + 10*a) + 5*b^2*sin(8*b*
x^3 + 8*a) + 5*b^2*sin(6*b*x^3 + 6*a) + 3*b^2*sin(4*b*x^3 + 4*a) + b^2*sin(
2*b*x^3 + 2*a))*sin(14*b*x^3 + 14*a) + 98*(3*b^2*sin(10*b*x^3 + 10*a) + 5*b
^2*sin(8*b*x^3 + 8*a) + 5*b^2*sin(6*b*x^3 + 6*a) + 3*b^2*sin(4*b*x^3 + 4*a)
 + b^2*sin(2*b*x^3 + 2*a))*sin(12*b*x^3 + 12*a) + 294*(5*b^2*sin(8*b*x^3 +
8*a) + 5*b^2*sin(6*b*x^3 + 6*a) + 3*b^2*sin(4*b*x^3 + 4*a) + b^2*sin(2*b*x^
3 + 2*a))*sin(10*b*x^3 + 10*a) + 490*(5*b^2*sin(6*b*x^3 + 6*a) + 3*b^2*sin(
4*b*x^3 + 4*a) + b^2*sin(2*b*x^3 + 2*a))*sin(8*b*x^3 + 8*a) + 490*(3*b^2*si
n(4*b*x^3 + 4*a) + b^2*sin(2*b*x^3 + 2*a))*sin(6*b*x^3 + 6*a))

mupad [B] time = 13.42, size = 730, normalized size = 6.64

$$-\frac{\frac{8\,{\mathrm{e}}^{1{}\mathrm{i}\,b\,x^3+a\,1{}\mathrm{i}}\,\left(15\,b\,x^3-8{}\mathrm{i}\right)}{315\,b^2}-\frac{8\,{\mathrm{e}}^{3{}\mathrm{i}\,b\,x^3+a\,3{}\mathrm{i}}\,\left(35\,b\,x^3-12{}\mathrm{i}\right)}{315\,b^2}}{5\,{\mathrm{e}}^{2{}\mathrm{i}\,b\,x^3+a\,2{}\mathrm{i}}+10\,{\mathrm{e}}^{4{}\mathrm{i}\,b\,x^3+a\,4{}\mathrm{i}}+10\,{\mathrm{e}}^{6{}\mathrm{i}\,b\,x^3+a\,6{}\mathrm{i}}+5\,{\mathrm{e}}^{8{}\mathrm{i}\,b\,x^3+a\,8{}\mathrm{i}}+{\mathrm{e}}^{10{}\mathrm{i}\,b\,x^3+a\,10{}\mathrm{i}}+1}+\frac{5\,\ln\left(x^2\,\left({\mathrm{e}}^{1{}\mathrm{i}\,b\,x^3+a\,1{}\mathrm{i}}-\mathrm{i}\right)\right)}{336\,b^2}-\frac{5\,\ln\left(x^2\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^5*tan(a + b*x^3))/cos(a + b*x^3)^7,x)
```

```
[Out] (5*log(x^2*(exp(a*1i + b*x^3*1i) - 1i)))/(336*b^2) - ((8*exp(a*1i + b*x^3*1
i)*(15*b*x^3 - 8i))/(315*b^2) - (8*exp(a*3i + b*x^3*3i)*(35*b*x^3 - 12i))/(
315*b^2))/(5*exp(a*2i + b*x^3*2i) + 10*exp(a*4i + b*x^3*4i) + 10*exp(a*6i +
 b*x^3*6i) + 5*exp(a*8i + b*x^3*8i) + exp(a*10i + b*x^3*10i) + 1) - (5*log(
x^2*(exp(a*1i + b*x^3*1i) + 1i)))/(336*b^2) - ((16*exp(a*3i + b*x^3*3i)*(5*
b*x^3 - 1i))/(63*b^2) - (16*exp(a*5i + b*x^3*5i)*(7*b*x^3 - 1i))/(63*b^2))/
(6*exp(a*2i + b*x^3*2i) + 15*exp(a*4i + b*x^3*4i) + 20*exp(a*6i + b*x^3*6i)
 + 15*exp(a*8i + b*x^3*8i) + 6*exp(a*10i + b*x^3*10i) + exp(a*12i + b*x^3*1
2i) + 1) - ((64*x^3*exp(a*5i + b*x^3*5i))/(21*b) - (64*x^3*exp(a*7i + b*x^3
*7i))/(21*b))/(7*exp(a*2i + b*x^3*2i) + 21*exp(a*4i + b*x^3*4i) + 35*exp(a*
6i + b*x^3*6i) + 35*exp(a*8i + b*x^3*8i) + 21*exp(a*10i + b*x^3*10i) + 7*ex
p(a*12i + b*x^3*12i) + exp(a*14i + b*x^3*14i) + 1) + (exp(a*1i + b*x^3*1i)*
1i)/(63*b^2*(3*exp(a*2i + b*x^3*2i) + 3*exp(a*4i + b*x^3*4i) + exp(a*6i + b
*x^3*6i) + 1)) + (exp(a*1i + b*x^3*1i)*5i)/(168*b^2*(exp(a*2i + b*x^3*2i) +
 1)) + (exp(a*1i + b*x^3*1i)*5i)/(252*b^2*(2*exp(a*2i + b*x^3*2i) + exp(a*4
i + b*x^3*4i) + 1)) + (2*exp(a*1i + b*x^3*1i)*(60*b*x^3 - 47i))/(315*b^2*(4
*exp(a*2i + b*x^3*2i) + 6*exp(a*4i + b*x^3*4i) + 4*exp(a*6i + b*x^3*6i) + e
xp(a*8i + b*x^3*8i) + 1))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^{5} \tan\left(a + bx^{3}\right) \sec^{7}\left(a + bx^{3}\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*sec(b*x**3+a)**7*tan(b*x**3+a),x)
```

```
[Out] Integral(x**5*tan(a + b*x**3)*sec(a + b*x**3)**7, x)
```

3.928 $\int \frac{\sec^2\left(\frac{1}{x}\right)}{x^2} dx$

Optimal. Leaf size=6

$$-\tan\left(\frac{1}{x}\right)$$

[Out] -tan(1/x)

Rubi [A] time = 0.02, antiderivative size = 6, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {4204, 3767, 8}

$$-\tan\left(\frac{1}{x}\right)$$

Antiderivative was successfully verified.

[In] Int[Sec[x^(-1)]^2/x^2,x]

[Out] -Tan[x^(-1)]

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 4204

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sec[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sec[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IGtQ[Simplify[(m +
1)/n], 0] && IntegerQ[p]
```

Rubi steps

$$\int \frac{\sec^2\left(\frac{1}{x}\right)}{x^2}\,dx = -\text{Subst}\left(\int \sec^2(x)\,dx, x, \frac{1}{x}\right)$$
$$= \text{Subst}\left(\int 1\,dx, x, -\tan\left(\frac{1}{x}\right)\right)$$
$$= -\tan\left(\frac{1}{x}\right)$$

Mathematica [A] time = 0.02, size = 6, normalized size = 1.00

$$-\tan\left(\frac{1}{x}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[x^(-1)]^2/x^2,x]
```

```
[Out] -Tan[x^(-1)]
```

fricas [A] time = 1.04, size = 12, normalized size = 2.00

$$-\frac{\sin\left(\frac{1}{x}\right)}{\cos\left(\frac{1}{x}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(1/x)^2/x^2,x, algorithm="fricas")
```

```
[Out] -sin(1/x)/cos(1/x)
```

giac [B] time = 0.13, size = 20, normalized size = 3.33

$$\frac{2\,\tan\left(\frac{1}{2\,x}\right)}{\tan\left(\frac{1}{2\,x}\right)^2 - 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(1/x)^2/x^2,x, algorithm="giac")
```

```
[Out] 2*tan(1/2/x)/(tan(1/2/x)^2 - 1)
```

maple [A] time = 0.04, size = 7, normalized size = 1.17

$$-\tan\left(\frac{1}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(1/x)^2/x^2,x)
```

```
[Out] -tan(1/x)
```

maxima [B] time = 0.38, size = 36, normalized size = 6.00

$$-\frac{2\,\sin\left(\frac{2}{x}\right)}{\cos\left(\frac{2}{x}\right)^2+\sin\left(\frac{2}{x}\right)^2+2\,\cos\left(\frac{2}{x}\right)+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(1/x)^2/x^2,x, algorithm="maxima")
```

```
[Out] -2*sin(2/x)/(cos(2/x)^2 + sin(2/x)^2 + 2*cos(2/x) + 1)
```

mupad [B] time = 3.03, size = 14, normalized size = 2.33

$$-\frac{2\mathrm{i}}{\mathrm{e}^{\frac{2\mathrm{i}}{x}}+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*cos(1/x)^2),x)
```

```
[Out] -2i/(exp(2i/x) + 1)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sec^2\left(\frac{1}{x}\right)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(1/x)**2/x**2,x)
```

```
[Out] Integral(sec(1/x)**2/x**2, x)
```

3.929 $\int 3x^2 \cos\left(x^3\right) dx$

Optimal. Leaf size=4

$$\sin\left(x^3\right)$$

```
[Out] sin(x^3)
```

Rubi [A] time = 0.01, antiderivative size = 4, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {12, 3380, 2637}

$$\sin\left(x^3\right)$$

Antiderivative was successfully verified.

```
[In] Int[3*x^2*Cos[x^3],x]
```

```
[Out] Sin[x^3]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3380

```
Int[((a_.) + Cos[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Cos[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rubi steps

$$\begin{aligned}
\int 3x^2 \cos\left(x^3\right) dx &= 3 \int x^2 \cos\left(x^3\right) dx \\
&= \text{Subst}\left(\int \cos(x)\, dx, x, x^3\right) \\
&= \sin\left(x^3\right)
\end{aligned}$$

Mathematica [A] time = 0.00, size = 4, normalized size = 1.00

$$\sin\left(x^3\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[3*x^2*Cos[x^3],x]
[Out] Sin[x^3]
```

fricas [A] time = 0.92, size = 4, normalized size = 1.00

$$\sin\left(x^3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(3*x^2*cos(x^3),x, algorithm="fricas")
[Out] sin(x^3)
```

giac [A] time = 0.13, size = 4, normalized size = 1.00

$$\sin\left(x^3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(3*x^2*cos(x^3),x, algorithm="giac")
[Out] sin(x^3)
```

maple [A] time = 0.00, size = 5, normalized size = 1.25

$$\sin\left(x^3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(3*x^2*cos(x^3),x)
[Out] sin(x^3)
```

maxima [A] time = 0.32, size = 4, normalized size = 1.00

$$\sin\left(x^3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(3*x^2*cos(x^3),x, algorithm="maxima")
```

```
[Out] sin(x^3)
```

mupad [B] time = 2.95, size = 4, normalized size = 1.00

$$\sin\left(x^3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(3*x^2*cos(x^3),x)
```

```
[Out] sin(x^3)
```

sympy [A] time = 0.27, size = 3, normalized size = 0.75

$$\sin\left(x^3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(3*x**2*cos(x**3),x)
```

```
[Out] sin(x**3)
```

3.930 $\int (1+2x)\sec^2(1+2x)\,dx$

Optimal. Leaf size=27

$$\frac{1}{2}(2x+1)\tan(2x+1)+\frac{1}{2}\log(\cos(2x+1))$$

```
[Out] 1/2*ln(cos(1+2*x))+1/2*(1+2*x)*tan(1+2*x)
```

Rubi [A] time = 0.02, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {4184, 3475}

$$\frac{1}{2}(2x+1)\tan(2x+1)+\frac{1}{2}\log(\cos(2x+1))$$

Antiderivative was successfully verified.

```
[In] Int[(1 + 2*x)*Sec[1 + 2*x]^2,x]
```

```
[Out] Log[Cos[1 + 2*x]]/2 + ((1 + 2*x)*Tan[1 + 2*x])/2
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 4184

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> -Sim
p[((c + d*x)^m*Cot[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Co
t[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rubi steps

$$\begin{aligned}\int (1+2x)\sec^2(1+2x)\,dx &= \frac{1}{2}(1+2x)\tan(1+2x)-\int \tan(1+2x)\,dx\\ &= \frac{1}{2}\log(\cos(1+2x))+\frac{1}{2}(1+2x)\tan(1+2x)\end{aligned}$$

Mathematica [A] time = 0.01, size = 30, normalized size = 1.11

$$x\tan(2x+1)+\frac{1}{2}\tan(2x+1)+\frac{1}{2}\log(\cos(2x+1))$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + 2*x)*Sec[1 + 2*x]^2,x]
```

```
[Out] Log[Cos[1 + 2*x]]/2 + Tan[1 + 2*x]/2 + x*Tan[1 + 2*x]
```

fricas [A] time = 0.83, size = 39, normalized size = 1.44

$$\frac{\cos(2x+1)\log(-\cos(2x+1)) + (2x+1)\sin(2x+1)}{2\cos(2x+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)*sec(1+2*x)^2,x, algorithm="fricas")
```

```
[Out] 1/2*(cos(2*x + 1)*log(-cos(2*x + 1)) + (2*x + 1)*sin(2*x + 1))/cos(2*x + 1)
```

giac [B] time = 0.37, size = 943, normalized size = 34.93

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)*sec(1+2*x)^2,x, algorithm="giac")
```

```
[Out] 1/4*(log(4*(tan(1/2)^4*tan(x)^8 - 8*tan(1/2)^3*tan(x)^7 - 2*tan(1/2)^2*tan(
x)^8 - 2*tan(1/2)^4*tan(x)^4 - 8*tan(1/2)^3*tan(x)^5 + 16*tan(1/2)^2*tan(x)
^6 + 8*tan(1/2)*tan(x)^7 + tan(x)^8 + 8*tan(1/2)^3*tan(x)^3 + 36*tan(1/2)^2
*tan(x)^4 + 8*tan(1/2)*tan(x)^5 + tan(1/2)^4 + 8*tan(1/2)^3*tan(x) + 16*tan
(1/2)^2*tan(x)^2 - 8*tan(1/2)*tan(x)^3 - 2*tan(x)^4 - 2*tan(1/2)^2 - 8*tan(
1/2)*tan(x) + 1)/(tan(1/2)^4 + 2*tan(1/2)^2 + 1))*tan(1/2)^2*tan(x)^2 - 8*x
*tan(1/2)^2*tan(x) - 8*x*tan(1/2)*tan(x)^2 - log(4*(tan(1/2)^4*tan(x)^8 - 8
*tan(1/2)^3*tan(x)^7 - 2*tan(1/2)^2*tan(x)^8 - 2*tan(1/2)^4*tan(x)^4 - 8*ta
n(1/2)^3*tan(x)^5 + 16*tan(1/2)^2*tan(x)^6 + 8*tan(1/2)*tan(x)^7 + tan(x)^8
 + 8*tan(1/2)^3*tan(x)^3 + 36*tan(1/2)^2*tan(x)^4 + 8*tan(1/2)*tan(x)^5 + t
an(1/2)^4 + 8*tan(1/2)^3*tan(x) + 16*tan(1/2)^2*tan(x)^2 - 8*tan(1/2)*tan(x
)^3 - 2*tan(x)^4 - 2*tan(1/2)^2 - 8*tan(1/2)*tan(x) + 1)/(tan(1/2)^4 + 2*ta
n(1/2)^2 + 1))*tan(1/2)^2 - 4*log(4*(tan(1/2)^4*tan(x)^8 - 8*tan(1/2)^3*tan
(x)^7 - 2*tan(1/2)^2*tan(x)^8 - 2*tan(1/2)^4*tan(x)^4 - 8*tan(1/2)^3*tan(x)
^5 + 16*tan(1/2)^2*tan(x)^6 + 8*tan(1/2)*tan(x)^7 + tan(x)^8 + 8*tan(1/2)^3
*tan(x)^3 + 36*tan(1/2)^2*tan(x)^4 + 8*tan(1/2)*tan(x)^5 + tan(1/2)^4 + 8*t
an(1/2)^3*tan(x) + 16*tan(1/2)^2*tan(x)^2 - 8*tan(1/2)*tan(x)^3 - 2*tan(x)^
4 - 2*tan(1/2)^2 - 8*tan(1/2)*tan(x) + 1)/(tan(1/2)^4 + 2*tan(1/2)^2 + 1))*
tan(1/2)*tan(x) - 4*tan(1/2)^2*tan(x) - log(4*(tan(1/2)^4*tan(x)^8 - 8*tan(
1/2)^3*tan(x)^7 - 2*tan(1/2)^2*tan(x)^8 - 2*tan(1/2)^4*tan(x)^4 - 8*tan(1/2
)^3*tan(x)^5 + 16*tan(1/2)^2*tan(x)^6 + 8*tan(1/2)*tan(x)^7 + tan(x)^8 + 8*
tan(1/2)^3*tan(x)^3 + 36*tan(1/2)^2*tan(x)^4 + 8*tan(1/2)*tan(x)^5 + tan(1/
```

```
2)^4 + 8*tan(1/2)^3*tan(x) + 16*tan(1/2)^2*tan(x)^2 - 8*tan(1/2)*tan(x)^3 -
 2*tan(x)^4 - 2*tan(1/2)^2 - 8*tan(1/2)*tan(x) + 1)/(tan(1/2)^4 + 2*tan(1/2
)^2 + 1))*tan(x)^2 - 4*tan(1/2)*tan(x)^2 + 8*x*tan(1/2) + 8*x*tan(x) + log(
4*(tan(1/2)^4*tan(x)^8 - 8*tan(1/2)^3*tan(x)^7 - 2*tan(1/2)^2*tan(x)^8 - 2*
tan(1/2)^4*tan(x)^4 - 8*tan(1/2)^3*tan(x)^5 + 16*tan(1/2)^2*tan(x)^6 + 8*ta
n(1/2)*tan(x)^7 + tan(x)^8 + 8*tan(1/2)^3*tan(x)^3 + 36*tan(1/2)^2*tan(x)^4
 + 8*tan(1/2)*tan(x)^5 + tan(1/2)^4 + 8*tan(1/2)^3*tan(x) + 16*tan(1/2)^2*t
an(x)^2 - 8*tan(1/2)*tan(x)^3 - 2*tan(x)^4 - 2*tan(1/2)^2 - 8*tan(1/2)*tan(
x) + 1)/(tan(1/2)^4 + 2*tan(1/2)^2 + 1)) + 4*tan(1/2) + 4*tan(x))/(tan(1/2)
^2*tan(x)^2 - tan(1/2)^2 - 4*tan(1/2)*tan(x) - tan(x)^2 + 1)
```

maple [A] time = 0.04, size = 24, normalized size = 0.89

$$\frac{\ln(\cos(1+2x))}{2}+\frac{(1+2x)\tan(1+2x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+2*x)*sec(1+2*x)^2,x)
```

```
[Out] 1/2*ln(cos(1+2*x))+1/2*(1+2*x)*tan(1+2*x)
```

maxima [B] time = 0.45, size = 98, normalized size = 3.63

$$\frac{\left(\cos(4x+2)^2+\sin(4x+2)^2+2\cos(4x+2)+1\right)\log\left(\cos(4x+2)^2+\sin(4x+2)^2+2\cos(4x+2)+1\right)}{4\left(\cos(4x+2)^2+\sin(4x+2)^2+2\cos(4x+2)+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)*sec(1+2*x)^2,x, algorithm="maxima")
```

```
[Out] 1/4*((cos(4*x + 2)^2 + sin(4*x + 2)^2 + 2*cos(4*x + 2) + 1)*log(cos(4*x + 2
)^2 + sin(4*x + 2)^2 + 2*cos(4*x + 2) + 1) + 4*(2*x + 1)*sin(4*x + 2))/(cos
(4*x + 2)^2 + sin(4*x + 2)^2 + 2*cos(4*x + 2) + 1)
```

mupad [B] time = 0.10, size = 23, normalized size = 0.85

$$\frac{\ln(\cos(2x+1))}{2}+\frac{\tan(2x+1)\ (2x+1)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x + 1)/cos(2*x + 1)^2,x)
```

```
[Out] log(cos(2*x + 1))/2 + (tan(2*x + 1)*(2*x + 1))/2
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (2x+1)\sec^2(2x+1)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)*sec(1+2*x)**2,x)
```

```
[Out] Integral((2*x + 1)*sec(2*x + 1)**2, x)
```

3.931 $\int \left(\frac{x^4}{b\sqrt{x^3+3\sin(a+bx)}}+\frac{x^2\cos(a+bx)}{\sqrt{x^3+3\sin(a+bx)}}+\frac{4x\sqrt{x^3+3\sin(a+bx)}}{3b}\right) d$

Optimal. Leaf size=26

$$\frac{2x^2\sqrt{3\sin(a+bx)+x^3}}{3b}$$

```
[Out] 2/3*x^2*(x^3+3*sin(b*x+a))^(1/2)/b
```

Rubi **[F]** time = 0.81, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \left(\frac{x^4}{b\sqrt{x^3+3\sin(a+bx)}}+\frac{x^2\cos(a+bx)}{\sqrt{x^3+3\sin(a+bx)}}+\frac{4x\sqrt{x^3+3\sin(a+bx)}}{3b}\right) dx$$

Verification is Not applicable to the result.

```
[In] Int[x^4/(b*Sqrt[x^3 + 3*Sin[a + b*x]]) + (x^2*Cos[a + b*x])/Sqrt[x^3 + 3*Si
n[a + b*x]] + (4*x*Sqrt[x^3 + 3*Sin[a + b*x]])/(3*b),x]
```

```
[Out] Defer[Int][x^4/Sqrt[x^3 + 3*Sin[a + b*x]], x]/b + Defer[Int][(x^2*Cos[a + b
*x])/Sqrt[x^3 + 3*Sin[a + b*x]], x] + (4*Defer[Int][x*Sqrt[x^3 + 3*Sin[a +
b*x]], x])/(3*b)
```

Rubi steps

$$\int \left(\frac{x^4}{b\sqrt{x^3+3\sin(a+bx)}}+\frac{x^2\cos(a+bx)}{\sqrt{x^3+3\sin(a+bx)}}+\frac{4x\sqrt{x^3+3\sin(a+bx)}}{3b}\right) dx = \frac{\int \frac{x^4}{\sqrt{x^3+3\sin(a+bx)}}\,dx}{b}+\frac{4\int x$$

Mathematica **[A]** time = 0.45, size = 26, normalized size = 1.00

$$\frac{2x^2\sqrt{3\sin(a+bx)+x^3}}{3b}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/(b*Sqrt[x^3 + 3*Sin[a + b*x]]) + (x^2*Cos[a + b*x])/Sqrt[x^3
+ 3*Sin[a + b*x]] + (4*x*Sqrt[x^3 + 3*Sin[a + b*x]])/(3*b),x]
```

```
[Out] (2*x^2*Sqrt[x^3 + 3*Sin[a + b*x]])/(3*b)
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/b/(x^3+3*sin(b*x+a))^(1/2)+x^2*cos(b*x+a)/(x^3+3*sin(b*x+a))^
(1/2)+4/3*x*(x^3+3*sin(b*x+a))^(1/2)/b,x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{\sqrt{x^3+3\,\sin\left(bx+a\right)}\,b}+\frac{x^2\cos\left(bx+a\right)}{\sqrt{x^3+3\,\sin\left(bx+a\right)}}+\frac{4\,\sqrt{x^3+3\,\sin\left(bx+a\right)}\,x}{3\,b}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/b/(x^3+3*sin(b*x+a))^(1/2)+x^2*cos(b*x+a)/(x^3+3*sin(b*x+a))^
(1/2)+4/3*x*(x^3+3*sin(b*x+a))^(1/2)/b,x, algorithm="giac")
```

```
[Out] integrate(x^4/(sqrt(x^3 + 3*sin(b*x + a))*b) + x^2*cos(b*x + a)/sqrt(x^3 +
3*sin(b*x + a)) + 4/3*sqrt(x^3 + 3*sin(b*x + a))*x/b, x)
```

maple [A] time = 1.04, size = 28, normalized size = 1.08

$$\frac{\sqrt{2x^3+6\sin\left(bx+a\right)}\,\sqrt{2}\,x^2}{3b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/b/(x^3+3*sin(b*x+a))^(1/2)+x^2*cos(b*x+a)/(x^3+3*sin(b*x+a))^(1/2)+
4/3*x*(x^3+3*sin(b*x+a))^(1/2)/b,x)
```

```
[Out] 1/3*(2*x^3+6*sin(b*x+a))^(1/2)/b*2^(1/2)*x^2
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{\sqrt{x^3+3\,\sin\left(bx+a\right)}\,b}+\frac{x^2\cos\left(bx+a\right)}{\sqrt{x^3+3\,\sin\left(bx+a\right)}}+\frac{4\,\sqrt{x^3+3\,\sin\left(bx+a\right)}\,x}{3\,b}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/b/(x^3+3*sin(b*x+a))^(1/2)+x^2*cos(b*x+a)/(x^3+3*sin(b*x+a))^
(1/2)+4/3*x*(x^3+3*sin(b*x+a))^(1/2)/b,x, algorithm="maxima")
```

```
[Out] integrate(x^4/(sqrt(x^3 + 3*sin(b*x + a))*b) + x^2*cos(b*x + a)/sqrt(x^3 +
3*sin(b*x + a)) + 4/3*sqrt(x^3 + 3*sin(b*x + a))*x/b, x)
```

mupad [B] time = 3.47, size = 22, normalized size = 0.85

$$\frac{2\,x^2\,\sqrt{3\,\sin\left(a+b\,x\right)+x^3}}{3\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(b*(3*sin(a + b*x) + x^3)^(1/2)) + (x^2*cos(a + b*x))/(3*sin(a + b*
x) + x^3)^(1/2) + (4*x*(3*sin(a + b*x) + x^3)^(1/2))/(3*b),x)
```

```
[Out] (2*x^2*(3*sin(a + b*x) + x^3)^(1/2))/(3*b)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{7x^4}{\sqrt{x^3+3\sin\left(a+bx\right)}}\,dx+\int \frac{12x\sin\left(a+bx\right)}{\sqrt{x^3+3\sin\left(a+bx\right)}}\,dx+\int \frac{3bx^2\cos\left(a+bx\right)}{\sqrt{x^3+3\sin\left(a+bx\right)}}\,dx}{3b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/b/(x**3+3*sin(b*x+a))**(1/2)+x**2*cos(b*x+a)/(x**3+3*sin(b*x
+a))**(1/2)+4/3*x*(x**3+3*sin(b*x+a))**(1/2)/b,x)
```

```
[Out] (Integral(7*x**4/sqrt(x**3 + 3*sin(a + b*x)), x) + Integral(12*x*sin(a + b*
x)/sqrt(x**3 + 3*sin(a + b*x)), x) + Integral(3*b*x**2*cos(a + b*x)/sqrt(x*
*3 + 3*sin(a + b*x)), x))/(3*b)
```

3.932 $\int \frac{x^2\cos(a+bx)}{\sqrt{x^3+3\sin(a+bx)}}\,dx$

Optimal. Leaf size=29

$$\text{Int}\left(\frac{x^2\cos(a+bx)}{\sqrt{3\sin(a+bx)+x^3}},x\right)$$

```
[Out] CannotIntegrate(x^2*cos(b*x+a)/(x^3+3*sin(b*x+a))^(1/2),x)
```

Rubi [A] time = 0.11, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{x^2\cos(a+bx)}{\sqrt{x^3+3\sin(a+bx)}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(x^2*Cos[a + b*x])/Sqrt[x^3 + 3*Sin[a + b*x]],x]
```

```
[Out] Defer[Int][(x^2*Cos[a + b*x])/Sqrt[x^3 + 3*Sin[a + b*x]], x]
```

Rubi steps

$$\int \frac{x^2\cos(a+bx)}{\sqrt{x^3+3\sin(a+bx)}}\,dx=\int \frac{x^2\cos(a+bx)}{\sqrt{x^3+3\sin(a+bx)}}\,dx$$

Mathematica [A] time = 7.57, size = 0, normalized size = 0.00

$$\int \frac{x^2\cos(a+bx)}{\sqrt{x^3+3\sin(a+bx)}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(x^2*Cos[a + b*x])/Sqrt[x^3 + 3*Sin[a + b*x]],x]
```

```
[Out] Integrate[(x^2*Cos[a + b*x])/Sqrt[x^3 + 3*Sin[a + b*x]], x]
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*cos(b*x+a)/(x^3+3*sin(b*x+a))^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2 \cos(bx+a)}{\sqrt{x^3 + 3 \sin(bx+a)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*cos(b*x+a)/(x^3+3*sin(b*x+a))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^2*cos(b*x + a)/sqrt(x^3 + 3*sin(b*x + a)), x)
```

maple [A] time = 1.10, size = 0, normalized size = 0.00

$$\int \frac{x^2 \cos(bx+a)}{\sqrt{x^3 + 3 \sin(bx+a)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*cos(b*x+a)/(x^3+3*sin(b*x+a))^(1/2),x)
```

```
[Out] int(x^2*cos(b*x+a)/(x^3+3*sin(b*x+a))^(1/2),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2 \cos(bx+a)}{\sqrt{x^3 + 3 \sin(bx+a)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*cos(b*x+a)/(x^3+3*sin(b*x+a))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^2*cos(b*x + a)/sqrt(x^3 + 3*sin(b*x + a)), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{x^2 \cos(a+b\,x)}{\sqrt{3 \sin(a+b\,x) + x^3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*cos(a + b*x))/(3*sin(a + b*x) + x^3)^(1/2),x)
```

```
[Out] int((x^2*cos(a + b*x))/(3*sin(a + b*x) + x^3)^(1/2), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2 \cos(a+bx)}{\sqrt{x^3+3\sin(a+bx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*cos(b*x+a)/(x**3+3*sin(b*x+a))**(1/2),x)
[Out] Integral(x**2*cos(a + b*x)/sqrt(x**3 + 3*sin(a + b*x)), x)
```

3.933 $\int \frac{\cos(x)+\sin(x)}{e^{-x}+\sin(x)}\,dx$

Optimal. Leaf size=9

$$\log\left(e^x\sin(x)+1\right)$$

```
[Out] ln(1+exp(x)*sin(x))
```

Rubi [F] time = 0.38, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\cos(x)+\sin(x)}{e^{-x}+\sin(x)}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(Cos[x] + Sin[x])/(E^(-x) + Sin[x]),x]
```

```
[Out] x + Log[Sin[x]] - Defer[Int][(1 + E^x*Sin[x])^(-1), x] - Defer[Int][Cot[x]/
(1 + E^x*Sin[x]), x]
```

Rubi steps

$$\begin{aligned}
\int \frac{\cos(x)+\sin(x)}{e^{-x}+\sin(x)}\,dx &= \int \left(1+\cot(x)-\frac{(1+\cot(x))\csc(x)}{e^x+\csc(x)}\right)dx\\
&= x+\int \cot(x)\,dx-\int \frac{(1+\cot(x))\csc(x)}{e^x+\csc(x)}\,dx\\
&= x+\log(\sin(x))-\int \left(\frac{1}{1+e^x\sin(x)}+\frac{\cot(x)}{1+e^x\sin(x)}\right)dx\\
&= x+\log(\sin(x))-\int \frac{1}{1+e^x\sin(x)}\,dx-\int \frac{\cot(x)}{1+e^x\sin(x)}\,dx
\end{aligned}$$

Mathematica [A] time = 0.12, size = 9, normalized size = 1.00

$$\log\left(e^x\sin(x)+1\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[x] + Sin[x])/(E^(-x) + Sin[x]),x]
```

```
[Out] Log[1 + E^x*Sin[x]]
```

fricas [A] time = 0.88, size = 10, normalized size = 1.11

$$x+\log\left(e^{(-x)}+\sin(x)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((cos(x)+sin(x))/(exp(-x)+sin(x)),x, algorithm="fricas")
```

```
[Out] x + log(e^(-x) + sin(x))
```

giac [B] time = 0.15, size = 83, normalized size = 9.22

$$x+\frac{1}{2}\log\left(\frac{4\left(e^{(-2x)}\tan\left(\frac{1}{2}x\right)^{4}+4e^{(-x)}\tan\left(\frac{1}{2}x\right)^{3}+2e^{(-2x)}\tan\left(\frac{1}{2}x\right)^{2}+4e^{(-x)}\tan\left(\frac{1}{2}x\right)+4\tan\left(\frac{1}{2}x\right)^{2}+e^{(-2x)}\right)}{\tan\left(\frac{1}{2}x\right)^{4}+2\tan\left(\frac{1}{2}x\right)^{2}+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((cos(x)+sin(x))/(exp(-x)+sin(x)),x, algorithm="giac")
```

```
[Out] x + 1/2*log(4*(e^(-2*x)*tan(1/2*x)^4 + 4*e^(-x)*tan(1/2*x)^3 + 2*e^(-2*x)*t
an(1/2*x)^2 + 4*e^(-x)*tan(1/2*x) + 4*tan(1/2*x)^2 + e^(-2*x))/(tan(1/2*x)^
4 + 2*tan(1/2*x)^2 + 1))
```

maple [B] time = 0.19, size = 57, normalized size = 6.33

$$\frac{x+x\left(\tan^{2}\left(\frac{x}{2}\right)\right)}{1+\tan^{2}\left(\frac{x}{2}\right)}-\ln\left(1+\tan^{2}\left(\frac{x}{2}\right)\right)+\ln\left(e^{-x}\left(\tan^{2}\left(\frac{x}{2}\right)\right)+e^{-x}+2\tan\left(\frac{x}{2}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(x)+sin(x))/(exp(-x)+sin(x)),x)
```

```
[Out] (x+x*tan(1/2*x)^2)/(1+tan(1/2*x)^2)-ln(1+tan(1/2*x)^2)+ln(exp(-x)*tan(1/2*x
)^2+exp(-x)+2*tan(1/2*x))
```

maxima [B] time = 0.56, size = 82, normalized size = 9.11

$$x+\frac{1}{2}\log\left(\left(\cos\left(2x\right)^{2}e^{(2x)}+4\cos(x)e^{x}\sin\left(2x\right)+e^{(2x)}\sin\left(2x\right)^{2}-2\left(2e^{x}\sin(x)+e^{(2x)}\right)\cos\left(2x\right)+4\cos(x)^{2}+4\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((cos(x)+sin(x))/(exp(-x)+sin(x)),x, algorithm="maxima")
```

```
[Out] x + 1/2*log((cos(2*x)^2*e^(2*x) + 4*cos(x)*e^x*sin(2*x) + e^(2*x)*sin(2*x)^
2 - 2*(2*e^x*sin(x) + e^(2*x))*cos(2*x) + 4*cos(x)^2 + 4*e^x*sin(x) + 4*sin
(x)^2 + e^(2*x))*e^(-2*x))
```

mupad [B] time = 2.95, size = 10, normalized size = 1.11

$$x+\ln\left(e^{-x}+\sin(x)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(x) + sin(x))/(exp(-x) + sin(x)),x)
```

```
[Out] x + log(exp(-x) + sin(x))
```

sympy [A] time = 0.29, size = 10, normalized size = 1.11

$$x + \log\left(\sin\left(x\right) + e^{-x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((cos(x)+sin(x))/(exp(-x)+sin(x)),x)
```

```
[Out] x + log(sin(x) + exp(-x))
```

3.934 $\int \sin(c+dx) \left(a\sin^2(c+dx)+b\sin^3(c+dx)\right) dx$

Optimal. Leaf size=77

$$\frac{a\cos^3(c+dx)}{3d}-\frac{a\cos(c+dx)}{d}-\frac{b\sin^3(c+dx)\cos(c+dx)}{4d}-\frac{3b\sin(c+dx)\cos(c+dx)}{8d}+\frac{3bx}{8}$$

```
[Out] 3/8*b*x-a*cos(d*x+c)/d+1/3*a*cos(d*x+c)^3/d-3/8*b*cos(d*x+c)*sin(d*x+c)/d-1
/4*b*cos(d*x+c)*sin(d*x+c)^3/d
```

Rubi [A] time = 0.13, antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.179, Rules used = {4393, 2748, 2633, 2635, 8}

$$\frac{a\cos^3(c+dx)}{3d}-\frac{a\cos(c+dx)}{d}-\frac{b\sin^3(c+dx)\cos(c+dx)}{4d}-\frac{3b\sin(c+dx)\cos(c+dx)}{8d}+\frac{3bx}{8}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]*(a*Sin[c + d*x]^2 + b*Sin[c + d*x]^3),x]
```

```
[Out] (3*b*x)/8 - (a*Cos[c + d*x])/d + (a*Cos[c + d*x]^3)/(3*d) - (3*b*Cos[c + d*
x]*Sin[c + d*x])/(8*d) - (b*Cos[c + d*x]*Sin[c + d*x]^3)/(4*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
```

```
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 4393

```
Int[(u_)*((a_)*(F_)[(c_.) + (d_.)*(x_)]^(p_.) + (b_.)*(F_)[(c_.) + (d_.)*(x
_)]^(q_.))^(n_.), x_Symbol] :> Int[ActivateTrig[u*F[c + d*x]^(n*p)*(a + b*F
[c + d*x]^(q - p))^n], x] /; FreeQ[{a, b, c, d, p, q}, x] && InertTrigQ[F]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\int \sin(c+dx)\left(a\sin^2(c+dx)+b\sin^3(c+dx)\right)\,dx &= \int \sin^3(c+dx)(a+b\sin(c+dx))\,dx\\ &= a\int \sin^3(c+dx)\,dx+b\int \sin^4(c+dx)\,dx\\ &= -\frac{b\cos(c+dx)\sin^3(c+dx)}{4d}+\frac{1}{4}(3b)\int \sin^2(c+dx)\,dx-\\ &= -\frac{a\cos(c+dx)}{d}+\frac{a\cos^3(c+dx)}{3d}-\frac{3b\cos(c+dx)\sin(c+\\ &= \frac{3bx}{8}-\frac{a\cos(c+dx)}{d}+\frac{a\cos^3(c+dx)}{3d}-\frac{3b\cos(c+dx)\sin\end{aligned}$$

Mathematica [A] time = 0.15, size = 76, normalized size = 0.99

$$-\frac{3a\cos(c+dx)}{4d}+\frac{a\cos(3(c+dx))}{12d}+\frac{3b(c+dx)}{8d}-\frac{b\sin(2(c+dx))}{4d}+\frac{b\sin(4(c+dx))}{32d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]*(a*Sin[c + d*x]^2 + b*Sin[c + d*x]^3),x]
```

```
[Out] (3*b*(c + d*x))/(8*d) - (3*a*Cos[c + d*x])/(4*d) + (a*Cos[3*(c + d*x)])/(12
*d) - (b*Sin[2*(c + d*x)])/(4*d) + (b*Sin[4*(c + d*x)])/(32*d)
```

fricas [A] time = 1.13, size = 60, normalized size = 0.78

$$\frac{8\,a\cos\left(dx+c\right)^3+9\,bdx-24\,a\cos\left(dx+c\right)+3\left(2\,b\cos\left(dx+c\right)^3-5\,b\cos\left(dx+c\right)\right)\sin\left(dx+c\right)}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)*(a*sin(d*x+c)^2+b*sin(d*x+c)^3),x, algorithm="fricas")
```

```
[Out] 1/24*(8*a*cos(d*x + c)^3 + 9*b*d*x - 24*a*cos(d*x + c) + 3*(2*b*cos(d*x + c
)^3 - 5*b*cos(d*x + c))*sin(d*x + c))/d
```

giac [A] time = 0.13, size = 62, normalized size = 0.81

$$\frac{3}{8}\,bx + \frac{a\cos\left(3\,dx + 3\,c\right)}{12\,d} - \frac{3\,a\cos\left(dx + c\right)}{4\,d} + \frac{b\sin\left(4\,dx + 4\,c\right)}{32\,d} - \frac{b\sin\left(2\,dx + 2\,c\right)}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)*(a*sin(d*x+c)^2+b*sin(d*x+c)^3),x, algorithm="giac")
```

```
[Out] 3/8*b*x + 1/12*a*cos(3*d*x + 3*c)/d - 3/4*a*cos(d*x + c)/d + 1/32*b*sin(4*d
*x + 4*c)/d - 1/4*b*sin(2*d*x + 2*c)/d
```

maple [A] time = 0.05, size = 60, normalized size = 0.78

$$\frac{b\left(-\frac{\left(\sin^3(dx+c)+\frac{3\sin(dx+c)}{2}\right)\cos(dx+c)}{4}+\frac{3dx}{8}+\frac{3c}{8}\right)-\frac{a\left(2+\sin^2(dx+c)\right)\cos(dx+c)}{3}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)*(a*sin(d*x+c)^2+b*sin(d*x+c)^3),x)
```

```
[Out] 1/d*(b*(-1/4*(sin(d*x+c)^3+3/2*sin(d*x+c))*cos(d*x+c)+3/8*d*x+3/8*c)-1/3*a*
(2+sin(d*x+c)^2)*cos(d*x+c))
```

maxima [A] time = 0.49, size = 57, normalized size = 0.74

$$\frac{32\left(\cos\left(dx + c\right)^{3} - 3\,\cos\left(dx + c\right)\right)a + 3\left(12\,dx + 12\,c + \sin\left(4\,dx + 4\,c\right) - 8\,\sin\left(2\,dx + 2\,c\right)\right)b}{96\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)*(a*sin(d*x+c)^2+b*sin(d*x+c)^3),x, algorithm="maxima")
```

```
[Out] 1/96*(32*(cos(d*x + c)^3 - 3*cos(d*x + c))*a + 3*(12*d*x + 12*c + sin(4*d*x
 + 4*c) - 8*sin(2*d*x + 2*c))*b)/d
```

mupad [B] time = 6.56, size = 111, normalized size = 1.44

$$\frac{3\,b\,x}{8}-\frac{-\frac{3\,b\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7}{4}-\frac{11\,b\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5}{4}+4\,a\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4+\frac{11\,b\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{4}+\frac{16\,a\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{3}+\frac{3\,b\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{4}+\frac{4\,a}{3}}{d\,{\left({\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2+1\right)}^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)*(a*sin(c + d*x)^2 + b*sin(c + d*x)^3),x)
```

```
[Out] (3*b*x)/8 - ((4*a)/3 + (3*b*tan(c/2 + (d*x)/2))/4 + (16*a*tan(c/2 + (d*x)/2
)^2)/3 + 4*a*tan(c/2 + (d*x)/2)^4 + (11*b*tan(c/2 + (d*x)/2)^3)/4 - (11*b*t
an(c/2 + (d*x)/2)^5)/4 - (3*b*tan(c/2 + (d*x)/2)^7)/4)/(d*(tan(c/2 + (d*x)/
2)^2 + 1)^4)
```

sympy [A] time = 0.97, size = 150, normalized size = 1.95

$$\begin{cases} -\frac{a\sin^2(c+dx)\cos(c+dx)}{d} - \frac{2a\cos^3(c+dx)}{3d} + \frac{3bx\sin^4(c+dx)}{8} + \frac{3bx\sin^2(c+dx)\cos^2(c+dx)}{4} + \frac{3bx\cos^4(c+dx)}{8} - \frac{5b\sin^3(c+dx)\cos(c+dx)}{8d} \cdot \\ x\left(a\sin^2(c) + b\sin^3(c)\right)\sin(c) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)*(a*sin(d*x+c)**2+b*sin(d*x+c)**3),x)
```

```
[Out] Piecewise((-a*sin(c + d*x)**2*cos(c + d*x)/d - 2*a*cos(c + d*x)**3/(3*d) +
3*b*x*sin(c + d*x)**4/8 + 3*b*x*sin(c + d*x)**2*cos(c + d*x)**2/4 + 3*b*x*c
os(c + d*x)**4/8 - 5*b*sin(c + d*x)**3*cos(c + d*x)/(8*d) - 3*b*sin(c + d*x
)*cos(c + d*x)**3/(8*d), Ne(d, 0)), (x*(a*sin(c)**2 + b*sin(c)**3)*sin(c),
True))
```

3.935 $\int \sin(c+dx) \left(a \sin^2(c+dx) + b \sin^3(c+dx)\right)^2 \, dx$

Optimal. Leaf size=161

$$-\frac{\left(a^2+3b^2\right)\cos^5(c+dx)}{5d}+\frac{\left(2a^2+3b^2\right)\cos^3(c+dx)}{3d}-\frac{\left(a^2+b^2\right)\cos(c+dx)}{d}-\frac{ab\sin^5(c+dx)\cos(c+dx)}{3d}-\frac{5ab\sin}{}$$

```
[Out] 5/8*a*b*x-(a^2+b^2)*cos(d*x+c)/d+1/3*(2*a^2+3*b^2)*cos(d*x+c)^3/d-1/5*(a^2+
3*b^2)*cos(d*x+c)^5/d+1/7*b^2*cos(d*x+c)^7/d-5/8*a*b*cos(d*x+c)*sin(d*x+c)/
d-5/12*a*b*cos(d*x+c)*sin(d*x+c)^3/d-1/3*a*b*cos(d*x+c)*sin(d*x+c)^5/d
```

Rubi [A] time = 0.27, antiderivative size = 161, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {4393, 2789, 2635, 8, 3013, 373}

$$-\frac{\left(a^2+3b^2\right)\cos^5(c+dx)}{5d}+\frac{\left(2a^2+3b^2\right)\cos^3(c+dx)}{3d}-\frac{\left(a^2+b^2\right)\cos(c+dx)}{d}-\frac{ab\sin^5(c+dx)\cos(c+dx)}{3d}-\frac{5ab\sin}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]*(a*Sin[c + d*x]^2 + b*Sin[c + d*x]^3)^2,x]
```

```
[Out] (5*a*b*x)/8 - ((a^2 + b^2)*Cos[c + d*x])/d + ((2*a^2 + 3*b^2)*Cos[c + d*x]^
3)/(3*d) - ((a^2 + 3*b^2)*Cos[c + d*x]^5)/(5*d) + (b^2*Cos[c + d*x]^7)/(7*d
) - (5*a*b*Cos[c + d*x]*Sin[c + d*x])/(8*d) - (5*a*b*Cos[c + d*x]*Sin[c + d
*x]^3)/(12*d) - (a*b*Cos[c + d*x]*Sin[c + d*x]^5)/(3*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 373

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> Int[ExpandIntegrand[(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b
, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2789

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)])^2, x_Symbol] :> Dist[(2*c*d)/b, Int[(b*Sin[e + f*x])^(m + 1), x], x] +
 Int[(b*Sin[e + f*x])^m*(c^2 + d^2*Sin[e + f*x]^2), x] /; FreeQ[{b, c, d, e
, f, m}, x]
```

Rule 3013

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((A_) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2),
 x_Symbol] :> -Dist[f^(-1), Subst[Int[(1 - x^2)^((m - 1)/2)*(A + C - C*x^2)
, x], x, Cos[e + f*x]], x] /; FreeQ[{e, f, A, C}, x] && IGtQ[(m + 1)/2, 0]
```

Rule 4393

```
Int[(u_)*((a_)*(F_)[(c_.) + (d_.)*(x_)]^(p_.) + (b_.)*(F_)[(c_.) + (d_.)*(x
_)]^(q_.))^(n_.), x_Symbol] :> Int[ActivateTrig[u*F[c + d*x]^(n*p)*(a + b*F
[c + d*x]^(q - p))^n], x] /; FreeQ[{a, b, c, d, p, q}, x] && InertTrigQ[F]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\int \sin(c+dx)\left(a\sin^2(c+dx)+b\sin^3(c+dx)\right)^2\,dx = \int \sin^5(c+dx)(a+b\sin(c+dx))^2\,dx$$
$$= (2ab)\int \sin^6(c+dx)\,dx + \int \sin^5(c+dx)\left(a^2+b^2\sin^2(c\right.$$
$$= -\frac{ab\cos(c+dx)\sin^5(c+dx)}{3d} + \frac{1}{3}(5ab)\int \sin^4(c+dx)\,dx$$
$$= -\frac{5ab\cos(c+dx)\sin^3(c+dx)}{12d} - \frac{ab\cos(c+dx)\sin^5(c+}{3d}$$
$$= -\frac{\left(a^2+b^2\right)\cos(c+dx)}{d} + \frac{\left(2a^2+3b^2\right)\cos^3(c+dx)}{3d} - \frac{\left(a^2\right.}{}$$
$$= \frac{5abx}{8} - \frac{\left(a^2+b^2\right)\cos(c+dx)}{d} + \frac{\left(2a^2+3b^2\right)\cos^3(c+dx)}{3d}$$

Mathematica [A] time = 0.21, size = 134, normalized size = 0.83

$$\frac{-525\left(8a^2+7b^2\right)\cos(c+dx)+35\left(20a^2+21b^2\right)\cos(3(c+dx))-84a^2\cos(5(c+dx))-3150ab\sin(2(c+dx))}{67}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]*(a*Sin[c + d*x]^2 + b*Sin[c + d*x]^3)^2,x]
```

```
[Out] (4200*a*b*c + 4200*a*b*d*x - 525*(8*a^2 + 7*b^2)*Cos[c + d*x] + 35*(20*a^2
+ 21*b^2)*Cos[3*(c + d*x)] - 84*a^2*Cos[5*(c + d*x)] - 147*b^2*Cos[5*(c + d
*x)] + 15*b^2*Cos[7*(c + d*x)] - 3150*a*b*Sin[2*(c + d*x)] + 630*a*b*Sin[4*
(c + d*x)] - 70*a*b*Sin[6*(c + d*x)])/(6720*d)
```

fricas [A] time = 0.91, size = 123, normalized size = 0.76

$$\frac{120\,b^2\cos\left(dx+c\right)^7-168\left(a^2+3\,b^2\right)\cos\left(dx+c\right)^5+525\,abdx+280\left(2\,a^2+3\,b^2\right)\cos\left(dx+c\right)^3-840\left(a^2+b^2\right)\mathrm{co}}{840\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)*(a*sin(d*x+c)^2+b*sin(d*x+c)^3)^2,x, algorithm="fricas
")
```

```
[Out] 1/840*(120*b^2*cos(d*x + c)^7 - 168*(a^2 + 3*b^2)*cos(d*x + c)^5 + 525*a*b*
d*x + 280*(2*a^2 + 3*b^2)*cos(d*x + c)^3 - 840*(a^2 + b^2)*cos(d*x + c) - 3
5*(8*a*b*cos(d*x + c)^5 - 26*a*b*cos(d*x + c)^3 + 33*a*b*cos(d*x + c))*sin(
d*x + c))/d
```

giac [A] time = 0.19, size = 143, normalized size = 0.89

$$\frac{5}{8}\,abx+\frac{b^2\cos\left(7\,dx+7\,c\right)}{448\,d}-\frac{ab\sin\left(6\,dx+6\,c\right)}{96\,d}+\frac{3\,ab\sin\left(4\,dx+4\,c\right)}{32\,d}-\frac{15\,ab\sin\left(2\,dx+2\,c\right)}{32\,d}-\frac{\left(4\,a^2+7\,b^2\right)\cos\left(5\,dx\right.}{320\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)*(a*sin(d*x+c)^2+b*sin(d*x+c)^3)^2,x, algorithm="giac")
```

```
[Out] 5/8*a*b*x + 1/448*b^2*cos(7*d*x + 7*c)/d - 1/96*a*b*sin(6*d*x + 6*c)/d + 3/
32*a*b*sin(4*d*x + 4*c)/d - 15/32*a*b*sin(2*d*x + 2*c)/d - 1/320*(4*a^2 + 7
*b^2)*cos(5*d*x + 5*c)/d + 1/192*(20*a^2 + 21*b^2)*cos(3*d*x + 3*c)/d - 5/6
4*(8*a^2 + 7*b^2)*cos(d*x + c)/d
```

maple [A] time = 0.08, size = 125, normalized size = 0.78

$$\frac{-\frac{b^2\left(\frac{16}{5}+\sin^6(dx+c)+\frac{6\left(\sin^4(dx+c)\right)}{5}+\frac{8\left(\sin^2(dx+c)\right)}{5}\right)\cos(dx+c)}{7}+2ab\left(-\frac{\left(\sin^5(dx+c)+\frac{5\left(\sin^3(dx+c)\right)}{4}+\frac{15\sin(dx+c)}{8}\right)\cos(dx+c)}{6}+\frac{5dx}{16}+\frac{5c}{16}\right)-\frac{a^2\left(\frac{8}{3}+\right.}{}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)*(a*sin(d*x+c)^2+b*sin(d*x+c)^3)^2,x)
```

```
[Out] 1/d*(-1/7*b^2*(16/5+sin(d*x+c)^6+6/5*sin(d*x+c)^4+8/5*sin(d*x+c)^2)*cos(d*x
+c)+2*a*b*(-1/6*(sin(d*x+c)^5+5/4*sin(d*x+c)^3+15/8*sin(d*x+c))*cos(d*x+c)+
5/16*d*x+5/16*c)-1/5*a^2*(8/3+sin(d*x+c)^4+4/3*sin(d*x+c)^2)*cos(d*x+c))
```

maxima [A] time = 0.33, size = 131, normalized size = 0.81

$$-\frac{224\left(3\cos(dx+c)^5-10\cos(dx+c)^3+15\cos(dx+c)\right)a^2-35\left(4\sin(2dx+2c)^3+60dx+60c+9\sin(4\right.}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)*(a*sin(d*x+c)^2+b*sin(d*x+c)^3)^2,x, algorithm="maxima
")
```

```
[Out] -1/3360*(224*(3*cos(d*x + c)^5 - 10*cos(d*x + c)^3 + 15*cos(d*x + c))*a^2 -
 35*(4*sin(2*d*x + 2*c)^3 + 60*d*x + 60*c + 9*sin(4*d*x + 4*c) - 48*sin(2*d
*x + 2*c))*a*b - 96*(5*cos(d*x + c)^7 - 21*cos(d*x + c)^5 + 35*cos(d*x + c)
^3 - 35*cos(d*x + c))*b^2)/d
```

mupad [B] time = 6.74, size = 210, normalized size = 1.30

$$\frac{5abx}{8}-\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^6\left(\frac{80a^2}{3}+32b^2\right)+\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2\left(\frac{112a^2}{15}+\frac{32b^2}{5}\right)+\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4\left(\frac{112a^2}{5}+\frac{96b^2}{5}\right)+\frac{32a^2\tan(}{3}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)*(a*sin(c + d*x)^2 + b*sin(c + d*x)^3)^2,x)
```

```
[Out] (5*a*b*x)/8 - (tan(c/2 + (d*x)/2)^6*((80*a^2)/3 + 32*b^2) + tan(c/2 + (d*x)
/2)^2*((112*a^2)/15 + (32*b^2)/5) + tan(c/2 + (d*x)/2)^4*((112*a^2)/5 + (96
*b^2)/5) + (32*a^2*tan(c/2 + (d*x)/2)^8)/3 + (16*a^2)/15 + (32*b^2)/35 + (2
5*a*b*tan(c/2 + (d*x)/2)^3)/3 + (283*a*b*tan(c/2 + (d*x)/2)^5)/12 - (283*a*
b*tan(c/2 + (d*x)/2)^9)/12 - (25*a*b*tan(c/2 + (d*x)/2)^11)/3 - (5*a*b*tan(
c/2 + (d*x)/2)^13)/4 + (5*a*b*tan(c/2 + (d*x)/2))/4)/(d*(tan(c/2 + (d*x)/2)
^2 + 1)^7)
```

sympy [A] time = 6.01, size = 326, normalized size = 2.02

$$\begin{cases}-\frac{a^2\sin^4(c+dx)\cos(c+dx)}{d}-\frac{4a^2\sin^2(c+dx)\cos^3(c+dx)}{3d}-\frac{8a^2\cos^5(c+dx)}{15d}+\frac{5abx\sin^6(c+dx)}{8}+\frac{15abx\sin^4(c+dx)\cos^2(c+dx)}{8}+\frac{15abx\sin}{} \\ x\left(a\sin^2(c)+b\sin^3(c)\right)^2\sin(c)\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)*(a*sin(d*x+c)**2+b*sin(d*x+c)**3)**2,x)
```

```
[Out] Piecewise((-a**2*sin(c + d*x)**4*cos(c + d*x)/d - 4*a**2*sin(c + d*x)**2*co
s(c + d*x)**3/(3*d) - 8*a**2*cos(c + d*x)**5/(15*d) + 5*a*b*x*sin(c + d*x)*
*6/8 + 15*a*b*x*sin(c + d*x)**4*cos(c + d*x)**2/8 + 15*a*b*x*sin(c + d*x)**
2*cos(c + d*x)**4/8 + 5*a*b*x*cos(c + d*x)**6/8 - 11*a*b*sin(c + d*x)**5*co
s(c + d*x)/(8*d) - 5*a*b*sin(c + d*x)**3*cos(c + d*x)**3/(3*d) - 5*a*b*sin(
c + d*x)*cos(c + d*x)**5/(8*d) - b**2*sin(c + d*x)**6*cos(c + d*x)/d - 2*b*
*2*sin(c + d*x)**4*cos(c + d*x)**3/d - 8*b**2*sin(c + d*x)**2*cos(c + d*x)*
*5/(5*d) - 16*b**2*cos(c + d*x)**7/(35*d), Ne(d, 0)), (x*(a*sin(c)**2 + b*s
in(c)**3)**2*sin(c), True))
```

3.936 $\int \sin(c+dx)\left(a\sin(c+dx)+b\sin^2(c+dx)+c\sin^3(c$

Optimal. Leaf size=89

$$-\frac{(4a+3c)\sin(c+dx)\cos(c+dx)}{8d}+\frac{1}{8}x(4a+3c)+\frac{b\cos^3(c+dx)}{3d}-\frac{b\cos(c+dx)}{d}-\frac{c\sin^3(c+dx)\cos(c+dx)}{4d}$$

```
[Out] 1/8*(4*a+3*c)*x-b*cos(d*x+c)/d+1/3*b*cos(d*x+c)^3/d-1/8*(4*a+3*c)*cos(d*x+c
)*sin(d*x+c)/d-1/4*c*cos(d*x+c)*sin(d*x+c)^3/d
```

Rubi [A] time = 0.11, antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {4237, 3023, 2748, 2635, 8, 2633}

$$-\frac{(4a+3c)\sin(c+dx)\cos(c+dx)}{8d}+\frac{1}{8}x(4a+3c)+\frac{b\cos^3(c+dx)}{3d}-\frac{b\cos(c+dx)}{d}-\frac{c\sin^3(c+dx)\cos(c+dx)}{4d}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]*(a*Sin[c + d*x] + b*Sin[c + d*x]^2 + c*Sin[c + d*x]^3),x]
```

```
[Out] ((4*a + 3*c)*x)/8 - (b*Cos[c + d*x])/d + (b*Cos[c + d*x]^3)/(3*d) - ((4*a +
 3*c)*Cos[c + d*x]*Sin[c + d*x])/(8*d) - (c*Cos[c + d*x]*Sin[c + d*x]^3)/(4
*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
```

```
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 4237

```
Int[(u_)*((A_.)*sin[(a_.) + (b_.)*(x_)]^(n_.) + (B_.)*sin[(a_.) + (b_.)*(x_
)]^(n1_) + (C_.)*sin[(a_.) + (b_.)*(x_)]^(n2_)), x_Symbol] :> Int[ActivateT
rig[u]*Sin[a + b*x]^n*(A + B*Sin[a + b*x] + C*Sin[a + b*x]^2), x] /; FreeQ[
{a, b, A, B, C, n}, x] && EqQ[n1, n + 1] && EqQ[n2, n + 2]
```

Rubi steps

$$\int \sin(c+dx)\left(a\sin(c+dx)+b\sin^2(c+dx)+c\sin^3(c+dx)\right)dx = \int \sin^2(c+dx)\left(a+b\sin(c+dx)+c\sin^2(c\right.$$
$$= -\frac{c\cos(c+dx)\sin^3(c+dx)}{4d}+\frac{1}{4}\int \sin^2(c+$$
$$= -\frac{c\cos(c+dx)\sin^3(c+dx)}{4d}+b\int \sin^3(c+$$
$$= -\frac{(4a+3c)\cos(c+dx)\sin(c+dx)}{8d}-\frac{c\cos(c}{}$$
$$= \frac{1}{8}(4a+3c)x-\frac{b\cos(c+dx)}{d}+\frac{b\cos^3(c+dx}{3d}$$

Mathematica [A] time = 0.15, size = 105, normalized size = 1.18

$$\frac{a(c+dx)}{2d}-\frac{a\sin(2(c+dx))}{4d}-\frac{3b\cos(c+dx)}{4d}+\frac{b\cos(3(c+dx))}{12d}+\frac{3c(c+dx)}{8d}-\frac{c\sin(2(c+dx))}{4d}+\frac{c\sin(4(c+dx))}{32d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]*(a*Sin[c + d*x] + b*Sin[c + d*x]^2 + c*Sin[c + d*x]^
3),x]
```

```
[Out] (a*(c + d*x))/(2*d) + (3*c*(c + d*x))/(8*d) - (3*b*Cos[c + d*x]])/(4*d) + (b
*Cos[3*(c + d*x)]])/(12*d) - (a*Sin[2*(c + d*x)]])/(4*d) - (c*Sin[2*(c + d*x)
])/(4*d) + (c*Sin[4*(c + d*x)]])/(32*d)
```

fricas [A] time = 0.88, size = 72, normalized size = 0.81

$$\frac{8\,b\cos\left(dx+c\right)^{3}+3\,(4\,a+3\,c)dx-24\,b\cos\left(dx+c\right)+3\left(2\,c\cos\left(dx+c\right)^{3}-(4\,a+5\,c)\cos\left(dx+c\right)\right)\sin\left(dx+c\right)}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)*(a*sin(d*x+c)+b*sin(d*x+c)^2+c*sin(d*x+c)^3),x, algori
thm="fricas")
```

```
[Out] 1/24*(8*b*cos(d*x + c)^3 + 3*(4*a + 3*c)*d*x - 24*b*cos(d*x + c) + 3*(2*c*c
os(d*x + c)^3 - (4*a + 5*c)*cos(d*x + c))*sin(d*x + c))/d
```

giac [A] time = 0.15, size = 70, normalized size = 0.79

$$\frac{1}{8}\,(4\,a+3\,c)x+\frac{b\cos\left(3\,dx+3\,c\right)}{12\,d}-\frac{3\,b\cos\left(dx+c\right)}{4\,d}+\frac{c\sin\left(4\,dx+4\,c\right)}{32\,d}-\frac{(a+c)\sin\left(2\,dx+2\,c\right)}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)*(a*sin(d*x+c)+b*sin(d*x+c)^2+c*sin(d*x+c)^3),x, algori
thm="giac")
```

```
[Out] 1/8*(4*a + 3*c)*x + 1/12*b*cos(3*d*x + 3*c)/d - 3/4*b*cos(d*x + c)/d + 1/32
*c*sin(4*d*x + 4*c)/d - 1/4*(a + c)*sin(2*d*x + 2*c)/d
```

maple [A] time = 0.08, size = 84, normalized size = 0.94

$$\frac{c\left(-\frac{\left(\sin^{3}(dx+c)+\frac{3\sin(dx+c)}{2}\right)\cos(dx+c)}{4}+\frac{3dx}{8}+\frac{3c}{8}\right)-\frac{b\left(2+\sin^{2}(dx+c)\right)\cos(dx+c)}{3}+a\left(-\frac{\sin(dx+c)\cos(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)*(a*sin(d*x+c)+b*sin(d*x+c)^2+c*sin(d*x+c)^3),x)
```

```
[Out] 1/d*(c*(-1/4*(sin(d*x+c)^3+3/2*sin(d*x+c))*cos(d*x+c)+3/8*d*x+3/8*c)-1/3*b*
(2+sin(d*x+c)^2)*cos(d*x+c)+a*(-1/2*sin(d*x+c)*cos(d*x+c)+1/2*d*x+1/2*c))
```

maxima [A] time = 0.46, size = 79, normalized size = 0.89

$$\frac{24\,(2\,dx+2\,c-\sin\left(2\,dx+2\,c\right))a+32\left(\cos\left(dx+c\right)^{3}-3\,\cos\left(dx+c\right)\right)b+3\,(12\,dx+12\,c+\sin\left(4\,dx+4\,c\right)-8\,s}{96\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)*(a*sin(d*x+c)+b*sin(d*x+c)^2+c*sin(d*x+c)^3),x, algori
thm="maxima")
```

```
[Out] 1/96*(24*(2*d*x + 2*c - sin(2*d*x + 2*c))*a + 32*(cos(d*x + c)^3 - 3*cos(d*
x + c))*b + 3*(12*d*x + 12*c + sin(4*d*x + 4*c) - 8*sin(2*d*x + 2*c))*c)/d
```

mupad [B] time = 3.18, size = 73, normalized size = 0.82

$$\frac{2\,b\,\cos\left(3\,c+3\,d\,x\right)-18\,b\,\cos\left(c+d\,x\right)-6\,a\,\sin\left(2\,c+2\,d\,x\right)-6\,c\,\sin\left(2\,c+2\,d\,x\right)+\frac{3\,c\,\sin\left(4\,c+4\,d\,x\right)}{4}+12\,a\,d\,x+9\,c\,d\,x}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)*(a*sin(c + d*x) + b*sin(c + d*x)^2 + c*sin(c + d*x)^3),x)
```

```
[Out] (2*b*cos(3*c + 3*d*x) - 18*b*cos(c + d*x) - 6*a*sin(2*c + 2*d*x) - 6*c*sin(
2*c + 2*d*x) + (3*c*sin(4*c + 4*d*x))/4 + 12*a*d*x + 9*c*d*x)/(24*d)
```

sympy [A] time = 1.03, size = 201, normalized size = 2.26

$$\begin{cases} \frac{ax\sin^2(c+dx)}{2} + \frac{ax\cos^2(c+dx)}{2} - \frac{a\sin(c+dx)\cos(c+dx)}{2d} - \frac{b\sin^2(c+dx)\cos(c+dx)}{d} - \frac{2b\cos^3(c+dx)}{3d} + \frac{3cx\sin^4(c+dx)}{8} + \frac{3cx\sin^2(c+dx)\cos^2(c+dx)}{4} \\ x\left(a\sin(c) + b\sin^2(c) + c\sin^3(c)\right)\sin(c) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)*(a*sin(d*x+c)+b*sin(d*x+c)**2+c*sin(d*x+c)**3),x)
```

```
[Out] Piecewise((a*x*sin(c + d*x)**2/2 + a*x*cos(c + d*x)**2/2 - a*sin(c + d*x)*c
os(c + d*x)/(2*d) - b*sin(c + d*x)**2*cos(c + d*x)/d - 2*b*cos(c + d*x)**3/
(3*d) + 3*c*x*sin(c + d*x)**4/8 + 3*c*x*sin(c + d*x)**2*cos(c + d*x)**2/4 +
 3*c*x*cos(c + d*x)**4/8 - 5*c*sin(c + d*x)**3*cos(c + d*x)/(8*d) - 3*c*sin
(c + d*x)*cos(c + d*x)**3/(8*d), Ne(d, 0)), (x*(a*sin(c) + b*sin(c)**2 + c*
sin(c)**3)*sin(c), True))
```

3.937 $\int \sin(c+dx)\left(a\sin(c+dx)+b\sin^2(c+dx)+c\sin^3(c\right.$

Optimal. Leaf size=288

$$\frac{a^2\cos^3(c+dx)}{3d}-\frac{a^2\cos(c+dx)}{d}-\frac{\left(2ac+b^2\right)\cos^5(c+dx)}{5d}+\frac{2\left(2ac+b^2\right)\cos^3(c+dx)}{3d}-\frac{\left(2ac+b^2\right)\cos(c+dx)}{d}-\frac{a}{}$$

```
[Out] 3/4*a*b*x+5/8*b*c*x-a^2*cos(d*x+c)/d-c^2*cos(d*x+c)/d-(2*a*c+b^2)*cos(d*x+c
)/d+1/3*a^2*cos(d*x+c)^3/d+c^2*cos(d*x+c)^3/d+2/3*(2*a*c+b^2)*cos(d*x+c)^3/
d-3/5*c^2*cos(d*x+c)^5/d-1/5*(2*a*c+b^2)*cos(d*x+c)^5/d+1/7*c^2*cos(d*x+c)^
7/d-3/4*a*b*cos(d*x+c)*sin(d*x+c)/d-5/8*b*c*cos(d*x+c)*sin(d*x+c)/d-1/2*a*b
*cos(d*x+c)*sin(d*x+c)^3/d-5/12*b*c*cos(d*x+c)*sin(d*x+c)^3/d-1/3*b*c*cos(d
*x+c)*sin(d*x+c)^5/d
```

Rubi [A] time = 0.40, antiderivative size = 288, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 5, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.132, Rules used = {4394, 3256, 2633, 2635, 8}

$$\frac{a^2\cos^3(c+dx)}{3d}-\frac{a^2\cos(c+dx)}{d}-\frac{\left(2ac+b^2\right)\cos^5(c+dx)}{5d}+\frac{2\left(2ac+b^2\right)\cos^3(c+dx)}{3d}-\frac{\left(2ac+b^2\right)\cos(c+dx)}{d}-\frac{a}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]*(a*Sin[c + d*x] + b*Sin[c + d*x]^2 + c*Sin[c + d*x]^3)^2,x
]
```

```
[Out] (3*a*b*x)/4 + (5*b*c*x)/8 - (a^2*Cos[c + d*x])/d - (c^2*Cos[c + d*x])/d - (
(b^2 + 2*a*c)*Cos[c + d*x])/d + (a^2*Cos[c + d*x]^3)/(3*d) + (c^2*Cos[c + d
*x]^3)/d + (2*(b^2 + 2*a*c)*Cos[c + d*x]^3)/(3*d) - (3*c^2*Cos[c + d*x]^5)/
(5*d) - ((b^2 + 2*a*c)*Cos[c + d*x]^5)/(5*d) + (c^2*Cos[c + d*x]^7)/(7*d) -
 (3*a*b*Cos[c + d*x]*Sin[c + d*x])/(4*d) - (5*b*c*Cos[c + d*x]*Sin[c + d*x]
)/(8*d) - (a*b*Cos[c + d*x]*Sin[c + d*x]^3)/(2*d) - (5*b*c*Cos[c + d*x]*Sin
[c + d*x]^3)/(12*d) - (b*c*Cos[c + d*x]*Sin[c + d*x]^5)/(3*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 3256

```
Int[sin[(d_.) + (e_.)*(x_)]^(m_.)*((a_.) + (b_.)*sin[(d_.) + (e_.)*(x_)]^(n
_.) + (c_.)*sin[(d_.) + (e_.)*(x_)]^(n2_.))^(p_), x_Symbol] :> Int[ExpandTr
ig[sin[d + e*x]^m*(a + b*sin[d + e*x]^n + c*sin[d + e*x]^(2*n))^p, x], x] /
; FreeQ[{a, b, c, d, e}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && Integ
ersQ[m, n, p]
```

Rule 4394

```
Int[(u_)*((a_)*(F_)[(d_.) + (e_.)*(x_)]^(p_.) + (b_.)*(F_)[(d_.) + (e_.)*(x
_)]^(q_.) + (c_.)*(F_)[(d_.) + (e_.)*(x_)]^(r_.))^(n_.), x_Symbol] :> Int[A
ctivateTrig[u*F[d + e*x]^(n*p)*(a + b*F[d + e*x]^(q - p) + c*F[d + e*x]^(r
- p))^n], x] /; FreeQ[{a, b, c, d, e, p, q, r}, x] && InertTrigQ[F] && Inte
gerQ[n] && PosQ[q - p] && PosQ[r - p]
```

Rubi steps

$$\int \sin(c+dx)\left(a\sin(c+dx)+b\sin^2(c+dx)+c\sin^3(c+dx)\right)^2\,dx = \int \sin^3(c+dx)\left(a+b\sin(c+dx)+c\sin^2(c\right.$$

$$= \int \left(a^2\sin^3(c+dx)+2ab\sin^4(c+dx)+\left(b^2\right.\right.$$

$$= a^2\int \sin^3(c+dx)\,dx+(2ab)\int \sin^4(c+dx$$

$$= -\frac{ab\cos(c+dx)\sin^3(c+dx)}{2d}-\frac{bc\cos(c+d}{}$$

$$= -\frac{a^2\cos(c+dx)}{d}-\frac{c^2\cos(c+dx)}{d}-\frac{\left(b^2+2\right.}{}$$

$$= \frac{3abx}{4}-\frac{a^2\cos(c+dx)}{d}-\frac{c^2\cos(c+dx)}{d}-$$

$$= \frac{3abx}{4}+\frac{5bcx}{8}-\frac{a^2\cos(c+dx)}{d}-\frac{c^2\cos(c+}{d}$$

Mathematica [A] time = 0.45, size = 167, normalized size = 0.58

$$\frac{-105\left(48a^2+80ac+40b^2+35c^2\right)\cos(c+dx)+35\left(16a^2+40ac+20b^2+21c^2\right)\cos(3(c+dx))-21\left(c(8a+7c)\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]*(a*Sin[c + d*x] + b*Sin[c + d*x]^2 + c*Sin[c + d*x]^
3)^2,x]
```

```
[Out] (840*b*(6*a + 5*c)*(c + d*x) - 105*(48*a^2 + 40*b^2 + 80*a*c + 35*c^2)*Cos[
c + d*x] + 35*(16*a^2 + 20*b^2 + 40*a*c + 21*c^2)*Cos[3*(c + d*x)] - 21*(4*
b^2 + c*(8*a + 7*c))*Cos[5*(c + d*x)] + 15*c^2*Cos[7*(c + d*x)] - 210*b*(16
*a + 15*c)*Sin[2*(c + d*x)] + 210*b*(2*a + 3*c)*Sin[4*(c + d*x)] - 70*b*c*S
in[6*(c + d*x)])/(6720*d)
```

fricas [A] time = 1.96, size = 162, normalized size = 0.56

$$\frac{120\,c^{2}\cos\left(dx+c\right)^{7}-168\left(b^{2}+2\,ac+3\,c^{2}\right)\cos\left(dx+c\right)^{5}+280\left(a^{2}+2\,b^{2}+4\,ac+3\,c^{2}\right)\cos\left(dx+c\right)^{3}+105\,(6\,a}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)*(a*sin(d*x+c)+b*sin(d*x+c)^2+c*sin(d*x+c)^3)^2,x, algo
rithm="fricas")
```

```
[Out] 1/840*(120*c^2*cos(d*x + c)^7 - 168*(b^2 + 2*a*c + 3*c^2)*cos(d*x + c)^5 +
280*(a^2 + 2*b^2 + 4*a*c + 3*c^2)*cos(d*x + c)^3 + 105*(6*a*b + 5*b*c)*d*x
- 840*(a^2 + b^2 + 2*a*c + c^2)*cos(d*x + c) - 35*(8*b*c*cos(d*x + c)^5 - 2
*(6*a*b + 13*b*c)*cos(d*x + c)^3 + 3*(10*a*b + 11*b*c)*cos(d*x + c))*sin(d*
x + c))/d
```

giac [A] time = 0.18, size = 186, normalized size = 0.65

$$\frac{1}{8}\,(6\,ab+5\,bc)x+\frac{c^{2}\cos\left(7\,dx+7\,c\right)}{448\,d}-\frac{bc\sin\left(6\,dx+6\,c\right)}{96\,d}-\frac{\left(4\,b^{2}+8\,ac+7\,c^{2}\right)\cos\left(5\,dx+5\,c\right)}{320\,d}+\frac{\left(16\,a^{2}+20\,b^{2}+4\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)*(a*sin(d*x+c)+b*sin(d*x+c)^2+c*sin(d*x+c)^3)^2,x, algo
rithm="giac")
```

```
[Out] 1/8*(6*a*b + 5*b*c)*x + 1/448*c^2*cos(7*d*x + 7*c)/d - 1/96*b*c*sin(6*d*x +
 6*c)/d - 1/320*(4*b^2 + 8*a*c + 7*c^2)*cos(5*d*x + 5*c)/d + 1/192*(16*a^2
+ 20*b^2 + 40*a*c + 21*c^2)*cos(3*d*x + 3*c)/d - 1/64*(48*a^2 + 40*b^2 + 80
*a*c + 35*c^2)*cos(d*x + c)/d + 1/32*(2*a*b + 3*b*c)*sin(4*d*x + 4*c)/d - 1
/32*(16*a*b + 15*b*c)*sin(2*d*x + 2*c)/d
```

maple [A] time = 0.09, size = 213, normalized size = 0.74

$$-\frac{c^2\left(\frac{16}{5}+\sin^6(dx+c)+\frac{6\left(\sin^4(dx+c)\right)}{5}+\frac{8\left(\sin^2(dx+c)\right)}{5}\right)\cos(dx+c)}{7}+2cb\left(-\frac{\left(\sin^5(dx+c)+\frac{5\left(\sin^3(dx+c)\right)}{4}+\frac{15\sin(dx+c)}{8}\right)\cos(dx+c)}{6}+\frac{5dx}{16}+\frac{5c}{16}\right)-\frac{2ac\left(\frac{8}{3}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)*(a*sin(d*x+c)+b*sin(d*x+c)^2+c*sin(d*x+c)^3)^2,x)
```

```
[Out] 1/d*(-1/7*c^2*(16/5+sin(d*x+c)^6+6/5*sin(d*x+c)^4+8/5*sin(d*x+c)^2)*cos(d*x
+c)+2*c*b*(-1/6*(sin(d*x+c)^5+5/4*sin(d*x+c)^3+15/8*sin(d*x+c))*cos(d*x+c)+
5/16*d*x+5/16*c)-2/5*a*c*(8/3+sin(d*x+c)^4+4/3*sin(d*x+c)^2)*cos(d*x+c)-1/5
*b^2*(8/3+sin(d*x+c)^4+4/3*sin(d*x+c)^2)*cos(d*x+c)+2*a*b*(-1/4*(sin(d*x+c)
^3+3/2*sin(d*x+c))*cos(d*x+c)+3/8*d*x+3/8*c)-1/3*a^2*(2+sin(d*x+c)^2)*cos(d
*x+c))
```

maxima [A] time = 0.40, size = 218, normalized size = 0.76

$$\frac{1120\left(\cos(dx+c)^3-3\cos(dx+c)\right)a^2+210\left(12\,dx+12\,c+\sin(4\,dx+4\,c)-8\sin(2\,dx+2\,c)\right)ab-224\left(3\cos\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)*(a*sin(d*x+c)+b*sin(d*x+c)^2+c*sin(d*x+c)^3)^2,x, algo
rithm="maxima")
```

```
[Out] 1/3360*(1120*(cos(d*x + c)^3 - 3*cos(d*x + c))*a^2 + 210*(12*d*x + 12*c + s
in(4*d*x + 4*c) - 8*sin(2*d*x + 2*c))*a*b - 224*(3*cos(d*x + c)^5 - 10*cos(
d*x + c)^3 + 15*cos(d*x + c))*b^2 - 448*(3*cos(d*x + c)^5 - 10*cos(d*x + c)
^3 + 15*cos(d*x + c))*a*c + 35*(4*sin(2*d*x + 2*c)^3 + 60*d*x + 60*c + 9*si
n(4*d*x + 4*c) - 48*sin(2*d*x + 2*c))*b*c + 96*(5*cos(d*x + c)^7 - 21*cos(d
*x + c)^5 + 35*cos(d*x + c)^3 - 35*cos(d*x + c))*c^2)/d
```

mupad [B] time = 4.52, size = 456, normalized size = 1.58

$$\frac{b\,\mathrm{atan}\left(\frac{b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)(6\,a+5\,c)}{4\left(\frac{3\,a\,b}{2}+\frac{5\,b\,c}{4}\right)}\right)(6\,a+5\,c)}{4\,d}-\frac{b\left(\mathrm{atan}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)-\frac{d\,x}{2}\right)(6\,a+5\,c)}{4\,d}-\frac{\frac{32\,a\,c}{15}-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{13}\left(\frac{3\,a\,b}{2}+\frac{5}{}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)*(a*sin(c + d*x) + b*sin(c + d*x)^2 + c*sin(c + d*x)^3)^2,x
)
```

```
[Out] (b*atan((b*tan(c/2 + (d*x)/2)*(6*a + 5*c))/(4*((3*a*b)/2 + (5*b*c)/4)))*(6*
a + 5*c))/(4*d) - (b*(atan(tan(c/2 + (d*x)/2)) - (d*x)/2)*(6*a + 5*c))/(4*d
) - ((32*a*c)/15 - tan(c/2 + (d*x)/2)^13*((3*a*b)/2 + (5*b*c)/4) + tan(c/2
+ (d*x)/2)^3*(10*a*b + (25*b*c)/3) - tan(c/2 + (d*x)/2)^11*(10*a*b + (25*b*
c)/3) + tan(c/2 + (d*x)/2)^5*((31*a*b)/2 + (283*b*c)/12) - tan(c/2 + (d*x)/
2)^9*((31*a*b)/2 + (283*b*c)/12) + 4*a^2*tan(c/2 + (d*x)/2)^10 + tan(c/2 +
(d*x)/2)^8*((64*a*c)/3 + (52*a^2)/3 + (32*b^2)/3) + tan(c/2 + (d*x)/2)^6*((
160*a*c)/3 + (88*a^2)/3 + (80*b^2)/3 + 32*c^2) + tan(c/2 + (d*x)/2)^2*((224
*a*c)/15 + (28*a^2)/3 + (112*b^2)/15 + (32*c^2)/5) + tan(c/2 + (d*x)/2)^4*(
(224*a*c)/5 + 24*a^2 + (112*b^2)/5 + (96*c^2)/5) + (4*a^2)/3 + (16*b^2)/15
+ (32*c^2)/35 + tan(c/2 + (d*x)/2)*((3*a*b)/2 + (5*b*c)/4))/(d*(7*tan(c/2 +
 (d*x)/2)^2 + 21*tan(c/2 + (d*x)/2)^4 + 35*tan(c/2 + (d*x)/2)^6 + 35*tan(c/
2 + (d*x)/2)^8 + 21*tan(c/2 + (d*x)/2)^10 + 7*tan(c/2 + (d*x)/2)^12 + tan(c
/2 + (d*x)/2)^14 + 1))
```

sympy [A] time = 6.81, size = 541, normalized size = 1.88

$$\begin{cases} -\frac{a^2\sin^2(c+dx)\cos(c+dx)}{d} - \frac{2a^2\cos^3(c+dx)}{3d} + \frac{3abx\sin^4(c+dx)}{4} + \frac{3abx\sin^2(c+dx)\cos^2(c+dx)}{2} + \frac{3abx\cos^4(c+dx)}{4} - \frac{5ab\sin^3(c+dx)\cos(c}{4d} \\ x\left(a\sin(c) + b\sin^2(c) + c\sin^3(c)\right)^2\sin(c) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)*(a*sin(d*x+c)+b*sin(d*x+c)**2+c*sin(d*x+c)**3)**2,x)
```

```
[Out] Piecewise((-a**2*sin(c + d*x)**2*cos(c + d*x)/d - 2*a**2*cos(c + d*x)**3/(3
*d) + 3*a*b*x*sin(c + d*x)**4/4 + 3*a*b*x*sin(c + d*x)**2*cos(c + d*x)**2/2
 + 3*a*b*x*cos(c + d*x)**4/4 - 5*a*b*sin(c + d*x)**3*cos(c + d*x)/(4*d) - 3
*a*b*sin(c + d*x)*cos(c + d*x)**3/(4*d) - 2*a*c*sin(c + d*x)**4*cos(c + d*x
)/d - 8*a*c*sin(c + d*x)**2*cos(c + d*x)**3/(3*d) - 16*a*c*cos(c + d*x)**5/
(15*d) - b**2*sin(c + d*x)**4*cos(c + d*x)/d - 4*b**2*sin(c + d*x)**2*cos(c
 + d*x)**3/(3*d) - 8*b**2*cos(c + d*x)**5/(15*d) + 5*b*c*x*sin(c + d*x)**6/
8 + 15*b*c*x*sin(c + d*x)**4*cos(c + d*x)**2/8 + 15*b*c*x*sin(c + d*x)**2*c
os(c + d*x)**4/8 + 5*b*c*x*cos(c + d*x)**6/8 - 11*b*c*sin(c + d*x)**5*cos(c
 + d*x)/(8*d) - 5*b*c*sin(c + d*x)**3*cos(c + d*x)**3/(3*d) - 5*b*c*sin(c +
 d*x)*cos(c + d*x)**5/(8*d) - c**2*sin(c + d*x)**6*cos(c + d*x)/d - 2*c**2*
sin(c + d*x)**4*cos(c + d*x)**3/d - 8*c**2*sin(c + d*x)**2*cos(c + d*x)**5/
(5*d) - 16*c**2*cos(c + d*x)**7/(35*d), Ne(d, 0)), (x*(a*sin(c) + b*sin(c)*
*2 + c*sin(c)**3)**2*sin(c), True))
```

3.938 $\int \sin(c+dx)\left(a+\frac{b}{\sqrt{\sin(c+dx)}}+c\sin(c+dx)\right)dx$

Optimal. Leaf size=61

$$-\frac{a\cos(c+dx)}{d}+\frac{2bE\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)}{d}-\frac{c\sin(c+dx)\cos(c+dx)}{2d}+\frac{cx}{2}$$

```
[Out] 1/2*c*x-a*cos(d*x+c)/d-2*b*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/
4*Pi+1/2*d*x)*EllipticE(cos(1/2*c+1/4*Pi+1/2*d*x),2^(1/2))/d-1/2*c*cos(d*x+
c)*sin(d*x+c)/d
```

Rubi [A] time = 0.29, antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.207, Rules used = {4395, 4401, 2639, 2638, 2635, 8}

$$-\frac{a\cos(c+dx)}{d}+\frac{2bE\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)}{d}-\frac{c\sin(c+dx)\cos(c+dx)}{2d}+\frac{cx}{2}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]*(a + b/Sqrt[Sin[c + d*x]] + c*Sin[c + d*x]),x]
```

```
[Out] (c*x)/2 - (a*Cos[c + d*x])/d + (2*b*EllipticE[(c - Pi/2 + d*x)/2, 2])/d - (
c*Cos[c + d*x]*Sin[c + d*x])/(2*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 4395

```
Int[(u_)*((a_) + (b_.)*(F_)[(d_.) + (e_.)*(x_)]^(p_.) + (c_.)*(F_)[(d_.) +
(e_.)*(x_)]^(q_.))^(n_.), x_Symbol] :> Int[ActivateTrig[u*F[d + e*x]^(n*p)*
(b + a/F[d + e*x]^p + c*F[d + e*x]^(q - p))^n], x] /; FreeQ[{a, b, c, d, e,
 p, q}, x] && InertTrigQ[F] && IntegerQ[n] && NegQ[p]
```

Rule 4401

```
Int[u_, x_Symbol] :> With[{v = ExpandTrig[u, x]}, Int[v, x] /; SumQ[v]] /;
 !InertTrigFreeQ[u]
```

Rubi steps

$$\begin{aligned}\int \sin(c+dx)\left(a+\frac{b}{\sqrt{\sin(c+dx)}}+c\sin(c+dx)\right)dx &= \int \sqrt{\sin(c+dx)}\left(b+a\sqrt{\sin(c+dx)}+c\sin^{\frac{3}{2}}(c+dx)\right)\\ &= \int \left(b\sqrt{\sin(c+dx)}+a\sin(c+dx)+c\sin^2(c+dx)\right)dx\\ &= a\int \sin(c+dx)\,dx+b\int \sqrt{\sin(c+dx)}\,dx+c\int \sin^2(\\ &= -\frac{a\cos(c+dx)}{d}+\frac{2bE\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)}{d}-\frac{c\cos(c+}{}\\ &= \frac{cx}{2}-\frac{a\cos(c+dx)}{d}+\frac{2bE\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)}{d}-\frac{c\cos(}{}\end{aligned}$$

Mathematica [A] time = 0.18, size = 55, normalized size = 0.90

$$\frac{-4a\cos(c+dx)-8bE\left(\frac{1}{4}(-2c-2dx+\pi)\middle|2\right)+c(-\sin(2(c+dx))+2c+2dx)}{4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]*(a + b/Sqrt[Sin[c + d*x]] + c*Sin[c + d*x]),x]
```

```
[Out] (-4*a*Cos[c + d*x] - 8*b*EllipticE[(-2*c + Pi - 2*d*x)/4, 2] + c*(2*c + 2*d
*x - Sin[2*(c + d*x)]))/(4*d)
```

fricas [F] time = 1.78, size = 0, normalized size = 0.00

$$\text{integral}\left(-c\cos\left(dx+c\right)^{2}+a\sin\left(dx+c\right)+b\sqrt{\sin\left(dx+c\right)}+c,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)*(a+c*sin(d*x+c)+b/sin(d*x+c)^(1/2)),x, algorithm="fric
as")
```

```
[Out] integral(-c*cos(d*x + c)^2 + a*sin(d*x + c) + b*sqrt(sin(d*x + c)) + c, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(c\sin\left(dx+c\right)+a+\frac{b}{\sqrt{\sin\left(dx+c\right)}}\right)\sin\left(dx+c\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)*(a+c*sin(d*x+c)+b/sin(d*x+c)^(1/2)),x, algorithm="giac
")
```

```
[Out] integrate((c*sin(d*x + c) + a + b/sqrt(sin(d*x + c)))*sin(d*x + c), x)
```

maple [A] time = 0.34, size = 136, normalized size = 2.23

$$cx-\frac{a\cos\left(dx+c\right)}{d}-\frac{c\left(\frac{\sin(dx+c)\cos(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)}{d}-\frac{b\sqrt{1+\sin\left(dx+c\right)}\,\sqrt{-2\sin\left(dx+c\right)+2}\,\sqrt{-\sin\left(dx+c\right)}\,\left(2\,\mathrm{El}\cdots\right.}{\cos\left(dx+c\cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)*(a+c*sin(d*x+c)+b/sin(d*x+c)^(1/2)),x)
```

```
[Out] c*x-a*cos(d*x+c)/d-c/d*(1/2*sin(d*x+c)*cos(d*x+c)+1/2*d*x+1/2*c)-b*(1+sin(d
*x+c))^(1/2)*(-2*sin(d*x+c)+2)^(1/2)*(-sin(d*x+c))^(1/2)*(2*EllipticE((1+si
n(d*x+c))^(1/2),1/2*2^(1/2))-EllipticF((1+sin(d*x+c))^(1/2),1/2*2^(1/2)))/c
os(d*x+c)/sin(d*x+c)^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{2\,cdx-4\,a\cos\left(dx+c\right)-2\,d\int\frac{\left(\left(b\cos\left(\frac{3}{2}\,dx+\frac{3}{2}\,c\right)-b\cos\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-b\sin\left(\frac{3}{2}\,dx+\frac{3}{2}\,c\right)-b\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)\cos\left(\frac{1}{2}\,\arctan(\sin(dx+c),-\cos(dx+c\cdots\right.}{\cdots}}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)*(a+c*sin(d*x+c)+b/sin(d*x+c)^(1/2)),x, algorithm="maxi
ma")
```

```
[Out] 1/4*(2*c*d*x - 4*a*cos(d*x + c) + 2*d*integrate(-(((b*cos(3/2*d*x + 3/2*c)
- b*cos(1/2*d*x + 1/2*c) - b*sin(3/2*d*x + 3/2*c) - b*sin(1/2*d*x + 1/2*c))
*cos(1/2*arctan2(sin(d*x + c), -cos(d*x + c) + 1)) - (b*cos(3/2*d*x + 3/2*c
) - b*cos(1/2*d*x + 1/2*c) + b*sin(3/2*d*x + 3/2*c) + b*sin(1/2*d*x + 1/2*c
))*sin(1/2*arctan2(sin(d*x + c), -cos(d*x + c) + 1)))*cos(1/2*arctan2(sin(d
*x + c), cos(d*x + c) + 1)) + ((b*cos(3/2*d*x + 3/2*c) - b*cos(1/2*d*x + 1/
2*c) + b*sin(3/2*d*x + 3/2*c) + b*sin(1/2*d*x + 1/2*c))*cos(1/2*arctan2(sin
(d*x + c), -cos(d*x + c) + 1)) + (b*cos(3/2*d*x + 3/2*c) - b*cos(1/2*d*x +
1/2*c) - b*sin(3/2*d*x + 3/2*c) - b*sin(1/2*d*x + 1/2*c))*sin(1/2*arctan2(s
in(d*x + c), -cos(d*x + c) + 1)))*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c
) + 1)))/((cos(d*x + c)^2 + sin(d*x + c)^2 + 2*cos(d*x + c) + 1)^(1/4)*(cos
(d*x + c)^2 + sin(d*x + c)^2 - 2*cos(d*x + c) + 1)^(1/4)), x) - c*sin(2*d*x
 + 2*c))/d
```

mupad [B] time = 3.25, size = 51, normalized size = 0.84

$$\frac{c\,x}{2}-\frac{c\,\sin\left(2\,c+2\,d\,x\right)}{4\,d}-\frac{a\,\cos\left(c+d\,x\right)}{d}+\frac{2\,b\,\mathrm{E}\left(\frac{c}{2}-\frac{\pi}{4}+\frac{d\,x}{2}\middle|2\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)*(a + c*sin(c + d*x) + b/sin(c + d*x)^(1/2)),x)
```

```
[Out] (c*x)/2 - (c*sin(2*c + 2*d*x))/(4*d) - (a*cos(c + d*x))/d + (2*b*ellipticE(
c/2 - pi/4 + (d*x)/2, 2))/d
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a\sqrt{\sin\left(c+dx\right)}+b+c\sin^{\frac{3}{2}}\left(c+dx\right)\right)\sqrt{\sin\left(c+dx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)*(a+c*sin(d*x+c)+b/sin(d*x+c)**(1/2)),x)
```

```
[Out] Integral((a*sqrt(sin(c + d*x)) + b + c*sin(c + d*x)**(3/2))*sqrt(sin(c + d*
x)), x)
```

3.939 $\int \sin(c+dx)\left(a+\frac{b}{\sqrt{\sin(c+dx)}}+c\sin(c+dx)\right)^2 dx$

Optimal. Leaf size=148

$$-\frac{a^2\cos(c+dx)}{d}+\frac{4abE\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)}{d}-\frac{ac\sin(c+dx)\cos(c+dx)}{d}+acx+b^2x+\frac{4bcF\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)}{3d}-\frac{4bc}{}$$

```
[Out] b^2*x+a*c*x-a^2*cos(d*x+c)/d-c^2*cos(d*x+c)/d+1/3*c^2*cos(d*x+c)^3/d-4*a*b*
(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticE(cos
(1/2*c+1/4*Pi+1/2*d*x),2^(1/2))/d-4/3*b*c*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/
2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticF(cos(1/2*c+1/4*Pi+1/2*d*x),2^(1/2))/d
-a*c*cos(d*x+c)*sin(d*x+c)/d-4/3*b*c*cos(d*x+c)*sin(d*x+c)^(1/2)/d
```

Rubi [A] time = 0.24, antiderivative size = 148, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.258, Rules used = {4395, 4401, 2639, 2638, 2635, 2641, 8, 2633}

$$-\frac{a^2\cos(c+dx)}{d}+\frac{4abE\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)}{d}-\frac{ac\sin(c+dx)\cos(c+dx)}{d}+acx+b^2x+\frac{4bcF\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)}{3d}-\frac{4bc}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]*(a + b/Sqrt[Sin[c + d*x]] + c*Sin[c + d*x])^2,x]
```

```
[Out] b^2*x + a*c*x - (a^2*Cos[c + d*x])/d - (c^2*Cos[c + d*x])/d + (c^2*Cos[c +
d*x]^3)/(3*d) + (4*a*b*EllipticE[(c - Pi/2 + d*x)/2, 2])/d + (4*b*c*Ellipti
cF[(c - Pi/2 + d*x)/2, 2])/(3*d) - (4*b*c*Cos[c + d*x]*Sqrt[Sin[c + d*x]])/
(3*d) - (a*c*Cos[c + d*x]*Sin[c + d*x])/d
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
```

```
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 4395

```
Int[(u_)*((a_) + (b_.)*(F_)[(d_.) + (e_.)*(x_)]^(p_.) + (c_.)*(F_)[(d_.) +
(e_.)*(x_)]^(q_.))^(n_.), x_Symbol] :> Int[ActivateTrig[u*F[d + e*x]^(n*p)*
(b + a/F[d + e*x]^p + c*F[d + e*x]^(q - p))^n], x] /; FreeQ[{a, b, c, d, e,
 p, q}, x] && InertTrigQ[F] && IntegerQ[n] && NegQ[p]
```

Rule 4401

```
Int[u_, x_Symbol] :> With[{v = ExpandTrig[u, x]}, Int[v, x] /; SumQ[v]] /;
 !InertTrigFreeQ[u]
```

Rubi steps

$$\int \sin(c+dx)\left(a+\frac{b}{\sqrt{\sin(c+dx)}}+c\sin(c+dx)\right)^2 dx = \int \left(b+a\sqrt{\sin(c+dx)}+c\sin^{\frac{3}{2}}(c+dx)\right)^2 dx$$
$$= \int \left(b^2+2ab\sqrt{\sin(c+dx)}+a^2\sin(c+dx)+2bc\sin^{\frac{3}{2}}(c$$
$$= b^2x+a^2\int \sin(c+dx)\,dx+(2ab)\int \sqrt{\sin(c+dx)}\,dx+$$
$$= b^2x-\frac{a^2\cos(c+dx)}{d}+\frac{4abE\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)}{d}-\frac{4bc}{}$$
$$= b^2x+acx-\frac{a^2\cos(c+dx)}{d}-\frac{c^2\cos(c+dx)}{d}+\frac{c^2\cos^3(c}{3d}$$

Mathematica [A] time = 0.28, size = 137, normalized size = 0.93

$$\frac{-12a^2\cos(c+dx)-48abE\left(\frac{1}{4}(-2c-2dx+\pi)\middle|2\right)+12ac^2+12acdx-6ac\sin(2(c+dx))+12b^2c+12b^2dx-16b}{12d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]*(a + b/Sqrt[Sin[c + d*x]] + c*Sin[c + d*x])^2,x]
```

```
[Out] (12*b^2*c + 12*a*c^2 + 12*b^2*d*x + 12*a*c*d*x - 12*a^2*Cos[c + d*x] - 9*c^
2*Cos[c + d*x] + c^2*Cos[3*(c + d*x)] - 48*a*b*EllipticE[(-2*c + Pi - 2*d*x
)/4, 2] - 16*b*c*EllipticF[(-2*c + Pi - 2*d*x)/4, 2] - 16*b*c*Cos[c + d*x]*
Sqrt[Sin[c + d*x]] - 6*a*c*Sin[2*(c + d*x)])/(12*d)
```

fricas [F] time = 1.46, size = 0, normalized size = 0.00

$$\text{integral}\left(-2\,ac\cos\left(dx+c\right)^2+b^2+2\,ac-\left(c^2\cos\left(dx+c\right)^2-a^2-c^2\right)\sin\left(dx+c\right)+2\left(bc\sin\left(dx+c\right)+ab\right)\sqrt{\sin}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)*(a+c*sin(d*x+c)+b/sin(d*x+c)^(1/2))^2,x, algorithm="fr
icas")
```

```
[Out] integral(-2*a*c*cos(d*x + c)^2 + b^2 + 2*a*c - (c^2*cos(d*x + c)^2 - a^2 -
c^2)*sin(d*x + c) + 2*(b*c*sin(d*x + c) + a*b)*sqrt(sin(d*x + c)), x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)*(a+c*sin(d*x+c)+b/sin(d*x+c)^(1/2))^2,x, algorithm="gi
ac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)
Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/t_n
ostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t
_nostep/2)Warning, choosing root of [1,0,%%%{-2,[2]%%%}+%%%{-2,[1]%%%}+%%%{
-2,[0]%%%},0,%%%{1,[4]%%%}+%%%{-2,[3]%%%}+%%%{3,[2]%%%}+%%%{-2,[1]%%%}+%%%{
1,[0]%%%}] at parameters values [93.1017843988]Warning, choosing root of [1
,0,%%%{-2,[2]%%%}+%%%{-2,[1]%%%}+%%%{-2,[0]%%%},0,%%%{1,[4]%%%}+%%%{-2,[3]%
%%}+%%%{3,[2]%%%}+%%%{-2,[1]%%%}+%%%{1,[0]%%%}] at parameters values [2.141
18046779]Warning, choosing root of [1,0,%%%{-2,[2]%%%}+%%%{-2,[1]%%%}+%%%{-
2,[0]%%%},0,%%%{1,[4]%%%}+%%%{-2,[3]%%%}+%%%{3,[2]%%%}+%%%{-2,[1]%%%}+%%%{1
,[0]%%%}] at parameters values [9.72821606882]int()  Error: Bad Argument Va
lue
```

maple [A] time = 0.32, size = 266, normalized size = 1.80

$$b^2x - \frac{a^2\cos(dx+c)}{d} - \frac{c^2\left(2+\sin^2(dx+c)\right)\cos(dx+c)}{3d} + \frac{2ac\left(-\frac{\sin(dx+c)\cos(dx+c)}{2} + \frac{dx}{2} + \frac{c}{2}\right)}{d} + \frac{2b\left(3a\sqrt{1+\sin(dx}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)*(a+c*sin(d*x+c)+b/sin(d*x+c)^(1/2))^2,x)
```

```
[Out] b^2*x-a^2*cos(d*x+c)/d-1/3*c^2/d*(2+sin(d*x+c)^2)*cos(d*x+c)+2*a*c/d*(-1/2*
sin(d*x+c)*cos(d*x+c)+1/2*d*x+1/2*c)+2/3*b*(3*a*(1+sin(d*x+c))^(1/2)*(-2*si
n(d*x+c)+2)^(1/2)*(-sin(d*x+c))^(1/2)*EllipticF((1+sin(d*x+c))^(1/2),1/2*2^
(1/2))+(1+sin(d*x+c))^(1/2)*(-2*sin(d*x+c)+2)^(1/2)*(-sin(d*x+c))^(1/2)*Ell
ipticF((1+sin(d*x+c))^(1/2),1/2*2^(1/2))*c-6*a*(1+sin(d*x+c))^(1/2)*(-2*sin
(d*x+c)+2)^(1/2)*(-sin(d*x+c))^(1/2)*EllipticE((1+sin(d*x+c))^(1/2),1/2*2^(
1/2))-2*cos(d*x+c)^2*sin(d*x+c)*c)/cos(d*x+c)/sin(d*x+c)^(1/2)/d
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)*(a+c*sin(d*x+c)+b/sin(d*x+c)^(1/2))^2,x, algorithm="ma
xima")
```

```
[Out] Timed out
```

mupad [B] time = 6.68, size = 129, normalized size = 0.87

$$b^2\,x-\frac{a^2\,\cos\left(c+d\,x\right)}{d}-\frac{a\,c\,\left(\sin\left(2\,c+2\,d\,x\right)-2\,d\,x\right)}{2\,d}+\frac{4\,a\,b\,\mathrm{E}\left(\frac{c}{2}-\frac{\pi}{4}+\frac{d\,x}{2}\middle|2\right)}{d}+\frac{c^2\,\cos\left(c+d\,x\right)\,\left({\cos\left(c+d\,x\right)}^2-3\right)}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)*(a + c*sin(c + d*x) + b/sin(c + d*x)^(1/2))^2,x)
```

```
[Out] b^2*x - (a^2*cos(c + d*x))/d - (a*c*(sin(2*c + 2*d*x) - 2*d*x))/(2*d) + (4*
a*b*ellipticE(c/2 - pi/4 + (d*x)/2, 2))/d + (c^2*cos(c + d*x)*(cos(c + d*x)
^2 - 3))/(3*d) - (2*b*c*cos(c + d*x)*sin(c + d*x)^(5/2)*hypergeom([-1/4, 1/
2], 3/2, cos(c + d*x)^2))/(d*(sin(c + d*x)^2)^(5/4))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a+\frac{b}{\sqrt{\sin\left(c+dx\right)}}+c\sin\left(c+dx\right)\right)^2\sin\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)*(a+c*sin(d*x+c)+b/sin(d*x+c)**(1/2))**2,x)
```

```
[Out] Integral((a + b/sqrt(sin(c + d*x)) + c*sin(c + d*x))**2*sin(c + d*x), x)
```

3.940 $\int f^{a+bx}(\cos(c+dx)+i\sin(c+dx))^n\,dx$

Optimal. Leaf size=34

$$\frac{f^{a+bx}\left(e^{i(c+dx)}\right)^n}{b\log(f)+idn}$$

```
[Out] exp(I*(d*x+c))^n*f^(b*x+a)/(I*d*n+b*ln(f))
```

Rubi [A] time = 0.10, antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {4614, 2281, 2287, 2194}

$$\frac{f^{a+bx}\left(e^{i(c+dx)}\right)^n}{b\log(f)+idn}$$

Antiderivative was successfully verified.

```
[In] Int[f^(a + b*x)*(Cos[c + d*x] + I*Sin[c + d*x])^n,x]
```

```
[Out] ((E^(I*(c + d*x)))^n*f^(a + b*x))/(I*d*n + b*Log[f])
```

Rule 2194

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rule 2281

```
Int[(u_.)*((a_.)*(F_)^(v_))^(n_), x_Symbol] :> Dist[(a*F^v)^n/F^(n*v), Int[
u*F^(n*v), x], x] /; FreeQ[{F, a, n}, x] &&  !IntegerQ[n]
```

Rule 2287

```
Int[(u_.)*(F_)^(v_)*(G_)^(w_), x_Symbol] :> With[{z = v*Log[F] + w*Log[G]},
 Int[u*NormalizeIntegrand[E^z, x], x] /; BinomialQ[z, x] || (PolynomialQ[z,
 x] && LeQ[Exponent[z, x], 2])] /; FreeQ[{F, G}, x]
```

Rule 4614

```
Int[(u_.)*(Cos[v_]*(a_.) + (b_.)*Sin[v_])^(n_.), x_Symbol] :> Int[u*(a/E^((
a*v)/b))^n, x] /; FreeQ[{a, b, n}, x] && EqQ[a^2 + b^2, 0]
```

Rubi steps

$$\begin{aligned}\int f^{a+bx}(\cos(c+dx)+i\sin(c+dx))^n\,dx &= \int \left(e^{i(c+dx)}\right)^n f^{a+bx}\,dx\\ &= \left(e^{-in(c+dx)}\left(e^{i(c+dx)}\right)^n\right)\int e^{in(c+dx)}f^{a+bx}\,dx\\ &= \left(e^{-in(c+dx)}\left(e^{i(c+dx)}\right)^n\right)\int e^{icn+a\log(f)+x(idn+b\log(f))}\,dx\\ &= \frac{\left(e^{i(c+dx)}\right)^n f^{a+bx}}{idn+b\log(f)}\end{aligned}$$

Mathematica [**A**] time = 0.10, size = 43, normalized size = 1.26

$$-\frac{if^{a+bx}(\cos(c+dx)+i\sin(c+dx))^n}{dn-ib\log(f)}$$

Antiderivative was successfully verified.

```
[In] Integrate[f^(a + b*x)*(Cos[c + d*x] + I*Sin[c + d*x])^n,x]
[Out] ((-I)*f^(a + b*x)*(Cos[c + d*x] + I*Sin[c + d*x])^n)/(d*n - I*b*Log[f])
```

fricas [**A**] time = 0.63, size = 30, normalized size = 0.88

$$\frac{f^{bx+a}e^{(i\,dnx+i\,cn)}}{i\,dn+b\log(f)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(b*x+a)*(cos(d*x+c)+I*sin(d*x+c))^n,x, algorithm="fricas")
[Out] f^(b*x + a)*e^(I*d*n*x + I*c*n)/(I*d*n + b*log(f))
```

giac [**A**] time = 0.62, size = 31, normalized size = 0.91

$$\frac{f^a e^{\left(i\,dnx+bx\log(f)+i\,cn\right)}}{i\,dn+b\log(f)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(b*x+a)*(cos(d*x+c)+I*sin(d*x+c))^n,x, algorithm="giac")
[Out] f^a*e^(I*d*n*x + b*x*log(f) + I*c*n)/(I*d*n + b*log(f))
```

maple [B] time = 0.44, size = 86, normalized size = 2.53

$$\frac{\mathrm{e}^{(bx+a)\ln(f)}\mathrm{e}^{n\ln\left(\frac{2i\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{1+\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)}+\frac{1-\left(\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{1+\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)}\right)}}{idn+b\ln(f)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(f^(b*x+a)*(cos(d*x+c)+I*sin(d*x+c))^n,x)
```

```
[Out] 1/(I*d*n+b*ln(f))*exp((b*x+a)*ln(f))*exp(n*ln(2*I*tan(1/2*d*x+1/2*c)/(1+tan
(1/2*d*x+1/2*c)^2)+(1-tan(1/2*d*x+1/2*c)^2)/(1+tan(1/2*d*x+1/2*c)^2)))
```

maxima [A] time = 0.81, size = 50, normalized size = 1.47

$$\frac{-i f^{bx} f^{a} \cos(dnx+cn)+f^{bx} f^{a} \sin(dnx+cn)}{dn-i b\log(f)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(b*x+a)*(cos(d*x+c)+I*sin(d*x+c))^n,x, algorithm="maxima")
```

```
[Out] (-I*f^(b*x)*f^a*cos(d*n*x + c*n) + f^(b*x)*f^a*sin(d*n*x + c*n))/(d*n - I*b
*log(f))
```

mupad [B] time = 3.46, size = 35, normalized size = 1.03

$$-\frac{f^{a+b\,x}\,{\left(\mathrm{e}^{c\,1\mathrm{i}+d\,x\,1\mathrm{i}}\right)}^n\,1\mathrm{i}}{d\,n-b\,\ln(f)\,1\mathrm{i}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(f^(a + b*x)*(cos(c + d*x) + sin(c + d*x)*1i)^n,x)
```

```
[Out] -(f^(a + b*x)*exp(c*1i + d*x*1i)^n*1i)/(d*n - b*log(f)*1i)
```

sympy [A] time = 6.71, size = 107, normalized size = 3.15

$$\begin{cases} \frac{f^{a} f^{bx} (i\sin(c+dx)+\cos(c+dx))^{n}}{b\log(f)+idn} & \text{for}\: b\neq-\frac{idn}{\log(f)} \\ f^{a} x (i\sin(c+dx)+\cos(c+dx))^{n} e^{-idnx} - \frac{i f^{a} (i\sin(c+dx)+\cos(c+dx))^{n} e^{-idnx}}{dn} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f**(b*x+a)*(cos(d*x+c)+I*sin(d*x+c))**n,x)
```

```
[Out] Piecewise((f**a*f**(b*x)*(I*sin(c + d*x) + cos(c + d*x))**n/(b*log(f) + I*d
*n), Ne(b, -I*d*n/log(f))), (f**a*x*(I*sin(c + d*x) + cos(c + d*x))**n*exp(
-I*d*n*x) - I*f**a*(I*sin(c + d*x) + cos(c + d*x))**n*exp(-I*d*n*x)/(d*n),
True))
```

3.941 $\int f^{a+bx}(\cos(c+dx) - i\sin(c+dx))^n\,dx$

Optimal. Leaf size=36

$$-\frac{f^{a+bx}\left(e^{-i(c+dx)}\right)^n}{-b\log(f)+idn}$$

[Out] -exp(-I*(d*x+c))^n*f^(b*x+a)/(I*d*n-b*ln(f))

Rubi [A] time = 0.10, antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {4614, 2281, 2287, 2194}

$$-\frac{f^{a+bx}\left(e^{-i(c+dx)}\right)^n}{-b\log(f)+idn}$$

Antiderivative was successfully verified.

[In] Int[f^(a + b*x)*(Cos[c + d*x] - I*Sin[c + d*x])^n,x]

[Out] -(((E^((-I)*(c + d*x)))^n*f^(a + b*x))/(I*d*n - b*Log[f]))

Rule 2194

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rule 2281

```
Int[(u_.)*((a_.)*(F_)^(v_))^(n_), x_Symbol] :> Dist[(a*F^v)^n/F^(n*v), Int[
u*F^(n*v), x], x] /; FreeQ[{F, a, n}, x] &&  !IntegerQ[n]
```

Rule 2287

```
Int[(u_.)*(F_)^(v_)*(G_)^(w_), x_Symbol] :> With[{z = v*Log[F] + w*Log[G]},
 Int[u*NormalizeIntegrand[E^z, x], x] /; BinomialQ[z, x] || (PolynomialQ[z,
 x] && LeQ[Exponent[z, x], 2])] /; FreeQ[{F, G}, x]
```

Rule 4614

```
Int[(u_.)*(Cos[v_]*(a_.) + (b_.)*Sin[v_])^(n_.), x_Symbol] :> Int[u*(a/E^((
a*v)/b))^n, x] /; FreeQ[{a, b, n}, x] && EqQ[a^2 + b^2, 0]
```

Rubi steps

$$\int f^{a+bx}(\cos(c+dx)-i\sin(c+dx))^n\,dx = \int \left(e^{-i(c+dx)}\right)^n f^{a+bx}\,dx$$
$$= \left(e^{in(c+dx)}\left(e^{-i(c+dx)}\right)^n\right)\int e^{-in(c+dx)}f^{a+bx}\,dx$$
$$= \left(e^{in(c+dx)}\left(e^{-i(c+dx)}\right)^n\right)\int \exp(-icn+a\log(f)-x(idn-b\log(f)))\,dx$$
$$= -\frac{\left(e^{-i(c+dx)}\right)^n f^{a+bx}}{idn-b\log(f)}$$

Mathematica [**A**] time = 0.09, size = 43, normalized size = 1.19

$$\frac{if^{a+bx}(\cos(c+dx)-i\sin(c+dx))^n}{dn+ib\log(f)}$$

Antiderivative was successfully verified.

```
[In] Integrate[f^(a + b*x)*(Cos[c + d*x] - I*Sin[c + d*x])^n,x]
[Out] (I*f^(a + b*x)*(Cos[c + d*x] - I*Sin[c + d*x])^n)/(d*n + I*b*Log[f])
```

fricas [**A**] time = 0.93, size = 30, normalized size = 0.83

$$\frac{f^{bx+a}e^{(-i\,dnx-i\,cn)}}{-i\,dn+b\log(f)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(b*x+a)*(cos(d*x+c)-I*sin(d*x+c))^n,x, algorithm="fricas")
[Out] f^(b*x + a)*e^(-I*d*n*x - I*c*n)/(-I*d*n + b*log(f))
```

giac [**A**] time = 0.83, size = 31, normalized size = 0.86

$$\frac{f^a e^{\left(-i\,dnx+bx\log(f)-i\,cn\right)}}{-i\,dn+b\log(f)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(b*x+a)*(cos(d*x+c)-I*sin(d*x+c))^n,x, algorithm="giac")
[Out] f^a*e^(-I*d*n*x + b*x*log(f) - I*c*n)/(-I*d*n + b*log(f))
```

maple [B] time = 0.39, size = 86, normalized size = 2.39

$$\frac{\mathrm{e}^{(bx+a)\ln(f)}\mathrm{e}^{n\ln\left(\frac{1-\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)}-\frac{2i\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)}\right)}}{-idn+b\ln(f)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(f^(b*x+a)*(cos(d*x+c)-I*sin(d*x+c))^n,x)
```

```
[Out] 1/(-I*d*n+b*ln(f))*exp((b*x+a)*ln(f))*exp(n*ln((1-tan(1/2*d*x+1/2*c)^2)/(1+
tan(1/2*d*x+1/2*c)^2)-2*I*tan(1/2*d*x+1/2*c)/(1+tan(1/2*d*x+1/2*c)^2)))
```

maxima [A] time = 0.49, size = 62, normalized size = 1.72

$$\frac{f^{bx}f^{a}\cos(dnx)-i\,f^{bx}f^{a}\sin(dnx)}{\left(-i\,dn+b\log(f)\right)\cos(cn)+\left(dn+i\,b\log(f)\right)\sin(cn)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(b*x+a)*(cos(d*x+c)-I*sin(d*x+c))^n,x, algorithm="maxima")
```

```
[Out] (f^(b*x)*f^a*cos(d*n*x) - I*f^(b*x)*f^a*sin(d*n*x))/((-I*d*n + b*log(f))*co
s(c*n) + (d*n + I*b*log(f))*sin(c*n))
```

mupad [B] time = 3.35, size = 35, normalized size = 0.97

$$-\frac{f^{a+b\,x}\,\left(\mathrm{e}^{-c\,1\mathrm{i}-d\,x\,1\mathrm{i}}\right)^n}{-b\,\ln(f)+d\,n\,1\mathrm{i}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(f^(a + b*x)*(cos(c + d*x) - sin(c + d*x)*1i)^n,x)
```

```
[Out] -(f^(a + b*x)*exp(- c*1i - d*x*1i)^n)/(d*n*1i - b*log(f))
```

sympy [A] time = 6.66, size = 107, normalized size = 2.97

$$\begin{cases} -\frac{f^{a}f^{bx}(-i\sin(c+dx)+\cos(c+dx))^{n}}{-b\log(f)+idn} & \text{for}\ b\neq\frac{idn}{\log(f)} \\ f^{a}x\left(-i\sin(c+dx)+\cos(c+dx)\right)^{n}e^{idnx}+\frac{if^{a}(-i\sin(c+dx)+\cos(c+dx))^{n}e^{idnx}}{dn} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f**(b*x+a)*(cos(d*x+c)-I*sin(d*x+c))**n,x)
```

```
[Out] Piecewise((-f**a*f**(b*x)*(-I*sin(c + d*x) + cos(c + d*x))**n/(-b*log(f) +
I*d*n), Ne(b, I*d*n/log(f))), (f**a*x*(-I*sin(c + d*x) + cos(c + d*x))**n*e
xp(I*d*n*x) + I*f**a*(-I*sin(c + d*x) + cos(c + d*x))**n*exp(I*d*n*x)/(d*n)
, True))
```

3.942 $\int \frac{\cos^5(a+bx)-\sin^5(a+bx)}{\cos^5(a+bx)+\sin^5(a+bx)} dx$

Optimal. Leaf size=120

$$-\frac{4\log\left(2\tan^2(a+bx)-\left(1-\sqrt{5}\right)\tan(a+bx)+2\right)}{5\left(1-\sqrt{5}\right)b}-\frac{4\log\left(2\tan^2(a+bx)-\left(1+\sqrt{5}\right)\tan(a+bx)+2\right)}{5\left(1+\sqrt{5}\right)b}+\frac{\log(\text{ta}}{}$$

```
[Out] ln(cos(b*x+a))/b+1/5*ln(1+tan(b*x+a))/b-4/5*ln(2-(-5^(1/2)+1)*tan(b*x+a)+2*
tan(b*x+a)^2)/b/(-5^(1/2)+1)-4/5*ln(2-(5^(1/2)+1)*tan(b*x+a)+2*tan(b*x+a)^2
)/b/(5^(1/2)+1)
```

Rubi [A] time = 0.70, antiderivative size = 120, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 39, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {2074, 260, 2086, 628}

$$-\frac{4\log\left(2\tan^2(a+bx)-\left(1-\sqrt{5}\right)\tan(a+bx)+2\right)}{5\left(1-\sqrt{5}\right)b}-\frac{4\log\left(2\tan^2(a+bx)-\left(1+\sqrt{5}\right)\tan(a+bx)+2\right)}{5\left(1+\sqrt{5}\right)b}+\frac{\log(\text{ta}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[a + b*x]^5 - Sin[a + b*x]^5)/(Cos[a + b*x]^5 + Sin[a + b*x]^5),x]
```

```
[Out] Log[Cos[a + b*x]]/b + Log[1 + Tan[a + b*x]]/(5*b) - (4*Log[2 - (1 - Sqrt[5]
)*Tan[a + b*x] + 2*Tan[a + b*x]^2])/(5*(1 - Sqrt[5])*b) - (4*Log[2 - (1 + S
qrt[5])*Tan[a + b*x] + 2*Tan[a + b*x]^2])/(5*(1 + Sqrt[5])*b)
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 2074

```
Int[(P_)^(p_)*(Q_)^(q_.), x_Symbol] :> With[{PP = Factor[P]}, Int[ExpandInt
egrand[PP^p*Q^q, x], x] /;  !SumQ[NonfreeFactors[PP, x]]] /; FreeQ[q, x] &&
 PolyQ[P, x] && PolyQ[Q, x] && IntegerQ[p] && NeQ[P, x]
```

Rule 2086

```
Int[(P3_)/((a_) + (b_.)*(x_) + (c_.)*(x_)^2 + (d_.)*(x_)^3 + (e_.)*(x_)^4),
 x_Symbol] :> With[{q = Sqrt[8*a^2 + b^2 - 4*a*c], A = Coeff[P3, x, 0], B =
 Coeff[P3, x, 1], C = Coeff[P3, x, 2], D = Coeff[P3, x, 3]}, Dist[1/q, Int[
(b*A - 2*a*B + 2*a*D + A*q + (2*a*A - 2*a*C + b*D + D*q)*x)/(2*a + (b + q)*
x + 2*a*x^2), x], x] - Dist[1/q, Int[(b*A - 2*a*B + 2*a*D - A*q + (2*a*A -
2*a*C + b*D - D*q)*x)/(2*a + (b - q)*x + 2*a*x^2), x], x]] /; FreeQ[{a, b,
c}, x] && PolyQ[P3, x, 3] && EqQ[a, e] && EqQ[b, d]
```

Rubi steps

$$\int \frac{\cos^5(a+bx)-\sin^5(a+bx)}{\cos^5(a+bx)+\sin^5(a+bx)}\,dx = \frac{\text{Subst}\left(\int \frac{1-x^5}{1+x^2+x^5+x^7}\,dx, x, \tan(a+bx)\right)}{b}$$

$$= \frac{\text{Subst}\left(\int \left(\frac{1}{5(1+x)}-\frac{x}{1+x^2}+\frac{2\left(2+x-4x^2+2x^3\right)}{5\left(1-x+x^2-x^3+x^4\right)}\right)dx, x, \tan(a+bx)\right)}{b}$$

$$= \frac{\log(1+\tan(a+bx))}{5b}+\frac{2\,\text{Subst}\left(\int \frac{2+x-4x^2+2x^3}{1-x+x^2-x^3+x^4}\,dx, x, \tan(a+bx)\right)}{5b}-\frac{\text{Subst}}{}$$

$$= \frac{\log(\cos(a+bx))}{b}+\frac{\log(1+\tan(a+bx))}{5b}-\frac{2\,\text{Subst}\left(\int \frac{-2\sqrt{5}+\left(10-2\sqrt{5}\right)x}{2+\left(-1-\sqrt{5}\right)x+2x^2}\,dx, x\right.}{5\sqrt{5}\,b}$$

$$= \frac{\log(\cos(a+bx))}{b}+\frac{\log(1+\tan(a+bx))}{5b}-\frac{4\log\left(2-\left(1-\sqrt{5}\right)\tan(a+bx)\right)}{5\left(1-\sqrt{5}\right)b}$$

Mathematica [A] time = 0.57, size = 73, normalized size = 0.61

$$\frac{-\left(\sqrt{5}-1\right)\log\left(\sin(2(a+bx))-\sqrt{5}+1\right)+\left(1+\sqrt{5}\right)\log\left(\sin(2(a+bx))+\sqrt{5}+1\right)+\log(\sin(a+bx)+\cos(a}{5b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[a + b*x]^5 - Sin[a + b*x]^5)/(Cos[a + b*x]^5 + Sin[a + b*x]^
5),x]
```

```
[Out] (Log[Cos[a + b*x] + Sin[a + b*x]] - (-1 + Sqrt[5])*Log[1 - Sqrt[5] + Sin[2*
(a + b*x)]] + (1 + Sqrt[5])*Log[1 + Sqrt[5] + Sin[2*(a + b*x)]])/(5*b)
```

fricas [A] time = 0.89, size = 150, normalized size = 1.25

$$\frac{2\sqrt{5}\log\left(-\frac{2\cos(bx+a)^4-2\left(\sqrt{5}+1\right)\cos(bx+a)\sin(bx+a)-2\cos(bx+a)^2-\sqrt{5}-3}{\cos(bx+a)^4-\cos(bx+a)^2-\cos(bx+a)\sin(bx+a)+1}\right)+2\log\left(\cos(bx+a)^4-\cos(bx+a)^2-\cos(b\right.}{10\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((cos(b*x+a)^5-sin(b*x+a)^5)/(cos(b*x+a)^5+sin(b*x+a)^5),x, algori
thm="fricas")
```

```
[Out] 1/10*(2*sqrt(5)*log(-(2*cos(b*x + a)^4 - 2*(sqrt(5) + 1)*cos(b*x + a)*sin(b
*x + a) - 2*cos(b*x + a)^2 - sqrt(5) - 3)/(cos(b*x + a)^4 - cos(b*x + a)^2
- cos(b*x + a)*sin(b*x + a) + 1)) + 2*log(cos(b*x + a)^4 - cos(b*x + a)^2 -
 cos(b*x + a)*sin(b*x + a) + 1) + log(2*cos(b*x + a)*sin(b*x + a) + 1))/b
```

giac [A] time = 0.53, size = 128, normalized size = 1.07

$$-\frac{2\sqrt{5}\log\left(-\frac{1}{2}\left(\sqrt{5}+1\right)\tan(bx+a)+\tan(bx+a)^2+1\right)-2\sqrt{5}\log\left(\frac{1}{2}\left(\sqrt{5}-1\right)\tan(bx+a)+\tan(bx+a)^2\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((cos(b*x+a)^5-sin(b*x+a)^5)/(cos(b*x+a)^5+sin(b*x+a)^5),x, algori
thm="giac")
```

```
[Out] -1/10*(2*sqrt(5)*log(-1/2*(sqrt(5) + 1)*tan(b*x + a) + tan(b*x + a)^2 + 1)
- 2*sqrt(5)*log(1/2*(sqrt(5) - 1)*tan(b*x + a) + tan(b*x + a)^2 + 1) - 2*lo
g(tan(b*x + a)^4 - tan(b*x + a)^3 + tan(b*x + a)^2 - tan(b*x + a) + 1) + 5*
log(tan(b*x + a)^2 + 1) - 2*log(abs(tan(b*x + a) + 1)))/b
```

maple [A] time = 0.74, size = 184, normalized size = 1.53

$$\frac{\ln\left(\tan(bx+a)\sqrt{5}+2\left(\tan^2(bx+a)\right)-\tan(bx+a)+2\right)\sqrt{5}}{5b}+\frac{\ln\left(\tan(bx+a)\sqrt{5}+2\left(\tan^2(bx+a)\right)-\tan(\right.}{5b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(b*x+a)^5-sin(b*x+a)^5)/(cos(b*x+a)^5+sin(b*x+a)^5),x)
```

```
[Out] 1/5/b*ln(tan(b*x+a)*5^(1/2)+2*tan(b*x+a)^2-tan(b*x+a)+2)*5^(1/2)+1/5/b*ln(t
an(b*x+a)*5^(1/2)+2*tan(b*x+a)^2-tan(b*x+a)+2)-1/5/b*ln(-tan(b*x+a)*5^(1/2)
+2*tan(b*x+a)^2-tan(b*x+a)+2)*5^(1/2)+1/5/b*ln(-tan(b*x+a)*5^(1/2)+2*tan(b*
x+a)^2-tan(b*x+a)+2)-1/2/b*ln(1+tan(b*x+a)^2)+1/5*ln(1+tan(b*x+a))/b
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos(bx+a)^5 - \sin(bx+a)^5}{\cos(bx+a)^5 + \sin(bx+a)^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((cos(b*x+a)^5-sin(b*x+a)^5)/(cos(b*x+a)^5+sin(b*x+a)^5),x, algori
thm="maxima")
```

```
[Out] integrate((cos(b*x + a)^5 - sin(b*x + a)^5)/(cos(b*x + a)^5 + sin(b*x + a)^
5), x)
```

mupad [B] time = 4.22, size = 226, normalized size = 1.88

$$\frac{\ln\left(\tan\left(\frac{a}{2}+\frac{b\,x}{2}\right)^2-2\tan\left(\frac{a}{2}+\frac{b\,x}{2}\right)-1\right)}{5\,b}-\frac{\ln\left(\tan\left(\frac{a}{2}+\frac{b\,x}{2}\right)^2+1\right)}{b}+\frac{\ln\left(2\tan\left(\frac{a}{2}+\frac{b\,x}{2}\right)^2-\tan\left(\frac{a}{2}+\frac{b\,x}{2}\right)+\tan\left(\frac{a}{2}+\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(a + b*x)^5 - sin(a + b*x)^5)/(cos(a + b*x)^5 + sin(a + b*x)^5),x)
```

```
[Out] log(tan(a/2 + (b*x)/2)^2 - 2*tan(a/2 + (b*x)/2) - 1)/(5*b) - log(tan(a/2 +
(b*x)/2)^2 + 1)/b + (log(2*tan(a/2 + (b*x)/2)^2 - tan(a/2 + (b*x)/2) + tan(
a/2 + (b*x)/2)^3 + tan(a/2 + (b*x)/2)^4 + 5^(1/2)*tan(a/2 + (b*x)/2) - 5^(1
/2)*tan(a/2 + (b*x)/2)^3 + 1)*(5^(1/2) + 1))/(5*b) - (log(2*tan(a/2 + (b*x)
/2)^2 - tan(a/2 + (b*x)/2) + tan(a/2 + (b*x)/2)^3 + tan(a/2 + (b*x)/2)^4 -
5^(1/2)*tan(a/2 + (b*x)/2) + 5^(1/2)*tan(a/2 + (b*x)/2)^3 + 1)*(5^(1/2) - 1
))/(5*b)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((cos(b*x+a)**5-sin(b*x+a)**5)/(cos(b*x+a)**5+sin(b*x+a)**5),x)
```

```
[Out] Timed out
```

3.943 $\int \frac{\cos^4(a+bx)-\sin^4(a+bx)}{\cos^4(a+bx)+\sin^4(a+bx)}\,dx$

Optimal. Leaf size=72

$$\frac{\log\left(\tan^2(a+bx)+\sqrt{2}\tan(a+bx)+1\right)}{2\sqrt{2}\,b}-\frac{\log\left(\tan^2(a+bx)-\sqrt{2}\tan(a+bx)+1\right)}{2\sqrt{2}\,b}$$

```
[Out] -1/4*ln(1-2^(1/2)*tan(b*x+a)+tan(b*x+a)^2)/b*2^(1/2)+1/4*ln(1+2^(1/2)*tan(b
*x+a)+tan(b*x+a)^2)/b*2^(1/2)
```

Rubi [A] time = 0.15, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 39, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.051, Rules used = {1165, 628}

$$\frac{\log\left(\tan^2(a+bx)+\sqrt{2}\tan(a+bx)+1\right)}{2\sqrt{2}\,b}-\frac{\log\left(\tan^2(a+bx)-\sqrt{2}\tan(a+bx)+1\right)}{2\sqrt{2}\,b}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[a + b*x]^4 - Sin[a + b*x]^4)/(Cos[a + b*x]^4 + Sin[a + b*x]^4),x]
```

```
[Out] -Log[1 - Sqrt[2]*Tan[a + b*x] + Tan[a + b*x]^2]/(2*Sqrt[2]*b) + Log[1 + Sqr
t[2]*Tan[a + b*x] + Tan[a + b*x]^2]/(2*Sqrt[2]*b)
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{\cos^4(a+bx)-\sin^4(a+bx)}{\cos^4(a+bx)+\sin^4(a+bx)}\,dx = \frac{\text{Subst}\left(\int \frac{1-x^2}{1+x^4}\,dx, x, \tan(a+bx)\right)}{b}$$

$$= -\frac{\text{Subst}\left(\int \frac{\sqrt{2}+2x}{-1-\sqrt{2}x-x^2}\,dx, x, \tan(a+bx)\right)}{2\sqrt{2}b} - \frac{\text{Subst}\left(\int \frac{\sqrt{2}-2x}{-1+\sqrt{2}x-x^2}\,dx, x, \tan(a$$

$$= -\frac{\log\left(1-\sqrt{2}\tan(a+bx)+\tan^2(a+bx)\right)}{2\sqrt{2}b} + \frac{\log\left(1+\sqrt{2}\tan(a+bx)+\tan\right.}{2\sqrt{2}b}$$

Mathematica [A] time = 0.03, size = 25, normalized size = 0.35

$$\frac{\tanh^{-1}\left(\frac{\sin(2a+2bx)}{\sqrt{2}}\right)}{\sqrt{2}b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[a + b*x]^4 - Sin[a + b*x]^4)/(Cos[a + b*x]^4 + Sin[a + b*x]^
4),x]
```

```
[Out] ArcTanh[Sin[2*a + 2*b*x]/Sqrt[2]]/(Sqrt[2]*b)
```

fricas [A] time = 0.97, size = 74, normalized size = 1.03

$$\frac{\sqrt{2}\log\left(-\frac{2\cos(bx+a)^4-2\sqrt{2}\cos(bx+a)\sin(bx+a)-2\cos(bx+a)^2-1}{2\cos(bx+a)^4-2\cos(bx+a)^2+1}\right)}{4b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((cos(b*x+a)^4-sin(b*x+a)^4)/(cos(b*x+a)^4+sin(b*x+a)^4),x, algori
thm="fricas")
```

```
[Out] 1/4*sqrt(2)*log(-(2*cos(b*x + a)^4 - 2*sqrt(2)*cos(b*x + a)*sin(b*x + a) -
2*cos(b*x + a)^2 - 1)/(2*cos(b*x + a)^4 - 2*cos(b*x + a)^2 + 1))/b
```

giac [A] time = 0.26, size = 48, normalized size = 0.67

$$-\frac{\sqrt{2}\log\left(\frac{\left|-2\sqrt{2}+2\sin(2bx+2a)\right|}{\left|2\sqrt{2}+2\sin(2bx+2a)\right|}\right)}{4b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((cos(b*x+a)^4-sin(b*x+a)^4)/(cos(b*x+a)^4+sin(b*x+a)^4),x, algori
thm="giac")
```

```
[Out] -1/4*sqrt(2)*log(abs(-2*sqrt(2) + 2*sin(2*b*x + 2*a))/abs(2*sqrt(2) + 2*sin
(2*b*x + 2*a)))/b
```

maple [A] time = 0.38, size = 108, normalized size = 1.50

$$\frac{\sqrt{2}\,\ln\left(\frac{1+\sqrt{2}\,\tan(bx+a)+\tan^2(bx+a)}{1-\sqrt{2}\,\tan(bx+a)+\tan^2(bx+a)}\right)}{8b}-\frac{\sqrt{2}\,\ln\left(\frac{1-\sqrt{2}\,\tan(bx+a)+\tan^2(bx+a)}{1+\sqrt{2}\,\tan(bx+a)+\tan^2(bx+a)}\right)}{8b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(b*x+a)^4-sin(b*x+a)^4)/(cos(b*x+a)^4+sin(b*x+a)^4),x)
```

```
[Out] 1/8/b*2^(1/2)*ln((1+2^(1/2)*tan(b*x+a)+tan(b*x+a)^2)/(1-2^(1/2)*tan(b*x+a)+
tan(b*x+a)^2))-1/8/b*2^(1/2)*ln((1-2^(1/2)*tan(b*x+a)+tan(b*x+a)^2)/(1+2^(1
/2)*tan(b*x+a)+tan(b*x+a)^2))
```

maxima [A] time = 0.46, size = 58, normalized size = 0.81

$$\frac{\sqrt{2}\log\left(\tan\left(bx+a\right)^2+\sqrt{2}\tan\left(bx+a\right)+1\right)-\sqrt{2}\log\left(\tan\left(bx+a\right)^2-\sqrt{2}\tan\left(bx+a\right)+1\right)}{4\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((cos(b*x+a)^4-sin(b*x+a)^4)/(cos(b*x+a)^4+sin(b*x+a)^4),x, algori
thm="maxima")
```

```
[Out] 1/4*(sqrt(2)*log(tan(b*x + a)^2 + sqrt(2)*tan(b*x + a) + 1) - sqrt(2)*log(t
an(b*x + a)^2 - sqrt(2)*tan(b*x + a) + 1))/b
```

mupad [B] time = 3.17, size = 23, normalized size = 0.32

$$\frac{\sqrt{2}\,\mathrm{atanh}\left(\frac{\sqrt{2}\,\sin(2\,a+2\,b\,x)}{2}\right)}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(a + b*x)^4 - sin(a + b*x)^4)/(cos(a + b*x)^4 + sin(a + b*x)^4),x)
```

```
[Out] (2^(1/2)*atanh((2^(1/2)*sin(2*a + 2*b*x))/2))/(2*b)
```

sympy [A] time = 5.59, size = 122, normalized size = 1.69

$$\begin{cases} -\frac{\sqrt{2}\log\left(4\sin^2(a+bx)-4\sqrt{2}\sin(a+bx)\cos(a+bx)+4\cos^2(a+bx)\right)}{4b}+\frac{\sqrt{2}\log\left(4\sin^2(a+bx)+4\sqrt{2}\sin(a+bx)\cos(a+bx)+4\cos^2(a+bx)\right)}{4b} & \text{for} \\ \frac{x\left(-\sin^4(a)+\cos^4(a)\right)}{\sin^4(a)+\cos^4(a)} & \text{oth} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((cos(b*x+a)**4-sin(b*x+a)**4)/(cos(b*x+a)**4+sin(b*x+a)**4),x)
```

```
[Out] Piecewise((-sqrt(2)*log(4*sin(a + b*x)**2 - 4*sqrt(2)*sin(a + b*x)*cos(a +
b*x) + 4*cos(a + b*x)**2)/(4*b) + sqrt(2)*log(4*sin(a + b*x)**2 + 4*sqrt(2)
*sin(a + b*x)*cos(a + b*x) + 4*cos(a + b*x)**2)/(4*b), Ne(b, 0)), (x*(-sin(
a)**4 + cos(a)**4)/(sin(a)**4 + cos(a)**4), True))
```

3.944 $\int \frac{\cos^3(a+bx)-\sin^3(a+bx)}{\cos^3(a+bx)+\sin^3(a+bx)} \, dx$

Optimal. Leaf size=55

$$-\frac{2\log\left(\tan^2(a+bx)-\tan(a+bx)+1\right)}{3b}+\frac{\log(\tan(a+bx)+1)}{3b}-\frac{\log(\cos(a+bx))}{b}$$

```
[Out] -ln(cos(b*x+a))/b+1/3*ln(1+tan(b*x+a))/b-2/3*ln(1-tan(b*x+a)+tan(b*x+a)^2)/
b
```

Rubi [A] time = 0.41, antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 39, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {2074, 260, 628}

$$-\frac{2\log\left(\tan^2(a+bx)-\tan(a+bx)+1\right)}{3b}+\frac{\log(\tan(a+bx)+1)}{3b}-\frac{\log(\cos(a+bx))}{b}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[a + b*x]^3 - Sin[a + b*x]^3)/(Cos[a + b*x]^3 + Sin[a + b*x]^3),x]
```

```
[Out] -(Log[Cos[a + b*x]]/b) + Log[1 + Tan[a + b*x]]/(3*b) - (2*Log[1 - Tan[a + b
*x] + Tan[a + b*x]^2])/(3*b)
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 2074

```
Int[(P_)^(p_)*(Q_)^(q_.), x_Symbol] :> With[{PP = Factor[P]}, Int[ExpandInt
egrand[PP^p*Q^q, x], x] /;  !SumQ[NonfreeFactors[PP, x]]] /; FreeQ[q, x] &&
 PolyQ[P, x] && PolyQ[Q, x] && IntegerQ[p] && NeQ[P, x]
```

Rubi steps

$$\int \frac{\cos^3(a+bx)-\sin^3(a+bx)}{\cos^3(a+bx)+\sin^3(a+bx)}\,dx = \frac{\text{Subst}\left(\int \frac{1-x^3}{1+x^2+x^3+x^5}\,dx, x, \tan(a+bx)\right)}{b}$$

$$= \frac{\text{Subst}\left(\int \left(\frac{1}{3(1+x)}+\frac{x}{1+x^2}-\frac{2(-1+2x)}{3(1-x+x^2)}\right)dx, x, \tan(a+bx)\right)}{b}$$

$$= \frac{\log(1+\tan(a+bx))}{3b} - \frac{2\,\text{Subst}\left(\int \frac{-1+2x}{1-x+x^2}\,dx, x, \tan(a+bx)\right)}{3b} + \frac{\text{Subst}\left(\int \frac{x}{1+}\right.}{}$$

$$= -\frac{\log(\cos(a+bx))}{b} + \frac{\log(1+\tan(a+bx))}{3b} - \frac{2\log\left(1-\tan(a+bx)+\tan^2(\right.}{3b}$$

Mathematica [A] time = 0.20, size = 42, normalized size = 0.76

$$\frac{\log(\sin(a+bx)+\cos(a+bx))}{3b} - \frac{2\log(2-\sin(2(a+bx)))}{3b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[a + b*x]^3 - Sin[a + b*x]^3)/(Cos[a + b*x]^3 + Sin[a + b*x]^
3),x]
```

```
[Out] Log[Cos[a + b*x] + Sin[a + b*x]]/(3*b) - (2*Log[2 - Sin[2*(a + b*x)]])/(3*b
)
```

fricas [A] time = 0.90, size = 42, normalized size = 0.76

$$\frac{\log\left(2\cos\left(bx+a\right)\sin\left(bx+a\right)+1\right)-4\log\left(-\cos\left(bx+a\right)\sin\left(bx+a\right)+1\right)}{6\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((cos(b*x+a)^3-sin(b*x+a)^3)/(cos(b*x+a)^3+sin(b*x+a)^3),x, algori
thm="fricas")
```

```
[Out] 1/6*(log(2*cos(b*x + a)*sin(b*x + a) + 1) - 4*log(-cos(b*x + a)*sin(b*x + a
) + 1))/b
```

giac [A] time = 0.29, size = 52, normalized size = 0.95

$$-\frac{4\log\left(\tan\left(bx+a\right)^2-\tan\left(bx+a\right)+1\right)-3\log\left(\tan\left(bx+a\right)^2+1\right)-2\log\left(\left|\tan\left(bx+a\right)+1\right|\right)}{6\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((cos(b*x+a)^3-sin(b*x+a)^3)/(cos(b*x+a)^3+sin(b*x+a)^3),x, algori
thm="giac")
```

```
[Out] -1/6*(4*log(tan(b*x + a)^2 - tan(b*x + a) + 1) - 3*log(tan(b*x + a)^2 + 1)
- 2*log(abs(tan(b*x + a) + 1)))/b
```

maple [A] time = 0.65, size = 56, normalized size = 1.02

$$-\frac{2\ln\left(1-\tan\left(bx+a\right)+\tan^{2}\left(bx+a\right)\right)}{3b}+\frac{\ln\left(1+\tan^{2}\left(bx+a\right)\right)}{2b}+\frac{\ln\left(1+\tan\left(bx+a\right)\right)}{3b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(b*x+a)^3-sin(b*x+a)^3)/(cos(b*x+a)^3+sin(b*x+a)^3),x)
```

```
[Out] -2/3*ln(1-tan(b*x+a)+tan(b*x+a)^2)/b+1/2/b*ln(1+tan(b*x+a)^2)+1/3*ln(1+tan(
b*x+a))/b
```

maxima [B] time = 0.43, size = 154, normalized size = 2.80

$$-\frac{2\,\log\left(-\frac{2\,\sin(bx+a)}{\cos(bx+a)+1}+\frac{2\,\sin(bx+a)^{2}}{(\cos(bx+a)+1)^{2}}+\frac{2\,\sin(bx+a)^{3}}{(\cos(bx+a)+1)^{3}}+\frac{\sin(bx+a)^{4}}{(\cos(bx+a)+1)^{4}}+1\right)-\log\left(-\frac{2\,\sin(bx+a)}{\cos(bx+a)+1}+\frac{\sin(bx+a)^{2}}{(\cos(bx+a)+1)^{2}}-1\right)-}{3\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((cos(b*x+a)^3-sin(b*x+a)^3)/(cos(b*x+a)^3+sin(b*x+a)^3),x, algori
thm="maxima")
```

```
[Out] -1/3*(2*log(-2*sin(b*x + a)/(cos(b*x + a) + 1) + 2*sin(b*x + a)^2/(cos(b*x
+ a) + 1)^2 + 2*sin(b*x + a)^3/(cos(b*x + a) + 1)^3 + sin(b*x + a)^4/(cos(b
*x + a) + 1)^4 + 1) - log(-2*sin(b*x + a)/(cos(b*x + a) + 1) + sin(b*x + a)
^2/(cos(b*x + a) + 1)^2 - 1) - 3*log(sin(b*x + a)^2/(cos(b*x + a) + 1)^2 +
1))/b
```

mupad [B] time = 3.30, size = 105, normalized size = 1.91

$$\frac{\ln\left(\tan\left(\frac{a}{2}+\frac{b\,x}{2}\right)^{2}+1\right)}{b}+\frac{\ln\left(\tan\left(\frac{a}{2}+\frac{b\,x}{2}\right)^{2}-2\tan\left(\frac{a}{2}+\frac{b\,x}{2}\right)-1\right)}{3\,b}-\frac{2\,\ln\left(\tan\left(\frac{a}{2}+\frac{b\,x}{2}\right)^{4}+2\tan\left(\frac{a}{2}+\frac{b\,x}{2}\right)^{3}+2\tan\right.}{3\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(a + b*x)^3 - sin(a + b*x)^3)/(cos(a + b*x)^3 + sin(a + b*x)^3),x)
```

```
[Out] log(tan(a/2 + (b*x)/2)^2 + 1)/b + log(tan(a/2 + (b*x)/2)^2 - 2*tan(a/2 + (b
*x)/2) - 1)/(3*b) - (2*log(2*tan(a/2 + (b*x)/2)^2 - 2*tan(a/2 + (b*x)/2) +
2*tan(a/2 + (b*x)/2)^3 + tan(a/2 + (b*x)/2)^4 + 1))/(3*b)
```

sympy [A] time = 1.02, size = 76, normalized size = 1.38

$$\begin{cases} \frac{\log(\sin(a+bx)+\cos(a+bx))}{3b} - \frac{2\log\left(\sin^2(a+bx)-\sin(a+bx)\cos(a+bx)+\cos^2(a+bx)\right)}{3b} & \text{for } b \neq 0 \\ \frac{x\left(-\sin^3(a)+\cos^3(a)\right)}{\sin^3(a)+\cos^3(a)} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((cos(b*x+a)**3-sin(b*x+a)**3)/(cos(b*x+a)**3+sin(b*x+a)**3),x)
```

```
[Out] Piecewise((log(sin(a + b*x) + cos(a + b*x))/(3*b) - 2*log(sin(a + b*x)**2 -
 sin(a + b*x)*cos(a + b*x) + cos(a + b*x)**2)/(3*b), Ne(b, 0)), (x*(-sin(a)
**3 + cos(a)**3)/(sin(a)**3 + cos(a)**3), True))
```

3.945 $\int \frac{\cos^2(a+bx)-\sin^2(a+bx)}{\cos^2(a+bx)+\sin^2(a+bx)}\,dx$

Optimal. Leaf size=16

$$\frac{\sin(a+bx)\cos(a+bx)}{b}$$

[Out] cos(b*x+a)*sin(b*x+a)/b

Rubi [A] time = 0.05, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, integrand size = 39, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {4380, 2635, 8}

$$\frac{\sin(a+bx)\cos(a+bx)}{b}$$

Antiderivative was successfully verified.

[In] Int[(Cos[a + b*x]^2 - Sin[a + b*x]^2)/(Cos[a + b*x]^2 + Sin[a + b*x]^2),x]

[Out] (Cos[a + b*x]*Sin[a + b*x])/b

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 4380

```
Int[(u_.)*((a_.) + cos[(d_.) + (e_.)*(x_)]^2*(b_.) + (c_.)*sin[(d_.) + (e_.
)*(x_)]^2)^(p_.), x_Symbol] :> Dist[(a + c)^p, Int[ActivateTrig[u], x], x]
/; FreeQ[{a, b, c, d, e, p}, x] && EqQ[b - c, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\cos^2(a+bx)-\sin^2(a+bx)}{\cos^2(a+bx)+\sin^2(a+bx)}\,dx &= \int \left(\cos^2(a+bx)-\sin^2(a+bx)\right)\,dx \\
&= \int \cos^2(a+bx)\,dx - \int \sin^2(a+bx)\,dx \\
&= \frac{\cos(a+bx)\sin(a+bx)}{b}
\end{aligned}
$$

Mathematica [B] time = 0.01, size = 33, normalized size = 2.06

$$
\frac{\sin(2a)\cos(2bx)}{2b}+\frac{\cos(2a)\sin(2bx)}{2b}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[a + b*x]^2 - Sin[a + b*x]^2)/(Cos[a + b*x]^2 + Sin[a + b*x]^
2),x]
```

```
[Out] (Cos[2*b*x]*Sin[2*a])/(2*b) + (Cos[2*a]*Sin[2*b*x])/(2*b)
```

fricas [A] time = 0.55, size = 16, normalized size = 1.00

$$
\frac{\cos\left(bx+a\right)\sin\left(bx+a\right)}{b}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((cos(b*x+a)^2-sin(b*x+a)^2)/(cos(b*x+a)^2+sin(b*x+a)^2),x, algori
thm="fricas")
```

```
[Out] cos(b*x + a)*sin(b*x + a)/b
```

giac [A] time = 0.18, size = 14, normalized size = 0.88

$$
\frac{\sin\left(2\,bx+2\,a\right)}{2\,b}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((cos(b*x+a)^2-sin(b*x+a)^2)/(cos(b*x+a)^2+sin(b*x+a)^2),x, algori
thm="giac")
```

```
[Out] 1/2*sin(2*b*x + 2*a)/b
```

maple [A] time = 0.23, size = 17, normalized size = 1.06

$$
\frac{\cos\left(bx+a\right)\sin\left(bx+a\right)}{b}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(b*x+a)^2-sin(b*x+a)^2)/(cos(b*x+a)^2+sin(b*x+a)^2),x)
```

```
[Out] cos(b*x+a)*sin(b*x+a)/b
```

maxima [A] time = 0.33, size = 22, normalized size = 1.38

$$\frac{\tan\left(bx+a\right)}{\left(\tan\left(bx+a\right)^{2}+1\right)b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((cos(b*x+a)^2-sin(b*x+a)^2)/(cos(b*x+a)^2+sin(b*x+a)^2),x, algori
thm="maxima")
```

```
[Out] tan(b*x + a)/((tan(b*x + a)^2 + 1)*b)
```

mupad [B] time = 3.03, size = 14, normalized size = 0.88

$$\frac{\sin\left(2\,a+2\,b\,x\right)}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(a + b*x)^2 - sin(a + b*x)^2)/(cos(a + b*x)^2 + sin(a + b*x)^2),x)
```

```
[Out] sin(2*a + 2*b*x)/(2*b)
```

sympy [B] time = 0.25, size = 32, normalized size = 2.00

$$\frac{\sin\left(a+bx\right)\cos\left(a+bx\right)}{b\sin^{2}\left(a+bx\right)+b\cos^{2}\left(a+bx\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((cos(b*x+a)**2-sin(b*x+a)**2)/(cos(b*x+a)**2+sin(b*x+a)**2),x)
```

```
[Out] sin(a + b*x)*cos(a + b*x)/(b*sin(a + b*x)**2 + b*cos(a + b*x)**2)
```

3.946 $\int \frac{\cos(a+bx)-\sin(a+bx)}{\cos(a+bx)+\sin(a+bx)} \, dx$

Optimal. Leaf size=18

$$\frac{\log(\sin(a+bx)+\cos(a+bx))}{b}$$

[Out] ln(cos(b*x+a)+sin(b*x+a))/b

Rubi [A] time = 0.03, antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.032, Rules used = {3133}

$$\frac{\log(\sin(a+bx)+\cos(a+bx))}{b}$$

Antiderivative was successfully verified.

[In] Int[(Cos[a + b*x] - Sin[a + b*x])/(Cos[a + b*x] + Sin[a + b*x]),x]

[Out] Log[Cos[a + b*x] + Sin[a + b*x]]/b

Rule 3133

```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])
/((a_.) + cos[(d_.) + (e_.)*(x_)]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)]), x
_Symbol] :> Simp[((b*B + c*C)*x)/(b^2 + c^2), x] + Simp[((c*B - b*C)*Log[a
+ b*Cos[d + e*x] + c*Sin[d + e*x]])/(e*(b^2 + c^2)), x] /; FreeQ[{a, b, c,
d, e, A, B, C}, x] && NeQ[b^2 + c^2, 0] && EqQ[A*(b^2 + c^2) - a*(b*B + c*C
), 0]
```

Rubi steps

$$\int \frac{\cos(a+bx)-\sin(a+bx)}{\cos(a+bx)+\sin(a+bx)} \, dx = \frac{\log(\cos(a+bx)+\sin(a+bx))}{b}$$

Mathematica [A] time = 0.04, size = 18, normalized size = 1.00

$$\frac{\log(\sin(a+bx)+\cos(a+bx))}{b}$$

Antiderivative was successfully verified.

[In] Integrate[(Cos[a + b*x] - Sin[a + b*x])/(Cos[a + b*x] + Sin[a + b*x]),x]

```
[Out] Log[Cos[a + b*x] + Sin[a + b*x]]/b
```

fricas [A] time = 1.16, size = 22, normalized size = 1.22

$$\frac{\log\left(2\,\cos\left(bx+a\right)\sin\left(bx+a\right)+1\right)}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((cos(b*x+a)-sin(b*x+a))/(cos(b*x+a)+sin(b*x+a)),x, algorithm="fri
cas")
```

```
[Out] 1/2*log(2*cos(b*x + a)*sin(b*x + a) + 1)/b
```

giac [A] time = 0.17, size = 29, normalized size = 1.61

$$-\frac{\log\left(\tan\left(bx+a\right)^{2}+1\right)-2\,\log\left(\left|\tan\left(bx+a\right)+1\right|\right)}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((cos(b*x+a)-sin(b*x+a))/(cos(b*x+a)+sin(b*x+a)),x, algorithm="gia
c")
```

```
[Out] -1/2*(log(tan(b*x + a)^2 + 1) - 2*log(abs(tan(b*x + a) + 1)))/b
```

maple [A] time = 0.25, size = 19, normalized size = 1.06

$$\frac{\ln\left(\cos\left(bx+a\right)+\sin\left(bx+a\right)\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(b*x+a)-sin(b*x+a))/(cos(b*x+a)+sin(b*x+a)),x)
```

```
[Out] ln(cos(b*x+a)+sin(b*x+a))/b
```

maxima [A] time = 0.31, size = 18, normalized size = 1.00

$$\frac{\log\left(\cos\left(bx+a\right)+\sin\left(bx+a\right)\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((cos(b*x+a)-sin(b*x+a))/(cos(b*x+a)+sin(b*x+a)),x, algorithm="max
ima")
```

```
[Out] log(cos(b*x + a) + sin(b*x + a))/b
```

mupad [B] time = 3.14, size = 50, normalized size = 2.78

$$\frac{2\,\mathrm{atanh}\left(\frac{128\tan\left(\frac{a}{2}+\frac{b\,x}{2}\right)+128}{16\tan\left(\frac{a}{2}+\frac{b\,x}{2}\right)^2+32\tan\left(\frac{a}{2}+\frac{b\,x}{2}\right)+48}-3\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(a + b*x) - sin(a + b*x))/(cos(a + b*x) + sin(a + b*x)),x)
```

```
[Out] (2*atanh((128*tan(a/2 + (b*x)/2) + 128)/(32*tan(a/2 + (b*x)/2) + 16*tan(a/2
 + (b*x)/2)^2 + 48) - 3))/b
```

sympy [A] time = 0.39, size = 31, normalized size = 1.72

$$\begin{cases} \frac{\log\left(\sin\left(a+bx\right)+\cos\left(a+bx\right)\right)}{b} & \text{for}\; b \neq 0 \\ \frac{x\left(-\sin\left(a\right)+\cos\left(a\right)\right)}{\sin\left(a\right)+\cos\left(a\right)} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((cos(b*x+a)-sin(b*x+a))/(cos(b*x+a)+sin(b*x+a)),x)
```

```
[Out] Piecewise((log(sin(a + b*x) + cos(a + b*x))/b, Ne(b, 0)), (x*(-sin(a) + cos
(a))/(sin(a) + cos(a)), True))
```

3.947 $\int \frac{-\csc(a+bx)+\sec(a+bx)}{\csc(a+bx)+\sec(a+bx)}\,dx$

Optimal. Leaf size=19

$$-\frac{\log(\sin(a+bx)+\cos(a+bx))}{b}$$

```
[Out] -ln(cos(b*x+a)+sin(b*x+a))/b
```

Rubi [A] time = 0.31, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.065, Rules used = {801, 260}

$$-\frac{\log(\sin(a+bx)+\cos(a+bx))}{b}$$

Antiderivative was successfully verified.

```
[In] Int[(-Csc[a + b*x] + Sec[a + b*x])/(Csc[a + b*x] + Sec[a + b*x]),x]
```

```
[Out] -(Log[Cos[a + b*x] + Sin[a + b*x]]/b)
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 801

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a + c*x^2), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rubi steps

$$\int \frac{-\csc(a+bx)+\sec(a+bx)}{\csc(a+bx)+\sec(a+bx)}\,dx = \frac{\text{Subst}\left(\int \frac{-1+x}{(1+x)(1+x^2)}\,dx, x, \tan(a+bx)\right)}{b}$$
$$= \frac{\text{Subst}\left(\int \left(\frac{1}{-1-x}+\frac{x}{1+x^2}\right)dx, x, \tan(a+bx)\right)}{b}$$
$$= -\frac{\log(1+\tan(a+bx))}{b} + \frac{\text{Subst}\left(\int \frac{x}{1+x^2}\,dx, x, \tan(a+bx)\right)}{b}$$
$$= -\frac{\log(\cos(a+bx))}{b} - \frac{\log(1+\tan(a+bx))}{b}$$

Mathematica [A] time = 0.06, size = 19, normalized size = 1.00

$$-\frac{\log(\sin(a+bx)+\cos(a+bx))}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-Csc[a + b*x] + Sec[a + b*x])/(Csc[a + b*x] + Sec[a + b*x]),x]
```

```
[Out] -(Log[Cos[a + b*x] + Sin[a + b*x]]/b)
```

fricas [A] time = 1.35, size = 22, normalized size = 1.16

$$-\frac{\log\left(2\cos\left(bx+a\right)\sin\left(bx+a\right)+1\right)}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-csc(b*x+a)+sec(b*x+a))/(csc(b*x+a)+sec(b*x+a)),x, algorithm="fr
icas")
```

```
[Out] -1/2*log(2*cos(b*x + a)*sin(b*x + a) + 1)/b
```

giac [A] time = 0.29, size = 29, normalized size = 1.53

$$\frac{\log\left(\tan\left(bx+a\right)^2+1\right)-2\,\log\left(|\tan\left(bx+a\right)+1|\right)}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-csc(b*x+a)+sec(b*x+a))/(csc(b*x+a)+sec(b*x+a)),x, algorithm="gi
ac")
```

```
[Out] 1/2*(log(tan(b*x + a)^2 + 1) - 2*log(abs(tan(b*x + a) + 1)))/b
```

maple [A] time = 0.75, size = 32, normalized size = 1.68

$$\frac{\ln\left(1+\tan^{2}\left(bx+a\right)\right)}{2b}-\frac{\ln\left(1+\tan\left(bx+a\right)\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-csc(b*x+a)+sec(b*x+a))/(csc(b*x+a)+sec(b*x+a)),x)
```

```
[Out] 1/2/b*ln(1+tan(b*x+a)^2)-ln(1+tan(b*x+a))/b
```

maxima [B] time = 0.41, size = 70, normalized size = 3.68

$$-\frac{\log\left(-\frac{2\sin(bx+a)}{\cos(bx+a)+1}+\frac{\sin(bx+a)^{2}}{(\cos(bx+a)+1)^{2}}-1\right)-\log\left(\frac{\sin(bx+a)^{2}}{(\cos(bx+a)+1)^{2}}+1\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-csc(b*x+a)+sec(b*x+a))/(csc(b*x+a)+sec(b*x+a)),x, algorithm="ma
xima")
```

```
[Out] -(log(-2*sin(b*x + a)/(cos(b*x + a) + 1) + sin(b*x + a)^2/(cos(b*x + a) + 1
)^2 - 1) - log(sin(b*x + a)^2/(cos(b*x + a) + 1)^2 + 1))/b
```

mupad [B] time = 3.37, size = 50, normalized size = 2.63

$$-\frac{2\,\mathrm{atanh}\left(\frac{128\tan\left(\frac{a}{2}+\frac{b\,x}{2}\right)+128}{16\tan\left(\frac{a}{2}+\frac{b\,x}{2}\right)^{2}+32\tan\left(\frac{a}{2}+\frac{b\,x}{2}\right)+48}-3\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/cos(a + b*x) - 1/sin(a + b*x))/(1/cos(a + b*x) + 1/sin(a + b*x)),x)
```

```
[Out] -(2*atanh((128*tan(a/2 + (b*x)/2) + 128)/(32*tan(a/2 + (b*x)/2) + 16*tan(a/
2 + (b*x)/2)^2 + 48) - 3))/b
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int\frac{\csc\left(a+bx\right)}{\csc\left(a+bx\right)+\sec\left(a+bx\right)}\,dx-\int\left(-\frac{\sec\left(a+bx\right)}{\csc\left(a+bx\right)+\sec\left(a+bx\right)}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-csc(b*x+a)+sec(b*x+a))/(csc(b*x+a)+sec(b*x+a)),x)
```

```
[Out] -Integral(csc(a + b*x)/(csc(a + b*x) + sec(a + b*x)), x) - Integral(-sec(a
+ b*x)/(csc(a + b*x) + sec(a + b*x)), x)
```

3.948 $\int \frac{-\csc^2(a+bx)+\sec^2(a+bx)}{\csc^2(a+bx)+\sec^2(a+bx)}\,dx$

Optimal. Leaf size=17

$$-\frac{\sin(a+bx)\cos(a+bx)}{b}$$

```
[Out] -cos(b*x+a)*sin(b*x+a)/b
```

Rubi [A] time = 0.17, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 39, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.026, Rules used = {383}

$$-\frac{\sin(a+bx)\cos(a+bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[(-Csc[a + b*x]^2 + Sec[a + b*x]^2)/(Csc[a + b*x]^2 + Sec[a + b*x]^2),x]
```

```
[Out] -((Cos[a + b*x]*Sin[a + b*x])/b)
```

Rule 383

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> S
imp[(c*x*(a + b*x^n)^(p + 1))/a, x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[
b*c - a*d, 0] && EqQ[a*d - b*c*(n*(p + 1) + 1), 0]
```

Rubi steps

$$\begin{aligned}\int \frac{-\csc^2(a+bx)+\sec^2(a+bx)}{\csc^2(a+bx)+\sec^2(a+bx)}\,dx &= \frac{\text{Subst}\left(\int \frac{-1+x^2}{\left(1+x^2\right)^2}\,dx, x, \tan(a+bx)\right)}{b}\\ &= -\frac{\cos(a+bx)\sin(a+bx)}{b}\end{aligned}$$

Mathematica [A] time = 0.01, size = 33, normalized size = 1.94

$$-\frac{\sin(2a)\cos(2bx)}{2b}-\frac{\cos(2a)\sin(2bx)}{2b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-Csc[a + b*x]^2 + Sec[a + b*x]^2)/(Csc[a + b*x]^2 + Sec[a + b*x]
^2),x]
```

```
[Out] -1/2*(Cos[2*b*x]*Sin[2*a])/b - (Cos[2*a]*Sin[2*b*x])/(2*b)
```

fricas [A] time = 1.41, size = 17, normalized size = 1.00

$$-\frac{\cos(bx+a)\sin(bx+a)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-csc(b*x+a)^2+sec(b*x+a)^2)/(csc(b*x+a)^2+sec(b*x+a)^2),x, algor
ithm="fricas")
```

```
[Out] -cos(b*x + a)*sin(b*x + a)/b
```

giac [A] time = 0.24, size = 14, normalized size = 0.82

$$-\frac{\sin(2\,bx+2\,a)}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-csc(b*x+a)^2+sec(b*x+a)^2)/(csc(b*x+a)^2+sec(b*x+a)^2),x, algor
ithm="giac")
```

```
[Out] -1/2*sin(2*b*x + 2*a)/b
```

maple [A] time = 0.34, size = 18, normalized size = 1.06

$$-\frac{\cos(bx+a)\sin(bx+a)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-csc(b*x+a)^2+sec(b*x+a)^2)/(csc(b*x+a)^2+sec(b*x+a)^2),x)
```

```
[Out] -cos(b*x+a)*sin(b*x+a)/b
```

maxima [A] time = 0.31, size = 23, normalized size = 1.35

$$-\frac{\tan(bx+a)}{\left(\tan(bx+a)^2+1\right)b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-csc(b*x+a)^2+sec(b*x+a)^2)/(csc(b*x+a)^2+sec(b*x+a)^2),x, algor
ithm="maxima")
```

```
[Out] -tan(b*x + a)/((tan(b*x + a)^2 + 1)*b)
```

mupad [B] time = 3.05, size = 14, normalized size = 0.82

$$-\frac{\sin\left(2\,a+2\,b\,x\right)}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/cos(a + b*x)^2 - 1/sin(a + b*x)^2)/(1/cos(a + b*x)^2 + 1/sin(a + b*x
)^2),x)
```

```
[Out] -sin(2*a + 2*b*x)/(2*b)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{\csc^2(a+bx)}{\csc^2(a+bx)+\sec^2(a+bx)}\,dx-\int\left(-\frac{\sec^2(a+bx)}{\csc^2(a+bx)+\sec^2(a+bx)}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-csc(b*x+a)**2+sec(b*x+a)**2)/(csc(b*x+a)**2+sec(b*x+a)**2),x)
```

```
[Out] -Integral(csc(a + b*x)**2/(csc(a + b*x)**2 + sec(a + b*x)**2), x) - Integra
l(-sec(a + b*x)**2/(csc(a + b*x)**2 + sec(a + b*x)**2), x)
```

3.949 $\int \frac{-\csc^3(a+bx)+\sec^3(a+bx)}{\csc^3(a+bx)+\sec^3(a+bx)} \, dx$

Optimal. Leaf size=54

$$\frac{2\log\left(\tan^2(a+bx)-\tan(a+bx)+1\right)}{3b}-\frac{\log(\tan(a+bx)+1)}{3b}+\frac{\log(\cos(a+bx))}{b}$$

```
[Out] ln(cos(b*x+a))/b-1/3*ln(1+tan(b*x+a))/b+2/3*ln(1-tan(b*x+a)+tan(b*x+a)^2)/b
```

Rubi [A] time = 0.53, antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 39, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {6725, 260, 628}

$$\frac{2\log\left(\tan^2(a+bx)-\tan(a+bx)+1\right)}{3b}-\frac{\log(\tan(a+bx)+1)}{3b}+\frac{\log(\cos(a+bx))}{b}$$

Antiderivative was successfully verified.

```
[In] Int[(-Csc[a + b*x]^3 + Sec[a + b*x]^3)/(Csc[a + b*x]^3 + Sec[a + b*x]^3),x]
```

```
[Out] Log[Cos[a + b*x]]/b - Log[1 + Tan[a + b*x]]/(3*b) + (2*Log[1 - Tan[a + b*x]
 + Tan[a + b*x]^2])/(3*b)
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$\int \frac{-\csc^3(a+bx)+\sec^3(a+bx)}{\csc^3(a+bx)+\sec^3(a+bx)}\,dx = \frac{\text{Subst}\left(\int \frac{-1+x^3}{(1+x^2)(1+x^3)}\,dx,x,\tan(a+bx)\right)}{b}$$
$$= \frac{\text{Subst}\left(\int \left(-\frac{1}{3(1+x)}-\frac{x}{1+x^2}+\frac{2(-1+2x)}{3(1-x+x^2)}\right)dx,x,\tan(a+bx)\right)}{b}$$
$$= -\frac{\log(1+\tan(a+bx))}{3b}+\frac{2\,\text{Subst}\left(\int \frac{-1+2x}{1-x+x^2}\,dx,x,\tan(a+bx)\right)}{3b}-\frac{\text{Subst}\left(\right.}{}$$
$$= \frac{\log(\cos(a+bx))}{b}-\frac{\log(1+\tan(a+bx))}{3b}+\frac{2\log\left(1-\tan(a+bx)+\tan^2\right.}{3b}$$

Mathematica [A] time = 0.23, size = 42, normalized size = 0.78

$$\frac{2\log(2-\sin(2(a+bx)))}{3b}-\frac{\log(\sin(a+bx)+\cos(a+bx))}{3b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-Csc[a + b*x]^3 + Sec[a + b*x]^3)/(Csc[a + b*x]^3 + Sec[a + b*x]
^3),x]
```

```
[Out] -1/3*Log[Cos[a + b*x] + Sin[a + b*x]]/b + (2*Log[2 - Sin[2*(a + b*x)]])/(3*
b)
```

fricas [A] time = 0.94, size = 42, normalized size = 0.78

$$-\frac{\log\left(2\,\cos\left(bx+a\right)\sin\left(bx+a\right)+1\right)-4\,\log\left(-\cos\left(bx+a\right)\sin\left(bx+a\right)+1\right)}{6\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-csc(b*x+a)^3+sec(b*x+a)^3)/(csc(b*x+a)^3+sec(b*x+a)^3),x, algor
ithm="fricas")
```

```
[Out] -1/6*(log(2*cos(b*x + a)*sin(b*x + a) + 1) - 4*log(-cos(b*x + a)*sin(b*x +
a) + 1))/b
```

giac [A] time = 0.39, size = 52, normalized size = 0.96

$$\frac{4\,\log\left(\tan\left(bx+a\right)^2-\tan\left(bx+a\right)+1\right)-3\,\log\left(\tan\left(bx+a\right)^2+1\right)-2\,\log\left(\left|\tan\left(bx+a\right)+1\right|\right)}{6\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-csc(b*x+a)^3+sec(b*x+a)^3)/(csc(b*x+a)^3+sec(b*x+a)^3),x, algor
ithm="giac")
```

```
[Out] 1/6*(4*log(tan(b*x + a)^2 - tan(b*x + a) + 1) - 3*log(tan(b*x + a)^2 + 1) -
 2*log(abs(tan(b*x + a) + 1)))/b
```

maple [A] time = 0.83, size = 56, normalized size = 1.04

$$\frac{2\ln\left(1-\tan\left(bx+a\right)+\tan^{2}\left(bx+a\right)\right)}{3b}-\frac{\ln\left(1+\tan^{2}\left(bx+a\right)\right)}{2b}-\frac{\ln\left(1+\tan\left(bx+a\right)\right)}{3b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-csc(b*x+a)^3+sec(b*x+a)^3)/(csc(b*x+a)^3+sec(b*x+a)^3),x)
```

```
[Out] 2/3*ln(1-tan(b*x+a)+tan(b*x+a)^2)/b-1/2/b*ln(1+tan(b*x+a)^2)-1/3*ln(1+tan(b
*x+a))/b
```

maxima [B] time = 0.44, size = 154, normalized size = 2.85

$$\frac{2\,\log\left(-\frac{2\,\sin(bx+a)}{\cos(bx+a)+1}+\frac{2\,\sin(bx+a)^{2}}{(\cos(bx+a)+1)^{2}}+\frac{2\,\sin(bx+a)^{3}}{(\cos(bx+a)+1)^{3}}+\frac{\sin(bx+a)^{4}}{(\cos(bx+a)+1)^{4}}+1\right)-\log\left(-\frac{2\,\sin(bx+a)}{\cos(bx+a)+1}+\frac{\sin(bx+a)^{2}}{(\cos(bx+a)+1)^{2}}-1\right)-3}{3\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-csc(b*x+a)^3+sec(b*x+a)^3)/(csc(b*x+a)^3+sec(b*x+a)^3),x, algor
ithm="maxima")
```

```
[Out] 1/3*(2*log(-2*sin(b*x + a)/(cos(b*x + a) + 1) + 2*sin(b*x + a)^2/(cos(b*x +
 a) + 1)^2 + 2*sin(b*x + a)^3/(cos(b*x + a) + 1)^3 + sin(b*x + a)^4/(cos(b*
x + a) + 1)^4 + 1) - log(-2*sin(b*x + a)/(cos(b*x + a) + 1) + sin(b*x + a)^
2/(cos(b*x + a) + 1)^2 - 1) - 3*log(sin(b*x + a)^2/(cos(b*x + a) + 1)^2 + 1
))/b
```

mupad [B] time = 3.23, size = 106, normalized size = 1.96

$$\frac{2\,\ln\left(\tan\left(\frac{a}{2}+\frac{b\,x}{2}\right)^{4}+2\tan\left(\frac{a}{2}+\frac{b\,x}{2}\right)^{3}+2\tan\left(\frac{a}{2}+\frac{b\,x}{2}\right)^{2}-2\tan\left(\frac{a}{2}+\frac{b\,x}{2}\right)+1\right)}{3\,b}-\frac{\ln\left(\tan\left(\frac{a}{2}+\frac{b\,x}{2}\right)^{2}-2\tan\left(\frac{a}{2}+\frac{b}{2}\right.\right.}{3\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/cos(a + b*x)^3 - 1/sin(a + b*x)^3)/(1/cos(a + b*x)^3 + 1/sin(a + b*x
)^3),x)
```

```
[Out] (2*log(2*tan(a/2 + (b*x)/2)^2 - 2*tan(a/2 + (b*x)/2) + 2*tan(a/2 + (b*x)/2)
^3 + tan(a/2 + (b*x)/2)^4 + 1))/(3*b) - log(tan(a/2 + (b*x)/2)^2 - 2*tan(a/
2 + (b*x)/2) - 1)/(3*b) - log(tan(a/2 + (b*x)/2)^2 + 1)/b
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-csc(b*x+a)**3+sec(b*x+a)**3)/(csc(b*x+a)**3+sec(b*x+a)**3),x)
```

```
[Out] Timed out
```

3.950 $\int \frac{-\csc^4(a+bx)+\sec^4(a+bx)}{\csc^4(a+bx)+\sec^4(a+bx)}\,dx$

Optimal. Leaf size=72

$$\frac{\log\left(\tan^2(a+bx)-\sqrt{2}\tan(a+bx)+1\right)}{2\sqrt{2}\,b}-\frac{\log\left(\tan^2(a+bx)+\sqrt{2}\tan(a+bx)+1\right)}{2\sqrt{2}\,b}$$

```
[Out] 1/4*ln(1-2^(1/2)*tan(b*x+a)+tan(b*x+a)^2)/b*2^(1/2)-1/4*ln(1+2^(1/2)*tan(b*
x+a)+tan(b*x+a)^2)/b*2^(1/2)
```

Rubi [A] time = 1.40, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 39, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.051, Rules used = {1165, 628}

$$\frac{\log\left(\tan^2(a+bx)-\sqrt{2}\tan(a+bx)+1\right)}{2\sqrt{2}\,b}-\frac{\log\left(\tan^2(a+bx)+\sqrt{2}\tan(a+bx)+1\right)}{2\sqrt{2}\,b}$$

Antiderivative was successfully verified.

```
[In] Int[(-Csc[a + b*x]^4 + Sec[a + b*x]^4)/(Csc[a + b*x]^4 + Sec[a + b*x]^4),x]
```

```
[Out] Log[1 - Sqrt[2]*Tan[a + b*x] + Tan[a + b*x]^2]/(2*Sqrt[2]*b) - Log[1 + Sqrt
[2]*Tan[a + b*x] + Tan[a + b*x]^2]/(2*Sqrt[2]*b)
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{-\csc^4(a+bx)+\sec^4(a+bx)}{\csc^4(a+bx)+\sec^4(a+bx)}\,dx = \frac{\text{Subst}\left(\int \frac{-1+x^2}{1+x^4}\,dx, x, \tan(a+bx)\right)}{b}$$
$$= \frac{\text{Subst}\left(\int \frac{\sqrt{2}+2x}{-1-\sqrt{2}x-x^2}\,dx, x, \tan(a+bx)\right)}{2\sqrt{2}b} + \frac{\text{Subst}\left(\int \frac{\sqrt{2}-2x}{-1+\sqrt{2}x-x^2}\,dx, x, \tan(\right.}{2\sqrt{2}b}$$
$$= \frac{\log\left(1-\sqrt{2}\tan(a+bx)+\tan^2(a+bx)\right)}{2\sqrt{2}b} - \frac{\log\left(1+\sqrt{2}\tan(a+bx)+\text{ta}\right.}{2\sqrt{2}b}$$

Mathematica [A] time = 0.02, size = 26, normalized size = 0.36

$$-\frac{\tanh^{-1}\left(\frac{\sin(2a+2bx)}{\sqrt{2}}\right)}{\sqrt{2}b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-Csc[a + b*x]^4 + Sec[a + b*x]^4)/(Csc[a + b*x]^4 + Sec[a + b*x]
^4),x]
```

```
[Out] -(ArcTanh[Sin[2*a + 2*b*x]/Sqrt[2]]/(Sqrt[2]*b))
```

fricas [A] time = 1.15, size = 74, normalized size = 1.03

$$\frac{\sqrt{2}\log\left(-\frac{2\cos(bx+a)^4+2\sqrt{2}\cos(bx+a)\sin(bx+a)-2\cos(bx+a)^2-1}{2\cos(bx+a)^4-2\cos(bx+a)^2+1}\right)}{4b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-csc(b*x+a)^4+sec(b*x+a)^4)/(csc(b*x+a)^4+sec(b*x+a)^4),x, algor
ithm="fricas")
```

```
[Out] 1/4*sqrt(2)*log(-(2*cos(b*x + a)^4 + 2*sqrt(2)*cos(b*x + a)*sin(b*x + a) -
2*cos(b*x + a)^2 - 1)/(2*cos(b*x + a)^4 - 2*cos(b*x + a)^2 + 1))/b
```

giac [A] time = 0.39, size = 48, normalized size = 0.67

$$\frac{\sqrt{2}\log\left(\frac{\left|-2\sqrt{2}+2\sin(2bx+2a)\right|}{\left|2\sqrt{2}+2\sin(2bx+2a)\right|}\right)}{4b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-csc(b*x+a)^4+sec(b*x+a)^4)/(csc(b*x+a)^4+sec(b*x+a)^4),x, algor
ithm="giac")
```

```
[Out] 1/4*sqrt(2)*log(abs(-2*sqrt(2) + 2*sin(2*b*x + 2*a))/abs(2*sqrt(2) + 2*sin(
2*b*x + 2*a)))/b
```

maple [A] time = 0.51, size = 108, normalized size = 1.50

$$-\frac{\sqrt{2}\,\ln\left(\frac{1+\sqrt{2}\,\tan(bx+a)+\tan^2(bx+a)}{1-\sqrt{2}\,\tan(bx+a)+\tan^2(bx+a)}\right)}{8b}+\frac{\sqrt{2}\,\ln\left(\frac{1-\sqrt{2}\,\tan(bx+a)+\tan^2(bx+a)}{1+\sqrt{2}\,\tan(bx+a)+\tan^2(bx+a)}\right)}{8b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-csc(b*x+a)^4+sec(b*x+a)^4)/(csc(b*x+a)^4+sec(b*x+a)^4),x)
```

```
[Out] -1/8/b*2^(1/2)*ln((1+2^(1/2)*tan(b*x+a)+tan(b*x+a)^2)/(1-2^(1/2)*tan(b*x+a)
+tan(b*x+a)^2))+1/8/b*2^(1/2)*ln((1-2^(1/2)*tan(b*x+a)+tan(b*x+a)^2)/(1+2^(
1/2)*tan(b*x+a)+tan(b*x+a)^2))
```

maxima [A] time = 0.43, size = 58, normalized size = 0.81

$$-\frac{\sqrt{2}\log\left(\tan\left(bx+a\right)^{2}+\sqrt{2}\tan\left(bx+a\right)+1\right)-\sqrt{2}\log\left(\tan\left(bx+a\right)^{2}-\sqrt{2}\tan\left(bx+a\right)+1\right)}{4\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-csc(b*x+a)^4+sec(b*x+a)^4)/(csc(b*x+a)^4+sec(b*x+a)^4),x, algor
ithm="maxima")
```

```
[Out] -1/4*(sqrt(2)*log(tan(b*x + a)^2 + sqrt(2)*tan(b*x + a) + 1) - sqrt(2)*log(
tan(b*x + a)^2 - sqrt(2)*tan(b*x + a) + 1))/b
```

mupad [B] time = 3.09, size = 23, normalized size = 0.32

$$-\frac{\sqrt{2}\,\mathrm{atanh}\left(\frac{\sqrt{2}\,\sin(2\,a+2\,b\,x)}{2}\right)}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/cos(a + b*x)^4 - 1/sin(a + b*x)^4)/(1/cos(a + b*x)^4 + 1/sin(a + b*x
)^4),x)
```

```
[Out] -(2^(1/2)*atanh((2^(1/2)*sin(2*a + 2*b*x))/2))/(2*b)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-csc(b*x+a)**4+sec(b*x+a)**4)/(csc(b*x+a)**4+sec(b*x+a)**4),x)
[Out] Timed out
```

Chapter 4

Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

4.0.1 Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)


(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*         GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)


GradeAntiderivative[result_,optimal_] :=
  If[ExpnType[result]<=ExpnType[optimal],
```

```
    If[FreeQ[result,Complex] || Not[FreeQ[optimal,Complex]],
      If[LeafCount[result]<=2*LeafCount[optimal],
        "A",
      "B"],
    "C"],
  If[FreeQ[result,Integrate] && FreeQ[result,Int],
    "C",
  "F"]]


(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
```

```
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
        Exp,Log,
        Sin,Cos,Tan,Cot,Sec,Csc,
        ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
        Sinh,Cosh,Tanh,Coth,Sech,Csch,
        ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
        Erf, Erfc, Erfi,
        FresnelS, FresnelC,
        ExpIntegralE, ExpIntegralEi, LogIntegral,
        SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
        Gamma, LogGamma, PolyGamma,
        Zeta, PolyLog, ProductLog,
        EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]


AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

4.0.2 Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
```

```
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems

GradeAntiderivative := proc(result,optimal)
local leaf_count_result, leaf_count_optimal,ExpnType_result,ExpnType_optimal,
   debug:=false;

      leaf_count_result:=leafcount(result);
      #do NOT call ExpnType() if leaf size is too large. Recursion problem
      if leaf_count_result > 500000 then
          return "B";
      fi;

      leaf_count_optimal:=leafcount(optimal);

      ExpnType_result:=ExpnType(result);
      ExpnType_optimal:=ExpnType(optimal);

      if debug then
         print("ExpnType_result",ExpnType_result," ExpnType_optimal=",
   ExpnType_optimal);
      fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
#       antiderivative
#   "A" if result can be considered optimal

  #This check below actually is not needed, since I only
  #call this grading only for passed integrals. i.e. I check
  #for "F" before calling this. But no harm of keeping it here.
  #just in case.


  if not type(result,freeof('int')) then
     return "F";
  end if;


  if ExpnType_result<=ExpnType_optimal then
      if debug then
           print("ExpnType_result<=ExpnType_optimal");
      fi;
```

```
      if is_contains_complex(result) then
         if is_contains_complex(optimal) then
            if debug then
               print("both result and optimal complex");
            fi;
            #both result and optimal complex
            if leaf_count_result<=2*leaf_count_optimal then
               return "A";
            else
               return "B";
            end if
         else #result contains complex but optimal is not
            if debug then
               print("result contains complex but optimal is not");
            fi;
            return "C";
         end if
      else # result do not contain complex
           # this assumes optimal do not as well
           if debug then
               print("result do not contain complex, this assumes optimal do not
    as well");
           fi;
           if leaf_count_result<=2*leaf_count_optimal then
              if debug then
                 print("leaf_count_result<=2*leaf_count_optimal");
              fi;
              return "A";
           else
              if debug then
                 print("leaf_count_result>2*leaf_count_optimal");
              fi;
              return "B";
           end if
      end if
  else #ExpnType(result) > ExpnType(optimal)
      if debug then
         print("ExpnType(result) > ExpnType(optimal)");
      fi;
      return "C";
  end if

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
```

```
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
```

```
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:
```

```
#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

4.0.3 Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#              Port of original Maple grading function by
#              Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#              added 'exp_polar'
from sympy import *

def leaf_count(expr):
    #sympy do not have leaf count function. This is approximation
    return round(1.7*count_ops(expr))

def is_sqrt(expr):
    if isinstance(expr,Pow):
        if expr.args[1] == Rational(1,2):
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
            asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
            asinh,acosh,atanh,acoth,asech,acsch
        ]

def is_special_function(func):
    return func in [ erf,erfc,erfi,
             fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
             gamma,loggamma,digamma,zeta,polylog,LambertW,
             elliptic_f,elliptic_e,elliptic_pi,exp_polar
         ]

def is_hypergeometric_function(func):
    return func in [hyper]

def is_appell_function(func):
    return func in [appellf1]
```

```
def is_atom(expn):
    try:
        if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
           return True
        else:
           return False

    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
       return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow):   #type(expn,'`^`')
        if isinstance(expn.args[1],Integer):  #type(op(2,expn),'integer')
            return expnType(expn.args[0])   #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational):  #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)
   ))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,
   ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type
   (expn,'`*`')
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):  #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))  #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,apply(max,map(ExpnType,[op(expn)])))
```

```
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,
   expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,
   Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        return "F"

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    return "A"
                else:
                    return "B"
            else: #result contains complex but optimal is not
                return "C"
        else: # result do not contain complex, this assumes optimal do not as
   well
            if leaf_count_result <= 2*leaf_count_optimal:
                return "A"
            else:
                return "B"
    else:
        return "C"
```

4.0.4 SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#              Albert Rich to use with Sagemath. This is used to
#              grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#              'arctan2','floor','abs','log_integral'

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    debug=False
    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
        if m:
```

```
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    debug=False
    if debug: print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
           'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','
    sinh_integral'
           'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
           'polylog','lambert_w','elliptic_f','elliptic_e',
           'elliptic_pi','exp_integral_e','log_integral']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m


def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U
    ']

def is_appell_function(func):
    return func.name() in ['hypergeometric']   #[appellf1] can't find this in
    sagemath

def is_atom(expn):

    debug=False
    if debug: print ("Enter is_atom")


    #thanks to answer at https://ask.sagemath.org/question/49179/what-is-
    sagemath-equivalent-to-atomic-type-in-maple/
    try:
        if expn.parent() is SR:
```

```
            return expn.operator() is None
        if expn.parent() in (ZZ, QQ, AA, QQbar):
            return expn in expn.parent() # Should always return True
        if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
            return expn in expn.parent().base_ring() or expn in expn.parent().
    gens()
        return False

    except AttributeError as error:
        return False


def expnType(expn):

    if debug:
        print (">>>>>Enter expnType, expn=", expn)
        print (">>>>>is_atom(expn)=", is_atom(expn))

    if is_atom(expn):
        return 1
    elif type(expn)==list:   #isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if  type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0],
    Rational):
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.
    args[0]))
    elif expn.operator() == operator.pow:   #isinstance(expn,Pow)
        if type(expn.operands()[1])==Integer:  #isinstance(expn.args[1],Integer)
            return expnType(expn.operands()[0])   #expnType(expn.args[0])
        elif type(expn.operands()[1])==Rational:  #isinstance(expn.args[1],
    Rational)
            if type(expn.operands()[0])==Rational: #isinstance(expn.args[0],
    Rational)
                return 1
            else:
                return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.
    args[0]))
        else:
            return max(3,expnType(expn.operands()[0]),expnType(expn.operands()
    [1])) #max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1]))
    elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #
    isinstance(expn,Add) or isinstance(expn,Mul)
        m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
        m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
```

```
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.operator()):  #is_elementary_function(expn.
    func)
        return max(3,expnType(expn.operands()[0]))
    elif is_special_function(expn.operator()): #is_special_function(expn.func)
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(
    expn.args)))
        return max(4,m1)   #max(4,m1)
    elif is_hypergeometric_function(expn.operator()): #
    is_hypergeometric_function(expn.func)
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(
    expn.args)))
        return max(5,m1)   #max(5,m1)
    elif is_appell_function(expn.operator()):
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(
    expn.args)))
        return max(6,m1)   #max(6,m1)
    elif str(expn).find("Integral") != -1: #this will never happen, since it
                #is checked before calling the grading function that is passed.
                #but kept it here.
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(
    expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    if debug: print ("Enter grade_antiderivative for sagemath")

    leaf_count_result  = tree_size(result) #leaf_count(result)
    leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

    if debug: print ("leaf_count_result=", leaf_count_result, "
    leaf_count_optimal=",leaf_count_optimal)


    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",
    expnType_optimal)

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
```

```
            if leaf_count_result <= 2*leaf_count_optimal:
                return "A"
            else:
                return "B"
        else: #result contains complex but optimal is not
            return "C"
    else: # result do not contain complex, this assumes optimal do not as
well
        if leaf_count_result <= 2*leaf_count_optimal:
            return "A"
        else:
            return "B"
else:
    return "C"
```